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A Unified Approach to Performance Analysis

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FADING CHANNEL CHARACTERIZATION AND MODELING

Radio-wave propagation through wireless channels is a complicated phenomenon characterized by various effects, such as multipath and shadowing. A precise mathematical description of this phenomenon is either unknown or too complex for tractable communications systems analyses. However, considerable efforts have been devoted to the statistical modeling and characterization of these different effects. The result is a range of relatively simple and accurate statistical models for fading channels which depend on the particular propagation environment and the underlying communication scenario.

The primary purpose of this chapter is to review briefly the principal characteristics and models for fading channels. More detailed treatment of this subject can be found in standard textbooks, such as Refs. 1,3. This chapter also introduces terminology and notation that are used throughout the book. The chapter is organized as follows. A brief qualitative description of the main characteristics of fading channels is presented in the next section. Models for frequency-flat fading channels, corresponding to narrowband transmission, are described in Section 2.2. Models for frequency-selective fading channels that characterize fading in wideband channels are described in Section 2.3.

2.1 MAIN CHARACTERISTICS OF FADING CHANNELS

2.1.1 Envelope and Phase Fluctuations

When a received signal experiences fading during transmission, both its envelope and phase fluctuate over time. For coherent modulations, the fading effects on the phase can severely degrade performance unless measures are taken to compensate for them at the receiver. Most often, analyses of systems employing such modulations assume that the phase effects due to fading are perfectly corrected

at the receiver, resulting in what is referred to as ideal coherent demodulation. For noncoherent modulations, phase information is not needed at the receiver and therefore the phase variation due to fading does not affect the performance. Hence performance analyses for both ideal coherent and noncoherent modulations over fading channels requires only knowledge of the fading envelope statistics and is the case most often considered in this book. Furthermore, for slow fading (discussed next), wherein the fading is at least constant over the duration of a symbol time, the fading envelope random process can be represented by a random variable (RV) over the symbol time.

2.1.2 Slow and Fast Fading

The distinction between slow and fast fading is important for the mathematical modeling of fading channels and for the performance evaluation of communication systems operating over these channels. This notion is related to the *coherence time* T_c of the channel, which measures the period of time over which the fading process is correlated (or equivalently, the period of time after which the correlation function of two samples of the channel response taken at the same frequency but different time instants drops below a certain predetermined threshold). The coherence time is also related to the channel *Doppler spread* f_d by

$$T_c \simeq \frac{1}{f_d} \quad (2.1)$$

The fading is said to be slow if the symbol time duration T_s is smaller than the channel's coherence time T_c ; otherwise, it is considered to be fast. In slow fading a particular fade level will affect many successive symbols, which leads to burst errors, whereas in fast fading the fading decorrelates from symbol to symbol. In the latter case and when the communication receiver decisions are made based on an observation of the received signal over two or more symbol times (such as differentially coherent or coded communications), it becomes necessary to consider the variation of the fading channel from one symbol interval to the next. This is done through a range of correlation models that depend essentially on the particular propagation environment and the underlying communication scenario. These various autocorrelation models and their corresponding power spectral density are tabulated in Table 2.1, in which for convenience the variance of the fast-fading process is normalized to unity.

2.1.3 Frequency-Flat and Frequency-Selective Fading

Frequency selectivity is also an important characteristic of fading channels. If all the spectral components of the transmitted signal are affected in a similar manner, the fading is said to be *frequency nonselective* or, equivalently, *frequency flat*. This is the case for *narrowband* systems in which the transmitted signal bandwidth is much smaller than the channel's *coherence bandwidth* f_c . This

TABLE 2.1 Correlation and Spectral Properties of Various Types of Fading Processes of Practical Interest

Type of Fading Spectrum	Fading Autocorrelation, ρ	Normalized PSD
Rectangular	$\frac{\sin(2\pi f_d T_s)}{2\pi f_d T_s}$	$(2f_d)^{-1}, \quad f \leq f_d$
Gaussian	$\exp[-(\pi f_d T_s)^2]$	$\exp\left[-\left(\frac{f}{f_d}\right)^2\right] (\sqrt{\pi} f_d)^{-1}$
Land mobile	$J_0(2\pi f_d T_s)$	$[\pi^2(f^2 - f_d^2)]^{-1/2}, \quad f \leq f_d$
First-order Butterworth	$\exp(-2\pi f_d T_s)$	$\left[\pi f_d \left(1 + \frac{f}{f_d}\right)^2\right]^{-1}$
Second-order Butterworth	$\exp\left(-\frac{\pi f_d T_s }{\sqrt{2}}\right) \times \left(\cos \frac{\pi f_d T_s}{\sqrt{2}} + \sin \frac{\pi f_d T_s }{\sqrt{2}}\right)$	$\left[1 + 16 \left(\frac{f}{f_d}\right)^4\right]^{-1}$

Source: Data from Mason [4].

*PSD is the power spectral density, f_d the Doppler spread, and T_s the symbol time.

bandwidth measures the frequency range over which the fading process is correlated and is defined as the frequency bandwidth over which the correlation function of two samples of the channel response taken at the same time but at different frequencies falls below a suitable value. In addition, the coherence bandwidth is related to the *maximum delay spread* $\tau_{\max}$ by

$$f_c \simeq \frac{1}{\tau_{\max}} \quad (2.2)$$

On the other hand, if the spectral components of the transmitted signal are affected by different amplitude gains and phase shifts, the fading is said to be *frequency selective*. This applies to *wideband* systems in which the transmitted bandwidth is bigger than the channel's coherence bandwidth.

2.2 MODELING OF FLAT FADING CHANNELS

When fading affects narrowband systems, the received carrier amplitude is modulated by the fading amplitude α , where α is a RV with mean-square value $\Omega = \overline{\alpha^2}$ and probability density function (PDF) $p_\alpha(\alpha)$, which is dependent on the nature of the radio propagation environment. After passing through the fading channel, the signal is perturbed at the receiver by additive white Gaussian noise (AWGN), which is typically assumed to be statistically independent of the fading amplitude α and which is characterized by a one-sided power spectral density N_0 (W/Hz). Equivalently, the received instantaneous signal power is modulated by α^2 . Thus we define the instantaneous signal-to-noise power ratio (SNR) per

symbol by $\gamma = \alpha^2 E_s / N_0$ and the average SNR per symbol by $\bar{\gamma} = \Omega E_s / N_0$, where E_s is the energy per symbol.¹ In addition, the PDF of γ is obtained by introducing a change of variables in the expression for the fading PDF $p_\alpha(\alpha)$ of α , yielding

$$p_\gamma(\gamma) = \frac{p_\alpha(\sqrt{\Omega\gamma/\bar{\gamma}})}{2\sqrt{\gamma\bar{\gamma}/\Omega}}. \quad (2.3)$$

The moment generating function (MGF) $M_\gamma(s)$ associated with the fading PDF $p_\gamma(\gamma)$ and defined by

$$M_\gamma(s) = \int_0^\infty p_\gamma(\gamma) e^{s\gamma} d\gamma \quad (2.4)$$

is another important statistical characteristic of fading channels, particularly in the context of this book. In addition, the amount of fading (AF), or “fading figure,” associated with the fading PDF is defined as

$$\text{AF} = \frac{\text{var}(\alpha^2)}{(E[\alpha^2])^2} = \frac{E[(\alpha^2 - \Omega)^2]}{\Omega^2} = \frac{E(\gamma^2) - (E[\gamma])^2}{(E[\gamma])^2} \quad (2.5)$$

with $E[\cdot]$ denoting statistical average and $\text{var}(\cdot)$ denoting variance. This figure was introduced by Charash [5, p. 29; 6] as a unified measure of the severity of the fading and is typically independent of the average fading power Ω .

We now present the various radio propagation effects involved in fading channels, their corresponding PDF's, MGF's, AF's, and their relation to physical channels. A summary of these properties is tabulated in Table 2.2.

2.2.1 Multipath Fading

Multipath fading is due to the constructive and destructive combination of randomly delayed, reflected, scattered, and diffracted signal components. This type of fading is relatively fast and is therefore responsible for the short-term signal variations. Depending on the nature of the radio propagation environment, there are different models describing the statistical behavior of the multipath fading envelope.

2.2.1.1 Rayleigh Model. The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. In this case the channel fading amplitude α is distributed according to

$$p_\alpha(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right), \quad \alpha \geq 0 \quad (2.6)$$

¹ Our performance evaluation of digital communications over fading channels will generally be a function of the average SNR per symbol $\bar{\gamma}$.

TABLE 2.2 Probability Density Function (PDF) and Moment Generating Function (MGF) of the SNR per Symbol γ for Some Common Fading Channels

Type of Fading	Fading Parameter	PDF, $p_{\gamma}(\gamma)$	MGF, $M_{\gamma}(s)$
Rayleigh		$\frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right)$	$(1 - s\bar{\gamma})^{-1}$
Nakagami- q (Hoyt)	$0 \leq q \leq 1$	$\frac{(1+q^2)}{2q\bar{\gamma}} \exp\left[-\frac{(1+q^2)^2\gamma}{4q^2\bar{\gamma}}\right] \times I_0\left[\frac{(1-q^2)\gamma}{4q^2\bar{\gamma}}\right]$	$\left[1 - 2s\bar{\gamma} + \frac{(2s\bar{\gamma})^2 q^2}{(1+q^2)^2}\right]^{-1/2}$
Nakagami- n (Rice)	$0 \leq n$	$\frac{(1+n^2)e^{-n^2}}{\bar{\gamma}} \exp\left[-\frac{(1+n^2)\gamma}{\bar{\gamma}}\right] \times I_0\left[2n\sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}}\right]$	$\frac{(1+n^2)}{(1+n^2) - s\bar{\gamma}} \exp\left[-\frac{n^2 s\bar{\gamma}}{(1+n^2) - s\bar{\gamma}}\right]$
Nakagami- m	$\frac{1}{2} \leq m$	$\frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right)$	$\left(1 - \frac{s\bar{\gamma}}{m}\right)^{-m}$
Log-normal shadowing	σ	$\frac{4.34}{\sqrt{2\pi}\sigma\gamma} \exp\left[-\frac{(10\log_{10}\gamma - \mu)^2}{2\sigma^2}\right]$	$\frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_b} H_{\kappa_n} \exp(10\sqrt{2\sigma\kappa_n + \mu})/10_s)$
Composite gamma/log-normal	m and $0 \leq \sigma$	$\int_0^\infty \frac{m^m \gamma^{m-1}}{w^m \Gamma(m)} \exp\left(-\frac{m\gamma}{w}\right) \times \frac{\xi}{\sqrt{2\pi}\sigma w} \exp\left[-\frac{(10\log_{10} w - \mu)^2}{2\sigma^2}\right] dw$	$\frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_b} H_{\kappa_n} (1 - 10^{\sqrt{2\sigma\kappa_n + \mu}/10_s/m})^{-m}$

and hence, following (2.3), the instantaneous SNR per symbol of the channel, γ , is distributed according to an exponential distribution given by

$$p_\gamma(\gamma) = \frac{1}{\bar{\gamma}} \exp\left(-\frac{\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (2.7)$$

The MGF corresponding to this fading model is given by

$$M_\gamma(s) = (1 - s\bar{\gamma})^{-1} \quad (2.8)$$

In addition, the moments associated with this fading model can be shown to be given by

$$E[\gamma^k] = \Gamma(1+k)\bar{\gamma}^k \quad (2.9)$$

where $\Gamma(\cdot)$ is the gamma function. The Rayleigh fading model therefore has an AF equal to 1 and typically agrees very well with experimental data for mobile systems, where no LOS path exists between the transmitter and receiver antennas [3]. It also applies to the propagation of reflected and refracted paths through the troposphere [7] and ionosphere [8,9] and to ship-to-ship [10] radio links.

2.2.1.2 Nakagami- q (Hoyt) Model. The Nakagami- q distribution, also referred to as the Hoyt distribution [11], is given in Nakagami [12, Eq. (52)] by

$$p_\alpha(\alpha) = \frac{(1+q^2)\alpha}{q\Omega} \exp\left[-\frac{(1+q^2)\alpha^2}{4q^2\Omega}\right] I_0\left(\frac{(1-q^4)\alpha^2}{4q^2\Omega}\right), \quad \alpha \geq 0 \quad (2.10)$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind, and q is the Nakagami- q fading parameter which ranges from 0 to 1. Using (2.3), it can be shown that the SNR per symbol of the channel, γ , is distributed according to

$$p_\gamma(\gamma) = \frac{1+q^2}{2q\bar{\gamma}} \exp\left[-\frac{(1+q^2)\gamma}{4q^2\bar{\gamma}}\right] I_0\left(\frac{(1-q^4)\gamma}{4q^2\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (2.11)$$

It can be shown that the MGF corresponding to (2.11) is given by

$$M_\gamma(s) = \left[1 - 2s\bar{\gamma} + \frac{(2s\bar{\gamma})^2 q^2}{(1+q^2)^2}\right]^{-1/2} \quad (2.12)$$

Also, the moments associated with this model are given by [12, Eq. (52)]

$$E(\gamma^k) = \Gamma(1+k) {}_2F_1\left(-\frac{k-1}{2}, -\frac{k}{2}; 1, \left(\frac{1-q^2}{1+q^2}\right)^2\right) \bar{\gamma}^k \quad (2.13)$$

where ${}_2F_1(\cdot, \cdot; \cdot, \cdot)$ is the Gauss hypergeometric function, and the AF of the Nakagami- q distribution is therefore given by

$$\text{AF}_q = \frac{2(1+q^4)}{(1+q^2)^2}, \quad 0 \leq q \leq 1 \quad (2.14)$$

and hence ranges between 1 ($q = 1$) and 2 ($q = 0$). The Nakagami- q distribution spans the range from one-sided Gaussian fading ($q = 0$) to Rayleigh fading ($q = 1$). It is typically observed on satellite links subject to strong ionospheric scintillation [13,14]. Note that one-sided Gaussian fading corresponds to the worst-case fading or, equivalently, the largest AF for all multipath distributions considered in our analyses.

2.2.1.3 Nakagami- n (Rice) Model. The Nakagami- n distribution is also known as the Rice distribution [15]. It is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. Here the channel fading amplitude follows the distribution [12, Eq. (50)]

$$p_\alpha(\alpha) = \frac{2(1+n^2)e^{-n^2}\alpha}{\Omega} \exp\left[-\frac{(1+n^2)\alpha^2}{\Omega}\right] I_0\left(2n\alpha\sqrt{\frac{1+n^2}{\Omega}}\right), \quad \alpha \geq 0 \quad (2.15)$$

where n is the Nakagami- n fading parameter which ranges from 0 to ∞ and which is related to the Rician K factor by $K = n^2$. Applying (2.3) shows that the SNR per symbol of the channel, γ , is distributed according to a noncentral chi-square distribution given by

$$p_\gamma(\gamma) = \frac{(1+n^2)e^{-n^2}}{\bar{\gamma}} \exp\left[-\frac{(1+n^2)\gamma}{\bar{\gamma}}\right] I_0\left(2n\sqrt{\frac{(1+n^2)\gamma}{\bar{\gamma}}}\right), \quad \gamma \geq 0 \quad (2.16)$$

It can also be shown that the MGF associated with this fading model is given by

$$M_\gamma(s) = \frac{(1+n^2)}{(1+n^2)-s\bar{\gamma}} \exp\left[\frac{n^2s\bar{\gamma}}{(1+n^2)-s\bar{\gamma}}\right] \quad (2.17)$$

and that the moments are given by [12, Eq. (50)]

$$E(\gamma^k) = \frac{\Gamma(1+k)}{(1+n^2)^k} {}_1F_1(-k, 1; -n^2)\bar{\gamma}^k \quad (2.18)$$

where ${}_1F_1(\cdot, \cdot; \cdot)$ is the Kummer confluent hypergeometric function. The AF of the Nakagami- n distribution is given by

$$\text{AF}_n = \frac{1+2n^2}{(1+n^2)^2}, \quad n \geq 0 \quad (2.19)$$

and hence ranges between 0 ($n = \infty$) and 1 ($n = 0$). The Nakagami- n distribution spans the range from Rayleigh fading ($n = 0$) to no fading (constant amplitude) ($n = \infty$). This type of fading is typically observed in the first resolvable LOS paths of microcellular urban and suburban land-mobile [16], picocellular indoor [17], and factory [18] environments. It also applies to the dominant LOS path of satellite [19,20] and ship-to-ship [10] radio links.

2.2.1.4 Nakagami- m Model. The Nakagami- m PDF is in essence a central chi-square distribution given by [12, Eq. (11)]

$$p_{\alpha}(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right), \quad \alpha \geq 0 \quad (2.20)$$

where m is the Nakagami- m fading parameter which ranges from $\frac{1}{2}$ to ∞ . Figure 2.1 shows the Nakagami- m PDF for $\Omega = 1$ and various values of the m parameter. Applying (2.3) shows that the SNR per symbol, γ , is distributed according to a gamma distribution given by

$$p_{\gamma}(\gamma) = \frac{m^m \gamma^{m-1}}{\bar{\gamma}^m \Gamma(m)} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right), \quad \gamma \geq 0 \quad (2.21)$$

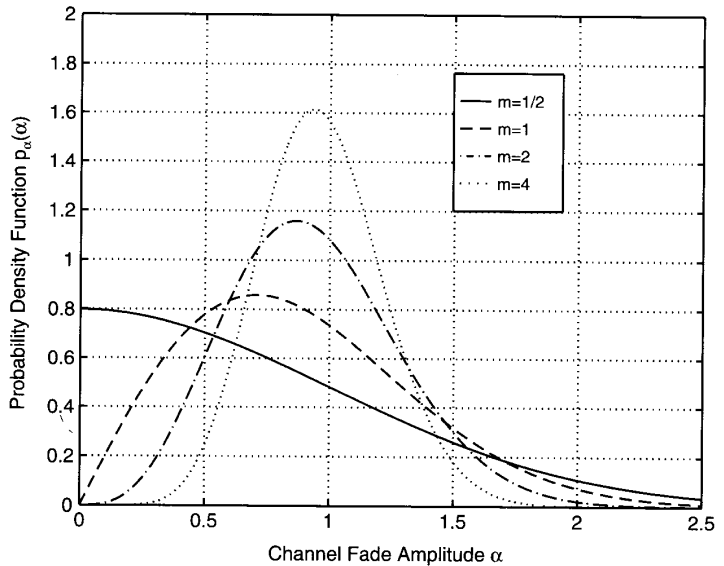


Figure 2.1. Nakagami PDF for $\Omega = 1$ and various values of the fading parameter m .

It can also be shown that the MGF is given in this case by

$$M_Y(s) = \left(1 - \frac{s\bar{Y}}{m}\right)^{-m} \quad (2.22)$$

and that the moments are given by [12, Eq. (65)]

$$E[\gamma^k] = \frac{\Gamma(m+k)}{\Gamma(m)m^k} \bar{\gamma}^k \quad (2.23)$$

which yields an AF of

$$\text{AF}_m = \frac{1}{m}, \quad m \geq \frac{1}{2} \quad (2.24)$$

Hence, the Nakagami- m distribution spans via the m parameter the widest range of AF (from 0 to 2) among all the multipath distributions considered in this book. For instance, it includes the one-sided Gaussian distribution ($m = \frac{1}{2}$) and the Rayleigh distribution ($m = 1$) as special cases. In the limit as $m \rightarrow +\infty$, the Nakagami- m fading channel converges to a nonfading AWGN channel. Furthermore, when $m < 1$, equating (2.14) and (2.24), we obtain a one-to-one mapping between the m parameter and the q parameter, allowing the Nakagami- m distribution to closely approximate the Nakagami- q (Hoyt) distribution, and this mapping is given by

$$m = \frac{(1+q^2)^2}{2(1+2q^4)}, \quad m \leq 1 \quad (2.25)$$

Similarly, when $m > 1$, equating (2.19) and (2.24) we obtain another one-to-one mapping between the m parameter and the n parameter (or, equivalently, the Rician K factor), allowing the Nakagami- m distribution to closely approximate the Nakagami- n (Rice) distribution, and this mapping is given by

$$\begin{aligned} m &= \frac{(1+n^2)^2}{1+2n^2}, \quad n \geq 0 \\ n &= \sqrt{\frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}}, \quad m \geq 1 \end{aligned} \quad (2.26)$$

Finally, the Nakagami- m distribution often gives the best fit to land-mobile [21–23] and indoor-mobile [24] multipath propagation, as well as scintillating ionospheric radio links [9,25–28].

2.2.2 Log-Normal Shadowing

In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level due to the shadowing from

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and that the moments are given by [12, Eq. (65)]

$$E[\gamma^k] = \frac{\Gamma(m+k)}{\Gamma(m)m^k} \bar{Y}^k \quad (2.23)$$

which yields an AF of

$$\text{AF}_m = \frac{1}{m}, \quad m \geq \frac{1}{2} \quad (2.24)$$

Hence, the Nakagami- m distribution spans via the m parameter the widest range of AF (from 0 to 2) among all the multipath distributions considered in this book. For instance, it includes the one-sided Gaussian distribution ($m = \frac{1}{2}$) and the Rayleigh distribution ($m = 1$) as special cases. In the limit as $m \rightarrow +\infty$, the Nakagami- m fading channel converges to a nonfading AWGN channel. Furthermore, when $m < 1$, equating (2.14) and (2.24), we obtain a one-to-one mapping between the m parameter and the q parameter, allowing the Nakagami- m distribution to closely approximate the Nakagami- q (Hoyt) distribution, and this mapping is given by

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Similarly, when $m > 1$, equating (2.19) and (2.24) we obtain another one-to-one mapping between the m parameter and the n parameter (or, equivalently, the Rician K factor), allowing the Nakagami- m distribution to closely approximate the Nakagami- n (Rice) distribution, and this mapping is given by

$$m = \frac{(1+n^2)^2}{1+2n^2}, \quad n \geq 0 \quad (2.26)$$

$$n = \sqrt{\frac{\sqrt{m^2-m}}{m-\sqrt{m^2-m}}}, \quad m \geq 1$$

Finally, the Nakagami- m distribution often gives the best fit to land-mobile [21–23] and indoor-mobile [24] multipath propagation, as well as scintillating ionospheric radio links [9,25–28].

2.2.2 Log-Normal Shadowing

In terrestrial and satellite land-mobile systems, the link quality is also affected by slow variation of the mean signal level due to the shadowing from

terrain, buildings, and trees. Communication system performance will depend on shadowing only if the radio receiver is able to average out the fast multipath fading or if an efficient microdiversity system is used to eliminate the effects of multipath. Based on empirical measurements, there is a general consensus that shadowing can be modeled by a log-normal distribution for various outdoor and indoor environments [21,29–33], in which case the path SNR per symbol γ has a PDF given by the standard log-normal expression

$$p_\gamma(\gamma) = \frac{\xi}{\sqrt{2\pi}\sigma\gamma} \exp \left[-\frac{(10 \log_{10} \gamma - \mu)^2}{2\sigma^2} \right] \quad (2.27)$$

where $\xi = 10/\ln 10 = 4.3429$, and μ (dB) and σ (dB) are the mean and standard deviation of $10 \log_{10} \gamma$, respectively.

The MGF associated with this slow-fading effect is given by

$$M_\gamma(s) \simeq \frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_p} H_{x_n} \exp(10^{(\sqrt{2}\sigma x_n + \mu)/10} s) \quad (2.28)$$

where x_n are the zeros of the N_p -order Hermite polynomial, and H_{x_n} are the weight factors of the N_p -order Hermite polynomial and are given by Table 25.10 of Ref. 50. In addition, the moments of (2.27) are given by

$$E[\gamma^k] = \exp \left[\frac{k}{\xi} \mu + \frac{1}{2} \left(\frac{k}{\xi} \right)^2 \sigma^2 \right] \quad (2.29)$$

yielding an AF of

$$\text{AF}_\sigma = \exp \left(\frac{\sigma^2}{\xi^2} \right) - 1 \quad (2.30)$$

From (2.30) the AF associated with a log-normal PDF can be arbitrarily high. However, as noted by Charash [5, p. 29], in practical situations the standard deviation of shadow fading does not exceed 9 dB [3, p. 88]. Hence, the AF of log-normal shadowing is bounded by 73. This number exceeds the maximal AF exhibited by the various multipath PDFs studied in Section 2.2.1 by several order of magnitudes.

2.2.3 Composite Multipath/Shadowing

A composite multipath/shadowed fading environment consists of multipath fading superimposed on log-normal shadowing. In this environment the receiver does not average out the envelope fading due to multipath but rather, reacts to the instantaneous composite multipath/shadowed signal [3, Sec. 2.4.2]. This is often the scenario in congested downtown areas with slow-moving pedestrians and

vehicles [21,34,35]. This type of composite fading is also observed in land-mobile satellite systems subject to vegetative and/or urban shadowing [36–40]. There are two approaches and various combinations suggested in the literature for obtaining the composite distribution. Here, as an example, we present the composite gamma/log-normal PDF introduced by Ho and Stüber [35]. This PDF arises in Nakagami- m shadowed environments and is obtained by averaging the gamma distributed signal power (or, equivalently, the SNR per symbol) of (2.21) over the conditional density of the log-normally distributed mean signal power (or equivalently, the average SNR per symbol) of (2.27), giving the following channel PDF:

$$p_Y(\gamma) = \int_0^\infty \frac{m^m \gamma^{m-1}}{w^m \Gamma(m)} \exp\left(-\frac{m\gamma}{w}\right) \frac{\xi}{\sqrt{2\pi}\sigma w} \exp\left[-\frac{(10 \log_{10} w - \mu)^2}{2\sigma^2}\right] dw \quad (2.31)$$

For the special case where the multipath is Rayleigh distributed ($m = 1$), (2.31) reduces to a composite exponential/log-normal PDF which was initially proposed by Hansen and Meno [34].

The MGF is given in this case by

$$M_Y(s) \simeq \frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_p} H_{x_n} (1 - 10^{(\sqrt{2}\alpha x_n + \mu)/10} s/m)^{-m} \quad (2.32)$$

and the moments associated with a gamma/log-normal PDF are given by

$$E[\gamma^k] = \frac{\Gamma(m+k)}{\Gamma(m)m^k} \exp\left[\frac{k}{\xi}\mu + \frac{1}{2}\left(\frac{k}{\xi}\right)^2 \sigma^2\right] \quad (2.33)$$

and the resulting AF is given by

$$\text{AF}_{m\sigma} = \frac{1+m}{m} \exp\left(\frac{\sigma^2}{\xi^2}\right) - 1 \quad (2.34)$$

Note that when shadowing is absent ($\sigma = 0$), (2.34) reduces to (2.24), as expected. Similarly, as the fading is reduced ($m \rightarrow \infty$), (2.34) reduces to (2.30), as expected.

2.2.4 Combined (Time-Shared) Shadowed/Unshadowed Fading

From their land-mobile satellite channel characterization experiments, Lutz et al. [39] and Barts and Stutzman [41] found that the overall fading process for land-mobile satellite systems is a convex combination of unshadowed multipath fading and a composite multipath/shadowed fading. Here, as an example, we present in more detail the Lutz et al. model [39]. When no shadowing is present,

the fading follows a Rice (Nakagami- n) PDF. On the other hand, when shadowing is present, it is assumed that no direct LOS path exists and the received signal power (or, equivalently, SNR per bit) is assumed to be an exponential/log-normal (Hansen–Meno) PDF [34]. The combination is characterized by the shadowing time-share factor, which is denoted by A , $0 \leq A \leq 1$; hence, the resulting combined PDF is given by

$$p_\gamma(\gamma) = (1-A) \frac{(1+K)e^{-K}}{\bar{\gamma}^\mu} \exp\left[-\frac{(1+K)\gamma}{\bar{\gamma}^\mu}\right] I_0\left[2\sqrt{\frac{K(1+K)\gamma}{\bar{\gamma}^\mu}}\right] \\ + A \int_0^\infty \frac{1}{w} \exp\left(-\frac{\gamma}{w}\right) \frac{\xi}{\sqrt{2\pi}\sigma^s w} \exp\left[-\frac{(10\log_{10} w - \mu^s)^2}{2(\sigma^s)^2}\right] dw \quad (2.35)$$

where $\bar{\gamma}^\mu$ is the average SNR per symbol during the unshadowed fraction of time, and μ^s and σ^s are the average and standard deviation of $10\log_{10} \gamma$ during the shadowed fraction of time, respectively. The overall average SNR per symbol, $\bar{\gamma}$, is then given by

$$\bar{\gamma} = (1-A)\bar{\gamma}^\mu + A \cdot 10^{\mu^s/10 + (\ln 10)(\sigma^s)^2/200} \quad (2.36)$$

Finally, the MGF can be shown to be given by

$$M_\gamma(s) \simeq (1-A) \frac{(1+K)}{1+K-s\bar{\gamma}^\mu} \exp\left[\frac{Ks\bar{\gamma}^\mu}{(1+K)-s\bar{\gamma}^\mu}\right] \\ + A \frac{1}{\sqrt{\pi}} \sum_{n=1}^{N_p} H_{x_n} (1 - 10^{(\sqrt{2}\sigma^s x_n + \mu^s)/10})^{-1} \quad (2.37)$$

2.3 MODELING OF FREQUENCY-SELECTIVE FADING CHANNELS

When wideband signals propagate through a frequency-selective channel, their spectrum is affected by the channel transfer function, resulting in a time dispersion of the waveform. This type of fading can be modeled as a linear filter characterized by the following complex-valued lowpass equivalent impulse response:

$$h(t) = \sum_{l=1}^{L_p} \alpha_l e^{-j\theta_l} \delta(t - \tau_l) \quad (2.38)$$

where $\delta(\cdot)$ is the Dirac delta function, l the channel index, and $\{\alpha_l\}_{l=1}^{L_p}$, $\{\theta_l\}_{l=1}^{L_p}$, and $\{\tau_l\}_{l=1}^{L_p}$ the random channel amplitudes, phases, and delays, respectively.

In (2.38) L_p is the number of resolvable paths (the first path being the reference path whose delay $\tau_1 = 0$) and is related to the ratio of the maximum delay spread to the symbol time. Under the slow-fading assumption, L_p is assumed to be constant over a certain period of time, and $\{\alpha_l\}_{l=1}^{L_p}$, $\{\theta_l\}_{l=1}^{L_p}$, and $\{\tau_l\}_{l=1}^{L_p}$ are all constant over a symbol interval. If the various paths of a given impulse response are generated by different scatterers, they tend to exhibit negligible correlations [33,42] and it is reasonable in that case to assume that the $\{\alpha_l\}_{l=1}^{L_p}$ are statistically independent RV's. Otherwise, the $\{\alpha_l\}_{l=1}^{L_p}$ have to be considered as correlated RV's and various fading correlation models of interest will be presented in Section 9.6.

Extending the flat fading notations, the fading amplitude α_l of the l th resolved path is assumed to be a RV whose mean-square value $\overline{\alpha_l^2}$ is denoted by Ω_l and whose PDF $p_{\alpha_l}(\alpha_l)$ can be any one of the PDFs presented above. Also as in the flat fading case, after passing through the fading channel, a wideband signal is perturbed by AWGN with a one-sided power spectral density N_0 (W/Hz). The AWGN is assumed to be independent of the fading amplitudes $\{\alpha_l\}_{l=1}^{L_p}$. Hence the instantaneous SNR per symbol of the l th channel is given by $\gamma_l = \alpha_l^2 E_s / N_0$, and the average SNR per symbol of the l th channel is given by $\bar{\gamma}_l = \Omega_l E_s / N_0$.

The first arriving path in the impulse response typically exhibits a lower amount of fading than subsequent paths, since it may contain the LOS path [16,23,42]. Furthermore, since the specular power component typically decreases with respect to delay, the last arriving paths exhibit higher amounts of fading [23,42]. The $\{\Omega_l\}_{l=1}^{L_p}$ are related to the channel's *power delay profile* (PDP), which is also referred to as the *multipath intensity profile* (MIP) and which is typically a decreasing function of the delay. The PDP model can assume various forms, depending on whether the model is for indoor or outdoor environments and for each environment, the general propagation conditions. PDP's for indoor partitioned office buildings, indoor factory buildings with heavy machinery, high-density office buildings in urban areas, low-density residential houses in suburban areas, open rural environment, hilly or mountainous regions, and maritime environment are described in Ref. 43. For example, experimental measurements indicate that the mobile radio channel is well characterized by an exponentially decaying PDP for indoor office buildings [33] and congested urban areas [29,44]:

$$\Omega_l = \Omega_1 e^{-\tau_l / \tau_{\max}}, \quad l = 1, 2, \dots, L_p \quad (2.39)$$

where Ω_1 is the average fading power corresponding to the first (reference) propagation path and $\tau_{\max}$ is the channel maximum delay spread. In the literature the delays are often assumed to be equally spaced ($\tau_{l+1} - \tau_l$ is constant and equal to the symbol time T_s) [1, Sec. 14-5-1; 45], and with this assumption, we get the equally spaced exponential profile given by

$$\Omega_l = \Omega_1 e^{-(l-1)\delta}, \quad \delta \geq 0 \quad \text{and} \quad l = 1, 2, \dots, L_p \quad (2.40)$$

where the parameter δ is the *power decay factor*, which reflects the rate at which the average fading power decays. Other idealized PDP profiles reported or used in the literature include the constant (flat) [46], the flat exponential [47], the double spike [46], the Gaussian [46], the power function (polynomial) [48], and other more complicated composite profiles [49].

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