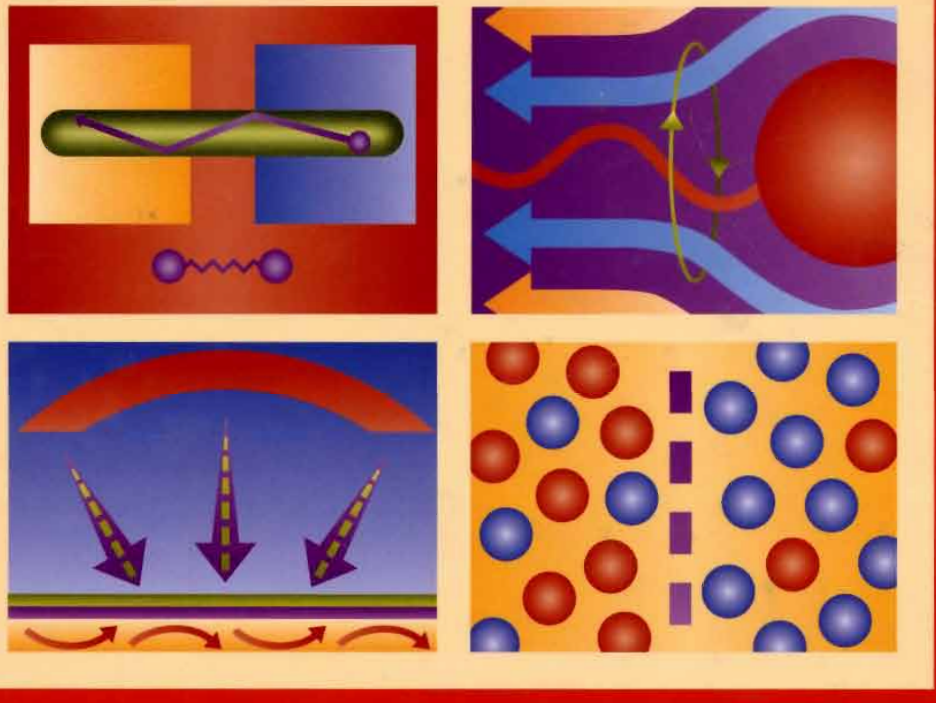
SIXTH
EDITION

Fundamentals of Heat and Mass Transfer



Incropera / DeWitt / Bergman / Lavine

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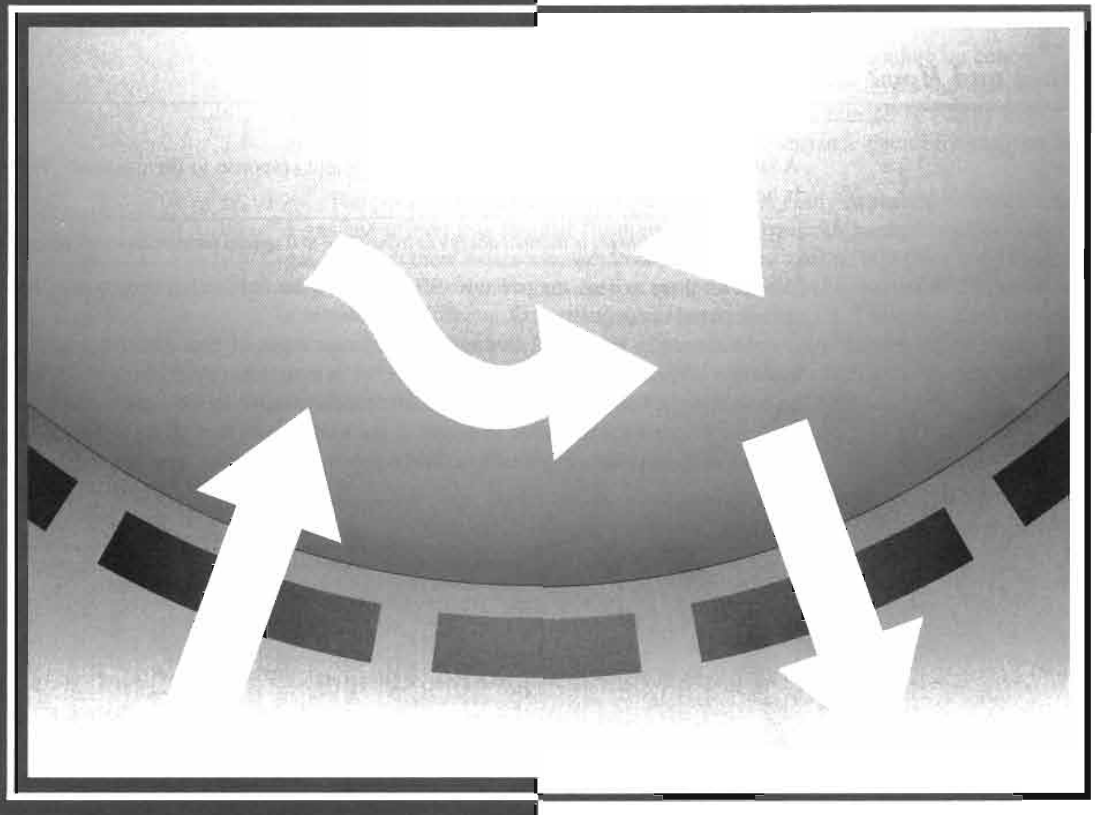
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CHAPTER 1

Introduction



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From the study of thermodynamics, you have learned that energy can be transferred by interactions of a system with its surroundings. These interactions are called work and heat. However, thermodynamics deals with the end states of the process during which an interaction occurs and provides no information concerning the nature of the interaction or the time rate at which it occurs. The objective of this text is to extend thermodynamic analysis through study of the *modes* of heat transfer and through development of relations to calculate heat transfer *rates*.

In this chapter we lay the foundation for much of the material treated in the text. We do so by raising several questions. *What is heat transfer? How is heat transferred? Why is it important?* One objective is to develop an appreciation for the fundamental concepts and principles that underlie heat transfer processes. A second objective is to illustrate the manner in which a knowledge of heat transfer may be used with the first law of thermodynamics (*conservation of energy*) to solve problems relevant to technology and society.

1.1 What and How?

A simple, yet general, definition provides sufficient response to the question: What is heat transfer?

> *Heat transfer (or heat) is thermal energy in transit due to a spatial temperature difference.*

Whenever there exists a temperature difference in a medium or between media, heat transfer must occur.

As shown in Figure 1.1, we refer to different types of heat transfer processes as *modes*. When a temperature gradient exists in a stationary medium, which may be a solid or a fluid, we use the term *conduction* to refer to the heat transfer that will occur across the medium. In contrast, the term *convection* refers to heat transfer that will occur between a surface and a moving fluid when they are at different

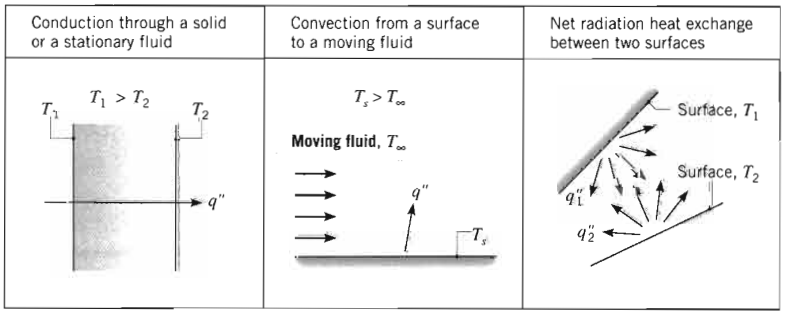


FIGURE 1.1 Conduction, convection, and radiation heat transfer modes.

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temperatures. The third mode of heat transfer is termed *thermal radiation.* All surfaces of finite temperature emit energy in the form of electromagnetic waves. Hence, in the absence of an intervening medium, there is net heat transfer by radiation between two surfaces at different temperatures.

1.2 *Physical Origins and Rate Equations*

As engineers it is important that we understand the *physical mechanisms* which underlie the heat transfer modes and that we be able to use the rate equations that quantify the amount of energy being transferred per unit time.

1.2.1 Conduction

At mention of the word *conduction*, we should immediately conjure up concepts of *atomic* and *molecular activity*, for it is processes at these levels that sustain this mode of heat transfer. Conduction may be viewed as the transfer of energy from the more energetic to the less energetic particles of a substance due to interactions between the particles.

The physical mechanism of conduction is most easily explained by considering a gas and using ideas familiar from your thermodynamics background. Consider a gas in which there exists a temperature gradient and assume that there is *no bulk, or macroscopic, motion.* The gas may occupy the space between two surfaces that are maintained at different temperatures, as shown in Figure 1.2. We associate the temperature at any point with the energy of gas molecules in proximity to the point. This energy is related to the random translational motion, as well as to the internal rotational and vibrational motions, of the molecules.

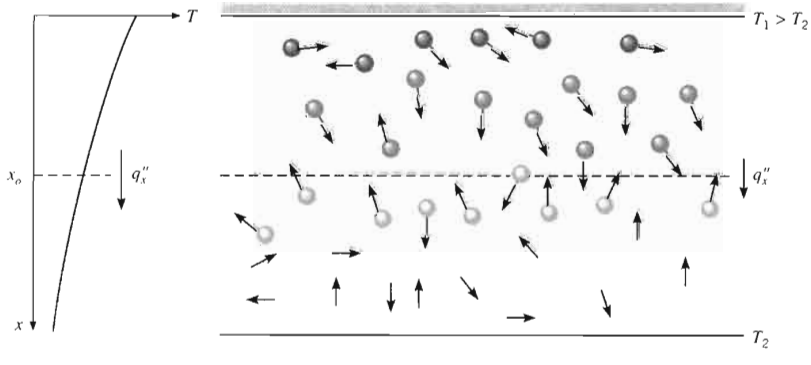


FIGURE 1.2 Association of conduction heat transfer with diffusion of energy due to molecular activity.

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Higher temperatures are associated with higher molecular energies, and when neighboring molecules collide, as they are constantly doing, a transfer of energy from the more energetic to the less energetic molecules must occur. In the presence of a temperature gradient, energy transfer by conduction must then occur in the direction of decreasing temperature. This would even be true in the absence of collisions, as is evident from Figure 1.2. The hypothetical plane at x_o is constantly being crossed by molecules from above and below due to their *random* motion. However, molecules from above are associated with a larger temperature than those from below, in which case there must be a *net* transfer of energy in the positive x direction. Collisions between molecules enhance this energy transfer. We may speak of the net transfer of energy by random molecular motion as a *diffusion* of energy.

The situation is much the same in liquids, although the molecules are more closely spaced and the molecular interactions are stronger and more frequent. Similarly, in a solid, conduction may be attributed to atomic activity in the form of lattice vibrations. The modern view is to ascribe the energy transfer to *lattice waves* induced by atomic motion. In an electrical nonconductor, the energy transfer is exclusively via these lattice waves; in a conductor it is also due to the translational motion of the free electrons. We treat the important properties associated with conduction phenomena in Chapter 2 and in Appendix A.

Examples of conduction heat transfer are legion. The exposed end of a metal spoon suddenly immersed in a cup of hot coffee will eventually be warmed due to the conduction of energy through the spoon. On a winter day there is significant energy loss from a heated room to the outside air. This loss is principally due to conduction heat transfer through the wall that separates the room air from the outside air.

It is possible to quantify heat transfer processes in terms of appropriate *rate equations*. These equations may be used to compute the amount of energy being transferred per unit time. For heat conduction, the rate equation is known as *Fourier's law*. For the one-dimensional plane wall shown in Figure 1.3, having a temperature distribution $T(x)$, the rate equation is expressed as

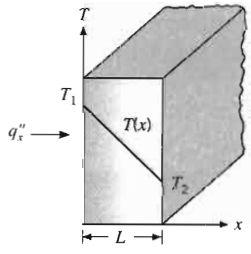


FIGURE 1.3
One-dimensional heat transfer by conduction (diffusion of energy).

$$q''_x = -k\frac{dT}{dx} \tag{1.1}$$

The *heat flux* q''_x (W/m^2) is the heat transfer rate in the x direction *per* unit area *perpendicular* to the direction of transfer, and it is proportional to the *temperature gradient*, dT/dx, in this direction. The parameter k is a *transport* property known as the *thermal conductivity* (W/m · K) and is a characteristic of the wall material. The minus sign is a consequence of the fact that heat is transferred in the direction of decreasing temperature. Under the *steady-state conditions* shown in Figure 1.3, where the temperature distribution is *linear*, the temperature gradient may be expressed as

$$\frac{dT}{dx} = \frac{T_2 - T_1}{L}$$

and the heat flux is then

$$q''_x = -k\frac{T_2 - T_1}{L}$$

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or

$$q''_x = k\frac{T_1 - T_2}{L} = k\frac{\Delta T}{L} \tag{1.2}$$

Note that this equation provides a *heat flux*, that is, the rate of heat transfer per *unit area*. The *heat rate* by conduction, q_x (W), through a plane wall of area A is then the product of the flux and the area, $q_x = q''_x \cdot A$.

EXAMPLE 1.1

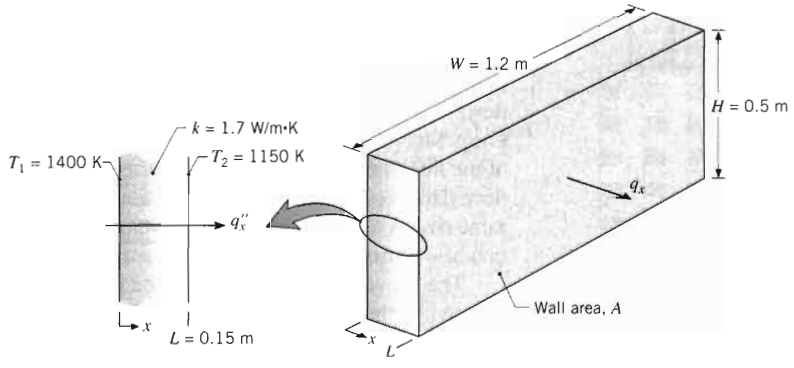
The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m · K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m by 1.2 m on a side?

SOLUTION

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity, and surface temperatures.

Find: Wall heat loss.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction through the wall.
3. Constant thermal conductivity.

Analysis: Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law. Using Equation 1.2, we have

$$q''_x = k\frac{\Delta T}{L} = 1.7\ \text{W/m} \cdot \text{K} \times \frac{250\ \text{K}}{0.15\ \text{m}} = 2833\ \text{W/m}^2$$

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The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area $A = H \times W$ is then

$$q_x = (HW)\, q_x'' = (0.5\ \text{m} \times 1.2\ \text{m})\ 2833\ \text{W/m}^2 = 1700\ \text{W} \qquad \triangleleft$$

Comments: Note the direction of heat flow and the distinction between heat flux and heat rate.

1.2.2 Convection

The convection heat transfer *mode* is comprised of *two mechanisms*. In addition to energy transfer due to *random molecular motion* (*diffusion*), energy is also transferred by the *bulk*, or *macroscopic, motion* of the fluid. This fluid motion is associated with the fact that, at any instant, large numbers of molecules are moving collectively or as aggregates. Such motion, in the presence of a temperature gradient, contributes to heat transfer. Because the molecules in the aggregate retain their random motion, the total heat transfer is then due to a superposition of energy transport by the random motion of the molecules and by the bulk motion of the fluid. It is customary to use the term *convection* when referring to this cumulative transport and the term *advection* when referring to transport due to bulk fluid motion.

We are especially interested in convection heat transfer, which occurs between a fluid in motion and a bounding surface when the two are at different temperatures. Consider fluid flow over the heated surface of Figure 1.4. A consequence of the fluid–surface interaction is the development of a region in the fluid through which the velocity varies from zero at the surface to a finite value u_∞ associated with the flow. This region of the fluid is known as the *hydrodynamic*, or *velocity, boundary layer*. Moreover, if the surface and flow temperatures differ, there will be a region of the fluid through which the temperature varies from T_s at $y = 0$ to T_∞ in the outer flow. This region, called the *thermal boundary layer*, may be smaller, larger, or the same size as that through which the velocity varies. In any case, if $T_s > T_\infty$, convection heat transfer will occur from the surface to the outer flow.

The convection heat transfer mode is sustained both by random molecular motion and by the bulk motion of the fluid within the boundary layer. The contribution due to random molecular motion (diffusion) dominates near the surface where the

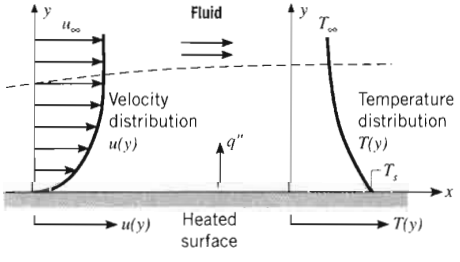


FIGURE 1.4
Boundary layer development in convection heat transfer.

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fluid velocity is low. In fact, at the interface between the surface and the fluid ($y = 0$), the fluid velocity is zero and heat is transferred by this mechanism only. The contribution due to bulk fluid motion originates from the fact that the boundary layer *grows* as the flow progresses in the x direction. In effect, the heat that is conducted into this layer is swept downstream and is eventually transferred to the fluid outside the boundary layer. Appreciation of boundary layer phenomena is essential to understanding convection heat transfer. It is for this reason that the discipline of fluid mechanics will play a vital role in our later analysis of convection.

Convection heat transfer may be classified according to the nature of the flow. We speak of *forced convection* when the flow is caused by external means, such as by a fan, a pump, or atmospheric winds. As an example, consider the use of a fan to provide forced convection air cooling of hot electrical components on a stack of printed circuit boards (Figure 1.5*a*). In contrast, for *free* (or *natural*) *convection* the flow is induced by buoyancy forces, which are due to density differences caused by temperature variations in the fluid. An example is the free convection heat transfer that occurs from hot components on a vertical array of circuit boards in air (Figure 1.5*b*). Air that makes contact with the components experiences an increase in temperature and hence a reduction in density. Since it is now lighter than the surrounding air, buoyancy forces induce a vertical motion for which warm air ascending from the boards is replaced by an inflow of cooler ambient air.

While we have presumed *pure* forced convection in Figure 1.5*a* and *pure* natural convection in Figure 1.5*b*, conditions corresponding to *mixed* (*combined*) *forced* and *natural convection* may exist. For example, if velocities associated with

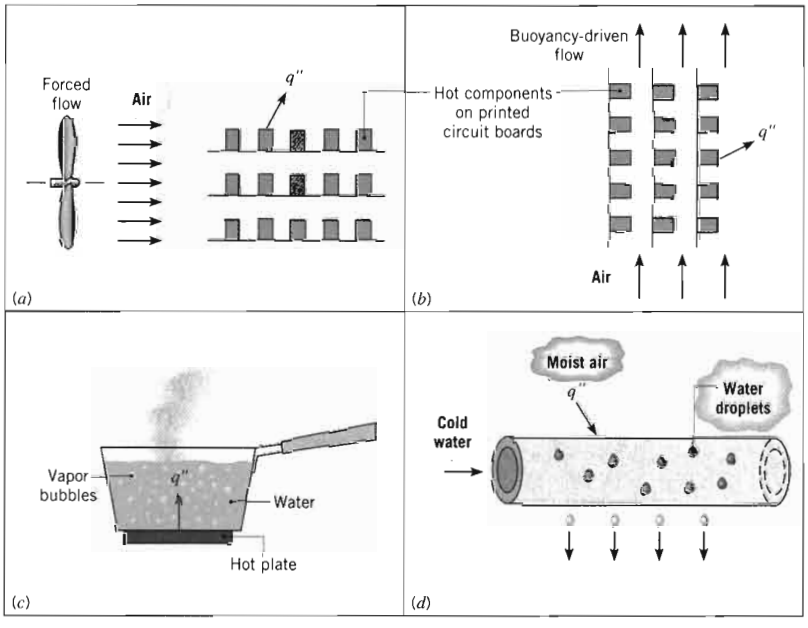


FIGURE 1.5 Convection heat transfer processes. (*a*) Forced convection. (*b*) Natural convection. (*c*) Boiling. (*d*) Condensation.

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the flow of Figure 1.5*a* are small and/or buoyancy forces are large, a secondary flow that is comparable to the imposed forced flow could be induced. In this case, the buoyancy-induced flow would be normal to the forced flow and could have a significant effect on convection heat transfer from the components. In Figure 1.5*b*, mixed convection would result if a fan were used to force air upward between the circuit boards, thereby assisting the buoyancy flow, or downward, thereby opposing the buoyancy flow.

We have described the convection heat transfer mode as energy transfer occurring within a fluid due to the combined effects of conduction and bulk fluid motion. Typically, the energy that is being transferred is the *sensible*, or internal thermal, energy of the fluid. However, there are convection processes for which there is, in addition, *latent* heat exchange. This latent heat exchange is generally associated with a phase change between the liquid and vapor states of the fluid. Two special cases of interest in this text are *boiling* and *condensation*. For example, convection heat transfer results from fluid motion induced by vapor bubbles generated at the bottom of a pan of boiling water (Figure 1.5*c*) or by the condensation of water vapor on the outer surface of a cold water pipe (Figure 1.5*d*).

Regardless of the particular nature of the convection heat transfer process, the appropriate rate equation is of the form

$$q'' = h(T_s - T_\infty) \tag{1.3a}$$

where q'', the convective *heat flux* (W/m^2), is proportional to the difference between the surface and fluid temperatures, T_s and T_∞, respectively. This expression is known as *Newton's law of cooling*, and the parameter h ($W/m^2 \cdot K$) is termed the *convection heat transfer coefficient*. It depends on conditions in the boundary layer, which are influenced by surface geometry, the nature of the fluid motion, and an assortment of fluid thermodynamic and transport properties.

Any study of convection ultimately reduces to a study of the means by which h may be determined. Although consideration of these means is deferred to Chapter 6, convection heat transfer will frequently appear as a boundary condition in the solution of conduction problems (Chapters 2 through 5). In the solution of such problems we presume h to be known, using typical values given in Table 1.1.

TABLE 1.1 Typical values of the convection heat transfer coefficient

Process	h ($W/m^2 \cdot K$)
Free convection	
Gases	2–25
Liquids	50–1000
Forced convection	
Gases	25–250
Liquids	100–20,000
Convection with phase change	
Boiling or condensation	2500–100,000

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When Equation 1.3a is used, the convection heat flux is presumed to be *positive* if heat is transferred *from* the surface ($T_s > T_\infty$) and *negative* if heat is transferred *to* the surface ($T_\infty > T_s$). However, if $T_\infty > T_s$, there is nothing to preclude us from expressing Newton's law of cooling as

$$q'' = h(T_\infty - T_s) \tag{1.3b}$$

in which case heat transfer is positive if it is to the surface.

1.2.3 Radiation

Thermal radiation is energy *emitted* by matter that is at a nonzero temperature. Although we will focus on radiation from solid surfaces, emission may also occur from liquids and gases. Regardless of the form of matter, the emission may be attributed to changes in the electron configurations of the constituent atoms or molecules. The energy of the radiation field is transported by electromagnetic waves (or alternatively, photons). While the transfer of energy by conduction or convection requires the presence of a material medium, radiation does not. In fact, radiation transfer occurs most efficiently in a vacuum.

Consider radiation transfer processes for the surface of Figure 1.6*a*. Radiation that is *emitted* by the surface originates from the thermal energy of matter bounded by the surface, and the rate at which energy is released per unit area (W/m^2) is termed the surface *emissive power E*. There is an upper limit to the emissive power, which is prescribed by the *Stefan–Boltzmann law*

$$E_b = \sigma T_s^4 \tag{1.4}$$

where T_s is the *absolute temperature* (K) of the surface and σ is the *Stefan–Boltzmann constant* ($\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$). Such a surface is called an ideal radiator or *blackbody*.

The heat flux emitted by a real surface is less than that of a blackbody at the same temperature and is given by

$$E = \varepsilon \sigma T_s^4 \tag{1.5}$$

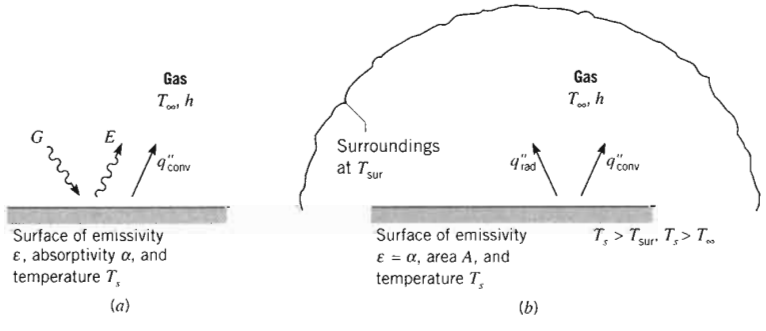


FIGURE 1.6 Radiation exchange: (*a*) at a surface and (*b*) between a surface and large surroundings.

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where ε is a radiative property of the surface termed the *emissivity*. With values in the range $0 \leq \varepsilon \leq 1$, this property provides a measure of how efficiently a surface emits energy relative to a blackbody. It depends strongly on the surface material and finish, and representative values are provided in Appendix A.

Radiation may also be *incident* on a surface from its surroundings. The radiation may originate from a special source, such as the sun, or from other surfaces to which the surface of interest is exposed. Irrespective of the source(s), we designate the rate at which all such radiation is incident on a unit area of the surface as the *irradiation* G (Figure 1.6*a*).

A portion, or all, of the irradiation may be *absorbed* by the surface, thereby increasing the thermal energy of the material. The rate at which radiant energy is absorbed per unit surface area may be evaluated from knowledge of a surface radiative property termed the *absorptivity* α. That is,

$$G_{abs} = \alpha G \tag{1.6}$$

where $0 \leq \alpha \leq 1$. If $\alpha < 1$ and the surface is *opaque*, portions of the irradiation are *reflected*. If the surface is *semitransparent*, portions of the irradiation may also be *transmitted*. However, while absorbed and emitted radiation increase and reduce, respectively, the thermal energy of matter, reflected and transmitted radiation have no effect on this energy. Note that the value of α depends on the nature of the irradiation, as well as on the surface itself. For example, the absorptivity of a surface to solar radiation may differ from its absorptivity to radiation emitted by the walls of a furnace.

In many engineering problems (a notable exception being problems involving solar radiation or radiation from other very high temperature sources), liquids can be considered opaque, and gases can be considered transparent, to radiation heat transfer. Solids can be opaque (as is the case for metals) or *semitransparent* (as is the case for thin sheets of some polymers and some semiconducting materials).

A special case that occurs frequently involves radiation exchange between a small surface at T_s and a much larger, isothermal surface that completely surrounds the smaller one (Figure 1.6*b*). The *surroundings* could, for example, be the walls of a room or a furnace whose temperature T_{sur} differs from that of an enclosed surface ($T_{sur} \neq T_s$). We will show in Chapter 12 that, for such a condition, the irradiation may be approximated by emission from a blackbody at T_{sur}, in which case $G = \sigma T_{sur}^4$. If the surface is assumed to be one for which $\alpha = \varepsilon$ (a *gray surface*), the *net* rate of radiation heat transfer *from* the surface, expressed per unit area of the surface, is

$$q''_{rad} = \frac{q}{A} = \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma(T_s^4 - T_{sur}^4) \tag{1.7}$$

This expression provides the difference between thermal energy that is released due to radiation emission and that which is gained due to radiation absorption.

There are many applications for which it is convenient to express the net radiation heat exchange in the form

$$q_{rad} = h_r A(T_s - T_{sur}) \tag{1.8}$$

where, from Equation 1.7, the *radiation heat transfer coefficient* h_r is

$$h_r \equiv \varepsilon\sigma(T_s + T_{sur})(T_s^2 + T_{sur}^2) \tag{1.9}$$

Here we have modeled the radiation mode in a manner similar to convection. In this sense we have *linearized* the radiation rate equation, making the heat rate proportional

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to a temperature difference rather than to the difference between two temperatures to the fourth power. Note, however, that h_r depends strongly on temperature, while the temperature dependence of the convection heat transfer coefficient h is generally weak.

The surfaces of Figure 1.6 may also simultaneously transfer heat by convection to an adjoining gas. For the conditions of Figure 1.6*b*, the total rate of heat transfer *from* the surface is then

$$q = q_{\text{conv}} + q_{\text{rad}} = hA(T_s - T_\infty) + \varepsilon A\sigma(T_s^4 - T_{\text{sur}}^4) \tag{1.10}$$

EXAMPLE 1.2

An uninsulated steam pipe passes through a room in which the air and walls are at 25°C. The outside diameter of the pipe is 70 mm, and its surface temperature and emissivity are 200°C and 0.8, respectively. What are the surface emissive power and irradiation? If the coefficient associated with free convection heat transfer from the surface to the air is 15 W/m^2 · K, what is the rate of heat loss from the surface per unit length of pipe?

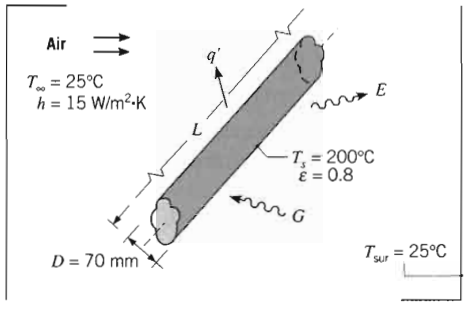
SOLUTION

Known: Uninsulated pipe of prescribed diameter, emissivity, and surface temperature in a room with fixed wall and air temperatures.

Find:

1. Surface emissive power and irradiation.
2. Pipe heat loss per unit length, q'.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Radiation exchange between the pipe and the room is between a small surface and a much larger enclosure.
3. The surface emissivity and absorptivity are equal.

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Analysis:

1. The surface emissive power may be evaluated from Equation 1.5, while the irradiation corresponds to $G = \sigma T_{sur}^4$. Hence

$$E = \varepsilon \sigma T_s^4 = 0.8(5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4)\,(473\ \text{K})^4 = 2270\ \text{W/m}^2$$

$$G = \sigma T_{sur}^4 = 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\,(298\ \text{K})^4 = 447\ \text{W/m}^2$$

2. Heat loss from the pipe is by convection to the room air and by radiation exchange with the walls. Hence, $q = q_{conv} + q_{rad}$ and from Equation 1.10, with $A = \pi DL$,

$$q = h(\pi DL)(T_s - T_\infty) + \varepsilon(\pi DL)\sigma(T_s^4 - T_{sur}^4)$$

The heat loss per unit length of pipe is then

$$q' = \frac{q}{L} = 15\ \text{W/m}^2 \cdot \text{K}(\pi \times 0.07\ \text{m})(200 - 25)^\circ\text{C}$$

$$+\ 0.8(\pi \times 0.07\ \text{m})\,5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\,(473^4 - 298^4)\ \text{K}^4$$

$$q' = 577\ \text{W/m} + 421\ \text{W/m} = 998\ \text{W/m} \quad \triangleleft$$

Comments:

1. Note that temperature may be expressed in units of °C or K when evaluating the temperature difference for a convection (or conduction) heat transfer rate. However, temperature must be expressed in kelvins (K) when evaluating a radiation transfer rate.
2. The net rate of radiation heat transfer from the pipe may be expressed as

$$q'_{rad} = \pi D\,(E - \alpha G)$$

$$q'_{rad} = \pi \times 0.07\ \text{m}\,(2270 - 0.8 \times 447)\ \text{W/m}^2 = 421\ \text{W/m}$$

3. In this situation the radiation and convection heat transfer rates are comparable because T_s is large compared to T_{sur} and the coefficient associated with free convection is small. For more moderate values of T_s and the larger values of h associated with forced convection, the effect of radiation may often be neglected. The radiation heat transfer coefficient may be computed from Equation 1.9, and for the conditions of this problem its value is $h_r = 11\ \text{W/m}^2 \cdot \text{K}$.
4. This example is provided as a tutorial session in the Interactive Heat Transfer (*IHT*) software accompanying your text.

1.2.4 Relationship to Thermodynamics

At this point it is appropriate to note the fundamental differences between heat transfer and thermodynamics. Although thermodynamics is concerned with the heat interaction and the vital role it plays in the first and second laws, it considers neither the mechanisms that provide for heat exchange nor the methods that exist for computing the *rate* of heat exchange. Thermodynamics is concerned with *equilibrium* states of matter, where an equilibrium state necessarily precludes the existence of a temperature gradient. Although thermodynamics may be used to determine the amount

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of energy required in the form of heat for a system to pass from one equilibrium state to another, it does not acknowledge that *heat transfer is inherently a nonequilibrium process.* For heat transfer to occur, there must be a temperature gradient and, hence, thermodynamic nonequilibrium. The discipline of heat transfer therefore seeks to do what thermodynamics is inherently unable to do, namely, to quantify the *rate* at which heat transfer occurs in terms of the degree of thermal nonequilibrium. This is done through the rate equations for the three modes, expressed, for example, by Equations 1.2, 1.3, and 1.7.

1.3 *The Conservation of Energy Requirement*

The subjects of thermodynamics and heat transfer are highly complementary. For example, because it treats the *rate* at which heat is transferred, the subject of heat transfer may be viewed as an extension of thermodynamics. Conversely, for many heat transfer problems, the *first law* of thermodynamics (the *law of conservation of energy*) provides a useful, often essential, tool. In anticipation of such problems, general formulations of the first law are now obtained.

1.3.1 Conservation of Energy for a Control Volume

At its heart, the first law of thermodynamics is simply a statement that the total energy of a system is conserved, and therefore the only way that the amount of energy in a system can change is if energy crosses its boundaries. The first law also addresses the ways in which energy can cross the boundaries of a system. For a closed system (a region of fixed mass), there are only two: heat transfer through the boundaries and work done on or by the system. This leads to the following statement of the first law for a closed system, which will be familiar if you have taken a course in thermodynamics:

$$\Delta E_{st}^{tot} = Q - W \tag{1.11a}$$

where ΔE_{st}^{tot} is the change in the total energy stored in the system, Q is the *net* heat transferred to the system, and W is the *net* work done by the system. This is schematically illustrated in Figure 1.7*a*.

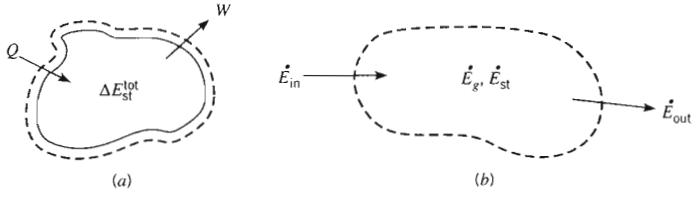


FIGURE 1.7 Conservation of energy: (*a*) for a closed system over a time interval, and (*b*) for a control volume at an instant.

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The first law can also be applied to a *control volume* (or *open system*), a region of space bounded by a *control surface* through which mass may pass. Mass entering and leaving the control volume carries energy with it; this process, termed energy advection, adds a third way in which energy can cross the boundaries of a control volume. To summarize, the first law of thermodynamics can be very simply stated as follows for both a control volume and a closed system.

First Law of Thermodynamics over a Time Interval (Δt)

The increase in the amount of energy stored in a control volume must equal the amount of energy that enters the control volume, minus the amount of energy that leaves the control volume.

In applying this principle, it is recognized that energy can enter and leave the control volume due to heat transfer through the boundaries, work done on or by the control volume, and energy advection.

The first law of thermodynamics addresses *total* energy, which consists of kinetic and potential energies (together known as mechanical energy) and internal energy. Internal energy can be further subdivided into thermal energy (which will be defined more carefully later) and other forms of internal energy such as chemical and nuclear energy. For the study of heat transfer, we wish to focus attention on the thermal and mechanical forms of energy. We must recognize that the sum of thermal and mechanical energy is *not* conserved, because there can be conversion between other forms of energy and thermal energy. For example, if a chemical reaction occurs that decreases the amount of chemical energy in the system, it will result in an increase in the thermal energy of the system. Thus, we can think of energy conversion as resulting in *thermal energy generation* (which can be either positive or negative). So, a statement of the first law that is well suited for heat transfer analysis is:

Thermal and Mechanical Energy Equation over a Time Interval (Δt)

The increase in the amount of thermal and mechanical energy stored in the control volume must equal the amount of thermal and mechanical energy that enters the control volume, minus the amount of thermal and mechanical energy that leaves the control volume, plus the amount of thermal energy that is generated within the control volume.

This expression applies over a *time interval* Δt, and all the energy terms are measured in joules. Since the first law must be satisfied at each and every *instant* of time t, we can also formulate the law on a *rate basis*. That is, at any instant, there must be a balance between all *energy rates*, as measured in joules per second (W). In words, this is expressed as:

Thermal and Mechanical Energy Equation at an Instant (t)

The rate of increase of thermal and mechanical energy stored in the control volume must equal the rate at which thermal and mechanical energy enter the control volume, minus the rate at which thermal and mechanical energy leave the control volume, plus the rate at which thermal energy is generated within the control volume.

If the inflow and generation of thermal and mechanical energy exceed the outflow, there must be an increase in the amount of thermal and mechanical energy stored

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(accumulated) in the control volume; if the converse is true, there will be a decrease in thermal and mechanical energy storage. If the inflow and generation equal the outflow, a *steady-state* condition must prevail such that there will be no change in the amount of thermal and mechanical energy stored in the control volume.

We will now define symbols for each of the energy terms so that the boxed statements can be rewritten as equations. We let E stand for the sum of thermal and mechanical energy (in contrast to the symbol E^{tot} for total energy). Using the subscript st to denote energy stored in the control volume, the change in thermal and mechanical energy stored over the time interval Δt is then ΔE_{st}. The subscripts *in* and *out* refer to energy entering and leaving the control volume. Finally, thermal energy generation is given the symbol E_g. Thus, the first boxed statement can be written as:

$$\Delta E_{st} = E_{in} - E_{out} + E_g \tag{1.11b}$$

Next, using a dot over a term to indicate a rate, the second boxed statement becomes:

$$\dot{E}_{st} \equiv \frac{dE_{st}}{dt} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \tag{1.11c}$$

This expression is illustrated schematically in Figure 1.7b.

Equations 1.11b,c provide important and, in some cases, essential tools for solving heat transfer problems. Every application of the first law must begin with the identification of an appropriate control volume and its control surface, to which an analysis is subsequently applied. The first step is to indicate the control surface by drawing a dashed line. The second step is to decide whether to perform the analysis for a time interval Δt (Equation 1.11b) or on a rate basis (Equation 1.11c). This choice depends on the objective of the solution and the way in which information is given in the problem. The next step is to identify the energy terms that are relevant in the problem you are solving. To develop your confidence in taking this last step, the remainder of this section is devoted to clarifying the following energy terms:

- Stored thermal and mechanical energy, E_{st}.
- Thermal energy generation, E_g.
- Thermal and mechanical energy transport across the control surfaces, that is, the inflow and outflow terms, E_{in} and E_{out}.

In the statement of the first law (Equation 1.11a), the total energy, E^{tot}, consists of kinetic energy (KE = $\frac{1}{2}mV^2$, where m and V are mass and velocity, respectively), potential energy (PE = mgz, where g is the gravitational acceleration and z is the vertical coordinate), and *internal energy* (U). Mechanical energy is defined as the sum of kinetic and potential energy. It will most often be the case in heat transfer problems that the changes in kinetic and potential energy are small and can be neglected. The internal energy consists of a *sensible component*, which accounts for the translational, rotational, and/or vibrational motion of the atoms/molecules comprising the matter; a *latent component*, which relates to intermolecular forces influencing phase change between solid, liquid, and vapor states; a *chemical component*, which accounts for energy stored in the chemical bonds between atoms; and a *nuclear component*, which accounts for the binding forces in the nucleus.

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For the study of heat transfer, we focus attention on the sensible and latent components of the internal energy (U_{sens} and U_{lat}, respectively), which are together referred to as *thermal energy*, U_t. The sensible energy is the portion that we associate mainly with changes in temperature (although it can also depend on pressure). The latent energy is the component we associate with changes in phase. For example, if the material in the control volume changes from solid to liquid (*melting*) or from liquid to vapor (*vaporization, evaporation, boiling*), the latent energy increases. Conversely, if the phase change is from vapor to liquid (*condensation*) or from liquid to solid (*solidification, freezing*), the latent energy decreases. Obviously, if there is no phase change occurring, there is no change in latent energy, and this term can be neglected.

Based on the above discussion, the *stored thermal and mechanical energy* is given by $E_{st} = KE + PE + U_t$, where $U_t = U_{sens} + U_{lat}$. In many problems, the only relevant energy term will be the sensible energy, that is, $E_{st} = U_{sens}$.

The *energy generation term* is associated with conversion from some other form of internal energy (chemical, electrical, electromagnetic, or nuclear) to thermal energy. It is a *volumetric phenomenon*. That is, it occurs within the control volume and is generally proportional to the magnitude of this volume. For example, an exothermic chemical reaction may be occurring, converting chemical energy to thermal energy. The net effect is an increase in the thermal energy of the matter within the control volume. Another source of thermal energy is the conversion from electrical energy that occurs due to resistance heating when an electric current is passed through a conductor. That is, if an electric current I passes through a resistance R in the control volume, electrical energy is dissipated at a rate I^2R, which corresponds to the rate at which thermal energy is generated (released) within the volume. In all applications of interest in this text, if chemical, electrical, or nuclear effects exist, they are treated as sources (or *sinks*, which correspond to negative sources) of thermal energy and hence are included in the generation terms of Equations 1.11b,c.

The inflow and outflow terms are *surface phenomena*. That is, they are associated exclusively with processes occurring at the control surface and are generally proportional to the surface area. As discussed previously, the energy inflow and outflow terms include heat transfer (which can be by conduction, convection, and/or radiation) and work interactions occurring at the system boundaries (e.g., due to displacement of a boundary, a rotating shaft, and/or electromagnetic effects). For cases in which mass crosses the control volume boundary (e.g., for situations involving fluid flow), the inflow and outflow terms also include energy (thermal and mechanical) advected (carried) by mass entering and leaving the control volume. For instance, if the mass flow rate entering through the boundary is $\dot{m}$, then the rate at which thermal and mechanical energy enters with the flow is $\dot{m}\,(u_t + \frac{1}{2}V^2 + gz)$, where u_t is the thermal energy per unit mass.

When the first law is applied to a control volume with fluid crossing its boundary, it is customary to divide the work term into two contributions. The first contribution, termed *flow work*, is associated with work done by pressure forces moving fluid through the boundary. For a *unit mass*, the amount of work is equivalent to the product of the pressure and the specific volume of the fluid (pv). The symbol $\dot{W}$ is traditionally used for the *remainder* of the work term (not including flow work). If operation is under steady-state conditions ($dE_{st}/dt = 0$) and there is no thermal energy generation, Equation 1.11c reduces to the following form of the steady-flow

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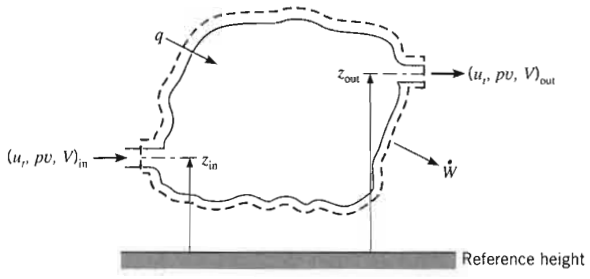


FIGURE 1.8 Conservation of energy for a steady-flow, open system.

energy equation (see Figure 1.8) which will be familiar if you have taken a thermodynamics course:

$$\dot{m}\,(u_t + pv + \tfrac{1}{2}V^2 + gz)_{in} - \dot{m}\,(u_t + pv + \tfrac{1}{2}V^2 + gz)_{out} + q - \dot{W} = 0 \qquad (1.11d)$$

Terms within the parentheses are expressed for a unit mass of fluid at the inflow and outflow locations. When multiplied by the mass flow rate $\dot{m}$, they yield the rate at which the corresponding form of the energy (thermal, flow work, kinetic, and potential) enters or leaves the control volume. The sum of thermal energy and flow work per unit mass may be replaced by the enthalpy per unit mass, $i = u_t + pv$.

In most open system applications of interest in this text, changes in latent energy between the inflow and outflow conditions of Equation 1.11d may be neglected, so the thermal energy reduces to only the sensible component. If the fluid is approximated as an *ideal gas* with *constant specific heats*, the difference in enthalpies (per unit mass) between the inlet and outlet flows may then be expressed as $(i_{in} - i_{out}) = c_p\,(T_{in} - T_{out})$, where c_p is the specific heat at constant pressure and T_{in} and T_{out} are the inlet and outlet temperatures, respectively. If the fluid is an *incompressible liquid*, its specific heats at constant pressure and volume are equal, $c_p = c_v \equiv c$, and for Equation 1.11d the change in sensible energy (per unit mass) reduces to $(u_{t,in} - u_{t,out}) = c(T_{in} - T_{out})$. Unless the pressure drop is extremely large, the difference in flow work terms, $(pv)_{in} - (pv)_{out}$, is negligible for a liquid.

Having already assumed steady-state conditions, no changes in latent energy, and no thermal energy generation, there are at least four cases in which further assumptions can be made to reduce Equation 1.11d to the *simplified steady-flow thermal energy equation*:

$$q = \dot{m}\,c_p(T_{out} - T_{in}) \qquad (1.11e)$$

The right-hand side of Equation 1.11e represents the net rate of outflow of enthalpy (thermal energy plus flow work) for an ideal gas or of thermal energy for an incompressible liquid.

The first two cases for which Equation 1.11e holds can readily be verified by examining Equation 1.11d. They are:

1. An ideal gas with negligible kinetic and potential energy changes and negligible work (other than flow work).

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2. An incompressible liquid with negligible kinetic and potential energy changes and negligible work, *including* flow work. As noted in the preceding discussion, flow work is negligible for an incompressible liquid provided the pressure variation is not too great.

The second pair of cases cannot be directly derived from Equation 1.11d but require further knowledge of how mechanical energy is converted into thermal energy. These cases are:

3. An ideal gas with negligible viscous dissipation and negligible pressure variation.
4. An incompressible liquid with negligible viscous dissipation.

Viscous dissipation is the conversion from mechanical energy to thermal energy associated with viscous forces acting in a fluid. It is only important in cases involving high-speed flow and/or highly viscous fluid. Since so many engineering applications satisfy one or more of the above four conditions, Equation 1.11e is commonly used for the analysis of heat transfer in moving fluids. It will be used in Chapter 8 in the study of convection heat transfer in internal flow.

The *mass flow rate* $\dot{m}$ of the fluid may be expressed as $\dot{m} = \rho V A_c$, where ρ is the fluid density and A_c is the cross-sectional area of the channel through which the fluid flows. The *volumetric flow rate* is simply $\dot{\forall} = V A_c = \dot{m}/\rho$.

EXAMPLE 1.3

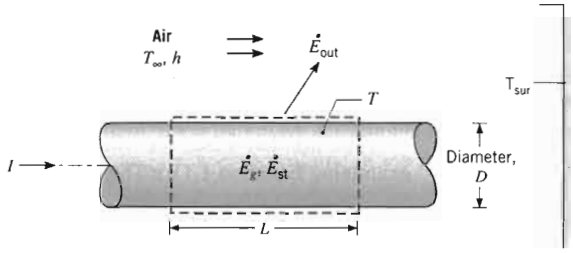
A long conducting rod of diameter D and electrical resistance per unit length R'_e is initially in thermal equilibrium with the ambient air and its surroundings. This equilibrium is disturbed when an electrical current I is passed through the rod. Develop an equation that could be used to compute the variation of the rod temperature with time during passage of the current.

SOLUTION

Known: Temperature of a rod of prescribed diameter and electrical resistance changes with time due to passage of an electrical current.

Find: Equation that governs temperature change with time for the rod.

Schematic:



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Assumptions:

1. At any time t the temperature of the rod is uniform.
2. Constant properties ($\rho, c, \varepsilon = \alpha$).
3. Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and a large enclosure.

Analysis: The first law of thermodynamics may often be used to determine an unknown temperature. In this case, relevant terms include heat transfer by convection and radiation from the surface, energy generation due to Ohmic heating within the conductor, and a change in thermal energy storage. Since we wish to determine the rate of change of the temperature, the first law should be applied at an instant of time. Hence, applying Equation 1.11c to a control volume of length L about the rod, it follows that

$$\dot{E}_g - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}$$

where energy generation is due to the electric resistance heating,

$$\dot{E}_g = I^2 R_e' L$$

Heating occurs uniformly within the control volume and could also be expressed in terms of a volumetric heat generation rate $\dot{q}$ (W/m^3). The generation rate for the entire control volume is then $\dot{E}_g = \dot{q}V$, where $\dot{q} = I^2 R_e'/(\pi D^2/4)$. Energy outflow is due to convection and net radiation from the surface, Equations 1.3a and 1.7, respectively,

$$\dot{E}_{\text{out}} = h(\pi DL)(T - T_\infty) + \varepsilon\sigma(\pi DL)(T^4 - T_{\text{sur}}^4)$$

and the change in energy storage is due to the temperature change,

$$\dot{E}_{\text{st}} = \frac{dU_t}{dt} = \frac{d}{dt}(\rho VcT)$$

The term $\dot{E}_{\text{st}}$ is associated with the rate of change in the internal thermal energy of the rod, where ρ and c are the mass density and the specific heat, respectively, of the rod material, and V is the volume of the rod, $V = (\pi D^2/4)L$. Substituting the rate equations into the energy balance, it follows that

$$I^2 R_e' L - h(\pi DL)(T - T_\infty) - \varepsilon\sigma(\pi DL)(T^4 - T_{\text{sur}}^4) = \rho c\left(\frac{\pi D^2}{4}\right)L\frac{dT}{dt}$$

Hence

$$\frac{dT}{dt} = \frac{I^2 R_e' - \pi Dh(T - T_\infty) - \pi D\varepsilon\sigma(T^4 - T_{\text{sur}}^4)}{\rho c(\pi D^2/4)} \quad \triangleleft$$

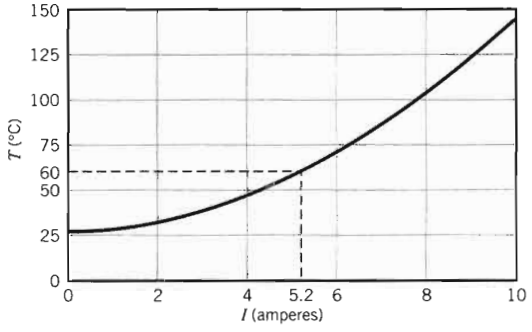
Comments:

1. The above equation could be solved for the time dependence of the rod temperature by integrating numerically. A steady-state condition would eventually be reached for which $dT/dt = 0$. The rod temperature is then determined by an algebraic equation of the form

$$\pi Dh(T - T_\infty) + \pi D\varepsilon\sigma(T^4 - T_{\text{sur}}^4) = I^2 R_e'$$

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2. For fixed environmental conditions (h, T_∞, T_{sur}), as well as a rod of fixed geometry (D) and properties (ε, R_e'), the steady-state temperature depends on the rate of thermal energy generation and hence on the value of the electric current. Consider an uninsulated copper wire ($D = 1$ mm, $\varepsilon = 0.8$, $R_e' = 0.4\ \Omega$/m) in a relatively large enclosure ($T_{sur} = 300$ K) through which cooling air is circulated ($h = 100\ \text{W/m}^2 \cdot \text{K}$, $T_\infty = 300$ K). Substituting these values into the foregoing equation, the rod temperature has been computed for operating currents in the range $0 \le I \le 10$ A and the following results were obtained:



3. If a maximum operating temperature of $T = 60$°C is prescribed for safety reasons, the current should not exceed 5.2 A. At this temperature, heat transfer by radiation (0.6 W/m) is much less than heat transfer by convection (10.4 W/m). Hence, if one wished to operate at a larger current while maintaining the rod temperature within the safety limit, the convection coefficient would have to be increased by increasing the velocity of the circulating air. For $h = 250\ \text{W/m}^2 \cdot \text{K}$, the maximum allowable current could be increased to 8.1 A.
4. The *Interactive Heat Transfer* (*IHT*) software accompanying your text is especially useful for solving equations, such as the energy balance in Comment 1, and generating the graphical results of Comment 2. To apply *IHT* for this purpose, key the energy balance into the *Workspace* using the following free-form symbol format.

$$pi*D*h*(T - Tinf) + pi*D*eps*sigma*(T\text{^}4 - Tsur\text{^}4) = I\text{^}2*Re'$$

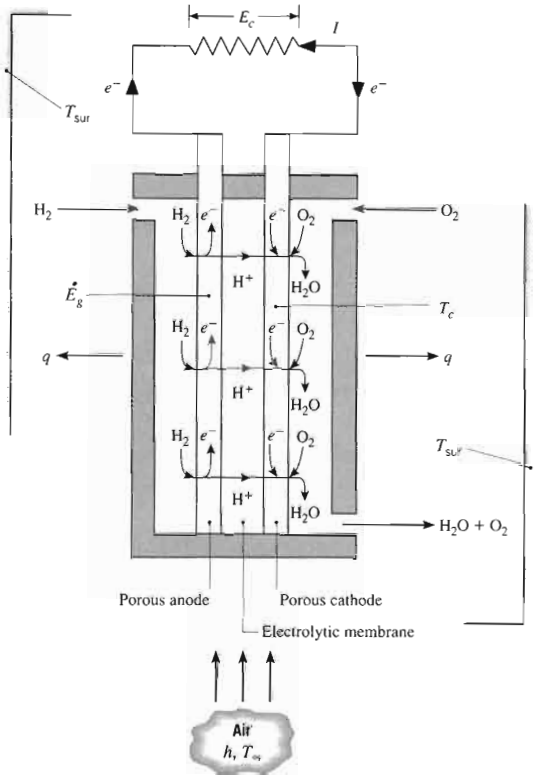
Next, key in the known input parameters, press *Solve*, and find T. To determine T for the operating current range, $0 \le I \le 10$ A, use the *Explore* feature to sweep over the variable I. You may plot T vs. I using the *Graph* feature and may label the plot to provide a professional appearance.

EXAMPLE 1.4

A *hydrogen-air* Proton Exchange Membrane (PEM) fuel cell is illustrated below. It consists of an *electrolytic membrane* sandwiched between porous *cathode* and *anode* materials, forming a very thin, three-layer *membrane electrode assembly*

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(MEA). At the anode, protons and electrons are generated ($2H_2 \rightarrow 4H^+ + 4e^-$), while at the cathode the protons and electrons recombine to form water ($O_2 + 4e^- + 4H^+ \rightarrow 2H_2O$). The overall reaction is then $2H_2 + O_2 \rightarrow 2H_2O$. The dual role of the electrolytic membrane is to transfer hydrogen ions and serve as a barrier to electron transfer, forcing the electrons to the electrical load that is external to the fuel cell.



The membrane must operate in a moist state in order to conduct ions. However, the presence of liquid water in the cathode material may block the oxygen from reaching the cathode reaction sites, resulting in failure of the fuel cell. Therefore, it is critical to control the temperature of the fuel cell, T_c, so that the cathode side contains saturated water vapor.

For a given set of H_2 and air inlet flow rates and use of a 50 mm × 50 mm MEA, the fuel cell generates $P = I \cdot E_c = 9$ W of electrical power, associated with a cell voltage $E_c = 0.6$ volts and electric current $I = 15$ A. Saturated vapor conditions exist in the fuel cell, corresponding to $T_c = T_{sat} = 56.4°C$. The overall electrochemical reaction is exothermic and the corresponding thermal generation rate of $\dot{E}_g = 11.25$ W must be removed from the fuel cell by convection and radiation. The ambient and surrounding temperatures are $T_\infty = T_{sur} = 25°C$, and the relationship between the cooling air velocity and the convection heat transfer coefficient, h, is

$$h = 10.9\ \text{W} \cdot \text{s}^{0.8}/\text{m}^{2.8} \cdot \text{K} \times V^{0.8}$$

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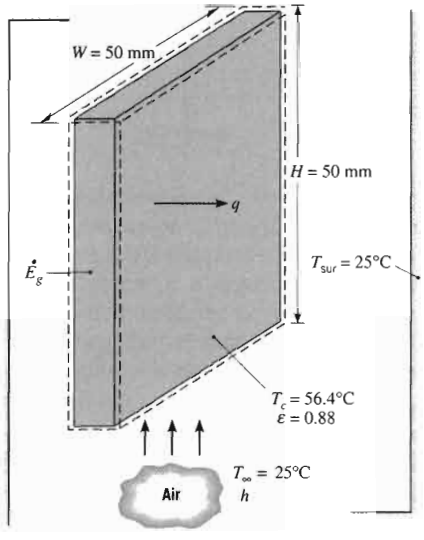
where V has units of m/s. The exterior surface of the fuel cell has an emissivity of $\varepsilon = 0.88$. Determine the value of the cooling air velocity needed to maintain steady-state operating conditions. Assume the edges of the fuel cell are well insulated.

SOLUTION

Known: Ambient and surrounding temperatures, fuel cell output voltage and electrical current, heat generated by the overall electrochemical reaction, and the desired fuel cell operating temperature.

Find: The required cooling air velocity, V, needed to maintain steady-state operation at $T_c \approx 56.4°C$.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Negligible temperature variations within the fuel cell.
3. Fuel cell is placed in large surroundings.
4. Edges of the fuel cell are well insulated.
5. Negligible energy entering or leaving the control volume due to gas or liquid flows.

Analysis: To determine the required cooling air velocity, we must first perform an energy balance on the fuel cell. With $\dot{E}_{in} = 0$ and $\dot{E}_{out} = \dot{E}_g$,

$$q_{conv} + q_{rad} = \dot{E}_g = 11.25 \text{ W}$$

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where

$$q_{rad} = \varepsilon A\sigma(T_c^4 - T_{sur}^4)$$
$$= 0.88 \times (2 \times 0.05\ \text{m} \times 0.05\ \text{m}) \times 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4 \times (329.4^4 - 298^4)\text{K}^4$$
$$= 0.97\ \text{W}$$

Therefore, we may find

$$q_{conv} = 11.25\ \text{W} - 0.97\ \text{W} = 10.28\ \text{W}$$
$$= hA(T_c - T_\infty)$$
$$= 10.9\ \text{W} \cdot \text{s}^{0.8}/\text{m}^{2.8} \cdot \text{K} \times V^{0.8} A(T_c - T_\infty)$$

which may be rearranged to yield

$$V = \left[\frac{10.28\ \text{W}}{10.9\ \text{W}\cdot\text{s}^{0.8}/\text{m}^{2.8}\cdot\text{K} \times (2 \times 0.05\ \text{m} \times 0.05\ \text{m}) \times (56.4 - 25)^\circ\text{C}}\right]^{1.25}$$

$$V = 9.4\ \text{m/s}$$

Comments:

1. Temperature and humidity of the MEA will vary from location to location within the fuel cell. Prediction of the *local* conditions within the fuel cell would require a more detailed analysis.
2. The required cooling air velocity is quite high. Decreased cooling velocities could be used if heat transfer enhancement devices were added to the exterior of the fuel cell.
3. The convective heat rate is significantly greater than the radiation heat rate.
4. The chemical energy (20.25 W) of the hydrogen and oxygen is converted to electrical (9 W) and thermal (11.25 W) energy. This fuel cell operates at a conversion efficiency of (9 W)/(20.25 W) × 100 = 44 percent.

EXAMPLE 1.5

Large PEM fuel cells, such as those used in automotive applications, often require internal cooling using pure liquid water to maintain their temperature at a desired level (see Example 1.4). In cold climates, the cooling water must be drained from the fuel cell to an adjoining container when the automobile is turned off so that harmful freezing does not occur within the fuel cell. Consider a mass M of ice that was frozen while the automobile was not being operated. The ice is at the fusion temperature ($T_f = 0°\text{C}$) and is enclosed in a cubical container of width W on a side. The container wall is of thickness L and thermal conductivity k. If the outer surface of the wall is heated to a temperature $T_1 > T_f$ to melt the ice, obtain an expression for the time needed to melt the entire mass of ice and, in turn, deliver cooling water to, and energize, the fuel cell.

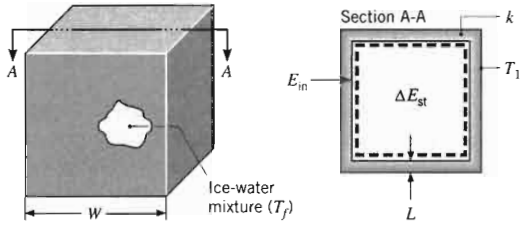
SOLUTION

Known: Mass and temperature of ice. Dimensions, thermal conductivity, and outer surface temperature of containing wall.

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Find: Expression for time needed to melt the ice.

Schematic:



Assumptions:

1. Inner surface of wall is at T_f throughout the process.
2. Constant properties.
3. Steady-state, one-dimensional conduction through each wall.
4. Conduction area of one wall may be approximated as W^2 ($L \ll W$).

Analysis: Since we must determine the melting time t_m, the first law should be applied over the time interval $\Delta t = t_m$. Hence, applying Equation 1.11b to a control volume about the ice–water mixture, it follows that

$$E_{in} = \Delta E_{st} = \Delta U_{lat}$$

where the increase in energy stored within the control volume is due exclusively to the change in latent energy associated with conversion from the solid to liquid state. Heat is transferred to the ice by means of conduction through the container wall, and since the temperature difference across the wall is assumed to remain at $(T_1 - T_f)$ throughout the melting process, the wall conduction rate is a constant

$$q_{cond} = k(6W^2)\frac{T_1 - T_f}{L}$$

and the amount of energy inflow is

$$E_{in} = \left[k(6W^2)\frac{T_1 - T_f}{L}\right] t_m$$

The amount of energy required to effect such a phase change per unit mass of solid is termed the *latent heat of fusion* h_{sf}. Hence the increase in energy storage is

$$\Delta E_{st} = Mh_{sf}$$

By substituting into the first law expression, it follows that

$$t_m = \frac{Mh_{sf}L}{6W^2k(T_1 - T_f)}$$ ◁

Comments:

1. Several complications would arise if the ice were initially subcooled. The storage term would have to include the change in sensible (internal thermal) energy required

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to take the ice from the subcooled to the fusion temperature. During this process, temperature gradients would develop in the ice.

2. Consider a cavity of width $W = 100$ mm on a side, wall thickness $L = 5$ mm, and thermal conductivity $k = 0.05$ W/m · K. The mass of the ice in the cavity is

$$M = \rho_s(W - 2L)^3 = 920 \text{ kg/m}^3 \times (0.100 - 0.01)^3 \text{ m}^3 = 0.67 \text{ kg}$$

If the outer surface temperature is $T_1 = 30°C$, the time required to melt the ice is

$$t_m = \frac{0.67 \text{ kg} \times 334{,}000 \text{ J/kg} \times 0.005 \text{ m}}{6(0.100 \text{ m})^2 \times 0.05 \text{ W/m} \cdot \text{K} \, (30 - 0)°C} = 1243 \text{ s} = 20.7 \text{ min}$$

The density and latent heat of fusion of the ice are $\rho_s = 920$ kg/m^3 and $h_{sf} = 334$ kJ/kg, respectively.

3. Note that the units of K and °C cancel each other in the foregoing expression for t_m. Such cancellation occurs frequently in heat transfer analysis and is due to both units appearing in the context of a *temperature difference*.

1.3.2 The Surface Energy Balance

We will frequently have occasion to apply the conservation of energy requirement at the surface of a medium. In this special case the control surfaces are located on either side of the physical boundary and enclose no mass or volume (see Figure 1.9). Accordingly, the generation and storage terms of the conservation expression, Equation 1.11c, are no longer relevant and it is only necessary to deal with surface phenomena. For this case the conservation requirement becomes

$$\dot{E}_{in} - \dot{E}_{out} = 0 \tag{1.12}$$

Even though thermal energy generation may be occurring in the medium, the process would not affect the energy balance at the control surface. Moreover, this conservation requirement holds for both *steady-state* and *transient* conditions.

In Figure 1.9 three heat transfer terms are shown for the control surface. On a unit area basis they are conduction from the medium *to* the control surface (q''_{cond}),

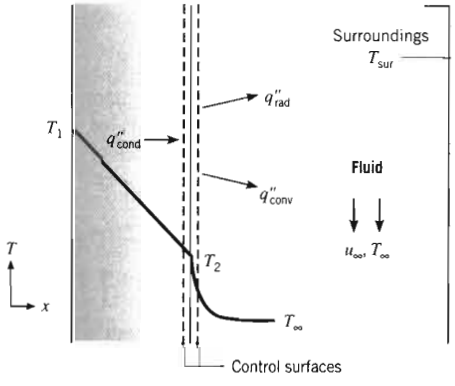


FIGURE 1.9
The energy balance for conservation of energy at the surface of a medium.

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convection *from* the surface to a fluid (q''_{conv}), and net radiation exchange from the surface to the surroundings (q''_{rad}). The energy balance then takes the form

$$q''_{cond} - q''_{conv} - q''_{rad} = 0 \tag{1.13}$$

and we can express each of the terms using the appropriate rate equations, Equations 1.2, 1.3a, and 1.7.

EXAMPLE 1.6

Humans are able to control their heat production rate and heat loss rate to maintain a nearly constant core temperature of $T_c = 37°C$ under a wide range of environmental conditions. This process is called *thermoregulation*. From the perspective of calculating heat transfer between a human body and its surroundings, we focus on a layer of skin and fat, with its outer surface exposed to the environment and its inner surface at a temperature slightly less than the core temperature, $T_i = 35°C = 308$ K. Consider a person with a skin/fat layer of thickness $L = 3$ mm and effective thermal conductivity $k = 0.3$ W/m · K. The person has a surface area $A = 1.8$ m^2 and is dressed in a bathing suit. The emissivity of the skin is $\varepsilon = 0.95$.

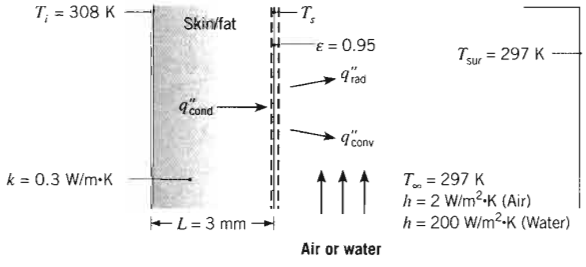
1. When the person is in still air at $T_\infty = 297$ K, what is the skin surface temperature and rate of heat loss to the environment? Convection heat transfer to the air is characterized by a free convection coefficient of $h = 2$ W/m^2 · K.
2. When the person is in water at $T_\infty = 297$ K, what is the skin surface temperature and heat loss rate? Heat transfer to the water is characterized by a convection coefficient of $h = 200$ W/m^2 · K.

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, emissivity, and surface area. Ambient conditions.

Find: Skin surface temperature and heat loss rate for the person in air and the person in water.

Schematic:



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Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat layer.
3. Thermal conductivity is uniform.
4. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
5. Liquid water is opaque to thermal radiation.
6. Bathing suit has no effect on heat loss from body.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

Analysis:

1. The skin surface temperature may be obtained by performing an energy balance at the skin surface. From Equation 1.12,

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

It follows that, on a unit area basis,

$$q''_{cond} - q''_{conv} - q''_{rad} = 0$$

or, rearranging and substituting from Equations 1.2, 1.3a, and 1.7,

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + \varepsilon\sigma(T_s^4 - T_{sur}^4)$$

The only unknown is T_s, but we cannot solve for it explicitly because of the fourth power dependence of the radiation term. Therefore, we must solve the equation iteratively, which can be done by hand or by using *IHT* or some other equation solver. To expedite a hand solution, we write the radiation heat flux in terms of the radiation heat transfer coefficient, using Equations 1.8 and 1.9:

$$k\frac{T_i - T_s}{L} = h(T_s - T_\infty) + h_r(T_s - T_{sur})$$

Solving for T_s, with $T_{sur} = T_\infty$, we have

$$T_s = \frac{\dfrac{kT_i}{L} + (h + h_r)T_\infty}{\dfrac{k}{L} + (h + h_r)}$$

We estimate h_r using Equation 1.9 with a guessed value of $T_s = 305$ K and $T_\infty = 297$ K, to yield $h_r = 5.9\ \text{W/m}^2 \cdot \text{K}$. Then, substituting numerical values into the above equation, we find

$$T_s = \frac{\dfrac{0.3\ \text{W/m} \cdot \text{K} \times 308\ \text{K}}{3 \times 10^{-3}\ \text{m}} + (2 + 5.9)\ \text{W/m}^2 \cdot \text{K} \times 297\ \text{K}}{\dfrac{0.3\ \text{W/m} \cdot \text{K}}{3 \times 10^{-3}\ \text{m}} + (2 + 5.9)\ \text{W/m}^2 \cdot \text{K}} = 307.2\ \text{K}$$

With this new value of T_s we can recalculate h_r and T_s, which are unchanged. Thus the skin temperature is 307.2 K ≅ 34°C. ◁

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The rate of heat loss can be found by evaluating the conduction through the skin/fat layer:

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2 \times \frac{(308 - 307.2)\ \text{K}}{3 \times 10^{-3}\ \text{m}} = 146\ \text{W} \quad \triangleleft$$

2. Since liquid water is opaque to thermal radiation, heat loss from the skin surface is by convection only. Using the previous expression with $h_r = 0$, we find

$$T_s = \frac{\dfrac{0.3\ \text{W/m} \cdot \text{K} \times 308\ \text{K}}{3 \times 10^{-3}\ \text{m}} + 200\ \text{W/m}^2 \cdot \text{K} \times 297\ \text{K}}{\dfrac{0.3\ \text{W/m} \cdot \text{K}}{3 \times 10^{-3}\ \text{m}} + 200\ \text{W/m}^2 \cdot \text{K}} = 300.7\ \text{K} \quad \triangleleft$$

and

$$q_s = kA\frac{T_i - T_s}{L} = 0.3\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2 \times \frac{(308 - 300.7)\ \text{K}}{3 \times 10^{-3}\ \text{m}} = 1320\ \text{W} \quad \triangleleft$$

Comments:

1. When using energy balances involving radiation exchange, the temperatures appearing in the radiation terms must be expressed in kelvins, and it is good practice to use kelvins in all terms to avoid confusion.
2. In part 1, heat losses due to convection and radiation are 37 W and 109 W, respectively. Thus, it would not have been reasonable to neglect radiation. Care must be taken to include radiation when the heat transfer coefficient is small (as it often is for natural convection to a gas), even if the problem statement does not give any indication of its importance.
3. A typical rate of metabolic heat generation is 100 W. If the person stayed in the water too long the core body temperature would begin to fall. The large heat loss in water is due to the higher heat transfer coefficient, which in turn is due to the much larger thermal conductivity of water compared to air.
4. The skin temperature of 34°C in part 1 is comfortable, but the skin temperature of 28°C in part 2 is uncomfortably cold.
5. By entering the energy balance and appropriate input parameters in the *IHT Workspace*, a *model* of the system may be developed for calculating T_s and q_s or any other system parameter. With this model, *parameter sensitivity* studies may be performed to explore, for example, the effect of changing h on T_s. Wherever possible, it is good practice to *validate* your model against a known solution, which in this case is shown in the foregoing analysis.

1.3.3 Application of the Conservation Laws: Methodology

In addition to being familiar with the transport rate equations described in Section 1.2, the heat transfer analyst must be able to work with the energy conservation requirements of Equations 1.11 and 1.12. The application of these balances is simplified if a few basic rules are followed.

1.4 Analysis Methodo

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1. The appropriate control volume must be defined, with the control surfaces represented by a dashed line or lines.
2. The appropriate time basis must be identified.
3. The relevant energy processes must be identified, and each process should be shown on the control volume by an appropriately labeled arrow.
4. The conservation equation must then be written, and appropriate rate expressions must be substituted for the relevant terms in the equation.

It is important to note that the energy conservation requirement may be applied to a *finite* control volume or a *differential* (infinitesimal) control volume. In the first case the resulting expression governs overall system behavior. In the second case a differential equation is obtained that can be solved for conditions at each point in the system. Differential control volumes are introduced in Chapter 2, and both types of control volumes are used extensively throughout the text.

1.4 *Analysis of Heat Transfer Problems: Methodology*

A major objective of this text is to prepare you to solve engineering problems that involve heat transfer processes. To this end numerous problems are provided at the end of each chapter. In working these problems you will gain a deeper appreciation for the fundamentals of the subject, and you will gain confidence in your ability to apply these fundamentals to the solution of engineering problems.

In solving problems, we advocate the use of a systematic procedure characterized by a prescribed format. We consistently employ this procedure in our examples, and we require our students to use it in their problem solutions. It consists of the following steps:

1. *Known:* After carefully reading the problem, state briefly and concisely what is known about the problem. Do not repeat the problem statement.
2. *Find:* State briefly and concisely what must be found.
3. *Schematic:* Draw a schematic of the physical system. If application of the conservation laws is anticipated, represent the required control surface or surfaces by dashed lines on the schematic. Identify relevant heat transfer processes by appropriately labeled arrows on the schematic.
4. *Assumptions:* List all pertinent simplifying assumptions.
5. *Properties:* Compile property values needed for subsequent calculations and identify the source from which they are obtained.
6. *Analysis:* Begin your analysis by applying appropriate conservation laws, and introduce rate equations as needed. Develop the analysis as completely as possible before substituting numerical values. Perform the calculations needed to obtain the desired results.
7. *Comments:* Discuss your results. Such a discussion may include a summary of key conclusions, a critique of the original assumptions, and an inference of

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trends obtained by performing additional *what-if* and *parameter sensitivity* calculations.

The importance of following steps 1 through 4 should not be underestimated. They provide a useful guide to thinking about a problem before effecting its solution. In step 7 we hope you will take the initiative to gain additional insights by performing calculations that may be computer-based. The software accompanying this text provides a suitable tool for effecting such calculations.

EXAMPLE 1.7

The coating on a plate is cured by exposure to an infrared lamp providing a uniform irradiation of 2000 W/m^2. It absorbs 80% of the irradiation and has an emissivity of 0.50. It is also exposed to an air flow and large surroundings for which temperatures are 20°C and 30°C, respectively.

1. If the convection coefficient between the plate and the ambient air is 15 W/m$^2 \cdot$ K, what is the cure temperature of the plate?
2. Final characteristics of the coating, including wear and durability, are known to depend on the temperature at which curing occurs. An air flow system is able to control the air velocity, and hence the convection coefficient, on the cured surface, but the process engineer needs to know how the temperature depends on the convection coefficient. Provide the desired information by computing and plotting the surface temperature as a function of h for $2 \leq h \leq 200$ W/m$^2 \cdot$ K. What value of h would provide a cure temperature of 50°C?

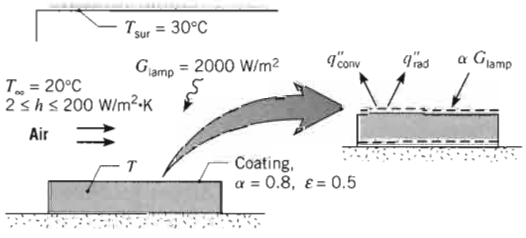
SOLUTION

Known: Coating with prescribed radiation properties is cured by irradiation from an infrared lamp. Heat transfer from the coating is by convection to ambient air and radiation exchange with the surroundings.

Find:

1. Cure temperature for $h = 15$ W/m$^2 \cdot$ K.
2. Effect of air flow on the cure temperature for $2 \leq h \leq 200$ W/m$^2 \cdot$ K. Value of h for which the cure temperature is 50°C.

Schematic:



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Assumptions:

1. Steady-state conditions.
2. Negligible heat loss from back surface of plate.
3. Plate is small object in large surroundings, and coating has an absorptivity of $\alpha_{sur} = \varepsilon = 0.5$ with respect to irradiation from the surroundings.

Analysis:

1. Since the process corresponds to steady-state conditions and there is no heat transfer at the back surface, the plate must be isothermal ($T_s = T$). Hence the desired temperature may be determined by placing a control surface about the exposed surface and applying Equation 1.12 or by placing the control surface about the entire plate and applying Equation 1.11c. Adopting the latter approach and recognizing that there is no internal energy generation ($\dot{E}_g = 0$), Equation 1.11c reduces to

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

where $\dot{E}_{st} = 0$ for steady-state conditions. With energy inflow due to absorption of the lamp irradiation by the coating and outflow due to convection and net radiation transfer to the surroundings, it follows that

$$(\alpha G)_{lamp} - q''_{conv} - q''_{rad} = 0$$

Substituting from Equations 1.3a and 1.7, we obtain

$$(\alpha G)_{lamp} - h(T - T_\infty) - \varepsilon\sigma(T^4 - T_{sur}^4) = 0$$

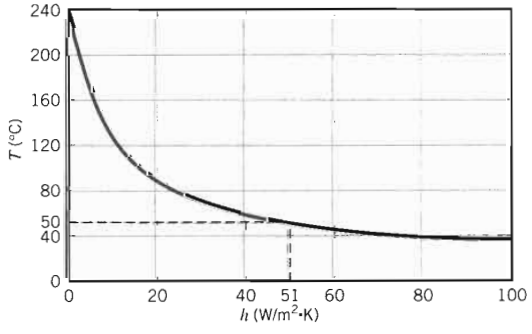
Substituting numerical values

$$0.8 \times 2000\ \text{W/m}^2 - 15\ \text{W/m}^2 \cdot \text{K}(T - 293)\ \text{K}$$
$$- 0.5 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4\,(T^4 - 303^4)\ \text{K}^4 = 0$$

and solving by trial-and-error, we obtain

$$T = 377\ \text{K} = 104°\text{C} \quad \triangleleft$$

2. Solving the foregoing energy balance for selected values of h in the prescribed range and plotting the results, we obtain



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If a cure temperature of 50°C is desired, the air flow must provide a convection coefficient of

$$h(T = 50°C) = 51.0\ W/m^2 \cdot K \quad \triangleleft$$

Comments:

1. The coating (plate) temperature may be reduced by decreasing T_∞ and T_{sur}, as well as by increasing the air velocity and hence the convection coefficient.
2. The relative contributions of convection and radiation to heat transfer from the plate vary greatly with h. For $h = 2\ W/m^2 \cdot K$, $T = 204°C$ and radiation dominates ($q''_{rad} \approx 1232\ W/m^2$, $q''_{conv} \approx 368\ W/m^2$). Conversely, for $h = 200\ W/m^2 \cdot K$, $T = 28°C$ and convection dominates ($q''_{conv} \approx 1606\ W/m^2$, $q''_{rad} \approx -6\ W/m^2$). In fact, for this condition the plate temperature is slightly less than that of the surroundings and net radiation exchange is *to* the plate.
3. This example is one of 15 ready-to-solve models in *IHT*, which may be accessed from *Examples* in the menu bar. As you begin each chapter, check the corresponding model examples. Each model includes *Exercises* that test your understanding of heat transfer concepts.

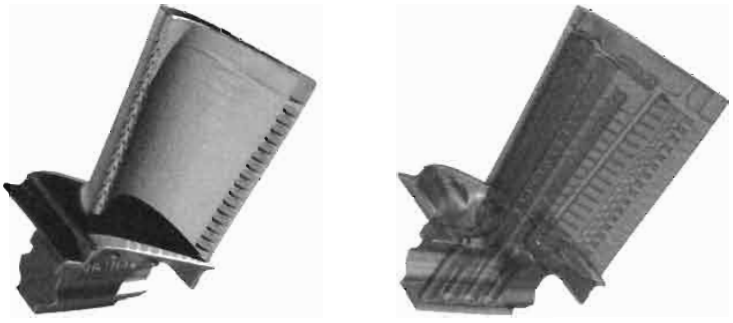
1.5 *Relevance of Heat Transfer*

We will devote much time to acquiring an understanding of heat transfer effects and to developing the skills needed to predict heat transfer rates and temperatures that evolve in certain situations. What is the value of this knowledge, and to what problems may it be applied? A few examples will serve to illustrate the rich breadth of applications in which heat transfer plays a critical role.

Heat transfer is a dominant aspect of nearly every energy conversion and production device. For example, the efficiency of a *gas turbine engine* increases with its operating temperature. Today, the temperatures of the *combustion gases* inside these engines far exceed the melting point of the exotic alloys used to manufacture the turbine blades and vanes. Safe operation is typically achieved by three means. First, relatively cool gases are injected through small holes at the leading edge of a turbine blade (Figure 1.10). These gases hug the blade as they are carried downstream and help insulate the blade from the hot combustion gases. Second, thin layers of a very low thermal conductivity, ceramic *thermal barrier coating* are applied to the blades and vanes to provide an extra layer of insulation. These coatings are produced by spraying *molten* ceramic powders onto the engine components using extremely high-temperature sources such as plasma spray guns that can operate in excess of 10,000 kelvins. Third, the blades and vanes are designed with intricate, internal cooling passages, all carefully configured by the heat transfer engineer to allow the gas turbine engine to operate under such extreme conditions.

Emerging energy conversion devices such as *fuel cells* generate power from environmentally benign fuels such as hydrogen. Major roadblocks hindering widespread adoption of fuel cells are their size, weight, and limited durability. As with the gas turbine engine, the efficiency of a fuel cell increases with temperature, but

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(*a*) (*b*)

FIGURE 1.10 Gas turbine blade. (*a*) External view showing holes for injection of cooling gases. (*b*) X ray view showing internal cooling passages. (Images courtesy of FarField Technology, Ltd., Christchurch, New Zealand.)

high operating temperatures and large internal temperature gradients can cause the delicate polymeric materials within the fuel cell to fail. The hydrogen fuel cell is the type that might eventually be used for automotive applications and is an *electrochemical reactor* that will cease operating if its internal components are contaminated with impurities. Water, in both the liquid and vapor phases, exists within every hydrogen fuel cell, but commonly used substances in internal combustion engines such as antifreeze cannot be utilized in a fuel cell. What are the heat transfer mechanisms that must be controlled to avoid freezing of pure water within the fuel cell engine when a vehicle of the future is parked overnight in a cold region? How might your knowledge of internal forced convection, evaporation, or condensation be applied to control the operating temperatures and enhance the durability of a fuel cell?

Due to the *information technology* revolution of the last two decades, strong industrial productivity growth has brought an improved quality of life worldwide. Many information technology breakthroughs have been enabled by advances in heat transfer engineering that have ensured the precise control of temperatures of systems ranging in size from nanoscale *integrated circuits*, to microscale *storage media* including compact discs, to large *data centers* filled with heat-generating equipment. As electronic devices become faster and incorporate greater functionality, they generate more thermal energy. Simultaneously, the devices have become smaller. Inevitably, heat fluxes (W/m^2) and volumetric energy generation rates (W/m^3) keep increasing; but the operating temperatures of the devices must be held to reasonably low values to ensure their reliability.

For *personal computers*, cooling fins (also known as *heat sinks*) are fabricated of a high thermal conductivity material (usually aluminum) and attached to the microprocessors to reduce their operating temperatures, as shown in Figure 1.11. Small fans are used to induce forced convection over the fins. The cumulative energy that is consumed worldwide, just to (1) power the small fans that provide the air flow over the fins and (2) manufacture the heat sinks for personal computers, is estimated to be over 10^9 kW · h per year [1]. How might your knowledge of conduction, convection, and radiation be used to, for example, eliminate the fan and minimize the size of the heat sink?

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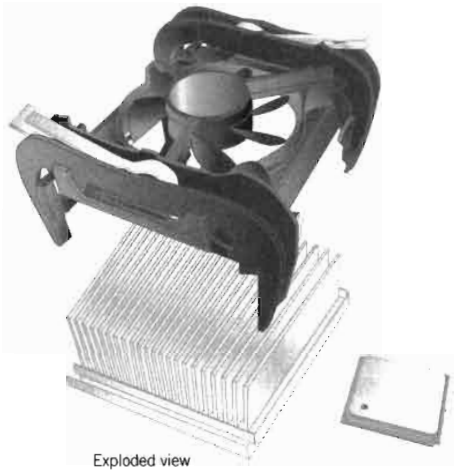


FIGURE 1.11
A finned heat sink and fan assembly (left) and microprocessor (right).

Further improvements in microprocessor technology are currently limited by our ability to cool these tiny devices. Policy makers have voiced concern about our ability to continually reduce the cost of computing and, in turn as a society, continue the growth in productivity that has marked the last 25 years, specifically citing the need to *enhance heat transfer* in *electronics cooling* [2]. How might your knowledge of heat transfer help ensure continued industrial productivity into the future?

Heat transfer is important not only in engineered systems but also in nature. Temperature regulates and triggers biological responses in all living systems and ultimately marks the boundary between sickness and health. Two common examples include *hypothermia*, which results from excessive cooling of the human body, and *heat stroke*, which is triggered in warm, humid environments. Both are deadly and are associated with *core temperatures* of the body exceeding physiological limits. Both are directly linked to the convection, radiation, and evaporation processes occurring at the surface of the body, the transport of heat within the body, and the metabolic energy generated volumetrically within the body.

Recent advances in *biomedical engineering*, such as *laser surgery*, have been enabled by successfully applying fundamental heat transfer principles [3,4]. While increased temperatures resulting from contact with hot objects may cause thermal *burns*, beneficial *hyperthermal treatments* are used to purposely destroy, for example, cancerous lesions. In a similar manner, very low temperatures might induce *frostbite*, but purposeful localized freezing can selectively destroy diseased tissue during *cryosurgery*. Many medical therapies and devices therefore operate by destructively heating or cooling diseased tissue, while simultaneously leaving the surrounding healthy tissue unaffected.

The ability to design many medical devices and to develop the appropriate protocol for their use hinges on the engineer's ability to predict and control the distribution of temperatures during thermal treatment and the distribution of chemical species in *chemotherapies*. The treatment of mammalian tissue is made complicated by the morphology of this tissue, as shown in Figure 1.12. The flow of blood within the venular and capillary structure of a thermally-treated area affects heat transfer through advection processes. Larger veins and arteries, which commonly exist in pairs throughout the body, carry blood at different temperatures and advect thermal energy at different

1.6
Units an

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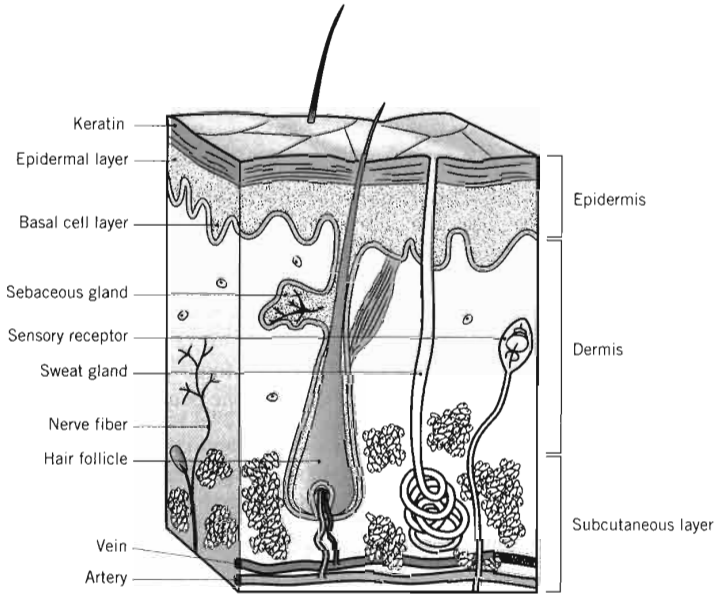


FIGURE 1.12 Morphology of human skin.

rates. Therefore, the veins and arteries exist in a *counterflow heat exchange* arrangement with warm, arteriolar blood exchanging thermal energy with the cooler, venular blood through the intervening solid tissue. Networks of smaller capillaries can also affect local temperatures by *perfusing* blood through the treated area.

In subsequent chapters, many example and homework problems will deal with the analysis of these and other *thermal systems*.

1.6 *Units and Dimensions*

The physical quantities of heat transfer are specified in terms of *dimensions,* which are measured in terms of units. Four *basic* dimensions are required for the development of heat transfer; they are length (L), mass (M), time (t), and temperature (T). All other physical quantities of interest may be related to these four basic dimensions.

In the United States it has been customary to measure dimensions in terms of an *English system of units*, for which the *base units* are

Dimension		Unit
Length (L)	→	foot (ft)
Mass (M)	→	pound mass (lb_m)
Time (t)	→	second (s)
Temperature (T)	→	degree Fahrenheit (°F)

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The units required to specify other physical quantities may then be inferred from this group. For example, the dimension of force is related to mass through Newton's second law of motion,

$$F = \frac{1}{g_c} Ma \tag{1.14}$$

where the acceleration a has units of feet per square second and g_c is a proportionality constant. If this constant is arbitrarily set equal to unity and made *dimensionless*, the dimensions of force are $(F) = (M) \cdot (L)/(t)^2$ and the unit of force is

$$1 \text{ poundal} = 1 \text{ lb}_m \cdot \text{ft/s}^2$$

Alternatively, one could work with a system of basic dimensions that includes both mass and force. However, in this case the proportionality constant must have the dimensions $(M) \cdot (L)/(F) \cdot (t)^2$. Moreover, if one defines the pound force (lb_f) as a unit of force that will accelerate one pound mass by 32.17 ft/s^2, the proportionality constant must be of the form

$$g_c = 32.17 \text{ lb}_m \cdot \text{ft/lb}_f \cdot \text{s}^2$$

The units of work may be inferred from its definition as the product of a force times a distance, in which case the units are $\text{ft} \cdot \text{lb}_f$. The units of work and energy are, of course, equivalent, although it is customary to use the British thermal unit (Btu) as the unit of thermal energy. One British thermal unit will raise the temperature of 1 lb_m of water at 68°F by 1°F. It is equivalent to $778.16 \text{ ft} \cdot \text{lb}_f$, which is termed the *mechanical equivalent of heat.*

In recent years there has been a strong trend toward worldwide usage of a standard set of units. In 1960 the SI (Système International d'Unités) system of units was defined by the Eleventh General Conference on Weights and Measures and recommended as a worldwide standard. In response to this trend, the American Society of Mechanical Engineers (ASME) has required the use of SI units in all of its publications since July 1, 1974. For this reason and because it is operationally more convenient than the English system, the SI system is used for calculations of this text. However, because for some time to come, engineers will also have to work with results expressed in the English system, you should be able to convert from one system to the other. For your convenience, conversion factors are provided on the inside back cover of the text.

The SI *base* units required for this text are summarized in Table 1.2. With regard to these units note that 1 mol is the amount of substance that has as many atoms or

TABLE 1.2 SI base and supplementary units

Quantity and Symbol	Unit and Symbol
Length (L)	meter (m)
Mass (M)	kilogram (kg)
Concentration (C)	mole (mol)
Time (t)	second (s)
Electric current (I)	ampere (A)
Thermodynamic temperature (T)	kelvin (K)
Plane angle[a] (θ)	radian (rad)
Solid angle[a] (ω)	steradian (sr)

[a]Supplementary unit.

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TABLE 1.3 SI derived units for selected quantities

Quantity	Name and Symbol	Formula	Expression in SI Base Units
Force	newton (N)	$m \cdot kg/s^2$	$m \cdot kg/s^2$
Pressure and stress	pascal (Pa)	N/m^2	$kg/m \cdot s^2$
Energy	joule (J)	$N \cdot m$	$m^2 \cdot kg/s^2$
Power	watt (W)	J/s	$m^2 \cdot kg/s^3$

molecules as there are atoms in 12 g of carbon-12 (^{12}C); this is the gram-mole (mol). Although the mole has been recommended as the unit quantity of matter for the SI system, it is more consistent to work with the kilogram-mol (kmol, kg-mol). One kmol is simply the amount of substance that has as many atoms or molecules as there are atoms in 12 kg of ^{12}C. As long as the use is consistent within a given problem, no difficulties arise in using either mol or kmol. The molecular weight of a substance is the mass associated with a mole or kilogram-mole. For oxygen, as an example, the molecular weight $\mathcal{M}$ is 16 g/mol or 16 kg/kmol.

Although the SI unit of temperature is the kelvin, use of the Celsius temperature scale remains widespread. Zero on the Celsius scale (0°C) is equivalent to 273.15 K on the thermodynamic scale,[1] in which case

$$T(\text{K}) = T(^\circ\text{C}) + 273.15$$

However, temperature *differences* are equivalent for the two scales and may be denoted as °C or K. Also, although the SI unit of time is the second, other units of time (minute, hour, and day) are so common that their use with the SI system is generally accepted.

The SI units comprise a coherent form of the metric system. That is, all remaining units may be derived from the base units using formulas that do not involve any numerical factors. *Derived* units for selected quantities are listed in Table 1.3. Note that force is measured in newtons, where a 1-N force will accelerate a 1-kg mass at

TABLE 1.4 Multiplying prefixes

Prefix	Abbreviation	Multiplier
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
hecto	h	10^{2}
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}

[1]The degree symbol is retained for designation of the Celsius temperature (°C) to avoid confusion with use of C for the unit of electrical charge (coulomb).

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$1\ m/s^2$. Hence $1\ N = 1\ kg \cdot m/s^2$. The unit of pressure (N/m^2) is often referred to as the pascal. In the SI system there is one unit of energy (thermal, mechanical, or electrical), called the joule (J), and $1\ J = 1\ N \cdot m$. The unit for energy rate, or power, is then J/s, where one joule per second is equivalent to one watt ($1\ J/s = 1\ W$). Since it is frequently necessary to work with extremely large or small numbers, a set of standard prefixes has been introduced to simplify matters (Table 1.4). For example, 1 megawatt (MW) = 10^6 W, and 1 micrometer (μm) = 10^{-6} m.

1.7 *Summary*

Although much of the material of this chapter will be discussed in greater detail, you should now have a reasonable overview of heat transfer. You should be aware of the several modes of transfer and their physical origins. Moreover, given a physical situation, you should be able to perceive the relevant transport phenomena. The importance of developing this facility must not be underestimated. You will be devoting much time to acquiring the tools needed to calculate heat transfer phenomena. However, before you can begin to use these tools to solve practical problems, you must have the intuition to determine what is happening physically. In short, you must be able to look at a problem and identify the pertinent transport phenomena. The example and problems at the end of this chapter should help you to begin developing this intuition.

You should also appreciate the significance of the rate equations and feel comfortable in using them to compute transport rates. These equations, summarized in Table 1.5, *should be committed to memory.* You must also recognize the importance of the conservation laws and the need to carefully identify control volumes. With the rate equations, the conservation laws may be used to solve numerous heat transfer problems.

Lastly, you should have begun to acquire an appreciation for the terminology and physical concepts that underpin the subject of heat transfer. Test your understanding

TABLE 1.5 Summary of heat transfer processes

Mode	Mechanism(s)	Rate Equation	Equation Number	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x\ (W/m^2) = -k\dfrac{dT}{dx}$	(1.1)	$k\ (W/m \cdot K)$
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(W/m^2) = h(T_s - T_\infty)$	(1.3a)	$h\ (W/m^2 \cdot K)$
Radiation	Energy transfer by electromagnetic waves	$q''(W/m^2) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q(W) = h_r A(T_s - T_{sur})$	(1.7) (1.8)	ε $h_r\ (W/m^2 \cdot K)$

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of the important terms and concepts introduced in this chapter by addressing the following questions.

- What are the *physical mechanisms* associated with heat transfer by *conduction*, *convection*, and *radiation*?
- What is the driving potential for heat transfer? What are analogs to this potential and to heat transfer itself for the transport of electric charge?
- What is the difference between a heat *flux* and a heat *rate*? What are their units?
- What is a *temperature gradient*? What are its units? What is the relationship of heat flow to a temperature gradient?
- What is the *thermal conductivity*? What are its units? What role does it play in heat transfer?
- What is *Fourier's law*? Can you write the equation from memory?
- If heat transfer by conduction through a medium occurs under *steady-state* conditions, will the temperature at a particular instant vary with location in the medium? Will the temperature at a particular location vary with time?
- What is the difference between *natural convection* and *forced convection*?
- What conditions are necessary for the development of a *hydrodynamic boundary layer*? A *thermal boundary layer*? What varies across a hydrodynamic boundary layer? A thermal boundary layer?
- If convection heat transfer for flow of a liquid or a vapor is not characterized by liquid/vapor phase change, what is the nature of the energy being transferred? What is it if there is such a phase change?
- What is *Newton's law of cooling*? Can you write the equation from memory?
- What role is played by the *convection heat transfer coefficient* in Newton's law of cooling? What are its units?
- What effect does convection heat transfer from or to a surface have on the solid bounded by the surface?
- What is predicted by the Stefan–Boltzmann law, and what unit of temperature must be used with the law? Can you write the equation from memory?
- What is the *emissivity*, and what role does it play in characterizing radiation transfer at a surface?
- What is *irradiation*, and what are its units?
- What two outcomes characterize the response of an *opaque* surface to incident radiation? Which outcome affects the thermal energy of the medium bounded by the surface and how? What property characterizes this outcome?
- What conditions are associated with use of the *radiation heat transfer coefficient*?
- Can you write the equation used to express net radiation exchange between a small isothermal surface and a large isothermal enclosure?
- Consider the surface of a solid that is at an elevated temperature and exposed to cooler surroundings. By what mode(s) is heat transferred from the surface if (1) it is in intimate (perfect) contact with another solid, (2) it is exposed to the flow of a liquid, (3) it is exposed to the flow of a gas, and (4) it is in an evacuated chamber?
- What is the inherent difference between application of conservation of energy over a *time interval* and at an *instant of time*?
- What is *thermal energy storage*? How does it differ from *thermal energy generation*? What role do the terms play in a surface energy balance?

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EXAMPLE 1.8

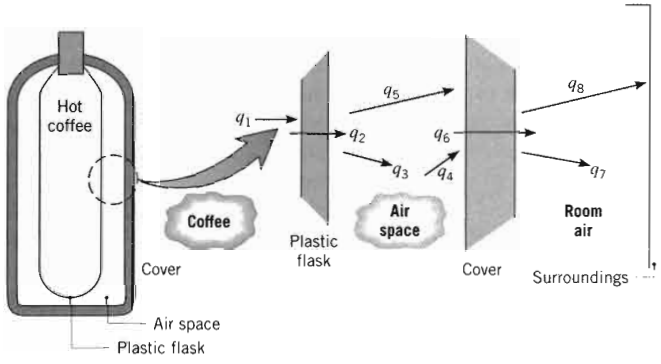
A closed container filled with hot coffee is in a room whose air and walls are at a fixed temperature. Identify all heat transfer processes that contribute to cooling of the coffee. Comment on features that would contribute to a superior container design.

SOLUTION

Known: Hot coffee is separated from its cooler surroundings by a plastic flask, an air space, and a plastic cover.

Find: Relevant heat transfer processes.

Schematic:



Pathways for energy transfer from the coffee are as follows:

- q_1: free convection from the coffee to the flask
- q_2: conduction through the flask
- q_3: free convection from the flask to the air
- q_4: free convection from the air to the cover
- q_5: net radiation exchange between the outer surface of the flask and the inner surface of the cover
- q_6: conduction through the cover
- q_7: free convection from the cover to the room air
- q_8: net radiation exchange between the outer surface of the cover and the surroundings

Comments: Design improvements are associated with (1) use of aluminized (low emissivity) surfaces for the flask and cover to reduce net radiation, and (2) evacuating the air space or using a filler material to retard free convection.

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Problems

Conduction

1.1 The thermal conductivity of a sheet of rigid, extruded insulation is reported to be $k = 0.029$ W/m · K. The measured temperature difference across a 20-mm-thick sheet of the material is $T_1 - T_2 = 10$°C.

(a) What is the heat flux through a 2 m × 2 m sheet of the insulation?

(b) What is the rate of heat transfer through the sheet of insulation?

1.2 A concrete wall, which has a surface area of 20 m^2 and is 0.30 m thick, separates conditioned room air from ambient air. The temperature of the inner surface of the wall is maintained at 25°C, and the thermal conductivity of the concrete is 1 W/m · K.

(a) Determine the heat loss through the wall for outer surface temperatures ranging from −15°C to 38°C, which correspond to winter and summer extremes, respectively. Display your results graphically.

(b) On your graph, also plot the heat loss as a function of the outer surface temperature for wall materials having thermal conductivities of 0.75 and 1.25 W/m · K. Explain the family of curves you have obtained.

1.3 The concrete slab of a basement is 11 m long, 8 m wide, and 0.20 m thick. During the winter, temperatures are nominally 17°C and 10°C at the top and bottom surfaces, respectively. If the concrete has a thermal conductivity of 1.4 W/m · K, what is the rate of heat loss through the slab? If the basement is heated by a gas furnace operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.01$/MJ, what is the daily cost of the heat loss?

1.4 The heat flux through a wood slab 50 mm thick, whose inner and outer surface temperatures are 40 and 20°C, respectively, has been determined to be 40 W/m^2. What is the thermal conductivity of the wood?

1.5 The inner and outer surface temperatures of a glass window 5 mm thick are 15 and 5°C. What is the heat loss through a window that is 1 m by 3 m on a side? The thermal conductivity of glass is 1.4 W/m · K.

1.6 A glass window of width $W = 1$ m and height $H = 2$ m is 5 mm thick and has a thermal conductivity of $k_g = 1.4$ W/m · K. If the inner and outer surface temperatures of the glass are 15°C and −20°C, respectively, on a cold winter day, what is the rate of heat loss through the glass? To reduce heat loss through windows, it is customary to use a double pane construction in which adjoining panes are separated by an air space. If the spacing is 10 mm and the glass surfaces in contact with the air have temperatures of 10°C and −15°C, what is the rate of heat loss from a 1 m × 2 m window? The thermal conductivity of air is $k_a = 0.024$ W/m · K.

1.7 A freezer compartment consists of a cubical cavity that is 2 m on a side. Assume the bottom to be perfectly insulated. What is the minimum thickness of styrofoam insulation ($k = 0.030$ W/m · K) that must be applied to the top and side walls to ensure a heat load of less than 500 W, when the inner and outer surfaces are −10 and 35°C?

1.8 An inexpensive food and beverage container is fabricated from 25-mm-thick polystyrene ($k = 0.023$ W/m · K) and has interior dimensions of 0.8 m × 0.6 m × 0.6 m. Under conditions for which an inner surface temperature of approximately 2°C is maintained by an ice-water mixture and an outer surface temperature of 20°C is maintained by the ambient, what is the heat flux through the container wall? Assuming negligible heat gain through the 0.8 m × 0.6 m base of the cooler, what is the total heat load for the prescribed conditions?

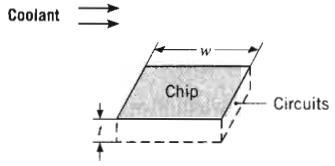
1.9 What is the thickness required of a masonry wall having thermal conductivity 0.75 W/m · K if the heat rate is to be 80% of the heat rate through a composite structural wall having a thermal conductivity of 0.25 W/m · K and a thickness of 100 mm? Both walls are subjected to the same surface temperature difference.

1.10 The 5-mm-thick bottom of a 200-mm-diameter pan may be made from aluminum ($k = 240$ W/m · K) or

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copper (k = 390 W/m · K). When used to boil water, the surface of the bottom exposed to the water is nominally at 110°C. If heat is transferred from the stove to the pan at a rate of 600 W, what is the temperature of the surface in contact with the stove for each of the two materials?

1.11 A square silicon chip (k = 150 W/m · K) is of width w = 5 mm on a side and of thickness t = 1 mm. The chip is mounted in a substrate such that its side and back surfaces are insulated, while the front surface is exposed to a coolant.

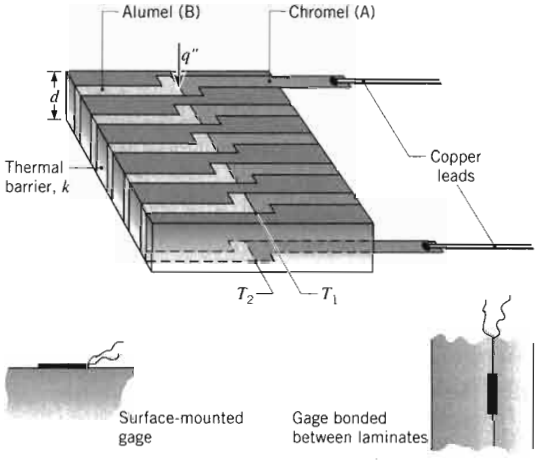


If 4 W are being dissipated in circuits mounted to the back surface of the chip, what is the steady-state temperature difference between back and front surfaces?

1.12 A gage for measuring heat flux to a surface or through a laminated material employs five thin-film, chromel/alumel (type K) thermocouples deposited on the upper and lower surfaces of a wafer with a thermal conductivity of 1.4 W/m · K and a thickness of 0.25 mm.

(a) Determine the heat flux q'' through the gage when the voltage output at the copper leads is 350 μV. The Seebeck coefficient of the type-K thermocouple materials is approximately 40 μV/°C.

(b) What precaution should you take in using a gage of this nature to measure heat flow through the laminated structure shown?



Convection

1.13 You've experienced convection cooling if you've ever extended your hand out the window of a moving vehicle or into a flowing water stream. With the surface of your hand at a temperature of 30°C, determine the convection heat flux for (a) a vehicle speed of 35 km/h in air at −5°C with a convection coefficient of 40 W/m² · K and (b) a velocity of 0.2 m/s in a water stream at 10°C with a convection coefficient of 900 W/m² · K. Which condition would *feel* colder? Contrast these results with a heat loss of approximately 30 W/m² under normal room conditions.

1.14 Air at 40°C flows over a long, 25-mm-diameter cylinder with an embedded electrical heater. In a series of tests, measurements were made of the power per unit length, P', required to maintain the cylinder surface temperature at 300°C for different freestream velocities V of the air. The results are as follows:

Air velocity, V (m/s)	1	2	4	8	12
Power, P' (W/m)	450	658	983	1507	1963

(a) Determine the convection coefficient for each velocity, and display your results graphically.

(b) Assuming the dependence of the convection coefficient on the velocity to be of the form $h = CV^n$, determine the parameters C and n from the results of part (a).

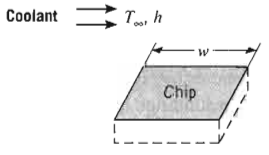
1.15 An electric resistance heater is embedded in a long cylinder of diameter 30 mm. When water with a temperature of 25°C and velocity of 1 m/s flows crosswise over the cylinder, the power per unit length required to maintain the surface at a uniform temperature of 90°C is 28 kW/m. When air, also at 25°C, but with a velocity of 10 m/s is flowing, the power per unit length required to maintain the same surface temperature is 400 W/m. Calculate and compare the convection coefficients for the flows of water and air.

1.16 A cartridge electrical heater is shaped as a cylinder of length L = 200 mm and outer diameter D = 20 mm. Under normal operating conditions the heater dissipates 2 kW while submerged in a water flow that is at 20°C and provides a convection heat transfer coefficient of h = 5000 W/m² · K. Neglecting heat transfer from the ends of the heater, determine its surface temperature T_s. If the water flow is inadvertently terminated while the heater continues to operate, the heater surface is exposed to air that is also at 20°C but for which h = 50 W/m² · K. What is the corresponding surface temperature? What are the consequences of such an event?

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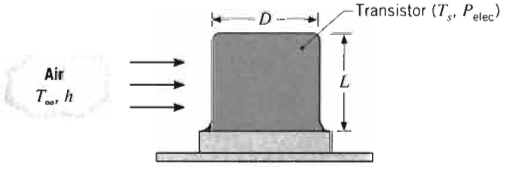
1.17 A common procedure for measuring the velocity of an air stream involves insertion of an electrically heated wire (called a *hot-wire anemometer*) into the air flow, with the axis of the wire oriented perpendicular to the flow direction. The electrical energy dissipated in the wire is assumed to be transferred to the air by forced convection. Hence, for a prescribed electrical power, the temperature of the wire depends on the convection coefficient, which, in turn, depends on the velocity of the air. Consider a wire of length $L = 20$ mm and diameter $D = 0.5$ mm, for which a calibration of the form, $V = 6.25 \times 10^{-5} h^2$, has been determined. The velocity V and the convection coefficient h have units of m/s and W/m$^2 \cdot$ K, respectively. In an application involving air at a temperature of $T_\infty = 25$°C, the surface temperature of the anemometer is maintained at $T_s = 75$°C with a voltage drop of 5 V and an electric current of 0.1 A. What is the velocity of the air?

1.18 A square isothermal chip is of width $w = 5$ mm on a side and is mounted in a substrate such that its side and back surfaces are well insulated, while the front surface is exposed to the flow of a coolant at $T_\infty = 15$°C. From reliability considerations, the chip temperature must not exceed $T = 85$°C.



If the coolant is air and the corresponding convection coefficient is $h = 200$ W/m$^2 \cdot$ K, what is the maximum allowable chip power? If the coolant is a dielectric liquid for which $h = 3000$ W/m$^2 \cdot$ K, what is the maximum allowable power?

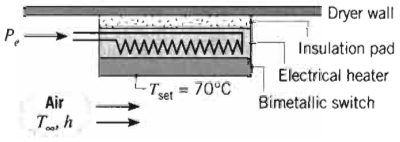
1.19 The case of a power transistor, which is of length $L = 10$ mm and diameter $D = 12$ mm, is cooled by an air stream of temperature $T_\infty = 25$°C.



Under conditions for which the air maintains an average convection coefficient of $h = 100$ W/m$^2 \cdot$ K on the surface of the case, what is the maximum allowable power dissipation if the surface temperature is not to exceed 85°C?

1.20 The use of impinging air jets is proposed as a means of effectively cooling high-power logic chips in a computer. However, before the technique can be implemented, the convection coefficient associated with jet impingement on a chip surface must be known. Design an experiment that could be used to determine convection coefficients associated with air jet impingement on a chip measuring approximately 10 mm by 10 mm on a side.

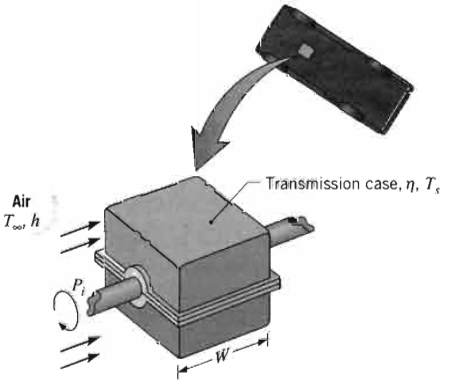
1.21 The temperature controller for a clothes dryer consists of a bimetallic switch mounted on an electrical heater attached to a wall-mounted insulation pad.



The switch is set to open at 70°C, the maximum dryer air temperature. In order to operate the dryer at a lower air temperature, sufficient power is supplied to the heater such that the switch reaches 70°C (T_{set}) when the air temperature T is less than T_{set}. If the convection heat transfer coefficient between the air and the exposed switch surface of 30 mm^2 is 25 W/m$^2 \cdot$ K, how much heater power P_e is required when the desired dryer air temperature is $T_\infty = 50$°C?

1.22 The free convection heat transfer coefficient on a thin hot vertical plate suspended in still air can be determined from observations of the change in plate temperature with time as it cools. Assuming the plate is isothermal and radiation exchange with its surroundings is negligible, evaluate the convection coefficient at the instant of time when the plate temperature is 225°C and the change in plate temperature with time (dT/dt) is -0.022 K/s. The ambient air temperature is 25°C and the plate measures 0.3 × 0.3 m with a mass of 3.75 kg and a specific heat of 2770 J/kg $\cdot$ K.

1.23 A transmission case measures $W = 0.30$ m on a side and receives a power input of $P_i = 150$ hp from the engine.



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If the transmission efficiency is $\eta = 0.93$ and air flow over the case corresponds to $T_\infty = 30°C$ and $h = 200\ W/m^2 \cdot K$, what is the surface temperature of the transmission?

Radiation

1.24 Under conditions for which the same room temperature is maintained by a heating or cooling system, it is not uncommon for a person to feel chilled in the winter but comfortable in the summer. Provide a plausible explanation for this situation (with supporting calculations) by considering a room whose air temperature is maintained at 20°C throughout the year, while the walls of the room are nominally at 27°C and 14°C in the summer and winter, respectively. The exposed surface of a person in the room may be assumed to be at a temperature of 32°C throughout the year and to have an emissivity of 0.90. The coefficient associated with heat transfer by natural convection between the person and the room air is approximately $2\ W/m^2 \cdot K$.

1.25 A spherical interplanetary probe of 0.5-m diameter contains electronics that dissipate 150 W. If the probe surface has an emissivity of 0.8 and the probe does not receive radiation from other surfaces, as, for example, from the sun, what is its surface temperature?

1.26 An instrumentation package has a spherical outer surface of diameter $D = 100$ mm and emissivity $\varepsilon = 0.25$. The package is placed in a large space simulation chamber whose walls are maintained at 77 K. If operation of the electronic components is restricted to the temperature range $40 \le T \le 85°C$, what is the range of acceptable power dissipation for the package? Display your results graphically, showing also the effect of variations in the emissivity by considering values of 0.20 and 0.30.

1.27 Consider the conditions of Problem 1.22. However, now the plate is in a vacuum with a surrounding temperature of 25°C. What is the emissivity of the plate? What is the rate at which radiation is emitted by the surface?

1.28 An overhead 25-m-long, uninsulated industrial steam pipe of 100 mm diameter is routed through a building whose walls and air are at 25°C. Pressurized steam maintains a pipe surface temperature of 150°C, and the coefficient associated with natural convection is $h = 10\ W/m^2 \cdot K$. The surface emissivity is $\varepsilon = 0.8$.

(a) What is the rate of heat loss from the steam line?

(b) If the steam is generated in a gas-fired boiler operating at an efficiency of $\eta_f = 0.90$ and natural gas is priced at $C_g = \$0.01$ per MJ, what is the annual cost of heat loss from the line?

1.29 If $T_s \approx T_{sur}$ in Equation 1.9, the radiation heat transfer coefficient may be approximated as

$$h_{r,a} = 4\varepsilon\sigma\bar{T}^3$$

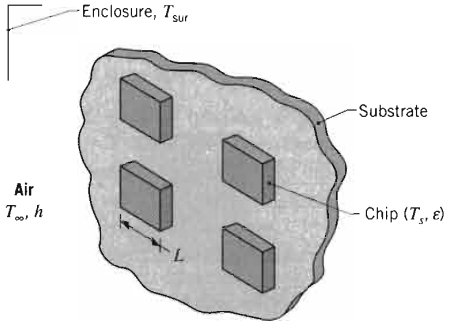
where $\bar{T} \equiv (T_s + T_{sur})/2$. We wish to assess the validity of this approximation by comparing values of h_r and $h_{r,a}$ for the following conditions. In each case represent your results graphically and comment on the validity of the approximation.

(a) Consider a surface of either polished aluminum ($\varepsilon = 0.05$) or black paint ($\varepsilon = 0.9$), whose temperature may exceed that of the surroundings ($T_{sur} = 25°C$) by 10 to 100°C. Also compare your results with values of the coefficient associated with free convection in air ($T_\infty = T_{sur}$), where $h\ (W/m^2 \cdot K) = 0.98\,\Delta T^{1/3}$.

(b) Consider initial conditions associated with placing a workpiece at $T_s = 25°C$ in a large furnace whose wall temperature may be varied over the range $100 \le T_{sur} \le 1000°C$. According to the surface finish or coating, its emissivity may assume values of 0.05, 0.2, and 0.9. For each emissivity, plot the relative error, $(h_r - h_{r,a})/h_r$, as a function of the furnace temperature.

1.30 Consider the conditions of Problem 1.18. With heat transfer by convection to air, the maximum allowable chip power is found to be 0.35 W. If consideration is also given to net heat transfer by radiation from the chip surface to large surroundings at 15°C, what is the percentage increase in the maximum allowable chip power afforded by this consideration? The chip surface has an emissivity of 0.9.

1.31 Chips of width $L = 15$ mm on a side are mounted to a substrate that is installed in an enclosure whose walls and air are maintained at a temperature of $T_{sur} = T_\infty = 25°C$. The chips have an emissivity of $\varepsilon = 0.60$ and a maximum allowable temperature of $T_s = 85°C$.

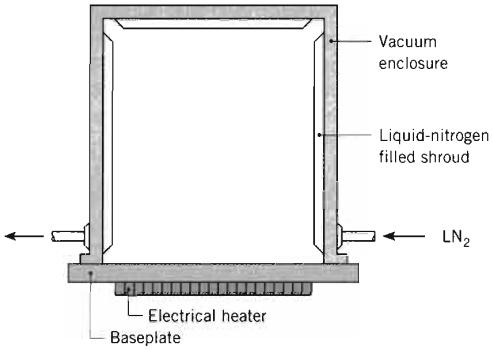


(a) If heat is rejected from the chips by radiation and natural convection, what is the maximum operating

power of each chip? The convection coefficient depends on the chip-to-air temperature difference and may be approximated as $h = C(T_s - T_\infty)^{1/4}$, where $C = 4.2\ \text{W/m}^2 \cdot \text{K}^{5/4}$.

(b) If a fan is used to maintain air flow through the enclosure and heat transfer is by forced convection, with $h = 250\ \text{W/m}^2 \cdot \text{K}$, what is the maximum operating power?

1.32 A vacuum system, as used in sputtering electrically conducting thin films on microcircuits, is comprised of a baseplate maintained by an electrical heater at 300 K and a shroud within the enclosure maintained at 77 K by a liquid-nitrogen coolant loop. The circular baseplate, insulated on the lower side, is 0.3 m in diameter and has an emissivity of 0.25.



(a) How much electrical power must be provided to the baseplate heater?

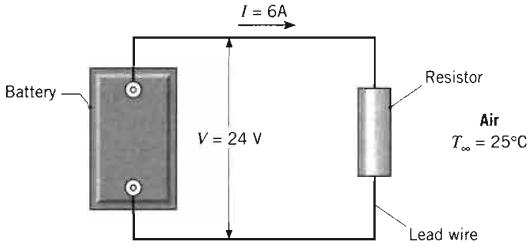
(b) At what rate must liquid nitrogen be supplied to the shroud if its heat of vaporization is 125 kJ/kg?

(c) To reduce the liquid-nitrogen consumption, it is proposed to bond a thin sheet of aluminum foil ($\varepsilon = 0.09$) to the baseplate. Will this have the desired effect?

1.33 Consider the transmission case of Problem 1.23, but now allow for radiation exchange with the ground/chassis, which may be approximated as large surroundings at $T_{sur} = 30°\text{C}$. If the emissivity of the case is $\varepsilon = 0.80$, what is the surface temperature?

Energy Balance and Multimode Effects

1.34 An electrical resistor is connected to a battery, as shown schematically. After a brief transient, the resistor assumes a nearly uniform, steady-state temperature of 95°C, while the battery and lead wires remain at the ambient temperature of 25°C. Neglect the electrical resistance of the lead wires.



(a) Consider the resistor as a system about which a control surface is placed and Equation 1.11c is applied. Determine the corresponding values of $\dot{E}_{in}$(W), $\dot{E}_g$(W), $\dot{E}_{out}$(W), and $\dot{E}_{st}$(W). If a control surface is placed about the entire system, what are the values of $\dot{E}_{in}$, $\dot{E}_g$, $\dot{E}_{out}$, and $\dot{E}_{st}$?

(b) If electrical energy is dissipated uniformly within the resistor, which is a cylinder of diameter $D = 60$ mm and length $L = 250$ mm, what is the volumetric heat generation rate, $\dot{q}$ (W/m^3)?

(c) Neglecting radiation from the resistor, what is the convection coefficient?

1.35 An aluminum plate 4 mm thick is mounted in a horizontal position, and its bottom surface is well insulated. A special, thin coating is applied to the top surface such that it absorbs 80% of any incident solar radiation, while having an emissivity of 0.25. The density ρ and specific heat c of aluminum are known to be 2700 kg/m^3 and 900 J/kg · K, respectively.

(a) Consider conditions for which the plate is at a temperature of 25°C and its top surface is suddenly exposed to ambient air at $T_\infty = 20°\text{C}$ and to solar radiation that provides an incident flux of 900 W/m^2. The convection heat transfer coefficient between the surface and the air is $h = 20\ \text{W/m}^2 \cdot \text{K}$. What is the initial rate of change of the plate temperature?

(b) What will be the equilibrium temperature of the plate when steady-state conditions are reached?

(c) The surface radiative properties depend on the specific nature of the applied coating. Compute and plot the steady-state temperature as a function of the emissivity for $0.05 \le \varepsilon \le 1$, with all other conditions remaining as prescribed. Repeat your calculations for values of $\alpha_S = 0.5$ and 1.0, and plot the results with those obtained for $\alpha_S = 0.8$. If the intent is to maximize the plate temperature, what is the most desirable combination of the plate emissivity and its absorptivity to solar radiation?

1.36 A blood warmer is to be used during the transfusion of blood to a patient. This device is to heat blood taken from the blood bank at 10°C to 37°C at a flow rate of 200 ml/min. The blood passes through tubing of length

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2 m, with a rectangular cross section 6.4 mm × 1.6 mm. At what rate must heat be added to the blood to accomplish the required temperature increase? If the fluid originates from a large tank with nearly zero velocity and flows vertically downward for its 2-m length, estimate the magnitudes of kinetic and potential energy changes. Assume the blood's properties are similar to those of water.

1.37 The energy consumption associated with a home water heater has two components: (i) the energy that must be supplied to bring the temperature of groundwater to the heater storage temperature, as it is introduced to replace hot water that has been used, and (ii) the energy needed to compensate for heat losses incurred while the water is stored at the prescribed temperature. In this problem, we will evaluate the first of these components for a family of four, whose daily hot water consumption is approximately 100 gallons. If groundwater is available at 15°C, what is the annual energy consumption associated with heating the water to a storage temperature of 55°C? For a unit electrical power cost of \$0.08/kW · h, what is the annual cost associated with supplying hot water by means of (a) electric resistance heating or (b) a heat pump having a COP of 3.

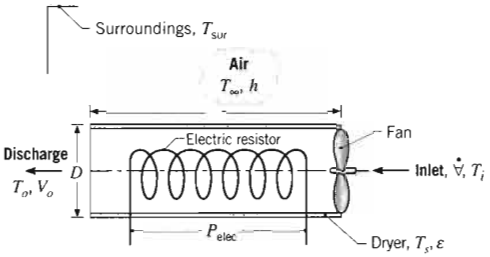
1.38 Three electric resistance heaters of length $L = 250$ mm and diameter $D = 25$ mm are submerged in a 10 gallon tank of water, which is initially at 295 K. The water may be assumed to have a density and specific heat of $\rho = 990$ kg/m^3 and $c = 4180$ J/kg · K.

(a) If the heaters are activated, each dissipating $q_1 = 500$ W, estimate the time required to bring the water to a temperature of 335 K.

(b) If the natural convection coefficient is given by an expression of the form $h = 370(T_s - T)^{1/3}$, where T_s and T are temperatures of the heater surface and water, respectively, what is the temperature of each heater shortly after activation and just before deactivation? Units of h and $(T_s - T)$ are W/m^2 · K and K, respectively.

(c) If the heaters are inadvertently activated when the tank is empty, the natural convection coefficient associated with heat transfer to the ambient air at $T_\infty = 300$ K may be approximated as $h = 0.70(T_s - T_\infty)^{1/3}$. If the temperature of the tank walls is also 300 K and the emissivity of the heater surface is $\varepsilon = 0.85$, what is the surface temperature of each heater under steady-state conditions?

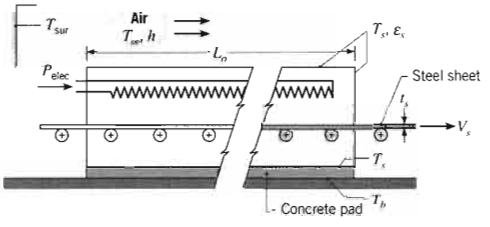
1.39 A hair dryer may be idealized as a circular duct through which a small fan draws ambient air and within which the air is heated as it flows over a coiled electric resistance wire.



(a) If a dryer is designed to operate with an electric power consumption of $P_{elec} = 500$ W and to heat air from an ambient temperature of $T_i = 20$°C to a discharge temperature of $T_o = 45$°C, at what volumetric flow rate $\dot{\forall}$ should the fan operate? Heat loss from the casing to the ambient air and the surroundings may be neglected. If the duct has a diameter of $D = 70$ mm, what is the discharge velocity V_o of the air? The density and specific heat of the air may be approximated as $\rho = 1.10$ kg/m^3 and $c_p = 1007$ J/kg · K, respectively.

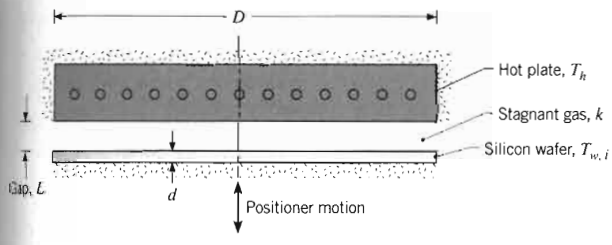
(b) Consider a dryer duct length of $L = 150$ mm and a surface emissivity of $\varepsilon = 0.8$. If the coefficient associated with heat transfer by natural convection from the casing to the ambient air is $h = 4$ W/m^2 · K and the temperature of the air and the surroundings is $T_\infty = T_{sur} = 20$°C, confirm that the heat loss from the casing is, in fact, negligible. The casing may be assumed to have an average surface temperature of $T_s = 40$°C.

1.40 In one stage of an annealing process, 304 stainless steel sheet is taken from 300 K to 1250 K as it passes through an electrically heated oven at a speed of $V_s = 10$ mm/s. The sheet thickness and width are $t_s = 8$ mm and $W_s = 2$ m, respectively, while the height, width, and length of the oven are $H_o = 2$ m, $W_o = 2.4$ m, and $L_o = 25$ m, respectively. The top and four sides of the oven are exposed to ambient air and large surroundings, each at 300 K, and the corresponding surface temperature, convection coefficient, and emissivity are $T_s = 350$ K, $h = 10$ W/m^2 · K, and $\varepsilon_s = 0.8$. The bottom surface of the oven is also at 350 K and rests on a 0.5-m thick concrete pad whose base is at 300 K. Estimate the required electric power input, P_{elec}, to the oven.



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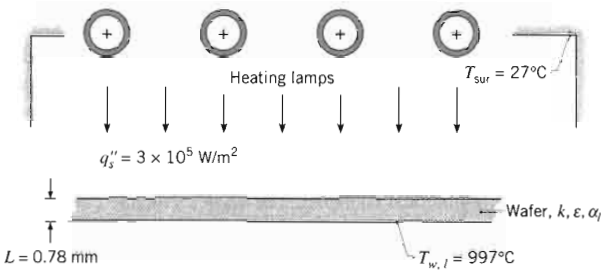
1.41 Annealing, an important step in semiconductor materials processing, can be accomplished by rapidly heating the silicon wafer to a high temperature for a short period of time. The schematic shows a method involving use of a hot plate operating at an elevated temperature T_h. The wafer, initially at a temperature of $T_{w,i}$, is suddenly positioned at a gap separation distance L from the hot plate. The purpose of the analysis is to compare the heat fluxes by conduction through the gas within the gap and by radiation exchange between the hot plate and the cool wafer. The initial time rate of change in the temperature of the wafer, $(dT_w/dt)_i$, is also of interest. Approximating the surfaces of the hot plate and the wafer as blackbodies and assuming their diameter D to be much larger than the spacing L, the radiative heat flux may be expressed as $q''_{rad} = \sigma(T_h^4 - T_w^4)$. The silicon wafer has a thickness of $d = 0.78$ mm, a density of 2700 kg/m^3, and a specific heat of 875 J/kg · K. The thermal conductivity of the gas in the gap is 0.0436 W/m · K.



(a) For $T_h = 600$°C and $T_{w,i} = 20$°C, calculate the radiative heat flux and the heat flux by conduction across a gap distance of $L = 0.2$ mm. Also determine the value of $(dT_w/dt)_i$, resulting from each of the heating modes.

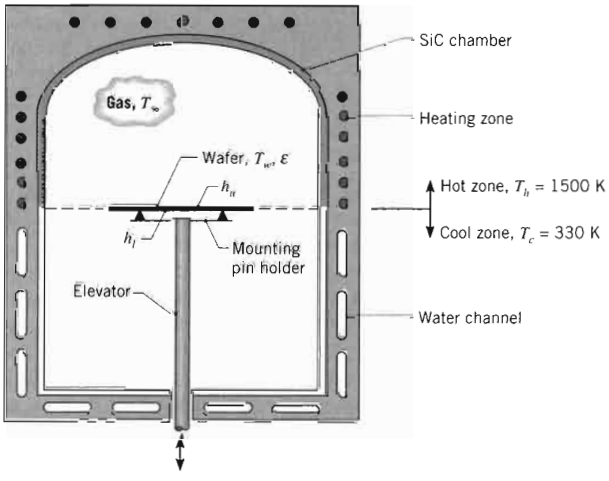
(b) For gap distances of 0.2, 0.5, and 1.0 mm, determine the heat fluxes and temperature-time change as a function of the hot plate temperature for $300 \le T_h \le 1300$°C. Display your results graphically. Comment on the relative importance of the two heat transfer modes and the effect of the gap distance on the heating process. Under what conditions could a wafer be heated to 900°C in less than 10 seconds?

1.42 In the thermal processing of semiconductor materials, annealing is accomplished by heating a silicon wafer according to a temperature-time recipe and then maintaining a fixed elevated temperature for a prescribed period of time. For the process tool arrangement shown as follows, the wafer is in an evacuated chamber whose walls are maintained at 27°C and within which heating lamps maintain a radiant flux q''_s at its upper surface. The wafer is 0.78 mm thick, has a thermal conductivity of 30 W/m · K, and an emissivity that equals its absorptivity to the radiant flux ($\varepsilon = \alpha_l = 0.65$). For $q''_s = 3.0 \times 10^5$ W/m^2, the temperature on its lower surface is measured by a radiation thermometer and found to have a value of $T_{w,l} = 997$°C.



To avoid warping the wafer and inducing slip planes in the crystal structure, the temperature difference across the thickness of the wafer must be less than 2°C. Is this condition being met?

1.43 A furnace for processing semiconductor materials is formed by a silicon carbide chamber that is zone heated on the top section and cooled on the lower section. With the elevator in the lowest position, a robot arm inserts the silicon wafer on the mounting pins. In a production operation, the wafer is rapidly moved toward the hot zone to achieve the temperature-time history required for the process recipe. In this position the top and bottom surfaces of the wafer exchange radiation with the hot and cool zones, respectively, of the chamber. The zone temperatures are $T_h = 1500$ K and $T_c = 330$ K, and the emissivity and thickness of the wafer are $\varepsilon = 0.65$ and $d = 0.78$ mm, respectively. With the ambient gas at $T_\infty = 700$ K, convection coefficients at the upper and lower surfaces of the wafer are 8 and 4 W/m^2 · K, respectively. The silicon wafer has a density of 2700 kg/m^3 and a specific heat of 875 J/kg · K.

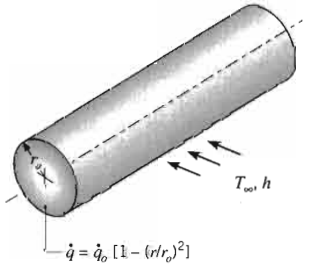


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(a) For an initial condition corresponding to a wafer temperature of $T_{w,i} = 300$ K and the position of the wafer shown schematically, determine the corresponding time rate of change of the wafer temperature, $(dT_w/dt)_i$.

(b) Determine the steady-state temperature reached by the wafer if it remains in this position. How significant is convection heat transfer for this situation? Sketch how you would expect the wafer temperature to vary as a function of vertical distance.

1.44 Radioactive wastes are packed in a long, thin-walled cylindrical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_o[1 - (r/r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Obtain an expression for the total rate at which energy is generated in a unit length of the container. Use this result to obtain an expression for the temperature T_s of the container wall.

1.45 Consider the conducting rod of Example 1.3 under steady-state conditions. As suggested in Comment 3, the temperature of the rod may be controlled by varying the speed of air flow over the rod, which, in turn, alters the convection heat transfer coefficient. To consider the effect of the convection coefficient, generate plots of T versus I for values of $h = 50$, 100, and 250 W/m$^2 \cdot$ K. Would variations in the surface emissivity have a significant effect on the rod temperature?

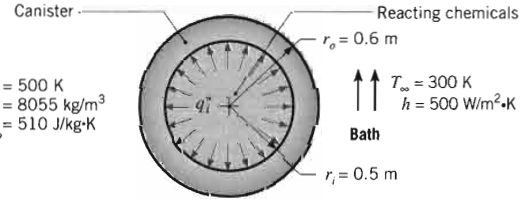
1.46 A long bus bar (cylindrical rod used for making electrical connections) of diameter D is installed in a large conduit having a surface temperature of 30°C and in which the ambient air temperature is $T_\infty = 30$°C. The electrical resistivity, $\rho_e(\mu\Omega \cdot \text{m})$, of the bar material is a function of temperature, $\rho_e = \rho_{e,o}\,[1 + \alpha(T - T_o)]$, where $\rho_{e,o} = 0.0171\ \mu\Omega \cdot$ m, $T_o = 25$°C, and $\alpha = 0.00396$ K^{-1}. The bar experiences free convection in the ambient air, and the convection coefficient depends on the bar diameter, as well as on the difference between the surface and ambient temperatures. The governing relation is of the form, $h = CD^{-0.25}(T - T_\infty)^{0.25}$, where $C = 1.21$ W $\cdot$ m$^{-1.75}$ $\cdot$ K$^{-1.25}$. The emissivity of the bar surface is $\varepsilon = 0.85$.

(a) Recognizing that the electrical resistance per unit length of the bar is $R'_e = \rho_e/A_c$, where A_c is its cross-sectional area, calculate the current-carrying capacity of a 20-mm-diameter bus bar if its temperature is not to exceed 65°C. Compare the relative importance of heat transfer by free convection and radiation exchange.

(b) To assess the trade-off between current-carrying capacity, operating temperature, and bar diameter, for diameters of 10, 20, and 40 mm, plot the bar temperature T as a function of current for the range $100 \le I \le 5000$ A. Also plot the ratio of the heat transfer by convection to the total heat transfer.

1.47 A small sphere of reference-grade iron with a specific heat of 447 J/kg $\cdot$ K and a mass of 0.515 kg is suddenly immersed in a water-ice mixture. Fine thermocouple wires suspend the sphere, and the temperature is observed to change from 15 to 14°C in 6.35 s. The experiment is repeated with a metallic sphere of the same diameter, but of unknown composition with a mass of 1.263 kg. If the same observed temperature change occurs in 4.59 s, what is the specific heat of the unknown material?

1.48 A spherical, stainless steel (AISI 302) canister is used to store reacting chemicals that provide for a uniform heat flux q''_i to its inner surface. The canister is suddenly submerged in a liquid bath of temperature $T_\infty < T_i$, where T_i is the initial temperature of the canister wall.



(a) Assuming negligible temperature gradients in the canister wall and a constant heat flux q''_i, develop an equation that governs the variation of the wall temperature with time during the transient process. What is the initial rate of change of the wall temperature if $q''_i = 10^5$ W/m^2?

(b) What is the steady-state temperature of the wall?

(c) The convection coefficient depends on the velocity associated with fluid flow over the canister and whether or not the wall temperature is large enough to induce boiling in the liquid. Compute and plot the steady-state temperature as a function of h for

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the range $100 \leq h \leq 10{,}000$ W/m$^2 \cdot$K. Is there a value of h below which operation would be unacceptable?

1.49 Liquid oxygen, which has a boiling point of 90 K and a latent heat of vaporization of 214 kJ/kg, is stored in a spherical container whose outer surface is of 500-mm diameter and at a temperature of -10°C. The container is housed in a laboratory whose air and walls are at 25°C.

(a) If the surface emissivity is 0.20 and the heat transfer coefficient associated with free convection at the outer surface of the container is 10 W/m$^2 \cdot$K, what is the rate, in kg/s, at which oxygen vapor must be vented from the system?

(b) Moisture in the ambient air will result in frost formation on the container, causing the surface emissivity to increase. Assuming the surface temperature and convection coefficient to remain at -10°C and 10 W/m$^2 \cdot$K, respectively, compute the oxygen evaporation rate (kg/s) as a function of surface emissivity over the range $0.2 \leq \varepsilon \leq 0.94$.

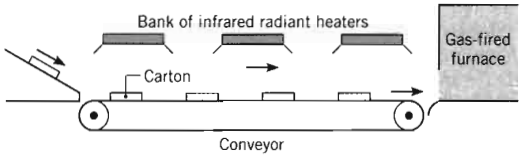
1.50 A freezer compartment is covered with a 2-mm-thick layer of frost at the time it malfunctions. If the compartment is in ambient air at 20°C and a coefficient of $h = 2$ W/m$^2 \cdot$K characterizes heat transfer by natural convection from the exposed surface of the layer, estimate the time required to completely melt the frost. The frost may be assumed to have a mass density of 700 kg/m^3 and a latent heat of fusion of 334 kJ/kg.

1.51 A vertical slab of Woods metal is joined to a substrate on one surface and is melted as it is uniformly irradiated by a laser source on the opposite surface. The metal is initially at its fusion temperature of $T_f = 72$°C, and the melt runs off by gravity as soon as it is formed. The absorptivity of the metal to the laser radiation is $\alpha_l = 0.4$, and its latent heat of fusion is $h_{sf} = 33$ kJ/kg.

(a) Neglecting heat transfer from the irradiated surface by convection or radiation exchange with the surroundings, determine the instantaneous rate of melting in kg/s$\cdot$m^2 if the laser irradiation is 5 kW/m^2. How much material is removed if irradiation is maintained for a period of 2 s?

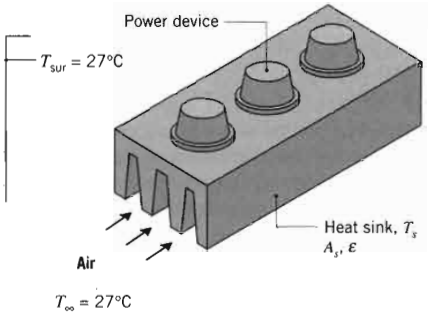
(b) Allowing for convection to ambient air, with $T_\infty = 20$°C and $h = 15$ W/m$^2 \cdot$K, and radiation exchange with large surroundings ($\varepsilon = 0.4$, $T_{sur} = 20$°C), determine the instantaneous rate of melting during irradiation.

1.52 Following the hot vacuum forming of a paper-pulp mixture, the product, an egg carton, is transported on a conveyor for 18 s toward the entrance of a gas-fired oven where it is dried to a desired final water content. It is observed that very little water evaporates during the travel time. So, to increase the productivity of the line, it is proposed that a bank of infrared radiation heaters, which provide a uniform radiant flux of 5000 W/m^2, be installed over the conveyor. The carton has an exposed area of 0.0625 m^2 and a mass of 0.220 kg, 75% of which is water after the forming process.



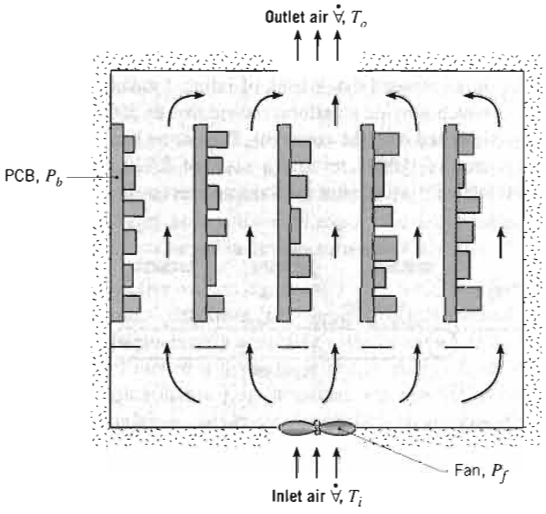
The chief engineer of your plant will approve the purchase of the heaters if they can reduce the water content by 10% of the total mass. Would you recommend the purchase? Assume the heat of vaporization of water is $h_{fg} = 2400$ kJ/kg.

1.53 Electronic power devices are mounted to a heat sink having an exposed surface area of 0.045 m^2 and an emissivity of 0.80. When the devices dissipate a total power of 20 W and the air and surroundings are at 27°C, the average sink temperature is 42°C. What average temperature will the heat sink reach when the devices dissipate 30 W for the same environmental condition?



1.54 A computer consists of an array of five printed circuit boards (PCBs), each dissipating $P_b = 20$ W of power. Cooling of the electronic components on a board is provided by the forced flow of air, equally distributed in passages formed by adjoining boards, and the convection coefficient associated with heat transfer from the components to the air is approximately $h = 200$ W/m$^2 \cdot$K. Air enters the computer console at a temperature of $T_i = 20$°C, and flow is driven by a fan whose power consumption is $P_f = 25$ W.

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(a) If the temperature rise of the air flow, $(T_o - T_i)$, is not to exceed 15°C, what is the minimum allowable volumetric flow rate $\dot{\forall}$ of the air? The density and specific heat of the air may be approximated as $\rho = 1.161$ kg/m^3 and $c_p = 1007$ J/kg · K, respectively.

(b) The component that is most susceptible to thermal failure dissipates 1 W/cm^2 of surface area. To minimize the potential for thermal failure, where should the component be installed on a PCB? What is its surface temperature at this location?

1.55 The roof of a car in a parking lot absorbs a solar radiant flux of 800 W/m^2, while the underside is perfectly insulated. The convection coefficient between the roof and the ambient air is 12 W/m^2 · K.

(a) Neglecting radiation exchange with the surroundings, calculate the temperature of the roof under steady-state conditions if the ambient air temperature is 20°C.

(b) For the same ambient air temperature, calculate the temperature of the roof if its surface emissivity is 0.8.

(c) The convection coefficient depends on airflow conditions over the roof, increasing with increasing air speed. Compute and plot the roof temperature as a function of h for $2 \leq h \leq 200$ W/m^2 · K.

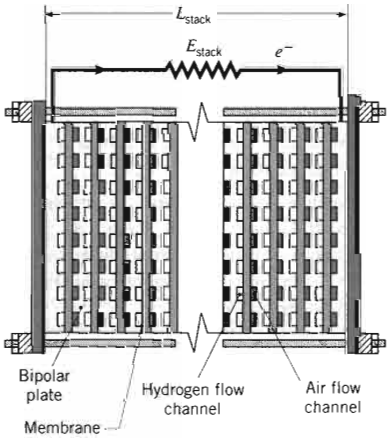
1.56 Consider the conditions of Problem 1.22, but the surroundings temperature is 25°C and radiation exchange with the surroundings is not negligible. If the convection coefficient is 6.4 W/m^2 · K and the emissivity of the plate is $\varepsilon = 0.42$, determine the time rate of change of the plate temperature, dT/dt, when the plate temperature is 225°C. Evaluate the heat loss by convection and the heat loss by radiation.

1.57 Most of the energy we consume as food is converted to thermal energy in the process of performing all our bodily functions and is ultimately lost as heat from our bodies. Consider a person who consumes 2100 kcal per day (note that what are commonly referred to as food calories are actually kilocalories), of which 2000 kcal is converted to thermal energy. (The remaining 100 kcal is used to do work on the environment.) The person has a surface area of 1.8 m^2 and is dressed in a bathing suit.

(a) The person is in a room at 20°C, with a convection heat transfer coefficient of 3 W/m^2 · K. At this air temperature, the person is not perspiring much. Estimate the person's average skin temperature.

(b) If the temperature of the environment were 33°C, what rate of perspiration would be needed to maintain a comfortable skin temperature of 33°C?

1.58 Single fuel cells such as the one of Example 1.4 can be scaled up by arranging them into a *fuel cell stack*. A stack consists of multiple electrolytic membranes that are sandwiched between electrically conducting *bipolar plates*. Air and hydrogen are fed to each membrane through *flow channels* within each bipolar plate, as shown in the sketch. With this stack arrangement, the individual fuel cells are connected in series, electrically, producing a stack voltage of $E_{stack} = N \times E_c$, where E_c is the voltage produced across each membrane and N is the number of membranes in the stack. The electrical current is the same for each membrane. The cell voltage, E_c, as well as the cell efficiency, increases with temperature (the air and hydrogen fed to the stack are humidified to allow operation at temperatures greater than in Example 1.4), but the membranes will fail at temperatures exceeding $T \approx 85$°C. Consider $L \times w$ membranes, where $L = w = 100$ mm, of thickness $t_m = 0.43$ mm that each produce $E_c = 0.6$ volts at $I = 60$ A, and $\dot{E}_{c,g} = 45$ W of thermal energy when operating at $T = 80$°C. The external surfaces of the stack are exposed to air at $T_\infty = 25$°C and surroundings at $T_{sur} = 30$°C, with $\varepsilon = 0.88$ and $h = 150$ W/m^2 · K.



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(a) Find the electrical power produced by a stack that is $L_{stack} = 200$ mm long, for bipolar plate thickness in the range 1 mm $< t_{bp} <$ 10 mm. Determine the total thermal energy generated by the stack.

(b) Calculate the surface temperature and explain whether the stack needs to be internally heated or cooled to operate at the optimal internal temperature of 80°C for various bipolar plate thicknesses.

(c) Identify how the internal stack operating temperature might be lowered or raised for a given bipolar plate thickness, and discuss design changes that would promote a more uniform temperature distribution within the stack. How would changes in the external air and surroundings temperature affect your answer? Which membrane in the stack is most likely to fail due to high operating temperature?

1.59 Consider Problem 1.1.

(a) If the exposed cold surface of the insulation is at $T_2 = 20$°C, what is the value of the convection heat transfer coefficient on the cold side of the insulation if the surroundings temperature is $T_{sur} = 320$ K, the ambient temperature is $T_\infty = 5$°C, and the emissivity is $\varepsilon = 0.95$? Express your results in units of W/m$^2 \cdot$ K and W/m$^2 \cdot$ °C.

(b) Using the convective heat transfer coefficient you calculated in part (a), determine the surface temperature, T_2, as the emissivity of the surface is varied over the range $0.05 \leq \varepsilon \leq 0.95$. The hot wall temperature of the insulation remains fixed at $T_1 =$ 30°C. Display your results graphically.

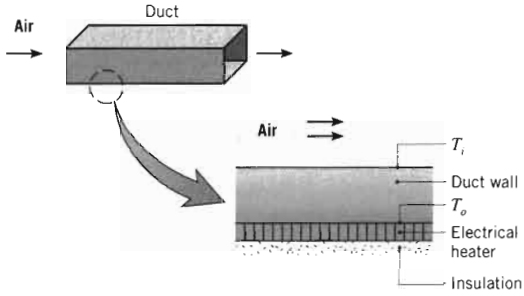
1.60 The wall of an oven used to cure plastic parts is of thickness $L = 0.05$ m and is exposed to large surroundings and air at its outer surface. The air and the surroundings are at 300 K.

(a) If the temperature of the outer surface is 400 K and its convection coefficient and emissivity are $h = 20$ W/m$^2 \cdot$ K and $\varepsilon = 0.8$, respectively, what is the temperature of the inner surface if the wall has a thermal conductivity of $k = 0.7$ W/m $\cdot$ K?

(b) Consider conditions for which the temperature of the inner surface is maintained at 600 K, while the air and large surroundings to which the outer surface is exposed are maintained at 300 K. Explore the effects of variations in k, h, and ε on (i) the temperature of the outer surface, (ii) the heat flux through the wall, and (iii) the heat fluxes associated with convection and radiation heat transfer from the outer surface. Specifically, compute and plot the foregoing dependent variables for parametric variations about baseline conditions of $k = 10$ W/m $\cdot$ K, $h = 20$ W/m$^2 \cdot$ K, and $\varepsilon = 0.5$. Suggested ranges of the independent variables are $0.1 \leq k \leq$ 400 W/m $\cdot$ K, $2 \leq h \leq 200$ W/m$^2 \cdot$ K, and $0.05 \leq \varepsilon \leq 1$. Discuss the physical implications of your results. Under what conditions will the temperature of the outer surface be less than 45°C, which is a reasonable upper limit to avoid burn injuries if contact is made?

1.61 An experiment to determine the convection coefficient associated with airflow over the surface of a thick stainless steel casting involves insertion of thermocouples in the casting at distances of 10 and 20 mm from the surface along a hypothetical line normal to the surface. The steel has a thermal conductivity of 15 W/m $\cdot$ K. If the thermocouples measure temperatures of 50 and 40°C in the steel when the air temperature is 100°C, what is the convection coefficient?

1.62 A thin electrical heating element provides a uniform heat flux q_o'' to the outer surface of a duct through which air flows. The duct wall has a thickness of 10 mm and a thermal conductivity of 20 W/m $\cdot$ K.



(a) At a particular location, the air temperature is 30°C and the convection heat transfer coefficient between the air and inner surface of the duct is 100 W/m$^2 \cdot$ K. What heat flux q_o'' is required to maintain the inner surface of the duct at $T_i = 85$°C?

(b) For the conditions of part (a), what is the temperature (T_o) of the duct surface next to the heater?

(c) With $T_i = 85$°C, compute and plot q_o'' and T_o as a function of the air-side convection coefficient h for the range $10 \leq h \leq 200$ W/m$^2 \cdot$ K. Briefly discuss your results.

1.63 A rectangular forced air heating duct is suspended from the ceiling of a basement whose air and walls are at a temperature of $T_\infty = T_{sur} = 5$°C. The duct is 15 m long, and its cross-section is 350 mm $\times$ 200 mm.

(a) For an uninsulated duct whose average surface temperature is 50°C, estimate the rate of heat loss from the duct. The surface emissivity and convection coefficient are approximately 0.5 and 4 W/m$^2 \cdot$ K, respectively.

(b) If heated air enters the duct at 58°C and a velocity of 4 m/s and the heat loss corresponds to the result of

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part (a), what is the outlet temperature? The density and specific heat of the air may be assumed to be $\rho = 1.10\ \text{kg/m}^3$ and $c_p = 1008\ \text{J/kg} \cdot \text{K}$, respectively.

1.64 Consider the steam pipe of Example 1.2. The facilities manager wants you to recommend methods for reducing the heat loss to the room, and two options are proposed. The first option would restrict air movement around the outer surface of the pipe and thereby reduce the convection coefficient by a factor of two. The second option would coat the outer surface of the pipe with a low emissivity ($\varepsilon = 0.4$) paint.

(a) Which of the foregoing options would you recommend?

(b) To prepare for a presentation of your recommendation to management, generate a graph of the heat loss q' as a function of the convection coefficient for $2 \leq h \leq 20\ \text{W/m}^2 \cdot \text{K}$ and emissivities of 0.2, 0.4, and 0.8. Comment on the relative efficacy of reducing heat losses associated with convection and radiation.

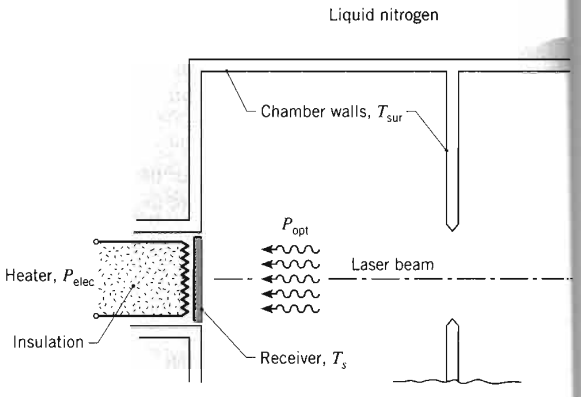
1.65 During its manufacture, plate glass at 600°C is cooled by passing air over its surface such that the convection heat transfer coefficient is $h = 5\ \text{W/m}^2 \cdot \text{K}$. To prevent cracking, it is known that the temperature gradient must not exceed 15°C/mm at any point in the glass during the cooling process. If the thermal conductivity of the glass is $1.4\ \text{W/m} \cdot \text{K}$ and its surface emissivity is 0.8, what is the lowest temperature of the air that can initially be used for the cooling? Assume that the temperature of the air equals that of the surroundings.

1.66 The curing process of Example 1.7 involves exposure of the plate to irradiation from an infrared lamp and attendant cooling by convection and radiation exchange with the surroundings. Alternatively, in lieu of the lamp, heating may be achieved by inserting the plate in an oven whose walls (the surroundings) are maintained at an elevated temperature.

(a) Consider conditions for which the oven walls are at 200°C, air flow over the plate is characterized by $T_\infty = 20°\text{C}$ and $h = 15\ \text{W/m}^2 \cdot \text{K}$, and the coating has an emissivity of $\varepsilon = 0.5$. What is the temperature of the plate?

(b) For ambient air temperatures of 20, 40, and 60°C, determine the plate temperature as a function of the oven wall temperature over the range from 150 to 250°C. Plot your results, and identify conditions for which acceptable curing temperatures between 100 and 110°C may be maintained.

1.67 The electrical-substitution radiometer shown schematically determines the optical (radiant) power of a beam by measuring the electrical power required to heat the receiver to the same temperature. With a beam, such as a laser of optical power P_{opt}, incident on the receiver, its temperature, T_s, increases above that of the chamber walls held at a uniform temperature, $T_{\text{sur}} = 77$ K. With the optical beam blocked, the heater on the backside of the receiver is energized and the electrical power, P_{elec}, required to reach the same value of T_s is measured. The purpose of your analysis is to determine the relationship between the electrical and optical power, considering heat transfer processes experienced by the receiver.



Consider a radiometer with a 15-mm-diameter receiver having a blackened surface with an emissivity of 0.95 and an absorptivity of 0.98 for the optical beam. When operating in the optical mode, conduction heat losses from the backside of the receiver are negligible. In the electrical mode, the loss amounts to 5% of the electrical power. What is the optical power of a beam when the indicated electrical power is 20.64 mW? What is the corresponding receiver temperature?

1.68 The diameter and surface emissivity of an electrically heated plate are $D = 300$ mm and $\varepsilon = 0.80$, respectively.

(a) Estimate the power needed to maintain a surface temperature of 200°C in a room for which the air and the walls are at 25°C. The coefficient characterizing heat transfer by natural convection depends on the surface temperature and, in units of $\text{W/m}^2 \cdot \text{K}$, may be approximated by an expression of the form $h = 0.80(T_s - T_\infty)^{1/3}$.

(b) Assess the effect of surface temperature on the power requirement, as well as on the relative contributions of convection and radiation to heat transfer from the surface.

1.69 Bus bars proposed for use in a power transmission station have a rectangular cross section of height $H = 600$ mm and width $W = 200$ mm. The electrical resistivity,

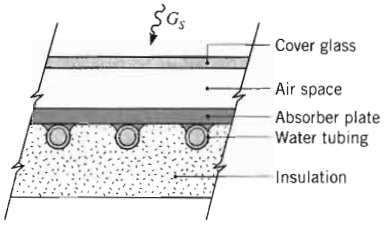
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$\rho_e(\mu\Omega \cdot \text{m})$, of the bar material is a function of temperature, $\rho_e = \rho_{e,o}[1 + \alpha(T - T_o)]$, where $\rho_{e,o} = 0.0828$ $\mu\Omega \cdot \text{m}$, $T_o = 25°\text{C}$, and $\alpha = 0.0040\ \text{K}^{-1}$. The emissivity of the bar's painted surface is 0.8, and the temperature of the surroundings is 30°C. The convection coefficient between the bar and the ambient air at 30°C is 10 $\text{W/m}^2 \cdot \text{K}$.

(a) Assuming the bar has a uniform temperature T, calculate the steady-state temperature when a current of 60,000 A passes through the bar.

(b) Compute and plot the steady-state temperature of the bar as a function of the convection coefficient for $10 \le h \le 100\ \text{W/m}^2 \cdot \text{K}$. What minimum convection coefficient is required to maintain a safe-operating temperature below 120°C? Will increasing the emissivity significantly affect this result?

1.70 A solar flux of 700 W/m^2 is incident on a flat-plate solar collector used to heat water. The area of the collector is 3 m^2, and 90% of the solar radiation passes through the cover glass and is absorbed by the absorber plate. The remaining 10% is reflected away from the collector. Water flows through the tube passages on the back side of the absorber plate and is heated from an inlet temperature T_i to an outlet temperature T_o. The cover glass, operating at a temperature of 30°C, has an emissivity of 0.94 and experiences radiation exchange with the sky at −10°C. The convection coefficient between the cover glass and the ambient air at 25°C is 10 $\text{W/m}^2 \cdot \text{K}$.

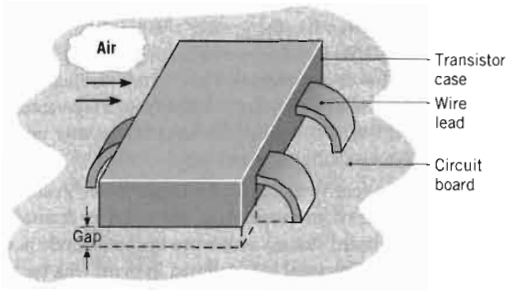


(a) Perform an overall energy balance on the collector to obtain an expression for the rate at which useful heat is collected per unit area of the collector, q''_u. Determine the value of q''_u.

(b) Calculate the temperature rise of the water, $T_o - T_i$, if the flow rate is 0.01 kg/s. Assume the specific heat of the water to be 4179 $\text{J/kg} \cdot \text{K}$.

(c) The collector efficiency η is defined as the ratio of the useful heat collected to the rate at which solar energy is incident on the collector. What is the value of η?

1.71 Consider a surface-mount type transistor on a circuit board whose temperature is maintained at 35°C. Air at 20°C flows over the upper surface of dimensions 4 mm by 8 mm with a convection coefficient of 50 $\text{W/m}^2 \cdot \text{K}$. Three wire leads, each of cross section 1 mm by 0.25 mm and length 4 mm, conduct heat from the case to the circuit board. The gap between the case and the board is 0.2 mm.



(a) Assuming the case is isothermal and neglecting radiation, estimate the case temperature when 150 mW is dissipated by the transistor and (i) stagnant air or (ii) a conductive paste fills the gap. The thermal conductivities of the wire leads, air, and conductive paste are 25, 0.0263, and 0.12 $\text{W/m} \cdot \text{K}$, respectively.

(b) Using the conductive paste to fill the gap, we wish to determine the extent to which increased heat dissipation may be accommodated, subject to the constraint that the case temperature not exceed 40°C. Options include increasing the air speed to achieve a larger convection coefficient h and/or changing the lead wire material to one of larger thermal conductivity. Independently considering leads fabricated from materials with thermal conductivities of 200 and 400 $\text{W/m} \cdot \text{K}$, compute and plot the maximum allowable heat dissipation for variations in h over the range $50 \le h \le 250\ \text{W/m}^2 \cdot \text{K}$.

Process Identification

1.72 In analyzing the performance of a thermal system, the engineer must be able to identify the relevant heat transfer processes. Only then can the system behavior be properly quantified. For the following systems identify the pertinent processes, designating them by appropriately labeled arrows on a sketch of the system. Answer additional questions that appear in the problem statement.

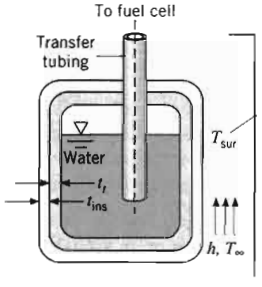
(a) Identify the heat transfer processes that determine the temperature of an asphalt pavement on a summer day. Write an energy balance for the surface of the pavement.

(b) Microwave radiation is known to be transmitted by plastics, glass, and ceramics, but to be absorbed by materials having polar molecules such as water. Water molecules exposed to microwave radiation align and reverse alignment with the microwave radiation at frequencies up to $10^9\ \text{s}^{-1}$, causing heat to be generated. Contrast cooking in a microwave

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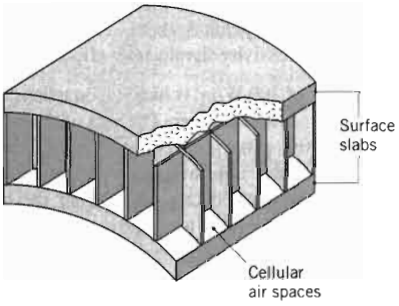
oven with cooking in a conventional radiant or convection oven. In each case what is the physical mechanism responsible for heating the food? Which oven has the greater energy utilization efficiency? Why? Microwave heating is being considered for drying clothes. How would operation of a microwave clothes dryer differ from a conventional dryer? Which is likely to have the greater energy utilization efficiency and why?

(c) To prevent freezing of the liquid water inside the fuel cell of an automobile, the water is drained to an on-board storage tank when the automobile is not in use. (The water is transferred from the tank back to the fuel cell when the automobile is turned on.) Consider a fuel cell–powered automobile that is parked outside on a very cold evening with $T_\infty = -20°C$. The storage tank is initially empty and at $T_{i,t} = -20°C$ when liquid water at atmospheric pressure and temperature $T_{i,w} = 50°C$ is introduced into the tank. The tank has a wall thickness t_t and is blanketed with insulation of thickness t_{ins}. Identify the heat transfer processes that will promote freezing of the water. Will the likelihood of freezing change as the insulation thickness is modified? Will the likelihood of freezing depend on the tank wall thickness and tank wall material? Would freezing of the water be more likely if plastic (low thermal conductivity) or stainless steel (moderate thermal conductivity) tubing is used to transfer the water to and from the tank? Is there an optimal tank shape that would minimize the probability of the water freezing? Would freezing be more likely or less likely to occur if a thin sheet of aluminum foil (high thermal conductivity, low emissivity) is applied to the outside of the insulation?

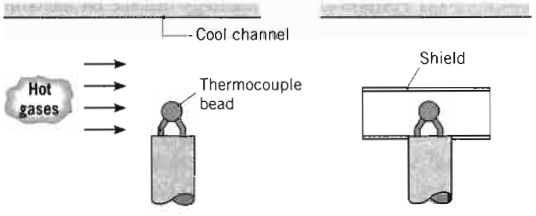


(d) Consider an incandescent light source that consists of a tungsten filament enclosed in a gas-filled glass bulb. Assuming steady-state operation with the filament at a temperature of approximately 2900 K, list all the pertinent heat transfer processes for (i) the filament and (ii) the glass bulb.

(e) There is considerable interest in developing building materials that have improved insulating qualities. The development of such materials would do much to enhance energy conservation by reducing space heating requirements. It has been suggested that superior structural and insulating qualities could be obtained by using the composite shown. The material consists of a honeycomb, with cells of square cross section, sandwiched between solid slabs. The cells are filled with air, and the slabs, as well as the honeycomb matrix, are fabricated from plastics of low thermal conductivity. For heat transfer normal to the slabs, identify all heat transfer processes pertinent to the performance of the composite. Suggest ways in which this performance could be enhanced.

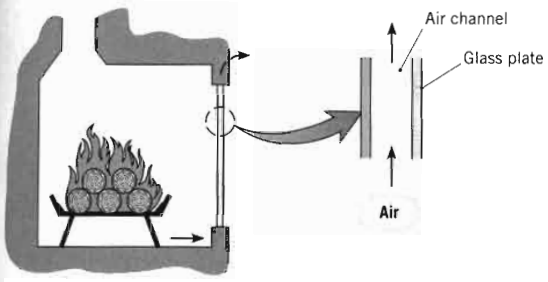


(f) A thermocouple junction (bead) is used to measure the temperature of a hot gas stream flowing through a channel by inserting the junction into the mainstream of the gas. The surface of the channel is cooled such that its temperature is well below that of the gas. Identify the heat transfer processes associated with the junction surface. Will the junction sense a temperature that is less than, equal to, or greater than the gas temperature? A radiation shield is a small, open-ended tube that encloses the thermocouple junction, yet allows for passage of the gas through the tube. How does use of such a shield improve the accuracy of the temperature measurement?



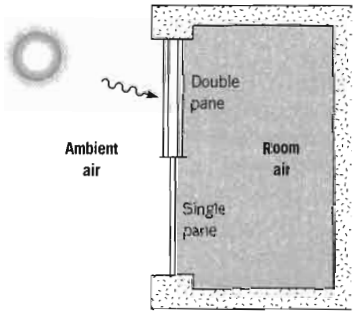
(g) A double-glazed, glass fire screen is inserted between a wood-burning fireplace and the interior of a room. The screen consists of two vertical glass plates that are separated by a space through which room air may flow (the space is open at the top and bottom). Identify the heat transfer processes associated with the fire screen.

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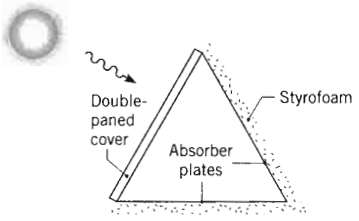
1.73 In considering the following problems involving heat transfer in the natural environment (outdoors), recognize that solar radiation is comprised of long and short wavelength components. If this radiation is incident on a *semitransparent medium*, such as water or glass, two things will happen to the nonreflected portion of the radiation. The long wavelength component will be absorbed at the surface of the medium, whereas the short wavelength component will be transmitted by the surface.

(a) The number of panes in a window can strongly influence the heat loss from a heated room to the outside ambient air. Compare the single- and double-paned units shown by identifying relevant heat transfer processes for each case.



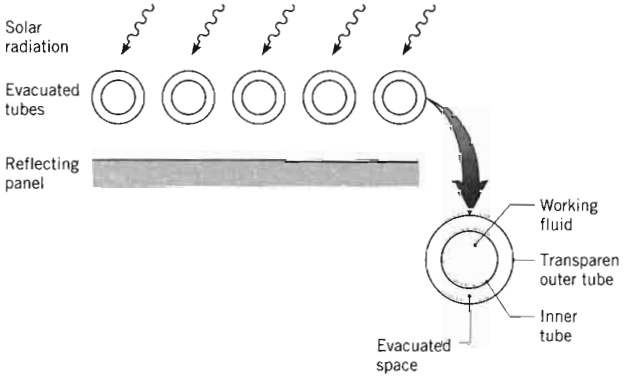
(b) In a typical flat-plate solar collector, energy is collected by a working fluid that is circulated through tubes that are in good contact with the back face of an absorber plate. The back face is insulated from the surroundings, and the absorber plate receives solar radiation on its front face, which is typically covered by one or more transparent plates. Identify the relevant heat transfer processes, first for the absorber plate with no cover plate and then for the absorber plate with a single cover plate.

(c) The solar energy collector design shown below has been used for agricultural applications. Air is blown through a long duct whose cross section is in the form of an equilateral triangle. One side of the triangle is comprised of a double-paned, semitransparent cover, while the other two sides are constructed from aluminum sheets painted flat black on the inside and covered on the outside with a layer of styrofoam insulation. During sunny periods, air entering the system is heated for delivery to either a greenhouse, grain drying unit, or a storage system.



Identify all heat transfer processes associated with the cover plates, the absorber plate(s), and the air.

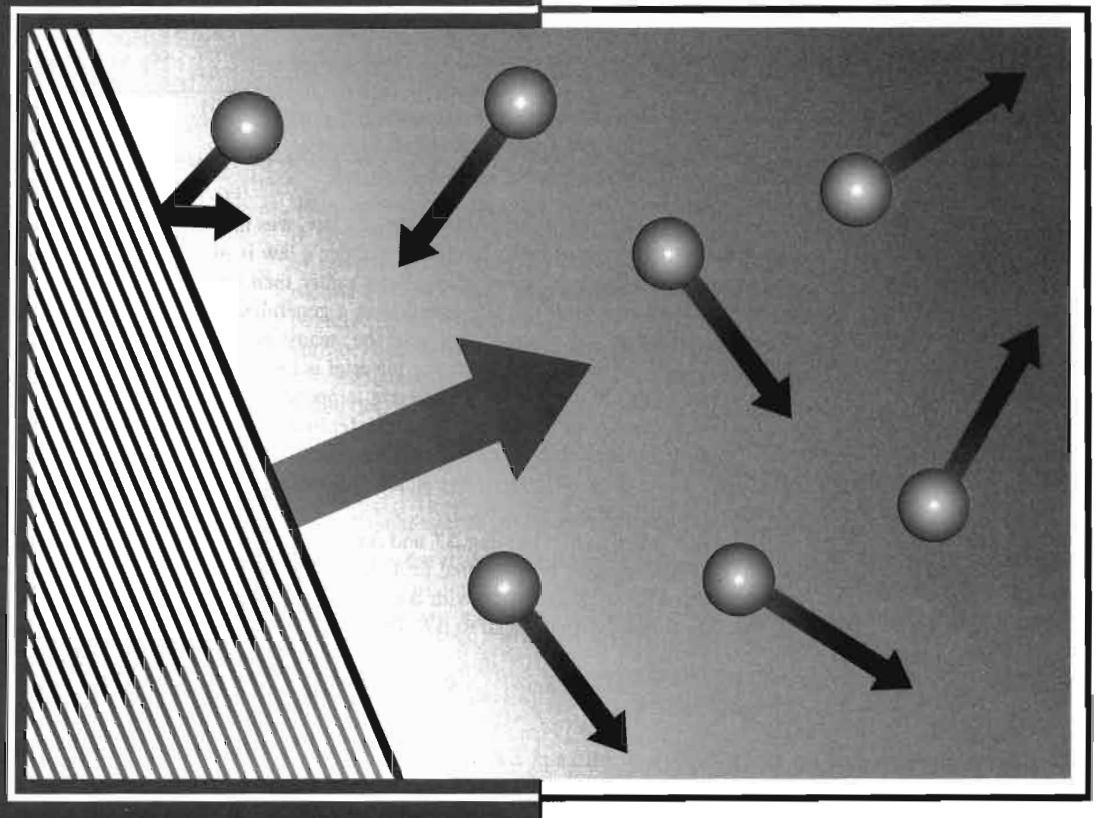
(d) Evacuated-tube solar collectors are capable of improved performance relative to flat-plate collectors. The design consists of an inner tube enclosed in an outer tube that is transparent to solar radiation. The annular space between the tubes is evacuated. The outer, opaque surface of the inner tube absorbs solar radiation, and a working fluid is passed through the tube to collect the solar energy. The collector design generally consists of a row of such tubes arranged in front of a reflecting panel. Identify all heat transfer processes relevant to the performance of this device.



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CHAPTER 2

Introduction to Conduction



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Recall that conduction refers to the transport of energy in a medium due to a temperature gradient, and the physical mechanism is one of random atomic or molecular activity. In Chapter 1 we learned that conduction heat transfer is governed by *Fourier's law* and that use of the law to determine the heat flux depends on knowledge of the manner in which temperature varies within the medium (the *temperature distribution*). By way of introduction, we restricted our attention to simplified conditions (one-dimensional, steady-state conduction in a plane wall) for which the temperature distribution is readily deduced to be linear. However, Fourier's law is applicable to transient, multidimensional conduction in complex geometries for which the nature of the temperature distribution is not evident.

The objectives of this chapter are twofold. First, we wish to develop a deeper understanding of Fourier's law. What are its origins? What form does it take for different geometries? How does its proportionality constant (the *thermal conductivity*) depend on the physical nature of the medium? Our second objective is to develop, from basic principles, the general equation, termed the *heat equation*, which governs the temperature distribution in a medium. It is the solution to this equation that provides knowledge of the temperature distribution, which may then be used with Fourier's law to determine the heat flux.

2.1 The Conduction Rate Equation

Although the conduction rate equation, Fourier's law, was introduced in Section 1.2, it is now appropriate to consider its origin. Fourier's law is *phenomenological*; that is, it is developed from observed phenomena rather than being derived from first principles. Hence, we view the rate equation as a generalization based on much experimental evidence. For example, consider the steady-state conduction experiment of Figure 2.1. A cylindrical rod of known material is insulated on its lateral surface, while its end faces are maintained at different temperatures, with $T_1 > T_2$. The temperature difference causes conduction heat transfer in the positive x direction. We are able to measure the heat transfer rate q_x, and we seek to determine how q_x depends on the following variables: ΔT, the temperature difference; Δx, the rod length; and A, the cross-sectional area.

We might imagine first holding ΔT and Δx constant and varying A. If we do so, we find that q_x is directly proportional to A. Similarly, holding ΔT and A constant, we observe that q_x varies inversely with Δx. Finally, holding A and Δx constant, we find that q_x is directly proportional to ΔT. The collective effect is then

$$q_x \propto A \frac{\Delta T}{\Delta x}$$

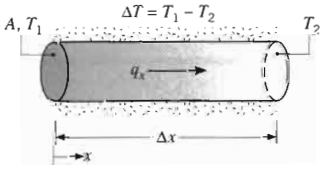


FIGURE 2.1
Steady-state heat conduction experiment.

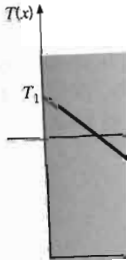


FIGURE 2.2
The relationsl
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in one dimen



FIGURE 2.
The heat fl
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In changing the material (e.g., from a metal to a plastic), we would find that the above proportionality remains valid. However, we would also find that, for equal values of A, Δx, and ΔT, the value of q_x would be smaller for the plastic than for the metal. This suggests that the proportionality may be converted to an equality by introducing a coefficient that is a measure of the material behavior. Hence, we write

$$q_x = kA\frac{\Delta T}{\Delta x}$$

where k, the *thermal conductivity* (W/m · K), is an important *property* of the material. Evaluating this expression in the limit as $\Delta x \to 0$, we obtain for the heat *rate*

$$q_x = -kA\frac{dT}{dx} \tag{2.1}$$

or for the heat *flux*

$$q''_x = \frac{q_x}{A} = -k\frac{dT}{dx} \tag{2.2}$$

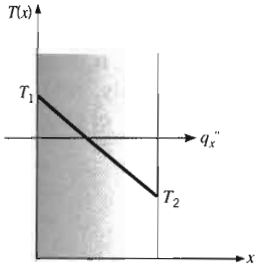


FIGURE 2.2
The relationship between coordinate system, heat flow direction, and temperature gradient in one dimension.

Recall that the minus sign is necessary because heat is always transferred in the direction of decreasing temperature.

Fourier's law, as written in Equation 2.2, implies that the heat flux is a directional quantity. In particular, the direction of q''_x is *normal* to the cross-sectional area A. Or, more generally, the direction of heat flow will always be normal to a surface of constant temperature, called an *isothermal* surface. Figure 2.2 illustrates the direction of heat flow q''_x in a plane wall for which the *temperature gradient* dT/dx is negative. From Equation 2.2, it follows that q''_x is positive. Note that the isothermal surfaces are planes normal to the x direction.

Recognizing that the heat flux is a vector quantity, we can write a more general statement of the conduction rate equation (*Fourier's law*) as follows:

$$\boldsymbol{q}'' = -k\nabla T = -k\left(\boldsymbol{i}\frac{\partial T}{\partial x} + \boldsymbol{j}\frac{\partial T}{\partial y} + \boldsymbol{k}\frac{\partial T}{\partial z}\right) \tag{2.3}$$

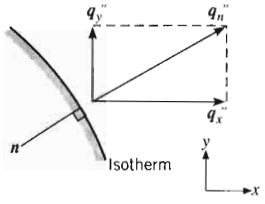


FIGURE 2.3
The heat flux vector normal to an isotherm in a two-dimensional coordinate system.

where ∇ is the three-dimensional del operator and $T(x, y, z)$ is the scalar temperature field. It is implicit in Equation 2.3 that the heat flux vector is in a direction perpendicular to the isothermal surfaces. An alternative form of Fourier's law is therefore

$$q''_n = -k\frac{\partial T}{\partial n} \tag{2.4}$$

where q''_n is the heat flux in a direction n, which is normal to an *isotherm*, as shown for the two-dimensional case in Figure 2.3. The heat transfer is sustained by a temperature gradient along $\boldsymbol{n}$. Note also that the heat flux vector can be resolved into components such that, in Cartesian coordinates, the general expression for $\boldsymbol{q}''$ is

$$\boldsymbol{q}'' = \boldsymbol{i}q''_x + \boldsymbol{j}q''_y + \boldsymbol{k}q''_z \tag{2.5}$$

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where, from Equation 2.3, it follows that

$$q''_x = -k\frac{\partial T}{\partial x} \qquad q''_y = -k\frac{\partial T}{\partial y} \qquad q''_z = -k\frac{\partial T}{\partial z} \tag{2.6}$$

Each of these expressions relates the heat flux *across a surface* to the temperature gradient in a direction perpendicular to the surface. It is also implicit in Equation 2.3 that the medium in which the conduction occurs is *isotropic*. For such a medium the value of the thermal conductivity is independent of the coordinate direction.

Fourier's law is the cornerstone of conduction heat transfer, and its key features are summarized as follows. It is *not* an expression that may be derived from first principles; it is instead a generalization based on experimental evidence. It is an expression that *defines* an important material property, the thermal conductivity. In addition, Fourier's law is a vector expression indicating that the heat flux is normal to an isotherm and in the direction of decreasing temperature. Finally, note that Fourier's law applies for all matter, regardless of its state (solid, liquid, or gas).

2.2 *The Thermal Properties of Matter*

To use Fourier's law, the thermal conductivity of the material must be known. This property, which is referred to as a *transport property,* provides an indication of the rate at which energy is transferred by the diffusion process. It depends on the physical structure of matter, atomic and molecular, which is related to the state of the matter. In this section we consider various forms of matter, identifying important aspects of their behavior and presenting typical property values.

2.2.1 Thermal Conductivity

From Fourier's law, Equation 2.6, the thermal conductivity associated with conduction in the x-direction is defined as

$$k_x \equiv -\frac{q''_x}{(\partial T/\partial x)}$$

Similar definitions are associated with thermal conductivities in the y- and z-directions (k_y, k_z), but for an isotropic material the thermal conductivity is independent of the direction of transfer, $k_x = k_y = k_z \equiv k$.

From the foregoing equation, it follows that, for a prescribed temperature gradient, the conduction heat flux increases with increasing thermal conductivity. In general, the thermal conductivity of a solid is larger than that of a liquid, which is larger than that of a gas. As illustrated in Figure 2.4, the thermal conductivity of a solid may be more than four orders of magnitude larger than that of a gas. This trend is due largely to differences in intermolecular spacing for the two states.

The Solid State In the modern view of materials, a solid may be comprised of free electrons and atoms bound in a periodic arrangement called the lattice.

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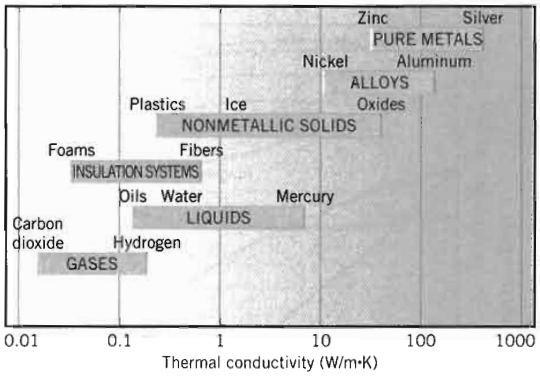


FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

Accordingly, transport of thermal energy may be due to two effects: the migration of free electrons and lattice vibrational waves. When viewed as a particle-like phenomenon, the lattice vibration quanta are termed *phonons*. In pure metals, the electron contribution to conduction heat transfer dominates, while in nonconductors and semiconductors, the phonon contribution is dominant.

Kinetic theory yields the following expression for the thermal conductivity [1]:

$$k = \frac{1}{3} C \bar{c} \lambda_{\text{mfp}} \tag{2.7}$$

For conducting materials such as metals, $C \equiv C_e$ is the electron specific heat per unit volume, $\bar{c}$ is the mean electron velocity, and $\lambda_{\text{mfp}} \equiv \lambda_e$ is the electron mean free path, which is defined as the average distance traveled by an electron before it collides with either an imperfection in the material or with a phonon. In nonconducting solids, $C \equiv C_{\text{ph}}$ is the phonon specific heat, $\bar{c}$ is the average speed of sound, and $\lambda_{\text{mfp}} \equiv \lambda_{\text{ph}}$ is the phonon mean free path, which again is determined by collisions with imperfections or other phonons. In all cases, the thermal conductivity increases as the mean free path of the *energy carriers* (electrons or phonons) is increased.

When electrons and phonons carry thermal energy leading to conduction heat transfer in a solid, the thermal conductivity may be expressed as

$$k = k_e + k_{\text{ph}} \tag{2.8}$$

To a first approximation, k_e is inversely proportional to the electrical resistivity, ρ_e. For pure metals, which are of low ρ_e, k_e is much larger than k_{ph}. In contrast, for alloys, which are of substantially larger ρ_e, the contribution of k_{ph} to k is no longer negligible. For nonmetallic solids, k is determined primarily by k_{ph}, which increases as the frequency of interactions between the atoms and the lattice decreases. The regularity of the lattice arrangement has an important effect on k_{ph}, with crystalline (well-ordered) materials like quartz having a higher thermal conductivity than amorphous materials like glass. In fact, for crystalline, nonmetallic solids such as diamond and beryllium oxide, k_{ph} can be quite large, exceeding values of k associated with good conductors, such as aluminum.

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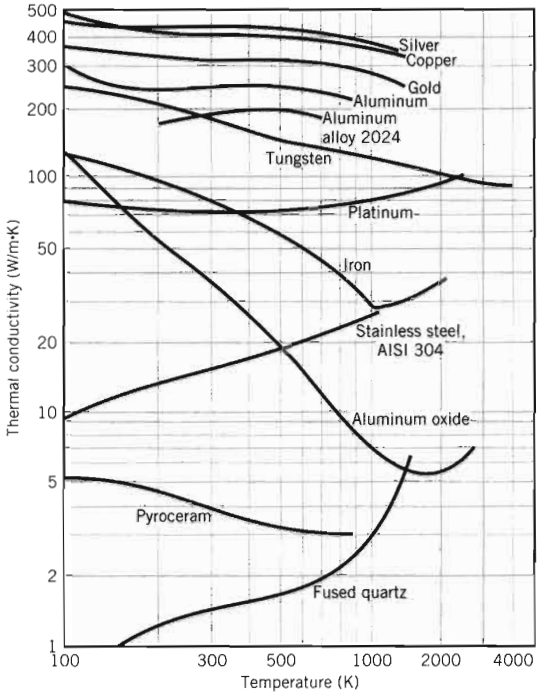


FIGURE 2.5 The temperature dependence of the thermal conductivity of selected solids.

The temperature dependence of k is shown in Figure 2.5 for representative metallic and nonmetallic solids. Values for selected materials of technical importance are also provided in Table A.1 (metallic solids) and Tables A.2 and A.3 (nonmetallic solids). More detailed treatments of thermal conductivity are available in the literature [2].

The Solid State: Micro- and Nanoscale Effects In the preceding discussion, the *bulk* thermal conductivity is described, and the thermal conductivity values listed in Tables A.1 through A.3 are appropriate for use when the physical dimensions of the material of interest are relatively large. This is the case in many commonplace engineering problems. However, in several areas of technology, such as microelectronics, the material's characteristic dimensions can be on the order of micrometers or nanometers, in which case care must be taken to account for the possible modifications of k that can occur as the physical dimensions become small.

Cross sections of *films* of the same material having thicknesses L_1 and L_2 are shown in Figure 2.6. Electrons or phonons that are associated with conduction of thermal energy are also shown qualitatively. Note that the physical boundaries of the film act to *scatter* the energy carriers and *redirect* their propagation. For large L/λ_{mfp} (Figure 2.6*a*), the effect of the boundaries on reducing the *average* energy carrier path length is minor, and conduction heat transfer occurs as described for bulk materials. However, as the film becomes thinner, the physical boundaries of the material can decrease the average *net* distance traveled by the energy carriers, as

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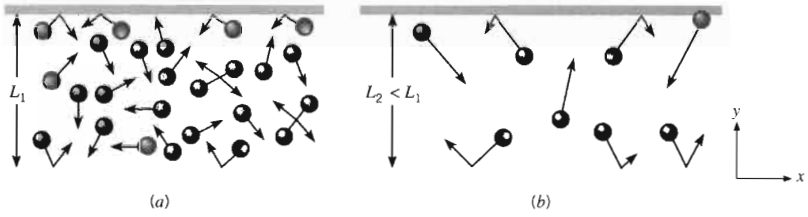


FIGURE 2.6 Electron or phonon trajectories in (*a*) a relatively thick film and (*b*) a relatively thin film with boundary effects.

shown in Figure 2.6*b*. Moreover, electrons and phonons moving in the thin *y*-direction (representing conduction in the *y*-direction) are affected by the boundaries to a more significant degree than energy carriers moving in the *x*-direction. As such, for films characterized by small L/λ_{mfp}, we find that $k_y < k_x < k$, where k is the bulk thermal conductivity of the film material.

For $L/\lambda_{mfp} \geq 1$, predicted values of k_x and k_y may be estimated to within 20% from the following expression [1]:

$$k_x/k = 1-2\lambda_{mfp}/(3\pi L) \tag{2.9a}$$

$$k_y/k = 1-\lambda_{mfp}/(3L) \tag{2.9b}$$

Equations 2.9a, b reveal that the values of k_x and k_y are within approximately 5% of the bulk thermal conductivity if $L/\lambda_{mfp} > 7$ (for k_y) and $L/\lambda_{mfp} > 4.5$ (for k_x). Values of the mean free path as well as critical film thicknesses below which microscale effects must be considered, L_{crit}, are included in Table 2.1 for several materials at $T \approx 300$ K. For films with $\lambda_{mfp} < L < L_{crit}$, k_x and k_y are reduced from the bulk value as indicated in Equations 2.9a, b. No general guidelines exist for predicting values of the thermal conductivities for $L/\lambda_{mfp} < 1$. Note that, in solids, the value of λ_{mfp} decreases as the temperature increases.

In addition to scattering from physical boundaries, as in the case of Figure 2.6*b*, energy carriers may be redirected by chemical *dopants* embedded within a material or by *grain boundaries* that separate individual clusters of material in otherwise homogeneous matter. *Nanostructured materials* are chemically identical to their conventional counterparts but are thermally processed to provide very small grain sizes in the final material and, from the heat transfer perspective, significantly increase the scattering and reflection of energy carriers at the grain boundaries.

TABLE 2.1 Mean free path and critical film thickness for various materials at $T \approx 300$ K [3,4]

Material	λ_{mfp} (nm)	$L_{crit,y}$ (nm)	$L_{crit,x}$ (nm)
Aluminum oxide	5.08	36	22
Diamond (IIa)	315	2200	1400
Gallium arsenide	23	160	100
Gold	31	220	140
Silicon	43	290	180
Silicon dioxide	0.6	4	3
Yttria-stabilized zirconia	25	170	110

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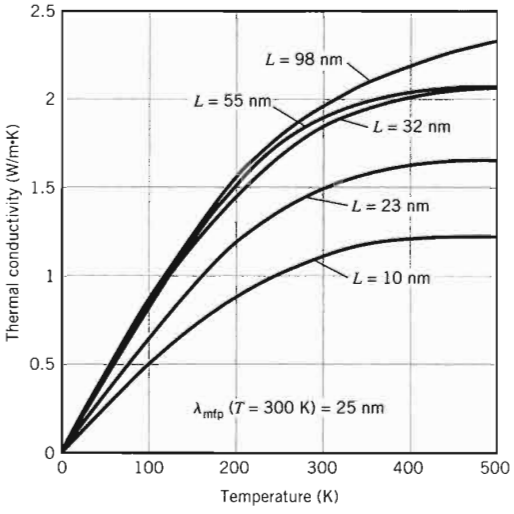


FIGURE 2.7 Measured thermal conductivity of yttria-stabilized zirconia as a function of temperature and mean grain size [3].

Measured values of the thermal conductivity of a bulk, nanostructured yttria-stabilized zirconia material are shown in Figure 2.7. This particular ceramic is widely used for insulation purposes in high-temperature combustion devices such as gas turbine engines. Conduction is dominated by phonon transfer, and the mean free path of the phonon energy carriers is, from Table 2.1, $\lambda_{mfp} = 25$ nm at 300 K. As the grain sizes are reduced to characteristic dimensions less than 25 nm (and more grain boundaries are introduced in the material per unit volume), significant reduction of the thermal conductivity occurs. Extrapolation of the results of Figure 2.7 to higher temperatures is not recommended, since the mean free path decreases with increasing temperature ($\lambda_{mfp} \approx 4$ nm at $T \approx 1525$ K) *and* grains of the material may coalesce, merge, and enlarge at elevated temperatures. Therefore, L/λ_{mfp} becomes larger at high temperatures, and reduction of k due to nanoscale effects becomes less pronounced.

The Fluid State The fluid state includes both liquids and gases. Since the intermolecular spacing is much larger and the motion of the molecules is more random for the fluid state than for the solid state, thermal energy transport is less effective. The thermal conductivity of gases and liquids is therefore generally smaller than that of solids.

The effect of temperature, pressure, and chemical species on the thermal conductivity of a gas may be explained in terms of the kinetic theory of gases [5]. From this theory it is known that the thermal conductivity is directly proportional to the density of the gas, the mean molecular speed $\bar{c}$, and the mean free path λ_{mfp}, which is the average distance traveled by an energy carrier (a molecule) before experiencing a collision.

$$k = \frac{1}{3} c_v \rho \bar{c} \lambda_{mfp} \tag{2.10}$$

Because $\bar{c}$ increases with increasing temperature and decreasing molecular weight, the thermal conductivity of a gas increases with increasing temperature and decreasing

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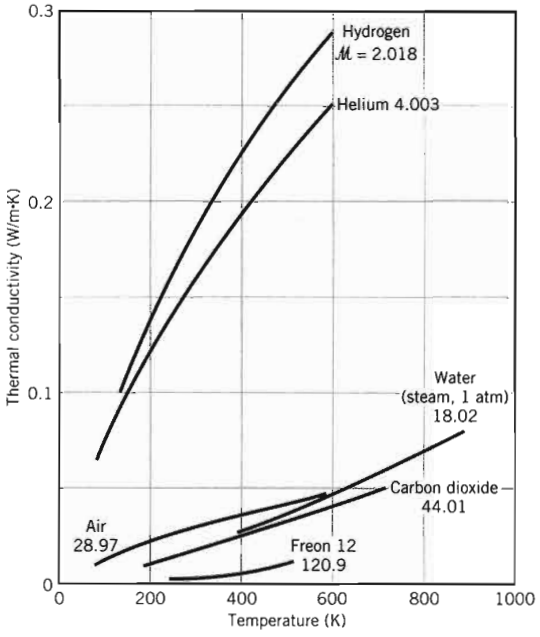


FIGURE 2.8 The temperature dependence of the thermal conductivity of selected gases at normal pressures. Molecular weights (M) of the gases are also shown.

molecular weight. These trends are shown in Figure 2.8. However, because ρ and λ_{mfp} are directly and inversely proportional to the gas pressure, respectively, the thermal conductivity is independent of pressure except in extreme cases as, for example, when conditions approach that of a perfect vacuum. Therefore, the assumption that k is independent of gas pressure for large volumes of gas is appropriate for the pressures of interest in this text. Accordingly, although the values of k presented in Table A.4 pertain to atmospheric pressure or the saturation pressure corresponding to the prescribed temperature, they may be used over a much wider pressure range.

Molecular conditions associated with the liquid state are more difficult to describe, and physical mechanisms for explaining the thermal conductivity are not well understood [6]. The thermal conductivity of nonmetallic liquids generally decreases with increasing temperature. As shown in Figure 2.9, water, glycerine, and engine oil are notable exceptions. The thermal conductivity of liquids is usually insensitive to pressure except near the critical point. Also, it is generally true that thermal conductivity decreases with increasing molecular weight. Values of the thermal conductivity are often tabulated as a function of temperature for the saturated state of the liquid. Tables A.5 and A.6 present such data for several common liquids.

Liquid metals are commonly used in high heat flux applications, such as occur in nuclear power plants. The thermal conductivity of such liquids is given in Table A.7. Note that the values are much larger than those of the nonmetallic liquids [7].

The Fluid State: Micro- and Nanoscale Effects As in the solid state, the bulk thermal conductivity may be modified as the characteristic dimensions of the

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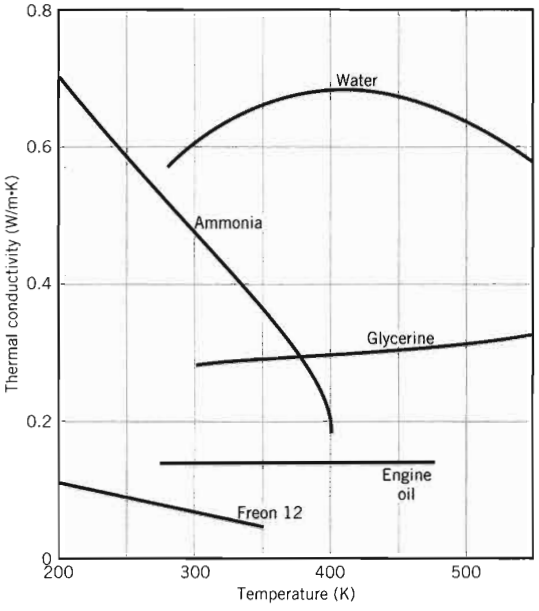


FIGURE 2.9 The temperature dependence of the thermal conductivity of selected nonmetallic liquids under saturated conditions.

system become small, in particular for small values of L/λ_{mfp}. As for the situation shown in Figure 2.6*b*, the mean free path of the molecules becomes restricted when the fluid is, for example, contained in an enclosure of small physical dimension.

Insulation Systems Thermal insulations consist of low thermal conductivity materials combined to achieve an even lower system thermal conductivity. In conventional *fiber-*, *powder-*, and *flake*-type insulations, the solid material is finely dispersed throughout an air space. Such systems are characterized by an *effective thermal conductivity*, which depends on the thermal conductivity and surface radiative properties of the solid material, as well as the nature and volumetric fraction of the air or void space. A special parameter of the system is its bulk density (solid mass/total volume), which depends strongly on the manner in which the material is packed.

If small voids or hollow spaces are formed by bonding or fusing portions of the solid material, a rigid matrix is created. When these spaces are sealed from each other, the system is referred to as a *cellular* insulation. Examples of such rigid insulations are *foamed* systems, particularly those made from plastic and glass materials. *Reflective* insulations are composed of multilayered, parallel, thin sheets or foils of high reflectivity, which are spaced to reflect radiant energy back to its source. The spacing between the foils is designed to restrict the motion of air, and in high-performance insulations, the space is evacuated. In all types of insulation, evacuation of the air in the void space will reduce the effective thermal conductivity of the system.

It is important to recognize that heat transfer through any of these insulation systems may include several modes: conduction through the solid materials; conduction

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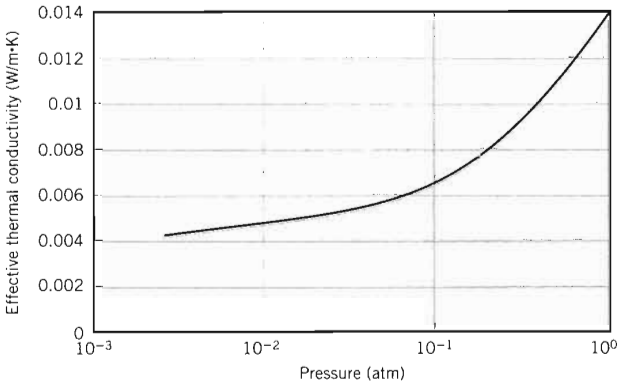


FIGURE 2.10 Measured thermal conductivity of carbon-doped silica aerogel as a function of pressure at $T \approx 300$ K [10].

or convection through the air in the void spaces; and radiation exchange between the surfaces of the solid matrix. The effective thermal conductivity accounts for all of these processes, and values for selected insulation systems are summarized in Table A.3. Additional background information and data are available in the literature [8, 9].

As with thin films, micro- and nanoscale effects can influence the effective thermal conductivity of insulating materials. The value of k for a nanostructured silica aerogel material that is composed of approximately 5% by volume solid material and 95% by volume air that is trapped within pores of $L \approx 20$ nm is shown in Figure 2.10. Note that at $T \approx 300$ K, the mean free path for air at atmospheric pressure is approximately 80 nm. As the gas pressure is reduced, λ_{mfp} would increase for an unconfined gas, but the molecular motion of the trapped air is restricted by the walls of the small pores and k is reduced to extremely small values relative to the thermal conductivities of conventional matter reported in Figure 2.4.

2.2.2 Other Relevant Properties

In our analysis of heat transfer problems, it will be necessary to use several properties of matter. These properties are generally referred to as *thermophysical* properties and include two distinct categories, *transport* and *thermodynamic* properties. The transport properties include the diffusion rate coefficients such as k, the thermal conductivity (for heat transfer), and ν, the kinematic viscosity (for momentum transfer). Thermodynamic properties, on the other hand, pertain to the equilibrium state of a system. Density (ρ) and specific heat (c_p) are two such properties used extensively in thermodynamic analysis. The product ρc_p (J/m^3 · K), commonly termed the *volumetric heat capacity*, measures the ability of a material to store thermal energy. Because substances of large density are typically characterized by small specific heats, many solids and liquids, which are very good energy storage media, have comparable heat capacities ($\rho c_p > 1$ MJ/m^3 · K). Because of their very small densities, however, gases are poorly suited for thermal energy storage ($\rho c_p \approx 1$ kJ/m^3 · K). Densities and specific heats are provided in the tables of Appendix A for a wide range of solids, liquids, and gases.

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In heat transfer analysis, the ratio of the thermal conductivity to the heat capacity is an important property termed the *thermal diffusivity* α, which has units of m^2/s:

$$\alpha = \frac{k}{\rho c_p}$$

It measures the ability of a material to conduct thermal energy relative to its ability to store thermal energy. Materials of large α will respond quickly to changes in their thermal environment, while materials of small α will respond more sluggishly, taking longer to reach a new equilibrium condition.

The accuracy of engineering calculations depends on the accuracy with which the thermophysical properties are known [11–13]. Numerous examples could be cited of flaws in equipment and process design or failure to meet performance specifications that were attributable to misinformation associated with the selection of key property values used in the initial system analysis. Selection of reliable property data is an integral part of any careful engineering analysis. The casual use of data that have not been well characterized or evaluated, as may be found in some literature or handbooks, is to be avoided. Recommended data values for many thermophysical properties can be obtained from Reference 14. This reference, available in most institutional libraries, was prepared by the Thermophysical Properties Research Center (TPRC) at Purdue University.

EXAMPLE 2.1

The thermal diffusivity α is the controlling transport property for transient conduction. Using appropriate values of k, ρ, and c_p from Appendix A, calculate α for the following materials at the prescribed temperatures: pure aluminum, 300 and 700 K; silicon carbide, 1000 K; paraffin, 300 K.

SOLUTION

Known: Definition of the thermal diffusivity α.

Find: Numerical values of α for selected materials and temperatures.

Properties: Table A.1, pure aluminum (300 K):

$$\left.\begin{array}{l}\rho = 2702\ \text{kg/m}^3 \\ c_p = 903\ \text{J/kg} \cdot \text{K} \\ k = 237\ \text{W/m} \cdot \text{K}\end{array}\right\} \alpha = \frac{k}{\rho c_p} = \frac{237\ \text{W/m} \cdot \text{K}}{2702\ \text{kg/m}^3 \times 903\ \text{J/kg} \cdot \text{K}}$$

$$= 97.1 \times 10^{-6}\ \text{m}^2/\text{s} \qquad \triangleleft$$

Table A.1, pure aluminum (700 K):

$$\begin{array}{lll}\rho = 2702\ \text{kg/m}^3 & & \text{at 300 K} \\ c_p = 1090\ \text{J/kg} \cdot \text{K} & & \text{at 700 K (by linear interpolation)} \\ k = 225\ \text{W/m} \cdot \text{K} & & \text{at 700 K (by linear interpolation)}\end{array}$$

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Hence

$$\alpha = \frac{k}{\rho c_p} = \frac{225 \text{ W/m} \cdot \text{K}}{2702 \text{ kg/m}^3 \times 1090 \text{ J/kg} \cdot \text{K}} = 76 \times 10^{-6} \text{ m}^2/\text{s} \quad \triangleleft$$

Table A.2, silicon carbide (1000 K):

$$\left.\begin{array}{ll} \rho = 3160 \text{ kg/m}^3 & \text{at 300 K} \\ c_p = 1195 \text{ J/kg} \cdot \text{K} & \text{at 1000 K} \\ k = 87 \text{ W/m} \cdot \text{K} & \text{at 1000 K} \end{array}\right\} \alpha = \frac{87 \text{ W/m} \cdot \text{K}}{3160 \text{ kg/m}^3 \times 1195 \text{ J/kg} \cdot \text{K}}$$

$$= 23 \times 10^{-6} \text{ m}^2/\text{s} \quad \triangleleft$$

Table A.3, paraffin (300 K):

$$\left.\begin{array}{l} \rho = 900 \text{ kg/m}^3 \\ c_p = 2890 \text{ J/kg} \cdot \text{K} \\ k = 0.24 \text{ W/m} \cdot \text{K} \end{array}\right\} \alpha = \frac{k}{\rho c_p} = \frac{0.24 \text{ W/m} \cdot \text{K}}{900 \text{ kg/m}^3 \times 2890 \text{ J/kg} \cdot \text{K}}$$

$$= 9.2 \times 10^{-8} \text{ m}^2/\text{s} \quad \triangleleft$$

Comments:

1. Note temperature dependence of the thermophysical properties of aluminum and silicon carbide. For example, for silicon carbide, $\alpha(1000 \text{ K}) \approx 0.1 \times \alpha(300 \text{ K})$; hence properties of this material have a strong temperature dependence.
2. The physical interpretation of α is that it provides a measure of heat transport (k) relative to energy storage (ρc_p). In general, metallic solids have higher α, while nonmetallics (e.g., paraffin) have lower values of α.
3. Linear interpolation of property values is generally acceptable for engineering calculations.
4. Use of the low-temperature (300 K) density at higher temperatures ignores thermal expansion effects but is also acceptable for engineering calculations.
5. The *IHT* software provides a library of thermophysical properties for selected solids, liquids, and gases that can be accessed from the toolbar button, *Properties*. The properties are expressed as intrinsic functions, as shown for the thermal conductivity of atmospheric air,

$$k = \text{k\_T("Air",T)} \qquad \text{//Thermal conductivity, W/m} \cdot \text{K}$$

and are based on values tabulated in Appendix A, with temperatures in kelvin units. You can create your own functions to represent property values or other input data by using the *User-Defined Function* feature in *IHT*, as described in the *Help* section.

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2.3 *The Heat Diffusion Equation*

A major objective in a conduction analysis is to determine the *temperature field* in a medium resulting from conditions imposed on its boundaries. That is, we wish to know the *temperature distribution*, which represents how temperature varies with position in the medium. Once this distribution is known, the conduction heat flux at any point in the medium or on its surface may be computed from Fourier's law. Other important quantities of interest may also be determined. For a solid, knowledge of the temperature distribution could be used to ascertain structural integrity through determination of thermal stresses, expansions, and deflections. The temperature distribution could also be used to optimize the thickness of an insulating material or to determine the compatibility of special coatings or adhesives used with the material.

We now consider the manner in which the temperature distribution can be determined. The approach follows the methodology described in Section 1.3.3 of applying the energy conservation requirement. In this case, we define a *differential control volume*, identify the relevant energy transfer processes, and introduce the appropriate rate equations. The result is a differential equation whose solution, for prescribed boundary conditions, provides the temperature distribution in the medium.

Consider a homogeneous medium within which there is no bulk motion (advection) and the temperature distribution $T(x, y, z)$ is expressed in Cartesian coordinates. Following the methodology of applying conservation of energy (Section 1.3.3), we first define an infinitesimally small (differential) control volume, $dx \cdot dy \cdot dz$, as shown in Figure 2.11. Choosing to formulate the first law at an instant of time, the second step is to consider the energy processes that are relevant to this control volume. If there are temperature gradients, conduction heat transfer will occur across

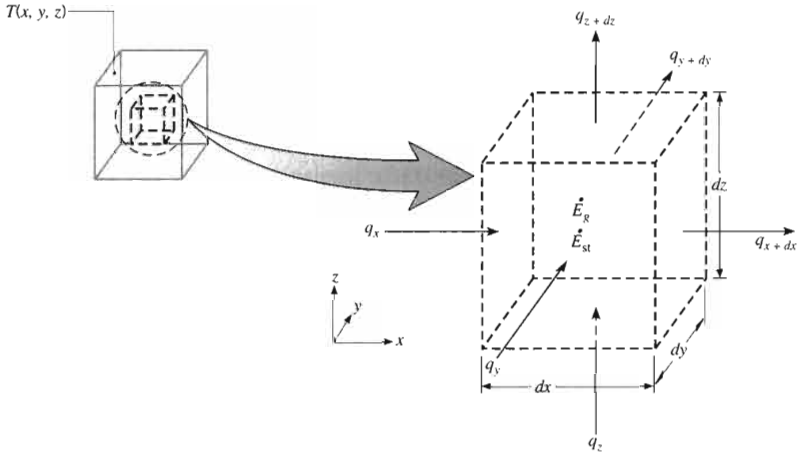


FIGURE 2.11 Differential control volume, $dx\ dy\ dz$, for conduction analysis in Cartesian coordinates.

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each of the control surfaces. The conduction heat rates perpendicular to each of the control surfaces at the x, y, and z coordinate locations are indicated by the terms q_x, q_y, and q_z, respectively. The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher order terms,

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx \tag{2.11a}$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy \tag{2.11b}$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz \tag{2.11c}$$

In words, Equation 2.11a simply states that the x component of the heat transfer rate at $x + dx$ is equal to the value of this component at x plus the amount by which it changes with respect to x times dx.

Within the medium there may also be an *energy source* term associated with the rate of thermal energy generation. This term is represented as

$$\dot{E}_g = \dot{q}\, dx\, dy\, dz \tag{2.12}$$

where $\dot{q}$ is the rate at which energy is generated per unit volume of the medium (W/m^3). In addition, there may occur changes in the amount of the internal thermal energy stored by the material in the control volume. If the material is not experiencing a change in phase, latent energy effects are not pertinent, and the *energy storage* term may be expressed as

$$\dot{E}_{st} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz \tag{2.13}$$

where $\rho c_p\, \partial T/\partial t$ is the time rate of change of the sensible (thermal) energy of the medium per unit volume.

Once again it is important to note that the terms $\dot{E}_g$ and $\dot{E}_{st}$ represent different physical processes. The energy generation term $\dot{E}_g$ is a manifestation of some energy conversion process involving thermal energy on one hand and some other form of energy, such as chemical, electrical, or nuclear, on the other. The term is positive (a *source*) if thermal energy is being generated in the material at the expense of some other energy form; it is negative (a *sink*) if thermal energy is being consumed. In contrast, the energy storage term $\dot{E}_{st}$ refers to the rate of change of thermal energy stored by the matter.

The last step in the methodology outlined in Section 1.3.3 is to express conservation of energy using the foregoing rate equations. On a *rate* basis, the general form of the conservation of energy requirement is

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st} \tag{1.11c}$$

Hence, recognizing that the conduction rates constitute the energy inflow, $\dot{E}_{in}$, and outflow, $\dot{E}_{out}$, and substituting Equations 2.12 and 2.13, we obtain

$$q_x + q_y + q_z + \dot{q}\, dx\, dy\, dz - q_{x+dx} - q_{y+dy} - q_{z+dz} = \rho c_p \frac{\partial T}{\partial t} dx\, dy\, dz \tag{2.14}$$

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Substituting from Equations 2.11, it follows that

$$-\frac{\partial q_x}{\partial x}dx - \frac{\partial q_y}{\partial y}dy - \frac{\partial q_z}{\partial z}dz + \dot{q}\,dx\,dy\,dz = \rho c_p \frac{\partial T}{\partial t}dx\,dy\,dz \tag{2.15}$$

The conduction heat rates may be evaluated from Fourier's law,

$$q_x = -k\,dy\,dz\frac{\partial T}{\partial x} \tag{2.16a}$$

$$q_y = -k\,dx\,dz\frac{\partial T}{\partial y} \tag{2.16b}$$

$$q_z = -k\,dx\,dy\frac{\partial T}{\partial z} \tag{2.16c}$$

where each heat flux component of Equation 2.6 has been multiplied by the appropriate control surface (differential) area to obtain the heat transfer rate. Substituting Equations 2.16 into Equation 2.15 and dividing out the dimensions of the control volume ($dx\,dy\,dz$), we obtain

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \tag{2.17}$$

Equation 2.17 is the general form, in Cartesian coordinates, of the *heat diffusion equation.* This equation, often referred to as the *heat equation*, provides the basic tool for heat conduction analysis. From its solution, we can obtain the temperature distribution $T(x, y, z)$ as a function of time. The apparent complexity of this expression should not obscure the fact that it describes an important physical condition, that is, conservation of energy. You should have a clear understanding of the physical significance of each term appearing in the equation. For example, the term $\partial(k\partial T/\partial x)/\partial x$ is related to the *net* conduction heat flux *into* the control volume for the x-coordinate direction. That is, multiplying by dx,

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx = q''_x - q''_{x+dx} \tag{2.18}$$

with similar expressions applying for the fluxes in the y and z directions. In words, the heat equation, Equation 2.17, therefore states that *at any point in the medium the net rate of energy transfer by conduction into a unit volume plus the volumetric rate of thermal energy generation must equal the rate of change of thermal energy stored within the volume.*

It is often possible to work with simplified versions of Equation 2.17. For example, if the thermal conductivity is constant, the heat equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{2.19}$$

where $\alpha = k/\rho c_p$ is the *thermal diffusivity.* Additional simplifications of the general form of the heat equation are often possible. For example, under *steady-state*

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conditions, there can be no change in the amount of energy storage; hence Equation 2.17 reduces to

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)+\dot{q}=0 \tag{2.20}$$

Moreover, if the heat transfer is *one-dimensional* (e.g., in the x direction) and there is *no energy generation*, Equation 2.20 reduces to

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right)=0 \tag{2.21}$$

The important implication of this result is that *under steady-state, one-dimensional conditions with no energy generation*, the heat flux is a constant in the direction of transfer ($dq_x''/dx = 0$).

The heat equation may also be expressed in cylindrical and spherical coordinates. The differential control volumes for these two coordinate systems are shown in Figures 2.12 and 2.13.

Cylindrical Coordinates When the del operator ∇ of Equation 2.3 is expressed in cylindrical coordinates, the general form of the heat flux vector, and hence of Fourier's law, is

$$\boldsymbol{q}''=-k\nabla T=-k\left(\boldsymbol{i}\frac{\partial T}{\partial r}+\boldsymbol{j}\frac{1}{r}\frac{\partial T}{\partial \phi}+\boldsymbol{k}\frac{\partial T}{\partial z}\right) \tag{2.22}$$

where

$$q_r''=-k\frac{\partial T}{\partial r}\qquad q_\phi''=-\frac{k}{r}\frac{\partial T}{\partial \phi}\qquad q_z''=-k\frac{\partial T}{\partial z} \tag{2.23}$$

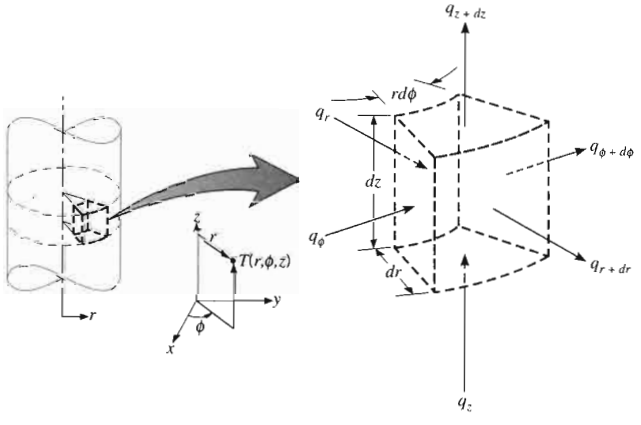


FIGURE 2.12 Differential control volume, $dr \cdot r\,d\phi \cdot dz$, for conduction analysis in cylindrical coordinates (r, ϕ, z).

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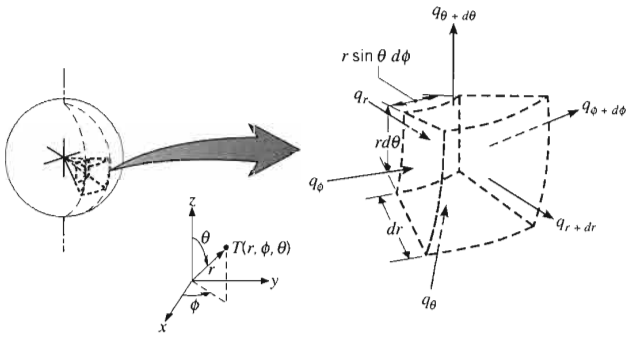


FIGURE 2.13 Differential control volume, $dr \cdot r\sin\theta\, d\phi \cdot r\, d\theta$, for conduction analysis in spherical coordinates (r, ϕ, θ).

are heat flux components in the radial, circumferential, and axial directions, respectively. Applying an energy balance to the differential control volume of Figure 2.12, the following general form of the heat equation is obtained:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p \frac{\partial T}{\partial t} \tag{2.24}$$

Spherical Coordinates In spherical coordinates the general form of the heat flux vector and Fourier's law is

$$\boldsymbol{q}'' = -k\nabla T = -k\left(\boldsymbol{i}\frac{\partial T}{\partial r} + \boldsymbol{j}\frac{1}{r}\frac{\partial T}{\partial \theta} + \boldsymbol{k}\frac{1}{r\sin\theta}\frac{\partial T}{\partial \phi}\right) \tag{2.25}$$

where

$$q_r'' = -k\frac{\partial T}{\partial r} \qquad q_\theta'' = -\frac{k}{r}\frac{\partial T}{\partial \theta} \qquad q_\phi'' = -\frac{k}{r\sin\theta}\frac{\partial T}{\partial \phi} \tag{2.26}$$

are heat flux components in the radial, polar, and azimuthal directions, respectively. Applying an energy balance to the differential control volume of Figure 2.13, the following general form of the heat equation is obtained:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t} \tag{2.27}$$

Since it is important that you be able to apply conservation principles to differential control volumes, you should attempt to derive Equation 2.24 or 2.27 (see Problems 2.35 and 2.36). Note that the temperature gradient in Fourier's law must have units of K/m. Hence, when evaluating the gradient for an angular coordinate,

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it must be expressed in terms of the differential change in arc *length*. For example, the heat flux component in the circumferential direction of a cylindrical coordinate system is $q''_\phi = -(k/r)(\partial T/\partial \phi)$, and *not* $q''_\phi = -k(\partial T/\partial \phi)$.

EXAMPLE 2.2

The temperature distribution across a wall 1 m thick at a certain instant of time is given as

$$T(x) = a + bx + cx^2$$

where T is in degrees Celsius and x is in meters, while $a = 900°C$, $b = -300°C/m$, and $c = -50°C/m^2$. A uniform heat generation, $\dot{q} = 1000\ W/m^3$, is present in the wall of area $10\ m^2$ having the properties $\rho = 1600\ kg/m^3$, $k = 40\ W/m \cdot K$, and $c_p = 4\ kJ/kg \cdot K$.

1. Determine the rate of heat transfer entering the wall ($x = 0$) and leaving the wall ($x = 1$ m).
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change at $x = 0$, 0.25, and 0.5 m.

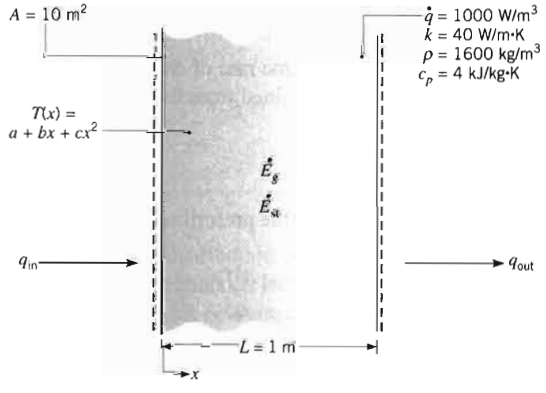
SOLUTION

Known: Temperature distribution $T(x)$ at an instant of time t in a one-dimensional wall with uniform heat generation.

Find:

1. Heat rates entering, q_{in} ($x = 0$), and leaving, q_{out} ($x = 1$ m), the wall.
2. Rate of change of energy storage in the wall, $\dot{E}_{st}$.
3. Time rate of temperature change at $x = 0$, 0.25, and 0.5 m.

Schematic:



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Assumptions:

1. One-dimensional conduction in the x direction.
2. Isotropic medium with constant properties.
3. Uniform internal heat generation, $\dot{q}$ (W/m^3).

Analysis:

1. Recall that once the temperature distribution is known for a medium, it is a simple matter to determine the conduction heat transfer rate at any point in the medium, or at its surfaces, by using Fourier's law. Hence the desired heat rates may be determined by using the prescribed temperature distribution with Equation 2.1. Accordingly,

$$q_{in} = q_x(0) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=0} = -kA(b + 2cx)_{x=0}$$

$$q_{in} = -bkA = 300°C/m \times 40\ W/m \cdot K \times 10\ m^2 = 120\ kW \quad \triangleleft$$

Similarly,

$$q_{out} = q_x(L) = -kA\left.\frac{\partial T}{\partial x}\right|_{x=L} = -kA(b + 2cx)_{x=L}$$

$$q_{out} = -(b + 2cL)kA = -[-300°C/m + 2(-50°C/m^2) \times 1\ m] \times 40\ W/m \cdot K \times 10\ m^2 = 160\ kW \quad \triangleleft$$

2. The rate of change of energy storage in the wall $\dot{E}_{st}$ may be determined by applying an overall energy balance to the wall. Using Equation 1.11c for a control volume about the wall,

$$\dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

where $\dot{E}_g = \dot{q}AL$, it follows that

$$\dot{E}_{st} = \dot{E}_{in} + \dot{E}_g - \dot{E}_{out} = q_{in} + \dot{q}AL - q_{out}$$

$$\dot{E}_{st} = 120\ kW + 1000\ W/m^3 \times 10\ m^2 \times 1\ m - 160\ kW$$

$$\dot{E}_{st} = -30\ kW \quad \triangleleft$$

3. The time rate of change of the temperature at any point in the medium may be determined from the heat equation, Equation 2.19, rewritten as

$$\frac{\partial T}{\partial t} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{\rho c_p}$$

From the prescribed temperature distribution, it follows that

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right)$$

$$= \frac{\partial}{\partial x}(b + 2cx) = 2c = 2(-50°C/m^2) = -100°C/m^2$$

2.4 Boundar

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Note that this derivative is independent of position in the medium. Hence the time rate of temperature change is also independent of position and is given by

$$\frac{\partial T}{\partial t} = \frac{40\ \text{W/m} \cdot \text{K}}{1600\ \text{kg/m}^3 \times 4\ \text{kJ/kg} \cdot \text{K}} \times (-100°\text{C/m}^2) + \frac{1000\ \text{W/m}^3}{1600\ \text{kg/m}^3 \times 4\ \text{kJ/kg} \cdot \text{K}}$$

$$\frac{\partial T}{\partial t} = -6.25 \times 10^{-4}°\text{C/s} + 1.56 \times 10^{-4}°\text{C/s}$$

$$= -4.69 \times 10^{-4}°\text{C/s} \quad \triangleleft$$

Comments:

1. From the above result it is evident that the temperature at every point within the wall is decreasing with time.
2. Fourier's law can always be used to compute the conduction heat rate from knowledge of the temperature distribution, even for unsteady conditions with internal heat generation.

Microscale Effects For most practical situations, the heat diffusion equations generated in this text may be used with confidence. However, these equations are based on use of Fourier's law to describe the conduction effects, which does not account for the finite speed at which thermal information is propagated within the medium by the various energy carriers. The consequences of the finite propagation speed may be neglected if the heat transfer events of interest occur over a sufficiently long time scale, Δt, such that

$$\frac{\lambda_{\text{mfp}}}{\bar{c}\Delta t} \ll 1 \tag{2.28}$$

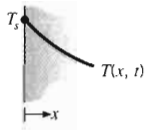
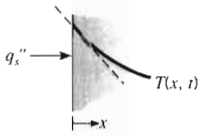
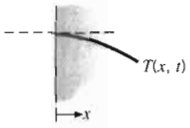
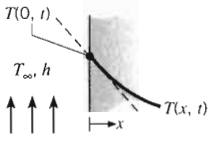
The heat diffusion equations of this text are likewise invalid for problems where boundary scattering must be explicitly considered. For example, the temperature distribution *within* the thin film of Figure 2.6*b* cannot be determined by applying the foregoing heat diffusion equations. Additional discussion of micro- and nanoscale heat transfer applications and analysis methods is available in the literature [1, 15].

2.4 *Boundary and Initial Conditions*

To determine the temperature distribution in a medium, it is necessary to solve the appropriate form of the heat equation. However, such a solution depends on the physical conditions existing at the *boundaries* of the medium and, if the situation is time dependent, on conditions existing in the medium at some *initial time*. With regard to the *boundary conditions,* there are several common possibilities that are

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TABLE 2.2 Boundary conditions for the heat diffusion equation at the surface ($x = 0$)

1. Constant surface temperature		
$T(0, t) = T_s$	(2.29)	
2. Constant surface heat flux		
(a) Finite heat flux		
$-k\left.\frac{\partial T}{\partial x}\right\|_{x=0} = q_s''$	(2.30)	
(b) Adiabatic or insulated surface		
$\left.\frac{\partial T}{\partial x}\right\|_{x=0} = 0$	(2.31)	
3. Convection surface condition		
$-k\left.\frac{\partial T}{\partial x}\right\|_{x=0} = h[T_\infty - T(0, t)]$	(2.32)	

simply expressed in mathematical form. Because the heat equation is second order in the spatial coordinates, two boundary conditions must be expressed for each coordinate needed to describe the system. Because the equation is first order in time, however, only one condition, termed the *initial condition*, must be specified.

Three kinds of boundary conditions commonly encountered in heat transfer are summarized in Table 2.2. The conditions are specified at the surface $x = 0$ for a one-dimensional system. Heat transfer is in the positive x direction with the temperature distribution, which may be time dependent, designated as $T(x, t)$. The first condition corresponds to a situation for which the surface is maintained at a fixed temperature T_s. It is commonly termed a *Dirichlet condition*, or a boundary condition of the *first kind*. It is closely approximated, for example, when the surface is in contact with a melting solid or a boiling liquid. In both cases there is heat transfer at the surface, while the surface remains at the temperature of the phase change process. The second condition corresponds to the existence of a fixed or constant heat flux q_s'' at the surface. This heat flux is related to the temperature gradient at the surface by Fourier's law, Equation 2.6, which may be expressed as

$$q_x''(0) = -k\left.\frac{\partial T}{\partial x}\right|_{x=0} = q_s''$$

It is termed a *Neumann condition*, or a boundary condition of the *second kind*, and may be realized by bonding a thin film electric heater to the surface. A special case of this condition corresponds to the *perfectly insulated*, or *adiabatic*, surface for

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which $\partial T/\partial x|_{x=0} = 0$. The boundary condition of the *third kind* corresponds to the existence of convection heating (or cooling) at the surface and is obtained from the surface energy balance discussed in Section 1.3.2.

EXAMPLE 2.3

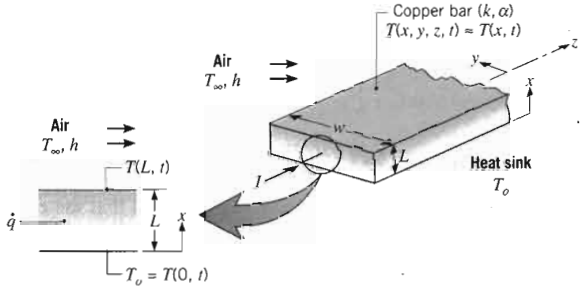
A long copper bar of rectangular cross section, whose width w is much greater than its thickness L, is maintained in contact with a heat sink at its lower surface, and the temperature throughout the bar is approximately equal to that of the sink, T_o. Suddenly, an electric current is passed through the bar and an airstream of temperature T_∞ is passed over the top surface, while the bottom surface continues to be maintained at T_o. Obtain the differential equation and the boundary and initial conditions that could be solved to determine the temperature as a function of position and time in the bar.

SOLUTION

Known: Copper bar initially in thermal equilibrium with a heat sink is suddenly heated by passage of an electric current.

Find: Differential equation and boundary and initial conditions needed to determine temperature as a function of position and time within the bar.

Schematic:



Assumptions:

1. Since the bar is long and $w \gg L$, end and side effects are negligible and heat transfer within the bar is primarily one dimensional in the x direction.
2. Uniform volumetric heat generation, $\dot{q}$.
3. Constant properties.

Analysis: The temperature distribution is governed by the heat equation (2.17), which, for the one-dimensional and constant property conditions of the present problem, reduces to

$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \qquad (1) \quad \triangleleft$$

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where the temperature is a function of position and time, $T(x, t)$. Since this differential equation is second order in the spatial coordinate x and first order in time t, there must be two boundary conditions for the x direction and one condition, termed the initial condition, for time. The boundary condition at the bottom surface corresponds to case 1 of Table 2.2. In particular, since the temperature of this surface is maintained at a value, T_o, which is fixed with time, it follows that

$$T(0, t) = T_o \tag{2}$$ ◁

The convection surface condition, case 3 of Table 2.2, is appropriate for the top surface. Hence

$$-k \left.\frac{\partial T}{\partial x}\right|_{x=L} = h[T(L, t) - T_\infty] \tag{3}$$ ◁

The initial condition is inferred from recognition that, before the change in conditions, the bar is at a uniform temperature T_o. Hence

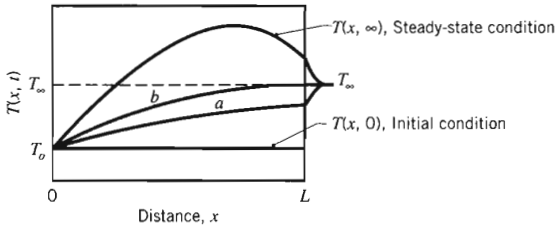
$$T(x, 0) = T_o \tag{4}$$ ◁

If T_o, T_∞, $\dot{q}$, and h are known, Equations 1 through 4 may be solved to obtain the time-varying temperature distribution $T(x, t)$ following imposition of the electric current.

2.5 Summary

Comments:

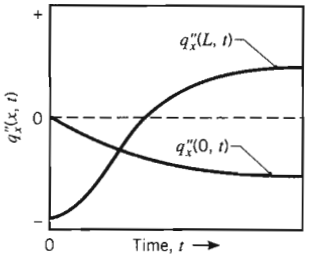
1. The *heat sink* at $x = 0$ could be maintained by exposing the surface to an ice bath or by attaching it to a *cold plate.* A cold plate contains coolant channels machined in a solid of large thermal conductivity (usually copper). By circulating a liquid (usually water) through the channels, the plate, and hence the surface to which it is attached, may be maintained at a nearly uniform temperature.
2. The temperature of the top surface $T(L, t)$ will change with time. This temperature is an unknown and may be obtained after finding $T(x, t)$.
3. We may use our physical intuition to sketch temperature distributions in the bar at selected times from the beginning to the end of the transient process. If we assume that $T_\infty > T_o$ and that the electric current is sufficiently large to heat the bar to temperatures in excess of T_∞, the following distributions would correspond to the initial condition ($t \leq 0$), the final (steady-state) condition ($t \to \infty$), and two intermediate times.



Note how the distributions comply with the initial and boundary conditions. What is a special feature of the distribution labeled (b)?

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4. Our intuition may also be used to infer the manner in which the heat flux varies with time at the surfaces ($x = 0, L$) of the bar. On $q''_x - t$ coordinates, the transient variations are as follows.



Convince yourself that the foregoing variations are consistent with the temperature distributions of Comment 3. For $t \to \infty$, how are $q''_x(0)$ and $q''_x(L)$ related to the volumetric rate of energy generation?

2.5 *Summary*

The primary objectives of this chapter were to improve your understanding of the conduction rate equation (Fourier's law) and to familiarize you with the heat equation. You may test your understanding of related concepts by addressing the following questions.

- In the general formulation of *Fourier's law* (applicable to any geometry), what are the vector and scalar quantities? Why is there a minus sign on the right-hand side of the equation?
- What is an *isothermal surface*? What can be said about the heat flux at any location on this surface?
- What form does *Fourier's law* take for each of the orthogonal directions of Cartesian, cylindrical, and spherical coordinate systems? In each case, what are the units of the temperature gradient? Can you write each equation from memory?
- An important property of matter is defined by *Fourier's law*. What is it? What is its physical significance? What are its units?
- What is an *isotropic* material?
- Why is the thermal conductivity of a solid generally larger than that of a liquid? Why is the thermal conductivity of a liquid larger than that of a gas?
- Why is the thermal conductivity of an electrically conducting solid generally larger than that of a nonconductor? Why are materials such as beryllium oxide, diamond, and silicon carbide (see Table A.2) exceptions to this rule?
- Is the *effective thermal conductivity* of an insulation system a true manifestation of the efficacy with which heat is transferred through the system by conduction alone?
- Why does the thermal conductivity of a gas increase with increasing temperature? Why is it approximately independent of pressure?
- What is the physical significance of the *thermal diffusivity*? How is it defined and what are its units?

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- What is the physical significance of each term appearing in the *heat equation*?
- Cite some examples of *thermal energy generation*. If the rate at which thermal energy is generated per unit volume, $\dot{q}$, varies with location in a medium of volume V, how can the rate of energy generation for the entire medium, $\dot{E}_g$, be determined from knowledge of $\dot{q}(x, y, z)$?
- For a chemically reacting medium, what kind of reaction provides a *source* of thermal energy ($\dot{q} > 0$)? What kind of reaction provides a *sink* for thermal energy ($\dot{q} < 0$)?
- To solve the *heat equation* for the temperature distribution in a medium, *boundary conditions* must be prescribed at the surfaces of the medium. What physical conditions are commonly suitable for this purpose?

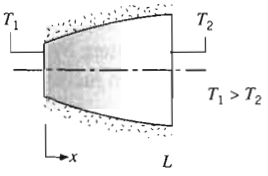
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Problems

Fourier's Law

2.1 Assume steady-state, one-dimensional heat conduction through the axisymmetric shape shown below.



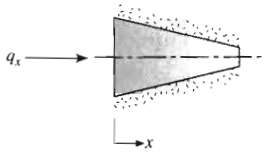
Assuming constant properties and no internal heat generation, sketch the temperature distribution on T–x coordinates. Briefly explain the shape of your curve.

2.2 A hot water pipe with outside radius r_1 has a temperature T_1. A thick insulation applied to reduce the heat loss has an outer radius r_2 and temperature T_2. On T–r coordinates, sketch the temperature distribution in the insulation for one-dimensional, steady-state heat transfer with constant properties. Give a brief explanation, justifying the shape of your curve.

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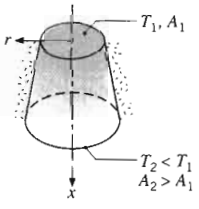
2.3 A spherical shell with inner radius r_1 and outer radius r_2 has surface temperatures T_1 and T_2, respectively, where $T_1 > T_2$. Sketch the temperature distribution on T–r coordinates assuming steady-state, one-dimensional conduction with constant properties. Briefly justify the shape of your curve.

2.4 Assume steady-state, one-dimensional heat conduction through the symmetric shape shown.



Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000$ W, where A is in square meters, T in kelvins, and x in meters.

2.5 A solid, truncated cone serves as a support for a system that maintains the top (truncated) face of the cone at a temperature T_1, while the base of the cone is at a temperature $T_2 < T_1$.



The thermal conductivity of the solid depends on temperature according to the relation $k = k_o - aT$, where a is a positive constant, and the sides of the cone are well insulated. Do the following quantities increase, decrease, or remain the same with increasing x: the heat transfer rate q_x, the heat flux q''_x, the thermal conductivity k, and the temperature gradient dT/dx?

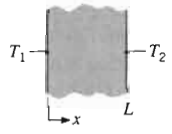
2.6 To determine the effect of the temperature dependence of the thermal conductivity on the temperature distribution in a solid, consider a material for which this dependence may be represented as

$$k = k_o + aT$$

where k_o is a positive constant and a is a coefficient that may be positive or negative. Sketch the steady-state temperature distribution associated with heat transfer in a plane wall for three cases corresponding to $a > 0$, $a = 0$, and $a < 0$.

2.7 A young engineer is asked to design a thermal protection barrier for a sensitive electronic device that might be exposed to irradiation from a high-powered infrared laser. Having learned as a student that a low thermal conductivity material provides good insulating characteristics, the engineer specifies use of a nanostructured aerogel, characterized by a thermal conductivity of $k_a = 0.005$ W/m · K, for the protective barrier. The engineer's boss questions the wisdom of selecting the aerogel *because* it has a low thermal conductivity. Consider the sudden laser irradiation of (a) pure aluminum, (b) glass, and (c) aerogel. The laser provides irradiation of $G = 10 \times 10^6$ W/m^2. The absorptivities of the materials are $\alpha = 0.2$, 0.9, and 0.8 for the aluminum, glass, and aerogel, respectively, and the initial temperature of the barrier is $T_i = 300$ K. Explain why the boss is concerned. *Hint:* All materials experience thermal expansion (or contraction), and local stresses that develop within a material are, to a first approximation, proportional to the local temperature gradient.

2.8 Consider steady-state conditions for one-dimensional conduction in a plane wall having a thermal conductivity $k = 50$ W/m · K and a thickness $L = 0.25$ m, with no internal heat generation.

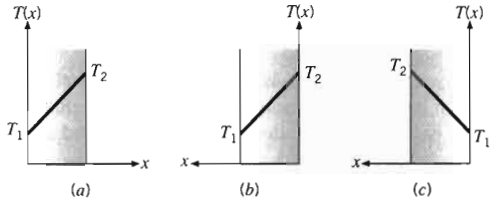


Determine the heat flux and the unknown quantity for each case and sketch the temperature distribution, indicating the direction of the heat flux.

Case	T_1(°C)	T_2(°C)	dT/dx (K/m)
1	50	−20	
2	−30	−10	
3	70		160
4		40	−80
5		30	200

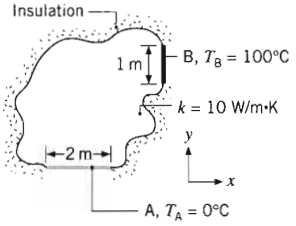
2.9 Consider a plane wall 100 mm thick and of thermal conductivity 100 W/m · K. Steady-state conditions are known to exist with $T_1 = 400$ K and $T_2 = 600$ K. Determine the heat flux q''_x and the temperature gradient dT/dx for the coordinate systems shown.

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2.10 A cylinder of radius r_o, length L, and thermal conductivity k is immersed in a fluid of convection coefficient h and unknown temperature T_∞. At a certain instant the temperature distribution in the cylinder is $T(r) = a + br^2$, where a and b are constants. Obtain expressions for the heat transfer rate at r_o and the fluid temperature.

2.11 In the two-dimensional body illustrated, the gradient at surface A is found to be $\partial T/\partial y = 30$ K/m. What are $\partial T/\partial y$ and $\partial T/\partial x$ at surface B?

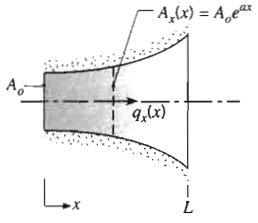


2.12 Sections of the trans-Alaska pipeline run above the ground and are supported by vertical steel shafts ($k = 25$ W/m · K) that are 1 m long and have a cross-sectional area of 0.005 m^2. Under normal operating conditions, the temperature variation along the length of a shaft is known to be governed by an expression of the form

$$T = 100 - 150x + 10x^2$$

where T and x have units of °C and meters, respectively. Temperature variations are small over the shaft cross section. Evaluate the temperature and conduction heat rate at the shaft–pipeline joint ($x = 0$) and at the shaft–ground interface ($x = 1$ m). Explain the difference in the heat rates.

2.13 Steady-state, one-dimensional conduction occurs in a rod of constant thermal conductivity k and variable cross-sectional area $A_x(x) = A_o e^{ax}$, where A_o and a are constants. The lateral surface of the rod is well insulated.



(a) Write an expression for the conduction heat rate, $q_x(x)$. Use this expression to determine the temperature distribution $T(x)$ and qualitatively sketch the distribution for $T(0) > T(L)$.

(b) Now consider conditions for which thermal energy is generated in the rod at a volumetric rate $\dot{q} = \dot{q}_o \exp(-ax)$, where $\dot{q}_o$ is a constant. Obtain an expression for $q_x(x)$ when the left face ($x = 0$) is well insulated.

Thermophysical Properties

2.14 Consider a 300 mm × 300 mm window in an aircraft. For a temperature difference of 80°C from the inner to the outer surface of the window, calculate the heat loss through $L = 10$-mm-thick polycarbonate, soda lime glass, and aerogel windows, respectively. The thermal conductivities of the aerogel and polycarbonate are $k_{ag} = 0.014$ W/m · K and $k_{pc} = 0.21$ W/m · K, respectively. Evaluate the thermal conductivity of the soda lime glass at 300 K. If the aircraft has 130 windows and the cost to heat the cabin air is \$1/kW · h, compare the costs associated with the heat loss through the windows for an 8-hour intercontinental flight.

2.15 Gold is commonly used in semiconductor packaging to form interconnections (also known as interconnects) that carry electrical signals between different devices in the package. In addition to being a good electrical conductor, gold interconnects are also effective at protecting the heat-generating devices to which they are attached by conducting thermal energy away from the devices to surrounding, cooler regions. Consider a thin film of gold that has a cross section of 60 nm × 250 nm.

(a) For an applied temperature difference of 20°C, determine the energy conducted along a 1-μm-long, thin-film interconnect. Evaluate properties at 300 K.

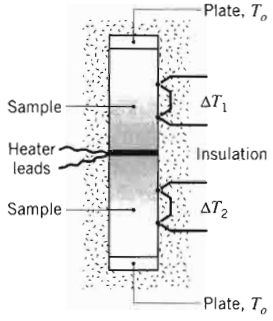
(b) Plot the lengthwise (in the 1-μm direction) and spanwise (in the thinnest direction) thermal conductivities of the gold film as a function of the film thickness, L, for $30 \leq L \leq 140$ nm.

2.16 A TV advertisement by a well-known insulation manufacturer states: it isn't the thickness of the insulating material that counts, it's the R-value. The ad shows that to obtain an R-value of 19, you need 18 ft of rock, 15 in. of wood, or just 6 in. of the manufacturer's insulation. Is this advertisement technically reasonable? If you are like most TV viewers, you don't know the R-value is defined as L/k, where L (in.) is the thickness of the insulation and k (Btu · in./hr · ft^2 · °F) is the thermal conductivity of the material.

2.17 An apparatus for measuring thermal conductivity employs an electrical heater sandwiched between two

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identical samples of diameter 30 mm and length 60 mm, which are pressed between plates maintained at a uniform temperature $T_o = 77°C$ by a circulating fluid. A conducting grease is placed between all the surfaces to ensure good thermal contact. Differential thermocouples are imbedded in the samples with a spacing of 15 mm. The lateral sides of the samples are insulated to ensure one-dimensional heat transfer through the samples.



(a) With two samples of SS316 in the apparatus, the heater draws 0.353 A at 100 V and the differential thermocouples indicate $\Delta T_1 = \Delta T_2 = 25.0°C$. What is the thermal conductivity of the stainless steel sample material? What is the average temperature of the samples? Compare your result with the thermal conductivity value reported for this material in Table A.1.

(b) By mistake, an Armco iron sample is placed in the lower position of the apparatus with one of the SS316 samples from part (a) in the upper portion. For this situation, the heater draws 0.601 A at 100 V and the differential thermocouples indicate $\Delta T_1 = \Delta T_2 = 15.0°C$. What are the thermal conductivity and average temperature of the Armco iron sample?

(c) What is the advantage in constructing the apparatus with two identical samples sandwiching the heater rather than with a single heater–sample combination? When would heat leakage out of the lateral surfaces of the samples become significant? Under what conditions would you expect $\Delta T_1 \neq \Delta T_2$?

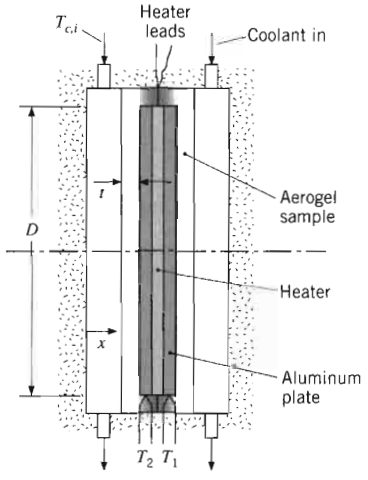
2.18 An engineer desires to measure the thermal conductivity of an aerogel material. It is expected that the aerogel will have an extremely small thermal conductivity.

(a) Explain why the apparatus of Problem 2.17 cannot be used to obtain an accurate measurement of the aerogel's thermal conductivity.

(b) The engineer designs a new apparatus for which an electric heater of diameter $D = 150$ mm is sandwiched between two thin plates of aluminum. The steady-state temperatures of the 5-mm-thick aluminum plates, T_1 and T_2, are measured with thermocouples. Aerogel sheets of thickness $t = 5$ mm are placed outside the aluminum plates, while a coolant with an inlet temperature of $T_{c,i} = 25°C$ maintains the exterior surfaces of the aerogel at a low temperature. The circular aerogel sheets are formed so that they encase the heater and aluminum sheets, providing insulation to minimize radial heat losses. At steady state, $T_1 = T_2 = 55°C$ and the heater draws 125 mA at 10 V. Determine the value of the aerogel thermal conductivity k_a.

(c) Calculate the temperature difference across the thickness of the 5-mm-thick aluminum plates. Comment on whether it is important to know the axial locations at which the temperatures of the aluminum plates are measured.

(d) If liquid water is used as the coolant with a total flow rate of $\dot{m} = 1$ kg/min (0.5 kg/min for each of the two streams), calculate the outlet temperature of the water, $T_{c,o}$.

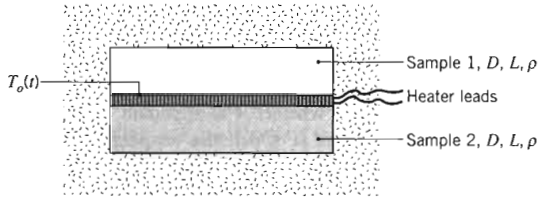


2.19 A method for determining the thermal conductivity k and the specific heat c_p of a material is illustrated in the sketch. Initially the two identical samples of diameter $D = 60$ mm and thickness $L = 10$ mm and the thin heater are at a uniform temperature of $T_i = 23.00°C$, while surrounded by an insulating powder. Suddenly the heater is energized to provide a uniform heat flux q''_o on each of the sample interfaces, and the heat flux is maintained constant for a period of time, Δt_o. A short time after sudden heating is initiated, the temperature at this interface T_o is related to the heat flux as

$$T_o(t) - T_i = 2q''_o\left(\frac{t}{\pi\rho c_p k}\right)^{1/2}$$

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For a particular test run, the electrical heater dissipates 15.0 W for a period of $\Delta t_o = 120$ s and the temperature at the interface is $T_o(30\text{ s}) = 24.57°\text{C}$ after 30 s of heating. A long time after the heater is deenergized, $t \gg \Delta t_o$, the samples reach the uniform temperature of $T_o(\infty) = 33.50°\text{C}$. The density of the sample materials, determined by measurement of volume and mass, is $\rho = 3965\ \text{kg/m}^3$.



Determine the specific heat and thermal conductivity of the test material. By looking at values of the thermophysical properties in Table A.1 or A.2, identify the test sample material.

The Heat Equation

2.20 At a given instant of time the temperature distribution within an infinite homogeneous body is given by the function

$$T(x, y, z) = x^2 - 2y^2 + z^2 - xy + 2yz$$

Assuming constant properties and no internal heat generation, determine the regions where the temperature changes with time.

2.21 A pan is used to boil water by placing it on a stove, from which heat is transferred at a fixed rate q_o. There are two stages to the process. In Stage 1, the water is taken from its initial (room) temperature T_i to the boiling point, as heat is transferred from the pan by natural convection. During this stage, a constant value of the convection coefficient h may be assumed, while the bulk temperature of the water increases with time, $T_\infty = T_\infty(t)$. In Stage 2, the water has come to a boil, and its temperature remains at a fixed value, $T_\infty = T_b$, as heating continues. Consider a pan bottom of thickness L and diameter D, with a coordinate system corresponding to $x = 0$ and $x = L$ for the surfaces in contact with the stove and water, respectively.

(a) Write the form of the heat equation and the boundary/initial conditions that determine the variation of temperature with position and time, $T(x,t)$, in the pan bottom during Stage 1. Express your result in terms of the parameters q_o, D, L, h, and T_∞, as well as appropriate properties of the pan material.

(b) During Stage 2, the surface of the pan in contact with the water is at a fixed temperature, $T(L, t) = T_L > T_b$. Write the form of the heat equation and boundary conditions that determine the temperature distribution, $T(x)$, in the pan bottom. Express your result in terms of the parameters q_o, D, L, and T_L, as well as appropriate properties of the pan material.

2.22 Uniform internal heat generation at $\dot{q} = 5 \times 10^7\ \text{W/m}^3$ is occurring in a cylindrical nuclear reactor fuel rod of 50-mm diameter, and under steady-state conditions the temperature distribution is of the form $T(r) = a + br^2$, where T is in degrees Celsius and r is in meters, while $a = 800°\text{C}$ and $b = -4.167 \times 10^5°\text{C/m}^2$. The fuel rod properties are $k = 30\ \text{W/m} \cdot \text{K}$, $\rho = 1100\ \text{kg/m}^3$, and $c_p = 800\ \text{J/kg} \cdot \text{K}$.

(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at $r = 25$ mm (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q}_2 = 10^8\ \text{W/m}^3$, what is the initial time rate of temperature change at $r = 0$ and $r = 25$ mm?

2.23 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity $50\ \text{W/m} \cdot \text{K}$ and thickness 50 mm is observed to be $T(°\text{C}) = a + bx^2$, where $a = 200°\text{C}$, $b = -2000°\text{C/m}^2$, and x is in meters.

(a) What is the heat generation rate $\dot{q}$ in the wall?

(b) Determine the heat fluxes at the two wall faces. In what manner are these heat fluxes related to the heat generation rate?

2.24 The temperature distribution across a wall 0.3 m thick at a certain instant of time is $T(x) = a + bx + cx^2$, where T is in degrees Celsius and x is in meters, $a = 200°\text{C}$, $b = -200°\text{C/m}$, and $c = 30°\text{C/m}^2$. The wall has a thermal conductivity of $1\ \text{W/m} \cdot \text{K}$.

(a) On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

(b) If the cold surface is exposed to a fluid at 100°C, what is the convection coefficient?

2.25 A plane wall of thickness $2L = 40$ mm and thermal conductivity $k = 5\ \text{W/m} \cdot \text{K}$ experiences uniform volumetric heat generation at a rate $\dot{q}$, while convection heat transfer occurs at both of its surfaces ($x = -L, +L$), each of which is exposed to a fluid of temperature $T_\infty = 20°\text{C}$. Under steady-state conditions, the temperature distribution in the wall is of the form $T(x) = a + bx + cx^2$, where $a = 82.0°\text{C}$, $b = -210°\text{C/m}$, $c = -2 \times 10^4°\text{C/m}^2$, and x is in meters. The origin of the x-coordinate is at the midplane of the wall.

(a) Sketch the temperature distribution and identify significant physical features.

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(b) What is the volumetric rate of heat generation $\dot{q}$ in the wall?

(c) Determine the surface heat fluxes, $q''_x(-L)$ and $q''_x(+L)$. How are these fluxes related to the heat generation rate?

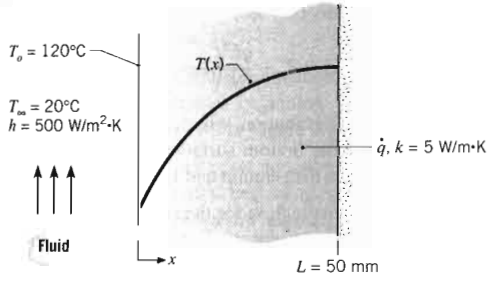
(d) What are the convection coefficients for the surfaces at $x = -L$ and $x = +L$?

(e) Obtain an expression for the heat flux distribution, $q''_x(x)$. Is the heat flux zero at any location? Explain any significant features of the distribution.

(f) If the source of the heat generation is suddenly deactivated ($\dot{q} = 0$), what is the rate of change of energy stored in the wall at this instant?

(g) What temperature will the wall eventually reach with $\dot{q} = 0$? How much energy must be removed by the fluid per unit area of the wall (J/m^2) to reach this state? The density and specific heat of the wall material are 2600 kg/m^3 and 800 J/kg · K, respectively.

2.26 One-dimensional, steady-state conduction with uniform internal energy generation occurs in a plane wall with a thickness of 50 mm and a constant thermal conductivity of 5 W/m · K. For these conditions, the temperature distribution has the form, $T(x) = a + bx + cx^2$. The surface at $x = 0$ has a temperature of $T(0) \equiv T_o = 120°C$ and experiences convection with a fluid for which $T_\infty = 20°C$ and $h = 500$ W/m^2 · K. The surface at $x = L$ is well insulated.



(a) Applying an overall energy balance to the wall, calculate the internal energy generation rate, $\dot{q}$.

(b) Determine the coefficients a, b, and c by applying the boundary conditions to the prescribed temperature distribution. Use the results to calculate and plot the temperature distribution.

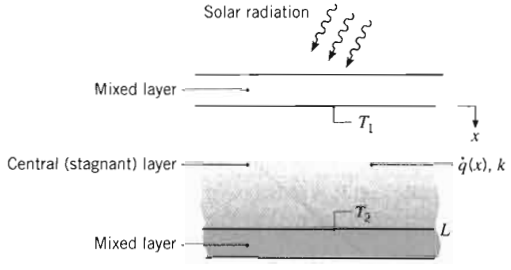
(c) Consider conditions for which the convection coefficient is halved, but the internal energy generation rate remains unchanged. Determine the new values of a, b, and c, and use the results to plot the temperature distribution. *Hint:* recognize that $T(0)$ is no longer 120°C.

(d) Under conditions for which the internal energy generation rate is doubled, and the convection coefficient remains unchanged ($h = 500$ W/m^2 · K), determine the new values of a, b, and c and plot the corresponding temperature distribution. Referring to the results of parts (b), (c), and (d) as Cases 1, 2, and 3, respectively, compare the temperature distributions for the three cases and discuss the effects of h and $\dot{q}$ on the distributions.

2.27 A salt-gradient solar pond is a shallow body of water that consists of three distinct fluid layers and is used to collect solar energy. The upper- and lower-most layers are well mixed and serve to maintain the upper and lower surfaces of the central layer at uniform temperatures T_1 and T_2, where $T_2 > T_1$. Although there is bulk fluid motion in the mixed layers, there is no such motion in the central layer. Consider conditions for which solar radiation absorption in the central layer provides nonuniform heat generation of the form $\dot{q} = Ae^{-ax}$, and the temperature distribution in the central layer is

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

The quantities A (W/m^3), a (1/m), B (K/m), and C (K) are known constants having the prescribed units, and k is the thermal conductivity, which is also constant.



(a) Obtain expressions for the rate at which heat is transferred per unit area from the lower mixed layer to the central layer and from the central layer to the upper mixed layer.

(b) Determine whether conditions are steady or transient.

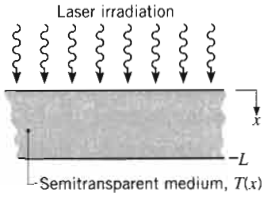
(c) Obtain an expression for the rate at which thermal energy is generated in the entire central layer, per unit surface area.

2.28 The steady-state temperature distribution in a semitransparent material of thermal conductivity k and thickness L exposed to laser irradiation is of the form

$$T(x) = -\frac{A}{ka^2}e^{-ax} + Bx + C$$

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where A, a, B, and C are known constants. For this situation, radiation absorption in the material is manifested by a distributed heat generation term, $\dot{q}(x)$.



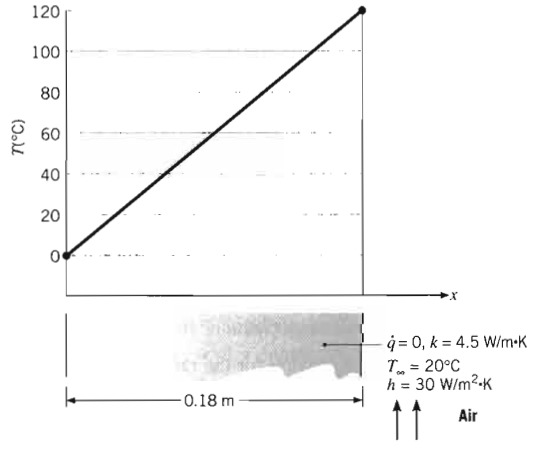
(a) Obtain expressions for the conduction heat fluxes at the front and rear surfaces.

(b) Derive an expression for $\dot{q}(x)$.

(c) Derive an expression for the rate at which radiation is absorbed in the entire material, per unit surface area. Express your result in terms of the known constants for the temperature distribution, the thermal conductivity of the material, and its thickness.

2.29 The steady-state temperature distribution in a one-dimensional wall of thermal conductivity k and thickness L is of the form $T = ax^3 + bx^2 + cx + d$. Derive expressions for the heat generation rate per unit volume in the wall and the heat fluxes at the two wall faces ($x = 0$, L).

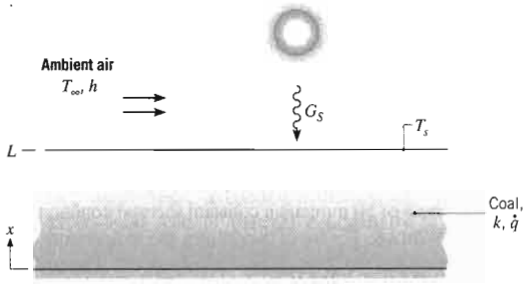
2.30 One-dimensional, steady-state conduction with no internal energy generation is occurring in a plane wall of constant thermal conductivity.



(a) Is the prescribed temperature distribution possible? Briefly explain your reasoning.

(b) With the temperature at $x = 0$ and the fluid temperature fixed at $T(0) = 0°C$ and $T_\infty = 20°C$, respectively, compute and plot the temperature at $x = L$, $T(L)$, as a function of h for $10 \le h \le 100$ W/m$^2 \cdot$ K. Briefly explain your results.

2.31 A plane layer of coal of thickness $L = 1$ m experiences uniform volumetric generation at a rate of $\dot{q} = 20$ W/m^3 due to slow oxidation of the coal particles. Averaged over a daily period, the top surface of the layer transfers heat by convection to ambient air for which $h = 5$ W/m$^2 \cdot$ K and $T_\infty = 25°C$, while receiving solar irradiation in the amount $G_S = 400$ W/m^2. Irradiation from the atmosphere may be neglected. The solar absorptivity and emissivity of the surface are each $\alpha_S = \varepsilon = 0.95$.



(a) Write the steady-state form of the heat diffusion equation for the layer of coal. Verify that this equation is satisfied by a temperature distribution of the form

$$T(x) = T_s + \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right)$$

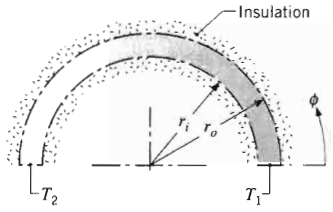
From this distribution, what can you say about conditions at the bottom surface ($x = 0$)? Sketch the temperature distribution and label key features.

(b) Obtain an expression for the rate of heat transfer by conduction per unit area at $x = L$. Applying an energy balance to a control surface about the top surface of the layer, obtain an expression for T_s. Evaluate T_s and $T(0)$ for the prescribed conditions.

(c) Daily average values of G_S and h depend on a number of factors such as time of year, cloud cover, and wind conditions. For $h = 5$ W/m$^2 \cdot$ K, compute and plot T_s and $T(0)$ as a function of G_S for $50 \le G_S \le 500$ W/m^2. For $G_S = 400$ W/m^2, compute and plot T_s and $T(0)$ as a function of h for $5 \le h \le 50$ W/m$^2 \cdot$ K.

2.32 The cylindrical system illustrated has negligible variation of temperature in the r and z directions. Assume that $\Delta r = r_o - r_i$ is small compared to r_i and denote the length in the z direction, normal to the page, as L.

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(a) Beginning with a properly defined control volume and considering energy generation and storage effects, derive the differential equation that prescribes the variation in temperature with the angular coordinate ϕ. Compare your result with Equation 2.24.

(b) For steady-state conditions with no internal heat generation and constant properties, determine the temperature distribution $T(\phi)$ in terms of the constants T_1, T_2, r_i, and r_o. Is this distribution linear in ϕ?

(c) For the conditions of part (b) write the expression for the heat rate q_ϕ.

2.33 Beginning with a differential control volume in the form of a cylindrical shell, derive the heat diffusion equation for a one-dimensional, cylindrical, radial coordinate system with internal heat generation. Compare your result with Equation 2.24.

2.34 Beginning with a differential control volume in the form of a spherical shell, derive the heat diffusion equation for a one-dimensional, spherical, radial coordinate system with internal heat generation. Compare your result with Equation 2.27.

2.35 Derive the heat diffusion equation, Equation 2.24, for cylindrical coordinates beginning with the differential control volume shown in Figure 2.12.

2.36 Derive the heat diffusion equation, Equation 2.27, for spherical coordinates beginning with the differential control volume shown in Figure 2.13.

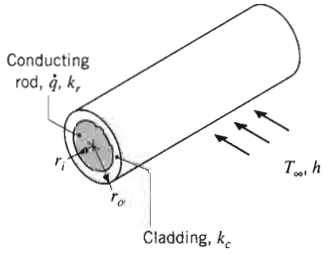
2.37 A steam pipe is wrapped with insulation of inner and outer radii, r_i and r_o, respectively. At a particular instant the temperature distribution in the insulation is known to be of the form

$$T(r) = C_1 \ln\left(\frac{r}{r_o}\right) + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.38 For a long circular tube of inner and outer radii r_1 and r_2, respectively, uniform temperatures T_1 and T_2 are maintained at the inner and outer surfaces, while thermal energy generation is occurring within the tube wall ($r_1 < r < r_2$). Consider steady-state conditions for which $T_1 > T_2$. Is it possible to maintain a *linear* radial temperature distribution in the wall? If so, what special conditions must exist?

2.39 Passage of an electric current through a long conducting rod of radius r_i and thermal conductivity k_r results in uniform volumetric heating at a rate of $\dot{q}$. The conducting rod is wrapped in an electrically nonconducting cladding material of outer radius r_o and thermal conductivity k_c, and convection cooling is provided by an adjoining fluid.



For steady-state conditions, write appropriate forms of the heat equations for the rod and cladding. Express appropriate boundary conditions for the solution of these equations.

2.40 Two-dimensional, steady-state conduction occurs in a hollow cylindrical solid of thermal conductivity $k = 16\ \text{W/m} \cdot \text{K}$, outer radius $r_o = 1$ m, and overall length $2z_o = 5$ m, where the origin of the coordinate system is located at the midpoint of the centerline. The inner surface of the cylinder is insulated, and the temperature distribution within the cylinder has the form $T(r, z) = a + br^2 + c \ln r + dz^2$, where $a = 20°\text{C}$, $b = 150°\text{C/m}^2$, $c = -12°\text{C}$, $d = -300°\text{C/m}^2$ and r and z are in meters.

(a) Determine the inner radius r_i of the cylinder.

(b) Obtain an expression for the volumetric rate of heat generation, $\dot{q}(\text{W/m}^3)$.

(c) Determine the axial distribution of the heat flux at the outer surface, $q_r''(r_o, z)$. What is the heat rate at the outer surface? Is it into or out of the cylinder?

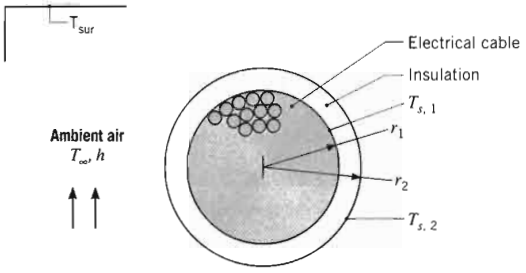
(d) Determine the radial distribution of the heat flux at the end faces of the cylinder, $q_z''(r, +z_o)$ and $q_z''(r, -z_o)$. What are the corresponding heat rates? Are they into or out of the cylinder?

(e) Verify that your results are consistent with an overall energy balance on the cylinder.

2.41 An electric cable of radius r_1 and thermal conductivity k_c is enclosed by an insulating sleeve whose outer surface is of radius r_2 and experiences convection heat transfer and radiation exchange with the adjoining air and large surroundings, respectively. When electric current passes

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through the cable, thermal energy is generated within the cable at a volumetric rate $\dot{q}$.



(a) Write the steady-state forms of the heat diffusion equation for the insulation and the cable. Verify that these equations are satisfied by the following temperature distributions:

$$\textit{Insulation: } T(r) = T_{s,2} + (T_{s,1} - T_{s,2})\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$$

$$\textit{Cable: } T(r) = T_{s,1} + \frac{\dot{q}r_1^2}{4k_c}\left(1 - \frac{r^2}{r_1^2}\right)$$

Sketch the temperature distribution, $T(r)$, in the cable and the sleeve, labeling key features.

(b) Applying Fourier's law, show that the rate of conduction heat transfer per unit length through the sleeve may be expressed as

$$q_r' = \frac{2\pi k_s(T_{s,1} - T_{s,2})}{\ln(r_2/r_1)}$$

Applying an energy balance to a control surface placed around the cable, obtain an alternative expression for q_r', expressing your result in terms of $\dot{q}$ and r_1.

(c) Applying an energy balance to a control surface placed around the outer surface of the sleeve, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) Consider conditions for which 250 A are passing through a cable having an electric resistance per unit length of $R_e' = 0.005\ \Omega/\text{m}$, a radius of $r_1 = 15$ mm, and a thermal conductivity of $k_c = 200\ \text{W/m} \cdot \text{K}$. For $k_s = 0.15\ \text{W/m} \cdot \text{K}$, $r_2 = 15.5$ mm, $h = 25\ \text{W/m}^2 \cdot \text{K}$, $\varepsilon = 0.9$, $T_\infty = 25°\text{C}$, and $T_{sur} = 35°\text{C}$, evaluate the surface temperatures, $T_{s,1}$ and $T_{s,2}$, as well as the temperature T_o at the centerline of the cable.

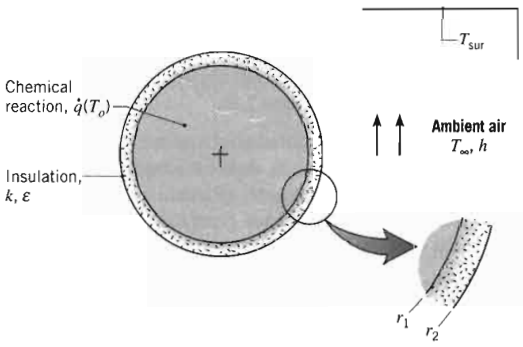
(e) With all other conditions remaining the same, compute and plot T_o, $T_{s,1}$, and $T_{s,2}$ as a function of r_2 for $15.5 \le r_2 \le 20$ mm.

2.42 A spherical shell of inner and outer radii r_i and r_o, respectively, contains heat-dissipating components, and at a particular instant the temperature distribution in the shell is known to be of the form

$$T(r) = \frac{C_1}{r} + C_2$$

Are conditions steady-state or transient? How do the heat flux and heat rate vary with radius?

2.43 A chemically reacting mixture is stored in a thin-walled spherical container of radius $r_1 = 200$ mm, and the exothermic reaction generates heat at a uniform, but temperature-dependent volumetric rate of $\dot{q} = \dot{q}_o \exp(-A/T_o)$, where $\dot{q}_o = 5000\ \text{W/m}^3$, $A = 75$ K, and T_o is the mixture temperature in kelvins. The vessel is enclosed by an insulating material of outer radius r_2, thermal conductivity k, and emissivity ε. The outer surface of the insulation experiences convection heat transfer and net radiation exchange with the adjoining air and large surroundings, respectively.



(a) Write the steady-state form of the heat diffusion equation for the insulation. Verify that this equation is satisfied by the temperature distribution

$$T(r) = T_{s,1} - (T_{s,1} - T_{s,2})\left[\frac{1 - (r_1/r)}{1 - (r_1/r_2)}\right]$$

Sketch the temperature distribution, $T(r)$, labeling key features.

(b) Applying Fourier's law, show that the rate of heat transfer by conduction through the insulation may be expressed as

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)}$$

Applying an energy balance to a control surface about the container, obtain an alternative expression for q_r, expressing your result in terms of $\dot{q}$ and r_1.

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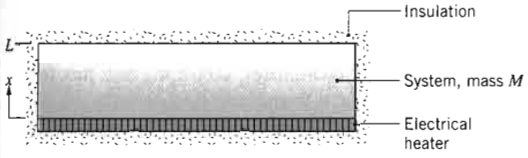
(c) Applying an energy balance to a control surface placed around the outer surface of the insulation, obtain an expression from which $T_{s,2}$ may be determined as a function of $\dot{q}$, r_1, h, T_∞, ε, and T_{sur}.

(d) The process engineer wishes to maintain a reactor temperature of $T_o = T(r_1) = 95°C$ under conditions for which $k = 0.05\ W/m \cdot K$, $r_2 = 208\ mm$, $h = 5\ W/m^2 \cdot K$, $\varepsilon = 0.9$, $T_\infty = 25°C$, and $T_{sur} = 35°C$. What is the actual reactor temperature and the outer surface temperature of the insulation, $T_{s,2}$?

(e) Compute and plot the variation of $T_{s,2}$ with r_2 for $201 \le r_2 \le 210\ mm$. The engineer is concerned about potential burn injuries to personnel who may come into contact with the exposed surface of the insulation. Is increasing the insulation thickness a practical solution to maintaining $T_{s,2} \le 45°C$? What other parameter could be varied to reduce $T_{s,2}$?

Graphical Representations

2.44 The one-dimensional system of mass M with constant properties and no internal heat generation shown in the figure is initially at a uniform temperature T_i. The electrical heater is suddenly energized providing a uniform heat flux q_o'' at the surface $x = 0$. The boundaries at $x = L$ and elsewhere are perfectly insulated.



(a) Write the differential equation and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the system.

(b) On $T - x$ coordinates, sketch the temperature distributions for the initial condition ($t \le 0$) and for several times after the heater is energized. Will a steady-state temperature distribution ever be reached?

(c) On $q_x''-t$ coordinates, sketch the heat flux $q_x''(x,t)$ at the planes $x = 0$, $x = L/2$, and $x = L$ as a function of time.

(d) After a period of time t_e has elapsed, the heater power is switched off. Assuming that the insulation is perfect, the system will eventually reach a final uniform temperature T_f. Derive an expression that can be used to determine T_f as a function of the parameters q_o'', t_e, T_i, and the system characteristics M, c_p, and A_s (the heater surface area).

2.45 A large plate of thickness $2L$ is at a uniform temperature of $T_i = 200°C$, when it is suddenly quenched by dipping it in a liquid bath of temperature $T_\infty = 20°C$. Heat transfer to the liquid is characterized by the convection coefficient h.

(a) If $x = 0$ corresponds to the midplane of the wall, on $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \le 0$), steady-state condition ($t \to \infty$), and two intermediate times.

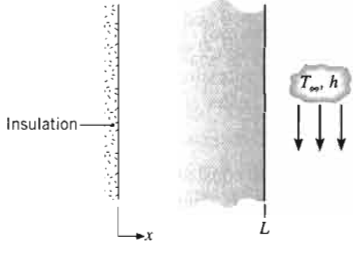
(b) On $q_x'' - t$ coordinates, sketch the variation with time of the heat flux at $x = L$.

(c) If $h = 100\ W/m^2 \cdot K$, what is the heat flux at $x = L$ and $t = 0$? If the wall has a thermal conductivity of $k = 50\ W/m \cdot K$, what is the corresponding temperature gradient at $x = L$?

(d) Consider a plate of thickness $2L = 20\ mm$ with a density of $\rho = 2770\ kg/m^3$ and a specific heat $c_p = 875\ J/kg \cdot K$. By performing an energy balance on the plate, determine the amount of energy per unit surface area of the plate (J/m^2) that is transferred to the bath over the time required to reach steady-state conditions.

(e) From other considerations, it is known that, during the quenching process, the heat flux at $x = +L$ and $x = -L$ decays exponentially with time according to the relation, $q'' = A \exp(-Bt)$, where t is in seconds, $A = 1.80 \times 10^4\ W/m^2$, and $B = 4.126 \times 10^{-3}\ s^{-1}$. Use this information to determine the energy per unit surface area of the plate that is transferred to the fluid during the quenching process.

2.46 The plane wall with constant properties and no internal heat generation shown in the figure is initially at a uniform temperature T_i. Suddenly the surface at $x = L$ is heated by a fluid at T_∞ having a convection heat transfer coefficient h. The boundary at $x = 0$ is perfectly insulated.



(a) Write the differential equation and identify the boundary and initial conditions that could be used to determine the temperature as a function of position and time in the wall.

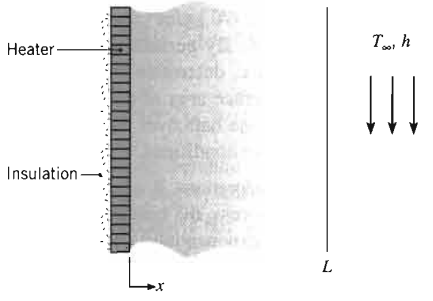
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(b) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \to \infty$), and two intermediate times.

(c) On $q''_x - t$ coordinates, sketch the heat flux at the locations $x = 0$ and $x = L$. That is, show qualitatively how $q''_x(0, t)$ and $q''_x(L, t)$ vary with time.

(d) Write an expression for the total energy transferred to the wall per unit volume of the wall (J/m^3).

2.47 A plane wall has constant properties, no internal heat generation, and is initially at a uniform temperature T_i. Suddenly, the surface at $x = L$ is heated by a fluid at T_∞ having a convection coefficient h. At the same instant, the electrical heater is energized, providing a constant heat flux q''_o at $x = 0$.



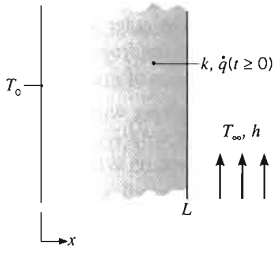
(a) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \to \infty$), and for two intermediate times.

(b) On $q''_x - x$ coordinates, sketch the heat flux corresponding to the four temperature distributions of part (a).

(c) On $q''_x - t$ coordinates, sketch the heat flux at the locations $x = 0$ and $x = L$. That is, show qualitatively how $q''_x(0, t)$ and $q''_x(L, t)$ vary with time.

(d) Derive an expression for the steady-state temperature at the heater surface, $T(0, \infty)$, in terms of q''_o, T_∞, k, h, and L.

2.48 A plane wall with constant properties is initially at a uniform temperature T_o. Suddenly, the surface at $x = L$ is exposed to a convection process with a fluid at T_∞ ($> T_o$) having a convection coefficient h. Also, suddenly the wall experiences a uniform internal volumetric heating $\dot{q}$ that is sufficiently large to induce a maximum steady-state temperature within the wall, which exceeds that of the fluid. The boundary at $x = 0$ remains at T_o.



(a) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \to \infty$), and for two intermediate times. Show also the distribution for the special condition when there is no heat flow at the $x = L$ boundary.

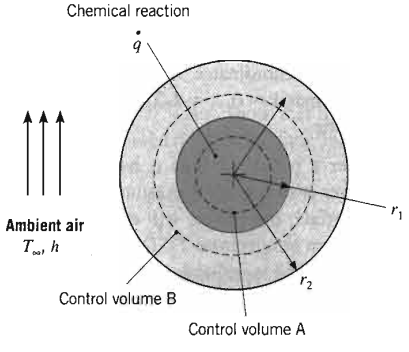
(b) On $q''_x - t$ coordinates, sketch the heat flux for the locations $x = 0$ and $x = L$, that is, $q''_x(0, t)$ and $q''_x(L, t)$, respectively.

2.49 Consider the conditions associated with Problem 2.48, but now with a convection process for which $T_\infty < T_o$.

(a) On $T - x$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \to \infty$), and for two intermediate times. Identify key features of the distributions, especially the location of the maximum temperature and the temperature gradient at $x = L$.

(b) On $q'' - t$ coordinates, sketch the heat flux for the locations $x = 0$ and $x = L$, that is, $q''_x(0, t)$ and $q''_x(L, t)$, respectively. Identify key features of the flux histories.

2.50 A spherical particle of radius r_1 experiences uniform thermal generation at a rate of $\dot{q}$. The particle is encapsulated by a spherical shell of outside radius r_2 that is cooled by ambient air. The thermal conductivities of the particle and shell are k_1 and k_2, respectively, where $k_1 = 2k_2$.



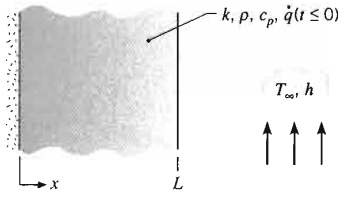
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(a) By applying the conservation of energy principle to spherical control volume A, which is placed at an arbitrary location within the sphere, determine a relationship between the temperature gradient, dT/dr, and the local radius, r, for $0 \le r \le r_1$.

(b) By applying the conservation of energy principle to spherical control volume B, which is placed at an arbitrary location within the spherical shell, determine a relationship between the temperature gradient, dT/dr, and the local radius, r, for $r_1 \le r \le r_2$.

(c) On $T - r$ coordinates, sketch the temperature distribution over the range $0 \le r \le r_2$.

2.51 A plane wall of thickness $L = 0.1$ m experiences uniform volumetric heating at a rate $\dot{q}$. One surface of the wall ($x = 0$) is insulated, while the other surface is exposed to a fluid at $T_\infty = 20°C$, with convection heat transfer characterized by $h = 1000$ W/m$^2 \cdot$ K. Initially, the temperature distribution in the wall is $T(x,0) = a + bx^2$, where $a = 300°C$, $b = -1.0 \times 10^4$°C/m^2 and x is in meters. Suddenly, the volumetric heat generation is deactivated ($\dot{q} = 0$ for $t \ge 0$), while convection heat transfer continues to occur at $x = L$. The properties of the wall are $\rho = 7000$ kg/m^3, $c_p = 450$ J/kg $\cdot$ K, and $k = 90$ W/m $\cdot$ K.



(a) Determine the magnitude of the volumetric energy generation rate $\dot{q}$ associated with the initial condition ($t < 0$).

(b) On $T - x$ coordinates, sketch the temperature distribution for the following conditions: initial condition ($t < 0$), steady-state condition ($t \to \infty$), and two intermediate conditions.

(c) On $q''_x - t$ coordinates, sketch the variation with time of the heat flux at the boundary exposed to the convection process, $q''_x(L, t)$. Calculate the corresponding value of the heat flux at $t = 0$, $q''_x(L, 0)$.

(d) Calculate the amount of energy removed from the wall per unit area (J/m^2) by the fluid stream as the wall cools from its initial to steady-state condition.

2.52 A plane wall that is insulated on one side ($x = 0$) is initially at a uniform temperature T_i, when its exposed surface at $x = L$ is suddenly raised to a temperature T_s.

(a) Verify that the following equation satisfies the heat equation and boundary conditions:

$$\frac{T(x,t) - T_s}{T_i - T_s} = C_1 \exp\left(-\frac{\pi^2}{4}\frac{\alpha t}{L^2}\right)\cos\left(\frac{\pi}{2}\frac{x}{L}\right)$$

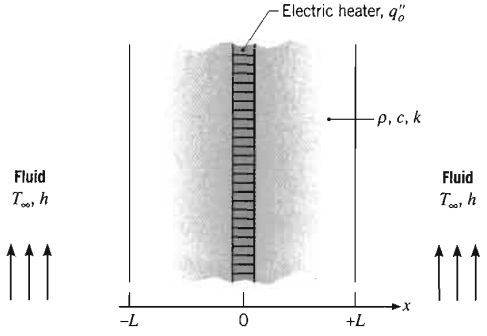
where C_1 is a constant and α is the thermal diffusivity.

(b) Obtain expressions for the heat flux at $x = 0$ and $x = L$.

(c) Sketch the temperature distribution $T(x)$ at $t = 0$, at $t \to \infty$, and at an intermediate time. Sketch the variation with time of the heat flux at $x = L$, $q''_L(t)$.

(d) What effect does α have on the thermal response of the material to a change in surface temperature?

2.53 A thin electrical heater dissipating 4000 W/m^2 is sandwiched between two 25-mm-thick plates whose exposed surfaces experience convection with a fluid for which $T_\infty = 20°C$ and $h = 400$ W/m$^2 \cdot$ K. The thermophysical properties of the plate material are $\rho = 2500$ kg/m^3, $c = 700$ J/kg$\cdot$ K, and $k = 5$ W/m$\cdot$ K.



(a) On $T - x$ coordinates, sketch the steady-state temperature distribution for $-L \le x \le +L$. Calculate values of the temperatures at the surfaces, $x = \pm L$, and the midpoint, $x = 0$. Label this distribution as Case 1, and explain its salient features.

(b) Consider conditions for which there is a loss of coolant and existence of a nearly adiabatic condition on the $x = +L$ surface. On the $T - x$ coordinates used for part (a), sketch the corresponding steady-state temperature distribution and indicate the temperatures at $x = 0, \pm L$. Label the distribution as Case 2, and explain its key features.

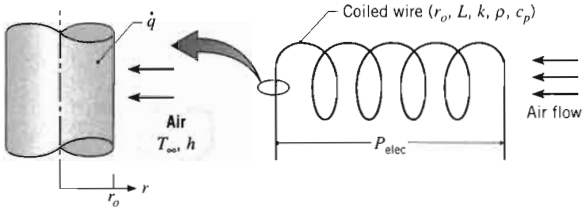
(c) With the system operating as described in part (b), the surface $x = -L$ also experiences a sudden loss of coolant. This dangerous situation goes undetected for 15 minutes, at which time the power to the heater is deactivated. Assuming no heat losses from the surfaces of the plates, what is the eventual ($t \to \infty$), uni-

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form, steady-state temperature distribution in the plates? Show this distribution as Case 3 on your sketch, and explain its key features. *Hint*: Apply the conservation of energy requirement on a time-interval basis, Eq. 1.11b, for the initial and final conditions corresponding to Case 2 and Case 3, respectively.

(d) On $T - t$ coordinates, sketch the temperature history at the plate locations $x = 0, \pm L$ during the transient period between the distributions for Cases 2 and 3. Where and when will the temperature in the system achieve a maximum value?

2.54 Typically, air is heated in a hair dryer by blowing it across a coiled wire through which an electric current is passed. Thermal energy is generated by electric resistance heating within the wire and is transferred by convection from the surface of the wire to the air. Consider conditions for which the wire is initially at room temperature, T_i, and resistance heating is concurrently initiated with air flow at $t = 0$.



(a) For a wire radius r_o, an air temperature T_∞, and a convection coefficient h, write the form of the heat equation and the boundary/initial conditions that govern the transient thermal response, $T(r, t)$, of the wire.

(b) If the length and radius of the wire are 500 mm and 1 mm, respectively, what is the volumetric rate of thermal energy generation for a power consumption of $P_{elec} = 500$ W? What is the convection heat flux under steady-state conditions?

(c) On $T - r$ coordinates, sketch the temperature distributions for the following conditions: initial condition ($t \leq 0$), steady-state condition ($t \to \infty$), and for two intermediate times.

(d) On $q''_r - t$ coordinates, sketch the variation of the heat flux with time for locations at $r = 0$ and $r = r_o$.



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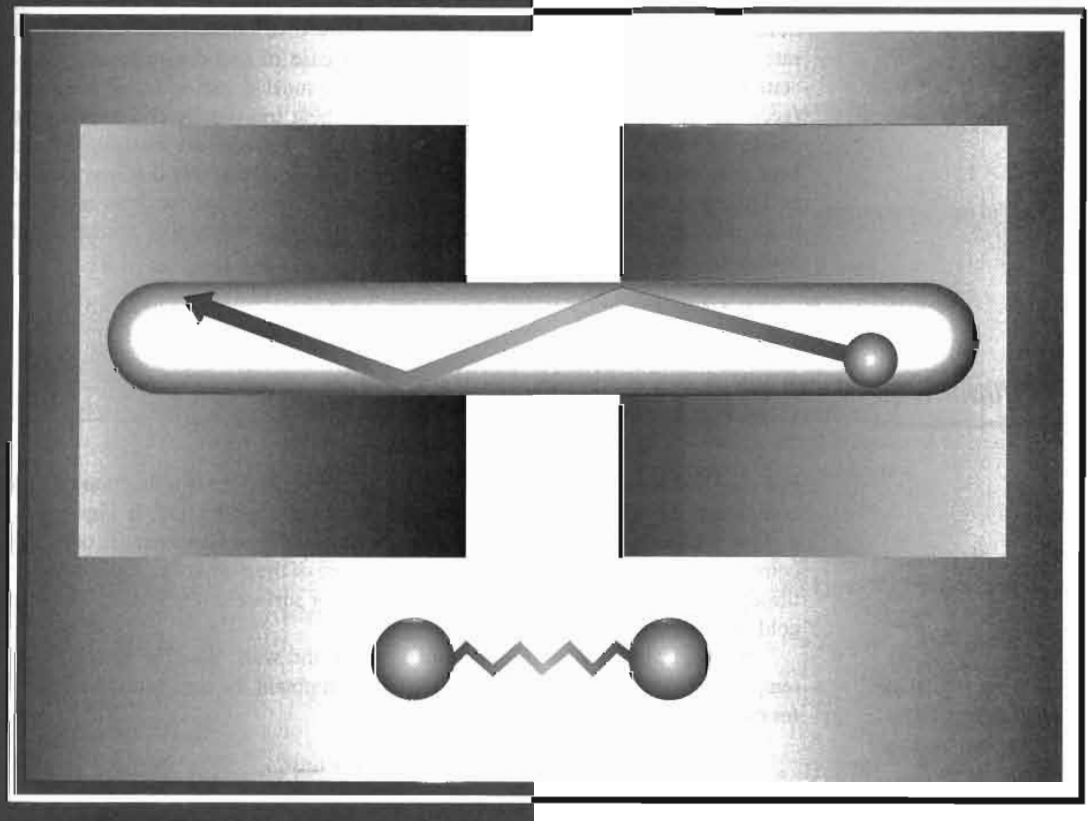
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CHAPTER 3

One-Dimensional, Steady-State Conduction



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In this chapter we treat situations for which heat is transferred by diffusion under *one-dimensional, steady-state* conditions. The term "one-dimensional" refers to the fact that only one coordinate is needed to describe the spatial variation of the dependent variables. Hence, in a *one-dimensional* system, temperature gradients exist along only a single coordinate direction, and heat transfer occurs exclusively in that direction. The system is characterized by *steady-state* conditions if the temperature at each point is independent of time. Despite their inherent simplicity, one-dimensional, steady-state models may be used to accurately represent numerous engineering systems.

We begin our consideration of one-dimensional, steady-state conduction by discussing heat transfer with no internal generation of thermal energy (Sections 3.1 through 3.4). The objective is to determine expressions for the temperature distribution and heat transfer rate in common (planar, cylindrical, and spherical) geometries. For such geometries, an additional objective is to introduce the concept of *thermal resistance* and to show how *thermal circuits* may be used to model heat flow, much as electrical circuits are used for current flow. The effect of internal heat generation is treated in Section 3.5, and again our objective is to obtain expressions for determining temperature distributions and heat transfer rates. In Section 3.6, we consider the special case of one-dimensional, steady-state conduction for *extended surfaces*. In their most common form, these surfaces are termed *fins* and are used to *enhance* heat transfer by convection to an adjoining fluid. In addition to determining related temperature distributions and heat rates, our objective is to introduce *performance parameters* that may be used to determine their efficacy. Finally, in Section 3.7 we present concepts and equations relating to heat transfer within the human body, including the effects of *metabolic heat generation* and *perfusion*.

3.1 *The Plane Wall*

For one-dimensional conduction in a plane wall, temperature is a function of the x coordinate only and heat is transferred exclusively in this direction. In Figure 3.1*a*, a plane wall separates two fluids of different temperatures. Heat transfer occurs by convection from the hot fluid at $T_{\infty,1}$ to one surface of the wall at $T_{s,1}$, by conduction through the wall, and by convection from the other surface of the wall at $T_{s,2}$ to the cold fluid at $T_{\infty,2}$.

We begin by considering conditions *within* the wall. We first determine the temperature distribution, from which we can then obtain the conduction heat transfer rate.

3.1.1 Temperature Distribution

The temperature distribution in the wall can be determined by solving the heat equation with the proper boundary conditions. For steady-state conditions with no

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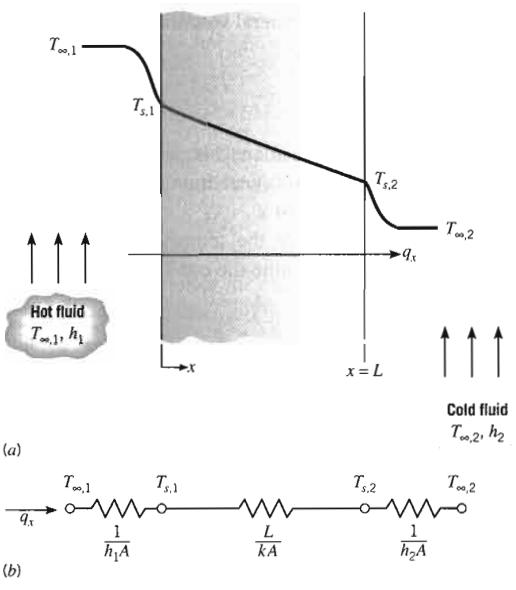


FIGURE 3.1 Heat transfer through a plane wall. (*a*) Temperature distribution. (*b*) Equivalent thermal circuit.

distributed source or sink of energy within the wall, the appropriate form of the heat equation is Equation 2.21

$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) = 0 \tag{3.1}$$

Hence, from Equation 2.2, it follows that, for *one-dimensional, steady-state conduction in a plane wall with no heat generation, the heat flux is a constant, independent of x.* If the thermal conductivity of the wall material is assumed to be constant, the equation may be integrated twice to obtain the *general solution*

$$T(x) = C_1 x + C_2 \tag{3.2}$$

To obtain the constants of integration, C_1 and C_2, boundary conditions must be introduced. We choose to apply conditions of the first kind at $x = 0$ and $x = L$, in which case

$$T(0) = T_{s,1} \qquad \text{and} \qquad T(L) = T_{s,2}$$

Applying the condition at $x = 0$ to the general solution, it follows that

$$T_{s,1} = C_2$$

Similarly, at $x = L$,

$$T_{s,2} = C_1 L + C_2 = C_1 L + T_{s,1}$$

in which case

$$\frac{T_{s,2} - T_{s,1}}{L} = C_1$$

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Substituting into the general solution, the temperature distribution is then

$$T(x) = (T_{s,2} - T_{s,1})\frac{x}{L} + T_{s,1} \tag{3.3}$$

From this result it is evident that, *for one-dimensional, steady-state conduction in a plane wall with no heat generation and constant thermal conductivity, the temperature varies linearly with x.*

Now that we have the temperature distribution, we may use Fourier's law, Equation 2.1, to determine the conduction heat transfer rate. That is,

$$q_x = -kA\frac{dT}{dx} = \frac{kA}{L}(T_{s,1} - T_{s,2}) \tag{3.4}$$

Note that A is the area of the wall *normal* to the direction of heat transfer and, for the plane wall, it is a constant independent of x. The heat flux is then

$$q''_x = \frac{q_x}{A} = \frac{k}{L}(T_{s,1} - T_{s,2}) \tag{3.5}$$

Equations 3.4 and 3.5 indicate that both the heat rate q_x and heat flux q''_x are constants, independent of x.

In the foregoing paragraphs we have used the *standard approach* to solving conduction problems. That is, the general solution for the temperature distribution is first obtained by solving the appropriate form of the heat equation. The boundary conditions are then applied to obtain the particular solution, which is used with Fourier's law to determine the heat transfer rate. Note that we have opted to prescribe surface temperatures at $x = 0$ and $x = L$ as boundary conditions, even though it is the fluid temperatures, and not the surface temperatures, that are typically known. However, since adjoining fluid and surface temperatures are easily related through a surface energy balance (see Section 1.3.2), it is a simple matter to express Equations 3.3 through 3.5 in terms of fluid, rather than surface, temperatures. Alternatively, equivalent results could be obtained directly by using the surface energy balances as boundary conditions of the third kind in evaluating the constants of Equation 3.2 (see Problem 3.1).

3.1.2 Thermal Resistance

At this point we note that, for the special case of one-dimensional heat transfer with no internal energy generation and with constant properties, a very important concept is suggested by Equation 3.4. In particular, there exists an analogy between the diffusion of heat and electrical charge. Just as an electrical resistance is associated with the conduction of electricity, a thermal resistance may be associated with the conduction of heat. Defining resistance as the ratio of a driving potential to the corresponding transfer rate, it follows from Equation 3.4 that the *thermal resistance for conduction* in a plane wall is

$$R_{t,\text{cond}} \equiv \frac{T_{s,1} - T_{s,2}}{q_x} = \frac{L}{kA} \tag{3.6}$$

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Similarly, for electrical conduction in the same system, Ohm's law provides an electrical resistance of the form

$$R_e = \frac{E_{s,1} - E_{s,2}}{I} = \frac{L}{\sigma A} \tag{3.7}$$

The analogy between Equations 3.6 and 3.7 is obvious. A thermal resistance may also be associated with heat transfer by convection at a surface. From Newton's law of cooling,

$$q = hA(T_s - T_\infty) \tag{3.8}$$

The *thermal resistance for convection* is then

$$R_{t,\text{conv}} \equiv \frac{T_s - T_\infty}{q} = \frac{1}{hA} \tag{3.9}$$

Circuit representations provide a useful tool for both conceptualizing and quantifying heat transfer problems. The *equivalent thermal circuit* for the plane wall with convection surface conditions is shown in Figure 3.1*b*. The heat transfer rate may be determined from separate consideration of each element in the network. Since q_x is constant throughout the network, it follows that

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{1/h_1 A} = \frac{T_{s,1} - T_{s,2}}{L/kA} = \frac{T_{s,2} - T_{\infty,2}}{1/h_2 A} \tag{3.10}$$

In terms of the *overall temperature difference*, $T_{\infty,1} - T_{\infty,2}$, and the *total thermal resistance*, R_{tot}, the heat transfer rate may also be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,2}}{R_{\text{tot}}} \tag{3.11}$$

Because the conduction and convection resistances are in series and may be summed, it follows that

$$R_{\text{tot}} = \frac{1}{h_1 A} + \frac{L}{kA} + \frac{1}{h_2 A} \tag{3.12}$$

Radiation exchange between the surface and surroundings may also be important if the convection heat transfer coefficient is small (as it often is for natural convection in a gas). A *thermal resistance for radiation* may be defined by reference to Equation 1.8:

$$R_{t,\text{rad}} = \frac{T_s - T_{\text{sur}}}{q_{\text{rad}}} = \frac{1}{h_r A} \tag{3.13}$$

For radiation between a surface and *large surroundings*, h_r is determined from Equation 1.9. Surface radiation and convection resistances act in parallel, and if $T_\infty = T_{\text{sur}}$, they may be combined to obtain a single, effective surface resistance.

3.1.3 The Composite Wall

Equivalent thermal circuits may also be used for more complex systems, such as *composite walls*. Such walls may involve any number of series and parallel thermal

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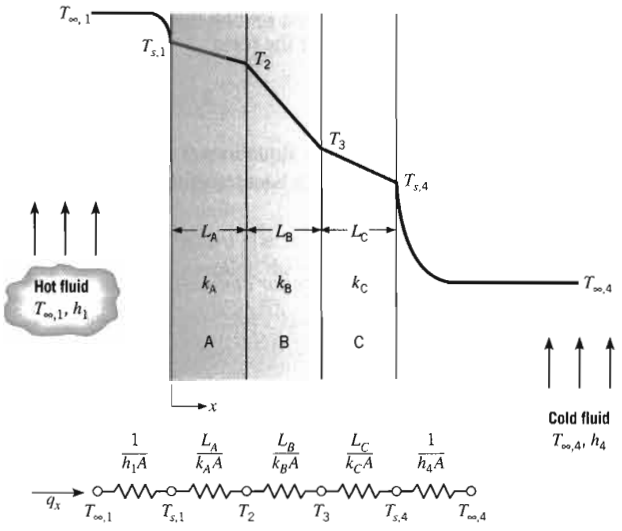


FIGURE 3.2 Equivalent thermal circuit for a series composite wall.

resistances due to layers of different materials. Consider the series composite wall of Figure 3.2. The one-dimensional heat transfer rate for this system may be expressed as

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{\Sigma R_t} \tag{3.14}$$

where $T_{\infty,1} - T_{\infty,4}$ is the *overall* temperature difference and the summation includes all thermal resistances. Hence

$$q_x = \frac{T_{\infty,1} - T_{\infty,4}}{[(1/h_1 A) + (L_A/k_A A) + (L_B/k_B A) + (L_C/k_C A) + (1/h_4 A)]} \tag{3.15}$$

Alternatively, the heat transfer rate can be related to the temperature difference and resistance associated with each element. For example,

$$q_x = \frac{T_{\infty,1} - T_{s,1}}{(1/h_1 A)} = \frac{T_{s,1} - T_2}{(L_A/k_A A)} = \frac{T_2 - T_3}{(L_B/k_B A)} = \cdots \tag{3.16}$$

With composite systems it is often convenient to work with an *overall heat transfer coefficient*, U, which is defined by an expression analogous to Newton's law of cooling. Accordingly,

$$q_x \equiv UA\,\Delta T \tag{3.17}$$

where ΔT is the overall temperature difference. The overall heat transfer coefficient is related to the total thermal resistance, and from Equations 3.14 and 3.17 we see that $UA = 1/R_{tot}$. Hence, for the composite wall of Figure 3.2,

$$U = \frac{1}{R_{tot} A} = \frac{1}{[(1/h_1) + (L_A/k_A) + (L_B/k_B) + (L_C/k_C) + (1/h_4)]} \tag{3.18}$$

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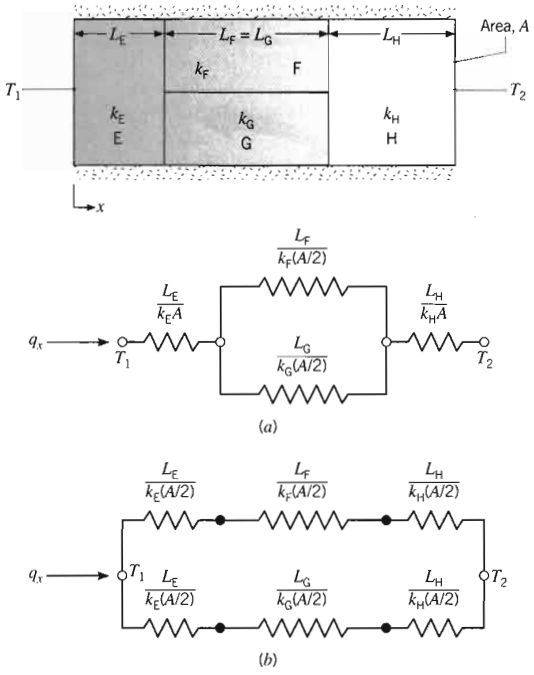


FIGURE 3.3 Equivalent thermal circuits for a series–parallel composite wall.

In general, we may write

$$R_{tot} = \sum R_t = \frac{\Delta T}{q} = \frac{1}{UA} \tag{3.19}$$

Composite walls may also be characterized by series–parallel configurations, such as that shown in Figure 3.3. Although the heat flow is now multidimensional, it is often reasonable to assume one-dimensional conditions. Subject to this assumption, two different thermal circuits may be used. For case (*a*) it is presumed that surfaces normal to the x direction are isothermal, while for case (*b*) it is assumed that surfaces parallel to the x direction are adiabatic. Different results are obtained for R_{tot}, and the corresponding values of q bracket the actual heat transfer rate. These differences increase with increasing $|k_F - k_G|$, as multidimensional effects become more significant.

3.1.4 Contact Resistance

Although neglected until now, it is important to recognize that, in composite systems, the temperature drop across the interface between materials may be appreciable. This temperature change is attributed to what is known as the *thermal contact*

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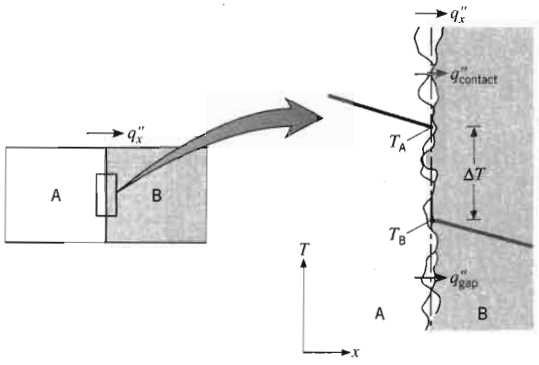


FIGURE 3.4 Temperature drop due to thermal contact resistance.

resistance, $R_{t,c}$. The effect is shown in Figure 3.4, and for a unit area of the interface, the resistance is defined as

$$R''_{t,c} = \frac{T_A - T_B}{q''_x} \tag{3.20}$$

The existence of a finite contact resistance is due principally to surface roughness effects. Contact spots are interspersed with gaps that are, in most instances, air filled. Heat transfer is therefore due to conduction across the actual contact area and to conduction and/or radiation across the gaps. The contact resistance may be viewed as two parallel resistances: that due to the contact spots and that due to the gaps. The contact area is typically small, and especially for rough surfaces, the major contribution to the resistance is made by the gaps.

For solids whose thermal conductivities exceed that of the interfacial fluid, the contact resistance may be reduced by increasing the area of the contact spots. Such an increase may be effected by increasing the joint pressure and/or by reducing the roughness of the mating surfaces. The contact resistance may also be reduced by selecting an interfacial fluid of large thermal conductivity. In this respect, no fluid (an evacuated interface) eliminates conduction across the gap, thereby increasing the

TABLE 3.1 Thermal contact resistance for (*a*) metallic interfaces under vacuum conditions and (*b*) aluminum interface (10-μm surface roughness, 10^5 N/m^2) with different interfacial fluids [1]

Thermal Resistance, $R''_{t,c} \times 10^4$ (m$^2 \cdot$ K/W)				
(*a*) Vacuum Interface			**(*b*) Interfacial Fluid**	
Contact pressure	100 kN/m^2	10,000 kN/m^2	Air	2.75
Stainless steel	6–25	0.7–4.0	Helium	1.05
Copper	1–10	0.1–0.5	Hydrogen	0.720
Magnesium	1.5–3.5	0.2–0.4	Silicone oil	0.525
Aluminum	1.5–5.0	0.2–0.4	Glycerine	0.265

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TABLE 3.2 Thermal resistance of representative solid/solid interfaces

Interface	$R''_{t,c} \times 10^4$ ($m^2 \cdot K/W$)	Source
Silicon chip/lapped aluminum in air (27–500 kN/m^2)	0.3–0.6	[2]
Aluminum/aluminum with indium foil filler (~100 kN/m^2)	~0.07	[1, 3]
Stainless/stainless with indium foil filler (~3500 kN/m^2)	~0.04	[1, 3]
Aluminum/aluminum with metallic (Pb) coating	0.01–0.1	[4]
Aluminum/aluminum with Dow Corning 340 grease (~100 kN/m^2)	~0.07	[1, 3]
Stainless/stainless with Dow Corning 340 grease (~3500 kN/m^2)	~0.04	[1, 3]
Silicon chip/aluminum with 0.02-mm epoxy	0.2–0.9	[5]
Brass/brass with 15-μm tin solder	0.025–0.14	[6]

contact resistance. Likewise, if the characteristic gap width, L, becomes small (as, for example, in the case of very smooth surfaces in contact), L/λ_{mfp} can approach values for which the thermal conductivity of the interfacial gas is reduced by microscale effects, as discussed in Section 2.2.

Although theories have been developed for predicting $R''_{t,c}$, the most reliable results are those that have been obtained experimentally. The effect of loading on metallic interfaces can be seen in Table 3.1*a*, which presents an approximate range of thermal resistances under vacuum conditions. The effect of interfacial fluid on the thermal resistance of an aluminum interface is shown in Table 3.1*b*.

Contrary to the results of Table 3.1, many applications involve contact between dissimilar solids and/or a wide range of possible interstitial (filler) materials (Table 3.2). Any interstitial substance that fills the gap between contacting surfaces and whose thermal conductivity exceeds that of air will decrease the contact resistance. Two classes of materials that are well suited for this purpose are soft metals and thermal greases. The metals, which include indium, lead, tin, and silver, may be inserted as a thin foil or applied as a thin coating to one of the parent materials. Silicon-based thermal greases are attractive on the basis of their ability to completely fill the interstices with a material whose thermal conductivity is as much as 50 times that of air.

Unlike the foregoing interfaces, which are not permanent, many interfaces involve permanently bonded joints. The joint could be formed from an epoxy, a soft solder rich in lead, or a hard solder such as a gold/tin alloy. Due to interface resistances between the parent and bonding materials, the actual thermal resistance of the joint exceeds the theoretical value (L/k) computed from the thickness L and thermal conductivity k of the joint material. The thermal resistance of epoxied and soldered joints is also adversely affected by voids and cracks, which may form during manufacture or as a result of thermal cycling during normal operation.

Comprehensive reviews of thermal contact resistance results and models are provided by Snaith et al. [3], Madhusudana and Fletcher [7], and Yovanovich [8].

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EXAMPLE 3.1

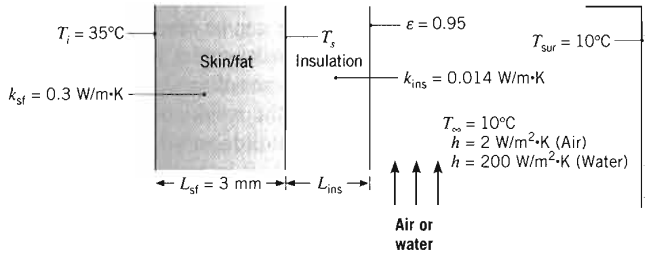
In Example 1.6, we calculated the heat loss rate from a human body in air and water environments. Now we consider the same conditions except that the surroundings (air or water) are at 10°C. To reduce the heat loss rate, the person wears special sporting gear (snow suit and wet suit) made from a nanostructured silica aerogel insulation with an extremely low thermal conductivity of 0.014 W/m · K. The emissivity of the outer surface of the snow and wet suits is 0.95. What thickness of aerogel insulation is needed to reduce the heat loss rate to 100 W (a typical metabolic heat generation rate) in air and water? What are the resulting skin temperatures?

SOLUTION

Known: Inner surface temperature of a skin/fat layer of known thickness, thermal conductivity, and surface area. Thermal conductivity and emissivity of snow and wet suits. Ambient conditions.

Find: Insulation thickness needed to reduce heat loss rate to 100 W and corresponding skin temperature.

Schematic:

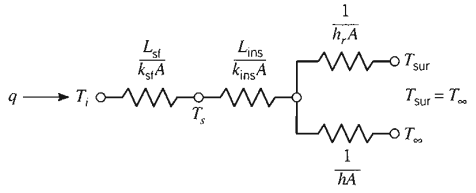


Assumptions:

1. Steady-state conditions.
2. One-dimensional heat transfer by conduction through the skin/fat and insulation layers.
3. Contact resistance is negligible.
4. Thermal conductivities are uniform.
5. Radiation exchange between the skin surface and the surroundings is between a small surface and a large enclosure at the air temperature.
6. Liquid water is opaque to thermal radiation.
7. Solar radiation is negligible.
8. Body is completely immersed in water in part 2.

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Analysis: The thermal circuit can be constructed by recognizing that resistance to heat flow is associated with conduction through the skin/fat and insulation layers and convection and radiation at the outer surface. Accordingly, the circuit and the resistances are of the following form (with $h_r = 0$ for water):



The total thermal resistance needed to achieve the desired heat loss rate is found from Equation 3.19,

$$R_{tot} = \frac{T_i - T_\infty}{q} = \frac{(35 - 10)\ \text{K}}{100\ \text{W}} = 0.25\ \text{K/W}$$

The total thermal resistance between the inside of the skin/fat layer and the cold surroundings includes conduction resistances for the skin/fat and insulation layers and an effective resistance associated with convection and radiation, which act in parallel. Hence,

$$R_{tot} = \frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_rA}\right)^{-1} = \frac{1}{A}\left(\frac{L_{sf}}{k_{sf}} + \frac{L_{ins}}{k_{ins}} + \frac{1}{h + h_r}\right)$$

This equation can be solved for the insulation thickness.

Air

The radiation heat transfer coefficient is approximated as having the same value as in Example 1.6: $h_r = 5.9\ \text{W/m}^2 \cdot \text{K}$.

$$L_{ins} = k_{ins}\left[AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h + h_r}\right]$$

$$= 0.014\ \text{W/m} \cdot \text{K}\left[1.8\ \text{m}^2 \times 0.25\ \text{K/W} - \frac{3 \times 10^{-3}\text{m}}{0.3\ \text{W/m} \cdot \text{K}} - \frac{1}{(2 + 5.9)\ \text{W/m}^2 \cdot \text{K}}\right]$$

$$= 0.0044\ \text{m} = 4.4\ \text{mm}$$ ◁

Water

$$L_{ins} = k_{ins}\left[AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h}\right]$$

$$= 0.014\ \text{W/m} \cdot \text{K}\left[1.8\ \text{m}^2 \times 0.25\ \text{K/W} - \frac{3 \times 10^{-3}\ \text{m}}{0.3\ \text{W/m} \cdot \text{K}} - \frac{1}{200\ \text{W/m}^2 \cdot \text{K}}\right]$$

$$= 0.0061\ \text{m} = 6.1\ \text{mm}$$ ◁

These required thicknesses of insulation material can easily be incorporated into the snow and wet suits.

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The skin temperature can be calculated by considering conduction through the skin/fat layer:

$$q = \frac{k_{sf} A(T_i - T_s)}{L_{sf}}$$

or solving for T_s,

$$T_s = T_i - \frac{qL_{sf}}{k_{sf}A} = 35°C - \frac{100\ \text{W} \times 3 \times 10^{-3}\ \text{m}}{0.3\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2} = 34.4°C$$

The skin temperature is the same in both cases because the heat loss rate and skin/fat properties are the same.

Comments:

1. The nanostructured silica aerogel is an extremely porous material that is only around 5% solid. Its thermal conductivity is less than the thermal conductivity of the gas that fills its pores. As explained in Section 2.2, the reason for this seemingly impossible result is that the pore size is only around 20 nm, which reduces the mean free path of the gas and hence decreases its thermal conductivity.
2. By reducing the heat loss rate to 100 W, a person could remain in the cold environments indefinitely without becoming chilled. The skin temperature of 34.4°C would feel comfortable.
3. In the water case, the thermal resistance of the insulation dominates and all other resistances can be neglected.
4. The convection heat transfer coefficient associated with the air depends on the wind conditions, and can vary over a broad range. As it changes, so will the outer surface temperature of the insulation layer. Since the radiation heat transfer coefficient depends on this temperature, it will also vary. We can perform a more complete analysis which takes this into account. The radiation heat transfer coefficient is given by Equation 1.9:

$$h_r = \varepsilon\sigma(T_{s,o} + T_{sur})(T_{s,o}^2 + T_{sur}^2) \tag{1}$$

Here $T_{s,o}$ is the outer surface temperature of the insulation layer, which can be calculated from

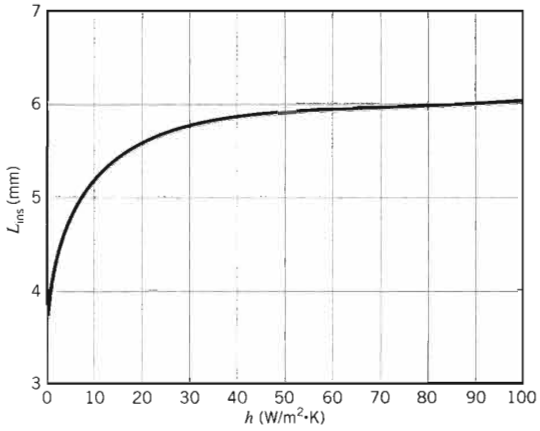
$$T_{s,o} = T_i - \frac{q}{\frac{L_{sf}}{k_{sf}A} + \frac{L_{ins}}{k_{ins}A}} \tag{2}$$

Since this depends on the insulation thickness, we also need the previous equation for L_{ins}:

$$L_{ins} = k_{ins}\left(AR_{tot} - \frac{L_{sf}}{k_{sf}} - \frac{1}{h + h_r}\right) \tag{3}$$

With all other values known, these three equations can be solved for the required insulation thickness. Using all the values from above, these equations have been solved for values of h in the range $0 \le h \le 100\ \text{W/m}^2 \cdot \text{K}$, and the results are represented graphically.

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Increasing h reduces the corresponding convection resistance, which then requires additional insulation to maintain the heat transfer rate at 100 W. Once the heat transfer coefficient exceeds approximately 60 W/m^2 · K, the convection resistance is negligible and further increases in h have little effect on the required insulation thickness.

The outer surface temperature and radiation heat transfer coefficient can also be calculated. As h increases from 0 to 100 W/m^2 · K, $T_{s,o}$ decreases from 294 to 284 K, while h_r decreases from 5.2 to 4.9 W/m^2 · K. The initial estimate of $h_r = 5.9$ W/m^2 · K was not highly accurate. Using this more complete model of the radiation heat transfer, with $h = 2$ W/m^2 · K, the radiation heat transfer coefficient is 5.1 W/m^2 · K and the required insulation thickness is 4.2 mm, close to the value calculated in the first part of the problem.

5. This example is provided as a ready-to-solve model in *IHT* under *Examples* in the menu bar. It could also be solved using the thermal resistance network builder, *Models/Resistance Networks*. As an exercise, enter Equations 1 through 3 from Comment 4 into the *IHT* workspace and solve for the required insulation thickness when the air convection heat transfer coefficient is 20 W/m^2 · K. All other conditions remain the same as in the problem statement. [Answer, 5.6 mm]

EXAMPLE 3.2

A thin silicon chip and an 8-mm-thick aluminum substrate are separated by a 0.02-mm-thick epoxy joint. The chip and substrate are each 10 mm on a side, and their exposed surfaces are cooled by air, which is at a temperature of 25°C and provides a convection coefficient of 100 W/m^2 · K. If the chip dissipates 10^4 W/m^2 under normal conditions, will it operate below a maximum allowable temperature of 85°C?

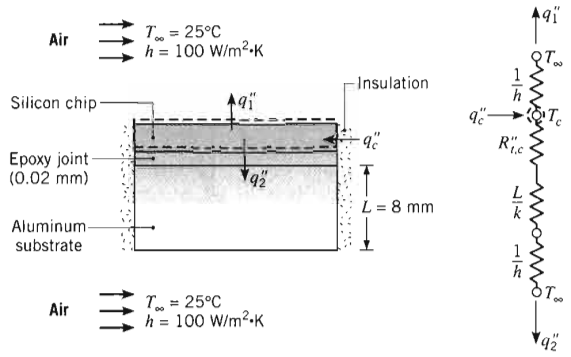
SOLUTION

Known: Dimensions, heat dissipation, and maximum allowable temperature of a silicon chip. Thickness of aluminum substrate and epoxy joint. Convection conditions at exposed chip and substrate surfaces.

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Find: Whether maximum allowable temperature is exceeded.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction (negligible heat transfer from sides of composite).
3. Negligible chip thermal resistance (an isothermal chip).
4. Constant properties.
5. Negligible radiation exchange with surroundings.

Properties: Table A.1, pure aluminum ($T \sim 350$ K): $k = 239$ W/m · K.

Analysis: Heat dissipated in the chip is transferred to the air directly from the exposed surface and indirectly through the joint and substrate. Performing an energy balance on a control surface about the chip, it follows that, on the basis of a unit surface area,

$$q_c'' = q_1'' + q_2''$$

or

$$q_c'' = \frac{T_c - T_\infty}{(1/h)} + \frac{T_c - T_\infty}{R_{t,c}'' + (L/k) + (1/h)}$$

To conservatively estimate T_c, the maximum possible value of $R_{t,c}'' = 0.9 \times 10^{-4}$ m² · K/W is obtained from Table 3.2. Hence

$$T_c = T_\infty + q_c''\left[h + \frac{1}{R_{t,c}'' + (L/k) + (1/h)}\right]^{-1}$$

or

$$T_c = 25°C + 10^4 \text{ W/m}^2 \times \left[100 + \frac{1}{(0.9 + 0.33 + 100) \times 10^{-4}}\right]^{-1} \text{m}^2 \cdot \text{K/W}$$

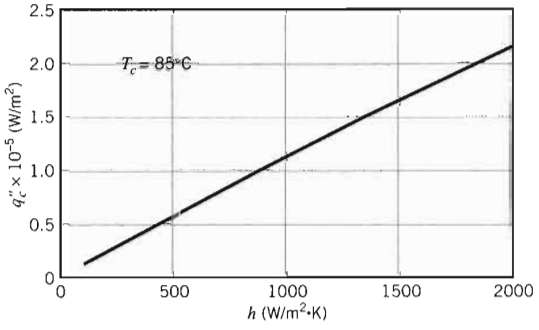
$$T_c = 25°C + 50.3°C = 75.3°C \quad \triangleleft$$

Hence the chip will operate below its maximum allowable temperature.

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Comments:

1. The joint and substrate thermal resistances are much less than the convection resistance. The joint resistance would have to increase to the unrealistically large value of 50×10^{-4} m$^2 \cdot$ K/W, before the maximum allowable chip temperature would be exceeded.
2. The allowable power dissipation may be increased by increasing the convection coefficients, either by increasing the air velocity and/or by replacing the air with a more effective heat transfer fluid. Exploring this option for $100 \leq h \leq 2000$ W/m$^2 \cdot$ K with $T_c = 85$°C, the following results are obtained.



As $h \to \infty$, $q_2'' \to 0$ and virtually all of the chip power is transferred directly to the fluid stream.

3. As calculated, the *difference* between the air temperature ($T_\infty = 25$°C) and the chip temperature ($T_c = 75.3$°C) is 50.3 K. Keep in mind that this is a temperature *difference* and therefore is the same as 50.3°C.
4. Consider conditions for which air flow over the chip (upper) or substrate (lower) surface ceases due to a blockage in the air supply channel. If heat transfer from either surface is negligible, what are the resulting chip temperatures for $q_c'' = 10^4$ W/m^2? [Answer, 126°C or 125°C]
5. Consider using *IHT* to calculate the allowable power dissipation for the prescribed air-flow conditions and $T_c = 85$°C. Plot the results and compare them with those shown graphically in Comment 2.

EXAMPLE 3.3

The thermal conductivity of a $D = 14$-nm-diameter carbon nanotube is measured with an instrument that is fabricated of a wafer of silicon nitride at a temperature of $T_\infty = 300$ K. The 20-μm-long nanotube rests on two 0.5-μm-thick, 10 μm $\times$ 10 μm square islands that are separated by a distance $s = 5$ μm. A thin layer of platinum is used as an electrical resistor on the *heated island* (at temperature T_h) to dissipate $q = 11.3$ μW of electrical power. On the *sensing island*, a similar layer of platinum is used to determine its temperature, T_s. The platinum's electrical resistance, $R(T_s) = E/I$, is found by measuring the voltage drop and electrical current across the platinum layer. The temperature of the sensing island, T_s, is then determined from the relationship of the platinum electrical resistance to its temperature. Each island is

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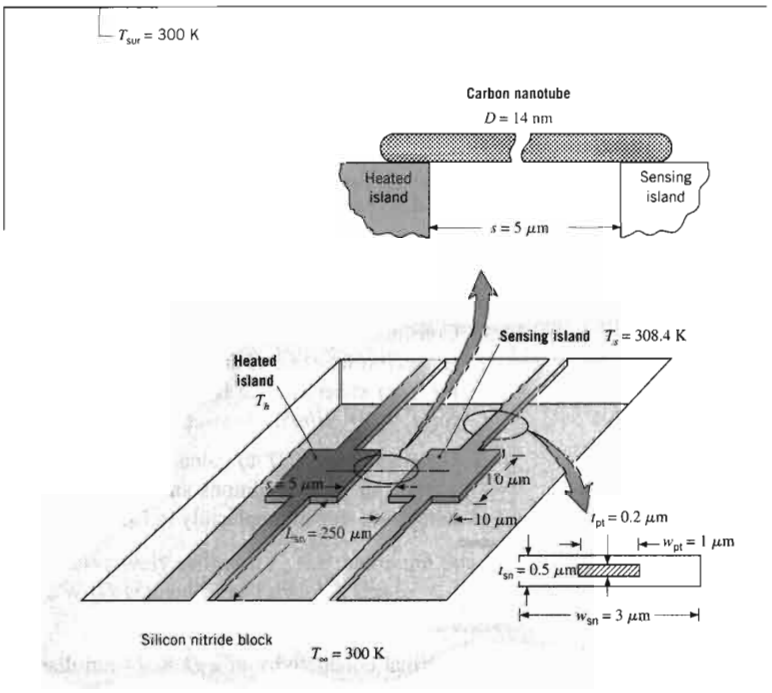
suspended by two L_{sn} = 250-μm-long silicon nitride beams that are w_{sn} = 3 μm wide and t_{sn} = 0.5 μm thick. A platinum line of width w_{pt} = 1 μm and thickness t_{pt} = 0.2 μm is deposited within each silicon nitride beam to power the heated island or detect the voltage drop associated with the determination of T_s. The entire experiment is performed in a vacuum with T_{sur} = 300 K and at steady state, T_s = 308.4 K. Estimate the thermal conductivity of the carbon nanotube.

SOLUTION

Known: Dimensions, heat dissipated at the heated island, and temperatures of the sensing island and surrounding silicon nitride wafer.

Find: The thermal conductivity of the carbon nanotube.

Schematic:



Assumptions:

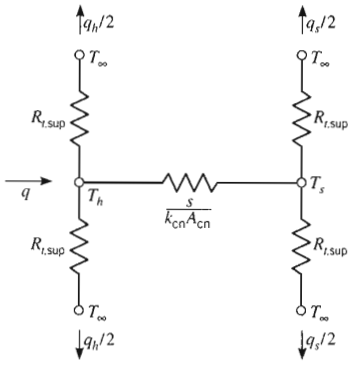
1. Steady-state conditions.
2. One-dimensional heat transfer.
3. The heated and sensing islands are isothermal.
4. Radiation exchange between the surfaces and the surroundings is negligible.
5. Convection losses are negligible.

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6. Ohmic heating in the platinum signal lines is negligible.
7. Constant properties.
8. Contact resistance between the nanotube and the islands is negligible.

Properties: Table A.1, platinum (325 K, assumed): $k_{pt} = 71.6$ W/m · K. Table A.2, silicon nitride (325 K, assumed): $k_{sn} = 15.5$ W/m · K.

Analysis: Energy that is dissipated at the heated island is transferred to the silicon nitride block through the support beams of the heated island, the carbon nanotube, and subsequently through the support beams of the sensing island. Therefore, the thermal circuit may be constructed as follows



where each supporting beam provides a thermal resistance, $R_{t,sup}$, that is composed of a resistance due to the silicon nitride (sn) in parallel with a resistance due to the platinum (pt) line.

The cross-sectional areas of the materials in the support beams are

$$A_{pt} = w_{pt}t_{pt} = (1 \times 10^{-6}\,\text{m}) \times (0.2 \times 10^{-6}\,\text{m}) = 2 \times 10^{-13}\,\text{m}^2$$

$$A_{sn} = w_{sn}t_{sn} - A_{pt} = (3 \times 10^{-6}\,\text{m}) \times (0.5 \times 10^{-6}\,\text{m}) - 2 \times 10^{-13}\,\text{m}^2 = 1.3 \times 10^{-12}\,\text{m}^2$$

while the cross-sectional area of the carbon nanotube is

$$A_{cn} = \pi D^2/4 = \pi(14 \times 10^{-9}\,\text{m})^2/4 = 1.54 \times 10^{-16}\,\text{m}^2$$

The thermal resistance of each support is

$$R_{t,sup} = \left[\frac{k_{pt}A_{pt}}{L_{pt}} + \frac{k_{sn}A_{sn}}{L_{sn}}\right]^{-1}$$

$$= \left[\frac{71.6\,\text{W/m} \cdot \text{K} \times 2 \times 10^{-13}\,\text{m}^2}{250 \times 10^{-6}\,\text{m}} + \frac{15.5\,\text{W/m} \cdot \text{K} \times 1.3 \times 10^{-12}\,\text{m}^2}{250 \times 10^{-6}\,\text{m}}\right]^{-1}$$

$$= 7.25 \times 10^6\,\text{K/W}$$

The combined heat loss through both sensing island supports is

$$q_s = 2(T_s - T_\infty)/R_{t,sup} = 2 \times (308.4\,\text{K} - 300\,\text{K})/(7.25 \times 10^6\,\text{K/W})$$

$$= 2.32 \times 10^{-6}\,\text{W} = 2.32\,\mu\text{W}$$

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It follows that

$$q_h = q - q_s = 11.3\ \mu\text{W} - 2.32\ \mu\text{W} = 8.98\ \mu\text{W}$$

and T_h attains a value of

$$T_h = T_\infty + \frac{1}{2} q_h R_{t,\text{sup}} = 300\ \text{K} + \frac{8.98 \times 10^{-6}\ \text{W} \times 7.25 \times 10^{6}\ \text{K/W}}{2} = 332.6\ \text{K}$$

For the portion of the thermal circuit connecting T_h and T_s,

$$q_s = \frac{T_h - T_s}{s/(k_{\text{cn}} A_{\text{cn}})}$$

from which

$$k_{\text{cn}} = \frac{q_s s}{A_{\text{cn}}(T_h - T_s)} = \frac{2.32 \times 10^{-6}\ \text{W} \times 5 \times 10^{-6}\ \text{m}}{1.54 \times 10^{-16}\ \text{m}^2 \times (332.6\ \text{K} - 308.4\ \text{K})}$$

$$k_{\text{cn}} = 3113\ \text{W/m} \cdot \text{K} \qquad \triangleleft$$

Comments:

1. The measured thermal conductivity is extremely large, as evident by comparing its value to the thermal conductivities of pure metals shown in Figure 2.4. Carbon nanotubes may be used to *dope* otherwise low thermal conductivity materials to improve heat transfer.
2. Contact resistances between the carbon nanotube and the heated and sensing islands were neglected because little is known about such resistances at the nanoscale. However, *if* a contact resistance were included in the analysis, the measured thermal conductivity of the carbon nanotube would be even higher than the predicted value.
3. The significance of radiation heat transfer may be estimated by approximating the heated island as a blackbody radiating to T_{sur} from both its top and bottom surfaces. Hence, $q_{\text{rad},b} \approx 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4 \times 2 \times (10 \times 10^{-6}\ \text{m})^2 \times (332.6^4 - 300^4)\text{K}^4 = 4.7 \times 10^{-8}\ \text{W} = 0.047\ \mu\text{W}$, and radiation is negligible.

3.2 An Alternative Conduction Analysis

The conduction analysis of Section 3.1 was performed using the *standard approach.* That is, the heat equation was solved to obtain the temperature distribution, Equation 3.3, and Fourier's law was then applied to obtain the heat transfer rate, Equation 3.4. However, an alternative approach may be used for the conditions presently of interest. Considering conduction in the system of Figure 3.5, we recognize that, for *steady-state conditions* with *no heat generation* and *no heat loss from the sides*, the heat transfer rate q_x must be a constant independent of x. That is, for any differential element dx, $q_x = q_{x+dx}$. This condition is, of course, a consequence of the energy conservation requirement, and it must apply even if the area varies

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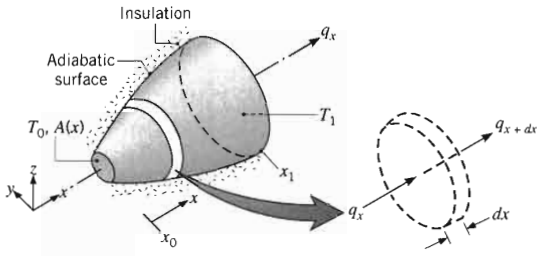


FIGURE 3.5 System with a constant conduction heat transfer rate.

with position, $A(x)$, and the thermal conductivity varies with temperature, $k(T)$. Moreover, even though the temperature distribution may be two dimensional, varying with x and y, it is often reasonable to neglect the y variation and to assume a *one-dimensional* distribution in x.

For the above conditions it is possible to work exclusively with Fourier's law when performing a conduction analysis. In particular, since the conduction rate is a *constant*, the rate equation may be *integrated*, even though neither the rate nor the temperature distribution is known. Consider Fourier's law, Equation 2.1, which may be applied to the system of Figure 3.5. Although we may have no knowledge of the value of q_x or the form of $T(x)$, we do know that q_x is a constant. Hence we may express Fourier's law in the integral form

$$q_x \int_{x_0}^{x} \frac{dx}{A(x)} = -\int_{T_0}^{T} k(T)\, dT \tag{3.21}$$

The cross-sectional area may be a known function of x, and the material thermal conductivity may vary with temperature in a known manner. If the integration is performed from a point x_0 at which the temperature T_0 is known, the resulting equation provides the functional form of $T(x)$. Moreover, if the temperature $T = T_1$ at some $x = x_1$ is also known, integration between x_0 and x_1 provides an expression from which q_x may be computed. Note that, if the area A is uniform and k is independent of temperature, Equation 3.21 reduces to

$$\frac{q_x \Delta x}{A} = -k\, \Delta T \tag{3.22}$$

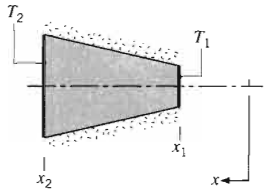
where $\Delta x = x_1 - x_0$ and $\Delta T = T_1 - T_0$.

We frequently elect to solve diffusion problems by working with integrated forms of the diffusion rate equations. However, the limiting conditions for which this may be done should be firmly fixed in our minds: *steady-state* and *one-dimensional* transfer with *no heat generation.*

EXAMPLE 3.4

The diagram shows a conical section fabricated from pyroceram. It is of circular cross section with the diameter $D = ax$, where $a = 0.25$. The small end is at $x_1 = 50$ mm and the large end at $x_2 = 250$ mm. The end temperatures are $T_1 = 400$ K and $T_2 = 600$ K, while the lateral surface is well insulated.

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1. Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.
2. Calculate the heat rate q_x through the cone.

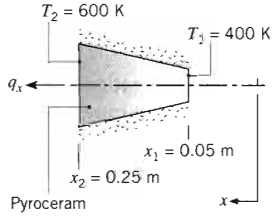
SOLUTION

Known: Conduction in a circular conical section having a diameter $D = ax$, where $a = 0.25$.

Find:

1. Temperature distribution $T(x)$.
2. Heat transfer rate q_x.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in the x direction.
3. No internal heat generation.
4. Constant properties.

Properties: Table A.2, pyroceram (500 K): $k = 3.46$ W/m · K.

Analysis:

1. Since heat conduction occurs under steady-state, one-dimensional conditions with no internal heat generation, the heat transfer rate q_x is a constant independent of x. Accordingly, Fourier's law, Equation 2.1, may be used to determine the temperature distribution

$$q_x = -kA\frac{dT}{dx}$$

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where $A = \pi D^2/4 = \pi a^2 x^2/4$. Separating variables,

$$\frac{4q_x\,dx}{\pi a^2 x^2} = -k\,dT$$

Integrating from x_1 to any x within the cone, and recalling that q_x and k are constants, it follows that

$$\frac{4q_x}{\pi a^2}\int_{x_1}^{x}\frac{dx}{x^2} = -k\int_{T_1}^{T} dT$$

Hence

$$\frac{4q_x}{\pi a^2}\left(-\frac{1}{x}+\frac{1}{x_1}\right) = -k(T - T_1)$$

or solving for T

$$T(x) = T_1 - \frac{4q_x}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x}\right)$$

Although q_x is a constant, it is as yet an unknown. However, it may be determined by evaluating the above expression at $x = x_2$, where $T(x_2) = T_2$. Hence

$$T_2 = T_1 - \frac{4q_x}{\pi a^2 k}\left(\frac{1}{x_1}-\frac{1}{x_2}\right)$$

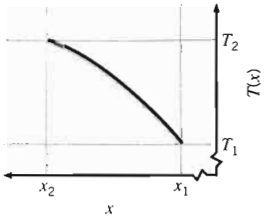
and solving for q_x

$$q_x = \frac{\pi a^2 k(T_1 - T_2)}{4[(1/x_1) - (1/x_2)]}$$

Substituting for q_x into the expression for $T(x)$, the temperature distribution becomes

$$T(x) = T_1 + (T_1 - T_2)\left[\frac{(1/x) - (1/x_1)}{(1/x_1) - (1/x_2)}\right] \quad \triangleleft$$

From this result, temperature may be calculated as a function of x and the distribution is as shown.



Note that, since $dT/dx = -4q_x/k\pi a^2 x^2$ from Fourier's law, it follows that the temperature gradient and heat flux decrease with increasing x.

2. Substituting numerical values into the foregoing result for the heat transfer rate, it follows that

$$q_x = \frac{\pi(0.25)^2 \times 3.46\ \text{W/m}\cdot\text{K}\,(400 - 600)\ \text{K}}{4\,(1/0.05\ \text{m} - 1/0.25\ \text{m})} = -2.12\ \text{W} \quad \triangleleft$$

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Comments: When the parameter a increases, the cross-sectional area changes more rapidly with distance, causing the one-dimensional assumption to become less appropriate.

3.3 *Radial Systems*

Cylindrical and spherical systems often experience temperature gradients in the radial direction only and may therefore be treated as one dimensional. Moreover, under steady-state conditions with no heat generation, such systems may be analyzed by using the *standard* method, which begins with the appropriate form of the heat equation, or the *alternative* method, which begins with the appropriate form of Fourier's law. In this section, the cylindrical system is analyzed by means of the standard method and the spherical system by means of the alternative method.

3.3.1 The Cylinder

A common example is the hollow cylinder, whose inner and outer surfaces are exposed to fluids at different temperatures (Figure 3.6). For steady-state conditions with no heat generation, the appropriate form of the heat equation, Equation 2.24, is

$$\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right) = 0 \tag{3.23}$$

where, for the moment, k is treated as a variable. The physical significance of this result becomes evident if we also consider the appropriate form of Fourier's law. The

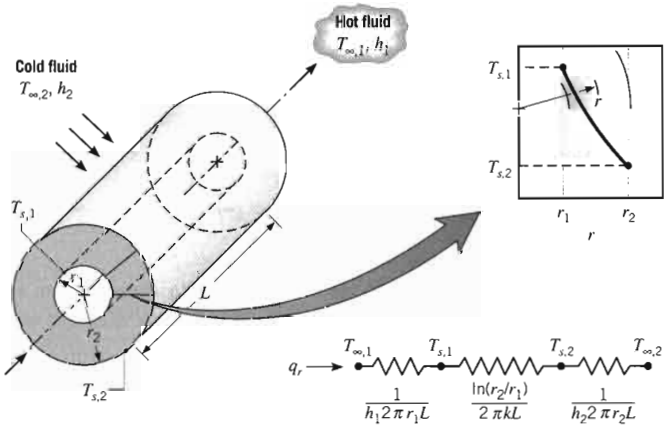


FIGURE 3.6 Hollow cylinder with convective surface conditions.

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rate at which energy is conducted across any cylindrical surface in the solid may be expressed as

$$q_r = -kA\frac{dT}{dr} = -k(2\pi rL)\frac{dT}{dr} \tag{3.24}$$

where $A = 2\pi rL$ is the area normal to the direction of heat transfer. Since Equation 3.23 dictates that the quantity $kr(dT/dr)$ is independent of r, it follows from Equation 3.24 that the conduction *heat transfer rate* q_r (*not* the heat flux q_r'') is a *constant in the radial direction.*

We may determine the temperature distribution in the cylinder by solving Equation 3.23 and applying appropriate boundary conditions. Assuming the value of k to be constant, Equation 3.23 may be integrated twice to obtain the general solution

$$T(r) = C_1 \ln r + C_2 \tag{3.25}$$

To obtain the constants of integration C_1 and C_2, we introduce the following boundary conditions:

$$T(r_1) = T_{s,1} \qquad \text{and} \qquad T(r_2) = T_{s,2}$$

Applying these conditions to the general solution, we then obtain

$$T_{s,1} = C_1 \ln r_1 + C_2 \qquad \text{and} \qquad T_{s,2} = C_1 \ln r_2 + C_2$$

Solving for C_1 and C_2 and substituting into the general solution, we then obtain

$$T(r) = \frac{T_{s,1} - T_{s,2}}{\ln (r_1/r_2)} \ln\left(\frac{r}{r_2}\right) + T_{s,2} \tag{3.26}$$

Note that the temperature distribution associated with radial conduction through a cylindrical wall is logarithmic, not linear, as it is for the plane wall under the same conditions. The logarithmic distribution is sketched in the inset of Figure 3.6.

If the temperature distribution, Equation 3.26, is now used with Fourier's law, Equation 3.24, we obtain the following expression for the heat transfer rate:

$$q_r = \frac{2\pi Lk(T_{s,1} - T_{s,2})}{\ln (r_2/r_1)} \tag{3.27}$$

From this result it is evident that, for radial conduction in a cylindrical wall, the thermal resistance is of the form

$$R_{t,\text{cond}} = \frac{\ln (r_2/r_1)}{2\pi Lk} \tag{3.28}$$

This resistance is shown in the series circuit of Figure 3.6. Note that since the value of q_r is independent of r, the foregoing result could have been obtained by using the alternative method, that is, by integrating Equation 3.24.

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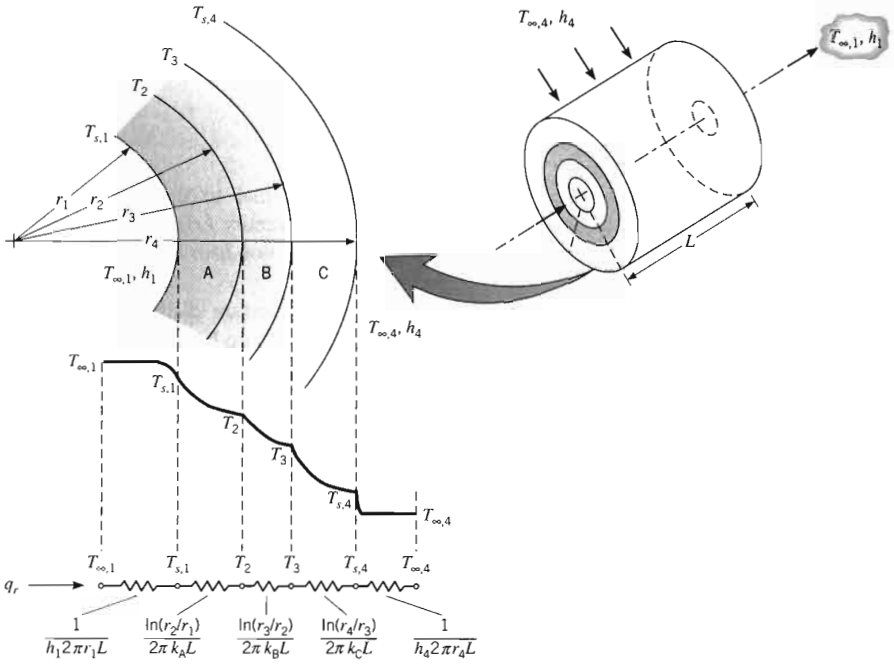


FIGURE 3.7 Temperature distribution for a composite cylindrical wall.

Consider now the composite system of Figure 3.7. Recalling how we treated the composite plane wall and neglecting the interfacial contact resistances, the heat transfer rate may be expressed as

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{\frac{1}{2\pi r_1 L h_1} + \frac{\ln(r_2/r_1)}{2\pi k_A L} + \frac{\ln(r_3/r_2)}{2\pi k_B L} + \frac{\ln(r_4/r_3)}{2\pi k_C L} + \frac{1}{2\pi r_4 L h_4}} \tag{3.29}$$

The foregoing result may also be expressed in terms of an overall heat transfer coefficient. That is,

$$q_r = \frac{T_{\infty,1} - T_{\infty,4}}{R_{tot}} = UA(T_{\infty,1} - T_{\infty,4}) \tag{3.30}$$

If U is defined in terms of the inside area, $A_1 = 2\pi r_1 L$, Equations 3.29 and 3.30 may be equated to yield

$$U_1 = \frac{1}{\frac{1}{h_1} + \frac{r_1}{k_A}\ln\frac{r_2}{r_1} + \frac{r_1}{k_B}\ln\frac{r_3}{r_2} + \frac{r_1}{k_C}\ln\frac{r_4}{r_3} + \frac{r_1}{r_4}\frac{1}{h_4}} \tag{3.31}$$

This definition is *arbitrary,* and the overall coefficient may also be defined in terms of A_4 or any of the intermediate areas. Note that

$$U_1 A_1 = U_2 A_2 = U_3 A_3 = U_4 A_4 = (\Sigma R_t)^{-1} \tag{3.32}$$

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and the specific forms of U_2, U_3, and U_4 may be inferred from Equations 3.29 and 3.30.

EXAMPLE 3.5

The possible existence of an optimum insulation thickness for radial systems is suggested by the presence of competing effects associated with an increase in this thickness. In particular, although the conduction resistance increases with the addition of insulation, the convection resistance decreases due to increasing outer surface area. Hence there may exist an insulation thickness that minimizes heat loss by maximizing the total resistance to heat transfer. Resolve this issue by considering the following system.

1. A thin-walled copper tube of radius r_i is used to transport a low-temperature refrigerant and is at a temperature T_i that is less than that of the ambient air at T_∞ around the tube. Is there an optimum thickness associated with application of insulation to the tube?
2. Confirm the above result by computing the total thermal resistance per unit length of tube for a 10-mm-diameter tube having the following insulation thicknesses: 0, 2, 5, 10, 20, and 40 mm. The insulation is composed of cellular glass, and the outer surface convection coefficient is 5 W/m$^2 \cdot$ K.

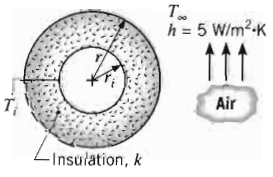
SOLUTION

Known: Radius r_i and temperature T_i of a thin-walled copper tube to be insulated from the ambient air.

Find:

1. Whether there exists an optimum insulation thickness that minimizes the heat transfer rate.
2. Thermal resistance associated with using cellular glass insulation of varying thickness.

Schematic:



Assumptions:

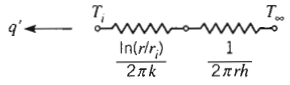
1. Steady-state conditions.
2. One-dimensional heat transfer in the radial (cylindrical) direction.
3. Negligible tube wall thermal resistance.
4. Constant properties for insulation.
5. Negligible radiation exchange between insulation outer surface and surroundings.

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Properties: Table A.3, cellular glass (285 K, assumed): $k = 0.055$ W/m · K.

Analysis:

1. The resistance to heat transfer between the refrigerant and the air is dominated by conduction in the insulation and convection in the air. The thermal circuit is therefore



where the conduction and convection resistances per unit length follow from Equations 3.28 and 3.9, respectively. The total thermal resistance per unit length of tube is then

$$R'_{tot} = \frac{\ln (r/r_i)}{2\pi k} + \frac{1}{2\pi rh}$$

where the rate of heat transfer per unit length of tube is

$$q' = \frac{T_\infty - T_i}{R'_{tot}}$$

An optimum insulation thickness would be associated with the value of r that minimized q' or maximized R'_{tot}. Such a value could be obtained from the requirement that

$$\frac{dR'_{tot}}{dr} = 0$$

Hence

$$\frac{1}{2\pi kr} - \frac{1}{2\pi r^2 h} = 0$$

or

$$r = \frac{k}{h}$$

To determine whether the foregoing result maximizes or minimizes the total resistance, the second derivative must be evaluated. Hence

$$\frac{d^2R'_{tot}}{dr^2} = -\frac{1}{2\pi kr^2} + \frac{1}{\pi r^3 h}$$

or, at $r = k/h$,

$$\frac{d^2R'_{tot}}{dr^2} = \frac{1}{\pi(k/h)^2}\left(\frac{1}{k} - \frac{1}{2k}\right) = \frac{1}{2\pi k^3/h^2} > 0$$

Since this result is always positive, it follows that $r = k/h$ is the insulation radius for which the total resistance is a minimum, not a maximum. Hence an *optimum* insulation thickness *does not exist.*

From the above result it makes more sense to think in terms of a *critical insulation radius*

$$r_{cr} \equiv \frac{k}{h}$$

which maximizes heat transfer, that is, below which q' increases with increasing r and above which q' decreases with increasing r.

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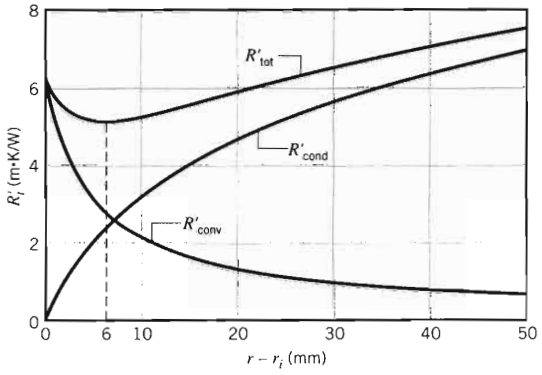
2. With $h = 5\ \text{W/m}^2 \cdot \text{K}$ and $k = 0.055\ \text{W/m} \cdot \text{K}$, the critical radius is

$$r_{cr} = \frac{0.055\ \text{W/m} \cdot \text{K}}{5\ \text{W/m}^2 \cdot \text{K}} = 0.011\ \text{m}$$

Hence $r_{cr} > r_i$ and heat transfer will increase with the addition of insulation up to a thickness of

$$r_{cr} - r_i = (0.011 - 0.005)\ \text{m} = 0.006\ \text{m}$$

The thermal resistances corresponding to the prescribed insulation thicknesses may be calculated and are plotted as follows:



Comments:

1. The effect of the critical radius is revealed by the fact that, even for 20 mm of insulation, the total resistance is not as large as the value for no insulation.
2. If $r_i < r_{cr}$, as it is in this case, the total resistance decreases and the heat rate therefore increases with the addition of insulation. This trend continues until the outer radius of the insulation corresponds to the critical radius. The trend is desirable for electrical current flow through a wire, since the addition of electrical insulation would aid in transferring heat dissipated in the wire to the surroundings. Conversely, if $r_i > r_{cr}$, any addition of insulation would increase the total resistance and therefore decrease the heat loss. This behavior would be desirable for steam flow through a pipe, where insulation is added to reduce heat loss to the surroundings.
3. For radial systems, the problem of reducing the total resistance through the application of insulation exists only for small diameter wires or tubes and for small convection coefficients, such that $r_{cr} > r_i$. For a typical insulation ($k \approx 0.03\ \text{W/m} \cdot \text{K}$) and free convection in air ($h \approx 10\ \text{W/m}^2 \cdot \text{K}$), $r_{cr} = (k/h) \approx 0.003$ m. Such a small value tells us that, normally, $r_i > r_{cr}$ and we need not be concerned with the effects of a critical radius.
4. The existence of a critical radius requires that the heat transfer area change in the direction of transfer, as for radial conduction in a cylinder (or a sphere). In a plane wall the area perpendicular to the direction of heat flow is constant and there is no critical insulation thickness (the total resistance always increases with increasing insulation thickness).

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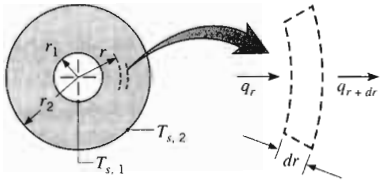


FIGURE 3.8
Conduction in a spherical shell.

3.3.2 The Sphere

Now consider applying the alternative method to analyzing conduction in the hollow sphere of Figure 3.8. For the differential control volume of the figure, energy conservation requires that $q_r = q_{r+dr}$ for steady-state, one-dimensional conditions with no heat generation. The appropriate form of Fourier's law is

$$q_r = -kA\frac{dT}{dr} = -k(4\pi r^2)\frac{dT}{dr} \tag{3.33}$$

where $A = 4\pi r^2$ is the area normal to the direction of heat transfer.

Acknowledging that q_r is a constant, independent of r, Equation 3.33 may be expressed in the integral form

$$\frac{q_r}{4\pi}\int_{r_1}^{r_2}\frac{dr}{r^2} = -\int_{T_{s,1}}^{T_{s,2}} k(T)\,dT \tag{3.34}$$

Assuming constant k, we then obtain

$$q_r = \frac{4\pi k(T_{s,1} - T_{s,2})}{(1/r_1) - (1/r_2)} \tag{3.35}$$

Remembering that the thermal resistance is defined as the temperature difference divided by the heat transfer rate, we obtain

$$R_{t,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \tag{3.36}$$

Note that the temperature distribution and Equations 3.35 and 3.36 could have been obtained by using the standard approach, which begins with the appropriate form of the heat equation.

Spherical composites may be treated in much the same way as composite walls and cylinders, where appropriate forms of the total resistance and overall heat transfer coefficient may be determined.

EXAMPLE 3.6

A spherical, thin-walled metallic container is used to store liquid nitrogen at 77 K. The container has a diameter of 0.5 m and is covered with an evacuated, reflective insulation composed of silica powder. The insulation is 25 mm thick, and its outer surface is exposed to ambient air at 300 K. The convection coefficient is

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known to be 20 W/m^2 · K. The latent heat of vaporization and the density of liquid nitrogen are 2×10^5 J/kg and 804 kg/m^3, respectively.

1. What is the rate of heat transfer to the liquid nitrogen?
2. What is the rate of liquid boil-off?

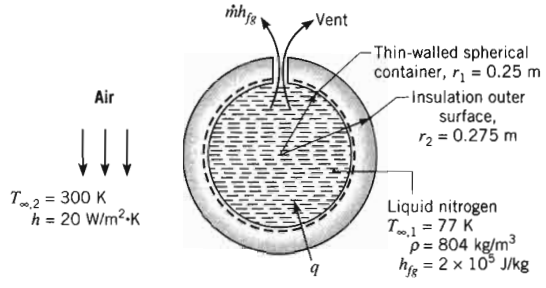
SOLUTION

Known: Liquid nitrogen is stored in a spherical container that is insulated and exposed to ambient air.

Find:

1. The rate of heat transfer to the nitrogen.
2. The mass rate of nitrogen boil-off.

Schematic:



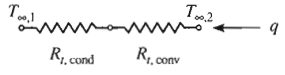
Assumptions:

1. Steady-state conditions.
2. One-dimensional transfer in the radial direction.
3. Negligible resistance to heat transfer through the container wall and from the container to the nitrogen.
4. Constant properties.
5. Negligible radiation exchange between outer surface of insulation and surroundings.

Properties: Table A.3, evacuated silica powder (300 K): $k = 0.0017$ W/m · K.

Analysis:

1. The thermal circuit involves a conduction and convection resistance in series and is of the form



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where, from Equation 3.36,

$$R_{t,\text{cond}} = \frac{1}{4\pi k}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

and from Equation 3.9

$$R_{t,\text{conv}} = \frac{1}{h4\pi r_2^2}$$

The rate of heat transfer to the liquid nitrogen is then

$$q = \frac{T_{\infty,2} - T_{\infty,1}}{(1/4\pi k)[(1/r_1) - (1/r_2)] + (1/h4\pi r_2^2)}$$

Hence,

$$q = [(300 - 77)\text{ K}] \div \left[\frac{1}{4\pi(0.0017\text{ W/m} \cdot \text{K})}\left(\frac{1}{0.25\text{ m}} - \frac{1}{0.275\text{ m}}\right) + \frac{1}{(20\text{ W/m}^2 \cdot \text{K})4\pi(0.275\text{ m})^2}\right]$$

$$q = \frac{223}{17.02 + 0.05}\text{ W} = 13.06\text{ W} \quad \triangleleft$$

2. Performing an energy balance for a control surface about the nitrogen, it follows from Equation 1.12 that

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0$$

where $\dot{E}_{\text{in}} = q$ and $\dot{E}_{\text{out}} = \dot{m}h_{fg}$ is associated with the loss of latent energy due to boiling. Hence

$$q - \dot{m}h_{fg} = 0$$

and the boil-off $\dot{m}$ is

$$\dot{m} = \frac{q}{h_{fg}}$$

$$\dot{m} = \frac{13.06\text{ J/s}}{2 \times 10^5\text{ J/kg}} = 6.53 \times 10^{-5}\text{ kg/s}$$

The loss per day is

$$\dot{m} = 6.53 \times 10^{-5}\text{ kg/s} \times 3600\text{ s/h} \times 24\text{ h/day}$$

$$\dot{m} = 5.64\text{ kg/day} \quad \triangleleft$$

or on a volumetric basis

$$\dot{\forall} = \frac{\dot{m}}{\rho} = \frac{5.64\text{ kg/day}}{804\text{ kg/m}^3} = 0.007\text{ m}^3\text{/day} = 7\text{ liters/day}$$

Comments:

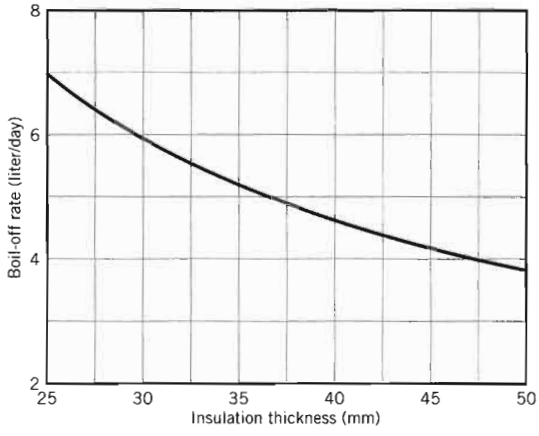
1. Since $R_{t,\text{conv}} \ll R_{t,\text{cond}}$, the dominant contribution to the total thermal resistance is that due to conduction in the insulation. Even if the convection coefficient

3.4 Summary Conducti

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were reduced by a factor of 10, thereby increasing the convection resistance by the same amount, the effect on the boil-off would be small.

2. With a container volume of $(4/3)(\pi r_1^3) = 0.065\ m^3 = 65$ liters, the daily loss amounts to (7 liters/65 liters) 100% = 10.8% of capacity.
3. Using the foregoing model, the boil-off rate has been calculated as a function of the insulation thickness, $\Delta r = (r_2 - r_1)$, where $r_1 = 0.25$ m.



As shown graphically, doubling the insulation thickness from 25 mm to 50 mm reduces the boil-off rate by 45%.

4. The designer of the storage container has been asked to reduce the daily boil-off rate from 7 to 4 liters/day. What is the required thickness of the silica powder insulation? Another option is to use an evacuated foil-mat blanket for cryogenic applications with a thermal conductivity of 0.00016 W/m · K and a thickness of 5 mm. What is the corresponding daily boil-off rate? [Answers: 47.5 mm, 3.1 liter/day]

3.4 *Summary of One-Dimensional Conduction Results*

Many important problems are characterized by one-dimensional, steady-state conduction in plane, cylindrical, or spherical walls without thermal energy generation. Key results for these three geometries are summarized in Table 3.3, where ΔT refers to the temperature difference, $T_{s,1} - T_{s,2}$, between the inner and outer surfaces identified in Figures 3.1, 3.6, and 3.8. In each case, beginning with the heat equation, you should be able to derive the corresponding expressions for the temperature distribution, heat flux, heat rate, and thermal resistance.

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TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\text{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

3.5 Conduction with Thermal Energy Generation

In the preceding section we considered conduction problems for which the temperature distribution in a medium was determined solely by conditions at the boundaries of the medium. We now want to consider the additional effect on the temperature distribution of processes that may be occurring *within* the medium. In particular, we wish to consider situations for which thermal energy is being *generated* due to *conversion* from some other energy form.

A common thermal energy generation process involves the conversion from *electrical to thermal energy* in a current-carrying medium (*Ohmic*, or *resistance*, or *Joule heating*). The rate at which energy is generated by passing a current I through a medium of electrical resistance R_e is

$$\dot{E}_g = I^2 R_e \tag{3.37}$$

If this power generation (W) occurs uniformly throughout the medium of volume V, the volumetric generation rate (W/m^3) is then

$$\dot{q} \equiv \frac{\dot{E}_g}{V} = \frac{I^2 R_e}{V} \tag{3.38}$$

Energy generation may also occur as a result of the deceleration and absorption of neutrons in the fuel element of a nuclear reactor or exothermic chemical reactions occurring within a medium. Endothermic reactions would, of course, have the inverse effect (a thermal energy sink) of converting thermal energy to chemical bonding energy. Finally, a conversion from electromagnetic to thermal energy may occur due to the absorption of radiation within the medium. The process occurs, for example, when gamma rays are absorbed in external nuclear reactor components (cladding,

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thermal shields, pressure vessels, etc.) or when visible radiation is absorbed in a semitransparent medium. Remember not to confuse energy generation with energy storage (Section 1.3.1).

3.5.1 The Plane Wall

Consider the plane wall of Figure 3.9*a*, in which there is *uniform* energy generation per unit volume ($\dot{q}$ is constant) and the surfaces are maintained at $T_{s,1}$ and $T_{s,2}$. For constant thermal conductivity k, the appropriate form of the heat equation, Equation 2.20, is

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0 \tag{3.39}$$

The general solution is

$$T = -\frac{\dot{q}}{2k}x^2 + C_1x + C_2 \tag{3.40}$$

where C_1 and C_2 are the constants of integration. For the prescribed boundary conditions,

$$T(-L) = T_{s,1} \qquad \text{and} \qquad T(L) = T_{s,2}$$

The constants may be evaluated and are of the form

$$C_1 = \frac{T_{s,2} - T_{s,1}}{2L} \qquad \text{and} \qquad C_2 = \frac{\dot{q}}{2k}L^2 + \frac{T_{s,1} + T_{s,2}}{2}$$

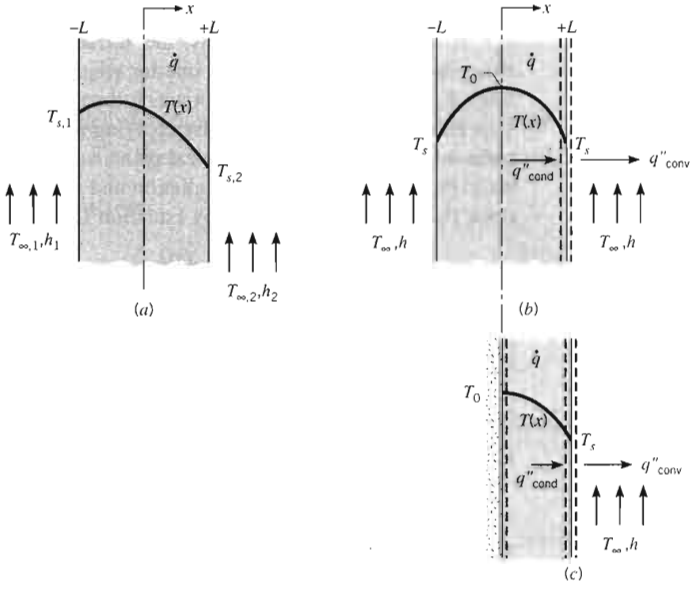


FIGURE 3.9 Conduction in a plane wall with uniform heat generation. (*a*) Asymmetrical boundary conditions. (*b*) Symmetrical boundary conditions. (*c*) Adiabatic surface at midplane.

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in which case the temperature distribution is

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \tag{3.41}$$

The heat flux at any point in the wall may, of course, be determined by using Equation 3.41 with Fourier's law. Note, however, that *with generation the heat flux is no longer independent of x.*

The preceding result simplifies when both surfaces are maintained at a common temperature, $T_{s,1} = T_{s,2} \equiv T_s$. The temperature distribution is then *symmetrical* about the midplane, Figure 3.9*b*, and is given by

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \tag{3.42}$$

The maximum temperature exists at the midplane

$$T(0) \equiv T_0 = \frac{\dot{q}L^2}{2k} + T_s \tag{3.43}$$

in which case the temperature distribution, Equation 3.42, may be expressed as

$$\frac{T(x) - T_0}{T_s - T_0} = \left(\frac{x}{L}\right)^2 \tag{3.44}$$

It is important to note that at the plane of symmetry in Figure 3.9*b*, the temperature gradient is zero, $(dT/dx)_{x=0} = 0$. Accordingly, there is no heat transfer across this plane, and it may be represented by the *adiabatic* surface shown in Figure 3.9*c*. One implication of this result is that Equation 3.42 also applies to plane walls that are perfectly insulated on one side ($x = 0$) and maintained at a fixed temperature T_s on the other side ($x = L$).

To use the foregoing results, the surface temperature(s) T_s must be known. However, a common situation is one for which it is the temperature of an adjoining fluid, T_∞, and not T_s, which is known. It then becomes necessary to relate T_s to T_∞. This relation may be developed by applying a surface energy balance. Consider the surface at $x = L$ for the symmetrical plane wall (Figure 3.9*b*) or the insulated plane wall (Figure 3.9*c*). Neglecting radiation and substituting the appropriate rate equations, the energy balance given by Equation 1.12 reduces to

$$-k\left.\frac{dT}{dx}\right|_{x=L} = h(T_s - T_\infty) \tag{3.45}$$

Substituting from Equation 3.42 to obtain the temperature gradient at $x = L$, it follows that

$$T_s = T_\infty + \frac{\dot{q}L}{h} \tag{3.46}$$

Hence T_s may be computed from knowledge of T_∞, $\dot{q}$, L, and h.

Equation 3.46 may also be obtained by applying an *overall* energy balance to the plane wall of Figure 3.9*b* or 3.9*c*. For example, relative to a control surface about the wall of Figure 3.9*c*, the rate at which energy is generated within the wall must be balanced by the rate at which energy leaves via convection at the boundary. Equation 1.11c reduces to

$$\dot{E}_g = \dot{E}_{out} \tag{3.47}$$

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or, for a unit surface area,

$$\dot{q}L = h(T_s - T_\infty) \tag{3.48}$$

Solving for T_s, Equation 3.46 is obtained.

Equation 3.46 may be combined with Equation 3.42 to eliminate T_s from the temperature distribution, which is then expressed in terms of the known quantities $\dot{q}$, L, k, h, and T_∞. The same result may be obtained directly by using Equation 3.45 as a boundary condition to evaluate the constants of integration appearing in Equation 3.40.

EXAMPLE 3.7

A plane wall is a composite of two materials, A and B. The wall of material A has uniform heat generation $\dot{q} = 1.5 \times 10^6$ W/m^3, $k_A = 75$ W/m · K, and thickness $L_A = 50$ mm. The wall material B has no generation with $k_B = 150$ W/m · K and thickness $L_B = 20$ mm. The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream with $T_\infty = 30$°C and $h = 1000$ W/m^2 · K.

1. Sketch the temperature distribution that exists in the composite under steady-state conditions.
2. Determine the temperature T_0 of the insulated surface and the temperature T_2 of the cooled surface.

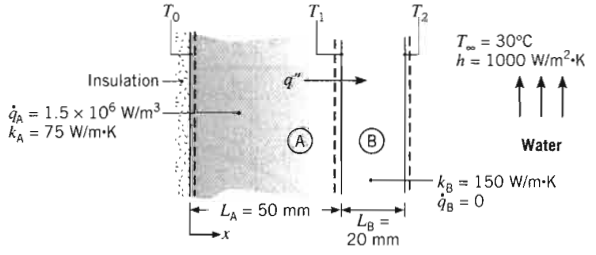
SOLUTION

Known: Plane wall of material A with internal heat generation is insulated on one side and bounded by a second wall of material B, which is without heat generation and is subjected to convection cooling.

Find:

1. Sketch of steady-state temperature distribution in the composite.
2. Inner and outer surface temperatures of the composite.

Schematic:



Assumptions:

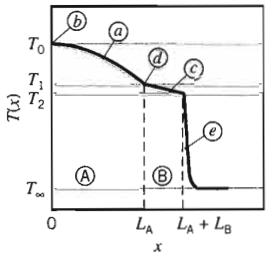
1. Steady-state conditions.
2. One-dimensional conduction in x-direction.
3. Negligible contact resistance between walls.

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4. Inner surface of A adiabatic.
5. Constant properties for materials A and B.

Analysis:

1. From the prescribed physical conditions, the temperature distribution in the composite is known to have the following features, as shown:
   (a) Parabolic in material A.
   (b) Zero slope at insulated boundary.
   (c) Linear in material B.
   (d) Slope change $= k_B/k_A = 2$ at interface.
   The temperature distribution in the water is characterized by
   (e) Large gradients near the surface.



2. The outer surface temperature T_2 may be obtained by performing an energy balance on a control volume about material B. Since there is no generation in this material, it follows that, for steady-state conditions and a unit surface area, the heat flux into the material at $x = L_A$ must equal the heat flux from the material due to convection at $x = L_A + L_B$. Hence

$$q'' = h(T_2 - T_\infty) \tag{1}$$

The heat flux q'' may be determined by performing a second energy balance on a control volume about material A. In particular, since the surface at $x = 0$ is adiabatic, there is no inflow and the rate at which energy is generated must equal the outflow. Accordingly, for a unit surface area,

$$\dot{q}L_A = q'' \tag{2}$$

Combining Equations 1 and 2, the outer surface temperature is

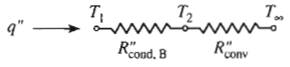
$$T_2 = T_\infty + \frac{\dot{q}L_A}{h}$$

$$T_2 = 30°C + \frac{1.5 \times 10^6\ W/m^3 \times 0.05\ m}{1000\ W/m^2 \cdot K} = 105°C \quad \triangleleft$$

From Equation 3.43 the temperature at the insulated surface is

$$T_0 = \frac{\dot{q}L_A^2}{2k_A} + T_1 \tag{3}$$

where T_1 may be obtained from the following thermal circuit:



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That is,

$$T_1 = T_\infty + (R''_{\text{cond,B}} + R''_{\text{conv}})\, q''$$

where the resistances for a unit surface area are

$$R''_{\text{cond, B}} = \frac{L_B}{k_B} \qquad R''_{\text{conv}} = \frac{1}{h}$$

Hence,

$$T_1 = 30°\text{C} + \left(\frac{0.02\ \text{m}}{150\ \text{W/m} \cdot \text{K}} + \frac{1}{1000\ \text{W/m}^2 \cdot \text{K}}\right) \times 1.5 \times 10^6\ \text{W/m}^3 \times 0.05\ \text{m}$$

$$T_1 = 30°\text{C} + 85°\text{C} = 115°\text{C}$$

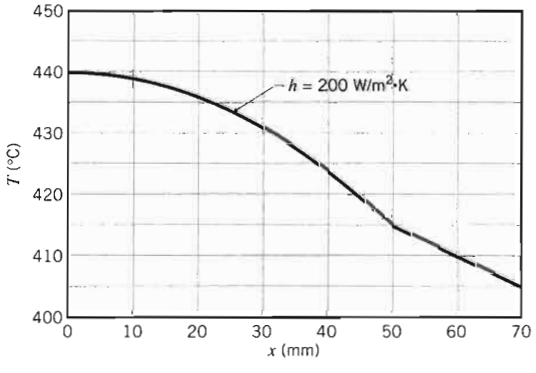
Substituting into Equation 3,

$$T_0 = \frac{1.5 \times 10^6\ \text{W/m}^3\ (0.05\ \text{m})^2}{2 \times 75\ \text{W/m} \cdot \text{K}} + 115°\text{C}$$

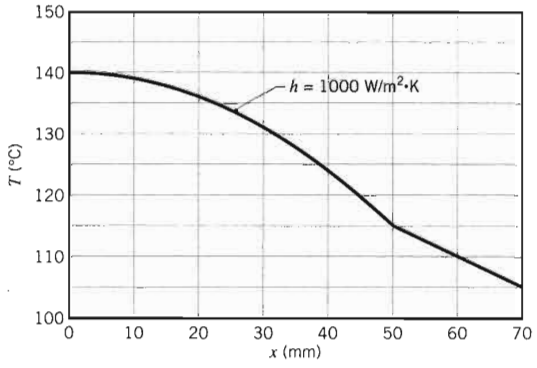
$$T_0 = 25°\text{C} + 115°\text{C} = 140°\text{C} \qquad \triangleleft$$

Comments:

1. Material A, having heat generation, cannot be represented by a thermal circuit element.
2. Since the resistance to heat transfer by convection is significantly larger than that due to conduction in material B, $R''_{\text{conv}}/R''_{\text{cond}} = 7.5$, the surface-to-fluid temperature difference is much larger than the temperature drop across material B, $(T_2 - T_\infty)/(T_1 - T_2) = 7.5$. This result is consistent with the temperature distribution plotted in part 1.
3. The surface and interface temperatures (T_0, T_1, and T_2) depend on the generation rate $\dot{q}$, the thermal conductivities k_A and k_B, and the convection coefficient h. Each material will have a maximum allowable operating temperature, which must not be exceeded if thermal failure of the system is to be avoided. We explore the effect of one of these parameters by computing and plotting temperature distributions for values of $h = 200$ and 1000 $\text{W/m}^2 \cdot \text{K}$, which would be representative of air and liquid cooling, respectively.



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For $h = 200$ W/m$^2 \cdot$ K, there is a significant increase in temperature throughout the system and, depending on the selection of materials, thermal failure could be a problem. Note the slight discontinuity in the temperature gradient, dT/dx, at $x = 50$ mm. What is the physical basis for this discontinuity? We have assumed negligible contact resistance at this location. What would be the effect of such a resistance on the temperature distribution throughout the system? Sketch a representative distribution. What would be the effect on the temperature distribution of an increase in $\dot{q}$, k_A, or k_B? Qualitatively sketch the effect of such changes on the temperature distribution.

4. This example is provided as two ready-to-solve models in *IHT*, which may be accessed in *Examples* on the *menu bar*. The first approach uses the model builder, *Models/1-D, Steady-State Conduction*, which solves for temperature distributions and heat rates in plane walls, cylinders, and spheres. The second approach demonstrates how to represent temperature distributions as *User-Defined Functions*, which for this situation represents two piece-wise distributions in materials A (quadratic) and B (linear).

3.5.2 Radial Systems

Heat generation may occur in a variety of radial geometries. Consider the long, solid cylinder of Figure 3.10, which could represent a current-carrying wire or a fuel

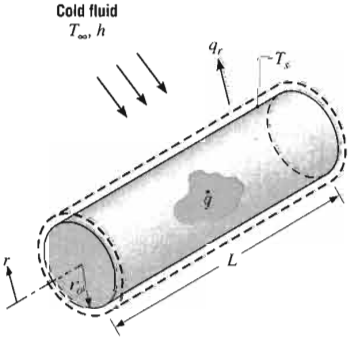


FIGURE 3.10
Conduction in a solid cylinder with uniform heat generation.

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element in a nuclear reactor. For steady-state conditions the rate at which heat is generated within the cylinder must equal the rate at which heat is convected from the surface of the cylinder to a moving fluid. This condition allows the surface temperature to be maintained at a fixed value of T_s.

To determine the temperature distribution in the cylinder, we begin with the appropriate form of the heat equation. For constant thermal conductivity k, Equation 2.24 reduces to

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{\dot{q}}{k}=0 \tag{3.49}$$

Separating variables and assuming uniform generation, this expression may be integrated to obtain

$$r\frac{dT}{dr}=-\frac{\dot{q}}{2k}r^2+C_1 \tag{3.50}$$

Repeating the procedure, the general solution for the temperature distribution becomes

$$T(r)=-\frac{\dot{q}}{4k}r^2+C_1\ln r+C_2 \tag{3.51}$$

To obtain the constants of integration C_1 and C_2, we apply the boundary conditions

$$\left.\frac{dT}{dr}\right|_{r=0}=0 \qquad \text{and} \qquad T(r_o)=T_s$$

The first condition results from the symmetry of the situation. That is, for the solid cylinder the centerline is a line of symmetry for the temperature distribution and the temperature gradient must be zero. Recall that similar conditions existed at the midplane of a wall having symmetrical boundary conditions (Figure 3.9*b*). From the symmetry condition at $r = 0$ and Equation 3.50, it is evident that $C_1 = 0$. Using the surface boundary condition at $r = r_o$ with Equation 3.51, we then obtain

$$C_2=T_s+\frac{\dot{q}}{4k}r_o^2 \tag{3.52}$$

The temperature distribution is therefore

$$T(r)=\frac{\dot{q}r_o^2}{4k}\left(1-\frac{r^2}{r_o^2}\right)+T_s \tag{3.53}$$

Evaluating Equation 3.53 at the centerline and dividing the result into Equation 3.53, we obtain the temperature distribution in nondimensional form,

$$\frac{T(r)-T_s}{T_o-T_s}=1-\left(\frac{r}{r_o}\right)^2 \tag{3.54}$$

where T_o is the centerline temperature. The heat rate at any radius in the cylinder may, of course, be evaluated by using Equation 3.53 with Fourier's law.

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To relate the surface temperature, T_s, to the temperature of the cold fluid, T_∞, either a surface energy balance or an overall energy balance may be used. Choosing the second approach, we obtain

$$\dot{q}(\pi r_o^2 L) = h(2\pi r_o L)(T_s - T_\infty)$$

or

$$T_s = T_\infty + \frac{\dot{q} r_o}{2h} \tag{3.55}$$

A convenient and systematic procedure for treating the different combinations of surface conditions, which may be applied to one-dimensional planar and radial (cylindrical and spherical) geometries with uniform thermal energy generation, is provided in Appendix C. From the tabulated results of this appendix, it is a simple matter to obtain distributions of the temperature, heat flux, and heat rate for boundary conditions of the *second kind* (a uniform surface heat flux) and the *third kind* (a surface heat flux that is proportional to a convection coefficient h or the overall heat transfer coefficient U). You are encouraged to become familiar with the contents of the appendix.

EXAMPLE 3.8

Consider a long solid tube, insulated at the outer radius r_2 and cooled at the inner radius r_1, with uniform heat generation $\dot{q}$ (W/m^3) within the solid.

1. Obtain the general solution for the temperature distribution in the tube.
2. In a practical application a limit would be placed on the maximum temperature that is permissible at the insulated surface ($r = r_2$). Specifying this limit as $T_{s,2}$, identify appropriate boundary conditions that could be used to determine the arbitrary constants appearing in the general solution. Determine these constants and the corresponding form of the temperature distribution.
3. Determine the heat removal rate per unit length of tube.
4. If the coolant is available at a temperature T_∞, obtain an expression for the convection coefficient that would have to be maintained at the inner surface to allow for operation at prescribed values of $T_{s,2}$ and $\dot{q}$.

SOLUTION

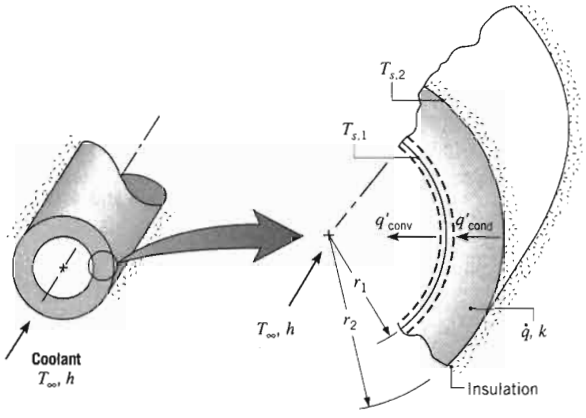
Known: Solid tube with uniform heat generation is insulated at the outer surface and cooled at the inner surface.

Find:

1. General solution for the temperature distribution $T(r)$.
2. Appropriate boundary conditions and the corresponding form of the temperature distribution.
3. Heat removal rate for specified maximum temperature.
4. Corresponding required convection coefficient at the inner surface.

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Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional radial conduction.
3. Constant properties.
4. Uniform volumetric heat generation.
5. Outer surface adiabatic.

Analysis:

1. To determine $T(r)$, the appropriate form of the heat equation, Equation 2.24, must be solved. For the prescribed conditions, this expression reduces to Equation 3.49, and the general solution is given by Equation 3.51. Hence, this solution applies in a cylindrical shell, as well as in a solid cylinder (Figure 3.10).
2. Two boundary conditions are needed to evaluate C_1 and C_2, and in this problem it is appropriate to specify both conditions at r_2. Invoking the prescribed temperature limit,

$$T(r_2) = T_{s,2} \tag{1}$$

and applying Fourier's law, Equation 3.24, at the adiabatic outer surface

$$\left.\frac{dT}{dr}\right|_{r_2} = 0 \tag{2}$$

Using Equations 3.51 and 1, it follows that

$$T_{s,2} = -\frac{\dot{q}}{4k} r_2^2 + C_1 \ln r_2 + C_2 \tag{3}$$

Similarly, from Equations 3.50 and 2

$$0 = -\frac{\dot{q}}{2k} r_2^2 + C_1 \tag{4}$$

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Hence, from Equation 4,

$$C_1 = \frac{\dot{q}}{2k} r_2^2 \tag{5}$$

and from Equation 3

$$C_2 = T_{s,2} + \frac{\dot{q}}{4k} r_2^2 - \frac{\dot{q}}{2k} r_2^2 \ln r_2 \tag{6}$$

Substituting Equations 5 and 6 into the general solution, Equation 3.51, it follows that

$$T(r) = T_{s,2} + \frac{\dot{q}}{4k}(r_2^2 - r^2) - \frac{\dot{q}}{2k} r_2^2 \ln \frac{r_2}{r} \tag{7}$$

3. The heat removal rate may be determined by obtaining the conduction rate at r_1 or by evaluating the total generation rate for the tube. From Fourier's law

$$q_r' = -k2\pi r \frac{dT}{dr}$$

Hence, substituting from Equation 7 and evaluating the result at r_1,

$$q_r'(r_1) = -k2\pi r_1 \left(-\frac{\dot{q}}{2k} r_1 + \frac{\dot{q}}{2k} \frac{r_2^2}{r_1} \right) = -\pi \dot{q}(r_2^2 - r_1^2) \tag{8}$$

Alternatively, because the tube is insulated at r_2, the rate at which heat is generated in the tube must equal the rate of removal at r_1. That is, for a control volume about the tube, the energy conservation requirement, Equation 1.11c, reduces to $\dot{E}_g - \dot{E}_{out} = 0$, where $\dot{E}_g = \dot{q}\pi(r_2^2 - r_1^2)L$ and $\dot{E}_{out} = q_{cond}' L = -q_r'(r_1)L$. Hence

$$q_r'(r_1) = -\pi \dot{q}(r_2^2 - r_1^2) \tag{9}$$

4. Applying the energy conservation requirement, Equation 1.12, to the inner surface, it follows that

$$q_{cond}' = q_{conv}'$$

or

$$\pi \dot{q}(r_2^2 - r_1^2) = h2\pi r_1 (T_{s,1} - T_\infty)$$

Hence

$$h = \frac{\dot{q}(r_2^2 - r_1^2)}{2r_1(T_{s,1} - T_\infty)} \tag{10}$$

where $T_{s,1}$ may be obtained by evaluating Equation 7 at $r = r_1$.

Comments:

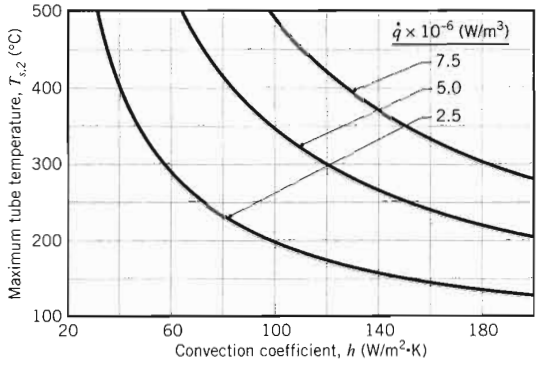
1. Note that, through application of Fourier's law in part 3, the sign on $q_r'(r_1)$ was found to be negative, Equation 8, implying that heat flow is in the negative r direction. However, in applying the energy balance, we acknowledged that heat flow was *out* of the wall. Hence we expressed q_{cond}' as $-q_r'(r_1)$ and we expressed q_{conv}' in terms of $(T_{s,1} - T_\infty)$, rather than $(T_\infty - T_{s,1})$.

2. Results of the foregoing analysis may be used to determine the convection coefficient required to maintain the maximum tube temperature, $T_{s,2}$, below a prescribed value. Consider a tube of thermal conductivity $k = 5$ W/m · K

3.6 Heat Tran

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and inner and outer radii of $r_1 = 20$ mm and $r_2 = 25$ mm, respectively, with a maximum allowable temperature of $T_{s,2} = 350°C$. The tube experiences heat generation at a rate of $\dot{q} = 5 \times 10^6$ W/m^3, and the coolant is at a temperature of $T_\infty = 80°C$. Obtaining $T(r_1) = T_{s,1} = 336.5°C$ from Equation 7 and substituting into Equation 10, the required convection coefficient is found to be $h = 110$ W/m$^2 \cdot$ K. Using the *IHT Workspace*, parametric calculations may be performed to determine the effects of the convection coefficient and the generation rate on the maximum tube temperature, and results are plotted as a function of h for three values of $\dot{q}$.



For each generation rate, the minimum value of h needed to maintain $T_{s,2} \leq 350°C$ may be determined from the figure.

3. The temperature distribution, Equation 7, may also be obtained by using the results of Appendix C. Applying a surface energy balance at $r = r_1$, with $q(r_1) = -\dot{q}\pi(r_2^2 - r_1^2)L$, $(T_{s,2} - T_{s,1})$ may be determined from Equation C.8 and the result substituted into Equation C.2 to eliminate $T_{s,1}$ and obtain the desired expression.

3.5.3 Application of Resistance Concepts

We conclude our discussion of heat generation effects with a word of caution. In particular, when such effects are present, the heat transfer rate is not a constant, independent of the spatial coordinate. Accordingly, it would be *incorrect* to use the conduction resistance concepts and the related heat rate equations developed in Sections 3.1 and 3.3.

3.6 *Heat Transfer from Extended Surfaces*

The term *extended surface* is commonly used to depict an important special case involving heat transfer by conduction within a solid and heat transfer by convection (and/or radiation) from the boundaries of the solid. Until now, we have considered heat transfer from the boundaries of a solid to be in the same direction as heat

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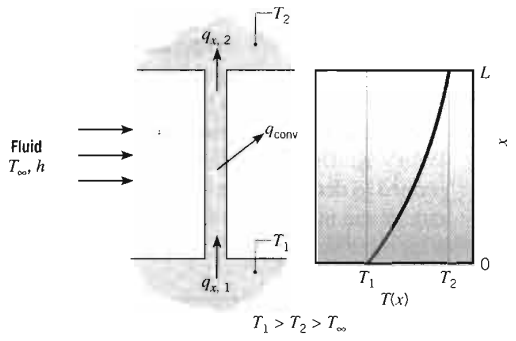


FIGURE 3.11 Combined conduction and convection in a structural element.

transfer by conduction in the solid. In contrast, for an extended surface, the direction of heat transfer from the boundaries is perpendicular to the principal direction of heat transfer in the solid.

Consider a strut that connects two walls at different temperatures and across which there is fluid flow (Figure 3.11). With $T_1 > T_2$, temperature gradients in the x-direction sustain heat transfer by conduction in the strut. However, with $T_1 > T_2 > T_\infty$, there is concurrent heat transfer by convection to the fluid, causing q_x, and hence the magnitude of the temperature gradient, $|dT/dx|$, to decrease with increasing x.

Although there are many different situations that involve such combined conduction–convection effects, the most frequent application is one in which an extended surface is used specifically to *enhance* heat transfer between a solid and an adjoining fluid. Such an extended surface is termed a *fin*.

Consider the plane wall of Figure 3.12*a*. If T_s is fixed, there are two ways in which the heat transfer rate may be increased. The convection coefficient h could be increased by increasing the fluid velocity, and/or the fluid temperature T_∞ could be reduced. However, there are many situations for which increasing h to the maximum possible value is either insufficient to obtain the desired heat transfer rate or the associated costs are prohibitive. Such costs are related to the blower or pump

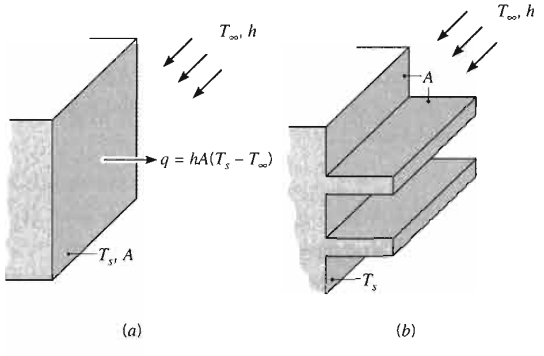


FIGURE 3.12 Use of fins to enhance heat transfer from a plane wall. (*a*) Bare surface. (*b*) Finned surface.

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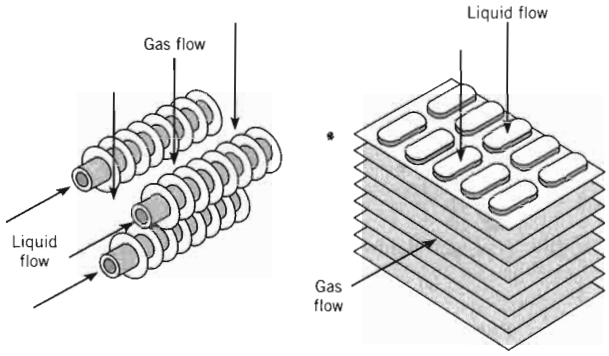


FIGURE 3.13 Schematic of typical finned-tube heat exchangers.

power requirements needed to increase h through increased fluid motion. Moreover, the second option of reducing T_∞ is often impractical. Examining Figure 3.12*b*, however, we see that there exists a third option. That is, the heat transfer rate may be increased by increasing the surface area across which the convection occurs. This may be done by employing *fins* that *extend* from the wall into the surrounding fluid. The thermal conductivity of the fin material can have a strong effect on the temperature distribution along the fin and therefore influences the degree to which the heat transfer rate is enhanced. Ideally, the fin material should have a large thermal conductivity to minimize temperature variations from its base to its tip. In the limit of infinite thermal conductivity, the entire fin would be at the temperature of the base surface, thereby providing the maximum possible heat transfer enhancement.

Examples of fin applications are easy to find. Consider the arrangement for cooling engine heads on motorcycles and lawn mowers or for cooling electric power transformers. Consider also the tubes with attached fins used to promote heat exchange between air and the working fluid of an air conditioner. Two common finned-tube arrangements are shown in Figure 3.13.

Different fin configurations are illustrated in Figure 3.14. A *straight fin* is any extended surface that is attached to a *plane wall.* It may be of uniform cross-sectional area, or its cross-sectional area may vary with the distance x from the wall. An *annular fin* is one that is circumferentially attached to a cylinder, and its cross section varies with radius from the wall of the cylinder. The foregoing fin types have rectangular cross sections, whose area may be expressed as a product of the fin thickness t and the width w for straight fins or the circumference $2\pi r$ for annular fins. In contrast a *pin fin,* or *spine,* is an extended surface of circular cross section. Pin fins may also be of uniform or nonuniform cross section. In any application, selection of a particular fin configuration may depend on space, weight, manufacturing, and cost considerations, as well as on the extent to which the fins reduce the surface convection coefficient and increase the pressure drop associated with flow over the fins.

3.6.1 A General Conduction Analysis

As engineers we are primarily interested in knowing the extent to which particular extended surfaces or fin arrangements could improve heat transfer from a surface

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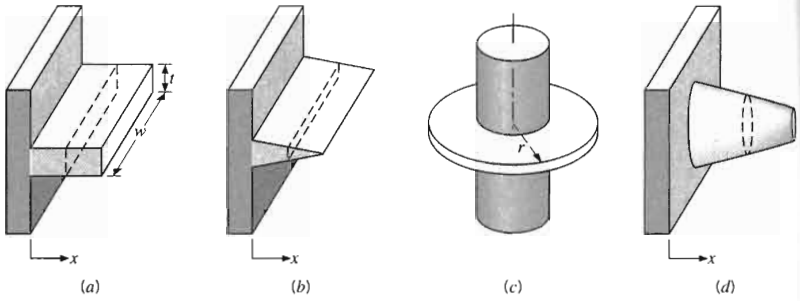


FIGURE 3.14 Fin configurations. (*a*) Straight fin of uniform cross section. (*b*) Straight fin of nonuniform cross section. (*c*) Annular fin. (*d*) Pin fin.

to the surrounding fluid. To determine the heat transfer rate associated with a fin, we must first obtain the temperature distribution along the fin. As we have done for previous systems, we begin by performing an energy balance on an appropriate differential element. Consider the extended surface of Figure 3.15. The analysis is simplified if certain assumptions are made. We choose to assume one-dimensional conditions in the longitudinal (x) direction, even though conduction within the fin is actually two dimensional. The rate at which energy is convected to the fluid from any point on the fin surface must be balanced by the net rate at which energy reaches that point due to conduction in the transverse (y, z) direction. However, in practice the fin is thin, and temperature changes in the transverse direction within the fin are small compared with the temperature difference between the fin and the environment. Hence, we may assume that the temperature is uniform across the fin thickness, that is, it is only a function of x. We will consider steady-state conditions and also assume that the thermal conductivity is constant, that radiation from the surface is negligible, that heat generation effects are absent, and that the convection heat transfer coefficient h is uniform over the surface.

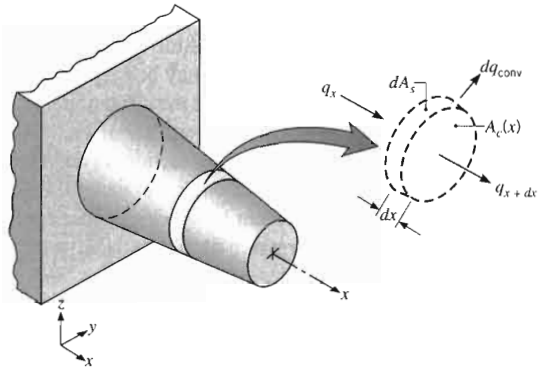
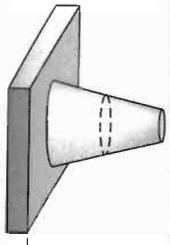


FIGURE 3.15 Energy balance for an extended surface.

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(*d*)

. (*b*) Straight fin of

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Applying the conservation of energy requirement, Equation 1.11c, to the differential element of Figure 3.15, we obtain

$$q_x = q_{x+dx} + dq_{\text{conv}} \tag{3.56}$$

From Fourier's law we know that

$$q_x = -kA_c \frac{dT}{dx} \tag{3.57}$$

where A_c is the *cross-sectional* area, which may vary with x. Since the conduction heat rate at $x + dx$ may be expressed as

$$q_{x+dx} = q_x + \frac{dq_x}{dx}\, dx \tag{3.58}$$

it follows that

$$q_{x+dx} = -kA_c \frac{dT}{dx} - k \frac{d}{dx}\left(A_c \frac{dT}{dx}\right) dx \tag{3.59}$$

The convection heat transfer rate may be expressed as

$$dq_{\text{conv}} = h\, dA_s (T - T_\infty) \tag{3.60}$$

where dA_s is the *surface* area of the differential element. Substituting the foregoing rate equations into the energy balance, Equation 3.56, we obtain

$$\frac{d}{dx}\left(A_c \frac{dT}{dx}\right) - \frac{h}{k}\frac{dA_s}{dx}(T - T_\infty) = 0$$

or

$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \left(\frac{1}{A_c}\frac{h}{k}\frac{dA_s}{dx}\right)(T - T_\infty) = 0 \tag{3.61}$$

This result provides a general form of the energy equation for an extended surface. Its solution for appropriate boundary conditions provides the temperature distribution, which may be used with Equation 3.57 to calculate the conduction rate at any x.

3.6.2 Fins of Uniform Cross-Sectional Area

To solve Equation 3.61 it is necessary to be more specific about the geometry. We begin with the simplest case of straight rectangular and pin fins of uniform cross section (Figure 3.16). Each fin is attached to a base surface of temperature $T(0) = T_b$ and extends into a fluid of temperature T_∞.

For the prescribed fins, A_c is a constant and $A_s = Px$, where A_s is the surface area measured from the base to x and P is the fin perimeter. Accordingly, with $dA_c/dx = 0$ and $dA_s/dx = P$, Equation 3.61 reduces to

$$\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0 \tag{3.62}$$

To simplify the form of this equation, we transform the dependent variable by defining an *excess temperature* θ as

$$\theta(x) \equiv T(x) - T_\infty \tag{3.63}$$

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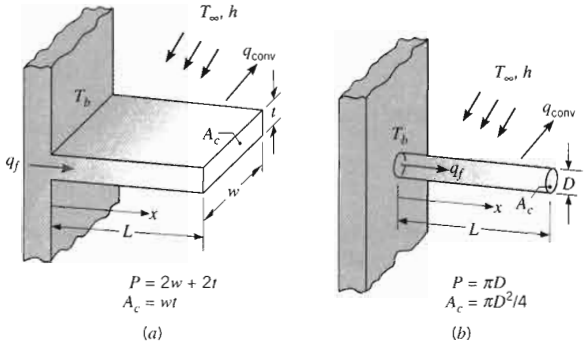


FIGURE 3.16 Straight fins of uniform cross section. (*a*) Rectangular fin. (*b*) Pin fin.

where, since T_∞ is a constant, $d\theta/dx = dT/dx$. Substituting Equation 3.63 into Equation 3.62, we then obtain

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0 \tag{3.64}$$

where

$$m^2 \equiv \frac{hP}{kA_c} \tag{3.65}$$

Equation 3.64 is a linear, homogeneous, second-order differential equation with constant coefficients. Its general solution is of the form

$$\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \tag{3.66}$$

By substitution it may readily be verified that Equation 3.66 is indeed a solution to Equation 3.64.

To evaluate the constants C_1 and C_2 of Equation 3.66, it is necessary to specify appropriate boundary conditions. One such condition may be specified in terms of the temperature at the *base* of the fin ($x = 0$)

$$\theta(0) = T_b - T_\infty \equiv \theta_b \tag{3.67}$$

The second condition, specified at the fin tip ($x = L$), may correspond to one of four different physical situations.

The first condition, Case A, considers convection heat transfer from the fin tip. Applying an energy balance to a control surface about this tip (Figure 3.17), we obtain

$$hA_c[T(L) - T_\infty] = -kA_c \left.\frac{dT}{dx}\right|_{x=L}$$

or

$$h\theta(L) = -k \left.\frac{d\theta}{dx}\right|_{x=L} \tag{3.68}$$

That is, the rate at which energy is transferred to the fluid by convection from the tip must equal the rate at which energy reaches the tip by conduction through

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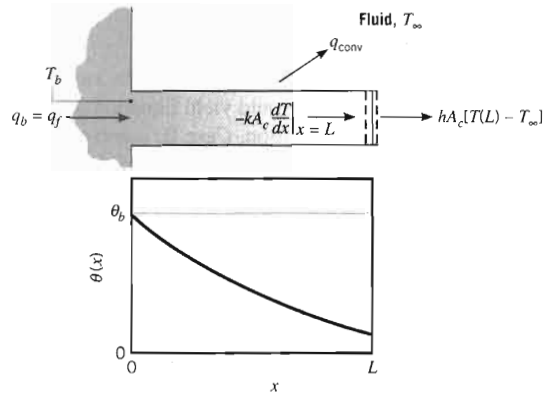


FIGURE 3.17 Conduction and convection in a fin of uniform cross section.

the fin. Substituting Equation 3.66 into Equations 3.67 and 3.68, we obtain, respectively,

$$\theta_b = C_1 + C_2 \tag{3.69}$$

and

$$h(C_1 e^{mL} + C_2 e^{-mL}) = km(C_2 e^{-mL} - C_1 e^{mL})$$

Solving for C_1 and C_2, it may be shown, after some manipulation, that

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L - x) + (h/mk) \sinh m(L - x)}{\cosh mL + (h/mk) \sinh mL} \tag{3.70}$$

The form of this temperature distribution is shown schematically in Figure 3.17. Note that the magnitude of the temperature gradient decreases with increasing x. This trend is a consequence of the reduction in the conduction heat transfer $q_x(x)$ with increasing x due to continuous convection losses from the fin surface.

We are particularly interested in the amount of heat transferred from the entire fin. From Figure 3.17 it is evident that the fin heat transfer rate q_f may be evaluated in two alternative ways, both of which involve use of the temperature distribution. The simpler procedure, and the one that we will use, involves applying Fourier's law at the fin base. That is,

$$q_f = q_b = -kA_c \frac{dT}{dx}\bigg|_{x=0} = -kA_c \frac{d\theta}{dx}\bigg|_{x=0} \tag{3.71}$$

Hence, knowing the temperature distribution, $\theta(x)$, q_f may be evaluated, giving

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\sinh mL + (h/mk) \cosh mL}{\cosh mL + (h/mk) \sinh mL} \tag{3.72}$$

However, conservation of energy dictates that the rate at which heat is transferred by convection from the fin must equal the rate at which it is conducted through the base of the fin. Accordingly, the alternative formulation for q_f is

$$q_f = \int_{A_f} h[T(x) - T_\infty]\, dA_s$$

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$$q_f = \int_{A_f} h\theta(x)\, dA_s \tag{3.73}$$

where A_f is the *total,* including the tip, *fin surface area.* Substitution of Equation 3.70 into Equation 3.73 would yield Equation 3.72.

The second tip condition, Case B, corresponds to the assumption that the convective heat loss from the fin tip is negligible, in which case the tip may be treated as adiabatic and

$$\left.\frac{d\theta}{dx}\right|_{x=L} = 0 \tag{3.74}$$

Substituting from Equation 3.66 and dividing by m, we then obtain

$$C_1 e^{mL} - C_2 e^{-mL} = 0$$

Using this expression with Equation 3.69 to solve for C_1 and C_2 and substituting the results into Equation 3.66, we obtain

$$\frac{\theta}{\theta_b} = \frac{\cosh m(L-x)}{\cosh mL} \tag{3.75}$$

Using this temperature distribution with Equation 3.71, the fin heat transfer rate is then

$$q_f = \sqrt{hPkA_c}\,\theta_b \tanh mL \tag{3.76}$$

In the same manner, we can obtain the fin temperature distribution and heat transfer rate for Case C, where the temperature is prescribed at the fin tip. That is, the second boundary condition is $\theta(L) = \theta_L$, and the resulting expressions are of the form

$$\frac{\theta}{\theta_b} = \frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL} \tag{3.77}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \frac{\cosh mL - \theta_L/\theta_b}{\sinh mL} \tag{3.78}$$

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\|_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$ (3.70)	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$ (3.72)
B	Adiabatic $d\theta/dx\|_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$ (3.75)	$M \tanh mL$ (3.76)
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$ (3.77)	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$ (3.78)
D	Infinite fin ($L \to \infty$): $\theta(L) = 0$	e^{-mx} (3.79)	M (3.80)

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$

$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\,\theta_b$

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The *very long fin,* Case D, is an interesting extension of these results. In particular, as $L \to \infty$, $\theta_L \to 0$ and it is easily verified that

$$\frac{\theta}{\theta_b} = e^{-mx} \tag{3.79}$$

$$q_f = \sqrt{hPkA_c}\,\theta_b \tag{3.80}$$

The foregoing results are summarized in Table 3.4. A table of hyperbolic functions is provided in Appendix B.1.

EXAMPLE 3.9

A very long rod 5 mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 W/m^2 · K.

1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of *infinite length* to yield an accurate estimate of the heat loss.

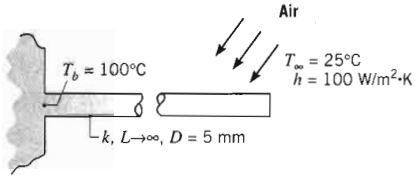
SOLUTION

Known: A long circular rod exposed to ambient air.

Find:

1. Temperature distribution and heat loss when rod is fabricated from copper, an aluminum alloy, or stainless steel.
2. How long rods must be to assume infinite length.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction along the rod.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform heat transfer coefficient.
6. Infinitely long rod.

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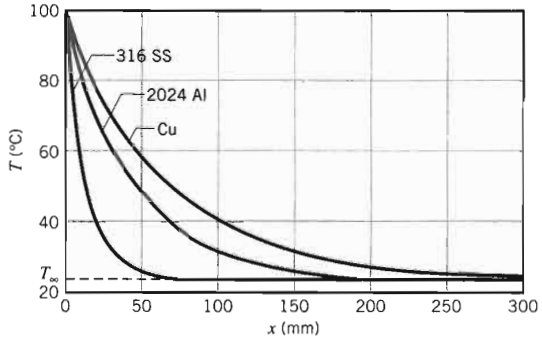
Properties: Table A.1, copper [$T = (T_b + T_\infty)/2 = 62.5°C \approx 335$ K]: $k = 398$ W/m · K. Table A.1, 2024 aluminum (335 K): $k = 180$ W/m · K. Table A.1, stainless steel, AISI 316 (335 K): $k = 14$ W/m · K.

Analysis:

1. Subject to the assumption of an infinitely long fin, the temperature distributions are determined from Equation 3.79, which may be expressed as

$$T = T_\infty + (T_b - T_\infty)e^{-mx}$$

where $m = (hP/kA_c)^{1/2} = (4h/kD)^{1/2}$. Substituting for h and D, as well as for the thermal conductivities of copper, the aluminum alloy, and the stainless steel, respectively, the values of m are 14.2, 21.2, and 75.6 m^{-1}. The temperature distributions may then be computed and plotted as follows.



From these distributions, it is evident that there is little additional heat transfer associated with extending the length of the rod much beyond 50, 200, and 300 mm, respectively, for the stainless steel, the aluminum alloy, and the copper.

From Equation 3.80 the heat loss is

$$q_f = \sqrt{hPkA_c}\,\theta_b$$

Hence for copper,

$$q_f = \left[100\ \text{W/m}^2 \cdot \text{K} \times \pi \times 0.005\ \text{m} \times 398\ \text{W/m} \cdot \text{K} \times \frac{\pi}{4}(0.005\ \text{m})^2\right]^{1/2} (100 - 25)°\text{C}$$

$$= 8.3\ \text{W} \quad \triangleleft$$

Similarly, for the aluminum alloy and stainless steel, respectively, the heat rates are $q_f = 5.6$ W and 1.6 W.

2. Since there is no heat loss from the tip of an infinitely long rod, an estimate of the validity of this approximation may be made by comparing Equations 3.76 and 3.80. To a satisfactory approximation, the expressions provide equivalent results if $\tanh mL \geq 0.99$ or $mL \geq 2.65$. Hence a rod may be assumed to be infinitely long if

$$L \geq L_\infty \equiv \frac{2.65}{m} = 2.65\left(\frac{kA_c}{hP}\right)^{1/2}$$

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For copper,

$$L_\infty = 2.65\left[\frac{398\ \text{W/m}\cdot\text{K}\times(\pi/4)(0.005\ \text{m})^2}{100\ \text{W/m}^2\cdot\text{K}\times\pi(0.005\ \text{m})}\right]^{1/2} = 0.19\ \text{m} \quad \triangleleft$$

Results for the aluminum alloy and stainless steel are $L_\infty = 0.13$ m and $L_\infty = 0.04$ m, respectively.

Comments:

1. The foregoing results suggest that the fin heat transfer rate may accurately be predicted from the infinite fin approximation if $mL \gtrsim 2.65$. However, if the infinite fin approximation is to accurately predict the temperature distribution $T(x)$, a larger value of mL would be required. This value may be inferred from Equation 3.79 and the requirement that the tip temperature be very close to the fluid temperature. Hence, if we require that $\theta(L)/\theta_b = \exp(-mL) < 0.01$, it follows that $mL > 4.6$, in which case $L_c \approx 0.33$, 0.23, and 0.07 m for the copper, aluminum alloy, and stainless steel, respectively. These results are consistent with the distributions plotted in part 1.
2. For the four tip conditions of Table 3.4, the *Models/Extended Surfaces* option of *IHT* provides ready-to-solve models for the heat rate and temperature distribution of straight fins and pin fins of uniform cross section. These models can save considerable time in solving more complicated problems, assuming of course, that you understand the relations and use them properly. See if you can use the *Pin Fins* model to generate the graph of this example. The example is also provided as a solved-model in the *Examples* section of *IHT*.

3.6.3 Fin Performance

Recall that fins are used to increase the heat transfer from a surface by increasing the effective surface area. However, the fin itself represents a conduction resistance to heat transfer from the original surface. For this reason, there is no assurance that the heat transfer rate will be increased through the use of fins. An assessment of this matter may be made by evaluating the *fin effectiveness* ε_f. It is defined as the *ratio of the fin heat transfer rate to the heat transfer rate that would exist without the fin.* Therefore

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} \tag{3.81}$$

where $A_{c,b}$ is the fin cross-sectional area at the base. In any rational design the value of ε_f should be as large as possible, and in general, the use of fins may rarely be justified unless $\varepsilon_f \gtrsim 2$.

Subject to any one of the four tip conditions that have been considered, the effectiveness for a fin of uniform cross section may be obtained by dividing the appropriate expression for q_f in Table 3.4 by $hA_{c,b}\theta_b$. Although the installation of fins will alter the surface convection coefficient, this effect is commonly neglected.

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Hence, assuming the convection coefficient of the finned surface to be equivalent to that of the unfinned base, it follows that, for the infinite fin approximation (Case D), the result is

$$\varepsilon_f = \left(\frac{kP}{hA_c}\right)^{1/2} \tag{3.82}$$

Several important trends may be inferred from this result. Obviously, fin effectiveness is enhanced by the choice of a material of high thermal conductivity. Aluminum alloys and copper come to mind. However, although copper is superior from the standpoint of thermal conductivity, aluminum alloys are the more common choice because of additional benefits related to lower cost and weight. Fin effectiveness is also enhanced by increasing the ratio of the perimeter to the cross-sectional area. For this reason, the use of *thin,* but closely spaced fins, is preferred, with the proviso that the fin gap not be reduced to a value for which flow between the fins is severely impeded, thereby reducing the convection coefficient.

Equation 3.82 also suggests that the use of fins can be better justified under conditions for which the convection coefficient h is small. Hence from Table 1.1 it is evident that the need for fins is stronger when the fluid is a gas rather than a liquid and when the surface heat transfer is by *free* convection. If fins are to be used on a surface separating a gas and a liquid, they are generally placed on the gas side, which is the side of lower convection coefficient. A common example is the tubing in an automobile radiator. Fins are applied to the outer tube surface, over which there is flow of ambient air (small h), and not to the inner surface, through which there is flow of water (large h). Note that, if $\varepsilon_f > 2$ is used as a criterion to justify the implementation of fins, Equation 3.82 yields the requirement that $(kP/hA_c) > 4$.

Equation 3.82 provides an upper limit to ε_f, which is reached as L approaches infinity. However, it is certainly not necessary to use very long fins to achieve near maximum heat transfer enhancement. As seen in Example 3.8, 99% of the maximum possible fin heat transfer rate is achieved for $mL = 2.65$. Hence, it would make no sense to extend the fins beyond $L = 2.65/m$.

Fin performance may also be quantified in terms of a thermal resistance. Treating the difference between the base and fluid temperatures as the driving potential, a *fin resistance* may be defined as

$$R_{t,f} = \frac{\theta_b}{q_f} \tag{3.83}$$

This result is extremely useful, particularly when representing a finned surface by a thermal circuit. Note that, according to the fin tip condition, an appropriate expression for q_f may be obtained from Table 3.4.

Dividing Equation 3.83 into the expression for the thermal resistance due to convection at the exposed base,

$$R_{t,b} = \frac{1}{hA_{c,b}} \tag{3.84}$$

and substituting from Equation 3.81, it follows that

$$\varepsilon_f = \frac{R_{t,b}}{R_{t,f}} \tag{3.85}$$

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Hence the fin effectiveness may be interpreted as a ratio of thermal resistances, and to increase ε_f it is necessary to reduce the conduction/convection resistance of the fin. If the fin is to enhance heat transfer, its resistance must not exceed that of the exposed base.

Another measure of fin thermal performance is provided by the *fin efficiency* η_f. The maximum driving potential for convection is the temperature difference between the base ($x = 0$) and the fluid, $\theta_b = T_b - T_\infty$. Hence the maximum rate at which a fin could dissipate energy is the rate that would exist *if* the entire fin surface were at the base temperature. However, since any fin is characterized by a finite conduction resistance, a temperature gradient must exist along the fin and the above condition is an idealization. A logical definition of fin efficiency is therefore

$$\eta_f \equiv \frac{q_f}{q_{\max}} = \frac{q_f}{hA_f\theta_b} \tag{3.86}$$

where A_f is the surface area of the fin. For a straight fin of uniform cross section and an adiabatic tip, Equations 3.76 and 3.86 yield

$$\eta_f = \frac{M \tanh mL}{hPL\theta_b} = \frac{\tanh mL}{mL} \tag{3.87}$$

Referring to Table B.1, this result tells us that η_f approaches its maximum and minimum values of 1 and 0, respectively, as L approaches 0 and ∞.

In lieu of the somewhat cumbersome expression for heat transfer from a straight rectangular fin with an active tip, Equation 3.72, it has been shown that approximate, yet accurate, predictions may be obtained by using the adiabatic tip result, Equation 3.76, with a corrected fin length of the form $L_c = L + (t/2)$ for a rectangular fin and $L_c = L + (D/4)$ for a pin fin [9]. The correction is based on assuming equivalence between heat transfer from the actual fin with tip convection and heat transfer from a longer, hypothetical fin with an adiabatic tip. Hence, with tip convection, the fin heat rate may be approximated as

$$q_f = M \tanh mL_c \tag{3.88}$$

and the corresponding efficiency as

$$\eta_f = \frac{\tanh mL_c}{mL_c} \tag{3.89}$$

Errors associated with the approximation are negligible if (ht/k) or $(hD/2k) \lesssim 0.0625$ [10].

If the width of a rectangular fin is much larger than its thickness, $w \gg t$, the perimeter may be approximated as $P = 2w$, and

$$mL_c = \left(\frac{hP}{kA_c}\right)^{1/2} L_c = \left(\frac{2h}{kt}\right)^{1/2} L_c$$

Multiplying numerator and denominator by $L_c^{1/2}$ and introducing a corrected fin profile area, $A_p = L_c t$, it follows that

$$mL_c = \left(\frac{2h}{kA_p}\right)^{1/2} L_c^{3/2} \tag{3.90}$$

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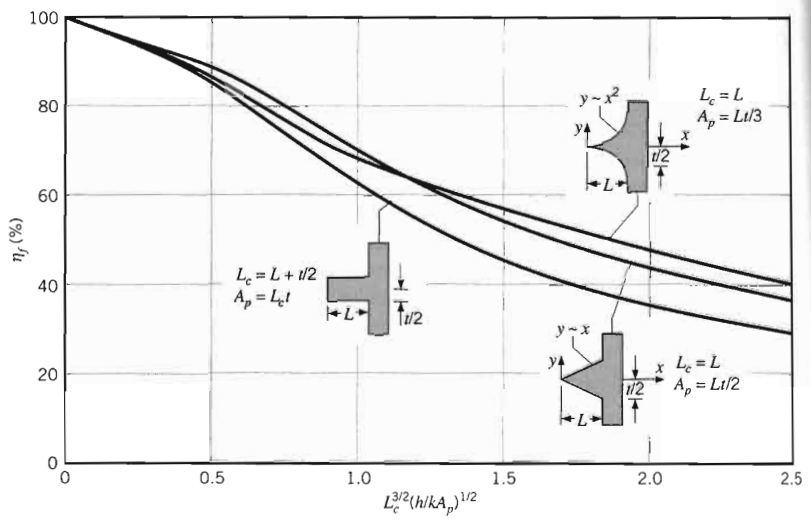


FIGURE 3.18 Efficiency of straight fins (rectangular, triangular, and parabolic profiles).

Hence, as shown in Figures 3.18 and 3.19, the efficiency of a rectangular fin with tip convection may be represented as a function of $L_c^{3/2}(h/kA_p)^{1/2}$.

3.6.4 Fins of Nonuniform Cross-Sectional Area

Analysis of fin thermal behavior becomes more complex if the fin is of nonuniform cross section. For such cases the second term of Equation 3.61 must be retained, and the solutions are no longer in the form of simple exponential or hyperbolic

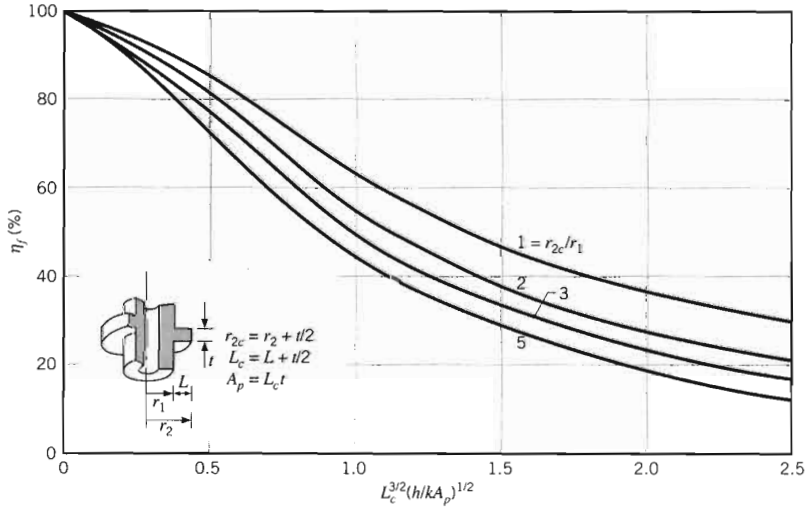
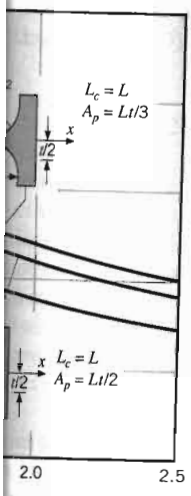


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

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abolic profiles).

ectangular fin with

n is of nonuniform
must be retained,
tial or hyperbolic



functions. As a special case, consider the annular fin shown in the inset of Figure 3.19. Although the fin thickness is uniform (t is independent of r), the cross-sectional area, $A_c = 2\pi rt$, varies with r. Replacing x by r in Equation 3.61 and expressing the surface area as $A_s = 2\pi(r^2 - r_1^2)$, the general form of the fin equation reduces to

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} - \frac{2h}{kt}(T - T_\infty) = 0$$

or, with $m^2 \equiv 2h/kt$ and $\theta \equiv T - T_\infty$,

$$\frac{d^2\theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - m^2\theta = 0$$

The foregoing expression is a *modified Bessel equation* of order zero, and its general solution is of the form

$$\theta(r) = C_1 I_0(mr) + C_2 K_0(mr)$$

where I_0 and K_0 are modified, zero-order Bessel functions of the first and second kinds, respectively. If the temperature at the base of the fin is prescribed, $\theta(r_1) = \theta_b$, and an adiabatic tip is presumed, $d\theta/dr|_{r_2} = 0$, C_1 and C_2 may be evaluated to yield a temperature distribution of the form

$$\frac{\theta}{\theta_b} = \frac{I_0(mr)K_1(mr_2) + K_0(mr)I_1(mr_2)}{I_0(mr_1)K_1(mr_2) + K_0(mr_1)I_1(mr_2)}$$

where $I_1(mr) = d[I_0(mr)]/d(mr)$ and $K_1(mr) = -d[K_0(mr)]/d(mr)$ are modified, first-order Bessel functions of the first and second kinds, respectively. The Bessel functions are tabulated in Appendix B.

With the fin heat transfer rate expressed as

$$q_f = -kA_{c,b}\left.\frac{dT}{dr}\right|_{r=r_1} = -k(2\pi r_1 t)\left.\frac{d\theta}{dr}\right|_{r=r_1}$$

it follows that

$$q_f = 2\pi k r_1 t \theta_b m \frac{K_1(mr_1)I_1(mr_2) - I_1(mr_1)K_1(mr_2)}{K_0(mr_1)I_1(mr_2) + I_0(mr_1)K_1(mr_2)}$$

from which the fin efficiency becomes

$$\eta_f = \frac{q_f}{h2\pi(r_2^2 - r_1^2)\theta_b} = \frac{2r_1}{m(r_2^2 - r_1^2)} \frac{K_1(mr_1)I_1(mr_2) - I_1(mr_1)K_1(mr_2)}{K_0(mr_1)I_1(mr_2) + I_0(mr_1)K_1(mr_2)} \quad (3.91)$$

This result may be applied for an active (convecting) tip, if the tip radius r_2 is replaced by a corrected radius of the form $r_{2c} = r_2 + (t/2)$. Results are represented graphically in Figure 3.19.

Knowledge of the thermal efficiency of a fin may be used to evaluate the fin resistance, where, from Equations 3.83 and 3.86, it follows that

$$R_{t,f} = \frac{1}{hA_f\eta_f} \quad (3.92)$$

Expressions for the efficiency and surface area of several common fin geometries are summarized in Table 3.5. Although results for the fins of uniform thickness or diameter were obtained by assuming an adiabatic tip, the effects of convection

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TABLE 3.5 Efficiency of common fin shapes

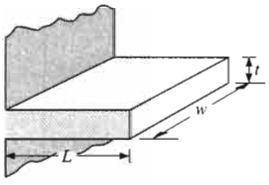
Straight Fins

Rectangular[a]

$A_f = 2wL_c$

$L_c = L + (t/2)$

$A_p = tL$

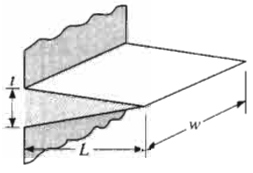


$$\eta_f = \frac{\tanh mL_c}{mL_c} \tag{3.89}$$

Triangular[a]

$A_f = 2w[L^2 + (t/2)^2]^{1/2}$

$A_p = (t/2)L$



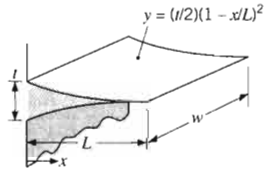
$$\eta_f = \frac{1}{mL}\frac{I_1(2mL)}{I_0(2mL)} \tag{3.93}$$

Parabolic[a]

$A_f = w[C_1L + (L^2/t)\ln(t/L + C_1)]$

$C_1 = [1 + (t/L)^2]^{1/2}$

$A_p = (t/3)L$



$$\eta_f = \frac{2}{[4(mL)^2 + 1]^{1/2} + 1} \tag{3.94}$$

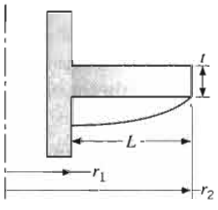
Circular Fin

Rectangular[a]

$A_f = 2\pi(r_{2c}^2 - r_1^2)$

$r_{2c} = r_2 + (t/2)$

$V = \pi(r_2^2 - r_1^2)t$



$$\eta_f = C_2\frac{K_1(mr_1)I_1(mr_{2c}) - I_1(mr_1)K_1(mr_{2c})}{I_0(mr_1)K_1(mr_{2c}) + K_0(mr_1)I_1(mr_{2c})} \tag{3.91}$$

$$C_2 = \frac{(2r_1/m)}{(r_{2c}^2 - r_1^2)}$$

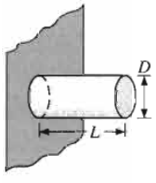
Pin Fins

Rectangular[b]

$A_f = \pi DL_c$

$L_c = L + (D/4)$

$V = (\pi D^2/4)L$

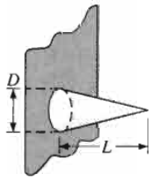


$$\eta_f = \frac{\tanh mL_c}{mL_c} \tag{3.95}$$

Triangular[b]

$A_f = \frac{\pi D}{2}[L^2 + (D/2)^2]^{1/2}$

$V = (\pi/12)D^2L$



$$\eta_f = \frac{2}{mL}\frac{I_2(2mL)}{I_1(2mL)} \tag{3.96}$$

TABLE 3.5 Co

Parabolic[b]

$A_f = \frac{\pi L^3}{8D}\{C_3C_4 -$

$\frac{L}{2D}\ln[(2DC_4$

$C_3 = 1 + 2(D/L)^2$

$C_4 = [1 + (D/L)^2]^1$

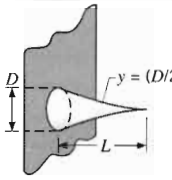
$V = (\pi/20)D^2L$

[a] $m = (2h/kt)^{1/2}$.

[b] $m = (4h/kD)^{1/2}$.

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TABLE 3.5 *Continued*

Parabolic[b]			
$A_f = \frac{\pi L^3}{8D}\{C_3C_4 - \frac{L}{2D}\ln[(2DC_4/L) + C_3]\}$		$\eta_f = \frac{2}{[4/9(mL)^2 + 1]^{1/2} + 1}$	(3.97)
$C_3 = 1 + 2(D/L)^2$			
$C_4 = [1 + (D/L)^2]^{1/2}$			
$V = (\pi/20)D^2L$			

[a] $m = (2h/kt)^{1/2}$.
[b] $m = (4h/kD)^{1/2}$.

may be treated by using a corrected length (Equations 3.89 and 3.95) or radius (Equation 3.91). The triangular and parabolic fins are of nonuniform thickness, which reduces to zero at the fin tip.

Expressions for the profile area, A_p, or the volume, V, of a fin are also provided in Table 3.5. The volume of a straight fin is simply the product of its width and profile area, $V = wA_p$.

Fin design is often motivated by a desire to minimize the fin material and/or related manufacturing costs required to achieve a prescribed cooling effectiveness. Hence, a straight *triangular* fin is attractive because, for equivalent heat transfer, it requires much less volume (fin material) than a rectangular profile. In this regard, heat dissipation per unit volume, $(q/V)_f$, is largest for a *parabolic* profile. However, since $(q/V)_f$ for the parabolic profile is only slightly larger than that for a triangular profile, its use can rarely be justified in view of its larger manufacturing costs. The *annular* fin of rectangular profile is commonly used to enhance heat transfer to or from circular tubes.

3.6.5 Overall Surface Efficiency

In contrast to the fin efficiency η_f, which characterizes the performance of a single fin, the *overall surface efficiency* η_o characterizes an *array* of fins and the base surface to which they are attached. Representative arrays are shown in Figure 3.20, where S designates the fin pitch. In each case the overall efficiency is defined as

$$\eta_o = \frac{q_t}{q_{\max}} = \frac{q_t}{hA_t\theta_b} \tag{3.98}$$

where q_t is the total heat rate from the surface area A_t associated with both the fins and the exposed portion of the base (often termed the *prime* surface). If there are N fins in the array, each of surface area A_f, and the area of the prime surface is designated as A_b, the total surface area is

$$A_t = NA_f + A_b \tag{3.99}$$

The maximum possible heat rate would result if the entire fin surface, as well as the exposed base, were maintained at T_b.

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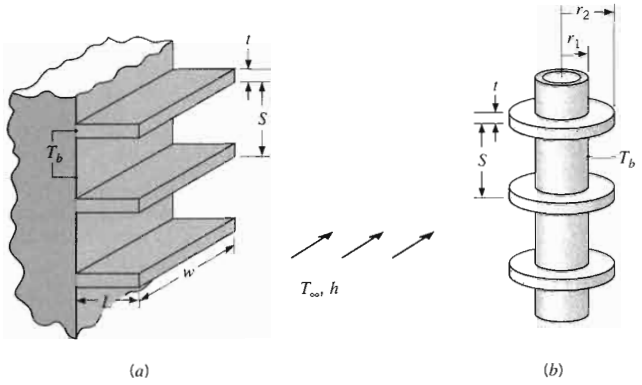


FIGURE 3.20 Representative fin arrays. (*a*) Rectangular fins. (*b*) Annular fins.

The total rate of heat transfer by convection from the fins and the prime (unfinned) surface may be expressed as

$$q_t = N\eta_f hA_f\theta_b + hA_b\theta_b \tag{3.100}$$

where the convection coefficient h is assumed to be equivalent for the finned and prime surfaces and η_f is the efficiency of a single fin. Hence

$$q_t = h[N\eta_f A_f + (A_t - NA_f)]\theta_b = hA_t\left[1 - \frac{NA_f}{A_t}(1 - \eta_f)\right]\theta_b \tag{3.101}$$

Substituting Equation (3.101) into (3.98), it follows that

$$\eta_o = 1 - \frac{NA_f}{A_t}(1 - \eta_f) \tag{3.102}$$

From knowledge of η_o, Equation 3.98 may be used to calculate the total heat rate for a fin array.

Recalling the definition of the fin thermal resistance, Equation 3.83, Equation 3.98 may be used to infer an expression for the thermal resistance of a fin array. That is,

$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o hA_t} \tag{3.103}$$

where $R_{t,o}$ is an effective resistance that accounts for parallel heat flow paths by conduction/convection in the fins and by convection from the prime surface. Figure 3.21 illustrates the thermal circuits corresponding to the parallel paths and their representation in terms of an effective resistance.

If fins are machined as an integral part of the wall from which they extend (Figure 3.21*a*), there is no contact resistance at their base. However, more commonly, fins are manufactured separately and are attached to the wall by a metallurgical or adhesive joint. Alternatively, the attachment may involve a *press fit*, for which the

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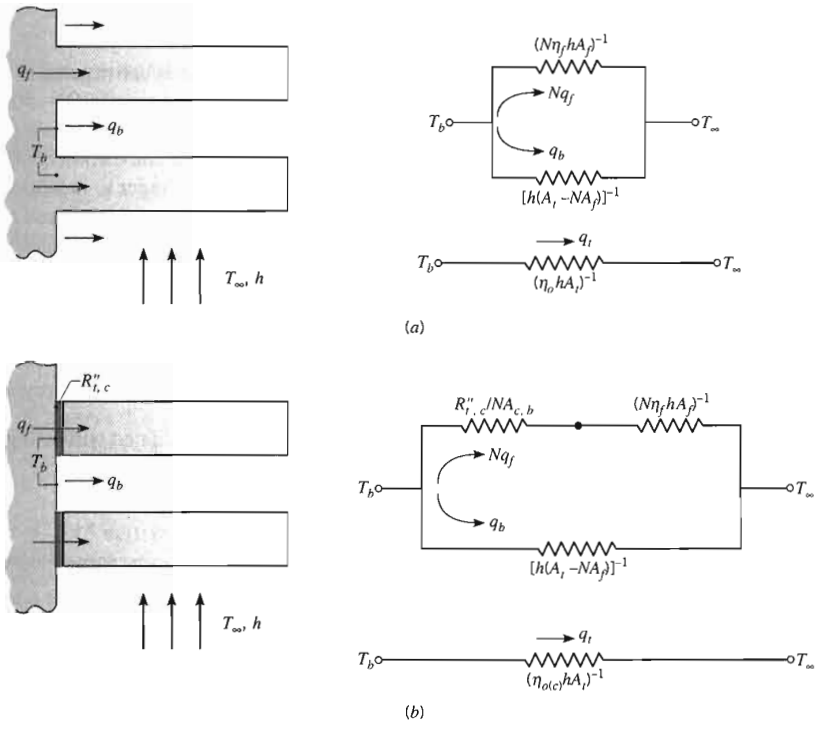


FIGURE 3.21 Fin array and thermal circuit. (*a*) Fins that are integral with the base. (*b*) Fins that are attached to the base.

fins are forced into slots machined on the wall material. In such cases (Figure 3.21*b*), there is a thermal contact resistance, $R_{t,c}$, which may adversely influence overall thermal performance. An effective circuit resistance may again be obtained, where, with the contact resistance,

$$R_{t,o(c)} = \frac{\theta_b}{q_t} = \frac{1}{\eta_{o(c)} h A_t} \tag{3.104}$$

It is readily shown that the corresponding overall surface efficiency is

$$\eta_{o(c)} = 1 - \frac{NA_f}{A_t}\left(1 - \frac{\eta_f}{C_1}\right) \tag{3.105a}$$

where

$$C_1 = 1 + \eta_f h A_f (R''_{t,c}/A_{c,b}) \tag{3.105b}$$

In manufacturing, care must be taken to render $R_{t,c} \ll R_{t,f}$.

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EXAMPLE 3.10

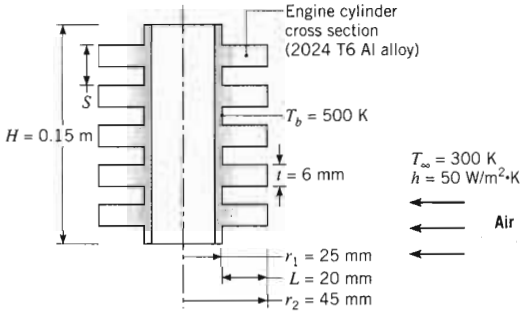
The engine cylinder of a motorcycle is constructed of 2024-T6 aluminum alloy and is of height $H = 0.15$ m and outside diameter $D = 50$ mm. Under typical operating conditions the outer surface of the cylinder is at a temperature of 500 K and is exposed to ambient air at 300 K, with a convection coefficient of 50 W/m$^2 \cdot$ K. Annular fins are integrally cast with the cylinder to increase heat transfer to the surroundings. Consider five such fins, which are of thickness $t = 6$ mm, length $L = 20$ mm, and equally spaced. What is the increase in heat transfer due to use of the fins?

SOLUTION

Known: Operating conditions of a finned motorcycle cylinder.

Find: Increase in heat transfer associated with using fins.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional radial conduction in fins.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform convection coefficient over outer surface (with or without fins).

Properties: Table A.1, 2024-T6 aluminum ($T = 400$ K): $k = 186$ W/m $\cdot$ K.

Analysis: With the fins in place, the heat transfer rate is given by Equation 3.101

$$q_t = hA_t\left[1 - \frac{NA_f}{A_t}(1 - \eta_f)\right]\theta_b$$

where $A_f = 2\pi(r_{2c}^2 - r_1^2) = 2\pi[(0.048\text{ m})^2 - (0.025\text{ m})^2] = 0.0105\text{ m}^2$ and, from Equation 3.99, $A_t = NA_f + 2\pi r_1(H - Nt) = 0.0527\text{ m}^2 + 2\pi(0.025\text{ m})\,[0.15\text{ m} - 0.03\text{ m}] = 0.0716\text{ m}^2$. With $r_{2c}/r_1 = 1.92$, $L_c = 0.023$ m, $A_p = 1.380 \times 10^{-4}$ m^2, we obtain $L_c^{3/2}(h/kA_p)^{1/2} = 0.15$. Hence, from Figure 3.19, the fin efficiency is $\eta_f \approx 0.95$.

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With the fins, the total heat transfer rate is then

$$q_t = 50\ \text{W/m}^2 \cdot \text{K} \times 0.0716\ \text{m}^2 \left[1 - \frac{0.0527\ \text{m}^2}{0.0716\ \text{m}^2}(0.05)\right] 200\ \text{K} = 690\ \text{W}$$

Without the fins, the convection heat transfer rate would be

$$q_{wo} = h(2\pi r_1 H)\theta_b = 50\ \text{W/m}^2 \cdot \text{K}(2\pi \times 0.025\ \text{m} \times 0.15\ \text{m})200\ \text{K} = 236\ \text{W}$$

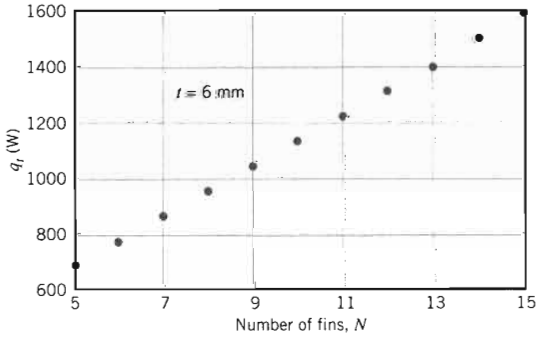
Hence

$$\Delta q = q_t - q_{wo} = 454\ \text{W}$$

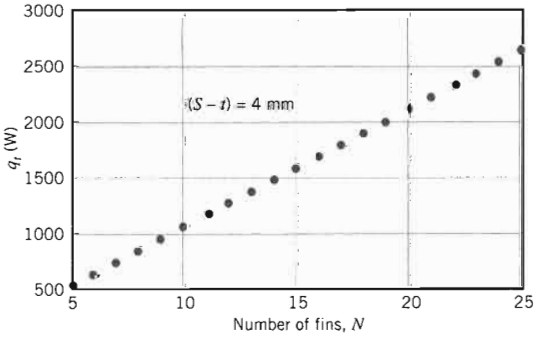
◁

Comments:

1. Although the fins significantly increase heat transfer from the cylinder, considerable improvement could still be obtained by increasing the number of fins. We assess this possibility by computing q_t as a function of N, first by fixing the fin thickness at $t = 6$ mm and increasing the number of fins by reducing the spacing between fins. Prescribing a fin clearance of 2 mm at each end of the array and a minimum fin gap of 4 mm, the maximum allowable number of fins is $N = H/S = 0.15\ \text{m}/(0.004 + 0.006)\ \text{m} = 15$. The parametric calculations yield the following variation of q_t with N:



The number of fins could also be increased by reducing the fin thickness. If the fin gap is fixed at $(S - t) = 4$ mm and manufacturing constraints dictate a minimum allowable fin thickness of 2 mm, up to $N = 25$ fins may be accommodated. In this case the parametric calculations yield



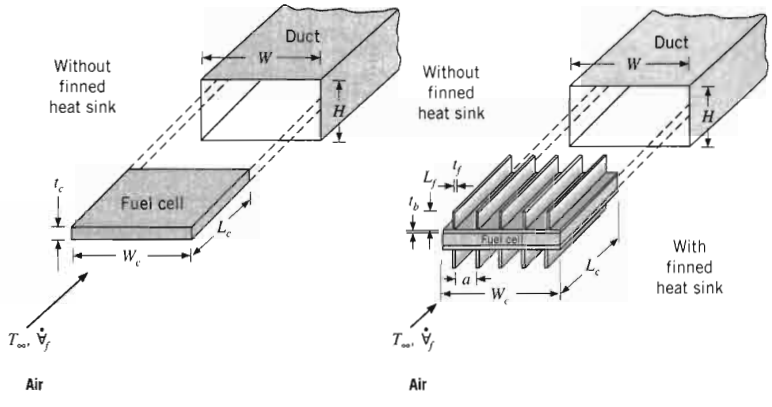
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The foregoing calculations are based on the assumption that h is not affected by a reduction in the fin gap. The assumption is reasonable as long as there is no interaction between boundary layers that develop on the opposing surfaces of adjoining fins. Note that, since $NA_f \gg 2\pi r_1(H - Nt)$ for the prescribed conditions, q_f increases nearly linearly with increasing N.

2. The *Models/Extended Surfaces* option of *IHT* provides ready-to-solve models for straight, pin, and circular fins, as well as for fin arrays. The models include the efficiency relations of Figures 3.18 and 3.19 and Table 3.5.

EXAMPLE 3.11

In Example 1.4, we saw that to generate an electrical power of $P = 9$ W, the temperature of the PEM fuel cell had to be maintained at $T_c \approx 56.4°C$, which required removal of 11.25 W from the fuel cell and a cooling air velocity of $V = 9.4$ m/s for $T_\infty = 25°C$. To provide these convective conditions, the fuel cell is centered in a 50 mm × 26 mm rectangular duct, with 10-mm gaps between the exterior of the 50 mm × 50 mm × 6 mm fuel cell and the top and bottom of the well-insulated duct wall. A small fan, powered by the fuel cell, is used to circulate the cooling air. Inspection of a particular fan vendor's data sheets suggests that the ratio of the fan power consumption to the fan's volumetric flow rate is $P_f/\dot{\forall}_f = C = 1000$ W/(m^3/s) for the range $10^{-4} \leq \dot{\forall}_f \leq 10^{-2}$ m^3/s.



1. Determine the *net* electric power produced by the fuel cell–fan system, $P_{net} = P - P_f$.
2. Consider the effect of attaching an aluminum ($k = 200$ W/m · K) *finned heat sink*, of identical top and bottom sections, onto the fuel cell body. The contact joint has a thermal resistance of $R''_{t,c} = 10^{-3}$ m^2 · K/W, and the base of the heat sink is of thickness $t_b = 2$ mm. Each of the N rectangular fins is of length $L_f = 8$ mm and thickness $t_f = 1$ mm, and spans the entire length of the fuel cell, $L_c = 50$ mm. With the heat sink in place, radiation losses are negligible and the convective heat transfer coefficient may be related to the size and geometry of a

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typical air channel by an expression of the form $h = 1.78\, k_{air}\,(L_f + a)/(L_f \cdot a)$, where a is the distance between fins. Draw an equivalent thermal circuit for part 2 and determine the total number of fins needed to reduce the fan power consumption to half of the value found in part 1.

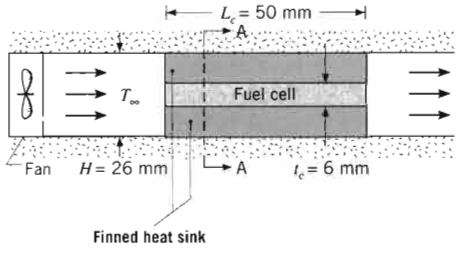
SOLUTION

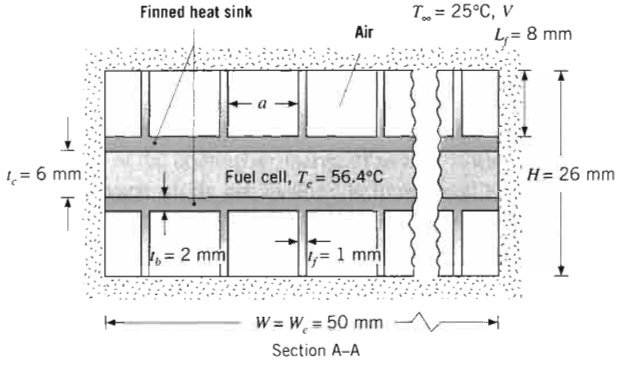
Known: Dimensions of a fuel cell and finned heat sink, fuel cell operating temperature, rate of thermal energy generation, power production. Relationship between power consumed by a cooling fan and the fan air flow rate. Relationship between the convection coefficient and the air channel dimensions.

Find:

1. The net power produced by the fuel cell–fan system when there is no heat sink.
2. The number of fins needed to reduce the fan power consumption found in part 1 by 50%.

Schematic:





Assumptions:

1. Steady-state conditions.
2. Negligible heat transfer from the edges of the fuel cell, as well as from the front and back faces of the finned heat sink.
3. One-dimensional heat transfer through the heat sink.

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4. Adiabatic fin tips.
5. Constant properties.
6. Negligible radiation when the heat sink is in place.

Properties: Table A.4. air ($\bar{T}$ = 300 K): k_{air} = 0.0263 W/m · K, c_p = 1007 J/kg · K, ρ = 1.1614 kg/m^3.

Analysis:

1. The volumetric flow rate of cooling air is $\dot{\forall}_f = VA_c$, where $A_c = W(H - t_c)$ is the cross-sectional area of the flow region between the duct walls and the unfinned fuel cell. Therefore,

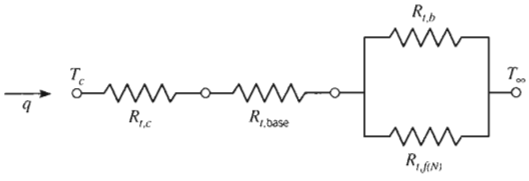
$$\dot{\forall}_f = V[W(H - t_c)] = 9.4 \text{ m/s} \times [0.05 \text{ m} \times (0.026 \text{ m} - 0.006 \text{ m})]$$
$$= 9.4 \times 10^{-3} \text{ m}^3/\text{s}$$

and

$$P_{net} = P - C\dot{\forall}_f = 9.0 \text{ W} - 1000 \text{ W}/(\text{m}^3/\text{s}) \times 9.4 \times 10^{-3} \text{ m}^3/\text{s} = -0.4 \text{ W} \quad \triangleleft$$

With this arrangement, the fan consumes more power than is generated by the fuel cell, and the system cannot produce net power.

2. To reduce the fan power consumption by 50%, the volumetric flow rate of air must be reduced to $\dot{\forall}_f = 4.7 \times 10^{-3}$ m^3/s. The thermal circuit includes resistances for the contact joint, conduction through the base of the finned heat sink, and resistances for the exposed base of the finned side of the heat sink, as well as the fins.



The thermal resistances for the contact joint and the base are

$$R_{t,c} = R''_{t,c}/2L_cW_c = (10^{-3} \text{ m}^2 \cdot \text{K/W})/(2 \times 0.05 \text{ m} \times 0.05 \text{ m}) = 0.2 \text{ K/W}$$

and

$$R_{t,base} = t_b/(2kL_cW_c) = (0.002 \text{ m})/(2 \times 200 \text{ W/m} \cdot \text{K} \times 0.05 \text{ m} \times 0.05 \text{ m})$$
$$= 0.002 \text{ K/W}$$

where the factors of two account for the two sides of the heat sink assembly. For the portion of the base exposed to the cooling air, the thermal resistance is

$$R_{t,b} = 1/[h(2W_c - Nt_f)L_c] = 1/[h \times (2 \times 0.05 \text{ m} - N \times 0.001 \text{ m}) \times 0.05 \text{ m}]$$

which cannot be evaluated until the total number of fins on both sides, N, and h are determined.

For a single fin, $R_{t,f} = \theta_b/q_f$, where, from Table 3.4 for a fin with an insulated fin tip, $R_{t,f} = (hPkA_c)^{-1/2}/\tanh(mL_f)$. In our case, $P = 2(L_c + t_f) = 2 \times (0.05 \text{ m} + 0.001 \text{ m}) = 0.102$ m, $A_c = L_ct_f = 0.05 \text{ m} \times 0.001 \text{ m} = 0.00005$ m^2, and

$$m = \sqrt{hP/kA_c} = [h \times 0.102 \text{ m}/(200 \text{ W/m} \cdot \text{K} \times 0.00005 \text{ m}^2)]^{1/2}$$

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Hence,

$$R_{t,f} = \frac{(h \times 0.102\ \text{m} \times 200\ \text{W/m} \cdot \text{K} \times 0.00005\ \text{m}^2)^{-1/2}}{\tanh(m \times 0.008\ \text{m})}$$

and for N fins, $R_{t,f(N)} = R_{t,f}/N$. As for $R_{t,b}$, $R_{t,f}$ cannot be evaluated until h and N are determined. Also, h depends on a, the distance between fins, which in turn depends on N, according to $a = (2W_c - Nt_f)/N = (2 \times 0.05\ \text{m} - N \times 0.001\ \text{m})/N$. Thus, specification of N will make it possible to calculate all resistances. From the thermal resistance network, the total thermal resistance is $R_{\text{tot}} = R_{t,c} + R_{t,\text{base}} + R_{\text{equiv}}$, where $R_{\text{equiv}} = [R_{t,b}^{-1} + R_{t,f(N)}^{-1}]^{-1}$.

The equivalent fin resistance, R_{equiv}, corresponding to the desired fuel cell temperature is found from the expression

$$q = \frac{T_c - T_\infty}{R_{\text{tot}}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,\text{base}} + R_{\text{equiv}}}$$

in which case,

$$R_{\text{equiv}} = \frac{T_c - T_\infty}{q} - (R_{t,c} + R_{t,\text{base}})$$
$$= (56.4°\text{C} - 25°\text{C})/11.25\ \text{W} - (0.2 + 0.002)\ \text{K/W} = 2.59\ \text{K/W}$$

For $N = 22$, the following values of the various parameters are obtained: $a = 0.0035$ m, $h = 19.1\ \text{W/m}^2 \cdot \text{K}$, $m = 13.9\ \text{m}^{-1}$, $R_{t,f(N)} = 2.94$ K/W, $R_{t,b} = 13.5$ K/W, $R_{\text{equiv}} = 2.41$ K/W, and $R_{\text{tot}} = 2.61$ K/W, resulting in a fuel cell temperature of 54.4°C. Fuel cell temperatures associated with $N = 20$ and $N = 24$ fins are $T_c = 58.9°\text{C}$ and 50.7°C, respectively.

The actual fuel cell temperature is closest to the desired value when $N = 22$. Therefore, a total of 22 fins, 11 on top and 11 on the bottom, should be specified, resulting in

$$P_{\text{net}} = P - P_f = 9.0\ \text{W} - 4.7\ \text{W} = 4.3\ \text{W} \quad \triangleleft$$

Comments:

1. The performance of the fuel cell–fan system is enhanced significantly by combining the finned heat sink with the fuel cell. Good thermal management can transform an impractical proposal into a viable concept.
2. The temperature of the cooling air increases as heat is transferred from the fuel cell. The temperature of the air leaving the finned heat sink may be calculated from an overall energy balance on the air flow, which yields $T_o = T_i + q/(\rho c_p \dot{\forall}_f)$. For part 1, $T_o = 25°\text{C} + 10.28\ \text{W}/(1.1614\ \text{kg/m}^3 \times 1007\ \text{J/kg} \cdot \text{K} \times 9.4 \times 10^{-3}\ \text{m}^3/\text{s}) = 25.9°\text{C}$. For part 2, the outlet air temperature is $T_o = 27.0°\text{C}$. Hence, the operating temperature of the fuel cell will be slightly higher than predicted under the assumption that the cooling air temperature is constant at 25°C and will be closer to the desired value.
3. For the conditions in part 2, the convection heat transfer coefficient does not vary with the air velocity. The insensitivity of the value of h to the fluid velocity occurs frequently in cases where the flow is confined within passages of small cross-sectional area, as will be discussed in detail in Chapter 8. The fin's influence on increasing or reducing the value of h relative to that of an unfinned surface should be taken into account in critical applications.

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4. A more detailed analysis of the system would involve prediction of the pressure drop associated with the fan-induced flow of air through the gaps between the fins.
5. The adiabatic fin tip assumption is valid since the duct wall is well insulated.

3.7 *The Bioheat Equation*

The topic of heat transfer within the human body is becoming increasingly important as new medical treatments are developed that involve extreme temperatures [11] and as we explore more adverse environments, such as the Arctic, underwater, or space. There are two main phenomena that make heat transfer in living tissues more complex than in conventional engineering materials: metabolic heat generation and the exchange of thermal energy between flowing blood and the surrounding tissue. Pennes [12] introduced a modification to the heat equation, now known as the Pennes or bioheat equation, to account for these effects. The bioheat equation is known to have limitations, but it continues to be a useful tool for understanding heat transfer in living tissues. In this section, we present a simplified version of the bioheat equation for the case of steady-state, one-dimensional heat transfer.

Both the metabolic heat generation and exchange of thermal energy with the blood can be viewed as effects of thermal energy generation. Therefore, we can rewrite Equation 3.39 to account for these two heat sources as

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_m + \dot{q}_p}{k} = 0 \tag{3.106}$$

where $\dot{q}_m$ and $\dot{q}_p$ are the *metabolic* and *perfusion* heat source terms, respectively. The perfusion term accounts for energy exchange between the blood and the tissue and is an energy source or sink according to whether heat transfer is from or to the blood, respectively. The thermal conductivity has been assumed constant in writing Equation 3.106.

Pennes proposed an expression for the perfusion term by assuming that within any small volume of tissue, the blood flowing in the small capillaries enters at an arterial temperature, T_a, and exits at the local tissue temperature, T. The rate at which heat is gained by the tissue is the rate at which heat is lost from the blood. If the perfusion rate is ω (m^3/s of volumetric blood flow per m^3 of tissue), the heat lost from the blood can be calculated from Equation 1.11e, or, on a unit volume basis,

$$\dot{q}_p = \omega\rho_b c_b(T_a - T) \tag{3.107}$$

where ρ_b and c_b are the blood density and specific heat, respectively. Note that $\omega\rho_b$ is the blood mass flow rate per unit volume of tissue.

Substituting Equation 3.107 into Equation 3.106, we find

$$\frac{d^2T}{dx^2} + \frac{\dot{q}_m + \omega\rho_b c_b(T_a - T)}{k} = 0 \tag{3.108}$$

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Drawing on our experience with extended surfaces, it is convenient to define an excess temperature of the form $\theta \equiv T - T_a - \dot{q}_m / \omega\rho_b c_b$. Then, if we assume that T_a, $\dot{q}_m$, ω, and the blood properties are all constant, Equation 3.108 can be rewritten as

$$\frac{d^2\theta}{dx^2} - \tilde{m}^2\theta = 0 \tag{3.109}$$

where $\tilde{m}^2 = \omega\rho_b c_b/k$. This equation is identical in form to Equation 3.64. Depending on the form of the boundary conditions, it may therefore be possible to use the results of Table 3.4 to estimate the temperature distribution within the living tissue.

EXAMPLE 3.12

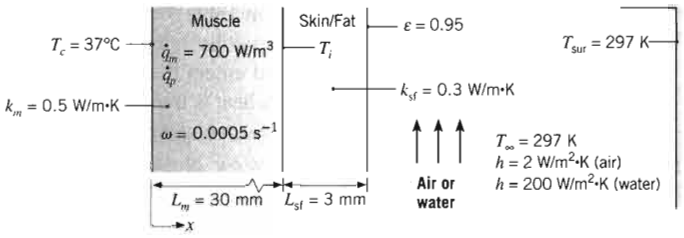
In Example 1.6, the temperature at the inner surface of the skin/fat layer was given as 35°C. In reality, this temperature depends on the existing heat transfer conditions, including phenomena occurring farther inside the body. Consider a region of muscle with a skin/fat layer over it. At a depth of $L_m = 30$ mm into the muscle, the temperature can be assumed to be at the core body temperature of $T_c = 37$°C. The muscle thermal conductivity is $k_m = 0.5$ W/m · K. The metabolic heat generation rate within the muscle is $\dot{q}_m = 700$ W/m^3. The perfusion rate is $\omega = 0.0005$ s^{-1}; the blood density and specific heat are $\rho_b = 1000$ kg/m^3 and $c_b = 3600$ J/kg · K, respectively, and the arterial blood temperature, T_a, is the same as the core body temperature. The thickness, emissivity, and thermal conductivity of the skin/fat layer are as given in Example 1.6; perfusion and metabolic heat generation within this layer can be neglected. We wish to predict the heat loss rate from the body and the temperature at the inner surface of the skin/fat layer for air and water environments of Example 1.6.

SOLUTION

Known: Dimensions and thermal conductivities of a muscle layer and a skin/fat layer. Skin emissivity and surface area. Metabolic heat generation rate and perfusion rate within the muscle layer. Core body and arterial temperatures. Blood density and specific heat. Ambient conditions.

Find: Heat loss rate from body and temperature at inner surface of the skin/fat layer.

Schematic:



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Assumptions:

1. Steady-state conditions.
2. One dimensional heat transfer through the muscle and skin/fat layers.
3. Metabolic heat generation rate, perfusion rate, arterial temperature, blood properties, and thermal conductivities are all uniform.
4. Radiation heat transfer coefficient is known from Example 1.6.
5. Solar irradiation is negligible.

Analysis: We will combine an analysis of the muscle layer with a treatment of heat transfer through the skin/fat layer and into the environment. The rate of heat transfer through the skin/fat layer and into the environment can be expressed in terms of a total resistance, R_{tot}, as

$$q = \frac{T_i - T_\infty}{R_{tot}} \tag{1}$$

As in Example 3.1 and for exposure of the skin to the air, R_{tot} accounts for conduction through the skin/fat layer in series with heat transfer by convection and radiation, which act in parallel with each other. Thus,

$$R_{tot} = \frac{L_{sf}}{k_{sf}A} + \left(\frac{1}{1/hA} + \frac{1}{1/h_rA}\right)^{-1} = \frac{1}{A}\left(\frac{L_{sf}}{k_{sf}} + \frac{1}{h + h_r}\right)$$

Using the values from Example 1.6 for air,

$$R_{tot} = \frac{1}{1.8\ \text{m}^2}\left(\frac{0.003\ \text{m}}{0.3\ \text{W/m} \cdot \text{K}} + \frac{1}{(2 + 5.9)\ \text{W/m}^2 \cdot \text{K}}\right) = 0.076\ \text{K/W}$$

For water, with $h_r = 0$ and $h = 200\ \text{W/m}^2 \cdot \text{K}$, $R_{tot} = 0.0083\ \text{W/m}^2 \cdot \text{K}$.

Heat transfer in the muscle layer is governed by Equation 3.109. The boundary conditions are specified in terms of the temperatures, T_c and T_i, where T_i is, as yet, unknown. In terms of the excess temperature θ, the boundary conditions are then

$$\theta(0) = T_c - T_a - \frac{\dot{q}_m}{\omega\rho_b c_b} = \theta_c \qquad \text{and} \qquad \theta(L_m) = T_i - T_a - \frac{\dot{q}_m}{\omega\rho_b c_b} = \theta_i$$

Since we have two boundary conditions involving prescribed temperatures, the solution for θ is given by case C of Table 3.4,

$$\frac{\theta}{\theta_c} = \frac{(\theta_i/\theta_c)\sinh \tilde{m}x + \sinh \tilde{m}(L_m - x)}{\sinh \tilde{m}L_m}$$

The value of q_f given in Table 3.4 would correspond to the heat transfer rate at $x = 0$, but this is not of particular interest here. Rather, we seek the rate at which heat leaves the muscle and enters the skin/fat layer so that we can equate this quantity with the rate at which heat is transferred through the skin/fat layer and into the environment. Therefore, we calculate the heat transfer rate at $x = L_m$ as

$$q\Big|_{x=L_m} = -k_m A \frac{dT}{dx}\Big|_{x=L_m} = -k_m A \frac{d\theta}{dx}\Big|_{x=L_m} = -k_m A \tilde{m}\theta_c \frac{(\theta_i/\theta_c)\cosh \tilde{m}L_m - 1}{\sinh \tilde{m}L_m} \tag{2}$$

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Combining Equations 1 and 2 yields

$$-k_m A\tilde{m}\theta_c \frac{(\theta_i/\theta_c)\cosh \tilde{m}L_m - 1}{\sinh \tilde{m}L_m} = \frac{T_i - T_\infty}{R_{tot}}$$

This expression can be solved for T_i, recalling that T_i also appears in θ_i.

$$T_i = \frac{T_\infty \sinh \tilde{m}L_m + k_m A\tilde{m}R_{tot}\left[\theta_c + \left(T_a + \frac{\dot{q}_m}{\omega\rho_b c_b}\right)\cosh \tilde{m}L_m\right]}{\sinh \tilde{m}L_m + k_m A\tilde{m}R_{tot}\cosh \tilde{m}L_m}$$

where

$$\tilde{m} = \sqrt{\omega\rho_b c_b/k_m} = [0.0005\ \text{s}^{-1} \times 1000\ \text{kg/m}^3 \times 3600\ \text{J/kg} \cdot \text{K}/0.5\ \text{W/m} \cdot \text{K}]^{1/2}$$
$$= 60\ \text{m}^{-1}$$

$$\sinh(\tilde{m}L_m) = \sinh(60\ \text{m}^{-1} \times 0.03\ \text{m}) = 2.94 \quad \text{and}$$

$$\cosh(\tilde{m}L_m) = \cosh(60\ \text{m}^{-1} \times 0.03\ \text{m}) = 3.11$$

$$\theta_c = T_c - T_a - \frac{\dot{q}_m}{\omega\rho_b c_b} = -\frac{\dot{q}_m}{\omega\rho_b c_b} = -\frac{700\ \text{W/m}^3}{0.0005\ \text{s}^{-1} \times 1000\ \text{kg/m}^3 \times 3600\ \text{J/kg} \cdot \text{K}}$$
$$= -0.389\ \text{K}$$

The excess temperature can be expressed in kelvins or degrees Celsius, since it is a temperature difference.

Thus, for air:

$$T_i = \frac{\{24°\text{C} \times 2.94 + 0.5\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2 \times 60\ \text{m}^{-1} \times 0.076\ \text{K/W}[-0.389°\text{C} + (37°\text{C} + 0.389°\text{C}) \times 3.11]\}}{2.94 + 0.5\ \text{W/m} \cdot \text{K} \times 1.8\ \text{m}^2 \times 60\ \text{m}^{-1} \times 0.076\ \text{K/W} \times 3.11} = 34.8°\text{C}$$

This result agrees well with the value of 35°C that was assumed for Example 1.6. Next we can find the heat loss rate:

$$q = \frac{T_i - T_\infty}{R_{tot}} = \frac{34.8°\text{C} - 24°\text{C}}{0.076\ \text{K/W}} = 142\ \text{W}$$

Again this agrees well with the previous result. Repeating the calculation for water, we find

$$T_i = 28.2°\text{C}$$

$$q = 514\ \text{W}$$

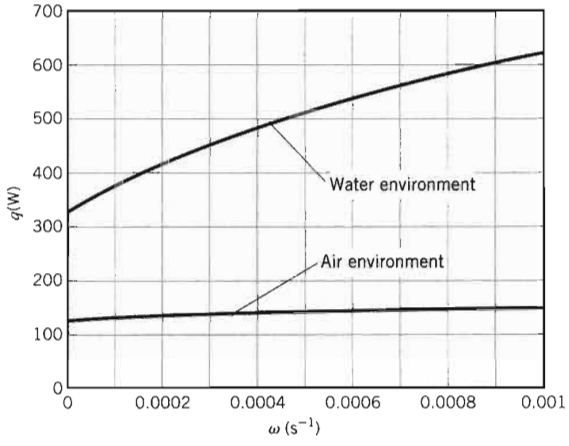
Here the calculation of Example 1.6 was not accurate because it incorrectly assumed that the inside of the skin/fat layer would be at 35°C. Furthermore, the skin temperature in this case would be only 25.4°C based on this more complete calculation.

Comments:

1. In reality, our bodies adjust in many ways to the thermal environment. For example, if we are too cold, we will shiver, which increases our metabolic heat generation rate. If we are too warm, the perfusion rate near the skin surface will increase, locally raising the skin temperature to increase heat loss to the environment.

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2. Measuring the true thermal conductivity of living tissue is very challenging, first because of the necessity of making *invasive* measurements in a living being, and second because it is difficult to experimentally separate the effects of heat conduction and perfusion. It is easier to measure an effective thermal conductivity that would account for the combined contributions of conduction and perfusion. However, this effective conductivity value necessarily depends on the perfusion rate, which in turn varies with the thermal environment and physical condition of the specimen.
3. The calculations can be repeated for a range of values of the perfusion rate, and the dependence of the heat loss rate on the perfusion rate is illustrated below. The effect is stronger for the case of the water environment, because the muscle temperature is lower and therefore the effect of perfusion by the warm arterial blood is more pronounced.



3.8 Summary

Despite its inherent mathematical simplicity, one-dimensional, steady-state heat transfer occurs in numerous engineering applications. Although one-dimensional, steady-state conditions may not apply exactly, the assumptions may often be made to obtain results of reasonable accuracy. You should therefore be thoroughly familiar with the means by which such problems are treated. In particular, you should be comfortable with the use of equivalent thermal circuits and with the expressions for the conduction resistances that pertain to each of the three common geometries. You should also be familiar with how the heat equation and Fourier's law may be used to obtain temperature distributions and the corresponding fluxes. The implications of an internally distributed source of energy should also be clearly understood. Finally, you should appreciate the important role

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that extended surfaces can play in the design of thermal systems and should have the facility to effect design and performance calculations for such surfaces.

You may test your understanding of this chapter's key concepts by addressing the following questions.

- Under what conditions may it be said that the *heat flux* is a constant, independent of the direction of heat flow? For each of these conditions, use physical considerations to convince yourself that the heat flux would not be independent of direction if the condition were not satisfied.
- For one-dimensional, steady-state conduction in a cylindrical or spherical shell without heat generation, is the radial heat flux independent of radius? Is the radial heat rate independent of radius?
- For one-dimensional, steady-state conduction without heat generation, what is the shape of the temperature distribution in a *plane wall*? In a *cylindrical shell*? In a *spherical shell*?
- What is the *thermal resistance*? How is it defined? What are its units?
- For conduction across a *plane wall*, can you write the expression for the thermal resistance from memory? Similarly, can you write expressions for the thermal resistance associated with conduction across *cylindrical* and *spherical* shells? From memory, can you express the thermal resistances associated with convection from a surface and net radiation exchange between the surface and large surroundings?
- What is the physical basis for existence of a *critical insulation radius*? How do the thermal conductivity and the convection coefficient affect its value?
- How is the conduction resistance of a solid affected by its thermal conductivity? How is the convection resistance at a surface affected by the convection coefficient? How is the radiation resistance affected by the surface emissivity?
- If heat is transferred from a surface by convection and radiation, how are the corresponding thermal resistances represented in a circuit?
- Consider steady-state conduction through a plane wall separating fluids of different temperatures, $T_{\infty,i}$ and $T_{\infty,o}$, adjoining the inner and outer surfaces, respectively. If the convection coefficient at the outer surface is five times larger than that at the inner surface, $h_o = 5h_i$, what can you say about relative proximity of the corresponding surface temperatures, $T_{s,o}$ and $T_{s,i}$, to their adjoining fluid temperatures?
- Can a thermal conduction resistance be applied to a *solid* cylinder or sphere?
- What is a *contact resistance*? How is it defined? What are its units for an interface of prescribed area? What are they for a unit area?
- How is the contact resistance affected by the roughness of adjoining surfaces?
- If the air in the contact region between two surfaces is replaced by helium, how is the thermal contact resistance affected? How is it affected if the region is evacuated?
- What is the *overall heat transfer coefficient*? How is it defined and how is it related to the *total thermal resistance*? What are its units?
- In a solid circular cylinder experiencing uniform volumetric heating and convection heat transfer from its surface, how does the heat flux vary with radius? How does the heat rate vary with radius?
- In a solid circular sphere experiencing uniform volumetric heating and convection heat transfer from its surface, how does the heat flux vary with radius? How does the heat rate vary with radius?

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- Is it possible to achieve steady-state conditions in a solid cylinder or sphere that is experiencing heat generation and whose surface is perfectly insulated? Explain.
- Can a material experiencing heat generation be represented by a thermal resistance and included in a circuit analysis? If so, why? If not, why not?
- What is the physical mechanism associated with cooking in a microwave oven? How do conditions differ from a conventional (convection or radiant) oven?
- If radiation is incident on the surface of a semitransparent medium and is absorbed as it propagates through the medium, will the corresponding volumetric rate of heat generation $\dot{q}$ be distributed uniformly in the medium? If not, how will $\dot{q}$ vary with distance from the surface?
- In what way is a plane wall that is of thickness $2L$ and experiences uniform volumetric heating and equivalent convection conditions at both surfaces similar to a plane wall that is of thickness L and experiences the same volumetric heating and convection conditions at one surface but whose opposite surface is well insulated?
- What purpose is served by attaching *fins* to a surface?
- In the derivation of the general form of the energy equation for an extended surface, why is the assumption of one-dimensional conduction an approximation? Under what conditions is it a good approximation?
- Consider a straight fin of uniform cross section (Figure 3. l4a). For an x location in the fin, sketch the temperature distribution in the transverse (y) direction, placing the origin of the coordinate at the midplane of the fin ($-t/2 \leq y \leq t/2$). What is the form of a *surface* energy balance applied at the location (x, $t/2$)?
- What is the *fin effectiveness*? What is its range of possible values? Under what conditions are fins most effective?
- What is the *fin efficiency*? What is its range of possible values? Under what conditions will the efficiency be large?
- What is the *fin resistance*? What are its units?
- How are the effectiveness, efficiency, and thermal resistance of a fin affected if its thermal conductivity is increased? If the convection coefficient is increased? If the length of the fin is increased? If the thickness (or diameter) of the fin is increased?
- Heat is transferred from hot water flowing through a tube to air flowing over the tube. To enhance the rate of heat transfer, should fins be installed on the tube interior or exterior surface?
- A fin may be manufactured as an integral part of a surface by using a casting or extrusion process, or it may be separately brazed or adhered to the surface. From thermal considerations, which option is preferred?
- Describe the physical origins of the two heat source terms in the bioheat equation. Under what conditions is the perfusion term a heat sink?

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Problems

Plane Wall

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Problems

Plane Wall

3.1 Consider the plane wall of Figure 3.1, separating hot and cold fluids at temperatures $T_{\infty,1}$ and $T_{\infty,2}$, respectively. Using surface energy balances as boundary conditions at $x = 0$ and $x = L$ (see Equation 2.32), obtain the temperature distribution within the wall and the heat flux in terms of $T_{\infty,1}$, $T_{\infty,2}$, h_1, h_2, k, and L.

3.2 The rear window of an automobile is defogged by passing warm air over its inner surface.

(a) If the warm air is at $T_{\infty,i} = 40°C$ and the corresponding convection coefficient is $h_i = 30$ W/m$^2 \cdot$ K, what are the inner and outer surface temperatures of 4-mm-thick window glass, if the outside ambient air temperature is $T_{\infty,o} = -10°C$ and the associated convection coefficient is $h_o = 65$ W/m$^2 \cdot$ K?

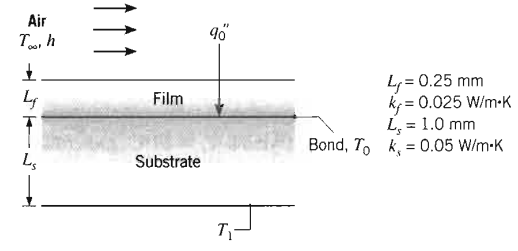
(b) In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2$, 65, and 100 W/m$^2 \cdot$ K, compute and plot the inner and outer surface temperatures as a function of $T_{\infty,o}$ for $-30 \le T_{\infty,o} \le 0°C$.

3.3 The rear window of an automobile is defogged by attaching a thin, transparent, film-type heating element to its inner surface. By electrically heating this element, a uniform heat flux may be established at the inner surface.

(a) For 4-mm-thick window glass, determine the electrical power required per unit window area to maintain an inner surface temperature of 15°C when the interior air temperature and convection coefficient are $T_{\infty,i} = 25°C$ and $h_i = 10$ W/m$^2 \cdot$ K, while the exterior (ambient) air temperature and convection coefficient are $T_{\infty,o} = -10°C$ and $h_o = 65$ W/m$^2 \cdot$ K.

(b) In practice $T_{\infty,o}$ and h_o vary according to weather conditions and car speed. For values of $h_o = 2$, 20, 65, and 100 W/m$^2 \cdot$ K, determine and plot the electrical power requirement as a function of $T_{\infty,o}$ for $-30 \le T_{\infty,o} \le 0°C$. From your results, what can you conclude about the need for heater operation at low values of h_o? How is this conclusion affected by the value of $T_{\infty,o}$? If $h \propto V^n$, where V is the vehicle speed and n is a positive exponent, how does the vehicle speed affect the need for heater operation?

3.4 In a manufacturing process, a transparent film is being bonded to a substrate as shown in the sketch. To cure the bond at a temperature T_0, a radiant source is used to provide a heat flux q_0'' (W/m^2), all of which is absorbed at the bonded surface. The back of the substrate is maintained at T_1 while the free surface of the film is exposed to air at T_∞ and a convection heat transfer coefficient h.



(a) Show the thermal circuit representing the steady-state heat transfer situation. Be sure to label *all* elements, nodes, and heat rates. Leave in symbolic form.

(b) Assume the following conditions: $T_\infty = 20°C$, $h = 50$ W/m$^2 \cdot$ K, and $T_1 = 30°C$. Calculate the heat flux q_0'' that is required to maintain the bonded surface at $T_0 = 60°C$.

(c) Compute and plot the required heat flux as a function of the film thickness for $0 \le L_f \le 1$ mm.

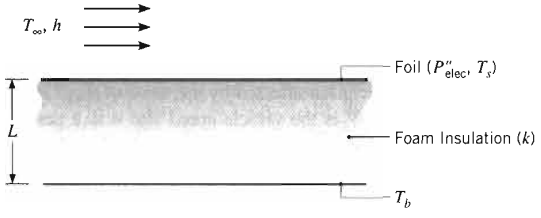
(d) If the film is not transparent and all of the radiant heat flux is absorbed at its upper surface, determine

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the heat flux required to achieve bonding. Plot your results as a function of L_f for $0 \leq L_f \leq 1$ mm.

3.5 The walls of a refrigerator are typically constructed by sandwiching a layer of insulation between sheet metal panels. Consider a wall made from fiberglass insulation of thermal conductivity $k_i = 0.046$ W/m · K and thickness $L_i = 50$ mm and steel panels, each of thermal conductivity $k_p = 60$ W/m · K and thickness $L_p = 3$ mm. If the wall separates refrigerated air at $T_{\infty,i} = 4°C$ from ambient air at $T_{\infty,o} = 25°C$, what is the heat gain per unit surface area? Coefficients associated with natural convection at the inner and outer surfaces may be approximated as $h_i = h_o = 5$ W/m^2 · K.

3.6 A technique for measuring convection heat transfer coefficients involves bonding one surface of a thin metallic foil to an insulating material and exposing the other surface to the fluid flow conditions of interest.



By passing an electric current through the foil, heat is dissipated uniformly within the foil and the corresponding flux, P''_{elec}, may be inferred from related voltage and current measurements. If the insulation thickness L and thermal conductivity k are known and the fluid, foil, and insulation temperatures (T_∞, T_s, T_b) are measured, the convection coefficient may be determined. Consider conditions for which $T_\infty = T_b = 25°C$, $P''_{elec} = 2000$ W/m^2, $L = 10$ mm, and $k = 0.040$ W/m · K.

(a) With water flow over the surface, the foil temperature measurement yields $T_s = 27°C$. Determine the convection coefficient. What error would be incurred by assuming all of the dissipated power to be transferred to the water by convection?

(b) If, instead, air flows over the surface and the temperature measurement yields $T_s = 125°C$, what is the convection coefficient? The foil has an emissivity of 0.15 and is exposed to large surroundings at 25°C. What error would be incurred by assuming all of the dissipated power to be transferred to the air by convection?

(c) Typically, heat flux gages are operated at a fixed temperature (T_s), in which case the power dissipation provides a direct measure of the convection coefficient. For $T_s = 27°C$, plot P''_{elec} as a function of h_o for $10 \leq h_o \leq 1000$ W/m^2 · K. What effect does h_o have on the error associated with neglecting conduction through the insulation?

3.7 The *wind chill*, which is experienced on a cold, windy day, is related to increased heat transfer from exposed human skin to the surrounding atmosphere. Consider a layer of fatty tissue that is 3 mm thick and whose interior surface is maintained at a temperature of 36°C. On a calm day the convection heat transfer coefficient at the outer surface is 25 W/m^2 · K, but with 30 km/h winds it reaches 65 W/m^2 · K. In both cases the ambient air temperature is −15°C.

(a) What is the ratio of the heat loss per unit area from the skin for the calm day to that for the windy day?

(b) What will be the skin outer surface temperature for the calm day? For the windy day?

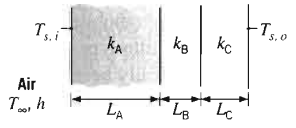
(c) What temperature would the air have to assume on the calm day to produce the same heat loss occurring with the air temperature at −15°C on the windy day?

3.8 A thermopane window consists of two pieces of glass 7 mm thick that enclose an air space 7 mm thick. The window separates room air at 20°C from outside ambient air at −10°C. The convection coefficient associated with the inner (room-side) surface is 10 W/m^2 · K.

(a) If the convection coefficient associated with the outer (ambient) air is $h_o = 80$ W/m^2 · K, what is the heat loss through a window that is 0.8 m long by 0.5 m wide? Neglect radiation, and assume the air enclosed between the panes to be stagnant.

(b) Compute and plot the effect of h_o on the heat loss for $10 \leq h_o \leq 100$ W/m^2 · K. Repeat this calculation for a triple-pane construction in which a third pane and a second air space of equivalent thickness are added.

3.9 The composite wall of an oven consists of three materials, two of which are of known thermal conductivity, $k_A = 20$ W/m · K and $k_C = 50$ W/m · K, and known thickness, $L_A = 0.30$ m and $L_C = 0.15$ m. The third material, B, which is sandwiched between materials A and C, is of known thickness, $L_B = 0.15$ m, but unknown thermal conductivity k_B.

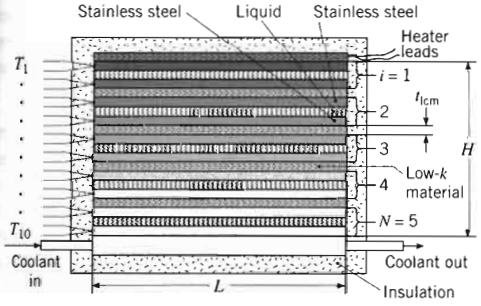


Under steady-state operating conditions, measurements reveal an outer surface temperature of $T_{s,o} = 20°C$, an inner surface temperature of $T_{s,i} = 600°C$, and an oven air temperature of $T_\infty = 800°C$. The inside convection coefficient h is known to be 25 W/m^2 · K. What is the value of k_B?

3.10 A testing lab is contracted to measure the thermal conductivity of various liquids as a function of the liquid temperature. Typically, the lab would measure the thermal conductivity and its temperature dependence

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by performing many time-consuming experiments at various operating temperatures. A new experimental design is proposed whereby the temperature dependence may be determined in one experiment. The proposed apparatus consists of multiple layers, with each layer consisting of a t_{lcm}-thick square sheet of low thermal conductivity material sandwiched between two t_{ss} = 1-mm-thick stainless steel plates with k_{ss} = 15 W/m · K. The resulting *stainless steel–low thermal conductivity–stainless steel* sandwiches then separate N = 5, t_l = 2-mm-thick layers of the liquid. The entire structure is heated from above to eliminate natural convection within the liquid, and cooled from below with a flowing liquid. The temperature of each stainless steel sheet is measured with a thermocouple, and the device is encased in insulation. The temperature range over which the thermal conductivity of a particular liquid is to be measured is 300 K ≤ T ≤ 400 K. To resolve the temperature dependence of the liquid's thermal conductivity, the temperature difference across each liquid layer is to be held to within ΔT = 2°C. The nominal thermal conductivity of the liquid is k_l = 0.8 W/m · K.

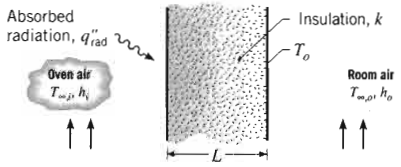


(a) Consider the low thermal conductivity material to be Bakelite. Determine the overall height, H, of the experimental apparatus.

(b) Consider replacing the Bakelite with an aerogel characterized by k_a = 0.0065 W/m · K. What is the overall height of the apparatus?

(c) To minimize heat losses through the sides of the device, the area of the heater (A_h) is made 10 times larger than the area of the sides (A_s) of the device. Compare the required heater area and required electrical power for devices constructed using Bakelite and aerogel low thermal conductivity materials.

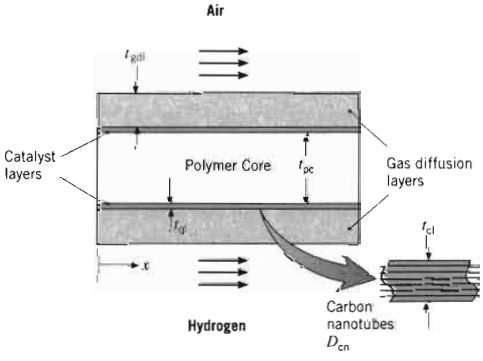
3.11 The wall of a drying oven is constructed by sandwiching an insulation material of thermal conductivity k = 0.05 W/m · K between thin metal sheets. The oven air is at $T_{\infty,i}$ = 300°C, and the corresponding convection coefficient is h_i = 30 W/m² · K. The inner wall surface absorbs a radiant flux of q''_{rad} = 100 W/m² from hotter objects within the oven. The room air is at $T_{\infty,o}$ = 25°C, and the overall coefficient for convection and radiation from the outer surface is h_o = 10 W/m² · K.



(a) Draw the thermal circuit for the wall and label all temperatures, heat rates, and thermal resistances.

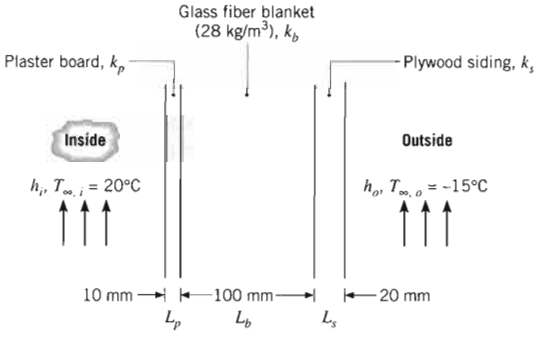
(b) What insulation thickness L is required to maintain the outer wall surface at a *safe-to-touch* temperature of T_o = 40°C?

3.12 The electrolytic membrane of the fuel cell in Example 1.4 is a thin composite structure consisting of sandwiched layers of delicate materials, as shown in the sketch. The thickness of the polymer core is t_{pc} = 0.20 mm, while the thickness of each of the catalyst layers is t_{cl} = 0.01 mm. The gas diffusion layers are each t_{gdl} = 0.1 mm thick. Since the membrane can soften and lose durability at temperatures exceeding 85°C, the materials engineer decides to strengthen the membrane by implanting long carbon nanotubes (diameter D_{cn} = 14 nm, k_{cn} = 3000 W/m · K) lengthwise within both catalyst layers. Determine the value of the *effective thermal conductivity*, $k_{eff,x}$, of the membrane assembly that is defined by the relation $q_x = k_{eff,x}\, wt\Delta T/L$, where L, w, and t are the length, width, and total thickness of the membrane assembly respectively; q_x is the heat transfer rate along the assembly; and ΔT is the temperature drop along a section of the assembly of length L. Determine the value of $k_{eff,x}$ for carbon nanotube loadings of f = 0, 10%, 20%, and 30%, where f is the volume fraction of carbon nanotubes in the catalyst layers. The thermal conductivity of the polymer core is k_{pc} = 0.25 W/m · K, and the thermal conductivities of the gas diffusion layers and catalyst layers are k_{gdl} = 1.3 W/m · K and k_{cl} = 1 W/m · K, respectively.



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3.13 A house has a composite wall of wood, fiberglass insulation, and plaster board, as indicated in the sketch. On a cold winter day the convection heat transfer coefficients are $h_o = 60$ W/m$^2 \cdot$K and $h_i = 30$ W/m$^2 \cdot$K. The total wall surface area is 350 m^2.



(a) Determine a symbolic expression for the total thermal resistance of the wall, including inside and outside convection effects for the prescribed conditions.

(b) Determine the total heat loss through the wall.

(c) If the wind were blowing violently, raising h_o to 300 W/m$^2 \cdot$K, determine the percentage increase in the heat loss.

(d) What is the controlling resistance that determines the amount of heat flow through the wall?

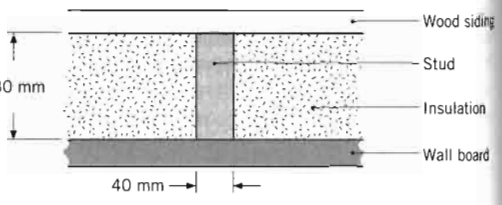
3.14 Consider the composite wall of Problem 3.13 under conditions for which the inside air is still characterized by $T_{\infty,i} = 20$°C and $h_i = 30$ W/m$^2 \cdot$K. However, use the more realistic conditions for which the outside air is characterized by a diurnal (time) varying temperature of the form

$$T_{\infty,o}(\text{K}) = 273 + 5 \sin\left(\frac{2\pi}{24} t\right) \qquad 0 \le t \le 12 \text{ h}$$

$$T_{\infty,o}(\text{K}) = 273 + 11 \sin\left(\frac{2\pi}{24} t\right) \qquad 12 \le t \le 24 \text{ h}$$

with $h_o = 60$ W/m$^2 \cdot$K. Assuming quasisteady conditions for which changes in energy storage within the wall may be neglected, estimate the daily heat loss through the wall if its total surface area is 200 m^2.

3.15 Consider a composite wall that includes an 8-mm-thick hardwood siding, 40-mm by 130-mm hardwood studs on 0.65-m centers with glass fiber insulation (paper faced, 28 kg/m^3), and a 12-mm layer of gypsum (vermiculite) wall board.



What is the thermal resistance associated with a wall that is 2.5 m high by 6.5 m wide (having 10 studs, each 2.5 m high)?

3.16 The thermal characteristics of a small, dormitory refrigerator are determined by performing two separate experiments, each with the door closed and the refrigerator placed in ambient air at $T_\infty = 25$°C. In one case, an electric heater is suspended in the refrigerator cavity, while the refrigerator is unplugged. With the heater dissipating 20 W, a steady-state temperature of 90°C is recorded within the cavity. With the heater removed and the refrigerator now in operation, the second experiment involves maintaining a steady-state cavity temperature of 5°C for a fixed time interval and recording the electrical energy required to operate the refrigerator. In such an experiment for which steady operation is maintained over a 12-hour period, the input electrical energy is 125,000 J. Determine the refrigerator's coefficient of performance (COP).

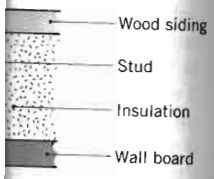
3.17 In the design of buildings, energy conservation requirements dictate that the exterior surface area, A_s, be minimized. This requirement implies that, for a desired floor space, there may be optimum values associated with the number of floors and horizontal dimensions of the building. Consider a design for which the total floor space, A_f, and the vertical distance between floors, H_f, are prescribed.

(a) If the building has a square cross section of width W on a side, obtain an expression for the value of W that would minimize heat loss to the surroundings. Heat loss may be assumed to occur from the four vertical side walls and from a flat roof. Express your result in terms of A_f and H_f.

(b) If $A_f = 32{,}768$ m^2 and $H_f = 4$ m, for what values of W and N_f (the number of floors) is the heat loss minimized? If the average overall heat transfer coefficient is $U = 1$ W/m$^2 \cdot$K and the difference between the inside and ambient air temperatures is 25°C, what is the corresponding heat loss? What is the percent reduction in heat loss compared with a building for $N_f = 2$?

3.18 When raised to very high temperatures, many conventional liquid fuels dissociate into hydrogen and other components. Thus the advantage of a *solid oxide fuel*

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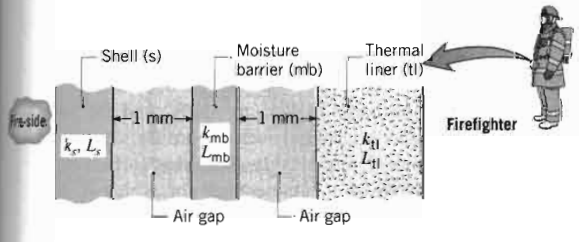
cell is that such a device can internally *reform* readily available liquid fuels into hydrogen that can then be used to produce electrical power in a manner similar to Example 1.4. Consider a portable solid oxide fuel cell, operating at a temperature of $T_{fc} = 800°C$. The fuel cell is housed within a cylindrical canister of diameter $D = 75$ mm and length $L = 120$ mm. The outer surface of the canister is insulated with a low-thermal-conductivity material. For a particular application, it is desired that the *thermal signature* of the canister be small, to avoid its detection by infrared sensors. The degree to which the canister can be detected with an infrared sensor may be estimated by equating the radiation heat flux emitted from the exterior surface of the canister (Equation 1.5; $E_s = \varepsilon_s \sigma T_s^4$) to the heat flux emitted from an *equivalent* black surface, ($E_b = \sigma T_b^4$). If the equivalent black surface temperature, T_b, is near the surroundings temperature, the thermal signature of the canister is too small to be detected—the canister is indistinguishable from the surroundings.

(a) Determine the required thickness of insulation to be applied to the cylindrical wall of the canister to ensure that the canister does not become highly visible to an infrared sensor (i.e., $T_b - T_{sur} < 5$ K). Consider cases where (i) the outer surface is covered with a very thin layer of dirt ($\varepsilon_s = 0.90$) and (ii) the outer surface is comprised of a very thin polished aluminum sheet ($\varepsilon_s = 0.08$). Calculate the required thicknesses for two types of insulating material, calcium silicate ($k = 0.09$ W/m · K) and aerogel ($k = 0.006$ W/m · K). The temperatures of the surroundings and the ambient are $T_{sur} = 300$ K and $T_\infty = 298$ K, respectively. The outer surface is characterized by a convective heat transfer coefficient of $h = 12$ W/m^2 · K.

(b) Calculate the outer surface temperature of the canister for the four cases (high and low thermal conductivity; high and low surface emissivity).

(c) Calculate the heat loss from the cylindrical walls of the canister for the four cases.

3.19 A firefighter's protective clothing, referred to as a *turnout coat*, is typically constructed as an ensemble of three layers separated by air gaps, as shown schematically.



Representative dimensions and thermal conductivities for the layers are as follows.

Layer	*Thickness* (mm)	k (W/m · K)
Shell (s)	0.8	0.047
Moisture barrier (mb)	0.55	0.012
Thermal liner (tl)	3.5	0.038

The air gaps between the layers are 1 mm thick, and heat is transferred by conduction and radiation exchange through the stagnant air. The linearized radiation coefficient for a gap may be approximated as, $h_{rad} = \sigma(T_1 + T_2)(T_1^2 + T_2^2) \approx 4\sigma T_{avg}^3$, where T_{avg} represents the average temperature of the surfaces comprising the gap, and the radiation flux across the gap may be expressed as $q''_{rad} = h_{rad}(T_1 - T_2)$.

(a) Represent the turnout coat by a thermal circuit, labeling all the thermal resistances. Calculate and tabulate the thermal resistances per unit area (m^2 · K/W) for each of the layers, as well as for the conduction and radiation processes in the gaps. Assume that a value of $T_{avg} = 470$ K may be used to approximate the radiation resistance of both gaps. Comment on the relative magnitudes of the resistances.

(b) For a *pre-flash-over* fire environment in which firefighters often work, the typical radiant heat flux on the fire-side of the turnout coat is 0.25 W/cm^2. What is the outer surface temperature of the turnout coat if the inner surface temperature is 66°C, a condition that would result in burn injury?

Contact Resistance

3.20 A composite wall separates combustion gases at 2600°C from a liquid coolant at 100°C, with gas- and liquid-side convection coefficients of 50 and 1000 W/m^2 · K. The wall is composed of a 10-mm-thick layer of beryllium oxide on the gas side and a 20-mm-thick slab of stainless steel (AISI 304) on the liquid side. The contact resistance between the oxide and the steel is 0.05 m^2 · K/W. What is the heat loss per unit surface area of the composite? Sketch the temperature distribution from the gas to the liquid.

3.21 Two stainless steel plates 10 mm thick are subjected to a contact pressure of 1 bar under vacuum conditions for which there is an overall temperature drop of 100°C across the plates. What is the heat flux through the plates? What is the temperature drop across the contact plane?

3.22 Consider a plane composite wall that is composed of two materials of thermal conductivities $k_A = 0.1$ W/m · K and

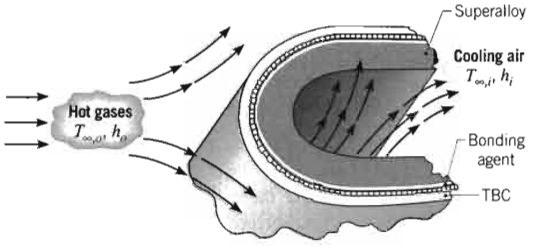
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$k_B = 0.04$ W/m · K and thicknesses $L_A = 10$ mm and $L_B = 20$ mm. The contact resistance at the interface between the two materials is known to be 0.30 m² · K/W. Material A adjoins a fluid at 200°C for which $h = 10$ W/m² · K, and material B adjoins a fluid at 40°C for which $h = 20$ W/m² · K.

(a) What is the rate of heat transfer through a wall that is 2 m high by 2.5 m wide?

(b) Sketch the temperature distribution.

3.23 The performance of gas turbine engines may be improved by increasing the tolerance of the turbine blades to hot gases emerging from the combustor. One approach to achieving high operating temperatures involves application of a *thermal barrier coating* (TBC) to the exterior surface of a blade, while passing cooling air through the blade. Typically, the blade is made from a high-temperature superalloy, such as Inconel ($k \approx 25$ W/m · K), while a ceramic, such as zirconia ($k \approx 1.3$ W/m · K), is used as a TBC.



Consider conditions for which hot gases at $T_{\infty,o} = 1700$ K and cooling air at $T_{\infty,i} = 400$ K provide outer and inner surface convection coefficients of $h_o = 1000$ W/m² · K and $h_i = 500$ W/m² · K, respectively. If a 0.5-mm-thick zirconia TBC is attached to a 5-mm-thick Inconel blade wall by means of a metallic bonding agent, which provides an interfacial thermal resistance of $R''_{t,c} = 10^{-4}$ m² · K/W, can the Inconel be maintained at a temperature that is below its maximum allowable value of 1250 K? Radiation effects may be neglected, and the turbine blade may be approximated as a plane wall. Plot the temperature distribution with and without the TBC. Are there any limits to the thickness of the TBC?

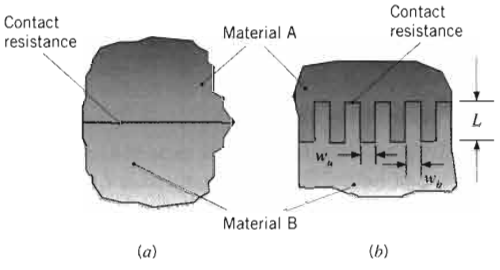
3.24 A commercial grade cubical freezer, 3 m on a side, has a composite wall consisting of an exterior sheet of 6.35-mm-thick plain carbon steel, an intermediate layer of 100-mm-thick cork insulation, and an inner sheet of 6.35-mm-thick aluminum alloy (2024). Adhesive interfaces between the insulation and the metallic strips are each characterized by a thermal contact resistance of $R''_{t,c} = 2.5 \times 10^{-4}$ m² · K/W. What is the steady-state cooling load that must be maintained by the refrigerator under conditions for which the outer and inner surface temperatures are 22°C and −6°C respectively?

3.25 Physicists have determined the theoretical value of the thermal conductivity of a carbon nanotube to be $k_{cn,T} = 5000$ W/m · K.

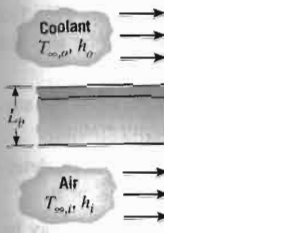
(a) Assuming the actual thermal conductivity of the carbon nanotube is the same as its theoretical value, find the thermal contact resistance, $R_{t,c}$, that exists between the carbon nanotube and the top surfaces of the heated and sensing islands in Example 3.3.

(b) Using the value of the thermal contact resistance calculated in part (a), plot the fraction of the total resistance between the heated and sensing islands that is due to the thermal contact resistances for island separation distances of $5\ \mu\text{m} \leq s \leq 20\ \mu\text{m}$.

3.26 In a particular application, it is desirable to minimize the effects of the thermal contact resistance between two plane mating surfaces as shown in part (*a*) of the schematic. An engineer suggests that the overall resistance to heat transfer can be reduced by cutting relatively deep linear grooves in each surface, resulting in the interlocking fin-like structure shown in part (*b*) of the schematic. If the grooves in material A are of the same width as the grooves in material B, evaluate the merit of the proposed scheme using an appropriate analysis.



3.27 Approximately 10^6 discrete electrical components can be placed on a single integrated circuit (chip), with electrical heat dissipation as high as 30,000 W/m². The chip, which is very thin, is exposed to a dielectric liquid at its outer surface, with $h_o = 1000$ W/m² · K and $T_{\infty,o} = 20$°C, and is joined to a circuit board at its inner surface. The thermal contact resistance between the chip and the board is 10^{-4} m² · K/W, and the board thickness and thermal conductivity are $L_b = 5$ mm and $k_b = 1$ W/m · K, respectively. The other surface of the board is exposed to ambient air for which $h_i = 40$ W/m² · K and $T_{\infty,i} = 20$°C.

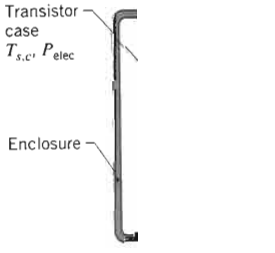


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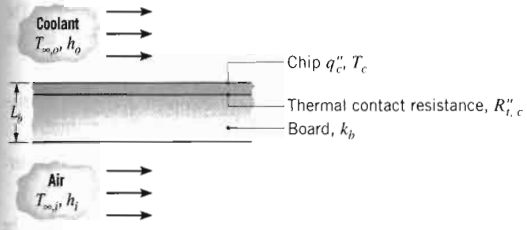
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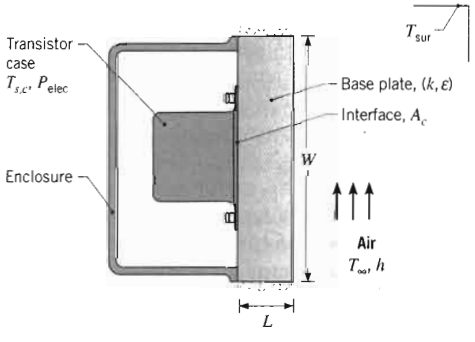


(a) Sketch the equivalent thermal circuit corresponding to steady-state conditions. In variable form, label appropriate resistances, temperatures, and heat fluxes.

(b) Under steady-state conditions for which the chip heat dissipation is $q_c'' = 30{,}000$ W/m^2, what is the chip temperature?

(c) The maximum allowable heat flux, $q_{c,m}''$, is determined by the constraint that the chip temperature must not exceed 85°C. Determine $q_{c,m}''$ for the foregoing conditions. If air is used in lieu of the dielectric liquid, the convection coefficient is reduced by approximately an order of magnitude. What is the value of $q_{c,m}''$ for $h_o = 100$ W/m$^2 \cdot$ K? With air cooling, can significant improvements be realized by using an aluminum oxide circuit board and/or by using a conductive paste at the chip/board interface for which $R_{t,c}'' = 10^{-5}$ m$^2 \cdot$ K/W?

3.28 Consider a power transistor encapsulated in an aluminum case that is attached at its base to a square aluminum plate of thermal conductivity $k = 240$ W/m $\cdot$ K, thickness $L = 6$ mm, and width $W = 20$ mm. The case is joined to the plate by screws that maintain a contact pressure of 1 bar, and the back surface of the plate transfers heat by natural convection and radiation to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The surface has an emissivity of $\varepsilon = 0.9$, and the convection coefficient is $h = 4$ W/m$^2 \cdot$ K. The case is completely enclosed such that heat transfer may be assumed to occur exclusively through the base plate.

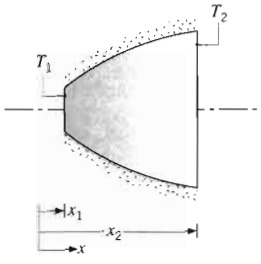


(a) If the air-filled aluminum-to-aluminum interface is characterized by an area of $A_c = 2 \times 10^{-4}$ m^2 and a roughness of 10 μm, what is the maximum allowable power dissipation if the surface temperature of the case, $T_{s,c}$, is not to exceed 85°C?

(b) The convection coefficient may be increased by subjecting the plate surface to a forced flow of air. Explore the effect of increasing the coefficient over the range $4 \le h \le 200$ W/m$^2 \cdot$ K.

Alternative Conduction Analysis

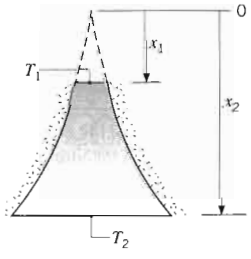
3.29 The diagram shows a conical section fabricated from pure aluminum. It is of circular cross section having diameter $D = ax^{1/2}$, where $a = 0.5$ m$^{1/2}$. The small end is located at $x_1 = 25$ mm and the large end at $x_2 = 125$ mm. The end temperatures are $T_1 = 600$ K and $T_2 = 400$ K, while the lateral surface is well insulated.



(a) Derive an expression for the temperature distribution $T(x)$ in symbolic form, assuming one-dimensional conditions. Sketch the temperature distribution.

(b) Calculate the heat rate q_x.

3.30 A truncated solid cone is of circular cross section, and its diameter is related to the axial coordinate by an expression of the form $D = ax^{3/2}$, where $a = 1.0$ m$^{-1/2}$.



The sides are well insulated, while the top surface of the cone at x_1 is maintained at T_1 and the bottom surface at x_2 is maintained at T_2.

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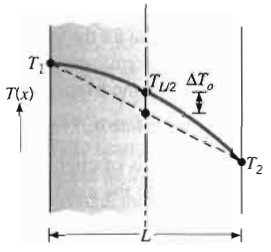
(a) Obtain an expression for the temperature distribution $T(x)$.

(b) What is the rate of heat transfer across the cone if it is constructed of pure aluminum with $x_1 = 0.075$ m, $T_1 = 100°C$, $x_2 = 0.225$ m, and $T_2 = 20°C$?

3.31 From Figure 2.5 it is evident that, over a wide temperature range, the temperature dependence of the thermal conductivity of many solids may be approximated by a linear expression of the form $k = k_o + aT$, where k_o is a positive constant and a is a coefficient that may be positive or negative. Obtain an expression for the heat flux across a plane wall whose inner and outer surfaces are maintained at T_0 and T_1, respectively. Sketch the forms of the temperature distribution corresponding to $a > 0$, $a = 0$, and $a < 0$.

3.32 Consider a tube wall of inner and outer radii r_i and r_o, whose temperatures are maintained at T_i and T_o, respectively. The thermal conductivity of the cylinder is temperature dependent and may be represented by an expression of the form $k = k_o(1 + aT)$, where k_o and a are constants. Obtain an expression for the heat transfer per unit length of the tube. What is the thermal resistance of the tube wall?

3.33 Measurements show that steady-state conduction through a plane wall without heat generation produced a convex temperature distribution such that the midpoint temperature was ΔT_o higher than expected for a linear temperature distribution.



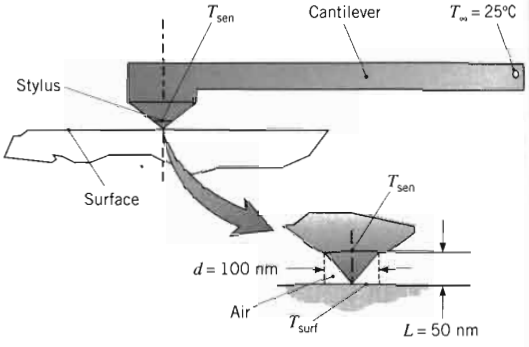
Assuming that the thermal conductivity has a linear dependence on temperature, $k = k_o(1 + \alpha T)$, where α is a constant, develop a relationship to evaluate α in terms of ΔT_o, T_1, and T_2.

3.34 A device used to measure the surface temperature of an object to within a spatial resolution of approximately 50 nm is shown in the schematic. It consists of an extremely sharp-tipped stylus and an extremely small cantilever that is scanned across the surface. The probe tip is of circular cross section and is fabricated of polycrystalline silicon dioxide. The ambient temperature is measured at the pivoted end of the cantilever as $T_\infty = 25°C$, and the device is equipped with a sensor to measure the temperature at the upper end of the sharp tip, T_{sen}. The thermal resistance between the sensing probe and the pivoted end is $R_t = 5 \times 10^6$ K/W.

(a) Determine the thermal resistance between the surface temperature and the sensing temperature.

(b) If the sensing temperature is $T_{sen} = 28.5°C$, determine the surface temperature.

Hint: Although nanoscale heat transfer effects may be important, assume that the conduction occurring in the air adjacent to the probe tip can be described by Fourier's law and the thermal conductivity found in Table A.4.



Cylindrical Wall

3.35 A steam pipe of 0.12-m outside diameter is insulated with a layer of calcium silicate.

(a) If the insulation is 20 mm thick and its inner and outer surfaces are maintained at $T_{s,1} = 800$ K and $T_{s,2} = 490$ K, respectively, what is the heat loss per unit length (q') of the pipe?

(b) We wish to explore the effect of insulation thickness on the heat loss q' and outer surface temperature $T_{s,2}$, with the inner surface temperature fixed at $T_{s,1} = 800$ K. The outer surface is exposed to an airflow ($T_\infty = 25°C$) that maintains a convection coefficient of $h = 25$ W/m$^2 \cdot$K and to large surroundings for which $T_{sur} = T_\infty = 25°C$. The surface emissivity of calcium silicate is approximately 0.8. Compute and plot the temperature distribution in the insulation as a function of the dimensionless radial coordinate, $(r - r_1)/(r_2 - r_1)$, where $r_1 = 0.06$ m and r_2 is a variable ($0.06 < r_2 \leq 0.20$ m). Compute and plot the heat loss as a function of the insulation thickness for $0 \leq (r_2 - r_1) \leq 0.14$ m.

3.36 Consider the water heater described in Problem 1.37. We now wish to determine the energy needed to compensate for heat losses incurred while the water is

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stored at the prescribed temperature of 55°C. The cylindrical storage tank (with flat ends) has a capacity of 100 gallons, and foamed urethane is used to insulate the side and end walls from ambient air at an annual average temperature of 20°C. The resistance to heat transfer is dominated by conduction in the insulation and by free convection in the air, for which $h \approx 2$ W/m$^2 \cdot$ K. If electric resistance heating is used to compensate for the losses and the cost of electric power is \$0.08/kW $\cdot$ h, specify tank and insulation dimensions for which the annual cost associated with the heat losses is less than \$50.

3.37 A thin electrical heater is wrapped around the outer surface of a long cylindrical tube whose inner surface is maintained at a temperature of 5°C. The tube wall has inner and outer radii of 25 and 75 mm, respectively, and a thermal conductivity of 10 W/m $\cdot$ K. The thermal contact resistance between the heater and the outer surface of the tube (per unit length of the tube) is $R'_{t,c} = 0.01$ m $\cdot$ K/W. The outer surface of the heater is exposed to a fluid with $T_\infty = -10$°C and a convection coefficient of $h = 100$ W/m$^2 \cdot$ K. Determine the heater power per unit length of tube required to maintain the heater at $T_o = 25$°C.

3.38 In the previous problem, the electrical power required to maintain the heater at $T_o = 25$°C depends on the thermal conductivity of the wall material k, the thermal contact resistance $R'_{t,c}$, and the convection coefficient h. Compute and plot the separate effect of changes in k ($1 \le k \le 200$ W/m $\cdot$ K), $R'_{t,c}$ ($0 \le R'_{t,c} \le 0.1$ m $\cdot$ K/W), and h ($10 \le h \le 1000$ W/m$^2 \cdot$ K) on the total heater power requirement, as well as the rate of heat transfer to the inner surface of the tube and to the fluid.

3.39 A stainless steel (AISI 304) tube used to transport a chilled pharmaceutical has an inner diameter of 36 mm and a wall thickness of 2 mm. The pharmaceutical and ambient air are at temperatures of 6°C and 23°C, respectively, while the corresponding inner and outer convection coefficients are 400 W/m$^2 \cdot$ K and 6 W/m$^2 \cdot$ K, respectively.

(a) What is the heat gain per unit tube length?

(b) What is the heat gain per unit length if a 10-mm-thick layer of calcium silicate insulation ($k_{ins} = 0.050$ W/m $\cdot$ K) is applied to the tube?

3.40 Superheated steam at 575°C is routed from a boiler to the turbine of an electric power plant through steel tubes ($k = 35$ W/m $\cdot$ K) of 300 mm inner diameter and 30 mm wall thickness. To reduce heat loss to the surroundings and to maintain a *safe-to-touch* outer surface temperature, a layer of calcium silicate insulation ($k = 0.10$ W/m $\cdot$ K) is applied to the tubes, while degradation of the insulation is reduced by wrapping it in a thin sheet of aluminum having an emissivity of $\varepsilon = 0.20$. The air and wall temperatures of the power plant are 27°C.

(a) Assuming that the inner surface temperature of a steel tube corresponds to that of the steam and the convection coefficient outside the aluminum sheet is 6 W/m$^2 \cdot$ K, what is the minimum insulation thickness needed to insure that the temperature of the aluminum does not exceed 50°C? What is the corresponding heat loss per meter of tube length?

(b) Explore the effect of the insulation thickness on the temperature of the aluminum and the heat loss per unit tube length.

3.41 A thin electrical heater is inserted between a long circular rod and a concentric tube with inner and outer radii of 20 and 40 mm. The rod (A) has a thermal conductivity of $k_A = 0.15$ W/m $\cdot$ K, while the tube (B) has a thermal conductivity of $k_B = 1.5$ W/m $\cdot$ K and its outer surface is subjected to convection with a fluid of temperature $T_\infty = -15$°C and heat transfer coefficient 50 W/m$^2 \cdot$ K. The thermal contact resistance between the cylinder surfaces and the heater is negligible.

(a) Determine the electrical power per unit length of the cylinders (W/m) that is required to maintain the outer surface of cylinder B at 5°C.

(b) What is the temperature at the center of cylinder A?

3.42 A wire of diameter $D = 2$ mm and uniform temperature T has an electrical resistance of 0.01 Ω/m and a current flow of 20 A.

(a) What is the rate at which heat is dissipated per unit length of wire? What is the heat dissipation per unit volume within the wire?

(b) If the wire is not insulated and is in ambient air and large surrounding for which $T_\infty = T_{sur} = 20$°C, what is the temperature T of the wire? The wire has an emissivity of 0.3, and the coefficient associated with heat transfer by natural convection may be approximated by an expression of the form, $h = C[(T - T_\infty)/D]^{1/4}$, where $C = 1.25$ W/m$^{7/4} \cdot$ K$^{5/4}$.

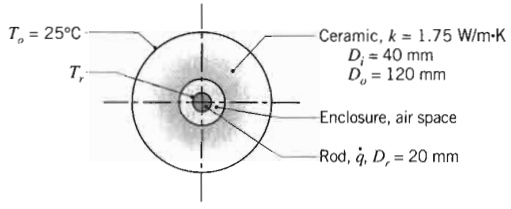
(c) If the wire is coated with plastic insulation of 2-mm thickness and a thermal conductivity of 0.25 W/m $\cdot$ K, what are the inner and outer surface temperatures of the insulation? The insulation has an emissivity of 0.9, and the convection coefficient is given by the expression of part (b). Explore the effect of the insulation thickness on the surface temperatures.

3.43 A 2-mm-diameter electrical wire is insulated by a 2-mm-thick rubberized sheath ($k = 0.13$ W/m $\cdot$ K), and the wire/sheath interface is characterized by a thermal

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contact resistance of $R''_{t,c} = 3 \times 10^{-4}\,\text{m}^2 \cdot \text{K/W}$. The convection heat transfer coefficient at the outer surface of the sheath is $10\ \text{W/m}^2 \cdot \text{K}$, and the temperature of the ambient air is 20°C. If the temperature of the insulation may not exceed 50°C, what is the maximum allowable electrical power that may be dissipated per unit length of the conductor? What is the critical radius of the insulation?

3.44 Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q} = 2 \times 10^6\ \text{W/m}^3$. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.



The thermal resistance per unit length due to radiation between the enclosure surfaces is $R'_{\text{rad}} = 0.30\ \text{m} \cdot \text{K/W}$, and the coefficient associated with free convection in the enclosure is $h = 20\ \text{W/m}^2 \cdot \text{K}$.

(a) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r. Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.

(b) Calculate the surface temperature of the rod for the prescribed conditions.

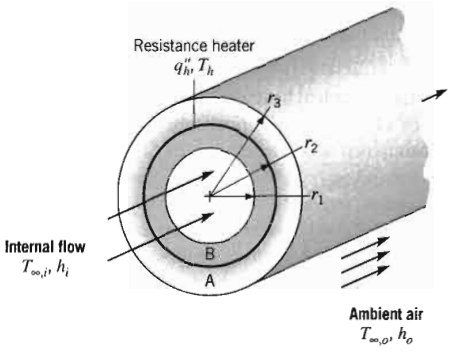
3.45 The evaporator section of a refrigeration unit consists of thin-walled, 10-mm-diameter tubes through which refrigerant passes at a temperature of −18°C. Air is cooled as it flows over the tubes, maintaining a surface convection coefficient of $100\ \text{W/m}^2 \cdot \text{K}$, and is subsequently routed to the refrigerator compartment.

(a) For the foregoing conditions and an air temperature of −3°C, what is the rate at which heat is extracted from the air per unit tube length?

(b) If the refrigerator's defrost unit malfunctions, frost will slowly accumulate on the outer tube surface. Assess the effect of frost formation on the cooling capacity of a tube for frost layer thicknesses in the range $0 \le \delta \le 4$ mm. Frost may be assumed to have a thermal conductivity of $0.4\ \text{W/m} \cdot \text{K}$.

(c) The refrigerator is disconnected after the defrost unit malfunctions and a 2-mm-thick layer of frost has formed. If the tubes are in ambient air for which $T_\infty = 20$°C and natural convection maintains a convection coefficient of $2\ \text{W/m}^2 \cdot \text{K}$, how long will it take for the frost to melt? The frost may be assumed to have a mass density of $700\ \text{kg/m}^3$ and a latent heat of fusion of 334 kJ/kg.

3.46 A composite cylindrical wall is composed of two materials of thermal conductivity k_A and k_B, which are separated by a very thin, electric resistance heater for which interfacial contact resistances are negligible.



Liquid pumped through the tube is at a temperature $T_{\infty,i}$ and provides a convection coefficient h_i at the inner surface of the composite. The outer surface is exposed to ambient air, which is at $T_{\infty,o}$ and provides a convection coefficient of h_o. Under steady-state conditions, a uniform heat flux of q''_h is dissipated by the heater.

(a) Sketch the equivalent thermal circuit of the system and express all resistances in terms of relevant variables.

(b) Obtain an expression that may be used to determine the heater temperature, T_h.

(c) Obtain an expression for the ratio of heat flows to the outer and inner fluids, q'_o/q'_i. How might the variables of the problem be adjusted to minimize this ratio?

3.47 An electrical current of 700 A flows through a stainless steel cable having a diameter of 5 mm and an electrical resistance of $6 \times 10^{-4}\ \Omega/\text{m}$ (i.e., per meter of cable length). The cable is in an environment having a temperature of 30°C, and the total coefficient associated with convection and radiation between the cable and the environment is approximately $25\ \text{W/m}^2 \cdot \text{K}$.

(a) If the cable is bare, what is its surface temperature?

(b) If a very thin coating of electrical insulation is applied to the cable, with a contact resistance of $0.02\ \text{m}^2 \cdot \text{K/W}$, what are the insulation and cable surface temperatures?

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(c) There is some concern about the ability of the insulation to withstand elevated temperatures. What thickness of this insulation ($k = 0.5$ W/m · K) will yield the lowest value of the maximum insulation temperature? What is the value of the maximum temperature when the thickness is used?

3.48 A 0.20-m-diameter, thin-walled steel pipe is used to transport saturated steam at a pressure of 20 bars in a room for which the air temperature is 25°C and the convection heat transfer coefficient at the outer surface of the pipe is 20 W/m² · K.

(a) What is the heat loss per unit length from the bare pipe (no insulation)? Estimate the heat loss per unit length if a 50-mm-thick layer of insulation (magnesia, 85%) is added. The steel and magnesia may each be assumed to have an emissivity of 0.8, and the steam-side convection resistance may be neglected.

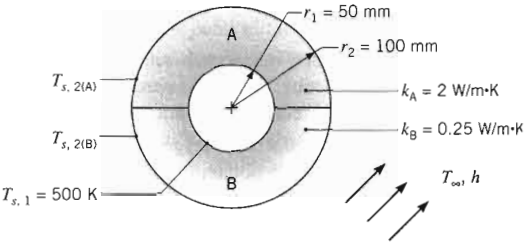
(b) The costs associated with generating the steam and installing the insulation are known to be \$4/10⁹ J and \$100/m of pipe length, respectively. If the steam line is to operate 7500 h/yr, how many years are needed to pay back the initial investment in insulation?

3.49 Steam at a temperature of 250°C flows through a steel pipe (AISI 1010) of 60-mm inside diameter and 75-mm outside diameter. The convection coefficient between the steam and the inner surface of the pipe is 500 W/m² · K, while that between the outer surface of the pipe and the surroundings is 25 W/m² · K. The pipe emissivity is 0.8, and the temperature of the air and the surroundings is 20°C. What is the heat loss per unit length of pipe?

3.50 We wish to determine the effect of adding a layer of magnesia insulation to the steam pipe of the foregoing problem. The convection coefficient at the outer surface of the insulation may be assumed to remain at 25 W/m² · K, and the emissivity is $\varepsilon = 0.8$. Determine and plot the heat loss per unit length of pipe and the outer surface temperature as a function of insulation thickness. If the cost of generating the steam is \$4/10⁹ J and the steam line operates 7000 h/yr, recommend an insulation thickness and determine the corresponding annual savings in energy costs. Plot the temperature distribution in the insulation for the recommended thickness.

3.51 An uninsulated, thin-walled pipe of 100-mm diameter is used to transport water to equipment that operates outdoors and uses the water as a coolant. During particularly harsh winter conditions, the pipe wall achieves a temperature of −15°C and a cylindrical layer of ice forms on the inner surface of the wall. If the mean water temperature is 3°C and a convection coefficient of 2000 W/m² · K is maintained at the inner surface of the ice, which is at 0°C, what is the thickness of the ice layer?

3.52 Steam flowing through a long, thin-walled pipe maintains the pipe wall at a uniform temperature of 500 K. The pipe is covered with an insulation blanket comprised of two different materials, A and B.



The interface between the two materials may be assumed to have an infinite contact resistance, and the entire outer surface is exposed to air for which $T_\infty = 300$ K and $h = 25$ W/m² · K.

(a) Sketch the thermal circuit of the system. Label (using the above symbols) all pertinent nodes and resistances.

(b) For the prescribed conditions, what is the total heat loss from the pipe? What are the outer surface temperatures $T_{s,2(A)}$ and $T_{s,2(B)}$?

3.53 A bakelite coating is to be used with a 10-mm-diameter conducting rod, whose surface is maintained at 200°C by passage of an electrical current. The rod is in a fluid at 25°C, and the convection coefficient is 140 W/m² · K. What is the critical radius associated with the coating? What is the heat transfer rate per unit length for the bare rod and for the rod with a coating of bakelite that corresponds to the critical radius? How much bakelite should be added to reduce the heat transfer associated with the bare rod by 25%?

Spherical Wall

3.54 A storage tank consists of a cylindrical section that has a length and inner diameter of $L = 2$ m and $D_i = 1$ m, respectively, and two hemispherical end sections. The tank is constructed from 20-mm-thick glass (Pyrex) and is exposed to ambient air for which the temperature is 300 K and the convection coefficient is 10 W/m² · K. The tank is used to store heated oil, which maintains the inner surface at a temperature of 400 K. Determine the electrical power that must be supplied to a heater submerged in the oil if the prescribed conditions are to be maintained. Radiation

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effects may be neglected, and the Pyrex may be assumed to have a thermal conductivity of 1.4 W/m · K.

3.55 Consider the liquid oxygen storage system and the laboratory environmental conditions of Problem 1.49. To reduce oxygen loss due to vaporization, an insulating layer should be applied to the outer surface of the container. Consider using a laminated aluminum foil/glass mat insulation, for which the thermal conductivity and surface emissivity are $k = 0.00016$ W/m · K and $\varepsilon = 0.20$, respectively.

(a) If the container is covered with a 10-mm-thick layer of insulation, what is the percentage reduction in oxygen loss relative to the uncovered container?

(b) Compute and plot the oxygen evaporation rate (kg/s) as a function of the insulation thickness t for $0 \leq t \leq 50$ mm.

3.56 In Example 3.5, an expression was derived for the critical insulation radius of an insulated, cylindrical tube. Derive the expression that would be appropriate for an insulated sphere.

3.57 A hollow aluminum sphere, with an electrical heater in the center, is used in tests to determine the thermal conductivity of insulating materials. The inner and outer radii of the sphere are 0.15 and 0.18 m, respectively, and testing is done under steady-state conditions with the inner surface of the aluminum maintained at 250°C. In a particular test, a spherical shell of insulation is cast on the outer surface of the sphere to a thickness of 0.12 m. The system is in a room for which the air temperature is 20°C and the convection coefficient at the outer surface of the insulation is 30 W/m² · K. If 80 W are dissipated by the heater under steady-state conditions, what is the thermal conductivity of the insulation?

3.58 A spherical tank for storing liquid oxygen on the space shuttle is to be made from stainless steel of 0.80-m outer diameter and 5-mm wall thickness. The boiling point and latent heat of vaporization of liquid oxygen are 90 K and 213 kJ/kg, respectively. The tank is to be installed in a large compartment whose temperature is to be maintained at 240 K. Design a thermal insulation system that will maintain oxygen losses due to boiling below 1 kg/day.

3.59 A spherical, cryosurgical probe may be imbedded in diseased tissue for the purpose of freezing, and thereby destroying, the tissue. Consider a probe of 3-mm diameter whose surface is maintained at −30°C when imbedded in tissue that is at 37°C. A spherical layer of frozen tissue forms around the probe, with a temperature of 0°C existing at the phase front (interface) between the frozen and normal tissue. If the thermal conductivity of frozen tissue is approximately 1.5 W/m · K and heat transfer at the phase front may be characterized by an effective convection coefficient of 50 W/m² · K, what is the thickness of the layer of frozen tissue (assuming negligible perfusion)?

3.60 A spherical vessel used as a reactor for producing pharmaceuticals has a 10-mm-thick stainless steel wall (k = 17 W/m · K) and an inner diameter of 1 m. The exterior surface of the vessel is exposed to ambient air (T_∞ = 25°C) for which a convection coefficient of 6 W/m² · K may be assumed.

(a) During steady-state operation, an inner surface temperature of 50°C is maintained by energy generated within the reactor. What is the heat loss from the vessel?

(b) If a 20-mm-thick layer of fiberglass insulation (k = 0.040 W/m · K) is applied to the exterior of the vessel and the rate of thermal energy generation is unchanged, what is the inner surface temperature of the vessel?

3.61 The wall of a spherical tank of 1-m diameter contains an exothermic chemical reaction and is at 200°C when the ambient air temperature is 25°C. What thickness of urethane foam is required to reduce the exterior temperature to 40°C, assuming the convection coefficient is 20 W/m² · K for both situations? What is the percentage reduction in heat rate achieved by using the insulation?

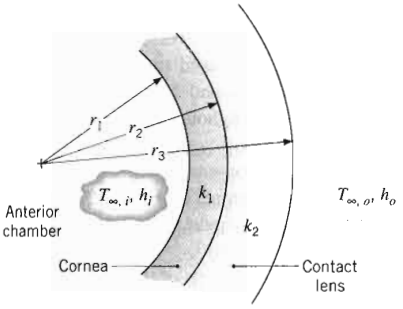
3.62 A composite spherical shell of inner radius $r_1 = 0.25$ m is constructed from lead of outer radius $r_2 = 0.30$ m and AISI 302 stainless steel of outer radius $r_3 = 0.31$ m. The cavity is filled with radioactive wastes that generate heat at a rate of $\dot{q} = 5 \times 10^5$ W/m³. It is proposed to submerge the container in oceanic waters that are at a temperature of $T_\infty = 10$°C and provide a uniform convection coefficient of $h = 500$ W/m² · K at the outer surface of the container. Are there any problems associated with this proposal?

3.63 As an alternative to storing radioactive materials in oceanic waters, it is proposed that the system of Problem 3.62 be placed in a large tank for which the flow of water, and hence the convection coefficient h, can be controlled. Compute and plot the maximum temperature of the lead, $T(r_1)$, as a function of h for $100 \leq h \leq 1000$ W/m² · K. If the temperature of the lead is not to exceed 500 K, what is the minimum allowable value of h? To improve system reliability, it is desirable to increase the thickness of the stainless steel shell. For h = 300, 500, and 1000 W/m² · K, compute and plot the maximum lead temperature as a function of shell thickness for $r_3 \geq 0.30$ m. What are the corresponding values of the maximum allowable thickness?

3.64 The energy transferred from the anterior chamber of the eye through the cornea varies considerably depending on whether a contact lens is worn. Treat the eye as a

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spherical system and assume the system to be at steady state. The convection coefficient h_o is unchanged with and without the contact lens in place. The cornea and the lens cover one-third of the spherical surface area.



Values of the parameters representing this situation are as follows:

$r_1 = 10.2$ mm	$r_2 = 12.7$ mm
$r_3 = 16.5$ mm	$T_{\infty,o} = 21°C$
$T_{\infty,i} = 37°C$	$k_2 = 0.80$ W/m · K
$k_1 = 0.35$ W/m · K	$h_o = 6$ W/m^2 · K
$h_i = 12$ W/m^2 · K	

(a) Construct the thermal circuits, labeling all potentials and flows for the systems excluding the contact lens and including the contact lens. Write resistance elements in terms of appropriate parameters.

(b) Determine the heat loss from the anterior chamber with and without the contact lens in place.

(c) Discuss the implication of your results.

3.65 The outer surface of a hollow sphere of radius r_2 is subjected to a uniform heat flux q''_2. The inner surface at r_1 is held at a constant temperature $T_{s,1}$.

(a) Develop an expression for the temperature distribution $T(r)$ in the sphere wall in terms of q''_2, $T_{s,1}$, r_1, r_2, and the thermal conductivity of the wall material k.

(b) If the inner and outer tube radii are $r_1 = 50$ mm and $r_2 = 100$ mm, what heat flux q''_2 is required to maintain the outer surface at $T_{s,2} = 50°C$, while the inner surface is at $T_{s,1} = 20°C$? The thermal conductivity of the wall material is $k = 10$ W/m · K.

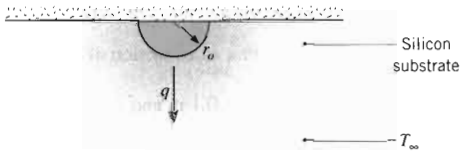
3.66 A spherical shell of inner and outer radii r_i and r_o, respectively, is filled with a heat-generating material that provides for a uniform volumetric generation rate (W/m^3) of $\dot{q}$. The outer surface of the shell is exposed to a fluid having a temperature T_∞ and a convection coefficient h. Obtain an expression for the steady-state temperature distribution $T(r)$ in the shell, expressing your result in terms of r_i, r_o, $\dot{q}$, h, T_∞, and the thermal conductivity k of the shell material.

3.67 A spherical tank of 3-m diameter contains a liquified-petroleum gas at −60°C. Insulation with a thermal conductivity of 0.06 W/m · K and thickness 250 mm is applied to the tank to reduce the heat gain.

(a) Determine the radial position in the insulation layer at which the temperature is 0°C when the ambient air temperature is 20°C and the convection coefficient on the outer surface is 6 W/m^2 · K.

(b) If the insulation is pervious to moisture from the atmospheric air, what conclusions can you reach about the formation of ice in the insulation? What effect will ice formation have on heat gain to the LP gas? How could this situation be avoided?

3.68 A transistor, which may be approximated as a hemispherical heat source of radius $r_o = 0.1$ mm, is embedded in a large silicon substrate ($k = 125$ W/m · K) and dissipates heat at a rate q. All boundaries of the silicon are maintained at an ambient temperature of $T_\infty = 27°C$, except for the top surface, which is well insulated.



Obtain a general expression for the substrate temperature distribution and evaluate the surface temperature of the heat source for $q = 4$ W.

3.69 One modality for destroying malignant tissue involves imbedding a small spherical heat source of radius r_o within the tissue and maintaining local temperatures above a critical value T_c for an extended period. Tissue that is well removed from the source may be assumed to remain at normal body temperature ($T_b = 37°C$). Obtain a general expression for the radial temperature distribution in the tissue under steady-state conditions for which heat is dissipated at a rate q. If $r_o = 0.5$ mm, what heat rate must be supplied to maintain a tissue temperature of $T \geq T_c = 42°C$ in the domain $0.5 \leq r \leq 5$ mm? The tissue thermal conductivity is approximately 0.5 W/m · K. Assume negligible perfusion.

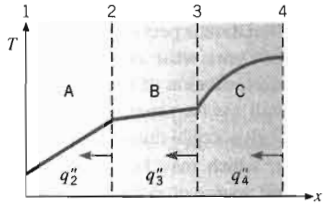
Conduction with Thermal Energy Generation

3.70 Consider cylindrical and spherical shells with inner and outer surfaces at r_1 and r_2 maintained at uniform

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temperatures $T_{s,1}$ and $T_{s,2}$, respectively. If there is uniform heat generation within the shells, obtain expressions for the steady-state, one-dimensional radial distributions of the temperature, heat flux, and heat rate. Contrast your results with those summarized in Appendix C.

3.71 The steady-state temperature distribution in a composite plane wall of three different materials, each of constant thermal conductivity, is shown as follows.



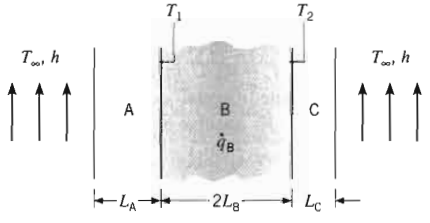
(a) Comment on the relative magnitudes of q_2'' and q_3'' and of q_3'' and q_4''.

(b) Comment on the relative magnitudes of k_A and k_B and of k_B and k_C.

(c) Sketch the heat flux as a function of x.

3.72 A plane wall of thickness 0.1 m and thermal conductivity 25 W/m · K having uniform volumetric heat generation of 0.3 MW/m^3 is insulated on one side, while the other side is exposed to a fluid at 92°C. The convection heat transfer coefficient between the wall and the fluid is 500 W/m^2 · K. Determine the maximum temperature in the wall.

3.73 Consider one-dimensional conduction in a plane composite wall. The outer surfaces are exposed to a fluid at 25°C and a convection heat transfer coefficient of 1000 W/m^2 · K. The middle wall B experiences uniform heat generation $\dot{q}_B$, while there is no generation in walls A and C. The temperatures at the interfaces are T_1 = 261°C and T_2 = 211°C.



k_A = 25 W/m·K L_A = 30 mm
k_C = 50 W/m·K L_B = 30 mm
L_C = 20 mm

(a) Assuming negligible contact resistance at the interfaces, determine the volumetric heat generation $\dot{q}_B$ and the thermal conductivity k_B.

(b) Plot the temperature distribution, showing its important features.

(c) Consider conditions corresponding to a *loss of coolant* at the exposed surface of material A (h = 0). Determine T_1 and T_2 and plot the temperature distribution throughout the system.

3.74 Consider a plane composite wall that is composed of three materials (materials A, B, and C are arranged left to right) of thermal conductivities k_A = 0.24 W/m · K, k_B = 0.13 W/m · K, and k_C = 0.50 W/m · K. The thicknesses of the three sections of the wall are L_A = 20 mm, L_B = 13 mm, and L_C = 20 mm. A contact resistance of $R''_{t,c}$ = 10^{-2} m^2 · K/W exists at the interface between materials A and B, as well as at the interface between materials B and C. The left face of the composite wall is insulated, while the right face is exposed to convective conditions characterized by h = 10 W/m^2 · K, T_∞ = 20°C. For Case 1, thermal energy is generated within material A at the rate $\dot{q}_A$ = 5000 W/m^3. For Case 2, thermal energy is generated within material C at the rate $\dot{q}_C$ = 5000 W/m^3.

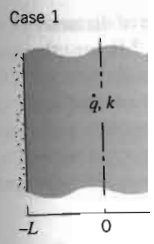
(a) Determine the maximum temperature within the composite wall under steady-state conditions for Case 1.

(b) Sketch the steady-state temperature distribution on $T - x$ coordinates for Case 1.

(c) Sketch the steady-state temperature distribution for Case 2 on the same $T - x$ coordinates used for Case 1.

3.75 Consider the composite wall of Example 3.7. In the Comments section, temperature distributions in the wall were determined assuming negligible contact resistance between materials A and B. Compute and plot the temperature distributions if the thermal contact resistance is $R''_{t,c}$ = 10^{-4} m^2 · K/W.

3.76 A plane wall of thickness 2L and thermal conductivity k experiences a uniform volumetric generation rate $\dot{q}$. As shown in the sketch for Case 1, the surface at $x = -L$ is perfectly insulated, while the other surface is maintained at a uniform, constant temperature T_o. For Case 2, a very thin dielectric strip is inserted at the midpoint of the wall (x = 0) in order to electrically isolate the two sections, A and B. The thermal resistance of the strip is R'_t = 0.0005 m^2 · K/W. The parameters associated with the wall are k = 50 W/m · K, L = 20 mm, $\dot{q}$ = 5 × 10^6 W/m^3, and T_o = 50°C.



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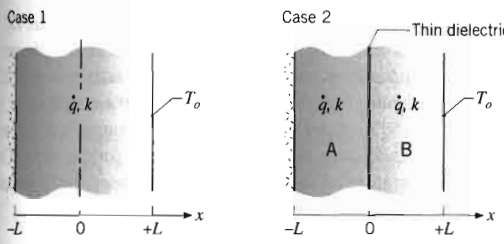
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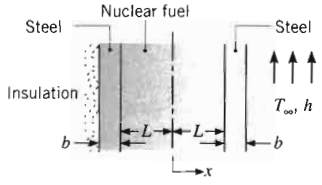
(a) Sketch the temperature distribution for case 1 on $T-x$ coordinates. Describe the key features of this distribution. Identify the location of the maximum temperature in the wall and calculate this temperature.

(b) Sketch the temperature distribution for Case 2 on the same $T - x$ coordinates. Describe the key features of this distribution.

(c) What is the temperature difference between the two walls at $x = 0$ for Case 2?

(d) What is the location of the maximum temperature in the composite wall of Case 2? Calculate this temperature.

3.77 A nuclear fuel element of thickness $2L$ is covered with a steel cladding of thickness b. Heat generated within the nuclear fuel at a rate $\dot{q}$ is removed by a fluid at T_∞, which adjoins one surface and is characterized by a convection coefficient h. The other surface is well insulated, and the fuel and steel have thermal conductivities of k_f and k_s, respectively.



(a) Obtain an equation for the temperature distribution $T(x)$ in the nuclear fuel. Express your results in terms of $\dot{q}$, k_f, L, b, k_s, h, and T_∞.

(b) Sketch the temperature distribution $T(x)$ for the entire system.

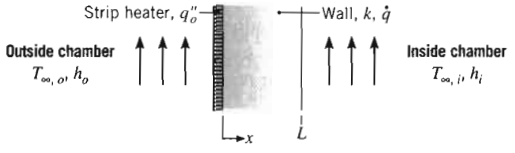
3.78 Consider the clad fuel element of Problem 3.77.

(a) Using appropriate relations from Tables C.1 and C.2, obtain an expression for the temperature distribution $T(x)$ in the fuel element. For $k_f = 60$ W/m · K, $L =$ 15 mm, $b = 3$ mm, $k_s = 15$ W/m · K, $h = 10{,}000$ W/m^2 · K, and $T_\infty = 200$°C, what are the largest and smallest temperatures in the fuel element if heat is generated uniformly at a volumetric rate of $\dot{q} = 2 \times 10^7$ W/m^3? What are the corresponding locations?

(b) If the insulation is removed and equivalent convection conditions are maintained at each surface, what is the corresponding form of the temperature distribution in the fuel element? For the conditions of part (a), what are the largest and smallest temperatures in the fuel? What are the corresponding locations?

(c) For the conditions of parts (a) and (b), plot the temperature distributions in the fuel element.

3.79 The air *inside* a chamber at $T_{\infty,i} = 50$°C is heated convectively with $h_i = 20$ W/m^2 · K by a 200-mm-thick wall having a thermal conductivity of 4 W/m · K and a uniform heat generation of 1000 W/m^3. To prevent any heat generated within the wall from being lost to the *outside* of the chamber at $T_{\infty,o} = 25$°C with $h_o = 5$ W/m^2 · K, a very thin electrical strip heater is placed on the outer wall to provide a uniform heat flux, q''_o.



(a) Sketch the temperature distribution in the wall on T–x coordinates for the condition where no heat generated within the wall is lost to the *outside* of the chamber.

(b) What are the temperatures at the wall boundaries, $T(0)$ and $T(L)$, for the conditions of part (a)?

(c) Determine the value of q''_o that must be supplied by the strip heater so that all heat generated within the wall is transferred to the *inside* of the chamber.

(d) If the heat generation in the wall were switched off while the heat flux to the strip heater remained constant, what would be the steady-state temperature, $T(0)$, of the outer wall surface?

3.80 In the previous problem, the strip heater acts to *guard* against heat losses from the wall to the outside, and the required heat flux q''_o depends on chamber operating conditions such as $\dot{q}$ and $T_{\infty,i}$. As a first step in designing a controller for the guard heater, compute and plot q''_o and $T(0)$ as a function of $\dot{q}$ for $200 \le \dot{q} \le 2000$ W/m^3 and $T_{\infty,i} = 30$, 50, and 70°C.

3.81 The exposed surface ($x = 0$) of a plane wall of thermal conductivity k is subjected to microwave radiation that causes volumetric heating to vary as

$$\dot{q}(x) = \dot{q}_o\left(1 - \frac{x}{L}\right)$$

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where $\dot{q}_o$ (W/m^3) is a constant. The boundary at $x = L$ is perfectly insulated, while the exposed surface is maintained at a constant temperature T_o. Determine the temperature distribution $T(x)$ in terms of x, L, k, $\dot{q}_o$, and T_o.

3.82 A quartz window of thickness L serves as a viewing port in a furnace used for annealing steel. The inner surface ($x = 0$) of the window is irradiated with a uniform heat flux q''_o due to emission from hot gases in the furnace. A fraction, β, of this radiation may be assumed to be absorbed at the inner surface, while the remaining radiation is partially absorbed as it passes through the quartz. The volumetric heat generation due to this absorption may be described by an expression of the form

$$\dot{q}(x) = (1 - \beta)q''_o\alpha e^{-\alpha x}$$

where α is the absorption coefficient of the quartz. Convection heat transfer occurs from the outer surface ($x = L$) of the window to ambient air at T_∞ and is characterized by the convection coefficient h. Convection and radiation emission from the inner surface may be neglected, along with radiation emission from the outer surface. Determine the temperature distribution in the quartz, expressing your result in terms of the foregoing parameters.

3.83 For the conditions described in Problem 1.44, determine the temperature distribution, $T(r)$, in the container, expressing your result in terms of $\dot{q}_o$, r_o, T_∞, h, and the thermal conductivity k of the radioactive wastes.

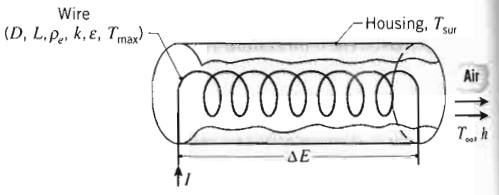
3.84 A cylindrical shell of inner and outer radii, r_i and r_o, respectively, is filled with a heat-generating material that provides a uniform volumetric generation rate (W/m^3) of $\dot{q}$. The inner surface is insulated, while the outer surface of the shell is exposed to a fluid at T_∞ and a convection coefficient h.

(a) Obtain an expression for the steady-state temperature distribution, $T(r)$, in the shell, expressing your result in terms of r_i, r_o, $\dot{q}$, h, T_∞, and the thermal conductivity k of the shell material.

(b) Determine an expression for the heat rate, $q'(r_o)$, at the outer radius of the shell in terms of $\dot{q}$ and shell dimensions.

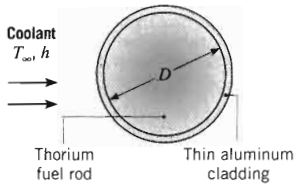
3.85 Consider the solid tube of Example 3.8. Using Equation (7) with Equation (10) in the *IHT* workspace, calculate and plot the temperature distributions for a tube of inner and outer radii, 50 mm and 100 mm, and a thermal conductivity of 5 W/m · K for volumetric generation rates of 1×10^5, 5×10^5, and 1×10^6 W/m^3. The inner surface is cooled by a fluid at 30°C with a convection coefficient of 1000 W/m^2 · K.

3.86 An air heater may be fabricated by coiling Nichrome wire and passing air in cross flow over the wire. Consider a heater fabricated from wire of diameter $D = 1$ mm, electrical resistivity $\rho_e = 10^{-6}\ \Omega \cdot$ m, thermal conductivity $k = 25$ W/m · K, and emissivity $\varepsilon = 0.20$. The heater is designed to deliver air at a temperature of $T_\infty = 50$°C under flow conditions that provide a convection coefficient of $h = 250$ W/m^2 · K for the wire. The temperature of the housing that encloses the wire and through which the air flows is $T_{sur} = 50$°C.



If the maximum allowable temperature of the wire is $T_{max} = 1200$°C, what is the maximum allowable electric current I? If the maximum available voltage is $\Delta E = 110$ V, what is the corresponding length L of wire that may be used in the heater and the power rating of the heater? *Hint:* In your solution, assume negligible temperature variations within the wire, but after obtaining the desired results, assess the validity of this assumption.

3.87 The cross section of a long cylindrical fuel element in a nuclear reactor is shown. Energy generation occurs uniformly in the thorium fuel rod, which is of diameter $D = 25$ mm and is wrapped in a thin aluminum cladding.



(a) It is proposed that, under steady-state conditions, the system operates with a generation rate of $\dot{q} = 7 \times 10^8$ W/m^3 and cooling system characteristics of $T_\infty = 95$°C and $h = 7000$ W/m^2 · K. Is this proposal satisfactory?

(b) Explore the effect of variations in $\dot{q}$ and h by plotting temperature distributions, $T(r)$, for a range of parameter values. Suggest an envelope of acceptable operating conditions.

3.88 A nuclear reactor fuel element consists of a solid cylindrical pin of radius r_1 and thermal conductivity k_f. The fuel pin is in good contact with a cladding material of outer radius r_2 and thermal conductivity k_c. Consider steady-state conditions for which uniform

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heat generation occurs within the fuel at a volumetric rate $\dot{q}$ and the outer surface of the cladding is exposed to a coolant that is characterized by a temperature T_∞ and a convection coefficient h.

(a) Obtain equations for the temperature distributions $T_f(r)$ and $T_c(r)$ in the fuel and cladding, respectively. Express your results exclusively in terms of the foregoing variables.

(b) Consider a uranium oxide fuel pin for which $k_f = 2$ W/m · K and $r_1 = 6$ mm and cladding for which $k_c = 25$ W/m · K and $r_2 = 9$ mm. If $\dot{q} = 2 \times 10^8$ W/m^3, $h = 2000$ W/m^2 · K, and $T_\infty = 300$ K, what is the maximum temperature in the fuel element?

(c) Compute and plot the temperature distribution, $T(r)$, for values of $h = 2000$, 5000, and 10,000 W/m^2 · K. If the operator wishes to maintain the centerline temperature of the fuel element below 1000 K, can she do so by adjusting the coolant flow and hence the value of h?

3.89 Consider the configuration of Example 3.8, where uniform volumetric heating within a stainless steel tube is induced by an electric current and heat is transferred by convection to air flowing through the tube. The tube wall has inner and outer radii of $r_1 = 25$ mm and $r_2 = 35$ mm, a thermal conductivity of $k = 15$ W/m · K, an electrical resistivity of $\rho_e = 0.7 \times 10^{-6}$ Ω · m, and a maximum allowable operating temperature of 1400 K.

(a) Assuming the outer tube surface to be perfectly insulated and the air flow to be characterized by a temperature and convection coefficient of $T_{\infty,1} = 400$ K and $h_1 = 100$ W/m^2 · K, determine the maximum allowable electric current I.

(b) Compute and plot the radial temperature distribution in the tube wall for the electric current of part (a) and three values of h_1 (100, 500, and 1000 W/m^2 · K). For each value of h_1, determine the rate of heat transfer to the air per unit length of tube.

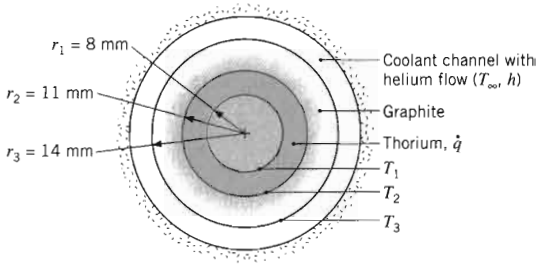
(c) In practice, even the best of insulating materials would be unable to maintain adiabatic conditions at the outer tube surface. Consider use of a refractory insulating material of thermal conductivity $k = 1.0$ W/m · K and neglect radiation exchange at its outer surface. For $h_1 = 100$ W/m^2 · K and the maximum allowable current determined in part (a), compute and plot the temperature distribution in the *composite* wall for two values of the insulation thickness ($\delta = 25$ and 50 mm). The outer surface of the insulation is exposed to room air for which $T_{\infty,2} = 300$ K and $h_2 = 25$ W/m^2 · K. For each insulation thickness, determine the rate of heat transfer per unit tube length to the inner air flow and the ambient air.

3.90 A homeowner, whose water pipes have frozen during a period of cold weather, decides to melt the ice by passing an electric current I through the pipe wall. The inner and outer radii of the wall are designated as r_1 and r_2, and its electrical resistance per unit length is designated as R'_e (Ω/m). The pipe is well insulated on the outside, and during melting the ice (and water) in the pipe remain at a constant temperature T_m associated with the melting process.

(a) Assuming that steady-state conditions are reached shortly after application of the current, determine the form of the steady-state temperature distribution $T(r)$ in the pipe wall during the melting process.

(b) Develop an expression for the time t_m required to completely melt the ice. Calculate this time for $I = 100$ A, $R'_e = 0.30$ Ω/m, and $r_1 = 50$ mm.

3.91 A high-temperature, gas-cooled nuclear reactor consists of a composite cylindrical wall for which a thorium fuel element ($k \approx 57$ W/m · K) is encased in graphite ($k \approx 3$ W/m · K) and gaseous helium flows through an annular coolant channel. Consider conditions for which the helium temperature is $T_\infty = 600$ K and the convection coefficient at the outer surface of the graphite is $h = 2000$ W/m^2 · K.



(a) If thermal energy is uniformly generated in the fuel element at a rate $\dot{q} = 10^8$ W/m^3, what are the temperatures T_1 and T_2 at the inner and outer surfaces, respectively, of the fuel element?

(b) Compute and plot the temperature distribution in the composite wall for selected values of $\dot{q}$. What is the maximum allowable value of $\dot{q}$?

3.92 A long cylindrical rod of diameter 200 mm with thermal conductivity of 0.5 W/m · K experiences uniform volumetric heat generation of 24,000 W/m^3. The rod is encapsulated by a circular sleeve having an outer diameter of 400 mm and a thermal conductivity of 4 W/m · K. The outer surface of the sleeve is exposed to cross flow of air at 27°C with a convection coefficient of 25 W/m^2 · K.

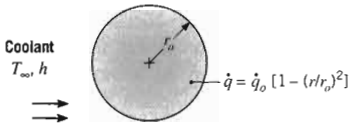
(a) Find the temperature at the interface between the rod and sleeve and on the outer surface.

(b) What is the temperature at the center of the rod?

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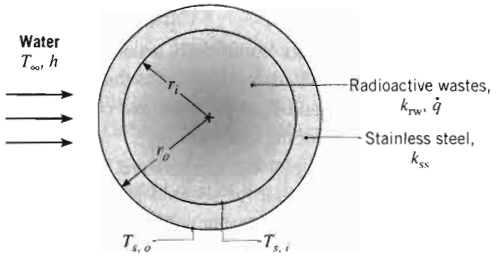
3.93 A radioactive material of thermal conductivity k is cast as a solid sphere of radius r_o and placed in a liquid bath for which the temperature T_∞ and convection coefficient h are known. Heat is uniformly generated within the solid at a volumetric rate of $\dot{q}$. Obtain the steady-state radial temperature distribution in the solid, expressing your result in terms of r_o, $\dot{q}$, k, h, and T_∞.

3.94 Radioactive wastes are packed in a thin-walled spherical container. The wastes generate thermal energy nonuniformly according to the relation $\dot{q} = \dot{q}_o[1 - (r/r_o)^2]$, where $\dot{q}$ is the local rate of energy generation per unit volume, $\dot{q}_o$ is a constant, and r_o is the radius of the container. Steady-state conditions are maintained by submerging the container in a liquid that is at T_∞ and provides a uniform convection coefficient h.



Determine the temperature distribution, $T(r)$, in the container. Express your result in terms of $\dot{q}_o$, r_o, T_∞, h, and the thermal conductivity k of the radioactive wastes.

3.95 Radioactive wastes ($k_{rw} = 20$ W/m · K) are stored in a spherical, stainless steel ($k_{ss} = 15$ W/m · K) container of inner and outer radii equal to $r_i = 0.5$ m and $r_o = 0.6$ m. Heat is generated volumetrically within the wastes at a uniform rate of $\dot{q} = 10^5$ W/m^3, and the outer surface of the container is exposed to a water flow for which $h = 1000$ W/m^2 · K and $T_\infty = 25$°C.



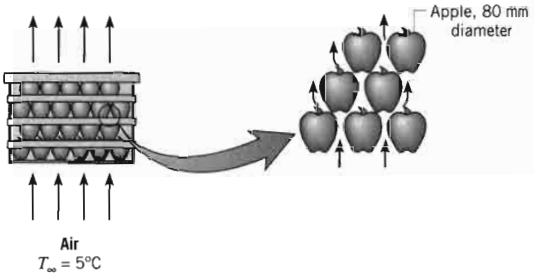
(a) Evaluate the steady-state outer surface temperature, $T_{s,o}$.

(b) Evaluate the steady-state inner surface temperature, $T_{s,i}$.

(c) Obtain an expression for the temperature distribution, $T(r)$, in the radioactive wastes. Express your result in terms of r_i, $T_{s,i}$, k_{rw}, and $\dot{q}$. Evaluate the temperature at $r = 0$.

(d) A proposed extension of the foregoing design involves storing waste materials having the same thermal conductivity but twice the heat generation ($\dot{q} = 2 \times 10^5$ W/m^3) in a stainless steel container of equivalent inner radius ($r_i = 0.5$ m). Safety considerations dictate that the maximum system temperature not exceed 475°C and that the container wall thickness be no less than $t = 0.04$ m and preferably at or close to the original design ($t = 0.1$ m). Assess the effect of varying the outside convection coefficient to a maximum achievable value of $h = 5000$ W/m^2 · K (by increasing the water velocity) and the container wall thickness. Is the proposed extension feasible? If so, recommend suitable operating and design conditions for h and t, respectively.

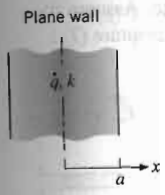
3.96 Unique characteristics of biologically active materials such as fruits, vegetables, and other products require special care in handling. Following harvest and separation from producing plants, glucose is catabolized to produce carbon dioxide, water vapor, and heat, with attendant internal energy generation. Consider a carton of apples, each of 80-mm diameter, which is ventilated with air at 5°C and a velocity of 0.5 m/s. The corresponding value of the heat transfer coefficient is 7.5 W/m^2 · K. Within each apple thermal energy is uniformly generated at a total rate of 4000 J/kg · day. The density and thermal conductivity of the apple are 840 kg/m^3 and 0.5 W/m · K, respectively.



(a) Determine the apple center and surface temperatures.

(b) For the stacked arrangement of apples within the crate, the convection coefficient depends on the velocity as $h = C_1 V^{0.425}$, where $C_1 = 10.1$ W/m^2 · K · (m/s)$^{0.425}$. Compute and plot the center and surface temperatures as a function of the air velocity for $0.1 \le V \le 1$ m/s.

3.97 Consider the plane wall, long cylinder, and sphere shown schematically, each with the same characteristic length a, thermal conductivity k, and uniform volumetric energy generation rate $\dot{q}$.



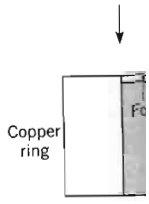
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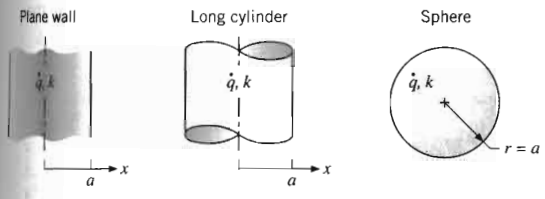
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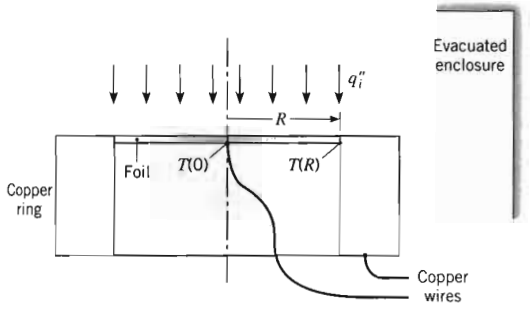
(a) On the same graph, plot the steady-state dimensionless temperature, $[T(x \text{ or } r) - T(a)]/[(\dot{q}a^2)/2k]$, versus the dimensionless characteristic length, x/a or r/a, for each shape.

(b) Which shape has the smallest temperature difference between the center and the surface? Explain this behavior by comparing the ratio of the volume-to-surface area.

(c) Which shape would be preferred for use as a nuclear fuel element? Explain why.

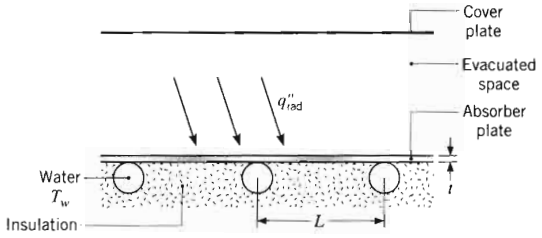
Extended Surfaces

3.98 The radiation heat gage shown in the diagram is made from constantan metal foil, which is coated black and is in the form of a circular disk of radius R and thickness t. The gage is located in an evacuated enclosure. The incident radiation flux absorbed by the foil, q_i'', diffuses toward the outer circumference and into the larger copper ring, which acts as a heat sink at the constant temperature $T(R)$. Two copper lead wires are attached to the center of the foil and to the ring to complete a thermocouple circuit that allows for measurement of the temperature difference between the foil center and the foil edge, $\Delta T = T(0) - T(R)$.



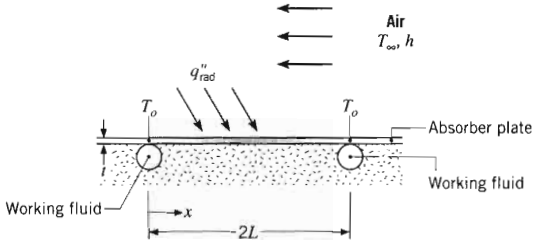
Obtain the differential equation that determines $T(r)$, the temperature distribution in the foil, under steady-state conditions. Solve this equation to obtain an expression relating ΔT to q_i''. You may neglect radiation exchange between the foil and its surroundings.

3.99 Copper tubing is joined to the absorber of a flat-plate solar collector as shown.



The aluminum alloy (2024-T6) absorber plate is 6 mm thick and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced a distance L of 0.20 m from each other, and water is circulated through the tubes to remove the collected energy. The water may be assumed to be at a uniform temperature of $T_w = 60°C$. Under steady-state operating conditions for which the *net* radiation heat flux to the surface is $q''_{rad} = 800 \text{ W/m}^2$, what is the maximum temperature on the plate and the heat transfer rate per unit length of tube? Note that q''_{rad} represents the net effect of solar radiation absorption by the absorber plate and radiation exchange between the absorber and cover plates. You may assume the temperature of the absorber plate directly above a tube to be equal to that of the water.

3.100 Copper tubing is joined to a solar collector plate of thickness t, and the working fluid maintains the temperature of the plate above the tubes at T_o. There is a uniform net radiation heat flux q''_{rad} to the top surface of the plate, while the bottom surface is well insulated. The top surface is also exposed to a fluid at T_∞ that provides for a uniform convection coefficient h.



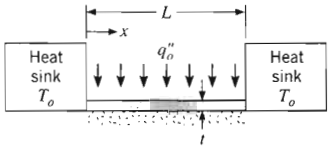
(a) Derive the differential equation that governs the temperature distribution $T(x)$ in the plate.

(b) Obtain a solution to the differential equation for appropriate boundary conditions.

3.101 A thin flat plate of length L, thickness t, and width $W \gg L$ is thermally joined to two large heat sinks that are maintained at a temperature T_o. The bottom of the

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plate is well insulated, while the net heat flux to the top surface of the plate is known to have a uniform value of q''_o.



(a) Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

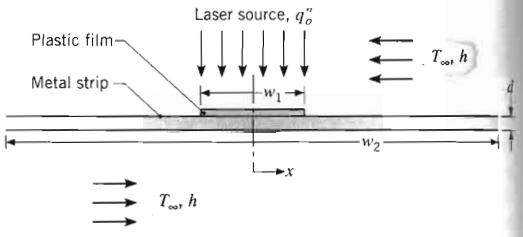
3.102 Consider the flat plate of Problem 3.101, but with the heat sinks at different temperatures, $T(0) = T_o$ and $T(L) = T_L$, and with the bottom surface no longer insulated. Convection heat transfer is now allowed to occur between this surface and a fluid at T_∞, with a convection coefficient h.

(a) Derive the differential equation that determines the steady-state temperature distribution $T(x)$ in the plate.

(b) Solve the foregoing equation for the temperature distribution, and obtain an expression for the rate of heat transfer from the plate to the heat sinks.

(c) For $q''_o = 20{,}000$ W/m^2, $T_o = 100$°C, $T_L = 35$°C, $T_\infty = 25$°C, $k = 25$ W/m · K, $h = 50$ W/m^2 · K, $L = 100$ mm, $t = 5$ mm, and a plate width of $W = 30$ mm, plot the temperature distribution and determine the sink heat rates, $q_x(0)$ and $q_x(L)$. On the same graph, plot three additional temperature distributions corresponding to changes in the following parameters, with the remaining parameters unchanged: (i) $q''_o = 30{,}000$ W/m^2, (ii) $h = 200$ W/m^2 · K, and (iii) the value of q''_o for which $q_x(0) = 0$ when $h = 200$ W/m^2 · K.

3.103 A bonding operation utilizes a laser to provide a constant heat flux, q''_o, across the top surface of a thin adhesive-backed, plastic film to be affixed to a metal strip as shown in the sketch. The metal strip has a thickness $d = 1.25$ mm and its width is large relative to that of the film. The thermophysical properties of the strip are $\rho = 7850$ kg/m^3, $c_p = 435$ J/kg · K, and $k = 60$ W/m · K. The thermal resistance of the plastic film of width $w_1 = 40$ mm is negligible. The upper and lower surfaces of the strip (including the plastic film) experience convection with air at 25°C and a convection coefficient of 10 W/m^2 · K. The strip and film are very long in the direction normal to the page. Assume the edges of the metal strip are at the air temperature (T_∞).



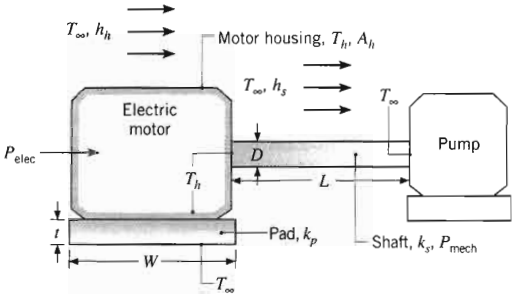
(a) Derive an expression for the temperature distribution in the portion of the steel strip with the plastic film ($-w_1/2 \leq x \leq +w_1/2$).

(b) If the heat flux provided by the laser is 10,000 W/m^2, determine the temperature of the plastic film at the center ($x = 0$) and its edges ($x = \pm w_1/2$).

(c) Plot the temperature distribution for the entire strip and point out its special features.

3.104 A thin metallic wire of thermal conductivity k, diameter D, and length $2L$ is annealed by passing an electrical current through the wire to induce a uniform volumetric heat generation $\dot{q}$. The ambient air around the wire is at a temperature T_∞, while the ends of the wire at $x = \pm L$ are also maintained at T_∞. Heat transfer from the wire to the air is characterized by the convection coefficient h. Obtain an expression for the steady-state temperature distribution $T(x)$ along the wire.

3.105 A motor draws electric power P_{elec} from a supply line and delivers mechanical power P_{mech} to a pump through a rotating copper shaft of thermal conductivity k_s, length L, and diameter D. The motor is mounted on a square pad of width W, thickness t, and thermal conductivity k_p. The surface of the housing exposed to ambient air at T_∞ is of area A_h, and the corresponding convection coefficient is h_h. Opposite ends of the shaft are at temperatures of T_h and T_∞, and heat transfer from the shaft to the ambient air is characterized by the convection coefficient h_s. The base of the pad is at T_∞.



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(a) Expressing your result in terms of P_{elec}, P_{mech}, k_s, L, D, W, t, k_ρ, A_h, h_h, and h_s, obtain an expression for $(T_h - T_\infty)$.

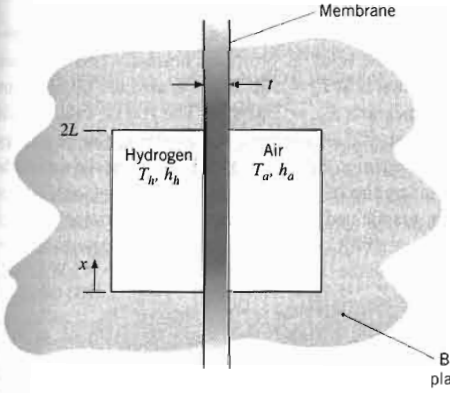
(b) What is the value of T_h if $P_{elec} = 25$ kW, $P_{mech} = 15$ kW, $k_s = 400$ W/m · K, $L = 0.5$ m, $D = 0.05$ m, $W = 0.7$ m, $t = 0.05$ m, $k_\rho = 0.5$ W/m · K, $A_h = 2$ m^2, $h_h = 10$ W/m^2 · K, $h_s = 300$ W/m^2 · K, and $T_\infty = 25$°C?

3.106 Consider the fuel cell stack of Problem 1.58. The $t = 0.42$ mm thick membranes have a nominal thermal conductivity of $k = 0.79$ W/m · K that can be increased to $k_{eff,x} = 15.1$ W/m · K by loading 10%, by volume, carbon nanotubes into the catalyst layers (see Problem 3.12). The membrane experiences uniform volumetric energy generation at a rate of $\dot{q} = 10 \times 10^6$ W/m^3. Air at $T_a = 80$°C provides a convection coefficient of $h_a = 35$ W/m^2 · K on one side of the membrane, while hydrogen at $T_h = 80$°C, $h_h = 235$ W/m^2 · K flows on the opposite side of the membrane. The flow channels are $2L = 3$ mm wide. The membrane is clamped between bipolar plates, each of which is at a temperature $T_{bp} = 80$°C.

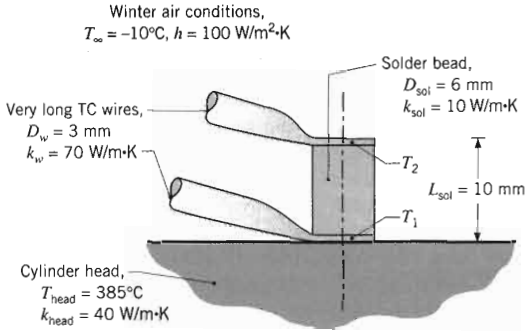
(a) Derive the differential equation that governs the temperature distribution $T(x)$ in the membrane.

(b) Obtain a solution to the differential equation, assuming the membrane is at the bipolar plate temperature at $x = 0$ and $x = 2L$.

(c) Plot the temperature distribution $T(x)$ from $x = 0$ to $x = L$ for carbon nanotube loadings of 0% and 10% by volume. Comment on the ability of the carbon nanotubes to keep the membrane below its softening temperature of 85°C.



3.107 The thermocouple (TC) installation on a snowmobile engine cylinder head is shown in the schematic. The TC wire leads are attached to the upper and lower surfaces of the cylindrically shaped solder bead. The base of the bead is attached to the cylinder head operating at 385°C, and there is a thermal resistance between the bead and the cylinder head due to the *constriction* of heat flow from the head to the bead. This *constriction resistance* may be expressed as $R_{con} = 1/(2k_{head}\, D_{sol})$. The TC wire leads are very long and experience heat loss to the winter air at −10°C with a convection coefficient of 100 W/m^2 · K. Values of other geometrical and thermal parameters are shown on the schematic.

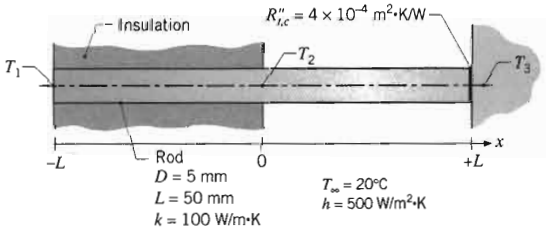


The objective of this problem is to develop a thermal model that can be used to determine the temperature difference $(T_1 - T_2)$ between the two intermediate-metal TC junctions. Assume that the solder bead does not experience any heat loss from its lateral surface.

(a) Sketch a thermal circuit of the installation, labeling temperatures, thermal resistances, and heat rates. Write expressions for each of the thermal resistances and calculate their values.

(b) Use your thermal circuit to evaluate $(T_1 - T_2)$ for the prescribed conditions. Comment on the assumptions made in building your model.

3.108 Consider a rod of diameter D, thermal conductivity k, and length $2L$ that is perfectly insulated over one portion of its length, $-L \le x \le 0$, and experiences convection with a fluid (T_∞, h) over the other portion, $0 \le x \le +L$. One end is maintained at T_1, while the other is separated from a heat sink at T_3 by an interfacial thermal contact resistance $R''_{t,c}$.



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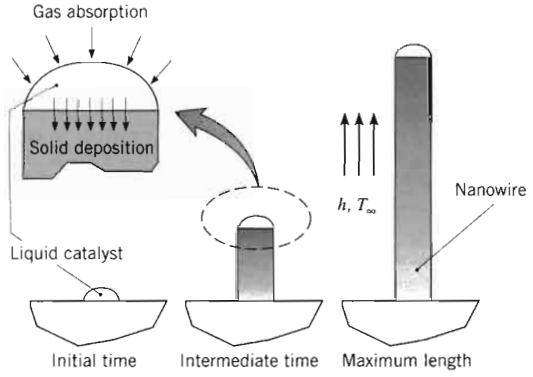
(a) Sketch the temperature distribution on $T-x$ coordinates and identify its key features. Assume that $T_1 > T_3 > T_\infty$.

(b) Derive an expression for the midpoint temperature T_2 in terms of the thermal and geometric parameters of the system.

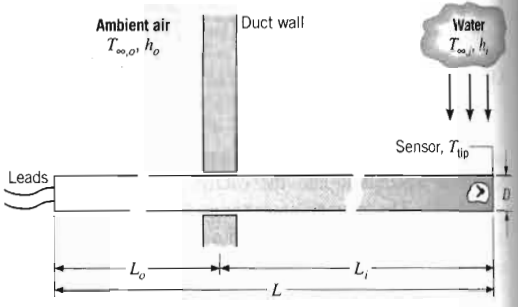
(c) For $T_1 = 200°C$, $T_3 = 100°C$, and the conditions provided in the schematic, calculate T_2 and plot the temperature distribution. Describe key features of the distribution and compare it to your sketch of part (a).

3.109 One method that is used to grow nanowires (nanotubes with solid cores) is to initially deposit a small droplet of a liquid catalyst onto a flat surface. The surface and catalyst are heated and simultaneously exposed to a higher-temperature, low-pressure gas that contains a mixture of chemical species from which the nanowire is to be formed. The catalytic liquid *slowly* absorbs the species from the gas through its top surface and converts these to a solid material that is deposited onto the underlying liquid-solid interface, resulting in construction of the nanowire. The liquid catalyst remains suspended at the tip of the nanowire.

Consider the growth of a 15-nm-diameter silicon carbide nanowire onto a silicon carbide surface. The surface is maintained at a temperature of $T_s = 2400$ K, and the particular liquid catalyst that is used must be maintained in the range $2400\ \text{K} \le T_c \le 3000\ \text{K}$ in order to perform its function. Determine the maximum length of a nanowire that may be grown for conditions characterized by $h = 10^5\ \text{W/m}^2 \cdot \text{K}$ and $T_\infty = 8000$ K. Assume properties of the nanowire are the same as for bulk silicon carbide.



3.110 A probe of overall length $L = 200$ mm and diameter $D = 12.5$ mm is inserted through a duct wall such that a portion of its length, referred to as the immersion length L_i, is in contact with the water stream whose temperature, $T_{\infty,i}$, is to be determined. The convection coefficients over the immersion and ambient-exposed lengths are $h_i = 1100\ \text{W/m}^2 \cdot \text{K}$ and $h_o = 10\ \text{W/m}^2 \cdot \text{K}$, respectively. The probe has a thermal conductivity of $177\ \text{W/m} \cdot \text{K}$ and is in poor thermal contact with the duct wall.

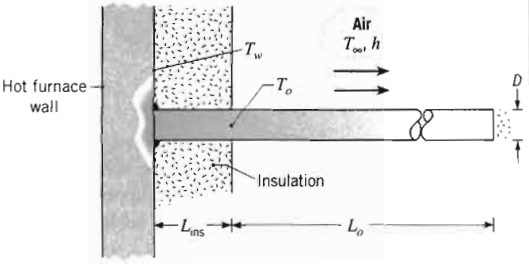


(a) Derive an expression for evaluating the measurement error, $\Delta T_{err} = T_{tip} - T_{\infty,i}$, which is the difference between the tip temperature, T_{tip}, and the water temperature, $T_{\infty,i}$. *Hint:* Define a coordinate system with the origin at the duct wall and treat the probe as two fins extending inward and outward from the duct, but having the same base temperature. Use Case A results from Table 3.4.

(b) With the water and ambient air temperatures at 80 and 20°C, respectively, calculate the measurement error, ΔT_{err}, as a function of immersion length for the conditions $L_i/L = 0.225$, 0.425, and 0.625.

(c) Compute and plot the effects of probe thermal conductivity and water velocity (h_i) on the measurement error.

3.111 A rod of diameter $D = 25$ mm and thermal conductivity $k = 60\ \text{W/m} \cdot \text{K}$ protrudes normally from a furnace wall that is at $T_w = 200°C$ and is covered by insulation of thickness $L_{ins} = 200$ mm. The rod is welded to the furnace wall and is used as a hanger for supporting instrumentation cables. To avoid damaging the cables, the temperature of the rod at its exposed surface, T_o, must be maintained below a specified operating limit of $T_{max} = 100°C$. The ambient air temperature is $T_\infty = 25°C$, and the convection coefficient is $h = 15\ \text{W/m}^2 \cdot \text{K}$.

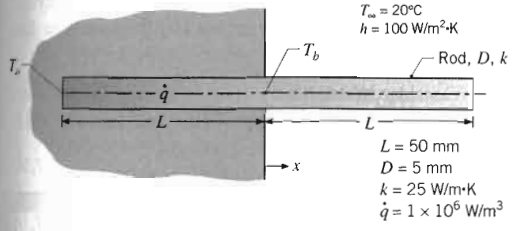


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(a) Derive an expression for the exposed surface temperature T_o as a function of the prescribed thermal and geometrical parameters. The rod has an exposed length L_o, and its tip is well insulated.

(b) Will a rod with $L_o = 200$ mm meet the specified operating limit? If not, what design parameters would you change? Consider another material, increasing the thickness of the insulation, and increasing the rod length. Also, consider how you might attach the base of the rod to the furnace wall as a means to reduce T_o.

3.112 A metal rod of length $2L$, diameter D, and thermal conductivity k is inserted into a perfectly insulating wall, exposing one-half of its length to an air stream that is of temperature T_∞ and provides a convection coefficient h at the surface of the rod. An electromagnetic field induces volumetric energy generation at a uniform rate $\dot{q}$ within the *embedded* portion of the rod.

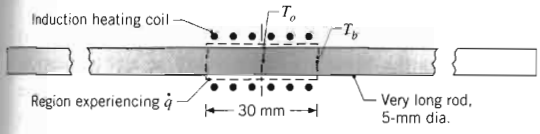


(a) Derive an expression for the steady-state temperature T_b at the base of the exposed half of the rod. The exposed region may be approximated as a very long fin.

(b) Derive an expression for the steady-state temperature T_o at the end of the embedded half of the rod.

(c) Using numerical values provided in the schematic, plot the temperature distribution in the rod and describe key features of the distribution. Does the rod behave as a very long fin?

3.113 A very long rod of 5-mm diameter and uniform thermal conductivity $k = 25$ W/m · K is subjected to a heat treatment process. The center, 30-mm-long portion of the rod within the induction heating coil experiences uniform volumetric heat generation of 7.5×10^6 W/m³.

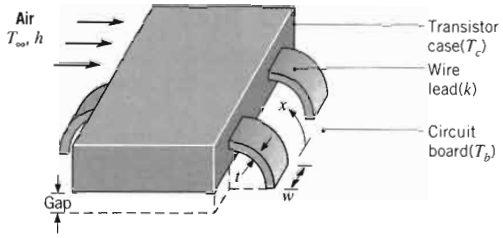


The unheated portions of the rod, which protrude from the heating coil on either side, experience convection with the ambient air at $T_\infty = 20°C$ and $h = 10$ W/m² · K. Assume that there is no convection from the surface of the rod within the coil.

(a) Calculate the steady-state temperature T_o of the rod at the midpoint of the heated portion in the coil.

(b) Calculate the temperature of the rod T_b at the edge of the heated portion.

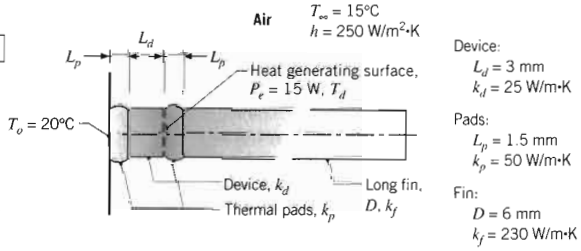
3.114 From Problem 1.71, consider the wire leads connecting the transistor to the circuit board. The leads are of thermal conductivity k, thickness t, width w, and length L. One end of a lead is maintained at a temperature T_c corresponding to the transistor case, while the other end assumes the temperature T_b of the circuit board. During steady-state operation, current flow through the leads provides for uniform volumetric heating in the amount $\dot{q}$, while there is convection cooling to air that is at T_∞ and maintains a convection coefficient h.



(a) Derive an equation from which the temperature distribution in a wire lead may be determined. List all pertinent assumptions.

(b) Determine the temperature distribution in a wire lead, expressing your results in terms of the prescribed variables.

3.115 A disk-shaped electronic device of thickness L_d, diameter D, and thermal conductivity k_d dissipates electrical power at a steady rate P_e along one of its surfaces. The device is bonded to a cooled base at T_o using a thermal pad of thickness L_p and thermal conductivity k_p. A long fin of diameter D and thermal conductivity k_f is bonded to the heat-generating surface of the device using an identical thermal pad. The fin is cooled by an air stream, which is at a temperature T_∞ and provides a convection coefficient h.



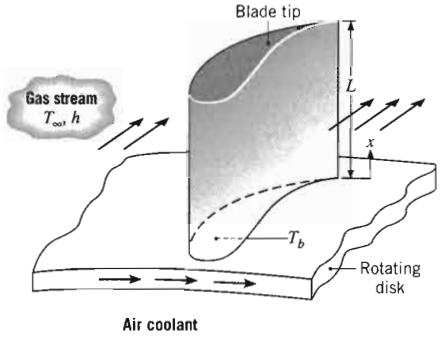
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(a) Construct a thermal circuit of the system.

(b) Derive an expression for the temperature T_d of the heat-generating surface of the device in terms of the circuit thermal resistances, T_o and T_∞. Express the thermal resistances in terms of appropriate parameters.

(c) Calculate T_d for the prescribed conditions.

3.116 Turbine blades mounted to a rotating disc in a gas turbine engine are exposed to a gas stream that is at $T_\infty = 1200°C$ and maintains a convection coefficient of $h = 250\ W/m^2 \cdot K$ over the blade.

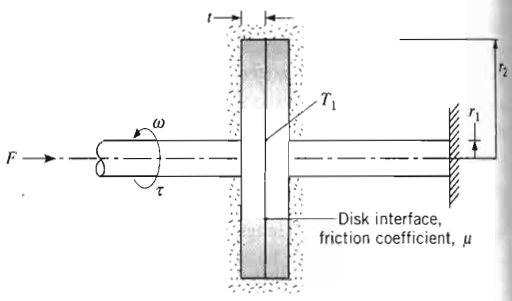


The blades, which are fabricated from Inconel, $k \approx 20\ W/m \cdot K$, have a length of $L = 50$ mm. The blade profile has a uniform cross-sectional area of $A_c = 6 \times 10^{-4}\ m^2$ and a perimeter of $P = 110$ mm. A proposed blade-cooling scheme, which involves routing air through the supporting disc, is able to maintain the base of each blade at a temperature of $T_b = 300°C$.

(a) If the maximum allowable blade temperature is 1050°C and the blade tip may be assumed to be adiabatic, is the proposed cooling scheme satisfactory?

(b) For the proposed cooling scheme, what is the rate at which heat is transferred from each blade to the coolant?

3.117 In a test to determine the friction coefficient, μ, associated with a disk brake, one disk and its shaft are rotated at a constant angular velocity ω, while an equivalent disk/shaft assembly is stationary. Each disk has an outer radius of $r_2 = 180$ mm, a shaft radius of $r_1 = 20$ mm, a thickness of $t = 12$ mm, and a thermal conductivity of $k = 15\ W/m \cdot K$. A known force F is applied to the system, and the corresponding torque τ required to maintain rotation is measured. The disk contact pressure may be assumed to be uniform (i.e., independent of location on the interface), and the disks may be assumed to be well insulated from the surroundings.

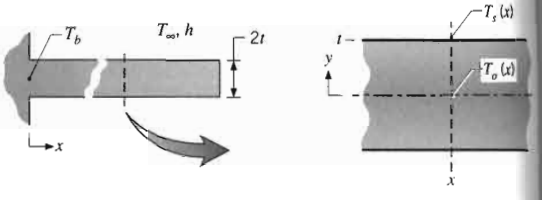


(a) Obtain an expression that may be used to evaluate μ from known quantities.

(b) For the region $r_1 \le r \le r_2$, determine the radial temperature distribution, $T(r)$, in the disk, where $T(r_1) = T_1$ is presumed to be known.

(c) Consider test conditions for which $F = 200$ N, $\omega = 40$ rad/s, $\tau = 8\ N \cdot m$, and $T_1 = 80°C$. Evaluate the friction coefficient and the maximum disk temperature.

3.118 Consider an extended surface of rectangular cross section with heat flow in the longitudinal direction.



In this problem we seek to determine conditions for which the transverse (y-direction) temperature difference within the extended surface is negligible compared to the temperature difference between the surface and the environment, such that the one-dimensional analysis of Section 3.6.1 is valid.

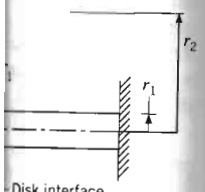
(a) Assume that the transverse temperature distribution is parabolic and of the form

$$\frac{T(y) - T_o(x)}{T_s(x) - T_o(x)} = \left(\frac{y}{t}\right)^2$$

where $T_s(x)$ is the surface temperature and $T_o(x)$ is the centerline temperature at any x-location. Using Fourier's law, write an expression for the conduction heat flux at the surface, $q''_y(t)$, in terms of T_s and T_o.

(b) Write an expression for the convection heat flux at the surface for the x-location. Equating the two expressions for the heat flux by conduction and

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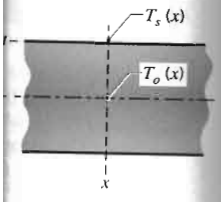
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convection, identify the parameter that determines the ratio $(T_o - T_s)/(T_s - T_\infty)$.

(c) From the foregoing analysis, develop a criterion for establishing the validity of the one-dimensional assumption used to model an extended surface.

Simple Fins

3.119 A long, circular aluminum rod is attached at one end to a heated wall and transfers heat by convection to a cold fluid.

(a) If the diameter of the rod is tripled, by how much would the rate of heat removal change?

(b) If a copper rod of the same diameter is used in place of the aluminum, by how much would the rate of heat removal change?

3.120 A brass rod 100 mm long and 5 mm in diameter extends horizontally from a casting at 200°C. The rod is in an air environment with $T_\infty = 20°C$ and $h = 30$ W/m$^2 \cdot$K. What is the temperature of the rod 25, 50, and 100 mm from the casting?

3.121 The extent to which the tip condition affects the thermal performance of a fin depends on the fin geometry and thermal conductivity, as well as the convection coefficient. Consider an alloyed aluminum ($k = 180$ W/m $\cdot$ K) rectangular fin of length $L = 10$ mm, thickness $t = 1$ mm, and width $w \gg t$. The base temperature of the fin is $T_b = 100°C$, and the fin is exposed to a fluid of temperature $T_\infty = 25°C$.

(a) Assuming a uniform convection coefficient of $h = 100$ W/m$^2 \cdot$K over the entire fin surface, determine the fin heat transfer rate per unit width q'_f, efficiency η_f, effectiveness ε_f, thermal resistance per unit width $R'_{t,f}$, and the tip temperature $T(L)$ for Cases A and B of Table 3.4. Contrast your results with those based on an *infinite fin* approximation.

(b) Explore the effect of variations in the convection coefficient on the heat rate for $10 < h < 1000$ W/m$^2 \cdot$K. Also consider the effect of such variations for a stainless steel fin ($k = 15$ W/m $\cdot$ K).

3.122 The extent to which the tip condition affects the thermal performance of a fin depends on the fin geometry and thermal conductivity, as well as the convection coefficient. Consider an alloyed aluminum ($k = 180$ W/m $\cdot$ K) rectangular fin whose base temperature is $T_b = 100°C$. The fin is exposed to a fluid of temperature $T_\infty = 25°C$, and a uniform convection coefficient of $h = 100$ W/m$^2 \cdot$K may be assumed for the fin surface.

(a) For a fin of length $L = 10$ mm, thickness $t = 1$ mm, and width $w \gg t$, determine the fin heat transfer rate per unit width q'_f, efficiency η_f, effectiveness ε_f, thermal resistance per unit width $R'_{t,f}$, and tip temperature $T(L)$ for Cases A and B of Table 3.4. Contrast your results with those based on an *infinite fin* approximation.

(b) Explore the effect of variations in L on the heat rate for $3 < L < 50$ mm. Also consider the effect of such variations for a stainless steel fin ($k = 15$ W/m $\cdot$ K).

3.123 A straight fin fabricated from 2024 aluminum alloy ($k = 185$ W/m $\cdot$ K) has a base thickness of $t = 3$ mm and a length of $L = 15$ mm. Its base temperature is $T_b = 100°C$, and it is exposed to a fluid for which $T_\infty = 20°C$ and $h = 50$ W/m$^2 \cdot$K. For the foregoing conditions and a fin of unit width, compare the fin heat rate, efficiency, and volume for rectangular, triangular, and parabolic profiles.

3.124 Two long copper rods of diameter $D = 10$ mm are soldered together end to end, with solder having a melting point of 650°C. The rods are in air at 25°C with a convection coefficient of 10 W/m$^2 \cdot$K. What is the minimum power input needed to effect the soldering?

3.125 Circular copper rods of diameter $D = 1$ mm and length $L = 25$ mm are used to enhance heat transfer from a surface that is maintained at $T_{s,1} = 100°C$. One end of the rod is attached to this surface (at $x = 0$), while the other end ($x = 25$ mm) is joined to a second surface, which is maintained at $T_{s,2} = 0°C$. Air flowing between the surfaces (and over the rods) is also at a temperature of $T_\infty = 0°C$, and a convection coefficient of $h = 100$ W/m$^2 \cdot$K is maintained.

(a) What is the rate of heat transfer by convection from a single copper rod to the air?

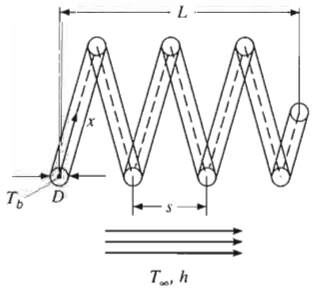
(b) What is the total rate of heat transfer from a 1-m by 1-m section of the surface at 100°C, if a bundle of the rods is installed on 4-mm centers?

3.126 During the initial stages of the growth of the nanowire of Problem 3.109, a slight perturbation of the liquid catalyst droplet can cause it to be suspended on the top of the nanowire in an off-center position. The resulting nonuniform deposition of solid at the solid-liquid interface can be manipulated to form engineered shapes such as a *nanospring*, that is characterized by a spring radius, r, spring pitch, s, overall chord length, L_c (length running along the spring), and end-to-end length, L, as shown in the sketch. Consider a silicon carbide nanospring of diameter $D = 15$ nm, $r = 30$ nm, $s = 25$ nm, and $L_c = 425$ nm. From experiments, it is known that the average spring pitch $\bar{s}$ varies with average temperature $\bar{T}$ by the relation $d\bar{s}/d\bar{T} = 0.1$ nm/K. Using this information, a student suggests that a *nanoactuator*

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can be constructed by connecting one end of the nanospring to a small heater and raising the temperature of that end of the nanospring above its initial value. Calculate the actuation distance, ΔL, for conditions where $h = 10^6$ W/m$^2 \cdot$ K, $T_\infty = T_i = 25$°C, with a base temperature of $T_b = 50$°C. If the base temperature can be controlled to within 1°C, calculate the accuracy to which the actuation distance can be controlled. *Hint:* Assume the spring radius does not change when the spring is heated. The overall spring length may be approximated by the formula,

$$L = \frac{\bar{s}}{2\pi} \frac{L_c}{\sqrt{r^2 + (\bar{s}/2\pi)^2}}$$



3.127 Consider two long, slender rods of the same diameter but different materials. One end of each rod is attached to a base surface maintained at 100°C, while the surfaces of the rods are exposed to ambient air at 20°C. By traversing the length of each rod with a thermocouple, it was observed that the temperatures of the rods were equal at the positions $x_A = 0.15$ m and $x_B = 0.075$ m, where x is measured from the base surface. If the thermal conductivity of rod A is known to be $k_A = 70$ W/m $\cdot$ K, determine the value of k_B for rod B.

3.128 A 40-mm-long, 2-mm-diameter pin fin is fabricated of an aluminum alloy ($k = 140$ W/m$^2 \cdot$ K).

(a) Determine the fin heat transfer rate for $T_b = 50$°C, $T_\infty = 25$°C, $h = 1000$ W/m$^2 \cdot$ K, and an adiabatic tip condition.

(b) An engineer suggests that by holding the fin tip at a low temperature, the fin heat transfer rate can be increased. For $T(x = L) = 0$°C, determine the new fin heat transfer rate. Other conditions are as in part (a).

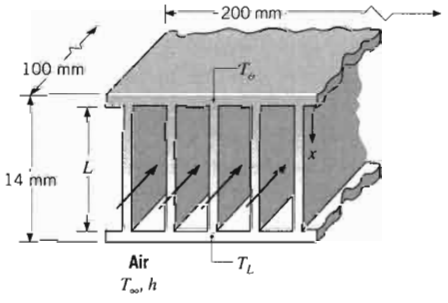
(c) Plot the temperature distribution, $T(x)$, over the range $0 \le x \le L$ for the adiabatic tip case and the prescribed tip temperature case. Also show the ambient temperature in your graph. Discuss relevant features of the temperature distribution.

(d) Plot the fin heat transfer rate over the range $0 \le h \le 1000$ W/m$^2 \cdot$ K for the adiabatic tip case and the prescribed tip temperature case. For the prescribed tip temperature case, what would the calculated fin heat transfer rate be if Equation 3.73 were used to determine q_f rather than Equation 3.71?

3.129 An experimental arrangement for measuring the thermal conductivity of solid materials involves the use of two long rods that are equivalent in every respect, except that one is fabricated from a standard material of known thermal conductivity k_A while the other is fabricated from the material whose thermal conductivity k_B is desired. Both rods are attached at one end to a heat source of fixed temperature T_b, are exposed to a fluid of temperature T_∞, and are instrumented with thermocouples to measure the temperature at a fixed distance x_1 from the heat source. If the standard material is aluminum, with $k_A = 200$ W/m $\cdot$ K, and measurements reveal values of $T_A = 75$°C and $T_B = 60$°C at x_1 for $T_b = 100$°C and $T_\infty = 25$°C, what is the thermal conductivity k_B of the test material?

Fin Systems and Arrays

3.130 Finned passages are frequently formed between parallel plates to enhance convection heat transfer in compact heat exchanger cores. An important application is in electronic equipment cooling, where one or more air-cooled stacks are placed between heat-dissipating electrical components. Consider a single stack of rectangular fins of length L and thickness t, with convection conditions corresponding to h and T_∞.



(a) Obtain expressions for the fin heat transfer rates, $q_{f,o}$ and $q_{f,L}$, in terms of the base temperatures, T_o and T_L.

(b) In a specific application, a stack that is 200 mm wide and 100 mm deep contains 50 fins, each of length $L = 12$ mm. The entire stack is made from aluminum, which is everywhere 1.0 mm thick. If temperature limitations associated with electrical

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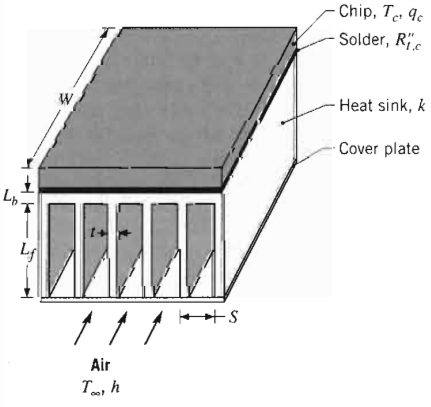
components joined to opposite plates dictate maximum allowable plate temperatures of $T_o = 400$ K and $T_L = 350$ K, what are the corresponding maximum power dissipations if $h = 150$ W/m$^2 \cdot$ K and $T_\infty = 300$ K?

3.131 The fin array of Problem 3.130 is commonly found in *compact heat exchangers,* whose function is to provide a large surface area per unit volume in transferring heat from one fluid to another. Consider conditions for which the second fluid maintains equivalent temperatures at the parallel plates, $T_o = T_L$, thereby establishing symmetry about the midplane of the fin array. The heat exchanger is 1 m long in the direction of the flow of air (first fluid) and 1 m wide in a direction normal to both the air flow and the fin surfaces. The length of the fin passages between adjoining parallel plates is $L = 8$ mm, while the fin thermal conductivity and convection coefficient are $k = 200$ W/m $\cdot$ K (aluminum) and $h = 150$ W/m$^2 \cdot$ K, respectively.

(a) If the fin thickness and pitch are $t = 1$ mm and $S = 4$ mm, respectively, what is the value of the thermal resistance $R_{t,o}$ for a one-half section of the fin array?

(b) Subject to the constraints that the fin thickness and pitch may not be less than 0.5 and 3 mm, respectively, assess the effect of changes in t and S.

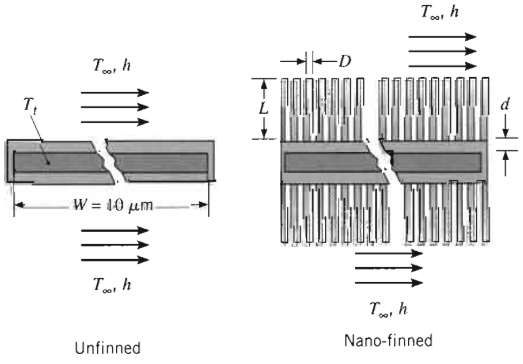
3.132 An isothermal silicon chip of width $W = 20$ mm on a side is soldered to an aluminum heat sink ($k = 180$ W/m $\cdot$ K) of equivalent width. The heat sink has a base thickness of $L_b = 3$ mm and an array of rectangular fins, each of length $L_f = 15$ mm. Air flow at $T_\infty = 20$°C is maintained through channels formed by the fins and a cover plate, and for a convection coefficient of $h = 100$ W/m$^2 \cdot$ K, a minimum fin spacing of 1.8 mm is dictated by limitations on the flow pressure drop. The solder joint has a thermal resistance of $R''_{t,c} = 2 \times 10^{-6}$ m$^2 \cdot$ K/W.



(a) Consider limitations for which the array has $N = 11$ fins, in which case values of the fin thickness $t = 0.182$ mm and pitch $S = 1.982$ mm are obtained from the requirements that $W = (N - 1)S + t$ and $S - t = 1.8$ mm. If the maximum allowable chip temperature is $T_c = 85$°C, what is the corresponding value of the chip power q_c? An adiabatic fin tip condition may be assumed, and air flow along the outer surfaces of the heat sink may be assumed to provide a convection coefficient equivalent to that associated with air flow through the channels.

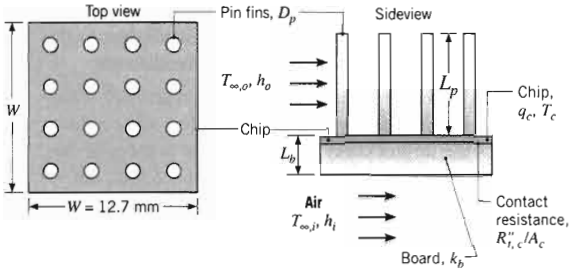
(b) With $(S - t)$ and h fixed at 1.8 mm and 100 W/m$^2 \cdot$ K, respectively, explore the effect of increasing the fin thickness by reducing the number of fins. With $N = 11$ and $S - t$ fixed at 1.8 mm, but relaxation of the constraint on the pressure drop, explore the effect of increasing the air flow, and hence the convection coefficient.

3.133 As seen in Problem 3.109, silicon carbide nanowires of diameter $D = 15$ nm can be grown onto a solid silicon carbide surface by carefully depositing droplets of catalyst liquid onto a flat silicon carbide substrate. Silicon carbide nanowires grow upward from the deposited drops, and if the drops are deposited in a pattern, an array of nanowire fins can be grown, forming a silicon carbide *nano-heat sink.* Consider finned and unfinned electronics packages in which an extremely small, 10 μm $\times$ 10 μm electronics device is sandwiched between two $d = 100$ nm thick silicon carbide sheets. In both cases, the coolant is a dielectric liquid at 20°C. A heat transfer coefficient of $h = 1 \times 10^5$ W/m$^2 \cdot$ K exists on the top and bottom of the unfinned package and on all surfaces of the exposed silicon carbide fins, which are each $L = 300$ nm long. Each nano-heat sink includes a 200 $\times$ 200 array of nanofins. Determine the maximum allowable heat rate that can be generated by the electronic device so that its temperature is maintained at $T_t < 85$°C for the unfinned and finned packages.



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3.134 As more and more components are placed on a single integrated circuit (chip), the amount of heat that is dissipated continues to increase. However, this increase is limited by the maximum allowable chip operating temperature, which is approximately 75°C. To maximize heat dissipation, it is proposed that a 4 × 4 array of copper pin fins be metallurgically joined to the outer surface of a square chip that is 12.7 mm on a side.

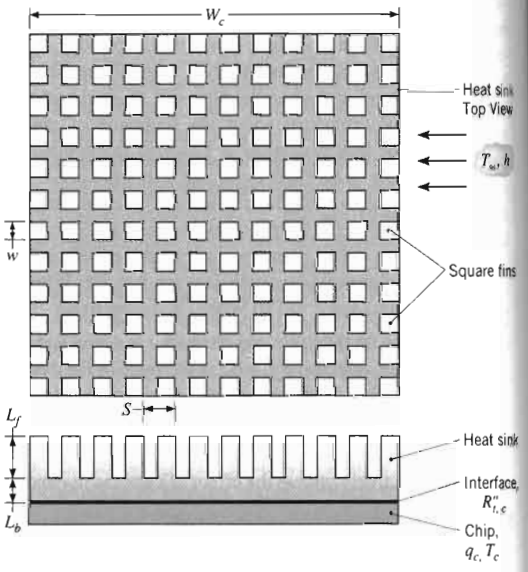


(a) Sketch the equivalent thermal circuit for the pin–chip–board assembly, assuming one-dimensional, steady-state conditions and negligible contact resistance between the pins and the chip. In variable form, label appropriate resistances, temperatures, and heat rates.

(b) For the conditions prescribed in Problem 3.27, what is the maximum rate at which heat can be dissipated in the chip when the pins are in place? That is, what is the value of q_c for $T_c = 75°C$? The pin diameter and length are $D_p = 1.5$ mm and $L_p = 15$ mm.

3.135 In Problem 3.134, the prescribed value of $h_o = 1000$ W/m$^2 \cdot$ K is large and characteristic of liquid cooling. In practice it would be far more preferable to use air cooling, for which a reasonable upper limit to the convection coefficient would be $h_o = 250$ W/m$^2 \cdot$ K. Assess the effect of changes in the pin fin geometry on the chip heat rate if the remaining conditions of Problem 3.134, including a maximum allowable chip temperature of 75°C, remain in effect. Parametric variations that may be considered include the total number of pins, N, in the square array, the pin diameter D_p, and the pin length L_p. However, the product $N^{1/2}D_p$ should not exceed 9 mm to ensure adequate air flow passage through the array. Recommend a design that enhances chip cooling.

3.136 As a means of enhancing heat transfer from high-performance logic chips, it is common to attach a *heat sink* to the chip surface in order to increase the surface area available for convection heat transfer. Because of the ease with which it may be manufactured (by taking orthogonal sawcuts in a block of material), an attractive option is to use a heat sink consisting of an array of square fins of width w on a side. The spacing between adjoining fins would be determined by the width of the sawblade, with the sum of this spacing and the fin width designated as the fin pitch S. The method by which the heat sink is joined to the chip would determine the interfacial contact resistance, $R''_{t,c}$.



Consider a square chip of width $W_c = 16$ mm and conditions for which cooling is provided by a dielectric liquid with $T_\infty = 25°C$ and $h = 1500$ W/m$^2 \cdot$ K. The heat sink is fabricated from copper ($k = 400$ W/m $\cdot$ K), and its characteristic dimensions are $w = 0.25$ mm, $S = 0.50$ mm, $L_f = 6$ mm, and $L_b = 3$ mm. The prescribed values of w and S represent minima imposed by manufacturing constraints and the need to maintain adequate flow in the passages between fins.

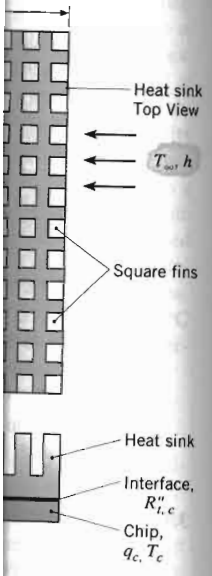
(a) If a metallurgical joint provides a contact resistance of $R''_{t,c} = 5 \times 10^{-6}$ m$^2 \cdot$ K/W and the maximum allowable chip temperature is 85°C, what is the maximum allowable chip power dissipation q_c? Assume all of the heat to be transferred through the heat sink.

(b) It may be possible to increase the heat dissipation by increasing w, subject to the constraint that $(S - w) \geq 0.25$ mm, and/or increasing L_f (subject to manufacturing constraints that $L_f \leq 10$ mm). Assess the effect of such changes.

3.137 Because of the large number of devices in today's PC chips, finned heat sinks are often used to maintain the chip at an acceptable operating temperature. Two fin

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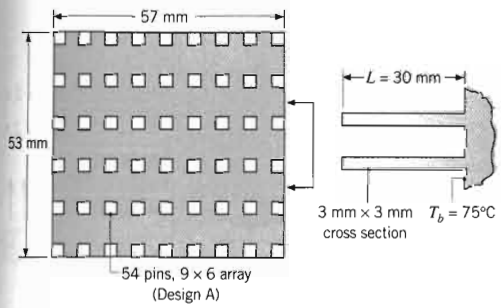
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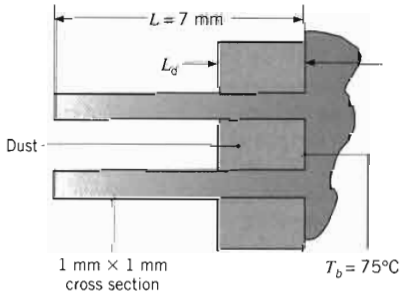
designs are to be evaluated, both of which have base (unfinned) area dimensions of 53 mm × 57 mm. The fins are of square cross section and fabricated from an extruded aluminum alloy with a thermal conductivity of 175 W/m · K. Cooling air may be supplied at 25°C, and the maximum allowable chip temperature is 75°C. Other features of the design and operating conditions are tabulated.

	Fin Dimensions			
Design	Cross Section $w \times w$ (mm)	Length L (mm)	Number of Fins in Array	Convection Coefficient (W/m$^2 \cdot$ K)
A	3 × 3	30	6 × 9	125
B	1 × 1	7	14 × 17	375



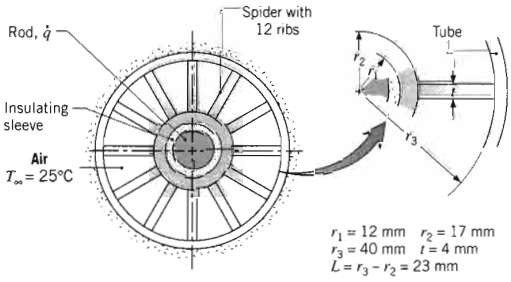
Determine which fin arrangement is superior. In your analysis, calculate the heat rate, efficiency, and effectiveness of a single fin, as well as the total heat rate and overall efficiency of the array. Since real estate inside the computer enclosure is important, compare the total heat rate per unit volume for the two designs.

3.138 Consider design B of Problem 3.137. Over time, dust can collect in the fine grooves that separate the fins. Consider the buildup of a dust layer of thickness L_d, as shown in the sketch. Calculate and plot the total heat rate for design B for dust layers in the range $0 \leq L_d \leq 5$ mm. The thermal conductivity of the dust can be taken as $k_d = 0.032$ W/m · K. Include the effects of convection off the fin tip.

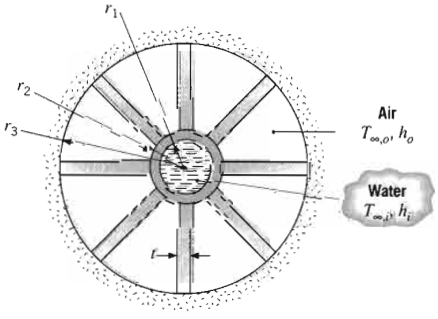


3.139 A long rod of 20-mm diameter and a thermal conductivity of 1.5 W/m · K has a uniform internal volumetric thermal energy generation of 10^6 W/m^3. The rod is covered with an electrically insulating sleeve of 2-mm thickness and thermal conductivity of 0.5 W/m · K. A spider with 12 ribs and dimensions as shown in the sketch has a thermal conductivity of 175 W/m · K, and is used to support the rod and to maintain concentricity with an 80-mm-diameter tube. Air at $T_\infty = 25$°C passes over the spider surface, and the convection coefficient is 20 W/m$^2 \cdot$ K. The outer surface of the tube is well insulated.

We wish to increase volumetric heating within the rod, while not allowing its centerline temperature to exceed 100°C. Determine the impact of the following changes, which may be effected independently or concurrently: (i) increasing the air speed and hence the convection coefficient; (ii) changing the number and/or thickness of the ribs; and (iii) using an electrically nonconducting sleeve material of larger thermal conductivity (e.g., amorphous carbon or quartz). Recommend a realistic configuration that yields a significant increase in $\dot{q}$.



3.140 An air heater consists of a steel tube ($k = 20$ W/m · K), with inner and outer radii of $r_1 = 13$ mm and $r_2 = 16$ mm, respectively, and eight integrally machined longitudinal fins, each of thickness $t = 3$ mm. The fins extend to a concentric tube, which is of radius $r_3 = 40$ mm and insulated on its outer surface. Water at a temperature $T_{\infty,i} = 90$°C flows through the inner tube, while air at $T_{\infty,o} = 25$°C flows through the annular region formed by the larger concentric tube.



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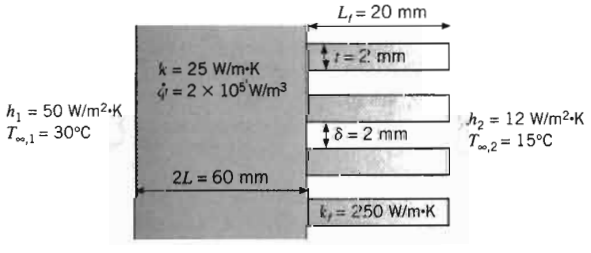
(a) Sketch the equivalent thermal circuit of the heater and relate each thermal resistance to appropriate system parameters.

(b) If $h_i = 5000$ W/m$^2 \cdot$ K and $h_o = 200$ W/m$^2 \cdot$ K, what is the heat rate per unit length?

(c) Assess the effect of increasing the number of fins N and/or the fin thickness t on the heat rate, subject to the constraint that $Nt < 50$ mm.

3.141 Determine the percentage increase in heat transfer associated with attaching aluminum fins of rectangular profile to a plane wall. The fins are 50 mm long, 0.5 mm thick, and are equally spaced at a distance of 4 mm (250 fins/m). The convection coefficient associated with the bare wall is 40 W/m$^2 \cdot$ K, while that resulting from attachment of the fins is 30 W/m$^2 \cdot$ K.

3.142 Heat is uniformly generated at the rate of 2×10^5 W/m^3 in a wall of thermal conductivity 25 W/m $\cdot$ K and thickness 60 mm. The wall is exposed to convection on both sides, with different heat transfer coefficients and temperatures as shown. There are straight rectangular fins on the right-hand side of the wall, with dimensions as shown and thermal conductivity of 250 W/m $\cdot$ K. What is the maximum temperature that will occur in the wall?



3.143 Aluminum fins of triangular profile are attached to a plane wall whose surface temperature is 250°C. The fin base thickness is 2 mm, and its length is 6 mm. The system is in ambient air at a temperature of 20°C, and the surface convection coefficient is 40 W/m$^2 \cdot$ K.

(a) What are the fin efficiency and effectiveness?

(b) What is the heat dissipated per unit width by a single fin?

3.144 An annular aluminum fin of rectangular profile is attached to a circular tube having an outside diameter of 25 mm and a surface temperature of 250°C. The fin is 1 mm thick and 10 mm long, and the temperature and the convection coefficient associated with the adjoining fluid are 25°C and 25 W/m$^2 \cdot$ K, respectively.

(a) What is the heat loss per fin?

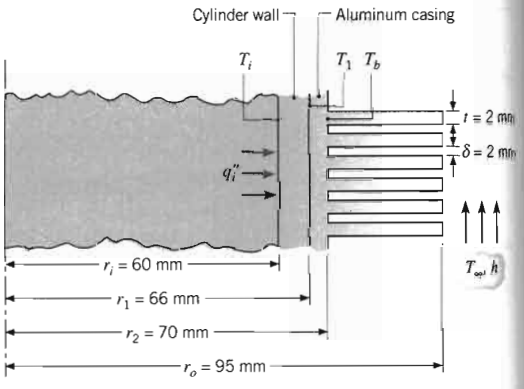
(b) If 200 such fins are spaced at 5-mm increments along the tube length, what is the heat loss per meter of tube length?

3.145 Annular aluminum fins of rectangular profile are attached to a circular tube having an outside diameter of 50 mm and an outer surface temperature of 200°C. The fins are 4 mm thick and 15 mm long. The system is in ambient air at a temperature of 20°C, and the surface convection coefficient is 40 W/m$^2 \cdot$ K.

(a) What are the fin efficiency and effectiveness?

(b) If there are 125 such fins per meter of tube length, what is the rate of heat transfer per unit length of tube?

3.146 It is proposed to air-cool the cylinders of a combustion chamber by joining an aluminum casing with annular fins ($k = 240$ W/m $\cdot$ K) to the cylinder wall ($k = 50$ W/m $\cdot$ K).



The air is at 320 K and the corresponding convection coefficient is 100 W/m$^2 \cdot$ K. Although heating at the inner surface is periodic, it is reasonable to assume steady-state conditions with a time-averaged heat flux of $q_i'' = 10^5$ W/m^2. Assuming negligible contact resistance between the wall and the casing, determine the wall inner temperature T_i, the interface temperature T_1, and the fin base temperature T_b. Determine these temperatures if the interface contact resistance is $R_{t,c}'' = 10^{-4}$ m$^2 \cdot$ K/W.

3.147 Consider the air-cooled combustion cylinder of Problem 3.146, but instead of imposing a uniform heat flux at the inner surface, consider conditions for which the time-averaged temperature of the combustion gases is $T_g = 1100$ K and the corresponding convection coefficient is $h_g = 150$ W/m$^2 \cdot$ K. All other conditions, including the cylinder/casing contact resistance, remain the same. Determine the heat rate per unit length of cylinder (W/m), as well as the cylinder inner temperature T_i, the interface temperatures $T_{1,i}$ and $T_{1,o}$, and the fin base temperature T_b. Subject to the constraint that the fin gap is fixed at $\delta = 2$ mm, assess the effect of increasing the fin thickness at the expense of reducing the number of fins.

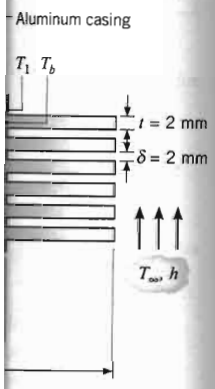
3.148 Heat transfer from a transistor may be enhanced by inserting it in an aluminum sleeve ($k = 200$ W/m $\cdot$ K)

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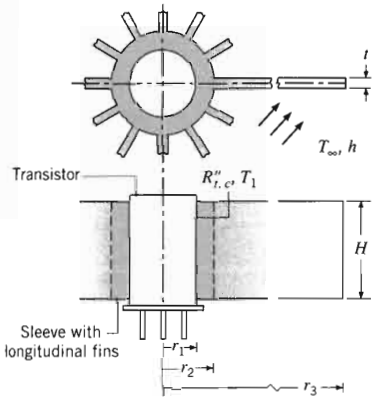


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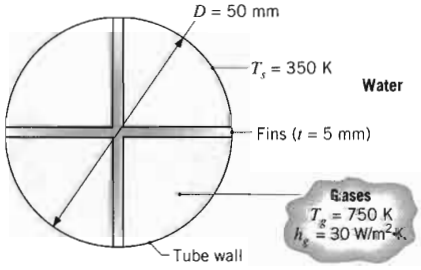
having 12 integrally machined longitudinal fins on its outer surface. The transistor radius and height are r_1 = 2.5 mm and H = 4 mm, respectively, while the fins are of length $L = r_3 - r_2$ = 8 mm and uniform thickness t = 0.8 mm. The thickness of the sleeve base is $r_2 - r_1$ = 1 mm, and the contact resistance of the sleeve-transistor interface is $R''_{t,c} = 0.6 \times 10^{-3}$ m$^2 \cdot$ K/W. Air at T_∞ = 20°C flows over the fin surface, providing an approximately uniform convection coeffficient of h = 30 W/m$^2 \cdot$ K.



(a) When the transistor case temperature is 80°C, what is the rate of heat transfer from the sleeve?

(b) Identify all of the measures that could be taken to improve design and/or operating conditions, such that heat dissipation may be increased while still maintaining a case temperature of 80°C. In words, assess the relative merits of each measure. Choose what you believe to be the three most promising measures, and numerically assess the effect of corresponding changes in design and/or operating conditions on thermal performance.

3.149 Water is heated by submerging 50-mm diameter, thin-walled copper tubes in a tank and passing hot combustion gases (T_g = 750 K) through the tubes. To enhance heat transfer to the water, four straight fins of uniform cross section, which form a cross, are inserted in each tube. The fins are 5 mm thick and are also made of copper (k = 400 W/m · K).



If the tube surface temperature is T_s = 350 K and the gas-side convection coefficient is h_g = 30 W/m$^2 \cdot$ K, what is the rate of heat transfer to the water per meter of pipe length?

3.150 Consider the conditions of Problem 3.149, but now allow for a tube wall thickness of 5 mm (inner and outer diameters of 50 and 60 mm), a fin-to-tube thermal contact resistance of 10^{-4} m$^2 \cdot$ K/W, and the fact that it is the water temperature, T_w = 350 K, rather than the tube surface temperature, that is known. The water-side convection coefficient is h_w = 2000 W/m$^2 \cdot$ K. Determine the rate of heat transfer per unit tube length (W/m) to the water. What would be the separate effect of each of the following design changes on the heat rate: (i) elimination of the contact resistance; (ii) increasing the number of fins from four to eight; and (iii) changing the tube wall and fin material from copper to AISI 304 stainless steel (k = 20 W/m · K)?

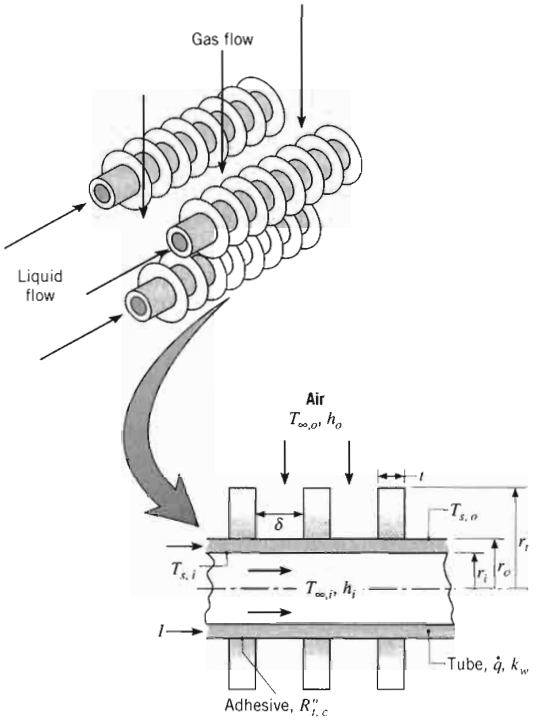
3.151 A scheme for concurrently heating separate water and air streams involves passing them through and over an array of tubes, respectively, while the tube wall is heated electrically. To enhance gas-side heat transfer, annular fins of rectangular profile are attached to the outer tube surface. Attachment is facilitated with a dielectric adhesive that electrically isolates the fins from the current-carrying tube wall.

(a) Assuming uniform volumetric heat generation within the tube wall, obtain expressions for the heat rate per unit tube length (W/m) at the inner (r_i) and outer (r_o) surfaces of the wall. Express your results in terms of the tube inner and outer surface temperatures, $T_{s,i}$ and $T_{s,o}$, and other pertinent parameters.

(b) Obtain expressions that could be used to determine $T_{s,i}$ and $T_{s,o}$ in terms of parameters associated with the water- and air-side conditions.

(c) Consider conditions for which the water and air are at $T_{\infty,i} = T_{\infty,o}$ = 300 K, with corresponding convection coefficients of h_i = 2000 W/m$^2 \cdot$ K and h_o = 100 W/m$^2 \cdot$ K. Heat is uniformly dissipated in a stainless steel tube (k_w = 15 W/m · K), having inner and outer radii of r_i = 25 mm and r_o = 30 mm, and aluminum fins ($t = \delta$ = 2 mm, r_t = 55 mm) are attached to the outer surface, with $R''_{t,c} = 10^{-4}$ m$^2 \cdot$ K/W. Determine the heat rates and temperatures at the inner and outer surfaces as a function of the rate of volumetric heating $\dot{q}$. The upper limit to $\dot{q}$ will be determined by the constraints that $T_{s,i}$ not exceed the boiling point of water (100°C) and $T_{s,o}$ not exceed the decomposition temperature of the adhesive (250°C).

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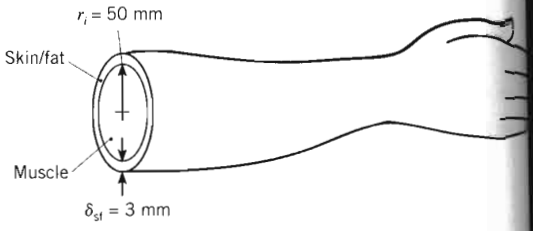


The Bioheat Equation

3.152 Consider the conditions of Example 3.12, except that the person is now exercising (in the air environment), which increases the metabolic heat generation rate by a factor of eight, to 5600 W/m^3. At what rate would the person have to perspire (in liters/s) to maintain the same skin temperature as in that example?

3.153 Consider the conditions of Example 3.12 for an air environment, except now the air and surroundings temperatures are both 15°C. Humans respond to cold by shivering, which increases the metabolic heat generation rate. What would the metabolic heat generation rate (per unit volume) have to be to maintain a comfortable skin temperature of 33°C under these conditions?

3.154 Consider heat transfer in a forearm, which can be approximated as a cylinder of muscle of radius 50 mm (neglecting the presence of bones), with an outer layer of skin and fat of thickness 3 mm. There is metabolic heat generation and perfusion within the muscle. The metabolic heat generation rate, perfusion rate, arterial temperature, and properties of blood, muscle, and skin/fat layer are identical to those in Example 3.12. The environment and surroundings are the same as for the air environment in Example 3.12.



(a) Write the bioheat transfer equation in radial coordinates. Write the boundary conditions that express symmetry at the centerline of the forearm and specified temperature at the outer surface of the muscle. Solve the differential equation and apply the boundary conditions to find an expression for the temperature distribution. Note that the derivatives of the modified Bessel functions are given in Section 3.6.4.

(b) Equate the heat flux at the outer surface of the muscle to the heat flux through the skin/fat layer and into the environment to determine the temperature at the outer surface of the muscle.

(c) Find the maximum forearm temperature.

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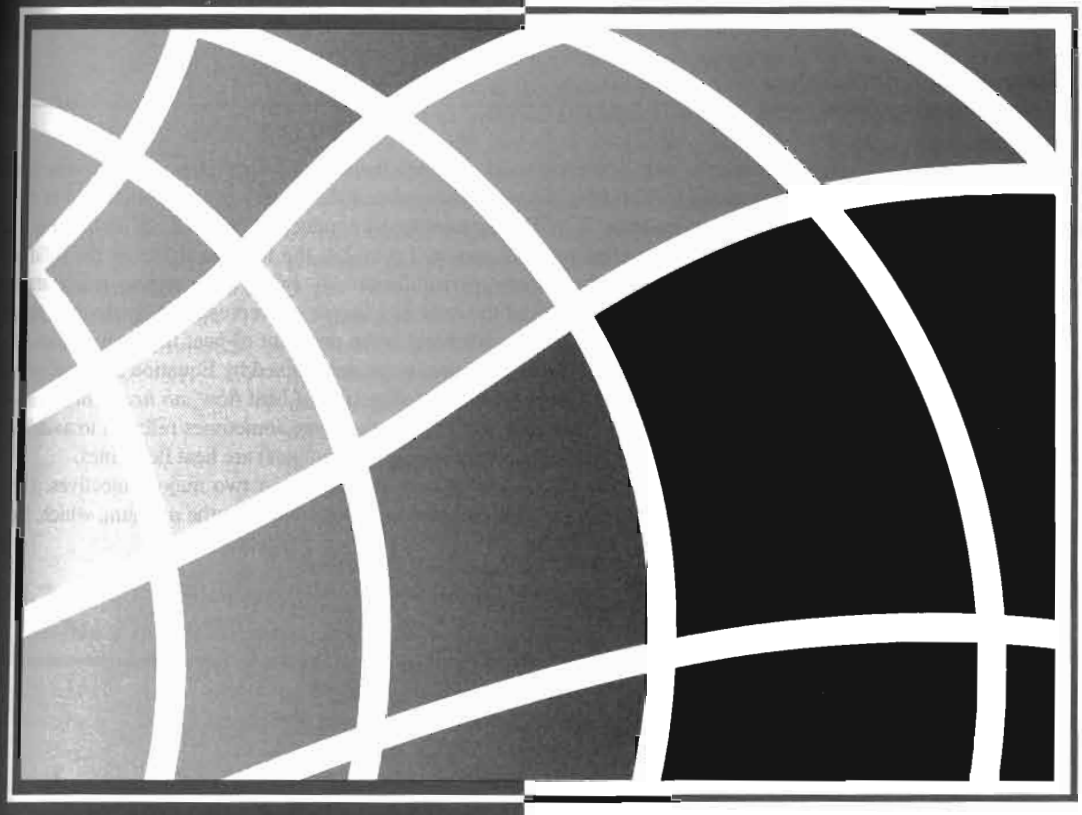
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CHAPTER 4

Two-Dimensional, Steady-State Conduction



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To this point we have restricted our attention to conduction problems in which the temperature gradient is significant for only one coordinate direction. However, in many cases such problems are grossly oversimplified if a one-dimensional treatment is used, and it is necessary to account for multidimensional effects. In this chapter, we consider several techniques for treating two-dimensional systems under steady-state conditions.

We begin our consideration of two-dimensional, steady-state conduction by briefly reviewing alternative approaches to determining temperatures and heat rates (Section 4.1). The approaches range from *exact solutions*, which may be obtained for idealized conditions, to *approximate methods* of varying complexity and accuracy. In Section 4.2 we consider some of the mathematical issues associated with obtaining an exact solution. In Section 4.3, we present compilations of existing exact solutions for a variety of simple geometries. Our objective in Sections 4.4 and 4.5 is to show how, with the aid of a computer, *numerical (finite-difference* or *finite-element) methods* may be used to accurately predict temperatures and heat rates within the medium and at its boundaries.

4.1 *Alternative Approaches*

Consider a long, prismatic solid in which there is two-dimensional heat conduction (Figure 4.1). With two surfaces insulated and the other surfaces maintained at different temperatures, $T_1 > T_2$, heat transfer by conduction occurs from surface 1 to 2. According to Fourier's law, Equation 2.3 or 2.4, the local heat flux in the solid is a vector that is everywhere perpendicular to lines of constant temperature (*isotherms*). The directions of the heat flux vector are represented by the *heat flow lines* of Figure 4.1, and the vector itself is the resultant of heat flux components in the x and y directions. These components are determined by Equation 2.6. Since the heat flow lines are, by definition, in the direction of heat flow, *no heat can be conducted across a heat flow line* and they are therefore sometimes referred to as *adiabats*. Conversely, adiabatic surfaces (or symmetry lines) are heat flow lines.

Recall that, in any conduction analysis, there exist two major objectives. The first objective is to determine the temperature distribution in the medium, which, for

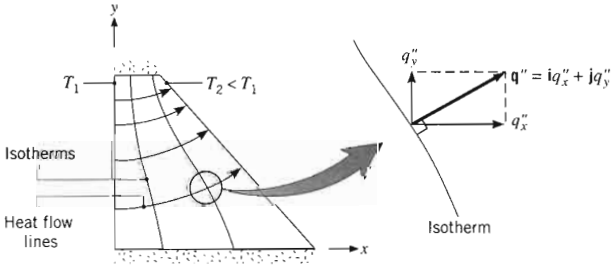


FIGURE 4.1 Two-dimensional conduction.

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the present problem, necessitates determining $T(x, y)$. This objective is achieved by solving the appropriate form of the heat equation. For two-dimensional, steady-state conditions with no generation and constant thermal conductivity, this form is, from Equation 2.20,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \tag{4.1}$$

If Equation 4.1 can be solved for $T(x, y)$, it is then a simple matter to satisfy the second major objective, which is to determine the heat flux components q''_x and q''_y by applying the rate equations (2.6). Methods for solving Equation 4.1 include the use of *analytical, graphical,* and *numerical* (*finite-difference, finite-element,* or *boundary-element*) approaches.

The analytical method involves effecting an exact mathematical solution to Equation 4.1. The problem is more difficult than those considered in Chapter 3, since it now involves a partial, rather than an ordinary, differential equation. Although several techniques are available for solving such equations, the solutions typically involve complicated mathematical series and functions and may be obtained for only a restricted set of simple geometries and boundary conditions [1–5]. Nevertheless, the solutions are of value, since the dependent variable T is determined as a continuous function of the independent variables (x, y). Hence the solution could be used to compute the temperature at *any* point of interest in the medium. To illustrate the nature and importance of analytical techniques, an exact solution to Equation 4.1 is obtained in Section 4.2 by the method of *separation of variables.* Conduction shape factors and dimensionless conduction heat rates (Section 4.3) are compilations of existing solutions for geometries that are commonly encountered in engineering practice.

In contrast to the analytical methods, which provide *exact* results at *any* point, graphical and numerical methods can provide only *approximate* results at *discrete* points. Although superseded by computer solutions based on numerical procedures, the graphical, or flux-plotting, method may be used to obtain a quick estimate of the temperature distribution. Its use is restricted to two-dimensional problems involving adiabatic and isothermal boundaries. The method is based on the fact that isotherms must be perpendicular to heat flow lines, as noted in Figure 4.1. Unlike the analytical or graphical approaches, numerical methods (Sections 4.4 and 4.5) may be used to obtain accurate results for complex, two- or three-dimensional geometries involving a wide variety of boundary conditions.

The graphical method is described, and its use is demonstrated, in Section 4S.1.

4.2 The Method of Separation of Variables

To appreciate how the method of separation of variables may be used to solve two-dimensional conduction problems, we consider the system of Figure 4.2. Three sides of a thin rectangular plate or a long rectangular rod are maintained at a constant temperature T_1, while the fourth side is maintained at a constant temperature $T_2 \neq T_1$. Assuming negligible heat transfer from the surfaces of the plate or the ends of the rod, temperature gradients normal to the x–y plane may be neglected ($\partial^2 T/\partial z^2 \approx 0$) and conduction heat transfer is primarily in the x and y directions.

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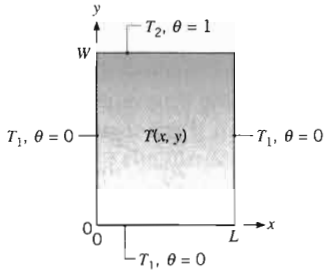


FIGURE 4.2
Two-dimensional conduction in a thin rectangular plate or a long rectangular rod.

We are interested in the temperature distribution $T(x, y)$, but to simplify the solution we introduce the transformation

$$\theta \equiv \frac{T - T_1}{T_2 - T_1} \tag{4.2}$$

Substituting Equation 4.2 into Equation 4.1, the transformed differential equation is then

$$\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0 \tag{4.3}$$

Since the equation is second order in both x and y, two boundary conditions are needed for each of the coordinates. They are

$$\theta(0, y) = 0 \quad \text{and} \quad \theta(x, 0) = 0$$
$$\theta(L, y) = 0 \quad \text{and} \quad \theta(x, W) = 1$$

Note that, through the transformation of Equation 4.2, three of the four boundary conditions are now homogeneous and the value of θ is restricted to the range from 0 to 1.

We now apply the separation of variables technique by assuming that the desired solution can be expressed as the product of two functions, one of which depends only on x while the other depends only on y. That is, we assume the existence of a solution of the form

$$\theta(x, y) = X(x) \cdot Y(y) \tag{4.4}$$

Substituting into Equation 4.3 and dividing by XY, we obtain

$$-\frac{1}{X}\frac{d^2X}{dx^2} = \frac{1}{Y}\frac{d^2Y}{dy^2} \tag{4.5}$$

and it is evident that the differential equation is, in fact, separable. That is, the left-hand side of the equation depends only on x and the right-hand side depends only on y. Hence the equality can apply in general (for any x or y) only if both sides are equal to the same constant. Identifying this, as yet unknown, *separation constant* as λ^2, we then have

$$\frac{d^2X}{dx^2} + \lambda^2 X = 0 \tag{4.6}$$

$$\frac{d^2Y}{dy^2} - \lambda^2 Y = 0 \tag{4.7}$$

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and the partial differential equation has been reduced to two ordinary differential equations. Note that the designation of λ^2 as a positive constant was not arbitrary. If a negative value were selected or a value of $\lambda^2 = 0$ was chosen, it is readily shown (Problem 4.1) that it would be impossible to obtain a solution that satisfies the prescribed boundary conditions.

The general solutions to Equations 4.6 and 4.7 are, respectively,

$$X = C_1 \cos \lambda x + C_2 \sin \lambda x$$
$$Y = C_3 e^{-\lambda y} + C_4 e^{+\lambda y}$$

in which case the general form of the two-dimensional solution is

$$\theta = (C_1 \cos \lambda x + C_2 \sin \lambda x)(C_3 e^{-\lambda y} + C_4 e^{\lambda y}) \tag{4.8}$$

Applying the condition that $\theta(0, y) = 0$, it is evident that $C_1 = 0$. In addition from the requirement that $\theta(x, 0) = 0$, we obtain

$$C_2 \sin \lambda x (C_3 + C_4) = 0$$

which may only be satisfied if $C_3 = -C_4$. Although the requirement could also be satisfied by having $C_2 = 0$, this would result in $\theta(x,y) = 0$, which does not satisfy the boundary condition $\theta(x,W) = 1$. If we now invoke the requirement that $\theta(L, y) = 0$, we obtain

$$C_2 C_4 \sin \lambda L (e^{\lambda y} - e^{-\lambda y}) = 0$$

The only way in which this condition may be satisfied (and still have a nonzero solution) is by requiring that λ assume discrete values for which $\sin \lambda L = 0$. These values must then be of the form

$$\lambda = \frac{n\pi}{L} \qquad n = 1, 2, 3, \ldots \tag{4.9}$$

where the integer $n = 0$ is precluded, since it implies $\theta(x,y) = 0$. The desired solution may now be expressed as

$$\theta = C_2 C_4 \sin \frac{n\pi x}{L} (e^{n\pi y/L} - e^{-n\pi y/L}) \tag{4.10}$$

Combining constants and acknowledging that the new constant may depend on n, we obtain

$$\theta(x, y) = C_n \sin \frac{n\pi x}{L} \sinh \frac{n\pi y}{L}$$

where we have also used the fact that $(e^{n\pi y/L} - e^{-n\pi y/L}) = 2 \sinh (n\pi y/L)$. In the above form we have really obtained an infinite number of solutions that satisfy the differential equation and boundary conditions. However, since the problem is linear, a more general solution may be obtained from a superposition of the form

$$\theta(x, y) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{L} \sinh \frac{n\pi y}{L} \tag{4.11}$$

To determine C_n we now apply the remaining boundary condition, which is of the form

$$\theta(x, W) = 1 = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi x}{L} \sinh \frac{n\pi W}{L} \tag{4.12}$$

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Although Equation 4.12 would seem to be an extremely complicated relation for evaluating C_n, a standard method is available. It involves writing an infinite series expansion in terms of *orthogonal functions.* An infinite set of functions $g_1(x), g_2(x), \ldots, g_n(x), \ldots$ is said to be orthogonal in the domain $a \leq x \leq b$ if

$$\int_a^b g_m(x) g_n(x)\, dx = 0 \qquad m \neq n \tag{4.13}$$

Many functions exhibit orthogonality, including the trigonometric functions $\sin(n\pi x/L)$ and $\cos(n\pi x/L)$ for $0 \leq x \leq L$. Their utility in the present problem rests with the fact that any function $f(x)$ may be expressed in terms of an infinite series of orthogonal functions

$$f(x) = \sum_{n=1}^{\infty} A_n g_n(x) \tag{4.14}$$

The form of the coefficients A_n in this series may be determined by multiplying each side of the equation by $g_m(x)$ and integrating between the limits a and b.

$$\int_a^b f(x) g_m(x)\, dx = \int_a^b g_m(x) \sum_{n=1}^{\infty} A_n g_n(x)\, dx \tag{4.15}$$

However, from Equation 4.13 it is evident that all but one of the terms on the right-hand side of Equation 4.15 must be zero, leaving us with

$$\int_a^b f(x) g_m(x)\, dx = A_m \int_a^b g_m^2(x)\, dx$$

Hence, solving for A_m, and recognizing that this holds for any A_n by switching m to n:

$$A_n = \frac{\displaystyle\int_a^b f(x) g_n(x)\, dx}{\displaystyle\int_a^b g_n^2(x)\, dx} \tag{4.16}$$

The properties of orthogonal functions may be used to solve Equation 4.12 for C_n by formulating an infinite series for the appropriate form of $f(x)$. From Equation 4.14 it is evident that we should choose $f(x) = 1$ and the orthogonal function $g_n(x) = \sin(n\pi x/L)$. Substituting into Equation 4.16 we obtain

$$A_n = \frac{\displaystyle\int_0^L \sin\frac{n\pi x}{L}\, dx}{\displaystyle\int_0^L \sin^2\frac{n\pi x}{L}\, dx} = \frac{2}{\pi}\frac{(-1)^{n+1}+1}{n}$$

Hence from Equation 4.14, we have

$$1 = \sum_{n=1}^{\infty} \frac{2}{\pi}\frac{(-1)^{n+1}+1}{n} \sin\frac{n\pi x}{L} \tag{4.17}$$

which is simply the expansion of unity in a Fourier series. Comparing Equations 4.12 and 4.17 we obtain

$$C_n = \frac{2[(-1)^{n+1}+1]}{n\pi \sinh(n\pi W/L)} \qquad n = 1, 2, 3, \ldots \tag{4.18}$$

4.3 The Conducti Dimensionless

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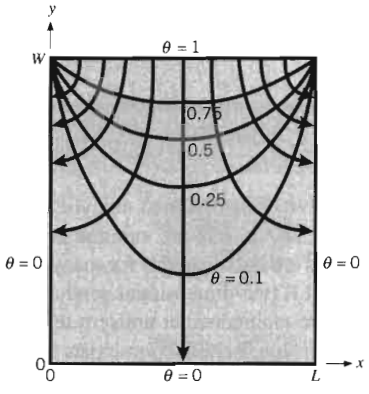


FIGURE 4.3
Isotherms and heat flow lines for two-dimensional conduction in a rectangular plate.

Substituting Equation 4.18 into Equation 4.11, we then obtain for the final solution

$$\theta(x, y) = \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}+1}{n}\sin\frac{n\pi x}{L}\frac{\sinh(n\pi y/L)}{\sinh(n\pi W/L)} \tag{4.19}$$

Equation 4.19 is a convergent series, from which the value of θ may be computed for any x and y. Representative results are shown in the form of isotherms for a schematic of the rectangular plate (Figure 4.3). The temperature T corresponding to a value of θ may be obtained from Equation 4.2, and components of the heat flux may be determined by using Equation 4.19 with Equation 2.6. The heat flux components determine the heat flow lines, which are shown in the figure. We note that the temperature distribution is symmetric about $x = L/2$, with $\partial T/\partial x = 0$ at that location. Hence, from Equation 2.6, we know the symmetry plane at $x = L/2$ is adiabatic and, therefore, is a heat flow line. However, note that the discontinuities prescribed at the upper corners of the plate are physically untenable. In reality, large temperature gradients could be maintained in proximity to the corners, but discontinuities could not exist.

Exact solutions have been obtained for many other geometries and boundary conditions, including cylindrical and spherical systems. Such solutions are presented in specialized books on conduction heat transfer [1–5].

4.3 *The Conduction Shape Factor and the Dimensionless Conduction Heat Rate*

In many instances, two-dimensional or three-dimensional conduction problems may be rapidly solved by utilizing existing solutions to the heat diffusion equation. These solutions are reported in terms of a *shape factor*, S, or a steady-state *dimensionless conduction heat rate*, q^*_{ss}. That is, the heat transfer rate may be expressed as

$$q = Sk\Delta T_{1-2} \tag{4.20}$$

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where ΔT_{1-2} is the temperature difference between boundaries, as shown in, for example, Figure 4.2. It also follows that a two-dimensional conduction resistance may be expressed as

$$R_{t,cond(2D)} = \frac{1}{Sk} \tag{4.21}$$

Shape factors for two-dimensional geometries may also be estimated with the graphical method that is described in Section 4S.1.

Shape factors have been obtained analytically for numerous two- and three-dimensional systems, and results are summarized in Table 4.1 for some common configurations. Results are also available for many other configurations [6–9]. In cases 1 through 8 and case 11, two-dimensional conduction is presumed to occur between the boundaries that are maintained at uniform temperatures, with $\Delta T_{1-2} = T_1 - T_2$. In case 9, three-dimensional conduction exists in the corner region, while in case 10, conduction occurs between an isothermal disk (T_1) and a semi-infinite medium of uniform temperature (T_2) at locations well removed from the disk. Shape factors may also be defined for one-dimensional geometries, and from the results of Table 3.3, it follows that for plane, cylindrical, and spherical walls, respectively, the shape factors are A/L, $2\pi L/\ln(r_2/r_1)$, and $4\pi r_1 r_2/(r_2 - r_1)$.

Cases 12 through 15 are associated with conduction from objects held at an isothermal temperature (T_1) that are embedded within an infinite medium of uniform temperature (T_2) at locations removed from the object. For the infinite medium cases, useful results may be obtained by defining a *characteristic length*

$$L_c \equiv (A_s/4\pi)^{1/2} \tag{4.22}$$

where A_s is the surface area of the object. Conduction heat transfer rates from the object to the infinite medium may then be reported in terms of a *dimensionless conduction heat rate* [10]

$$q^*_{ss} \equiv qL_c/kA_s(T_1 - T_2) \tag{4.23}$$

From Table 4.1*b*, it is evident that the values of q^*_{ss}, which have been obtained analytically and numerically, are similar for a wide range of geometrical configurations. As a consequence of this similarity, values of q^*_{ss} may be *estimated* for configurations that are *similar* to those for which q^*_{ss} is known. For example, dimensionless conduction heat rates from cuboid shapes (case 15) over the range $0.1 \leq d/D \leq 10$ may be closely approximated by interpolating the values of q^*_{ss} reported in Table 4.1. Additional procedures that may be exploited to estimate values of q^*_{ss} for other geometries are explained in [10]. Note that results for q^*_{ss} in Table 4.1*b* may be converted to expressions for S listed in Table 4.1*a*. For example, the shape factor of case 10 may be derived from the dimensionless conduction heat rate of case 13 (recognizing that the infinite medium can be viewed as two adjacent semi-infinite media).

The shape factors and dimensionless conduction heat rates reported in Table 4.1 are associated with objects that are held at uniform temperatures. For uniform heat flux conditions, the object's temperature is no longer uniform but varies spatially with the coolest temperatures located near the periphery of the heated object. Hence, the temperature difference that is used to define S or q^*_{ss} is replaced by a temperature difference involving the *spatially averaged* surface temperature of the object ($\overline{T}_1 - T_2$), or the difference between the *maximum* surface temperature of the

TABLE 4.1 Co
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(*a*) Shape factors [*q*

System
Case 1 Isothermal sphere bu a semi-infinite mediu
Case 2 Horizontal isotherma cylinder of length L in a semi-infinite me
Case 3 Vertical cylinder in a semi-infinite mediun
Case 4 Conduction between cylinders of length L infinite medium
Case 5 Horizontal circular c of length L midway l parallel planes of equ length and infinite w
Case 6 Circular cylinder of centered in a square
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length

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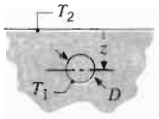
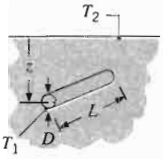
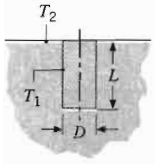
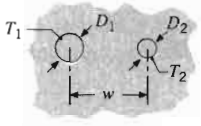
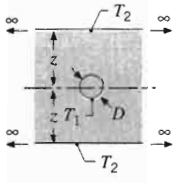
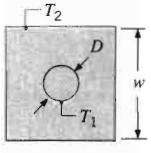
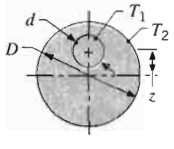
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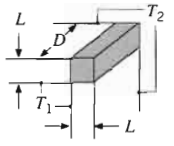
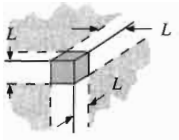
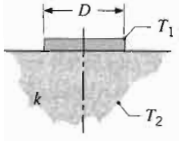
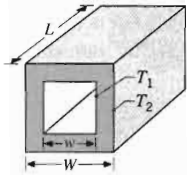
TABLE 4.1 Conduction shape factors and dimensionless conduction heat rates for selected systems.

(a) Shape factors [$q = Sk(T_1 - T_2)$]

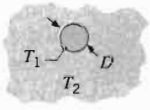
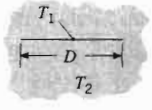
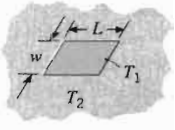
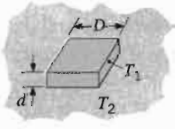
System	Schematic	Restrictions	Shape Factor
Case 1 Isothermal sphere buried in a semi-infinite medium		$z > D/2$	$\dfrac{2\pi D}{1 - D/4z}$
Case 2 Horizontal isothermal cylinder of length L buried in a semi-infinite medium		$L \gg D$ $L \gg D$ $z > 3D/2$	$\dfrac{2\pi L}{\cosh^{-1}(2z/D)}$ $\dfrac{2\pi L}{\ln(4z/D)}$
Case 3 Vertical cylinder in a semi-infinite medium		$L \gg D$	$\dfrac{2\pi L}{\ln(4L/D)}$
Case 4 Conduction between two cylinders of length L in infinite medium		$L \gg D_1, D_2$ $L \gg w$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{4w^2 - D_1^2 - D_2^2}{2D_1D_2}\right)}$
Case 5 Horizontal circular cylinder of length L midway between parallel planes of equal length and infinite width		$z \gg D/2$ $L \gg z$	$\dfrac{2\pi L}{\ln(8z/\pi D)}$
Case 6 Circular cylinder of length L centered in a square solid of equal length		$w > D$ $L \gg w$	$\dfrac{2\pi L}{\ln(1.08\, w/D)}$
Case 7 Eccentric circular cylinder of length L in a cylinder of equal length		$D > d$ $L \gg D$	$\dfrac{2\pi L}{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}$

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TABLE 4.1 *Continued*

System	Schematic	Restrictions	Shape Factor
Case 8 Conduction through the edge of adjoining walls		$D > 5L$	$0.54D$
Case 9 Conduction through corner of three walls with a temperature difference ΔT_{1-2} across the walls		$L \ll$ length and width of wall	$0.15L$
Case 10 Disk of diameter D and temperature T_1 on a semi-infinite medium of thermal conductivity k and temperature T_2		None	$2D$
Case 11 Square channel of length L		$\frac{W}{w} < 1.4$ $\frac{W}{w} > 1.4$ $L \gg W$	$\frac{2\pi L}{0.785 \ln (W/w)}$ $\frac{2\pi L}{0.930 \ln (W/w) - 0.050}$

(*b*) Dimensionless conduction heat rates [$q = q^*_{ss} k A_s (T_1 - T_2)/L_c$; $L_c \equiv (A_s/4\pi)^{1/2}$]

System	Schematic	Active Area, A_s	q^*_{ss}
Case 12 Isothermal sphere of diameter D and temperature T_1 in an infinite medium of temperature T_2		πD^2	1
Case 13 Infinitely thin, isothermal disk of diameter D and temperature T_1 in an infinite medium of temperature T_2		$\frac{\pi D^2}{2}$	$\frac{2\sqrt{2}}{\pi} = 0.900$
Case 14 Infinitely thin rectangle of length L, width w, and temperature T_1 in an infinite medium of temperature T_2		$2wL$	0.932
Case 15 Cuboid shape of height d with a square footprint of width D and temperature T_1 in an infinite medium of temperature T_2		$2D^2 + 4Dd$	d/D / q^*_{ss} 0.1 / 0.943 1.0 / 0.956 2.0 / 0.961 10 / 1.111

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Shape Factor	q^*_{ss}
$0.54D$	1
$0.15L$	$\frac{2\sqrt{2}}{\pi} = 0.900$
$2D$	0.932
$\frac{2\pi L}{0.785 \ln (W/w)}$	
$\frac{2\pi L}{0 \ln (W/w) - 0.050}$	

W/D	q^*_{ss}
.1	0.943
.0	0.956
.0	0.961
0	1.111

heated object and the far field temperature of the surrounding medium, $(T_{1,max} - T_2)$. For the *uniformly heated* geometry of case 10 (a disk of diameter D in contact with a semi-infinite medium of thermal conductivity k and temperature T_2), the values of S are $3\pi^2 D/16$ and $\pi D/2$ for temperature differences based on the average and maximum disk temperatures, respectively.

EXAMPLE 4.1

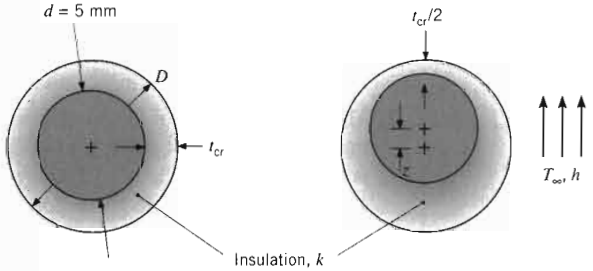
A metallic electrical wire of diameter $d = 5$ mm is to be coated with insulation of thermal conductivity $k = 0.35$ W/m · K. It is expected that, for the typical installation, the coated wire will be exposed to conditions for which the total coefficient associated with convection and radiation is $h = 15$ W/m^2 · K. To minimize the temperature rise of the wire due to ohmic heating, the insulation thickness is specified so that the *critical insulation radius* is achieved (see Example 3.4). During the wire coating process, however, the insulation thickness sometimes varies around the periphery of the wire, resulting in eccentricity of the wire relative to the coating. Determine the change in the thermal resistance of the insulation due to an eccentricity that is 50% of the critical insulation thickness.

SOLUTION

Known: Wire diameter, convective conditions, and insulation thermal conductivity.

Find: Thermal resistance of the wire coating associated with peripheral variations in the coating thickness.

Schematic:



(*a*) Concentric wire (*b*) Eccentric wire

Assumptions:

1. Steady-state conditions.
2. Two-dimensional conduction.
3. Constant properties.
4. Both the exterior and interior surfaces of the coating are at uniform temperatures.

Analysis: From Example 3.4, the critical insulation radius is

$$r_{cr} = \frac{k}{h} = \frac{0.35 \text{ W/m} \cdot \text{K}}{15 \text{ W/m}^2 \cdot \text{K}} = 0.023 \text{ m} = 23 \text{ mm}$$

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Therefore, the critical insulation thickness is

$$t_{cr} = r_{cr} - d/2 = 0.023\ \text{m} - \frac{0.005\ \text{m}}{2} = 0.021\ \text{m} = 21\ \text{mm}$$

The thermal resistance of the coating associated with the concentric wire may be evaluated using Equation 3.28 and is

$$R'_{t,\text{cond}} = \frac{\ln[r_{cr}/(d/2)]}{2\pi k} = \frac{\ln[0.023\ \text{m}/(0.005\ \text{m}/2)]}{2\pi(0.35\ \text{W/m} \cdot \text{K})} = 1.0\ \text{m} \cdot \text{K/W}$$

For the eccentric wire, the thermal resistance of the insulation may be evaluated using case 7 of Table 4.1, where the eccentricity is $z = 0.5 \times t_{cr} = 0.5 \times 0.021\ \text{m} = 0.010\ \text{m}$

$$R'_{t,\text{cond(2D)}} = \frac{1}{Sk} = \frac{\cosh^{-1}\left(\dfrac{D^2 + d^2 - 4z^2}{2Dd}\right)}{2\pi k}$$

$$= \frac{\cosh^{-1}\left(\dfrac{(2 \times 0.023\ \text{m})^2 + (0.005\ \text{m})^2 - 4(0.010\ \text{m})^2}{2 \times (2 \times 0.023\ \text{m}) \times 0.005\ \text{m}}\right)}{2\pi \times 0.35\ \text{W/m} \cdot \text{K}}$$

$$= 0.91\ \text{m} \cdot \text{K/W}$$

Therefore, the reduction in the thermal resistance of the insulation is 0.10 m · K/W, or 10%. ◁

Comments:

1. Reduction in the local insulation thickness leads to a smaller local thermal resistance of the insulation. Conversely, locations associated with thicker coatings have increased local thermal resistances. These effects offset each other, but not exactly; the maximum resistance is associated with the concentric wire case. For this application, eccentricity of the wire relative to the coating provides *enhanced* thermal performance relative to the concentric wire case.
2. The interior surface of the coating will be at nearly uniform temperature if the thermal conductivity of the wire is high relative to that of the insulation. Such is the case for metallic wire. However, the exterior surface temperature of the coating will not be perfectly uniform due to the variation in the local insulation thickness.

4.4 *Finite-Difference Equations*

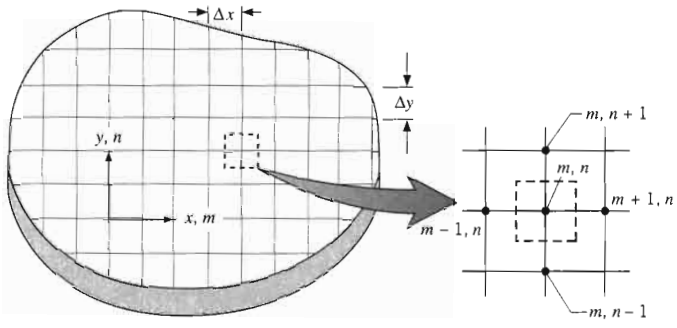
As discussed in Sections 4.1 and 4.2, analytical methods may be used, in certain cases, to effect exact mathematical solutions to steady, two-dimensional conduction problems. These solutions have been generated for an assortment of simple geometries and boundary conditions and are well documented in the literature [1–5]. However,

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more often than not, two-dimensional problems involve geometries and/or boundary conditions that preclude such solutions. In these cases, the best alternative is often one that uses a *numerical* technique such as the *finite-difference, finite-element,* or *boundary-element* method. Another strength of numerical methods is that they can be readily extended to three-dimensional problems. Because of its ease of application, the finite-difference method is well suited for an introductory treatment of numerical techniques.

4.4.1 The Nodal Network

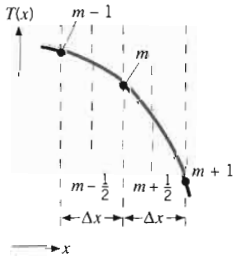
In contrast to an analytical solution, which allows for temperature determination at *any* point of interest in a medium, a numerical solution enables determination of the temperature at only *discrete* points. The first step in any numerical analysis must therefore be to select these points. Referring to Figure 4.4, this may be done by subdividing the medium of interest into a number of small regions and assigning to each a reference point that is at its center. The reference point is frequently termed a *nodal point* (or simply a *node*), and the aggregate of points is termed a *nodal network, grid,* or *mesh.* The nodal points are designated by a numbering scheme that, for a two-dimensional system, may take the form shown in Figure 4.4*a*. The x and y locations are designated by the m and n indices, respectively.



(*a*)

$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n} = \frac{T_{m,n} - T_{m-1,n}}{\Delta x}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n} = \frac{T_{m+1,n} - T_{m,n}}{\Delta x}$$



(*b*)

FIGURE 4.4 Two-dimensional conduction. (*a*) Nodal network. (*b*) Finite-difference approximation.

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Each node represents a certain region, and its temperature is a measure of the *average* temperature of the region. For example, the temperature of the node m, n of Figure 4.4*a* may be viewed as the average temperature of the surrounding shaded area. The selection of nodal points is rarely arbitrary, depending often on matters such as geometric convenience and the desired accuracy. The numerical accuracy of the calculations depends strongly on the number of designated nodal points. If this number is large (a *fine mesh*), accurate solutions can be obtained.

4.4.2 Finite-Difference Form of the Heat Equation

Determination of the temperature distribution numerically dictates that an appropriate conservation equation be written for *each* of the nodal points of unknown temperature. The resulting set of equations may then be solved simultaneously for the temperature at each node. For *any interior* node of a two-dimensional system with no generation and uniform thermal conductivity, the *exact* form of the energy conservation requirement is given by the heat equation, Equation 4.1. However, if the system is characterized in terms of a nodal network, it is necessary to work with an *approximate,* or *finite-difference,* form of this equation.

A finite-difference equation that is suitable for the interior nodes of a two-dimensional system may be inferred directly from Equation 4.1. Consider the second derivative, $\partial^2 T/\partial x^2$. From Figure 4.4*b*, the value of this derivative at the m, n nodal point may be approximated as

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \approx \frac{\partial T/\partial x|_{m+1/2,n} - \partial T/\partial x|_{m-1/2,n}}{\Delta x} \tag{4.24}$$

The temperature gradients may in turn be expressed as a function of the nodal temperatures. That is,

$$\left.\frac{\partial T}{\partial x}\right|_{m+1/2,n} \approx \frac{T_{m+1,n} - T_{m,n}}{\Delta x} \tag{4.25}$$

$$\left.\frac{\partial T}{\partial x}\right|_{m-1/2,n} \approx \frac{T_{m,n} - T_{m-1,n}}{\Delta x} \tag{4.26}$$

Substituting Equations 4.25 and 4.26 into 4.24, we obtain

$$\left.\frac{\partial^2 T}{\partial x^2}\right|_{m,n} \approx \frac{T_{m+1,n} + T_{m-1,n} - 2T_{m,n}}{(\Delta x)^2} \tag{4.27}$$

Proceeding in a similar fashion, it is readily shown that

$$\left.\frac{\partial^2 T}{\partial y^2}\right|_{m,n} \approx \frac{\partial T/\partial y|_{m,n+1/2} - \partial T/\partial y|_{m,n-1/2}}{\Delta y}$$

$$\approx \frac{T_{m,n+1} + T_{m,n-1} - 2T_{m,n}}{(\Delta y)^2} \tag{4.28}$$

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Using a network for which $\Delta x = \Delta y$ and substituting Equations 4.27 and 4.28 into Equation 4.1, we obtain

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0 \tag{4.29}$$

Hence for the m, n node the heat equation, which is an *exact differential equation,* is reduced to an *approximate algebraic equation.* This approximate, *finite-difference form of the heat equation* may be applied to any interior node that is equidistant from its four neighboring nodes. It simply requires that the temperature of an interior node be equal to the average of the temperatures of the four neighboring nodes.

4.4.3 The Energy Balance Method

In many cases, it is desirable to develop the finite-difference equations by an alternative method called the *energy balance method.* As will become evident, this approach enables one to analyze many different phenomena such as problems involving multiple materials, embedded heat sources, or exposed surfaces that do not align with an axis of the coordinate system. In the energy balance method, the finite-difference equation for a node is obtained by applying conservation of energy to a control volume about the nodal region. Since the actual direction of heat flow (into or out of the node) is often unknown, it is convenient to formulate the energy balance by *assuming* that *all* the heat flow is *into the node.* Such a condition is, of course, impossible, but if the rate equations are expressed in a manner consistent with this assumption, the correct form of the finite-difference equation is obtained. For steady-state conditions with generation, the appropriate form of Equation 1.11c is then

$$\dot{E}_{\text{in}} + \dot{E}_g = 0 \tag{4.30}$$

Consider applying Equation 4.30 to a control volume about the interior node m, n of Figure 4.5. For two-dimensional conditions, energy exchange is influenced by

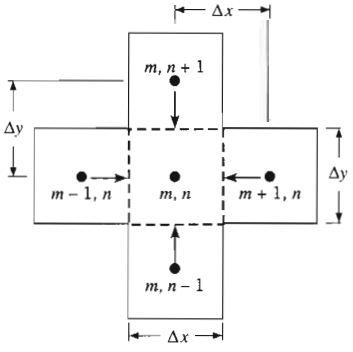


FIGURE 4.5
Conduction to an interior node from its adjoining nodes.

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conduction between m, n and its four adjoining nodes, as well as by generation. Hence Equation 4.30 reduces to

$$\sum_{i=1}^{4} q_{(i)\rightarrow(m,n)} + \dot{q}(\Delta x \cdot \Delta y \cdot 1) = 0$$

where i refers to the neighboring nodes, $q_{(i)\rightarrow(m,n)}$ is the conduction rate between nodes, and unit depth is assumed. To evaluate the conduction rate terms, we *assume* that conduction transfer occurs exclusively through *lanes* that are oriented in either the x or y direction. Simplified forms of Fourier's law may therefore be used. For example, the rate at which energy is transferred by conduction from node $m-1, n$ to m, n may be expressed as

$$q_{(m-1,n)\rightarrow(m,n)} = k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} \tag{4.31}$$

The quantity $(\Delta y \cdot 1)$ is the heat transfer area, and the term $(T_{m-1,n} - T_{m,n})/\Delta x$ is the finite-difference approximation to the temperature gradient at the boundary between the two nodes. The remaining conduction rates may be expressed as

$$q_{(m+1,n)\rightarrow(m,n)} = k(\Delta y \cdot 1)\frac{T_{m+1,n} - T_{m,n}}{\Delta x} \tag{4.32}$$

$$q_{(m,n+1)\rightarrow(m,n)} = k(\Delta x \cdot 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} \tag{4.33}$$

$$q_{(m,n-1)\rightarrow(m,n)} = k(\Delta x \cdot 1)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} \tag{4.34}$$

Note that in evaluating each conduction rate, we have subtracted the temperature of the m, n node from the temperature of its adjoining node. This convention is necessitated by the assumption of heat flow into m, n and is consistent with the direction of the arrows shown in Figure 4.5. Substituting Equations 4.31 through 4.34 into the energy balance and remembering that $\Delta x = \Delta y$, it follows that the finite-difference equation for an interior node with generation is

$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} + \frac{\dot{q}(\Delta x)^2}{k} - 4T_{m,n} = 0 \tag{4.35}$$

If there is no internally distributed source of energy ($\dot{q} = 0$), this expression reduces to Equation 4.29.

It is important to note that a finite-difference equation is needed for each nodal point at which the temperature is unknown. However, it is not always possible to classify all such points as interior and hence to use Equation 4.29 or 4.35. For example, the temperature may be unknown at an insulated surface or at a surface that is exposed to convective conditions. For points on such surfaces, the finite-difference equation must be obtained by applying the energy balance method.

To further illustrate this method, consider the node corresponding to the internal corner of Figure 4.6. This node represents the three-quarter shaded section and exchanges energy by convection with an adjoining fluid at T_∞. Conduction to the

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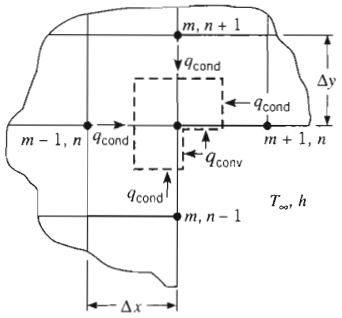


FIGURE 4.6
Formulation of the finite-difference equation for an internal corner of a solid with surface convection.

nodal region (m, n) occurs along four different lanes from neighboring nodes in the solid. The conduction heat rates q_{cond} may be expressed as

$$q_{(m-1,n)\to(m,n)} = k(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x} \tag{4.36}$$

$$q_{(m,n+1)\to(m,n)} = k(\Delta x \cdot 1)\frac{T_{m,n+1} - T_{m,n}}{\Delta y} \tag{4.37}$$

$$q_{(m+1,n)\to(m,n)} = k\left(\frac{\Delta y}{2} \cdot 1\right)\frac{T_{m+1,n} - T_{m,n}}{\Delta x} \tag{4.38}$$

$$q_{(m,n-1)\to(m,n)} = k\left(\frac{\Delta x}{2} \cdot 1\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y} \tag{4.39}$$

Note that the areas for conduction from nodal regions $(m - 1, n)$ and $(m, n + 1)$ are proportional to Δy and Δx, respectively, whereas conduction from $(m + 1, n)$ and $(m, n - 1)$ occurs along lanes of width $\Delta y/2$ and $\Delta x/2$, respectively.

Conditions in the nodal region m, n are also influenced by convective exchange with the fluid, and this exchange may be viewed as occurring along half-lanes in the x and y directions. The total convection rate q_{conv} may be expressed as

$$q_{(\infty)\to(m,n)} = h\left(\frac{\Delta x}{2} \cdot 1\right)(T_\infty - T_{m,n}) + h\left(\frac{\Delta y}{2} \cdot 1\right)(T_\infty - T_{m,n}) \tag{4.40}$$

Implicit in this expression is the assumption that the exposed surfaces of the corner are at a uniform temperature corresponding to the nodal temperature $T_{m,n}$. This assumption is consistent with the concept that the entire nodal region is characterized by a single temperature, which represents an average of the actual temperature distribution in the region. In the absence of transient, three-dimensional, and generation effects, conservation of energy, Equation 4.30, requires that the sum of Equations 4.36 through 4.40 be zero. Summing these equations and rearranging, we therefore obtain

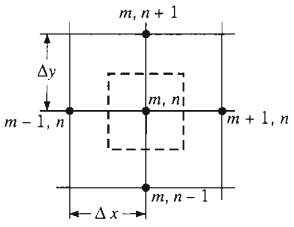
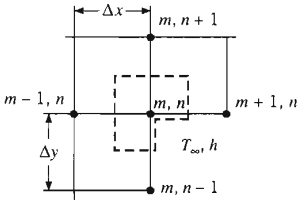
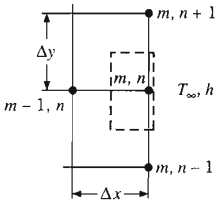
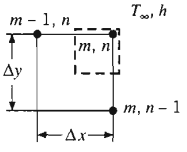
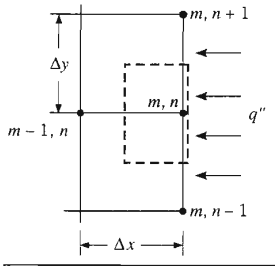
$$T_{m-1,n} + T_{m,n+1} + \frac{1}{2}(T_{m+1,n} + T_{m,n-1}) + \frac{h\Delta x}{k}T_\infty - \left(3 + \frac{h\Delta x}{k}\right)T_{m,n} = 0 \tag{4.41}$$

where again the mesh is such that $\Delta x = \Delta y$.

Nodal energy balance equations pertinent to several common geometries for situations where there is no internal energy generation are presented in Table 4.2.

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TABLE 4.2 Summary of nodal finite-difference equations

Configuration	Finite-Difference Equation for $\Delta x = \Delta y$
	$$T_{m,n+1} + T_{m,n-1} + T_{m+1,n} + T_{m-1,n} - 4T_{m,n} = 0 \quad (4.29)$$ **Case 1.** Interior node
	$$2(T_{m-1,n} + T_{m,n+1}) + (T_{m+1,n} + T_{m,n-1}) + 2\frac{h\,\Delta x}{k}T_\infty - 2\left(3 + \frac{h\,\Delta x}{k}\right)T_{m,n} = 0 \quad (4.41)$$ **Case 2.** Node at an internal corner with convection
	$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 2\right)T_{m,n} = 0 \quad (4.42)^a$$ **Case 3.** Node at a plane surface with convection
	$$(T_{m,n-1} + T_{m-1,n}) + 2\frac{h\,\Delta x}{k}T_\infty - 2\left(\frac{h\,\Delta x}{k} + 1\right)T_{m,n} = 0 \quad (4.43)$$ **Case 4.** Node at an external corner with convection
	$$(2T_{m-1,n} + T_{m,n+1} + T_{m,n-1}) + \frac{2q''\,\Delta x}{k} - 4T_{m,n} = 0 \quad (4.44)^b$$ **Case 5.** Node at a plane surface with uniform heat flux

[a,b] To obtain the finite-difference equation for an adiabatic surface (or surface of symmetry), simply set h or q'' equal to zero.

EXAMPLE 4.2

Using the energy balance method, derive the finite-difference equation for the m, n nodal point located on a plane, insulated surface of a medium with uniform heat generation.

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(4.29)

(4.41)

(4.42)[a]

(4.43)

(4.44)[b]

zero.

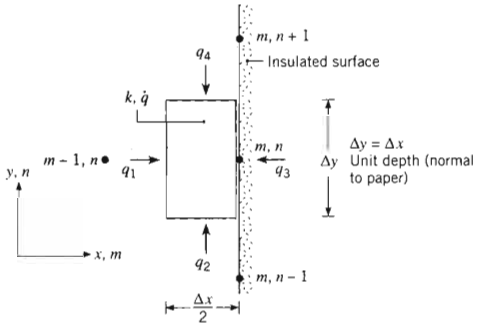
ıation for the m, n
with uniform heat

SOLUTION

Known: Network of nodal points adjoining an insulated surface.

Find: Finite-difference equation for the surface nodal point.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Two-dimensional conduction.
3. Constant properties.
4. Uniform internal heat generation.

Analysis: Applying the energy conservation requirement, Equation 4.30, to the control surface about the region $(\Delta x/2 \cdot \Delta y \cdot 1)$ associated with the m, n node, it follows that, with volumetric heat generation at a rate $\dot{q}$,

$$q_1 + q_2 + q_3 + q_4 + \dot{q}\left(\frac{\Delta x}{2} \cdot \Delta y \cdot 1\right) = 0$$

where

$$q_1 = k\,(\Delta y \cdot 1)\frac{T_{m-1,n} - T_{m,n}}{\Delta x}$$

$$q_2 = k\left(\frac{\Delta x}{2} \cdot 1\right)\frac{T_{m,n-1} - T_{m,n}}{\Delta y}$$

$$q_3 = 0$$

$$q_4 = k\left(\frac{\Delta x}{2} \cdot 1\right)\frac{T_{m,n+1} - T_{m,n}}{\Delta y}$$

Substituting into the energy balance and dividing by $k/2$, it follows that

$$2T_{m-1,n} + T_{m,n-1} + T_{m,n+1} - 4T_{m,n} + \frac{\dot{q}(\Delta x \cdot \Delta y)}{k} = 0$$

◁

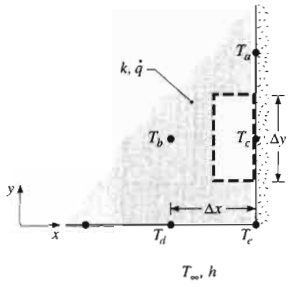
Comments:

1. The same result could be obtained by using the symmetry condition, $T_{m+1,n} = T_{m-1,n}$, with the finite-difference equation (Equation 4.35) for an interior nodal

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point. If $\dot{q} = 0$, the desired result could also be obtained by setting $h = 0$ in Equation 4.42 (Table 4.2).

2. As an application of the foregoing finite-difference equation, consider the following two-dimensional system within which thermal energy is uniformly generated at an unknown rate $\dot{q}$. The thermal conductivity of the solid is known, as are convection conditions at one of the surfaces. In addition, temperatures have been measured at locations corresponding to the nodal points of a finite-difference mesh.



$T_a = 235.9°C$ $T_b = 227.6°C$

$T_c = 230.9°C$ $T_d = 220.1°C$

$T_e = 222.4°C$ $T_\infty = 200.0°C$

$h = 50\ W/m^2 \cdot K$ $k = 1\ W/m \cdot K$

$\Delta x = 10\ mm$ $\Delta y = 10\ mm$

The generation rate can be determined by applying the finite-difference equation to node c.

$$2T_b + T_e + T_a - 4T_c + \frac{\dot{q}(\Delta x \cdot \Delta y)}{k} = 0$$

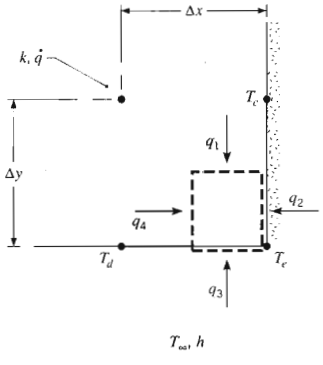
$$(2 \times 227.6 + 222.4 + 235.9 - 4 \times 230.9)°C + \frac{\dot{q}(0.01m)^2}{1\ W/m \cdot K} = 0$$

$$\dot{q} = 1.01 \times 10^5\ W/m^3$$

From the prescribed thermal conditions and knowledge of $\dot{q}$, we can also determine whether the conservation of energy requirement is satisfied for node *e*. Applying an energy balance to a control volume about this node, it follows that

$$q_1 + q_2 + q_3 + q_4 + \dot{q}(\Delta x/2 \cdot \Delta y/2 \cdot 1) = 0$$

$$k(\Delta x/2 \cdot 1)\frac{T_c - T_e}{\Delta y} + 0 + h(\Delta x/2 \cdot 1)(T_\infty - T_e) + k(\Delta y/2 \cdot 1)\frac{T_d - T_e}{\Delta x} + \dot{q}(\Delta x/2 \cdot \Delta y/2 \cdot 1) = 0$$



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If the energy balance is satisfied, the left-hand side of this equation will be identically equal to zero. Substituting values, we obtain

$$1\text{W/m} \cdot \text{K}(0.005\ \text{m}^2)\frac{(230.9 - 222.4)^\circ\text{C}}{0.010\ \text{m}}$$
$$+ 0 + 50\ \text{W/m}^2 \cdot \text{K}(0.005\ \text{m}^2)(200 - 222.4)^\circ\text{C}$$
$$1\text{W/m} \cdot \text{K}(0.005\ \text{m}^2)\frac{(220.1 - 222.4)^\circ\text{C}}{0.010\ \text{m}} + 1.01 \times 10^5\ \text{W/m}^3(0.005)^2\ \text{m}^3 = 0(?)$$
$$4.250\ \text{W} + 0 - 5.600\ \text{W} - 1.150\ \text{W} + 2.525\ \text{W} = 0(?)$$
$$0.025\ \text{W} \approx 0$$

The inability to precisely satisfy the energy balance is attributable to temperature measurement errors, the approximations employed in developing the finite-difference equations, and the use of a relatively coarse mesh.

It is useful to note that heat rates between adjoining nodes may also be formulated in terms of the corresponding thermal resistances. Referring, for example, to Figure 4.6, the rate of heat transfer by conduction from node $(m - 1, n)$ to (m, n) may be expressed as

$$q_{(m-1,n)\to(m,n)} = \frac{T_{m-1,n} - T_{m,n}}{R_{t,\text{cond}}} = \frac{T_{m-1,n} - T_{m,n}}{\Delta x/k\,(\Delta y \cdot 1)}$$

yielding a result that is equivalent to that of Equation 4.36. Similarly, the rate of heat transfer by convection to (m, n) may be expressed as

$$q_{(\infty)\to(m,n)} = \frac{T_\infty - T_{m,n}}{R_{t,\text{conv}}} = \frac{T_\infty - T_{m,n}}{\{h[(\Delta x/2) \cdot 1 + (\Delta y/2) \cdot 1]\}^{-1}}$$

which is equivalent to Equation 4.40.

As an example of the utility of resistance concepts, consider an interface that separates two dissimilar materials and is characterized by a thermal contact resistance $R''_{t,c}$ (Figure 4.7). The rate of heat transfer from node (m, n) to $(m, n - 1)$ may be expressed as

$$q_{(m,n)\to(m,n-1)} = \frac{T_{m,n} - T_{m,n-1}}{R_{\text{tot}}} \tag{4.45}$$

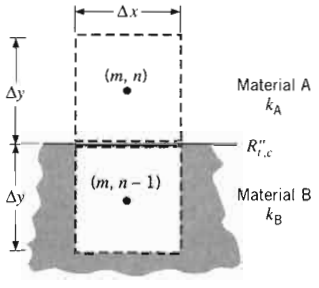


FIGURE 4.7
Conduction between adjoining, dissimilar materials with an interface contact resistance.

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where, for a unit depth,

$$R_{tot} = \frac{\Delta y/2}{k_A(\Delta x \cdot 1)} + \frac{R''_{t,c}}{\Delta x \cdot 1} + \frac{\Delta y/2}{k_B(\Delta x \cdot 1)} \tag{4.46}$$

4.5 *Solving the Finite-Difference Equations*

Once the nodal network has been established and an appropriate finite-difference equation has been written for each node, the temperature distribution may be determined. The problem reduces to one of solving a system of linear, algebraic equations. Numerous methods are available for this purpose and may be classified according to whether they are *direct* or *iterative*. Direct methods involve a fixed and predetermined number of arithmetic operations and are suitable for use when the number of equations (unknown nodal temperatures) is small. However, such methods are associated with larger computer memory and time requirements, and it is sometimes more efficient to use an iterative technique.

In this section we consider matrix inversion and Gauss–Seidel iteration as examples of the direct and iterative methods, respectively. More detailed descriptions of such procedures, as well as related algorithms, may be found in the literature [11, 12].

4.5.1 The Matrix Inversion Method

Consider a system of N finite-difference equations corresponding to N unknown temperatures. Identifying the nodes by a single integer subscript, rather than by the double subscript (m, n), the procedure for performing a matrix inversion begins by expressing the equations as

$$\begin{aligned}
a_{11}T_1 + a_{12}T_2 + a_{13}T_3 + \cdots + a_{1N}T_N &= C_1 \\
a_{21}T_1 + a_{22}T_2 + a_{23}T_3 + \cdots + a_{2N}T_N &= C_2 \\
\vdots \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots \quad\quad \vdots & \\
a_{N1}T_1 + a_{N2}T_2 + a_{N3}T_3 + \cdots + a_{NN}T_N &= C_N
\end{aligned} \tag{4.47}$$

where the quantities $a_{11}, a_{12}, \ldots, C_1, \ldots$ are known coefficients and constants involving quantities such as Δx, k, h, and T_∞. Using matrix notation, these equations may be expressed as

$$[A][T] = [C] \tag{4.48}$$

where

$$A \equiv \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1N} \\ a_{21} & a_{22} & \cdots & a_{2N} \\ \vdots & \vdots & & \vdots \\ a_{N1} & a_{N2} & \cdots & a_{NN} \end{bmatrix}, \quad T \equiv \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_N \end{bmatrix}, \quad C \equiv \begin{bmatrix} C_1 \\ C_2 \\ \vdots \\ C_N \end{bmatrix}$$

The *coefficient matrix* $[A]$ is square $(N \times N)$, and its *elements* are designated by a double subscript notation, for which the first and second subscripts refer to rows and columns, respectively. The matrices $[T]$ and $[C]$ have a single column and are

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known as *column vectors.* Typically, they are termed the *solution* and *right-hand side vectors,* respectively. If the matrix multiplication implied by the left-hand side of Equation 4.48 is performed, Equations 4.47 are obtained.

The solution vector may now be expressed as

$$[T] = [A]^{-1}[C] \tag{4.49}$$

where $[A]^{-1}$ is the inverse of $[A]$ and is defined as

$$[A]^{-1} \equiv \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1N} \\ b_{21} & b_{22} & \cdots & b_{2N} \\ \vdots & \vdots & & \vdots \\ b_{N1} & b_{N2} & \cdots & b_{NN} \end{bmatrix}$$

Evaluating the right-hand side of Equation 4.49, it follows that

$$\begin{aligned} T_1 &= b_{11}C_1 + b_{12}C_2 + \cdots + b_{1N}C_N \\ T_2 &= b_{21}C_1 + b_{22}C_2 + \cdots + b_{2N}C_N \\ &\vdots \\ T_N &= b_{N1}C_1 + b_{N2}C_2 + \cdots + b_{NN}C_N \end{aligned} \tag{4.50}$$

and the problem reduces to one of determining $[A]^{-1}$. That is, if $[A]$ is inverted, its elements $b_{11}, b_{12}, \ldots$ may be determined and the unknown temperatures may be computed from the above expressions.

Matrix inversion may readily be performed on a programmable calculator or a personal computer, depending on the size of the matrix. Hence the method provides a convenient means of solving two-dimensional conduction problems.

4.5.2 Gauss–Seidel Iteration

Application of the Gauss–Seidel method to the system of equations represented by Equation 4.47 is facilitated by the following procedure.

1. To whatever extent possible, the equations should be reordered to provide diagonal elements whose magnitudes are larger than those of other elements in the same row. That is, it is desirable to sequence the equations such that $|a_{11}| > |a_{12}|, |a_{13}|, \ldots, |a_{1N}|$; $|a_{22}| > |a_{21}|, |a_{23}|, \ldots, |a_{2N}|$; and so on.
2. After reordering, each of the N equations should be written in explicit form for the temperature associated with its diagonal element. Each temperature in the solution vector would then be of the form

$$T_i^{(k)} = \frac{C_i}{a_{ii}} - \sum_{j=1}^{i-1} \frac{a_{ij}}{a_{ii}} T_j^{(k)} - \sum_{j=i+1}^{N} \frac{a_{ij}}{a_{ii}} T_j^{(k-1)} \tag{4.51}$$

   where $i = 1, 2, \ldots, N$. The superscript k refers to the level of the iteration.
3. An initial ($k = 0$) value is assumed for each temperature T_i. Subsequent computations may be reduced by selecting values based on rational estimates of the actual solution.
4. Setting $k = 1$ in Equation 4.51, values of $T_i^{(1)}$ are then calculated by substituting assumed (second summation, $k - 1 = 0$) or new (first summation, $k = 1$) values of T_j into the right-hand side. This step is the first ($k = 1$) iteration.

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5. Using Equation 4.51, the iteration procedure is continued by calculating new values of $T_i^{(k)}$ from the $T_j^{(k)}$ values of the current iteration, where $1 \leq j \leq i-1$, and the $T_j^{(k-1)}$ values of the previous iteration, where $i+1 \leq j \leq N$.
6. The iteration is terminated when a prescribed *convergence criterion* is satisfied. The criterion may be expressed as

$$|T_i^{(k)} - T_i^{(k-1)}| \leq \varepsilon \tag{4.52}$$

where ε represents an error in the temperature, which is considered to be acceptable.

If step 1 can be accomplished for each equation, the resulting system is said to be *diagonally dominant,* and the rate of convergence is maximized (the number of required iterations is minimized). However, convergence may also be achieved in many situations for which diagonal dominance cannot be obtained, although the rate of convergence is slowed. The manner in which new values of T_i are computed (steps 4 and 5) should also be noted. Because the T_i for a particular iteration are calculated sequentially, each value can be computed by using the *most recent estimates* of the other T_i. This feature is implicit in Equation 4.51, where the value of each unknown is updated as soon as possible, that is, for $1 \leq j \leq i-1$.

EXAMPLE 4.3

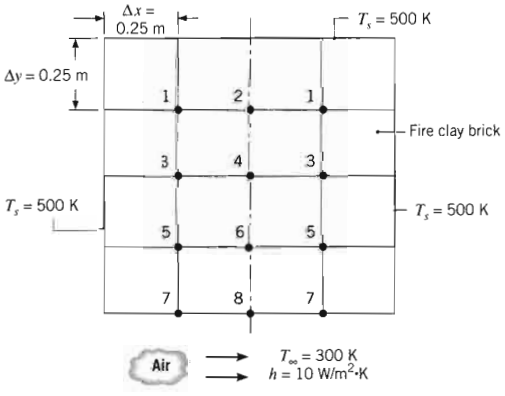
A large industrial furnace is supported on a long column of fireclay brick, which is 1 m by 1 m on a side. During steady-state operation, installation is such that three surfaces of the column are maintained at 500 K while the remaining surface is exposed to an airstream for which $T_\infty = 300$ K and $h = 10$ W/m$^2 \cdot$ K. Using a grid of $\Delta x = \Delta y = 0.25$ m, determine the two-dimensional temperature distribution in the column and the heat rate to the airstream per unit length of column.

SOLUTION

Known: Dimensions and surface conditions of a support column.

Find: Temperature distribution and heat rate per unit length.

Schematic:



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Assumptions:

1. Steady-state conditions.
2. Two-dimensional conduction.
3. Constant properties.
4. No internal heat generation.

Properties: Table A.3, fireclay brick ($T \approx 478$ K): $k = 1$ W/m · K.

Analysis: The prescribed grid consists of 12 nodal points at which the temperature is unknown. However, the number of unknowns is reduced to 8 through symmetry, in which case the temperature of nodal points to the left of the symmetry line must equal the temperature of those to the right.

Nodes 1, 3, and 5 are interior points for which the finite-difference equations may be inferred from Equation 4.29. Hence

$$\text{Node 1:} \quad T_2 + T_3 + 1000 - 4T_1 = 0$$
$$\text{Node 3:} \quad T_1 + T_4 + T_5 + 500 - 4T_3 = 0$$
$$\text{Node 5:} \quad T_3 + T_6 + T_7 + 500 - 4T_5 = 0$$

Equations for points 2, 4, and 6 may be obtained in a like manner or, since they lie on a symmetry adiabat, by using Equation 4.42 with $h = 0$. Hence

$$\text{Node 2:} \quad 2T_1 + T_4 + 500 - 4T_2 = 0$$
$$\text{Node 4:} \quad T_2 + 2T_3 + T_6 - 4T_4 = 0$$
$$\text{Node 6:} \quad T_4 + 2T_5 + T_8 - 4T_6 = 0$$

From Equation 4.42 and the fact that $h\,\Delta x/k = 2.5$, it also follows that

$$\text{Node 7:} \quad 2T_5 + T_8 + 2000 - 9T_7 = 0$$
$$\text{Node 8:} \quad 2T_6 + 2T_7 + 1500 - 9T_8 = 0$$

Having the required finite-difference equations, a matrix inversion solution may be obtained by ordering them from node 1 to node 8 as follows:

$$\begin{aligned}
-4T_1 + T_2 + T_3 + 0 + 0 + 0 + 0 + 0 &= -1000 \\
2T_1 - 4T_2 + 0 + T_4 + 0 + 0 + 0 + 0 &= -500 \\
T_1 + 0 - 4T_3 + T_4 + T_5 + 0 + 0 + 0 &= -500 \\
0 + T_2 + 2T_3 - 4T_4 + 0 + T_6 + 0 + 0 &= 0 \\
0 + 0 + T_3 + 0 - 4T_5 + T_6 + T_7 + 0 &= -500 \\
0 + 0 + 0 + T_4 + 2T_5 - 4T_6 + 0 + T_8 &= 0 \\
0 + 0 + 0 + 0 + 2T_5 + 0 - 9T_7 + T_8 &= -2000 \\
0 + 0 + 0 + 0 + 0 + 2T_6 + 2T_7 - 9T_8 &= -1500
\end{aligned}$$

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In matrix notation, following Equation 4.48, these equations are of the form $[A][T] = [C]$, where

$$[A] = \begin{bmatrix} -4 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 2 & -4 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -4 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 2 & -4 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & -4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & -4 & 0 & 1 \\ 0 & 0 & 0 & 0 & 2 & 0 & -9 & 1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & -9 \end{bmatrix} \quad [C] = \begin{bmatrix} -1000 \\ -500 \\ -500 \\ 0 \\ -500 \\ 0 \\ -2000 \\ -1500 \end{bmatrix}$$

Using a standard matrix inversion routine, it is a simple matter to find the inverse of $[A]$, $[A]^{-1}$, giving

$$[T] = [A]^{-1}[C]$$

where

$$[T] = \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{bmatrix} = \begin{bmatrix} 489.30 \\ 485.15 \\ 472.07 \\ 462.01 \\ 436.95 \\ 418.74 \\ 356.99 \\ 339.05 \end{bmatrix} \text{K} \quad \triangleleft$$

The heat rate from the column to the airstream may be computed from the expression

$$\left(\frac{q}{L}\right) = 2h\left[\left(\frac{\Delta x}{2}\right)(T_s - T_\infty) + \Delta x\,(T_7 - T_\infty) + \left(\frac{\Delta x}{2}\right)(T_8 - T_\infty)\right]$$

where the factor of 2 outside the brackets originates from the symmetry condition. Hence

$$\left(\frac{q}{L}\right) = 2 \times 10\ \text{W/m}^2 \cdot \text{K}[(0.125\ \text{m}\ (200\ \text{K}) + 0.25\ \text{m}\ (56.99\ \text{K}) + 0.125\ \text{m}\ (39.05\ \text{K})] = 883\ \text{W/m} \quad \triangleleft$$

Comments:

1. To ensure that no errors have been made in formulating the finite-difference equations or in effecting their solution, a check should be made to verify that the results satisfy conservation of energy for the nodal network. For steady-state conditions, the requirement dictates that the rate of energy inflow be balanced by the rate of outflow *for a control surface about those nodal regions whose temperatures have been evaluated.*

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ions are of the form

$$[C] = \begin{bmatrix} -1000 \\ -500 \\ -500 \\ 0 \\ -500 \\ 0 \\ -2000 \\ -1500 \end{bmatrix}$$

to find the inverse of

◁

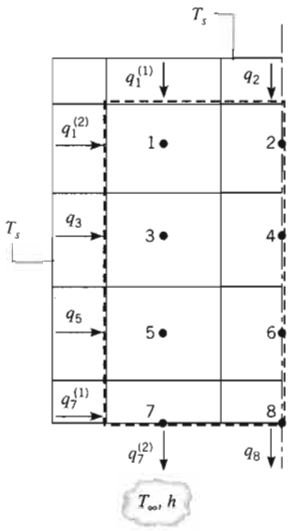
from the expression

$T_8 - T_\infty)\Big]$

symmetry condition.

3 W/m ◁

the finite-difference
made to verify that
etwork. For steady-
ergy inflow be bal-
hose nodal regions



For the one-half symmetrical section shown schematically above, it follows that conduction into the nodal regions must be balanced by convection from the regions. Hence

$$q_1^{(1)} + q_1^{(2)} + q_2 + q_3 + q_5 + q_7^{(1)} = q_7^{(2)} + q_8$$

The cumulative conduction rate is then

$$\frac{q_{\text{cond}}}{L} = k\left[\Delta x \frac{(T_s - T_1)}{\Delta y} + \Delta y \frac{(T_s - T_1)}{\Delta x} + \frac{\Delta x}{2}\frac{(T_s - T_2)}{\Delta y} + \Delta y \frac{(T_s - T_3)}{\Delta x} + \Delta y \frac{(T_s - T_5)}{\Delta x} + \frac{\Delta y}{2}\frac{(T_s - T_7)}{\Delta x}\right]$$
$$= 191.31 \text{ W/m}$$

and the convection rate is

$$\frac{q_{\text{conv}}}{L} = h\left[\Delta x(T_7 - T_\infty) + \frac{\Delta x}{2}(T_8 - T_\infty)\right] = 191.29 \text{ W/m}$$

Agreement between the conduction and convection rates is excellent (within round-off error), confirming that mistakes have not been made in formulating and solving the finite-difference equations. Note that convection transfer from the entire bottom surface (883 W/m) is obtained by adding transfer from the edge node at 500 K (250 W/m) to that from the interior nodes (191.3 W/m) and multiplying by 2 from symmetry.

2. Although the computed temperatures satisfy the finite-difference equations, they do not provide us with the exact temperature field. Remember that the equations are approximations whose accuracy may be improved by reducing the grid size (increasing the number of nodal points).
3. The temperature distribution may also be determined by using the Gauss–Seidel iteration method. Referring to the arrangement of finite-difference equations, it

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is evident that the order is already characterized by diagonal dominance. This behavior is typical of finite-difference solutions to conduction problems. We therefore begin with step 2 and express the equations in explicit form

$$T_1^{(k)} = 0.25T_2^{(k-1)} + 0.25T_3^{(k-1)} + 250$$
$$T_2^{(k)} = 0.50T_1^{(k)} + 0.25T_4^{(k-1)} + 125$$
$$T_3^{(k)} = 0.25T_1^{(k)} + 0.25T_4^{(k-1)} + 0.25T_5^{(k-1)} + 125$$
$$T_4^{(k)} = 0.25T_2^{(k)} + 0.50T_3^{(k)} + 0.25T_6^{(k-1)}$$
$$T_5^{(k)} = 0.25T_3^{(k)} + 0.25T_6^{(k-1)} + 0.25T_7^{(k-1)} + 125$$
$$T_6^{(k)} = 0.25T_4^{(k)} + 0.50T_5^{(k)} + 0.25T_8^{(k-1)}$$
$$T_7^{(k)} = 0.2222T_5^{(k)} + 0.1111T_8^{(k-1)} + 222.22$$
$$T_8^{(k)} = 0.2222T_6^{(k)} + 0.2222T_7^{(k)} + 166.67$$

Having the finite-difference equations in the required form, the iteration procedure may be implemented by using a table that has one column for the iteration (step) number and a column for each of the nodes labeled as T_i. The calculations proceed as follows:

1. For each node, the initial temperature estimate is entered on the row for $k = 0$. Values are selected rationally to reduce the number of required iterations.
2. Using the N finite-difference equations and values of T_i from the first and second rows, the new values of T_i are calculated for the first iteration ($k = 1$). These new values are entered on the second row.
3. This procedure is repeated to calculate $T_i^{(k)}$ from the previous values of $T_i^{(k-1)}$ and the current values of $T_i^{(k)}$, until the temperature difference between iterations meets the prescribed criterion, $\varepsilon \leq 0.2$ K, at every nodal point.

k	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	480	470	440	430	400	390	370	350
1	477.5	471.3	451.9	441.3	428.0	411.8	356.2	337.3
2	480.8	475.7	462.5	453.1	432.6	413.9	355.8	337.7
3	484.6	480.6	467.6	457.4	434.3	415.9	356.2	338.3
4	487.0	482.9	469.7	459.6	435.5	417.2	356.6	338.6
5	488.1	484.0	470.8	460.7	436.1	417.9	356.7	338.8
6	488.7	484.5	471.4	461.3	436.5	418.3	356.9	338.9
7	489.0	484.8	471.7	461.6	436.7	418.5	356.9	339.0
8	489.1	485.0	471.9	461.8	436.8	418.6	356.9	339.0

The results given in row 8 are in excellent agreement with those obtained by matrix inversion, although better agreement could be obtained by reducing the value of ε. However, given the approximate nature of the finite-difference equations, the results still represent approximations to the actual temperatures. The accuracy of the approximation may be improved by using a finer grid (increasing the number of nodes).

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4. Note that the *IHT* solver can be used to solve the foregoing system of eight finite-difference equations for the nodal temperatures. As a skill-building exercise, key the equations into the *IHT Workspace* and press *Solve*. Compare your results with those in the foregoing table. For more complicated applications, consider using the equation-builder in the *Tools* menu, *Finite-Difference Equations* for *1-D* or *2-D*, *Steady-State* or *Transient* conditions. The nodal arrangements include those shown in Table 4.2, as well as those for extended surfaces, and allow for inclusion of volumetric generation and/or uniform heating at a boundary.
5. A second software package accompanying this text, *Finite-Element Heat Transfer (FEHT)*, may also be used to solve one- and two-dimensional forms of the heat equation. As in application of the finite-difference method, the solid is discretized, but in terms of triangular nodal elements on which energy balances are formulated. This example is provided in *FEHT* as a solved model, which may be accessed through the *Toolbar* menu *Examples*. The input screen summarizes key pre-processing and post-processing steps, as well as the results for different mesh sizes. As an exercise, press *Run* to solve for the nodal temperatures, and in the *View* menu, select *Temperature Contours* to represent the temperature distribution by a group of isotherms. For a tutorial on how to develop a model in *FEHT*, refer to the booklet *Software Tools and User's Guides,* which accompanies the IHT and FEHT software.

4.5.3 Some Precautions

As previously noted, it is good practice to verify that a numerical solution has been correctly formulated by performing an energy balance on a control surface about the nodal regions whose temperatures have been evaluated. The temperatures should be substituted into the energy balance equation, and if the balance is not satisfied to a high degree of precision, the finite-difference equations should be checked for errors.

Even when the finite-difference equations have been properly formulated and solved, the results may still represent a coarse approximation to the actual temperature field. This behavior is a consequence of the finite spacings (Δx, Δy) between nodes and of finite-difference approximations, such as $k(\Delta y \cdot 1)(T_{m-1,n} - T_{m,n})/\Delta x$, to Fourier's law of conduction, $-k(dy \cdot 1)dT/dx$. We have previously indicated that the finite-difference approximations become more accurate as the nodal network is refined (Δx and Δy are reduced). Hence, if accurate results are desired, grid studies should be performed, whereby results obtained for a fine grid are compared with those obtained for a coarse grid. One could, for example, reduce Δx and Δy by a factor of 2, thereby increasing the number of nodes and finite-difference equations by a factor of 4. If the agreement is unsatisfactory, further grid refinements could be made until the computed temperatures no longer depend significantly on the choice of Δx and Δy. Such *grid-independent* results would provide an accurate solution to the physical problem.

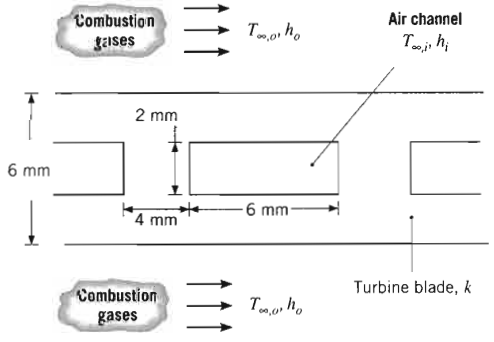
Another option for validating a numerical solution involves comparing results with those obtained from an exact solution. For example, a finite-difference solution

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of the physical problem described in Figure 4.2 could be compared with the exact solution given by Equation 4.19. However, this option is limited by the fact that we seldom seek numerical solutions to problems for which there exist exact solutions. Nevertheless, if we seek a numerical solution to a complex problem for which there is no exact solution, it is often useful to test our finite-difference procedures by applying them to a simpler version of the problem.

EXAMPLE 4.4

A major objective in advancing gas turbine engine technologies is to increase the temperature limit associated with operation of the gas turbine blades. This limit determines the permissible turbine gas inlet temperature, which, in turn, strongly influences overall system performance. In addition to fabricating turbine blades from special, high-temperature, high-strength superalloys, it is common to use internal cooling by machining flow channels within the blades and routing air through the channels. We wish to assess the effect of such a scheme by approximating the blade as a rectangular solid in which rectangular channels are machined. The blade, which has a thermal conductivity of $k = 25$ W/m · K, is 6 mm thick, and each channel has a 2 mm × 6 mm rectangular cross section, with a 4-mm spacing between adjoining channels.



Under operating conditions for which $h_o = 1000$ W/m^2 · K, $T_{\infty,o} = 1700$ K, $h_i = 200$ W/m^2 · K, and $T_{\infty,i} = 400$ K, determine the temperature field in the turbine blade and the rate of heat transfer per unit length to the channel. At what location is the temperature a maximum?

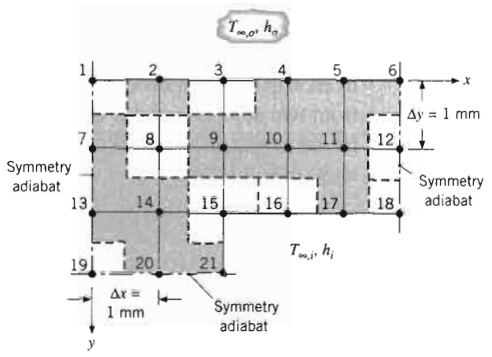
SOLUTION

Known: Dimensions and operating conditions for a gas turbine blade with embedded channels.

Find: Temperature field in the blade, including a location of maximum temperature. Rate of heat transfer per unit length to the channel.

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Schematic:



Assumptions:

1. Steady-state, two-dimensional conduction.
2. Constant properties.

Analysis: Adopting a grid space of $\Delta x = \Delta y = 1$ mm and identifying the three lines of symmetry, the foregoing nodal network is constructed. The corresponding finite-difference equations may be obtained by applying the energy balance method to nodes 1, 6, 18, 19, and 21 and by using the results of Table 4.2 for the remaining nodes.

Heat transfer to node 1 occurs by conduction from nodes 2 and 7, as well as by convection from the outer fluid. Since there is no heat transfer from the region beyond the symmetry adiabat, application of an energy balance to the one-quarter section associated with node 1 yields a finite-difference equation of the form

$$\text{Node 1:} \quad T_2 + T_7 - \left(2 + \frac{h_o \Delta x}{k}\right) T_1 = -\frac{h_o \Delta x}{k} T_{\infty,o}$$

A similar result may be obtained for nodal region 6, which is characterized by equivalent surface conditions (2 conduction, 1 convection, 1 adiabatic). Nodes 2 to 5 correspond to case 3 of Table 4.2, and choosing node 3 as an example, it follows that

$$\text{Node 3:} \quad T_2 + T_4 + 2T_9 - 2\left(\frac{h_o \Delta x}{k} + 2\right) T_3 = -\frac{2h_o \Delta x}{k} T_{\infty,o}$$

Nodes 7, 12, 13, and 20 correspond to case 5 of Table 4.2, with $q'' = 0$, and choosing node 12 as an example, it follows that

$$\text{Node 12:} \quad T_6 + 2T_{11} + T_{18} - 4T_{12} = 0$$

Nodes 8 to 11 and 14 are interior nodes (case 1), in which case the finite-difference equation for node 8 is

$$\text{Node 8:} \quad T_2 + T_7 + T_9 + T_{14} - 4T_8 = 0$$

Node 15 is an internal corner (case 2) for which

$$\text{Node 15:} \quad 2T_9 + 2T_{14} + T_{16} + T_{21} - 2\left(3 + \frac{h_i \Delta x}{k}\right) T_{15} = -2\frac{h_i \Delta x}{k} T_{\infty,i}$$

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while nodes 16 and 17 are situated on a plane surface with convection (case 3):

$$\text{Node 16:} \quad 2T_{10} + T_{15} + T_{17} - 2\left(\frac{h_i \Delta x}{k} + 2\right)T_{16} = -\frac{2h_i \Delta x}{k} T_{\infty,i}$$

In each case heat transfer to nodal regions 18 and 21 is characterized by conduction from two adjoining nodes and convection from the internal flow, with no heat transfer occurring from an adjoining adiabat. Performing an energy balance for nodal region 18, it follows that

$$\text{Node 18:} \quad T_{12} + T_{17} - \left(2 + \frac{h_i \Delta x}{k}\right) T_{18} = -\frac{h_i \Delta x}{k} T_{\infty,i}$$

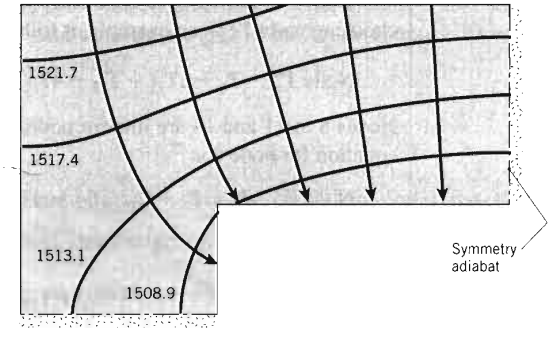
The last special case corresponds to nodal region 19, which has two adiabatic surfaces and experiences heat transfer by conduction across the other two surfaces.

$$\text{Node 19:} \quad T_{13} + T_{20} - 2T_{19} = 0$$

The 21 finite-difference equations may be solved for the unknown temperatures, and for the prescribed conditions, the following results are obtained:

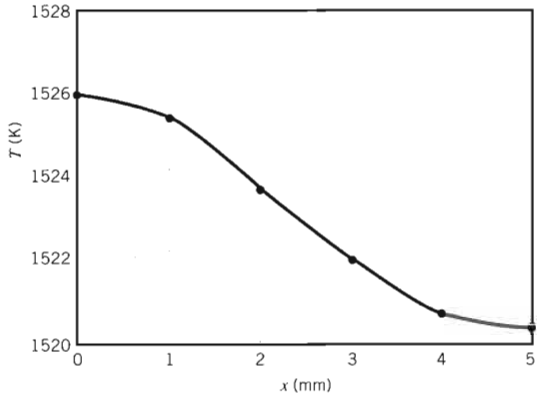
T_1	T_2	T_3	T_4	T_5	T_6
1526.0K	1525.3 K	1523.6 K	1521.9 K	1520.8 K	1520.5 K
T_7	T_8	T_9	T_{10}	T_{11}	T_{12}
1519.7 K	1518.8K	1516.5 K	1514.5 K	1513.3 K	1512.9 K
T_{13}	T_{14}	T_{15}	T_{16}	T_{17}	T_{18}
1515.1 K	1513.7 K	1509.2 K	1506.4 K	1505.0 K	1504.5 K
T_{19}	T_{20}	T_{21}			
1513.4 K	1511.7 K	1506.0 K			

The temperature field may also be represented in the form of isotherms, and four such lines of constant temperature are shown schematically. Also shown are heat flux lines that have been carefully drawn so that they are everywhere perpendicular to the isotherms and coincident with the symmetry adiabat. The surfaces that are exposed to the combustion gases and air are not isothermal, and therefore the heat flow lines are not perpendicular to these boundaries.



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As expected, the maximum temperature exists at the location furthest removed from the coolant, which corresponds to node 1. Temperatures along the surface of the turbine blade exposed to the combustion gases are of particular interest, and using an interpolation scheme with the finite-difference predictions, the following distribution is obtained:



The rate of heat transfer per unit length of channel may be expressed as

$$q' = 4h_i[(\Delta y/2)(T_{21} - T_{\infty,i}) + (\Delta y/2 + \Delta x/2)(T_{15} - T_{\infty,i}) + (\Delta x)(T_{16} - T_{\infty,i}) + \Delta x(T_{17} - T_{\infty,i}) + (\Delta x/2)(T_{18} - T_{\infty,i})]$$

or, alternatively, as

$$q' = 4h_o[(\Delta x/2)(T_{\infty,o} - T_1) + (\Delta x)(T_{\infty,o} - T_2) + (\Delta x)(T_{\infty,o} - T_3) + (\Delta x)(T_{\infty,o} - T_4) + (\Delta x)(T_{\infty,o} - T_5) + (\Delta x/2)(T_{\infty,o} - T_6)]$$

where the factor of 4 originates from the symmetry conditions. In both cases, we obtain

$$q' = 3540.6 \text{ W/m} \quad \triangleleft$$

Comments:

1. The accuracy of the finite-difference solution may be improved by refining the grid. If, for example, we halve the grid spacing ($\Delta x = \Delta y = 0.5$ mm), thereby increasing the number of unknown nodal temperatures to 65, we obtain the following results for selected temperatures and the heat rate:

$$T_1 = 1525.9 \text{ K}, \quad T_6 = 1520.5 \text{ K}, \quad T_{15} = 1509.2 \text{ K},$$
$$T_{18} = 1504.5 \text{ K}, \quad T_{19} = 1513.5 \text{ K}, \quad T_{21} = 1505.7 \text{ K},$$
$$q' = 3539.9 \text{ W/m}$$

   Agreement between the two sets of results is excellent. Of course, use of the finer mesh increases set up and computation time, and in many cases results obtained from a coarse grid are satisfactory. Selection of the appropriate grid is a judgment that must be made by the engineer.

2. In the gas turbine industry, there is great interest in adopting measures that reduce blade temperatures. Such measures could include use of a different alloy

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of larger thermal conductivity and/or increasing coolant flow through the channel, thereby increasing h_i. Using the finite-difference solution with $\Delta x = \Delta y = 1$ mm, the following results are obtained for parametric variations of k and h_i:

k (W/m · K)	h_i (W/m² · K)	T_1 (K)	q' (W/m)
25	200	1526.0	3540.6
50	200	1523.4	3563.3
25	1000	1154.5	11,095.5
50	1000	1138.9	11,320.7

Why do increases in k and h_i reduce temperature in the blade? Why is the effect of the change in h_i more significant than that of k?

3. Note that, because the exterior surface of the blade is at an extremely high temperature, radiation losses to its surroundings may be significant. In the finite-difference analysis, such effects could be considered by linearizing the radiation rate equation (see Equations 1.8 and 1.9) and treating radiation in the same manner as convection. However, because the radiation coefficient h_r depends on the surface temperature, an iterative finite-difference solution would be necessary to ensure that the resulting surface temperatures correspond to the temperatures at which h_r is evaluated at each nodal point.
4. The *IHT* solver can be used to solve the system of 19 finite-difference equations for the nodal temperatures. The equations may be formulated by accessing the equation-builder in the *Finite-Difference Equations, 2-D, Steady-State* section of the *Tools* menu. The menu provides schematics of the control volumes (interior nodes, corner nodes, etc.), and boxes are provided to enter identifiers or subscripts for the temperatures and other parameters.
5. This example is provided as a solved model in *FEHT* and may be accessed through *Examples* on the *Toolbar*. The input screen summarizes pre- and post-processing steps, as well as the results for different nodal spacings of 1 and 0.125 mm. As an exercise, press *Run* to compute the nodal temperatures, and in the *View* menu, select *Temperature Contours* to represent the temperature field in the form of isotherms. For a tutorial on how to develop a model in *FEHT*, refer to the booklet, *Software Tools and User's Guides,* which accompanies the IHT and FEHT software.

4.6 Summary

The primary objective of this chapter was to develop an appreciation for the nature of a two-dimensional conduction problem and the methods that are available for its solution. When confronted with a two-dimensional problem, one should first determine whether an exact solution is known. This may be done by examining some of the excellent references in which exact solutions to the heat equation are obtained [1–5]. One may also want to determine whether the shape factor or dimensionless conduction heat rate is known for the system of interest [6–10]. However, often,

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conditions are such that the use of a shape factor, dimensionless conduction heat rate, or an exact solution is not possible, and it is necessary to use a finite-difference or finite-element solution. You should appreciate the inherent nature of the *discretization process* and know how to formulate and solve the finite-difference equations for the discrete points of a nodal network. You may test your understanding of related concepts by addressing the following questions.

- What is an *isotherm*? What is a *heat flow line*? How are the two lines related geometrically?
- What is an *adiabat*? How is it related to a line of symmetry? How is it intersected by an isotherm?
- What parameters characterize the effect of geometry on the relationship between the heat rate and the overall temperature difference for steady conduction in a two-dimensional system? How are these parameters related to the conduction resistance?
- What is represented by the temperature of a *nodal point*, and how does the accuracy of a nodal temperature depend on prescription of the *nodal network*?

References

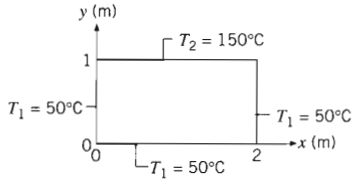
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Problems

Exact Solutions

4.1 In the method of separation of variables (Section 4.2) for two-dimensional, steady-state conduction, the separation constant λ^2 in Equations 4.6 and 4.7 must be a positive constant. Show that a negative or zero value of λ^2 will result in solutions that cannot satisfy the prescribed boundary conditions.

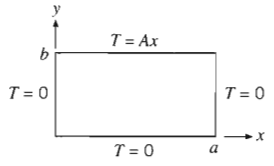
4.2 A two-dimensional rectangular plate is subjected to prescribed boundary conditions. Using the results of the exact solution for the heat equation presented in Section 4.2, calculate the temperature at the midpoint (1, 0.5) by considering the first five nonzero terms of the infinite series that must be evaluated. Assess the error resulting from using only the first three terms of the infinite series. Plot the temperature distributions $T(x, 0.5)$ and $T(1.0, y)$.



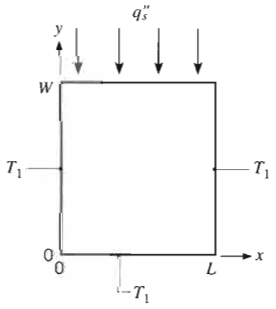
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4.3 Consider the two-dimensional rectangular plate of Problem 4.2 having a thermal conductivity of 50 W/m · K. Beginning with the exact solution for the temperature distribution, derive an expression for the heat transfer rate per unit thickness from the plate along the lower surface ($0 \le x \le 2$, $y = 0$). Evaluate the heat rate considering the first five nonzero terms of the infinite series.

4.4 A two-dimensional rectangular plate is subjected to the boundary conditions shown. Derive an expression for the steady-state temperature distribution $T(x, y)$.



4.5 A two-dimensional rectangular plate is subjected to prescribed temperature boundary conditions on three sides and a uniform heat flux *into* the plate at the top surface. Using the general approach of Section 4.2, derive an expression for the temperature distribution in the plate.



Shape Factors and Dimensionless Conduction Heat Rates

4.6 Using the thermal resistance relations developed in Chapter 3, determine shape factor expressions for the following geometries:

(a) Plane wall, cylindrical shell, and spherical shell.

(b) Isothermal sphere of diameter D buried in an infinite medium.

4.7 Consider Problem 4.5 for the case where the plate is of square cross section, $W = L$.

(a) Derive an expression for the shape factor, S_{max}, associated with the *maximum* top surface temperature, such that $q = S_{max}\, k\, (T_{2,max} - T_1)$ where $T_{2,max}$ is the maximum temperature along $y = W$.

(b) Derive an expression for the shape factor, S_{avg}, associated with the *average* top surface temperature, $q = S_{avg} k(\overline{T}_2 - T_1)$ where $\overline{T}_2$ is the average temperature along $y = W$.

(c) Evaluate the shape factors that can be used to determine the maximum and average temperatures along $y = W$. Evaluate the maximum and average temperatures for $T_1 = 0°C$, $L = W = 10$ mm, $k = 20$ W/m · K, and $q_s'' = 1000$ W/m^2.

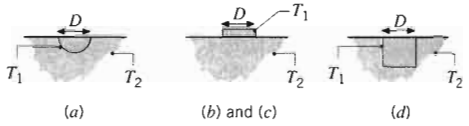
4.8 Based on the dimensionless conduction heat rates for cases 12–15 in Table 4.1*b*, find shape factors for the following objects having temperature T_1, located at the surface of a semi-infinite medium having temperature T_2. The surface of the semi-infinite medium is adiabatic.

(a) A buried hemisphere, flush with the surface.

(b) A disk on the surface. Compare your result to Table 4.1*a*, case 10.

(c) A square on the surface.

(d) A buried cube, flush with the surface.



(*a*) (*b*) and (*c*) (*d*)

4.9 Radioactive wastes are temporarily stored in a spherical container, the center of which is buried a distance of 10 m below the earth's surface. The outside diameter of the container is 2 m, and 500 W of heat are released as a result of the radioactive decay process. If the soil surface temperature is 20°C, what is the outside surface temperature of the container under steady-state conditions? On a sketch of the soil–container system drawn to scale, show representative isotherms and heat flow lines in the soil.

4.10 A pipeline, used for the transport of crude oil, is buried in the earth such that its centerline is a distance of 1.5 m below the surface. The pipe has an outer diameter of 0.5 m and is insulated with a layer of cellular glass 100 mm thick. What is the heat loss per unit length of pipe under conditions for which heated oil at 120°C flows through the pipe and the surface of the earth is at a temperature of 0°C?

4.11 A long power transmission cable is buried at a depth (ground to cable centerline distance) of 2 m. The cable is encased in a thin-walled pipe of 0.1-m diameter, and to render the cable *superconducting* (essentially zero power dissipation), the space between the cable and pipe is filled with liquid nitrogen at 77 K. If the pipe is covered with a superinsulator ($k_i = 0.005$ W/m · K) of 0.05-m thickness and the surface of the earth ($k_g = 1.2$ W/m · K) is at 300 K, what is the cooling load in W/m that must be maintained by a cryogenic refrigerator per unit pipe length?

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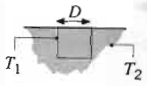
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4.12 An electrical heater 100 mm long and 5 mm in diameter is inserted into a hole drilled normal to the surface of a large block of material having a thermal conductivity of 5 W/m · K. Estimate the temperature reached by the heater when dissipating 50 W with the surface of the block at a temperature of 25°C.

4.13 Two parallel pipelines spaced 0.5 m apart are buried in soil having a thermal conductivity of 0.5 W/m · K. The pipes have outer diameters of 100 and 75 mm with surface temperatures of 175°C and 5°C, respectively. Estimate the heat transfer rate per unit length between the two pipelines.

4.14 A tube of diameter 50 mm having a surface temperature of 85°C is embedded in the center plane of a concrete slab 0.1 m thick with upper and lower surfaces at 20°C. Using the appropriate tabulated relation for this configuration, find the shape factor. Determine the heat transfer rate per unit lengh of the tube.

4.15 Pressurized steam at 450 K flows through a long, thin-walled pipe of 0.5-m diameter. The pipe is enclosed in a concrete casing that is of square cross section and 1.5 m on a side. The axis of the pipe is centered in the casing, and the outer surfaces of the casing are maintained at 300 K. What is the heat loss per unit length of pipe?

4.16 Hot water at 85°C flows through a thin-walled copper tube of 30 mm diameter. The tube is enclosed by an eccentric cylindrical shell that is maintained at 35°C and has a diameter of 120 mm. The eccentricity, defined as the separation between the centers of the tube and shell, is 20 mm. The space between the tube and shell is filled with an insulating material having a thermal conductivity of 0.05 W/m · K. Calculate the heat loss per unit length of the tube and compare the result with the heat loss for a concentric arrangement.

4.17 A furnace of cubical shape, with external dimensions of 0.35 m, is constructed from a refractory brick (fireclay). If the wall thickness is 50 mm, the inner surface temperature is 600°C, and the outer surface temperature is 75°C, calculate the heat loss from the furnace.

4.18 The temperature distribution in laser-irradiated materials is determined by the power, size, and shape of the laser beam, along with the properties of the material being irradiated. The beam shape is typically Gaussian, and the local beam irradiation flux (often referred to as the laser *fluence*) is

$$q''(x, y) = q''(x = y = 0)\exp(-x/r_b)^2 \exp(-y/r_b)^2$$

The x and y coordinates determine the location of interest on the surface of the irradiated material. Consider the case where the center of the beam is located at $x = y = r = 0$. The beam is characterized by a radius, r_b, defined as the radial location where the local fluence is $q''(r_b) = q''(r = 0)/e \approx 0.368q''(r = 0)$.

A shape factor for Gaussian heating is $S = 2\pi^{1/2}r_b$, where S is defined in terms of $T_{1,\max} - T_2$ [Nissin, Y. I., A. Lietoila, R. G. Gold, and J. F. Gibbons, *J.Appl. Phys.*, **51**, 274, 1980]. Calculate the maximum steady-state surface temperature associated with irradiation of a material of thermal conductivity $k = 27$ W/m · K and absorptivity $\alpha = 0.45$ by a Gaussian beam with $r_b = 0.1$ mm and power $P = 1$ W. Compare your result with the maximum temperature that would occur if the irradiation was from a circular beam of the same diameter and power, but characterized by a uniform fluence (a *flat* beam). Also calculate the average temperature of the irradiated surface for the uniform fluence case. The temperature far from the irradiated spot is $T_2 = 25$°C.

4.19 Laser beams are used to thermally process materials in a wide range of applications. Often, the beam is scanned along the surface of the material in a desired pattern. Consider the laser heating process of Problem 4.18, except now the laser beam scans the material at a scanning velocity of U. A dimensionless maximum surface temperature can be well correlated by an expression of the form [Nissin, Y. I., A. Lietoila, R. G. Gold, and J. F. Gibbons, *J. Appl. Phys.*, **51**, 274, 1980]

$$\frac{T_{1,\max,U=0} - T_2}{T_{1,\max,U\neq 0} - T_2} = 1 + 0.301Pe - 0.0108Pe^2$$

for the range $0 < Pe < 10$ where Pe is a dimensionless velocity known as the Peclet number. For this problem, $Pe = Ur_b/\sqrt{2}\alpha$ where α is the thermal diffusivity of the material. The maximum material temperature does not occur directly below the laser beam, but a lag distance, δ, behind the center of the moving beam. The dimensionless lag distance can be correlated to Pe by [Sheng, I. C., and Y. Chen, *J. Thermal Stresses*, **14**, 129, 1991]

$$\frac{\delta U}{\alpha} = 0.944Pe^{1.55}$$

(a) For the laser beam size and shape, and material of Problem 4.18, determine the laser power required to achieve $T_{1,\max} = 200$°C for $U = 2$ m/s. The density and specific heat of the material are $\rho = 2000$ kg/m^3 and $c = 800$ J/kg · K, respectively.

(b) Determine the lag distance, δ, associated with $U = 2$ m/s.

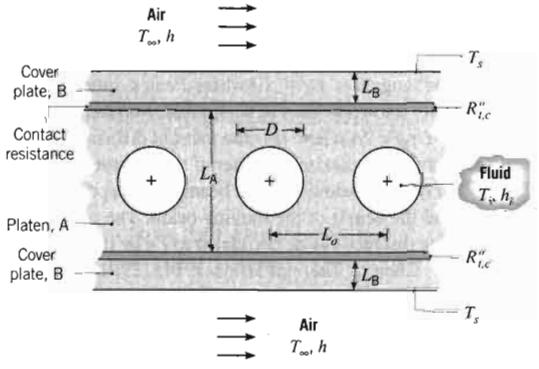
(c) Plot the required laser power to achieve $T_{\max,1} = 200$°C for $0 \leq U \leq 2$ m/s.

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Shape Factors with Thermal Circuits

4.20 A cubical glass melting furnace has exterior dimensions of width $W = 5$ m on a side and is constructed from refractory brick of thickness $L = 0.35$ m and thermal conductivity $k = 1.4$ W/m · K. The sides and top of the furnace are exposed to ambient air at 25°C, with free convection characterized by an average coefficient of $h = 5$ W/m² · K. The bottom of the furnace rests on a framed platform for which much of the surface is exposed to the ambient air, and a convection coefficient of $h = 5$ W/m² · K may be assumed as a first approximation. Under operating conditions for which combustion gases maintain the inner surfaces of the furnace at 1100°C, what is the heat loss from the furnace?

4.21 A hot fluid passes through circular channels of a cast iron platen (A) of thickness $L_A = 30$ mm which is in poor contact with the cover plates (B) of thickness $L_B = 7.5$ mm. The channels are of diameter $D = 15$ mm with a centerline spacing of $L_o = 60$ mm. The thermal conductivities of the materials are $k_A = 20$ W/m · K and $k_B = 75$ W/m · K, while the contact resistance between the two materials is $R''_{t,c} = 2.0 \times 10^{-4}$ m² · K/W. The hot fluid is at $T_i = 150$°C, and the convection coefficient is 1000 W/m² · K. The cover plate is exposed to ambient air at $T_\infty = 25$°C with a convection coefficient of 200 W/m² · K. The shape factor between one channel and the platen top and bottom surfaces is 4.25.

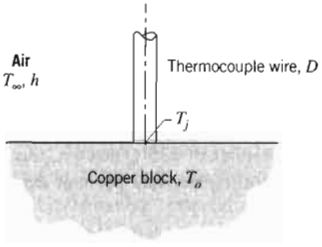


(a) Determine the heat rate from a single channel per unit length of the platen normal to the page, q'_i.

(b) Determine the outer surface temperature of the cover plate, T_s.

(c) Comment on the effects that changing the centerline spacing will have on q'_i and T_s. How would insulating the lower surface affect q'_i and T_s?

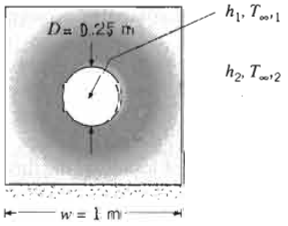
4.22 A long constantan wire of 1-mm diameter is butt welded to the surface of a large copper block, forming a thermocouple junction. The wire behaves as a fin, permitting heat to flow from the surface, thereby depressing the sensing junction temperature T_j below that of the block T_o.



(a) If the wire is in air at 25°C with a convection coefficient of 10 W/m² · K, estimate the measurement error $(T_j - T_o)$ for the thermocouple when the block is at 125°C.

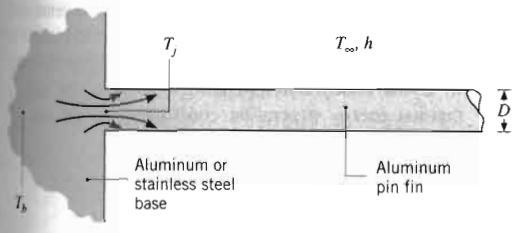
(b) For convection coefficients of 5, 10, and 25 W/m² · K, plot the measurement error as a function of the thermal conductivity of the block material over the range 15 to 400 W/m · K. Under what circumstances is it advantageous to use smaller diameter wire?

4.23 A hole of diameter $D = 0.25$ m is drilled through the center of a solid block of square cross section with $w = 1$ m on a side. The hole is drilled along the length, $l = 2$ m, of the block, which has a thermal conductivity of $k = 150$ W/m · K. The outer surfaces are exposed to ambient air, with $T_{\infty,2} = 25$°C and $h_2 = 4$ W/m² · K, while hot oil flowing through the hole is characterized by $T_{\infty,1} = 300$°C and $h_1 = 50$ W/m² · K. Determine the corresponding heat rate and surface temperatures.



4.24 In Chapter 3 we assumed that, whenever fins are attached to a base material, the base temperature is unchanged. What in fact happens is that, if the temperature of the base material exceeds the fluid temperature, attachment of a fin depresses the junction temperature T_j below the original temperature of the base, and heat flow from the base material to the fin is two-dimensional.

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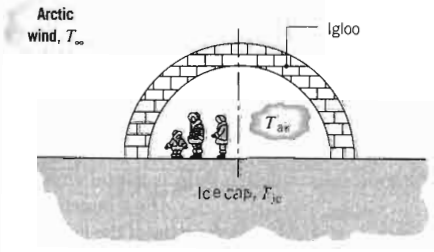
Consider conditions for which a long aluminum pin fin of diameter $D = 5$ mm is attached to a base material whose temperature far from the junction is maintained at $T_b = 100°C$. Fin convection conditions correspond to $h = 50$ W/m$^2 \cdot$ K and $T_\infty = 25°C$.

(a) What are the fin heat rate and junction temperature when the base material is (i) aluminum ($k = 240$ W/m $\cdot$ K) and (ii) stainless steel ($k = 15$ W/m $\cdot$ K)?

(b) Repeat the foregoing calculations if a thermal contact resistance of $R''_{t,j} = 3 \times 10^{-5}$ m$^2 \cdot$ K/W is associated with the method of joining the pin fin to the base material.

(c) Considering the thermal contact resistance, plot the heat rate as a function of the convection coefficient over the range $10 \leq h \leq 100$ W/m$^2 \cdot$ K for each of the two materials.

4.25 An igloo is built in the shape of a hemisphere, with an inner radius of 1.8 m and walls of compacted snow that are 0.5 m thick. On the inside of the igloo the surface heat transfer coefficient is 6 W/m$^2 \cdot$ K; on the outside, under normal wind conditions, it is 15 W/m$^2 \cdot$ K. The thermal conductivity of compacted snow is 0.15 W/m $\cdot$ K. The temperature of the ice cap on which the igloo sits is $-20°C$ and has the same thermal conductivity as the compacted snow.

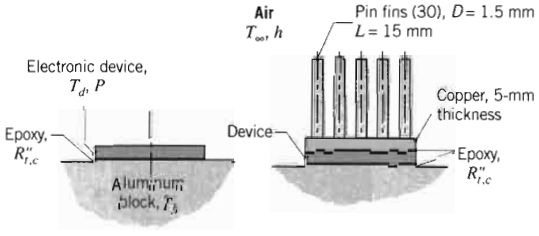


(a) Assuming that the occupants' body heat provides a continuous source of 320 W within the igloo, calculate the inside air temperature when the outside air temperature is $T_\infty = -40°C$. Be sure to consider heat losses through the floor of the igloo.

(b) Using the thermal circuit of part (a), perform a parameter sensitivity analysis to determine which variables have a significant effect on the inside air temperature. For instance, for very high wind conditions, the outside convection coefficient could double or even triple. Does it make sense to construct the igloo with walls half or twice as thick?

4.26 Consider the thin integrated circuit (chip) of Problem 3.136. Instead of attaching the heat sink to the chip surface, an engineer suggests that sufficient cooling might be achieved by mounting the top of the chip onto a large copper ($k = 400$ W/m $\cdot$ K) surface that is located nearby. The metallurgical joint between the chip and the substrate provides a contact resistance of $R''_{t,c} = 5 \times 10^{-6}$ m$^2 \cdot$ K/W, and the maximum allowable chip temperature is 85°C. If the large substrate temperature is $T_2 = 25°C$ at locations far from the chip, what is the maximum allowable chip power dissipation q_c?

4.27 An electronic device, in the form of a disk 20 mm in diameter, dissipates 100 W when mounted flush on a large aluminum alloy (2024) block whose temperature is maintained at 27°C. The mounting arrangement is such that a contact resistance of $R''_{t,c} = 5 \times 10^{-5}$ m$^2 \cdot$ K/W exists at the interface between the device and the block.



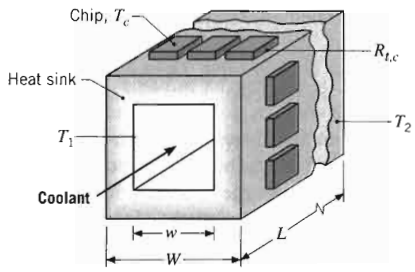
(a) Calculate the temperature the device will reach, assuming that all the power generated by the device must be transferred by conduction to the block.

(b) In order to operate the device at a higher power level, a circuit designer proposes to attach a finned heat sink to the top of the device. The pin fins and base material are fabricated from copper ($k = 400$ W/m $\cdot$ K) and are exposed to an airstream at 27°C for which the convection coefficient is 1000 W/m$^2 \cdot$ K. For the device temperature computed in part (a), what is the permissible operating power?

4.28 An aluminum heat sink ($k = 240$ W/m$\cdot$K) used to cool an array of electronic chips consists of a square channel of inner width $w = 25$ mm, through which liquid flow may be assumed to maintain a uniform surface temperature of

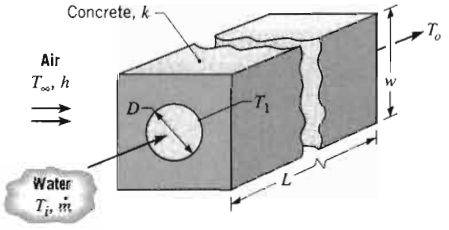
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$T_1 = 20°C$. The outer width and length of the channel are $W = 40$ mm and $L = 160$ mm, respectively.



If $N = 120$ chips attached to the outer surfaces of the heat sink maintain an approximately uniform surface temperature of $T_2 = 50°C$ and all of the heat dissipated by the chips is assumed to be transferred to the coolant, what is the heat dissipation per chip? If the contact resistance between each chip and the heat sink is $R_{t,c} = 0.2$ K/W, what is the chip temperature?

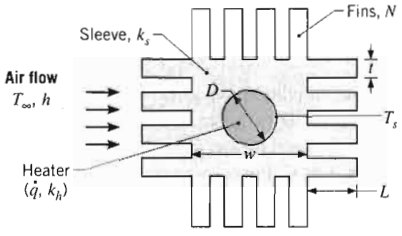
4.29 Hot water is transported from a cogeneration power station to commercial and industrial users through steel pipes of diameter $D = 150$ mm, with each pipe centered in concrete ($k = 1.4$ W/m · K) of square cross section ($w = 300$ mm). The outer surfaces of the concrete are exposed to ambient air for which $T_\infty = 0°C$ and $h = 25$ W/m^2 · K.



(a) If the inlet temperature of water flowing through the pipe is $T_i = 90°C$, what is the heat loss per unit length of pipe in proximity to the inlet? The temperature of the pipe T_1 may be assumed to be that of the inlet water.

(b) If the difference between the inlet and outlet temperatures of water flowing through a 100-m-long pipe is not to exceed 5°C, estimate the minimum allowable flow rate $\dot{m}$. A value of $c = 4207$ J/kg · K may be used for the specific heat of the water.

4.30 The elemental unit of an air heater consists of a long circular rod of diameter D, which is encapsulated by a finned sleeve and in which thermal energy is generated by Ohmic heating. The N fins of thickness t and length L are integrally fabricated with the square sleeve of width w. Under steady-state operating conditions, the rate of thermal energy generation corresponds to the rate of heat transfer to air flow over the sleeve.



(a) Under conditions for which a uniform surface temperature T_s is maintained around the circumference of the heater and the temperature T_∞ and convection coefficient h of the air flow are known, obtain an expression for the rate of heat transfer per unit length to the air. Evaluate the heat rate for $T_s = 300°C$, $D = 20$ mm, an aluminum sleeve ($k_s = 240$ W/m · K), $w = 40$ mm, $N = 16$, $t = 4$ mm, $L = 20$ mm, $T_\infty = 50°C$, and $h = 500$ W/m^2 · K.

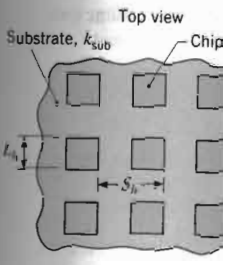
(b) For the foregoing heat rate and a copper heater of thermal conductivity $k_h = 400$ W/m·K, what is the required volumetric heat generation within the heater and its corresponding centerline temperature?

(c) With all other quantities unchanged, explore the effect of variations in the fin parameters (N, L, t) on the heat rate, subject to the constraint that the fin thickness and the spacing between fins cannot be less than 2 mm.

4.31 For a small heat source attached to a large substrate, the *spreading* resistance associated with multidimensional conduction in the substrate may be approximated by the expression (Yovanovich, M. M. and V. W. Antonetti, *Adv. Thermal Modeling Elec. Comp. and Systems*, Vol. I, A. Bar-Cohen and A. D. Kraus, Eds., Hemisphere, NY, 79–128, 1988)

$$R_{t(sp)} = \frac{1 - 1.410A_r + 0.344A_r^3 + 0.043A_r^5 + 0.034A_r^7}{4k_{sub}A_{s,h}^{1/2}}$$

where $A_r = A_{s,h}/A_{s,sub}$ is the ratio of the heat source area to the substrate area. Consider application of the expression to an in-line array of square chips of width $L_h = 5$ mm on a side and pitch $S_h = 10$ mm. The interface between the chips and a large substrate of thermal conductivity $k_{sub} = 80$ W/m·K is characterized by a thermal contact resistance of $R''_{t,c} = 0.5 \times 10^{-4}$ m^2 · K/W.



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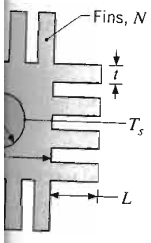
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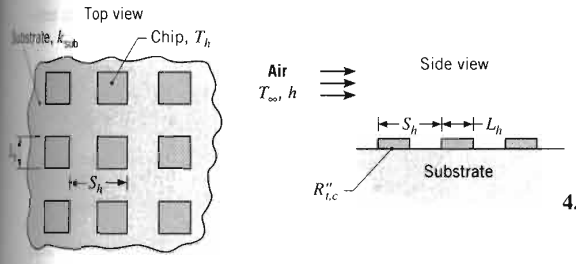


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If a convection heat transfer coefficient of h = 100 W/m$^2 \cdot$K is associated with air flow (T_∞ = 15°C) over the chips and substrate, what is the maximum allowable chip power dissipation if the chip temperature is not to exceed T_h = 85°C?

Finite-Difference Equations: Derivations

4.32 Consider nodal configuration 2 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The horizontal boundary of the internal corner is perfectly insulated and the vertical boundary is subjected to the convection process (T_∞, h).

(b) Both boundaries of the internal corner are perfectly insulated. How does this result compare with Equation 4.41?

4.33 Consider nodal configuration 3 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The boundary is insulated. Explain how Equation 4.42 can be modified to agree with your result.

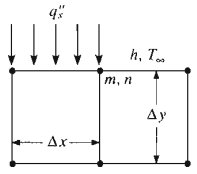
(b) The boundary is subjected to a constant heat flux.

4.34 Consider nodal configuration 4 of Table 4.2. Derive the finite-difference equations under steady-state conditions for the following situations.

(a) The upper boundary of the external corner is perfectly insulated and the side boundary is subjected to the convection process (T_∞, h).

(b) Both boundaries of the external corner are perfectly insulated. How does this result compare with Equation 4.43?

4.35 One of the strengths of numerical methods is their ability to handle complex boundary conditions. In the sketch, the boundary condition changes from specified heat flux, q''_s (into the domain), to convection, at the location of the node m,n. Write the steady-state, two-dimensional finite difference equation at this node.

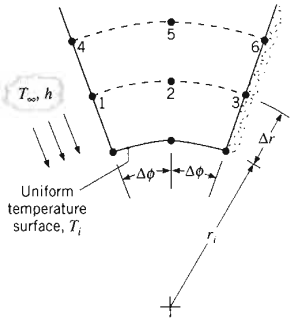


4.36 Consider heat transfer in a one-dimensional (radial) cylindrical coordinate system under steady-state conditions with volumetric heat generation.

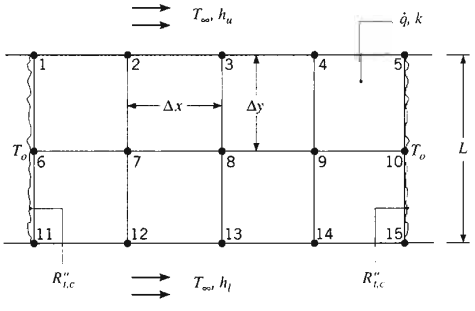
(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for the node n located at the external boundary subjected to the convection process (T_∞, h).

4.37 In a two-dimensional cylindrical configuration, the radial (Δr) and angular ($\Delta\phi$) spacings of the nodes are uniform. The boundary at $r = r_i$ is of uniform temperature T_i. The boundaries in the radial direction are adiabatic (insulated) and exposed to surface convection (T_∞, h), as illustrated. Derive the finite-difference equations for (a) node 2, (b) node 3, and (c) node 1.



4.38 Upper and lower surfaces of a bus bar are convectively cooled by air at T_∞, with $h_u \neq h_l$. The sides are cooled by maintaining contact with heat sinks at T_o, through a thermal contact resistance of $R''_{t,c}$. The bar is of thermal conductivity k, and its width is twice its thickness L.

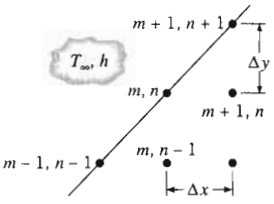


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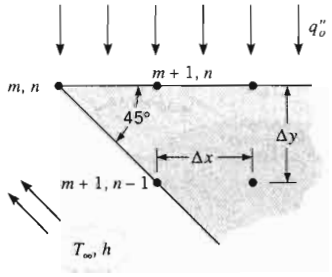
Consider steady-state conditions for which heat is uniformly generated at a volumetric rate $\dot{q}$ due to passage of an electric current. Using the energy balance method, derive finite-difference equations for nodes 1 and 13.

4.39 Derive the nodal finite-difference equations for the following configurations.

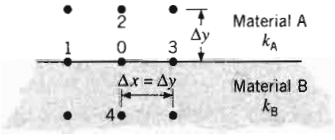
(a) Node m, n on a diagonal boundary subjected to convection with a fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.



(b) Node m, n at the tip of a cutting tool with the upper surface exposed to a constant heat flux q''_o, and the diagonal surface exposed to a convection cooling process with the fluid at T_∞ and a heat transfer coefficient h. Assume $\Delta x = \Delta y$.

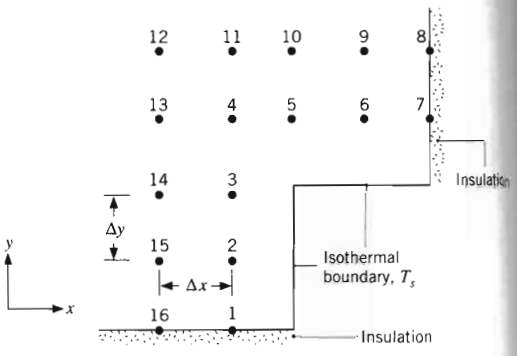


4.40 Consider the nodal point 0 located on the boundary between materials of thermal conductivity k_A and k_B.



Derive the finite-difference equation, assuming no internal generation.

4.41 Consider the two-dimensional grid ($\Delta x = \Delta y$) representing steady-state conditions with no internal volumetric generation for a system with thermal conductivity k. One of the boundaries is maintained at a constant temperature T_s while the others are adiabatic.



Derive an expression for the heat rate per unit length normal to the page crossing the isothermal boundary (T_s).

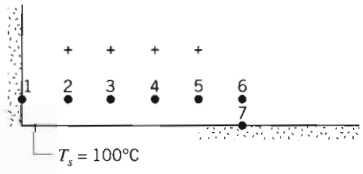
4.42 Consider a one-dimensional fin of uniform cross-sectional area, insulated at its tip, $x = L$. (See Table 3.4, case B). The temperature at the base of the fin T_b and of the adjoining fluid T_∞, as well as the heat transfer coefficient h and the thermal conductivity k, are known.

(a) Derive the finite-difference equation for any interior node m.

(b) Derive the finite-difference equation for a node n located at the insulated tip.

Finite-Difference Equations: Analysis

4.43 Consider the network for a two-dimensional system without internal volumetric generation having nodal temperatures shown below. If the grid space is 125 mm and the thermal conductivity of the material is 50 W/m · K, calculate the heat rate per unit length normal to the page from the isothermal surface (T_s).

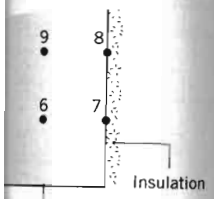


Node	T_j (°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

4.44 Consider the square channel shown in the sketch operating under steady-state conditions. The inner surface of the channel is at a uniform temperature of 600 K, while the outer surface is exposed to convection with a fluid at 300 K and a convection coefficient of 50 W/m^2 · K. From a symmetrical element of the channel, a two-dimensional

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ained at a constant tem-
adiabatic.



Isothermal
boundary, T_s

Insulation

t rate per unit length nor-
hermal boundary (T_s).

fin of uniform cross-
, $x = L$. (See Table 3.4,
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equation for any inte-

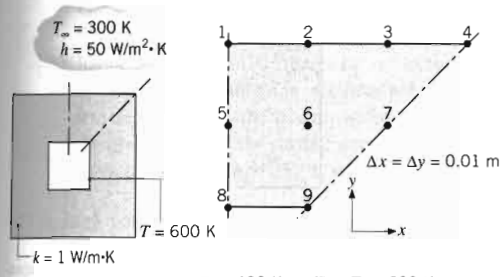
equation for a node n

Analysis

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gth normal to the page

Node	T_i (°C)
1	120.55
2	120.64
3	121.29
4	123.89
5	134.57
6	150.49
7	147.14

vn in the sketch oper-
s. The inner surface of
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of 50 W/m² · K. From
el, a two-dimensional grid has been constructed and the nodes labeled. The temperatures for nodes 1, 3, 6, 8, and 9 are identified.

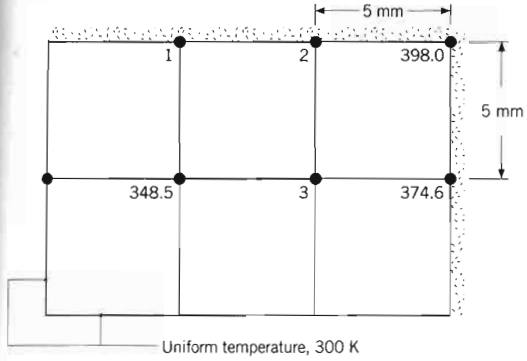


T_1 = 430 K $T_8 = T_9$ = 600 K
T_3 = 394 K T_6 = 492 K

(a) Beginning with properly defined control volumes, derive the finite-difference equations for nodes 2, 4, and 7 and determine the temperatures T_2, T_4, and T_7 (K).

(b) Calculate the heat loss per unit length from the channel.

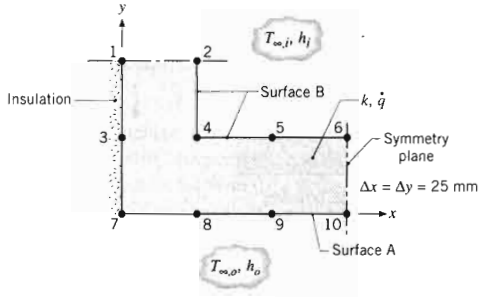
4.45 Steady-state temperatures (K) at three nodal points of a long rectangular rod are as shown. The rod experiences a uniform volumetric generation rate of 5×10^7 W/m³ and has a thermal conductivity of 20 W/m · K. Two of its sides are maintained at a constant temperature of 300 K, while the others are insulated.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit length (W/m) from the rod using the nodal temperatures. Compare this result with the heat rate calculated from knowledge of the volumetric generation rate and the rod dimensions.

4.46 Steady-state temperatures at selected nodal points of the symmetrical section of a flow channel are known to be $T_2 = 95.47$°C, $T_3 = 117.3$°C, $T_5 = 79.79$°C, $T_6 = 77.29$°C, $T_8 = 87.28$°C, and $T_{10} = 77.65$°C. The wall experiences uniform volumetric heat generation of $\dot{q} = 10^6$ W/m³ and has a thermal conductivity of $k = 10$ W/m · K. The inner and outer surfaces of the channel experience convection with fluid temperatures of $T_{\infty,i} = 50$°C and $T_{\infty,o} = 25$°C and convection coefficients of $h_i = 500$ W/m²·K and $h_o = 250$ W/m² · K.



(a) Determine the temperatures at nodes 1, 4, 7, and 9.

(b) Calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

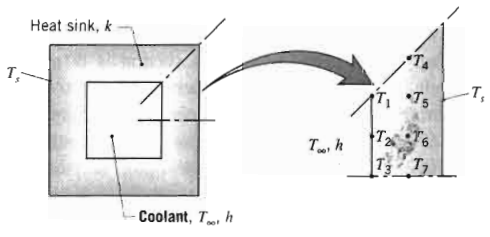
(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

4.47 Consider an aluminum heat sink ($k = 240$ W/m · K), such as that shown schematically in Problem 4.28. The inner and outer widths of the square channel are $w = 20$ mm and $W = 40$ mm, respectively, and an outer surface temperature of $T_s = 50$°C is maintained by the array of electronic chips. In this case, it is not the inner surface temperature that is known, but conditions (T_∞, h) associated with coolant flow through the channel, and we wish to determine the rate of heat transfer to the coolant per unit length of channel. For this purpose, consider a symmetrical section of the channel and a two-dimensional grid with $\Delta x = \Delta y = 5$ mm.

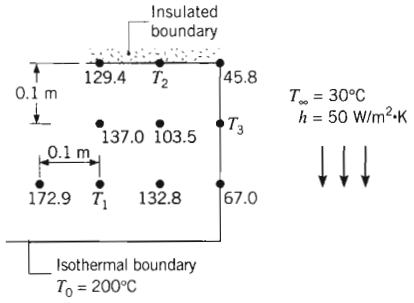
(a) For $T_\infty = 20$°C and $h = 5000$ W/m² · K, determine the unknown temperatures, $T_1, \ldots, T_7$, and the rate of heat transfer per unit length of channel, q'.

(b) Assess the effect of variations in h on the unknown temperatures and the heat rate.



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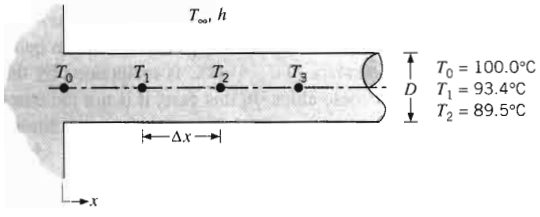
4.48 The steady-state temperatures (°C) associated with selected nodal points of a two-dimensional system having a thermal conductivity of 1.5 W/m · K are shown on the accompanying grid.



(a) Determine the temperatures at nodes 1, 2, and 3.

(b) Calculate the heat transfer rate per unit thickness normal to the page from the system to the fluid.

4.49 A steady-state, finite-difference analysis has been performed on a cylindrical fin with a diameter of 12 mm and a thermal conductivity of 15 W/m·K. The convection process is characterized by a fluid temperature of 25°C and a heat transfer coefficient of 25 W/m² · K.

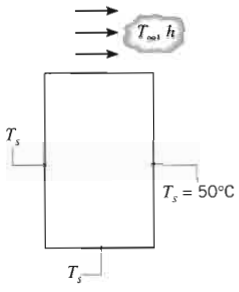


(a) The temperatures for the first three nodes, separated by a spatial increment of $x = 10$ mm, are given in the sketch. Determine the fin heat rate.

(b) Determine the temperature at node 3, T_3.

Solving the Finite-Difference Equations

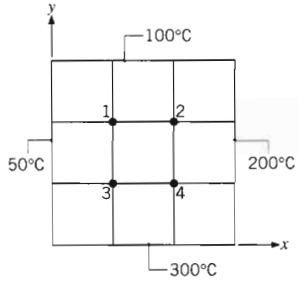
4.50 A long bar of rectangular cross section is 60 mm by 90 mm on a side and has a thermal conductivity of 1 W/m·K. One surface is exposed to a convection process with air at 100°C and a convection coefficient of 100 W/m² · K, while the remaining surfaces are maintained at 50°C.



(a) Using a grid spacing of 30 mm and the Gauss-Seidel iteration method, determine the nodal temperatures and the heat rate per unit length normal to the page into the bar from the air.

(b) Determine the effect of grid spacing on the temperature field and heat rate. Specifically, consider a grid spacing of 15 mm. For this grid, explore the effect of changes in h on the temperature field and the isotherms.

4.51 Consider two-dimensional, steady-state conduction in a square cross section with prescribed surface temperatures.



(a) Determine the temperatures at nodes 1, 2, 3, and 4. Estimate the midpoint temperature.

(b) Reducing the mesh size by a factor of 2, determine the corresponding nodal temperatures. Compare your results with those from the coarser grid.

(c) From the results for the finer grid, plot the 75, 150, and 250°C isotherms.

4.52 Consider a long bar of square cross section (0.8 m to the side) and of thermal conductivity 2 W/m·K. Three of these sides are maintained at a uniform temperature of 300°C. The fourth side is exposed to a fluid at 100°C for which the convection heat transfer coefficient is 10 W/m² · K.

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(a) Using an appropriate numerical technique with a grid spacing of 0.2 m, determine the midpoint temperature and heat transfer rate between the bar and the fluid per unit length of the bar.

(b) Reducing the grid spacing by a factor of 2, determine the midpoint temperature and heat transfer rate. Plot the corresponding temperature distribution across the surface exposed to the fluid. Also, plot the 200 and 250°C isotherms.

4.53 A long conducting rod of rectangular cross section (20 mm × 30 mm) and thermal conductivity k = 20 W/m · K experiences uniform heat generation at a rate $\dot{q} = 5 \times 10^7$ W/m^3, while its surfaces are maintained at 300 K.

(a) Using a finite-difference method with a grid spacing of 5 mm, determine the temperature distribution in the rod.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

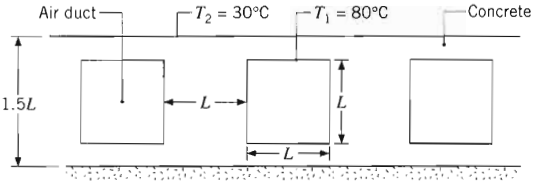
4.54 A flue passing hot exhaust gases has a square cross section, 300 mm to a side. The walls are constructed of refractory brick 150 mm thick with a thermal conductivity of 0.85 W/m · K. Calculate the heat loss from the flue per unit length when the interior and exterior surfaces are maintained at 350 and 25°C, respectively. Use a grid spacing of 75 mm.

4.55 Consider the system of Problem 4.54. The interior surface is exposed to hot gases at 350°C with a convection coefficient of 100 W/m^2 · K, while the exterior surface experiences convection with air at 25°C and a convection coefficient of 5 W/m^2 · K.

(a) Using a grid spacing of 75 mm, calculate the temperature field within the system and determine the heat loss per unit length by convection from the outer surface of the flue to the air. Compare this result with the heat gained by convection from the hot gases to the air.

(b) Determine the effect of grid spacing on the temperature field and heat loss per unit length to the air. Specifically, consider a grid spacing of 25 mm and plot appropriately spaced isotherms on a schematic of the system. Explore the effect of changes in the convection coefficients on the temperature field, and heat loss.

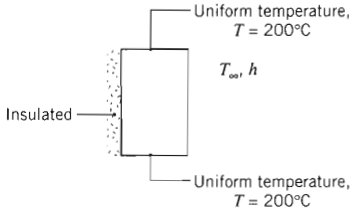
4.56 A common arrangement for heating a large surface area is to move warm air through rectangular ducts below the surface. The ducts are square and located midway between the top and bottom surfaces that are exposed to room air and insulated, respectively.



For the condition when the floor and duct temperatures are 30 and 80°C, respectively, and the thermal conductivity of concrete is 1.4 W/m · K, calculate the heat rate from each duct, per unit length of duct. Use a grid spacing with $\Delta x = 2\,\Delta y$, where $\Delta y = 0.125L$ and L = 150 mm.

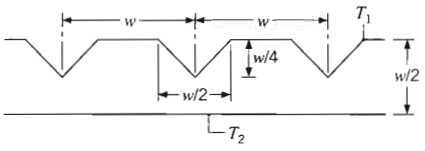
4.57 Consider the gas turbine cooling scheme of Example 4.4. In Problem 3.23, advantages associated with applying a *thermal barrier coating* (TBC) to the exterior surface of a turbine blade are described. If a 0.5-mm-thick zirconia coating (k=1.3 W/m · K, $R''_{t,c} = 10^{-4}$ m^2 · K/W) is applied to the outer surface of the air-cooled blade, determine the temperature field in the blade for the operating conditions of Example 4.4.

4.58 A long bar of rectangular cross section, 0.4 m × 0.6 m on a side and having a thermal conductivity of 1.5 W/m · K, is subjected to the boundary conditions shown below. Two of the sides are maintained at a uniform temperature of 200°C. One of the sides is adiabatic, and the remaining side is subjected to a convection process with T_∞ = 30°C and h = 50 W/m^2 · K. Using an appropriate numerical technique with a grid spacing of 0.1 m, determine the temperature distribution in the bar and the heat transfer rate between the bar and the fluid per unit length of the bar.



4.59 The top surface of a plate, including its grooves, is maintained at a uniform temperature of T_1 = 200°C. The lower surface is at T_2 = 20°C, the thermal conductivity is 15 W/m · K, and the groove spacing is 0.16 m.

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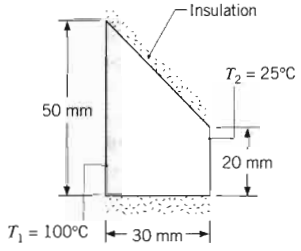


(a) Using a finite-difference method with a mesh size of $\Delta x = \Delta y = 40$ mm, calculate the unknown nodal temperatures and the heat transfer rate per width of groove spacing (w) and per unit length normal to the page.

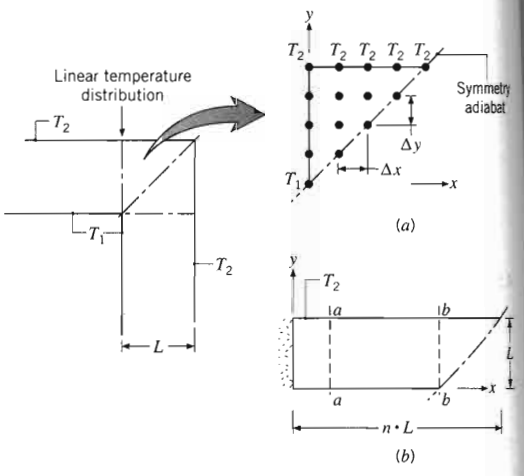
(b) With a mesh size of $\Delta x = \Delta y = 10$ mm, repeat the foregoing calculations, determining the temperature field and the heat rate. Also, consider conditions for which the bottom surface is not at a uniform temperature T_2 but is exposed to a fluid at $T_\infty = 20°C$. With $\Delta x = \Delta y = 10$ mm, determine the temperature field and heat rate for values of $h = 5$, 200, and 1000 W/m^2·K, as well as for $h \to \infty$.

4.60 Refer to the two-dimensional rectangular plate of Problem 4.2. Using an appropriate numerical method with $\Delta x = \Delta y = 0.25$ m, determine the temperature at the midpoint (1, 0.5).

4.61 A long trapezoidal bar is subjected to uniform temperatures on two surfaces, while the remaining surfaces are well insulated. If the thermal conductivity of the material is 20 W/m·K, estimate the heat transfer rate per unit length of the bar using a finite-difference method. Use the Gauss–Seidel method of solution with a space increment of 10 mm.



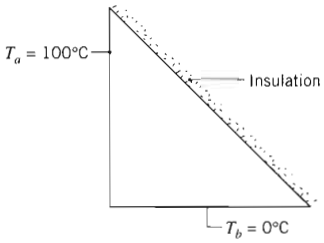
4.62 The shape factor for conduction through the edge of adjoining walls for which $D > L/5$, where D and L are the wall depth and thickness, respectively, is shown in Table 4.1. The two-dimensional symmetrical element of the edge, which is represented by inset (*a*), is bounded by the diagonal symmetry adiabat and a section of the wall thickness over which the temperature distribution is assumed to be linear between T_1 and T_2.



(a) Using the nodal network of inset (*a*) with $L = 40$ mm, determine the temperature distribution in the element for $T_1 = 100°C$ and $T_2 = 0°C$. Evaluate the heat rate per unit depth ($D = 1$ m) if $k = 1$ W/m·K. Determine the corresponding shape factor for the edge and compare your result with that from Table 4.1.

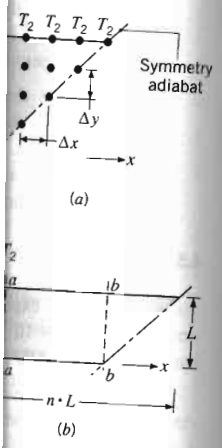
(b) Choosing a value of $n = 1$ or $n = 1.5$, establish a nodal network for the trapezoid of inset (*b*) and determine the corresponding temperature field. Assess the validity of assuming linear temperature distributions across sections *a*–*a* and *b*–*b*.

4.63 The diagonal of a long triangular bar is well insulated, while sides of equivalent length are maintained at uniform temperatures T_a and T_b.



(a) Establish a nodal network consisting of five nodes along each of the sides. For one of the nodes on the diagonal surface, define a suitable control volume and derive the corresponding finite-difference equation. Using this form for the diagonal nodes and appropriate equations for the interior nodes, find the temperature distribution for the bar. On a scale drawing of the shape, show the 25, 50, and 75°C isotherms.

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inset (*a*) with $L = 40$
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$T_2 = 0°C$. Evaluate
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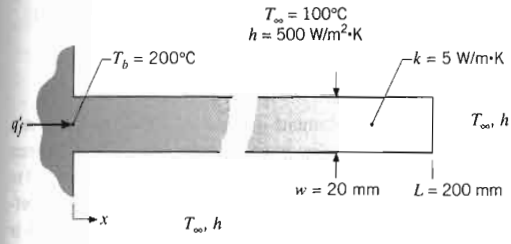
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(b) An alternate and simpler procedure to obtain the finite-difference equations for the diagonal nodes follows from recognizing that the insulated diagonal surface is a symmetry plane. Consider a square 5×5 nodal network and represent its diagonal as a symmetry line. Recognize which nodes on either side of the diagonal have identical temperatures. Show that you can treat the diagonal nodes as "interior" nodes and write the finite-difference equations by inspection.

Finite-Element Solutions

4.64 A straight fin of uniform cross section is fabricated from a material of thermal conductivity $k = 5\,W/m \cdot K$, thickness $w = 20\,mm$, and length $L = 200\,mm$. The fin is very long in the direction normal to the page. The base of the fin is maintained at $T_b = 200°C$, and the tip condition allows for convection (case A of Table 3.4), with $h = 500\,W/m^2 \cdot K$ and $T_\infty = 25°C$.



(a) Assuming one-dimensional heat transfer in the fin, calculate the fin heat rate, q_f' (W/m), and the tip temperature T_L. Calculate the Biot number for the fin to determine whether the one-dimensional assumption is valid.

(b) Using the finite-element method of *FEHT*, perform a two-dimensional analysis on the fin to determine the fin heat rate and tip temperature. Compare your results with those from the one-dimensional, analytical solution of part (a). Use the *View/Temperature Contours* option to display isotherms, and discuss key features of the corresponding temperature field and heat flow pattern. *Hint*: In drawing the outline of the fin, take advantage of symmetry. Use a fine mesh near the base and a coarser mesh near the tip. Why?

(c) Validate your *FEHT* model by comparing predictions with the analytical solution for a fin with thermal conductivities of $k = 50\,W/m \cdot K$ and $500\,W/m \cdot K$. Is the one-dimensional heat transfer assumption valid for these conditions?

4.65 Consider the long rectangular bar of Problem 4.50 with the prescribed boundary conditions.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the bar to the air stream.

(c) Explore the effect on the heat rate of increasing the convection coefficient by factors of two and three. Explain why the change in the heat rate is not proportional to the change in the convection coefficient.

4.66 Consider the long rectangular rod of Problem 4.53, which experiences uniform heat generation while its surfaces are maintained at a fixed temperature.

(a) Using the finite-element method of *FEHT*, determine the temperature distribution. Use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the distribution.

(b) With the boundary conditions unchanged, what heat generation rate will cause the midpoint temperature to reach 600 K?

4.67 Consider the symmetrical section of the flow channel of Problem 4.46, with the prescribed values of $\dot{q}$, $k, T_{\infty,i}, T_{\infty,o}, h_i$ and h_o. Use the finite-element method of *FEHT* to obtain the following results.

(a) Determine the temperature distribution in the symmetrical section, and use the *View/Temperature Contours* command to represent the isotherms. Identify significant features of the temperature distribution, including the hottest and coolest regions and the region with the steepest gradients. Describe the heat flow field.

(b) Using the *View/Heat Flows* command, calculate the heat rate per unit length (W/m) from the outer surface A to the adjacent fluid.

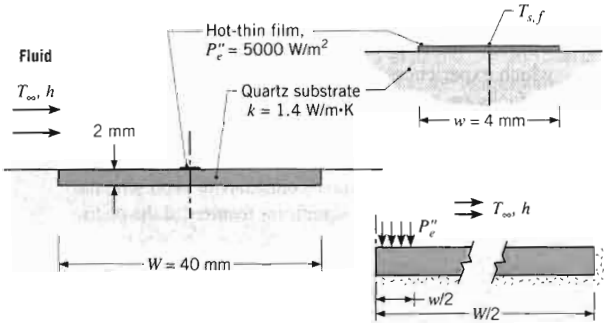
(c) Calculate the heat rate per unit length from the inner fluid to surface B.

(d) Verify that your results are consistent with an overall energy balance on the channel section.

4.68 The hot-film heat flux gage shown schematically may be used to determine the convection coefficient of an adjoining fluid stream by measuring the electric power dissipation per unit area, $P_e''(W/m^2)$, and the average surface temperature, $T_{s,f}$, of the film. The power dissipated in the film is transferred

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directly to the fluid by convection, as well as by conduction into the substrate. If substrate conduction is negligible, the gage measurements can be used to determine the convection coefficient without application of a correction factor. Your assignment is to perform a two-dimensional, steady-state conduction analysis to estimate the fraction of the power dissipation that is conducted into a 2-mm-thick quartz substrate of width $W = 40$ mm and thermal conductivity $k = 1.4$ W/m · K. The thin, hot-film gage has a width of $w = 4$ mm and operates at a uniform power dissipation of 5000 W/m^2. Consider cases for which the fluid temperature is 25°C and the convection coefficient is 500, 1000, and 2000 W/m^2 · K.

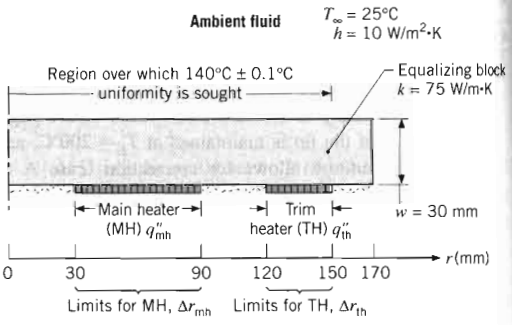


Use the finite-element method of *FEHT* to analyze a symmetrical half-section of the gage and the quartz substrate. Assume that the lower and end surfaces of the substrate are perfectly insulated, while the upper surface experiences convection with the fluid.

(a) Determine the temperature distribution and the conduction heat rate into the region below the hot film for the three values of h. Calculate the fractions of electric power dissipation represented by these rates. *Hint*: Use the *View/Heat Flow* command to find the heat rate across the boundary elements.

(b) Use the *View/Temperature Contours* command to view the isotherms and heat flow patterns. Describe the heat flow paths, and comment on features of the gage design that influence the paths. What limitations on applicability of the gage have been revealed by your analysis?

4.69 A semiconductor industry roadmap for microlithography processing requires that a 300-mm-diameter silicon wafer be maintained at a steady-state temperature of 140°C to within a uniformity of 0.1°C. The design of a hot-plate tool to hopefully meet this requirement is shown schematically. An *equalizing block* (EB), on which the wafer would be placed, is fabricated from an aluminum alloy of thermal conductivity $k = 75$ W/m·K and is heated by two ring-shaped electrical heaters. The two-zone heating arrangement allows for independent control of a main heater (MH) and a trim heater (TH), which is used to improve the uniformity of the surface temperature for the EB. Your assignment is to size the heaters, MH and TH, by specifying their applied heat fluxes, q''_{mh} and q''_{th}(W/m^2), and their radial extents, Δr_{mh} and Δr_{th}. The constraints on radial positioning of the heaters are imposed by manufacturing considerations and are shown in the schematic.



Use the finite-element method of *FEHT* to perform a conduction analysis on an axisymmetric EB of 340-mm diameter. The upper and lateral surfaces are exposed to the ambient fluid at $T_\infty = 25$°C with a convection coefficient of 10 W/m^2·K. The lower surface of the EB is adiabatic, except for the ring sectors with the uniform applied heat fluxes, q''_{mh} and q''_{th}.

(a) For an upper surface of 140°C, perform an overall energy balance on the EB to obtain an initial estimate for the applied heater fluxes. Assume that $q''_{mh} = q''_{th}$ and that each heater extends fully over the radial limits indicated schematically. Use this estimate as a boundary condition in your *FEHT* model; determine the temperature distribution; and using the *View/Temperature Contours* command, examine the isotherms and the temperature distribution across the upper surface of the EB. Did you achieve the desired uniformity?

(b) Rerun your *FEHT* model with different values of the heater fluxes, until you obtain the best uniformity possible within the imposed constraints. *Hint*: If you want to know the specific nodal temperatures, see the *View/Tabular Output* page. You may obtain a plot of the surface temperature distribution by highlighting the appropriate nodal data, using the *Copy* command, and pasting the data into *Excel* for graphing.

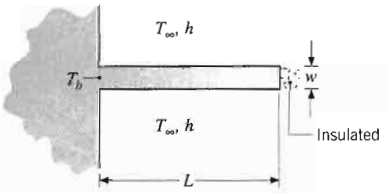
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(c) In what manner would a nonuniform distribution of the convection coefficient across the upper surface of the EB affect the temperature uniformity? For the downward flowing gas stream used in the microlithography process, a representative distribution of the convection coefficient on the upper surface of the EB is $h(r) = h_o[1+a(r/r_o)^n]$, where $h_o = 5.4\,W/m^2 \cdot K$ and $a = n = 1.5$. For this distribution and retention of a value of $h = 10\,W/m^2 \cdot K$ at the lateral surface of the EB, can you adjust the trim heater flux to obtain improved uniformity of the surface temperature?

(d) What changes to the design would you propose for improving the surface temperature uniformity?

Special Applications

4.70 A straight fin of uniform cross section is fabricated from a material of thermal conductivity 50 W/m·K, thickness $w = 6$ mm, and length $L = 48$ mm, and is very long in the direction normal to the page. The convection heat transfer coefficient is 500 $W/m^2 \cdot K$ with an ambient air temperature of $T_\infty = 30°C$. The base of the fin is maintained at $T_b = 100°C$, while the fin tip is well insulated.

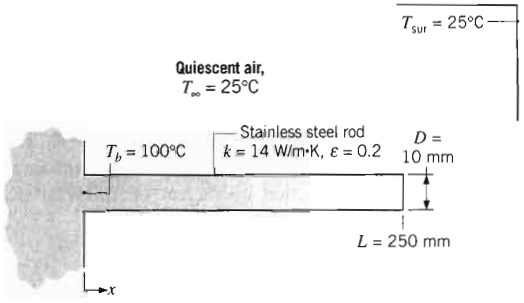


(a) Using a finite-difference method with a space increment of 4 mm, estimate the temperature distribution within the fin. Is the assumption of one-dimensional heat transfer reasonable for this fin?

(b) Estimate the fin heat transfer rate per unit length normal to the page. Compare your result with the one-dimensional, analytical solution, Equation 3.76.

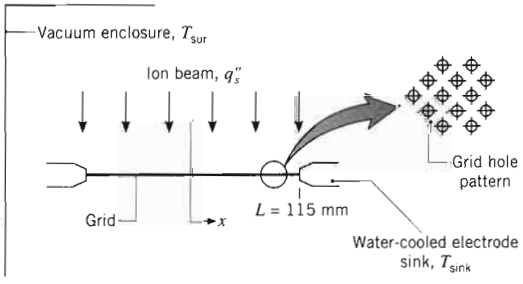
(c) Using the finite-difference mesh of part (a), compute and plot the fin temperature distribution for values of $h = 10$, 100, 500, and 1000 $W/m^2 \cdot K$. Determine and plot the fin heat transfer rate as a function of h.

4.71 A rod of 10-mm diameter and 250-mm length has one end maintained at 100°C. The surface of the rod experiences free convection with the ambient air at 25°C and a convection coefficient that depends on the difference between the temperature of the surface and the ambient air. Specifically, the coefficient is prescribed by a correlation of the form, $h_{fc} = 2.89[0.6 + 0.624(T - T_\infty)^{1/6}]^2$, where the units are h_{fc} ($W/m^2 \cdot K$) and T (K). The surface of the rod has an emissivity $\varepsilon = 0.2$ and experiences radiation exchange with the surroundings at $T_{sur} = 25°C$. The fin tip also experiences free convection and radiation exchange.



Assuming one-dimensional conduction and using a finite-difference method representing the fin by five nodes, estimate the temperature distribution for the fin. Determine also the fin heat rate and the relative contributions of free convection and radiation exchange. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node. Similarly, for the convection rate equation associated with each node, the free convection coefficient h_{fc} must be evaluated for each node.

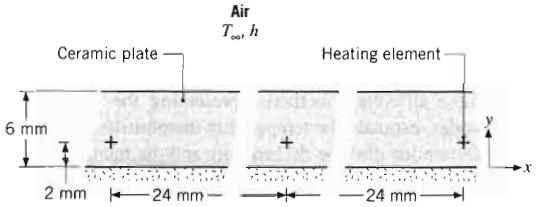
4.72 A thin metallic foil of thickness 0.25 mm with a pattern of extremely small holes serves as an acceleration grid to control the electrical potential of an ion beam. Such a grid is used in a chemical vapor deposition (CVD) process for the fabrication of semiconductors. The top surface of the grid is exposed to a uniform heat flux caused by absorption of the ion beam, $q_s'' = 600\,W/m^2$. The edges of the foil are thermally coupled to water-cooled sinks maintained at 300 K. The upper and lower surfaces of the foil experience radiation exchange with the vacuum enclosure walls maintained at 300 K. The effective thermal conductivity of the foil material is 40 W/m·K and its emissivity is 0.45.



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Assuming one-dimensional conduction and using a finite-difference method representing the grid by ten nodes in the x direction, estimate the temperature distribution for the grid. *Hint:* For each node requiring an energy balance, use the linearized form of the radiation rate equation, Equation 1.8, with the radiation coefficient h_r, Equation 1.9, evaluated for each node.

4.73 Small diameter electrical heating elements dissipating 50 W/m (length normal to the sketch) are used to heat a ceramic plate of thermal conductivity 2 W/m · K. The upper surface of the plate is exposed to ambient air at 30°C with a convection coefficient of 100 W/m² · K, while the lower surface is well insulated.



(a) Using the Gauss–Seidel method with a grid spacing of $\Delta x = 6$ mm and $\Delta y = 2$ mm, obtain the temperature distribution within the plate.

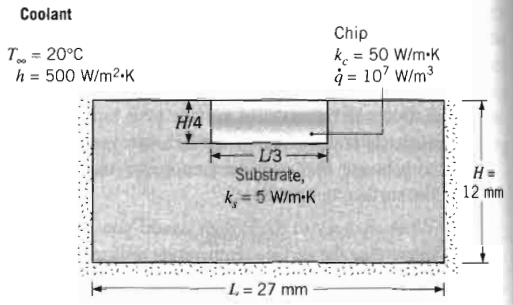
(b) Using the calculated nodal temperatures, sketch four isotherms to illustrate the temperature distribution in the plate.

(c) Calculate the heat loss by convection from the plate to the fluid. Compare this value with the element dissipation rate.

(d) What advantage, if any, is there in not making $\Delta x = \Delta y$ for this situation?

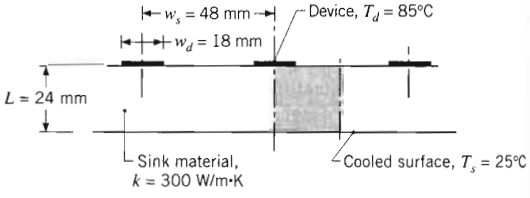
(e) With $\Delta x = \Delta y = 2$ mm, calculate the temperature field within the plate and the rate of heat transfer from the plate. Under no circumstances may the temperature at any location in the plate exceed 400°C. Would this limit be exceeded if the air flow were terminated and heat transfer to the air was by natural convection with $h = 10$ W/m² · K?

4.74 A simplified representation for cooling in very large-scale integration (VLSI) of microelectronics is shown in the sketch. A silicon chip is mounted in a dielectric substrate, and one surface of the system is convectively cooled, while the remaining surfaces are well insulated from the surroundings. The problem is rendered two-dimensional by assuming the system to be very long in the direction perpendicular to the paper. Under steady-state operation, electric power dissipation in the chip provides for uniform volumetric heating at a rate of $\dot{q}$. However, the heating rate is limited by restrictions on the maximum temperature that the chip is allowed to assume.



For the conditions shown on the sketch, will the maximum temperature in the chip exceed 85°C, the maximum allowable operating temperature set by industry standards? A grid spacing of 3 mm is suggested.

4.75 Electronic devices dissipating electrical power can be cooled by conduction to a heat sink. The lower surface of the sink is cooled, and the spacing of the devices w_s, the width of the device w_d, and the thickness L and thermal conductivity k of the heat sink material each affect the thermal resistance between the device and the cooled surface. The function of the heat sink is to *spread* the heat dissipated in the device throughout the sink material.



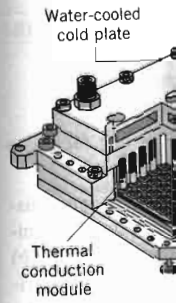
(a) Beginning with the shaded symmetrical element, use a coarse (5 × 5) nodal network to estimate the thermal resistance per unit depth between the device and lower surface of the sink, $R'_{t,d-s}$ (m · K/W). How does this value compare with thermal resistances based on the assumption of one-dimensional conduction in rectangular domains of (i) width w_d and length L and (ii) width w_s and length L?

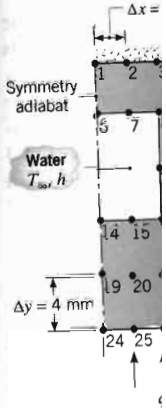
(b) Using nodal networks with grid spacings three and five times smaller than that in part (a), determine the effect of grid size on the precision of the thermal resistance calculation.

(c) Using the finer nodal network developed for part (b), determine the effect of device width on the

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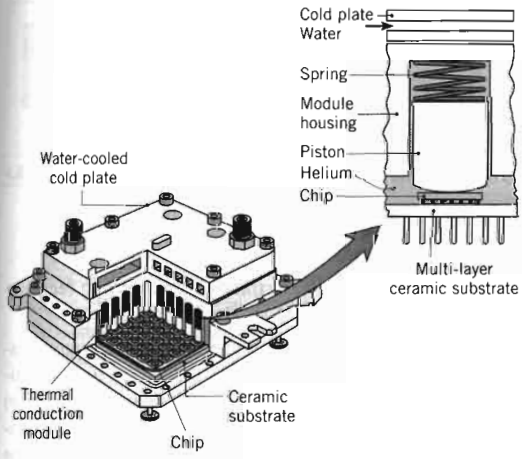


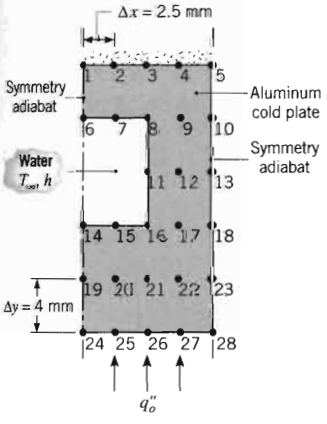
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thermal resistance. Specifically, keeping w_s and L fixed, find the thermal resistance for values of $w_d/w_s = 0.175, 0.275, 0.375$, and 0.475.

4.76 A major problem in packaging very large-scale integrated (VLSI) circuits concerns cooling of the circuit elements. The problem results from increasing levels of power dissipation within a chip, as well as from packing chips closer together in a module. A novel technique for cooling multichip modules has been developed by IBM. Termed the thermal conduction module (TCM), the chips are soldered to a multilayer ceramic substrate, and heat dissipated in each chip is conducted through a spring-loaded aluminum piston to a water-cooled *cold plate.*

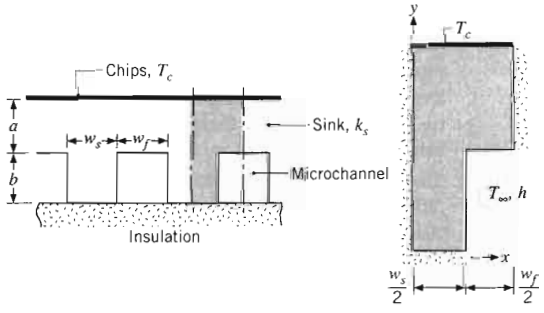




(a) Consider a cold plate fabricated from aluminum ($k = 190\,\text{W/m}\cdot\text{K}$) with regularly spaced rectangular channels through which the water is passed. Under normal operation, power dissipation within the chips results in a uniform heat flux of $q_o'' = 10^5\,\text{W/m}^2$ at the base of the cold plate, while water flow provides a temperature of $T_\infty = 15°\text{C}$ and a convection coefficient of $h = 5000\,\text{W/m}^2\cdot\text{K}$ within the channels. We are interested in obtaining the steady-state temperature distribution within the cold plate, and from symmetry considerations, we may confine our attention to the prescribed nodal network. Assuming the top surface of the cold plate to be well insulated, determine the nodal temperatures.

(b) Although there is interest in operating at higher power levels, system reliability considerations dictate that the maximum cold plate temperature must not exceed 40°C. Using the prescribed cold plate geometry and nodal network, assess the effect of changes in operating or design conditions intended to increase the operating heat flux q_o''. Estimate the upper limit for the heat flux.

4.77 A heat sink for cooling computer chips is fabricated from copper ($k_s = 400\,\text{W/m}\cdot\text{K}$), with machined microchannels passing a cooling fluid for which $T_\infty = 25°\text{C}$ and $h = 30{,}000\,\text{W/m}^2\cdot\text{K}$. The lower side of the sink experiences no heat removal, and a preliminary heat sink design calls for dimensions of $a = b = w_s = w_f = 200\,\mu\text{m}$. A symmetrical element of the heat path from the chip to the fluid is shown in the inset.



(a) Using the symmetrical element with a square nodal network of $\Delta x = \Delta y = 100\,\mu\text{m}$, determine the corresponding temperature field and the heat rate q' to the coolant per unit channel length (W/m) for a maximum allowable chip temperature $T_{c,\max} = 75°\text{C}$. Estimate the corresponding thermal resistance between the chip surface and the fluid, $R'_{t,c-f}(\text{m}\cdot\text{K/W})$. What is the maximum allowable heat dissipation for a chip that measures 10 mm × 10 mm on a side?

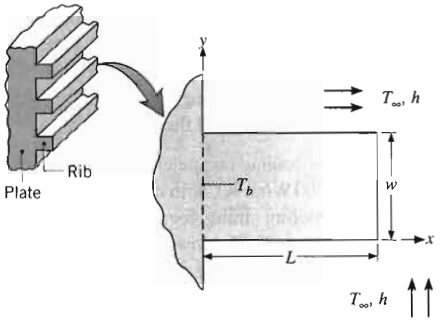
(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for the temperature distribution and heat removal rate.

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Investigate the effect of grid spacing by considering spatial increments of 50 and 25 μm.

(c) Consistent with the requirement that $a + b = 400\ \mu$m, can the heat sink dimensions be altered in a manner that reduces the overall thermal resistance?

4.78 A plate ($k = 10$ W/m·K) is stiffened by a series of longitudinal ribs having a rectangular cross section with length $L = 8$ mm and width $w = 4$ mm. The base of the plate is maintained at a uniform temperature $T_b = 45°C$, while the rib surfaces are exposed to air at a temperature of $T_\infty = 25°C$ and a convection coefficient of $h = 600$ W/m^2·K.

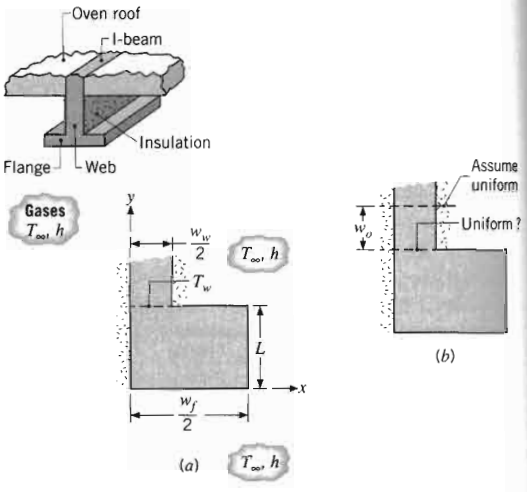


(a) Using a finite-difference method with $\Delta x = \Delta y = 2$ mm and a total of 5×3 nodal points and regions, estimate the temperature distribution and the heat rate from the base. Compare these results with those obtained by assuming that heat transfer in the rib is one-dimensional, thereby approximating the behavior of a fin.

(b) The grid spacing used in the foregoing finite-difference solution is coarse, resulting in poor precision for estimates of temperatures and the heat rate. Investigate the effect of grid refinement by reducing the nodal spacing to $\Delta x = \Delta y = 1$ mm (a 9×3 grid) considering symmetry of the center line.

(c) Investigate the nature of two-dimensional conduction in the rib and determine a criterion for which the one-dimensional approximation is reasonable. Do so by extending your finite-difference analysis to determine the heat rate from the base as a function of the length of the rib for the range $1.5 \leq L/w \leq 10$, keeping the length L constant. Compare your results with those determined by approximating the rib as a fin.

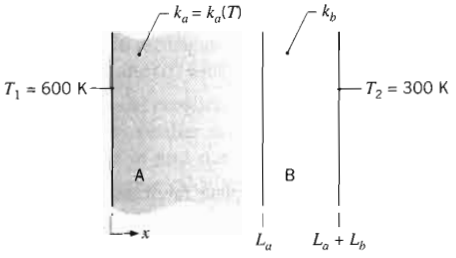
4.79 The bottom half of an I-beam providing support for a furnace roof extends into the heating zone. The web is well insulated, while the flange surfaces experience convection with hot gases at $T_\infty = 400°C$ and a convection coefficient of $h = 150$ W/m^2 · K. Consider the symmetrical element of the flange region (inset *a*), assuming that the temperature distribution across the web is uniform at $T_w = 100°C$. The beam thermal conductivity is 10 W/m · K, and its dimensions are $w_f = 80$ mm, $w_w = 30$ mm, and $L = 30$ mm.



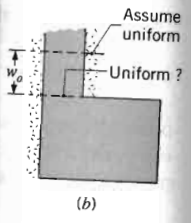
(a) Calculate the heat transfer rate per unit length to the beam using a 5×4 nodal network.

(b) Is it reasonable to assume that the temperature distribution across the web–flange interface is uniform? Consider the L-shaped domain of inset (*b*) and use a fine grid to obtain the temperature distribution across the web–flange interface. Make the distance $w_o \geq w_w/2$.

4.80 Consider one-dimensional conduction in a plane composite wall. The exposed surfaces of materials A and B are maintained at $T_1 = 600$ K and $T_2 = 300$ K respectively. Material A, of thickness $L_a = 20$ mm, has a temperature-dependent thermal conductivity of $k_a = k_o[1 + \alpha(T - T_o)]$, where $k_o = 4.4$ W/m·K, $\alpha = 0.008\ \text{K}^{-1}$, $T_o = 300$ K and T is in kelvins. Material B is of thickness $L_b = 5$ mm and has a thermal conductivity of $k_b = 1$ W/m·K.



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(*b*)

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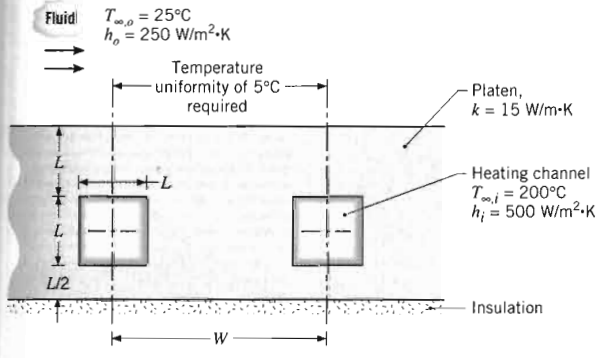


(a) Calculate the heat flux through the composite wall by assuming material A to have a uniform thermal conductivity evaluated at the average temperature of the section.

(b) Using a space increment of 1 mm, obtain the finite-difference equations for the internal nodes and calculate the heat flux considering the temperature-dependent thermal conductivity for Material A. If the *IHT* software is employed, call-up functions from *Tools/Finite-Difference Equations* may be used to obtain the nodal equations. Compare your result with that obtained in part (a).

(c) As an alternative to the finite-difference method of part (b), use the finite-element method of *FEHT* to calculate the heat flux, and compare the result with that from part (a). *Hint*: In the *Specify/Material Properties* box, properties may be entered as a function of temperature (T), the space coordinates (x, y), or time (t). See the *Help* section for more details.

4.81 A platen of thermal conductivity $k = 15\,\text{W/m} \cdot \text{K}$ is heated by flow of a hot fluid through channels of width $L = 20\,\text{mm}$, with $T_{\infty,i} = 200°\text{C}$ and $h_i = 500\,\text{W/m}^2 \cdot \text{K}$. The upper surface of the platen is used to heat a process fluid at $T_{\infty,o} = 25°\text{C}$ with a convection coefficient of $h_o = 250\,\text{W/m}^2 \cdot \text{K}$. The lower surface of the platen is insulated. To heat the process fluid uniformly, the temperature of the platen's upper surface must be uniform to within 5°C. Use a finite-difference method, such as that of *IHT*, or the finite-element method of *FEHT* to obtain the following results.



(a) Determine the maximum allowable spacing, W, between the channel centerlines that will satisfy the specified temperature uniformity requirement.

(b) What is the corresponding heat rate per unit length from a flow channel?

4.82 Consider the cooling arrangement for the very large-scale integration (VLSI) chip of Problem 4.75. Use the finite-element method of *FEHT* to obtain the following results.

(a) Determine the temperature distribution in the chip-substrate system. Will the maximum temperature exceed 85°C?

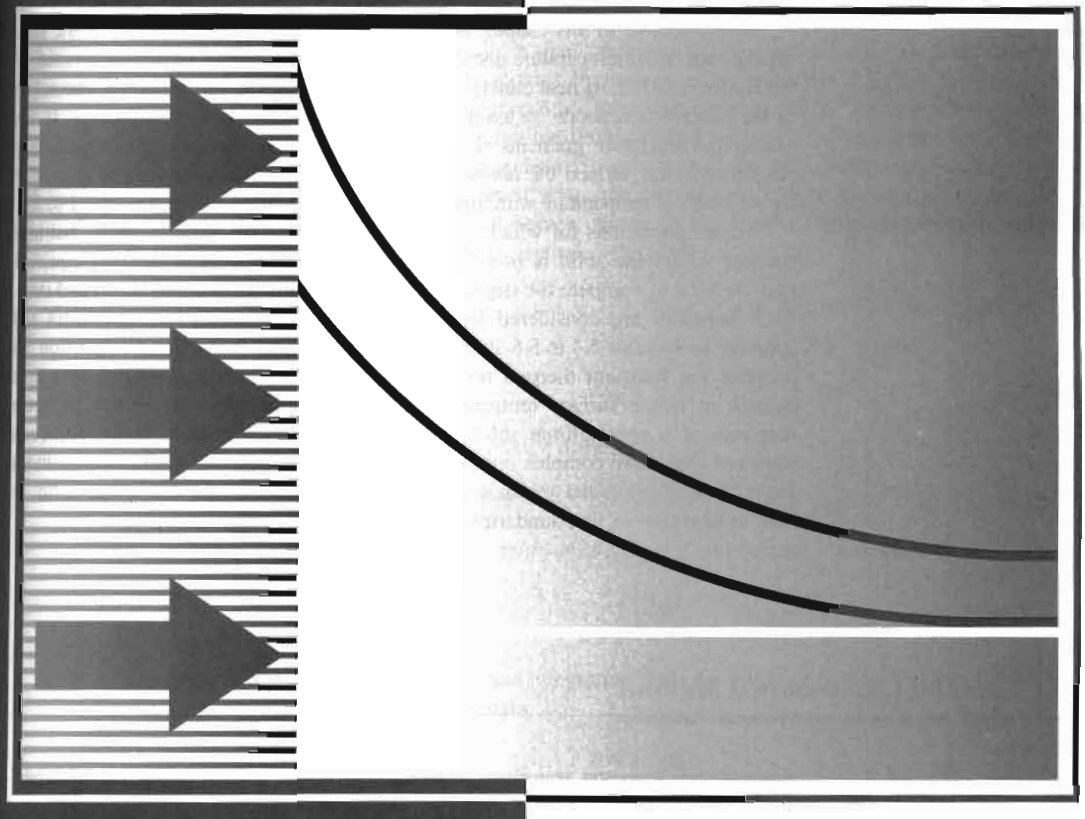
(b) Using the *FEHT* model developed for part (a), determine the volumetric heating rate that yields a maximum temperature of 85°C.

(c) What effect would reducing the substrate thickness have on the maximum operating temperature? For a volumetric generation rate of $\dot{q} = 10^7\ \text{W/m}^3$, reduce the thickness of the substrate from 12 mm to 6 mm, keeping all other dimensions unchanged. What is the maximum system temperature for these conditions? What fraction of the chip power generation is removed by convection directly from the chip surface?

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CHAPTER 5

Transient Conduction



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In our treatment of conduction we have gradually considered more complicated conditions. We began with the simple case of one-dimensional, steady-state conduction with no internal generation, and we subsequently considered more realistic situations involving multidimensional and generation effects. However, we have not yet considered situations for which conditions change with time.

We now recognize that many heat transfer problems are time dependent. Such *unsteady,* or *transient,* problems typically arise when the boundary conditions of a system are changed. For example, if the surface temperature of a system is altered, the temperature at each point in the system will also begin to change. The changes will continue to occur until a *steady-state* temperature distribution is reached. Consider a hot metal billet that is removed from a furnace and exposed to a cool airstream. Energy is transferred by convection and radiation from its surface to the surroundings. Energy transfer by conduction also occurs from the interior of the metal to the surface, and the temperature at each point in the billet decreases until a steady-state condition is reached. The final properties of the metal will depend significantly on the time-temperature history that results from heat transfer. Controlling the heat transfer is one key to fabricating new materials with enhanced properties.

Our objective in this chapter is to develop procedures for determining the time dependence of the temperature distribution within a solid during a transient process, as well as for determining heat transfer between the solid and its surroundings. The nature of the procedure depends on assumptions that may be made for the process. If, for example, temperature gradients within the solid may be neglected, a comparatively simple approach, termed the *lumped capacitance method,* may be used to determine the variation of temperature with time. The method is developed in Sections 5.1 to 5.3.

Under conditions for which temperature gradients are not negligible, but heat transfer within the solid is one-dimensional, exact solutions to the heat equation may be used to compute the dependence of temperature on both location and time. Such solutions are considered for *finite solids* (plane walls, long cylinders and spheres) in Sections 5.4 to 5.6 and for *semi-infinite solids* in Section 5.7. Section 5.8 presents the transient thermal response of a variety of objects subject to a step change in either surface temperature or surface heat flux. In Section 5.9, the response of a semi-infinite solid to periodic heating conditions at its surface is explored. For more complex conditions, finite-difference or finite-element methods must be used to predict the time dependence of temperatures within the solid, as well as heat rates at its boundaries (Section 5.10).

5.1 *The Lumped Capacitance Method*

A simple, yet common, transient conduction problem is one for which a solid experiences a sudden change in its thermal environment. Consider a hot metal forging that is initially at a uniform temperature T_i and is quenched by immersing it in a liquid of lower temperature $T_\infty < T_i$ (Figure 5.1). If the quenching is said to begin at time $t = 0$, the temperature of the solid will decrease for time $t > 0$, until it eventually

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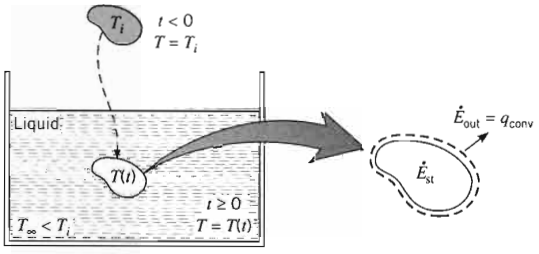


FIGURE 5.1 Cooling of a hot metal forging.

reaches T_∞. This reduction is due to convection heat transfer at the solid–liquid interface. The essence of the lumped capacitance method is the assumption that the temperature of the solid is *spatially uniform* at any instant during the transient process. This assumption implies that temperature gradients within the solid are negligible.

From Fourier's law, heat conduction in the absence of a temperature gradient implies the existence of infinite thermal conductivity. Such a condition is clearly impossible. However, the condition is closely approximated if the resistance to conduction within the solid is small compared with the resistance to heat transfer between the solid and its surroundings. For now we assume that this is, in fact, the case.

In neglecting temperature gradients within the solid, we can no longer consider the problem from within the framework of the heat equation. Instead, the transient temperature response is determined by formulating an overall energy balance on the solid. This balance must relate the rate of heat loss at the surface to the rate of change of the internal energy. Applying Equation 1.11c to the control volume of Figure 5.1, this requirement takes the form

$$-\dot{E}_{out} = \dot{E}_{st} \tag{5.1}$$

or

$$-hA_s(T - T_\infty) = \rho Vc\frac{dT}{dt} \tag{5.2}$$

Introducing the temperature difference

$$\theta \equiv T - T_\infty \tag{5.3}$$

and recognizing that $(d\theta/dt) = (dT/dt)$ if T_∞ is constant, it follows that

$$\frac{\rho Vc}{hA_s}\frac{d\theta}{dt} = -\theta$$

Separating variables and integrating from the initial condition, for which $t = 0$ and $T(0) = T_i$, we then obtain

$$\frac{\rho Vc}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt$$

where

$$\theta_i \equiv T_i - T_\infty \tag{5.4}$$

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Evaluating the integrals, it follows that

$$\frac{\rho Vc}{hA_s}\ln\frac{\theta_i}{\theta} = t \tag{5.5}$$

or

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right] \tag{5.6}$$

Equation 5.5 may be used to determine the time required for the solid to reach some temperature T, or, conversely, Equation 5.6 may be used to compute the temperature reached by the solid at some time t.

The foregoing results indicate that the difference between the solid and fluid temperatures must decay exponentially to zero as t approaches infinity. This behavior is shown in Figure 5.2. From Equation 5.6 it is also evident that the quantity $(\rho Vc/hA_s)$ may be interpreted as a *thermal time constant* expressed as

$$\tau_t = \left(\frac{1}{hA_s}\right)(\rho Vc) = R_t C_t \tag{5.7}$$

where R_t is the resistance to convection heat transfer and C_t is the *lumped thermal capacitance* of the solid. Any increase in R_t or C_t will cause a solid to respond more slowly to changes in its thermal environment. This behavior is analogous to the voltage decay that occurs when a capacitor is discharged through a resistor in an electrical RC circuit.

To determine the total energy transfer Q occurring up to some time t, we simply write

$$Q = \int_0^t q\,dt = hA_s\int_0^t \theta\,dt$$

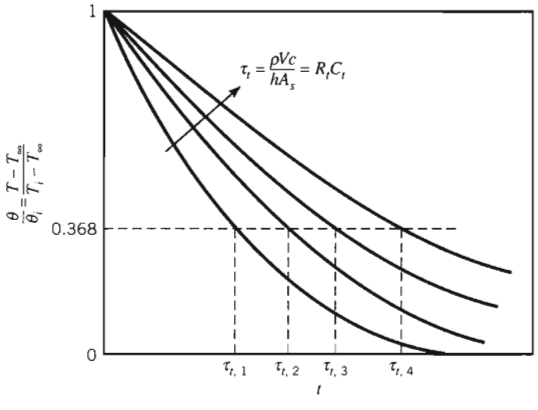


FIGURE 5.2 Transient temperature response of lumped capacitance solids for different thermal time constants τ_t.

5.2 Validity of th

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Substituting for θ from Equation 5.6 and integrating, we obtain

$$Q = (\rho Vc)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right] \tag{5.8a}$$

The quantity Q is, of course, related to the change in the internal energy of the solid, and from Equation 1.11b

$$-Q = \Delta E_{st} \tag{5.8b}$$

For quenching Q is positive and the solid experiences a decrease in energy. Equations 5.5, 5.6, and 5.8a also apply to situations where the solid is heated ($\theta < 0$), in which case Q is negative and the internal energy of the solid increases.

5.2 *Validity of the Lumped Capacitance Method*

From the foregoing results it is easy to see why there is a strong preference for using the lumped capacitance method. It is certainly the simplest and most convenient method that can be used to solve transient heating and cooling problems. Hence it is important to determine under what conditions it may be used with reasonable accuracy.

To develop a suitable criterion consider steady-state conduction through the plane wall of area A (Figure 5.3). Although we are assuming steady-state conditions, this criterion is readily extended to transient processes. One surface is maintained at a temperature $T_{s,1}$ and the other surface is exposed to a fluid of temperature $T_\infty < T_{s,1}$. The temperature of this surface will be some intermediate value, $T_{s,2}$, for which $T_\infty < T_{s,2} < T_{s,1}$. Hence under steady-state conditions the surface energy balance, Equation 1.12, reduces to

$$\frac{kA}{L}(T_{s,1} - T_{s,2}) = hA(T_{s,2} - T_\infty)$$

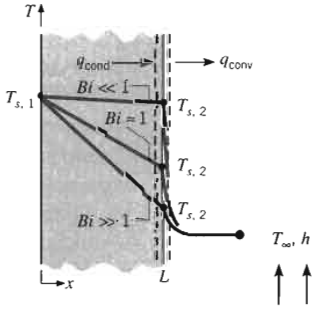


FIGURE 5.3
Effect of Biot number on steady-state temperature distribution in a plane wall with surface convection.

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where k is the thermal conductivity of the solid. Rearranging, we then obtain

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{(L/kA)}{(1/hA)} = \frac{R_{cond}}{R_{conv}} = \frac{hL}{k} \equiv Bi \tag{5.9}$$

The quantity (hL/k) appearing in Equation 5.9 is a *dimensionless parameter.* It is termed the *Biot number,* and it plays a fundamental role in conduction problems that involve surface convection effects. According to Equation 5.9 and as illustrated in Figure 5.3, the Biot number provides a measure of the temperature drop in the solid relative to the temperature difference between the surface and the fluid. Note especially the conditions corresponding to $Bi \ll 1$. The results suggest that, for these conditions, it is reasonable to *assume* a uniform temperature distribution within a solid at any time during a transient process. This result may also be associated with interpretation of the Biot number as a ratio of thermal resistances, Equation 5.9. *If* $Bi \ll 1$, *the resistance to conduction within the solid is much less than the resistance to convection across the fluid boundary layer.* Hence the assumption of a uniform temperature distribution is reasonable.

We have introduced the Biot number because of its significance to transient conduction problems. Consider the plane wall of Figure 5.4, which is initially at a uniform temperature T_i and experiences convection cooling when it is immersed in a fluid of $T_\infty < T_i$. The problem may be treated as one-dimensional in x, and we are interested in the temperature variation with position and time, $T(x, t)$. This variation is a strong function of the Biot number, and three conditions are shown in Figure 5.4. For $Bi \ll 1$ the temperature gradient in the solid is small and $T(x, t) \approx T(t)$. Virtually all the temperature difference is between the solid and the fluid, and the solid temperature remains nearly uniform as it decreases to T_∞. For moderate to large values of the Biot number, however, the temperature gradients within the solid are significant. Hence $T = T(x, t)$. Note that for $Bi \gg 1$, the temperature difference across the solid is much larger than that between the surface and the fluid.

We conclude this section by emphasizing the importance of the lumped capacitance method. Its inherent simplicity renders it the preferred method for solving

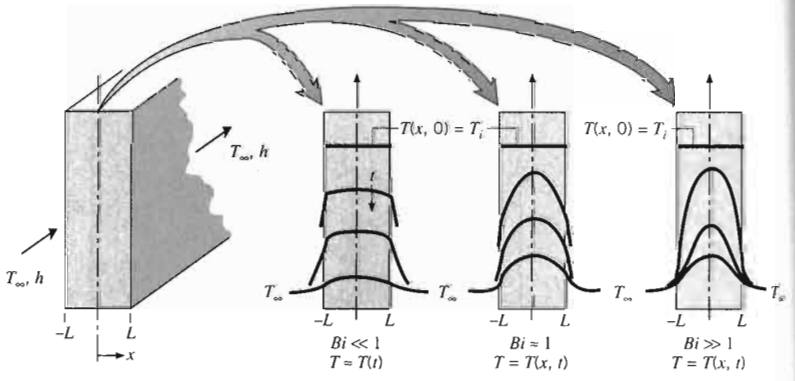


FIGURE 5.4 Transient temperature distributions for different Biot numbers in a plane wall symmetrically cooled by convection.

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transient heating and cooling problems. Hence, when confronted with such a problem, *the very first thing that one should do is calculate the Biot number.* If the following condition is satisfied

$$Bi = \frac{hL_c}{k} < 0.1 \tag{5.10}$$

the error associated with using the lumped capacitance method is small. For convenience, it is customary to define the *characteristic length* of Equation 5.10 as the ratio of the solid's volume to surface area, $L_c \equiv V/A_s$. Such a definition facilitates calculation of L_c for solids of complicated shape and reduces to the half-thickness L for a plane wall of thickness $2L$ (Figure 5.4), to $r_o/2$ for a long cylinder, and to $r_o/3$ for a sphere. However, if one wishes to implement the criterion in a conservative fashion, L_c should be associated with the length scale corresponding to the maximum spatial temperature difference. Accordingly, for a symmetrically heated (or cooled) plane wall of thickness $2L$, L_c would remain equal to the half-thickness L. However, for a long cylinder or sphere, L_c would equal the actual radius r_o, rather than $r_o/2$ or $r_o/3$.

Finally, we note that, with $L_c \equiv V/A_s$, the exponent of Equation 5.6 may be expressed as

$$\frac{hA_s t}{\rho V c} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2}$$

or

$$\frac{hA_s t}{\rho V c} = Bi \cdot Fo \tag{5.11}$$

where

$$Fo \equiv \frac{\alpha t}{L_c^2} \tag{5.12}$$

is termed the Fourier number. It is a *dimensionless time,* which, with the Biot number, characterizes transient conduction problems. Substituting Equation 5.11 into 5.6, we obtain

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp(-Bi \cdot Fo) \tag{5.13}$$

EXAMPLE 5.1

A thermocouple junction, which may be approximated as a sphere, is to be used for temperature measurement in a gas stream. The convection coefficient between the junction surface and the gas is $h = 400\ \text{W/m}^2 \cdot \text{K}$, and the junction thermophysical properties are $k = 20\ \text{W/m} \cdot \text{K}$, $c = 400\ \text{J/kg} \cdot \text{K}$, and $\rho = 8500\ \text{kg/m}^3$. Determine the junction diameter needed for the thermocouple to have a time constant of 1 s. If the junction is at 25°C and is placed in a gas stream that is at 200°C, how long will it take for the junction to reach 199°C?

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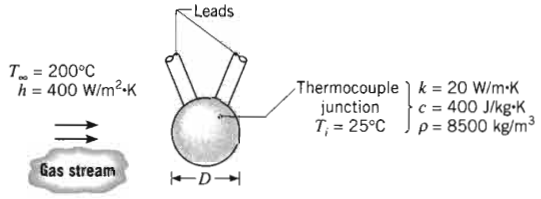
SOLUTION

Known: Thermophysical properties of thermocouple junction used to measure temperature of a gas stream.

Find:

1. Junction diameter needed for a time constant of 1 s.
2. Time required to reach 199°C in gas stream at 200°C.

Schematic:



Assumptions:

1. Temperature of junction is uniform at any instant.
2. Radiation exchange with the surroundings is negligible.
3. Losses by conduction through the leads are negligible.
4. Constant properties.

Analysis:

1. Because the junction diameter is unknown, it is not possible to begin the solution by determining whether the criterion for using the lumped capacitance method, Equation 5.10, is satisfied. However, a reasonable approach is to use the method to find the diameter and to then determine whether the criterion is satisfied. From Equation 5.7 and the fact that $A_s = \pi D^2$ and $V = \pi D^3/6$ for a sphere, it follows that

$$\tau_t = \frac{1}{h\pi D^2} \times \frac{\rho \pi D^3}{6} c$$

Rearranging and substituting numerical values,

$$D = \frac{6h\tau_t}{\rho c} = \frac{6 \times 400\ \text{W/m}^2 \cdot \text{K} \times 1\ \text{s}}{8500\ \text{kg/m}^3 \times 400\ \text{J/kg} \cdot \text{K}} = 7.06 \times 10^{-4}\ \text{m}$$

With $L_c = r_o/3$ it then follows from Equation 5.10 that

$$Bi = \frac{h(r_o/3)}{k} = \frac{400\ \text{W/m}^2 \cdot \text{K} \times 3.53 \times 10^{-4}\ \text{m}}{3 \times 20\ \text{W/m} \cdot \text{K}} = 2.35 \times 10^{-3}$$

Accordingly, Equation 5.10 is satisfied (for $L_c = r_o$, as well as for $L_c = r_o/3$) and the lumped capacitance method may be used to an excellent approximation.

5.3 General Lu

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2. From Equation 5.5 the time required for the junction to reach $T = 199°C$ is

$$t = \frac{\rho(\pi D^3/6)c}{h(\pi D^2)} \ln \frac{T_i - T_\infty}{T - T_\infty} = \frac{\rho D c}{6h} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

$$t = \frac{8500\ \text{kg/m}^3 \times 7.06 \times 10^{-4}\ \text{m} \times 400\ \text{J/kg} \cdot \text{K}}{6 \times 400\ \text{W/m}^2 \cdot \text{K}} \ln \frac{25 - 200}{199 - 200}$$

$$t = 5.2\ \text{s} \approx 5\tau_t \qquad \triangleleft$$

Comments: Heat transfer due to radiation exchange between the junction and the surroundings and conduction through the leads would affect the time response of the junction and would, in fact, yield an equilibrium temperature that differs from T_∞.

5.3 General Lumped Capacitance Analysis

Although transient conduction in a solid is commonly initiated by convection heat transfer to or from an adjoining fluid, other processes may induce transient thermal conditions within the solid. For example, a solid may be separated from large surroundings by a gas or vacuum. If the temperatures of the solid and surroundings differ, radiation exchange could cause the internal thermal energy, and hence the temperature, of the solid to change. Temperature changes could also be induced by applying a heat flux at a portion, or all, of the surface and/or by initiating thermal energy generation within the solid. Surface heating could, for example, be applied by attaching a film or sheet electrical heater to the surface, while thermal energy could be generated by passing an electrical current through the solid.

Figure 5.5 depicts a situation for which thermal conditions within a solid may be influenced simultaneously by convection, radiation, an applied surface heat flux, and internal energy generation. It is presumed that, initially ($t = 0$), the temperature of the solid (T_i) differs from that of the fluid, T_∞, and the surroundings, T_{sur}, and that both surface and volumetric heating (q_s'' and $\dot{q}$) are initiated. The imposed heat flux

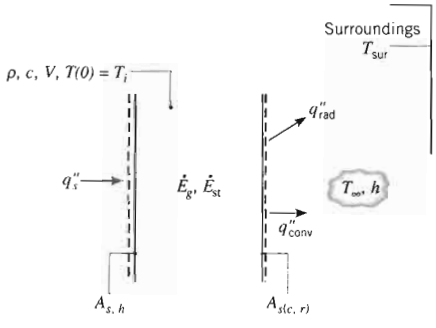


FIGURE 5.5
Control surface for general lumped capacitance analysis.

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q_s'' and the convection–radiation heat transfer occur at mutually exclusive portions of the surface, $A_{s(h)}$ and $A_{s(c,r)}$, respectively, and convection–radiation transfer is presumed to be *from* the surface. Moreover, although convection and radiation have been prescribed for the same surface, the surfaces may, in fact, differ ($A_{s,c} \neq A_{s,r}$). Applying conservation of energy at any instant t, it follows from Equation 1.11c that

$$q_s'' A_{s,h} + \dot{E}_g - (q''_{\text{conv}} + q''_{\text{rad}}) A_{s(c,r)} = \rho V c \frac{dT}{dt} \tag{5.14}$$

or, from Equations 1.3a and 1.7,

$$q_s'' A_{s,h} + \dot{E}_g - [h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{\text{sur}}^4)] A_{s(c,r)} = \rho V c \frac{dT}{dt} \tag{5.15}$$

Equation 5.15 is a nonlinear, first-order, nonhomogeneous, ordinary differential equation that cannot be integrated to obtain an exact solution.[1] However, exact solutions may be obtained for simplified versions of the equation. For example, if there is no imposed heat flux or generation and convection is either nonexistent (a vacuum) or negligible relative to radiation, Equation 5.15 reduces to

$$\rho V c \frac{dT}{dt} = -\varepsilon A_{s,r} \sigma (T^4 - T_{\text{sur}}^4) \tag{5.16}$$

Separating variables and integrating from the initial condition to any time t, it follows that

$$\frac{\varepsilon A_{s,r} \sigma}{\rho V c} \int_0^t dt = \int_{T_i}^{T} \frac{dT}{T_{\text{sur}}^4 - T^4} \tag{5.17}$$

Evaluating both integrals and rearranging, the time required to reach the temperature T becomes

$$t = \frac{\rho V c}{4\varepsilon A_{s,r} \sigma T_{\text{sur}}^3} \left\{ \ln \left| \frac{T_{\text{sur}} + T}{T_{\text{sur}} - T} \right| - \ln \left| \frac{T_{\text{sur}} + T_i}{T_{\text{sur}} - T_i} \right| + 2 \left[\tan^{-1} \left(\frac{T}{T_{\text{sur}}} \right) - \tan^{-1} \left(\frac{T_i}{T_{\text{sur}}} \right) \right] \right\} \tag{5.18}$$

This expression cannot be used to evaluate T explicitly in terms of t, T_i, and T_{sur}, nor does it readily reduce to the limiting result for $T_{\text{sur}} = 0$ (radiation to deep space). However, returning to Equation 5.17, its solution for $T_{\text{sur}} = 0$ yields

$$t = \frac{\rho V c}{3\varepsilon A_{s,r} \sigma} \left(\frac{1}{T^3} - \frac{1}{T_i^3} \right) \tag{5.19}$$

An exact solution to Equation 5.15 may also be obtained if radiation may be neglected and h is independent of time. Introducing a temperature difference $\theta \equiv T - T_\infty$, where $d\theta/dt = dT/dt$, Equation 5.15 reduces to a linear, first-order, nonhomogeneous differential equation of the form

$$\frac{d\theta}{dt} + a\theta - b = 0 \tag{5.20}$$

[1] An approximate, finite-difference solution may be obtained by *discretizing* the time derivative (Section 5.10) and *marching* the solution out in time.

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where $a \equiv (hA_{s,c}/\rho Vc)$ and $b \equiv [(q''_s A_{s,h} + \dot{E}_g)/\rho Vc]$. Although Equation 5.20 may be solved by summing its homogeneous and particular solutions, an alternative approach is to eliminate the nonhomogeneity by introducing the transformation

$$\theta' \equiv \theta - \frac{b}{a} \tag{5.21}$$

Recognizing that $d\theta'/dt = d\theta/dt$, Equation 5.21 may be substituted into (5.20) to yield

$$\frac{d\theta'}{dt} + a\theta' = 0 \tag{5.22}$$

Separating variables and integrating from 0 to t (θ'_i to θ'), it follows that

$$\frac{\theta'}{\theta'_i} = \exp(-at) \tag{5.23}$$

or substituting for θ' and θ,

$$\frac{T - T_\infty - (b/a)}{T_i - T_\infty - (b/a)} = \exp(-at) \tag{5.24}$$

Hence

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}[1 - \exp(-at)] \tag{5.25}$$

As it must, Equation 5.25 reduces to Equation 5.6 when $b = 0$ and yields $T = T_i$ at $t = 0$. As $t \to \infty$, Equation 5.25 reduces to $(T - T_\infty) = (b/a)$, which could also be obtained by performing an energy balance on the control surface of Figure 5.5 for steady-state conditions.

EXAMPLE 5.2

Consider the thermocouple and convection conditions of Example 5.1, but now allow for radiation exchange with the walls of a duct that encloses the gas stream. If the duct walls are at 400°C and the emissivity of the thermocouple bead is 0.9, calculate the steady-state temperature of the junction. Also, determine the time for the junction temperature to increase from an initial condition of 25°C to a temperature that is within 1°C of its steady-state value.

SOLUTION

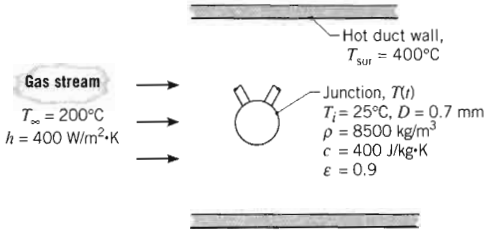
Known: Thermophysical properties and diameter of the thermocouple junction used to measure temperature of a gas stream passing through a duct with hot walls.

Find:

1. Steady-state temperature of the junction.
2. Time required for the thermocouple to reach a temperature that is within 1°C of its steady-state value.

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Schematic:



Assumptions: Same as Example 5.1, but radiation transfer is no longer treated as negligible and is approximated as exchange between a small surface and large surroundings.

Analysis:

1. For steady-state conditions, the energy balance on the thermocouple junction has the form

$$\dot{E}_{in} - \dot{E}_{out} = 0$$

Recognizing that net radiation to the junction must be balanced by convection from the junction to the gas, the energy balance may be expressed as

$$\left[\varepsilon\sigma(T_{sur}^4 - T^4) - h(T - T_\infty)\right]A_s = 0$$

Substituting numerical values, we obtain

$$T = 218.7°C \quad \triangleleft$$

2. The temperature-time history, $T(t)$, for the junction, initially at $T(0) = T_i = 25°C$, follows from the energy balance for transient conditions,

$$\dot{E}_{in} - \dot{E}_{out} = \dot{E}_{st}$$

From Equation 5.15, the energy balance may be expressed as

$$-\left[h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)\right]A_s = \rho V c \frac{dT}{dt}$$

The solution to this first-order differential equation can be obtained by numerical integration, giving the result, $T(4.9\text{ s}) = 217.7°C$. Hence, the time required to reach a temperature that is within 1°C of the steady-state value is

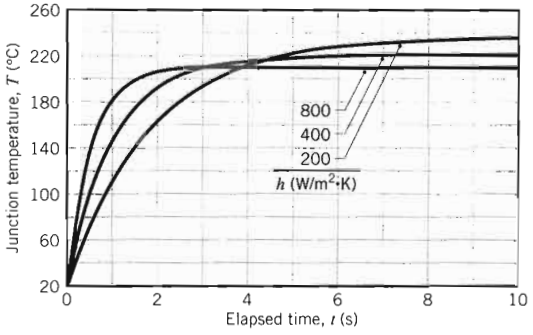
$$t = 4.9\text{ s}. \quad \triangleleft$$

Comments:

1. The effect of radiation exchange with the hot duct walls is to increase the junction temperature, such that the thermocouple indicates an erroneous gas stream temperature that exceeds the actual temperature by 18.7°C. The time required to reach a temperature that is within 1°C of the steady-state value is slightly less than the result of Example 5.1, which only considers convection heat transfer. Why is this so?

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2. The response of the thermocouple and the indicated gas stream temperature depend on the velocity of the gas stream, which in turn affects the magnitude of the convection coefficient. Temperature-time histories for the thermocouple junction are shown in the following graph for values of $h = 200$, 400, and 800 W/m$^2 \cdot$ K.



The effect of increasing the convection coefficient is to cause the junction to indicate a temperature closer to that of the gas stream. Further, the effect is to reduce the time required for the junction to reach the near-steady-state condition. What physical explanation can you give for these results?

3. The *IHT* software accompanying this text includes an integral function, Der(T, t), that can be used to represent the temperature-time derivative and to integrate first-order differential equations. For this problem, an energy balance relation of the following form may be entered into the *Workspace*,

```
(-h*(T - Tinf) - eps*sigma*(T^4 - Tsur^4))*As = rho*V*c*Der(T, t)
```

After hitting the *Solve* button, the *Diff/Integral Equations* pad will appear, identifying the independent variable t and providing boxes (*Start*, *Stop*, and *Step*) for entering the integration limits and the time increment Δt, respectively, as well as the initial condition (*IC*). Alternatively, the *IHT Lumped Capacitance Model* can be used to build a model that includes the phenomena of this problem and to perform the numerical integration.

EXAMPLE 5.3

A 3-mm-thick panel of aluminum alloy ($k = 177$ W/m $\cdot$ K, $c = 875$ J/kg $\cdot$ K, and $\rho = 2770$ kg/m^3) is finished on both sides with an epoxy coating that must be cured at or above $T_c = 150$°C for at least 5 min. The production line for the curing operation involves two steps: (1) heating in a large oven with air at $T_{\infty,o} = 175$°C and a convection coefficient of $h_o = 40$ W/m$^2 \cdot$ K, and (2) cooling in a large chamber with air at $T_{\infty,c} = 25$°C and a convection coefficient of $h_c = 10$ W/m$^2 \cdot$ K. The heating portion of the process is conducted over a time interval t_e, which exceeds the time t_c required to reach 150°C by 5 min ($t_e = t_c + 300$ s). The coating has an emissivity of $\varepsilon = 0.8$, and the temperatures of the oven and chamber walls are 175°C and 25°C, respectively. If the panel is placed in the oven at an initial temperature of 25°C and

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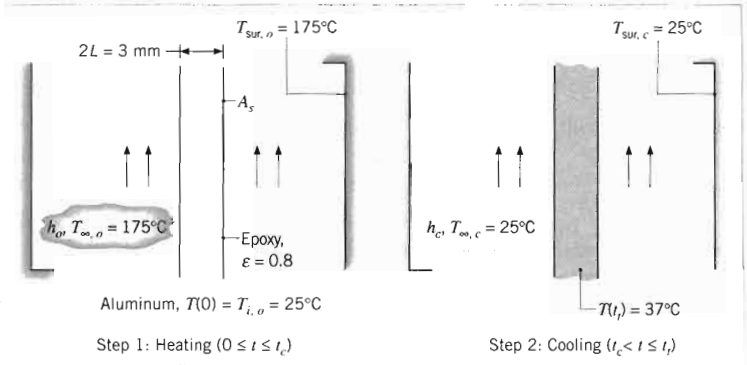
removed from the chamber at a *safe-to-touch* temperature of 37°C, what is the total elapsed time for the two-step curing operation?

SOLUTION

Known: Operating conditions for a two-step heating/cooling process in which a coated aluminum panel is maintained at or above a temperature of 150°C for at least 5 min.

Find: Total time t_f required for the two-step process.

Schematic:



Assumptions:

1. Panel temperature is uniform at any instant.
2. Thermal resistance of epoxy is negligible.
3. Constant properties.

Analysis: To assess the validity of the lumped capacitance approximation, we begin by calculating Biot numbers for the heating and cooling processes.

$$Bi_h = \frac{h_o L}{k} = \frac{(40\ \text{W/m}^2 \cdot \text{K})(0.0015\ \text{m})}{177\ \text{W/m} \cdot \text{K}} = 3.4 \times 10^{-4}$$

$$Bi_c = \frac{h_c L}{k} = \frac{(10\ \text{W/m}^2 \cdot \text{K})(0.0015\ \text{m})}{177\ \text{W/m} \cdot \text{K}} = 8.5 \times 10^{-5}$$

Hence the lumped capacitance approximation is excellent.

To determine whether radiation exchange between the panel and its surroundings should be considered, the radiation heat transfer coefficient is determined from Equation 1.9. A representative value of h_r for the heating process is associated with the cure condition, in which case

$$\begin{aligned} h_{r,o} &= \varepsilon\sigma(T_c + T_{sur,o})(T_c^2 + T_{sur,o}^2) \\ &= 0.8 \times 5.67 \times 10^{-8}\ \text{W/m}^2 \cdot \text{K}^4(423 + 448)\text{K}(423^2 + 448^2)\text{K}^2 \\ &= 15\ \text{W/m}^2 \cdot \text{K} \end{aligned}$$

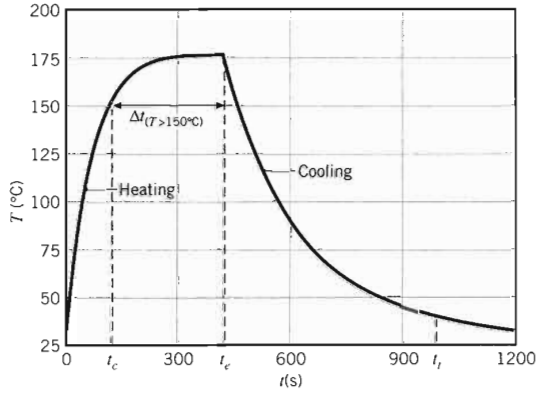
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Using $T_c = 150°C$ with $T_{sur,c} = 25°C$ for the cooling process, we also obtain $h_{r,c} = 8.8\ W/m^2 \cdot K$. Since the values of $h_{r,o}$ and $h_{r,c}$ are comparable to those of h_o and h_c, respectively, radiation effects must be considered.

With $V = 2LA_s$ and $A_{s,c} = A_{s,r} = 2A_s$, Equation 5.15 may be expressed as

$$\int_{T_i}^{T} dT = T(t) - T_i = -\frac{1}{\rho c L}\int_0^t [h(T - T_\infty) + \varepsilon\sigma(T^4 - T_{sur}^4)]dt$$

Selecting a suitable time increment Δt, the right-hand side of this equation may be evaluated numerically to obtain the panel temperature at $t = \Delta t$, $2\Delta t$, $3\Delta t$, and so on. At each new step of the calculation, the value of T computed from the previous time step is used in the integrand. Selecting $\Delta t = 10$ s, calculations for the heating process are extended to $t_e = t_c + 300$ s, which is 5 min beyond the time required for the panel to reach $T_c = 150°C$. At t_e the cooling process is initiated and continued until the panel temperature reaches 37°C at $t = t_f$. The integration was performed using a fourth-order Runge-Kutta scheme, and results of the calculations are plotted as follows:



The total time for the two-step process is

$$t_f = 989\ s \qquad \triangleleft$$

with intermediate times of $t_c = 124$ s and $t_e = 424$ s.

Comments:

1. Generally, the accuracy of a numerical integration improves with decreasing Δt, but at the expense of increased computation time. In this case, however, results obtained for $\Delta t = 1$ s are virtually identical to those obtained for $\Delta t = 10$ s, indicating that the larger time interval is sufficient to accurately depict the temperature history.
2. The duration of the two-step process may be reduced by increasing the convection coefficients and/or by reducing the period of extended heating. The second option is made possible by the fact that, during a portion of the cooling period, the panel temperature remains above 150°C. Hence, to satisfy the cure requirement, it is not necessary to extend heating for as much as 5 min from $t = t_c$. If the convection coefficients are increased to $h_o = h_c = 100\ W/m^2 \cdot K$ and an

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extended heating period of 300 s is maintained, the numerical integration yields $t_c = 58$ s and $t_t = 445$ s. The corresponding time interval over which the panel temperature exceeds 150°C is $\Delta t_{(T>150°C)} = 306$ s (58 s $\leq t \leq$ 364 s). If the extended heating period is reduced to 294 s, the numerical integration yields $t_c = 58$ s, $t_t = 439$ s, and $\Delta t_{(T>150°C)} = 300$ s. Hence the total process time is reduced, while the curing requirement is still satisfied.

3. The solution for this example is provided as a ready-to-solve model in *IHT*, with annotations on how the code is written and solved. Look under *Examples* on the *Toolbar*. Note how the *User-Defined Function* feature of *IHT* (*.udf) is used to represent the time dependence of the ambient temperature in the two-step heating and cooling process, thereby allowing use of a single form of the energy balance relation for the entire process. The model can be used to check the results of Comment 2 or to independently explore modifications of the cure process.

5.4 *Spatial Effects*

Situations frequently arise for which the lumped capacitance method is inappropriate, and alternative approaches must be used. Regardless of the particular method, we must now cope with the fact that temperature gradients within the medium are no longer negligible.

In their most general form, transient conduction problems are described by the heat equation, Equation 2.17, for rectangular coordinates or Equations 2.24 and 2.27, respectively, for cylindrical and spherical coordinates. The solutions to these partial differential equations provide the variation of temperature with both time and the spatial coordinates. However, in many problems, such as the plane wall of Figure 5.4, only one spatial coordinate is needed to describe the internal temperature distribution. With no internal generation and the assumption of constant thermal conductivity, Equation 2.17 then reduces to

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{5.26}$$

To solve Equation 5.26 for the temperature distribution $T(x, t)$, it is necessary to specify an *initial* condition and two *boundary conditions*. For the typical transient conduction problem of Figure 5.4, the initial condition is

$$T(x, 0) = T_i \tag{5.27}$$

and the boundary conditions are

$$\left.\frac{\partial T}{\partial x}\right|_{x=0} = 0 \tag{5.28}$$

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and

$$-k\left.\frac{\partial T}{\partial x}\right|_{x=L} = h[T(L,t) - T_\infty] \tag{5.29}$$

Equation 5.27 presumes a uniform temperature distribution at time $t = 0$; Equation 5.28 reflects the *symmetry requirement* for the midplane of the wall; and Equation 5.29 describes the surface condition experienced for time $t > 0$. From Equations 5.26 through 5.29, it is evident that, in addition to depending on x and t, temperatures in the wall also depend on a number of physical parameters. In particular

$$T = T(x, t, T_i, T_\infty, L, k, \alpha, h) \tag{5.30}$$

The foregoing problem may be solved analytically or numerically. These methods will be considered in subsequent sections, but first it is important to note the advantages that may be obtained by *nondimensionalizing* the governing equations. This may be done by arranging the relevant variables into suitable *groups*. Consider the dependent variable T. If the temperature difference $\theta \equiv T - T_\infty$ is divided by the *maximum possible temperature difference* $\theta_i \equiv T_i - T_\infty$, a dimensionless form of the dependent variable may be defined as

$$\theta^* \equiv \frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} \tag{5.31}$$

Accordingly, θ^* must lie in the range $0 \le \theta^* \le 1$. A dimensionless spatial coordinate may be defined as

$$x^* \equiv \frac{x}{L} \tag{5.32}$$

where L is the half-thickness of the plane wall, and a dimensionless time may be defined as

$$t^* \equiv \frac{\alpha t}{L^2} \equiv Fo \tag{5.33}$$

where t^* is equivalent to the dimensionless *Fourier number*, Equation 5.12.

Substituting the definitions of Equations 5.31 through 5.33 into Equations 5.26 through 5.29, the heat equation becomes

$$\frac{\partial^2 \theta^*}{\partial x^{*2}} = \frac{\partial \theta^*}{\partial Fo} \tag{5.34}$$

and the initial and boundary conditions become

$$\theta^*(x^*, 0) = 1 \tag{5.35}$$

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=0} = 0 \tag{5.36}$$

and

$$\left.\frac{\partial \theta^*}{\partial x^*}\right|_{x^*=1} = -Bi\,\theta^*(1, t^*) \tag{5.37}$$

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where the *Biot number* is $Bi \equiv hL/k$. In dimensionless form the functional dependence may now be expressed as

$$\theta^* = f(x^*, Fo, Bi) \tag{5.38}$$

Recall that a similar functional dependence, without the x^* variation, was obtained for the lumped capacitance method, as shown in Equation 5.13.

Although the Fourier number may be viewed as a dimensionless time, it has an important physical interpretation when used in problems for which heat transfer by conduction through a solid is concurrent with thermal energy storage by the solid. With a characteristic length of L, the temperature gradient across a solid and the cross-sectional area perpendicular to conduction heat transfer may be approximated as $\Delta T/L$ and L^2, respectively. Hence, as a first-order approximation, the heat rate may be expressed as $q \sim kL^2\Delta T/L$. Similarly, with the volume of the solid represented as $V \sim L^3$, the rate of change of thermal energy storage by the solid may be approximated as $\dot{E}_{st} \sim \rho L^3 c\Delta T/t$. It follows that $(q/\dot{E}_{st}) \sim kt/\rho cL^2 = \alpha t/L^2 = Fo$. Hence, the Fourier number provides a measure of the relative effectiveness with which a solid conducts and stores thermal energy.

Comparing Equations 5.30 and 5.38, the considerable advantage associated with casting the problem in dimensionless form becomes apparent. Equation 5.38 implies that *for a prescribed geometry, the transient temperature distribution is a universal function of x^*, Fo, and Bi.* That is, the *dimensionless solution* has a prescribed form that does not depend on the particular value of T_i, T_∞, L, k, α, or h. Since this generalization greatly simplifies the presentation and utilization of transient solutions, the dimensionless variables are used extensively in subsequent sections.

5.5 The Plane Wall with Convection

Exact, analytical solutions to transient conduction problems have been obtained for many simplified geometries and boundary conditions and are well documented [1–4]. Several mathematical techniques, including the method of separation of variables (Section 4.2), may be used for this purpose, and typically the solution for the dimensionless temperature distribution, Equation 5.38, is in the form of an infinite series. However, except for very small values of the Fourier number, this series may be approximated by a single term.

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Section 5S.

5.5.1 Exact Solution

Consider the *plane wall* of thickness $2L$ (Figure 5.6*a*). If the thickness is small relative to the width and height of the wall, it is reasonable to assume that conduction occurs exclusively in the x direction. If the wall is initially at a uniform temperature, $T(x, 0) = T_i$, and is suddenly immersed in a fluid of $T_\infty \neq T_i$, the resulting temperatures may be obtained by solving Equation 5.34 subject to the conditions of Equations 5.35 through 5.37. Since the convection conditions for the surfaces at

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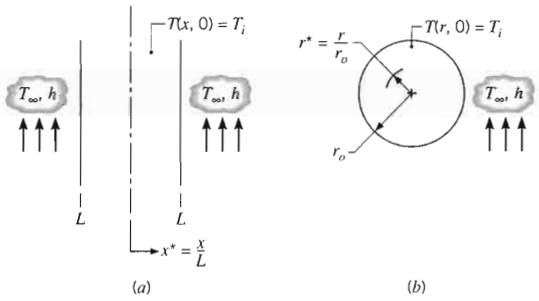


FIGURE 5.6 One-dimensional systems with an initial uniform temperature subjected to sudden convection conditions. (*a*) Plane wall. (*b*) Infinite cylinder or sphere.

$x^* = \pm 1$ are the same, the temperature distribution at any instant must be symmetrical about the midplane ($x^* = 0$). An exact solution to this problem is of the form [2]

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo) \cos(\zeta_n x^*) \tag{5.39a}$$

where $Fo = \alpha t/L^2$, the coefficient C_n is

$$C_n = \frac{4 \sin \zeta_n}{2\zeta_n + \sin(2\zeta_n)} \tag{5.39b}$$

and the discrete values of ζ_n (*eigenvalues*) are positive roots of the transcendental equation

$$\zeta_n \tan \zeta_n = Bi \tag{5.39c}$$

The first four roots of this equation are given in Appendix B.3.

5.5.2 Approximate Solution

It can be shown (Problem 5.34) that for values of $Fo > 0.2$, the infinite series solution, Equation 5.39a, can be approximated by the first term of the series. Invoking this approximation, the dimensionless form of the temperature distribution becomes

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo) \cos(\zeta_1 x^*) \tag{5.40a}$$

or

$$\theta^* = \theta_o^* \cos(\zeta_1 x^*) \tag{5.40b}$$

where $\theta_o^* \equiv (T_o - T_\infty)/(T_i - T_\infty)$ represents the midplane ($x^* = 0$) temperature

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo) \tag{5.41}$$

An important implication of Equation 5.40b is that *the time dependence of the temperature at any location within the wall is the same as that of the midplane temperature.* The coefficients C_1 and ζ_1 are evaluated from Equations 5.39b and 5.39c, respectively, and are given in Table 5.1 for a range of Biot numbers.

Graphical representations of the one-term approximations are presented in Section 5S.1.

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TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

5.5.3 Total Energy Transfer

In many situations it is useful to know the total energy that has left (or entered) the wall up to any time t in the transient process. The conservation of energy requirement,

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Equation 1.11b, may be applied for the time interval bounded by the initial condition ($t = 0$) and any time $t > 0$

$$E_{in} - E_{out} = \Delta E_{st} \tag{5.42}$$

Equating the energy transferred from the wall Q to E_{out} and setting $E_{in} = 0$ and $\Delta E_{st} = E(t) - E(0)$, it follows that

$$Q = -[E(t) - E(0)] \tag{5.43a}$$

or

$$Q = -\int \rho c[T(x, t) - T_i]dV \tag{5.43b}$$

where the integration is performed over the volume of the wall. It is convenient to nondimensionalize this result by introducing the quantity

$$Q_o = \rho cV(T_i - T_\infty) \tag{5.44}$$

which may be interpreted as the initial internal energy of the wall relative to the fluid temperature. It is also the *maximum* amount of energy transfer that could occur if the process were continued to time $t = \infty$. Hence, assuming constant properties, the ratio of the total energy transferred from the wall over the time interval t to the maximum possible transfer is

$$\frac{Q}{Q_o} = \int \frac{-[T(x, t) - T_i]}{T_i - T_\infty} \frac{dV}{V} = \frac{1}{V}\int (1 - \theta^*)dV \tag{5.45}$$

Employing the approximate form of the temperature distribution for the plane wall, Equation 5.40b, the integration prescribed by Equation 5.45 can be performed to obtain

$$\frac{Q}{Q_o} = 1 - \frac{\sin \zeta_1}{\zeta_1}\theta_o^* \tag{5.46}$$

where θ_o^* can be determined from Equation 5.41, using Table 5.1 for values of the coefficients C_1 and ζ_1.

5.5.4 Additional Considerations

Because the mathematical problem is precisely the same, the foregoing results may also be applied to a plane wall of thickness L, which is insulated on one side ($x^* = 0$) and experiences convective transport on the other side ($x^* = +1$). This equivalence is a consequence of the fact that, regardless of whether a symmetrical or an adiabatic requirement is prescribed at $x^* = 0$, the boundary condition is of the form $\partial\theta^*/\partial x^* = 0$.

It should also be noted that the foregoing results may be used to determine the transient response of a plane wall to a sudden change in *surface* temperature. The process is equivalent to having an infinite convection coefficient, in which case the Biot number is infinite ($Bi = \infty$) and the fluid temperature T_∞ is replaced by the prescribed surface temperature T_s.

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nduction

Sphere	
ζ_1 (rad)	C_1
.1730	1.0030
.2445	1.0060
.2991	1.0090
3450	1.0120
3854	1.0149
4217	1.0179
4551	1.0209
4860	1.0239
5150	1.0268
5423	1.0298
5609	1.0445
5593	1.0592
5447	1.0737
5208	1.0880
5528	1.1164
5556	1.1441
5544	1.1713
5525	1.1978
5520	1.2236
544	1.2488
508	1.2732
488	1.4793
489	1.6227
556	1.7202
594	1.7870
567	1.8338
565	1.8673
54	1.8920
54	1.9106
53	1.9249
57	1.9781
52	1.9898
52	1.9942
53	1.9962
51	1.9990
	2.0000

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5.6 *Radial Systems with Convection*

For an infinite cylinder or sphere of radius r_o (Figure 5.6*b*), which is at an initial uniform temperature and experiences a change in convective conditions, results similar to those of Section 5.5 may be developed. That is, an exact series solution may be obtained for the time dependence of the radial temperature distribution, and a one-term approximation may be used for most conditions. The infinite cylinder is an idealization that permits the assumption of one-dimensional conduction in the radial direction. It is a reasonable approximation for cylinders having $L/r_o \gtrsim 10$.

Graphical rep tations of the term approxi are presented Section 5S.1.

5.6.1 Exact Solutions

For a uniform initial temperature and convective boundary conditions, the exact solutions [2] are as follows.

Infinite Cylinder In dimensionless form, the temperature is

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo) J_0(\zeta_n r^*) \tag{5.47a}$$

Graphical re tations of the term approxi are presented Section 5S.1.

where $Fo = \alpha t/r_o^2$,

$$C_n = \frac{2}{\zeta_n} \frac{J_1(\zeta_n)}{J_0^2(\zeta_n) + J_1^2(\zeta_n)} \tag{5.47b}$$

and the discrete values of ζ_n are positive roots of the transcendental equation

$$\zeta_n \frac{J_1(\zeta_n)}{J_0(\zeta_n)} = Bi \tag{5.47c}$$

where $Bi = hr_o/k$. The quantities J_1 and J_0 are Bessel functions of the first kind and their values are tabulated in Appendix B.4. Roots of the transcendental equation (5.47c) are tabulated by Schneider [2].

Sphere Similarly, for the sphere

$$\theta^* = \sum_{n=1}^{\infty} C_n \exp(-\zeta_n^2 Fo) \frac{1}{\zeta_n r^*} \sin(\zeta_n r^*) \tag{5.48a}$$

where $Fo = \alpha t/r_o^2$,

$$C_n = \frac{4[\sin(\zeta_n) - \zeta_n \cos(\zeta_n)]}{2\zeta_n - \sin(2\zeta_n)} \tag{5.48b}$$

and the discrete values of ζ_n are positive roots of the transcendental equation

$$1 - \zeta_n \cot \zeta_n = Bi \tag{5.48c}$$

where $Bi = hr_o/k$. Roots of the transcendental equation are tabulated by Schneider [2].

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5.6.2 Approximate Solutions

For the infinite cylinder and sphere, the foregoing series solutions can again be approximated by a single term for $Fo > 0.2$. Hence, as for the case of the plane wall, the time dependence of the temperature at any location within the radial system is the same as that of the centerline or centerpoint.

Graphical representations of the one-term approximations are presented in Section 5S.1.

Infinite Cylinder The one-term approximation to Equation 5.47a is

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo) J_0(\zeta_1 r^*) \tag{5.49a}$$

or

$$\theta^* = \theta_o^* J_0(\zeta_1 r^*) \tag{5.49b}$$

where θ_o^* represents the centerline temperature and is of the form

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo) \tag{5.49c}$$

Values of the coefficients C_1 and ζ_1 have been determined and are listed in Table 5.1 for a range of Biot numbers.

Graphical representations of the one-term approximations are presented in Section 5S.1.

Sphere From Equation 5.48a, the one-term approximation is

$$\theta^* = C_1 \exp(-\zeta_1^2 Fo) \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*) \tag{5.50a}$$

or

$$\theta^* = \theta_o^* \frac{1}{\zeta_1 r^*} \sin(\zeta_1 r^*) \tag{5.50b}$$

where θ_o^* represents the center temperature and is of the form

$$\theta_o^* = C_1 \exp(-\zeta_1^2 Fo) \tag{5.50c}$$

Values of the coefficients C_1 and ζ_1 have been determined and are listed in Table 5.1 for a range of Biot numbers.

5.6.3 Total Energy Transfer

As in Section 5.5.3, an energy balance may be performed to determine the total energy transfer from the infinite cylinder or sphere over the time interval $\Delta t = t$. Substituting from the approximate solutions, Equations 5.49b and 5.50b, and introducing Q_o from Equation 5.44, the results are as follows.

Infinite Cylinder

$$\frac{Q}{Q_o} = 1 - \frac{2\theta_o^*}{\zeta_1} J_1(\zeta_1) \tag{5.51}$$

Sphere

$$\frac{Q}{Q_o} = 1 - \frac{3\theta_o^*}{\zeta_1^3} [\sin(\zeta_1) - \zeta_1 \cos(\zeta_1)] \tag{5.52}$$

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Values of the center temperature θ_o^* are determined from Equation 5.49c or 5.50c, using the coefficients of Table 5.1 for the appropriate system.

5.6.4 Additional Considerations

As for the plane wall, the foregoing results may be used to predict the transient response of long cylinders and spheres subjected to a sudden change in *surface* temperature. Namely, an infinite Biot number is prescribed, and the fluid temperature T_∞ is replaced by the constant surface temperature T_s.

EXAMPLE 5.4

Consider a steel pipeline (AISI 1010) that is 1 m in diameter and has a wall thickness of 40 mm. The pipe is heavily insulated on the outside, and before the initiation of flow, the walls of the pipe are at a uniform temperature of $-20°C$. With the initiation of flow, hot oil at 60°C is pumped through the pipe, creating a convective condition corresponding to $h = 500\ W/m^2 \cdot K$ at the inner surface of the pipe.

1. What are the appropriate Biot and Fourier numbers 8 min after the initiation of flow?
2. At $t = 8$ min, what is the temperature of the exterior pipe surface covered by the insulation?
3. What is the heat flux q'' (W/m^2) to the pipe from the oil at $t = 8$ min?
4. How much energy per meter of pipe length has been transferred from the oil to the pipe at $t = 8$ min?

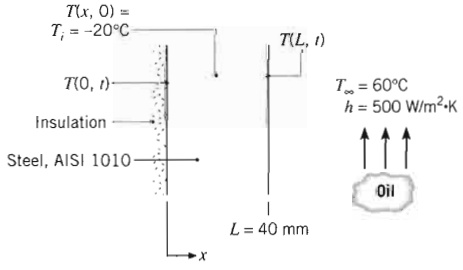
SOLUTION

Known: Wall subjected to sudden change in convective surface condition.

Find:

1. Biot and Fourier numbers after 8 min.
2. Temperature of exterior pipe surface after 8 min.
3. Heat flux to the wall at 8 min.
4. Energy transferred to pipe per unit length after 8 min.

Schematic:



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Assumptions:

1. Pipe wall can be approximated as plane wall, since thickness is much less than diameter.
2. Constant properties.
3. Outer surface of pipe is adiabatic.

Properties: Table A.1, steel type AISI 1010 [$T = (-20 + 60)°C/2 \approx 300$ K]: $\rho = 7832$ kg/m^3, $c = 434$ J/kg · K, $k = 63.9$ W/m · K, $\alpha = 18.8 \times 10^{-6}$ m^2/s.

Analysis:

1. At $t = 8$ min, the Biot and Fourier numbers are computed from Equations 5.10 and 5.12, respectively, with $L_c = L$. Hence

$$Bi = \frac{hL}{k} = \frac{500\ \text{W/m}^2 \cdot \text{K} \times 0.04\ \text{m}}{63.9\ \text{W/m} \cdot \text{K}} = 0.313 \quad \triangleleft$$

$$Fo = \frac{\alpha t}{L^2} = \frac{18.8 \times 10^{-6}\ \text{m}^2/\text{s} \times 8\ \text{min} \times 60\ \text{s/min}}{(0.04\ \text{m})^2} = 5.64 \quad \triangleleft$$

2. With $Bi = 0.313$, use of the lumped capacitance method is inappropriate. However, since $Fo > 0.2$ and transient conditions in the insulated pipe wall of thickness L correspond to those in a plane wall of thickness $2L$ experiencing the same surface condition, the desired results may be obtained from the one-term approximation for a plane wall. The midplane temperature can be determined from Equation 5.41

$$\theta_o^* = \frac{T_o - T_\infty}{T_i - T_\infty} = C_1 \exp(-\zeta_1^2 Fo)$$

where, with $Bi = 0.313$, $C_1 = 1.047$ and $\zeta_1 = 0.531$ rad from Table 5.1. With $Fo = 5.64$,

$$\theta_o^* = 1.047 \exp[-(0.531\ \text{rad})^2 \times 5.64] = 0.214$$

Hence after 8 min, the temperature of the exterior pipe surface, which corresponds to the midplane temperature of a plane wall, is

$$T(0, 8\ \text{min}) = T_\infty + \theta_o^*(T_i - T_\infty) = 60°C + 0.214(-20 - 60)°C = 42.9°C \quad \triangleleft$$

3. Heat transfer to the inner surface at $x = L$ is by convection, and at any time t the heat flux may be obtained from Newton's law of cooling. Hence at $t = 480$ s,

$$q_x''(L, 480\ \text{s}) \equiv q_L'' = h[T(L, 480\ \text{s}) - T_\infty]$$

Using the one-term approximation for the surface temperature, Equation 5.40b with $x^* = 1$ has the form

$$\theta^* = \theta_o^* \cos(\zeta_1)$$

$$T(L, t) = T_\infty + (T_i - T_\infty)\theta_o^* \cos(\zeta_1)$$

$$T(L, 8\ \text{min}) = 60°C + (-20 - 60)°C \times 0.214 \times \cos(0.531\ \text{rad})$$

$$T(L, 8\ \text{min}) = 45.2°C$$

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The heat flux at $t = 8$ min is then

$$q''_L = 500\ \text{W/m}^2 \cdot \text{K}\,(45.2 - 60)^\circ\text{C} = -7400\ \text{W/m}^2$$

4. The energy transfer to the pipe wall over the 8-min interval may be obtained from Equations 5.44 and 5.46. With

$$\frac{Q}{Q_o} = 1 - \frac{\sin(\zeta_1)}{\zeta_1}\theta_o^*$$

$$\frac{Q}{Q_o} = 1 - \frac{\sin(0.531\ \text{rad})}{0.531\ \text{rad}} \times 0.214 = 0.80$$

it follows that

$$Q = 0.80\,\rho c V(T_i - T_\infty)$$

or with a volume per unit pipe length of $V' = \pi DL$,

$$Q' = 0.80\,\rho c \pi DL(T_i - T_\infty)$$

$$Q' = 0.80 \times 7832\ \text{kg/m}^3 \times 434\ \text{J/kg} \cdot \text{K} \times \pi \times 1\ \text{m} \times 0.04\ \text{m}\,(-20 - 60)^\circ\text{C}$$

$$Q' = -2.73 \times 10^7\ \text{J/m}$$

Comments:

1. The minus sign associated with q'' and Q' simply implies that the direction of heat transfer is from the oil to the pipe (into the pipe wall).

2. The foregoing results may be obtained using the *IHT Models, Transient Conduction, Plane Wall* option. The model provides for calculation of the temperature distribution, $T(x, t)$, the heat flux distribution, $q''_x(x, t)$, and the amount of energy transferred from the wall, $Q(t)$. Substituting the appropriate input parameters for the wall geometry, thermophysical properties and thermal conditions, the model yields the following results:

$T(0, 8\ \text{min}) = 43.1^\circ\text{C}$	$q''(L, 8\ \text{min}) = q''_L = -7305\ \text{W/m}^2$
$T(\text{L}, 8\ \text{min}) = 45.4^\circ\text{C}$	$Q'(8\ \text{min}) = -2.724 \times 10^7\ \text{J/m}$

Since the *IHT* model uses a multiple-term approximation to the series solution, the results are more accurate than those obtained from the foregoing one-term approximation. The complete solution for this example is provided as a ready-to-solve model in *IHT*, with annotations on how the code is written and solved (see *Examples*). *IHT Models* for *Transient Conduction* are also provided for the radial systems treated in Section 5.6.

EXAMPLE 5.5

A new process for treatment of a special material is to be evaluated. The material, a sphere of radius $r_o = 5$ mm, is initially in equilibrium at 400°C in a furnace. It is suddenly removed from the furnace and subjected to a two-step cooling process.

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Step 1 Cooling in air at 20°C for a period of time t_a until the center temperature reaches a critical value, $T_a(0, t_a) = 335°C$. For this situation, the convective heat transfer coefficient is $h_a = 10\ W/m^2 \cdot K$.

After the sphere has reached this critical temperature, the second step is initiated.

Step 2 Cooling in a well-stirred water bath at 20°C, with a convective heat transfer coefficient of $h_w = 6000\ W/m^2 \cdot K$.

The thermophysical properties of the material are $\rho = 3000\ kg/m^3$, $k = 20\ W/m \cdot K$, $c = 1000\ J/kg \cdot K$, and $\alpha = 6.66 \times 10^{-6}\ m^2/s$.

1. Calculate the time t_a required for step 1 of the cooling process to be completed.
2. Calculate the time t_w required during step 2 of the process for the center of the sphere to cool from 335°C (the condition at the completion of step 1) to 50°C.

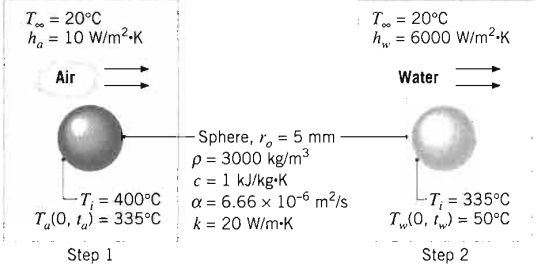
SOLUTION

Known: Temperature requirements for cooling a sphere.

Find:

1. Time t_a required to accomplish desired cooling in air.
2. Time t_w required to complete cooling in water bath.

Schematic:



Assumptions:

1. One-dimensional conduction in r.
2. Constant properties.

Analysis:

1. To determine whether the lumped capacitance method can be used, the Biot number is calculated. From Equation 5.10, with $L_c = r_o/3$,

$$Bi = \frac{h_a r_o}{3k} = \frac{10\ W/m^2 \cdot K \times 0.005\ m}{3 \times 20\ W/m \cdot K} = 8.33 \times 10^{-4}$$

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Accordingly, the lumped capacitance method may be used, and the temperature is nearly uniform throughout the sphere. From Equation 5.5 it follows that

$$t_a = \frac{\rho V c}{h_a A_s} \ln \frac{\theta_i}{\theta_a} = \frac{\rho r_o c}{3h_a} \ln \frac{T_i - T_\infty}{T_a - T_\infty}$$

where $V = (4/3)\pi r_o^3$ and $A_s = 4\pi r_o^2$. Hence

$$t_a = \frac{3000\ \text{kg/m}^3 \times 0.005\ \text{m} \times 1000\ \text{J/kg} \cdot \text{K}}{3 \times 10\ \text{W/m}^2 \cdot \text{K}} \ln \frac{400 - 20}{335 - 20} = 94\ \text{s}$$

2. To determine whether the lumped capacitance method may also be used for the second step of the cooling process, the Biot number is again calculated. In this case

$$Bi = \frac{h_w r_o}{3k} = \frac{6000\ \text{W/m}^2 \cdot \text{K} \times 0.005\ \text{m}}{3 \times 20\ \text{W/m} \cdot \text{K}} = 0.50$$

and the lumped capacitance method is not appropriate. However, to an excellent approximation, the temperature of the sphere is uniform at $t = t_a$ and the one-term approximation may be used for the calculations. The time t_w at which the center temperature reaches 50°C, that is, $T(0, t_w) = 50°\text{C}$, can be obtained by rearranging Equation 5.50c

$$Fo = -\frac{1}{\zeta_1^2} \ln \left[\frac{\theta_o^*}{C_1} \right] = -\frac{1}{\zeta_1^2} \ln \left[\frac{1}{C_1} \times \frac{T(0, t_w) - T_\infty}{T_i - T_\infty} \right]$$

where $t_w = Fo\, r_o^2/\alpha$. With the Biot number now defined as

$$Bi = \frac{h_w r_o}{k} = \frac{6000\ \text{W/m}^2 \cdot \text{K} \times 0.005\ \text{m}}{20\ \text{W/m} \cdot \text{K}} = 1.50$$

Table 5.1 yields $C_1 = 1.376$ and $\zeta_1 = 1.800$ rad. It follows that

$$Fo = -\frac{1}{(1.800\ \text{rad})^2} \ln \left[\frac{1}{1.376} \times \frac{(50 - 20)°\text{C}}{(335 - 20)°\text{C}} \right] = 0.82$$

and

$$t_w = Fo \frac{r_o^2}{\alpha} = 0.82 \frac{(0.005\ \text{m})^2}{6.66 \times 10^{-6}\ \text{m}^2/\text{s}} = 3.1\ \text{s}$$

Note that, with $Fo = 0.82$, use of the one-term approximation is justified.

Comments:

1. If the temperature distribution in the sphere at the conclusion of step 1 were not uniform, the one-term approximation could not be used for the calculations of step 2.

2. The surface temperature of the sphere at the conclusion of step 2 may be obtained from Equation 5.50b. With $\theta_o^* = 0.095$ and $r^* = 1$,

$$\theta^*(r_o) = \frac{T(r_o) - T_\infty}{T_i - T_\infty} = \frac{0.095}{1.800\ \text{rad}} \sin(1.800\ \text{rad}) = 0.0514$$

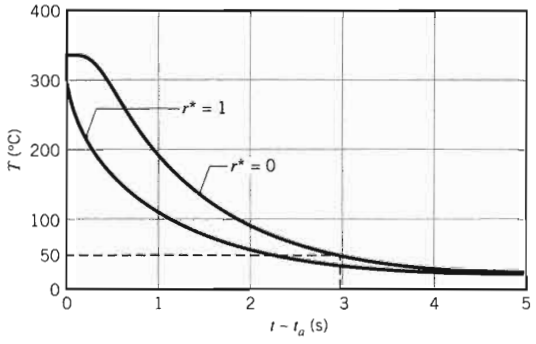
5.7 The Semi-In

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and

$$T(r_o) = 20°C + 0.0514(335 - 20)°C = 36°C$$

The infinite series, Equation 5.48a, and its one-term approximation, Equation 5.50b, may be used to compute the temperature at any location in the sphere and at any time $t > t_a$. For $(t - t_a) < 0.2(0.005\ m)^2/6.66 \times 10^{-6}\ m^2/s = 0.75$ s, a sufficient number of terms must be retained to ensure convergence of the series. For $(t - t_a) > 0.75$ s, satisfactory convergence is provided by the one-term approximation. Computing and plotting the temperature histories for $r = 0$ and $r = r_o$, we obtain the following results for $0 \leq (t - t_a) \leq 5$ s:



3. The *IHT Models*, *Transient Conduction*, *Sphere* option could be used to analyze the cooling processes experienced by the sphere in air and water, steps 1 and 2. The *IHT Models*, *Lumped Capacitance* option may only be used to analyze the air-cooling process, step 1.

5.7 The Semi-Infinite Solid

Another simple geometry for which analytical solutions may be obtained is the *semi-infinite solid*. Since, in principle, such a solid extends to infinity in all but one direction, it is characterized by a single identifiable surface (Figure 5.7). If a sudden change of conditions is imposed at this surface, transient, one-dimensional conduction will occur within the solid. The semi-infinite solid provides a *useful idealization* for many practical problems. It may be used to determine transient heat transfer near the surface of the earth or to approximate the transient response of a finite solid, such as a thick slab. For this second situation the approximation would be reasonable for the early portion of the transient, during which temperatures in the slab interior (well removed from the surface) are essentially uninfluenced by the change in surface conditions.

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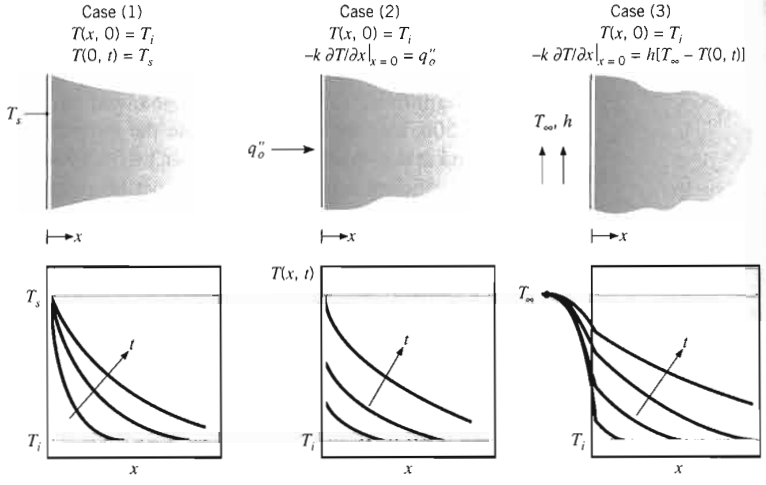


FIGURE 5.7 Transient temperature distributions in a semi-infinite solid for three surface conditions: constant surface temperature, constant surface heat flux, and surface convection.

The heat equation for transient conduction in a semi-infinite solid is given by Equation 5.26. The initial condition is prescribed by Equation 5.27, and the interior boundary condition is of the form

$$T(x \to \infty, t) = T_i \tag{5.53}$$

Closed-form solutions have been obtained for three important surface conditions, instantaneously applied at $t = 0$ [1, 2]. These conditions are shown in Figure 5.7. They include application of a constant surface temperature $T_s \neq T_i$, application of a constant surface heat flux q''_o, and exposure of the surface to a fluid characterized by $T_\infty \neq T_i$ and the convection coefficient h.

The solution for case 1 may be obtained by recognizing the existence of a *similarity variable* η, through which the heat equation may be transformed from a partial differential equation, involving two independent variables (x and t), to an ordinary differential equation expressed in terms of the single similarity variable. To confirm that such a requirement is satisfied by $\eta \equiv x/(4\alpha t)^{1/2}$, we first transform the pertinent differential operators, such that

$$\frac{\partial T}{\partial x} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial x} = \frac{1}{(4\alpha t)^{1/2}}\frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{d}{d\eta}\left[\frac{\partial T}{\partial x}\right]\frac{\partial \eta}{\partial x} = \frac{1}{4\alpha t}\frac{d^2T}{d\eta^2}$$

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta}\frac{\partial \eta}{\partial t} = -\frac{x}{2t(4\alpha t)^{1/2}}\frac{dT}{d\eta}$$

Substituting into Equation 5.26, the heat equation becomes

$$\frac{d^2T}{d\eta^2} = -2\eta\frac{dT}{d\eta} \tag{5.54}$$

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With $x = 0$ corresponding to $\eta = 0$, the surface condition may be expressed as

$$T(\eta = 0) = T_s \tag{5.55}$$

and with $x \to \infty$, as well as $t = 0$, corresponding to $\eta \to \infty$, both the initial condition and the interior boundary condition correspond to the single requirement that

$$T(\eta \to \infty) = T_i \tag{5.56}$$

Since the transformed heat equation and the initial/boundary conditions are independent of x and t, $\eta \equiv x/(4\alpha t)^{1/2}$ is, indeed, a similarity variable. Its existence implies that, irrespective of the values of x and t, the temperature may be represented as a unique function of η.

The specific form of the temperature dependence, $T(\eta)$, may be obtained by separating variables in Equation 5.54, such that

$$\frac{d(dT/d\eta)}{(dT/d\eta)} = -2\eta\, d\eta$$

Integrating, it follows that

$$\ln(dT/d\eta) = -\eta^2 + C_1'$$

or

$$\frac{dT}{d\eta} = C_1 \exp(-\eta^2)$$

Integrating a second time, we obtain

$$T = C_1 \int_0^\eta \exp(-u^2)\, du + C_2$$

where u is a dummy variable. Applying the boundary condition at $\eta = 0$, Equation 5.55, it follows that $C_2 = T_s$ and

$$T = C_1 \int_0^\eta \exp(-u^2)\, du + T_s$$

From the second boundary condition, Equation 5.56, we obtain

$$T_i = C_1 \int_0^\infty \exp(-u^2)\, du + T_s$$

or, evaluating the definite integral,

$$C_1 = \frac{2(T_i - T_s)}{\pi^{1/2}}$$

Hence the temperature distribution may be expressed as

$$\frac{T - T_s}{T_i - T_s} = (2/\pi^{1/2}) \int_0^\eta \exp(-u^2)\, du \equiv \operatorname{erf} \eta \tag{5.57}$$

where the *Gaussian error function,* erf η, is a standard mathematical function that is tabulated in Appendix B. Note that erf(η) asymptotically approaches unity as η becomes infinite. Thus, at any nonzero time, temperatures everywhere are predicted to have changed from T_i (become closer to T_s). The infinite speed at which

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boundary-condition information propagates into the semi-infinite solid is physically unrealistic, but this limitation of Fourier's law is not important except at extremely small time scales, as discussed in Section 2.3. The surface heat flux may be obtained by applying Fourier's law at $x = 0$, in which case

$$q_s'' = -k\frac{\partial T}{\partial x}\bigg|_{x=0} = -k(T_i - T_s)\frac{d(\text{erf }\eta)}{d\eta}\frac{\partial \eta}{\partial x}\bigg|_{\eta=0}$$

$$q_s'' = k(T_s - T_i)(2/\pi^{1/2})\exp(-\eta^2)(4\alpha t)^{-1/2}\big|_{\eta=0}$$

$$q_s'' = \frac{k(T_s - T_i)}{(\pi\alpha t)^{1/2}} \tag{5.58}$$

Analytical solutions may also be obtained for the case 2 and case 3 surface conditions, and results for all three cases are summarized as follows.

Case 1 Constant Surface Temperature: $T(0, t) = T_s$

$$\frac{T(x,t) - T_s}{T_i - T_s} = \text{erf}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{5.57}$$

$$q_s''(t) = \frac{k(T_s - T_i)}{\sqrt{\pi\alpha t}} \tag{5.58}$$

Case 2 Constant Surface Heat Flux: $q_s'' = q_o''$

$$T(x,t) - T_i = \frac{2q_o''(\alpha t/\pi)^{1/2}}{k}\exp\left(\frac{-x^2}{4\alpha t}\right) - \frac{q_o'' x}{k}\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) \tag{5.59}$$

Case 3 Surface Convection: $-k\dfrac{\partial T}{\partial x}\bigg|_{x=0} = h[T_\infty - T(0,t)]$

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \left[\exp\left(\frac{hx}{k} + \frac{h^2\alpha t}{k^2}\right)\right]\left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right] \tag{5.60}$$

The *complementary error function,* erfc w, is defined as erfc $w \equiv 1 - \text{erf } w$.

Temperature histories for the three cases are shown in Figure 5.7, and distinguishing features should be noted. With a step change in the surface temperature, case 1, temperatures within the medium monotonically approach T_s with increasing t, while the magnitude of the surface temperature gradient, and hence the surface heat flux, decreases as $t^{-1/2}$. In contrast, for a fixed surface heat flux (case 2), Equation 5.59 reveals that $T(0, t) = T_s(t)$ increases monotonically as $t^{1/2}$. For surface convection (case 3), the surface temperature and temperatures within the medium approach the fluid temperature T_∞ with increasing time. As T_s approaches T_∞, there is, of course, a reduction in the surface heat flux, $q_s''(t) = h[T_\infty - T_s(t)]$. Specific temperature histories computed from Equation 5.60 are plotted in Figure 5.8. The result corresponding to $h = \infty$ is equivalent to that associated with a sudden change

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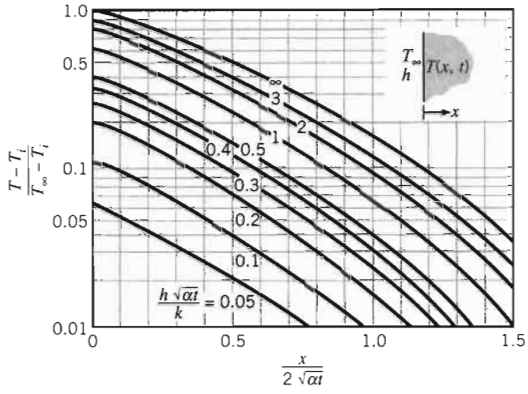


FIGURE 5.8
Temperature histories in a semi-infinite solid with surface convection [2]. Adapted with permission.

in surface temperature, case 1. That is, for $h = \infty$, the surface instantaneously achieves the imposed fluid temperature ($T_s = T_\infty$), and with the second term on the right-hand side of Equation 5.60 reducing to zero, the result is equivalent to Equation 5.57.

An interesting permutation of case 1 results when two semi-infinite solids, initially at uniform temperatures $T_{A,i}$ and $T_{B,i}$, are placed in contact at their free surfaces (Figure 5.9). If the contact resistance is negligible, the requirement of thermal equilibrium dictates that, at the instant of contact ($t = 0$), both surfaces must assume the same temperature T_s, for which $T_{B,i} < T_s < T_{A,i}$. Since T_s does not change with increasing time, it follows that the transient thermal response and the surface heat flux of each of the solids are determined by Equations 5.57 and 5.58, respectively.

The equilibrium surface temperature of Figure 5.9 may be determined from a surface energy balance, which requires that

$$q''_{s,A} = q''_{s,B} \tag{5.61}$$

Substituting from Equation 5.58 for $q''_{s,A}$ and $q''_{s,B}$ and recognizing that the x coordinate of Figure 5.9 requires a sign change for $q''_{s,A}$, it follows that

$$\frac{-k_A(T_s - T_{A,i})}{(\pi\alpha_A t)^{1/2}} = \frac{k_B(T_s - T_{B,i})}{(\pi\alpha_B t)^{1/2}} \tag{5.62}$$

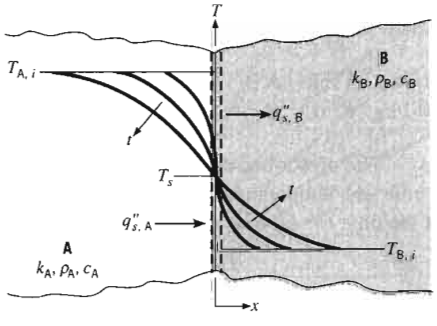


FIGURE 5.9
Interfacial contact between two semi-infinite solids at different initial temperatures.

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or, solving for T_s,

$$T_s = \frac{(k\rho c)_A^{1/2} T_{A,i} + (k\rho c)_B^{1/2} T_{B,i}}{(k\rho c)_A^{1/2} + (k\rho c)_B^{1/2}} \tag{5.63}$$

Hence the quantity $m \equiv (k\rho c)^{1/2}$ is a weighting factor that determines whether T_s will more closely approach $T_{A,i} (m_A > m_B)$ or $T_{B,i} (m_B > m_A)$.

EXAMPLE 5.6

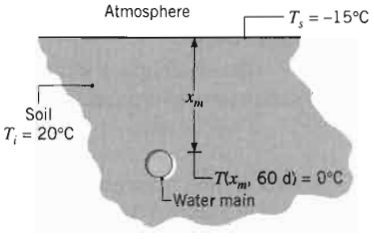
In laying water mains, utilities must be concerned with the possibility of freezing during cold periods. Although the problem of determining the temperature in soil as a function of time is complicated by changing surface conditions, reasonable estimates can be based on the assumption of a constant surface temperature over a prolonged period of cold weather. What minimum burial depth x_m would you recommend to avoid freezing under conditions for which soil, initially at a uniform temperature of 20°C, is subjected to a constant surface temperature of −15°C for 60 days?

SOLUTION

Known: Temperature imposed at the surface of soil initially at 20°C.

Find: The depth x_m to which the soil has frozen after 60 days.

Schematic:



Assumptions:

1. One-dimensional conduction in x.
2. Soil is a semi-infinite medium.
3. Constant properties.

Properties: Table A.3, soil (300 K): $\rho = 2050\ kg/m^3$, $k = 0.52\ W/m \cdot K$, $c = 1840\ J/kg \cdot K$, $\alpha = (k/\rho c) = 0.138 \times 10^{-6}\ m^2/s$.

Analysis: The prescribed conditions correspond to those of case 1 of Figure 5.7, and the transient temperature response of the soil is governed by Equation 5.57. Hence at the time $t = 60$ days after the surface temperature change,

$$\frac{T(x_m, t) - T_s}{T_i - T_s} = \text{erf}\left(\frac{x_m}{2\sqrt{\alpha t}}\right)$$

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(5.63)

etermines whether T_s

ossibility of freezing
emperature in soil as
ons, reasonable esti-
nperature over a pro-
uld you recommend
niform temperature
C for 60 days?

t 20°C.

= 0.52 W/m · K,

1 of Figure 5.7,
Equation 5.57.

or

$$\frac{0-(-15)}{20-(-15)} = 0.429 = \text{erf}\left(\frac{x_m}{2\sqrt{\alpha t}}\right)$$

Hence, from Appendix B.2

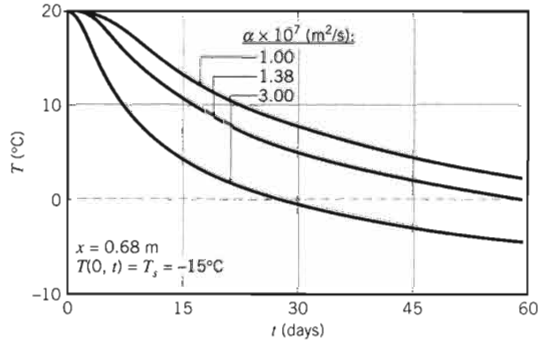
$$\frac{x_m}{2\sqrt{\alpha t}} = 0.40$$

and

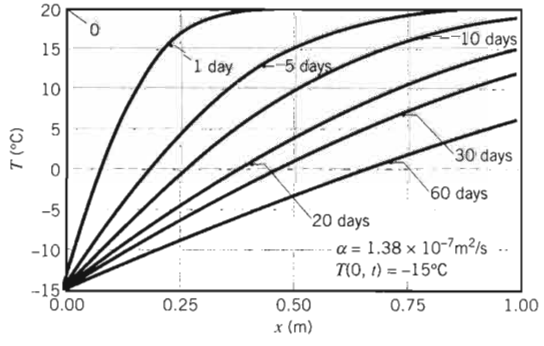
$$x_m = 0.80(\alpha t)^{1/2} = 0.80(0.138 \times 10^{-6}\ \text{m}^2/\text{s} \times 60\ \text{days} \times 24\ \text{h/day} \times 3600\ \text{s/h})^{1/2} = 0.68\ \text{m} \quad \triangleleft$$

Comments:

1. The properties of soil are highly variable, depending on the nature of the soil and its moisture content, and a representative range of thermal diffusivities is $1 \times 10^{-7} < \alpha < 3 \times 10^{-7}\ \text{m}^2/\text{s}$. To assess the effect of soil properties on freezing conditions, we use Equation 5.57 to compute temperature histories at $x_m = 0.68$ m for $\alpha \times 10^7 = 1.0$, 1.38, and 3.0 m^2/s.



If $\alpha > 1.38 \times 10^{-7}\ \text{m}^2/\text{s}$, the design criterion is not achieved at $x_m = 0.68$ m and freezing would occur. It is also instructional to examine temperature distributions in the soil at representative times during the cooling period. Using Equation 5.57 with $\alpha = 1.38 \times 10^{-7}\ \text{m}^2/\text{s}$, the following results are obtained:



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As thermal penetration increases with increasing time, the temperature gradient at the surface, $\partial T/\partial x|_{x=0}$, and hence the rate of heat extraction from the soil, decreases.

2. The *IHT Models*, *Transient Conduction*, *Semi-infinite Solid* option may be used to generate the foregoing numerical results. The option provides model builders for the three boundary conditions of *constant surface temperature*, *constant surface heat flux*, and *surface convection*.

5.8 *Objects with Constant Surface Temperatures or Surface Heat Fluxes*

In Sections 5.5 and 5.6, the transient thermal response of plane walls, cylinders, and spheres to an applied convection boundary condition was considered in detail. It was pointed out that the solutions in those sections may be used for cases involving a step change in surface temperature by allowing the Biot number to be infinite. In Section 5.7, the response of a semi-infinite solid to a step change in surface temperature, or to an applied constant heat flux, was determined. This section further explores the transient thermal response of a variety of objects to a constant surface temperature or a constant surface heat flux boundary condition.

5.8.1 Constant Temperature Boundary Conditions

In the following discussion, the transient thermal response of objects to a step change in surface temperature is considered.

Semi-Infinite Solid Insight into the thermal response of objects to an applied constant temperature boundary condition may be obtained by casting the heat flux in Equation 5.58 into the nondimensional form

$$q^* \equiv \frac{q_s'' L_c}{k(T_s - T_i)} \tag{5.64}$$

where L_c is a *characteristic length* and q^* is the *dimensionless conduction heat rate* that was introduced in Section 4.3.3. Substituting Equation 5.64 into Equation 5.58 yields

$$q^* = \frac{1}{\sqrt{\pi Fo}} \tag{5.65}$$

where the Fourier number is defined as $Fo \equiv \alpha t/L_c^2$. Note that the value of q_s'' is independent of the choice of the characteristic length, as it must be for a semi-infinite solid. Equation 5.65 is plotted in Figure 5.10*a*, and since $q^* \propto Fo^{-1/2}$, the slope of the line is $-1/2$ on the log-log plot.

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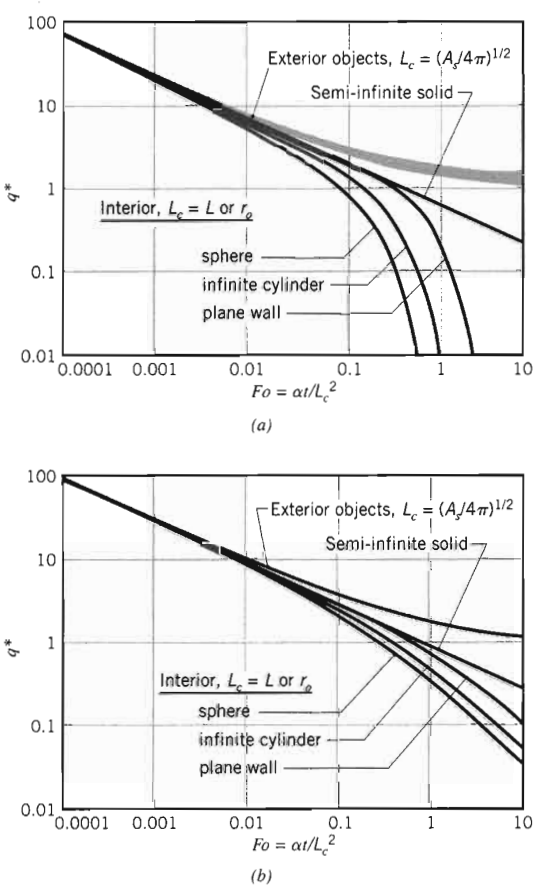


FIGURE 5.10 Transient dimensionless conduction heat rates for a variety of geometries. (*a*) Constant surface temperature. Results for the geometries of Table 4.1 lie within the shaded region and are from [5]. (*b*) Constant surface heat flux.

Interior Heat Transfer: Plane Wall, Cylinder, and Sphere Results for heat transfer to the *interior* of a plane wall, cylinder, and sphere are also shown in Figure 5.10*a*. These results are generated by using Fourier's law in conjunction with Equations 5.39, 5.47, and 5.48 for $Bi \rightarrow \infty$. As in Sections 5.5 and 5.6, the characteristic length is $L_c = L$ or r_o for a plane wall of thickness $2L$ or a cylinder (or sphere) of radius r_o, respectively. For each geometry, q^* initially follows the semi-infinite solid solution but at some point decreases rapidly as the objects approach their equilibrium temperature and $q_s'' \, (t \rightarrow \infty) \rightarrow 0$. The value of q^* is expected to decrease more rapidly for geometries that possess large surface area to volume ratios, and this trend is evident in Figure 5.10*a*.

Exterior Heat Transfer: Various Geometries Additional results are shown in Figure 5.10*a* for objects that are embedded in an exterior (surrounding) medium

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of infinite extent. The infinite medium is initially at temperature T_i, and the surface temperature of the object is suddenly changed to T_s. For the exterior cases, L_c is the characteristic length used in Section 4.3.3, namely $L_c = (A_s/4\pi)^{1/2}$. For the sphere in a surrounding infinite medium, the exact solution for $q^*(Fo)$ is [5]

$$q^* = \frac{1}{\sqrt{\pi Fo}} + 1 \tag{5.66}$$

As seen in the figure, for all of the *exterior cases* q^* closely mimics that of the sphere when the appropriate length scale is used in its definition, regardless of the object's shape. In a manner consistent with the interior cases, q^* initially follows the semi-infinite solid solution. In contrast to the interior cases, q^* eventually reaches the nonzero, steady-state value of q^*_{ss} that is listed in Table 4.1. Note that q''_s in Equation 5.64 is the *average* surface heat flux for geometries that have nonuniform surface heat flux.

As seen in Figure 5.10*a*, all of the thermal responses collapse to that of the semi-infinite solid for early times, that is, for Fo less than approximately 10^{-3}. This remarkable consistency reflects the fact that temperature variations are confined to thin layers adjacent to the surface of any object at early times, regardless of whether internal or external heat transfer is of interest. At early times, therefore, Equations 5.57 and 5.58 may be used to predict the temperatures and heat transfer rates within the thin regions adjacent to the boundaries of any object. For example, predicted local heat fluxes and local dimensionless temperatures using the semi-infinite solid solutions are within approximately 5% of the predictions obtained from the exact solutions for the interior and exterior heat transfer cases involving spheres when $Fo \leq 10^{-3}$.

5.8.2 Constant Heat Flux Boundary Conditions

When a constant surface heat flux is applied to an object, the resulting surface temperature history is often of interest. In this case, the heat flux in the numerator of Equation 5.64 is now constant, and the temperature difference in the denominator, $T_s - T_i$, increases with time.

Semi-Infinite Solid In the case of a semi-infinite solid, the surface temperature history can be found by evaluating Equation 5.59 at $x = 0$, which may be rearranged and combined with Equation 5.64 to yield

$$q^* = \frac{1}{2}\sqrt{\frac{\pi}{Fo}} \tag{5.67}$$

As for the constant temperature case, $q^* \propto Fo^{-1/2}$, but with a different coefficient. Equation 5.67 is presented in Figure 5.10*b*.

Interior Heat Transfer: Plane Wall, Cylinder, and Sphere A second set of results is shown in Figure 5.10*b* for the *interior* cases of the plane wall, cylinder, and sphere. As for the constant surface temperature results of Figure 5.10*a*, q^* initially follows the semi-infinite solid solution and subsequently decreases more rapidly, with the decrease occurring first for the sphere, then the cylinder, and finally the plane wall. Compared to the constant surface temperature case, the rate at which q^* decreases is not as dramatic, since steady-state conditions are never reached; the surface temperature must continue to increase with time. At *late* times (large Fo), the surface temperature increases linearly with time, yielding $q^* \propto Fo^{-1}$, with a slope of -1 on the log-log plot.

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Exterior Heat Transfer: Various Geometries Results for heat transfer between a sphere and an exterior infinite medium are also presented in Figure 5.10*b*. The exact solution for the embedded sphere is

$$q^* = [1 - \exp(Fo)\,\mathrm{erfc}(Fo^{1/2})]^{-1} \tag{5.68}$$

As in the constant surface temperature case of Figure 5.10*a*, this solution approaches steady-state conditions, with $q^*_{ss} = 1$. For objects of other shapes that are embedded within an infinite medium, q^* would follow the semi-infinite solid solution at small Fo. At larger Fo, q^* must asymptotically approach the value of q^*_{ss} given in Table 4.1 where T_s in Equation 5.64 is the *average* surface temperature for geometries that have nonuniform surface temperatures.

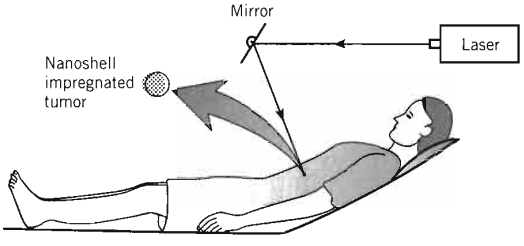
5.8.3 Approximate Solutions

Simple expressions have been developed for $q^*(Fo)$. These expressions may be used to approximate all the results included in Figure 5.10 over the *entire range of Fo*. These expressions are listed in Table 5.2, along with the corresponding exact solutions. Table 5.2*a* is for the constant surface temperature case, while Table 5.2*b* is for the constant surface heat flux situation. For each of the geometries listed in the left-hand column, the tables provide the length scale to be used in the definition of both Fo and q^*, the exact solution for $q^*(Fo)$, the approximation solutions for early times ($Fo < 0.2$) and late times ($Fo \geq 0.2$), and the maximum percentage error associated with use of the approximations (which occurs at $Fo \approx 0.2$ for all results except the external sphere with constant heat flux).

EXAMPLE 5.7

A proposed cancer treatment utilizes small, composite *nanoshells* whose size and composition are carefully engineered so that the particles efficiently absorb laser irradiation at particular wavelengths [6]. Prior to treatment, antibodies are attached to the nanoscale particles. The doped particles are then injected into the patient's bloodstream and are distributed throughout the body. The antibodies are attracted to malignant sites, and therefore carry and adhere the nanoshells only to cancerous tissue. After the particles have come to rest within the tumor, a laser beam penetrates through the tissue between the skin and the cancer, is absorbed by the nanoshells, and, in turn, heats and destroys the cancerous tissues.

Consider an approximately spherical tumor of diameter $D_t = 3$ mm that is uniformly infiltrated with nanoshells that are highly absorptive of incident radiation from a laser located outside the patient's body.



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TABLE 5.2*a* Summary of transient heat transfer results for constant surface temperature cases[a]

Geometry	Length Scale, L_c	$q^*(Fo)$			Maximum Error (%)
		Exact Solutions	Approximate Solutions		
			$Fo < 0.2$	$Fo \geq 0.2$	
Semi-infinite	L (arbitrary)	$\frac{1}{\sqrt{\pi Fo}}$	Use exact solution.		none
Interior Cases					
Plane wall of thickness $2L$	L	$2\sum_{n=1}^{\infty} \exp(-\zeta_n^2 Fo) \quad \zeta_n = (n-\frac{1}{2})\pi$	$\frac{1}{\sqrt{\pi Fo}}$	$2\exp(-\zeta_1^2 Fo) \quad \zeta_1 = \pi/2$	1.7
Infinite cylinder	r_o	$2\sum_{n=1}^{\infty} \exp(-\zeta_n^2 Fo) \quad J_0(\zeta_n) = 0$	$\frac{1}{\sqrt{\pi Fo}} - 0.50 - 0.65\,Fo$	$2\exp(-\zeta_1^2 Fo) \quad \zeta_1 = 2.4050$	0.8
Sphere	r_o	$2\sum_{n=1}^{\infty} \exp(-\zeta_n^2 Fo) \quad \zeta_n = n\pi$	$\frac{1}{\sqrt{\pi Fo}} - 1$	$2\exp(-\zeta_1^2 Fo) \quad \zeta_1 = \pi$	6.3
Exterior Cases					
Sphere	r_o	$\frac{1}{\sqrt{\pi Fo}} + 1$	Use exact solution.		none
Various shapes (Table 4.1, cases 12–15)	$(A_s/4\pi)^{1/2}$	none	$\frac{1}{\sqrt{\pi Fo}} + q_s^*, \quad q_s^*$ from Table 4.1		7.1

[a] $q^* \equiv q_s'' L_c / k(T_s - T_i)$ and $Fo \equiv \alpha t / L_c^2$ where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

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TABLE 5.2*b* Summary of transient heat transfer results for constant surface heat flux cases[a]

Geometry	Length Scale, L_c	$q^*(Fo)$ Exact Solutions	Approximate Solutions $Fo < 0.2$	Approximate Solutions $Fo \geq 0.2$	Maximum Error (%)
Semi-infinite	L (arbitrary)	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}}$	Use exact solution.		none
Interior Cases					
Plane wall of thickness $2L$	L	$\left[Fo + \frac{1}{3} - 2\sum_{n=1}^{\infty}\frac{\exp(-\zeta_n^2 Fo)}{\zeta_n^2}\right]^{-1}$ $\zeta_n = n\pi$	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}}$	$\left[Fo + \frac{1}{3}\right]^{-1}$	5.3
Infinite cylinder	r_o	$\left[2Fo + \frac{1}{4} - 2\sum_{n=1}^{\infty}\frac{\exp(-\zeta_n^2 Fo)}{\zeta_n^2}\right]^{-1}$ $J_1(\zeta_n) = 0$	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}} - \frac{\pi}{8}$	$\left[2Fo + \frac{1}{4}\right]^{-1}$	2.1
Sphere	r_o	$\left[3Fo + \frac{1}{5} - 2\sum_{n=1}^{\infty}\frac{\exp(-\zeta_n^2 Fo)}{\zeta_n^2}\right]^{-1}$ $\tan(\zeta_n) = \zeta_n$	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}} - \frac{\pi}{4}$	$\left[3Fo + \frac{1}{5}\right]^{-1}$	4.5
Exterior Cases					
Sphere	r_o	$[1 - \exp(Fo)\text{erfc}(Fo^{1/2})]^{-1}$	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}} + \frac{\pi}{4}$	$\frac{0.77}{\sqrt{Fo}} + 1$	3.2
Various shapes (Table 4.1, cases 12–15)	$(A_s/4\pi)^{1/2}$	none	$\frac{1}{2}\sqrt{\frac{\pi}{Fo}} + \frac{\pi}{4}$	$\frac{0.77}{\sqrt{Fo}} + q^*_{ss}$	unknown

[a] $q^* \equiv q''_s L_c/k(T_s - T_i)$ and $Fo \equiv \alpha t/L_c^2$, where L_c is the length scale given in the table, T_s is the object surface temperature, and T_i is (a) the initial object temperature for the interior cases and (b) the temperature of the infinite medium for the exterior cases.

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1. Estimate the heat transfer rate from the tumor to the surrounding healthy tissue for a steady-state treatment temperature of $T_{t,ss} = 55°C$ at the surface of the tumor. The thermal conductivity of healthy tissue is approximately $k = 0.5$ W/m · K, and the body temperature is $T_b = 37°C$.
2. Find the laser power necessary to sustain the tumor surface temperature at $T_{t,ss} = 55°C$ if the tumor is located $d = 20$ mm beneath the surface of the skin, and the laser heat flux decays exponentially, $q''_l(x) = q''_{l,o}(1 - \rho)\, e^{-\kappa x}$, between the surface of the body and the tumor. In the preceding expression, $q''_{l,o}$ is the laser heat flux outside the body, $\rho = 0.05$ is the reflectivity of the skin surface, and $\kappa = 0.02$ mm^{-1} is the *extinction coefficient* of the tissue between the tumor and the surface of the skin. The laser beam has a diameter of $D_l = 5$ mm.
3. Neglecting heat transfer to the surrounding tissue, estimate the time at which the tumor temperature is within 3°C of $T_{t,ss} = 55°C$ for the laser power found in part 2. Assume the tissue's density and specific heat are that of water.
4. Neglecting the thermal mass of the tumor but accounting for heat transfer to the surrounding tissue, estimate the time needed for the surface temperature of the tumor to reach $T_t = 52°C$.

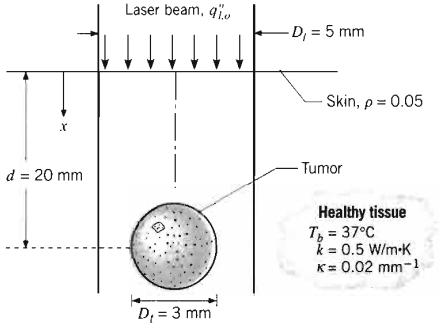
SOLUTION

Known: Size of a small sphere; thermal conductivity, reflectivity, and extinction coefficient of tissue; depth of sphere below the surface of the skin.

Find:

1. Heat transferred from the tumor to maintain its surface temperature at $T_{t,ss} = 55°C$.
2. Laser power needed to sustain the tumor surface temperature at $T_{t,ss} = 55°C$.
3. Time for the tumor to reach $T_t = 52°C$ when heat transfer to the surrounding tissue is neglected.
4. Time for the tumor to reach $T_t = 52°C$ when heat transfer to the surrounding tissue is considered and the thermal mass of the tumor is neglected.

Schematic:



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Assumptions:

1. One-dimensional conduction in the radial direction.
2. Constant properties.
3. Healthy tissue can be treated as an infinite medium.
4. The treated tumor absorbs all irradiation incident from the laser.
5. Lumped capacitance behavior for the tumor.
6. Neglect potential nanoscale heat transfer effects.
7. Neglect the effect of perfusion.

Properties: Table A.6, water (320 K, assumed): $\rho = v_f^{-1} = 989.1$ kg/m^3, $c_p = 4180$ J/kg · K.

Analysis:

1. The steady-state heat loss from the spherical tumor may be determined by evaluating the dimensionless heat rate from the expression for case 12 of Table 4.1:

$$q = 2\pi k D_t (T_{t,ss} - T_b) = 2 \times \pi \times 0.5 \text{ W/m} \cdot \text{K} \times 3 \times 10^{-3} \text{ m} \times (55 - 37)^\circ\text{C}$$
$$= 0.170 \text{ W} \quad \triangleleft$$

2. The laser irradiation will be absorbed over the projected area of the tumor, $\pi D_t^2/4$. To determine the laser power corresponding to $q = 0.170$ W, we first write an energy balance for the sphere. For a control surface about the sphere, the energy absorbed from the laser irradiation is offset by heat conduction to the healthy tissue, $q = 0.170 \text{ W} \approx q_l''(x = d)\pi D_t^2/4$, where, $q_l''(x = d) = q_{l,o}''(1 - \rho)e^{-\kappa d}$ and the laser power is $P_l = q_{l,o}''\pi D_t^2/4$. Hence,

$$P_l = qD_t^2 e^{\kappa d}/[(1 - \rho)D_t^2]$$
$$= 0.170 \text{ W} \times (5 \times 10^{-3} \text{ m})^2 \times e^{(0.02 \text{ mm}^{-1} \times 20 \text{ mm})}/[(1 - 0.05) \times (3 \times 10^{-3} \text{ m})^2]$$
$$= 0.74 \text{ W} \quad \triangleleft$$

3. The general lumped capacitance energy balance, Equation 5.14, may be written

$$q_l''(x = d)\pi D_t^2/4 = q = \rho V c \frac{dT}{dt}$$

Separating variables and integrating between appropriate limits,

$$\frac{q}{\rho V c} \int_{t=0}^{t} dt = \int_{T_b}^{T_t} dT$$

yields

$$t = \frac{\rho V c}{q}(T_t - T_b) = \frac{989.1 \text{ kg/m}^3 \times (\pi/6) \times (3 \times 10^{-3} \text{ m})^3 \times 4180 \text{ J/kg} \cdot \text{K}}{0.170 \text{ W}}$$
$$\times (52^\circ\text{C} - 37^\circ\text{C})$$

or

$$t = 5.16 \text{ s} \quad \triangleleft$$

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4. Using Equation 5.68,

$$q/2\pi kD_t(T_t - T_b) = q^* = [1 - \exp(Fo)\text{erfc}(Fo^{1/2})]^{-1}$$

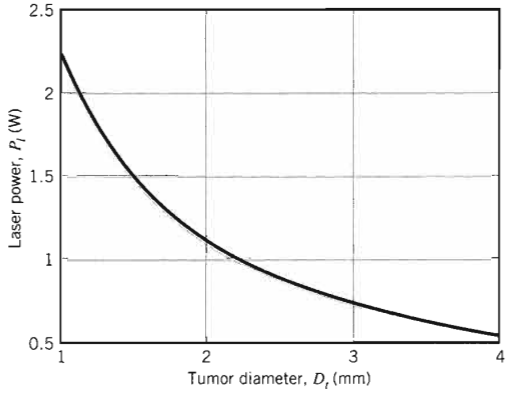
which may be solved by trial-and-error to yield $Fo = 10.3 = 4\alpha t/D_t^2$. Then, with $\alpha = k/\rho c = 0.50$ W/m · K/(989.1 kg/m^3 × 4180 J/kg · K) = 1.21×10^{-7} m^2/s, we find

$$t = FoD_t^2/4\alpha = 10.3 \times (3 \times 10^{-3}\text{ m})^2/(4 \times 1.21 \times 10^{-7}\text{ m}^2/\text{s}) = 192\text{ s} \quad \triangleleft$$

Comments:

1. The analysis does not account for blood perfusion. The flow of blood would lead to advection of warmed fluid away from the tumor (and relatively cool blood to the vicinity of the tumor), increasing the power needed to reach the desired treatment temperature.

2. The laser power needed to treat various-sized tumors, calculated as in parts 1 and 2 of the problem solution, is shown below. Note that as the tumor becomes smaller, a higher-powered laser is needed, which may seem counterintuitive. The power required to heat the tumor, which is the same as the heat loss calculated in part 1, increases in direct proportion to the diameter, as might be expected. However, since the laser power flux remains constant, a smaller tumor cannot absorb as much energy (the energy absorbed has a D_t^2 dependence). Less of the overall laser power is utilized to heat the tumor, and the required laser power increases for smaller tumors.



3. To determine the *actual* time needed for the tumor temperature to approach steady-state conditions, a numerical solution of the heat diffusion equation applied to the surrounding tissue, *coupled* with a solution for the temperature history within the tumor, would be required. However, we see that significantly more time is needed for the surrounding tissue to reach steady-state conditions than to increase the temperature of the isolated spherical tumor. This is due to the fact that higher temperatures propagate into a large volume when heating of the surrounding tissue is considered, while in contrast the thermal mass of the tumor is limited by the tumor's size. Hence, the *actual* time to heat *both* the tumor and the surrounding tissue will be slightly greater than 192 s.

5.9 Periodic Hea

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4. Since temperatures are likely to increase at a considerable distance from the tumor, the assumption that the surroundings are of infinite size would need to be checked by inspecting results of the proposed numerical solution described in Comment 3.

5.9 *Periodic Heating*

Periodic heating is used in various applications, such as thermal processing of materials using pulsed lasers, and occurs naturally in situations such as those involving the collection of solar energy.

Consider, for example, the semi-infinite solid of Figure 5.11*a*. For a surface temperature history described by $T(0,t) = T_i + \Delta T \sin \omega t$, the solution of Equation 5.26 subject to the interior boundary condition given by Equation 5.53 is

$$\frac{T(x,t) - T_i}{\Delta T} = \exp[-x\sqrt{\omega/2\alpha}]\sin[\omega t - x\sqrt{\omega/2\alpha}] \tag{5.69}$$

This solution applies after sufficient time has passed to yield a *quasi* steady state for which all temperatures fluctuate periodically about a time-invariant mean value. At locations in the solid, the fluctuations have a time lag relative to the surface temperature. In addition, the amplitude of the fluctuations within the material decays exponentially with distance from the surface. For $x > \delta_p \equiv 4\sqrt{\alpha/\omega}$, the amplitude of the temperature fluctuation is reduced by approximately 90% relative to that of the surface. The term δ_p is a *thermal penetration depth*, which is an indication of the extent to which significant temperature effects propagate within a medium. The heat flux at the surface may be determined by applying Fourier's law at $x = 0$, yielding

$$q_s''(t) = k\Delta T\sqrt{\omega/\alpha}\,\sin(\omega t + \pi/4) \tag{5.70}$$

Equation 5.70 reveals that the surface heat flux is periodic, with a time-averaged value of zero.

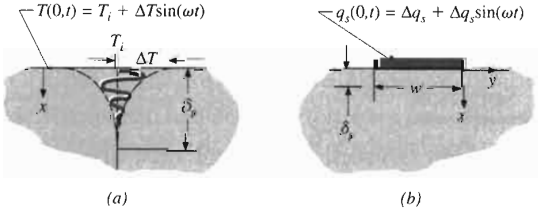


FIGURE 5.11 Schematic of (*a*) a periodically heated, one-dimensional semi-infinite solid and (*b*) a periodically heated strip attached to a semi-infinite solid.

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Periodic heating can also occur in two- or three-dimensional arrangements, as shown in Figure 5.11*b*. Recall that for this geometry, a steady state can be attained with constant heating of the strip placed upon a semi-infinite solid (Table 4.1, case 13). In a similar manner, a quasi steady state may be achieved when sinusoidal heating ($q_s = \Delta q_s + \Delta q_s \sin \omega t$) is applied to the strip. Again, a quasi steady state is achieved for which all temperatures fluctuate about a time-invariant mean value.

The solution of the two-dimensional, transient heat diffusion equation for the two-dimensional configuration shown in Figure 5.11*b* has been obtained, and the relationship between the amplitude of the applied sinusoidal heating and the amplitude of the temperature response of the heated strip can be approximated as [7]

$$\Delta T \approx \frac{\Delta q_s}{L\pi k}\left[-\frac{1}{2}\ln(\omega/2) - \ln(w^2/4\alpha) + C_1\right] = \frac{\Delta q_s}{L\pi k}\left[-\frac{1}{2}\ln(\omega/2) + C_2\right] \quad (5.71)$$

where the constant C_1 depends on the thermal contact resistance at the interface between the heated strip and the underlying material. Note that the amplitude of the temperature fluctuation, ΔT, corresponds to the spatially averaged temperature of the rectangular strip of length L and width w. The heat flux from the strip to the semi-infinite medium is assumed to be spatially uniform. The approximation is valid for $L \gg w$. For the system of Figure 5.11*b*, the thermal penetration depth is smaller than that of Figure 5.11*a* because of the lateral spreading of thermal energy and is $\delta_p \approx \sqrt{\alpha/\omega}$.

EXAMPLE 5.8

A novel nanostructured dielectric material has been fabricated, and the following method is used to measure its thermal conductivity. A long metal strip 3000 angstroms thick, $w = 100$ μm wide, and $L = 3.5$ mm long is deposited by a photolithography technique on the top surface of a $d = 300$-μm thick sample of the new material. The strip is heated periodically by an electric current supplied through two connector pads. The heating rate is $q_s(t) = \Delta q_s + \Delta q_s \sin(\omega t)$, where Δq_s is 3.5 mW. The instantaneous, spatially averaged temperature of the metal strip is found experimentally by measuring the time variation of its electrical resistance, $R(t) = E(t)/I(t)$, and by knowing how the electrical resistance of the metal varies with temperature. The measured temperature of the metal strip is periodic; it has an amplitude of $\Delta T = 1.37$ K at a relatively low heating frequency of $\omega = 2\pi$ rad/s and 0.71 K at a frequency of 200π rad/s. Determine the thermal conductivity of the nanostructured dielectric material. The density and specific heats of the conventional version of the material are 3100 kg/m^3 and 820 J/kg · K, respectively.

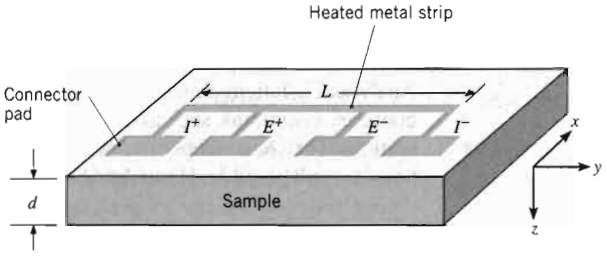
SOLUTION

Known: Dimensions of a thin metal strip, the frequency and amplitude of the electric power dissipated within the strip, the amplitude of the induced oscillating strip temperature, and the thickness of the underlying nanostructured material.

Find: The thermal conductivity of the nanostructured material.

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Schematic:



Assumptions:

1. Two-dimensional transient conduction in the x- and z-directions.
2. Constant properties.
3. Negligible radiation and convection losses from the metal strip and top surface of the sample.
4. The nanostructured material sample is a semi-infinite solid.
5. Uniform heat flux at the interface between the heated strip and the nanostructured material.

Analysis: Substitution of $\Delta T = 1.37$ K at $\omega = 2\pi$ rad/s and $\Delta T = 0.71$ K at $\omega = 200\pi$ rad/s into Equation 5.71 results in two equations that may be solved simultaneously to yield

$$C_2 = 5.35 \qquad k = 1.11 \text{ W/m} \cdot \text{K}$$

The thermal diffusivity is $\alpha = 4.37 \times 10^{-7}$ m^2/s, while the thermal penetration depths are estimated by $\delta_p \approx \sqrt{\alpha/\omega}$, resulting in $\delta_p = 260$ μm and $\delta_p = 26$ μm at $\omega = 2\pi$ rad/s and $\omega = 200\pi$ rad/s, respectively.

Comments:

1. The foregoing experimental technique, which is widely used to measure the thermal conductivity of microscale devices and nanostructured materials, is referred to as the *3 ω method* [7].
2. Because this technique is based on measurement of a temperature that fluctuates about a mean value that is approximately the same as the temperature of the surroundings, the measured value of k is relatively insensitive to radiation heat transfer losses from the top of the metal strip. Likewise, the technique is insensitive to thermal contact resistances that may exist at the interface between the sensing strip and the underlying material since these effects cancel when measurements are made at two different excitation frequencies [7].
3. The specific heat and density are not strongly dependent on the nanostructure of most solids, and properties of conventional material may be used.
4. The thermal penetration depth is less than the sample thickness. Therefore, treating the sample as a semi-infinite solid is a valid approach. Thinner samples could be used if higher heating frequencies were employed.

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5.10 *Finite-Difference Methods*

Analytical solutions for some simple two- and three-dimensional geometries are found in Section 5S.2.

Analytical solutions to transient problems are restricted to simple geometries and boundary conditions, such as the one-dimensional cases considered in the preceding sections. For some simple two- and three-dimensional geometries, analytical solutions are still possible. However, in many cases the geometry and/or boundary conditions preclude the use of analytical techniques, and recourse must be made to *finite-difference* (or *finite-element*) methods. Such methods, introduced in Section 4.4 for steady-state conditions, are readily extended to transient problems. In this section we consider *explicit* and *implicit* forms of finite-difference solutions to transient conduction problems.

5.10.1 Discretization of the Heat Equation: The Explicit Method

Once again consider the two-dimensional system of Figure 4.4. Under transient conditions with constant properties and no internal generation, the appropriate form of the heat equation, Equation 2.19, is

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \tag{5.72}$$

To obtain the finite-difference form of this equation, we may use the *central-difference* approximations to the spatial derivatives prescribed by Equations 4.27 and 4.28. Once again the m and n subscripts may be used to designate the x and y locations of *discrete nodal points*. However, in addition to being discretized in space, the problem must be discretized in time. The integer p is introduced for this purpose, where

$$t = p\,\Delta t \tag{5.73}$$

and the finite-difference approximation to the time derivative in Equation 5.72 is expressed as

$$\left.\frac{\partial T}{\partial t}\right|_{m,n} \approx \frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t} \tag{5.74}$$

The superscript p is used to denote the time dependence of T, and the time derivative is expressed in terms of the difference in temperatures associated with the *new* $(p + 1)$ and *previous* (p) times. Hence calculations must be performed at successive times separated by the interval Δt, and just as a finite-difference solution restricts temperature determination to discrete points in space, it also restricts it to discrete points in time.

If Equation 5.74 is substituted into Equation 5.72, the nature of the finite-difference solution will depend on the specific time at which temperatures are evaluated in the finite-difference approximations to the spatial derivatives. In the *explicit method* of solution, these temperatures are evaluated at the *previous* (p) time. Hence Equation 5.74 is considered to be a *forward-difference* approximation to the time derivative.

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Evaluating terms on the right-hand side of Equations 4.27 and 4.28 at p and substituting into Equation 5.72, the explicit form of the finite-difference equation for the interior node m, n is

$$\frac{1}{\alpha}\frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t} = \frac{T_{m+1,n}^{p} + T_{m-1,n}^{p} - 2T_{m,n}^{p}}{(\Delta x)^2} + \frac{T_{m,n+1}^{p} + T_{m,n-1}^{p} - 2T_{m,n}^{p}}{(\Delta y)^2} \quad (5.75)$$

Solving for the nodal temperature at the new $(p + 1)$ time and assuming that $\Delta x = \Delta y$, it follows that

$$T_{m,n}^{p+1} = Fo(T_{m+1,n}^{p} + T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p}) + (1 - 4Fo)T_{m,n}^{p} \quad (5.76)$$

where Fo is a finite-difference form of the Fourier number

$$Fo = \frac{\alpha\,\Delta t}{(\Delta x)^2} \quad (5.77)$$

This approach can easily be extended to one- or three-dimensional systems. If the system is one-dimensional in x, the explicit form of the finite-difference equation for an interior node m reduces to

$$T_{m}^{p+1} = Fo(T_{m+1}^{p} + T_{m-1}^{p}) + (1 - 2Fo)T_{m}^{p} \quad (5.78)$$

Equations 5.76 and 5.78 are *explicit* because *unknown* nodal temperatures for the new time are determined exclusively by *known* nodal temperatures at the previous time. Hence calculation of the unknown temperatures is straightforward. Since the temperature of each interior node is known at $t = 0$ $(p = 0)$ from prescribed initial conditions, the calculations begin at $t = \Delta t$ $(p = 1)$, where Equation 5.76 or 5.78 is applied to each interior node to determine its temperature. With temperatures known for $t = \Delta t$, the appropriate finite-difference equation is then applied at each node to determine its temperature at $t = 2\,\Delta t$ $(p = 2)$. In this way, the transient temperature distribution is obtained by *marching out in time,* using intervals of Δt.

The accuracy of the finite-difference solution may be improved by decreasing the values of Δx and Δt. Of course, the number of interior nodal points that must be considered increases with decreasing Δx, and the number of time intervals required to carry the solution to a prescribed final time increases with decreasing Δt. Hence the computation time increases with decreasing Δx and Δt. The choice of Δx is typically based on a compromise between accuracy and computational requirements. Once this selection has been made, however, the value of Δt may not be chosen independently. It is, instead, determined by *stability* requirements.

An undesirable feature of the explicit method is that it is not unconditionally *stable.* In a transient problem, the solution for the nodal temperatures should continuously approach final (steady-state) values with increasing time. However, with the explicit method, this solution may be characterized by numerically induced oscillations, which are physically impossible. The oscillations may become *unstable,* causing the solution to diverge from the actual steady-state conditions. To prevent such erroneous results, the prescribed value of Δt must be maintained below a certain limit, which depends on Δx and other parameters of the system. This dependence is termed a *stability criterion,* which may be obtained mathematically or demonstrated from a thermodynamic argument (see Problem 5.92). For the problems of interest in this text, *the criterion is determined by requiring that the coefficient associated with the node of interest at the previous*

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time is greater than or equal to zero. In general, this is done by collecting all terms involving $T_{m,n}^{p}$ to obtain the form of the coefficient. This result is then used to obtain a limiting relation involving Fo, from which the maximum allowable value of Δt may be determined. For example, with Equations 5.76 and 5.78 already expressed in the desired form, it follows that the stability criterion for a one-dimensional interior node is $(1 - 2Fo) \geq 0$, or

$$Fo \leq \frac{1}{2} \tag{5.79}$$

and for a two-dimensional node, it is $(1 - 4Fo) \geq 0$, or

$$Fo \leq \frac{1}{4} \tag{5.80}$$

For prescribed values of Δx and α, these criteria may be used to determine upper limits to the value of Δt.

Equations 5.76 and 5.78 may also be derived by applying the energy balance method of Section 4.4.3 to a control volume about the interior node. Accounting for changes in thermal energy storage, a general form of the energy balance equation may be expressed as

$$\dot{E}_{\text{in}} + \dot{E}_g = \dot{E}_{\text{st}} \tag{5.81}$$

In the interest of adopting a consistent methodology, it is again assumed that all heat flow is *into* the node.

To illustrate application of Equation 5.81, consider the surface node of the one-dimensional system shown in Figure 5.12. To more accurately determine thermal conditions near the surface, this node has been assigned a thickness that is one-half that of the interior nodes. Assuming convection transfer from an adjoining fluid and no generation, it follows from Equation 5.81 that

$$hA(T_\infty - T_0^p) + \frac{kA}{\Delta x}(T_1^p - T_0^p) = \rho cA\,\frac{\Delta x}{2}\,\frac{T_0^{p+1} - T_0^p}{\Delta t}$$

or, solving for the surface temperature at $t + \Delta t$,

$$T_0^{p+1} = \frac{2h\,\Delta t}{\rho c\,\Delta x}(T_\infty - T_0^p) + \frac{2\alpha\,\Delta t}{\Delta x^2}(T_1^p - T_0^p) + T_0^p$$

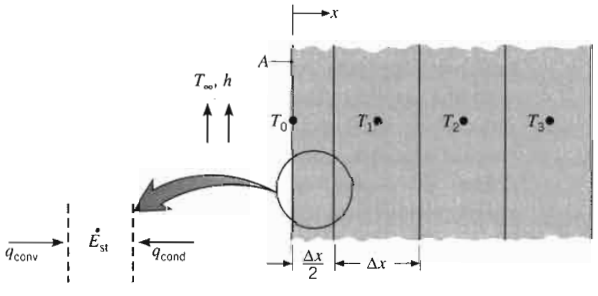


FIGURE 5.12 Surface node with convection and one-dimensional transient conduction.

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Recognizing that $(2h\ \Delta t/\rho c\ \Delta x) = 2(h\ \Delta x/k)(\alpha\ \Delta t/\Delta x^2) = 2\ Bi\ Fo$ and grouping terms involving T_0^p, it follows that

$$T_0^{p+1} = 2Fo(T_1^p + Bi\,T_\infty) + (1 - 2Fo - 2Bi\,Fo)T_0^p \tag{5.82}$$

The finite-difference form of the Biot number is

$$Bi = \frac{h\,\Delta x}{k} \tag{5.83}$$

Recalling the procedure for determining the stability criterion, we require that the coefficient for T_0^p be greater than or equal to zero. Hence

$$1 - 2Fo - 2Bi\,Fo \geq 0$$

or

$$Fo(1 + Bi) \leq \frac{1}{2} \tag{5.84}$$

Since the complete finite-difference solution requires the use of Equation 5.78 for the interior nodes, as well as Equation 5.82 for the surface node, Equation 5.84 must be contrasted with Equation 5.79 to determine which requirement is more stringent. Since $Bi \geq 0$, it is apparent that the limiting value of Fo for Equation 5.84 is less than that for Equation 5.79. To ensure stability for all nodes, Equation 5.84 should therefore be used to select the maximum allowable value of Fo, and hence Δt, to be used in the calculations.

Forms of the explicit finite-difference equation for several common geometries are presented in Table 5.3(*a*). Each equation may be derived by applying the energy balance method to a control volume about the corresponding node. To develop confidence in your ability to apply this method, you should attempt to verify at least one of these equations.

EXAMPLE 5.9

A fuel element of a nuclear reactor is in the shape of a plane wall of thickness $2L = 20$ mm and is convectively cooled at both surfaces, with $h = 1100\ \text{W/m}^2 \cdot \text{K}$ and $T_\infty = 250°\text{C}$. At normal operating power, heat is generated uniformly within the element at a volumetric rate of $\dot{q}_1 = 10^7\ \text{W/m}^3$. A departure from the steady-state conditions associated with normal operation will occur if there is a change in the generation rate. Consider a sudden change to $\dot{q}_2 = 2 \times 10^7\ \text{W/m}^3$, and use the explicit finite-difference method to determine the fuel element temperature distribution after 1.5 s. The fuel element thermal properties are $k = 30\ \text{W/m} \cdot \text{K}$ and $\alpha = 5 \times 10^{-6}\ \text{m}^2/\text{s}$.

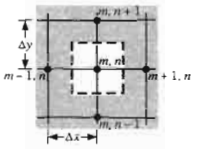
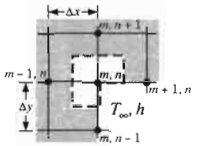
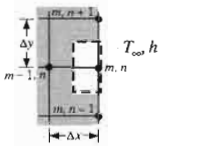
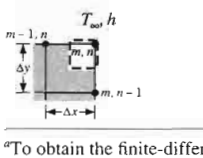
SOLUTION

Known: Conditions associated with heat generation in a rectangular fuel element with surface cooling.

Find: Temperature distribution 1.5 s after a change in operating power.

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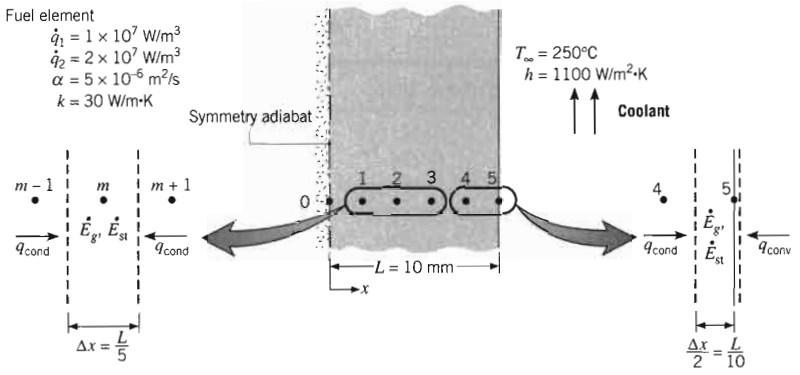
TABLE 5.3 Transient, two-dimensional finite-difference equations ($\Delta x = \Delta y$)

Configuration	(*a*) Explicit Method: Finite-Difference Equation	Stability Criterion	(*b*) Implicit Method
	$T_{m,n}^{p+1} = Fo(T_{m+1,n}^{p} + T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p}) + (1 - 4Fo)T_{m,n}^{p}$ (5.76) 1. Interior node	$Fo \le \frac{1}{4}$ (5.80)	$(1 + 4Fo)T_{m,n}^{p+1} - Fo(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p}$ (5.92)
	$T_{m,n}^{p+1} = \frac{2}{3}Fo(T_{m+1,n}^{p} + 2T_{m-1,n}^{p} + 2T_{m,n+1}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - \frac{4}{3}Bi\,Fo)T_{m,n}^{p}$ (5.85) 2. Node at interior corner with convection	$Fo(3 + Bi) \le \frac{3}{4}$ (5.86)	$(1 + 4Fo(1 + \frac{1}{3}Bi))T_{m,n}^{p+1} - \frac{2}{3}Fo \cdot (T_{m+1,n}^{p+1} + 2T_{m-1,n}^{p+1} + 2T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + \frac{4}{3}Bi\,Fo\,T_{\infty}$ (5.95)
	$T_{m,n}^{p+1} = Fo(2T_{m-1,n}^{p} + T_{m,n+1}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - 2Bi\,Fo)T_{m,n}^{p}$ (5.87) 3. Node at plane surface with convection[a]	$Fo(2 + Bi) \le \frac{1}{2}$ (5.88)	$(1 + 2Fo(2 + Bi))T_{m,n}^{p+1} - Fo(2T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + 2Bi\,Fo\,T_{\infty}$ (5.96)
	$T_{m,n}^{p+1} = 2Fo(T_{m-1,n}^{p} + T_{m,n-1}^{p} + 2Bi\,T_{\infty}) + (1 - 4Fo - 4Bi\,Fo)T_{m,n}^{p}$ (5.89) 4. Node at exterior corner with convection	$Fo(1 + Bi) \le \frac{1}{4}$ (5.90)	$(1 + 4Fo(1 + Bi))T_{m,n}^{p+1} - 2Fo(T_{m-1,n}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} + 4Bi\,Fo\,T_{\infty}$ (5.97)

[a]To obtain the finite-difference equation and/or stability criterion for an adiabatic surface (or surface of symmetry), simply set *Bi* equal to zero.

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Schematic:



Assumptions:

1. One-dimensional conduction in x.
2. Uniform generation.
3. Constant properties.

Analysis: A numerical solution will be obtained using a space increment of $\Delta x = 2$ mm. Since there is symmetry about the midplane, the nodal network yields six unknown nodal temperatures. Using the energy balance method, Equation 5.81, an explicit finite-difference equation may be derived for any interior node m.

$$kA\frac{T_{m-1}^p - T_m^p}{\Delta x} + kA\frac{T_{m+1}^p - T_m^p}{\Delta x} + \dot{q}A\,\Delta x = \rho A\,\Delta x\,c\frac{T_m^{p+1} - T_m^p}{\Delta t}$$

Solving for T_m^{p+1} and rearranging,

$$T_m^{p+1} = Fo\left[T_{m-1}^p + T_{m+1}^p + \frac{\dot{q}(\Delta x)^2}{k}\right] + (1 - 2Fo)T_m^p \tag{1}$$

This equation may be used for node 0, with $T_{m-1}^p = T_{m+1}^p$, as well as for nodes 1, 2, 3, and 4. Applying energy conservation to a control volume about node 5,

$$hA(T_\infty - T_5^p) + kA\frac{T_4^p - T_5^p}{\Delta x} + \dot{q}A\frac{\Delta x}{2} = \rho A\frac{\Delta x}{2}c\frac{T_5^{p+1} - T_5^p}{\Delta t}$$

or

$$T_5^{p+1} = 2Fo\left[T_4^p + Bi\,T_\infty + \frac{\dot{q}(\Delta x)^2}{2k}\right] + (1 - 2Fo - 2Bi\,Fo)T_5^p \tag{2}$$

Since the most restrictive stability criterion is associated with Equation 2, we select Fo from the requirement that

$$Fo(1 + Bi) \leq \frac{1}{2}$$

[5] To obtain the finite-difference equation and/or stability criterion for an adiabatic surface (or surface of symmetry), simply set Bi equal to zero.

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Hence, with

$$Bi = \frac{h\,\Delta x}{k} = \frac{1100\ \text{W/m}^2 \cdot \text{K}\ (0.002\ \text{m})}{30\ \text{W/m} \cdot \text{K}} = 0.0733$$

it follows that

$$Fo \leq 0.466$$

or

$$\Delta t = \frac{Fo(\Delta x)^2}{\alpha} \leq \frac{0.466(2 \times 10^{-3}\ \text{m})^2}{5 \times 10^{-6}\ \text{m}^2/\text{s}} \leq 0.373\ \text{s}$$

To be well within the stability limit, we select $\Delta t = 0.3$ s, which corresponds to

$$Fo = \frac{5 \times 10^{-6}\ \text{m}^2/\text{s}(0.3\ \text{s})}{(2 \times 10^{-3}\ \text{m})^2} = 0.375$$

Substituting numerical values, including $\dot{q} = \dot{q}_2 = 2 \times 10^7\ \text{W/m}^3$, the nodal equations become

$$T_0^{p+1} = 0.375(2T_1^p + 2.67) + 0.250T_0^p$$

$$T_1^{p+1} = 0.375(T_0^p + T_2^p + 2.67) + 0.250T_1^p$$

$$T_2^{p+1} = 0.375(T_1^p + T_3^p + 2.67) + 0.250T_2^p$$

$$T_3^{p+1} = 0.375(T_2^p + T_4^p + 2.67) + 0.250T_3^p$$

$$T_4^{p+1} = 0.375(T_3^p + T_5^p + 2.67) + 0.250T_4^p$$

$$T_5^{p+1} = 0.750(T_4^p + 19.67) + 0.195T_5^p$$

To begin the marching solution, the initial temperature distribution must be known. This distribution is given by Equation 3.42, with $\dot{q} = \dot{q}_1$. Obtaining $T_s = T_5$ from Equation 3.46,

$$T_5 = T_\infty + \frac{\dot{q}L}{h} = 250°\text{C} + \frac{10^7\ \text{W/m}^3 \times 0.01\ \text{m}}{1100\ \text{W/m}^2 \cdot \text{K}} = 340.91°\text{C}$$

it follows that

$$T(x) = 16.67\left(1 - \frac{x^2}{L^2}\right) + 340.91°\text{C}$$

Computed temperatures for the nodal points of interest are shown in the first row of the accompanying table.

Using the finite-difference equations, the nodal temperatures may be sequentially calculated with a time increment of 0.3 s until the desired final time is reached. The results are illustrated in rows 2 through 6 of the table and may be contrasted

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with the new steady-state condition (row 7), which was obtained by using Equations 3.42 and 3.46 with $\dot{q} = \dot{q}_2$:

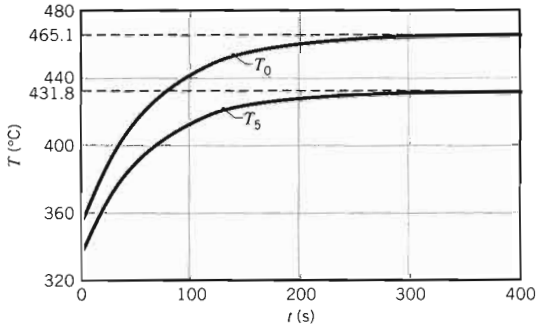
Tabulated Nodal Temperatures

p	t(s)	T_0	T_1	T_2	T_3	T_4	T_5
0	0	357.58	356.91	354.91	351.58	346.91	340.91
1	0.3	358.08	357.41	355.41	352.08	347.41	341.41
2	0.6	358.58	357.91	355.91	352.58	347.91	341.88
3	0.9	359.08	358.41	356.41	353.08	348.41	342.35
4	1.2	359.58	358.91	356.91	353.58	348.89	342.82
5	1.5	360.08	359.41	357.41	354.07	349.37	343.27
∞	∞	465.15	463.82	459.82	453.15	443.82	431.82

Comments:

1. It is evident that at 1.5 s, the wall is in the early stages of the transient process and that many additional calculations would have to be made to reach steady-state conditions with the finite-difference solution. The computation time could be reduced slightly by using the maximum allowable time increment ($\Delta t = 0.373$ s), but with some loss of accuracy. In the interest of maximizing accuracy, the time interval should be reduced until the computed results become independent of further reductions in Δt.

   Extending the finite-difference solution, the time required to achieve the new steady-state condition may be determined, with temperature histories computed for the midplane (0) and surface (5) nodes having the following forms:



   With steady-state temperatures of $T_0 = 465.15$°C and $T_5 = 431.82$°C, it is evident that the new equilibrium condition is reached within 250 s of the step change in operating power.

2. The *IHT* option entitled *Tools*, *Finite-Difference Equations*, *One-Dimensional*, *Transient* can be used to express the nodal equations according to the implicit method of solution. In solving the equations, the user is prompted to enter the initial temperature of each node on the *Initial Condition* (*IC*) pad. The problem may also be solved using *FEHT*, with the initial temperature distribution determined by applying the *Steady-State* command of the *Set-up* menu for

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$\dot{q}_1 = 10^7$ W/m^3. The generation rate may then be changed to $\dot{q}_2 = 2 \times 10^7$ W/m^3, and after enabling the *Transient* command of the *Set-up* menu, the *Continue* command of the *Solve* menu may be activated to obtain the transient solution. For details on application of *IHT* or *FEHT* to transient conduction problems, refer to the booklet *Software Tools and User's Guides,* which accompanies the software.

5.10.2 Discretization of the Heat Equation: The Implicit Method

In the *explicit* finite-difference scheme, the temperature of any node at $t + \Delta t$ may be calculated from knowledge of temperatures at the same and neighboring nodes for the *preceding time t.* Hence determination of a nodal temperature at some time is *independent* of temperatures at other nodes for the *same time.* Although the method offers computational convenience, it suffers from limitations on the selection of Δt. For a given space increment, the time interval must be compatible with stability requirements. Frequently, this dictates the use of extremely small values of Δt, and a very large number of time intervals may be necessary to obtain a solution.

A reduction in the amount of computation time may often be realized by employing an *implicit,* rather than explicit, finite-difference scheme. The implicit form of a finite-difference equation may be derived by using Equation 5.74 to approximate the time derivative, while evaluating all other temperatures at the *new* $(p + 1)$ time, instead of the previous (p) time. Equation 5.74 is then considered to provide a *backward-difference* approximation to the time derivative. In contrast to Equation 5.75, the implicit form of the finite-difference equation for the interior node of a two-dimensional system is then

$$\frac{1}{\alpha}\frac{T_{m,n}^{p+1} - T_{m,n}^{p}}{\Delta t} = \frac{T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta x)^2} + \frac{T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1} - 2T_{m,n}^{p+1}}{(\Delta y)^2} \tag{5.91}$$

Rearranging and assuming $\Delta x = \Delta y$, it follows that

$$(1 + 4Fo)T_{m,n}^{p+1} - Fo(T_{m+1,n}^{p+1} + T_{m-1,n}^{p+1} + T_{m,n+1}^{p+1} + T_{m,n-1}^{p+1}) = T_{m,n}^{p} \tag{5.92}$$

From Equation 5.92 it is evident that the *new* temperature of the m, n node depends on the *new* temperatures of its adjoining nodes, which are, in general, unknown. Hence, to determine the unknown nodal temperatures at $t + \Delta t$, the corresponding nodal equations must be *solved simultaneously.* Such a solution may be effected by using Gauss–Seidel iteration or matrix inversion, as discussed in Section 4.5. The *marching solution* would then involve simultaneously solving the nodal equations at each time $t = \Delta t, 2\Delta t, \ldots$, until the desired final time was reached.

Relative to the explicit method, the implicit formulation has the important advantage of being *unconditionally stable.* That is, the solution remains stable for all space and time intervals, in which case there are no restrictions on Δx and Δt.

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Since larger values of Δt may therefore be used with an implicit method, computation times may often be reduced, with little loss of accuracy. Nevertheless, to maximize accuracy, Δt should be sufficiently small to ensure that the results are independent of further reductions in its value.

The implicit form of a finite-difference equation may also be derived from the energy balance method. For the surface node of Figure 5.12, it is readily shown that

$$(1 + 2Fo + 2Fo\,Bi)T_0^{p+1} - 2Fo\,T_1^{p+1} = 2Fo\,Bi\,T_\infty + T_0^p \tag{5.93}$$

For any interior node of Figure 5.12, it may also be shown that

$$(1 + 2Fo)T_m^{p+1} - Fo\,(T_{m-1}^{p+1} + T_{m+1}^{p+1}) = T_m^p \tag{5.94}$$

Forms of the implicit finite-difference equation for other common geometries are presented in Table 5.3(*b*). Each equation may be derived by applying the energy balance method.

EXAMPLE 5.10

A thick slab of copper initially at a uniform temperature of 20°C is suddenly exposed to radiation at one surface such that the net heat flux is maintained at a constant value of 3×10^5 W/m^2. Using the explicit and implicit finite-difference techniques with a space increment of $\Delta x = 75$ mm, determine the temperature at the irradiated surface and at an interior point that is 150 mm from the surface after 2 min have elapsed. Compare the results with those obtained from an appropriate analytical solution.

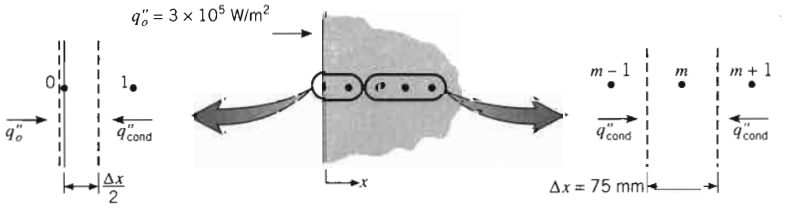
SOLUTION

Known: Thick slab of copper, initially at a uniform temperature, is subjected to a constant net heat flux at one surface.

Find:

1. Using the explicit finite-difference method, determine temperatures at the surface and 150 mm from the surface after an elapsed time of 2 min.
2. Repeat the calculations using the implicit finite-difference method.
3. Determine the same temperatures analytically.

Schematic:



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Assumptions:

1. One-dimensional conduction in x.
2. Thick slab may be approximated as a semi-infinite medium with constant surface heat flux.
3. Constant properties.

Properties: Table A.1, copper (300 K): $k = 401$ W/m · K, $\alpha = 117 \times 10^{-6}$ m²/s.

Analysis:

1. An explicit form of the finite-difference equation for the surface node may be obtained by applying an energy balance to a control volume about the node.

$$q_o''A + kA\frac{T_1^p - T_0^p}{\Delta x} = \rho A\frac{\Delta x}{2}c\frac{T_0^{p+1} - T_0^p}{\Delta t}$$

or

$$T_0^{p+1} = 2Fo\left(\frac{q_o''\Delta x}{k} + T_1^p\right) + (1 - 2Fo)T_0^p$$

The finite-difference equation for any interior node is given by Equation 5.78. Both the surface and interior nodes are governed by the stability criterion

$$Fo \leq \frac{1}{2}$$

Noting that the finite-difference equations are simplified by choosing the maximum allowable value of Fo, we select $Fo = \frac{1}{2}$. Hence

$$\Delta t = Fo\frac{(\Delta x)^2}{\alpha} = \frac{1}{2}\frac{(0.075\text{ m})^2}{117 \times 10^{-6}\text{ m}^2/\text{s}} = 24\text{ s}$$

With

$$\frac{q_o''\Delta x}{k} = \frac{3 \times 10^5\text{ W/m}^2\,(0.075\text{ m})}{401\text{ W/m} \cdot \text{K}} = 56.1°\text{C}$$

the finite-difference equations become

$$T_0^{p+1} = 56.1°\text{C} + T_1^p \qquad \text{and} \qquad T_m^{p+1} = \frac{T_{m+1}^p + T_{m-1}^p}{2}$$

for the surface and interior nodes, respectively. Performing the calculations, the results are tabulated as follows:

Explicit Finite-Difference Solution for $Fo = \frac{1}{2}$

p	t(s)	T_0	T_1	T_2	T_3	T_4
0	0	20	20	20	20	20
1	24	76.1	20	20	20	20
2	48	76.1	48.1	20	20	20
3	72	104.2	48.1	34.1	20	20
4	96	104.2	69.1	34.1	27.1	20
5	120	125.3	69.1	48.1	27.1	20

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After 2 min, the surface temperature and the desired interior temperature are $T_0 = 125.3°C$ and $T_2 = 48.1°C$.

Note that calculation of identical temperatures at successive times for the same node is an idiosyncrasy of using the maximum allowable value of Fo with the explicit finite-difference technique. The actual physical condition is, of course, one in which the temperature changes continuously with time. The idiosyncrasy is eliminated and the accuracy of the calculations is improved by reducing the value of Fo.

To determine the extent to which the accuracy may be improved by reducing Fo, let us redo the calculations for $Fo = \frac{1}{4}(\Delta t = 12\text{ s})$. The finite-difference equations are then of the form

$$T_0^{p+1} = \frac{1}{2}(56.1°C + T_1^p) + \frac{1}{2}T_0^p$$

$$T_m^{p+1} = \frac{1}{4}(T_{m+1}^p + T_{m-1}^p) + \frac{1}{2}T_m^p$$

and the results of the calculations are tabulated as follows:

Explicit Finite-Difference Solution for $Fo = \frac{1}{4}$

p	t(s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	0	20	20	20	20	20	20	20	20	20
1	12	48.1	20	20	20	20	20	20	20	20
2	24	62.1	27.0	20	20	20	20	20	20	20
3	36	72.6	34.0	21.8	20	20	20	20	20	20
4	48	81.4	40.6	24.4	20.4	20	20	20	20	20
5	60	89.0	46.7	27.5	21.3	20.1	20	20	20	20
6	72	95.9	52.5	30.7	22.6	20.4	20.0	20	20	20
7	84	102.3	57.9	34.1	24.1	20.8	20.1	20.0	20	20
8	96	108.1	63.1	37.6	25.8	21.5	20.3	20.0	20.0	20
9	108	113.7	68.0	41.0	27.6	22.2	20.5	20.1	20.0	20.0
10	120	118.9	72.6	44.4	29.6	23.2	20.8	20.2	20.0	20.0

After 2 min, the desired temperatures are $T_0 = 118.9°C$ and $T_2 = 44.4°C$. Comparing the above results with those obtained for $Fo = \frac{1}{2}$, it is clear that by reducing Fo we have eliminated the problem of recurring temperatures. We have also predicted greater thermal penetration (to node 6 instead of node 3). An assessment of the improvement in accuracy must await a comparison with results based on an exact solution.

2. Performing an energy balance on a control volume about the surface node, the implicit form of the finite-difference equation is

$$q_o'' + k\frac{T_1^{p+1} - T_0^{p+1}}{\Delta x} = \rho\frac{\Delta x}{2}c\frac{T_0^{p+1} - T_0^p}{\Delta t}$$

or

$$(1 + 2Fo)T_0^{p+1} - 2FoT_1^{p+1} = \frac{2\alpha q_o''\Delta t}{k\,\Delta x} + T_0^p$$

Arbitrarily choosing $Fo = \frac{1}{2}(\Delta t = 24\text{ s})$, it follows that

$$2T_0^{p+1} - T_1^{p+1} = 56.1 + T_0^p$$

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From Equation 5.94, the finite-difference equation for any interior node is then of the form

$$-T_{m-1}^{p+1} + 4T_m^{p+1} - T_{m+1}^{p+1} = 2T_m^p$$

Since we are dealing with a semi-infinite solid, the number of nodes is, in principle, infinite. In practice, however, the number may be limited to the nodes that are affected by the change in the boundary condition for the time of interest. From the results of the explicit method, it is evident that we are safe in choosing nine nodes corresponding to $T_0, T_1, \ldots, T_8$. We are thereby assuming that, at $t = 120$ s, there has been no change in T_8.

We now have a set of nine equations that must be solved simultaneously for each time increment. Using the matrix inversion method, we express the equations in the form $[A][T] = [C]$, where

$$[A] = \begin{bmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 4 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 4 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 4 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 4 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 4 \end{bmatrix}$$

$$[C] = \begin{bmatrix} 56.1 + T_0^p \\ 2T_1^p \\ 2T_2^p \\ 2T_3^p \\ 2T_4^p \\ 2T_5^p \\ 2T_6^p \\ 2T_7^p \\ 2T_8^p + T_9^{p+1} \end{bmatrix}$$

Note that numerical values for the components of $[C]$ are determined from previous values of the nodal temperatures. Note also how the finite-difference equation for node 8 appears in matrices $[A]$ and $[C]$.

A table of nodal temperatures may be compiled, beginning with the first row ($p = 0$) corresponding to the prescribed initial condition. To obtain nodal temperatures for subsequent times, the inverse of the coefficient matrix $[A]^{-1}$ must first be found. At each time $p + 1$, it is then multiplied by the column vector $[C]$, which is evaluated at p, to obtain the temperatures $T_0^{p+1}, T_1^{p+1}, \ldots, T_8^{p+1}$. For example, multiplying $[A]^{-1}$ by the column vector corresponding to $p = 0$,

$$[C]_{p=0} = \begin{bmatrix} 76.1 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 40 \\ 60 \end{bmatrix}$$

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the second row of the table is obtained. Updating $[C]$, the process is repeated four more times to determine the nodal temperatures at 120 s. The desired temperatures are $T_0 = 114.7°C$ and $T_2 = 44.2°C$.

Implicit Finite-Difference Solution for $Fo = \frac{1}{2}$

p	t(s)	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8
0	0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0	20.0
1	24	52.4	28.7	22.3	20.6	20.2	20.0	20.0	20.0	20.0
2	48	74.0	39.5	26.6	22.1	20.7	20.2	20.1	20.0	20.0
3	72	90.2	50.3	32.0	24.4	21.6	20.6	20.2	20.1	20.0
4	96	103.4	60.5	38.0	27.4	22.9	21.1	20.4	20.2	20.1
5	120	114.7	70.0	44.2	30.9	24.7	21.9	20.8	20.3	20.1

3. Approximating the slab as a semi-infinite medium, the appropriate analytical expression is given by Equation 5.59, which may be applied to any point in the slab.

$$T(x,t) - T_i = \frac{2q_o''(\alpha t/\pi)^{1/2}}{k} \exp\left(-\frac{x^2}{4\alpha t}\right) - \frac{q_o'' x}{k} \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)$$

At the surface, this expression yields

$$T(0, 120\,\text{s}) - 20°C = \frac{2 \times 3 \times 10^5\,\text{W/m}^2}{401\,\text{W/m} \cdot \text{K}} (117 \times 10^{-6}\,\text{m}^2/\text{s} \times 120\,\text{s}/\pi)^{1/2}$$

or

$$T(0, 120\,\text{s}) = 120.0°C \quad \triangleleft$$

At the interior point ($x = 0.15$ m)

$$T(0.15\,\text{m}, 120\,\text{s}) - 20°C = \frac{2 \times 3 \times 10^5\,\text{W/m}^2}{401\,\text{W/m} \cdot \text{K}} \times (117 \times 10^{-6}\,\text{m}^2/\text{s} \times 120\,\text{s}/\pi)^{1/2} \times \exp\left[-\frac{(0.15\,\text{m})^2}{4 \times 117 \times 10^{-6}\,\text{m}^2/\text{s} \times 120\,\text{s}}\right] - \frac{3 \times 10^5\,\text{W/m}^2 \times 0.15\,\text{m}}{401\,\text{W/m} \cdot \text{K}} \times \left[1 - \operatorname{erf}\left(\frac{0.15\,\text{m}}{2\sqrt{117 \times 10^{-6}\,\text{m}^2/\text{s} \times 120\,\text{s}}}\right)\right]$$

$$T(0.15\,\text{m}, 120\,\text{s}) = 45.4°C \quad \triangleleft$$

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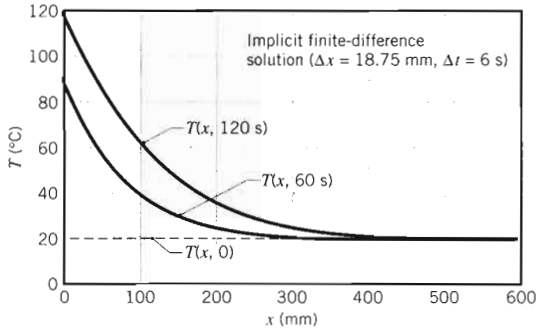
1. Comparing the exact results with those obtained from the three approximate solutions, it is clear that the explicit method with $Fo = \frac{1}{4}$ provides the most accurate predictions.

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Method	$T_0 = T(0, 120\ \text{s})$	$T_2 = T(0.15\ \text{m}, 120\ \text{s})$
Explicit ($Fo = \frac{1}{2}$)	125.3	48.1
Explicit ($Fo = \frac{1}{4}$)	118.9	44.4
Implicit ($Fo = \frac{1}{2}$)	114.7	44.2
Exact	120.0	45.4

This is not unexpected, since the corresponding value of Δt is 50% smaller than that used in the other two methods. Although computations are simplified by using the maximum allowable value of Fo in the explicit method, the accuracy of the results is seldom satisfactory.

2. The accuracy of the foregoing calculations is adversely affected by the coarse grid ($\Delta x = 75$ mm), as well as by the large time steps ($\Delta t = 24$ s, 12 s). Applying the implicit method with $\Delta x = 18.75$ mm and $\Delta t = 6$ s ($Fo = 2.0$), the solution yields $T_0 = T(0,\ 120\ \text{s}) = 119.2°\text{C}$ and $T_2 = T(0.15\ \text{m},\ 120\ \text{s}) = 45.3°\text{C}$, both of which are in good agreement with the exact solution. Complete temperature distributions may be plotted at any of the discrete times, and results obtained at $t = 60$ and 120 s are as follows:



Note that, at $t = 120$ s, the assumption of a semi-infinite medium would remain valid if the thickness of the slab exceeded approximately 500 mm.

3. Note that the coefficient matrix $[A]$ is *tridiagonal*. That is, all elements are zero except those that are on, or to either side of, the main diagonal. Tridiagonal matrices are associated with one-dimensional conduction problems.

4. A more general radiative heating condition would be one in which the surface is suddenly exposed to large surroundings at an elevated temperature T_{sur} (Problem 5.107). The net rate at which radiation is transferred to the surface may then be calculated from Equation 1.7. Allowing for convection heat transfer to the surface, application of conservation of energy to the surface node yields an explicit finite-difference equation of the form

$$\varepsilon\sigma[T_{sur}^4 - (T_0^p)^4] + h(T_\infty - T_0^p) + k\frac{T_1^p - T_0^p}{\Delta x} = \rho\frac{\Delta x}{2}c\frac{T_0^{p+1} - T_0^p}{\Delta t}$$

Use of this finite-difference equation in a numerical solution is complicated by the fact that it is *nonlinear*. However, the equation may be *linearized* by introducing

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the radiation heat transfer coefficient h_r defined by Equation 1.9, and the finite-difference equation is

$$h_r^p(T_{sur} - T_0^p) + h(T_\infty - T_0^p) + k\frac{T_1^p - T_0^p}{\Delta x} = \rho\frac{\Delta x}{2}c\frac{T_0^{p+1} - T_0^p}{\Delta t}$$

The solution may proceed in the usual manner, although the effect of a radiative Biot number ($Bi_r \equiv h_r\,\Delta x/k$) must be included in the stability criterion and the value of h_r must be updated at each step in the calculations. If the implicit method is used, h_r is calculated at $p + 1$, in which case an iterative calculation must be made at each time step.

5. The *IHT Tools*, *Finite-Difference Equations*, *One-Dimensional*, *Transient* option can be used to formulate the nodal equations for the implicit method. A schematic of the pertinent node and its control volume appears in the tool pad, with provision for entering indices associated with the node of interest and relevant adjoining nodes. The system of equations, with comments, has this form:

```
//Node 0

rho * cp * der(T0,t) = fd_1d_sur_w(T0,T1,k,qdot,deltax,Tinf,h,q"a0)
q"a0 = 3e5          // Applied heat flux, W/m^2
Tinf = 20           // Arbitrary value for convection process
h = 1e-20           // Makes convection process negligible

// Interior nodes, 1 - 8
rho*cp*der(T1,t) = fd_1d_int(T1,T2,T0,k,qdot,deltax)
rho*cp*der(T2,t) = fd_1d_int(T2,T3,T1,k,qdot,deltax)
rho*cp*der(T3,t) = fd_1d_int(T3,T4,T2,k,qdot,deltax)
rho*cp*der(T4,t) = fd_1d_int(T4,T5,T3,k,qdot,deltax)
rho*cp*der(T5,t) = fd_1d_int(T5,T6,T4,k,qdot,deltax)
rho*cp*der(T6,t) = fd_1d_int(T6,T7,T5,k,qdot,deltax)
rho*cp*der(T7,t) = fd_1d_int(T7,T8,T6,k,qdot,deltax)
rho*cp*der(T8,t) = fd_1d_int(T8,T9,T7,k,qdot,deltax)
```

Note that the prescribed convection coefficient renders convection negligible, as required by the problem statement. Using $\Delta x = 75$ mm and $\Delta t = 12$ s, the solution yields $T_0 = T(0,120\text{ s}) = 116.0°\text{C}$ and $T_2 = T(150\text{ mm}, 120\text{ s}) = 44.2°\text{C}$. Contrast these results with those summarized in Comment 1.

6. This example is provided in *FEHT* as a solved model accessed through the *Toolbar* menu, *Examples*. The input screen summarizes key pre- and post-processing steps, as well as results for nodal spacings of 1 and 0.125 mm. As an exercise, press *Run* to solve for the nodal temperatures, and in the *View* menu, select *Temperature Contours* to represent the temperature field in the form of isotherms.

5.11 Summary

Transient conduction occurs in numerous engineering applications and may be treated using different methods. There is certainly much to be said for simplicity, in which case, when confronted with a transient problem, the first thing you should do

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is calculate the Biot number. If this number is much less than unity, you may use the lumped capacitance method to obtain accurate results with minimal computational requirements. However, if the Biot number is not much less than unity, spatial effects must be considered, and some other method must be used. Analytical results are available in convenient graphical and equation form for the plane wall, the infinite cylinder, the sphere, and the semi-infinite solid. You should know when and how to use these results. If geometrical complexities and/or the form of the boundary conditions preclude their use, recourse must be made to an approximate numerical technique, such as the finite-difference method.

You may test your understanding of key concepts by addressing the following questions.

- Under what conditions may the *lumped capacitance method* be used to predict the transient response of a solid to a change in its thermal environment?
- What is the physical interpretation of the *Biot number*?
- Is the lumped capacitance method of analysis likely to be more applicable for a hot solid being cooled by forced convection in air or in water? By forced convection in air or natural convection in air?
- Is the lumped capacitance method of analysis likely to be more applicable for cooling of a hot solid made of copper or aluminum? For silicon nitride or glass?
- What parameters determine the *time constant* associated with the transient thermal response of a lumped capacitance solid? Is this response accelerated or decelerated by an increase in the convection coefficient? By an increase in the density or specific heat of the solid?
- For one-dimensional, transient conduction in a plane wall, a long cylinder or a sphere with surface convection, what dimensionless parameters may be used to simplify the representation of thermal conditions? How are these parameters defined?
- Why is the semi-infinite solution applicable to any geometry at early times?
- What is the physical interpretation of the *Fourier number*?
- What requirement must be satisfied for use of a *one-term approximation* to determine the transient thermal response of a plane wall, a long cylinder or a sphere experiencing one-dimensional conduction due to a change in surface conditions? At what stage of a transient process is the requirement not satisfied?
- What does transient heating or cooling of a plane wall with equivalent convection conditions at opposite surfaces have in common with a plane wall heated or cooled by convection at one surface and well insulated at the other surface?
- How may a one-term approximation be used to determine the transient thermal response of a plane wall, long cylinder, or sphere subjected to a sudden change in surface temperature?
- For one-dimensional, transient conduction, what is implied by the idealization of a *semi-infinite* solid? Under what conditions may the idealization be applied to a plane wall?
- What differentiates an *explicit*, finite-difference solution to a transient conduction problem from an *implicit* solution?
- What is meant by characterization of the implicit finite-difference method *as unconditionally stable*? What constraint is placed on the explicit method to ensure a stable solution?

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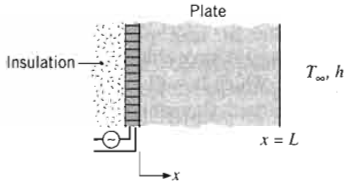
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Qualitative Considerations

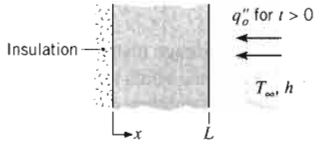
5.1 Consider a thin electrical heater attached to a plate and backed by insulation. Initially, the heater and plate are at the temperature of the ambient air, T_∞. Suddenly, the power to the heater is activated, yielding a constant heat flux q_o'' (W/m^2) at the inner surface of the plate.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q_x''(L, t)$ as a function of time.

5.2 The inner surface of a plane wall is insulated while the outer surface is exposed to an airstream at T_∞. The wall is at a uniform temperature corresponding to that of the airstream. Suddenly, a radiation heat source is switched on applying a uniform flux q_o'' to the outer surface.



(a) Sketch and label, on T–x coordinates, the temperature distributions: initial, steady-state, and at two intermediate times.

(b) Sketch the heat flux at the outer surface $q_x''(L, t)$ as a function of time.

5.3 A microwave oven operates on the principle that application of a high-frequency field causes electrically polarized molecules in food to oscillate. The net effect is a nearly *uniform generation* of thermal energy within the food. Consider the process of cooking a slab of beef of thickness $2L$ in a microwave oven and compare it with cooking in a conventional oven, where *each side* of the slab is *heated by radiation.* In each case the meat is to be heated from 0°C to a *minimum* temperature of 90°C. Base your comparison on a sketch of the temperature distribution at selected times for each of the cooking processes. In particular, consider the time t_0 at which heating is initiated, a time t_1 during the heating process, the time t_2 corresponding to the conclusion of heating, and a time t_3 well into the subsequent cooling process.

5.4 A plate of thickness $2L$, surface area A_s, mass M, and specific heat c_p, initially at a uniform temperature T_i, is suddenly heated on both surfaces by a convection process (T_∞, h) for a period of time t_o, following which the plate is insulated. Assume that the midplane temperature does not reach T_∞ within this period of time.

(a) Assuming $Bi \gg 1$ for the heating process, sketch and label, on T–x coordinates, the following temperature distributions: initial, steady-state ($t \to \infty$), $T(x, t_o)$, and at two intermediate times between $t = t_o$ and $t \to \infty$.

(b) Sketch and label, on T–t coordinates, the midplane and exposed surface temperature distributions.

(c) Repeat parts (a) and (b) assuming $Bi \ll 1$ for the plate.

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(d) Derive an expression for the steady-state temperature $T(x, \infty) = T_f$, leaving your result in terms of plate parameters (M, c_p), thermal conditions (T_i, T_∞, h), the surface temperature $T(L, t)$, and the heating time t_o.

Lumped Capacitance Method

5.5 Steel balls 12 mm in diameter are annealed by heating to 1150 K and then slowly cooling to 400 K in an air environment for which $T_\infty = 325$ K and $h = 20$ W/m$^2 \cdot$K. Assuming the properties of the steel to be $k = 40$ W/m $\cdot$ K, $\rho = 7800$ kg/m^3, and $c = 600$ J/kg $\cdot$ K, estimate the time required for the cooling process.

5.6 Consider the steel balls of Problem 5.5, except now the air temperature increases with time as $T_\infty(t) = 325$ K + at where $a = 0.1875$ K/s.

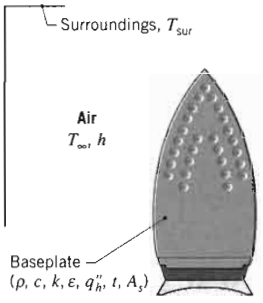
(a) Sketch the ball temperature versus time for $0 \le t \le 1$ h. Also show the ambient temperature, T_∞, in your graph. Explain special features of the ball temperature behavior.

(b) Find an expression for the ball temperature as a function of time, $T(t)$, and plot the ball temperature for $0 \le t \le 1$ h. Was your sketch correct?

5.7 The heat transfer coefficient for air flowing over a sphere is to be determined by observing the temperature–time history of a sphere fabricated from pure copper. The sphere, which is 12.7 mm in diameter, is at 66°C before it is inserted into an airstream having a temperature of 27°C. A thermocouple on the outer surface of the sphere indicates 55°C 69 s after the sphere is inserted in the airstream. Assume, and then justify, that the sphere behaves as a spacewise isothermal object and calculate the heat transfer coefficient.

5.8 A solid steel sphere (AISI 1010), 300 mm in diameter, is coated with a dielectric material layer of thickness 2 mm and thermal conductivity 0.04 W/m $\cdot$ K. The coated sphere is initially at a uniform temperature of 500°C and is suddenly quenched in a large oil bath for which $T_\infty = 100$°C and $h = 3300$ W/m$^2 \cdot$K. Estimate the time required for the coated sphere temperature to reach 140°C. *Hint:* Neglect the effect of energy storage in the dielectric material, since its thermal capacitance (ρcV) is small compared to that of the steel sphere.

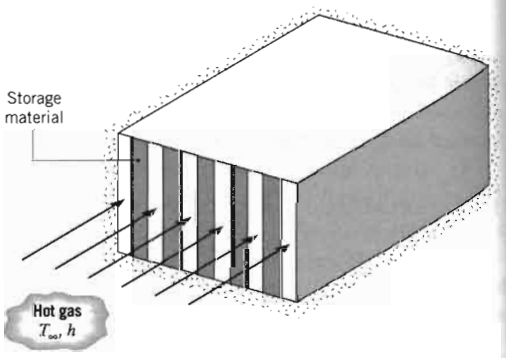
5.9 The base plate of an iron has a thickness of $L = 7$ mm and is made from an aluminum alloy ($\rho = 2800$ kg/m^3, $c = 900$ J/kg $\cdot$ K, $k = 180$ W/m $\cdot$ K, $\varepsilon = 0.80$). An electric resistance heater is attached to the inner surface of the plate, while the outer surface is exposed to ambient air and large surroundings at $T_\infty = T_{sur} = 25$°C. The areas of both the inner and outer surfaces are $A_s = 0.040$ m^2.



If an approximately uniform heat flux of $q_h'' = 1.25 \times 10^4$ W/m^2 is applied to the inner surface of the base plate and the convection coefficient at the outer surface is $h = 10$ W/m$^2 \cdot$K, estimate the time required for the plate to reach a temperature of 135°C. *Hint:* Numerical integration is suggested in order to solve the problem.

5.10 Carbon steel (AISI 1010) shafts of 0.1-m diameter are heat treated in a gas-fired furnace whose gases are at 1200 K and provide a convection coefficient of 100 W/m$^2 \cdot$K. If the shafts enter the furnace at 300 K, how long must they remain in the furnace to achieve a centerline temperature of 800 K?

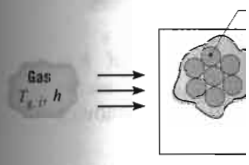
5.11 A thermal energy storage unit consists of a large rectangular channel, which is well insulated on its outer surface and encloses alternating layers of the storage material and the flow passage.



Each layer of the storage material is an aluminum slab of width $W = 0.05$ m, which is at an initial temperature of 25°C. Consider conditions for which the storage unit is charged by passing a hot gas through the passages, with the gas temperature and the convection coefficient assumed to have constant values of $T_\infty = 600$°C and $h = 100$ W/m$^2 \cdot$K throughout the channel. How long will it take to achieve 75% of the maximum possible

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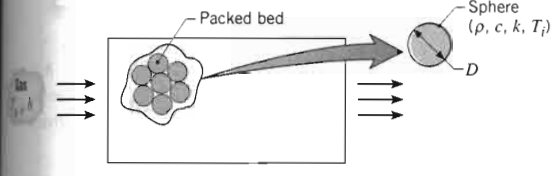
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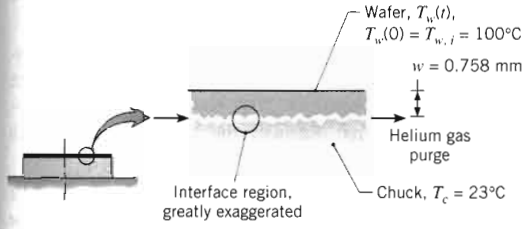
energy storage? What is the temperature of the aluminum at this time?

5.12 Thermal energy storage systems commonly involve a *packed bed* of solid spheres, through which a hot gas flows if the system is being charged, or a cold gas if it is being discharged. In a charging process, heat transfer from the hot gas increases thermal energy stored within the colder spheres; during discharge, the stored energy decreases as heat is transferred from the warmer spheres to the cooler gas.



Consider a packed bed of 75-mm-diameter aluminum spheres ($\rho = 2700$ kg/m^3, $c = 950$ J/kg · K, $k = 240$ W/m · K) and a charging process for which gas enters the storage unit at a temperature of $T_{g,i} = 300$°C. If the initial temperature of the spheres is $T_i = 25$°C and the convection coefficient is $h = 75$ W/m^2 · K, how long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere? Is there any advantage to using copper instead of aluminum?

5.13 A tool used for fabricating semiconductor devices consists of a chuck (thick metallic, cylindrical disk) onto which a very thin silicon wafer ($\rho = 2700$ kg/m^3, $c = 875$ J/kg · K, $k = 177$ W/m · K) is placed by a robotic arm. Once in position, an electric field in the chuck is energized, creating an electrostatic force that holds the wafer firmly to the chuck. To ensure a reproducible thermal contact resistance between the chuck and the wafer from cycle-to-cycle, pressurized helium gas is introduced at the center of the chuck and flows (very slowly) radially outward between the asperities of the interface region.



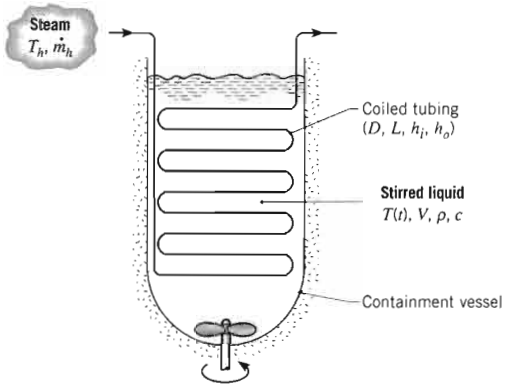
An experiment has been performed under conditions for which the wafer, initially at a uniform temperature $T_{w,i} = 100$°C, is suddenly placed on the chuck, which is at a uniform and constant temperature $T_c = 23$°C. With the wafer in place, the electrostatic force and the helium gas flow are applied. After 15 seconds, the temperature of the wafer is determined to be 33°C. What is the thermal contact resistance $R''_{t,c}$ (m^2 · K/W) between the wafer and chuck? Will the value of $R''_{t,c}$ increase, decrease, or remain the same if air, instead of helium, is used as the purge gas?

5.14 A spherical vessel used as a reactor for producing pharmaceuticals has a 5-mm-thick stainless steel wall ($k = 17$ W/m · K) and an inner diameter of $D_i = 1.0$ m. During production, the vessel is filled with reactants for which $\rho = 1100$ kg/m^3 and $c = 2400$ J/kg · K, while exothermic reactions release energy at a volumetric rate of $\dot{q} = 10^4$ W/m^3. As first approximations, the reactants may be assumed to be well stirred and the thermal capacitance of the vessel may be neglected.

(a) The exterior surface of the vessel is exposed to ambient air ($T_\infty = 25$°C) for which a convection coefficient of $h = 6$ W/m^2 · K may be assumed. If the initial temperature of the reactants is 25°C, what is the temperature of the reactants after five hours of process time? What is the corresponding temperature at the outer surface of the vessel?

(b) Explore the effect of varying the convection coefficient on transient thermal conditions within the reactor.

5.15 Batch processes are often used in chemical and pharmaceutical operations to achieve a desired chemical composition for the final product and typically involve a transient heating operation to take the product from room temperature to the desired process temperature.

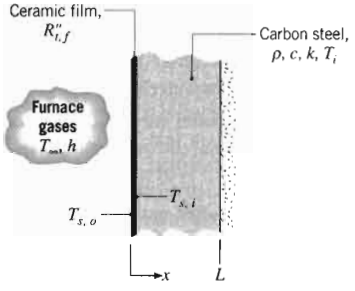


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Consider a situation for which a chemical of density $\rho = 1200$ kg/m^3 and specific heat $c = 2200$ J/kg · K occupies a volume of $V = 2.25$ m^3 in an insulated vessel. The chemical is to be heated from room temperature, $T_i = 300$ K, to a process temperature of $T = 450$ K by passing saturated steam at $T_h = 500$ K through a coiled, thin-walled, 20-mm-diameter tube in the vessel. Steam condensation within the tube maintains an interior convection coefficient of $h_i = 10{,}000$ W/m^2 · K, while the highly agitated liquid in the stirred vessel maintains an outside convection coefficient of $h_o = 2000$ W/m^2 · K.

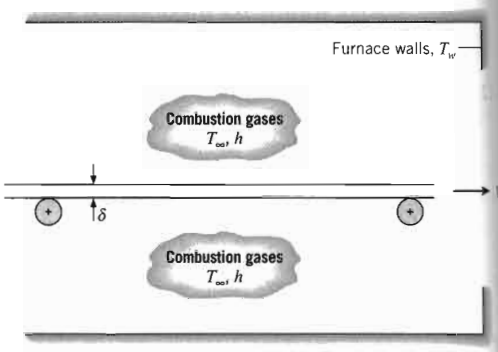
If the chemical is to be heated from 300 to 450 K in 60 minutes, what is the required length L of the submerged tubing?

5.16 A plane wall of a furnace is fabricated from plain carbon steel ($k = 60$ W/m · K, $\rho = 7850$ kg/m^3, $c = 430$ J/kg · K) and is of thickness $L = 10$ mm. To protect it from the corrosive effects of the furnace combustion gases, one surface of the wall is coated with a thin ceramic film that, for a unit surface area, has a thermal resistance of $R''_{t,f} = 0.01$ m^2 · K/W. The opposite surface is well insulated from the surroundings.



At furnace start-up the wall is at an initial temperature of $T_i = 300$ K, and combustion gases at $T_\infty = 1300$ K enter the furnace, providing a convection coefficient of $h = 25$ W/m^2 · K at the ceramic film. Assuming the film to have negligible thermal capacitance, how long will it take for the inner surface of the steel to achieve a temperature of $T_{s,i} = 1200$ K? What is the temperature $T_{s,o}$ of the exposed surface of the ceramic film at this time?

5.17 A steel strip of thickness $\delta = 12$ mm is annealed by passing it through a large furnace whose walls are maintained at a temperature T_w corresponding to that of combustion gases flowing through the furnace ($T_w = T_\infty$). The strip, whose density, specific heat, thermal conductivity, and emissivity are $\rho = 7900$ kg/m^3, $c_p = 640$ J/kg · K, $k = 30$ W/m · K, and $\varepsilon = 0.7$, respectively, is to be heated from 300°C to 600°C.



(a) For a uniform convection coefficient of $h = 100$ W/m^2 · K and $T_w = T_\infty = 700$°C, determine the time required to heat the strip. If the strip is moving at 0.5 m/s, how long must the furnace be?

(b) The annealing process may be accelerated (the strip speed increased) by increasing the environmental temperatures. For the furnace length obtained in part (a), determine the strip speed for $T_w = T_\infty = 850$°C and $T_w = T_\infty = 1000$°C. For each set of environmental temperatures (700, 850, and 1000°C), plot the strip temperature as a function of time over the range 25°C ≤ T ≤ 600°C. Over this range, also plot the radiation heat transfer coefficient, h_r, as a function of time.

5.18 A long wire of diameter $D = 1$ mm is submerged in an oil bath of temperature $T_\infty = 25$°C. The wire has an electrical resistance per unit length of $R'_e = 0.01$ Ω/m. If a current of $I = 100$ A flows through the wire and the convection coefficient is $h = 500$ W/m^2 · K, what is the steady-state temperature of the wire? From the time the current is applied, how long does it take for the wire to reach a temperature that is within 1°C of the steady-state value? The properties of the wire are $\rho = 8000$ kg/m^3, $c = 500$ J/kg · K, and $k = 20$ W/m · K.

5.19 Consider the system of Problem 5.1 where the temperature of the plate is spacewise isothermal during the transient process.

(a) Obtain an expression for the temperature of the plate as a function of time $T(t)$ in terms of q''_o, T_∞, h, L, and the plate properties ρ and c.

(b) Determine the thermal time constant and the steady-state temperature for a 12-mm-thick plate of pure copper when $T_\infty = 27$°C, $h = 50$ W/m^2 · K, and $q''_o = 5000$ W/m^2. Estimate the time required to reach steady-state conditions.

(c) For the conditions of part (b), as well as for $h = 100$ and 200 W/m^2 · K, compute and plot the

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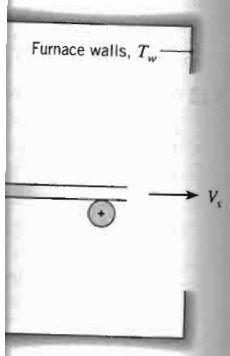
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5.20 An electronic device, such as a power transistor mounted on a finned heat sink, can be modeled as a spatially isothermal object with internal heat generation and an external convection resistance.

(a) Consider such a system of mass M, specific heat c, and surface area A_s, which is initially in equilibrium with the environment at T_∞. Suddenly, the electronic device is energized such that a constant heat generation $\dot{E}_g$ (W) occurs. Show that the temperature response of the device is

$$\frac{\theta}{\theta_i} = \exp\left(-\frac{t}{RC}\right)$$

where $\theta \equiv T - T(\infty)$ and $T(\infty)$ is the steady-state temperature corresponding to $t \to \infty$; $\theta_i = T_i - T(\infty)$; T_i = initial temperature of device; R = thermal resistance $1/\bar{h}A_s$; and C = thermal capacitance Mc.

(b) An electronic device, which generates 60 W of heat, is mounted on an aluminum heat sink weighing 0.31 kg and reaches a temperature of 100°C in ambient air at 20°C under steady-state conditions. If the device is initially at 20°C, what temperature will it reach 5 min after the power is switched on?

5.21 Before being injected into a furnace, pulverized coal is preheated by passing it through a cylindrical tube whose surface is maintained at T_{sur} = 1000°C. The coal pellets are suspended in an airflow and are known to move with a speed of 3 m/s. If the pellets may be approximated as spheres of 1-mm diameter and it may be assumed that they are heated by radiation transfer from the tube surface, how long must the tube be to heat coal entering at 25°C to a temperature of 600°C? Is the use of the lumped capacitance method justified?

5.22 A metal sphere of diameter D, which is at a uniform temperature T_i, is suddenly removed from a furnace and suspended from a fine wire in a large room with air at a uniform temperature T_∞ and the surrounding walls at a temperature T_{sur}.

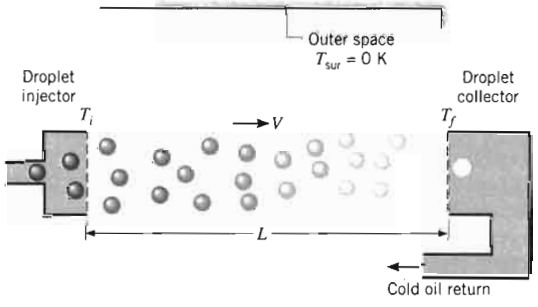
(a) Neglecting heat transfer by radiation, obtain an expression for the time required to cool the sphere to some temperature T.

(b) Neglecting heat transfer by convection, obtain an expression for the time required to cool the sphere to the temperature T.

(c) How would you go about determining the time required for the sphere to cool to the temperature T if both convection and radiation are of the same order of magnitude?

(d) Consider an anodized aluminum sphere (ε = 0.75) 50 mm in diameter, which is at an initial temperature of T_i = 800 K. Both the air and surroundings are at 300 K, and the convection coefficient is $10\ W/m^2 \cdot K$. For the conditions of parts (a), (b), and (c), determine the time required for the sphere to cool to 400 K. Plot the corresponding temperature histories. Repeat the calculations for a polished aluminum sphere (ε = 0.1).

5.23 As permanent space stations increase in size, there is an attendant increase in the amount of electrical power they dissipate. To keep station compartment temperatures from exceeding prescribed limits, it is necessary to transfer the dissipated heat to space. A novel heat rejection scheme that has been proposed for this purpose is termed a Liquid Droplet Radiator (LDR). The heat is first transferred to a high vacuum oil, which is then injected into outer space as a stream of small droplets. The stream is allowed to traverse a distance L, over which it cools by radiating energy to outer space at absolute zero temperature. The droplets are then collected and routed back to the space station.



Consider conditions for which droplets of emissivity ε = 0.95 and diameter D = 0.5 mm are injected at a temperature of T_i = 500 K and a velocity of V = 0.1 m/s. Properties of the oil are $\rho = 885\ kg/m^3$, $c = 1900\ J/kg \cdot K$, and $k = 0.145\ W/m \cdot K$. Assuming each drop to radiate to deep space at T_{sur} = 0 K, determine the distance L required for the droplets to impact the collector at a final temperature of T_f = 300 K. What is the amount of thermal energy rejected by each droplet?

5.24 In a material processing experiment conducted aboard the space shuttle, a coated niobium sphere of 10-mm diameter is removed from a furnace at 900°C and cooled to a temperature of 300°C. Although properties of the niobium vary over this temperature range, constant values may be assumed to a reasonable approximation, with $\rho = 8600\ kg/m^3$, $c = 290\ J/kg \cdot K$, and $k = 63\ W/m \cdot K$.

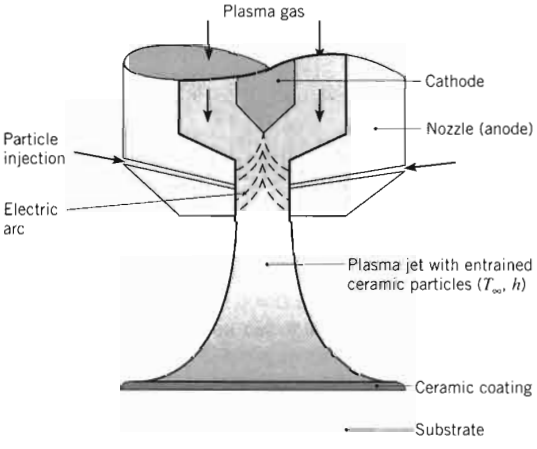
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(a) If cooling is implemented in a large evacuated chamber whose walls are at 25°C, determine the time required to reach the final temperature if the coating is polished and has an emissivity of $\varepsilon = 0.1$. How long would it take if the coating is oxidized and $\varepsilon = 0.6$?

(b) To reduce the time required for cooling, consideration is given to immersion of the sphere in an inert gas stream for which $T_\infty = 25°C$ and $h = 200$ W/m$^2 \cdot$ K. Neglecting radiation, what is the time required for cooling?

(c) Considering the effect of both radiation and convection, what is the time required for cooling if $h = 200$ W/m$^2 \cdot$ K and $\varepsilon = 0.6$? Explore the effect on the cooling time of independently varying h and ε.

5.25 Plasma spray-coating processes are often used to provide surface protection for materials exposed to hostile environments, which induce degradation through factors such as wear, corrosion, or outright thermal failure. *Ceramic* coatings are commonly used for this purpose. By injecting ceramic powder through the nozzle (anode) of a plasma torch, the particles are entrained by the plasma jet, within which they are then accelerated and heated.



During their *time-in-flight,* the ceramic particles must be heated to their melting point and experience complete conversion to the liquid state. The coating is formed as the molten droplets impinge (*splat*) on the substrate material and experience rapid solidification. Consider conditions for which spherical alumina (Al_2O_3) particles of diameter $D_p = 50$ μm, density $\rho_p = 3970$ kg/m^3, thermal conductivity $k_p = 10.5$ W/m $\cdot$ K, and specific heat $c_p = 1560$ J/kg $\cdot$ K are injected into an arc plasma, which is at $T_\infty = 10{,}000$ K and provides a coefficient of $h = 30{,}000$ W/m$^2 \cdot$ K for convective heating of the particles. The melting point and latent heat of fusion of alumina are $T_{mp} = 2318$ K and $h_{if} = 3577$ kJ/kg, respectively.

(a) Neglecting radiation, obtain an expression for the time-in-flight, t_{i-f}, required to heat a particle from its initial temperature T_i to its melting point T_{mp}, and, once at the melting point, for the particle to experience complete melting. Evaluate t_{i-f} for $T_i = 300$ K and the prescribed heating conditions.

(b) Assuming alumina to have an emissivity of $\varepsilon_p = 0.4$ and the particles to exchange radiation with large surroundings at $T_{sur} = 300$ K, assess the validity of neglecting radiation.

5.26 Thin film coatings characterized by high resistance to abrasion and fracture may be formed by using microscale composite particles in a plasma spraying process. A spherical particle typically consists of a *ceramic core*, such as tungsten carbide (WC), and a *metallic shell*, such as cobalt (Co). The ceramic provides the thin film coating with its desired hardness at elevated temperatures, while the metal serves to coalesce the particles on the coated surface and to inhibit crack formation. In the plasma spraying process, the particles are injected into a plasma gas jet that heats them to a temperature above the melting point of the metallic casing and melts the casing before the particles impact the surface.

Consider spherical particles comprised of a WC core of diameter $D_i = 16$ μm, which is encased in a Co shell of outer diameter $D_o = 20$ μm. If the particles flow in a plasma gas at $T_\infty = 10{,}000$ K and the coefficient associated with convection from the gas to the particles is $h = 20{,}000$ W/m$^2 \cdot$ K, how long does it take to heat the particles from an initial temperature of $T_i = 300$ K to the melting point of cobalt, $T_{mp} = 1770$ K? The density and specific heat of WC (the core of the particle) are $\rho_c = 16{,}000$ kg/m^3 and $c_c = 300$ J/kg $\cdot$ K, while the corresponding values for Co (the outer shell) are $\rho_s = 8900$ kg/m^3 and $c_s = 750$ J/kg $\cdot$ K. Once having reached the melting point, how much additional time is required to completely melt the cobalt if its latent heat of fusion is $h_{sf} = 2.59 \times 10^5$ J/kg? You may use the lumped capacitance method of analysis and neglect radiation exchange between the particle and its surroundings.

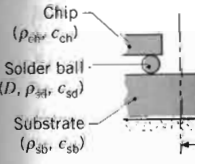
5.27 A chip that is of length $L = 5$ mm on a side and thickness $t = 1$ mm is encased in a ceramic substrate, and its exposed surface is convectively cooled by a dielectric liquid for which $h = 150$ W/m$^2 \cdot$ K and $T_\infty = 20°C$.



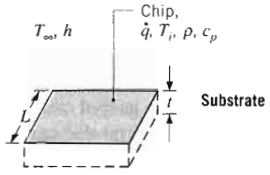
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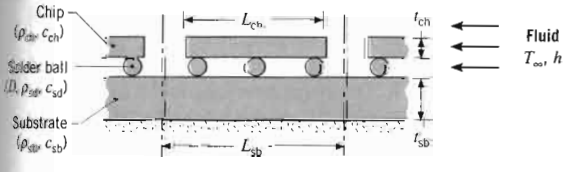
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In the off-mode the chip is in thermal equilibrium with the coolant ($T_i = T_\infty$). When the chip is energized, however, its temperature increases until a new steady-state is established. For purposes of analysis, the energized chip is characterized by uniform volumetric heating with $\dot{q} = 9 \times 10^6$ W/m^3. Assuming an infinite contact resistance between the chip and substrate and negligible conduction resistance within the chip, determine the steady-state chip temperature T_f. Following activation of the chip, how long does it take to come within 1°C of this temperature? The chip density and specific heat are $\rho = 2000$ kg/m^3 and $c = 700$ J/kg · K, respectively.

5.28 Consider the conditions of Problem 5.27. In addition to treating heat transfer by convection directly from the chip to the coolant, a more realistic analysis would account for indirect transfer from the chip to the substrate and then from the substrate to the coolant. The total thermal resistance associated with this indirect route includes contributions due to the chip–substrate interface (a contact resistance), multidimensional conduction in the substrate, and convection from the surface of the substrate to the coolant. If this total thermal resistance is $R_t = 200$ K/W, what is the steady-state chip temperature T_f? Following activation of the chip, how long does it take to come within 1°C of this temperature?

5.29 Thermal stress testing is a common procedure used to assess the reliability of an electronic package. Typically, thermal stresses are induced in soldered or wired connections to reveal mechanisms that could cause failure and must therefore be corrected before the product is released. As an example of the procedure, consider an array of silicon chips ($\rho_{ch} = 2300$ kg/m^3, $c_{ch} = 710$ J/kg · K) joined to an alumina substrate ($\rho_{sb} = 4000$ kg/m^3, $c_{sb} = 770$ J/kg · K) by solder balls ($\rho_{sd} = 11{,}000$ kg/m^3, $c_{sd} = 130$ J/kg · K). Each chip of width L_{ch} and thickness t_{ch} is joined to a unit substrate section of width L_{sb} and thickness t_{sb} by solder balls of diameter D.

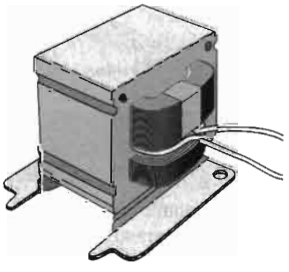


A thermal stress test begins by subjecting the multichip module, which is initially at room temperature, to a hot fluid stream and subsequently cooling the module by exposing it to a cold fluid stream. The process is repeated for a prescribed number of cycles to assess the integrity of the soldered connections.

(a) As a first approximation, assume that there is negligible heat transfer between the components (chip/solder/substrate) of the module and that the thermal response of each component may be determined from a lumped capacitance analysis involving the same convection coefficient h. Assuming no reduction in surface area due to contact between a solder ball and the chip or substrate, obtain expressions for the thermal time constant of each component. Heat transfer is to all surfaces of a chip, but to only the top surface of the substrate. Evaluate the three time constants for $L_{ch} = 15$ mm, $t_{ch} = 2$ mm, $L_{sb} = 25$ mm, $t_{sb} = 10$ mm, $D = 2$ mm, and a value of $h = 50$ W/m^2 · K, which is characteristic of an air stream. Compute and plot the temperature histories of the three components for the heating portion of a cycle, with $T_i = 20$°C and $T_\infty = 80$°C. At what time does each component experience 99% of its maximum possible temperature rise, that is, $(T - T_i)/(T_\infty - T_i) = 0.99$? If the maximum stress on a solder ball corresponds to the maximum difference between its temperature and that of the chip or substrate, when will this maximum occur?

(b) To reduce the time required to complete a stress test, a dielectric liquid could be used in lieu of air to provide a larger convection coefficient of $h = 200$ W/m^2 · K. What is the corresponding savings in time for each component to achieve 99% of its maximum possible temperature rise?

5.30 The objective of this problem is to develop thermal models for estimating the steady-state temperature and the transient temperature history of the electrical transformer shown below.



The external transformer geometry is approximately cubical, with a length of 32 mm to a side. The combined

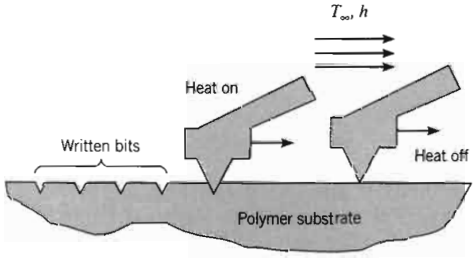
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mass of the iron and copper in the transformer is 0.28 kg, and its weighted-average specific heat is 400 J/kg · K. The transformer dissipates 4.0 W and is operating in ambient air at $T_\infty = 20°C$, with a convection coefficient of 10 W/m² · K. List and justify the assumptions made in your analysis, and discuss limitations of the models.

(a) Beginning with a properly defined control volume, develop a model for estimating the steady-state temperature of the transformer, $T(\infty)$. Evaluate $T(\infty)$ for the prescribed operating conditions.

(b) Develop a model for estimating the thermal response (temperature history) of the transformer if it is initially at a temperature of $T_i = T_\infty$ and power is suddenly applied. Determine the time required for the transformer to come within 5°C of its steady-state operating temperature.

5.31 In thermomechanical data storage, a processing head, consisting of M heated cantilevers, is used to write data onto an underlying polymer storage medium. Electrical resistance heaters are microfabricated onto each cantilever, which continually travel over the surface of the medium. The resistance heaters are turned on and off by controlling electrical current to each cantilever. As a cantilever goes through a complete heating and cooling cycle, the underlying polymer is softened, and one bit of data is written in the form of a *surface pit* in the polymer. A track of individual data bits (pits), each separated by approximately 50 nm, can be fabricated. Multiple tracks of bits, also separated by approximately 50 nm, are subsequently fabricated into the surface of the storage medium. Consider a single cantilever that is fabricated primarily of silicon with a mass of 50×10^{-18} kg and a surface area of 600×10^{-15} m². The cantilever is initially at $T_i = T_\infty = 300$ K, and the heat transfer coefficient between the cantilever and the ambient is 200×10^3 W/m² · K.



(a) Determine the ohmic heating required to raise the cantilever temperature to $T = 1000$ K within a heating time of $t_h = 1$ μs. *Hint*: See Problem 5.20.

(b) Find the time required to cool the cantilever from 1000 K to 400 K (t_c) and the thermal processing time required for one complete heating and cooling cycle, $t_p = t_h + t_c$.

(c) Determine how many bits (N) can be written onto a 1 mm × 1 mm polymer storage medium. If $M = 100$ cantilevers are ganged onto a single processing head, determine the total thermal processing time needed to write the data.

5.32 The melting of water initially at the fusion temperature, $T_f = 0°C$, was considered in Example 1.5. Freezing of water often occurs at 0°C. However, pure liquids that undergo a cooling process can remain in a *supercooled* liquid state well below their equilibrium freezing temperature, T_f, particularly when the liquid is not in contact with any solid material. Droplets of liquid water in the atmosphere have a supercooled freezing temperature, $T_{f,sc}$, that can be well correlated to the droplet diameter by the expression $T_{f,sc} = -28 + 0.87 \ln(D_p)$ in the diameter range $10^{-7} < D_p < 10^{-2}$ m, where $T_{f,sc}$ has units of degrees Celsius and D_p is expressed in units of meters. For a droplet of diameter $D = 50$ μm and initial temperature $T_i = 10°C$ subject to ambient conditions of $T_\infty = -40°C$ and $h = 900$ W/m² · K, compare the time needed to completely solidify the droplet for case A, when the droplet solidifies at $T_f = 0°C$, and case B, when the droplet starts to freeze at $T_{f,sc}$. Sketch the temperature histories from the initial time to the time when the droplets are completely solid. *Hint:* When the droplet reaches $T_{f,sc}$ in case B, rapid solidification occurs during which the latent energy released by the freezing water is absorbed by the remaining liquid in the drop. As soon as any ice is formed within the droplet, the remaining liquid is in contact with a solid (the ice) and the freezing temperature immediately shifts from $T_{f,sc}$ to $T_f = 0°C$.

5.33 As noted in Problem 5.3, microwave ovens operate by rapidly aligning and reversing water molecules within the food, resulting in volumetric energy generation and, in turn, cooking of the food. When the food is initially frozen, however, the water molecules do not readily oscillate in response to the microwaves, and the volumetric generation rates are between one and two orders of magnitude lower than if the water were in liquid form. (Microwave power that is not absorbed in the food is reflected back to the microwave generator, where it must be dissipated in the form of heat to prevent damage to the generator.)

(a) Consider a frozen, 1-kg spherical piece of ground beef at an initial temperature of $T_i = -20°C$ placed in a microwave oven with $T_\infty = 30°C$ and $h = 15$ W/m² · K. Determine how long it will take the beef to reach a uniform temperature of $T = 0°C$, with all the water in the form of ice. Assume the properties

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2	0.40	25×10^{-6}

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of the beef are the same as ice, and assume 3% of the oven power (P = 1 kW total) is absorbed in the food.

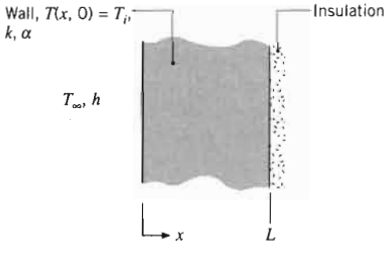
(b) After all the ice is converted to liquid, determine how long it will take to heat the beef to T_f = 80°C if 95% of the oven power is absorbed in the food. Assume the properties of the beef are the same as liquid water.

(c) When thawing food in microwave ovens, one may observe that some of the food may still be frozen while other parts of the food are overcooked. Explain why this occurs. Explain why most microwave ovens have thaw cycles that are associated with very low oven powers.

One-Dimensional Conduction: The Plane Wall

5.34 Consider the series solution, Equation 5.39, for the plane wall with convection. Calculate midplane ($x^* = 0$) and surface ($x^* = 1$) temperatures θ^* for $Fo = 0.1$ and 1, using $Bi = 0.1$, 1, and 10. Consider only the first four eigenvalues. Based on these results discuss the validity of the approximate solutions, Equations 5.40 and 5.41.

5.35 Consider the one-dimensional wall shown in the sketch, which is initially at a uniform temperature T_i and is suddenly subjected to the convection boundary condition with a fluid at T_∞.

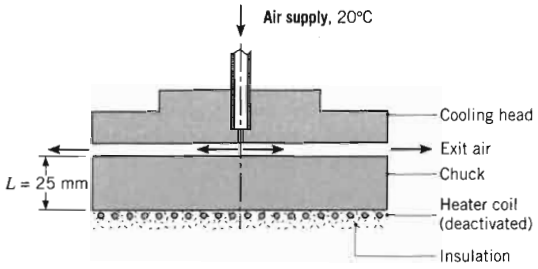


For a particular wall, case 1, the temperature at $x = L_1$ after t_1 = 100 s is $T_1(L_1, t_1)$ = 315°C. Another wall, case 2, has different thickness and thermal conditions as shown below.

Case	L (m)	α (m²/s)	k (W/m · K)	T_i (°C)	T_∞ (°C)	h (W/m² · K)
1	0.10	15×10^{-6}	50	300	400	200
2	0.40	25×10^{-6}	100	30	20	100

How long will it take for the second wall to reach 28.5°C at the position $x = L_2$? Use as the basis for analysis, the dimensionless functional dependence for the transient temperature distribution expressed in Equation 5.38.

5.36 Referring to the semiconductor processing tool of Problem 5.13, it is desired at some point in the manufacturing cycle to cool the chuck, which is made of aluminum alloy 2024. The proposed cooling scheme passes air at 20°C between the air-supply head and the chuck surface.



(a) If the chuck is initially at a uniform temperature of 100°C, calculate the time required for its lower surface to reach 25°C, assuming a uniform convection coefficient of 50 W/m² · K at the head–chuck interface.

(b) Generate a plot of the time-to-cool as a function of the convection coefficient for the range $10 \le h \le 2000$ W/m² · K. If the lower limit represents a free convection condition without any head present, comment on the effectiveness of the head design as a method for cooling the chuck.

5.37 Annealing is a process by which steel is reheated and then cooled to make it less brittle. Consider the reheat stage for a 100-mm-thick steel plate (ρ = 7830 kg/m³, c = 550 J/kg · K, k = 48 W/m · K), which is initially at a uniform temperature of T_i = 200°C and is to be heated to a minimum temperature of 550°C. Heating is effected in a gas-fired furnace, where products of combustion at T_∞ = 800°C maintain a convection coefficient of h = 250 W/m² · K on both surfaces of the plate. How long should the plate be left in the furnace?

5.38 Consider the heavily insulated pipe of Example 5.4, which is suddenly subjected to the flow of hot oil. Use the *Transient Conduction, Plane Wall* model of *IHT* to obtain the following solutions.

(a) Calculate the temperature of the inner and outer surfaces of the pipe, the heat flux at the inner surface, and the energy transferred to the wall after 8 min. Compare your results with those obtained in the example.

(b) At what time will the outer surface temperature of the pipe, $T(0, t)$, reach 25°C?

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(c) Using the *Explore* and *Graph* options of *IHT*, calculate and plot on a single graph the temperature distributions, $T(x, t)$, for the initial condition, the final condition, and intermediate times of 4 and 8 min. Explain key features of the distributions.

(d) Calculate and plot the temperature histories, $T(x, t)$, at the inner ($x = 0$) and outer ($x = L$) pipe surfaces for $0 \leq t \leq 16$ min.

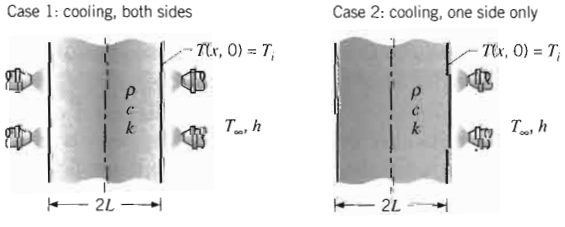
5.39 The 150-mm-thick wall of a gas-fired furnace is constructed of fire-clay brick ($k = 1.5$ W/m · K, $\rho = 2600$ kg/m^3, $c_p = 1000$ J/kg · K) and is well insulated at its outer surface. The wall is at a uniform initial temperature of 20°C, when the burners are fired and the inner surface is exposed to products of combustion for which $T_\infty = 950$°C and $h = 100$ W/m^2 · K.

(a) How long does it take for the outer surface of the wall to reach a temperature of 750°C?

(b) Plot the temperature distribution in the wall at the foregoing time, as well as at several intermediate times.

5.40 Steel is sequentially heated and cooled (*annealed*) to relieve stresses and to make it less brittle. Consider a 100-mm-thick plate ($k = 45$ W/m · K, $\rho = 7800$ kg/m^3, $c_p = 500$ J/kg · K) that is initially at a uniform temperature of 300°C and is heated (on both sides) in a gas-fired furnace for which $T_\infty = 700$°C and $h = 500$ W/m^2 · K. How long will it take for a minimum temperature of 550°C to be reached in the plate?

5.41 A plate of thickness $2L = 25$ mm at a temperature of 600°C is removed from a hot pressing operation and must be cooled rapidly in order to achieve the required physical properties. The process engineer plans to use air jets to control the rate of cooling, but she is uncertain whether it is necessary to cool both sides (case 1) or only one side (case 2) of the plate. The concern is not just for the *time-to-cool*, but also for the maximum temperature difference within the plate. If this temperature difference is too large, the plate can experience significant warping.



The air supply is at 25°C, and the convection coefficient on the surface is 400 W/m^2 · K. The thermophysical properties of the plate are $\rho = 3000$ kg/m^3, $c = 750$ J/kg · K, and $k = 15$ W/m · K.

(a) Using the *IHT* software, calculate and plot on one graph the temperature histories for cases 1 and 2 for a 500-s cooling period. Compare the times required for the maximum temperature in the plate to reach 100°C. Assume no heat loss from the unexposed surface of case 2.

(b) For both cases, calculate and plot on one graph the variation with time of the maximum temperature difference in the plate. Comment on the relative magnitudes of the temperature gradients within the plate as a function of time.

5.42 During transient operation, the steel nozzle of a rocket engine must not exceed a maximum allowable operating temperature of 1500 K when exposed to combustion gases characterized by a temperature of 2300 K and a convection coefficient of 5000 W/m^2 · K. To extend the duration of engine operation, it is proposed that a ceramic *thermal barrier coating* ($k = 10$ W/m · K, $\alpha = 6 \times 10^{-6}$ m^2/s) be applied to the interior surface of the nozzle.

(a) If the ceramic coating is 10 mm thick and at an initial temperature of 300 K, obtain a conservative estimate of the maximum allowable duration of engine operation. The nozzle radius is much larger than the combined wall and coating thickness.

(b) Compute and plot the inner and outer surface temperatures of the coating as a function of time for $0 \leq t \leq 150$ s. Repeat the calculations for a coating thickness of 40 mm.

5.43 In a tempering process, glass plate, which is initially at a uniform temperature T_i, is cooled by suddenly reducing the temperature of both surfaces to T_s. The plate is 20 mm thick, and the glass has a thermal diffusivity of 6×10^{-7} m^2/s.

(a) How long will it take for the midplane temperature to achieve 50% of its maximum possible temperature reduction?

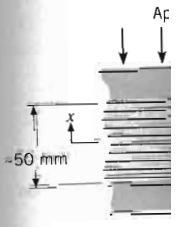
(b) If $(T_i - T_s) = 300$°C, what is the maximum temperature gradient in the glass at the above time?

5.44 The strength and stability of tires may be enhanced by heating both sides of the rubber ($k = 0.14$ W/m · K, $\alpha = 6.35 \times 10^{-8}$ m^2/s) in a steam chamber for which $T_\infty = 200$°C. In the heating process, a 20-mm-thick rubber wall (assumed to be untreaded) is taken from an initial temperature of 25°C to a midplane temperature of 150°C.

(a) If steam flow over the tire surfaces maintains a convection coefficient of $h = 200$ W/m^2 · K, how long will it take to achieve the desired midplane temperature?

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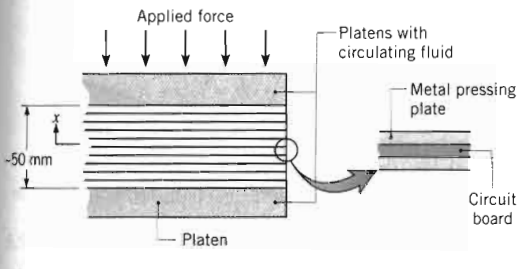
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(b) To accelerate the heating process, it is recommended that the steam flow be made sufficiently vigorous to maintain the tire surfaces at 200°C throughout the process. Compute and plot the midplane and surface temperatures for this case, as well as for the conditions of part (a).

5.45 Copper-coated, epoxy-filled fiberglass circuit boards are treated by heating a stack of them under high pressure as shown in the sketch. The purpose of the pressing–heating operation is to cure the epoxy that bonds the fiberglass sheets, imparting stiffness to the boards. The stack, referred to as a *book,* is comprised of 10 *boards* and 11 *pressing plates,* which prevent epoxy from flowing between the boards and impart a smooth finish to the cured boards. In order to perform simplified thermal analyses, it is reasonable to approximate the book as having an effective thermal conductivity (k) and an effective thermal capacitance (ρc_p). Calculate the effective properties if each of the boards and plates has a thickness of 2.36 mm and the following thermophysical properties: board (b) $\rho_b = 1000$ kg/m^3, $c_{p,b} = 1500$ J/kg · K, $k_b = 0.30$ W/m · K; plate (p) $\rho_p = 8000$ kg/m^3, $c_{p,p} = 480$ J/kg · K, $k_p = 12$ W/m · K.

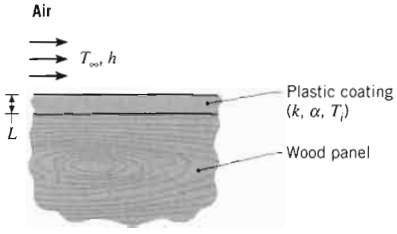


5.46 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.45. The platens at the top and bottom of the stack are maintained at a uniform temperature by a circulating fluid. The purpose of the pressing–heating operation is to cure the epoxy, which bonds the fiberglass sheets, and impart stiffness to the boards. The cure condition is achieved when the epoxy has been maintained at or above 170°C for at least 5 min. The effective thermophysical properties of the stack or *book* (boards and metal pressing plates) are $k = 0.613$ W/m · K and $\rho c_p = 2.73 \times 10^6$ J/m^3 · K.

(a) If the book is initially at 15°C and, following application of pressure, the platens are suddenly brought to a uniform temperature of 190°C, calculate the elapsed time t_e required for the midplane of the book to reach the cure temperature of 170°C.

(b) If, at this instant of time, $t = t_e$, the platen temperature were reduced suddenly to 15°C, how much energy would have to be removed from the book by the coolant circulating in the platen, in order to return the stack to its initial uniform temperature?

5.47 A plastic coating is applied to wood panels by first depositing molten polymer on a panel and then cooling the surface of the polymer by subjecting it to air flow at 25°C. As first approximations, the heat of reaction associated with solidification of the polymer may be neglected and the polymer/wood interface may be assumed to be adiabatic.



If the thickness of the coating is $L = 2$ mm and it has an initial uniform temperature of $T_i = 200$°C, how long will it take for the surface to achieve a *safe-to-touch* temperature of 42°C if the convection coefficient is $h = 200$ W/m^2 · K? What is the corresponding value of the interface temperature? The thermal conductivity and diffusivity of the plastic are $k = 0.25$ W/m · K and $\alpha = 1.20 \times 10^{-7}$ m^2/s, respectively.

One-Dimensional Conduction: The Long Cylinder

5.48 A long rod of 60-mm diameter and thermophysical properties $\rho = 8000$ kg/m^3, $c = 500$ J/kg · K, and $k = 50$ W/m · K is initially at a uniform temperature and is heated in a forced convection furnace maintained at 750 K. The convection coefficient is estimated to be 1000 W/m^2 · K.

(a) What is the centerline temperature of the rod when the surface temperature is 550 K?

(b) In a heat-treating process, the centerline temperature of the rod must be increased from $T_i = 300$ K to $T = 500$ K. Compute and plot the centerline temperature histories for $h = 100$, 500, and 1000 W/m^2 · K. In each case the calculation may be terminated when $T = 500$ K.

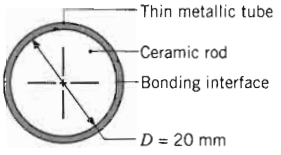
5.49 A long cylinder of 30-mm diameter, initially at a uniform temperature of 1000 K, is suddenly quenched in a large, constant-temperature oil bath at 350 K. The cylinder properties are $k = 1.7$ W/m · K, $c = 1600$ J/kg · K, and $\rho = 400$ kg/m^3, while the convection coefficient is 50 W/m^2 · K.

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(a) Calculate the time required for the surface of the cylinder to reach 500 K.

(b) Compute and plot the surface temperature history for $0 \leq t \leq 300$ s. If the oil were agitated, providing a convection coefficient of 250 W/m$^2 \cdot$ K, how would the temperature history change?

5.50 A long pyroceram rod of diameter 20 mm is clad with a very thin metallic tube for mechanical protection. The bonding between the rod and the tube has a thermal contact resistance of $R'_{t,c} = 0.12$ m $\cdot$ K/W.



(a) If the rod is initially at a uniform temperature of 900 K and is suddenly cooled by exposure to an airstream for which $T_\infty = 300$ K and $h = 100$ W/m$^2 \cdot$ K, at which time will the centerline reach 600 K?

(b) Cooling may be accelerated by increasing the airspeed and hence the convection coefficient. For values of $h = 100$, 500, and 1000 W/m$^2 \cdot$ K, compute and plot the centerline and surface temperatures of the pyroceram as a function of time for $0 \leq t \leq 300$ s. Comment on the implications of achieving enhanced cooling solely by increasing h.

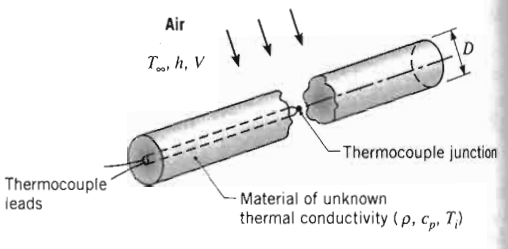
5.51 A long rod 40 mm in diameter, fabricated from sapphire (aluminum oxide) and initially at a uniform temperature of 800 K, is suddenly cooled by a fluid at 300 K having a heat transfer coefficient of 1600 W/m$^2 \cdot$ K. After 35 s, the rod is wrapped in insulation and experiences no heat losses. What will be the temperature of the rod after a long period of time?

5.52 A long plastic rod of 30-mm diameter ($k = 0.3$ W/m $\cdot$ K and $\rho c_p = 1040$ kJ/m$^3 \cdot$ K) is uniformly heated in an oven as preparation for a pressing operation. For best results, the temperature in the rod should not be less than 200°C. To what uniform temperature should the rod be heated in the oven if, for the worst case, the rod sits on a conveyor for 3 min while exposed to convection cooling with ambient air at 25°C and with a convection coefficient of 8 W/m$^2 \cdot$ K? A further condition for good results is a maximum–minimum temperature difference of less than 10°C. Is this condition satisfied and, if not, what could you do to satisfy it?

5.53 As part of a heat treatment process, cylindrical, 304 stainless steel rods of 100-mm diameter are cooled from an initial temperature of 500°C by suspending them in an oil bath at 30°C. If a convection coefficient of 500 W/m$^2 \cdot$ K is maintained by circulation of the oil, how long does it take for the centerline of a rod to reach a temperature of 50°C, at which point it is withdrawn from the bath? If 10 rods of length $L = 1$ m are processed per hour, what is the nominal rate at which energy must be extracted from the bath (the cooling load)?

5.54 In a manufacturing process, long rods of different diameters are at a uniform temperature of 400°C in a curing oven, from which they are removed and cooled by forced convection in air at 25°C. One of the line operators has observed that it takes 280 seconds for a 40-mm-diameter rod to cool to an *safe-to-handle* temperature of 60°C. For an equivalent convection coefficient, how long will it take for an 80-mm-diameter rod to cool to the same temperature? The thermophysical properties of the rod are $\rho = 2500$ kg/m^3, $c = 900$ J/kg $\cdot$ K, and $k = 15$ W/m $\cdot$ K. Comment on your result. Did you anticipate this outcome?

5.55 The density and specific heat of a particular material are known ($\rho = 1200$ kg/m^3, $c_p = 1250$ J/kg $\cdot$ K), but its thermal conductivity is unknown. To determine the thermal conductivity, a long cylindrical specimen of diameter $D = 40$ mm is machined, and a thermocouple is inserted through a small hole drilled along the centerline.



The thermal conductivity is determined by performing an experiment in which the specimen is heated to a uniform temperature of $T_i = 100$°C and then cooled by passing air at $T_\infty = 25$°C in cross flow over the cylinder. For the prescribed air velocity, the convection coefficient is $h = 55$ W/m$^2 \cdot$ K.

(a) If a centerline temperature of $T(0, t) = 40$°C is recorded after $t = 1136$ s of cooling, verify that the material has a thermal conductivity of $k = 0.30$ W/m $\cdot$ K.

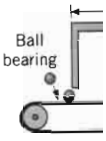
(b) For air in cross-flow over the cylinder, the prescribed value of $h = 55$ W/m$^2 \cdot$ K corresponds to a velocity of $V = 6.8$ m/s. If $h = CV^{0.618}$, where the constant C has units of W $\cdot$ s$^{0.618}$/m$^{2.618} \cdot$ K, how does the centerline temperature at $t = 1136$ s vary with velocity for $3 \leq V \leq 20$ m/s? Determine the centerline temperature histories for $0 \leq t \leq 1500$ s and velocities of 3, 10, and 20 m/s.

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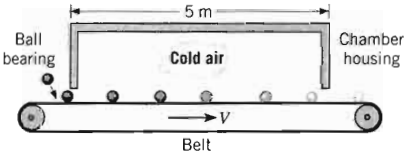
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5.56 In Section 5.2 we noted that the value of the Biot number significantly influences the nature of the temperature distribution in a solid during a transient conduction process. Reinforce your understanding of this important concept by using the *IHT* model for one-dimensional transient conduction to determine radial temperature distributions in a 30-mm-diameter, stainless steel rod ($k = 15$ W/m · K, $\rho = 8000$ kg/m^3, $c_p = 475$ J/kg · K), as it is cooled from an initial uniform temperature of 325°C by a fluid at 25°C. For the following values of the convection coefficient and the designated times, determine the radial temperature distribution: $h = 100$ W/m^2 · K ($t = 0,\ 100,\ 500$ s); $h = 1000$ W/m^2 · K ($t = 0, 10, 50$ s); $h = 5000$ W/m^2 · K ($t = 0, 1, 5, 25$ s). Prepare a separate graph for each convection coefficient, on which temperature is plotted as a function of dimensionless radius at the designated times.

One-Dimensional Conduction: The Sphere

5.57 In heat treating to harden steel ball bearings ($c = 500$ J/kg · K, $\rho = 7800$ kg/m^3, $k = 50$ W/m · K), it is desirable to increase the surface temperature for a short time without significantly warming the interior of the ball. This type of heating can be accomplished by sudden immersion of the ball in a molten salt bath with $T_\infty = 1300$ K and $h = 5000$ W/m^2 · K. Assume that any location within the ball whose temperature exceeds 1000 K will be hardened. Estimate the time required to harden the outer millimeter of a ball of diameter 20 mm, if its initial temperature is 300 K.

5.58 A cold air chamber is proposed for quenching steel ball bearings of diameter $D = 0.2$ m and initial temperature $T_i = 400$°C. Air in the chamber is maintained at -15°C by a refrigeration system, and the steel balls pass through the chamber on a conveyor belt. Optimum bearing production requires that 70% of the initial thermal energy content of the ball above -15°C be removed. Radiation effects may be neglected, and the convection heat transfer coefficient within the chamber is 1000 W/m^2 · K. Estimate the residence time of the balls within the chamber, and recommend a drive velocity of the conveyor. The following properties may be used for the steel: $k = 50$ W/m · K, $\alpha = 2 \times 10^{-5}$ m^2/s, and $c = 450$ J/kg · K.



5.59 Stainless steel (AISI 304) ball bearings, which have uniformly been heated to 850°C, are hardened by quenching them in an oil bath that is maintained at 40°C. The ball diameter is 20 mm, and the convection coefficient associated with the oil bath is 1000 W/m^2 · K.

(a) If quenching is to occur until the surface temperature of the balls reaches 100°C, how long must the balls be kept in the oil? What is the center temperature at the conclusion of the cooling period?

(b) If 10,000 balls are to be quenched per hour, what is the rate at which energy must be removed by the oil bath cooling system in order to maintain its temperature at 40°C?

5.60 A sphere 30 mm in diameter initially at 800 K is quenched in a large bath having a constant temperature of 320 K with a convection heat transfer coefficient of 75 W/m^2 · K. The thermophysical properties of the sphere material are: $\rho = 400$ kg/m^3, $c = 1600$ J/kg · K, and $k = 1.7$ W/m · K.

(a) Show, in a qualitative manner on T–t coordinates, the temperatures at the center and at the surface of the sphere as a function of time.

(b) Calculate the time required for the surface of the sphere to reach 415 K.

(c) Determine the heat flux (W/m^2) at the outer surface of the sphere at the time determined in part (b).

(d) Determine the energy (J) that has been lost by the sphere during the process of cooling to the surface temperature of 415 K.

(e) At the time determined by part (b), the sphere is quickly removed from the bath and covered with perfect insulation, such that there is no heat loss from the surface of the sphere. What will be the temperature of the sphere after a long period of time has elapsed?

(f) Compute and plot the center and surface temperature histories over the period $0 \le t \le 150$ s. What effect does an increase in the convection coefficient to $h = 200$ W/m^2 · K have on the foregoing temperature histories? For $h = 75$ and 200 W/m^2 · K, compute and plot the surface heat flux as a function of time for $0 \le t \le 150$ s.

5.61 Spheres A and B are initially at 800 K, and they are simultaneously quenched in large constant temperature baths, each having a temperature of 320 K. The following parameters are associated with each of the spheres and their cooling processes.

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	Sphere A	Sphere B
Diameter (mm)	300	30
Density (kg/m^3)	1600	400
Specific heat (kJ/kg · K)	0.400	1.60
Thermal conductivity (W/m · K)	170	1.70
Convection coefficient (W/m^2 · K)	5	50

(a) Show in a qualitative manner, on T versus t coordinates, the temperatures at the center and at the surface for each sphere as a function of time. Briefly explain the reasoning by which you determine the relative positions of the curves.

(b) Calculate the time required for the surface of each sphere to reach 415 K.

(c) Determine the energy that has been gained by each of the baths during the process of the spheres cooling to 415 K.

5.62 Spheres of 40-mm diameter heated to a uniform temperature of 400°C are suddenly removed from the oven and placed in a forced-air bath operating at 25°C with a convection coefficient of 300 W/m^2 · K on the sphere surfaces. The thermophysical properties of the sphere material are $\rho = 3000$ kg/m^3, $c = 850$ J/kg · K, and $k = 15$ W/m · K.

(a) How long must the spheres remain in the air bath for 80% of the thermal energy to be removed?

(b) The spheres are then placed in a packing carton that prevents further heat transfer to the environment. What uniform temperature will the spheres eventually reach?

5.63 Consider the packed bed operating conditions of Problem 5.12, but with Pyrex ($\rho = 2225$ kg/m^3, $c = 835$ J/kg · K, $k = 1.4$ W/m · K) used instead of aluminum. How long does it take a sphere near the inlet of the system to accumulate 90% of the maximum possible thermal energy? What is the corresponding temperature at the center of the sphere?

5.64 The convection coefficient for flow over a solid sphere may be determined by submerging the sphere, which is initially at 25°C, into the flow, which is at 75°C, and measuring its surface temperature at some time during the transient heating process.

(a) If the sphere has a diameter of 0.1 m, a thermal conductivity of 15 W/m · K, and a thermal diffusivity of 10^{-5} m^2/s, at what time will a surface temperature of 60°C be recorded if the convection coefficient is 300 W/m^2 · K?

(b) Assess the effect of thermal diffusivity on the thermal response of the material by computing center and surface temperature histories for $\alpha = 10^{-5}$, 10^{-5}, and 10^{-4} m^2/s. Plot your results for the period $0 \le t \le 300$ s. In a similar manner, assess the effect of thermal conductivity by considering values of $k = 1.5$, 15, and 150 W/m · K.

5.65 Consider the sphere of Example 5.5, which is initially at a uniform temperature when it is suddenly removed from the furnace and subjected to a two-step cooling process. Use the *Transient Conduction*, *Sphere* model of *IHT* to obtain the following solutions.

(a) For step 1, calculate the time required for the center temperature to reach $T(0, t) = 335$°C, while cooling in air at 20°C with a convection coefficient of 10 W/m^2 · K. What is the Biot number for this cooling process? Do you expect radial temperature gradients to be appreciable? Compare your results to those of the example.

(b) For step 2, calculate the time required for the center temperature to reach $T(0, t) = 50$°C, while cooling in a water bath at 20°C with a convection coefficient of 6000 W/m^2 · K.

(c) For the step 2 cooling process, calculate and plot the temperature histories, $T(r, t)$, for the center and surface of the sphere. Identify and explain key features of the histories. When do you expect the temperature gradients in the sphere to be the largest?

Semi-Infinite Media

5.66 Two large blocks of different materials, such as copper and concrete, have been sitting in a room (23°C) for a very long time. Which of the two blocks, if either, will feel colder to the touch? Assume the blocks to be semi-infinite solids and your hand to be at a temperature of 37°C.

5.67 A plane wall of thickness 0.6 m ($L = 0.3$ m) is made of steel ($k = 30$ W/m · K, $\rho = 7900$ kg/m^3, $c = 640$ J/kg · K). It is initially at a uniform temperature and is then exposed to air on both surfaces. Consider two different convection conditions: natural convection, characterized by $h = 10$ W/m^2 · K, and forced convection, with $h = 100$ W/m^2 · K. You are to calculate the surface temperature at three different times—$t = 2.5$ min, 25 min, and 250 min—for a total of six different cases.

(a) For each of these six cases, calculate the nondimensional surface temperature, $\theta_s^* = (T_s - T_\infty)/(T_i - T_\infty)$, using four different methods: exact solution, first-term-of-the-series solution, lumped capacitance, and semi-infinite solid. Present your results in a table.

(b) Briefly explain the conditions for which (i) the first-term solution is a good approximation to the exact solu
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exact solution, (ii) the lumped capacitance solution is a good approximation, (iii) the semi-infinite solid solution is a good approximation.

5.68 Asphalt pavement may achieve temperatures as high as 50°C on a hot summer day. Assume that such a temperature exists throughout the pavement, when suddenly a rainstorm reduces the surface temperature to 20°C. Calculate the total amount of energy (J/m^2) that will be transferred from the asphalt over a 30-min period in which the surface is maintained at 20°C.

5.69 A thick steel slab ($\rho = 7800$ kg/m^3, $c = 480$ J/kg · K, $k = 50$ W/m · K) is initially at 300°C and is cooled by water jets impinging on one of its surfaces. The temperature of the water is 25°C, and the jets maintain an extremely large, approximately uniform convection coefficient at the surface. Assuming that the surface is maintained at the temperature of the water throughout the cooling, how long will it take for the temperature to reach 50°C at a distance of 25 mm from the surface?

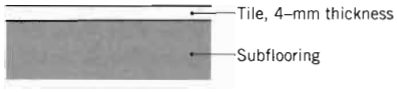
5.70 Consider the water main of Example 5.6, which is buried in soil initially at 20°C and is suddenly subjected to a constant surface temperature of −15°C for 60 days. Use the *Transient Conduction/Semi-Infinite Solid* model of *IHT* to obtain the following solutions. Compare your results with those in the Comments section of the example.

(a) Calculate and plot the temperature history at the burial depth of 0.68 m for thermal diffusivities of $\alpha \times 10^7 = 1.0$, 1.38, and 3.0 m^2/s.

(b) For $\alpha = 1.38 \times 10^{-7}$ m^2/s, plot the temperature distribution over the depth $0 \le x \le 1.0$ m for times of 1, 5, 10, 30, and 60 days.

(c) For $\alpha = 1.38 \times 10^{-7}$ m^2/s, show that the heat flux from the soil decreases with increasing time by plotting $q''_x(0, t)$ as a function of time for the 60-day period. On this graph, also plot the heat flux at the depth of the buried main, $q''_x(0.68\text{ m}, t)$.

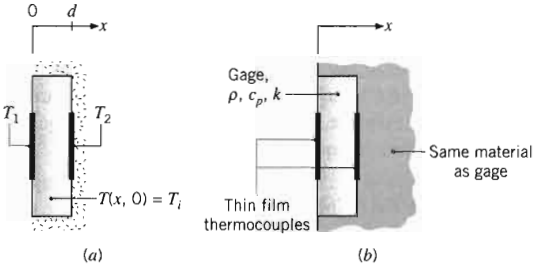
5.71 A tile-iron consists of a massive plate maintained at 150°C by an imbedded electrical heater. The iron is placed in contact with a tile to soften the adhesive, allowing the tile to be easily lifted from the subflooring. The adhesive will soften sufficiently if heated above 50°C for at least 2 min, but its temperature should not exceed 120°C to avoid deterioration of the adhesive. Assume the tile and subfloor to have an initial temperature of 25°C and to have equivalent thermophysical properties of $k = 0.15$ W/m · K and $\rho c_p = 1.5 \times 10^6$ J/m^3 · K.



(a) How long will it take a worker using the tile-iron to lift a tile? Will the adhesive temperature exceed 120°C?

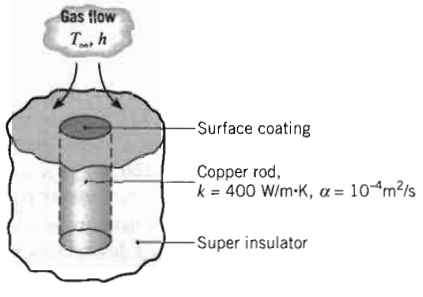
(b) If the tile-iron has a square surface area 254 mm to the side, how much energy has been removed from it during the time it has taken to lift the tile?

5.72 The manufacturer of a heat flux gage like that illustrated in Problem 1.12 claims the time constant for a 63.2% response to be $\tau = (4d^2\rho c_p)/\pi^2 k$, where ρ, c_p, and k are the thermophysical properties of the gage material and d is its thickness. Not knowing the origin of this relation, your task is to model the gage considering the two extreme cases illustrated below. In both cases, the gage, initially at a uniform temperature T_i, is exposed to a sudden change in surface temperature, $T(0, t) = T_s$. For case (*a*) the backside of the gage is insulated, and for case (*b*) the gage is imbedded in a semi-infinite solid having the same thermophysical properties as those of the gage.



Develop relationships for predicting the time constant of the gage for the two cases and compare them to the manufacturer's relation. What conclusion can you draw from this analysis regarding the transient response of gages for different applications?

5.73 A simple procedure for measuring surface convection heat transfer coefficients involves coating the surface with a thin layer of material having a precise melting point temperature. The surface is then heated and, by determining the time required for melting to occur, the



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convection coefficient is determined. The following experimental arrangement uses the procedure to determine the convection coefficient for gas flow normal to a surface. Specifically, a long copper rod is encased in a super insulator of very low thermal conductivity, and a very thin coating is applied to its exposed surface.

If the rod is initially at 25°C and gas flow for which $h = 200\ \text{W/m}^2 \cdot \text{K}$ and $T_\infty = 300°\text{C}$ is initiated, what is the melting point temperature of the coating if melting is observed to occur at $t = 400$ s?

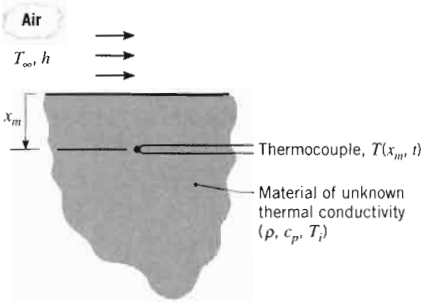
5.74 An insurance company has hired you as a consultant to improve their understanding of burn injuries. They are especially interested in injuries induced when a portion of a worker's body comes into contact with machinery that is at elevated temperatures in the range of 50 to 100°C. Their medical consultant informs them that irreversible thermal injury (cell death) will occur in any living tissue that is maintained at $T \geq 48°\text{C}$ for a duration $\Delta t \geq 10$ s. They want information concerning the extent of irreversible tissue damage (as measured by distance from the skin surface) as a function of the machinery temperature and the time during which contact is made between the skin and the machinery. Assume that living tissue has a normal temperature of 37°C, is isotropic, and has constant properties equivalent to those of liquid water.

(a) To assess the seriousness of the problem, compute locations in the tissue at which the temperature will reach 48°C after 10 s of exposure to machinery at 50°C and 100°C.

(b) For a machinery temperature of 100°C and $0 \leq t \leq 30$ s, compute and plot temperature histories at tissue locations of 0.5, 1, and 2 mm from the skin.

5.75 A procedure for determining the thermal conductivity of a solid material involves embedding a thermocouple in a thick slab of the solid and measuring the response to a prescribed change in temperature at one surface. Consider an arrangement for which the thermocouple is embedded 10 mm from a surface that is suddenly brought to a temperature of 100°C by exposure to boiling water. If the initial temperature of the slab was 30°C and the thermocouple measures a temperature of 65°C, 2 min after the surface is brought to 100°C, what is its thermal conductivity? The density and specific heat of the solid are known to be 2200 kg/m^3 and 700 J/kg · K.

5.76 The density and specific heat of a plastic material are known ($\rho = 950\ \text{kg/m}^3$, $c_p = 1100\ \text{J/kg} \cdot \text{K}$), but its thermal conductivity is unknown. To determine the thermal conductivity, an experiment is performed in which a thick sample of the material is heated to a uniform temperature of 100°C and then cooled by passing air at 25°C over one surface. A thermocouple embedded a distance of $x_m = 10$ mm below the surface records the thermal response of the plastic during cooling.



If the convection coefficient associated with the air flow is $h = 200\ \text{W/m}^2 \cdot \text{K}$ and a temperature of 60°C is recorded 5 min after the onset of cooling, what is the thermal conductivity of the material?

5.77 A very thick slab with thermal diffusivity 5.6×10^{-6} m^2/s and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is $100\ \text{W/m}^2 \cdot \text{K}$.

(a) Determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

(b) Compute and plot temperature histories ($0 \leq t \leq 300$ s) at $x = 0$ and $x = 45$ mm for the following parametric variations: (i) $\alpha = 5.6 \times 10^{-7}$, 5.6×10^{-6}, and 5.6×10^{-5} m^2/s; and (ii) $k = 2$, 20, and 200 W/m · K.

5.78 A thick oak wall, initially at 25°C, is suddenly exposed to combustion products for which $T_\infty = 800°\text{C}$ and $h = 20\ \text{W/m}^2 \cdot \text{K}$.

(a) Determine the time of exposure required for the surface to reach the ignition temperature of 400°C.

(b) Plot the temperature distribution $T(x)$ in the medium at $t = 325$ s. The distribution should extend to a location for which $T \approx 25°\text{C}$.

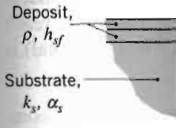
5.79 Standards for firewalls may be based on their thermal response to a prescribed radiant heat flux. Consider a 0.25-m-thick concrete wall ($\rho = 2300\ \text{kg/m}^3$, $c = 880\ \text{J/kg} \cdot \text{K}$, $k = 1.4\ \text{W/m} \cdot \text{K}$), which is at an initial temperature of $T_i = 25°\text{C}$ and irradiated at one surface by lamps that provide a uniform heat flux of $q_s'' = 10^4\ \text{W/m}^2$. The absorptivity of the surface to the irradiation is $\alpha_s = 1.0$. If building code requirements dictate that the temperatures of the irradiated and back

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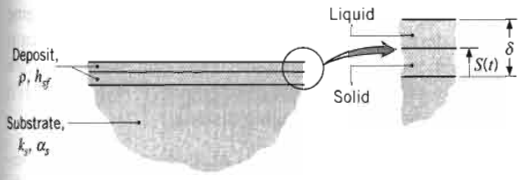
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surfaces must not exceed 325°C and 25°C, respectively, after 30 min of heating, will the requirements be met?

5.80 It is well known that, although two materials are at the same temperature, one may feel cooler to the touch than the other. Consider thick plates of copper and glass, each at an initial temperature of 300 K. Assuming your finger to be at an initial temperature of 310 K and to have thermophysical properties of $\rho = 1000$ kg/m^3, $c = 4180$ J/kg · K, and $k = 0.625$ W/m · K, determine whether the copper or the glass will feel cooler to the touch.

5.81 Two stainless steel plates ($\rho = 8000$ kg/m^3, $c = 500$ J/kg · K, $k = 15$ W/m · K), each 20 mm thick and insulated on one surface, are initially at 400 and 300 K when they are pressed together at their uninsulated surfaces. What is the temperature of the insulated surface of the hot plate after 1 min has elapsed?

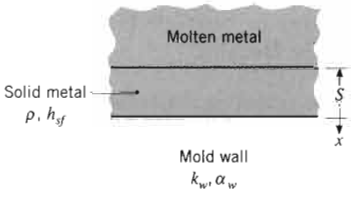
5.82 Special coatings are often formed by depositing thin layers of a molten material on a solid substrate. Solidification begins at the substrate surface and proceeds until the thickness S of the solid layer becomes equal to the thickness δ of the deposit.



(a) Consider conditions for which molten material at its fusion temperature T_f is deposited on a *large* substrate that is at an initial uniform temperature T_i. With $S = 0$ at $t = 0$, develop an expression for estimating the time t_d required to completely solidify the deposit if it remains at T_f throughout the solidification process. Express your result in terms of the substrate thermal conductivity and thermal diffusivity (k_s, α_s), the density and latent heat of fusion of the deposit (ρ, h_{sf}), the deposit thickness δ, and the relevant temperatures (T_f, T_i).

(b) The plasma spray deposition process of Problem 5.25 is used to apply a thin ($\delta = 2$ mm) alumina coating on a thick tungsten substrate. The substrate has a uniform initial temperature of $T_i = 300$ K, and its thermal conductivity and thermal diffusivity may be approximated as $k_s = 120$ W/m · K and $\alpha_s = 4.0 \times 10^{-5}$ m^2/s, respectively. The density and latent heat of fusion of the alumina are $\rho = 3970$ kg/m^3 and $h_{sf} = 3577$ kJ/kg, respectively, and the alumina solidifies at its fusion temperature ($T_f = 2318$ K). Assuming that the molten layer is instantaneously deposited on the substrate, estimate the time required for the deposit to solidify.

5.83 When a molten metal is cast in a mold that is a poor conductor, the dominant resistance to heat flow is within the mold wall. Consider conditions for which a liquid metal is solidifying in a thick-walled mold of thermal conductivity k_w and thermal diffusivity α_w. The density and latent heat of fusion of the metal are designated as ρ and h_{sf}, respectively, and in both its molten and solid states, the thermal conductivity of the metal is very much larger than that of the mold.



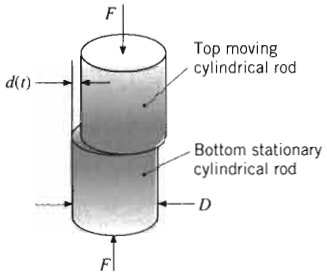
Just before the start of solidification ($S = 0$), the mold wall is everywhere at an initial uniform temperature T_i and the molten metal is everywhere at its fusion (melting point) temperature of T_f. Following the start of solidification, there is conduction heat transfer into the mold wall and the thickness of the solidified metal, S, increases with time t.

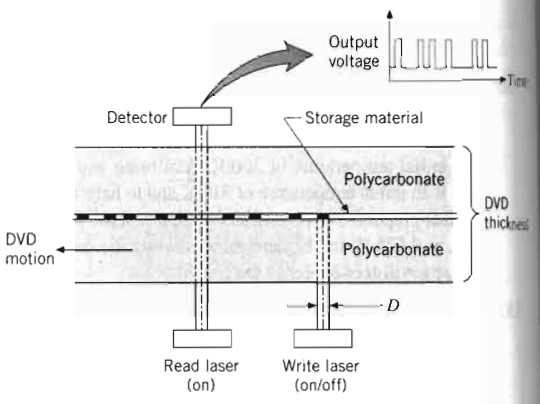
(a) Sketch the one-dimensional temperature distribution, $T(x)$, in the mold wall and the metal at $t = 0$ and at two subsequent times during the solidification. Clearly indicate any underlying assumptions.

(b) Obtain a relation for the variation of the solid layer thickness S with time t, expressing your result in terms of appropriate parameters of the system.

5.84 Joints of high quality can be formed by friction welding. Consider the friction welding of two 40-mm-diameter Inconel rods. The bottom rod is stationary, while the top rod is forced into a back-and-forth linear motion characterized by an instantaneous horizontal displacement, $d(t) = a\cos(\omega t)$ where $a = 2$ mm and $\omega = 1000$ rad/s. The coefficient of sliding friction between the two pieces is $\mu = 0.3$. Determine the compressive force that must be applied in order to heat the joint to the Inconel melting point within $t = 3$ s, starting from an initial temperature of 20°C. *Hint*: The frequency of the motion and resulting heat rate are very high. The temperature response can be approximated as if the heating rate were constant in time, equal to its average value.

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Objects with Constant Surface Temperatures or Surface Heat Fluxes and Periodic Heating

5.85 An above-ground circular swimming pool is heated so that it can be used in cool weather. The ground has a temperature of 10°C far from the pool. The heater is turned on and quickly brings the pool water to a comfortable 20°C; assume this is the temperature of the circular region of the ground beneath the swimming pool. The pool has a diameter of 5 m.

(a) Calculate the rate of heat transfer from the pool to the ground, ten hours after the heater is turned on. *Hint:* Based on symmetry considerations, the pool footprint can be viewed as a heated disk in *infinite* surroundings.

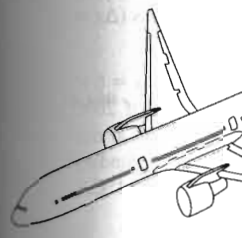
(b) Calculate the time it would take for the heat transfer rate to reach within 10% of its steady-state value.

5.86 A rewritable optical disc (DVD) is formed by sandwiching a 15-nm-thick binary compound storage material between two 1-mm-thick polycarbonate sheets. Data are *written* to the opaque storage medium by irradiating it from below with a relatively high-powered laser beam of diameter 0.4 μm and power 1 mW, resulting in rapid heating of the compound material (the polycarbonate is transparent to the laser irradiation). If the temperature of the storage medium exceeds 900 K, a noncrystalline, amorphous material forms at the heated spot when the laser irradiation is curtailed and the spot is allowed to cool rapidly. The resulting spots of amorphous material have a different reflectivity from the surrounding crystalline material, so they can subsequently be *read* by irradiating them with a second, low-power laser and detecting the changes in laser radiation transmitted through the entire DVD thickness. Determine the irradiation (write) time needed to raise the storage medium temperature from an initial value of 300 K to 1000 K. The absorptivity of the storage medium is 0.8. The polycarbonate properties are $\rho = 1200$ kg/m^3, $k = 0.21$ W/m · K, and $c_p = 1260$ J/kg · K.

5.87 To enable cooking a wider range of foods in microwave ovens, thin, metallic packaging materials have been developed that will readily absorb microwave energy. As the packaging material is heated by the microwaves, conduction simultaneously occurs from the hot packaging material to the cold food. Consider the spherical piece of frozen ground beef of Problem 5.33 that is now wrapped in the thin microwave-absorbing packaging material. Determine the time needed for the beef that is immediately adjacent to the packaging material to reach $T = 0$°C if 50% of the oven power ($P = 1$ kW total) is absorbed in the packaging material.

5.88 The structural components of modern aircraft are commonly fabricated of high-performance composite materials. These materials are fabricated by impregnating mats of extremely strong fibers that are held within a form with an epoxy or thermoplastic liquid. After the liquid cures or cools, the resulting component is of extremely high strength and low weight. Periodically, these components must be inspected to ensure that the fiber mats and bonding material do not become delaminated and, in turn, the component loses its airworthiness. One inspection method involves application of a uniform, constant radiation heat flux to the surface being inspected. The thermal response of the surface is measured with an infrared imaging system, which captures the emission from the surface and converts it to a color-coded map of the surface temperature distribution. Consider the case where a uniform flux of 5 kW/m^2 is applied to the top skin of an airplane wing initially at 20°C. The opposite side of the 15-mm-thick skin is adjacent to stagnant air and can be treated as well insulated. The density and specific heat of the skin material are 1200 kg/m^3 and 1200 J/kg · K, respectively. The effective thermal conductivity of the intact skin material is $k_1 = 1.6$ W/m · K. Contact resistances develop internal to the structure as a result of delamination

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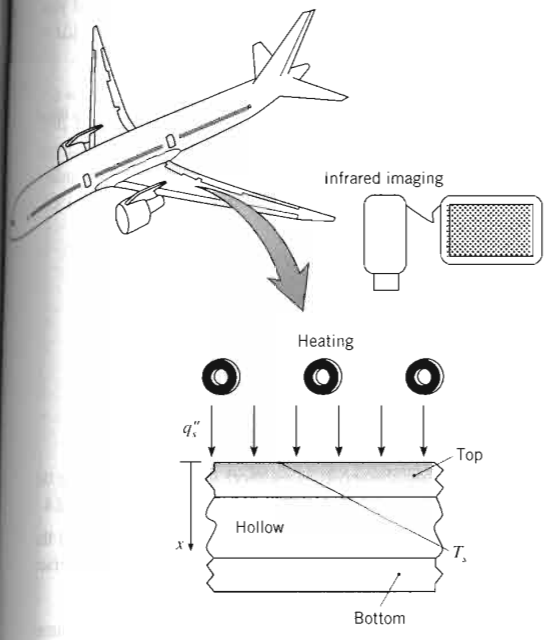
5.89 Problem 4.9
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between the fiber mats and the bonding material, leading to a reduced effective thermal conductivity of $k_2 = 1.1$ W/m · K. Determine the surface temperature of the component after 10 and 100 seconds of irradiation for (a) an area where the material is structurally intact and (b) an adjacent area where delamination has occurred within the wing.



5.89 Problem 4.9 addressed radioactive wastes stored underground in a spherical container. Because of uncertainty in the thermal properties of the soil, it is desired to measure the steady-state temperature using a test container (identical to the real container) that is equipped with internal electrical heaters. Estimate how long it will take the test container to come within 10°C of its steady-state value, assuming it is buried very far underground. Use the soil properties from Table A.3 in your analysis.

5.90 A thermal storage medium consists of a large mass of limestone. Ambient air is pumped through a 30-mm-high, 5-m-long, 5-m-wide fissure within the limestone at a velocity of 2 m/s, providing a heat transfer coefficient between the air and the limestone of $h = 10$ W/m^2 · K. Over a 24-hour time period, the temperature of the surface of the limestone adjacent to the air stream near the inlet exhibits periodic behavior with maximum and minimum values of 302 and 298 K, respectively.

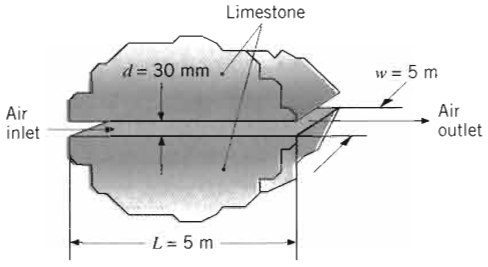
(a) Determine the minimum and maximum values of the temperature of the air near the inlet.

(b) Determine the maximum heat flux to the air during a 24-hour time period near the inlet. Determine the air inlet and outlet temperatures at the time corresponding to the maximum air heat flux.

(c) Determine the air outlet temperatures at the times corresponding to the maximum and minimum air inlet temperatures.

(d) Plot the inlet air temperature, the limestone surface temperature, and the heat transfer rate to the limestone over 48 hours.

(e) What is the required thickness of limestone necessary to ensure that the limestone may be viewed as a semi-infinite medium?



5.91 Consider the experimental measurement of Example 5.8. It is desired to measure the thermal conductivity of an extremely thin sample of the same nanostructured material having the same length and width. To minimize experimental uncertainty, the experimenter wishes to keep the amplitude of the temperature response, ΔT, above a value of 0.1°C. What is the minimum sample thickness that can be measured? Assume the properties of the thin sample and the magnitude of the applied heating rate are the same as those measured and used in Example 5.8.

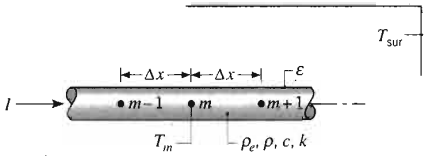
Finite-Difference Equations: Derivations

5.92 The stability criterion for the explicit method requires that the coefficient of the T_m^p term of the one-dimensional, finite-difference equation be zero or positive. Consider the situation for which the temperatures at the two neighboring nodes (T_{m-1}^p, T_{m+1}^p) are 100°C while the center node (T_m^p) is at 50°C. Show that for values of $Fo > \frac{1}{2}$ the finite-difference equation will predict a value of T_m^{p+1} that violates the second law of thermodynamics.

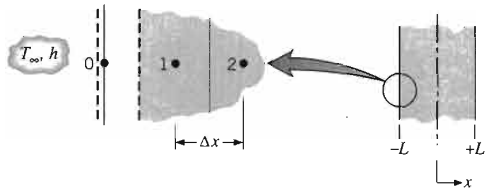
5.93 A thin rod of diameter D is initially in equilibrium with its surroundings, a large vacuum enclosure at temperature T_{sur}. Suddenly an electrical current I (A) is passed through the rod having an electrical resistivity ρ_e and emissivity ε. Other pertinent thermophysical properties

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are identified in the sketch. Derive the transient, finite-difference equation for node m.

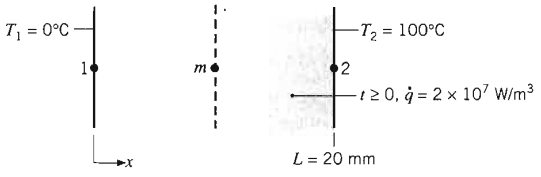


5.94 A one-dimensional slab of thickness $2L$ is initially at a uniform temperature T_i. Suddenly, electric current is passed through the slab causing a uniform volumetric heating $\dot{q}$ (W/m^3). At the same time, both outer surfaces ($x = \pm L$) are subjected to a convection process at T_∞ with a heat transfer coefficient h.



Write the finite-difference equation expressing conservation of energy for node 0 located on the outer surface at $x = -L$. Rearrange your equation and identify any important dimensionless coefficients.

5.95 A plane wall ($\rho = 4000$ kg/m^3, $c_p = 500$ J/kg · K, $k = 10$ W/m · K) of thickness $L = 20$ mm initially has a linear, steady-state temperature distribution with boundaries maintained at $T_1 = 0$°C and $T_2 = 100$°C. Suddenly, an electric current is passed through the wall, causing uniform energy generation at a rate $\dot{q} = 2 \times 10^7$ W/m^3. The boundary conditions T_1 and T_2 remain fixed.



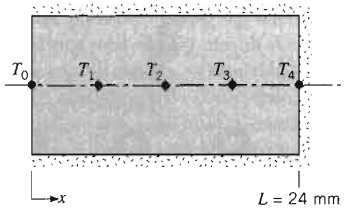
(a) On T–x coordinates, sketch temperature distributions for the following cases: (i) initial condition ($t \leq 0$); (ii) steady-state conditions ($t \to \infty$), assuming that the maximum temperature in the wall exceeds T_2; and (iii) for two intermediate times. Label all important features of the distributions.

(b) For the system of three nodal points shown schematically (1, m, 2), define an appropriate control volume for node m and, identifying all relevant processes, derive the corresponding finite-difference equation using either the *explicit* or *implicit* method.

(c) With a time increment of $\Delta t = 5$ s, use the finite-difference method to obtain values of T_m for the first 45 s of elapsed time. Determine the corresponding heat fluxes at the boundaries, that is, q_x'' (0, 45 s) and q_x'' (20 mm, 45 s).

(d) To determine the effect of mesh size, repeat your analysis using grids of 5 and 11 nodal points ($\Delta x = 5.0$ and 2.0 mm, respectively).

5.96 A round solid cylinder made of a plastic material ($\alpha = 6 \times 10^{-7}$ m^2/s) is initially at a uniform temperature of 20°C and is well insulated along its lateral surface and at one end. At time $t = 0$, heat is applied to the left boundary causing T_0 to increase linearly with time at a rate of 1°C/s.

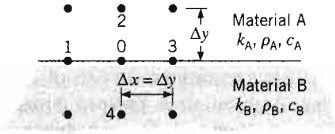


(a) Using the explicit method with $Fo = \frac{1}{2}$, derive the finite-difference equations for nodes 1, 2, 3, and 4.

(b) Format a table with headings of p, t (s), and the nodal temperatures T_0 to T_4. Determine the surface temperature T_0 when $T_4 = 35$°C.

5.97 Derive the explicit finite-difference equation for an interior node for three-dimensional transient conduction. Also determine the stability criterion. Assume constant properties and equal grid spacing in all three directions.

5.98 Derive the transient, two-dimensional finite-difference equation for the temperature at nodal point 0 located on the boundary between two different materials.



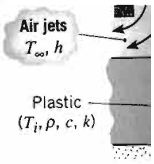
Finite-Difference Solutions: One-Dimensional Systems

5.99 A wall 0.12 m thick having a thermal diffusivity of 1.5×10^{-6} m^2/s is initially at a uniform temperature of 85°C. Suddenly one face is lowered to a temperature of 20°C, while the other face is perfectly insulated.

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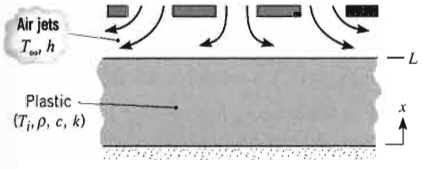
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(a) Using the explicit finite-difference technique with space and time increments of 30 mm and 300 s, respectively, determine the temperature distribution at $t = 45$ min.

(b) With $\Delta x = 30$ mm and $\Delta t = 300$ s, compute $T(x, t)$ for $0 \le t \le t_{ss}$, where t_{ss} is the time required for the temperature at each nodal point to reach a value that is within 1°C of the steady-state temperature. Repeat the foregoing calculations for $\Delta t = 75$ s. For each value of Δt, plot temperature histories for each face and the midplane.

5.100 A molded plastic product ($\rho = 1200$ kg/m^3, $c = 1500$ J/kg · K, $k = 0.30$ W/m · K) is cooled by exposing one surface to an array of air jets, while the opposite surface is well insulated. The product may be approximated as a slab of thickness $L = 60$ mm, which is initially at a uniform temperature of $T_i = 80$°C. The air jets are at a temperature of $T_\infty = 20$°C and provide a uniform convection coefficient of $h = 100$ W/m^2 · K at the cooled surface.



Using a finite-difference solution with a space increment of $\Delta x = 6$ mm, determine temperatures at the cooled and insulated surfaces after 1 hour of exposure to the gas jets.

5.101 The plane wall of Problem 2.48 ($k = 50$ W/m · K, $\alpha = 1.5 \times 10^{-6}$ m^2/s) has a thickness of $L = 40$ mm and an initial uniform temperature of $T_o = 25$°C. Suddenly, the boundary at $x = L$ experiences heating by a fluid for which $T_\infty = 50$°C and $h = 1000$ W/m^2 · K, while heat is uniformly generated within the wall at $\dot{q} = 1 \times 10^7$ W/m^3. The boundary at $x = 0$ remains at T_o.

(a) With $\Delta x = 4$ mm and $\Delta t = 1$ s, plot temperature distributions in the wall for (i) the initial condition, (ii) the steady-state condition, and (iii) two intermediate times.

(b) On q''_x–t coordinates, plot the heat flux at $x = 0$ and $x = L$. At what elapsed time is there zero heat flux at $x = L$?

5.102 Consider the fuel element of Example 5.9. Initially, the element is at a uniform temperature of 250°C with no heat generation. Suddenly, the element is inserted into the reactor core causing a uniform volumetric heat generation rate of $\dot{q} = 10^8$ W/m^3. The surfaces are convectively cooled with $T_\infty = 250$°C and $h = 1100$ W/m^2 · K. Using the explicit method with a space increment of 2 mm, determine the temperature distribution 1.5 s after the element is inserted into the core.

5.103 Consider the fuel element of Example 5.9, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7$ W/m^3, until the generation rate suddenly changes to $\dot{q}_2 = 2 \times 10^7$ W/m^3. Use the *Finite-Difference Equations*, *One-Dimensional*, *Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the 6 nodes, with $\Delta x = 2$ mm, as shown in the example.

(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example.

(b) Use the *Explore* and *Graph* options of *IHT* to calculate and plot temperature histories at the midplane (00) and surface (05) nodes for $0 \le t \le 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

5.104 Consider the fuel element of Example 5.9, which operates at a uniform volumetric generation rate of $\dot{q}_1 = 10^7$ W/m^3, until the generation rate suddenly changes to $\dot{q}_2 = 2 \times 10^7$ W/m^3. Use the finite-element software *FEHT* to obtain the following solutions.

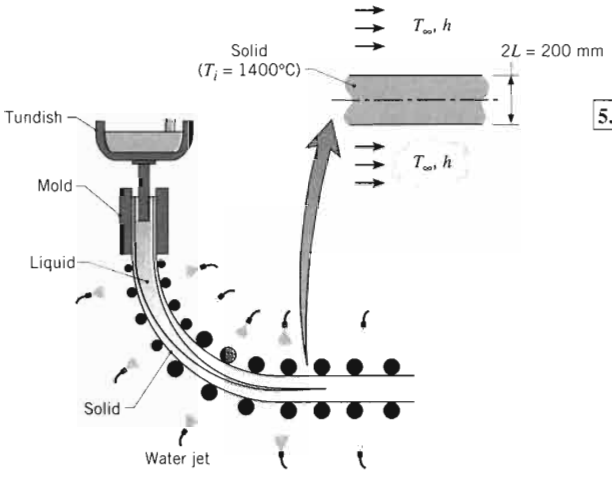
(a) Calculate the temperature distribution 1.5 s after the change in operating power and compare your results with those tabulated in the example. *Hint*: First determine the steady-state temperature distribution for $\dot{q}_1$, which represents the initial condition for the transient temperature distribution after the step change in power to $\dot{q}_2$. Next, in the *Setup* menu, click on *Transient*: in the *Specify/Internal Generation* box, change the value to $\dot{q}_2$; and in the *Run* command, click on *Continue* (not *Calculate*). See the *Run* menu in the *FEHT* Help section for background information on the *Continue* option.

(b) Use your *FEHT* model to plot temperature histories at the midplane and surface for $0 \le t \le 400$ s. What are the steady-state temperatures, and approximately how long does it take to reach the new equilibrium condition after the step change in operating power?

5.105 In a thin-slab, continuous casting process, molten steel leaves a mold with a thin solid shell, and the molten material solidifies as the slab is quenched by water jets en route to a section of rollers. Once fully solidified,

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the slab continues to cool as it is brought to an acceptable handling temperature. It is this portion of the process that is of interest.



Consider a 200-mm-thick solid slab of steel (ρ = 7800 kg/m^3, c = 700 J/kg · K, k = 30 W/m · K), initially at a uniform temperature of T_i = 1400°C. The slab is cooled at its top and bottom surfaces by water jets (T_∞ = 50°C), which maintain an approximately uniform convection coefficient of h = 5000 W/m^2 · K at both surfaces. Using a finite-difference solution with a space increment of Δx = 1 mm, determine the time required to cool the surface of the slab to 200°C. What is the corresponding temperature at the midplane of the slab? If the slab moves at a speed of V = 15 mm/s, what is the required length of the cooling section?

5.106 A very thick plate with thermal diffusivity 5.6 × 10^{-6} m^2/s and thermal conductivity 20 W/m · K is initially at a uniform temperature of 325°C. Suddenly, the surface is exposed to a coolant at 15°C for which the convection heat transfer coefficient is 100 W/m^2 · K. Using the finite-difference method with a space increment of Δx = 15 mm and a time increment of 18 s, determine temperatures at the surface and at a depth of 45 mm after 3 min have elapsed.

5.107 Referring to Example 5.10, Comment 4, consider a sudden exposure of the surface to large surroundings at an elevated temperature (T_{sur}) and to convection (T_∞, h).

(a) Derive the explicit, finite-difference equation for the surface node in terms of Fo, Bi, and Bi_r.

(b) Obtain the stability criterion for the surface node. Does this criterion change with time? Is the criterion more restrictive than that for an interior node?

(c) A thick slab of material (k = 1.5 W/m · K, α = 7 × 10^{-7} m^2/s, ε = 0.9), initially at a uniform temperature of 27°C, is suddenly exposed to large surroundings at 1000 K. Neglecting convection and using a space increment of 10 mm, determine temperatures at the surface and 30 mm from the surface after an elapsed time of 1 min.

5.108 Consider the thick slab of copper in Example 5.10, which is initially at a uniform temperature of 20°C and is suddenly exposed to a net radiant flux of 3 × 10^5 W/m^2. Use the *Finite-Difference Equations/One-Dimensional/Transient* conduction model builder of *IHT* to obtain the implicit form of the finite-difference equations for the interior nodes. In your analysis, use a space increment of Δx = 37.5 mm with a total of 17 nodes (00–16), and a time increment of Δt = 1.2 s. For the surface node 00, use the finite-difference equation derived in Section 2 of the Example.

(a) Calculate the 00 and 04 nodal temperatures at t = 120 s, that is, T(0, 120 s) and T(0.15 m, 120 s), and compare the results with those given in Comment 1 for the exact solution. Will a time increment of 0.12 s provide more accurate results?

(b) Plot temperature histories for x = 0, 150, and 600 mm, and explain key features of your results.

5.109 Consider the thick slab of copper in Example 5.10, which is initially at a uniform temperature of 20°C and is suddenly exposed to large surroundings at 1000°C (instead of a prescribed heat flux).

(a) For a surface emissivity of 0.94, calculate the temperatures T(0, 120 s) and T(0.15 m, 120 s) using the finite-element software *FEHT*. *Hint:* In the *Convection Coefficient* box of the *Specify Boundary Conditions* menu of *FEHT*, enter the linearized radiation coefficient (see Equation 1.9) for the surface (x = 0). Enter the temperature of the surroundings in the *Fluid Temperature* box. See also the *Help* section on *Entering Equations*. Click on *Setup/Temperatures in K* to enter all temperatures in kelvins.

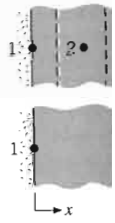
(b) Plot the temperature histories for x = 0, 150, and 600 mm, and explain key features of your results.

5.110 Consider the composite wall of Problem 2.53. In part (d), you are asked to sketch the temperature histories at x = 0, L during the transient period between cases 2 and 3. Calculate and plot these histories using the finite-element method of *FEHT*, the finite-difference method of *IHT* (with Δx = 5 mm and Δt = 1.2 s), and/or an alternative procedure of your choice. If you

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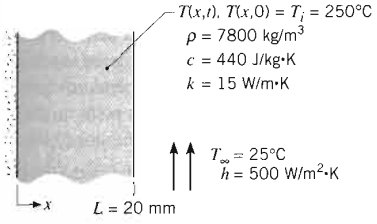
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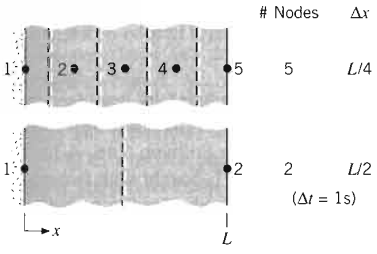
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use more than one method, compare the respective results. Note that, in using *FEHT* or *IHT*, a look-up table must be created for prescribing the variation of the heater flux with time (see the appropriate *Help* section for guidance).

5.111 In Section 5.5, the one-term approximation to the series solution for the temperature distribution was developed for a plane wall of thickness $2L$ that is initially at a uniform temperature and suddenly subjected to convection heat transfer. If $Bi < 0.1$, the wall can be approximated as isothermal and represented as a lumped capacitance (Equation 5.7). For the conditions shown schematically, we wish to compare predictions based on the one-term approximation, the lumped capacitance method, and a finite-difference solution.



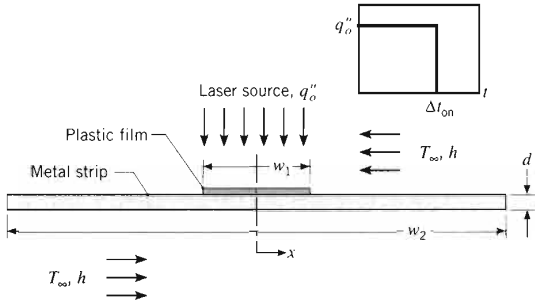


(a) Determine the midplane, $T(0, t)$, and surface, $T(L, t)$, temperatures at $t = 100$, 200, and 500 s using the one-term approximation to the series solution, Equation 5.40. What is the Biot number for the system?

(b) Treating the wall as a lumped capacitance, calculate the temperatures at $t = 50$, 100, 200, and 500 s. Did you expect these results to compare favorably with those from part (a)? Why are the temperatures considerably higher?

(c) Consider the 2- and 5-node networks shown schematically. Write the implicit form of the finite-difference equations for each network, and determine the temperature distributions for $t = 50$, 100, 200, and 500 s using a time increment of $\Delta t = 1$ s. You may use *IHT* to solve the finite-difference equations by representing the rate of change of the nodal temperatures by the intrinsic function, Der(T, t). Prepare a table summarizing the results of parts (a), (b), and (c). Comment on the relative differences of the predicted temperatures. *Hint*: See the *Solver/Intrinsic Functions* section of *IHT/Help* or the *IHT Examples* menu (Example 5.2) for guidance on using the Der(T, t) function.

5.112 Consider the bonding operation described in Problem 3.103, which was analyzed under steady-state conditions. In this case, however, the laser will be used to heat the film for a prescribed period of time, creating the transient heating situation shown in the sketch.



The strip is initially at 25°C and the laser provides a uniform flux of 85,000 W/m^2 over a time interval of $\Delta t_{on} = 10$ s. The system dimensions and thermophysical properties remain the same, but the convection coefficient to the ambient air at 25°C is now 100 W/m$^2 \cdot$ K and $w_1 = 44$ mm.

Using an implicit finite-difference method with $\Delta x = 4$ mm and $\Delta t = 1$ s, obtain temperature histories for $0 \le t \le 30$ s at the center and film edge, $T(0, t)$ and $T(w_1/2, t)$, respectively, to determine if the adhesive is satisfactorily cured above 90°C for 10 s and if its degradation temperature of 200°C is exceeded.

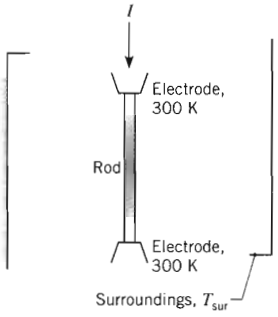
5.113 One end of a stainless steel (AISI 316) rod of diameter 10 mm and length 0.16 m is inserted into a fixture maintained at 200°C. The rod, covered with an insulating sleeve, reaches a uniform temperature throughout its length. When the sleeve is removed, the rod is subjected to ambient air at 25°C such that the convection heat transfer coefficient is 30 W/m$^2 \cdot$ K.

(a) Using the explicit finite-difference technique with a space increment of $\Delta x = 0.016$ m, estimate the time required for the midlength of the rod to reach 100°C.

(b) With $\Delta x = 0.016$ m and $\Delta t = 10$ s, compute $T(x, t)$ for $0 \le t \le t_1$, where t_1 is the time required for the midlength of the rod to reach 50°C. Plot the temperature distribution for $t = 0$, 200 s, 400 s, and t_1.

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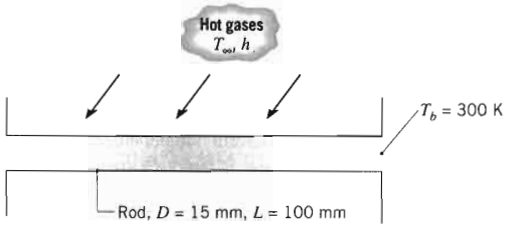
5.114 A tantalum rod of diameter 3 mm and length 120 mm is supported by two electrodes within a large vacuum enclosure. Initially the rod is in equilibrium with the electrodes and its surroundings, which are maintained at 300 K. Suddenly, an electrical current, $I = 80$ A, is passed through the rod. Assume the emissivity of the rod is 0.1 and the electrical resistivity is $95 \times 10^{-8}\ \Omega \cdot$ m. Use Table A.1 to obtain the other thermophysical properties required in your solution. Use a finite-difference method with a space increment of 10 mm.



(a) Estimate the time required for the midlength of the rod to reach 1000 K.

(b) Determine the steady-state temperature distribution and estimate approximately how long it will take to reach this condition.

5.115 A support rod ($k = 15$ W/m · K, $\alpha = 4.0 \times 10^{-6}$ m²/s) of diameter $D = 15$ mm and length $L = 100$ mm spans a channel whose walls are maintained at a temperature of $T_b = 300$ K. Suddenly, the rod is exposed to a cross flow of hot gases for which $T_\infty = 600$ K and $h = 75$ W/m² · K. The channel walls are cooled and remain at 300 K.



(a) Using an appropriate numerical technique, determine the thermal response of the rod to the convective heating. Plot the midspan temperature as a function of elapsed time. Using an appropriate analytical model of the rod, determine the steady-state temperature distribution and compare the result with that obtained numerically for very long elapsed times.

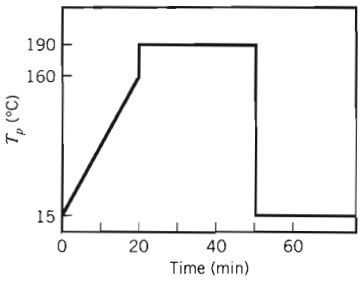
(b) After the rod has reached steady-state conditions, the flow of hot gases is suddenly terminated, and the rod cools by free convection to ambient air at $T_\infty = 300$ K and by radiation exchange with large surroundings at $T_{sur} = 300$ K. The free convection coefficient can be expressed as h (W/m² · K) = $C \Delta T^n$, where $C = 4.4$ W/m² · K$^{1.188}$ and $n = 0.188$. The emissivity of the rod is 0.5. Determine the subsequent thermal response of the rod. Plot the midspan temperature as a function of cooling time, and determine the time required for the rod to reach a *safe-to-touch* temperature of 315 K.

5.116 Consider the acceleration-grid foil ($k = 40$ W/m · K, $\alpha = 3 \times 10^{-5}$ m²/s, $\varepsilon = 0.45$) of Problem 4.72. Develop an implicit, finite-difference model of the foil, which can be used for the following purposes.

(a) Assuming the foil to be at a uniform temperature of 300 K when the ion beam source is activated, obtain a plot of the midspan temperature–time history. At what elapsed time does this point on the foil reach a temperature within 1 K of the steady-state value?

(b) The foil is operating under steady-state conditions when, suddenly, the ion beam is deactivated. Obtain a plot of the subsequent midspan temperature-time history. How long does it take for the hottest point on the foil to cool to 315 K, a *safe-to-touch* condition?

5.117 Circuit boards are treated by heating a stack of them under high pressure as illustrated in Problem 5.45 and described further in Problem 5.46. A finite-difference method of solution is sought with two additional considerations. First, the book is to be treated as having distributed, rather than lumped, characteristics, by using a grid spacing of $\Delta x = 2.36$ mm with nodes at the center of the individual circuit board or plate. Second, rather than bringing the platens to 190°C in one sudden change, the heating schedule $T_p(t)$ shown below is to be used in order to minimize excessive thermal stresses induced by rapidly changing thermal gradients in the vicinity of the platens.

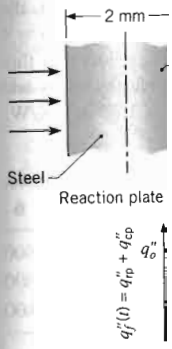


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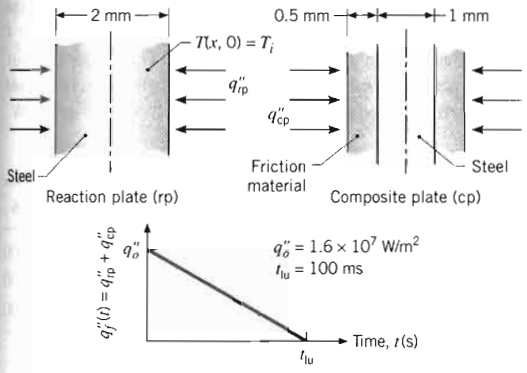


(a) Using a time increment of Δt = 60 s and the implicit method, find the temperature history of the midplane of the book and determine whether curing will occur (170°C for 5 min).

(b) Following the reduction of the platen temperatures to 15°C (t = 50 min), how long will it take for the midplane of the book to reach 37°C, a safe temperature at which the operator can begin unloading the press?

(c) Validate your program code by using the heating schedule of a sudden change of platen temperature from 15 to 190°C and compare results with those from an appropriate analytical solution (see Problem 5.46).

5.118 Common transmission failures result from the glazing of clutch surfaces by deposition of oil oxidation and decomposition products. Both the oxidation and decomposition processes depend on temperature histories of the surfaces. Because it is difficult to measure these surface temperatures during operation, it is useful to develop models to predict clutch-interface thermal behavior. The relative velocity between mating clutch plates, from the initial engagement to the zero-sliding (*lock-up*) condition, generates heat that is transferred to the plates. The relative velocity decreases at a constant rate during this period, producing a heat flux that is initially very large and decreases linearly with time, until lock-up occurs. Accordingly, $q''_f = q''_o[1 - (t/t_{lu})]$, where $q''_o = 1.6 \times 10^7$ W/m^2 and t_{lu} = 100 ms is the lock-up time. The plates have an initial uniform temperature of T_i = 40°C, when the prescribed frictional heat flux is suddenly applied to the surfaces. The reaction plate is fabricated from steel, while the composite plate has a thinner steel center section bonded to low-conductivity friction material layers. The thermophysical properties are ρ_s = 7800 kg/m^3, c_s = 500 J/kg $\cdot$ K, and k_s = 40 W/m $\cdot$ K for the steel and ρ_{fm} = 1150 kg/m^3, c_{fm} = 1650 J/kg $\cdot$ K, and k_{fm} = 4 W/m $\cdot$ K for the friction material.



(a) On $T - t$ coordinates, sketch the temperature history at the midplane of the reaction plate, at the interface between the clutch pair, and at the midplane of the composite plate. Identify key features.

(b) Perform an energy balance on the clutch pair over the time interval $\Delta t = t_{lu}$ to determine the steady-state temperature resulting from clutch engagement. Assume negligible heat transfer from the plates to the surroundings.

(c) Compute and plot the three temperature histories of interest using the finite-element method of *FEHT* or the finite-difference method of *IHT* (with Δx = 0.1 mm and Δt = 1 ms). Calculate and plot the frictional heat fluxes to the reaction and composite plates, q''_{rp} and q''_{cp}, respectively, as a function of time. Comment on features of the temperature and heat flux histories. Validate your model by comparing predictions with the results from part (b). *Note*: Use of both *FEHT* and *IHT* requires creation of a look-up data table for prescribing the heat flux as a function of time.

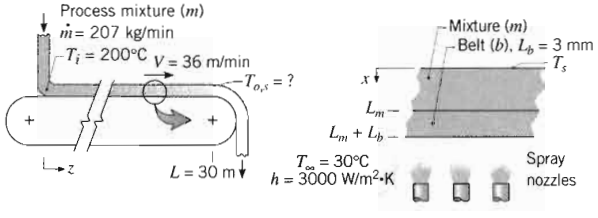
5.119 Heat transfer is not an intuitive process muses the Curious Cook. Does doubling the thickness of a hamburger approximately double the cooking time? What effect does the initial temperature have on cooking time? To answer these questions, develop a model to do virtual cooking of meat of thickness 2L in a double-sided grill. The meat is initially at 20°C when it is placed in the grill and both sides experience convection heat transfer characterized by an ambient temperature of 100°C and a convection coefficient of 5000 W/m$^2 \cdot$ K. Assume the meat to have the properties of liquid water at 300 K and to be properly cooked when the center temperature is 60°C.

(a) For hamburgers of thickness 2L = 10, 20, and 30 mm, calculate the time for the center to reach the required cooking temperature of 60°C. Determine a relationship between the cooking time and the thickness. For your solution, use the finite-element method of *FEHT*, the ready-to-solve model in the *Models/Transient Conduction/Plane Wall* section of *IHT*, or a numerical procedure of your choice. For one of the thicknesses, use an appropriate analytical solution to validate your numerical results.

(b) Without performing a detailed numerical solution, but drawing on the results of part (a), what can you say about the effect on the cooking time of changing the initial temperature of the meat from 20°C to 5°C? You may use your numerical model from part (a) to confirm your assessment.

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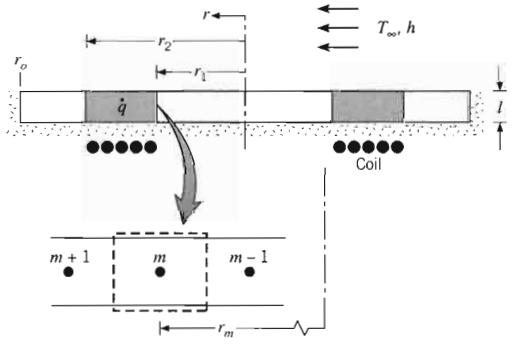
5.120 A process mixture at 200°C flows at a rate of 207 kg/min onto a conveyor belt of 3-mm thickness, 1-m width, and 30-m length traveling with a velocity of 36 m/min. The underside of the belt is cooled by a water spray at a temperature of 30°C, and the convection coefficient is 3000 W/m^2 · K. The thermophysical properties of the process mixture are $\rho_m = 960$ kg/m^3, $c_m = 1700$ J/kg · K, and $k_m = 1.5$ W/m · K, while the properties for the conveyor (metallic) belt are $\rho_b = 8000$ kg/m^3, $c_b = 460$ J/kg · K, and $k_b = 15$ W/m · K.



Using the finite-difference method of *IHT* ($\Delta x = 0.5$ mm, $\Delta t = 0.05$ s), the finite-element method of *FEHT*, or a numerical procedure of your choice, calculate the surface temperature of the mixture at the end of the conveyor belt, $T_{o,s}$. Assume negligible heat transfer to the ambient air by convection or by radiation to the surroundings.

Finite-Difference Equations: Cylindrical Coordinates

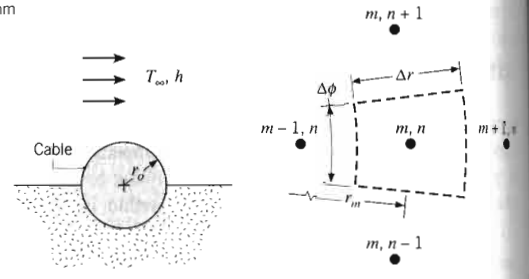
5.121 A thin circular disk is subjected to induction heating from a coil, the effect of which is to provide a uniform heat generation within a ring section as shown. Convection occurs at the upper surface, while the lower surface is well insulated.



(a) Derive the transient, finite-difference equation for node m, which is within the region subjected to induction heating.

(b) On T–r coordinates sketch, in a qualitative manner, the steady-state temperature distribution, identifying important features.

5.122 An electrical cable, experiencing a uniform volumetric generation $\dot{q}$, is half buried in an insulating material while the upper surface is exposed to a convection process (T_∞, h).



(a) Derive the explicit, finite-difference equations for an interior node (m, n), the center node ($m = 0$), and the outer surface nodes (M, n) for the convection and insulated boundaries.

(b) Obtain the stability criterion for each of the finite-difference equations. Identify the most restrictive criterion.

Finite-Difference Solutions: Two-Dimensional Systems

5.123 Two very long (in the direction normal to the page) bars having the prescribed initial temperature distributions are to be soldered together. At time $t = 0$, the $m = 3$ face of the copper (pure) bar contacts the $m = 4$ face of the steel (AISI 1010) bar. The solder and flux act as an interfacial layer of negligible thickness and effective contact resistance $R''_{t,c} = 2 \times 10^{-5}$ m^2 · K/W.

Initial Temperatures (K)

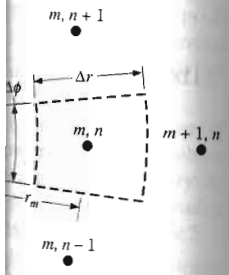
n/m	1	2	3	4	5	6
1	700	700	700	1000	900	800
2	700	700	700	1000	900	800
3	700	700	700	1000	900	800

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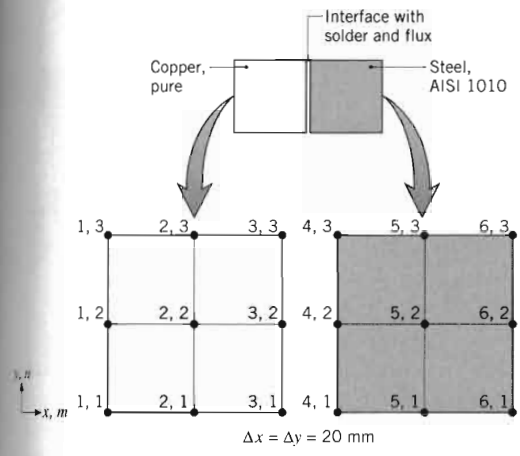


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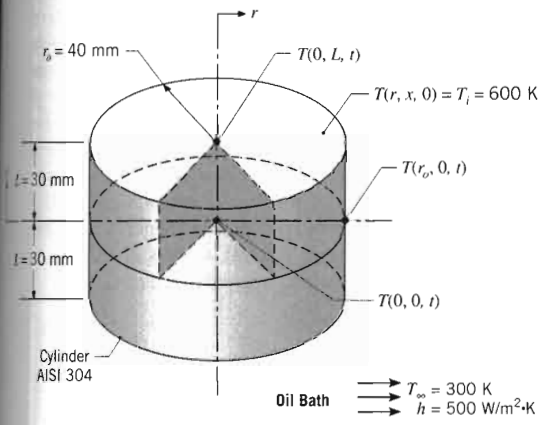
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1000	900	800
1000	900	800



(a) Derive the explicit, finite-difference equation in terms of Fo and $Bi_c = \Delta x/kR''_{t,c}$ for $T_{4,2}$ and determine the corresponding stability criterion.

(b) Using $Fo = 0.01$, determine $T_{4,2}$ one time step after contact is made. What is Δt? Is the stability criterion satisfied?

5.124 In a manufacturing process, stainless steel cylinders (AISI 304) initially at 600 K are quenched by submersion in an oil bath maintained at 300 K with $h = 500$ W/m² · K. Each cylinder is of length $2L = 60$ mm and diameter $D = 80$ mm. Use the *ready-to-solve* model in the *Examples* menu of *FEHT* to obtain the following solutions.



(a) Calculate the temperatures, $T(r, x, t)$, after 3 min at the cylinder center, $T(0, 0, 3$ min), at the center of a circular face, $T(0, L, 3$ min), and the midheight of the side, $T(r_o, 0, 3$ min).

(b) Plot the temperature history at the center, $T(0, 0, t)$, and at the midheight of the side, $T(r_o, 0, t)$, for $0 \le t \le 10$ min using the *View/Temperatures vs. Time* command. Comment on the gradients occurring at these locations and what effect they might have on phase transformations and thermal stresses.

(c) Having solved the model for a total integration time of 10 min in part (b), now use the *View/Temperature Contours* command with the shaded band option for the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the cylinder cools during the quench process. Describe the major features of the cooling process revealed by this display. Use other options of this command to create a 10-isotherm temperature distribution for $t = 3$ min.

(d) For the location of part (a), calculate the temperatures after 3 min if the convection coefficient is doubled ($h = 1000$ W/m² · K). Also, for convection coefficients of 500 and 1000 W/m² · K, determine how long the cylinder needs to remain in the oil bath to achieve a *safe-to-touch* surface temperature of 316 K. Tabulate and comment on the results of parts (a) and (d).

5.125 Consider the system of Problem 4.55. Initially with no flue gases flowing, the walls ($\alpha = 5.5 \times 10^{-7}$ m²/s) are at a uniform temperature of 25°C. Using the implicit, finite-difference method with a time increment of 1 h, find the temperature distribution in the wall 5, 10, 50, and 100 h after introduction of the flue gases.

5.126 Consider the system of Problem 4.73. Initially, the ceramic plate ($\alpha = 1.5 \times 10^{-6}$ m²/s) is at a uniform temperature of 30°C, and suddenly the electrical heating elements are energized. Using the implicit, finite-difference method, estimate the time required for the difference between the surface and initial temperatures to reach 95% of the difference for steady-state conditions. Use a time increment of 2 s.

5.127 Consider the thermal conduction module and operating conditions of Problem 4.76. To evaluate the transient response of the cold plate, which has a thermal diffusivity of $\alpha = 75 \times 10^{-6}$ m²/s, assume that, when the module is activated at $t = 0$, the initial temperature of the cold plate is $T_i = 15$°C and a uniform heat flux of $q''_o = 10^5$ W/m² is applied at its base. Using the implicit finite-difference method and a time increment of $\Delta t = 0.1$ s, compute the designated nodal temperatures as a function of time. From the temperatures

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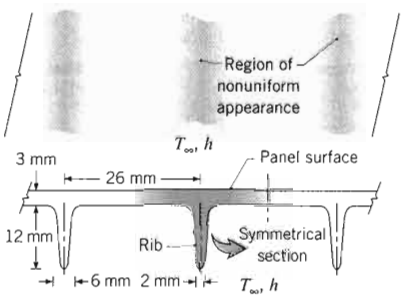
computed at a particular time, evaluate the ratio of the rate of heat transfer by convection to the water to the heat input at the base. Terminate the calculations when this ratio reaches 0.99. Print the temperature field at 5-s intervals and at the time for which the calculations are terminated.

5.128 The operations manager for a metals processing plant anticipates the need to repair a large furnace and has come to you for an estimate of the time required for the furnace interior to cool to a safe working temperature. The furnace is cubical with a 16-m interior dimension and 1-m thick walls for which $\rho = 2600$ kg/m^3, $c = 960$ J/kg · K, and $k = 1$ W/m · K. The operating temperature of the furnace is 900°C, and the outer surface experiences convection with ambient air at 25°C and a convection coefficient of 20 W/m^2 · K.

(a) Use a numerical procedure to estimate the time required for the inner surface of the furnace to cool to a safe working temperature of 35°C. *Hint*: Consider a two-dimensional cross-section of the furnace, and perform your analysis on the smallest symmetrical section.

(b) Anxious to reduce the furnace downtime, the operations manager also wants to know what effect circulating ambient air through the furnace would have on the cool-down period. Assume equivalent convection conditions for the inner and outer surfaces.

5.129 The door panel of an automobile is fabricated by a plastic hot-extrusion process resulting in the ribbed cross section shown schematically. Following a process involving air-cooling, painting, and baking, the panel is ready for assembly on the vehicle. However, upon visual inspection, the rib pattern is evident on the outer surface. In regions over the ribs, the paint has an "orange peel" appearance, making the door panel unacceptable for use. The apparent reason for this defect is the variable microfinish caused by differential cooling rates at the surface of the panel, which affect adherence of the paint. Your assignment is to estimate the panel surface temperature distribution as a function of cooling time.



Use the finite-element software *FEHT* to obtain the temperature distribution in the symmetrical section shown schematically. The panel is ejected from the extrusion press at a uniform temperature of 275°C and is allowed to cool on a transport table, where the air temperature is 25°C and the convection coefficient is 10 W/m^2 · K. The thermophysical properties of the extruded plastic are $\rho = 1050$ kg/m^3, $c = 800$ J/kg · K, and $k = 0.5$ W/m · K.

(a) Using the *View/Temperature vs. Time* command, plot temperature histories for selected locations on the panel surface. Is there noticeable differential cooling in the region above the rib? If so, what is the cause? *Hint*: When you draw the shape outline in *FEHT*, represent the fillet region by six or seven segments that approximate the fillet radius. Then draw element lines to form triangular elements.

(b) Use the *View/Temperature Contours* command with the shaded band option to plot the isotherm contours. Select the *From Start to Stop* time option, and view the temperature contours as the panel cools. Describe major features of the cooling process observed from the display. Use other options of this command to create a 10-isotherm temperature distribution for a time that illustrates some of the foregoing important features. How would you redesign the ribbed panel to reduce this thermally induced defect, while still retaining the stiffening function required of the ribs?

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T_∞, h

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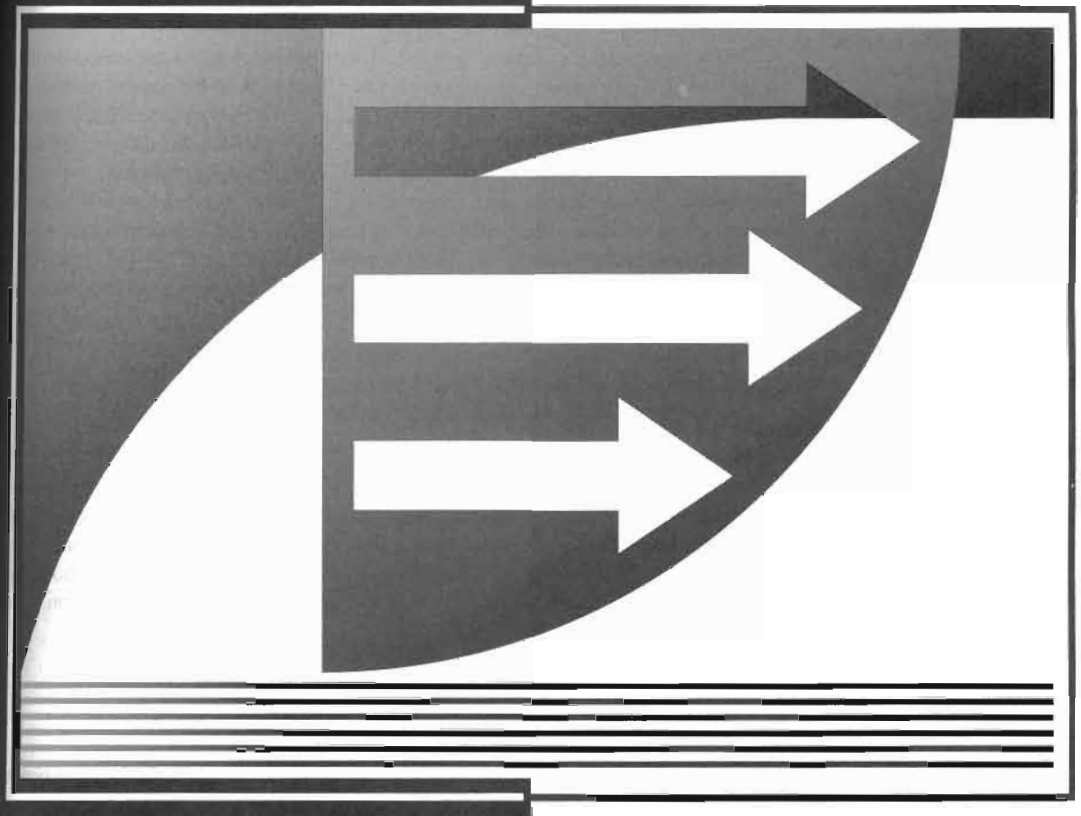
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CHAPTER 6

Introduction to Convection



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Thus far we have focused on heat transfer by conduction and have considered convection only to the extent that it provides a possible boundary condition for conduction problems. In Section 1.2.2 we used the term *convection* to describe energy transfer between a surface and a fluid moving over the surface. Convection includes energy transfer by both the bulk fluid motion (advection) and the random motion of fluid molecules (conduction or diffusion).

In our treatment of convection, we have two major objectives. In addition to obtaining an understanding of the physical mechanisms that underlie convection transfer, we wish to develop the means to perform convection transfer calculations. This chapter and the supplementary material of Appendix D are devoted primarily to achieving the former objective. Physical origins are discussed, and relevant dimensionless parameters, as well as important analogies, are developed.

A unique feature of this chapter is the manner in which convection mass transfer effects are introduced by analogy to those of convection heat transfer. In mass transfer by convection, gross fluid motion combines with diffusion to promote the transfer of a species for which there exists a concentration gradient. In this text, we focus on convection mass transfer that occurs at the surface of a volatile solid or liquid due to motion of a gas over the surface.

With conceptual foundations established, subsequent chapters are used to develop useful tools for quantifying convection effects. Chapters 7 and 8 present methods for computing the coefficients associated with forced convection in external and internal flow configurations, respectively. Chapter 9 describes methods for determining these coefficients in free convection, and Chapter 10 considers the problem of convection with phase change (boiling and condensation). Chapter 11 develops methods for designing and evaluating the performance of heat exchangers, devices that are widely used in engineering practice to effect heat transfer between fluids.

Accordingly, we begin by developing our understanding of the nature of convection.

6.1 *The Convection Boundary Layers*

The concept of boundary layers is central to the understanding of convection heat and mass transfer between a surface and a fluid flowing past it. In this section, velocity, thermal, and concentration boundary layers are described, and their relationships to the friction coefficient, convection heat transfer coefficient, and convection mass transfer coefficient are introduced.

6.1.1 The Velocity Boundary Layer

To introduce the concept of a boundary layer, consider flow over the flat plate of Figure 6.1. When fluid particles make contact with the surface, they assume zero velocity. These particles then act to retard the motion of particles in the adjoining fluid layer, which act to retard the motion of particles in the next layer, and so on until, at a distance $y = \delta$ from the surface, the effect becomes negligible. This retardation of fluid motion is

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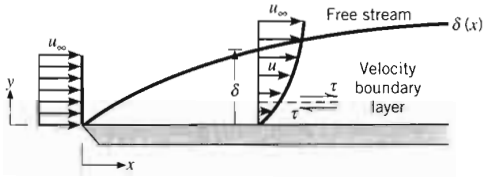


FIGURE 6.1
Velocity boundary layer development on a flat plate.

associated with *shear stresses* τ acting in planes that are parallel to the fluid velocity (Figure 6.1). With increasing distance y from the surface, the x velocity component of the fluid, u, must then increase until it approaches the free stream value u_∞. The subscript ∞ is used to designate conditions in the *free stream* outside the boundary layer.

The quantity δ is termed the *boundary layer thickness*, and it is typically defined as the value of y for which $u = 0.99u_\infty$. The *boundary layer velocity profile* refers to the manner in which u varies with y through the boundary layer. Accordingly, the fluid flow is characterized by two distinct regions, a thin fluid layer (the boundary layer) in which velocity gradients and shear stresses are large and a region outside the boundary layer in which velocity gradients and shear stresses are negligible. With increasing distance from the leading edge, the effects of viscosity penetrate further into the free stream and the boundary layer grows (δ increases with x).

Because it pertains to the fluid velocity, the foregoing boundary layer may be referred to more specifically as the *velocity boundary layer*. It develops whenever there is fluid flow over a surface, and it is of fundamental importance to problems involving convection transport. In fluid mechanics its significance to the engineer stems from its relation to the surface shear stress τ_s, and hence to surface frictional effects. For external flows it provides the basis for determining the local *friction coefficient*

$$C_f \equiv \frac{\tau_s}{\rho u_\infty^2/2} \tag{6.1}$$

a key dimensionless parameter from which the surface frictional drag may be determined. Assuming a *Newtonian fluid*, the surface shear stress may be evaluated from knowledge of the velocity gradient at the surface

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{6.2}$$

where μ is a fluid property known as the *dynamic viscosity*. In a velocity boundary layer, the velocity gradient at the surface depends on the distance x from the leading edge of the plate. Therefore, the surface shear stress and friction coefficient also depend on x.

6.1.2 The Thermal Boundary Layer

Just as a velocity boundary layer develops when there is fluid flow over a surface, a *thermal boundary layer* must develop if the fluid free stream and surface temperatures differ. Consider flow over an isothermal flat plate (Figure 6.2). At the leading edge the *temperature profile* is uniform, with $T(y) = T_\infty$. However, fluid particles that come into contact with the plate achieve thermal equilibrium at the plate's surface

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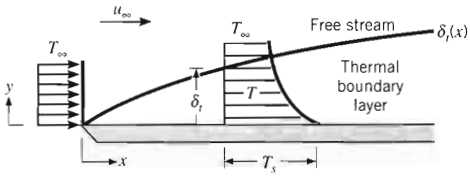


FIGURE 6.2
Thermal boundary layer development on an isothermal flat plate.

temperature. In turn, these particles exchange energy with those in the adjoining fluid layer, and temperature gradients develop in the fluid. The region of the fluid in which these temperature gradients exist is the thermal boundary layer, and its thickness δ_t is typically defined as the value of y for which the ratio $[(T_s - T)/(T_s - T_\infty)] = 0.99$. With increasing distance from the leading edge, the effects of heat transfer penetrate further into the free stream and the thermal boundary layer grows.

The relation between conditions in this boundary layer and the convection heat transfer coefficient may readily be demonstrated. At any distance x from the leading edge, the *local* surface heat flux may be obtained by applying Fourier's law to the *fluid* at $y = 0$. That is,

$$q''_s = -k_f \left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{6.3}$$

The subscript s has been used to emphasize that this is the surface heat flux, but it will be dropped in later sections. This expression is appropriate because, *at the surface, there is no fluid motion and energy transfer occurs only by conduction*. Recalling Newton's law of cooling, we see that

$$q''_s = h(T_s - T_\infty) \tag{6.4}$$

and combining this with Equation 6.3, we obtain

$$h = \frac{-k_f \, \partial T/\partial y|_{y=0}}{T_s - T_\infty} \tag{6.5}$$

Hence, conditions in the thermal boundary layer, which strongly influence the wall temperature gradient $\partial T/\partial y|_{y=0}$, determine the rate of heat transfer across the boundary layer. Since $(T_s - T_\infty)$ is a constant, independent of x, while δ_t increases with increasing x, temperature gradients in the boundary layer must decrease with increasing x. Accordingly, the magnitude of $\partial T/\partial y|_{y=0}$ decreases with increasing x, and it follows that q''_s and h decrease with increasing x.

6.1.3 The Concentration Boundary Layer

When air moves past the surface of a pool of water, the liquid water will evaporate, and water vapor will be transferred into the air stream. This is an example of convection mass transfer. More generally, consider a *binary mixture* of chemical species A and B that flows over a surface (Figure 6.3). The molar concentration (kmol/m^3) of species A at the surface is $C_{A,s}$, and in the free stream it is $C_{A,\infty}$. If $C_{A,s}$ differs from $C_{A,\infty}$, transfer of species A by convection will occur. For example, species A could

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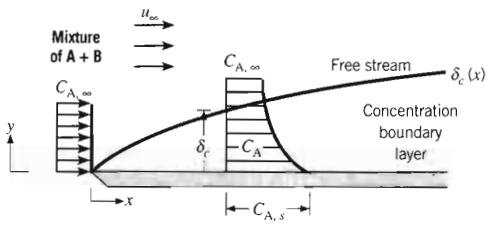


FIGURE 6.3
Species concentration boundary layer development on a flat plate.

be a vapor that is transferred into a gas stream (species B) due to *evaporation* at a liquid surface (as in the water example) or due to *sublimation* at a solid surface. In this situation, a *concentration boundary layer* will develop that is similar to the velocity and thermal boundary layers. The concentration boundary layer is the region of the fluid in which concentration gradients exist, and its thickness δ_c is typically defined as the value of y for which $[(C_{A,s} - C_A)/(C_{A,s} - C_{A,\infty})] = 0.99$. With increasing distance from the leading edge, the effects of species transfer penetrate further into the free stream and the concentration boundary layer grows.

Species transfer by convection between the surface and the free stream fluid is determined by conditions in the boundary layer, and we are interested in determining the rate at which this transfer occurs. In particular, we are interested in the molar flux of species A, N''_A (kmol/s · m^2). It is helpful to recognize that the molar flux associated with species transfer *by diffusion* is determined by an expression that is analogous to Fourier's law. For the conditions of interest in this chapter, the expression, which is termed *Fick's law*, has the form

$$N''_A = -D_{AB}\frac{\partial C_A}{\partial y} \tag{6.6}$$[1]

where D_{AB} is a property of the binary mixture known as the *binary diffusion coefficient*. At any point corresponding to $y > 0$ in the concentration boundary layer of Figure 6.3, species transfer is due to both bulk fluid motion (*advection*) and diffusion. However, at $y = 0$ there is no fluid motion (neglecting the often small velocity normal to the surface caused by the species transfer process itself, as is discussed in Chapter 14), and thus species transfer is by diffusion only. Applying Fick's law at $y = 0$, we see that the molar flux at the surface at any distance from the leading edge is then

$$N''_{A,s} = -D_{AB}\left.\frac{\partial C_A}{\partial y}\right|_{y=0} \tag{6.7}$$

The subscript s has been used to emphasize that this is the molar flux at the surface, but it will be dropped in later sections. Analogous to Newton's law of cooling, an equation can be written that relates the molar flux to the concentration difference across the boundary layer, as

$$N''_{A,s} = h_m(C_{A,s} - C_{A,\infty}) \tag{6.8}$$

[1]This expression is an approximation of a more general form of Fick's law of diffusion (Section 14.1.3) when the total molar concentration of the mixture, $C = C_A + C_B$, is a constant.

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where h_m (m/s) is the *convection mass transfer coefficient*, analogous to the convection heat transfer coefficient. Combining Equations 6.7 and 6.8, it follows that

$$h_m = \frac{-D_{AB}\partial C_A/\partial y|_{y=0}}{C_{A,s} - C_{A,\infty}} \tag{6.9}$$

Therefore, conditions in the concentration boundary layer, which strongly influence the surface concentration gradient $\partial C_A/\partial y|_{y=0}$, also influence the convection mass transfer coefficient and hence the rate of species transfer in the boundary layer.

6.1.4 Significance of the Boundary Layers

For flow over any surface, there will always exist a velocity boundary layer and hence surface friction. Likewise, a thermal boundary layer, and hence convection heat transfer, will always exist if the surface and free stream temperatures differ. Similarly, a concentration boundary layer and convection mass transfer will exist if the surface concentration of a species differs from its free stream concentration. The velocity boundary layer is of extent $\delta(x)$ and is characterized by the presence of velocity gradients and shear stresses. The thermal boundary layer is of extent $\delta_t(x)$ and is characterized by temperature gradients and heat transfer. Finally, the concentration boundary layer is of extent $\delta_c(x)$ and is characterized by concentration gradients and species transfer. Situations can arise in which all three boundary layers are present. In such cases, the boundary layers rarely grow at the same rate, and the values of δ, δ_t, and δ_c at a given location are not the same.

For the engineer, the principal manifestations of the three boundary layers are, respectively, *surface friction*, *convection heat transfer*, and *convection mass transfer*. The key boundary layer parameters are then the *friction coefficient* C_f and the *heat* and *mass transfer convection coefficients* h and h_m, respectively. We now turn our attention to examining these key parameters, which are central to the analysis of convection heat and mass transfer problems.

6.2 *Local and Average Convection Coefficients*

6.2.1 Heat Transfer

Consider the conditions of Figure 6.4*a*. A fluid of velocity V and temperature T_∞ flows over a surface of arbitrary shape and of area A_s. The surface is presumed to be at a uniform temperature, T_s, and if $T_s \neq T_\infty$, we know that convection heat transfer will occur. From Section 6.1.2, we also know that the surface heat flux and convection heat transfer coefficient both vary along the surface. The *total heat transfer rate* q may be obtained by integrating the local flux over the entire surface. That is,

$$q = \int_{A_s} q'' dA_s \tag{6.10}$$

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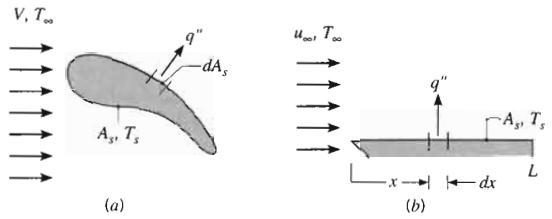


FIGURE 6.4 Local and total convection heat transfer. (*a*) Surface of arbitrary shape. (*b*) Flat plate.

or, from Equation 6.4,

$$q = (T_s - T_\infty)\int_{A_s} h dA_s \tag{6.11}$$

Defining an *average convection coefficient* $\bar{h}$ for the entire surface, the total heat transfer rate may also be expressed as

$$q = \bar{h} A_s (T_s - T_\infty) \tag{6.12}$$

Equating Equations 6.11 and 6.12, it follows that the average and local convection coefficients are related by an expression of the form

$$\bar{h} = \frac{1}{A_s}\int_{A_s} h dA_s \tag{6.13}$$

Note that for the special case of flow over a flat plate (Figure 6.4*b*), h varies only with the distance x from the leading edge and Equation 6.13 reduces to

$$\bar{h} = \frac{1}{L}\int_0^L h dx \tag{6.14}$$

6.2.2 Mass Transfer

Similar results may be obtained for convection mass transfer. If a fluid of species molar concentration $C_{A,\infty}$ flows over a surface at which the species concentration is maintained at some uniform value $C_{A,s} \neq C_{A,\infty}$ (Figure 6.5*a*), transfer of the species by convection will occur. From Section 6.1.3, we know that the surface molar flux and convection mass transfer coefficient both vary along the surface. The total molar transfer rate for an entire surface, N_A (kmol/s), may then be expressed as

$$N_A = \bar{h}_m A_s (C_{A,s} - C_{A,\infty}) \tag{6.15}$$

where the average and local mass transfer convection coefficients are related by an equation of the form

$$\bar{h}_m = \frac{1}{A_s}\int_{A_s} h_m dA_s \tag{6.16}$$

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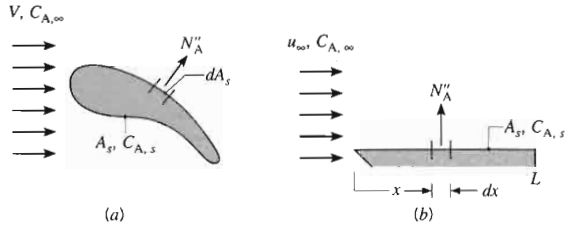


FIGURE 6.5 Local and total convection species transfer. (*a*) Surface of arbitrary shape. (*b*) Flat plate.

For the flat plate of Figure 6.5*b*, it follows that

$$\bar{h}_m = \frac{1}{L}\int_0^L h_m dx \tag{6.17}$$

Species transfer may also be expressed as a mass flux, n''_A (kg/s · m^2), or as a mass transfer rate, n_A (kg/s), by multiplying both sides of Equations 6.8 and 6.15, respectively, by the molecular weight $\mathcal{M}_A$ (kg/kmol) of species A. Accordingly,

$$n''_A = h_m(\rho_{A,s} - \rho_{A,\infty}) \tag{6.18}$$

and

$$n_A = \bar{h}_m A_s(\rho_{A,s} - \rho_{A,\infty}) \tag{6.19}$$

where ρ_A (kg/m^3) is the mass density of species A.[2] We can also write Fick's law on a mass basis by multiplying Equation 6.7 by $\mathcal{M}_A$ to yield

$$n''_{A,s} = -D_{AB}\left.\frac{\partial \rho_A}{\partial y}\right|_{y=0} \tag{6.20}$$

Furthermore, multiplying the numerator and denominator of Equation 6.9 by $\mathcal{M}_A$ yields an alternative expression for h_m:

$$h_m = \frac{-D_{AB}\partial\rho_A/\partial y|_{y=0}}{\rho_{A,s} - \rho_{A,\infty}} \tag{6.21}$$

To perform a convection mass transfer calculation, it is necessary to determine the value of $C_{A,s}$ or $\rho_{A,s}$. Such a determination may be made by assuming thermodynamic equilibrium at the interface between the gas and the liquid or solid phase. One implication of equilibrium is that the temperature of the vapor at the interface is equal to the surface temperature T_s. A second implication is that the vapor is in a *saturated state*, in which case thermodynamic tables, such as Table A.6 for water, may be used to obtain its density from knowledge of T_s. To a good approximation, the molar concentration of the vapor at the surface may also be determined from the vapor pressure through application of the equation of state for an ideal gas. That is,

[2]Although the foregoing nomenclature is well suited for characterizing mass transfer processes of interest in this text, there is by no means a standard nomenclature, and it is often difficult to reconcile the results from different publications. A review of the different ways in which driving potentials, fluxes, and convection coefficients may be formulated is provided by Webb [1].

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$$C_{A,s} = \frac{p_{sat}(T_s)}{\mathcal{R}T_s} \tag{6.22}$$

where $\mathcal{R}$ is the universal gas constant and $p_{sat}(T_s)$ is the vapor pressure corresponding to saturation at T_s. Note that the vapor mass density and molar concentration are related by $\rho_A = \mathcal{M}_A C_A$.

6.2.3 The Problem of Convection

The local flux and/or the total transfer rate are of paramount importance in any convection problem. These quantities may be determined from the rate equations, Equations 6.4, 6.8, 6.12, and 6.15, which depend on knowledge of the local (h or h_m) and average ($\bar{h}$ or $\bar{h}_m$) convection coefficients. It is for this reason that determination of these coefficients is viewed as *the problem of convection*. However, the problem is not a simple one, for in addition to depending on numerous *fluid properties* such as density, viscosity, thermal conductivity, and specific heat, the coefficients depend on the *surface geometry* and the *flow conditions*. This multiplicity of independent variables is attributable to the dependence of convection transfer on the boundary layers that develop on the surface.

EXAMPLE 6.1

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

where a is a coefficient (W/m$^{1.9}$ · K) and x (m) is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ for a plate of length x to the local heat transfer coefficient h_x at x.
2. Show, in a qualitative manner, the variation of h_x and $\bar{h}_x$ as a function of x.

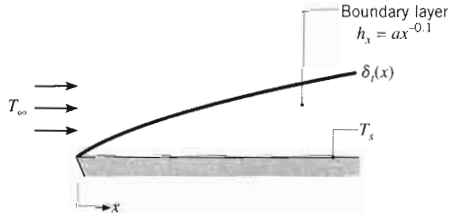
SOLUTION

Known: Variation of the local heat transfer coefficient, $h_x(x)$.

Find:

1. The ratio of the average heat transfer coefficient $\bar{h}(x)$ to the local value $h_x(x)$.
2. Sketch of the variation of h_x and $\bar{h}_x$ with x.

Schematic:



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Analysis:

1. From Equation 6.14 the average value of the convection heat transfer coefficient over the region from 0 to x is

$$\bar{h}_x = \bar{h}_x(x) = \frac{1}{x}\int_0^x h_x(x)\,dx$$

Substituting the expression for the local heat transfer coefficient

$$h_x(x) = ax^{-0.1}$$

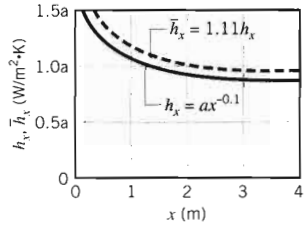
and integrating, we obtain

$$\bar{h}_x = \frac{1}{x}\int_0^x ax^{-0.1}\,dx = \frac{a}{x}\int_0^x x^{-0.1}\,dx = \frac{a}{x}\left(\frac{x^{+0.9}}{0.9}\right) = 1.11ax^{-0.1}$$

or

$$\bar{h}_x = 1.11h_x$$

2. The variation of h_x and $\bar{h}_x$ with x is as follows:



Comments: Boundary layer development causes both the local and average coefficients to decrease with increasing distance from the leading edge. The average coefficient up to x must therefore exceed the local value at x.

EXAMPLE 6.2

A long circular cylinder 20 mm in diameter is fabricated from solid naphthalene, a common moth repellant, and is exposed to an airstream that provides for an average convection mass transfer coefficient of $\bar{h}_m = 0.05$ m/s. The molar concentration of naphthalene vapor at the cylinder surface is 5×10^{-6} kmol/m^3, and its molecular weight is 128 kg/kmol. What is the mass sublimation rate per unit length of cylinder?

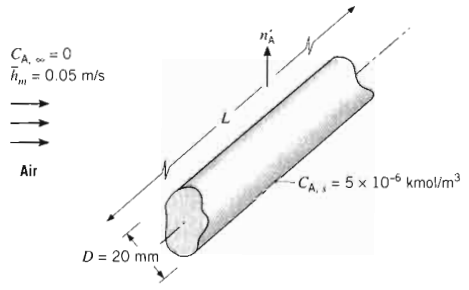
SOLUTION

Known: Saturated vapor concentration of naphthalene.

Find: Sublimation rate per unit length, n'_A (kg/s · m).

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Schematic:



Assumptions:

1. Steady-state conditions.
2. Negligible concentration of naphthalene in free stream of air.

Analysis: Naphthalene is transported to the air by convection, and from Equation 6.15, the molar transfer rate for the cylinder is

$$N_A = \bar{h}_m \pi DL(C_{A,s} - C_{A,\infty})$$

With $C_{A,\infty} = 0$ and $N'_A = N_A/L$, it follows that

$$N'_A = (\pi D)\bar{h}_m C_{A,s} = \pi \times 0.02\ \text{m} \times 0.05\ \text{m/s} \times 5 \times 10^{-6}\ \text{kmol/m}^3$$

$$N'_A = 1.57 \times 10^{-8}\ \text{kmol/s} \cdot \text{m}$$

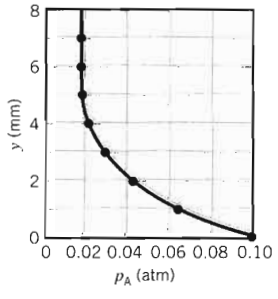
The mass sublimation rate is then

$$n'_A = \mathcal{M}_A N'_A = 128\ \text{kg/kmol} \times 1.57 \times 10^{-8}\ \text{kmol/s} \cdot \text{m}$$

$$n'_A = 2.01 \times 10^{-6}\ \text{kg/s} \cdot \text{m}$$ ◁

EXAMPLE 6.3

At some location on the surface of a pan of water, measurements of the partial pressure of water vapor p_A (atm) are made as a function of the distance y from the surface, and the results are as follows:



Determine the convection mass transfer coefficient $h_{m,x}$ at this location.

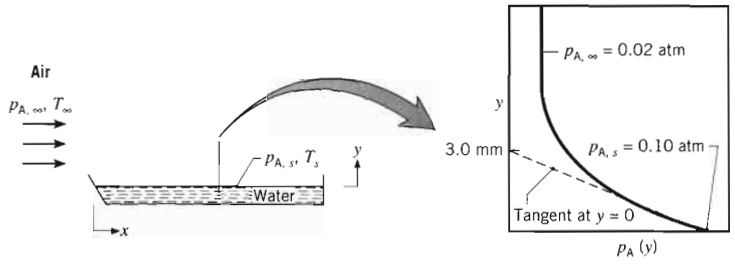
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SOLUTION

Known: Partial pressure p_A of water vapor as a function of distance y at a particular location on the surface of a water layer.

Find: Convection mass transfer coefficient at the prescribed location.

Schematic:



Assumptions:

1. Water vapor may be approximated as an ideal gas.
2. Conditions are isothermal.

Properties: Table A.6, saturated vapor (0.1 atm = 0.101 bar): T_s = 319 K. Table A.8, water vapor–air (319 K): D_{AB} (319 K) = D_{AB} (298 K) × (319 K/298 K)$^{3/2}$ = 0.288×10^{-4} m^2/s.

Analysis: From Equation 6.21 the local convection mass transfer coefficient is

$$h_{m,x} = \frac{-D_{AB}\,\partial\rho_A/\partial y\big|_{y=0}}{\rho_{A,s} - \rho_{A,\infty}}$$

or, approximating the vapor as an ideal gas

$$p_A = \rho_A RT$$

with constant T (isothermal conditions),

$$h_{m,x} = \frac{-D_{AB}\,\partial p_A/\partial y\big|_{y=0}}{p_{A,s} - p_{A,\infty}}$$

From the measured vapor pressure distribution

$$\left.\frac{\partial p_A}{\partial y}\right|_{y=0} = \frac{(0 - 0.1)\text{ atm}}{(0.003 - 0)\text{ m}} = -33.3\text{ atm/m}$$

Hence

$$h_{m,x} = \frac{-0.288 \times 10^{-4}\text{ m}^2\text{/s}\,(-33.3\text{ atm/m})}{(0.1 - 0.02)\text{ atm}} = 0.0120\text{ m/s}$$

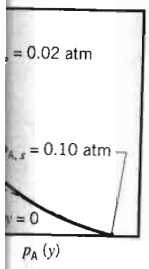
Comments: From thermodynamic equilibrium at the liquid–vapor interface, the interfacial temperature was determined from Table A.6.

6.3 Laminar and

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stance y at a partic-

cation.



T_s = 319 K. Table
19 K/298 K)$^{3/2}$ =

coefficient is

m/s

or interface, the

6.3 *Laminar and Turbulent Flow*

In the discussion of convection so far, we have not addressed the significance of the *flow conditions*. An essential first step in the treatment of any convection problem is to determine whether the boundary layer is *laminar* or *turbulent*. Surface friction and the convection transfer rates depend strongly on which of these conditions exists.

6.3.1 Laminar and Turbulent Velocity Boundary Layers

Boundary layer *development* on a flat plate is illustrated in Figure 6.6. In many cases, laminar and turbulent flow conditions both occur, with the laminar section preceding the turbulent section. For either condition, the fluid motion is characterized by velocity components in the x- and y-directions. Fluid motion away from the surface is necessitated by the slowing of the fluid near the wall as the boundary layer grows in the x-direction. Figure 6.6 shows that there are sharp differences between laminar and turbulent flow conditions, as described in the following paragraphs.

In the laminar boundary layer, the fluid flow is highly ordered and it is possible to identify streamlines along which fluid particles move. From Section 6.1.1 we know that the boundary layer thickness grows and that velocity gradients at $y = 0$ decrease in the streamwise (increasing x) direction. From Equation 6.2, we see that the local surface shear stress τ_s also decreases with increasing x. The highly ordered behavior continues until a *transition* zone is reached, across which a conversion from laminar to turbulent conditions occurs. Conditions within the transition zone change with time, with the flow sometimes exhibiting laminar behavior and sometimes exhibiting the characteristics of turbulent flow.

Flow in the fully turbulent boundary layer is, in general, highly irregular and is characterized by random, three-dimensional motion of relatively large parcels of fluid. Mixing within the boundary layer carries high-speed fluid toward the solid surface and transfers slower-moving fluid farther into the free stream. Much of the

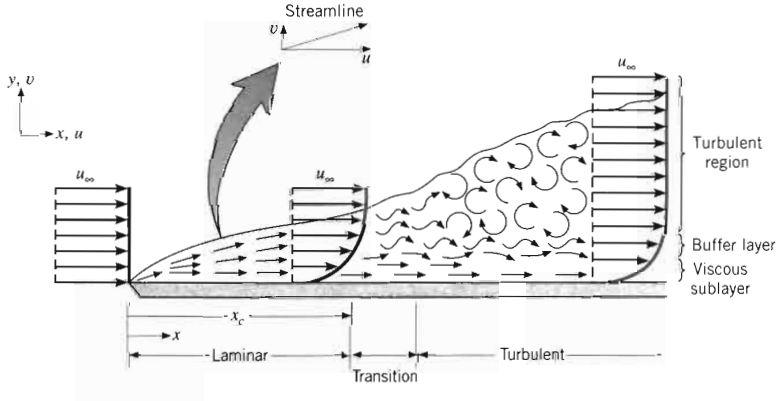


FIGURE 6.6 Velocity boundary layer development on a flat plate.

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mixing is promoted by streamwise vortices called *streaks* that are generated intermittently near the flat plate, where they rapidly grow and decay. Recent analytical and experimental studies have suggested that these and other *coherent structures* within the turbulent flow can travel in *waves* at velocities that can exceed u_∞, interact nonlinearly, and spawn the chaotic conditions that characterize turbulent flow [2].

As a result of the interactions that lead to chaotic flow conditions, velocity and pressure fluctuations occur at any point within the turbulent boundary layer. Three different regions may be delineated within the turbulent boundary layer as a function of distance from the surface. We may speak of a *viscous sublayer* in which transport is dominated by diffusion and the velocity profile is nearly linear. There is an adjoining *buffer layer* in which diffusion and turbulent mixing are comparable, and there is a *turbulent zone* in which transport is dominated by turbulent mixing. A comparison of the laminar and turbulent boundary layer profiles for the x-component of the velocity, provided in Figure 6.7, shows that the turbulent velocity profile is relatively flat due to the mixing that occurs within the buffer layer and turbulent region, giving rise to large velocity gradients within the viscous sublayer. Hence, τ_s is generally larger in the turbulent portion of the boundary layer of Figure 6.6 than in the laminar portion.

The transition from laminar to turbulent flow is ultimately due to *triggering mechanisms*, such as the interaction of unsteady flow structures that develop naturally within the fluid or small disturbances that exist within many typical boundary layers. These disturbances may originate from fluctuations in the free stream, or they may be induced by surface roughness or minute surface vibrations. The onset of turbulence depends on whether the triggering mechanisms are amplified or attenuated in the direction of fluid flow, which in turn depends on a dimensionless grouping of parameters called the *Reynolds number*,

$$Re_x = \frac{\rho u_\infty x}{\mu} \tag{6.23}$$

where, for a flat plate, the characteristic length is x, the distance from the leading edge. It will be shown later that the Reynolds number represents the ratio of the inertia to viscous forces. If the Reynolds number is small, inertia forces are insignificant relative to viscous forces. The disturbances are then dissipated, and the flow remains laminar. For a large Reynolds number, however, the inertia forces can be sufficient to amplify the triggering mechanisms, and a transition to turbulence occurs.

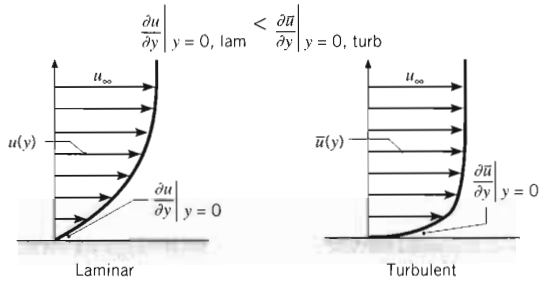


FIGURE 6.7 Comparison of laminar and turbulent velocity boundary layer profiles for the same free stream velocity.[3]

[3]Since velocity fluctuates with time in turbulent flow, the time-averaged velocity, $\bar{u}$, is plotted in Figure 6.7.

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In calculating boundary layer behavior, it is frequently reasonable to assume that transition begins at some location x_c, as shown in Figure 6.6. This location is determined by the *critical* Reynolds number, $Re_{x,c}$. For flow over a flat plate, $Re_{x,c}$ is known to vary from approximately 10^5 to 3×10^6, depending on surface roughness and the turbulence level of the free stream. A representative value of

$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} = 5 \times 10^5 \tag{6.24}$$

is often assumed for boundary layer calculations and, unless otherwise noted, is used for the calculations of this text that involve a flat plate.

6.3.2 Laminar and Turbulent Thermal and Species Concentration Boundary Layers

Since the velocity distribution determines the advective component of thermal energy or chemical species transfer within the boundary layer, the nature of the flow also has a profound effect on convective heat and mass transfer rates. Similar to the laminar velocity boundary layer, the thermal and species boundary layers grow in the streamwise (increasing x) direction, temperature and species concentration gradients in the fluid at $y = 0$ decrease in the streamwise direction, and, from Equations 6.5 and 6.9, the heat and mass transfer coefficients also decrease with increasing x.

Just as it induces large velocity gradients at $y = 0$, turbulent mixing promotes large temperature and species concentration gradients adjacent to the solid surface as well as a corresponding increase in the heat and mass transfer coefficients across the transition region. These effects are illustrated in Figure 6.8 for the velocity boundary layer thickness δ and the local convection heat transfer coefficient h. Because turbulence induces mixing, which in turn reduces the importance of conduction and diffusion in determining the thermal and species boundary layer thicknesses, *differences* in the thicknesses of the velocity, thermal, and species boundary layers tend to be much smaller in turbulent flow than in laminar flow. As is evident in Equation 6.24, the presence of heat and/or mass transfer can affect the location of the transition from laminar to turbulent flow x_c if the density or dynamic viscosity of the fluid is dependent on temperature or species concentration.

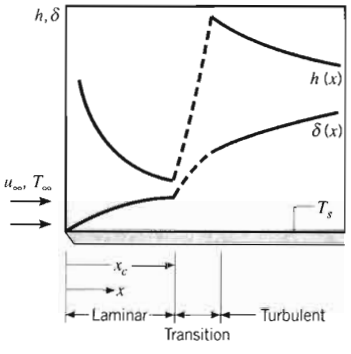


FIGURE 6.8
Variation of velocity boundary layer thickness δ and the local heat transfer coefficient h for flow over an isothermal flat plate.

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EXAMPLE 6.4

Water flows at a velocity $u_\infty = 1$ m/s over a flat plate of length $L = 0.6$ m. Consider two cases, one for which the water temperature is approximately 300 K and the other for an approximate water temperature of 350 K. In the laminar and turbulent regions, experimental measurements show that the local convection coefficients are well described by

$$h_{lam}(x) = C_{lam}x^{-0.5} \qquad h_{turb}(x) = C_{turb}x^{-0.2}$$

where x has units of m. At 300 K,

$$C_{lam,300} = 395 \text{ W/m}^{1.5} \cdot \text{K} \qquad C_{turb,300} = 2330 \text{ W/m}^{1.8} \cdot \text{K}$$

while at 350 K,

$$C_{lam,350} = 477 \text{ W/m}^{1.5} \cdot \text{K} \qquad C_{turb,350} = 3600 \text{ W/m}^{1.8} \cdot \text{K}$$

As is evident, the constant, C, depends on the nature of the flow as well as the water temperature because of the thermal dependence of various properties of the fluid.

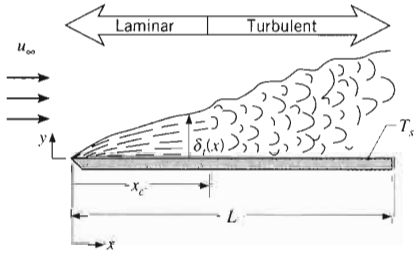
Determine the average convection coefficient, $\bar{h}$, over the entire plate for the two water temperatures.

SOLUTION

Known: Water flow over a flat plate; expressions for the dependence of the local convection coefficient with distance from the plate's leading edge, x; and approximate temperature of the water.

Find: Average convection coefficient, $\bar{h}$.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Transition occurs at a critical Reynolds number of $Re_{x,c} = 5 \times 10^5$.

Properties: Table A.6, water ($\bar{T} \approx 300$ K): $\rho = v_f^{-1} = 997$ kg/m^3, $\mu = 855 \times 10^{-6}$ N · s/m^2. Table A.6 ($\bar{T} \approx 350$ K): $\rho = v_f^{-1} = 974$ kg/m^3, $\mu = 365 \times 10^{-6}$ N · s/m^2.

Analysis: The local convection coefficient is highly dependent on whether laminar or turbulent conditions exist. Therefore, we first determine the extent to which these conditions exist by finding the location where transition occurs, x_c. From Equation 6.24, we know that at 300 K,

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$$x_c = \frac{Re_{x,c}\mu}{\rho u_\infty} = \frac{5 \times 10^5 \times 855 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2}{997\ \text{kg/m}^3 \times 1\ \text{m/s}} = 0.43\ \text{m}$$

while at 350 K,

$$x_c = \frac{Re_{x,c}\mu}{\rho u_\infty} = \frac{5 \times 10^5 \times 365 \times 10^{-6}\ \text{N} \cdot \text{s/m}^2}{974\ \text{kg/m}^3 \times 1\ \text{m/s}} = 0.19\ \text{m}$$

From Equation 6.14 we know that

$$\bar{h} = \frac{1}{L}\int_0^L h\,dx = \frac{1}{L}\left[\int_0^{x_c} h_{\text{lam}}\,dx + \int_{x_c}^L h_{\text{turb}}\,dx\right]$$

or

$$\bar{h} = \frac{1}{L}\left[\frac{C_{\text{lam}}}{0.5}x^{0.5}\Big|_0^{x_c} + \frac{C_{\text{turb}}}{0.8}x^{0.8}\Big|_{x_c}^{L}\right]$$

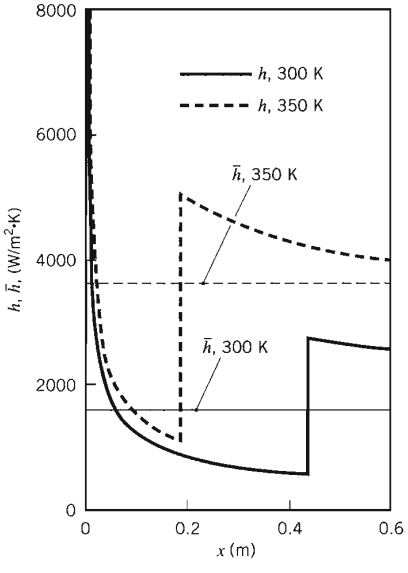
At 300 K,

$$\bar{h} = \frac{1}{0.6\ \text{m}}\left[\frac{395\ \text{W/m}^{1.5} \cdot \text{K}}{0.5} \times (0.43^{0.5})\ \text{m}^{0.5} + \frac{2330\ \text{W/m}^{1.8} \cdot \text{K}}{0.8} \times (0.6^{0.8} - 0.43^{0.8})\ \text{m}^{0.8}\right] = 1620\ \text{W/m}^2 \cdot \text{K} \quad \triangleleft$$

while at 350 K,

$$\bar{h} = \frac{1}{0.6\ \text{m}}\left[\frac{477\ \text{W/m}^{1.5} \cdot \text{K}}{0.5} \times (0.19^{0.5})\ \text{m}^{0.5} + \frac{3600\ \text{W/m}^{1.8} \cdot \text{K}}{0.8} \times (0.6^{0.8} - 0.19^{0.8})\ \text{m}^{0.8}\right] = 3710\ \text{W/m}^2 \cdot \text{K} \quad \triangleleft$$

The local and average convection coefficient distributions for the plate are shown in the figure below.



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Comments:

1. The average convection coefficient at $T \approx 350$ K is over twice as large as the value at $T \approx 300$ K. This strong temperature dependence is due primarily to the significant shift of x_c that is associated with the smaller viscosity of the water at the higher temperature. Careful consideration of the temperature dependence of fluid properties is *crucial* when performing a convection heat transfer analysis.
2. Spatial variations in the local convection coefficient are significant. The largest local convection coefficients occur at the leading edge of the flat plate, where the laminar thermal boundary layer is extremely thin, and just downstream of x_c, where the turbulent boundary layer is thinnest.

These equation derived in Secti

6.4 *The Boundary Layer Equations*

We can improve our understanding of the physical effects that determine boundary layer behavior and further illustrate its relevance to convection transport by considering the equations that govern boundary layer conditions, such as those illustrated in Figure 6.9.

The velocity boundary layer results from the difference between the free stream velocity and the zero velocity at the wall, while the thermal boundary layer results from a difference between the free stream and surface temperatures. The fluid is considered to be a binary mixture of species A and B, and the concentration boundary layer originates from a difference between the free stream and surface concentrations ($C_{A,\infty} \neq C_{A,s}$). Illustration of the relative thicknesses ($\delta_t > \delta_c > \delta$) in Figure 6.9 is arbitrary, for the moment, and the factors that influence relative boundary layer development are discussed later in this chapter.

Our objective in this section is to obtain the differential equations that govern the velocity, temperature, and species concentration fields that are applicable to

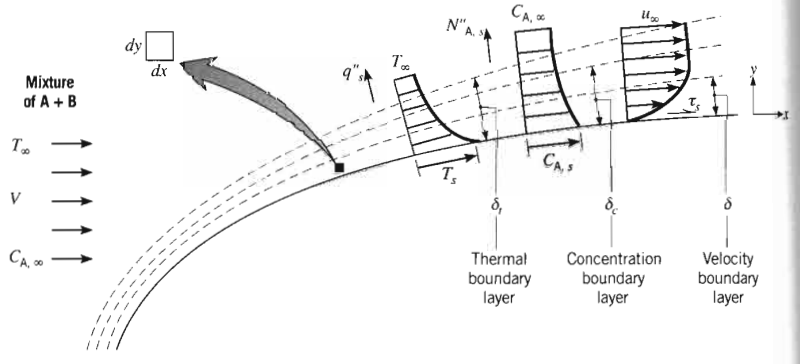


FIGURE 6.9 Development of the velocity, thermal, and concentration boundary layers for an arbitrary surface.

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boundary layer flow with heat and species transfer. Section 6.4.1 presents the laminar boundary layer equations, and Appendix E gives the corresponding equations for turbulent conditions.

6.4.1 Boundary Layer Equations for Laminar Flow

These equations are derived in Section 6S.1.

Motion of a fluid in which there are coexisting velocity, temperature, and concentration gradients must comply with several *fundamental laws of nature*. In particular, at each point in the fluid, *conservation of mass*, *energy*, and *chemical species*, as well as *Newton's second law of motion*, must be satisfied. Equations representing these requirements are derived by applying the laws to a differential control volume situated in the flow. The resulting equations, in Cartesian coordinates, for the *steady*, *two-dimensional flow* of an *incompressible fluid* with *constant properties* are given in Appendix D. These equations serve as starting points for our analysis of laminar boundary layers. Note that turbulent flows are inherently unsteady, and the equations governing them are presented in Appendix E.

We begin by restricting attention to applications for which *body forces are negligible* ($X = Y = 0$), there is *no thermal energy generation* ($\dot{q} = 0$), and the *flow is nonreacting* ($N_A = 0$). Additional simplifications may be made by invoking approximations pertinent to conditions in the velocity, thermal, and concentration boundary layers. The boundary layer thicknesses are typically very small relative to the size of the object upon which they form, and the x-direction velocity, temperature, and concentration must change from their surface to their free stream values over these very small distances. Therefore, gradients normal to the object's surface are much larger than those along the surface. As a result, we can neglect terms that represent x-direction diffusion of momentum, thermal energy, and chemical species, relative to their y-direction counterparts. That is [3, 4]:

$$\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial^2 T}{\partial x^2} \ll \frac{\partial^2 T}{\partial y^2} \qquad \frac{\partial^2 C_A}{\partial x^2} \ll \frac{\partial^2 C_A}{\partial y^2} \tag{6.25}$$

By neglecting the x-direction terms, we are assuming that the corresponding shear stress, conduction flux, and species diffusion flux are negligible.

Furthermore, because the boundary layer is so thin, the x-direction pressure gradient within the boundary layer can be approximated as the free stream pressure gradient:

$$\frac{\partial p}{\partial x} \approx \frac{dp_\infty}{dx} \tag{6.26}$$

The form of $p_\infty(x)$ depends on the surface geometry and may be obtained from a separate consideration of flow conditions in the free stream. Hence, the pressure gradient may be treated as a known quantity.

With the foregoing simplifications and approximations, the overall continuity equation is unchanged from Equation D.1:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{6.27}$$

This equation is an outgrowth of applying conservation of mass to the differential, $dx \cdot dy \cdot 1$ control volume shown in Figure 6.9. The two terms represent the *net* outflow (outflow minus inflow) of mass in the x- and y-directions, the sum of which must be zero for steady flow.

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The x-momentum equation (Equation D.2) reduces to:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \tag{6.28}$$

This equation results from application of Newton's second law of motion in the x-direction to the $dx \cdot dy \cdot 1$ differential control volume in the fluid. The left-hand side represents the net rate at which x-momentum leaves the control volume due to fluid motion across its boundaries. The first term on the right-hand side represents the net pressure force, and the second term represents the net force due to viscous shear stresses.

The energy equation (Equation D.4) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{6.29}$$

This equation results from application of conservation of energy to a differential control volume in the flowing fluid. Terms on the left-hand side account for the net rate at which thermal energy leaves the control volume due to bulk fluid motion (advection). The first term on the right-hand side accounts for the net inflow of thermal energy due to y-direction conduction. The last term on the right-hand side is what remains of the viscous dissipation, Equation D.5, when it is acknowledged that, in a boundary layer, the velocity component in the direction along the surface, u, is much larger than that normal to the surface, v, and gradients normal to the surface are much larger than those along the surface. In many situations this term may be neglected relative to those that account for advection and conduction. However, aerodynamic heating that accompanies high-speed (especially supersonic) flight is a noteworthy situation in which this term is important.

The species conservation equation (Equation D.6) reduces to

$$u\frac{\partial C_A}{\partial x} + v\frac{\partial C_A}{\partial y} = D_{AB}\frac{\partial^2 C_A}{\partial y^2} \tag{6.30}$$

This equation is obtained by applying conservation of chemical species to a differential control volume in the flow. Terms on the left-hand side account for net transport of species A due to bulk fluid motion (advection), while the right-hand side represents the net inflow due to y-direction diffusion.

Equations 6.27 through 6.30 may be solved to determine the spatial variations of u, v, T, and C_A in the different laminar boundary layers. For incompressible, constant property flow, Equations 6.27 and 6.28 are *uncoupled* from Equations 6.29 and 6.30. That is, Equations 6.27 and 6.28 may be solved for the *velocity field*, $u(x,y)$ and $v(x,y)$, without consideration of Equations 6.29 and 6.30. From knowledge of $u(x,y)$, the velocity gradient $(\partial u/\partial y)|_{y=0}$ could then be evaluated, and the wall shear stress could be obtained from Equation 6.2. In contrast, through the appearance of u and v in Equations 6.29 and 6.30, the temperature and species concentration are *coupled* to the velocity field. Hence $u(x,y)$ and $v(x,y)$ must be known before Equations 6.29 and 6.30 may be solved for $T(x,y)$ and $C_A(x,y)$. Once $T(x,y)$ and $C_A(x,y)$ have been obtained from such solutions, the convection heat and mass transfer coefficients may

6.5 Boundary L
Boundary L

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be determined from Equations 6.5 and 6.9, respectively. It follows that these coefficients depend strongly on the velocity field.[4]

Because boundary layer solutions generally involve mathematics beyond the scope of this text, our treatment of such solutions will be restricted to the analysis of laminar parallel flow over a flat plate (Section 7.2 and Appendix F). However, other analytical solutions are discussed in advanced texts on convection [6–8], and detailed boundary layer solutions may be obtained by using numerical (finite-difference or finite-element) techniques [9]. It is also essential to recognize that a wide array of situations of engineering relevance involve turbulent convective heat transfer, which is both mathematically and physically more complex than laminar convection. The boundary layer equations for turbulent flow are included in Appendix E.

It is important to stress that we have not developed the laminar boundary layer equations solely for the purpose of obtaining solutions. In fact, we have been motivated primarily by two other considerations. One motivation has been to obtain an appreciation for the physical processes that occur in boundary layers. These processes affect wall friction as well as energy and species transfer in the boundary layers. A second important motivation arises from the fact that the equations may be used to identify key *boundary layer similarity parameters*, as well as important *analogies* between *momentum, heat*, and *mass* transfer. The laminar governing equations will be used for this purpose in Sections 6.5 through 6.7, but the same key parameters and analogies hold true for turbulent conditions as well.

6.5 Boundary Layer Similarity: The Normalized Boundary Layer Equations

If we examine Equations 6.28, 6.29, and 6.30 we note a strong similarity. In fact, if the pressure gradient appearing in Equation 6.28 and the viscous dissipation term of Equation 6.29 are negligible, the three equations are of the same form. *Each equation is characterized by advection terms on the left-hand side and a diffusion term on the right-hand side.* This situation describes *low-speed, forced convection flows,* which are

[4]Special attention must be given to the effect of species transfer on the velocity boundary layer. Recall that velocity boundary layer development is generally characterized by the existence of zero fluid velocity *at the surface*. This condition pertains to the velocity component v normal to the surface, as well as to the velocity component u along the surface. However, if there is simultaneous mass transfer to or from the surface, it is evident that v can no longer be zero at the surface. Nevertheless, for the mass transfer problems of interest in this text, it is reasonable to assume that $v = 0$ at the surface, which is equivalent to assuming that mass transfer has a negligible effect on the velocity boundary layer. The assumption is appropriate for many problems involving evaporation or sublimation from gas–liquid or gas–solid interfaces, respectively. It is not appropriate, however, for *mass transfer cooling* problems that involve large surface mass transfer rates [5]. In addition, we note that, with mass transfer, the boundary layer fluid is a binary mixture of species A and B, and its properties should be those of the mixture. However, in all problems in this text, $C_A \ll C_B$ and it is reasonable to assume that the boundary layer properties (such as k, μ, c_p, etc.) are those of species B.

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found in many engineering applications. Implications of this similarity may be developed in a rational manner by first *nondimensionalizing* the governing equations.

6.5.1 Boundary Layer Similarity Parameters

The boundary layer equations are normalized by first defining dimensionless independent variables of the forms

$$x^* \equiv \frac{x}{L} \qquad \text{and} \qquad y^* \equiv \frac{y}{L} \tag{6.31}$$

where L is a *characteristic length* for the surface of interest (e.g., the length of a flat plate). Moreover, dependent dimensionless variables may also be defined as

$$u^* \equiv \frac{u}{V} \qquad \text{and} \qquad v^* \equiv \frac{v}{V} \tag{6.32}$$

where V is the velocity upstream of the surface (Figure 6.9), and as

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s} \tag{6.33}$$

$$C_A^* \equiv \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}} \tag{6.34}$$

Equations 6.31 through 6.34 may be substituted into Equations 6.28, 6.29, and 6.30 to obtain the dimensionless forms of the conservation equations shown in Table 6.1. Note that viscous dissipation has been neglected and that $p^* \equiv (p_\infty/\rho V^2)$ is a dimensionless pressure. The y-direction boundary conditions required to solve the equations are also shown in the table.

Three very important dimensionless *similarity parameters* are introduced in Table 6.1. They are the Reynolds number, Re_L; Prandtl number, Pr; and Schmidt number, Sc. Similarity parameters are important because they allow us to apply results obtained for a surface experiencing one set of convective conditions to *geometrically similar* surfaces experiencing entirely different conditions. These conditions may vary, for example, with the nature of the fluid, the fluid velocity, and/or the size of the surface, as described by the characteristic length, L. As long as the similarity parameters and dimensionless boundary conditions are the same for the two sets of conditions, the solutions of the differential equations for the nondimensional velocity, temperature, and species concentration will also be the same. This concept will be expanded upon in the remainder of this section.

6.5.2 Functional Form of the Solutions

Equations 6.35 through 6.43 in Table 6.1 are extremely useful from the standpoint of suggesting how important boundary layer results may be simplified and generalized. The momentum equation (6.35) suggests that, although conditions in the velocity boundary layer depend on the fluid properties ρ and μ, the velocity V, and the length scale L, this dependence may be simplified by grouping these variables

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TABLE 6.1 The boundary layer equations and their *y*-direction boundary conditions in nondimensional form

Boundary Layer	Conservation Equation		Boundary Conditions: Wall	Boundary Conditions: Free Stream		Similarity Parameter(s)	
Velocity	$u^*\frac{\partial u^*}{\partial x^*} + v^*\frac{\partial u^*}{\partial y^*} = -\frac{dp^*}{dx^*} + \frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}}$	(6.35)	$u^*(x^*,0) = 0$ $v^*(x^*,0) = 0$	$u^*(x^*,\infty) = \frac{u_\infty(x^*)}{V}$	(6.38)	$Re_L = \frac{VL}{\nu}$	(6.41)
Thermal	$u^*\frac{\partial T^*}{\partial x^*} + v^*\frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}}$	(6.36)	$T^*(x^*,0) = 0$	$T^*(x^*,\infty) = 1$	(6.39)	$Re_L, Pr = \frac{\nu}{\alpha}$	(6.42)
Concentration	$u^*\frac{\partial C_A^*}{\partial x^*} + v^*\frac{\partial C_A^*}{\partial y^*} = \frac{1}{Re_L Sc}\frac{\partial^2 C_A^*}{\partial y^{*2}}$	(6.37)	$C_A^*(x^*,0) = 0$	$C_A^*(x^*,\infty) = 1$	(6.40)	$Re_L, Sc = \frac{\nu}{D_{AB}}$	(6.43)

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in the form of the Reynolds number. We therefore anticipate that the solution to Equation 6.35 will be of the functional form

$$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right) \tag{6.44}$$

Since the pressure distribution $p^*(x^*)$ depends on the surface geometry and may be obtained independently by considering flow conditions in the free stream, the appearance of dp^*/dx^* in Equation 6.44 represents the influence of geometry on the velocity distribution.

From Equation 6.2, the shear stress at the surface, $y^* = 0$, may be expressed as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \left(\frac{\mu V}{L}\right) \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0}$$

and from Equations 6.1 and 6.41, it follows that the friction coefficient is

$$C_f = \frac{\tau_s}{\rho V^2/2} = \frac{2}{Re_L} \left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} \tag{6.45}$$

From Equation 6.44 we also know that

$$\left.\frac{\partial u^*}{\partial y^*}\right|_{y^*=0} = f\left(x^*, Re_L, \frac{dp^*}{dx^*}\right)$$

Hence, *for a prescribed geometry* Equation 6.45 may be expressed as

$$C_f = \frac{2}{Re_L} f(x^*, Re_L) \tag{6.46}$$

The significance of this result should not be overlooked. Equation 6.46 states that the friction coefficient, a dimensionless parameter of considerable importance to the engineer, may be expressed exclusively in terms of a dimensionless space coordinate and the Reynolds number. Hence, for a prescribed geometry we expect the function that relates C_f to x^* and Re_L to be *universally* applicable. That is, we expect it to apply to different fluids and over a wide range of values for V and L.

Similar results may be obtained for the convection coefficients of heat and mass transfer. Intuitively, we might anticipate that h depends on the fluid properties (k, c_p, μ, and ρ), the fluid velocity V, the length scale L, and the surface geometry. However, Equation 6.36 suggests the manner in which this dependence may be simplified. In particular, the solution to this equation may be expressed in the form

$$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right) \tag{6.47}$$

where the dependence on dp^*/dx^* originates from the influence of the geometry on the fluid motion (u^* and v^*), which, in turn, affects the thermal conditions. Once again the term dp^*/dx^* represents the effect of surface geometry. From the definition of the convection coefficient, Equation 6.5, and the dimensionless variables, Equations 6.31 and 6.33, we also obtain

$$h = -\frac{k_f}{L}\frac{(T_\infty - T_s)}{(T_s - T_\infty)} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0} = +\frac{k_f}{L} \left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=0}$$

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This expression suggests defining a dependent dimensionless parameter termed the Nusselt number.

Nusselt Number:

$$Nu \equiv \frac{hL}{k_f} = +\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} \tag{6.48}$$

This parameter is equal to the dimensionless temperature gradient at the surface, and it provides a measure of the convection heat transfer occurring at the surface. From Equation 6.47 it follows that, *for a prescribed geometry,*

$$Nu = f(x^*, Re_L, Pr) \tag{6.49}$$

The Nusselt number is to the thermal boundary layer what the friction coefficient is to the velocity boundary layer. Equation 6.49 implies that for a given geometry, the Nusselt number must be some *universal function* of x^*, Re_L, and Pr. If this function were known, it could be used to compute the value of Nu for different fluids and for different values of V and L. From knowledge of Nu, the local convection coefficient h may be found and the *local* heat flux may then be computed from Equation 6.4. Moreover, since the *average* heat transfer coefficient is obtained by integrating over the surface of the body, it must be independent of the spatial variable x^*. Hence the functional dependence of the *average* Nusselt number is

$$\overline{Nu} = \frac{\bar{h}L}{k_f} = f(Re_L, Pr) \tag{6.50}$$

Similarly, it may be argued that, for mass transfer in a gas flow over an evaporating liquid or a sublimating solid, the convection mass transfer coefficient h_m depends on the properties D_{AB}, ρ, and μ, the velocity V, and the characteristic length L. However, Equation 6.37 suggests that this dependence may be simplified. The solution to this equation must be of the form

$$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right) \tag{6.51}$$

where the dependence on dp^*/dx^* again originates from the influence of the fluid motion. From the definition of the convection coefficient, Equation 6.9, and the dimensionless variables, Equation 6.31 and 6.34, we know that

$$h_m = -\frac{D_{AB}}{L}\frac{(C_{A,\infty} - C_{A,s})}{(C_{A,s} - C_{A,\infty})}\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{D_{AB}}{L}\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0}$$

Hence we may define a dependent dimensionless parameter termed the Sherwood number (Sh).

Sherwood Number:

$$Sh \equiv \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\bigg|_{y^*=0} \tag{6.52}$$

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This parameter is equal to the dimensionless concentration gradient at the surface, and it provides a measure of the convection mass transfer occurring at the surface. From Equation 6.51 it follows that, *for a prescribed geometry*,

$$Sh = f(x^*, Re_L, Sc) \tag{6.53}$$

The Sherwood number is to the concentration boundary layer what the Nusselt number is to the thermal boundary layer, and Equation 6.53 implies that it must be a universal function of x^*, Re_L, and Sc. As for the Nusselt number, it is also possible to work with an *average* Sherwood number that depends on only Re_L and Sc.

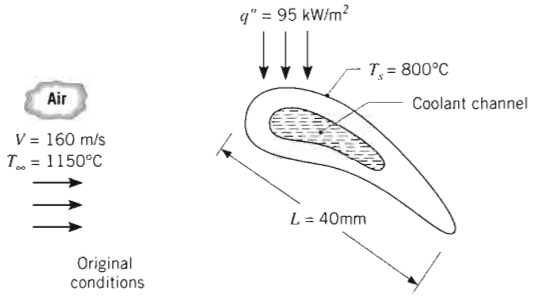
$$\overline{Sh} = \frac{\bar{h}_m L}{D_{AB}} = f(Re_L, Sc) \tag{6.54}$$

From the foregoing development we have obtained the relevant dimensionless parameters for low-speed, forced-convection boundary layers. We have done so by nondimensionalizing the differential equations that describe the physical processes occurring within the boundary layers. An alternative approach could have involved the use of dimensional analysis in the form of the Buckingham pi theorem [10]. However, the success of that method depends on one's ability to select, largely from intuition, the various parameters that influence a problem. For example, knowing beforehand that $\bar{h} = f(k, c_p, \rho, \mu, V, L)$, one could use the Buckingham pi theorem to obtain Equation 6.50. However, having begun with the differential form of the conservation equations, we have eliminated the guesswork and have established the similarity parameters in a rigorous fashion.

The value of an expression such as Equation 6.50 should be fully appreciated. It states that many convection heat transfer results, whether obtained theoretically or experimentally, can be represented in terms of three dimensionless groups, instead of the original seven parameters. The convenience afforded by such simplifications is evident. Moreover, once the form of the functional dependence of Equation 6.50 has been obtained for a particular surface geometry, let us say from laboratory measurements, it is known to be *universally* applicable. By this we mean that it may be applied for different fluids, velocities, and length scales, as long as the assumptions implicit in the originating boundary layer equations remain valid (e.g., negligible viscous dissipation and body forces).

EXAMPLE 6.5

Experimental tests on a portion of the turbine blade shown indicate a heat flux to the blade of $q'' = 95{,}000$ W/m^2. To maintain a steady-state surface temperature of 800°C, heat transferred to the blade is removed by circulating a coolant inside the blade.



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1. Determine the heat flux to the blade if its temperature is reduced to $T_{s,1} = 700°C$ by increasing the coolant flow.
2. Determine the heat flux at the same dimensionless location for a similar turbine blade having a chord length of $L = 80$ mm, when the blade operates in an airflow at $T_\infty = 1150°C$ and $V = 80$ m/s, with $T_s = 800°C$.

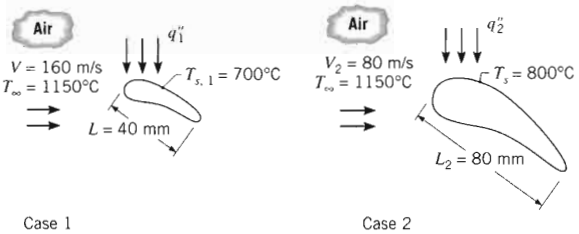
SOLUTION

Known: Operating conditions of an internally cooled turbine blade.

Find:

1. Heat flux to the blade when the surface temperature is reduced.
2. Heat flux to a larger turbine blade of the same shape with reduced air velocity.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Constant air properties.

Analysis:

1. When the surface temperature is 800°C, the convection heat transfer coefficient between the surface and the air can be obtained from Newton's law of cooling:

$$q'' = h(T_\infty - T_s)$$

Thus,

$$h = \frac{q''}{(T_\infty - T_s)}$$

We proceed without calculating the value for now. From Equation 6.49, it follows that, for the prescribed geometry,

$$Nu = \frac{hL}{k} = f(x^*, Re_L, Pr)$$

Hence, since there is no change in x^*, Re_L, or Pr associated with a change in T, for constant properties, the local Nusselt number is unchanged. Moreover, since L and k are also unchanged, the local convection coefficient remains the same. Thus, when the surface temperature is reduced to 700°C, the heat flux may be obtained from Newton's law of cooling, using the same convection coefficient:

$$q_1'' = h(T_\infty - T_{s,1}) = \frac{q''}{(T_\infty - T_s)}(T_\infty - T_{s,1}) = \frac{95{,}000 \text{ W/m}^2}{(1150 - 800)°C}(1150 - 700)°C$$

$$= 122{,}000 \text{ W/m}^2 \quad \triangleleft$$

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2. To determine the heat flux associated with the larger blade and the reduced airflow (case 2), we first note that, although L has increased by a factor of 2, the velocity has decreased by the same factor and the Reynolds number has not changed. That is,

$$Re_{L,2} = \frac{V_2 L_2}{\nu} = \frac{VL}{\nu} = Re_L$$

Accordingly, since x^* and Pr are also unchanged, the local Nusselt number remains the same.

$$Nu_2 = Nu$$

Because the characteristic length is different, however, the convection coefficient changes, where

$$\frac{h_2 L_2}{k} = \frac{hL}{k} \qquad \text{or} \qquad h_2 = h\frac{L}{L_2} = \frac{q''}{(T_\infty - T_s)}\frac{L}{L_2}$$

The heat flux is then

$$q_2'' = h_2(T_\infty - T_s) = q''\frac{(T_\infty - T_s)}{(T_\infty - T_s)}\frac{L}{L_2}$$

$$q_2'' = 95{,}000\ \text{W/m}^2 \times \frac{0.04\ \text{m}}{0.08\ \text{m}} = 47{,}500\ \text{W/m}^2$$ ◁

Comments: If the Reynolds number for the two situations of part 2 differed, that is, $Re_{L,2} \neq Re_L$, the heat flux q_2'' could only be obtained if the particular functional dependence of Equation 6.49 were known. Such forms are provided for many different shapes in subsequent chapters.

6.6 *Physical Significance of the Dimensionless Parameters*

All of the foregoing dimensionless parameters have physical interpretations that relate to conditions in the flow, not only for boundary layers but also for other flow types as well, such as the internal flows we will see in Chapter 8. Consider the *Reynolds number*, Re_L (Equation 6.41), which may be interpreted as the *ratio of inertia to viscous forces* in a region of characteristic dimension L. Inertia forces are associated with an increase in the momentum of a moving fluid. From Equation 6.28, it is evident that these forces (per unit mass) are of the form $u\partial u/\partial x$, in which case an order-of-magnitude approximation gives $F_I \approx V^2/L$. Similarly, the net shear force (per unit mass) is found on the right-hand side of Equation 6.28 as $\nu(\partial^2 u/\partial y^2)$ and may be approximated as $F_s \approx \nu V/L^2$. Therefore, the ratio of forces is

$$\frac{F_I}{F_s} \approx \frac{\rho V^2 / L}{\mu V / L^2} = \frac{\rho VL}{\mu} = Re_L$$

We therefore expect inertia forces to dominate for large values of Re_L and viscous forces to dominate for small values of Re_L.

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There are several important implications of this result. Recall that the Reynolds number determines the existence of laminar or turbulent flow. In any flow there exist small disturbances that can be amplified to produce turbulent conditions. For small Re_L, however, viscous forces are sufficiently large relative to inertia forces to prevent this amplification. Hence, laminar flow is maintained. But, with increasing Re_L, viscous effects become progressively less important relative to inertia effects, and small disturbances may be amplified to a point where transition occurs. We should also expect the magnitude of the Reynolds number to influence the velocity boundary layer thickness δ. With increasing Re_L at a fixed location on a surface, we expect viscous forces to become less influential relative to inertia forces. Hence the effects of viscosity do not penetrate as far into the free stream, and the value of δ diminishes.

The physical interpretation of the *Prandtl number* follows from its definition as a ratio of the momentum diffusivity ν to the thermal diffusivity α. The Prandtl number provides a *measure of the relative effectiveness of momentum and energy transport by diffusion in the velocity and thermal boundary layers*, respectively. From Table A.4 we see that the Prandtl number of gases is near unity, in which case energy and momentum transfer by diffusion are comparable. In a liquid metal (Table A.7), $Pr \ll 1$ and the energy diffusion rate greatly exceeds the momentum diffusion rate. The opposite is true for oils (Table A.5), for which $Pr \gg 1$. From this interpretation it follows that the value of Pr strongly influences the relative growth of the velocity and thermal boundary layers. In fact for laminar boundary layers (in which transport by diffusion is *not* overshadowed by turbulent mixing) it is reasonable to expect that

$$\frac{\delta}{\delta_t} \approx Pr^n \tag{6.55}$$

where n is a positive exponent. Hence for a gas $\delta_t \approx \delta$; for a liquid metal $\delta_t \gg \delta$; for an oil $\delta_t \ll \delta$.

Similarly, the *Schmidt number,* which is defined by Equation 6.43, provides *a measure of the relative effectiveness of momentum and mass transport by diffusion in the velocity and concentration boundary layers*, respectively. For convection mass transfer in laminar flows, it therefore determines the relative velocity and concentration boundary layer thicknesses, where

$$\frac{\delta}{\delta_c} \approx Sc^n \tag{6.56}$$

Another parameter, which is related to Pr and Sc, is the *Lewis number* (Le). It is defined as

$$Le = \frac{\alpha}{D_{AB}} = \frac{Sc}{Pr} \tag{6.57}$$

and is relevant to any situation involving simultaneous heat and mass transfer by convection. From Equations 6.55 through 6.57 it then follows that

$$\frac{\delta_t}{\delta_c} \approx Le^n \tag{6.58}$$

The Lewis number is therefore a measure of the relative thermal and concentration boundary layer thicknesses. For most applications it is reasonable to assume a value of $n = 1/3$ in Equations 6.55, 6.56, and 6.58.

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Table 6.2 lists the dimensionless groups that appear frequently in heat and mass transfer. The list includes groups already considered, as well as those yet to be introduced for special conditions. As a new group is confronted, its definition and interpretation should be committed to memory. Note that the *Grashof number* provides a measure of the ratio of buoyancy forces to viscous forces in the velocity

TABLE 6.2 Selected dimensionless groups of heat and mass transfer

Group	Definition	Interpretation
Biot number (Bi)	$\frac{hL}{k_s}$	Ratio of the internal thermal resistance of a solid to the boundary layer thermal resistance.
Mass transfer Biot number (Bi_m)	$\frac{h_m L}{D_{AB}}$	Ratio of the internal species transfer resistance to the boundary layer species transfer resistance.
Bond number (Bo)	$\frac{g(\rho_l - \rho_v)L^2}{\sigma}$	Ratio of gravitational and surface tension forces.
Coefficient of friction (C_f)	$\frac{\tau_s}{\rho V^2/2}$	Dimensionless surface shear stress.
Eckert number (Ec)	$\frac{V^2}{c_p(T_s - T_\infty)}$	Kinetic energy of the flow relative to the boundary layer enthalpy difference.
Fourier number (Fo)	$\frac{\alpha t}{L^2}$	Ratio of the heat conduction rate to the rate of thermal energy storage in a solid. Dimensionless time.
Mass transfer Fourier number (Fo_m)	$\frac{D_{AB} t}{L^2}$	Ratio of the species diffusion rate to the rate of species storage. Dimensionless time.
Friction factor (f)	$\frac{\Delta p}{(L/D)(\rho u_m^2/2)}$	Dimensionless pressure drop for internal flow.
Grashof number (Gr_L)	$\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$	Measure of the ratio of buoyancy forces to viscous forces.
Colburn j factor (j_H)	$St\, Pr^{2/3}$	Dimensionless heat transfer coefficient.
Colburn j factor (j_m)	$St_m\, Sc^{2/3}$	Dimensionless mass transfer coefficient.
Jakob number (Ja)	$\frac{c_p(T_s - T_{sat})}{h_{fg}}$	Ratio of sensible to latent energy absorbed during liquid–vapor phase change.
Lewis number (Le)	$\frac{\alpha}{D_{AB}}$	Ratio of the thermal and mass diffusivities.
Nusselt number (Nu_L)	$\frac{hL}{k_f}$	Ratio of convection to pure conduction heat transfer.
Peclet number (Pe_L)	$\frac{VL}{\alpha} = Re_L Pr$	Ratio of advection to conduction heat transfer rates.
Prandtl number (Pr)	$\frac{c_p \mu}{k} = \frac{\nu}{\alpha}$	Ratio of the momentum and thermal diffusivities.

6.7 Boundary Lay

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TABLE 6.2 *Continued*

Group	Definition	Interpretation
Reynolds number (Re_L)	$\frac{VL}{\nu}$	Ratio of the inertia and viscous forces.
Schmidt number (Sc)	$\frac{\nu}{D_{AB}}$	Ratio of the momentum and mass diffusivities.
Sherwood number (Sh_L)	$\frac{h_m L}{D_{AB}}$	Dimensionless concentration gradient at the surface.
Stanton number (St)	$\frac{h}{\rho V c_p} = \frac{Nu_L}{Re_L Pr}$	Modified Nusselt number.
Mass transfer Stanton number (St_m)	$\frac{h_m}{V} = \frac{Sh_L}{Re_L Sc}$	Modified Sherwood number.
Weber number (We)	$\frac{\rho V^2 L}{\sigma}$	Ratio of inertia to surface tension forces.

boundary layer. Its role in free convection (Chapter 9) is much the same as that of the Reynolds number in forced convection. The *Eckert number* provides a measure of the kinetic energy of the flow relative to the enthalpy difference across the thermal boundary layer. It plays an important role in high-speed flows for which viscous dissipation is significant. Note also that, although similar in form, the Nusselt and Biot numbers differ in both definition and interpretation. Whereas the Nusselt number is defined in terms of the thermal conductivity of the fluid, the Biot number is based on the solid thermal conductivity, Equation 5.9.

6.7 *Boundary Layer Analogies*

As engineers, our interest in boundary layer behavior is directed principally toward the dimensionless parameters C_f, Nu, and Sh. From knowledge of these parameters, we may compute the wall shear stress and the convection heat and mass transfer rates. It is therefore understandable that expressions that relate C_f, Nu, and Sh to each other can be useful tools in convection analysis. Such expressions are available in the form of *boundary layer analogies.*

6.7.1 The Heat and Mass Transfer Analogy

If two or more processes are governed by dimensionless equations of the same form, the processes are said to be *analogous.* Clearly, then, from Equations 6.36 and 6.37 and the boundary conditions, Equations 6.39 and 6.40, of Table 6.1, convection heat and mass transfer are analogous. Each of the differential equations is composed of advection and diffusion terms of the same form. Moreover, as shown in

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TABLE 6.3 Functional relations pertinent to the boundary layer analogies

Fluid Flow		Heat Transfer		Mass Transfer	
$u^* = f\left(x^*, y^*, Re_L, \frac{dp^*}{dx^*}\right)$	(6.44)	$T^* = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$	(6.47)	$C_A^* = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$	(6.51)
$C_f = \frac{2}{Re_L}\frac{\partial u^*}{\partial y^*}\Big\|_{y^*=0}$	(6.45)	$Nu = \frac{hL}{k} = +\frac{\partial T^*}{\partial y^*}\Big\|_{y^*=0}$	(6.48)	$Sh = \frac{h_m L}{D_{AB}} = +\frac{\partial C_A^*}{\partial y^*}\Big\|_{y^*=0}$	(6.52)
$C_f = \frac{2}{Re_L} f(x^*, Re_L)$	(6.46)	$Nu = f(x^*, Re_L, Pr)$	(6.49)	$Sh = f(x^*, Re_L, Sc)$	(6.53)
		$\overline{Nu} = f(Re_L, Pr)$	(6.50)	$\overline{Sh} = f(Re_L, Sc)$	(6.54)

Equations 6.36 and 6.37, each equation is related to the velocity field through Re_L and the parameters Pr and Sc assume analogous roles. One implication of this analogy is that dimensionless relations that govern thermal boundary layer behavior must be of the same form as those that govern the concentration boundary layer. Hence the boundary layer temperature and concentration profiles must also be of the same functional form if the applied boundary conditions are analogous.

Recalling the discussion of Section 6.5.2, features of which are summarized in Table 6.3, an important result of the heat and mass transfer analogy may be obtained. From the foregoing paragraph, it follows that Equation 6.47 must be of the same functional form as Equation 6.51. From Equations 6.48 and 6.52, it then follows that the dimensionless temperature and concentration gradients evaluated at the surface, and therefore the values of Nu and Sh, are analogous. Similarly, expressions for the average Nusselt and Sherwood numbers, Equations 6.50 and 6.54, respectively, are also of the same form. *Accordingly, heat and mass transfer relations for a particular geometry are interchangeable.* If, for example, one has performed a set of heat transfer experiments to determine the functional form of Equation 6.49 for a particular surface geometry, the results may be used for convection mass transfer involving the same geometry, simply by replacing Nu with Sh and Pr with Sc.

The analogy may also be used to directly relate the two convection coefficients. In subsequent chapters we will find that Nu and Sh are generally proportional to Pr^n and Sc^n, respectively, where n is a positive exponent less than 1. Anticipating this dependence, we use Equations 6.49 and 6.53 to obtain

$$Nu = f(x^*, Re_L)Pr^n \qquad \text{and} \qquad Sh = f(x^*, Re_L)Sc^n$$

in which case, with equivalent functions, $f(x^*, Re_L)$,

$$\frac{Nu}{Pr^n} = \frac{Sh}{Sc^n} \tag{6.59}$$

Substituting from Equations 6.48 and 6.52 we then obtain

$$\frac{hL/k}{Pr^n} = \frac{h_m L/D_{AB}}{Sc^n}$$

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or, from Equation 6.57,

$$\frac{h}{h_m} = \frac{k}{D_{AB}Le^n} = \rho c_p Le^{1-n} \tag{6.60}$$

This result may often be used to determine one convection coefficient, for example, h_m, from knowledge of the other coefficient. The same relation may be applied to the average coefficients $\bar{h}$ and $\bar{h}_m$, and it may be used in turbulent, as well as laminar, flow. For most applications it is reasonable to assume a value of $n = \frac{1}{3}$.

EXAMPLE 6.6

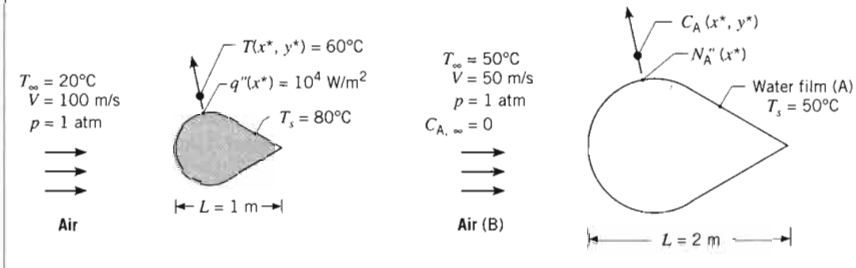
A solid of arbitrary shape is suspended in atmospheric air having a free stream temperature and velocity of 20°C and 100 m/s, respectively. The solid has a characteristic length of 1 m, and its surface is maintained at 80°C. Under these conditions measurements of the heat flux at a particular point (x^*) on the surface and of the temperature in the boundary layer above this point (x^*, y^*) reveal values of 10^4 W/m^2 and 60°C, respectively. A mass transfer operation is to be effected for a second solid having the same shape but a characteristic length of 2 m. In particular, a thin film of water on the solid is to be evaporated in dry atmospheric air having a free stream velocity of 50 m/s, with the air and the solid both at a temperature of 50°C. What are the molar concentration and the species molar flux of the water vapor at a location (x^*, y^*) corresponding to the point at which the temperature and heat flux measurements were made in the first case?

SOLUTION

Known: A boundary layer temperature and heat flux at a location on a solid in an airstream of prescribed temperature and velocity.

Find: Water vapor concentration and flux associated with the same location on a larger surface of the same shape.

Schematic:



Case 1: heat transfer

Case 2: mass transfer

Assumptions:

1. Steady-state, two-dimensional, incompressible boundary layer behavior; constant properties.

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2. Boundary layer approximations are valid.
3. Negligible viscous dissipation.
4. Mole fraction of water vapor in concentration boundary layer is much less than unity.

Properties: Table A.4, air (50°C): $\nu = 18.2 \times 10^{-6}$ m²/s, $k = 28 \times 10^{-3}$ W/m · K, $Pr = 0.70$. Table A.6, saturated water vapor (50°C): $\rho_{A,sat} = v_g^{-1} = 0.082$ kg/m³. Table A.8, water vapor–air (50°C): $D_{AB} \approx 0.26 \times 10^{-4}$ m²/s.

Analysis: The desired molar concentration and flux may be determined by invoking the analogy between heat and mass transfer. From Equations 6.47 and 6.51, we know that

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s} = f\left(x^*, y^*, Re_L, Pr, \frac{dp^*}{dx^*}\right)$$

and

$$C_A^* \equiv \frac{C_A - C_{A,s}}{C_{A,\infty} - C_{A,s}} = f\left(x^*, y^*, Re_L, Sc, \frac{dp^*}{dx^*}\right)$$

However, for case 1

$$Re_{L,1} = \frac{V_1 L_1}{\nu} = \frac{100 \text{ m/s} \times 1 \text{ m}}{18.2 \times 10^{-6} \text{ m}^2/\text{s}} = 5.5 \times 10^6, \qquad Pr = 0.70$$

while for case 2

$$Re_{L,2} = \frac{V_2 L_2}{\nu} = \frac{50 \text{ m/s} \times 2 \text{ m}}{18.2 \times 10^{-6} \text{ m}^2/\text{s}} = 5.5 \times 10^6$$

$$Sc = \frac{\nu}{D_{AB}} = \frac{18.2 \times 10^{-6} \text{ m}^2/\text{s}}{26 \times 10^{-6} \text{ m}^2/\text{s}} = 0.70$$

Since $Re_{L,1} = Re_{L,2}$, $Pr = Sc$, $x_1^* = x_2^*$, $y_1^* = y_2^*$, and the surface geometries are the same, it follows that the temperature and concentration distributions have the same functional form. Hence

$$\frac{C_A(x^*, y^*) - C_{A,s}}{C_{A,\infty} - C_{A,s}} = \frac{T(x^*, y^*) - T_s}{T_\infty - T_s} = \frac{60 - 80}{20 - 80} = 0.33$$

or, with $C_{A,\infty} = 0$,

$$C_A(x^*, y^*) = C_{A,s}(1 - 0.33) = 0.67 C_{A,s}$$

With

$$C_{A,s} = C_{A,sat}(50°\text{C}) = \frac{\rho_{A,sat}}{\mathcal{M}_A} = \frac{0.082 \text{ kg/m}^3}{18 \text{ kg/kmol}} = 0.0046 \text{ kmol/m}^3$$

it follows that

$$C_A(x^*, y^*) = 0.67\ (0.0046 \text{ kmol/m}^3) = 0.0031 \text{ kmol/m}^3 \quad \triangleleft$$

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The molar flux may be obtained from Equation 6.8

$$N''_A(x^*) = h_m(C_{A,s} - C_{A,\infty})$$

with h_m evaluated from the analogy. From Equations 6.49 and 6.53 we know that, since $x_1^* = x_2^*$, $Re_{L,1} = Re_{L,2}$, and $Pr = Sc$, the corresponding functional forms are equivalent. Hence

$$Sh = \frac{h_m L_2}{D_{AB}} = Nu = \frac{hL_1}{k}$$

With $h = q''/(T_s - T_\infty)$ from Newton's law of cooling,

$$h_m = \frac{L_1}{L_2} \times \frac{D_{AB}}{k} \times \frac{q''}{(T_s - T_\infty)} = \frac{1}{2} \times \frac{0.26 \times 10^{-4}\ \text{m}^2/\text{s}}{0.028\ \text{W/m} \cdot \text{K}} \times \frac{10^4\ \text{W/m}^2}{(80 - 20)°\text{C}}$$

$$h_m = 0.077\ \text{m/s}$$

Hence

$$N''_A(x^*) = 0.077\ \text{m/s}\ (0.0046 - 0.0)\ \text{kmol/m}^3$$

or

$$N''_A(x^*) = 3.54 \times 10^{-4}\ \text{kmol/s} \cdot \text{m}^2 \quad \triangleleft$$

Comments:

1. Recognize that, since the mole fraction of water vapor in the concentration boundary layer is small, the kinematic viscosity of air (ν_B) may be used to evaluate $Re_{L,2}$.
2. The properties of air (and a variety of other substances) can be very quickly and easily found using *IHT* by selecting *Properties* from the *Tools* menu, and then "Air." The *Viewpad* window will list property function calls for the properties available for air. Copy the equations to the *Workspace* as needed, and add an equation that specifies the temperature at which properties are to be evaluated, namely the *film temperature*, $T_f = (T_s + T_\infty)/2$. The resulting lines of *IHT* are shown below. Click the *Solve* button, accept the initial guess, and the properties will be displayed in the *Data Browser*.

```
//Evaluate air properties at the film temperature
T = 50 + 273

//Air property functions : From Table A.4
//Units: T(K); 1 atm pressure
nu = nu_T("Air",T)      //Kinematic viscosity, m^2/s
k = k_T("Air",T)        //Thermal conductivity, W/m · K
Pr = Pr_T("Air",T)      //Prandtl number
```

6.7.2 Evaporative Cooling

An important application of the heat and mass transfer analogy is to the process of *evaporative cooling*, which occurs when a gas flows over a liquid (Figure 6.10). Evaporation occurs from the liquid surface, and the energy associated with the phase change is the latent heat of vaporization of the liquid. Evaporation occurs

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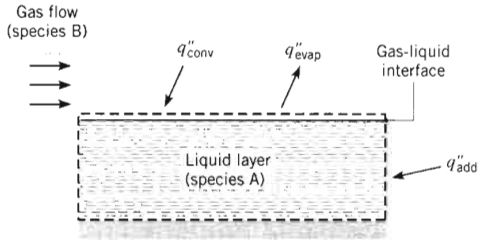


FIGURE 6.10
Latent and sensible heat exchange at a gas–liquid interface.

when liquid molecules near the surface experience collisions that increase their energy above that needed to overcome the surface binding energy. The energy required to sustain the evaporation must come from the internal energy of the liquid, which would then experience a reduction in temperature (the cooling effect). However, if steady-state conditions are to be maintained, the latent energy lost by the liquid because of evaporation must be replenished by energy transfer to the liquid from its surroundings. Neglecting radiation effects, this transfer may be due to the convection of sensible energy from the gas or to heat addition by other means, as, for example, by an electrical heater submerged in the liquid. Applying conservation of energy to a control surface about the liquid (Equation 1.11c), it follows that, for a unit surface area,

$$q''_{conv} + q''_{add} = q''_{evap} \tag{6.61}$$

where q''_{evap} may be approximated as the product of the evaporative mass flux and the latent heat of vaporization

$$q''_{evap} = n''_A h_{fg} \tag{6.62}$$

If there is no heat addition by other means, Equation 6.61 reduces to a balance between convection heat transfer from the gas and the evaporative heat loss from the liquid. Substituting from Equations 6.4, 6.18, and 6.62, Equation 6.61 may then be expressed as

$$h(T_\infty - T_s) = h_{fg} h_m [\rho_{A,sat}(T_s) - \rho_{A,\infty}] \tag{6.63}$$

where the vapor density at the surface is that associated with saturated conditions at T_s. Hence the magnitude of the cooling effect may be expressed as

$$T_\infty - T_s = h_{fg}\left(\frac{h_m}{h}\right)[\rho_{A,sat}(T_s) - \rho_{A,\infty}] \tag{6.64}$$

Substituting for (h_m/h) from Equation 6.60 and for the vapor densities from the ideal gas law, the cooling effect may also be expressed as

$$(T_\infty - T_s) = \frac{\mathcal{M}_A h_{fg}}{\mathcal{R}\rho c_p Le^{2/3}}\left[\frac{p_{A,sat}(T_s)}{T_s} - \frac{p_{A,\infty}}{T_\infty}\right] \tag{6.65}$$

In the interest of accuracy, the gas (species B) properties ρ, c_p, and Le should be evaluated at the arithmetic mean temperature of the thermal boundary layer,

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$T_{am} = (T_s + T_\infty)/2$. A representative value of $n = \frac{1}{3}$ has been assumed for the Pr and Sc exponent of Equation 6.60.

Numerous environmental and industrial applications of the foregoing results arise for situations in which the gas is *air* and the liquid is *water*.

EXAMPLE 6.7

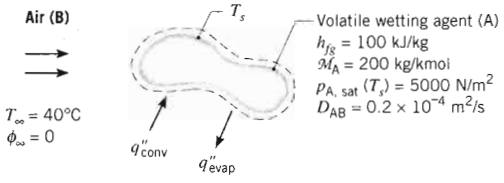
A container, which is wrapped in a fabric that is continually moistened with a highly volatile liquid, may be used to keep beverages cool in hot arid regions. Suppose that the container is placed in dry ambient air at 40°C, with heat and mass transfer between the wetting agent and the air occurring by forced convection. The wetting agent is known to have a molecular weight of 200 kg/kmol and a latent heat of vaporization of 100 kJ/kg. Its saturated vapor pressure for the prescribed conditions is approximately 5000 N/m^2, and the diffusion coefficient of the vapor in air is 0.2×10^{-4} m^2/s. What is the steady-state temperature of the beverage?

SOLUTION

Known: Properties of wetting agent used to evaporatively cool a beverage container.

Find: Steady-state temperature of beverage.

Schematic:



Assumptions:

1. Heat and mass transfer analogy is applicable.
2. Vapor displays ideal gas behavior.
3. Radiation effects are negligible.
4. Air properties may be evaluated at a mean boundary layer temperature assumed to be 300 K.

Properties: Table A.4, air (300 K): $\rho = 1.16$ kg/m^3, $c_p = 1.007$ kJ/kg · K, $\alpha = 22.5 \times 10^{-6}$ m^2/s.

Analysis: Subject to the foregoing assumptions, the evaporative cooling effect is given by Equation 6.65.

$$(T_\infty - T_s) = \frac{\mathcal{M}_A h_{fg}}{\mathcal{R}\rho c_p Le^{2/3}}\left[\frac{p_{A,sat}(T_s)}{T_s} - \frac{p_{A,\infty}}{T_\infty}\right]$$

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Setting $p_{A,\infty} = 0$ and rearranging, it follows that

$$T_s^2 - T_\infty T_s + B = 0$$

where the coefficient B is

$$B = \frac{\mathcal{M}_A h_{fg} p_{A,sat}}{\mathcal{R} \rho c_p Le^{2/3}}$$

or

$$B = [200\ \text{kg/kmol} \times 100\ \text{kJ/kg} \times 5000\ \text{N/m}^2 \times 10^{-3}\ \text{kJ/N} \cdot \text{m}] \div \left[8.315\ \text{kJ/kmol} \cdot \text{K} \times 1.16\ \text{kg/m}^3 \times 1.007\ \text{kJ/kg} \cdot \text{K} \times \left(\frac{22.5 \times 10^{-6}\ \text{m}^2/\text{s}}{20 \times 10^{-6}\ \text{m}^2/\text{s}} \right)^{2/3} \right] = 9518\ \text{K}^2$$

Hence

$$T_s = \frac{T_\infty \pm \sqrt{T_\infty^2 - 4B}}{2} = \frac{313\ \text{K} \pm \sqrt{(313)^2 - 4(9514)}\ \text{K}}{2}$$

Rejecting the minus sign on physical grounds (T_s must equal T_∞ if there is no evaporation, in which case $p_{A,sat} = 0$ and $B = 0$), it follows that

$$T_s = 278.9\ \text{K} = 5.9°\text{C} \quad \triangleleft$$

Comments: The result is independent of the shape of the container as long as the heat and mass transfer analogy may be used.

6.7.3 The Reynolds Analogy

A second boundary layer analogy may be obtained by noting from Table 6.1 that, for $dp^*/dx^* = 0$ and $Pr = Sc = 1$, the boundary layer equations, Equations 6.35 through 6.37, are of precisely the same form. For a flat plate parallel to the incoming flow, we have $dp^*/dx^* = 0$ and there is no variation in the free stream velocity outside the boundary layer. With $u_\infty = V$, Equations 6.38 through 6.40 also have the same form. Hence the functional forms of the solutions for u^*, T^*, and C_A^*, Equations 6.44, 6.47, and 6.51, must be equivalent. From Equations 6.45, 6.48, and 6.52, it follows that

$$C_f \frac{Re_L}{2} = Nu = Sh \tag{6.66}$$

Replacing *Nu* and *Sh* by the *Stanton number* (*St*) and the *mass transfer Stanton number* (St_m), respectively,

$$St \equiv \frac{h}{\rho V c_p} = \frac{Nu}{Re\ Pr} \tag{6.67}$$

$$St_m \equiv \frac{h_m}{V} = \frac{Sh}{Re\ Sc} \tag{6.68}$$

Equation 6.66 may also be expressed in the form

$$\frac{C_f}{2} = St = St_m \tag{6.69}$$

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Equation 6.69 is known as the *Reynolds analogy,* and it relates the key engineering parameters of the velocity, thermal, and concentration boundary layers. If the velocity parameter is known, the analogy may be used to obtain the other parameters, and vice versa. However, there are restrictions associated with using this result. In addition to relying on the validity of the boundary layer approximations, the accuracy of Equation 6.69 depends on having Pr and $Sc \approx 1$ and $dp^*/dx^* \approx 0$. However, it has been shown that the analogy may be applied over a wide range of Pr and Sc, if certain corrections are added. In particular the *modified Reynolds,* or *Chilton–Colburn, analogies* [11, 12], have the form

$$\frac{C_f}{2} = St\, Pr^{2/3} \equiv j_H \qquad 0.6 < Pr < 60 \tag{6.70}$$

$$\frac{C_f}{2} = St_m\, Sc^{2/3} \equiv j_m \qquad 0.6 < Sc < 3000 \tag{6.71}$$

where j_H and j_m are the *Colburn j factors* for heat and mass transfer, respectively. For laminar flow Equations 6.70 and 6.71 are only appropriate when $dp^*/dx^* \approx 0$, but in turbulent flow, conditions are less sensitive to the effect of pressure gradients and these equations remain approximately valid. If the analogy is applicable at every point on a surface, it may be applied to the surface average coefficients.

6.8 *The Convection Coefficients*

In this chapter we have considered several fundamental issues related to convection transport phenomena. In the process, however, you should not lose sight of what remains *the problem of convection.* Our primary objective is still one of developing means to determine the convection coefficients h and h_m. Although these coefficients may be obtained by solving the boundary layer equations, it is only for simple flow situations that such solutions are readily effected. The more practical approach frequently involves calculating h and h_m from empirical relations of the form given by Equations 6.49 and 6.53. The particular form of these equations is obtained by *correlating* measured convection heat and mass transfer results in terms of appropriate dimensionless groups. It is this approach that is emphasized in the chapters that follow.

6.9 *Summary*

In this chapter, attempts have been made to develop, in a logical fashion, the mathematical and physical bases of convection transport. To test your comprehension of the material, you should challenge yourself with appropriate questions.

- What is the difference between a *local* convection heat transfer coefficient and an *average* coefficient? What are their units? What is the difference between local and average convection coefficients for species transfer? What are their units?

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- What are the forms of *Newton's law of cooling* for a *heat flux* and a *heat rate*? What are the analogous forms for convection mass transfer, expressed in molar and mass units?
- Provide some examples for which species transfer by convection is pertinent.
- What is *Fick's law*?
- What are the *velocity*, *thermal*, and *concentration boundary layers*? Under what conditions do they develop?
- What quantities change with location in a *velocity boundary layer*? A *thermal boundary layer*? A *concentration boundary layer*?
- Recognizing that convection heat (mass) transfer is strongly influenced by conditions associated with fluid flow over a surface, how is it that we may determine the convection heat (species) flux by applying Fourier's (Fick's) law to the fluid at the surface?
- Do we expect heat and mass transfer to change with transition from a laminar to a turbulent boundary layer? If so, how?
- What laws of nature are embodied by the *convection transfer equations*?
- What physical processes are represented by the terms of the x-momentum equation (6.28)? By the energy equation (6.29)? By the species conservation equation (6.30)?
- What special approximations may be made for conditions within *thin* velocity, thermal, and concentration boundary layers?
- How is the *Reynolds number* defined? What is its physical interpretation? What role is played by the *critical Reynolds number*?
- What is the definition of the *Prandtl number*? How does its value affect relative growth of the velocity and thermal boundary layers for laminar flow over a surface? What are representative room-temperature values of the Prandtl number for a liquid metal, a gas, water, and an oil?
- What is the definition of the *Schmidt number*? The *Lewis number*? What are their physical interpretations, and how do they influence relative velocity, thermal, and concentration boundary layer development for laminar flow over a surface?
- What is the *coefficient of friction*? The *Nusselt number*? The *Sherwood number*? For flow over a prescribed geometry, what are the independent parameters that determine local and average values of these quantities?
- Under what conditions may velocity, thermal, and concentration boundary layers be termed *analogous*? What is the physical basis of analogous behavior?
- What important boundary layer parameters are linked by the *heat and mass transfer analogy*?
- What is the physical basis of the *evaporative cooling effect*? Have you ever experienced the effect?
- What important boundary layer parameters are linked by the *Reynolds analogy*?
- What physical features distinguish a turbulent flow from a laminar flow?

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Problems

Boundary Lay

6.1 In flow over a are of the form

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6.2 Water at a ten the surfaces o ture is $T_{s,1}$ = other surface state conditio ciated with t gradient in th with the wall wall and in th

6.3 In a particular surface, the b be approximat

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Heat Transfer

6.4 For laminar fer coefficie the distance plate. What between the plate to the

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Problems

Boundary Layer Profiles

6.1 In flow over a surface, velocity and temperature profiles are of the forms

$$u(y) = Ay + By^2 - Cy^3 \qquad \text{and}$$
$$T(y) = D + Ey + Fy^2 - Gy^3$$

where the coefficients A through G are constants. Obtain expressions for the friction coefficient C_f and the convection coefficient h in terms of u_∞, T_∞, and appropriate profile coefficients and fluid properties.

6.2 Water at a temperature of $T_\infty = 25°C$ flows over one of the surfaces of a steel wall (AISI 1010) whose temperature is $T_{s,1} = 40°C$. The wall is 0.35 m thick, and its other surface temperature is $T_{s,2} = 100°C$. For steady-state conditions what is the convection coefficient associated with the water flow? What is the temperature gradient in the wall and in the water that is in contact with the wall? Sketch the temperature distribution in the wall and in the adjoining water.

6.3 In a particular application involving airflow over a heated surface, the boundary layer temperature distribution may be approximated as

$$\frac{T - T_s}{T_\infty - T_s} = 1 - \exp\left(-Pr\,\frac{u_\infty y}{\nu}\right)$$

where y is the distance normal to the surface and the Prandtl number, $Pr = c_p\mu/k = 0.7$, is a dimensionless fluid property. If $T_\infty = 400$ K, $T_s = 300$ K, and $u_\infty/\nu = 5000\ \text{m}^{-1}$, what is the surface heat flux?

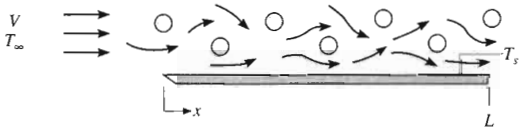
Heat Transfer Coefficients

6.4 For laminar flow over a flat plate, the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge ($x = 0$) of the plate. What is the ratio of the average coefficient between the leading edge and some location x on the plate to the local coefficient at x?

6.5 For laminar free convection from a heated vertical surface, the local convection coefficient may be expressed as $h_x = Cx^{-1/4}$, where h_x is the coefficient at a distance x from the leading edge of the surface and the quantity C, which depends on the fluid properties, is independent of x. Obtain an expression for the ratio $\bar{h}_x/h_x$, where $\bar{h}_x$ is the average coefficient between the leading edge ($x = 0$) and the x location. Sketch the variation of h_x and $\bar{h}_x$ with x.

6.6 A circular, hot gas jet at T_∞ is directed normal to a circular plate that has radius r_o and is maintained at a uniform temperature T_s. Gas flow over the plate is axisymmetric, causing the local convection coefficient to have a radial dependence of the form $h(r) = a + br^n$, where a, b, and n are constants. Determine the rate of heat transfer to the plate, expressing your result in terms of T_∞, T_s, r_o, a, b, and n.

6.7 Parallel flow of atmospheric air over a flat plate of length $L = 3$ m is disrupted by an array of stationary rods placed in the flow path over the plate.



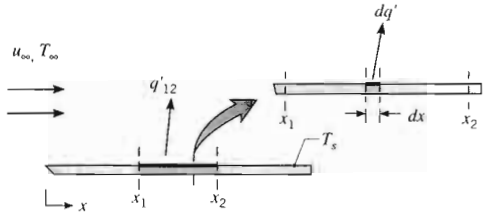
Laboratory measurements of the local convection coefficient at the surface of the plate are made for a prescribed value of V and $T_s > T_\infty$. The results are correlated by an expression of the form $h_x = 0.7 + 13.6x - 3.4x^2$, where h_x has units of $\text{W/m}^2 \cdot \text{K}$ and x is in meters. Evaluate the average convection coefficient $\bar{h}_L$ for the entire plate and the ratio $\bar{h}_L/h_L$ at the trailing edge.

6.8 Air at a free stream temperature of $T_\infty = 20°C$ is in parallel flow over a flat plate of length $L = 5$ m and temperature $T_s = 90°C$. However, obstacles placed in the flow intensify mixing with increasing distance x from the leading edge, and the spatial variation of temperatures

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measured in the boundary layer is correlated by an expression of the form $T(°C) = 20 + 70 \exp(-600xy)$, where x and y are in meters. Determine and plot the manner in which the local convection coefficient h varies with x. Evaluate the average convection coefficient $\bar{h}$ for the plate.

6.9 The heat transfer rate per unit width (normal to the page) from a longitudinal section, $x_2 - x_1$, can be expressed as $q'_{12} = \bar{h}_{12}(x_2 - x_1)(T_s - T_\infty)$, where $\bar{h}_{12}$ is the average coefficient for the section of length $(x_2 - x_1)$. Consider laminar flow over a flat plate with a uniform temperature T_s. The spatial variation of the local convection coefficient is of the form $h_x = Cx^{-1/2}$, where C is a constant.



(a) Beginning with the convection rate equation in the form $dq' = h_x\,dx(T_s - T_\infty)$, derive an expression for $\bar{h}_{12}$ in terms of C, x_1, and x_2.

(b) Derive an expression for $\bar{h}_{12}$ in terms of x_1, x_2, and the average coefficients $\bar{h}_1$ and $\bar{h}_2$, corresponding to lengths x_1 and x_2, respectively.

6.10 Experiments have been conducted to determine local heat transfer coefficients for flow perpendicular to a long, isothermal bar of rectangular cross section. The bar is of width c, parallel to the flow, and height d, normal to the flow. For Reynolds numbers in the range $10^4 \leq Re_d \leq 5 \times 10^4$, the *face-averaged* Nusselt numbers are well correlated by an expression of the form

$$Nu_d = hd/k = CRe_d^m Pr^{1/3}$$

The values of C and m for the front face, side faces, and back face of the rectangular rod are found to be the following:

Face	c/d	C	m
Front	$0.33 \leq c/d \leq 1.33$	0.674	1/2
Side	0.33	0.153	2/3
Side	1.33	0.107	2/3
Back	0.33	0.174	2/3
Back	1.33	0.153	2/3

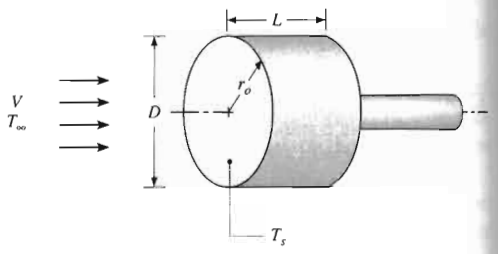
Determine the value of the average heat transfer coefficient for the entire exposed surface (that is, averaged over all four faces) of a $c = 40$-mm wide, $d = 30$-mm tall rectangular rod. The rod is exposed to air in cross flow at $V = 10$ m/s, $T_\infty = 300$ K. Provide a plausible explanation of the relative values of the face-averaged heat transfer coefficients on the front, side, and back faces.

6.11 Experiments to determine the local convection heat transfer coefficient for uniform flow normal to a heated circular disk have yielded a radial Nusselt number distribution of the form

$$Nu_D = \frac{h(r)D}{k} = Nu_o\left[1 + a\left(\frac{r}{r_o}\right)^n\right]$$

where both n and a are positive. The Nusselt number at the stagnation point is correlated in terms of the Reynolds ($Re_D = VD/\nu$) and Prandtl numbers

$$Nu_o = \frac{h(r=0)D}{k} = 0.814Re_D^{1/2} Pr^{0.36}$$



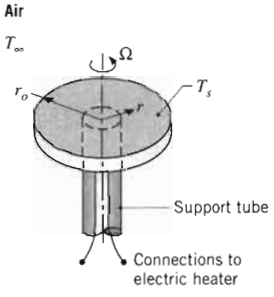
Obtain an expression for the average Nusselt number, $\overline{Nu}_D = \bar{h}D/k$, corresponding to heat transfer from an isothermal disk. Typically, boundary layer development from a stagnation point yields a decaying convection coefficient with increasing distance from the stagnation point. Provide a plausible explanation for why the opposite trend is observed for the disk.

6.12 An experimental procedure for validating results of the foregoing problem involves preheating a copper disk to an initial elevated temperature T_i and recording its temperature history $T(t)$ as it is subsequently cooled by the impinging flow to a final temperature T_f. The measured temperature decay may then be compared with predictions based on the correlation for $\overline{Nu}_D$. Assume that values of $a = 0.30$ and $n = 2$ are associated with the correlation.

Consider experimental conditions for which a disk of diameter $D = 50$ mm and length $L = 25$ mm is preheated to $T_i = 1000$ K and cooled to $T_f = 400$ K by an impinging airflow at $T_\infty = 300$ K. The cooled surface of

the disk has an emissivity of $\varepsilon = 0.8$ and is exposed to large, isothermal surroundings for which $T_{sur} = T_\infty$. The remaining surfaces of the disk are well insulated, and heat transfer through the supporting rod may be neglected. Using results from the preceding problem, compute and plot temperature histories corresponding to air velocities of $V = 4$, 20, and 50 m/s. Constant properties may be assumed for the copper ($\rho = 8933$ kg/m^3, $c_p = 425$ J/kg · K, $k = 386$ W/m · K) and air ($\nu = 38.8 \times 10^{-6}$ m^2/s, $k = 0.0407$ W/m · K, $Pr = 0.684$).

6.13 If laminar flow is induced at the surface of a disk due to rotation about its axis, the local convection coefficient is known to be a constant, $h = C$, independent of radius. Consider conditions for which a disk of radius $r_o = 100$ mm is rotating in stagnant air at $T_\infty = 20°C$ and a value of $C = 20$ W/m^2 · K is maintained.



If an embedded electric heater maintains a surface temperature of $T_s = 50°C$, what is the local heat flux at the surface of the disk? What is the total electric power requirement? What can you say about the nature of boundary layer development on the disk?

Boundary Layer Transition

6.14 Consider airflow over a flat plate of length $L = 1$ m under conditions for which transition occurs at $x_c = 0.5$ m based on the critical Reynolds number, $Re_{x,c} = 5 \times 10^5$.

(a) Evaluating the thermophysical properties of air at 350 K, determine the air velocity.

(b) In the laminar and turbulent regions, the local convection coefficients are, respectively,

$$h_{lam}(x) = C_{lam}\, x^{-0.5} \quad \text{and} \quad h_{turb} = C_{turb}\, x^{-0.2}$$

where, at $T = 350$ K, $C_{lam} = 8.845$ W/m$^{3/2}$ · K, $C_{turb} = 49.75$ W/m$^{1.8}$ · K, and x has units of m. Develop an expression for the average convection coefficient, $\bar{h}_{lam}(x)$, as a function of distance from the leading edge, x, for the laminar region, $0 \le x \le x_c$.

(c) Develop an expression for the average convection coefficient, $\bar{h}_{turb}(x)$, as a function of distance from the leading edge, x, for the turbulent region, $x_c \le x \le L$.

(d) On the same coordinates, plot the local and average convection coefficients, h_x and $\bar{h}_x$, respectively, as a function of x for $0 \le x \le L$.

6.15 A fan that can provide air speeds up to 50 m/s is to be used in a low-speed wind tunnel with atmospheric air at 25°C. If one wishes to use the wind tunnel to study flat-plate boundary layer behavior up to Reynolds numbers of $Re_x = 10^8$, what is the minimum plate length that should be used? At what distance from the leading edge would transition occur if the critical Reynolds number were $Re_{x,c} = 5 \times 10^5$?

6.16 Assuming a transition Reynolds number of 5×10^5, determine the distance from the leading edge of a flat plate at which transition will occur for each of the following fluids when $u_\infty = 1$ m/s: atmospheric air, engine oil, and mercury. In each case, calculate the transition location for fluid temperatures of 27°C and 77°C.

6.17 To a good approximation, the dynamic viscosity μ, the thermal conductivity k, and the specific heat c_p are independent of pressure. In what manner do the kinematic viscosity ν and thermal diffusivity α vary with pressure for an incompressible liquid and an ideal gas? Determine α of air at 350 K for pressures of 1, 5, and 10 atm. Assuming a transition Reynolds number of 5×10^5, determine the distance from the leading edge of a flat plate at which transition will occur for air at 350 K at pressures of 1, 5, and 10 atm with $u_\infty = 2$ m/s.

Similarity and Dimensionless Parameters

6.18 An object of irregular shape has a characteristic length of $L = 1$ m and is maintained at a uniform surface temperature of $T_s = 400$ K. When placed in atmospheric air at a temperature of $T_\infty = 300$ K and moving with a velocity of $V = 100$ m/s, the average heat flux from the surface to the air is 20,000 W/m^2. If a second object of the same shape, but with a characteristic length of $L = 5$ m, is maintained at a surface temperature of $T_s = 400$ K and is placed in atmospheric air at $T_\infty = 300$ K, what will the value of the average convection coefficient be if the air velocity is $V = 20$ m/s?

6.19 Experiments have shown that, for airflow at $T_\infty = 35°C$ and $V_1 = 100$ m/s, the rate of heat transfer from a turbine blade of characteristic length $L_1 = 0.15$ m and surface temperature $T_{s,1} = 300°C$ is $q_1 = 1500$ W. What would be the heat transfer rate from a second turbine blade of characteristic length $L_2 = 0.3$ m operating at $T_{s,2} = 400°C$ in airflow of $T_\infty = 35°C$ and $V_2 = 50$ m/s?

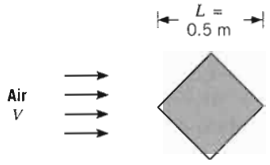
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The surface area of the blade may be assumed to be directly proportional to its characteristic length.

6.20 Experimental measurements of the convection heat transfer coefficient for a square bar in cross flow yielded the following values:

$\bar{h}_1 = 50\ \text{W/m}^2 \cdot \text{K}$ when $V_1 = 20\ \text{m/s}$

$\bar{h}_2 = 40\ \text{W/m}^2 \cdot \text{K}$ when $V_2 = 15\ \text{m/s}$



Assume that the functional form of the Nusselt number is $\overline{Nu} = C\,Re^m\,Pr^n$, where C, m, and n are constants.

(a) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 15$ m/s?

(b) What will be the convection heat transfer coefficient for a similar bar with $L = 1$ m when $V = 30$ m/s?

(c) Would your results be the same if the side of the bar, rather than its diagonal, were used as the characteristic length?

6.21 Experimental results for heat transfer over a flat plate with an extremely rough surface were found to be correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.9}\,Pr^{1/3}$$

where Nu_x is the local value of the Nusselt number at a position x measured from the leading edge of the plate. Obtain an expression for the ratio of the average heat transfer coefficient $\bar{h}_x$ to the local coefficient h_x.

6.22 Consider conditions for which a fluid with a free stream velocity of $V = 1$ m/s flows over a surface with a characteristic length of $L = 1$ m, providing an average convection heat transfer coefficient of $\bar{h} = 100\ \text{W/m}^2 \cdot \text{K}$. Calculate the dimensionless parameters $\overline{Nu}_L$, Re_L, Pr, and $\bar{j}_H$ for the following fluids: air, engine oil, mercury, and water. Assume the fluids to be at 300 K.

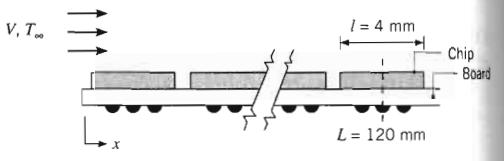
6.23 For flow over a flat plate of length L, the local heat transfer coefficient h_x is known to vary as $x^{-1/2}$, where x is the distance from the leading edge of the plate. What is the ratio of the average Nusselt number for the entire plate ($\overline{Nu}_L$) to the local Nusselt number at $x = L$ (Nu_L)?

6.24 For laminar boundary layer flow over a flat plate with air at 20°C and 1 atm, the thermal boundary layer thickness δ_t is approximately 13% larger than the velocity boundary layer thickness δ. Determine the ratio δ/δ_t if the fluid is ethylene glycol under the same flow conditions.

6.25 Sketch the variation of the velocity and thermal boundary layer thicknesses with distance from the leading edge of a flat plate for the laminar flow of air, water, engine oil, and mercury. For each case assume a mean fluid temperature of 300 K.

6.26 Forced air at $T_\infty = 25$°C and $V = 10$ m/s is used to cool electronic elements on a circuit board. One such element is a chip, 4 mm by 4 mm, located 120 mm from the leading edge of the board. Experiments have revealed that flow over the board is disturbed by the elements and that convection heat transfer is correlated by an expression of the form

$$Nu_x = 0.04 Re_x^{0.85}\,Pr^{1/3}$$



Estimate the surface temperature of the chip if it is dissipating 30 mW.

6.27 Consider the electronic elements that are cooled by forced convection in Problem 6.26. The cooling system is designed and tested at sea level ($P \approx 1$ atm), but the circuit board is sold to a customer in Mexico City, with an elevation of 2250 m and atmospheric pressure of 76.5 kPa.

(a) Estimate the surface temperature of the chip located 120 mm from the leading edge of the board when the board is operated in Mexico City. The dependence of various thermophysical properties upon pressure is noted in Problem 6.17.

(b) It is highly desirable for the chip operating temperature to be independent of the location of the customer. What air velocity is required for operation in Mexico City if the chip temperature is to be the same as at sea level?

6.28 Consider the chip on the circuit board of Problem 6.26. To ensure reliable operation over extended periods, the chip temperature should not exceed 85°C. Assuming the availability of forced air at $T_\infty = 25$°C and applicability of the prescribed heat transfer correlation, compute and plot the maximum allowable chip power dissipation P_c as a function of air velocity for $1 \le V \le 25$ m/s. If the chip surface has an emissivity of 0.80 and the board is mounted in a large enclosure whose walls are at 25°C, what is the effect of radiation on the P_c–V plot?

6.29 A major contributor to product defects in electronic modules relates to stresses induced during thermal

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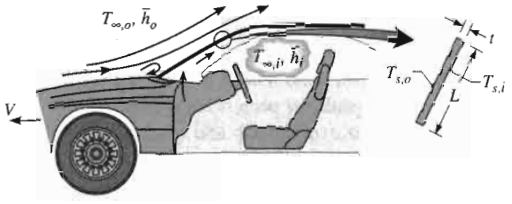
cycling (intermittent heating and cooling). For example, in circuit cards having active and passive components with materials of different thermal expansion coefficients, thermal stresses are the principal source of failure in component joints, such as soldered and wired connections. Although concern is generally for fatigue failure resulting from numerous excursions during the life of a product, it is possible to identify defective joints by performing accelerated thermal stress tests before the product is released to the customer. In such cases, it is important to achieve rapid thermal cycling to minimize disruptions to production schedules.

A manufacturer of circuit cards wishes to develop an apparatus for imposing rapid thermal transients on the cards by subjecting them to forced convection characterized by a relation of the form $\overline{Nu}_L = C\,Re_L^m\,Pr^n$, where $m = 0.8$ and $n = 0.33$. However, she does not know whether to use air ($k = 0.026$ W/m · K, $\nu = 1.6\times10^{-5}$ m²/s, $Pr = 0.71$) or a dielectric liquid ($k = 0.064$ W/m · K, $\nu = 10^{-6}$ m²/s, $Pr = 25$) as the working fluid. Assuming equivalent air and liquid velocities and validity of the lumped capacitance model for the components, obtain a quantitative estimate of the ratio of the thermal time constants for the two fluids. What fluid provides the faster thermal response?

6.30 To assess the efficacy of different liquids for cooling an object of given size and shape by forced convection, it is convenient to introduce a *figure of merit*, F_F, which combines the influence of all pertinent fluid properties on the convection coefficient. If the Nusselt number is governed by an expression of the form, $Nu_L \sim Re_L^m Pr^n$, obtain the corresponding relationship between F_F and the fluid properties. For representative values of $m = 0.80$ and $n = 0.33$, calculate values of F_F for air ($k = 0.026$ W/m · K, $\nu = 1.6 \times 10^{-5}$ m²/s, $Pr = 0.71$), water ($k = 0.600$ W/m · K, $\nu = 10^{-6}$ m²/s, $Pr = 5.0$), and a dielectric liquid ($k = 0.064$ W/m · K, $\nu = 10^{-6}$ m²/s, $Pr = 25$). What fluid is the most effective cooling agent?

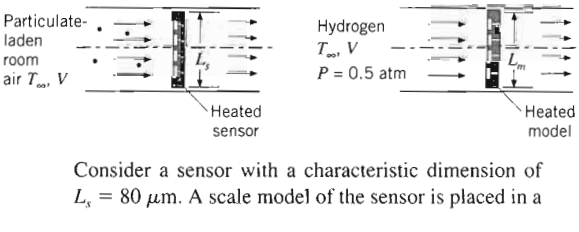
6.31 Gases are often used instead of liquids to cool electronics in avionics applications because of weight considerations. The cooling systems are often *closed* so that coolants other than air may be used. Gases with high figures of merit (see Problem 6.30) are desired. For representative values of $m = 0.85$ and $n = 0.33$ in the expression of Problem 6.30, determine the figures of merit for air, pure helium, pure xenon ($k = 0.006$ W/m · K, $\mu = 24.14 \times 10^{-6}$ N · s/m²), and an ideal He-Xe mixture containing 0.75 mole fraction of helium ($k = 0.0713$ W/m · K, $\mu = 25.95 \times 10^{-6}$ N · s/m²). Evaluate properties at 300 K and atmospheric pressure. For monatomic gases such as helium and xenon and their mixtures, the specific heat at constant pressure is well described by the relation $c_p = (5/2)\mathcal{R}/\mathcal{M}$.

6.32 The defroster of an automobile functions by discharging warm air on the inner surface of the windshield. To prevent condensation of water vapor on the surface, the temperature of the air and the surface convection coefficient ($T_{\infty,i}$, $\bar{h}_i$) must be large enough to maintain a surface temperature $T_{s,i}$ that is at least as high as the dewpoint ($T_{s,i} \geq T_{dp}$).



Consider a windshield of length $L = 800$ mm and thickness $t = 6$ mm and driving conditions for which the vehicle moves at a velocity of $V = 70$ mph in ambient air at $T_{\infty,o} = -15$°C. From laboratory experiments performed on a model of the vehicle, the average convection coefficient on the outer surface of the windshield is known to be correlated by an expression of the form $\overline{Nu}_L = 0.030\,Re_L^{0.8}\,Pr^{1/3}$, where $Re_L \equiv VL/\nu$. Properties of the ambient air may be approximated as $k = 0.023$ W/m · K, $\nu = 12.5\times10^{-6}$ m²/s, and $Pr = 0.71$. If $T_{dp} = 10$°C and $T_{\infty,i} = 50$°C, what is the smallest value of $\bar{h}_i$ required to prevent condensation on the inner surface?

6.33 A microscale detector monitors a steady flow ($T_\infty = 27$°C, $V = 10$ m/s) of air for the possible presence of small, hazardous particulate matter that may be suspended in the room. The sensor is heated to a slightly higher temperature in order to induce a chemical reaction associated with certain substances of interest that might impinge on the sensor's active surface. The active surface produces an electric current if such surface reactions occur; the electric current is then sent to an alarm. To maximize the sensor head's surface area and, in turn, the probability of capturing and detecting a particle, the sensor head is designed with a very complex shape. The value of the average heat transfer coefficient associated with the heated sensor must be known so that the required electrical power to the sensor can be determined.



Consider a sensor with a characteristic dimension of $L_s = 80$ μm. A scale model of the sensor is placed in a

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recirculating (closed) wind tunnel using hydrogen as the working fluid. If the wind tunnel operates at a hydrogen absolute pressure of 0.5 atm and velocity of $V = 0.5$ m/s, find the required hydrogen temperature and characteristic dimension of the scale model, L_m.

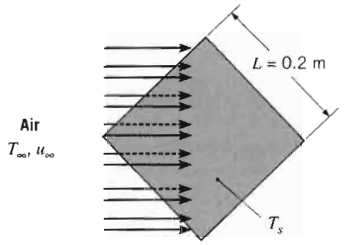
Reynolds Analogy

6.34 A thin, flat plate that is 0.2 m by 0.2 m on a side is oriented parallel to an atmospheric airstream having a velocity of 40 m/s. The air is at a temperature of $T_\infty = 20°C$, while the plate is maintained at $T_s = 120°C$. The air flows over the top and bottom surfaces of the plate, and measurement of the drag force reveals a value of 0.075 N. What is the rate of heat transfer from both sides of the plate to the air?

6.35 Atmospheric air is in parallel flow ($u_\infty = 15$ m/s, $T_\infty = 15°C$) over a flat heater surface that is to be maintained at a temperature of 140°C. The heater surface area is 0.25 m^2, and the airflow is known to induce a drag force of 0.25 N on the heater. What is the electrical power needed to maintain the prescribed surface temperature?

6.36 For flow over a flat plate with an extremely rough surface, convection heat transfer effects are known to be correlated by the expression of Problem 6.21. For airflow at 50 m/s, what is the surface shear stress at $x = 1$ m from the leading edge of the plate? Assume the air to be at a temperature of 300 K.

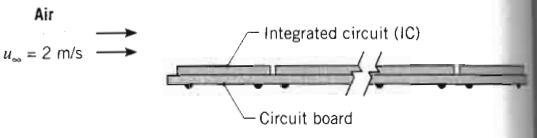
6.37 A thin, flat plate that is 0.2 m by 0.2 m on a side with extremely rough top and bottom surfaces is placed in a wind tunnel so that its surfaces are parallel to an atmospheric air stream having a velocity of 30 m/s. The air is at a temperature of $T_\infty = 20°C$ while the plate is maintained at $T_s = 80°C$. The plate is rotated 45° about its center point, as shown in the schematic. Air flows over the top and bottom surfaces of the plate, and measurement of the heat transfer rate is 2000 W. What is the drag force on the plate?



Top view of thin, flat plate

6.38 As a means of preventing ice formation on the wings of a small, private aircraft, it is proposed that electric resistance heating elements be installed within the wings. To determine representative power requirements, consider nominal flight conditions for which the plane moves at 100 m/s in air that is at a temperature of −23°C and has properties of $k = 0.022$ W/m · K, $Pr = 0.72$, and $\nu = 16.3 \times 10^{-6}$ m^2/s. If the characteristic length of the airfoil is $L = 2$ m and wind tunnel measurements indicate an average friction coefficient of $\overline{C}_f = 0.0025$ for the nominal conditions, what is the average heat flux needed to maintain a surface temperature of $T_s = 5°C$?

6.39 A circuit board with a dense distribution of integrated circuits (ICs) and dimensions of 120 mm by 120 mm on a side is cooled by the parallel flow of atmospheric air with a velocity of 2 m/s.



From wind tunnel tests under the same flow conditions, the average frictional shear stress on the upper surface is determined to be 0.0625 N/m^2. What is the allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed the ambient air temperature by more than 25°C? Evaluate the thermophysical properties of air at 300 K.

Mass Transfer Coefficients

6.40 On a summer day the air temperature is 27°C and the relative humidity is 30%. Water evaporates from the surface of a lake at a rate of 0.10 kg/h per square meter of water surface area. The temperature of the water is also 27°C. Determine the value of the convection mass transfer coefficient.

6.41 It is observed that a 230-mm-diameter pan of water at 23°C has a mass loss rate of 1.5×10^{-5} kg/s when the ambient air is dry and at 23°C.

(a) Determine the convection mass transfer coefficient for this situation.

(b) Estimate the evaporation mass loss rate when the ambient air has a relative humidity of 50%.

(c) Estimate the evaporation mass loss rate when the water and ambient air temperatures are 47°C, assuming that the convection mass transfer coefficient remains unchanged and the ambient air is dry.

6.42 The rate at which water is lost because of evaporation from the surface of a body of water may be determined

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by measuring the surface recession rate. Consider a summer day for which the temperature of both the water and the ambient air is 305 K and the relative humidity of the air is 40%. If the surface recession rate is known to be 0.1 mm/h, what is the rate at which mass is lost because of evaporation per unit surface area? What is the convection mass transfer coefficient?

6.43 Photosynthesis, as it occurs in the leaves of a green plant, involves transport of carbon dioxide (CO_2) from the atmosphere to the chloroplasts of the leaves, and the rate of photosynthesis may be quantified in terms of the rate of CO_2 assimilation by the chloroplasts. This assimilation is strongly influenced by CO_2 transfer through the boundary layer that develops on the leaf surface. Under conditions for which the density of CO_2 is 6×10^{-4} kg/m^3 in the air and 5×10^{-4} kg/m^3 at the leaf surface and the convection mass transfer coefficient is 10^{-2} m/s, what is the rate of photosynthesis in terms of kilograms of CO_2 assimilated per unit time and area of leaf surface?

6.44 Species A is evaporating from a flat surface into species B. Assume that the concentration profile for species A in the concentration boundary layer is of the form $C_A(y) = Dy^2 + Ey + F$, where D, E, and F are constants at any x location and y is measured along a normal from the surface. Develop an expression for the mass transfer convection coefficient h_m in terms of these constants, the concentration of A in the free stream $C_{A,\infty}$ and the mass diffusivity D_{AB}. Write an expression for the molar flux of mass transfer by convection for species A.

Similarity and Heat–Mass Transfer Analogy

6.45 Consider cross flow of gas X over an object having a characteristic length of $L = 0.1$ m. For a Reynolds number of 1×10^4, the average heat transfer coefficient is 25 W/m$^2 \cdot$ K. The same object is then impregnated with liquid Y and subjected to the same flow conditions. Given the following thermophysical properties, what is the average convection mass transfer coefficient?

	ν (m^2/s)	k (W/m $\cdot$ K)	α (m^2/s)
Gas X	21×10^{-6}	0.030	29×10^{-6}
Liquid Y	3.75×10^{-7}	0.665	1.65×10^{-7}
Vapor Y	4.25×10^{-5}	0.023	4.55×10^{-5}
Mixture of gas X–vapor Y:		$Sc = 0.72$	

6.46 Consider conditions for which a fluid with a free stream velocity of $V = 1$ m/s flows over an evaporating or subliming surface with a characteristic length of $L = 1$ m, providing an average mass transfer convection coefficient of $\bar{h}_m = 10^{-2}$ m/s. Calculate the dimensionless parameters $\overline{Sh}_L$, Re_L, Sc, and $\bar{j}_m$ for the following combinations: airflow over water, airflow over naphthalene, and warm glycerol over ice. Assume a fluid temperature of 300 K and a pressure of 1 atm.

6.47 An object of irregular shape has a characteristic length of $L = 1$ m and is maintained at a uniform surface temperature of $T_s = 325$ K. It is suspended in an airstream that is at atmospheric pressure ($p = 1$ atm) and has a velocity of $V = 100$ m/s and a temperature of $T_\infty = 275$ K. The average heat flux from the surface to the air is 12,000 W/m^2. Referring to the foregoing situation as case 1, consider the following cases and determine whether conditions are analogous to those of case 1. Each case involves an object of the same shape, which is suspended in an airstream in the same manner. Where analogous behavior does exist, determine the corresponding value of the average convection coefficient.

(a) The values of T_s, T_∞, and p remain the same, but $L = 2$ m and $V = 50$ m/s.

(b) The values of T_s and T_∞ remain the same, but $L = 2$ m, $V = 50$ m/s, and $p = 0.2$ atm.

(c) The surface is coated with a liquid film that evaporates into the air. The entire system is at 300 K, and the diffusion coefficient for the air–vapor mixture is $D_{AB} = 1.12 \times 10^{-4}$ m^2/s. Also, $L = 2$ m, $V = 50$ m/s, and $p = 1$ atm.

(d) The surface is coated with another liquid film for which $D_{AB} = 1.12 \times 10^{-4}$ m^2/s, and the system is at 300 K. In this case $L = 2$ m, $V = 250$ m/s, and $p = 0.2$ atm.

6.48 On a cool day in April a scantily clothed runner is known to lose heat at a rate of 500 W because of convection to the surrounding air at $T_\infty = 10$°C. The runner's skin remains dry and at a temperature of $T_s = 30$°C. Three months later, the runner is moving at the same speed, but the day is warm and humid with a temperature of $T_\infty = 30$°C and a relative humidity of $\phi_\infty = 60\%$. The runner is now drenched in sweat and has a uniform surface temperature of 35°C. Under both conditions constant air properties may be assumed with $\nu = 1.6 \times 10^{-5}$ m^2/s, $k = 0.026$ W/m $\cdot$ K, $Pr = 0.70$, and D_{AB} (water vapor–air) $= 2.3 \times 10^{-5}$ m^2/s.

(a) What is the rate of water loss due to evaporation on the summer day?

(b) What is the total convective heat loss on the summer day?

6.49 An object of irregular shape 1 m long maintained at a constant temperature of 100°C is suspended in an airstream having a free stream temperature of 0°C, a

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pressure of 1 atm, and a velocity of 120 m/s. The air temperature measured at a point near the object in the airstream is 80°C. A second object having the same shape is 2 m long and is suspended in an airstream in the same manner. The air free stream velocity is 60 m/s. Both the air and the object are at 50°C, and the total pressure is 1 atm. A plastic coating on the surface of the object is being dried by this process. The molecular weight of the vapor is 82 and the saturation pressure at 50°C for the plastic material is 0.0323 atm. The mass diffusivity for the vapor in air at 50°C is 2.60×10^{-5} m^2/s.

(a) For the second object, at a location corresponding to the point of measurement on the first object, determine the vapor concentration and partial pressure.

(b) If the average heat flux q'' is 2000 W/m^2 for the first object, determine the average mass flux n''_A (kg/s · m^2) for the second object.

6.50 An industrial process involves the evaporation of water from a liquid film that forms on a contoured surface. Dry air is passed over the surface, and from laboratory measurements the convection heat transfer correlation is of the form

$$\overline{Nu}_L = 0.43 Re_L^{0.58} Pr^{0.4}$$

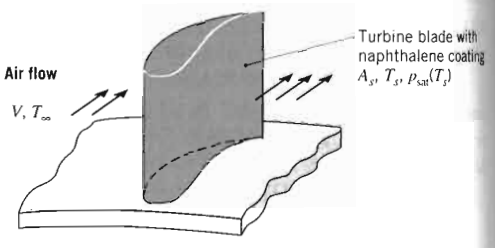
(a) For an air temperature and velocity of 27°C and 10 m/s, respectively, what is the rate of evaporation from a surface of 1-m^2 area and characteristic length $L = 1$ m? Approximate the density of saturated vapor as $\rho_{A,sat} = 0.0077$ kg/m^3.

(b) What is the steady-state temperature of the liquid film?

6.51 The *naphthalene sublimation technique* involves use of a mass transfer experiment coupled with an analysis based on the heat and mass transfer analogy to obtain local or average convection heat transfer coefficients for complex surface geometries. A coating of naphthalene, which is a volatile solid at room temperature, is applied to the surface and is then subjected to airflow in a wind tunnel. Alternatively, solid objects may be cast from liquid naphthalene. Over a designated time interval, Δt, there is a discernible loss of naphthalene due to sublimation, and by measuring the surface recession at locations of interest or the mass loss of the sample, local or average mass transfer coefficients may be determined.

Consider a rectangular rod of naphthalene exposed to air in cross flow at $V = 10$ m/s, $T_\infty = 300$ K, as in Problem 6.10, except now $c = 10$ mm and $d = 30$ mm. Determine the change in mass of the $L = 500$ mm long rod over a time period of $\Delta t = 30$ min. Naphthalene has a molecular weight of $\mathcal{M}_A = 128.16$ kg/kmol, and its solid–vapor saturation pressure at 27°C and 1 atm is $p_{A,sat} = 1.33 \times 10^{-4}$ bar.

6.52 Consider application of the naphthalene sublimation technique (Problem 6.51) to a gas turbine blade that is coated with naphthalene and has a surface area of $A_s = 0.05$ m^2.



To determine the average convection heat transfer coefficient for a representative operating condition, an experiment is performed in which the coated blade is exposed for 30 min to atmospheric air at the desired velocity and a temperature of $T_\infty = 27$°C. During the experiment the surface temperature is $T_s = 27$°C, and at its conclusion the mass of the blade is reduced by $\Delta m = 8$ g. What is the average convection heat transfer coefficient associated with the operating condition?

6.53 A manufacturer of ski equipment wishes to develop headgear that will offer enhanced thermal protection for skiers on cold days at the slopes. Headgear can be made with good thermal insulating characteristics, but it tends to be bulky and cumbersome. Skiers prefer comfortable, lighter gear that offers good visibility, but such gear tends to have poor thermal insulating characteristics. The manufacturer decides to take a new approach to headgear design by concentrating the insulation in areas about the head that are prone to the highest heat losses from the skier and minimizing use of insulation in other locations. Hence, the manufacturer must determine the local heat transfer coefficients associated with the human head with a velocity of $V = 10$ m/s directed normal to the face and an air temperature of -13°C. A young engineer is assigned the task of building the heat transfer experiment but soon finds the experimental challenges to be daunting. She decides to make use of the heat and mass transfer analogy and the naphthalene sublimation technique (see Problem 6.51) and casts head shapes of solid naphthalene with characteristic dimensions that are half-scale (i.e., the models are half as large as the full-scale head).

(a) What wind tunnel velocity ($T_\infty = 300$ K) is needed in order to apply the experimental results to the human head associated with $V = 10$ m/s?

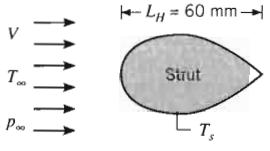
(b) A wind tunnel experiment is performed for $\Delta t = 120$ min, $T_\infty = 27$°C. The engineer finds that the

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naphthalene has receded by $\delta_1 = 0.1$ mm at the back of the head, $\delta_2 = 0.32$ mm in the middle of the forehead, and $\delta_3 = 0.64$ mm on the ear. Determine the heat transfer coefficients at these locations for the full-scale head at $-13°C$. The density of solid naphthalene is $\rho_{A,sol} = 1025$ kg/m^3.

(c) After the new headgear is designed, the models are fitted with the new gear (half-scale) and the experiments are repeated. Some areas of the model that were found to have extremely small local heat transfer coefficients are left uncovered since insulating these areas would have little overall benefit in reducing heat losses during skiing. Would you expect the local heat transfer coefficients for these exposed areas to remain the same as before fitting the model with the headgear? Explain why.

6.54 A streamlined strut supporting a bearing housing is exposed to a hot airflow from an engine exhaust. It is necessary to run experiments to determine the average convection heat transfer coefficient $\bar{h}$ from the air to the strut in order to be able to cool the strut to the desired surface temperature T_s. It is decided to run mass transfer experiments on an object of the same shape and to obtain the desired heat transfer results by using the heat and mass transfer analogy.



The mass transfer experiments were conducted using a half-size model strut constructed from naphthalene exposed to an airstream at 27°C. Mass transfer measurements yielded these results:

Re_L	$\overline{Sh}_L$
60,000	282
120,000	491
144,000	568
288,000	989

(a) Using the mass transfer experimental results, determine the coefficients C and m for a correlation of the form $\overline{Sh}_L = C\,Re_L^m\,Sc^{1/3}$.

(b) Determine the average convection heat transfer coefficient $\bar{h}$ for the full-sized strut, $L_H = 60$ mm, when exposed to a free stream airflow with $V = 60$ m/s, $T_\infty = 184°C$, and $p_\infty = 1$ atm when $T_s = 70°C$.

(c) The surface area of the strut can be expressed as $A_s = 2.2L_H \cdot l$, where l is the length normal to the page. For the conditions of part (b), what is the change in the rate of heat transfer to the strut if the characteristic length L_H is doubled?

6.55 Consider the conditions of Problem 6.3, but with a thin film of water on the surface. If the air is dry and the Schmidt number Sc is 0.6, what is the evaporative mass flux? Is there net energy transfer to or from the water?

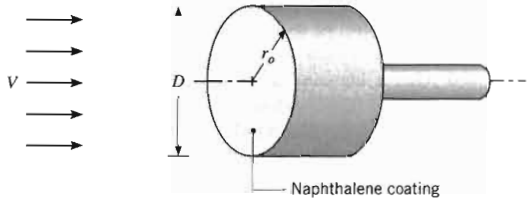
6.56 Consider the conditions of Problem 6.7, for which a heat transfer experiment yielded the prescribed distribution of the local convection coefficient, $h_x(x)$. The experiment was performed for surface and free stream temperatures of 310 and 290 K, respectively. Now consider repeating the experiment under conditions for which the surface is coated with a thin layer of naphthalene and both the surface and air are at 300 K. What is the corresponding value of the average convection mass transfer coefficient, $\bar{h}_{mL}$?

6.57 Using the naphthalene sublimation technique, the radial distribution of the local convection mass transfer coefficient for uniform flow normal to a circular disk has been correlated by an expression of the form

$$Sh_D \equiv \frac{h_m(r)D}{D_{AB}} = Sh_o\left[1 + a\left(\frac{r}{r_o}\right)^n\right]$$

The stagnation point Sherwood number (Sh_o) depends on the Reynolds ($Re_D = VD/\nu$) and Schmidt ($Sc = \nu/D_{AB}$) numbers, and data have been correlated by the following expression:

$$Sh_o = \frac{h_m(r=0)D}{D_{AB}} = 0.814 Re_D^{1/2}\,Sc^{0.36}$$

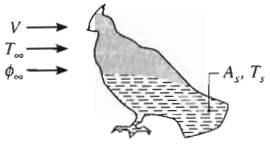


Obtain an expression for the average Nusselt number ($\overline{Nu}_D = \bar{h}D/k$) corresponding to heat transfer from an isothermal disk exposed to the foregoing flow conditions. If $a = 1.2$ and $n = 5.5$, what is the rate of heat transfer from a disk of diameter $D = 20$ mm and surface temperature $T_s = 125°C$ to an airstream for which $Re_D = 5 \times 10^4$ and $T_\infty = 25°C$? Typically, boundary layer development from a stagnation point yields a decaying convection coefficient with increasing

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distance from the stagnation point. Provide a plausible explanation for why the opposite trend is observed for the disk.

6.58 To reduce the threat of predators, the sand grouse, a bird of Kenya, will lay its eggs in locations well removed from sources of groundwater. To bring water to its chicks, the grouse will then fly to the nearest source and, by submerging the lower part of its body, will entrain water within its plumage. The grouse will then return to its nest, and the chicks will imbibe water from the plumage. Of course, if the time of flight is too long, evaporative losses could cause a significant reduction in the water content of the plumage and the chicks could succumb to dehydration.



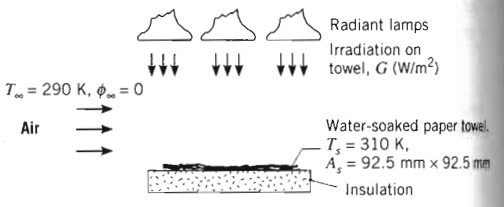
To gain a better understanding of convective transfer during flight, wind tunnel studies were performed using molded models of the grouse. By heating that portion of the model which corresponds to the water-encapsulating plumage, an average convection heat transfer coefficient was determined. Results for different air speeds and model sizes were then used to develop an empirical correlation of the form

$$\overline{Nu}_L = 0.034 Re_L^{4/5} Pr^{1/3}$$

The effective surface area of the water-encapsulating portion of the plumage is designated as A_s, and the characteristic length is defined as $L = (A_s)^{1/2}$.

Consider conditions for which a grouse has entrained 0.05 kg of water within plumage of $A_s = 0.04$ m^2 and is returning to its nest at a constant speed of $V = 30$ m/s. The ambient air is stagnant and at a temperature and relative humidity of $T_\infty = 37°C$ and $\phi_\infty = 25\%$, respectively. If, throughout the flight, the surface A_s is covered with a liquid water film at $T_s = 32°C$, what is the maximum allowable distance of the nest from the water source, if the bird must return with at least 50% of its initial water supply? Properties of the air and the air–vapor mixture may be taken to be $\nu = 16.7 \times 10^{-6}$ m^2/s and $D_{AB} = 26.0 \times 10^{-6}$ m^2/s.

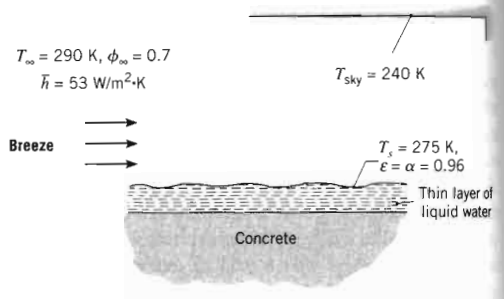
6.59 A laboratory experiment involves simultaneous heat and mass transfer from a water-soaked towel experiencing irradiation from a bank of radiant lamps and parallel flow of air over its surface. Using a convection correlation to be introduced in Chapter 7, the average heat transfer convection coefficient is estimated to be $\bar{h} = 28.7$ W/m$^2 \cdot$ K. Assume that the radiative properties of the towel are those of water, for which $\alpha = \varepsilon = 0.96$, and that the surroundings are at 300 K.



(a) Determine the rate at which water evaporates from the towel, n_A (kg/s).

(b) Perform an energy balance on the towel to determine the net rate of radiation transfer, q_{rad} (W), to the towel. Determine the irradiation G (W/m^2).

6.60 In the spring, concrete surfaces such as sidewalks and driveways are sometimes very wet in the morning, even when it has not rained during the night. Typical nighttime conditions are shown in the sketch.



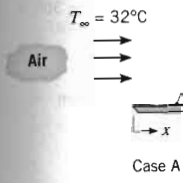
(a) Determine the heat fluxes associated with convection, q''_{conv}, evaporation, q''_{evap}, and radiation exchange with the sky, q''_{rad}.

(b) Do your calculations suggest why the concrete is wet instead of dry? Explain briefly.

(c) Is heat flowing from the liquid layer to the concrete? Or from the concrete to the liquid layer? Determine the heat flux by conduction into or out of the concrete.

6.61 Dry air at 32°C flows over a wetted (water) plate of 0.2 m^2 area. The average convection coefficient is $\bar{h} = 20$ W/m$^2 \cdot$ K, and the heater power required to maintain the plate at a temperature of 27°C is 432 W. Estimate the power required to maintain the wetted plate at a temperature of 37°C in dry air at 32°C if the convection coefficients remain unchanged.

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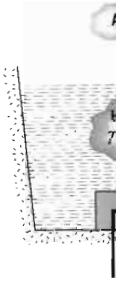
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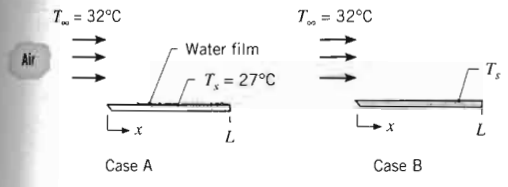
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6.62 Dry air at 32°C flows over a wetted plate of length 200 mm and width 1 m (case A). An imbedded electrical heater supplies 432 W and the surface temperature is 27°C.



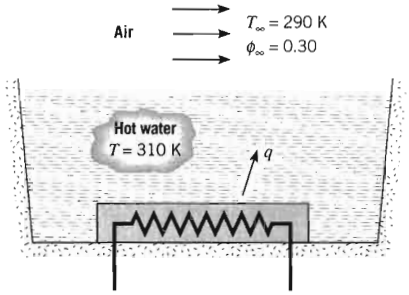
(a) What is the evaporation rate of water from the plate (kg/h)?

(b) After a long period of operation, all the water is evaporated from the plate and its surface is dry (case B). For the same free stream conditions and the same heater power as case A, estimate the temperature of the plate, T_s.

Evaporative Cooling

6.63 A 20-mm diameter sphere is suspended in a dry air stream with a temperature of 22°C. The power supplied to an embedded electrical heater within the sphere is 2.51 W when the surface temperature is 32°C. How much power is required to maintain the sphere at 32°C if its outer surface has a thin porous covering saturated with water? Evaluate the properties of air and the diffusion coefficient of the air–water vapor mixture at 300 K.

6.64 A successful California engineer has installed a circular hot tub in his back yard and finds that for the typical operating conditions prescribed below, water must be added at a rate of 0.001 kg/s in order to maintain a fixed water level in the tub.



If the tub is well insulated on its sides and bottom and the temperature of the makeup water is equal to that of the tub water, at what rate must electrical heaters supply energy to maintain the tub water at 310 K?

6.65 It is known that on clear nights the air temperature need not drop below 0°C before a thin layer of water on the ground will freeze. Consider such a layer of water on a clear night for which the effective sky temperature is −30°C and the convection heat transfer coefficient due to wind motion is $h = 25\ \text{W/m}^2 \cdot \text{K}$. The water may be assumed to have an emissivity of 1.0 and to be insulated from the ground as far as conduction is concerned.

(a) Neglecting evaporation, determine the lowest temperature the air can have without the water freezing.

(b) For the conditions given, estimate the mass transfer coefficient for water evaporation h_m (m/s).

(c) Accounting now for the effect of evaporation, what is the lowest temperature the air can have without the water freezing? Assume the air to be dry.

6.66 An expression for the actual water vapor partial pressure in terms of wet-bulb and dry-bulb temperatures, referred to as the Carrier equation, is given as

$$p_v = p_{gw} - \frac{(p - p_{gw})(T_{db} - T_{wb})}{1810 - T_{wb}}$$

where p_v, p_{gw}, and p are the actual partial pressure, the saturation pressure at the wet-bulb temperature, and the total pressure (all in bars), while T_{db} and T_{wb} are the dry- and wet-bulb temperatures in kelvins. Consider air at 1 atm and 37.8°C flowing over a wet-bulb thermometer that indicates 21.1°C.

(a) Using Carrier's equation, calculate the partial pressure of the water vapor in the free stream. What is the relative humidity?

(b) Refer to a psychrometric chart and obtain the relative humidity directly for the conditions indicated. Compare the result with part (a).

(c) Use Equation 6.65 to determine the relative humidity. Compare the result to parts (a) and (b).

6.67 A wet-bulb thermometer consists of a mercury-in-glass thermometer covered with a wetted (water) fabric. When suspended in a stream of air, the steady-state thermometer reading indicates the wet-bulb temperature T_{wb}. Obtain an expression for determining the relative humidity of the air from knowledge of the air temperature (T_∞), the wet-bulb temperature, and appropriate air and water vapor properties. If T_∞ = 45°C and T_{wb} = 25°C, what is the relative humidity of the airstream?

6.68 An industrial process involves evaporation of a thin water film from a contoured surface by heating it from

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below and forcing air across it. Laboratory measurements for this surface have provided the following heat transfer correlation:

$$\overline{Nu}_L = 0.43 Re_L^{0.58} Pr^{0.4}$$

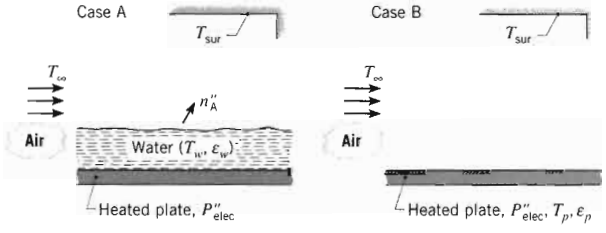
The air flowing over the surface has a temperature of 290 K, a velocity of 10 m/s, and is completely dry ($\phi_\infty = 0$). The surface has a length of 1 m and a surface area of 1 m^2. Just enough energy is supplied to maintain its steady-state temperature at 310 K.

(a) Determine the heat transfer coefficient and the rate at which the surface loses heat by convection.

(b) Determine the mass transfer coefficient and the evaporation rate (kg/h) of the water on the surface.

(c) Determine the rate at which heat must be supplied to the surface for these conditions.

6.69 A 2-mm-thick layer of water on an electrically heated plate is maintained at a temperature of $T_w = 340$ K, as dry air at $T_\infty = 300$ K flows over the surface of the water (case A). The arrangement is in large surroundings that are also at 300 K.

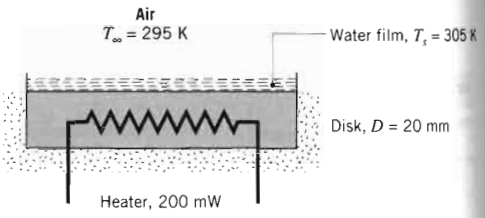


(a) If the evaporative flux from the surface of the water to the air is $n''_A = 0.030$ kg/s · m^2, what is the corresponding value of the convection mass transfer coefficient? How long will it take for the water to completely evaporate?

(b) What is the corresponding value of the convection heat transfer coefficient and the rate at which electrical power must be supplied per unit area of the plate to maintain the prescribed temperature of the water? Property values of the air are $\rho = 1.08$ kg/m^3, $c_p = 1008$ J/kg · K, and $k = 0.028$ W/m · K. The binary diffusion coefficient for air and water vapor is $D_{AB} = 0.29 \times 10^{-4}$ m^2/s, and the emissivity of water is $\varepsilon_w = 0.95$.

(c) If the electrical power determined in part (b) is maintained after complete evaporation of the water (case B), what is the resulting temperature of the plate, whose emissivity is $\varepsilon_p = 0.60$?

6.70 A disk of 20-mm diameter is covered with a water film. Under steady-state conditions, a heater power of 200 mW is required to maintain the disk–water film at 305 K in dry air at 295 K and the observed evaporation rate is 2.55×10^{-4} kg/h.



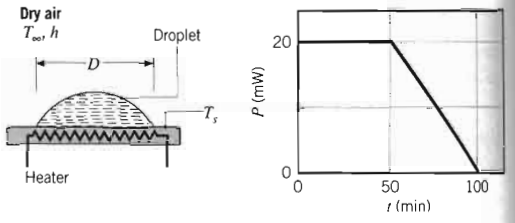
(a) Calculate the average mass transfer convection coefficient, $\bar{h}_m$, for the evaporation process.

(b) Calculate the average heat transfer convection coefficient, $\bar{h}$.

(c) Do the values of $\bar{h}_m$ and $\bar{h}$ satisfy the heat–mass analogy?

(d) If the relative humidity of the ambient air at 295 K were increased from 0 (dry) to 0.50, but the power supplied to the heater was maintained at 200 mW, would the evaporation rate increase or decrease? Would the disk temperature increase or decrease?

6.71 An experiment is conducted to determine the average mass transfer convection coefficient of a small droplet using a heater controlled to operate at a constant temperature. The power history required to completely evaporate the droplet at a temperature of 37°C is shown in the sketch. It was observed that, as the droplet dried, its wetted diameter on the heater surface remained nearly constant at a value of 4 mm.



(a) Calculate the average mass transfer convection coefficient based on the wetted area during the evaporation process when the droplet, heater, and the *dry* ambient air are at 37°C.

(b) How much energy will be required to evaporate the droplet if the *dry* ambient air temperature is 27°C, while the droplet–heater temperature remains at 37°C?

6.72 It is desired to develop a simple model for predicting the temperature–time history of a plate during the drying

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cycle in a dishwasher. Following the wash cycle the plate is at $T_p(t) = T_p(0) = 65°C$ and the air in the dishwasher is completely saturated ($\phi_\infty = 1.0$) at $T_\infty = 55°C$. The values of the plate surface area A_s, mass M, and specific heat c are such that $Mc/A_s = 1600$ J/m$^2 \cdot$ K.

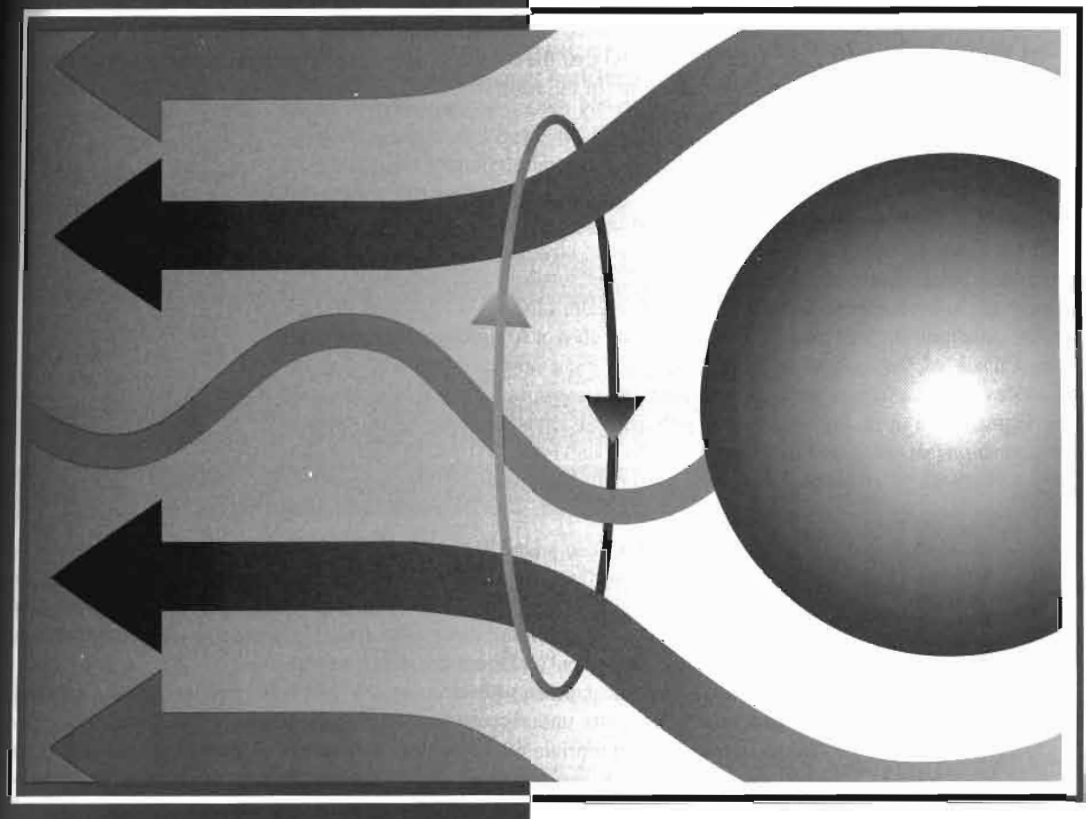
(a) Assuming the plate is completely covered by a thin film of water and neglecting the thermal resistances of the film and plate, derive a differential equation for predicting the plate temperature as a function of time.

(b) For the initial conditions ($t = 0$) estimate the change in plate temperature with time, dT/dt (°C/s), assuming that the average heat transfer coefficient on the plate is 3.5 W/m$^2 \cdot$ K.

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CHAPTER 7

External Flow



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In this chapter we focus on the problem of computing heat and mass transfer rates to or from a surface in *external flow*. In such a flow boundary layers develop freely, without constraints imposed by adjacent surfaces. Accordingly, there will always exist a region of the flow outside the boundary layer in which velocity, temperature, and/or concentration gradients are negligible. Examples include fluid motion over a flat plate (inclined or parallel to the free stream velocity) and flow over curved surfaces such as a sphere, cylinder, airfoil, or turbine blade.

For the moment we confine our attention to problems of *low-speed, forced convection* with *no phase change* occurring within the fluid. In addition, we will not consider potential micro- or nanoscale effects within the fluid, as described in Section 2.2, in this chapter. In *forced* convection, the relative motion between the fluid and the surface is maintained by external means, such as a fan or a pump, and not by buoyancy forces due to temperature gradients in the fluid (*natural* convection). *Internal flows, natural convection,* and *convection with phase change* are treated in Chapters 8, 9, and 10, respectively.

Our primary objective is to determine convection coefficients for different flow geometries. In particular, we wish to obtain specific forms of the functions that represent these coefficients. By nondimensionalizing the boundary layer equations in Chapter 6, we found that the local and average convection coefficients may be correlated by equations of the form

Heat Transfer:

$$Nu_x = f(x^*, Re_x, Pr) \tag{6.49}$$

$$\overline{Nu}_x = f(Re_x, Pr) \tag{6.50}$$

Mass Transfer:

$$Sh_x = f(x^*, Re_x, Sc) \tag{6.53}$$

$$\overline{Sh}_x = f(Re_x, Sc) \tag{6.54}$$

The subscript x has been added to emphasize our interest in conditions at a particular location on the surface. The overbar indicates an average from $x^* = 0$, where the boundary layer begins to develop, to the location of interest. Recall that *the problem of convection* is one of obtaining these functions. There are two approaches that we could take, one theoretical and the other experimental.

The *experimental* or *empirical approach* involves performing heat or mass transfer measurements under controlled laboratory conditions and correlating the data in terms of appropriate dimensionless parameters. A general discussion of the approach is provided in Section 7.1. It has been applied to many different geometries and flow conditions, and important results are presented in Sections 7.2 through 7.8.

The *theoretical approach* involves solving the boundary layer equations for a particular geometry. For example, obtaining the temperature profile T^* from such a

7.1 The Empirica

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solution, Equation 6.48 may be used to evaluate the local Nusselt number Nu_x, and therefore the local convection coefficient h_x. With knowledge of how h_x varies over the surface, Equation 6.13 may then be used to determine the average convection coefficient $\bar{h}_x$, and therefore the Nusselt number $\overline{Nu}_x$. In Section 7.2.1 this approach is illustrated by using the *similarity method* to obtain an *exact solution* of the boundary layer equations for a flat plate in parallel, laminar flow [1–3]. An *approximate solution* to the same problem is obtained in Appendix F by using the *integral method* [4].

7.1 The Empirical Method

The manner in which a convection heat transfer correlation may be obtained experimentally is illustrated in Figure 7.1. If a prescribed geometry, such as the flat plate in parallel flow, is heated electrically to maintain $T_s > T_\infty$, convection heat transfer occurs from the surface to the fluid. It would be a simple matter to measure T_s and T_∞, as well as the electrical power, $E \cdot I$, which is equal to the total heat transfer rate q. The convection coefficient $\bar{h}_L$, which is an average associated with the entire plate, could then be computed from Newton's law of cooling, Equation 6.12. Moreover, from knowledge of the characteristic length L and the fluid properties, the Nusselt, Reynolds, and Prandtl numbers could be computed from their definitions, Equations 6.50, 6.41, and 6.42, respectively.

The foregoing procedure could be repeated for a variety of test conditions. We could vary the velocity u_∞ and the plate length L, as well as the nature of the fluid, using, for example, air, water, and engine oil, which have substantially different Prandtl numbers. We would then be left with many different values of the Nusselt number corresponding to a wide range of Reynolds and Prandtl numbers, and the results could be plotted on a *log–log* scale, as shown in Figure 7.2*a*. Each symbol represents a unique set of test conditions. As is often the case, the results associated with a given fluid, and hence a fixed Prandtl number, fall close to a straight line, indicating a power law dependence of the Nusselt number on the Reynolds number. Considering all the fluids, the data may then be represented by an algebraic expression of the form

$$\overline{Nu}_L = C\, Re_L^m\, Pr^n \tag{7.1}$$

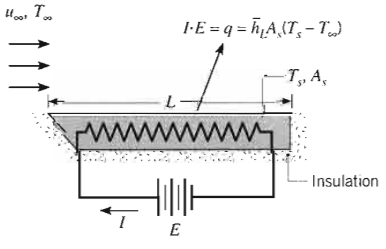


FIGURE 7.1 Experiment for measuring the average convection heat transfer coefficient $\bar{h}_L$.

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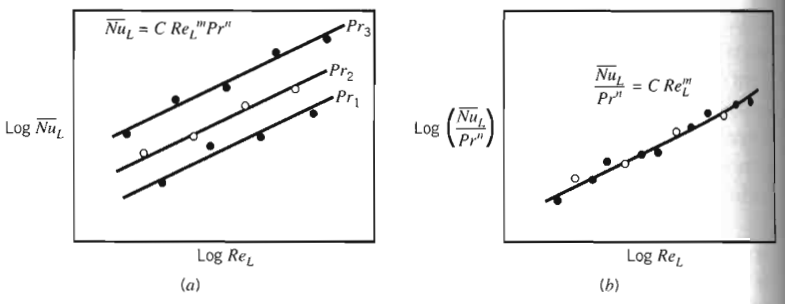


FIGURE 7.2 Dimensionless representation of convection heat transfer measurements.

Since the values of C, m, and n are often independent of the nature of the fluid, the family of straight lines corresponding to different Prandtl numbers can be collapsed to a single line by plotting the results in terms of the ratio, $\overline{Nu}_L/Pr^n$, as shown in Figure 7.2*b*.

Because Equation 7.1 is inferred from experimental measurements, it is termed an *empirical correlation.* The specific values of the coefficient C and the exponents m and n vary with the nature of the surface geometry and the type of flow.

We will use expressions of the form given by Equation 7.1 for many special cases, and it is important to note that the assumption of *constant fluid properties* is often implicit in the results. However, we know that the fluid properties vary with temperature across the boundary layer and that this variation can certainly influence the heat transfer rate. This influence may be handled in one of two ways. In one method, Equation 7.1 is used with all properties evaluated at a mean boundary layer temperature T_f, termed the *film temperature.*

$$T_f \equiv \frac{T_s + T_\infty}{2} \tag{7.2}$$

The alternate method is to evaluate all properties at T_∞ and to multiply the right-hand side of Equation 7.1 by an additional parameter to account for the property variations. The parameter is commonly of the form $(Pr_\infty/Pr_s)^r$ or $(\mu_\infty/\mu_s)^r$, where the subscripts ∞ and s designate evaluation of the properties at the free stream and surface temperatures, respectively. Both methods are used in the results that follow.

Finally, we note that experiments may also be performed to obtain convection mass transfer correlations. However, under conditions for which the heat and mass transfer analogy (Section 6.7.1) may be applied, the mass transfer correlation assumes the same form as the corresponding heat transfer correlation. Accordingly, we anticipate correlations of the form

$$\overline{Sh}_L = C\,Re_L^m\,Sc^n \tag{7.3}$$

where, for a given geometry and flow condition, the values of C, m, and n are the same as those appearing in Equation 7.1.

7.2 The Flat Plat

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7.2 *The Flat Plate in Parallel Flow*

Despite its simplicity, parallel flow over a flat plate (Figure 7.3) occurs in numerous engineering applications. As discussed in Section 6.3, laminar boundary layer development begins at the leading edge ($x = 0$) and transition to turbulence may occur at a downstream location (x_c) for which a critical Reynolds number $Re_{x,c}$ is achieved. We begin by considering conditions in the laminar boundary layer.

7.2.1 Laminar Flow over an Isothermal Plate: A Similarity Solution

The major convection parameters may be obtained by solving the appropriate form of the boundary layer equations. Assuming *steady, incompressible, laminar* flow with *constant fluid properties* and *negligible viscous dissipation* and recognizing that $dp/dx = 0$, the boundary layer equations (6.27, 6.28, 6.29, and 6.30) reduce to

Continuity:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{7.4}$$

Momentum:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{7.5}$$

Energy:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{7.6}$$

Species:

$$u\frac{\partial \rho_A}{\partial x} + v\frac{\partial \rho_A}{\partial y} = D_{AB}\frac{\partial^2 \rho_A}{\partial y^2} \tag{7.7}$$

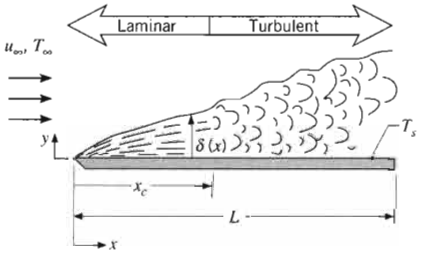


FIGURE 7.3
The flat plate in parallel flow.

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Solution of these equations is simplified by the fact that for constant properties, conditions in the velocity (hydrodynamic) boundary layer are independent of temperature and species concentration. Hence we may begin by solving the hydrodynamic problem, Equations 7.4 and 7.5, to the exclusion of Equations 7.6 and 7.7. Once the hydrodynamic problem has been solved, solutions to Equations 7.6 and 7.7, which depend on u and v, may be obtained.

The hydrodynamic solution follows the method of Blasius [1, 2]. The velocity components are defined in terms of a stream function $\psi(x, y)$,

$$u \equiv \frac{\partial \psi}{\partial y} \quad \text{and} \quad v \equiv -\frac{\partial \psi}{\partial x} \tag{7.8}$$

such that Equation 7.4 is automatically satisfied and hence is no longer needed. New dependent and independent variables, f and η, respectively, are then defined such that

$$f(\eta) \equiv \frac{\psi}{u_\infty \sqrt{\nu x / u_\infty}} \tag{7.9}$$

$$\eta \equiv y \sqrt{u_\infty / \nu x} \tag{7.10}$$

As we will find, use of these variables simplifies matters by reducing the partial differential equation, Equation 7.5, to an ordinary differential equation.

The Blasius solution is termed a *similarity solution,* and η is a *similarity variable.* This terminology is used because, despite growth of the boundary layer with distance x from the leading edge, the velocity profile u/u_∞ remains *geometrically similar.* This similarity is of the functional form

$$\frac{u}{u_\infty} = \phi\left(\frac{y}{\delta}\right)$$

where δ is the boundary layer thickness. We will find from the Blasius solution that δ varies as $(\nu x/u_\infty)^{1/2}$; thus, it follows that

$$\frac{u}{u_\infty} = \phi(\eta) \tag{7.11}$$

Hence the velocity profile is uniquely determined by the similarity variable η, which depends on both x and y.

From Equations 7.8 through 7.10 we obtain

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = u_\infty \sqrt{\frac{\nu x}{u_\infty}}\frac{df}{d\eta}\sqrt{\frac{u_\infty}{\nu x}} = u_\infty \frac{df}{d\eta} \tag{7.12}$$

and

$$v = -\frac{\partial \psi}{\partial x} = -\left(u_\infty \sqrt{\frac{\nu x}{u_\infty}}\frac{\partial f}{\partial x} + \frac{u_\infty}{2}\sqrt{\frac{\nu}{u_\infty x}} f\right)$$

$$v = \frac{1}{2}\sqrt{\frac{\nu u_\infty}{x}}\left(\eta \frac{df}{d\eta} - f\right) \tag{7.13}$$

By differentiating the velocity components, it may also be shown that

$$\frac{\partial u}{\partial x} = -\frac{u_\infty}{2x}\eta \frac{d^2 f}{d\eta^2} \tag{7.14}$$

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$$\frac{\partial u}{\partial y} = u_\infty \sqrt{\frac{u_\infty}{\nu x}} \frac{d^2 f}{d\eta^2} \tag{7.15}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{u_\infty^2}{\nu x} \frac{d^3 f}{d\eta^3} \tag{7.16}$$

Substituting these expressions into Equation 7.5, we then obtain

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 0 \tag{7.17}$$

Hence the hydrodynamic boundary layer problem is reduced to one of solving a nonlinear, third-order ordinary differential equation. The appropriate boundary conditions are

$$u(x, 0) = v(x, 0) = 0 \quad \text{and} \quad u(x, \infty) = u_\infty$$

or, in terms of the similarity variables,

$$\left.\frac{df}{d\eta}\right|_{\eta=0} = f(0) = 0 \quad \text{and} \quad \left.\frac{df}{d\eta}\right|_{\eta\to\infty} = 1 \tag{7.18}$$

The solution to Equation 7.17, subject to the conditions of Equations 7.18, may be obtained by a series expansion [2] or by numerical integration [3]. Selected results are presented in Table 7.1, from which useful information may be extracted. We first note that, to a good approximation, $(u/u_\infty) = 0.99$ for $\eta = 5.0$. Defining the

TABLE 7.1 Flat plate laminar boundary layer functions [3]

$\eta = y\sqrt{\frac{u_\infty}{\nu x}}$	f	$\frac{df}{d\eta} = \frac{u}{u_\infty}$	$\frac{d^2 f}{d\eta^2}$
0	0	0	0.332
0.4	0.027	0.133	0.331
0.8	0.106	0.265	0.327
1.2	0.238	0.394	0.317
1.6	0.420	0.517	0.297
2.0	0.650	0.630	0.267
2.4	0.922	0.729	0.228
2.8	1.231	0.812	0.184
3.2	1.569	0.876	0.139
3.6	1.930	0.923	0.098
4.0	2.306	0.956	0.064
4.4	2.692	0.976	0.039
4.8	3.085	0.988	0.022
5.2	3.482	0.994	0.011
5.6	3.880	0.997	0.005
6.0	4.280	0.999	0.002
6.4	4.679	1.000	0.001
6.8	5.079	1.000	0.000

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boundary layer thickness δ as that value of y for which $(u/u_\infty) = 0.99$, it follows from Equation 7.10 that

$$\delta = \frac{5.0}{\sqrt{u_\infty/\nu x}} = \frac{5x}{\sqrt{Re_x}} \tag{7.19}$$

From Equation 7.19 it is clear that δ increases with increasing x and ν but decreases with increasing u_∞ (the larger the free stream velocity, the *thinner* the boundary layer). In addition, from Equation 7.15 the wall shear stress may be expressed as

$$\tau_s = \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu u_\infty \sqrt{u_\infty/\nu x} \left.\frac{d^2 f}{d\eta^2}\right|_{\eta=0}$$

Hence from Table 7.1

$$\tau_s = 0.332 u_\infty \sqrt{\rho \mu u_\infty / x}$$

The *local* friction coefficient is then

$$C_{f,x} \equiv \frac{\tau_{s,x}}{\rho u_\infty^2/2} = 0.664 Re_x^{-1/2} \tag{7.20}$$

From knowledge of conditions in the velocity boundary layer, the energy and species continuity equations may now be solved. To solve Equation 7.6 we introduce the dimensionless temperature $T^* \equiv [(T - T_s)/(T_\infty - T_s)]$ and assume a similarity solution of the form $T^* = T^*(\eta)$. Making the necessary substitutions, Equation 7.6 reduces to

$$\frac{d^2 T^*}{d\eta^2} + \frac{Pr}{2} f \frac{dT^*}{d\eta} = 0 \tag{7.21}$$

Note the dependence of the thermal solution on hydrodynamic conditions through appearance of the variable f in Equation 7.21. The appropriate boundary conditions are

$$T^*(0) = 0 \quad \text{and} \quad T^*(\infty) = 1 \tag{7.22}$$

Subject to the conditions of Equation 7.22, Equation 7.21 may be solved by numerical integration for different values of the Prandtl number. One important consequence of this solution is that, for $Pr \gtrsim 0.6$, results for the surface temperature gradient $dT^*/d\eta|_{\eta=0}$ may be correlated by the following relation:

$$\left.\frac{dT^*}{d\eta}\right|_{\eta=0} = 0.332 Pr^{1/3}$$

Expressing the local convection coefficient as

$$h_x = \frac{q_s''}{T_s - T_\infty} = -\frac{T_\infty - T_s}{T_s - T_\infty} k \left.\frac{\partial T^*}{\partial y}\right|_{y=0}$$

$$h_x = k \left(\frac{u_\infty}{\nu x}\right)^{1/2} \left.\frac{dT^*}{d\eta}\right|_{\eta=0}$$

it follows that the *local* Nusselt number is of the form

$$Nu_x \equiv \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \qquad Pr \gtrsim 0.6 \tag{7.23}$$

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From the solution to Equation 7.21, it also follows that the ratio of the velocity to thermal boundary layer thickness is

$$\frac{\delta}{\delta_t} \approx Pr^{1/3} \tag{7.24}$$

where δ is given by Equation 7.19.

The species boundary layer equation, Equation 7.7, is of the same form as the energy boundary layer equation, Equation 7.6, with D_{AB} replacing α. Introducing a normalized species density $\rho_A^* = [(\rho_A - \rho_{A,s})/(\rho_{A,\infty} - \rho_{A,s})]$ and noting that, for a fixed surface species concentration

$$\rho_A^*(0) = 0 \quad \text{and} \quad \rho_A^*(\infty) = 1 \tag{7.25}$$

we also see that the species boundary conditions are of the same form as the temperature boundary conditions given in Equation 7.22. Therefore, as discussed in Section 6.7.1, the heat and mass transfer analogy may be applied since the differential equation and boundary conditions for the species concentration are of the same form as for temperature. Hence, with reference to Equation 7.23,

$$Sh_x \equiv \frac{h_{m,x}x}{D_{AB}} = 0.332Re_x^{1/2}\,Sc^{1/3} \qquad Sc \gtrsim 0.6 \tag{7.26}$$

By analogy to Equation 7.24, it also follows that the ratio of boundary layer thicknesses is

$$\frac{\delta}{\delta_c} \approx Sc^{1/3} \tag{7.27}$$

The foregoing results may be used to compute important *laminar* boundary layer parameters for $0 < x < x_c$, where x_c is the distance from the leading edge at which transition begins. Equations 7.20, 7.23, and 7.26 imply that $\tau_{s,x}$, h_x, and $h_{m,x}$ are, in principle, infinite at the leading edge and decrease as $x^{-1/2}$ in the flow direction. Equations 7.24 and 7.27 also imply that, for values of Pr and Sc close to unity, which is the case for most gases, the three boundary layers experience nearly identical growth.

From the foregoing local results, average boundary layer parameters may be determined. With the average friction coefficient defined as

$$\overline{C}_{f,x} \equiv \frac{\overline{\tau}_{s,x}}{\rho u_\infty^2/2} \tag{7.28}$$

where

$$\overline{\tau}_{s,x} \equiv \frac{1}{x}\int_0^x \tau_{s,x}\,dx$$

the form of $\tau_{s,x}$ may be substituted from Equation 7.20 and the integration performed to obtain

$$\overline{C}_{f,x} = 1.328Re_x^{-1/2} \tag{7.29}$$

Moreover, from Equations 6.14 and 7.23, the *average* heat transfer coefficient for laminar flow is

$$\overline{h}_x = \frac{1}{x}\int_0^x h_x\,dx = 0.332\left(\frac{k}{x}\right)Pr^{1/3}\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^x \frac{dx}{x^{1/2}}$$

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Integrating and substituting from Equation 7.23, it follows that $\overline{h}_x = 2h_x$. Hence

$$\overline{Nu}_x \equiv \frac{\overline{h}_x x}{k} = 0.664 Re_x^{1/2} Pr^{1/3} \quad Pr \gtrsim 0.6 \tag{7.30}$$

Employing the heat and mass transfer analogy, it follows that

$$\overline{Sh}_x \equiv \frac{\overline{h}_{m,x} x}{D_{AB}} = 0.664 Re_x^{1/2} Sc^{1/3} \quad Sc \gtrsim 0.6 \tag{7.31}$$

If the flow is laminar over the entire surface, the subscript x may be replaced by L, and Equations 7.29 through 7.31 may be used to predict average conditions for the entire surface.

From the foregoing expressions we see that, for laminar flow over a flat plate, the *average* friction and convection coefficients from the leading edge to a point x on the surface are *twice* the *local* coefficients at that point. We also note that, in using these expressions, the effect of variable properties can be treated by evaluating all properties at the *film temperature,* Equation 7.2.

For fluids of small Prandtl number, namely, *liquid metals,* Equation 7.23 does not apply. However, for this case the thermal boundary layer development is much more rapid than that of the velocity boundary layer ($\delta_t \gg \delta$), and it is reasonable to assume uniform velocity ($u = u_\infty$) throughout the thermal boundary layer. From a solution to the thermal boundary layer equation based on this assumption [5], it may then be shown that

$$Nu_x = 0.565 Pe_x^{1/2} \quad Pr \lesssim 0.05, \quad Pe_x \gtrsim 100 \tag{7.32}$$

where $Pe_x \equiv Re_x\, Pr$ is the *Peclet number* (Table 6.2). Despite the corrosive and reactive nature of liquid metals, their unique properties (low melting point and vapor pressure, as well as high thermal capacity and conductivity) render them attractive as coolants in applications requiring high heat transfer rates.

A single correlating equation, which applies for all Prandtl numbers, has been recommended by Churchill and Ozoe [6]. For laminar flow over an isothermal plate, the local convection coefficient may be obtained from

$$Nu_x = \frac{0.3387 Re_x^{1/2} Pr^{1/3}}{[1 + (0.0468/Pr)^{2/3}]^{1/4}} \quad Pe_x \gtrsim 100 \tag{7.33}$$

with $\overline{Nu}_x = 2Nu_x$.

7.2.2 Turbulent Flow over an Isothermal Plate

From experiment [2] it is known that, for turbulent flows with Reynolds numbers up to approximately 10^8, the *local* friction coefficient is correlated to within 15% accuracy by an expression of the form

$$C_{f,x} = 0.0592 Re_x^{-1/5} \qquad Re_{x,c} \lesssim Re_x \lesssim 10^8 \tag{7.34}$$

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Moreover, it is known that, to a reasonable approximation, the velocity boundary layer thickness may be expressed as

$$\delta = 0.37x\,Re_x^{-1/5} \tag{7.35}$$

Comparing these results with those for the laminar boundary layer, Equations 7.19 and 7.20, we see that turbulent boundary layer growth is much more rapid (δ varies as $x^{4/5}$ in contrast to $x^{1/2}$ for laminar flow) and that the decay in the friction coefficient is more gradual ($x^{-1/5}$ versus $x^{-1/2}$). For turbulent flow, boundary layer development is influenced strongly by random fluctations in the fluid and not by molecular diffusion. Hence relative boundary layer growth does not depend on the value of Pr or Sc, and Equation 7.35 may be used to obtain the thermal and concentration, as well as the velocity, boundary layer thicknesses. That is, for turbulent flow, $\delta \approx \delta_t \approx \delta_c$.

Using Equation 7.34 with the modified Reynolds, or Chilton–Colburn, analogy, Equations 6.70 and 6.71, the *local* Nusselt number for turbulent flow is

$$Nu_x = St\,Re_x\,Pr = 0.0296Re_x^{4/5}\,Pr^{1/3} \quad 0.6 \lesssim Pr \lesssim 60 \tag{7.36}$$

and the *local* Sherwood number is

$$Sh_x = St_m Re_x Sc = 0.0296Re_x^{4/5}\,Sc^{1/3} \quad 0.6 \lesssim Sc \lesssim 3000 \tag{7.37}$$

Enhanced mixing causes the turbulent boundary layer to grow more rapidly than the laminar boundary layer and to have larger friction and convection coefficients.

Expressions for the average coefficients may now be determined. However, since the turbulent boundary layer is generally preceded by a laminar boundary layer, we first consider *mixed* boundary layer conditions.

7.2.3 Mixed Boundary Layer Conditions

For laminar flow over the entire plate, Equations 7.29 through 7.31 may be used to compute the average coefficients. Moreover, if transition occurs toward the rear of the plate, for example, in the range $0.95 \lesssim (x_c/L) \lesssim 1$, these equations may be used to compute the average coefficients to a reasonable approximation. However, when transition occurs sufficiently upstream of the trailing edge, $(x_c/L) \lesssim 0.95$, the surface average coefficients will be influenced by conditions in both the laminar and turbulent boundary layers.

In the mixed boundary layer situation (Figure 7.3), Equation 6.14 may be used to obtain the average convection heat transfer coefficient for the entire plate. Integrating over the laminar region ($0 \le x \le x_c$) and then over the turbulent region ($x_c < x \le L$), this equation may be expressed as

$$\bar{h}_L = \frac{1}{L}\left(\int_0^{x_c} h_{\text{lam}}\,dx + \int_{x_c}^{L} h_{\text{turb}}\,dx\right)$$

where it is assumed that transition occurs abruptly at $x = x_c$. Substituting from Equations 7.23 and 7.36 for h_{lam} and h_{turb}, respectively, we obtain

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$$\bar{h}_L = \left(\frac{k}{L}\right)\left[0.332\left(\frac{u_\infty}{\nu}\right)^{1/2}\int_0^{x_c}\frac{dx}{x^{1/2}} + 0.0296\left(\frac{u_\infty}{\nu}\right)^{4/5}\int_{x_c}^{L}\frac{dx}{x^{1/5}}\right]Pr^{1/3}$$

Integrating, we then obtain

$$\overline{Nu}_L = (0.037\,Re_L^{4/5} - A)\,Pr^{1/3} \qquad (7.38)$$
$$\left[\begin{array}{l} 0.6 \lesssim Pr \lesssim 60 \\ Re_{x,c} \lesssim Re_L \lesssim 10^8 \end{array}\right]$$

where the bracketed relations indicate the range of applicability and the constant A is determined by the value of the critical Reynolds number, $Re_{x,c}$. That is,

$$A = 0.037Re_{x,c}^{4/5} - 0.664Re_{x,c}^{1/2} \qquad (7.39)$$

Similarly, the average friction coefficient may be found using the expression

$$\bar{C}_{f,L} = \frac{1}{L}\left(\int_0^{x_c} C_{f,x,\text{lam}}\,dx + \int_{x_c}^{L} C_{f,x,\text{turb}}\,dx\right)$$

Substituting expressions for $C_{f,x,\text{lam}}$ and $C_{f,x,\text{turb}}$ from Equations 7.20 and 7.34, respectively, and carrying out the integration provides an expression of the form

$$\bar{C}_{f,L} = 0.074Re_L^{-1/5} - \frac{2A}{Re_L} \qquad (7.40)$$
$$[Re_{x,c} \lesssim Re_L \lesssim 10^8]$$

Applying the heat and mass transfer analogy to Equation 7.38 yields

$$\overline{Sh}_L = (0.037Re_L^{4/5} - A)\,Sc^{1/3} \qquad (7.41)$$
$$\left[\begin{array}{l} 0.6 \lesssim Sc \lesssim 60 \\ Re_{x,c} \lesssim Re_L \lesssim 10^8 \end{array}\right]$$

For a completely turbulent boundary layer ($Re_{x,c} = 0$), $A = 0$. Such a condition may be realized by *tripping* the boundary layer at the leading edge, using a fine wire or some other turbulence promoter. For a transition Reynolds number of $Re_{x,c} = 5 \times 10^5$, $A = 871$.

All of the foregoing correlations require evaluation of the fluid properties at the film temperature, Equation 7.2.

7.2.4 Unheated Starting Length

All the foregoing Nusselt number expressions are restricted to situations for which the surface temperature T_s is uniform. A common exception involves existence of an *unheated starting length* ($T_s = T_\infty$) upstream of a heated section ($T_s \neq T_\infty$). As shown in Figure 7.4, velocity boundary layer growth begins at $x = 0$, while thermal boundary layer development begins at $x = \xi$. Hence there is no heat transfer for $0 \leq x \leq \xi$. Through use of an integral boundary layer solution [5], it is known that, for laminar flow,

$$Nu_x = \frac{Nu_x\big|_{\xi=0}}{[1 - (\xi/x)^{3/4}]^{1/3}} \qquad (7.42)$$

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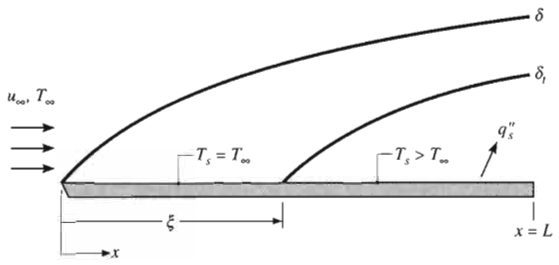


FIGURE 7.4 Flat plate in parallel flow with unheated starting length.

where $Nu_x|_{\xi=0}$, is given by Equation 7.23. In both Nu_x and $Nu_x|_{\xi=0}$, the characteristic length x is measured from the leading edge of the unheated starting length. It has also been found that, for turbulent flow,

$$Nu_x = \frac{Nu_x|_{\xi=0}}{[1-(\xi/x)^{9/10}]^{1/9}} \tag{7.43}$$

where $Nu_x|_{\xi=0}$ is given by Equation 7.36. Analogous mass transfer results are obtained by replacing (Nu_x, Pr) with (Sh_x, Sc).

By using Equation 6.14 with local convection coefficients given by the foregoing relations, expressions may be obtained for the *average Nusselt number* of an isothermal plate with an unheated starting length [7]. For a plate of total length L, with laminar *or* turbulent flow over the entire surface, the expressions are of the form

$$\overline{Nu}_L = \overline{Nu}_L|_{\xi=0}\frac{L}{L-\xi}[1-(\xi/L)^{(p+1)/(p+2)}]^{p/(p+1)} \tag{7.44}$$

where $p = 2$ for laminar flow and $p = 8$ for turbulent flow. The quantity $\overline{Nu}_L|_{\xi=0}$ is the average Nusselt number for a plate of length L when heating starts at the leading edge of the plate. For laminar flow, it can be obtained from Equation 7.30 (with x replaced by L); for turbulent flow it is given by Equation 7.38 with $A = 0$ (assuming turbulent flow over the entire surface). Note that $\overline{Nu}_L$ is equal to $\overline{h}L/k$, where $\overline{h}$ is averaged over the heated portion of the plate only, which is of length $(L - \xi)$. The corresponding value of $\overline{h}_L$ must therefore be multiplied by the area of the heated section to determine the total heat rate from the plate.

7.2.5 Flat Plates with Constant Heat Flux Conditions

It is also possible to have a uniform surface heat flux, rather than a uniform temperature, imposed at the plate. For laminar flow, it may be shown that [5]

$$Nu_x = 0.453Re_x^{1/2}Pr^{1/3} \qquad Pr \gtrsim 0.6 \tag{7.45}$$

while for turbulent flow

$$Nu_x = 0.0308Re_x^{4/5}Pr^{1/3} \qquad 0.6 \lesssim Pr \lesssim 60 \tag{7.46}$$

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Hence the Nusselt number is 36% and 4% larger than the constant surface temperature result for laminar and turbulent flow, respectively. Correction for the effect of an unheated starting length may be made by using Equations 7.45 and 7.46 with Equations 7.42 and 7.43, respectively. If the heat flux is known, the convection coefficient may be used to determine the local surface temperature

$$T_s(x) = T_\infty + \frac{q_s''}{h_x} \tag{7.47}$$

Since the total heat rate is readily determined from the product of the uniform flux and the surface area, $q = q_s'' A_s$, it is not necessary to introduce an average convection coefficient for the purpose of determining q. However, one may still wish to determine an *average surface temperature* from an expression of the form

$$\overline{(T_s - T_\infty)} = \frac{1}{L}\int_0^L (T_s - T_\infty)dx = \frac{q_s''}{L}\int_0^L \frac{x}{k\,Nu_x}dx$$

where Nu_x is obtained from the appropriate convection correlation. Substituting from Equation 7.45, it follows that

$$\overline{(T_s - T_\infty)} = \frac{q_s'' L}{k\overline{Nu}_L} \tag{7.48}$$

where

$$\overline{Nu}_L = 0.680 Re_L^{1/2} Pr^{1/3} \tag{7.49}$$

This result is only 2% larger than that obtained by evaluating Equation 7.30 at $x = L$. Differences are even smaller for turbulent flow, suggesting that any of the $\overline{Nu}_L$ results obtained for a uniform surface temperature may be used with Equation 7.48 to evaluate $\overline{(T_s - T_\infty)}$. Expressions for the average temperature of a plate that is subjected to a uniform heat flux downstream of an unheated starting section have been obtained by Ameel [7].

7.2.6 Limitations on Use of Convection Coefficients

Although the equations of this section are suitable for most engineering calculations, in practice they rarely provide exact values for the convection coefficients. Conditions vary according to free stream turbulence and surface roughness, and errors as large as 25% may be incurred by using the expressions. A detailed description of free stream turbulence effects is provided by Blair [8].

7.3 *Methodology for a Convection Calculation*

Although we have only discussed correlations for parallel flow over a flat plate, selection and application of a convection correlation for *any flow situation* are facilitated by following a few simple rules.

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1. *Become immediately cognizant of the flow geometry.* Does the problem involve flow over a flat plate, a sphere, or a cylinder? The specific form of the convection correlation depends, of course, on the geometry.
2. *Specify the appropriate reference temperature and evaluate the pertinent fluid properties at that temperature.* For moderate boundary layer temperature differences, the film temperature, Equation 7.2, may be used for this purpose. However, we will consider correlations that require property evaluation at the free stream temperature and include a property ratio to account for the nonconstant property effect.
3. *In mass transfer problems the pertinent fluid properties are those of species B.* In our treatment of convection mass transfer, we are only concerned with *dilute, binary mixtures.* That is, the problems involve transport of some species A for which $x_A \ll 1$. To a good approximation the properties of the mixture may then be assumed to be the properties of species B. The Schmidt number, for example, would be $Sc = \nu_B/D_{AB}$ and the Reynolds number would be $Re_L = (VL/\nu_B)$.
4. *Calculate the Reynolds number.* Boundary layer conditions are strongly influenced by this parameter. If the geometry is a flat plate in parallel flow, determine whether the flow is laminar or turbulent.
5. *Decide whether a local or surface average coefficient is required.* Recall that for constant surface temperature or vapor density, the local coefficient is used to determine the flux at a particular point on the surface, whereas the average coefficient determines the transfer rate for the entire surface.
6. *Select the appropriate correlation.*

EXAMPLE 7.1

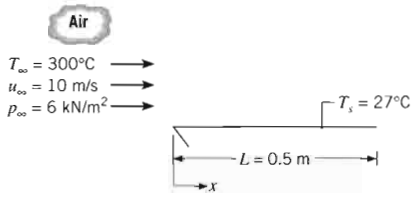
Air at a pressure of 6 kN/m^2 and a temperature of 300°C flows with a velocity of 10 m/s over a flat plate 0.5 m long. Estimate the cooling rate per unit width of the plate needed to maintain it at a surface temperature of 27°C.

SOLUTION

Known: Airflow over an isothermal flat plate.

Find: Cooling rate per unit width of the plate, q' (W/m).

Schematic:



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Assumptions:

1. Steady-state conditions.
2. Negligible radiation effects.

Properties: Table A.4, air ($T_f = 437$ K, $p = 1$ atm): $\nu = 30.84 \times 10^{-6}$ m^2/s, $k = 36.4 \times 10^{-3}$ W/m · K, $Pr = 0.687$. Properties such as k, Pr, and μ may be assumed to be independent of pressure to an excellent approximation. However, for a gas, the kinematic viscosity $\nu = \mu/\rho$ will vary with pressure through its dependence on density. From the ideal gas law, $\rho = p/RT$, it follows that the ratio of kinematic viscosities for a gas at the same temperature but at different pressures, p_1 and p_2, is $(\nu_1/\nu_2) = (p_2/p_1)$. Hence the kinematic viscosity of air at 437 K and $p_\infty = 6 \times 10^3$ N/m^2 is

$$\nu = 30.84 \times 10^{-6}\ \text{m}^2/\text{s} \times \frac{1.0133 \times 10^5\ \text{N/m}^2}{6 \times 10^3\ \text{N/m}^2} = 5.21 \times 10^{-4}\ \text{m}^2/\text{s}$$

Analysis: For a plate of unit width, it follows from Newton's law of cooling that the rate of convection heat transfer *to* the plate is

$$q' = \bar{h}L(T_\infty - T_s)$$

To determine the appropriate convection correlation for computing $\bar{h}$, the Reynolds number must first be determined

$$Re_L = \frac{u_\infty L}{\nu} = \frac{10\ \text{m/s} \times 0.5\ \text{m}}{5.21 \times 10^{-4}\ \text{m}^2/\text{s}} = 9597$$

Hence the flow is laminar over the entire plate, and the appropriate correlation is given by Equation 7.30.

$$\overline{Nu}_L = 0.664 Re_L^{1/2}\, Pr^{1/3} = 0.664(9597)^{1/2}(0.687)^{1/3} = 57.4$$

The average convection coefficient is then

$$\bar{h} = \frac{\overline{Nu}_L k}{L} = \frac{57.4 \times 0.0364\ \text{W/m} \cdot \text{K}}{0.5\ \text{m}} = 4.18\ \text{W/m}^2 \cdot \text{K}$$

and the required cooling rate per unit width of plate is

$$q' = 4.18\ \text{W/m}^2 \cdot \text{K} \times 0.5\ \text{m}\,(300 - 27)^\circ\text{C} = 570\ \text{W/m}$$

Comments: The results of Table A.4 apply to gases at atmospheric pressure. Except for the kinematic viscosity, mass density, and thermal diffusivity, they may generally be used at other pressures without correction. The kinematic viscosity and thermal diffusivity for pressures other than 1 atm may be obtained by dividing the tabulated value by the pressure (atm).

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EXAMPLE 7.2

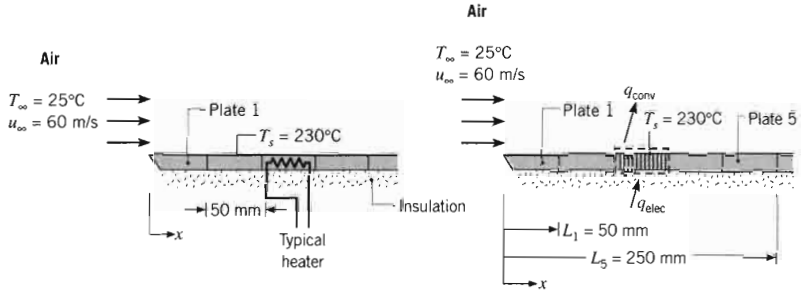
A flat plate of width $w = 1$ m is maintained at a uniform surface temperature, $T_s = 230°C$, by using independently controlled, electrical strip heaters, each of which is 50 mm long. If atmospheric air at 25°C flows over the plate at a velocity of 60 m/s, at what heater is the electrical input a maximum? What is the value of this input?

SOLUTION

Known: Airflow over a flat plate with segmented heaters.

Find: Maximum heater power requirement.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Negligible radiation effects.
3. Bottom surface of plate adiabatic.

Properties: Table A.4, air ($T_f = 400$ K, $p = 1$ atm): $\nu = 26.41 \times 10^{-6}$ m^2/s, $k = 0.0338$ W/m · K, $Pr = 0.690$.

Analysis: The location of the heater requiring the maximum electrical power may be determined by first finding the point of boundary layer transition. The Reynolds number based on the length L_1 of the first heater is

$$Re_1 = \frac{u_\infty L_1}{\nu} = \frac{60 \text{ m/s} \times 0.05 \text{ m}}{26.41 \times 10^{-6} \text{ m}^2/\text{s}} = 1.14 \times 10^5$$

If the transition Reynolds number is assumed to be $Re_{x,c} = 5 \times 10^5$, it follows that transition will occur on the fifth heater, or more specifically at

$$x_c = \frac{\nu}{u_\infty} Re_{x,c} = \frac{26.41 \times 10^{-6} \text{ m}^2/\text{s}}{60 \text{ m/s}} 5 \times 10^5 = 0.22 \text{ m}$$

The heater requiring the maximum electrical power is that for which the average convection coefficient is largest. Knowing how the local convection coefficient

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varies with distance from the leading edge, we conclude that there are three possibilities:

1. Heater 1, since it corresponds to the largest local, laminar convection coefficient.
2. Heater 5, since it corresponds to the largest local, turbulent convection coefficient.
3. Heater 6, since turbulent conditions exist over the entire heater.

For each of these heaters, conservation of energy requires that

$$q_{elec} = q_{conv}$$

For the first heater,

$$q_{conv,1} = \bar{h}_1 L_1 w(T_s - T_\infty)$$

where $\bar{h}_1$ is determined from Equation 7.30,

$$\overline{Nu}_1 = 0.664 Re_1^{1/2} Pr^{1/3} = 0.664(1.14 \times 10^5)^{1/2}(0.69)^{1/3} = 198$$

Hence

$$\bar{h}_1 = \frac{\overline{Nu}_1 k}{L_1} = \frac{198 \times 0.0338\ \text{W/m} \cdot \text{K}}{0.05\ \text{m}} = 134\ \text{W/m}^2 \cdot \text{K}$$

and

$$q_{conv,1} = 134\ \text{W/m}^2 \cdot \text{K}(0.05 \times 1)\ \text{m}^2(230 - 25)^\circ\text{C} = 1370\ \text{W}$$

The power requirement for the fifth heater may be obtained by subtracting the total heat loss associated with the first four heaters from that associated with the first five heaters. Accordingly,

$$q_{conv,5} = \bar{h}_{1-5} L_5 w(T_s - T_\infty) - \bar{h}_{1-4} L_4 w(T_s - T_\infty)$$

$$q_{conv,5} = (\bar{h}_{1-5} L_5 - \bar{h}_{1-4} L_4) w(T_s - T_\infty)$$

The value of $\bar{h}_{1-4}$ may be obtained from Equation 7.30, where

$$\overline{Nu}_4 = 0.664 Re_4^{1/2} Pr^{1/3}$$

With $Re_4 = 4Re_1 = 4.56 \times 10^5$,

$$\overline{Nu}_4 = 0.664(4.56 \times 10^5)^{1/2}(0.69)^{1/3} = 396$$

Hence

$$\bar{h}_{1-4} = \frac{\overline{Nu}_4 k}{L_4} = \frac{396 \times 0.0338\ \text{W/m} \cdot \text{K}}{0.2\ \text{m}} = 67\ \text{W/m}^2 \cdot \text{K}$$

In contrast, the fifth heater is characterized by mixed boundary layer conditions, and $\bar{h}_{1-5}$ must be obtained from Equation 7.38, with $A = 871$. With $Re_5 = 5Re_1 = 5.70 \times 10^5$,

$$\overline{Nu}_5 = (0.037 Re_5^{4/5} - 871) Pr^{1/3}$$

$$\overline{Nu}_5 = [0.037(5.70 \times 10^5)^{4/5} - 871](0.69)^{1/3} = 546$$

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Hence

$$\bar{h}_{1-5} = \frac{\overline{Nu}_5 k}{L_5} = \frac{546 \times 0.0338\ \text{W/m} \cdot \text{K}}{0.25\ \text{m}} = 74\ \text{W/m}^2 \cdot \text{K}$$

The rate of heat transfer from the fifth heater is then

$$q_{\text{conv},5} = (74\ \text{W/m}^2 \cdot \text{K} \times 0.25\ \text{m} - 67\ \text{W/m}^2 \cdot \text{K} \times 0.20\ \text{m}) \times 1\ \text{m}\,(230 - 25)^\circ\text{C}$$
$$q_{\text{conv},5} = 1050\ \text{W}$$

Similarly, the power requirement for the sixth heater may be obtained by subtracting the total heat loss associated with the first five heaters from that associated with the first six heaters. Hence

$$q_{\text{conv},6} = (\bar{h}_{1-6} L_6 - \bar{h}_{1-5} L_5) w (T_s - T_\infty)$$

where $\bar{h}_{1-6}$ may be obtained from Equation 7.38. With $Re_6 = 6Re_1 = 6.84 \times 10^5$,

$$\overline{Nu}_6 = [0.037(6.84 \times 10^5)^{4/5} - 871](0.69)^{1/3} = 753$$

Hence

$$\bar{h}_{1-6} = \frac{\overline{Nu}_6 k}{L_6} = \frac{753 \times 0.0338\ \text{W/m} \cdot \text{K}}{0.30\ \text{m}} = 85\ \text{W/m}^2 \cdot \text{K}$$

and

$$q_{\text{conv},6} = (85\ \text{W/m}^2 \cdot \text{K} \times 0.30\ \text{m} - 74\ \text{W/m}^2 \cdot \text{K} \times 0.25\ \text{m}) \times 1\ \text{m}\,(230 - 25)^\circ\text{C}$$
$$q_{\text{conv},6} = 1440\ \text{W}$$ ◁

Hence $q_{\text{conv},6} > q_{\text{conv},1} > q_{\text{conv},5}$, and the sixth plate has the largest power requirement.

Comments:

1. An alternative, less accurate method of finding the convection heat transfer rate from a particular plate involves estimating an average local convection coefficient for the surface. For example, Equation 7.36 could be used to evaluate the local convection coefficient at the midpoint of the sixth plate. With $x = 0.275$ m, $Re_x = 6.25 \times 10^5$, $Nu_x = 1130$, and $h_x = 139\ \text{W/m}^2 \cdot \text{K}$, the convection heat transfer rate from the sixth plate is

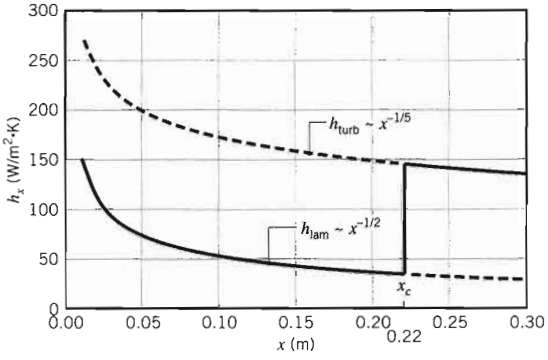
   $$q_{\text{conv},6} = h_x (L_6 - L_5) w (T_s - T_\infty)$$
   $$q_{\text{conv},6} = 139\ \text{W/m}^2 \cdot \text{K}\,(0.30 - 0.25)\ \text{m} \times 1\ \text{m}\,(230 - 25)^\circ\text{C} = 1430\ \text{W}$$

   This procedure must be used with great caution and only when the variation of the local convection coefficient with distance is gradual, such as in turbulent flow. It could lead to significant error when used for a surface that experiences transition.

2. The variation of the local convection coefficient along the flat plate may be determined from Equations 7.23 and 7.36 for laminar and turbulent flow,

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respectively, and the results are represented by the solid curves of the following schematic:



The $x^{-1/2}$ decay of the laminar convection coefficient is presumed to conclude abruptly at $x_c = 0.22$ m, where transition yields more than a fourfold increase in the local convection coefficient. For $x > x_c$, the decay in the convection coefficient is more gradual ($x^{-1/5}$). The dashed lines represent extensions of the distributions, which would apply if the value of x_c were shifted. For example, if the free stream turbulence were to increase and/or the surface were to be roughened, $Re_{x,c}$ would decrease. The smaller value of x_c would cause the laminar and turbulent distributions, respectively, to extend over smaller and larger portions of the plate. A similar effect may be achieved by increasing u_∞. In this case larger values of h_x would be associated with the laminar and turbulent distributions ($h_{lam} \sim u_\infty^{1/2}$, $h_{turb} \sim u_\infty^{4/5}$).

3. This example is provided as a ready-to-solve model in *IHT*, which may be accessed in *Examples* from the menu. This model demonstrates how to use the *Correlations* and *Properties* tools, which can facilitate performing convection calculations.

EXAMPLE 7.3

Drought conditions in the southwestern United States have prompted officials to question whether the operation of residential swimming pools should be permitted. As the chief engineer of a city that has a large number of pools, you have been asked to estimate the *daily* water loss due to pool evaporation. For representative conditions, you may assume water and ambient air temperatures of 25°C and an ambient relative humidity of 50%. Typical pool surface dimensions are 6 m by 12 m. There is a 1.5-m-wide deck around the pool, which is raised relative to the surrounding ground. Wind blows in the direction of the long side of the pool, with velocity 2 m/s. You may assume the free stream turbulence of the air to be negligible, the surface of the water to be smooth and level with the pool deck, and the deck to be dry. What is the water loss for the pool in kilograms per day?

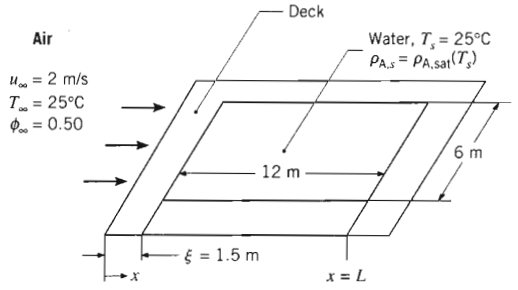
SOLUTION

Known: Ambient air conditions above a swimming pool, pool and deck dimensions.

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Find: Daily evaporative water loss.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Smooth water surface and negligible free stream turbulence.
3. Deck is dry.
4. Heat and mass transfer analogy applicable.
5. Flow is tripped to turbulence by the leading edge of the deck.
6. Ideal gas behavior for water vapor in free stream.

Properties: Table A.4, air (25°C): $\nu = 15.7 \times 10^{-6}$ m^2/s. Table A.8, water vapor–air (25°C): $D_{AB} = 0.26 \times 10^{-4}$ m^2/s, $Sc = \nu/D_{AB} = 0.60$. Table A.6, saturated water vapor (25°C): $\rho_{A,\,sat} = v_g^{-1} = 0.0226$ kg/m^3.

Analysis: The leading edge of the velocity boundary layer is at the edge of the deck; therefore, the trailing edge of the pool is at a distance of $L = 13.5$ m from the leading edge. The Reynolds number at that point is

$$Re_L = \frac{u_\infty L}{\nu} = \frac{2 \text{ m/s} \times 13.5 \text{ m}}{15.7 \times 10^{-6} \text{m}^2/\text{s}} = 1.72 \times 10^6$$

Applying the heat and mass transfer analogy to Equation 7.44 yields

$$\overline{Sh}_L = \overline{Sh}_L|_{\xi=0} \frac{L}{L-\xi}[1 - (\xi/L)^{(p+1)/(p+2)}]^{p/(p+1)} \tag{1}$$

The average Sherwood number, $\overline{Sh}_L|_{\xi=0}$, is evaluated from Equation 7.41 with $A = 0$ because the boundary layer is tripped to turbulent conditions by the leading edge of the deck

$$\overline{Sh}_L|_{\xi=0} = 0.037 Re_L^{4/5} Sc^{1/3}$$

$$\overline{Sh}_L|_{\xi=0} = 0.037(1.72 \times 10^6)^{4/5} \times (0.60)^{1/3} = 3040$$

With $p = 8$ for turbulent flow, Equation 1 may be evaluated as

$$\overline{Sh}_L = 3040 \frac{13.5 \text{ m}}{(13.5 \text{ m} - 1.5 \text{ m})}[1 - (1.5 \text{ m}/13.5 \text{ m})^{(8+1)/(8+2)}]^{8/(8+1)} = 2990$$

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It follows that

$$\bar{h}_{m,L} = \overline{Sh}_L\left(\frac{D_{AB}}{L}\right) = 2990\,\frac{0.26 \times 10^{-4}\ \text{m}^2/\text{s}}{13.5\ \text{m}} = 5.77 \times 10^{-3}\ \text{m/s}$$

The evaporation rate for the pool is then

$$n_A = \bar{h}_m A(\rho_{A,s} - \rho_{A,\infty})$$

where A is the pool area (not including the deck). With the free stream vapor assumed to be an ideal gas,

$$\phi_\infty = \frac{\rho_{A,\infty}}{\rho_{A,sat}(T_\infty)}$$

and with $\rho_{A,s} = \rho_{A,sat}(T_s)$,

$$n_A = \bar{h}_m A[\rho_{A,sat}(T_s) - \phi_\infty \rho_{A,sat}(T_\infty)]$$

Since $T_s = T_\infty = 25°\text{C}$, it follows that

$$n_A = \bar{h}_m A \rho_{A,sat}(25°\text{C})[1 - \phi_\infty]$$

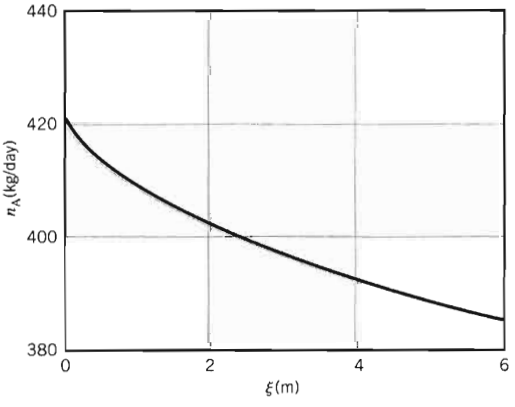
Hence

$$n_A = 5.77 \times 10^{-3}\ \text{m/s} \times 72\ \text{m}^2 \times 0.0226\ \text{kg/m}^3 \times 0.5 \times 86{,}400\ \text{s/day}$$

$$n_A = 405\ \text{kg/day}$$

Comments:

1. The water surface temperature is likely to be slightly less than the air temperature because of the evaporative cooling effect.
2. The volume loss, with a water density of 996 kg/m^3, is $n_A/\rho = 0.4$ m^3/day. This would mean a drop in the pool level of 6 mm per day. Of course the loss would be greater in summer when the air temperature is higher.
3. The influence of the deck length on the daily evaporation rate is shown below. As the length of the deck is increased, the overall evaporation rate is reduced due to the extension of the leading edge of the velocity boundary layer farther away from the pool.



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7.4 *The Cylinder in Cross Flow*

7.4.1 Flow Considerations

Another common external flow involves fluid motion normal to the axis of a circular cylinder. As shown in Figure 7.5, the free stream fluid is brought to rest at the *forward stagnation point,* with an accompanying rise in pressure. From this point, the pressure decreases with increasing x, the streamline coordinate, and the boundary layer develops under the influence of a *favorable pressure gradient* ($dp/dx < 0$). However, the pressure must eventually reach a minimum, and toward the rear of the cylinder further boundary layer development occurs in the presence of an *adverse pressure gradient* ($dp/dx > 0$).

In Figure 7.5 the distinction between the upstream velocity V and the free stream velocity u_∞ should be noted. Unlike conditions for the flat plate in parallel flow, these velocities differ, with u_∞ now depending on the distance x from the stagnation point. From Euler's equation for an inviscid flow [9], $u_\infty(x)$ must exhibit behavior opposite to that of $p(x)$. That is, from $u_\infty = 0$ at the stagnation point, the fluid accelerates because of the favorable pressure gradient ($du_\infty/dx > 0$ when $dp/dx < 0$), reaches a maximum velocity when $dp/dx = 0$, and decelerates because of the adverse pressure gradient ($du_\infty/dx < 0$ when $dp/dx > 0$). As the fluid decelerates, the velocity gradient at the surface, $\partial u/\partial y|_{y=0}$, eventually becomes zero (Figure 7.6). At this location, termed the *separation point,* fluid near the surface lacks sufficient momentum to overcome the pressure gradient, and continued downstream movement is impossible. Since the oncoming fluid also precludes flow back upstream, *boundary layer separation* must occur. This is a condition for which the boundary layer detaches from the surface, and a *wake* is formed in the downstream region. Flow in this region is characterized by vortex formation and is highly irregular. The *separation point* is the location for which $\partial u/\partial y|_{y=0} = 0$. An excellent review of flow conditions in the wake of a circular cylinder is provided by Coutanceau and Defaye [10].

The occurrence of *boundary layer transition,* which depends on the Reynolds number, strongly influences the position of the separation point. For the circular

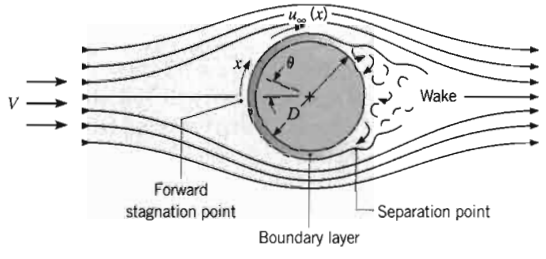


FIGURE 7.5 Boundary layer formation and separation on a circular cylinder in cross flow.

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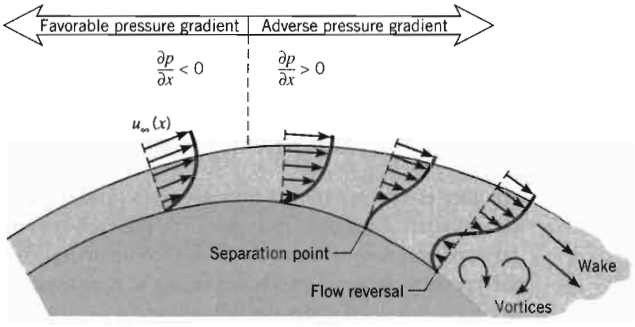


FIGURE 7.6 Velocity profile associated with separation on a circular cylinder in cross flow.

cylinder the characteristic length is the diameter, and the Reynolds number is defined as

$$Re_D \equiv \frac{\rho VD}{\mu} = \frac{VD}{\nu}$$

Since the momentum of fluid in a turbulent boundary layer is larger than in the laminar boundary layer, it is reasonable to expect transition to delay the occurrence of separation. If $Re_D \lesssim 2 \times 10^5$, the boundary layer remains laminar, and separation occurs at $\theta \approx 80°$ (Figure 7.7). However, if $Re_D \gtrsim 2 \times 10^5$, boundary layer transition occurs, and separation is delayed to $\theta \approx 140°$.

The foregoing processes strongly influence the drag force, F_D, acting on the cylinder. This force has two components, one of which is due to the boundary layer surface shear stress (*friction drag*). The other component is due to a pressure differential in the flow direction resulting from formation of the wake (*form,* or *pressure, drag*). A dimensionless *drag coefficient* C_D may be defined as

$$C_D \equiv \frac{F_D}{A_f(\rho V^2/2)} \tag{7.50}$$

where A_f is the cylinder frontal area (the area projected perpendicular to the free stream velocity). The drag coefficient is a function of the Reynolds number and

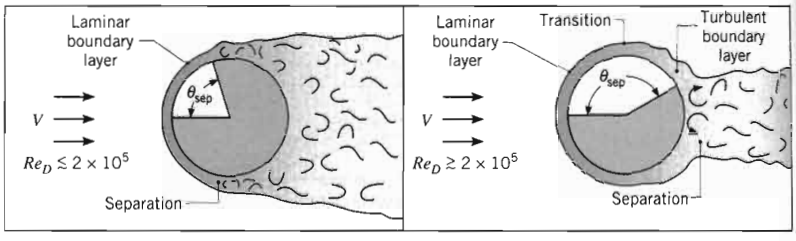


FIGURE 7.7 The effect of turbulence on separation.

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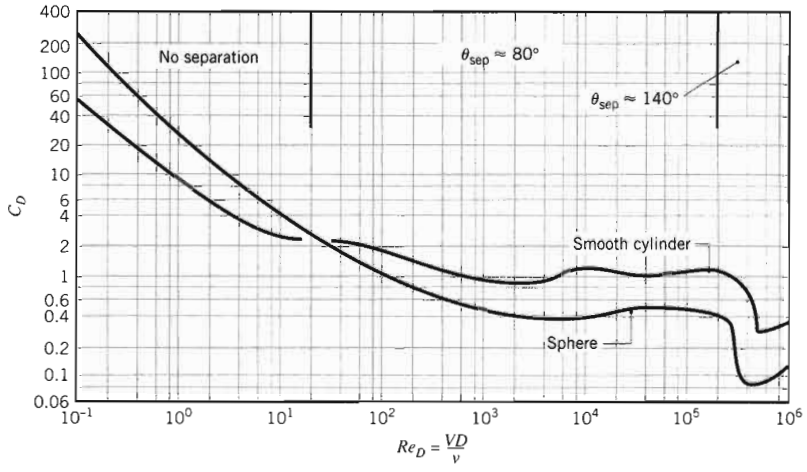


FIGURE 7.8 Drag coefficients for a smooth circular cylinder in cross flow and for a sphere [2]. Boundary layer separation angles are for a cylinder. Adapted with permission.

results are presented in Figure 7.8. For $Re_D \lesssim 2$ separation effects are negligible, and conditions are dominated by friction drag. However, with increasing Reynolds number, the effect of separation, and therefore form drag, becomes more important. The large reduction in C_D that occurs for $Re_D \gtrsim 2 \times 10^5$ is due to boundary layer transition, which delays separation, thereby reducing the extent of the wake region and the magnitude of the form drag.

7.4.2 Convection Heat and Mass Transfer

Experimental results for the variation of the local Nusselt number with θ are shown in Figure 7.9 for the cylinder in a cross flow of air. Not unexpectedly, the results are strongly influenced by the nature of boundary layer development on the surface. Consider conditions for $Re_D \lesssim 10^5$. Starting at the stagnation point, Nu_θ decreases with increasing θ as a result of laminar boundary layer development. However, a minimum is reached at $\theta \approx 80°$, where separation occurs and Nu_θ increases with θ due to mixing associated with vortex formation in the wake. In contrast, for $Re_D \gtrsim 10^5$ the variation of Nu_θ with θ is characterized by two minima. The decline in Nu_θ from the value at the stagnation point is again due to laminar boundary layer development, but the sharp increase that occurs between 80° and 100° is now due to boundary layer transition to turbulence. With further development of the turbulent boundary layer, Nu_θ again begins to decline. Eventually separation occurs ($\theta \approx 140°$), and Nu_θ increases as a result of mixing in the wake region. The increase in Nu_θ with increasing Re_D is due to a corresponding reduction in the boundary layer thickness.

Correlations may be obtained for the local Nusselt number, and at the forward stagnation point for $Pr \gtrsim 0.6$, boundary layer analysis [5] yields an expression of the following form, which is most accurate at low Reynolds number:

$$Nu_D(\theta = 0) = 1.15 Re_D^{1/2} Pr^{1/3} \tag{7.51}$$

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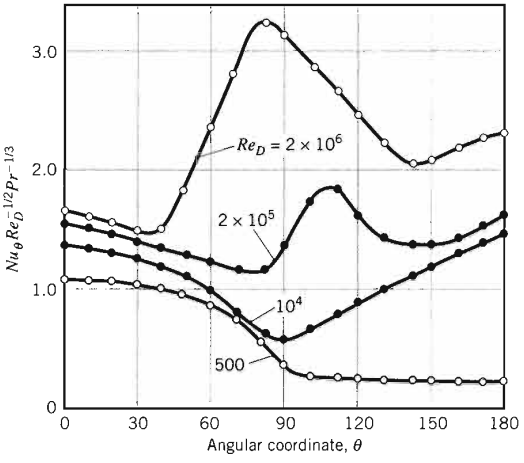


FIGURE 7.9 Local Nusselt number for airflow normal to a circular cylinder. Adapted with permission from Zukauskas, A., "Convective Heat Transfer in Cross Flow," in S. Kakac, R. K. Shah, and W. Aung, Eds., ***Handbook of Single-Phase Convective Heat Transfer***, Wiley, New York, 1987.

However, from the standpoint of engineering calculations, we are more interested in overall average conditions. The empirical correlation due to Hilpert [11]

$$\overline{Nu}_D \equiv \frac{\bar{h}D}{k} = C\, Re_D^m\, Pr^{1/3} \tag{7.52}$$

is widely used for $Pr \gtrsim 0.7$, where the constants C and m are listed in Table 7.2. Equation 7.52 may also be used for flow over cylinders of noncircular cross section, with the characteristic length D and the constants obtained from Table 7.3. In working with Equations 7.51 and 7.52 all properties are evaluated at the film temperature.

Other correlations have been suggested for the circular cylinder in cross flow [14, 15, 16]. The correlation due to Zukauskas [15] is of the form

TABLE 7.2 Constants of Equation 7.52 for the circular cylinder in cross flow [11, 12]

Re_D	C	m
0.4–4	0.989	0.330
4–40	0.911	0.385
40–4000	0.683	0.466
4000–40,000	0.193	0.618
40,000–400,000	0.027	0.805

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TABLE 7.3 Constants of Equation 7.52 for noncircular cylinders in cross flow of a gas [13]

Geometry	Re_D	C	m
Square			
$V \rightarrow$  D	5×10^3–10^5	0.246	0.588
$V \rightarrow$  D	5×10^3–10^5	0.102	0.675
Hexagon			
$V \rightarrow$  D	5×10^3–1.95×10^4	0.160	0.638
	1.95×10^4–10^5	0.0385	0.782
$V \rightarrow$  D	5×10^3–10^5	0.153	0.638
Vertical plate			
$V \rightarrow$  D	4×10^3–1.5×10^4	0.228	0.731

$$\overline{Nu}_D = C\,Re_D^m\,Pr^n\left(\frac{Pr}{Pr_s}\right)^{1/4} \qquad (7.53)$$

$$\left[\begin{array}{l} 0.7 \lesssim Pr \lesssim 500 \\ 1 \lesssim Re_D \lesssim 10^6 \end{array}\right]$$

where all properties are evaluated at T_∞, except Pr_s, which is evaluated at T_s. Values of C and m are listed in Table 7.4. If $Pr \lesssim 10$, $n = 0.37$; if $Pr \gtrsim 10$, $n = 0.36$. Churchill and Bernstein [16] have proposed a single comprehensive equation that covers the entire range of Re_D for which data are available, as well as a wide range of Pr. The equation is recommended for all $Re_D\,Pr \gtrsim 0.2$ and has the form

$$\overline{Nu}_D = 0.3 + \frac{0.62Re_D^{1/2}\,Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5} \qquad (7.54)$$

where all properties are evaluated at the film temperature.

Again we caution the reader not to view any of the foregoing correlations as sacrosanct. Each correlation is reasonable over a certain range of conditions, but for most engineering calculations one should not expect accuracy to much better than 20%. Because they are based on more recent results encompassing a wide range of conditions, Equations 7.53 and 7.54 are used for the calculations of this text. A detailed review of the many correlations that have been developed for the circular cylinder is provided by Morgan [17].

Finally, we note that by invoking the heat and mass transfer analogy, Equations 7.51 through 7.54 may be applied to problems involving convection mass transfer from

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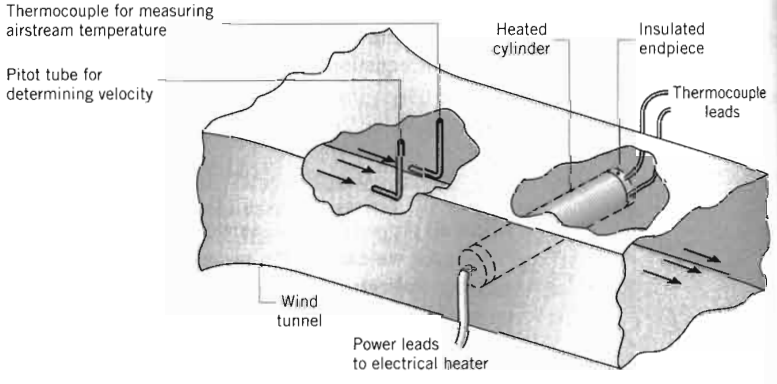
TABLE 7.4 Constants of Equation 7.53 for the circular cylinder in cross flow [16]

Re_D	C	m
1–40	0.75	0.4
40–1000	0.51	0.5
10^3–2×10^5	0.26	0.6
2×10^5–10^6	0.076	0.7

a cylinder in cross flow. It is simply a matter of replacing $\overline{Nu}_D$ by $\overline{Sh}_D$ and Pr by Sc. In mass transfer problems, boundary layer property variations are typically small. Hence, when using the mass transfer analog of Equation 7.53, the property ratio, which accounts for nonconstant property effects, may be neglected.

EXAMPLE 7.4

Experiments have been conducted on a metallic cylinder 12.7 mm in diameter and 94 mm long. The cylinder is heated internally by an electrical heater and is subjected to a cross flow of air in a low-speed wind tunnel. Under a specific set of operating conditions for which the upstream air velocity and temperature were maintained at $V = 10$ m/s and 26.2°C, respectively, the heater power dissipation was measured to be $P = 46$ W, while the average cylinder surface temperature was determined to be $T_s = 128.4$°C. It is estimated that 15% of the power dissipation is lost through the cumulative effect of surface radiation and conduction through the endpieces.



1. Determine the convection heat transfer coefficient from the experimental observations.
2. Compare the experimental result with the convection coefficient computed from an appropriate correlation.

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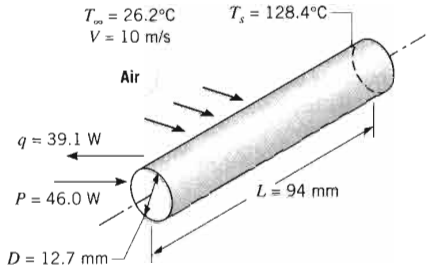
SOLUTION

Known: Operating conditions for a heated cylinder.

Find:

1. Convection coefficient associated with the operating conditions.
2. Convection coefficient from an appropriate correlation.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Uniform cylinder surface temperature.

Properties: Table A.4, air ($T_\infty = 26.2°C \approx 300$ K): $\nu = 15.89 \times 10^{-6}$ m^2/s, $k = 26.3 \times 10^{-3}$ W/m · K, $Pr = 0.707$. Table A.4, air ($T_f \approx 350$ K): $\nu = 20.92 \times 10^{-6}$ m^2/s, $k = 30 \times 10^{-3}$ W/m · K, $Pr = 0.700$. Table A.4, air ($T_s = 128.4°C = 401$ K): $Pr = 0.690$.

Analysis:

1. The convection heat transfer coefficient may be determined from the data by using Newton's law of cooling. That is,

$$\bar{h} = \frac{q}{A(T_s - T_\infty)}$$

With $q = 0.85P$ and $A = \pi DL$, it follows that

$$\bar{h} = \frac{0.85 \times 46\ \text{W}}{\pi \times 0.0127\ \text{m} \times 0.094\ \text{m}\ (128.4 - 26.2)°\text{C}} = 102\ \text{W/m}^2 \cdot \text{K} \qquad \triangleleft$$

2. Working with the Zukauskas relation, Equation 7.53,

$$\overline{Nu}_D = C\,Re_D^m\,Pr^n \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

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all properties, except Pr_s, are evaluated at T_∞. Accordingly,

$$Re_D = \frac{VD}{\nu} = \frac{10\ \text{m/s} \times 0.0127\ \text{m}}{15.89 \times 10^{-6}\ \text{m}^2/\text{s}} = 7992$$

Hence, from Table 7.4, $C = 0.26$ and $m = 0.6$. Also, since $Pr < 10$, $n = 0.37$. It follows that

$$\overline{Nu}_D = 0.26(7992)^{0.6}(0.707)^{0.37}\left(\frac{0.707}{0.690}\right)^{0.25} = 50.5$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 50.5\,\frac{0.0263\ \text{W/m} \cdot \text{K}}{0.0127\ \text{m}} = 105\ \text{W/m}^2 \cdot \text{K}$$

Comments:

1. Using the Churchill relation, Equation 7.54,

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2}\, Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}}\left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

With all properties evaluated at T_f, $Pr = 0.70$ and

$$Re_D = \frac{VD}{\nu} = \frac{10\ \text{m/s} \times 0.0127\ \text{m}}{20.92 \times 10^{-6}\ \text{m}^2/\text{s}} = 6071$$

Hence the Nusselt number and the convection coefficient are

$$\overline{Nu}_D = 0.3 + \frac{0.62(6071)^{1/2}(0.70)^{1/3}}{[1 + (0.4/0.70)^{2/3}]^{1/4}}\left[1 + \left(\frac{6071}{282{,}000}\right)^{5/8}\right]^{4/5} = 40.6$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 40.6\,\frac{0.030\ \text{W/m} \cdot \text{K}}{0.0127\ \text{m}} = 96.0\ \text{W/m}^2 \cdot \text{K}$$

Alternatively, from the Hilpert correlation, Equation 7.52,

$$\overline{Nu}_D = C\, Re_D^m\, Pr^{1/3}$$

With all properties evaluated at the film temperature, $Re_D = 6071$ and $Pr = 0.70$. Hence, from Table 7.2, $C = 0.193$ and $m = 0.618$. The Nusselt number and the convection coefficient are then

$$\overline{Nu}_D = 0.193(6071)^{0.618}(0.700)^{0.333} = 37.3$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 37.3\,\frac{0.030\ \text{W/m} \cdot \text{K}}{0.0127\ \text{m}} = 88\ \text{W/m}^2 \cdot \text{K}$$

2. Uncertainties associated with measuring the air velocity, estimating the heat loss from cylinder ends, and averaging the cylinder surface temperature, which varies axially and circumferentially, render the experimental result accurate to no better than 15%. Accordingly, calculations based on each of the three correlations are within the experimental uncertainty of the measured result.
3. Recognize the importance of using the proper temperature when evaluating fluid properties.

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EXAMPLE 7.5

Because the molecular weight of hydrogen is very small, storing significant amounts in its gaseous form requires very large, high-pressure containers. In situations where use of such high-pressure storage is not feasible, such as in automotive applications, the H_2 is typically stored by *adsorbing* it into a metal hydride powder. The hydrogen is subsequently *desorbed* as needed, by heating the metal hydride throughout its volume.

Gaseous, desorbed hydrogen is present within the interstitial regions of the powder at a pressure that depends on the metal hydride temperature as

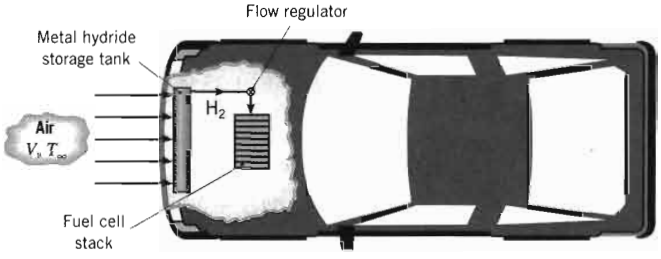
$$p_{H_2} = \exp^{(-3550/T+12.9)}$$

where p_{H_2} is the hydrogen pressure in atmospheres and T is the metal hydride temperature in kelvins. The desorption process is an *endothermic* chemical reaction corresponding to a thermal generation rate expressed as

$$\dot{E}_g = -\dot{m}_{H_2} \times (29.5 \times 10^3 \text{ kJ/kg})$$

where $\dot{m}_{H_2}$ is the hydrogen desorption rate (kg/s). Thermal energy must be supplied to the metal hydride in order to maintain a sufficiently high operating temperature. The operating temperature is determined by the requirement that the hydrogen pressure remain above 1 atm so that hydrogen can be delivered to the fuel cell, which operates at $p_{fc} = 1$ atm.

At a steady-state cruising speed of $V = 25$ m/s, a fuel cell–powered automobile consumes $\dot{m}_{H_2} = 1.35 \times 10^{-4}$ kg/s of hydrogen, which is supplied from a cylindrical, stainless steel canister with inside diameter $D_i = 0.1$ m, length $L = 0.8$ m, and wall thickness $t = 0.5$ mm. The canister, which is loaded with metal hydride powder, is installed in the vehicle so that it is subject to air in cross flow at $V = 25$ m/s, $T_\infty = 23°C$. Determine how much additional heating, beyond that supplied by convection from the warm air, should be supplied to the canister so that $p_{H_2} > p_{fc}$.



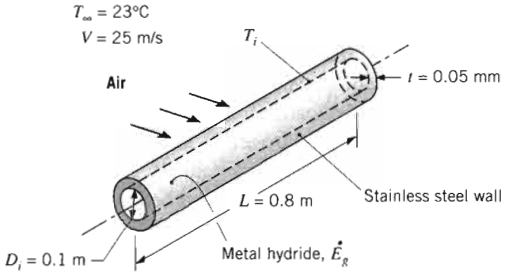
SOLUTION

Known: Size of a hydrogen storage canister, hydrogen desorption rate, required hydrogen operating pressure, velocity and temperature of air in cross flow.

Find: The convective heat transfer to the canister and the additional heating needed to sustain $p_{H_2} > p_{fc}$.

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Schematic:



Assumptions:

1. Steady-state conditions.
2. Uniform cylinder surface temperature.
3. Negligible heat gain through the ends of the cylinder.
4. Uniform metal hydride temperature.
5. Negligible contact resistance between the canister wall and the metal hydride.

Properties: Table A.4, air ($T_f \approx 285$ K): $\nu = 14.56 \times 10^{-6}$ m^2/s, $k = 25.2 \times 10^{-3}$ W/m · K·, $Pr = 0.712$. Table A.1, AISI 316 stainless steel ($T_{ss} \approx 300$ K): $k_{ss} = 13.4$ W/m · K.

Analysis: We begin by finding the minimum allowable operating temperature of the metal hydride, T_{min}, corresponding to $p_{H_2,min} = 1$ atm. The relationship between the operating temperature and pressure may be rearranged to yield

$$T_{min} = \frac{-3550}{\ln(p_{H_2,min}) - 12.9} = \frac{-3550}{\ln(1) - 12.9} = 275.2 \text{ K}$$

The thermal energy generation rate associated with the desorption of hydrogen from the metal hydride at the required flow rate is

$$\dot{E}_g = -(1.35 \times 10^{-4} \text{ kg/s}) \times (29.5 \times 10^6 \text{ J/kg}) = -3982 \text{ W}$$

To determine the convective heat transfer rate, we begin by calculating the Reynolds number:

$$Re_D = \frac{V(D_i + 2t)}{\nu} = \frac{23 \text{ m/s} \times (0.1 \text{ m} + 2 \times 0.005 \text{ m})}{14.56 \times 10^{-6} \text{ m}^2/\text{s}} = 173{,}760$$

7.5 The Sphere

Use of Equation 7.54

$$\overline{Nu}_D = 0.3 + \frac{0.62 Re_D^{1/2} Pr^{1/3}}{[1 + (0.4/Pr)^{2/3}]^{1/4}} \left[1 + \left(\frac{Re_D}{282{,}000}\right)^{5/8}\right]^{4/5}$$

yields

$$\overline{Nu}_D = 0.3 + \frac{0.62(173{,}760)^{1/2}(0.712)^{1/3}}{[1 + (0.4/0.712)^{2/3}]^{1/4}} \left[1 + \left(\frac{173{,}760}{282{,}000}\right)^{5/8}\right]^{4/5} = 315.8$$

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Therefore, the average convective heat transfer coefficient is

$$\bar{h} = \overline{Nu}_D \frac{k}{(D_i + 2t)} = 315.8 \times \frac{25.3 \times 10^{-3}\ \text{W/m} \cdot \text{K}}{(0.1\ \text{m} + 2 \times 0.005\ \text{m})} = 72.6\ \text{W/m}^2 \cdot \text{K}$$

Simplifying Equation 3.29, we find

$$q_{\text{conv}} = \frac{T_\infty - T_i}{\dfrac{1}{\pi L(D_i + 2t)\bar{h}} + \dfrac{\ln[(D_i + 2t)/D_i]}{2\pi k_{ss} L}}$$

or, substituting values,

$$q_{\text{conv}} = \frac{296\ \text{K} - 275.2\ \text{K}}{\dfrac{1}{\pi(0.8\ \text{m})(0.1\ \text{m} + 2 \times 0.005\ \text{m})(72.6\ \text{W/m}^2 \cdot \text{K})} + \dfrac{\ln[(0.1\ \text{m} + 2 \times 0.005\ \text{m})/0.1\ \text{m}]}{2\pi(13.4\ \text{W/m} \cdot \text{K})(0.8\ \text{m})}}$$

$$= 406\ \text{W}$$

The additional thermal energy, q_{add}, that must be supplied to the canister to maintain the steady-state operating temperature may be found from an energy balance, $q_{\text{add}} + q_{\text{conv}} + \dot{E}_g = 0$. Therefore,

$$q_{\text{add}} = -q_{\text{conv}} - \dot{E}_g = -406\text{W} + 3982\ \text{W} = 3576\ \text{W}$$ ◁

Comments:

1. Additional heating will occur due to radiation, conduction from the canister mounting hardware and fuel lines, and possibly condensation of water vapor on the cool canister. Waste heat from the fuel cell (see Example 3.11) might also be used as a source of thermal energy for the hydrogen storage canister.
2. The thermal resistances associated with conduction in the canister wall and convection are 0.0014 K/W and 0.053 K/W, respectively. The convection resistance dominates and can be reduced by adding fins to the exterior of the canister.
3. The amount of additional heating that is required will increase if the automobile is operated at a higher speed, since the hydrogen consumption scales as V^3, while the convective heat transfer coefficient increases as $V^{0.7}$ to $V^{0.8}$. Additional heating is also needed when the automobile is operated in a cooler climate.

7.5 *The Sphere*

Boundary layer effects associated with flow over a sphere are much like those for the circular cylinder, with transition and separation playing prominent roles. Results for the drag coefficient, which is defined by Equation 7.50, are presented in Figure 7.8. In the limit of very small Reynolds numbers (*creeping flow*), the coefficient is

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inversely proportional to the Reynolds number and the specific relation is termed *Stokes' law*

$$C_D = \frac{24}{Re_D} \qquad Re_D \lesssim 0.5 \tag{7.55}$$

Numerous heat transfer correlations have been proposed, and Whitaker [14] recommends an expression of the form

$$\overline{Nu}_D = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4}\left(\frac{\mu}{\mu_s}\right)^{1/4} \tag{7.56}$$

$$\begin{bmatrix} 0.71 \lesssim Pr \lesssim 380 \\ 3.5 \lesssim Re_D \lesssim 7.6 \times 10^4 \\ 1.0 \lesssim (\mu/\mu_s) \lesssim 3.2 \end{bmatrix}$$

All properties except μ_s are evaluated at T_∞, and the result may be applied to mass transfer problems simply by replacing $\overline{Nu}_D$ and Pr with $\overline{Sh}_D$ and Sc, respectively. A special case of convection heat and mass transfer from spheres relates to transport from freely falling liquid drops, and the correlation of Ranz and Marshall [18] is often used

$$\overline{Nu}_D = 2 + 0.6Re_D^{1/2}\,Pr^{1/3} \tag{7.57}$$

In the limit $Re_D \to 0$, Equations 7.56 and 7.57 reduce to $\overline{Nu}_D = 2$, which corresponds to heat transfer by conduction from a spherical surface to a stationary, infinite medium around the surface, as may be derived from Case 1 of Table 4.1.

EXAMPLE 7.6

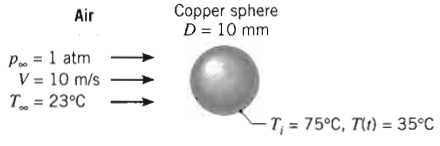
The decorative plastic film on a copper sphere of 10-mm diameter is cured in an oven at 75°C. Upon removal from the oven, the sphere is subjected to an airstream at 1 atm and 23°C having a velocity of 10 m/s. Estimate how long it will take to cool the sphere to 35°C.

SOLUTION

Known: Sphere cooling in an airstream.

Find: Time t required to cool from $T_i = 75°C$ to $T(t) = 35°C$.

Schematic:



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Assumptions:

1. Negligible thermal resistance and capacitance for the plastic film.
2. Spatially isothermal sphere.
3. Negligible radiation effects.

Properties: Table A.1, copper ($T \approx 328$ K): $\rho = 8933$ kg/m^3, $k = 399$ W/m · K, $c_p = 388$ J/kg · K. Table A.4, air ($T_\infty = 296$ K): $\mu = 182.6 \times 10^{-7}$ N · s/m^2, $\nu = 15.53 \times 10^{-6}$ m^2/s, $k = 0.0251$ W/m · K, $Pr = 0.708$. Table A.4, air ($T_s \approx 328$ K): $\mu = 197.8 \times 10^{-7}$ N · s/m^2.

Analysis: The time required to complete the cooling process may be obtained from results for a lumped capacitance. In particular, from Equations 5.4 and 5.5

$$t = \frac{\rho V c_p}{\bar{h} A_s} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

or, with $V = \pi D^3/6$ and $A_s = \pi D^2$,

$$t = \frac{\rho c_p D}{6\bar{h}} \ln \frac{T_i - T_\infty}{T - T_\infty}$$

From Equation 7.56

$$\overline{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{0.4} \left(\frac{\mu}{\mu_s}\right)^{1/4}$$

where

$$Re_D = \frac{VD}{\nu} = \frac{10 \text{ m/s} \times 0.01 \text{ m}}{15.53 \times 10^{-6} \text{ m}^2/\text{s}} = 6440$$

Hence the Nusselt number and the convection coefficient are

$$\overline{Nu}_D = 2 + [0.4(6440)^{1/2} + 0.06(6440)^{2/3}](0.708)^{0.4} \times \left(\frac{182.6 \times 10^{-7} \text{ N} \cdot \text{s/m}^2}{197.8 \times 10^{-7} \text{ N} \cdot \text{s/m}^2}\right)^{1/4} = 47.1$$

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 47.1 \frac{0.0251 \text{ W/m} \cdot \text{K}}{0.01 \text{ m}} = 118 \text{ W/m}^2 \cdot \text{K}$$

The time required for cooling is then

$$t = \frac{8933 \text{ kg/m}^3 \times 387 \text{ J/kg} \cdot \text{K} \times 0.01 \text{ m}}{6 \times 118 \text{ W/m}^2 \cdot \text{K}} \ln\left(\frac{75 - 23}{35 - 23}\right) = 71.8 \text{ s} \quad \triangleleft$$

Comments:

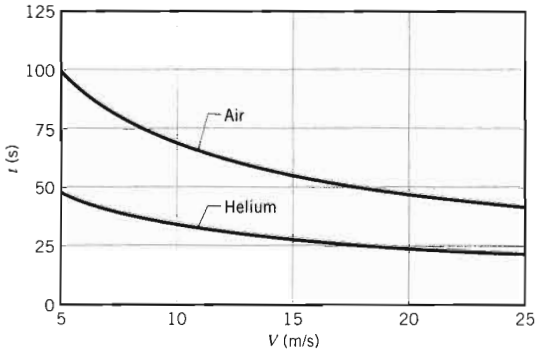
1. The validity of the lumped capacitance method may be determined by calculating the Biot number. From Equation 5.10

$$Bi = \frac{\bar{h} L_c}{k_s} = \frac{\bar{h}(r_o/3)}{k_s} = \frac{118 \text{ W/m}^2 \cdot \text{K} \times 0.005 \text{ m}/3}{399 \text{ W/m} \cdot \text{K}} = 4.9 \times 10^{-4}$$

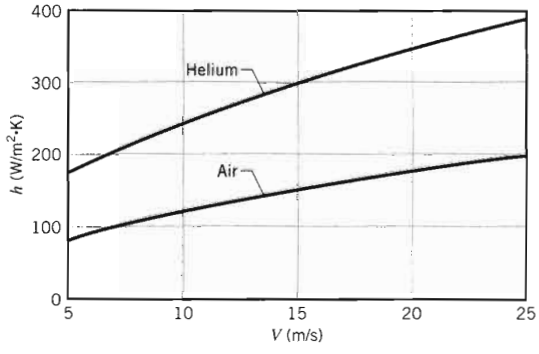
and the criterion is satisfied.

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2. Although their definitions are similar, the Nusselt number is defined in terms of the thermal conductivity of the fluid, whereas the Biot number is defined in terms of the thermal conductivity of the solid.
3. Options for enhancing production rates include accelerating the cooling process by increasing the fluid velocity and/or using a different fluid. Applying the foregoing procedures, the cooling time is computed and plotted for air and helium over the range of velocities, $5 \leq V \leq 25$ m/s.



Although Reynolds numbers for He are much smaller than those for air, the thermal conductivity is much larger and, as shown below, convection heat transfer is enhanced.



Hence production rates could be increased by substituting helium for air, albeit with a significant increase in cost.

7.6 *Flow across Banks of Tubes*

Heat transfer to or from a bank (or bundle) of tubes in cross flow is relevant to numerous industrial applications, such as steam generation in a boiler or air cooling in the coil of an air conditioner. The geometric arrangement is shown schematically

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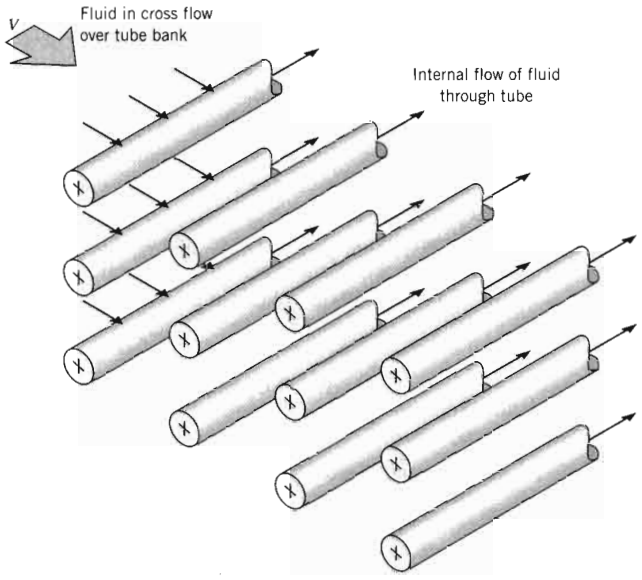


FIGURE 7.10 Schematic of a tube bank in cross flow.

in Figure 7.10. Typically, one fluid moves over the tubes, while a second fluid at a different temperature passes through the tubes. In this section we are specifically interested in the convection heat transfer associated with cross flow over the tubes.

The tube rows of a bank are either *staggered* or *aligned* in the direction of the fluid velocity V (Figure 7.11). The configuration is characterized by the tube diameter D and by the *transverse pitch* S_T and *longitudinal pitch* S_L measured between tube centers. Flow conditions within the bank are dominated by boundary layer separation effects and by wake interactions, which in turn influence convection heat transfer.

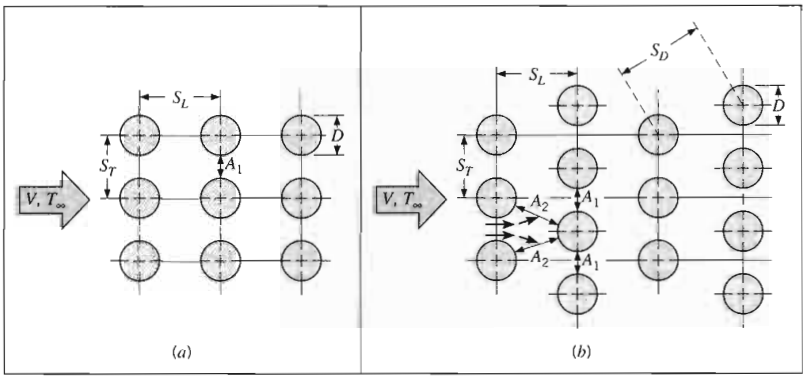


FIGURE 7.11 Tube arrangements in a bank. (*a*) Aligned. (*b*) Staggered.

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The heat transfer coefficient associated with a tube is determined by its position in the bank. The coefficient for a tube in the first row is approximately equal to that for a single tube in cross flow, whereas larger heat transfer coefficients are associated with tubes of the inner rows. The tubes of the first few rows act as a turbulence-generating grid, which increases the heat transfer coefficient for tubes in the following rows. In most configurations, however, heat transfer conditions stabilize, such that little change occurs in the convection coefficient for a tube beyond the fourth or fifth row.

Generally, we wish to know the *average* heat transfer coefficient for the *entire* tube bundle. For airflow across tube bundles composed of *10 or more rows* ($N_L \geq 10$), Grimison [19] has obtained a correlation of the form

$$\overline{Nu}_D = C_1 Re_{D,\max}^m \quad \begin{bmatrix} N_L \geq 10 \\ 2000 \lesssim Re_{D,\max} \lesssim 40{,}000 \\ Pr = 0.7 \end{bmatrix} \tag{7.58}$$

where C_1 and m are listed in Table 7.5 and

$$Re_{D,\max} \equiv \frac{\rho V_{\max} D}{\mu} \tag{7.59}$$

It has become common practice to extend this result to other fluids through insertion of the factor $1.13Pr^{1/3}$, in which case

TABLE 7.5 Constants of Equations 7.58 and 7.60 for airflow over a tube bank of 10 or more rows [19]

	S_T/D							
	1.25		1.5		2.0		3.0	
S_L/D	C_1	m	C_1	m	C_1	m	C_1	m
Aligned								
1.25	0.348	0.592	0.275	0.608	0.100	0.704	0.0633	0.752
1.50	0.367	0.586	0.250	0.620	0.101	0.702	0.0678	0.744
2.00	0.418	0.570	0.299	0.602	0.229	0.632	0.198	0.648
3.00	0.290	0.601	0.357	0.584	0.374	0.581	0.286	0.608
Staggered								
0.600	—	—	—	—	—	—	0.213	0.636
0.900	—	—	—	—	0.446	0.571	0.401	0.581
1.000	—	—	0.497	0.558	—	—	—	—
1.125	—	—	—	—	0.478	0.565	0.518	0.560
1.250	0.518	0.556	0.505	0.554	0.519	0.556	0.522	0.562
1.500	0.451	0.568	0.460	0.562	0.452	0.568	0.488	0.568
2.000	0.404	0.572	0.416	0.568	0.482	0.556	0.449	0.570
3.000	0.310	0.592	0.356	0.580	0.440	0.562	0.428	0.574

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$$\overline{Nu}_D = 1.13 C_1 Re_{D,\max}^m Pr^{1/3} \qquad (7.60)$$

$$\begin{bmatrix} N_L \gtrsim 10 \\ 2000 \lesssim Re_{D,\max} \lesssim 40{,}000 \\ Pr \gtrsim 0.7 \end{bmatrix}$$

All properties appearing in the above equations are evaluated at the film temperature. If $N_L < 10$, a correction factor may be applied such that

$$\overline{Nu}_D\big|_{(N_L<10)} = C_2 \overline{Nu}_D\big|_{(N_L\geq 10)} \qquad (7.61)$$

where C_2 is given in Table 7.6.

The Reynolds number $Re_{D,\max}$ for the foregoing correlations is based on the *maximum fluid velocity* occurring within the tube bank. For the aligned arrangement, $V_{\max}$ occurs at the transverse plane A_1 of Figure 7.11*a*, and from the mass conservation requirement for an incompressible fluid

$$V_{\max} = \frac{S_T}{S_T - D} V \qquad (7.62)$$

For the staggered configuration, the maximum velocity may occur at either the transverse plane A_1 or the diagonal plane A_2 of Figure 7.11*b*. It will occur at A_2 if the rows are spaced such that

$$2(S_D - D) < (S_T - D)$$

The factor of 2 results from the bifurcation experienced by the fluid moving from the A_1 to the A_2 planes. Hence $V_{\max}$ occurs at A_2 if

$$S_D = \left[S_L^2 + \left(\frac{S_T}{2} \right)^2 \right]^{1/2} < \frac{S_T + D}{2}$$

in which case it is given by

$$V_{\max} = \frac{S_T}{2(S_D - D)} V \qquad (7.63)$$

If $V_{\max}$ occurs at A_1 for the staggered configuration, it may again be computed from Equation 7.62.

TABLE 7.6 Correction factor C_2 of Equation 7.61 for $N_L < 10$ [20]

N_L	1	2	3	4	5	6	7	8	9
Aligned	0.64	0.80	0.87	0.90	0.92	0.94	0.96	0.98	0.99
Staggered	0.68	0.75	0.83	0.89	0.92	0.95	0.97	0.98	0.99

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TABLE 7.7 Constants of Equation 7.64 for the tube bank in cross flow [15]

Configuration	$Re_{D,max}$	C	m
Aligned	$10–10^2$	0.80	0.40
Staggered	$10–10^2$	0.90	0.40
Aligned	$10^2–10^3$	Approximate as a single (isolated) cylinder	
Staggered	$10^2–10^3$		
Aligned ($S_T/S_L > 0.7$)[a]	$10^3–2 \times 10^5$	0.27	0.63
Staggered ($S_T/S_L < 2$)	$10^3–2 \times 10^5$	$0.35(S_T/S_L)^{1/5}$	0.60
Staggered ($S_T/S_L > 2$)	$10^3–2 \times 10^5$	0.40	0.60
Aligned	$2 \times 10^5–2 \times 10^6$	0.021	0.84
Staggered	$2 \times 10^5–2 \times 10^6$	0.022	0.84

[a]For $S_T/S_L < 0.7$, heat transfer is inefficient and aligned tubes should not be used.

More recent results have been obtained and Zukauskas [15] has proposed a correlation of the form

$$\overline{Nu}_D = C\,Re_{D,max}^m\,Pr^{0.36}\left(\frac{Pr}{Pr_s}\right)^{1/4} \qquad (7.64)$$

$$\begin{bmatrix} N_L \geq 20 \\ 0.7 \leq Pr \leq 500 \\ 1000 \leq Re_{D,max} \leq 2 \times 10^6 \end{bmatrix}$$

where all properties except Pr_s are evaluated at the arithmetic mean of the fluid inlet and outlet temperatures and the constants C and m are listed in Table 7.7. The need to evaluate fluid properties at the arithmetic mean of the inlet ($T_i = T_\infty$) and outlet (T_o) temperatures is dictated by the fact that the fluid temperature will decrease or increase, respectively, due to heat transfer to or from the tubes. If the fluid temperature change, $|T_i - T_o|$, is large, significant error could result from evaluation of the properties at the inlet temperature. If $N_L < 20$, a correction factor may be applied such that

$$\overline{Nu}_D\big|_{(N_L<20)} = C_2\,\overline{Nu}_D\big|_{(N_L\geq 20)} \qquad (7.65)$$

where C_2 is given in Table 7.8.

TABLE 7.8 Correction factor C_2 of Equation 7.65 for $N_L < 20$ ($Re_{D,max} \gtrsim 10^3$) [15]

N_L	1	2	3	4	5	7	10	13	16
Aligned	0.70	0.80	0.86	0.90	0.92	0.95	0.97	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.92	0.95	0.97	0.98	0.99

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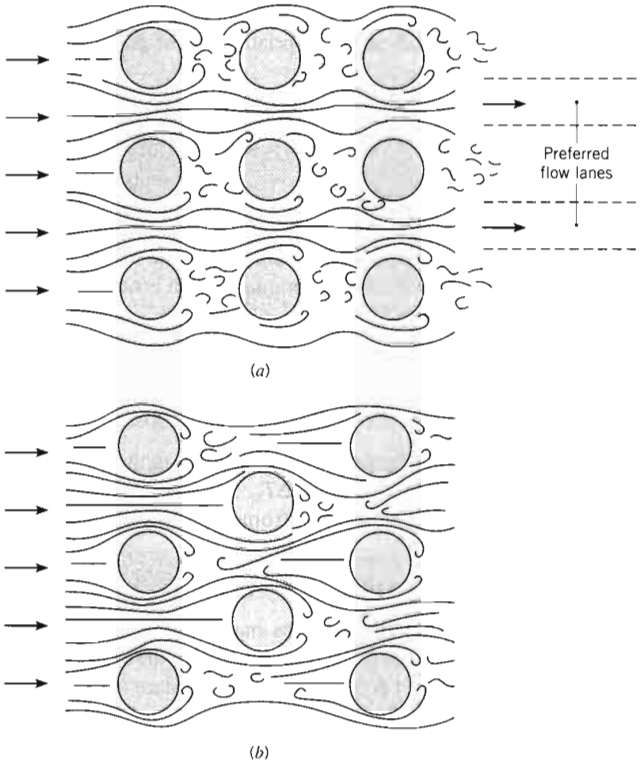


FIGURE 7.12 Flow conditions for (*a*) aligned and (*b*) staggered tubes.

Flow around tubes in the first row of a tube bank corresponds to that for a single (isolated) cylinder in cross flow. However, for subsequent rows, flow depends strongly on the tube bank arrangement (Figure 7.12). Aligned tubes beyond the first row are in the turbulent wakes of upstream tubes, and for moderate values of S_L convection coefficients associated with downstream rows are enhanced by turbulation of the flow. Typically, the convection coefficient of a row increases with increasing row number until approximately the fifth row, after which there is little change in the turbulence and hence in the convection coefficient. However, for small values of S_T/S_L, upstream rows, in effect, shield downstream rows from much of the flow, and heat transfer is adversely affected. That is, the preferred flow path is in lanes between the tubes and much of the tube surface is not exposed to the main flow. For this reason, operation of aligned tube banks with $S_T/S_L < 0.7$ (Table 7.7) is undesirable. For the staggered array, however, the path of the main flow is more tortuous and a greater portion of the surface area of downstream tubes remains in this path. In general, heat transfer enhancement is favored by the more tortuous flow of a staggered arrangement, particularly for small Reynolds number ($Re_D \lesssim 100$).

Since the fluid may experience a large change in temperature as it moves through the tube bank, the heat transfer rate could be significantly overpredicted by using $\Delta T = T_s - T_\infty$ as the temperature difference in Newton's law of cooling. As

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the fluid moves through the bank, its temperature approaches T_s and $|\Delta T|$ decreases. In Chapter 8 the appropriate form of ΔT is shown to be a *log-mean temperature difference,*

$$\Delta T_{lm} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} \tag{7.66}$$

where T_i and T_o are temperatures of the fluid as it enters and leaves the bank, respectively. The outlet temperature, which is needed to determine ΔT_{lm}, may be estimated from

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right) \tag{7.67}$$

where N is the total number of tubes in the bank and N_T is the number of tubes in the transverse plane. Once ΔT_{lm} is known, the heat transfer rate per unit length of the tubes may be computed from

$$q' = N(\bar{h}\pi D \Delta T_{lm}) \tag{7.68}$$

The foregoing results may be used to determine mass transfer rates associated with evaporation or sublimation from the surfaces of a bank of cylinders in cross flow. Once again it is only necessary to replace $\overline{Nu}_D$ and Pr by $\overline{Sh}_D$ and Sc, respectively.

We close by recognizing that there is generally as much interest in the pressure drop associated with flow across a tube bank as in the overall heat transfer rate. The power required to move the fluid across the bank is often a major operating expense and is directly proportional to the pressure drop, which may be expressed as [15]

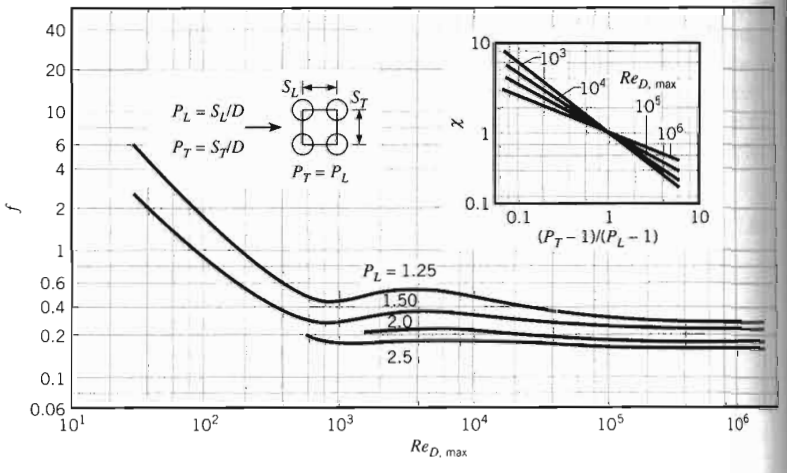


FIGURE 7.13 Friction factor f and correction factor χ for Equation 7.69. In-line tube bundle arrangement [15]. Used with permission.

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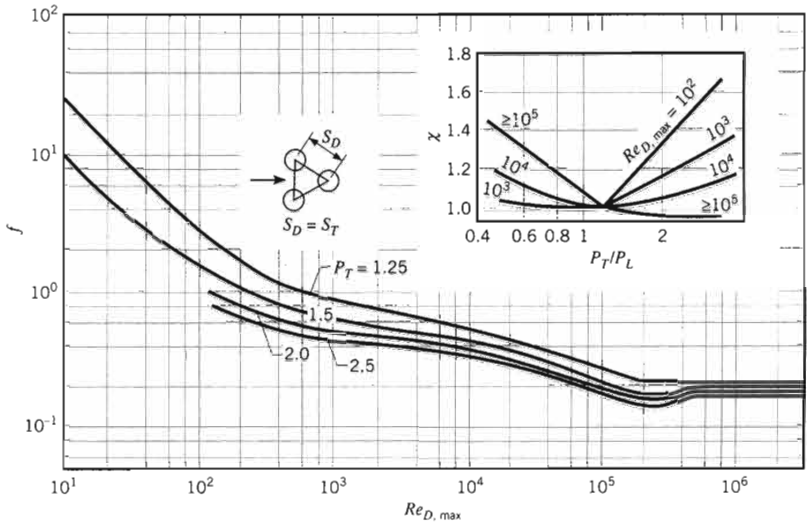


FIGURE 7.14 Friction factor f and correction factor χ for Equation 7.69. Staggered tube bundle arrangement [15]. Used with permission.

$$\Delta p = N_L \chi \left(\frac{\rho V_{\max}^2}{2}\right) f \tag{7.69}$$

The friction factor f and the correction factor χ are plotted in Figures 7.13 and 7.14. Figure 7.13 pertains to a square, in-line tube arrangement for which the dimensionless longitudinal and transverse pitches, $P_L \equiv S_L/D$ and $P_T \equiv S_T/D$, respectively, are equal. The correction factor χ, plotted in the inset, is used to apply the results to other in-line arrangements. Similarly, Figure 7.14 applies to a staggered arrangement of tubes in the form of an equilateral triangle ($S_T = S_D$), and the correction factor enables extension of the results to other staggered arrangements. Note that the Reynolds number appearing in Figures 7.13 and 7.14 is based on the maximum fluid velocity $V_{\max}$.

EXAMPLE 7.7

Pressurized water is often available at elevated temperatures and may be used for space heating or industrial process applications. In such cases it is customary to use a tube bundle in which the water is passed through the tubes, while air is passed in cross flow over the tubes. Consider a staggered arrangement for which the tube outside diameter is 16.4 mm and the longitudinal and transverse pitches are $S_L = 34.3$ mm and $S_T = 31.3$ mm. There are seven rows of tubes in the airflow direction and eight tubes per row. Under typical operating conditions the cylinder surface temperature is at 70°C, while the air upstream temperature and velocity are 15°C and 6 m/s, respectively. Determine the air-side convection coefficient and the rate of heat transfer for the tube bundle. What is the air-side pressure drop?

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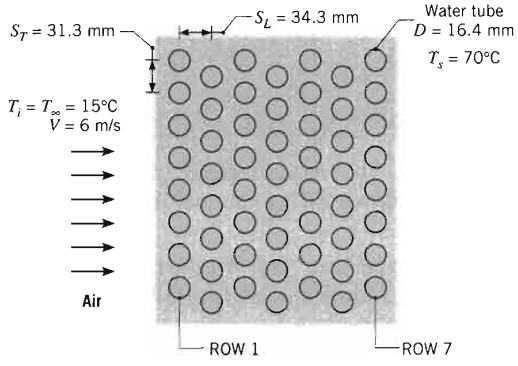
SOLUTION

Known: Geometry and operating conditions of a tube bank.

Find:

1. Air-side convection coefficient and heat rate.
2. Pressure drop.

Schematic:



Assumptions:

1. Steady-state conditions.
2. Negligible radiation effects.
3. Negligible effect of change in air temperature across tube bank on air properties.

Properties: Table A.4, air ($T_\infty = 15°C$): $\rho = 1.217$ kg/m^3, $c_p = 1007$ J/kg·K, $\nu = 14.82 \times 10^{-6}$ m^2/s, $k = 0.0253$ W/m·K, $Pr = 0.710$. Table A.4, air ($T_s = 70°C$): $Pr = 0.701$. Table A.4, air ($T_f = 43°C$): $\nu = 17.4 \times 10^{-6}$ m^2/s, $k = 0.0274$ W/m·K, $Pr = 0.705$.

Analysis:

1. From Equations 7.64 and 7.65, the air-side Nusselt number is

$$\overline{Nu}_D = C_2 C\, Re_{D,\max}^m\, Pr^{0.36} \left(\frac{Pr}{Pr_s}\right)^{1/4}$$

Since $S_D = [S_L^2 + (S_T/2)^2]^{1/2} = 37.7$ mm is greater than $(S_T + D)/2$, the maximum velocity occurs on the transverse plane, A_1, of Figure 7.11. Hence from Equation 7.62

$$V_{\max} = \frac{S_T}{S_T - D} V = \frac{31.3 \text{ mm}}{(31.3 - 16.4) \text{ mm}} 6 \text{ m/s} = 12.6 \text{ m/s}$$

With

$$Re_{D,\max} = \frac{V_{\max} D}{\nu} = \frac{12.6 \text{ m/s} \times 0.0164 \text{ m}}{14.82 \times 10^{-6} \text{ m}^2/\text{s}} = 13{,}943$$

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and

$$\frac{S_T}{S_L} = \frac{31.3\text{ mm}}{34.3\text{ mm}} = 0.91 < 2$$

it follows from Tables 7.7 and 7.8 that

$$C = 0.35\left(\frac{S_T}{S_L}\right)^{1/5} = 0.34, \quad m = 0.60, \quad \text{and} \quad C_2 = 0.95$$

Hence

$$\overline{Nu}_D = 0.95 \times 0.34(13{,}943)^{0.60}(0.71)^{0.36}\left(\frac{0.710}{0.701}\right)^{0.25} = 87.9$$

and

$$\bar{h} = \overline{Nu}_D \frac{k}{D} = 87.9 \times \frac{0.0253\text{ W/m} \cdot \text{K}}{0.0164\text{ m}} = 135.6\text{ W/m}^2 \cdot \text{K} \quad \triangleleft$$

From Equation 7.67

$$T_s - T_o = (T_s - T_i)\exp\left(-\frac{\pi D N \bar{h}}{\rho V N_T S_T c_p}\right)$$

$$T_s - T_o = (55°\text{C})\exp\left(-\frac{\pi(0.0164\text{ m})\,56\,(135.6\text{ W/m}^2 \cdot \text{K})}{1.217\text{ kg/m}^3\,(6\text{ m/s})\,8\,(0.0313\text{ m})\,1007\text{ J/kg} \cdot \text{K}}\right)$$

$$T_s - T_o = 44.5°\text{C}$$

Hence from Equations 7.66 and 7.68

$$\Delta T_{\text{lm}} = \frac{(T_s - T_i) - (T_s - T_o)}{\ln\left(\frac{T_s - T_i}{T_s - T_o}\right)} = \frac{(55 - 44.5)°\text{C}}{\ln\left(\frac{55}{44.5}\right)} = 49.6°\text{C}$$

and

$$q' = N(\bar{h}\pi D\Delta T_{\text{lm}}) = 56\pi \times 135.6\text{ W/m}^2 \cdot \text{K} \times 0.0164\text{ m} \times 49.6°\text{C}$$

$$q' = 19.4\text{ kW/m} \quad \triangleleft$$

2. The pressure drop may be obtained from Equation 7.69.

$$\Delta p = N_L \chi\left(\frac{\rho V_{\max}^2}{2}\right) f$$

With $Re_{D,\max} = 13{,}943$, $P_T = (S_T/D) = 1.91$, and $(P_T/P_L) = 0.91$, it follows from Figure 7.14 that $\chi \approx 1.04$ and $f \approx 0.35$. Hence with $N_L = 7$

$$\Delta p = 7 \times 1.04\left[\frac{1.217\text{ kg/m}^3(12.6\text{ m/s})^2}{2}\right]0.35$$

$$\Delta p = 246\text{ N/m}^2 = 2.46 \times 10^{-3}\text{ bars} \quad \triangleleft$$

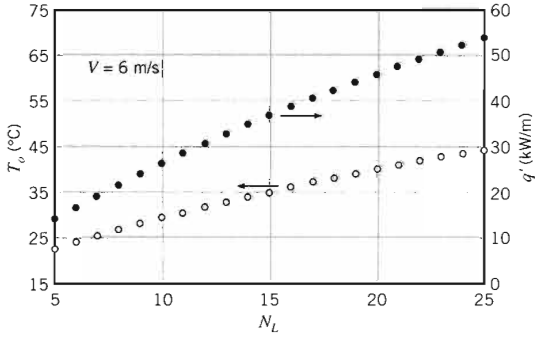
Comments:

1. With properties evaluated at T_f, $Re_{D,\max} = 11{,}876$. With $S_T/D \approx 2$ and $S_L/D \approx 2$, it follows from Tables 7.5 and 7.6 that $C_1 = 0.482$, $m = 0.556$, and $C_2 = 0.97$. From

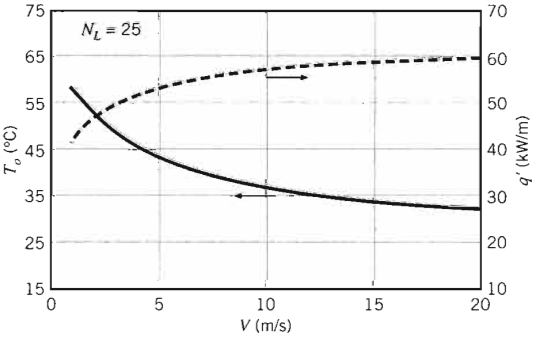
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Equations 7.60 and 7.61, the Nusselt number is then $\overline{Nu}_D = 86.7$, and $\bar{h} = 144.8$ W/m$^2 \cdot$ K. Values of $\bar{h}$ obtained from Equations 7.60 and 7.64 therefore agree to within 7%, which is well within their uncertainties.

2. Had $\Delta T_i \equiv T_s - T_i$ been used in lieu of ΔT_{lm} in Equation 7.68, the heat rate would have been overpredicted by 11%.
3. Since the air temperature is predicted to increase by only 10.5°C, evaluation of the air properties at $T_i = 15$°C is a reasonable approximation. However, if improved accuracy is desired, the calculations could be repeated with the properties reevaluated at $(T_i + T_o)/2 = 20.25$°C. An exception pertains to the density ρ in the exponential term of Equation 7.67. As it appears in the denominator of this term, ρ is matched with the inlet velocity to provide a product (ρV) that is linked to the mass flow rate of air entering the tube bank. Hence, in this term, ρ should be evaluated at T_i.
4. The air outlet temperature and heat rate may be increased by increasing the number of tube rows, and for a fixed number of rows, they may be varied by adjusting the air velocity. For $5 \leq N_L \leq 25$ and $V = 6$ m/s, parametric calculations based on Equations 7.64 through 7.68 yield the following results:



The air outlet temperature would asymptotically approach the surface temperature with increasing N_L, at which point the heat rate approaches a constant value and there is no advantage to adding more tube rows. Note that Δp increases linearly with increasing N_L. For $N_L = 25$ and $1 \leq V \leq 20$ m/s, we obtain



7.7 Impinging Jets

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Although the heat rate increases with increasing V, the air outlet temperature decreases, approaching T_i as $V \longrightarrow \infty$.

7.7 Impinging Jets

A single gas jet or an array of such jets, impinging normally on a surface, may be used to achieve enhanced coefficients for convective heating, cooling, or drying. Applications include tempering of glass plate, annealing of metal sheets, drying of textile and paper products, cooling of heated components in gas turbine engines, and deicing of aircraft systems.

7.7.1 Hydrodynamic and Geometric Considerations

As shown in Figure 7.15, gas jets are typically discharged into a quiescent ambient from a round nozzle of diameter D or a slot (rectangular) nozzle of width W. Typically, the jet is turbulent and, at the nozzle exit, is characterized by a uniform velocity profile. However, with increasing distance from the exit, momentum exchange between the jet and the ambient causes the free boundary of the jet to broaden and the *potential core,* within which the uniform exit velocity is retained, to contract. Downstream of the potential core the velocity profile is nonuniform over the entire

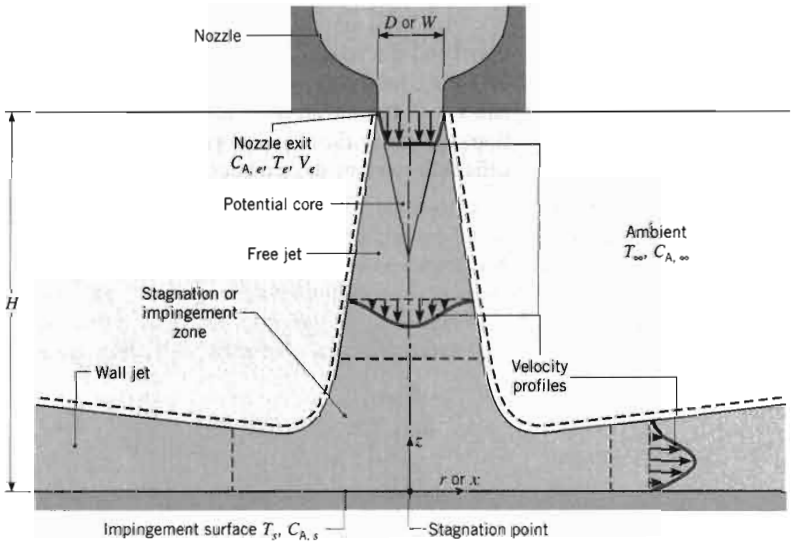


FIGURE 7.15 Surface impingement of a single round or slot gas jet.

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jet cross section and the maximum (center) velocity decreases with increasing distance from the nozzle exit. The region of the flow over which conditions are unaffected by the impingement (target) surface is termed the *free jet.*

Within the *stagnation* or *impingement zone,* flow is influenced by the target surface and is decelerated and accelerated in the normal (z) and transverse (r or x) directions, respectively. However, since the flow continues to entrain zero momentum fluid from the ambient, transverse acceleration cannot continue indefinitely and accelerating flow in the stagnation zone is transformed to a decelerating *wall jet.* Hence, with increasing r or x, velocity components parallel to the surface increase from a value of zero to some maximum and subsequently decay to zero. Velocity profiles within the wall jet are characterized by zero velocity at both the impingement and free surfaces. If $T_s \neq T_e$ and/or $C_{A,s} \neq C_{A,e}$, convection heat and/or mass transfer occurs in both the stagnation and wall jet regions.

Many impingement heat (mass) transfer schemes involve an array of jets, as, for example, the array of slot jets shown in Figure 7.16. In addition to flow from each nozzle exhibiting free jet, stagnation, and wall jet regions, secondary stagnation zones result from the interaction of adjoining wall jets. In many such schemes the jets are discharged into a restricted volume bounded by the target surface and the nozzle plate from which the jets originate. The overall rate of heat (mass) transfer depends strongly on the manner in which *spent gas,* whose temperature (species concentration) is between values associated with the nozzle exit and the impingement surface, is vented from the system. For the configuration of Figure 7.16, spent gas cannot flow upward between the nozzles but must instead flow symmetrically in the $\pm y$ directions. As the temperature (surface cooling) or species concentration (surface evaporation) of the gas increases with increasing $|y|$, the local surface-to-gas temperature or concentration difference decreases, causing a reduction in local convection fluxes. A preferable situation is one for which the space between adjoining nozzles is open to the ambient, thereby permitting continuous upflow and direct discharge of the spent gas.

Plan (top) views of single round and slot nozzles, as well as regular arrays of round and slot nozzles, are shown in Figure 7.17. For the isolated nozzles (Figures 7.17*a*, *d*), local and average convection coefficients are associated with any $r > 0$ and $x > 0$. For the arrays, with discharge of the spent gas in the vertical (z) direction, symmetry dictates equivalent local and average values for each of the unit cells delineated by dashed lines. For a large number of square-in-line (Figure 7.17*b*)

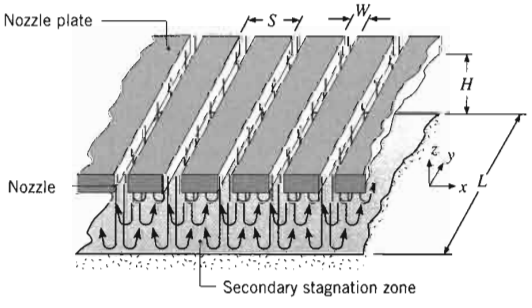


FIGURE 7.16 Surface impingement of an array of slot jets.

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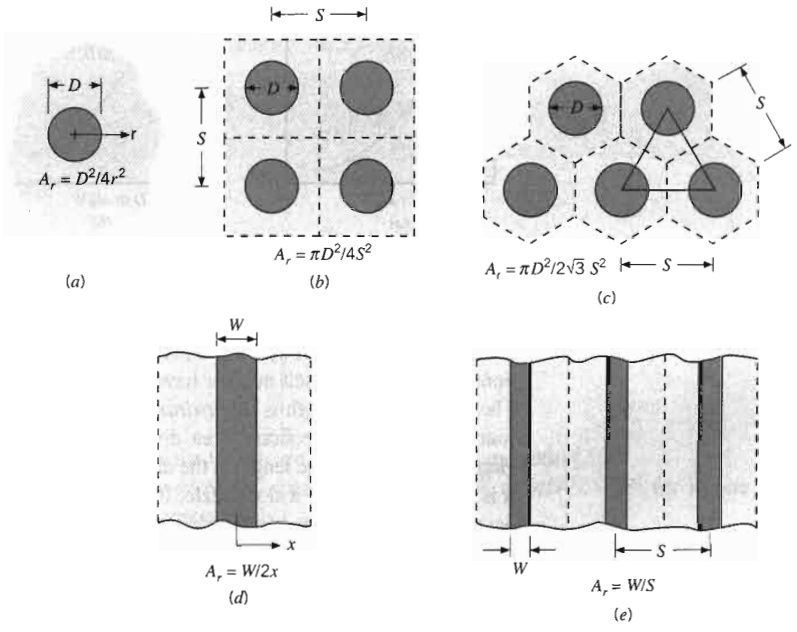


FIGURE 7.17 Plan view of pertinent geometrical features for (*a*) single round jet, (*b*) in-line array of round jets, (*c*) staggered array of round jets, (*d*) single slot jet, and (*e*) array of slot jets.

or equilaterally staggered (Figure 7.17*c*) round jets, the unit cells correspond to a square or hexagon, respectively. A pertinent geometric parameter is the relative nozzle area, which is defined as the ratio of the nozzle exit cross-sectional area to the surface area of the cell ($A_r \equiv A_{c,e}/A_{\text{cell}}$). In each case, S represents the pitch of the array.

7.7.2 Convection Heat and Mass Transfer

In the results that follow, it is presumed that the gas jet exits its nozzle with a uniform velocity V_e, temperature T_e, and species concentration $C_{A,e}$. Thermal and compositional equilibrium with the ambient are presumed ($T_e = T_\infty$, $C_{A,e} = C_{A,\infty}$), while convection heat and/or mass transfer may occur at an impingement surface of uniform temperature and/or species composition ($T_s \neq T_e$, $C_{A,s} \neq C_{A,e}$). Newton's law of cooling and its mass transfer analog are then

$$q'' = h(T_s - T_e) \tag{7.70}$$

$$N''_A = h_m(C_{A,s} - C_{A,e}) \tag{7.71}$$

Conditions are presumed to be uninfluenced by the level of turbulence at the nozzle exit, and the surface is presumed to be stationary. However, this requirement may be relaxed for surface velocities which are much less than the jet impact velocity.

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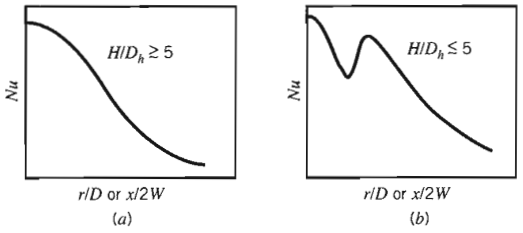


FIGURE 7.18
Distribution of local Nusselt number associated with a single round or slot nozzle for (*a*) large and (*b*) small relative nozzle-to-plate spacings.

An extensive review of available convection coefficient data for impinging gas jets has been performed by Martin [21], and for a single round or slot nozzle, distributions of the *local* Nusselt number have the characteristic forms shown in Figure 7.18. The characteristic length is the *hydraulic diameter* of the nozzle, which is defined as four times its cross-sectional area divided by its wetted perimeter ($D_h \equiv 4A_{c,e}/P$). Hence the characteristic length is the diameter of a round nozzle, and assuming $L \gg W$, it is twice the width of a slot nozzle. It follows that $Nu = hD/k$ for a round nozzle and $Nu = h(2W/k)$ for a slot nozzle. For large nozzle-to-plate separations, Figure 7.18*a*, the distribution is characterized by a bell-shaped curve for which Nu monotonically decays from a maximum value at the *stagnation point*, r/D or $x/2W$ equal to zero.

For small separations, Figure 7.18*b*, the distribution is characterized by a second maximum, whose value increases with increasing jet Reynolds number and may exceed that of the first maximum. The threshold separation of $H/D \approx 5$, below which there is a second maximum, is loosely associated with the length of the potential core (Figure 7.15). Appearance of the second maximum is attributed to a sharp rise in the turbulence level which accompanies the transition from an accelerating stagnation region flow to a decelerating wall jet [21]. Additional maxima have been observed and attributed to the formation of vortices in the stagnation zone, as well as transition to a turbulent wall jet [22].

Secondary maxima in Nu are also associated with the interaction of adjoining wall jets for an array [21, 23]. However, distributions are two-dimensional, exhibiting, for example, variations with both x and y for the slot jet array of Figure 7.16. Variations with x could be expected to yield maxima at the jet centerline and halfway between adjoining jets, while constraint of the exhaust flow to the $\pm y$ direction would induce acceleration with increasing $|y|$ and hence a monotonically increasing Nu with $|y|$. However, variations with y decrease with increasing cross-sectional area of the outflow and may be neglected if $S \times H \gtrsim W \times L$ [21].

Average Nusselt numbers may be obtained by integrating local results over the appropriate surface area. The resulting correlations are reported in the form

$$\overline{Nu} = f(Re, Pr, A_r, H/D_h) \tag{7.72}$$

where

$$\overline{Nu} \equiv \frac{\bar{h}D_h}{k} \tag{7.73}$$

$$Re = \frac{V_e D_h}{\nu} \tag{7.74}$$

and $D_h = D$ (round nozzle) or $D_h = 2W$ (slot nozzle).

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Round Nozzles Having assessed data from several sources, Martin [21] recommends the following correlation for a *single round nozzle* ($A_r = D^2/4r^2$)

$$\frac{\overline{Nu}}{Pr^{0.42}} = G\left(A_r, \frac{H}{D}\right)[2Re^{1/2}(1 + 0.005Re^{0.55})^{1/2}] \tag{7.75}$$

where

$$G = 2A_r^{1/2}\frac{1 - 2.2A_r^{1/2}}{1 + 0.2(H/D - 6)A_r^{1/2}} \tag{7.76}$$

The ranges of validity are

$$\begin{bmatrix} 2000 \lesssim Re \lesssim 400{,}000 \\ 2 \lesssim H/D \lesssim 12 \\ 0.004 \lesssim A_r \lesssim 0.04 \end{bmatrix}$$

For $A_r \gtrsim 0.04$, results for $\overline{Nu}$ are available in graphical form [21].

For an *array of round nozzles* ($A_r = \pi D^2/4S^2$ or $\pi D^2/2\sqrt{3}S^2$ for in-line and staggered arrays, respectively),

$$\frac{\overline{Nu}}{Pr^{0.42}} = 0.5K\left(A_r, \frac{H}{D}\right)G\left(A_r, \frac{H}{D}\right)Re^{2/3} \tag{7.77}$$

where

$$K = \left[1 + \left(\frac{H/D}{0.6/A_r^{1/2}}\right)^6\right]^{-0.05} \tag{7.78}$$

and G is the single nozzle function given by Equation 7.76. The function K accounts for the fact that, for $H/D \gtrsim 0.6/A_r^{1/2}$, the average Nusselt number for the array decays more rapidly with increasing H/D than that for the single nozzle. The correlation is valid over the ranges

$$\begin{bmatrix} 2000 \lesssim Re \lesssim 100{,}000 \\ 2 \lesssim H/D \lesssim 12 \\ 0.004 \lesssim A_r \lesssim 0.04 \end{bmatrix}$$

Slot Nozzles For a *single slot nozzle* ($A_r = W/2x$), the recommended correlation is

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{3.06}{0.5/A_r + H/W + 2.78}Re^m \tag{7.79}$$

where

$$m = 0.695 - \left[\left(\frac{1}{4A_r}\right) + \left(\frac{H}{2W}\right)^{1.33} + 3.06\right]^{-1} \tag{7.80}$$

and the ranges of validity are

$$\begin{bmatrix} 3000 \lesssim Re \lesssim 90{,}000 \\ 2 \lesssim H/W \lesssim 10 \\ 0.025 \lesssim A_r \lesssim 0.125 \end{bmatrix}$$

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As a *first approximation*, Equation 7.79 may be used for $A_r \gtrsim 0.125$, yielding predictions for the stagnation point ($x = 0, A_r \to \infty$) that are within 40% of measured values.

For an *array of slot nozzles* ($A_r = W/S$), the recommended correlation is

$$\frac{\overline{Nu}}{Pr^{0.42}} = \frac{2}{3} A_{r,o}^{3/4} \left(\frac{2Re}{A_r/A_{r,o} + A_{r,o}/A_r} \right)^{2/3} \tag{7.81}$$

where

$$A_{r,o} = \left[60 + 4\left(\frac{H}{2W} - 2 \right)^2 \right]^{-1/2} \tag{7.82}$$

The correlation pertains to conditions for which the outflow of spent gas is restricted to the $\pm$ y-directions of Figure 7.16 and the outflow area is large enough to satisfy the requirement $(S \times H)/(W \times L) \gtrsim 1$. Additional restrictions are

$$\begin{bmatrix} 1500 \lesssim Re \lesssim 40{,}000 \\ 2 \lesssim H/W \lesssim 80 \\ 0.008 \lesssim A_r \lesssim 2.5A_{r,o} \end{bmatrix}$$

An *optimal* arrangement of nozzles would be one for which the values of H, S, and D_h yielded the largest value of $\overline{Nu}$ for a prescribed total gas flow rate per unit surface area of the target. For fixed H and for arrays of both round and slot nozzles, optimal values of D_h and S have been found to be [21]

$$D_{h,\text{op}} \approx 0.2H \tag{7.83}$$

$$S_{\text{op}} \approx 1.4H \tag{7.84}$$

The optimum value of $(D_h/H)^{-1} \approx 5$ coincides approximately with the length of the potential core. Beyond the potential core, the midline jet velocity decays, causing an attendant reduction in convection coefficients.

Invoking the heat and mass transfer analogy by substituting $\overline{Sh}/Sc^{0.42}$ for $\overline{Nu}/Pr^{0.42}$, the foregoing correlations may also be applied to convection mass transfer. However, for both heat and mass transfer, application of the equations should be restricted to conditions for which they were developed. For example, in their present form, the correlations may not be used if the jets emanate from sharp-edged orifices instead of bell-shaped nozzles. The orifice jet is strongly affected by a flow contraction phenomenon that alters convection heat or mass transfer [21, 22]. In the case of convection heat transfer, conditions are also influenced by differences between the jet exit and ambient temperatures ($T_e \neq T_\infty$). The exit temperature is then an inappropriate temperature in Newton's law of cooling, Equation 7.70, and should be replaced by what is commonly termed the recovery, or adiabatic wall, temperature [24, 25].

7.8 *Packed Beds*

Gas flow through a *packed bed* of solid particles (Figure 7.19) is relevant to many industrial processes, which include the transfer and storage of thermal energy, heterogeneous catalytic reactions, and drying. The term *packed bed* refers to a

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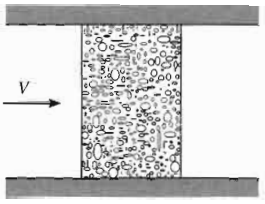


FIGURE 7.19
Gas flow through a packed bed of solid particles.

condition for which the position of the particles is *fixed.* In contrast, a *fluidized bed* is one for which the particles are in motion due to advection with the fluid.

For a packed bed a large amount of heat or mass transfer surface area can be obtained in a small volume, and the irregular flow that exists in the voids of the bed enhances transport through turbulent mixing. Many correlations that have been developed for different particle shapes, sizes, and packing densities are described in the literature [26–29]. One such correlation, which has been recommended for gas flow in a bed of spheres, is of the form

$$\varepsilon \bar{j}_H = \varepsilon \bar{j}_m = 2.06 Re_D^{-0.575} \quad \begin{bmatrix} Pr\,(\text{or } Sc) \approx 0.7 \\ 90 \lesssim Re_D \lesssim 4000 \end{bmatrix} \tag{7.85}$$

where $\bar{j}_H$ and $\bar{j}_m$ are the Colburn j factors defined by Equations 6.70 and 6.71. The Reynolds number $Re_D = VD/\nu$ is defined in terms of the sphere diameter and the upstream velocity V that would exist in the empty channel without the packing. The quantity ε is the *porosity,* or *void fraction,* of the bed (volume of void space per unit volume of bed), and its value typically ranges from 0.30 to 0.50. The correlation may be applied to packing materials other than spheres by multiplying the right-hand side by an appropriate correction factor. For a bed of uniformly sized cylinders, with length-to-diameter ratio of 1, the factor is 0.79; for a bed of cubes it is 0.71.

In using Equation 7.85, properties should be evaluated at the arithmetic mean of the fluid temperatures entering and leaving the bed. If the particles are at a uniform temperature T_s, the heat transfer rate for the bed may be computed from

$$q = \bar{h} A_{p,t} \Delta T_{\text{lm}} \tag{7.86}$$

where $A_{p,t}$ is the total surface area of the particles and ΔT_{lm} is the log-mean temperature difference defined by Equation 7.66. The outlet temperature, which is needed to compute ΔT_{lm}, may be estimated from

$$\frac{T_s - T_o}{T_s - T_i} = \exp\left(- \frac{\bar{h} A_{p,t}}{\rho V A_{c,b} c_p} \right) \tag{7.87}$$

where ρ and V are the inlet density and velocity, respectively, and $A_{c,b}$ is the bed (channel) cross-sectional area.

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7.9 Summary

In this chapter we have considered *forced convection* heat and mass transfer for an important class of problems involving *external flow* at low-to-moderate speeds. Consideration was given to several common geometries, for which convection coefficients depend on the nature of boundary layer development. You should test your understanding of related concepts by addressing the following questions.

- What is an *external flow*?
- How does the velocity boundary layer thickness vary with distance from the leading edge for *laminar flow* over a flat plate? For *turbulent flow*? What determines the relative velocity, thermal, and concentration boundary layer thicknesses for laminar flow? For turbulent flow?
- How does the *local* convection heat or mass transfer coefficient vary with distance from the leading edge for *laminar flow* over a flat plate? For *turbulent flow*? For flow in which *transition* to turbulence occurs on the plate?
- How is local heat transfer from the surface of a flat plate affected by the existence of an *unheated starting length*?
- What are the manifestations of *boundary layer separation* from the surface of a circular cylinder in cross flow? How is separation influenced by whether the upstream flow is laminar or turbulent?
- How is variation of the local convection coefficient on the surface of a circular cylinder in cross flow affected by boundary layer separation? By boundary layer transition? Where do local maxima and minima in the convection coefficient occur on the surface?
- How does the average convection coefficient of a tube vary with its location in a tube bank?
- For jet impingement on a surface, what are distinguishing features of the *free jet*? The *potential core*? The *impingement zone*? The *wall jet*?
- At what location on the surface of an impinging jet will a maximum in the convection coefficient always exist? Under what conditions will there be a secondary maximum?
- For an *array* of impinging jets, how are flow and heat transfer affected by the manner in which *spent fluid* is discharged from the system?
- What is the difference between a *packed bed* and a *fluidized bed* of solid particles?
- What is an empirical heat or mass transfer correlation? What are the inherent dimensionless parameters for forced convection?
- What is the *film temperature*?
- What temperature difference must be used when computing the total rate of heat transfer from a bank of tubes or a packed bed?

In this chapter we have also compiled convection correlations that may be used to estimate convection transfer rates for a variety of external flow conditions. For simple surface geometries these results may be derived from a boundary layer analysis, but in most cases they are obtained from generalizations based on experiment.

TABLE 7.9 Sum

Correlation
$\delta = 5x\,Re_x^{-1/2}$
$C_{f,x} = 0.664Re_x^{-1/2}$
$Nu_x = 0.332Re_x^{1/2}\,Pr^{1/3}$
$\delta_t = \delta Pr^{-1/3}$
$\overline{C}_{f,x} = 1.328Re_x^{-1/2}$
$\overline{Nu}_x = 0.664Re_x^{1/2}\,Pr^{1/3}$
$Nu_x = 0.565Pe_x^{1/2}$
$C_{f,x} = 0.0592Re_x^{-1/5}$
$\delta = 0.37x\,Re_x^{-1/5}$
$Nu_x = 0.0296Re_x^{4/5}\,Pr^{1/3}$
$\overline{C}_{f,L} = 0.074Re_L^{-1/5} - 17$
$\overline{Nu}_L = (0.037Re_L^{4/5} - 87$
$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$ (Table 7.2)
$\overline{Nu}_D = C\,Re_D^m\,Pr^n(Pr/Pr$ (Table 7.4)
$\overline{Nu}_D = 0.3 + [0.62Re_D^{1/2}$ $\times [1 + (0.4/Pr)^2$ $\times [1 + (Re_D/282$
$\overline{Nu}_D = 2 + (0.4Re_D^{1/2}$ $+ 0.06Re_D^{2/3})Pr^{0.}$ $\times (\mu/\mu_s)^{1/4}$
$\overline{Nu}_D = 2 + 0.6Re_D^{1/2}\,Pr^{1}$
$\overline{Nu}_D = 1.13C_1C_2\,Re_{D,ma}^m$ (Tables 7.5, 7.6)
$\overline{Nu}_D = CC_2\,Re_{D,\max}^m\,Pr^{0.}$ (Tables 7.7, 7.8)

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You should know when and how to use the various expressions, and you should be familiar with the general methodology of a convection calculation. To facilitate their use, the correlations are summarized in Table 7.9.

TABLE 7.9 Summary of convection heat transfer correlations for external flow[a, b]

Correlation		Geometry	Conditions[c]
$\delta = 5x\,Re_x^{-1/2}$	(7.19)	Flat plate	Laminar, T_f
$C_{fx} = 0.664Re_x^{-1/2}$	(7.20)	Flat plate	Laminar, local, T_f
$Nu_x = 0.332Re_x^{1/2}\,Pr^{1/3}$	(7.23)	Flat plate	Laminar, local, T_f, $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$	(7.24)	Flat plate	Laminar, T_f
$\overline{C}_{fx} = 1.328Re_x^{-1/2}$	(7.29)	Flat plate	Laminar, average, T_f
$\overline{Nu}_x = 0.664Re_x^{1/2}\,Pr^{1/3}$	(7.30)	Flat plate	Laminar, average, T_f, $Pr \gtrsim 0.6$
$Nu_x = 0.565Pe_x^{1/2}$	(7.32)	Flat plate	Laminar, local, T_f, $Pr \lesssim 0.05$, $Pe_x \gtrsim 100$
$C_{fx} = 0.0592Re_x^{-1/5}$	(7.34)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$
$\delta = 0.37x\,Re_x^{-1/5}$	(7.35)	Flat plate	Turbulent, T_f, $Re_x \lesssim 10^8$
$Nu_x = 0.0296Re_x^{4/5}\,Pr^{1/3}$	(7.36)	Flat plate	Turbulent, local, T_f, $Re_x \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{C}_{fL} = 0.074Re_L^{-1/5} - 1742Re_L^{-1}$	(7.40)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$
$\overline{Nu}_L = (0.037Re_L^{4/5} - 871)Pr^{1/3}$	(7.38)	Flat plate	Mixed, average, T_f, $Re_{x,c} = 5 \times 10^5$, $Re_L \lesssim 10^8$, $0.6 \lesssim Pr \lesssim 60$
$\overline{Nu}_D = C\,Re_D^m\,Pr^{1/3}$ (Table 7.2)	(7.52)	Cylinder	Average, T_f, $0.4 \lesssim Re_D \lesssim 4 \times 10^5$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = C\,Re_D^m\,Pr^n(Pr/Pr_s)^{1/4}$ (Table 7.4)	(7.53)	Cylinder	Average, T_∞, $1 \lesssim Re_D \lesssim 10^6$, $0.7 \lesssim Pr \lesssim 500$
$\overline{Nu}_D = 0.3 + [0.62Re_D^{1/2}\,Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}] \times [1 + (Re_D/282{,}000)^{5/8}]^{4/5}$	(7.54)	Cylinder	Average, T_f, $Re_D\,Pr \gtrsim 0.2$
$\overline{Nu}_D = 2 + (0.4Re_D^{1/2} + 0.06Re_D^{2/3})Pr^{0.4} \times (\mu/\mu_s)^{1/4}$	(7.56)	Sphere	Average, T_∞, $3.5 \lesssim Re_D \lesssim 7.6 \times 10^4$, $0.71 \lesssim Pr \lesssim 380$
$\overline{Nu}_D = 2 + 0.6Re_D^{1/2}\,Pr^{1/3}$	(7.57)	Falling drop	Average, T_∞
$\overline{Nu}_D = 1.13C_1C_2\,Re_{D,\max}^m\,Pr^{1/3}$ (Tables 7.5, 7.6)	(7.60), (7.61)	Tube bank[d]	Average, $\overline{T}_f$, $2000 \lesssim Re_{D,\max} \lesssim 4 \times 10^4$, $Pr \gtrsim 0.7$
$\overline{Nu}_D = CC_2\,Re_{D,\max}^m\,Pr^{0.36}(Pr/Pr_s)^{1/4}$ (Tables 7.7, 7.8)	(7.64), (7.65)	Tube bank[d]	Average, $\overline{T}$, $1000 \lesssim Re_D \lesssim 2 \times 10^6$, $0.7 \lesssim Pr \lesssim 500$

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TABLE 7.9 *(Continued)*

Correlation		Geometry	Conditions
Single round nozzle	(7.75)	Impinging jet	Average, T_f, $2000 \lesssim Re \lesssim 4 \times 10^5$, $2 \lesssim (H/D) \lesssim 12$, $2.5 \lesssim (r/D) \lesssim 7.5$
Single slot nozzle	(7.79)	Impinging jet	Average, T_f, $3000 \lesssim Re \lesssim 9 \times 10^4$, $2 \lesssim (H/W) \lesssim 10$, $4 \lesssim (x/W) \lesssim 20$
Array of round nozzles	(7.77)	Impinging jet	Average, T_f, $2000 \lesssim Re \lesssim 10^5$, $2 \lesssim (H/D) \lesssim 12$, $0.004 \lesssim A_r \lesssim 0.04$
Array of slot nozzles	(7.81)	Impinging jet	Average, T_f, $1500 \lesssim Re \lesssim 4 \times 10^4$, $2 \lesssim (H/W) \lesssim 80$, $0.008 \lesssim A_r \lesssim 2.5A_{r,s}$
$\varepsilon \bar{j}_H = \varepsilon \bar{j}_m = 2.06 Re_D^{-0.575}$	(7.85)	Packed bed of spheres[d]	Average, $\overline{T}$, $90 \lesssim Re_D \lesssim 4000$, Pr (or Sc) ≈ 0.7

[a]Correlations in this table pertain to isothermal surfaces; for special cases involving an unheated starting length or a uniform surface heat flux, see Section 7.2.4 or 7.2.5.
[b]When the heat and mass transfer analogy is applicable, the corresponding mass transfer correlations may be obtained by replacing Nu and Pr by Sh and Sc, respectively.
[c]The temperature listed under "Conditions" is the temperature at which properties should be evaluated.
[d]For tube banks and packed beds, properties are evaluated at the average fluid temperature, $\overline{T} = (T_i + T_o)/2$, or the average film temperature, $\overline{T}_f = (T_s + \overline{T})/2$.

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Problems

Flat Plate in Par

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7.2 Engine oil at 100°
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7.3 Consider steady,
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Problems

Flat Plate in Parallel Flow

7.1 Consider the following fluids at a film temperature of 300 K in parallel flow over a flat plate with velocity of 1 m/s: atmospheric air, water, engine oil, and mercury.

(a) For each fluid, determine the velocity and thermal boundary layer thicknesses at a distance of 40 mm from the leading edge.

(b) For each of the prescribed fluids and on the same coordinates, plot the boundary layer thicknesses as a function of distance from the leading edge to a plate length of 40 mm.

7.2 Engine oil at 100°C and a velocity of 0.1 m/s flows over both surfaces of a 1-m-long flat plate maintained at 20°C. Determine:

(a) The velocity and thermal boundary layer thicknesses at the trailing edge.

(b) The local heat flux and surface shear stress at the trailing edge.

(c) The total drag force and heat transfer per unit width of the plate.

(d) Plot the boundary layer thicknesses and local values of the surface shear stress, convection coefficient, and heat flux as a function of x for $0 \leq x \leq 1$ m.

7.3 Consider steady, parallel flow of atmospheric air over a flat plate. The air has a temperature and free stream velocity of 300 K and 25 m/s.

(a) Evaluate the boundary layer thickness at distances of $x = 1$, 10, and 100 mm from the leading edge. If a second plate were installed parallel to and at a distance of 3 mm from the first plate, what is the distance from the leading edge at which boundary layer merger would occur?

(b) Evaluate the surface shear stress and the y-velocity component at the outer edge of the boundary layer for the single plate at $x = 1$, 10, and 100 mm.

(c) Comment on the validity of the boundary layer approximations.

7.4 Consider a liquid metal ($Pr \ll 1$), with free stream conditions u_∞ and T_∞, in parallel flow over an isothermal flat plate at T_s. Assuming that $u = u_\infty$ throughout the thermal boundary layer, write the corresponding form of the boundary layer energy equation. Applying appropriate initial ($x = 0$) and boundary conditions, solve this equation for the boundary layer temperature field, $T(x, y)$. Use the result to obtain an expression for the local Nusselt number Nu_x. *Hint:* This problem is analogous to one-dimensional heat transfer in a semi-infinite medium with a sudden change in surface temperature.

7.5 Consider the velocity boundary layer profile for flow over a flat plate to be of the form $u = C_1 + C_2 y$. Applying appropriate boundary conditions, obtain an expression for the velocity profile in terms of the boundary layer thickness δ and the free stream velocity u_∞. Using the integral form of the boundary layer momentum equation (Appendix F), obtain expressions for the boundary layer thickness and the local friction coefficient, expressing your result in terms of the local Reynolds number. Compare your results with those obtained from the exact solution (Section 7.2.1) and the integral solution with a cubic profile (Appendix F).

7.6 Consider a steady, turbulent boundary layer on an isothermal flat plate of temperature T_s. The boundary layer is "tripped" at the leading edge $x = 0$ by a fine wire. Assume constant physical properties and velocity and temperature profiles of the form

$$\frac{u}{u_\infty} = \left(\frac{y}{\delta}\right)^{1/7} \quad \text{and} \quad \frac{T - T_\infty}{T_s - T_\infty} = 1 - \left(\frac{y}{\delta_t}\right)^{1/7}$$

(a) From experiment it is known that the surface shear stress is related to the boundary layer thickness by an expression of the form

$$\tau_s = 0.0228 \rho u_\infty^2 \left(\frac{u_\infty \delta}{\nu}\right)^{-1/4}$$

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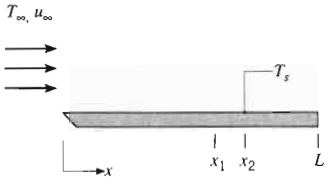
Beginning with the momentum integral equation (Appendix F), show that

$$\delta/x = 0.376Re_x^{-1/5}.$$

Determine the average friction coefficient $\overline{C}_{f,x}$.

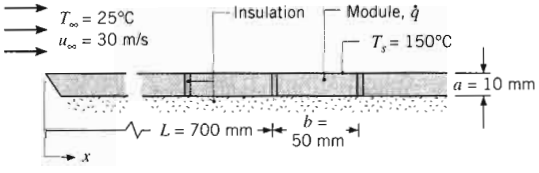
(b) Beginning with the energy integral equation, obtain an expression for the local Nusselt number Nu_x and use this result to evaluate the average Nusselt number $\overline{Nu}_x$.

7.7 Consider flow over a flat plate for which it is desired to determine the average heat transfer coefficient over the short span x_1 to x_2, $\bar{h}_{1-2}$, where $(x_2 - x_1) \ll L$.



Provide three different expressions that can be used to evaluate $\bar{h}_{1-2}$ in terms of (a) the local coefficient at $x = (x_1 + x_2)/2$, (b) the local coefficients at x_1 and x_2, and (c) the average coefficients at x_1 and x_2. Indicate which of the expressions is approximate. Considering whether the flow is laminar, turbulent, or mixed, indicate when it is appropriate or inappropriate to use each of the equations.

7.8 A flat plate of width 1 m is maintained at a uniform surface temperature of $T_s = 150°C$ by using independently controlled, heat-generating rectangular modules of thickness $a = 10$ mm and length $b = 50$ mm. Each module is insulated from its neighbors, as well as on its back side. Atmospheric air at 25°C flows over the plate at a velocity of 30 m/s. The thermophysical properties of the module are $k = 5.2$ W/m · K, $c_p = 320$ J/kg · K, and $\rho = 2300$ kg/m^3.



(a) Find the required power generation, $\dot{q}$ (W/m^3), in a module positioned at a distance 700 mm from the leading edge.

(b) Find the maximum temperature T_{max} in the heat-generating module.

7.9 An electric air heater consists of a horizontal array of thin metal strips that are each 10 mm long in the direction of an airstream that is in parallel flow over the top of the strips. Each strip is 0.2 m wide, and 25 strips are arranged side by side, forming a continuous and smooth surface over which the air flows at 2 m/s. During operation, each strip is maintained at 500°C and the air is at 25°C.

(a) What is the rate of convection heat transfer from the first strip? The fifth strip? The tenth strip? All the strips?

(b) For air velocities of 2, 5, and 10 m/s, determine the convection heat rates for all the locations of part (a). Represent your results in tabular or bar graph form.

(c) Repeat part (b), but under conditions for which the flow is fully turbulent over the entire array of strips.

7.10 Consider atmospheric air at 25°C and a velocity of 25 m/s flowing over both surfaces of a 1-m-long flat plate that is maintained at 125°C. Determine the rate of heat transfer per unit width from the plate for values of the critical Reynolds number corresponding to 10^5, 5×10^5, and 10^6.

7.11 Consider a flat plate subject to parallel flow (top and bottom) characterized by $u_\infty = 5$ m/s, $T_\infty = 20°C$.

(a) Determine the average convective heat transfer coefficient, convective heat transfer rate, and drag force associated with a $L = 2$ m long, $w = 2$ m wide flat plate for air flow and surface temperatures of $T_s = 50°C$ and 80°C.

(b) Determine the average convective heat transfer coefficient, convective heat transfer rate, and drag force associated with a $L = 0.1$ m long, $w = 0.1$ m wide flat plate for water flow and surface temperatures of $T_s = 50°C$ and 80°C.

7.12 Consider water at 27°C in parallel flow over an isothermal, 1-m-long flat plate with a velocity of 2 m/s.

(a) Plot the variation of the local heat transfer coefficient, $h_x(x)$, with distance along the plate for three flow conditions corresponding to transition Reynolds numbers of (i) 5×10^5, (ii) 3×10^5, and (iii) 0 (the flow is fully turbulent).

(b) Plot the variation of the average heat transfer coefficient, $\bar{h}_x(x)$, with distance for the three flow conditions of part (a).

(c) What are the average heat transfer coefficients for the entire plate, $\bar{h}_L$, for the three flow conditions of part (a)?

7.13 Explain under what conditions the total rate of heat transfer from an isothermal flat plate of dimensions L by $2L$ would be the same, independent of whether parallel flow over the plate is directed along the side of length L or $2L$. With a critical Reynolds number of 5×10^5, for what val
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7.14 In fuel cell stacks, it is
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5×10^5, for what values of Re_L would the total heat transfer be independent of orientation?

7.14 In fuel cell stacks, it is desirable to operate under conditions that promote uniform surface temperatures for the electrolytic membranes. This is especially true in high-temperature fuel cells where the membrane is constructed of a brittle ceramic material. Electrochemical reactions in the electrolytic membranes generate thermal energy, while gases flowing above and below the membranes cool it. The stack designer may specify top and bottom flows that are in the same, opposite, or orthogonal directions. A preliminary study of the effect of the relative flow directions is conducted whereby a 150 mm × 150 mm *thin* sheet of material, producing a uniform heat flux of 100 W/m^2, is cooled (top and bottom) by air with a free stream temperature and velocity of 25°C and 2 m/s, respectively.

(a) Determine the minimum and maximum local membrane temperatures for top and bottom flows that are in the same, opposite, and orthogonal directions. Which flow configuration minimizes the membrane temperature? *Hint*: For the opposite and orthogonal flow cases, the boundary layers are subject to boundary conditions that are neither uniform temperature nor uniform heat flux. It is, however, reasonable to expect that the resulting temperatures would be *bracketed* by your answers based on the constant heat flux and constant temperature boundary conditions.

(b) Plot the surface temperature distribution $T(x)$ for the cases involving flow in the opposite and same directions. Thermal stresses are undesirable and are related to the spatial temperature gradient along the membrane. Which configuration minimizes spatial temperature gradients?

7.15 Air at a pressure of 1 atm and a temperature of 50°C is in parallel flow over the top surface of a flat plate that is heated to a uniform temperature of 100°C. The plate has a length of 0.20 m (in the flow direction) and a width of 0.10 m. The Reynolds number based on the plate length is 40,000. What is the rate of heat transfer from the plate to the air? If the free stream velocity of the air is doubled and the pressure is increased to 10 atm, what is the rate of heat transfer?

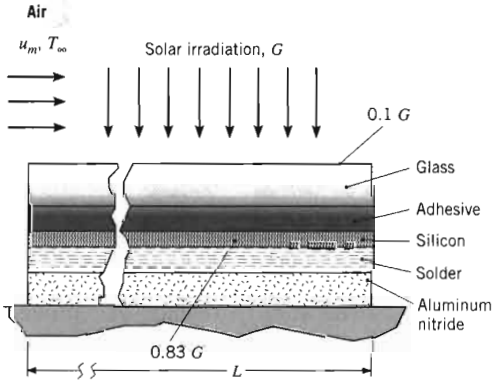
7.16 Consider a rectangular fin that is used to cool a motorcycle engine. The fin is 0.15 m long and at a temperature of 250°C, while the motorcycle is moving at 80 km/h in air at 27°C. The air is in parallel flow over both surfaces of the fin, and turbulent flow conditions may be assumed to exist throughout.

(a) What is the rate of heat removal per unit width of the fin?

(b) Generate a plot of the heat removal rate per unit width of the fin for motorcycle speeds ranging from 10 to 100 km/h.

7.17 The Weather channel reports that it is a hot, muggy day with an air temperature of 90°F, a 10 mph breeze out of the southwest, and bright sunshine with a solar insolation of 400 W/m^2. Consider the wall of a metal building over which the prevailing wind blows. The length of the wall in the wind direction is 10 m, and the emissivity is 0.93. Assume that all the solar irradiation is absorbed, that irradiation from the sky is negligible, and that flow is fully turbulent over the wall. Estimate the average wall temperature.

7.18 A photovoltaic solar panel consists of a sandwich of (top to bottom) a 3-mm-thick ceria-doped glass (k_g = 1.4 W/m · K), a 0.1-mm-thick optical grade adhesive (k_a = 145 W/m · K), a *very thin* silicon semiconducting material, a 0.1-mm-thick solder layer (k_s = 50 W/m · K) and a 2-mm-thick aluminum nitride substrate (k_{an} = 120 W/m · K). The solar-to-electrical conversion efficiency within the semiconductor depends on the silicon temperature, T_{si}, and is described by the expression $\eta = 0.28 - 0.001T_{si}$, where T_{si} is in °C, for 25°C ≤ T_{si} ≤ 250°C. Ten percent of the solar irradiation is absorbed at the top surface of the glass, while 83% of the solar irradiation is transmitted to and absorbed by the silicon (the remaining 7% is reflected away from the cell). The glass has an emissivity of 0.90.



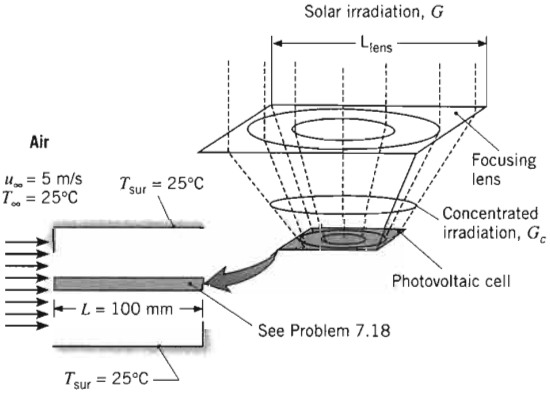
(a) Consider an L = 1 m long, w = 0.1 m wide solar cell that is placed on an insulated surface. Determine the silicon temperature and the electric power produced by the solar cell for an air velocity of 4 m/s parallel to the long direction, with air and surroundings temperatures of 25°C. The solar irradiation is 700 W/m^2. The boundary layer is tripped to a turbulent condition at the leading edge of the panel.

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(b) Repeat part (a), except now the panel is oriented with its short side parallel to the air flow, that is, $L = 0.1$ m and $w = 1$ m.

(c) Plot the electric power output and the silicon temperature versus air velocity over the range $0 \leq u_m \leq 10$ m/s for the $L = 0.1$ m and $w = 1$ m case.

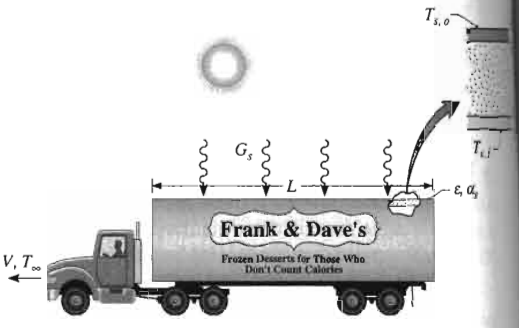
7.19 Concentration of sunlight onto photovoltaic cells is desired since the concentrating mirrors and lenses are less expensive than the photovoltaic material. Consider the solar photovoltaic cell of Problem 7.18. A 100 mm × 100 mm photovoltaic cell is irradiated with concentrated solar energy. Since the concentrating lens is glass, it absorbs 10% of the irradiation instead of the top surface of the solar cell, as in Problem 7.18. The remaining irradiation is reflected from the system (7%) or is absorbed in the silicon semiconductor material of the photovoltaic cell (83%). The photovoltaic cell is cooled by air directed parallel to its top and bottom surfaces. The air temperature and velocity are 25°C and 5 m/s, respectively, and the bottom surface is coated with a high-emissivity paint, $\varepsilon_b = 0.95$.



(a) Determine the electric power produced by the photovoltaic cell and the silicon temperature for a square concentrating lens with $L_{lens} = 400$ mm, which focuses the irradiation falling on the lens to the smaller area of the photovoltaic cell. Assume the concentrating lens temperature is 25°C and does not interfere with boundary layer development over the photovoltaic cell's top surface. The top and bottom boundary layers are both tripped to turbulent conditions at the leading edge of the photovoltaic material.

(b) Determine the electric power output of the photovoltaic cell and the silicon temperature over the range 100 mm $\leq L_{lens} \leq$ 600 mm.

7.20 The roof of a refrigerated truck compartment is of composite construction, consisting of a layer of foamed urethane insulation ($t_2 = 50$ mm, $k_i = 0.026$ W/m·K) sandwiched between aluminum alloy panels ($t_1 = 5$ mm, $k_p = 180$ W/m · K). The length and width of the roof are $L = 10$ m and W = 3.5 m, respectively, and the temperature of the inner surface is $T_{s,i} = -10$°C. Consider conditions for which the truck is moving at a speed of $V = 105$ km/h, the air temperature is $T_\infty = 32$°C, and the solar irradiation is $G_S = 750$ W/m^2. Turbulent flow may be assumed over the entire length of the roof.

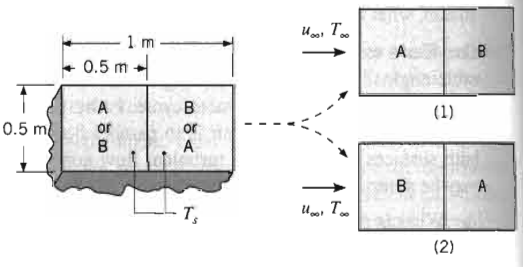


(a) For equivalent values of the solar absorptivity and the emissivity of the outer surface ($\alpha_S = \varepsilon = 0.5$), estimate the average temperature $T_{s,o}$ of the outer surface. What is the corresponding heat load imposed on the refrigeration system?

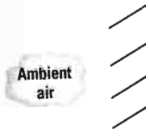
(b) A special finish ($\alpha_S = 0.15$, $\varepsilon = 0.8$) may be applied to the outer surface. What effect would such an application have on the surface temperature and the heat load?

(c) If, with $\alpha_S = \varepsilon = 0.5$, the roof is not insulated ($t_2 = 0$), what are the corresponding values of the surface temperature and the heat load?

7.21 The top surface of a heated compartment consists of very smooth (A) and highly roughened (B) portions, and the surface is placed in an atmospheric airstream. In the interest of minimizing total convection heat transfer from the surface, which orientation, (1) or (2), is preferred? If $T_s = 100$°C, $T_\infty = 20$°C, and $u_\infty = 20$ m/s, what is the convection heat transfer from the entire surface for this orientation?



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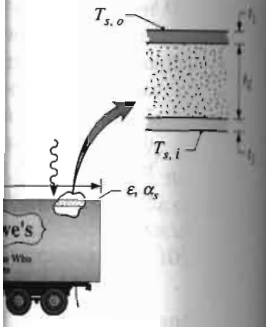
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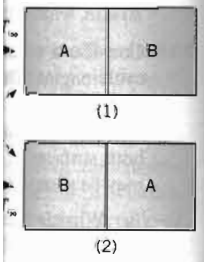


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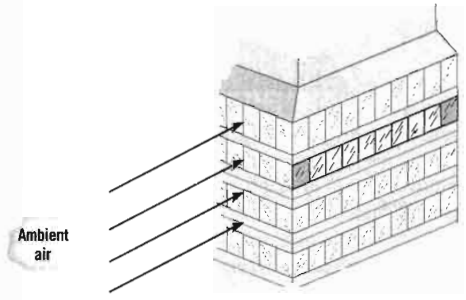
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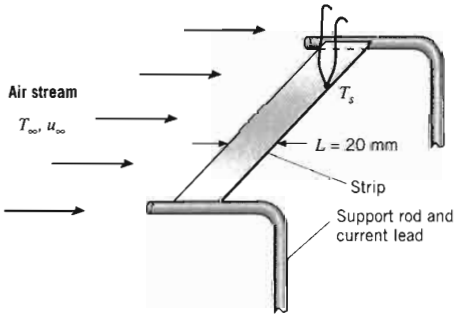
7.22 Consider weather conditions for which the prevailing wind blows past the penthouse tower on a tall building. The tower length in the wind direction is 10 m and there are 10 window panels.



(a) Calculate the average convection coefficient for the first, third, and tenth window panels when the wind speed is 5 m/s. Use a film temperature of 300 K to evaluate the thermophysical properties required of the correlation. Would this be a suitable value of the film temperature for ambient air temperatures in the range $-15 \le T_\infty \le 38$°C?

(b) For the first, third, and tenth windows, on one graph, plot the variation of the average convection coefficient with wind speed for the range $5 \le u_\infty \le 100$ km/h. Explain the major features of each curve and their relative magnitudes.

7.23 The proposed design for an anemometer to determine the velocity of an airstream in a wind tunnel is comprised of a thin metallic strip whose ends are supported by stiff rods serving as electrodes for passage of current used to heat the strip. A fine-wire thermocouple is attached to the trailing edge of the strip and serves as the sensor for a system that controls the power to maintain the strip at a constant operating temperature for variable airstream velocities. Design conditions pertain to an airstream at $T_\infty = 25$°C and $1 \le u_\infty \le 50$ m/s, with a strip temperature of $T_s = 35$°C.



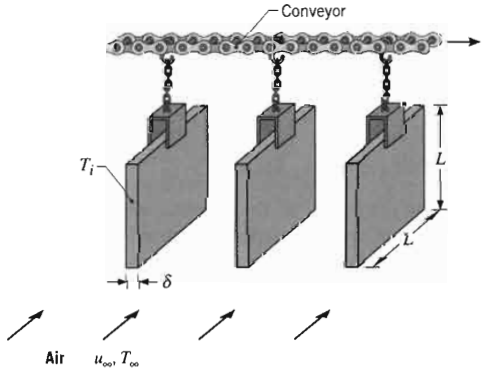
(a) Determine the relationship between the electrical power dissipation per unit width of the strip in the transverse direction, P' (mW/mm), and the airstream velocity. Show this relationship graphically for the specified range of u_∞.

(b) If the accuracy with which the temperature of the operating strip can be measured and maintained constant is ±0.2°C, what is the uncertainty in the airstream velocity?

(c) The proposed design operates in a strip constant-temperature mode for which the airstream velocity is related to the measured power. Consider now an alternative mode wherein the strip is provided with a constant power, say, 30 mW/mm, and the airstream velocity is related to the measured strip temperature T_s. For this mode of operation, show the graphical relationship between the strip temperature and airstream velocity. If the temperature can be measured with an uncertainty of ±0.2°C, what is the uncertainty in the airstream velocity?

(d) Compare the features associated with each of the anemometer operating modes.

7.24 Steel (AISI 1010) plates of thickness $\delta = 6$ mm and length $L = 1$ m on a side are conveyed from a heat treatment process and are concurrently cooled by atmospheric air of velocity $u_\infty = 10$ m/s and $T_\infty = 20$°C in parallel flow over the plates.

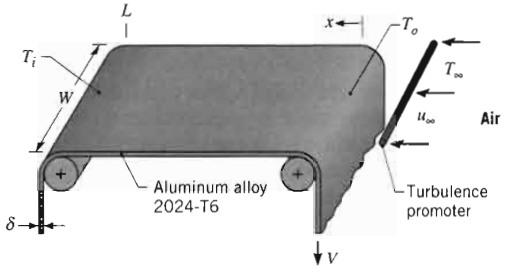


For an initial plate temperature of $T_i = 300$°C, what is the rate of heat transfer from the plate? What is the corresponding rate of change of the plate temperature? The velocity of the air is much larger than that of the plate.

7.25 In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment to the customer. Typically, the process is continuous, with a sheet of thickness δ and width W cooled as it transits the

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distance L between two rollers at a velocity V. In this problem, we consider cooling of an aluminum alloy (2024-T6) by an air stream moving at a velocity u_∞ in counter flow over the top surface of the sheet. A turbulence promoter is used to provide turbulent boundary layer development over the entire surface.

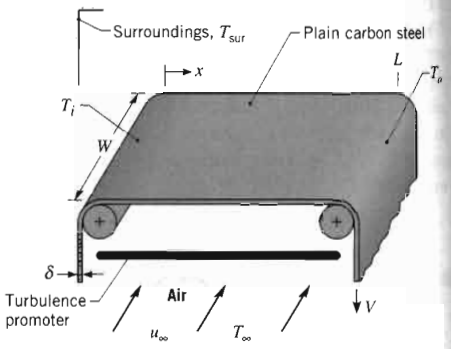


(a) By applying conservation of energy to a differential control surface of length dx, which either moves with the sheet *or* is stationary and through which the sheet passes, derive a differential equation that governs the temperature distribution along the sheet. Because of the low emissivity of the aluminum, radiation effects may be neglected. Express your result in terms of the velocity, thickness, and properties of the sheet (V, δ, ρ, c_p), the local convection coefficient h_x associated with the counter flow, and the air temperature. For a known temperature of the sheet (T_i) at the onset of cooling and a negligible effect of the sheet velocity on boundary layer development, solve the equation to obtain an expression for the outlet temperature T_o.

(b) For $\delta = 2$ mm, $V = 0.10$ m/s, $L = 5$ m, $W = 1$ m, $u_\infty = 20$ m/s, $T_\infty = 20$°C, and $T_i = 300$°C, what is the outlet temperature T_o?

7.26 In the production of sheet metals or plastics, it is customary to cool the material before it leaves the production process for storage or shipment to the customer. Typically, the process is continuous, with a sheet of thickness δ and width W cooled as it transits the distance L between two rollers at a velocity V. In this problem, we consider cooling of plain carbon steel by an air stream moving at a velocity u_∞ in cross flow over the top and bottom surfaces of the sheet. A turbulence promoter is used to provide turbulent boundary layer development over the entire surface.

(a) By applying conservation of energy to a differential control surface of length dx, which either moves with the sheet *or* is stationary and through which the sheet passes, and assuming a uniform sheet temperature in the direction of air flow, derive a differential equation that governs the temperature

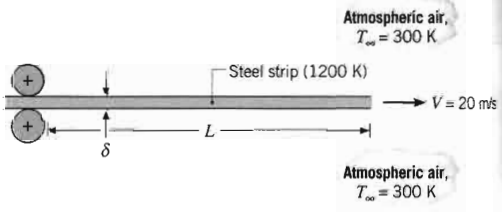


distribution, $T(x)$, along the sheet. Consider the effects of radiation, as well as convection, and express your result in terms of the velocity, thickness, and properties of the sheet (V, δ, ρ, c_p, ε), the average convection coefficient $\bar{h}_W$ associated with the cross flow, and the environmental temperatures (T_∞, T_{sur}).

(b) Neglecting radiation, obtain a closed form solution to the foregoing equation. For $\delta = 3$ mm, $V = 0.10$ m/s, $L = 10$ m, $W = 1$ m, $u_\infty = 20$ m/s, $T_\infty = 20$°C, and a sheet temperature of $T_i = 500$°C at the onset of cooling, what is the outlet temperature T_o? Assume a negligible effect of the sheet velocity on boundary layer development in the direction of air flow. The density and specific heat of the steel are $\rho = 7850$ kg/m^3 and $c_p = 620$ J/kg · K, while properties of the air may be taken to be $k = 0.044$ W/m · K, $\nu = 4.5 \times 10^{-5}$ m^2/s, $Pr = 0.68$.

(c) Accounting for the effects of radiation, with $\varepsilon = 0.70$ and $T_{sur} = 20$°C, numerically integrate the differential equation derived in part (a) to determine the temperature of the sheet at $L = 10$ m. Explore the effect of V on the temperature distribution along the sheet.

7.27 A steel strip emerges from the hot roll section of a steel mill at a speed of 20 m/s and a temperature of 1200 K. Its length and thickness are $L = 100$ m and $\delta = 0.003$ m, respectively, and its density and specific heat are 7900 kg/m^3 and 640 J/kg · K, respectively.



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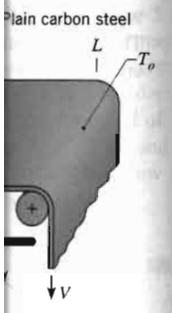
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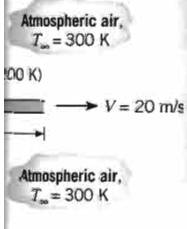


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Accounting for heat transfer from the top and bottom surfaces and neglecting radiation and strip conduction effects, determine the time rate of change of the strip temperature at a distance of 1 m from the leading edge and at the trailing edge. Determine the distance from the leading edge at which the minimum cooling rate is achieved.

7.28 In Problem 7.23, an anemometer design was explored, and the assumption was made that the strip temperature was uniform. This is a good assumption when the heat transfer coefficient is low or the strip thermal conductivity high, because then conduction within the strip redistributes the generated heat and makes the strip temperature uniform. However, as the heat transfer coefficient increases or strip thermal conductivity decreases, heat generated at a point in the strip leaves the surface close to that point, and the thermal condition is closer to one of uniform surface heat flux.

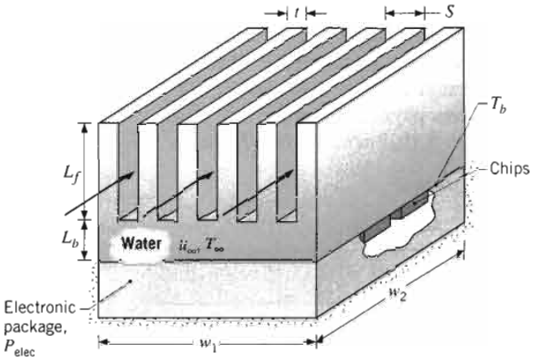
(a) Develop the calibration equations for both the constant surface temperature and constant heat flux conditions, that is, find the equations that predict the velocity as a function of the power per unit strip width, P' (mW/mm), and the temperature measured at the trailing edge (as in Problem 7.23). Assume laminar flow conditions.

(b) If the true condition is uniform surface heat flux, but the uniform surface temperature calibration is used, what percentage error will be incurred in the velocity determination?

(c) Where could the thermocouple be placed so that the calibration is insensitive to whether the thermal condition is uniform surface temperature or uniform surface heat flux?

7.29 An array of electronic chips is mounted within a sealed rectangular enclosure, and cooling is implemented by attaching an aluminum heat sink (k = 180 W/m · K). The base of the heat sink has dimensions of $w_1 = w_2$ = 100 mm, while the 6 fins are of thickness t = 10 mm and pitch S = 18 mm. The fin length is L_f = 50 mm, and the base of the heat sink has a thickness of L_b = 10 mm.

If cooling is implemented by water flow through the heat sink, with u_∞ = 3 m/s and T_∞ = 17°C, what is the base temperature T_b of the heat sink when power dissipation by the chips is P_{elec} = 1800 W? The average convection coefficient for surfaces of the fins and the exposed base may be estimated by assuming parallel flow over a flat plate. Properties of the water may be approximated as k = 0.62 W/m · K, $\nu = 7.73 \times 10^{-7}$ m²/s, and Pr = 5.2.



7.30 The heat sink of Problem 7.29 is considered for an application in which the power dissipation is only 70 W, and the engineer proposes to use air at T_∞ = 20°C for cooling. If the base temperature, T_b, must remain below 70°C, what air velocity is required? Properties of the air may be approximated as k = 0.027 W/m · K, $\nu = 16.4 \times 10^{-6}$ m²/s, and Pr = 0.706. *Hint:* An iterative solution is required. One approach is to assume the fin efficiency is unity to initiate the iterations.

7.31 Consider the concentrating photovoltaic apparatus of Problem 7.19. The apparatus is to be installed in a desert environment, so the space between the concentrating lens and top of the photovoltaic cell is enclosed to protect the cell from sand abrasion in windy conditions. Since convection cooling from the top of the cell is reduced by the enclosure, an engineer proposes to cool the photovoltaic cell by attaching an aluminum heat sink to its bottom surface. The heat sink dimensions and material are the same as those of Problem 7.29. A contact resistance of 0.5×10^{-4} m² · K/W exists at the photovoltaic cell/heat sink interface and a dielectric liquid (k = 0.064 W/m · K, $\nu = 10^{-6}$ m²/s, Pr = 25) flows between the heat sink fins at u_∞ = 3 m/s, T_∞ = 25°C.

(a) Determine the electric power produced by the photovoltaic cell and the silicon temperature for a square concentrating lens with L_{lens} = 400 mm.

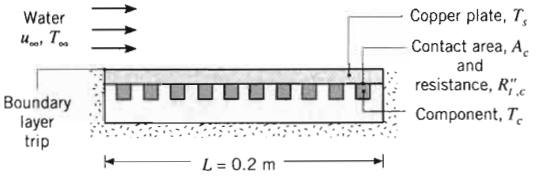
(b) Compare the electric power produced by the photovoltaic cell with the heat sink in place and with the bottom surface cooled directly by the dielectric fluid (i.e., no heat sink) for L_{lens} = 1.5 m.

(c) Determine the electric power output and the silicon temperature over the range 100 mm ≤ L_{lens} < 3000 mm with the aluminum heat sink in place.

7.32 One hundred electrical components, each dissipating 25 W, are attached to one surface of a square (0.2 m × 0.2 m) copper plate, and all the dissipated energy is transferred to water in parallel flow over the opposite surface.

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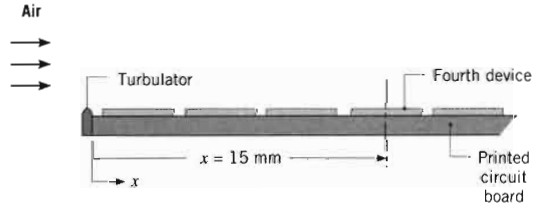
A protuberance at the leading edge of the plate acts to *trip* the boundary layer, and the plate itself may be assumed to be isothermal. The water velocity and temperature are $u_\infty = 2$ m/s and $T_\infty = 17°C$, and the water's thermophysical properties may be approximated as $\nu = 0.96 \times 10^{-6}$ m^2/s, $k = 0.620$ W/m · K, and $Pr = 5.2$.



(a) What is the temperature of the copper plate?

(b) If each component has a plate contact surface area of 1 cm^2 and the corresponding contact resistance is 2×10^{-4} m^2 · K/W, what is the component temperature? Neglect the temperature variation across the thickness of the copper plate.

7.33 Air at 27°C with a free stream velocity of 10 m/s is used to cool electronic devices mounted on a printed circuit board. Each device, 4 mm by 4 mm, dissipates 40 mW, which is removed from the top surface. A turbulator is located at the leading edge of the board, causing the boundary layer to be turbulent.

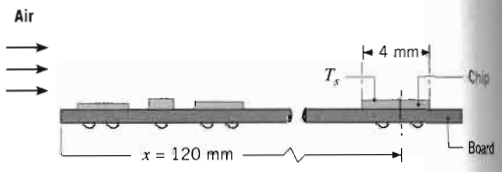


(a) Estimate the surface temperature of the fourth device located 15 mm from the leading edge of the board.

(b) Generate a plot of the surface temperature of the first four devices as a function of the free stream velocity for $5 \leq u_\infty \leq 15$ m/s.

(c) What is the minimum free stream velocity if the surface temperature of the hottest device is not to exceed 80°C?

7.34 Forced air at 25°C and 10 m/s is used to cool electronic elements mounted on a circuit board. Consider a chip of length 4 mm and width 4 mm located 120 mm from the leading edge. Because the board surface is irregular, the flow is disturbed and the appropriate convection correlation is of the form $Nu_x = 0.04 Re_x^{0.85} Pr^{0.33}$.



Estimate the surface temperature of the chip, T_s, if its heat dissipation rate is 30 mW.

7.35 Air at atmospheric pressure and a temperature of 25°C is in parallel flow at a velocity of 5 m/s over a 1-m-long flat plate that is heated with a uniform heat flux of 1250 W/m^2. Assume the flow is fully turbulent over the length of the plate.

(a) Calculate the plate surface temperature, $T_s(L)$, and the local convection coefficient, $h_x(L)$, at the trailing edge, $x = L$.

(b) Calculate the average temperature of the plate surface, $\overline{T}_s$.

(c) Plot the variation of the surface temperature, $T_s(x)$, and the convection coefficient, $h_x(x)$, with distance on the same graph. Explain the key features of these distributions.

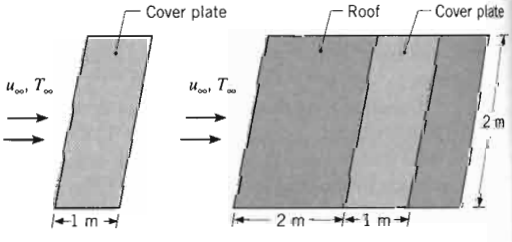
7.36 Consider atmospheric air at $u_\infty = 2$ m/s and $T_\infty = 300$ K in parallel flow over an isothermal flat plate of length $L = 1$ m and temperature $T_s = 350$ K.

(a) Compute the local convection coefficient at the leading and trailing edges of the heated plate with and without an unheated starting length of $\xi = 1$ m.

(b) Compute the average convection coefficient for the plate for the same conditions as part (a).

(c) Plot the variation of the local convection coefficient over the plate with and without an unheated starting length.

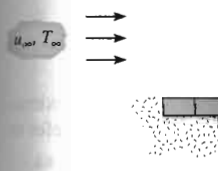
7.37 The cover plate of a flat-plate solar collector is at 15°C, while ambient air at 10°C is in parallel flow over the plate, with $u_\infty = 2$ m/s.



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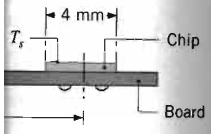
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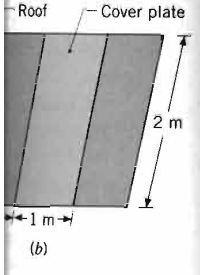
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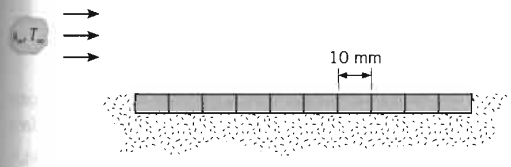


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(b) If the plate is installed 2 m from the leading edge of a roof and flush with the roof surface, what is the rate of convective heat loss?

7.38 An array of 10 silicon chips, each of length L = 10 mm on a side, is insulated on one surface and cooled on the opposite surface by atmospheric air in parallel flow with T_∞ = 24°C and u_∞ = 40 m/s. When in use, the same electrical power is dissipated in each chip, maintaining a uniform heat flux over the entire cooled surface.

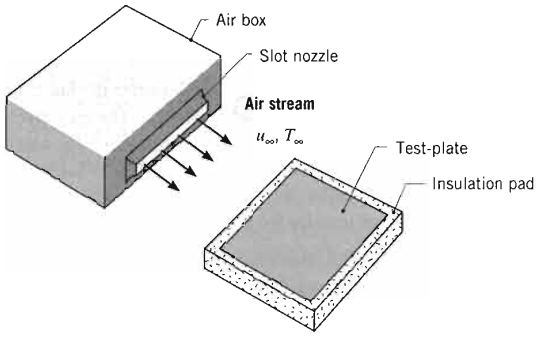


If the temperature of each chip may not exceed 80°C, what is the maximum allowable power per chip? What is the maximum allowable power if a turbulence promoter is used to trip the boundary layer at the leading edge? Would it be preferable to orient the array normal, instead of parallel, to the airflow?

7.39 A square (10 mm × 10 mm) silicon chip is insulated on one side and cooled on the opposite side by atmospheric air in parallel flow at u_∞ = 20 m/s and T_∞ = 24°C. When in use, electrical power dissipation within the chip maintains a uniform heat flux at the cooled surface. If the chip temperature may not exceed 80°C at any point on its surface, what is the maximum allowable power? What is the maximum allowable power if the chip is flush mounted in a substrate that provides for an unheated starting length of 20 mm?

7.40 Working in groups of two, our students design and perform experiments on forced convection phenomena using the general arrangement shown schematically. The air box consists of two muffin fans, a plenum chamber, and flow straighteners discharging a nearly uniform airstream over the flat *test-plate*. The objectives of one experiment were to measure the heat transfer coefficient and to compare the results with standard convection correlations. The velocity of the airstream was measured using a thermistor-based anemometer, and thermocouples were used to determine the temperatures of the airstream and the *test-plate*.

With the airstream from the box fully stabilized at T_∞ = 20°C, an aluminum plate was preheated in a convection oven and quickly mounted in the test-plate holder. The subsequent temperature history of the plate was determined from thermocouple measurements, and histories obtained for airstream velocities of 3 and 9 m/s were fitted by the following polynomial:



$$T(t) = a + bt + ct^2 + dt^3 + et^4$$

The temperature T and time t have units of °C and s, respectively, and values of the coefficients appropriate for the time interval of the experiments are tabulated as follows:

Velocity (m/s)	**3**	**9**
Elapsed Time (s)	300	160
a (°C)	56.87	57.00
b (°C/s)	−0.1472	−0.2641
c (°C/s^2)	3×10^{-4}	9×10^{-4}
d (°C/s^3)	-4×10^{-7}	-2×10^{-6}
e (°C/s^4)	2×10^{-10}	1×10^{-9}

The plate is square, 133 mm to a side, with a thickness of 3.2 mm, and is made from a highly polished aluminum alloy (ρ = 2770 kg/m^3, c = 875 J/kg · K, k = 177 W/m · K).

(a) Determine the heat transfer coefficients for the two cases, assuming the plate behaves as a spacewise isothermal object.

(b) Evaluate the coefficients C and m for a correlation of the form

$$\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$$

Compare this result with a standard flat-plate correlation. Comment on the *goodness* of the comparison and explain any differences.

Cylinder in Cross Flow

7.41 Consider the following fluids, each with a velocity of V = 5 m/s and a temperature of T_∞ = 20°C, in cross flow over a 10-mm diameter cylinder maintained at 50°C: atmospheric air, saturated water, and engine oil.

(a) Calculate the rate of heat transfer per unit length, q', using the Churchill–Bernstein correlation.

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(b) Generate a plot of q' as a function of fluid velocity for $0.5 \leq V \leq 10$ m/s.

7.42 A circular pipe of 25-mm outside diameter is placed in an airstream at 25°C and 1-atm pressure. The air moves in cross flow over the pipe at 15 m/s, while the outer surface of the pipe is maintained at 100°C. What is the drag force exerted on the pipe per unit length? What is the rate of heat transfer from the pipe per unit length?

7.43 A long, cylindrical, electrical heating element of diameter $D = 10$ mm, thermal conductivity $k = 240$ W/m · K, density $\rho = 2700$ kg/m^3, and specific heat $c_p = 900$ J/kg · K is installed in a duct for which air moves in cross flow over the heater at a temperature and velocity of 27°C and 10 m/s, respectively.

(a) Neglecting radiation, estimate the steady-state surface temperature when, per unit length of the heater, electrical energy is being dissipated at a rate of 1000 W/m.

(b) If the heater is activated from an initial temperature of 27°C, estimate the time required for the surface temperature to come within 10°C of its steady-state value.

7.44 Consider the conditions of Problem 7.43, but now allow for radiation exchange between the surface of the heating element ($\varepsilon = 0.8$) and the walls of the duct, which form a large enclosure at 27°C.

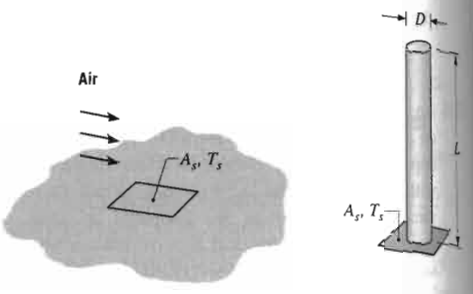
(a) Evaluate the steady-state surface temperature.

(b) If the heater is activated from an initial temperature of 27°C, estimate the time required for the surface temperature to come within 10°C of the steady-state value.

(c) To guard against overheating due to unanticipated excursions in the blower output, the heater controller is designed to maintain a fixed surface temperature of 275°C. Determine the power dissipation required to maintain this temperature for air velocities in the range $5 \leq V \leq 10$ m/s.

7.45 A pin fin of 10-mm diameter dissipates 30 W by forced convection to air in cross flow with a Reynolds number of 4000. If the diameter of the fin is doubled and all other conditions remain the same, estimate the fin heat rate. Assume the pin to be infinitely long.

7.46 Air at 27°C and a velocity of 5 m/s passes over the small region A_s (20 mm × 20 mm) on a large surface, which is maintained at $T_s = 127$°C. For these conditions, 0.5 W is removed from the surface A_s. In order to increase the heat removal rate, a stainless steel (AISI 304) pin fin of diameter 5 mm is affixed to A_s, which is assumed to remain at $T_s = 127$°C.



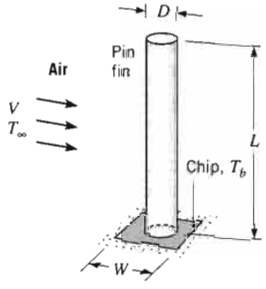
(a) Determine the maximum possible heat removal rate through the fin.

(b) What fin length would provide a close approximation to the heat rate found in part (a)? *Hint*: Refer to Example 3.9.

(c) Determine the fin effectiveness, ε_f.

(d) What is the percentage increase in the heat rate from A_s due to installation of the fin?

7.47 To enhance heat transfer from a silicon chip of width $W = 4$ mm on a side, a copper pin fin is brazed to the surface of the chip. The pin length and diameter are $L = 12$ mm and $D = 2$ mm, respectively, and atmospheric air at $V = 10$ m/s and $T_\infty = 300$ K is in cross flow over the pin. The surface of the chip, and hence the base of the pin, are maintained at a temperature of $T_b = 350$ K.



(a) Assuming the chip to have a negligible effect on flow over the pin, what is the average convection coefficient for the surface of the pin?

(b) Neglecting radiation and assuming the convection coefficient at the pin tip to equal that calculated in part (a), determine the pin heat transfer rate.

(c) Neglecting radiation and assuming the convection coefficient at the exposed chip surface to equal that calculated in part (a), determine the total rate of heat transfer from the chip.

(d) Independently determine and plot the effect of increasing velocity ($10 \leq V \leq 40$ m/s) and pin

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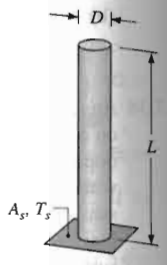
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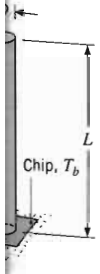


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diameter ($2 \leq D \leq 4$ mm) on the total rate of heat transfer from the chip. What is the heat rate for $V = 40$ m/s and $D = 4$ mm?

7.48 Consider the Nichrome wire ($D = 1$ mm, $\rho_e = 10^{-6}$ $\Omega \cdot$ m, $k = 25$ W/m $\cdot$ K, $\varepsilon = 0.20$) used to fabricate the air heater of Problem 3.86, but now under conditions for which the convection heat transfer coefficient must be determined.

(a) For atmospheric air at 50°C and a cross-flow velocity of 5 m/s, what are the surface and centerline temperatures of the wire when it carries a current of 25 A and the housing of the heater is also at 50°C?

(b) Explore the effect of variations in the flow velocity and electrical current on the surface and centerline temperatures of the wire.

7.49 A fine wire of diameter D is positioned across a passage to determine flow velocity from heat transfer characteristics. Current is passed through the wire to heat it, and the heat is dissipated to the flowing fluid by convection. The resistance of the wire is determined from electrical measurements, and the temperature is known from the resistance.

(a) For a fluid of arbitrary Prandtl number, develop an expression for its velocity in terms of the difference between the temperature of the wire and the free stream temperature of the fluid.

(b) What is the velocity of an airstream at 1 atm and 25°C, if a wire of 0.5-mm diameter achieves a temperature of 40°C while dissipating 35 W/m?

7.50 To determine air velocity changes, it is proposed to measure the electric current required to maintain a platinum wire of 0.5-mm diameter at a constant temperature of 77°C in a stream of air at 27°C.

(a) Assuming Reynolds numbers in the range $40 < Re_D < 1000$, develop a relationship between the wire current and the velocity of the air that is in cross flow over the wire. Use this result to establish a relation between fractional changes in the current, $\Delta I/I$, and the air velocity, $\Delta V/V$.

(b) Calculate the current required when the air velocity is 10 m/s and the electrical resistivity of the platinum wire is 17.1×10^{-5} $\Omega \cdot$ m.

7.51 A computer code is being developed to analyze a temperature sensor of 12.5-mm diameter experiencing cross flow of water with a free stream temperature of 80°C and variable velocity. Derive an expression for the convection heat transfer coefficient as a function of the sensor surface temperature T_s for the range $20 < T_s < 80$°C and for velocities V in the range $0.005 < V < 0.20$ m/s. Use the Zukauskas correlation for the range $40 < Re_D < 1000$ and assume that the Prandtl number of water has a linear temperature dependence.

7.52 A 25-mm-diameter, high-tension line has an electrical resistance of 10^{-4} Ω/m and is transmitting a current of 1000 A.

(a) If ambient air at 10°C and 5 m/s is in cross flow over the line, what is its surface temperature?

(b) If the line may be approximated as a solid copper rod, what is its centerline temperature?

(c) Generate a plot that depicts the variation of the surface temperature with air velocity for $1 \leq V \leq 10$ m/s.

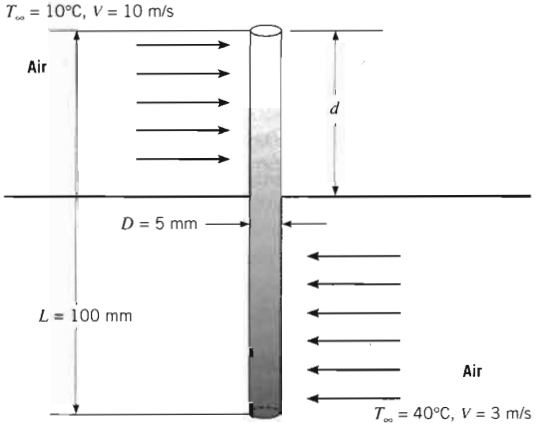
7.53 An aluminum transmission line with a diameter of 20 mm has an electrical resistance of $R'_{elec} = 2.636 \times 10^{-4}$ Ω/m and carries a current of 700 A. The line is subjected to frequent and severe cross winds, increasing the probability of contact between adjacent lines, thereby causing sparks and creating a potential fire hazard for nearby vegetation. The remedy is to insulate the line, but with the adverse effect of increasing the conductor operating temperature.

(a) Calculate the conductor temperature when the air temperature is 20°C and the line is subjected to cross flow with a velocity of 10 m/s.

(b) Calculate the conductor temperature for the same conditions, but with a 2-mm-thick insulation having a thermal conductivity of 0.15 W/m $\cdot$ K.

(c) Calculate and plot the temperatures of the bare and insulated conductors for wind velocities in the range from 2 to 20 m/s. Comment on features of the curves and the effect of the wind velocity on the conductor temperatures.

7.54 To augment heat transfer between two flowing fluids, it is proposed to insert a 100-mm-long, 5-mm-diameter



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2024 aluminum pin fin through the wall separating the two fluids. The pin is inserted to a depth of d into fluid 1. Fluid 1 is air with a mean temperature of 10°C and nearly uniform velocity of 10 m/s. Fluid 2 is air with a mean temperature of 40°C and mean velocity of 3 m/s.

(a) Determine the rate of heat transfer from the warm air to the cool air through the pin fin for $d = 50$ mm.

(b) Plot the variation of the heat transfer rate with the insertion distance, d. Does an optimal insertion distance exist?

7.55 Repeat Problem 7.54, except now fluid 1 is ethylene glycol with a mean temperature of 10°C and nearly uniform velocity of 10 m/s. Fluid 2 is water with a mean temperature of 40°C and mean velocity of 3 m/s.

7.56 Hot water at 50°C is routed from one building in which it is generated to an adjoining building in which it is used for space heating. Transfer between the buildings occurs in a steel pipe ($k = 60$ W/m · K) of 100-mm outside diameter and 8-mm wall thickness. During the winter, representative environmental conditions involve air at $T_\infty = -5$°C and $V = 3$ m/s in cross flow over the pipe.

(a) If the cost of producing the hot water is $0.05 per kW · h, what is the representative daily cost of heat loss from an uninsulated pipe to the air per meter of pipe length? The convection resistance associated with water flow in the pipe may be neglected.

(b) Determine the savings associated with application of a 10-mm-thick coating of urethane insulation ($k = 0.026$ W/m · K) to the outer surface of the pipe.

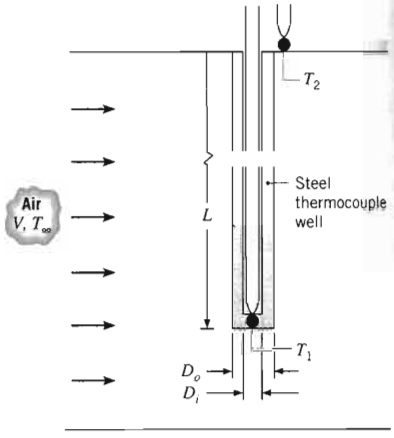
7.57 An uninsulated steam pipe is used to transport high-temperature steam from one building to another. The pipe is of 0.5-m diameter, has a surface temperature of 150°C, and is exposed to ambient air at −10°C. The air moves in cross flow over the pipe with a velocity of 5 m/s.

(a) What is the heat loss per unit length of pipe?

(b) Consider the effect of insulating the pipe with a rigid urethane foam ($k = 0.026$ W/m · K). Evaluate and plot the heat loss as a function of the thickness δ of the insulation layer for $0 \leq \delta \leq 50$ mm.

7.58 A thermocouple is inserted into a hot air duct to measure the air temperature. The thermocouple (T_1) is soldered to the tip of a steel *thermocouple well* of length $L = 0.15$ m and inner and outer diameters of $D_i = 5$ mm and $D_o = 10$ mm. A second thermocouple (T_2) is used to measure the duct wall temperature.

Consider conditions for which the air velocity in the duct is $V = 3$ m/s and the two thermocouples register temperatures of $T_1 = 450$ K and $T_2 = 375$ K. Neglecting radiation, determine the air temperature T_∞. Assume that,



for steel, $k = 35$ W/m · K, and, for air, $\rho = 0.774$ kg/m^3, $\mu = 251 \times 10^{-7}$ N · s/m^2, $k = 0.0373$ W/m · K, and $Pr = 0.686$.

7.59 Consider conditions for which a mercury-in-glass thermometer of 4-mm diameter is inserted to a length L through the wall of a duct in which air at 77°C is flowing. If the stem of the thermometer at the duct wall is at the wall temperature $T_w = 15$°C, conduction heat transfer through the glass causes the bulb temperature to be lower than that of the airstream.

(a) Develop a relationship for the *immersion* error, $\Delta T_i = T(L) - T_\infty$, as a function of air velocity, thermometer diameter, and insertion length L.

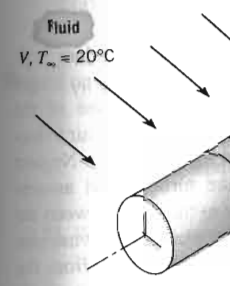
(b) To what length L must the thermometer be inserted if the immersion error is not to exceed 0.25°C when the air velocity is 10 m/s?

(c) Using the insertion length determined in part (b), calculate and plot the immersion error as a function of air velocity for the range 2 to 20 m/s.

(d) For a given insertion length, will the immersion error increase or decrease if the diameter of the thermometer is increased? Is the immersion error more sensitive to the diameter or air velocity?

7.60 Fluid velocities can be measured using hot-film sensors, and a common design is one for which the sensing element forms a thin film about the circumference of a quartz rod. The film is typically comprised of a thin (~100 nm) layer of platinum, whose electrical resistance is proportional to its temperature. Hence, when submerged in a fluid stream, an electric current may be passed through the film to maintain its temperature above that of the fluid. The temperature of the film is

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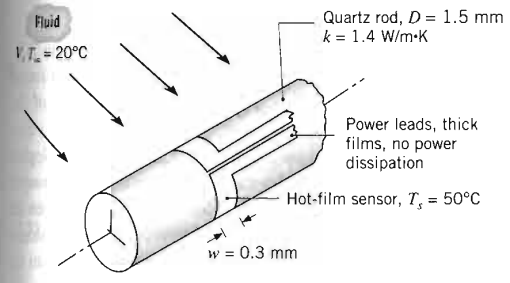
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controlled by monitoring its electric resistance, and with concurrent measurement of the electric current, the power dissipated in the film may be determined.



Proper operation is assured only if the heat generated in the film is transferred to the fluid, rather than conducted from the film into the quartz rod. Thermally, the film should therefore be strongly coupled to the fluid and weakly coupled to the quartz rod. This condition is satisfied if the Biot number is very large, $Bi = \bar{h}D/2k \gg 1$, where $\bar{h}$ is the convection coefficient between the fluid and the film and k is the thermal conductivity of the rod.

(a) For the following fluids and velocities, calculate and plot the convection coefficient as a function of velocity: (i) water, $0.5 \leq V \leq 5$ m/s; (ii) air, $1 \leq V \leq 20$ m/s.

(b) Comment on the suitability of using this hot-film sensor for the foregoing conditions.

7.61 Consider use of the hot-film sensor described in Problem 7.60 to determine the velocity of water entering the cooling system of an electric power plant from an adjoining lake. The sensor is mounted within an intake pipe, and its controls are set to maintain an average hot-film temperature that is 5°C larger than the fluid temperature ($T_{s,hf} - T_\infty = 5°C$).

(a) If an independent measurement of the water temperature yields a value of $T_\infty = 17°C$, use the Churchill–Bernstein correlation to estimate the velocity of the water under conditions for which the power input to the sensor maintains a heat flux of $q''_{hf} = 4 \times 10^4$ W/m^2 from the film to the water.

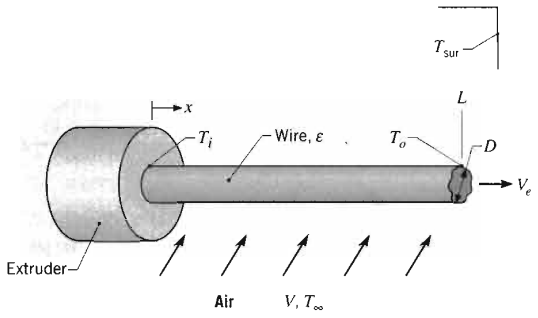
(b) If the sensor is exposed to the water for an extended period, its surface will be *fouled* by an accumulation of deposits from the water. Consider conditions for which the deposits form a 0.1-mm-thick shell around the sensor and have a thermal conductivity of $k_d = 2$ W/m · K. For $T_\infty = 17°C$ and the flow velocity determined in part (a), what heat flux must be supplied to the sensor to maintain its temperature at $T_{s,hf} = 22°C$? What is the corresponding error in the velocity measurement? *Note:* conduction across the deposit may be approximated as that across a plane wall.

7.62 In a manufacturing process, a long coated plastic rod ($\rho = 2200$ kg/m^3, $c = 800$ J/kg · K, $k = 1$ W/m · K) of diameter $D = 20$ mm is initially at a uniform temperature of 25°C and is suddenly exposed to a cross flow of air at $T_\infty = 350°C$ and $V = 50$ m/s.

(a) How long will it take for the surface of the rod to reach 175°C, the temperature above which the special coating will cure?

(b) Generate a plot of the time-to-reach 175°C as a function of air velocity for $5 \leq V \leq 50$ m/s.

7.63 In an extrusion process, copper wire emerges from the extruder at a velocity V_e and is cooled by convection heat transfer to air in cross flow over the wire, as well as by radiation to the surroundings.



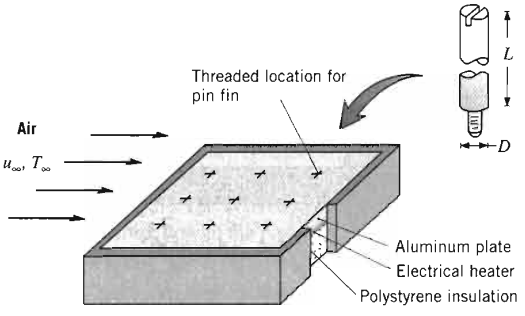
(a) By applying conservation of energy to a differential control surface of length dx, which either moves with the wire *or* is stationary and through which the wire passes, derive a differential equation that governs the temperature distribution, $T(x)$, along the wire. In your derivation, the effect of axial conduction along the wire may be neglected. Express your result in terms of the velocity, diameter, and properties of the wire (V_e, D, ρ, c_p, ε), the convection coefficient associated with the cross flow ($\bar{h}$), and the environmental temperatures (T_∞, T_{sur}).

(b) Neglecting radiation, obtain a closed form solution to the foregoing equation. For $V_e = 0.2$ m/s, $D = 5$ mm, $V = 5$ m/s, $T_\infty = 25°C$, and an initial wire temperature of $T_i = 600°C$, compute the temperature T_o of the wire at $x = L = 5$ m. The density and specific heat of the copper are $\rho = 8900$ kg/m^3 and $c_p = 400$ J/kg · K, while properties of the air may be taken to be $k = 0.037$ W/m · K, $\nu = 3 \times 10^{-5}$ m^2/s, and $Pr = 0.69$.

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(c) Accounting for the effects of radiation, with $\varepsilon = 0.55$ and $T_{sur} = 25°C$, numerically integrate the differential equation derived in part (a) to determine the temperature of the wire at $L = 5$ m. Explore the effects of V_e and ε on the temperature distribution along the wire.

7.64 The objective of an experiment performed by our students is to determine the effect of pin fins on the thermal resistance between a flat plate and an airstream. A 25.9-mm-square polished aluminum plate is subjected to an airstream in parallel flow at $T_\infty = 20°C$ and $u_\infty = 6$ m/s. An electrical heating patch is attached to the backside of the plate and dissipates 15.5 W under all conditions. Pin fins of diameter $D = 4.8$ mm and length $L = 25.4$ mm are fabricated from brass and can be firmly attached to the plate at various locations over its surface. Thermocouples are attached to the plate surface and the tip of one of the fins.



Measured temperatures for five pin-fin configurations are tabulated.

Number of Pin Fins	Temperature (°C)	
	Fin Tip	Plate Base
0	—	70.2
1	40.6	67.4
2	39.5	64.7
5	36.4	57.4
8	34.2	52.1

(a) Using the experimental observations and neglecting the effect of flow interactions between pins, determine the thermal resistance between the plate and the airstream for the five configurations.

(b) Develop a model of the plate–pin fin system and using appropriate convection correlations, predict the thermal resistances for the five configurations. Compare your predictions with the observations and explain any differences.

(c) Use your model to predict the thermal resistances when the airstream velocity is doubled.

7.65 Consider the enclosure and heat sink of Problem 7.30. The heat sink is rotated 90° so that the air is no longer directed between the fins, but normal to one of the $(L_b + L_f) \times w_2$ surfaces. Estimate the required air velocity if the height of the enclosure is $L_e = 10$ mm. Neglect heat transfer from the enclosure surfaces and assume isothermal fins, no heat transfer in the gaps between the fins, and negligible heat transfer from the downstream $(L_b + L_f) \times w_2$ surface. Include heat transfer from the $(L_b + L_f) \times w_1$ sides and the fin tips.

Spheres

7.66 Air at 25°C flows over a 10-mm-diameter sphere with a velocity of 25 m/s, while the surface of the sphere is maintained at 75°C.

(a) What is the drag force on the sphere?

(b) What is the rate of heat transfer from the sphere?

(c) Generate a plot of the heat transfer from the sphere as a function of the air velocity for the range 1 to 25 m/s.

7.67 Consider a sphere with a diameter of 20 mm and a surface temperature of 60°C that is immersed in a fluid at a temperature of 30°C and a velocity of 2.5 m/s. Calculate the drag force and the heat rate when the fluid is (a) water and (b) air at atmospheric pressure. Explain why the results for the two fluids are so different.

7.68 Atmospheric air at 25°C and a velocity of 0.5 m/s flows over a 50-W incandescent bulb whose surface temperature is at 140°C. The bulb may be approximated as a sphere of 50-mm diameter. What is the rate of heat loss by convection to the air?

7.69 Consider the material processing experiment of Problem 5.24, with atmospheric nitrogen used to implement cooling by convection. However, instead of using a prescribed value of the convection coefficient, compute the coefficient from an appropriate correlation.

(a) Neglecting radiation, determine the time required to cool the sphere from 900°C to 300°C if the velocity and temperature of the nitrogen are $V = 5$ m/s and $T_\infty = 25°C$.

(b) Accounting for the effects of both convection and radiation, with $\varepsilon = 0.6$ and $T_{sur} = 25°C$, determine the time required to cool the sphere. Explore the effects of the flow velocity on your result.

7.70 A spherical, under
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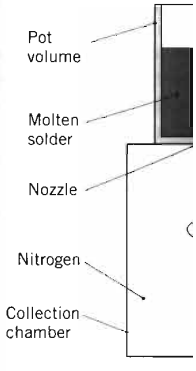
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7.70 A spherical, underwater instrument pod used to make soundings and to measure conditions in the water has a diameter of 85 mm and dissipates 300 W.

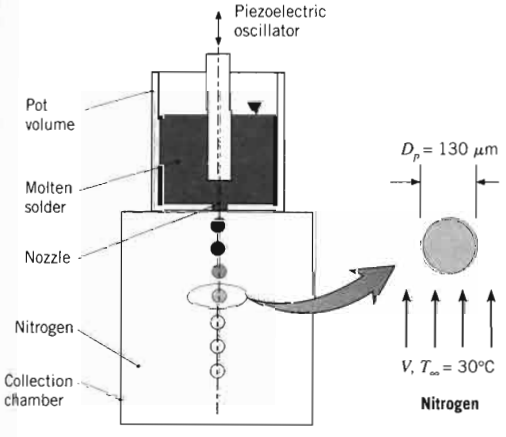
(a) Estimate the surface temperature of the pod when suspended in a bay where the current is 1 m/s and the water temperature is 15°C.

(b) Inadvertently, the pod is hauled out of the water and suspended in ambient air without deactivating the power. Estimate the surface temperature of the pod if the air temperature is 15°C and the wind speed is 3 m/s.

7.71 Worldwide, over a billion solder balls must be manufactured daily for assembling electronics packages. The *uniform droplet spray* method uses a piezoelectric device to vibrate a shaft in a pot of molten solder that, in turn, ejects small droplets of solder through a precision-machined nozzle. As they traverse a collection chamber, the droplets cool and solidify. The collection chamber is flooded with an inert gas such as nitrogen to prevent oxidation of the solder ball surfaces.

(a) Molten solder droplets of diameter 130 μm are ejected at a velocity of 2 m/s at an initial temperature of 225°C into gaseous nitrogen that is at 30°C and slightly above atmospheric pressure. Determine the terminal velocity of the particles and the distance the particles have traveled when they become completely solidified. The solder properties are $\rho = 8230$ kg/m^3, $c = 240$ J/kg · K, $k = 38$ W/m · K, $h_{sf} = 42$ kJ/kg. The solder's melting temperature is 183°C.

(b) The piezoelectric device oscillates at 1.8 kHz, producing 1800 particles per second. Determine the separation distance between the particles as they traverse the nitrogen gas and the pot volume needed in order to produce the solder balls continuously for one week.



7.72 A spherical workpiece of pure copper with a diameter of 15 mm and an emissivity of 0.5 is suspended in a large furnace with walls at a uniform temperature of 600°C. Air flows over the workpiece at a temperature of 900°C and a velocity of 7.5 m/s.

(a) Determine the steady-state temperature of the workpiece.

(b) Estimate the time required for the workpiece to come within 5°C of the steady-state temperature if it is at an initial, uniform temperature of 25°C.

(c) In order to decrease the time to heat the workpiece, the air velocity is doubled, with all other conditions remaining the same. Determine the steady-state temperature of the workpiece and the time required for it to come within 5°C of this value. Plot on the same graph the workpiece temperature histories for the two velocities.

7.73 Copper spheres of 20-mm diameter are quenched by being dropped into a tank of water that is maintained at 280 K. The spheres may be assumed to reach the terminal velocity on impact and to drop freely through the water. Estimate the terminal velocity by equating the drag and gravitational forces acting on the sphere. What is the approximate height of the water tank needed to cool the spheres from an initial temperature of 360 K to a center temperature of 320 K?

7.74 For the conditions of Problem 7.73, what are the terminal velocity and the tank height if engine oil at 300 K, rather than water, is used as the coolant?

7.75 Consider the plasma spray coating process of Problem 5.25. In addition to the prescribed conditions, the argon plasma jet is known to have a mean velocity of $V = 400$ m/s, while the *initial* velocity of the injected alumina particles may be approximated as zero. The nozzle exit and the substrate are separated by a distance of $L = 100$ mm, and pertinent properties of the argon plasma may be approximated as $k = 0.671$ W/m · K, $c_p = 1480$ J/kg · K, $\mu = 2.70 \times 10^{-4}$ kg/s · m, and $\nu = 5.6 \times 10^{-3}$ m^2/s.

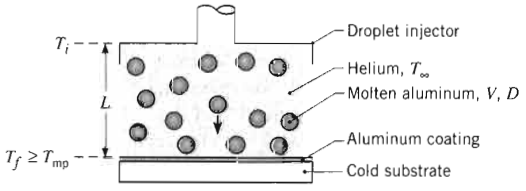
(a) Assuming the motion of particles entrained by the plasma jet to be governed by Stokes' law, derive expressions for the particle velocity, $V_p(t)$, and its distance of travel from the nozzle exit, $x_p(t)$, as a function of time, t, where $t = 0$ corresponds to particle injection. Evaluate the time-in-flight required for a particle to traverse the separation distance, $x_p = L$, and the velocity V_p at this time.

(b) Assuming an average relative velocity of $(\overline{V - V_p}) = 315$ m/s during the time-of-flight, estimate the convection coefficient associated with heat transfer from the plasma to the particle. Using this coefficient and assuming an initial particle temperature of $T_i = 300$

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K, estimate the time-in-flight required to heat a particle to its melting point, T_{mp}, and, once at T_{mp}, for the particle to experience complete melting. Is the prescribed value of L sufficient to ensure complete particle melting before surface impact?

7.76 Highly reflective aluminum coatings may be formed on the surface of a substrate by impacting the surface with molten drops of aluminum. The droplets are discharged from an injector, proceed through an inert gas (helium), and must still be in a molten state at the time of impact.



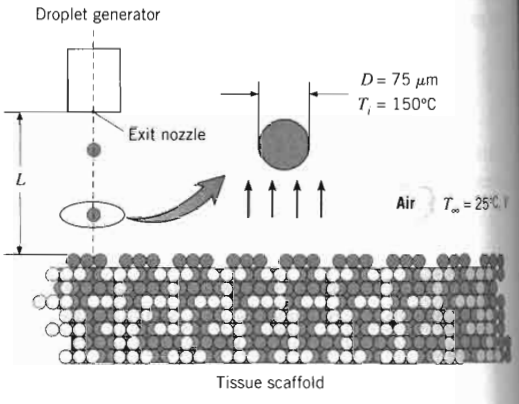
Consider conditions for which droplets with a diameter, velocity, and initial temperature of $D = 500\ \mu m$, $V = 3$ m/s, and $T_i = 1100$ K, respectively, traverse a stagnant layer of atmospheric helium that is at a temperature of $T_\infty = 300$ K. What is the maximum allowable thickness of the helium layer needed to ensure that the temperature of droplets impacting the substrate is greater than or equal to the melting point of aluminum ($T_f \geq T_{mp} = 933$ K)? Properties of the molten aluminum may be approximated as $\rho = 2500$ kg/m^3, $c = 1200$ J/kg · K, and $k = 200$ W/m · K.

7.77 Tissue engineering involves the development of biological substitutes that restore or improve tissue function. Once manufactured, engineered organs can be implanted and grow within the patient, obviating chronic shortages of natural organs that arise when traditional organ transplant procedures are used. Artificial organ manufacture involves two major steps. First, a porous *scaffold* is fabricated with a specific pore size and pore distribution, as well as overall shape and size. Second, the top surface of the scaffold is seeded with human cells that grow into the pores of the scaffold. The scaffold material is biodegradable and is eventually replaced with healthy tissue. The artificial organ is then ready to be implanted in the patient.

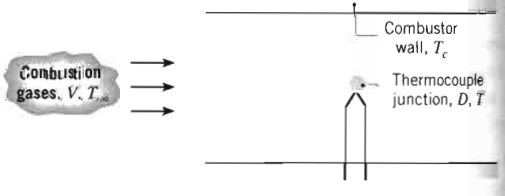
The complex pore shapes, small pore sizes, and unusual organ shapes preclude use of traditional manufacturing methods to fabricate the scaffolds. A method that has been used with success is a *solid freeform fabrication* technique whereby small spherical drops are directed to a substrate. The drops are initially molten and solidify when they impact the room-temperature substrate. By controlling the location of the droplet deposition, complex scaffolds can be built up, one drop at a time. A device similar to that of Problem 7.71 is used to generate uniform, 75-μm-diameter drops at an initial temperature of $T_i = 150$°C. The particles are sent through quiescent air at $T_\infty = 25$°C. The droplet properties are $\rho = 2200$ kg/m^3, $c = 700$ J/kg · K.

(a) It is desirable for the droplets to exit the nozzle at their terminal velocity. Determine the terminal velocity of the drops.

(b) It is desirable for the droplets to impact the structure at a temperature of $T_2 = 120$°C. What is the required distance between the exit nozzle and the structure, L?



7.78 A spherical thermocouple junction 1.0 mm in diameter is inserted in a combustion chamber to measure the temperature T_∞ of the products of combustion. The hot gases have a velocity of $V = 5$ m/s.



(a) If the thermocouple is at room temperature, T_i, when it is inserted in the chamber, estimate the time required for the temperature difference, $T_\infty - T$, to reach 2% of the initial temperature difference, $T_\infty - T_i$. Neglect radiation and conduction through the leads. Properties of the thermocouple junction are approximated as $k = 100$ W/m · K, $c = 385$ J/kg · K, and $\rho = 8920$ kg/m^3, while those of the combustion gases may be approximated as $k = 0.05$ W/m · K, $\nu = 50 \times 10^{-6}$ m^2/s, and $Pr = 0.69$.

(b) If the thermocouple junction has an emissivity of 0.5 and the cooled walls of the combustor are at

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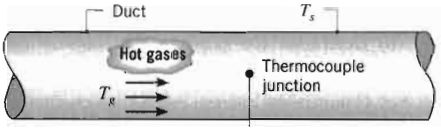
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$T_c = 400$ K, what is the steady-state temperature of the thermocouple junction if the combustion gases are at 1000 K? Conduction through the lead wires may be neglected.

(c) To determine the influence of the gas velocity on the thermocouple measurement error, compute the steady-state temperature of the thermocouple junction for velocities in the range $1 \leq V \leq 25$ m/s. The emissivity of the junction can be controlled through application of a thin coating. To reduce the measurement error, should the emissivity be increased or decreased? For $V = 5$ m/s, compute the steady-state junction temperature for emissivities in the range $0.1 \leq \varepsilon \leq 1.0$.

7.79 A thermocouple junction is inserted in a large duct to measure the temperature of hot gases flowing through the duct.



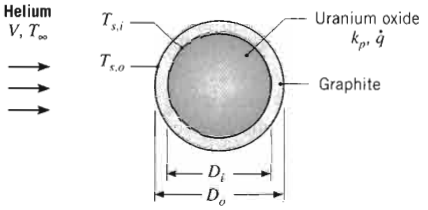
(a) If the duct surface temperature T_s is less than the gas temperature T_g, will the thermocouple sense a temperature that is less than, equal to, or greater than T_g? Justify your answer on the basis of a simple analysis.

(b) A thermocouple junction in the shape of a 2-mm-diameter sphere with a surface emissivity of 0.60 is placed in a gas stream moving at 3 m/s. If the thermocouple senses a temperature of 320°C when the duct surface temperature is 175°C, what is the actual gas temperature? The gas may be assumed to have the properties of air at atmospheric pressure.

(c) How would changes in velocity and emissivity affect the temperature measurement error? Determine the measurement error for velocities in the range $1 \leq V \leq 25$ m/s ($\varepsilon = 0.6$) and for emissivities in the range $0.1 \leq \varepsilon \leq 1.0$ ($V = 3$ m/s).

7.80 Consider temperature measurement in a gas stream using the thermocouple junction described in Problem 7.79 ($D = 2$ mm, $\varepsilon = 0.60$). If the gas velocity and temperature are 3 m/s and 500°C, respectively, what temperature will be indicated by the thermocouple if the duct surface temperature is 200°C? The gas may be assumed to have the properties of atmospheric air. What temperature will be indicated by the thermocouple if the gas pressure is doubled and all other conditions remain the same?

7.81 A high-temperature gas reactor (HTGR) consists of *spherical*, uranium oxide fuel elements in which there is uniform volumetric heating ($\dot{q}$). Each fuel element is embedded in a graphite spherical shell, which is cooled by a helium gas flow at 1 atm.



Consider steady-state conditions for which radiation effects may be neglected, the gas velocity and temperature are $V = 20$ m/s and $T_\infty = 500$ K, the pellet and shell diameters are $D_i = 10$ mm and $D_o = 12$ mm, and the shell surface temperature is $T_{s,o} = 1300$ K. The uranium oxide and graphite each have a thermal conductivity of $k_p = k_g = 2$ W/m · K.

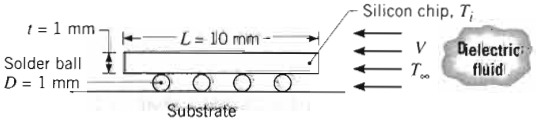
(a) What is the rate of heat transfer from a single pellet to the gas stream?

(b) What is the volumetric rate of heat generation in the pellet and what is the temperature at the pellet–graphite interface ($T_{s,i}$)?

(c) Obtain an expression for the radial temperature distribution, $T(r)$, in the pellet, expressing your result in terms of the temperature at the center of the pellet, $T(0)$. Evaluate $T(0)$ for the prescribed conditions.

(d) Determine $T_{s,o}$, $T_{s,i}$ and $T(0)$ as a function of the gas velocity for $5 \leq V \leq 20$ m/s and $\dot{q} = 1.50 \times 10^8$ W/m^3.

7.82 A silicon chip ($k = 150$ W/m · K, $\rho = 2300$ kg/m^3, $c_p = 700$ J/kg · K), 10 mm on a side and 1 mm thick, is connected to a substrate by solder balls ($k = 40$ W/m · K, $\rho = 10{,}000$ kg/m^3, $c_p = 150$ J/kg · K) of 1 mm diameter, and during an accelerated thermal stress test, the system is exposed to the flow of a dielectric liquid ($k = 0.064$ W/m · K, $\nu = 10^{-6}$ m^2/s, $Pr = 25$). As first approximations, treat the top and bottom surfaces of the chip as flat plates in turbulent, parallel flow and assume the substrate and lower chip surfaces to have a negligible effect on flow over the solder balls. Also assume point contact between the chip and the solder, thereby neglecting heat transfer by conduction between the components.



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(a) The stress test begins with the components at ambient temperature ($T_i = 20°C$) and proceeds with heating by the fluid at $T_\infty = 80°C$. If the fluid velocity is $V = 0.2$ m/s, estimate the ratio of the time constant of the chip to that of a solder ball. Which component responds more rapidly to the heating process?

(b) The thermal stress acting on the solder joint is proportional to the chip-to-solder temperature difference. What is this temperature difference 0.25 s after the start of heating?

Tube Banks

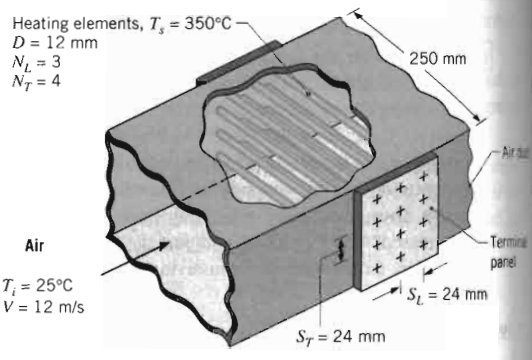
7.83 Repeat Example 7.7 for a more compact tube bank in which the longitudinal and transverse pitches are $S_L = S_T = 20.5$ mm. All other conditions remain the same.

7.84 A preheater involves the use of condensing steam at 100°C on the inside of a bank of tubes to heat air that enters at 1 atm and 25°C. The air moves at 5 m/s in cross flow over the tubes. Each tube is 1 m long and has an outside diameter of 10 mm. The bank consists of 196 tubes in a square, aligned array for which $S_T = S_L = 15$ mm. What is the total rate of heat transfer to the air? What is the pressure drop associated with the airflow?

7.85 Consider the in-line tube bank of Problem 7.84 ($D = 10$ mm, $L = 1$ m, and $S_T = S_L = 15$ mm), with condensing steam used to heat atmospheric air entering the tube bank at $T_i = 25°C$ and $V = 5$ m/s. In this case, however, the desired outlet temperature, not the number of tube rows, is known. What is the minimum value of N_L needed to achieve an outlet temperature of $T_o \geq 75°C$? What is the corresponding pressure drop across the tube bank?

7.86 A tube bank uses an aligned arrangement of 10-mm-diameter tubes with $S_T = S_L = 20$ mm. There are 10 rows of tubes with 50 tubes in each row. Consider an application for which cold water flows through the tubes, maintaining the outer surface temperature at 27°C, while flue gases at 427°C and a velocity of 5 m/s are in cross flow over the tubes. The properties of the flue gas may be approximated as those of atmospheric air at 427°C. What is the total rate of heat transfer per unit length of the tubes in the bank?

7.87 An air duct heater consists of an aligned array of electrical heating elements in which the longitudinal and transverse pitches are $S_L = S_T = 24$ mm. There are 3 rows of elements in the flow direction ($N_L = 3$) and 4 elements per row ($N_T = 4$). Atmospheric air with an upstream velocity of 12 m/s and a temperature of 25°C moves in cross flow over the elements, which have a diameter of 12 mm, a length of 250 mm, and are maintained at a surface temperature of 350°C.



(a) Determine the total heat transfer to the air and the temperature of the air leaving the duct heater.

(b) Determine the pressure drop across the element bank and the fan power requirement.

(c) Compare the average convection coefficient obtained in your analysis with the value for an isolated (single) element. Explain the difference between the results.

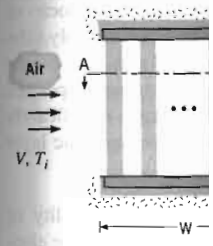
(d) What effect would increasing the longitudinal and transverse pitches to 30 mm have on the exit temperature of the air, the total heat rate, and the pressure drop?

7.88 A tube bank uses an aligned arrangement of 30-mm-diameter tubes with $S_T = S_L = 60$ mm and a tube length of 1 m. There are 10 tube rows in the flow direction ($N_L = 10$) and 7 tubes per row ($N_T = 7$). Air with upstream conditions of $T_\infty = 27°C$ and $V = 15$ m/s is in cross flow over the tubes, while a tube wall temperature of 100°C is maintained by steam condensation inside the tubes. Determine the temperature of air leaving the tube bank, the pressure drop across the bank, and the fan power requirement.

7.89 Electrical components mounted to each of two isothermal plates are cooled by passing atmospheric air between the plates, and an in-line array of aluminum pin fins is used to enhance heat transfer to the air.

The pins are of diameter $D = 2$ mm, length $L = 100$ mm, and thermal conductivity $k = 240$ W/m·K. The longitudinal and transverse pitches are $S_L = S_T = 4$ mm, with a square array of 625 pins ($N_T = N_L = 25$) mounted to square plates that are each of width $W = 100$ mm on a side. Air enters the pin array with a velocity of 10 m/s and a temperature of 300 K.

(a) Evaluating air properties at 300 K, estimate the average convection coefficient for the array of pin fins.

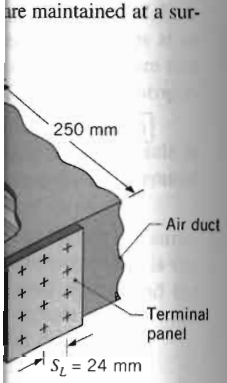


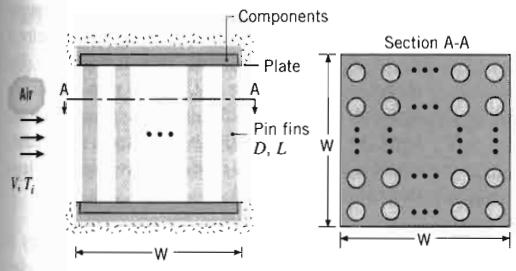
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(b) Assuming a uniform convection coefficient over all heat transfer surfaces (plates and pins), use the result of part (a) to determine the air outlet temperature and total heat rate when the plates are maintained at 350 K. *Hint:* The air outlet temperature is governed by an exponential relation of the form $[(T_s - T_o)/(T_s - T_i)] = \exp[-(\bar{h}A_t\eta_o)/\dot{m}c_p]$, where $\dot{m} = \rho VLN_TS_T$ is the mass flow rate of air passing through the array, A_t is the total heat transfer surface area (plates and fins), and η_o is the overall surface efficiency defined by Equation 3.102.

7.90 Consider the chip cooling scheme of Problem 3.134, but with an insulated top wall placed at the pin tips to force airflow across the pin array. Air enters the array at 20°C and with a velocity V that may be varied but cannot exceed 10 m/s due to pressure drop considerations. The pin fin geometry, which includes the number of pins in the $N \times N$ square array, as well as the pin diameter D_p and length L_p, may also be varied, subject to the constraint that the product ND_p not exceed 9 mm. Neglecting heat transfer through the board, assess the effect of changes in air velocity, and hence h_o, as well as pin fin geometry, on the air outlet temperature and the chip heat rate, if the remaining conditions of Problems 3.134 and 3.27, including a maximum allowable chip temperature of 75°C, remain in effect. Recommend design and operating conditions for which chip cooling is enhanced. *Hint:* The air outlet temperature is governed by a relation of the form $[(T_s - T_o)/(T_s - T_i)] = \exp[-(\bar{h}A_t\eta_o)/\dot{m}c_p]$, where $\dot{m}$ is the mass flow rate of air passing through the array, A_t is the total heat transfer surface area (chip and pins), and η_o is the overall surface efficiency defined by Equation 3.102.

7.91 An air-cooled steam condenser is operated with air in cross flow over a square, in-line array of 400 tubes ($N_L = N_T = 20$), with an outside tube diameter of 20 mm and longitudinal and transverse pitches of $S_L = 60$ mm and $S_T = 30$ mm, respectively. Saturated steam at a pressure of 2.455 bars enters the tubes, and a uniform tube outer surface temperature of $T_s = 390$ K may be assumed to be maintained as condensation occurs within the tubes.

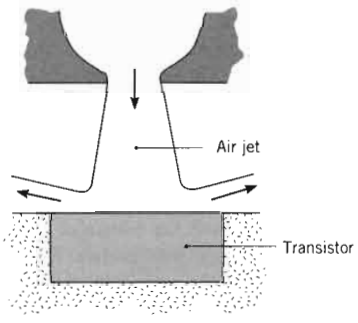
(a) If the temperature and velocity of the air upstream of the array are $T_i = 300$ K and $V = 4$ m/s, what is the temperature T_o of the air that leaves the array? As a first approximation, evaluate the properties of air at 300 K.

(b) If the tubes are 2 m long, what is the total heat transfer rate for the array? What is the rate at which steam is condensed in kg/s?

(c) Assess the effect of increasing N_L by a factor of 2, while reducing S_L to 30 mm. For this configuration, explore the effect of changes in the air velocity.

Impinging Jets

7.92 A circular transistor of 10-mm diameter is cooled by impingement of an air jet exiting a 2-mm-diameter round nozzle with a velocity of 20 m/s and a temperature of 15°C. The jet exit and the exposed surface of the transistor are separated by a distance of 10 mm.



If the transistor is well insulated at all but its exposed surface and the surface temperature is not to exceed 85°C, what is the transistor's maximum allowable operating power?

7.93 A long rectangular plate of AISI 304 stainless steel is initially at 1200 K and is cooled by an array of slot jets (see Figure 7.16). The nozzle width and pitch are $W = 10$ mm and $S = 100$ mm, respectively, and the nozzle-to-plate separation is $H = 200$ mm. The plate thickness and width are $t = 8$ mm and $L = 1$ m, respectively. If air exits the nozzles at a temperature of 400 K and a velocity of 30 m/s, what is the initial cooling rate of the plate?

7.94 Air at 10 m/s and 15°C is used to cool a square hot molded plastic plate 0.5 m to a side having a surface temperature of 140°C. To increase the throughput of the

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production process, it is proposed to cool the plate using an array of slotted nozzles with width and pitch of 4 mm and 56 mm, respectively, and a nozzle-to-plate separation of 40 mm. The air exits the nozzle at a temperature of 15°C and a velocity of 10 m/s.

(a) Determine the improvement in cooling rate that can be achieved using the slotted nozzle arrangement in lieu of turbulated air at 10 m/s and 15°C in parallel flow over the plate.

(b) Would the heat rates for both arrangements change significantly if the air velocities were increased by a factor of 2?

(c) What is the air mass rate requirement for the slotted nozzle arrangement?

7.95 Consider Problem 7.94 in which the improvement in performance of slot-jet cooling over parallel-flow cooling was demonstrated. Design an optimal round nozzle array, using the same air jet velocity and temperature, 10 m/s and 15°C, respectively, and compare the cooling rates and supply air requirements. Discuss the features associated with each of the three methods relevant to selecting one for this application of cooling the plastic part.

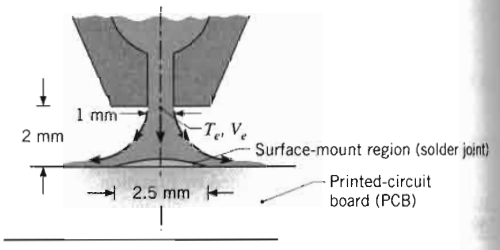
7.96 Consider the plasma spraying process of Problems 5.25 and 7.75. For a nozzle exit diameter of $D = 10$ mm and a substrate radius of $r = 25$ mm, *estimate* the rate of heat transfer by convection, q_{conv}, from the argon plasma to the substrate, if the substrate temperature is maintained at 300 K. Energy transfer to the substrate is also associated with the release of latent heat, q_{lat}, which occurs during solidification of the impacted molten droplets. If the mass rate of droplet impingement is $\dot{m}_p = 0.02$ kg/s · m^2, estimate the rate of latent heat release.

7.97 Consider the plasma jet of Problem 7.96. When new ceramic coating materials are developed, *spray trials* are performed using small, circular substrates called *coupons* that can be coated easily and subsequently tested for the new coating material's wear or corrosion resistance. Consider a 50-mm-diameter, 3-mm-thick 304 stainless steel coupon to be used in a spray trial with the goal of building up a 100-μm-thick ceramic coating.

(a) For a coupon that is insulated on its backside, determine how long the plasma torch can be in place before the coupon melts. How thick would the sprayed ceramic layer be when the coupon melts? The plasma heat transfer coefficient is 609 W/m^2 · K, and the heat transfer rate due to the impinging droplets is 140 W. The initial temperature of the coupon is 300 K.

(b) It is proposed to cool the backside of the coupon with an air jet at an exit temperature and velocity of $T_{e,a} = 300$ K and $V_{e,a} = 30$ m/s, respectively. The air jet nozzle diameter is $D_a = 10$ mm. The nozzle and the coupon's backside surface are separated by $L_a = 50$ mm. Will this scheme significantly increase the thickness of the sprayed ceramic layer that can be deposited?

7.98 You have been asked to determine the feasibility of using an impinging jet in a soldering operation for electronic assemblies. The schematic illustrates the use of a single, round nozzle to direct high velocity, hot air to a location where a *surface mount* joint is to be formed.



For your study, consider a round nozzle with a diameter of 1 mm located a distance of 2 mm from the region of the surface mount, which has a diameter of 2.5 mm.

(a) For an air jet velocity of 70 m/s and a temperature of 500°C, estimate the average convection coefficient over the area of the surface mount.

(b) Assume that the surface mount region on the printed circuit board (PCB) can be modeled as a semi-infinite medium, which is initially at a uniform temperature of 25°C and suddenly experiences convective heating by the jet. Estimate the time required for the surface to reach 183°C. The thermophysical properties of a typical solder are $\rho = 8333$ kg/m^3, $c_p = 188$ J/kg · K, and $k = 51$ W/m · K.

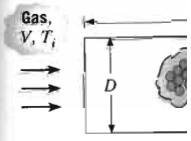
(c) For three air jet temperatures of 500, 600, and 700°C, calculate and plot the surface temperature as a function of time for $0 \le t \le 150$ s. On this plot, identify important temperature limits for the soldering process: the lower limit corresponding to the solder's eutectic temperature, $T_{sol} = 183$°C, and the upper limit corresponding to the glass transition temperature, $T_{gl} = 250$°C, at which the PCB becomes plastic. Comment on the outcome of your study, the appropriateness of the assumptions, and the feasibility of using the jet for a soldering application.

Packed Beds

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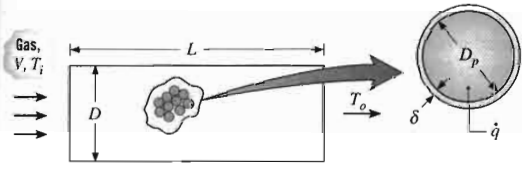
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Packed Beds

7.99 Consider the packed bed of aluminum spheres described in Problem 5.12 under conditions for which the bed is charged by hot air with an inlet velocity of $V = 1$ m/s and temperature of $T_{g,i} = 300°C$, but for which the convection coefficient is not prescribed. If the porosity of the bed is $\varepsilon = 0.40$ and the initial temperature of the spheres is $T_i = 25°C$, how long does it take a sphere near the inlet of the bed to accumulate 90% of its maximum possible energy?

7.100 The use of rock pile thermal energy storage systems has been considered for solar energy and industrial process heat applications. A particular system involves a cylindrical container, 2 m long by 1 m in diameter, in which nearly spherical rocks of 0.03-m diameter are packed. The bed has a void space of 0.42, and the density and specific heat of the rock are $\rho = 2300$ kg/m^3 and $c_p = 879$ J/kg · K, respectively. Consider conditions for which atmospheric air is supplied to the rock pile at a steady flow rate of 1 kg/s and a temperature of 90°C. The air flows in the axial direction through the container. If the rock is at a temperature of 25°C, what is the total rate of heat transfer from the air to the rock pile?

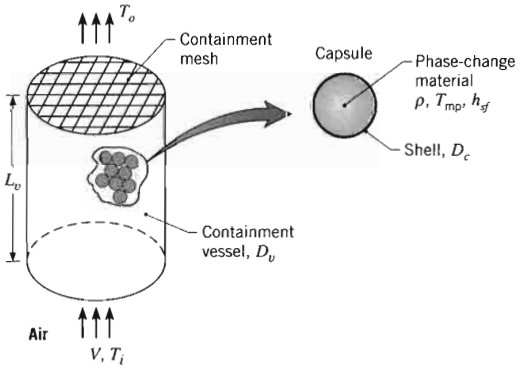
7.101 The cylindrical chamber of a *pebble bed nuclear reactor* is of length $L = 10$ m and diameter $D = 3$ m. The chamber is filled with spherical uranium oxide pellets of core diameter $D_p = 50$ mm. Each pellet generates thermal energy in its core at a rate of E_g and is coated with a layer of non-heat–generating graphite, which is of uniform thickness $\delta = 5$ mm, to form a pebble. The uranium oxide and graphite each have a thermal conductivity of 2 W/m · K. The packed bed has a porosity of $\varepsilon = 0.4$. Pressurized helium at 40 bars is used to absorb the thermal energy from the pebbles. The helium enters the packed bed at $T_i = 450°C$ with a velocity of 3.2 m/s. The properties of the helium may be assumed to be $c_p = 5193$ J/kg · K, $k = 0.3355$ W/m · K, $\rho = 2.1676$ kg/m^3, $\mu = 4.214 \times 10^{-5}$ kg/s · m, $Pr = 0.654$.



(a) For a desired overall thermal energy transfer rate of $q = 125$ MW, determine the mean outlet temperature of the helium leaving the bed, T_o, and the amount of thermal energy generated by each pellet, $\dot{E}_g$.

(b) The amount of energy generated by the fuel decreases if a maximum operating temperature of approximately 2100°C is exceeded. Determine the maximum internal temperature of the hottest pellet in the packed bed. For Reynolds numbers in the range $4000 \leq Re_D \leq 10{,}000$, Equation 7.85 may be replaced by $\varepsilon j_H = 2.876\, Re_D^{-1} + 0.3023\, Re_D^{-0.35}$.

7.102 *Latent heat capsules* consist of a thin-walled spherical shell within which a solid-liquid, phase-change material (PCM) of melting point T_{mp} and latent heat of fusion h_{sf} is enclosed. As shown schematically, the capsules may be packed in a cylindrical vessel through which there is fluid flow. If the PCM is in its solid state and $T_{mp} < T_i$, heat is transferred from the fluid to the capsules and latent energy is stored in the PCM as it melts. Conversely, if the PCM is a liquid and $T_{mp} > T_i$, energy is released from the PCM as it freezes and heat is transferred to the fluid. In either situation, all of the capsules within the packed bed would remain at T_{mp} through much of the phase change process, in which case the fluid outlet temperature would remain at a fixed value T_o.



Consider an application for which air is chilled by passing it through a packed bed ($\varepsilon = 0.5$) of capsules ($D_c = 50$ mm) containing an organic compound with a melting point of $T_{mp} = 4°C$. The air enters a cylindrical vessel ($L_v = D_v = 0.40$ m) at $T_i = 25°C$ and $V = 1.0$ m/s.

(a) If the PCM in each capsule is in the solid state at T_{mp} as melting occurs within the capsule, what is the outlet temperature of the air? If the density and latent heat of fusion of the PCM are $\rho = 1200$ kg/m^3 and $h_{sf} = 165$ kJ/kg, what is the mass rate (kg/s) at which the PCM is converted from solid to liquid in the vessel?

(b) Explore the effect of the inlet air velocity and capsule diameter on the outlet temperature.

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(c) At what location in the vessel will complete melting of the PCM in a capsule first occur? Once complete melting begins to occur, how will the outlet temperature vary with time and what is its asymptotic value?

7.103 Consider the packed bed ($\varepsilon = 0.5$) of latent heat capsules ($D_c = 50$ mm) described in Problem 7.102, but now for an application in which ambient air is to be heated by passing it through the bed. In this case the capsules contain an organic compound with a melting point of $T_{mp} = 50°C$, and the air enters the vessel ($L_v = D_v = 0.40$ m) at $T_i = 20°C$ and $V = 1.0$ m/s.

(a) If the PCM in each capsule is in the liquid state at T_{mp} as solidification occurs within the capsule what is the outlet temperature of the air? If the density and latent heat of fusion of the PCM are $\rho = 900$ kg/m^3 and $h_{sf} = 200$ kJ/kg, what is the mass rate (kg/s) at which the PCM is converted from liquid to solid in the vessel?

(b) Explore the effect of the inlet air velocity and capsule diameter on the outlet temperature.

(c) At what location in the vessel will complete freezing of the PCM in a capsule first occur? Once complete freezing begins to occur, how will the outlet temperature vary with time and what is its asymptotic value?

Heat and Mass Transfer

7.104 Consider mass loss from a smooth wet flat plate due to forced convection at atmospheric pressure. The plate is 0.5 m long and 3 m wide. Dry air at 300 K and a free stream velocity of 35 m/s flows over the surface, which is also at a temperature of 300 K. Estimate the average mass transfer coefficient $\bar{h}_m$ and determine the water vapor mass loss rate (kg/s) from the plate.

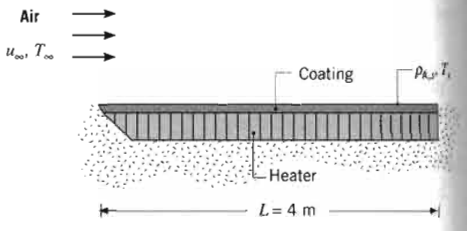
7.105 Consider dry, atmospheric air in parallel flow over a 0.5-m-long plate whose surface is wetted. The air velocity is 35 m/s, and the air and water are each at a temperature of 300 K.

(a) Estimate the heat loss and evaporation rate per unit width of the plate, q' and n'_A, respectively.

(b) Assuming the air temperature remains at 300 K, generate plots of q' and n'_A for a range of water temperatures from 300 to 350 K, with air velocities of 10, 20, and 35 m/s.

(c) For the air velocities and air temperature of part (b), determine the water temperatures for which the heat loss will be zero.

7.106 A flat plate coated with a volatile substance (species A) is exposed to dry, atmospheric air in parallel flow with $T_\infty = 20°C$ and $u_\infty = 8$ m/s. The plate is maintained at a constant temperature of 134°C by an electrical heating element, and the substance evaporates from the surface. The plate has a width of 0.25 m (normal to the plane of the sketch) and is well insulated on the bottom.



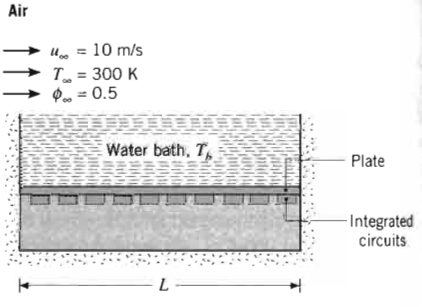
The molecular weight and the latent heat of vaporization of species A are $M_A = 150$ kg/kmol and $h_{fg} = 5.44 \times 10^6$ J/kg, respectively, and the mass diffusivity is $D_{AB} = 7.75 \times 10^{-7}$ m^2/s. If the saturated vapor pressure of the substance is 0.12 atm at 134°C, what is the electrical power required to maintain steady-state conditions?

7.107 Dry air at atmospheric pressure and 350 K, with a free stream velocity of 25 m/s, flows over a smooth, porous plate 1 m long.

(a) Assuming the plate to be saturated with liquid water at 350 K, estimate the mass rate of evaporation per unit width of the plate, n'_A (kg/s · m).

(b) For air and liquid water temperatures of 300, 325, and 350 K, generate plots of n'_A as a function of velocity for the range from 1 to 25 m/s.

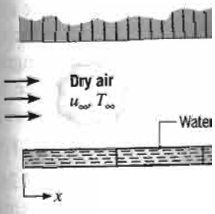
7.108 A scheme for dissipating heat from an array of $N = 100$ integrated circuits involves joining the circuits to the bottom of a plate and exposing the top of the plate to a water bath. The water container is of length $L = 100$ mm on a side and is exposed to airflow at its top surface. The flow is turbulated by the protruding lip of the side wall.



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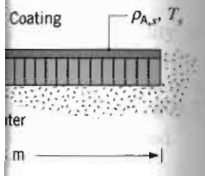
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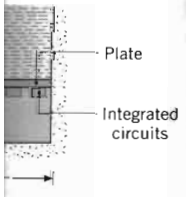


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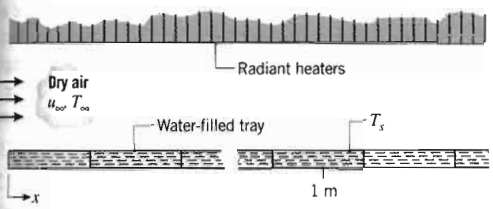
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dissipated in each circuit, at what rate may heat be dissipated from each circuit when the water temperature is maintained at $T_b = 350$ K?

7.109 A series of water-filled trays, each 222 mm long, experiences an evaporative drying process. Dry air at $T_\infty = 300$ K flows over the trays with a velocity of 15 m/s, while radiant heaters maintain the surface temperature at $T_s = 330$ K.



(a) What is the evaporative flux (kg/s · m²) at a distance 1 m from the leading edge?

(b) What is the irradiation (W/m²) that should be supplied to the tray surface at this location to maintain the water temperature at 330 K?

(c) Assuming the water temperature is uniform over the tray at this location, what is the evaporation rate (kg/s · m) from the tray per unit width of the tray?

(d) What irradiation should be applied to each of the first four trays such that the corresponding evaporation rates are identical to that found in part (c)?

7.110 Consider the physical system of Problem 7.109 (a series of water-filled trays heated radiatively), but under operating conditions for which each tray is 0.25 m long by 1 m wide and is uniformly irradiated, with $G = 10^4$ W/m². Dry air at $T_\infty = 300$ K continues to flow over the trays at a velocity of 15 m/s.

(a) What is the rate of water loss (kg/s) from the first, third, and fourth trays?

(b) Estimate the temperature of the water in each of the designated trays.

7.111 The apparatus described in Problem 7.40 is used by our students to experimentally determine convection heat and mass transfer coefficients, to confirm the heat–mass analogy, and to compare measured results with predictions based on standard correlations. The velocity, V, of the airstream is measured using a thermistor-based anemometer, and its relative humidity is determined from measurements of the wet and dry bulb temperatures, T_{wb} and T_{db}, respectively. Thermocouples are attached to the *test-plate*, which is covered with a sheet of wet paper in the mass transfer experiments.

(a) *Convection heat transfer coefficient.* Using the data provided in Problem 7.40, determine the heat transfer coefficients for the two velocities, assuming the plate to behave as a spacewise isothermal object. Evaluate the coefficients C and m for a correlation of the form $\overline{Nu}_L = C\,Re^m\,Pr^{1/3}$. Compare this result with a standard flat-plate correlation. Comment on the *goodness* of the comparison and provide reasons for any differences.

(b) *Convection mass transfer coefficient.* A sheet of water-saturated paper, 133 mm to a side, was used as the test surface and its mass measured at two different times, $m(t)$ and $m(t + \Delta t)$. Thermocouples were used to monitor the paper temperature as a function of time, from which the average temperature, $\overline{T}_s$, was determined. The wet and dry bulb temperatures were $T_{wb} = 13$°C and $T_{db} = 27$°C, and data recorded for two airstream velocities are as follows:

		Water Mass Loss Observations		
V (m/s)	$\overline{T}_s$(°C)	$m(t)$ (g)	$m(t + \Delta t)$ (g)	Δt (s)
3	15.3	55.62	54.45	475
9	16.0	55.60	54.50	240

Determine the convection mass transfer coefficients for the two flow conditions. Evaluate the coefficients C and m for a correlation of the form $\overline{Sh}_L = C\,Re^m\,Sc^{1/3}$.

(c) Using the heat–mass analogy, compare the experimental results with each other and against standard correlations. Comment on the *goodness* of the comparison and provide reasons for any differences.

7.112 Dry air at 35°C and a velocity of 20 m/s flows over a wetted plate of length 500 mm and width 150 mm. An imbedded electrical heater supplies power to maintain the plate surface temperature at 20°C.

(a) What is the evaporation rate (kg/h) of water from the plate? What electrical power is required to maintain steady-state conditions?

(b) After a long period of operation, all the water is evaporated from the plate and its surface is dry. For the same free stream conditions and heater power of part (a), estimate the temperature of the plate.

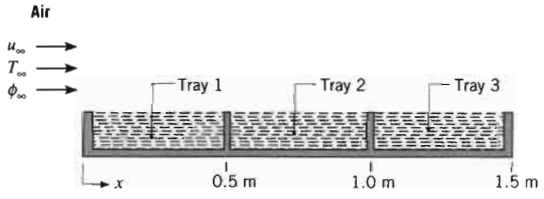
7.113 A minivan traveling 90 km/h has just passed through a thunderstorm that left a film of water 0.1 mm thick on the top of the van. The top of the van can be assumed

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to be a flat plate 6 m long. Assume isothermal conditions at 27°C, an ambient air relative humidity of 80%, and turbulent flow over the entire surface. What location on the van top will be the last to dry? What is the water evaporation rate per unit area (kg/s · m^2) at the trailing edge of the van top?

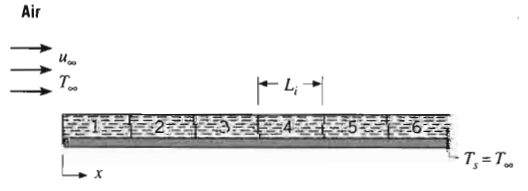
7.114 Benzene, a known carcinogen, has been spilled on the laboratory floor and has spread to a length of 2 m. If a film 1 mm deep is formed, how long will it take for the benzene to completely evaporate? Ventilation in the laboratory provides for airflow parallel to the surface at 1 m/s, and the benzene and air are both at 25°C. The mass densities of benzene in the saturated vapor and liquid states are known to be 0.417 and 900 kg/m^3, respectively.

7.115 Atmospheric air of 40% relative humidity and temperature $T_\infty = 300$ K is in parallel flow over a series of water-filled trays, with $u_\infty = 12$ m/s.



What is the rate at which energy must be supplied to each of the first three trays to maintain the water at 300 K?

7.116 A stream of atmospheric air is used to dry a series of biological samples on plates that are each of length $L_i = 0.25$ m in the direction of the airflow. The air is dry and at a temperature equal to that of the plates ($T_\infty = T_s = 50$°C). The air speed is $u_\infty = 9.1$ m/s.



(a) Sketch the variation of the local convection mass transfer coefficient $h_{m,x}$ with distance x from the leading edge. Indicate the specific nature of the x dependence.

(b) Which of the plates will dry the fastest? Calculate the drying rate per meter of width for this plate (kg/s · m).

(c) At what rate would heat have to be supplied to the fastest drying plate to maintain it at $T_s = 50$°C during the drying process?

7.117 Condenser cooling water for a power plant is stored in a cooling pond that is 1000 m long by 500 m wide. However, because of evaporative losses, it is necessary to periodically add "makeup" water to the pond in order to maintain a suitable water level. Assuming isothermal conditions at 27°C for the water and the air, that the free stream air is dry and moving at a velocity of 2 m/s in the direction of the 1000-m pond length, and that the boundary layer on the water surface is everywhere turbulent, determine the amount of makeup water that should be added to the pond daily.

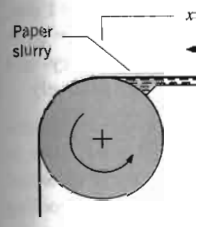
7.118 Consider the plate conveyor system of Problem 7.24, but now under conditions for which the plates are being transported from a liquid bath used for surface cleaning. The initial plate temperature is $T_i = 40$°C, and the surfaces are covered with a thin liquid film. If the air velocity and temperature are $u_\infty = 1$ m/s and $T_\infty = 20$°C, respectively, what is the initial rate of heat transfer from the plate? What is the corresponding rate of change of the plate temperature? The latent heat of vaporization of the solvent, the diffusion coefficient associated with transport of its vapor in air, and its saturated vapor density at 40°C are $h_{fg} = 900$ kJ/kg, $D_{AB} = 10^{-5}$ m^2/s, and $\rho_{A,sat} = 0.75$ kg/m^3, respectively. The velocity of the conveyor can be neglected relative to that of the air.

7.119 In a paper drying process, the paper moves on a conveyor belt at 0.2 m/s, while dry air from an in-line array of round jets (Figure 7.17*b*) impinges normal to its surface. The nozzle diameter and pitch are $D = 20$ mm and $S = 100$ mm, respectively, and the nozzle-to-paper separation is $H = 200$ mm. Air exits the nozzle at a velocity and temperature of 20 m/s and 300 K, while the wet paper is maintained at 300 K. In kg/s · m^2, what is the average drying rate of the paper?

7.120 In a paper mill drying process, a sheet of paper slurry (water–fiber mixture) has a linear velocity of 5 m/s as it is rolled. Radiant heaters maintain a sheet temperature of $T_s = 330$ K, as evaporation occurs to dry, ambient air at 300 K above and below the sheet.

(a) What is the evaporative flux at a distance of $x = 1$ m from the leading edge of the roll? What is the corresponding value of the radiant flux (irradiation, G) that must be supplied to the sheet to maintain its temperature at 330 K? The sheet has an absorptivity of $\alpha = 1$.

(b) To accelerate the drying and paper production processes, the velocity and temperature of the strip are increased to 10 m/s and 340 K, respectively.



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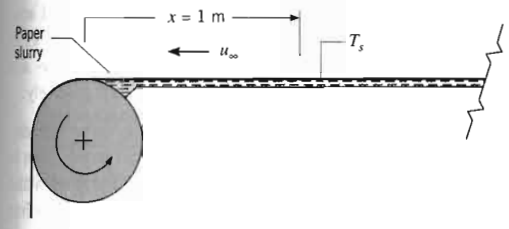
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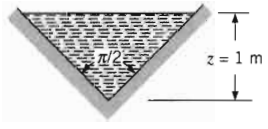
7.123 Dry air at 1-atm
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To maintain a uniform strip temperature, the irradiation G must be varied with x along the strip. For $0 \leq x \leq 1$ m, compute and plot the variations $h_{m,x}(x)$, $N''_A(x)$, and $G(x)$.

7.121 A channel of triangular cross section, which is 25 m long and 1 m deep, is used for the storage of water.



The water and the surrounding air are each at a temperature of 25°C, and the relative humidity of the air is 50%.

(a) If the air moves at a velocity of 5 m/s along the length of the channel, what is the rate of water loss due to evaporation from the surface?

(b) Obtain an expression for the rate at which the water depth would decrease with time due to evaporation. For the above conditions, how long would it take for all the water to evaporate?

7.122 Mass transfer experiments have been conducted on a naphthalene cylinder of 18.4-mm diameter and 88.9-mm length subjected to a cross flow of air in a low-speed wind tunnel. After exposure for 39 min to the airstream at a temperature of 26°C and a velocity of 12 m/s, it was determined that the cylinder mass decreased by 0.35 g. The barometric pressure was recorded at 750.6 mm Hg. The saturation pressure p_{sat} of naphthalene vapor in equilibrium with solid naphthalene is given by the relation $p_{sat} = p \times 10^E$, where $E = 8.67 - (3766/T)$, with T (K) and p (bar) being the temperature and pressure of air. Naphthalene has a molecular weight of 128.16 kg/kmol.

(a) Determine the convection mass transfer coefficient from the experimental observations.

(b) Compare this result with an estimate from an appropriate correlation for the prescribed flow conditions.

7.123 Dry air at 1-atm pressure and a velocity of 15 m/s is to be humidified by passing it in cross flow over a porous cylinder of diameter $D = 40$ mm, which is saturated with water.

(a) Assuming the water and air to be at 300 K, calculate the mass rate of water evaporated under steady-state conditions from the cylindrical medium per unit length.

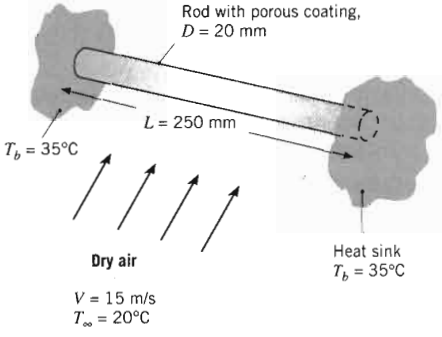
(b) How will the evaporation rate change if the air and water are maintained at a higher temperature? Generate a plot for the temperature range 300 to 350 K to illustrate the effect of temperature on the evaporation rate.

7.124 Dry air at 35°C and a velocity of 15 m/s flows over a long cylinder of 20-mm diameter. The cylinder is covered with a thin porous coating saturated with water, and an imbedded electrical heater supplies power to maintain the coating surface temperature at 20°C.

(a) What is the evaporation rate of water from the cylinder per unit length (kg/h · m)? What electrical power per unit length of the cylinder (W/m) is required to maintain steady-state conditions?

(b) After a long period of operation, all the water is evaporated from the coating and its surface is dry. For the same free stream conditions and heater power of part (a), estimate the temperature of the surface.

7.125 Dry air at 20°C and a velocity of 15 m/s flows over a 20-mm-diameter rod covered with a thin porous coating that is saturated with water. The rod ($k = 175$ W/m · K) is 250 mm long and its ends are attached to heat sinks maintained at 35°C.



Perform a steady-state, finite-difference analysis of the rod–porous coating system, considering conduction in the rod as well as energy transfer from the surface by convection heat and mass transfer. Use the analysis to estimate the temperature at the midspan of the rod and the evaporation rate from the surface. (*Suggestions:* Use 10 nodes to represent the half-length of the system.

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Estimate the overall average convection heat transfer coefficient based on an average film temperature for the system, and use the heat–mass transfer analogy to determine the average convection mass transfer coefficient. Validate your code by using it to predict a temperature distribution that agrees with the analytical solution for a fin without evaporation.)

7.126 Approximate the human form as an unclothed vertical cylinder of 0.3-m diameter and 1.75-m length with a surface temperature of 30°C.

(a) Calculate the heat loss in a 10-m/s wind at 20°C.

(b) What is the heat loss if the skin is covered with a thin layer of water at 30°C and the relative humidity of the air is 60%?

7.127 It has been suggested that heat transfer from a surface can be augmented by wetting it with water. As a specific example, consider a horizontal tube that is exposed to a transverse stream of dry air. You may assume that the tube, which is maintained at a temperature $T_s > T_\infty$, is completely wetted on the outside with a thin film of water. Derive an equation to determine the extent of heat transfer enhancement due to wetting. Evaluate the enhancement for $V = 10$ m/s, $D = 10$ mm, $T_s = 320$ K, and $T_\infty = 300$ K.

7.128 In the first stage of a paper drying process, a cylinder of diameter 0.15 m is covered by moisture-soaked paper. The temperature of the paper is maintained at 70°C by imbedded electrical heaters. Dry air at a velocity of 10 m/s and temperature of 20°C flows over the cylinder.

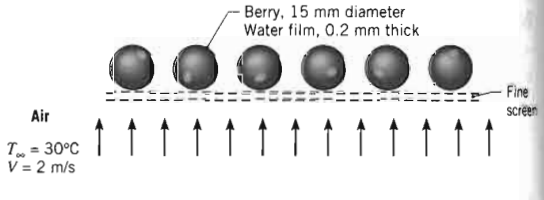
(a) Calculate the required electrical power and the evaporation rate per unit length of the cylinder, q' and n'_A, respectively.

(b) Generate plots of q' and n'_A as a function of the dry air velocity for $5 \leq V \leq 20$ m/s and for paper temperatures of 65, 70, and 75°C.

7.129 Cylindrical dry-bulb and wet-bulb thermometers are installed in a large-diameter duct to obtain the temperature T_∞ and the relative humidity ϕ_∞ of moist air flowing through the duct at a velocity V. The dry-bulb thermometer has a bare glass surface of diameter D_{db} and emissivity ε_g. The wet-bulb thermometer is covered with a thin wick that is saturated with water flowing continuously by capillary action from a bottom reservoir. Its diameter and emissivity are designated as D_{wb} and ε_w. The duct inside surface is at a known temperature T_s, which is less than T_∞. Develop expressions that may be used to obtain T_∞ and ϕ_∞ from knowledge of the dry-bulb and wet-bulb temperatures T_{db} and T_{wb} and the foregoing parameters. Determine T_∞ and ϕ_∞ when $T_{db} = 45$°C, $T_{wb} = 25$°C, $T_s = 35$°C, $p = 1$ atm, $V = 5$ m/s, $D_{db} = 3$ mm, $D_{wb} = 4$ mm, and $\varepsilon_g = \varepsilon_w = 0.95$. As a first approximation, evaluate the dry- and wet-bulb air properties at 45 and 25°C, respectively.

7.130 The thermal pollution problem is associated with discharging warm water from an electrical power plant or from an industrial source to a natural body of water. Methods for alleviating this problem involve cooling the warm water before allowing the discharge to occur. Two such methods, involving wet cooling towers or spray ponds, rely on heat transfer from the warm water in droplet form to the surrounding atmosphere. To develop an understanding of the mechanisms that contribute to this cooling, consider a spherical droplet of diameter D and temperature T, which is moving at a velocity V relative to air at a temperature T_∞ and relative humidity ϕ_∞. The surroundings are characterized by the temperature T_{sur}. Develop expressions for the droplet evaporation and cooling rates. Calculate the evaporation rate (kg/s) and cooling rate (K/s) when $D = 3$ mm, $V = 7$ m/s, $T = 40$°C, $T_\infty = 25$°C, $T_{sur} = 15$°C, and $\phi_\infty = 0.60$. The emissivity of water is $\varepsilon_w = 0.96$.

7.131 Cranberries are harvested by flooding the bogs in which they are grown and raking them into troughs for transport. At the processing plant, the surface moisture on the berries is removed as they roll over a fine screen through which warm air is blown. The berries have an average diameter of 15 mm, and the thickness of the water layer is 0.2 mm.



If the velocity and temperature of the heated air are 2 m/s and 30°C, respectively, estimate the time required to dry the berries. Assume that the water film on the berries is also at 30°C.

7.132 A spherical drop of water, 0.5 mm in diameter, is falling at a velocity of 2.15 m/s through dry still air at 1-atm pressure. Estimate the instantaneous rate of evaporation from the drop if the drop surface is at 60°C and the air is at 100°C.

7.133 A spherical droplet of alcohol, 0.5 mm in diameter, is falling freely through quiescent air at a velocity of 1.8 m/s. The concentration of alcohol vapor at the surface

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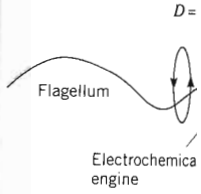
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7.135 Motile bacteria
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of the droplet is 0.0573 kg/m^3, and the diffusion coefficient for alcohol in air is 10^{-5} m^2/s. Neglecting radiation and assuming steady-state conditions, calculate the surface temperature of the droplet if the ambient air temperature is 300 K. The latent heat of vaporization is 8.42×10^5 J/kg.

7.134 As described in Problem 7.77, the second step in tissue engineering is to seed the top surface of the scaffold with human cells that subsequently grow into the pores of the scaffold. A seeding method that has been proposed is to use a droplet generator similar to that of Problem 7.77 to generate $D_p = 50$ μm diameter drops. The material in the droplet generator is a slurry consisting of a mixture of a host liquid and human liver cells. The host liquid has properties similar to water, and the liver cells are spherical with a diameter of $D_{lc} = 20$ μm and density $\rho_{lc} = 2400$ kg/m^3. Droplets are injected into atmospheric air with a relative humidity and temperature of $\phi = 0.50$ and $T_\infty = 25$°C, respectively. The particles are injected with an initial temperature of $T_i = 25$°C.

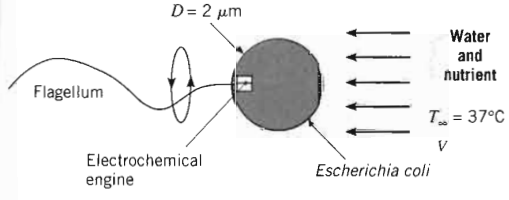
(a) It is desirable for each drop to contain one liver cell. Determine the volume fraction, f, of liver cells in the slurry and the terminal velocity for a drop containing one liver cell.

(b) The droplet containing one liver cell is injected at its terminal velocity. Determine the time of flight for a distance between the ejector nozzle and the scaffold of $L = 4$ mm.

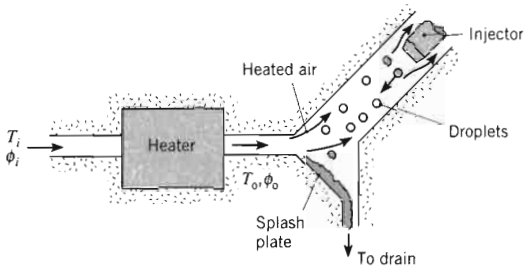
(c) Determine the initial evaporation rate from the droplet.

(d) The tissue engineer is concerned that evaporation will change the mass of the droplet and, in turn, will affect its time of flight and the precision with which the seeds can be placed on the scaffold. Estimate the maximum change in mass due to evaporation during the time of flight. Compare the variation of mass due to evaporation to the variation associated with there being one to three liver cells per droplet. Does evaporation or the liver cell population per droplet influence the variability of the droplet mass most significantly?

7.135 Motile bacteria are equipped with flagella that are rotated by tiny, biological electrochemical engines which, in turn, propel the bacteria through a host liquid. Consider a nominally spherical *Escherichia coli* bacterium that is of diameter $D = 2$ μm. The bacterium is in a water-based solution at 37°C containing a nutrient which is characterized by a binary diffusion coefficient of $D_{AB} = 0.7 \times 10^{-9}$ m^2/s and a food energy value of $\mathcal{N} = 16{,}000$ kJ/kg. There is a nutrient density difference between the fluid and the shell of the bacterium of $\Delta\rho_A = 860 \times 10^{-12}$ kg/m^3. Assuming a propulsion efficiency of $\eta = 0.5$, determine the maximum speed of the *E. coli*. Report your answer in body diameters per second.

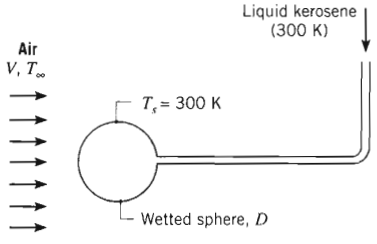


7.136 In a home furnace humidification system, water droplets of diameter D are discharged in a direction opposing the motion of warm air emerging from the heater. The air is humidified by evaporation from the droplets, and the excess water is collected on a *splash plate,* from which it is routed to a drain.



Consider conditions for which air enters the heater at a temperature and relative humidity of 17°C and 70%, respectively, and leaves the heater at a temperature of 47°C. The droplet diameter is 1 mm, and the relative velocity between the droplets and the heated air is 15 m/s. During the time-of-flight, the change in droplet diameter may be neglected and the droplet temperature may be assumed to remain at 47°C. What is the rate of evaporation from a single droplet?

7.137 Evaporation of liquid fuel droplets is often studied in the laboratory by using a porous sphere technique in which the fuel is supplied at a rate just sufficient to maintain a completely wetted surface on the sphere.



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Consider the use of kerosene at 300 K with a porous sphere of 1-mm diameter. At this temperature the kerosene has a saturated vapor density of 0.015 kg/m^3 and a latent heat of vaporization of 300 kJ/kg. The mass diffusivity for the vapor–air mixture is 10^{-5} m^2/s. If dry, atmospheric air at $V = 15$ m/s and $T_\infty = 300$ K flows over the sphere, what is the minimum mass rate at which kerosene must be supplied to maintain a wetted surface? For this condition, by how much must T_∞ actually exceed T_s to maintain the wetted surface at 300 K?

7.138 Consider an air conditioning system composed of a bank of tubes arranged normal to air flowing in a duct at a mass rate of $\dot{m}_a$ (kg/s). A coolant flowing through the tubes is able to maintain the surface temperature of the tubes at a constant value of $T_s < T_{a,i}$, where $T_{a,i}$ is the inlet air temperature (upstream of the tube bank). It has been suggested that air cooling may be enhanced if a thin, uniform film of water is maintained on the outer surface of each of the tubes.

(a) Assuming the water film to be at the temperature T_s, develop an expression for the ratio of the amount of cooling that occurs with the water film to the amount of cooling that occurs without the film. The amount of cooling may be defined as $T_{a,i} - T_{a,o}$, where $T_{a,o}$ is the outlet air temperature (downstream of the tube bank). The upstream air may be assumed to be dry, and the driving potentials for convection heat and mass transfer may be approximated as $(T_{a,i} - T_s)$ and $\rho_{A,sat}(T_s)$, respectively. *Note:* The total rate of heat loss from the air may be expressed as $q = \dot{m}_a c_{p,a}(T_{a,i} - T_{a,o})$. Estimate the value of this ratio under conditions for which $T_{a,i} = 35$°C and $T_s = 10$°C.

(b) Consider a tube bank that is 5 rows deep, with 12 tubes in a row. Each tube is 0.5 m long, with an outside diameter of 8 mm, and a staggered arrangement is used for which $S_T = S_L = 24$ mm. Under conditions for which $\dot{m}_a = 0.5$ kg/s, $V = 3$ m/s, $T_{a,i} = 35$°C, and $T_s = 10$°C, what is the value of $T_{a,o}$ if the tubes are wetted? What is the specific humidity of the air leaving the tube bank?

7.139 In a paper-drying process, the paper moves on a conveyor belt at 0.2 m/s, while dry air from an array of slot jets (Figure 7.16) impinges normal to its surface. The nozzle width and pitch are $W = 10$ mm and $S = 100$ mm, respectively, and the nozzle-to-plate separation is $H = 200$ mm. The wet paper is of width $L = 1$ m and is maintained at 300 K, while the air exits the nozzles at a temperature of 300 K and a velocity of 20 m/s. In kg/s · m^2, what is the average drying rate per unit surface area of the paper?



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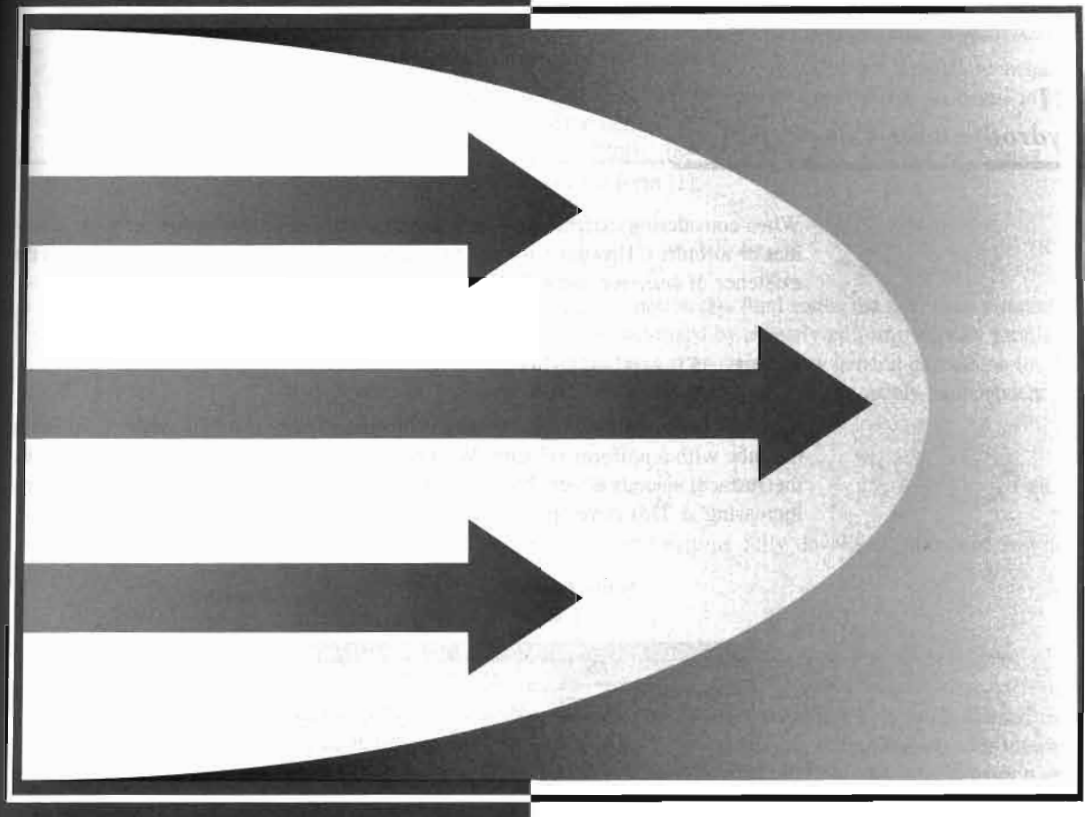
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CHAPTER 8

Internal Flow



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Having acquired the means to compute convection transfer rates for external flow, we now consider the convection transfer problem for *internal flow*. Recall that an external flow is one for which boundary layer development on a surface is allowed to continue without external constraints, as for the flat plate of Figure 6.6. In contrast, an internal flow, such as flow in a pipe, is one for which the fluid is *confined* by a surface. Hence the boundary layer is unable to develop without eventually being constrained. The internal flow configuration represents a convenient geometry for heating and cooling fluids used in chemical processing, environmental control, and energy conversion technologies.

Our objectives are to develop an appreciation for the physical phenomena associated with internal flow and to obtain convection coefficients for flow conditions of practical importance. We begin by considering velocity (hydrodynamic) effects pertinent to internal flows, focusing on certain unique features of boundary layer development. Thermal boundary layer effects are considered next, and an overall energy balance is applied to determine fluid temperature variations in the flow direction. Finally, correlations for estimating the convection heat transfer coefficient are presented for a variety of internal flow conditions.

8.1 *Hydrodynamic Considerations*

When considering external flow, it is necessary to ask only whether the flow is laminar or turbulent. However, for an internal flow we must also be concerned with the existence of *entrance* and *fully developed* regions.

8.1.1 Flow Conditions

Consider laminar flow in a circular tube of radius r_o (Figure 8.1), where fluid enters the tube with a uniform velocity. We know that when the fluid makes contact with the surface, viscous effects become important, and a boundary layer develops with increasing x. This development occurs at the expense of a shrinking inviscid flow

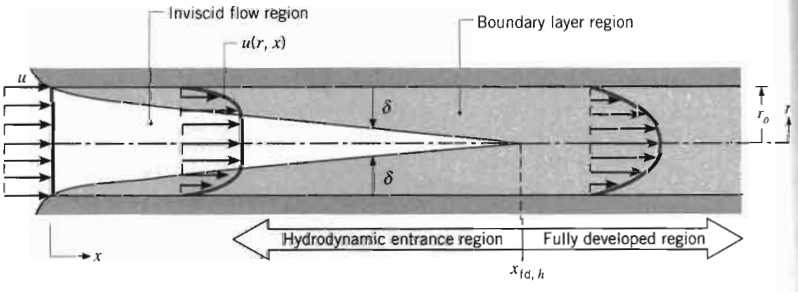


FIGURE 8.1 Laminar, hydrodynamic boundary layer development in a circular tube.

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region and concludes with boundary layer merger at the centerline. Following this merger, viscous effects extend over the entire cross section and the velocity profile no longer changes with increasing x. The flow is then said to be *fully developed,* and the distance from the entrance at which this condition is achieved is termed the *hydrodynamic entry length,* $x_{fd,h}$. As shown in Figure 8.1, the *fully developed velocity profile* is parabolic for laminar flow in a circular tube. For turbulent flow, the profile is *flatter* due to turbulent mixing in the radial direction.

When dealing with internal flows, it is important to be cognizant of the extent of the entry region, which depends on whether the flow is laminar or turbulent. The Reynolds number for flow in a circular tube is defined as

$$Re_D \equiv \frac{\rho u_m D}{\mu} = \frac{u_m D}{\nu} \tag{8.1}$$

where u_m is the mean fluid velocity over the tube cross section and D is the tube diameter. In a fully developed flow, the critical Reynolds number corresponding to the *onset* of turbulence is

$$Re_{D,c} \approx 2300 \tag{8.2}$$

although much larger Reynolds numbers ($Re_D \approx 10{,}000$) are needed to achieve fully turbulent conditions. The transition to turbulence is likely to begin in the developing boundary layer of the entrance region.

For laminar flow ($Re_D \lesssim 2300$), the hydrodynamic entry length may be obtained from an expression of the form [1]

$$\left(\frac{x_{fd,h}}{D}\right)_{lam} \approx 0.05\, Re_D \tag{8.3}$$

This expression is based on the presumption that fluid enters the tube from a rounded converging nozzle and is hence characterized by a nearly uniform velocity profile at the entrance (Figure 8.1). Although there is no satisfactory general expression for the entry length in turbulent flow, we know that it is approximately independent of Reynolds number and that, as a first approximation [2],

$$10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{turb} \lesssim 60 \tag{8.4}$$

For the purposes of this text, we shall assume fully developed turbulent flow for $(x/D) > 10$.

8.1.2 The Mean Velocity

Because the velocity varies over the cross section and there is no well-defined free stream, it is necessary to work with a mean velocity u_m when dealing with internal flows. This velocity is defined such that, when multiplied by the fluid density ρ and the cross-sectional area of the tube A_c, it provides the rate of mass flow through the tube. Hence

$$\dot{m} = \rho u_m A_c \tag{8.5}$$

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For steady, incompressible flow in a tube of uniform cross-sectional area, $\dot{m}$ and u_m are constants independent of x. From Equations 8.1 and 8.5 it is evident that, for flow in a *circular tube* ($A_c = \pi D^2/4$), the Reynolds number reduces to

$$Re_D = \frac{4\dot{m}}{\pi D \mu} \tag{8.6}$$

Since the mass flow rate may also be expressed as the integral of the mass flux (ρu) over the cross section

$$\dot{m} = \int_{A_c} \rho u(r, x)\, dA_c \tag{8.7}$$

it follows that, for *incompressible* flow in a *circular* tube,

$$u_m = \frac{\int_{A_c} \rho u(r, x)\, dA_c}{\rho A_c} = \frac{2\pi\rho}{\rho\pi r_o^2} \int_0^{r_o} u(r, x) r\, dr = \frac{2}{r_o^2} \int_0^{r_o} u(r, x) r\, dr \tag{8.8}$$

The foregoing expression may be used to determine u_m at any axial location x from knowledge of the velocity profile $u(r)$ at that location.

8.1.3 Velocity Profile in the Fully Developed Region

The form of the velocity profile may readily be determined for the *laminar flow* of an *incompressible, constant property fluid* in the *fully developed region* of a *circular tube*. An important feature of hydrodynamic conditions in the fully developed region is that both the radial velocity component v and the gradient of the axial velocity component ($\partial u/\partial x$) are everywhere zero.

$$v = 0 \quad \text{and} \quad \left(\frac{\partial u}{\partial x}\right) = 0 \tag{8.9}$$

Hence the axial velocity component depends only on r, $u(x, r) = u(r)$.

The radial dependence of the axial velocity may be obtained by solving the appropriate form of the x-momentum equation. This form is determined by first recognizing that, for the conditions of Equation 8.9, the net momentum flux is everywhere zero in the fully developed region. Hence the momentum conservation requirement reduces to a simple balance between shear and pressure forces in the flow. For the annular differential element of Figure 8.2, this force balance may be expressed as

$$\tau_r(2\pi r\, dx) - \left\{\tau_r(2\pi r\, dx) + \frac{d}{dr}[\tau_r(2\pi r\, dx)]\, dx\right\}$$
$$+ p(2\pi r\, dr) - \left\{p(2\pi r\, dr) + \frac{d}{dx}[p(2\pi r\, dr)]\, dx\right\} = 0$$

which reduces to

$$-\frac{d}{dr}(r\tau_r) = r\frac{dp}{dx} \tag{8.10}$$

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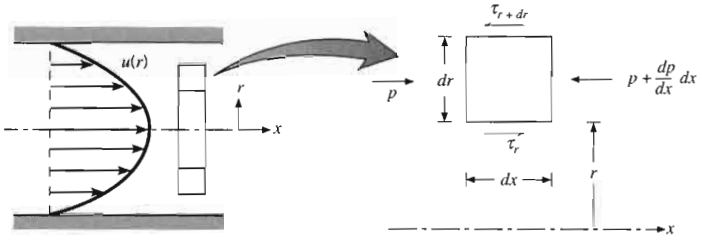


FIGURE 8.2 Force balance on a differential element for laminar, fully developed flow in a circular tube.

With $y = r_o - r$, Newton's law of viscosity, Equation 6S.10, assumes the form

$$\tau_r = -\mu \frac{du}{dr} \tag{8.11}$$

and Equation 8.10 becomes

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dp}{dx} \tag{8.12}$$

Since the axial pressure gradient is independent of r, Equation 8.12 may be solved by integrating twice to obtain

$$r\frac{du}{dr} = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{2} + C_1$$

and

$$u(r) = \frac{1}{\mu}\left(\frac{dp}{dx}\right)\frac{r^2}{4} + C_1 \ln r + C_2$$

The integration constants may be determined by invoking the boundary conditions

$$u(r_o) = 0 \qquad \text{and} \qquad \left.\frac{\partial u}{\partial r}\right|_{r=0} = 0$$

which, respectively, impose the requirements of zero slip at the tube surface and radial symmetry about the centerline. It is a simple matter to evaluate the constants, and it follows that

$$u(r) = -\frac{1}{4\mu}\left(\frac{dp}{dx}\right) r_o^2 \left[1 - \left(\frac{r}{r_o}\right)^2\right] \tag{8.13}$$

Hence the fully developed velocity profile is *parabolic*. Note that the pressure gradient must always be negative.

The foregoing result may be used to determine the mean velocity of the flow. Substituting Equation 8.13 into Equation 8.8 and integrating, we obtain

$$u_m = -\frac{r_o^2}{8\mu}\frac{dp}{dx} \tag{8.14}$$

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Substituting this result into Equation 8.13, the velocity profile is then

$$\frac{u(r)}{u_m} = 2\left[1 - \left(\frac{r}{r_o}\right)^2\right] \tag{8.15}$$

Since u_m can be computed from knowledge of the mass flow rate, Equation 8.14 can be used to determine the pressure gradient.

8.1.4 Pressure Gradient and Friction Factor in Fully Developed Flow

The engineer is frequently interested in the pressure drop needed to sustain an internal flow because this parameter determines pump or fan power requirements. To determine the pressure drop, it is convenient to work with the *Moody* (or Darcy) *friction factor,* which is a dimensionless parameter defined as

$$f \equiv \frac{-(dp/dx)D}{\rho u_m^2/2} \tag{8.16}$$

This quantity is not to be confused with the *friction coefficient,* sometimes called the Fanning friction factor, which is defined as

$$C_f \equiv \frac{\tau_s}{\rho u_m^2/2} \tag{8.17}$$

Since $\tau_s = -\mu(du/dr)_{r=r_o}$, it follows from Equation 8.13 that

$$C_f = \frac{f}{4} \tag{8.18}$$

Substituting Equations 8.1 and 8.14 into 8.16, it follows that, for fully developed laminar flow,

$$f = \frac{64}{Re_D} \tag{8.19}$$

For fully developed turbulent flow, the analysis is much more complicated, and we must ultimately rely on experimental results. Friction factors for a wide Reynolds number range are presented in the *Moody diagram* of Figure 8.3. In addition to depending on the Reynolds number, the friction factor is a function of the tube surface condition. It is a minimum for *smooth* surfaces and increases with increasing surface roughness, e. Correlations that reasonably approximate the smooth surface condition are of the form

$$f = 0.316Re_D^{-1/4} \qquad Re_D \lesssim 2 \times 10^4 \tag{8.20a}$$

$$f = 0.184Re_D^{-1/5} \qquad Re_D \gtrsim 2 \times 10^4 \tag{8.20b}$$

Alternatively, a single correlation that encompasses a large Reynolds number range has been developed by Petukhov [4] and is of the form

$$f = (0.790 \ln Re_D - 1.64)^{-2} \qquad 3000 \lesssim Re_D \lesssim 5 \times 10^6 \tag{8.21}$$

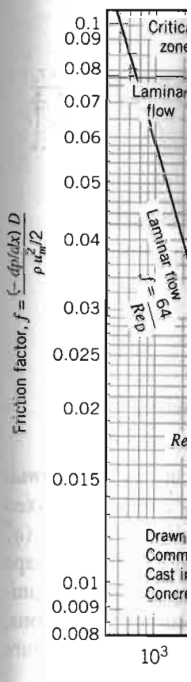


FIGURE 8.3 Friction

8.2 Thermal Cons

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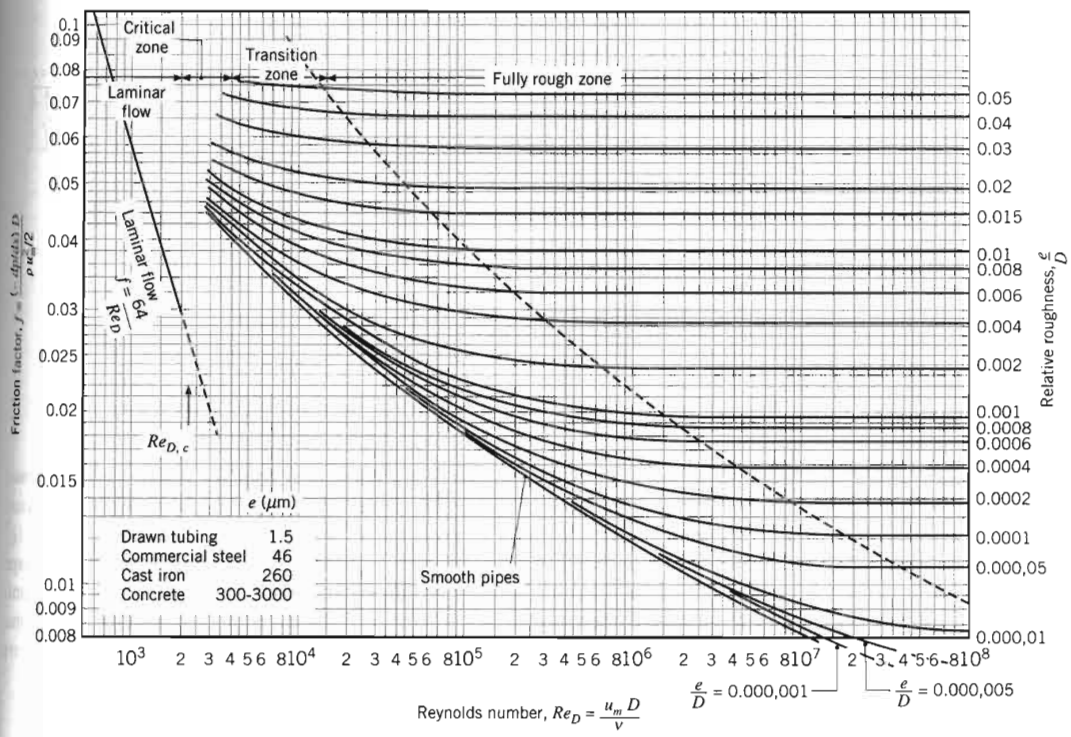


FIGURE 8.3 Friction factor for fully developed flow in a circular tube [3]. Used with permission.

Note that f, hence dp/dx, is a constant in the fully developed region. From Equation 8.16 the pressure drop $\Delta p = p_1 - p_2$ associated with fully developed flow from the axial position x_1 to x_2 may then be expressed as

$$\Delta p = -\int_{p_1}^{p_2} dp = f\frac{\rho u_m^2}{2D}\int_{x_1}^{x_2} dx = f\frac{\rho u_m^2}{2D}(x_2 - x_1) \tag{8.22a}$$

where f is obtained from Figure 8.3 or from Equation 8.19 for laminar flow and from Equation 8.20 or 8.21 for turbulent flow in smooth tubes. The pump or fan power required to overcome the resistance to flow associated with this pressure drop may be expressed as

$$P = (\Delta p)\dot{\forall} \tag{8.22b}$$

where the volumetric flow rate $\dot{\forall}$ may, in turn, be expressed as $\dot{\forall} = \dot{m}/\rho$ for an incompressible fluid.

8.2 *Thermal Considerations*

Having reviewed the fluid mechanics of internal flow, we now consider thermal effects. If fluid enters the tube of Figure 8.4 at a uniform temperature $T(r, 0)$ that is

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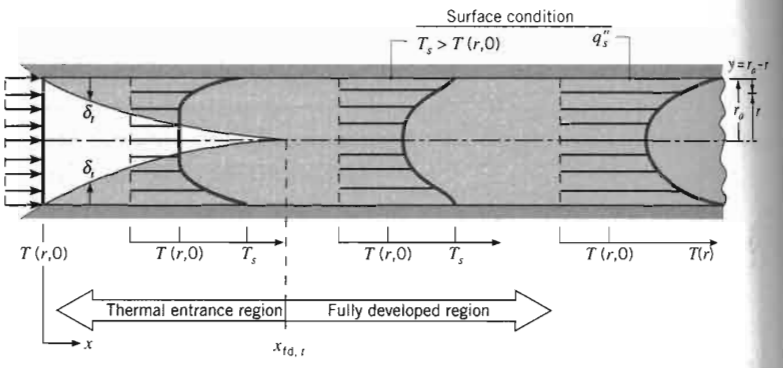


FIGURE 8.4 Thermal boundary layer development in a heated circular tube.

less than the surface temperature, convection heat transfer occurs and a *thermal boundary layer* begins to develop. Moreover, if the tube *surface* condition is fixed by imposing either a uniform temperature (T_s is constant) or a uniform heat flux (q_s'' is constant), a *thermally fully developed condition* is eventually reached. The shape of the fully developed temperature profile $T(r, x)$ differs according to whether a uniform surface temperature or heat flux is maintained. For both surface conditions, however, the amount by which fluid temperatures exceed the entrance temperature increases with increasing x.

For laminar flow the *thermal entry length* may be expressed as [2]

$$\left(\frac{x_{fd,t}}{D}\right)_{lam} \approx 0.05 Re_D Pr \tag{8.23}$$

Comparing Equations 8.3 and 8.23, it is evident that, if $Pr > 1$, the hydrodynamic boundary layer develops more rapidly than the thermal boundary layer ($x_{fd,h} < x_{fd,t}$), while the inverse is true for $Pr < 1$. For extremely large Prandtl number fluids, such as oils ($Pr \gtrsim 100$), $x_{fd,h}$ is very much smaller than $x_{fd,t}$ and it is reasonable to assume a fully developed velocity profile throughout the thermal entry region. In contrast, for turbulent flow, conditions are nearly independent of Prandtl number, and to a first approximation, we shall assume $(x_{fd,t}/D) = 10$.

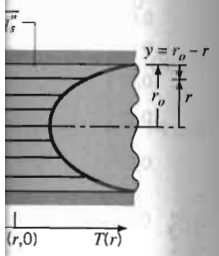
Thermal conditions in the fully developed region are characterized by several interesting and useful features. Before we can consider these features (Section 8.2.3), however, it is necessary to introduce the concept of a mean temperature and the appropriate form of Newton's law of cooling.

8.2.1 The Mean Temperature

Just as the absence of a free stream velocity requires use of a mean velocity to describe an internal flow, the absence of a fixed free stream temperature necessitates using a *mean* (or *bulk*) *temperature*. To provide a definition of the mean temperature, we begin by returning to Equation 1.11e:

$$q = \dot{m} c_p (T_{out} - T_{in}) \tag{1.11e}$$

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Recall that the terms on the right-hand side represent the thermal energy for an incompressible liquid or the enthalpy (thermal energy plus flow work) for an ideal gas, which is carried by the fluid. In developing this equation, it was implicitly assumed that the temperature was uniform across the inlet and outlet cross-sectional areas. In reality, this is not true if convection heat transfer occurs, and we *define* the mean temperature so that the term $\dot{m}c_pT_m$ is equal to the true rate of thermal energy (or enthalpy) advection integrated over the cross section. This true advection rate may be obtained by integrating the product of mass flux (ρu) and the thermal energy (or enthalpy) per unit mass, c_pT, over the cross section. Therefore, we define T_m from

$$\dot{m}c_pT_m = \int_{A_c} \rho u c_p T dA_c \tag{8.24}$$

or

$$T_m = \frac{\int_{A_c} \rho u c_p T dA_c}{\dot{m}c_p} \tag{8.25}$$

For flow in a circular tube with constant ρ and c_p, it follows from Equations 8.5 and 8.25 that

$$T_m = \frac{2}{u_m r_o^2} \int_0^{r_o} uTrdr \tag{8.26}$$

It is important to note that, when multiplied by the mass flow rate and the specific heat, T_m provides the rate at which thermal energy (or enthalpy) is advected with the fluid as it moves along the tube.

8.2.2 Newton's Law of Cooling

The mean temperature T_m is a convenient reference temperature for internal flows, playing much the same role as the free stream temperature T_∞ for external flows. Accordingly, Newton's law of cooling may be expressed as

$$q_s'' = h(T_s - T_m) \tag{8.27}$$

where h is the *local* convection heat transfer coefficient. However, there is an essential difference between T_m and T_∞. Whereas T_∞ is constant in the flow direction, T_m must vary in this direction. That is, dT_m/dx is never zero if heat transfer is occurring. The value of T_m increases with x if heat transfer is from the surface to the fluid ($T_s > T_m$); it decreases with x if the opposite is true ($T_s < T_m$).

8.2.3 Fully Developed Conditions

Since the existence of convection heat transfer between the surface and the fluid dictates that the fluid temperature must continue to change with x, one might legitimately question whether fully developed thermal conditions can ever be reached. The situation is certainly different from the hydrodynamic case, for which $(\partial u/\partial x) = 0$ in the fully developed region. In contrast, if there is heat transfer, (dT_m/dx), as well as $(\partial T/\partial x)$ at any radius r, is not zero. Accordingly, the temperature profile $T(r)$ is

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continuously changing with x, and it would seem that a fully developed condition could never be reached. This apparent contradiction may be reconciled by working with a dimensionless form of the temperature.

Analyses may be simplified by working with dimensionless temperature differences, as for transient conduction (Chapter 5) and the energy conservation equation (Chapter 6). Introducing a dimensionless temperature difference of the form $(T_s - T)/(T_s - T_m)$, conditions for which this ratio becomes independent of x are known to exist [2]. That is, although the temperature profile $T(r)$ continues to change with x, the *relative* shape of the profile no longer changes and the flow is said to be *thermally fully developed.* The requirement for such a condition is formally stated as

$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r,x)}{T_s(x) - T_m(x)}\right]_{\text{fd},t} = 0 \tag{8.28}$$

where T_s is the tube surface temperature, T is the local fluid temperature, and T_m is the mean temperature of the fluid over the cross section of the tube.

The condition given by Equation 8.28 is eventually reached in a tube for which there is either a *uniform surface heat flux* (q_s'' is constant) *or a uniform surface temperature* (T_s is constant). These surface conditions arise in many engineering applications. For example, a constant surface heat flux would exist if the tube wall were heated electrically or if the outer surface were uniformly irradiated. In contrast, a constant surface temperature would exist if a phase change (due to boiling or condensation) were occurring at the outer surface. Note that it is impossible to *simultaneously* impose the conditions of constant surface heat flux and constant surface temperature. If q_s'' is constant, T_s must vary with x; conversely, if T_s is constant, q_s'' must vary with x.

Several important features of thermally developed flow may be inferred from Equation 8.28. Since the temperature ratio is independent of x, the derivative of this ratio with respect to r must also be independent of x. Evaluating this derivative at the tube surface (note that T_s and T_m are constants insofar as differentiation with respect to r is concerned), we then obtain

$$\frac{\partial}{\partial r}\left(\frac{T_s - T}{T_s - T_m}\right)\bigg|_{r=r_o} = \frac{-\partial T/\partial r\big|_{r=r_o}}{T_s - T_m} \neq f(x)$$

Substituting for $\partial T/\partial r$ from Fourier's law, which, from Figure 8.4, is of the form

$$q_s'' = -k\frac{\partial T}{\partial y}\bigg|_{y=0} = k\frac{\partial T}{\partial r}\bigg|_{r=r_o}$$

and for q_s'' from Newton's law of cooling, Equation 8.27, we obtain

$$\frac{h}{k} \neq f(x) \tag{8.29}$$

Hence *in the thermally fully developed flow* of a fluid *with constant properties,* the *local convection coefficient is a constant, independent of x.*

Equation 8.28 is not satisfied in the entrance region, where h varies with x, as shown in Figure 8.5. Because the thermal boundary layer thickness is zero at the tube entrance, the convection coefficient is extremely large at $x = 0$. However, h decays rapidly as the thermal boundary layer develops, until the constant value associated with fully developed conditions is reached.

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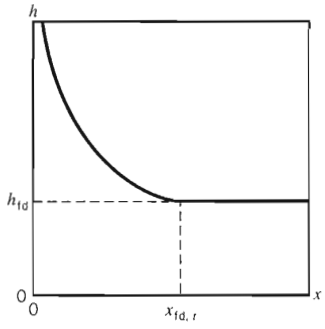


FIGURE 8.5
Axial variation of the convection heat transfer coefficient for flow in a tube.

Additional simplifications are associated with the special case of *uniform surface heat flux*. Since both h and q_s'' are constant in the fully developed region, it follows from Equation 8.27 that

$$\left.\frac{dT_s}{dx}\right|_{fd,t} = \left.\frac{dT_m}{dx}\right|_{fd,t} \qquad q_s'' = \text{constant} \tag{8.30}$$

If we expand Equation 8.28 and solve for $\partial T/\partial x$, it also follows that

$$\left.\frac{\partial T}{\partial x}\right|_{fd,t} = \left.\frac{dT_s}{dx}\right|_{fd,t} - \frac{(T_s - T)}{(T_s - T_m)}\left.\frac{dT_s}{dx}\right|_{fd,t} + \frac{(T_s - T)}{(T_s - T_m)}\left.\frac{dT_m}{dx}\right|_{fd,t} \tag{8.31}$$

Substituting from Equation 8.30, we then obtain

$$\left.\frac{\partial T}{\partial x}\right|_{fd,t} = \left.\frac{dT_m}{dx}\right|_{fd,t} \qquad q_s'' = \text{constant} \tag{8.32}$$

Hence the axial temperature gradient is independent of the radial location. For the case of *constant surface temperature* ($dT_s/dx = 0$), it also follows from Equation 8.31 that

$$\left.\frac{\partial T}{\partial x}\right|_{fd,t} = \left.\frac{(T_s - T)}{(T_s - T_m)}\frac{dT_m}{dx}\right|_{fd,t} \qquad T_s = \text{constant} \tag{8.33}$$

in which case the value of $\partial T/\partial x$ depends on the radial coordinate.

From the foregoing results, it is evident that the mean temperature is a very important variable for internal flows. To describe such flows, its variation with x must be known. This variation may be obtained by applying an *overall energy balance* to the flow, as will be shown in the next section.

EXAMPLE 8.1

For flow of a liquid metal through a circular tube, the velocity and temperature profiles at a particular axial location may be approximated as being uniform and parabolic, respectively. That is, $u(r) = C_1$ and $T(r) - T_s = C_2[1 - (r/r_o)^2]$, where C_1 and C_2 are constants. What is the value of the Nusselt number Nu_D at this location?

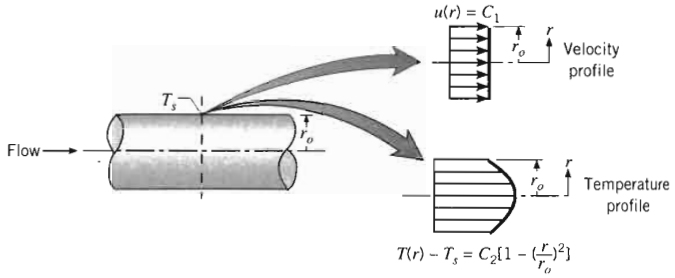
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SOLUTION

Known: Form of the velocity and temperature profiles at a particular axial location for flow in a circular tube.

Find: Nusselt number at the prescribed location.

Schematic:



Assumptions: Incompressible, constant property flow.

Analysis: The Nusselt number may be obtained by first determining the convection coefficient, which, from Equation 8.27, is given as

$$h = \frac{q_s''}{T_s - T_m}$$

From Equation 8.26, the mean temperature is

$$T_m = \frac{2}{u_m r_o^2}\int_0^{r_o} uTr\,dr = \frac{2C_1}{u_m r_o^2}\int_0^{r_o}\left\{T_s + C_2\left[1 - \left(\frac{r}{r_o}\right)^2\right]\right\} r\,dr$$

or, since $u_m = C_1$ from Equation 8.8,

$$T_m = \frac{2}{r_o^2}\int_0^{r_o}\left\{T_s + C_2\left[1 - \left(\frac{r}{r_o}\right)^2\right]\right\} r\,dr$$

$$T_m = \frac{2}{r_o^2}\left[T_s\frac{r^2}{2} + C_2\frac{r^2}{2} - \frac{C_2}{4}\frac{r^4}{r_o^2}\right]\Bigg|_0^{r_o}$$

$$T_m = \frac{2}{r_o^2}\left(T_s\frac{r_o^2}{2} + \frac{C_2}{2}r_o^2 - \frac{C_2}{4}r_o^2\right) = T_s + \frac{C_2}{2}$$

The heat flux may be obtained from Fourier's law, in which case

$$q_s'' = k\frac{\partial T}{\partial r}\bigg|_{r=r_o} = -kC_2 2\frac{r}{r_o^2}\bigg|_{r=r_o} = -2C_2\frac{k}{r_o}$$

Hence

$$h = \frac{q_s''}{T_s - T_m} = \frac{-2C_2(k/r_o)}{-C_2/2} = \frac{4k}{r_o}$$

8.3 The Energy B

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