

Digital Communications

Fourth Edition

John G. Proakis

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Fourth Edition

Proakis



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ISBN 0-07-232111-3



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DIGITAL COMMUNICATIONS

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This book is printed on acid-free paper.

4 5 6 7 8 9 0 DOC/DOC 0 9 8 7 6 5 4 3

ISBN 0-07-232111-3

Publisher: *Thomas Casson*
Sponsoring editor: *Catherine Fields Shultz*
Developmental editor: *Emily J. Gray*
Marketing manager: *John Wannemacher*
Project manager: *Craig S. Leonard*
Production supervisor: *Rose Hepburn*
Senior designer: *Kiera Cunningham*
New media: *Christopher Styles*
Compositor: *Interactive Composition Corporation*
Typeface: *10.5/12 Times Roman*
Printer: *Quebecor World Fairfield, Inc.*

Library of Congress Cataloging-in-Publication Data

Proakis, John G.

Digital communications / John G. Proakis.—4th ed.
p. cm.

ISBN 0-07-232111-3

1. Digital communications. I. Title.

TK5103.7.P76 2000

621.382—dc21

00-025305

www.mhhe.com

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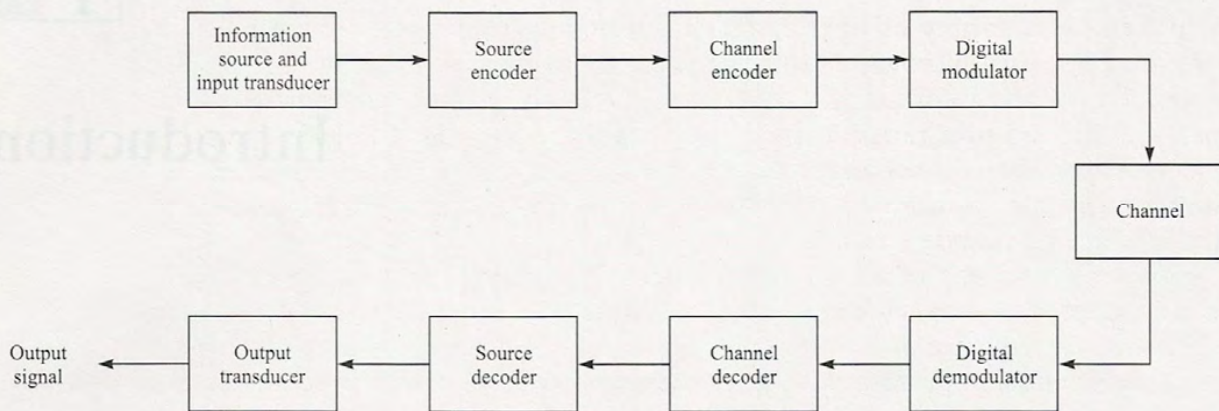


FIGURE 1.1-1

Basic elements of a digital communication system.

through the channel. Thus, the added redundancy serves to increase the reliability of the received data and improves the fidelity of the received signal. In effect, redundancy in the information sequence aids the receiver in decoding the desired information sequence. For example, a (trivial) form of encoding of the binary information sequence is simply to repeat each binary digit m times, where m is some positive integer. More sophisticated (nontrivial) encoding involves taking k information bits at a time and mapping each k -bit sequence into a unique n -bit sequence, called a *code word*. The amount of redundancy introduced by encoding the data in this manner is measured by the ratio n/k . The reciprocal of this ratio, namely k/n , is called the rate of the code or, simply, the *code rate*.

The binary sequence at the output of the channel encoder is passed to the *digital modulator*, which serves as the interface to the communication channel. Since nearly all the communication channels encountered in practice are capable of transmitting electrical signals (waveforms), the primary purpose of the digital modulator is to map the binary information sequence into signal waveforms. To elaborate on this point, let us suppose that the coded information sequence is to be transmitted one bit at a time at some uniform rate R bits per second (bits/s). The digital modulator may simply map the binary digit 0 into a waveform $s_0(t)$ and the binary digit 1 into a waveform $s_1(t)$. In this manner, each bit from the channel encoder is transmitted separately. We call this *binary modulation*. Alternatively, the modulator may transmit b coded information bits at a time by using $M = 2^b$ distinct waveforms $s_i(t)$, $i = 0, 1, \dots, M - 1$, one waveform for each of the 2^b possible b -bit sequences. We call this *M-ary modulation* ($M > 2$). Note that a new b -bit sequence enters the modulator every b/R seconds. Hence, when the channel bit rate R is fixed, the amount of time available to transmit one of the M waveforms corresponding to a b -bit sequence is b times the time period in a system that uses binary modulation.

The *communication channel* is the physical medium that is used to send the signal from the transmitter to the receiver. In wireless transmission, the channel may be the atmosphere (free space). On the other hand, telephone channels

When the mapping from the digital sequence $\{a_n\}$ to waveforms is performed under the constraint that a waveform transmitted in any time interval depends on one or more previously transmitted waveforms, the modulator is said to have *memory*. On the other hand, when the mapping from the sequence $\{a_n\}$ to the waveforms $\{s_m(t)\}$ is performed without any constraint on previously transmitted waveforms, the modulator is called *memoryless*.

In addition to classifying the modulator as either memoryless or having memory, we may classify it as either *linear* or *non-linear*. Linearity of a modulation method requires that the principle of superposition applies in the mapping of the digital sequence into successive waveforms. In non-linear modulation, the superposition principle does not apply to signals transmitted in successive time intervals. We shall begin by describing memoryless modulation methods.

4.3.1 Memoryless Modulation Methods

As indicated above, the modulator in a digital communication system maps a sequence of binary digits into a set of corresponding signal waveforms. These waveforms may differ in either amplitude or in phase or in frequency, or some combination of two or more signal parameters. We consider each of these signal types separately, beginning with digital pulse amplitude modulation (PAM). In all cases, we assume that the sequence of binary digits at the input to the modulator occurs at a rate of R bits/s.

Pulse-amplitude-modulated (PAM) signals. In digital PAM, the signal waveforms may be represented as

$$\begin{aligned} s_m(t) &= \text{Re}[A_m g(t) e^{j2\pi f_c t}] \\ &= A_m g(t) \cos 2\pi f_c t, \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T \end{aligned} \quad (4.3-1)$$

where $\{A_m, 1 \leq m \leq M\}$ denote the set of M possible amplitudes corresponding to $M = 2^k$ possible k -bit blocks of *symbols*. The signal amplitudes A_m take the discrete values (levels)

$$A_m = (2m - 1 - M)d, \quad m = 1, 2, \dots, M \quad (4.3-2)$$

where $2d$ is the ^{difference} distance between adjacent signal amplitudes. The waveform $g(t)$ is a real-valued signal pulse whose shape influences the spectrum of the transmitted signal, as we shall observe later. The symbol rate for the PAM signal is R/k . This is the rate at which changes occur in the amplitude of the carrier to reflect the transmission of new information. The time interval $T_b = 1/R$ is called the *bit interval* and the time interval $T = k/R = kT_b$ is called the *symbol interval*.

The M PAM signals have energies

$$\begin{aligned}\mathcal{E}_m &= \int_0^T s_m^2(t) dt \\ &= \frac{1}{2} A_m^2 \int_0^T g^2(t) dt \\ &= \frac{1}{2} A_m^2 \mathcal{E}_g\end{aligned}\quad (4.3-3)$$

where $\mathcal{E}_g$ denotes the energy in the pulse $g(t)$. Clearly, these signals are one-dimensional ($N = 1$), and, hence, are represented by the general form

$$s_m(t) = s_m f(t) \quad (4.3-4)$$

where $f(t)$ is defined as the unit-energy signal waveform given as

$$f(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos 2\pi f_c t \quad (4.3-5)$$

and

$$s_m = A_m \sqrt{\frac{1}{2} \mathcal{E}_g}, \quad m = 1, 2, \dots, M \quad (4.3-6)$$

The corresponding signal space diagrams for $M = 2$, $M = 4$, and $M = 8$ are shown in Figure 4.3-1. Digital PAM is also called *amplitude-shift keying* (ASK).

The mapping or assignment of k information bits to the $M = 2^k$ possible signal amplitudes may be done in a number of ways. The preferred assignment is one in which the adjacent signal amplitudes differ by one binary digit as illustrated in Figure 4.3-1. This mapping is called *Gray encoding*. It is important in the demodulation of the signal because the most likely errors caused by noise involve the erroneous selection of an adjacent amplitude to the transmitted signal amplitude. In such a case, only a single bit error occurs in the k -bit sequence.

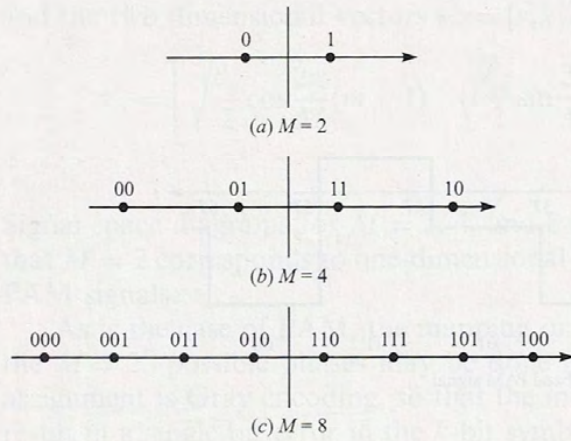
We note that the Euclidean distance between any pair of signal points is

$$\begin{aligned}d_{mn}^{(e)} &= \sqrt{(s_m - s_n)^2} \\ &= \sqrt{\frac{1}{2} \mathcal{E}_g} |A_m - A_n| \\ &= d \sqrt{2 \mathcal{E}_g} |m - n|\end{aligned}\quad (4.3-7)$$

Hence, the distance between a pair of adjacent signal points, i.e., the minimum Euclidean distance, is

$$d_{\min}^{(e)} = d \sqrt{2 \mathcal{E}_g} \quad (4.3-8)$$

The carrier-modulated PAM signal represented by Equation 4.3-1 is a double-sideband (DSB) signal and requires twice the channel bandwidth of the equivalent low-pass signal for transmission. Alternatively, we may use single-sideband (SSB) PAM, which has the representation (lower or upper sideband):

**FIGURE 4.3-1**

Signal space diagram for digital PAM signals.

$$s_m(t) = \text{Re}\{A_m[g(t) \pm j\hat{g}(t)]e^{j2\pi f_c t}\}, \quad m = 1, 2, \dots, M \quad (4.3-9)$$

where $\hat{g}(t)$ is the Hilbert transform of $g(t)$. Thus, the bandwidth of the SSB signal is half that of the DSB signal.

The digital PAM signal is also appropriate for transmission over a channel that does not require carrier modulation. In this case, the signal waveform may be simply represented as

$$s_m(t) = A_m g(t), \quad m = 1, 2, \dots, M \quad (4.3-10)$$

This is now called a *baseband* signal. For example a four-amplitude level baseband PAM signal is illustrated in Figure 4.3-2a. The carrier-modulated version of the signal is shown in Figure 4.3-2b.

In the special case of $M = 2$ signals, the binary PAM waveforms have the special property that

$$s_1(t) = -s_2(t)$$

Hence, these two signals have the same energy and a cross-correlation coefficient of -1 . Such signals are called *antipodal*.

➤ **Phase-modulated signals.** In digital phase modulation, the M signal waveforms are represented as

$$\begin{aligned} s_m(t) &= \text{Re}[g(t)e^{j2\pi(m-1)/M}e^{j2\pi f_c t}], \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T \\ &= g(t) \cos\left[2\pi f_c t + \frac{2\pi}{M}(m-1)\right] \\ &= g(t) \cos \frac{2\pi}{M}(m-1) \cos 2\pi f_c t - g(t) \sin \frac{2\pi}{M}(m-1) \sin 2\pi f_c t \end{aligned} \quad (4.3-11)$$

where $g(t)$ is the signal pulse shape and $\theta_m = 2\pi(m-1)/M$, $m = 1, 2, \dots, M$, are the M possible phases of the carrier that convey the transmitted information. Digital phase modulation is usually called phase-shift keying (PSK).

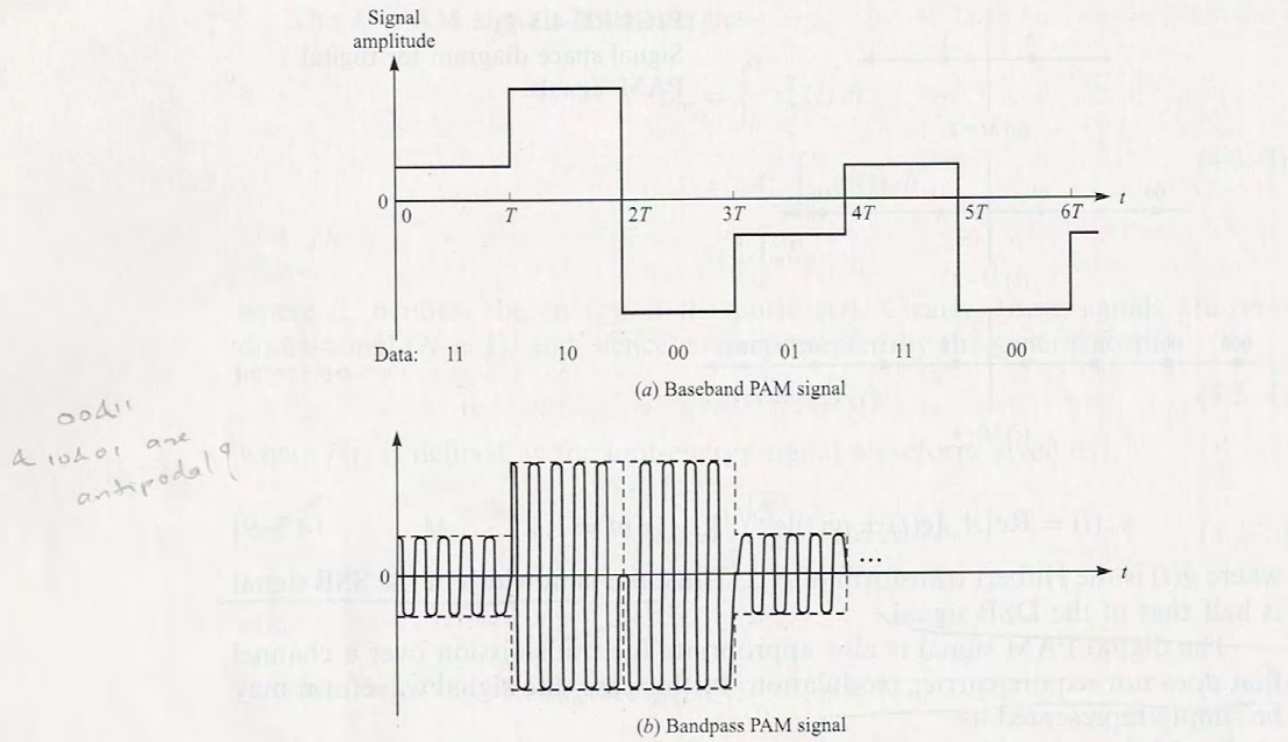


FIGURE 4.3-2
Baseband and band-pass PAM signals.

We note that these signal waveforms have equal energy, i.e.,

$$\begin{aligned}\mathcal{E} &= \int_0^T s_m^2(t) dt \\ &= \frac{1}{2} \int_0^T g^2(t) dt = \frac{1}{2} \mathcal{E}_g\end{aligned}\quad (4.3-12)$$

Furthermore, the signal waveforms may be represented as a linear combination of two orthonormal signal waveforms, $f_1(t)$ and $f_2(t)$, i.e.,

$$s_m(t) = s_{m1}f_1(t) + s_{m2}f_2(t) \quad (4.3-13)$$

where

$$f_1(t) = \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos 2\pi f_c t \quad (4.3-14)$$

$$f_2(t) = -\sqrt{\frac{2}{\mathcal{E}_g}} g(t) \sin 2\pi f_c t \quad (4.3-15)$$

and the two-dimensional vectors $\mathbf{s}_m = [s_{m1} \ s_{m2}]$ are given by

$$\mathbf{s}_m = \left[\sqrt{\frac{\mathcal{E}_g}{2}} \cos \frac{2\pi}{M}(m-1) \quad \sqrt{\frac{\mathcal{E}_g}{2}} \sin \frac{2\pi}{M}(m-1) \right], \quad m = 1, 2, \dots, M \quad (4.3-16)$$

Signal space diagrams for $M = 2, 4$, and 8 are shown in Figure 4.3-3. We note that $M = 2$ corresponds to one-dimensional signals, which are identical to binary PAM signals.

As is the case of PAM, the mapping or assignment of k information bits to the $M = 2^k$ possible phases may be done in a number of ways. The preferred assignment is Gray encoding, so that the most likely errors caused by noise will result in a single bit error in the k -bit symbol.

The Euclidean distance between signal points is

$$\begin{aligned} d_{mn}^{(e)} &= \|\mathbf{s}_m - \mathbf{s}_n\| \\ &= \left\{ \mathcal{E}_g \left[1 - \cos \frac{2\pi}{M}(m-n) \right] \right\}^{1/2} \end{aligned} \quad (4.3-17)$$

The minimum Euclidean distance corresponds to the case in which $|m-n| = 1$, i.e., adjacent signal phases. In this case,

$$d_{\min}^{(e)} = \sqrt{\mathcal{E}_g \left(1 - \cos \frac{2\pi}{M} \right)} \quad (4.3-18)$$

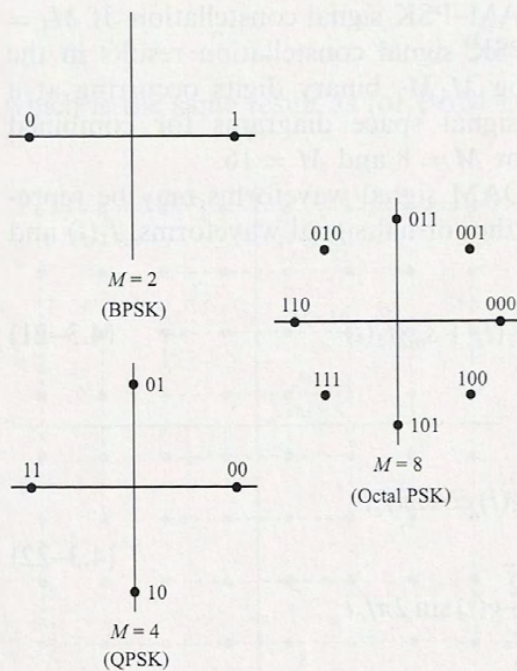


FIGURE 4.3-3

Signal space diagrams for PSK signals.

A variant of 4-phase PSK (QPSK), called $\pi/4$ -QPSK is obtained by introducing an additional $\pi/4$ phase shift in the carrier phase in each symbol interval. This phase shift facilitates symbol synchronization.

Quadrature amplitude modulation. The bandwidth efficiency of PAM/SSB can also be obtained by simultaneously impressing two separate k -bit symbols from the information sequence $\{a_n\}$ on two quadrature carriers $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$. The resulting modulation technique is called quadrature PAM or QAM, and the corresponding signal waveforms may be expressed as

$$\begin{aligned} s_m(t) &= \text{Re}[(A_{mc} + jA_{ms})g(t)e^{j2\pi f_c t}], \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T \\ &= A_{mc}g(t) \cos 2\pi f_c t - A_{ms}g(t) \sin 2\pi f_c t \end{aligned} \quad (4.3-19)$$

where A_{mc} and A_{ms} are the information-bearing signal amplitudes of the quadrature carriers and $g(t)$ is the signal pulse.

Alternatively, the QAM signal waveforms may be expressed as

$$\begin{aligned} s_m(t) &= \text{Re}[V_m e^{j\theta_m} g(t) e^{j2\pi f_c t}] \\ &= V_m g(t) \cos(2\pi f_c t + \theta_m) \end{aligned} \quad (4.3-20)$$

where $V_m = \sqrt{A_{mc}^2 + A_{ms}^2}$ and $\theta_m = \tan^{-1}(A_{ms}/A_{mc})$. From this expression, it is apparent that the QAM signal waveforms may be viewed as combined amplitude and phase modulation.

In fact, we may select any combination of M_1 -level PAM and M_2 -phase PSK to construct an $M = M_1 M_2$ combined PAM-PSK signal constellation. If $M_1 = 2^n$ and $M_2 = 2^m$, the combined PAM-PSK signal constellation results in the simultaneous transmission of $m + n = \log M_1 M_2$ binary digits occurring at a symbol rate $R/(m + n)$. Examples of signal space diagrams for combined PAM-PSK are shown in Figure 4.3-4, for $M = 8$ and $M = 16$.

As in the case of PSK signals, the QAM signal waveforms may be represented as a linear combination of two orthonormal signal waveforms, $f_1(t)$ and $f_2(t)$, i.e.,

$$s_m(t) = s_{m1}f_1(t) + s_{m2}f_2(t) \quad (4.3-21)$$

where

$$\begin{aligned} f_1(t) &= \sqrt{\frac{2}{\mathcal{E}_g}} g(t) \cos 2\pi f_c t \\ f_2(t) &= -\sqrt{\frac{2}{\mathcal{E}_g}} g(t) \sin 2\pi f_c t \end{aligned} \quad (4.3-22)$$

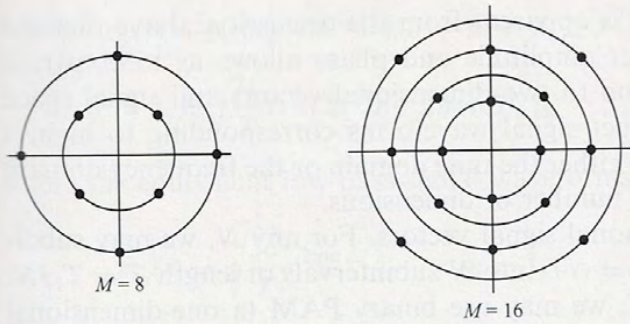


FIGURE 4.3-4
Examples of combined PAM-
PSK signal space diagrams.

and

$$\begin{aligned} \mathbf{s}_m &= [s_{m1} \quad s_{m2}] \\ &= [A_{mc}\sqrt{\frac{1}{2}\mathcal{E}_g} \quad A_{ms}\sqrt{\frac{1}{2}\mathcal{E}_g}] \end{aligned} \quad (4.3-23)$$

where $\mathcal{E}_g$ is the energy of the signal pulse $g(t)$.

The Euclidean distance between any pair of signal vectors is

$$\begin{aligned} d_{mn}^{(e)} &= \|\mathbf{s}_m - \mathbf{s}_n\| \\ &= \sqrt{\frac{1}{2}\mathcal{E}_g[(A_{mc} - A_{nc})^2 + (A_{ms} - A_{ns})^2]} \end{aligned} \quad (4.3-24)$$

In the special case where the signal amplitudes take the set of discrete values $\{(2m-1-M)d, m=1, 2, \dots, M\}$, the signal space diagram is rectangular, as shown in Figure 4.3-5. In this case, the Euclidean distance between adjacent points, i.e., the minimum distance, is

$$d_{\min}^{(e)} = d\sqrt{2\mathcal{E}_g} \quad (4.3-25)$$

which is the same result as for PAM.

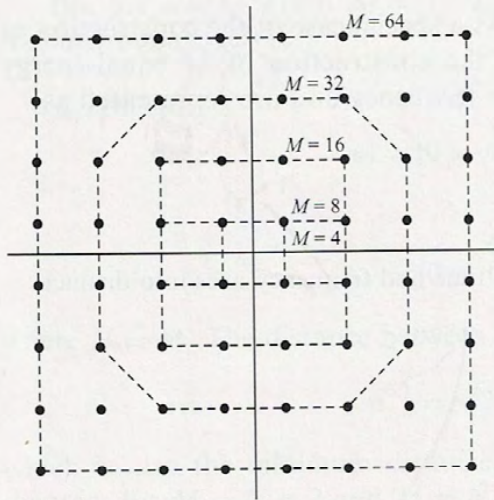


FIGURE 4.3-5
Several signal space diagrams for
rectangular QAM.

Multidimensional signals. It is apparent from the discussion above that the digital modulation of the carrier amplitude and phase allows us to construct signal waveforms that correspond to two-dimensional vectors and signal space diagrams. If we wish to construct signal waveforms corresponding to higher-dimensional vectors, we may use either the time domain or the frequency domain or both in order to increase the number of dimensions.

Suppose we have N -dimensional signal vectors. For any N , we may subdivide a time interval of length $T_1 = NT$ into N subintervals of length $T = T_1/N$. In each subinterval of length T , we may use binary PAM (a one-dimensional signal) to transmit an element of the N -dimensional signal vector. Thus, the N time slots are used to transmit the N -dimensional signal vector. If N is even, a time slot of length T may be used to simultaneously transmit two components of the N -dimensional vector by modulating the amplitude of quadrature carriers independently by the corresponding components. In this manner, the N -dimensional signal vector is transmitted in $\frac{1}{2}NT$ seconds ($\frac{1}{2}N$ time slots).

Alternatively, a frequency band of width $N\Delta f$ may be subdivided into N frequency slots each of width Δf . An N -dimensional signal vector can be transmitted over the channel by simultaneously modulating the amplitude of N carriers, one in each of the N frequency slots. Care must be taken to provide sufficient frequency separation Δf between successive carriers so that there is no cross-talk interference among the signals on the N carriers. If quadrature carriers are used in each frequency slot, the N -dimensional vector (even N) may be transmitted in $\frac{1}{2}N$ frequency slots, thus reducing the channel bandwidth utilization by a factor of 2.

More generally, we may use both the time and frequency domains jointly to transmit an N -dimensional signal vector. For example, Figure 4.3-6 illustrates a subdivision of the time and frequency axes into 12 slots. Thus, an $N = 12$ -dimensional signal vector may be transmitted by PAM or an $N = 24$ -dimensional signal vector may be transmitted by use of two quadrature carriers (QAM) in each slot.

Orthogonal multidimensional signals. As a special case of the construction of multidimensional signals, let us consider the construction of M equal-energy orthogonal signal waveforms that differ in frequency and are represented as

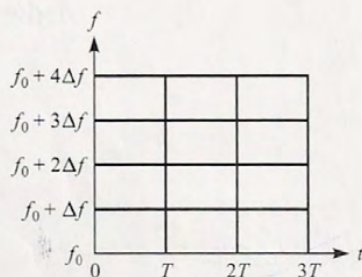


FIGURE 4.3-6

Subdivision of time and frequency axes into distinct slots.

$$\begin{aligned}
 s_m(t) &= \text{Re}[s_{lm}(t)e^{j2\pi f_c t}], \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T \\
 &= \sqrt{\frac{2\mathcal{E}}{T}} \cos[2\pi f_c t + 2\pi m \Delta f t]
 \end{aligned} \tag{4.3-26}$$

where the equivalent low-pass signal waveforms are defined as

$$s_{lm}(t) = \sqrt{\frac{2\mathcal{E}}{T}} e^{j2\pi m \Delta f t}, \quad m = 1, 2, \dots, M, \quad 0 \leq t \leq T \tag{4.3-27}$$

This type of frequency modulation is called *frequency-shift keying* (FSK).

These waveforms are characterized as having equal energy and cross-correlation coefficients

$$\begin{aligned}
 \rho_{km} &= \frac{2\mathcal{E}/T}{2\mathcal{E}} \int_0^T e^{j2\pi(m-k)\Delta f t} dt \\
 &= \frac{\sin \pi T(m-k)\Delta f}{\pi T(m-k)\Delta f} e^{j\pi T(m-k)\Delta f}
 \end{aligned} \tag{4.3-28}$$

The real part of ρ_{km} is

$$\begin{aligned}
 \rho_r &\equiv \text{Re}(\rho_{km}) = \frac{\sin[\pi T(m-k)\Delta f]}{\pi T(m-k)\Delta f} \cos[\pi T(m-k)\Delta f] \\
 &= \frac{\sin[2\pi T(m-k)\Delta f]}{2\pi T(m-k)\Delta f}
 \end{aligned} \tag{4.3-29}$$

First, we observe that $\text{Re}(\rho_{km}) = 0$ when $\Delta f = 1/2T$ and $m \neq k$. Since $|m - k| = 1$ corresponds to adjacent frequency slots, $\Delta f = 1/2T$ represents the minimum frequency separation between adjacent signals for orthogonality of the M signals. Plots of $\text{Re}(\rho_{km})$ versus Δf and $|\rho_{km}|$ versus Δf are shown in Figure 4.3-7. Note that $|\rho_{km}| = 0$ for multiples of $1/T$ whereas $\text{Re}(\rho_{km}) = 0$ for multiples of $1/2T$.

For the case in which $\Delta f = 1/2T$, the M FSK signals are equivalent to the N -dimensional vectors

$$\begin{aligned}
 \mathbf{s}_1 &= [\sqrt{\mathcal{E}} \quad 0 \quad 0 \quad \dots \quad 0 \quad 0] \\
 \mathbf{s}_2 &= [0 \quad \sqrt{\mathcal{E}} \quad 0 \quad \dots \quad 0 \quad 0] \\
 &\vdots \\
 \mathbf{s}_N &= [0 \quad 0 \quad 0 \quad \dots \quad 0, \sqrt{\mathcal{E}}]
 \end{aligned} \tag{4.3-30}$$

where $N = M$. The distance between pairs of signals is

$$d_{km}^{(e)} = \sqrt{2\mathcal{E}}, \quad \text{for all } m, k \tag{4.3-31}$$

which is also the minimum distance. Figure 4.3-8 illustrates the signal space diagram for $M = N = 2$ and $M = N = 3$.

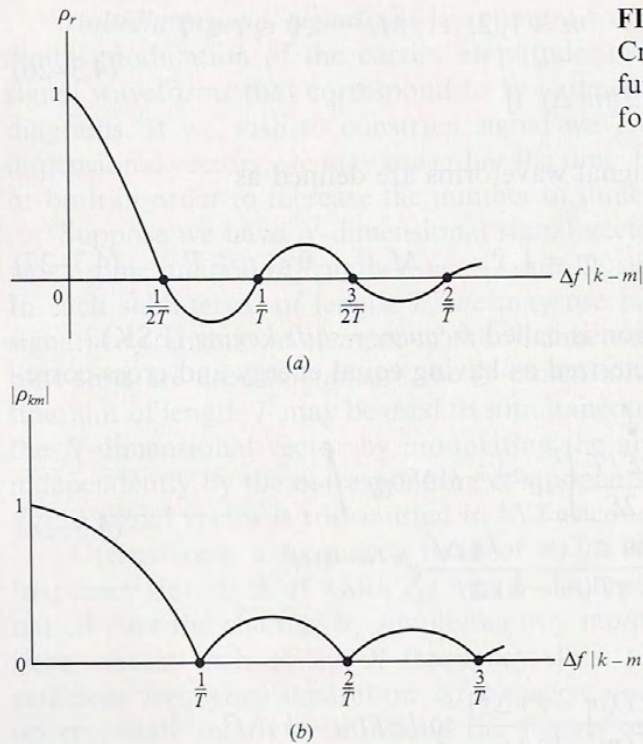


FIGURE 4.3-7
Cross-correlation coefficient as a function of frequency separation for FSK signals.

Biorthogonal Signals. A set of M biorthogonal signals can be constructed from $\frac{1}{2}M$ orthogonal signals by simply including the negatives of the orthogonal signals. Thus, we require $N = \frac{1}{2}M$ dimensions for the construction of a set of M biorthogonal signals. Figure 4.3-9 illustrates the biorthogonal signals for $M = 4$ and 6.

We note that the correlation between any pair of waveforms is either $\rho_r = -1$ or 0. The corresponding distances are $d = 2\sqrt{\mathcal{E}}$ or $\sqrt{2\mathcal{E}}$, with the latter being the minimum distance.

Simplex signals. Suppose we have a set of M orthogonal waveforms $\{s_m(t)\}$ or, equivalently, their vector representation $\{\mathbf{s}_m\}$. Their mean is

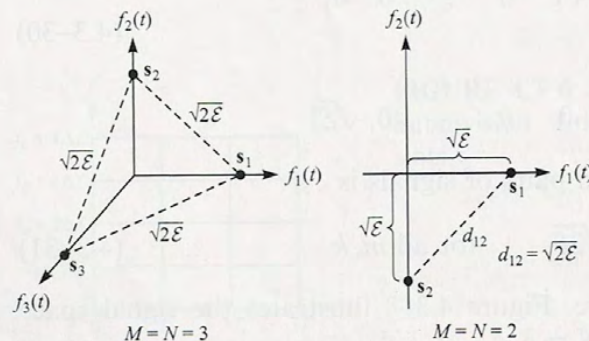
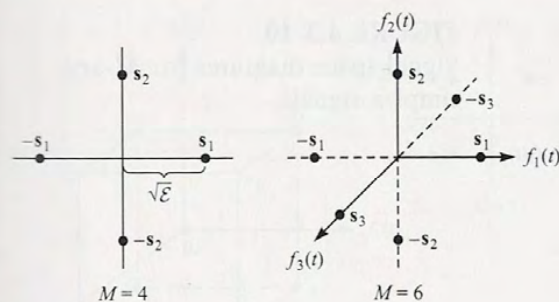


FIGURE 4.3-8
Orthogonal signals for $M = N = 3$ and $M = N = 2$.

**FIGURE 4.3-9**

Signal space diagrams for $M = 4$ and $M = 6$ biorthogonal signals.

$$\bar{s} = \frac{1}{M} \sum_{m=1}^M s_m \quad (4.3-32)$$

Now, let us construct another set of M signals by subtracting the mean from each of the M orthogonal signals. Thus,

$$s'_m = s_m - \bar{s}, \quad m = 1, 2, \dots, M \quad (4.3-33)$$

The effect of the subtraction is to translate the origin of the m orthogonal signals to the point $\bar{s}$.

The resulting signal waveforms are called *simplex signals* and have the following properties. First, the energy per waveform is

$$\begin{aligned} \|s'_m\|^2 &= \|s_m - \bar{s}\|^2 \\ &= \mathcal{E} - \frac{2}{M}\mathcal{E} + \frac{1}{M}\mathcal{E} \\ &= \mathcal{E}\left(1 - \frac{1}{M}\right) \end{aligned} \quad (4.3-34)$$

Second, the cross correlation of any pair of signals is

$$\begin{aligned} \text{Re}(\rho_{mn}) &= \frac{s'_m \cdot s'_n}{\|s'_m\| \|s'_n\|} \\ &= \frac{-1/M}{1 - 1/M} = -\frac{1}{M-1} \end{aligned} \quad (4.3-35)$$

for all m, n . Hence, the set of simplex waveforms is *equally correlated* and requires less energy, by the factor $1 - 1/M$, than the set of orthogonal waveforms. Since only the origin was translated, the distance between any pair of signal points is maintained at $d = \sqrt{2\mathcal{E}}$, which is the same as the distance between any pair of orthogonal signals.

Figure 4.3-10 illustrates the simplex signals for $M = 2, 3$, and 4 . Note that the signal dimensionality is $N = M - 1$.

Signal waveforms from binary codes. A set of M signaling waveforms can be generated from a set of M binary code words of the form

$$C_m = [c_{m1} \ c_{m2} \ \dots \ c_{mN}], \quad m = 1, 2, \dots, M \quad (4.3-36)$$

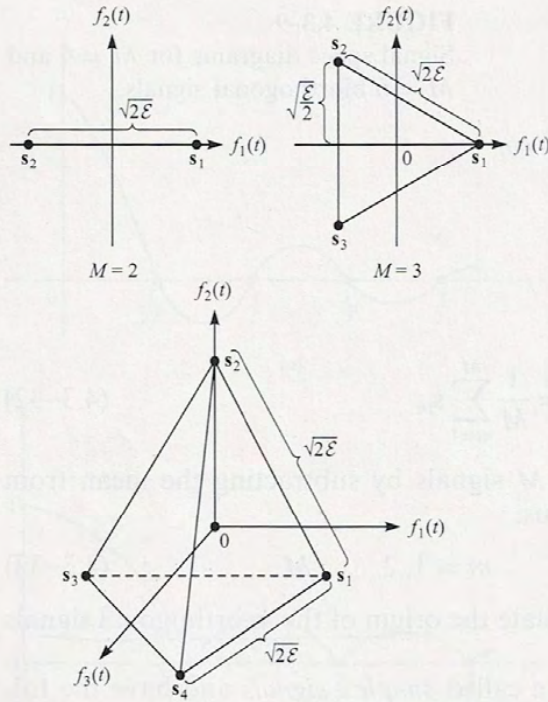


FIGURE 4.3-10
Signal space diagrams for M -ary
simplex signals.

where $c_{mj} = 0$ or 1 for all m and j . Each component of a code word is mapped into an elementary binary PSK waveform as follows:

$$\begin{aligned} c_{mj} = 1 &\Rightarrow s_{mj}(t) = \sqrt{\frac{2\mathcal{E}_c}{T_c}} \cos 2\pi f_c t, & 0 \leq t \leq T_c \\ c_{mj} = 0 &\Rightarrow s_{mj}(t) = -\sqrt{\frac{2\mathcal{E}_c}{T_c}} \cos 2\pi f_c t, & 0 \leq t \leq T_c \end{aligned} \quad (4.3-37)$$

where $T_c = T/N$ and $\mathcal{E}_c = \mathcal{E}/N$. Thus, the M code words $\{C_m\}$ are mapped into a set of M waveforms $\{s_m(t)\}$.

The waveforms can be represented in vector form as

$$\mathbf{s}_m = [s_{m1} \ s_{m2} \ \cdots \ s_{mN}], \quad m = 1, 2, \dots, M \quad (4.3-38)$$

where $s_{mj} = \pm\sqrt{\mathcal{E}/N}$ for all m and j . N is called the block length of the code, and it is also the dimension of the M waveforms.

We note that there are 2^N possible waveforms that can be constructed from the 2^N possible binary code words. We may select a subset of $M < 2^N$ signal waveforms for transmission of the information. We also observe that the 2^N possible signal points correspond to the vertices of an N -dimensional hypercube with its center at the origin. Figure 4.3-11 illustrates the signal points in $N = 2$ and 3 dimensions.

Each of the M waveforms has energy $\mathcal{E}$. The cross correlation between any pair of waveforms depends on how we select the M waveforms from the 2^N

On the other hand, four-phase DPSK is approximately 2.3 dB poorer in performance than four-phase PSK at large SNR. Consequently the choice between these two four-phase systems is not as clear-cut. One must weigh the 2.3 dB loss against the reduction in implementation complexity.

5.2.9 Probability of Error for QAM

Recall from Section 4.3 that QAM signal waveforms may be expressed as

$$s_m(t) = A_{mc}g(t)\cos 2\pi f_c t - A_{ms}g(t)\sin 2\pi f_c t, \quad 0 \leq t \leq T \quad (5.2-72)$$

where A_{mc} and A_{ms} are the information-bearing signal amplitudes of the quadrature carriers and $g(t)$ is the signal pulse. The vector representation of these waveforms is

$$\mathbf{s}_m = [A_{mc}\sqrt{\frac{1}{2}\mathcal{E}_g} \quad A_{ms}\sqrt{\frac{1}{2}\mathcal{E}_g}] \quad (5.2-73)$$

To determine the probability of error for QAM, we must specify the signal point constellation. We begin with QAM signal sets that have $M = 4$ points. Figure 5.2-14 illustrates two four-point signal sets. The first is a four-phase modulated signal and the second is a QAM signal with two amplitude levels, labeled A_1 and A_2 , and four phases. Because the probability of error is dominated by the minimum distance between pairs of signal points, let us impose the condition that $d_{\min}^{(e)} = 2A$ for both signal constellations and let us evaluate the average transmitter power, based on the premise that all signal points are equally probable. For the four-phase signal, we have

$$P_{av} = \frac{1}{4}(4)2A^2 = 2A^2 \quad (5.2-74)$$

For the two-amplitude, four-phase QAM, we place the points on circles of radii A and $\sqrt{3}A$. Thus, $d_{\min}^{(e)} = 2A$, and

$$P_{av} = \frac{1}{2}[2(3A^2) + 2A^2] = 2A^2 \quad (5.2-75)$$

which is the same average power as the $M = 4$ -phase signal constellation. Hence, for all practical purposes, the error rate performance of the two signal sets is the same. In other words, there is no advantage of the two-amplitude QAM signal set over $M = 4$ -phase modulation.

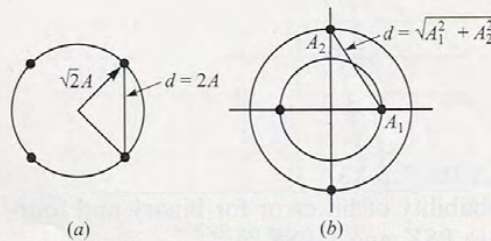


FIGURE 5.2-14
Two four-point signal constellations.

Next, let us consider $M = 8$ QAM. In this case, there are many possible signal constellations. We shall consider the four signal constellations shown in Figure 5.2-15, all of which consist of two amplitudes and have a minimum distance between signal points of $2A$. The coordinates (A_{mc}, A_{ms}) for each signal point, normalized by A , are given in the figure. Assuming that the signal points are equally probable, the average transmitted signal power is

$$\begin{aligned} P_{av} &= \frac{1}{M} \sum_{m=1}^M (A_{mc}^2 + A_{ms}^2) \\ &= \frac{A^2}{M} \sum_{m=1}^M (a_{mc}^2 + a_{ms}^2) \end{aligned} \quad (5.2-76)$$

where (a_{mc}, a_{ms}) are the coordinates of the signal points, normalized by A .

The two signal sets (a) and (c) in Figure 5.2-15 contain signal points that fall on a rectangular grid and have $P_{av} = 6A^2$. The signal set (b) requires an average transmitted power $P_{av} = 6.83A^2$, and (d) requires $P_{av} = 4.73A^2$. Therefore, the fourth signal set requires approximately 1 dB less power than the first two and 1.6 dB less power than the third to achieve the same probability of error. This signal constellation is known to be the best eight-point QAM constellation because it requires the least power for a given minimum distance between signal points.

For $M \geq 16$, there are many more possibilities for selecting the QAM signal points in the two-dimensional space. For example, we may choose a circular multi-amplitude constellation for $M = 16$, as shown in Figure 4.3-4. In this case, the signal points at a given amplitude level are phase-rotated by $\frac{1}{4}\pi$ relative

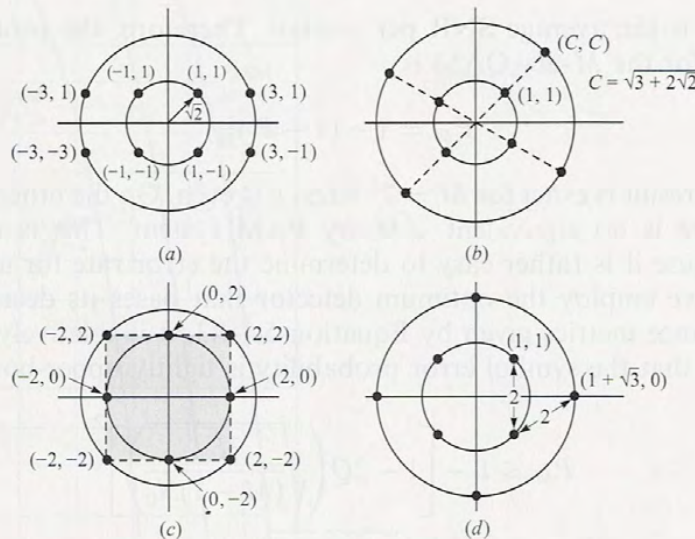


FIGURE 5.2-15
Four eight-point QAM signal constellations.

to the signal points at adjacent amplitude levels. This 16-QAM constellation is a generalization of the optimum 8-QAM constellation. However, the circular 16-QAM constellation is not the best 16-point QAM signal constellation for the AWGN channel.

Rectangular QAM signal constellations have the distinct advantage of being easily generated as two PAM signals impressed on phase-quadrature carriers. In addition, they are easily demodulated. Although they are not the best M -ary QAM signal constellations for $M \geq 16$, the average transmitted power required to achieve a given minimum distance is only slightly greater than the average power required for the best M -ary QAM signal constellation. For these reasons, rectangular M -ary QAM signals are most frequently used in practice.

For rectangular signal constellations in which $M = 2^k$ where k is even, the QAM signal constellation is equivalent to two PAM signals on quadrature carriers, each having $\sqrt{M} = 2^{k/2}$ signal points. Since the signals in the phase-quadrature components can be perfectly separated at the demodulator, the probability of error for QAM is easily determined from the probability of error for PAM. Specifically, the probability of a correct decision for the M -ary QAM system is

$$P_c = (1 - P_{\sqrt{M}})^2 \quad (5.2-77)$$

where $P_{\sqrt{M}}$ is the probability of error of an $\sqrt{M}$ -ary PAM with one-half the average power in each quadrature signal of the equivalent QAM system. By appropriately modifying the probability of error for M -ary PAM, we obtain

$$P_{\sqrt{M}} = 2 \left(1 - \frac{1}{\sqrt{M}} \right) Q \left(\sqrt{\frac{3}{M-1} \frac{\mathcal{E}_{av}}{N_0}} \right) \quad (5.2-78)$$

where $\mathcal{E}_{av}/N_0$ is the average SNR per symbol. Therefore, the probability of a symbol error for the M -ary QAM is

$$P_M = 1 - (1 - P_{\sqrt{M}})^2 \quad (5.2-79)$$

Note that this result is exact for $M = 2^k$ when k is even. On the other hand, when k is odd, there is no equivalent $\sqrt{M}$ -ary PAM system. This is no problem, however, because it is rather easy to determine the error rate for a rectangular signal set. If we employ the optimum detector that bases its decisions on the optimum distance metrics given by Equation 5.1-43, it is relatively straightforward to show that the symbol error probability is tightly upper-bounded as

$$\begin{aligned} P_M &\leq 1 - \left[1 - 2Q \left(\sqrt{\frac{3\mathcal{E}_{av}}{(M-1)N_0}} \right) \right]^2 \\ &\leq 4Q \left(\sqrt{\frac{3k\mathcal{E}_{bav}}{(M-1)N_0}} \right) \end{aligned} \quad (5.2-80)$$