Doc Code: TRACK1.REQ
Document Description: TrackOne Request

PTO/AIA/424 (04-14)

CERTIFICATION AND REQUEST FOR PRIORITIZED EXAMINATION UNDER 37 CFR 1.102(e) (Page 1 of 1)

First Named Inventor:	Maged F. Barsoum	Nonprovisional Application Number (if known):	
Title of Invention:	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations		

APPLICANT HEREBY CERTIFIES THE FOLLOWING AND REQUESTS PRIORITIZED EXAMINATION FOR THE ABOVE-IDENTIFIED APPLICATION.

1. The processing fee set forth in 37 CFR 1.17(i)(1) and the prioritized examination fee set forth in 37 CFR 1.17(c) have been filed with the request. The publication fee requirement is met because that fee, set forth in 37 CFR 1.18(d), is currently $0. The basic filing fee, search fee, and examination fee are filed with the request or have been already been paid. I understand that any required excess claims fees or application size fee must be paid for the application.

2. I understand that the application may not contain, or be amended to contain, more than four independent claims, more than thirty total claims, or any multiple dependent claims, and that any request for an extension of time will cause an outstanding Track I request to be dismissed.

3. The applicable box is checked below:

I. ☑ Original Application (Track One) - Prioritized Examination under § 1.102(e)(1)

i. (a) The application is an original nonprovisional utility application filed under 35 U.S.C. 111(a). This certification and request is being filed with the utility application via EFS-Web.

---OR---

(b) The application is an original nonprovisional plant application filed under 35 U.S.C. 111(a). This certification and request is being filed with the plant application in paper.

ii. An executed inventor's oath or declaration under 37 CFR 1.63 or 37 CFR 1.64 for each inventor, or the application data sheet meeting the conditions specified in 37 CFR 1.53(f)(3)(i) is filed with the application.

II. ☐ Request for Continued Examination - Prioritized Examination under § 1.102(e)(2)

i. A request for continued examination has been filed with, or prior to, this form.
ii. If the application is a utility application, this certification and request is being filed via EFS-Web.
iii. The application is an original nonprovisional utility application filed under 35 U.S.C. 111(a), or is a national stage entry under 35 U.S.C. 371.
iv. This certification and request is being filed prior to the mailing of a first Office action responsive to the request for continued examination.
v. No prior request for continued examination has been granted prioritized examination status under 37 CFR 1.102(e)(2).

Signature /David J. Bailey/	Date December 27, 2019
Name (Print/Typed) David J. Bailey	Practitioner Registration Number 59,632

Note: *This form must be signed in accordance with 37 CFR 1.33. See 37 CFR 1.4(d) for signature requirements and certifications. Submit multiple forms if more than one signature is required.**

☑ *Total of 1 forms are submitted.

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Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

The application data sheet is part of the provisional or nonprovisional application for which it is being submitted. The following form contains the bibliographic data arranged in a format specified by the United States Patent and Trademark Office as outlined in 37 CFR 1.76.
This document may be completed electronically and submitted to the Office in electronic format using the Electronic Filing System (EFS) or the document may be printed and included in a paper filed application.

Secrecy Order 37 CFR 5.2:

☐ Portions or all of the application associated with this Application Data Sheet may fall under a Secrecy Order pursuant to 37 CFR 5.2 (Paper filers only. Applications that fall under Secrecy Order may not be filed electronically.)

Inventor Information:

Inventor 1 Remove

Legal Name

Prefix	Given Name	Middle Name	Family Name	Suffix
	Maged	F.	Barsoum	

Residence Information (Select One) ◉ US Residency ○ Non US Residency ○ Active US Military Service

City	San Jose	State/Province	CA	Country of Residence	US

Mailing Address of Inventor:

Address 1	1468 Kinsport Lane		
Address 2			
City	San Jose	State/Province	CA
Postal Code	95120	Country	US

Inventor 2 Remove

Legal Name

Prefix	Given Name	Middle Name	Family Name	Suffix
	Christopher	R.	Jones	

Residence Information (Select One) ◉ US Residency ○ Non US Residency ○ Active US Military Service

City	Pacific Palisades	State/Province	CA	Country of Residence	US

Mailing Address of Inventor:

Address 1	1563 Calle Patricia		
Address 2			
City	Pacific Palisades	State/Province	CA
Postal Code	90272	Country	US

All Inventors Must Be Listed - Additional Inventor Information blocks may be generated within this form by selecting the **Add** button. Add

Correspondence Information:

Enter either Customer Number or complete the Correspondence Information section below.
For further information see 37 CFR 1.33(a).

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Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

☐ An Address is being provided for the correspondence Information of this application.			
Customer Number	71897		
Email Address	pair@kppb.com	Add Email	Remove Email

Application Information:

Title of the Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations		
Attorney Docket Number	C44-00999.CON15	**Small Entity Status Claimed**	☐
Application Type	Nonprovisional		
Subject Matter	Utility		
Total Number of Drawing Sheets (if any)	43	**Suggested Figure for Publication (if any)**	1

Filing By Reference:

Only complete this section when filing an application by reference under 35 U.S.C. 111(c) and 37 CFR 1.57(a). Do not complete this section if application papers including a specification and any drawings are being filed. Any domestic benefit or foreign priority information must be provided in the appropriate section(s) below (i.e., "Domestic Benefit/National Stage Information" and "Foreign Priority Information").

For the purposes of a filing date under 37 CFR 1.53(b), the description and any drawings of the present application are replaced by this reference to the previously filed application, subject to conditions and requirements of 37 CFR 1.57(a).

Application number of the previously filed application	Filing date (YYYY-MM-DD)	Intellectual Property Authority or Country

Publication Information:

☐ Request Early Publication (Fee required at time of Request 37 CFR 1.219)

☐ **Request Not to Publish.** I hereby request that the attached application not be published under 35 U.S.C. 122(b) and certify that the invention disclosed in the attached application **has not and will not** be the subject of an application filed in another country, or under a multilateral international agreement, that requires publication at eighteen months after filing.

Representative Information:

Representative information should be provided for all practitioners having a power of attorney in the application. Providing this information in the Application Data Sheet does not constitute a power of attorney in the application (see 37 CFR 1.32).
Either enter Customer Number or complete the Representative Name section below. If both sections are completed the customer Number will be used for the Representative Information during processing.

Please Select One:	◉ Customer Number	○ US Patent Practitioner	○ Limited Recognition (37 CFR 11.9)
Customer Number	71897		

Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

Domestic Benefit/National Stage Information:

This section allows for the applicant to either claim benefit under 35 U.S.C. 119(e), 120, 121, 365(c), or 386(c) or indicate National Stage entry from a PCT application. Providing benefit claim information in the Application Data Sheet constitutes the specific reference required by 35 U.S.C. 119(e) or 120, and 37 CFR 1.78.
When referring to the current application, please leave the "Application Number" field blank.

Prior Application Status	Pending		Remove
Application Number	Continuity Type	Prior Application Number	Filing or 371(c) Date (YYYY-MM-DD)
	Continuation of	16/206991	2018-11-30

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
16/206991	Continuation of	15/682475	2017-08-21	10149179	2018-12-04

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
15/682475	Continuation of	15/200800	2016-07-01	9743292	2017-08-22

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
15/200800	Continuation of	14/491731	2014-09-19	9385832	2016-07-05

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
14/491731	Continuation of	13/618630	2012-09-14	8842761	2014-09-23

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
13/618630	Continuation of	13/118921	2011-05-31	8270511	2012-09-18

Prior Application Status	Patented				Remove
Application Number	Continuity Type	Prior Application Number	Filing Date (YYYY-MM-DD)	Patent Number	Issue Date (YYYY-MM-DD)
13/118921	Continuation of	12/156989	2008-06-05	7978777	2011-07-12

Prior Application Status	Expired		Remove
Application Number	Continuity Type	Prior Application Number	Filing or 371(c) Date (YYYY-MM-DD)
12/156989	Claims benefit of provisional	60/933319	2007-06-05

Additional Domestic Benefit/National Stage Data may be generated within this form by selecting the **Add** button.

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Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

Foreign Priority Information:

This section allows for the applicant to claim priority to a foreign application. Providing this information in the application data sheet constitutes the claim for priority as required by 35 U.S.C. 119(b) and 37 CFR 1.55. When priority is claimed to a foreign application that is eligible for retrieval under the priority document exchange program (PDX)[i] the information will be used by the Office to automatically attempt retrieval pursuant to 37 CFR 1.55(i)(1) and (2). Under the PDX program, applicant bears the ultimate responsibility for ensuring that a copy of the foreign application is received by the Office from the participating foreign intellectual property office, or a certified copy of the foreign priority application is filed, within the time period specified in 37 CFR 1.55(g)(1).

Remove

Application Number	Country[i]	Filing Date (YYYY-MM-DD)	Access Code[i] (if applicable)

Additional Foreign Priority Data may be generated within this form by selecting the **Add** button.

Statement under 37 CFR 1.55 or 1.78 for AIA (First Inventor to File) Transition Applications

☐ This application (1) claims priority to or the benefit of an application filed before March 16, 2013 and (2) also contains, or contained at any time, a claim to a claimed invention that has an effective filing date on or after March 16, 2013.
NOTE: By providing this statement under 37 CFR 1.55 or 1.78, this application, with a filing date on or after March 16, 2013, will be examined under the first inventor to file provisions of the AIA.

Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

Authorization or Opt-Out of Authorization to Permit Access:

When this Application Data Sheet is properly signed and filed with the application, applicant has provided written authority to permit a participating foreign intellectual property (IP) office access to the instant application-as-filed (see paragraph A in subsection 1 below) and the European Patent Office (EPO) access to any search results from the instant application (see paragraph B in subsection 1 below).

Should applicant choose not to provide an authorization identified in subsection 1 below, applicant **must opt-out** of the authorization by checking the corresponding box A or B or both in subsection 2 below.

NOTE: This section of the Application Data Sheet is **ONLY** reviewed and processed with the **INITIAL** filing of an application. After the initial filing of an application, an Application Data Sheet cannot be used to provide or rescind authorization for access by a foreign IP office(s). Instead, Form PTO/SB/39 or PTO/SB/69 must be used as appropriate.

1. Authorization to Permit Access by a Foreign Intellectual Property Office(s)

A. Priority Document Exchange (PDX) - Unless box A in subsection 2 (opt-out of authorization) is checked, the undersigned hereby **grants the USPTO authority** to provide the European Patent Office (EPO), the Japan Patent Office (JPO), the Korean Intellectual Property Office (KIPO), the State Intellectual Property Office of the People's Republic of China (SIPO), the World Intellectual Property Organization (WIPO), and any other foreign intellectual property office participating with the USPTO in a bilateral or multilateral priority document exchange agreement in which a foreign application claiming priority to the instant patent application is filed, access to: (1) the instant patent application-as-filed and its related bibliographic data, (2) any foreign or domestic application to which priority or benefit is claimed by the instant application and its related bibliographic data, and (3) the date of filing of this Authorization. See 37 CFR 1.14(h)(1).

B. Search Results from U.S. Application to EPO - Unless box B in subsection 2 (opt-out of authorization) is checked, the undersigned hereby **grants the USPTO authority** to provide the EPO access to the bibliographic data and search results from the instant patent application when a European patent application claiming priority to the instant patent application is filed. See 37 CFR 1.14(h)(2).

The applicant is reminded that the EPO's Rule 141(1) EPC (European Patent Convention) requires applicants to submit a copy of search results from the instant application without delay in a European patent application that claims priority to the instant application.

2. Opt-Out of Authorizations to Permit Access by a Foreign Intellectual Property Office(s)

☐ A. Applicant **DOES NOT** authorize the USPTO to permit a participating foreign IP office access to the instant application-as-filed. If this box is checked, the USPTO will not be providing a participating foreign IP office with any documents and information identified in subsection 1A above.

☐ B. Applicant **DOES NOT** authorize the USPTO to transmit to the EPO any search results from the instant patent application. If this box is checked, the USPTO will not be providing the EPO with search results from the instant application.

NOTE: Once the application has published or is otherwise publicly available, the USPTO may provide access to the application in accordance with 37 CFR 1.14.

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Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

Applicant Information:

Providing assignment information in this section does not substitute for compliance with any requirement of part 3 of Title 37 of CFR to have an assignment recorded by the Office.

Applicant 1

If the applicant is the inventor (or the remaining joint inventor or inventors under 37 CFR 1.45), this section should not be completed. The information to be provided in this section is the name and address of the legal representative who is the applicant under 37 CFR 1.43; or the name and address of the assignee, person to whom the inventor is under an obligation to assign the invention, or person who otherwise shows sufficient proprietary interest in the matter who is the applicant under 37 CFR 1.46. If the applicant is an applicant under 37 CFR 1.46 (assignee, person to whom the inventor is obligated to assign, or person who otherwise shows sufficient proprietary interest) together with one or more joint inventors, then the joint inventor or inventors who are also the applicant should be identified in this section.

Clear

◉ Assignee	○ Legal Representative under 35 U.S.C. 117	○ Joint Inventor
○ Person to whom the inventor is obligated to assign.	○ Person who shows sufficient proprietary interest	

If applicant is the legal representative, indicate the authority to file the patent application, the inventor is:

Name of the Deceased or Legally Incapacitated Inventor:

If the Applicant is an Organization check here. ☒

Organization Name	Constellation Designs, LLC

Mailing Address Information For Applicant:

Address 1	2190 S. Towne Centre Place		
Address 2	Suite 300		
City	Anaheim	**State/Province**	CA
Country	US	Postal Code	92806
Phone Number		Fax Number	
Email Address			

Additional Applicant Data may be generated within this form by selecting the Add button.

Assignee Information including Non-Applicant Assignee Information:

Providing assignment information in this section does not substitute for compliance with any requirement of part 3 of Title 37 of CFR to have an assignment recorded by the Office.

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Application Data Sheet 37 CFR 1.76	Attorney Docket Number	C44-00999.CON15
	Application Number	
Title of Invention	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	

Assignee 1

Complete this section if assignee information, including non-applicant assignee information, is desired to be included on the patent application publication. An assignee-applicant identified in the "Applicant Information" section will appear on the patent application publication as an applicant. For an assignee-applicant, complete this section only if identification as an assignee is also desired on the patent application publication.

If the Assignee or Non-Applicant Assignee is an Organization check here. ☒

Organization Name	Constellation Designs, LLC

Mailing Address Information For Assignee including Non-Applicant Assignee:

Address 1	2190 S. Towne Centre Place		
Address 2	Suite 300		
City	Anaheim	**State/Province**	CA
Country	US	Postal Code	92806
Phone Number		Fax Number	
Email Address			

Additional Assignee or Non-Applicant Assignee Data may be generated within this form by selecting the Add button.

Signature:

NOTE: This Application Data Sheet must be signed in accordance with 37 CFR 1.33(b). **However, if this Application Data Sheet is submitted with the INITIAL filing of the application and either box A or B is not checked in subsection 2 of the "Authorization or Opt-Out of Authorization to Permit Access" section, then this form must also be signed in accordance with 37 CFR 1.14(c).**

This Application Data Sheet **must** be signed by a patent practitioner if one or more of the applicants is a **juristic entity** (e.g., corporation or association). If the applicant is two or more joint inventors, this form must be signed by a patent practitioner, **all** joint inventors who are the applicant, or one or more joint inventor-applicants who have been given power of attorney (e.g., see USPTO Form PTO/AIA/81) on behalf of **all** joint inventor-applicants.

See 37 CFR 1.4(d) for the manner of making signatures and certifications.

Signature	/David J. Bailey/			Date (YYYY-MM-DD)	2019-12-27
First Name	David	Last Name	Bailey	Registration Number	59,632

Additional Signature may be generated within this form by selecting the Add button.

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DECLARATION (37 CFR 1.63) FOR UTILITY OR DESIGN APPLICATION USING AN APPLICATION DATA SHEET (37 CFR 1.76)

Title of Invention	Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellations

As the below named inventor, I hereby declare that:

This declaration is directed to: ☐ The attached application, or

☒ United States application or PCT international application number 15/200,800

filed on July 1, 2016.

The above-identified application was made or authorized to be made by me.

I believe that I am the original inventor or an original joint inventor of a claimed invention in the application.

I hereby acknowledge that any willful false statement made in this declaration is punishable under 18 U.S.C. 1001 by fine or imprisonment of not more than five (5) years, or both.

WARNING:

Petitioner/applicant is cautioned to avoid submitting personal information in documents filed in a patent application that may contribute to identity theft. Personal information such as social security numbers, bank account numbers, or credit card numbers (other than a check or credit card authorization form PTO-2038 submitted for payment purposes) is never required by the USPTO to support a petition or an application. If this type of personal information is included in documents submitted to the USPTO, petitioners/applicants should consider redacting such personal information from the documents before submitting them to the USPTO. Petitioner/applicant is advised that the record of a patent application is available to the public after publication of the application (unless a non-publication request in compliance with 37 CFR 1.213(a) is made in the application) or issuance of a patent. Furthermore, the record from an abandoned application may also be available to the public if the application is referenced in a published application or an issued patent (see 37 CFR 1.14). Checks and credit card authorization forms PTO-2038 submitted for payment purposes are not retained in the application file and therefore are not publicly available.

LEGAL NAME OF INVENTOR

Inventor: Maged F. Barsoum Date (Optional): _____

Signature: /Maged F. Barsoum/

Note: An application data sheet (PTO/SB/14 or equivalent), including naming the entire inventive entity, must accompany this form or must have been previously filed. Use an additional PTO/AIA/01 form for each additional inventor.

This collection of information is required by 35 U.S.C. 115 and 37 CFR 1.63. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.11 and 1.14. This collection is estimated to take 1 minute to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: **Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.**

If you need assistance in completing this form, call 1-800-PTO-9199 and select option 2.

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DECLARATION (37 CFR 1.63) FOR UTILITY OR DESIGN APPLICATION USING AN APPLICATION DATA SHEET (37 CFR 1.76)

Title of Invention	Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellations

As the below named inventor, I hereby declare that:

This declaration is directed to: ☐ The attached application, or

☒ United States application or PCT international application number 15/200,800

filed on July 1, 2016.

The above-identified application was made or authorized to be made by me.

I believe that I am the original inventor or an original joint inventor of a claimed invention in the application.

I hereby acknowledge that any willful false statement made in this declaration is punishable under 18 U.S.C. 1001 by fine or imprisonment of not more than five (5) years, or both.

WARNING:

Petitioner/applicant is cautioned to avoid submitting personal information in documents filed in a patent application that may contribute to identity theft. Personal information such as social security numbers, bank account numbers, or credit card numbers (other than a check or credit card authorization form PTO-2038 submitted for payment purposes) is never required by the USPTO to support a petition or an application. If this type of personal information is included in documents submitted to the USPTO, petitioners/applicants should consider redacting such personal information from the documents before submitting them to the USPTO. Petitioner/applicant is advised that the record of a patent application is available to the public after publication of the application (unless a non-publication request in compliance with 37 CFR 1.213(a) is made in the application) or issuance of a patent. Furthermore, the record from an abandoned application may also be available to the public if the application is referenced in a published application or an issued patent (see 37 CFR 1.14). Checks and credit card authorization forms PTO-2038 submitted for payment purposes are not retained in the application file and therefore are not publicly available.

LEGAL NAME OF INVENTOR

Inventor: Christopher R. Jones Date (Optional): 6/5/17

Signature: /Christopher R. Jones/

Note: An application data sheet (PTO/SB/14 or equivalent), including naming the entire inventive entity, must accompany this form or must have been previously filed. Use an additional PTO/AIA/01 form for each additional inventor.

This collection of information is required by 35 U.S.C. 115 and 37 CFR 1.63. The information is required to obtain or retain a benefit by the public which is to file (and by the USPTO to process) an application. Confidentiality is governed by 35 U.S.C. 122 and 37 CFR 1.11 and 1.14. This collection is estimated to take 1 minute to complete, including gathering, preparing, and submitting the completed application form to the USPTO. Time will vary depending upon the individual case. Any comments on the amount of time you require to complete this form and/or suggestions for reducing this burden, should be sent to the Chief Information Officer, U.S. Patent and Trademark Office, U.S. Department of Commerce, P.O. Box 1450, Alexandria, VA 22313-1450. DO NOT SEND FEES OR COMPLETED FORMS TO THIS ADDRESS. SEND TO: **Commissioner for Patents, P.O. Box 1450, Alexandria, VA 22313-1450.**

If you need assistance in completing this form, call 1-800-PTO-9199 and select option 2.

Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations

RELATED APPLICATIONS

[0001] This application is a continuation of application number 16/206,991 filed November 30, 2018, which application is a continuation of application number 15/682,475 filed August 21, 2017 and issued on December 4, 2018 as U.S. Patent No. 10,149,179, which application is a continuation of application number 15/200,800 filed July 1, 2016 and issued on August 22, 2017 as U.S. Patent No. 9,743,292, which application is a continuation of application number 14/491,731 filed September 19, 2014 and issued on July 5, 2016 as U.S. Patent No. 9,385,832, which application is a continuation of application number 13/618,630 filed September 14, 2012 and issued on September 23, 2014 as U.S. Patent No. 8,842,761, which application is a continuation of application number 13/118,921 filed May 31, 2011 and issued on September 18, 2012 as U.S. Patent No. 8,270,511, which application is a continuation of application number 12/156,989 filed June 5, 2008 and issued on July 12, 2011 as U.S. Patent No. 7,978,777, which application claimed priority to U.S. Provisional Application 60/933,319 filed June 5, 2007, the disclosures of which are incorporated herein by reference.

STATEMENT OF FEDERALLY SPONSORED RESEARCH

[0002] This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

[0003] The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

[0004] The term “constellation” is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a

constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

[0005] Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

[0006] Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

[0007] Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

[0008] Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon

Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

[0009] One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

[0010] A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the

demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

[0011] Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

[0012] A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

[0013] Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

[0014] A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

[0015] Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

[0016] FIG. 1 is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

[0017] FIG. 2 is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

[0018] FIG. 3 is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

[0019] FIG. 4a is a conceptual illustration of the joint capacity of a channel.

[0020] FIG. 4b is a conceptual illustration of the parallel decoding capacity of a channel.

[0021] FIG. 5 is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

[0022] FIG. 6a is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

[0023] FIG. 6b is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8,16,32.

[0024] FIG. 7 is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2,4,8,16,32 constellations.

[0025] FIG. 8a is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0026] FIG. 8b is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0027] FIG. 9 is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

[0028] FIGS. 10a – 10d are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0029] FIGS. 11a and 11b are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

[0030] FIGS. 12a – 12d are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0031] FIGS. 13a and 13b are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

[0032] FIGS. 14a – 14d are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0033] FIGS. 15a and 15b are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

[0034] FIGS. 16a – 16d are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0035] FIGS. 17a and 17b are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

[0036] FIG. 18 is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

[0037] FIG. 19 is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

[0038] FIG. 20 is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

[0039] FIG. 21 illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

[0040] FIGS. 22a and 22b are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0041] FIGS. 23a and 23b are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0042] FIGS. 24a and 24b are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

[0043] Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by.a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

[0044] In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

COMMUNICATION SYSTEMS

[0045] A communication system in accordance with an embodiment of the invention is shown in FIG. 1. The communication system 10 includes a source 12 that provides user bits to a transmitter 14. The transmitter transmits symbols over a channel to a receiver 16 using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

[0046] A transmitter in accordance with an embodiment of the invention is shown in FIG. 2. The transmitter 14 includes a coder 20 that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper 22 is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator 24, which modulates the symbols for transmission via the channel.

[0047] A receiver in accordance with an embodiment of the invention is illustrated in FIG. 3. The receiver 16 includes a demodulator 30 that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper 32 provides the likelihoods to a decoder 34 that decodes the encoded bit stream to provide a sequence of received bits to a sink.

GEOMETRICALLY SHAPED CONSTELLATIONS

[0048] Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in

accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

SELECTION OF A GEOMETRICALLY SHAPED CONSTELLATION

[0049] Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

UPGRADING EXISTING COMMUNICATION SYSTEMS

[0050] A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

JOINT CAPACITY AND PARALLEL DECODING CAPACITY

[0051] A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. 4a. The portions of the communication system that are considered part of the channel are indicated by the ghost line 40. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{l-1} I(X_i;Y)$$

where X_i is the ith bit of the l-bits transmitted symbol, and Y is the received symbol, and $I(A;B)$ denotes the mutual information between random variables A and B.

[0052] Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

[0053] With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4b. The portions of the communication system that are considered part of the channel are indicated by the ghost line 42. The joint capacity of the channel is given by:

$$C_{JOINT} = I(X;Y)$$

[0054] Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_G \geq C_{\text{JOINT}} \geq C_{\text{PD}}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

SELECTING A CONSTELLATION HAVING AN OPTIMAL CAPACITY

[0055] Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. 5. The process 50 commences with the selection (52) of an appropriate constellation size *M* and a desired capacity per dimension r_j. In the illustrated embodiment, the process involves a check (52) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the *M*-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known *a priori*. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be

used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

[0056] In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An *M*-ary constellation is then obtained by optimizing (56) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

[0057] The SNR at which the optimized *M*-ary constellation provides the desired capacity per dimension η_j (SNR_{out}) is determined (57). A determination (58) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (55). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

[0058] The process illustrated in FIG. 5 can maximize PD capacity or joint capacity of an *M*-ary constellation for a given SNR. Although the process illustrated in FIG. 5 shows selecting an *M*-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an *M*-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR

gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify *M*-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

[0059] We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q) , or with multi-stage decoding.. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

OPTIMIZING THE CAPACITY OF AN *M*-ARY CONSTELLATION AT A GIVEN SNR

[0060] Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an *M*-ary constellation at a given SNR. An optimization process, such as the optimization process 56 shown in FIG. 5, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged..

[0061] The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. 5.

[0062] Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

[0063] The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

GAINS ACHIEVED BY OPTIMIZED GEOMETRICALLY SPACED CONSTELLATIONS

[0064] The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}log_2(1+SNR)$$

[0065] Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

[0066] The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. 6a and 6b. FIG. 6a includes a chart 60 showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}, Gaps 62 exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. 6b includes a chart 64 showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32

constellations that maximize d_{min}, Gaps 66 exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity..

[0067] In order to gain a better view of the differences between the curves shown in FIGS. 6a and 6b at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. 7. It is interesting to note from the chart 70 in FIG. 7 that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

[0068] FIGS. 8a and 8b summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. 8a shows the SNR gap for each constellation optimized for PD capacity. FIG. 8b shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

[0069] Referring to FIG. 8a, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

[0070] The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. 9. A chart 90 includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. 8a.

CAPACITY OPTIMIZED PAM CONSTELLATIONS

[0071] Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. 10a – 17b. The locus plots and design tables show PAM-4,8,16,32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

[0072] In FIG.10a, a locus plot 100 shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot 105 showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. 10b. In FIG. 10c. the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. 10d the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. 11a and FIG. 11b where vertical slices of FIGS. 10ab and 10cd are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities = {0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol

(bps). Given that PAM-4 can provide at most $\log_2(4) = 2$ bps, these design points represent systems with information code rates R = {1/4, 3/8, 1/2, 5/8, 3/4} respectively.

[0073] FIGS. 12ab and 12cd present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. 13a and 13b provide slices from these plots at SNRs corresponding to achievable capacities r_i = {0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate R = r_i bps / $\log_2(8)$, resulting in R ={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. 13b which shows a PD capacity optimized PAM-8 constellation optimized for SNR = **9.00 dB**, or 1.5 bps. We next examine the plot provided in FIG. **8a** and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

[0074] Similar information is presented in FIGS. 14abcd, and 15ab which provide loci plots and *design* tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. 16abcd, 17ab provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

CAPACITY OPTIMIZED PSK CONSTELLATIONS

[0075] Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart 180 comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. 18 where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4,8,16,32 and for PD capacaity optimized PSK-4,8,16,32.

[0076] The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. 19, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. 20. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR = 8.4 dB optimal PSK-32 is the same as traditional PSK-8,

at SNR = 14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR = 2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

[0077] We note now that the locus of points for PD optimized PSK-32 in FIG. 19 in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. 18 implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8,16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. 19. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. 18.

ADAPTIVE RATE DESIGN

[0078] In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation / code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper / decoder configuration in order to match the new transmit-side configuration.

[0079] Conversely, the receiver could implement a quality of performance based optimized constellation / code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation / code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper / decoder modules.

GEOMETRICALLY SHAPED QAM CONSTELLATIONS

[0080] Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

[0081] In FIG. 21 we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. 13b). The label-point pairs in this PAM-8 constellation are {(000, -1.72), (001, -0.81), (010, 1.72), (011,-0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, -0.02)}. Examination of FIG. 21 shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8x8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-DIMENSIONAL CONSTELLATION OPTIMIZATION

[0082] Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

CAPACITY OPTIMIZED CONSTELALTIONS FOR FADING CHANNELS

[0083] Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y = a(t).X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and $a(t)$ is the fading distribution, which is a function of time.

[0084] In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

[0085] Many fading channels follow a Rayleigh distribution. FIGS. 22a – 24b are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

WHAT IS CLAIMED IS:

1. A communication system, comprising:

a receiver capable of receiving signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator capable of demodulating a received signal into a demodulated signal;

a demapper, coupled to the demodulator, capable of determining likelihoods using the demodulated signal and a non-uniform symbol constellation that includes at least two constellation points having identical locations and different labels; and

a decoder, coupled to the demapper, capable of using the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code.

2. The communication system of claim 1, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations;

the plurality of non-uniform symbol constellations includes a plurality of non-uniform constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where non-uniform symbol constellations of the first type include at least two constellation points having identical locations and different labels;

the receiver is capable of selecting an LDPC code rate and the non-uniform symbol constellation as a pair from a plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs; and

each of the plurality of non-uniform symbol constellations is only included in one of the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

3. The communication system of claim 2, wherein the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs includes:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the first type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the first type.

4. The communication system of claim 3, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

equal to or less than 15.42dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point non-uniform symbol constellation of the first type; and

equal to or less than 21.52dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 7/10 and a one-thousand-twenty-four-point non-uniform symbol constellations of the first type.

5. The communication system of claim 4, wherein:

the plurality of non-uniform symbol constellations include a plurality of non-uniform constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different

two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within a non-uniform symbol constellation of the second type share an identical location;

the demapper is also capable of determining likelihoods using the demodulated signal and a non-uniform symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs include:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11dB and 9.25dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

between 9.00dB and 15.93dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

between 18.72dB and 22.13dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

between 24.79dB and 28.20dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type.

6. The communication system of claim 1, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of non-uniform symbol constellations is capable of providing greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of non-uniform symbol constellations when operated at the same SNR.

7. The communication system of claim 6, wherein the symbol constellation operating SNR is a channel SNR where the receiver is capable of using the respective non-uniform symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

8. The communication system of claim 1, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

9. The communication system of claim 1, wherein the non-uniform symbol constellation is an orthogonalized non-uniform Pulse Amplitude Modulated (PAM) constellation.

10. The communication system of claim 1, further comprising a transmitter capable of transmitting signals via the communication channel, where the transmitter comprises:

a coder capable of receiving bits and outputting encoded bits using a Low Density Parity Check (LDPC) code;

a mapper, coupled to the coder, capable of mapping the encoded bits to symbols in the non-uniform symbol constellation; and

a modulator, coupled to the mapper, capable of generating a signal for transmission via the communication channel based upon symbols selected by the mapper.

11. A communication system, comprising:

a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver comprises:

a demodulator that demodulates a received signal into a demodulated signal;

a demapper that determines likelihoods using the demodulated signal and a non-uniform symbol constellation that includes at least two constellation points having identical locations and different labels; and

a decoder that uses the likelihoods determined by the demapper to provide a sequence of received bits based upon a low density parity check (LDPC) code;

wherein the demapper is interposed between the demodulator and the decoder and the demapper receives information from the demodulator and provides information to the decoder.

12. The communication system of claim 11, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations;

the plurality of non-uniform symbol constellations includes a plurality of non-uniform constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where non-uniform symbol constellations of the first type include at least two constellation points having identical locations and different labels;

the receiver selects an LDPC code rate and the non-uniform symbol constellation as a pair from a plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs; and

each of the plurality of non-uniform symbol constellations is only included in one of the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

13. The communication system of claim 12, wherein the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs includes:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the first type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the first type.

14. The communication system of claim 13, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

equal to or less than 15.42dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point non-uniform symbol constellation of the first type; and

equal to or less than 21.52dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 7/10 and a one-thousand-twenty-four-point non-uniform symbol constellations of the first type.

15. The communication system of claim 14, wherein:

the plurality of non-uniform symbol constellations includes a plurality of non-uniform constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within a non-uniform symbol constellation of the second type share an identical location;

the demapper also determines likelihoods using the demodulated signal and a non-uniform symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs includes:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11dB and 9.25dBdB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

between 9.00dB and 15.93dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

between 18.72dB and 22.13dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

between 24.79dB and 28.20dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type.

16. The communication system of claim 11, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of non-uniform symbol constellations provides greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of non-uniform symbol constellations when operated at the same SNR.

17. The communication system of claim 16, wherein the symbol constellation operating SNR is a channel SNR where the receiver uses the respective non-uniform symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

18. The communication system of claim 11, wherein

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

19. The communication system of claim 11, wherein the non-uniform symbol constellation is an orthogonalized non-uniform Pulse Amplitude Modulated (PAM) constellation.

20. The communication system of claim 11, further comprising a transmitter that transmits signals via the communication channel, where the transmitter comprises:

a coder that receives bits and outputs encoded bits using a Low Density Parity Check (LDPC) code;

a mapper that maps the encoded bits to symbols in the selected symbol constellation; and

a modulator that generates a signal for transmission via the communication channel based upon symbols selected by the mapper;

wherein the mapper is interposed between the modulator and the coder and the mapper receives information from the coder and provides information to the modulator.

21. The communication system of claim 11, further comprising a transmitter that transmits signals via the communication channel, where the transmitter uses the non-uniform symbol constellation to transform encoded bits into the transmitted signals.

22. A communication system, comprising a receiver that receives signals via a communication channel having a channel signal-to-noise ratio (SNR), wherein the receiver uses a non-uniform symbol constellation that includes at least two constellation points having identical locations and different labels to transform the received signals into received bits.

23. The communication system of claim 22, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations;

the plurality of non-uniform symbol constellations includes a plurality of non-uniform constellations of a first type that comprise multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations, where non-uniform symbol constellations of the first type include at least two constellation points having identical locations and different labels;

the receiver selects an LDPC code rate and the non-uniform symbol constellation as a pair from a plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs; and

each of the plurality of non-uniform symbol constellations is only included in one of the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs.

24. The communication system of claim 23, wherein the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs includes:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 5/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the first type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate below 7/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the first type.

25. The communication system of claim 24, wherein the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

equal to or less than 5.27dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate that is below 1/3 and a sixty-four-point non-uniform symbol constellation of the first type;

equal to or less than 15.42dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 5/8 and a two-hundred-fifty-six-point non-uniform symbol constellation of the first type; and

equal to or less than 21.52dB while using an LDPC code rate and non-uniform symbol constellation pair that includes an LDPC code rate below 7/10 and

a one-thousand-twenty-four-point non-uniform symbol constellations of the first type.

26. The communication system of claim 25, wherein:

the plurality of non-uniform symbol constellations includes a plurality of non-uniform constellations of a second type that comprises multiple different sixteen-point symbol constellations, multiple different sixty-four-point symbol constellations, multiple different two-hundred-fifty-six-point symbol constellations, and multiple different one-thousand-twenty-four-point symbol constellations;

no two constellation points within a non-uniform symbol constellation of the second type share an identical location;

the receiver also transforms received signals into received bits using a non-uniform symbol constellation of the second type; and

the plurality of predetermined LDPC code rate and non-uniform symbol constellation pairs include:

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than

or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type;

the receiver is capable of decoding the signals received over the communication channel when the SNR of the communication channel is:

between 3.11dB and 9.25dBdB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/8 and less than or equal to 6/8 and a sixteen-point non-uniform symbol constellation of the second type;

between 9.00dB and 15.93dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 3/6 and less than or equal to 5/6 and a sixty-four-point non-uniform symbol constellation of the second type;

between 18.72dB and 22.13dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 6/8 and less than or equal to 7/8 and a two-hundred-and-fifty-six-point non-uniform symbol constellation of the second type; and

between 24.79dB and 28.20dB while using the multiple LDPC code rate and non-uniform symbol constellation pairs that each include an LDPC code rate that is equal to or greater than 8/10 and less than or equal to 9/10 and a one-thousand-twenty-four-point non-uniform symbol constellation of the second type.

27. The communication system of claim 22, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of the constellation points such that each of the plurality of non-uniform symbol constellations provides greater parallel decoding capacity when operated at a symbol constellation operating SNR than the other plurality of non-uniform symbol constellations when operated at the same SNR.

28. The communication system of claim 27, wherein the symbol constellation operating SNR is a channel SNR where the receiver uses the respective non-uniform symbol constellation to receive data at a frame error rate (FER) of 10^{-2} or lower.

29. The communication system of claim 22, wherein:

the non-uniform symbol constellation is selected from a plurality of non-uniform symbol constellations; and

each of the plurality of non-uniform symbol constellations is characterized by the assignment of labels and spacing of constellation points so as to maximize parallel decoding capacity at a symbol constellation operating SNR subject to at least one constraint.

30. The communication system of claim 22, wherein at least one of the plurality of non-uniform symbol constellations is an orthogonalized non-uniform Pulse Amplitude Modulated (PAM) constellation.

ABSTRACT

Communication systems are described that use unequally spaced constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. One embodiment is a digital communications system including a transmitter transmitting signals via a communication channel, the transmitter including a coder capable of receiving user bits and outputting encoded bits at a rate, a mapper capable of mapping encoded bits to symbols in a constellation, and a modulator capable of generating a modulated signal for transmission via the communication channel using symbols generated by the mapper, wherein the constellation is unequally spaced and characterizable by assignment of locations and labels of constellation points to maximize parallel decode capacity of the constellation at a given signal-to-noise ratio so that the constellation provides a given capacity at a reduced signal-to-noise ratio compared to a uniform constellation that maximizes the minimum distance between constellation points of the uniform constellation.

1 / 43

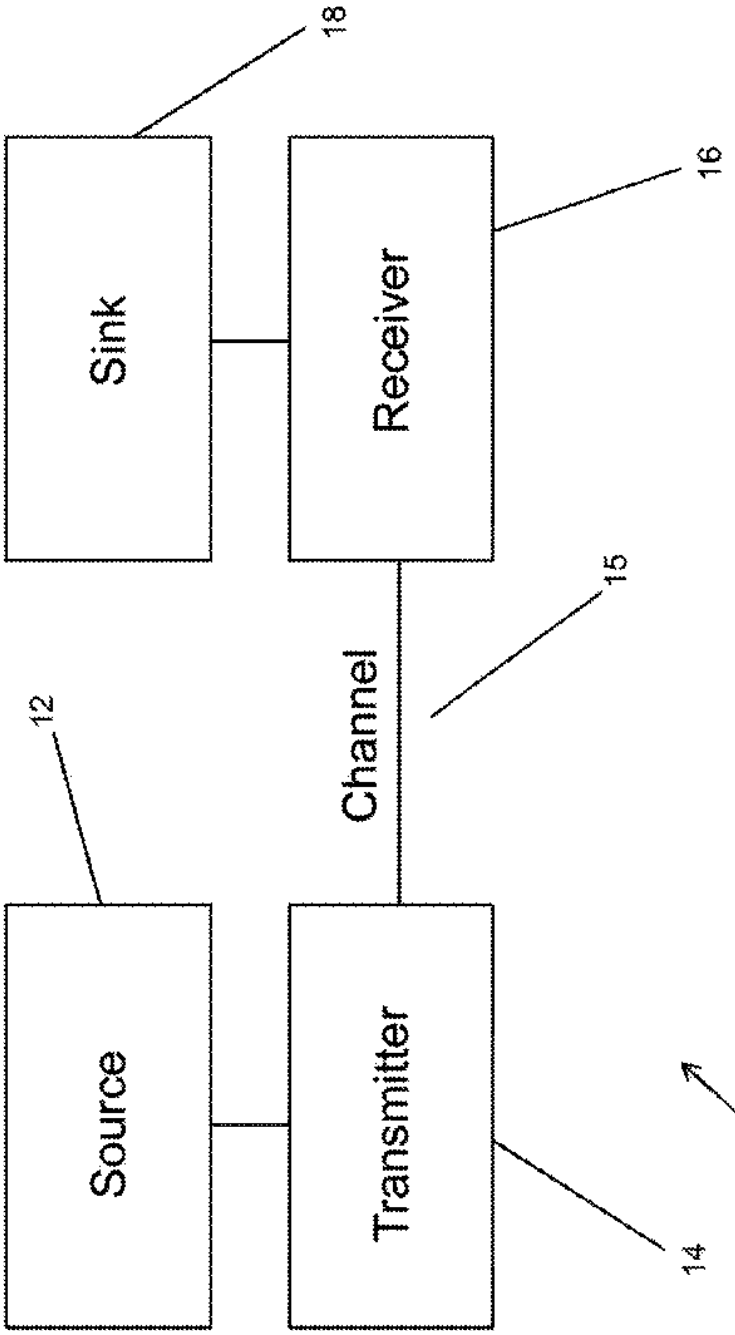


FIG. 1

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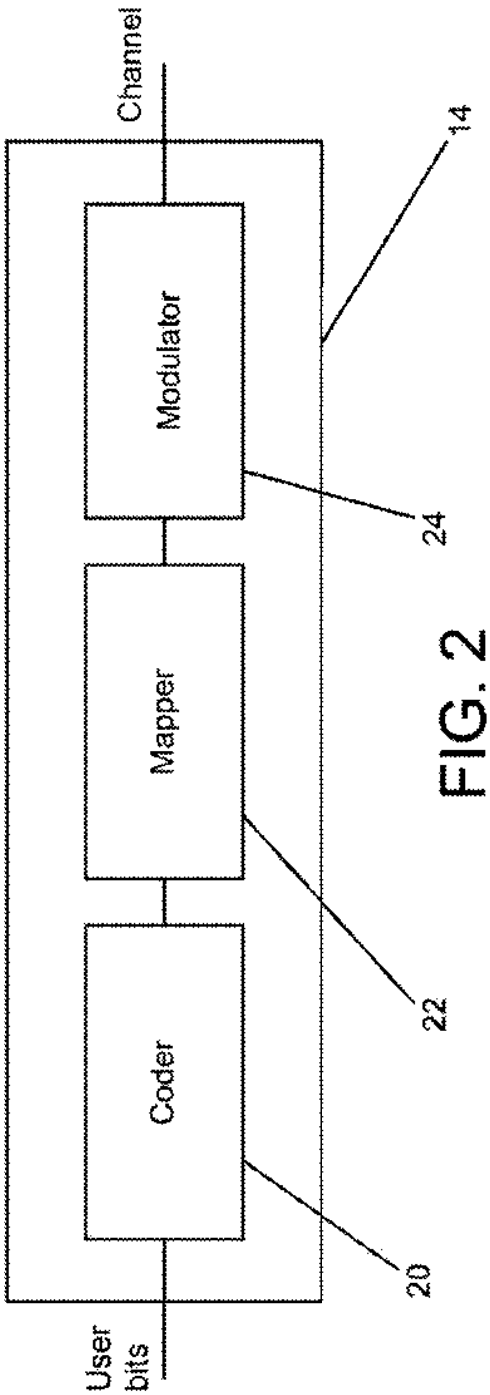


FIG. 2

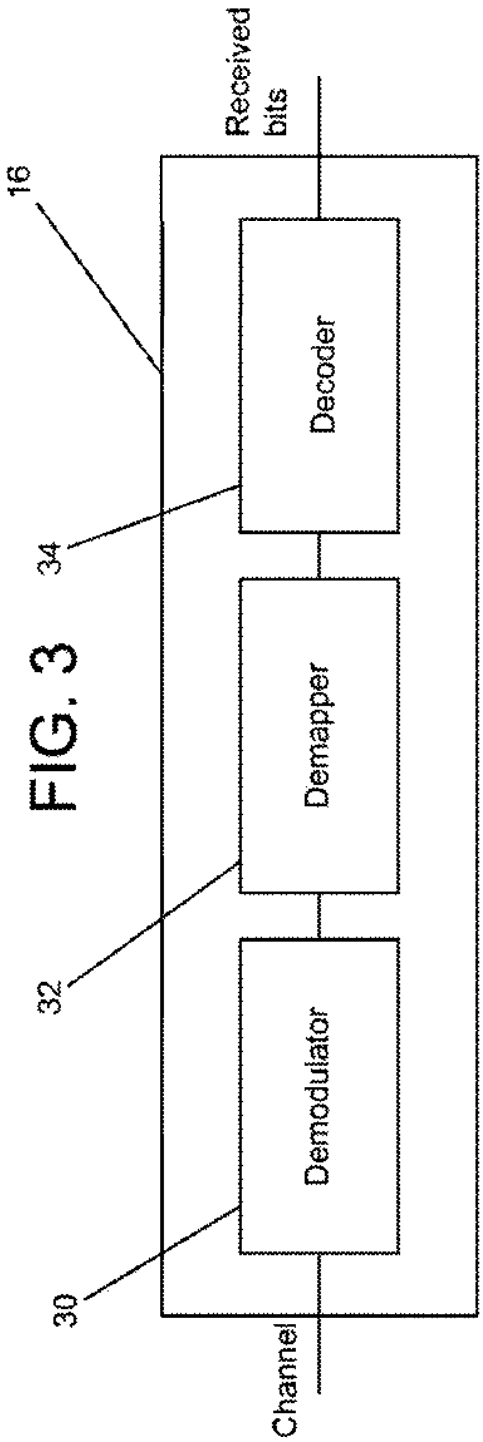


FIG. 3

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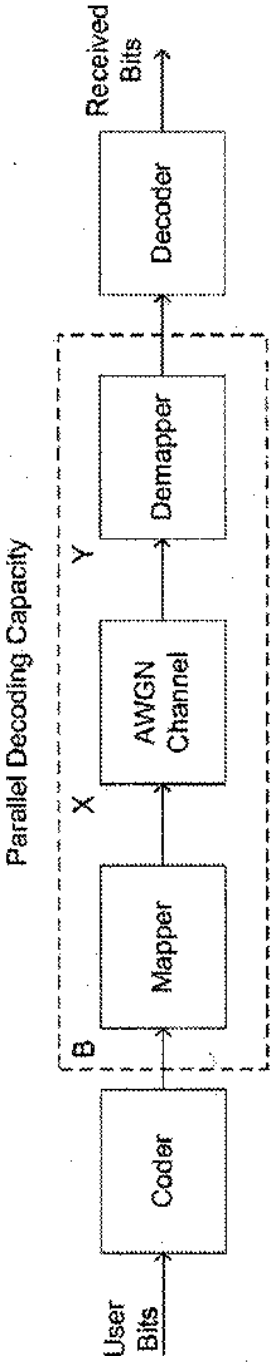


FIG. 4a

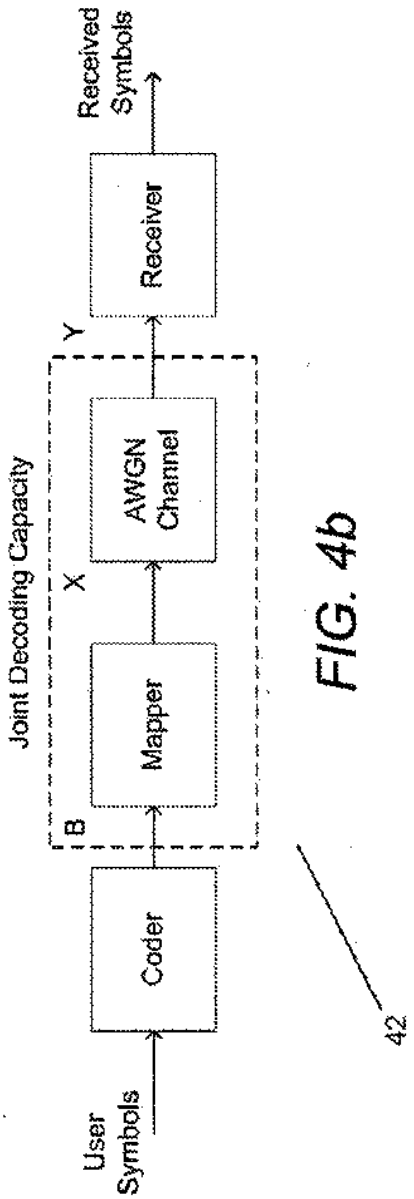


FIG. 4b

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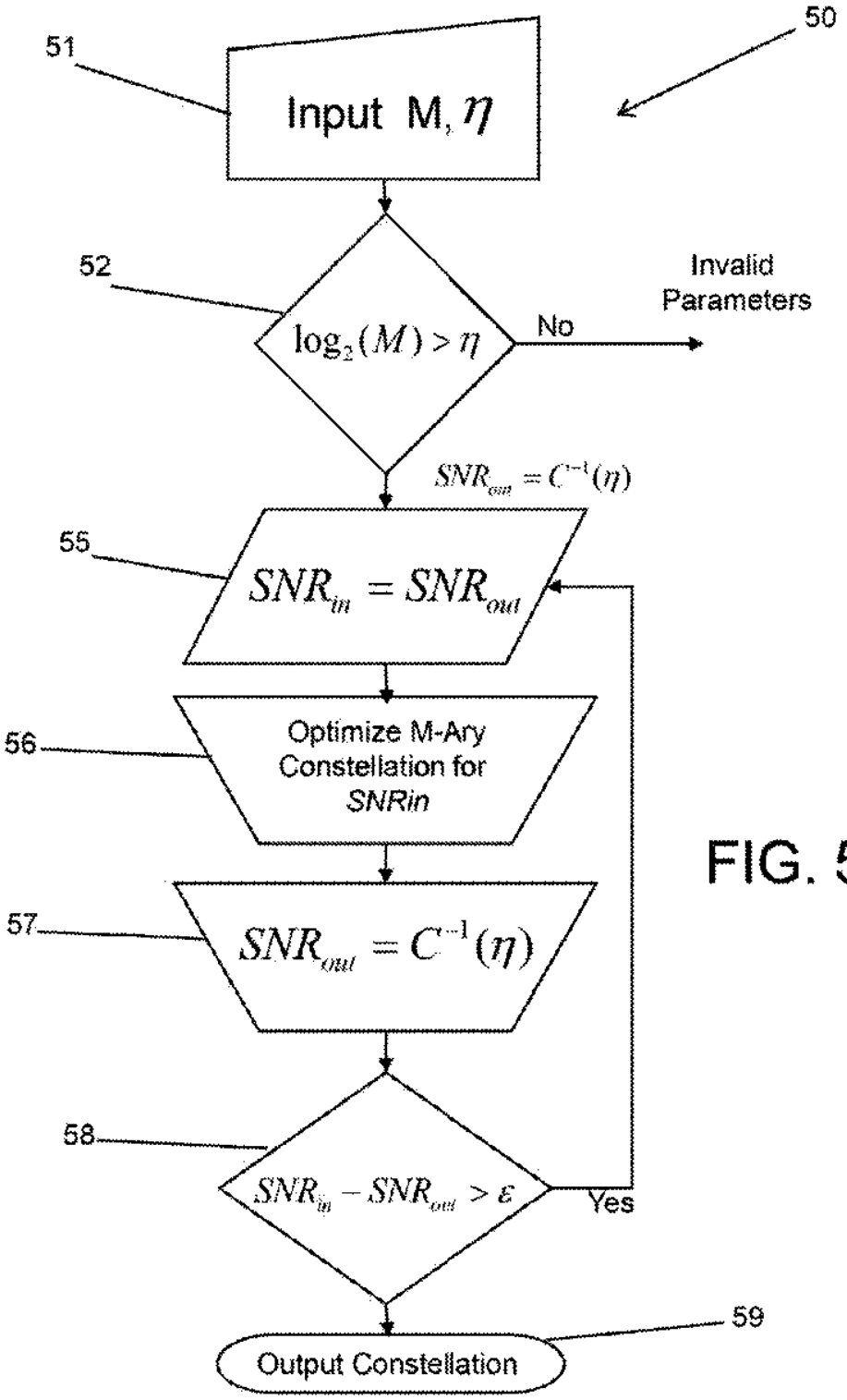

50
51
Input M, η
52
$\log_2(M) > \eta$
No
Invalid Parameters
$SNR_{out} = C^{-1}(\eta)$
55
$SNR_{in} = SNR_{out}$
56
Optimize M-Ary Constellation for SNRin
57
$SNR_{out} = C^{-1}(\eta)$
58
$SNR_{in} - SNR_{out} > \varepsilon$
Yes
59
Output Constellation

FIG. 5

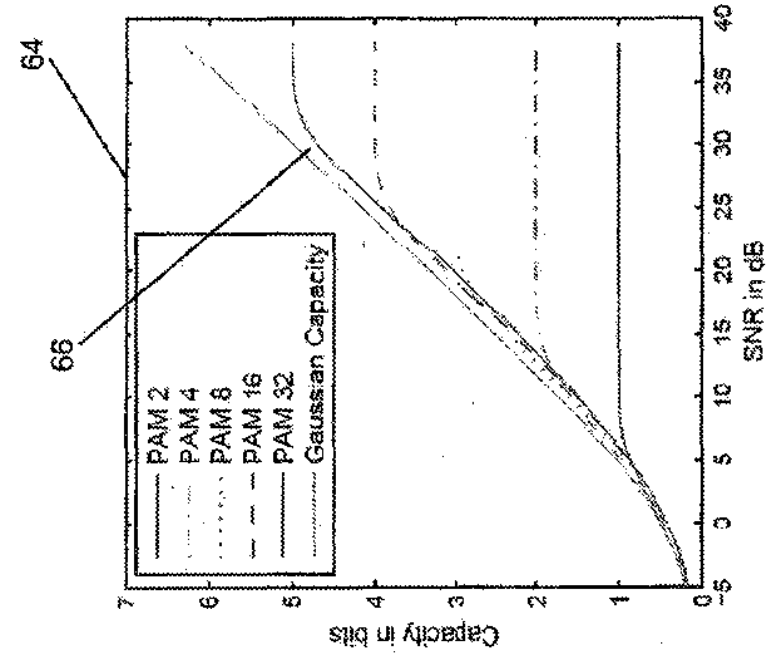

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

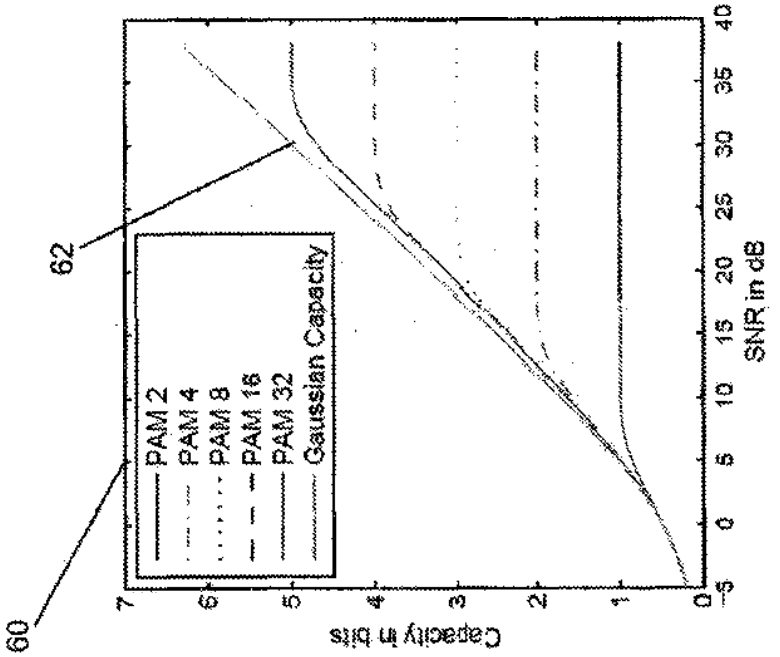

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

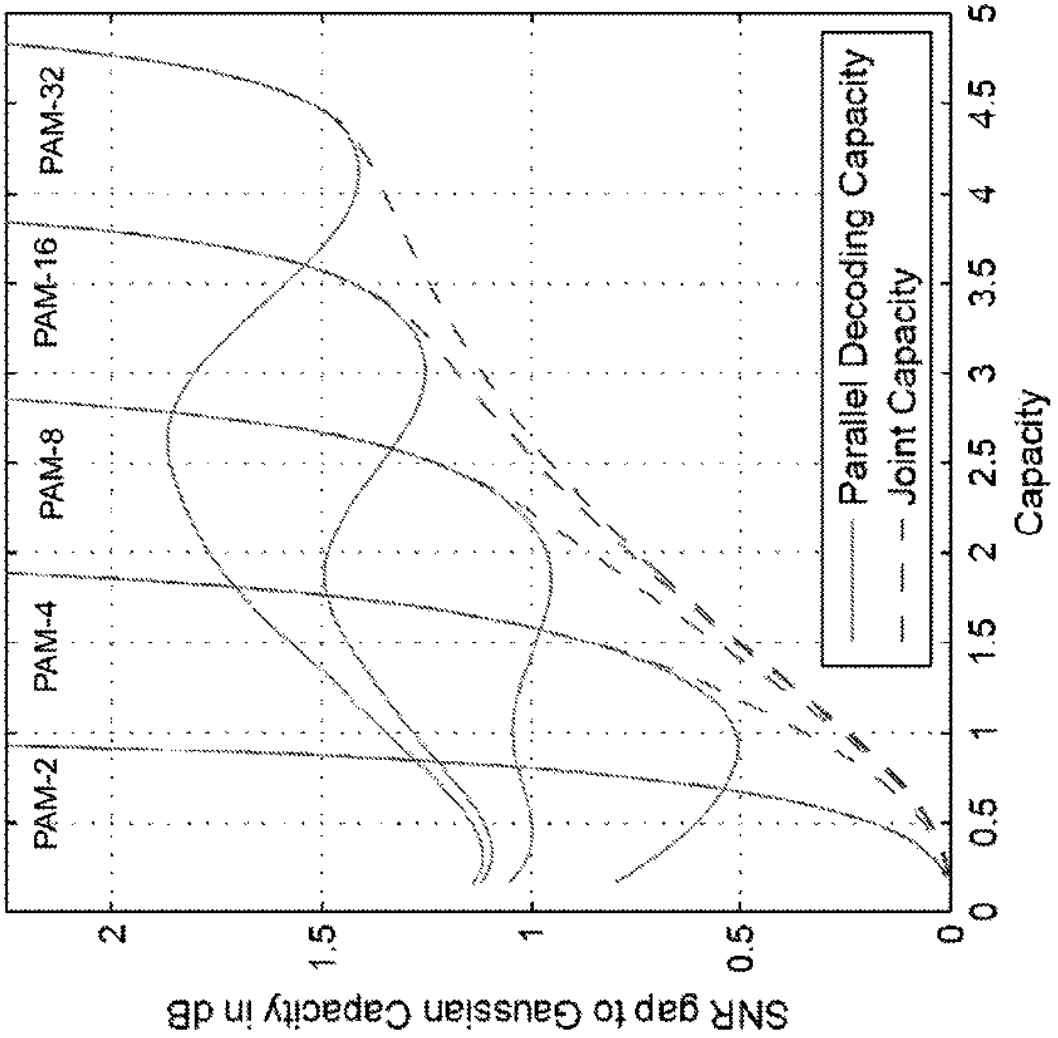

SNR gap to Gaussian Capacity in dB
Capacity
Parallel Decoding Capacity
Joint Capacity
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

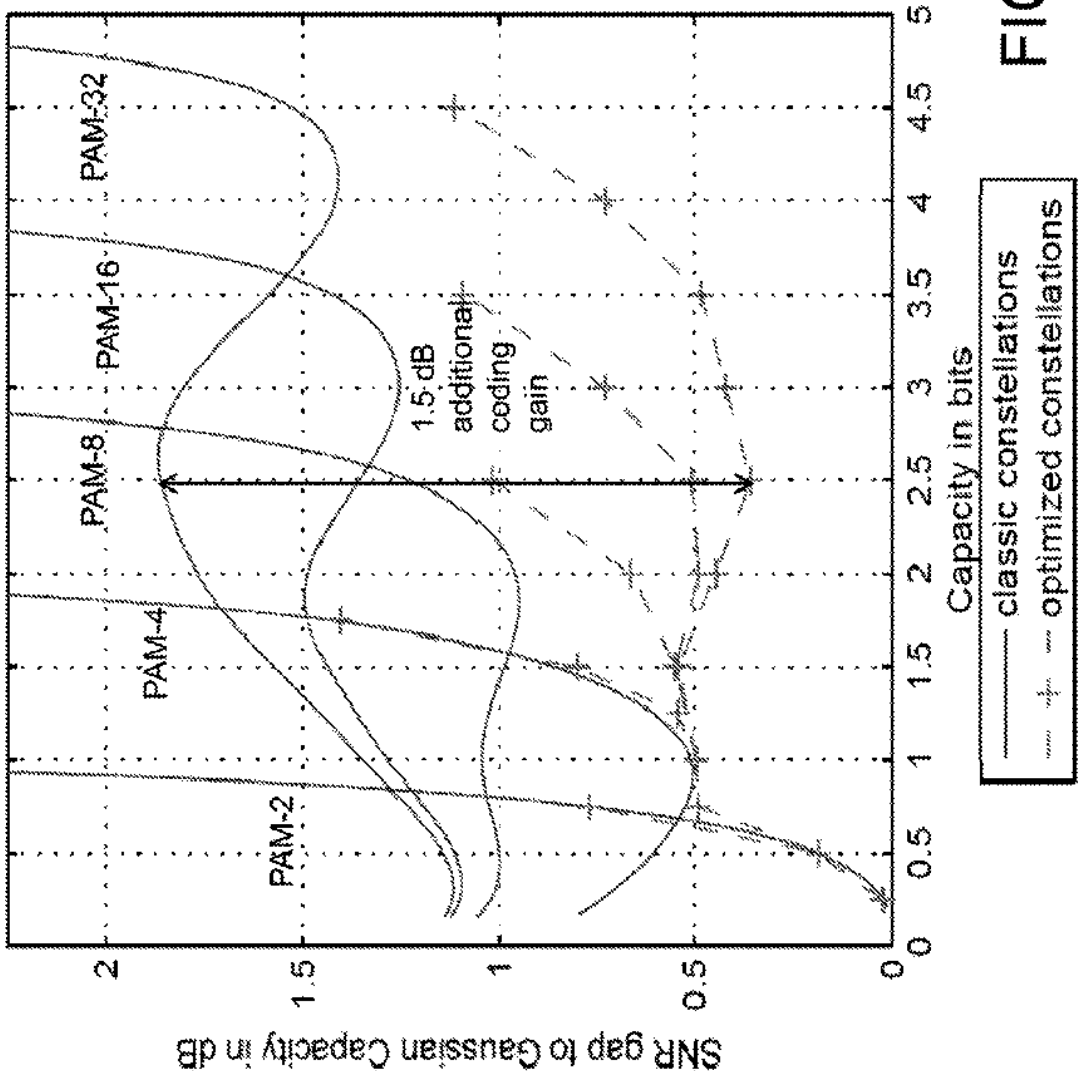

SNR gap to Gaussian Capacity in dB
Capacity in bits
PAM-32
PAM-16
PAM-8
PAM-4
PAM-2
1.5 dB additional coding gain
classic constellations
optimized constellations

FIG. 8a

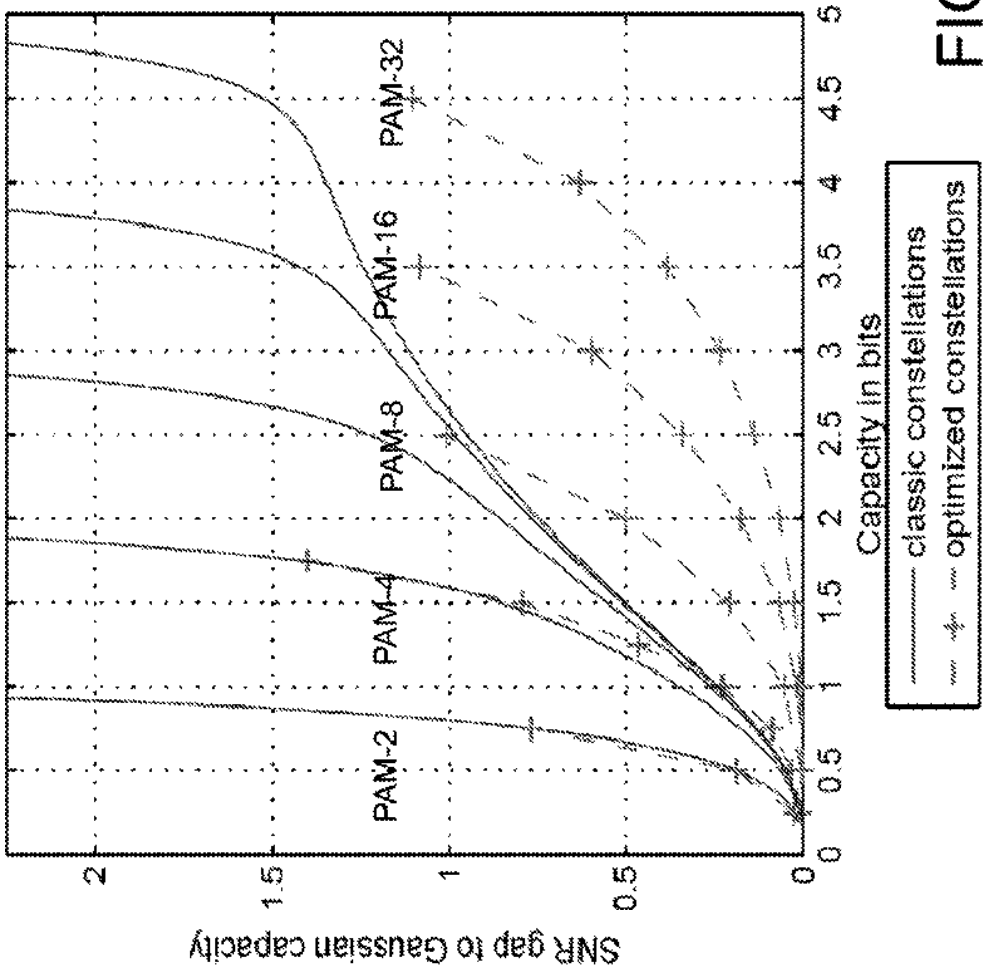

SNR gap to Gaussian capacity
Capacity in bits
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
classic constellations
optimized constellations
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 8b

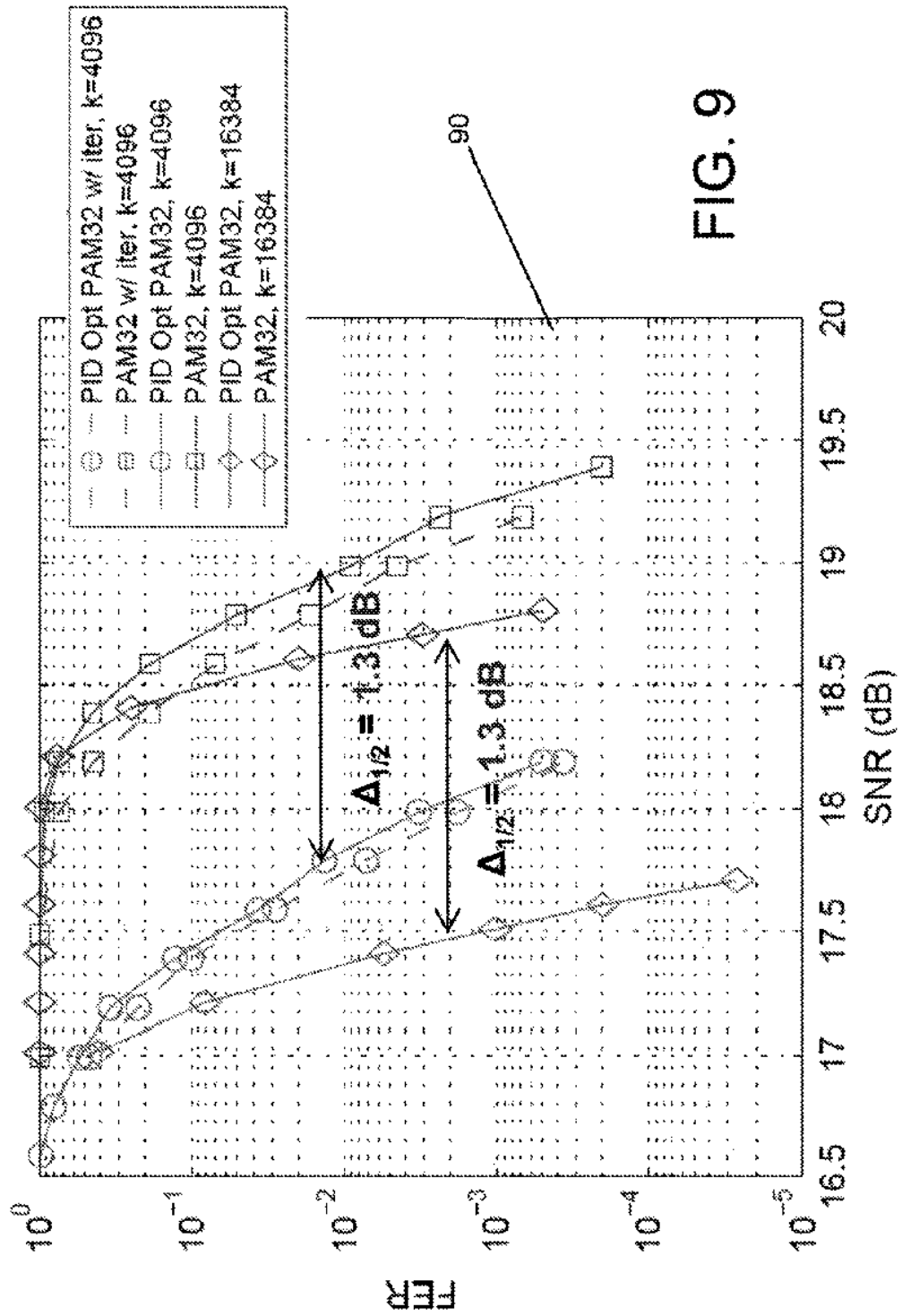

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

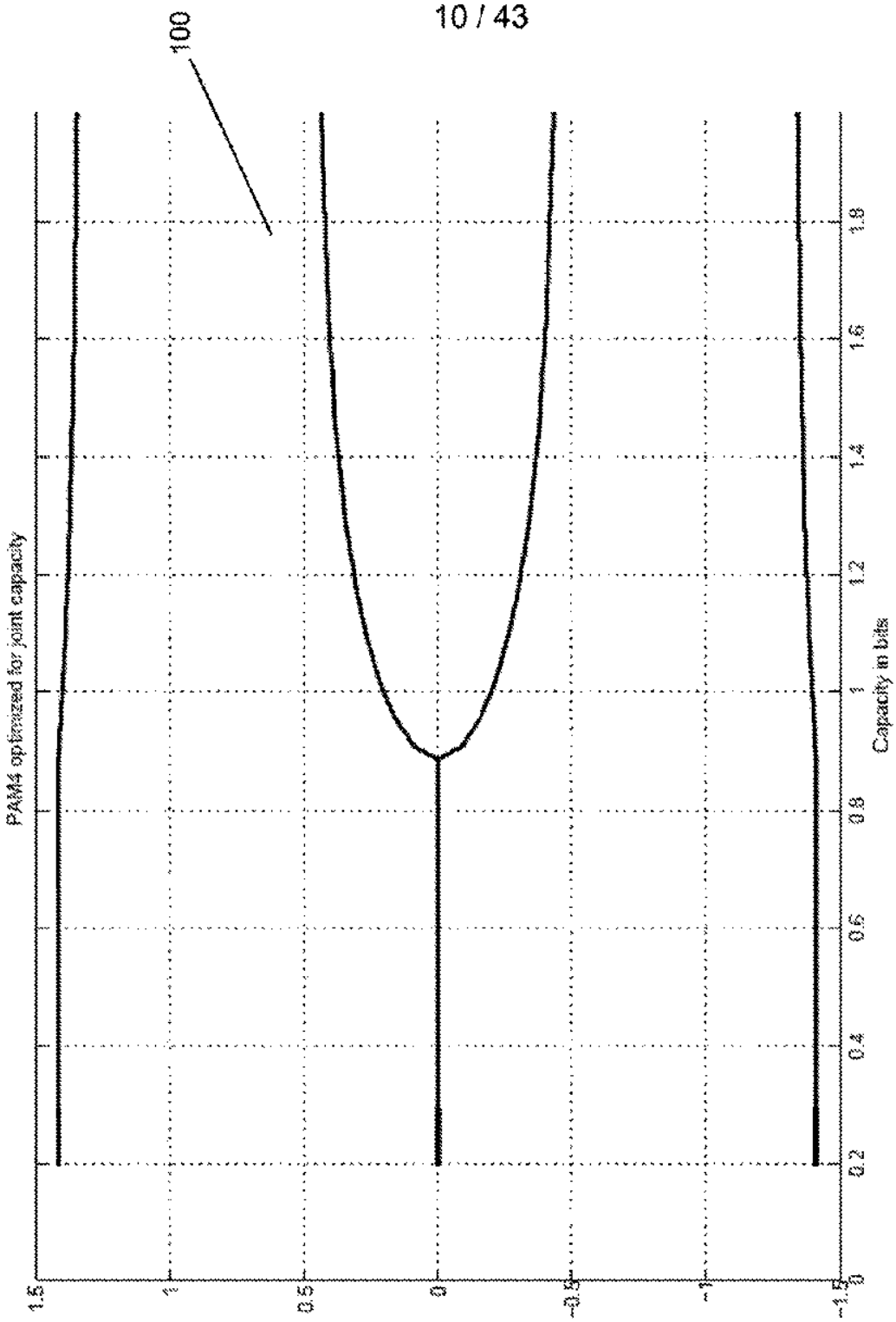

100
PAM4 optimized for joint capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10a

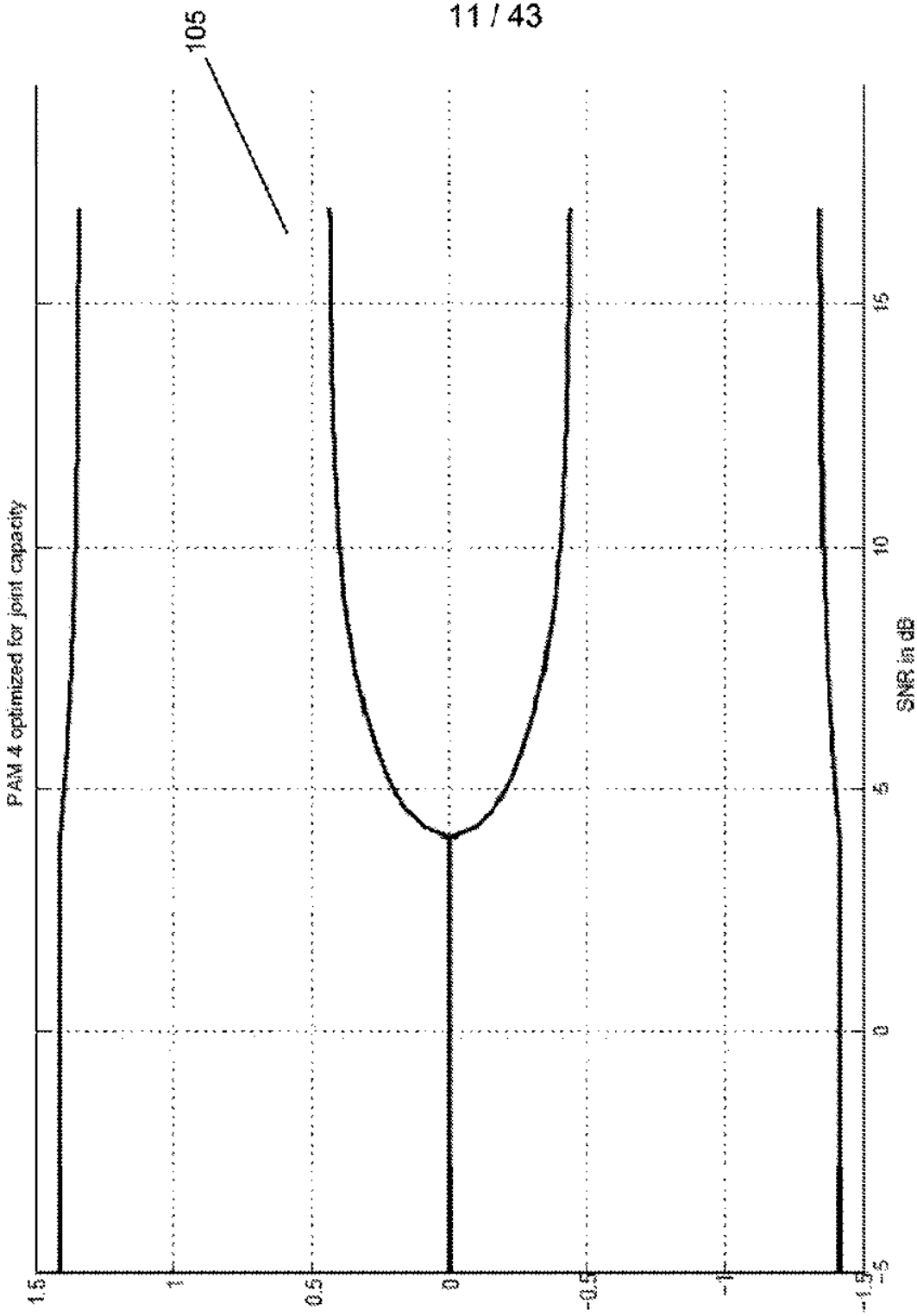

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
15
10
5
0
-5

FIG. 10b

12 / 43

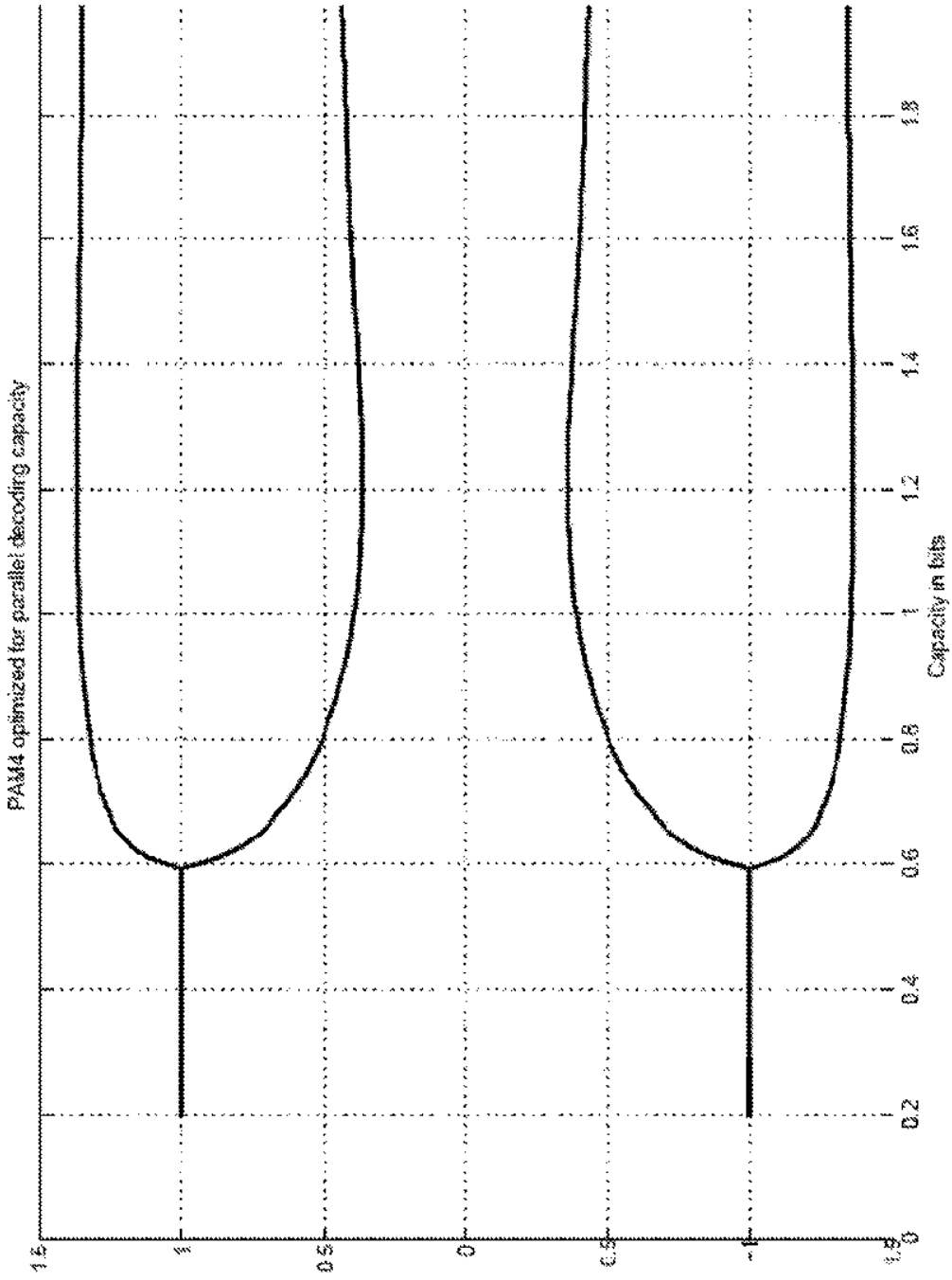

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

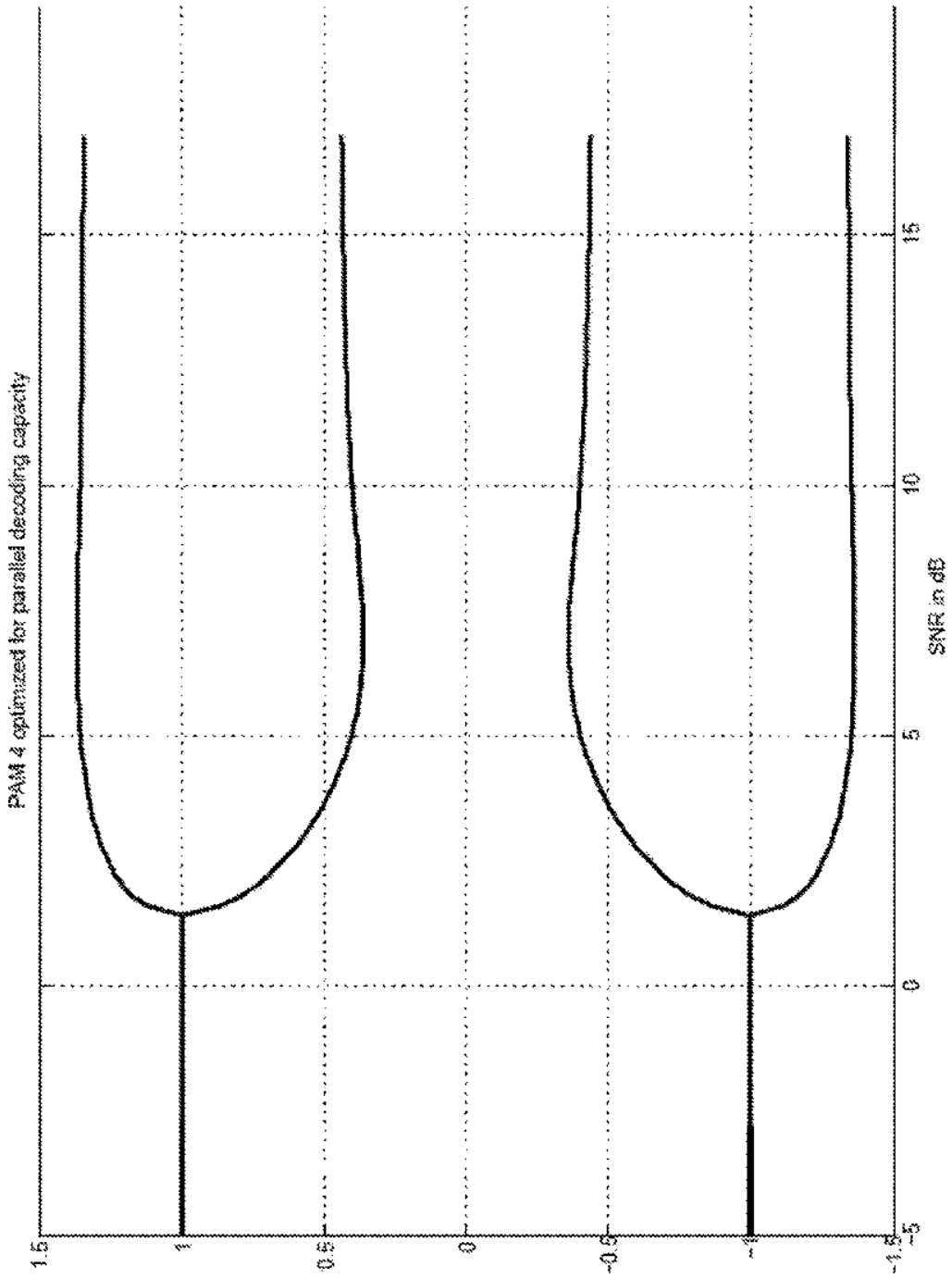

PAM 4 optimized for parallel decoding capacity
SNR in dB
15
10
5
0
-5
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

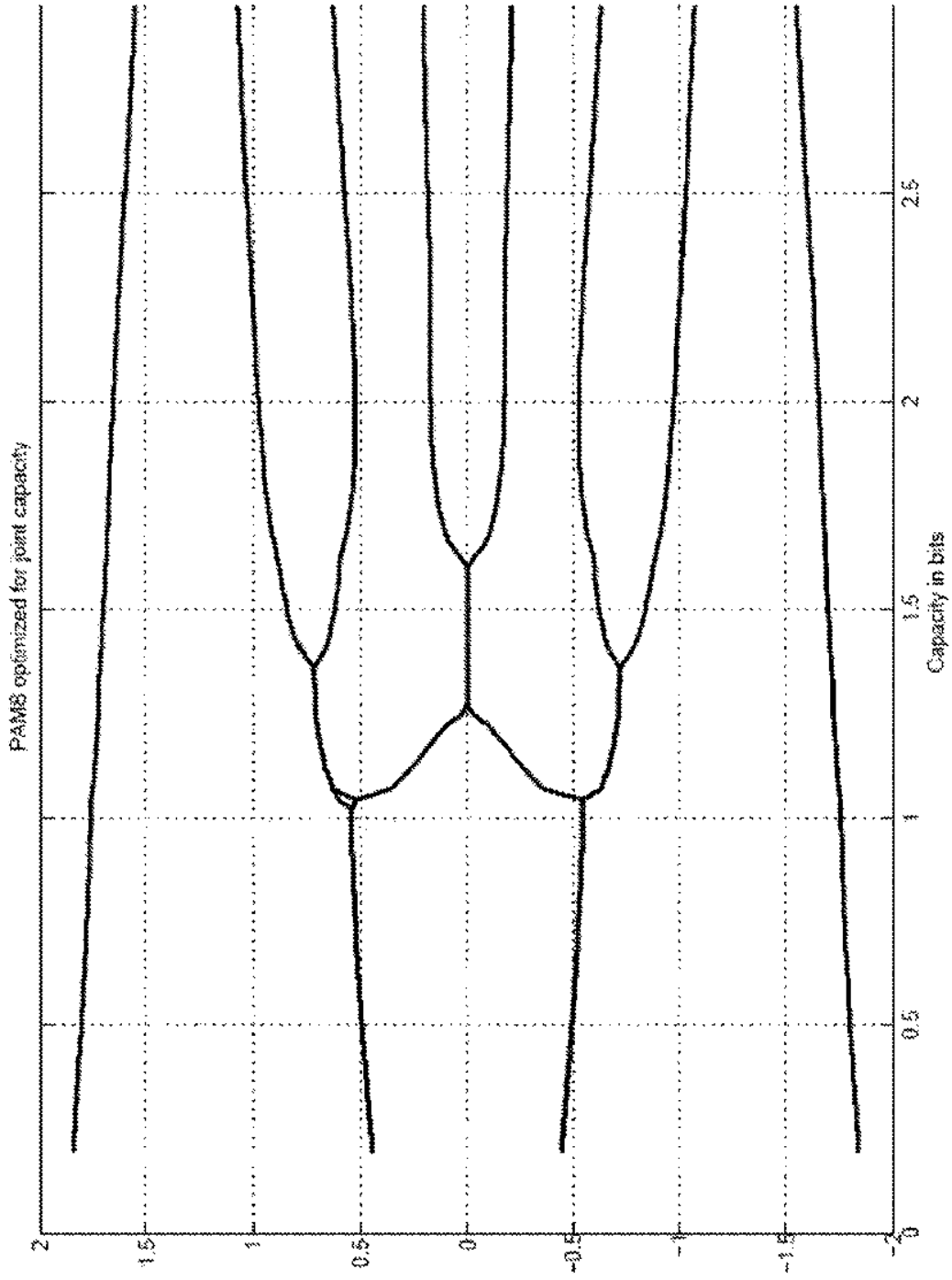

PAM8 optimized for joint capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12a

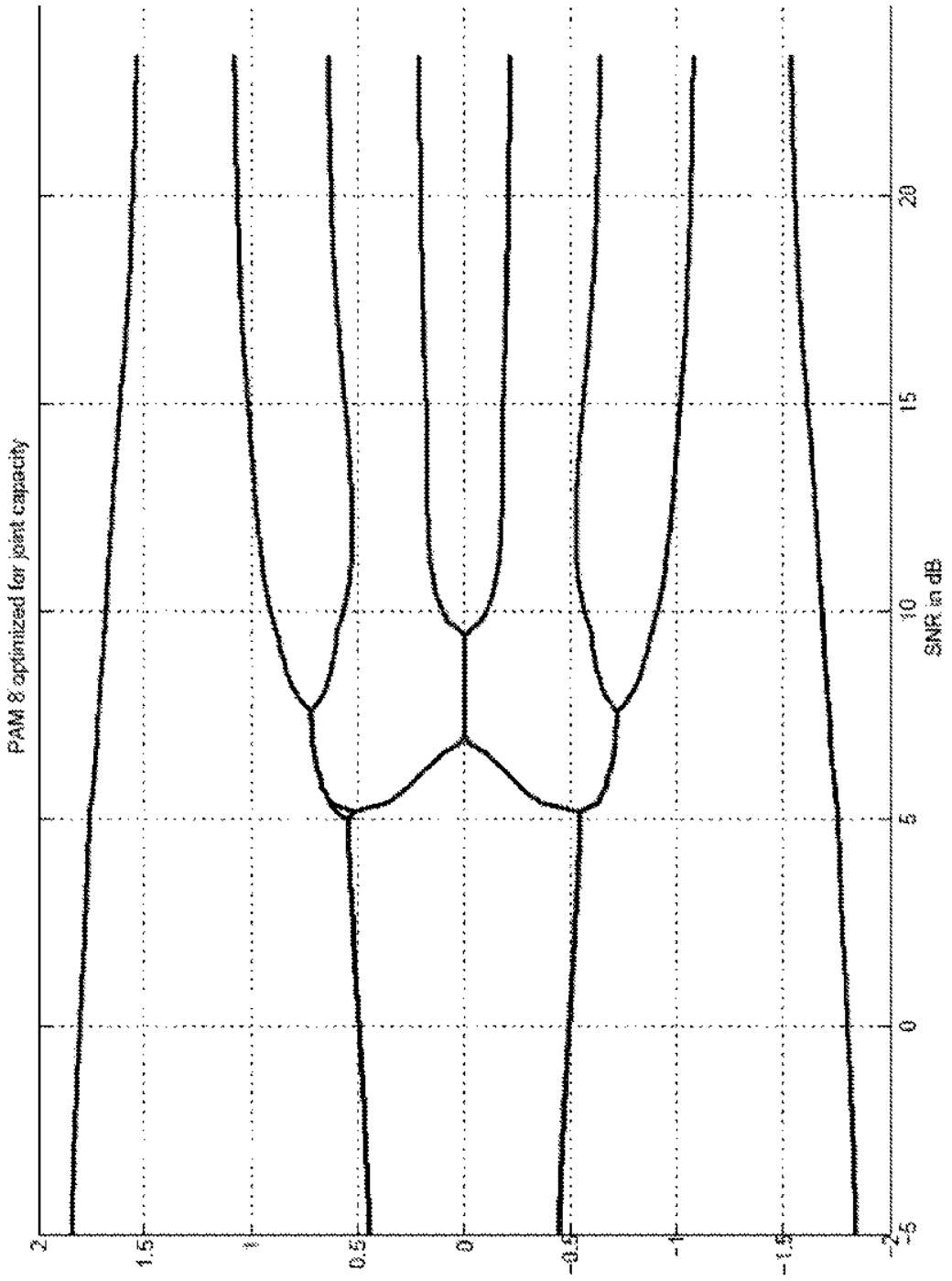

PAM 8 optimized for joint capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12b

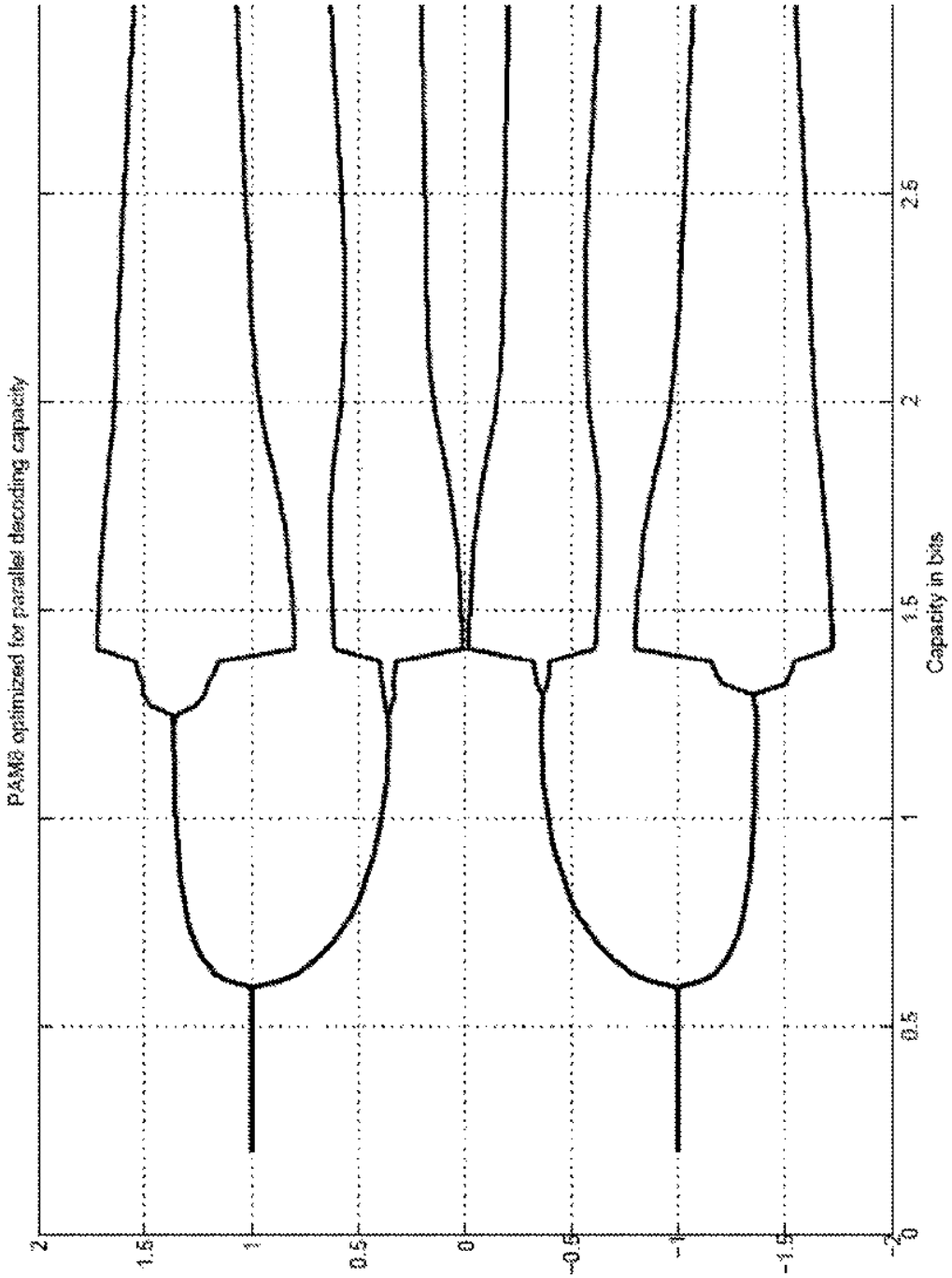

PAM8 optimized for parallel decoding capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12c

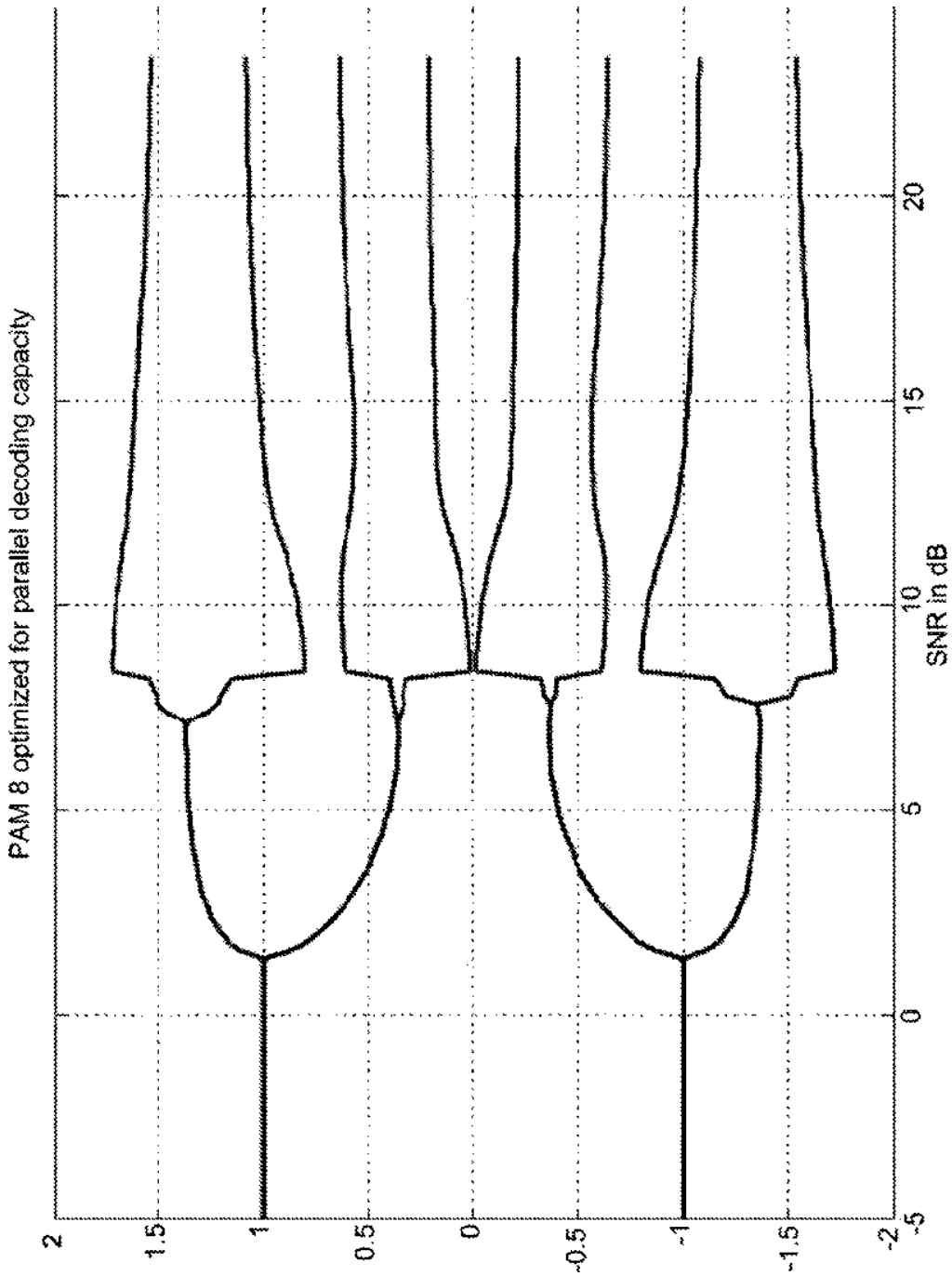

PAM 8 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5.27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

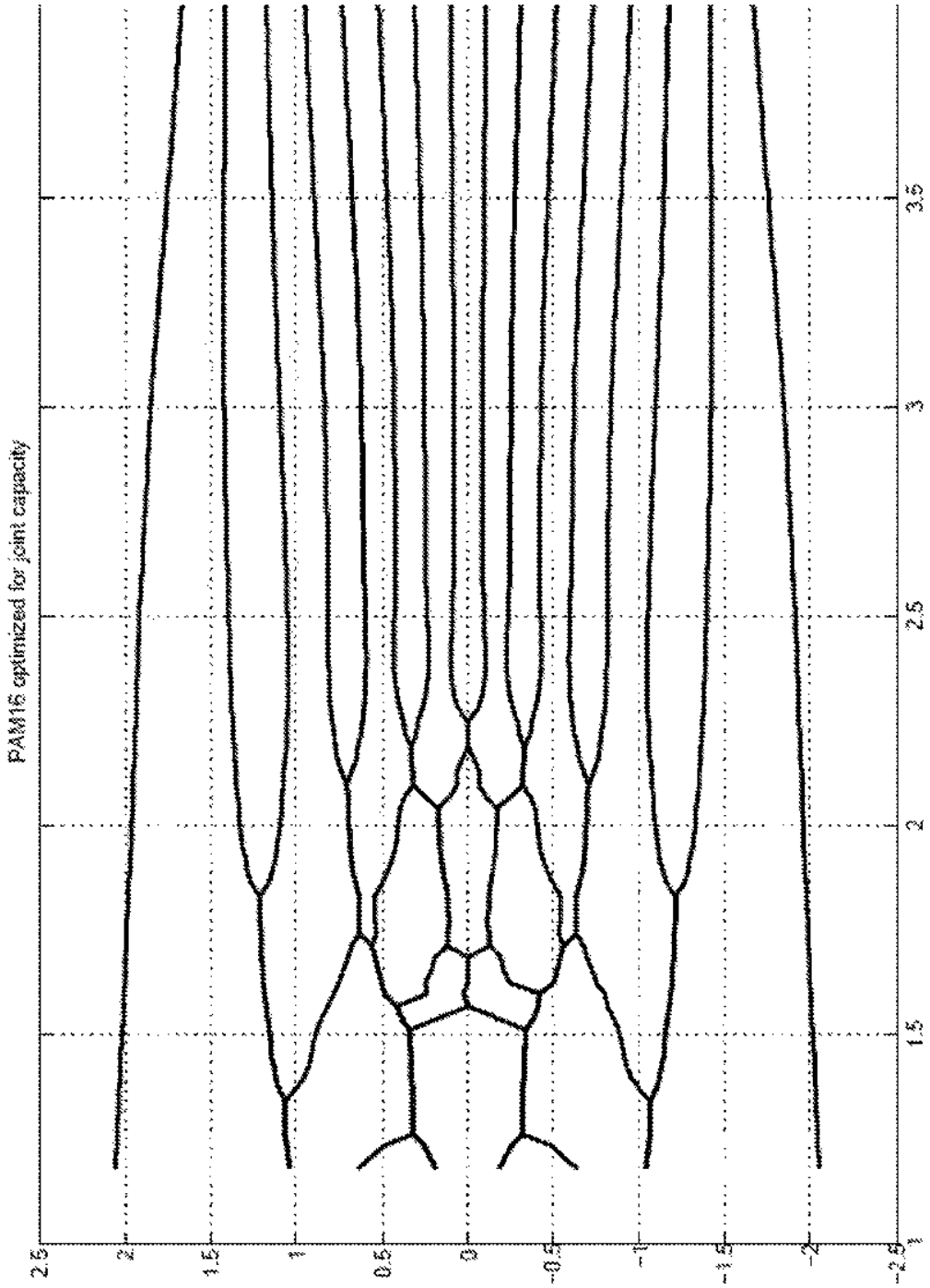

PAM16 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.5
2
2.5
3
3.5

FIG. 14a

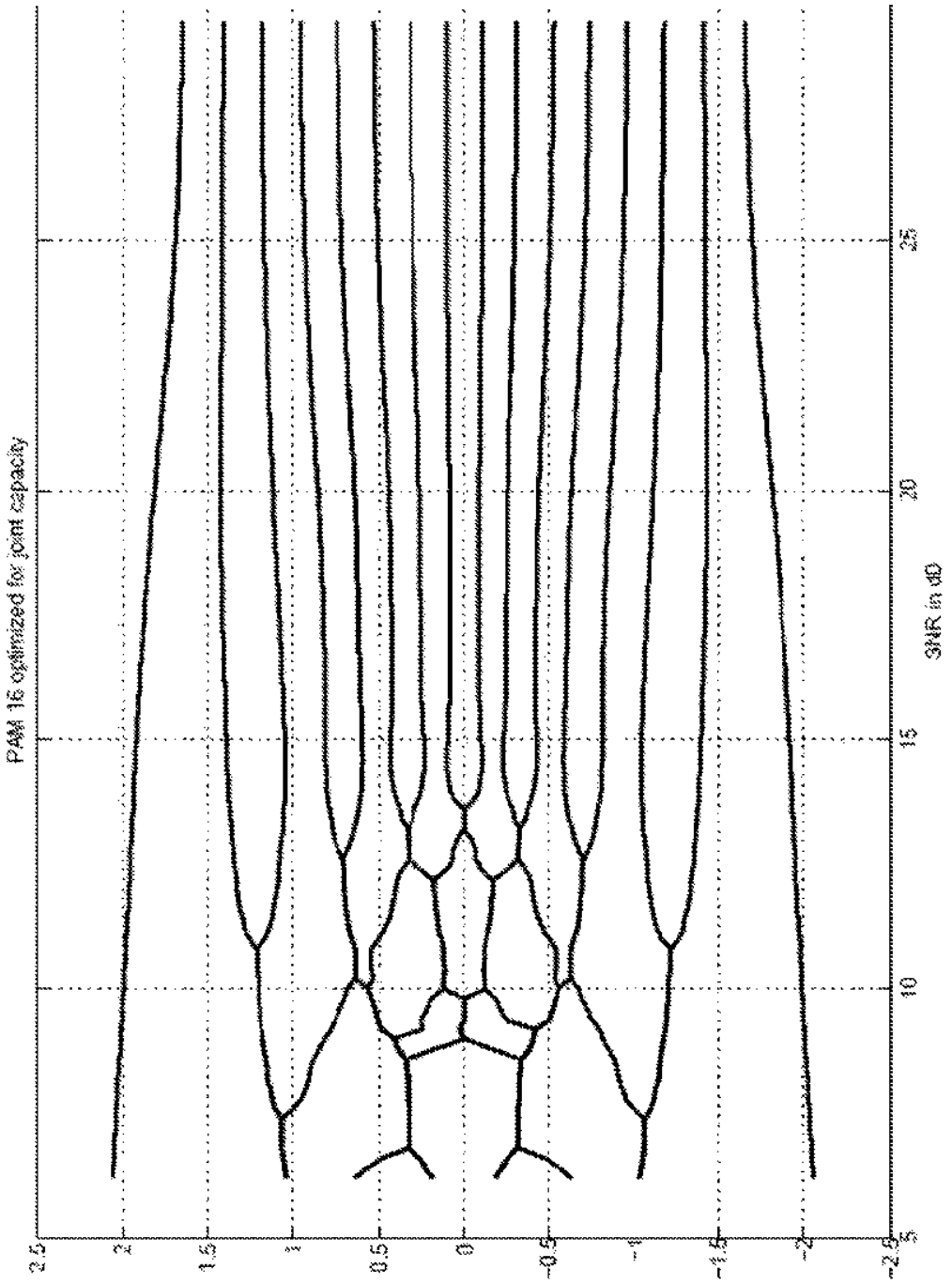

PAM 16 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14b

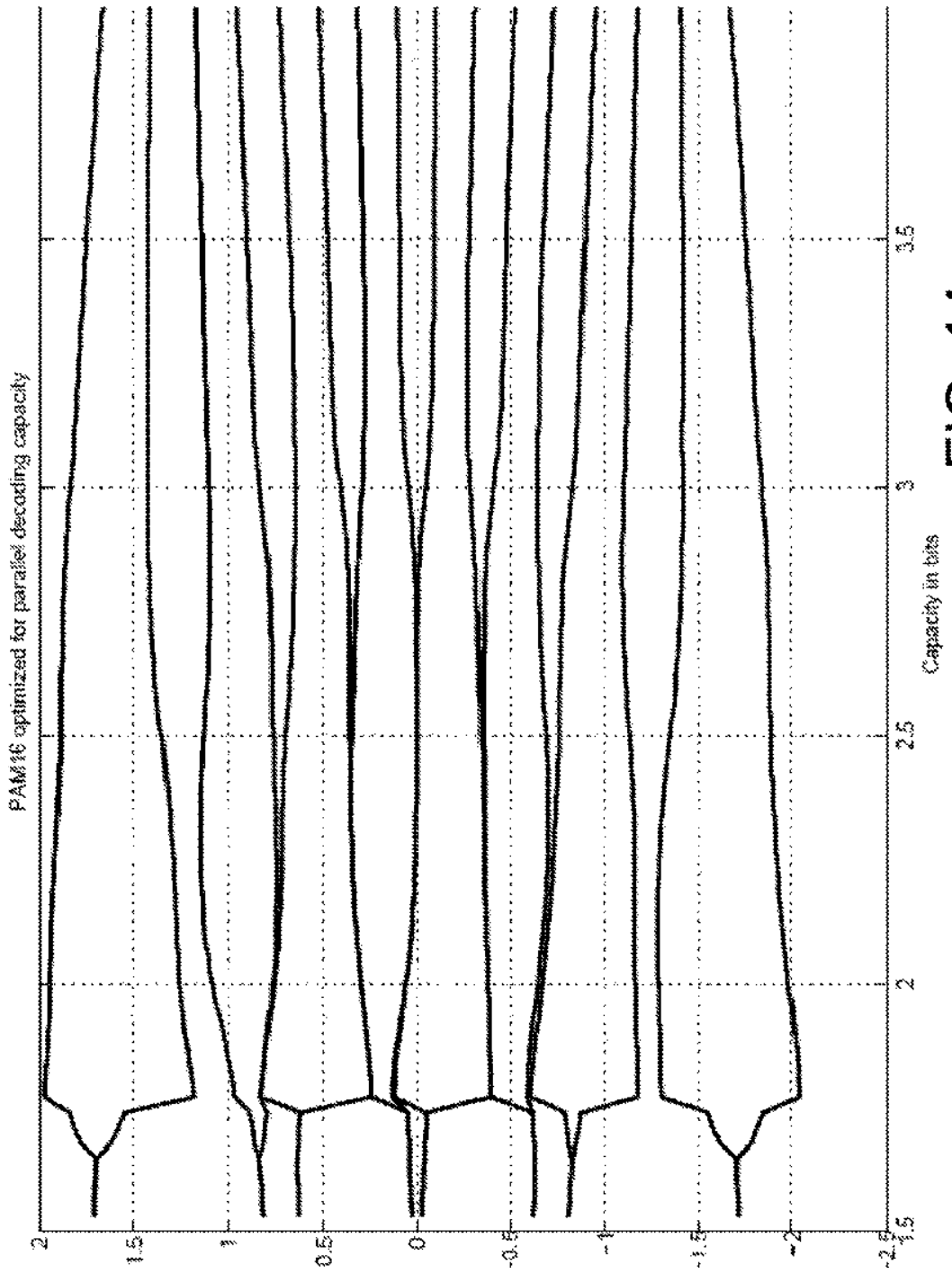

PAM16 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5

FIG. 14c

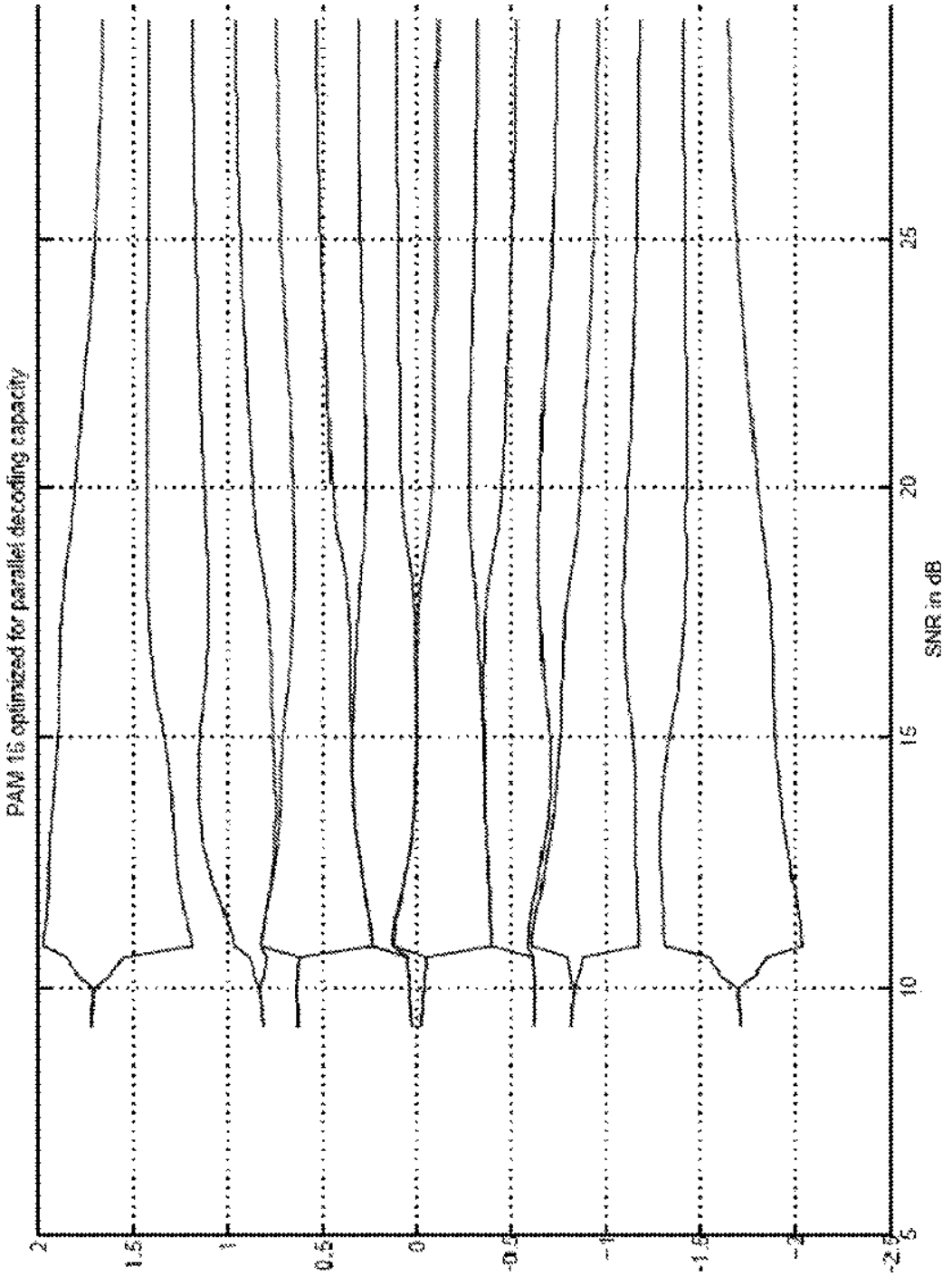

PAM 16 optimized for parallel decoding capacity
SNR in dB
25
20
15
10
5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

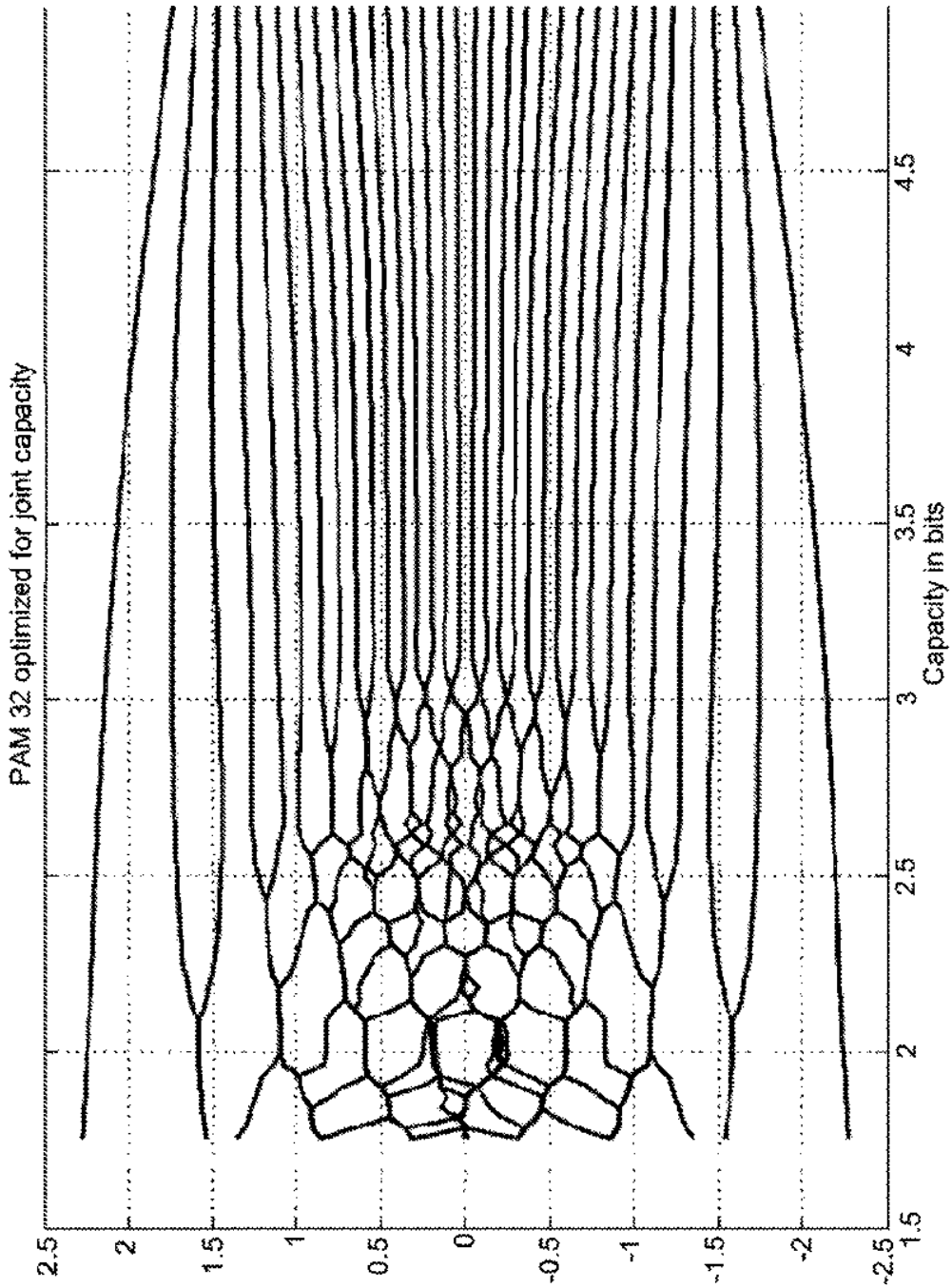

PAM 32 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16a

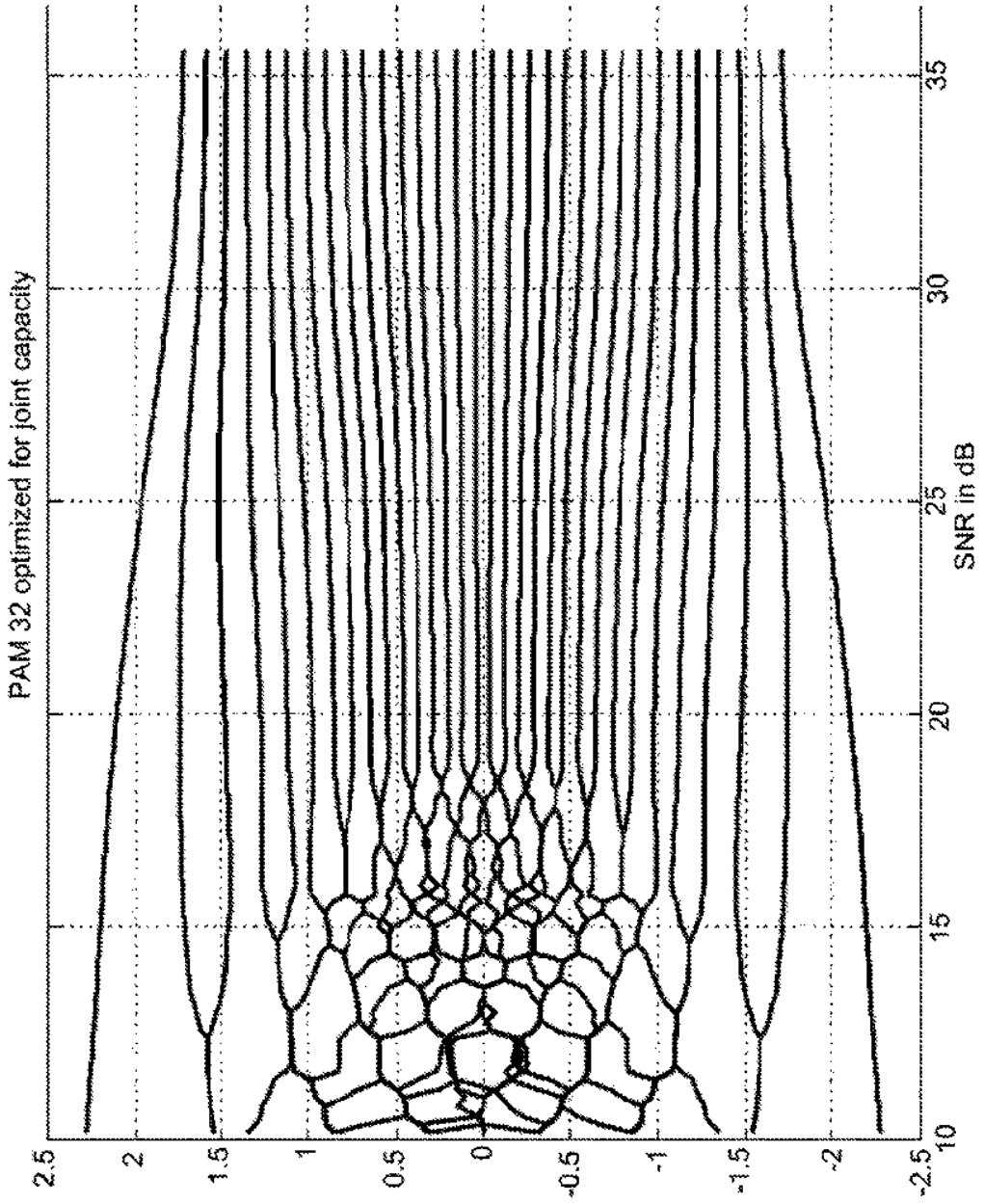

PAM 32 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35

FIG. 16b

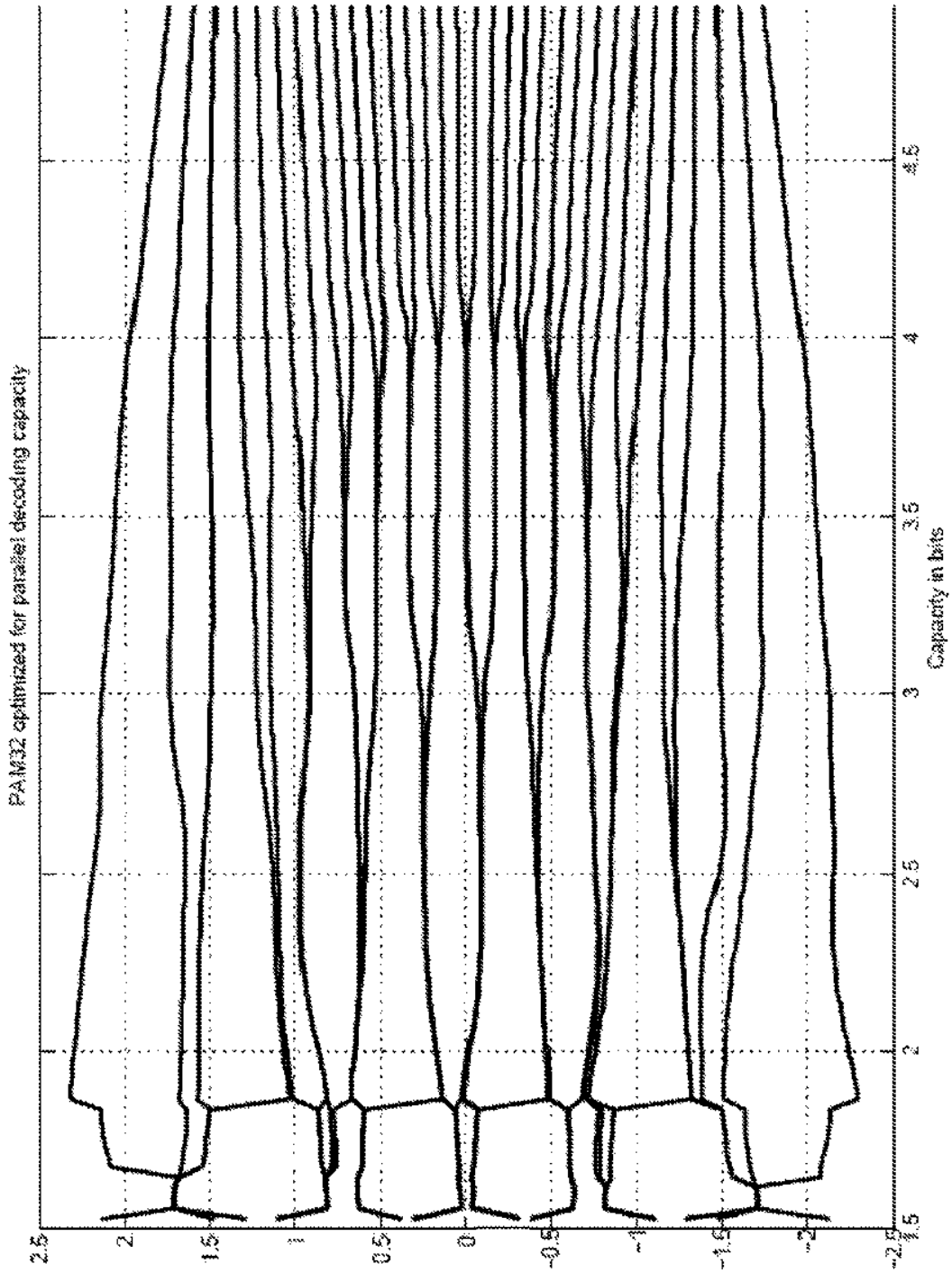

PAM32 optimized for parallel decoding capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16c

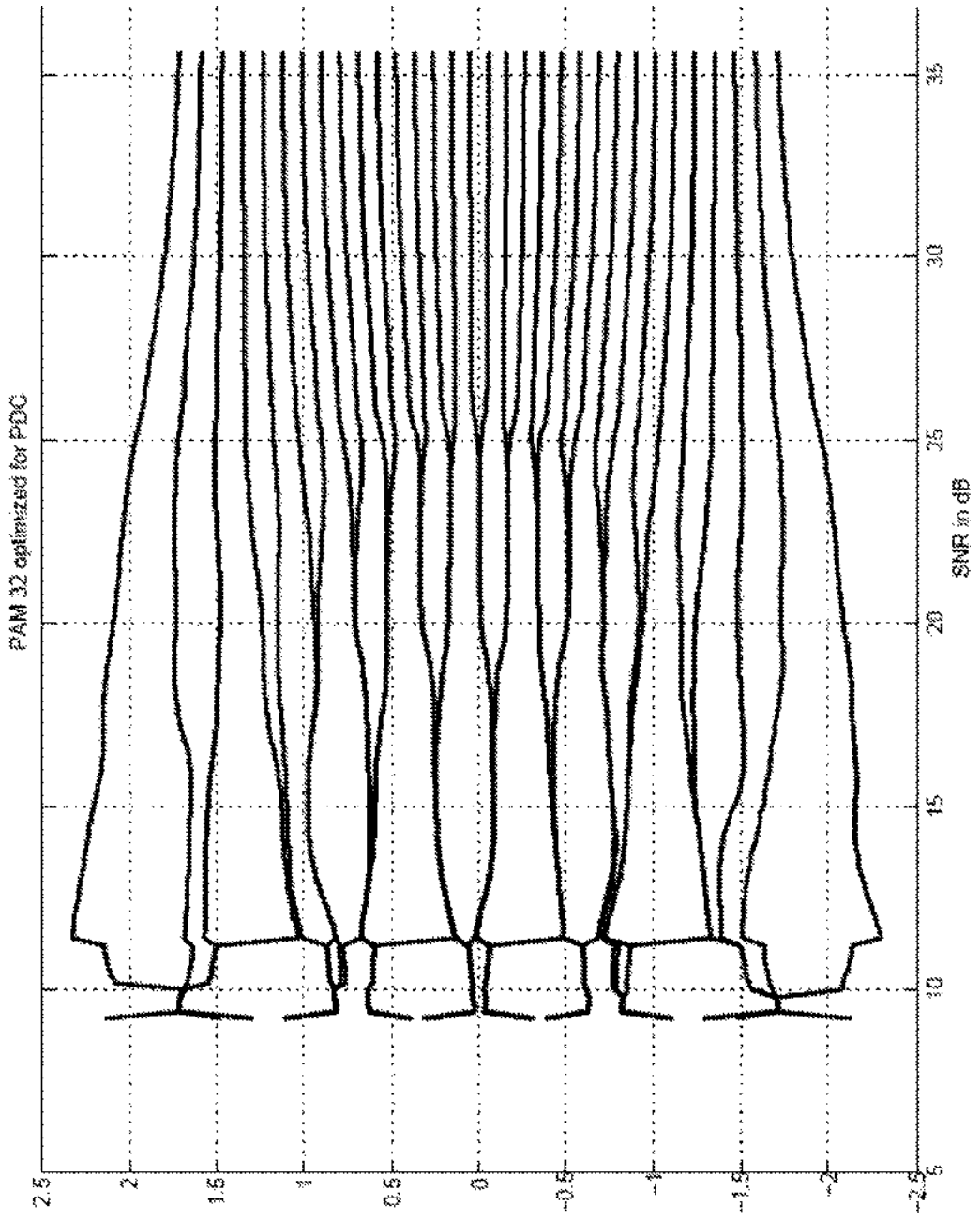

PAM 32 optimized for PDC
SNR in dB
35
30
25
20
15
10
5
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

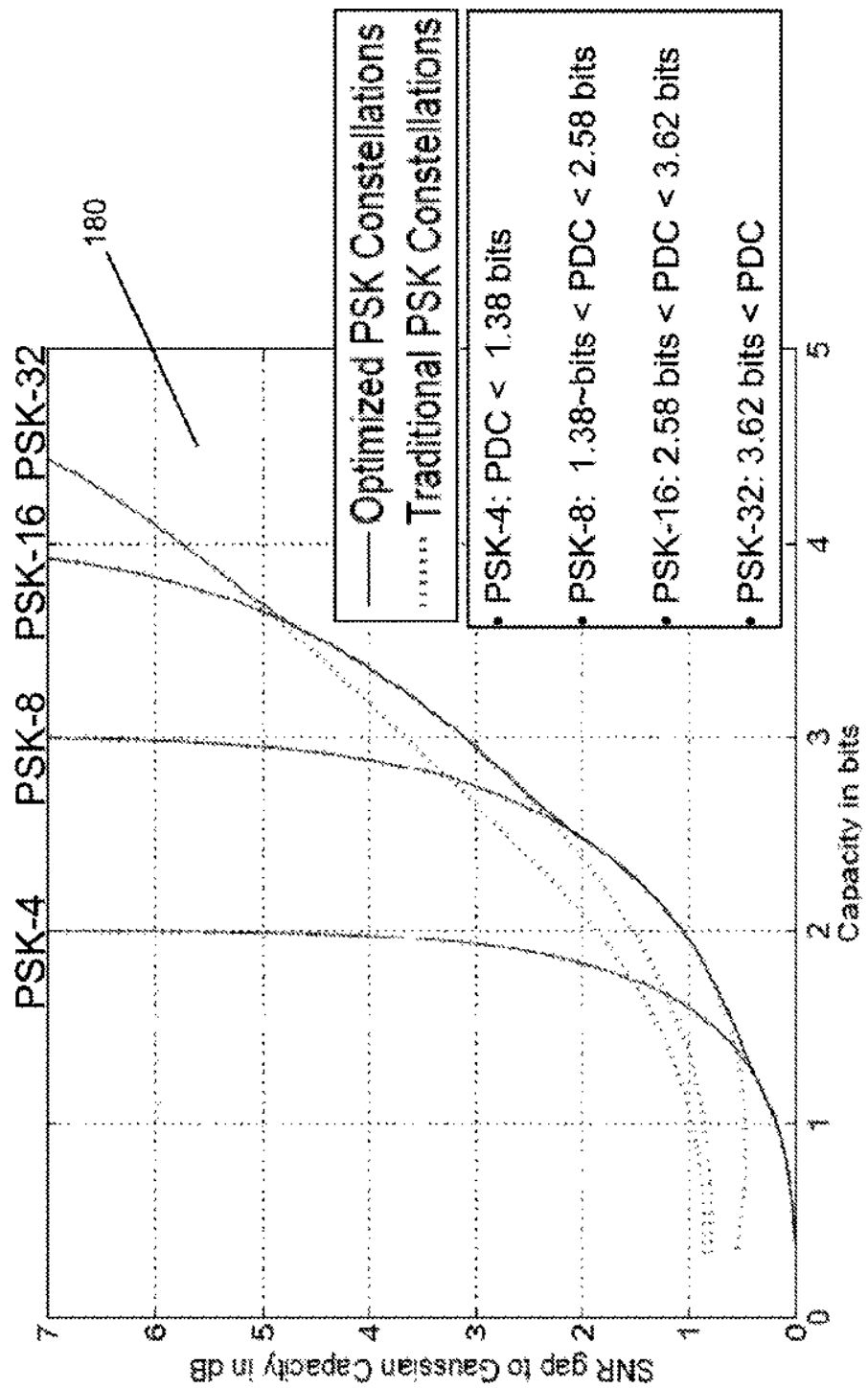

180
PSK-4
PSK-8
PSK-16
PSK-32
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38~bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

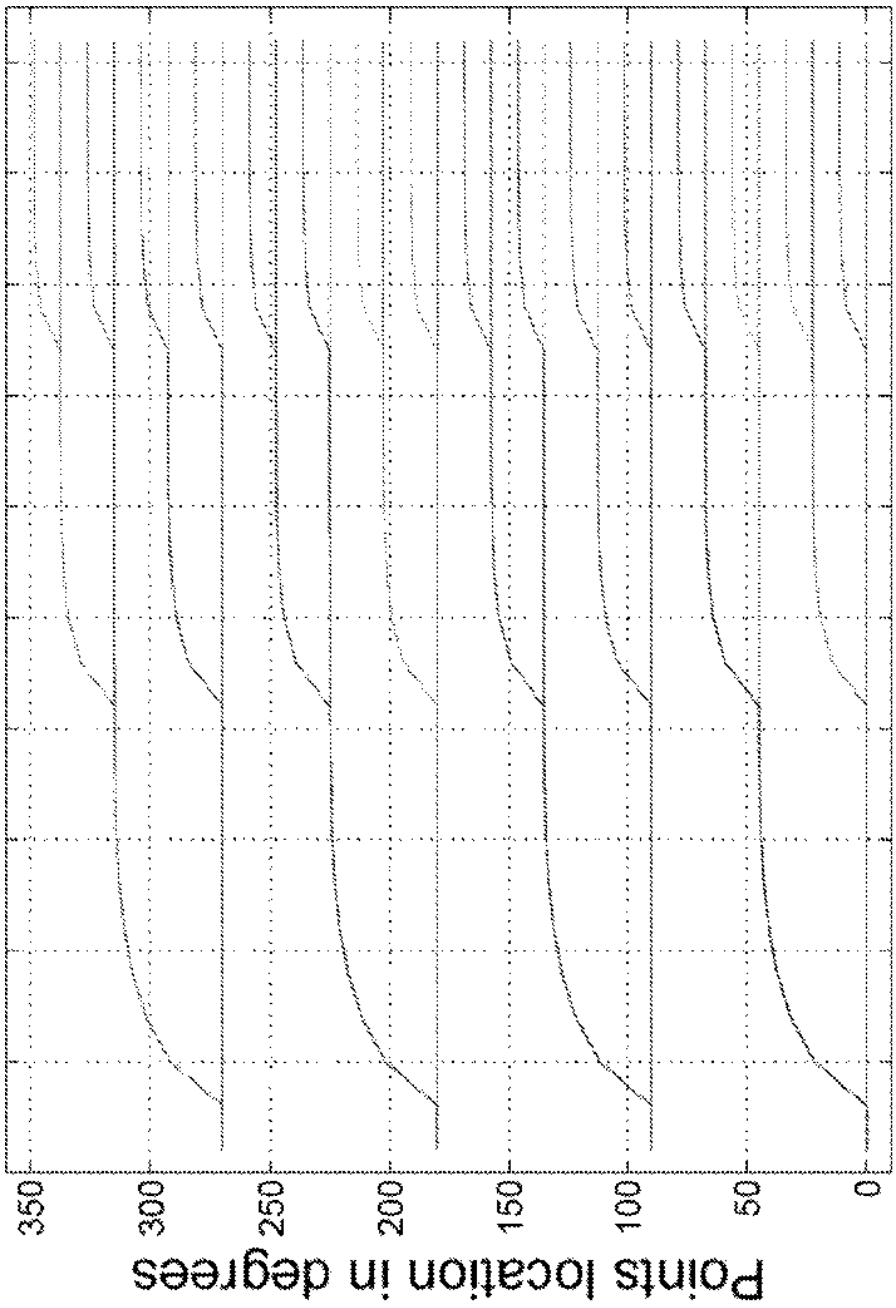

Points location in degrees
350
300
250
200
150
100
50
0
0
2
4
6
8
10
12
14
16
18
20
SNR in dB

FIG. 19

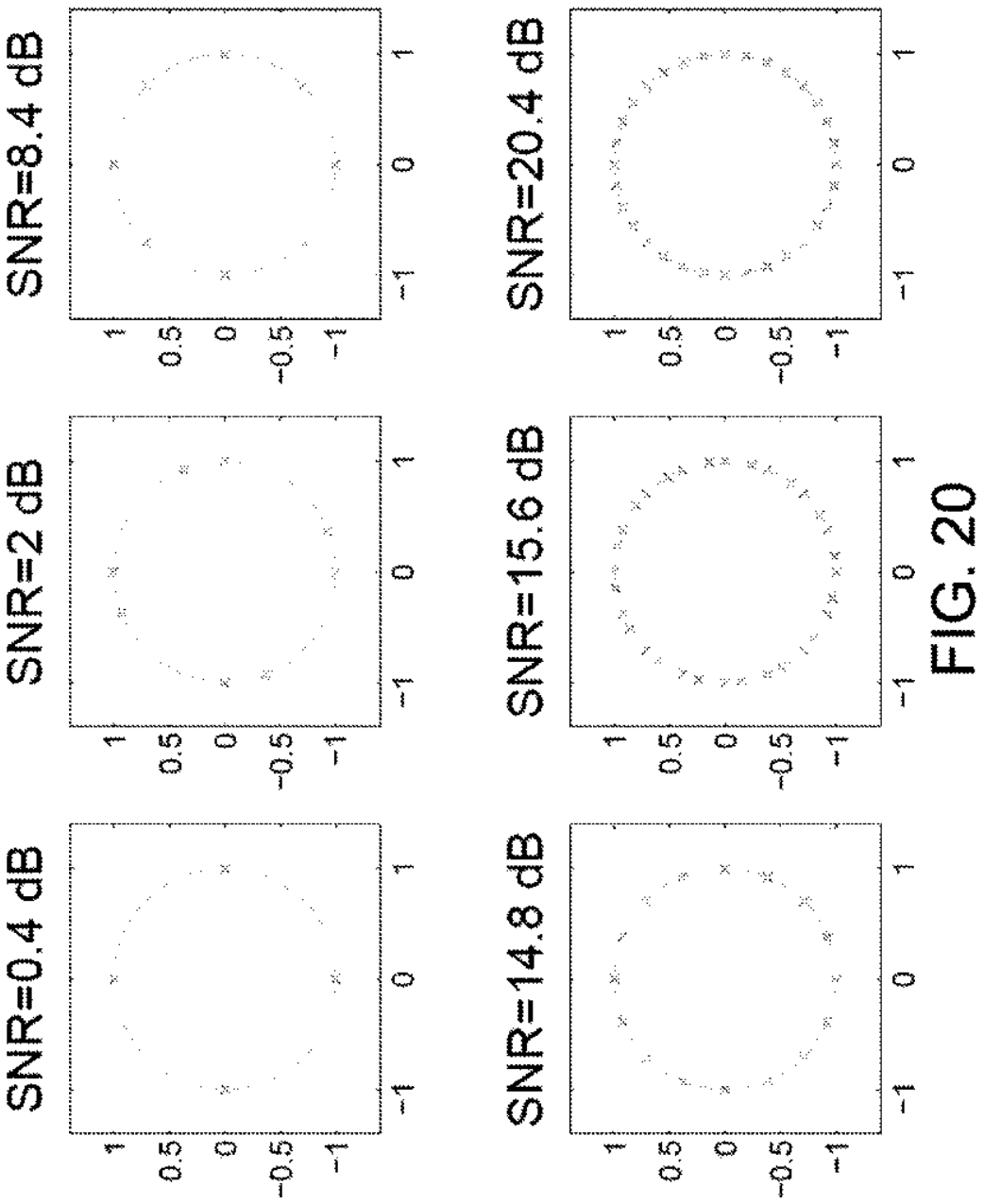

SNR=8.4 dB
SNR=20.4 dB
SNR=2 dB
SNR=15.6 dB
SNR=0.4 dB
SNR=14.8 dB
1
0.5
0
−0.5
−1

FIG. 20

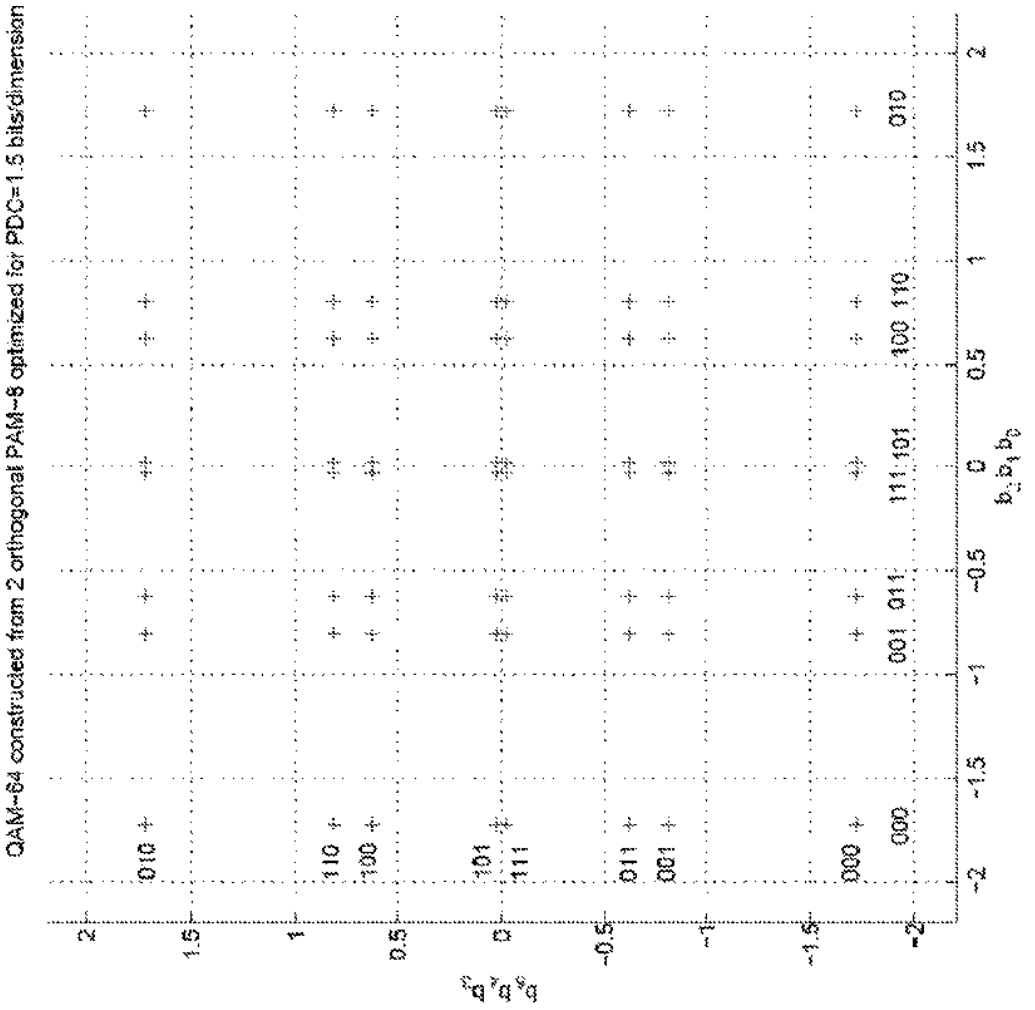

QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
$b_5 b_4 b_3$
$b_2 b_1 b_0$
010
110
100
101
111
011
001
000
001
011
111
101
100
110
010
000

FIG. 21

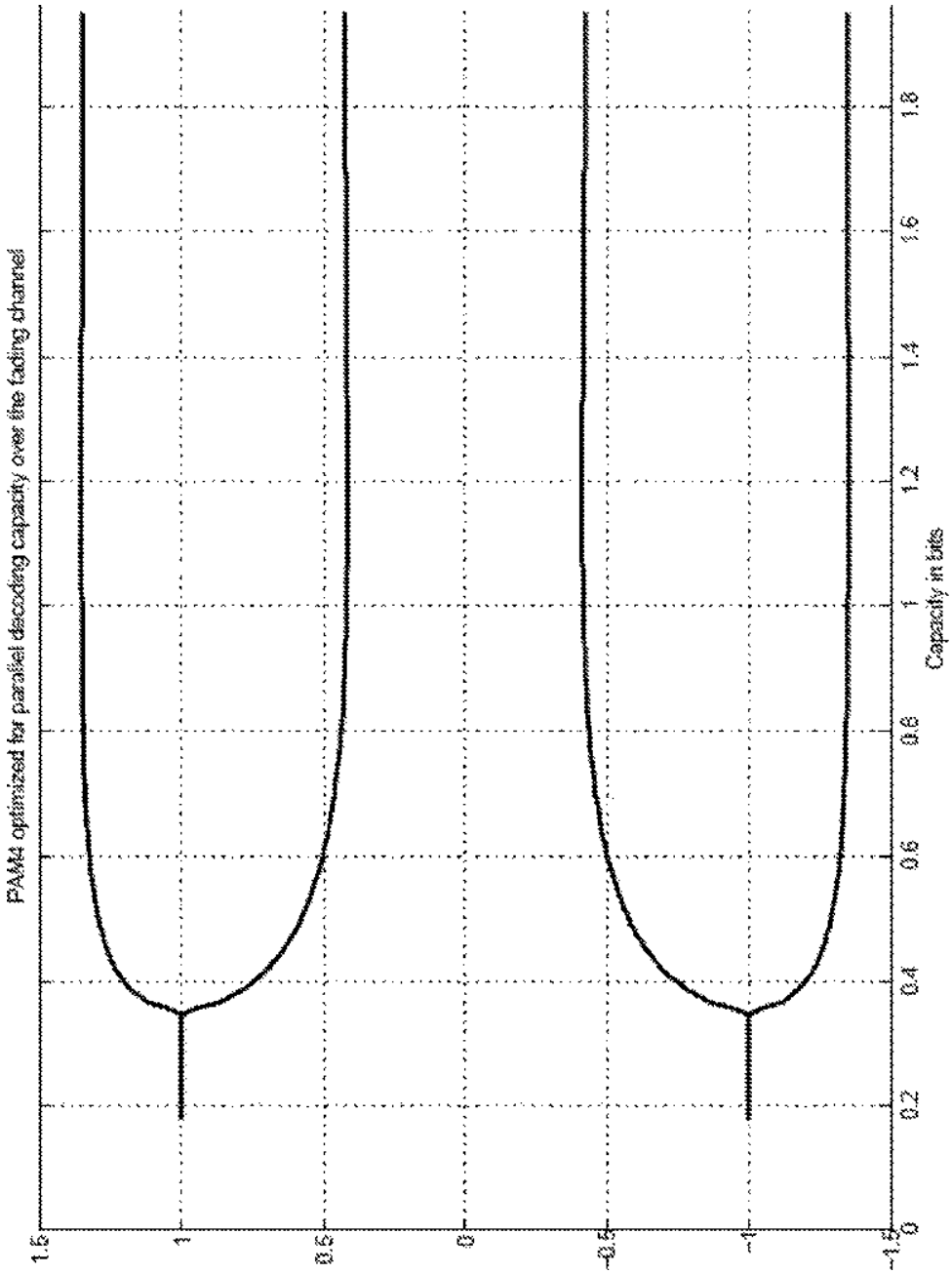

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 22a

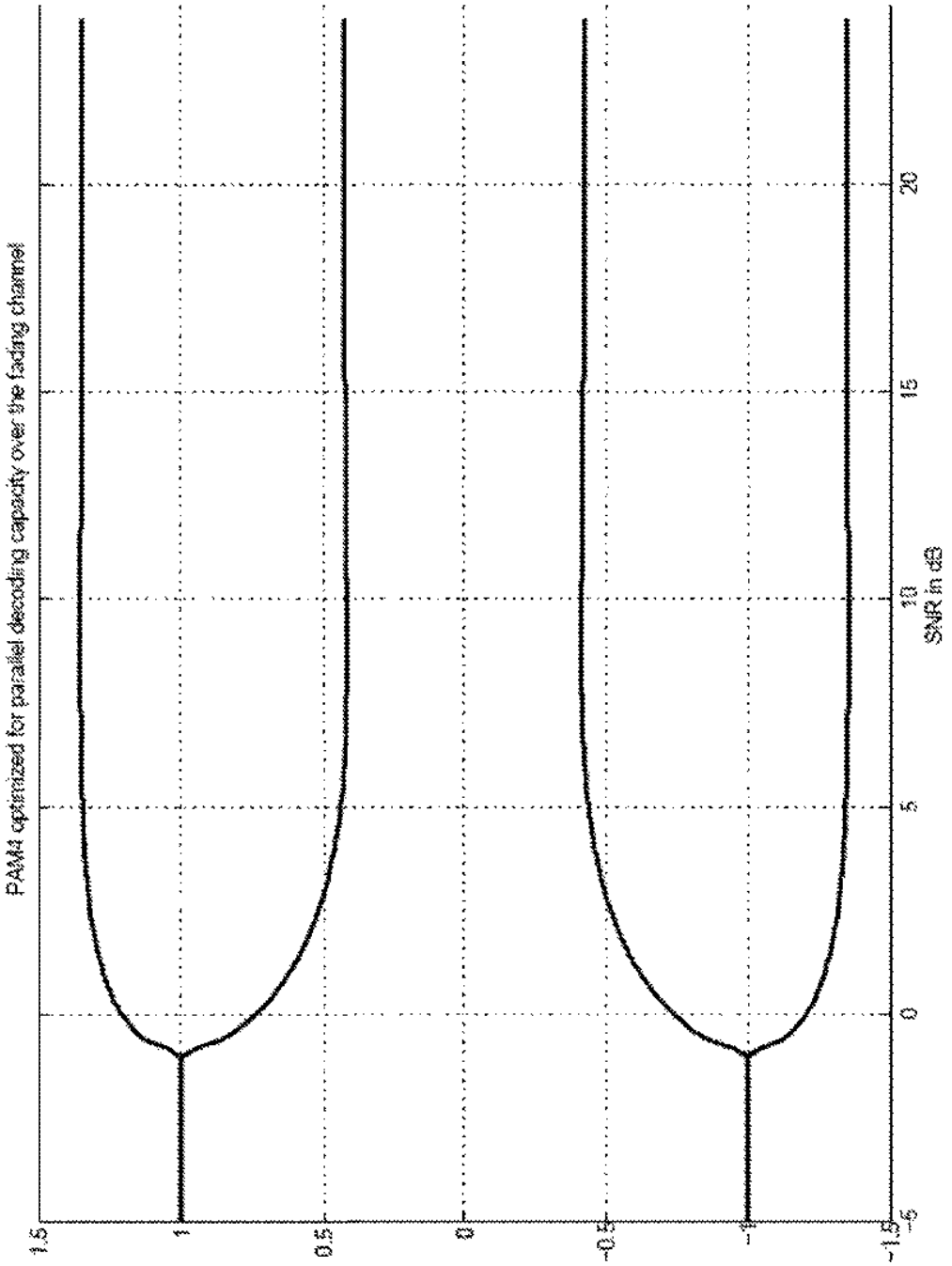

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15
20

FIG. 22b

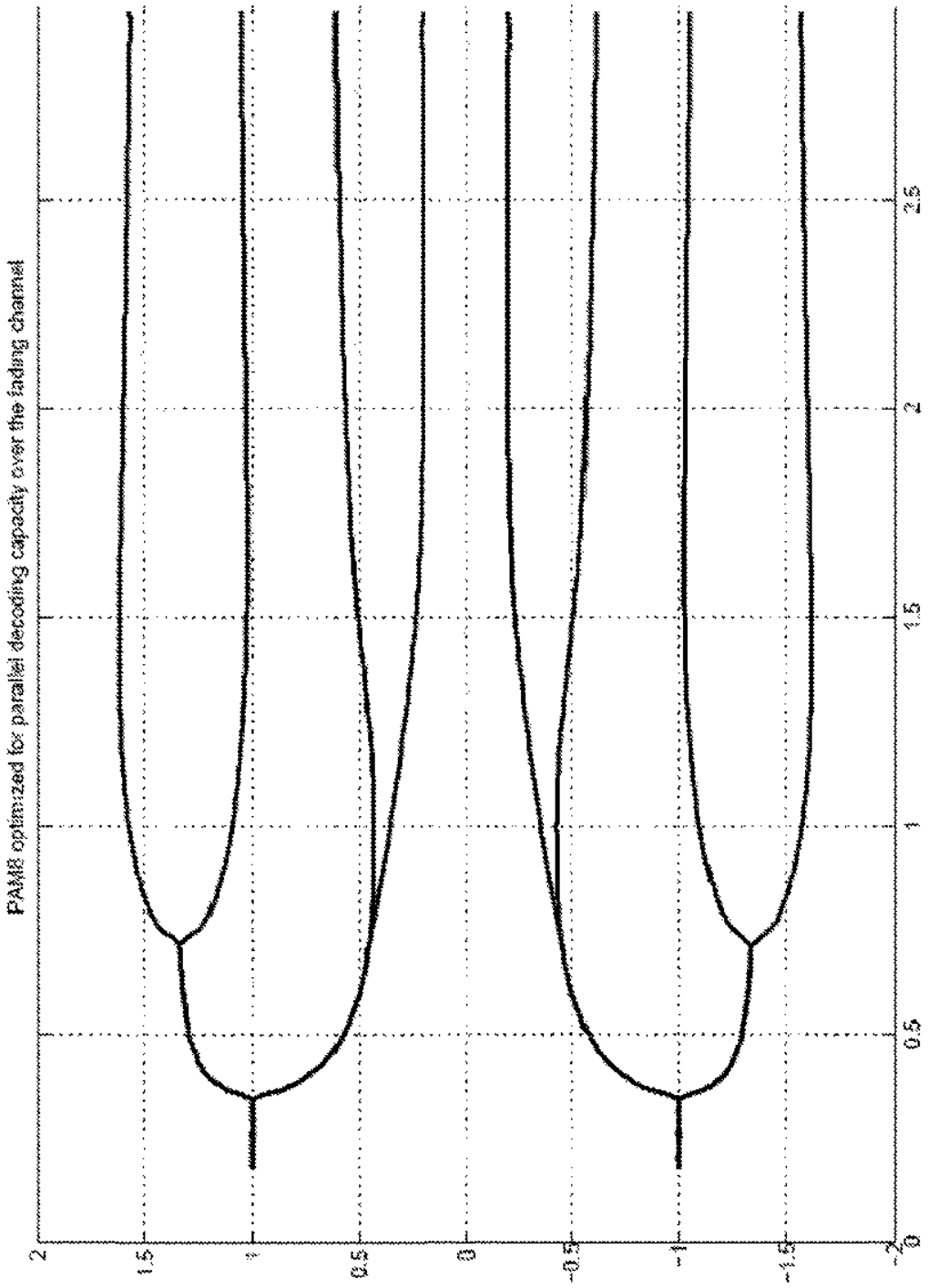

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
0
0.5
1
1.5
2
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 23a

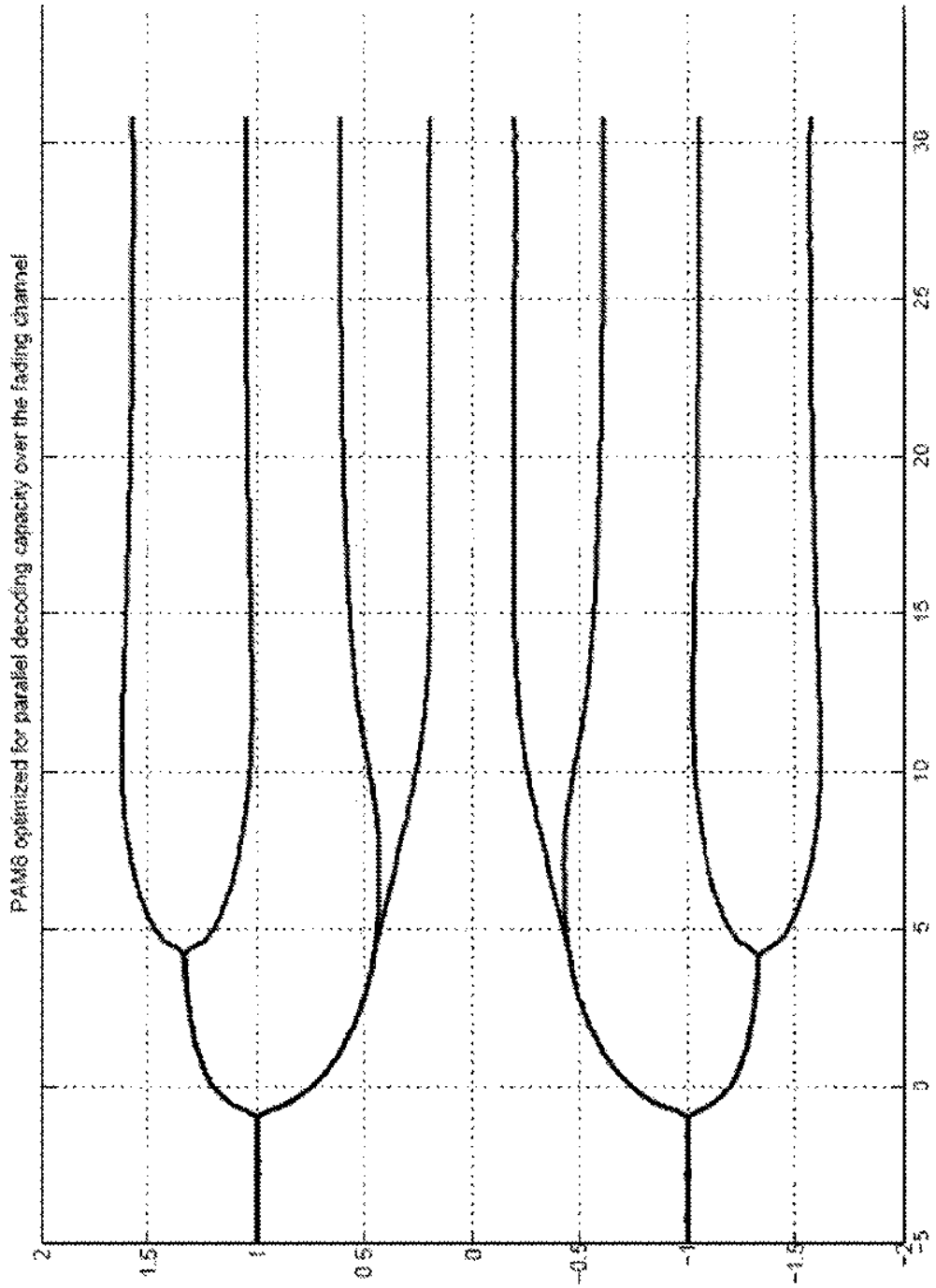

PAM8 optimized for parallel decoding capacity over the fading channel
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20
25
30

FIG. 23b

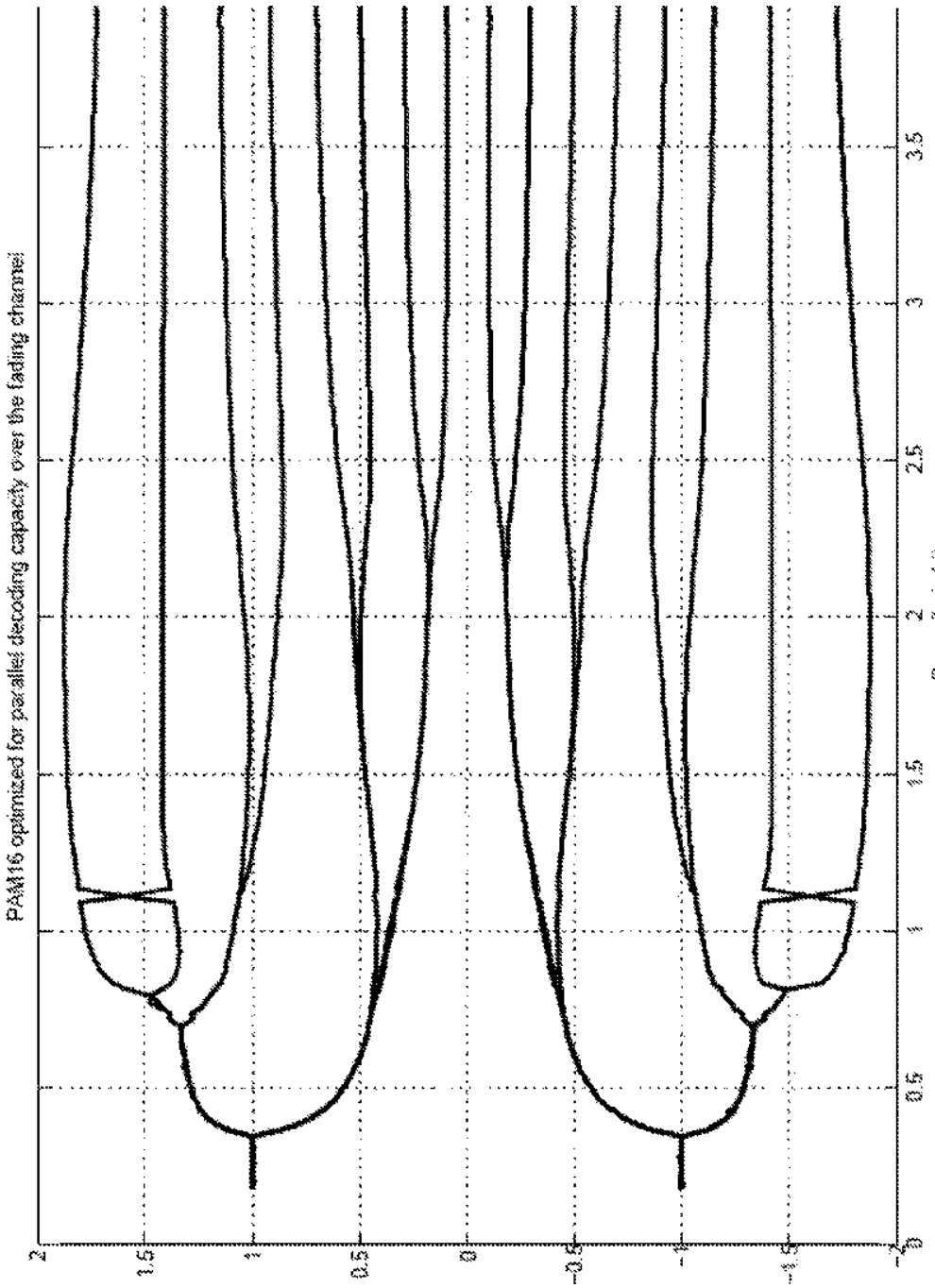

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5

FIG. 24a

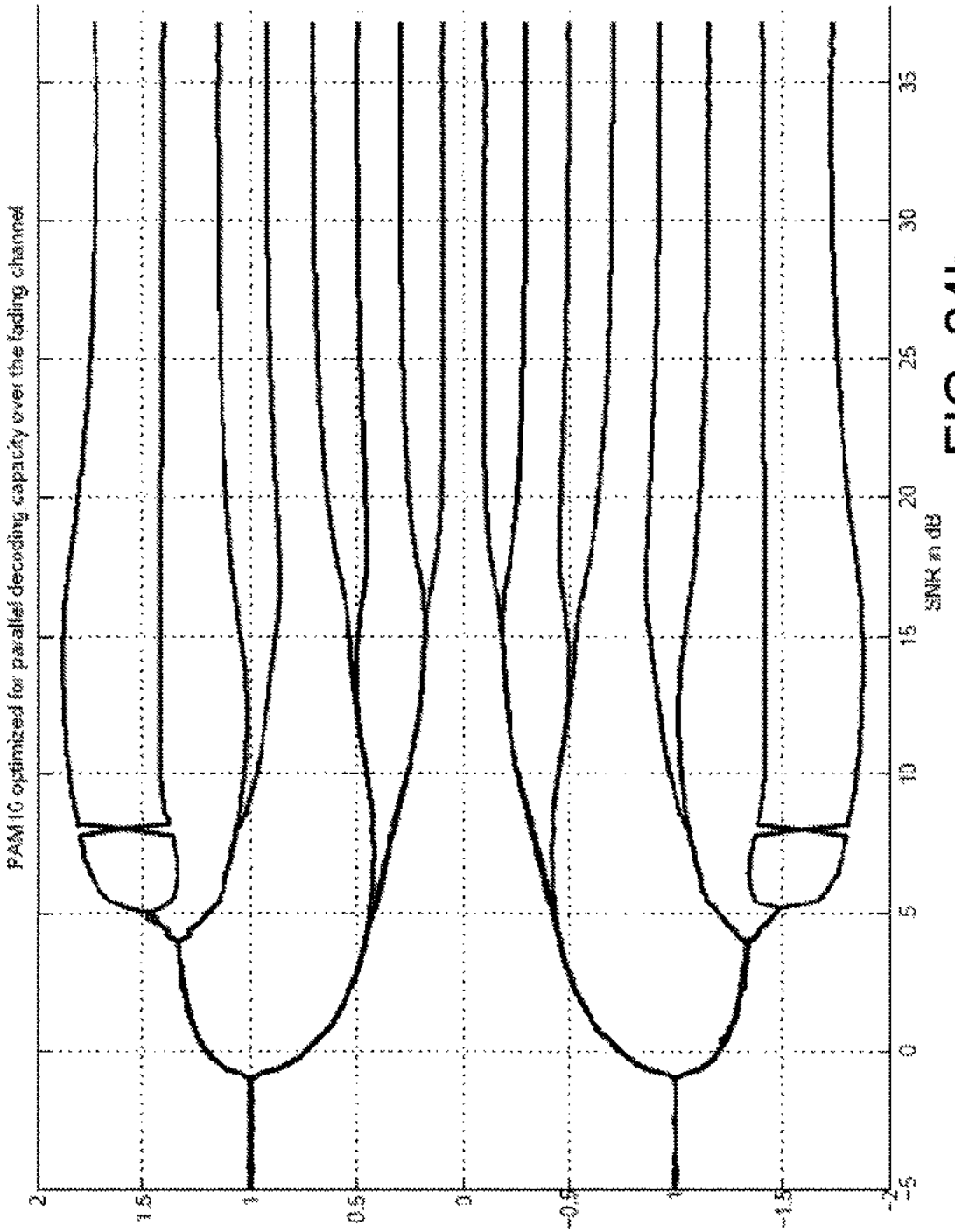

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20
25
30
35

FIG. 24b

Electronic Patent Application Fee Transmittal	
Application Number:	
Filing Date:	
Title of Invention:	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations
First Named Inventor/Applicant Name:	Maged F. Barsoum
Filer:	David J. Bailey/Agnes Poon
Attorney Docket Number:	C44-00999.CON15
Filed as Large Entity	

Filing Fees for Track I Prioritized Examination - Nonprovisional Application under 35 USC 111(a)

Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Basic Filing:				
UTILITY APPLICATION FILING	1011	1	300	300
UTILITY SEARCH FEE	1111	1	660	660
UTILITY EXAMINATION FEE	1311	1	760	760
REQUEST FOR PRIORITIZED EXAMINATION	1817	1	4000	4000
Pages:				
Claims:				
CLAIMS IN EXCESS OF 20	1202	10	100	1000
Miscellaneous-Filing:				

Description	Fee Code	Quantity	Amount	Sub-Total in USD($)
Petition:				
Patent-Appeals-and-Interference:				
Post-Allowance-and-Post-Issuance:				
Extension-of-Time:				
Miscellaneous:				
		Total in USD ($)		**6720**

Electronic Acknowledgement Receipt	
EFS ID:	38143591
Application Number:	16728384
International Application Number:	
Confirmation Number:	6350
Title of Invention:	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations
First Named Inventor/Applicant Name:	Maged F. Barsoum
Customer Number:	71897
Filer:	David J. Bailey/Agnes Poon
Filer Authorized By:	David J. Bailey
Attorney Docket Number:	C44-00999.CON15
Receipt Date:	27-DEC-2019
Filing Date:	
Time Stamp:	13:46:41
Application Type:	Utility under 35 USC 111(a)

Payment information:

Submitted with Payment	yes
Payment Type	CARD
Payment was successfully received in RAM	$6720
RAM confirmation Number	E2019BQD47144065
Deposit Account	504407
Authorized User	Agnes Poon
The Director of the USPTO is hereby authorized to charge indicated fees and credit any overpayment as follows: 37 CFR 1.16 (National application filing, search, and examination fees) 37 CFR 1.17 (Patent application and reexamination processing fees)	

37 CFR 1.19 (Document supply fees)

37 CFR 1.20 (Post Issuance fees)

37 CFR 1.21 (Miscellaneous fees and charges)

File Listing:

Document Number	Document Description	File Name	File Size(Bytes)/ Message Digest	Multi Part /.zip	Pages (if appl.)
1	TrackOne Request	00999con15track1req.pdf	125721 65ad7feaadac22b9cb8798f6968271c486de43b7	no	1
Warnings:					
Information:					
2	Application Data Sheet	00999con15ads.pdf	102386 98fb5f8c11922b3c12db4b40680fa31a3d222c03	no	7
Warnings:					
Information:					
This is not an USPTO supplied ADS fillable form					
3		00999con15dec.pdf	1643558 cb9e47fedf385f91cfc505adbcffeb6d4d564f42	yes	11

	Multipart Description/PDF files in .zip description		
	Document Description	**Start**	**End**
	Transmittal Letter	1	9
	Oath or Declaration filed	10	11

Warnings:					
Information:					
4		00999con15spec.pdf	256891 6fb457ae519f6691665b79d3db167e7d31bdf666	yes	35

	Multipart Description/PDF files in .zip description		
	Document Description	**Start**	**End**
	Specification	1	20
	Claims	21	34

	Abstract		35	35	
Warnings:					
Information:					
5	Drawings-only black and white line drawings	00999con15drawings.pdf	7564696 a3ee66b85bb8b0b408d515a8300f37c8138fe4da	no	43
Warnings:					
Information:					
6	Fee Worksheet (SB06)	fee-info.pdf	38892 6fbfbbcd8ead0b9a416ad7b7f1101d0ddb06c122	no	2
Warnings:					
Information:					
		Total Files Size (in bytes):	9732144		

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New Applications Under 35 U.S.C. 111
If a new application is being filed and the application includes the necessary components for a filing date (see 37 CFR 1.53(b)-(d) and MPEP 506), a Filing Receipt (37 CFR 1.54) will be issued in due course and the date shown on this Acknowledgement Receipt will establish the filing date of the application.
National Stage of an International Application under 35 U.S.C. 371
If a timely submission to enter the national stage of an international application is compliant with the conditions of 35 U.S.C. 371 and other applicable requirements a Form PCT/DO/EO/903 indicating acceptance of the application as a national stage submission under 35 U.S.C. 371 will be issued in addition to the Filing Receipt, in due course.
New International Application Filed with the USPTO as a Receiving Office
If a new international application is being filed and the international application includes the necessary components for an international filing date (see PCT Article 11 and MPEP 1810), a Notification of the International Application Number and of the International Filing Date (Form PCT/RO/105) will be issued in due course, subject to prescriptions concerning national security, and the date shown on this Acknowledgement Receipt will establish the international filing date of the application.

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

CERTIFICATE OF EFS WEB TRANSMISSION

I hereby certify that this correspondence is being electronically filed with the United States Patent and Trademark Office on December 27, 2019 at or before 11:59 p.m. Pacific Time under the Rules of 37 C.F.R. 1.8.

/Agnes Poon/
Agnes Poon

Appl No.	: Filed Herewith	Confirmation No.
Applicant	: Constellation Designs, LLC	
Filed	: December 27, 2019	
Title	: Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	
TC/A.U.	: Unassigned	
Examiner	: Unassigned	
Docket No.	: C44-00999.CON15	
Customer No.	: 71897	

INVENTOR'S OATH OR DECLARATION

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

2190 S. Towne Centre Place, Suite 300
Anaheim, CA 92806
December 27, 2019

Commissioner:

The instant application claims priority in accordance with 35 U.S.C. § 120 to applications including U.S. Patent Application No. 15/200,800, filed July 1, 2016. Applicant respectfully notes that declarations for inventors Maged F. Barsoum and Christopher R. Jones were filed on July 13, 2017 in U.S. Patent Application No. 15/200,800 with EFS ID 29780896 in accordance with 35 U.S.C. § 115(a), a copy of which is enclosed. Pursuant to 35 U.S.C. § 115(g)(1)(A), Applicant respectfully submits that Applicant need not file Inventor's Oath or Declarations in the instant application as declarations meeting the requirements of U.S.C. § 115(a) were executed by the inventors and filed in connection with the earlier-filed application. In accordance with 37 C.F.R. 1.63(d)(1), Applicant submits copies of the declarations previously filed in U.S. Patent Application No. 15/200,800.

Inventor's Oath or Declaration

The Commissioner is authorized to charge any fee under 37 CFR 1.16 and 1.17 that may be required during the **entire pendency** of the application to Deposit Account No. 50-4407. Please show our docket number with any transaction to the Deposit Account.

Respectfully submitted,
KPPB LLP

By /David J. Bailey/
David J. Bailey
Registration No. 59,632
949.852.0000

DJB/ap

Electronic Acknowledgement Receipt

EFS ID:	29780896
Application Number:	15200800
International Application Number:	
Confirmation Number:	5631
Title of Invention:	Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellations
First Named Inventor/Applicant Name:	Maged F. Barsoum
Customer Number:	71897
Filer:	Isaac M. Fine/Katrina Lee
Filer Authorized By:	Isaac M. Fine
Attorney Docket Number:	JB-00999.CON4
Receipt Date:	13-JUL-2017
Filing Date:	01-JUL-2016
Time Stamp:	21:32:42
Application Type:	Utility under 35 USC 111(a)

Payment information:

Submitted with Payment	yes
Payment Type	CARD
Payment was successfully received in RAM	$570
RAM confirmation Number	071417INTEFSW21332800
Deposit Account	504407
Authorized User	Katrina Lee
The Director of the USPTO is hereby authorized to charge indicated fees and credit any overpayment as follows: 37 CFR 1.16 (National application filing, search, and examination fees) 37 CFR 1.17 (Patent application and reexamination processing fees)	

37 CFR 1.19 (Document supply fees)

37 CFR 1.20 (Post issuance fees)

37 CFR 1.21 (Miscellaneous fees and charges)

File Listing:

Document Number	Document Description	File Name	File Size(Bytes)/ Message Digest	Multi Part /.zip	Pages (if appl.)
1	Non Patent Literature	agrell.pdf	426943	no	14
Warnings:					
Information:					
2	Non Patent Literature	choi.pdf	867970	no	2
Warnings:					
Information:					
3	Non Patent Literature	fabregaserror.pdf	256716	no	7
Warnings:					
Information:					
4	Non Patent Literature	fabvegas.pdf	241995	no	15
Warnings:					
Information:					
5	Non Patent Literature	fragouli.pdf	1798623	no	5
Warnings:					
Information:					
6	Non Patent Literature	khoo.pdf	210467	no	5
Warnings:					
Information:					

7	Non Patent Literature	lee.pdf	146113	no	3
Warnings:					
Information:					
8	Non Patent Literature	ngo.pdf	435781	no	7
Warnings:					
Information:					
9	Non Patent Literature	ngoperformanceofnon.pdf	59924	no	2
Warnings:					
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10	Non Patent Literature	raphaelianimproved.pdf	230842	no	6
Warnings:					
Information:					
11	Non Patent Literature	goff.pdf	608813	no	2
Warnings:					
Information:					
12	Non Patent Literature	goffsignal.pdf	533386	no	7
Warnings:					
Information:					
13	Non Patent Literature	schreckenbach.pdf	209659	no	10
Warnings:					
Information:					

14	Non Patent Literature	soutoiterative.pdf	330376	no	6
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15	Non Patent Literature	souto.pdf	142767	no	5
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Information:					
16	Non Patent Literature	theodorakopoulos comp.pdf	964630	no	6
Warnings:					
Information:					
17	Non Patent Literature	vitthaladevuni.pdf	1046325	no	11
Warnings:					
Information:					
18	Non Patent Literature	yuan.pdf	294190	no	3
Warnings:					
Information:					
19	Transmittal Letter	00999con4decxtml.pdf	112373	no	1
Warnings:					
Information:					
20	Oath or Declaration filed	00999con4decs.pdf	138902	no	2
Warnings:					
Information:					

21	Power of Attorney	00999con4poa.pdf	259183	no	2
Warnings:					
Information:					
22	Transmittal Letter	00999con4idsxtml.pdf	118179	no	2
Warnings:					
Information:					
23	Information Disclosure Statement (IDS) Form (SB08)	00999con4ids1449.pdf	114306	no	3
Warnings:					
Information:					
This is not an USPTO supplied IDS fillable form					
24	Issue Fee Payment (PTO-85B)	00999con4issuefeextml.pdf	100195	no	2
Warnings:					
Information:					
25	Fee Worksheet (SB06)	fee-info.pdf	32331	no	2
Warnings:					
Information:					
		Total Files Size (in bytes):	9760899		

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New Applications Under 35 U.S.C. 111
If a new application is being filed and the application includes the necessary components for a filing date (see 37 CFR 1.53(b)-(d) and MPEP 506), a Filing Receipt (37 CFR 1.54) will be issued in due course and the date shown on this Acknowledgement Receipt will establish the filing date of the application.
National Stage of an International Application under 35 U.S.C. 371
If a timely submission to enter the national stage of an international application is compliant with the conditions of 35 U.S.C. 371 and other applicable requirements a Form PCT/DO/EO/903 indicating acceptance of the application as a national stage submission under 35 U.S.C. 371 will be issued in addition to the Filing Receipt, in due course.
New International Application Filed with the USPTO as a Receiving Office
If a new international application is being filed and the international application includes the necessary components for an international filing date (see PCT Article 11 and MPEP 1810), a Notification of the International Application Number and of the International Filing Date (Form PCT/RO/105) will be issued in due course, subject to prescriptions concerning national security, and the date shown on this Acknowledgement Receipt will establish the international filing date of the application.

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

CERTIFICATE OF EFS WEB TRANSMISSION

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/Katina Lee/
Katina Lee

Appl No.	: 15/200,800	Confirmation No. 5831
Applicant	: Constellation Designs, Inc.	
Filed	: July 1, 2016	
Title	: Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellations	
TC/A.U.	: 2634	
Examiner	: Tesfaldet Bocure	
Docket No.	: J3-00999.CON4	
Customer No.	: 71897	

TRANSMITTAL OF INVENTOR DECLARATIONS

Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

2400 E. Katella, Suite 1050
Anaheim, CA 92806
July 13, 2017

Commissioner:

Enclosed are the signed Declarations of the inventors listed in this case.

The Commissioner is authorized to charge any fee under 37 CFR 1.16 and 1.17 which may be required during the **entire pendency** of the application to Deposit Account No. 50-4407. Please show our docket number with any transaction to the Deposit Account.

Respectfully submitted,

KPPB LLP

By /Isaac M. Fine/
Isaac M. Fine
Registration No. 74,274
949.852.0000

IMF/kl

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Application Number: 16728384 Document Date: 12/27/2019

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U.S. Patent and Trademark Office; U.S. DEPARTMENT OF COMMERCE

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INFORMATION DISCLOSURE STATEMENT BY APPLICANT (Not for submission under 37 CFR 1.99)		
	Application Number	16/728,384
	Filing Date	Dec 27, 2019
	First Named Inventor	Maged F. Barsoum
	Art Unit	Unassigned
	Examiner Name	Unassigned
	Attorney Docket Number	C44-00999.CON15

U.S.PATENTS

Examiner Initial*	Cite No	Patent Number	Kind Code [1]	Issue Date	Name of Patentee or Applicant of cited Document	Pages, Columns, Lines, Where Relevant Passages or Relevant Figures Appear
		5289501		1994-02-22	Seshadri et al.	
		5862179		1999-01-19	Goldstein et al.	
		6084915	A	2000-07-04	Williams et al.	
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		6157678		2000-12-05	Wei	
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	Art Unit	Unassigned
	Examiner Name	Unassigned
	Attorney Docket Number	C44-00999.CON15

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	Attorney Docket Number	C44-00999.CON15

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EXAMINER SIGNATURE

Examiner Signature		Date Considered	

*EXAMINER: Initial if reference considered, whether or not citation is in conformance with MPEP 609. Draw line through a citation if not in conformance and not considered. Include copy of this form with next communication to applicant.

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[4] Kind of document by the appropriate symbols as indicated on the document under WIPO Standard ST.16 if possible. [5] Applicant is to place a check mark here if English language translation is attached.

PATENT

IN THE UNITED STATES PATENT AND TRADEMARK OFFICE

CERTIFICATE OF EFS WEB TRANSMISSION

I hereby certify that this correspondence is being electronically filed with the United States Patent and Trademark Office on January 9, 2020 at or before 11:59 p.m. Pacific Time under the Rules of 37 C.F.R. 1.8.

/Katrina Lee/
Katrina Lee

Appl No.	: 16/728,384	Confirmation No. 6350
Applicant	: Constellation Designs, LLC	
Filed	: December 27, 2019	
Title	: Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations	
TC/A.U.	: Unassigned	
Examiner	: Unassigned	
Docket No.	: C44-00999.CON15	
Customer No.	: 71897	

INFORMATION DISCLOSURE STATEMENT § 1.97(b)

MAIL STOP AMENDMENT
Commissioner for Patents
P.O. Box 1450
Alexandria, VA 22313-1450

2190 S. Towne Centre Place, Suite 300
Anaheim, CA 92806
January 9, 2020

Commissioner

In compliance with the duty of disclosure under 37 C.F.R. §§ 1.56, 1.97 and 1.98, and in accordance with the provisions in the MPEP §§ 609 and 707.05(b), enclosed is FORM PTO/SB/08A/B with a listing of references that are known to applicant. Copies of each of the references listed under “Foreign Patent Documents” and “Non-Patent Literature Documents" are enclosed.

It is respectfully requested that these references be considered in the examination of this application and identified on the list of references cited on the patent issuing on this application. Applicant also requests that an initialed copy of said FORM PTO/SB/08A/B be entered in the application file, and returned to the Applicant with the next official communication from the Office in accordance with MPEP § 609.

It is believed that there are no fees due in connection with this filing. If there are any fees due in connection with this filing, the Commissioner is authorized to charge Deposit Account 50-4407. Please show our docket number with any charge to the Deposit Account.

Respectfully submitted,

KPPB LLP

/Isaac M. Fine/
Isaac M. Fine
Registration No. 74,274
949.852.0000

IMF/kl

Enclosures

Electronic Acknowledgement Receipt	
EFS ID:	38241709
Application Number:	16728384
International Application Number:	
Confirmation Number:	6350
Title of Invention:	Receivers Incorporating Non-Uniform Constellations with Overlapping Constellation Point Locations
First Named Inventor/Applicant Name:	Maged F. Barsoum
Customer Number:	71897
Filer:	Isaac M. Fine/Katrina Lee
Filer Authorized By:	Isaac M. Fine
Attorney Docket Number:	C44-00999.CON15
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File Listing:

Document Number	Document Description	File Name	File Size(Bytes)/ Message Digest	Multi Part /.zip	Pages (if appl.)
1	Foreign Reference	JP09130438A.pdf	315397 0816ff6e0d2853d068011a7c2632c5f6148baebc	no	6
Warnings:					

Information:					
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21	Other Reference-Patent/App/Search documents	writtenopinion65994.pdf	383790 6070fbe3932beec276a68e20413c91a2a5edd683	no	4
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Information:					
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Warnings:					
Information:					
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27	Non Patent Literature	agrellonthebicm.pdf	687059 07e2913217fdb98efa8f55e26a0f538060734553	no	53
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Information:					
29	Non Patent Literature	agrelloptimal.pdf	740198 288f03f4e203acda12f5852fd7f3def28b7e7362	no	24
Warnings:					
Information:					

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If a new application is being filed and the application includes the necessary components for a filing date (see 37 CFR 1.53(b)-(d) and MPEP 506), a Filing Receipt (37 CFR 1.54) will be issued in due course and the date shown on this Acknowledgement Receipt will establish the filing date of the application.

National Stage of an International Application under 35 U.S.C. 371

If a timely submission to enter the national stage of an international application is compliant with the conditions of 35 U.S.C. 371 and other applicable requirements a Form PCT/DO/EO/903 indicating acceptance of the application as a national stage submission under 35 U.S.C. 371 will be issued in addition to the Filing Receipt, in due course.

New International Application Filed with the USPTO as a Receiving Office

If a new international application is being filed and the international application includes the necessary components for an international filing date (see PCT Article 11 and MPEP 1810), a Notification of the International Application Number and of the International Filing Date (Form PCT/RO/105) will be issued in due course, subject to prescriptions concerning national security, and the date shown on this Acknowledgement Receipt will establish the international filing date of the application.

Espacenet

Bibliographic data: JPH09130438 (A) — 1997-05-16

RADIO EQUIPMENT

Inventor(s): TOMABECHI AKITAKA; KANDA JUN; OTSUBO TETSUO ± (TOMABECHI AKITAKA, ; KANDA JUN, ; OTSUBO TETSUO)

Applicant(s): MATSUSHITA ELECTRIC IND CO LTD ± (MATSUSHITA ELECTRIC IND CO LTD)

Classification: - **international:** ***H03M7/16; H04L25/497; H04L27/18;*** (IPC1-7): H03M7/16; H04L25/497; H04L27/18

- **cooperative:**

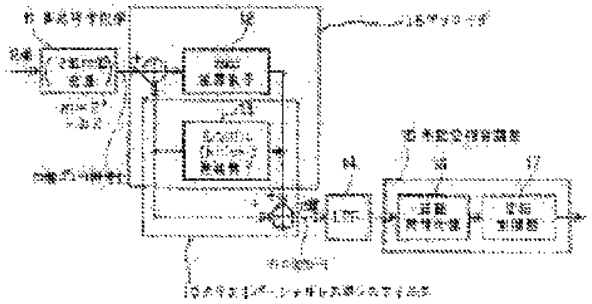
Application number: JP19950280259 19951027

Priority number(s): JP19950280259 19951027

Also published as: JP3273880 (B2)

Abstract of JPH09130438 (A)

PROBLEM TO BE SOLVED: To obtain a radio equipment whose transmission band is made narrow and reception sensitivity is improved by adopting the constant envelope differential phase modulation system and optimizing the phase assignment in partial response. SOLUTION: A multiple value coding section 11 applies m-value of gray coding to binary data in k-bit each. A mod arithmetic element 12 and a 1-symbol delay element 13 form a pre-coder 18 for class 1 partial response. Furthermore, a 1-symbol delay element 13 forms a class 1 partial response filter 19 converting the m-value into an n-value symbol. The frequency band of the output is limited by an LPF 14 and differential-coded by a differential coder 16 and a phase modulator 17 provides an output of a phase modulation signal.



(19)日本国特許庁（ＪＰ）　(12) **公開特許公報**（Ａ）　(11)特許出願公開番号

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(43)公開日　平成９年(1997)５月16日

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Ｈ０４Ｌ　25/497		9199 -5K	Ｈ０４Ｌ　25/497	

技術表示箇所

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(71)出願人　000005821
松下電器産業株式会社
大阪府門真市大字門真1006番地

(72)発明者　苫　米　地　明　孝
神奈川県横浜市港北区綱島東四丁目３番１号　松下通信工業株式会社内

(72)発明者　神　山　潤
神奈川県横浜市港北区綱島東四丁目３番１号　松下通信工業株式会社内

(72)発明者　大　坪　哲　郎
神奈川県横浜市港北区綱島東四丁目３番１号　松下通信工業株式会社内

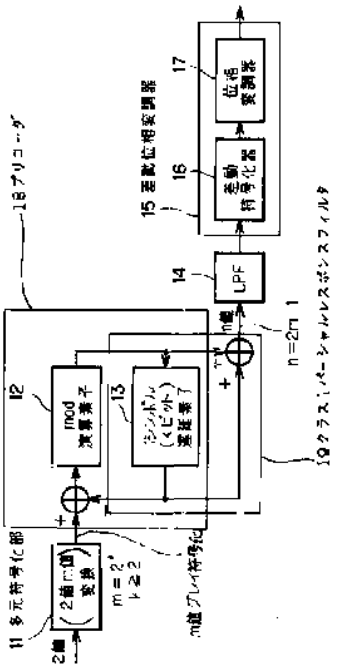
(74)代理人　弁理士　蔵合　正博

(54)【発明の名称】　無線装置

(57)【要約】

【課題】　定包絡差動位相変調方式を使用し、パーシャルレスポンスの位相割り当てを最適とすることにより、狭帯域化とともに受信感度の向上を図る。

【解決手段】　２値をｋビット毎のグレイ符号化されたｍ値に変換する多元符号化部１１と、クラス１パーシャルレスポンスを行なうプリコーダ１８と、ｍ値をｎ値（ｎ＝２ｍ－１）のシンボルに変換するためのクラス１パーシャルレスポンスフィルタ１９と、帯域制限を行なうＬＰＦ１４と、非線形の位相割り当てを行なう差動位相変調器１５とから構成する。



【特許請求の範囲】
【請求項１】　連続的に入力された２値系列をｋビット毎のグレイ符号化されたｍ値系列（ｍ＝2^k）に変換する多元符号化部と、クラス１パーシャルレスポンスを行なうためのプリコーダと、ｍ値をｎ値（ｎ－２ｍ－１）のシンボルに変換するためのクラス１パーシャルレスポンスフィルタと、帯域制限を行なうＬＰＦと、非線形の位相割り当てを行なう差動符号化器および位相変調器からなる差動位相変調器とを備えた無線装置。
【請求項２】　差動位相変調器による差動位相変調において、発生頻度の高いシンボルはシンボル間距離を大きくし、発生頻度の低いシンボルはシンボル間距離を小さくする非線形の位相割り当てを行なう請求項１記載の無線装置。
【請求項３】　非線形の位相割り当てにおいて、信号間距離が等差数列となるように位相割り当てを行なう請求項２記載の無線装置。
【発明の詳細な説明】
【０００１】
【発明の属する技術分野】本発明は、狭帯域化を実現するための定包絡差動位相変調を行なうデジタル式の無線装置に関する。
【０００２】
【従来の技術】従来、この種の定包絡差動位相変調を行なう無線装置としては、４相ＰＳＫ変調方式やデュオバイナリが広く利用されてきた。そして、より狭帯域化を行なうために、８相ＰＳＫ変調方式やデュオカテナリを利用するといった方法も提案されている。
【０００３】
【発明が解決しようとする課題】しかしながら、上記従来の８相ＰＳＫ変調方式やデュオカテナリといった方式は、同一ビットレートで狭帯域化を図る場合、受信感度が著しく劣化するという問題があった。
【０００４】本発明は、このような従来の問題を解決するものであり、同一ビットレートで狭帯域化を行なっても、８相ＰＳＫ変調方式やデュオカテナリといった方式に比べて、受信感度を向上できる優れた無線装置を提供することを目的とする。
【０００５】
【課題を解決するための手段】本発明は、上記目的を達成するために、２値をｋビット毎のグレイ符号化されたｍ値に変換する多元符号化部と、クラス１パーシャルレスポンスを行なうためのプリコーダと、ｍ値をｎ値のシンボルに変換するクラス１パーシャルレスポンスフィルタと、帯域制限を行なうＬＰＦと、非線形の位相割り当てを行なう差動位相変調器とを備えたものであり、これにより、受信感度をあまり劣化させずに同一ビットレートでの狭帯域化、または同一伝送帯域での伝送速度の高速化を図ることができる。
【０００６】
【発明の実施の形態】本発明の請求項１に記載の発明は、連続的に入力された２値系列をｋビット毎のグレイ符号化されたｍ値系列（ｍ＝2^k）に変換する多元符号化部と、クラス１パーシャルレスポンスを行なうためのプリコーダと、ｍ値をｎ値（ｎ－２ｍ－１）のシンボルに変換するためのクラス１パーシャルレスポンスフィルタと、帯域制限を行なうＬＰＦと、非線形の位相割り当てを行なう差動符号化器および位相変調器からなる差動位相変調器とを備えたものであり、２値をｍ値に多元符号化する際にグレイ符号化し、パーシャルレスポンスにおいてはバイナリ配置とすることにより、発生頻度の高いシンボルは位相遷移を小さく、発生頻度の低いシンボルは位相遷移を大きくすることができるので、平均位相遷移を小さくすることができ、送信スペクトルの狭帯域化を図ることができる。また、グレイ符号化によって、隣接シンボルへのビット誤りが１ビット誤りのみとなるので、ＢＥＲ（Ｂｉｔ　Ｅｒｒｏｒ　Ｒａｔｅ）を向上させることができる。
【０００７】本発明の請求項２に記載の発明は、差動位相変調器による差動位相変調において、発生頻度の高いシンボルはシンボル間距離を大きくし、発生頻度の低いシンボルはシンボル間距離を小さくする非線形の位相割り当てを行なうものであり、シンボルの発生頻度に応じてシンボル間距離を非線形に割り当てることにより、発生頻度の低いシンボルのＢＥＲは劣化するが、発生頻度の高いシンボルのＢＥＲは向上するので、平均ＢＥＲを向上させることができる。
【０００８】本発明の請求項３に記載の発明は、非線形の位相割り当てにおいて、信号間距離が等差数列となるように位相割り当てを行なうものであり、装置構成を容易にすることができる。
【０００９】（実施の形態）図１は本発明の一実施の形態を示すものである。図１において、１１は多元符号化部であり、２値のデータをｋビット毎にｍ値（ｍ＝2^k）のグレイ符号化を行なう。１２はｍｏｄ演算素子であり、１３は１シンボル（ｋビット遅延素子である。１２および１３はクラス１パーシャルレスポンスを行うためのプリコーダ１８を構成し、また１３はｍ値をｎ値（ｎ＝２ｍ　１）のシンボルに変換するクラス１パーシャルレスポンスフィルタ１９を構成する。１４は帯域制限を行なうＬＰＦ（ローパスフィルタ）であり、１５は差動符号化器１６および位相変調器１７からなる差動位相変調器である。
【００１０】次に上記実施の形態における動作について説明する。多元符号化部１１は、連続的に入力された２値のデータをｋビット毎にｍ値（ｍ＝2^k）のグレイ符号化を行ない、結果を出力する。プリコーダ１８におけるｍｏｄ演算素子１２には、多元符号化部１１からｍ値のシンボルが連続的に入力され、時刻ｔに入力されたシンボルから時刻ｔ－１に入力されたシンボルの減算を行

ない、ｍｏｄ　ｍ演算したものを出力する。この出力は、ｍｏｄ　ｍ演算を行なっているため、ｍ値のデータである。図２はｍｏｄ演算素子１２が、ｍ＝４でｍｏｄ演算を行なった場合の加算および減算の結果を示している。次に、クラス１パーシャルレスポンスフィルタ１９における１シンボル遅延素子１３に、ｍｏｄ演算素子１２から連続的にｍ値のシンボルが入力されて１シンボルが遅延され、次いで時刻ｔに入力されたシンボルと時刻ｔ－１に入力されたシンボルの加算を行ない、ｎ値（ｎ＝２ｍ　１）のシンボルが出力される。このｎ値は、ＬＰＦ１４により帯域制限され、差動符号化器１６により差動符号化され、位相変調器１７により位相変調信号が出力される。

【００１１】図３は差動位相変調器１５による差動位相変調において、ｋ＝２の場合のｎ値シンボルの非線形位相配置の例を示している。図３（ａ）に示すように、ｋ＝２の場合、ｎ値のシンボルの発生頻度は３が最も高く、次いで２および４、１および５、０および６の順となっている。そこで、発生頻度の最も高い３において隣接シンボルとのシンボル間距離が最も大きくなるようにし、発生頻度に応じてシンボル間距離を小さくしていく。すなわち、図３（ｂ）のように、発生頻度の高いシンボルはシンボル間距離を大きくし、発生頻度の低いシンボルはシンボル間距離を小さくする非線形の位相割り当てを行なう。ちなみに、ｋ＝２の場合のｎ値の線形位相配置は図３（ｃ）のようになる。

【００１２】非線形の位相割り当てにおいて、図３（ｂ）に示すように．信号間距離が等差数列となるように位相割り当てを行なうことにより、装置構成を容易にすることができる。

【００１３】

【発明の効果】本発明は、上記実施の形態から明らかなように、２値をｋビット毎のグレイ符号化されたｍ値に変換する多元符号化部と、クラス１パーシャルレスポンスを行なうためのプリコーダと、ｍ値をｎ値のシンボルに変換するためのクラス１パーシャルレスポンスフィルタと、帯域制限を行なうＬＰＦと、非線形の位相割り当てを行なう差動位相変調器とを備えているので、受信感度をあまり劣化させることなく伝送帯域の狭帯域化を図ることができ、周波数利用効率を向上させることができる。

【図面の簡単な説明】

【図１】本発明の一実施の形態を示す無線装置の要部ブロック図

【図２】本発明の一実施の形態におけるｍｏｄ　ｍ演算結果の例を示す一覧図

【図３】（ａ）本発明の一実施の形態におけるｎ値シンボルの発生頻度例を示す模式図

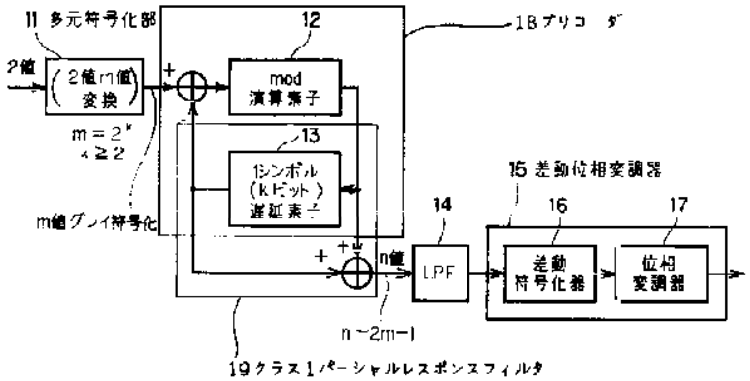
（ｂ）本発明の一実施の形態におけるｎ値シンボルの非線形位相配置例を示す模式図

（ｃ）従来例におけるｎ値シンボルの線形位相配置例を示す模式図

【符号の説明】

１１　多元符号化部
１２　ｍｏｄ演算素子
１３　１シンボル（ｋビット）ルタ）
１４　ＬＰＦ（ローパスフィルタ）
１５　差動位相変調器
１６　差動符号化器
１７　位相変調器
１８　プリコーダ
１９　クラス１パーシャルレスポンスフィルタ

【図１】



【図2】

(a)

加算 A+B

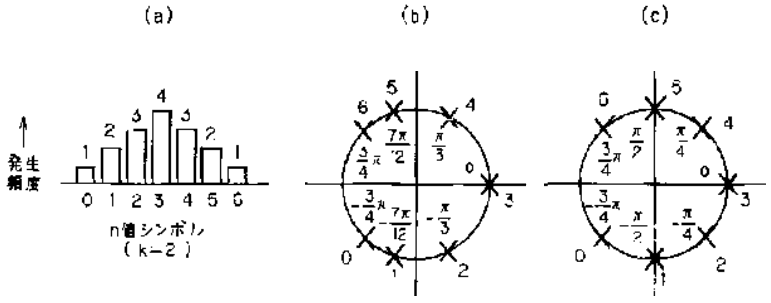
B＼A	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

(b)

減算 A -B

B＼A	0	1	2	3
0	0	1	2	3
1	3	0	1	2
2	2	3	0	1
3	1	2	3	0

【図3】



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(71) Applicant: MOTOROLA, INC. [US/US]; 1303 East Algonquin Road, Schaumburg, IL 60196 (US).

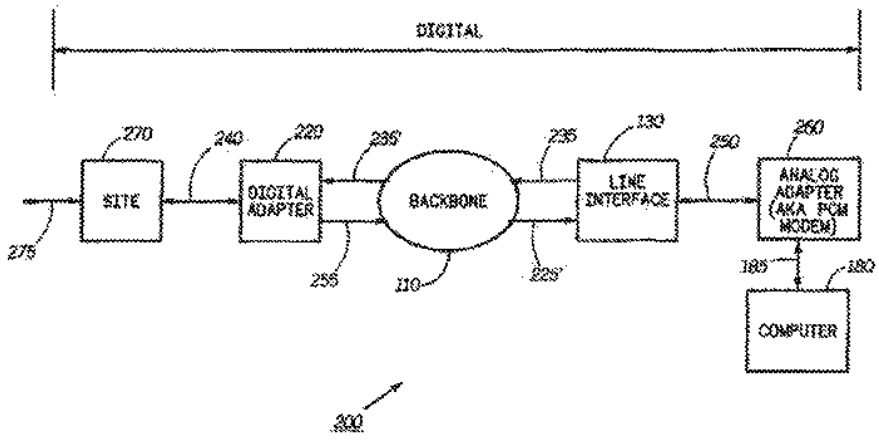
(72) Inventor: FORNEY, George, D., Jr.; 6 Coolidge Hill Road, Cambridge, MA 02138 (US).

(74) Agents: WOOD, L. Ray et al.; Motorola Inc., Intellectual Property Dept., 1303 East Algonquin Road, Schaumburg, IL 60196 (US).

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(54) Title: SYSTEM AND DEVICE FOR, AND METHOD OF, COMMUNICATING ACCORDING TO A COMPOSITE CODE



(57) Abstract

System and device (200) for, and method of, communicating according to a trellis code (300) of baseband signals chosen from a fixed set of baseband signal points. A first set of signal points define a first subconstellation, and a second set of signal points define a second subconstellation. The first subconstellation is characterized with a different minimum distance between adjacent signal points than the minimum distance for the second subconstellation. Under a PCM modem arrangement (200) the first and second constellations respectively include set of signal points selected from the PSTN quantization rules, and the first subconstellation has a smaller minimum distance than the second.

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SYSTEM AND DEVICE FOR, AND METHOD OF, COMMUNICATING ACCORDING TO A COMPOSITE CODE

Cross-Reference to Related Applications

This application is related to the following U.S. applications, all of which are owned by the same assignee and all of which are incorporated by reference in their entirety:

Device, System and Method for Spectrally Shaping Transmitted Data Signals, to Vedat Eyuboglu and Pierre Humblet, filed on October 15, 1996, Ser. No. 08/730,434;

Device, System and Method for Adaptive Self-Noise Cancellation for Decision-Directed Timing Recovery to Jian Yang, filed on October 15, 1996, Ser. No. 08/720,992;

System and Device for, and Method of, Detecting, Characterizing, and Mitigating Deterministic Distortion in a Communication System, to Vedat Eyuboglu, Arthur Barabell, and Pierre Humblet, filed on October 15, 1996, Ser. No. 08/730,433; and

System and Device for, and Method of, Processing Baseband Signals to Combat ISI and Nonlinearities in a Communication System, to Pierre Humblet and Vedat Eyuboglu, filed on October 15, 1996, Ser. No. 08/720,988.

Background

1. Field of the Invention

The invention relates generally to communication systems and, more particularly, to communicating information using a new type of code called a composite code, for example, on a PCM modem connection using the public switched telephone network (PSTN).

2. Discussion of Related Art

There is an increasing demand for data communications and, in particular, for communication systems with increasingly higher transmission rates. With the advent of the Internet, this demand is expected to increase.

To date, the most popular form of data communications is accomplished using voiceband modems connected to local loops of a conventional PSTN. Voiceband modems are very popular largely because of their leverage of existing telephone network infrastructure. In short, a user needs to make only a relatively small investment for a modem and has to pay relatively modest line charges.

A conventional PSTN arrangement 100 is shown in Fig. 1. What are typically interpreted as analog signals enter and exit the network 100 at "local loops" 140 and 150. Each signal on loop 140 and 150 is received by a corresponding line interface 120 and 130, or local switch, and each line interface communicates with another via a backbone digital network 110.

Under conventional operation, an information signal 175 is sent to a first site 170, which emits an analog signal, for example, representative of a voice signal or binary information, on the local loop 140. The line interface 120 filters, samples, and quantizes the analog signal and outputs a sequence of octets 125, representative of the analog signal 140. The PCM quantization rules, used for the quantizing, are known and pre-specified, e.g., μ-law or A-law. (Appendix A lists the values for μ-law).

The backbone 110 receives the octet sequence 125 and transmits and routes the data to various line interfaces, e.g., 130. Eventually an octet sequence 125', which is similar but not necessarily identical to the original octet sequence 125, is transmitted to the line interface 130 corresponding to the destination site 160.

The line interface 130 essentially inverse quantizes and further processes the received octet sequence 125' to create on loop 150 an analog signal that is an "approximate representation" of the originally-transmitted signal 140. Signal 150 is then transmitted to the site 160, where it may be used to recreate a voice signal or the binary information.

To meet user demand for higher transmission rates, communication standards for voiceband modems have evolved with each generation including capabilities to support increasingly higher transmission rates. Current state-of-the-art voiceband modems attain transmission rates of about 30 kb/s, nearly the conventionally-accepted view of the telephone channel's capacity.

Unfortunately, the rate of growth of the transmission rates of modems has slowed, but users still desire higher transmission rates. Consequently, users who want higher transmission rates are currently forced to use alternative technologies, such as ISDN connections into their homes and offices. Though these alternative arrangements provide higher transmission rates, the equipment and line charges are high.

Currently, various proposals have been made and communication standards are being pursued that promise higher transmission rates using the conventional PSTN. Hereinafter these proposed modems and

arrangements are called "PCM modems" and "PCM Modem arrangements."

Exemplary PCM modems and PCM modem arrangements are described in the related applications, identified and incorporated above. Referring to Fig. 2, in the PCM modem arrangement, the backbone 110, line interface 130, and computer 180, remain unchanged from the conventional components, outlined above. A first site 270, such as an Internet server site, communicates with a digital adapter 220, or digital modem, by sending signals over a high-speed link 240. The digital adapter 220 sends a sequence of octets 225 to backbone 110, which, in turn, sends a sequence of octets 225' to conventional line interface 130. Octets 225' may differ from octets 225 because of robbed bit signaling, for example. Line interface 130 then inverse quantizes octets 225' and transmits line interface signals on downstream, user-end loop 250. Analog adapter 260 (also known as PCM modem) receives the line interface signals and may, in turn, possibly equalize and sample the received signals, detect the binary information in the demodulated signal, and send the results to computer 180. A reverse path from analog adapter 260 to digital adapter 220 may be constructed using conventional modem techniques, for example, V.34 technology, or other technologies can be employed.

Though the conventional arrangement of Fig. 1 considers the operations of line interface 130 as inverse quantization, the PCM modem arrangement considers the same operations as a form of baseband modulation. That is, the line interface 130 produces a signal on the loop whose amplitude varies depending on the information received by the line interface. In this sense, the

modulation technique is akin to PAM, but unlike PAM, and as further described below, the signal constellation involved is unevenly-spaced. This uneven spacing is a consequence of the PCM quantization rules being non-linear.

Possibly the most desirable feature of the PCM modem arrangement is that, unlike the conventional arrangement of Fig. 1, the conventional local switch 120 is avoided and, therefore, its capacity-limiting, quantization noise is also avoided. Thus, higher transmission rates may be attainable.

Under the paradigm of Fig. 2, a system is theoretically capable of transmitting data at rates of about 64,000 b/s, i.e., 8,000 octets per second, if noise and other distortion were not considered. To approach the 64,000 b/s rate, however, the baseband modulation would require a signal constellation using all of the quantization levels defined by the corresponding PCM quantization rules.

Unfortunately, if all levels were used the minimum distance d_{min} between levels is too small to be practical. (d_{min} is a known parameter for characterizing the performance of a signal constellations.) Those no serious proposals of PCM modems can attain 64,000 b/s in "real world" contexts having noisy local loops 250 and the like.

A subset of the μ-law or A-law quantization levels may be straightforwardly selected to create a signal constellation for an uncoded system that would avoid the smallest spacings and attain noise resistance improvements over the complete constellation. For example, if the objective is to send 56 kb/s, or 7 bits per signal point, then an uncoded constellation with 128 levels is required. It is straightforward to show that the maximum minimum distance that

can be achieved with a 128-point subconstellation of the μ-law constellation has d_{min} = 52. For example, the following set of 128 levels has d_{min} = 52.

Exemplary constellation for uncoded modulation

1 level at 30;
1 level at 85;
2 levels at 139 and 195;
4 levels evenly spaced with d = 64 between 247 and 439;
8 levels evenly spaced with d = 64 between 495 and 943;
16 levels evenly spaced with d = 64 between 1023 and 1983;
16 levels evenly spaced with d = 128 between 2079 and 3999;
16 levels evenly spaced with d = 256 between 4191 and 8031; and
64 negative levels have the same magnitude but opposite sign.

This uncoded constellation with d_{min} = 52 will be used later for comparative purposes as a baseline system for a data rate of 56 kb/s.

To date, nearly all of the proposals involve uncoded modulation. With uncoded modulation, each transmitted signal level is detected independently of other signals at the receiver. The performance of an uncoded system is therefore governed by the minimum distance d_{min} between the set of levels that may be transmitted. The set of levels that may be transmitted is the "signal constellation."

In conventional telephone-line modems, it has been found advantageous to use coded modulation. In a coded modulation system, dependencies are introduced between transmitted signal levels. Thus, a sequence of signals carries the information, rather than isolated

signals. Moreover, the dependencies are constructed such that the minimum Euclidean distance d_{min} between sequences is greater than the minimum distance d_{min} within the signal constellation. At the receiver, a decoder such as the well-known Viterbi algorithm uses these dependencies in sequence decoding to achieve a "coding gain." For example, V.32 modems use a two-dimensional (2D) trellis code to achieve a coding gain of about 3.6 dB, and V.34 modems use one of three four-dimensional (4D) trellis codes to achieve a coding gain of from about 4.2 to 4.7 dB. (Trellis coding may be found in the literature, for example, EZIO BIGLIERI ET AL., INTRODUCTION TO TRELLIS-CODED MODULATION WITH APPLICATIONS (MacMillan 1991) and JOHN G. PROAKIS, DIGITAL COMMUNICATIONS 511-526 (McGraw Hill 1995).)

It is desirable to achieve similar coding gains by the use of coded modulation in PCM modems. However, known coding schemes such as the V.32 and V.34 trellis codes cannot be straightforwardly applied because the signal levels are restricted to the finite, nonuniform μ-law or A-law constellations.

Summary

A method, adapters, and a system are provided for communicating information. A first set of signal points define a first subconstellation, and a second set of signal points define a second subconstellation, wherein the first subconstellation is characterized with a different minimum distance between adjacent signal points than the minimum distance for the second subconstellation. Information to be communicated is mapped to one of the first and second subconstellations. If the information maps to the first subconstellation, the information is transmitted according

to a first transmission scheme; and if the information maps to the second subconstellation, the information is transmitted according to a second transmission scheme.

Under one aspect of the invention, information mapped to the first constellation may be transmitted as coded information, for example, using a trellis code.

Under another aspect of the invention, information mapped to the second constellation may be transmitted as uncoded information.

According to another aspect of the invention, the first and second signal constellation are constructed using signal points from PCM quantization rules for PSTNs. From the pre-specified constellation, at least two subconstellations are chosen. One, the "outer subconstellation," includes a subset of the PSTN quantization levels. At least one other "inner subconstellation" includes another subset of the PSTN quantization levels. The outer subconstellation includes signal points sufficiently separated so that coding is not needed for their transmission.

According to another aspect of the invention, n-dimensional trellis codes are used for transmitting information mapped to the first subconstellation.

Another aspect of the invention provides mechanisms for recovering from lost synchronism when decoding an n-dimensional code.

Another aspect of the invention provides mechanisms for avoiding catastrophic effects from lost synchronism when decoding an n-dimensional code.

Brief Description of the Drawing

In the Drawing,

Fig. 1 shows a conventional PSTN having local loops;

Fig. 2 is an architectural diagram of a PCM modem arrangement according to an exemplary embodiment of the invention;

Fig. 3 is a trellis diagram associated with a trellis code used for inner signal points of a composite code according to an exemplary embodiment of the invention;

Fig. 4 is another trellis diagram associated with a trellis code used for inner signal points and having state transitions for outer signal points of a composite code according to an exemplary embodiment of the invention;

Fig. 5 is an architectural diagram of the relevant part of an analog adapter according to an exemplary embodiment of the invention; and

Fig. 6 is an architectural diagram of the relevant part of a digital adapter according to an exemplary embodiment of the invention.

Detailed Description

An exemplary embodiment of the invention uses a new type of code called a composite code. Under the principles of the invention, a transmitter, or a digital adapter 220, and a PCM modem, or an analog adapter 260, may be implemented to encode and decode information using the exemplary code. As will be explained below, exemplary embodiments of the invention can attain the same transmission rate as uncoded transmission with improved noise resistance. For example, an exemplary embodiment attains a 1.8 dB nominal coding

gain over the baseline uncoded constellation above at 56 kb/s, and up to 4.8 dB may be attainable with more complexity.

In a PCM modem, the designer must use the pre-specified μ-law or A-law constellation, rather than being able to choose the signal constellation freely. The new type of code described in this specification was designed in response to this new constraint as well as the nonuniform nature of the pre-specified constellation.

In a composite code, some signal points are transmitted in coded fashion and others may not. More specifically, the composite codes used in exemplary embodiments use subsets of the μ-law or A-law quantization levels that are grouped to form at least two subconstellations. Each subconstellation is then associated with a corresponding transmission and decoding scheme tailored to the characteristics of that subconstellation.

I. Composite Codes Involving 1D Trellis Codes

One embodiment forms a composite code by combining one-dimensional trellis coded modulation and uncoded modulation in the following manner. A subset of the 255 detectable μ-law levels are used to create two subconstellations: a subconstellation of outer points with minimum distance d_o, and a subconstellation of inner points with minimum distance $d_i < d_o$. All outer points are chosen to have larger magnitude than all inner points, and as a consequence of the PCM quantization rules, the outer points also have larger distances to adjacent outer points than the distances between adjacent inner points. The separation between the maximum inner point and minimum outer point is chosen to be at least d_o. Finally, the

inner subconstellation is chosen to approximate a normal evenly-spaced pulse amplitude modulation (PAM) constellation.

For example, a 118-point outer subconstellation with $d_o = 64$ and a 20-point inner subconstellation with $d_i = 22$ may be chosen as follows:

Exemplary inner subconstellation:

1 level at 12;

3 levels evenly spaced with d = 24 between 37 and 85;

5 levels evenly spaced with d = 24 between 107 and 203;

1 level at 231; and

10 negative levels with the same magnitudes and opposite sign.

Exemplary outer subconstellation:

3 levels evenly spaced with d = 64 between 295 and 423;

8 levels evenly spaced with d = 64 between 495 and 943;

16 levels evenly spaced with d = 64 between 1023 and 1983;

16 levels evenly spaced with d = 128 between 2079 and 3999;

16 levels evenly spaced with d = 256 between 4191 and 8031; and

59 negative levels with the same magnitudes and opposite sign.

Known trellis codes may be used with such an inner subconstellation to achieve a large minimum distance between coded sequences. In this case, a simple one-dimensional (1D) 4-state Ungerboeck trellis code may be used.

For example, the inner subconstellation may be 4-way partitioned into four subsets A, B, C, D in a periodic ABCDABCD . . . pattern as follows:

231 A	-231 D
203 B	-203 C
179 C	-179 B
155 D	-155 A
131 A	-131 D
107 B	-107 C
85 C	-85 B
61 D	-61 A
37 A	-37 D
12 B	-12 C

With this partition, the minimum distance within a subset is at least $4d_i = 88$, and the minimum distance between points in subsets A or C and points in subsets B or D is at least $2d_i = 44$.

The digital adapter 220 and analog adapter 260 could use the exemplary 4-state trellis diagram shown in Fig. 3 in conjunction with the above constellations. The minimum squared distance between trellis code sequences is easily shown to be $(44)^2 + (22)^2 + (44)^2 = (66)^2$, so the minimum distance between trellis code sequences is 66.

In order to use both the inner and outer subconstellations to send 7 bits per sample, the encoder operates as follows. The digital adapter 220 receives information on link 240 and maps 7 bits of the information, at a time, into one of 128 values, of which

- 118 values are associated with unique outer points; and

* 10 values are associated with two inner points, one in A or C and one in B or D, in such a way that each of the 20 inner points is associated with a unique value.

Note that, in the above, the negative of an A point is a D point, and the negative of a B point is a C point. Let the 10 of the 128-bit 7-tuples that designate an inner point designate one of the 10 inner subconstellation magnitudes. As shown in Fig. 3, the state of the trellis decoder then allows either only A and C points, or alternatively B and D points to be sent; thus the state can be used together with the magnitude to select the sign of a signal point with that magnitude to satisfy the constraints of the code trellis. In this way, the data 7-tuple and the trellis state together select one of these 20 inner points to be transmitted. If on the other hand the input 7-tuple corresponds to one of the 118 outer point values, then the corresponding outer point is simply sent uncoded.

Since the minimum distance of the trellis code is 66, the minimum distance of the uncoded outer subconstellation is 64, and the minimum distance between the inner and outer subconstellations is 64, the minimum distance between any two sequences in the composite coding scheme is 64. This means that the nominal coding gain of the composite code is a factor of $(64/52)^2 = 1.515$ (1.8 dB). (64 being the distance of the exemplary composite code, and 52 being the distance of the constellation for the baseline uncoded system, outlined above.)

The above composite coding arrangement creates some delay and buffering issues not found in conventional trellis coding and uncoded transmission schemes. More particularly, if the analog

adapter 260 detects a signal level in the outer subconstellation, an immediate decision can be made, since the outer subconstellation is uncoded. However, if the analog adapter 260 detects a signal level in the inner subconstellation, then the received signal magnitude must be decoded according to the inner trellis code (e.g., using the Viterbi algorithm). Such decoding inherently introduces some delay. Therefore, in order to put out the decoded received bits in the same order in which they were transmitted, the receiver needs to:

- Establish a fixed decoding delay between the time each received sample arrives and the time the corresponding data bits are put out; and
- Force the inner subconstellation decoder to make decisions in time to meet this delay constraint.

The latter condition will sometimes force the decoding of the inner subconstellation point to make a decision prematurely, i.e., sooner than an unconstrained decoder would. This increases the decoder error rate over that of an unconstrained decoder. However, this performance loss may be made arbitrarily small by choosing large enough.

Another way of visualizing the composite code and the effect of the fixed decoding delay constraint is as follows. A composite code trellis diagram may be constructed by starting with a conventional trellis diagram for the known trellis code, whose branches correspond to transmission of inner points, and adding branches corresponding to outer points that do not cause the encoder state to change.

For example, by starting with a trellis diagram for inner points, such as the one of Fig. 3, the trellis diagram of Fig. 4 may be constructed. The thin-line branches correspond to inner points; for clarity the labels are omitted. The thick-line branches correspond to outer points. As readily seen, the modified trellis diagram indicates that transmissions of outer points do not change the encoder state.

A transmitter that encodes using this trellis diagram 400 and a receiver that simply decodes this trellis code using the standard Viterbi algorithm with a fixed decoding delay will be equivalent to the composite encoder and decoder described previously.

II. Composite Codes involving nD Trellis Codes.

The "constellation expansion" of the exemplary 1D trellis code embodiment, described above, is a factor of 2. This constrains the amount of gain that can be achieved with 1D codes in this application. By using a 2D or 4D trellis code, the constellation expansion due to trellis coding can be reduced to about 50% or 25%, respectively. (For example, Ungerboeck 2D code, a Wei 4D code, or Ungerboeck- or Wei-type codes such as those used in V.32 or V.34 may be used)

Exemplary embodiments involving 2D Ungerboeck trellis code may be constructed as follows. For these embodiments, a 64-point outer subconstellation with $d_o = 128$ and a 92-point inner subconstellation with $d_i = 32$ are chosen as follows:

Exemplary inner subconstellation for nD embodiment

2 levels at 33 and 65;

4 levels evenly spaced with d = 32 between 99 and 195;

8 levels evenly spaced with d = 32 between 231 and 455;

16 levels evenly spaced with d = 32 between 495 and 975; 16 levels evenly spaced with d = 64 between 1023 and 1983; and

46 negative levels with the same magnitudes and opposite signs.

Exemplary outer subconstellation for nD embodiment

16 levels evenly spaced with d = 128 between 2079 and 3999; 16 levels evenly spaced with d = 256 between 4191 and 8031; and

32 negative levels with the same magnitudes and opposite signs.

Now since $\log_2 92 > 6.5$, or equivalently $(92)^2 > 2^{13}$, we can use trellis codes such as the two-dimensional (2D) Ungerboeck codes that introduce 1 redundant coded bit per two dimensions, or 1/2 b/D, to support a data rate of 6 bits per sample with the inner subconstellation, using a 2^{13}-point subset of the set of all pairs of inner points. In other words, there are $92x92 = 8464 > 8192 = 2^{13}$ possible pairs of points from a 92-point inner constellation. Therefore, a 8192-point may be selected and each pair may be assigned a unique 13-bit label, to send 13 bits per two dimensions. Conventionally one would select the least-energy 8192 points.

With the Ungerboeck 2D trellis codes, it is known that a distance gain of a factor of d can be achieved, where d is given as a function of the number s of states in the trellis code by the table below. In this application, relative to the baseline uncoded constellation with $d_{min} = 52$, this yields a nominal coding gain of a factor of $(32/52)^2 d = 0.379d$, which is also shown in the table.

s	d	nominal coding gain
4	4	1.51 (1.8 dB)
8	5	1.89 (2.8 dB)
16	6	2.27 (3.6 dB)
64	7	2.65 (4.2 dB)
128	8	3.02 (4.8 dB)

Note that even with the most powerful scheme, the code distance of 32 8 = 90.5 is less than the distance d = 96 between inner and outer subconstellations, and *a fortiori* less than the distance d = 128 within the outer subconstellation.

Now in order to support a data rate of 7 bits per sample, the uncoded outer subconstellation and the coded inner subconstellation may be used as follows. One bit per sample determines whether we use the inner or outer subconstellation. If outer, then we transmit the remaining 6 bits using the 64-point uncoded outer subconstellation. If inner, then we transmit the remaining 6 bits using the 92-point coded inner subconstellation. The simplicity of this scheme results from the inner and outer scheme both supporting the same number (6) of bits per sample.

The delay and buffering issues that arise with this kind of composite coding scheme are addressed as before. However, here there is another important issue that has to do with the effect of an error in deciding whether a received level is an inner or outer subconstellation point. Such errors will be rare because the distance between inner and outer constellations is d = 96, greater than the code distance, but nonetheless are possible and must not lead to

catastrophic effects. When such a decoding error occurs, with an n-dimensional (nD) trellis code with n > 1 there will in general be a loss of n-bit block synchronization. Unless this loss of synchronization is addressed, further decoding of subsequent signals will almost surely be incorrect.

Several methods for dealing with such a loss of synchronization may be used:

1. Detect loss of block synchronization, and then resynchronize to the proper one out of the possible n block phases, for example, by a trial-and-error procedure. (This technique is known in the context of initially synchronizing nD block or trellis codes.)

2. Implement n parallel decoders at all times, and always choose the path with the best metric. (Implementing n parallel decoders is straightforward for those skilled in the art.) More elegantly, this may be realized by a novel single Viterbi algorithm search through a "super-trellis" containing one state for each of the n possible block phases and each of the states in a 1D trellis for the original code. While this approach will resynchronize faster and more seamlessly than the one first mentioned, it will of course take about n times as much computation in normal operation.

3. Constrain the transmit sequence so that a multiple of n inner and outer points occur per N dimensions, where N is a multiple of n. Such a constraint may also assure that there are at least n

inner points per N dimensions. However, such a constraint will necessarily incur a cost in redundancy (overhead). (There are known constrained coding methods for straightforwardly implementing such constraints, but as far as we know this suggestion is novel for this type of application.)

While an error in distinguishing between an inner or outer point will in general disrupt the inner decoder by inserting or deleting a random state transition, error propagation can be limited by any of the above methods. Furthermore, while such an error will cause a series of output errors, it will not lead to input bits being delivered out of order, because both subconstellations support an equal number of bits per sample. Some of the techniques detect and recover from the problem of lost synchronism, while others ensure that the problem will not lead to catastrophic effects.

III. Implementation

The implementation of encoders given a trellis diagram and corresponding signal constellation is straightforward. Thus, for example, those skilled in the art will appreciate that the above description for composite codes involving 1D trellis codes specifies that the digital adapter 220 is to include encoding logic implementing the trellis 800 and the relevant constellations listed above. The digital adapter 220 would thus send octets 225 listed above.

The relevant part of a receiver, or analog adapter, 260 is shown in Fig. 5. Most of the elements shown are described in the related applications, identified and incorporated above. For clarity, that description is not repeated here.

In material part, the elements of Fig. 5 that differ from the elements described in the related applications is the decoder logic 570 within the analog adapter 260. This decoder logic will implement Viterbi decoding of the trellises of Figs. 3-4. The decoder logic may be separated into distinct decoders to correspond to the decoding of signal points in the first and second subconstellations. Moreover, under some embodiments using nD trellis codes, n parallel decoders may be used to recover from decoding errors. The decoding logic will also include delay logic, described above, to ensure that the decoded outputs correspond to the sequence of inputs.

Implementers will appreciate that the analog adapter 260 receives a baseband signal 250 that is an equivalent of the transmitted octet 225 and that the above description referred to them as signal points regardless of a binary or baseband representation. Those skilled in the art will further appreciate the applicability of the above logic and new code to other receiver techniques, e.g., sub-optimum decoders.

In analogous fashion, the relevant part of a transmitter, or digital adapter, 220 is shown in Fig. 6. Information received from link 240 is mapped by mapper 610 to a relevant subconstellation (in the diagram the transmitter includes two modulators, one for an inner subconstellation and one for an outer subconstellation). Then, depending on which subconstellation is determined as associated with the signal, the signal is modulated by a corresponding modulation technique. For example, for signal points mapped to the outer constellation, the modulation 620 would be for uncoded information, according to the exemplary embodiments described

above. For signal points mapped to the inner constellation, the modulation 630 would be for coded information. According to the exemplary embodiments described above, this coding could be 1D or nD trellis coding, but the principles of the invention are not limited to just these codes. Control logic 640 besides controlling the above may implement various control functions. For example, in the case of nD codes and depending upon the embodiment of the invention, control logic 640 may constrain the first and second modulators in the above described manners to avoid catastrophic effects from occurring as a result of decoding errors at the analog adapter 260.

Many aspects of the digital adapter and analog adapter are not shown because they are known in the art. For example, the digital adapter may include logic for grouping information received from link 240 into a pre-specified number of bits, e.g., 7 bits. This logic is known and well understood. Analogously, the trellis coding logic described above is well understood, though the combination and use of them with the particular subconstellations is novel.

IV. Further Embodiments

Though specific 1D and nD schemes for coding inner points are discussed above, other embodiments are within the scope of the invention.

The use of known trellis codes with less redundancy, such as the Wei 4D trellis codes, may often be attractive for these highly bandlimited applications.

The signal constellation may be partitioned into more than two subconstellations. For example, the μ-law constellation may be partitioned into three subconstellations:

- an inner-inner subconstellation consisting of the inner 48 points from the d = 32 inner subconstellation above;
- an outer-inner subconstellation consisting of 36 levels with d = 64 (879, 943, 1023-1983);
- the same 64-point outer subconstellation with d = 128 as above.

Note that the distance between subconstellations is still d = 96. The inner-inner constellation can then support 5 bits per sample with a 2D Ungerboeck-type trellis code; the outer-inner subconstellation can support 5 bits per sample with a simple mod-2 lattice code based on a binary single-parity-check code such as the (6, 5, 2) or (8, 7, 2) code; and the outer subconstellation can support 6 bits per sample, as before. Selection may be made by using one bit per sample to choose the outer subconstellation or not, and, if not, a second bit to choose between the inner-inner and outer-inner subconstellations. In this example there is no increase in nominal coding gain over the composite codes of the previous section, but the example does indicate a possible direction for generalizations.

These codes may be straightforwardly modified to include spectral shaping methods or for robbed-bit signaling protection such as those disclosed in the related applications identified and incorporated above. They could also be used with constellation shaping methods known in the art.

This type of code may also be used when the system is power-constrained. In this case the highest-energy levels of the μ-law of A-law constellation may not be used at all.

Although novel composite codes are discussed with respect to the system of Fig. 2, it will be apparent to those skilled in the art that the same techniques can be used and applied to other systems in which codes are constructed from a fixed set of signal points. It will be appreciated by those skilled in the art that the division into inner and outer subconstellations shown in the exemplary embodiments above and the election to code inner points but not outer points are due to the particular nonuniform character of the μ-law and A-law constellations, and might be modified for other kinds of constellations.

The present invention may be embodied in other specific forms without departing from the spirit or essential characteristics. The described embodiments are to be considered in all respects only as illustrative and not restrictive.

Appendix A

(μ-law values)

1 level at 0;

15 positive levels evenly spaced with d = 2 between 2 and 30 (i.e., 2, 4, 6, ..., 30);

16 positive levels evenly spaced with d = 4 between 33 and 93;

16 positive levels evenly spaced with d = 8 between 99 and 219;

16 positive levels evenly spaced with d = 16 between 231 and 471;

16 positive levels evenly spaced with d = 32 between 495 and 975;

16 positive levels evenly spaced with d = 64 between 1023 and 1983;

16 positive levels evenly spaced with d = 128 between 2079 and 3999; and

16 positive levels evenly spaced with d = 256 between 4191 and 8031

plus 127 symmetric negative levels

CLAIMS

1. A method of communicating information comprising the steps of:

grouping a first set of signal points together to form a first subconstellation;

grouping a second set of signal points together to form a second subconstellation, wherein the first subconstellation is characterized with a different minimum distance between adjacent signal points than the minimum distance for the second subconstellation;

receiving an information signal and mapping the information signal to one of the first and second subconstellations;

transmitting the information inherent in the information signal according to a first transmission scheme if the information signal maps to the first subconstellation; and

transmitting the information inherent in the information signal according to a second transmission scheme if the information signal maps to the second subconstellation.

2. The method of claim 1 further comprising the steps of

receiving a transmitted signal;

comparing the received signal to the first and to the second subconstellation to determine which subconstellation is most likely associated with the received signal;

decoding the information in the received signal according to a first decoding scheme that corresponds to the first transmission scheme if the comparing step determines that the first subconstellation was the most likely subconstellation; and

decoding the information in the received signal according to a second decoding scheme that corresponds to the second transmission scheme if the comparing step determines that the second subconstellation was the most likely subconstellation.

3. The method of claim 1 wherein the first and second subconstellations are each subsets of the signal points available from PCM quantization rules for a PSTN and wherein the first subconstellation has a smaller minimum distance than the second subconstellation.

4. The method of claim 3 wherein the step of transmitting the information according to a second transmission scheme transmits the information as uncoded information.

5. The method of claim 4 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a one-dimensional trellis code.

6. The method of claim 5 wherein the step of transmitting the information according to a one-dimensional trellis code transmits the information according to a four state trellis.

7. The method of claim 4 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a multi-dimensional trellis code.

8. The method of claim 4 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a two-dimensional trellis code.

9. The method of claim 5 wherein the step of grouping a first set of signal points selects a subset of 20 signal points available from the PCM quantization rules to form a first subconstellation having a minimum distance of 22.

10. The method of claim 9 further comprising the step of organizing the 20 signal points into a first, second, third, and fourth set, associated with a four state trellis code associated used in the first transmission scheme;

the first set comprising the signal points 231, 131, 37, -61, -155;

the second set comprising the signal points 203, 107, 12, -85, -179;

the third set comprising the signal points 179, 85, -12, -107, -203; and

the fourth set comprising the signal points 155, 61, -37, -131, -231.

11. The method of claim 10 wherein the first transmission scheme and the second transmission scheme transmit octets to carry the information inherent in the information signal and wherein each octet carries 7 bits of information.

12. The method of claim 11 further comprising the step of mapping 7 bits of information, provided by the information signal, into one of 128 values, wherein 118 of the values are the values of the signal points of the second subconstellation and wherein the remaining 10 values are the magnitudes of the values in the first subconstellation, and wherein if the 7 bits of information maps on to one of the remaining 10 values, the state of the four state trellis of the first transmission scheme together with the mapped magnitude is used to determine the signal point to be transmitted according to the four state trellis code of the first transmission scheme.

13. The method of claim 5 wherein the step of grouping a second set of signal points selects a subset of 118 signal points available from the PCM quantization rules to form a second subconstellation having a minimum distance of 64.

14. The method of claim 7 wherein the step of grouping a first set of signal points selects a subset of 92 signal points available from the PCM quantization rules to form a first subconstellation having a minimum distance of 32.

15. The method of claim 7 wherein the step of grouping a second set of signal points selects a subset of 64 signal points available from the PCM quantization rules to form a second subconstellation having a minimum distance of 128.

16. The method of claim 2 wherein the first and second subconstellations are each subsets of the signal points available from PCM quantization rules for a PSTN, wherein the first subconstellation has a smaller minimum distance than the second subconstellation, wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a trellis code, wherein the step of transmitting the information according to a second transmission scheme transmits the information as uncoded information, and

wherein the step of decoding according to the first decoding scheme decodes the received signal according to a fixed decoding delay, defining the time between when the received signal is received and when the decoding step provides decoded information, and

wherein the step of decoding according to the second decoding scheme introduces a delay before providing decoded information, the delay equaling the fixed decoding delay so that the decoding steps provide decoded information corresponding to the received signal in the order of the received signals.

17. The method of claim 13 wherein the decoding steps operate according to a composite trellis in which decoding operations according to the first decoding scheme cause the decoding to change states defined by the trellis code associated with the first transmission scheme and in which decoding operations according to the second decoding scheme cause the decoding to remain in the same trellis state.

18. The method of claim 2 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a n-dimensional trellis code and wherein the method further comprises the steps of

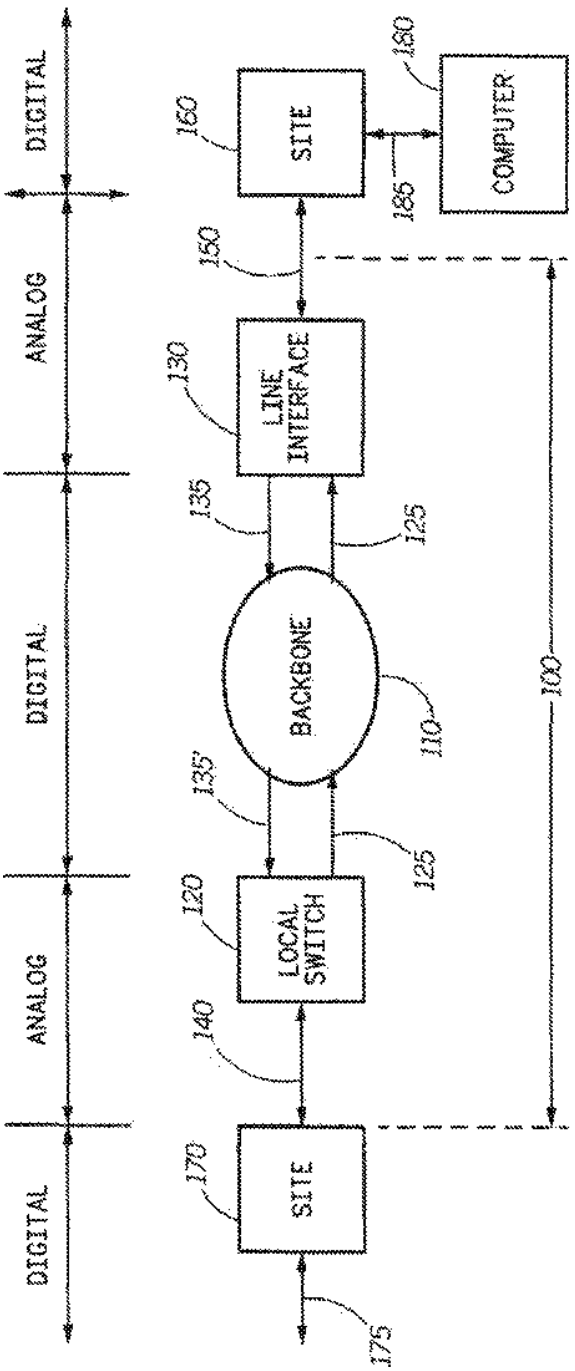
detecting whether the first and the second decoding scheme have lost block synchronization;

resynchronizing the first and second decoding scheme to one of the possible n block phases.

19. The method of claim 2 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a n-dimensional trellis code and wherein the first decoding scheme uses n parallel decoders and wherein the method further comprises the step of

choosing the decoder path with the best decoder metric, the decoder paths and metrics being defined by the state of the n decoders.

20. The method of 4 wherein the step of transmitting the information according to a first transmission scheme transmits the information according to a n-dimensional trellis code, and wherein the first and second transmission schemes are together constrained so that a multiple of n first signal points and second signal points occur per N dimensions, where N is a multiple of n.



PRIOR ART

FIG. 1

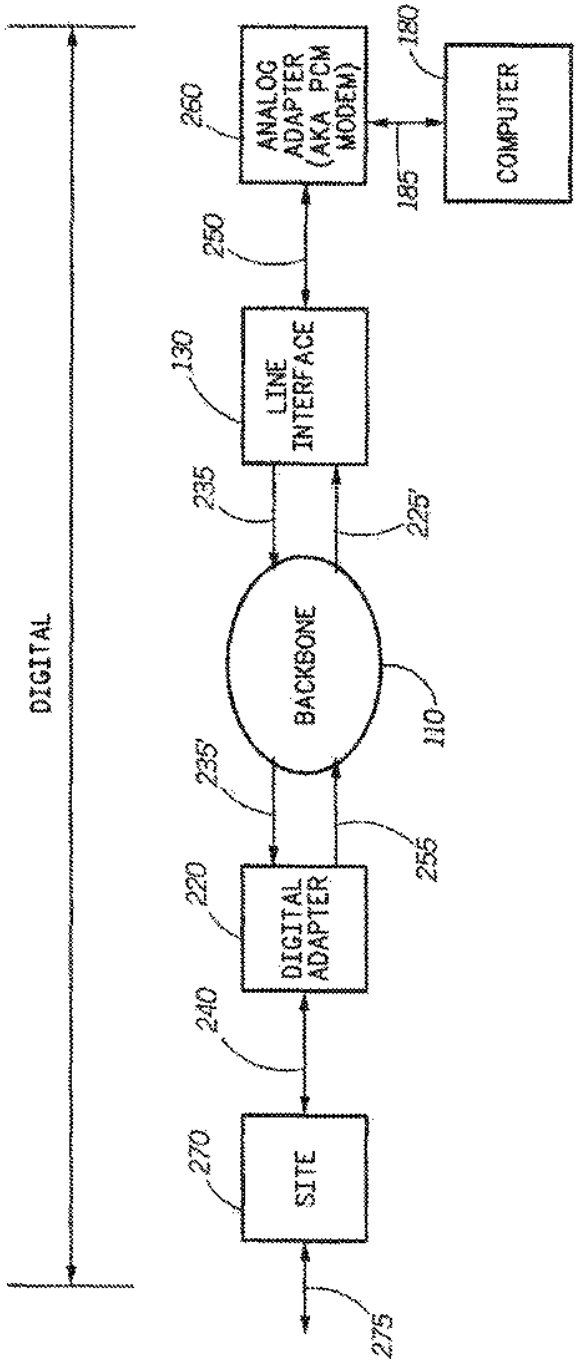

DIGITAL
ANALOG ADAPTER (AKA PCM MODEM)
260
180
COMPUTER
185
250
LINE INTERFACE
130
235
225
BACKBONE
110
235'
255
DIGITAL ADAPTER
220
240
SITE
270
275
200

FIG. 2

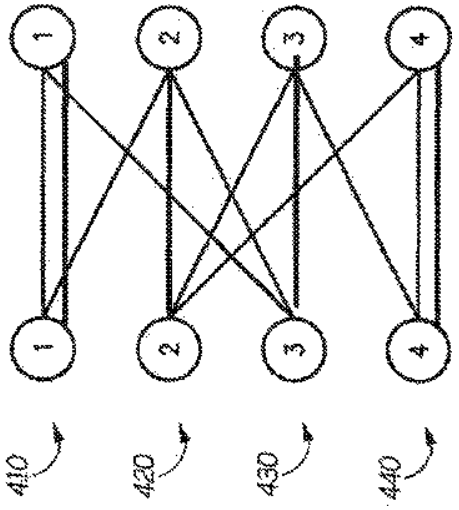

1
2
3
4
1
2
3
4
410
420
430
440

FIG. 4

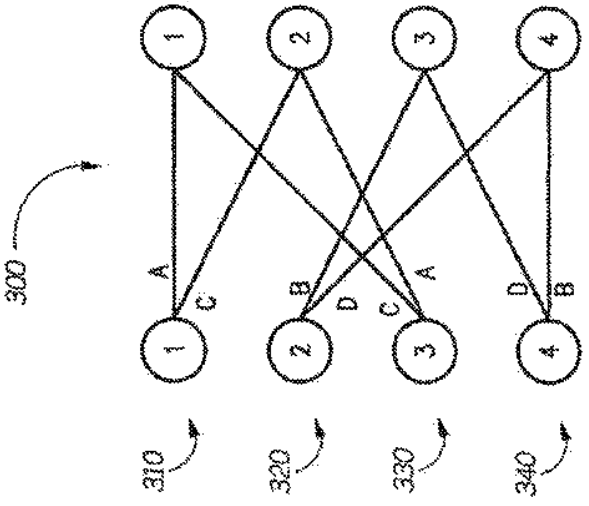

300
1
2
3
4
A
C
B
D
C
A
D
B
1
2
3
4
310
320
330
340

FIG. 3

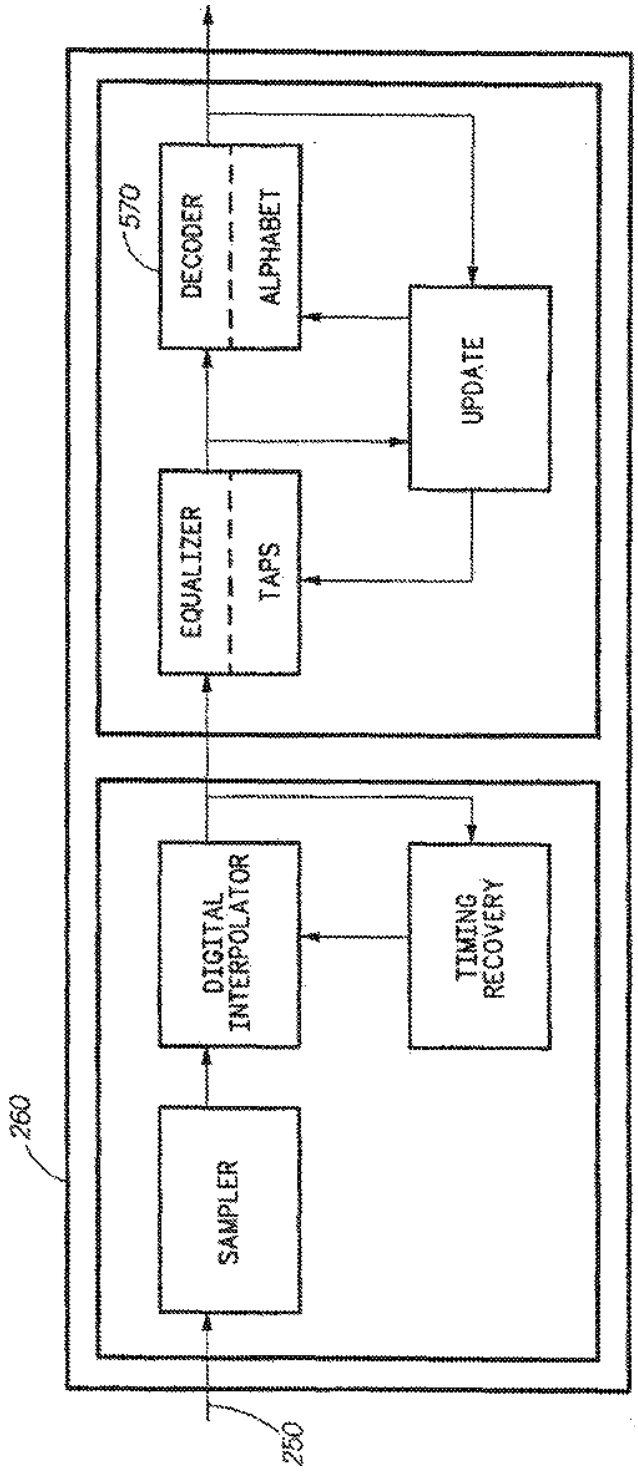

260
250
SAMPLER
DIGITAL INTERPOLATOR
TIMING RECOVERY
570
DECODER
ALPHABET
EQUALIZER
TAPS
UPDATE

FIG. 5

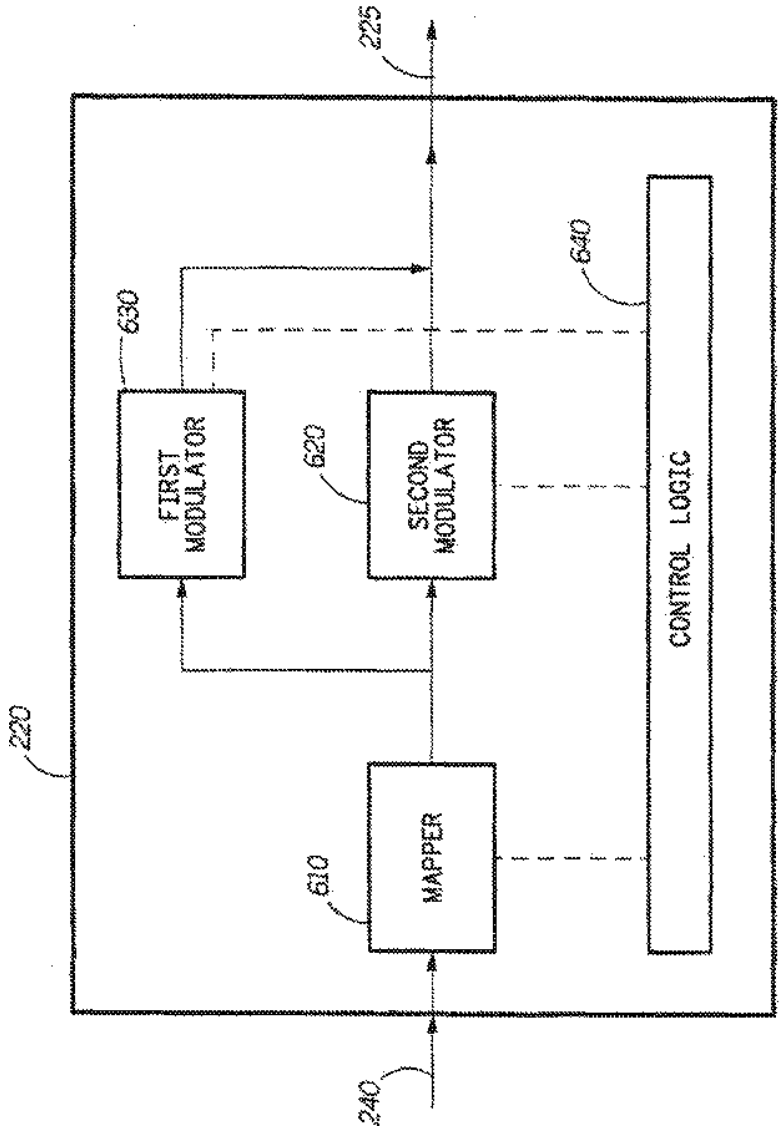

225
640
630
FIRST MODULATOR
620
SECOND MODULATOR
CONTROL LOGIC
220
610
MAPPER
240

FIG. 6

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US97/22920

A. CLASSIFICATION OF SUBJECT MATTER
IPC(6): H04L 5/12, 23/02; G06F 11/10; H03M 13/12
US CL: 375/261, 265; 371/43.4
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
U.S. : 375/261, 262, 265, 298, 341; 370/ 207; 371/43.4

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
APS searched terms: PCM and constellation#

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 4,891,806 A (FARIAS et al) 02 January 1990, figures 1-2 and 10 and column 5, line 31 to column 25, line 18.	1-20
X	US 5,363,408 A (PAIK et al) 08 November 1994, figures 1-2, 6 and 10, columnn 6, line 4 to column 12, line 68.	1-20
X	US 5,377,194 A (CALDERBANK) 27 December 1994, figures 1-3, 5, 7 and 9 and column 3, line 27 to column 8, line 33.	1-9 and 13-20
X	US 4,937,844 A (KAO) 26 June 1990, figures 1-7 and column 3, line 56 to column 7, line 13.	1-9 and 13-20
A	US 4,495,619 A (ACAMPORA) 22 January 1985, abstract and figures 1-2.	1-20

[X] Further documents are listed in the continuation of Box C. [ ] See patent family annex.

*	Special categories of cited documents:	"T"	later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"A"	document defining the general state of the art which is not considered to be of particular relevance	"X"	document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"E"	earlier document published on or after the international filing date	"Y"	document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"L"	document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)	"&"	document member of the same patent family
"O"	document referring to an oral disclosure, use, exhibition or other means		
"P"	document published prior to the international filing date but later than the priority date claimed		

Date of the actual completion of the international search	Date of mailing of the international search report
17 APRIL 1998	19 JUN 1998
Name and mailing address of the ISA/US Commissioner of Patents and Trademarks Box PCT Washington, D.C. 20231 Facsimile No. (703) 305-3230	Authorized officer DON N. VO Telephone No. (703) 305-4885

Form PCT/ISA/210 (second sheet)(July 1992)*

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US97/22920

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	US 5,343,500 A (BETTS et al) 30 August 1994, abstract and figures 3-6.	1-20

Form PCT/ISA/210 (continuation of second sheet)(July 1992)*

Espacenet

Bibliographic data: JP2003229835 (A) — 2003-08-15

MULTIPLE ACCESS SYSTEM AND MULTIPLE ACCESS METHOD

Inventor(s): SUZUKI MITSUHIRO ± (SUZUKI MITSUHIRO)

Applicant(s): SONY CORP ± (SONY CORP)

Classification: - international: ***H03M13/29; H04J13/00; H04J99/00; H04L1/00;*** (IPC1-7): H03M13/29; H04J13/00; H04L1/00

- cooperative:

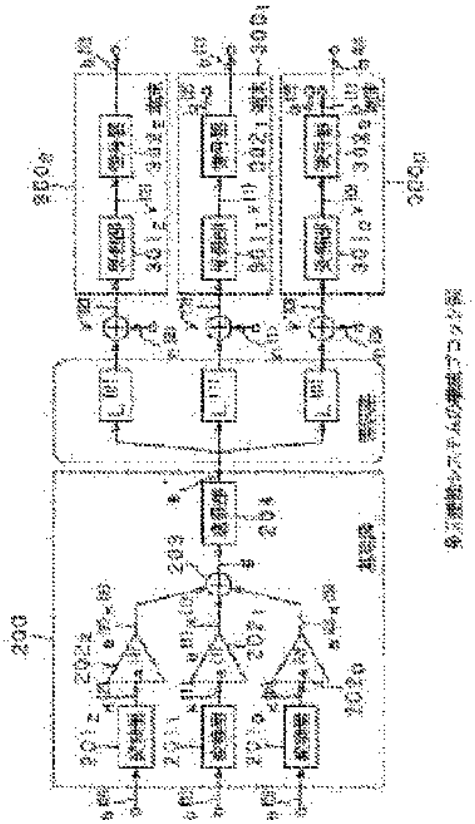
Application number: JP20020028020 20020205

Priority number(s): JP20020028020 20020205

Also published as: JP3979105 (B2)

Abstract of JP2003229835 (A)

PROBLEM TO BE SOLVED: To propose a new multiple access system capable of accommodating many users by effectively utilizing limited resources. ;SOLUTION: In this multiple access system, a base station 200 in a downlink is provided with three converters 201 for converting information bit sequences b<SP>(0)</SP>, b<SP>(1)</SP>and b<SP>(2)</SP>into encoded sequences x<SP>(0)</SP>, x<SP>(1)</SP>and x<SP>(2)</SP>composed of M numerical values, three multipliers 202 for multiplying the encoded sequences x<SP>(0)</SP>, x<SP>(1)</SP>and x<SP>(2)</SP>by constants a<SP>(0)</SP>, a<SP>(1)</SP>and a<SP>(2)</SP>, and an adder 203 for adding obtained constant-multiple encoded sequences a<SP>(0)</SP>x<SP>(0)</SP>, a<SP>(1)</SP>x<SP>(1)</SP>and a<SP>(2)</SP>x<SP>(2)</SP>in each element.; Meanwhile, terminals 300<SB>0</SB>, 300<SB>1</SB>and 300<SB>2</SB>are respectively provided with a decoder 302 for decoding at least one information bit sequence among information bit sequences b<SP>(0)</SP>, b<SP>(1)</SP>and b<SP>(2)</SP>on the basis of reception values y<SP>(0)</SP>, y<SP>(1)</SP>and y<SP>(2)</SP>and for outputting a needed information bit sequence. ;COPYRIGHT: (C)2003,JPO



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（P2003－229835A）
(43)公開日　平成15年８月15日(2003.8.15)

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Ｈ０４Ｌ　1/00		Ｈ０４Ｊ　13/00	Ｚ	５Ｋ０２２

審査請求　未請求　請求項の数40　ＯＬ　（全 43 頁）

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(71)出願人　000002185
ソニー株式会社
東京都品川区北品川６丁目７番35号

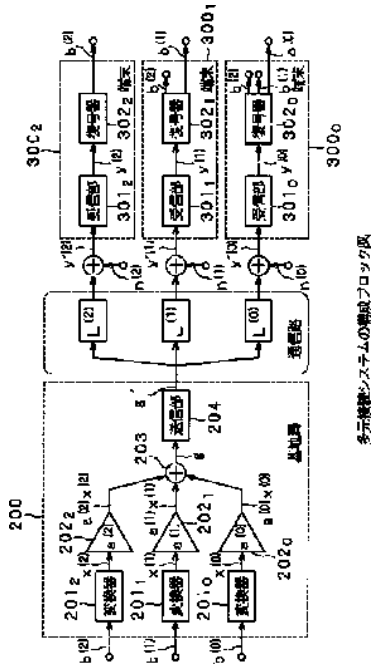
(72)発明者　鈴木　三博
東京都品川区北品川６丁目７番35号　ソニー株式会社内

(74)代理人　100110434
弁理士　佐藤　勝

Ｆターム(参考)　5J065 AA01 AA03 AB01 AC02 AD10 AD06 AF03 AG05 AG06 AH02
5K014 AA01 BA10 EA02 FA16 HA06 HA10
5K022 EE14 EE22 EE32

(54)【発明の名称】　多元接続システム及び多元接続方法

(57)【要約】
【課題】　限られたリソースを有効利用してより多くのユーザを収容することができる新たな多元接続方式を提案する。
【解決手段】　多元接続システムにおいて、下りリンクにおける基地局２００は、情報ビット系列ｂ(０)，ｂ(１)，ｂ(２)をＭ個の数値からなる符号化系列ｘ(０)，ｘ(１)，ｘ(２)に変換する３つの変換器２０１と、符号化系列ｘ(０)，ｘ(１)，ｘ(２)に対して定数ａ(０)，ａ(１)，ａ(２)を乗算する３つの乗算器２０２と、得られた定数倍符号化系列ａ(０)ｘ(０)，ａ(１)ｘ(１)，ａ(２)ｘ(２)を要素毎に加算する加算器２０３とを備える。一方、端末３００$_0$，３００$_1$，３００$_2$は、それぞれ、受信値ｙ(０)，ｙ(１)，ｙ(２)に基づいて、情報ビット系列ｂ(０)，ｂ(１)，ｂ(２)のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号器３０２を備える。



【特許請求の範囲】

【請求項１】　基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、

各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける上記基地局は、

所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する第１の変換手段と、

上記第１の変換手段によって変換されて得られた上記第１の符号化系列に対して第１の定数を乗算する第１の乗算手段と、

所定のビットからなる第２のユーザの第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの第２の変換手段と、

上記第２の変換手段によって変換されて得られた上記第２の符号化系列に対して第２の定数を乗算する少なくとも１つの第２の乗算手段と、

上記第１の乗算手段によって乗算されて得られた第１の定数倍符号化系列と、上記第２の乗算手段によって乗算されて得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する加算手段と、

上記加算符号化系列を送信信号として送信する送信手段とを備え、

上記基地局によって伝送された上記加算符号化系列に対して所定の雑音が加算された受信信号を受信する上記端末は、それぞれ、

上記受信信号を入力する受信手段と、

上記受信手段から供給された上記受信値に基づいて、上記第１の情報ビット系列又は上記第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備えることを特徴とする多元接続システム。

【請求項２】　上記第１の乗算手段は、雑音が加算される通信路を介して上記第１の符号化系列を伝送するものとみなしたとき、上記第１の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第１の定数を、上記第１の符号化系列に対して乗算し、

上記第２の乗算手段は、上記雑音と、上記第１の定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して上記第２の符号化系列を伝送するものとみなしたとき、上記第２の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第２の定数を、上記第２の符号化系列に対して乗算することを特徴とする請求項１記載の多元接続システム。

【請求項３】　上記第１の乗算手段は、雑音が加算される通信路を介して上記第１の符号化系列を伝送するものとみなしたとき、上記第１の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第１の定数を、上記第１の符号化系列に対して乗算し、

上記第２の乗算手段は、上記雑音に比して所定量だけ大きい雑音と、上記第１の定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して上記第２の符号化系列を伝送するものとみなしたとき、上記第２の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第２の定数を、上記第２の符号化系列に対して乗算することを特徴とする請求項１記載の多元接続システム。

【請求項４】　上記基地局は、伝送ロスの大きなユーザに対して割り当てた上記第２の符号化系列に対する上記第２の定数として、マージンを多くとるように決定し、伝送ロスの小さなユーザに対して割り当てた上記第１の符号化系列に対する上記第１の定数として、マージンを少なくとるように決定することを特徴とする請求項３記載の多元接続システム。

【請求項５】　上記第１の変換手段は、

上記第１の情報ビット系列に対して所定の符号化を施す第１の符号化手段と、

上記第１の符号化手段によって得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなる上記第１の符号化系列を生成する第１の変調手段とを有し、

上記第２の変換手段は、

上記第２の情報ビット系列に対して所定の符号化を施す第２の符号化手段と、

上記第２の符号化手段によって得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなるからなる上記第２の符号化系列を生成する第２の変調手段とを有することを特徴とする請求項１記載の多元接続システム。

【請求項６】　上記第１の符号化手段及び／又は上記第２の符号化手段は、入力したデータの順序を所定の置換位置情報に基づいて並べ替えるチャネル用のチャネル・インターリーブ手段を有し、

上記第１の変調手段及び／又は上記第２の変調手段は、上記チャネル・インターリーブ手段から供給されたインターリーブデータに対して所定の変調方式に基づく信号点のマッピングを行うことを特徴とする請求項５記載の多元接続システム。

【請求項７】　上記復号手段は、少なくとも出力する情報ビット系列よりも高次の情報ビット系列を変換する変換手段に対応した復号を行うことを特徴とする請求項１記載の多元接続システム。

【請求項８】　上記復号手段は、上記受信値から受信シンボルに関する尤度を算出する尤度算出手段を有することを特徴とする請求項７記載の多元接続システム。

【請求項９】　上記復号手段は、上記加算手段によって

最後に加算された最高次の上記第２の情報ビット系列から順次復号を行うことを特徴とする請求項１記載の多元接続システム。

【請求項１０】　上記基地局は、伝送ロスの大きなユーザに対しては、高次の情報ビット系列として割り当てることを特徴とする請求項１記載の多元接続システム。

【請求項１１】　基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、

各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける上記基地局によって所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する工程と、

上記基地局によって上記第１の情報ビット系列を変換して得られた上記第１の符号化系列に対して第１の定数を乗算する工程と、

上記基地局によって所定のビットからなる第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの工程と、

上記基地局によって上記第２の情報ビット系列を変換して得られた上記第２の符号化系列に対して第２の定数を乗算する少なくとも１つの工程と、

上記基地局によって上記第１の符号化系列に対して上記第１の定数を乗算して得られた第１の定数倍符号化系列と、上記第２の符号化系列に対して上記第２の定数を乗算して得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する工程と、

上記基地局によって上記加算符号化系列を送信信号として送信する工程と、

上記基地局によって伝送された上記加算符号化系列に対して所定の雑音が加算された受信信号を受信する上記端末によって上記受信信号を入力する工程と、

上記端末によって受信された上記受信値に基づいて、上記第１の情報ビット系列又は上記第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備えることを特徴とする多元接続方法。

【請求項１２】　上記第１の符号化系列に対して上記第１の定数を乗算する工程では、雑音が加算される通信路を介して上記第１の符号化系列を伝送するものとみなしたとき、上記第１の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第１の定数が、上記第１の符号化系列に対して乗算され、

上記第２の符号化系列に対して上記第２の定数を乗算する工程では、上記雑音と、上記第１の定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して上記第２の符号化系列を伝送するものとみなしたとき、上記第２の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第２の定数が、上記第２の符号化系列に対して乗算されることを特徴とする請求項１１記載の多元接続方法。

【請求項１３】　上記第１の符号化系列に対して上記第１の定数を乗算する工程では、雑音が加算される通信路を介して上記第１の符号化系列を伝送するものとみなしたとき、上記第１の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第１の定数が、上記第１の符号化系列に対して乗算され、

上記第２の符号化系列に対して上記第２の定数を乗算する工程では、上記雑音に比して所定量だけ大きい雑音と、上記第１の定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して上記第２の符号化系列を伝送するものとみなしたとき、上記第２の情報ビット系列に対するビットエラーレートが十分小さくなるように設定された上記第２の定数が、上記第２の符号化系列に対して乗算されることを特徴とする請求項１１記載の多元接続方法。

【請求項１４】　上記基地局は、伝送ロスの大きなユーザに対して割り当てた上記第２の符号化系列に対する上記第２の定数として、マージンを多くとるように決定し、伝送ロスの小さなユーザに対して割り当てた上記第１の符号化系列に対する上記第１の定数として、マージンを少なくとるように決定することを特徴とする請求項１３記載の多元接続方法。

【請求項１５】　上記第１の情報ビット系列を変換する工程は、

上記第１の情報ビット系列に対して所定の符号化を施す工程と、

上記第１の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなる上記第１の符号化系列を生成する工程とを有し、

上記第２の情報ビット系列を変換する工程は、

上記第２の情報ビット系列に対して所定の符号化を施す工程と、

上記第２の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなる上記第２の符号化系列を生成する工程とを有することを特徴とする請求項１１記載の多元接続方法。

【請求項１６】　上記第１の情報ビット系列に対して所定の符号化を施す工程及び／又は上記第２の情報ビット系列に対して所定の符号化を施す工程は、入力したデータの順序を所定の置換位置情報に基づいて並べ替えてインターリーブデータを生成するチャネル用の工程を有し、

上記第１の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行う工程及び／又は上記第２の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッ

ピングを行う工程では、上記インターリーブデータに対して所定の変調方式に基づく信号点のマッピングが行われることを特徴とする請求項１５記載の多元接続方法。

【請求項１７】　上記少なくとも１つの情報ビット系列に対する復号を行う工程では、少なくとも出力する情報ビット系列よりも高次の情報ビット系列を変換する工程に対応した復号が行われることを特徴とする請求項１１記載の多元接続方法。

【請求項１８】　上記少なくとも１つの情報ビット系列に対する復号を行う工程は、上記受信値から受信シンボルに関する尤度を算出する工程を有することを特徴とする請求項１７記載の多元接続方法。

【請求項１９】　上記少なくとも１つの情報ビット系列に対する復号を行う工程では、上記加算符号化系列を生成する工程にて最後に加算された最高次の上記第２の情報ビット系列から順次復号が行われることを特徴とする請求項１１記載の多元接続方法。

【請求項２０】　上記基地局は、伝送ロスの大きなユーザに対しては、高次の情報ビット系列として割り当てることを特徴とする請求項１１記載の多元接続方法。

【請求項２１】　基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、

情報を所定の形式に変換して伝送する上りリンクにおける上記端末は、それぞれ、

所定のビットからなる情報ビット系列をM個の数値からなる符号化系列に変換する変換手段と、

上記変換手段によって変換されて得られた上記符号化系列に対して所定の定数を乗算する乗算手段とを備え、

複数の上記端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する上記基地局は、

上記受信信号を入力する受信手段と、

上記受信手段から供給された上記受信値に基づいて、上記端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備えることを特徴とする多元接続システム。

【請求項２２】　低次の情報ビット系列が割り当てられた第１の端末における上記乗算手段は、雑音が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数を、符号化系列に対して乗算し、

高次の情報ビット系列が割り当てられた第２の端末における上記乗算手段は、上記雑音と、上記第１の端末から伝送された定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数を、符号化系列に対して乗算することを特徴とする請求項２１記載の多元接続システム。

【請求項２３】　低次の情報ビット系列が割り当てられた第１の端末における上記乗算手段は、雑音が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数を、符号化系列に対して乗算し、

高次の情報ビット系列が割り当てられた第２の端末における上記乗算手段は、上記雑音に比して所定量だけ大きい雑音と、上記第１の端末から伝送された定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数を、符号化系列に対して乗算することを特徴とする請求項２１記載の多元接続システム。

【請求項２４】　上記端末は、それぞれ、伝送ロスに基づいて送信パワーを制御し、各ユーザ間の送信信号の振幅が適切となるように上記定数を設定することを特徴とする請求項２３記載の多元接続システム。

【請求項２５】　低次の情報ビット系列が割り当てられた第１の端末における上記変換手段は、

情報ビット系列に対して所定の符号化を施す第１の符号化手段と、

上記第１の符号化手段によって得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、M個の数値からなる符号化系列を生成する第１の変調手段とを有し、

高次の情報ビット系列が割り当てられた第２の端末における上記変換手段は、

情報ビット系列に対して所定の符号化を施す第２の符号化手段と、

上記第２の符号化手段によって得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、M個の数値からなるからなる符号化系列を生成する第２の変調手段とを有することを特徴とする請求項２１記載の多元接続システム。

【請求項２６】　上記第１の符号化手段及び／又は上記第２の符号化手段は、入力したデータの順序を所定の置換位置情報に基づいて並べ替えるチャネル用のチャネル・インターリーブ手段を有し、

上記第１の変調手段及び／又は上記第２の変調手段は、上記チャネル・インターリーブ手段から供給されたインターリーブデータに対して所定の変調方式に基づく信号点のマッピングを行うことを特徴とする請求項２５記載の多元接続システム。

【請求項２７】　上記復号手段は、

低次の情報ビット系列が割り当てられた第１の端末における上記変換手段に対応した復号を行う第１の復号手段

と、
高次の情報ビット系列が割り当てられた第２の端末における上記変換手段に対応した復号を行う少なくとも１つの第２の復号手段とを有することを特徴とする請求項２１記載の多元接続システム。
【請求項２８】　上記第１の復号手段及び上記第２の復号手段は、それぞれ、上記受信値から受信シンボルに関する尤度を算出する尤度算出手段を有することを特徴とする請求項２７記載の多元接続システム。
【請求項２９】　上記復号手段は、最高次の情報ビット系列から順次復号を行うことを特徴とする請求項２１記載の多元接続システム。
【請求項３０】　伝送ロスの小さなユーザの上記端末ほど、高次の情報ビット系列として割り当てられることを特徴とする請求項２１記載の多元接続システム。
【請求項３１】　基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、
情報を所定の形式に変換して伝送する上りリンクにおける上記端末のそれぞれによって所定のビットからなる情報ビット系列をＭ個の数値からなる符号化系列に変換する工程と、
上記端末のそれぞれによって上記情報ビット系列を変換して得られた上記符号化系列に対して所定の定数を乗算する工程と、
複数の上記端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する上記基地局によって上記受信信号を入力する工程と、
上記基地局によって受信された上記受信値に基づいて、上記端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備えることを特徴とする多元接続方法。
【請求項３２】　低次の情報ビット系列が割り当てられた第１の端末によって符号化系列に対して定数を乗算する工程では、雑音が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数が、符号化系列に対して乗算され、
高次の情報ビット系列が割り当てられた第２の端末によって符号化系列に対して定数を乗算する工程では、上記雑音と、上記第１の端末から伝送された定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数が、符号化系列に対して乗算されることを特徴とする請求項３１記載の多元接続方法。
【請求項３３】　低次の情報ビット系列が割り当てられた第１の端末によって符号化系列に対して定数を乗算する工程では、雑音が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数が、符号化系列に対して乗算され、
高次の情報ビット系列が割り当てられた第２の端末によって符号化系列に対して定数を乗算する工程では、上記雑音に比して所定量だけ大きい雑音と、上記第１の端末から伝送された定数倍符号化系列と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列を伝送するものとみなしたとき、情報ビット系列に対するビットエラーレートが十分小さくなるように設定された定数が、符号化系列に対して乗算されることを特徴とする請求項３１記載の多元接続方法。
【請求項３４】　上記端末は、それぞれ、伝送ロスに基づいて送信パワーを制御し、各ユーザ間の送信信号の振幅が適切となるように上記定数を設定することを特徴とする請求項３３記載の多元接続方法。
【請求項３５】　低次の情報ビット系列が割り当てられた第１の端末によって第１の情報ビット系列を変換する工程は、
上記第１の情報ビット系列に対して所定の符号化を施す工程と、
上記第１の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなる第１の符号化系列を生成する工程とを有し、
高次の情報ビット系列が割り当てられた第２の端末によって第２の情報ビット系列を変換する工程は、
上記第２の情報ビット系列に対して所定の符号化を施す工程と、
上記第２の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行い、Ｍ個の数値からなる第２の符号化系列を生成する工程とを有することを特徴とする請求項３１記載の多元接続方法。
【請求項３６】　上記第１の情報ビット系列に対して所定の符号化を施す工程及び／又は上記第２の情報ビット系列に対して所定の符号化を施す工程は、入力したデータの順序を所定の置換位置情報に基づいて並べ替えてインターリーブデータを生成するチャネル用の工程を有し、
上記第１の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行う工程及び／又は上記第２の情報ビット系列に対して所定の符号化を施す工程にて得られた系列に対して所定の変調方式に基づく信号点のマッピングを行う工程では、上記インターリーブデータに対して所定の変調方式に基づく信号点のマッピングが行われることを特徴とする請求項３５記載の多元接続方法。

【請求項３７】　上記少なくとも１つの情報ビット系列に対する復号を行う工程は、
低次の情報ビット系列が割り当てられた第１の端末によって第１の情報ビット系列を変換する工程に対応した復号を行う工程と、
高次の情報ビット系列が割り当てられた第２の端末によって第２の情報ビット系列を変換する工程に対応した復号を行う少なくとも１つの工程とを有することを特徴とする請求項３１記載の多元接続方法。
【請求項３８】　上記第１の端末によって上記第１の情報ビット系列を変換する工程に対応した復号を行う工程及び上記第２の端末によって上記第２の情報ビット系列を変換する工程に対応した復号を行う少なくとも１つの工程は、それぞれ、上記受信値から受信シンボルに関する尤度を算出する工程を有することを特徴とする請求項３７記載の多元接続方法。
【請求項３９】　上記少なくとも１つの情報ビット系列に対する復号を行う工程では、最高次の上記第２の情報ビット系列から順次復号が行われることを特徴とする請求項３１記載の多元接続方法。
【請求項４０】　伝送ロスの小さなユーザの上記端末ほど、高次の情報ビット系列として割り当てられることを特徴とする請求項３１記載の多元接続方法。
【発明の詳細な説明】
【０００１】
【発明の属する技術分野】本発明は、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システム及び多元接続方法に関する。
【０００２】
【従来の技術】近年、例えば、移動体通信や深宇宙通信といった通信分野、及び地上波又は衛星ディジタル放送といった放送分野の研究が著しく進められているが、それに伴い、誤り訂正符号化及び復号の効率化を目的として符号理論に関する研究も盛んに行われている。
【０００３】符号性能の理論的限界としては、いわゆるシャノン（C. E. Shannon）の通信路符号化定理によって与えられるシャノン限界が知られている。
【０００４】符号理論に関する研究は、このシャノン限界に近い性能を示す符号を開発することを目的として行われている。近年では、シャノン限界に近い性能を示す符号化方法として、例えば、いわゆるターボ符号と称される並列連接畳み込み符号（Parallel Concatenated Convolutional Codes；以下、ＰＣＣＣという。）や縦列連接畳み込み符号（Serially Concatenated Convolutional Codes；以下、ＳＣＣＣという。）が開発されている。
【０００５】一方、近年では、これらの符号に対する復号方法についても研究が盛んに行われている。具体的には、連接符号における内符号の復号出力や繰り返し復号法における各繰り返し復号動作の出力を軟出力とすることで、シンボル誤り率を小さくする研究がなされており、それに適した復号方法に関する研究が盛んに行われている。例えば畳み込み符号等の所定の符号を復号した際のシンボル誤り率を最小にする方法としては、「Bahl, Cocke, Jelinek and Raviv, "Optimal decoding of linear codes for minimizing symbol error rate", IEEE Trans. Inf. Theory, vol. IT-20, pp. 284-287, Mar. 1974」に記載されているＢＣＪＲアルゴリズムや、このＢＣＪＲアルゴリズムを改良した「Robertson, Villebrun and Hoeher, "A comparison of optimal and sub-optimal MAP decoding algorithmsoperating in the domain", IEEE Int. Conf. on Communications, pp. 1009-1013, June 1995」に記載されているＭａｘ－Ｌｏｇ　ＭＡＰアルゴリズム及びＬｏｇ　ＭＡＰアルゴリズム（以下、Ｍａｘ－Ｌｏｇ－ＢＣＪＲアルゴリズム及びＬｏｇ－ＢＣＪＲアルゴリズムという。）が知られている。上述したＰＣＣＣ又はＳＣＣＣを復号する際には、これらのＢＣＪＲアルゴリズム、Ｍａｘ－Ｌｏｇ－ＢＣＪＲアルゴリズム又はＬｏｇ－ＢＣＪＲアルゴリズムに基づく最大事後確率（Maximum A Posteriori probability；ＭＡＰ）復号を行う複数の復号器の間で、いわゆる繰り返し復号を行うことになる。
【０００６】
【発明が解決しようとする課題】ところで、符号化を行う場合には、符号化して得られたビットデータに対して、例えば、８相位相変調方式（8-Phase Shift Keying；以下、８ＰＳＫ変調方式という。）、１６相直交振幅変調方式（16-Quadrature Amplitude Modulation；以下、１６ＱＡＭ変調方式という。）又は６４相直交振幅変調方式（64-Quadrature Amplitude Modulation；以下、６４ＱＡＭ変調方式という。）といった予め定められた多値変調方式に基づく信号点のマッピングが行われる。
【０００７】しかしながら、符号化においては、予め定められた多値変調方式に基づく信号点のマッピングを用いた場合には、符号化したデータの雑音に対する余裕とマッピングによって算出される雑音に対する余裕とを完全に一致させることはできず、ビットエラーレート等の伝送特性を劣化させていた。
【０００８】一方、移動体通信等の通信システムにおいては、所定の符号化が施された複数系統の情報系列が多元接続され、各基地局又は端末によって復号される。この移動体通信等において用いられる多元接続方式としては、いわゆる時分割多元接続（Time Division Multiple Access；ＴＤＭＡ）やいわゆる周波数分割多元接続（Frequency Division Multiple Access；ＦＤＭＡ）が用いられている。これらの多元接続方式は、互いに直交関係となるように時間又は周波数等のリソースを分割し、それぞれのリソースに対して異なるユーザの信号を割り当てることによって実現される。

【０００９】しかしながら、これらの多元接続方式においては、リソースが限定された場合には、多重化できるユーザの信号数が制限されることになる。そのため、特に移動体通信のユーザ数が増加の一途を辿る実情を克服するための新たな多元接続方式が待望されている。

【００１０】本発明は、このような実情に鑑みてなされたものであり、多値変調方式等を伴う符号化において、高い性能での符号化を容易に実現することができる理論的に最適と思われる符号化方式を新たに開発するとともに、この符号化を適用した新たな多元接続方式を提案し、限られたリソースを有効利用してより多くのユーザを収容することができる多元接続システム及び多元接続方法を提供することを目的とする。

【００１１】

【課題を解決するための手段】上述した目的を達成する本発明にかかる多元接続システムは、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける基地局は、所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する第１の変換手段と、この第１の変換手段によって変換されて得られた第１の符号化系列に対して第１の定数を乗算する第１の乗算手段と、所定のビットからなる第２のユーザの第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの第２の変換手段と、この第２の変換手段によって変換されて得られた第２の符号化系列に対して第２の定数を乗算する少なくとも１つの第２の乗算手段と、第１の乗算手段によって乗算されて得られた第１の定数倍符号化系列と、第２の乗算手段によって乗算されて得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する加算手段と、加算符号化系列を送信信号として送信する送信手段とを備え、基地局によって伝送された加算符号化系列に対して所定の雑音が加算された受信信号を受信する端末は、それぞれ、受信信号を入力する受信手段と、この受信手段から供給された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備えることを特徴としている。

【００１２】このような本発明にかかる多元接続システムは、第１の符号化系列に対して第１の乗算手段によって第１の定数が乗算された第１の定数倍符号化系列と、第２の符号化系列に対して第２の乗算手段によって第２の定数が乗算された第２の定数倍符号化系列とを加算手段によって加算して加算符号化系列を生成して基地局から伝送し、この加算符号化系列に対して所定の雑音が加算された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を各端末における復号手段によって行う。

【００１３】また、上述した目的を達成する本発明にかかる多元接続方法は、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける基地局によって所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する工程と、基地局によって第１の情報ビット系列を変換して得られた第１の符号化系列に対して第１の定数を乗算する工程と、基地局によって所定のビットからなる第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの工程と、基地局によって第２の情報ビット系列を変換して得られた第２の符号化系列に対して第２の定数を乗算する少なくとも１つの工程と、基地局によって第１の符号化系列に対して第１の定数を乗算して得られた第１の定数倍符号化系列と、第２の符号化系列に対して第２の定数を乗算して得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する工程と、基地局によって加算符号化系列を送信信号として送信する工程と、基地局によって伝送された加算符号化系列に対して所定の雑音が加算された受信信号を受信する端末によって受信信号を入力する工程と、端末によって受信された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備えることを特徴としている。

【００１４】このような本発明にかかる多元接続方法は、第１の符号化系列に対して第１の定数が乗算された第１の定数倍符号化系列と、第２の符号化系列に対して第２の定数が乗算された第２の定数倍符号化系列とを基地局によって加算して加算符号化系列を生成して伝送し、この加算符号化系列に対して所定の雑音が加算された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を各端末によって行う。

【００１５】さらに、上述した目的を達成する本発明にかかる多元接続システムは、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、情報を所定の形式に変換して伝送する上りリンクにおける端末は、それぞれ、所定のビットからなる情報ビット系列をＭ個の数値からなる符号化系列に変換する変換手段と、この変換手段によって変換されて得られた符号化系列に対して所定の定数を乗算する乗算手段とを備え、複数の端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する基地局は、受信信号を入力する受信手段と、この受信手段から

供給された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも1つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備えることを特徴としている。

【0016】このような本発明にかかる多元接続システムは、符号化系列に対して乗算手段によって所定の定数が乗算された定数倍符号化系列が各端末から伝送され、これらの定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも1つの情報ビット系列に対する復号を基地局における復号手段によって行う。

【0017】さらにまた、上述した目的を達成する本発明にかかる多元接続方法は、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、情報を所定の形式に変換して伝送する上りリンクにおける端末のそれぞれによって所定のビットからなる情報ビット系列をM個の数値からなる符号化系列に変換する工程と、端末のそれぞれによって情報ビット系列を変換して得られた符号化系列に対して所定の定数を乗算する工程と、複数の端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する基地局によって受信信号を入力する工程と、基地局によって受信された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも1つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備えることを特徴としている。

【0018】このような本発明にかかる多元接続方法は、符号化系列に対して所定の定数が乗算された定数倍符号化系列が各端末から伝送され、これらの定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも1つの情報ビット系列に対する復号を基地局によって行う。

【0019】

【発明の実施の形態】以下、本発明を適用した具体的な実施の形態について図面を参照しながら詳細に説明する。

【0020】この実施の形態は、ディジタル情報を図示しない送信装置によって符号化し、その出力を雑音のある通信路を介して図示しない受信装置に入力して、この受信装置によって復号するチャネルモデルに適用したデータ送受信システムを、移動体通信等の基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムに適用したものである。この多元接続システムにおいて、データを送信する側は、後述する伝送率の小さな符号を用いて伝送率が大きな符号を構成して伝送することができるものであって、既存の信号点のマッピング方法の概念を変える符号化を行うものである。一方、多元接続システムにおいて、データを受信する側は、このような送信側によって符号化がなされた符号を受信し、この符号の復号を高精度且つ容易に行うものである。

【0021】まず、多元接続システムの説明に先立って、本発明における"符号化"について、以下のように定義する。

【0022】すなわち、ここで述べる符号化は、いわゆる通信路符号化を意味するが、広義に解し、ある情報が存在するとき、与えられた通信路に対して信号を変換することを意味するものとする。通信路として最も重要であるものは、白色ガウス雑音が加えられるユークリッド空間で表されるものである。通常、データ伝送分野においては、伝送速度［ビット／ｓ］や占有帯域が問題となる。しかしながら、サンプリング定理により、1［ｓ］×1［Ｈｚ］内の信号は、2個の実数又は1個の複素数で記述されることから、時間や周波数の概念は、ここで述べる符号化においては考慮する必要がなく、単純にベクトル空間における次元数に置換することができる。すなわち、ここで述べる符号化とは、図1に示すように、「Ｎビットからなる論理情報を2^N個の符号語としてＭ個の数値からなる符号化系列に変換する」ことであると定義することができる。

【0023】ここで、符号化の対象、すなわち、変換の対象となる論理情報のビット数Ｎと、変換先のベクトル空間における次元数Ｍとを用いて、次式（1）に示すように、伝送率（Transmission Rate）Ｃというパラメータを定義する。

【0024】

【数1】

$$C = N/M \qquad \cdots(1)$$

【0025】すなわち、伝送率Ｃは、実数1次元あたりに伝送されるビット数を意味するものである。ここで、符号理論においては、情報ビット数を"k"、符号長を"n"として、次式（2）に示すように、符号化率Ｒが定義される。伝送率Ｃは、例えば2相位相変調方式（Binary Phase Shift Keying；以下、ＢＰＳＫ変調方式という。）のように1つの符号ビットを1個の実数にマッピングする場合には、符号化率Ｒと同一のものとなる。

【0026】

【数2】

$$R = k/n \qquad \cdots(2)$$

【0027】伝送率Ｃは、符号化率Ｒと比較すると、いわゆる符号化変調（coded modulation）を含む一般的な符号化を考える場合には、符号化率Ｒよりも意義があることが多いパラメータといえる。この伝送率Ｃと等価な次元を有するパラメータとして、周波数あたりの伝送速度Ｕ［ビット／ｓ／Ｈｚ］がある。ここで、サンプリング定理により、1［Ｈｚ］あたりで2個の実数が単位時

間で伝送されることから、伝送率Cと伝送速度Uとの間には、次式（３）に示す関係が成立する。伝送率Cは、伝送速度Uと比較した場合であっても、上述したように、符号化においては時間や周波数の概念が不要であることから、伝送速度Uよりも意義があるパラメータといえる。

【００２８】

【数３】

$$C = U/2 \qquad \cdots(3)$$

【００２９】ここで、シャノン（C. E. Shannon）の通信路符号化定理における最大通信路容量C_{max}［ビット］は、次式（４）で表される。

【００３０】

【数４】

$$C_{max} = \frac{1}{2}\log_2\left(1+\frac{S}{N}\right) \qquad \cdots(4)$$

【００３１】上式（４）は、加法的白色ガウス雑音（Additive White Gaussian Noise；以下、AWGNという。）が加えられて信号対雑音比が"S／N"である通信路において、１個の実数あたりにC_{max}［ビット］の情報を誤りなく伝送できることを意味している。また、１ビットあたりの情報のエネルギは、通常、"E_b［J］"と表される。すなわち、１個の実数あたりにC_{max}［ビット］の情報を伝送する場合には、１個の実数あたりのエネルギは、$C_{max}\cdot E_b$［J］で表される。雑音電力密度n_0［J］のAWGNチャネルにおいては、１個の実数あたりに加算される雑音のエネルギは、"$n_0/2$［J］"となることから、最大通信路容量C_{max}は、通信路容量の限界式に当てはめると、次式（５）に示すように表される。

【００３２】

【数５】

$$C_{max} = \frac{1}{2}\log_2\left(1+2C_{max}\frac{E_b}{n_0}\right) \qquad \cdots(5)$$

【００３３】一方、１個の実数あたりにC［ビット］の情報を伝送する際に必要となる最小の１ビットあたりの信号対雑音電力比E_b/n_0の値を"ξ_{min}"とすると、通信路容量C［ビット］は、次式（６）で表され、これを"ξ_{min}"について解くと、次式（７）で表される。

【００３４】

【数６】

$$C = \frac{1}{2}\log_2\left(1+2C\xi_{min}\right) \qquad \cdots(6)$$

【００３５】

【数７】

$$\xi_{min} = \frac{1}{2C}\left[2^{2C}-1\right] \qquad \cdots(7)$$

【００３６】さて、以下では、以上のように定義される符号化を行う送信装置と、この送信装置によって符号化がなされた符号の復号を行う受信装置とを備えるデータ送受信システムについて説明する。なお、本発明の実施の形態として示す多元接続システムは、後述するように、このデータ送受信システムにおける送信装置及び受信装置を、それぞれ、下りリンクと上りリンクとに応じて、基地局又は端末に適用して実現される。

【００３７】まず、データ送受信システムにおける送信装置について説明する。この送信装置は、上述した伝送率の小さな符号を用いて、順次伝送率が大きな符号を構成するものである。ここではまず、送信装置の現実的な具体的構成の説明に先立って、送信装置が行う符号化の基本的な原理について説明する。

【００３８】L個の情報ビット系列{$b^{(0)}$, $b^{(1)}$, ・・・, $b^{(L-1)}$}があり、l番目の情報ビット系列$b^{(l)}$は、次式（８）に示すように、Nビットの情報ビット$b_n^{(l)}$（n＝0，1，・・・，N－1）から構成されるものとする。

【００３９】

【数８】

$$b^{(l)} = \left\{b_0^{(l)}, b_1^{(l)} \cdots\cdots, b_{N-1}^{(l)}\right\}^T \qquad \cdots(8)$$

【００４０】そして、上式（８）で表される情報ビット$b_n^{(l)}$を情報ビット系列毎に写像する符号化を行って得られる次式（９）に示す実数値系列$x^{(\cdot)}$を考える。なお、以下では、実数値系列$x^{(\cdot)}$を符号化系列$x^{(l)}$と称するものとする。

【００４１】

【数９】

$$x^{(l)} = x^{(l)}\left(b^{(l)}\right) \qquad \cdots(9)$$

【００４２】ここで、符号化系列$x^{(\cdot)}$は、系列によらずM個の数値からなるM次元実数ベクトルであり、次式（１０）で表される。

【００４３】

【数１０】

$$x^{(l)} = \left\{x_0^{(l)}, x_1^{(l)} \cdots\cdots, x_{M-1}^{(l)}\right\}^T \qquad \cdots(10)$$

【００４４】このとき、伝送率$C^{(l)}$は、次式（１１）で表される。

【００４５】

【数１１】

$$C^{(l)} = N^{(l)}/M \qquad \cdots(11)$$

【００４６】この符号化において、伝送誤り率を"0"又は"0"に十分近い値にするために必要な最小の１ビットあたりの信号対雑音電力比E_b/n_0の値を"$\xi^{(l)}$"とする。また、以下では、説明の便宜上、符号xは、次式（１２）に示すように、M次元ガウス分布をなすもの

とする。

【0047】

【数12】

$$p\left(x^{(i)}\right)=\frac{1}{\left(2\pi C^{(i)}E_b\right)^{M/2}}\exp\left[-\frac{1}{2C^{(i)}E_b}\left|x^{(i)}\right|^2\right] \quad \cdots(12)$$

【0048】情報ビット系列b(0)に対する符号化x(0)=x(0)(b(0))によって得られた符号化系列x(0)を、雑音電力密度n_0[J]のAWGNチャネルで正確に伝送するために必要となる1ビットあたりのエネルギ$E_b^{(0)}$[J]は、次式(13)に示すように、1個の実数あたりの雑音の分散"$n_0/2$"の2ξ倍となる。

【0049】

【数13】

$$E_b^{(0)}=2\xi^{(0)}n_0/2=\xi^{(0)}n_0 \quad \cdots(13)$$

【0050】このとき、1個の実数あたりの信号の分散ν(0)は、次式(14)に示すように算出される。

【0051】

【数14】

$$v^{(0)}=C^{(0)}\xi^{(0)}n_0 \quad \cdots(14)$$

【0052】ここで、情報ビット系列b(1)に対する符号化x(1)=x(1)(b(1))によって得られた符号化系列x(1)を、この伝送系に加算して正確に伝送することを考える。符号化系列x(0)は、符号化系列x(1)にとっては無関係であり、雑音としてしかみえないはずであることから、符号化系列x(1)を正確に伝送するために必要となる1ビットあたりのエネルギ$E_b^{(1)}$[J]を、次式(15)に示すように、元々の雑音と符号x(1)のエネルギの和に対してξ(1)倍に設定する。

【0053】

【数15】

$$E_b^{(1)}=2\xi^{(1)}\left(n_0/2+C^{(0)}\xi^{(0)}n_0\right)=n_0\xi^{(1)}\left(1+2C^{(0)}\xi^{(0)}\right) \quad \cdots(15)$$

【0054】このとき、1個の実数あたりの信号の分散ν(1)は、次式(16)に示すように算出される。

【0055】

【数16】

$$v^{(1)}=C^{(1)}\xi^{(1)}n_0\left(1+2C^{(0)}\xi^{(0)}\right) \quad \cdots(16)$$

【0056】さらに、同様の操作として、情報ビット系列b(2)に対する符号化x(2)=x(2)(b(2))によって得られた符号化系列x(2)を、この伝送系に加算して正確に伝送することを考える。符号化系列x(0),x(1)は、符号化系列x(2)にとっては無関係であり、雑音としてしか見えないはずであることから、符号化系列x(2)を正確に伝送するために必要となる1ビットあたりのエネルギ$E_b^{(2)}$[J]を、次式(17)に示すように、元々の雑音と符号x(0),x(1)のエネルギの和に対してξ(2)倍に設定する。また、このときの1個の実数あたりの信号の分散ν(2)は、次式(18)に示すように算出される。

【0057】

【数17】

$$\begin{aligned}E_b^{(2)}&=2\xi^{(2)}\left\{n_0/2+C^{(0)}\xi^{(0)}n_0+C^{(1)}\xi^{(1)}n_0\left(1+2C^{(0)}\xi^{(0)}\right)\right\}\\&=\xi^{(2)}n_0\left(1+2C^{(0)}\xi^{(0)}\right)\left(1+2C^{(1)}\xi^{(1)}\right)\end{aligned} \quad \cdots(17)$$

【0058】

【数18】

$$v^{(2)}=C^{(2)}\xi^{(2)}n_0\left(1+2C^{(0)}\xi^{(0)}\right)\left(1+2C^{(1)}\xi^{(1)}\right) \quad \cdots(18)$$

【0059】以後、同様の操作を情報ビット系列b(L-1)まで行うと、符号化系列x(L-1)を正確に伝送するために必要となる1ビットあたりのエネルギ$E_b^{(L-1)}$[J]は、次式(19)に示すようになり、このときの1個の実数あたりの信号の分散ν(L-1)は、次式(20)に示すように算出される。

【0060】

【数19】

$$E_b^{(L-1)}=\xi^{(L-1)}n_0\prod_{l=0}^{L-2}\left(1+2C^{(l)}\xi^{(l)}\right) \quad \cdots(19)$$

【0061】

【数20】

$$v^{(L-1)}=C^{(L-1)}\xi^{(L-1)}n_0\prod_{l=0}^{L-2}\left(1+2C^{(l)}\xi^{(l)}\right) \quad \cdots(20)$$

【００６２】以上の操作を１つの伝送系としてとらえ、各情報ビット系列に対する平均振幅を"ａ（ｉ）"とすると、情報ビット系列｛ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１）｝に対する符号化系列は、次式（２１）及び次式（２２）で表される。

【００６３】

【数２１】

$$g\left(b^{(0)},b^{(1)},\cdots,b^{(L-1)}\right)=\sum_{i=0}^{L-1}a^{(i)}x^{(i)} \quad \cdots(21)$$
$$=a^{(0)}x^{(0)}+a^{(1)}x^{(1)}+\cdots+a^{(L-1)}x^{(L-1)}$$

【００６４】

【数２２】

$$a^{(i)}=\sqrt{\frac{\nu^{(i)}}{n_0/2}}=\sqrt{2C^{(L-1)}\xi^{(L-1)}\prod_{i=0}^{L-2}\left(1+2C^{(i)}\xi^{(i)}\right)} \quad \cdots(22)$$

【００６５】なお、以下では、符号化系列ｘ（ｉ）をそれぞれ定数ａ（ｉ）倍した系列を定数倍符号化系列と称し、符号化系列ｇ（ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１））を加算符号化系列と称するものとする。なお、この加算符号化系列ｇ（ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１））は、数値系列を２つずつ組み合わせた複素数値系列であってもよい。

【００６６】送信装置は、このような原理に基づいて、入力した複数の情報ビット系列｛ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１）｝に対する符号化及び／又は変調を含む変換処理を行い、加算符号化系列ｇ（ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１））に変換し、通信路に伝送する。すなわち、送信装置は、入力した複数の情報ビット系列｛ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１）｝のそれぞれに対して所定の変換処理を行い、得られた符号化系列｛ｘ（０），ｘ（１），・・・，ｘ（Ｌ－１）｝のそれぞれに対して定数｛ａ（０），ａ（１），・・・，ａ（Ｌ－１）｝を乗算し、得られた定数倍符号化系列を要素毎に加算して加算符号化系列ｇ（ｂ（０），ｂ（１），・・・，ｂ（Ｌ－１））を生成する。

【００６７】このとき、送信装置は、基本的には、上式（２２）に示したように、任意の符号化系列ｘ（ｉ）に対して乗算する定数ａ（ｉ）として、雑音と、それ以前に加算された定数倍符号化系列ａ（０）ｘ（０），ａ（１）ｘ（１），・・・，ａ（ｉ－１）ｘ（ｉ－１）と同一の統計的性質を有する系列との和が加算される通信路を介して、符号化系列ｘ（ｉ）が正確に伝送されるように、すなわち、情報ビット系列ｂ（ｉ）に対するビットエラーレートが十分小さくなるように設定する。なお、ここでの定数倍符号化系列ａ（ｉ）ｘ（ｉ）が有する統計的性質とは、分散、確率密度関数及びパワースペクトルの形状等を示すものである。

【００６８】具体的には、定数ａ（０）は、雑音が加算される通信路を介して符号化系列ｘ（０）を伝送するものとみなしたとき、情報ビット系列ｂ（０）に対するビットエラーレートが十分小さくなるように設定される。また、定数ａ（１）は、雑音と、定数倍符号化系列ａ（０）ｘ（０）と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列ｘ（・）を伝送するものとみなしたとき、情報ビット系列ｂ（ｉ）に対するビットエラーレートが十分小さくなるように設定される。さらに、定数ａ（２）は、雑音と、定数倍符号化系列ａ（０）ｘ（０），ａ（１）ｘ（１）と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列ｘ（２）を伝送するものとみなしたとき、情報ビット系列ｂ（２）に対するビットエラーレートが十分小さくなるように設定される。

【００６９】ここで、情報ビット系列ｂ（ｉ）に対するビットエラーレートが十分小さくなるか否かの基準は、理論的考察によって求めてもよく、シミュレーションによって求めてもよい。なお、十分に小さいビットエラーレートとしては、最終的にシステムとして必要とするビットエラーレートよりも低い値とするのが順当であり、例えば"１０－５"程度が望ましい。

【００７０】このように、送信装置は、各情報ビット系列ｂ（ｉ）に対して等しい重みを与えながら定数ａ（ｉ）を設定することができる。

【００７１】また、送信装置は、符号毎に雑音に対するマージンを変化させて定数ａ（ｉ）を設定するようにしてもよい。

【００７２】すなわち、送信装置は、任意の符号化系列ｘ（ｉ）に対して乗算する定数ａ（ｉ）として、想定される雑音に比してＧ（ｉ）［ｄＢ］だけ大きな雑音と、それ以前に加算された定数倍符号化系列ａ（０）ｘ（０），ａ（１）ｘ（１），・・・，ａ（ｉ－１）ｘ（ｉ－１）と同一の統計的性質を有する系列との和が加算される通信路を介して、符号化系列ｘ（ｉ）が正確に伝送されるように、すなわち、情報ビット系列ｂ（ｉ）に対するビットエラーレートが十分小さくなるように設定する。

【００７３】具体的には、定数ａ（０）は、上述したように、雑音が加算される通信路を介して符号化系列ｘ（０）を伝送するものとみなしたとき、情報ビット系列ｂ（０）に対するビットエラーレートが十分小さくなるように設定される。また、定数ａ（１）は、想定される

雑音に比してG(1)［dB］だけ大きい雑音と、定数倍符号化系列a(0)x(0)と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列x(1)を伝送するものとみなしたとき、情報ビット系列b(1)に対するビットエラーレートが十分小さくなるように設定される。さらに、定数a(2)は、想定される雑音に比してG(1)［dB］だけ大きい雑音と、定数倍符号化系列a(0)x(0)，a(1)x(1)と同一の統計的性質を有する系列との和が加算される通信路を介して符号化系列x(2)を伝送するものとみなしたとき、情報ビット系列b(2)に対するビットエラーレートが十分小さくなるように設定される。

【0074】このように、送信装置は、各情報ビット系列b(1)に対して異なる重みを与えながら定数a(1)を容易に設定することができる。この設定方法は、後述するように、各情報ビット系列b(1)の重要度が異なる場合に有効であり、重要な情報ビット系列b(1)ほど、大きい定数a(1)が設定されることになる。

【0075】なお、送信装置は、例えばレイリーフェージング・チャネル（Rayleigh fading channel）といった変動する通信路を介した伝送を行う場合であっても、上述した方法と同様の操作によって定数a(1)を設定することができる。情報ビット系列b(1)に対するビットエラーレートが十分小さくなるか否かの基準は、所望のビットエラーレートを実現するために必要となるエネルギE_b［J］が通信路の状態によって変化することから、想定するチャネルモデルを用いた考察やシミュレーションによって判断することができる。また、送信装置は、後述するように、通信路の状態変化に適応的に対応して定数a(1)を設定することもできる。

【0076】いずれにせよ、送信装置は、所望のビットエラーレートを実現するエネルギE_b［J］で情報ビット系列b(1)を伝送するために、定数a(1)を符号化系列x(1)に対して乗算する。

【0077】ここで、このような送信装置によって符号化されて得られる加算符号化系列gの信号点配置について考える。なお、ここでは、説明の便宜上、符号化系列x(1)を生成する際に、系列によらず、BPSK変調方式に基づく信号点のマッピングを行うものとする。

【0078】符号化系列x(0)は、BPSK変調方式が施されていることから、いわゆるIQ平面におけるI軸上の"1"，"−1"に信号点が配置される。したがって、定数倍符号化系列a(0)x(0)は、図2中黒丸で示すように、I軸上の"a(0)"，"−a(0)"に信号点が配置されたものとなる。ここで、"a(0)"の絶対値は、系列によって異なるものであることはいうまでもない。

【0079】したがって、このようなBPSK変調方式が施されて得られる定数倍符号化系列a(0)x(0)，a(1)x(1)を加算することによって得られる加算符号化系列a(0)x(0)＋a(1)x(1)の信号点配置は、図3中黒丸で示すように、$2^2=4$個の信号点がI軸上に配置された振幅変調方式（AmplitudeShift Keying；以下、ASK変調方式という。）に類似したものとなる。同様に、定数倍符号化系列a(0)x(0)，a(1)x(1)，a(2)x(2)を加算することによって得られる加算符号化系列a(0)x(0)＋a(1)x(1)＋a(2)x(2)の信号点配置は、図4中黒丸で示すように、$2^3=8$個の信号点がI軸上に配置されたASK変調方式に類似したものとなる。最終的には、定数倍符号化系列a(0)x(0)，a(1)x(1)，a(2)x(2)，・・・，a(L−1)x(L−1)を加算することによって得られる加算符号化系列g（−a(0)x(0)−a(1)x(1)＋a(2)x(2)＋・・・＋a(L−1)x(L−1)）の信号点配置は、2^L個の信号点がI軸上に配置されたASK変調方式に類似したものとなる。

【0080】ここで、通常のASK変調方式においては、図5に4ASK変調方式による場合を示すように、信号点が等間隔に配置される。しかしながら、ここで提案する符号化においては、上述したように、所望のビットエラーレートを実現するエネルギE_b［J］で情報ビット系列b(1)が伝送されるように定数a(1)を設定しており、これに基づいて信号点を配置した場合には、加算符号化系列gにおける信号点が必ずしも等間隔になるとは限らず、むしろ図3及び図4に示したように非等間隔になる。この正当性について、4ASK変調方式を例に挙げて説明する。

【0081】信号点が等間隔に配置される必然性を探るために、4ASK変調方式における各信号点を、次式（23）に示すように与え、xを変化させて情報量を求める。ここで、分散は、"1"になるようにしてある。

【0082】

【数23】

$$\{x_0,x_1,x_2,x_3\}=\left\{\sqrt{2-x^2},x,-x,-\sqrt{2-x^2}\right\} \quad \cdots(23)$$

【0083】送信する信号の情報量H［ビット］は、x!＝1である場合には、xを変化させるまでもなく次式（24）で表され、2ビットとなる。

【0084】

【数24】

$$H = -\sum_{i=0}^{3} P(x_i)\log(P(x_i)) - \sum_{i=0}^{3}\frac{1}{4}\log\left(\frac{1}{4}\right) = 2 \qquad \cdots(24)$$

【0085】一方、通信路が雑音電力密度n_0のAWGNチャネルであるものとすると、受信側で受信する情報量Iは、受信値を"y"として、次式（25）で表される。なお、次式（25）における"p（y|x_i）"は、次式（26）で表される。

【0086】

【数25】

$$I(x,y) = H(y) - H(y|x) \cdot H(y) - H(n)$$
$$\cdot\cdot - \int_{-\infty}^{\infty} p(y)\log_2(p(y))dy - \frac{1}{2}\log_2(\pi e n_0)$$
$$= -\int_{-\infty}^{\infty}\sum_{i=0}^{3} p(y|x_i)P(x_i)\log_2\left(\sum_{i=0}^{3} p(y|x_i)P(x_i)\right)dy - \frac{1}{2}\log_2(\pi e n_0)$$
$$\cdots(25)$$

【0087】

【数26】

$$p(y|x_i) = \frac{1}{\sqrt{\pi n_0}}\exp\left[-\frac{1}{n_0}|y - x_i|^2\right] \qquad \cdots(26)$$

【0088】信号対雑音比S／Nを"12|dB|"としたときのxの配置に対する情報量を算出すると、図6に示すように、2点の極大値をとる曲線で表される。同図から明らかなように、情報量は、x－1．34，0．45の場合に極大となり、2ビット近傍になる。この状態は、次式（27）に示す信号点が等間隔である通常の4ASK変調方式の場合に相当する。これにより、信号点が等間隔に配置される正当性が示される。なお、同図から明らかなように、情報量は、x＝1．0の場合にはBPSK変調方式と同様となり、1ビット以上伝送できないことになる。

【0089】

【数27】

$$\{x_0, x_1, x_2, x_3\} = \{3/\sqrt{5}, 1/\sqrt{5}, -1/\sqrt{5}, -3/\sqrt{5}\} = \{1.34, 0.45, -0.45, -1.34\}$$
$$\cdots(27)$$

【0090】続いて、この状態から信号対雑音比S／Nを低下させ、極大となる情報量が1ビットになる信号対雑音比S／Nを求める。すなわち、上述した伝送率Cが"1．0"となる状態を求める。信号対雑音比S／Nを"1．96［dB］"としたときのxの配置に対する情報量は、図7に示すように、xの変化に対して1ビット近傍で緩やかに変化するものとなる。同図における情報量が1ビット近傍である領域を拡大すると、図8に示すように、2点の極大値をとる曲線で表される。同図から明らかなように、情報量は、x＝1．4，0．2の場合に極大となり、情報量を最大にする信号点配置は、次式（28）に示すように、非等間隔のものであることがわかる。

【0091】

【数28】

$$\{x_0, x_1, x_2, x_3\} = \{1.4, 0.2, -0.2, -1.4\} \qquad \cdots(28)$$

【0092】これらの事実は、最適な信号点配置が信号対雑音比S／Nに依存して決定されることを示している。すなわち、ここで提案する符号化においては、所望のビットエラーレートを実現するエネルギE_b［J］で情報ビット系列b（i）が伝送されるように定数a（i）を設定する際に、信号点が等間隔に配置される既存のマッピング方式に拘泥することなく、信号点が非等間隔に配置されるマッピングを行う方が特性が向上することになる。なお、信号対雑音比S／Nを極度に低下させた場合には、信号点配置は、次式（29）に示すように、いわば3ASK変調方式といえる状態になることを確認した。

【0093】

【数29】

$$\{x_0, x_1, x_2, x_3\} = \{1.414, 0.0, -0.0, -1.414\} \qquad \cdots(29)$$

【0094】このように、送信装置は、所望のビットエラーレートを実現するエネルギE_b［J］で情報ビット系列b（i）を伝送するために、定数a（i）を符号化系列x（i）に対して乗算し、得られた定数倍符号化系

列a(1)x(1)を加算することにより、信号点が非等間隔に配置された加算符号化系列gを生成する。勿論、送信装置は、信号点が等間隔に配置された加算符号化系列gを生成することもあり、これは、上述したように、信号対雑音比S/Nに依存して決定される。

【0095】ここで、L個の情報ビット系列{b(0),b(1),・・・,b(L-1)}が伝送されることによる合計の伝送率を"C'"とすると、伝送率C'は、次式(30)で表される。

【0096】

【数30】

$$C' = \sum_{l=0}^{L-1} C^{(l)} \qquad \cdots(30)$$

【0097】そして、情報ビット系列{b(0),b(1),・・・,b(L-1)}が要している平均の1ビットあたりの情報のエネルギを"$E_{b\cdot ave}$[J]"とすると、このエネルギ$E_{b\cdot ave}$[J]は、次式(31)で表される。

【0098】

【数31】

$$E_{b\cdot ave} = \frac{\sum_{l=0}^{L-1} N^{(l)} E_b^{(l)}}{\sum_{l=0}^{L-1} N^{(l)}} = \frac{\sum_{l=0}^{L-1} MC^{(l)} E_b^{(l)}}{\sum_{l=0}^{L-1} MC^{(l)}} = \frac{\sum_{l=0}^{L-1} C^{(l)} E_b^{(l)}}{\sum_{l=0}^{L-1} C^{(l)}} = \frac{n_0}{C'} \sum_{l=0}^{L-1} C^{(l)} \xi^{(l)} \prod_{l=0}^{l-1} \left(1 + 2C^{(l)} \xi^{(l)}\right) \qquad \cdots(31)$$

【0099】このとき、エラーレートを"0"とするために必要となる最小の1ビットあたりの信号対雑音電力比$E_{b\cdot ave}/n_0$の値を"ξ_{ave}'"とすると、このξ_{ave}'は、次式(32)で表される。

【0100】

【数32】

$$\xi'_{ave} = \frac{1}{C'} \sum_{l=0}^{L-1} C^{(l)} \xi^{(l)} \prod_{l=0}^{l-1} \left(1 + 2C^{(l)} \xi^{(l)}\right) \qquad \cdots(32)$$

【0101】ここで、L個の符号化系列{x(0),x(1),・・・,x(L-1)}が次式(33)で表されるシャノンの限界式を満たしているものとする。

【0102】

【数33】

$$\xi^{(l)} = \frac{1}{2C^{(l)}} \left[2^{2C^{(l)}} - 1\right] \qquad \cdots(33)$$

【0103】したがって、上式(33)を上式(32)に代入すると、次式(34)が得られる。

【0104】

【数34】

$$\begin{aligned}
\xi'_{ave} &= \frac{1}{2C'} \sum_{l=0}^{L-1} \left(2^{2C^{(l)}} - 1\right) \prod_{l=0}^{l-1} \left(2^{2C^{(l)}}\right) \\
&= \frac{1}{2C'} \sum_{l=0}^{L-1} \left(2^{2C^{(l)}} - 1\right) 2^{2\sum_{i=0}^{l-1} C^{(i)}} = \frac{1}{2C'} \sum_{l=0}^{L-1} \left(2^{2\sum_{i=0}^{l} C^{(i)}} - 2^{2\sum_{i=0}^{l-1} C^{(i)}}\right) \\
&= \frac{1}{2C'} \left(2^{2\sum_{i=0}^{L-1} C^{(i)}} - 2^{2\sum_{i=0}^{L-2} C^{(i)}} + 2^{2\sum_{i=0}^{L-2} C^{(i)}} - 2^{2\sum_{i=0}^{L-3} C^{(i)}} + \cdots\cdots + 2^{2C^{(0)}} - 1\right) \\
&= \frac{1}{2C'} \left(2^{2C'} - 1\right)
\end{aligned} \qquad \cdots(34)$$

【0105】これは、L個の符号化系列{x(0),x(1),・・・,x(L-1)}がシャノンの限界式を満たしている場合には、送信装置によって最終的に生成された符号も同様にシャノンの限界式を満たすものとなることを示している。

【0106】このように、送信装置は、シャノンの限界式を満たす符号を生成することができるものである。

【0107】さて、以下では、このような符号化を行う送信装置の現実的な具体的構成について詳述する。なお、ここでは、説明の便宜上、送信装置は、3つの情報

ビット系列{b(0), b(1), b(2)}を入力するものとする。

【0108】例えば図9に示すように、送信装置10は、情報ビット系列b(i)を入力して符号化系列x(i)へと変換する3つの変換器11_0, 11_1, 11_2と、これらの変換器11_0, 11_1, 11_2のそれぞれによって変換されて得られた符号化系列x(i)に対して定数a(i)を乗算する3つの乗算器12_0, 12_1, 12_2と、乗算器12_0によって乗算されて得られた定数倍符号化系列a(0)x(0)と乗算器12_1によって乗算されて得られた定数倍符号化系列a(1)x(1)とを要素毎に加算する加算器13_0と、この加算器13_0によって加算されて得られた加算符号化系列a(0)x(0)+a(1)x(1)と乗算器12_2によって乗算されて得られた定数倍符号化系列a(2)x(2)とを要素毎に加算する加算器13_1と、この加算器13_1によって加算されて得られた加算符号化系列g(=a(0)x(0)-a(1)x(1)+a(2)x(2))を外部へと送信する送信部14とを備える。

【0109】なお、入力される情報ビット系列{b(0), b(1), b(2)}は、互いに独立した3チャンネルの情報であってもよく、1つの情報ビット系列を3つに分割したものであってもよい。また、これらの情報ビット系列{b(0), b(1), b(2)}は、同一ビット数であっても互いに異なるビット数であってもよく、各ビット数をN_0, N_1, N_2とする。

【0110】変換器11_0, 11_1, 11_2は、それぞれ、ここでは図示しないが、所定の符号化器と変調器とを有し、入力したN_0, N_1, N_2ビットからなる情報ビット系列b(0), b(1), b(2)を"1", "-1"からなるユークリッド空間における信号へと変換する。変換器11_0, 11_1, 11_2は、それぞれ、この変換処理として、いかなる符号化及び変調を行ってよく、極言すれば、情報ビット系列b(0), b(1), b(2)を符号化することなく変調してもよい。いずれにせよ、変換器11_0, 11_1, 11_2は、それぞれ、N_0, N_1, N_2ビットからなる情報ビット系列b(0), b(1), b(2)をM個の数値からなる符号化系列に変換する。

【0111】このような変換器11_0, 11_1, 11_2としては、例えば図10に示すように、いわゆるターボ符号と称される並列連接畳み込み符号(Parallel Concatenated Convolutional Codes;以下、PCCCという。)及びBPSK変調方式を行うものが考えられる。

【0112】この変換器11_jは、同図に示すように、例えば畳み込み演算を行う2つの要素符号化器21_0, 21_1と、入力したデータの順序を並べ替えるインターリーバ22と、入力したデータを適宜間引くパンクチャ器23と、入力したデータの順序を並べ替えるチャネル用のチャネル・インターリーバ24と、BPSK変調方式に基づいて信号点のマッピングを行うBPSKマッピング器25とを有する。

【0113】要素符号化器21_0, 21_1は、例えば再帰的な畳み込み演算を行うものとして構成される。要素符号化器21_0, 21_1は、互いに同一のものであっても異なるものであってもよい。要素符号化器21_0, 21_1としては、例えば図11に示すように、2つの排他的論理和回路31_0, 31_1と、2つのシフトレジスタ32_0, 32_1とを有する要素符号化器21_jが考えられる。

【0114】この要素符号化器21_jにおいて、排他的論理和回路31_0は、インターリーバ22が要する処理時間と同時間だけ遅延された情報ビット系列b(i)を構成する情報ビット$b_n(i)$又はインターリーバ22から供給されたインターリーブデータと、シフトレジスタ32_0, 32_1から供給されるデータとを用いて排他的論理和演算を行い、演算結果を排他的論理和回路31_1及びシフトレジスタ32_0に供給する。

【0115】また、排他的論理和回路31_1は、排他的論理和回路31_0から供給されるデータとシフトレジスタ32_1から供給されるデータとを用いて排他的論理和演算を行い、演算結果を出力データとして外部に出力する。

【0116】さらに、シフトレジスタ32_0は、保持している1ビットのデータを排他的論理和回路31_0及びシフトレジスタ32_1に供給し続ける。そして、シフトレジスタ32_0は、クロックに同期させて、排他的論理和回路31_0から供給される1ビットのデータを新たに保持し、このデータを排他的論理和回路31_0及びシフトレジスタ32_1に新たに供給する。

【0117】さらにまた、シフトレジスタ32_1は、保持している1ビットのデータを排他的論理和回路31_0及び排他的論理和回路31_1に供給し続ける。そして、シフトレジスタ32_1は、クロックに同期させて、シフトレジスタ32_0から供給される1ビットのデータを新たに保持し、このデータを排他的論理和回路31_0及び排他的論理和回路31_1に新たに供給する。

【0118】このような要素符号化器21_jからなる要素符号化器21_0は、情報ビット系列$b_n(i)$を入力すると、各情報ビット$b_n(i)$に対して畳み込み演算を行い、演算結果を1ビットの出力データDaとして後段のパンクチャ器23に出力する。また、要素符号化器21_1も、要素符号化器21_0と同様に、インターリーバ22から供給されるインターリーブデータDbを入力すると、各ビットデータに対して畳み込み演算を行い、演算結果を1ビットの出力データDcとして後段のパンクチャ器23に出力する。

【0119】インターリーバ22は、情報ビット系列b(i)を入力し、この情報ビット系列b(i)を構成する情報ビット$b_n(i)$の順序を予め格納している置換

位置情報に基づいて並べ替え、インターリーブデータDbを生成する。インターリーバ22は、生成したインターリーブデータDbを要素符号化器21_1に供給する。

【0120】パンクチャ器23は、要素符号化器21_0，21_1から供給された2系列の出力データDa、Dcを所定の規則に基づいて択一的に選択することによって間引き、ビット数が削減されたパンクチャデータDdとしてチャネル・インターリーバ24に供給する。

【0121】チャネル・インターリーバ24は、要素符号化器21_1、インターリーバ22及びパンクチャ器23が要する処理時間と同時間だけ遅延された情報ビット系列b(i)と、パンクチャ器23から供給されたパンクチャデータDdとを入力し、これらの情報ビット系列b(i)を構成する情報ビットb_n(i)及びパンクチャデータDdを構成する各ビットデータの順序を予め格納している置換位置情報に基づいて並べ替え、MビットからなるインターリーブデータD_eを生成する。チャネル・インターリーバ24は、生成したインターリーブデータD_eをBPSKマッピング器25に供給する。なお、このチャネル・インターリーバ24は、必ずしも必要なものではなく、例えばバースト的に生じる誤りを分散させることによって特性を向上させることを主目的として設けられるものである。

【0122】BPSKマッピング器25は、チャネル・インターリーバ24から供給されたインターリーブデータD_eを、クロックに同期させて、BPSK変調方式の伝送シンボルにマッピングする。マッピング器25は、生成した伝送シンボルを符号化系列x(i)として外部に出力する。

【0123】このような変換器11_jは、情報ビット系列b(i)を入力すると、この情報ビット系列b(i)を組織成分データとして、チャネル・インターリーバ24に供給するとともに、要素符号化器21_0による情報ビット系列b(i)の畳み込み演算の結果得られる出力データDaと、要素符号化器21_1によるインターリーブデータDbの畳み込み演算の結果得られる出力データDcとをパンクチャしてチャネル・インターリーバ24に供給する。そして、変換器11_jは、チャネル・インターリーバ24から供給されたインターリーブデータD_eをBPSK変調方式の伝送シンボルにマッピングし、符号化系列x(i)として外部に出力する。

【0124】以下では、説明の便宜上、変換器11_0，11_1，11_2は、同一のものであるものとし、同図に示す変換器11_jであるものとして説明する。

【0125】すなわち、このような変換器11_jからなる変換器$_0$は、入力したN_0ビットからなる情報ビット系列b(0)をM次元実数ベクトル空間に配置するように変換して得られたM個の数値からなる符号化系列x(0)を乗算器12_0に供給する。また、変換器11_jからなる変換器$_1$は、入力したN_1ビットからなる情報ビット系列b(1)をM次元実数ベクトル空間に配置するように変換して得られたM個の数値からなる符号化系列x(1)を乗算器12_1に供給する。さらに、変換器11_jからなる変換器$_2$は、入力したN_2ビットからなる情報ビット系列b(2)をM次元実数ベクトル空間に配置するように変換して得られたM個の数値からなる符号化系列x(2)を乗算器12_2に供給する。

【0126】乗算器12_0は、変換器11_0から供給された符号化系列x(0)に対して、上述した方法に基づいて設定された定数a(0)を乗算する。乗算器12_0は、乗算して得られた定数倍符号化系列a(0)x(0)を加算器13_0に供給する。

【0127】乗算器12_1は、乗算器12_0と同様に、変換器11_1から供給された符号化系列x(1)に対して、上述した方法に基づいて設定された定数a(1)を乗算する。乗算器12_1は、乗算して得られた定数倍符号化系列a(1)x(1)を加算器13_0に供給する。

【0128】乗算器12_2は、乗算器12_0，12_1と同様に、変換器11_2から供給された符号化系列x(2)に対して、上述した方法に基づいて設定された定数a(2)を乗算する。乗算器12_2は、乗算して得られた定数倍符号化系列a(2)x(2)を加算器13_1に供給する。

【0129】加算器13_0は、乗算器12_0から供給された定数倍符号化系列a(0)x(0)と、乗算器12_1から供給された定数倍符号化系列a(1)x(1)とを要素毎にユークリッド的に加算する。加算器13_0は、加算して得られた加算符号化系列a(0)x(0)+a(1)x(1)を加算器13_1に供給する。

【0130】加算器13_1は、加算器13_0から供給された加算符号化系列a(0)x(0)+a(1)x(1)と、乗算器12_2から供給された定数倍符号化系列a(2)x(2)とを要素毎にユークリッド的に加算する。加算器13_1は、加算して得られた最終的な加算符号化系列g(=a(0)x(0)+a(1)x(1)+a(2)x(2))を送信部14に供給する。

【0131】送信部14は、外部へとデータを送信するインターフェースである。送信部14は、加算器13_1から供給された加算符号化系列gを送信信号g'として、外部へと送信する。

【0132】このような送信装置10は、3つの情報ビット系列{b(0)，b(1)，b(2)}を入力すると、これらの情報ビット系列{b(0)，b(1)，b(2)}に対して所定の符号化を施し、所望のビットエラーレートを実現するエネルギE_s[J]で情報ビット系列{b(0)，b(1)，b(2)}をそれぞれ伝送するために、得られた符号化系列{x(0)，x(1)，x(2)}に対して定数{a(0)，a(1)，a(2)}を乗算する。そして、送信装置10は、得られた定数倍符号化系列{a(0)x(0)，a

(1)x(1),a(2)x(2)}を加算して加算符号化系列gを生成する。この送信装置10によって得られた加算符号化系列gからなる送信信号は、通信路を伝送する際に雑音nが加算され、後述する受信装置へと到達する。

【0133】つぎに、データ送受信システムにおける受信装置について説明する。この受信装置は、次式(35)に示すように、送信装置によって伝送された加算符号化系列gからなる送信信号に対して雑音nが加算された受信値y(L)を受信し、少なくとも1つの情報ビット系列b(1)に対する復号を可能とするものである。特に、受信装置は、受信値y(L)を受信すると、送信装置によって符号化された情報ビット系列b(1)のうち、少なくとも、最後に加算されて最も大きい情報ビットエネルギE_bで伝送された系列である最高次の情報ビット系列b(L-1)に対する復号を可能とするものである。

【0134】

【数35】

$$y^{(L)} = g\left(b^{(0)},b^{(1)},\cdots,b^{(L-1)}\right)+n = \sum_{i=0}^{L-1} a^{(i)}x^{(i)}\left(b^{(i)}\right)+n \quad \cdots(35)$$

【0135】この受信装置は、一般的には図12に示す構成にしたがって実現することができる。すなわち、同図に示す受信装置50は、上述した送信装置における各変換器に対応するL個の復号器51_{L-1},51_{L-2},・・・,51_0と、上述した送信装置における各変換器と同一のL-1個の変換器52_{L-1},52_{L-2},・・・,51_1と、上述した送信装置における各乗算器と同一のL-1個の乗算器53_{L-1},53_{L-2},・・・,53_1と、L-1個の差分器54_{L-1},54_{L-2},・・・,54_1とを備えるものである。

【0136】受信装置50は、まず最初に情報ビット系列b(L-1)に対する復号を行う。情報ビットエネルギE_b(L-1)は、上述したように、雑音nと符号x(0)から符号x(L-2)までの電力密度の和に対してs倍に設定されていることから、受信装置50は、エラーレートが"0"又は"0"に十分近い値で復号することができる。受信装置50は、図示しない受信部を介して入力した受信値y(L)を用いて、復号器51_{L-1}によって情報ビット系列b(L-1)に対する復号を正確に行ったものとすると、復号の結果得られた情報ビット系列b(L-1)を変換器52_{L-1}によって再符号化し、得られた符号化系列x(L-1)に対して乗算器53_{L-1}によって定数a(L-1)を乗算する。そして、受信装置50は、乗算器53_{L-1}から出力された定数倍符号化系列a(L-1)x(L-1)を受信値y(L)から差分器54_{L-1}によって要素毎に差分する。これにより、受信装置50は、次式(36)に示すように、送信装置によって情報ビット系列b(L-2)まで符号化して得られた符号を受信した場合における受信値と等価な情報y(L-1)を得ることができる。

【0137】

【数36】

$$y^{(L-1)} = y^{(L)} - a^{(L-1)}x\left(b^{(L-1)}\right) = \sum_{i=0}^{L-2} a^{(i)}x^{(i)}\left(b^{(i)}\right)+n \quad \cdots(36)$$

【0138】同様に、受信装置50は、情報y(L-1)を用いて、復号器51_{L-2}によって情報ビット系列b(L-2)に対する復号を正確に行ったものとすると、復号の結果得られた情報ビット系列b(L-2)を変換器52_{L-2}によって再符号化し、得られた符号化系列x(L-2)に対して乗算器53_{L-2}によって定数a(L-2)を乗算する。そして、受信装置50は、乗算器53_{L-2}から出力された定数倍符号化系列a(L-2)x(L-2)を情報y(L-1)から差分器54_{L-2}によって要素毎に差分する。これにより、受信装置50は、情報ビット系列b(L-2)に対する復号を行うことができるとともに、次式(37)に示すように、送信装置によって情報ビット系列b(L-3)まで符号化して得られた符号を受信した場合における受信値と等価な情報y(L-2)を得ることができる。

【0139】

【数37】

$$y^{(L-2)} = y^{(L-1)} - a^{(L-2)}x\left(b^{(L-2)}\right) = \sum_{i=0}^{L-3} a^{(i)}x^{(i)}\left(b^{(i)}\right)+n \quad \cdots(37)$$

【0140】そして、受信装置50は、同様の操作を繰り返すことによって順次情報ビット系列b(i)に対する復号を行うことができ、最後の情報ビット系列b(0)については、次式(38)に示すように、送信装置によって情報ビット系列b(0)まで符号化して得られた符号を受信した場合における受信値と等価な情報y(1)を得ることができ、情報ビット系列b(0)に対する復号を行うことができる。

【0141】

【数38】

$$y^{(L)} = x^{(0)}\left(b^{(0)}\right) + n \qquad \cdots(38)$$

【0142】このように、受信装置50は、受信値y(L)を入力すると、最高次の情報ビット系列b(L-1)に対する復号から行うとともに、この情報ビット系列b(L-1)を再符号化し、得られた符号化系列x(L-1)に対して定数a(L-1)を乗算し、さらに、受信値y(L)から定数倍符号化系列a(L-1)x(L-1)を差し引くことにより、次の次数の情報ビット系列b(L-2)に対する復号を行うことができる。受信装置50は、このような操作を最後の情報ビット系列b(0)まで繰り返すことによって全ての情報ビット系列{b(0), b(1), ・・・, b(L-1)}に対する復号を行うことができる。

【0143】また、受信装置50は、全ての情報ビット系列{b(0), b(1), ・・・, b(L-1)}に対する復号を行うのではなく、少なくとも最高次の情報ビット系列b(L-1)に対する復号のみを行うといったように、最高次から所定次数までの情報ビット系列b(l)に対する復号のみを行うようにしてもよい。

【0144】これは、例えば、種々の解像度を有する画像データを送信装置によって符号化して伝送し、受信装置50によって復号して表示するアプリケーションを想定した場合等に有効である。すなわち、送信装置は、同一内容である画像データについて複数の解像度のものを用意し、これらの複数の画像データのそれぞれを、複数の情報ビット系列b(l)として符号化する。このとき、送信装置は、最も解像度の高い画像データを最も次数の低い情報ビット系列b(0)として符号化し、順次解像度が低くなる順序で符号化する。すなわち、送信装置は、解像度が低い画像データほど重要度が高いものとし、振幅を大きくした状態で伝送する。

【0145】これに応じて、受信装置50は、受信値y(l)を入力すると、情報ビット系列b(l)に対する復号を順次行い、当該受信装置50が有する表示器の解像度に応じた画像データを表す情報ビット系列b(i)に対する復号が終了すると、それよりも低次の情報ビット系列b(l)に対する復号を行うことなく処理を終了する。これにより、受信装置50は、自己が表示可能な画像データのみを選択的に復号して表示することが可能となる。

【0146】このように、受信装置50は、全ての情報ビット系列{b(0), b(1), ・・・, b(L-1)}に対する復号を行うのではなく、重要度の高い高次の情報ビット系列b(L-1)に対する復号のみを行うこともできる。

【0147】ところで、シャノン限界を満足する符号は、ある信号対雑音電力比E_b/n_0を境として誤りが完全になくなり、この信号対雑音電力比E_b/n_0を下回ると急激に誤りを生じることが知られている。ここで、符号全体として所要とする信号対雑音電力比E_b/n_0を少しでも下回る通信路を介して伝送を行った場合には、受信装置は、最初の復号である最高次の情報ビット系列b(L-1)に対する復号を行う際に壊滅的な誤りを引き起こし、それよりも低次の情報ビット系列b(L-2), ・・・, 情報ビット系列b(0)については、全く復号することができなくなる。一方、符号全体として所要とする信号対雑音電力比E_b/n_0を少しでも上回る通信路を介して伝送を行った場合には、受信装置は、全ての情報ビット系列b(l)に対してエラーレートを"0"として復号を行うことができる。

【0148】しかしながら、現存する実際の符号においては、これほど急激な特性を得ることはできないのが現状である。ただし、このような場合であっても、原符号がいわゆる最大事後確率(Maximum A Posteriori probability;以下、MAPという。)復号又はこれに準ずる復号を行えるものであれば、誤りが多少なりとも残存してしまう現実の符号に対する復号を行うことができる。

【0149】そこで、データ送受信システムにおいては、現実的な受信装置として、MAP復号を利用したものを提案する。

【0150】MAP復号においては、送信信号の候補に対する受信値の尤度(likelihood)を入力として、受信値から推測される情報ビットの事後確率情報(a posteriori probability information)が求められる。データ送受信システムにおける伝送系は、次式(39)で表される。MAP復号においては、情報ビット系列b(i)に対する復号を行うとき、この情報ビット系列b(i)に対する尤度は、次式(40)に示す条件付き確率P(y(L)|b(i))で表される。

【0151】

【数39】

$$y^{(L)} = g\left(b^{(0)}, b^{(1)}, \cdots, b^{(L-1)}\right) + n = x + n \qquad \cdots(39)$$

【0152】

【数40】

$$Likelihood^{(i)} = p\left(y^{(L)} \mid b^{(i)}\right) \propto \sum_{\substack{b^{(0)} \\ L:i}} P\left(b^{(0)}\right)P\left(b^{(1)}\right)\cdots\cdots P\left(b^{(L-1)}\right)\exp\left[-\frac{1}{n_0}\left|y^{(L)} - g\left(b^{(0)},b^{(1)},\cdots\cdots b^{(L-1)}\right)\right|^2\right] \quad \cdots(40)$$

【0153】ここで、上式（40）における"Σ"は、i段目を除く全ての1段目で、全ての情報ビット系列b（l）の候補に関して和をとることを示す演算子である。また、上式（40）における"P（b（j））"は、情報ビット系列b（j）が生じる確率である。なお、復号を開始する段階においては、どの符号が受信されるのかが不明であることから、確率P（b（j））の初期値は、次式（41）で表される。

【0154】

【数41】

$$P\left(b^{(j)}\right) = 1/2^N \quad \cdots(41)$$

【0155】受信装置においては、高次の符号から復号することにより、MAP復号を行うことによって得られる事後確率情報は、次の復号の尤度算出に利用することができる。そのため、受信装置においては、復号結果に多少のあやふやな誤りが残存している場合であっても、次段の復号時にそれを加味した復号を行うことができる。このとき、受信装置においては、次段の復号時においては前段の復号による悪影響が受継されるが、特性の劣化が緩やかであることから壊滅的な影響にはならない。

【0156】上式（40）に示した系列全体についての尤度からM次元の要素毎の尤度に分解すると、尤度は、次式（42）で表される。

【0157】

【数42】

$$Likelihood^{(i)} = p\left(y^{(L)} \mid b^{(i)}\right) = p\left(y^{(L)} \mid x\left(b^{(i)}\right)\right) = \prod_{k=0}^{M-1} p\left(y_k^{(L)} \mid x_k^{(j)}\right) \propto \prod_{k=0}^{M-1}\sum_{\substack{x_k^{(j)} \\ L:i}} P\left(x_k^{(0)}\right)P\left(x_k^{(1)}\right)\cdots\cdots P\left(x_k^{(L-1)}\right)\exp\left[-\frac{1}{n_0}\left|y_k^{(L)} - \sum_{j=0}^{\lambda-1} a^{(j)}x_k^{(j)}\right|^2\right] \quad \cdots(42)$$

【0158】ここで、上式（42）における"P（x_k（l））"は、情報ビット系列b（j）に対する符号語のM次元ベクトルのk番目の要素が"x_k（i）"となる確率である。また、上式（42）における"Σ"は、i段目を除く全ての1段目で、起こり得る全ての要素x_k（l）に関して和をとることを示す演算子である。なお、前段で全く曖昧さを残存させずに復号できた場合には、ある特定の要素x_k（j）についてP（x_k（j））＝1となることから、この演算は、再符号化してキャンセルすることと等価である。

【0159】このように、受信装置は、原符号がMAP復号を行えるものであれば、ある情報ビット系列b（l）に対するMAP復号結果を他の情報ビット系列に対する尤度算出に反映させることにより、全ての情報ビット系列｛b（0），b（1），・・・，b（L－1）｝に対する復号を行うことができる。

【0160】なお、レイリーフェージング・チャネルといった変動する通信路を介した伝送を行う場合には、原符号の所要とする信号対雑音電力比E_b/n_0は、AWGNチャネルに比べ、若干劣化する。この場合、送信側においては、対応する通信路で求めた所要とする信号対雑音電力比$E_b/(n_0/2\nu)$で符号を構成すればよい。ただし、高次の符号に関する雑音源は、上述したように、熱雑音とより低次の符号であることから、低次の符号は、高次の符号とともにレベル変動を生じている。そのため、高次の符号の変動の偏差（deviation）は、小さくなり、所要とする信号対雑音電力比$E_b/(n_0+2\nu)$は、小さくなる傾向にある。この結果、符号全体として所要とする信号対雑音電力比$E_b/(n_0/2\nu)$は、単一の符号としての信号対雑音電力比E_b/n_0の劣化よりも小さくなることが期待される。そこで、変動する通信路での振幅fを次式（43）で表すと、受信値yは、次式（44）で表される。したがって、尤度は、次式（45）で表される。

【0161】

【数43】

$$f = \left\{f_0, f_1, \cdots\cdots, f_{M-1}\right\}^T \quad \cdots(43)$$

【0162】

【数44】

$$y = f^T \cdot g + n \qquad \cdots(44)$$

【0163】
【数45】

$$Likelihood^{(i)} = p\left(y^{(i)} \mid b^{(i)}\right)$$
$$= p\left(y^{(i)} \mid x\left(b^{(i)}\right)\right) \cdot \prod_{k=0}^{M-1} p\left(y_k^{(i)} \mid x_k^{(i)}\right)$$
$$\propto \prod_{k=0}^{M-1} \sum_{\substack{x_k^{(i)} \\ l=i}} P\left(x_k^{(0)}\right) P\left(x_k^{(1)}\right) \cdots P\left(x_k^{(L-1)}\right) \exp\left[-\frac{1}{n_0}\left|y_k^{(i)} - f_k \cdot \sum_{j=0}^{L-1} a^{(j)} x_k^{(j)}\right|^2\right]$$

$$\cdots(45)$$

【0164】さて、以下では、このような復号を行う受信装置の現実的な具体的構成について詳述する。なお、ここでは、説明の便宜上、受信装置は、上述した送信装置10によって符号化されて伝送された送信信号g'に対して雑音nが加算された受信値yからなる受信信号y'を受信するものとする。すなわち、受信装置は、3つの情報ビット系列{b(0)，b(1)，b(2)}の軟判定（soft-decision）値を復号して得るものとする。

【0165】例えば図13に示すように、受信装置60は、外部から伝送されてきた受信信号y'を入力する受信部61と、上述した送信装置10における変換器11_0，11_1，11_2によるPCCCに対応したターボ復号を行う3つの復号器62_0，62_1，62_2とを備える。

【0166】受信部61は、外部からデータを受信するインターフェースである。受信部61は、受信信号y'を入力すると、受信値yとして、復号器62_0，62_1，62_2に供給する。

【0167】復号器62_0，62_1，62_2は、それぞれ、送信装置10における変換器11_0，11_1，11_2に対応して備えられるものである。復号器62_0，62_1，62_2は、それぞれ、受信値yから受信シンボルに関する尤度を算出する尤度算出部63_0，63_1，63_2を有し、上述した送信装置10における変換器11_0，11_1，11_2によるPCCCに対応したターボ復号を行うことによって情報ビットに対する事後確率情報を求める。これらの復号器62_0，62_1，62_2については、後に詳述するものとする。

【0168】このような受信装置60は、復号器62_0，62_1，62_2のそれぞれから符号化系列に対する事後確率情報P(x(0)　y)，P(x(1)|y)，P(x(2)|y)が出力され、これらの事後確率情報P(x(0)|y)，P(x(1)|y)，P(x(2)|y)が、それぞれ、各符号化系列{x(0)，x(1)，x(2)}に対する事前確率情報（a priori probability information）P(x(0))，P(x(1))，P(x(2))として、他の復号器に対して入力されることに特徴を有するものである。ここで、初期状態においては、復号器62_0，62_1，62_2のそれぞれに入力される事前確率情報P(x(0))，P(x(1))，P(x(2))は、確率が未知であることを示す値に初期化される。ここでは、符号化系列{x(0)，x(1)，x(2)}の各要素は、送信装置10によってBPSK変調方式に基づく信号点のマッピングが施されていることから、P(x(1)=1　y)=P(x(1)=1)=0.5，P(x(1)=−1|y)=P(x(1)=−1)=0.5とする。

【0169】受信装置60は、受信値yを入力すると、まず最初にこの受信値yを復号器62_2に供給する。また、受信装置60においては、復号器62_2には、符号化系列x(0)に対する事前確率情報P(x(0))と符号化系列x(1)に対する事前確率情報P(x(1))とが入力される。受信装置60においては、復号器62_2により、これらの受信値y及び事前確率情報P(x(0))，P(x(1))を用いてターボ復号を行い、符号化系列x(2)に対する事後確率情報P(x(2)|y)及び情報ビット系列b(2)に対する事後確率情報P(b(2)|y)を生成する。そして、受信装置60においては、事後確率情報P(x(2)　y)が復号器62_2から復号器62_0，62_1に対して、符号化系列x(2)に対する事前確率情報P(x(2))として供給されるとともに、事後確率情報P(b(2)|y)が軟出力（soft-output）として外部に出力される。

【0170】続いて、受信装置60は、復号器62_2が要する処理時間と同時間だけ遅延された受信値yを復号器62_1に供給する。また、受信装置60においては、復号器62_1には、符号化系列x(0)に対する事前確率情報P(x(0))と復号器62_2によって生成された符号化系列x(2)に対する事前確率情報P(x(2))とが入力される。受信装置60においては、復号器62_1により、これらの受信値y及び事前確率情報P(x(0))，P(x(2))を用いてターボ復号を行い、符号化系列x(1)に対する事後確率情報P(x

(1)|y)及び情報ビット系列b(1)に対する事後確率情報P(b(1) y)を生成する。そして、受信装置60においては、事後確率情報P(x(1) y)が復号器62_1から復号器62_0及び必要に応じて復号器62_2に対して、符号化系列x(1)に対する事前確率情報P(x(1))として供給されるとともに、事後確率情報P(b(1) y)が軟出力として外部に出力される。

【0171】そして、受信装置60は、復号器62_2及び復号器62_1が要する処理時間と同時間だけ遅延された受信値yを復号器62_0に供給する。また、受信装置60においては、復号器62_0には、復号器62_1によって生成された符号化系列x(2)に対する事前確率情報P(x(2))と復号器62_2によって生成された符号化系列x(2)に対する事前確率情報P(x(2))とが入力される。受信装置60においては、復号器62_0により、これらの受信値y及び事前確率情報P(x(1))，P(x(2))を用いてターボ復号を行い、符号化系列x(0)に対する事後確率情報P(x(0)|y)及び情報ビット系列b(0)に対する事後確率情報P(b(0)|y)を生成する。そして、受信装置60においては、事後確率情報P(x(0)|y)が復号器62_0から必要に応じて復号器62_1，62_2に対して、符号化系列x(0)に対する事前確率情報P(x(0))として供給されるとともに、事後確率情報P(b(0)|y)が軟出力として外部に出力される。

【0172】受信装置60は、このような動作を行うことにより、事後確率情報P(b(2)|y)，P(b(1)|y)，P(b(0) y)の順序で復号することができる。受信装置60は、これらの事後確率情報P(b(2)|y)，P(b(1)|y)，P(b(0)|y)を図示しない硬判定(hard-decision)器によって2値化することにより、情報ビット系列b(2)，b(1)，b(0)を得ることができる。なお、受信装置60は、基本的には、高次の情報ビット系列b(2)から順次1回のみ復号を行うが、いわゆるジグザグ復号や繰り返し復号を行うこともできる。これについては、後述するものとする。

【0173】以下、ターボ復号を行う復号器62_0，62_1，62_2について説明する。まず、受信装置60に適用する復号器62_0，62_1，62_2の特徴を明確化すべく、1つの符号化系列についてターボ復号を行う通常のターボ復号器について説明する。なお、ここでは、説明の便宜上、先に図10に示した変換器11_jに対応するターボ復号器について説明する。

【0174】図14に示すように、通常のターボ復号器70は、入力したデータの順序を元に戻すチャネル用のチャネル・デインターリーバ71と、間引かれたデータを回復するデパンクチャ器72と、入力したデータの順序を並べ替える2つのインターリーバ73，75と、MAP復号を行う2つのMAP復号器74，76と、入力したデータの順序を元に戻すデインターリーバ77とを有する。

【0175】チャネル・デインターリーバ71は、上述した変換器11_jのようにチャネル・インターリーバ24が設けられた場合に対応して設けられるものである。チャネル・デインターリーバ71は、受信値yを入力し、変換器11_jにおけるチャネル・インターリーバ24によってインターリーブされたインターリーブデータDeのビット配列を、それぞれ、元の情報ビット系列b(i)及びパンクチャデータDdのビット配列に戻すように、受信値yにデインターリーブを施す。チャネル・デインターリーバ71は、デインターリーブして得られた情報ビット系列b(i)に対応する系列Dfをインターリーバ73及びMAP復号器74に供給するとともに、得られたパンクチャデータDdに対応する系列Dgをデパンクチャ器72に供給する。

【0176】デパンクチャ器72は、チャネル・デインターリーバ71から供給された系列Dgに対して、変換器11_jにおけるパンクチャ器23によって間引きされたビットに相当する箇所に例えば"0．0"を挿入することによって回復し、変換器11_jにおける要素符号化器21_0，21_1から出力された2系列の出力データDa，Dcに対応する2つの系列Dh，Diを生成する。デパンクチャ器72は、生成した系列DhをMAP復号器74に供給するとともに、系列DiをMAP復号器76に供給する。

【0177】インターリーバ73は、チャネル・デインターリーバ71から供給された系列Dfを入力し、この系列Dfに対して、変換器11_jにおけるインターリーバ22と同一の置換位置情報に基づいたインターリーブを施す。インターリーバ73は、生成した系列DjをMAP復号器76に供給する。

【0178】MAP復号器74は、変換器11_jにおける要素符号化器21_0に対応して設けられるものである。MAP復号器74は、チャネル・デインターリーバ71から供給された情報ビット系列b(i)に対応する軟入力(soft-input)の系列Df及びデパンクチャ器72から供給された軟入力の系列Dhを入力するとともに、デインターリーバ77から供給された軟入力の情報ビットに対する事前確率情報Apr_0を入力し、これらの系列Df，Dh及び事前確率情報Apr_0を用いて、MAP復号を行う。そして、MAP復号器74は、符号の拘束条件によって求められる情報ビット系列に対するいわゆる外部情報(extrinsic information)Ext_0を生成し、この外部情報Ext_0をインターリーバ75に軟出力として出力する。なお、この外部情報Ext_0は、尤度の増分を示すものである。

【0179】インターリーバ75は、MAP復号器74から供給された軟入力である情報ビット系列に対する外

部情報Ext_0に対して、変換器11_2におけるインターリーバ22と同一の置換位置情報に基づいたインターリーブを施す。インターリーバ75は、生成したインターリーブデータをMAP復号器76における情報ビットに対する事前確率情報Apr_1として出力する。

【0180】MAP復号器76は、変換器11_1における要素符号化器21_1に対応して設けられるものである。MAP復号器76は、インターリーバ73から供給された情報ビット系列b(i)に対応する軟入力の系列Dj及びデパンクチャ器72から供給された軟入力の系列Diを入力するとともに、インターリーバ75から供給された軟入力の情報ビットに対する事前確率情報Apr_1を入力し、これらの系列Di，Dj及び事前確率情報Apr_1を用いて、MAP復号を行う。そして、MAP復号器76は、符号の拘束条件によって求められる情報ビット系列に対する外部情報Ext_1を生成し、この外部情報Ext_1をデインターリーバ77に軟出力として出力する。なお、この外部情報Ext_1は、外部情報Ext_0と同様に、尤度の増分を示すものである。また、MAP復号器76は、所定の繰り返し回数での繰り返し復号の結果得られた軟出力の外部情報に基づいて、情報ビットに対する事後確率情報P(b|y)を生成し、復号データとして出力する。

【0181】デインターリーバ77は、変換器11_2におけるインターリーバ22によってインターリーブされたインターリーブデータDbのビット配列を、元の情報ビット系列b(i)のビット配列に戻すように、MAP復号器76から供給された軟入力の外部情報Ext_2にデインターリーブを施す。デインターリーバ77は、生成したデインターリーブデータをMAP復号器74における情報ビットに対する事前確率情報Apr_0として出力する。

【0182】このようなターボ復号器70において、MAP復号器74，76は、情報ビット系列b(i)及びパンクチャデータDdに対応する軟入力の系列と軟入力の情報ビットに対する事前確率情報Apr_0，Apr_1とを用いて、時刻jにおいて情報ビットが"b_j"である確率、すなわち、事後確率情報$Apo(b_j)$を算出する。MAP復号器74，76は、次式(46)に示すように、事後確率情報$Apo(b_j)$を算出する。

【0183】

【数46】

$$
\begin{aligned}
Apo(b_j) &= P(b_j \mid y) \\
&= \sum_{\Sigma b_j} P(b_j, b \mid y) = \sum_{\Sigma b_j} \frac{p(y \mid b_j, b) P(b_j, b)}{p(y)} \\
&\propto \sum_{\Sigma b_j} p(y \mid b_j, b) P(b_j, b) = \sum_{\Sigma b_j} p(y \mid b) P(b) \\
&= \sum_{\Sigma b_j} p(y \mid x) P(b) = \sum_{\Sigma b_j} \prod_{k=0}^{N-1} p(y_k \mid x_k) P(b_k) \\
&= \sum_{\Sigma b_j} \prod_{k=0}^{N-1} p(y_k \mid x_k) Apr(b_k)
\end{aligned}
\qquad \cdots (46)
$$

【0184】なお、上式(46)においては、事前確率情報$Apr(b_k)$を次式(47)のように定義している。

【0185】

【数47】

$$
Apr(b_k) = P(b_k) \qquad \cdots (47)
$$

【0186】符号xは、情報ビットbが決定されればx=x(b)で表される符号化によって一意に生成されることから、上式(46)は、第2行目から第3行目にかけて示すように変換される。また、上式(46)における"Σ"は、$c=\{c_0, c_1, \cdots, c_{N-1}\}$が符号語であり、且つ、時刻jにおいて情報ビットが"b_j"となっている全ての系列に関して和をとることを示す演算子である。

【0187】ここで、上式(46)における第5行目は、次式(48)に示すように、事後確率情報$Apo(b_j)$に比例しているだけであることから、事後確率情報$Apo(b_j)$は、最終的には次式(49)に示すように正規化する必要がある。

【0188】

【数48】

$$
Apo(b_j) \propto \sum_{\Sigma b_j} \prod_{k=0}^{N-1} p(y_k \mid x_k) Apr(b_k) \qquad \cdots (48)
$$

【0189】

【数49】

$$
\sum_{b_j} Apo(b_j) = 1 \qquad \cdots (49)
$$

【0190】このように、MAP復号器74，76は、

事前確率情報$Apr(b_k)=P(b_k)$と送信シンボルの候補に対する受信シンボルの尤度$L-p(y_k|x_k)$とを用いることにより、事後確率情報$Apo(b_j)$を算出することができる。なお、尤度Lは、例えば通信路がAWGNチャネルである場合には、雑音の確率密度関数を次式（５０）で表すものとすれば、次式（５１）に示すように求められる。

【０１９１】

【数５０】

$$p_n(n)=\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{1}{2\sigma_n^2}|n|^2\right] \quad \cdots(50)$$

【０１９２】

【数５１】

$$L=p(y_k|x_k)\propto\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{1}{2\sigma_n^2}|y_k-x_k|^2\right] \quad \cdots(51)$$

【０１９３】ターボ復号器７０において、ＭＡＰ復号器７４，７６は、このようにして算出した事後確率情報$Apo(b_j)$と事前確率情報Apr_0，Apr_1との差分を上述した外部情報Ext_0，Ext_1として出力する。

【０１９４】このようなＭＡＰ復号器７４，７６を備えるターボ復号器７０は、受信値ｙを受信すると、所定の繰り返し回数での繰り返し復号を行い、この復号動作の結果得られた軟出力の外部情報に基づいて、ＭＡＰ復号器７６から情報ビットに対する事後確率情報$P(b|y)$を復号データとして出力する。

【０１９５】さて、上述した受信装置６０においては、このようなターボ復号器７０を改良して復号器62_0，62_1，62_2を構成する。なお、ここでは、説明の便宜上、先に図１０に示した変換器11_jに対応するターボ復号を行う復号器62_0について説明する。

【０１９６】図１５に示すように、復号器62_0は、入力したデータの順序を元に戻すチャネル用のチャネル・デインターリーバ８１と、間引かれたデータを回復するデパンクチャ器８２と、入力したデータの順序を並べ替える２つのインターリーバ８３，８５と、ＭＡＰ復号を行う２つのＭＡＰ復号器８４，８６と、入力したデータの順序を元に戻すデインターリーバ８７と、入力したデータを適宜間引くパンクチャ器８８と、入力したデータの順序を並べ替えるチャネル用のチャネル・インターリーバ８９とを有する。

【０１９７】チャネル・デインターリーバ８１は、上述したターボ復号器７０におけるチャネル・デインターリーバ７１と同様に、上述した変換器11_jにおけるチャネル・インターリーバ２４に対応して設けられるものである。チャネル・デインターリーバ８１は、受信値ｙ並びに符号化系列$x^{(1)}$に対する事前確率情報$P(x^{(1)})$及び符号化系列$x^{(2)}$に対する事前確率情報$P(x^{(2)})$を入力し、変換器11_jにおけるチャネル・インターリーバ２４によってインターリーブされたインターリーブデータＤｅのビット配列を、それぞれ、元の情報ビット系列$b^{(i)}$及びパンクチャデータＤｄのビット配列に戻すように、受信値ｙ及び事前確率情報$P(x^{(1)})$，$P(x^{(2)})$にデインターリーブを施す。チャネル・デインターリーバ８１は、デインターリーブして得られた情報ビット系列$b^{(1)}$に対応する受信値ｙ及び事前確率情報$P(x^{(1)})$，$P(x^{(2)})$のそれぞれについての３つの系列Dk_y，Dk_1，Dk_2をインターリーバ８３及びＭＡＰ復号器８４に供給するとともに、得られたパンクチャデータＤｄに対応する受信値ｙ及び事前確率情報$P(x^{(1)})$，$P(x^{(2)})$のそれぞれについての３つの系列Dl_y，Dl_1，Dl_2をデパンクチャ器８２に供給する。

【０１９８】デパンクチャ器８２は、上述したターボ復号器７０におけるデパンクチャ器７２と同様に、チャネル・デインターリーバ８１から供給された系列Dl_yに対して、変換器11_jにおけるパンクチャ器２３によって間引きされたビットに相当する箇所に例えば"０．０"を挿入することによって回復し、変換器11_jにおける要素符号化器21_0から出力された出力データＤａに対応する受信値ｙについての系列Dm_yを生成するとともに、要素符号化器21_1から出力された出力データＤｃに対応する受信値ｙについての系列Dn_yを生成する。また、デパンクチャ器８２は、チャネル・デインターリーバ８１から供給された系列Dl_1，Dl_2に対して、変換器11_jにおけるパンクチャ器２３によって間引きされたビットに相当する箇所を$P(x^{(1)}=1)=0.5$，$P(x^{(1)}=-1)=0.5$として挿入することによって回復し、変換器11_jにおける要素符号化器21_0から出力された出力データＤａに対応する事前確率情報$P(x^{(1)})$，$P(x^{(2)})$のそれぞれについての２つの系列Dm_1，Dm_2を生成するとともに、要素符号化器21_1から出力された出力データＤｃに対応する事前確率情報$P(x^{(1)})$，$P(x^{(2)})$のそれぞれについての２つの系列Dn_1，Dn_2を生成する。デパンクチャ器８２は、生成した系列Dm_y，Dm_1，Dm_2をＭＡＰ復号器８４に供給するとともに、系列Dn_y，Dn_1，Dn_2をＭＡＰ復号器８６に供給する。

【０１９９】インターリーバ８３は、上述したターボ復号器７０におけるインターリーバ７３と同様に、チャネル・デインターリーバ８１から供給された系列Dk_y，Dk_1，Dk_2を入力し、これらの系列Dk_y，Dk_1，Dk_2に対して、変換器11_jにおけるインターリーバ２２と同一の置換位置情報に基づいたインターリ

ーブを施す。インターリーバ83は、生成した3つの系列Do_y，Do_1，Do_2をMAP復号器86に供給する。

【0200】MAP復号器84は、上述したターボ復号器70におけるMAP復号器74と同様に、変換器11_jにおける要素符号化器21_0に対応して設けられるものである。MAP復号器84は、チャネル・デインターリーバ81から供給された情報ビット系列$b(i)$に対応する軟入力の系列Dk_y，Dk_1，Dk_2及びデパンクチャ器82から供給された軟入力の系列Dm_y，Dm_1，Dm_2を入力するとともに、デインターリーバ87から供給された軟入力の情報ビットに対する事前確率情報Apr_0を入力し、これらの系列Dk_y，Dk_1，Dk_2，Dm_y，Dm_1，Dm_2及び事前確率情報Apr_0を用いて、MAP復号を行う。そして、MAP復号器84は、符号の拘束条件によって求められる情報ビット系列に対する外部情報Ext_0を生成し、この外部情報Ext_0をインターリーバ85に軟出力として出力する。また、MAP復号器84は、所定の繰り返し回数での繰り返し復号の結果得られた軟出力の外部情報に基づいて、情報ビットに対する事後確率情報Apo_{0i}を生成するとともに、変換器11_jにおける要素符号化器21_0から出力された出力データDaに相当する符号化ビットに対する事後確率情報Apo_{0z}を生成する。MAP復号器84は、生成した事後確率情報Apo_{0i}をチャネル・インターリーバ89に供給するとともに、事後確率情報Apo_{0c}をパンクチャ器88に供給する。なお、MAP復号器84は、上述したターボ復号器70におけるMAP復号器74と同様に尤度Lを算出するが、MAP復号器74とは異なる演算を行う。これについては、後述するものとする。

【0201】インターリーバ85は、上述したターボ復号器70におけるインターリーバ75と同様に、MAP復号器84から供給された軟入力である情報ビット系列に対する外部情報Ext_0に対して、変換器11_jにおけるインターリーバ22と同一の置換位置情報に基づいたインターリーブを施す。インターリーバ85は、生成したインターリーブデータをMAP復号器86における情報ビットに対する事前確率情報Apr_1として出力する。

【0202】MAP復号器86は、上述したターボ復号器70におけるMAP復号器76と同様に、変換器11_jにおける要素符号化器21_1に対応して設けられるものである。MAP復号器86は、インターリーバ83から供給された情報ビット系列$b(i)$に対応する軟入力の系列Do_y，Do_1，Do_2及びデパンクチャ器82から供給された軟入力の系列Dn_y，Dn_1，Dn_2を入力するとともに、インターリーバ85から供給された軟入力の情報ビットに対する事前確率情報Apr_1を入力し、これらの系列Dn_y，Dn_1，Dn_2，Do_y，Do_1，Do_2及び事前確率情報Apr_1を用いて、MAP復号を行う。そして、MAP復号器86は、符号の拘束条件によって求められる情報ビットに対する外部情報Ext_1を生成し、この外部情報Ext_1をデインターリーバ87に軟出力として出力する。また、MAP復号器86は、所定の繰り返し回数での繰り返し復号の結果得られた軟出力の外部情報に基づいて、情報ビット系列$b(i)$に対する事後確率情報$P(b(i)|y)$を生成し、復号データとして外部へと出力するとともに、変換器11_jにおける要素符号化器21_1から出力された出力データDcに相当する符号化ビットに対する事後確率情報Apo_{1c}を生成する。MAP復号器86は、生成した事後確率情報Apo_{1c}をパンクチャ器88に供給する。なお、MAP復号器86は、上述したターボ復号器70におけるMAP復号器76と同様に尤度Lを算出するが、MAP復号器76とは異なる演算を行う。これについては、後述するものとする。

【0203】デインターリーバ87は、上述したターボ復号器70におけるデインターリーバ77と同様に、変換器11_jにおけるインターリーバ22によってインターリーブされたインターリーブデータDbのビット配列を、元の情報ビット系列$b(i)$のビット配列に戻すように、MAP復号器86から供給された軟入力の外部情報Ext_1にデインターリーブを施す。デインターリーバ87は、生成したデインターリーブデータをMAP復号器84における情報ビットに対する事前確率情報Apr_0として出力する。

【0204】パンクチャ器88は、MAP復号器84から供給された符号化ビットに対する事後確率情報Apo_{0z}と、MAP復号器86から供給された符号化ビットに対する事後確率情報Apo_{1z}とを、変換器11_jにおけるパンクチャ器23と同一の規則に基づいて間引き、ビット数が削減されたパンクチャデータDpとしてチャネル・インターリーバ89に供給する。

【0205】チャネル・インターリーバ89は、パンクチャ器88が要する処理時間と同時間だけ遅延されてMAP復号器84から供給された事後確率情報Apo_{0i}と、パンクチャ器88から供給されたパンクチャデータDpとを入力し、これらの事後確率情報Apo_{0i}及びパンクチャデータDpを構成する各ビットデータの順序を、変換器11_jにおけるチャネル・インターリーバ24と同一の置換位置情報に基づいて並べ替え、符号化系列$x(i)$に対する事後確率情報$P(x(i)|y)$を生成する。チャネル・インターリーバ89は、生成した事後確率情報$P(x(i)|y)$を外部へと出力する。

【0206】ところで、このような復号器62_0においては、送信装置10による符号化が複数の符号化系列が加算されたものを出力するものであることから、上式(51)に示したように単純に尤度を算出することができない。そこで、復号器62_0において、MAP復号器

84，86は、以下のようにして尤度を算出する。

【0207】符号化系列x(0)に対する受信系列の時間的にk番目の要素が"1"，" 1"であるとしたときの尤度$L_k^{(0)}(+1)$，$L_k^{(0)}(-1)$は、それぞれ、他の符号化系列x(1)，x(2)についての確率が得られているものとすると、次式（52）及び次式（53）で表される。なお、尤度$L_k^{(0)}(+1)$，$L_k^{(0)}(-1)$は、最終的にはそれらの和が"1"となるように正規化される。

【0208】

【数52】

$$L_k^{(0)}(+1) = p(y_k \mid x_k^{(0)} = +1)$$
$$\propto P(x_k^{(1)} = +1)P(x_k^{(2)} = +1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(+1)+a^{(1)}(+1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = -1)P(x_k^{(2)} = +1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(+1)+a^{(1)}(-1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = +1)P(x_k^{(2)} = -1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(+1)+a^{(1)}(+1)+a^{(2)}(-1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = -1)P(x_k^{(2)} = -1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(+1)+a^{(1)}(-1)+a^{(2)}(-1)\right)\right|^2\right]$$

・・・(52)

【0209】

【数53】

$$L_k^{(0)}(-1) = p(y_k \mid x_k^{(0)} = -1)$$
$$\propto P(x_k^{(1)} = +1)P(x_k^{(2)} = +1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(-1)+a^{(1)}(+1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = -1)P(x_k^{(2)} = +1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(-1)+a^{(1)}(-1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = +1)P(x_k^{(2)} = -1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(-1)+a^{(1)}(+1)+a^{(2)}(-1)\right)\right|^2\right]$$
$$+P(x_k^{(1)} = -1)P(x_k^{(2)} = -1)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}(-1)+a^{(1)}(-1)+a^{(2)}(-1)\right)\right|^2\right]$$

・・・(53)

【0210】すなわち、尤度$L_k^{(0)}(+1)$，$L_k^{(0)}(-1)$としては、それぞれ、可能性のある全ての送信シンボルの候補に対して、受信シンボルを比較したときの尤もらしさを求め、その送信シンボルが送信される確率で重み付けして加算したものが用いられる。ここで、送信シンボルが送信される確率は、符号化系列x(0)の要素が送信される確率を乗算して求めることができる。また、符号化系列x(0)の要素が送信される確率としては、以前にその符号が復号されている場合には、その確率が用いられ、復号されていない場合には、確率が未知であることを示す値として反映される。

【0211】なお、振幅に変動がある通信路を介した伝送が行われた場合には、尤度$L_k^{(0)}(-1)$，$L_k^{(0)}(-1)$は、それぞれ、次式（54）及び次式（55）を用いて算出される。なお、次式（54）及び次式（55）における"f_k"は、通信路におけるk番目の要素の振幅を示すものである。

【0212】

【数54】

$$L_k^{(0)}(+1) = p\left(y_k \mid x_k^{(0)} = +1\right)$$
$$\propto P\left(x_k^{(1)} = +1\right)P\left(x_k^{(2)} = +1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(+1)+a^{(1)}(+1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = -1\right)P\left(x_k^{(2)} = +1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(+1)+a^{(1)}(-1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = +1\right)P\left(x_k^{(2)} = -1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(+1)+a^{(1)}(+1)+a^{(2)}(-1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = -1\right)P\left(x_k^{(2)} = -1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(+1)+a^{(1)}(-1)+a^{(2)}(-1)\right)\right|^2\right]$$

・・・(54)

【0213】

【数55】

$$L_k^{(0)}(-1) = p\left(y_k \mid x_k^{(0)} = -1\right)$$
$$\propto P\left(x_k^{(1)} = +1\right)P\left(x_k^{(2)} = +1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(-1)+a^{(1)}(+1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = -1\right)P\left(x_k^{(2)} = +1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(-1)+a^{(1)}(-1)+a^{(2)}(+1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = +1\right)P\left(x_k^{(2)} = -1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(-1)+a^{(1)}(+1)+a^{(2)}(-1)\right)\right|^2\right]$$
$$+P\left(x_k^{(1)} = -1\right)P\left(x_k^{(2)} = -1\right)\frac{1}{\sqrt{2\pi}\sigma_n}\exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - f_k\cdot\left(a^{(0)}(-1)+a^{(1)}(-1)+a^{(2)}(-1)\right)\right|^2\right]$$

・・・(55)

【0214】復号器62_0において、MAP復号器84，86は、このようにして算出した尤度$L_k(0)(+1)$，$L_k(0)(-1)$を用いて事後確率情報を算出する。

【0215】このようなMAP復号器84，86を備える復号器62_0は、変換器11_jにおける要素符号化器21_0，21_1のそれぞれに対応するMAP復号器84，86を備えることにより、復号複雑度が高い符号を複雑度の小さい要素に分解し、MAP復号器84，86の間の相互作用によって特性を逐次的に向上させることができる。復号器62_0は、受信値yを受信すると、所定の繰り返し回数での繰り返し復号を行い、この復号動作の結果得られた軟出力の外部情報に基づいて、MAP復号器86から情報ビット系列b(0)に対する事後確率情報P(b(0)|y)を復号データとして出力するとともに、符号化系列x(0)に対する事後確率情報P(x(0)|y)を、必要に応じて、他の復号器62_1，62_2に供給する。

【0216】受信装置60においては、復号器62_1，62_2についても復号器62_0と同様に構成される。すなわち、復号器62_1は、上式(52)及び上式(53)又は上式(54)及び上式(55)に示したように算出した尤度$L_k(1)(+1)$，$L_k(1)(-1)$を用いて、情報ビット系列b(1)に対する事後確率情報P(b(1) y)及び符号化系列x(1)に対する事後確率情報P(x(1)|y)を生成し、事後確率情報P(b(1) y)を復号データとして出力するとともに、事後確率情報P(x(1)|y)を、必要に応じて、他の復号器62_0，62_2に供給する。また、復号器62_2についても同様に、上式(52)及び上式(53)又は上式(54)及び上式(55)に示したように算出した尤度$L_k(2)(+1)$，$L_k(2)(-1)$を用いて、情報ビット系列b(2)に対する事後確率情報P(b(2) y)及び符号化系列x(2)に対する事後確率情報P(x(2)|y)を生成し、事後確率情報P(b(2) y)を復号データとして出力するとともに、事後確率情報P(x(2)|y)を、必要に応じて、他の復号器62_0，62_1に供給する。

【0217】このような復号器62_0，62_1，62_2を備える受信装置60は、情報ビット系列b(2)，b(1)，b(0)に対する事後確率情報P(b(2)|y)，P(b(1)|y)，P(b(0) y)を順次求めることにより、情報ビット系列b(2)，b(1)，b(0)の順序で復号することができる。

【0218】また、受信装置60は、高次の情報ビット系列b(2)から順次1回のみ復号を行うのではなく、

上述したように、ジグザグ復号や繰り返し復号を行うこともできる。

【0219】具体的には、受信装置60は、復号器62_2，62_1，62_0による復号処理を、それぞれ、"A"，"B"，"C"で表すものとすると、基本的には上述したように、"A"，"B"，"C"の順序で復号を行うが、"A"，"B"が終了した段階で、得られた符号化系列$x^{(1)}$に対する事後確率情報$P(x^{(1)}|y)$を符号化系列$x^{(1)}$に対する事前確率情報$P(x^{(1)})$として復号器62_2に供給して再度"A"を行うこともできる。これにより、受信装置60は、符号化系列$x^{(2)}$に対する復号の信頼性を向上させることができ、これに応じて、より低次の符号化系列$x^{(1)}$，$x^{(0)}$に対する復号の信頼性も向上させることができる。同様に、受信装置60は、"A"，"B"，"C"が終了した段階で、得られた符号化系列$x^{(0)}$に対する事後確率情報$P(x^{(0)}|y)$を符号化系列$x^{(0)}$に対する事前確率情報$P(x^{(0)})$として復号器62_1に供給して再度"B"を行い、符号化系列$x^{(1)}$に対する復号の信頼性をより向上させることもできる。すなわち、受信装置60は、"A"，"B"，"A"，"B"，"C"，"B"，"C"の順序で復号を行うこともできる。

【0220】また、受信装置60は、"A"，"B"，"C"が終了した段階で、同様の復号動作を複数回繰り返し、"A"，"B"，"C"，"A"，"B"，"C"，・・・の順序で復号を行ったり、"A"，"B"，"A"，"B"，"C"，"B"，"C"が終了した段階で、同様の復号動作を複数回繰り返し、"A"，"B"，"A"，"B"，"C"，"B"，"C"，"A"，"B"，"A"，"B"，"C"，"B"，"C"，・・・の順序で復号を行うようにしてもよい。

【0221】このように、受信装置60は、高次の情報ビット系列$b^{(2)}$から順次1回のみ復号を行うのではなく、所定の規則に基づいたジグザグ復号や繰り返し復号を行うことができる。

【0222】また、受信装置60は、尤度を求める際に、以下のような方法を採用することにより、演算の簡略化を図ることもできる。

【0223】まず、第1の方法は、復号が終了した符号化系列がある場合に、次式（56）に示すように、k番目の要素に対する事後確率情報$P(x_k^{(i)})$を最大にする要素$x_k^{(i)}$を最良の候補として選出し、この最良の候補を事前確率情報$x_{k\cdot best}^{(i)}$として他の符号化系列に対する尤度を求めるものである。すなわち、この第1の方法は、復号が終了した符号化系列については硬判定を行うものである。

【0224】

【数56】

$$x_{k\cdot best}^{(i)} = \max_{x_k^{(i)}} \left[P\left(x_k^{(i)}\right) \right] \qquad \cdots (56)$$

【0225】例えば、上述した復号器62_0，62_1，62_2による復号処理"A"，"B"，"C"のうち、復号処理"A"，"C"が終了しているものとした場合、復号器62_1は、要素$x_k^{(1)}$の尤度を、次式（57）に示すように算出する。すなわち、復号器62_1は、復号が終了した符号化系列$x^{(0)}$，$x^{(2)}$における最良の候補に対する事前確率情報を"1"とみなし、他の要素に対する事前確率情報を"0"とみなすことにより、上式（52）及び上式（53）を簡略化することができる。

【0226】

【数57】

$$p\left(y_k \mid x_k^{(1)}\right) \propto \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[\frac{-1}{2\sigma_n^2} \left| y_k - \left(a^{(0)} x_{k\cdot best}^{(0)} + a^{(1)} x_k^{(1)} + a^{(2)} x_{k\cdot best}^{(2)} \right) \right|^2 \right]$$

$$\cdots (57)$$

【0227】また、第2の方法は、復号が終了した符号化系列がある場合に、次式（58）に示すように、軟判定を行ったk番目の要素に対する期待値を事後確率情報として求め、この事後確率情報を事前確率情報$x_{k\cdot exp}^{(1)}$として他の符号化系列に対する尤度を求めるものである。

【0228】

【数58】

$$x_{k\cdot exp}^{(i)} = \sum_{x_k^{(i)}} P\left(x_k^{(i)}\right) x_k^{(i)} \qquad \cdots (58)$$

【0229】例えば、上述した復号器62_0，62_1，62_2による復号処理"A"，"B"，"C"のうち、復号処理"A"，"C"が終了しているものとした場合、復号器62_1は、要素$x_k^{(1)}$の尤度を、次式（59）に示すように算出することにより、上式（52）及び上式（53）を簡略化することができる。

【0230】

【数59】

$$p\left(y_k \mid x_k^{(1)}\right) \propto \frac{1}{\sqrt{2\pi}\sigma_n} \exp\left[\frac{-1}{2\sigma_n^2}\left|y_k - \left(a^{(0)}x_{k\text{-exp}}^{(0)} + a^{(1)}x_k^{(1)} + a^{(2)}x_{k\text{-exp}}^{(2)}\right)\right|^2\right] \quad \cdots(59)$$

【0231】さらに、第3の方法は、ある符号化系列を復号しようとする際に、他の符号化系列が復号されていない場合、これらの復号されていない符号化系列を電力の等しいガウス雑音とみなすものである。

【0232】例えば、復号器62_2は、上述した復号器62_2による復号処理"C"を行う場合には、要素x_k(2)の尤度を、次式(60)に示すように算出することにより、上式(52)及び上式(53)を簡略化することができる。なお、この場合、仮に復号が終了した符号化系列がある場合には、これらの符号化系列については上述した第1の方法又は第2の方法を適用することもできる。

【0233】

【数60】

$$p\left(y_k \mid x_k^{(2)}\right) \propto \frac{1}{\sqrt{2\pi\left(\sigma_n^2 + a^{(1)^2} + a^{(0)^2}\right)}} \exp\left[\frac{-1}{2\left(\sigma_n^2 + a^{(1)^2} + a^{(0)^2}\right)}\left|y_k - a^{(2)}x_k^{(2)}\right|^2\right] \quad \cdots(60)$$

【0234】さて、復号するにあたって通信路にゲイン又はロスがある場合には、受信装置60は、この値を把握する必要が生じる。ここでは、通信路の推定方法について説明する。

【0235】通常、通信路を推定するには、符号化系列とは別途設けられたパイロット信号を用いて、ゲイン又はロスの大きさを把握することができる。しかしながら、この方法は、多くのパイロット信号を用いた場合には、これを伝送するためのエネルギが過大となり望ましくない。

【0236】そこで、受信装置60は、図16に示すように、チャネル推定部90を少なくとも復号器62_2に付設し、通信路を推定することができる。

【0237】すなわち、受信装置60は、最も振幅が大きい符号化系列x(2)の復号に十分な精度の振幅fについては、推定器91によってパイロット信号を用いる方法や受信値yの大きさを判別する方法等を用いて推定した後、復号器62_2によって符号化系列x(2)の復号を行う。続いて、受信装置60は、この符号化系列x(2)の復号が終了した際に、次式(61)及び次式(62)に示すように、復号結果である情報ビット系列b(2)に対する事後確率情報p(b(2)|y)を用いて、変換器92によって再符号化gを行い、符号化系列x(2)の推定値である硬判定値系列x(2)'を求める。

【0238】

【数61】

$$b^{(2)'} = \max_b\left[p\left(b^{(2)} \mid y\right)\right] \quad \cdots(61)$$

【0239】

【数62】

$$x^{(2)'} = g\left(b^{(2)'}\right) \quad \cdots(62)$$

【0240】そして、受信装置60は、相関算出器93によって硬判定値系列x(2)'と受信値yとの相関を算出する。通信路の振幅を"f"とすると、受信値yは、次式(63)で表されることから、硬判定値系列x(2)'に誤りがないものとすると、相関値f'は、次式(64)によって求めることができる。なお、次式(64)における"・"は、内積を示す演算子である。また、次式(64)における分母は、大きさを正規化するためのものであって、|x(2)|2は、符号化系列x(2)がM次元ベクトルであることから"M"に置換できる。

【0241】

【数63】

$$y = f\left(a^{(0)}x^{(0)} + a^{(1)}x^{(1)} + a^{(2)}x^{(2)}\right) + n \quad \cdots(63)$$

【0242】

【数64】

$$f' = \frac{y \bullet x^{(2)}}{a^{(2)}\left|x^{(2)}\right|^2} = \frac{f \cdot a^{(0)}x^{(0)} \bullet x^{(2)} + f \cdot a^{(1)}x^{(1)} \bullet x^{(2)} + f \cdot a^{(2)}x^{(2)} \bullet x^{(2)} + nx^{(2)}}{a^{(2)}M}$$
$$= \frac{f \cdot a^{(0)}x^{(0)} \bullet x^{(2)} + f \cdot a^{(1)}x^{(1)} \bullet x^{(2)} + f \cdot a^{(2)}M + nx^{(2)}}{a^{(2)}M}$$
$$= f + \frac{f \cdot a^{(0)}x^{(0)} \bullet x^{(2)} + f \cdot a^{(1)}x^{(1)} \bullet x^{(2)} + nx^{(2)}}{a^{(2)}M}$$
$$\rightarrow f \quad (M \rightarrow \infty)$$

・・・(64)

【0243】すなわち、受信装置60は、符号化系列x(0)，x(2)が符号化系列x(2)とは独立であることから、上式(64)によって求められる相関値f'において"M"を大きな値とすることにより、通信路の振幅fを正確に推定することができる。

【0244】また、受信装置60は、図17に示すように、チャネル推定部100を少なくとも復号器62_2に付設し、通信路を推定することもできる。

【0245】すなわち、受信装置60は、チャネル推定部90と同様に、最も振幅が大きい符号化系列x(2)の復号に十分な精度の振幅fについては、推定器101によってパイロット信号を用いる方法や受信値yの大きさを判別する方法等を用いて推定した後、復号器62_2によって符号化系列x(2)の復号を行う。続いて、受信装置60は、この符号化系列x(2)の復号が終了した際に、次式(65)に示すように、符号化系列x(2)に対する事後確率情報p(x(2)|y)を用いて、符号化系列推定器102によって符号化系列x(2)の推定値である硬判定値系列x(2)'を求める。

【0246】

【数65】

$$x^{(2)'} = \max_{x}\left[p\left(x^{(2)} \mid y\right)\right] \qquad \cdots(65)$$

【0247】そして、受信装置60は、チャネル推定部90と同様に、相関算出器103によって硬判定値系列x(2)'と受信値yとの相関を算出し、通信路の振幅fを正確に推定することができる。

【0248】受信装置60は、このようにして推定した振幅fを用いて、復号器62_1，62_0によって符号化系列x(1)，x(0)の復号を行う。また、受信装置60は、推定した振幅fを用いて、再度、復号器62_2によって符号化系列x(2)の復号を行うようにしてもよい。

【0249】このような符号化を行う送信装置10と復号を行う受信装置60とを備えるデータ送受信システムにおいては、以下のようにして、通信路の状態変化に適応的に対応して定数a(1)を設定することができる。すなわち、例えば移動通信においては、移動機の移動速度によってチャネルモデルが変化し、静止時にはスタティックチャネルであり、高速移動時にはレイリーチャネルに遷移する。このような場合、データ送受信システムにおいては、送信装置10によって符号化のパラメータである各符号化系列x(1)の振幅、すなわち、定数a(1)を、通信路に応じて変化させた方が良好な特性が得られる。

【0250】データ送受信システムにおいては、チャネルモデルが変化する場合、通信路の状態を同定し、その通信路に最適な定数a(1)を求め、この定数a(1)に基づいて適応的に符号化を行う。

【0251】このような適応的符号化としては、いわゆるドゥプレックス(duplex)通信を行っている場合に、受信側が通信路の状態を同定し、その情報を送信側にフィードバックし、このフィードバックされた情報に基づいて送信側が適応的に符号化する方法が考えられる。

【0252】具体的には、データ送受信システムにおいては、例えば図18に概略を示すように、チャネル推定部を備える受信装置60によって通信路の状態を同定し、その結果に基づいてコントローラ150_2によって符号化のパラメータ、すなわち、定数a(1)を決定する。データ送受信システムにおいては、決定された定数a(1)を現時刻のパラメータPR_Cとして用いて、復号器によって復号を行うとともに、決定された定数a(1)を次時刻のパラメータPR_Nとして送信する。なお、ここでの受信装置60における復号器とは、上述した復号器62_0，62_1，62_2に相当するものである。

【0253】そして、データ送受信システムにおいては、通信路を介して受信装置60から送信されてきた次時刻のパラメータPR_Nを受信すると、コントローラ150_2を介してこのパラメータPR_Nを現時刻のパラメータPR_Cとして用いて、符号化器によって符号化を行い、通信路を介して受信装置60へと送信する。なお、ここでの送信装置10における符号化器とは、上述した変換器11_0，11_1，11_2及び乗算器12_0，12_1，12_2並びに加算器13_0，13_1に相当するものである。

【0254】このように、データ送受信システムにおい

ては、ドゥプレックス通信を行っている場合に、受信側が通信路の状態を同定して送信側にフィードバックし、このフィードバックされた情報に基づいて送信側が適応的に符号化することができ、特性を向上させることができる。

【0255】また、適応的符号化としては、送信側が受信している通信路の状態を同定し、送信する通信路の状態が受信している通信路の状態と等しいものとみなして、受信している通信路の状態を示す情報に基づいて送信側が適応的に符号化する方法が考えられる。

【0256】具体的には、データ送受信システムにおいては、パラメータの変更を行う際に、その変更の予告を事前に行う場合には、例えば図19に概略を示すように、チャネル推定部を備える送信装置10によって通信路の状態を同定し、その結果に基づいてコントローラ150_1によって符号化のパラメータ、すなわち、定数$a^{(1)}$を決定する。データ送受信システムにおいては、決定された定数$a^{(1)}$を次時刻のパラメータPR_Nとして、マルチプレクサによって情報ビット系列に多重させ、さらに、コントローラ150_2から与えられている定数$a^{(1)}$を現時刻のパラメータPR_Cとして用いて、符号化器によって多重化されたデータに対する符号化を行い、送信する。なお、ここでの送信装置10における符号化器とは、上述した変換器11_0，11_1，11_2及び乗算器12_0，12_1，12_2並びに加算器13_0，13_1に相当するものである。

【0257】そして、データ送受信システムにおいては、通信路を介して送信装置10から送信されてきたデータを受信装置60が受信すると、復号器によって復号を行う。このとき、データ送受信システムにおいては、コントローラ150_2から与えられるパラメータPR_Cを用いて復号が行われる。データ送受信システムにおいては、復号されたデータからデマルチプレクサによって送信装置10によって多重されたパラメータPR_Nを分離してコントローラ150_2に与え、パラメータを変更させる。これにより、データ送受信システムにおいては、次時刻からの復号に際しては、新たなパラメータPR_Nが現時刻のパラメータPR_Cとして用いられ、復号が行われる。なお、ここでの受信装置60における復号器とは、上述した復号器62_0，62_1，62_2に相当するものである。

【0258】このように、データ送受信システムにおいては、送信側が受信している通信路の状態を同定し、送信する通信路の状態が受信している通信路の状態と等しいものとみなして、受信している通信路の状態を示す情報に基づいて送信側が適応的に符号化することができ、受信側の処理負担を軽減することができる。

【0259】なお、この適応的符号化としては、パラメータの変更の予告を事前に行うようにしている。これは、適応的符号化においては、基本的には、次時刻のデータの復号に用いるパラメータを予め受信装置に対して教える必要があるからである。しかしながら、データ送受信システムにおいては、以下のようにして、現時刻のパラメータを現時刻のデータに含ませて伝送することもできる。

【0260】具体的には、データ送受信システムにおいては、例えば図20に概略を示すように、先に図19に示したデータ送受信システムと同様に構成される。このとき、受信装置60は、パラメータが多重化されたデータを復号する必要がある。ここで、最高次の符号化系列は、低次の符号化系列の電力比に比較的影響を受けることなく復号することができることに着目する。

【0261】データ送受信システムにおいては、チャネル推定部を備える送信装置10によって通信路の状態を同定し、その結果に基づいてコントローラ150_1によって符号化のパラメータ、すなわち、定数$a^{(1)}$を決定すると、この決定された定数$a^{(1)}$を現時刻のパラメータPR_Cとして、マルチプレクサによって最高次の情報ビット系列に多重させるとともに、決定された定数$a^{(1)}$を現時刻のパラメータPR_Cとして用いて、符号化器によって多重化されたデータに対する符号化を行い、送信する。

【0262】そして、データ送受信システムにおいては、通信路を介して送信装置10から送信されてきたデータを受信装置60が受信すると、復号器によって最高次の符号化系列から復号を行う。このとき、データ送受信システムにおいては、最高次の符号化系列の復号に際しては、コントローラ150_2から与えられる所定のパラメータPR_Cを用いて復号が行われる。データ送受信システムにおいては、復号されたデータからデマルチプレクサによって送信装置10によって多重されたパラメータPR_Cを分離してコントローラ150_2に与え、パラメータを変更させる。これにより、データ送受信システムにおいては、最高次よりも低次の符号化系列の復号に際しては、新たなパラメータPR_Cを用いた復号を行うことができる。

【0263】このように、データ送受信システムにおいては、送信側が受信している通信路の状態を同定し、送信する通信路の状態が受信している通信路の状態と等しいものとみなして、受信している通信路の状態を示す情報に基づいて送信側が適応的に符号化する際に、現時刻のパラメータを現時刻の最高次の符号化系列に含ませることができ、受信側にパラメータを予告することなく符号化を行うことができる。

【0264】さて、このようなデータ送受信システムにおける性能を評価するために、符号性能を示す上で一般的に用いられるビットエラーレートBERと1ビットあたりの信号対雑音電力比E_b/n_0との関係で示される特性を、AWGNチャネルとレイリーチャネルのそれぞれについてシミュレーションによって求めた。

【0265】このシミュレーションにおいては、原符号として、生成多項式G(D)が次式(66)で表されるターボ符号を用い、インターリーバとして、各変換器間で異なるランダムインターリーブを施すものを用い、伝送率をC=1/2とし、パンクチャパタンを次式(67)に示すものとし、さらに、各変換器間で異なるランダムインターリーブを施すチャネル・インターリーバを備えるものとした。そして、このシミュレーションにおいては、各変換器に入力する情報ビット数をN=20000、すなわち、各変換器によって生成される符号化系列の要素数をM=40000とし、各復号器によるターボ復号における繰り返し回数を20回とした。

【0266】

【数66】

$$G(D)=\left(\frac{1}{\frac{1+D^4}{1+D+D^2+D^3+D^4}}\right) \quad \cdots(66)$$

【0267】

【数67】

$$\begin{pmatrix} \text{情報} \\ \text{パリティ}_0 \\ \text{パリティ}_1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \cdots(67)$$

【0268】このような条件のもと、AWGNチャネルにおける特性を求めると、図21に示すようにまとめられた結果が得られるとともに、図22に示す特性曲線が得られた。

【0269】すなわち、このターボ符号の単独のAWGN特性として、雑音の確率密度関数を次式(68)で表してビットエラーレートを求めると、図22に示す特性曲線のうち、最左側に位置する実線で表される特性曲線が得られた。ここで、このチャネルに関する特性から符号として所要とする信号対雑音電力比$E_b/n_0=\xi(0)$を"0.8[dB]"として選出すると、図21に示すように、定数$a(0)$は、"0.775"となる。

【0270】

【数68】

$$p(n)=\frac{1}{\sqrt{\pi n_0}}\exp\left[-\frac{1}{n_0}n^2\right] \quad \cdots(68)$$

【0271】次に、チャネルモデルとして、雑音と、定数倍符号化系列$a(0)x(0)$と同一の統計的性質を有する系列との和が加算される通信路を確率密度関数で表すと、次式(69)で表され、図22に示す特性曲線のうち、最左側に位置する破線で表される特性曲線が得られた。ここで、このチャネルに関する特性から符号として所要とする信号対雑音電力比$E_b/(n_0+2\nu(0))=\xi(1)$を"0.7[dB]"として選出すると、定数$a(1)$は、"1.137"となる。

【0272】

【数69】

$$p(n)=\frac{1}{\sqrt{\pi n_0}}\exp\left[-\frac{1}{n_0}n^2\right]\otimes\frac{1}{2}\left(\delta\left(n+a^{(0)}\right)+\delta\left(n-a^{(0)}\right)\right)$$
$$=\frac{1}{2\sqrt{\pi n_0}}\exp\left[-\frac{1}{n_0}\left(n+a^{(0)}\right)^2\right]+\frac{1}{2\sqrt{\pi n_0}}\exp\left[-\frac{1}{n_0}\left(n-a^{(0)}\right)^2\right] \quad \cdots(69)$$

【0273】さらに、チャネルモデルとして、雑音と、定数倍符号化系列$a(0)x(0)$、$a(1)x(1)$と同一の統計的性質を有する系列との和が加算される通信路を想定すると、図22に示す特性曲線のうち、最左側に位置する一点鎖線で表される特性曲線が得られた。ここで、このチャネルに関する特性から符号として所要とする信号対雑音電力比$E_b/(n_0+2\nu(1))=\xi(2)$を"0.6[dB]"とすると、定数$a(2)$は、"1.658"となる。

【0274】同様に、チャネルモデルとして、雑音と、定数倍符号化系列$a(0)x(0)$、$a(1)x(1)$、$a(2)x(2)$と同一の統計的性質を有する系列との和が加算される通信路を想定すると、図22に示す特性曲線のうち、最左側に位置する二点鎖線で表される特性曲線が得られた。ここで、このチャネルに関する特性から符号として所要とする信号対雑音電力比$E_b/(n_0+2\nu(2))=\xi(3)$を"0.6[dB]"とすると、定数$a(3)$は、"2.130"となる。

【0275】これらの定数$a(i)$を用いることにより、符号として、2系列の定数倍符号化系列$a(0)x(0)$、$a(1)x(1)$を加算した伝送率がC=2/2である加算符号化系列と、3系列の定数倍符号化系列$a(0)x(0)$、$a(1)x(1)$、$a(2)x(2)$を加算した伝送率がC=3/2である加算符号化系列と、4系列の定数倍符号化系列$a(0)x(0)$、$a(1)x(1)$、$a(2)x(2)$、$a(3)x(3)$を加算した伝送率がC=4/2である加算符号化

系列とを構成することができる。これらをまとめたものが、図21である。なお、ここでは、各定数倍符号化系列$a^{(0)}x^{(0)}$，$a^{(1)}x^{(1)}$，$a^{(2)}x^{(2)}$，$a^{(3)}x^{(3)}$を、それぞれ、0段、1段、2段、3段と称するものとする。なお、同図においては、各加算符号化系列を伝送する際に期待される所要とする平均の信号対雑音電力比$E_b/n_0=\xi_{ave}$'も示している。

【0276】このように設計された符号を用いて雑音電力密度n_0を変化させ、AWGNチャネルにおける各系列の特性を求めた結果、図22に示す特性曲線が得られた。

【0277】また、同様の条件のもと、通信路推定が完全であるものとして、フリー・インターリーブド・レイリーチャネル（fully-interleaved Rayleigh Channel）における特性も求めた。図23に示すように、符号として所要とする信号対雑音電力比$E_b/n_0=\xi^{(0)}$，$E_b/(n_0+2\nu^{(0)})=\xi^{(1)}$，$E_b/(n_0+2\nu^{(1)})=\xi^{(2)}$，$E_b/(n_0+2\nu^{(2)})=\xi^{(3)}$を、それぞれ、"2.7[dB]"，"1.9[dB]"，"1.1[dB]"，"1.0[dB]"として選出すると、定数$a^{(0)}$，$a^{(1)}$，$a^{(2)}$，$a^{(3)}$は、それぞれ、"0.965"，"1.489"，"2.167"，"3.242"となる。

【0278】このように設計された符号を用いて雑音電力密度n_0を変化させ、レイリーチャネルにおける各系列の特性を求めた結果、図24に示す特性曲線が得られた。

【0279】これらの図22及び図24から明らかなように、所要とする信号対雑音電力比$E_b/(n_0-2\nu)$の値は、加算する定数倍符号化系列の数が増加するほど、すなわち、段数が増加するほど小さくなることがわかる。これは、加算する定数倍符号化系列の数が増加すると、通信路がガウス分布の状態ではなくなっていくことによるものである。また、この加算する定数倍符号化系列の数の増加に伴う信号対雑音電力比$E_b/(n_0+2\nu)$の値の減少の度合いは、AWGNチャネルの場合よりもレイリーチャネルの場合の方が顕著である。これは、加算する定数倍符号化系列の数が増加すると、通信路がガウス分布の状態ではなくなっていくことによるものの他、上述したように、符号の変動の偏差が小さくなることによるものである。なお、シミュレーション結果からは、高次の符号の復号結果の曖昧さが低次の符号の復号に影響を与えていることもわかる。そのため、高次の符号に対する定数$a^{(i)}$としては、マージンを多くとることが有効だと思われる。

【0280】このように、データ送受信システムは、伝送率の小さな符号を用いて、伝送率が大きな符号を容易に構成することができ、高い性能で容易に復号することができるものであることがわかる。

【0281】なお、このシミュレーションにおいては、通信路がスタティックであるか又はレイリーであるかによって定数$a^{(i)}$等の符号化のパラメータを最適化したが、仮にレイリーチャネル用の符号化のパラメータを用いてスタティックチャネルにおける特性を予測すると、図25に示すようになる。すなわち、この場合、1段以上の符号においては、スタティックチャネルよりもレイリーチャネルの方が所要とする信号対雑音電力比$E_b/(n_0+2\nu)$の値が大きくなり、また、0段の符号の所要とする信号対雑音電力比E_b/n_0は、図21に示した0.8[dB]よりも1.9[dB]だけ大きい2.7[dB]となることから、スタティックチャネルにおける特性は、レイリーチャネルにおける特性よりも少なくとも1.9[dB]だけ低い信号対雑音電力比$E_b/(n_0+2\nu)$となることが予測される。

【0282】以上のように、データ送受信システムにおいては、送信装置により、複数の情報ビット系列$\{b^{(0)},b^{(1)},\cdots,b^{(L-1)}\}$に対して所定の符号化及び／又は変調を含む変換処理を施し、得られた符号化系列$\{x^{(0)},x^{(1)},\cdots,x^{(L-1)}\}$に対して定数$\{a^{(0)},a^{(1)},\cdots,a^{(L-1)}\}$を乗算し、さらに、得られた定数倍符号化系列$\{a^{(0)}x^{(0)},a^{(1)}x^{(1)},\cdots,a^{(L-1)}x^{(L-1)}\}$を加算して加算符号化系列gを生成して伝送することにより、基本となる原符号に制限が少なく、符号設計の自由度を格段に向上させた高い性能での符号化を容易に実現して、情報ビット系列$\{b^{(0)},b^{(1)},\cdots,b^{(L-1)}\}$に対して最適な符号化を行うことができ、ビットエラーレートを十分に低くするための所要信号対雑音電力比E_b/n_0の値を小さくすることができる。

【0283】また、データ送受信システムにおいては、受信装置により、最高次の情報ビット系列$b^{(L-1)}$から順次復号を行うことにより、少なくとも1つの情報ビット系列$b^{(i)}$に対する復号を高精度且つ容易に行うことができる。特に、データ送受信システムにおいては、受信装置により、MAP復号又はこれに準ずる復号を行うことにより、現実の符号に対する復号を行うことができる。また、データ送受信システムにおいては、受信装置による任意の符号化系列の復号の際に、他の符号化系列に対する情報を利用することにより、ビットエラーレートを十分に低くするための所要信号対雑音電力比E_b/n_0の値を小さくすることができる。

【0284】したがって、データ送受信システムは、帯域が制限されている場合等、限られた個数の実数でデータを伝送する必要がある場合に、高い伝送率での符号化を行いたいといった要求に十分に応えられるものであり、ユーザにとって優れた利便を提供することができるものである。

【0285】なお、上述した実施の形態では、基本的には、原符号の振幅がガウス分布をなすものとして説明したが、通常の符号の振幅は、ガウス分布でないことが多く、特に、符号化率が低い符号ほど、BPSK変調方式が施されている。このような場合には、符号化の過程で、以前に加算した符号のエネルギに基づいて符号のエネルギを求めるのは望ましくない。

【0286】ここで、BPSK変調方式による信号点の2値の分布のエントロピは"1"であり、ガウス分布のエントロピ$(1/2)\log_2(\pi e)=1.65$よりも低い値となる。したがって、各情報ビット系列に対する符号化系列の振幅がガウス分布を呈していなくても、フェージングチャネルを除いては、これにランダムな正規直交変換を施すことによって見かけ上ガウス分布にすることは可能であることから、各符号がガウス分布であるものと仮定した結果よりも悪い結果が得られることはないのは明らかである。

【0287】そこで、原符号の振幅が非ガウス分布を呈する場合における所要信号対雑音電力比$E_b/(n_0+2\nu)$は、符号化器のモデルを反映した非ガウスチャネルで測定すれば求めることができる。また、復号においては、尤度を符号化器のモデルに即して正確に計算するのが最適である。

【0288】さて、以下では、このようなデータ送受信システムを適用した多元接続システムについて説明する。

【0289】この多元接続システムは、上述したデータ送受信システムにおける各レベルの情報ビット系列$b^{(i)}$を多元接続における各端末の情報系列とするものである。

【0290】具体的には、多元接続システムにおける下りリンクは、3人のユーザからのデータからなる3つの情報ビット系列$\{b^{(0)},b^{(1)},b^{(2)}\}$を多元接続するものとすると、図26に示すように構成される。すなわち、多元接続システムは、上述した送信装置10が適用された基地局200と、各情報ビット系列$\{b^{(0)},b^{(1)},b^{(2)}\}$を出力する上述した受信装置60が適用された3つの端末300_0，300_1，300_2とを備える。

【0291】基地局200は、上述した変換器11_0，11_1，11_2に相当する3つの変換器201_0，201_1，201_2と、上述した乗算器12_0，12_1，12_2に相当する3つの乗算器202_0，202_1，202_2と、上述した加算器13_0，13_1に相当する加算器203と、上述した送信部14に相当する送信部204とを有する。ここで、加算器203は、同図においては1つのみを示しているが、基地局200においては、上述した送信装置10のように2つの加算器としてもよい。

【0292】一方、端末300_0，300_1，300_2は、それぞれ、上述した受信部61に相当する受信部301_0，301_1，301_2と、復号器302_0，302_1，302_2とを有する。ここで、復号器302_0は、上述した3つの復号器62_0，62_1，62_2に相当するものであり、復号器302_1は、少なくとも上述した2つの復号器62_1，62_2に相当するものであり、復号器302_2は、少なくとも上述した1つの復号器62_2に相当するものである。

【0293】このような多元接続システムにおいては、基地局200によって上述した送信装置10と同様の処理を行うことにより、3つの情報ビット系列$\{b^{(0)},b^{(1)},b^{(2)}\}$を符号化して送信信号$g'$に変換し、この送信信号$g'$を外部へと送信する。

【0294】多元接続システムにおいては、この送信信号g'が、3人のユーザのそれぞれに対応する伝送ロス$L^{(0)}$，$L^{(1)}$，$L^{(2)}$を有する通信路を介して伝送されることにより、送信信号g'に対して雑音$n^{(0)}$，$n^{(1)}$，$n^{(2)}$がそれぞれ加算され、この信号が受信信号$y'^{(0)}$，$y'^{(1)}$，$y'^{(2)}$として端末300_0，300_1，300_2のそれぞれによって受信される。なお、伝送ロス$L^{(0)}$，$L^{(1)}$，$L^{(2)}$は、値が小さいほどロスが大きいことを示すものである。

【0295】そして、多元接続システムにおいては、端末300_0，300_1，300_2のそれぞれによって上述した受信装置60と同様の処理を行うことにより、3つの情報ビット系列$\{b^{(0)},b^{(1)},b^{(2)}\}$のそれぞれに対する事後確率情報$P(b^{(0)}|y)$，$P(b^{(1)}|y)$，$P(b^{(2)}|y)$を硬判定した情報ビット系列$\{b^{(0)},b^{(1)},b^{(2)}\}$を得る。

【0296】このとき、端末300_2は、受信値$y^{(2)}$に基づいて少なくとも情報ビット系列$b^{(2)}$を復号して得ればよいことから、上述したように、少なくとも上述した1つの復号器62_2に相当する復号器302_2を有していればよい。また、端末300_1は、受信値$y^{(1)}$に基づいて少なくとも情報ビット系列$b^{(1)}$，$b^{(2)}$を復号して得た後、必要な情報ビット系列$b^{(1)}$のみを出力することになることから、上述したように、少なくとも上述した2つの復号器62_1，62_2に相当する復号器302_1を有する必要がある。さらに、端末300_0は、受信値$y^{(0)}$に基づいて情報ビット系列$b^{(0)}$，$b^{(1)}$，$b^{(2)}$の全てを復号して得た後、必要な情報ビット系列$b^{(0)}$のみを出力することになることから、上述したように、上述した3つの復号器62_0，62_1，62_2の全てに相当する復号器302_0を有する必要がある。

【0297】このように、多元接続システムは、基地局200として上述した送信装置10を適用し、端末300_0，300_1，300_2として上述した受信装置60

を適用することにより、下りリンクを構成することができる。

【0298】なお、多元接続システムにおいては、基地局200と、各端末300_0，300_1，300_2との間での伝送ロスに差異があることが想定されることから、伝送ロスの大きなユーザに対しては、上述したように、高次の情報ビット系列として割り当てるのが有効である。また、多元接続システムにおいては、伝送ロスの大きなユーザのデータほど相対的に信号対雑音比S/Nが低い値で受信されることから、上述したように、伝送ロスの大きなユーザに対して割り当てた高次の符号に対する定数a(1)として、マージンを多くとるように決定してもよい。同図においては、高次から順に符号化系列x(2)，x(1)，x(0)が割り当てられているが、これは、対応する伝送ロスL(2)，L(1)，L(0)の値の関係が、L(2)<L(1)<L(0)となっていることに相当する。

【0299】一方、このようなドりリンクに対する多元接続システムにおける上りリンクは、同じく3人のユーザからのデータからなる3つの情報ビット系列{b(0)，b(1)，b(2)}を多元接続するものとすると、図27に示すように構成される。すなわち、多元接続システムは、上述した受信装置60が適用された3つの端末400_0，400_1，400_2と、情報ビット系列{b(0)，b(1)，b(2)}を出力する上述した送信装置10が適用された基地局500とを備える。

【0300】端末400_0，400_1，400_2は、それぞれ、上述した変換器11_0，11_1，11_2に相当する変換器401_0，401_1，401_2と、上述した乗算器12_0，12_1，12_2に相当する乗算器402_0，402_1，402_2と、上述した送信部14に相当する送信部403_0，403_1，403_2とを有する。

【0301】一方、基地局500は、上述した受信部61に相当する受信部501と、上述した3つの復号器62_0，62_1，62_2に相当する復号器502とを有する。

【0302】このような多元接続システムにおいては、端末400_0，400_1，400_2のそれぞれによって上述した送信装置10と同様の処理を行うことにより、各情報ビット系列{b(0)，b(1)，b(2)}をそれぞれ符号化して送信信号g'(0)，g'(1)，g'(2)に変換し、これらの送信信号g'(0)，g'(1)，g'(2)を外部へと送信する。ここで、定数G(0)，G(1)，G(2)は、それぞれ、通信路上で発生する3人のユーザのそれぞれに対応する伝送ロスL(0)，L(1)，L(2)に基づいて送信パワーが制御され、G(0)×L(0)，G(1)×L(1)，G(2)×L(2)のそれぞれが上述したように適切な振幅となるように設定される。

【0303】多元接続システムにおいては、これらの送信信号g'(0)，g'(1)，g'(2)のそれぞれが、3人のユーザのそれぞれに対応する伝送ロスL(0)，L(1)，L(2)を有する通信路を介して互いに多重化されて伝送されることにより、互いに多重化された送信信号g'(0)，g'(1)，g'(2)に対して雑音nが加算され、この信号が受信信号y'として基地局500によって受信される。なお、伝送ロスL(0)，L(1)，L(2)は、上述したように、値が小さいほどロスが大きいことを示すものである。

【0304】そして、多元接続システムにおいては、基地局500によって上述した受信装置60と同様の処理を行うことにより、3つの情報ビット系列{b(0)，b(1)，b(2)}のそれぞれに対する事後確率情報P(b(0)|y)，P(b(1)|y)，P(b(2)|y)を硬判定した情報ビット系列{b(0)，b(1)，b(2)}を得る。

【0305】このように、多元接続システムは、端末400_0，400_1，400_2として上述した送信装置10を適用し、基地局500として上述した受信装置60を適用することにより、上りリンクを構成することができる。

【0306】なお、多元接続システムにおいては、各端末400_0，400_1，400_2と、基地局500との間での伝送ロスに差異があることが想定されることから、伝送ロスの小さなユーザに対しては、上述したように、高次の情報ビット系列として割り当てるのが有効である。また、多元接続システムにおいては、伝送ロスの大きなユーザのデータほど相対的に送信パワーに余裕があることから、上述したように、伝送ロスの小さなユーザに対して割り当てた高次の符号に対する定数G(i)として、マージンを多くとるように決定してもよい。同図においては、高次から順に符号化系列x(2)，x(1)，x(0)が割り当てられているが、これは、対応する伝送ロスL(2)，L(1)，L(0)の値の関係が、L(2)>L(1)>L(0)となっていることに相当する。

【0307】以上説明したように、多元接続システムにおいては、上述したデータ送受信システムにおける各レベルの情報ビット系列b(i)を多元接続における各端末の情報系列とすることにより、限られたリソースでより多くのユーザを収容することができる。また、多元接続システムにおいては、各ユーザの伝送ロスに応じた適切な多重化を行うことができることから、リソースを有効利用することができる。

【0308】なお、本発明は、上述した実施の形態に限定されるものではない。例えば、上述した実施の形態では、移動体通信における基地局と端末との間でデータを送受信する際の多元接続システムについて説明したが、本発明は、いわゆる無線LAN(Local Area Network)

における制御局又はアクセスポイントと端末との間でデータを送受信する際の多元接続システムといった各種システムにも適用することができる。すなわち、上述した基地局は、移動体通信における基地局のみならず、無線ＬＡＮやその他各種の通信システムにおいて同等の機能を有する装置も含む概念である。

【０３０９】また、上述した実施の形態では、送信装置における変換器による符号化としてターボ符号を例として説明したが、本発明は、例えばリード・ソロモン符号（Reed-Solomon code）やＢＣＨ符号（Bose-Chaudhuri-Hocquenghem code）といったいかなる符号でも適用することができる。

【０３１０】さらに、上述した実施の形態では、送信装置における変換器による符号化としてＢＰＳＫ変調方式を行うものとして説明したが、本発明は、例えば４相位相変調方式（Quadrature Phase Shift Keying：ＱＰＳＫ）といった変調方式でも適用することができる。なお、ＱＰＳＫ変調方式を適用した場合であっても、加算符号化系列ｇにおける信号点は、上述したように、信号対雑音比Ｓ／Ｎに依存して配置され、例えば図２８に示すように、非等間隔に配置される。

【０３１１】さらにまた、上述した実施の形態では、送信装置における１つの変換器に対して、１系列の情報ビット系列ｂ（１）が入力されるものとして説明したが、この情報ビット系列ｂ（１）は、例えば、１系列の２０００ビットの情報ビットではなく、２系列の１０００ビットの情報ビットとして構成されてもよい。すなわち、本発明は、各変換器から出力される各符号化系列ｘ（１）が全てＭ個の数値からなるものであればよい。

【０３１２】また、上述した実施の形態では、受信装置における復号器による復号としてＭＡＰ復号を例として説明したが、本発明は、例えばビタビ復号（Viterbi decoding）等も適用することができる。なお、ビタビ復号を適用した場合には、復号器は、尤度を入力することになるが、復号結果としては軟出力の事後確率情報を出力することはない。そのため、この場合、復号器は、復号結果が確率が"１"で正しいものであるとして復号することになる。

【０３１３】このように、本発明は、その趣旨を逸脱しない範囲で適宜変更が可能であることはいうまでもない。

【０３１４】

【発明の効果】以上詳細に説明したように、本発明にかかる多元接続システムは、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける基地局は、所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する第１の変換手段と、この第１の変換手段によって変換されて得られた第１の符号化系列に対して第１の定数を乗算する第１の乗算手段と、所定のビットからなる第２のユーザの第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの第２の変換手段と、この第２の変換手段によって変換されて得られた第２の符号化系列に対して第２の定数を乗算する少なくとも１つの第２の乗算手段と、第１の乗算手段によって乗算されて得られた第１の定数倍符号化系列と、第２の乗算手段によって乗算されて得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する加算手段と、加算符号化系列を送信信号として送信する送信手段とを備え、基地局によって伝送された加算符号化系列に対して所定の雑音が加算された受信信号を受信する端末は、それぞれ、受信信号を入力する受信手段と、この受信手段から供給された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備える。

【０３１５】したがって、本発明にかかる多元接続システムは、第１の符号化系列に対して第１の乗算手段によって第１の定数が乗算された第１の定数倍符号化系列と、第２の符号化系列に対して第２の乗算手段によって第２の定数が乗算された第２の定数倍符号化系列とを加算手段によって加算して加算符号化系列を生成して基地局から伝送し、この加算符号化系列に対して所定の雑音が加算された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を各端末における復号手段によって行うことにより、符号設計の自由度を格段に向上させた高い性能での符号化を容易に実現し、限られたリソースを有効利用してより多くのユーザを収容することができる。

【０３１６】また、本発明にかかる多元接続方法は、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、各ユーザの情報を所定の形式に変換して伝送する下りリンクにおける基地局によって所定のビットからなる第１のユーザの第１の情報ビット系列をＭ個の数値からなる第１の符号化系列に変換する工程と、基地局によって第１の情報ビット系列を変換して得られた第１の符号化系列に対して第１の定数を乗算する工程と、基地局によって所定のビットからなる第２の情報ビット系列をＭ個の数値からなる第２の符号化系列に変換する少なくとも１つの工程と、基地局によって第２の情報ビット系列を変換して得られた第２の符号化系列に対して第２の定数を乗算する少なくとも１つの工程と、基地局によって第１の符号化系列に対して第１の定数を乗算して得られた第１の定数倍符号化系列と、第２の符号化系列に対して第２の定数を乗算して得られた第２の定数倍符号化系列とを要素毎に加算して加算符号化系列を生成する工程と、基地局に

よって加算符号化系列を送信信号として送信する工程と、基地局によって伝送された加算符号化系列に対して所定の雑音が加算された受信信号を受信する端末によって受信信号を入力する工程と、端末によって受信された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備える。

【０３１７】したがって、本発明にかかる多元接続方法は、第１の符号化系列に対して第１の定数が乗算された第１の定数倍符号化系列と、第２の符号化系列に対して第２の定数が乗算された第２の定数倍符号化系列とを基地局によって加算して加算符号化系列を生成して伝送し、この加算符号化系列に対して所定の雑音が加算された受信値に基づいて、第１の情報ビット系列又は第２の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を各端末によって行うことにより、符号設計の自由度を格段に向上させた高い性能での符号化を容易に実現し、限られたリソースを有効利用してより多くのユーザを収容することが可能となる。

【０３１８】さらに、本発明にかかる多元接続システムは、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続システムであって、情報を所定の形式に変換して伝送する上りリンクにおける端末は、それぞれ、所定のビットからなる情報ビット系列をＭ個の数値からなる符号化系列に変換する変換手段と、この変換手段によって変換されて得られた符号化系列に対して所定の定数を乗算する乗算手段とを備え、複数の端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する基地局は、受信信号を入力する受信手段と、この受信手段から供給された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する復号手段とを備える。

【０３１９】したがって、本発明にかかる多元接続システムは、符号化系列に対して乗算手段によって所定の定数が乗算された定数倍符号化系列が各端末から伝送され、これらの定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を基地局における復号手段によって行うことにより、符号設計の自由度を格段に向上させた高い性能での符号化を容易に実現し、限られたリソースを有効利用してより多くのユーザを収容することができる。

【０３２０】さらにまた、本発明にかかる多元接続方法は、基地局と複数の端末との間で多元接続を行うことによってデータの送受信を行う多元接続方法であって、情報を所定の形式に変換して伝送する上りリンクにおける端末のそれぞれによって所定のビットからなる情報ビット系列をＭ個の数値からなる符号化系列に変換する工程と、端末のそれぞれによって情報ビット系列を変換して得られた符号化系列に対して所定の定数を乗算する工程と、複数の端末のそれぞれによって伝送された定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信信号を受信する基地局によって受信信号を入力する工程と、基地局によって受信された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を行い、必要な情報ビット系列を出力する工程とを備える。

【０３２１】したがって、本発明にかかる多元接続方法は、符号化系列に対して所定の定数が乗算された定数倍符号化系列が各端末から伝送され、これらの定数倍符号化系列が多重化された信号に対して所定の雑音が加算された受信値に基づいて、端末のそれぞれに対応する複数の情報ビット系列のうち、少なくとも１つの情報ビット系列に対する復号を基地局によって行うことにより、符号設計の自由度を格段に向上させた高い性能での符号化を容易に実現し、限られたリソースを有効利用してより多くのユーザを収容することが可能となる。

【図面の簡単な説明】

【図１】本発明における符号化の定義を説明するための図である。

【図２】本発明の実施の形態として示す多元接続システムに適用されるデータ送受信システムにおける送信装置が行う符号化の過程で得られる１つの定数倍符号化系列における信号点の配置を説明する図である。

【図３】同データ送受信システムにおける送信装置が行う符号化の過程で得られる２つの定数倍符号化系列を加算することによって得られる加算符号化系列における信号点の配置を説明する図である。

【図４】同データ送受信システムにおける送信装置が行う符号化の過程で得られる３つの定数倍符号化系列を加算することによって得られる加算符号化系列における信号点の配置を説明する図である。

【図５】通常の４ＡＳＫ変調方式による信号点の配置を説明する図である。

【図６】信号対雑音比を"１２［ｄＢ］"としたときの信号点の配置による情報量を説明する図である。

【図７】信号対雑音比を"１．９６［ｄＢ］"としたときの信号点の配置による情報量を説明する図である。

【図８】図７における情報量が１ビット近傍である領域を拡大した場合の信号点の配置による情報量を説明する図である。

【図９】同データ送受信システムにおける送信装置の現実的な具体的構成を説明するブロック図である。

【図１０】同送信装置が有する変換器の具体的構成を説

明するブロック図である。

【図11】同変換器が有する要素符号化器の具体的構成を説明するブロック図である。

【図12】本発明における復号を実現する一般的な受信装置の構成を説明するブロック図である。

【図13】同データ送受信システムにおける受信装置の現実的な具体的構成を説明するブロック図である。

【図14】通常のターボ復号器の構成を説明するブロック図である。

【図15】同受信装置が有する復号器の具体的構成を説明するブロック図である。

【図16】同受信装置が有するチャネル推定部の構成を説明するブロック図であって、情報ビット系列に対する事後確率情報を用いて硬判定値系列を求めるチャネル推定部の構成を説明するブロック図である。

【図17】同受信装置が有するチャネル推定部の他の構成を説明するブロック図であって、符号化系列に対する事後確率情報を用いて硬判定値系列を求めるチャネル推定部の構成を説明するブロック図である。

【図18】適応的符号化として、受信側が通信路の状態を同定するデータ送受信システムの概略構成を説明するブロック図である。

【図19】適応的符号化として、送信側が通信路の状態を同定するデータ送受信システムの概略構成を説明するブロック図であって、符号化のパラメータの変更の予告を事前に行うデータ送受信システムの概略構成を説明するブロック図である。

【図20】適応的符号化として、送信側が通信路の状態を同定するデータ送受信システムの概略構成を説明するブロック図であって、現時刻の符号化のパラメータを現時刻のデータに含ませて伝送するデータ送受信システムの概略構成を説明するブロック図である。

【図21】AWGNチャネルにおける特性を求めるシミュレーションで用いた符号構成を説明する図である。

【図22】同シミュレーションで求めたAWGNチャネルにおける特性を示す特性曲線を説明する図である。

【図23】フリー・インターリーブド・レイリーチャネルにおける特性を求めるシミュレーションで用いた符号構成を説明する図である。

【図24】同シミュレーションで求めたフリー・インターリーブド・レイリーチャネルにおける特性を示す特性曲線を説明する図である。

【図25】レイリーチャネル用の符号化のパラメータを用いてスタティックチャネルにおける特性を予測した結果の符号構成を説明する図である。

【図26】同多元接続システムにおける下りリンクの構成を説明するブロック図である。

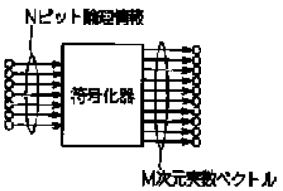
【図27】同多元接続システムにおける上りリンクの構成を説明するブロック図である。

【図28】同送信装置における変換器による符号化としてQPSK変調方式を適用した場合に得られる加算符号化系列における信号点の配置を説明する図である。

【符号の説明】

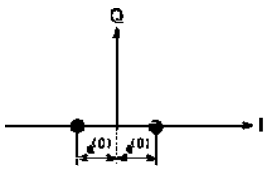
10，10_1，10_2 送信装置、 11_0，11_1，11_2，11_3，52_{L-1}，52_{L-2}，・・・，51_1，92，201_0，201_1，201_2，401_0，401_1，401_2 変換器、 12_0，12_1，12_2，53_{L-1}，53_{L-2}，・・・，53_1，202_0，202_1，202_2，402_0，402_1，402_2 乗算器、 13_0，13_1，203 加算器、 14，204，403_0，403_1，403_2 送信部、 21_0，21_1 要素符号化器、 22，83，85 インターリーバ、 23，88 パンクチャ器、 24，89 チャネル・インターリーバ、 25 BPSKマッピング器、 50，60，60_1，60_2 受信装置、 51_{L-1}，51_{L-2}，・・・，51_0，62_0，62_1，62_2，302_0，302_1，302_2，502 復号器、 54_{L-1}，54_{L-2}，・・・，54_1 差分器、 61，301_0，301_1，301_2，501 受信部、 63_0，63_1，63_2 尤度算出部、81 チャネル・デインターリーバ、 82 デパンクチャ器、 84，86MAP復号器、 87 デインターリーバ、 90，100 チャネル推定部、 91，101 推定器、 93，103 相関算出器、 102 符号化系列推定器、 150_1，150_2 コントローラ、 200，500 基地局、300_0，300_1，300_2，400_0，400_1，400_2 端末

【図1】



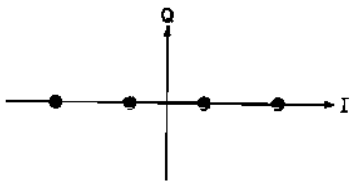
符号化の定義の説明図

【図2】



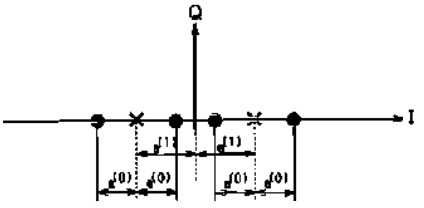
2倍符号化系列における信号点の配置の説明図

【図5】



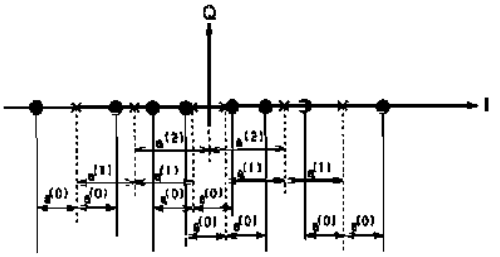
4ASK変調方式による信号点の配置の説明図

【図3】



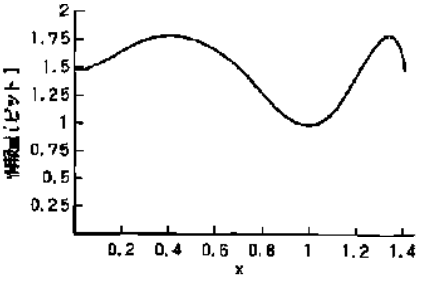
加算符号化系列における信号点の配置の説明図

【図4】



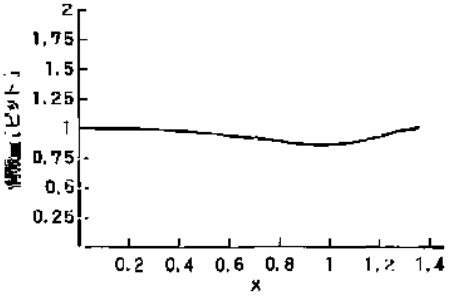
加算符号化系列における信号点の配置の説明図

【図6】



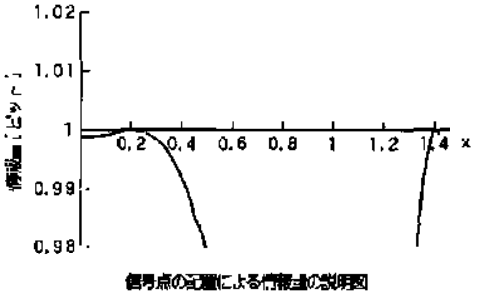
信号点の配置による情報量の説明図

【図7】



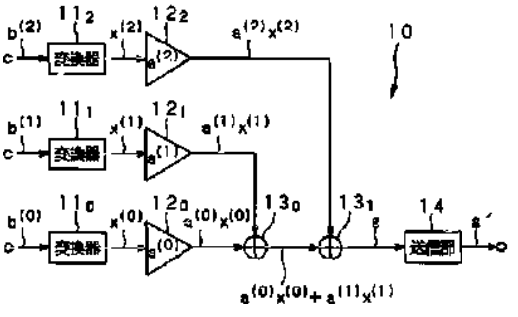
信号点の配置による情報量の説明図

【図8】



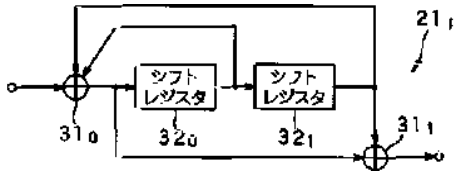
信号点の配置による情報量の説明図

【図9】



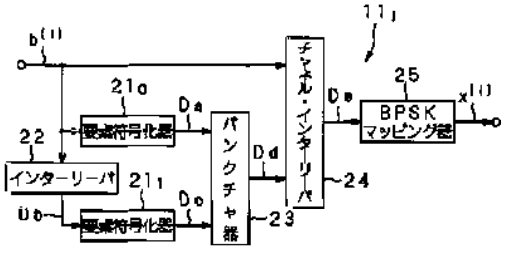
送信装置の構成ブロック図

【図11】



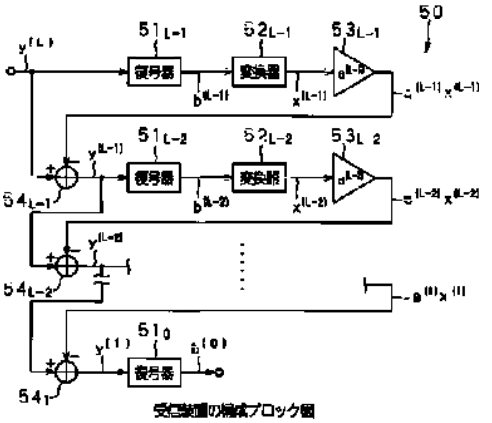
要素符号化器の構成ブロック図

【図10】



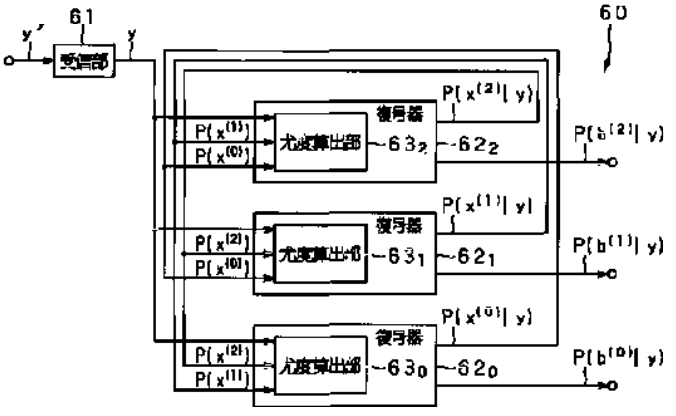
変換器の構成ブロック図

【図12】



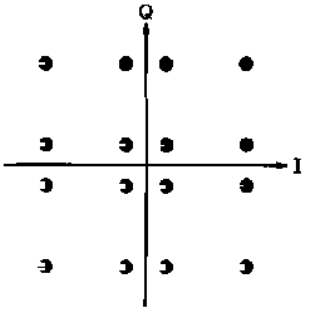
受信装置の構成ブロック図

【図13】



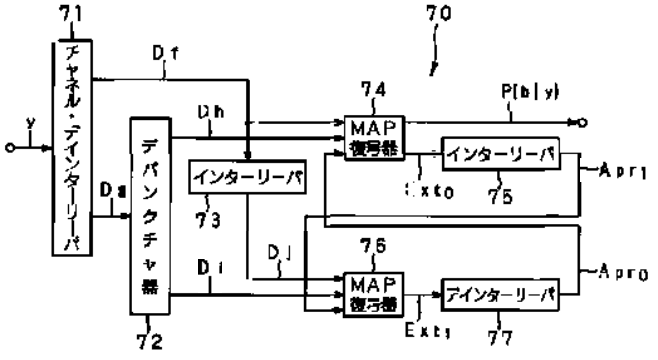
受信装置の構成ブロック図

【図28】



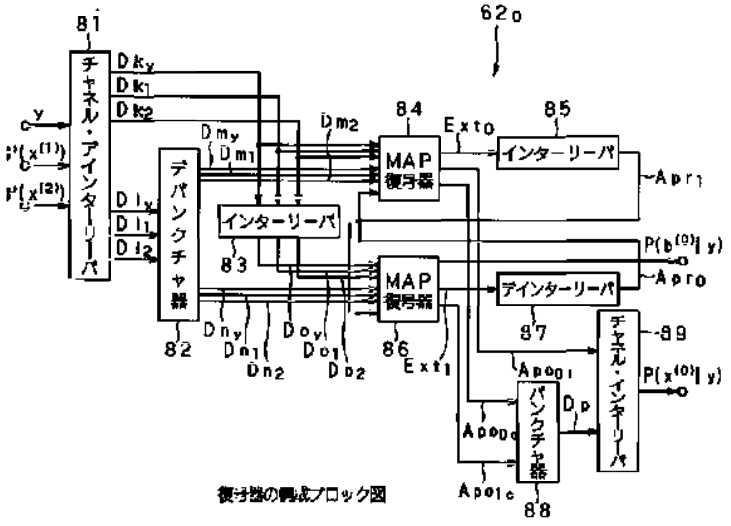
拡張符号化系列における信号点の配置の説明図

【図14】



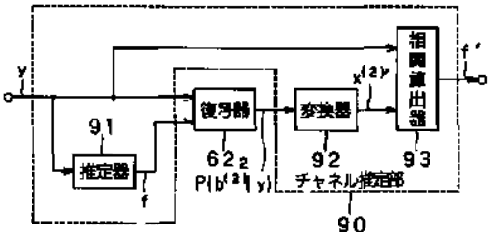
通常のターボ復号器の構成ブロック図

【図15】



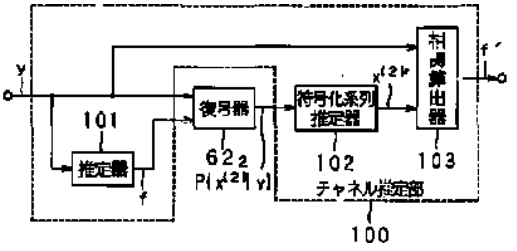
復号器の構成ブロック図

【図16】



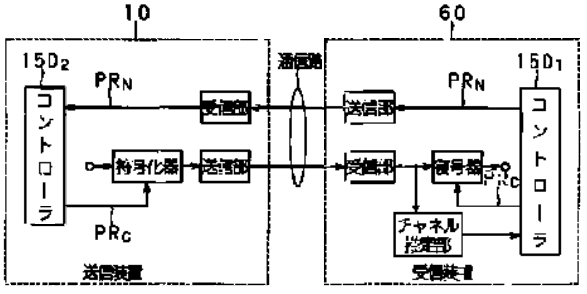
チャネル推定部の構成ブロック図

【図17】



チャネル推定部の構成ブロック図

【図18】



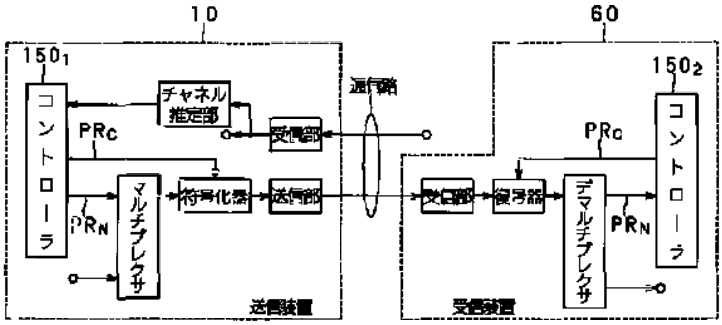
データ送受信システムの構成ブロック図

【図21】

各段における所要 $E_b/(n_0+2\nu)$	各段の定数 (各段の符号の規則)	その段まで用いたときの伝送率	その段まで用いたときの期待される平均所要 E_b/n_0
$\xi^{(0)}$=0.8[dB]	$a^{(0)}$=0.775	C=1/2	ξ_{ave}'=0.800[dB]
$\xi^{(1)}$=0.7[dB]	$a^{(1)}$=1.137	C=2/2	ξ_{ave}'=2.773[dB]
$\xi^{(2)}$=0.6[dB]	$a^{(2)}$=1.658	C=3/2	ξ_{ave}'=4.908[dB]
$\xi^{(3)}$=0.8[dB]	$a^{(3)}$=2.430	C=4/2	ξ_{ave}'=7.221[dB]

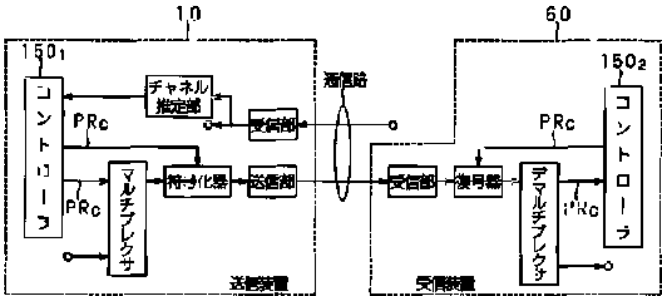
符号構成の説明図

【図19】



データ送受信システムの構成ブロック図

【図20】



データ送受信システムの構成ブロック図

【図23】

各段における所要 $E_b/(n_0+2\nu)$	各段の定数（各段の符号の振幅）	その段まで用いたときの伝送率	その段まで用いたときの期待される平均所要 E_b/n_0
$\xi^{(0)}$≒2.7[dB]	$a^{(0)}$=0.965	C=1/2	ξ_{ave}'=2.700[dB]
$\xi^{(1)}$≒1.9[dB]	$a^{(1)}$=1.489	C=2/2	ξ_{ave}'=4.979[dB]
$\xi^{(2)}$≒1.1[dB]	$a^{(2)}$=2.167	C=3/2	ξ_{ave}'=7.185[dB]
$\xi^{(3)}$≒1.0[dB]	$a^{(3)}$=3.242	C=4/2	ξ_{ave}'=9.626[dB]

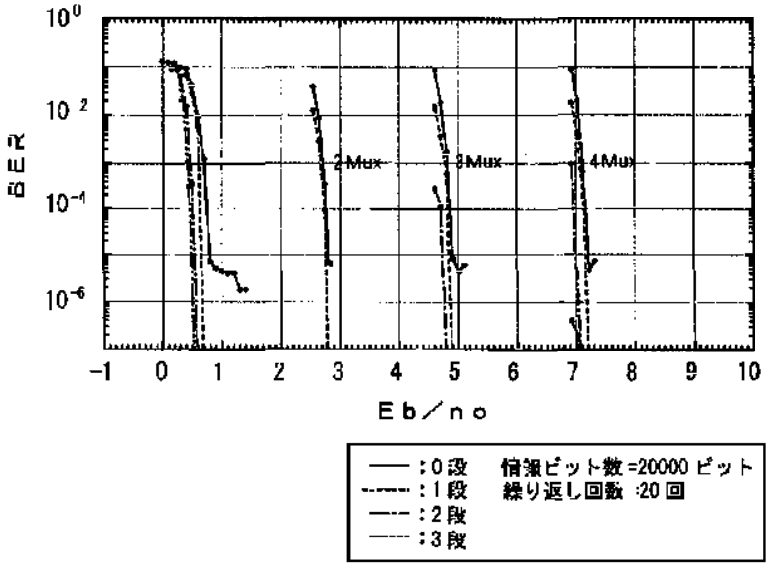
符号構成の説明図

【図25】

その段まで用いたときの伝送率	レイリーチャネルでその段まで用いたときの期待される平均所要 E_b/n_0	レイリーチャネル用のパラメータのままスタティックチャネルで最低限期待される平均所要 E_b/n_0	スタティックチャネルに最適化したとき期待される平均所要 E_b/n_0
C=1/2	ξ_{ave}'=2.700[dB]	ξ_{ave}'=0.800[dB]	ξ_{ave}'=0.800[dB]
C=2/2	ξ_{ave}'=4.979[dB]	ξ_{ave}'=3.079[dB]	ξ_{ave}'=2.773[dB]
C=3/2	ξ_{ave}'=7.185[dB]	ξ_{ave}'=5.285[dB]	ξ_{ave}'=4.908[dB]
C=4/2	ξ_{ave}'=9.626[dB]	ξ_{ave}'=7.726[dB]	ξ_{ave}'=7.221[dB]

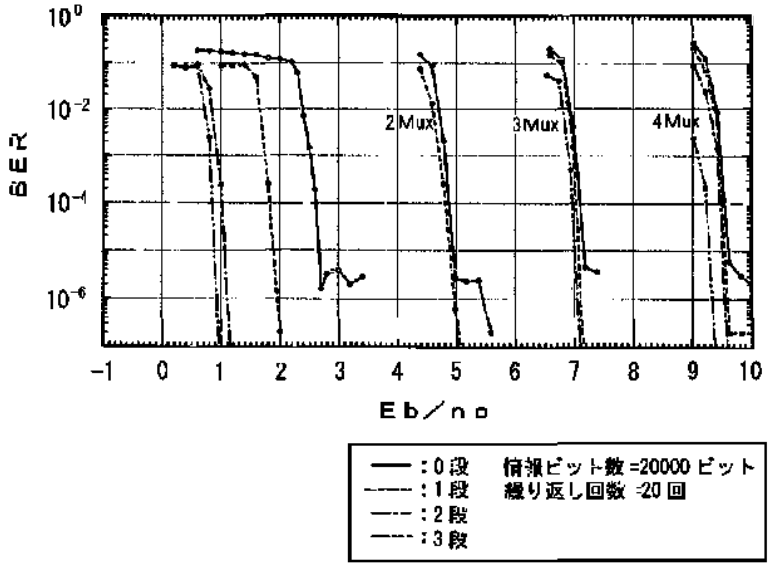
符号構成の説明図

【図22】



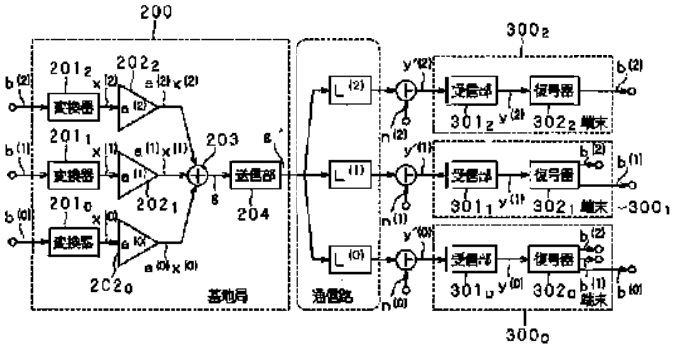
特性曲線の説明図

【図24】



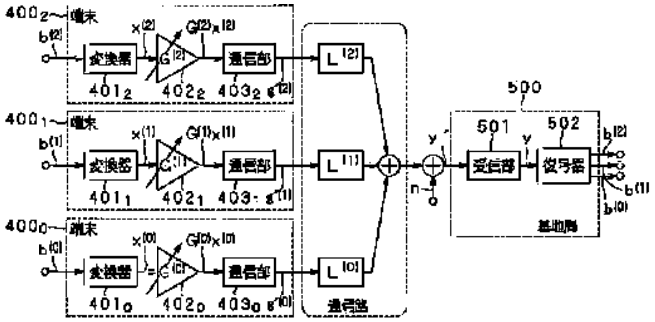
特性曲線の説明図

【図26】



多元接続システムの構成ブロック図

【図27】



多元接続システムの構成ブロック図

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(71) Applicant: **NEC CORPORATION**
Tokyo (JP)

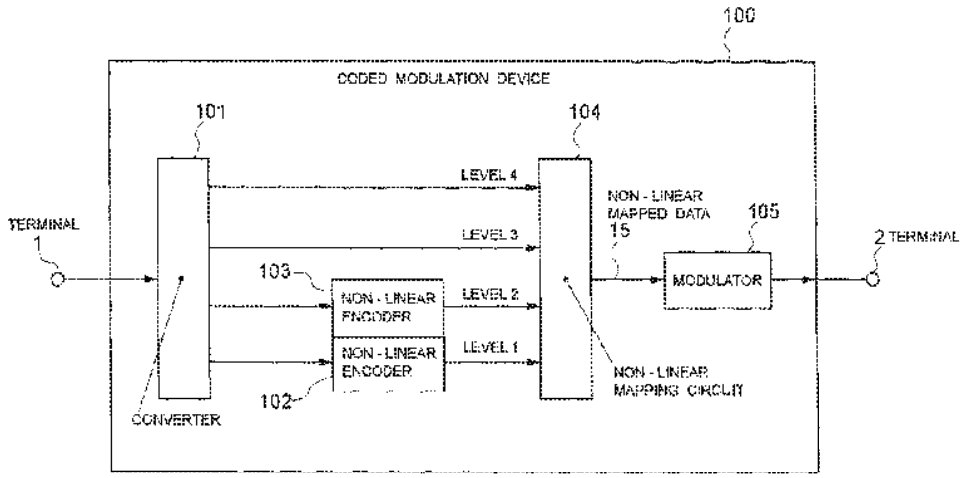
(72) Inventor: **Mitsutani, Naoki**
Tokyo (JP)

(74) Representative: **Betten & Resch**
Patentanwälte,
Theatinerstrasse 8
80333 München (DE)

(54) Coded modulation device and method

(57) The present invention realizes a high coding gain employing a non-linear coding and a non-linear mapping. A converter converts one row of data that is inputted from a terminal into plural rows of data. The non-linear encoders non-linearly encode lower level two rows of data in plural rows of converted data, and outputs them as data of level 1 and level 2 to a non-linear mapping circuit. The non-linear mapping circuit maps the data non-linearly encoded by the non-linear encoders and the uncoded data so that each codeword distance may be optimal. A modulator modulates a carrier wave with two-dimensional data that are mapped by the non-linear mapping circuit.

FIG. 1



EP 1 578 021 A1

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Description

BACKGROUND OF THE INVENTION

Field of the Invention

[0001] The present invention relates to a coded modulation device and method, and more particularly to a coded modulation device and method for increasing the coding gain by using the non-linear code and non-linear mapping.

Description of the Prior Art

[0002] In recent years, in a digital microwave communications system, a coded modulation system with higher error-correcting ability is employed to achieve the higher transmission quality

[0003] The coded modulation system integrates the mapping with signal point set partitioning and an error correction coding technique, and has an excellent error-correcting ability.

[0004] As the coding scheme, a Trellis Coded Modulation (TCM) system and a Multi Level Coded Modulation (MLCM) system are generally known.

[0005] In the Trellis coded modulation (TCM) system, a two-dimensional vector representation corresponds one-to-one to each signal point of a two-dimensional or multi-dimensional modulation system having 2^m signal points in two dimensions. And the Trellis coded modulation system gives one two-dimensional or multi-dimensional signal point by a combination of one convolutional encoder output and uncoded bits.

[0006] On the other hand, the multi level coded modulation system uses a plurality of block encoders having a different error correcting ability, each encoder coding data at each level corresponding to each component (bit position) of a two-dimensional vector, independently of each other. Each of codes for use with the multi level coded modulation method are referred to as a component code. For each component code, the required error correcting ability is decided based on the minimal signal point-to-point distance at the level to which it is applied (e.g., refer to Patent Document 1).

[0007] FIG. 6 is a block diagram showing a conventional coded modulation device.

[0008] Particularly, a case of the Trellis coded modulation (TCM) system is shown.

[0009] Referring to FIG. 6, in a coded modulation device 200, one row of data inputted from a terminal 3 is converted into three rows by a converter 201. Among them, one row is encoded into two rows of signals by a convolutional encoder 202. The two rows of signals are supplied as signals of level 1 and level 2 to a mapping circuit 203. Also, other two rows of signals outputted from the converter 201 are supplied as uncoded signals of levels to the mapping circuit 203.

[0010] In the mapping circuit 203, the coordinates of a signal point corresponding to a signal input are outputted with a signal point arrangement based on a signal point set partitioning. This signal point coordinate output is inputted into a modulator 204 for 16 QAM modulation, and outputted from a terminal 4.

[0011] This TCM uses a single convolutional code, with a very small degree of freedom in setting a coding ratio. Also, since the convolutional code is a kind of linear code, there is some limit on the coding gain of TCM.

[0012] FIG. 7 is a block diagram showing another conventional coded modulation device.

[0013] Particularly a case of the multi-level coded modulation (MLCM) method is shown.

[0014] Referring to FIG. 7, in a coded modulation device 300, one row of data inputted from a terminal 5 is converted into four rows by a converter 301. Among them, two rows of data at the lower level are encoded by a linear encoder 302 and a liner encoder 303, and outputted as signals of level 1 and level 2. The linearly coded signals of level 1 and level 2 and the uncoded signals of level 3 and level 4 are supplied to a mapping circuit 304.

[0015] In the mapping circuit 304, the coordinates of a signal point corresponding to a signal input are outputted with a signal point arrangement based on a signal point set partitioning. This signal point coordinate output is inputted into a modulator 305 for 16 QAM modulation, and outputted from a terminal 6.

[0016] This MLCM employs a different code at each level in the signal point set partitioning, with a larger degree of freedom in setting the coding ratio than the TCM. However, since the code for use at each level is a linear code, such as Hamming code, BCH (Bose Chaudhuri Hocquenghem) code, or RS (Reed Solomon) code, there is some limit on the coding gain.

[0017] Also, employing the convolutional code, the error correcting ability of coded modulation may be enhanced under the burst error environment such as fading (e. g. , refer to Patent Document 2).

[Patent Document 1] Japanese Patent Publication No. 2-291743 (pages 2 to 4)
[Patent Document 2] Japanese Patent Publication No. 9-93295 (pages 2 and 3, FIG. 2)

[0018] For above-mentioned conventional coded modulation device and method, the Trellis coded modulation (TCM) and the multi-level coded modulation (MLCM) are provided.

[0019] Since this TCM employs a single convolutional code, the degree of freedom in setting the coding ratio is very small. Since the convolutional code is a kind of linear code, there is some limit on the coding gain of the TCM.

[0020] Also, since the MLCM employs a different code at each level in the signal point set partitioning, there is a larger degree of freedom in setting the coding ratio than the TCM. However, since the code for use at each

level is a linear code such as Hamming code, BCH code or RS code, there is some limit on the coding gain.

BRIEF SUMMARY OF THE INVENTION

[0021] It is an object of the present invention to provide a coded modulation device and method for converting the transmission data into plural rows, making the non-linear coding for the lower level data, and making the non-linear mapping of mapping the coded data to signal points having a codeword distance of unequal interval on a two-dimensional plane to achieve a higher coding gain.

[0022] The present invention provides a first coded modulation method for converting the transmission data into plural rows, non-linearly encoding a part in plural rows of converted data, and non-linearly mapping the non-linearly coded data to signal points having a codeword distance of unequal interval on a two-dimensional plane.

[0023] The invention provides a second coded modulation method for converting the transmission data into plural rows, and non-linearly mapping the lower level data being non-linearly coded in plural rows of converted data, and the upper level data in the plural rows of converted data to signal points having a codeword distance of unequal interval on a two-dimensional plane.

[0024] The invention provides a first coded modulation device comprising a data converter for converting the transmission data into plural rows, a non-linear encoder for non-linearly encoding the lower level data in plural rows of data that are outputted from the data converter, a non-linear mapping part for non-linearly mapping the non-linearly coded data to signal points having a codeword distance of unequal interval on a two-dimensional plane, and a modulator for modulating a carrier wave with data mapped by the non-linear mapping part.

[0025] The invention provides a second coded modulation device comprising a data converter for converting the transmission data into plural rows, a non-linear encoder for non-linearly encoding the lower level data in plural rows of data that are outputted from the data converter, a non-linear mapping part for non-linearly mapping the non-linearly coded data and the upper level data in the plural rows of converted data to signal points having a codeword distance of unequal interval on a two-dimensional plane, and a modulator for modulating a carrier wave with data mapped by the non-linear mapping part.

[0026] The invention provides a third coded modulation device comprising a converter, a first non-linear encoder and a second non-linear encoder,a non-linear mapping circuit,and a modulator, wherein the converter converts one row of data that is inputted from a first terminal into plural rows of data, the first and second non-linear encoders non-linearly encode lower level two rows of data in plural rows of data converted by the converter, and output them as data of level 1 and level 2 to the non-linear mapping circuit, the non-linear mapping circuit non-linearly maps the data of level 1 and level 2 that are non-linearly encoded by the first and second non-linear encoders and the uncoded data of level 3 and level 4 to signal points having a codeword distance of unequal interval on a two-dimensional plane, and the modulator modulates a carrier wave with two-dimensional data that are non-linearly mapped by the non-linear mapping circuit.

[0027] The invention provides a third coded modulation method comprising converting one row of data that is inputted from a first terminal into plural rows of data, non-linearly encoding lower level two rows of data in plural rows of converted data, and outputting them as data of level 1 and level 2, non-linearly mapping the data of level 1 and level 2 that are non-linearly encoded and the uncoded data of level 3 and level 4 to signal points having a codeword distance of unequal interval on a two-dimensional plane, and modulating a carrier wave with two-dimensional data that are non-linearly mapped by the non-linear mapping.

[0028] The invention provides the fourth coding modulation method,

wherein the non-linear coding is generated by a non-linear equation of simultaneous multi-dimensional polynomials.

[0029] The invention provides the fifth coding modulation method,

wherein the non-linear coding is generated by a non-linear connection of generator polynomials.

[0030] The invention provides the fourth coding modulation device,

wherein the non-linear encoder is composed of a generation circuit that is generated by a non-linear equation of simultaneous multi-dimensional polynomials.

[0031] The invention provides the fifth coding modulation device,

wherein the non-linear encoder is generated by a non-linear connecting circuit of generator polynomials.

[0032] The coded modulation device and method of the invention makes the coded modulation employing the non-linear coding and non-linear mapping. Therefore, there is the effect that the higher coding gain is achieved because the codeword distance is secured larger than the coded modulation system employing the linear code.

BRIEF DESCRIPTION OF THE DRAWINGS

[0033]

FIG. 1 is a block diagram showing one embodiment of a coded modulation device and method according to one embodiment of the present invention;

FIG. 2 is a view showing an example of mapping of the invention;

FIG. 3 is a view showing one example of a signal

point set partitioning operation as shown in FIG. 2;
FIG. 4 is a view showing another example of mapping of the invention;
FIG. 5 is a view showing one example of a signal point set partitioning operation as shown in FIG. 4;
FIG. 6 is a block diagram showing a conventional coded modulation device; and
FIG. 7 is a block diagram showing another conventional coded modulation device.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

[0034] The preferred embodiments of the present invention will be described below with reference to the accompanying drawings.

[0035] FIG. 1 is a block diagram showing a coded modulation device and method according to one embodiment of the invention.

[0036] The coded modulation device 100 comprises a converter 101, a non-linear encoder 102, a non-linear encoder 103, a non-linear mapping circuit 104, and a modulator 105, as shown in FIG. 1.

[0037] The converter 101 is a serial-parallel converter that converts one row of data inputted from a terminal 1 into plural rows of data (parallel data). The non-linear encoder 102 and the non-linear encoder 103 non-linearly encode the lower level two rows in plural rows of data converted by the converter 101, and output them as data of level 1 and data of level 2 to the non-linear mapping circuit 104. The representation of "lower level" is idiomatic, in which the levels 1 to 4 are discriminated depending on whether to encode or not.

[0038] The non-linear mapping circuit 104 maps non-linearly coded data of level 1 and level 2 and the uncoded data of level 3 and level 4 that are outputs from the converter 101 to signal points on a two-dimensional plane so that each codeword distance may be unequal interval, and outputs them as non-linearly mapped data 15.

[0039] The modulator 105 modulates a carrier wave with two-dimensional, non-linearly mapped data 15 arranged by the non-linear mapping circuit 104.

[0040] Herein, the non-linear code for use in the non-linear encoders 102, 103 is any other code than the linear codes such as convolutional code, Hamming code, BCH code, and Read Solomon code. It is the code represented in the form of a non-linear equation of simultaneousmulti-dimensional polynomials,the code represented in the form of a non-linear connection of generator polynomials, or the code not obtained by the linear operation rules.

[0041] Herein, the non-linear coding or non-linear encoder is composed of a generation circuit that is generated by the non-linear equation of simultaneous multi-dimensional polynomials, or by the non-liner connecting circuit of generator polynomials

[0042] Also, the non-linear mapping made by the non-linear mapping circuit 104 of FIG. 1 does not arrange the signal points at the distance of equal interval between adjacent signal points on the two-dimensional plane or space, such as Phase Shift Keying (PSK) or quadrature amplitude modulation (QAM). It means such mapping that the signal points are non-linearly arranged, and the distance between adjacent signal points is not equal interval (unequal interval).

[0043] Referring to FIG. 1, the operation will be described below.

[0044] The transmission data is inputted from the terminal 1, and converted into plural rows (here four rows) by the converter 101. Among four rows of converted data, the lower two rows of data are only subjected to non-linear coding by the non-linear encoder 102 and the non-linear encoder 103. The non-linearly coded data is supplied as the data of level 1 and level 2 to the non-linear mapping circuit 104. Other data of level 3 and level 4 are not processed at all, and supplied to the non-linear mapping circuit 104.

[0045] FIG. 2 is a view showing an example of mapping of the invention. This signal point arrangement is involved in the quadrature amplitude modulation, in which the transverse axis represents Ich and the longitudinal axis represents Qch

[0046] The data of level 1 and level 2 inputted into the non-linear mapping circuit 104 and the data of level 3 and level 4 outputted from the converter 101 are non-linearly mapped onto the two-dimensional plane, as shown in FIG. 2.

[0047] Since four bits of digital signal of level 1 to level 4 are inputted into the non-linear mapping circuit 104, there are 2^4=16 signs (as indicated by black circle) of signal points. Each signal point of black circle represents four-bit sign data from (0000) to (0101) to (1010) to (1101) to (1111).

[0048] The signal point of FIG. 2 is mapped according to a signal point set partitioning as shown in FIG. 3

[0049] FIG. 3 is a view showing one example of the signal point set partitioning operation of FIG. 2.

[0050] For example, if the digital signal of level 1 to level 4 is data of "0010", it is mapped as the third figure <0010> from the left at the level 4 by the mapping of FIG. 3.

[0051] The data of level 1 in FIG. 1 is a binary signal of "1", "0" that is non-linearly encoded by the non-linear encoder 102, in which the first set of 16 signal points in FIG. 3 is partitioned into every eight points. Furthermore, it is partitioned into sets of every four signal points by a binary signal of "1", "0" at level 2 that is non-linearly encoded by the non-linear encoder 103 of FIG. 1. Then, it is partitioned into sets of every two signal points by a binary signal of "1", "0" at level 3 in FIG. 1, and partitioned into one signal point by a binary signal of "1", "0" at level 4

[0052] The signal point distance or sign distance between signal point sets partitioned at level 1 and level 2 is very closer than the signal point distance (sign dis-

tance) partitioned at level 3 and level 4. However, because the data at level 1 and level 2 are encoded by the non-linear coding, there is a very strong ability of identifying the signal point. That is, for identification of signal points with closer sign distance, the ability of identifying the signal point by non-linear coding is enhanced to reduce the identification error. Furthermore, the signal point set partitioned at level 3 and level 4 keeps a full signal point distance, whereby there is no danger that the signal point is mistaken.

[0053] FIG. 4 is a view showing another example of mapping of the invention.

[0054] Since four bits of digital signal of level 1 to level 4 are inputted into the non-linear mapping circuit 104, there are 2^4=16 signs (as indicated by black circle) of signal points. Each signal point of black circle indicates four-bit sign data from (0000) to (0101) to (1010) to (1101) to (1111).

[0055] A case of FIG. 4 is the non-linear mapping with a phase modulation system of PSK, in which signal points are arranged at unequal intervals on the same circumference of a circle. On the other hand, a case of FIG. 2 involves the quadrature amplitude modulation system, employing the phase and amplitude components, in which the signal points are arranged on the lattice intervals.

[0056] FIG. 5 is a view showing one example of the signal point set partitioning operation of FIG. 4.

[0057] For example, if the digital signal of level 1 to level 4 is data of "0010", it is mapped as the third figure <0010> from the left at level 4 by the mapping of FIG. 5.

[0058] The data of level 1 in FIG. 1 is a binary signal of "1", "0" that is non-linearly encoded by the non-linear encoder 102, in which the first set of 16 signal points in FIG. 5 is partitioned into every eight points. Furthermore, it is partitioned into sets of every four signal points by a binary signal of "1", "0" at level 2 that is non-linearly encoded by the non-linear encoder 103 of FIG. 1. Then, it is partitioned into sets of every two signal points by a binary signal of "1", "0" at level 3 in FIG. 1, and partitioned into one signal point by a binary signal of "1", "0" at level 4.

[0059] The non-linear mapped data 15 that is non-linearly mapped on the two-dimensional plane by the non-linear mapping circuit 104 is sent to the modulator 105. The modulator 106 modulates the carrier wave with the non-linearly mapped data 15 and outputs the modulated output data from the terminal 2.

[0060] As described above, the present invention enhances the coding gain, employing the non-linear code and the non-linear mapping.

[0061] That is, one row of input data is converted into plural rows by the converter 101. The lower level data in plural rows of converted data are non-linearly encoded by the non-linear encoders 102 and 103, and outputted to the non-linear mapping circuit 104. The non-linearly coded data are mapped to signal points on the two-dimensional plane by the non-linear mapping circuit 104 so that each codeword distance may be unequal interval. The two-dimensional data arranged by the non-linear mapping circuit 104 are subjected to modulation of carrier wave by the modulator 105.

[0062] The non-linearly coded data are set in the optimal two-dimensional arrangement by the non-linear mapping, and subjected to modulation of carrier wave, so that the coding gain becomes very high. Therefore, data can be transmitted without error even in the transmission path with bad line quality such as radio path.

[0063] Owing to high degree of freedom of mapping, the invention is beneficial for the multi-value PSK system greatly affected by amplitude distortion, instead of the multi-value QAM. In addition, owing to high degree of freedom of mapping, there is an advantage that multi-value PSK mapping is made to reduce the influence of the non-linear distortion of amplitude.

Claims

1. A coded modulation method for converting the transmission data into plural rows, non-linearly encoding a part in plural rows of converted data, and non-linearlymappingthenon-linearly coded data to signal points having a codeword distance of unequal interval on a two-dimensional plane.

2. A coded modulation method for converting the transmission data into plural rows, and non-linearly mapping the lower level data being non-linearly coded in plural rows of converted data, and the upper level data in said plural rows of converted data to signal points having a codeword distance of unequal interval on a two-dimensional plane.

3. A coded modulation device comprising:

   a data converter for converting the transmission data into plural rows;
   a non-linear encoder for non-linearly encoding the lower level data in plural rows of data that are outputted from said data converter;
   a non-linear mapping part for non-linearly mapping said non-linearly coded data to signal points having a codeword distance of unequal interval on a two-dimensional plane; and
   a modulator for modulating a carrier wave with data mapped by said non-linear mapping part.

4. A coded modulation device comprising:

   a data converter for converting the transmission data into plural rows;
   a non-linear encoder for non-linearly encoding the lower level data in plural rows of data that are outputted from said data converter;
   a non-linear mapping part for non-linearly map-

ping said non-linearly coded data and the upper level data in said plural rows of converted data to signal points having a codeword distance of unequal interval on a two-dimensional plane; and

a modulator for modulating a carrier wave with data mapped by said non-linear mapping part.

5. A coded modulation device comprising a converter, a first non-linear encoder and a second non-linear encoder, a non-linear mapping circuit, and a modulator, wherein

said converter converts one row of data that is inputted from a first terminal into plural rows of data;

said first and second non-linear encoders non-linearly encode lower level two rows of data in plural rows of data converted by said converter, and output them as data of level 1 and level 2 to said non-linear mapping circuit;

said non-linear mapping circuit non-linearly maps said data of level 1 and level 2 that are non-linearly encoded by said first and second non-linear encoders and the uncoded data of level 3 and level 4 to signal points having a codeword distance of unequal interval on a two-dimensional plane; and

said modulator modulates a carrier wave with two-dimensional data that are non-linearly mapped by said non-linear mapping circuit.

6. A coded modulation method comprising:

converting one row of data that is inputted from a first terminal into plural rows of data;

non-linearly encoding lower level two rows of data in plural rows of converted data, and outputting them as data of level 1 and level 2;

non-linearly mapping said data of level 1 and level 2 that are non-linearly encoded and the uncoded data of level 3 and level 4 to signal points having a codeword distance of unequal interval on a two-dimensional plane, and

modulating a carrier wave with two-dimensional data that are non-linearly mapped by said non-linear mapping.

7. The coding modulation method according to claim 1, wherein said non-linear coding is generated by a non-linear equation of simultaneous multi-dimensional polynomials.

8. The coding modulation method according to claim 1, wherein said non-linear coding is generated by a non-linear connection of generator polynomials.

9. The coding modulation device according to claim 3, wherein said non-linear encoder is composed of a generation circuit that is generated by a non-linear equation of simultaneous multi-dimensional polynomials.

10. The coding modulation device according to claim 3, wherein said non-linear encoder is generated by a non-linear connecting circuit of generator polynomials.

FIG. 1

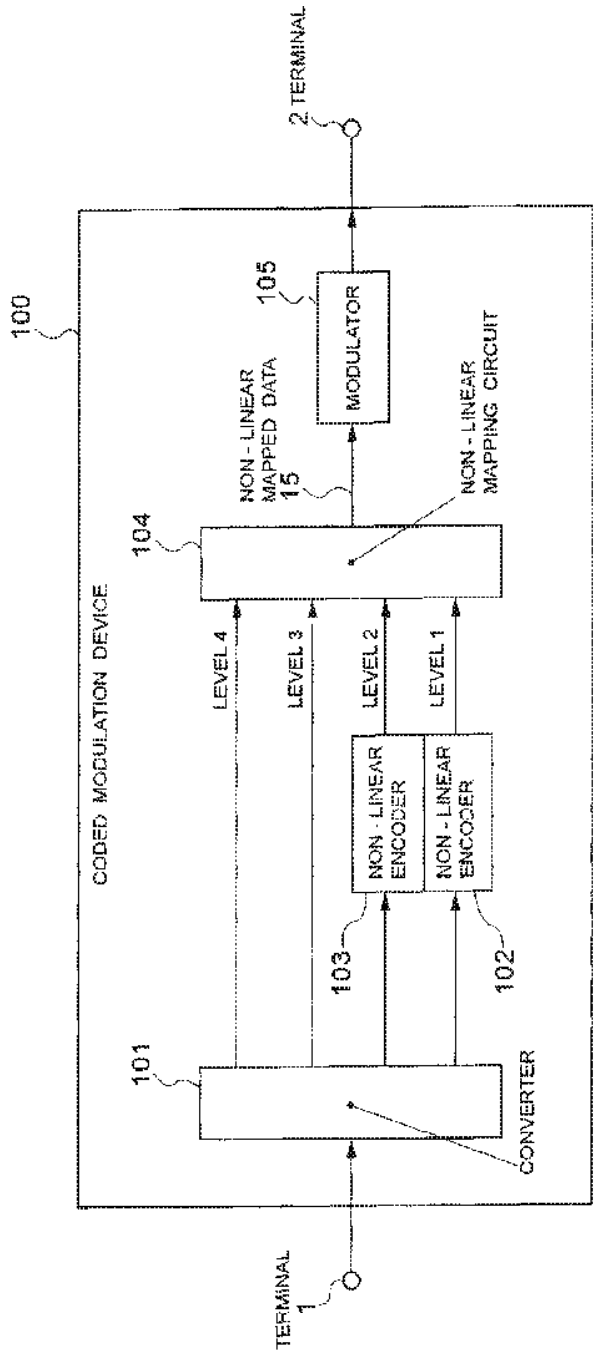

2 TERMINAL
100
105
MODULATOR
NON - LINEAR MAPPED DATA
15
NON - LINEAR MAPPING CIRCUIT
104
CODED MODULATION DEVICE
LEVEL 4
LEVEL 3
LEVEL 2
LEVEL 1
NON - LINEAR ENCODER
NON - LINEAR ENCODER
103
102
101
CONVERTER
TERMINAL
1

FIG. 2

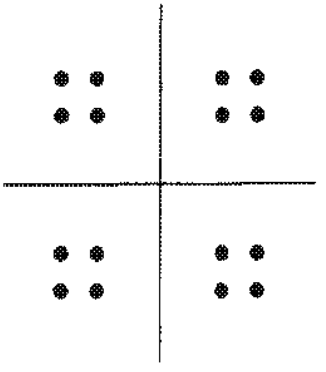


FIG. 3

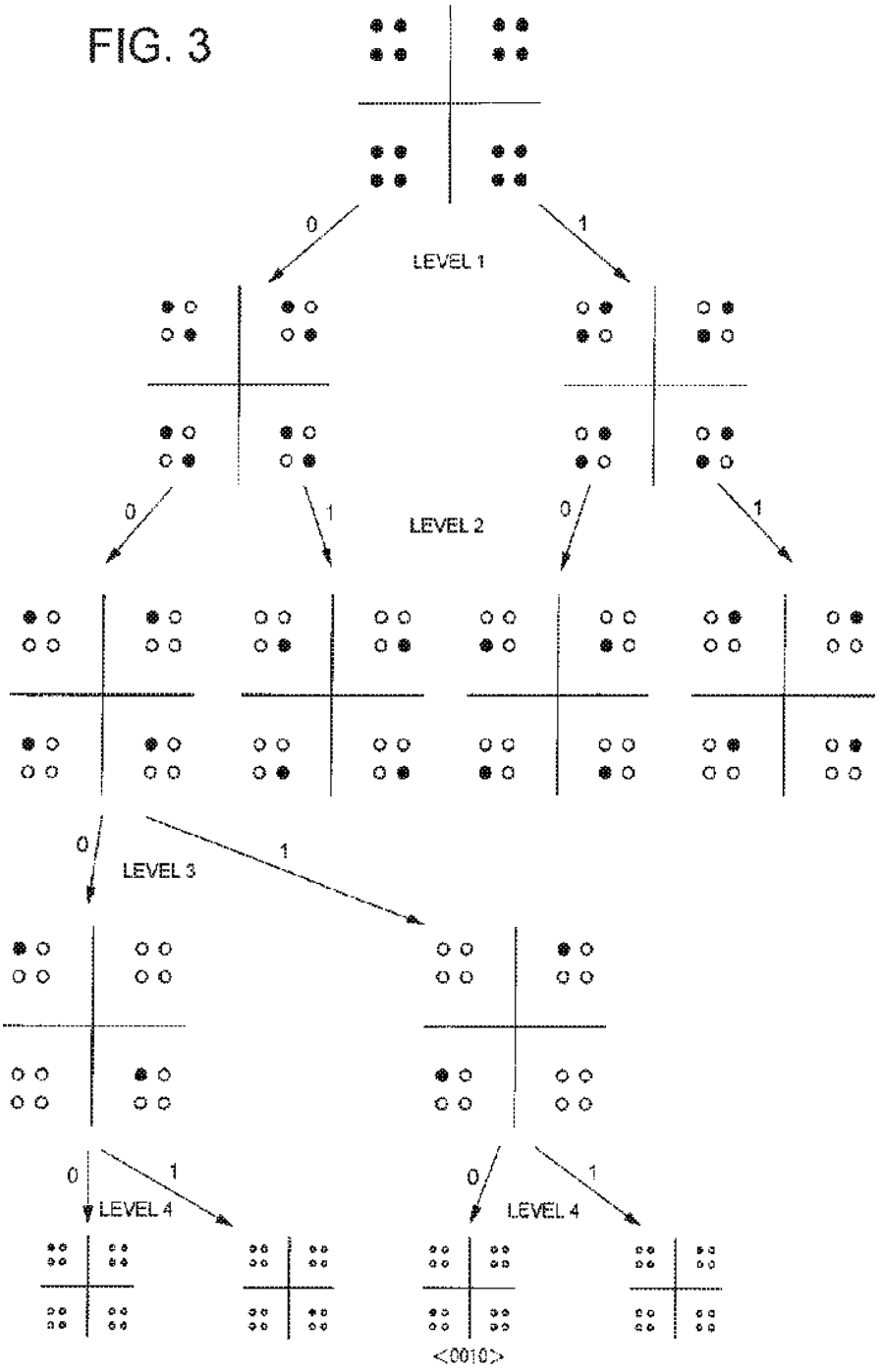


FIG. 4

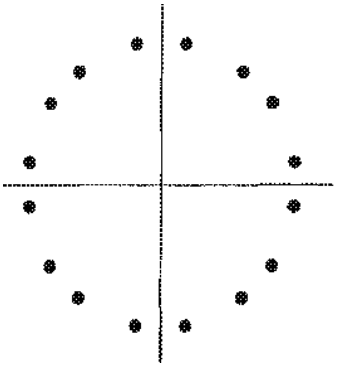


FIG. 5

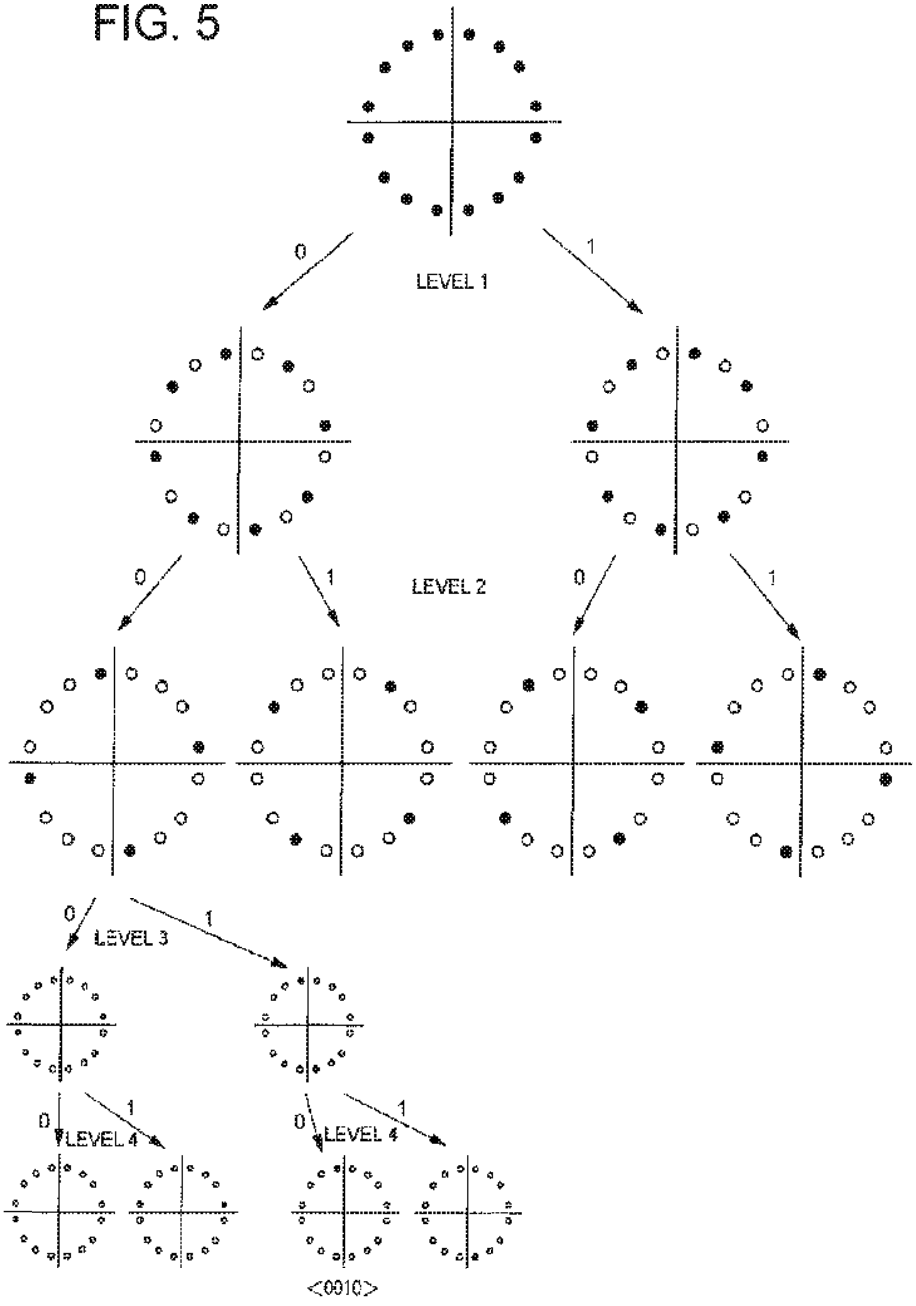

0
1
LEVEL 1
0
1
LEVEL 2
0
1
0
1
LEVEL 3
0
1
0
1
LEVEL 4
0
1
0
1
LEVEL 4
<0010>

FIG. 6

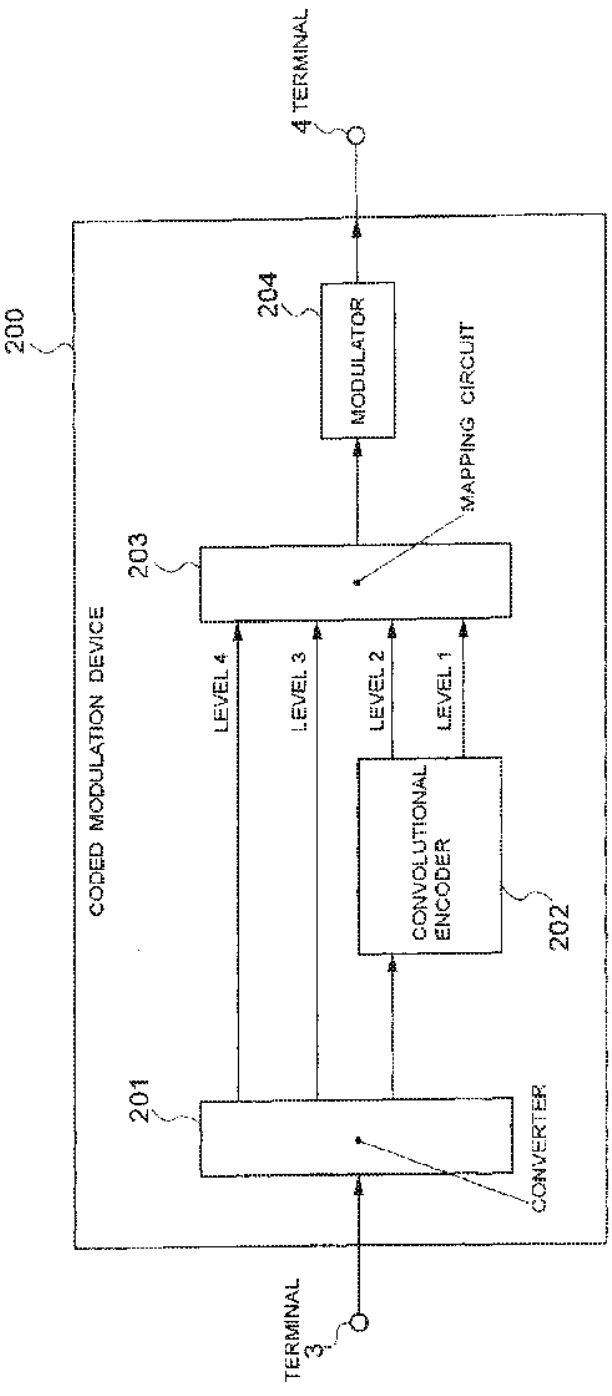
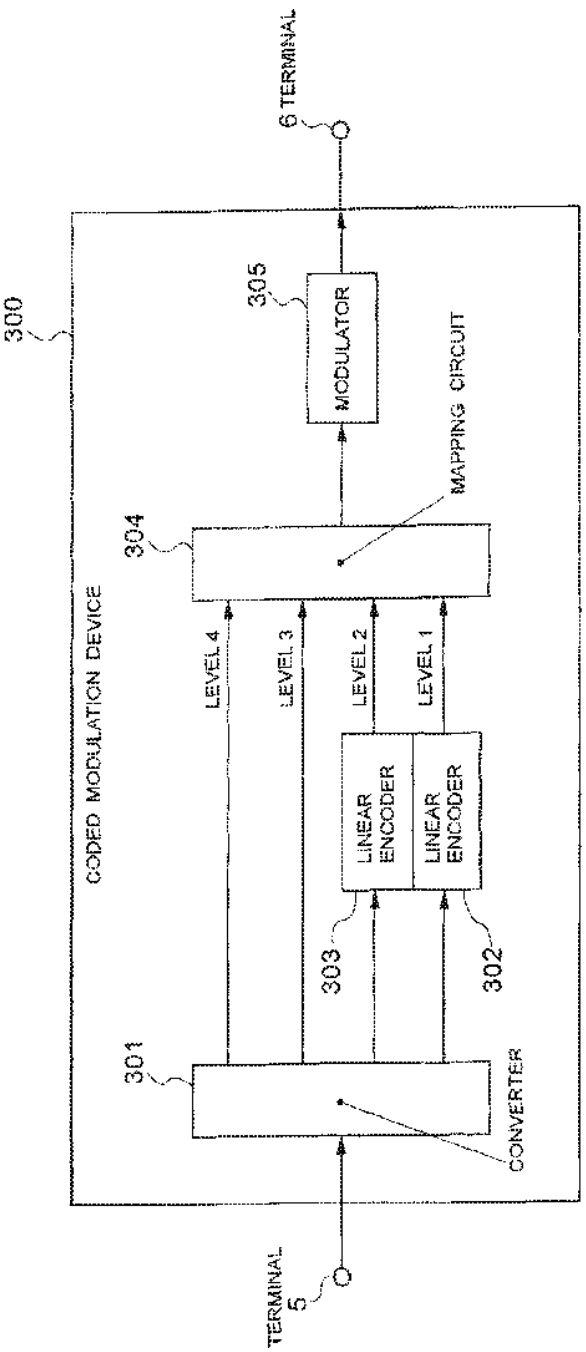

4 TERMINAL
200
204
MODULATOR
MAPPING CIRCUIT
203
CODED MODULATION DEVICE
LEVEL 4
LEVEL 3
LEVEL 2
LEVEL 1
CONVOLUTIONAL ENCODER
202
201
CONVERTER
TERMINAL 3

FIG. 7


6 TERMINAL
300
305
MODULATOR
MAPPING CIRCUIT
304
CODED MODULATION DEVICE
LEVEL 4
LEVEL 3
LEVEL 2
LEVEL 1
LINEAR ENCODER
LINEAR ENCODER
303
302
301
CONVERTER
TERMINAL 5

European Patent Office

EUROPEAN SEARCH REPORT

Application Number
EP 05 10 2138

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Category	Citation of document with indication, where appropriate, of relevant passages	Relevant to claim	CLASSIFICATION OF THE APPLICATION (Int.Cl.7)
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Y	UNGERBOECK G: "TRELLIS-CODED MODULATION WITH REDUNDANT SIGNAL SETS PART II: STATE OF THE ART" IEEE COMMUNICATIONS MAGAZINE, IEEE SERVICE CENTER. PISCATAWAY, N.J, US, vol. 25, no. 2, 1 February 1987 (1987-02-01), pages 12-21, XP000565140 ISSN: 0163-6804 * figures 6,7 *	1-10	
A	BERG L ET AL: "DESIGN PROCEDURE FOR OPTIMUM OR ROTATIONALLY INVARIANT TRELLIS CODES" INTERNATIONAL CONFERENCE ON COMMUNICATIONS. INCLUDING SUPERCOMM TECHNICAL SESSIONS. ATLANTA, APR. 15 - 19, 1990, NEW YORK, IEEE, US, vol. VOL. 2, 15 April 1990 (1990-04-15), pages 607-613, XP000146134 * abstract * * figure 1 *	1-10	

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TECHNICAL FIELDS SEARCHED (Int.Cl.7)
H03M
H04L

The present search report has been drawn up for all claims

Place of search	Date of completion of the search	Examiner
The Hague	19 July 2005	Ogor, M

CATEGORY OF CITED DOCUMENTS
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European Patent Office

EUROPEAN SEARCH REPORT

Application Number
EP 05 10 2138

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A	SNELLING W E ED - INSTITUTE OF ELECTRICAL AND ELECTRONICS ENGINEERS: "Evolutionary coded modulation: a coded-modulation extension of turbo codes designed with genetic algorithms" MILCOM 2001. PROCEEDINGS. COMMUNICATIONS FOR NETWORK-CENTRIC OPERATIONS: CREATING THE INFORMATION FORCE. MCLEAN, VA, OCT. 28 - 30, 2001, IEEE MILITARY COMMUNICATIONS CONFERENCE, NEW YORK, NY : IEEE, US, vol. VOL. 1 OF 2, 28 October 2001 (2001-10-28), pages 1133-1140, XP010579180 ISBN: 0-7803-7225-5 * the whole document * -----	1-10	

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The present search report has been drawn up for all claims

Place of search	Date of completion of the search	Examiner
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E : earlier patent document, but published on, or after the filing date
D : document cited in the application
L : document cited for other reasons

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EPO FORM 1503 03.82 (P04C01)

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For more details about this annex : see Official Journal of the European Patent Office, No. 12/82

Espacenet

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CODING MODULATION APPARATUS AND METHOD

Inventor(s): MITSUYA NAOKI ± (MITSUYA NAOKI)

Applicant(s): NEC CORP ± (NEC CORP)

Classification: - international: ***H03M13/21; H03M13/25; H04L25/49; H04L27/00; H04L27/18; H04L27/34; H04L27/36;*** (IPC1-7): H03M13/21; H03M13/25; H04L27/00; H04L27/18; H04L27/36

- cooperative: **H03M13/25; H03M13/256**

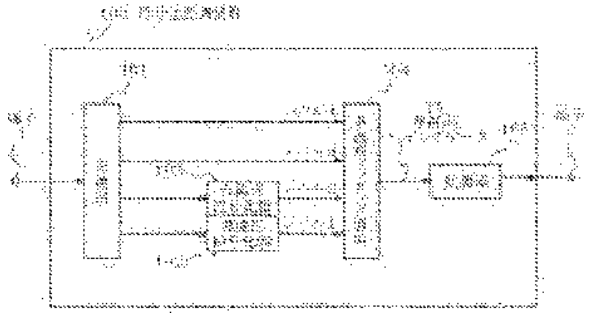
Application number: JP20040078971 20040318

Priority number(s): JP20040078971 20040318

Also published as: JP4622276 (B2) EP1578021 (A1) US2005207507 (A1)

Abstract of JP2005269258 (A)

PROBLEM TO BE SOLVED: To provide a coding modulation apparatus and method for realizing a high coding gain by employing nonlinear coding and nonlinear mapping. ;SOLUTION: A converter 101 converts data of a sequence received from a terminal 1 into data of a plurality of sequences. Nonlinear coders 102, 103 apply nonlinear coding to the data of two sequences at subordinate levels among data of each sequence converted into data of a plurality of sequences and provide outputs of the nonlinearly coded data to a nonlinear mapping circuit 104 as the data of levels 1, 2. The nonlinear mapping circuit 104 maps the data subjected to nonlinear coding by the nonlinear coders 102, 103 with the data that are not coded in a way of optimizing respective inter-code word distances. A modulator 105 modulates carriers with two-dimensional data arranged by the nonlinear mapping circuit 104. ;COPYRIGHT: (C)2005,JPO&NCIPI



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特開2005-269258
(P2005-269258A)
(43)公開日　平成17年9月29日(2005.9.29)

(51)Int.Cl.[7]	F I		テーマコード（参考）
H03M 13/25	H03M 13/25		5J065
H03M 13/21	H03M 13/21		5K004
H04L 27/00	H04L 27/18	B	
H04L 27/18	H04L 27/00	B	
H04L 27/36	H04L 27/00	F	

審査請求 未請求　請求項の数 10　ＯＬ　（全 11 頁）

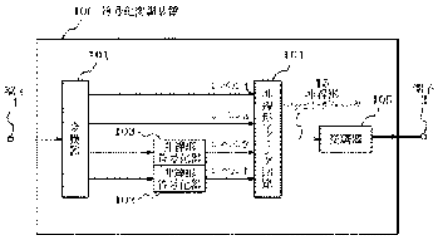
(21)出願番号　特願2004-78971(P2004-78971)
(22)出願日　平成16年3月18日(2004.3.18)

(71)出願人　000004237
日本電気株式会社
東京都港区芝五丁目７番１号
(74)代理人　100109313
弁理士　机　昌彦
(74)代理人　100136814
弁理士　工藤　雅司
(74)代理人　100111637
弁理士　谷澤　靖久
(72)発明者　光谷　直樹
東京都港区芝五丁目７番１号
日本電気株式会社内

Ｆターム(参考)　5J065 AA01 AB01 AC02 AD10 AE06
AG01 AH09 AH22
5K004 AA05 AA08 FD05 FF02 JF01

(54)【発明の名称】符号化変調装置および方法

(57)【要約】
【課題】非線形符号化と非線形マッピングにより高い符号化利得を実現する。
【解決手段】変換器１０１は端子１から入力された一列のデータを複数列のデータに変換する。非線形符号化器１０２、１０３は、複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、各々レベル１のデータ、レベル２のデータとして非線形マッピング回路１０４に出力する。非線形マッピング回路１０４は、非線形符号化器１０２、１０３により非線形符号化されたデータと符号化が行われていないデータとを、各々の符号語間距離が最適になるようにマッピングする。変調器１０５は、非線形マッピング回路１０４により配置された２次元のデータにより搬送波に変調をかける。
【選択図】　図１



【特許請求の範囲】
【請求項1】
伝送するデータを複数列に変換し、変換した複数列のデータの一部に非線形符号化を行い、非線形符号化されたデータを２次元平面上に符号語間距離が非等間隔になる信号点に配置する非線形マッピングを行うことを特徴とする符号化変調方法。
【請求項2】
伝送するデータを複数列に変換し、変換した複数列のデータの下位レベルのデータに非線形符号化を行った非線形符号化データ及び前記変換した複数列のデータの上位レベルのデータを、２次元平面上に符号語間距離が非等間隔になる信号点に配置する非線形マッピングを行うことを特徴とする符号化変調方法。
【請求項3】
伝送するデータを複数列に変換するデータ変換器と；
このデータ変換器から出力される複数列のデータのうち、下位レベルのデータに非線形符合化を行う非線形符合化器と；
前記非線形符合化されたデータを２次元平面上に符号語間距離が非等間隔になるように信号点を配置する非線形マッピング部と；
この非線形マッピング部により配置されたデータにより搬送波に変調をかける変調器と；
を備えたことを特徴とする符号化変調装置。
【請求項4】
伝送するデータを複数列に変換するデータ変換器と；
このデータ変換器から出力される複数列のデータのうち、下位レベルのデータに非線形符合化を行う非線形符合化器と；
前記非線形符合化されたデータ及び前記変換した複数列のデータの上位レベルのデータを２次元平面上に符号語間距離が非等間隔になるように信号点を配置する非線形マッピング部と；
この非線形マッピング部により配置されたデータにより搬送波に変調をかける変調器と；
を備えたことを特徴とする符号化変調装置。
【請求項5】
変換器と、第１の非線形符号化器及び第２の非線形符号化器と、非線形マッピング回路と、変調器とを備えた符号化変調装置であって、
前記変換器は、
第１の端子から入力された一列のデータを複数列のデータに変換し；
前記第１及び第２の非線形符号化器は、
前記変換器により複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、各々レベル１のデータ、レベル２のデータとして非線形マッピング回路に出力し；
前記非線形マッピング回路は、
前記第１及び第２の非線形符号化器により非線形符号化された前記レベル１、レベル２のデータと、符号化が行われていないレベル３、レベル４のデータとを、各々の符号語間距離が非等間隔になるように信号点を２次元平面上に配置し；
前記変調器は、前記非線形マッピング回路により配置された２次元の非線形マッピングデータにより搬送波に変調をかけることを特徴とする符号化変調装置。
【請求項6】
第１の端子から入力された一列のデータを複数列のデータに変換し；
複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、各々レベル１のデータ、レベル２のデータとして出力し；
非線形符号化された前記レベル１、レベル２のデータと、符号化が行われていないレベ

ル３、レベル４のデータとを、各々の符号語間距離が非等間隔になるように信号点を２次元平面上に配置する非線形マッピングを行い；

前記非線形マッピングにより配置された２次元の非線形マッピングデータにより搬送波に変調をかけることを特徴とする符号化変調方法。

【請求項7】

前記非線形符号化は、連立多次元多項式の非線形方程式で生成されることを特徴とする請求項１、２又は６記載の符号化変調方法。

【請求項8】

前記非線形符号化は、生成多項式の非線形結合により生成されることを特徴とする請求項１、２又は６記載の符号化変調方法。

【請求項9】

前記非線形符号化器は、連立多次元多項式の非線形方程式で生成される生成回路を備えたことを特徴とする請求項３、４又は５記載の符号化変調装置。

【請求項10】

前記非線形符号化器は、生成多項式の非線形結合回路により生成されることを特徴とする請求項３、４又は５記載の符号化変調装置。

【発明の詳細な説明】

【技術分野】

【0001】

本発明は符号化変調装置および方法に関し、特に非線形符号と非線形マッピングを用いることにより符号化利得を向上する符号化変調装置および方法に関する。

【背景技術】

【0002】

近年、ディジタルマイクロ波通信方式では、伝送特性の高品質化を目的に、より誤り訂正能力の高い符号化変調方式が使用されている。

【0003】

符号化変調方式は、信号点集合分割によるマッピングと誤り訂正符号化技術を一体化したもので、優れた誤り訂正能力を有している。

【0004】

符号化の構成法として、トレリス符号化変調（ＴＣＭ）方式と多レベル符号化変調（ＭＬＣＭ）方式が一般に知られている。

【0005】

トレリス符号化変調（ＴＣＭ）方式は、２次元当り2^m個の信号点を有する２次元または多次元変調方式の各信号点に２元ベクトル表現を１対１に対応させる。そして、一つの畳み込み符号器出力と非符号化ビットの組合せにより、一つの２次元または多次元信号点を与える符号化変調方式である。

【0006】

一方、多レベル符号化変調方式は、誤り訂正能力の異なる複数のブロック符号器を用いて、各々の符号器で2元ベクトルの各成分（ビット位置）に相当する各レベルを１レベルずつ独立に符号化する符号化変調方式である。多レベル符号化変調方式で用いる複数の符号は各々構成符号と称される。各構成符号は、それを適用するレベルの最小信号点間距離に基づいて必要な誤り訂正能力が決定される（例えば、特許文献１参照。）。

【0007】

図６は従来の符号化変調装置を示すブロック図である。

【0008】

特にトレリス符号化変調方式（ＴＣＭ）の場合を示す。

【0009】

図６を参照すると、符号化変調装置２００では、端子３から入力された１列のデータは変換器２０１で３列に変換される。そのうちの１列が畳み込み符号化器２０２で符号化され２列の信号となる。２列の信号はマッピング回路２０３にレベル１、レベル２の信号として供給される。また、変換器２０１から出力される他の２列の信号は非符号化レベルの

信号としてマッピング回路２０３に供給される。

【0010】

マッピング回路２０３では、信号点集合分割に基づく信号点配置により、信号入力に対応した信号点の座標が出力される。この信号点座標出力が変調器２０４に入力され、１６ＱＡＭ変調が行われ端子４から出力される。

【0011】

このＴＣＭは単一の畳み込み符号を用いるため符号化率の設定に関する自由度が非常に小さい。また畳み込み符号は線形符号の一種であるため、ＴＣＭの符号化利得はある程度の限界がある。

【0012】

図７は従来の他の符号化変調装置を示すブロック図である。

【0013】

特に多レベル符号化変調（ＭＬＣＭ）の場合を示す。

【0014】

図７を参照すると、符号化変調装置３００では、端子５から入力された一列のデータは変換器３０１で４列に変換される。そのうちの下位２列のデータがそれぞれ線形符号化器３０２、線形符号化器３０３により符号化され、レベル１、レベル２として出力される。線形符号化されたレベル１、レベル２の信号と符号化されないレベル３、レベル４の信号はマッピング回路３０４に供給される。

【0015】

マッピング回路３０４では、信号点集合分割に基づく信号点配置により、信号入力に対応した信号点の座標が出力される。この信号点座標出力が変調器３０５に入力され、１６ＱＡＭ変調が行われ端子６から出力される。

【0016】

このＭＬＣＭは、信号点集合分割における各レベルにそれぞれ異なる符号を用いるため、ＴＣＭに比べ符号化率の設定の自由度は大きい。しかし、各レベルに用いる符号はハミング符号やＢＣＨ（Ｂｏｓｅ　Ｃｈａｕｄｈｕｒｉ　Ｈｏｃｑｕｅｎｇｈｅｍ）符号、ＲＳ（Ｒｅｅｄ　Ｓｏｌｏｍｏｎ）符号などの線形符号であるため、符号化利得には限界がある。

【0017】

また、畳み込み符号を用いることで、フェージング等のバースト誤り環境下での符号化変調の誤り訂正能力を高めているものもある（例えば、特許文献２参照。）。

【0018】

【特許文献1】特開平２－２９１７４３号公報（第２－４頁）

【特許文献2】特開平９－９３２９５号公報（第２－３頁、図２）

【発明の開示】

【発明が解決しようとする課題】

【0019】

上述した従来の符号化変調装置および方法として、トレリス符号化変調（ＴＣＭ）と多レベル符号化変調（ＭＬＣＭ）がある。

【0020】

このＴＣＭは単一の畳み込み符号を用いるため、符号化率の設定に関する自由度が非常に小さい。畳み込み符号は線形符号の一種であるため、ＴＣＭの符号化利得はある程度の限界があるという欠点を有している。

【0021】

また、ＭＬＣＭは信号点集合分割における各レベルごとにそれぞれ異なる符号を用いるため、ＴＣＭに比べ符号化率の設定の自由度は大きい。しかし、各レベルに用いる符号はハミング符号やＢＣＨ符号、ＲＳ符号などの線形符号であるため、符号化利得には限界があるという欠点を有している。

【0022】

本発明の目的は、伝送するデータを複数列に変換し、下位レベルのデータに非線形符号化を行い、符号化されたデータを２次元平面上に各符号語間距離が非等間隔になるように信号点を配置する非線形マッピングを行い、より高い符号化利得を実現する符号化変調装置および方法を提供することにある。

【課題を解決するための手段】

【0023】

本発明の第１の符号化変調方法は、

伝送するデータを複数列に変換し、変換した複数列のデータの一部に非線形符号化を行い、非線形符号化されたデータを２次元平面上に符号語間距離が非等間隔になる信号点に配置する非線形マッピングを行うことを特徴としている。

【0024】

本発明の第２の符号化変調方法は、

伝送するデータを複数列に変換し、変換した複数列のデータの下位レベルのデータに非線形符号化を行った非線形符号化データ及び前記変換した複数列のデータの上位レベルのデータを、２次元平面上に符号語間距離が非等間隔になる信号点に配置する非線形マッピングを行うことを特徴としている。

【0025】

本発明の第１の符号化変調装置は、

伝送するデータを複数列に変換するデータ変換器と；

このデータ変換器から出力される複数列のデータのうち、下位レベルのデータに非線形符合化を行う非線形符合化器と；

前記非線形符合化されたデータを２次元平面上に符号語間距離が非等間隔になるように信号点を配置する非線形マッピング部と；

この非線形マッピング部により配置されたデータにより搬送波に変調をかける変調器と；

を備えたことを特徴としている。

【0026】

本発明の第２の符号化変調装置は、

伝送するデータを複数列に変換するデータ変換器と；

このデータ変換器から出力される複数列のデータのうち、下位レベルのデータに非線形符合化を行う非線形符合化器と；

前記非線形符合化されたデータ及び前記変換した複数列のデータの上位レベルのデータを２次元平面上に符号語間距離が非等間隔になるように信号点を配置する非線形マッピング部と；

この非線形マッピング部により配置されたデータにより搬送波に変調をかける変調器と；

を備えたことを特徴としている。

【0027】

本発明の第３の符号化変調装置は、

変換器と、第１の非線形符号化器及び第２の非線形符号化器と、非線形マッピング回路と、変調器とを備えた符号化変調装置であって、

前記変換器は、

第１の端子から入力された一列のデータを複数列のデータに変換し；

前記第１及び第２の非線形符号化器は、

前記変換器により複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、各々レベル１のデータ、レベル２のデータとして非線形マッピング回路に出力し；

前記非線形マッピング回路は、

前記第１及び第２の非線形符号化器により非線形符号化された前記レベル１、レベル２のデータと、符号化が行われていないレベル３、レベル４のデータとを、各々の符号語間

距離が非等間隔になるように信号点を２次元平面上に配置し；
　前記変調器は、前記非線形マッピング回路により配置された２次元の非線形マッピングデータにより搬送波に変調をかけることを特徴としている。
【0028】
　本発明の第３の符号化変調方法は、
　第１の端子から入力された一列のデータを複数列のデータに変換し；
　複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、各々レベル１のデータ、レベル２のデータとして出力し；
　非線形符号化された前記レベル１、レベル２のデータと、符号化が行われていないレベル３、レベル４のデータとを、各々の符号語間距離が非等間隔になるように信号点を２次元平面上に配置する非線形マッピングを行い；
　前記非線形マッピングにより配置された２次元の非線形マッピングデータにより搬送波に変調をかけることを特徴としている。
【0029】
　本発明の第４の符号化変調方法は、前記第１、２または３の符号化変調方法において、
　例えば前記非線形符号化は、連立多次元多項式の非線形方程式で生成されることを特徴としている。
【0030】
　本発明の第５の符号化変調方法は、前記第１、２または３の符号化変調方法において、
　前記非線形符号化は、生成多項式の非線形結合により生成されることを特徴としている。
【0031】
　本発明の第４の符号化変調装置は、前記第１、２または３の符号化変調装置において、
　前記非線形符号化器は、連立多次元多項式の非線形方程式で生成される生成回路を備えたことを特徴としている。
【0032】
　本発明の第５の符号化変調装置は、前記第１、２または３の符号化変調装置において、
　前記非線形符号化器は、生成多項式の非線形結合回路により生成されることを特徴としている。
【発明の効果】
【0033】
　本発明の符号化変調装置および方法は、非線形符号と非線形マッピングをもちいて符号化変調を行っている。このため、線形符号を用いた符号化変調方式より符号間距離が大きくとれるので、高い符号化利得が実現できるという効果を有している。
【発明を実施するための最良の形態】
【0034】
　次に、本発明の実施の形態について図面を参照して説明する。
【0035】
　図１は本発明の符号化変調装置および方法の一つの実施の形態を示すブロック図である。
【0036】
　図１に示す符号化変調装置１００は、変換器１０１と、非線形符号化器１０２および非線形符号化器１０３と、非線形マッピング回路１０４と、変調器１０５とから構成されている。
【0037】
　変換器１０１は端子１から入力された一列のデータを複数列のデータ（並列データ）に変換する直並列変換器である。非線形符号化器１０２および非線形符号化器１０３は、変換器１０１により複数列に変換された各列のデータのうち、下位レベルの２列のデータを非線形符号化し、レベル１のデータ、レベル２のデータとして非線形マッピング回路１０４に出力する。なお、下位レベル等の表現は慣用的なもので、符号化するか符号化しない

かによりレベル１～４を識別している。

【0038】

非線形マッピング回路１０４は、非線形符号化されたレベル１、レベル２のデータと符号化が行われていない変換器１０１の出力であるレベル３、レベル４のデータとを、各々の符号語間距離が非等間隔になるように信号点を２次元平面上に配置し、非線形マッピングデータ１５として出力する。

【0039】

変調器１０５は、非線形マッピング回路１０４で配置された２次元の非線形マッピングデータ１５により搬送波に変調をかける。

【0040】

ここで、非線形符号化器１０２，１０３で行われる非線形符号とは、畳み込み符号やハミング符号、ＢＣＨ符号、リードソロモン符号と云った線形符号以外の符号である。連立多次元多項式の非線形方程式の形で表現される符号、生成多項式の非線形結合の形で表現される符号、または線形演算則で求めることができない符号のことを示す。

【0041】

ここで、非線形符号化や非線形符号化器は、連立多次元多項式の非線形方程式で生成される生成回路により構成される。また、生成多項式の非線形結合回路により生成される。

【0042】

また、図１の非線形マッピング回路１０４で行われる非線形マッピングとは、位相変調（ＰＳＫ：Ｐｈａｓｅ　Ｓｈｉｆｔ　Ｋｅｙｉｎｇ）、直交振幅変調（ＱＡＭ）のように、２次元平面空間上に隣接する信号点間距離が等間隔になるように信号点を配置するものではない。信号点を非線形に配置し、隣接する信号点間距離は等間隔ではない（非等間隔）マッピングを意味する。

【0043】

次に図１を参照して動作を説明する。

【0044】

伝送されるデータは端子１から入力され、変換器１０１にて複数列（ここでは４列）に変換される。４列のデータ列に変換されたデータのうち、下位２列のデータのみがそれぞれ非線形符号化器１０２と非線形符号化器１０３により非線形符号化が行われる。非線形符号化されたデータは各々、レベル１、レベル２のデータとして非線形マッピング回路１０４に供給される。その他のレベル３、レベル４のデータは何も処理されずに非線形マッピング回路１０４に供給される。

【0045】

図２は本発明の第１のレベルマッピングを示す図である。直交振幅変調における信号点配置であり、横軸はＩｃｈ、縦軸はＱｃｈを示す。

【0046】

非線形マッピング回路１０４に入力されたレベル１、レベル２のデータ及び変換器１０１出力のレベル３、レベル４のデータは、例えば図２に示すように２次元平面上に非線形マッピングされる。

【0047】

レベル１～レベル４のディジタル信号４ビットが非線形マッピング回路１０４に入力されるので、2^4＝１６個の各信号点（黒丸で表示）の符号が存在する。各黒丸の信号点は、例えば（００００）～（０１０１）～（１０１０）～（１１０１）～（１１１１）の４ビットの符号データを示している。

【0048】

この図２の信号点は図３に示す信号点集合分割に従ってマッピングされる。

【0049】

図３は図２の信号点分割動作の一例を示す図である。

【0050】

例えば、レベル１～レベル４のディジタル信号が“００１０”のデータならば、図３の

マッピングによりレベル４の左から３番目の図<００１０>のようにマッピングされる。
【0051】
図１のレベル１のデータは非線形符号化器１０２により非線形符号化された“１”，“０”の２値信号であり、図３の最初の１６個の信号点集合を８個ずつに分割する。さらに図１の非線形符号化器１０３により非線形符号化されたレベル２の“１”，“０”の２値信号により、４個ずつの信号点集合に分割する。次に、図１のレベル３の“１”，“０”の２値信号により、２個ずつの信号点集合に分割し、さらにレベル４の“１”，“０”の２値信号により、１個の信号点に分割する，
【0052】
レベル１、レベル２で分割される信号点集合間の信号点間距離すなわち符号間距離は、他のレベル３、レベル４で分割される信号点間距離（符号間距離）よりも非常に近くなっている。しかしながら、レベル１、レベル２は非線形符合化により符合化が行われているため、非常に強力な信号点識別能力を有している。つまり、符号間距離が近い信号点の識別には、非線形の符号化により信号点の識別能力を高め識別エラーを低減させている。さらにレベル３、レベル４で分割される信号点集合は、十分に信号点間距離が確保されており、信号点を誤る恐れは全くない。
【0053】
また、図４は本発明の他のレベルマッピングを示す図である，
【0054】
レベル１～レベル４のディジタル信号４ビットが非線形マッピング回路１０４に入力されるので、$2^4 = 16$の各信号点（黒丸で表示）の符号が存在する。各黒丸の信号点は、例えば（００００）～（０１０１）～（１０１０）～（１１０１）～（１１１１）の４ビットの符号データを示している，
【0055】
図４の場合、ＰＳＫによる位相変調方式の非線形マッピングとなっており、同一円周上に非等間隔に信号点が配置される。一方、図２の場合は直交振幅変調の場合で、位相と振幅成分を用いた変調方式であり、格子間隔上に信号点が配置される。
【0056】
図５は図４の信号点分割動作の一例を示す図である。
【0057】
例えば、レベル１～レベル４のディジタル信号が“００１０”のデータならば、図５のマッピングによりレベル４の左から３番目の図<００１０>のようにマッピングされる。
【0058】
図１のレベル１のデータは非線形符号化器１０２により非線形符号化された“１”，“０”の２値信号であり、図５の最初の１６個の信号点集合を８個ずつに分割する。さらに図１の非線形符号化器１０３により非線形符号化されたレベル２の“１”，“０”の２値信号により、４個ずつの信号点集合に分割する。次に、図１のレベル３の“１”，“０”の２値信号により、２個ずつの信号点集合に分割し、さらにレベル４の“１”，“０”の２値信号により、１個の信号点に分割する，
【0059】
非線形マッピング回路１０４により、２次元平面上に非線形マッピングされた非線形マッピングデータ１５は変調器１０５に送られる。変調器１０５は非線形マッピングデータ１５により、搬送波に変調をかけて変調出力データを端子２から出力する。
【0060】
上述の通り、本発明は、非線形符号と非線形マッピングを利用し符号化利得を向上している，
【0061】
すなわち、一列の入力データが変換器１０１により複数列に変換される。複数列に変換された各列のデータのうち下位レベルデータが、非線形符号化器１０２，１０３により非線形符号化され非線形マッピング回路１０４に出力される。非線形符号化されたデータは

、２次元平面上に各符号語間距離が非等間隔になるように、信号点を非線形マッピング回路１０４により配置される。非線形マッピング回路１０４で配置された２次元のデータは、変調器１０５により搬送波に変調をかけることになる。

【0062】

非線形符号化されたデータをさらに非線形マッピングで最適な２次元配置を行い搬送波に変調かけるので、符号化利得が非常に高くなる。このため、無線のような回線品質の悪い伝送路においてもデータを誤ることなく伝送することが可能となる。

【0063】

なお、マッピングの自由度が高いため、多値ＱＡＭの代わりにアンプの振幅歪の影響が無い多値ＰＳＫ方式に有用である。加えてマッピングの自由度が非常に高いため、例えば多値ＰＳＫのマッピングを行えばアンプの非線形歪などの影響が少なくなると云った利点がある。

【図面の簡単な説明】

【0064】

【図1】本発明の符号化変調装置および方法の一つの実施の形態を示すブロック図である。

【図2】本発明の第１のレベルマッピングを示す図である。

【図3】図２の信号点分割動作の一例を示す図である。

【図4】本発明の他のレベルマッピングを示す図である。

【図5】図４の信号点分割動作の一例を示す図である。

【図6】従来の符号化変調装置を示すブロック図である。

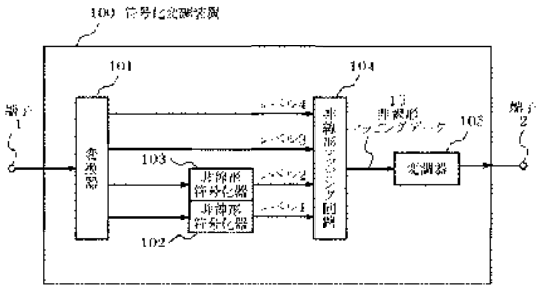
【図7】従来の他の符号化変調装置を示すブロック図である。

【符号の説明】

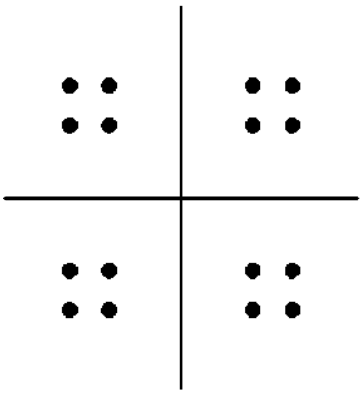
【0065】

１，２，３，４，５，６　　端子
１５　　非線形マッピングデータ
１００　　符号化変調装置
１０１　　変換器
１０２，１０３　　非線形符号化器
１０４　　非線形マッピング回路
１０５　　変調器
２００，３００　　符号化変調装置
２０１，３０１　　変換器
２０２　　畳み込み符号化器
２０３　　マッピング回路
２０４　　変調器
３０２，３０３　　線形符号化器
３０４　　マッピング回路
３０５　　変調器

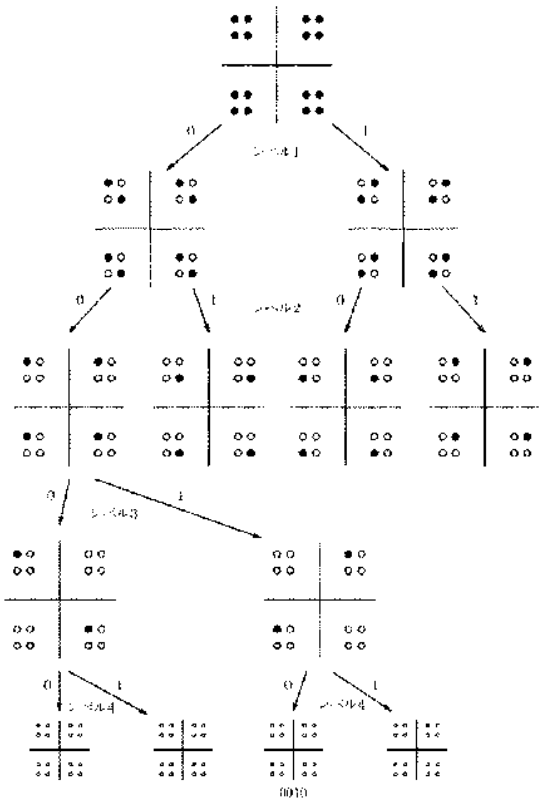
【図1】



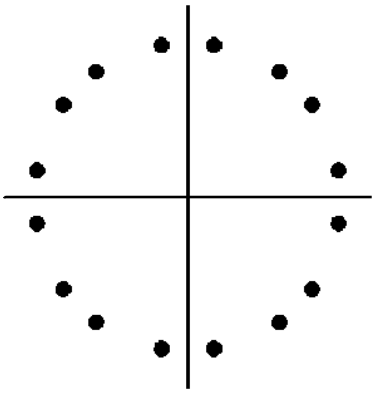
【図2】



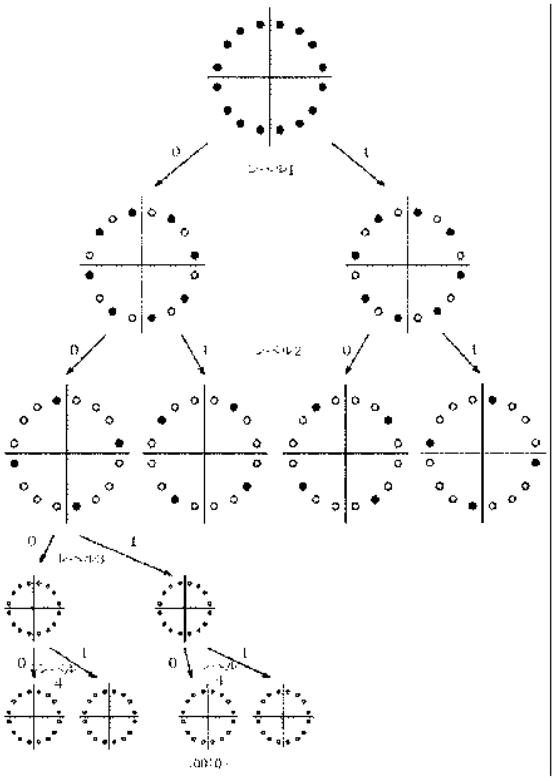
【図3】



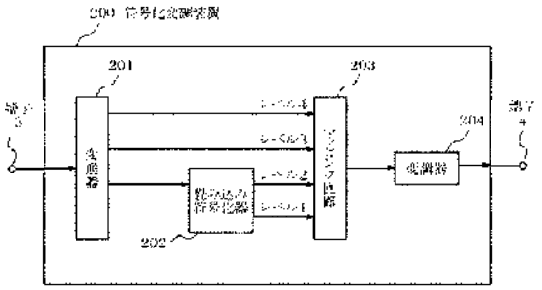
【図4】



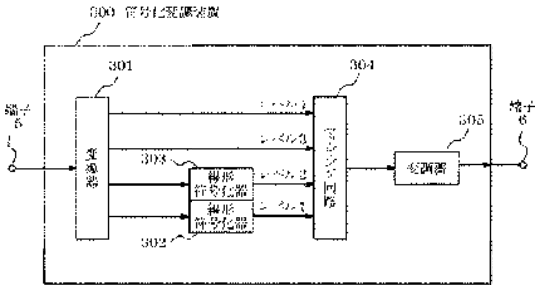
【図5】



【図6】



【図7】



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(71) 出願人 (米国を除く全ての指定国について): 富士通株式会社 (FUJITSU LIMITED) [JP/JP]; 〒2118588 神奈川県川崎市中原区上小田中 4 丁目 1 番 1 号 Kanagawa (JP).

(72) 発明者; および

(75) 発明者/出願人 (米国についてのみ): 中川 貴夫 (NAKAGAWA, Takao) [JP/JP]; 〒2118588 神奈川県川崎市中原区上小田中 4 丁目 1 番 1 号 富士通株式会社内 Kanagawa (JP). 矢野 哲也 (YANO, Tetsuya) [JP/JP]; 〒2118588 神奈川県川崎市中原区上小田中 4 丁目 1 番 1 号 富士通株式会社内 Kanagawa (JP). 大渕 一央 (OBUCHI, Kazuhisa) [JP/JP]; 〒2118588 神奈川県川崎市中原区上小田中 4 丁目 1 番 1 号 富士通株式会社内 Kanagawa (JP).

(74) 代理人: 真田 有 (SANADA, Tamotsu); 〒1800004 東京都武蔵野市吉祥寺本町 1 丁目 10 番 3 1 号吉祥寺広瀬ビル 5 階 Tokyo (JP).

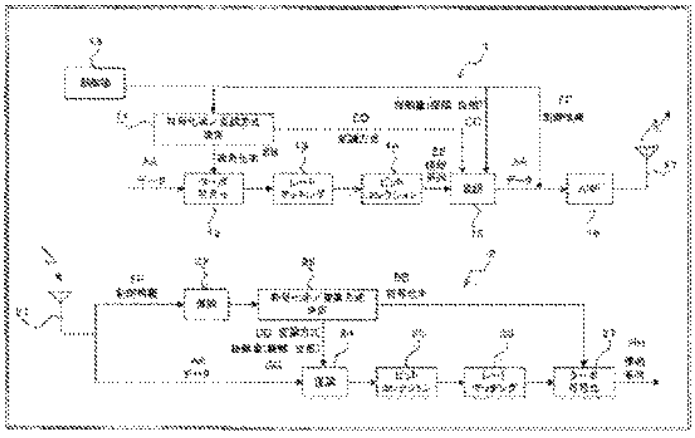
(81) 指定国 (表示のない限り、全ての種類の国内保護が可能): AE, AG, AL, AM, AT, AU, AZ, BA, BB, BG, BR, BW, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DZ, EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, HR, HU, ID, IL, IN, IS, JP, KE, KG, KM, KN, KP, KR, KZ, LC, LK, LR, LS, LT, LU, LV, LY, MA, MD, MG, MK, MN, MW, MX, MZ, NA, NG, NI, NO, NZ, OM, PG, PH, PL, PT, RO, RU, SC, SD, SE, SG, SK, SL, SM, SY, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VC, VN, YU, ZA, ZM, ZW.

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/続葉有/

(54) Title: DIGITAL WIRELESS COMMUNICATION METHOD, TRANSMITTER AND RECEIVER USING MULTILEVEL MODULATION SCHEME

(54) 発明の名称: 多値変調方式を用いたディジタル無線通信方法並びに送信機及び受信機



(57) Abstract: A transmitter (1), which is used in a digital wireless communication system, comprises a bit collection means (15) that controls the bit arrangement in such a manner that out of a plurality of codes obtained by an encoding process, codes having higher significance levels are assigned, on a priority basis, to bits the likelihoods of which are apt to become large during a symbol determination at a receiver (2); a

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KG, KZ, MD, RU, TJ, TM), ヨーロッパ (AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HU, IE, IS, IT, LT, LU, LV, MC, NL, PL, PT, RO, SE, SI, SK, TR), OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

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multilevel modulating means (15) that assigns the codes to the bits in accordance with the bit arrangement and performs a multilevel modulation with a predetermined symbol arrangement; and a symbol arrangement control means (18) that controls, in accordance with the ratio of the codes having different significance levels, the symbol arrangement in a direction in which the symbol arrangement is changed from an equal-interval arrangement. Thus, the symbol arrangement is changed, thereby enhancing the effect of the bit collection to improve the error rate at the receiver (2).

(57) 要約: ディジタル無線通信システムに用いられる送信機（１）において、受信機（２）でのシンボル判定時に尤度が大きくなる傾向にあるビットに対して、符号化により得られた複数の符号のうち重要度の高い符号が優先的に割り当てられるようビット配置を制御するビットコレクション手段（１４）と、前記ビット配置に従って、前記複数ビットへの前記符号の割り当てを行なって所定のシンボル配置で多値変調する多値変調手段（１５）と、前記重要度の異なる符号の割合に応じて、前記シンボル配置を等間隔配置から変更する方向に制御するシンボル配置制御手段（１８）とをそなえるように構成する。このように、シンボル配置を変更することにより、ビットコレクションの効果を大きくして、受信機（２）での誤り率を改善することが可能となる。

明 細 書

多値変調方式を用いたディジタル無線通信方法並びに送信機及び受信機

技術分野

[0001] 本発明は、多値変調方式を用いたディジタル無線通信方法並びに送信機及び受信機に関し、特に、ビットコレクション処理を併用するディジタル無線通信システムに用いて好適な技術に関する。

背景技術

[0002] 近年のディジタル無線通信システムでは、限られた無線周波数帯を有効に利用しつつ高速伝送を可能とするために、例えば、QPSK(Quadrature Phase Shift Keying)や8PSK、16QAM(Quadrature Amplitude Modulation)等の多値変調方式が採用されている。即ち、QPSKでは1シンボルで2ビット、8PSKでは1シンボルで3ビット、16QAMでは1シンボルで4ビットをそれぞれ伝送することができ、このようにして多値数を増やすことにより、1シンボルで伝送する情報量を増加させて高速伝送を実現している。

[0003] このような多値変調方式では、受信側でのシンボル判定誤り(符号誤り率)をできるだけ少なくする観点から、実軸(I軸)と虚軸(Q軸)の直交座標、即ち、複素平面(IQ平面)において、シンボル(信号点)間距離が等間隔になるようにシンボル配置される。例えば、16QAMでは、図36に示すように、複素平面において16シンボルをそれぞれ等間隔で格子状に配置するタイプ(以下、「格子状16QAM」と称する)や、図37に示すように、複素平面における2重の同心円のうち内側の円上に4シンボル、外側の円上に12シンボルをそれぞれ同一円上で等間隔に配置する同心円タイプ[以下、「(4, 12)サーキュラー(circular)16QAM」と称する]、図38に示すように、複素平面における2重の同心円のそれぞれに8シンボルずつを同一円上で等間隔に配置する同心円タイプ[以下、「(8, 8)スター(star)16QAM」と称する]が知られている。また、8QPSKでは、例えば図39に示すように、複素平面における円上に8シンボルが等間隔で配置される。

[0004] なお、図36に示すシンボル配置では、I軸、Q軸に近いシンボルと遠いシンボルとの間のシンボル間距離の比を1:3としており、図37及び図38に示すシンボル配置では内円と外円の半径比を1:rとしているが、勿論、これらに限られるものではない。また、図37に示す(4, 12)サーキュラー16QAM及び図38に示す(8, 8)スター16QAMについては、以前から研究が行なわれており、例えば、後記非特許文献1～4に記述がある。

[0005] [A]多値変調方式におけるシンボルへのビット割り当て(マッピング)方法

ここで、上記の格子状16QAM、(4, 12)サーキュラー16QAM、(8, 8)スター16QAM、8PSKのそれぞれにおけるシンボルへのビット割り当て(マッピング)方法について、図43～図47を用いて説明する。なお、図43～図46において、白丸がビット(符号)"0"を、黒丸がビット"1"を表している。

[0006] (A1)格子状16QAM

格子状16QAMの場合は、例えば図43に示すようなビット割り当てが行なわれる。

即ち、1ビット目は、図43の(1)に示すように、IQ平面におけるQ軸(虚軸)を中心とした象限判定により、Q軸の紙面左側(I軸(実軸)の負の領域)の8シンボルにビット"1"が、Q軸の紙面右側(I軸の正の領域)の8シンボルにビット"0"がそれぞれ割り当てられる。2ビット目は、図43の(2)に示すように、I軸を中心とした象限判定により、I軸の紙面上側(Q軸の正の領域)の8シンボルにビット"0"が、I軸の紙面下側(Q軸の負の領域)の8シンボルにビット"1"がそれぞれ割り当てられる。

[0007] さらに、3ビット目は、図43の(3)に示すように、Q軸からの距離判定により、Q軸に近い8シンボルにビット"0"が、Q軸から遠い8シンボルにビット"1"がそれぞれ割り当てられ、4ビット目は、図43の(4)に示すように、I軸からの距離判定により、I軸に近い8シンボルにビット"0"が、I軸から遠い8シンボルにビット"1"がそれぞれ割り当てられる。

[0008] このような方法でシンボルにビットをマッピングすることで、各シンボルに割り当てられるビット系列は、例えば図47に示すようになり、隣り合うシンボルのビット系列は1ビットしか違わなくなる。こうすることにより、雑音によって受信側で誤って隣のシンボルを受信したとしても、誤差を1ビットにとどめることができる。なお、このように隣接シン

ボル間で1ビットしか異ならないようなマッピング方法をグレイマッピング(Gray-Mapping)と呼ぶ。

[0009] (A2)(4, 12)サーキュラー16QAM

サーキュラー16QAMの場合は、例えば図44に示すようなビット割り当てが行なわれる。

即ち、1ビット目は、図44の(1)に示すように、IQ平面におけるQ軸を中心とした象限判定により、Q軸の紙面左側(I軸の負の領域)の各円上の8シンボルにビット"1"が、Q軸の紙面右側(I軸の正の領域)の各円上の8シンボルにビット"0"がそれぞれ割り当てられる。2ビット目は、図44の(2)に示すように、I軸を中心とした象限判定により、I軸の紙面上側(Q軸の正の領域)の各円上の8シンボルにビット"0"が、I軸の紙面下側(Q軸の負の領域)の各円上の8シンボルにビット"1"がそれぞれ割り当てられる。

[0010] さらに、3ビット目は、図44の(3)に示すように、Q軸からの距離判定により、Q軸に近い各円上の8シンボルにビット"0"が、Q軸から遠い各円上の8シンボルにビット"1"がそれぞれ割り当てられ、4ビット目は、図44の(4)に示すように、I軸からの距離判定により、I軸に近い各円上の8シンボルにビット"0"が、I軸から遠い8シンボルにビット"1"がそれぞれ割り当てられる。

[0011] つまり、上記の格子状16QAMおよび(4, 12)サーキュラー16QAMでは、1ビット目及び2ビット目はI軸、Q軸を中心とした象限判定により、3ビット目及び4ビット目はI軸、Q軸を中心とした距離判定によりそれぞれビット割り当てが行なわれ、1ビット目及び2ビット目並びに3ビット目及び4ビット目はいずれも同じ尤度(品質)をもつが、1ビット目及び2ビット目と3ビット目及び4ビット目とでは尤度が異なることになる(1ビット目及び2ビット目に比べて尤度が低い)。

[0012] (A3)(8, 8)スター16QAM

(8, 8)スター16QAMの場合は、例えば図45に示すようなビット割り当てが行なわれる。

即ち、1ビット目は、図45の(1)に示すように、IQ平面におけるQ軸を中心とした象限判定により、Q軸の紙面左側(I軸の負の領域)の各円上の8シンボルにビット"1"が

, Q軸の紙面右側(I軸の正の領域)の各円上の8シンボルにビット"0"がそれぞれ割り当てられ、2ビット目は、図45の(2)に示すように、I軸を中心とした象限判定により、I軸の紙面上側(Q軸の正の領域)の各円上の8シンボルにビット"0"が、I軸の紙面下側(Q軸の負の領域)の各円上の8シンボルにビット"1"がそれぞれ割り当てられる。

[0013] さらに、3ビット目は、図45の(3)に示すように、後述する8PSKの場合の3ビット目と同じ判定基準、具体的には、I軸及びQ軸を45度回転(例えば右回転)させたときの対角の象限判定により、第1象限及び第3象限における各円上の8シンボルにビット"0"が、第2象限及び第4象限における各円上の8シンボルにビット"1"がそれぞれ割り当てられ、4ビット目は、図45の(4)に示すように、原点からの距離判定により、原点に近い内側の円上の8シンボルにビット"0"が、原点から遠い外側の円上の8シンボルにビット"1"がそれぞれ割り当てられる。

[0014] したがって、(8, 8)スター16QAMの場合は、1ビット目及び2ビット目が同じ尤度を有するとともに、1ビット目及び2ビット目と3ビット目及び4ビット目とで異なる尤度(1ビット目及び2ビット目に比べて尤度が低い)を有し、かつ、3ビット目と4ビット目とでも異なる尤度を有する、つまり、3段階の尤度を有することになる。

(A4)8PSK

8PSK(3ビットで1シンボルを送信)の場合は、例えば図46に示すようなビット割り当てが行なわれる。

[0015] 即ち、1ビット目は、図46の(1)に示すように、IQ平面におけるQ軸を中心とした象限判定により、Q軸の紙面左側(I軸の負の領域)の円上の8シンボルにビット"1"が、Q軸の紙面右側(I軸の正の領域)の円上の8シンボルにビット"0"がそれぞれ割り当てられ、2ビット目は、図46の(2)に示すように、I軸を中心とした象限判定により、I軸の紙面上側(Q軸の正の領域)の円上の8シンボルにビット"0"が、I軸の紙面下側(Q軸の負の領域)の円上の8シンボルにビット"1"がそれぞれ割り当てられる。

[0016] そして、3ビット目は、図46の(3)に示すように、I軸及びQ軸を45度回転(例えば右回転)させたときの対角の象限判定により、第1象限及び第3象限における各円上の8シンボルにビット"0"が、第2象限及び第4象限における各円上の8シンボルにビット"1"がそれぞれ割り当てられる。

(B)ターボ符号及びビットコレクション方法を用いたシステム

上述したように、多値変調方式ではシンボルへのマッピング処理において、1シンボルにおけるシンボル判定基準の相違に応じてビット間に尤度(品質)の高低、つまり、誤りにくいビットと誤りやすいビットとが生じる(一般に、I軸又はQ軸中心の象限判定による1ビット目及び2ビット目に比べて3ビット目及び4ビット目の方が誤りやすい)。したがって、後記特許文献1でも述べられているように、各ビットの品質を考慮しないでマッピングを行なうと、符号化方式によっては受信側の復号結果の性能が劣化する場合がある。

[0017] 例えば、ターボ符号器では、図40に示すように、符号化しない情報系列[組織ビット(組織符号)]Sと、符号化したビット系列[パリティビット(パリティ符号)]、例えば、要素符号器101により符号化したビット系列(パリティビットP1)及びインターリーバ103でインターリーブした情報系列を要素符号器102により符号化したビット系列(パリティビットP2)とを伝送するが、特許文献1でも述べられているように、これらの組織ビットとパリティビットとを上述のごとく1シンボルにおける各ビットの品質を考慮しないでマッピングすると、組織ビットの品質がパリティビットの品質よりも復号結果の性能への影響が大きい(重要度が高い)にも関わらず、組織ビットの品質が劣化して復号結果の性能を劣化させてしまう。

[0018] 即ち、受信側(ターボ復号器)では、例えば図41に示すように、要素復号器201で組織ビットSと一方のパリティビットP1とを用いて復号(軟判定)を行ない、その結果をインターリーバ204でインターリーブした上で、もう1つの要素復号器202で組織ビットSと他方のパリティビットP2とを用いて復号を行ない、その復号結果をディンターリーバ205でディンターリーブした上で要素復号器201へフィードバックすることを所定回数繰り返すことにより、ターボ符号の復号処理を行ない、最終的に、要素復号器202の復号結果をディンターリーバ206でディンターリーブして復号系列を得るため、組織ビットSは、2つの要素復号器201及び202で用いられることになり、パリティビットの品質よりも復号結果の性能への影響が大きいのである。なお、ターボ符号を用いるシステムについては後記特許文献2にも記述がある。

[0019] そこで、ターボ符号を用いるシステムでは、通常、パリティビットPよりも重要度の高

い組織ビットSが1シンボルの中でも品質の良いビットにマッピングされるように、ビットコレクションと呼ばれる処理を行なって復号特性を改善させるのが好ましい(例えば、後記非特許文献5のChapter 4.5.4.4 (HARQ bit collection)参照)。即ち、上述した各種マッピング方法の16QAMの場合は、例えば図42に示すように、1シンボルにおける1ビット目及び2ビット目が象限判定なので3ビット目及び4ビット目に比べて品質が良いため、1ビット目及び2ビット目に組織ビットSが優先的に割り当てられるように、点線矢印で示すごとく紙面一番左の列から紙面縦方向に(最上位ビットから最下位ビットに向けて)、順次、組織ビットS及びパリティビットPが割り当てられる。なお、この図42は符号化率＝1／3のターボ符号化を行なう場合の例を示しており、組織ビットSはS1～S12、一方の(例えば、要素符号器101により得られた)パリティビットはP1－1～P1－12、他方の(要素符号器102により得られた)パリティビットはP2－1～P2－12と表記している(以降、同様)。

特許文献1:特開2002－171298号公報

特許文献2:特開2003－78419号公報

非特許文献1:大下、近藤、「16値振幅位相変調方式の最適信号点配置」、信学論文誌、1989年8月

非特許文献2:町田、半田、大下、「16値振幅位相変調方式における(4,12)型同心円信号点配置とその受信特性」、信学論文誌、1997年10月

非特許文献3:F.Adachi, M.Sawahashi, "PERFORMANCE ANALYSIS OF VARIOUS 16 LEVEL MODULATION SCHEMES UNDER RAYLEIGH FADING"

非特許文献4:鈴木、水野、「差動符号化振幅変調信号の多シンボル遅延検波方式とその16DAPSKへの適用」、信学論文誌、1994年12月

非特許文献5:3GPP TS 25.212 V5.2.0 (2002-09)

発明の開示

発明が解決しようとする課題

[0020] 上述したごとく、(4, 12)サーキュラー16QAMや、(8, 8)スター16QAMの同心円信号点配置は、同一円上の隣り合うシンボルが等間隔になるように配置すると、シンボル間距離の点で最適である。格子状16QAMの場合も、16点(シンボル)を等間

隔に配置することにより、シンボル間距離の観点から最適となる。

しかしながら、上述したターボ符号及びビットコレクション処理を組み合わせて用いるシステムの場合は、このような等間隔の信号点配置が誤り率の観点から必ずしも最適とはいえない。また、ビットコレクション処理も必ずしも最適とはいえない。

[0021] 本発明は、このような課題に鑑み創案されたもので、従来の多値変調方式における信号点(シンボル)配置を制御して送信を行なうことによって、ビットコレクション処理の効果を最大限に活かして、誤り率の改善を図ることを目的とする。

課題を解決するための手段

[0022] 上記の目的を達成するために、本発明では、下記の多値変調方式を用いたディジタル無線通信方法並びに送信機及び受信機を用いることを特徴としている。即ち、

(1)本発明の多値変調方式を用いたディジタル無線通信方法は、複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムにおいて、該送信機が、送信すべき情報系列を符号化して重要度の異なる複数の符号を生成する符号化過程と、前記複数ビットのうち、該受信機でのシンボル判定時に尤度が大きくなる傾向にあるビットに対して、前記符号化過程で得られた前記複数の符号のうち重要度の高い符号が優先的に割り当てられるようビット配置を制御するビットコレクション過程と、前記ビットコレクション過程で得られたビット配置に従って、前記複数ビットへの前記符号の割り当てを行なって所定のシンボル配置で前記情報系列を多値変調する多値変調過程と、前記ビットコレクション過程における前記重要度の異なる符号の割合に応じて、前記シンボル配置を等間隔配置から変更する方向に制御するシンボル配置制御過程とを有するとともに、該受信機が、該送信機から受信した信号を前記変更したシンボル配置に関する情報に基づいて復調する復調過程と、前記復調過程で復調された信号を復号して前記情報系列を得る復号過程とを有することを特徴としている。

[0023] (2)ここで、該送信機の前記シンボル配置制御過程において、前記複数ビットのうち前記重要度の高い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

(3)また、該送信機の前記シンボル配置制御過程において、前記複数ビットのうち前記重要度の低い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0024] (4)さらに、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0025] (5)また、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビット及び第2ビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0026] (6)さらに、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記第1～第3ビットに前記重要度の高い符号が割り当てられることになる場合に、当該第3ビット及び前記第4ビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0027] (7)また、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記第4ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第4ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0028] (8)さらに、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0029] (9)また、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットと第2ビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0030] (10)さらに、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、前記シンボル配置制御過程において、前記ビットコレクション過程によるビット配置により前記第3ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第3ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0031] (11)なお、前記シンボル配置制御過程において、前記シンボル配置の変更前後で送信電力が変化しないように前記シンボル配置制御を行なうのが好ましい。

(12)また、前記ビットコレクション過程において、前記複数の符号のうち重要度の低い複数種類の符号の比率が前記複数ビット毎に均等となるように前記ビット配置を制御してもよい。

[0032] (13)さらに、該送信機が、前記シンボル配置制御過程で変更したシンボル配置に関する情報を事前に該受信機へ通知する過程をさらに有していてもよい。

(14)また、該受信機が、前記重要度の異なる符号の割合に対する前記シンボル配置に関する情報を予めメモリに保持しておき、該送信機が、前記割合に関する情報を該受信機へ通知する過程を有するとともに、該受信機が、該送信機から通知された前記割合に関する情報に基づいて前記シンボル配置に関する情報を該メモリから読み出し、読み出した情報に基づいて前記復調を行なう過程を有していてもよい。

[0033] (15)さらに、前記重要度の高い符号がターボ符号における組織符号であり、前記重要度の低い符号が前記ターボ符号におけるパリティ符号であってもよい。

(16)また、本発明の送信機は、複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムに用いられる前記送信機であって、送信すべき情報系列を符号化して重要度の異なる複数の符号を生成

する符号化手段と、前記複数ビットのうち、該受信機でのシンボル判定時に尤度が大きくなる傾向にあるビットに対して、前記符号化手段で得られた前記複数の符号のうち重要度の高い符号が優先的に割り当てられるようビット配置を制御するビットコレクション手段と、該ビットコレクション手段で得られたビット配置に従って、前記複数ビットへの前記符号の割り当てを行なって所定のシンボル配置で前記情報系列を多値変調する多値変調手段と、該ビットコレクション手段における前記重要度の異なる符号の割合に応じて、前記シンボル配置を等間隔配置から変更する方向に制御するシンボル配置制御手段とをそなえたことを特徴としている。

[0034] (17)ここで、該シンボル配置制御手段は、前記複数ビットのうち前記重要度の高い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

(18)また、該シンボル配置制御手段は、前記複数ビットのうち前記重要度の低い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0035] (19)さらに、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0036] (20)また、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビット及び第2ビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0037] (21)さらに、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記第1～第3ビットに前記重要度の高い符号が割り当てられることになる場合に、当該第3ビット及び前記第4ビットの尤度が大きくなる方向に前記シン

ボル配置を制御してもよい。

[0038] (22)また、前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記第4ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第4ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0039] (23)さらに、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0040] (24)また、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットと第2ビットの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0041] (25)さらに、前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、該シンボル配置制御手段が、該ビットコレクション手段によるビット配置により前記第3ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第3ビットのみの尤度が大きくなる方向に前記シンボル配置を制御してもよい。

[0042] (26)なお、該シンボル配置制御手段は、前記シンボル配置の変更前後で送信電力が変化しない方向に前記シンボル配置制御を行なうのが好ましい。

(27)また、該ビットコレクション手段は、前記複数の符号のうち重要度の低い複数種類の符号の比率が前記複数ビット毎に均等となるように前記ビット配置を制御するのが好ましい。

[0043] (28)さらに、前記送信機は、該シンボル配置制御手段により変更したシンボル配置に関する情報を事前に該受信機へ通知する通知手段をさらにそなえていてもよい

。

(29)また、前記重要度の高い符号はターボ符号における組織符号であり、前記重要度の低い符号は前記ターボ符号におけるパリティ符号であってもよい。

(30)さらに、本発明の受信機は、複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムに用いられる前記受信機であって、該送信機から受信した信号を、該送信機において前記重要度の異なる符号の割合に応じて等間隔配置から変更されたシンボル配置に関する情報に基づいて復調する復調手段と、該復調手段で復調された信号を復号して該送信機の送信した情報系列を得る復号手段とをそなえたことを特徴としている。

[0044] (31)ここで、本受信機は、前記シンボル配置に関する情報を該送信機から受信するシンボル配置情報受信手段をさらにそなえていてもよい。

(32)また、本受信機は、前記重要度の異なる符号の割合に対する前記シンボル配置に関する情報を保持するメモリをさらにそなえ、該復調手段が、該送信機から通知される前記割合に関する情報に基づいて前記シンボル配置に関する情報を該メモリから読み出し、読み出した情報に基づいて前記復調を行なってもよい。

発明の効果

[0045] 上記本発明によれば、少なくとも以下のいずれかの効果ないし利点が得られる。

(1)送信すべき情報系列の符号化により得られる重要度の異なる符号の割合(例えば、符号化率に応じて変化する)に応じて、シンボル配置を等間隔配置から変更する方向に制御することで、重要度の高い符号(あるいは、低い符号)を誤りにくくすることができるので、ビットコレクション処理による効果を十分に発揮させて、多値変調方式の誤り率を改善することができる。

[0046] (2)また、前記シンボル配置の変更前後で送信電力が変化しないように行なえば、シンボル配置変更に伴う送信機における平均送信電力増加による負担増(例えば、必要な増幅特性)を緩和することができる。

(3)さらに、前記ビットコレクション過程において、前記複数の符号のうち重要度の低い複数種類の符号の比率が前記複数ビット毎に均等となるように前記ビット配置を

制御すれば、さらに誤り率の改善効果を得ることができる。

図面の簡単な説明

[0047] [図1]本発明で用いる横方向のビットコレクション方法によるビット配置例を示す図である。

[図2]本発明の第1実施形態に係るターボ符号及びビットコレクションを用いたディジタル無線通信システムの構成を示すブロック図である。

[図3]第1実施形態に係るビットコレクション方法によるビット配置例(符号化率＝1／9の場合)を示す図である。

[図4]第1実施形態の第1態様に係る縦方向のビットコレクション方法によるビット配置例(符号化率＝1／18の場合)を示す図である。

[図5]第1実施形態の第1態様に係る横方向のビットコレクション方法によるビット配置例(符号化率＝1／18の場合)を示す図である。

[図6]第1実施形態の第1態様において格子状16QAMの場合のシンボル配置制御(1ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図7]第1実施形態の第1態様において(4, 12)サーキュラー16QAMの場合のシンボル配置制御(1ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図8]第1実施形態の第1態様において(8, 8)スター16QAMの場合のシンボル配置制御(1ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図9]第1実施形態の第2態様に係る縦方向のビットコレクション方法によるビット配置例を示す図である。

[図10]第1実施形態の第2態様において格子状16QAMの場合のシンボル配置制御(1, 2ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図11]第1実施形態の第2態様において(4, 12)サーキュラー16QAMの場合のシンボル配置制御(1, 2ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図12]図11に示すシンボル配置制御によるシミュレーション結果(シンボル移動量に

対する平均S／N特性(@BLER＝0．1))の一例を示す図である。

[図13]第1実施形態の第2態様において(8，8)スター16QAMでの縦方向のビットコレクション方法によるビット配置例を示す図である。

[図14]第1実施形態の第2態様において(8，8)スター16QAMでの横方向のビットコレクション方法によるビット配置例を示す図である。

[図15]第1実施形態の第2態様において(8，8)スター16QAMの場合のシンボル配置制御(1，2ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図16]図15に示すシンボル配置制御によるシミュレーション結果(シンボル移動量に対する平均S／N特性(@BLER＝0．1))の一例を示す図である。

[図17]第1実施形態の第3態様に係る縦方向のビットコレクション方法によるビット配置例を示す図である。

[図18]第1実施形態の第3態様において格子状16QAMの場合のシンボル配置制御(3，4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図19]第1実施形態の第3態様において(4，12)サーキュラー16QAMの場合のシンボル配置制御(3，4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図20]第1実施形態の第3態様において(8，8)スター16QAMでの縦方向のビットコレクション方法によるビット配置例を示す図である。

[図21]第1実施形態の第3態様において(8，8)スター16QAMでの横方向のビットコレクション方法によるビット配置例を示す図である。

[図22]第1実施形態の第3態様において(4，12)サーキュラー16QAMの場合のシンボル配置制御(3，4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図23]第1実施形態の第4態様に係る縦方向のビットコレクション方法によるビット配置例を示す図である。

[図24]第1実施形態の第4態様において格子状16QAMの場合のシンボル配置制御(4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図25]第1実施形態の第4態様において(4, 12)サーキュラー16QAMの場合のシンボル配置制御(4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図26]第1実施形態の第4態様において(8, 8)スター16QAMの場合のシンボル配置制御(4ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図27]第1実施形態の第5態様に係る横方向のビットコレクションによるビット配置例を示す図である。

[図28]第1実施形態の第5態様において8PSKの場合のシンボル配置制御(1ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図29]第1実施形態の第5態様に係る横方向のビットコレクションによるビット配置例を示す図である。

[図30]第1実施形態の第5態様において8PSKの場合のシンボル配置制御(1, 2ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図31]第1実施形態の第5態様に係る横方向のビットコレクションによるビット配置例を示す図である。

[図32]第1実施形態の第5態様において8PSKの場合のシンボル配置制御(3ビット目の尤度を大きくする場合)を説明するためのシンボル配置図である。

[図33]第1実施形態において時間に応じてシンボル移動制御を変える例を説明すべくビット配置及びシンボル配置の一例を示す図である。

[図34]本発明の第2実施形態に係るターボ符号及びビットコレクションを用いたディジタル無線通信システムの構成を示すブロック図である。

[図35]図34に示す振幅／角度計算部で用いる変換テーブルの一例を示す図である。

[図36]従来の格子状16QAMのシンボル配置図である。

[図37]従来の(4, 12)サーキュラー16QAMのシンボル配置図である。

[図38]従来の(8, 8)スター16QAMのシンボル配置図である。

[図39]従来の8PSKのシンボル配置図である。

[図40]従来のターボ符号器の構成を示すブロック図である。

[図41]従来のターボ復号器の構成を示すブロック図である。

[図42]従来のビットコレクション方法によるビット配置例を示す図である。

[図43]従来の格子状16QAMでのビット割り当て(マッピング)方法を説明するためのシンボル配置図である。

[図44]従来の(4, 12)サーキュラー16QAMでのビット割り当て(マッピング)方法を説明するためのシンボル配置図である。

[図45]従来の(8, 8)スター16QAMでのビット割り当て(マッピング)方法を説明するためのシンボル配置図である。

[図46]従来の8PSKでのビット割り当て(マッピング)方法を説明するためのシンボル配置図である。

[図47]従来の格子状16QAMでのグレイマッピングを説明するためのシンボル配置図である。

符号の説明

[0048] 1 送信機

11 符号化率/変調方式決定部(送信パラメータ決定部)

12 ターボ符号化部(符号化手段)

13 レートマッチング部

14 ビットコレクション部(ビットコレクション手段)

15 変調部(多値変調手段)

16 増幅器(AMP)

17 送信アンテナ

18 制御部(シンボル配置制御手段)

19 振幅/角度計算部(シンボル移動量計算部)

2 受信機

21 受信アンテナ

22 復調部(シンボル配置情報受信手段)

23 符号化率/変調方式決定部(送信パラメータ決定部)

24 復調部(復調手段)

25 ビットコレクション部

26 レートマッチング部

27 ターボ復号部(復号手段)

28 振幅／角度計算部(シンボル移動量計算部)

191 変換テーブル

発明を実施するための最良の形態

[0049] 以下、図面を参照して本発明の実施の形態を説明する。

(1)概要説明

多値変調方式を採用するディジタル無線通信システムにおいて、符号化後に行なうビットコレクション処理の効果を大きくすることを目的として、従来は前述したごとく等間隔で並べられているシンボル配置を移動させて(ずらして)受信側での誤り率を改善する。例えば、誤り耐性を強めたいビットの尤度が大きくなるようにIQ平面においてシンボルを等間隔配置から変更(移動)させる(ずらす)制御を行なう。

[0050] また、符号化率や変調方式によっては従来のビットコレクション方法も変更する。例えば、ターボ符号を用いる場合、1シンボルにおける各ビットに対する複数種類のパリティビットの割り当ての割合が均等となるようにビットコレクション処理を制御する。一例として、従来は図42により前述したごとく縦方向(1シンボルにおける1ビット目(最上位ビット)から最下位ビットに向けて)に割り当てていた処理を、例えば図1に示すごとく横方向(点線矢印参照)に変更する。これにより、ビットコレクションのレベルが変調方式に適したものにすることができる(複数種類のパリティビットを1シンボルのビット毎に均等化できる)。

[0051] (2)第1実施形態の説明

図2は本発明の第1実施形態に係るターボ符号及びビットコレクションを用いたディジタル無線通信システムの構成を示すブロック図で、この図2に示すディジタル無線通信システムは、送信機1と受信機2とをそなえて構成され、送信機1は例えば無線基地局装置(BTS:Base Transceiving Station)に、受信機2は移動端末(MS:Mobile Station)に適用することができる。

[0052] そして、それぞれの受信に基づくと、送信機11は、例えば、符号化率／変調方式決定部11と、データ符号化部12と、パーキャプティング部13と、ビットインタリーバ部14と、変調部15と、増幅器(AMP)16と、送信アンテナ17と、情報部18とを有なえて構成され、受信機21は、例えば、受信アンテナ21と、復調部22と、符号化率／変調方式決定部23と、復調部24と、ビットインタリーバ部25と、パーキャプティング部26と、データ復号部27とをもえて構成されている、なお、図4Oによる側面したデータ符号器は上記送信機11における下一ド符号化部12に、図41による上側したデータ復号器は上記受信機21における下一ド復号部27に用いることができる。

[0053] ここで、送信機11において、符号化率／変調方式決定部(送信パラメータ決定部)11は、制御部18からの制御情報に基づいてデータ符号化部12での符号化率および変調部15での変調方式(例えば、QPSK、16QAM)を含む送信パラメータを決定するものであり、データ符号化部(符号化手段)12は、この符号化率／変調方式決定部11で決定した符号化率に基づいて送信データをデータ符号化するので、例えば上記データ符号器により、組織ビット(S)と2種類のパリティビット(P1, P2)が得られるとになっている。なお、これら組織ビット(S)とパリティビット(P1, P2)の生成割合は基本的に上記符号化率に依存するが、下記インタリーバ部13によるパンクチャリング処理にも依存する。

[0054] インタリーバ部13は、データ符号化部12により符号化されたビット系列について、ビット様の並び換え、あるいは、パンクチャリング(間引き)処理を行なうことにより、上記ビット系列を無難リングの所定チャネルのビット数に合わせるインタリーバを行なうものである。

ビットコレクション部(ビットコレクション手段)14は、前述した通り、上記インタリーバ後のビット系列について、データム推定(モード)の場合、1シンボルにおける変調(品質)の高いビット目及び2ビット目に組織ビット(S)が優先的に割り当てられるようにビットコレクションを行なうものである。ただし、後述するように、主旋のみいビットコレクションを行なうモード(区転モード)も存在する。

[0055] 変調部(多値変調手段)15は、上記符号化率／変調方式決定部11で決定した変

調方式で上記ビット系列を変調する(シンボルにマッピング)するものである。ただし、QPSKの場合については、上記ビットコレクションは行なわれない(基本的に、適応変調において16QAMが選択された場合に行なわれる)。

増幅器16は、上記変調部15により得られた変調データ[ただし、図示しないディジタル/アナログ変換器及び周波数変換器(アップコンバータ)等により得られるアナログ無線信号]を所要の送信信号レベルに増幅するものであり、送信アンテナ17は、増幅後の送信信号を伝搬路へ送信するものである。

[0056] そして、制御部(シンボル配置制御手段)18は、上記符号化率/変調方式決定部11、ビットコレクション部14、変調部15の動作を制御するもので、本実施形態では、上述した符号化率及び変調方式の決定要素となる情報(制御情報)を符号化率/変調方式決定部11に与えるほか、次のような機能も有している。

即ち、ビットコレクション部14でのビットコレクション処理を、組織ビット(S)とパリティビット(P1, P2)の割合(配分比)(主として、前記符号化率に依存する)に応じて制御してパリティビットの割り当て方法を制御するとともに、変調部15でのマッピング処理を前記符号化率に応じて制御して複素平面(IQ平面)におけるシンボル配置を等間隔配置からシンボル単位で所要量だけ移動制御(シンボル配置制御)することができ、前記符号化率に応じたビットコレクション制御情報をビットコレクション部14に、シンボル移動量情報(振幅、位相)を変調部15にそれぞれ与えることができる。

[0057] なお、ビットコレクション制御及びシンボル移動制御の詳細については後述する。また、上記のビットコレクション制御情報及びシンボル移動量情報は、受信機2への制御情報に含めることで受信機2に通知することができる。

例えば、3GPP(3rd Generation Partnership Project)のHSDPA(High Speed Downlink Packet Access)仕様では、パケットを送信する前に制御チャネルにて受信機2へシグナリングを行ない、これによって、送信機1側で適応的に選択した符号化率や変調方式に関する情報(送信パラメータ)を予め送信側から受信側に伝えておくようになっているので、これを利用して上記制御情報の通知が可能である。

[0058] 即ち、HSDPAの場合、HS-PDSCH(High Speed-Physical Downlink Shared Channel)で送信するデータに関する情報をHS-SCCH(High Speed-Shared Control Channel

)で予め送信側から受信側へ伝えておくので、変調方式等の情報と同様に、送信機1から上述したシンボル配置やビットコレクション方法が変化する情報をシグナリングパケットで受信機2へ送信すれば、受信機2はこれらの変化を認識してその変化に応じた復調、復号処理が可能となる。

[0059] 一方、受信機2において、受信アンテナ21は、上記送信機1からの無線信号を受信するものであり、復調部22は、この受信アンテナ21で受信された信号のうち制御情報を送信機1での変調方式に対応した復調方式で復調するものである。つまり、この復調部22は、送信機1側で変更したシンボル配置に関する情報を送信機1から受信するシンボル配置情報受信手段としての機能を果たすことになる。なお、受信信号をベースバンド信号に周波数変換する周波数変換器(ダウンコンバータ)やアナログ信号をディジタル信号に変換するディジタル/アナログ変換器等の図示は省略している。

[0060] 符号化率/変調方式決定部(送信パラメータ決定部)23は、上記復調部22で復調された制御情報に基づいて送信機1側での符号化率及び変調方式に関する情報(送信パラメータ)を同定して復調部24に通知するもので、本例では、上述したシンボル移動量情報も併せて復調部24に通知できるようになっている。

復調部(復調手段)24は、上記符号化率/変調方式決定部23から通知される情報に基づいて、受信信号のうち上記制御情報以外のデータ(ユーザデータ)を送信機1での変調方式に対応した復調方式で上記シンボル移動量情報(シンボル配置に関する情報)を加味して(基づいて)復調するものである。

[0061] ビットコレクション部25は、上記復調部24で復調されたデータを送信機1側のビットコレクション部14でのビットコレクションとは逆の操作(ビットコレクション)を行なうことにより、復調データを送信機1でのビットコレクション前に相当するデータに戻すためのものであり、レートマッチング部26は、同様に、送信機1側のレートマッチング処理13とは逆の処理(パンクチャリング又はビット繰り返し)を行なうことにより、送信機1でのレートマッチング前に相当するビット系列を得るためのものである。

[0062] ターボ復号部(復号手段)27は、上記レートマッチング後のビット系列についてターボ復号処理を施して、元の送信データ(情報系列)を復号するものである。

上述のごとく構成された本実施形態のシステムでは、送信機1において、符号化率／変調方式決定部11で決定された符号化率で送信データがターボ符号化部12によりターボ符号化され(符号化過程)、レートマッチング部13及びビットコレクション部14にてそれぞれレートマッチング処理及びビットコレクション処理を施された後(ビットコレクション過程)、当該ビットコレクション処理によるビット配置に従って、変調部15にて符号化率／変調方式決定部11で決定された変調方式(例えば、16QAM)で変調処理(シンボルへのマッピング処理)が施される(多値変調過程)。

[0063] その際、制御部18によって、組織ビット(S)とパリティビット(P1, P2)の割合に応じてマッピング処理が制御されて、IQ平面におけるシンボル配置がシンボル単位でIQ平面における等間隔配置位置から所要量だけ移動制御される(シンボル配置制御過程)。なお、その移動量(振幅や位相)に関する情報(即ち、シンボル配置に関する情報)は前記制御情報(例えば、前記HS-SCCHのシグナリングパケット)に含められて受信機2へ事前に通知される。

[0064] 変調部15により得られた変調データは、アナログ無線信号に変換された上で、増幅器16にて所要の送信信号レベルに増幅された後、送信アンテナ17を通じて伝播路へ送信される。

一方、受信機2では、受信アンテナ21にて、上記送信機1からの無線信号が受信され、当該受信信号に含まれる制御情報が復調部22にて復調され、当該制御情報に基づいて符号化率／変調方式決定部23にて送信機1で用いられた符号化率及び変調方式が同定され、変調方式に関する情報が復調部24へ、符号化率に関する情報がターボ復号部27へそれぞれ通知される。なお、上記制御情報にシンボル移動量情報が含まれている場合は、当該情報も復調部24へ通知される。

[0065] 一方、受信信号のうちユーザデータは、復調部24にて符号化率／変調方式決定部23から通知された情報(変調方式に関する情報及びシンボル移動量情報)に基づいて送信機1での変調方式に対応する復調方式で復調され(復調過程)、ビットコレクション部25及びレートマッチング部26にてそれぞれビットコレクション処理及びレートマッチング処理が施された後、ターボ復号部にてターボ復号され、これにより、送信機1の送信した情報系列が得られる(復号過程)。

[0066] 次に、以下では、本実施形態の要旨である上記制御部18によるビットコレクション制御及びシンボル移動制御について詳述する。なお、以下の説明において、シンボルの移動方向は複素平面(IQ平面)において、必ずしも円上や水平方向、垂直方向に限られない。符号化率に応じて所望の尤度を得たい1シンボル内のビットについて尤度が大きくなる方向に移動制御すればよい。

[0067] (2.1)第1態様(1ビット目の尤度のみを大きくする場合)

例えば、符号化率が1／4未満で、図3に示すように、ビットコレクション処理において組織ビット(S1～S4)が1ビット目(第1ビット)のみに割り当てられる(網かけ部参照)ことになる場合は、組織ビット(S1～S4)が割り当てられる1ビット目の尤度のみを大きくする。この場合は、シンボル配置を変えることに加えて、ビットコレクション部14でのビットコレクション方法も変えることで、より最適化を行なうことができる。

[0068] 即ち、この図3中に点線矢印で示すごとくパリティビット(P1－1～P1－12, P2－1～P2－12;以下、これらを区別しない場合はそれぞれP1, P2と表記する)を紙面横方向に敷き詰めて(割り当てて)いくとパリティビットP1とパリティビットP2がビット毎に均等な配分になる。このようにビットコレクション方法を変更することによる改善効果は、組織ビット(S)の数が少ないほど大きい(従来のビットコレクション方法では、組織ビット(S)が少ないほどビット毎のパリティビット(以下、単に「パリティ」と略記することもある)P1とパリティビットP2の偏りが大きくなる)。

[0069] 仮に、符号化率＝1／18で従来のビットコレクション方法によりビットコレクション処理を行なったとすると、図4に示すようになる。即ち、1シンボルにおける各ビットに対するパリティP1, P2の配分は次表1に示すようになる。

[0070] [表1]

縦方向のビットコレクションによるパリティ配分(符号化率＝1/18)

1シンボル内のビット位置	パリティＰ１の数	パリティＰ２の数
１ビット目	0	7
２ビット目	6	1
３ビット目	1	6
４ビット目	6	1

[0071] このように、符号化率が1／4未満の場合の中でも特に符号化率が小さい場合、従

来通りのビットコレクション方法ではパリティP1とパリティP2の配分は偏りが大きくなってしまう。

これに対して、ビットコレクション方法を変更した場合（パリティP1，P2を横方向に敷き詰める場合）は、図5に示すようになる。即ち、1シンボルにおける各ビットに対するパリティP1，P2の配分は次表2に示すようになる。

[0072] [表2]

横方向のビットコレクションによるパリティ配分（符号化率＝1/18）

1シンボル内のビット位置	パリティP1の数	パリティP2の数
1ビット目	3	4
2ビット目	5	4
3ビット目	4	5
4ビット目	3	4

[0073] このように、パリティP1，P2を横方向に割り当てていくことで，各ビットのパリティP1とパリティP2の配分がより均等に近くなることが分かる。これにより，従来法に比して、受信機2での誤り率の改善効果向上が期待できる。なお、以下では、従来法によるビットコレクションを「縦方向のビットコレクション」、本例でのビットコレクションを「横方向のビットコレクション」と称する。

[0074] このような横方向のビットコレクションをビットコレクション部14にて行なった上で、制御部18は，変調部15にて、組織ビット（S）が割り当てられている1ビット目（第1ビット）の尤度を大きくする方向にシンボルを移動させる。

(2.1.1)格子状16QAMの場合

この場合、制御部18は、例えば図6に示すように、IQ平面において、虚軸（Q軸）に最も近い8シンボル（符号5参照）がお互いに離れる方向に移動させる（シンボルの振幅、位相を変化させることを意味する。以下、同じ）制御を行なう（矢印7参照）。ここで、電力（送信電力：以下、同じ）を一定とする（つまり、シンボル配置変更前後で電力が変わらないようにする。以下、同じ）場合には、虚軸から遠い外側の8シンボル（符号6参照）については内側（虚軸に近づく方向）に移動させる（矢印8参照）。

[0075] このようなシンボル移動を行なうことにより、Q軸を中心とする象限判定である1ビット目の尤度は大きくなり、Q軸からの距離判定である3ビット目の尤度はシンボル密度

が高くなるので小さくなる。シンボルをこのように移動させるにつれて、1ビット目の尤度が大きくなるので、全体の誤り率は少しずつ改善してゆくが、ある量まで移動させると特性の改善は止まる。さらに、それ以上シンボルを移動させると、今度は3ビット目の尤度が小さくなり過ぎてしまうので全体の誤り率特性が劣化することになる。なお、2ビット目、4ビット目の尤度は変わらない。

[0076] また、上述のごとく前記シンボル配置の変更前後で電力一定となるようにシンボル移動制御を行なうことにより、シンボル配置変更に伴う送信機1における平均送信電力増加による負担増、例えば、PAPRを緩和して、増幅器16に必要な増幅特性を緩和することができる。なお、この点は、以降に説明する各種態様において電力一定となるようにシンボル配置制御を行なう場合についても同様である。本発明とPAPRとの関係については後述する。

[0077] (2.1.2)(4, 12)サーキュラー16QAMの場合

この場合、制御部18は、例えば図7に示すように、虚軸(Q軸)に最も近い4シンボル(符号5参照)がお互いに離れる方向(Q軸から離れる方向)に移動させる制御を行なう。その際、電力を一定とする場合には、移動対象のシンボルを同一円に沿って移動させる(矢印7参照)。

[0078] なお、最適な移動量は内円と外円の半径比によって変わる。即ち、サーキュラー状の変調方式である(4, 12)サーキュラー16QAMと(8, 8)スター16QAMは、内側の円と外側の円の半径比がパラメータとして特性に影響を与える。そして、ターボ符号を用いる場合、最適な半径比は符号化率と関係がある。つまり、ある符号化率が与えられたとき(伝送路の状態で決まる)に、最適な半径比が一つ求まる。例えば、符号化率が小さい程(組織ビットが少ない程)、最適な半径比も小さくなり、逆に、符号化率が大きい程(組織ビットが多い程)、最適な半径比も大きくなる。その理由は、次の通りである。

[0079] 符号化率が小さい場合というのは、組織ビットが少ないことを意味する。この場合、組織ビットの尤度を強くするように半径の比を調節することで、最適な特性を得ることができる。組織ビットは1ビット目から順に割り当てられるので、1ビット目の尤度が強くなるように半径比を決めることを考えると、半径比が小さい(内側の円と外側の円の半

径が近くなる)場合に最適となる。

[0080] このように半径比を小さくすると、(4, 12)サーキュラー16QAMの場合は3, 4ビット目の尤度が小さくなり、(8, 8)スター16QAMの場合は4ビット目の尤度が小さくなるが、それらに割り当てられているビットはパリティビットである。符号化率が小さい場合は、たくさんあるパリティビットを犠牲にしてでも数少ない組織ビットの尤度を大きくすることで最適な特性が得られる。

[0081] 符号化率が大きくなると、逆にパリティビットの尤度を大きくするようにすることで最適な特性を得ることができる。つまり、最適な半径の比は大きくなる。ただし、符号化率が大きくなってゆくということは、符号化を行なわない場合に近づくということを意味するので、最終的には符号化を行なわない場合に最適な半径比(=Raw BER(Bit Error Rate)特性での最適な半径比であり、全16シンボルができるだけ等間隔に並ぶ状態)に近づく。

[0082] また、シンボルを移動させて特性を改善させることが本発明のアイディアだが、この移動量(円に沿って移動させる場合には θ)の最適値も半径比と関係がある。即ち、内側の円と外側の円の半径比が小さい程、最適な移動量は大きくなり、逆に、内側の円と外側の円の半径比が大きい程、最適な移動量は小さくなる。

以上より、符号化率と最適な移動量の関係も考えることができる。即ち、符号化率が小さい程、最適な半径比が小さくなるので、最適な移動量は大きくなる。逆に、符号化率が大きい程、最適な半径比が大きくなるので、最適な移動量は小さくなる。

[0083] よって、組織ビットの量によって最適な半径比と最適な移動量は異なる。この点は以降の説明においても特に断らない限り同様である。

(2.1.3)(8, 8)スター16QAMの場合

この場合、制御部18は、例えば図8に示すように、虚軸(Q軸)に最も近い8シンボル(符号8参照)がお互いに離れる方向(Q軸から離れる方向)に移動させる制御を行なう。ここで、電力を一定とする場合には、移動対象のシンボルは同一円に沿って移動させる(矢印7参照)。なお、この場合も、最適な移動量は内円と外円の半径比によって変わる。

[0084] (2.2)第2態様(1ビット目と2ビット目の尤度を大きくする場合)

例えば、符号化率が1／4以上、1／2未満の場合で、図9に示すように、ビットコレクション処理において組織ビット(S1～S12)が1ビット目(第1ビット)と2ビット目(第2ビット)に割り当てられる(網かけ部参照)ことになる場合は、組織ビット(S1～S12)が割り当てられる1ビット目と2ビット目の尤度を大きくする。

[0085] この場合、制御部18は、(8, 8)スター16QAMの場合についてのみビットコレクション部14でのビットコレクションの方法を横方向のビットコレクションに変更する。なお、格子状16QAMと(4, 12)サーキュラー16QAMの場合については、ビットコレクション方法を横方向に変更しても、パリティP1, P2の配分に大きな差は生じない(特性は変化しない)ので、図9に示すごとく、従来通りの縦方向のビットコレクションでよい。

[0086] (2.2.1)格子状16QAMの場合

この場合、制御部18は、例えば図10に示すように、16シンボルすべてを移動制御する。即ち、組織ビット(S)に関係する内側の(I軸及びQ軸に近い)シンボルは外方向(I軸及びQ軸から離れる方向)へ移動させるため、電力を一定とする場合は外側のシンボルを内側方向(I軸及びQ軸に近づく方向)へ移動させる。これにより、I軸又はQ軸を中心とする象限判定である1ビット目、2ビット目(第1, 第2ビット)の尤度は大きくなる(I軸又はQ軸からの距離判定である3ビット目(第3ビット)、4ビット目(第4ビット)の尤度はシンボル密度が高くなるので小さくなる)。

[0087] このようにシンボルを移動させるにつれて、ある量までは組織ビット(S)の尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、3ビット目と4ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化することになる。

(2.2.2)(4, 12)サーキュラー16QAMの場合

この場合、制御部18は、例えば図11に示すように、外側の円上で実軸(I軸)に近い4シンボル(符号6参照)及び虚軸(Q軸)に近い4シンボル(符号5参照)の計8シンボルを移動制御する。その際、電力を一定とする場合、移動対象のシンボルは同一円に沿って移動させる(矢印7参照)。これにより、I軸又はQ軸を中心とする象限判定である1ビット目、2ビット目の尤度が大きくなる(I軸又はQ軸からの距離判定である3ビット目、4ビット目の尤度は小さくなる)。

[0088] この場合も、シンボルを移動させるにつれて、ある移動量までは組織ビット(S)の尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、3ビット目と4ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。なお、最適な移動量は内円と外円の半径比によって変わる。例えば、符号化率＝1／3の場合、最も特性の良い内円と外円の半径比は1:1．9である。ただし、符号化率が変わると、最適な半径比も変わる(即ち、最適なθ(円に沿って移動させる場合の角度)も変わる)。

[0089] 図12に、内円と外円の半径比＝1:1．9とし、図11に示すように円に沿ってシンボルを角度θだけ移動させた場合のθに対するBLER＝0．1を満たす所要平均S／N特性を示す。この図12から、シンボルを移動させない場合(θ＝0°)(つまり、等間隔配置)に比べて、最大で約0．2dB(θ＝6°)の改善が得られることが分かる。

このように、最適なシンボル移動量を設定することで、ビットコレクション処理の効果の方が大きくなり、全体としては復号後の特性を改善できる。

[0090] (2.2.3)(8，8)スター16QAMの場合

この場合、制御部18は、前記第1態様と同様にシンボル配置を変えることに加えて、ビットコレクション部14でのビットコレクション方法も変更することで、より最適化を行なうことができる。

ここで、(8，8)スター16QAMの場合は、3ビット目と4ビット目の尤度が異なるため、3ビット目で均等にパリティP1とパリティP2を割り当て、4ビット目も同様に均等にパリティP1，P2を割り当てる。格子状16QAMと(4，12)サーキュラー16QAMでは、3ビット目と4ビット目の尤度は等しいので、3ビット目と4ビット目を合計した中でパリティP1とパリティP2が均等な数になっていれば良い。故に、格子状16QAMと(4，12)サーキュラー16QAMについては、従来通りのビットコレクション方法でよい。

[0091] 仮に、従来通りの縦方向のビットコレクション方法を行なうとすると、図13に示すようになる。即ち、1シンボルにおける各ビットに対するパリティP1，P2の配分は次表3に示すようになる。

[0092] [表3]

縦方向のビットコレクションによるパリティ配分

1シンボル内のビット位置	パリティP1の数	パリティP2の数
2ビット目	3	3
3ビット目	3	6
4ビット目	6	3

[0093] このように、従来のビットコレクション方法では3ビット目と4ビット目のパリティはパリティP1とパリティP2とで均等にならない。

これに対して、横方向のビットコレクションを行なうと、図14に示すようになり、1シンボルにおける各ビットに対するパリティP1，P2の配分は次表4に示すようになる。

[0094] [表4]

横方向のビットコレクションによるパリティ配分

1シンボル内のビット位置	パリティP1の数	パリティP2の数
2ビット目	3	3
3ビット目	4	5
4ビット目	5	4

[0095] このように、従来のビットコレクション方法よりもパリティP1，P2の配分を均等にすることができる。

(2.2.4)(8，8)スター16QAMで移動させるシンボル

制御部18は、例えば図15に示すように、16シンボル全てを移動制御する。即ち、I軸に近い8シンボル(符号6参照)はお互いに離れる方向(I軸から離れる方向)に移動させるとともに、Q軸に近い8シンボル(符号5参照)もお互いに離れる方向(Q軸から離れる方向)に移動させる。その際、電力を一定とする場合、移動対象のシンボルは同一円に沿って移動させる(矢印7参照)。これにより、1ビット目、2ビット目の尤度が大きくなる(3ビット目の尤度はシンボル密度が高くなるので小さくなる)。なお、シンボルを同一円に沿って移動させる場合は4ビット目の尤度は変わらない。

[0096] このようにシンボルを移動させるにつれて、ある量までは組織ビットの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、3ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。この場合も、前記理由により、最適な移動量は内円と外円の半径比(4ビット目の尤度)によっても変わる。ただし、符号化率＝1／3の場合、(8，8)スター16QAMは半径比による特性の変

化がほとんどない。よって、一例として、図16に、半径比＝1：1．5の場合に図15に示すようにシンボルを円に沿って角度θだけ移動させた場合のシミュレーション結果、即ち、θに対するBLER＝0．1を満たす所要平均S／N特性を示す。この図16から、θ＝4°の場合に最も良い特性が得られることが分かる。

[0097] なお、図16では、図12に示した(4，12)サーキュラー16QAMの場合のグラフに合わせる形で符号化率＝1／3としたが、符号化率が異なる場合（例えば、符号化率＝1／2等）には半径比による特性の変化がみられるため、最適な半径比を同定することも可能である。

(2.3)第3態様(3ビット目と4ビット目の尤度を大きくする場合)

例えば、符号化率が1／2以上、3／4未満の場合で、図17に示すように、ビットコレクション処理において組織ビット(S1～S22)が1～3ビット目（第1～第3ビット)の途中まで割り当てられる(網がけ部参照)ことになる場合は、受信機2では、数少ないパリティビット(P1，P2)の重要度が高くなるので，パリティビットP1，P2が割り当てられる3ビット目と4ビット目の尤度を大きくする。

[0098] この場合も、第2態様と同様に(8，8)スター16QAMの場合についてのみビットコレクション方法を変更する。格子状16QAMと(4，12)サーキュラー16QAMの場合は、横方向のビットコレクション方法に変更しても特性は変化しないので、図17に示すように、従来通りの縦方向のビットコレクション方法でよい。

(2.3.1)格子状16QAMの場合

この場合、制御部18は、図18に示すように、16シンボルの全てを移動制御する。即ち、I軸及びQ軸に近い内側の8シンボルはI軸及びQ軸に近づく方向に移動させる。その際、電力を一定とする場合、内側の(I軸、Q軸に近い)シンボルを内方向へ移動させるため、外側の(I軸、Q軸から遠い)シンボルは外方向へ移動させる。これにより、3ビット目、4ビット目の尤度が大きくなる(1ビット目，2ビット目の尤度は小さくなる)。

[0099] 本例においても、上記のようにシンボルを移動させるにつれて，ある量まではパリティの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると，1ビット目と2ビット目の尤度が小さくなり過ぎるため、逆に特性は劣

化する。

(2.3.2)(4, 12)サーキュラー16QAMの場合

この場合，制御部18は、図19に示すように、実軸(I軸)に近い4シンボル(符号6参照)及び虚軸(Q軸)に近い4シンボル(符号5参照)の計8シンボルをそれぞれI軸、Q軸に近づく方向に移動制御する。その際、電力を一定とする場合、移動対象のシンボルは同一円に沿って移動させる(矢印7参照)。これにより，3ビット目、4ビット目の尤度は大きくなる(1ビット目、2ビット目の尤度はシンボル密度が高くなるので小さくなる)。

[0100] この場合も、上記のようにシンボルを移動させるにつれて、ある量まではパリティビットの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、1ビット目と2ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。

(2.3.3)(8, 8)スター16QAMの場合

この場合，制御部18は、シンボルの配置を均等配置から変更することに加えて、第1態様と同様にビットコレクション方法も変更することで、より最適化を行なうことができる。仮に、従来通りの縦方向のビットコレクションを行なうとすると、図20に示すようになる。即ち、1シンボルにおける各ビットに対するパリティP1，P2の配分は次表5に示すようになる。

[0101] [表5]

縦方向のビットコレクションによるパリティ配分

1シンボル内のビット位置	パリティP1の数	パリティP2の数
3ビット目	0	8
4ビット目	7	9

[0102] このように、従来のビットコレクション方法では3ビット目と4ビット目のパリティはパリティP1とパリティP2とで均等にならない。

これに対して、横方向のビットコレクションを行なうと、図21に示すようになり、1シンボルにおける各ビットに対するパリティP1，P2の配分は次表6に示すようになる。

[0103] [表6]

横方向のビットコレクションによるパリティ配分

1シンボル内のビット位置	パリティP1の数	パリティP2の数
3ビット目	2	2
4ビット目	6	4

[0104] このように、従来のビットコレクション方法よりもパリティP1, P2の配分を均等にすることができる。

(2.3.4)(8, 8)スター16QAMで移動させるシンボル

制御部18は、図22に示すように、16シンボル全てを移動制御する。即ち、I軸に近い8シンボル(符号6参照)はお互いに近づく方向(I軸に近づく方向)に、Q軸に近い8シンボル(符号5参照)もお互いに近づく方向(Q軸に近づく方向)にそれぞれ移動させる。その際、電力を一定とする場合、移動対象のシンボルは同一円に沿って移動させる(矢印7参照)。これにより、3ビット目の尤度が大きくなる(1ビット目と2ビット目の尤度は小さくなる)。また、内側の円と外側の円の半径の比を大きくすれば(矢印8参照)、4ビット目の尤度を大きくすることができる。

[0105] このようにシンボルを移動させるにつれて、ある量まではパリティビットの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、1ビット目と2ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。

(2.4)第4態様(4ビット目の尤度だけを大きくする場合)

例えば、符号化率が3／4以上の場合で、例えば図23に示すように、ビットコレクション処理において組織ビット(S1～S27)が1～3ビット目のすべてに割り当てられる(網かけ部参照)ことになる場合は、パリティビット(P1, P2)が割り当てられる4ビット目のみの尤度を大きくする。

[0106] この場合は、ビットコレクション部14でのビットコレクション方法を横方向のビットコレクション方法に変更しても従来の縦方向のビットコレクション方法と効果(ビット配置)は変わらないので、従来通りの縦方向のビットコレクション方法でよい。

(2.4.1)格子状16QAMの場合

この場合、制御部18は、図24に示すように、16シンボルの全てを移動制御する。即ち、I軸に近い8シンボルを内側(I軸に近づく方向)に移動させるとともに、I軸から遠い8シンボルを外側(I軸から離れる方向)に移動させる。これにより、4ビット目の尤

度が大きくなる(2ビット目の尤度は小さくなる)。なお、1ビット目と3ビット目の尤度は変わらない。

[0107] この場合も、上記のようにシンボルを移動させるにつれて、ある量まではパリティの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、2ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。

(2.4.2)(4, 12)サーキュラー16QAMの場合

この場合、制御部18は、図25に示すように、実軸(I軸)及び虚軸(Q軸)にそれぞれ近い8シンボルをI軸及びQ軸に近づく方向に移動制御する。その際、電力を一定とする場合、移動対象のシンボルは同一円に沿って移動させる。これにより、4ビット目の尤度が大きくなる(1ビット目、2ビット目の尤度は小さくなる)。

[0108] この場合も、上記のようにシンボルを移動させるにつれて、ある量まではパリティビットの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、1ビット目と2ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。

(2.4.3)(8, 8)スター16QAMの場合

この場合、制御部18は、図26に示すように、内円と外円の半径比を小さくする制御を行なう(矢印9参照)。その際、電力を一定とする場合、内円の半径だけでなく外円の半径も変化させる。これにより、4ビット目の尤度が大きくなる(1ビット目、2ビット目、3ビット目の尤度は小さくなる)。

[0109] このように半径比を変化させるにつれて、ある量まではパリティの尤度が大きくなる効果のために全体の誤り率は少しずつ改善していくが、それ以上移動させると、1ビット目、2ビット目、3ビット目の尤度が小さくなり過ぎるため、逆に特性は劣化する。

(2.5)第5態様(8PSKの場合)

(2.5.1)1ビット目の尤度を大きくしたい場合

8PSKの場合も、制御部18は、ビットコレクション部14でのビットコレクション方法を変更する(横方向のビットコレクションを行なう)(図27参照)。また、制御部18は、図28に示すように、虚軸(Q軸)に最も近い4シンボル(符号5参照)がお互いに離れる方向に移動制御する。その際、電力を一定とする場合、移動対象のシンボルは円に沿

って移動させる(矢印7参照)。

[0110] (2.5.2)2ビット目の途中まで組織ビットが割り当てられる場合(図29参照)

この場合も、制御部18は、図29に示すように、ビットコレクション部14でのビットコレクション方法を変更する(横方向のビットコレクションを行なう)。また、制御部18は、図30に示すように、8シンボルの全てを移動制御する。即ち、虚軸(Q軸)に近い4シンボル(符号5参照)をQ軸から離れる方向に、実軸(I軸)に近い4シンボル(符号6参照)をI軸から離れる方向にそれぞれ移動させる。その際、電力を一定とする場合、移動対象のシンボルは円に沿って移動させる(矢印7参照)。

[0111] (2.5.3)3ビット目の途中まで組織ビットが割り当てられる場合(図31参照)

この場合、ビットコレクション方法を変更しても意味がない(同じ割り当てになる)ので、従来通りの縦方向のビットコレクション方法でよい。制御部18は、図32に示すように、8シンボル全てを移動制御する。即ち、Q軸に近い4シンボル(符号5参照)はQ軸に近づく方向に、I軸に近い4シンボル(符号6参照)はI軸に近づく方向に移動させる。その際、電力を一定とする場合、移動するシンボルは円に沿って移動させる。

[0112] (2.6)時間に応じてシンボル移動制御を変える場合[一例として、(8, 8)スター16QAMの場合]

上述した第1～第4態様までの方法を時間によって適応的に変化させることで、より最適化することができる。即ち、制御部18は、例えば図33に示すように、初めの4シンボル(太実線両矢印31参照)については前記第4態様を適用して図32中の(1)で示すごとく内円と外円の半径の比を大きくするようにシンボルを移動させ、その後に続く5シンボル(太実線両矢印32参照)については、例えば前記第2態様を適用して図33中の(2)で示すごとく組織ビットSの誤り耐性を強めるようにシンボルを移動させる。

[0113] なお、図33では(8, 8)スター16QAMの場合を示しているが、(4, 12)サーキュラー16QAM、格子状16QAM、8PSKの場合についても同様に実施可能であり、また、組織ビットが1ビット目の途中まで割り当てられる場合、2ビット目の途中まで割り当てられる場合、4ビット目の途中まで割り当てられる場合などについても、上記と同様に実施可能である。

[0114] (2.7)組織ビットとパリティビットが入れ替わる場合

上述した例では、組織ビットは1ビット目から順に配置されることを前提としており、組織ビットが増えるにつれて、順に、2ビット目、3ビット目にも組織ビットが割り当てられるが、組織ビットとパリティビットとの関係を反転して、1ビット目にパリティビットから割り当てていく方法も考えられる(例えば、前記非特許文献5のChapter 4.5.4.3, Chapter 4.5.7, Chapter 4.6.2.1等参照)。

[0115] その場合も上述した例と同じ方法で対処可能である。即ち、パリティビットの数が少ない場合はパリティビットに割り当てられたビットの尤度が大きくなるようにシンボルを移動させ、パリティビットの数が多い場合は組織ビットの尤度が大きくなるようにシンボルを移動させればよい。

以上のように、符号化率(組織ビットとパリティビットの割合)に応じて、シンボル配置を等間隔配置から変更する方向に制御することで、ターボ符号を用いる際の16値又は8値変調方式の誤り率を改善することができる。これはビットコレクション処理を行なう場合に、従来の16点のシンボルが等間隔に並ぶシンボル(信号点)配置では十分にビットコレクション処理の効果を得られなかったが、シンボルを等間隔配置から移動させて、所望のビットの尤度を大きくし重要度の高い組織ビット(又はパリティ)を誤りにくくすることで、ビットコレクション処理の効果が大きくなるためである。

[0116] (3)第2実施形態の説明

図34は本発明の第2実施形態に係るターボ符号及びビットコレクションを用いたディジタル無線通信システムの構成を示すブロック図で、この図34に示すディジタル無線通信システムも、送信機1と受信機2とをそなえて構成され、送信機1は例えば無線基地局装置(BTS)に、受信機2は移動端末(MS)に適用することができ、図2に示す構成に比して、送信機1に振幅/角度計算部19が付加されるとともに、受信機2に振幅/角度計算部28が付加されている点が異なる。なお、他の既述の符号と同一符号を付したものは、特に断らない限り、既述のものと同一若しくは同様のものである。

[0117] ここで、送信機1の振幅/角度計算部(シンボル移動量計算部)19は、例えば図35に示すように、符号化率{R(0)～R(1), R(1)～R(2), …, R(n－1)～R(n):nは

2以上の整数]（送信ブロックサイズ）やRVパラメータに応じた振幅比a(i)及び角度θ(i)（ただし、i=1～n）をテーブル形式のデータ191（以下、変換テーブル191と表記する）として予め図示しないメモリ等に記憶しておき、符号化率／変調方式決定部11で決定した符号化率に対するシンボル移動量として振幅比a(i)及び角度θ(i)をこの変換テーブル191から読み出して出力（変調部15に通知）することができるものである。

[0118] なお、「RVパラメータ」とは、前述したビットコレクションにおいて組織ビットとパリティビットとの関係を反転する（つまり、パリティビットを1シンボル内の1ビット目から優先的に割り当ててゆく）モードや、レートマッチング部13において組織ビットを優先的にパンクチャするモード、パリティビットを優先的にパンクチャするモード等のモード設定情報を表し、これらのモード設定に応じてビットコレクション処理における組織ビットとパリティビットの割合（比）が変化することになる。

[0119] したがって、組織ビットを1シンボル内の1ビット目から順に割り当ててゆき、かつ、パリティビットを優先的にパンクチャするデフォルトモードでは、図34に示すごとく単純に符号化率に振幅比a(i)及び角度θ(i)を対応付けた変換テーブル191を用意すればよいが、デフォルトモード以外では、RVパラメータも考慮する必要がある。結局、組織ビットとパリティビットの割合に応じて振幅比a(i)及び角度θ(i)を対応付けた変換テーブル191を用意しておけばよいことになる。

[0120] 一方、受信機2の振幅／角度計算部（シンボル移動量計算部）28も、送信機1における上記振幅／角度計算部19と同様に、送信機1側と同じ変換テーブル191を図示しないメモリ等に予め記憶しておき、符号化率／変調方式決定部23で決定した符号化率に対するシンボル移動量情報として振幅比a(i)及び角度θ(i)をこの変換テーブル191から読み出して出力（復調部24に通知）することができるものである。つまり、前記メモリは、重要度の異なる符号、即ち、組織ビットとパリティビットの割合に対するシンボル配置に関する情報を保持していることになる。

[0121] 換言すれば、復調部24は、送信機1から制御情報により通知される符号化率（組織ビットとパリティビットの割合）に関する情報に基づいて、上記メモリ（変換テーブル191）からシンボル移動量情報を読み出し、読み出した情報に基づいて復調処理を行

なうことになる。

つまり、本実施形態では、第1実施形態のように振幅比、角度等の送信機1側でのシンボル移動量情報を受信機2へ送信される制御情報(シグナリングパケット)に含めるのではなく、予め送信機1及び受信機2の双方で同じ変換テーブル191を具備しておくことで、制御情報量を増やすことなく、第1実施形態と同等の処理を可能としているのである。

[0122] したがって、上述した第1実施形態と同等の効果ないし利点が得られるほか、第1実施形態に比して、送信機1及び受信機2間の無線リソースの有効利用を図りつつ、受信機2において送信機1側でのビットコレクション及びシンボル配置の変化に応じた復調、復号処理を確実かつ高速に実行することが可能となる。

(4)その他

・各変調方式の誤り率とPAPR(Peak-to-Average Power Ratio)の関係について

復号後のBER特性とBLER特性は、符号化率=1/3の場合、格子状16QAMでは、Raw BER特性、復号後のBER特性、BLER特性のいずれも良いが、(8, 8)スター16QAM及び(4, 12)サーキュラー16QAMに比べてPAPRは大きい。(8, 8)スター16QAMでは、Raw BER特性は上記3方式の中で最も悪いが、復号後のBER特性及びBLER特性は、格子状16QAMと同等程度の特性が得られる。つまり、符号化の効果を最も得ることができる方式であるといえる。(4, 12)サーキュラー16QAMでは、Raw BER特性、復号後のBER特性、BLER特性は、いずれも格子状16QAMよりも悪いが、PAPRは上記3方式の中で最も小さい。

[0123] つまり、下記に示すような関係になる。

[Raw BER]

格子状16QAM>(4, 12)サーキュラー16QAM>(8, 8)スター16QAM

優(良)←……→劣(悪)

[復号後の BER及びBLER (符号化率=1/3)]

格子状16QAM≒(8, 8)スター16QAM>(4, 12)サーキュラー16QAM

優(良)←……→劣(悪)

[PAPR]

(4, 12)サーキュラー16QAM＞(8, 8)スター16QAM＞格子状16QAM

優(良)←――――――――――――――――――――――――――――――――――→劣(悪)

・本発明のアイディアとPAPRへの影響について

格子状16QAMでは、符号化率が1／2よりも小さい場合は、図9により前述したように、外側に配置されたシンボルが内側方向に集まるように移動させるので、PAPRが小さくなる。これに対し、符号化率が1／2よりも大きい場合は、図17により前述したように、逆に外側のシンボルは中心から遠ざかるように移動させるのでPAPRは大きくなる。

[0124] また、(8, 8)スター16QAM及び(4, 12)サーキュラー16QAMでは、円に沿ってシンボルを移動させる場合はPAPRに変化はないが、2重円の半径比を変化させるとPAPRに影響する。即ち、半径比が小さくなるとPAPRも小さくなり、半径比が大きくなるとPAPRも大きくなる。

なお、本発明は上述した各実施形態に限定されず、本発明の趣旨を逸脱しない範囲で種々変形して実施できることはいうまでもない。

産業上の利用可能性

[0125] 以上詳述したように、本発明によれば、ビットコレクションにおける重要度の異なる符号の割合に応じて、多値変調方式のシンボル配置を等間隔配置から変更する方向に制御するので、ビットコレクションによる効果を最大限に発揮させて、受信側での誤り率を従来よりも改善することができ、ディジタル無線通信技術分野において極めて有用と考えられる。

請求の範囲

[1] 複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムにおいて、

該送信機が、

送信すべき情報系列を符号化して重要度の異なる複数の符号を生成する符号化過程と、

前記複数ビットのうち、該受信機でのシンボル判定時に尤度が大きくなる傾向にあるビットに対して、前記符号化過程で得られた前記複数の符号のうち重要度の高い符号が優先的に割り当てられるようビット配置を制御するビットコレクション過程と、

前記ビットコレクション過程で得られたビット配置に従って、前記複数ビットへの前記符号の割り当てを行なって所定のシンボル配置で前記情報系列を多値変調する多値変調過程と、

前記ビットコレクション過程における前記重要度の異なる符号の割合に応じて、前記シンボル配置を等間隔配置から変更する方向に制御するシンボル配置制御過程とを有するとともに、

該受信機が、

該送信機から受信した信号を前記変更したシンボル配置に関する情報に基づいて復調する復調過程と、

前記復調過程で復調された信号を復号して前記情報系列を得る復号過程とを有することを特徴とする、多値変調方式を用いたディジタル無線通信方法。

[2] 該送信機の前記シンボル配置制御過程において、

前記複数ビットのうち前記重要度の高い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項1記載の多値変調方式を用いたディジタル無線通信方法。

[3] 該送信機の前記シンボル配置制御過程において、

前記複数ビットのうち前記重要度の低い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項1記載の多値変

調方式を用いたディジタル無線通信方法。

[4] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項2記載の多値変調方式を用いたディジタル無線通信方法。

[5] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビット及び第2ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項2記載の多値変調方式を用いたディジタル無線通信方法。

[6] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記第1～第3ビットに前記重要度の高い符号が割り当てられることになる場合に、当該第3ビット及び前記第4ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項2記載の多値変調方式を用いたディジタル無線通信方法。

[7] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記第4ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第4ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項3記載の多値変調

方式を用いたディジタル無線通信方法。

[8] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項2記載の多値変調方式を用いたディジタル無線通信方法。

[9] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットと第2ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項2記載の多値変調方式を用いたディジタル無線通信方法。

[10] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

前記シンボル配置制御過程において、

前記ビットコレクション過程によるビット配置により前記第3ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第3ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項3記載の多値変調方式を用いたディジタル無線通信方法。

[11] 前記シンボル配置制御過程において、

前記シンボル配置の変更前後で送信電力が変化しない方向に前記シンボル配置制御を行なうことを特徴とする、請求項1～10のいずれか1項に記載の多値変調方式を用いたディジタル無線通信方法。

[12] 前記ビットコレクション過程において、

前記複数の符号のうち重要度の低い複数種類の符号の比率が前記複数ビット毎に

均等となるように前記ビット配置を制御することを特徴とする、請求項1記載の多値変調方式を用いたディジタル無線通信方法。

[13] 該送信機が、

前記シンボル配置制御過程で変更したシンボル配置に関する情報を事前に該受信機へ通知する過程をさらに有することを特徴とする、請求項1記載の多値変調方式を用いたディジタル無線通信方法。

[14] 該受信機が、前記重要度の異なる符号の割合に対する前記シンボル配置に関する情報を予めメモリに保持しておき、

該送信機が、

前記割合に関する情報を該受信機へ通知する過程を有するとともに、

該受信機が、

該送信機から通知された前記割合に関する情報に基づいて前記シンボル配置に関する情報を該メモリから読み出し、読み出した情報に基づいて前記復調を行なう過程を有することを特徴とする、請求項1記載の多値変調方式を用いたディジタル無線通信方法。

[15] 前記重要度の高い符号がターボ符号における組織符号であり、前記重要度の低い符号が前記ターボ符号におけるパリティ符号であることを特徴とする、請求項1～14のいずれか1項に記載の多値変調方式を用いたディジタル無線通信方法。

[16] 複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムに用いられる前記送信機であって、

送信すべき情報系列を符号化して重要度の異なる複数の符号を生成する符号化手段と、

前記複数ビットのうち、該受信機でのシンボル判定時に尤度が大きくなる傾向にあるビットに対して、該符号化手段で得られた前記複数の符号のうち重要度の高い符号が優先的に割り当てられるようビット配置を制御するビットコレクション手段と、

該ビットコレクション手段で得られたビット配置に従って、前記複数ビットへの前記符号の割り当てを行なって所定のシンボル配置で前記情報系列を多値変調する多値

変調手段と、

該ビットコレクション手段における前記重要度の異なる符号の割合に応じて、前記シンボル配置を等間隔配置から変更する方向に制御するシンボル配置制御手段とをそなえたことを特徴とする、送信機。

[17] 該シンボル配置制御手段が、

前記複数ビットのうち前記重要度の高い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項16記載の送信機。

[18] 該シンボル配置制御手段が、

前記複数ビットのうち前記重要度の低い符号が割り当てられるビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項16記載の送信機。

[19] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項17記載の送信機。

[20] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビット及び第2ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項17記載の送信機。

[21] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記第1～第3ビットに前記重要度の高い符号が割り当てられることになる場合に、当該第3ビット及び前記第4ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項17記載の送信機。

[22] 前記多値変調方式が第1～第4ビットの4ビットで1シンボルを送信する16値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記第4ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第4ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項18記載の送信機。

[23] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記第1ビットのみに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットのみの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項17記載の送信機。

[24] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記の第1ビットと第2ビットとに前記重要度の高い符号が割り当てられることになる場合に、当該第1ビットと第2ビットの尤度が大きくなる方向に前記シンボル配置を制御することを特徴とする、請求項17記載の送信機。

[25] 前記多値変調方式が第1～第3ビットの3ビットで1シンボルを送信する8値変調方式であり、

該シンボル配置制御手段が、

該ビットコレクション手段によるビット配置により前記第3ビットのみに前記重要度の低い符号が割り当てられることになる場合に、当該第3ビットのみの尤度が大きくなる

方向に前記シンボル配置を制御することを特徴とする、請求項18記載の送信機。

[26] 該シンボル配置制御手段が、

前記シンボル配置の変更前後で送信電力が変化しない方向に前記シンボル配置制御を行なうことを特徴とする、請求項16～25のいずれか1項に記載の送信機。

[27] 該ビットコレクション手段が、

前記複数の符号のうち重要度の低い複数種類の符号の比率が前記複数ビット毎に均等となるように前記ビット配置を制御することを特徴とする、請求項16記載の送信機。

[28] 該シンボル配置制御手段により変更したシンボル配置に関する情報を事前に該受信機へ通知する通知手段をさらにそなえたことを特徴とする、請求項16記載の送信機。

[29] 前記重要度の高い符号がターボ符号における組織符号であり、前記重要度の低い符号が前記ターボ符号におけるパリティ符号であることを特徴とする、請求項16～27のいずれか1項に記載の送信機。

[30] 複数ビットで1シンボルを送信する多値変調方式により信号を変調して送信する送信機と、該送信機からの受信信号をシンボル判定して復調する受信機とをそなえたディジタル無線通信システムに用いられる前記受信機であって、

該送信機から受信した信号を、該送信機において前記重要度の異なる符号の割合に応じて等間隔配置から変更されたシンボル配置に関する情報に基づいて復調する復調手段と、

該復調手段で復調された信号を復号して該送信機の送信した情報系列を得る復号手段とをそなえたことを特徴とする、受信機。

[31] 前記シンボル配置に関する情報を該送信機から受信するシンボル配置情報受信手段をさらにそなえたことを特徴とする、請求項30記載の受信機。

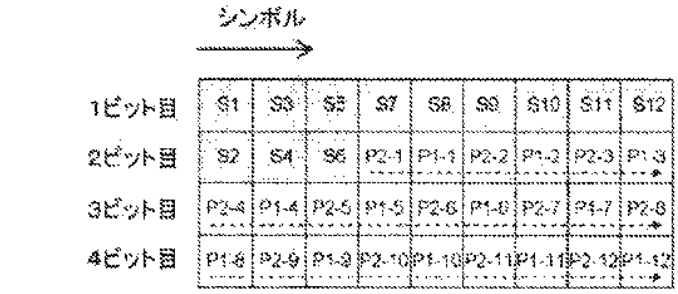
[32] 前記重要度の異なる符号の割合に対する前記シンボル配置に関する情報を保持するメモリをそなえ、

該復調手段が、

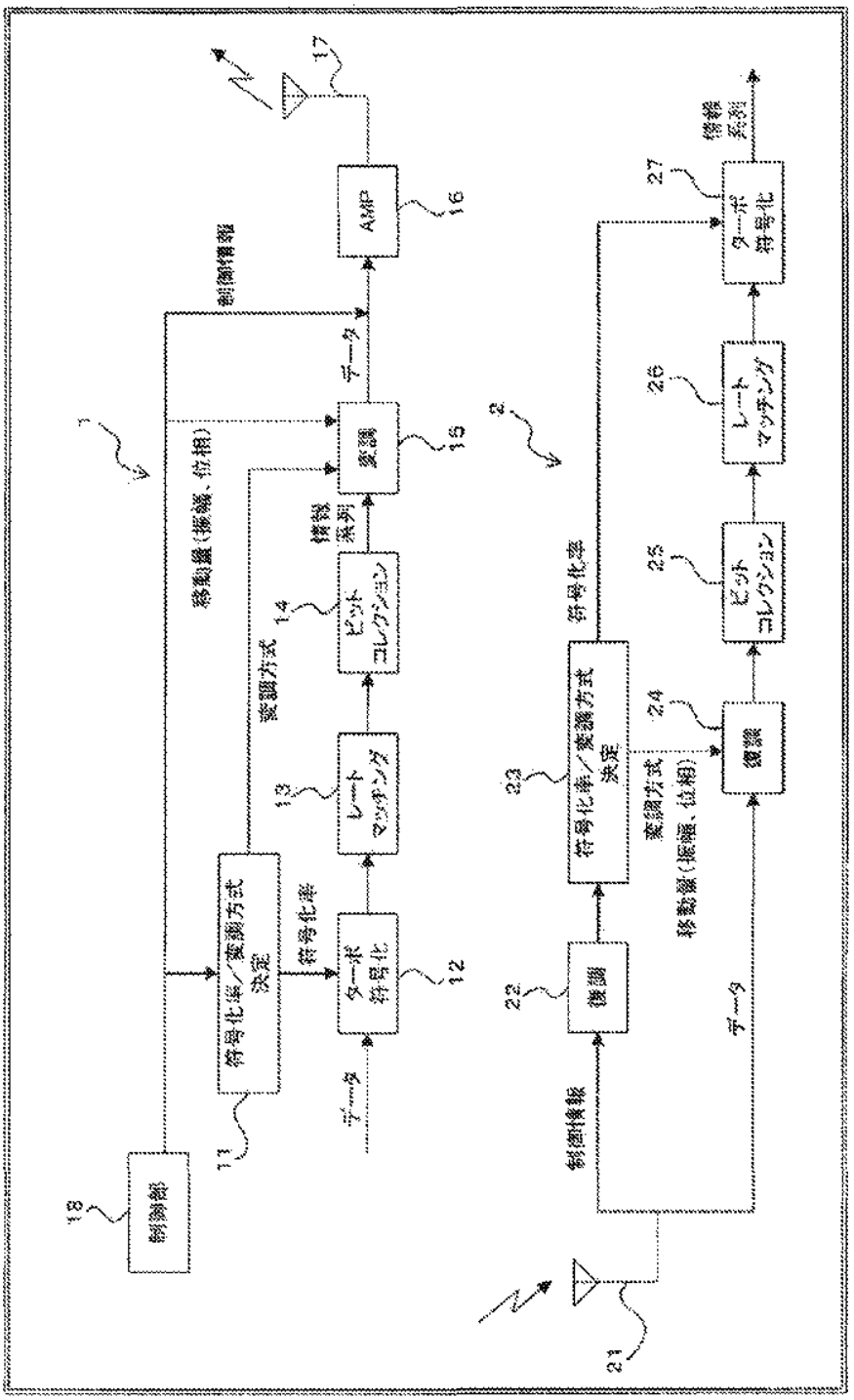
該送信機から通知される前記割合に関する情報に基づいて前記シンボル配置に

関する情報を該メモリから読み出し、読み出した情報に基づいて前記復調を行なうことを特徴とする、請求項30記載の受信機。

[図1]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S3 S5 S7 S8 S9 S10 S11 S12
S2 S4 S6 P2-1 P1-1 P2-2 P1-2 P2-3 P1-3
P2-4 P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7 P2-8
P1-8 P2-9 P1-9 P2-10 P1-10 P2-11 P1-11 P2-12 P1-12

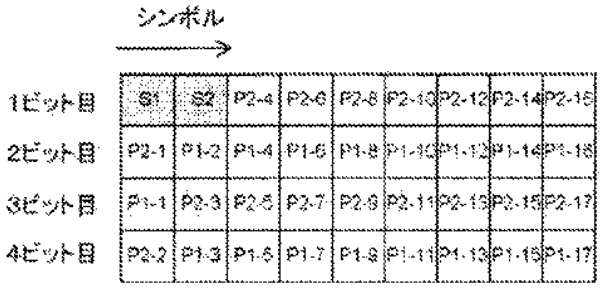
[図2]


1
2
11
12
13
14
15
16
17
18
21
22
23
24
25
26
27
制御部
符号化率／変調方式決定
ターボ符号化
レートマッチング
ビットコレクション
変調
AMP
制御情報
データ
移動量（振幅、位相）
変調方式
符号化率
情報系列
復調
ターボ符号化
ビットコレクション
レートマッチング

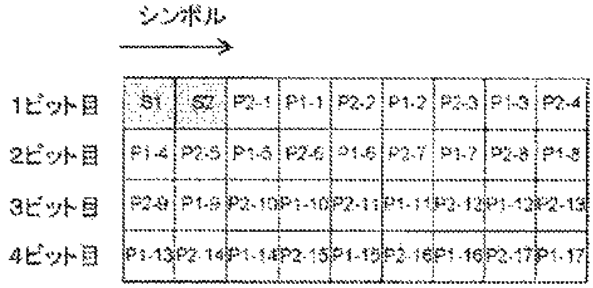
[図3]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S2 S3 S4 P2-1 P1-1 P2-2 P1-2 P2-3
P1-3 P2-4 P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7
P2-8 P1-8 P2-9 P1-9 P2-10 P1-10 P2-11 P1-11 P2-12
P1-12 P2-13 P1-13 P2-14 P1-14 P2-15 P1-15 P2-16 P1-16

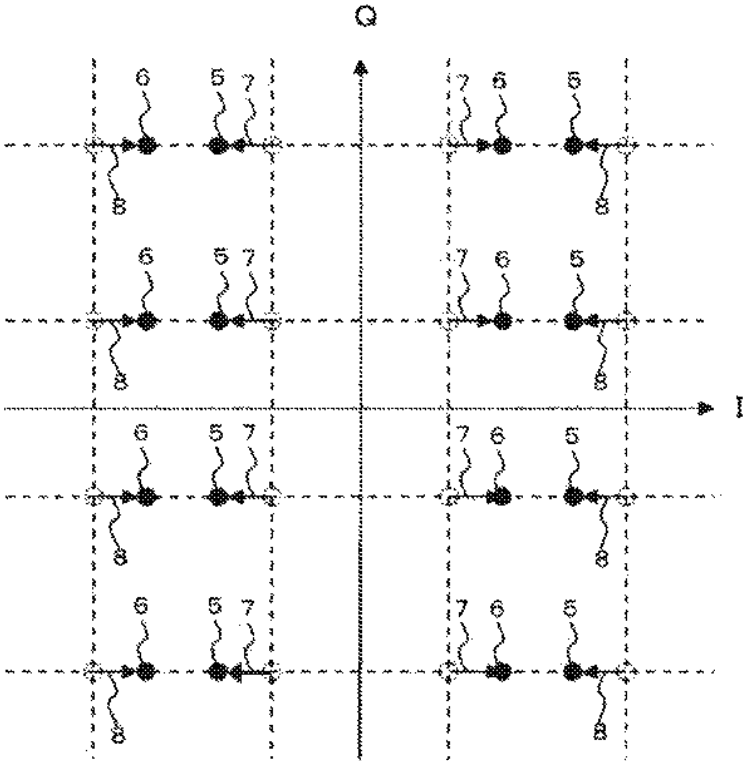
[図4]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S2 P2-4 P2-6 P2-8 P2-10 P2-12 P2-14 P2-16
P2-1 P1-2 P1-4 P1-6 P1-8 P1-10 P1-12 P1-14 P1-16
P1-1 P2-3 P2-5 P2-7 P2-9 P2-11 P2-13 P2-15 P2-17
P2-2 P1-3 P1-5 P1-7 P1-9 P1-11 P1-13 P1-15 P1-17

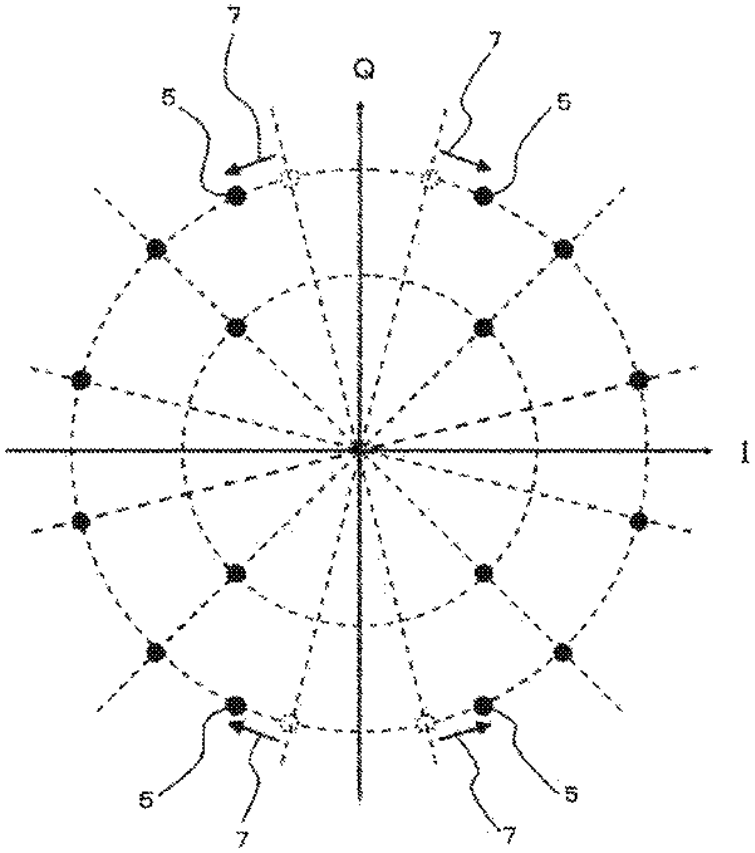
[図5]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S2 P2-1 P1-1 P2-2 P1-2 P2-3 P1-3 P2-4
P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7 P2-8 P1-8
P2-9 P1-9 P2-10 P1-10 P2-11 P1-11 P2-12 P1-12 P2-13
P1-13 P2-14 P1-14 P2-15 P1-15 P2-16 P1-16 P2-17 P1-17

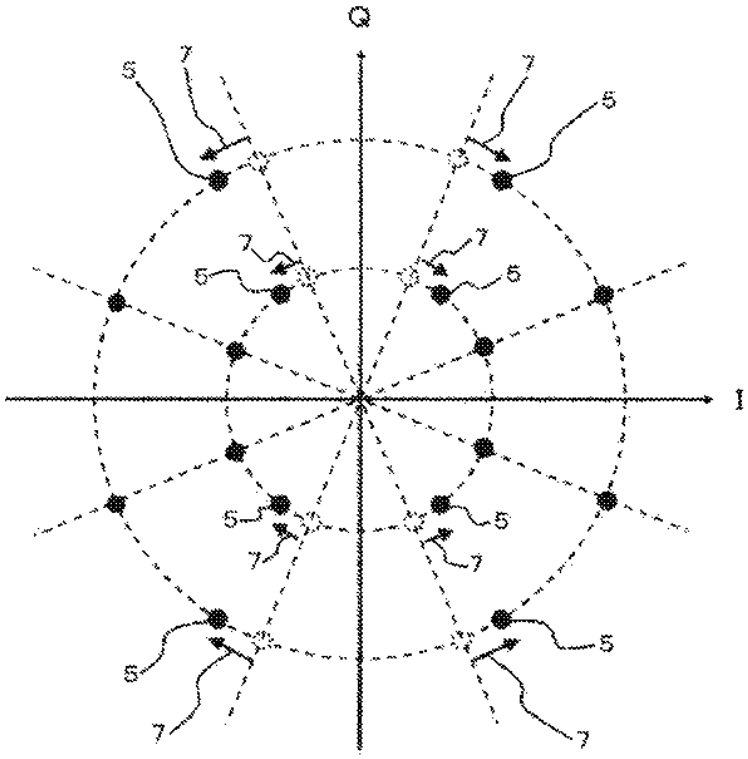
[図6]



[図7]



[図8]

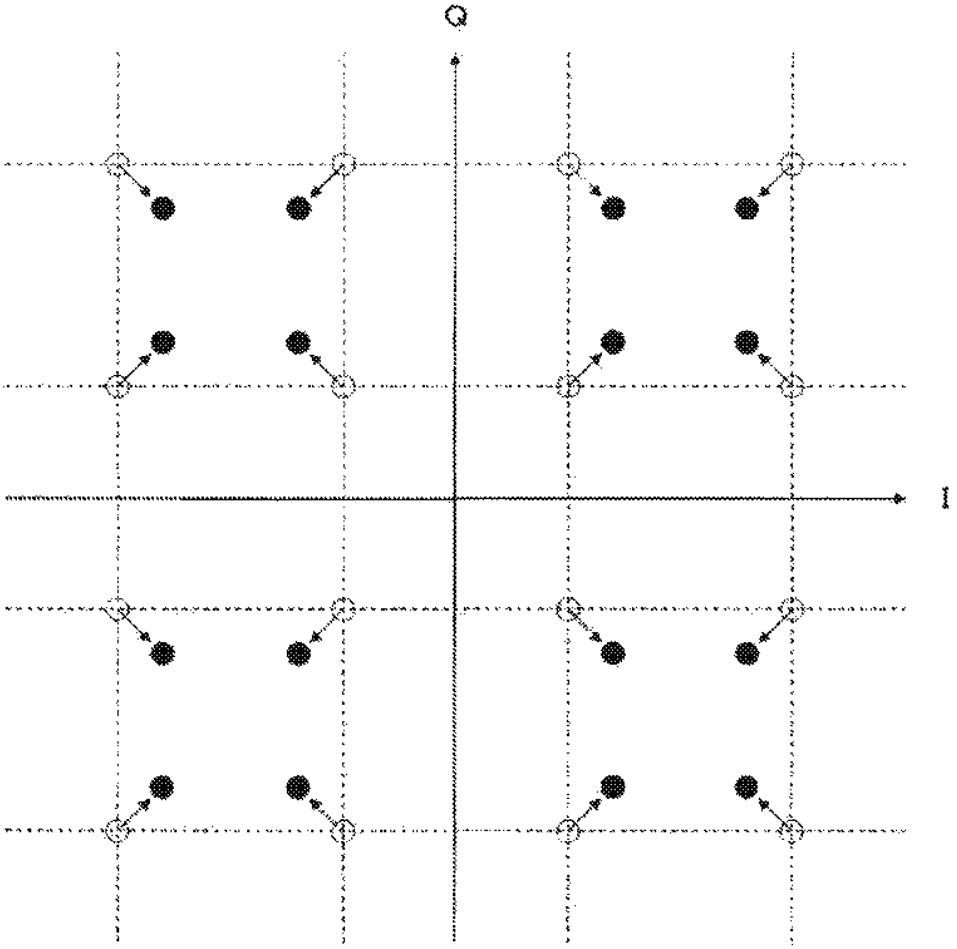


[図9]

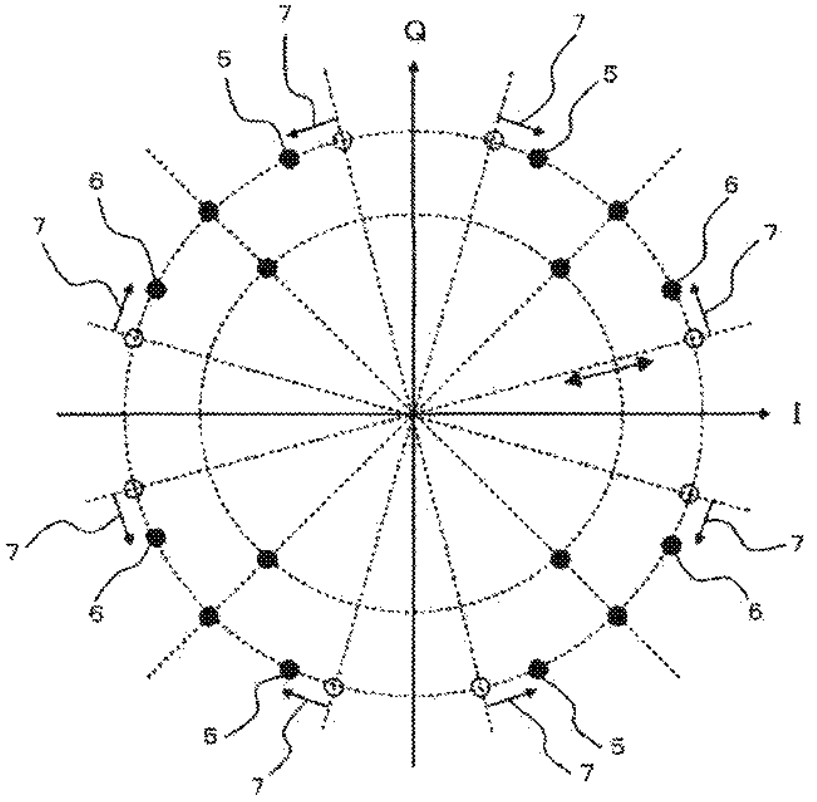
シンボル →

1ビット目	S1	S3	S5	S7	S9	S9	S10	S11	S12		
2ビット目	S2	S4	S6	P2-4	P1-5	P2-7	P1-8	P2-10	P1-11		
3ビット目	P2-1	P2-2	P2-3	P1-4	P2-6	P1-7	P2-9	P1-10	P2-12		
4ビット目	P1-1	P1-2	P1-3	P2-5	P1-5	P2-8	P1-9	P2-11	P1-12		

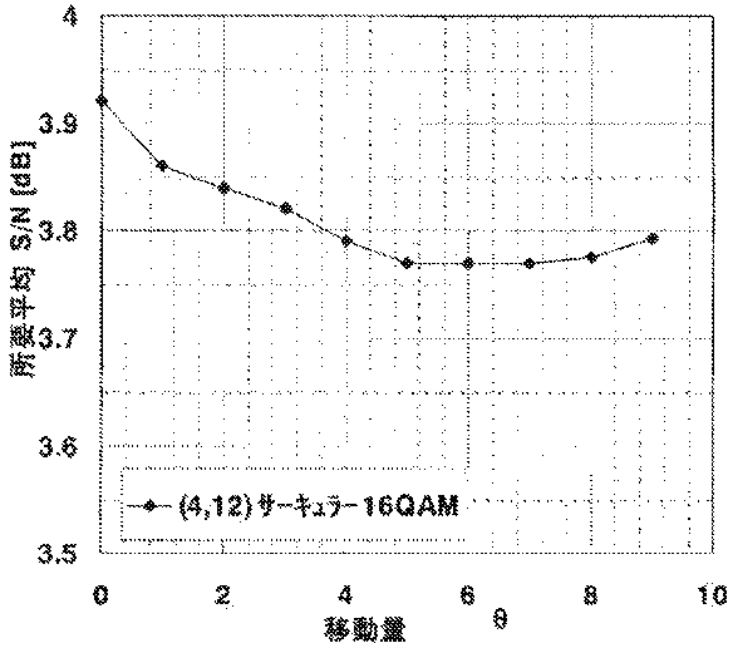
[図10]



[図11]



[図12]



[図13]

シンボル →

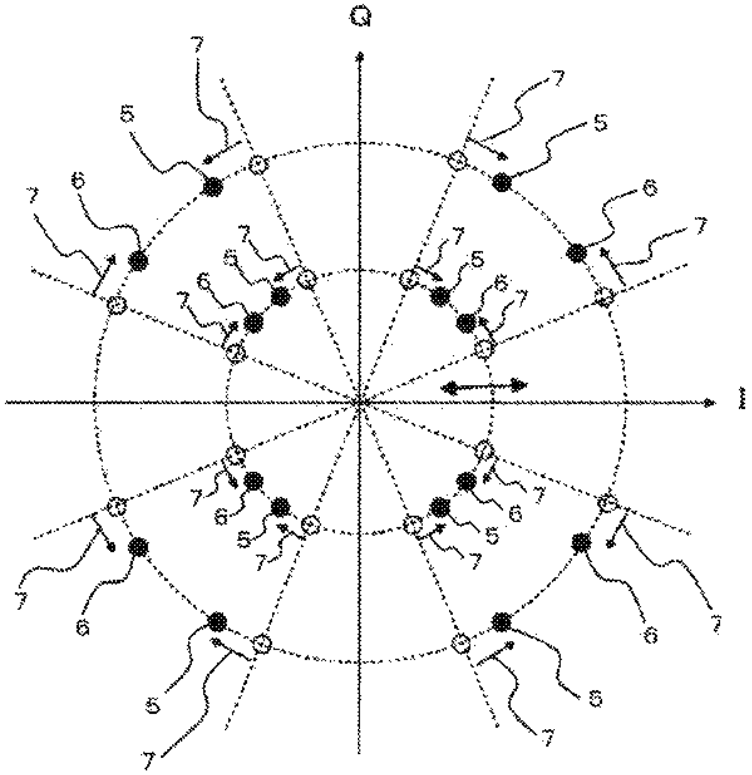
1ビット目	S1	S3	S5	S7	S8	S9	S10	S11	S12
2ビット目	S2	S4	S6	P2-4	P1-5	P2-7	P1-8	P2-10	P1-11
3ビット目	P2-1	P2-2	P2-3	P1-4	P2-6	P1-7	P2-9	P1-10	P2-12
4ビット目	P1-1	P1-2	P1-3	P2-5	P1-6	P2-8	P1-9	P2-11	P1-12

[図14]

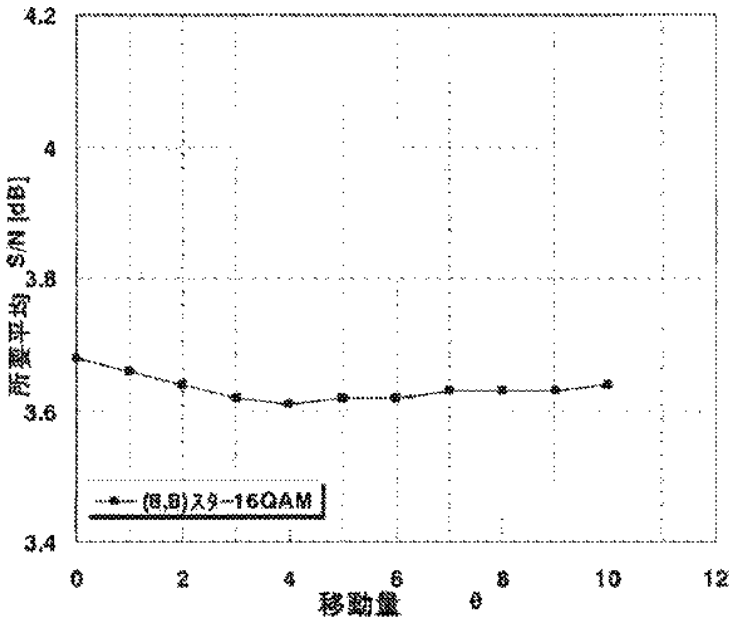
シンボル →

1ビット目	S1	S3	S5	S7	S8	S9	S10	S11	S12
2ビット目	S2	S4	S6	P2-1	P1-1	P2-2	P1-2	P2-3	P1-3
3ビット目	P2-4	P1-4	P2-5	P1-5	P2-6	P1-6	P2-7	P1-7	P2-8
4ビット目	P1-8	P2-9	P1-9	P2-10	P1-10	P2-11	P1-11	P2-12	P1-12

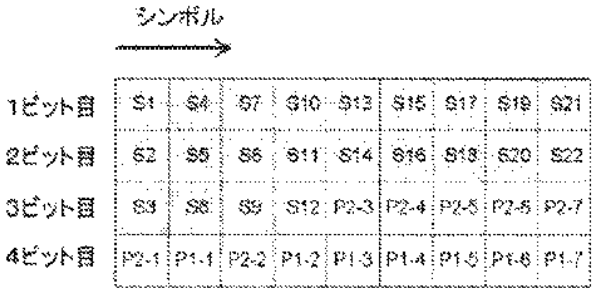
[図15]



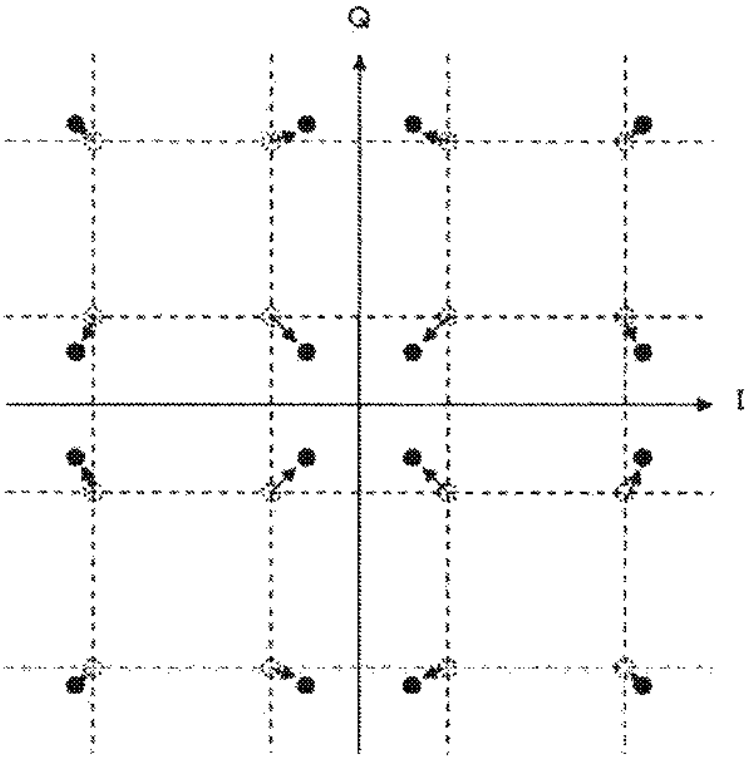
[図16]


4.2
4
3.8
3.6
3.4
所要平均S/N [dB]
(8,8)スタ-16QAM
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6
8
10
12
移動量 θ

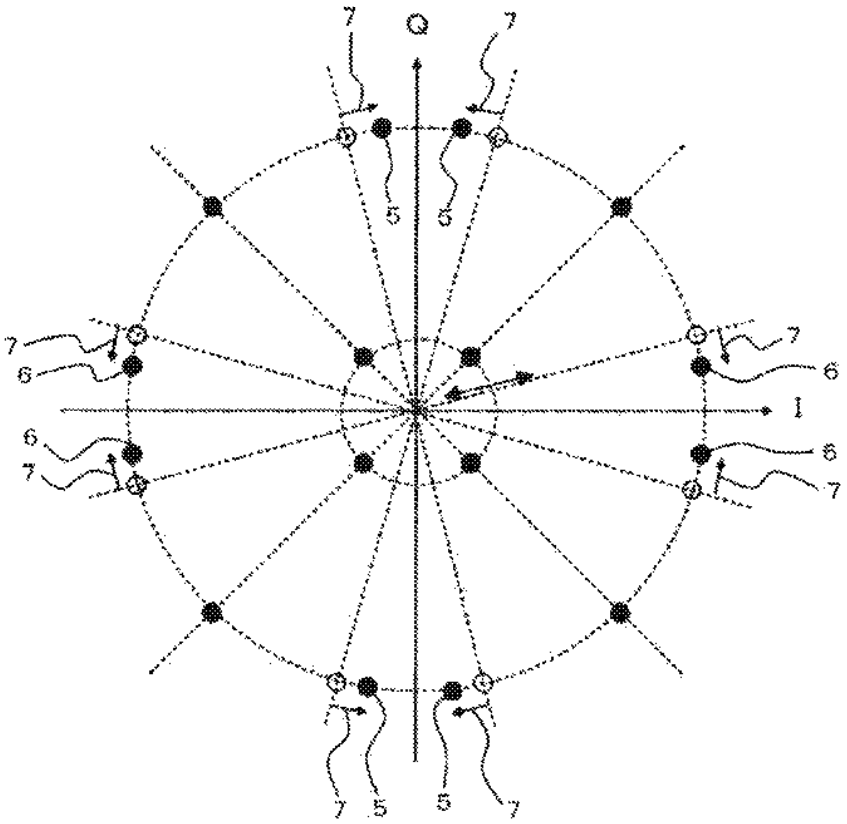
[図17]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S4 S7 S10 S13 S15 S17 S19 S21
S2 S5 S6 S11 S14 S16 S18 S20 S22
S3 S8 S9 S12 P2-3 P2-4 P2-5 P2-6 P2-7
P2-1 P1-1 P2-2 P1-2 P1-3 P1-4 P1-5 P1-6 P1-7

[図18]



[図19]



[図20]

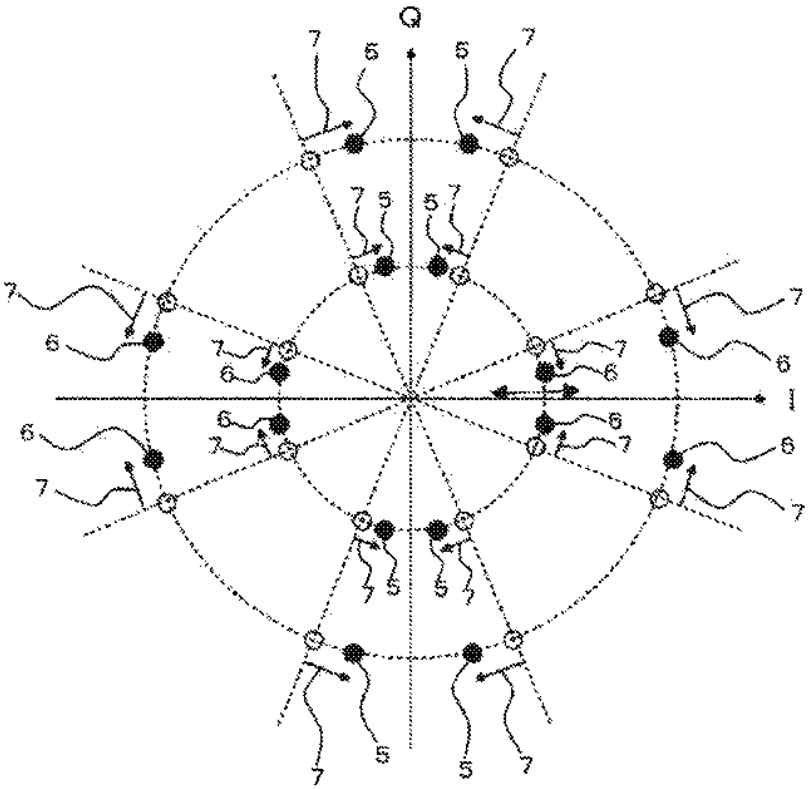
	シンボル →								
1ビット目	S1	S4	S7	S10	S13	S15	S17	S19	S21
2ビット目	S2	S5	S8	S11	S14	S16	S18	S20	S22
3ビット目	S3	S6	S9	S12	P2-3	P2-4	P2-5	P2-6	P2-7
4ビット目	P2-1	P1-1	P2-2	P1-2	P1-3	P1-4	P1-5	P1-6	P1-7

[図21]

シンボル →

1ビット目	S1	S4	S7	S10	S13	S15	S17	S19	S21
2ビット目	S2	S5	S8	S11	S14	S16	S18	S20	S22
3ビット目	S3	S6	S9	S12	P2-1	P1-1	P2-2	P1-2	P2-3
4ビット目	P1-3	P2-4	P1-4	P2-5	P1-5	P2-6	P1-6	P2-7	P1-7

[図22]

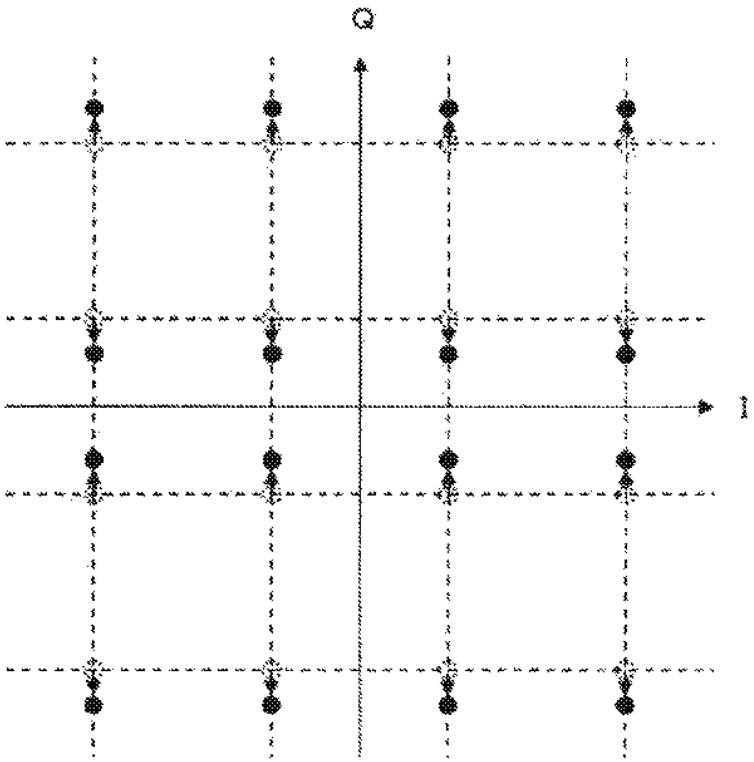


[図23]

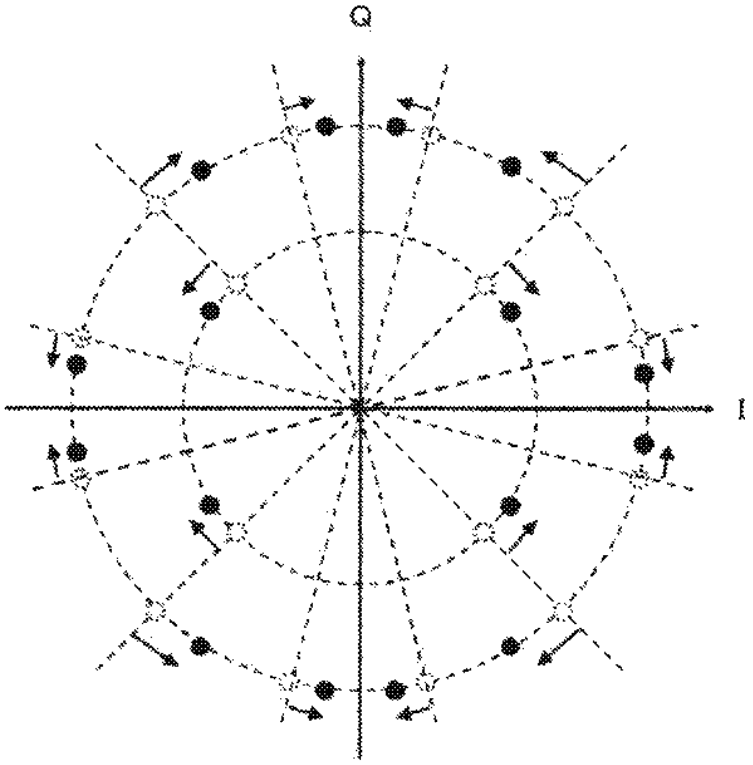
シンボル →

1ビット目	S1	S4	S7	S10	S13	S16	S19	S22	S25
2ビット目	S2	S5	S8	S11	S14	S17	S20	S23	S26
3ビット目	S3	S6	S9	S12	S15	S18	S21	S24	S27
4ビット目	P2-1	P1-1	P2-2	P1-2	P2-3	P1-3	P2-4	P1-4	P2-5

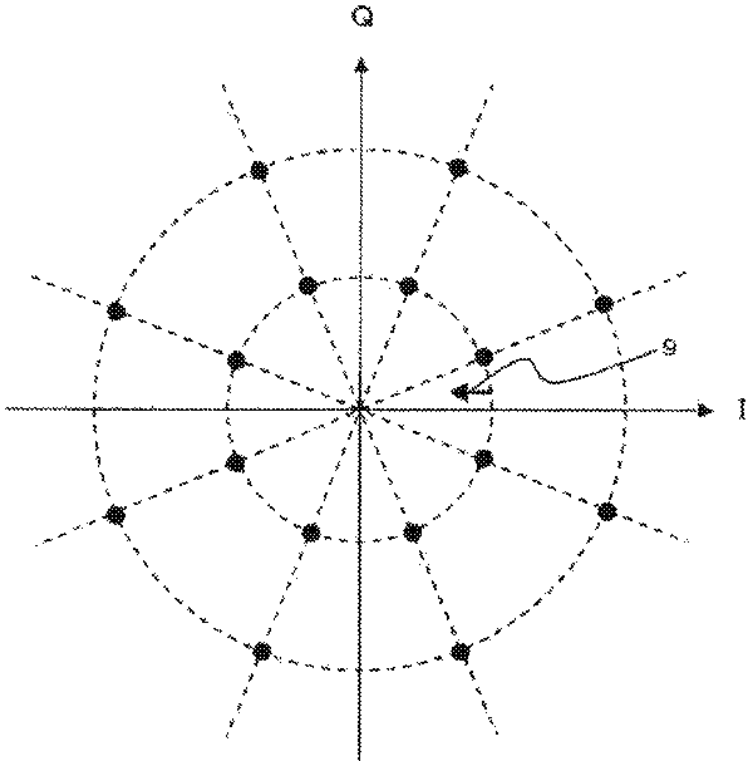
[図24]



[図25]



[図26]

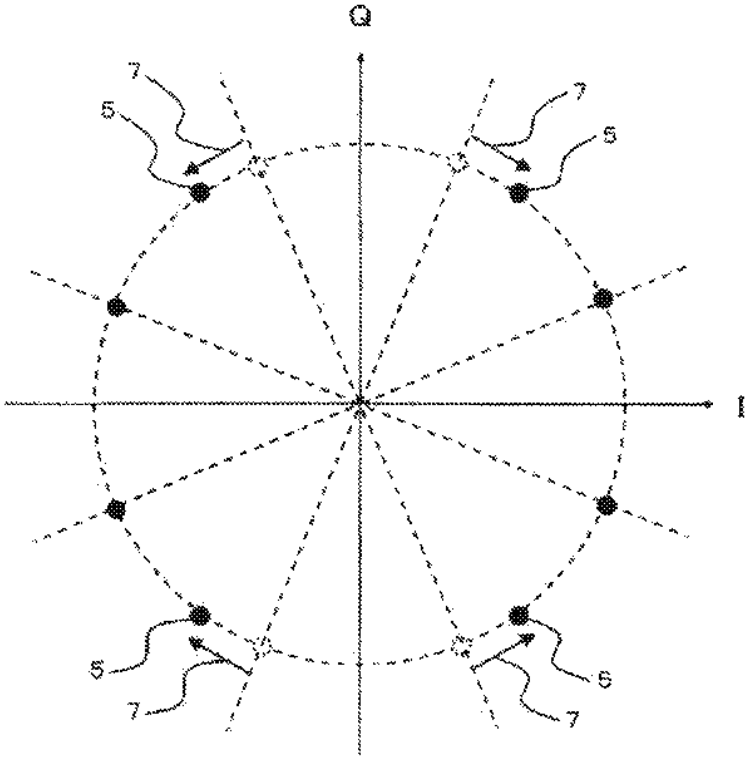


[図27]

シンボル →

1ビット目	S1	S2	S3	S4	S5	P2-1	P1-1	P2-2	P1-2
2ビット目	P2-3	P1-3	P2-4	P1-4	P2-5	P1-5	P2-6	P1-6	P2-7
3ビット目	P1-7	P2-8	P1-8	P2-9	P1-9	P2-10	P1-10	P2-11	P1-11

[図28]

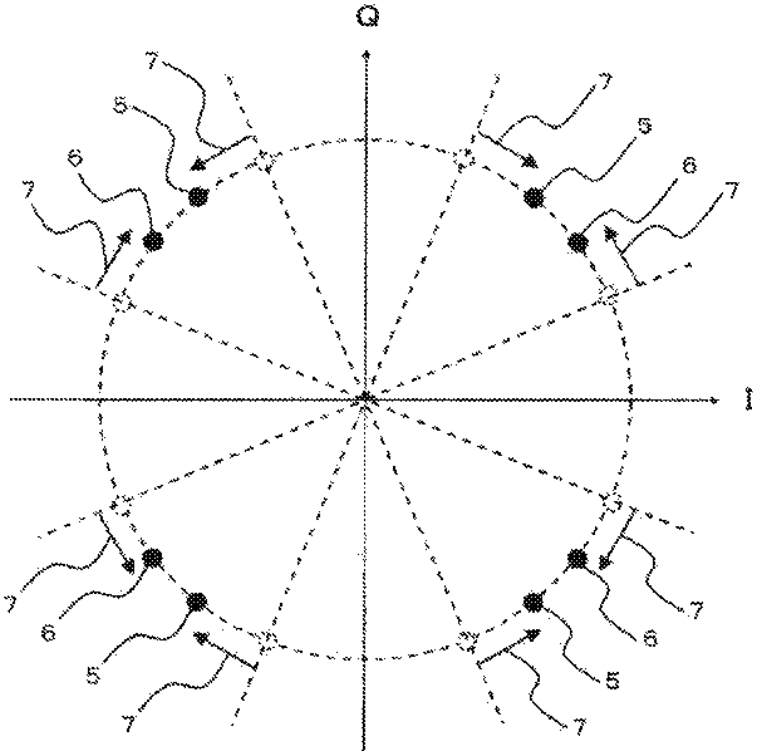


[図29]

シンボル →

1ビット目	S1	S3	S5	S7	S9	S9	S10	S11	S12
2ビット目	S2	S4	S6	P2-1	P1-1	P2-2	P1-2	P2-3	P1-3
3ビット目	P2-4	P1-4	P2-5	P1-5	P2-6	P1-6	P2-7	P1-7	P2-8

[図30]

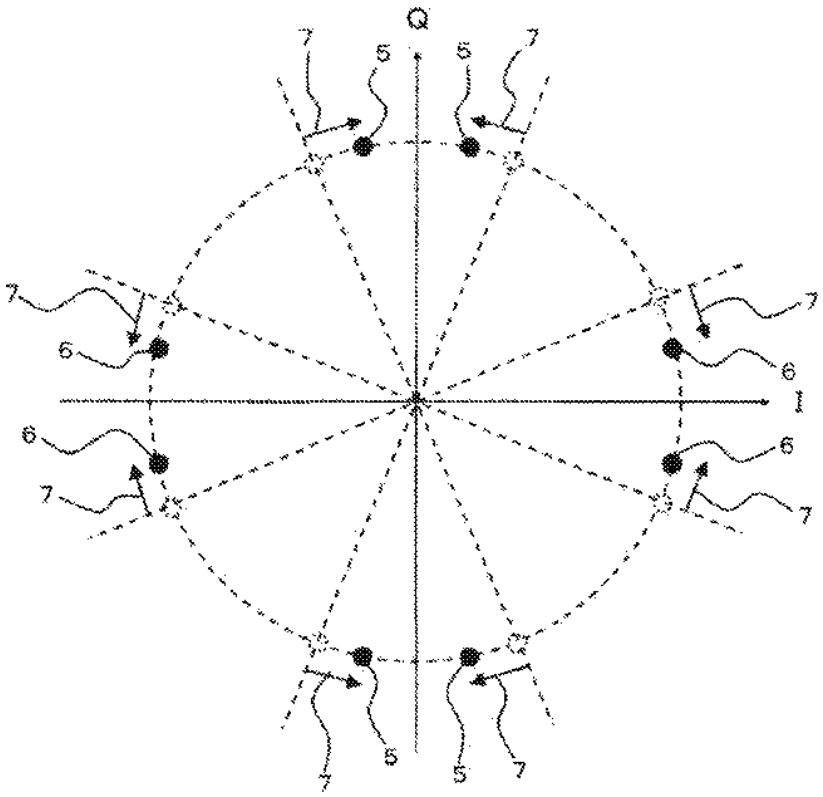


[図31]

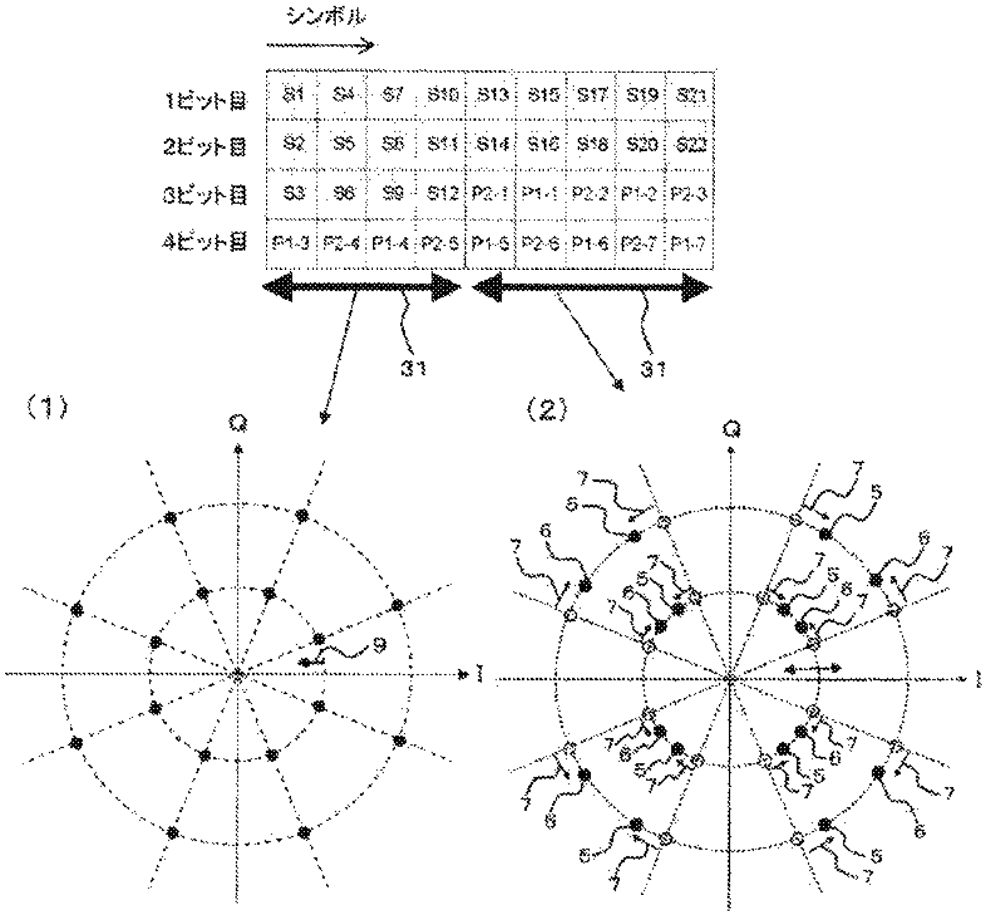
シンボル →

1ビット目	S1	S4	S7	S10	S13	S16	S17	S19	S21
2ビット目	S2	S5	S8	S11	S14	S18	S18	S20	S22
3ビット目	S3	S6	S9	S12	P2-1	P1-1	P2-2	P1-2	P2-3

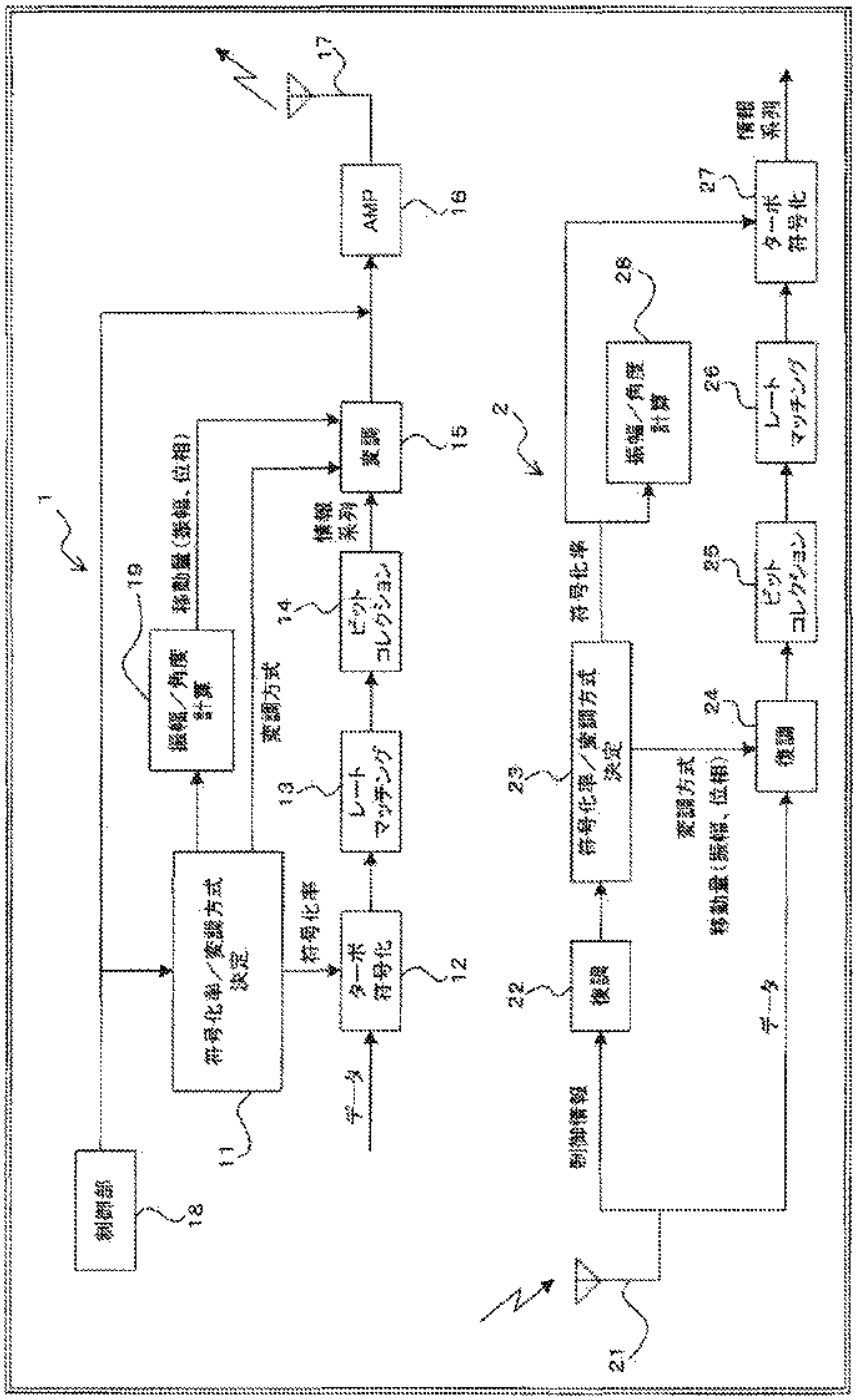
[図32]



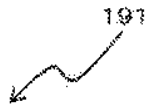
[図33]


シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S4 S7 S10 S13 S15 S17 S19 S21
S2 S5 S8 S11 S14 S16 S18 S20 S22
S3 S6 S9 S12 P2-1 P1-1 P2-2 P1-2 P2-3
P1-3 P2-4 P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7
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[図34]

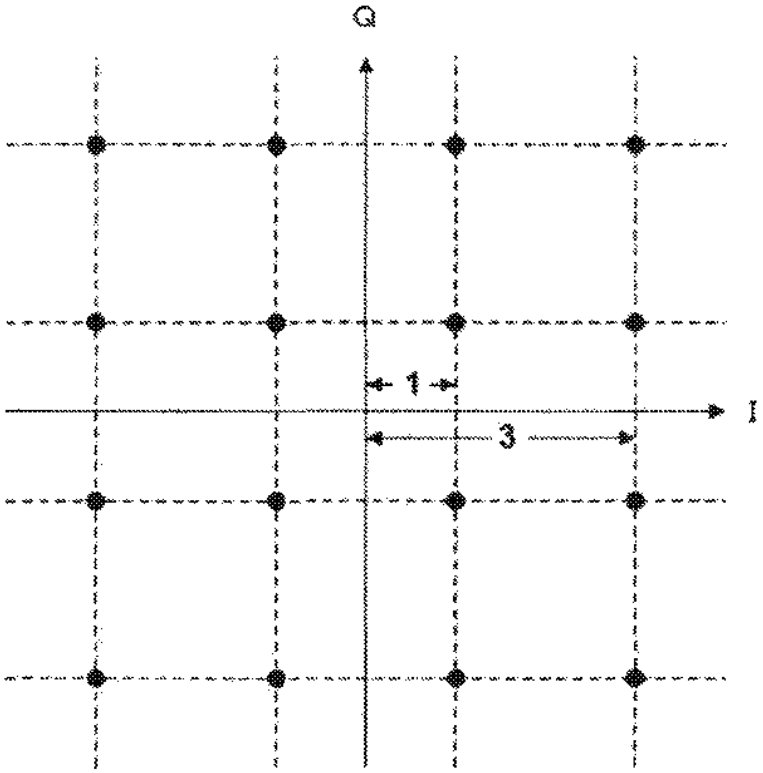

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AMP
制御部
符号化率／変調方式決定
ターボ符号化
レートマッチング
ビットコレクション
変調
振幅／角度計算
データ
制御情報
復調
符号化率／変調方式決定
ビットコレクション
レートマッチング
ターボ符号化
情報系列
移動量（振幅、位相）
変調方式
符号化率

[図35]

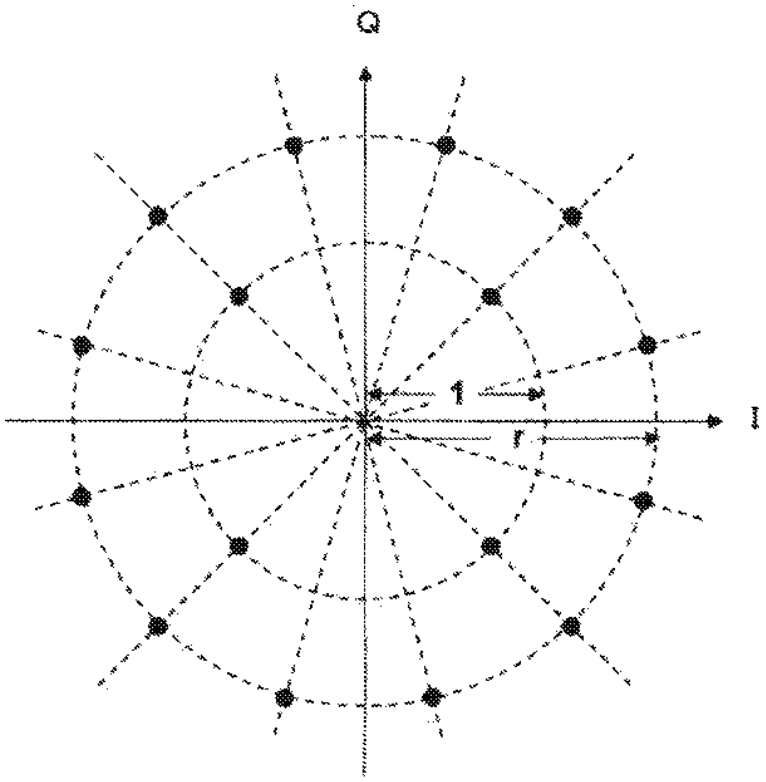


(組織ビットとパリティビットの割合)	振幅比	角度
R(0)～R(1)	a(1)	θ(1)
R(1)～R(2)	a(2)	θ(2)
⋮	⋮	⋮
⋮	⋮	⋮
R(n-1)～R(n)	a(n)	θ(n)

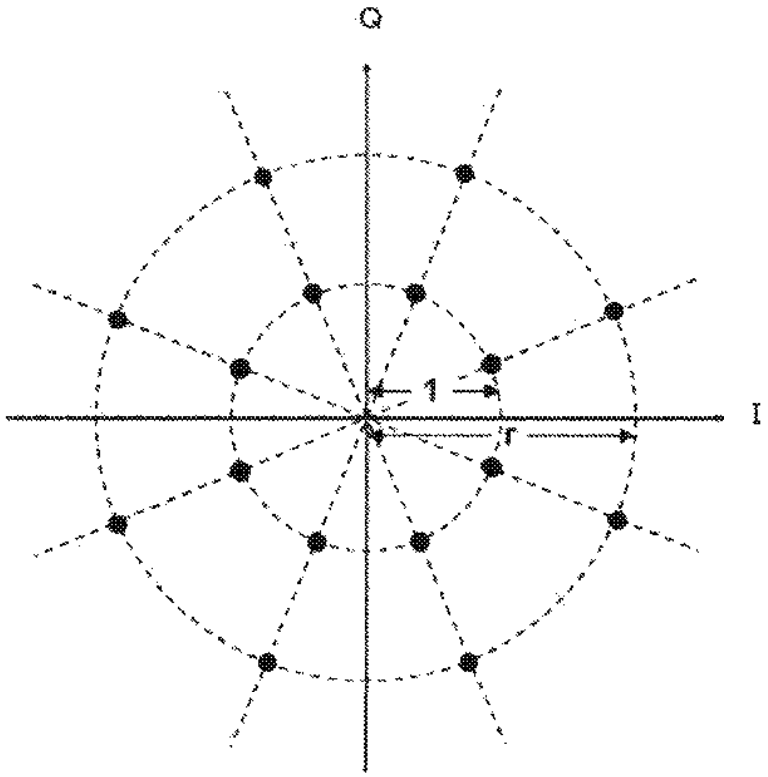
[図36]



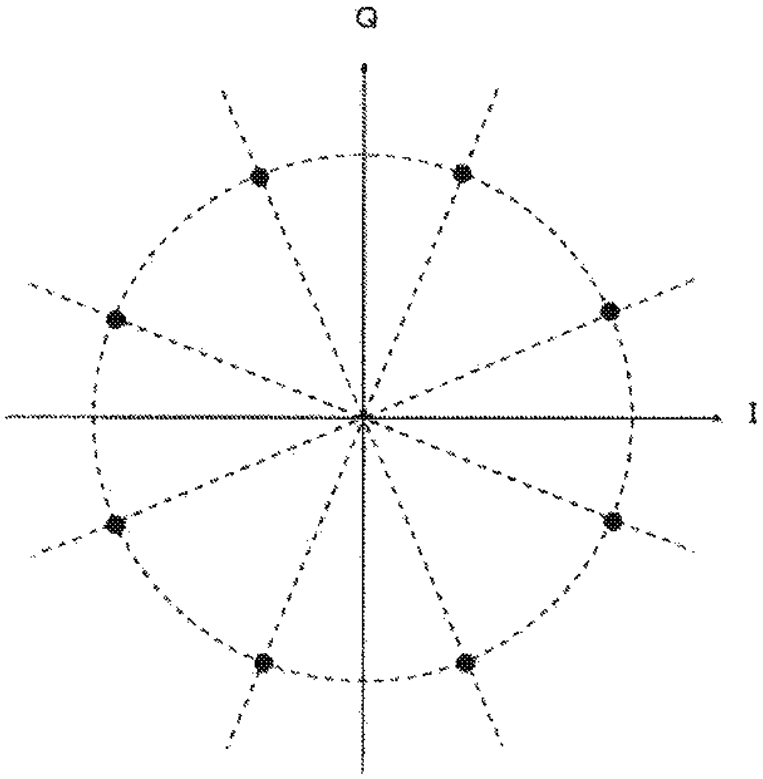
[図37]



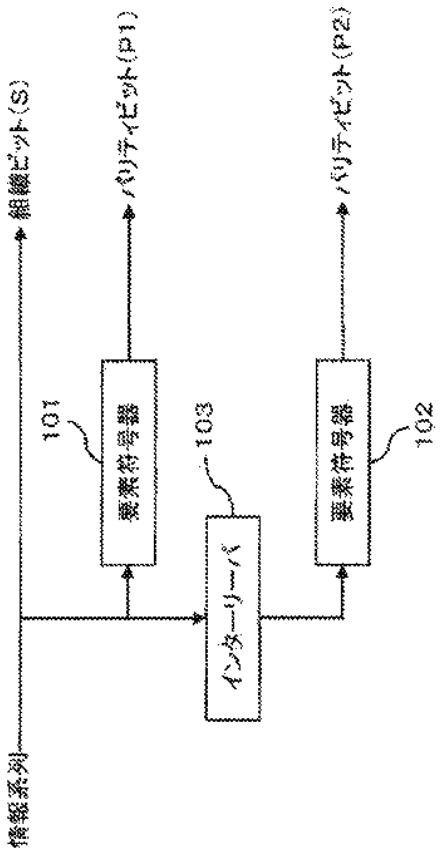
[図38]



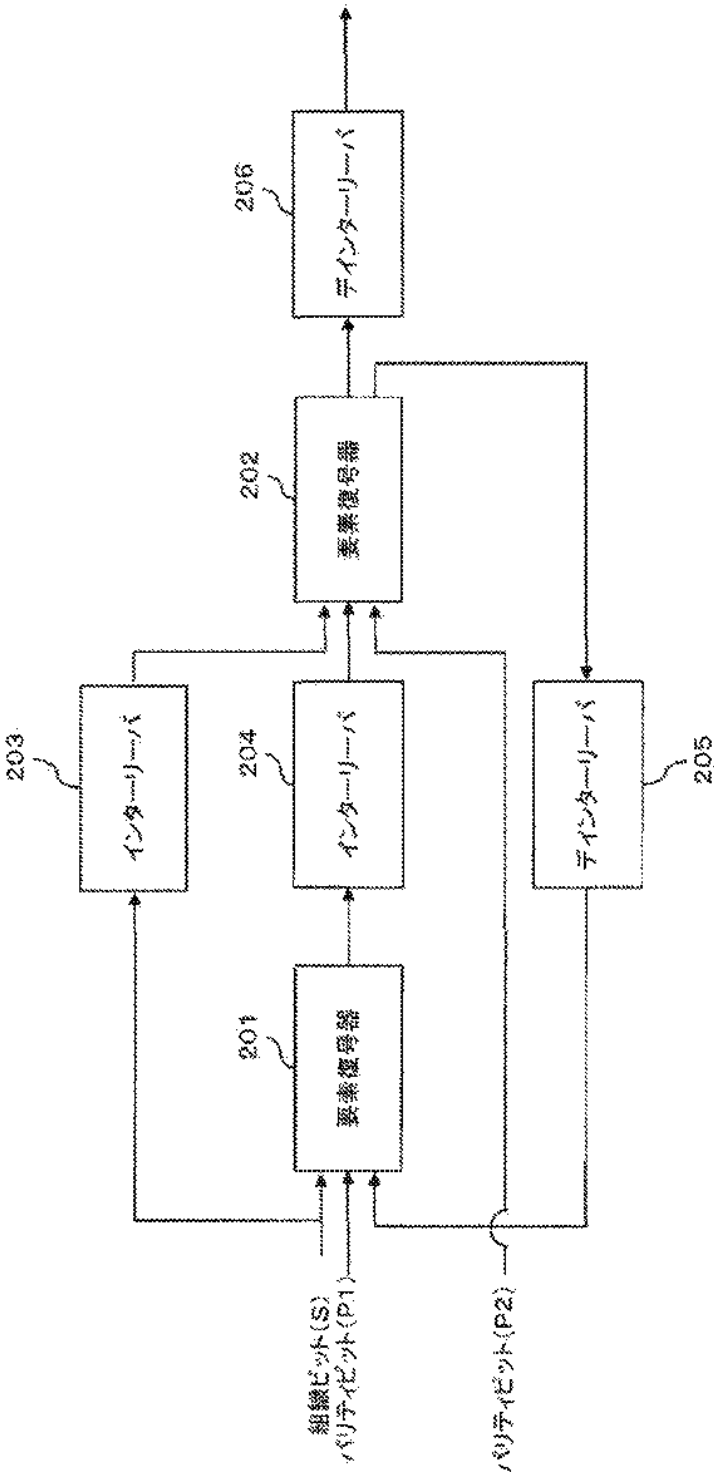
[図39]



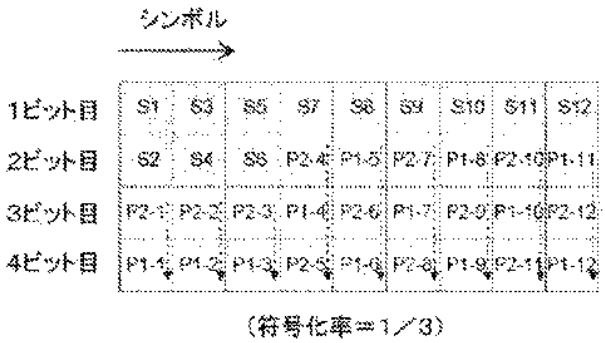
[図40]


組織ビット(S)
パリティビット(P1)
パリティビット(P2)
要素符号器
要素符号器
インターリーバ
101
102
103
情報系列

[図41]


206
デインターリーバ
202
要素復号器
203
インターリーバ
204
インターリーバ
205
デインターリーバ
201
要素復号器
組織ビット(S)
パリティビット(P1)
パリティビット(P2)

[図42]

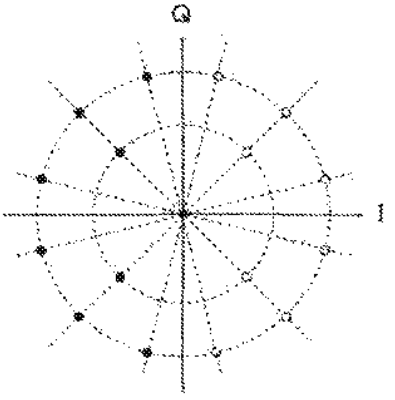

シンボル
1ビット目
2ビット目
3ビット目
4ビット目
S1 S3 S5 S7 S8 S9 S10 S11 S12
S2 S4 S6 P2-4 P1-5 P2-7 P1-8 P2-10 P1-11
P2-1 P2-2 P2-3 P1-4 P2-6 P1-7 P2-9 P1-10 P2-12
P1-1 P1-2 P1-3 P2-5 P1-6 P2-8 P1-9 P2-11 P1-12

（符号化率＝1／3）

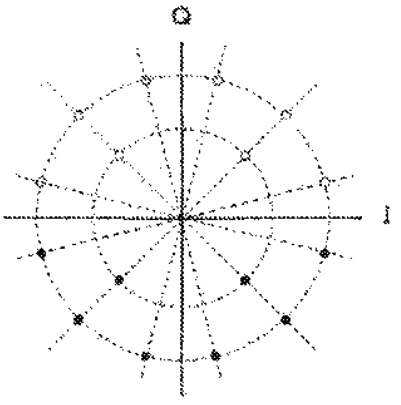
[図43]

[図44]

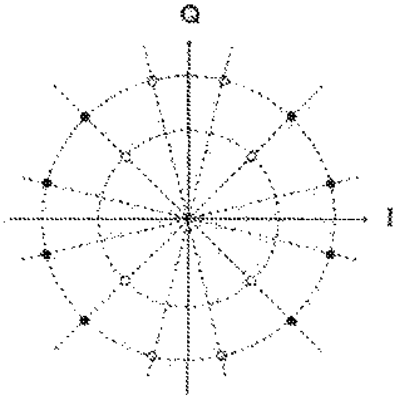
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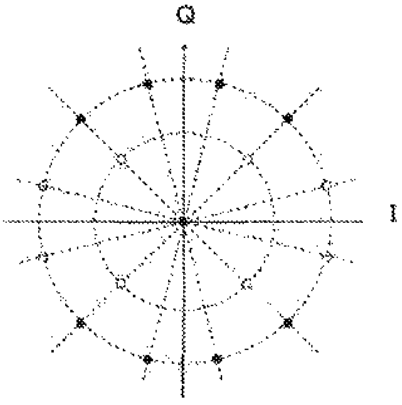
(2)



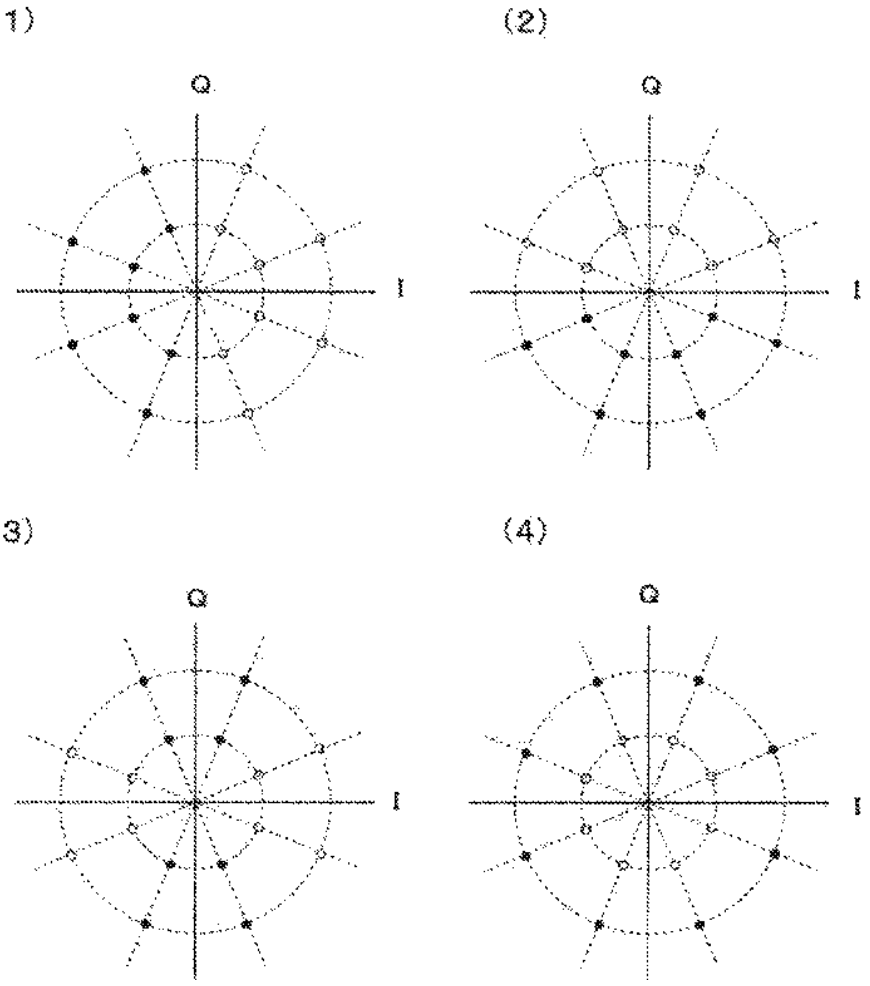
(3)



(4)

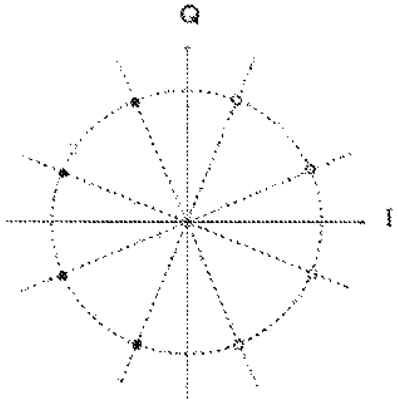


[図46]

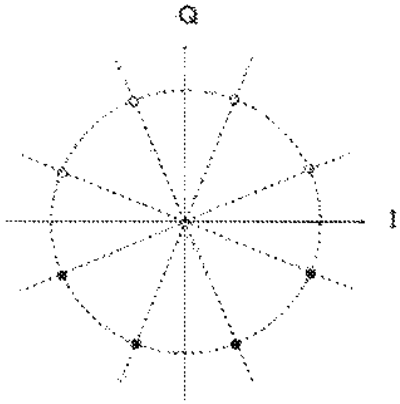


[図46]

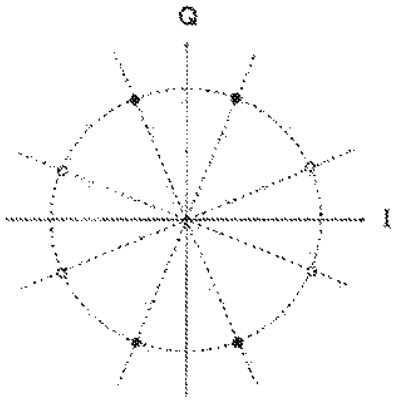
(1)



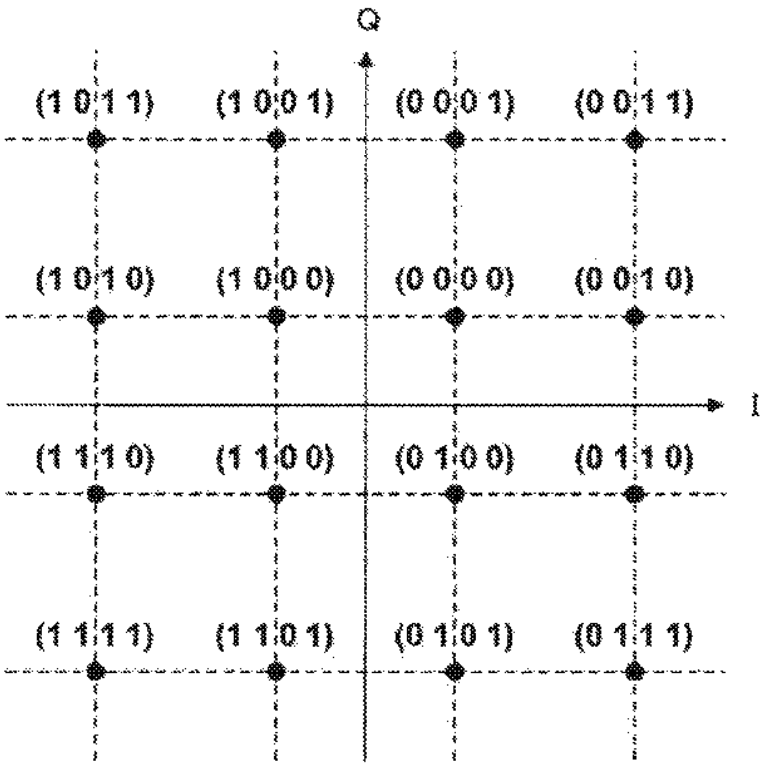
(2)



(3)



[図47]


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(1 0 0 1)
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INTERNATIONAL SEARCH REPORT

International application No.
PCT/JP2005/023956

A. CLASSIFICATION OF SUBJECT MATTER
H04L27/34(2006.01), *H03M13/29*(2006.01)

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H04L27/34(2006.01), *H03M13/29*(2006.01)

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Jitsuyo Shinan Koho 1922-1996 Jitsuyo Shinan Toroku Koho 1996-2006
Kokai Jitsuyo Shinan Koho 1971-2006 Toroku Jitsuyo Shinan Koho 1994-2006

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	JP 2003-143041 A (Samsung Electronics Co., Ltd.), 16 May, 2003 (16.05.03), Par. No. [0008]; Fig. 1 (Family: none)	1-32
A	JP 2004-040661 A (Matsushita Electric Industrial Co., Ltd.), 05 February, 2004 (05.02.04), Par. No. [0024] & US 2005/0166780 A1 & EP 1531560 A1 & WO 2004/006469 A1 & CN 1666442 A	1-32

☒ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

* Special categories of cited documents:
"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier application or patent but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed
"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
28 March, 2006 (28.03.06)	18 April, 2006 (18.04.06)
Name and mailing address of the ISA/ Japanese Patent Office	Authorized officer
Facsimile No.	Telephone No.

Form PCT/ISA/210 (second sheet) (April 2005)

INTERNATIONAL SEARCH REPORT

International application No.
PCT/JP2005/022956

C (Continuation). DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No
A	JP 2004-201131 A (Hitachi Kokusai Electric Inc.), 15 July, 2004 (15.07.04), Par. Nos. [0021] to [0023]; Figs. 1, 5 (Family: none)	1-32
A	JP 8-79325 A (Hitachi, Ltd.), 22 March, 1996 (22.03.96), Par. Nos. [0021] to [0023]; Fig. 1 (Family: none)	1-32

Form PCT/ISA/210 (continuation of second sheet) (April 2005)

国際調査報告

国際出願番号 PCT/JP2005/023956

A. 発明の属する分野の分類（国際特許分類（IPC））

Int.Cl. *H04L27/34(2006.01), H03M13/29(2006.01)*

B. 調査を行った分野

調査を行った最小限資料（国際特許分類（IPC））

Int.Cl. *H04L27/34(2006.01), H03M13/29(2006.01)*

最小限資料以外の資料で調査を行った分野に含まれるもの

日本国実用新案公報	1922－1996年
日本国公開実用新案公報	1971－2006年
日本国実用新案登録公報	1996－2006年
日本国登録実用新案公報	1994－2006年

国際調査で使用した電子データベース（データベースの名称、調査に使用した用語）

C. 関連すると認められる文献

引用文献のカテゴリー*	引用文献名 及び一部の箇所が関連するときは、その関連する箇所の表示	関連する請求の範囲の番号
A	JP 2005-143041 A (三星電子株式会社) 2005.05.16, 段落【0006】, 図1 (ファミリーなし)	1-32
A	JP 2004-040661 A (松下電器産業株式会社) 2004.02.05, 段落【0024】 & US 2005/0164740 A1 & EP 1531560 A1 & WO 2004/006469 A1 & CN 1666442 A	1-32

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☐ パテントファミリーに関する別紙を参照。

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国際調査を完了した日	国際調査報告の発送日
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国際調査機関の名称及びあて先	特許庁審査官（権限のある職員）		
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国際調査報告	国際出願番号 PCT/JP2005/023956

C（続き）． 関連すると認められる文献

引用文献のカテゴリー*	引用文献名 及び一部の箇所が関連するときは、その関連する箇所の表示	関連する請求の範囲の番号
A	JP 2004-201131 A （株式会社日立国際電気） 2004.07.15, 段落【0021】-【0023】, 図1, 図5 （ファミリーなし）	1-32
A	JP 8-79325 A （株式会社日立製作所） 1996.03.22, 段落【0021】-【0023】, 図1 （ファミリーなし）	1-32

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(71) Applicant: Fujitsu Limited
Kawasaki-shi, Kanagawa 211-8588 (JP)

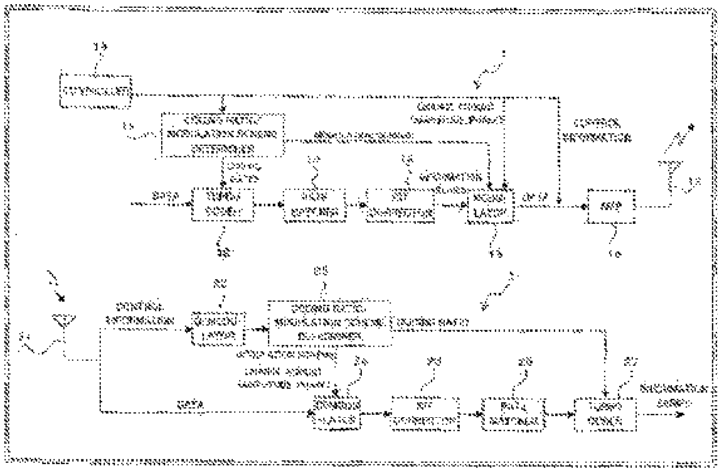
(72) Inventors:
- NAKAGAWA, Takao
Kawasaki-shi,
Kanagawa 2118588 (JP)
- YANO, Tetsuya
Kawasaki-shi,
Kanagawa 2118588 (JP)
- OBUCHI, Kazuhisa
Kawasaki-shi,
Kanagawa 2118588 (JP)

(74) Representative: Weihs, Bruno Konrad
HASELTINE LAKE
5th Floor, Lincoln House
300 High Holborn
London WC1V 7JH (GB)

(54) DIGITAL WIRELESS COMMUNICATION METHOD, TRANSMITTER AND RECEIVER USING MULTILEVEL MODULATION SCHEME

(57) A transmitter (1) for use in digital radio communications systems includes: a bit corrector (14) controls bit arrangement in such a manner that a code having high significance, out of multiple codes obtained by coding, is allocated with high priority to a bit having a tendency that the likelihood enlarges at the time of symbol decision on a receiver (2) ; a multi-level modulator (15) allocates the code to the multiple bits in accordance with a predetermined symbol arrangement; and a symbol arrangement controller (18) controls the symbol arrangement from equal distance arrangement to another arrangement in accordance with a ratio of the codes different in significance. To control symbol arrangement increases the effect of bit correction and improves an error rate on the receiver (2) .

FIG. 2



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Description

TECHNICAL FIELD

[0001] The present invention relates to a digital radio communications system employing the multi-level modulation scheme and also to a transmitter and a receiver. The invention relates, for example, to the technology suitable for use in a digital radio communications system using bit correction processing.

BACKGROUND ART

[0002] Recent digital radio communications systems employ the multi-level modulation scheme such as QPSK (Quadrature Phase Shift Keying), 8PSK, and 16QAM (Quadrature Amplitude Modulation) in order to enable high-speed transmission while effectively using limited radio frequency bands. That is, QPSEK is capable of transmitting two bits by a single symbol; 8PSK is capable of transmitting three bits by a single symbol; 16QAM is capable of transmitting four bits by a single symbol. In this manner, by means of increasing the number of such multi-levels, the amount of information transmitted by a single symbol is increased, high-speed transmission being thereby realized.

[0003] Such multi-level modulation schemes arranges symbols in such a manner that distances among symbols (signal points) are equal with respect to rectangular coordinates of a real axis (I axis) and an imaginary axis (Q axis), that is, distances among symbols (signal points) are equal on a complex plane (IQ plane), in terms of lowering a symbol evaluation error (code error rate) on the receiver end as much as possible. For example, as to 16QAM, the following types are well known: a grid-like arrangement of 16 symbols at equal intervals in a grid-like arrangement (hereinafter will be called "grid-like 16QAM") as shown in FIG. 36; a concentric type arrangement in which four symbols are arranged on the inner circle, of the double circles on a complex plane, and 12 symbols are arranged on the outer circle at the same intervals (hereinafter will be called "(4, 12) circular 16QAM") as shown in FIG. 37; and another concentric type arrangement in which eight symbols are arranged at the same intervals on the respective same circles, the double concentric circles, on a complex plane (hereinafter will be called (8, 8) star 16QAM") as shown in FIG. 38. Further, 8QPSK arranges eight symbols at the same intervals on a circle on a complex plane as shown in FIG. 39.

[0004] In this instance, in the symbol arrangement shown in FIG. 36, a distance ratio between a symbol close to the I axis and the Q axis to a symbol apart from those axes are given as 1:3; a radius ratio of the inner circle to the outer circle shown in FIG. 37 and FIG. 38 is given as 1 : r. Such ratios should by no means be limited to these. Further, as to (4, 12) circular 16QAM shown in FIG. 37 and (8, 8) star 16QAM shown in FIG. 38, researches thereof have been proceeded from theretofore, and descriptions of such researches are made in the following non-patent documents 1 through 4.

[0005]

(A) Bit Allocation (Mapping) Method to Symbols in the Multi-level Modulation Scheme:

Here, a description will be made of a bit allocation (mapping) method in the above described grid-like 16QAM, (4,12) circular 16QAM, (8, 8) star 16QAM, and 8PSK, with reference to FIG. 43 through FIG. 47. In this instance, in FIG. 43 through FIG. 46, a white dot indicates a bit (code) of "0"; a black dot indicates a bit of "1".

(A1) Grid-like 16QAM:

[0006] In a case of grid-like 16QAM, bit allocation as shown in FIG. 43, for example, is performed. That is, as to the first bit, a bit of "1" is allocated to the first bit of each of the eight symbols arranged in the area of the left side of the Q axis (the negative area of the I axis (real axis)) on the paper, by means of quadrant decision with respect to the Q axis (imaginary axis) on the IQ plane, and a bit of "0" is allocated to the first bit of each of the eight symbols arranged on the right side (the positive area of the I axis) on the paper as shown in (1) of FIG. 43. As to the second bit, a bit of "0" is allocated to the second bit of each of the eight symbols of the upper side area of the I axis (the positive area of the Q axis) on the paper, and a bit of "1" is allocated to the second bit of each of the eight symbols of the lower side area of the I axis (the negative area of the Q axis) on the paper by means of quadrant decision with respect to the I axis as shown in (2) of FIG. 43.

[0007] Further, as to the third bit, a bit of "0" is allocated to the third bit of each of the eight symbols close to the Q axis and a bit of "1" is allocated to the third bit of each of the eight symbols apart from the Q axis, by means of distance decision from the Q axis as shown in (3) of FIG. 43. As to the fourth bit, a bit of "0" is allocated to the fourth bit of each of the eight symbols close from the I axis and a bit of "1" is allocated to the third bit of each of the eight symbols apart from the I axis, by means of distance decision from the I axis as shown in (4) of FIG. 43.

[0008] To map bits to symbols with such a method brings about a bit series allocated to each symbol as shown FIG.

47, for example, so that the bit series of adjacent symbols differ from each other only by one bit. This makes it possible to suppress a probable error down to one bit, even if the receiver end erroneously receives the adjacent symbol due to noise. In this instance, such a mapping method in which adjacent symbols differ from each other by only one bit is called "gray-mapping".

(A2) (4, 12) Circular 16QAM:

[0009] In a case of circular 16QAM, bit allocation as shown in FIG. 44, for example, is performed.
That is, as to the first bit, a bit of "1" is allocated to the first bit of each of the eight symbols arranged on each of the circles in the area of the left side of the Q axis (the negative area of the I axis) on the paper, by means of quadrant decision with respect to the Q axis on an IQ plane, and a bit of "0" is allocated to the first bit of each of the eight symbols arranged on each of the circles of the right side area of the Q axis (the positive area of the I axis) on the paper as shown in (1) of FIG. 44. As to the second bit, a bit of "0" is allocated to the second bit of each of the eight symbols on each of the circles in the upper area of the I axis (the positive area of the Q axis) on the paper, and a bit of "1" is allocate to the second bit of each of the eight symbols on each of the circles in the lower area of the I axis (the negative area of the Q axis) on the paper, by means of quadrant decision with respect to the I axis as shown in (2) of FIG. 44.
[0010] Further, as to the third bit, a bit of "0" is allocated to the third bit of each of the eight symbols on the circle close to the Q axis, and a bit of "1" is allocated to the third bit of each of the eight symbols on the circle apart from the Q axis, by means of distance decision from the Q axis as shown in (3) of FIG. 44. As to the fourth bit, a bit of "0" is allocated to the fourth bit of each of the eight bits on the circle close from the I axis and a bit of "1" is allocated to the fourth bit of each of the eight symbols on the circle apart from the I axis, by means of distance decision from the I axis as shown in (4) of FIG. 44.
[0011] That is, in the above described grid-like 16QAM and (4, 2) circular 16QAM, bit allocation is performed to the first bits and to the second bits by means of quadrature decision with respect to the I axis and the Q axis, and bit allocation to the third bit and the fourth bit is performed by means of distance evaluation with respect to the I axis and the Q axis. The first bit and the second bit, and the third bit and the fourth bit, each have the same likelihood (quality), but the likelihood of the first bit and the second bit differs from the likelihood of the third bit and the fourth bit (the likelihood is lower than that of the first bit and the second bit).

(A3) (8, 8) Star 16QAM:

[0012] In a case of (8, 8) star 16QAM, bit allocation as shown in FIG. 45, for example, is performed.
That is, as to the first bit, a bit of "1" is allocated to the first bit of each of the eight symbols arranged on each of the circles in the area of the left side of the Q axis (the negative area of the I axis) on the paper, by means of quadrant decision with respect to the Q axis on the IQ plane, and a bit of "0" is allocated to the first bit of each of the eight symbols arranged on each of the circles of the right side area of the Q axis (the positive area of the I axis) on the paper by means of quadrant decision with respect to the I axis as shown in (1) of FIG. 45. As to the second bit, a bit of "0" is allocated to the second bit of each of the eight symbols on each of the circles in the upper area of the I axis (the positive area of the Q axis) on the paper, by means of quadrant decision with respect to the I axis as shown in (2) of FIG. 45, and a bit of "1" is allocate to the second bit of each of the eight symbols on each of the circles in the lower area of the I axis (the negative area of the Q axis) on the paper, by means of quadrant decision with respect to the I axis as shown in (2) of FIG. 45.
[0013] Further, as to the third bit, a bit of "0" is allocated to the third bit of each of the eight symbols on each of the circles arranged in the first quadrant and the third quadrant, and a bit of "1" is allocated to the third bit of each of the eight symbols on each of the circles arranged in the second quadrant and the fourth quadrant, with the evaluation reference the same as that of the third bit in a case of 8PSK, which will be described later, that is, by means of quadrant decision of a diagonal angle obtained by rotating (for example, right-handed rotation) the I axis and the Q axis 45 degrees as shown in (3) of FIG. 45. As to the fourth bit, a bit of "0" is allocated to the fourth bit of each of the eight symbols on the inner circle close to the origin point, and bit "1" is allocated to the fourth bit of each of the eight symbols on the outer circle apart from the origin point, by means of evaluation of a distance from the origin point, as shown in (4) of FIG. 45.
[0014] Accordingly, in a case of (8, 8) star 16QAM, the likelihood of the first bit is the same as the likelihood of the second bit, and the likelihood of the first bit and the second bit differs from the likelihood of the third bit and the fourth bit (lower than that of the first bit and the second bit). In addition, the likelihood of the third bit differs from that of the fourth bit. That is, (8, 8) likelihood has a three-stage likelihood.

(A4) 8PSK:

[0015] In a case of 8PSK (a single symbol is transmitted using three bits), bit allocation as shown in FIG. 46, for example, is performed

[0016] That is, as to the first bit, a bit of "1" is allocated to the first bit of each of the eight symbols arranged on the circle in the area of the left side of the Q axis (the negative area of the I axis) on the paper, by means of quadrant decision with respect to the Q axis on the IQ plane, and a bit of "0" is allocated to the first bit of each of the eight symbols arranged on the circle of the area of the right side of the Q axis (the positive area of the I axis) on the paper as shown in (1) of FIG. 46. As to the second bit, a bit of "0" is allocated to the second bit of each of the eight symbols on the circle in the upper area of the I axis (the positive area of the Q axis) on the paper, and bit "1" is allocate to the second bit of each of the eight symbols on the circle in the lower area of the I axis (the negative area of the Q axis) on the paper, by means of quadrant decision with respect to the I axis as shown in (2) of FIG. 46.

[0017] Further, as to the third bit, a bit of "0" is allocated to the third bit of each of the eight symbols on the circle arranged in the first quadrant and the third quadrant, and a bit of "1" is allocated to the third bit of each of the eight symbols on the circle in the second quadrant and the fourth quadrant, as shown in (3) of FIG. 46, by means of quadrant decision of a diagonal angle obtained by rotating (for example, right-handed rotation) the I axis and the Q axis 45 degrees as shown in (3) of FIG. 46.

[B] System Using Turbo Code and Bit Correction Method

[0018] As described above, in the multi-level modulation scheme, a difference of likelihood (quality) corresponding to a difference of a symbol evaluation reference in a single symbol is generated among the bits, that is, bits vulnerable to an error and bits resistant to an error are generated (generally speaking, in comparison with the first bit and the second bit allocated by means of quadrant decision with respect to the I axis and the Q axis, the third bit and the fourth bit are vulnerable to an error). Hence, as described in the following patent document 1, mapping performed without paying consideration to the quality of each bit can deteriorate the performance of decoding result on the receiver end, depending on the coding scheme.

[0019] For example, as shown in FIG. 40, a turbo coder transmits an information series [systematic bits (systematic codes)] S which is not coded and also a coded bit series [parity bits (parity codes)], such as a bit series (parity bits P1) coded by an element coder 101 and a bit series (parity bits P2) obtained by coding an information series interleaved by an interleaver 103 by an element coder 102. As described in the following patent document 1, mapping of these systematic bits and parity bits without paying consideration to the quality of each of the bits in a single symbol results in deterioration of the quality of the systematic bits in spite of the fact that the quality of systematic bits exhibits a larger effect (significance is high) to the performance of the decoding result than the quality of parity bits, so that the performance of the decoding result is deteriorated.

[0020] That is, on the receiver end (turbo decoder), as shown in FIG. 41, for example, the element decoder 201 performs decoding (soft decision) using systematic bits S and one kind of parity bits P1. After the interleaver 204 interleaves the obtained result, another element decoder 202 performs decoding thereof using systematic bits S and the other kind of parity bits P2. The deinterleaver 205 deinterleaves the decoding result, and then feedbacks the obtained result to the element decoder 201. This process is repeated a predetermined number of times, thereby performing decoding processing of turbo codes. Then, in order to finally obtain a decoded series as a result of deinterleaving the decoded result obtained by the element decoder 202 by use of the deinterleaver 206, the systematic bits *S* are used by the two element decoders 201 and 202, so that the effect of systematic bits S on the performance of the decoding result is greater than the effect of the quality of parity bits. In this instance, a system employing turbo coding is also described in the following patent document 2.

[0021] Thus, in systems using turbo coding, it is normally preferable that processing called bit correction be performed for improving decoding characteristic. It makes possible for systematic bits *S*, whose significance is higher than that of parity bits *P*, to be mapped to bits whose quality is high in a symbol [For example, see Chapter 4.6.4.4.4 (HARQ bit correction) of the following non-patent document 5]. That is, in 16QAM, of the above mentioned various types of mapping methods, as shown in FIG. 42, for example, the first bit and the second bit are superior in quality to the third bit and the fourth bit because the first and the second bit are quadrant decision in a symbol. Thus, systematic bits S and parity bits P are sequentially allocated in the vertical direction (beginning from the highest order bit toward the lowest order bit) of the paper sheet of FIG. 42, beginning from the left end row of the paper sheet as indicated by the dotted arrow so that systematic bits are allocated to the first and the second bits with high priority. In this instance, this FIG. 42 illustrates an example in which turbo coding is performed at a coding ratio = 1/3. Systematic bits S are shown as S1 through S12; one kind of parity bits (obtained by the element coder 101; for example) are showed as P1-1 through P1-12; another kind of parity bits (obtained by the element coder 102) are showed as P2-1 through P2-12 (the same goes for the description hereinafter).

Patent Document 1: Japanese Patent Application Laid-open No. 2002-171288;
Patent Document 2: Japanese Patent Application Laid-open No. 2003-78419;
Non-patent Document 1: Oshita and Kondo, *"Optimal Signal Point Arrangement in the 16-level Amplitude Phase*

Modulation Scheme", IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences, August, 1999;
Non-patent Document 2: Madoka, Handa, and Oshita, *"(4, 12) -Type Concentric Circle Signal Arrangement and Reception Characteristic thereof in the 16-ary Amplitude Phase Modulation Scheme"*, IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences, October, 1997;
Non-patent Document 3: F. Adachi, M. Sawahashi, *"Performance Analysis of Various 16 Level Modulation Schemes under Rayleigh Fading"*
Non-patent Document 4: Suzuki and Mizuno, *"Multi-symbol DelayWave Detection Scheme for Differential Motion Coding Amplitude Modulation Signals and Application thereof to 16DAPSK"*, IEICE TRANSACTIONS on Fundamentals of Electronics, Communications and Computer Sciences, December, 1994; and
Non-patent Document 5: 3GPP TS 25.212 V5.2.0 (2002-09)

DISCLOSURE OF THE INVENTION ISSUES TO BE SOLVED BY THE PRESET INVENTION

[0022] As already described, the concentric circle signal point arrangements of (4, 12) circular 16QAM and (8, 8) star 16QAM in which neighbouring symbols on the same circle are positioned at equal intervals are optimum in terms of distances among symbols. In a case of the grid-like 16QAM, an arrangement of 16 points (symbols) at equal intervals is optimal in terms of distances among symbols.
However, in a case in which the above described turbo coding and bit correction processing are used in combination, it cannot be said that such signal point arrangement at equal intervals are necessarily optimal in terms of an error rate. In addition, bit correction processing cannot be said to be necessarily optimal.
[0023] The present invention is proposed in view of these issues. One object of the preset invention is to improve an error rate by means of controlling signal points (symbols) arrangement with the previous multi-level modulation scheme for performing transmission so as to make the best use of the effect of bit correction processing.

MEANS TO SOLVE SHE TISSUES

[0024] In order to accomplish the above object, the present invention provides the following digital radio communications method which employs the following multi-levelmodulationscheme,transmitter,and receiver.

(1) As a generic aspect, there provided is a digital radio communications method using the multi-level modulation scheme for use in a digital radio communications system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which demodulates the reception signal received from the transmitter by means of performing symbol evaluation, the radio communications method comprising: on the transmitter, performing a coding process of coding an information series to be transmitted, to generate multiple codes which are different in significance; performing a bit correction process of controlling bit arrangement in such a manner that a code whose significance is high, out of the multiple codes obtained by the coding process, is allocated with high priority to a bit having a tendency such that the likelihood of the bit at the symbol evaluation performed on the receiver becomes large; performing a multi-level modulation process of allocating the code to the plurality of bits in accordance with a bit arrangement obtained in the bit correction process to perform multi-level modulation of the information series at a predetermined symbol arrangement; and performing a symbol arrangement control process of controlling the symbol arrangement in a direction such that the symbol arrangement is changed from an equal distance arrangement in accordance with a ratio of the codes different in significance in the bit correction process, and on the receiver, demodulating a signal received from the transmitter based on information relating to the changed symbol arrangement; and performing a decoding process of decoding the demodulated signal to obtain the information series.

[0025]

(2) As a preferred aspect, the method may comprise: in the symbol arrangement control process performed by the transmitter, controlling the symbol arrangement in a direction such that the likelihood of a bit to which a code high in significance is allocated, out of the plurality of bits, becomes large.
(3) As another preferred aspect, the method may comprise: in the symbol arrangement control process performed by the transmitter, controlling the symbol arrangement in a direction such that the likelihood of a bit to which a code low in significance is allocated, out of the plurality of bits, becomes large.

[0026]

(4) As yet another preferred aspect, the digital radio communications method, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed in the bit correction process, the likelihood of only the first bit becomes large.

[0027]

(5) As still another preferred aspect, the digital radio communications method, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, may comprise : in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed in the bit correction process, the likelihood of the first bit and the second bit becomes large.

[0028]

(6) As a further preferred aspect, the digital radio communications method, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit through the third bit by the bit arrangement performed in the bit correction process, the likelihood of the third bit and the fourth bit becomes large.

[0029]

(7) As a yet further preferred aspect, the digital radio communications method, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the fourth bit by the bit arrangement performed in the bit correction process, the likelihood of only the fourth bit becomes large.

[0030]

(8) As a still further preferred aspect, digital radio communications method, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed in the bit correction process, the likelihood of only the first bit becomes large.

[0031]

(9) As another preferred aspect, the digital radio communications method, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed in the bit correction process, the likelihood of the first bit and the second bit becomes large.

[0032]

(10) As yet another preferred aspect, the digital radio communications method, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, may comprise: in the symbol arrangement control process, controlling the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the third bit by the bit arrangement performed in the bit correction process, the likelihood of only the fourth bit becomes large.

[0033]

(11) As still another preferred aspect, the method may comprise. in the on the symbol arrangement control process, controlling the symbol arrangement in a direction such that a transmission power is not changed before and after

the change of the symbol arrangement.

(12) As a further preferred aspect, the method may comprise: in the bit correction process, controlling the bit arrangement in such a manner that a ratio of a plurality of types of codes low in significance, out of the multiple codes, is equal for every one of the plurality of bits.

[0034]

(13) As a yet further preferred aspect, the transmitter may further comprise a process of notifying the receiver beforehand of information relating to a symbol arrangement changed in the symbol arrangement control process.

(14) As a still further preferred aspect, the receiver may hold information beforehand relating to the symbol arrangement with respect to a ratio of codes different in significance in a memory; the transmitter is operable to perform the process of notifying the receiver of information relating to the ratio; and the receiver is operable to perform the process of reading the information relating to the symbol arrangement based on information relating to the ratio notified from the transmitter, and the process of executing the demodulation based on the read out information.

[0035]

(15) As another preferred aspect, the code high in significance may be a systematic code in a turbo code, and the code low in significance may be a parity code in the turbo code.

(16) As another generic aspect, there provided is transmitter for use in a digital radio communications system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which performs symbol evaluation of a reception signal from the transmitter and demodulates the reception signal received from the transmitter, the transmitter comprising: a coding means which codes an information series to be transmitted, to generate multiple codes different in significance; a bit correction means which controls bit arrangement in such a manner that a code whose significance is high, out of the multiple codes, is allocated with high priority to a bit having a tendency such that the likelihood of the bit at the symbol evaluation performed on the receiver becomes large; a multi-level modulation means which allocates the code to the plurality of bits in accordance with a bit arrangement obtained by the bit correction means to perform multi-level modulation of the information series with a predetermined symbol arrangement; and a symbol arrangement controlling means which controls the symbol arrangement in a direction such that the symbol arrangement is changed from an equal distance arrangement in accordance with a ratio of codes different in significance in the bit correction means.

[0036]

(17) As a preferred aspect, the symbol arrangement controlling means may control the symbol arrangement in a direction such that the likelihood of a bit to which a code high in significance is allocated, out of the plurality of bits, becomes large.

(18) As another preferred aspect, the symbol arrangement controlling means may control the symbol arrangement in a direction such that the likelihood of a bit, out of the plurality of bits, to be allocated with low priority becomes large.

[0037]

(19) As yet another preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed by the bit correction means, the likelihood of only the first bit becomes large.

[0038]

(20) As still another preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 16-ary modulation scheme that transmits a single symbol using four bits, a first bit through a fourth bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed by the bit correction means, the likelihood of the first bit and the second bit becomes large.

[0039]

(21) As a further preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code with high priority is allocated to the first bit through the third bit by the bit arrangement performed by the bit correction means, the likelihood of the third bit and the fourth bit becomes large.

[0040]

(22) As a yet further preferred aspect, the transmitter employs the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the fourth bit by the bit arrangement performed by the bit correction means, the likelihood of only the fourth bit becomes large.

[0041]

(23) As a still further preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 8-arymodulationmethod that transmits a single symbol using three bits, a first bit through a third bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed by the bit correction means, the likelihood of only the first bit becomes large.

[0042]

(24) As another preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 8-ary modulation method that transmits a single symbol using three bits, a first bit through a third bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed by the bit correction means, the likelihood of the first bit and the second bit becomes large.

[0043]

(25) As yet another preferred aspect, when the transmitter employs the multi-level modulation scheme which is the 8-arymodulationmethod that transmits a single symbol using three bits, a first bit through a third bit, the symbol arrangement controlling means may control the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the third bit by the bit arrangement performed by the bit correction means, the likelihood of only the fourth bit becomes large.

[0044]

(26) As still another preferred aspect, the symbol arrangement controlling means may control the symbol arrangement in a direction such that a transmission power is not changed before and after the change of the symbol arrangement.
(27) As a further preferred aspect, the bit correction means may control the bit arrangement in a direction such that a ratio of a plurality of types of codes low in significance, out of the multiple codes, is equal for every one of the plurality of bits.

[0045]

(28) As a yet further preferred aspect, the transmitter may further comprise a notifying means which notifies the receiver beforehand of information relating to a symbol arrangement changed by the symbol arrangement controlling means.
(29) As a still further preferred aspect, the code high in significance may be a systematic code in a turbo code, and the code low in significance may be a parity code in the turbo code.
(30) As yet another generic aspect, there provided is a receiver for use in a digital radio communication system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which performs symbol evaluation of a reception signal from the transmitter and demodulates the reception signal received from the transmitter, the transmitter comprising: a demodulating means which demodulates a signal received from the transmitter based on information relating to

the changed symbol arrangement that has been obtained by changing thereof, on the transmitter, from an equal distance arrangement in accordance with a ratio of the codes different in significance; and a decoding means which decodes the signal demodulated by the demodulation means to obtain an information series transmitted by the transmitter.

[0046]

(31) As a preferred aspect, the receiver may further comprise a symbol arrangement information receiving means which receives information relating to the symbol arrangement from the transmitter.

(32) As another preferred aspect, the receiver may further comprise a memory which holds information relating to the symbol arrangement with respect to the ratio of codes different in significance, and the demodulating means reads information relating to the symbol arrangement from the memory based information relating to the ratio notified from the transmitter, and performs the demodulation based on the read out information.

EFFECTS AND BENEFITS OF THE INVENTION

[0047] The above described aspect realizes at least any one of the following effects and benefits.

(1) Since symbol arrangement is controlled to be changed from an equal interval arrangement to another arrangement in accordance with a ratio (for example, changes in accordance with a coding ratio) of codes different in significance obtained as a result of coding a information series to be transmitted, it is possible to make a code of high significance (or a code of low significance) resistant to an error. In consequence, the effect of bit correction processing can be exhibited sufficiently, so that an error rate of the multi-level modulation scheme is improved.

[0048]

(2) Further, when a transmission electric power is made not to be changed before and after the change of symbol arrangement, it is possible to moderate increase in load (e.g. required amplification characteristic) due to average transmission power increase accompanying change of symbol arrangement in a transmitter.

(3) Furthermore, to control the bit arrangement in such a manner that a ratio of multiple types of codes low in significance, of the above mentioned multiple codes, becomes equal for each of the multiple bits in the bit correction process, contributes to further improvement of an error rate.

BRIEF DESCRIPTION OF THE DRAWINGS

[0049]

FIG. 1 is a diagram illustrating an example of a bit arrangement with a bit correction method in the horizontal direction;
FIG. 2 is a block diagram illustrating a construction of a digital radio communication system using turbo coding and bit correction according to a first embodiment;
FIG. 3 is a diagram illustrating an example of bit arrangement (a coding ratio = 1/9) with the bit correction method according to the first embodiment;
FIG. 4 is a diagram illustrating an example of a bit arrangement (a coding ratio = 1/18) with a vertical bit correction method according to a first aspect of the first embodiment;
FIG. 5 is a diagram illustrating an example of a bit arrangement (a coding ratio = 1/18) with a horizontal bit correction method according to the first aspect of the first embodiment;
FIG. 6 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first bit is made to be large) in a case of grid-like 16QAM in the first aspect of the first embodiment;
FIG. 7 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first bit is made to be large) in a case of (4, 12) circular 16QAM in the first aspect of the first embodiment;
FIG. 8 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first bit is set to be large) in a case of (8, 8) star 16QAM in the first aspect of the first embodiment;
FIG. 9 is a diagram illustrating an example of a bit arrangement with a vertical bit correction method according to a second aspect of the first embodiment;
FIG. 10 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first and a second bit is made to be large) in a case of grid-like 16 QAM in the second aspect of the first embodiment;
FIG. 11 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first bit and a second bit is made to be large) in a case of (4, 12) circular 16 QAM in the second aspect of the first

embodiment;
FIG. 12 is a graph indicating an example of a simulation result (an average SIN characteristic (@BLER = 0.1) with respect to the symbol control amount) as a result of symbol arrangement control shown in FIG. 11;
FIG. 13 is a diagram illustrating an example of a bit arrangement with a vertical bit correction method in a case of (8, 8) star 16QAM in the second aspect of the first embodiment;
FIG. 14 is a diagram illustrating an example of bit arrangement with a horizontal bit correction method in a case of (8, 8) star 16QAM in the second aspect of the first embodiment;
FIG. 15 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first and a second bit is made to be large) in a case of (8, 8) star 16QAM in the second aspect of the first embodiment;
FIG. 16 is a graph indicating an example of a simulation result (an average S/N characteristic (@BLER = 0.1) with respect to the symbol control amount) as a result of symbol arrangement control shown in FIG. 15;
FIG. 17 is a diagram illustrating an example of a bit arrangement with a bit correction method in the vertical direction according to a third aspect of the first embodiment;
FIG. 18 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a third bit and a fourth bit is made to be large) in a case of grid-like 16QAM in the third aspect of the first embodiment;
FIG. 19 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a third and a fourth bit is made to be large) in a case of (4, 12) circular 16QAM in the third aspect of the first embodiment;
FIG. 20 is a diagram illustrating an example of a bit arrangement with a vertical bit correction method in a case of (8, 8) star 16QAM in the third aspect of the first embodiment;
FIG. 21 is a diagram illustrating an example of bit arrangement with a horizontal bit correction method in a case of (8, 8) star 16QAM in the third aspect of the first embodiment;
FIG. 22 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a third and a fourth bit is made to be large) in a case of (4, 12) circular 16QAM in the third aspect of the first embodiment;
FIG. 23 is a diagram illustrating an example of a bit arrangement with a bit correction method in the vertical direction according to a fourth aspect of the first modification;
FIG. 24 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a fourth bit is made to be large) in a case of grid-like 16QAM in the fourth aspect of the first embodiment;
FIG. 25 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a fourth bit is made to be large) in a case of (4, 12) circular 16QAM in the fourth aspect of the first embodiment;
FIG. 26 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a fourth bit is made to be large) in a case of (8, 8) star 16QAM in the fourth aspect of the first embodiment;
FIG. 27 is a diagram illustrating an example of a bit arrangement with a horizontal bit correction method according to a fifth aspect of the first embodiment;
FIG. 28 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first is made to be large) in a case of 8PSK in the fifth aspect of the first embodiment;
FIG. 2 9 is a symbol arrangement diagram for describing an example of symbol arrangement with a horizontal bit correction in the fifth aspect of the first embodiment;
FIG. 30 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a first and a second bit is made to be large) in a case of 8PSK in the fifth aspect of the first embodiment;
FIG. 31 is a diagram illustrating an example of a bit arrangement with a horizontal bit correction method according to the fifth aspect of the first embodiment;
FIG. 32 is a symbol arrangement diagram for describing symbol arrangement control (when the likelihood of a third bit is made to be large) in a case of 8PSK in the fifth aspect of the first embodiment;
FIG. 33 is a diagram illustrating an example of bit arrangement and a symbol arrangement for describing an example in which symbol arrangement control is changed in accordance with time.
FIG. 34 is a block diagram illustrating a construction of a digital radio communications system using turbo coding and bit correction according to a second embodiment;
FIG. 35 is an example of a conversion table for use in an amplitude/angle calculator shown in FIG. 34;
FIG. 36 is a diagram illustrating a symbol arrangement under previous grid-like 16QAM;
FIG. 37 is a diagram illustrating a symbol arrangement under (4, 12) circular 16QAM;
FIG. 38 is a diagram illustrating a symbol arrangement under (8, 8) star 16QAM;
FIG. 39 is a diagram illustrating a symbol arrangement under 8PSK;
FIG. 40 is a block diagram illustrating a construction of a turbo coder;
FIG. 41 is a block diagram illustrating a construction of a turbo decoder;
FIG. 42 is a diagram illustrating an example of a bit arrangement with a bit correction method;
FIG. 43 is a diagram illustrating a symbol arrangement for describing a method of bit allocation (mapping) under grid-like 16QAM;
FIG. 44 is a diagram illustrating a symbol arrangement for describing a method of bit allocation (mapping) under (4,

12) circular 16QAM;
FIG. 45 is a diagram illustrating a symbol arrangement for describing a bit allocation (mapping) under (8, 8) star16QAM;
FIG. 46 is a diagram illustrating a symbol arrangement for describing a bit allocation (mapping) under 8PSK; and
FIG. 47 is a diagram illustrating a symbol arrangement for describing gray mapping under grid-like 16QAM.

DESCRIPTION OF PREFERENCE CHARACTERS

[0050]

1... transmitter
11... coding ratio/modulation scheme determiner (transmission parameter determiner)
12... turbo coder (turbo coding means)
13... rate matcher
14... bit corrector (bit correction means)
15... modulator (multi-level modulating means)
16... amplifier (AMP)
17... transmitter antenna
18... controller (symbol arrangement controlling means)
19...amplitude/angle calculator (symbol control amount calculating unit)
2...receiver
21 ...receiver antenna
22...demodulator (symbol arrangement information receiving means)
23...coding ratio/modulation scheme determiner (transmission parameter determining unit)
24 ...demodulator (demodulating means)
25...bit corrector
26...rate matcher
27...turbo decoder (decoding means)
28...amplitude/angle calculator (symbol control amount calculating unit)
191...conversion table

BEST MODE FOR CARRYING OUT THE INVENTION

[0051] Embodiments will now be described with reference to the relevant accompanying drawings.

[1] Description of an Overview

[0052] The present method improves an error rate on a receiver end by means of moving (shifting) the positions of symbols, which are arranged at equal intervals in an existing method as described above, in a digital radio communications system employing the multi-level modulation scheme, for the purpose of increasing the effect of bit correction processing performed after coding. For example, control is performed in such a manner that the positions of symbols on the IQ plane are changed (moved/shifted) from the positions arranged at equal intervals so that the likelihood of a bit whose error resistance is expected to be increased.
[0053] In addition, the existing method of bit correction is also changed, depending on a coding ratio or a modulation scheme. For example, in a case where turbo codes are used, bit correction processing is controlled in such a manner that a ratio of allocation of multiple types of parity bits to each of the bits in a single symbol is averaged. As an example, the processing in which allocation is performed in the vertical direction (beginning from the first bit (the highest order bit) toward the lowest order bit in one symbol) as already described with reference to FIG. 42 is changed to the processing in which allocation is performed in the horizontal direction (see the dotted arrow) as shown, for example, in FIG. 1. This makes it possible to optimize the level of bit correction to a modulation scheme (it is possible to equalize the multiple types of parity bits for the bits of a single symbol).

[2] First Embodiment:

[0054] FIG. 2 is a block diagram illustrating a construction of a digital radio communication system using turbo coding and bit correction according to a first embodiment of the present invention. The digital radio communications system of FIG. 2 includes a transmitter 1 and a receiver 2. The transmitter 1 is applicable, for example, to a radio base transceiving station (BTS); the receiver 2 is applicable, for example, to a mobile station (MS).
[0055] Then, with attention paid to important parts thereof, the transmitter 1 includes, for example, a coding ratio/

modulation scheme determiner 11; a turbo coder 12; a rate matcher 13; a bit corrector 14; a modulator 15; an amplifier (AMP) 16; a transmitter antenna 17; and a controller 18. The receiver 2 includes, for example: a receiver antenna 21; a demodulator 22; a coding ratio/modulation scheme determiner 23; a demodulator 24; a bit corrector 25; a rate matcher 26; and a turbo decoder 27. In this instance, the turbo coder described above with reference to FIG. 40 can be used as the turbo coder 12 of the transmitter 1, and the turbo coder described above with reference to FIG. 41 can be used as the turbo decoder 27 in the receiver 2.

[0056] Here, in the transmitter 1, the coding ratio/modulation scheme determiner (transmission parameter determining unit) 11 determines transmission parameters including a coding ratio used by the turbo coder 12 and a modulation scheme used by the modulator 15 in accordance with control information from the controller 18. The turbo coder (coding means) 12 performs turbo coding of transmission data in accordance with the coding ratio determined by the coding ratio/modulation scheme determiner 11. For example, the above turbo coder is operable to obtain a systematic bit (S) and two types of parity bits (P1 and P2). In this instance, a ration of generation of such a systematic bit (S) and parity bits (P1 and P2) basically depend on not only the above coding ratio but also puncturing processing performed by the rate matcher 13 described below.

[0057] The rate matcher 13 performs bit repetition processing or puncturing (thinning-out) processing to a bit series coded by the turbo coder 12, thereby executing rate matching, in which the above bit series are made to agree to the number of bits of a predetermined channel of a radio link.

The bit corrector (bit correction means) 14, as described above, performs bit correction in such a manner that systematic bits (S) are allocated with high priority to the first bit and the second bit with the high likelihood (quality) thereof in a single symbol, in a case of default setting (mode) with respect to the bit series after being subjected to the rate matching. Here, as described later, there also is a mode (inverse mode) in which parity bits (P1 and P2) are allocated with high priority to the first and the second bit with the high likelihood thereof.

[0058] The modulator (multi-level modulating means) 15 modulates (maps to a symbol) the above mentioned bit series with a modulation scheme determined by the coding ratio/modulation scheme determiner 11. Here, in a case of QPSK, the above mentioned bit correction is not performed (basically, performed in a case where 16QAM is selected in adaptive modulation).

The amplifier 16 amplifies the modulated data [an analogue radio signal obtained by a non-illustrated digital to analogue converter and a non-illustrated frequency converter (up-converter), or the like] obtained by the modulator 15 up to a required transmission signal level. The transmitter antenna 17 transmits the amplified transmission signal to a propagation path.

[0059] Then, the controller (symbol arrangement controlling means) 18 controls the coding ratio/modulation scheme determiner 11, the bit corrector 14, and the modulator 15. According to the present embodiment, the controller 18 gives information (control information) which works as an element for determining a coding ratio and a modulation scheme to the coding ratio/modulation scheme determiner 11, and also has the following functions.

That is, the controller 18 controls bit correction processing performed by the bit corrector 14 in accordance with a ratio (distribution ratio) of systematic bits (S) to parity bits (P1 and P2) (mainly depends on the above mentioned coding ratio), to control the method of allocation of parity bits. Further, the controller 18 controls mapping processing by the modulator 15 in accordance with the coding ratio, so as to perform change (symbol arrangement control) of the positions of symbols arranged at equal intervals on the complex plane (IQ plane) to different positions by a predetermined amount by the unit of a symbol. Furthermore, the controller 18 is capable of giving bit correction control information corresponding to the coding ratio and the symbol control amount information (amplitude and phase) to the bit corrector 14 and the modulator 15, respectively.

[0060] In this instance, a detailed description of bit correction control and symbol arrangement control will be made later, in addition, control information to the receiver 2 can contain the bit correction control information and the symbol control amount information, such information thereby being able to be notified to the receiver 2.

For example, according to HSDPA (High Speed Downlink Packet Access) specification of 3GPP (3rd Generation Partnership Project), signaling is executed to the receiver 2 through a control channel before packet transmission, a coding ratio adaptively selected on the transmitter 1 end and information (transmission parameters) relating to a modulation scheme being thereby notified from the transmitter end to the receiver end. This makes possible notification of the control information.

[0061] That is, in a case of HSDPA, since information relating to data to be transmitted on an HS-PDSCH (High Speed-Physical Downlink Shared Channel) is notified from the transmitter end to the receiver end beforehand on an HS-SCCH (High Speed-Shared Control Channel), if information bringing about change in the symbol arrangement and the bit correction method is transmitted from the transmitter 1 to the receiver 2 with a signaling packet, as in the case of information of the modulation scheme, etc., the receiver 2 is capable of recognizing such changes and performing demodulation and decoding processing in accordance with the thus recognized changes.

[0062] On the other hand, in the receiver 2, the receiver antenna 21 receives a radio signal from the above mentioned transmitter 1. The demodulator 22 demodulates control information, out of signals received by the receiver antenna 21,

with the demodulation scheme corresponding to the modulation scheme used by the transmitter 1. That is, the demodulator 22 functions as a symbol arrangement information receiving means which receives from the transmitter 1 information relating to the symbol arrangement changed on the transmitter end. In this instance, illustration of a frequency converter (down converter), which performs frequency conversion of a reception signal into a base band signal, and of a digital to analogue converter, which converts an analogue signal into a digital signal, or the like, are omitted.

[0063] The coding ratio/modulation scheme determiner (transmission parameter determining unit) 23 identifies information (transmission parameters) relating to the coding ratio and the modulation scheme on the transmitter 1, based on the control information demodulated by the demodulator 22, and notifies the identified information to the demodulator 24. In the present example, it is possible for the coding ratio/modulation scheme determiner 23 to notify the demodulator 24 also of the above described symbol control amount information together.

On the basis of the information notified from the coding ratio/modulation scheme determiner 23, the demodulator (demodulating means) 24 demodulates data (user data) other than the above control information, out of a reception signal, with the demodulation scheme corresponding to the modulation scheme used on the transmitter 1 in consideration of (based on) the symbol control amount information (information relating to symbol arrangement).

[0064] The bit corrector 25 performs procedures (bit correction) inverse to the bit correction performed by the bit corrector 14 on the transmitter 1 end to the data demodulated by the demodulator 24, thereby restoring the data corresponding to the demodulation data before being subjected to bit correction on the transmitter 1. Likewise, the rate matcher 26 performs processing (puncturing or bit repetition) inverse of that which is performed by the rate matcher 13 on the transmitter 1 end, thereby obtaining a bit series corresponding to that before being subjected to rate matching on the transmitter 1.

[0065] The turbo decoder (decoding means) 27 performs turbo decoding processing to a bit series after rate matching thereof, to decode the original transmission data (information series).

In the system of the present embodiment with the construction described above, the transmission data is turbo coded (coding process) by the turbo coder 12 at a coding ratio determined by the coding ratio/modulation scheme determiner 11, and the coded transmission data is then subjected to rate matching processing and bit correction processing by the rate matcher 13 and the bit corrector 14, respectively (bit correction process). After that, the transmission data is subjected to modulation processing (mapping processing to symbols) performed by the modulator 15 with a modulation scheme (for, example, 16QAM) determined by the coding ratio/modulation scheme determiner 11 (multi-level modulating process).

[0066] At that time, the controller 18 controls mapping processing in accordance with a ratio of systematic bits (S) to parity bits (P1 and P2) so that the positions of symbols on the IQ plane which are allocated at equal intervals are controlled to be changed by a predetermined amount from the original positions thereof (symbol arrangement control process). In this instance, information (that is, relating to symbol arrangement) relating to such an amount of change (amplitude and phase) is contained in the above mentioned control information (for example, a signaling packet on the HS-SCCH) and is notified to the receiver 2 beforehand.

[0067] The modulation data obtained by the modulator 15 is converted into an analogue signal, and is then amplified up to a required transmission signal level by the amplifier 16. After that, the amplified analogue signal is transmitted to a propagation path through the transmitter antenna 17.

On the other hand, in the turbo coder 12, the receiver antenna 21 receives the radio signal transmitted from the transmitter 1, and the demodulator 22 demodulates the control information contained in the received signal. On the basis of the control information, the coding ratio/modulation scheme determiner 23 identifies the coding ratio and the modulation scheme used in the transmitter 1. The information relating to the modulation scheme and the information relating to the coding ratio are notified to the demodulator 24 and to the turbo decoder 27, respectively. In this instance, if the control information contains symbol control amount information, such information is notified also to the demodulator 24.

[0068] On the other hand, user data, out of the reception signal, is demodulated by the demodulator 24 with the demodulation scheme corresponding to the modulation scheme used by the transmitter 1 based on the information (information relating to the modulation scheme and symbol control amount information) notified from the coding ratio/modulation scheme determiner 23 (demodulation process). After being subjected to bit correction processing performed by the bit corrector 25 and rate matching processing performed by the rate matcher 26, the user data is turbo decoded by the turbo decoder 27. As a result, the information series transmitted by the transmitter 1 is obtained (decoding process).

[0069] Next, a detailed description will be made hereinafter of bit correction control and symbol arrangement control performed by the controller 18, which are main points of the present embodiment. In this instance, in the following description, the directions of control of symbols should by no means be always limited to a circular one, a horizontal one, or a vertical one. The control can be performed in a direction such that the likelihood of the bits in a single symbol, a required likelihood thereof is expected to be obtained corresponding to a coding ratio, becomes large.

(2.1) First Aspect (in a Case of Enlarging the Likelihood Only of the First Bit):

[0070] For example, as shown in FIG. 3, in a case where the coding ratio is smaller than 1/4 and where systematic bit (S1 through S4 are allocated only to the leading bit (first bit) (see the shaded part), the likelihood only of the first bit to which the systematic bits (S1 through S4) are allocated is enlarged. In this case, to change the bit correction method used by the bit corrector 14 in addition to changing the symbol arrangement involves better optimization.

[0071] That is, as indicated by the dotted arrow of FIG. 3, parity bits (P1-1 through P1-12, and P2-1 through P2-12; hereinafter will be referred to as P1, P2 unless any distinction therearnong is made) are spread (allocated) over in the horizontal direction of the paper, so that the parity bits P1 and the parity bits P2 are equally distributed for each bit. The change of the bit correction method in this manner exhibits greater improvement effects with the smaller number of systematic bits (S) (in the previous bit correction, the smaller the number of systematic bits (S), the greater the deflection between parity bits (hereinafter, also simply called "parities") P1 and parity bits P2 in each bit).

[0072] Supposing that bit correction processing is performed with the present bit correction method at a coding ratio = 1/18, the result shown in FIG. 4 is obtained. That is, the distribution of parities P1 and P2 to each bit in a single symbol becomes that which is shown in the following table 1.

[0073]

Table 1: Parity Distribution with Bit Correction in the Vertical Direction (Coding Ratio = 1/18)

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
First Bit	0	7
Second Bit	8	1
Third Bit	1	8
Fourth Bit	8	1

[0074] In this manner, even when the coding ratio is smaller than 1/4, in a case of the coding ratio is particularly small, the previous bit correction method raises an issue of a large deflection in the distribution of parities P1 and parities P2. In contrast, in a case where the bit correction method is changed (parities P1 and parities P2 are spread over in the horizontal direction), the result shown in FIG. 5 is obtained. That is, the distribution of parities P1 and parities P2 to each bit in a single symbol becomes that which is shown in the following table 2.

[0075]

Table 2: Parity Distribution with Bit Correction in the Horizontal Direction (Coding Ratio = 1/18)

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
First Bit	3	4
Second Bit	5	4
Third Bit	4	5
Fourth Bit	5	4

[0076] In this manner, it is understandable that the distribution of parities P1 and parities P2 of each bit becomes closer to being averaged by means of allocating parities P1 and P2 in the horizontal direction. As a result, it can be expected that the effect of improvement of an error rate in the receiver 2 be increased more in comparison with the previous method. In this instance, hereinafter, bit correction with the previous method will be called "bit correction in the vertical Direction"; bit correction in the present example will be called "bit correction in the horizontal direction".

[0077] After such bit correction in the horizontal direction being performed by the bit corrector 14, the controller 18 controls the modulator 15 to control symbols in a direction such that the likelihood of the leading bit (first bit) to which systematic bit (S) has been allocated becomes larger.

(2.1.1) In a Case of Grid-like 16QAM:

[0078] In this case, as shown in FIG. 6, for example, the controller 18 performs control in such a manner that the positions of eight symbols (see the reference character 6) the closest to the imaginary axis (Q axis) are moved in a direction so as to separate from each other with respect to the Q axis (it means that the amplitude and the phase of the

symbols are changed; the same goes for the following description) (see the arrow 7) on the IQ plane. Here, in a case where the power (transmission power; the same goes for the following description) should maintain a fixed level (that is, making the power equal before and after change of symbol arrangement; the same goes for the following description), the eight symbols (see the reference character 6) are controlled inwardly (in the direction in which the symbols approach the imaginary axis (see the arrow 8).

[0079] The use of such symbol control enlarges the likelihood of the first bit, which indicates a quadrant decision with respect to the Q axis, and diminishes the third bit, which indicates a distance evaluation from the Q axis, because of increase in symbol density. With the symbols being controlled in such a manner, the likelihood of the first bit becomes larger, so that an error rate of the whole is generally improved, but the improvement of characteristic stops upon control made to a certain amount. In addition, since further control of the symbols makes the likelihood of the third bit too small, the characteristics of an error rate of the whole is deteriorated. In this instance, the likelihood of the second and the fourth bit is not changed.

[0080] Further, symbol control such that the power becomes equal before and after change of the symbol arrangement as described above, moderates increase in the load due to increase in the average transmission power in the transmitter 1 accompanying this symbol arrangement change, for example, PAPR, thereby realizing a lessening of amplification characteristic necessary for the amplifier 16. In this instance, this matter goes for in a case where symbol arrangement control is performed so as to make the power keep a fixed level in the various modifications described hereinafter. The relationship between the present invention and PAPR will be described later.

(2.1.2) in a Case of (4, 12) Circular 16QAM:

[0081] In this case, as shown in FIG. 7, for example, the controller 18 performs control in such a manner that the positions of four symbols (see the reference character 5) the closest to the imaginary axis (Qaxis) are controlled in a direction so as to separate from each other with respect to the Q axis. At that time, in a case where the power maintains a fixed level, the symbols to be controlled are controlled along the same circle (see the arrow 7).

[0082] In this instance, the optimal control amount depends on a radius ratio of the inner circle to the outer circle. That is, in (4, 12) circular 16QAM and (8, 8) star 16QAM, which are circular modulation schemes, the radius ratio of the inner circle to the outer circle exhibits effects to characteristics as parameters. Then, when turbo codes are used, the optimal radius ratio relates to the coding ratio. That is, at the time a certain coding ratio is given (depending on the condition of a propagation path), a single optimal radius ratio is obtained. For example, the smaller the coding ratio (the fewer the systematic bits), the smaller the optimal radius ratio. In contrary, the larger the coding ratio (the more the systematic bits), the larger the optimal radius ratio. That is because of the following reason.

[0083] A small coding ratio means that the number of systematic bits is small. In this case, adjustment of the radius ratio such that the likelihood of systematic bits becomes strong provides the optimal characteristic. Since the systematic bits are sequentially allocated beginning from the first bit, in consideration of determining the radius ratio such that the likelihood of the first bit is strong, the small radius ratio (the radius of the inner circle and that of the outer circle becomes close) becomes the optimal.

[0084] As a result of diminishing the radius ratio in such a manner, the likelihood of the third and the fourth bit becomes small in a case of (4, 12) circular 16QAM, and the likelihood of the fourth bit becomes small in a case of (8, 8) star 16QAM, and bits allocated to such bits are parity bits. When the coding ratio is small, the optimal characteristic can be obtained by means of enlarging the likelihood of systematic bits few in number even with parity bits large in number being sacrificed.

[0085] With the coding ratio enlarged, the optimal characteristic can contrarily be obtained by means of enlarging the likelihood of parity bits. That is, the optical radius becomes large. Here, since enlarging of the coding ratio means becoming close to a case in which coding is not performed, the radius ratio eventually becomes close to a radius ratio which is optimal in a case where coding is not executed (equals to a radius ratio optimal for the Raw BER (Bit Error Rate) characteristic, and the condition in which all of the 16 symbols are arranged at intervals as equal as possible.

[0086] Further, an idea of the embodiment is to improve characteristics by means of arranging symbols at positions different from those of the previous art. The optimal value of this control amount (if in a case where the symbols are moved along circles) has a relationship with a radius ratio. That is, the smaller the radius ratio of the inner circle to the outer circle, the larger the optimal control amount; in contrary, the larger the radius ratio of the inner circle to the outer circle, the smaller the optimal control amount.

With what is described the above, a relationship between the coding ratio and the optimal control amount is considerable. That is, the smaller the coding ratio, the smaller becomes the optimal radius ratio, so that the optimal control amount becomes large. Contrarily, the larger the coding ratio, the larger becomes the optimal radius, so that the optimal control amount becomes small.

[0087] Hence, the optimal radius ratio and the optimal control amount depend on the amount of systematic bits. This matter goes for the following description unless any particular description is made.

(2.1.3) In a Case of (8, 8) Star 16QAM:

[0088] In this case, as shown in FIG. 8, for example, the controller 18 performs control in such a manner that the positions of eight symbols (see the reference character 5) the closest to the imaginary axis (Q axis) are moved in a direction so as to separate from each other (from the Q axis) . At that time, in a case where the electric power should maintain a fixed level, the symbols to be moved are moved along the same circle (see the arrow 7). In this instance, in this case, also, the optimal control amount depends on a radius ratio of the inner circle to the outer circle.

(2.2) Second Aspect (in a Case Where the Likelihood of the First and the Second Bit is Enlarged:

[0089] For example, as shown in FIG. 19, when systematic bits (S1 through S12) are allocated (see the shaded part) to the first bit and the second bit in bit correction processing at a coding ratio not smaller than 1/4 and not larger than 1/2, the likelihood of the first bit and the second bit to which the systematic bits (S1 through S12) are allocated is enlarged.

[0090] In this case, the controller 18 changes the bit correction method used by the bit corrector 14 into bit correction in the horizontal direction only in a case of (8, 8) star 16QAM. In this instance, since change of the bit correction method in the horizontal direction does not cause a significant difference in the distribution of parities P1 and P2 (characteristic is not changed) in grid-like 16QAM and (4, 12) circular 16QAM, the previous bit correction in the vertical direction is available as shown in FIG. 19.

(2.2.1) In a Case of Grid-like 16QAM

[0091] In this case, as shown in FIG. 10, for example, the controller 18 performs symbol control to all the 16 symbols . That is, since inner (close to the I axis and the Q axis) the symbols relating to systematic bits (S) are controlled outwardly (in a direction in which the symbols separates from the I axis and the Q axis), the outer symbols are controlled in the inner direction (in a direction in which symbols approach the I axis and the Q axis). This makes the likelihood of the first bit and the second bit, which are used quadrant decision with respect to the I axis or the Q axis, large (the likelihood of the third bit and the fourth bit, which are used distance evaluation from the I axis or the Q axis, becomes small because of high symbol density).

[0092] In this manner, with the symbols being controlled, an error rate of the whole is generally improved up to a certain level because of the effect of enlarging of the likelihoodof systematicbit (S) . However, further symbol movement deteriorates characteristic because the likelihood of the third bit and the fourth bit becomes too small.

(2.2.2) In a Case of (4, 12) Circular 16QAM:

[0093] In this case, as shown in FIG. 11, for example, the controller 18 performs symbol control to a total of eight symbols, four symbols (see the reference character 6) close to the real axis (I axis) on the outer circle and four symbols (see the reference character 5) close to the imaginary axis (Q axis). At that time, if the power is made to be fixed, the symbols to be controlled are controlled along the same circle (see arrow 7). This enlarges the likelihood of the first bit and the second bit, which are used quadrant decision with respect to the I axis and the Q axis (the likelihood of the third bit and the fourth bit, which is distance evaluation from the I axis and the Q axis becomes small).

[0094] In this case, also, with the symbols being controlled, an error rate of the whole is generally improved up to a certain level because of the effect of enlarging of the likelihoodof systematic bit (S) . However, further symbol control will contrarily deteriorate characteristic because the likelihood of the third bit and the fourth bit becomes too small. In this instance, the optimal control amount depends on a diameter ratio of the inner circle to the outer circle. For example, when the coding ratio = 1/3, the optimal radius ratio of the inner circle to the outer circle is 1:1.9. Here, change in the coding ratio causes a change of the optimal radius ratio (that is, the optimal θ (an angle with which the symbols are controlled along the circles)).

[0095] FIG. 12 shows a graph indicating a required average S/N characteristic which satisfies BLER = 0.1 with respect to θ in a case where the symbols are controlled by an angle of θ along the circle as shown in FIG. 11. From this FIG. 12, it is understandable that an improvement of about 0.2 dB (θ = 6°) at the maximum can be obtained in comparison with a case where the symbols are not controlled (θ = 0°) (that is, an equal interval arrangement).

In this manner, to set the optimal symbol control amount enhances the effect of bit correction processing, so that characteristics after decoding performed is improved as a whole.

(2.2.3) In a Case of (8, 8) Star 16QAM:

[0096] In this case, the controller 18 is capable of performing an improved optimization by means of changing the bit correction method used by the bit corrector 14 also, in addition to changing the symbol arrangement as in the case of

the first modification.

Here, in a case of (8, 8) star 16QAM, since the likelihood of the third bit differs from that of the fourth bit, parities P1 and parities P2 are equally allocated to the third bit, and similarly, parities P1 and parities P2 are equally allocated to the fourth bit. In the grid-like 16QAM and (4, 12) circular 16QAM , the likelihood of the third bit equals to that of the fourth bit, so that the number of parities P1 and the number of parities P2 should be equal in the sum of the third bit and the fourth bit. Thus, as to the grid-like 16QAM and the (4, 12) circular 16QAM, the vertical bit correction method is available.

[0097] Provisionally, if the vertical bit correctionmethod is performed, the result becomes that which is shown in FIG. 13. That is, the distribution of parities P1 and P2 to each bit in a single symbol becomes what is shown in the following table 3.

[0098]

Table 3: Parity Distribution with Bit Correction in the Vertical Direction

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
Second Bit	3	3
Third Bit	3	6
Fourth Bit	6	3

[0099] As indicated in the above table, the number of parities of the third bit and that of the fourth bit are not averaged by use of the vertical bit correction method.

In contrast, bit correction in the horizontal direction obtains the distribution of parities P1 and parities P2 with respect to each bit in a single symbol becomes that which is shown in FIG. 14. The distribution of parities P1 and P2 in a single bit becomes what is shown in the following table 4.

[0100]

Table 4: Parity Distribution with Bit Correction in the Horizontal Direction

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
Second Bit	3	3
Third Bit	4	5
Fourth Bit	5	4

[0101] As indicated in the above table, it is possible to realize better averaging of the distribution of parities P1 and P2 in comparison with the previous bit correction method.

(2, 2, 4) Symbols to be Controlled in (8, 8) Star 16QAM:

[0102] In this case, as shown in FIG. 15, for example, the controller 18 performs symbol control to all of the 16 symbols. That is, the eight symbols (see the reference character 6) close to the I axis are controlled in a direction such that the symbols separate from each other (in the direction such that the symbols separate from the I axis; the eight symbols (see the reference character 5) close to the Q axis are also controlled in a direction such that the symbols separate from each other (in the direction such that the symbols separate from the Q axis) . At that time, when the power should keep a fixed level, the symbols to be controlled are controlled along the same circle (see arrow 7) . This enlarges the likelihood of the first bit and the second bit (the likelihood of the third bit becomes small because of a high symbol density). In this instance, to control the symbols along the same circle will not change the likelihood of the fourth bit.

[0103] With the symbols being controlled in such a manner, the likelihood of the third bit becomes larger, so that an error rate of the whole is generally improved up to a certain level, but further control of the symbols will contrarily deteriorate the likelihood of the third bit because of the likelihood of the third bit becoming too small. In this case, also, because of the above described reason, the optimal control amount depends on a radius ratio (the likelihood of the fourth bit) of the inner circle to the outer circle. Here, when the coding ratio = 1/3, (8, 8) star 16QAM hardly shows any change in characteristics due to a radius ratio. Hence, as an example, a result of simulation in which symbols are controlled along the circles by an angle of θ at a radius ratio = 1:1.6 as shown in FIG. 15 is shown in FIG. 16. That is, the simulation result indicates a required average S/N characteristic satisfying BLER = 0.1 with respect to θ. From this FIG. 16, it is understandable that the best characteristic can be obtained in a case where θ = 4°.

[0104] In this instance, in FIG. 16, a coding ratio = 1/3 is given for making agree to the graph of FIG. 12 in a case of

(4, 12) circular 16QAM. However, it is also possible to specify the optimal radius ratio because change of characteristics due to a radius ratio in a case where the coding ratio takes a different value (for example, the coding ratio = 1/2, or the like).

(2.3) Third Aspect (in a Case Where the Likelihood of the Third and the Fourth Bit is Enlarged):

[0105] For example, as shown in FIG. 17, when systematic bits (S1 through S22) are allocated (see the shaded part) to the middle of the first bit through the third bit in bit correction processing at a coding ratio not smaller than 1/2 and not larger than 3/4, the likelihood of the third bit and the fourth bit to which parity bits P1 and P2 (S1 through S12) are allocated is enlarged on the receiver 2, since the significance of parity bits (P1 and P2), few in number, is increased.

[0106] In this case, also, as in the case of the second modification, the bit correction method is changed only in a case of (8, 8) star 16QAM. Since change of the bit correction method in the horizontal direction does not bring about a significant difference in characteristics in a case of grid-like 16QAM and (4, 12) circular 16QAM, the vertical bit correction is available.

(2.3.1) In a Case of the Grid-like 16QAM:

[0107] In this case, as shown in FIG. 18, for example, the controller 18 performs symbol control to all the 16 symbols. That is, the eight inner symbols close to the I axis and the Q axis are controlled in a direction such that the symbols approach the I axis and the Q axis. At that time, when the power should keep a fixed level, the outer (apart from the I axis and the Q axis) symbols are controlled outwardly since the inner symbols (close to the I axis and the Q axis) are controlled inwardly. This enlarges the likelihood of the third bit and the fourth bit (the likelihood of the first and the second bit becomes small).

[0108] In the present example, with the symbols being controlled as described above, an error rate of the whole is generally improved to a certain level according to the control amount of the symbols due to the effect of enlarging of the likelihood of the parities. However, further symbol control will contrarily deteriorate characteristic because the likelihood of the first bit and the second bit becomes too small.

(2.3.2) In a Case of (4, 12) Circular 16QAM:

[0109] In this case, as shown in FIG. 19, for example, the controller 18 performs symbol control to a total of eight symbols, the four symbols (see the reference character 6) close to the real axis (I) and he four symbols (see the reference character 5) close to the imaginary axis (Q axis), in such a manner that the symbols approach the I axis and the Q axis, respectively. At that time, when the power should keep a fixed level, the symbols to be controlled are controlled along the same circle (see the arrow 7). This enlarges the likelihood of the third bit and the fourth bit (the likelihood of the first bit and the second bit is made to be small because of a high symbol density).

[0110] In this case, also, with the symbols being moved as described above, an error rate of the whole is generally improved to a certain level according to the control amount of the symbols due to the effect of enlarging of the likelihood of the parities. However, further symbol control will deteriorate characteristics because the likelihood of the first bit and the second bit becomes too small.

(2.3.3) In a Case of (8, 8) Star 16QAM:

[0111] In this case, the controller 18 changes the bit correction method, in addition to changing the symbol arrangement from an averaged arrangement as in the case of the first aspect, so that a better optimization can be realized. Provisionally, if the vertical bit correction is performed, the result becomes that which is shown in FIG. 20. That is, the distribution of parities P1 and P2 for each bit in a single symbol becomes what is shown in the following table 5.

[0112]

Table 6: Parity Distribution with Bit Correction in the Vertical Direction

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
Third Bit	0	5
Fourth Bit	7	2

[0113] As shown in the above table, the vertical bit correction method is incapable of averaging the number of parities of the third bit and the fourth bit between the parity P1 and the parity P2.

In contrast, the horizontal bit correction exhibits a result shown in FIG. 21, and the distribution of the parities P1 and P2

for each bit in a single symbol becomes that which is shown in the following table 6.

[0114]

Table 6: Parity Distribution with Bit Correction in the Horizontal Direction

Bit Position in 1 Symbol	The Number of Parities P1	The Number of Parities P2
Third Bit	2	3
Fourth Bit	5	4

[0115] As shown in the above table, it is possible to realize better averaging of the distribution of the parities P1 and P2 in comparison with the vertical bit correction method.

(2.3.4) Symbols Moved in (8, 8) Star 16QAM:

[0116] As shown in FIG. 22, the controller 18 performs symbol control to all the 16 symbols. That is, the eight symbols (see the reference character 6) close to the I axis are controlled in a direction such that the symbols approach each other (in a direction such that the symbols become closer to the I axis) and also the eight symbols (see the reference character 6) close to the Q axis are controlled in a direction such that the symbols approach each other (in a direction such that the symbols approach the Q axis. At that time, when the electric power should keep a fixed level, the symbols to be controlled are controlled along the respective same circles (see the arrow 7). This enlarges the likelihood of the third bit (the likelihood of the first and the second bit becomes small) . In addition, enlarging a radius ratio of the inner circle to the outer circle (see the arrow 8) enables enlargement of the fourth bit.

[0117] As described above, with the symbols being controlled, an error rate of the whole is generally improved up to a certain level according to the control amount of the symbols due to the effect of enlarging of the likelihood of the parities. However, further symbol control will contrarily deteriorate characteristics because the likelihood of the first and the second bit becomes too small.

(2.4) Fourth Aspect (In a Case Where the Likelihood Only of the Fourth Bit Is Enlarged)

[0118] For example, as shown in FIG. 23, when systematic bits (S1 through S27) are allocated (see the shaded part) to all the bits, the first bit through the third bit, in bit correction processing at a coding ratio of not smaller than 3/4, the likelihood only of the fourth bit to which the parity bits (P1 and P2) are allocated is enlarged.

[0119] In this case, the vertical bit correction is available since changing the bit correction method into in the horizontal direction does not bring about a significant difference in effect (bit arrangement) in comparison with the vertical bit correction method.

(2.4.1) In a Case of Grid-like 16QAM:

[0120] In this case, as shown in FIG. 24, the controller 18 performs symbol control to all the 16 symbols. That is, the eight symbols close to the I axis are controlled inwardly (in a direction such that the symbols become closer to the I axis) and also the eight symbols apart from the Q axis are controlled outwardly (in a direction such that the symbols separates from the I axis) . This enlarges the likelihood of the fourth bit (the likelihood of the second bit becomes small). In this instance, the likelihood of the first and the third bit is not changed.

[0121] In this case, also, with the symbols being controlled, an error rate of the whole is generally improved up to a certain level according to the symbol control amount due to the effect of enlarging of the likelihood of the parity bits. However, further symbol control will contrarily deteriorate characteristics because the likelihood of the second bit becomes too small.

(2.4.2) In a Case of (4, 12) Circular 16QAM:

[0122] In this case, as shown in FIG. 25, the controller 18 performs symbol control such that the eight symbols close to the real axis (I axis) and the eight symbols close to the imaginary axis (Q axis) come closer to the I axis and the Q axis, respectively. At that time, when the power should keep a fixed level, the symbols to be controlled are controlled along the respective same circles. This enlarges the likelihood of the fourth bit (the likelihood of the first and the second bit becomes small).

[0123] In this case, also, with the symbols being controlled, an error rate of the whole is generally improved up to a certain amount level according to the symbol controlled amount due to the effect of enlarging of the likelihood of the

parity bits. However, further symbol control will deteriorate characteristics because the likelihood of the first bit and the second bit becomes too small.

(2.4.3) In a Case of (8, 8) Star 16QAM:

[0124] In this case, the controller 18 performs control such that a radius ratio of the inner circle to the outer circle becomes small (see the arrow 9). At that time, when the power should keep a fixed level, not only the radius of the inner circle but also that of the outer circle is changed. This enlarges the likelihood of the fourth bit (the likelihood of the first, the second bit, and the third bit becomes small).

[0125] With the radius ratio being changed in such a manner, an error rate of the whole is gradually improved because of the effect of enlarging of the likelihood of the parities up to a certain level according to the change of the radius ratio. However, further control will make the likelihood of the first, the second, and the third bit too small, so that characteristics will contrarily deteriorate.

(2.5) Fifth Aspect (In a Case of 8PSK):

(2.5.1) For Enlarging the Likelihood of the First Bit:

[0126] In a case of 8PSK, also, the controller 18 changes the bit correction method used by the bit corrector 14 (performs a horizontal bit correction) (see FIG. 27). Further, as shown FIG. 28, the controller 18 performs symbol control of the four symbols (see the reference character 5) the closest to the imaginary axis (Q axis) in a direction such that the symbols separate from each other. At that time, when the power should keep a fixed level, the symbols to be controlled are controlled along the circle (see the arrow 7).

(2.5.2) In a Case Where Systematic Bits are Allocated to the Middle of the Second Bit:

[0127] In this case, also, as shown in FIG. 29, the controller 18 changes the bit correction method used by the bit corrector 14 (horizontal bit correction is performed). Further, the controller 18 performs symbol control to all the eight symbols as shown in FIG. 30. That is, the four symbols (see the reference character 5) close to the imaginary axis (Q axis) are controlled in a direction so as to separate from the Q axis; the four symbols (see the reference character 6) close to the real axis (I axis) are controlled so as to seperate from the I axis. At that time, in a case where the power should maintain a fixed level, the symbols to be controlled are controlled along the circle (see the arrow 7).

(2.5.3) In a Case where Systematic Bits are Allocated to the middle of the Third Bit (see FIG. 31):

[0128] In this case, the vertical bit correction method is available since there is no point in changing the bit correction method (allocation becomes the same). The controller 18, as shown in FIG. 32, performs symbol control to all the eight symbols. That is, the four symbols (see the reference character 5) close to the Q axis are controlled in a direction so as to come closer to the Q axis; the four symbols (see the reference character 6) close to the I axis are moved so as to come closer to the I axis. At that time, in a case where the power should maintain a fixed level, the symbols to be controlled are controlled along the circle.

(2.6) In a Case where Symbol Control is Changed in Accordance with Time [As an Example, in a Case of (8, 8) Star 16QAM]:

[0129] Adaptively changing the above described methods of the first through the fourth aspects with elapse of time makes possible a better optimization. That is, the controller 18, as shown in FIG. 33, controls the beginning four symbols (see the both of the thick solid arrows 31) in such a manner that a radius ratio of the inner circle to the outer circle becomes large as shown in (1) of FIG. 32 by means of applying the above described fourth aspect. As to the subsequent five symbols (see the thick solid arrow 32), the controller 18 applies the second aspect, for example, to control the symbols, as shown in (2) of FIG. 33, so as to increase an error resistance of the systematic bits S.

[0130] In this instance, although FIG. 33 shows a case of (8, 8) star 16QAM, the same operation can be similarly executed in the cases of (4, 12) circular 16QAM, grid-like 16QAM, and 8PSK. In addition, the same operation can be carried out in a case where the systematic bits are allocated to the middle of the first bit, in a case where those are allocated to the middle of the second bit, and in a case where those are allocated to the middle of the fourth bit.

(2.7) In a Case Where Systematic Bits and Parity Bits are Exchanged:

[0131] The above described examples are described on the assumption that systematic bits are sequentially arranged beginning from the first bit. With such systematic bits being increased, systematic bits are allocated also to the second bit and the third bit. However, it is considerable that bit allocation is performed beginning from parity bits of the first bit, inversing the relationship between systematic bits and parity bits (for example, see Chapter 4.5.4.3, Chapter 4.5.7, and Chapter 4.6.2.1, etc., of the above non-patent document 5)

[0132] The above described methods can support also such a case. That is, when the number of parity bits is small, the symbols can be controlled in such a manner that the likelihood of the bits which are allocated to parity bits becomes large. When the number of parity bits is large, the symbols can be controlled in such a manner that the likelihood of the systematic bits becomes large.

In this manner, the symbol arrangement is controlled to be changed from an equal interval arrangement in accordance with the coding ratio (a ratio of systematic bits to parity bits), an error rate in the 16-level or the 8-level modulation scheme with use of turbo codes being thereby improved. This is because, in bit correction processing, sufficient effect of bit correction processing cannot be obtained with the previous symbol (signal point) arrangement, in which 16 symbols are arranged at equal intervals, but the effect of bit correction processing becomes large as a result of controlling the symbols from the equal interval arrangement, thereby enlarging the likelihood of the demanded bits so that the systematic bits (or parities) of high significance are hardly received erroneously. As a result, the effect of bit correction becomes large.

[3] Second Embodiment:

[0133] FIG. 34 is a block diagram illustrating a construction of a digital radio communications system using turbo coding and bit correction according to a second embodiment. As in the case of the first embodiment, the digital radio communications system shown in FIG. 34 includes a transmitter 1 and a receiver 2. The transmitter 1 is applicable to, for example, a radio base transceiving station (BTS) ; the receiver 2 is applicable to, for example, a mobile station (MS). The second embodiment differs from the construction shown in FIG. 2 in that an amplitude/angle calculator 19 is added to the transmitter 1 and that the amplitude/angle calculator 29 is added to the receiver 2. In this instance, elements with reference characters the same as those already described are the same as or similar to the elements already described.

[0134] Here, as shown in FIG. 35, for example, the amplitude/angle calculator (symbol control amount calculating unit) 19 of the transmitter 1 stores beforehand the amplitude ratios *a* (*i*) and the angles θ (*i*) (here, i = 1 through n) corresponding to the coding ratios [R(0) through R (1), R (1) through R (2), ..., R (n-1) through R (n) ; n is an integer equal to or larger than 2] (transmission block size) and an RV parameter, as data 191 in a form of a table (hereinafter, will be called the conversion table 191), in a memory or the like, which is not illustrated. The amplitude/angle calculator 19 is capable of reading from this conversion table 191 an amplitude ratio *a* (*i*) and an angle θ (*i*) as a symbol control amount corresponding to a coding ratio determined by the coding ratio/modulation scheme determiner 11, and then outputting the read out data 191 (notifying to the modulator 15).

[0135] In this instance, the "RV parameter" is mode setting information for setting a mode such as a mode in which the relationship between systematic bits and parity bits are inversed (that is, parity bits are allocated with high priority to the bits in a single symbol beginning from the first bit), a mode in which the rate matcher 13 punctures systematic bits with high priority, and a mode in which parity bits are punctured with high priority. In accordance with such mode setting, a ratio between the systematic bits and the parity bits in bit correction processing is changed.

[0136] Accordingly, systematic bits are sequentially allocated beginning from the first bit in a single symbol, and in a default mode, in which parity bits are punctured with high priority, it is only necessary that a conversion table 191 be prepared in which amplitude ratios *a* (*i*) and angles θ (*i*) are simply associated with coding ratios. However, in modes other than the default mode, it is necessary to pay consideration to the RV parameter. In consequence, a conversion table 191 is necessary in which amplitude ratios *a* (*i*) and angles θ (*i*) are associated with each other corresponding to ratios of systematic bits to parity bits.

[0137] On the other hand, as in the case of the amplitude/angle calculator 19 of the transmitter 1, the amplitude/angle calculator (symbol control amount calculating unit) 28 of the receiver 2 stores beforehand the same conversion table 191 in a non-illustrated memory or the like as that provided for the transmitter 1 end. The amplitude/angle calculator 28 reads an amplitude ratio *a* (*i*) and an angle θ (*i*) from the conversion table 191 as symbol control amount information for a coding ratio determined by the coding ratio/modulation scheme determiner 23, and outputs the read out data (notifies to the demodulator 24). That is, the above mentioned memory holds codes of different significance, that is, information relating to symbol arrangements corresponding to ratios between systematic bits and parity bits.

[0138] In other words, the demodulator 24 reads symbol control amount information from the above memory (conversion table 191) based on information relating to a coding ratio (a ratio between systematic bits and parity bits) which is notified from the transmitter 1 as control information, and on the basis of the read out information, the demodulator 24 performs demodulation processing.

That is, according to the present embodiment, different from the first embodiment in which symbol control amount information, such as an amplitude ratio and an angle, on the transmitter 1 end is contained in control information (signaling packet) transmitted to the receiver 2, the same conversion tables 191 are provided beforehand for the both of the transmitter 1 and the receiver 2. This makes possible equivalent processing of the first embodiment, without causing increase in an amount of control information.

[0139] Accordingly, in addition to an advantage of obtaining the effects and the benefits equivalent to those in the first embodiment, another advantage can be obtained that the receiver 2 is capable of performing, with high accuracy and speed, modulation and demodulation processing corresponding to change of bit correction and symbol arrangement on the transmitter 1 end, while attempting to realize efficient use of radio resources between the transmitter 1 and the receiver 2 in comparison with the first embodiment.

[4] Others

a) Relationship between Error Rate of Each Modulation Scheme and PAPR (Peak-to-Average Ratio):

[0140] When a coding ratio = 1/3, the BER characteristic after decoding and the BLER characteristic may be any of the Raw BER characteristic, the BER characteristic after decoding, and the BLER characteristic. However, PAPR is large in comparison with (8, 8) star 16 QAM and (4, 12) circular 16QAM. The Raw BER characteristic in (8, 8) star 16QAM is the worst among those in the above three schemes, but the BER characteristic after decoding and the BLER characteristic are equivalent to those obtained in grid-like 16QAM. That is, it can be said that (8, 8) star 16QAM is a scheme in which the greatest effect of coding can be obtained. (4, 12) circular 16QAM exhibits the Raw BER characteristic, the BER characteristicafterdecoding, and the BLER characteristic worse than those in the grid-like 16QAM, but exhibits the smallest PAPR among that in the above mentioned three schemes.

[0141] That is, the relationshipbecomes what is shown below.

[Raw BER]

[0142]

Grid-like 16QAM > (4, 12) circular 16QAM > (8, 8) star 16QAM

Superior (good) ⟷ Inferior (bad)

[BER after decoding and BLER (coding ratio = 1/3)]

[0143]

Grid-like 16QAM ≈ (8, 8) star 16QAM > (4, 12) circular 16QAM

Superior (good) ⟷ Inferior (bad)

[PAPR]

[0144]

(4,12) circular 16QAM > (8, 8) star 16QAM > grid-like 16QAM

Superior (good) ⟷ Inferior (bad)

b) Idea of the Embodiments and its Effects to PAPR:

[0145] In grid-like 16QAM, when a coding ratio is smaller than 1/2, the outer symbols are controlled inwardly so as to

collect in the inner direction as already described with reference to FIG. 8, so that PAPR becomes small. In contrast, when the coding ratio is larger than 1/2, the outer symbols are controlled so as to get apart from the center, so that PAPR becomes large as already described with reference to Fig. 17.

[0146]

Further, in (8, 8) star 16QAM and (4, 12) circular 16QAM, control of symbols along the circles does not cause any change in PAPR, but change of the radius ratio of the double circles effects PAPR. More specifically, when the radius ratio becomes smaller, PAPR also becomes smaller; when the radius ratio becomes larger, PAPR also becomes larger.
In this instance, the present invention should by no means be limited to the above-illustrated embodiment, and various changes or modifications may be suggested without departing from the gist of the invention.

INDUSTRIAL APPLICABILITY

[0147] As detailed above, according to the present invention, control is performed in such a manner that symbol arrangement with the multi-level modulation scheme is changed from an symbol arrangement at equal intervals thereof to other arrangements in accordance with a ratio of codes of different significance in bit correction, so that the effect of bit correction can be exhibited at maximum, an error rate on the receiver end being thereby improved in comparison with the previous art. The invention thus is considered to be greatly useful in the field of the digital radio communications technology.

Claims

1. A digital radio communications method using the multi-level modulation scheme for use in a digital radio communications system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which demodulates the reception signal received from the transmitter by means of performing symbol evaluation, said radio communications method comprising:

   on the transmitter,
   performing a coding process of coding an information series to be transmitted, to generate multiple codes which are different in significance;
   performing a bit correction process of controlling bit arrangement in such a manner that a code having high significance, out of said multiple codes obtained by said coding process, is allocated with high priority to a bit having a tendency such that the likelihood of the bit at the symbol evaluation performed on the receiver becomes large;
   performing a multi-level modulation process of allocating the code to said plurality of bits in accordance with a bit arrangement obtained in said bit correction process to perform multi-level modulation of the information series at a predetermined symbol arrangement; and
   performing a symbol arrangement control process of controlling the symbol arrangement in a direction such that the symbol arrangement is changed from an equal distance arrangement in accordance with a ratio of the codes different in significance in said bit correction process,
   on the receiver,
   demodulating a signal received from the transmitter based on information relating to the controlled symbol arrangement; and
   performing decoding the demodulated signal to obtain the information series.

2. The method of claim 1, comprising:

   in said symbol arrangement control process performed by the transmitter,
   controlling the symbol arrangement in a direction such that the likelihood of a bit to which a code high in significance is allocated, out of said plurality of bits, becomes large.

3. The method of claim 1, comprising:

   in said symbol arrangement control process performed by the transmitter,
   controlling the symbol arrangement in a direction such that the likelihood of a bit to which a code low in significance is allocated, out of said plurality of bits, becomes large.

4. The method of claim 2, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed in said bit correction process, the likelihood of only the first bit becomes large.

5. The method of claim 2, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed in said bit correction process, the likelihood of the first bit and the second bit becomes large.

6. The method of claim 2, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit through the third bit by the bit arrangement performed in said bit correction process, the likelihood of the third bit and the fourth bit becomes large.

7. The method of claim 3, which is the 16-ary modulation scheme in which a single symbol is transmitted using four bits, a first through a fourth bit, said digital radio communications method comprising:

in said symbol arrangement control, process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the fourth bit by the bit arrangement performed in said bit correction process, the likelihood of only the fourth bit becomes large.

8. The method of claim 2, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed in said bit correction process, the likelihood of only the first bit becomes large.

9. The method of claim 2, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed in said bit correction process, the likelihood of the first bit and the second bit becomes large.

10. The method of claim 3, which is the 8-ary modulation scheme in which a single symbol is transmitted using three bits, a first through a third bit, said digital radio communications method comprising:

in said symbol arrangement control process,
controlling the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the third bit by the bit arrangement performed in said bit correction process, the likelihood of only the fourth bit becomes large.

11. The method of any one of claim 1 through claim 10, comprising:

in said on said symbol arrangement control process,
controlling the symbol arrangement in a direction such that a transmission power is not changed before and after the control of the symbol arrangement.

12. The method of claim 1, comprising:

in said bit correction process,
controlling the bit arrangement in such a manner that a ratio of a plurality of types of codes low in significance, out of said multiple codes, is equal for every one of said plurality of bits.

13. The method of claim 1, wherein the transmitter further comprises a process of notifying the receiver beforehand of information relating to a symbol arrangement controlled in said symbol arrangement control process.

14. The method of claim 1,
wherein the receiver holds information before hand relating to the symbol arrangement with respect to a ratio of codes different in significance in a memory,
wherein the transmitter is operable to perform the process of notifying the receiver of information relating to the ratio, and
wherein the receiver is operable to perform the process of reading the information relating to the symbol arrangement based on information relating to the ratio notified from the transmitter, and the process of executing the demodulation based on the read out information.

15. The method of any one of claim 1 through claim 14,
wherein the code high in significance is a systematic code in a turbo code, and
wherein the code low in significance is a parity code in the turbo code.

16. A transmitter for use in a digital radio communications system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which performs symbol evaluation of a reception signal from said transmitter and demodulates the reception signal received from the transmitter, said transmitter comprising:

coding means which codes an information series to be transmitted, to generate multiple codes different in significance;
bit correction means which controls bit arrangement in such a manner that a code having high significance, out of said multiple codes, is allocated with high priority to a bit having a tendency such that the likelihood of the bit at the symbol evaluation performed on the receiver becomes large;
multi-level modulation means which allocates the code to said plurality of bits in accordance with a bit arrangement obtained by said bit correction means to perform multi-level modulation of the information series with a predetermined symbol arrangement; and
symbol arrangement controllingmeans which controls the symbol arrangement in a direction such that the symbol arrangement is changed from an equal distance arrangement, in accordance with a ratio of codes different in significance in said bit correction means.

17. The transmitter of claim 16, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that the likelihood of a bit to which a code high in significance is allocated, out of said plurality of bits, becomes large.

18. The transmitter of claim 16, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that the likelihood of a bit, out of said plurality of bits, to be allocated with low priority becomes large.

19. The transmitter of claim 17, employing the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed by said bit correction means, the likelihood of only the first bit becomes large.

20. The transmitter of claim 17, employing the multi-level modulation scheme which is the 16-ary modulation scheme that transmits a single symbol using four bits, a first bit through a fourth bit, wherein said symbol arrangement

controlling means controls the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed by said bit correction means, the likelihood of the first bit and the second bit becomes large.

21. The transmitter of claim 17, employing the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code with high priority is allocated to the first bit through the third bit by the bit arrangement performed by said bit correction means, the likelihood of the third bit and the fourth bit becomes large.

22. The transmitter of claim 18, employing the multi-level modulation scheme which is the 16-ary modulation method that transmits a single symbol using four bits, a first bit through a fourth bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the fourth bit by the bit arrangement performed by said bit correction means, the likelihood of only the fourth bit becomes large.

23. The transmitter of claim 17, employing the multi-level modulation scheme which is the 8-ary modulation method that transmits a single symbol using three bits, a first bit through a third bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code high in significance is allocated only to the first bit by the bit arrangement performed by said bit correction means, the likelihood of only the first bit becomes large.

24. The transmitter of claim 17, employing the multi-level modulation scheme which is the 8-ary modulation method that transmits a single symbol using three bits, a first bit through a third bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code high in significance is allocated to the first bit and to the second bit by the bit arrangement performed by said bit correction means, the likelihood of the first bit and the second bit becomes large.

25. The transmitter of claim 18, employing the multi-level modulation scheme which is the 8-ary modulation method that transmits a single symbol using three bits, a first bit through a third bit, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that, in a case where the code low in significance is allocated only to the third bit by the bit arrangement performed by said bit correction means, the likelihood of only the fourth bit becomes large.

26. The transmitter of any one of claim 16 through claim 25, wherein said symbol arrangement controlling means controls the symbol arrangement in a direction such that a transmission power is not changed before and after the control of the symbol arrangement.

27. The transmitter of claim 16, wherein said bit correction means controls the bit arrangement in a direction such that a ratio of a plurality of types of codes low in significance, out of said multiple codes, is equal for every one of said plurality of bits.

28. The transmitter of claim 16, further comprising notifying means which notifies the receiver beforehand of information relating to a symbol arrangement controlled by said symbol arrangement controlling means.

29. The transmitter of any one of claim 16 through claim 27,
wherein the code high in significance is a systematic code in a turbo code, and
wherein the code low in significance is a parity code in the turbo code.

30. A receiver for use in a digital radio communication system including: a transmitter which transmits a signal modulated with the multi-level modulation scheme in which a single symbol is transmitted using a plurality of bits; and a receiver which performs symbol evaluation of a reception signal from said transmitter and demodulates the reception signal received from the transmitter, said transmitter comprising:

demodulating means which demodulates a signal received from the transmitter based on information relating to the controlled symbol arrangement that has been obtained by controlling thereof, on the transmitter, from an equal distance arrangement in accordance with a ratio of the codes different in significance; and
decoding means which decodes the signal demodulated by said receiver means to obtain an information series

transmitted by the transmitter.

31. The receiver of claim 30, further comprising symbol arrangement information receiving means which receives information relating to the symbol arrangement from the transmitter.

32. The receiver of claim 30, further comprising a memory which holds information relating to the symbol arrangement with respect to the ratio of codes different in significance,
said demodulating means reading information relating to the controlled, symbol arrangement from said memory based information relating to the ratio notified from the transmitter, and performing the demodulation based on the read out information.

FIG. 1

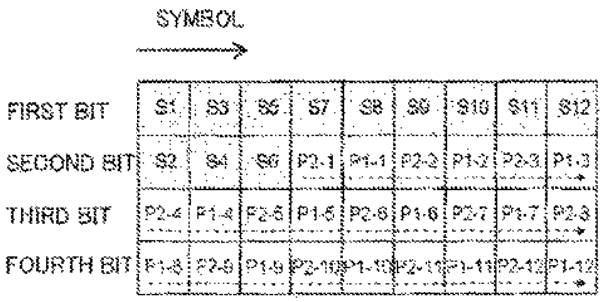

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S3
S5
S7
S8
S9
S10
S11
S12
S2
S4
S6
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5
P1-5
P2-6
P1-6
P2-7
P1-7
P2-8
P1-8
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P2-10
P1-10
P2-11
P1-11
P2-12
P1-12

FIG. 2

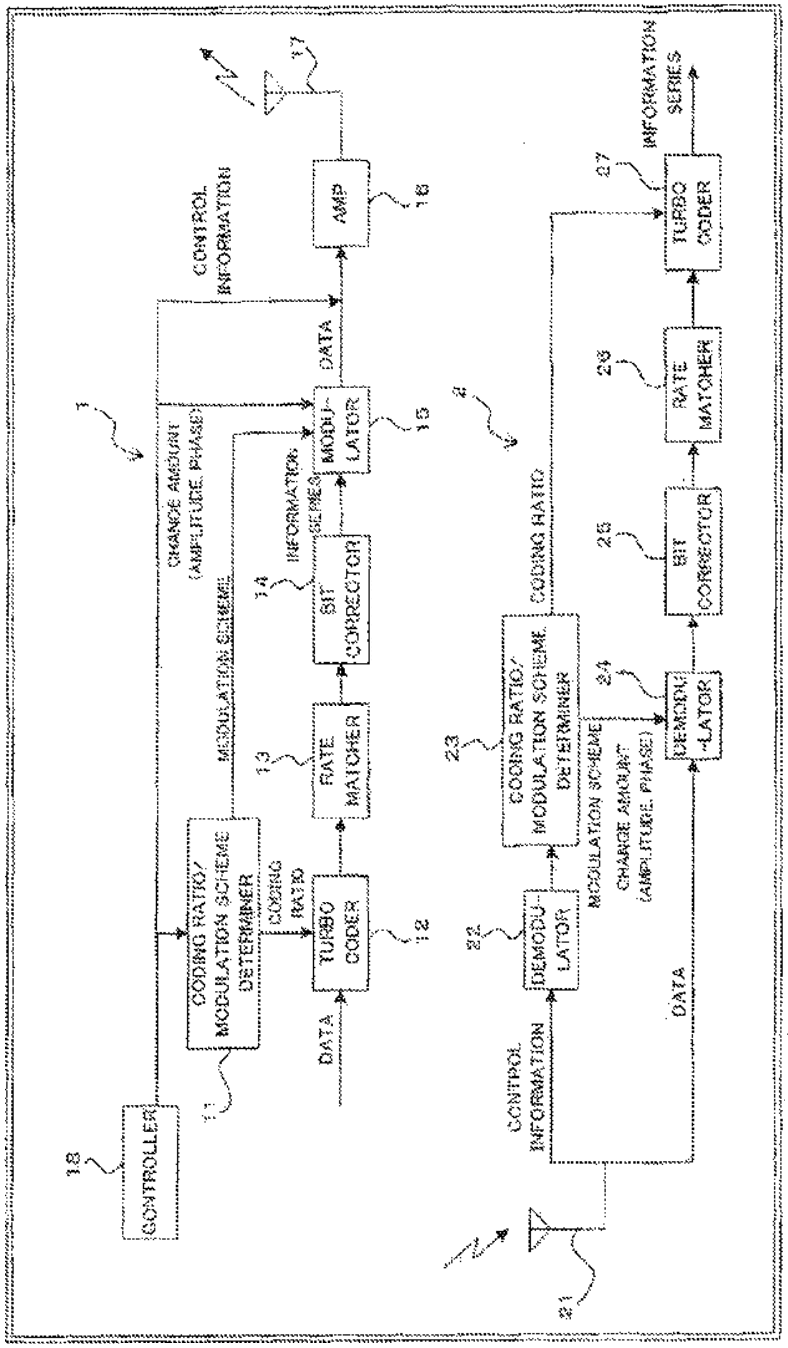

CONTROLLER
18
CODING RATIO/ MODULATION SCHEME DETERMINER
11
DATA
TURBO CODER
12
CODING RATIO
RATE MATCHER
13
MODULATION SCHEME
CHANGE AMOUNT (AMPLITUDE, PHASE)
CONTROL INFORMATION
1
SIT CORRECTOR
14
INFORMATION SERIES
MODU-LATOR
15
DATA
AMP
16
17
CONTROL INFORMATION
DEMODU-LATOR
22
CODING RATIO/ MODULATION SCHEME DETERMINER
23
MODULATION SCHEME
CHARGE AMOUNT (AMPLITUDE, PHASE)
DEMODU-LATOR
24
SIT CORRECTOR
25
CODING RATIO
RATE MATCHER
26
TURBO CODER
27
INFORMATION SERIES
DATA
2
21

FIG. 3

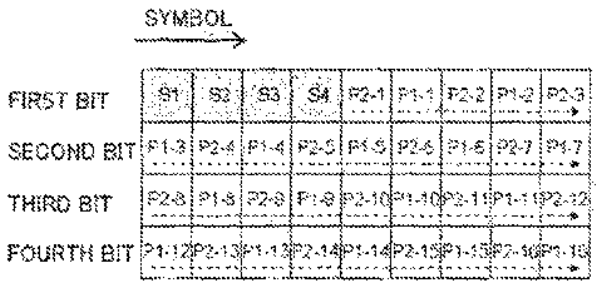

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S2
S3
S4
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5
P1-5
P2-6
P1-6
P2-7
P1-7
P2-8
P1-8
P2-9
P1-9
P2-10
P1-10
P2-11
P1-11
P2-12
P1-12
P2-13
P1-13
P2-14
P1-14
P2-15
P1-15
P2-16
P1-16

FIG. 4

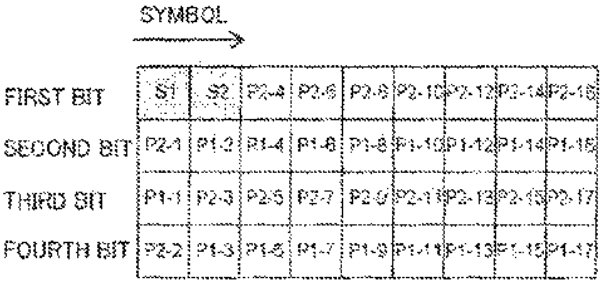

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S2
P2-4
P2-6
P2-8
P2-10
P2-12
P2-14
P2-16
P2-1
P1-2
P1-4
P1-6
P1-8
P1-10
P1-12
P1-14
P1-16
P1-1
P2-3
P2-5
P2-7
P2-9
P2-11
P2-13
P2-15
P2-17
P2-2
P1-3
P1-5
P1-7
P1-9
P1-11
P1-13
P1-15
P1-17

FIG. 5

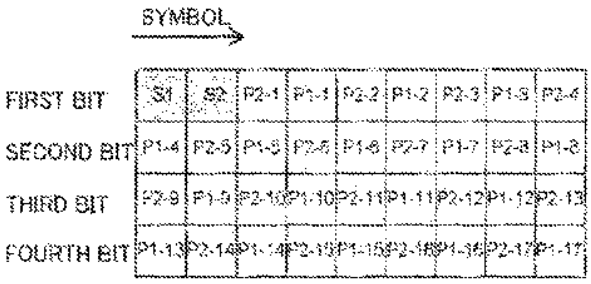

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S2
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5
P1-5
P2-6
P1-6
P2-7
P1-7
P2-8
P1-8
P2-9
P1-9
P2-10
P1-10
P2-11
P1-11
P2-12
P1-12
P2-13
P1-13
P2-14
P1-14
P2-15
P1-15
P2-16
P1-16
P2-17
P1-17

FIG. 6

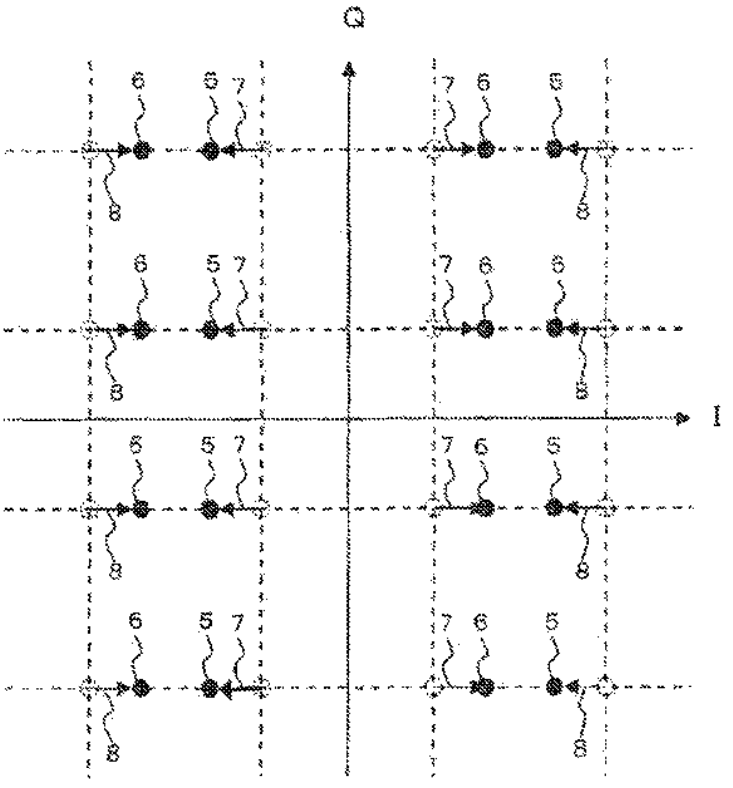


FIG. 7

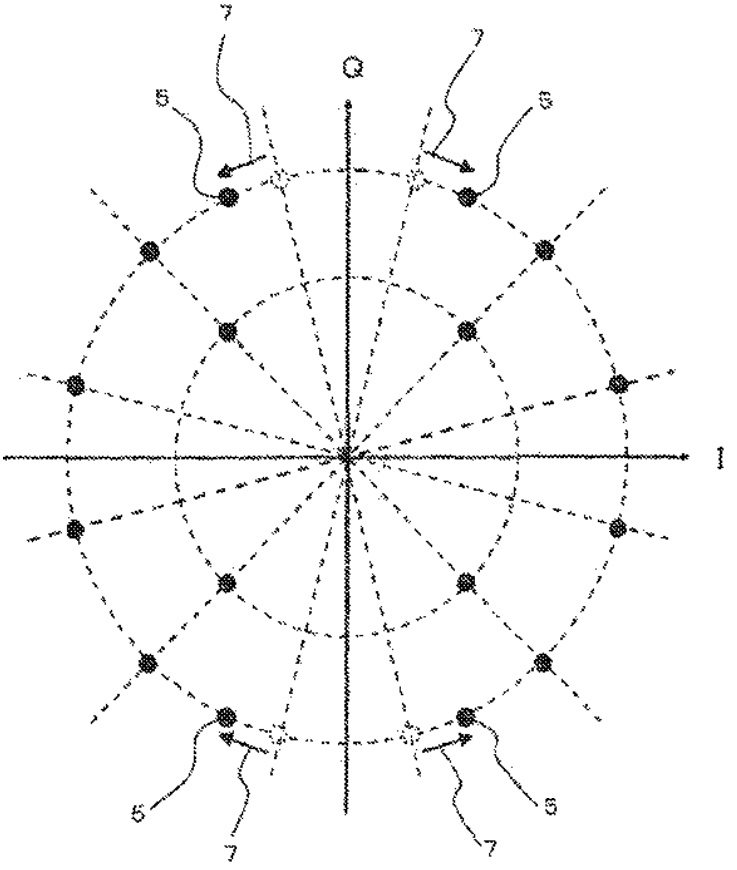


FIG. 8

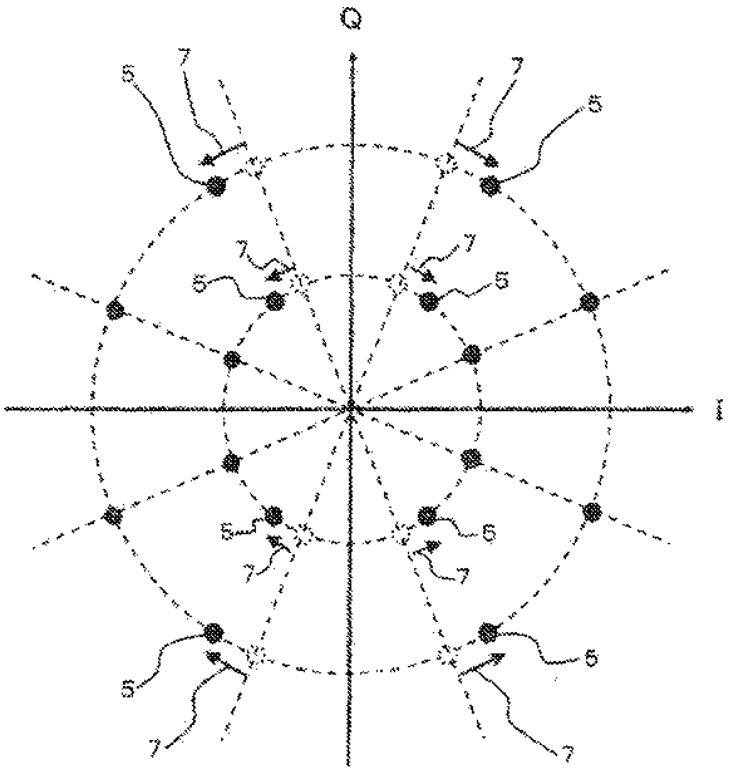


FIG. 9

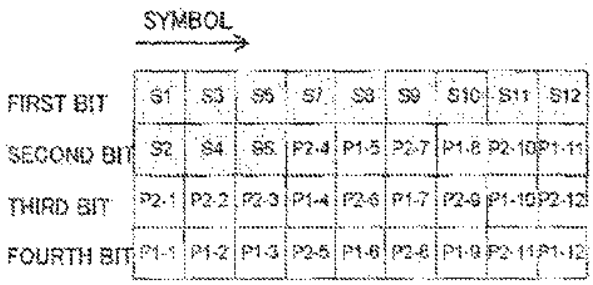

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S3
S5
S7
S9
S10
S11
S12
S2
S4
S6
P2-4
P1-5
P2-7
P1-8
P2-10
P1-11
P2-1
P2-2
P2-3
P1-4
P2-6
P1-7
P2-9
P1-10
P2-12
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P1-6
P2-8
P1-9
P2-11
P1-12

FIG. 10

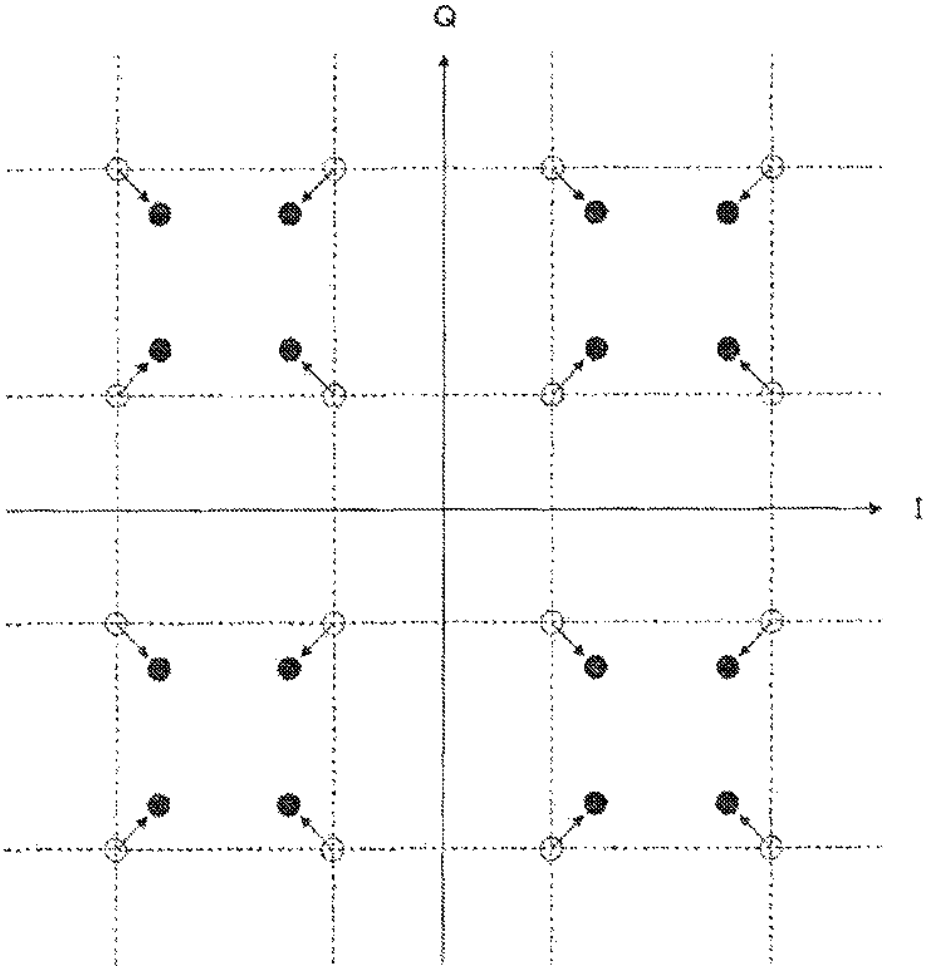


FIG. 11

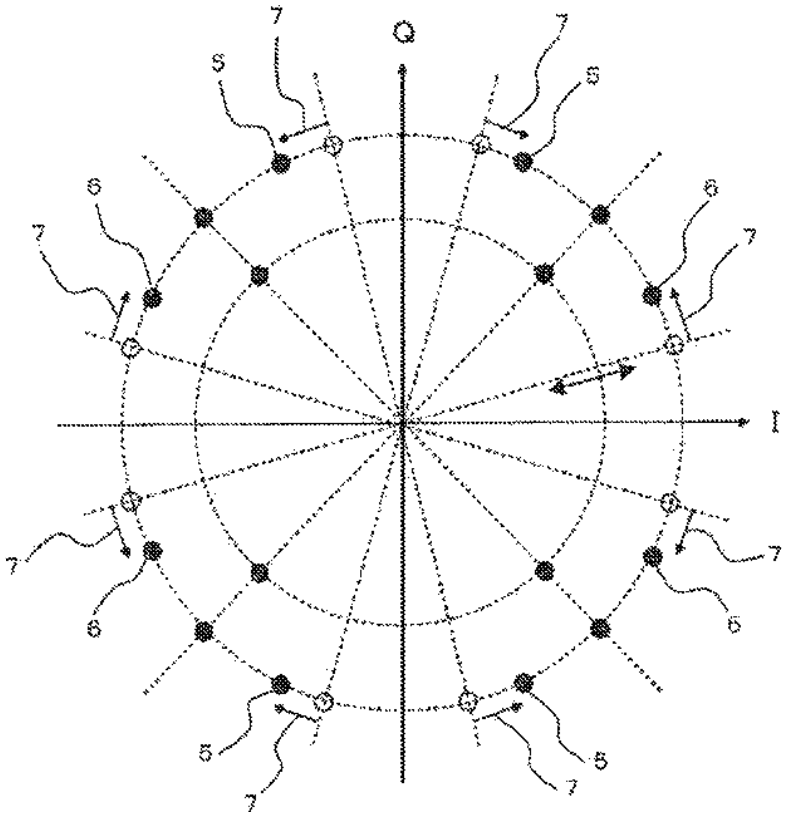


FIG. 12

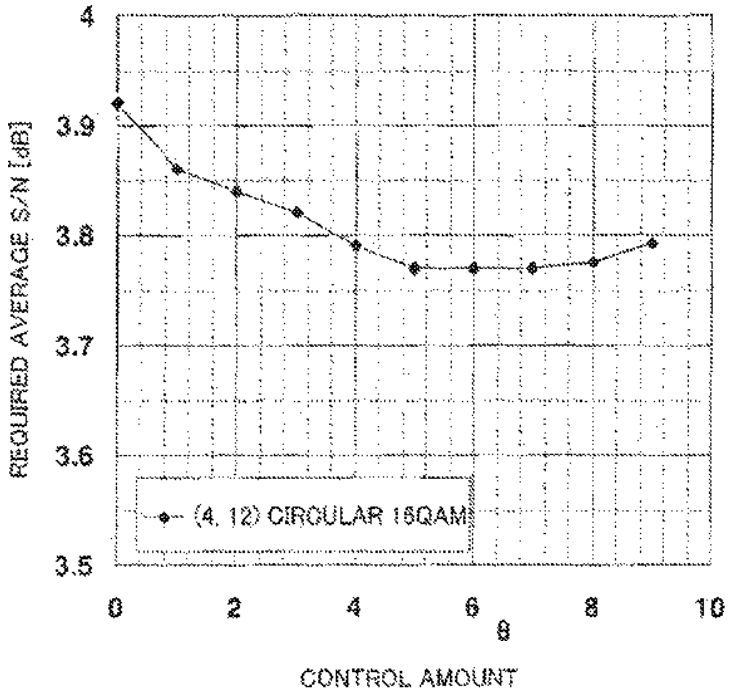

4
3.9
3.8
3.7
3.6
3.5
REQUIRED AVERAGE S/N [dB]
(4, 12) CIRCULAR 16QAM
0
2
4
6
8
10
θ
CONTROL AMOUNT

FIG. 13

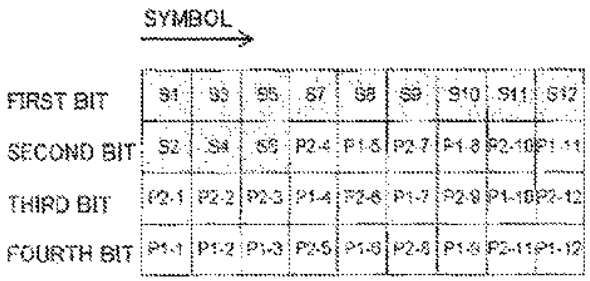

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1 S3 S5 S7 S8 S9 S10 S11 S12
S2 S4 S6 P2-4 P1-5 P2-7 P1-8 P2-10 P1-11
P2-1 P2-2 P2-3 P1-4 P2-6 P1-7 P2-9 P1-10 P2-12
P1-1 P1-2 P1-3 P2-5 P1-6 P2-8 P1-9 P2-11 P1-12

FIG. 14

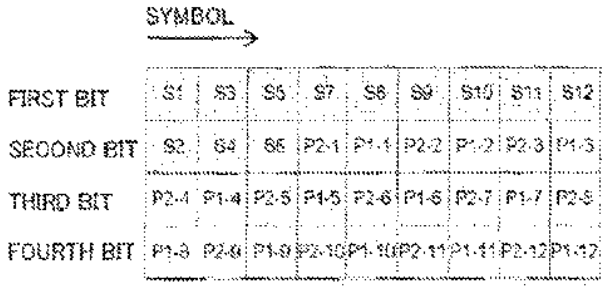

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1 S3 S5 S7 S8 S9 S10 S11 S12
S2 S4 S6 P2-1 P1-1 P2-2 P1-2 P2-3 P1-3
P2-4 P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7 P2-8
P1-8 P2-9 P1-9 P2-10 P1-10 P2-11 P1-11 P2-12 P1-12

FIG. 15

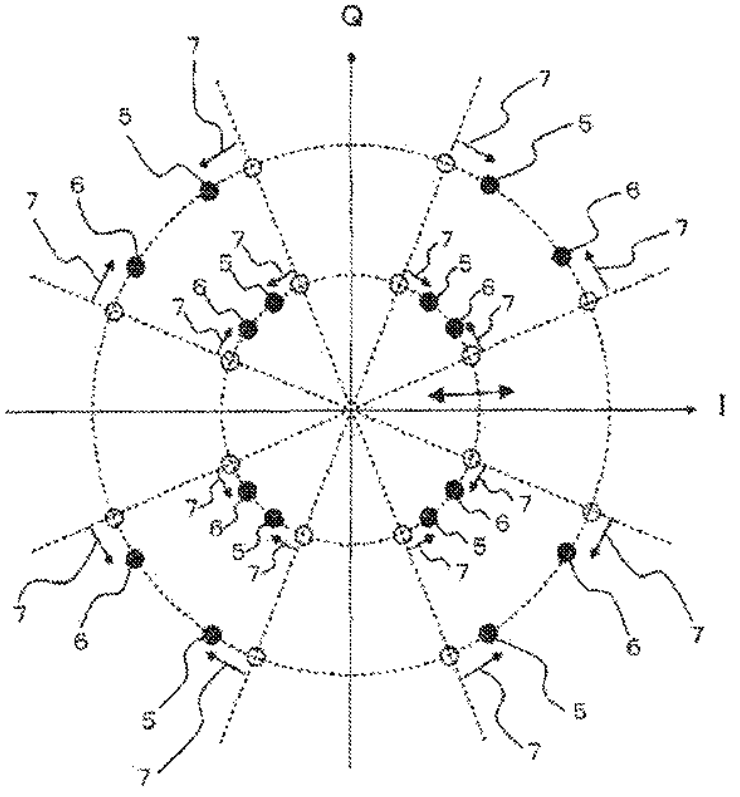


FIG. 16

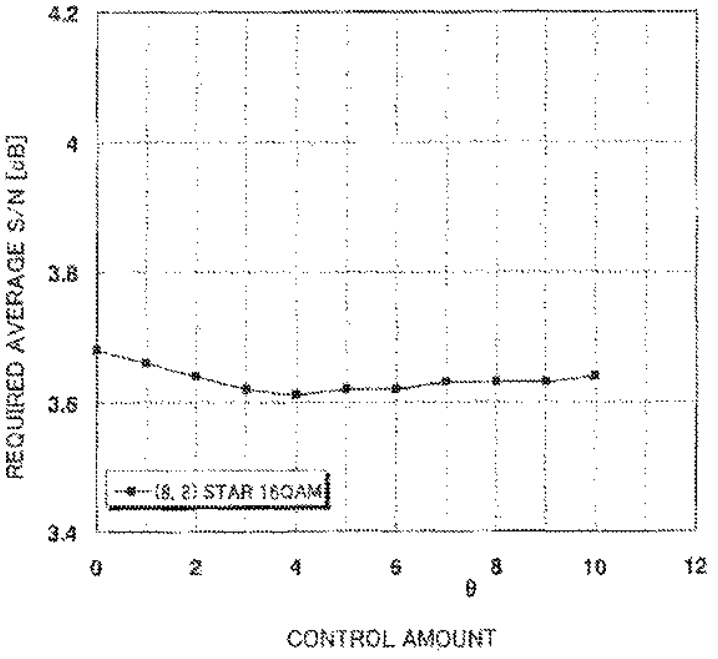

4.2
4
3.8
3.6
3.4
REQUIRED AVERAGE S/N [dB]
(8, 2) STAR 16QAM
0
2
4
6
8
10
12
θ
CONTROL AMOUNT

FIG. 17

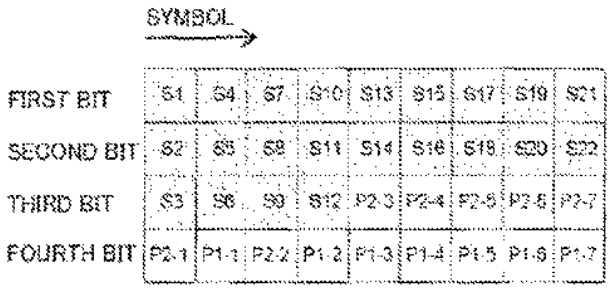

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1 S4 S7 S10 S13 S15 S17 S19 S21
S2 S5 S8 S11 S14 S16 S18 S20 S22
S3 S6 S9 S12 P2-3 P2-4 P2-5 P2-6 P2-7
P2-1 P1-1 P2-2 P1-2 P1-3 P1-4 P1-5 P1-6 P1-7

FIG. 18

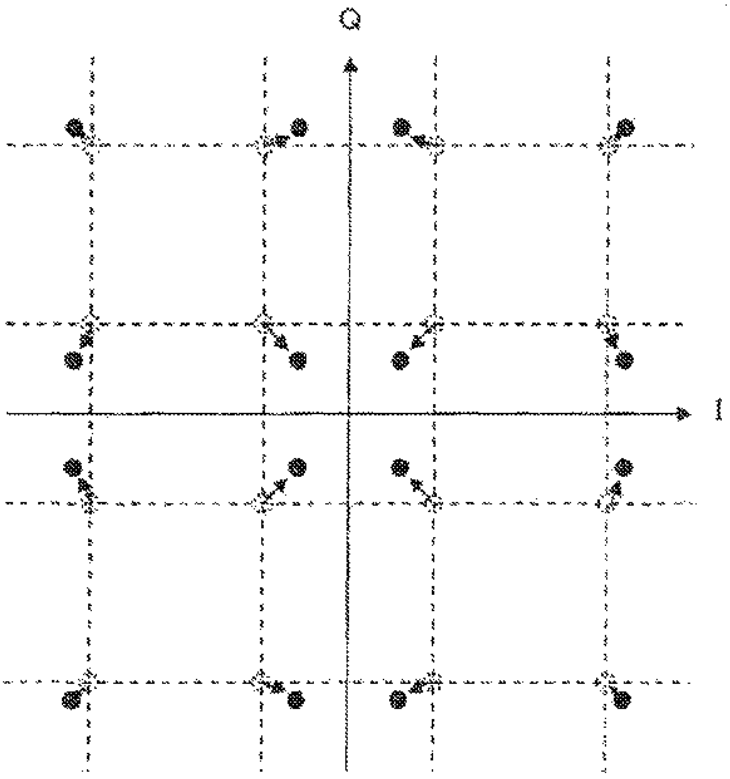


FIG. 19

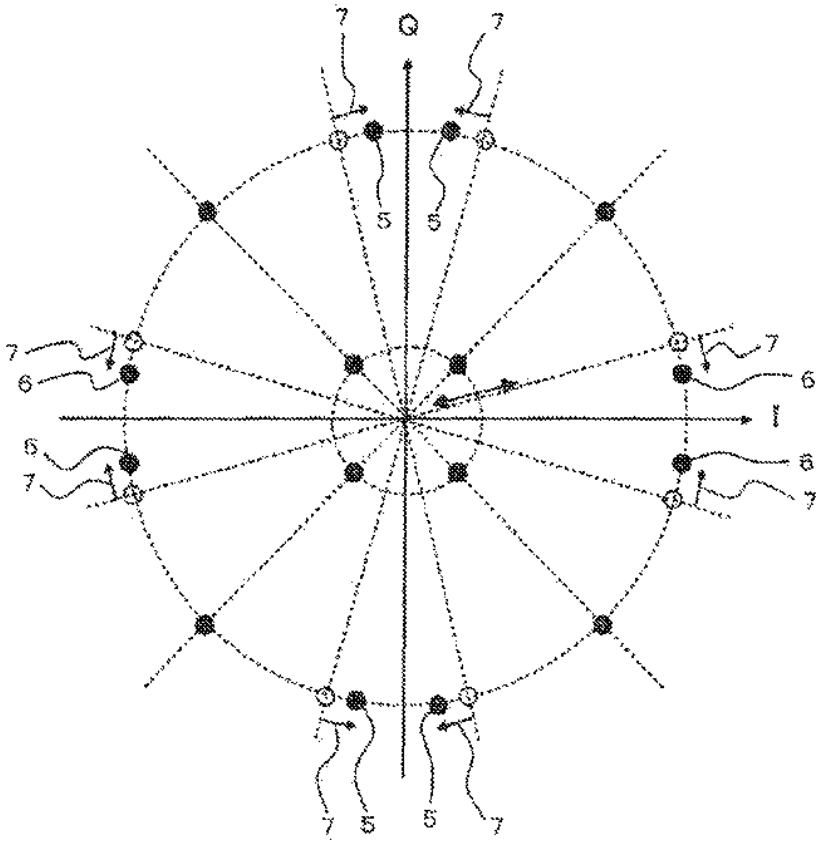


FIG. 20

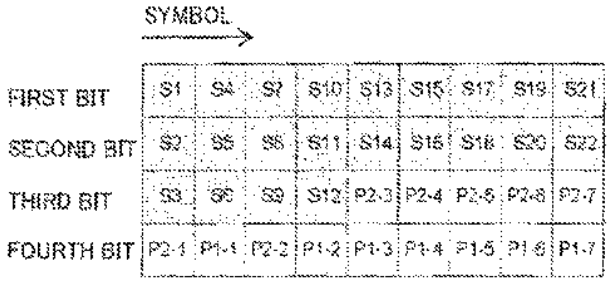

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1 S4 S7 S10 S13 S15 S17 S19 S21
S2 S5 S8 S11 S14 S16 S18 S20 S22
S3 S6 S9 S12 P2-3 P2-4 P2-5 P2-6 P2-7
P2-1 P1-1 P2-2 P1-2 P1-3 P1-4 P1-5 P1-6 P1-7

FIG. 21

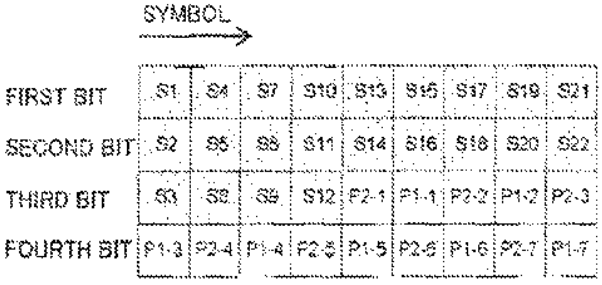

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1 S4 S7 S10 S13 S15 S17 S19 S21
S2 S5 S8 S11 S14 S16 S18 S20 S22
S3 S6 S9 S12 P2-1 P1-1 P2-2 P1-2 P2-3
P1-3 P2-4 P1-4 P2-5 P1-5 P2-6 P1-6 P2-7 P1-7

FIG. 22

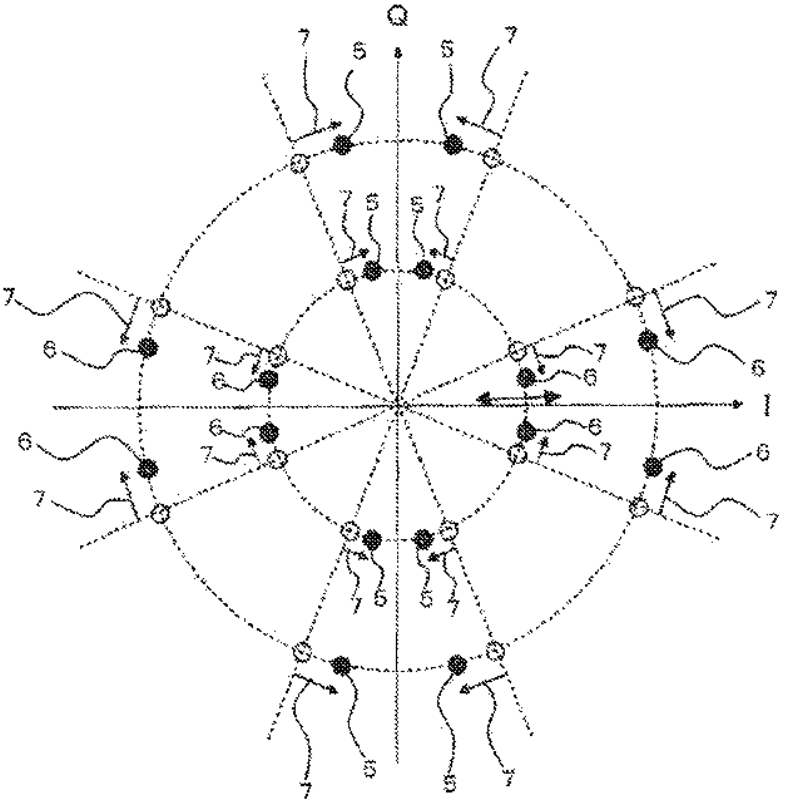


FIG. 23

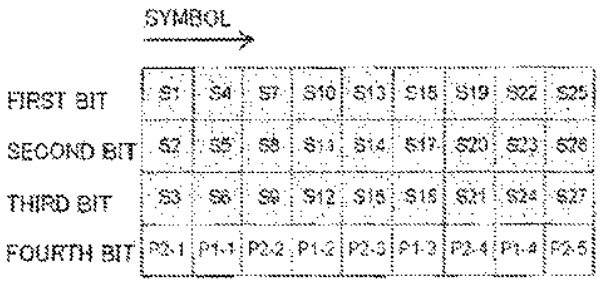

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
FOURTH BIT
S1
S4
S7
S10
S13
S16
S19
S22
S25
S2
S5
S8
S11
S14
S17
S20
S23
S26
S3
S6
S9
S12
S15
S18
S21
S24
S27
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5

FIG. 24

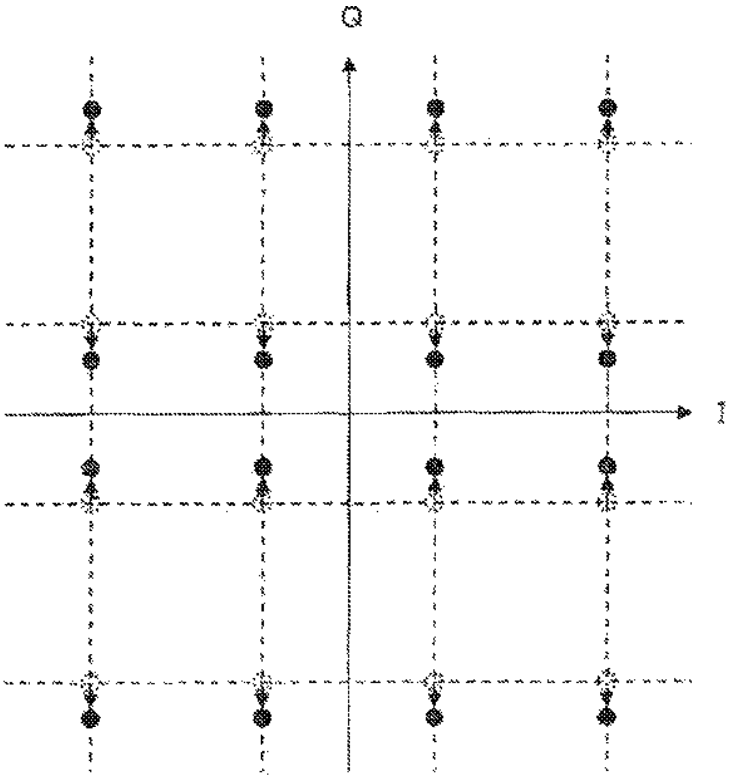


FIG. 25

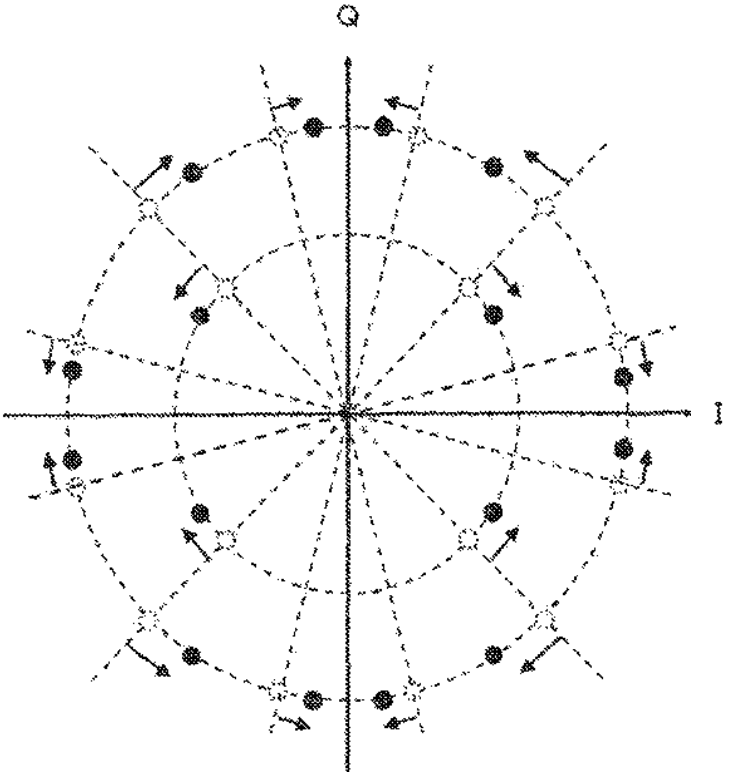


FIG. 26

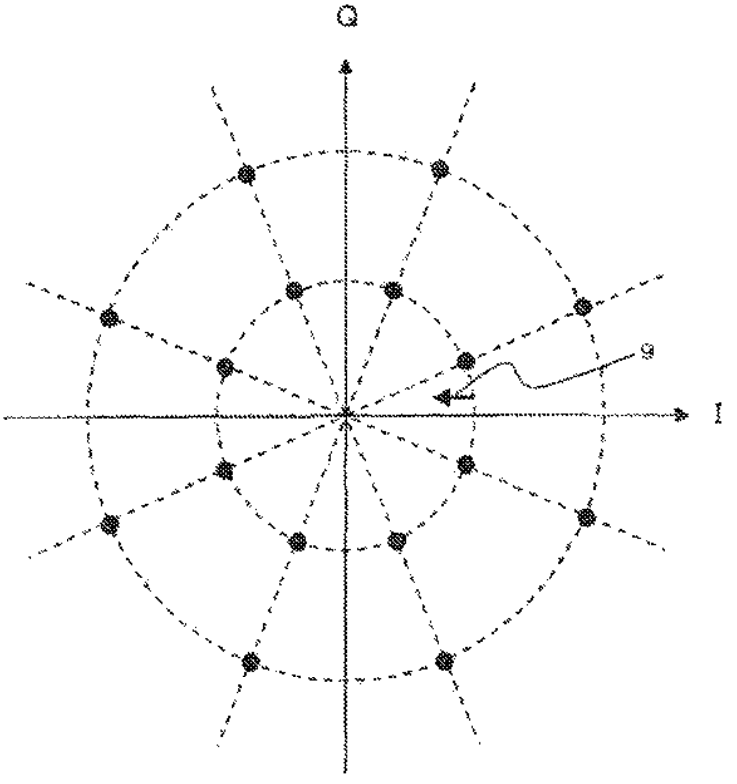


FIG. 27

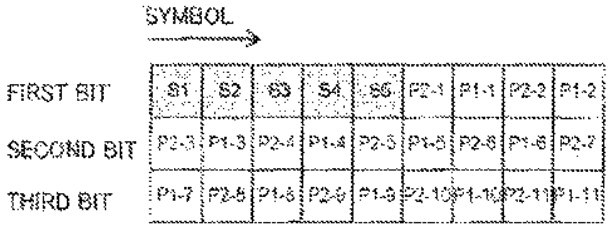

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
S1
S2
S3
S4
S5
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5
P1-5
P2-6
P1-6
P2-7
P1-7
P2-8
P1-8
P2-9
P1-9
P2-10
P1-10
P2-11
P1-11

FIG. 28

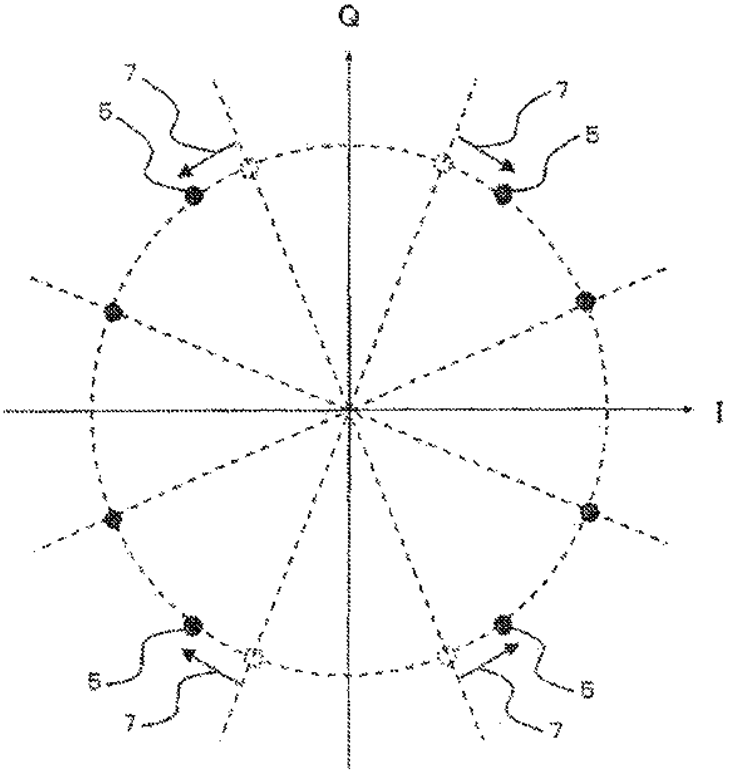


FIG. 29

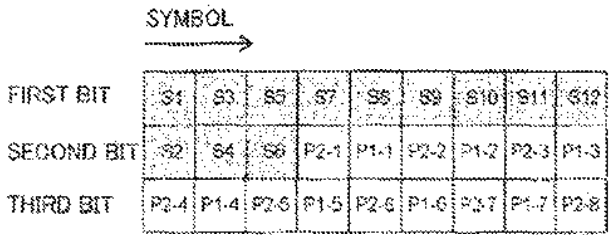

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
S1
S3
S5
S7
S9
S10
S11
S12
S2
S4
S6
P2-1
P1-1
P2-2
P1-2
P2-3
P1-3
P2-4
P1-4
P2-5
P1-5
P2-6
P1-6
P2-7
P1-7
P2-8

FIG. 30

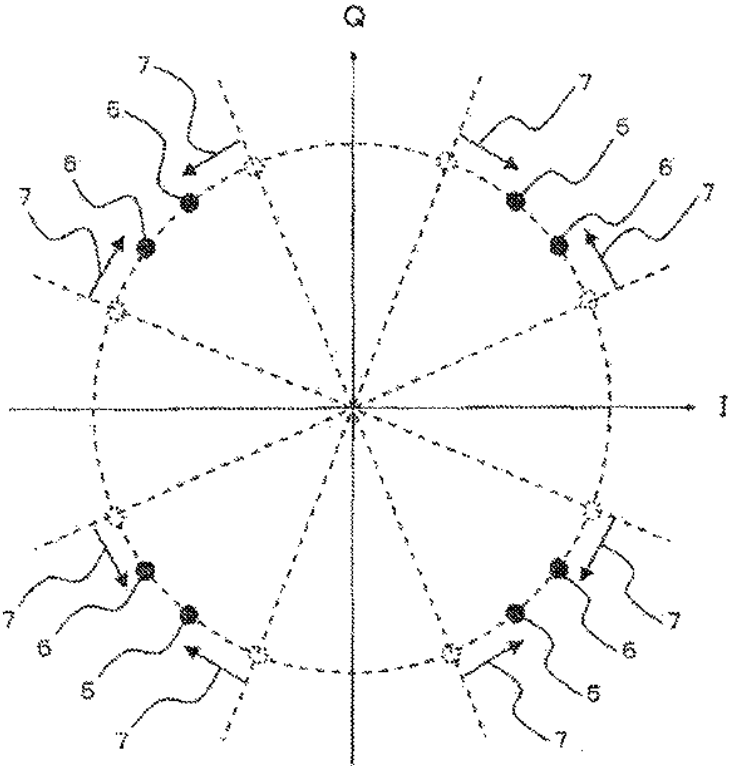


FIG. 31

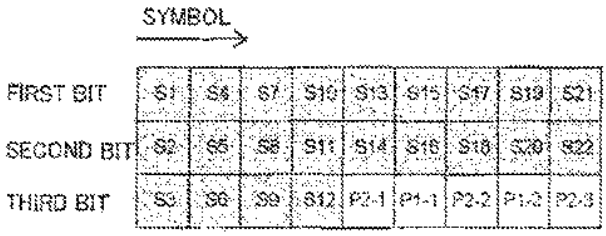

SYMBOL
FIRST BIT
SECOND BIT
THIRD BIT
S1
S4
S7
S10
S13
S15
S17
S19
S21
S2
S5
S8
S11
S14
S16
S18
S20
S22
S3
S6
S9
S12
P2-1
P1-1
P2-2
P1-2
P2-3

FIG. 32

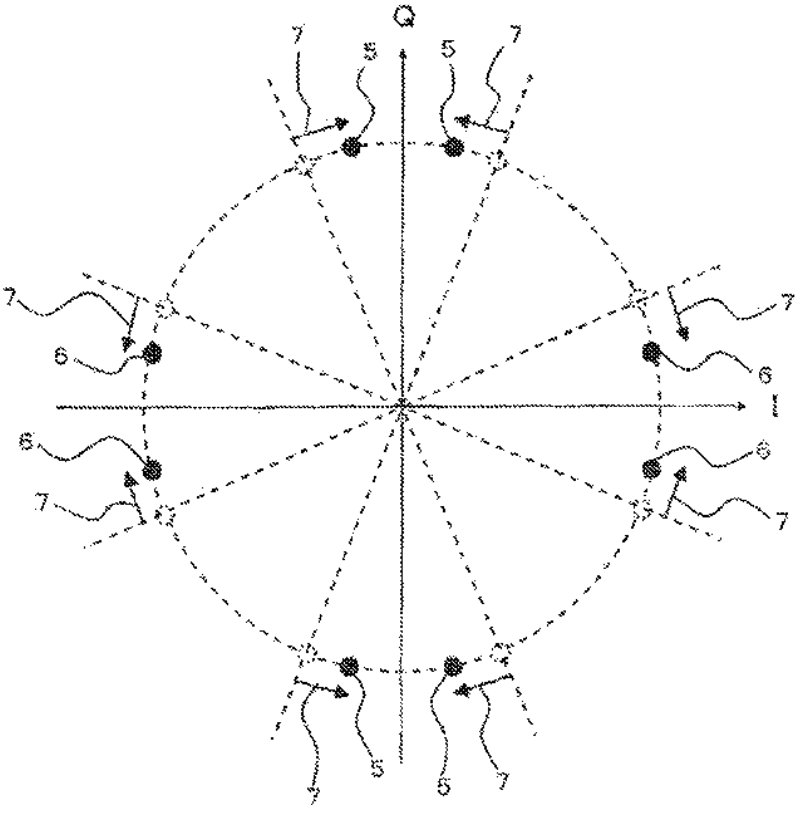


FIG. 33

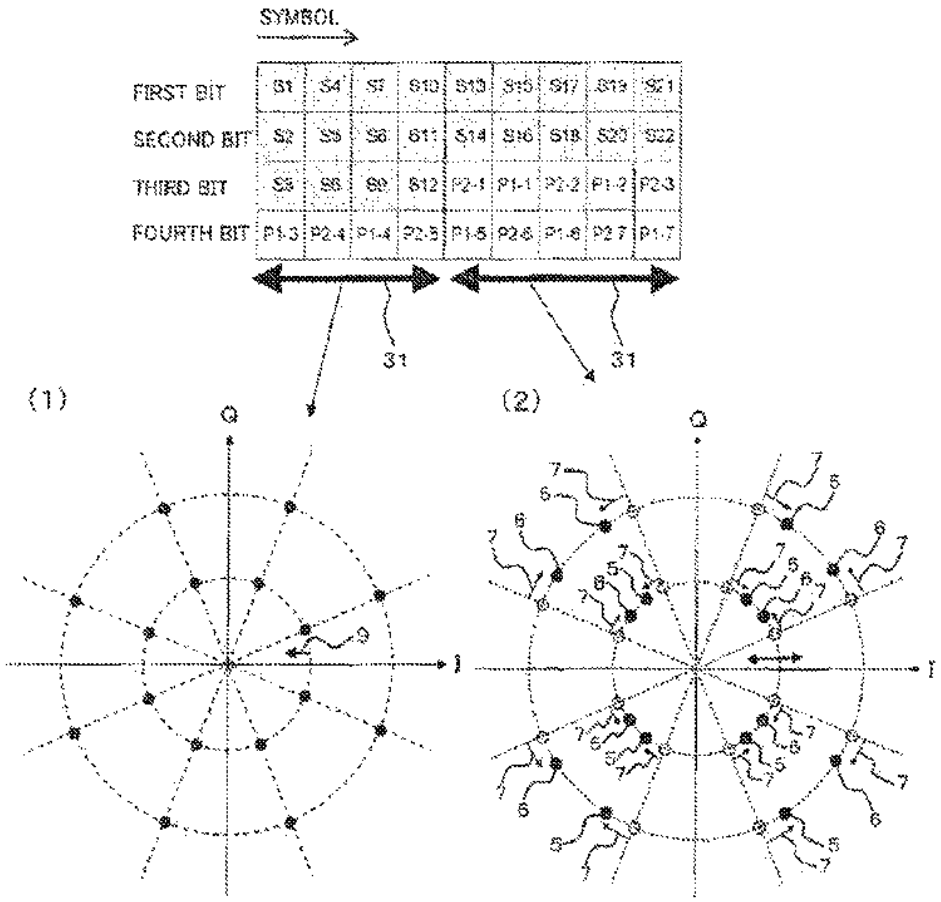


FIG. 34

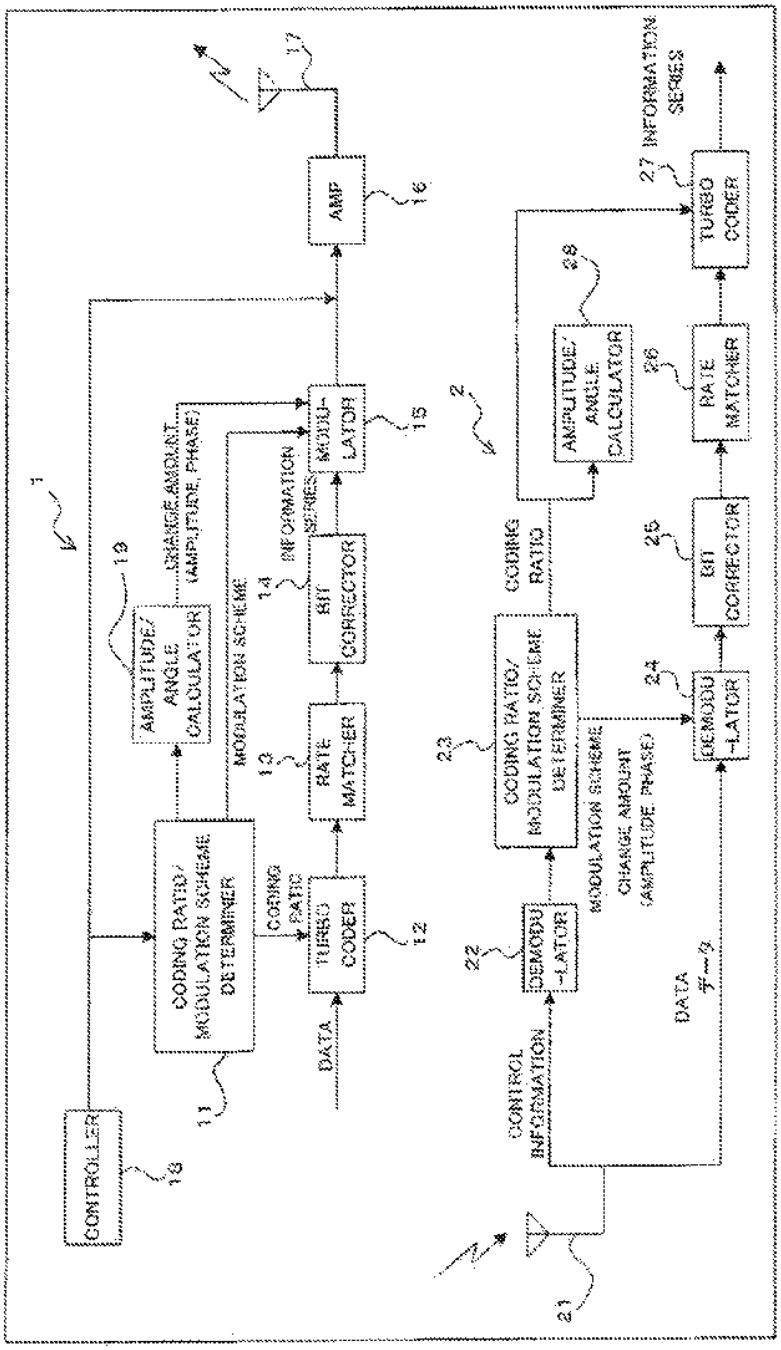
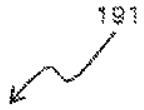

1
CONTROLLER
18
CODING RATIO/ MODULATION SCHEME DETERMINER
11
DATA
TURBO CODER
12
CODING RATE
RATE MATCHER
12
MODULATION SCHEME
AMPLITUDE/ ANGLE CALCULATOR
13
CHANGE AMOUNT (AMPLITUDE, PHASE)
BIT CORRECTOR
14
INFORMATION SERIES
MODU-LATOR
15
AMP
16
17
2
21
DEMODU-LATOR
22
CONTROL INFORMATION
CODING RATIO/ MODULATION SCHEME DETERMINER
23
CODING RATIO
MODULATION SCHEME
CHANGE AMOUNT (AMPLITUDE, PHASE)
DEMODU-LATOR
24
BIT CORRECTOR
25
RATE MATCHER
26
AMPLITUDE/ ANGLE CALCULATOR
28
DATA
TURBO CODER
27
INFORMATION SERIES

FIG. 35



(RATIO BETWEEN SYSTEMATIC BITS AND PARITY BITS)	AMPLITUDE RATIO	ANGLE
R(0)~R(1)	a(1)	θ(1)
R(1)~R(2)	a(2)	θ(2)
⋮	⋮	⋮
⋮	⋮	⋮
R(n-1)~R(n)	a(n)	θ(n)

FIG. 36

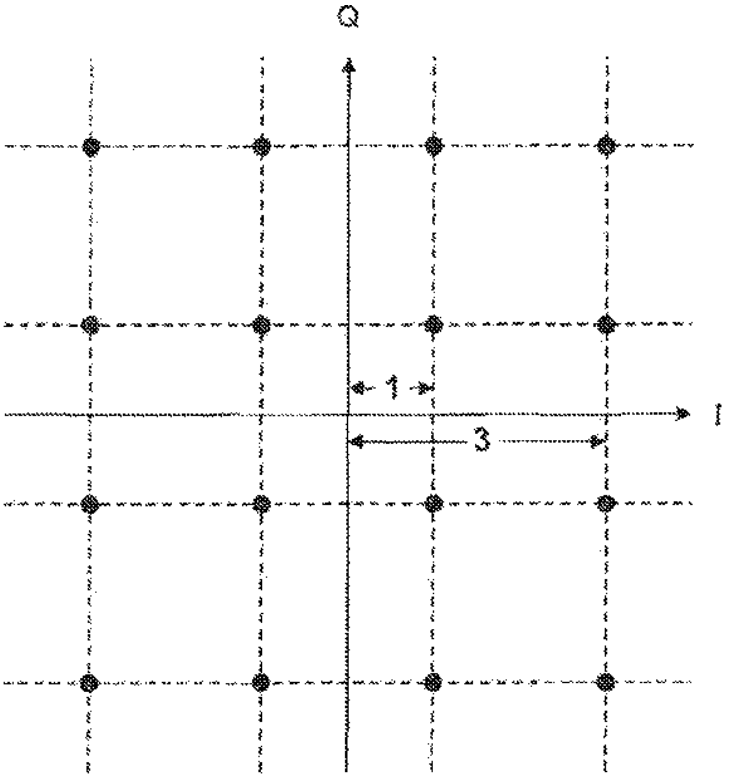


FIG. 37

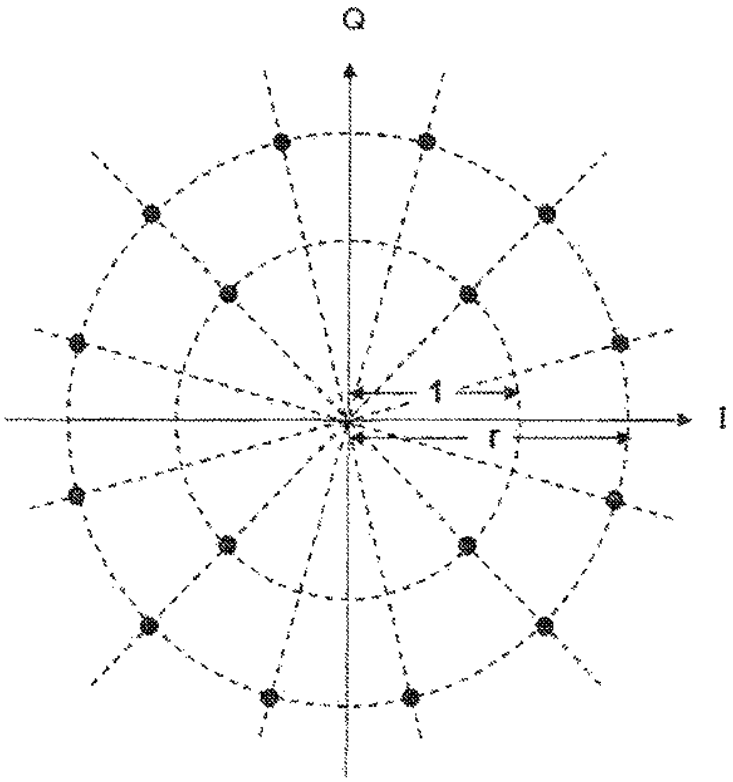


FIG. 38

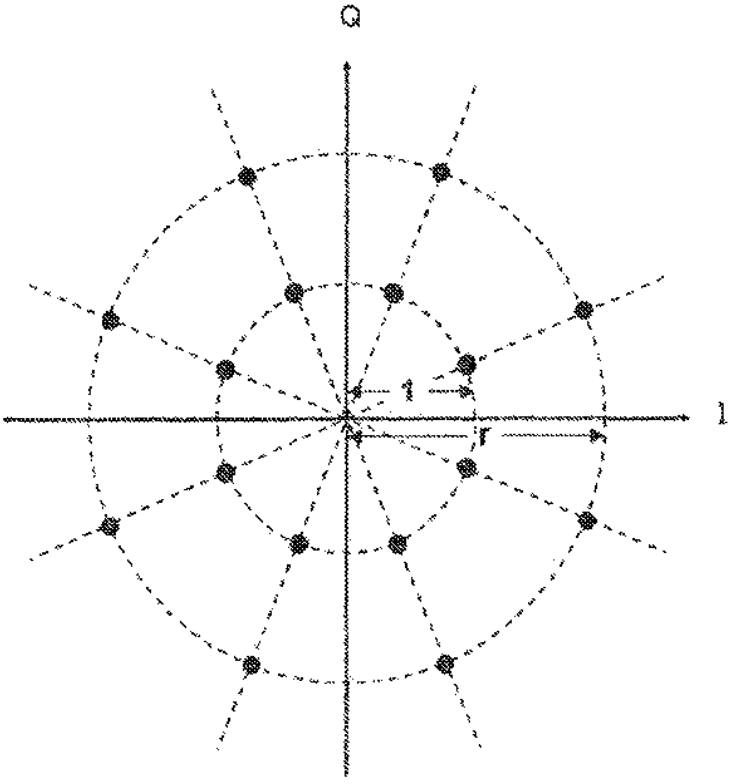


FIG. 39

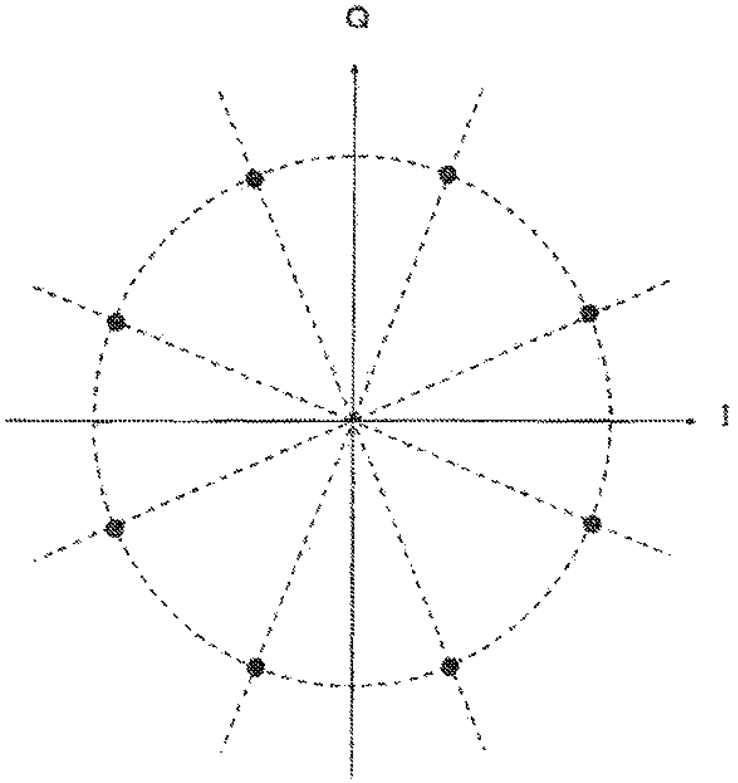


FIG. 40

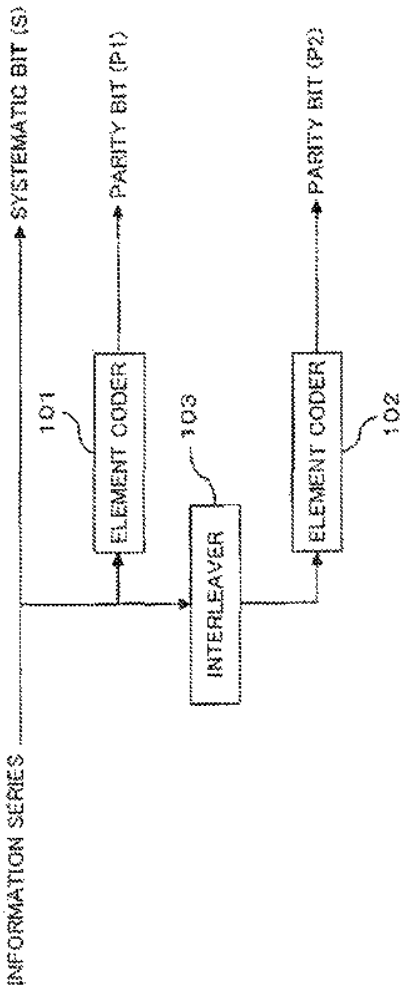

INFORMATION SERIES
SYSTEMATIC BIT (S)
PARITY BIT (P1)
PARITY BIT (P2)
ELEMENT CODER
101
INTERLEAVER
103
ELEMENT CODER
102

FIG. 41

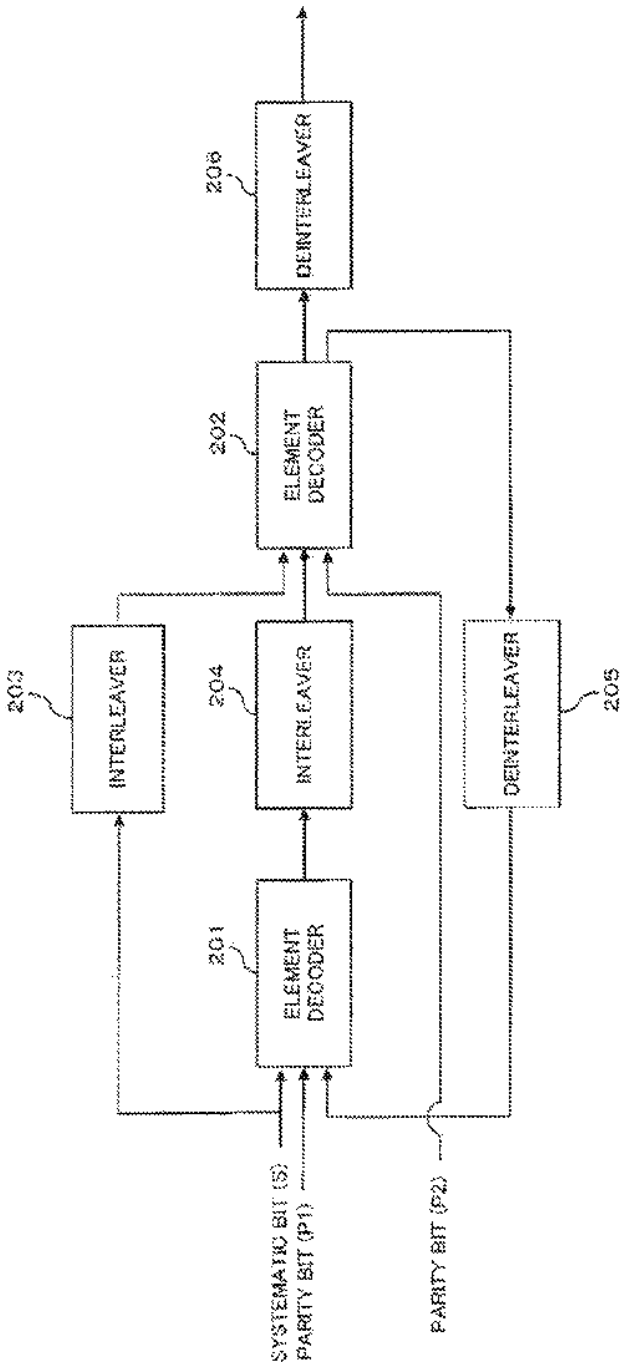
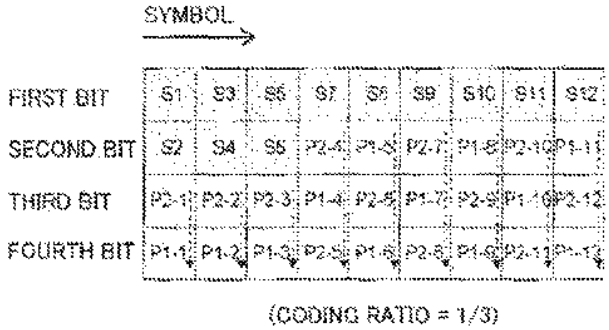

206
DEINTERLEAVER
202
ELEMENT DECODER
203
INTERLEAVER
204
INTERLEAVER
205
DEINTERLEAVER
201
ELEMENT DECODER
SYSTEMATIC BIT (S)
PARITY BIT (P1)
PARITY BIT (P2)

FIG. 42



(CODING RATIO = 1/3)

FIG. 43

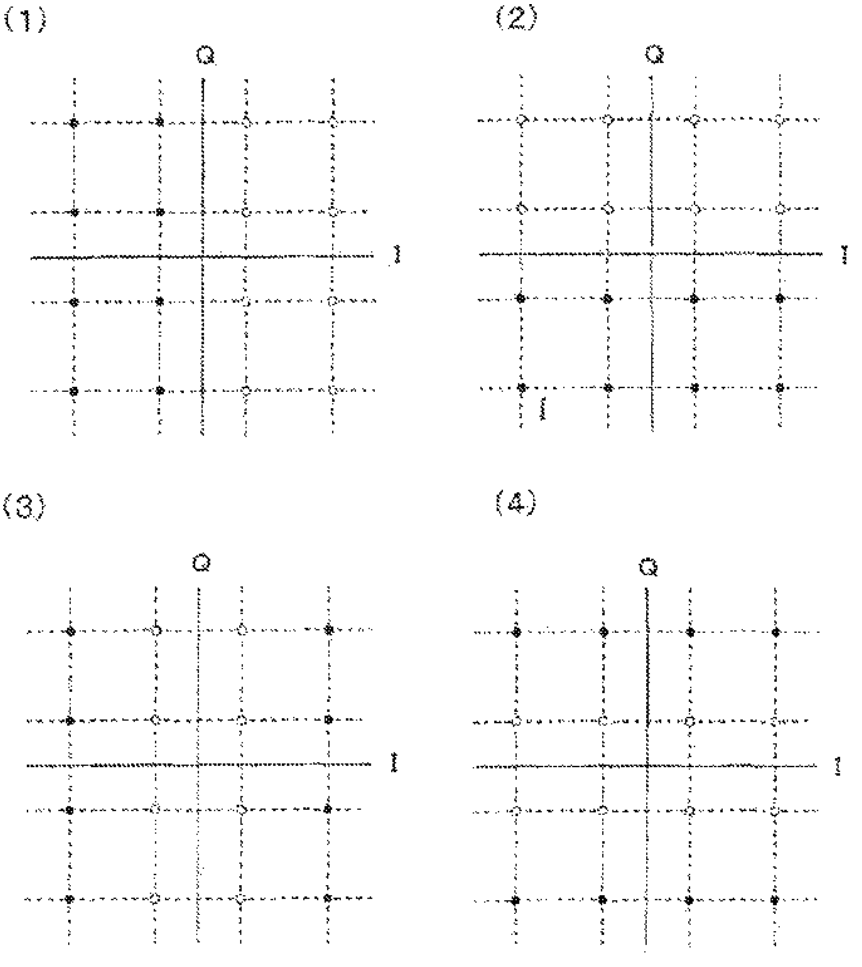


FIG. 44

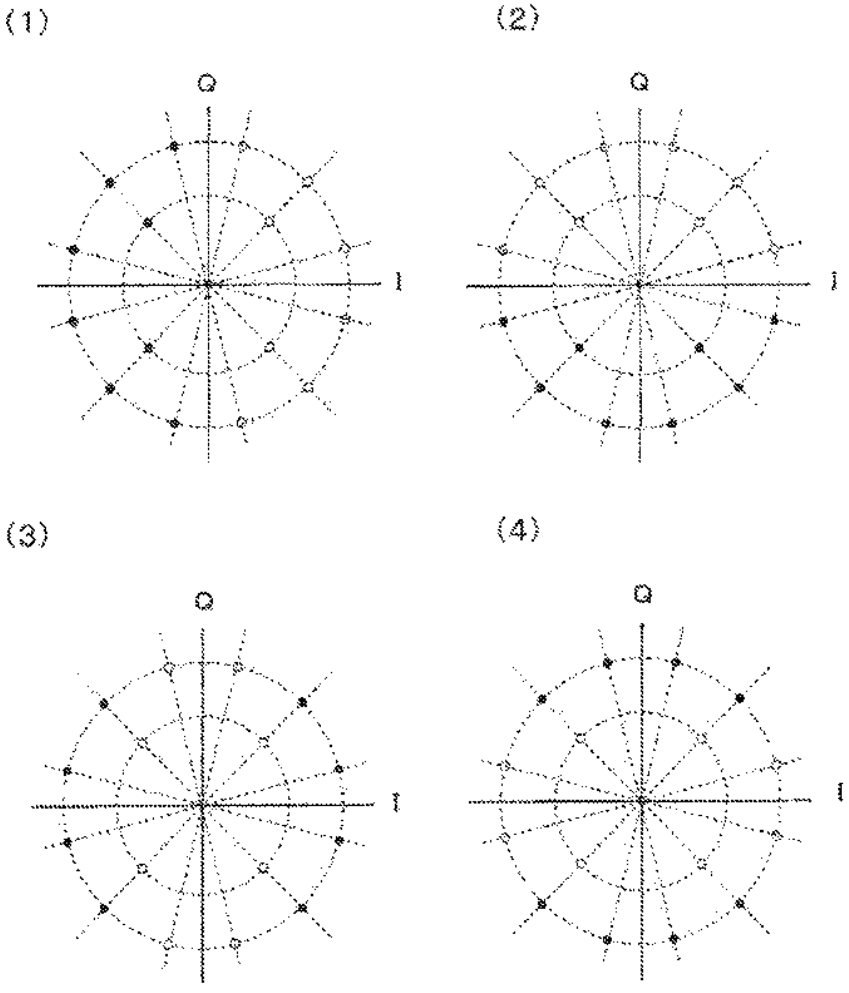


FIG. 45

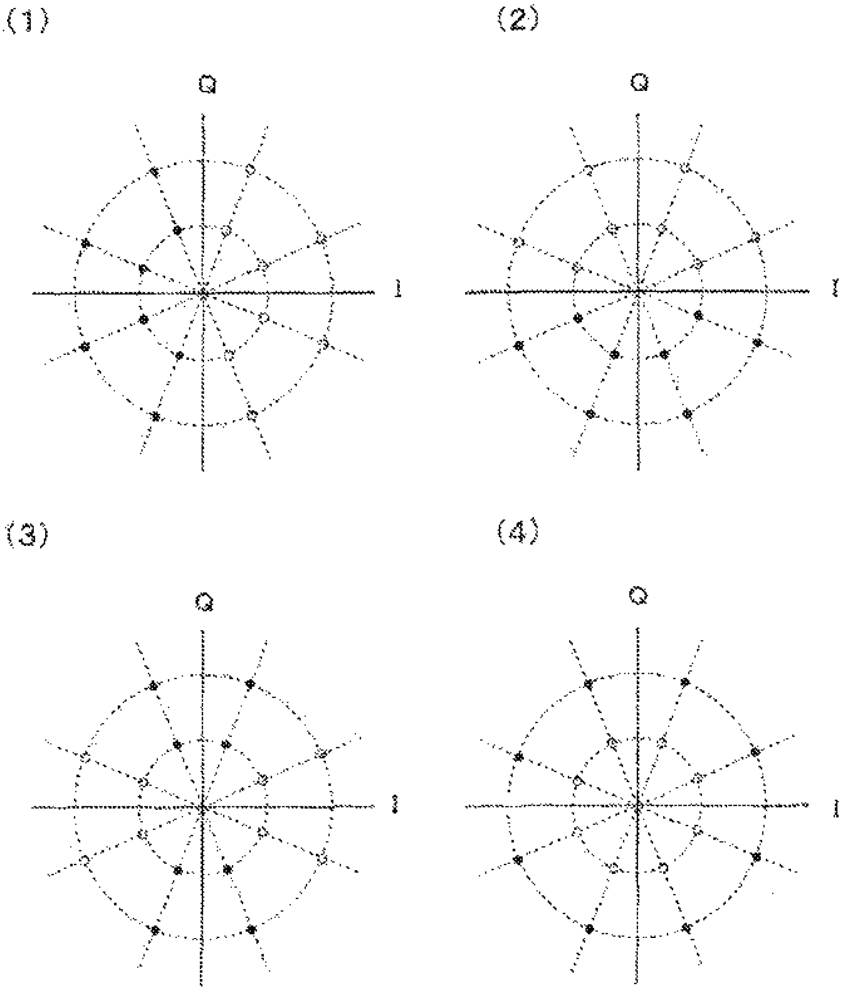


FIG. 46

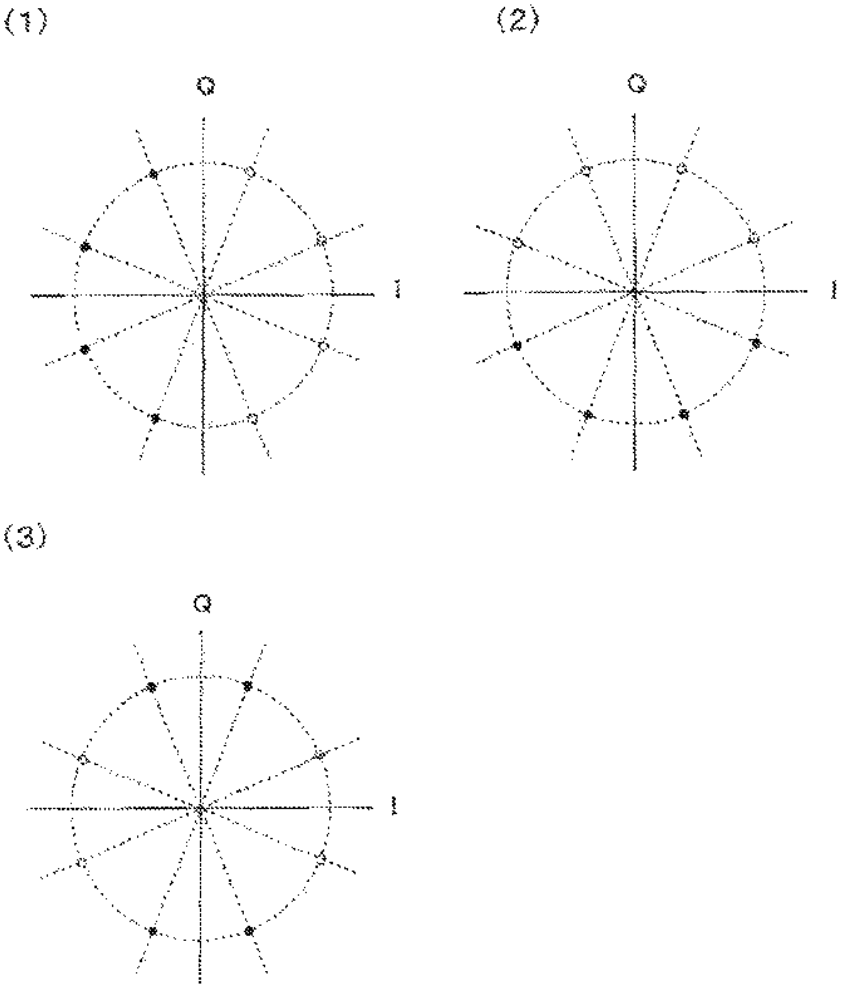
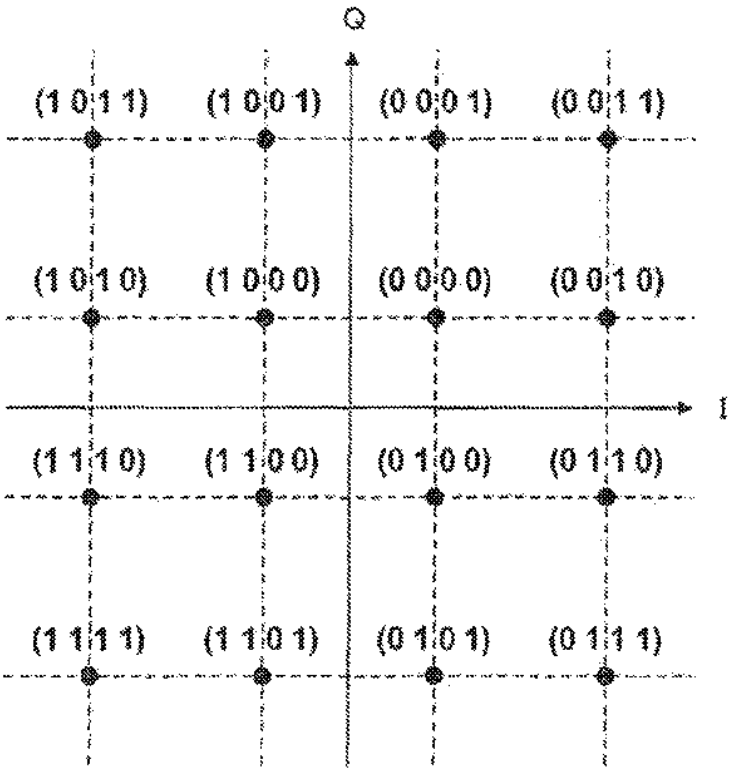


FIG. 47


Q
I
(1 0 1 1)
(1 0 0 1)
(0 0 0 1)
(0 0 1 1)
(1 0 1 0)
(1 0 0 0)
(0 0 0 0)
(0 0 1 0)
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INTERNATIONAL SEARCH REPORT

International application No.
PCT/JP2005/022956

A. CLASSIFICATION OF SUBJECT MATTER
H04L27/34(2006.01), *H03M13/29*(2006.01)

According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
H04L27/34(2006.01), *H03M13/29*(2006.01)

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
Jitsuyo Shinan Koho 1922-1996 Jitsuyo Shinan Toroku Koho 1996-2006
Kokai Jitsuyo Shinan Koho 1971-2006 Toroku Jitsuyo Shinan Koho 1994-2006

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	JP 2003-143041 A (Samsung Electronics Co., Ltd.), 16 May, 2003 (16.05.03), Par. No. [0008]; Fig. 1 (Family: none)	1-32
A	JP 2004-040661 A (Matsushita Electric Industrial Co., Ltd.), 05 February, 2004 (05.02.04), Par. No. [0024] & US 2005/0164740 A1 & EP 1521360 A1 & WO 2004/006469 A1 & CN 1666442 A	1-32

☒ Further documents are listed in the continuation of Box C. ☐ See patent family annex.

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"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
28 March, 2006 (28.03.06)	18 April, 2006 (18.04.06)
Name and mailing address of the ISA/ Japanese Patent Office	Authorized officer
Facsimile No.	Telephone No.

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INTERNATIONAL SEARCH REPORT

International application No.
PCT/JP2006/023956

C (Continuation) DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
A	JP 2004-201131 A (Hitachi Kokusai Electric Inc.), 15 July, 2004 (15.07.04), Par. Nos. [0023] to [0029]; Figs. 1, 5 (Family: none)	1-32
A	JP 8-79325 A (Hitachi, Ltd.), 22 March, 1996 (22.03.96), Par. Nos. [0021] to [0025]; Fig. 1 (Family: none)	1-32

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Patent documents cited in the description

- JP 2002171298 A [0021]
- JP 2003078419 A [0021]

(12) INTERNATIONAL APPLICATION PUBLISHED UNDER THE PATENT COOPERATION TREATY (PCT)

(19) World Intellectual Property Organization
International Bureau

PCT

(43) International Publication Date
11 December 2008 (11.12.2008)

(10) International Publication Number
WO 2008/151308 A1

(51) International Patent Classification:
H04J 11/00 (2006.01)

(21) International Application Number:
PCT/US2008/065994

(22) International Filing Date: 5 June 2008 (05.06.2008)

(25) Filing Language: English

(26) Publication Language: English

(30) Priority Data:
60/933,519 5 June 2007 (05.06.2007) US

(71) Applicants and
(72) Inventors: **BARSOUM, Maged, F.** [US/US]; 374 Durango Aisle, Irvine, CA 92606 (US). **JONES, Christopher, Richard** [US/US]; 1564 Calle Patricia, Pacific Palisades, CA 90272 (US).

(74) Agent: **BAILEY, David, L.**; Kaufi, Pomeroy, Peck & Bailey LLP, P.O. Box 19152, Irvine, CA 92623 (US).

(81) Designated States *(unless otherwise indicated, for every kind of national protection available)*: AE, AG, AL, AM, AO, AT, AU, AZ, BA, BB, BG, BH, BR, BW, BY, BZ, CA, CH, CN, CO, CR, CU, CZ, DE, DK, DM, DO, DZ, EC, EE, EG, ES, FI, GB, GD, GE, GH, GM, GT, HN, HR, HU, ID, IL, IN, IS, JP, KE, KG, KM, KN, KP, KR, KZ, LA, LC, LK, LR, LS, LT, LU, LY, MA, MD, ME, MG, MK, MN, MW, MX, MY, MZ, NA, NG, NI, NO, NZ, OM, PG, PH, PL, PT, RO, RS, RU, SC, SD, SE, SG, SK, SL, SM, SV, SY, TJ, TM, TN, TR, TT, TZ, UA, UG, US, UZ, VC, VN, ZA, ZM, ZW.

(84) Designated States *(unless otherwise indicated, for every kind of regional protection available)*: ARIPO (BW, GH, GM, KE, LS, MW, MZ, NA, SD, SL, SZ, TZ, UG, ZM, ZW), Eurasian (AM, AZ, BY, KG, KZ, MD, RU, TJ, TM), European (AT, BE, BG, CH, CY, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HR, HU, IE, IS, IT, LT, LU, LV, MC, MT, NL, NO, PL, PT, RO, SE, SI, SK, TR), OAPI (BF, BJ, CF, CG, CI, CM, GA, GN, GQ, GW, ML, MR, NE, SN, TD, TG).

Published:
— *with international search report*
— *before the expiration of the time limit for amending the claims and to be republished in the event of receipt of amendments*

(54) Title: DESIGN METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS

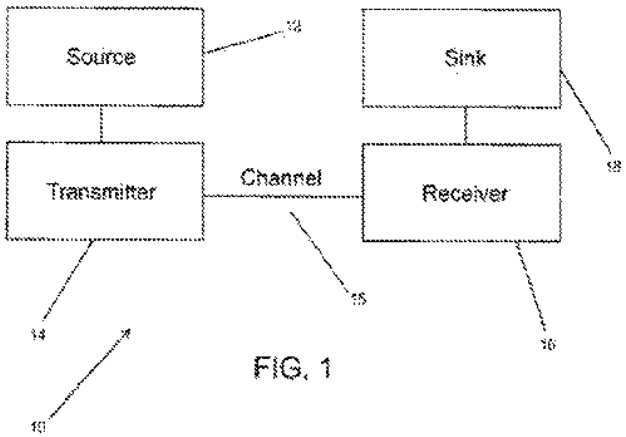


FIG. 1

(57) Abstract: Communication systems are described that use geometrically shaped constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. In several embodiments, the geometrically shaped is optimized based upon a capacity measure such as parallel decoding capacity or joint capacity. In many embodiments, a capacity optimized geometrically shaped constellation can be used to replace a conventional constellation as part of a firmware upgrade to transmitters and receivers within a communication system. In a number of embodiments, the geometrically shaped constellation is optimized for an Additive White Gaussian Noise channel or a fading channel. In numerous embodiments, the communication uses adaptive rate encoding and the location of points within the geometrically shaped constellation changes as the code rate changes. One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

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Design Methodology and Method and Apparatus for Signaling with Capacity Optimized Constellations

STATEMENT OF FEDERALLY SPONSORED RESEARCH

[0001] This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

[0002] The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

[0003] The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

[0004] Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation..

[0005] Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempts to mimic a Gaussian distribution.

[0006] Other approaches increases the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria..

SUMMARY OF THE INVENTION

[0007] Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

[0008] One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol

constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

[0009] A further embodiment of the invention includes encoding the bits of user information using a coding scheme, mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}, modulating the symbols in accordance with a modulation scheme, transmitting the modulated signal via the communication channel, receiving a modulated signal, demodulating the modulated signal in accordance with the modulation scheme, demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods, and decoding the likelihoods to obtain an estimate of the decoded bits.

[0010] Another embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

[0011] A still further embodiment of the invention includes selecting an appropriate constellation size and a desired capacity per dimension, estimating an initial SNR at which the system is likely to operate, and iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

[0012] Still another embodiment of the invention includes selecting an appropriate constellation size and a desired SNR, and optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

[0013] A yet further embodiment of the invention includes obtaining a geometrically shaped PAM constellation with a constellation size that is the square root of said given constellation size, where the geometrically shaped PAM constellation has a capacity greater than that of a PAM constellation that maximizes d_{min}, creating an orthogonalized PAM constellation using the

geometrically shaped PAM constellation, and combining the geometrically shaped PAM constellation and the orthogonalized PAM constellation to produce a geometrically shaped QAM constellation.

[0014] Another further embodiment of the invention includes transmitting information over a channel using a geometrically shaped symbol constellation, and modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

BRIEF DESCRIPTION OF DRAWINGS

[0015] FIG. 1 is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

[0016] FIG. 2 is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

[0017] FIG. 3 is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

[0018] FIG. 4a is a conceptual illustration of the joint capacity of a channel.

[0019] FIG. 4b is a conceptual illustration of the parallel decoding capacity of a channel.

[0020] FIG. 5 is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

[0021] FIG. 6a is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

[0022] FIG. 6b is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8,16,32.

[0023] FIG. 7 is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2,4,8,16,32 constellations.

[0024] FIG. 8a is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0025] FIG. 8b is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0026] FIG. 9 is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

[0027] FIGS. 10a – 10d are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0028] FIGS. 11a and 11b are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

[0029] FIGS. 12a – 12d are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0030] FIGS. 13a and 13b are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

[0031] FIGS. 14a – 14d are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0032] FIGS. 15a and 15b are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

[0033] FIGS. 16a – 16d are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0034] FIGS. 17a and 17b are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

[0035] FIG. 18 is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

[0036] FIG. 19 is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

[0037] FIG. 20 is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

[0038] FIG. 21 illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

[0039] FIGS. 22a and 22b are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0040] FIGS. 23a and 23b are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0041] FIGS. 24a and 24b are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

DETAILED DESCRIPTION OF THE INVENTION

[0042] Turning now to the drawings, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In several embodiments, the locations of geometrically shaped points are designed to provide a given capacity measure at a reduced signal-to-noise ratio (SNR) compared to the SNR required by a constellation that maximizes d_{min}. In many embodiments, the constellations are selected to provide increased capacity at a predetermined range of channel signal-to-noise ratios (SNR). Capacity measures that can be used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

[0043] In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve

gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. In a number of embodiments, the communication systems can adapt location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

COMMUNICATION SYSTEMS

[0044] A communication system in accordance with an embodiment of the invention is shown in FIG. 1. The communication system 10 includes a source 12 that provides user bits to a transmitter 14. The transmitter transmits symbols over a channel to a receiver 16 using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

[0045] A transmitter in accordance with an embodiment of the invention is shown in FIG. 2. The transmitter 14 includes a coder 20 that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper 22 is connected to the coder. The mapper maps the bits output by the coder to a symbol within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator 24, which modulates the symbols for transmission via the channel.

[0046] A receiver in accordance with an embodiment of the invention is illustrated in FIG. 3. The receiver 16 includes a demodulator 30 that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper 32 provides the likelihoods to a decoder 34 that decodes the encoded bit stream to provide a sequence of received bits to a sink.

GEOMETRICALLY SHAPED CONSTELLATIONS

[0047] Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

SELECTION OF A GEOMETRICALLY SHAPED CONSTELLATIONS

[0048] Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

UPGRADING EXISTING COMMUNICATION SYSTEMS

[0049] A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation

having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

JOINT CAPACITY AND PARALLEL DECODING CAPACITY

[0050] A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. 4a. The portions of the communication system that are considered part of the channel are indicated by the ghost line 40. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{i-1} I(X_i; Y)$$

where X_i is the ith bit of the i-bits transmitted symbol, and Y is the received symbol, and $I(A;B)$ denotes the mutual information between random variables A and B.

[0051] Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

[0052] With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4b.

The portions of the communication system that are considered part of the channel are indicated by the ghost line 42. The joint capacity of the channel is given by:

$$C_{JOINT} = I(X;Y)$$

[0053] Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_Q \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

SELECTING A CONSTELLATION HAVING AN OPTIMAL CAPACITY

[0054] Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. 5. The process 50 commences with the selection (52) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (52) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known *a priori*. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target

user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

[0055] In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An *M*-ary constellation is then obtained by optimizing (56) the constellation to maximize a selected capacity measure at the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

[0056] The SNR at which the optimized *M*-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (57). A determination (58) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (55). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellation at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

[0057] The process illustrated in FIG. 5 can maximize PD capacity or joint capacity of an *M*-ary constellation for a given SNR. Although the process illustrated in FIG. 5 shows selecting an *M*-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an *M*-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example

0.5 db. Alternatively, other processes that identify M-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

[0058] We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over GF(q) , or with multi-stage decoding.. Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

OPTIMIZING THE CAPACITY OF AN M-ARY CONSTELLATION AT A GIVEN SNR

[0059] Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an M-ary constellation at a given SNR. An optimization process, such as the optimization process 56 shown in FIG. 5, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the Joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged..

[0060] The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. 5.

[0061] Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting

constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

[0062] The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

GAINS ACHIEVED BY OPTIMIZED GEOMETRICALLY SPACED CONSTELLATIONS

[0063] The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}log_2(1 + SNR)$$

[0064] Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

[0065] The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. 6a and 6b. FIG. 6a includes a chart 60 showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps 62 exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. 6b includes a chart 64 showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps 66 exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These

gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity,.

[0066] In order to gain a better view of the differences between the curves shown in FIGS. 6a and 6b at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. 7. It is interesting to note from the chart 70 in FIG. 7 that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

[0067] FIGS. 8a and 8b summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. 8a shows the SNR gap for each constellation optimized for PD capacity. FIG. 8b shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

[0068] Referring to FIG. 8a, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

[0069] The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. 9. A chart 90

includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. 8a.

CAPACITY OPTIMIZED PAM CONSTELLATIONS

[0070] Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. 10a – 17b. The locus plots and design tables show PAM-4,8,16,32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

[0071] In FIG.10a, a locus plot 100 shows the location of the points of PAM-4 constellations optimized for Joint capacity plotted against achieved capacity. A similar locus plot 105 showing the location of the points of Joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. 10b. In FIG. 10c, the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. 10d the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the Joint optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. 11a and FIG. 11b where vertical slices of FIGS. 10ab and 10cd are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities = {0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4) = 2$ bps, these design points represent systems with information code rates R = {1/4, 3/8, 1/2, 5/8, 3/4} respectively.

[0072] FIGS. 12ab and 12cd present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. 13a and 13b provide slices from these plots at SNRs corresponding to achievable capacities η = {0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate $R = \eta$ bps / $\log_2(8)$, resulting

in R ={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. 13b which shows a PD capacity optimized PAM-8 constellation optimized for SNR = 9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. 8a and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

[0073] Similar information is presented in FIGS. 14abcd, and 15ab which provide loci plots and *design* tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. 16abcd, 17ab provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

CAPACITY OPTIMIZED PSK CONSTELLATIONS

[0074] Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart 180 comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. 18 where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4,8,16,32 and for PD capacity optimized PSK-4,8,16,32.

[0075] The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. 19, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. 20. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR = 8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR = 14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR = 2 and 15. dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

[0076] We note now that the locus of points for PD optimized PSK-32 in FIG. 19 in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. 18 implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8,16 by using only the optimized PSK-32 in

conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. 19. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. 18.

ADAPTIVE RATE DESIGN

[0077] In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation / code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper / decoder configuration in order to match the new transmit-side configuration.

[0078] Conversely, the receiver could implement a quality of performance based optimized constellation / code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation / code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper / decoder modules.

GEOMETRICALLY SHAPED QAM CONSTELLATIONS

[0079] Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM inphase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

[0080] In FIG. 21 we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. 13b). The label-point pairs in this PAM-8 constellation are {(000, -1.72), (001, -0.81), (010, 1.72), (011,-0.62), (100, 0.62), (101, 0.02), (110, 0.81), (111, -0.02)}. Examination of FIG. 21 shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8x8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-DIMENSIONAL CONSTELLATION OPTIMIZATION

[0081] Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

CAPACITY OPTIMIZED CONSTELALTIONS FOR FADING CHANNELS

[0082] Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y = a(t).X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and $a(t)$ is the fading distribution, which is a function of time.

[0083] In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

[0084] Many fading channels follow a Rayleigh distribution. FIGS. 22a – 24b are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

WHAT IS CLAIMED IS:

1. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate;

a mapper configured to map encoded bits to symbols in a symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper; and

wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min}.

2. The communication system of claim 1, wherein the geometrically spaced symbol constellation is capacity optimized subject to additional constraints.

3. The communication system of claim 1, wherein the geometrically shaped symbol constellation is optimized for capacity using parallel decode capacity.

4. The communication system of claim 1, wherein the geometrically shaped symbol constellation is optimized for capacity using joint capacity.

5. The communication system of claim 1, wherein the channel is an AWGN channel.

6. The communication system of claim 1, wherein the channel is a fading channel.

7. The communication system of claim 1, wherein the constellations points of the geometrically spaced symbol constellation are variable depending on the user rate.

8. A method of transmitting bits of user information over a communication channel, comprising:

encoding the bits of user information using a coding scheme;

mapping the encoded bits of user information to a symbol constellation, wherein the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes d_{min};

modulating the symbols in accordance with a modulation scheme;

transmitting the modulated signal via the communication channel;

receiving a modulated signal;

demodulating the modulated signal in accordance with the modulation scheme;

demapping the demodulated signal using the geometrically shaped signal constellation to produce likelihoods;

decoding the likelihoods to obtain an estimate of the decoded bits.

9. A method of selecting the points of a geometrically shaped symbol constellation used in a communication system, comprising:

selecting an appropriate constellation size and a desired capacity per dimension;

estimating an initial SNR at which the system is likely to operate; and

iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

10. A geometrically shaped symbol constellation used in a communication system, obtained by a process of:

selecting an appropriate constellation size and a desired capacity per dimension;

estimating an initial SNR at which the system is likely to operate; and

iteratively optimizing the location of the points of the constellation to maximize a capacity measure until a predetermined improvement in the SNR performance of the constellation relative to a constellation that maximizes d_{min} has been achieved.

11. A geometrically shaped symbol constellation used in a communication system, obtained by a process of:

selecting an appropriate constellation size and a desired SNR; and

optimizing the location of the points of the constellation to maximize a capacity measure of the constellation.

12. A method of transmitting information, comprising:

transmitting information over a channel using a geometrically shaped symbol constellation; and

modifying the location of points within the geometrically shaped symbol constellation to change the target user data rate.

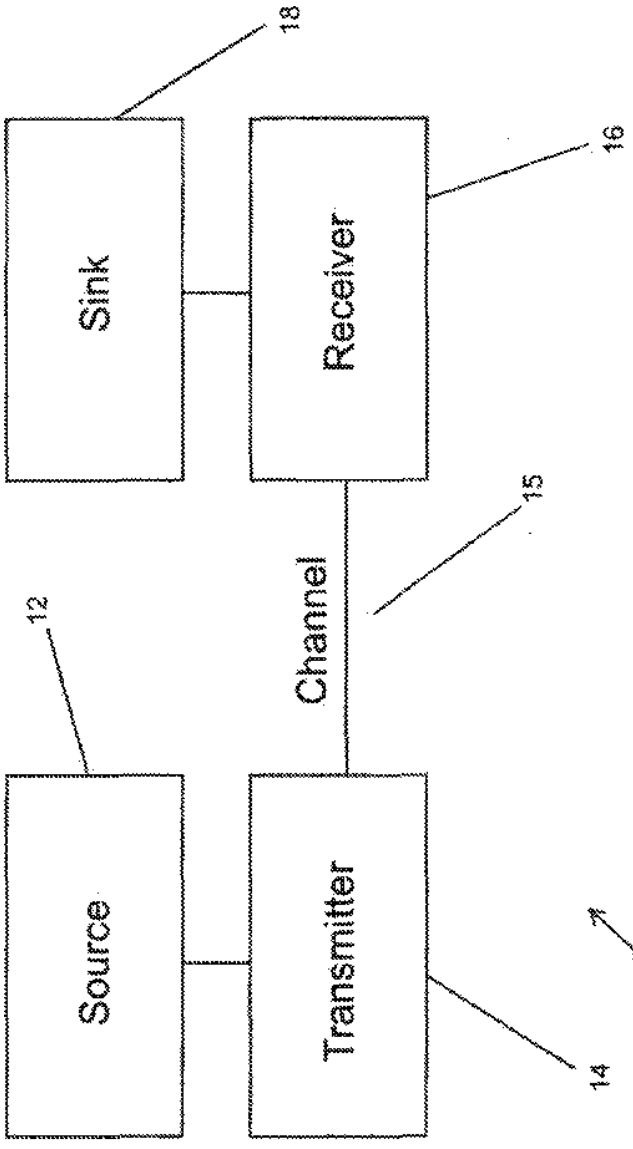


FIG. 1

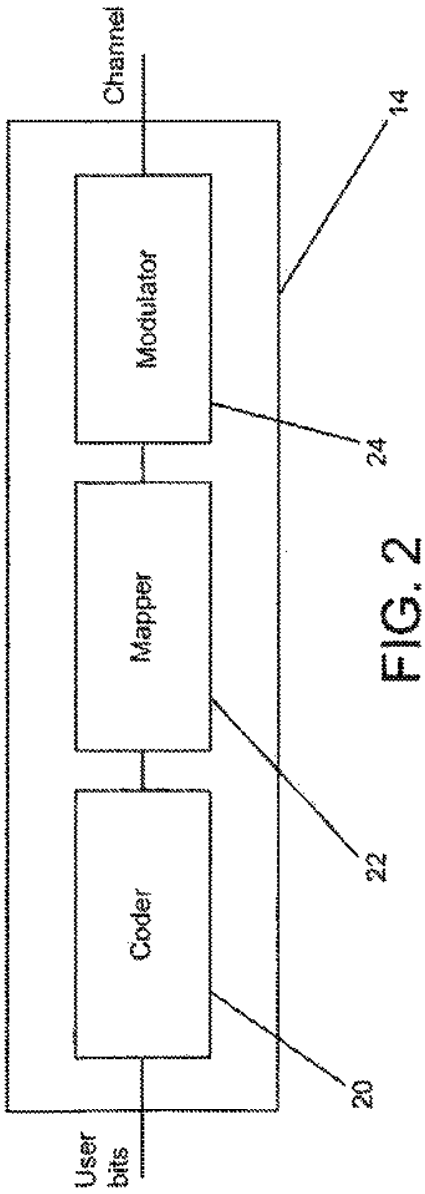

Channel
Modulator
Mapper
Coder
User bits
14
24
22
20

FIG. 2

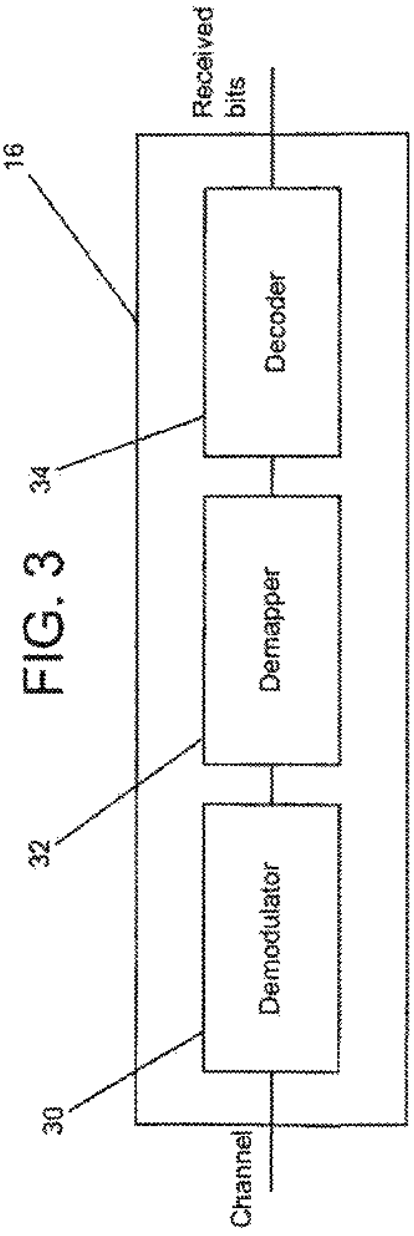

Received bits
Decoder
Demapper
Demodulator
Channel
16
34
32
30

FIG. 3

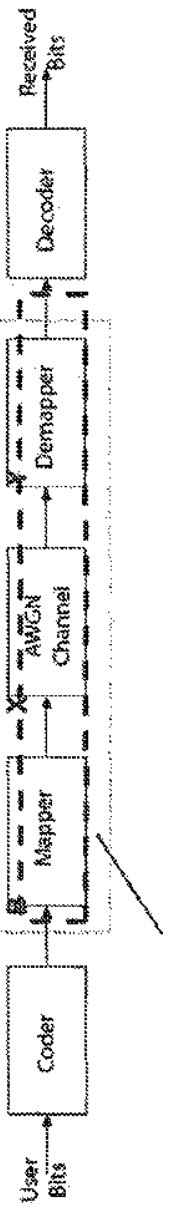

User Bits
Coder
B
Mapper
X
AWGN Channel
Y
Demapper
Decoder
Received Bits
Parallel Decoding Capacity
40

FIG. 4a

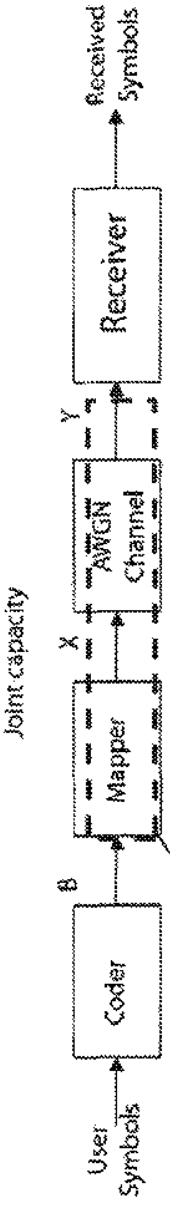

User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
Joint capacity
42

FIG. 4b

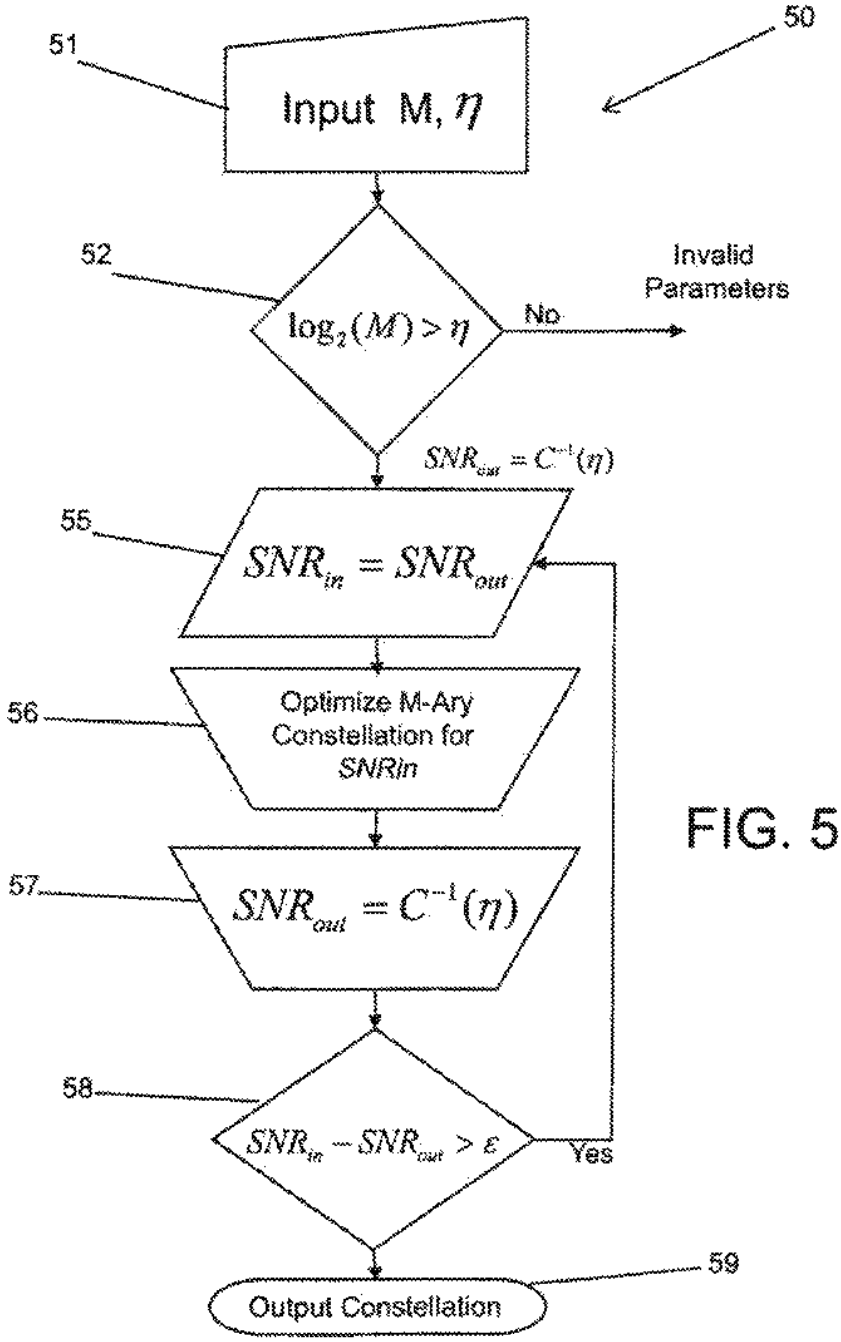

50
51
Input M, η
52
log2(M) > η
No
Invalid Parameters
SNRout = C-1(η)
55
SNRin = SNRout
56
Optimize M-Ary Constellation for SNRin
57
SNRout = C-1(η)
58
SNRin − SNRout > ε
Yes
59
Output Constellation

FIG. 5

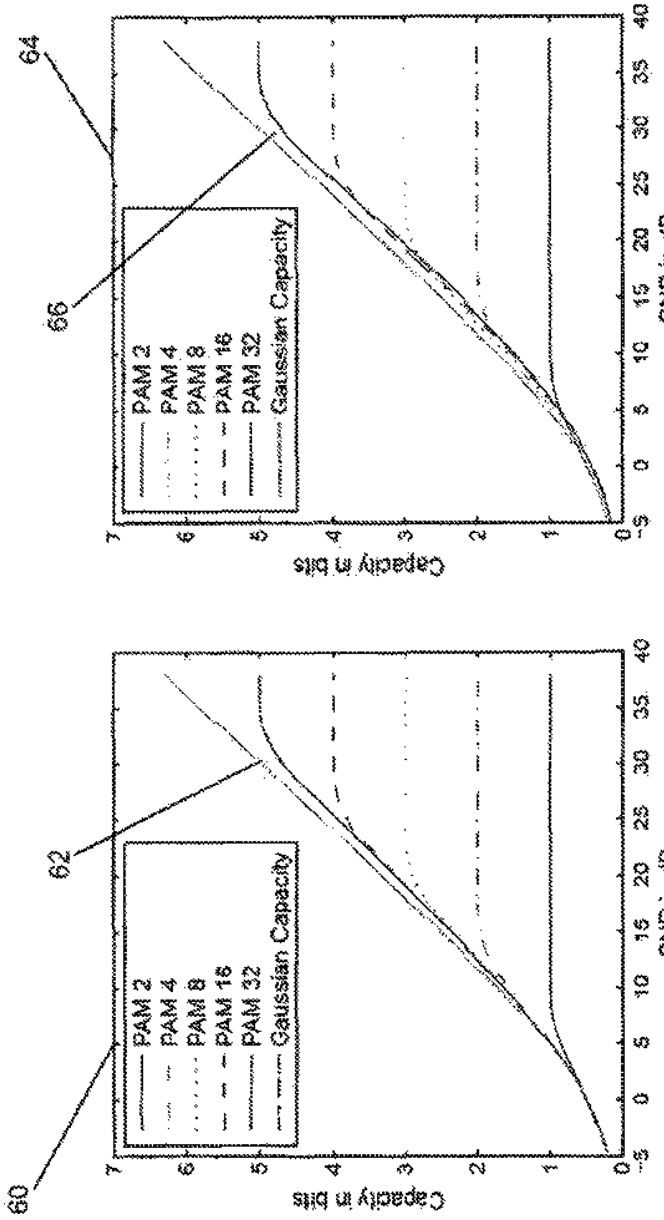

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)
60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6b

FIG. 6a

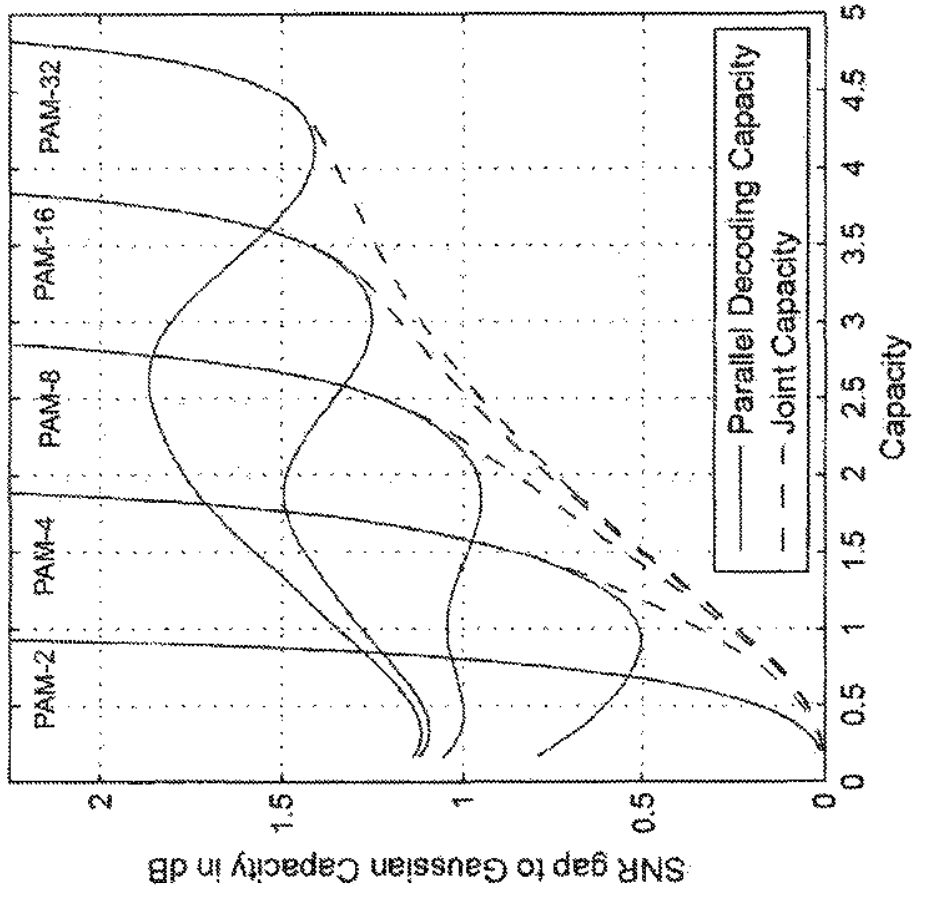

SNR gap to Gaussian Capacity in dB
Capacity
Parallel Decoding Capacity
Joint Capacity
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

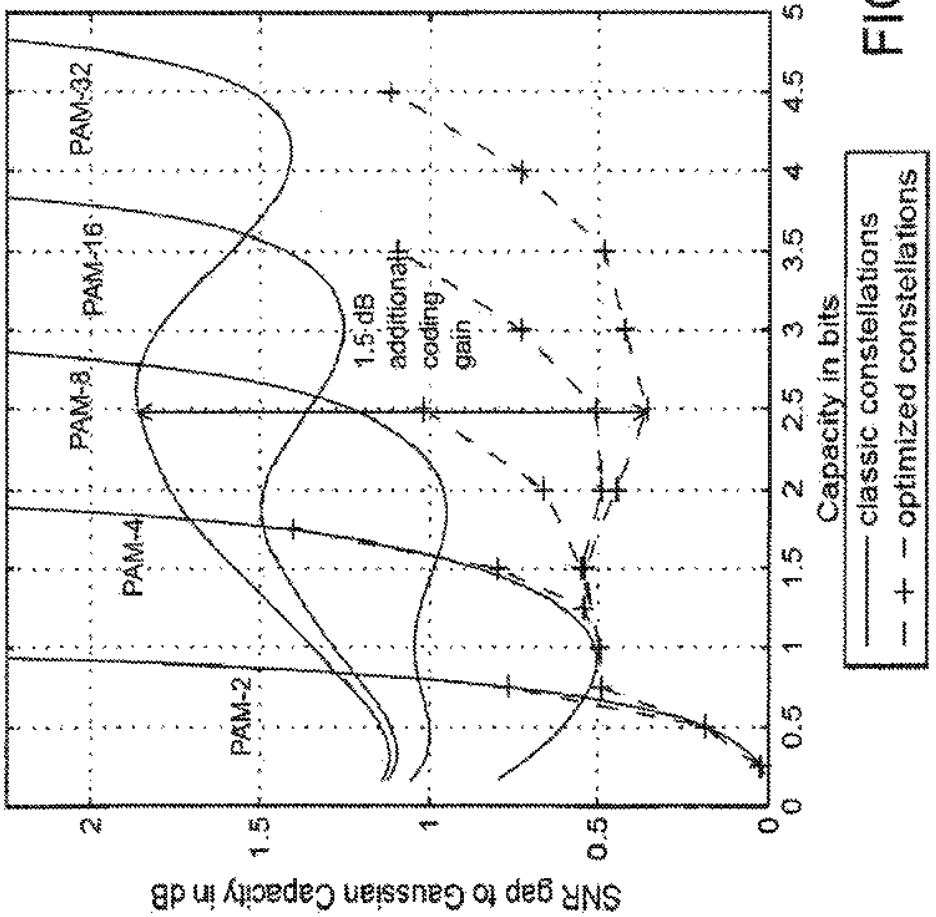

SNR gap to Gaussian Capacity in dB
Capacity in bits
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5 dB additional coding gain
classic constellations
optimized constellations

FIG. 8a

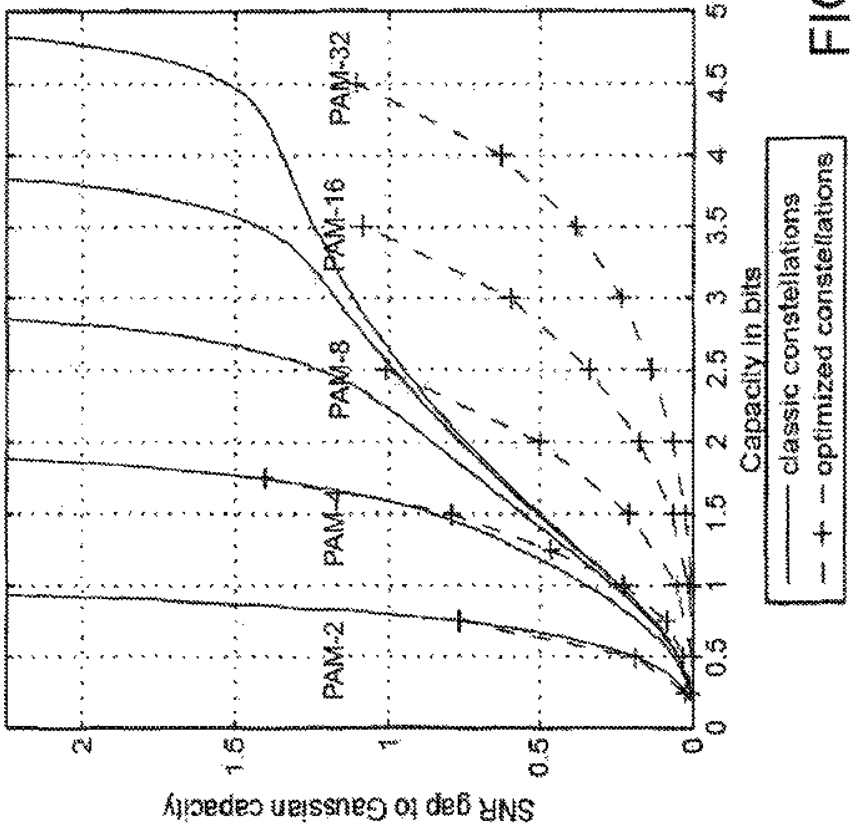

SNR gap to Gaussian capacity
Capacity in bits
classic constellations
optimized constellations
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 8b

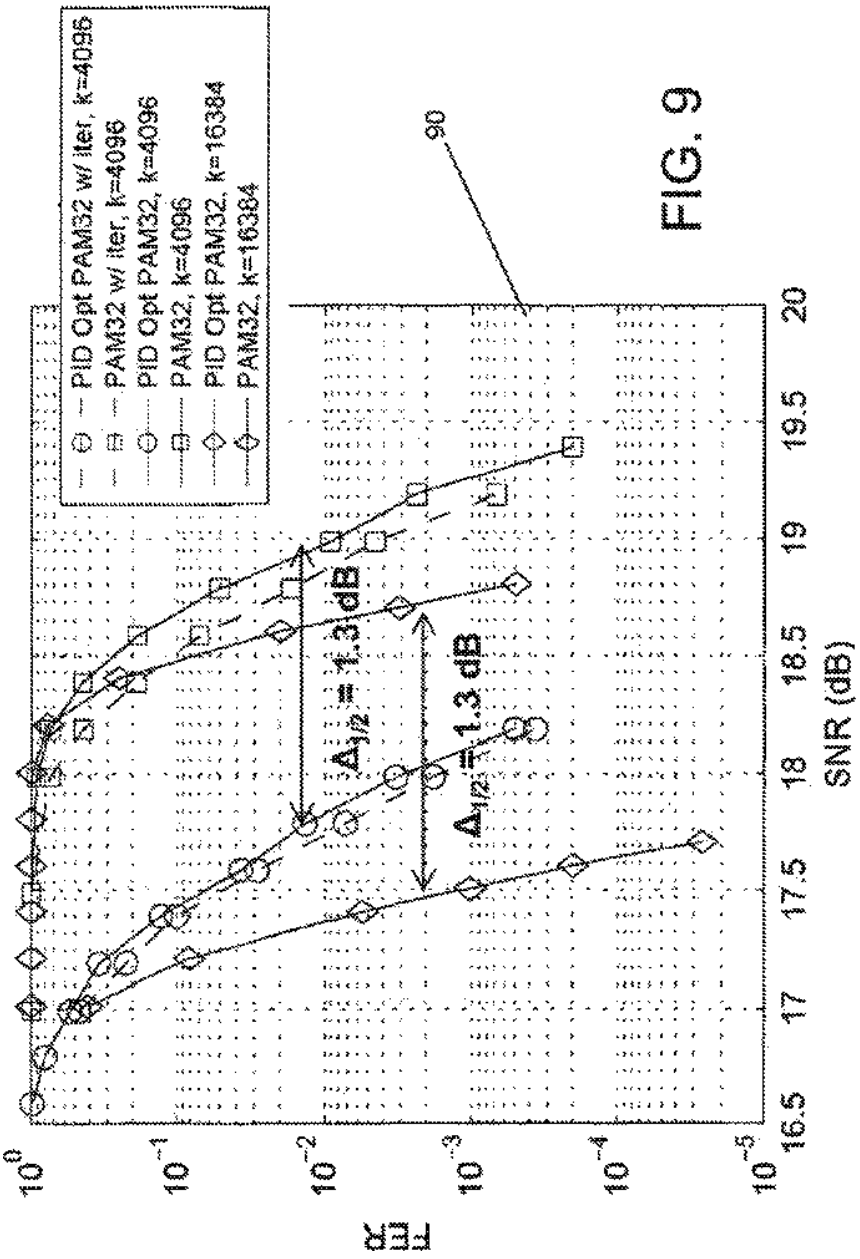

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/2}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

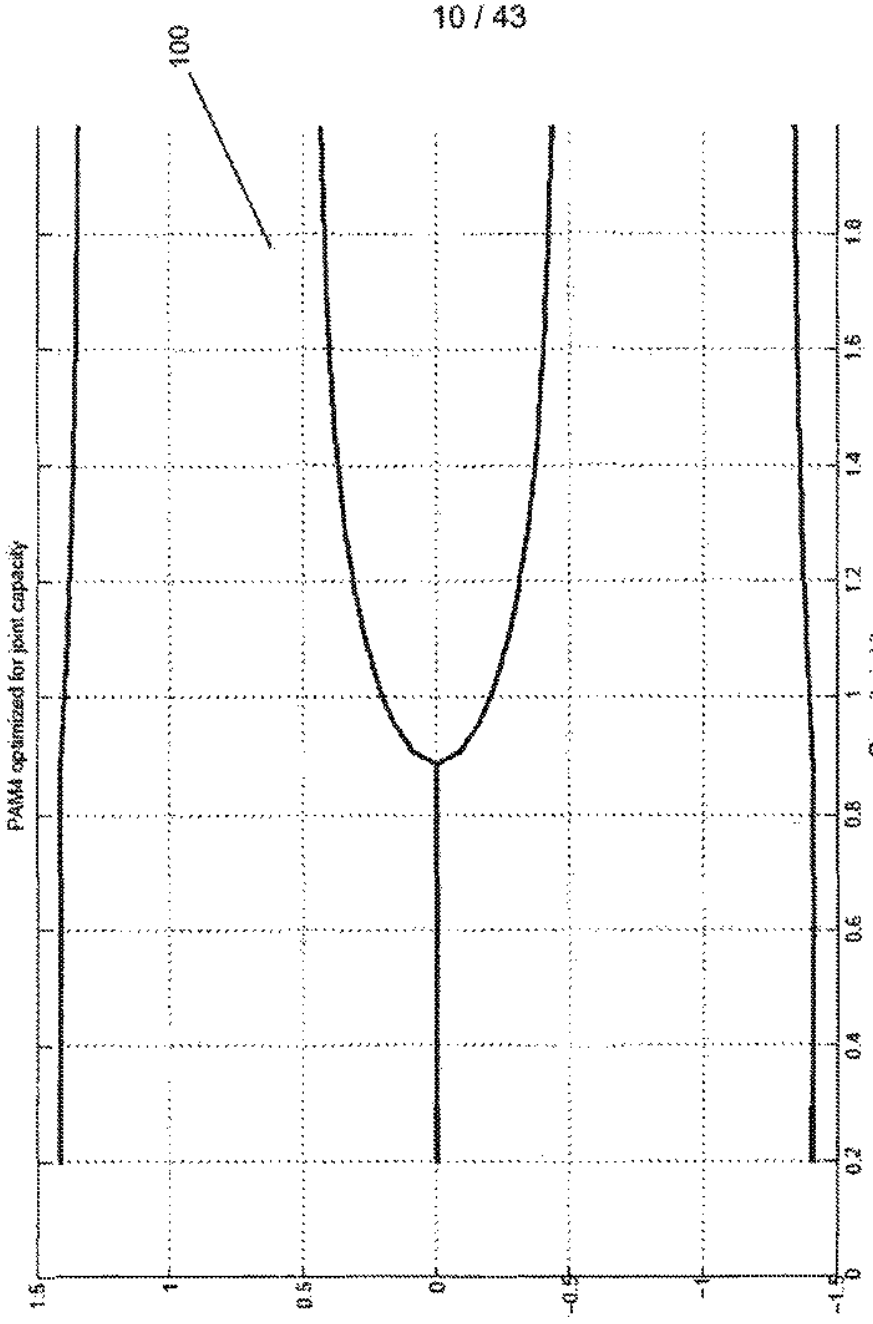

100
PAMA optimized for joint capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10a

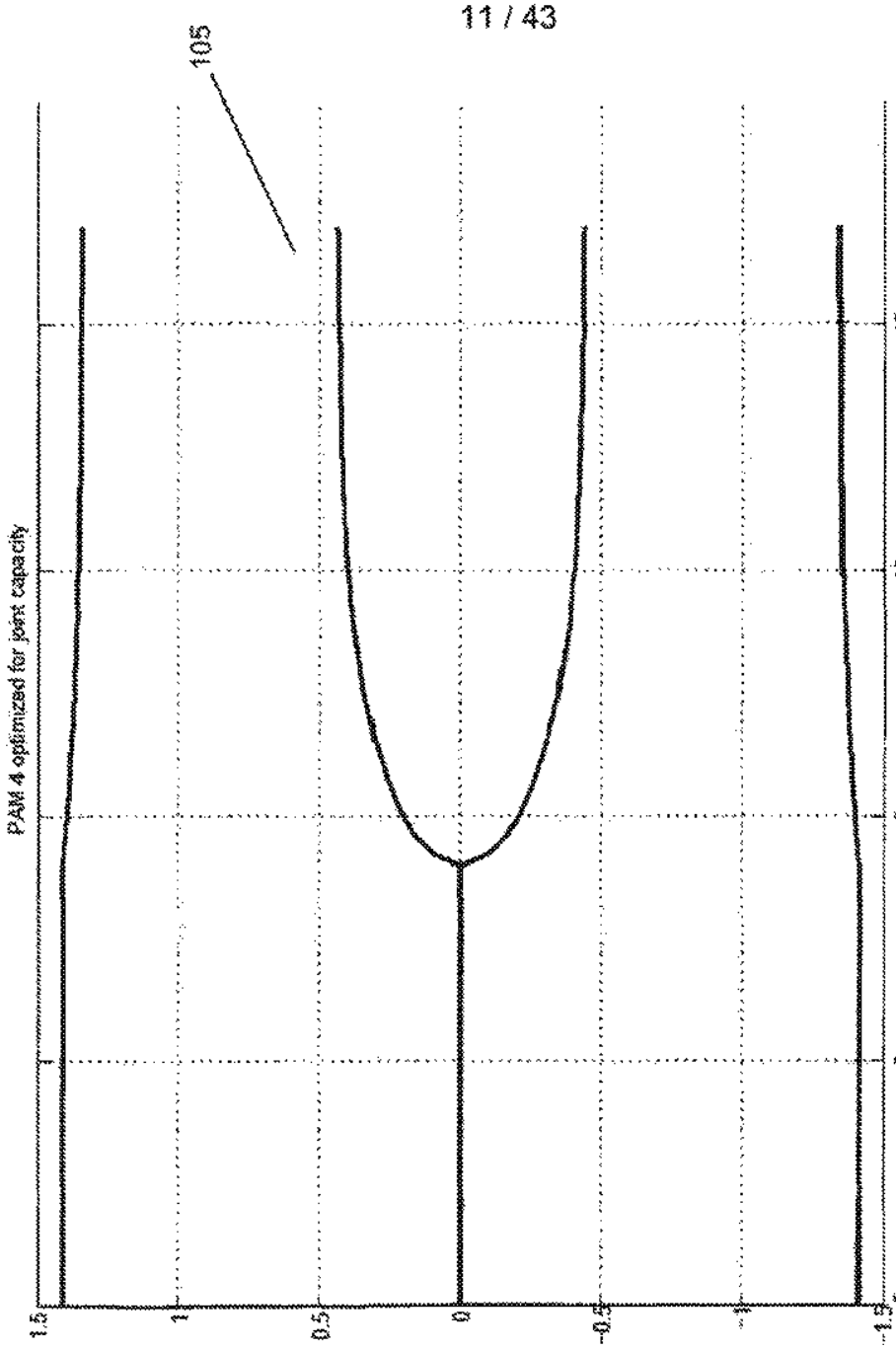

105
PAM 4 optimized for joint capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10b

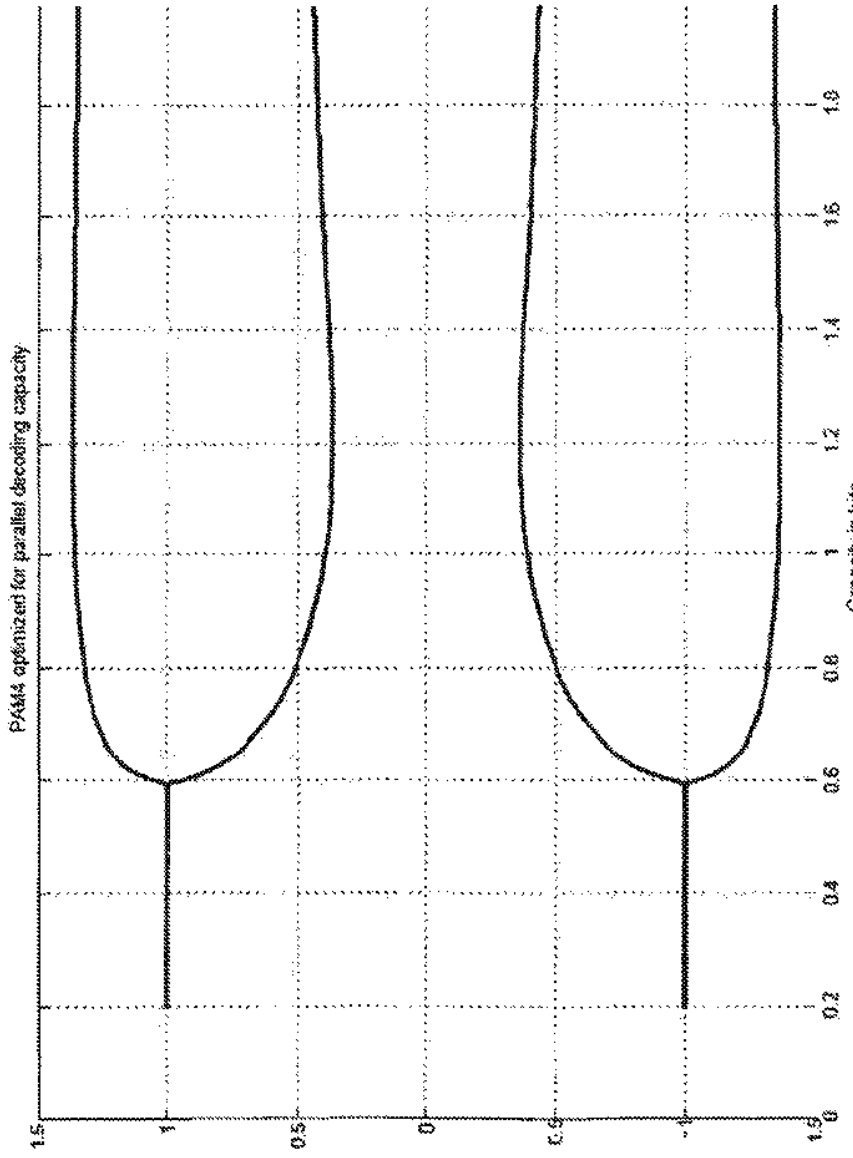

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

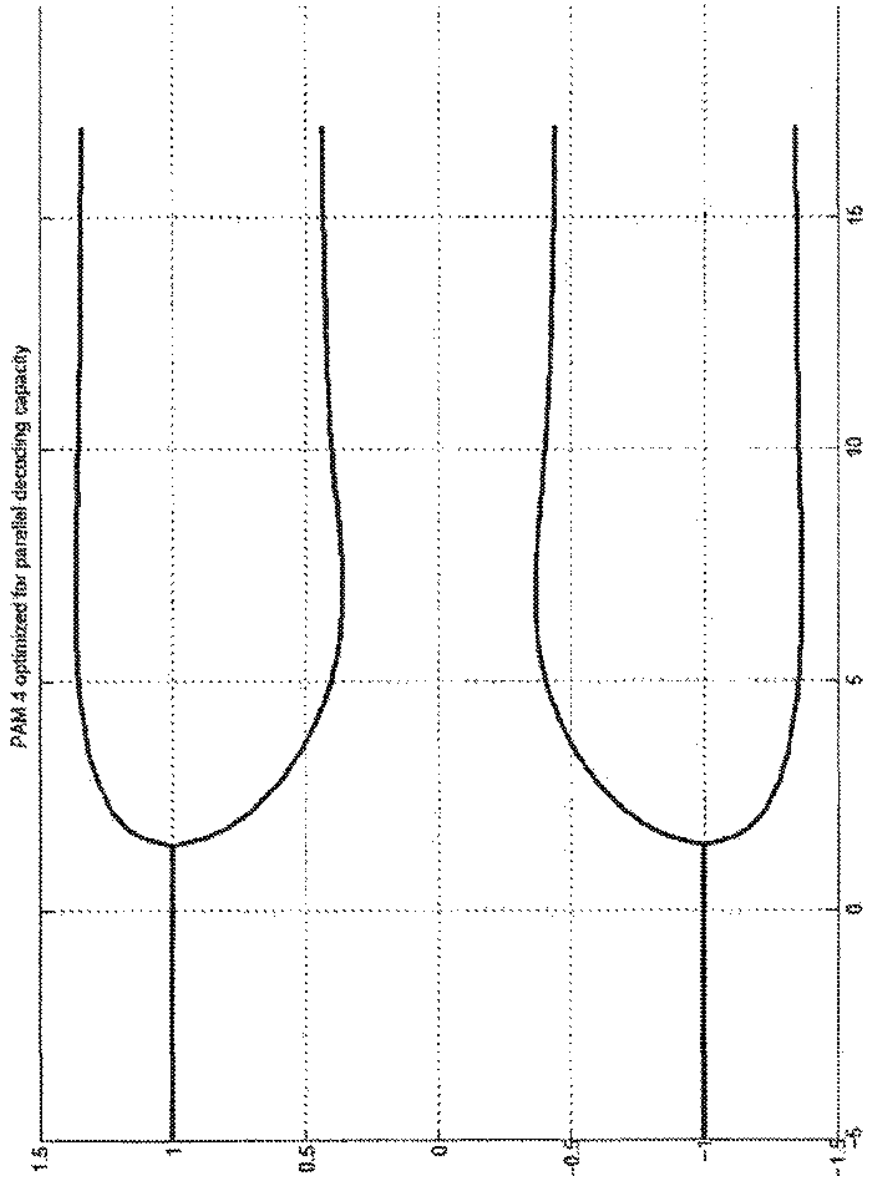

PAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
x_0	-1.41	-1.41	-1.40	-1.37	-1.36
x_1	0.00	0.00	-0.20	-0.33	-0.39
x_2	0.00	0.00	0.20	0.33	0.39
x_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.56	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

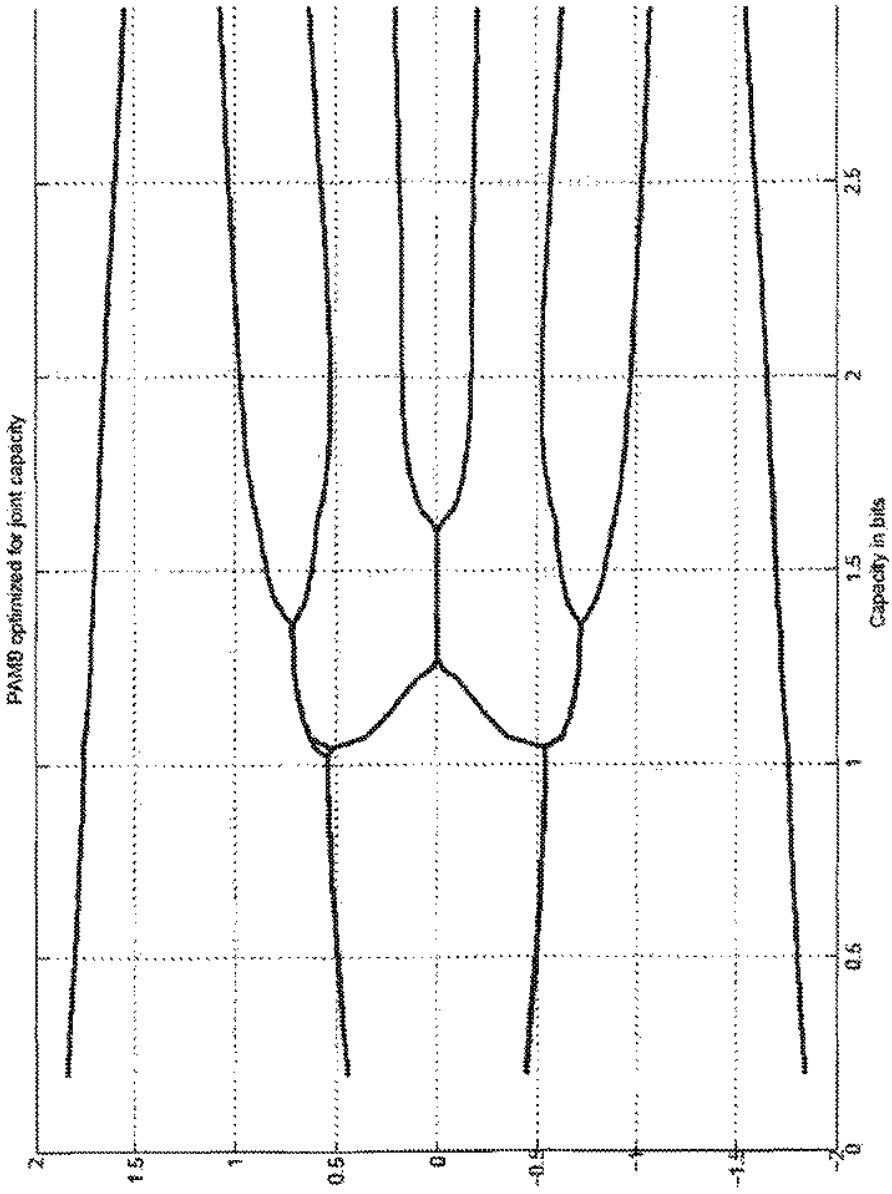

PAM9 optimized for joint capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12a

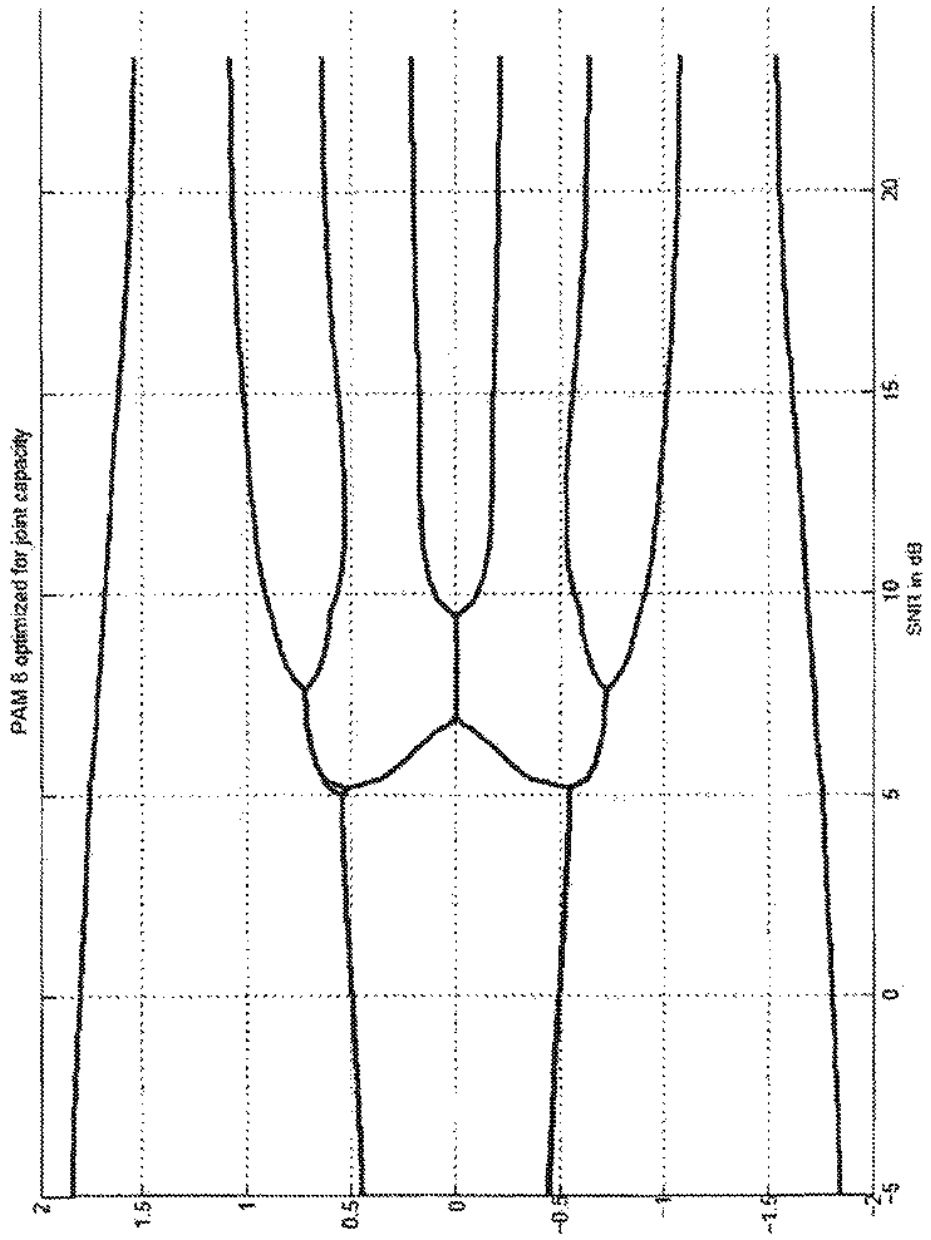

PAM 8 optimized for joint capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12b

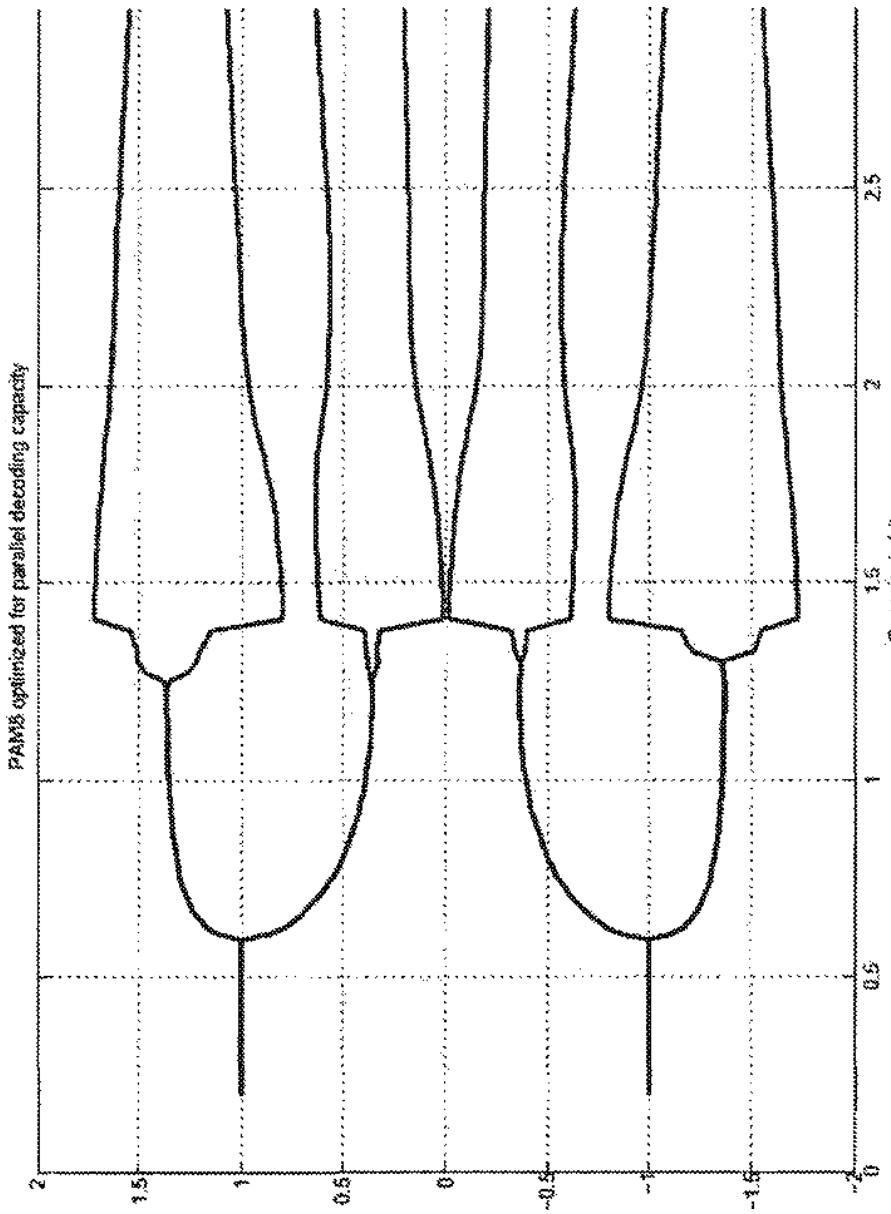

PAM8 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 12c

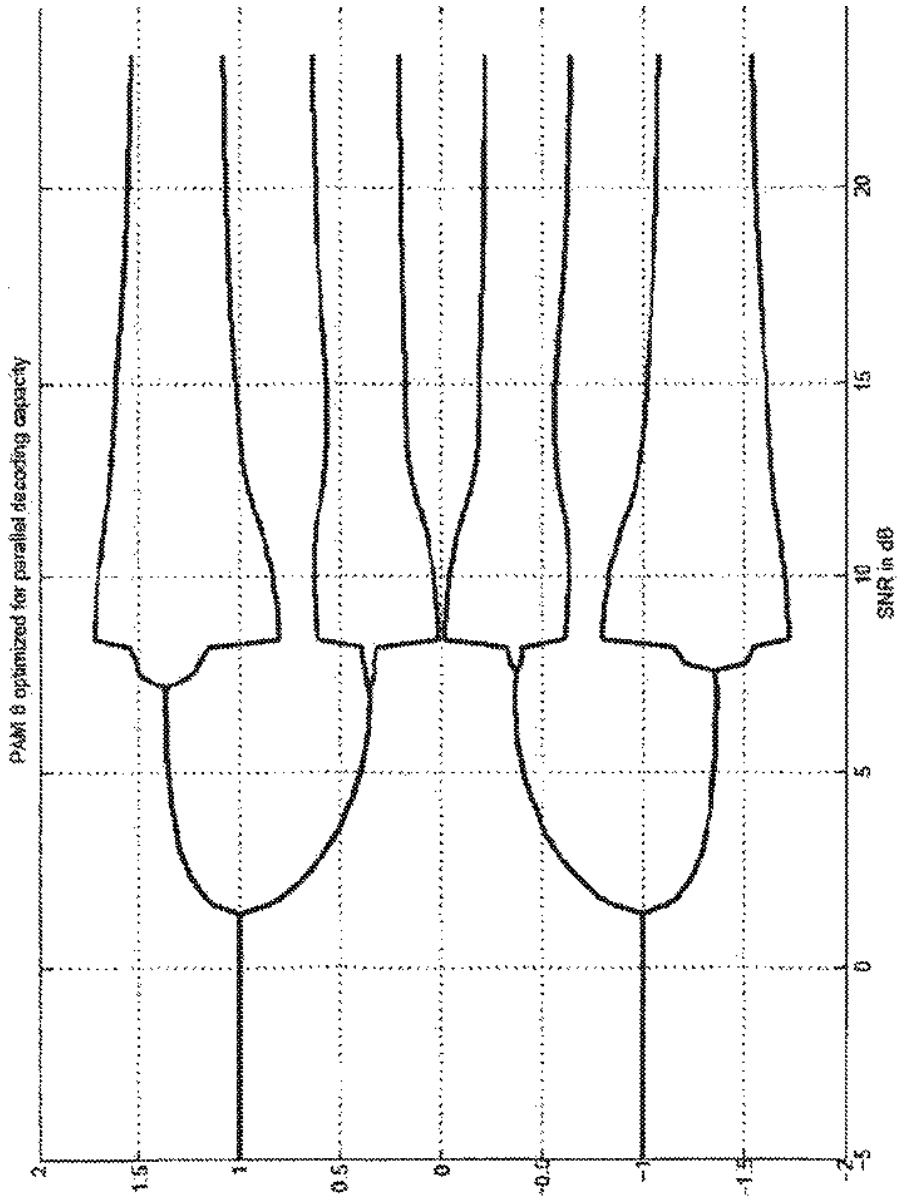

PAR 6 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20

FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5.27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

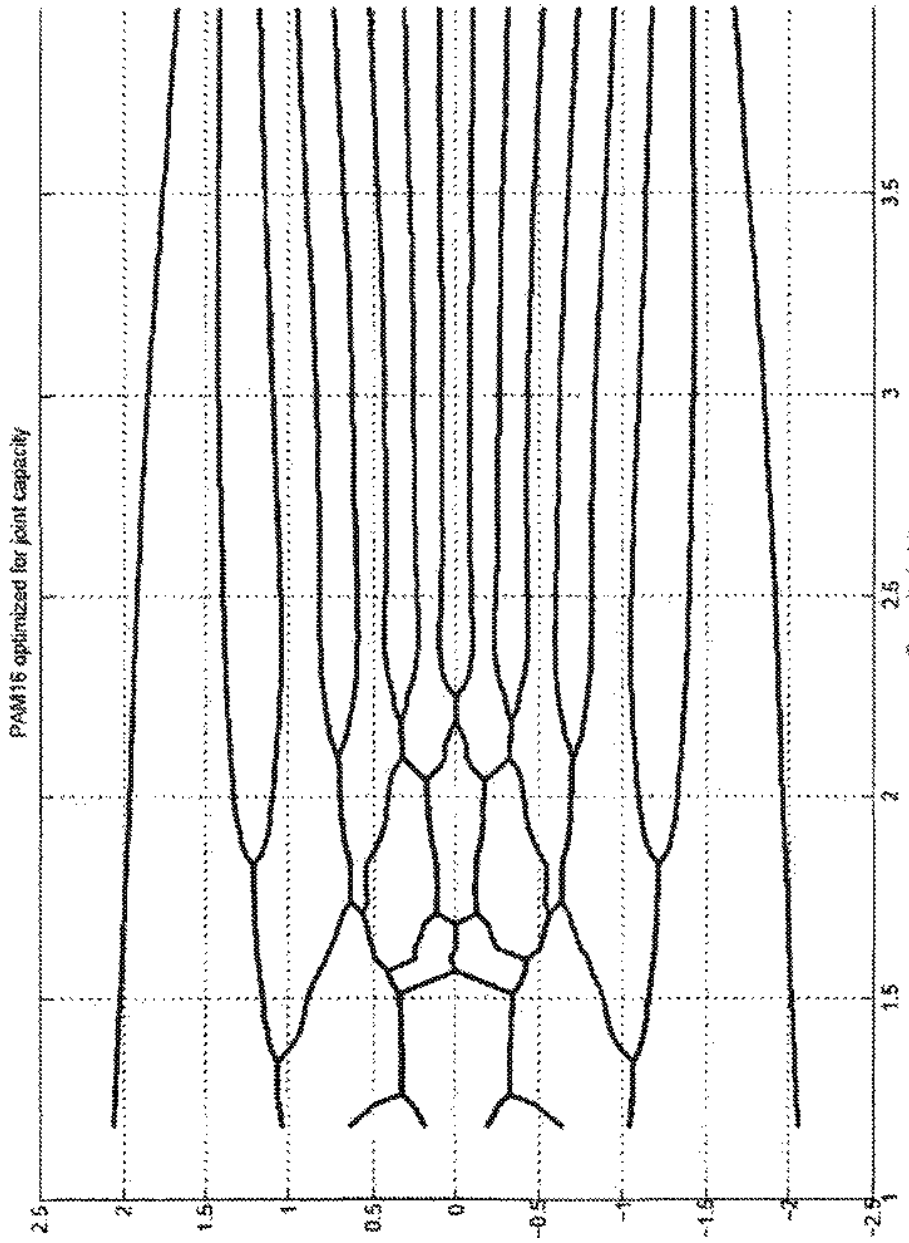

PAM16 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1
1.5
2
2.5
3
3.5

FIG. 14a

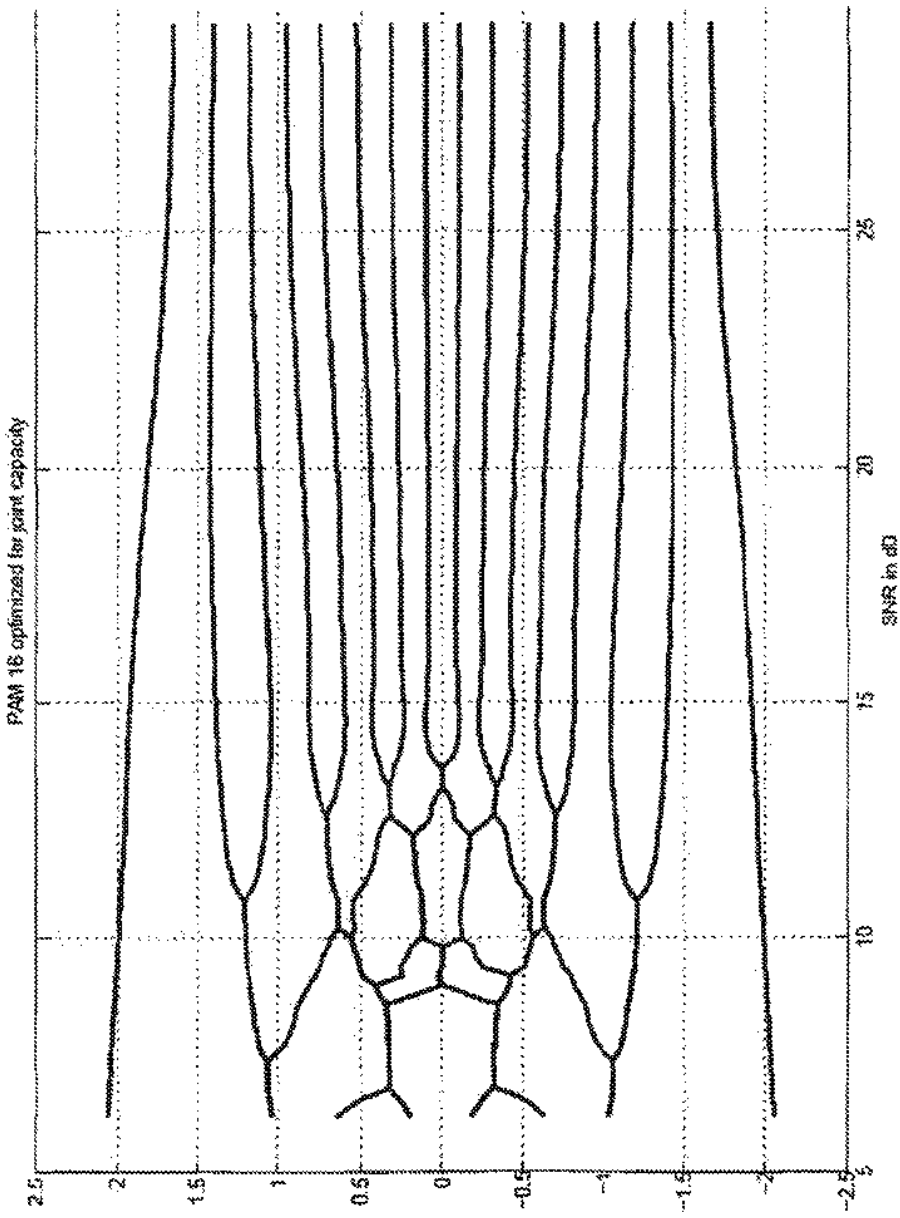

PAM 16 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14b

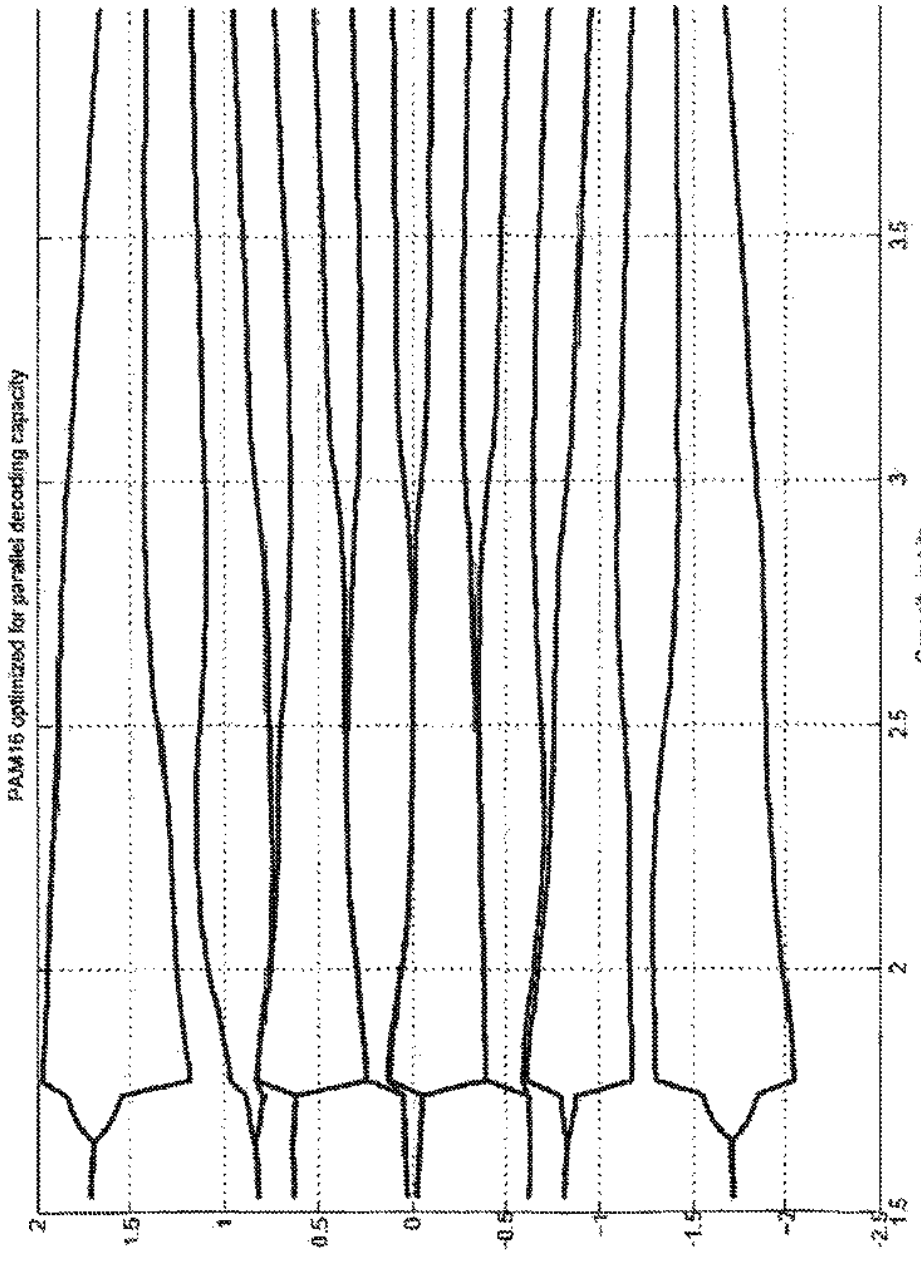

PAM16 optimized for parallel decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
1.5
2
2.5
3
3.5

FIG. 14c

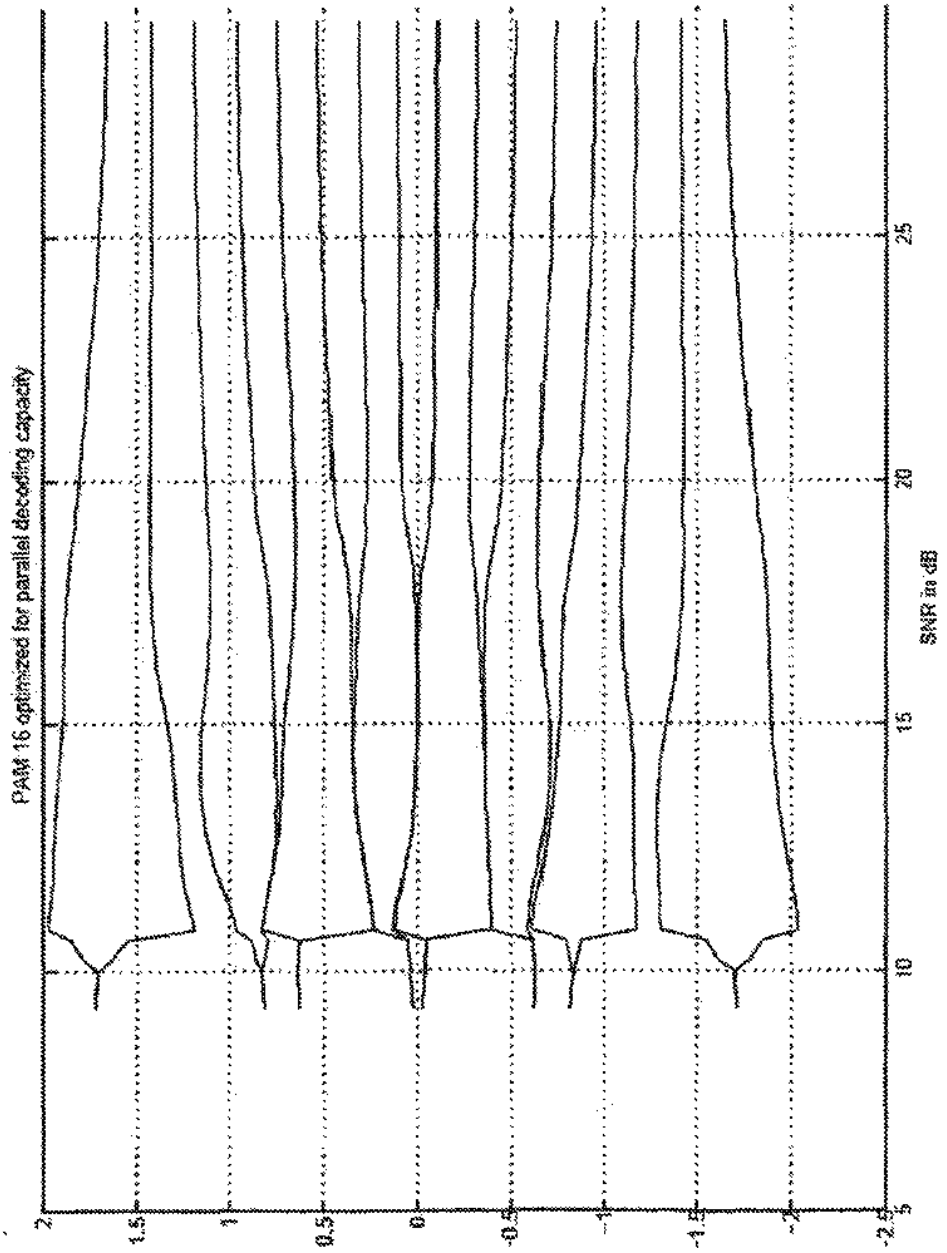

PAM 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

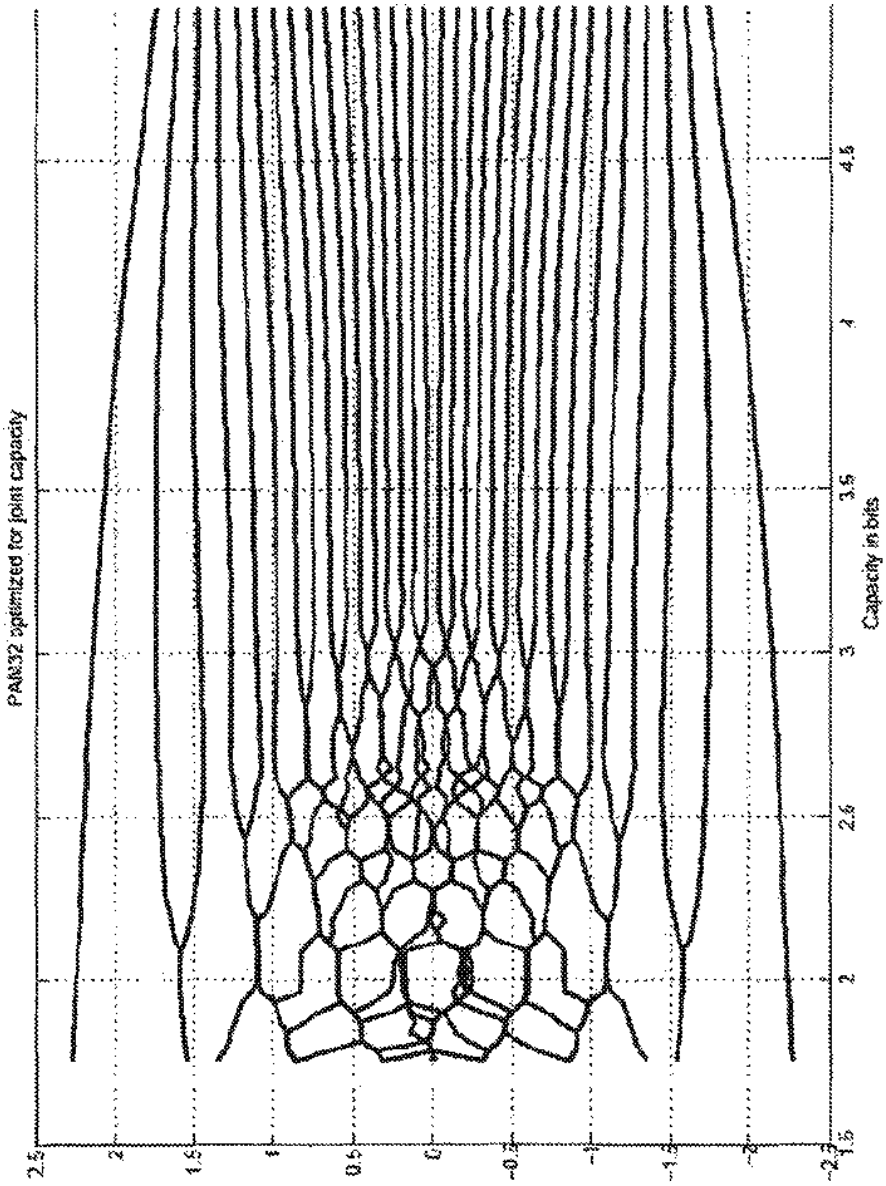

PAM32 optimized for joint capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16a

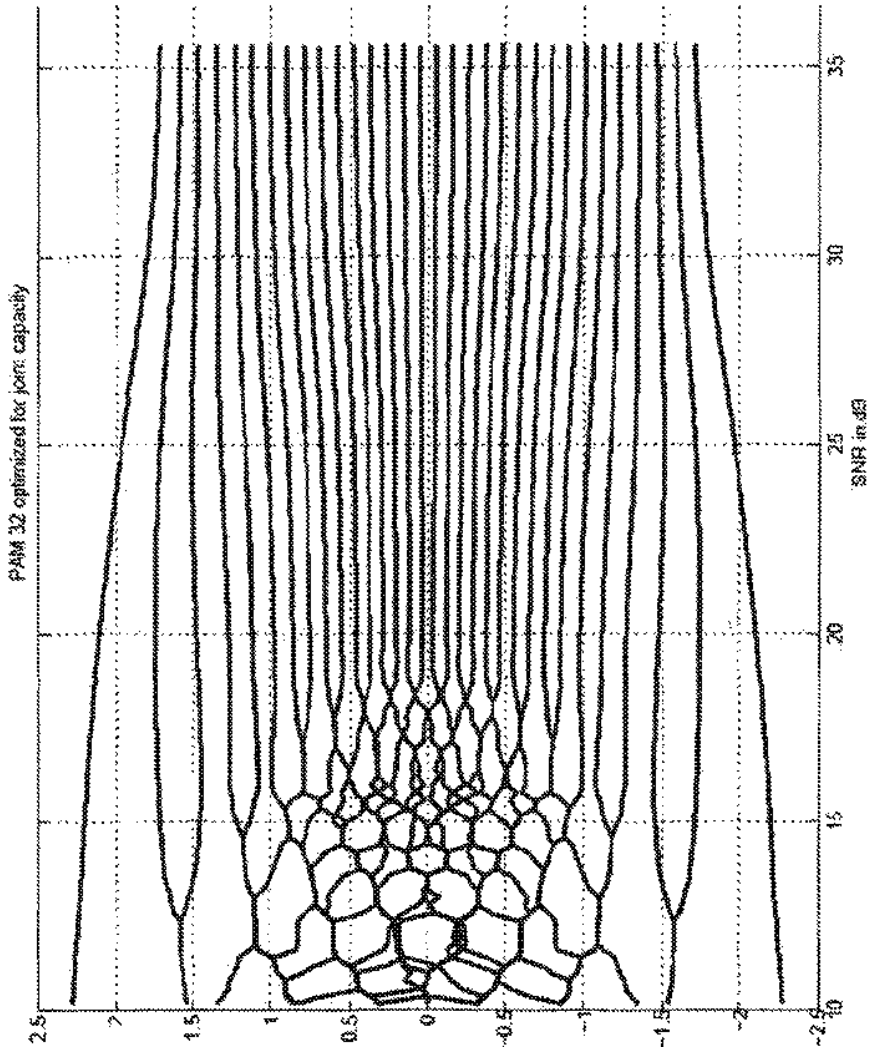

PAM 32 optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
10
15
20
25
30
35

FIG. 16b

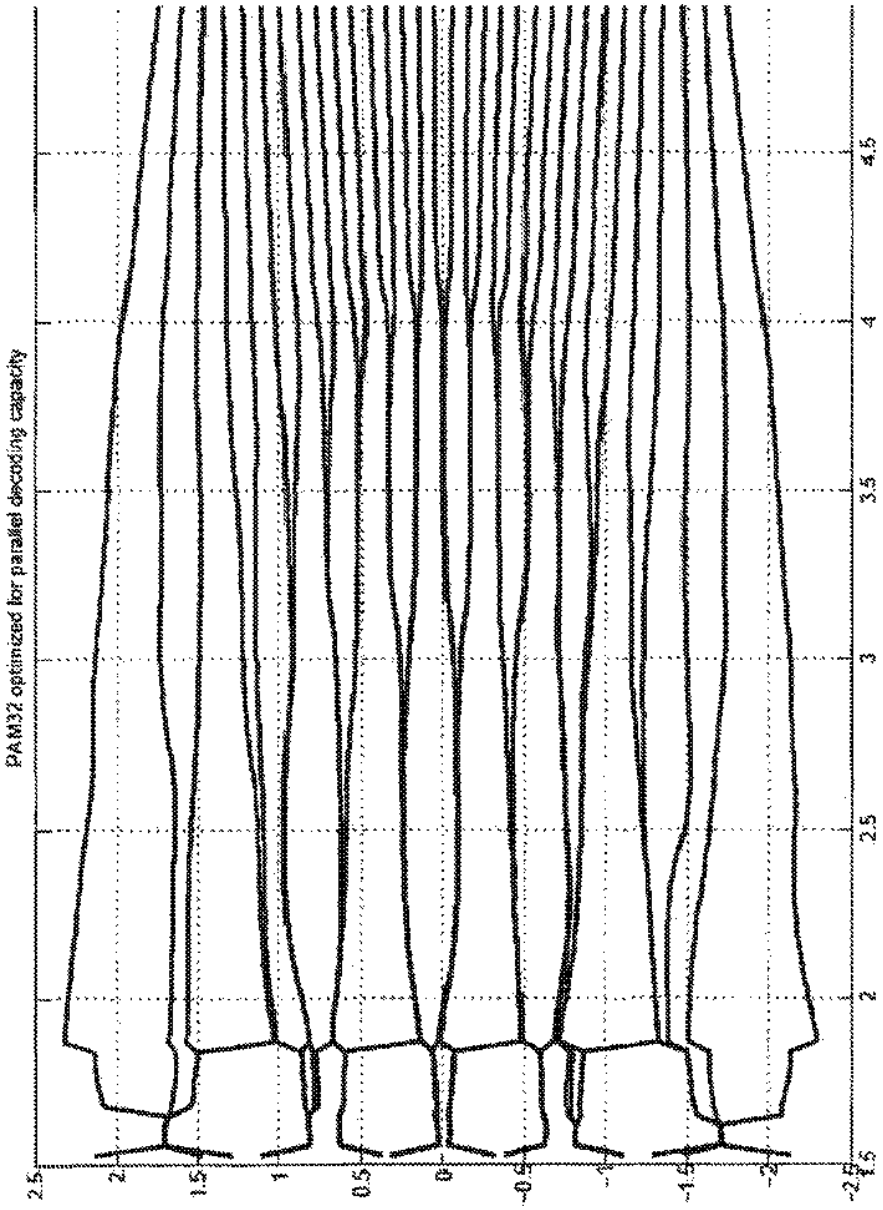

PAM32 optimized for parallel decoding capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
1.5
2
2.5
3
3.5
4
4.5

FIG. 16c

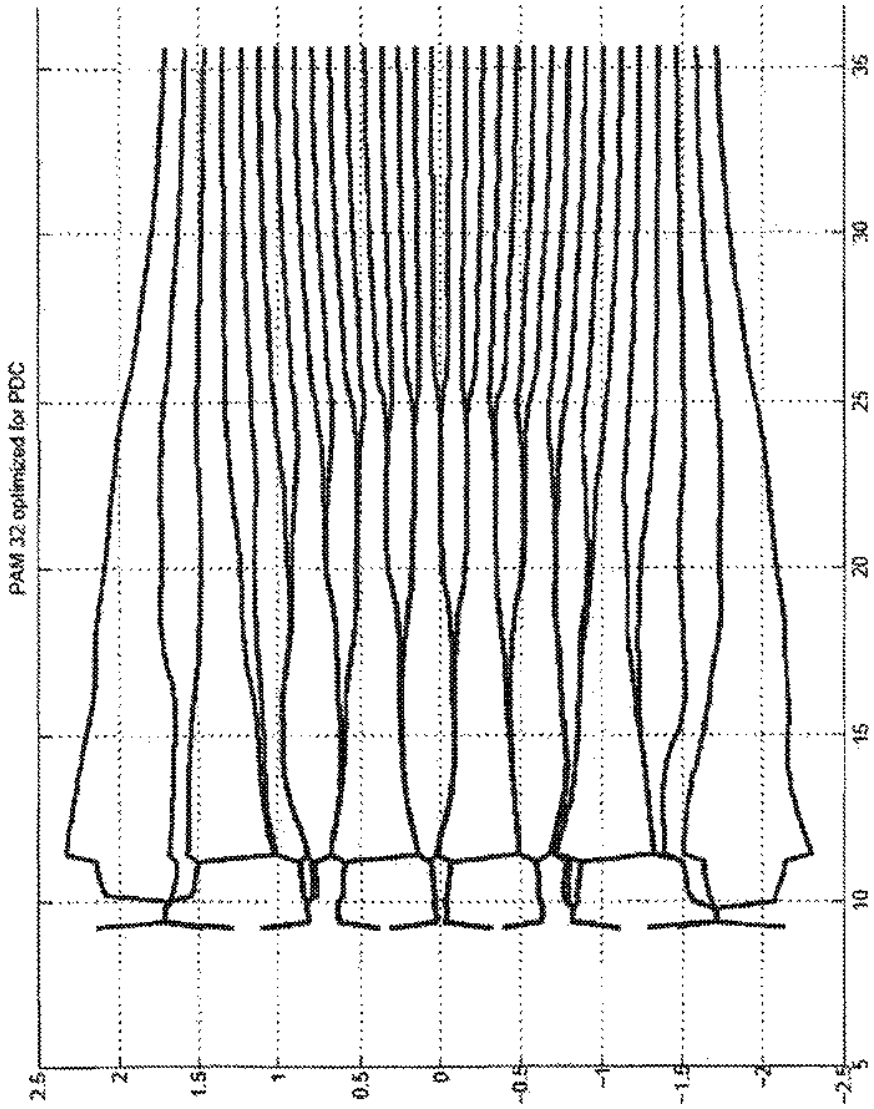

PAM 32 optimized for PDC
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25
30
35

FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.58	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.10	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.10	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.58	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.00	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.00	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.58	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.58	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellations optimized for parallel decoding capacity at differen

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	12.21	15.27	18.42	21.52	24.79	28.20
x_0	-2.25	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.19	-1.31	2.05	1.97	-1.35
x_3	-1.39	-1.48	-1.43	-1.49	-1.51	-1.49
x_4	1.56	1.54	2.14	-0.96	-1.03	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.67	1.65	-1.07	-0.91	-0.90	-1.21
x_7	-1.31	-1.24	-1.04	-1.28	-1.33	-1.08
x_8	-0.48	-0.43	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.43	-0.62	-0.17	-0.15	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.42	-0.29	-0.71	-0.67	-0.33
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.84
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.48	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.66	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.66	0.63	0.77	1.15	1.17	0.96
x_{20}	1.07	1.13	1.31	0.72	0.77	1.35
x_{21}	0.66	0.59	1.43	0.91	0.90	1.49
x_{22}	1.05	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.03	0.62
x_{24}	-0.01	-0.08	0.02	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.14
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.29	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.36	0.52	0.55	0.42
x_{31}	0.17	0.25	0.36	0.34	0.31	0.52

FIG. 17b

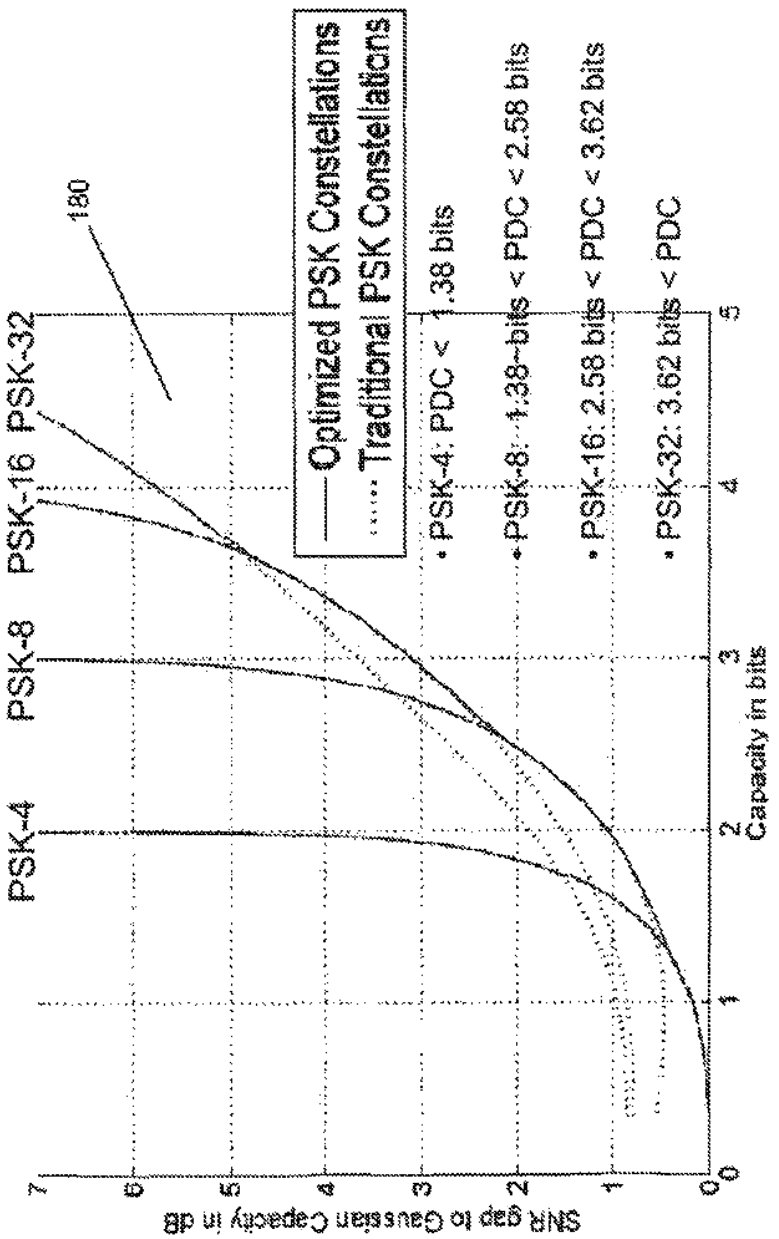

180
PSK-4
PSK-8
PSK-16
PSK-32
Optimized PSK Constellations
Traditional PSK Constellations
• PSK-4: PDC < 1.38 bits
• PSK-8: 1.38 bits < PDC < 2.58 bits
• PSK-16: 2.58 bits < PDC < 3.62 bits
• PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

Points location in degrees
0
50
100
150
200
250
300
350
SNR in dB
0
2
4
6
8
10
12
14
16
18
20

FIG. 19

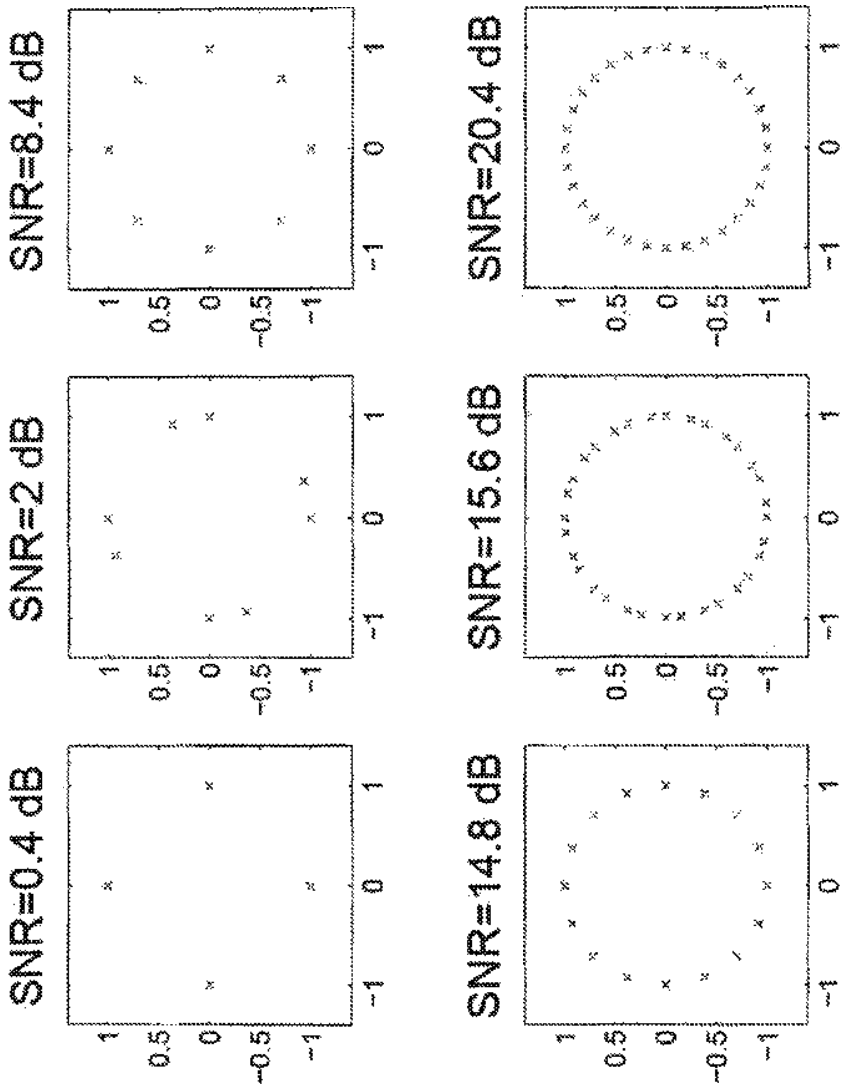


FIG. 20

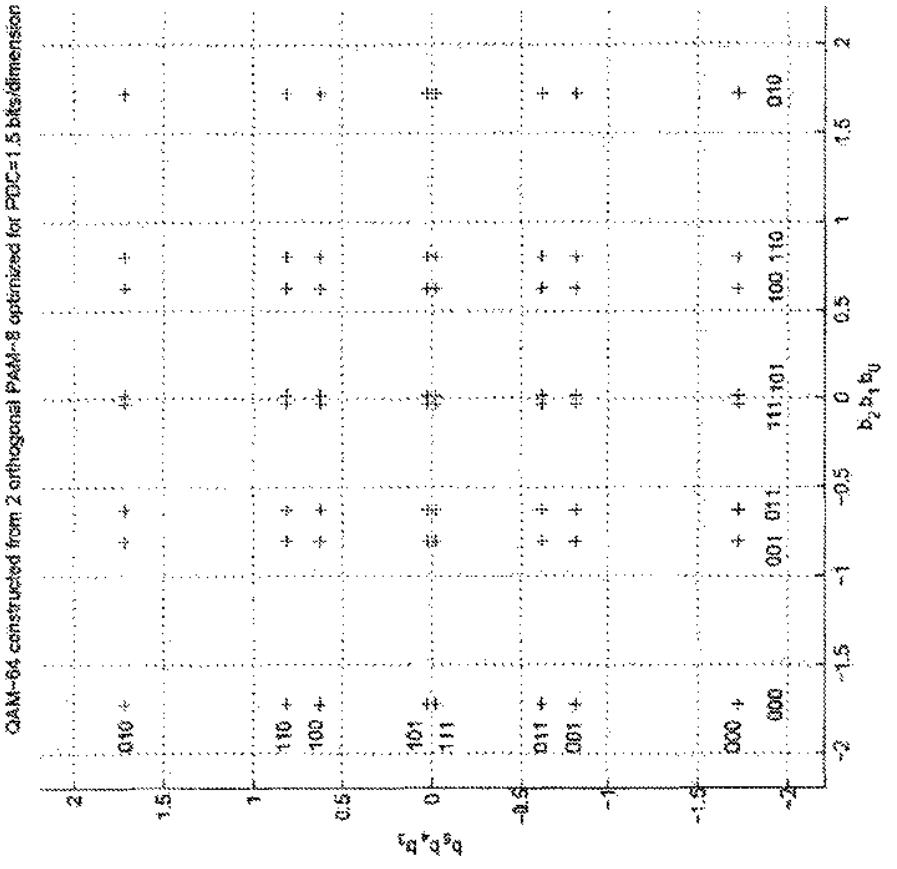

QAM-64 constructed from 2 orthogonal PAM-8 optimized for PDC=1.5 bits/dimension
$b_2 b_1 b_0$
$b_5 b_4 b_3$
000
001
011
010
110
100
101
111

FIG. 21

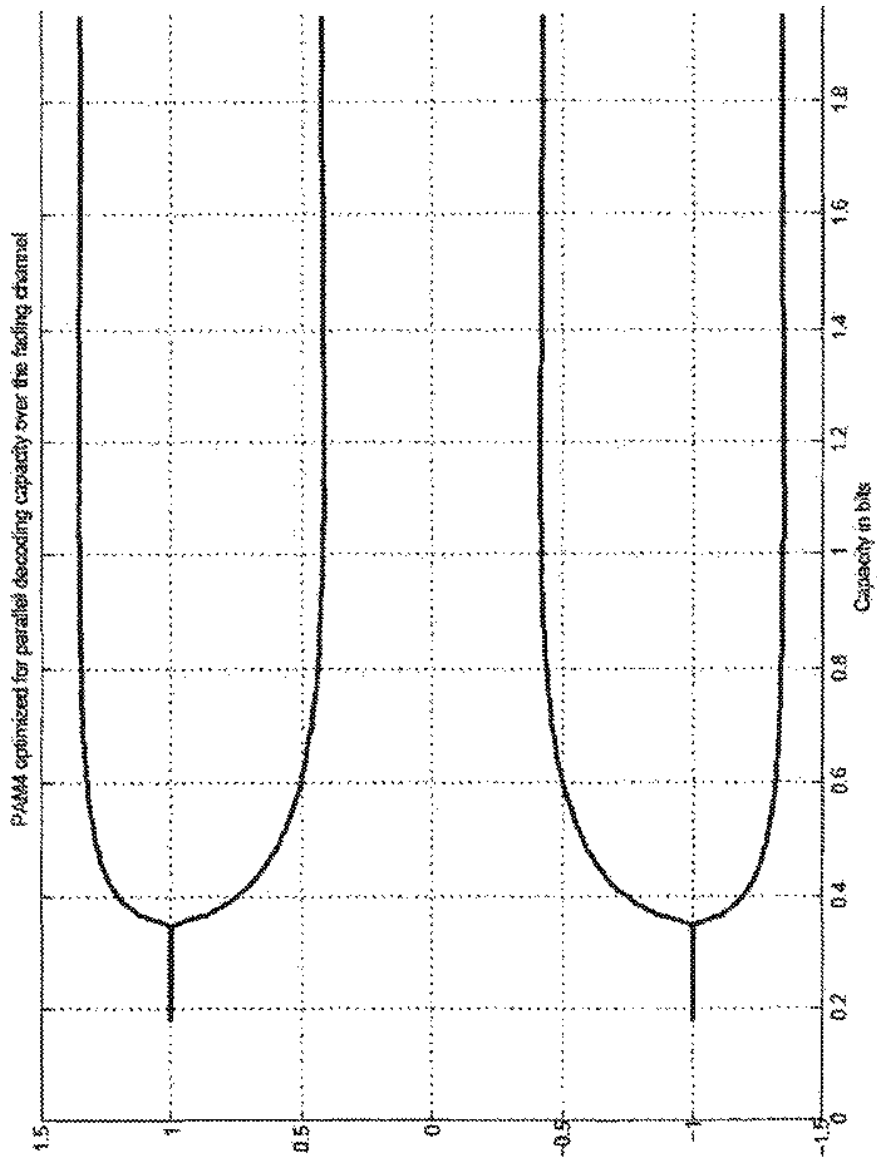

PAM4 optimized for parallel decoding capacity over the fading channel
Capacity in bits
1.5
1
0.5
0
-0.5
-1
-1.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 22a

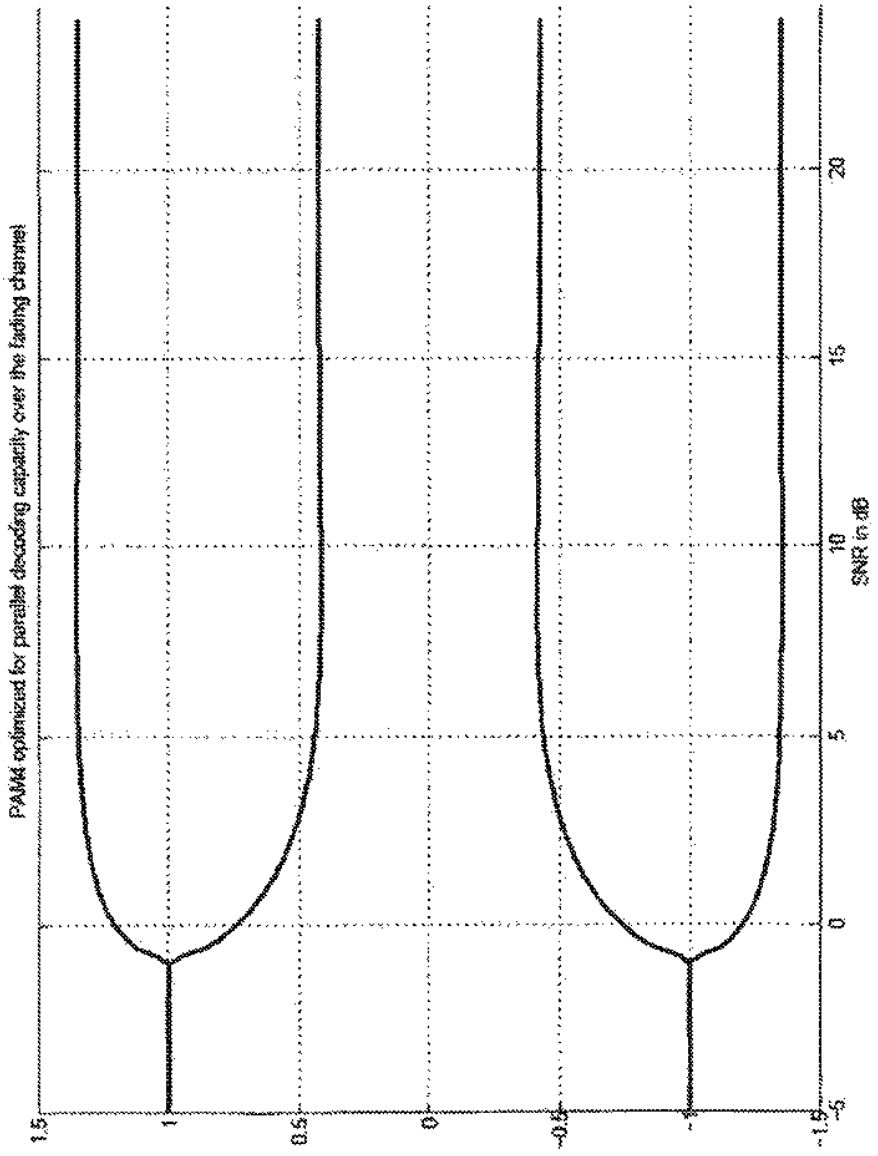

PAM4 optimized for parallel decoding capacity over the fading channel
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
-5
0
5
10
15
20

FIG. 22b

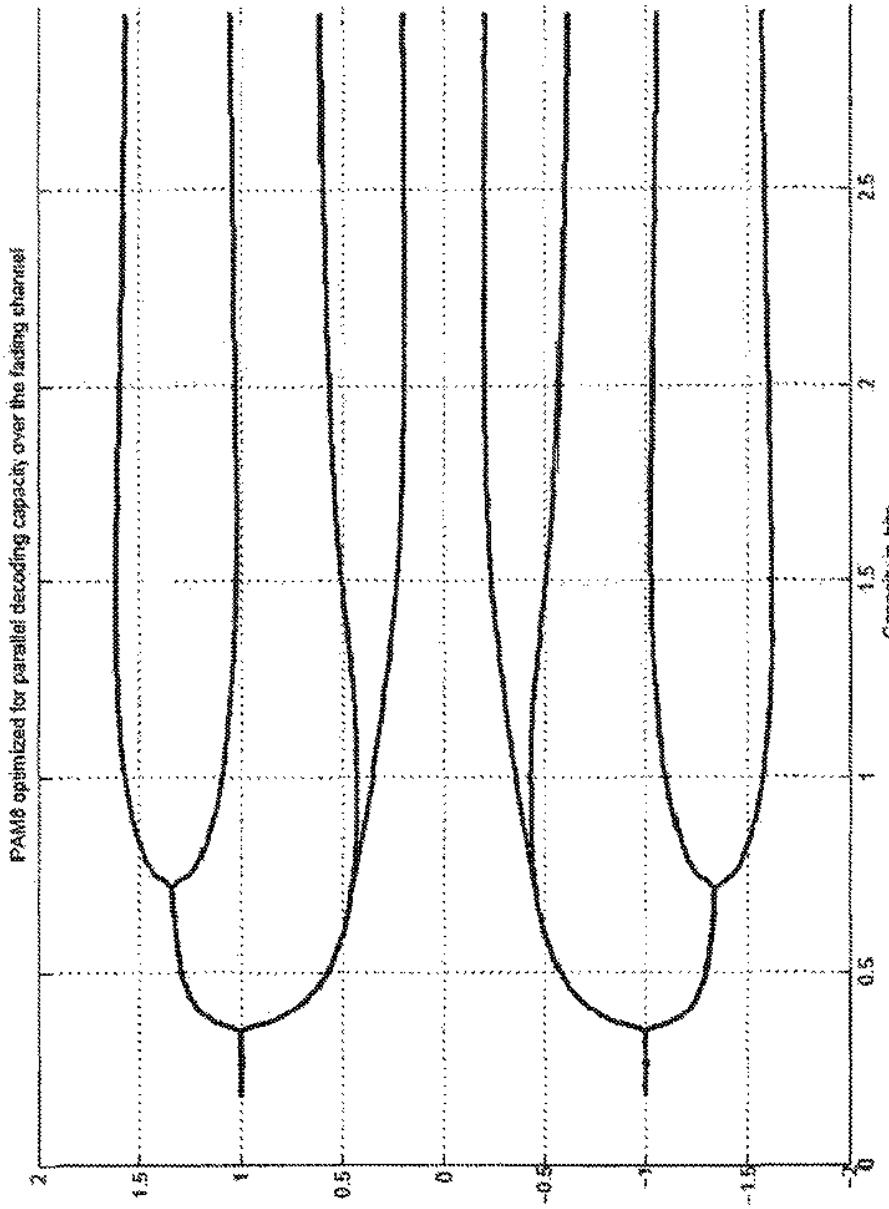

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5

FIG. 23a

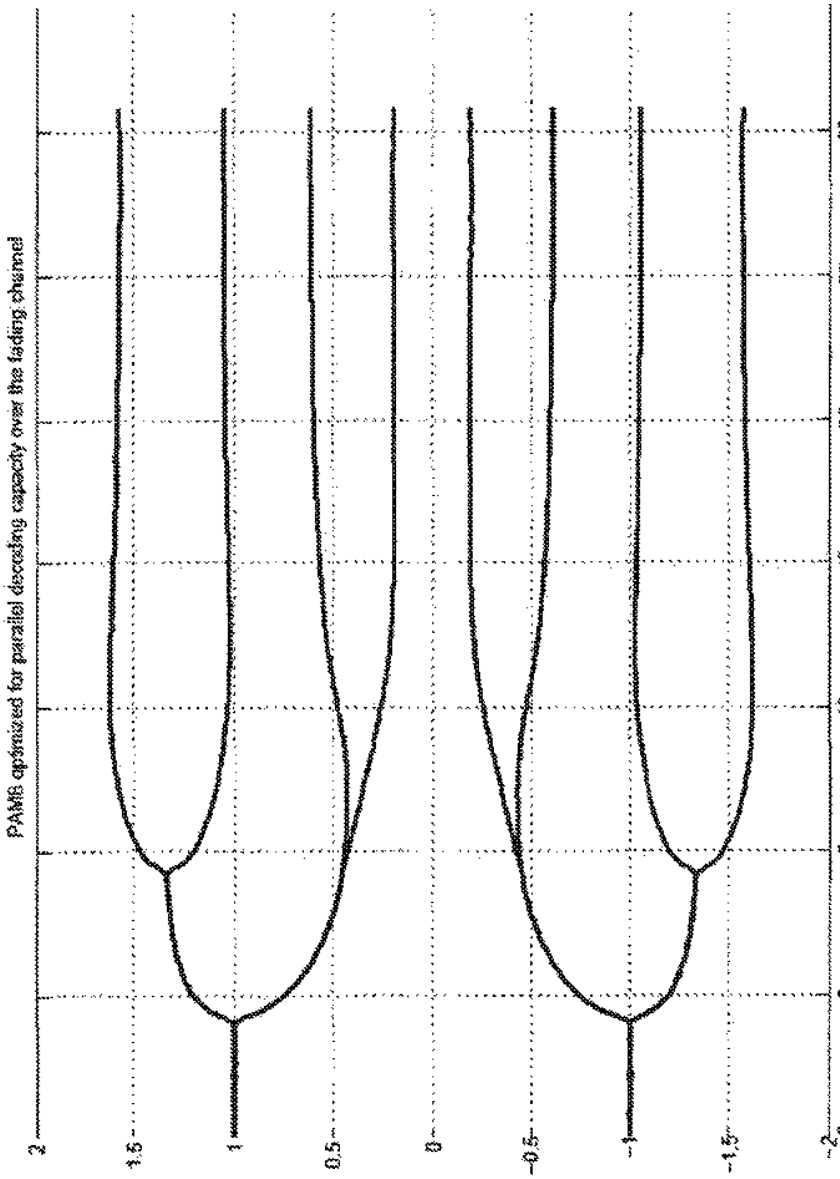

PAMS optimized for parallel decoding capacity over the fading channel
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-5
0
5
10
15
20
25
30
SNR in dB

FIG. 23b

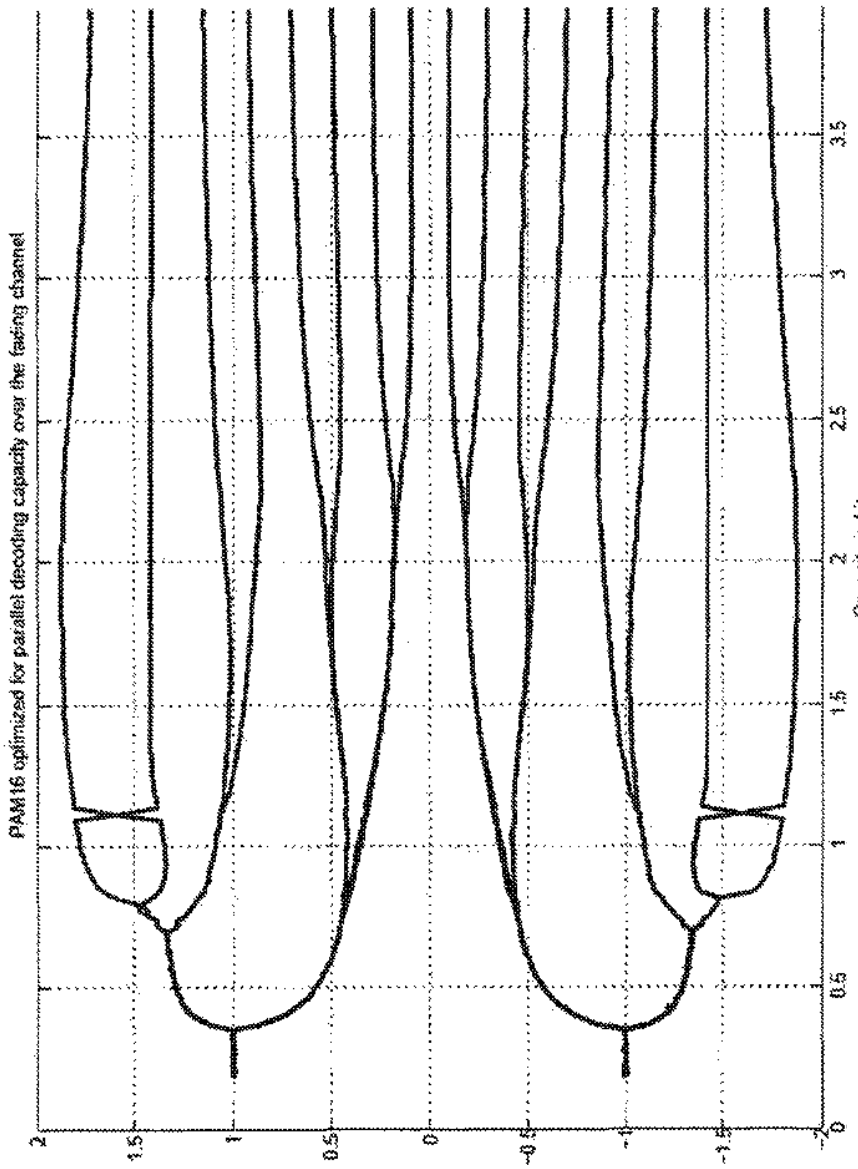

PAM16 optimized for parallel decoding capacity over the fading channel
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.5
1
1.5
2
2.5
3
3.5

FIG. 24a

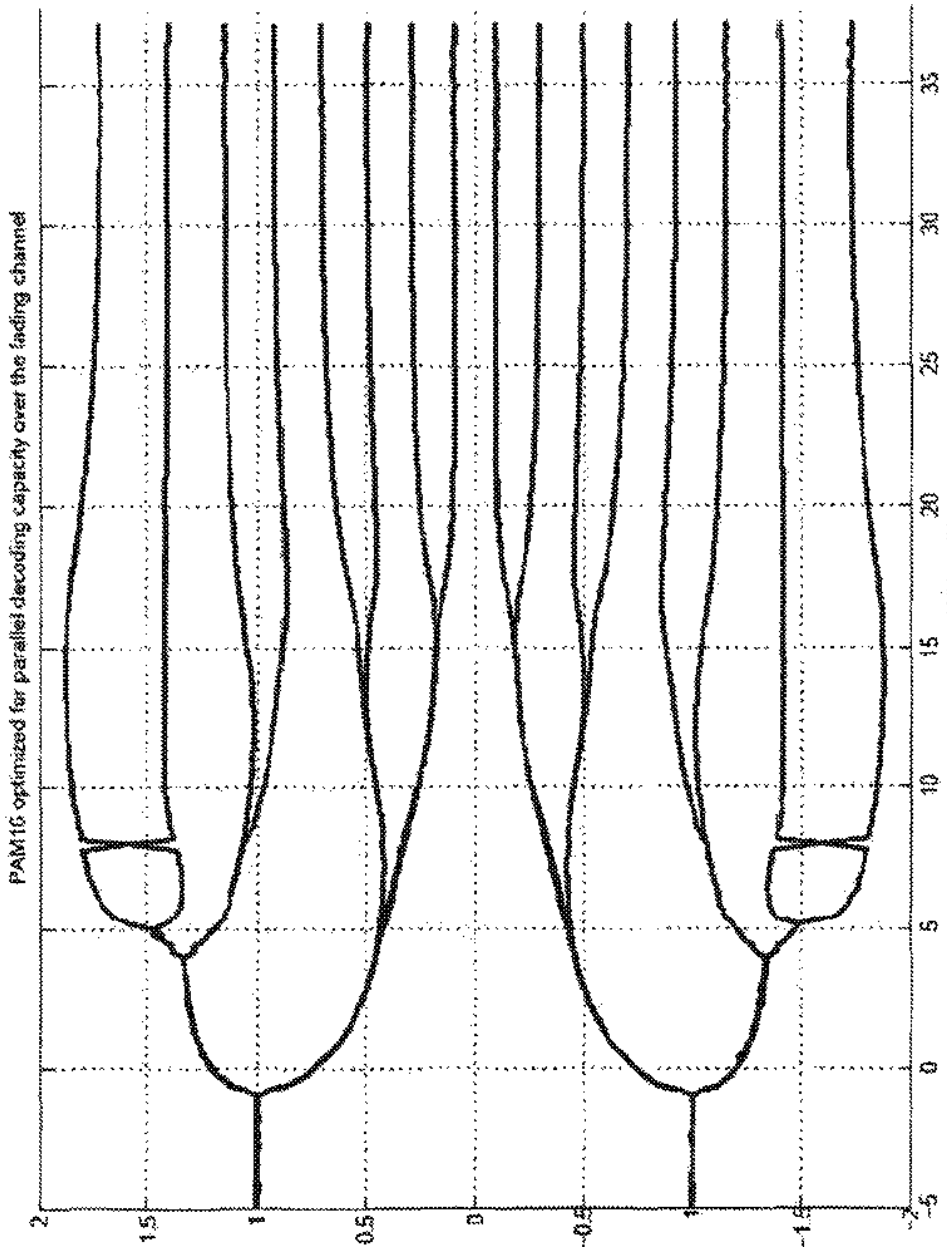

PAM16 optimized for parallel decoding capacity over the fading channel
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
35
30
25
20
15
10
5
0
-5

FIG. 24b

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US2008/055994

A. CLASSIFICATION OF SUBJECT MATTER
IPC(8) - H04J 11/00 (2008.04)
USPC - 370/207
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC(8) - H04J 11/00 (2008.04)
USPC - 370/207

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
PatBase, Google Patents

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X --- Y	US 2004/0022179 A1 (GIANNAKIS et al) 05 February 2004 (05.02.2004) entire document	9-11 --- 1-8, 12
Y	US 2007/0116161 A1 (TOKORO et al) 24 May 2007 (24.05.2007) entire document	1-8, 12
Y	US 6,603,801 B1 (ANDREN et al) 05 August 2003 (05.08.2003) entire document	4
Y	US 2005/0156643 A1 (GLASS et al) 13 July 2005 (13.07.2005) entire document	3

☐ Further documents are listed in the continuation of Box C. ☐

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"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
03 October 2008	22 OCT 2008
Name and mailing address of the ISA/US Mail Stop PCT, Attn: ISA/US, Commissioner for Patents P.O. Box 1450, Alexandria, Virginia 22313-1450 Facsimile No. 571-273-3201	Authorized officer: Blaine R. Copenheaver PCT Helpdesk: 571-272-4300 PCT OSP: 571-272-7774

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(71) Applicant *(for all designated States except US)*: **CONSTELLATION DESIGNS, INC.** [US/US]; 1563 Calle Patricia, Pacific Palisades, CA 90272 (US).

(72) Inventors; and

(75) Inventors/Applicants *(for US only)*: **BARSOUM, Maged, F.** [US/US]; 374 Durness Aisle, Irvine, CA 92606 (US). **JONES, Christopher, Richard** [US/US]; 1563 Calle Patricia, Pacific Palisades, CA 90272 (US).

(74) Agent: **BAILEY, David, L.**; Knobbe, Martens, Peck & Bailey LLP, 2855 Michelle Drive, Suite 100, Irvine, CA 92606 (US).

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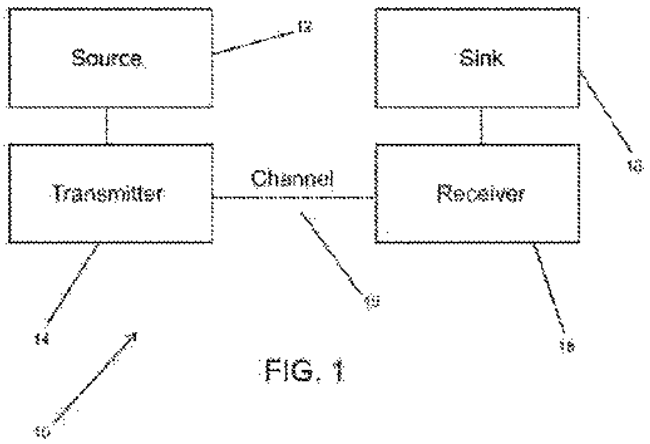


FIG. 1

(57) Abstract: Communication systems are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In many embodiments, the communication systems use specific geometric constellations that are capacity optimized at a specific SNR. In addition, ranges within which the constellation points of a capacity optimized constellation can be perturbed and are still likely to achieve a given percentage of the optimal capacity increase compared to a constellation that maximizes d_{min} are also described. Capacity measures that are used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

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Methods and Apparatuses for Signaling with Geometric Constellations

STATEMENT OF FEDERALLY SPONSORED RESEARCH

[0001] This invention was made with Government support under contract NAS7-03001 awarded by NASA. The Government has certain rights in this invention.

BACKGROUND

[0002] The present invention generally relates to bandwidth and/or power efficient digital transmission systems and more specifically to the use of unequally spaced constellations having increased capacity.

[0003] The term "constellation" is used to describe the possible symbols that can be transmitted by a typical digital communication system. A receiver attempts to detect the symbols that were transmitted by mapping a received signal to the constellation. The minimum distance (d_{min}) between constellation points is indicative of the capacity of a constellation at high signal-to-noise ratios (SNRs). Therefore, constellations used in many communication systems are designed to maximize d_{min}. Increasing the dimensionality of a constellation allows larger minimum distance for constant constellation energy per dimension. Therefore, a number of multi-dimensional constellations with good minimum distance properties have been designed.

[0004] Communication systems have a theoretical maximum capacity, which is known as the Shannon limit. Many communication systems attempt to use codes to increase the capacity of a communication channel. Significant coding gains have been achieved using coding techniques such as turbo codes and LDPC codes. The coding gains achievable using any coding technique are limited by the constellation of the communication system. The Shannon limit can be thought of as being based upon a theoretical constellation known as a Gaussian distribution, which is an infinite constellation where symbols at the center of the constellation are transmitted more frequently than symbols at the edge of the constellation. Practical constellations are finite and transmit symbols with equal likelihoods, and therefore have capacities that are less than the Gaussian capacity. The capacity of a constellation is thought to represent a limit on the gains that can be achieved using coding when using that constellation.

[0005] Prior attempts have been made to develop unequally spaced constellations. For example, a system has been proposed that uses unequally spaced constellations that are optimized to minimize the error rate of an uncoded system. Another proposed system uses a constellation with equiprobable but unequally spaced symbols in an attempt to mimic a Gaussian distribution.

[0006] Other approaches increase the dimensionality of a constellation or select a new symbol to be transmitted taking into consideration previously transmitted symbols. However, these constellation were still designed based on a minimum distance criteria.

SUMMARY OF THE INVENTION

[0007] Systems and methods are described for constructing a modulation such that the constrained capacity between a transmitter and a receiver approaches the Gaussian channel capacity limit first described by Shannon [ref Shannon 1948]. Traditional communications systems employ modulations that leave a significant gap to Shannon Gaussian capacity. The modulations of the present invention reduce, and in some cases, nearly eliminate this gap. The invention does not require specially designed coding mechanisms that tend to transmit some points of a modulation more frequently than others but rather provides a method for locating points (in a one or multiple dimensional space) in order to maximize capacity between the input and output of a bit or symbol mapper and demapper respectively. Practical application of the method allows systems to transmit data at a given rate for less power or to transmit data at a higher rate for the same amount of power.

[0008] One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a

PAM-8 symbol constellation having constellation points within at least one of the ranges specified in FIGS. 25 - 46.

[0009] In a further embodiment, the code is a Turbo code. In another embodiment, the code is a LDPC code.

[0010] In a still further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0011] In still another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 15% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0012] In a yet further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 30% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0013] In yet another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 45% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0014] A further embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0015] In another embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

[0016] In a further additional embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0017] In another additional embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 40% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0018] In a still yet further embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 50% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0019] In still yet another embodiment, the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0020] In a still further embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 70% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0021] In still another embodiment again, the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

[0022] A still further additional embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a PAM-16 symbol constellation having constellation points within at least one of the ranges specified in FIGS. 47 - 84.

[0023] Still another additional embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate

likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a PAM-32 symbol constellation having constellation points within at least one of the ranges specified in FIGS. 85 - 148.

[0024] Another further embodiment includes a transmitter configured to transmit signals to a receiver via a communication channel, where the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper, where the receiver, includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, and a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. In addition, the symbol constellation is a N-Dimensional symbol constellation, where the constellation points in at least one dimension are within at least one of the ranges specified in FIGS. 25 - 167.

BRIEF DESCRIPTION OF DRAWINGS

[0025] FIG. 1 is a conceptual illustration of a communication system in accordance with an embodiment of the invention.

[0026] FIG. 2 is a conceptual illustration of a transmitter in accordance with an embodiment of the invention.

[0027] FIG. 3 is a conceptual illustration of a receiver in accordance with an embodiment of the invention.

[0028] FIG. 4a is a conceptual illustration of the joint capacity of a channel.

[0029] FIG. 4b is a conceptual illustration of the parallel decoding capacity of a channel.

[0030] FIG. 5 is a flow chart showing a process for obtaining a constellation optimized for capacity for use in a communication system having a fixed code rate and modulation scheme in accordance with an embodiment of the invention.

[0031] FIG. 6a is a chart showing a comparison of Gaussian capacity and PD capacity for traditional PAM-2,4,8,16,32.

[0032] FIG. 6b is a chart showing a comparison between Gaussian capacity and joint capacity for traditional PAM-2,4,8,16,32.

[0033] FIG. 7 is a chart showing the SNR gap to Gaussian capacity for the PD capacity and joint capacity of traditional PAM-2,4,8,16,32 constellations.

[0034] FIG. 8a is a chart comparing the SNR gap to Gaussian capacity of the PD capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0035] FIG. 8b is a chart comparing the SNR gap to Gaussian capacity of the joint capacity for traditional and optimized PAM-2,4,8,16,32 constellations.

[0036] FIG. 9 is a chart showing Frame Error Rate performance of traditional and PD capacity optimized PAM-32 constellations in simulations involving several different length LDPC codes.

[0037] FIGS. 10a – 10d are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0038] FIGS. 11a and 11b are design tables of PD capacity and joint capacity optimized PAM-4 constellations in accordance with embodiments of the invention.

[0039] FIGS. 12a – 12d are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0040] FIGS. 13a and 13b are design tables of PD capacity and joint capacity optimized PAM-8 constellations in accordance with embodiments of the invention.

[0041] FIGS. 14a – 14d are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0042] FIGS. 15a and 15b are design tables of PD capacity and joint capacity optimized PAM-16 constellations in accordance with embodiments of the invention.

[0043] FIGS. 16a – 16d are locus plots showing the location of constellation points of a PAM-32 constellation optimized for PD capacity and joint capacity versus user bit rate per dimension and versus SNR.

[0044] FIGS. 17a and 17b are design tables of PD capacity and joint capacity optimized PAM-32 constellations in accordance with embodiments of the invention.

[0045] FIG. 18 is a chart showing the SNR gap to Gaussian capacity for traditional and capacity optimized PSK constellations.

[0046] FIG. 19 is a chart showing the location of constellation points of PD capacity optimized PSK-32 constellations.

[0047] FIG. 20 is a series of PSK-32 constellations optimized for PD capacity at different SNRs in accordance with embodiments of the invention.

[0048] FIG. 21 illustrates a QAM-64 constructed from orthogonal Cartesian product of two PD optimized PAM-8 constellations in accordance with an embodiment of the invention.

[0049] FIGS. 22a and 22b are locus plots showing the location of constellation points of a PAM-4 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0050] FIGS. 23a and 23b are locus plots showing the location of constellation points of a PAM-8 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0051] FIGS. 24a and 24b are locus plots showing the location of constellation points of a PAM-16 constellation optimized for PD capacity over a fading channel versus user bit rate per dimension and versus SNR.

[0052] FIGS. 25 – 28 are tables showing the performance of geometric PAM-8 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

[0053] FIGS. 29 – 32 are tables listing the constellation points corresponding to the geometric PAM-8 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 25 - 28.

[0054] FIGS. 33 – 36 are tables showing maximum ranges for the geometric PAM-8 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 25 – 28.

[0055] FIGS. 37 – 40 are tables showing the performance of geometric PAM-8 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

[0056] FIGS. 41 – 44 are tables listing the constellation points corresponding to the geometric PAM-8 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. 37 - 40.

[0057] FIGS. 45 – 46 are tables showing maximum ranges for the geometric PAM-8 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. 37 - 40.

[0058] FIGS. 47 – 51 are tables showing the performance of geometric PAM-16 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

[0059] FIGS. 52 – 61 are tables listing the constellation points corresponding to the geometric PAM-16 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 47 - 51.

[0060] FIGS. 62 – 66 are tables showing maximum ranges for the geometric PAM-16 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 47 - 51.

[0061] FIGS. 67 – 71 are tables showing the performance of geometric PAM-16 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

[0062] FIGS. 72 – 81 are tables listing the constellation points corresponding to the geometric PAM-16 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. 67 - 71.

[0063] FIGS. 82 – 84 are tables showing maximum ranges for the geometric PAM-16 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. 67 - 71.

[0064] FIGS. 85 – 90 are tables showing the performance of geometric PAM-32 constellations optimized for Joint Capacity at specific SNRs in accordance with embodiments of the invention.

[0065] FIGS. 91 – 114 are tables listing the constellation points corresponding to the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 85 - 90.

[0066] FIGS. 115 – 120 are tables showing maximum ranges for the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 85 - 90.

[0067] FIGS. 121 – 125 are tables showing the performance of geometric PAM-32 constellations optimized for PD Capacity at specific SNRs in accordance with embodiments of the invention.

[0068] FIGS. 126 – 145 are tables listing the constellation points corresponding to the geometric PAM-32 constellation designs optimized for PD Capacity at specific SNRs listed in FIGS. 121 - 125.

[0069] FIGS. 146 – 148 are tables showing maximum ranges for the geometric PAM-32 constellation designs optimized for Joint Capacity at specific SNRs listed in FIGS. 121 – 125.

DETAILED DESCRIPTION OF THE INVENTION

[0070] Turning now to the detailed description of the invention, communication systems in accordance with embodiments of the invention are described that use signal constellations, which have unequally spaced (i.e. 'geometrically' shaped) points. In many embodiments, the communication systems use specific geometric constellations that are capacity optimized at a specific SNR. In addition, ranges within which the constellation points of a capacity optimized constellation can be perturbed and are still likely to achieve a given percentage of the optimal capacity increase compared to a constellation that maximizes d_{min}, are also described. Capacity measures that are used in the selection of the location of constellation points include, but are not limited to, parallel decode (PD) capacity and joint capacity.

[0071] In many embodiments, the communication systems utilize capacity approaching codes including, but not limited to, LDPC and Turbo codes. As is discussed further below, direct optimization of the constellation points of a communication system utilizing a capacity approaching channel code, can yield different constellations depending on the SNR for which they are optimized. Therefore, the same constellation is unlikely to achieve the same coding gains applied across all code rates; that is, the same constellation will not enable the best possible performance across all rates. In many instances, a constellation at one code rate can achieve gains that cannot be achieved at another code rate. Processes for selecting capacity optimized constellations to achieve increased coding gains based upon a specific coding rate in accordance with embodiments of the invention are described below. Constellations points for geometric PAM-8, PAM-16, and PAM-32 constellations that are optimized for joint capacity or PD

capacity at specific SNRs are also provided. Additional geometric PAM-8, PAM-16, and PAM-32 constellations that are probabilistically likely to provide performance gains compared to constellations that maximize d_{min}, which were identified by perturbing the constellation points of geometric PAM-8, PAM-16, and PAM-32 constellations optimized for joint capacity or PD capacity, are also described. The constellations are described as being probabilistically likely to provide performance gains because all possible constellations within the ranges have not been exhaustively searched. Within each disclosed range, a large number of constellations were selected at random, and it was verified that all the selected constellations provided a gain that exceeds the given percentage of the optimal capacity increase achieved by the optimized constellations relative to a constellation that maximizes d_{min} (i.e. a PAM equally spaced constellation). In this way, ranges that are probabilistically likely to provide a performance gain that is at least a predetermined percentage of the optimal increase in capacity can be identified and a specific geometric constellation can be compared against the ranges as a guide to the increase in capacity that is likely to be achieved. In a number of embodiments, the communication systems can adapt the location of points in a constellation in response to channel conditions, changes in code rate and/or to change the target user data rate.

COMMUNICATION SYSTEMS

[0072] A communication system in accordance with an embodiment of the invention is shown in FIG. 1. The communication system 10 includes a source 12 that provides user bits to a transmitter 14. The transmitter transmits symbols over a channel to a receiver 16 using a predetermined modulation scheme. The receiver uses knowledge of the modulation scheme, to decode the signal received from the transmitter. The decoded bits are provided to a sink device that is connected to the receiver.

[0073] A transmitter in accordance with an embodiment of the invention is shown in FIG. 2. The transmitter 14 includes a coder 20 that receives user bits from a source and encodes the bits in accordance with a predetermined coding scheme. In a number of embodiments, a capacity approaching code such as a turbo code or a LDPC code is used. In other embodiments, other coding schemes can be used to providing a coding gain within the communication system. A mapper 22 is connected to the coder. The mapper maps the bits output by the coder to a symbol

within a geometrically distributed signal constellation stored within the mapper. The mapper provides the symbols to a modulator 24, which modulates the symbols for transmission via the channel.

[0074] A receiver in accordance with an embodiment of the invention is illustrated in FIG. 3. The receiver 16 includes a demodulator 30 that demodulates a signal received via the channel to obtain symbol or bit likelihoods. The demapper uses knowledge of the geometrically shaped symbol constellation used by the transmitter to determine these likelihoods. The demapper 32 provides the likelihoods to a decoder 34 that decodes the encoded bit stream to provide a sequence of received bits to a sink.

GEOMETRICALLY SHAPED CONSTELLATIONS

[0075] Transmitters and receivers in accordance with embodiments of the invention utilize geometrically shaped symbol constellations. In several embodiments, a geometrically shaped symbol constellation is used that optimizes the capacity of the constellation. In many embodiments, geometrically shaped symbol constellations, which include constellation points within predetermined ranges of the constellation points of a capacity optimized constellation, and that provide improved capacity compared to constellations that maximize d_{min} are used. Various geometrically shaped symbol constellations that can be used in accordance with embodiments of the invention, techniques for deriving geometrically shaped symbol constellations are described below.

SELECTION OF A GEOMETRICALLY SHAPED CONSTELLATIONS

[0076] Selection of a geometrically shaped constellation for use in a communication system in accordance with an embodiment of the invention can depend upon a variety of factors including whether the code rate is fixed. In many embodiments, a geometrically shaped constellation is used to replace a conventional constellation (i.e. a constellation maximized for d_{min}) in the mapper of transmitters and the demapper of receivers within a communication system. Upgrading a communication system involves selection of a constellation and in many instances the upgrade can be achieved via a simple firmware upgrade. In other embodiments, a geometrically shaped constellation is selected in conjunction with a code rate to meet specific

performance requirements, which can for example include such factors as a specified bit rate, a maximum transmit power. Processes for selecting a geometric constellation when upgrading existing communication systems and when designing new communication systems are discussed further below.

UPGRADING EXISTING COMMUNICATION SYSTEMS

[0077] A geometrically shaped constellation that provides a capacity, which is greater than the capacity of a constellation maximized for d_{min}, can be used in place of a conventional constellation in a communication system in accordance with embodiments of the invention. In many instances, the substitution of the geometrically shaped constellation can be achieved by a firmware or software upgrade of the transmitters and receivers within the communication system. Not all geometrically shaped constellations have greater capacity than that of a constellation maximized for d_{min}. One approach to selecting a geometrically shaped constellation having a greater capacity than that of a constellation maximized for d_{min} is to optimize the shape of the constellation with respect to a measure of the capacity of the constellation for a given SNR. Another approach is to select a constellation from a range that is probabilistically likely to yield a constellation having at least a predetermined percentage of the optimal capacity increase. Such an approach can prove useful in circumstances, for example, where an optimized constellation is unable to be implemented. Capacity measures that can be used in the optimization process can include, but are not limited to, joint capacity or parallel decoding capacity.

JOINT CAPACITY AND PARALLEL DECODING CAPACITY

[0078] A constellation can be parameterized by the total number of constellation points, M, and the number of real dimensions, N_{dim}. In systems where there are no belief propagation iterations between the decoder and the constellation demapper, the constellation demapper can be thought of as part of the channel. A diagram conceptually illustrating the portions of a communication system that can be considered part of the channel for the purpose of determining PD capacity is shown in FIG. 4a. The portions of the communication system that are considered

part of the channel are indicated by the ghost line 40. The capacity of the channel defined as such is the parallel decoding (PD) capacity, given by:

$$C_{PD} = \sum_{i=0}^{L-1} I(X_i; Y)$$

where X_i is the ith bit of the L-bits transmitted symbol, and Y is the received symbol, and $I(A;B)$ denotes the mutual information between random variables A and B.

[0079] Expressed another way, the PD capacity of a channel can be viewed in terms of the mutual information between the output bits of the encoder (such as an LDPC encoder) at the transmitter and the likelihoods computed by the demapper at the receiver. The PD capacity is influenced by both the placement of points within the constellation and by the labeling assignments.

[0080] With belief propagation iterations between the demapper and the decoder, the demapper can no longer be viewed as part of the channel, and the joint capacity of the constellation becomes the tightest known bound on the system performance. A diagram conceptually illustrating the portions of a communication system that are considered part of the channel for the purpose of determining the joint capacity of a constellation is shown in FIG. 4b. The portions of the communication system that are considered part of the channel are indicated by the ghost line 42. The joint capacity of the channel is given by:

$$C_{JOINT} = I(X;Y)$$

[0081] Joint capacity is a description of the achievable capacity between the input of the mapper on the transmit side of the link and the output of the channel (including for example AWGN and Fading channels). Practical systems must often 'demap' channel observations prior to decoding. In general, the step causes some loss of capacity. In fact it can be proven that $C_{\infty} \geq C_{JOINT} \geq C_{PD}$. That is, C_{JOINT} upper bounds the capacity achievable by C_{PD}. The methods of the present invention are motivated by considering the fact that practical limits to a given communication system capacity are limited by C_{JOINT} and C_{PD}. In several embodiments of the invention, geometrically shaped constellations are selected that maximize these measures.

SELECTING A CONSTELLATION HAVING AN OPTIMAL CAPACITY

[0082] Geometrically shaped constellations in accordance with embodiments of the invention can be designed to optimize capacity measures including, but not limited to PD capacity or joint capacity. A process for selecting the points, and potentially the labeling, of a geometrically shaped constellation for use in a communication system having a fixed code rate in accordance with an embodiment of the invention is shown in FIG. 5. The process 50 commences with the selection (52) of an appropriate constellation size M and a desired capacity per dimension η. In the illustrated embodiment, the process involves a check (52) to ensure that the constellation size can support the desired capacity. In the event that the constellation size could support the desired capacity, then the process iteratively optimizes the M-ary constellation for the specified capacity. Optimizing a constellation for a specified capacity often involves an iterative process, because the optimal constellation depends upon the SNR at which the communication system operates. The SNR for the optimal constellation to give a required capacity is not known *a priori*. Throughout the description of the present invention SNR is defined as the ratio of the average constellation energy per dimension to the average noise energy per dimension. In most cases the capacity can be set to equal the target user bit rate per symbol per dimension. In some cases adding some implementation margin on top of the target user bit rate could result in a practical system that can provide the required user rate at a lower rate. The margin is code dependent. The following procedure could be used to determine the target capacity that includes some margin on top of the user rate. First, the code (e.g. LDPC or Turbo) can be simulated in conjunction with a conventional equally spaced constellation. Second, from the simulation results the actual SNR of operation at the required error rate can be found. Third, the capacity of the conventional constellation at that SNR can be computed. Finally, a geometrically shaped constellation can be optimized for that capacity.

[0083] In the illustrated embodiment, the iterative optimization loop involves selecting an initial estimate of the SNR at which the system is likely to operate (i.e. SNR_{in}). In several embodiments the initial estimate is the SNR required using a conventional constellation. In other embodiments, other techniques can be used for selecting the initial SNR. An M-ary constellation is then obtained by optimizing (56) the constellation to maximize a selected capacity measure at

the initial SNR_{in} estimate. Various techniques for obtaining an optimized constellation for a given SNR estimate are discussed below.

[0084] The SNR at which the optimized *M*-ary constellation provides the desired capacity per dimension η (SNR_{out}) is determined (57). A determination (58) is made as to whether the SNR_{out} and SNR_{in} have converged. In the illustrated embodiment convergence is indicated by SNR_{out} equaling SNR_{in}. In a number of embodiments, convergence can be determined based upon the difference between SNR_{out} and SNR_{in} being less than a predetermined threshold. When SNR_{out} and SNR_{in} have not converged, the process performs another iteration selecting SNR_{out} as the new SNR_{in} (55). When SNR_{out} and SNR_{in} have converged, the capacity measure of the constellation has been optimized. As is explained in more detail below, capacity optimized constellations at low SNRs are geometrically shaped constellations that can achieve significantly higher performance gains (measured as reduction in minimum required SNR) than constellations that maximize d_{min}.

[0085] The process illustrated in FIG. 5 can maximize PD capacity or joint capacity of an *M*-ary constellation for a given SNR. Although the process illustrated in FIG. 5 shows selecting an *M*-ary constellation optimized for capacity, a similar process could be used that terminates upon generation of an *M*-ary constellation where the SNR gap to Gaussian capacity at a given capacity is a predetermined margin lower than the SNR gap of a conventional constellation, for example 0.5 db. Alternatively, other processes that identify *M*-ary constellations having capacity greater than the capacity of a conventional constellation can be used in accordance with embodiments of the invention. For example, the effect of perturbations on the constellation points of optimized constellations can be used to identify ranges in which predetermined performance improvements are probabilistically likely to be obtained. The ranges can then be used to select geometrically shaped constellations for use in a communication system. A geometrically shaped constellation in accordance with embodiments of the invention can achieve greater capacity than the capacity of a constellation that maximizes d_{min} without having the optimal capacity for the SNR range within which the communication system operates.

[0086] We note that constellations designed to maximize joint capacity may also be particularly well suited to codes with symbols over $GF(q)$, or with multi-stage decoding.

Conversely constellations optimized for PD capacity could be better suited to the more common case of codes with symbols over GF(2)

OPTIMIZING THE CAPACITY OF AN *M*-ARY CONSTELLATION AT A GIVEN SNR

[0087] Processes for obtaining a capacity optimized constellation often involve determining the optimum location for the points of an *M*-ary constellation at a given SNR. An optimization process, such as the optimization process 56 shown in FIG. 5, typically involves unconstrained or constrained non-linear optimization. Possible objective functions to be maximized are the joint or PD capacity functions. These functions may be targeted to channels including but not limited to Additive White Gaussian Noise (AWGN) or Rayleigh fading channels. The optimization process gives the location of each constellation point identified by its symbol labeling. In the case where the objective is joint capacity, point bit labelings are irrelevant meaning that changing the bit labelings doesn't change the joint capacity as long as the set of point locations remains unchanged..

[0088] The optimization process typically finds the constellation that gives the largest PD capacity or joint capacity at a given SNR. The optimization process itself often involves an iterative numerical process that among other things considers several constellations and selects the constellation that gives the highest capacity at a given SNR. In other embodiments, the constellation that requires the least SNR to give a required PD capacity or joint capacity can also be found. This requires running the optimization process iteratively as shown in FIG. 5.

[0089] Optimization constraints on the constellation point locations may include, but are not limited to, lower and upper bounds on point location, peak to average power of the resulting constellation, and zero mean in the resulting constellation. It can be easily shown that a globally optimal constellation will have zero mean (no DC component). Explicit inclusion of a zero mean constraint helps the optimization routine to converge more rapidly. Except for cases where exhaustive search of all combinations of point locations and labelings is possible it will not necessarily always be the case that solutions are provably globally optimal. In cases where exhaustive search is possible, the solution provided by the non-linear optimizer is in fact globally optimal.

[0090] The processes described above provide examples of the manner in which a geometrically shaped constellation having an increased capacity relative to a conventional capacity can be obtained for use in a communication system having a fixed code rate and modulation scheme. The actual gains achievable using constellations that are optimized for capacity compared to conventional constellations that maximize d_{min} are considered below.

GAINS ACHIEVED BY OPTIMIZED GEOMETRICALLY SPACED CONSTELLATIONS

[0091] The ultimate theoretical capacity achievable by any communication method is thought to be the Gaussian capacity, C_G which is defined as:

$$C_G = \frac{1}{2}log_2(1 + SNR)$$

[0092] Where signal-to-noise (SNR) is the ratio of expected signal power to expected noise power. The gap that remains between the capacity of a constellation and C_G can be considered a measure of the quality of a given constellation design.

[0093] The gap in capacity between a conventional modulation scheme in combination with a theoretically optimal coder can be observed with reference to FIGS. 6a and 6b. FIG. 6a includes a chart 60 showing a comparison between Gaussian capacity and the PD capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps 62 exist between the plot of Gaussian capacity and the PD capacity of the various PAM constellations. FIG. 6b includes a chart 64 showing a comparison between Gaussian capacity and the joint capacity of conventional PAM-2, 4, 8, 16, and 32 constellations that maximize d_{min}. Gaps 66 exist between the plot of Gaussian capacity and the joint capacity of the various PAM constellations. These gaps in capacity represent the extent to which conventional PAM constellations fall short of obtaining the ultimate theoretical capacity i.e. the Gaussian capacity.

[0094] In order to gain a better view of the differences between the curves shown in FIGS. 6a and 6b at points close to the Gaussian capacity, the SNR gap to Gaussian capacity for different values of capacity for each constellation are plotted in FIG. 7. It is interesting to note from the chart 70 in FIG. 7 that (unlike the joint capacity) at the same SNR, the PD capacity does not necessarily increase with the number of constellation points. As is discussed further below, this is not the case with PAM constellations optimized for PD capacity.

[0095] FIGS. 8a and 8b summarize performance of constellations for PAM-4, 8, 16, and 32 optimized for PD capacity and joint capacity (it should be noted that BPSK is the optimal PAM-2 constellation at all code rates). The constellations are optimized for PD capacity and joint capacity for different target user bits per dimension (i.e. code rates). The optimized constellations are different depending on the target user bits per dimension, and also depending on whether they have been designed to maximize the PD capacity or the joint capacity. All the PD optimized PAM constellations are labeled using a gray labeling which is not always the binary reflective gray labeling. It should be noted that not all gray labels achieve the maximum possible PD capacity even given the freedom to place the constellation points anywhere on the real line. FIG. 8a shows the SNR gap for each constellation optimized for PD capacity. FIG. 8b shows the SNR gap to Gaussian capacity for each constellation optimized for joint capacity. Again, it should be emphasized that each '+' on the plot represents a different constellation.

[0096] Referring to FIG. 8a, the coding gain achieved using a constellation optimized for PD capacity can be appreciated by comparing the SNR gap at a user bit rate per dimension of 2.5 bits for PAM-32. A user bit rate per dimension of 2.5 bits for a system transmitting 5 bits per symbol constitutes a code rate of 1/2. At that code rate the constellation optimized for PD capacity provides an additional coding gain of approximately 1.5 dB when compared to the conventional PAM-32 constellation.

[0097] The SNR gains that can be achieved using constellations that are optimized for PD capacity can be verified through simulation. The results of a simulation conducted using a rate 1/2 LDPC code in conjunction with a conventional PAM-32 constellation and in conjunction with a PAM-32 constellation optimized for PD capacity are illustrated in FIG. 9. A chart 90 includes plots of Frame Error Rate performance of the different constellations with respect to SNR and using different length codes (i.e. k=4,096 and k=16,384). Irrespective of the code that is used, the constellation optimized for PD capacity achieves a gain of approximately 1.3 dB, which closely approaches the gain predicted from FIG. 8a.

CAPACITY OPTIMIZED PAM CONSTELLATIONS

[0098] Using the processes outlined above, locus plots of PAM constellations optimized for capacity can be generated that show the location of points within PAM constellations versus

SNR. Locus plots of PAM-4, 8, 16, and 32 constellations optimized for PD capacity and joint capacity and corresponding design tables at various typical user bit rates per dimension are illustrated in FIGS. 10a – 17b. The locus plots and design tables show PAM-4, 8, 16, and 32 constellation point locations and labelings from low to high SNR corresponding to a range of low to high spectral efficiency.

[0099] In FIG.10a, a locus plot 100 shows the location of the points of PAM-4 constellations optimized for joint capacity plotted against achieved capacity. A similar locus plot 105 showing the location of the points of joint capacity optimized PAM-4 constellations plotted against SNR is included in FIG. 10b. In FIG. 10c, the location of points for PAM-4 optimized for PD capacity is plotted against achievable capacity and in FIG. 10d the location of points for PAM-4 for PD capacity is plotted against SNR. At low SNRs, the PD capacity optimized PAM-4 constellations have only 2 unique points, while the joint capacity optimized constellations have 3. As SNR is increased, each optimization eventually provides 4 unique points. This phenomenon is explicitly described in FIG. 11a and FIG. 11b where vertical slices of FIGS. 10ab and 10cd are captured in tables describing some PAM-4 constellations designs of interest. The SNR slices selected represent designs that achieve capacities = {0.5, 0.75, 1.0, 1.25, 1.5} bits per symbol (bps). Given that PAM-4 can provide at most $\log_2(4) = 2$ bps, these design points represent systems with information code rates R = {1/4, 3/8, 1/2, 5/8, 3/4} respectively.

[00100] FIGS. 12ab and 12cd present locus plots of PD capacity and joint capacity optimized PAM-8 constellation points versus achievable capacity and SNR. FIGS. 13a and 13b provide slices from these plots at SNRs corresponding to achievable capacities η = {0.5, 1.0, 1.5, 2.0, 2.5} bps. Each of these slices correspond to systems with code rate R = η bps / $\log_2(8)$, resulting in R ={1/6, 1/3, 1/2, 2/3, 5/6}. As an example of the relative performance of the constellations in these tables, consider FIG. 13b which shows a PD capacity optimized PAM-8 constellation optimized for SNR = 9.00 dB, or 1.5 bps. We next examine the plot provided in FIG. 8a and see that the gap of the optimized constellation to the ultimate, Gaussian, capacity (CG) is approximately 0.5 dB. At the same spectral efficiency, the gap of the traditional PAM-8 constellation is approximately 1.0 dB. The advantage of the optimized constellation is 0.5 dB for the same rate (in this case R=1/2). This gain can be obtained by only changing the mapper and demapper in the communication system and leaving all other blocks the same.

[00101] Similar information is presented in FIGS. 14a - 14d, and 15a - 15b which provide loci plots and *design* tables for PAM-16 PD capacity and joint capacity optimized constellations. Likewise FIGS. 16a - 16d, 17a and 17b provide loci plots and design tables for PAM-32 PD capacity and joint capacity optimized constellations.

CAPACITY OPTIMIZED PSK CONSTELLATIONS

[00102] Traditional phase shift keyed (PSK) constellations are already quite optimal. This can be seen in the chart 180 comparing the SNR gaps of tradition PSK with capacity optimized PSK constellations shown in FIG. 18 where the gap between PD capacity and Gaussian capacity is plotted for traditional PSK-4, 8, 16, and 32 and for PD capacity optimized PSK-4, 8, 16, and 32.

[00103] The locus plot of PD optimized PSK-32 points across SNR is shown in FIG. 19, which actually characterizes all PSKs with spectral efficiency $\eta \leq 5$. This can be seen in FIG. 20. Note that at low SNR (0.4 dB) the optimal PSK-32 design is the same as traditional PSK-4, at SNR = 8.4 dB optimal PSK-32 is the same as traditional PSK-8, at SNR = 14.8 dB optimal PSK-32 is the same as traditional PSK-16, and finally at SNRs greater than 20.4 dB optimized PSK-32 is the same as traditional PSK-32. There are SNRs between these discrete points (for instance SNR = 2 and 15 dB) for which optimized PSK-32 provides superior PD capacity when compared to traditional PSK constellations.

[00104] We note now that the locus of points for PD optimized PSK-32 in FIG. 19 in conjunction with the gap to Gaussian capacity curve for optimized PSK-32 in FIG. 18 implies a potential design methodology. Specifically, the designer could achieve performance equivalent or better than that enabled by traditional PSK-4,8, and 16 by using only the optimized PSK-32 in conjunction with a single tuning parameter that controlled where the constellation points should be selected from on the locus of FIG. 19. Such an approach would couple a highly rate adaptive channel code that could vary its rate, for instance, rate 4/5 to achieve and overall (code plus optimized PSK-32 modulation) spectral efficiency of 4 bits per symbol, down to 1/5 to achieve an overall spectral efficiency of 1 bit per symbol. Such an adaptive modulation and coding system could essentially perform on the optimal continuum represented by the rightmost contour of FIG. 18.

ADAPTIVE RATE DESIGN

[00105] In the previous example spectrally adaptive use of PSK-32 was described. Techniques similar to this can be applied for other capacity optimized constellations across the link between a transmitter and receiver. For instance, in the case where a system implements quality of service it is possible to instruct a transmitter to increase or decrease spectral efficiency on demand. In the context of the current invention a capacity optimized constellation designed precisely for the target spectral efficiency can be loaded into the transmit mapper in conjunction with a code rate selection that meets the end user rate goal. When such a modulation / code rate change occurred a message could propagated to the receiver so that the receiver, in anticipation of the change, could select a demapper / decoder configuration in order to match the new transmit-side configuration.

[00106] Conversely, the receiver could implement a quality of performance based optimized constellation / code rate pair control mechanism. Such an approach would include some form of receiver quality measure. This could be the receiver's estimate of SNR or bit error rate. Take for example the case where bit error rate was above some acceptable threshold. In this case, via a backchannel, the receiver could request that the transmitter lower the spectral efficiency of the link by swapping to an alternate capacity optimized constellation / code rate pair in the coder and mapper modules and then signaling the receiver to swap in the complementary pairing in the demapper / decoder modules.

GEOMETRICALLY SHAPED QAM CONSTELLATIONS

[00107] Quadrature amplitude modulation (QAM) constellations can be constructed by orthogonalizing PAM constellations into QAM in phase and quadrature components. Constellations constructed in this way can be attractive in many applications because they have low-complexity demappers.

[00108] In FIG. 21 we provide an example of a Quadrature Amplitude Modulation constellation constructed from a Pulse Amplitude Modulation constellation. The illustrated embodiment was constructed using a PAM-8 constellation optimized for PD capacity at user bit rate per dimension of 1.5 bits (corresponds to an SNR of 9.0 dB) (see FIG. 13b). The label-point pairs in this PAM-8 constellation are: {(000, -1.72), (001, -0.81), (010, 1.72), (011,-0.62), (100,

0.62), (101, 0.02), (110, 0.81), (111, -0.02)}. Examination of FIG. 21 shows that the QAM constellation construction is achieved by replicating a complete set of PAM-8 points in the quadrature dimension for each of the 8 PAM-8 points in the in-phase dimension. Labeling is achieved by assigning the PAM-8 labels to the LSB range on the in-phase dimension and to the MSB range on the quadrature dimension. The resulting 8x8 outer product forms a highly structured QAM-64 for which very low-complexity de-mappers can be constructed. Due to the orthogonality of the in-phase and quadrature components the capacity characteristics of the resulting QAM-64 constellation are identical to that of the PAM-8 constellation on a per-dimension basis.

N-DIMENSIONAL CONSTELLATION OPTIMIZATION

[00109] Rather than designing constellations in 1-D (PAM for instance) and then extending to 2-D (QAM), it is possible to take direct advantage in the optimization step of the additional degree of freedom presented by an extra spatial dimension. In general it is possible to design N-dimensional constellations and associated labelings. The complexity of the optimization step grows exponentially in the number of dimensions as does the complexity of the resulting receiver de-mapper. Such constructions constitute embodiments of the invention and simply require more 'run-time' to produce.

CAPACITY OPTIMIZED CONSTELLATIONS FOR FADING CHANNELS

[00110] Similar processes to those outlined above can be used to design capacity optimized constellations for fading channels in accordance with embodiments of the invention. The processes are essentially the same with the exception that the manner in which capacity is calculated is modified to account for the fading channel. A fading channel can be described using the following equation:

$$Y = a(t)X+N$$

where X is the transmitted signal, N is an additive white Gaussian noise signal and $a(t)$ is the fading distribution, which is a function of time.

[00111] In the case of a fading channel, the instantaneous SNR at the receiver changes according to a fading distribution. The fading distribution is Rayleigh and has the property that

the average SNR of the system remains the same as in the case of the AWGN channel, $E[X^2]/E[N^2]$. Therefore, the capacity of the fading channel can be computed by taking the expectation of AWGN capacity, at a given average SNR, over the Rayleigh fading distribution of a that drives the distribution of the instantaneous SNR.

[00112] Many fading channels follow a Rayleigh distribution. FIGS. 22a – 24b are locus plots of PAM-4, 8, and 16 constellations that have been optimized for PD capacity on a Rayleigh fading channel. Locus plots versus user bit rate per dimension and versus SNR are provided. Similar processes can be used to obtain capacity optimized constellations that are optimized using other capacity measures, such as joint capacity, and/or using different modulation schemes.

GEOMETRIC PAM-8, PAM-16, and PAM-32 CONSTELLATIONS

[00113] As described above, geometric constellations can be obtained that are optimized for joint or PD capacity at specific SNRs. In addition, ranges can be specified for the constellation points of a geometric constellation that are probabilistically likely to result in geometric constellations that provide at least a predetermined performance improvement relative to a constellation that maximizes d_{min}. Turning now to FIGS. 25 – 167, geometric PAM-8, PAM-16, and PAM-32 constellations optimized for joint and PD capacity over the Additive White Gaussian Noise (AWGN) channelat specific SNRs are listed. The performances of the optimal constellations are compared to the performances of traditional constellations that maximize d_{min}. Ranges for the constellation points are also defined at specific SNRs, where constellations having constellation points selected from within the ranges are probabilistically likely (with probability close to one) to result in at least a predetermined performance improvement at the specified SNR relative to a traditional constellation that maximizes d_{min}.

[00114] The geometric constellations disclosed in FIGS. 25 – 167 are defined by points $y(i)$ such that $y(i)=k(x(i)+r(i))+c$. Values for $x(i)$ and bounds on $r(i)$ are provided in FIGS. 25 – 167 for PAM-8, PAM-16, and PAM-32 optimized for joint and PD capacity. For PAM-8 $0 \le i \le 7$, PAM-16 $0 \le i \le 15$, and for PAM-32 $0 \le i \le 31$. To achieve optimal power efficiency, c should be set to zero. In addition to optimized constellations, FIGS. 25 – 167 specify ranges for the points of a geometric constellation, where selecting the points of a constellation from within the ranges is probabilistically likely to provide a geometric constellation having at least a

predetermined performance improvement relative to a constellation that maximizes d_{sqin}. The ranges are expressed as a maximum value for the constellation range parameter, $r(i)$, which specifies the amount by which the point $x(i)$ in the constellation is perturbed relative to the location of the corresponding point in the optimal constellation. A communication system using a constellation formed from constellations points selected from within the ranges specified by the maximum value (i.e. $-r_{max} \leq r(i) \leq r_{max}$) is probabilistically likely to achieve a predetermined performance improvement relative to a constellation that maximizes d_{sqin}. The predetermined performance improvements associated with the ranges specified in FIGS. 25 – 167 are expressed in terms of a percentage of the increase in capacity achieved by the optimized constellation relative to a constellation that maximizes d_{sqin}. Constellations formed from constellation points selected from within the ranges are probabilistically likely to achieve an increase in capacity at least as great as the indicated percentage.

[00115] With regard to the specific tables shown in FIGS. 25 - 167, each table is one of three different types of table. A first set of tables shows the performance of specific geometric constellations optimized for joint capacity or PD capacity. These tables include 6 columns. The first column enumerates a design number. The second column provides the SNR at which the constellation was optimized for the design defined by the entry in the first column. The third column provides the capacity achieved by the optimized constellation (Opt. Cap) at the SNR given in the second column. The fourth column provides the capacity achieved (Std. Cap) by a traditional uniformly spaced constellation i.e. a PAM constellation that maximizes d_{sqin} (with the same number of points as the optimized constellation and where binary reflective gray labeling is assumed) at the SNR given in the second column. The fifth column shows the gain in bits per transmission provided by the optimized constellation over a constellation that maximizes d_{sqin}. The sixth column shows the percentage gain in capacity provided by the optimized constellation over the capacity provided by the traditional uniformly spaced constellation.

[00116] A second set of tables lists the constellation points of the designs indicated in the first set of tables. These tables contain 9 columns. The fist column enumerates a design number. The remaining 8 columns describe a constellation point $x(i)$ enumerated by label in the second row of the table. Labels are given in decimal number format. With PAM 8 as an example, a label of 011 is given as the decimal number 3.

[00117] The third set of tables specifies maximum perturbation ranges for the capacity optimized constellations indicated in the first set of tables, where the maximum ranges correspond to a high probabilistic likelihood of at least a predetermined performance improvement relative to a constellation that maximizes d_{min}. These tables contain 8 columns. The first enumerates a design number (corresponding to a design from one of the aforementioned tables). The second column provides the SNR for the design defined by the entry in the first column. The remaining 5 columns describe parameter r_{max} which is the maximum amount any point in the designed constellation may be perturbed (in either the positive or negative direction) and still retain, with probability close to unity, at least the gain noted by each column header of the joint or PD capacity as a percentage of the gain provided by the corresponding optimized point design over a traditional constellation that maximizes d_{min} (all at the given SNR). Each table has a last column showing that if 100% of the gain afforded by the optimized constellation is desired, then parameter $r(l)$ must be equal to zero (no deviation from designed points described in the point specification tables).

EXAMPLE OF PERFORMANCE ACHIEVED BY CONSTELLATION WITHIN PREDETERMINED RANGES

[00118] By way of example, a constellation can be selected using the ranges specified with respect to the constellation points of a geometric PAM-8 constellation optimized with respect to PD capacity at SNR = 9dB. The optimized constellation points are as follows:

-7.8780 -3.7100 7.8780 -2.8590 2.8590 0.0990 3.7100 -0.0990

[00119] The PD capacity of the above constellation at 9 dB = 1.4999 bits. FIGS 26 – 167 define a range around each constellation r_{max} of 0.47 that is probabilistically likely to result in a constellation that can be used by a communication system to achieve at least 5% of the gain of the optimized constellation (compared to an equally spaced constellation).

[00120] An example of a PAM-8 constellation formed using constellation points selected from within the specified ranges is as follows:

-7.8462 -3.9552 7.7361 -3.2614 2.9395 0.5152 3.3867 0.0829

[00121] The distance between each of the constellation points and the constellation points of the optimized constellation are as follows:

0.0318 -0.2452 -0.1419 -0.4024 0.0805 0.4162 -0.3233 0.1819

[00122] The magnitude of each of the distances is less than r_{min} at 9 dB (i.e. 0.47). The capacity of the selected constellation = 1.4884. The capacity of a constellation that maximizes d_{min} at 9 dB = 1.435 bits. Therefore, the selected constellation achieves 82% of the gain made possible by the optimal constellation (i.e. at least 5%).

LABELLING OF CONSTELLATIONS USING CYCLICALLY ROTATED BINARY REFLECTIVE GRAY LABELS

[00123] In performing optimization with respect to PD capacity, a conjecture can be made that constraining the optimization process to the subsequently described class of labelings results in no or negligible loss in PD capacity (the maximum observed loss is 0.005 bits, but in many cases there is no loss at all). Use of this labeling constraint speeds the optimization process considerably. We note that joint capacity optimization is invariant to choice of labeling. Specifically, joint capacity depends only on point locations whereas PD capacity depends on point locations and respective labelings.

[00124] The class of cyclically rotated binary reflective gray labels can be used. The following example, using constellations with cardinality 8, describes the class of cyclically rotated binary reflective gray labels. Given for example the standard gray labeling scheme for PAM-8:

000, 001, 011, 010, 110, 111, 101, 100

Application of a cyclic rotation, one step left, yields:

001, 011, 010, 110, 111, 101, 100, 000

Application of a cyclic rotation, two steps left, yields:

011, 010, 110, 111, 101, 100, 000, 001

[00125] For a constellation with cardinality 8, cyclic rotations of 0 to 7 steps can be applied. It should be noted that within this class of labelings, some labelings perform better than others. It should also be noted that different rotations may yield labelings that are equivalent (through trivial column swapping and negation operations). In general, labelings can be expressed in different but equivalent forms through trivial operations such as column swapping and negation operations. For example the binary reflective gray labels with one step rotation:
001, 011, 010, 110, 111, 101, 100, 000

Can be shown to be equivalent to:
000, 001, 011, 111, 101, 100, 110, 010

[00126] The above equivalence can be shown by the following steps of trivial operations:
1) Negate the third column. This gives
000, 010, 011, 111, 110, 100, 101, 001

2) Swap the second and third columns. This gives
000, 001, 011, 111, 101, 100, 110, 010

The two labelings are considered equivalent because they yield the same PD Capacity as long as the constellation points locations are the same.

[00127] In the constellation point specifications shown in FIGS. 25 – 167, a labeling can be interchanged by any equivalent labeling without affecting the performance parameters. A labeling used in the specifications may not directly appear to be a cyclically rotated binary reflective gray labeling, but it can be shown to be equivalent to one or more cyclically rotated binary reflective gray labelings.

PRIOR ART GEOMETRIC CONSTELLATIONS

[00128] Geometric constellations have been specified in the prior art in attempts to achieve performance gains relative to constellations that maximize d_{min}. Examples of such constellations are disclosed in Sommer and Feltweis, "Signal Shaping by Non-Uniform QAM for AWGN

Channels and Applications Using Turbo Coding" *ITG Conference Source and Channel Coding*, p. 81 – 86, 2000. The specific constellations disclosed by Sommer and Fettweis for PAM-8, PAM-16, and PAM-32 are as follows:

PAM-8:

-1.6630 -0.9617 -0.5298 -0.1705 0.1705 0.5298 0.9617 1.6630

PAM-16:

-1.9382 -1.3714 -1.0509 -0.8079 -0.6026 -0.4185 -0.2468 -0.0816 0.0816

0.2468 0.4185 0.6026 0.8079 1.0509 1.3714 1.9382

PAM-32:

-2.1970 -1.7095 -1.4462 -1.2545 -1.0991 -0.9657 -0.8471 -0.7390 -0.6386

-0.5441 -0.4540 -0.3673 -0.2832 -0.2010 -0.1201 -0.0400 0.0400 0.1201

0.2010 0.2832 0.3673 0.4540 0.5441 0.6386 0.7390 0.8471 0.9657

1.0991 1.2545 1.4462 1.7095 2.1970

[00129] Another class of geometric constellations is disclosed in Long et al., "Approaching the AWGN Channel Capacity without Active Shaping" *Proceedings of International Symposium on Information Theory*, p. 374, 1997. The specific PAM-8, PAM-16, and PAM-32 constellations disclosed by Long et al. are as follows:

PAM-8:

-3 -1 -1 -1 1 1 1 3

PAM-16:

-4 -2 -2 -2 -2 0 0 0 0 0 0 2 2 2 2 4

PAM-32:

-5 -3 -3 -3 -3 -3 -1 -1 -1 -1 -1 -1 -1 -1 -1 -1
1 1 1 1 1 1 1 1 1 1 3 3 3 3 3 5

[00130] The above prior art constellations are geometric and can provide performance improvements at some SNRs relative to constellations that maximize d_{min}. The performance of the constellations varies with SNR and at certain SNRs the constellations are proximate to capacity optimized constellations. Therefore, the ranges specified in FIGS. 25 – 167 are defined so that prior art constellations are excluded at the specific SNRs at which these constellations are proximate to a capacity optimized constellation.

CONSTRUCTING MULTIDIMENSIONAL CONSTELLATIONS

[00131] The tables shown in FIGS. 25 – 167 can be used to identify optimal N-dimensional constellations. The optimized multi-dimensional constellation can be determined by finding the Cartesian power X^n and the resulting labeling constructed by finding the corresponding Cartesian power of L^n. Ranges within which the multi-dimensional constellation points can be selected (i.e. perturbed), can then be defined with respect to each constellation point of the constructed multi-dimensional constellation, using an n-dimensional perturbation vector, such that each component of the perturbation vector has a magnitude that is less than r_{min} defined by the range tables.

EXAMPLE OF A QAM CONSTELLATION

[00132] The optimized constellation points for a PAM-8 constellation optimized for PD capacity at SNR=9 dB are as follows:

-7.8780 -3.7100 7.8780 -2.8590 2.8590 0.0990 3.7100 -0.0990

[00133] The labelings corresponding to the above PAM-8 constellation points are:

[00134] 000 001 010 011 100 101 110 111

[00135] Using this PAM-8 constellation, it is possible to construct a QAM-64 constellation. While PAM-8 maps 3 bits to one dimension, QAM-64 maps 6 bits to two dimensions. The first three bits will determine the location in the X-dimension and the second three bits will determine the location in the Y-dimension. The resulting QAM-64 constellation for example will map the bits 000 010 to the two dimensional constellation point (-7.878, 7.878), and 111 110 to the two dimensional constellation point (-0.099, 3.71). The points corresponding to the remaining labels can be derived in a similar manner.

[00136] The ranges shown in FIGS. 25 – 167 can be utilized to select QAM constellations in a similar manner to that outlined above with respect to the selection of a PAM-8 constellation based upon ranges specified with respect to a PAM-8 constellation optimized for PD capacity at 9dB. A range of 0.47 can be applied to every component of each two dimensional constellation point. For example, the two points (-7.787, 8.078) and (0.201, 3.31) are within the ranges as they are spaced distances (-0.1, 0.2) and (0.3, -0.4) respectively from the optimized constellation points. In this way, the ranges can be used to identify constellations that are probabilistically likely to result in a performance improvement relative to a constellation that maximizes d_{min}.

[00137] The same procedure can apply to a constellation optimized for joint capacity. However, the choice of labeling does not affect joint capacity. The above procedure can similarly be applied to an N-dimensional constellation constructed from a PAM constellation.

[00138] Although the present invention has been described in certain specific embodiments, many additional modifications and variations would be apparent to those skilled in the art. It is therefore to be understood that the present invention may be practiced otherwise than specifically described, without departing from the scope and spirit of the present invention. Thus, embodiments of the present invention should be considered in all respects as illustrative and not restrictive.

WHAT IS CLAIMED IS:

1. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate;

a mapper configured to map encoded bits to symbols in a symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper; and

wherein the symbol constellation is a PAM-8 symbol constellation and at least one of the constellation points is a distance greater than 0 and less than the maximum distance specified in FIGS. 25 – 46 from the constellation point of a constellation optimized for capacity at a specific SNR.

2. The communication system of claim 1, wherein the code is a Turbo code.

3. The communication system of claim 1, wherein the code is a LDPC code.

4. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

5. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 15% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

6. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 30% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

7. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 45% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

8. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

9. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

10. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

11. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 40% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

12. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 50% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

13. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

14. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 70% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

15. The communication system of claim 1, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

16. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate;

a mapper configured to map encoded bits to symbols in a symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper; and

wherein the symbol constellation is a PAM-16 symbol constellation and at least one of the constellation points is a distance greater than 0 and less than the maximum distance specified in FIGS. 47 – 84 from the constellation point of a constellation optimized for capacity at a specific SNR.

17. The communication system of claim 16, wherein the code is a Turbo code.

18. The communication system of claim 16, wherein the code is a LDPC code.

19. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

20. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 15% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

21. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 30% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

22. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 45% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

23. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

24. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

25. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

26. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 40% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

27. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 50% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

28. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

30. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 70% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

31. The communication system of claim 16, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

32. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate;

a mapper configured to map encoded bits to symbols in a symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper; and

wherein the symbol constellation is a PAM-32 symbol constellation and at least one of the constellation points is a distance greater than 0 and less than the maximum distance specified in FIGS. 85 – 148 from the constellation point of a constellation optimized for capacity at a specific SNR.

33. The communication system of claim 32, wherein the code is a Turbo code.

34. The communication system of claim 32, wherein the code is a LDPC code.

35. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

36. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 15% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

37. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 30% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

38. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 45% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

39. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

40. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for joint capacity at the predetermined SNR.

41. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 5% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

42. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 40% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

43. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 50% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

44. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 60% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

45. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 70% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

46. The communication system of claim 32, wherein the constellation provides an increase in capacity at a predetermined SNR that is at least 100% of the gain in capacity achieved by a constellation optimized for PD capacity at the predetermined SNR.

47. A digital communication system, comprising:

a transmitter configured to transmit signals to a receiver via a communication channel;

wherein the transmitter, comprises:

a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate;

a mapper configured to map encoded bits to symbols in a symbol constellation;

a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper;

wherein the receiver, comprises:

a demodulator configured to demodulate the received signal via the communication channel;

a demapper configured to estimate likelihoods from the demodulated signal;

a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper; and

wherein the symbol constellation is a N-Dimensional symbol constellation, where the constellation points in at least one dimension are within at least one of the ranges specified in FIGS. 25 – 167.

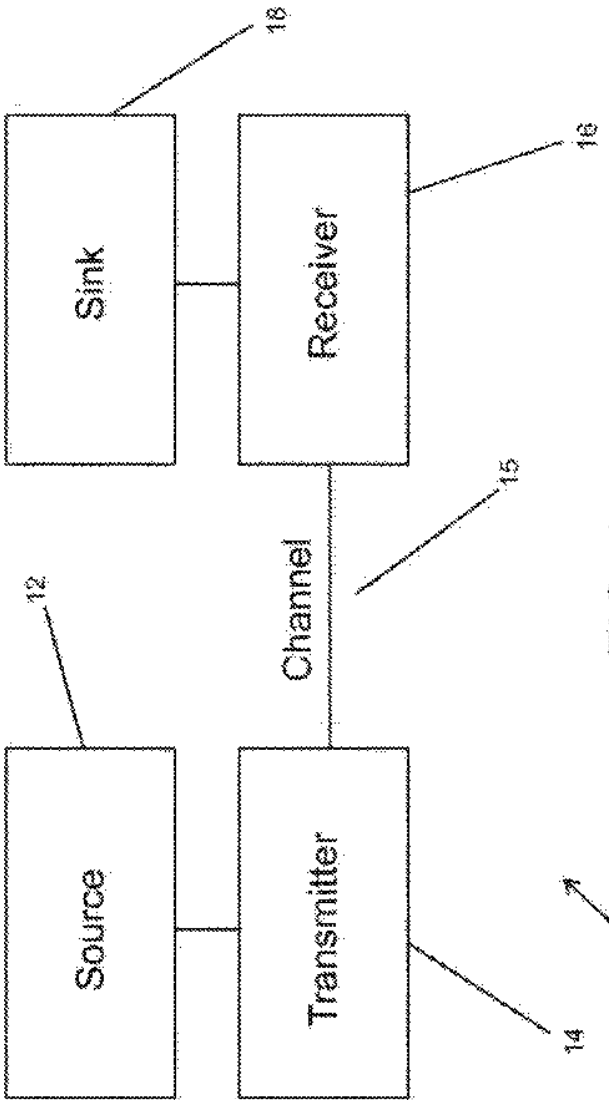


FIG. 1

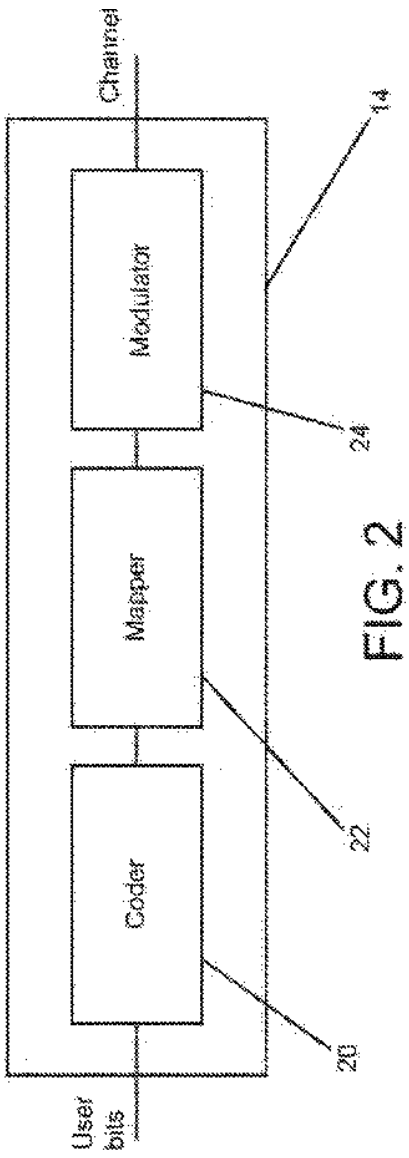

Channel
Modulator
Mapper
Coder
User bits
14
24
22
20

FIG. 2

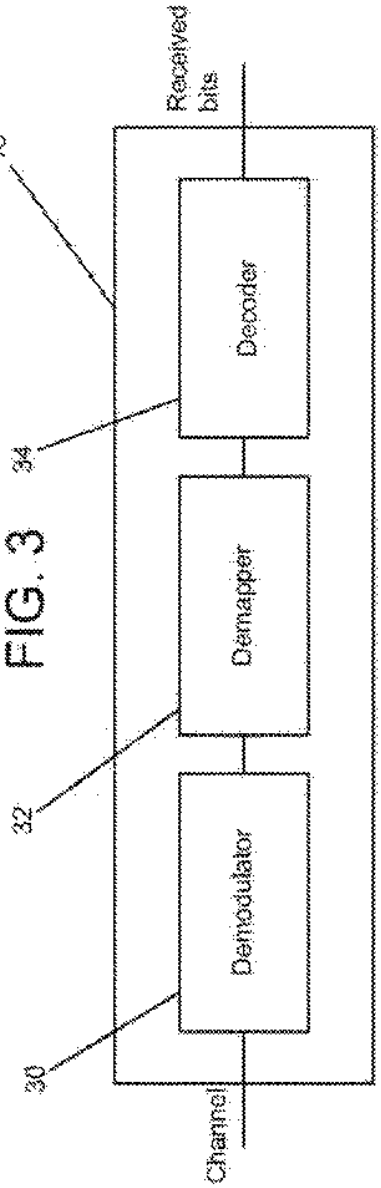

Received bits
Decoder
Demapper
Demodulator
Channel
16
34
32
30

FIG. 3

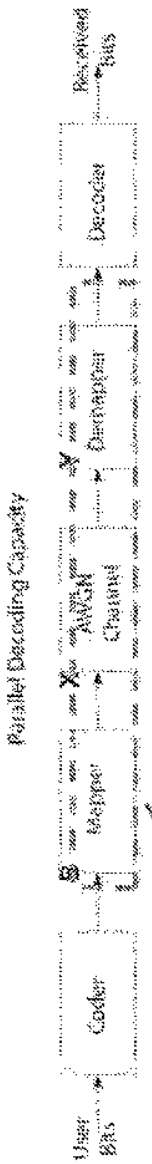

Parallel Decoding Capacity
User Bits
Coder
B
Mapper
X
AWGN Channel
Demapper
Decoder
Received bits
40

FIG. 4a

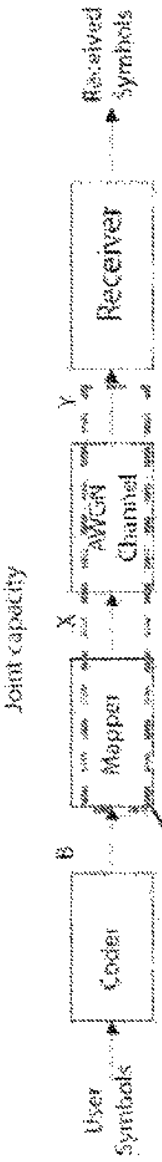

Joint capacity
User Symbols
Coder
B
Mapper
X
AWGN Channel
Y
Receiver
Received Symbols
42

FIG. 4b

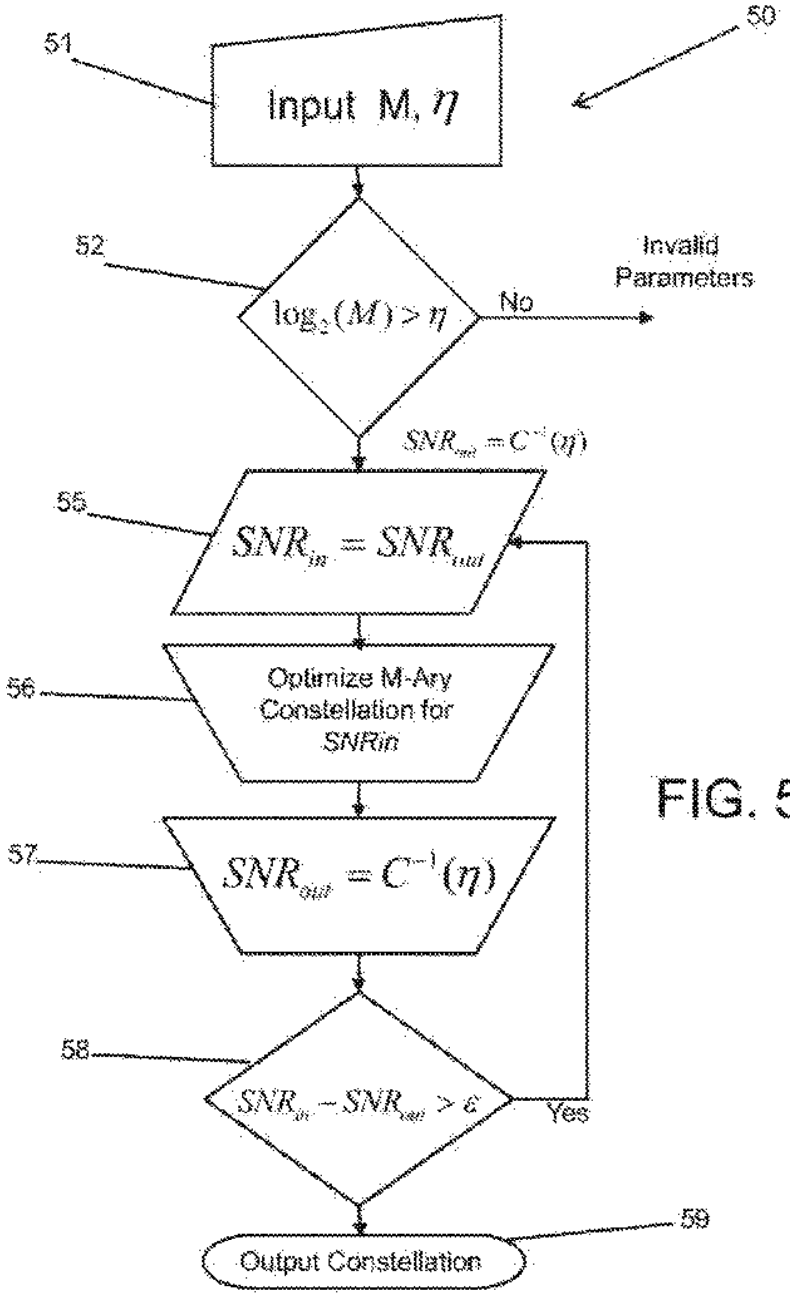

50
51
Input M, η
52
log2(M) > η
No
Invalid Parameters
SNRout = C-1(η)
55
SNRin = SNRout
56
Optimize M-Ary Constellation for SNRin
57
SNRout = C-1(η)
58
SNRin − SNRout > ε
Yes
59
Output Constellation

FIG. 5

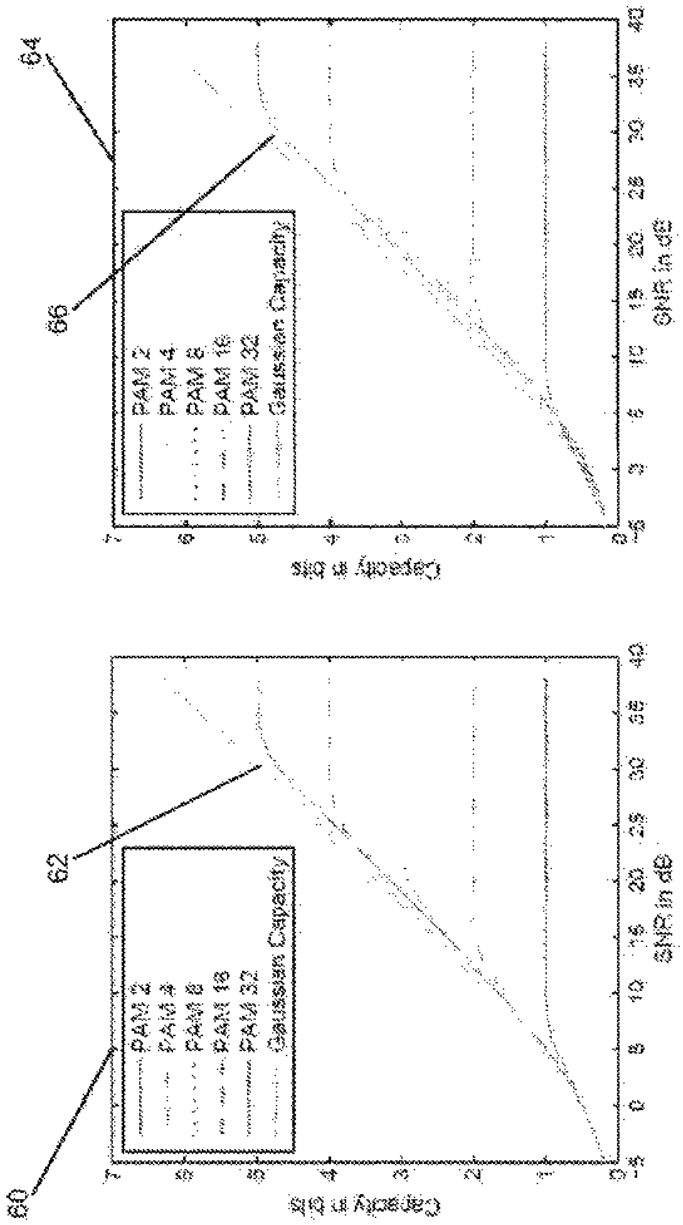

64
66
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(b)

FIG. 6b

60
62
PAM 2
PAM 4
PAM 8
PAM 16
PAM 32
Gaussian Capacity
Capacity in bits
SNR in dB
(a)

FIG. 6a

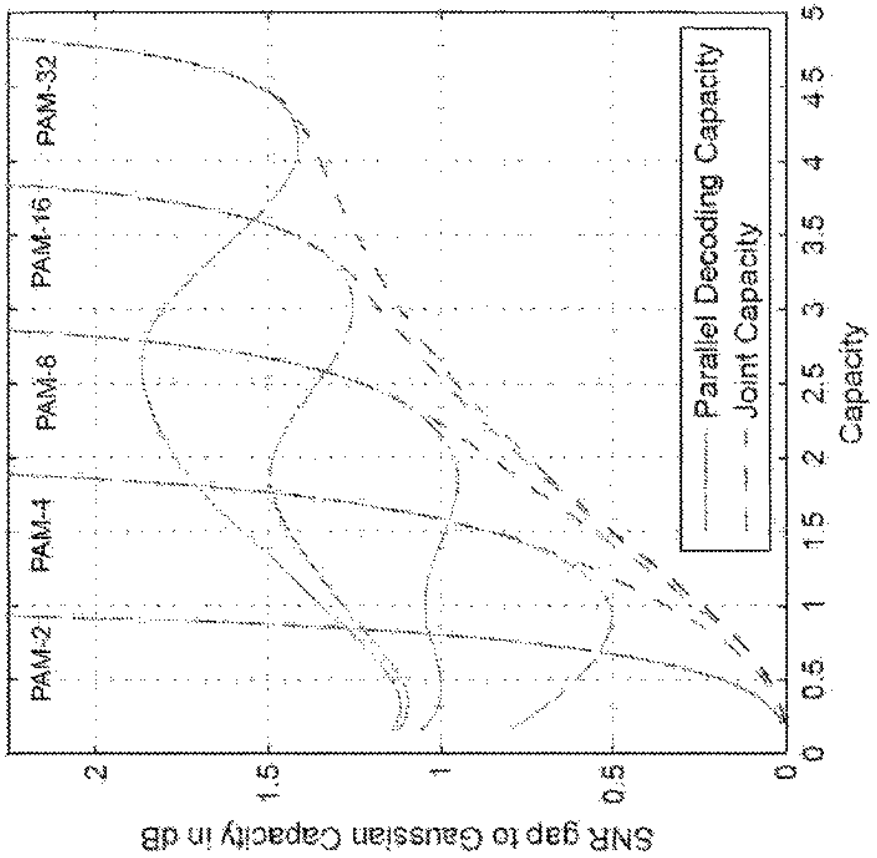

SNR gap to Gaussian Capacity in dB
Capacity
Parallel Decoding Capacity
Joint Capacity
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

FIG. 7

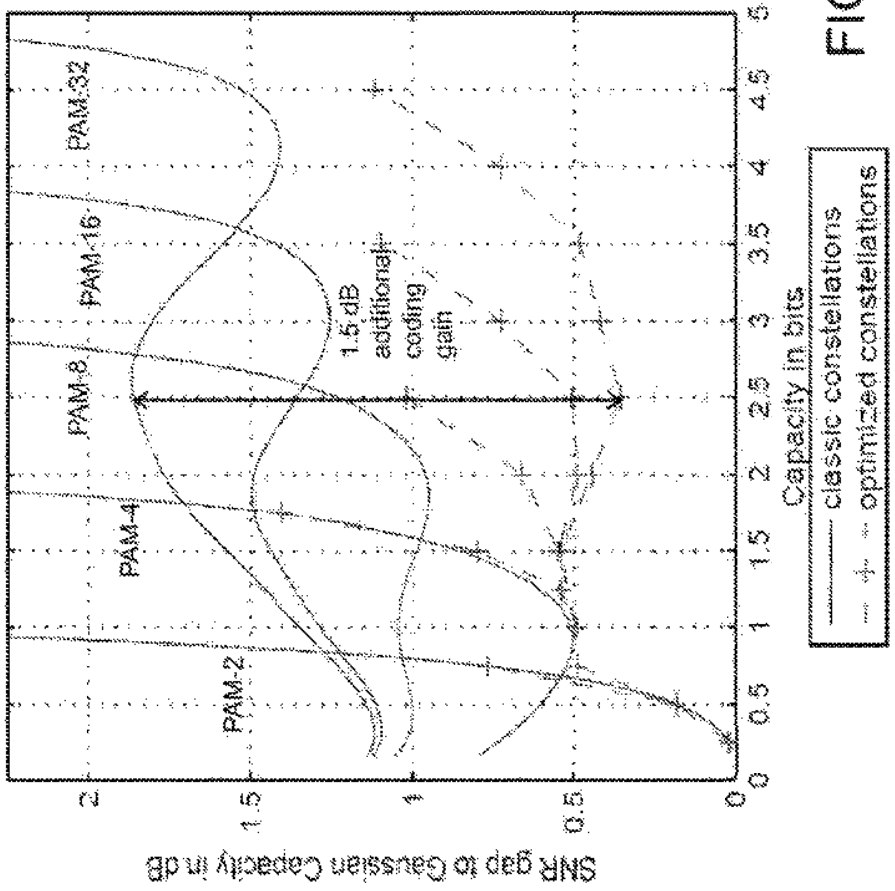

SNR gap to Gaussian Capacity in dB
Capacity in bits
classic constellations
optimized constellations
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
1.5 dB additional coding gain
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0.5
1
1.5
2

FIG. 8a

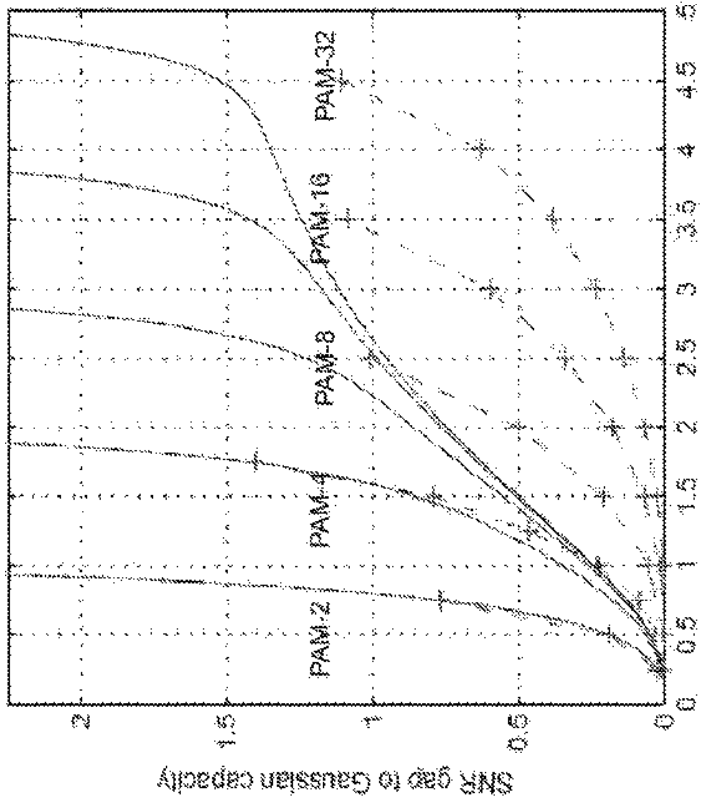

SNR gap to Gaussian capacity
Capacity in bits
classic constellations
optimized constellations
PAM-2
PAM-4
PAM-8
PAM-16
PAM-32
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0.5
1
1.5
2

FIG. 8b

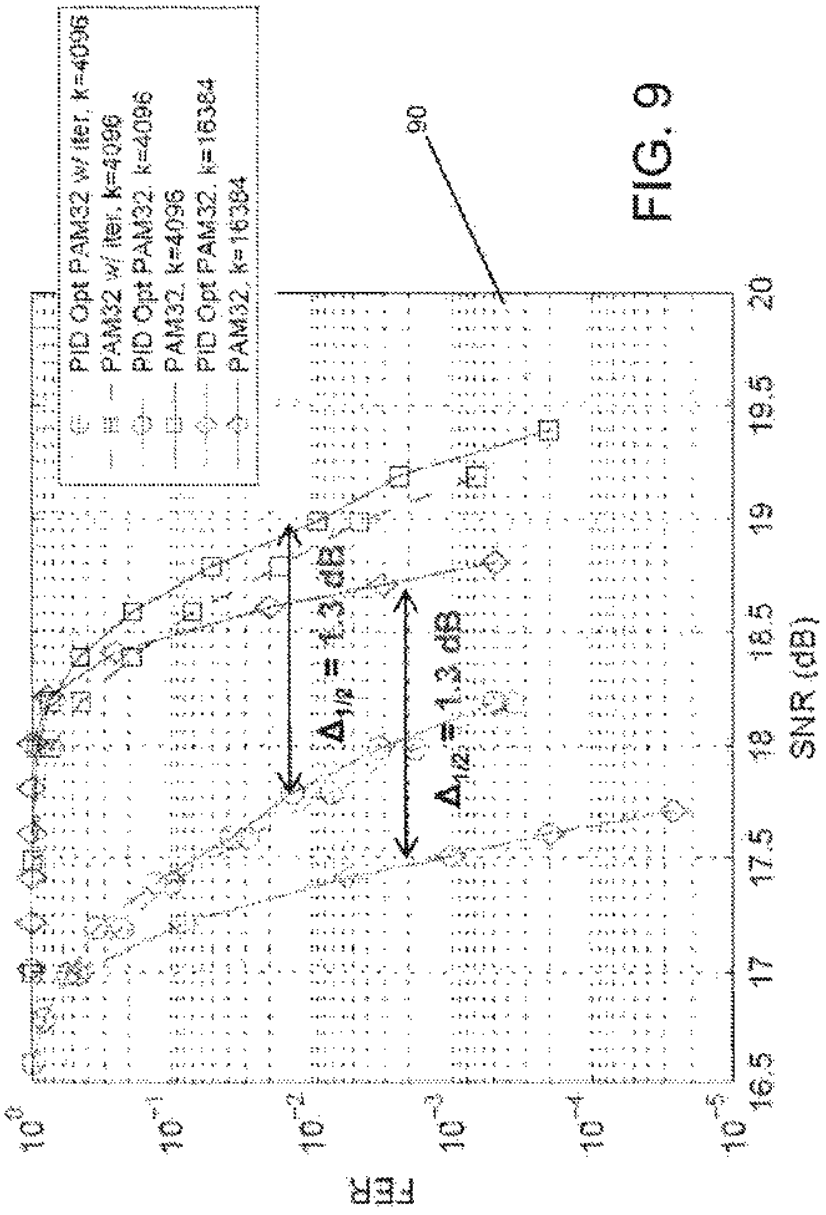

PID Opt PAM32 w/ iter, k=4096
PAM32 w/ iter, k=4096
PID Opt PAM32, k=4096
PAM32, k=4096
PID Opt PAM32, k=16384
PAM32, k=16384
90
$\Delta_{1/8}$ = 1.3 dB
$\Delta_{1/2}$ = 1.3 dB
FER
SNR (dB)
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
16.5
17
17.5
18
18.5
19
19.5
20

FIG. 9

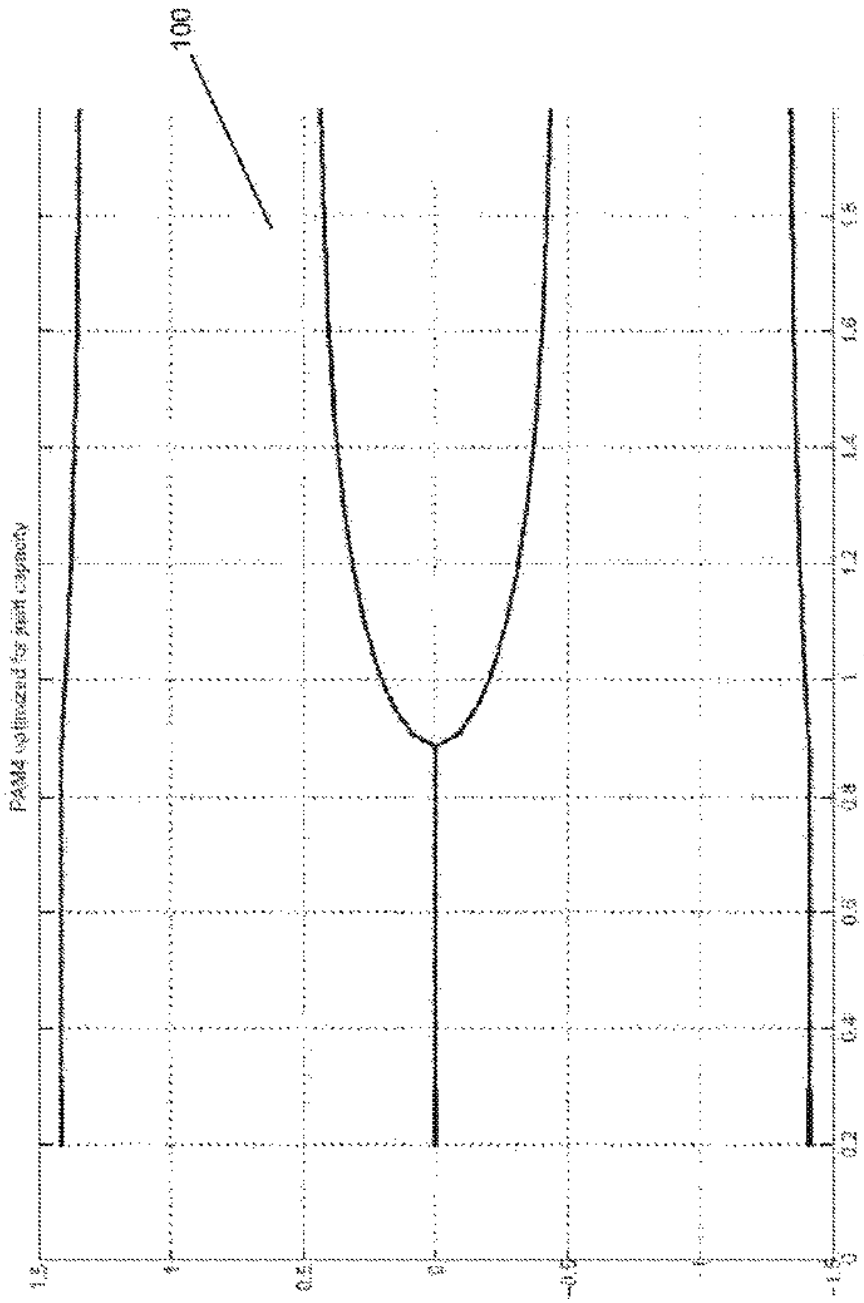

100
PAM4 optimized for peak capacity
Capacity in bits
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0.5
0
-0.5
-1

FIG. 10a

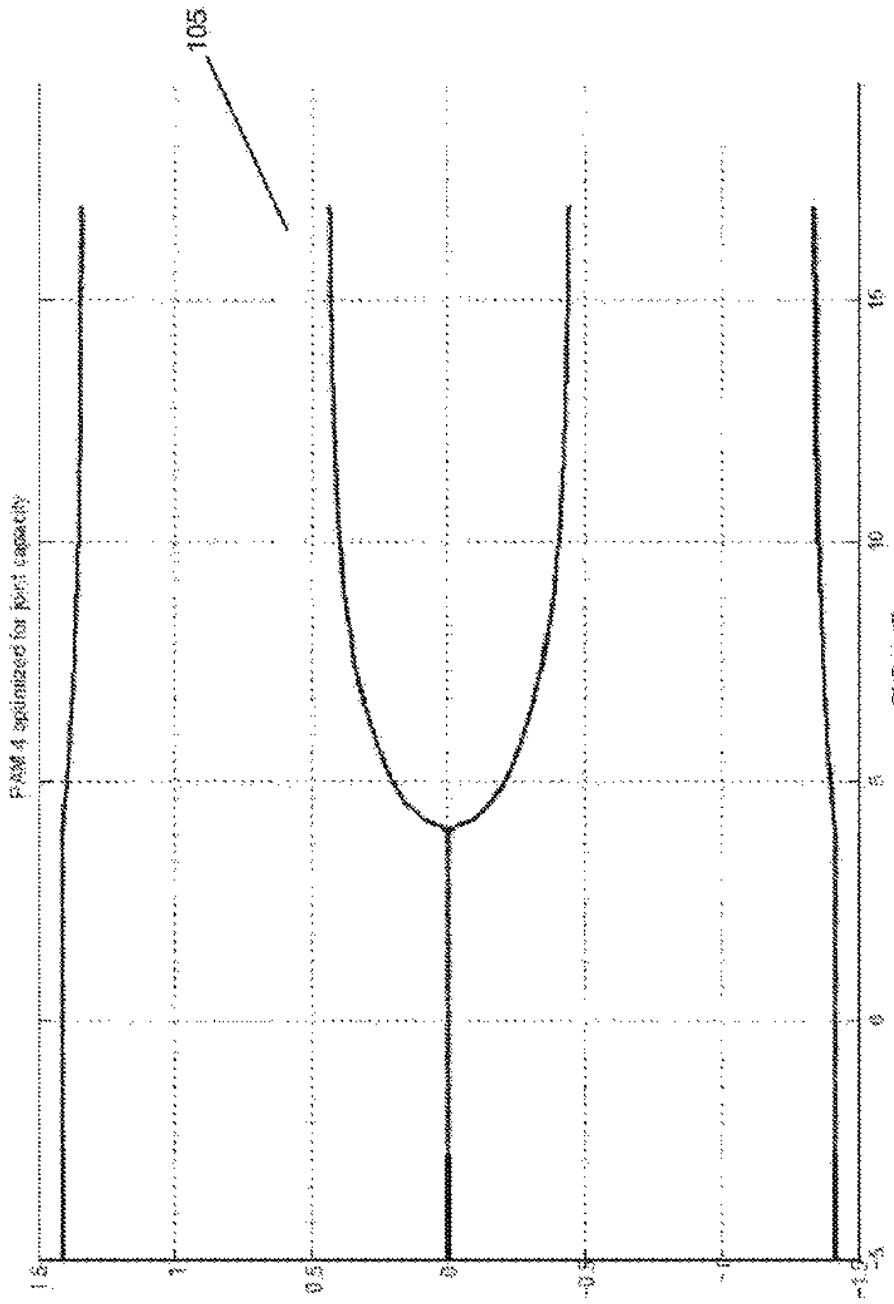

105
FAM 4 optimized for joint capacity
SNR in dB
15
10
5
0
-5
1.5
1
0.5
0
-0.5
-1
-1.5

FIG. 10b

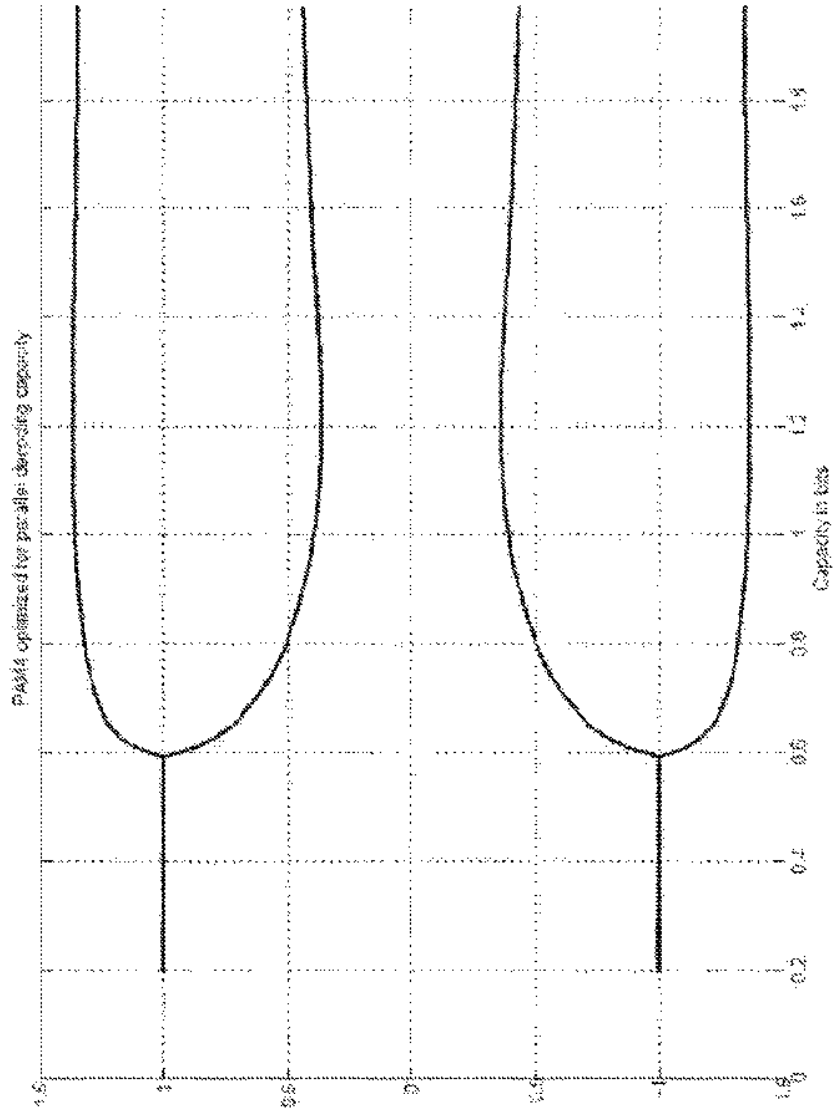

PAM4 optimized for parallel decoding capacity
Capacity in bits
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8

FIG. 10c

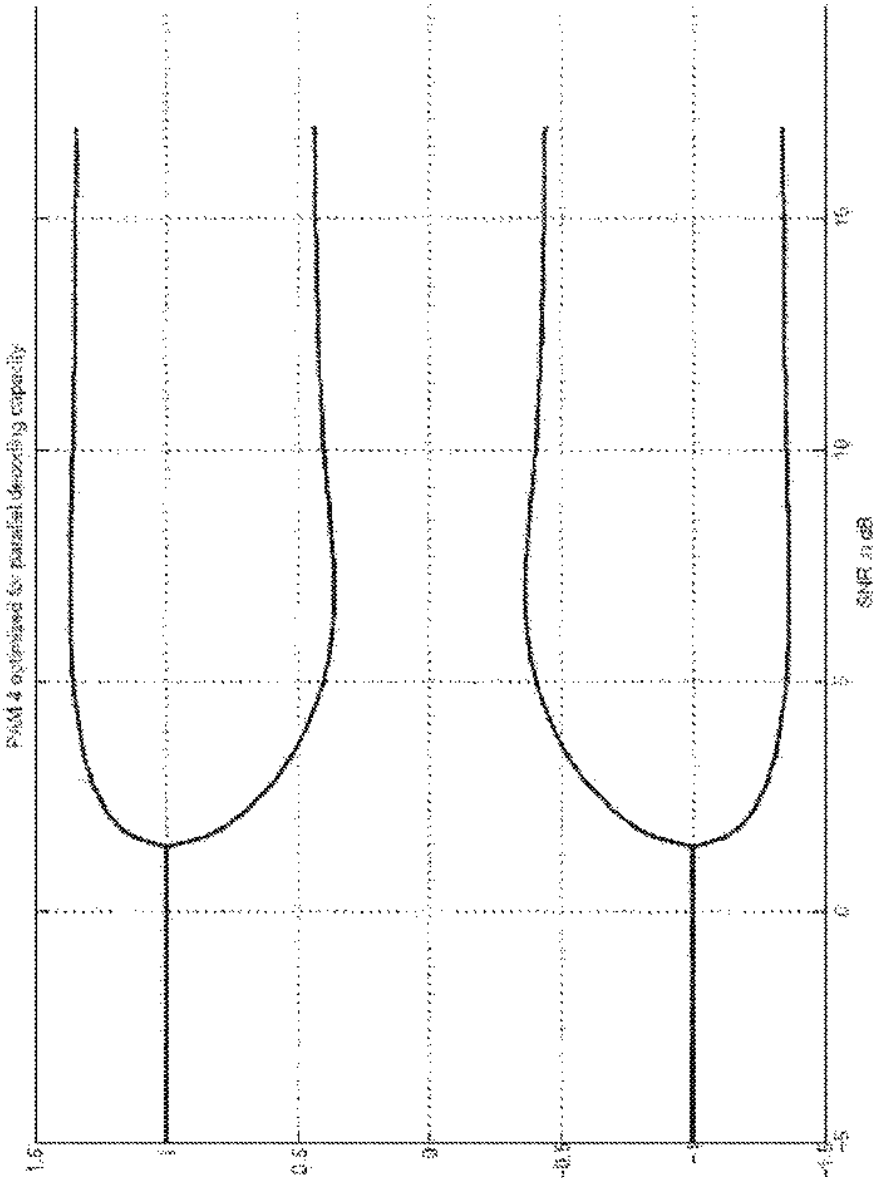

PSAM 4 optimized for parallel decoding capacity
SNR in dB
1.5
1
0.5
0
-0.5
-1
-1.5
15
10
5
0
-5

FIG. 10d

PAM-4 constellations optimized for joint capacity at different rates

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
α_0	-1.41	-1.41	-1.40	-1.37	-1.36
α_1	0.00	0.00	-0.20	-0.33	-0.39
α_2	0.00	0.00	0.20	0.33	0.39
α_3	1.41	1.41	1.40	1.37	1.36

FIG. 11a

PAM-4 constellations optimized for parallel decoding capacity at different

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.26	7.22	9.25
x_0	-1.00	-1.30	-1.36	-1.37	-1.36
x_1	-1.00	-0.36	-0.39	-0.33	-0.39
x_2	1.00	1.30	1.36	0.33	1.36
x_3	1.00	0.56	0.39	1.37	0.39

FIG. 11b

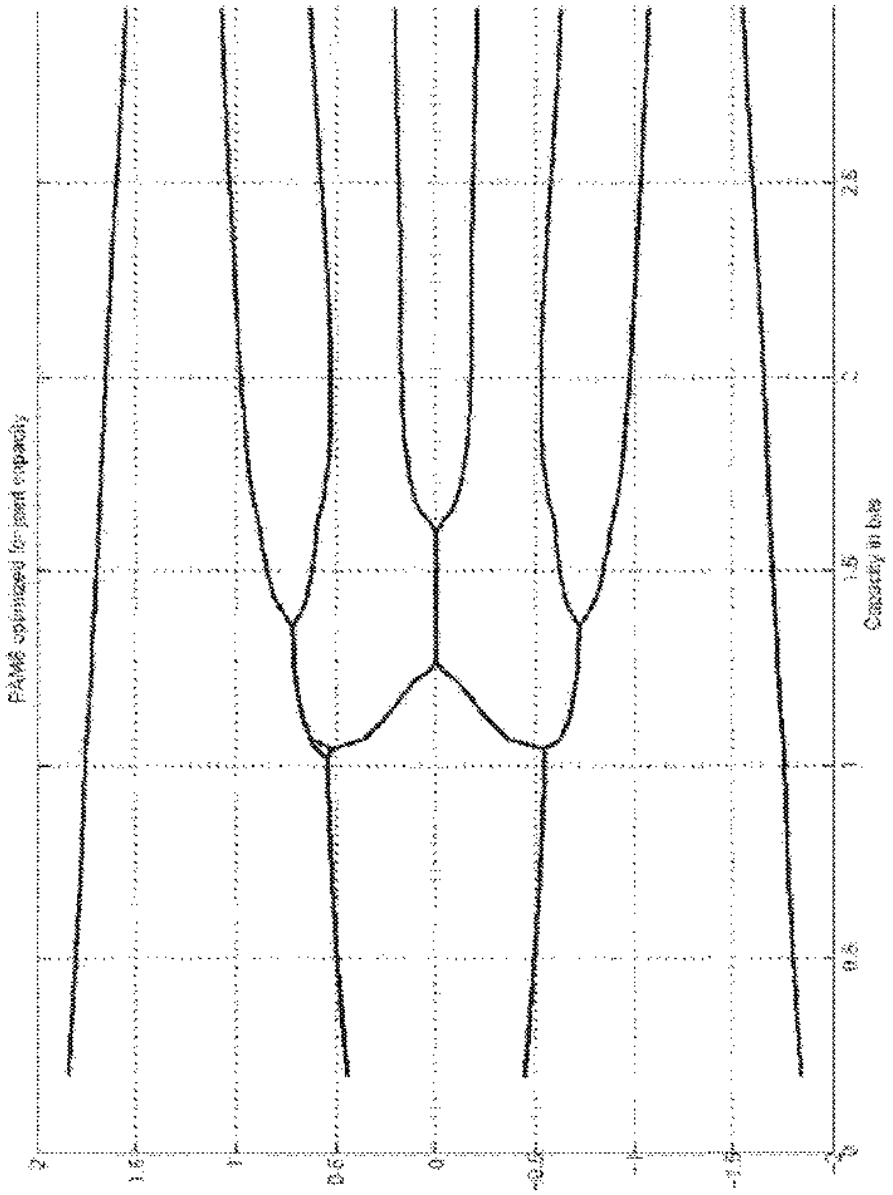

PAMR optimized for peak capacity
Capacity in kas
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
2.5
2
1.5
1
0.5

FIG. 12a

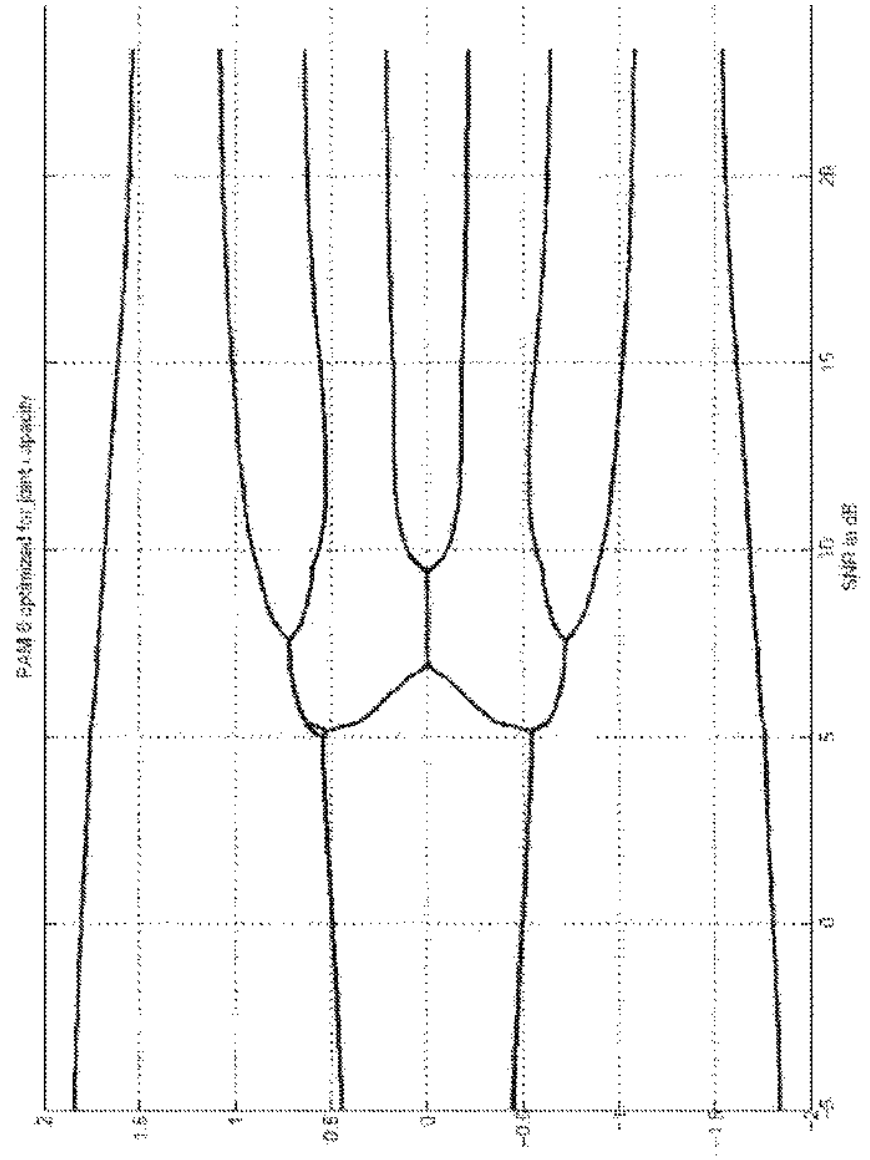

P-MM 6 optimized for plant 1 specificity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
5
10
15
20

FIG. 12b

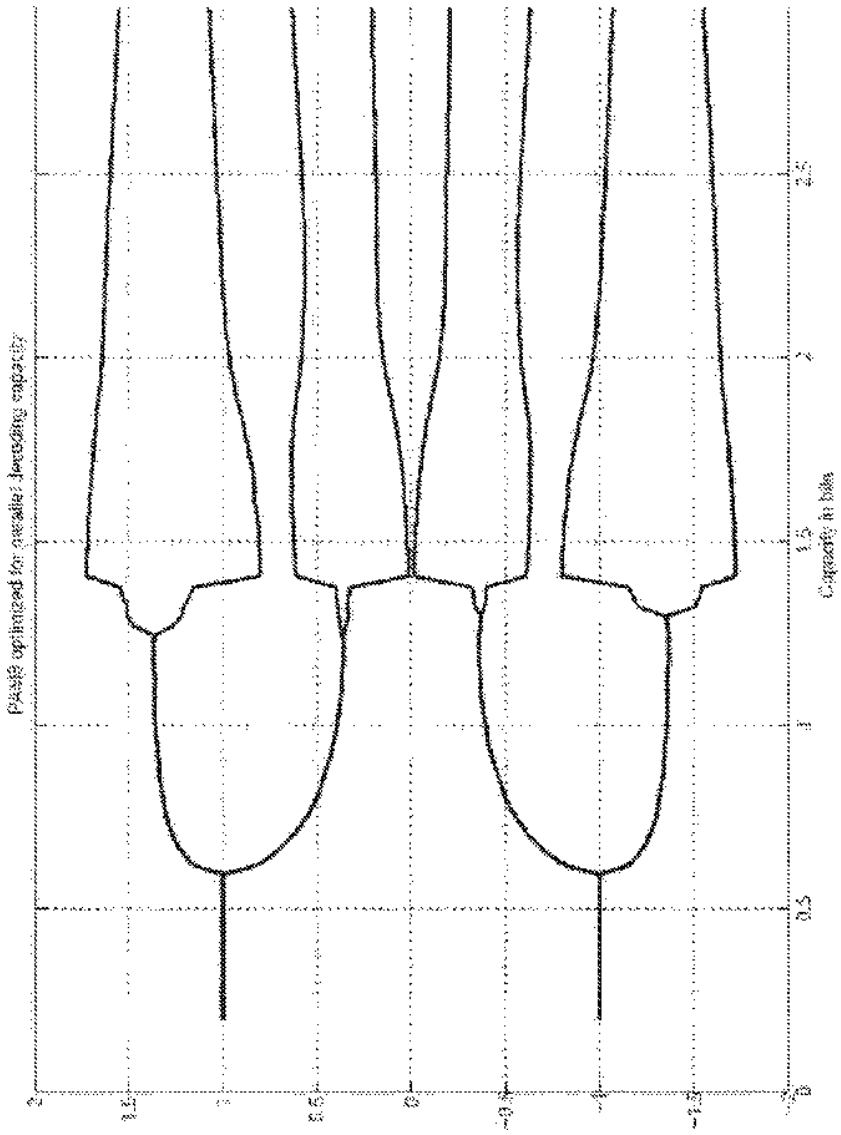

Passes optimized for metallic bedding capacity
Capacity in bits
2.5
2
1.5
1
0.5
0
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2

FIG. 12c

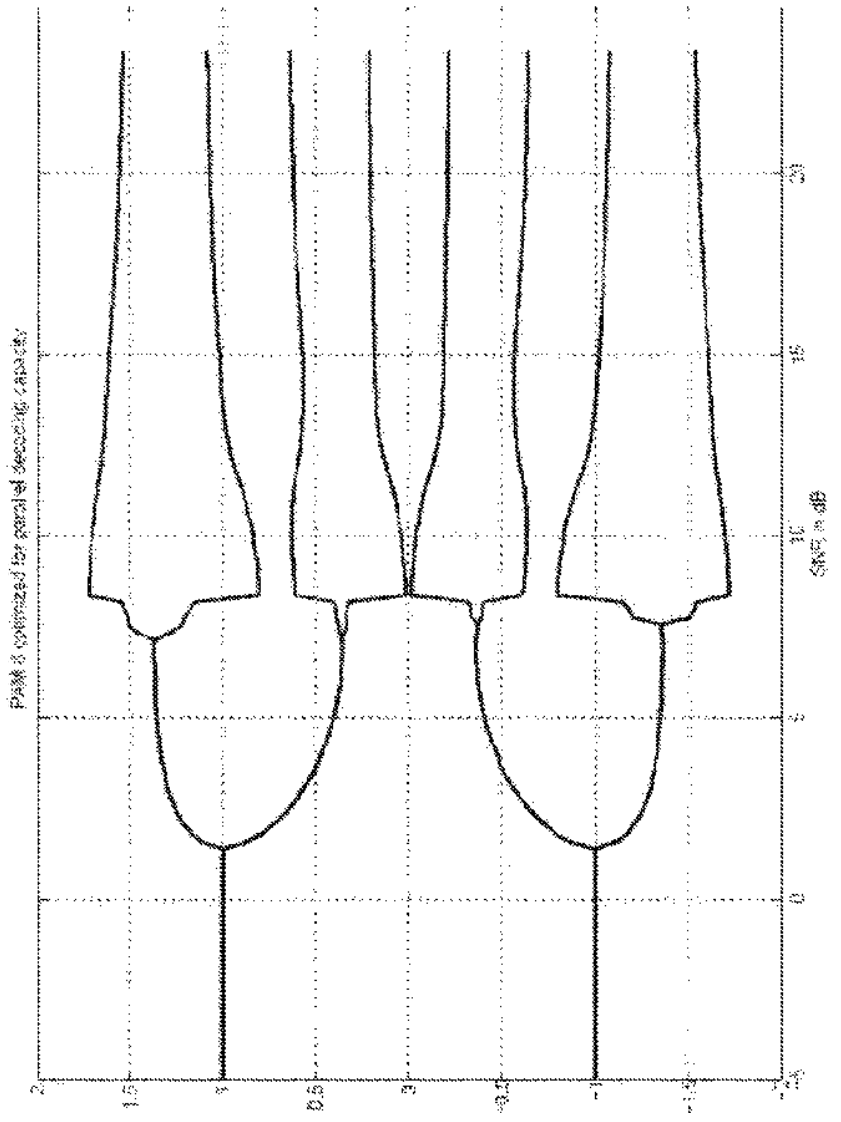


FIG. 12d

PAM-8 constellations optimized for joint capacity at different rates

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
x_0	-1.81	-1.76	-1.70	-1.66	-1.60
x_1	-0.50	-0.55	-0.84	-0.97	-1.03
x_2	-0.50	-0.55	-0.63	-0.53	-0.58
x_3	-0.50	-0.55	-0.00	-0.17	-0.19
x_4	0.50	0.55	0.00	0.17	0.19
x_5	0.50	0.55	0.63	0.53	0.58
x_6	0.50	0.55	0.84	0.97	1.03
x_7	1.81	1.76	1.70	1.66	1.60

FIG. 13a

PAM-8 constellations optimized for parallel decoding capacity at different

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	3.27	9.00	12.42	15.93
x_0	-1.00	-1.36	-1.72	-1.64	-1.60
x_1	-1.00	-1.36	-0.81	-0.97	-1.03
x_2	-1.00	-0.39	1.72	1.64	-0.19
x_3	-1.00	-0.39	-0.62	-0.58	-0.58
x_4	1.00	1.36	0.62	0.58	1.60
x_5	1.00	1.36	0.02	0.15	1.03
x_6	1.00	0.39	0.81	0.97	0.19
x_7	1.00	0.39	-0.02	-0.15	0.58

FIG. 13b

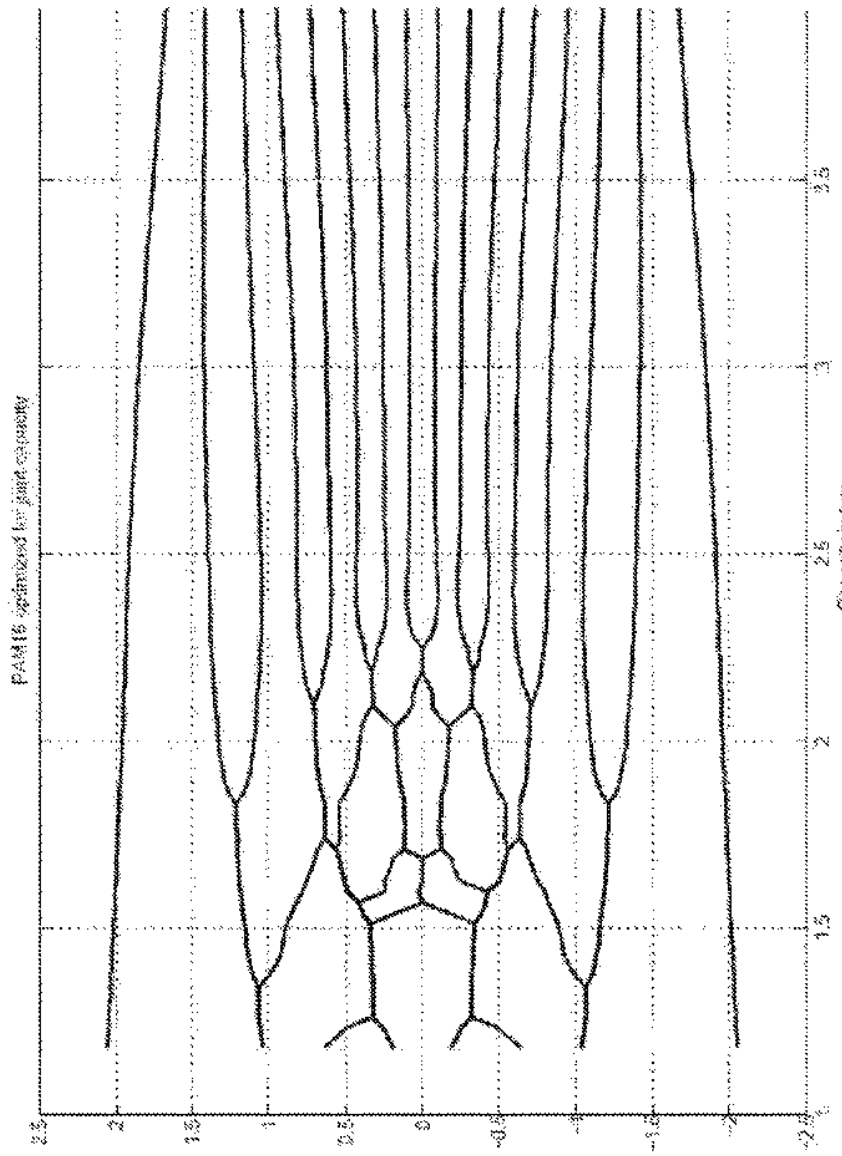

PAM16 optimized for plant capacity
Capacity in tons
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 14a

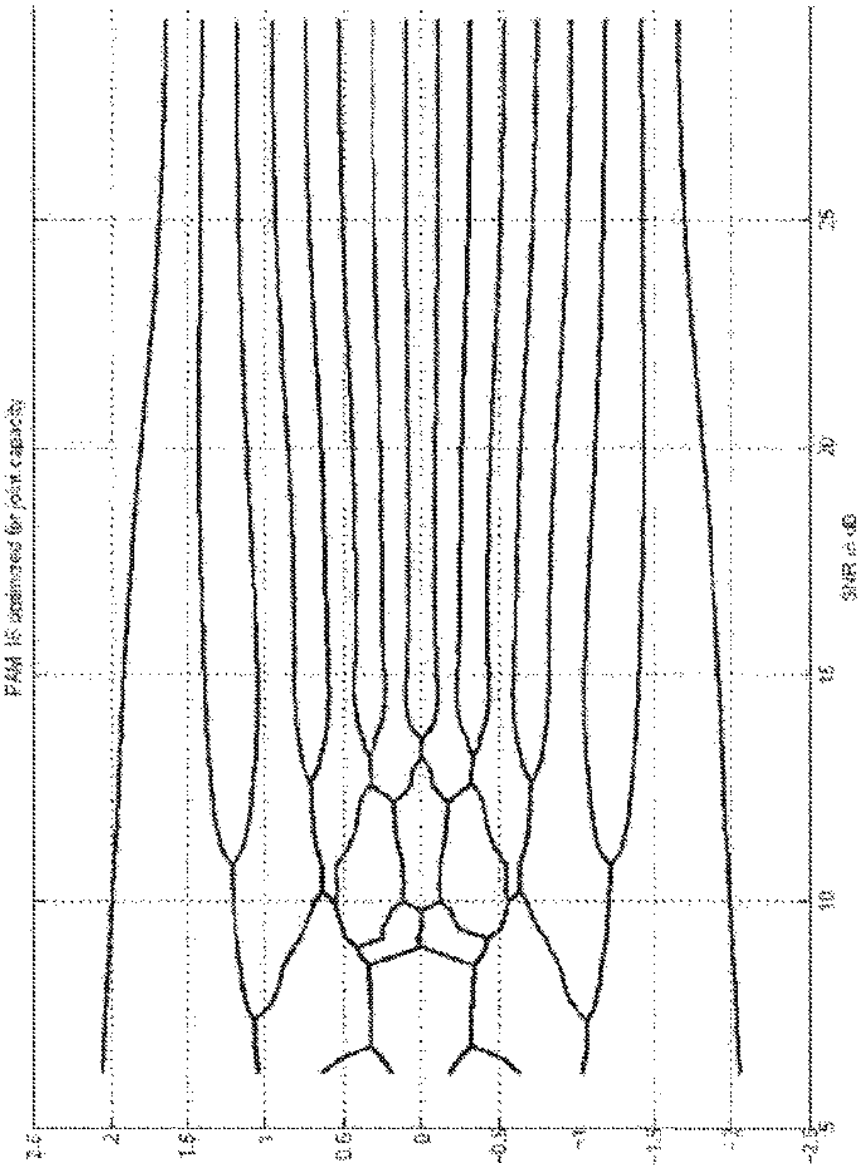

PAM IS optimized for joint capacity
SNR in dB
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14b

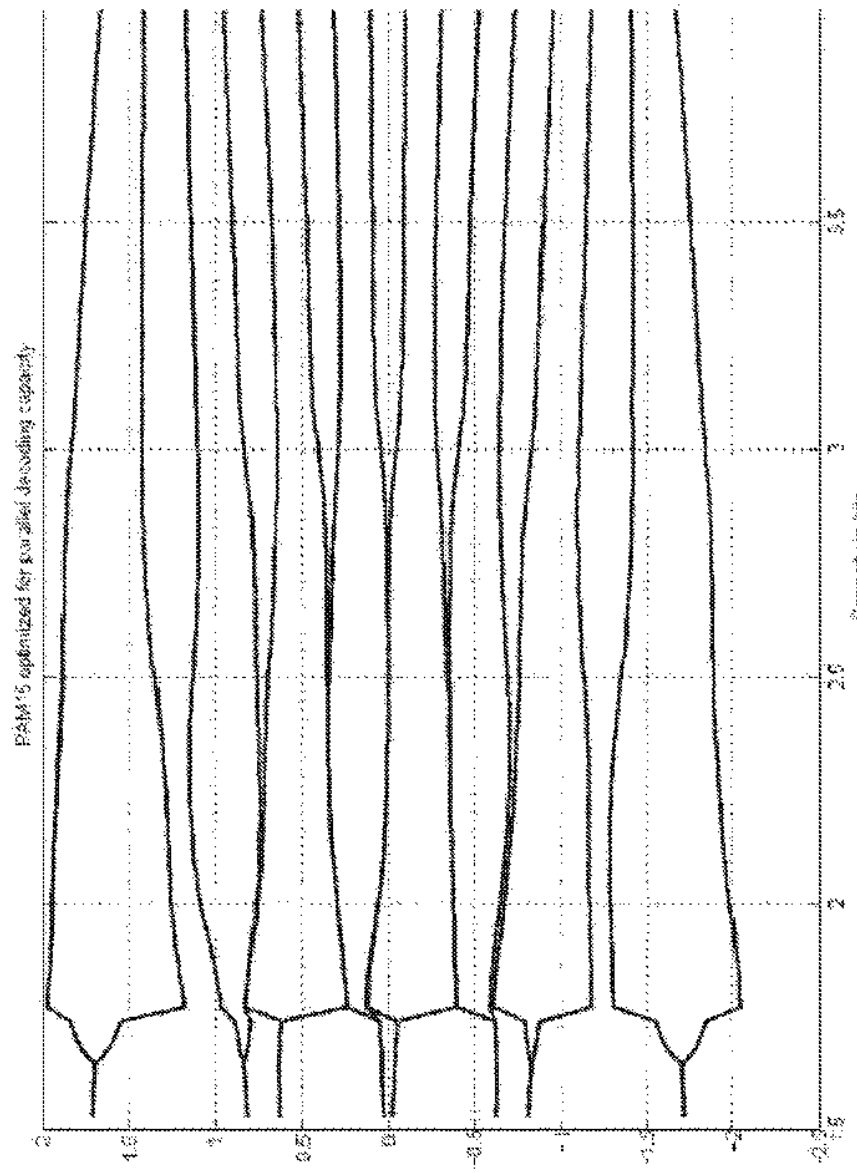

PAM15 optimized for gaussian decoding capacity
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0.5
1
1.5
2
2.5

FIG. 14c

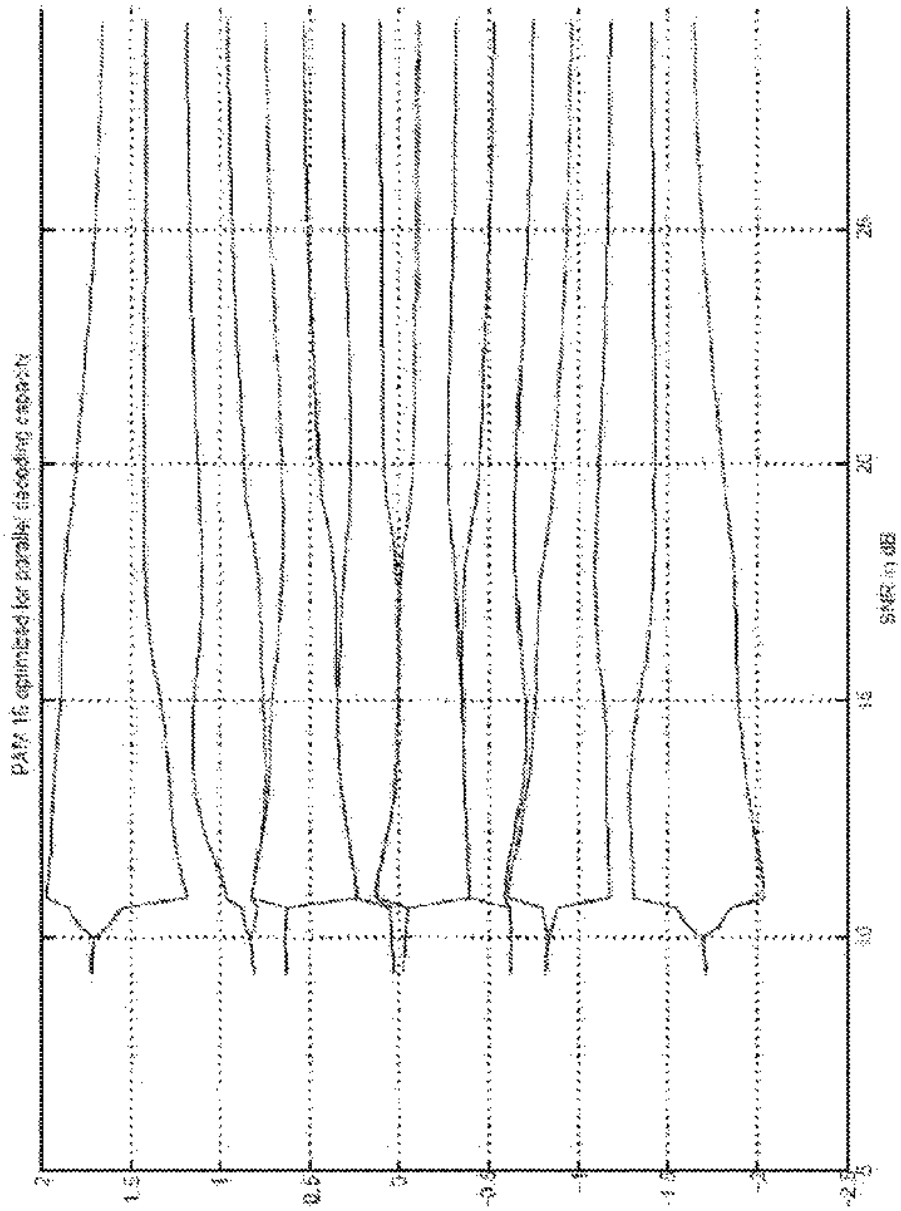

PAPR 16 optimized for parallel decoding capacity
SNR in dB
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
5
10
15
20
25

FIG. 14d

PAM-16 constellations optimized for joint capacity at different rates

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.94	15.25	18.60	22.12
x_0	-2.02	-1.96	-1.91	-1.85	-1.76
x_1	-1.16	-1.33	-1.40	-1.42	-1.42
x_2	-1.16	-1.10	-1.05	-1.10	-1.15
x_3	-0.90	-0.69	-0.82	-0.84	-0.90
x_4	-0.34	-0.69	-0.60	-0.62	-0.68
x_5	-0.34	-0.40	-0.43	-0.43	-0.47
x_6	-0.34	-0.17	-0.24	-0.26	-0.28
x_7	-0.34	-0.17	-0.09	-0.08	-0.09
x_8	0.34	0.17	0.09	0.08	0.09
x_9	0.34	0.17	0.24	0.26	0.28
x_{10}	0.34	0.40	0.43	0.43	0.47
x_{11}	0.34	0.69	0.60	0.62	0.68
x_{12}	0.90	0.69	0.82	0.84	0.90
x_{13}	1.16	1.10	1.05	1.10	1.15
x_{14}	1.16	1.33	1.40	1.42	1.42
x_{15}	2.02	1.96	1.91	1.85	1.76

FIG. 15a

PAM-16 constellations optimized for parallel decoding capacity at different

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.00	12.25	15.42	18.72	22.13
x_0	-1.72	-1.98	-1.89	-1.84	-1.75
x_1	-1.72	-1.29	-1.36	-1.42	-1.42
x_2	-0.81	1.94	1.89	1.84	1.75
x_3	-0.81	-1.17	-1.14	-1.11	-1.15
x_4	1.72	-0.38	-0.35	-0.40	-0.47
x_5	1.72	-0.65	-0.70	-0.65	-0.68
x_6	-0.62	-0.38	-0.34	-0.29	-0.28
x_7	-0.62	-0.68	-0.76	-0.83	-0.90
x_8	0.62	1.09	1.13	1.11	1.15
x_9	0.62	0.76	0.76	0.84	0.90
x_{10}	0.02	1.26	1.35	1.42	1.42
x_{11}	0.02	0.76	0.70	0.65	0.68
x_{12}	0.81	0.06	0.00	0.05	0.09
x_{13}	0.81	0.29	0.34	0.29	0.28
x_{14}	-0.02	0.06	0.00	-0.05	-0.09
x_{15}	-0.02	0.29	0.35	0.40	0.47

FIG. 15b

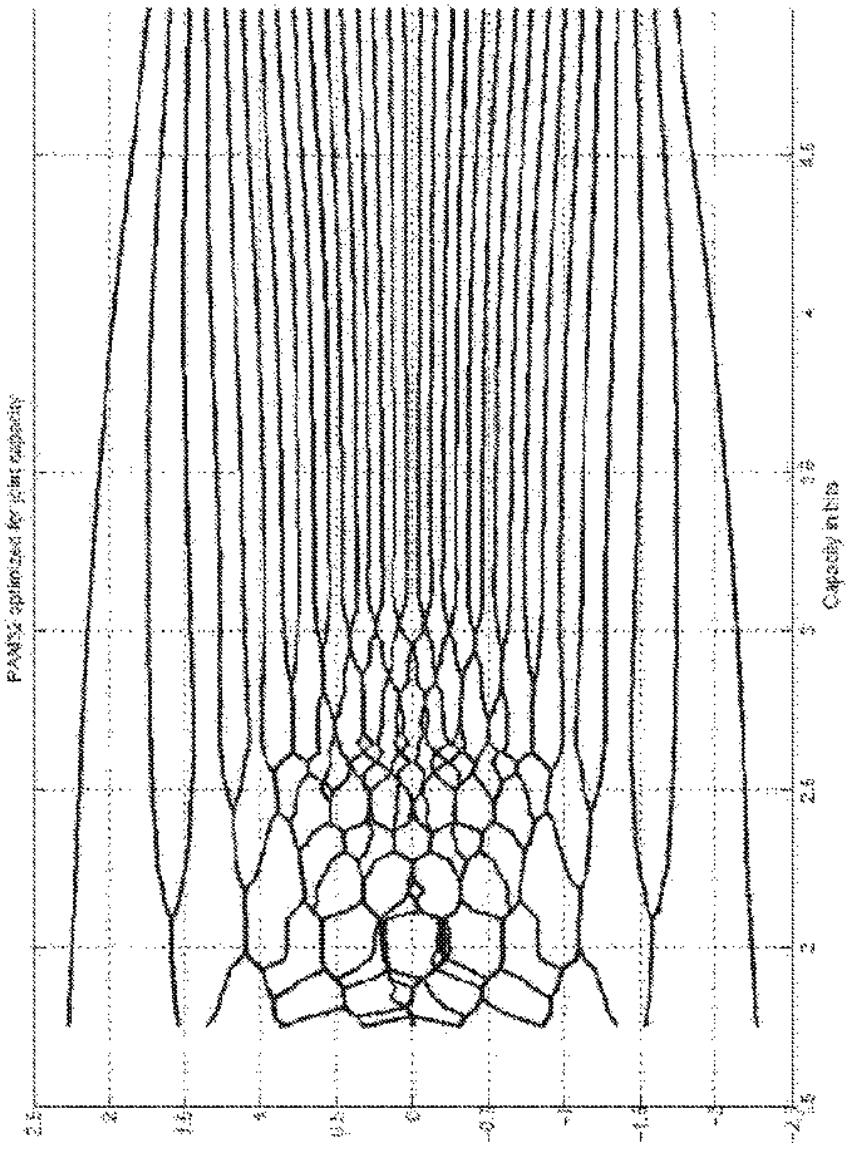


FIG. 16a

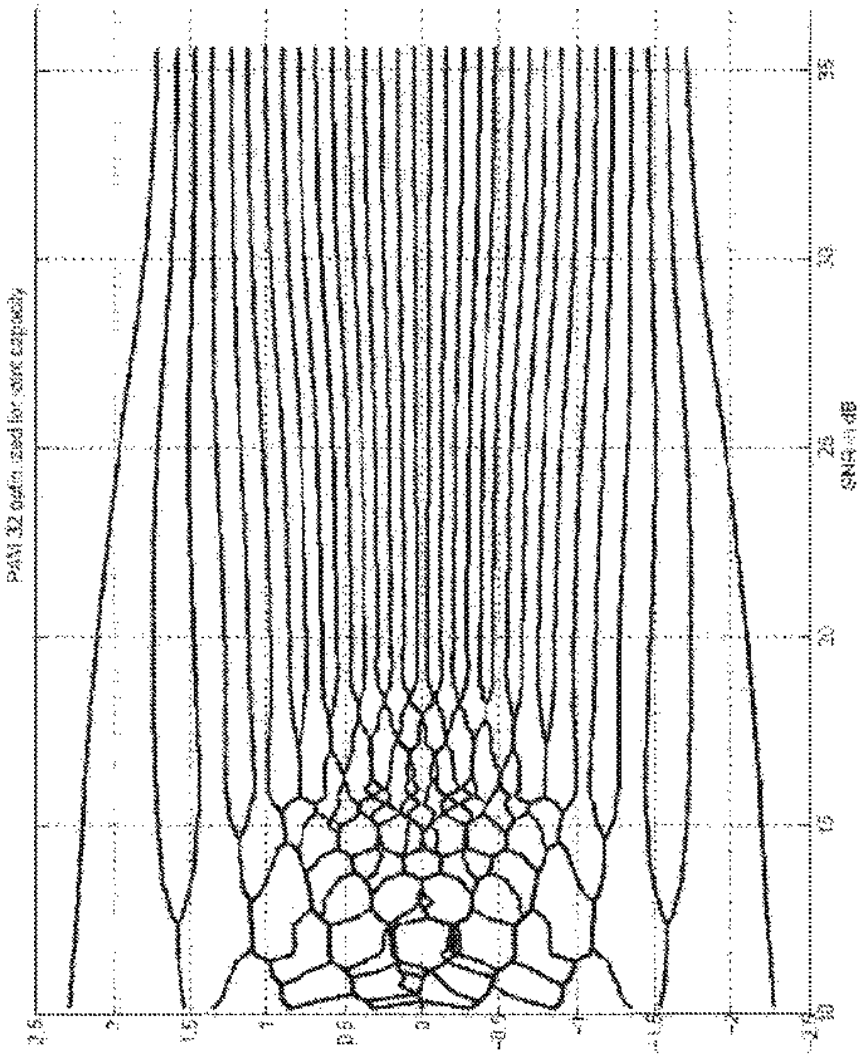


FIG. 16b

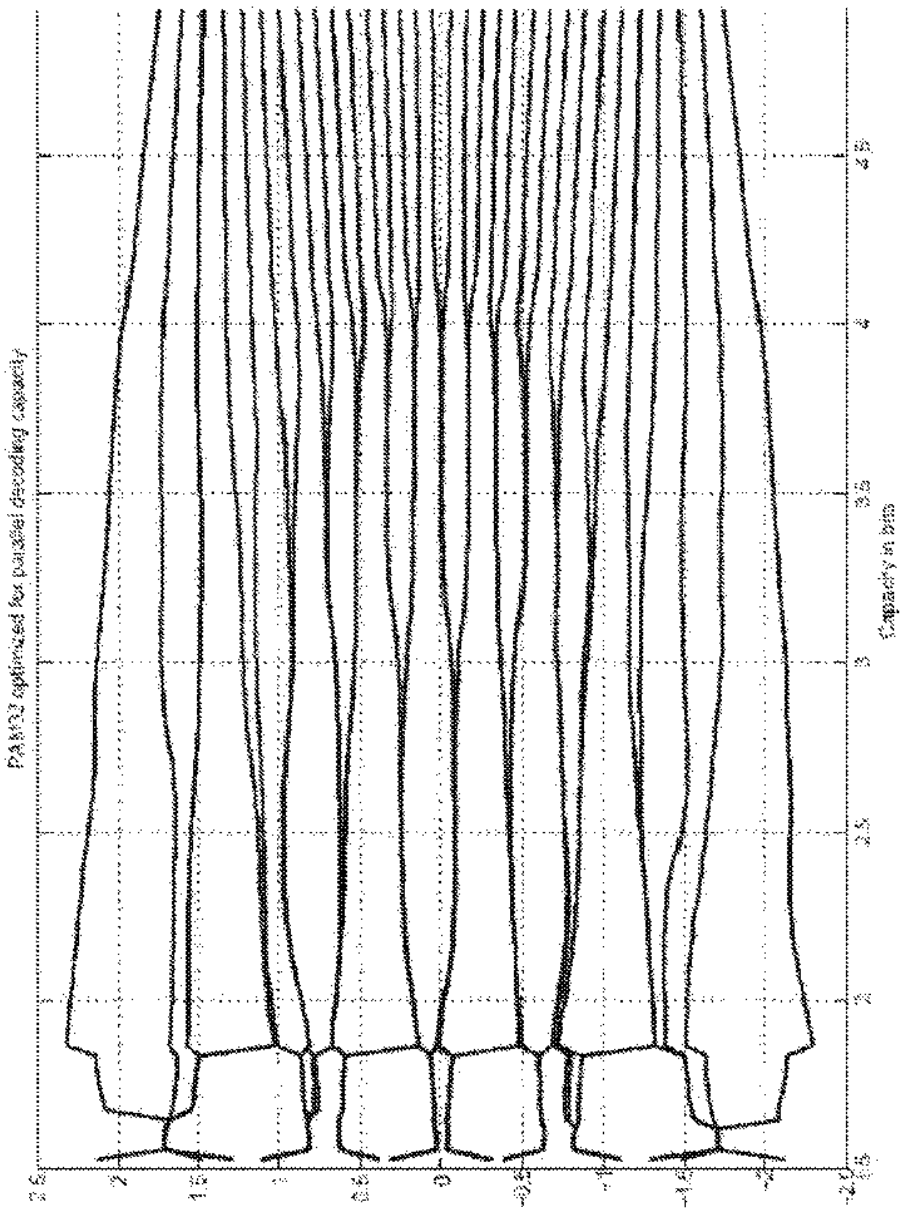

PAM32 optimized for parallel decoding capacity
Capacity in bits
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5

FIG. 16c

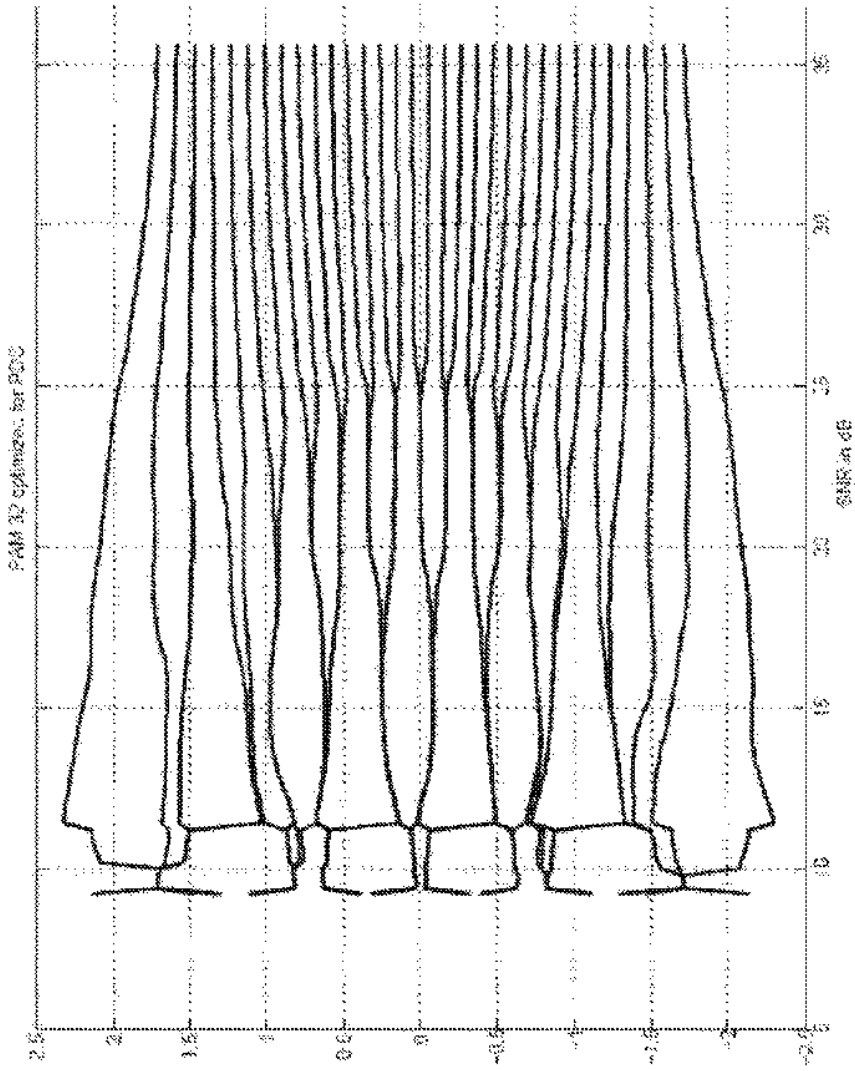


FIG. 16d

PAM-32 constellations optimized for joint capacity at different rates

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR)	11.83	15.05	18.23	21.42	24.69	28.19
x_0	-2.25	-2.19	-2.14	-2.07	-1.97	-1.85
x_1	-1.98	-1.71	-1.74	-1.74	-1.72	-1.66
x_2	-1.58	-1.46	-1.46	-1.49	-1.51	-1.50
x_3	-1.19	-1.23	-1.27	-1.29	-1.33	-1.35
x_4	-1.16	-1.13	-1.11	-1.13	-1.17	-1.21
x_5	-1.10	-0.90	-0.98	-0.99	-1.02	-1.08
x_6	-0.83	-0.90	-0.85	-0.87	-0.90	-0.95
x_7	-0.60	-0.75	-0.75	-0.76	-0.78	-0.84
x_8	-0.60	-0.58	-0.63	-0.65	-0.67	-0.73
x_9	-0.60	-0.55	-0.57	-0.56	-0.57	-0.62
x_{10}	-0.60	-0.49	-0.42	-0.46	-0.48	-0.52
x_{11}	-0.24	-0.29	-0.40	-0.38	-0.39	-0.42
x_{12}	-0.21	-0.28	-0.24	-0.29	-0.30	-0.32
x_{13}	-0.20	-0.28	-0.24	-0.21	-0.21	-0.23
x_{14}	-0.20	-0.09	-0.09	-0.12	-0.13	-0.14
x_{15}	-0.16	-0.09	-0.07	-0.04	-0.04	-0.05
x_{16}	0.16	0.09	0.07	0.04	0.04	0.05
x_{17}	0.19	0.09	0.09	0.12	0.13	0.14
x_{18}	0.21	0.28	0.24	0.21	0.21	0.23
x_{19}	0.22	0.28	0.24	0.29	0.30	0.32
x_{20}	0.23	0.28	0.41	0.38	0.39	0.42
x_{21}	0.60	0.49	0.42	0.46	0.48	0.52
x_{22}	0.60	0.58	0.57	0.56	0.57	0.62
x_{23}	0.60	0.56	0.62	0.65	0.67	0.73
x_{24}	0.60	0.75	0.75	0.76	0.78	0.84
x_{25}	0.83	0.90	0.85	0.87	0.90	0.95
x_{26}	1.10	0.90	0.98	0.99	1.02	1.08
x_{27}	1.10	1.13	1.11	1.13	1.17	1.21
x_{28}	1.10	1.23	1.27	1.29	1.33	1.35
x_{29}	1.58	1.46	1.46	1.49	1.51	1.50
x_{30}	1.98	1.71	1.74	1.74	1.72	1.66
x_{31}	2.25	2.19	2.14	2.07	1.97	1.85

FIG. 17a

PAM-32 constellation optimized for parallel decoding capacity at different

(bps)	2.0	2.5	3.0	3.5	4.0	4.5
(SNR:	12.21	15.27	18.42	21.53	24.79	28.20
x_0	-2.28	-2.16	-2.14	-2.05	-1.97	-1.85
x_1	-1.52	-1.64	-1.75	-1.74	-1.72	-1.66
x_2	2.30	2.39	-1.51	2.05	1.97	1.35
x_3	-1.39	-1.48	-1.43	-1.40	-1.56	-1.46
x_4	1.59	1.53	2.14	-0.96	-1.02	1.85
x_5	-1.31	-1.23	1.75	-1.15	-1.17	1.66
x_6	1.47	1.68	-1.07	-0.91	-0.60	-1.21
x_7	-1.31	-1.24	-1.04	-1.26	-1.33	-1.08
x_8	-0.48	-0.33	-0.36	-0.17	-0.17	-0.42
x_9	-0.72	-0.76	-0.36	-0.34	-0.31	-0.52
x_{10}	-0.48	-0.48	-0.62	-0.17	-0.13	-0.73
x_{11}	-0.73	-0.76	-0.62	-0.34	-0.35	-0.62
x_{12}	-0.48	-0.82	-0.29	-0.71	-0.67	-0.83
x_{13}	-0.76	-0.86	-0.29	-0.52	-0.55	-0.23
x_{14}	-0.48	-0.42	-0.77	-0.72	-0.77	-0.64
x_{15}	-0.76	-0.86	-0.77	-0.52	-0.88	-0.96
x_{16}	0.87	0.98	1.07	1.49	1.51	1.21
x_{17}	0.61	0.63	1.04	1.28	1.33	1.08
x_{18}	0.87	0.98	0.77	1.74	1.72	0.84
x_{19}	0.60	0.63	0.75	1.15	1.17	0.96
x_{20}	1.07	1.13	1.51	0.72	0.79	1.35
x_{21}	0.68	0.59	1.45	0.91	0.96	1.49
x_{22}	1.08	1.10	0.62	0.71	0.67	0.73
x_{23}	0.66	0.60	0.62	0.96	1.05	0.62
x_{24}	-0.01	-0.08	0.00	0.00	0.01	0.05
x_{25}	0.17	0.25	0.02	0.17	0.15	0.13
x_{26}	-0.01	-0.08	0.29	0.00	-0.01	0.33
x_{27}	0.17	0.25	0.20	0.17	0.17	0.23
x_{28}	-0.01	-0.08	-0.02	-0.52	0.48	-0.05
x_{29}	0.17	0.25	-0.02	-0.34	0.35	-0.14
x_{30}	-0.01	-0.08	0.56	0.52	0.55	0.42
x_{31}	0.17	0.25	-0.56	0.34	0.31	-0.52

FIG. 17b

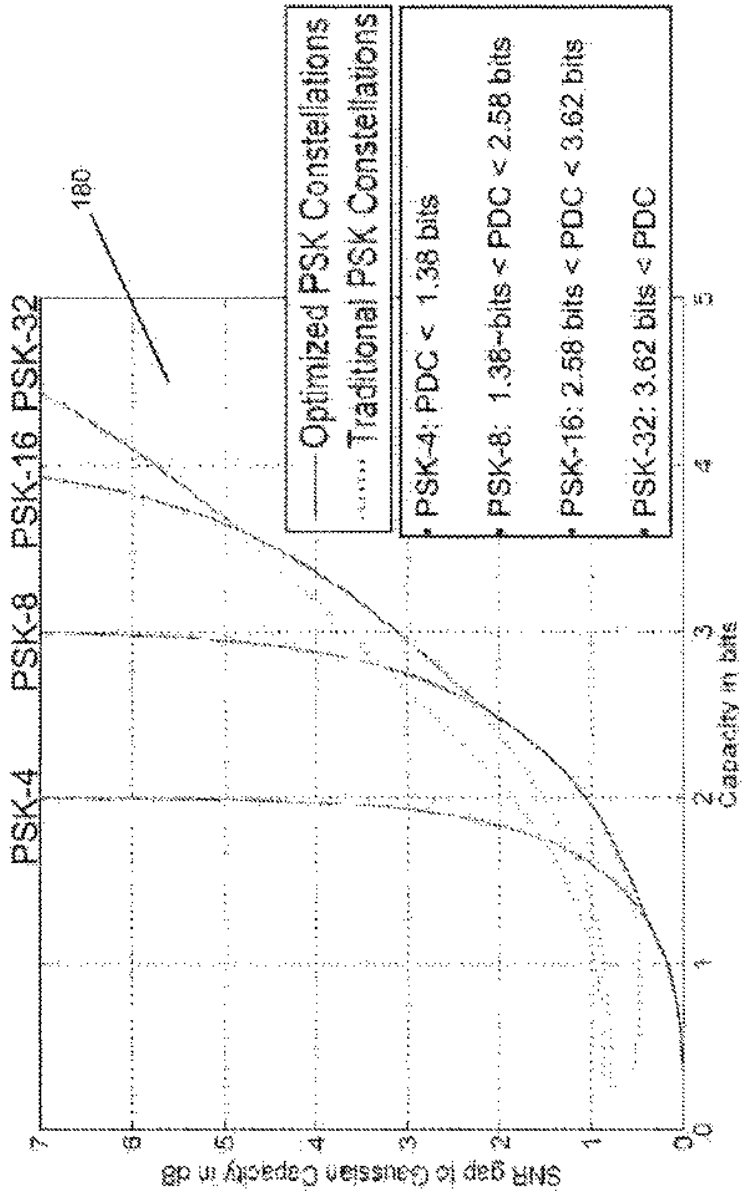

180
PSK-4
PSK-8
PSK-16
PSK-32
Optimized PSK Constellations
Traditional PSK Constellations
PSK-4: PDC < 1.38 bits
PSK-8: 1.38<bits < PDC < 2.58 bits
PSK-16: 2.58 bits < PDC < 3.62 bits
PSK-32: 3.62 bits < PDC
SNR gap to Gaussian Capacity in dB
Capacity in bits
0
1
2
3
4
5
6
7

FIG. 18

Points location in degrees
0
50
100
150
200
250
300
350
SNR in dB
0
2
4
6
8
10
12
14
16
18
20

FIG. 19

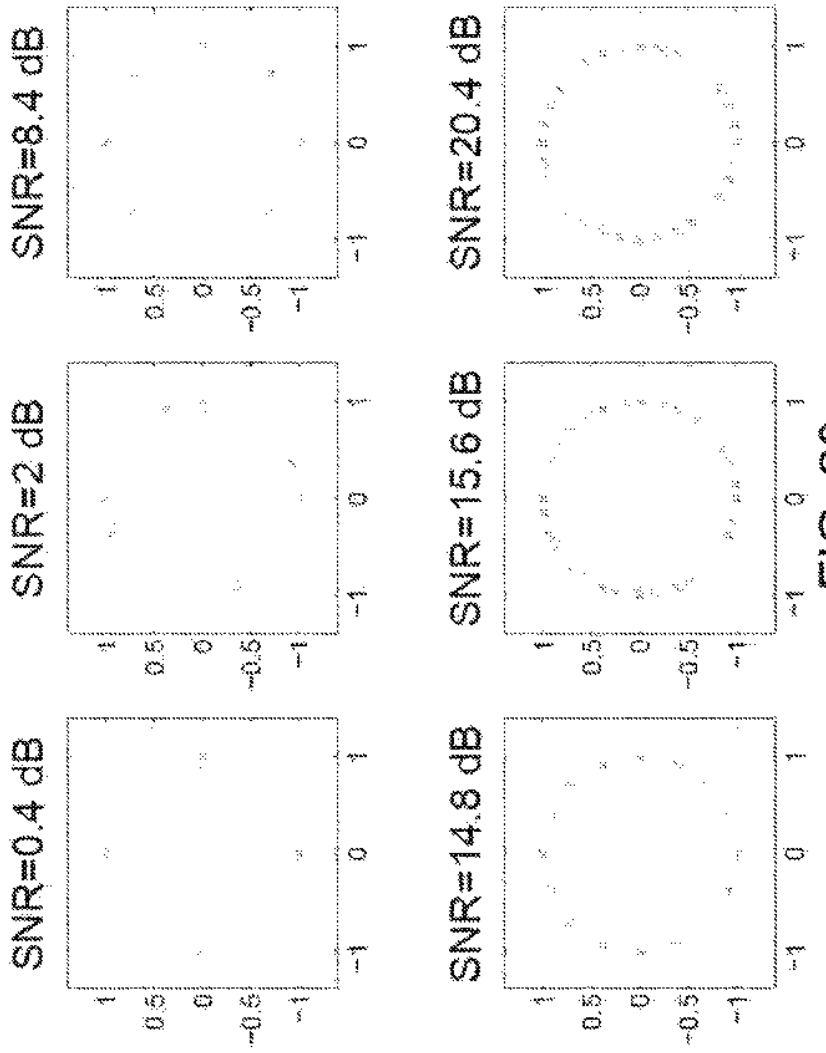


FIG. 20

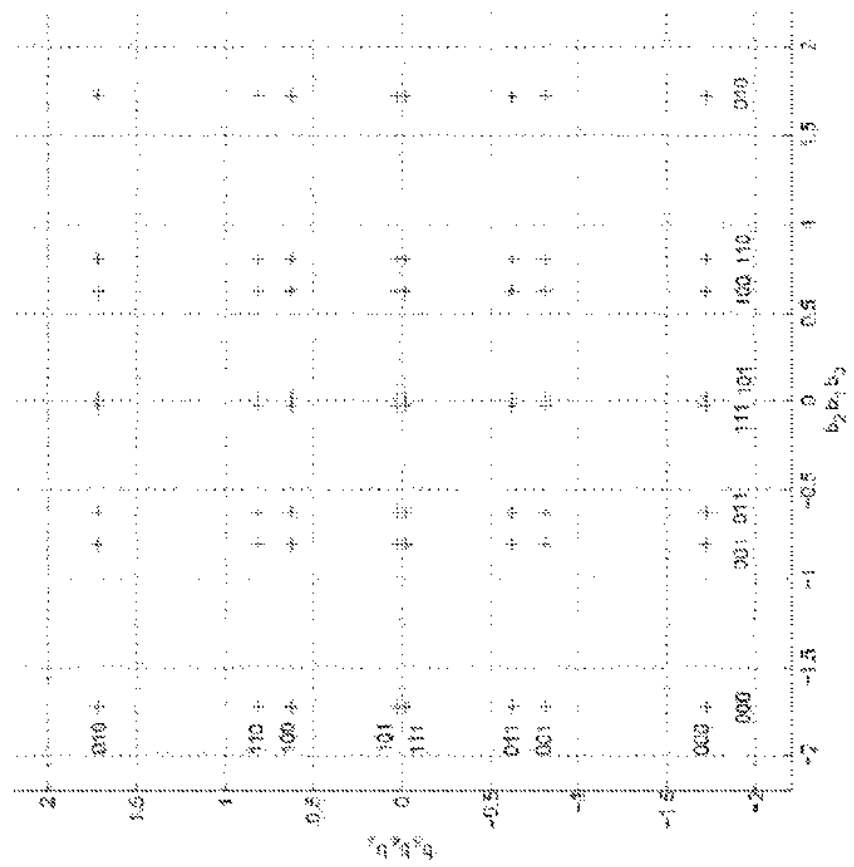

GAM-64 constructed from 2 orthogonal PA39-8 optimized for PEGet 2 basis/dimension
010
110
100
101
111
011
001
000
010
110
100
101
111
011
001
000
b_2 b_1 b_3

FIG. 21

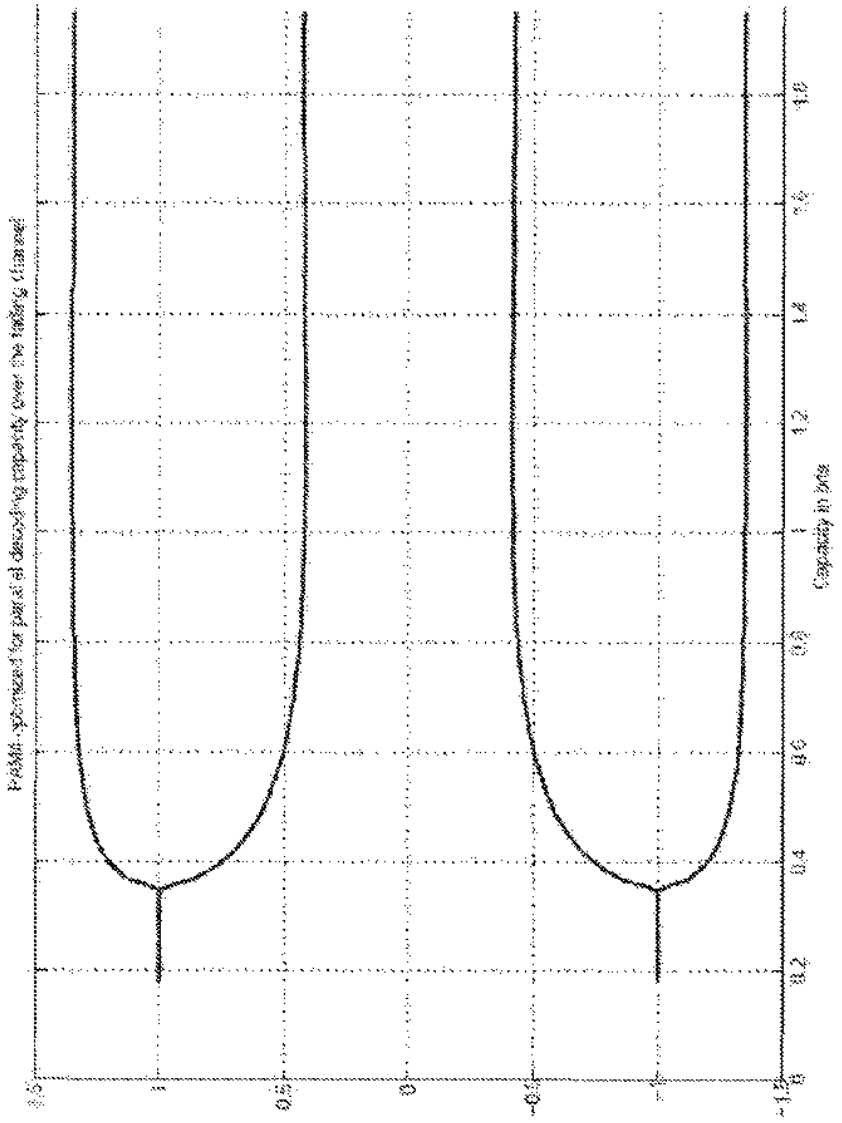


FIG. 22a

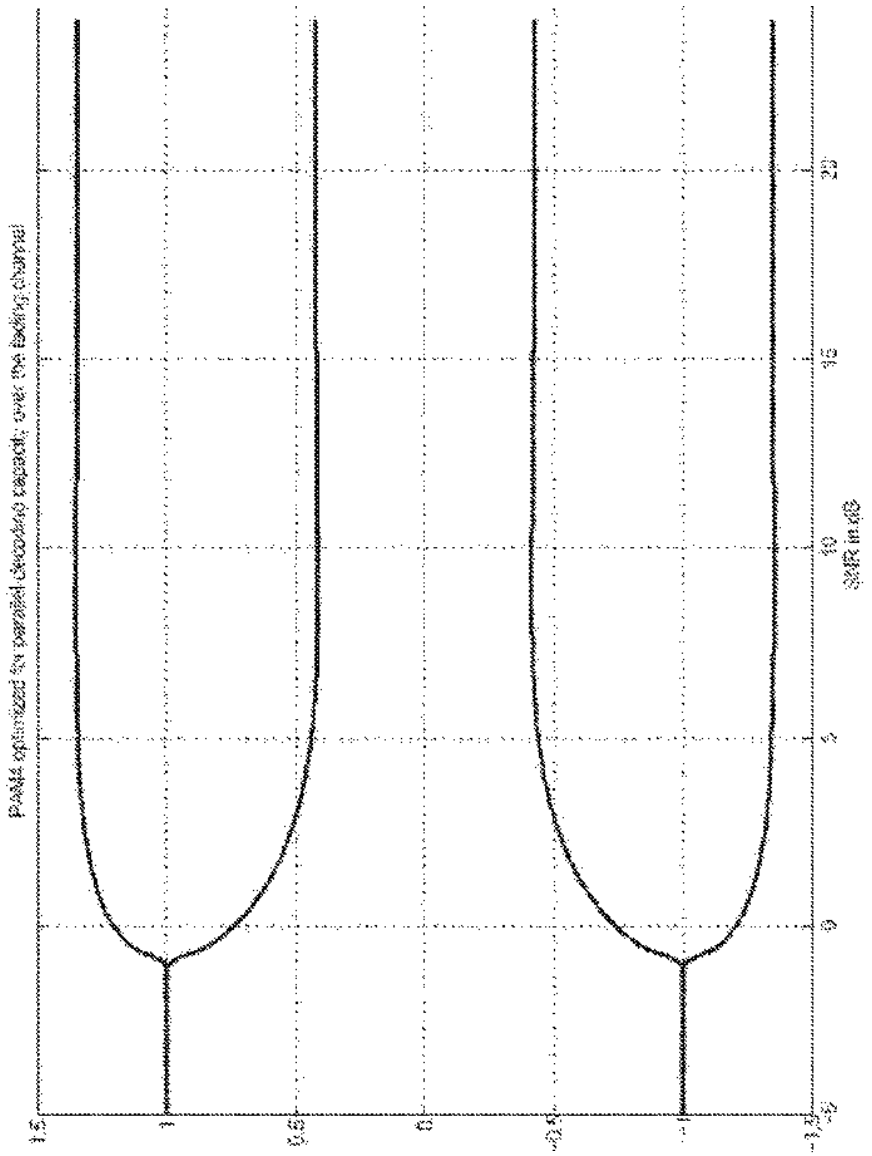

Packet optimized for parallel decoding capacity over the fading channel
SNR in dB

FIG. 22b

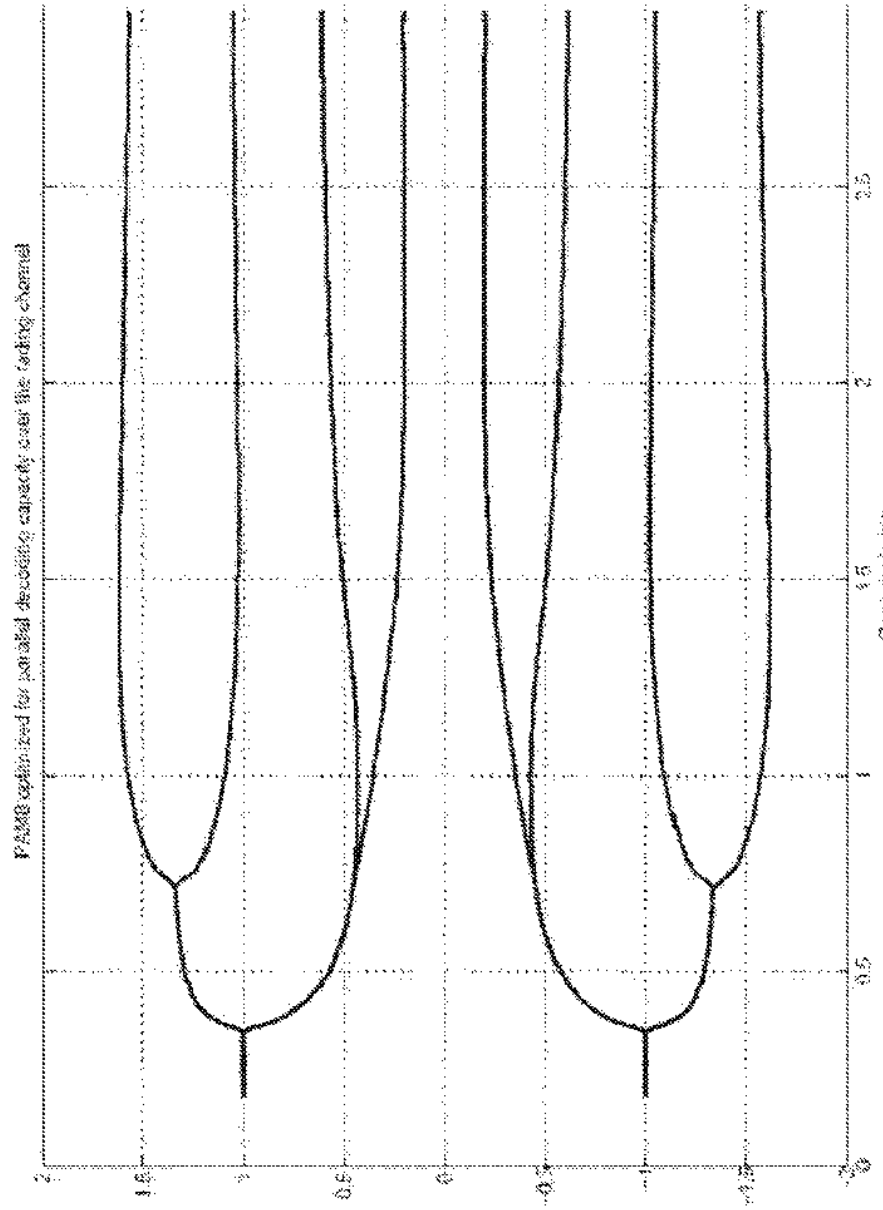

PAM8 optimized for parallel decoding capacity over the fading channel
Capacity in bits
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
2.5
2
1.5
1
0.5
0

FIG. 23a

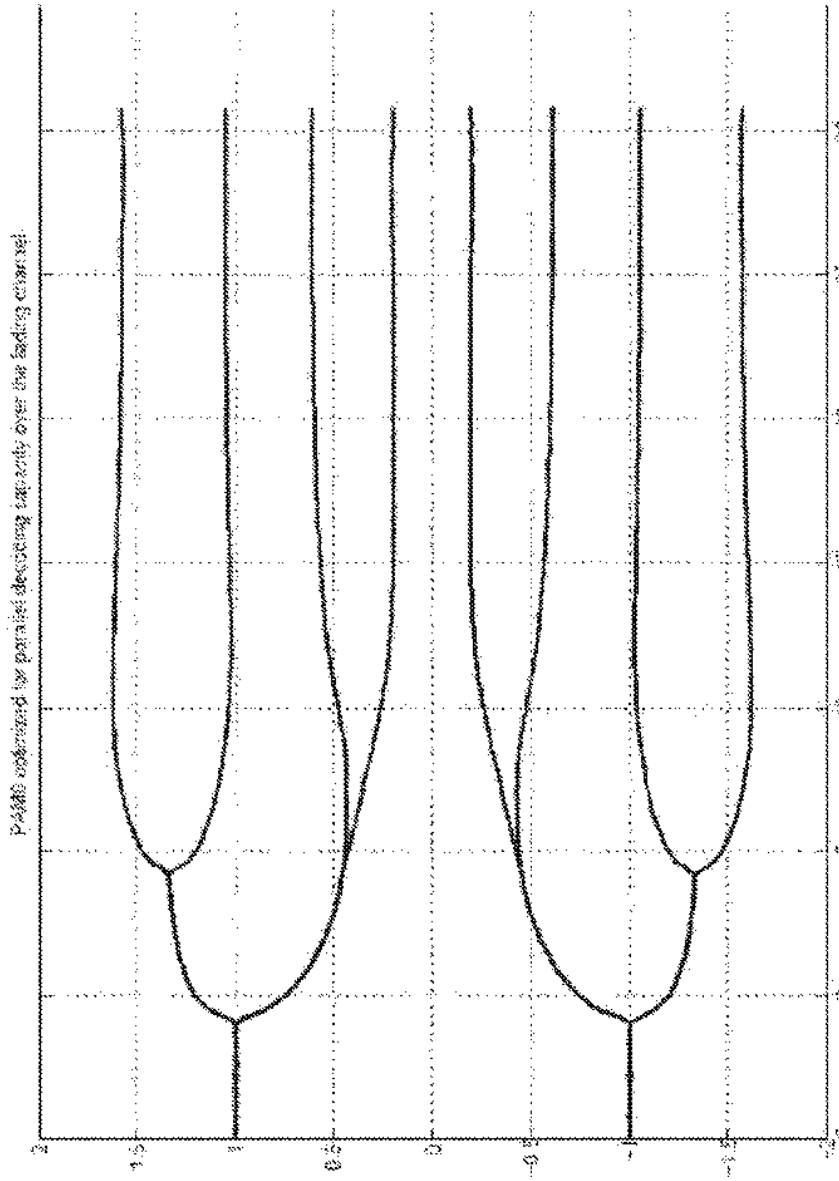

P-400b codeword bit pairwise decoding capacity over the fading channel
Eb/No in dB

FIG. 23b

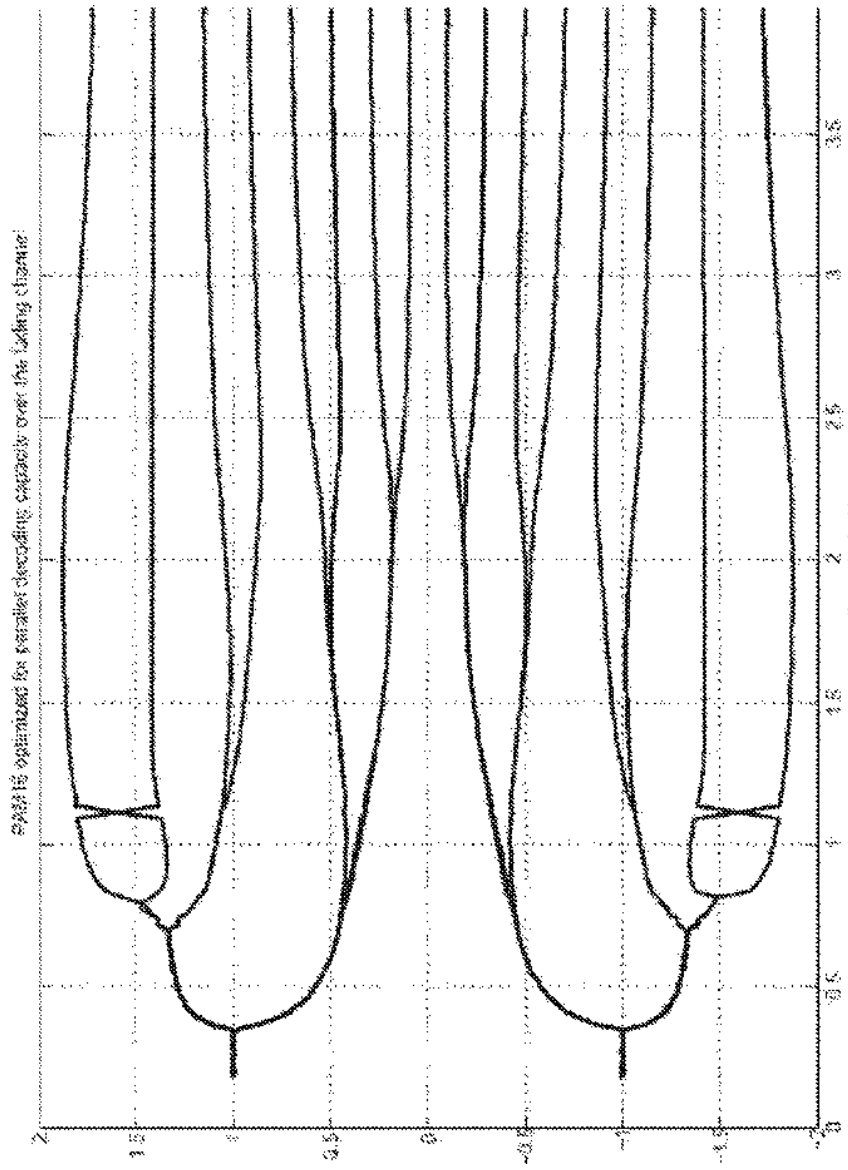


FIG. 24a

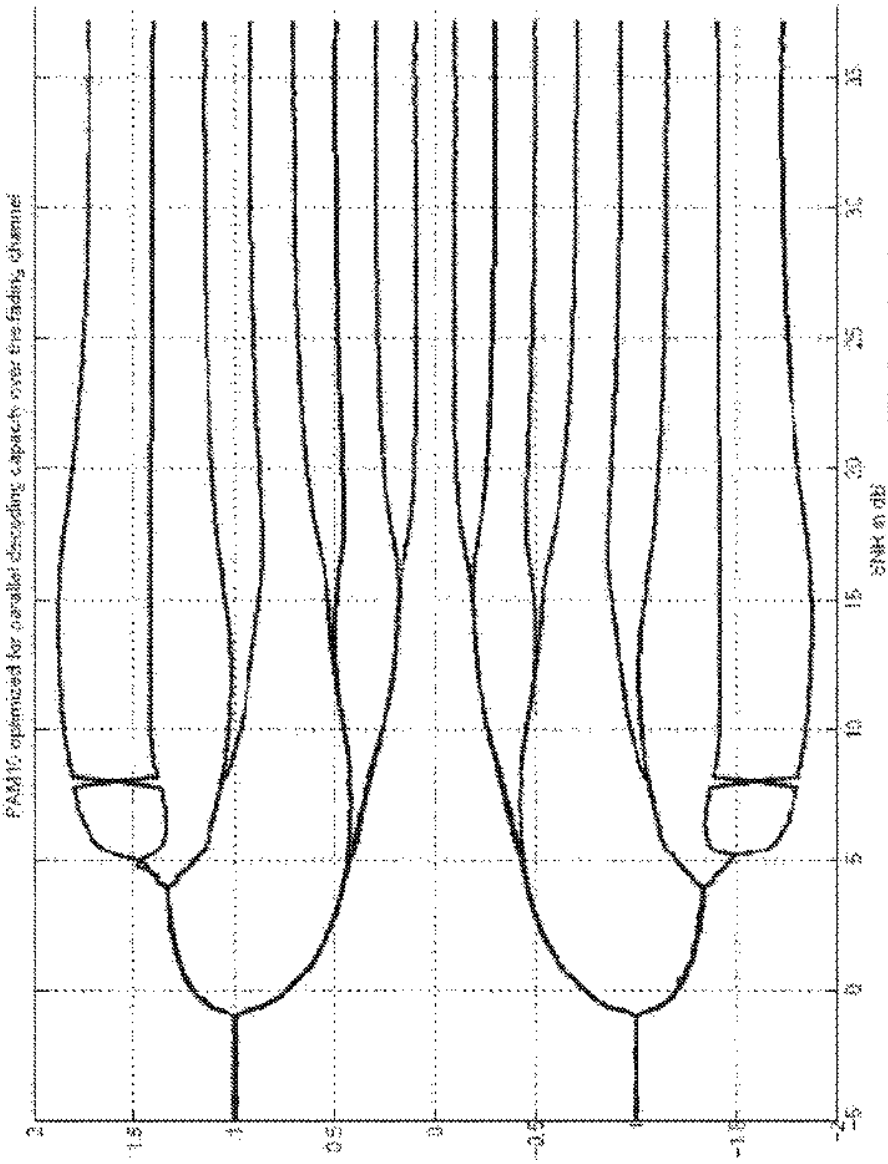

PAM16: optimized for parallel decoding capacity over the fading channel
SNR in dB

FIG. 24b

Design #	SNR	Opt. Cap	Std. Cap	Gain (bits)	Gain %
1	-5.0000	0.1982	0.1980	0.0002	0.0833
2	-4.8000	0.2063	0.2061	0.0002	0.0924
3	-4.6000	0.2147	0.2145	0.0002	0.1024
4	-4.4000	0.2234	0.2232	0.0003	0.1133
5	-4.2000	0.2324	0.2321	0.0003	0.1252
6	-4.0000	0.2417	0.2414	0.0003	0.1382
7	-3.8000	0.2513	0.2510	0.0004	0.1523
8	-3.6000	0.2613	0.2608	0.0004	0.1677
9	-3.4000	0.2715	0.2710	0.0005	0.1845
10	-3.2000	0.2821	0.2815	0.0006	0.2026
11	-3.0000	0.2930	0.2924	0.0006	0.2222
12	-2.8000	0.3043	0.3035	0.0007	0.2433
13	-2.6000	0.3159	0.3150	0.0008	0.2662
14	-2.4000	0.3278	0.3269	0.0010	0.2907
15	-2.2000	0.3401	0.3390	0.0011	0.3171
16	-2.0000	0.3528	0.3516	0.0012	0.3453
17	-1.8000	0.3658	0.3644	0.0014	0.3755
18	-1.6000	0.3792	0.3776	0.0015	0.4078
19	-1.4000	0.3929	0.3912	0.0017	0.4422
20	-1.2000	0.4070	0.4051	0.0019	0.4788
21	-1.0000	0.4215	0.4194	0.0022	0.5176
22	-0.8000	0.4364	0.4340	0.0024	0.5587
23	-0.6000	0.4516	0.4489	0.0027	0.6021
24	-0.4000	0.4672	0.4642	0.0030	0.6479
25	-0.2000	0.4832	0.4799	0.0033	0.6961
26	0.0000	0.4996	0.4959	0.0037	0.7468
27	0.0050	0.5000	0.4963	0.0037	0.7481
28	0.2000	0.5163	0.5122	0.0041	0.7999
29	0.4000	0.5334	0.5289	0.0045	0.8564
30	0.6000	0.5509	0.5459	0.0050	0.9133
31	0.8000	0.5688	0.5833	0.0055	0.9736
32	1.0000	0.5870	0.5810	0.0060	1.0363
33	1.2000	0.6056	0.5990	0.0066	1.1012
34	1.4000	0.6245	0.6173	0.0072	1.1684
35	1.6000	0.6439	0.6360	0.0079	1.2377
36	1.8000	0.6635	0.6550	0.0086	1.3090
37	2.0000	0.6835	0.6742	0.0093	1.3822
38	2.2000	0.7039	0.6938	0.0101	1.4572
39	2.4000	0.7246	0.7137	0.0109	1.5338
40	2.6000	0.7457	0.7339	0.0118	1.6117

Table 1

FIG. 25

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7570	0.7543	0.0128	1.6809
42	3.0000	0.7887	0.7750	0.0137	1.7709
43	3.2000	0.8107	0.7960	0.0147	1.8516
44	3.4000	0.8331	0.8173	0.0158	1.9326
45	3.6000	0.8557	0.8368	0.0169	2.0134
46	3.8000	0.8786	0.8606	0.0180	2.0936
47	4.0000	0.9018	0.8826	0.0192	2.1731
48	4.2000	0.9252	0.9049	0.0204	2.2508
49	4.4000	0.9490	0.9274	0.0216	2.3263
50	4.6000	0.9729	0.9501	0.0228	2.3989
51	4.8000	0.9971	0.9731	0.0240	2.4677
52	4.8237	1.0000	0.9758	0.0242	2.4785
53	5.0000	1.0215	0.9963	0.0252	2.5318
54	5.2000	1.0461	1.0197	0.0264	2.5905
55	5.4000	1.0710	1.0433	0.0276	2.6491
56	5.6000	1.0960	1.0672	0.0289	2.7048
57	5.8000	1.1213	1.0912	0.0301	2.7584
58	6.0000	1.1468	1.1154	0.0313	2.8099
59	6.2000	1.1724	1.1398	0.0326	2.8593
60	6.4000	1.1983	1.1644	0.0339	2.9064
61	6.6000	1.2243	1.1892	0.0351	2.9511
62	6.8000	1.2505	1.2142	0.0363	2.9934
63	7.0000	1.2769	1.2393	0.0376	3.0322
64	7.2000	1.3034	1.2646	0.0388	3.0683
65	7.4000	1.3300	1.2901	0.0399	3.0952
66	7.6000	1.3567	1.3157	0.0410	3.1187
67	7.8000	1.3836	1.3415	0.0421	3.1367
68	8.0000	1.4105	1.3674	0.0431	3.1506
69	8.2000	1.4376	1.3935	0.0440	3.1605
70	8.4000	1.4647	1.4197	0.0450	3.1665
71	8.6000	1.4919	1.4461	0.0458	3.1682
72	8.6592	1.5000	1.4539	0.0461	3.1678
73	8.8000	1.5192	1.4726	0.0466	3.1655
74	9.0000	1.5466	1.4992	0.0474	3.1584
75	9.2000	1.5740	1.5260	0.0480	3.1466
76	9.4000	1.6015	1.5529	0.0486	3.1299
77	9.6000	1.6290	1.5799	0.0491	3.1084
78	9.8000	1.6565	1.6070	0.0496	3.0839
79	10.0000	1.6843	1.6343	0.0500	3.0574
80	10.2000	1.7120	1.6616	0.0503	3.0290

Table 2

FIG. 26

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7396	1.6891	0.0505	2.9896
82	10.6000	1.7676	1.7167	0.0509	2.9662
83	10.8000	1.7955	1.7443	0.0511	2.9317
84	11.0000	1.8234	1.7721	0.0513	2.8951
85	11.2000	1.8514	1.8000	0.0514	2.8562
86	11.4000	1.8794	1.8279	0.0515	2.8149
87	11.6000	1.9074	1.8560	0.0514	2.7710
88	11.8000	1.9354	1.8841	0.0513	2.7244
89	12.0000	1.9634	1.9123	0.0512	2.6750
90	12.2000	1.9915	1.9406	0.0509	2.6226
91	12.2611	2.0000	1.9492	0.0508	2.6059
92	12.4000	2.0195	1.9689	0.0505	2.5670
93	12.6000	2.0474	1.9973	0.0501	2.5084
94	12.8000	2.0754	2.0258	0.0496	2.4466
95	13.0000	2.1033	2.0544	0.0489	2.3816
96	13.2000	2.1311	2.0829	0.0482	2.3135
97	13.4000	2.1589	2.1116	0.0473	2.2424
98	13.6000	2.1867	2.1402	0.0464	2.1683
99	13.8000	2.2145	2.1688	0.0454	2.0915
100	14.0000	2.2419	2.1976	0.0442	2.0121
101	14.2000	2.2693	2.2263	0.0430	1.9306
102	14.4000	2.2967	2.2550	0.0417	1.8471
103	14.6000	2.3239	2.2837	0.0402	1.7622
104	14.8000	2.3510	2.3122	0.0388	1.6762
105	15.0000	2.3779	2.3407	0.0372	1.5896
106	15.2000	2.4047	2.3691	0.0356	1.5030
107	15.4000	2.4313	2.3973	0.0340	1.4168
108	15.6000	2.4579	2.4253	0.0323	1.3317
109	15.8000	2.4837	2.4531	0.0306	1.2481
110	15.9256	2.5000	2.4704	0.0296	1.1965
111	16.0000	2.5096	2.4806	0.0289	1.1684
112	16.2000	2.5351	2.5078	0.0273	1.0871
113	16.4000	2.5603	2.5347	0.0256	1.0105
114	16.6000	2.5851	2.5611	0.0240	0.9369
115	16.8000	2.6095	2.5871	0.0224	0.8664
116	17.0000	2.6334	2.6125	0.0209	0.7992
117	17.2000	2.6568	2.6374	0.0194	0.7354
118	17.4000	2.6796	2.6616	0.0180	0.6750
119	17.6000	2.7017	2.6851	0.0166	0.6179
120	17.9000	2.7233	2.7080	0.0153	0.5642

Table 3

FIG. 27

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.7441	2.7300	0.0140	0.5137
122	18.2000	2.7841	2.7713	0.0128	0.4604
123	18.4000	2.7834	2.7717	0.0117	0.4222
124	18.6000	2.8016	2.7912	0.0106	0.3811
125	18.8000	2.8194	2.8097	0.0096	0.3428
126	19.0000	2.8360	2.8274	0.0087	0.3072
127	19.2000	2.8518	2.8440	0.0078	0.2744
128	19.4000	2.8667	2.8597	0.0070	0.2442
129	19.6000	2.8806	2.8743	0.0062	0.2164
130	19.8000	2.8936	2.8880	0.0055	0.1910
131	20.0000	2.9055	2.9007	0.0049	0.1678
132	20.2000	2.9167	2.9125	0.0043	0.1468
133	20.4000	2.9269	2.9232	0.0037	0.1277
134	20.6000	2.9363	2.9330	0.0032	0.1104
135	20.8000	2.9448	2.9420	0.0028	0.0951
136	21.0000	2.9524	2.9500	0.0024	0.0813
137	21.2000	2.9593	2.9572	0.0020	0.0692
138	21.4000	2.9654	2.9636	0.0017	0.0584
139	21.6000	2.9708	2.9693	0.0015	0.0491
140	21.8000	2.9756	2.9743	0.0012	0.0409
141	22.0000	2.9797	2.9786	0.0010	0.0338
142	22.2000	2.9832	2.9824	0.0008	0.0278
143	22.4000	2.9863	2.9856	0.0007	0.0226
144	22.6000	2.9889	2.9884	0.0005	0.0183
145	22.8000	2.9911	2.9907	0.0004	0.0145
146	23.0000	2.9929	2.9926	0.0003	0.0115
147	23.2000	2.9944	2.9942	0.0003	0.0092
148	23.4000	2.9957	2.9955	0.0002	0.0071

Table 4

FIG. 28

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-8.450	-2.051	-2.051	-2.048	2.035	2.057	2.057	8.450
2	-8.446	-2.061	-2.059	-2.046	2.055	2.055	2.055	8.446
3	-8.441	-2.064	-2.064	-2.057	2.061	2.061	2.062	8.441
4	-8.436	-2.067	-2.063	-2.068	2.067	2.067	2.069	8.436
5	-8.432	-2.075	-2.076	-2.072	2.074	2.075	2.074	8.432
6	-8.426	-2.082	-2.081	-2.081	2.081	2.081	2.081	8.426
7	-8.421	-2.089	-2.089	-2.089	2.089	2.089	2.089	8.421
8	-8.416	-2.097	-2.097	-2.095	2.096	2.095	2.097	8.416
9	-8.410	-2.104	-2.104	-2.104	2.102	2.105	2.105	8.410
10	-8.404	-2.112	-2.112	-2.112	2.111	2.112	2.112	8.404
11	-8.397	-2.120	-2.120	-2.120	2.120	2.120	2.120	8.397
12	-8.391	-2.129	-2.129	-2.129	2.129	2.129	2.129	8.391
13	-8.384	-2.138	-2.138	-2.138	2.138	2.138	2.138	8.384
14	-8.377	-2.147	-2.147	-2.147	2.147	2.147	2.147	8.377
15	-8.370	-2.156	-2.156	-2.156	2.156	2.156	2.156	8.370
16	-8.362	-2.166	-2.166	-2.166	2.166	2.166	2.166	8.362
17	-8.354	-2.176	-2.176	-2.176	2.176	2.176	2.176	8.354
18	-8.347	-2.186	-2.186	-2.186	2.186	2.186	2.186	8.347
19	-8.338	-2.196	-2.196	-2.196	2.196	2.196	2.196	8.338
20	-8.330	-2.207	-2.207	-2.207	2.207	2.207	2.207	8.330
21	-8.322	-2.218	-2.218	-2.218	2.217	2.218	2.218	8.322
22	-8.313	-2.228	-2.228	-2.228	2.228	2.228	2.228	8.313
23	-8.304	-2.239	-2.239	-2.239	2.239	2.239	2.239	8.304
24	-8.295	-2.250	-2.250	-2.250	2.250	2.250	2.250	8.295
25	-8.286	-2.261	-2.261	-2.261	2.261	2.261	2.261	8.286
26	-8.277	-2.273	-2.273	-2.272	2.273	2.273	2.273	8.277
27	-8.277	-2.273	-2.273	-2.273	2.273	2.273	2.272	8.277
28	-8.268	-2.284	-2.284	-2.284	2.284	2.284	2.284	8.268
29	-8.258	-2.295	-2.295	-2.295	2.295	2.295	2.295	8.258
30	-8.249	-2.306	-2.306	-2.306	2.306	2.306	2.306	8.249
31	-8.240	-2.317	-2.317	-2.317	2.317	2.317	2.317	8.240
32	-8.230	-2.328	-2.328	-2.326	2.328	2.328	2.328	8.230
33	-8.221	-2.339	-2.339	-2.339	2.339	2.339	2.339	8.221
34	-8.212	-2.350	-2.350	-2.350	2.350	2.350	2.350	8.212
35	-8.202	-2.361	-2.361	-2.361	2.361	2.361	2.361	8.202
36	-8.193	-2.372	-2.372	-2.372	2.372	2.372	2.372	8.193
37	-8.184	-2.382	-2.382	-2.382	2.382	2.382	2.382	8.184
38	-8.175	-2.392	-2.392	-2.392	2.392	2.392	2.392	8.175
39	-8.166	-2.402	-2.402	-2.402	2.402	2.402	2.402	8.166
40	-8.157	-2.412	-2.412	-2.412	2.412	2.412	2.412	8.157

Table 5

FIG. 29

	Label							
Design #	0	1	2	3	4	5	6	7
41	-8.149	-2.422	-2.422	-2.422	2.422	2.422	2.422	8.149
42	-8.141	-2.431	-2.431	-2.431	2.431	2.431	2.431	8.141
43	-8.132	-2.440	-2.440	-2.440	2.440	2.440	2.440	8.132
44	-8.124	-2.449	-2.449	-2.449	2.449	2.449	2.449	8.124
45	-8.117	-2.458	-2.458	-2.458	2.458	2.458	2.458	8.117
46	-8.109	-2.466	-2.466	-2.466	2.466	2.466	2.466	8.109
47	-8.102	-2.474	-2.474	-2.474	2.474	2.474	2.474	8.102
48	-8.095	-2.481	-2.481	-2.481	2.481	2.481	2.481	8.095
49	-8.088	-2.489	-2.489	-2.489	2.489	2.489	2.489	8.088
50	-8.082	-2.496	-2.496	-2.496	2.496	2.496	2.496	8.082
51	-8.075	-2.503	-2.503	-2.503	2.503	2.503	2.503	8.075
52	-8.075	-2.503	-2.503	-2.503	2.503	2.503	2.503	8.075
53	-8.069	-2.509	-2.509	-2.509	2.509	2.509	2.509	8.069
54	-8.063	-2.513	-2.513	-2.513	2.391	2.391	2.761	8.061
55	-8.031	-2.900	-2.900	-1.639	1.639	2.900	2.900	8.031
56	-8.012	-2.987	-2.987	-1.403	1.403	2.987	2.987	8.012
57	-7.993	-3.055	-3.055	-1.199	1.199	3.055	3.055	7.993
58	-7.976	-3.111	-3.111	-1.012	1.012	3.111	3.111	7.976
59	-7.960	-3.158	-3.158	-0.832	0.832	3.158	3.158	7.960
60	-7.945	-3.199	-3.199	-0.649	0.649	3.199	3.199	7.945
61	-7.930	-3.234	-3.234	-0.445	0.445	3.234	3.234	7.930
62	-7.916	-3.265	-3.265	-0.123	0.123	3.265	3.265	7.916
63	-7.906	-3.279	-3.279	0.000	0.000	3.279	3.279	7.906
64	-7.896	-3.291	-3.291	0.000	0.000	3.291	3.291	7.896
65	-7.885	-3.303	-3.303	0.000	0.000	3.303	3.303	7.885
66	-7.875	-3.316	-3.316	0.000	0.000	3.316	3.316	7.875
67	-7.862	-3.486	-3.166	0.000	0.000	3.166	3.486	7.862
68	-7.847	-3.621	-3.052	0.000	0.000	3.052	3.621	7.847
69	-7.833	-3.711	-2.979	0.000	0.000	2.979	3.711	7.833
70	-7.819	-3.782	-2.925	0.000	0.000	2.925	3.782	7.819
71	-7.806	-3.843	-2.881	0.000	0.000	2.881	3.843	7.806
72	-7.802	-3.860	-2.870	0.000	0.000	2.870	3.860	7.802
73	-7.793	-3.897	-2.844	0.000	0.000	2.844	3.897	7.793
74	-7.780	-3.945	-2.812	0.000	0.000	2.812	3.945	7.780
75	-7.768	-3.990	-2.783	0.000	0.000	2.783	3.990	7.768
76	-7.756	-4.031	-2.756	0.000	0.000	2.756	4.031	7.756
77	-7.744	-4.075	-2.720	-0.164	0.164	2.720	4.075	7.744
78	-7.730	-4.131	-2.686	-0.362	0.362	2.686	4.131	7.730
79	-7.717	-4.178	-2.603	-0.471	0.471	2.603	4.178	7.717
80	-7.704	-4.219	-2.560	-0.549	0.549	2.560	4.219	7.704

Table 6

FIG. 30

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-7.692	-4.255	-2.524	-0.608	0.608	2.524	4.255	7.692
82	-7.680	-4.286	-2.494	-0.654	0.654	2.494	4.286	7.680
83	-7.669	-4.314	-2.471	-0.690	0.690	2.471	4.314	7.669
84	-7.657	-4.340	-2.452	-0.718	0.718	2.452	4.340	7.657
85	-7.647	-4.363	-2.438	-0.740	0.740	2.438	4.363	7.647
86	-7.636	-4.385	-2.428	-0.757	0.757	2.428	4.385	7.636
87	-7.625	-4.405	-2.422	-0.770	0.770	2.422	4.405	7.625
88	-7.614	-4.424	-2.418	-0.780	0.780	2.418	4.424	7.614
89	-7.603	-4.441	-2.417	-0.787	0.787	2.417	4.441	7.603
90	-7.592	-4.458	-2.419	-0.792	0.792	2.419	4.458	7.592
91	-7.580	-4.464	-2.420	-0.793	0.793	2.420	4.464	7.569
92	-7.581	-4.475	-2.423	-0.795	0.795	2.423	4.475	7.581
93	-7.570	-4.491	-2.428	-0.797	0.797	2.428	4.491	7.570
94	-7.558	-4.507	-2.435	-0.799	0.799	2.435	4.507	7.558
95	-7.546	-4.522	-2.444	-0.800	0.800	2.444	4.522	7.546
96	-7.533	-4.538	-2.453	-0.800	0.800	2.453	4.538	7.533
97	-7.521	-4.553	-2.464	-0.801	0.801	2.464	4.553	7.521
98	-7.508	-4.568	-2.475	-0.802	0.802	2.475	4.568	7.508
99	-7.494	-4.583	-2.486	-0.804	0.804	2.489	4.583	7.494
100	-7.481	-4.597	-2.501	-0.805	0.805	2.501	4.597	7.481
101	-7.467	-4.612	-2.514	-0.808	0.808	2.514	4.612	7.467
102	-7.453	-4.626	-2.529	-0.811	0.811	2.529	4.626	7.453
103	-7.439	-4.641	-2.543	-0.815	0.815	2.543	4.641	7.439
104	-7.424	-4.655	-2.559	-0.819	0.819	2.559	4.655	7.424
105	-7.409	-4.669	-2.574	-0.824	0.824	2.574	4.669	7.409
106	-7.395	-4.682	-2.590	-0.830	0.830	2.590	4.682	7.395
107	-7.380	-4.695	-2.607	-0.836	0.836	2.607	4.695	7.380
108	-7.365	-4.708	-2.623	-0.842	0.842	2.623	4.708	7.366
109	-7.350	-4.721	-2.640	-0.849	0.849	2.640	4.721	7.350
110	-7.340	-4.729	-2.651	-0.853	0.853	2.651	4.729	7.340
111	-7.336	-4.733	-2.656	-0.855	0.855	2.656	4.733	7.336
112	-7.322	-4.745	-2.672	-0.862	0.862	2.672	4.745	7.322
113	-7.308	-4.756	-2.687	-0.869	0.869	2.687	4.756	7.308
114	-7.294	-4.767	-2.703	-0.875	0.875	2.703	4.767	7.294
115	-7.281	-4.778	-2.717	-0.882	0.882	2.717	4.778	7.281
116	-7.269	-4.788	-2.731	-0.888	0.888	2.731	4.788	7.269
117	-7.256	-4.798	-2.744	-0.893	0.893	2.744	4.798	7.256
118	-7.245	-4.807	-2.757	-0.899	0.899	2.757	4.807	7.245
119	-7.233	-4.816	-2.769	-0.904	0.904	2.769	4.816	7.233
120	-7.223	-4.824	-2.781	-0.909	0.908	2.781	4.824	7.223

Table 7

FIG. 31

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-7.213	-4.832	-2.792	-0.914	0.914	2.792	4.832	7.213
122	-7.203	-4.840	-2.802	-0.918	0.918	2.802	4.840	7.203
123	-7.194	-4.847	-2.812	-0.922	0.922	2.812	4.847	7.194
124	-7.185	-4.854	-2.821	-0.926	0.926	2.821	4.854	7.185
125	-7.176	-4.861	-2.830	-0.930	0.930	2.830	4.861	7.176
126	-7.168	-4.867	-2.838	-0.933	0.933	2.838	4.867	7.168
127	-7.161	-4.873	-2.846	-0.937	0.937	2.846	4.873	7.161
128	-7.154	-4.879	-2.853	-0.940	0.940	2.853	4.879	7.154
129	-7.147	-4.885	-2.860	-0.943	0.943	2.860	4.885	7.147
130	-7.140	-4.890	-2.867	-0.945	0.945	2.867	4.890	7.140
131	-7.134	-4.895	-2.873	-0.948	0.948	2.873	4.895	7.134
132	-7.128	-4.900	-2.879	-0.950	0.950	2.879	4.900	7.128
133	-7.122	-4.904	-2.885	-0.953	0.953	2.885	4.904	7.122
134	-7.116	-4.909	-2.890	-0.955	0.955	2.890	4.909	7.116
135	-7.111	-4.913	-2.895	-0.957	0.957	2.895	4.913	7.111
136	-7.106	-4.917	-2.900	-0.959	0.959	2.900	4.917	7.106
137	-7.102	-4.920	-2.904	-0.961	0.961	2.904	4.920	7.102
138	-7.097	-4.924	-2.909	-0.963	0.963	2.909	4.924	7.097
139	-7.093	-4.927	-2.913	-0.965	0.965	2.913	4.927	7.093
140	-7.089	-4.931	-2.917	-0.966	0.966	2.917	4.931	7.089
141	-7.084	-4.934	-2.921	-0.968	0.968	2.921	4.934	7.084
142	-7.081	-4.937	-2.924	-0.969	0.969	2.924	4.937	7.081
143	-7.077	-4.940	-2.928	-0.971	0.971	2.928	4.940	7.077
144	-7.074	-4.942	-2.931	-0.972	0.972	2.931	4.942	7.074
145	-7.071	-4.945	-2.934	-0.973	0.973	2.934	4.945	7.071
146	-7.068	-4.947	-2.937	-0.974	0.974	2.937	4.947	7.068
147	-7.064	-4.950	-2.941	-0.976	0.976	2.941	4.950	7.064
148	-7.062	-4.952	-2.942	-0.977	0.977	2.942	4.952	7.062

Table 8

FIG. 32

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.12	0.11	0.1	0.09	0.06	0
2	-4.8	0.12	0.11	0.1	0.09	0.06	0
3	-4.6	0.13	0.12	0.11	0.09	0.06	0
4	-4.4	0.13	0.12	0.1	0.09	0.07	0
5	-4.2	0.14	0.13	0.11	0.1	0.07	0
6	-4	0.15	0.14	0.12	0.11	0.08	0
7	-3.8	0.15	0.14	0.12	0.11	0.06	0
8	-3.6	0.17	0.15	0.14	0.12	0.09	0
9	-3.4	0.16	0.16	0.15	0.13	0.09	0
10	-3.2	0.18	0.16	0.15	0.13	0.1	0
11	-3	0.2	0.18	0.16	0.14	0.1	0
12	-2.8	0.19	0.17	0.16	0.14	0.1	0
13	-2.6	0.21	0.19	0.17	0.15	0.11	0
14	-2.4	0.23	0.21	0.19	0.17	0.12	0
15	-2.2	0.23	0.2	0.18	0.17	0.12	0
16	-2	0.25	0.22	0.2	0.18	0.13	0
17	-1.8	0.27	0.24	0.22	0.2	0.13	0
18	-1.6	0.26	0.24	0.21	0.19	0.14	0
19	-1.4	0.29	0.26	0.23	0.21	0.15	0
20	-1.2	0.28	0.25	0.23	0.2	0.15	0
21	-1	0.3	0.27	0.25	0.22	0.16	0
22	-0.8	0.33	0.3	0.27	0.24	0.18	0
23	-0.6	0.32	0.29	0.26	0.23	0.17	0
24	-0.4	0.35	0.31	0.28	0.25	0.18	0
25	-0.2	0.34	0.3	0.27	0.25	0.2	0
26	0	0.33	0.3	0.27	0.24	0.19	0
27	0.01	0.33	0.3	0.27	0.24	0.19	0
28	0.2	0.32	0.29	0.26	0.23	0.21	0
29	0.4	0.31	0.28	0.25	0.23	0.2	0
30	0.6	0.3	0.27	0.24	0.22	0.2	0
31	0.8	0.29	0.26	0.23	0.21	0.19	0
32	1	0.28	0.25	0.23	0.2	0.19	0
33	1.2	0.27	0.24	0.22	0.2	0.18	0
34	1.4	0.26	0.24	0.21	0.19	0.17	0
35	1.6	0.25	0.23	0.2	0.18	0.17	0
36	1.8	0.24	0.22	0.2	0.18	0.16	0
37	2	0.23	0.21	0.19	0.17	0.15	0
38	2.2	0.22	0.2	0.18	0.16	0.15	0
39	2.4	0.22	0.19	0.17	0.16	0.14	0
40	2.6	0.21	0.19	0.17	0.15	0.14	0

Table 9

FIG. 33

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
41	2.8	0.2	0.18	0.16	0.14	0.13	0
42	3	0.19	0.17	0.15	0.14	0.13	0
43	3.2	0.18	0.16	0.15	0.13	0.12	0
44	3.4	0.17	0.16	0.14	0.13	0.11	0
45	3.6	0.17	0.15	0.14	0.12	0.11	0
46	3.8	0.16	0.14	0.13	0.12	0.11	0
47	4	0.15	0.14	0.12	0.11	0.1	0
48	4.2	0.15	0.13	0.12	0.11	0.1	0
49	4.4	0.14	0.13	0.11	0.1	0.09	0
50	4.6	0.13	0.12	0.11	0.1	0.09	0
51	4.8	0.13	0.11	0.1	0.09	0.08	0
52	4.82	0.13	0.11	0.1	0.09	0.08	0
53	5	0.12	0.11	0.1	0.09	0.08	0
54	5.2	0.2	0.18	0.16	0.15	0.13	0
55	5.4	0.57	0.51	0.46	0.41	0.27	0
56	5.6	0.56	0.51	0.46	0.41	0.3	0
57	5.8	0.53	0.47	0.43	0.38	0.28	0
58	6	0.56	0.51	0.46	0.41	0.3	0
59	6.2	0.54	0.49	0.44	0.39	0.29	0
60	6.4	0.55	0.49	0.44	0.4	0.29	0
61	6.6	0.55	0.43	0.39	0.35	0.31	0
62	6.8	0.56	0.34	0.31	0.28	0.25	0
63	7	0.66	0.3	0.27	0.25	0.22	0
64	7.2	0.56	0.3	0.27	0.24	0.22	0
65	7.4	0.56	0.29	0.26	0.23	0.21	0
66	7.6	0.57	0.28	0.25	0.23	0.2	0
67	7.8	0.53	0.27	0.24	0.22	0.2	0
68	8	0.54	0.26	0.24	0.21	0.19	0
69	8.2	0.53	0.26	0.23	0.21	0.19	0
70	8.4	0.53	0.25	0.23	0.2	0.18	0
71	8.6	0.53	0.24	0.23	0.2	0.18	0
72	8.66	0.53	0.24	0.22	0.2	0.18	0
73	8.8	0.49	0.24	0.21	0.19	0.17	0
74	9	0.47	0.23	0.21	0.19	0.17	0
75	9.2	0.47	0.22	0.2	0.18	0.16	0
76	9.4	0.47	0.22	0.2	0.18	0.16	0
77	9.6	0.46	0.25	0.23	0.2	0.18	0
78	9.8	0.36	0.29	0.26	0.24	0.21	0
79	10	0.26	0.23	0.21	0.19	0.17	0
80	10.2	0.2	0.18	0.16	0.14	0.13	0

Table 18

FIG. 34

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	0.15	0.13	0.12	0.11	0.1	0
82	10.6	0.11	0.1	0.09	0.08	0.07	0
83	10.8	0.08	0.07	0.06	0.06	0.05	0
84	11	0.05	0.05	0.04	0.04	0.04	0
85	11.2	0.04	0.03	0.03	0.03	0.02	0
86	11.4	0.02	0.02	0.02	0.02	0.01	0
87	11.6	0.01	0.01	0.01	0.01	0.01	0
88	11.8	0.01	0.01	0	0	0	0
89	12	0.03	0.02	0.01	0.01	0.01	0
90	12.2	0.04	0.03	0.02	0.02	0.02	0
91	12.36	0.04	0.04	0.03	0.02	0.02	0
92	12.4	0.05	0.05	0.03	0.03	0.02	0
93	12.6	0.06	0.06	0.04	0.03	0.03	0
94	12.8	0.06	0.07	0.05	0.04	0.04	0
95	13	0.09	0.08	0.05	0.05	0.04	0
96	13.2	0.1	0.09	0.06	0.05	0.05	0
97	13.4	0.12	0.1	0.07	0.06	0.06	0
98	13.6	0.13	0.12	0.08	0.07	0.07	0
99	13.8	0.14	0.13	0.09	0.08	0.08	0
100	14	0.15	0.14	0.1	0.09	0.08	0
101	14.2	0.17	0.15	0.11	0.1	0.09	0
102	14.4	0.18	0.16	0.13	0.11	0.1	0
103	14.6	0.19	0.17	0.14	0.13	0.11	0
104	14.8	0.21	0.19	0.15	0.14	0.11	0
105	15	0.2	0.18	0.16	0.14	0.1	0
106	15.2	0.21	0.19	0.17	0.14	0.1	0
107	15.4	0.18	0.16	0.14	0.13	0.09	0
108	15.6	0.17	0.15	0.14	0.12	0.09	0
109	15.8	0.16	0.14	0.13	0.12	0.08	0
110	15.93	0.16	0.15	0.13	0.12	0.09	0
111	16	0.15	0.13	0.12	0.11	0.08	0
112	16.2	0.14	0.13	0.11	0.1	0.07	0
113	16.4	0.13	0.12	0.11	0.1	0.07	0
114	16.6	0.13	0.11	0.1	0.09	0.07	0
115	16.8	0.12	0.11	0.1	0.09	0.06	0
116	17	0.12	0.1	0.09	0.08	0.06	0
117	17.2	0.11	0.1	0.09	0.08	0.06	0
118	17.4	0.1	0.09	0.08	0.08	0.06	0
119	17.6	0.1	0.09	0.08	0.07	0.06	0
120	17.8	0.09	0.09	0.08	0.07	0.06	0

Table 11

FIG. 35

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.09	0.08	0.07	0.07	0.05	0
122	18.2	0.09	0.09	0.07	0.06	0.05	0
123	18.4	0.08	0.07	0.07	0.06	0.04	0
124	18.5	0.08	0.07	0.06	0.06	0.04	0
125	18.8	0.07	0.07	0.06	0.05	0.04	0
126	19	0.07	0.06	0.06	0.05	0.04	0
127	19.2	0.07	0.06	0.05	0.05	0.04	0
128	19.4	0.06	0.06	0.05	0.05	0.03	0
129	19.6	0.06	0.06	0.05	0.04	0.03	0
130	19.8	0.06	0.05	0.05	0.04	0.03	0
131	20	0.06	0.05	0.05	0.04	0.03	0
132	20.2	0.06	0.05	0.04	0.04	0.03	0
133	20.4	0.05	0.05	0.04	0.04	0.02	0
134	20.6	0.05	0.04	0.04	0.04	0.02	0
135	20.8	0.04	0.04	0.03	0.03	0.02	0
136	21	0.04	0.04	0.04	0.03	0.02	0
137	21.2	0.04	0.04	0.03	0.03	0.02	0
138	21.4	0.04	0.03	0.03	0.03	0.02	0
139	21.6	0.03	0.03	0.03	0.03	0.02	0
140	21.8	0.03	0.03	0.03	0.02	0.02	0
141	22	0.03	0.03	0.03	0.02	0.02	0
142	22.2	0.03	0.03	0.02	0.02	0.02	0
143	22.4	0.03	0.03	0.02	0.02	0.02	0
144	22.6	0.03	0.02	0.02	0.02	0.01	0
145	22.8	0.03	0.02	0.02	0.02	0.01	0
146	23	0.02	0.02	0.02	0.02	0.01	0
147	23.2	0.02	0.02	0.02	0.02	0.01	0
148	23.4	0.02	0.02	0.02	0.02	0.01	0

Table 12

FIG. 36

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1977	0.1598	0.0379	23.7349
2	-4.8000	0.2058	0.1666	0.0391	23.4911
3	-4.6000	0.2141	0.1737	0.0404	23.2414
4	-4.4000	0.2227	0.1811	0.0416	22.9856
5	-4.2000	0.2316	0.1887	0.0429	22.7233
6	-4.0000	0.2407	0.1966	0.0441	22.4546
7	-3.8000	0.2502	0.2048	0.0454	22.1792
8	-3.6000	0.2600	0.2133	0.0467	21.8969
9	-3.4000	0.2700	0.2220	0.0480	21.6075
10	-3.2000	0.2804	0.2311	0.0493	21.3107
11	-3.0000	0.2910	0.2405	0.0506	21.0063
12	-2.8000	0.3020	0.2502	0.0518	20.6939
13	-2.6000	0.3133	0.2602	0.0530	20.3732
14	-2.4000	0.3248	0.2706	0.0542	20.0439
15	-2.2000	0.3367	0.2813	0.0554	19.7054
16	-2.0000	0.3489	0.2923	0.0566	19.3575
17	-1.8000	0.3613	0.3037	0.0577	18.9996
18	-1.6000	0.3741	0.3154	0.0588	18.6313
19	-1.4000	0.3872	0.3274	0.0598	18.2519
20	-1.2000	0.4005	0.3398	0.0607	17.8609
21	-1.0000	0.4141	0.3526	0.0615	17.4577
22	-0.8000	0.4280	0.3657	0.0623	17.0415
23	-0.6000	0.4421	0.3791	0.0630	16.5117
24	-0.4000	0.4565	0.3930	0.0635	16.1675
25	-0.2000	0.4711	0.4072	0.0640	15.7080
26	0.0000	0.4859	0.4217	0.0642	15.2325
27	0.1871	0.5000	0.4356	0.0644	14.7724
28	0.2000	0.5010	0.4366	0.0644	14.7399
29	0.4000	0.5162	0.4519	0.0643	14.2295
30	0.6000	0.5316	0.4675	0.0641	13.7002
31	0.8000	0.5471	0.4835	0.0636	13.1510
32	1.0000	0.5626	0.4999	0.0629	12.5809
33	1.2000	0.5786	0.5166	0.0619	11.9887
34	1.4000	0.5944	0.5337	0.0607	11.3736
35	1.6000	0.6104	0.5511	0.0593	10.7597
36	1.8000	0.6270	0.5689	0.0581	10.2106
37	2.0000	0.6442	0.5871	0.0571	9.7259
38	2.2000	0.6619	0.6056	0.0563	9.3017
39	2.4000	0.6802	0.6244	0.0558	8.9342
40	2.6000	0.6991	0.6437	0.0555	8.6193

Table 13

FIG. 37

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7186	0.6632	0.0554	8.3527
42	3.0000	0.7385	0.6831	0.0555	8.1300
43	3.2000	0.7593	0.7034	0.0559	7.9468
44	3.4000	0.7804	0.7240	0.0565	7.7983
45	3.6000	0.8021	0.7449	0.0572	7.6801
46	3.8000	0.8243	0.7662	0.0581	7.5872
47	4.0000	0.8470	0.7878	0.0592	7.5148
48	4.2000	0.8702	0.8098	0.0604	7.4581
49	4.4000	0.8938	0.8321	0.0617	7.4120
50	4.6000	0.9178	0.8548	0.0630	7.3718
51	4.8000	0.9421	0.8778	0.0644	7.3328
52	5.0000	0.9668	0.9011	0.0657	7.2903
53	5.2000	0.9917	0.9248	0.0670	7.2406
54	5.2657	1.0000	0.9326	0.0674	7.2219
55	5.4000	1.0169	0.9488	0.0681	7.1793
56	5.6000	1.0423	0.9732	0.0691	7.1031
57	5.8000	1.0678	0.9979	0.0699	7.0087
58	6.0000	1.0934	1.0229	0.0705	6.8934
59	6.2000	1.1191	1.0483	0.0708	6.7559
60	6.4000	1.1448	1.0740	0.0708	6.5919
61	6.6000	1.1704	1.1000	0.0704	6.4032
62	6.8000	1.1960	1.1263	0.0697	6.1889
63	7.0000	1.2216	1.1530	0.0686	5.9490
64	7.2000	1.2470	1.1799	0.0671	5.6843
65	7.4000	1.2724	1.2072	0.0652	5.4035
66	7.6000	1.2980	1.2348	0.0632	5.1173
67	7.8000	1.3238	1.2626	0.0612	4.8477
68	8.0000	1.3500	1.2907	0.0593	4.5905
69	8.2000	1.3780	1.3191	0.0589	4.4669
70	8.4000	1.4086	1.3477	0.0609	4.5190
71	8.6000	1.4392	1.3766	0.0626	4.5447
72	8.8000	1.4696	1.4057	0.0639	4.5449
73	9.0000	1.4999	1.4350	0.0649	4.5208
74	9.0008	1.5000	1.4351	0.0649	4.5207
75	9.2000	1.5300	1.4645	0.0655	4.4740
76	9.4000	1.5600	1.4942	0.0658	4.4056
77	9.6000	1.5899	1.5241	0.0658	4.3170
78	9.8000	1.6195	1.5541	0.0654	4.2101
79	10.0000	1.6490	1.5843	0.0647	4.0864
80	10.2000	1.6783	1.6145	0.0637	3.9475

Table 14

FIG. 38

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7074	1.6449	0.0624	3.7953
82	10.6000	1.7363	1.6754	0.0609	3.6327
83	10.8000	1.7650	1.7060	0.0591	3.4625
84	11.0000	1.7937	1.7396	0.0571	3.2884
85	11.2000	1.8223	1.7672	0.0551	3.1185
86	11.4000	1.8509	1.7979	0.0530	2.9491
87	11.6000	1.8797	1.8286	0.0511	2.7943
88	11.8000	1.9086	1.8593	0.0494	2.6548
89	12.0000	1.9378	1.8899	0.0478	2.5317
90	12.2000	1.9671	1.9206	0.0466	2.4250
91	12.4000	1.9966	1.9511	0.0455	2.3327
92	12.4227	2.0000	1.9546	0.0454	2.3230
93	12.6000	2.0263	1.9816	0.0446	2.2522
94	12.8000	2.0559	2.0121	0.0439	2.1810
95	13.0000	2.0856	2.0424	0.0432	2.1167
96	13.2000	2.1153	2.0726	0.0426	2.0569
97	13.4000	2.1448	2.1028	0.0421	1.9998
98	13.6000	2.1742	2.1328	0.0415	1.9437
99	13.8000	2.2035	2.1627	0.0408	1.8875
100	14.0000	2.2326	2.1925	0.0401	1.8301
101	14.2000	2.2614	2.2221	0.0394	1.7711
102	14.4000	2.2901	2.2516	0.0385	1.7096
103	14.6000	2.3184	2.2809	0.0375	1.6457
104	14.8000	2.3465	2.3100	0.0365	1.5792
105	15.0000	2.3743	2.3390	0.0353	1.5104
106	15.2000	2.4018	2.3678	0.0341	1.4396
107	15.4000	2.4290	2.3963	0.0328	1.3670
108	15.6000	2.4559	2.4246	0.0314	1.2934
109	15.8000	2.4825	2.4526	0.0299	1.2192
110	15.9336	2.5000	2.4711	0.0289	1.1703
111	16.0000	2.5087	2.4802	0.0284	1.1457
112	16.2000	2.5344	2.5076	0.0269	1.0722
113	16.4000	2.5598	2.5345	0.0253	1.0000
114	16.6000	2.5848	2.5610	0.0238	0.9296
115	16.8000	2.6093	2.5870	0.0223	0.8615
116	17.0000	2.6332	2.6124	0.0208	0.7960
117	17.2000	2.6567	2.6373	0.0193	0.7333
118	17.4000	2.6795	2.6616	0.0179	0.6737
119	17.6000	2.7017	2.6851	0.0166	0.6171
120	17.8000	2.7232	2.7080	0.0153	0.5637

Table 15

FIG. 39

Design #	SNR	Opt. Cap	Std. Cap	Gain (bits)	Gain %
121	18.0000	2.7440	2.7300	0.0140	0.5134
122	18.2000	2.7841	2.7713	0.0128	0.4663
123	18.4000	2.7834	2.7717	0.0117	0.4222
124	18.6000	2.8018	2.7912	0.0106	0.3810

Table 16

FIG. 40

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.583	4.583
2	-4.584	-4.584	-4.581	-4.582	4.584	4.582	4.582	4.582
3	-4.584	-4.582	-4.582	-4.582	4.584	4.584	4.579	4.584
4	-4.584	-4.582	-4.582	-4.582	4.583	4.583	4.582	4.583
5	-4.583	-4.583	-4.583	-4.583	4.586	4.586	4.579	4.579
6	-4.586	-4.582	-4.582	-4.582	4.584	4.584	4.580	4.583
7	-4.587	-4.587	-4.578	-4.578	4.583	4.583	4.582	4.583
8	-4.586	-4.586	-4.580	-4.580	4.583	4.583	4.583	4.583
9	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.583
10	-4.585	-4.584	-4.581	-4.581	4.583	4.583	4.583	4.583
11	-4.583	-4.583	-4.583	-4.583	4.584	4.584	4.581	4.581
12	-4.583	-4.583	-4.582	-4.582	4.585	4.582	4.582	4.582
13	-4.587	-4.581	-4.581	-4.581	4.587	4.582	4.581	4.581
14	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.582	4.583
15	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.583
16	-4.583	-4.583	-4.582	-4.583	4.583	4.583	4.583	4.583
17	-4.587	-4.582	-4.580	-4.581	4.587	4.582	4.581	4.582
18	-4.583	-4.583	-4.582	-4.583	4.584	4.584	4.581	4.581
19	-4.583	-4.583	-4.582	-4.582	4.584	4.584	4.581	4.582
20	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
21	-4.583	-4.583	-4.582	-4.583	4.583	4.584	4.582	4.582
22	-4.584	-4.584	-4.580	-4.583	4.587	4.588	4.576	4.579
23	-4.589	-4.586	-4.578	-4.578	4.588	4.581	4.581	4.581
24	-4.584	-4.584	-4.581	-4.581	4.583	4.583	4.582	4.583
25	-4.586	-4.582	-4.582	-4.582	4.584	4.583	4.582	4.582
26	-4.583	-4.583	-4.583	-4.583	4.583	4.583	4.582	4.583
27	-4.584	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
28	-4.583	-4.583	-4.583	-4.583	4.583	4.583	4.582	4.582
29	-4.583	-4.582	-4.582	-4.582	4.583	4.583	4.583	4.583
30	-4.587	-4.587	-4.578	-4.579	4.587	4.587	4.579	4.579
31	-4.584	-4.583	-4.582	-4.582	4.584	4.584	4.581	4.582
32	-4.583	-4.583	-4.582	-4.582	4.583	4.583	4.582	4.582
33	-4.584	-4.584	-4.578	-4.584	4.584	4.584	4.580	4.582
34	-4.584	-4.584	-4.578	-4.584	4.584	4.584	4.578	4.584
35	-5.110	-5.110	-3.986	-3.986	5.110	5.110	3.986	3.986
36	-5.356	-5.354	-3.651	-3.651	5.355	5.355	3.650	3.650
37	-5.515	-5.514	-3.404	-3.404	5.515	5.515	3.404	3.404
38	-5.638	-5.638	-3.196	-3.196	5.640	5.637	3.196	3.196
39	-5.729	-5.729	-3.029	-3.029	5.729	5.729	3.029	3.029
40	-5.808	-5.807	-2.876	-2.876	5.808	5.808	2.876	2.876

Table 17

FIG. 41

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-5.874	-5.874	-2.739	-2.739	5.874	5.872	2.740	2.740
42	-5.929	-5.929	-2.617	-2.617	5.929	5.929	2.617	2.617
43	-5.978	-5.975	-2.506	-2.506	5.978	5.976	2.505	2.505
44	-6.018	-6.018	-2.405	-2.405	6.018	6.018	2.405	2.405
45	-6.055	-6.054	-2.312	-2.312	6.054	6.054	2.312	2.312
46	-6.087	-6.085	-2.227	-2.227	6.087	6.086	2.227	2.227
47	-6.113	-6.113	-2.151	-2.151	6.113	6.113	2.151	2.151
48	-6.138	-6.138	-2.081	-2.081	6.138	6.137	2.081	2.081
49	-6.159	-6.159	-2.017	-2.017	6.159	6.159	2.017	2.017
50	-6.179	-6.176	-1.960	-1.960	6.178	6.178	1.959	1.959
51	-6.194	-6.193	-1.907	-1.907	6.194	6.194	1.907	1.907
52	-6.208	-6.208	-1.860	-1.860	6.208	6.208	1.860	1.860
53	-6.221	-6.220	-1.819	-1.819	6.221	6.219	1.819	1.820
54	-6.224	-6.224	-1.807	-1.807	6.224	6.224	1.807	1.807
55	-6.231	-6.230	-1.783	-1.783	6.231	6.231	1.783	1.783
56	-6.240	-6.240	-1.752	-1.752	6.242	6.237	1.752	1.752
57	-6.247	-6.247	-1.725	-1.725	6.247	6.247	1.725	1.725
58	-6.254	-6.253	-1.703	-1.703	6.253	6.252	1.703	1.703
59	-6.260	-6.256	-1.686	-1.686	6.260	6.256	1.686	1.686
60	-6.261	-6.261	-1.673	-1.673	6.261	6.261	1.673	1.673
61	-6.263	-6.263	-1.665	-1.665	6.263	6.263	1.665	1.665
62	-6.264	-6.264	-1.661	-1.661	6.264	6.264	1.661	1.661
63	-6.265	-6.265	-1.660	-1.660	6.265	6.264	1.660	1.660
64	-6.264	-6.264	-1.663	-1.663	6.291	6.237	1.658	1.667
65	-6.734	-5.796	-1.570	-1.737	6.243	6.244	1.674	1.674
66	-6.781	-5.697	-1.582	-1.750	6.781	5.697	1.582	1.750
67	-6.914	-5.533	-1.550	-1.781	6.915	5.533	1.549	1.781
68	-7.016	-5.403	-1.517	-1.809	7.017	5.403	1.517	1.809
69	-7.923	-3.658	7.923	-2.800	2.800	0.035	3.659	-0.035
70	-7.913	-3.668	7.913	-2.816	2.816	0.052	3.668	-0.052
71	-7.902	-3.680	7.902	-2.831	2.831	0.068	3.680	-0.068
72	-7.890	-3.694	7.890	-2.845	2.845	0.084	3.694	-0.084
73	-7.878	-3.710	7.878	-2.859	2.859	0.099	3.710	-0.099
74	-7.878	-3.710	7.878	-2.859	2.859	0.099	3.710	-0.099
75	-7.864	-3.728	7.864	-2.872	2.872	0.115	3.728	-0.115
76	-7.850	-3.749	7.850	-2.883	2.883	0.131	3.749	-0.131
77	-7.836	-3.772	7.835	-2.893	2.893	0.148	3.772	-0.148
78	-7.819	-3.798	7.819	-2.902	2.902	0.166	3.798	-0.166
79	-7.802	-3.827	7.802	-2.908	2.908	0.185	3.827	-0.185
80	-7.784	-3.859	7.784	-2.911	2.911	0.206	3.859	-0.206

Table 18

FIG. 42

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-7.765	-3.896	7.765	-2.911	2.911	0.230	3.896	-0.230
82	-7.745	-3.937	7.745	-2.907	2.907	0.257	3.937	-0.257
83	-7.724	-3.983	7.724	-2.897	2.897	0.290	3.983	-0.290
84	-7.701	-4.036	7.701	-2.880	2.880	0.328	4.036	-0.328
85	-7.677	-4.095	7.677	-2.857	2.857	0.374	4.095	-0.374
86	-7.651	-4.157	7.651	-2.828	2.829	0.425	4.157	-0.425
87	-7.625	-4.220	7.625	-2.797	2.797	0.479	4.220	-0.479
88	-7.600	-4.280	7.600	-2.765	2.765	0.533	4.280	-0.533
89	-7.576	-4.334	7.576	-2.734	2.734	0.585	4.334	-0.585
90	-7.555	-4.382	7.555	-2.707	2.707	0.632	4.382	-0.632
91	-7.536	-4.423	7.536	-2.682	2.682	0.672	4.423	-0.672
92	-7.534	-4.428	7.534	-2.680	2.680	0.677	4.428	-0.677
93	-7.519	-4.459	7.519	-2.662	2.662	0.707	4.459	-0.707
94	-7.504	-4.489	7.504	-2.644	2.644	0.736	4.489	-0.736
95	-7.491	-4.515	7.491	-2.630	2.630	0.761	4.515	-0.761
96	-7.479	-4.538	7.479	-2.618	2.618	0.780	4.538	-0.780
97	-7.469	-4.558	7.469	-2.609	2.609	0.796	4.558	-0.796
98	-7.459	-4.576	7.459	-2.603	2.603	0.808	4.576	-0.808
99	-7.449	-4.593	7.449	-2.598	2.599	0.818	4.593	-0.817
100	-7.439	-4.608	7.439	-2.597	2.597	0.825	4.608	-0.825
101	-7.430	-4.623	7.430	-2.597	2.597	0.831	4.623	-0.830
102	-7.420	-4.636	7.420	-2.599	2.599	0.835	4.636	-0.835
103	-7.410	-4.650	7.410	-2.603	2.603	0.838	4.650	-0.838
104	-7.399	-4.663	7.399	-2.608	2.608	0.841	4.663	-0.841
105	-7.389	-4.675	7.388	-2.615	2.615	0.844	4.675	-0.844
106	-7.377	-4.688	7.377	-2.624	2.624	0.847	4.688	-0.847
107	-7.366	-4.700	7.366	-2.634	2.633	0.850	4.700	-0.850
108	-7.354	-4.712	7.354	-2.644	2.644	0.854	4.712	-0.854
109	-7.342	-4.724	7.342	-2.656	2.656	0.858	4.724	-0.858
110	-7.334	-4.732	-0.862	-2.661	7.334	4.732	0.862	2.661
111	-7.330	-4.736	-0.863	-2.666	7.329	4.736	0.863	2.666
112	-7.317	-4.747	-0.868	-2.679	7.317	4.747	0.868	2.679
113	-7.304	-4.758	-0.873	-2.693	7.304	4.758	0.873	2.693
114	-7.292	-4.768	-0.878	-2.707	7.292	4.768	0.878	2.707
115	-7.279	-4.778	-0.884	-2.720	7.279	4.778	0.884	2.720
116	-7.267	-4.788	-0.889	-2.733	7.267	4.788	0.889	2.733
117	-7.255	-4.798	-0.894	-2.746	7.256	4.798	0.894	2.746
118	-7.244	-4.807	-0.899	-2.758	7.244	4.807	0.899	2.758
119	-7.233	-4.816	-0.904	-2.770	7.233	4.816	0.904	2.770
120	-7.223	-4.824	-0.909	-2.781	7.223	4.824	0.909	2.781

Table 19

FIG. 43

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-7.212	-4.832	-0.914	-2.792	7.212	4.832	0.914	2.792
122	-7.203	-4.840	-0.918	-2.802	7.203	4.840	0.918	2.802
123	-7.194	-4.847	-0.922	-2.812	7.194	4.847	0.922	2.812
124	-7.185	-4.854	-0.926	-2.821	7.185	4.854	0.926	2.821

Table 28

FIG. 44

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
65	7.4	0.62	0.22	0.2	0.18	0.16	0
66	7.6	0.56	0.21	0.19	0.17	0.15	0
67	7.8	0.57	0.26	0.23	0.21	0.19	0
68	8	0.5	0.3	0.27	0.24	0.22	0
69	8.2	0.46	0.34	0.3	0.27	0.25	0
70	8.4	0.45	0.33	0.3	0.27	0.24	0
71	8.6	0.44	0.33	0.3	0.27	0.24	0
72	8.8	0.48	0.32	0.29	0.26	0.24	0
73	9	0.47	0.32	0.29	0.26	0.23	0
74	9	0.47	0.32	0.29	0.26	0.23	0
75	9.2	0.46	0.31	0.28	0.25	0.23	0
76	9.4	0.45	0.3	0.27	0.25	0.22	0
77	9.6	0.43	0.3	0.27	0.24	0.22	0
78	9.8	0.42	0.29	0.26	0.23	0.21	0
79	10	0.41	0.28	0.25	0.23	0.21	0
80	10.2	0.4	0.27	0.25	0.22	0.2	0
81	10.4	0.39	0.27	0.24	0.22	0.19	0
82	10.6	0.42	0.26	0.23	0.21	0.19	0
83	10.8	0.38	0.26	0.23	0.21	0.19	0
84	11	0.37	0.25	0.23	0.2	0.18	0
85	11.2	0.37	0.25	0.22	0.2	0.18	0
86	11.4	0.33	0.26	0.22	0.2	0.18	0
87	11.6	0.29	0.24	0.22	0.2	0.18	0
88	11.8	0.24	0.22	0.2	0.18	0.16	0
89	12	0.22	0.2	0.18	0.16	0.14	0
90	12.2	0.2	0.18	0.17	0.15	0.13	0
91	12.4	0.19	0.17	0.16	0.13	0.12	0
92	12.42	0.19	0.17	0.15	0.13	0.12	0
93	12.6	0.18	0.16	0.14	0.12	0.11	0
94	12.8	0.17	0.16	0.14	0.13	0.11	0
95	13	0.17	0.15	0.13	0.12	0.11	0
96	13.2	0.16	0.14	0.13	0.12	0.11	0
97	13.4	0.16	0.14	0.13	0.11	0.1	0
98	13.6	0.15	0.14	0.13	0.11	0.1	0
99	13.8	0.16	0.15	0.13	0.12	0.11	0
100	14	0.17	0.16	0.14	0.13	0.11	0
101	14.2	0.19	0.17	0.16	0.14	0.12	0
102	14.4	0.2	0.18	0.16	0.14	0.12	0
103	14.6	0.21	0.19	0.17	0.15	0.11	0
104	14.8	0.2	0.18	0.16	0.14	0.1	0

Table 21

FIG. 45

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
105	15	0.19	0.17	0.15	0.14	0.1	0
106	15.2	0.19	0.16	0.16	0.14	0.1	0
107	15.4	0.18	0.16	0.15	0.13	0.1	0
108	15.6	0.17	0.15	0.14	0.13	0.09	0
109	15.8	0.16	0.14	0.13	0.12	0.09	0
110	15.93	0.15	0.13	0.12	0.11	0.08	0
111	16	0.15	0.14	0.12	0.11	0.08	0
112	16.2	0.14	0.12	0.11	0.1	0.07	0
113	16.4	0.13	0.12	0.11	0.1	0.07	0
114	16.6	0.13	0.11	0.1	0.09	0.07	0
115	16.8	0.12	0.11	0.1	0.09	0.06	0
116	17	0.11	0.1	0.09	0.08	0.06	0
117	17.2	0.11	0.1	0.09	0.08	0.06	0
118	17.4	0.1	0.09	0.08	0.08	0.06	0
119	17.6	0.1	0.09	0.08	0.07	0.05	0
120	17.8	0.09	0.09	0.08	0.07	0.05	0
121	18	0.09	0.08	0.07	0.07	0.05	0
122	18.2	0.09	0.08	0.07	0.06	0.05	0
123	18.4	0.08	0.07	0.07	0.06	0.04	0
124	18.6	0.08	0.07	0.06	0.06	0.04	0

Table 22

FIG. 46

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1982	0.1980	0.0002	0.0812
2	-4.8000	0.2063	0.2061	0.0002	0.0902
3	-4.6000	0.2147	0.2145	0.0002	0.1001
4	-4.4000	0.2234	0.2232	0.0002	0.1110
5	-4.2000	0.2324	0.2321	0.0003	0.1229
6	-4.0000	0.2417	0.2414	0.0003	0.1359
7	-3.8000	0.2514	0.2510	0.0004	0.1501
8	-3.6000	0.2613	0.2609	0.0004	0.1656
9	-3.4000	0.2715	0.2710	0.0005	0.1825
10	-3.2000	0.2821	0.2816	0.0006	0.2009
11	-3.0000	0.2930	0.2924	0.0006	0.2209
12	-2.8000	0.3043	0.3036	0.0007	0.2425
13	-2.6000	0.3159	0.3151	0.0008	0.2659
14	-2.4000	0.3279	0.3269	0.0010	0.2911
15	-2.2000	0.3402	0.3391	0.0011	0.3183
16	-2.0000	0.3529	0.3516	0.0012	0.3476
17	-1.8000	0.3659	0.3645	0.0014	0.3791
18	-1.6000	0.3793	0.3777	0.0016	0.4128
19	-1.4000	0.3930	0.3913	0.0018	0.4488
20	-1.2000	0.4072	0.4052	0.0020	0.4872
21	-1.0000	0.4217	0.4195	0.0022	0.5281
22	-0.8000	0.4366	0.4341	0.0025	0.5715
23	-0.6000	0.4519	0.4491	0.0028	0.6176
24	-0.4000	0.4675	0.4644	0.0031	0.6663
25	-0.2000	0.4835	0.4801	0.0034	0.7177
26	0.0000	0.4999	0.4961	0.0038	0.7719
27	0.0008	0.5000	0.4962	0.0038	0.7658
28	0.2000	0.5167	0.5125	0.0042	0.8289
29	0.4000	0.5339	0.5292	0.0047	0.8886
30	0.6000	0.5514	0.5462	0.0052	0.9511
31	0.8000	0.5694	0.5636	0.0057	1.0164
32	1.0000	0.5877	0.5814	0.0063	1.0844
33	1.2000	0.6063	0.5994	0.0069	1.1552
34	1.4000	0.6254	0.6178	0.0076	1.2288
35	1.6000	0.6448	0.6365	0.0083	1.3040
36	1.8000	0.6646	0.6555	0.0091	1.3820
37	2.0000	0.6847	0.6748	0.0099	1.4625
38	2.2000	0.7052	0.6945	0.0107	1.5451
39	2.4000	0.7260	0.7144	0.0116	1.6297
40	2.6000	0.7472	0.7346	0.0126	1.7162

Table 23

FIG. 47

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7688	0.7551	0.0136	1.8046
42	3.0000	0.7906	0.7759	0.0147	1.8944
43	3.2000	0.8128	0.7970	0.0158	1.9858
44	3.4000	0.8354	0.8184	0.0170	2.0783
45	3.6000	0.8582	0.8400	0.0182	2.1719
46	3.8000	0.8814	0.8619	0.0195	2.2663
47	4.0000	0.9049	0.8840	0.0209	2.3612
48	4.2000	0.9288	0.9064	0.0223	2.4564
49	4.4000	0.9527	0.9290	0.0237	2.5516
50	4.6000	0.9771	0.9519	0.0252	2.6465
51	4.7854	1.0000	0.9734	0.0266	2.7327
52	4.8000	1.0017	0.9750	0.0267	2.7407
53	5.0000	1.0266	0.9983	0.0283	2.8339
54	5.2000	1.0517	1.0218	0.0299	2.9256
55	5.4000	1.0771	1.0456	0.0315	3.0153
56	5.6000	1.1028	1.0696	0.0332	3.1025
57	5.8000	1.1286	1.0938	0.0349	3.1868
58	6.0000	1.1547	1.1182	0.0366	3.2689
59	6.2000	1.1810	1.1427	0.0383	3.3517
60	6.4000	1.2076	1.1675	0.0401	3.4318
61	6.6000	1.2343	1.1925	0.0419	3.5103
62	6.8000	1.2613	1.2176	0.0437	3.5868
63	7.0000	1.2885	1.2430	0.0455	3.6606
64	7.2000	1.3158	1.2685	0.0473	3.7311
65	7.4000	1.3433	1.2941	0.0491	3.7978
66	7.6000	1.3710	1.3200	0.0510	3.8609
67	7.8000	1.3988	1.3460	0.0528	3.9211
68	8.0000	1.4268	1.3722	0.0546	3.9786
69	8.2000	1.4549	1.3985	0.0564	4.0328
70	8.4000	1.4832	1.4250	0.0582	4.0837
71	8.5186	1.5000	1.4408	0.0592	4.1088
72	8.6000	1.5116	1.4518	0.0600	4.1311
73	8.8000	1.5401	1.4784	0.0617	4.1748
74	9.0000	1.5688	1.5053	0.0635	4.2162
75	9.2000	1.5976	1.5324	0.0652	4.2547
76	9.4000	1.6265	1.5596	0.0669	4.2906
77	9.6000	1.6555	1.5869	0.0686	4.3237
78	9.8000	1.6846	1.6143	0.0703	4.3538
79	10.0000	1.7136	1.6419	0.0719	4.3805
80	10.2000	1.7431	1.6696	0.0735	4.4034

Table 24

FIG. 48

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7725	1.6974	0.0751	4.4223
82	10.6000	1.8019	1.7254	0.0765	4.4366
83	10.8000	1.8314	1.7534	0.0780	4.4462
84	11.0000	1.8609	1.7816	0.0793	4.4514
85	11.2000	1.8905	1.8099	0.0806	4.4542
86	11.4000	1.9202	1.8383	0.0819	4.4546
87	11.6000	1.9499	1.8667	0.0831	4.4534
88	11.8000	1.9797	1.8953	0.0843	4.4497
89	11.9363	2.0000	1.9149	0.0851	4.4441
90	12.0000	2.0095	1.9240	0.0855	4.4438
91	12.2000	2.0394	1.9528	0.0866	4.4356
92	12.4000	2.0694	1.9817	0.0877	4.4249
93	12.6000	2.0994	2.0107	0.0887	4.4121
94	12.8000	2.1294	2.0397	0.0897	4.3969
95	13.0000	2.1595	2.0689	0.0906	4.3798
96	13.2000	2.1896	2.0981	0.0915	4.3610
97	13.4000	2.2197	2.1274	0.0923	4.3403
98	13.6000	2.2499	2.1568	0.0931	4.3178
99	13.8000	2.2801	2.1863	0.0939	4.2934
100	14.0000	2.3104	2.2158	0.0946	4.2671
101	14.2000	2.3406	2.2454	0.0952	4.2389
102	14.4000	2.3709	2.2751	0.0958	4.2085
103	14.6000	2.4012	2.3048	0.0963	4.1761
104	14.8000	2.4314	2.3347	0.0967	4.1414
105	15.0000	2.4617	2.3646	0.0971	4.1045
106	15.2000	2.4920	2.3946	0.0974	4.0654
107	15.2531	2.5000	2.4026	0.0974	4.0539
108	15.4000	2.5222	2.4247	0.0976	4.0241
109	15.6000	2.5525	2.4547	0.0977	3.9805
110	15.8000	2.5827	2.4849	0.0978	3.9347
111	16.0000	2.6129	2.5151	0.0978	3.8868
112	16.2000	2.6430	2.5454	0.0977	3.8366
113	16.4000	2.6732	2.5757	0.0975	3.7842
114	16.6000	2.7033	2.6061	0.0972	3.7296
115	16.8000	2.7333	2.6365	0.0968	3.6729
116	17.0000	2.7634	2.6670	0.0964	3.6139
117	17.2000	2.7933	2.6975	0.0958	3.5527
118	17.4000	2.8233	2.7281	0.0952	3.4893
119	17.6000	2.8531	2.7587	0.0944	3.4237
120	17.8000	2.8829	2.7893	0.0936	3.3558

Table 25

FIG. 49

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.9127	2.8200	0.0927	3.2856
122	18.2000	2.9424	2.8508	0.0916	3.2130
123	18.4000	2.9720	2.8815	0.0904	3.1382
124	18.5898	3.0000	2.9108	0.0892	3.0644
125	18.6000	3.0015	2.9123	0.0891	3.0610
126	18.8000	3.0309	2.9432	0.0877	2.9814
127	19.0000	3.0603	2.9740	0.0862	2.8995
128	19.2000	3.0895	3.0049	0.0846	2.8153
129	19.4000	3.1187	3.0358	0.0828	2.7288
130	19.6000	3.1477	3.0667	0.0810	2.6400
131	19.8000	3.1766	3.0977	0.0790	2.5491
132	20.0000	3.2054	3.1286	0.0768	2.4563
133	20.2000	3.2341	3.1594	0.0746	2.3616
134	20.4000	3.2626	3.1903	0.0723	2.2653
135	20.6000	3.2909	3.2211	0.0698	2.1677
136	20.8000	3.3191	3.2518	0.0673	2.0691
137	21.0000	3.3471	3.2824	0.0647	1.9700
138	21.2000	3.3749	3.3129	0.0620	1.8706
139	21.4000	3.4025	3.3433	0.0592	1.7714
140	21.6000	3.4298	3.3734	0.0564	1.6730
141	21.8000	3.4569	3.4033	0.0536	1.5758
142	22.0000	3.4837	3.4329	0.0508	1.4802
143	22.1229	3.5000	3.4510	0.0490	1.4199
144	22.2000	3.5102	3.4622	0.0480	1.3867
145	22.4000	3.5364	3.4911	0.0452	1.2956
146	22.6000	3.5621	3.5196	0.0425	1.2074
147	22.8000	3.5874	3.5476	0.0398	1.1222
148	23.0000	3.6123	3.5751	0.0372	1.0403
149	23.2000	3.6366	3.6019	0.0346	0.9619
150	23.4000	3.6603	3.6281	0.0322	0.8870
151	23.6000	3.6835	3.6537	0.0298	0.8158
152	23.8000	3.7059	3.6784	0.0275	0.7482
153	24.0000	3.7277	3.7023	0.0253	0.6842
154	24.2000	3.7487	3.7254	0.0232	0.6239
155	24.4000	3.7689	3.7478	0.0213	0.5672
156	24.6000	3.7882	3.7688	0.0194	0.5139
157	24.8000	3.8067	3.7891	0.0176	0.4641
158	25.0000	3.8242	3.8083	0.0159	0.4176
159	25.2000	3.8409	3.8266	0.0143	0.3744
160	25.4000	3.8566	3.8437	0.0129	0.3344

Table 26

FIG. 50

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	26.6000	3.8713	3.8598	0.0115	0.2974
162	26.8000	3.8651	3.8749	0.0102	0.2634
163	26.0000	3.8979	3.8889	0.0090	0.2323
164	26.2000	3.9097	3.9018	0.0079	0.2037
165	26.4000	3.9207	3.9137	0.0070	0.1779
166	26.6000	3.9306	3.9246	0.0061	0.1545
167	26.8000	3.9397	3.9345	0.0052	0.1334
168	27.0000	3.9479	3.9434	0.0045	0.1145
169	27.2000	3.9553	3.9515	0.0039	0.0977
170	27.4000	3.9619	3.9587	0.0033	0.0829
171	27.6000	3.9678	3.9650	0.0028	0.0698
172	27.8000	3.9729	3.9706	0.0023	0.0584
173	28.0000	3.9775	3.9755	0.0019	0.0485
174	28.2000	3.9814	3.9798	0.0016	0.0399
175	28.4000	3.9847	3.9834	0.0013	0.0326
176	28.6000	3.9876	3.9865	0.0011	0.0264
177	28.8000	3.9900	3.9892	0.0008	0.0212
178	29.0000	3.9920	3.9914	0.0007	0.0169
179	29.2000	3.9937	3.9932	0.0005	0.0134
180	29.4000	3.9951	3.9947	0.0004	0.0103

Table 17

FIG. 51

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-20.036	-6.369	-6.322	-6.315	-6.314	-6.302	-6.294	-6.240
2	-20.035	-6.309	-6.309	-6.309	-6.309	-6.309	-6.309	-6.309
3	-20.032	-6.337	-6.280	-6.271	-6.349	-6.289	-6.366	-6.282
4	-20.029	-6.329	-6.311	-6.311	-6.311	-6.311	-6.307	-6.307
5	-20.026	-6.323	-6.323	-6.323	-6.305	-6.305	-6.305	-6.305
6	-20.024	-6.335	-6.335	-6.335	-6.335	-6.335	-6.260	-6.260
7	-20.021	-6.315	-6.316	-6.316	-6.315	-6.317	-6.317	-6.313
8	-20.017	-6.320	-6.320	-6.320	-6.320	-6.320	-6.320	-6.300
9	-20.014	-6.341	-6.329	-6.329	-6.329	-6.300	-6.300	-6.300
10	-20.011	-6.308	-6.320	-6.316	-6.307	-6.343	-6.326	-6.318
11	-20.008	-6.363	-6.326	-6.316	-6.318	-6.318	-6.306	-6.300
12	-20.004	-6.333	-6.326	-6.326	-6.323	-6.323	-6.323	-6.310
13	-20.001	-6.338	-6.333	-6.320	-6.326	-6.317	-6.316	-6.323
14	-19.997	-6.323	-6.331	-6.323	-6.331	-6.327	-6.322	-6.325
15	-19.993	-6.328	-6.328	-6.328	-6.328	-6.328	-6.328	-6.328
16	-19.990	-6.332	-6.331	-6.331	-6.332	-6.324	-6.328	-6.326
17	-19.986	-6.331	-6.331	-6.331	-6.331	-6.331	-6.331	-6.331
18	-19.983	-6.332	-6.332	-6.332	-6.332	-6.332	-6.332	-6.332
19	-19.979	-6.339	-6.339	-6.336	-6.336	-6.330	-6.330	-6.330
20	-19.976	-6.332	-6.330	-6.336	-6.336	-6.340	-6.332	-6.339
21	-19.972	-6.362	-6.362	-6.362	-6.333	-6.315	-6.315	-6.312
22	-19.950	-6.582	-6.589	-6.596	-6.580	-6.574	-6.584	-4.691
23	-19.919	-6.734	-6.725	-6.725	-6.725	-6.724	-6.730	-3.421
24	-19.889	-6.822	-6.823	-6.821	-6.829	-6.826	-6.831	-2.221
25	-19.862	-6.899	-6.898	-6.901	-6.903	-6.894	-6.895	-0.968
26	-19.850	-6.922	-6.913	-6.913	-6.913	-6.911	-6.911	0.144
27	-19.840	-7.180	-7.160	-7.150	-7.140	-7.140	-6.430	-0.150
28	-19.823	-6.964	-6.992	-6.967	-6.955	-6.965	-6.961	0.939
29	-19.796	-7.238	-7.238	-7.236	-7.238	-7.233	-5.505	1.455
30	-19.756	-7.424	-7.426	-7.426	-7.426	-7.425	-4.126	1.219
31	-19.713	-7.576	-7.574	-7.574	-7.574	-7.574	-0.998	-0.906
32	-19.683	-7.631	-7.628	-7.630	-7.628	-7.627	-0.542	-0.543
33	-19.663	-7.660	-7.660	-7.660	-7.659	-7.658	0.000	0.000
34	-19.654	-7.668	-7.668	-7.668	-7.664	-7.665	-0.001	0.000
35	-19.597	-8.156	-8.156	-8.154	-8.154	-6.289	-0.161	-0.159
36	-19.601	-7.739	-7.739	-7.737	-7.737	-7.734	0.324	0.324
37	-19.507	-8.421	-8.421	-8.421	-8.421	-3.082	-0.540	-0.540
38	-19.551	-7.804	-7.804	-7.804	-7.804	-7.804	1.020	1.020
39	-19.445	-8.528	-8.529	-8.528	-8.528	-0.978	-0.977	-0.977
40	-19.418	-8.562	-8.562	-8.562	-8.562	-0.916	-0.916	-0.916

Table 28

FIG. 52

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	6.314	6.351	6.351	6.315	6.320	6.236	6.269	20.036
2	6.159	6.287	6.287	6.287	6.381	6.381	6.381	20.034
3	6.316	6.282	6.286	6.298	6.298	6.296	6.414	20.032
4	6.248	6.309	6.309	6.326	6.328	6.328	6.328	20.029
5	6.303	6.303	6.303	6.303	6.326	6.326	6.326	20.026
6	6.200	6.267	6.267	6.267	6.398	6.398	6.398	20.023
7	6.309	6.312	6.318	6.316	6.320	6.320	6.313	20.020
8	6.273	6.321	6.321	6.321	6.321	6.330	6.330	20.017
9	6.302	6.302	6.308	6.329	6.329	6.329	6.329	20.014
10	6.312	6.312	6.326	6.311	6.325	6.325	6.326	20.011
11	6.321	6.321	6.321	6.321	6.321	6.321	6.321	20.007
12	6.320	6.320	6.320	6.324	6.326	6.326	6.326	20.004
13	6.317	6.322	6.322	6.323	6.327	6.323	6.335	20.001
14	6.323	6.329	6.328	6.324	6.323	6.324	6.332	19.997
15	6.324	6.324	6.324	6.324	6.326	6.333	6.337	19.993
16	6.324	6.323	6.326	6.325	6.327	6.336	6.341	19.990
17	6.328	6.328	6.330	6.332	6.332	6.332	6.334	19.986
18	6.331	6.331	6.331	6.331	6.333	6.333	6.338	19.983
19	6.330	6.331	6.333	6.333	6.337	6.337	6.337	19.979
20	6.333	6.336	6.339	6.339	6.334	6.328	6.340	19.975
21	6.313	6.313	6.313	6.355	6.355	6.356	6.355	19.972
22	4.681	6.578	6.671	6.582	6.587	6.603	6.582	19.950
23	3.421	6.716	6.716	6.722	6.737	6.738	6.734	19.919
24	2.221	6.820	6.831	6.822	6.830	6.826	6.822	19.889
25	0.059	6.893	6.894	6.902	6.901	6.901	6.899	19.862
26	0.144	5.429	7.147	7.147	7.154	7.154	7.169	19.843
27	-0.140	6.900	6.900	6.900	6.910	6.920	6.950	19.850
28	0.895	2.960	7.406	7.403	7.400	7.399	7.399	19.796
29	1.467	1.467	7.466	7.466	7.466	7.466	7.466	19.770
30	1.219	1.225	7.521	7.521	7.521	7.521	7.521	19.741
31	-0.906	2.985	7.536	7.537	7.537	7.537	7.546	19.719
32	-0.546	1.662	7.624	7.621	7.621	7.624	7.620	19.685
33	0.000	0.000	7.659	7.660	7.660	7.660	7.660	19.663
34	0.000	0.000	7.663	7.663	7.663	7.685	7.668	19.654
35	-0.159	-0.159	7.703	7.703	7.703	7.703	7.703	19.627
36	0.324	0.327	4.201	8.309	8.309	8.309	8.309	19.551
37	-0.540	-0.540	7.771	7.771	7.771	7.771	7.771	19.576
38	1.020	1.020	1.021	8.501	8.501	8.501	8.501	19.468
39	-0.977	-0.977	8.441	8.121	8.121	8.121	8.121	19.517
40	-0.915	-0.915	5.622	8.286	8.286	8.286	8.287	19.476

Table 29

FIG. 53

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-19.438	-8.393	-8.393	-8.393	-8.393	-5.030	0.855	0.855
42	-19.365	-8.624	-8.624	-8.624	-8.624	-0.796	-0.796	-0.795
43	-19.369	-8.542	-8.542	-8.542	-8.542	-4.089	0.735	0.735
44	-19.315	-8.681	-8.681	-8.681	-8.681	-0.676	-0.676	-0.675
45	-19.280	-8.706	-8.707	-8.707	-8.707	-0.617	-0.617	-0.617
46	-19.267	-8.731	-8.731	-8.731	-8.731	-0.559	-0.559	-0.558
47	-19.252	-8.724	-8.724	-8.724	-8.724	-2.617	0.500	0.500
48	-19.221	-8.776	-8.776	-8.776	-8.776	-0.444	-0.444	-0.444
49	-19.199	-8.796	-8.796	-8.796	-8.795	-0.388	-0.388	-0.388
50	-19.181	-8.805	-8.805	-8.805	-8.805	-1.696	0.329	0.329
51	-19.160	-8.820	-8.820	-8.820	-8.820	-1.430	0.280	0.280
52	-19.158	-8.831	-8.831	-8.831	-8.831	-0.285	-0.276	-0.276
53	-19.136	-8.846	-8.847	-8.846	-8.846	-0.241	-0.227	-0.241
54	-19.119	-8.860	-8.860	-8.860	-8.860	-0.702	-0.066	-0.066
55	-19.100	-8.873	-8.873	-8.873	-8.873	-0.232	-0.232	-0.212
56	-19.081	-8.884	-8.884	-8.884	-8.884	-0.360	-0.342	0.160
57	-19.043	-9.266	-9.256	-9.265	-7.801	-1.129	-1.129	0.534
58	-18.985	-9.478	-9.478	-9.478	-6.560	-1.335	-1.335	-1.335
59	-18.952	-9.606	-9.606	-9.606	-5.932	-1.717	-1.717	-1.715
60	-18.914	-9.697	-9.697	-9.697	-5.244	-2.057	-2.057	-2.056
61	-18.879	-9.767	-9.767	-9.767	-4.469	-2.412	-2.412	-2.412
62	-18.848	-9.816	-9.816	-9.816	-2.987	-2.987	-2.987	-2.987
63	-18.824	-9.832	-9.832	-9.832	-2.985	-2.985	-2.985	-2.985
64	-18.800	-9.848	-9.846	-9.848	-2.985	-2.986	-2.980	-2.990
65	-18.775	-9.862	-9.862	-9.862	-2.988	-2.988	-2.988	-2.989
66	-18.742	-10.161	-10.161	-9.276	-3.006	-3.006	-3.006	-3.006
67	-18.705	-10.362	-10.362	-8.862	-3.036	-3.036	-3.036	-3.036
68	-18.671	-10.482	-10.482	-8.612	-3.061	-3.061	-3.061	-3.061
69	-18.638	-10.573	-10.573	-8.428	-3.083	-3.083	-3.083	-3.083
70	-18.608	-10.645	-10.645	-8.285	-3.102	-3.102	-3.102	-3.102
71	-18.600	-10.660	-10.660	-8.250	-3.110	-3.110	-3.110	-3.110
72	-18.579	-10.705	-10.705	-8.170	-3.118	-3.118	-3.118	-3.118
73	-18.545	-10.787	-10.787	-7.881	-3.559	-3.559	-3.559	-1.874
74	-18.512	-10.880	-10.880	-7.584	-3.788	-3.788	-3.788	0.000
75	-18.481	-10.932	-10.932	-7.425	-3.867	-3.867	-3.867	0.259
76	-18.450	-11.005	-11.005	-6.996	-4.647	-4.647	-2.290	0.000
77	-18.421	-11.052	-11.052	-6.725	-4.860	-4.861	-1.978	0.000
78	-18.394	-11.089	-11.089	-6.487	-5.027	-5.027	-1.764	0.000
79	-18.366	-11.120	-11.120	-6.261	-5.182	-5.181	-1.148	-1.075
80	-18.339	-11.146	-11.146	-5.849	-5.849	-4.951	-1.079	-1.078

Table 30

FIG. 54

	Label							
Design #	8	9	10	11	12	13	14	15
41	0.855	0.855	0.856	8.594	8.594	8.594	8.594	19.391
42	-0.795	-0.792	4.532	8.475	8.475	8.475	8.476	19.403
43	0.735	0.735	0.737	8.653	8.653	8.653	8.663	19.340
44	-0.675	-0.675	3.084	8.599	8.599	8.599	8.599	19.338
45	-0.617	-0.617	3.307	8.646	8.646	8.646	8.647	19.308
46	-0.558	-0.558	2.953	8.688	8.688	8.688	8.688	19.279
47	0.500	0.501	0.502	8.754	8.754	8.754	8.754	19.243
48	-0.443	-0.443	2.296	8.755	8.755	8.755	8.755	19.227
49	-0.387	-0.387	1.989	8.782	8.782	8.782	8.782	19.203
50	0.335	0.335	0.336	8.814	8.814	8.814	8.814	19.178
51	0.280	0.290	0.290	8.830	8.830	8.830	8.830	19.160
52	-0.278	-0.278	1.416	8.826	8.826	8.826	8.826	19.159
53	-0.208	-0.224	1.152	8.843	8.844	8.844	8.844	19.139
54	0.002	0.146	0.663	8.860	8.860	8.860	8.860	19.119
55	-0.212	0.445	0.445	8.873	8.873	8.873	8.873	19.100
56	0.181	0.181	0.241	8.884	8.884	8.885	8.885	19.081
57	0.534	0.543	0.619	7.675	9.250	9.250	9.250	19.044
58	1.335	1.335	1.335	6.660	9.478	9.478	9.478	18.995
59	1.716	1.716	1.717	5.932	9.606	9.606	9.606	18.952
60	2.056	2.056	2.056	5.244	9.697	9.697	9.697	18.914
61	2.411	2.412	2.412	4.469	9.767	9.767	9.767	18.879
62	2.987	2.987	2.987	2.987	9.816	9.816	9.816	18.848
63	2.985	2.985	2.985	2.985	9.832	9.832	9.832	18.824
64	2.986	2.986	2.986	2.986	9.848	9.848	9.848	18.800
65	2.988	2.988	2.988	2.988	9.862	9.862	9.862	18.775
66	3.006	3.006	3.006	3.006	9.278	10.161	10.161	18.742
67	3.036	3.036	3.036	3.036	8.862	10.362	10.362	18.705
68	3.061	3.061	3.061	3.061	8.612	10.482	10.482	18.671
69	3.083	3.083	3.083	3.083	8.428	10.573	10.573	18.638
70	3.102	3.102	3.102	3.102	8.285	10.645	10.645	18.608
71	3.118	3.118	3.118	3.118	8.170	10.705	10.705	18.579
72	3.118	3.118	3.116	3.118	8.170	10.705	10.705	18.579
73	1.674	3.559	3.559	3.559	7.981	10.797	10.797	18.545
74	0.000	3.788	3.788	3.788	7.584	10.880	10.880	18.512
75	0.259	2.208	4.527	4.527	7.200	10.956	10.956	18.481
76	0.000	2.290	4.647	4.647	6.997	11.005	11.005	18.450
77	0.001	1.977	4.861	4.861	5.724	11.052	11.052	18.421
78	0.000	1.764	5.027	5.027	5.487	11.089	11.089	18.394
79	1.111	1.111	5.182	5.182	6.261	11.120	11.120	18.366
80	1.078	1.078	4.951	5.848	5.849	11.146	11.146	18.339

Table 31

FIG. 55

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-18.310	-11.166	-11.166	-5.820	-5.820	-5.038	-1.059	-1.059
82	-18.281	-11.184	-11.184	-5.829	-5.829	-5.042	-1.064	-1.064
83	-18.250	-11.200	-11.200	-5.869	-5.869	-4.977	-1.088	-1.088
84	-18.210	-11.625	-10.769	-6.029	-5.028	-4.637	-1.196	-1.196
85	-18.183	-11.856	-10.518	-6.167	-6.167	-4.318	-1.321	-1.321
86	-18.151	-12.003	-10.359	-6.260	-6.260	-4.096	-1.419	-1.419
87	-18.121	-12.114	-10.244	-6.327	-6.327	-3.928	-1.497	-1.497
88	-18.092	-12.202	-10.156	-6.378	-6.378	-3.806	-1.559	-1.559
89	-18.070	-12.260	-10.110	-6.410	-6.410	-3.730	-1.590	-1.590
90	-18.064	-12.276	-10.088	-6.417	-6.417	-3.703	-1.607	-1.607
91	-18.037	-12.339	-10.034	-6.448	-6.448	-3.630	-1.644	-1.643
92	-18.011	-12.397	-9.985	-6.483	-6.493	-3.412	-2.368	-0.967
93	-17.985	-12.447	-9.947	-6.510	-6.510	-2.960	-2.960	-0.633
94	-17.960	-12.496	-9.906	-6.787	-6.251	-2.977	-2.977	-0.578
95	-17.935	-12.548	-9.844	-7.024	-6.017	-3.029	-3.029	-0.316
96	-17.911	-12.592	-9.800	-7.137	-5.915	-3.056	-3.056	0.000
97	-17.887	-12.632	-9.760	-7.242	-5.805	-3.365	-2.785	0.000
98	-17.863	-12.668	-9.730	-7.318	-5.727	-3.481	-2.701	0.000
99	-17.839	-12.702	-9.704	-7.391	-5.829	-3.688	-2.476	-0.486
100	-17.814	-12.732	-9.684	-7.450	-5.545	-3.844	-2.283	-0.732
101	-17.789	-12.759	-9.675	-7.487	-5.497	-3.924	-2.181	-0.851
102	-17.763	-12.784	-9.674	-7.511	-5.476	-3.962	-2.136	-0.902
103	-17.736	-12.807	-9.675	-7.526	-5.473	-3.975	-2.126	-0.914
104	-17.708	-12.829	-9.687	-7.535	-5.482	-3.975	-2.140	-0.903
105	-17.679	-12.849	-9.700	-7.541	-5.496	-3.966	-2.162	-0.882
106	-17.650	-12.869	-9.716	-7.544	-5.514	-3.955	-2.187	-0.858
107	-17.640	-12.870	-9.720	-7.550	-5.520	-3.950	-2.190	-0.850
108	-17.619	-12.888	-9.734	-7.547	-5.533	-3.944	-2.212	-0.835
109	-17.588	-12.906	-9.753	-7.560	-5.552	-3.934	-2.234	-0.815
110	-17.556	-12.925	-9.774	-7.554	-5.571	-3.926	-2.253	-0.799
111	-17.523	-12.942	-9.795	-7.558	-5.588	-3.921	-2.269	-0.787
112	-17.490	-12.960	-9.818	-7.563	-5.604	-3.918	-2.282	-0.777
113	-17.456	-12.977	-9.841	-7.570	-5.619	-3.917	-2.293	-0.771
114	-17.421	-12.993	-9.864	-7.576	-5.633	-3.919	-2.301	-0.767
115	-17.385	-13.009	-9.888	-7.587	-5.646	-3.922	-2.308	-0.765
116	-17.349	-13.025	-9.913	-7.598	-5.659	-3.927	-2.314	-0.765
117	-17.312	-13.040	-9.938	-7.611	-5.671	-3.933	-2.320	-0.765
118	-17.274	-13.054	-9.963	-7.625	-5.683	-3.940	-2.325	-0.766
119	-17.235	-13.067	-9.989	-7.641	-5.695	-3.948	-2.329	-0.768
120	-17.196	-13.080	-10.015	-7.658	-5.708	-3.956	-2.334	-0.770

Table 32

FIG. 56

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	1.059	1.059	5.038	5.820	5.820	11.166	11.166	18.310
82	1.064	1.064	5.042	5.829	5.829	11.184	11.184	18.281
83	1.088	1.088	4.977	5.869	5.869	11.200	11.200	18.250
84	1.196	1.196	4.037	6.028	6.028	10.769	11.625	18.216
85	1.321	1.321	4.318	6.167	6.167	10.518	11.856	18.183
86	1.419	1.419	4.096	6.260	6.260	10.358	12.093	18.151
87	1.497	1.497	3.928	6.327	6.327	10.244	12.114	18.121
88	1.559	1.559	3.800	6.378	6.376	10.156	12.202	18.092
89	1.590	1.590	3.730	6.410	6.419	10.110	12.250	18.070
90	1.607	1.607	3.703	6.417	6.417	10.089	12.276	18.064
91	1.644	1.644	3.630	6.448	6.448	10.034	12.339	18.037
92	0.967	2.369	3.412	6.483	6.483	9.985	12.397	18.011
93	0.633	2.960	2.960	6.510	6.510	9.947	12.447	17.985
94	0.578	2.977	2.977	6.251	6.787	9.905	12.496	17.960
95	0.316	3.029	3.029	6.017	7.024	9.844	12.548	17.936
96	0.000	3.056	3.056	5.915	7.137	9.800	12.592	17.911
97	0.000	2.765	3.365	5.805	7.242	9.760	12.632	17.887
98	0.000	2.709	3.481	5.727	7.318	9.730	12.668	17.863
99	0.486	2.476	3.686	5.620	7.391	9.704	12.702	17.839
100	0.732	2.283	3.844	5.545	7.450	9.684	12.732	17.814
101	0.851	2.181	3.924	5.497	7.487	9.675	12.756	17.789
102	0.902	2.136	3.962	5.476	7.511	9.674	12.784	17.763
103	0.914	2.128	3.975	5.473	7.528	9.678	12.807	17.736
104	0.903	2.140	3.975	5.482	7.535	9.687	12.829	17.708
105	0.882	2.162	3.966	5.496	7.541	9.700	12.849	17.679
106	0.858	2.187	3.955	5.514	7.544	9.716	12.869	17.650
107	0.850	2.190	3.950	5.520	7.550	9.720	12.870	17.640
108	0.835	2.212	3.944	5.533	7.547	9.734	12.868	17.619
109	0.815	2.234	3.934	5.552	7.550	9.753	12.906	17.588
110	0.799	2.253	3.926	5.571	7.554	9.774	12.925	17.556
111	0.787	2.269	3.921	5.588	7.558	9.795	12.942	17.523
112	0.777	2.282	3.918	5.604	7.563	9.818	12.960	17.490
113	0.771	2.293	3.917	5.619	7.570	9.841	12.977	17.456
114	0.767	2.301	3.919	5.633	7.578	9.864	12.993	17.421
115	0.765	2.308	3.922	5.646	7.587	9.888	13.009	17.385
116	0.765	2.314	3.927	5.659	7.598	9.913	13.025	17.349
117	0.765	2.320	3.933	5.671	7.611	9.936	13.040	17.312
118	0.766	2.325	3.940	5.683	7.625	9.963	13.054	17.274
119	0.768	2.329	3.948	5.695	7.641	9.989	13.067	17.235
120	0.770	2.334	3.956	5.708	7.658	10.015	13.080	17.196

Table 33

FIG. 57

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-17.156	-13.092	-10.041	-7.676	-5.720	-3.986	-2.339	-0.772
122	-17.115	-13.104	-10.068	-7.696	-5.734	-3.975	-2.344	-0.774
123	-17.074	-13.114	-10.095	-7.718	-5.748	-3.966	-2.349	-0.776
124	-17.030	-13.120	-10.120	-7.740	-5.760	-4.000	-2.350	-0.780
125	-17.031	-13.124	-10.122	-7.741	-5.762	-3.997	-2.355	-0.778
126	-16.988	-13.132	-10.150	-7.765	-5.778	-4.008	-2.362	-0.780
127	-16.944	-13.140	-10.177	-7.791	-5.795	-4.020	-2.368	-0.783
128	-16.900	-13.147	-10.205	-7.817	-5.814	-4.032	-2.376	-0.785
129	-16.854	-13.152	-10.232	-7.845	-5.833	-4.046	-2.383	-0.788
130	-16.809	-13.157	-10.259	-7.875	-5.855	-4.060	-2.392	-0.790
131	-16.762	-13.161	-10.286	-7.905	-5.877	-4.075	-2.401	-0.793
132	-16.715	-13.163	-10.313	-7.936	-5.901	-4.091	-2.410	-0.796
133	-16.667	-13.165	-10.339	-7.967	-5.927	-4.109	-2.420	-0.800
134	-16.619	-13.166	-10.365	-8.000	-5.955	-4.128	-2.431	-0.803
135	-16.570	-13.165	-10.390	-8.033	-5.983	-4.148	-2.443	-0.807
136	-16.521	-13.164	-10.415	-8.066	-6.014	-4.170	-2.456	-0.811
137	-16.471	-13.162	-10.439	-8.100	-6.045	-4.193	-2.470	-0.816
138	-16.422	-13.159	-10.462	-8.134	-6.077	-4.217	-2.484	-0.821
139	-16.372	-13.156	-10.484	-8.167	-6.111	-4.243	-2.500	-0.826
140	-16.323	-13.151	-10.506	-8.201	-6.144	-4.270	-2.517	-0.832
141	-16.273	-13.146	-10.527	-8.234	-6.179	-4.297	-2.534	-0.838
142	-16.225	-13.140	-10.547	-8.266	-6.213	-4.326	-2.552	-0.844
143	-16.190	-13.140	-10.560	-8.290	-6.230	-4.340	-2.560	-0.850
144	-16.176	-13.134	-10.566	-8.298	-6.247	-4.354	-2.571	-0.850
145	-16.129	-13.126	-10.585	-8.329	-6.281	-4.383	-2.589	-0.857
146	-16.083	-13.122	-10.603	-8.359	-6.313	-4.411	-2.608	-0.863
147	-16.038	-13.116	-10.620	-8.388	-6.345	-4.439	-2.627	-0.870
148	-15.994	-13.110	-10.636	-8.416	-6.376	-4.466	-2.645	-0.876
149	-15.952	-13.104	-10.652	-8.443	-6.406	-4.492	-2.662	-0.882
150	-15.911	-13.098	-10.667	-8.469	-6.435	-4.517	-2.679	-0.888
151	-15.872	-13.093	-10.681	-8.494	-6.462	-4.541	-2.695	-0.894
152	-15.834	-13.087	-10.695	-8.517	-6.488	-4.564	-2.710	-0.899
153	-15.798	-13.083	-10.708	-8.540	-6.513	-4.585	-2.725	-0.904
154	-15.763	-13.078	-10.721	-8.561	-6.536	-4.606	-2.739	-0.909
155	-15.730	-13.074	-10.733	-8.582	-6.558	-4.625	-2.752	-0.913
156	-15.698	-13.070	-10.745	-8.601	-6.579	-4.643	-2.764	-0.918
157	-15.668	-13.066	-10.756	-8.619	-6.599	-4.660	-2.776	-0.922
158	-15.639	-13.062	-10.767	-8.637	-6.618	-4.677	-2.787	-0.926
159	-15.611	-13.059	-10.777	-8.654	-6.636	-4.692	-2.797	-0.929
160	-15.584	-13.056	-10.787	-8.669	-6.653	-4.707	-2.807	-0.933

Table 34

FIG. 58

	Label							
Design #	8	9	10	11	12	13	14	15
121	0.772	2.339	3.986	5.720	7.670	10.041	13.092	17.156
122	0.774	2.344	3.975	5.734	7.696	10.068	13.104	17.115
123	0.776	2.349	3.986	5.748	7.715	10.095	13.114	17.074
124	0.780	2.350	4.000	5.760	7.740	10.120	13.120	17.030
125	0.778	2.355	3.997	5.762	7.741	10.122	13.124	17.031
126	0.780	2.362	4.008	5.778	7.765	10.150	13.132	16.988
127	0.783	2.368	4.020	5.795	7.791	10.177	13.140	16.944
128	0.785	2.376	4.032	5.814	7.817	10.205	13.147	16.900
129	0.788	2.383	4.046	5.833	7.845	10.232	13.152	16.854
130	0.790	2.392	4.060	5.855	7.875	10.259	13.157	16.809
131	0.793	2.401	4.075	5.877	7.905	10.286	13.161	16.762
132	0.796	2.410	4.091	5.901	7.936	10.313	13.163	16.715
133	0.800	2.420	4.109	5.927	7.967	10.339	13.165	16.667
134	0.803	2.431	4.128	5.955	8.000	10.365	13.166	16.619
135	0.807	2.443	4.148	5.983	8.033	10.390	13.166	16.570
136	0.811	2.456	4.170	6.014	8.066	10.415	13.164	16.521
137	0.816	2.470	4.193	6.045	8.100	10.439	13.162	16.471
138	0.821	2.484	4.217	6.077	8.134	10.462	13.159	16.422
139	0.826	2.500	4.243	6.111	8.167	10.484	13.155	16.372
140	0.832	2.517	4.270	6.144	8.201	10.506	13.151	16.323
141	0.838	2.534	4.297	6.179	8.234	10.527	13.146	16.273
142	0.844	2.552	4.326	6.213	8.268	10.547	13.140	16.225
143	0.850	2.560	4.340	6.230	8.290	10.560	13.140	16.190
144	0.850	2.571	4.354	6.247	8.298	10.566	13.134	16.178
145	0.857	2.589	4.383	6.281	8.329	10.585	13.128	16.129
146	0.863	2.608	4.411	6.313	8.359	10.603	13.122	16.083
147	0.870	2.627	4.439	6.345	8.388	10.620	13.116	16.038
148	0.876	2.645	4.466	6.376	8.418	10.636	13.110	15.994
149	0.882	2.662	4.492	6.406	8.443	10.652	13.104	15.952
150	0.888	2.679	4.517	6.435	8.469	10.667	13.098	15.911
151	0.894	2.695	4.541	6.462	8.494	10.681	13.093	15.872
152	0.898	2.710	4.564	6.488	8.517	10.695	13.087	15.834
153	0.904	2.725	4.585	6.513	8.540	10.708	13.083	15.798
154	0.909	2.739	4.606	6.536	8.561	10.721	13.078	15.763
155	0.913	2.752	4.625	6.558	8.582	10.733	13.074	15.730
156	0.918	2.764	4.643	6.579	8.601	10.745	13.070	15.698
157	0.922	2.776	4.660	6.599	8.619	10.756	13.066	15.668
158	0.926	2.787	4.677	6.618	8.637	10.767	13.062	15.639
159	0.929	2.797	4.692	6.636	8.654	10.777	13.059	15.611
160	0.933	2.807	4.707	6.653	8.669	10.787	13.056	15.584

Table 35

FIG. 59

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-15.559	-13.053	-10.796	-8.684	-6.670	-4.721	-2.816	-0.936
162	-15.534	-13.050	-10.805	-8.699	-6.685	-4.734	-2.825	-0.939
163	-15.512	-13.047	-10.813	-8.712	-6.699	-4.747	-2.833	-0.942
164	-15.489	-13.045	-10.822	-8.726	-6.714	-4.758	-2.841	-0.945
165	-15.468	-13.042	-10.828	-8.738	-6.727	-4.770	-2.848	-0.947
166	-15.447	-13.040	-10.837	-8.750	-6.739	-4.781	-2.856	-0.950
167	-15.428	-13.038	-10.844	-8.761	-6.751	-4.791	-2.862	-0.952
168	-15.409	-13.036	-10.851	-8.772	-6.763	-4.801	-2.869	-0.954
169	-15.392	-13.035	-10.857	-8.782	-6.773	-4.809	-2.874	-0.956
170	-15.374	-13.033	-10.864	-8.792	-6.784	-4.818	-2.880	-0.958
171	-15.359	-13.032	-10.870	-8.801	-6.793	-4.826	-2.886	-0.960
172	-15.344	-13.030	-10.875	-8.810	-6.802	-4.834	-2.891	-0.962
173	-15.328	-13.028	-10.881	-8.819	-6.812	-4.842	-2.896	-0.964
174	-15.312	-13.027	-10.887	-8.827	-6.821	-4.850	-2.901	-0.966
175	-15.300	-13.025	-10.891	-8.835	-6.829	-4.858	-2.906	-0.967
176	-15.287	-13.024	-10.896	-8.842	-6.836	-4.862	-2.910	-0.969
177	-15.275	-13.023	-10.900	-8.849	-6.843	-4.868	-2.913	-0.970
178	-15.261	-13.022	-10.905	-8.856	-6.851	-4.875	-2.918	-0.971
179	-15.252	-13.022	-10.909	-8.861	-6.856	-4.879	-2.921	-0.973
180	-15.240	-13.020	-10.913	-8.869	-6.865	-4.887	-2.926	-0.974

Table 36

FIG. 60

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	0.938	2.818	4.721	6.670	6.684	10.796	13.053	15.559
162	0.939	2.825	4.734	6.685	6.699	10.806	13.060	15.534
163	0.942	2.833	4.747	6.699	6.712	10.813	13.047	15.512
164	0.945	2.841	4.758	6.714	6.726	10.822	13.046	15.489
165	0.947	2.848	4.770	6.727	6.738	10.829	13.042	15.468
166	0.950	2.856	4.781	6.739	6.750	10.837	13.040	15.447
167	0.952	2.862	4.791	6.751	6.761	10.844	13.038	15.428
168	0.954	2.869	4.801	6.763	6.772	10.851	13.036	15.409
169	0.956	2.874	4.809	6.773	6.782	10.857	13.035	15.392
170	0.958	2.880	4.818	6.784	6.792	10.864	13.033	15.374
171	0.960	2.885	4.826	6.793	6.801	10.870	13.032	15.359
172	0.962	2.891	4.834	6.802	6.810	10.875	13.030	15.344
173	0.964	2.896	4.842	6.812	6.819	10.881	13.028	15.328
174	0.966	2.901	4.850	6.821	6.827	10.887	13.027	15.312
175	0.967	2.906	4.856	6.829	6.835	10.891	13.026	15.300
176	0.969	2.909	4.862	6.836	6.842	10.896	13.024	15.287
177	0.970	2.913	4.868	6.843	6.849	10.900	13.023	15.275
178	0.971	2.918	4.875	6.851	6.856	10.905	13.022	15.261
179	0.973	2.921	4.879	6.856	6.861	10.909	13.022	15.252
180	0.974	2.926	4.887	6.865	6.869	10.913	13.019	15.240

Table 37

FIG. 61

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.27	0.23	0.2	0.17	0.14	0
2	-4.8	0.31	0.26	0.22	0.19	0.16	0
3	-4.6	0.36	0.32	0.27	0.23	0.18	0
4	-4.4	0.34	0.29	0.25	0.21	0.16	0
5	-4.2	0.34	0.29	0.25	0.21	0.18	0
6	-4	0.36	0.32	0.27	0.23	0.2	0
7	-3.8	0.38	0.32	0.27	0.23	0.2	0
8	-3.6	0.42	0.36	0.3	0.25	0.22	0
9	-3.4	0.38	0.32	0.27	0.23	0.2	0
10	-3.2	0.47	0.4	0.34	0.29	0.24	0
11	-3	0.52	0.4	0.34	0.26	0.22	0
12	-2.8	0.52	0.44	0.38	0.32	0.27	0
13	-2.6	0.47	0.4	0.34	0.29	0.24	0
14	-2.4	0.52	0.44	0.38	0.32	0.27	0
15	-2.2	0.56	0.49	0.42	0.36	0.3	0
16	-2	0.59	0.49	0.42	0.36	0.27	0
17	-1.8	0.66	0.56	0.47	0.4	0.34	0
18	-1.6	0.66	0.56	0.47	0.4	0.34	0
19	-1.4	0.72	0.61	0.52	0.44	0.37	0
20	-1.2	0.66	0.56	0.47	0.4	0.34	0
21	-1	0.71	0.61	0.52	0.44	0.37	0
22	-0.8	0.8	0.68	0.56	0.49	0.42	0
23	-0.6	0.74	0.63	0.53	0.45	0.38	0
24	-0.4	0.83	0.7	0.6	0.51	0.43	0
25	-0.2	0.93	0.79	0.67	0.57	0.44	0
26	0	0.93	0.79	0.67	0.57	0.49	0
27	0	0.93	0.79	0.67	0.57	0.44	0
28	0.2	1.04	0.89	0.75	0.64	0.49	0
29	0.4	0.97	0.82	0.7	0.59	0.45	0
30	0.6	0.97	0.82	0.7	0.59	0.6	0
31	0.8	0.97	0.82	0.7	0.6	0.51	0
32	1	0.97	0.83	0.7	0.6	0.51	0
33	1.2	0.96	0.83	0.71	0.6	0.51	0
34	1.4	1.09	0.92	0.79	0.67	0.57	0
35	1.6	1.09	0.93	0.79	0.67	0.57	0
36	1.8	1.32	1.04	0.88	0.75	0.64	0
37	2	1.23	1.04	0.89	0.75	0.64	0
38	2.2	1.39	1.05	0.89	0.76	0.64	0
39	2.4	1.26	1.07	0.91	0.77	0.86	0

Table 38

FIG. 62

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
40	2.6	1.25	1.06	0.9	0.77	0.65	0
41	2.8	1.25	1.06	0.9	0.77	0.65	0
42	3	1.38	1.18	1	0.85	0.65	0
43	3.2	1.38	1.18	1	0.85	0.65	0
44	3.4	1.38	1.18	1	0.85	0.72	0
45	3.6	1.34	1.14	0.97	0.83	0.7	0
46	3.8	1.2	1.02	0.87	0.74	0.63	0
47	4	1.46	1.24	1.05	0.9	0.69	0
48	4.2	1.42	1.21	1.03	0.87	0.74	0
49	4.4	1.92	1.29	1.1	0.93	0.71	0
50	4.6	1.44	1.29	1.04	0.89	0.68	0
51	4.79	1.22	1.04	0.88	0.75	0.64	0
52	4.8	1.2	1.02	0.87	0.74	0.63	0
53	5	0.98	0.83	0.71	0.6	0.51	0
54	5.2	0.6	0.51	0.43	0.37	0.31	0
55	5.4	0.38	0.33	0.28	0.24	0.2	0
56	5.6	0.37	0.32	0.27	0.23	0.19	0
57	5.8	1.09	0.93	0.79	0.67	0.57	0
58	6	1.45	1.23	1.05	0.89	0.76	0
59	6.2	1.46	1.24	1.06	0.9	0.69	0
60	6.4	1.4	1.19	1.01	0.86	0.73	0
61	6.6	1.41	1.2	1.02	0.86	0.74	0
62	6.8	1.42	1.2	1.02	0.87	0.67	0
63	7	1.42	1.21	1.02	0.87	0.67	0
64	7.2	1.42	1.21	1.03	0.87	0.67	0
65	7.4	1.28	1.09	0.92	0.79	0.67	0
66	7.6	1.42	1.21	1.02	0.87	0.67	0
67	7.8	1.29	1.1	0.93	0.79	0.67	0
68	8	1.3	1.11	0.94	0.8	0.68	0
69	8.2	1.31	1.11	0.95	0.8	0.68	0
70	8.4	1.32	1.12	0.95	0.81	0.69	0
71	8.62	1.32	1.12	0.95	0.81	0.67	0
72	8.8	1.32	1.13	0.96	0.81	0.69	0
73	8.8	1.29	1.09	0.93	0.79	0.67	0
74	9	1.35	1.15	0.97	0.83	0.7	0
75	9.2	1.38	1.17	0.99	0.85	0.66	0
76	9.4	1.24	1.05	0.89	0.76	0.65	0
77	9.6	1.24	1.06	0.89	0.76	0.65	0
78	9.8	1.25	1.06	0.89	0.76	0.64	0
79	10	1.25	1.06	0.89	0.76	0.64	0
80	10.2	1.18	1	0.85	0.72	0.61	0

Table 39

FIG. 63

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	1.2	1.02	0.86	0.73	0.62	0
82	10.6	1.2	1.02	0.87	0.74	0.63	0
83	10.8	1.19	1.01	0.86	0.73	0.56	0
84	11	1.08	0.92	0.6	0.51	0.43	0
85	11.2	0.92	0.78	0.44	0.37	0.32	0
86	11.4	0.82	0.7	0.36	0.31	0.26	0
87	11.6	0.78	0.67	0.37	0.31	0.27	0
88	11.8	0.78	0.67	0.39	0.33	0.28	0
89	11.94	0.79	0.67	0.4	0.34	0.29	0
90	12	0.79	0.67	0.39	0.33	0.28	0
91	12.2	0.8	0.68	0.41	0.35	0.3	0
92	12.4	0.8	0.68	0.58	0.49	0.42	0
93	12.6	0.81	0.68	0.59	0.49	0.42	0
94	12.8	0.68	0.58	0.49	0.42	0.36	0
95	13	0.66	0.56	0.48	0.41	0.35	0
96	13.2	0.67	0.57	0.48	0.41	0.35	0
97	13.4	0.61	0.52	0.44	0.38	0.32	0
98	13.6	0.61	0.52	0.44	0.38	0.32	0
99	13.8	0.22	0.19	0.16	0.13	0.11	0
100	14	0.06	0.05	0.05	0.04	0.03	0
101	14.2	0.09	0.07	0.06	0.05	0.04	0
102	14.4	0.13	0.11	0.09	0.06	0.07	0
103	14.6	0.14	0.12	0.1	0.08	0.07	0
104	14.8	0.15	0.13	0.11	0.09	0.08	0
105	15	0.17	0.14	0.12	0.1	0.09	0
106	15.2	0.19	0.16	0.14	0.12	0.1	0
107	15.25	0.19	0.17	0.14	0.12	0.1	0
108	15.4	0.21	0.18	0.15	0.13	0.11	0
109	15.6	0.23	0.2	0.17	0.14	0.12	0
110	15.8	0.25	0.21	0.18	0.15	0.13	0
111	16	0.27	0.23	0.2	0.17	0.14	0
112	16.2	0.29	0.25	0.21	0.18	0.15	0
113	16.4	0.31	0.26	0.22	0.19	0.16	0
114	16.6	0.33	0.28	0.24	0.2	0.17	0
115	16.8	0.35	0.3	0.25	0.22	0.18	0
116	17	0.37	0.32	0.27	0.23	0.19	0
117	17.2	0.39	0.33	0.28	0.24	0.21	0
118	17.4	0.41	0.35	0.3	0.26	0.22	0
119	17.6	0.43	0.37	0.31	0.27	0.23	0
120	17.8	0.45	0.39	0.33	0.28	0.24	0

Table 40

FIG. 64

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.47	0.4	0.34	0.29	0.25	0
122	18.2	0.49	0.42	0.36	0.3	0.25	0
123	18.4	0.46	0.39	0.33	0.28	0.24	0
124	18.59	0.48	0.41	0.31	0.27	0.23	0
125	18.6	0.48	0.41	0.31	0.27	0.23	0
126	18.8	0.45	0.36	0.3	0.25	0.21	0
127	19	0.46	0.39	0.26	0.24	0.21	0
128	19.2	0.43	0.37	0.28	0.24	0.21	0
129	19.4	0.4	0.34	0.28	0.24	0.2	0
130	19.6	0.41	0.35	0.28	0.24	0.2	0
131	19.8	0.36	0.3	0.25	0.22	0.18	0
132	20	0.33	0.28	0.24	0.2	0.17	0
133	20.2	0.34	0.29	0.25	0.21	0.18	0
134	20.4	0.32	0.27	0.23	0.2	0.17	0
135	20.6	0.3	0.26	0.22	0.18	0.16	0
136	20.8	0.31	0.26	0.22	0.19	0.15	0
137	21	0.26	0.22	0.19	0.16	0.14	0
138	21.2	0.27	0.23	0.19	0.17	0.14	0
139	21.4	0.25	0.21	0.18	0.15	0.13	0
140	21.6	0.24	0.2	0.17	0.15	0.11	0
141	21.8	0.21	0.18	0.15	0.13	0.11	0
142	22	0.2	0.17	0.14	0.12	0.1	0
143	22.12	0.22	0.18	0.16	0.13	0.1	0
144	22.2	0.19	0.16	0.14	0.12	0.1	0
145	22.4	0.18	0.18	0.13	0.11	0.1	0
146	22.6	0.18	0.15	0.13	0.11	0.09	0
147	22.8	0.17	0.14	0.12	0.1	0.09	0
148	23	0.14	0.12	0.1	0.09	0.08	0
149	23.2	0.15	0.13	0.11	0.09	0.07	0
150	23.4	0.13	0.11	0.09	0.08	0.07	0
151	23.6	0.14	0.12	0.1	0.09	0.07	0
152	23.8	0.13	0.11	0.1	0.08	0.07	0
153	24	0.13	0.11	0.09	0.08	0.06	0
154	24.2	0.11	0.09	0.08	0.07	0.06	0
155	24.4	0.1	0.09	0.08	0.06	0.05	0
156	24.6	0.1	0.08	0.07	0.06	0.05	0
157	24.8	0.09	0.08	0.07	0.06	0.05	0
158	25	0.09	0.08	0.07	0.06	0.05	0
159	25.2	0.09	0.07	0.06	0.05	0.05	0
160	25.4	0.08	0.07	0.06	0.05	0.04	0

Table 41

FIG. 65

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
161	25.6	0.09	0.07	0.06	0.05	0.04	0
162	25.8	0.08	0.06	0.06	0.05	0.04	0
163	26	0.07	0.06	0.05	0.04	0.04	0
164	26.2	0.07	0.06	0.05	0.04	0.04	0
165	26.4	0.07	0.06	0.05	0.04	0.03	0
166	26.6	0.06	0.05	0.05	0.04	0.03	0
167	26.8	0.06	0.05	0.04	0.03	0.03	0
168	27	0.06	0.05	0.04	0.04	0.03	0
169	27.2	0.06	0.05	0.04	0.03	0.03	0
170	27.4	0.05	0.04	0.04	0.03	0.03	0
171	27.6	0.05	0.04	0.04	0.03	0.03	0
172	27.8	0.05	0.04	0.03	0.03	0.02	0
173	28	0.04	0.04	0.03	0.03	0.02	0
174	28.2	0.04	0.04	0.03	0.03	0.02	0
175	28.4	0.04	0.04	0.03	0.03	0.02	0
176	28.6	0.04	0.03	0.03	0.02	0.02	0
177	28.8	0.03	0.03	0.03	0.02	0.02	0
178	29	0.04	0.03	0.03	0.02	0.02	0
179	29.2	0.04	0.03	0.03	0.02	0.02	0
180	29.4	0.03	0.03	0.02	0.02	0.02	0

Table 42

FIG. 66

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1977	0.1576	0.0401	25.4629
2	-4.8000	0.2058	0.1643	0.0415	25.2281
3	-4.6000	0.2141	0.1713	0.0428	24.9841
4	-4.4000	0.2227	0.1785	0.0442	24.7360
5	-4.2000	0.2316	0.1860	0.0455	24.4818
6	-4.0000	0.2407	0.1938	0.0469	24.2216
7	-3.8000	0.2502	0.2019	0.0484	23.9548
8	-3.6000	0.2600	0.2102	0.0498	23.6816
9	-3.4000	0.2700	0.2188	0.0512	23.4016
10	-3.2000	0.2804	0.2277	0.0526	23.1144
11	-3.0000	0.2910	0.2370	0.0541	22.8200
12	-2.8000	0.3020	0.2465	0.0556	22.5180
13	-2.6000	0.3133	0.2563	0.0569	22.2080
14	-2.4000	0.3248	0.2665	0.0583	21.8896
15	-2.2000	0.3367	0.2770	0.0597	21.5628
16	-2.0000	0.3489	0.2878	0.0611	21.2267
17	-1.8000	0.3613	0.2989	0.0624	20.8808
18	-1.6000	0.3741	0.3104	0.0637	20.5252
19	-1.4000	0.3872	0.3222	0.0650	20.1587
20	-1.2000	0.4005	0.3344	0.0661	19.7810
21	-1.0000	0.4141	0.3469	0.0673	19.3912
22	-0.8000	0.4280	0.3597	0.0683	18.9893
23	-0.6000	0.4421	0.3728	0.0693	18.5736
24	-0.4000	0.4565	0.3864	0.0701	18.1444
25	-0.2000	0.4711	0.4003	0.0708	17.7001
26	0.0000	0.4859	0.4145	0.0715	17.2401
27	0.2000	0.5010	0.4291	0.0719	16.7635
28	0.4000	0.5162	0.4440	0.0722	16.2693
29	0.6000	0.5316	0.4592	0.0724	15.7567
30	0.8000	0.5471	0.4748	0.0723	15.2244
31	1.0000	0.5628	0.4908	0.0720	14.6717
32	1.2000	0.5786	0.5071	0.0715	14.0974
33	1.4000	0.5944	0.5237	0.0707	13.5004
34	1.6000	0.6104	0.5407	0.0698	12.9055
35	1.8000	0.6270	0.5580	0.0691	12.3775
36	2.0000	0.6442	0.5756	0.0686	11.9162
37	2.2000	0.6619	0.5936	0.0684	11.5177
38	2.4000	0.6802	0.6118	0.0684	11.1782
39	2.6000	0.6991	0.6305	0.0687	10.8936
40	2.8000	0.7186	0.6494	0.0692	10.6597

Table 43

FIG. 67

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	3.0000	0.7386	0.6686	0.0700	10.4722
42	3.2000	0.7593	0.6882	0.0711	10.3285
43	3.4000	0.7804	0.7081	0.0724	10.2182
44	3.6000	0.8021	0.7282	0.0739	10.1425
45	3.8000	0.8243	0.7487	0.0756	10.0945
46	4.0000	0.8470	0.7695	0.0775	10.0696
47	4.2000	0.8702	0.7906	0.0796	10.0628
48	4.4000	0.8938	0.8120	0.0818	10.0689
49	4.6000	0.9178	0.8337	0.0841	10.0835
50	4.8000	0.9421	0.8557	0.0864	10.1016
51	5.0000	0.9668	0.8780	0.0888	10.1186
52	5.2000	0.9917	0.9005	0.0912	10.1305
53	5.4000	1.0169	0.9234	0.0936	10.1329
54	5.6000	1.0423	0.9465	0.0958	10.1224
55	5.8000	1.0678	0.9699	0.0979	10.0957
56	6.0000	1.0934	0.9936	0.0999	10.0497
57	6.2000	1.1191	1.0175	0.1016	9.9824
58	6.4000	1.1448	1.0417	0.1030	9.8921
59	6.6000	1.1704	1.0662	0.1042	9.7773
60	6.8000	1.1960	1.0909	0.1051	9.6380
61	7.0000	1.2216	1.1159	0.1057	9.4739
62	7.2000	1.2470	1.1411	0.1060	9.2859
63	7.4000	1.2725	1.1665	0.1060	9.0866
64	7.6000	1.2988	1.1922	0.1067	8.9456
65	7.8000	1.3259	1.2180	0.1079	8.8548
66	8.0000	1.3535	1.2441	0.1094	8.7928
67	8.2000	1.3814	1.2704	0.1110	8.7338
68	8.4000	1.4094	1.2969	0.1125	8.6745
69	8.6000	1.4376	1.3236	0.1140	8.6134
70	8.8000	1.4696	1.3505	0.1191	8.8172
71	9.0000	1.4999	1.3776	0.1223	8.8779
72	9.0009	1.5000	1.3777	0.1223	8.8771
73	9.2000	1.5300	1.4048	0.1252	8.9121
74	9.4000	1.5600	1.4323	0.1278	8.9202
75	9.6000	1.5899	1.4599	0.1300	8.9029
76	9.8000	1.6195	1.4877	0.1318	8.8608
77	10.0000	1.6490	1.5157	0.1333	8.7947
78	10.2000	1.6784	1.5439	0.1345	8.7124
79	10.4000	1.7079	1.5722	0.1357	8.8296
80	10.6000	1.7376	1.6008	0.1368	8.5474

Table 44

FIG. 68

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.8000	1.7719	1.6296	0.1424	8.7355
82	11.0000	1.8033	1.6586	0.1447	8.7246
83	11.2000	1.8346	1.6877	0.1468	8.6985
84	11.4000	1.8659	1.7172	0.1487	8.6597
85	11.6000	1.8972	1.7468	0.1504	8.6109
86	11.8000	1.9287	1.7767	0.1520	8.5555
87	12.0000	1.9603	1.8068	0.1535	8.4959
88	12.2000	1.9920	1.8371	0.1549	8.4335
89	12.2496	2.0000	1.8447	0.1553	8.4167
90	12.4000	2.0240	1.8677	0.1563	8.3688
91	12.6000	2.0561	1.8985	0.1576	8.2996
92	12.8000	2.0883	1.9296	0.1587	8.2232
93	13.0000	2.1205	1.9608	0.1596	8.1376
94	13.2000	2.1527	1.9925	0.1602	8.0402
95	13.4000	2.1848	2.0242	0.1605	7.9297
96	13.6000	2.2167	2.0563	0.1605	7.8050
97	13.8000	2.2486	2.0885	0.1601	7.6656
98	14.0000	2.2803	2.1209	0.1593	7.5115
99	14.2000	2.3117	2.1536	0.1581	7.3434
100	14.4000	2.3430	2.1864	0.1566	7.1620
101	14.6000	2.3741	2.2194	0.1547	6.9689
102	14.8000	2.4050	2.2526	0.1524	6.7660
103	15.0000	2.4358	2.2859	0.1499	6.5558
104	15.2000	2.4664	2.3194	0.1471	6.3410
105	15.4000	2.4971	2.3529	0.1441	6.1245
106	15.4152	2.5000	2.3562	0.1438	6.1030
107	15.6000	2.5276	2.3866	0.1410	5.9087
108	15.8000	2.5582	2.4204	0.1379	5.6954
109	16.0000	2.5888	2.4542	0.1346	5.4854
110	16.2000	2.6194	2.4881	0.1313	5.2791
111	16.4000	2.6500	2.5220	0.1280	5.0762
112	16.6000	2.6806	2.5560	0.1246	4.8762
113	16.8000	2.7111	2.5899	0.1212	4.6790
114	17.0000	2.7415	2.6238	0.1177	4.4842
115	17.2000	2.7718	2.6577	0.1141	4.2915
116	17.4000	2.8020	2.6916	0.1104	4.1009
117	17.6000	2.8321	2.7254	0.1066	3.9130
118	17.8000	2.8620	2.7591	0.1029	3.7285
119	18.0000	2.8919	2.7928	0.0991	3.5488
120	18.2000	2.9218	2.8263	0.0954	3.3762

Table 45

FIG. 69

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.4000	2.9517	2.8598	0.0919	3.2131
122	18.6000	2.9517	2.8931	0.0886	3.0622
123	18.7214	3.0000	2.9133	0.0867	2.9760
124	18.8000	3.0119	2.9263	0.0856	2.9259
125	19.0000	3.0423	2.9593	0.0830	2.8046
126	19.2000	3.0729	2.9922	0.0807	2.6964
127	19.4000	3.1035	3.0249	0.0786	2.5978
128	19.6000	3.1341	3.0575	0.0766	2.5058
129	19.8000	3.1645	3.0899	0.0747	2.4178
130	20.0000	3.1949	3.1221	0.0728	2.3321
131	20.2000	3.2250	3.1541	0.0709	2.2472
132	20.4000	3.2548	3.1859	0.0689	2.1625
133	20.6000	3.2844	3.2176	0.0668	2.0772
134	20.8000	3.3137	3.2490	0.0647	1.9911
135	21.0000	3.3427	3.2802	0.0625	1.9039
136	21.2000	3.3714	3.3112	0.0601	1.8158
137	21.4000	3.3997	3.3420	0.0577	1.7270
138	21.6000	3.4276	3.3724	0.0552	1.6377
139	21.8000	3.4553	3.4026	0.0527	1.5483
140	22.0000	3.4826	3.4324	0.0501	1.4593
141	22.1304	3.5000	3.4516	0.0484	1.4022
142	22.2000	3.5093	3.4618	0.0475	1.3712
143	22.4000	3.5357	3.4909	0.0448	1.2844
144	22.6000	3.5618	3.5194	0.0422	1.1994
145	22.8000	3.5871	3.5475	0.0396	1.1167
146	23.0000	3.6120	3.5750	0.0371	1.0366
147	23.2000	3.6364	3.6019	0.0346	0.9594
148	23.4000	3.6602	3.6281	0.0321	0.8854
149	23.6000	3.6834	3.6536	0.0298	0.8148
150	23.8000	3.7059	3.6784	0.0275	0.7476
151	24.0000	3.7277	3.7023	0.0253	0.6839
152	24.2000	3.7487	3.7254	0.0232	0.6237
153	24.4000	3.7689	3.7476	0.0213	0.5670
154	24.6000	3.7882	3.7688	0.0194	0.5139
155	24.8000	3.8067	3.7891	0.0176	0.4641
156	25.0000	3.8242	3.8083	0.0159	0.4176
157	25.2000	3.8409	3.8266	0.0143	0.3744
158	25.4000	3.8566	3.8437	0.0129	0.3344
159	25.6000	3.8713	3.8598	0.0115	0.2974
160	25.8000	3.8851	3.8749	0.0102	0.2634

Table 4a

FIG. 70

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	26.0000	3.8979	3.8889	0.0090	0.2323
162	26.2000	3.9097	3.9018	0.0079	0.2037
163	26.4000	3.9207	3.9137	0.0070	0.1779
164	26.6000	3.9306	3.9246	0.0061	0.1545
165	26.8000	3.9397	3.9345	0.0052	0.1334
166	27.0000	3.9479	3.9434	0.0045	0.1145
167	27.2000	3.9553	3.9516	0.0039	0.0977
168	27.4000	3.9619	3.9587	0.0033	0.0829
169	27.6000	3.9678	3.9650	0.0028	0.0698
170	27.8000	3.9729	3.9706	0.0023	0.0584
171	28.0000	3.9775	3.9755	0.0019	0.0486
172	28.2000	3.9814	3.9796	0.0016	0.0399
173	28.4000	3.9847	3.9834	0.0013	0.0326
174	28.6000	3.9876	3.9865	0.0011	0.0264
175	28.8000	3.9900	3.9892	0.0008	0.0212
176	29.0000	3.9920	3.9914	0.0007	0.0169
177	29.2000	3.9937	3.9932	0.0005	0.0134
178	29.4000	3.9951	3.9947	0.0004	0.0103

Table 47

FIG. 71

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-9.221	-9.221	-9.221	-9.221	-9.216	-9.216	-9.221	-9.221
2	-9.253	-9.252	-9.200	-9.252	-9.200	-9.200	-9.200	-9.200
3	-9.232	-9.222	-9.216	-9.222	-9.216	-9.216	-9.216	-9.216
4	-9.224	-9.220	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219
5	-9.226	-9.226	-9.226	-9.226	-9.195	-9.219	-9.220	-9.220
6	-9.230	-9.225	-9.218	-9.218	-9.217	-9.217	-9.217	-9.217
7	-9.229	-9.224	-9.224	-9.224	-9.214	-9.214	-9.214	-9.214
8	-9.225	-9.225	-9.225	-9.225	-9.192	-9.219	-9.225	-9.219
9	-9.225	-9.224	-9.219	-9.219	-9.215	-9.218	-9.219	-9.218
10	-9.227	-9.227	-9.227	-9.227	-9.193	-9.200	-9.227	-9.227
11	-9.221	-9.221	-9.221	-9.221	-9.214	-9.218	-9.221	-9.220
12	-9.220	-9.220	-9.220	-9.220	-9.217	-9.217	-9.220	-9.220
13	-9.228	-9.228	-9.215	-9.228	-9.215	-9.215	-9.215	-9.215
14	-9.221	-9.221	-9.221	-9.221	-9.216	-9.216	-9.221	-9.220
15	-9.220	-9.220	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
16	-9.220	-9.220	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
17	-9.232	-9.232	-9.232	-9.232	-9.189	-9.189	-9.231	-9.219
18	-9.224	-9.224	-9.224	-9.224	-9.211	-9.212	-9.224	-9.212
19	-9.222	-9.220	-9.220	-9.220	-9.219	-9.219	-9.220	-9.219
20	-9.223	-9.220	-9.219	-9.220	-9.217	-9.219	-9.219	-9.219
21	-9.226	-9.220	-9.220	-9.220	-9.219	-9.219	-9.220	-9.220
22	-9.222	-9.222	-9.220	-9.220	-9.218	-9.219	-9.219	-9.219
23	-9.229	-9.223	-9.219	-9.219	-9.217	-9.217	-9.217	-9.217
24	-9.225	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219	-9.219
25	-9.224	-9.222	-9.220	-9.221	-9.217	-9.217	-9.217	-9.217
26	-9.223	-9.223	-9.223	-9.223	-9.210	-9.212	-9.222	-9.222
27	-9.222	-9.221	-9.221	-9.221	-9.217	-9.217	-9.220	-9.218
28	-9.235	-9.235	-9.230	-9.235	-9.203	-9.206	-9.207	-9.207
29	-9.226	-9.220	-9.219	-9.219	-9.218	-9.218	-9.218	-9.218
30	-9.236	-9.236	-9.211	-9.236	-9.207	-9.210	-9.210	-9.210
31	-9.225	-9.225	-9.216	-9.225	-9.216	-9.216	-9.216	-9.216
32	-9.226	-9.226	-9.224	-9.226	-9.214	-9.214	-9.214	-9.214
33	-9.221	-9.221	-9.220	-9.220	-9.219	-9.219	-9.219	-9.219
34	-10.280	-10.279	-10.279	-10.279	-8.021	-8.021	-8.021	-8.021
35	-10.773	-10.773	-10.772	-10.772	-7.346	-7.346	-7.346	-7.346
36	-11.094	-11.094	-11.094	-11.094	-6.850	-6.850	-6.850	-6.850
37	-11.336	-11.336	-11.336	-11.336	-6.442	-6.442	-6.442	-6.442
38	-11.527	-11.527	-11.527	-11.527	-6.094	-6.094	-6.094	-6.094
39	-11.684	-11.684	-11.684	-11.684	-5.786	-5.786	-5.786	-5.786
40	-11.817	-11.816	-11.816	-11.816	-5.512	-5.512	-5.512	-5.512

Table 48

FIG. 72

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	9.220	9.220	9.220	9.220	9.220	9.220	9.220	9.220
2	9.220	9.220	9.220	9.220	9.218	9.218	9.220	9.220
3	9.232	9.233	9.232	9.232	9.205	9.205	9.210	9.207
4	9.223	9.223	9.223	9.223	9.193	9.223	9.223	9.223
5	9.226	9.219	9.219	9.219	9.219	9.219	9.219	9.219
6	9.220	9.220	9.220	9.220	9.218	9.220	9.220	9.220
7	9.225	9.225	9.225	9.225	9.197	9.208	9.225	9.225
8	9.220	9.220	9.219	9.219	9.219	9.219	9.219	9.219
9	9.221	9.221	9.219	9.221	9.218	9.218	9.219	9.219
10	9.227	9.221	9.221	9.221	9.214	9.215	9.221	9.217
11	9.229	9.226	9.214	9.229	9.214	9.214	9.214	9.214
12	9.224	9.224	9.217	9.224	9.217	9.217	9.217	9.217
13	9.226	9.221	9.217	9.221	9.217	9.217	9.217	9.217
14	9.221	9.219	9.219	9.219	9.219	9.219	9.219	9.219
15	9.222	9.220	9.220	9.220	9.219	9.219	9.220	9.219
16	9.233	9.218	9.218	9.218	9.218	9.218	9.218	9.218
17	9.232	9.225	9.223	9.223	9.213	9.213	9.214	9.214
18	9.226	9.224	9.224	9.224	9.214	9.214	9.218	9.214
19	9.222	9.222	9.221	9.222	9.217	9.218	9.218	9.218
20	9.223	9.219	9.219	9.219	9.219	9.219	9.219	9.219
21	9.246	9.246	9.247	9.246	9.193	9.193	9.193	9.193
22	9.222	9.220	9.220	9.220	9.219	9.219	9.219	9.219
23	9.226	9.226	9.217	9.217	9.217	9.217	9.217	9.217
24	9.223	9.223	9.219	9.219	9.218	9.218	9.218	9.218
25	9.224	9.224	9.220	9.224	9.214	9.214	9.220	9.214
26	9.224	9.224	9.224	9.224	9.199	9.217	9.223	9.224
27	9.222	9.219	9.219	9.219	9.219	9.219	9.219	9.219
28	9.235	9.228	9.226	9.228	9.210	9.210	9.210	9.210
29	9.220	9.220	9.220	9.220	9.219	9.219	9.219	9.219
30	9.227	9.225	9.224	9.225	9.214	9.214	9.214	9.214
31	9.222	9.222	9.221	9.221	9.218	9.218	9.218	9.218
32	9.229	9.229	9.214	9.229	9.214	9.214	9.214	9.214
33	9.222	9.222	9.222	9.222	9.217	9.217	9.218	9.217
34	10.279	10.279	10.279	10.279	8.021	8.021	8.022	8.021
35	10.773	10.773	10.773	10.773	7.345	7.345	7.345	7.345
36	11.096	11.090	11.089	11.096	6.850	6.850	6.850	6.850
37	11.336	11.334	11.334	11.334	6.443	6.443	6.443	6.443
38	11.527	11.527	11.527	11.527	6.094	6.094	6.094	6.094
39	11.684	11.684	11.684	11.684	5.786	5.786	5.786	5.786
40	11.817	11.816	11.815	11.815	5.512	5.512	5.512	5.512

Table 49

FIG. 73

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-11.928	-11.928	-11.928	-11.928	-5.265	-5.265	-5.265	-5.265
42	-12.025	-12.025	-12.023	-12.025	-5.042	-5.042	-5.042	-5.042
43	-12.108	-12.108	-12.107	-12.108	-4.838	-4.838	-4.838	-4.838
44	-12.181	-12.180	-12.180	-12.180	-4.652	-4.652	-4.652	-4.652
45	-12.244	-12.244	-12.244	-12.244	-4.482	-4.482	-4.482	-4.482
46	-12.300	-12.300	-12.300	-12.300	-4.327	-4.327	-4.327	-4.327
47	-12.348	-12.348	-12.348	-12.348	-4.187	-4.187	-4.187	-4.187
48	-12.391	-12.391	-12.390	-12.390	-4.058	-4.058	-4.058	-4.058
49	-12.429	-12.429	-12.428	-12.428	-3.942	-3.942	-3.942	-3.942
50	-12.461	-12.461	-12.461	-12.461	-3.837	-3.837	-3.837	-3.837
51	-12.491	-12.490	-12.489	-12.489	-3.743	-3.743	-3.743	-3.743
52	-12.515	-12.515	-12.514	-12.514	-3.660	-3.660	-3.660	-3.660
53	-12.541	-12.541	-12.517	-12.522	-3.592	-3.592	-3.594	-3.594
54	-12.556	-12.552	-12.552	-12.552	-3.524	-3.524	-3.524	-3.524
55	-12.568	-12.568	-12.568	-12.568	-3.471	-3.471	-3.471	-3.471
56	-12.586	-12.586	-12.569	-12.582	-3.426	-3.426	-3.427	-3.426
57	-12.595	-12.595	-12.562	-12.587	-3.391	-3.391	-3.393	-3.392
58	-12.605	-12.605	-12.575	-12.602	-3.366	-3.366	-3.366	-3.366
59	-12.005	-12.605	-12.595	-12.602	-3.349	-3.349	-3.350	-3.349
60	-12.606	-12.606	-12.593	-12.605	-3.340	-3.341	-3.343	-3.342
61	-12.615	-12.605	-12.596	-12.596	-3.339	-3.340	-3.340	-3.340
62	-12.669	-12.669	-12.531	-12.538	-3.334	-3.334	-3.357	-3.356
63	-16.220	-12.625	-10.823	-10.823	-2.815	-3.060	-3.520	-3.520
64	-12.374	-12.374	-12.374	-12.374	-3.419	-3.419	-3.419	-3.419
65	-16.429	-12.067	-10.510	-10.510	-2.915	-3.151	-3.545	-3.545
66	-16.646	-11.953	-10.412	-10.412	-2.868	-3.118	-3.554	-3.554
67	-16.808	-11.873	-10.335	-10.335	-2.829	-3.086	-3.564	-3.564
68	-16.934	-11.816	-10.270	-10.270	-2.793	-3.054	-3.575	-3.575
69	-17.032	-11.780	-10.213	-10.213	-2.763	-3.021	-3.588	-3.588
70	-15.874	-15.874	-7.431	-7.431	15.874	15.874	-5.725	-5.725
71	-15.852	-15.845	-7.463	-7.464	15.852	15.845	-5.752	-5.752
72	-15.850	-15.850	-7.460	-7.460	15.850	15.850	-5.750	-5.750
73	-15.822	-15.820	-7.500	-7.500	15.822	15.822	-5.777	-5.777
74	-15.793	-15.792	-7.542	-7.542	15.794	15.791	-5.800	-5.800
75	-15.765	-15.761	-7.587	-7.588	15.765	15.759	-5.821	-5.821
76	-15.756	-15.702	-7.633	-7.648	15.757	15.703	-5.837	-5.837
77	-15.736	-15.684	-7.688	-7.710	15.738	15.654	-5.850	-5.850
78	-16.426	-14.932	-7.512	-7.920	16.426	14.932	-5.820	-5.820
79	-16.777	-14.559	-7.409	-8.036	16.778	14.559	-5.775	-5.775
80	-17.021	-14.296	-7.335	-8.129	17.021	14.296	-5.726	-5.726

Table 50

FIG. 74

	Label							
	8	9	10	11	12	13	14	15
Design #								
41	11.928	11.928	11.928	11.928	5.265	5.265	5.265	5.265
42	12.025	12.025	12.023	12.023	5.042	5.042	5.042	5.042
43	12.108	12.108	12.108	12.108	4.838	4.838	4.838	4.838
44	12.181	12.181	12.181	12.181	4.652	4.652	4.652	4.652
45	12.244	12.244	12.244	12.244	4.482	4.482	4.482	4.482
46	12.299	12.299	12.299	12.299	4.328	4.328	4.328	4.328
47	12.348	12.348	12.348	12.348	4.187	4.187	4.187	4.187
48	12.391	12.392	12.390	12.391	4.058	4.058	4.058	4.058
49	12.429	12.429	12.429	12.429	3.941	3.941	3.941	3.941
50	12.461	12.461	12.461	12.461	3.837	3.837	3.837	3.837
51	12.491	12.491	12.489	12.489	3.743	3.743	3.743	3.743
52	12.515	12.515	12.514	12.514	3.660	3.660	3.660	3.660
53	12.540	12.540	12.540	12.540	3.583	3.583	3.585	3.583
54	12.556	12.556	12.547	12.565	3.523	3.523	3.524	3.523
55	12.569	12.569	12.568	12.569	3.470	3.470	3.471	3.471
56	12.581	12.581	12.579	12.579	3.427	3.427	3.427	3.427
57	12.595	12.588	12.587	12.587	3.392	3.393	3.393	3.393
58	12.599	12.596	12.595	12.595	3.366	3.367	3.367	3.367
59	12.600	12.600	12.600	12.600	3.351	3.351	3.351	3.351
60	12.604	12.604	12.603	12.604	3.341	3.341	3.341	3.341
61	12.607	12.607	12.597	12.607	3.336	3.338	3.339	3.338
62	12.602	12.602	12.602	12.602	3.345	3.345	3.345	3.345
63	12.424	12.424	12.424	12.424	3.403	3.403	3.403	3.403
64	16.734	12.329	10.663	10.663	2.735	3.014	3.518	3.518
65	16.429	12.066	10.511	10.511	2.915	3.151	3.545	3.545
66	16.646	11.953	10.412	10.412	2.868	3.118	3.554	3.554
67	16.808	11.873	10.335	10.335	2.826	3.086	3.564	3.564
68	16.933	11.816	10.270	10.270	2.793	3.054	3.575	3.575
69	17.033	11.780	10.213	10.213	2.762	3.022	3.588	3.588
70	5.725	5.725	0.168	0.168	7.431	7.431	-0.168	-0.168
71	5.752	5.752	0.200	0.200	7.464	7.465	-0.200	-0.200
72	5.750	5.750	0.200	0.200	7.460	7.460	-0.200	-0.200
73	5.777	5.777	0.231	0.231	7.501	7.501	-0.232	-0.232
74	5.800	5.800	0.264	0.264	7.542	7.542	-0.264	-0.264
75	5.821	5.821	0.297	0.297	7.587	7.589	-0.297	-0.297
76	5.837	5.837	0.333	0.333	7.633	7.648	-0.333	-0.333
77	5.850	5.850	0.372	0.372	7.687	7.710	-0.371	-0.371
78	5.820	5.820	0.403	0.403	7.512	7.920	-0.403	-0.403
79	5.775	5.775	0.435	0.435	7.409	8.036	-0.435	-0.435
80	5.726	5.726	0.469	0.469	7.335	8.129	-0.469	-0.469

Table 51

FIG. 75

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-18.783	-12.062	18.197	-10.830	-3.578	-5.451	-3.578	-5.945
82	-18.750	-12.045	18.134	-10.833	-3.574	-5.494	-3.574	-5.700
83	-18.705	-12.028	18.079	-10.832	-3.567	-5.547	-3.567	-5.764
84	-18.646	-12.005	18.034	-10.825	-3.556	-5.611	-3.556	-5.837
85	-18.574	-11.980	17.998	-10.814	-3.540	-5.687	-3.540	-5.920
86	-18.490	-11.954	17.970	-10.798	-3.520	-5.774	-3.520	-6.011
87	-18.398	-11.926	17.948	-10.779	-3.496	-5.869	-3.496	-6.108
88	-18.302	-11.901	17.929	-10.761	-3.470	-5.966	-3.470	-6.205
89	-18.280	-11.900	17.920	-10.766	-3.460	-5.990	-3.460	-6.230
90	-18.207	-11.880	17.909	-10.744	-3.444	-6.061	-3.444	-6.299
91	-18.120	-11.867	17.886	-10.731	-3.418	-6.149	-3.418	-6.389
92	-18.040	-11.861	17.859	-10.721	-3.393	-6.227	-3.393	-6.465
93	-17.969	-11.862	17.829	-10.715	-3.371	-6.294	-3.371	-6.537
94	-17.899	-11.872	17.796	-10.711	-3.351	-6.352	-3.351	-6.600
95	-17.837	-11.888	17.761	-10.709	-3.333	-6.400	-3.333	-6.656
96	-17.780	-11.911	17.724	-10.708	-3.317	-6.440	-3.317	-6.709
97	-17.724	-11.941	17.685	-10.705	-3.304	-6.472	-3.304	-6.756
98	-17.680	-11.982	17.639	-10.701	-3.291	-6.494	-3.291	-6.798
99	-17.625	-12.022	17.609	-10.691	-3.282	-6.514	-3.281	-6.839
100	-17.580	-12.076	17.569	-10.677	-3.277	-6.522	-3.265	-6.877
101	-17.536	-12.137	17.532	-10.655	-3.274	-6.524	-3.250	-6.913
102	-17.500	-12.211	17.494	-10.623	-3.270	-6.513	-3.233	-6.947
103	-17.466	-12.295	17.462	-10.581	-3.267	-6.491	-3.214	-6.979
104	-17.441	-12.388	17.434	-10.529	-3.263	-6.456	-3.192	-7.008
105	-17.421	-12.484	17.414	-10.470	-3.259	-6.413	-3.168	-7.036
106	-17.420	-12.490	17.410	-10.460	-3.260	-6.410	-3.170	-7.030
107	-17.407	-12.577	17.398	-10.411	-3.255	-6.363	-3.142	-7.046
108	-17.395	-12.683	17.388	-10.354	-3.252	-6.313	-3.115	-7.061
109	-17.385	-12.740	17.378	-10.304	-3.251	-6.266	-3.089	-7.074
110	-17.374	-12.807	17.370	-10.261	-3.252	-6.224	-3.063	-7.087
111	-17.363	-12.865	17.359	-10.225	-3.256	-6.186	-3.039	-7.101
112	-17.346	-12.915	17.346	-10.197	-3.264	-6.156	-3.017	-7.119
113	-17.335	-12.960	17.333	-10.172	-3.273	-6.124	-2.993	-7.137
114	-17.317	-12.998	17.316	-10.153	-3.287	-6.098	-2.970	-7.162
115	-17.296	-13.031	17.295	-10.140	-3.305	-6.074	-2.945	-7.193
116	-17.272	-13.058	17.272	-10.131	-3.328	-6.061	-2.919	-7.232
117	-17.244	-13.082	17.244	-10.127	-3.356	-6.029	-2.888	-7.279
118	-17.211	-13.101	17.211	-10.128	-3.393	-6.006	-2.852	-7.338
119	-17.172	-13.115	17.172	-10.134	-3.438	-5.963	-2.811	-7.406
120	-17.126	-13.125	17.126	-10.146	-3.496	-5.964	-2.765	-7.483

Table 52

FIG. 76

	Label							
Design #	8	9	10	11	12	13	14	15
81	8.945	7.590	10.694	7.590	1.248	2.169	1.131	2.166
82	9.040	7.532	11.014	7.532	1.186	2.227	1.081	2.227
83	9.154	7.464	11.131	7.464	1.111	2.291	1.022	2.291
84	9.291	7.388	11.242	7.389	1.022	2.360	0.950	2.360
85	9.452	7.304	11.344	7.304	0.918	2.436	0.865	2.436
86	9.629	7.215	11.435	7.215	0.803	2.516	0.768	2.516
87	9.809	7.127	11.512	7.127	0.683	2.601	0.664	2.601
88	9.978	7.065	11.580	7.036	0.566	2.685	0.569	2.685
89	10.020	7.040	11.600	7.010	0.540	2.710	0.546	2.710
90	10.127	7.007	11.643	6.939	0.461	2.766	0.461	2.766
91	10.247	6.969	11.696	6.864	0.372	2.841	0.372	2.841
92	10.344	6.939	11.742	6.805	0.297	2.909	0.297	2.909
93	10.423	6.918	11.784	6.758	0.234	2.968	0.234	2.968
94	10.486	6.904	11.823	6.722	0.183	3.019	0.183	3.019
95	10.536	6.897	11.861	6.694	0.141	3.063	0.141	3.063
96	10.575	6.897	11.898	6.673	0.108	3.099	0.108	3.099
97	10.604	6.901	11.939	6.655	0.082	3.128	0.082	3.128
98	10.626	6.911	11.977	6.646	0.065	3.153	0.065	3.153
99	10.633	6.924	12.029	6.627	0.045	3.166	0.045	3.173
100	10.633	6.941	12.082	6.613	0.034	3.175	0.034	3.193
101	10.623	6.961	12.144	6.595	0.025	3.178	0.025	3.207
102	10.600	6.983	12.213	6.572	0.019	3.178	0.019	3.220
103	10.566	7.004	12.293	6.541	0.015	3.172	0.015	3.228
104	10.518	7.024	12.381	6.500	0.012	3.160	0.012	3.234
105	10.464	7.041	12.474	6.451	0.009	3.143	0.009	3.237
106	10.460	7.040	12.480	6.450	0.010	3.140	0.010	3.240
107	10.407	7.065	12.566	6.397	0.006	3.123	0.006	3.238
108	10.352	7.067	12.653	6.342	0.010	3.101	0.002	3.239
109	10.303	7.079	12.731	6.291	0.015	3.077	-0.006	3.241
110	10.260	7.090	12.800	6.244	0.021	3.054	-0.015	3.244
111	10.225	7.103	12.860	6.202	0.029	3.031	-0.025	3.250
112	10.197	7.121	12.913	6.167	0.039	3.010	-0.037	3.259
113	10.172	7.139	12.958	6.134	0.050	2.987	-0.048	3.269
114	10.153	7.163	12.997	6.105	0.063	2.965	-0.063	3.284
115	10.140	7.194	13.030	6.080	0.079	2.942	-0.079	3.303
116	10.131	7.232	13.058	6.056	0.098	2.918	-0.099	3.326
117	10.127	7.280	13.082	6.032	0.122	2.895	-0.122	3.355
118	10.128	7.338	13.101	6.008	0.151	2.850	-0.152	3.392
119	10.135	7.407	13.115	5.985	0.188	2.809	-0.189	3.436
120	10.147	7.484	13.125	5.985	0.235	2.763	-0.236	3.496

Table 53

FIG. 77

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-17.073	-13.130	17.073	-10.163	-3.570	-5.950	-2.716	-7.565
122	-17.011	-13.129	17.012	-10.183	-3.661	-5.947	-2.667	-7.649
123	-16.970	-13.130	16.970	-10.200	-3.720	-5.950	-2.640	-7.700
124	-16.945	-13.126	16.945	-10.205	-3.765	-5.954	-2.624	-7.729
125	-16.879	-13.119	16.878	-10.228	-3.867	-5.968	-2.591	-7.798
126	-16.817	-13.115	16.817	-10.249	-3.954	-5.984	-2.568	-7.856
127	-16.763	-13.114	16.763	-10.271	-4.022	-5.999	-2.551	-7.903
128	-16.713	-13.115	16.713	-10.292	-4.074	-6.013	-2.539	-7.942
129	-16.667	-13.117	16.667	-10.313	-4.116	-6.026	-2.531	-7.975
130	-16.624	-13.120	16.624	-10.334	-4.149	-6.039	-2.526	-8.005
131	-16.582	-13.123	16.582	-10.356	-4.176	-6.053	-2.524	-8.033
132	-16.540	-13.127	16.541	-10.376	-4.199	-6.067	-2.524	-8.060
133	-16.499	-13.129	16.500	-10.400	-4.220	-6.083	-2.525	-8.087
134	-16.459	-13.132	16.458	-10.422	-4.238	-6.100	-2.528	-8.113
135	-16.418	-13.134	16.417	-10.444	-4.256	-6.119	-2.532	-8.140
136	-16.375	-13.134	16.376	-10.466	-4.275	-6.139	-2.539	-8.167
137	-16.335	-13.135	16.332	-10.487	-4.293	-6.162	-2.545	-8.195
138	-16.291	-13.134	16.290	-10.508	-4.312	-6.186	-2.555	-8.223
139	-16.249	-13.132	16.247	-10.526	-4.332	-6.212	-2.565	-8.251
140	-16.204	-13.129	16.204	-10.548	-4.354	-6.239	-2.577	-8.280
141	-16.160	-13.130	16.160	-10.560	-4.370	-6.260	-2.590	-8.300
142	-16.160	-13.126	16.161	-10.567	-4.377	-6.267	-2.590	-8.309
143	-16.117	-13.122	16.117	-10.585	-4.400	-6.296	-2.604	-8.337
144	-16.074	-13.117	16.074	-10.603	-4.424	-6.324	-2.619	-8.365
145	-16.031	-13.112	16.032	-10.620	-4.448	-6.353	-2.635	-8.393
146	-15.989	-13.107	15.989	-10.636	-4.472	-6.382	-2.650	-8.419
147	-15.948	-13.102	15.949	-10.652	-4.497	-6.410	-2.666	-8.445
148	-15.910	-13.096	15.909	-10.667	-4.519	-6.437	-2.681	-8.470
149	-15.872	-13.092	15.869	-10.682	-4.542	-6.463	-2.696	-8.495
150	-15.833	-13.087	15.833	-10.695	-4.565	-6.489	-2.712	-8.518
151	-15.797	-13.082	15.797	-10.708	-4.586	-6.513	-2.726	-8.540
152	-15.763	-13.078	15.763	-10.721	-4.606	-6.536	-2.739	-8.562
153	-15.730	-13.073	15.730	-10.733	-4.625	-6.559	-2.752	-8.582
154	-15.698	-13.069	15.698	-10.745	-4.643	-6.580	-2.764	-8.601
155	-15.668	-13.066	15.669	-10.756	-4.660	-6.599	-2.776	-8.619
156	-15.639	-13.062	15.639	-10.767	-4.677	-6.618	-2.787	-8.637
157	-15.610	-13.059	15.610	-10.777	-4.692	-6.637	-2.797	-8.654
158	-15.584	-13.056	15.584	-10.787	-4.707	-6.654	-2.807	-8.670
159	-15.559	-13.053	15.559	-10.796	-4.721	-6.670	-2.816	-8.685
160	-15.535	-13.050	15.536	-10.805	-4.734	-6.685	-2.825	-8.699

Table 84

FIG. 78

	Label							
	8	9	10	11	12	13	14	15
Design #								
121	10.183	7.568	13.130	5.951	0.294	2.713	-0.295	3.570
122	10.184	7.650	13.130	5.947	0.368	2.668	-0.369	3.661
123	10.200	7.709	13.130	5.960	0.420	2.640	-0.420	3.720
124	10.206	7.729	13.125	5.954	0.452	2.623	-0.452	3.765
125	10.228	7.799	13.120	5.969	0.534	2.590	-0.534	3.867
126	10.250	7.858	13.115	5.985	0.604	2.567	-0.604	3.954
127	10.271	7.903	13.114	6.000	0.658	2.550	-0.659	4.022
128	10.292	7.942	13.115	6.013	0.700	2.539	-0.700	4.074
129	10.313	7.975	13.117	6.026	0.732	2.531	-0.732	4.116
130	10.334	8.005	13.120	6.039	0.757	2.526	-0.757	4.149
131	10.356	8.033	13.123	6.053	0.776	2.524	-0.776	4.176
132	10.378	8.060	13.127	6.067	0.791	2.523	-0.791	4.199
133	10.400	8.087	13.130	6.083	0.803	2.525	-0.803	4.220
134	10.422	8.113	13.132	6.100	0.813	2.528	-0.813	4.238
135	10.444	8.140	13.133	6.119	0.822	2.533	-0.821	4.257
136	10.466	8.167	13.135	6.139	0.828	2.538	-0.828	4.274
137	10.487	8.195	13.134	6.163	0.835	2.547	-0.834	4.294
138	10.508	8.223	13.134	6.186	0.840	2.555	-0.840	4.312
139	10.528	8.251	13.131	6.212	0.846	2.566	-0.845	4.333
140	10.548	8.280	13.129	6.239	0.851	2.577	-0.850	4.354
141	10.568	8.308	13.130	6.260	0.850	2.590	-0.850	4.370
142	10.587	8.309	13.126	6.287	0.856	2.590	-0.856	4.376
143	10.585	8.337	13.122	6.296	0.861	2.604	-0.861	4.400
144	10.603	8.365	13.117	6.324	0.867	2.619	-0.866	4.424
145	10.620	8.392	13.113	6.353	0.872	2.634	-0.872	4.448
146	10.636	8.419	13.107	6.382	0.878	2.650	-0.878	4.472
147	10.652	8.446	13.102	6.410	0.883	2.666	-0.884	4.496
148	10.667	8.471	13.096	6.437	0.890	2.682	-0.888	4.520
149	10.681	8.495	13.091	6.464	0.895	2.697	-0.894	4.543
150	10.695	8.518	13.087	6.489	0.899	2.711	-0.900	4.565
151	10.709	8.540	13.082	6.513	0.904	2.726	-0.904	4.586
152	10.721	8.561	13.078	6.537	0.909	2.739	-0.909	4.606
153	10.733	8.582	13.073	6.559	0.914	2.752	-0.914	4.625
154	10.745	8.601	13.070	6.580	0.918	2.764	-0.918	4.643
155	10.756	8.619	13.066	6.599	0.922	2.776	-0.922	4.661
156	10.767	8.637	13.062	6.618	0.926	2.787	-0.926	4.677
157	10.777	8.654	13.059	6.637	0.929	2.797	-0.929	4.692
158	10.787	8.670	13.056	6.654	0.933	2.807	-0.933	4.707
159	10.796	8.685	13.053	6.670	0.936	2.816	-0.936	4.721
160	10.805	8.699	13.050	6.685	0.939	2.825	-0.939	4.734

Table 55

FIG. 79

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-15.512	-13.048	15.512	-10.814	-4.746	-6.700	-2.883	-8.713
162	-15.489	-13.045	15.488	-10.822	-4.755	-6.713	-2.841	-8.726
163	-15.465	-13.043	15.466	-10.829	-4.770	-6.727	-2.848	-8.738
164	-15.447	-13.040	15.447	-10.837	-4.781	-6.739	-2.856	-8.750
165	-15.428	-13.038	15.428	-10.844	-4.790	-6.751	-2.862	-8.761
166	-15.408	-13.036	15.409	-10.851	-4.801	-6.763	-2.869	-8.773
167	-15.393	-13.034	15.392	-10.857	-4.809	-6.773	-2.875	-8.782
168	-15.375	-13.033	15.374	-10.864	-4.818	-6.784	-2.880	-8.792
169	-15.358	-13.032	15.358	-10.870	-4.826	-6.793	-2.886	-8.801
170	-15.344	-13.030	15.344	-10.875	-4.834	-6.802	-2.891	-8.810
171	-15.328	-13.028	15.328	-10.881	-4.842	-6.811	-2.896	-8.819
172	-15.312	-13.026	15.313	-10.886	-4.850	-6.821	-2.902	-8.827
173	-15.299	-13.025	15.299	-10.891	-4.856	-6.829	-2.906	-8.835
174	-15.288	-13.024	15.288	-10.895	-4.862	-6.836	-2.910	-8.841
175	-15.276	-13.024	15.276	-10.901	-4.867	-6.843	-2.912	-8.849
176	-15.263	-13.023	15.261	-10.905	-4.874	-6.850	-2.917	-8.856
177	-15.252	-13.022	15.251	-10.909	-4.879	-6.856	-2.921	-8.862
178	-15.242	-13.021	15.241	-10.913	-4.885	-6.863	-2.923	-8.869

Table 56

FIG. 80

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	10.813	6.712	13.047	6.699	0.942	2.833	-0.942	4.747
162	10.822	6.726	13.045	6.713	0.945	2.841	-0.945	4.759
163	10.829	6.735	13.042	6.727	0.947	2.848	-0.947	4.770
164	10.837	6.750	13.040	6.740	0.950	2.856	-0.950	4.781
165	10.844	6.761	13.038	6.751	0.952	2.862	-0.952	4.790
166	10.852	6.773	13.037	6.763	0.954	2.869	-0.955	4.801
167	10.857	6.782	13.035	6.773	0.956	2.874	-0.957	4.809
168	10.864	6.792	13.033	6.784	0.959	2.881	-0.958	4.818
169	10.870	6.801	13.032	6.793	0.960	2.888	-0.960	4.826
170	10.875	6.810	13.030	6.802	0.962	2.891	-0.962	4.834
171	10.881	6.819	13.029	6.811	0.964	2.896	-0.964	4.842
172	10.887	6.828	13.027	6.821	0.965	2.901	-0.967	4.849
173	10.891	6.835	13.025	6.829	0.967	2.906	-0.967	4.856
174	10.896	6.842	13.025	6.836	0.968	2.909	-0.969	4.862
175	10.899	6.848	13.023	6.843	0.971	2.914	-0.969	4.868
176	10.905	6.856	13.022	6.851	0.972	2.918	-0.971	4.875
177	10.909	6.861	13.021	6.857	0.973	2.921	-0.972	4.880
178	10.911	6.867	13.019	6.863	0.975	2.926	-0.972	4.886

Table 57

FIG. 81

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
63	7.4	1.36	1.23	1.1	0.99	0.66	0
64	7.6	1.35	1.22	1.1	0.99	0.66	0
65	7.8	1.24	1.11	1	0.9	0.53	0
66	8	1.17	1.06	0.95	0.85	0.56	0
67	8.2	1.14	1.02	0.92	0.83	0.54	0
68	8.4	1.12	1.01	0.91	0.82	0.59	0
69	8.6	1.23	1.1	0.99	0.6	0.53	0
70	8.8	1.17	0.66	0.58	0.53	0.47	0
71	9	1.17	0.64	0.57	0.52	0.47	0
72	9	1.17	0.64	0.57	0.52	0.47	0
73	9.2	1.17	0.63	0.56	0.51	0.46	0
74	9.4	1.16	0.61	0.55	0.49	0.45	0
75	9.6	1.16	0.6	0.54	0.48	0.44	0
76	9.8	1.15	0.59	0.53	0.48	0.43	0
77	10	1.14	0.58	0.52	0.47	0.42	0
78	10.2	1.18	0.79	0.71	0.64	0.57	0
79	10.4	1.1	0.91	0.82	0.73	0.54	0
80	10.6	1.09	0.97	0.87	0.78	0.57	0
81	10.8	1.04	0.93	0.84	0.66	0.5	0
82	11	1	0.9	0.81	0.73	0.53	0
83	11.2	1.07	0.96	0.87	0.7	0.51	0
84	11.4	1.03	0.92	0.83	0.75	0.55	0
85	11.6	1.09	0.98	0.88	0.71	0.52	0
86	11.8	1.03	0.92	0.83	0.76	0.46	0
87	12	0.97	0.87	0.78	0.7	0.51	0
88	12.2	1.01	0.9	0.81	0.73	0.48	0
89	12.25	0.99	0.89	0.8	0.72	0.52	0
90	12.4	0.94	0.84	0.76	0.68	0.5	0
91	12.6	0.97	0.89	0.79	0.71	0.47	0
92	12.8	0.93	0.83	0.75	0.67	0.49	0
93	13	0.89	0.8	0.72	0.65	0.47	0
94	13.2	0.86	0.77	0.7	0.63	0.46	0
95	13.4	0.93	0.83	0.75	0.66	0.44	0
96	13.6	0.9	0.81	0.73	0.66	0.43	0
97	13.8	0.88	0.79	0.71	0.64	0.42	0
98	14	0.87	0.78	0.69	0.62	0.45	0
99	14.2	0.77	0.68	0.62	0.56	0.45	0
100	14.4	0.83	0.76	0.63	0.57	0.41	0
101	14.6	0.73	0.66	0.59	0.53	0.39	0
102	14.9	0.72	0.65	0.58	0.5	0.4	0
103	15	0.71	0.64	0.52	0.47	0.38	0

Table 58

FIG. 82

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
104	15.2	0.7	0.63	0.49	0.44	0.38	0
105	15.4	0.68	0.61	0.47	0.43	0.34	0
106	15.42	0.68	0.61	0.47	0.42	0.39	0
107	15.6	0.67	0.6	0.47	0.42	0.34	0
108	15.8	0.72	0.65	0.46	0.41	0.33	0
109	16	0.63	0.57	0.45	0.41	0.33	0
110	16.2	0.61	0.55	0.44	0.4	0.32	0
111	16.4	0.6	0.54	0.43	0.39	0.32	0
112	16.6	0.58	0.53	0.42	0.38	0.31	0
113	16.8	0.57	0.52	0.41	0.37	0.3	0
114	17	0.56	0.5	0.4	0.36	0.26	0
115	17.2	0.49	0.44	0.38	0.35	0.28	0
116	17.4	0.53	0.47	0.37	0.33	0.27	0
117	17.6	0.51	0.46	0.35	0.31	0.26	0
118	17.8	0.48	0.43	0.32	0.29	0.23	0
119	18	0.46	0.41	0.34	0.3	0.25	0
120	18.2	0.43	0.39	0.35	0.31	0.23	0
121	18.4	0.4	0.36	0.33	0.29	0.21	0
122	18.6	0.42	0.38	0.34	0.31	0.2	0
123	18.72	0.39	0.35	0.31	0.28	0.21	0
124	18.8	0.39	0.36	0.32	0.29	0.21	0
125	19	0.37	0.33	0.3	0.27	0.2	0
126	19.2	0.39	0.35	0.3	0.27	0.2	0
127	19.4	0.37	0.33	0.28	0.25	0.18	0
128	19.6	0.34	0.31	0.26	0.24	0.19	0
129	19.8	0.36	0.32	0.28	0.23	0.18	0
130	20	0.33	0.3	0.27	0.24	0.16	0
131	20.2	0.34	0.31	0.26	0.23	0.16	0
132	20.4	0.32	0.29	0.26	0.23	0.17	0
133	20.6	0.3	0.27	0.24	0.22	0.16	0
134	20.8	0.27	0.25	0.22	0.2	0.15	0
135	21	0.28	0.25	0.23	0.21	0.14	0
136	21.2	0.26	0.24	0.21	0.19	0.14	0
137	21.4	0.26	0.23	0.21	0.19	0.14	0
138	21.6	0.25	0.22	0.2	0.18	0.12	0
139	21.8	0.24	0.21	0.19	0.17	0.13	0
140	22	0.23	0.21	0.19	0.17	0.11	0
141	22.13	0.2	0.18	0.16	0.15	0.11	0
142	22.2	0.2	0.18	0.16	0.14	0.11	0
143	22.4	0.19	0.17	0.15	0.14	0.1	0
144	22.6	0.18	0.17	0.15	0.13	0.1	0

Table 59

FIG. 83

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
145	22.8	0.18	0.16	0.14	0.13	0.09	0
146	23	0.17	0.15	0.14	0.12	0.09	0
147	23.2	0.16	0.14	0.13	0.12	0.08	0
148	23.4	0.15	0.14	0.12	0.11	0.08	0
149	23.6	0.13	0.12	0.11	0.1	0.07	0
150	23.8	0.14	0.13	0.11	0.1	0.07	0
151	24	0.12	0.11	0.1	0.09	0.06	0
152	24.2	0.12	0.1	0.09	0.08	0.06	0
153	24.4	0.11	0.1	0.09	0.09	0.06	0
154	24.6	0.11	0.09	0.09	0.08	0.06	0
155	24.8	0.1	0.09	0.08	0.07	0.05	0
156	25	0.1	0.09	0.08	0.07	0.05	0
157	25.2	0.09	0.08	0.07	0.07	0.05	0
158	25.4	0.09	0.08	0.07	0.06	0.05	0
159	25.6	0.08	0.08	0.07	0.06	0.04	0
160	25.8	0.08	0.07	0.06	0.06	0.04	0
161	26	0.08	0.07	0.06	0.06	0.04	0
162	26.2	0.07	0.07	0.06	0.05	0.04	0
163	26.4	0.07	0.06	0.06	0.05	0.04	0
164	26.6	0.07	0.06	0.05	0.05	0.04	0
165	26.8	0.06	0.06	0.05	0.05	0.03	0
166	27	0.06	0.05	0.05	0.04	0.03	0
167	27.2	0.05	0.05	0.04	0.04	0.03	0
168	27.4	0.06	0.05	0.05	0.04	0.03	0
169	27.6	0.05	0.05	0.04	0.04	0.03	0
170	27.8	0.05	0.04	0.04	0.03	0.02	0
171	28	0.05	0.04	0.04	0.04	0.03	0
172	28.2	0.05	0.04	0.04	0.03	0.02	0
173	28.4	0.04	0.04	0.03	0.03	0.02	0
174	28.6	0.04	0.04	0.03	0.03	0.02	0
175	28.8	0.04	0.03	0.03	0.03	0.02	0
176	29	0.03	0.03	0.03	0.03	0.02	0
177	29.2	0.03	0.03	0.03	0.02	0.02	0
178	29.4	0.03	0.03	0.03	0.02	0.02	0

Table 60

FIG. 84

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.000	0.1982	0.1980	0.0002	0.0804
2	-4.800	0.2063	0.2061	0.0002	0.0894
3	-4.600	0.2147	0.2145	0.0002	0.0992
4	-4.400	0.2234	0.2232	0.0002	0.1089
5	-4.200	0.2324	0.2322	0.0003	0.1217
6	-4.000	0.2417	0.2414	0.0003	0.1346
7	-3.800	0.2514	0.2510	0.0004	0.1480
8	-3.600	0.2613	0.2609	0.0004	0.1642
9	-3.400	0.2715	0.2711	0.0005	0.1809
10	-3.200	0.2821	0.2816	0.0006	0.1992
11	-3.000	0.2931	0.2924	0.0006	0.2190
12	-2.800	0.3043	0.3036	0.0007	0.2405
13	-2.600	0.3159	0.3151	0.0008	0.2637
14	-2.400	0.3279	0.3269	0.0009	0.2888
15	-2.200	0.3402	0.3391	0.0011	0.3159
16	-2.000	0.3529	0.3516	0.0012	0.3451
17	-1.800	0.3659	0.3645	0.0014	0.3764
18	-1.600	0.3793	0.3777	0.0015	0.4100
19	-1.400	0.3931	0.3913	0.0017	0.4460
20	-1.200	0.4072	0.4052	0.0020	0.4845
21	-1.000	0.4217	0.4195	0.0022	0.5255
22	-0.800	0.4366	0.4341	0.0025	0.5692
23	-0.600	0.4519	0.4491	0.0028	0.6155
24	-0.400	0.4675	0.4644	0.0031	0.6647
25	-0.200	0.4836	0.4801	0.0034	0.7167
26	0.000	0.5000	0.4962	0.0038	0.7716
27	0.200	0.5168	0.5125	0.0043	0.8294
28	0.400	0.5340	0.5292	0.0047	0.8902
29	0.600	0.5515	0.5463	0.0052	0.9540
30	0.800	0.5695	0.5637	0.0058	1.0207
31	1.000	0.5878	0.5814	0.0063	1.0903
32	1.200	0.6065	0.5995	0.0070	1.1629
33	1.400	0.6255	0.6179	0.0077	1.2383
34	1.600	0.6450	0.6366	0.0084	1.3165
35	1.800	0.6648	0.6556	0.0092	1.3973
36	2.000	0.6850	0.6750	0.0100	1.4807
37	2.200	0.7055	0.6946	0.0109	1.5665
38	2.400	0.7264	0.7146	0.0118	1.6545
39	2.600	0.7476	0.7348	0.0128	1.7448
40	2.800	0.7692	0.7553	0.0139	1.8370

Table 61

FIG. 85

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	3.000	0.7912	0.7762	0.0150	1.9309
42	3.200	0.8134	0.7973	0.0162	2.0285
43	3.400	0.8360	0.8186	0.0174	2.1237
44	3.600	0.8589	0.8403	0.0187	2.2223
45	3.800	0.8822	0.8622	0.0200	2.3216
46	4.000	0.9057	0.8843	0.0214	2.4217
47	4.200	0.9296	0.9067	0.0229	2.5225
48	4.400	0.9538	0.9294	0.0244	2.6237
49	4.600	0.9782	0.9523	0.0260	2.7253
50	4.776	1.0000	0.9726	0.0274	2.8172
51	4.800	1.0030	0.9754	0.0276	2.8268
52	5.000	1.0280	0.9988	0.0292	2.9281
53	5.200	1.0533	1.0223	0.0310	3.0290
54	5.400	1.0789	1.0461	0.0327	3.1291
55	5.600	1.1047	1.0702	0.0345	3.2281
56	5.800	1.1308	1.0944	0.0364	3.3258
57	6.000	1.1571	1.1188	0.0383	3.4217
58	6.200	1.1836	1.1434	0.0402	3.5157
59	6.400	1.2104	1.1682	0.0422	3.6083
60	6.600	1.2374	1.1932	0.0441	3.6994
61	6.800	1.2646	1.2184	0.0462	3.7888
62	7.000	1.2920	1.2438	0.0482	3.8764
63	7.200	1.3197	1.2694	0.0503	3.9615
64	7.400	1.3475	1.2951	0.0524	4.0439
65	7.600	1.3755	1.3210	0.0545	4.1236
66	7.800	1.4037	1.3471	0.0566	4.2007
67	8.000	1.4320	1.3733	0.0587	4.2754
68	8.200	1.4605	1.3997	0.0609	4.3475
69	8.400	1.4892	1.4262	0.0630	4.4169
70	8.475	1.5000	1.4362	0.0638	4.4423
71	8.600	1.5180	1.4529	0.0651	4.4836
72	8.800	1.5470	1.4797	0.0673	4.5478
73	9.000	1.5762	1.5067	0.0695	4.6094
74	9.200	1.6054	1.5338	0.0716	4.6679
75	9.400	1.6348	1.5611	0.0737	4.7231
76	9.600	1.6643	1.5885	0.0758	4.7749
77	9.800	1.6940	1.6160	0.0779	4.8235
78	10.000	1.7237	1.6437	0.0800	4.8693
79	10.200	1.7536	1.6715	0.0821	4.9121
80	10.400	1.7835	1.6994	0.0842	4.9523

Table 62

FIG. 86

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.600	1.8136	1.7274	0.0862	4.9903
82	10.800	1.8437	1.7555	0.0882	5.0256
83	11.000	1.8740	1.7838	0.0902	5.0582
84	11.200	1.9043	1.8121	0.0922	5.0883
85	11.400	1.9348	1.8406	0.0942	5.1158
86	11.600	1.9653	1.8692	0.0961	5.1407
87	11.800	1.9959	1.8979	0.0980	5.1630
88	11.927	2.0000	1.9018	0.0982	5.1635
89	12.000	2.0265	1.9267	0.0998	5.1824
90	12.200	2.0572	1.9555	0.1017	5.1989
91	12.400	2.0880	1.9845	0.1034	5.2121
92	12.600	2.1188	2.0136	0.1052	5.2225
93	12.800	2.1496	2.0428	0.1069	5.2307
94	13.000	2.1805	2.0720	0.1085	5.2365
95	13.200	2.2115	2.1014	0.1101	5.2400
96	13.400	2.2425	2.1308	0.1117	5.2417
97	13.600	2.2736	2.1603	0.1132	5.2416
98	13.800	2.3047	2.1899	0.1147	5.2398
99	14.000	2.3358	2.2196	0.1162	5.2362
100	14.200	2.3670	2.2494	0.1177	5.2310
101	14.400	2.3983	2.2792	0.1191	5.2241
102	14.600	2.4296	2.3091	0.1204	5.2155
103	14.800	2.4609	2.3391	0.1218	5.2052
104	15.000	2.4922	2.3692	0.1230	5.1934
105	15.050	2.5000	2.3766	0.1234	5.1923
106	15.200	2.5236	2.3993	0.1243	5.1802
107	15.400	2.5550	2.4295	0.1255	5.1654
108	15.600	2.5864	2.4598	0.1267	5.1492
109	15.800	2.6179	2.4901	0.1278	5.1316
110	16.000	2.6493	2.5205	0.1289	5.1124
111	16.200	2.6808	2.5509	0.1299	5.0918
112	16.400	2.7123	2.5814	0.1309	5.0696
113	16.600	2.7439	2.6120	0.1318	5.0459
114	16.800	2.7753	2.6426	0.1327	5.0207
115	17.000	2.8068	2.6733	0.1335	4.9940
116	17.200	2.8383	2.7041	0.1343	4.9658
117	17.400	2.8698	2.7348	0.1350	4.9361
118	17.600	2.9013	2.7657	0.1357	4.9051
119	17.800	2.9329	2.7966	0.1363	4.8727
120	18.000	2.9643	2.8275	0.1368	4.8389

Table 63

FIG. 87

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.200	2.9956	2.8585	0.1373	4.8038
122	18.226	3.0000	2.8626	0.1374	4.7998
123	18.400	3.0273	2.8896	0.1378	4.7674
124	18.600	3.0588	2.9206	0.1381	4.7297
125	18.800	3.0902	2.9518	0.1385	4.6907
126	19.000	3.1217	2.9830	0.1387	4.6506
127	19.200	3.1531	3.0142	0.1389	4.6090
128	19.400	3.1845	3.0454	0.1391	4.5663
129	19.600	3.2159	3.0767	0.1391	4.5223
130	19.800	3.2472	3.1081	0.1392	4.4772
131	20.000	3.2786	3.1395	0.1391	4.4307
132	20.200	3.3099	3.1709	0.1390	4.3831
133	20.400	3.3411	3.2023	0.1388	4.3342
134	20.600	3.3723	3.2338	0.1385	4.2841
135	20.800	3.4035	3.2653	0.1382	4.2328
136	21.000	3.4347	3.2969	0.1378	4.1802
137	21.200	3.4658	3.3285	0.1373	4.1266
138	21.400	3.4969	3.3601	0.1368	4.0714
139	21.420	3.5000	3.3633	0.1367	4.0645
140	21.600	3.5279	3.3917	0.1362	4.0151
141	21.800	3.5589	3.4234	0.1355	3.9576
142	22.000	3.5898	3.4551	0.1347	3.8988
143	22.200	3.6207	3.4869	0.1338	3.8386
144	22.400	3.6515	3.5186	0.1329	3.7772
145	22.600	3.6823	3.5504	0.1318	3.7144
146	22.800	3.7130	3.5823	0.1308	3.6503
147	23.000	3.7437	3.6141	0.1296	3.5848
148	23.200	3.7742	3.6460	0.1283	3.5179
149	23.400	3.8047	3.6779	0.1269	3.4495
150	23.600	3.8351	3.7098	0.1254	3.3797
151	23.800	3.8655	3.7417	0.1238	3.3084
152	24.000	3.8958	3.7737	0.1221	3.2356
153	24.200	3.9259	3.8056	0.1203	3.1611
154	24.400	3.9560	3.8376	0.1184	3.0851
155	24.600	3.9860	3.8696	0.1164	3.0074
156	24.694	4.0000	3.8846	0.1154	2.9707
157	24.800	4.0159	3.9016	0.1142	2.9280
158	25.000	4.0457	3.9337	0.1120	2.8470
159	25.200	4.0753	3.9657	0.1096	2.7642
160	25.400	4.1049	3.9978	0.1071	2.6798

Table 64

FIG. 88

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	26.600	4.1343	4.0298	0.1045	2.5936
162	26.800	4.1635	4.0618	0.1018	2.5059
163	26.000	4.1926	4.0938	0.0989	2.4166
164	26.200	4.2218	4.1258	0.0960	2.3259
165	26.400	4.2507	4.1578	0.0929	2.2339
166	26.600	4.2793	4.1896	0.0897	2.1408
167	26.800	4.3079	4.2214	0.0864	2.0470
168	27.000	4.3362	4.2531	0.0830	1.9525
169	27.200	4.3643	4.2847	0.0796	1.8579
170	27.400	4.3922	4.3161	0.0761	1.7634
171	27.600	4.4199	4.3473	0.0726	1.6694
172	27.800	4.4473	4.3782	0.0690	1.5763
173	28.000	4.4744	4.4089	0.0655	1.4846
174	28.192	4.5000	4.4380	0.0620	1.3970
175	28.200	4.5011	4.4392	0.0619	1.3945
176	28.400	4.5276	4.4692	0.0584	1.3065
177	28.600	4.5536	4.4987	0.0549	1.2209
178	28.800	4.5793	4.5277	0.0515	1.1379
179	29.000	4.6044	4.5562	0.0482	1.0577
180	29.200	4.6290	4.5841	0.0450	0.9806
181	29.400	4.6531	4.6113	0.0418	0.9067
182	29.600	4.6765	4.6378	0.0388	0.8361
183	29.800	4.6993	4.6635	0.0359	0.7687
184	30.000	4.7214	4.6884	0.0330	0.7046
185	30.200	4.7427	4.7124	0.0304	0.6442
186	30.400	4.7632	4.7354	0.0278	0.5870
187	30.600	4.7829	4.7575	0.0254	0.5332
188	30.800	4.8017	4.7786	0.0231	0.4826
189	31.000	4.8196	4.7987	0.0209	0.4352
190	31.200	4.8366	4.8177	0.0188	0.3910
191	31.400	4.8528	4.8358	0.0169	0.3500
192	31.600	4.8676	4.8525	0.0151	0.3119
193	31.800	4.8817	4.8682	0.0135	0.2768
194	32.000	4.8948	4.8828	0.0119	0.2445
195	32.200	4.9069	4.8964	0.0105	0.2149
196	32.400	4.9181	4.9088	0.0092	0.1880
197	32.600	4.9283	4.9203	0.0080	0.1636
198	32.800	4.9376	4.9307	0.0070	0.1415
199	33.000	4.9461	4.9401	0.0060	0.1217
200	33.200	4.9537	4.9485	0.0051	0.1041

Table 65

FIG. 89

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
201	33.400	4.9605	4.9561	0.0044	0.0884
202	33.600	4.9665	4.9628	0.0037	0.0746
203	33.800	4.9718	4.9687	0.0031	0.0625
204	34.000	4.9765	4.9739	0.0026	0.0520
205	34.200	4.9806	4.9784	0.0021	0.0429
206	34.400	4.9840	4.9823	0.0017	0.0351
207	34.600	4.9870	4.9856	0.0014	0.0285
208	34.800	4.9895	4.9884	0.0011	0.0229
209	35.000	4.9916	4.9907	0.0009	0.0183
210	35.200	4.9934	4.9927	0.0007	0.0144
211	35.400	4.9948	4.9943	0.0006	0.0112
212	35.600	4.9960	4.9956	0.0004	0.0087

Table 66

FIG. 90

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-45.343	-24.394	-22.729	-22.548	-19.985	-18.269	-17.863	-14.325
2	-45.283	-24.883	-23.866	-21.906	-18.916	-18.279	-17.226	-15.598
3	-45.204	-23.374	-22.109	-19.415	-20.810	-22.694	-19.681	-19.323
4	-45.358	-20.100	-20.409	-20.348	-20.339	-20.192	-20.138	-20.147
5	-45.157	-29.835	-16.217	-16.186	-16.191	-16.174	-16.180	-16.171
6	-46.052	-27.227	-26.705	-14.338	-14.246	-14.277	-14.367	-14.108
7	-45.255	-18.839	-23.099	-23.230	-19.668	-21.696	-21.424	-13.256
8	-24.362	-46.114	-22.442	-22.568	-19.567	-15.644	3.330	-1.541
9	-45.303	-21.093	-20.986	-19.939	-20.641	-20.205	-20.253	-19.419
10	-45.073	-21.391	-21.412	-21.316	-21.414	-21.401	-21.433	-21.295
11	-45.215	-21.016	-21.354	-21.801	-21.401	-21.841	-20.198	-16.773
12	-45.219	-19.542	-19.569	-20.712	-21.194	-20.510	-20.220	-21.196
13	-17.864	-46.095	-24.540	-22.942	-23.784	-18.452	-13.887	-17.940
14	-45.156	-20.672	-20.650	-20.689	-20.103	-20.425	-20.291	-20.430
15	-45.118	-20.368	-20.347	-20.452	-20.577	-20.637	-20.511	-20.575
16	-45.172	-20.834	-19.751	-20.049	-20.844	-19.309	-20.258	-21.056
17	-45.108	-20.531	-20.445	-20.446	-20.492	-20.508	-20.508	-20.518
18	-45.094	-22.132	-22.129	-21.433	-21.433	-21.450	-16.562	-16.560
19	-45.032	-20.649	-20.648	-20.648	-20.650	-20.558	-20.552	-20.553
20	-45.043	-20.498	-20.561	-20.740	-20.838	-20.664	-20.223	-20.605
21	-44.984	-20.581	-20.757	-20.602	-20.753	-20.530	-20.581	-20.568
22	-44.957	-20.684	-20.684	-20.667	-20.646	-20.650	-20.652	-20.652
23	-44.931	-20.682	-20.682	-20.682	-20.682	-20.682	-20.682	-20.672
24	-44.914	-20.681	-20.681	-20.681	-20.681	-20.681	-20.681	-20.681
25	-44.884	-20.700	-20.710	-20.704	-20.706	-20.689	-20.706	-20.712
26	-44.850	-20.750	-20.740	-20.740	-20.740	-20.740	-20.740	-20.740
27	-44.829	-20.756	-20.756	-20.758	-20.756	-20.758	-20.756	-20.756
28	-44.817	-20.760	-20.760	-20.760	-20.762	-20.762	-20.762	-20.752
29	-44.805	-20.765	-20.764	-20.764	-20.764	-20.762	-20.761	-20.764
30	-44.786	-20.777	-20.781	-20.775	-20.772	-20.794	-20.766	-20.790
31	-44.736	-21.134	-21.019	-21.139	-21.139	-21.111	-21.114	-21.123
32	-44.702	-21.199	-21.183	-21.183	-21.183	-21.183	-21.183	-21.183
33	-44.668	-21.250	-21.250	-21.247	-21.249	-21.253	-21.255	-21.248
34	-44.635	-21.306	-21.306	-21.306	-21.306	-21.306	-21.301	-21.300
35	-44.604	-21.363	-21.363	-21.363	-21.363	-21.361	-21.361	-21.361
36	-44.573	-21.391	-21.391	-21.391	-21.391	-21.395	-21.395	-21.395
37	-44.543	-21.431	-21.430	-21.430	-21.430	-21.430	-21.428	-21.428
38	-44.486	-21.617	-21.657	-21.541	-21.590	-21.608	-21.546	-21.516
39	-44.404	-21.764	-21.767	-21.777	-21.785	-21.779	-21.774	-21.770
40	-44.300	-21.767	-21.794	-21.780	-21.780	-21.768	-21.789	-21.799

Table 67

FIG. 91

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	-13.636	-11.934	-8.649	-6.824	-5.114	-4.635	-3.002	-1.841
2	-12.831	-10.336	-9.442	-8.114	-6.012	-3.798	-3.272	-2.251
3	-4.779	-3.027	-3.096	-2.272	-0.919	-1.470	-2.114	-0.429
4	-20.325	-7.492	-2.105	-1.838	-1.625	-1.116	-1.116	-1.127
5	-16.199	-16.203	-13.514	-11.005	-8.561	-7.350	-4.689	-3.190
6	-14.280	-14.036	-14.051	-14.398	-12.854	-1.255	-0.433	1.263
7	-11.272	-14.648	-4.877	-6.884	-0.261	-3.154	-4.705	-2.791
8	2.680	-0.839	-22.596	-17.542	-6.498	-9.135	-3.610	-5.056
9	-16.872	-6.248	-1.868	-1.112	-1.283	-1.245	-1.367	-0.633
10	-4.409	-4.056	-3.087	-3.109	-3.056	-2.997	-2.974	-2.994
11	-15.579	-10.362	-7.645	-2.175	-1.624	-0.887	-1.247	-1.441
12	-19.696	-8.370	-2.717	-2.138	-0.283	-0.277	-0.303	-1.010
13	-15.407	-15.606	-3.399	-2.573	-3.270	-3.907	2.693	0.560
14	-20.141	-4.751	-2.071	-1.303	-0.418	-0.959	-1.139	-0.930
15	-20.770	-2.574	-0.053	-1.009	-1.099	-0.986	-1.226	-0.812
16	-20.343	-12.201	0.377	1.140	-0.046	1.530	0.760	0.663
17	-20.495	-7.614	-1.738	-1.162	-0.583	-0.540	-0.526	-0.545
18	-16.560	-16.560	-0.513	0.198	0.226	0.659	1.095	1.087
19	-20.592	-1.210	-0.755	-0.754	-0.754	-0.756	-0.756	-0.756
20	-20.469	-8.598	1.013	1.057	0.913	0.724	0.947	0.744
21	-20.712	-1.009	-0.885	-1.016	-0.934	-0.885	-0.996	-0.906
22	-20.652	-0.569	-0.569	-0.569	-0.569	-0.569	-0.569	-0.532
23	-20.673	-0.459	-0.460	-0.459	-0.459	-0.459	-0.459	-0.459
24	-20.681	-5.138	0.198	0.198	0.198	0.198	0.198	0.198
25	-20.742	-4.073	0.302	0.292	0.297	0.291	0.302	0.283
26	-20.730	-0.420	-0.380	-0.360	-0.320	-0.300	-0.280	-0.230
27	-20.756	-0.082	0.006	0.006	0.006	0.006	0.006	0.006
28	-20.752	-0.007	-0.007	-0.007	-0.010	0.001	-0.010	0.000
29	-20.754	-0.001	-0.002	-0.001	-0.002	-0.003	-0.004	-0.002
30	-20.715	0.011	0.013	0.011	0.014	0.012	0.011	0.010
31	-18.293	0.011	-0.004	0.004	-0.005	0.008	-0.020	-0.001
32	-17.778	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	0.003
33	-17.315	-0.003	0.000	-0.004	0.001	0.000	-0.002	0.005
34	-16.932	-0.005	-0.005	0.001	0.001	0.001	0.001	0.001
35	-16.587	-0.013	-0.010	-0.008	-0.009	0.004	0.004	0.004
36	-16.391	-0.017	-0.011	-0.011	0.004	0.004	0.004	0.004
37	-16.041	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004	-0.004
38	-13.874	-3.119	-3.130	-2.370	0.046	0.224	0.372	-0.010
39	-4.540	-4.609	-4.579	-4.575	-4.642	-4.538	-4.465	-4.463
40	-4.516	-4.521	-4.550	-4.546	-4.540	-4.557	-4.537	-4.536

Table 68

FIG. 92

	Label							
	16	17	18	19	20	21	22	23
Design #								
1	0.559	0.878	1.628	7.378	8.322	13.022	13.872	13.904
2	-1.704	3.042	3.725	7.812	9.088	9.252	9.824	13.966
3	-3.795	-5.863	-7.934	-8.868	18.716	18.996	20.161	15.141
4	-1.369	-0.980	-0.902	-0.750	-8.105	11.883	15.567	15.460
5	6.661	7.690	12.424	12.446	12.429	12.433	12.427	12.454
6	1.629	3.323	3.412	4.223	6.954	8.333	13.210	16.400
7	0.423	-0.188	1.768	1.979	5.146	12.533	19.507	17.546
8	-5.326	-5.776	-6.925	8.076	-0.329	7.538	21.080	19.904
9	-1.159	-2.404	-1.143	-1.834	16.015	-1.591	16.255	16.368
10	-3.042	-3.091	-3.066	-2.633	4.068	9.019	19.530	19.493
11	1.066	1.431	1.863	2.221	3.083	5.760	8.956	15.215
12	-0.833	-1.102	1.022	0.334	1.654	6.346	6.910	19.145
13	-1.453	-0.557	-1.031	4.251	-1.363	19.205	23.735	17.787
14	-0.890	-0.809	-0.582	0.426	0.057	2.811	14.025	15.501
15	-0.834	-0.404	-0.640	-0.676	-0.608	0.390	12.322	20.810
16	1.366	1.543	0.606	0.849	0.368	0.937	0.269	20.383
17	-0.104	-0.019	0.225	0.802	2.289	3.473	5.851	20.505
18	1.097	1.095	1.102	1.095	1.095	1.095	1.090	19.720
19	-0.756	-0.755	-0.494	-0.414	-0.413	-0.174	9.614	20.336
20	0.904	0.792	0.627	0.791	0.617	-0.641	-0.520	20.637
21	-1.020	-0.917	-0.884	-1.015	5.030	5.165	20.457	20.734
22	-0.589	-0.532	-0.546	-0.557	-0.542	-0.072	7.122	20.594
23	-0.459	-0.459	-0.460	-0.459	-0.459	-0.459	8.213	20.636
24	0.238	0.249	0.653	0.668	0.668	0.668	0.668	20.696
25	0.303	0.298	0.313	0.318	0.321	0.324	0.351	20.731
26	-0.190	-0.070	-0.030	-0.030	-0.010	0.010	2.590	20.730
27	0.006	0.006	0.006	0.006	0.006	0.006	0.006	20.734
28	-0.008	0.005	0.010	0.005	0.006	0.011	0.009	20.762
29	-0.003	-0.003	-0.003	-0.003	-0.003	0.014	0.016	20.734
30	0.009	0.006	0.008	0.014	0.012	0.010	0.016	18.707
31	-0.001	0.017	-0.021	0.023	-0.011	21.115	21.080	-0.032
32	0.003	0.003	0.003	0.003	0.003	0.003	0.003	17.791
33	-0.001	0.000	0.001	0.000	0.001	0.001	0.003	17.320
34	0.001	0.001	0.001	0.001	0.001	0.001	0.002	16.920
35	0.004	0.004	0.004	0.004	0.004	0.004	0.004	16.590
36	0.004	0.003	0.003	0.003	0.003	0.003	0.003	16.298
37	-0.004	-0.004	-0.004	-0.004	0.015	0.015	0.015	16.041
38	-0.001	-0.168	-0.092	0.249	2.273	2.831	3.034	13.943
39	4.450	4.538	4.602	4.675	4.557	4.565	4.550	4.485
40	4.533	4.547	4.536	4.524	4.524	4.538	4.568	4.532

Table 69

FIG. 93

	Label							
	24	25	26	27	28	29	30	31
Design #								
1	16.228	16.944	17.064	17.457	18.036	23.620	26.682	45.294
2	17.928	17.882	18.192	19.553	20.125	20.783	26.101	45.344
3	19.782	17.858	12.125	20.371	5.939	19.767	20.586	45.539
4	18.190	18.596	18.715	18.637	18.711	22.562	24.189	45.372
5	12.433	12.435	12.444	12.434	12.433	28.408	28.406	44.777
6	18.299	18.306	18.313	18.364	18.214	18.234	27.926	45.223
7	16.584	18.303	17.321	14.171	18.208	19.779	26.888	45.258
8	14.532	23.670	18.114	16.877	22.057	16.800	17.548	45.369
9	17.625	16.316	16.764	16.767	17.047	21.556	26.663	45.231
10	19.360	19.413	19.361	19.445	19.349	19.391	19.391	45.431
11	19.561	21.712	21.119	21.344	20.929	21.212	19.875	45.222
12	21.093	20.712	20.654	19.848	20.639	19.954	20.537	45.233
13	25.021	18.153	20.806	16.034	13.962	13.485	12.121	45.159
14	19.945	21.365	21.592	20.221	20.547	20.507	20.601	45.201
15	20.279	20.327	20.400	20.168	20.126	20.118	20.137	45.198
16	20.621	20.677	20.467	20.558	20.568	20.490	20.580	45.102
17	20.519	20.515	20.448	20.613	20.475	20.513	20.566	45.098
18	20.462	20.637	20.639	20.641	20.665	20.633	21.019	45.074
19	20.448	20.519	20.513	20.511	20.509	20.514	20.507	45.077
20	20.696	20.484	20.699	20.605	20.653	20.514	20.663	45.008
21	20.547	20.590	20.655	20.626	20.600	8.532	20.590	45.000
22	20.596	20.604	20.603	20.607	20.627	20.628	20.636	44.979
23	20.637	20.636	20.636	20.651	20.661	20.661	20.661	44.947
24	20.696	20.697	20.697	20.697	20.697	20.707	20.707	44.905
25	20.710	20.714	20.719	20.714	20.717	20.743	20.708	44.879
26	20.730	20.724	20.736	20.742	20.742	20.738	20.746	44.852
27	20.747	20.758	20.758	20.758	20.758	20.762	20.768	44.829
28	20.757	20.757	20.759	20.756	20.760	20.762	20.757	44.817
29	20.733	20.760	20.760	20.760	20.761	20.760	20.830	44.805
30	21.012	21.041	21.063	21.047	21.040	21.063	21.058	44.767
31	18.509	21.040	21.141	21.071	20.994	-0.005	21.154	44.740
32	21.184	21.184	21.184	21.184	21.184	21.184	21.184	44.702
33	21.240	21.248	21.251	21.252	21.254	21.250	21.252	44.668
34	21.305	21.305	21.305	21.305	21.305	21.305	21.309	44.635
35	21.360	21.362	21.362	21.362	21.363	21.362	21.362	44.604
36	21.379	21.387	21.395	21.399	21.400	21.398	21.394	44.573
37	21.427	21.427	21.427	21.432	21.432	21.432	21.432	44.543
38	21.535	21.638	21.561	21.528	21.540	21.589	21.575	44.487
39	21.768	21.771	21.771	21.773	21.761	21.796	21.773	44.404
40	21.783	21.784	21.779	21.783	21.786	21.784	21.787	44.380

Table 78

FIG. 94

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-44.331	-22.050	-22.075	-22.065	-22.069	-22.039	-22.053	-20.292
42	-44.215	-22.590	-22.587	-22.587	-22.587	-22.587	-22.586	-15.296
43	-44.114	-22.822	-22.823	-22.822	-22.822	-22.821	-22.821	-7.777
44	-44.082	-22.838	-22.838	-22.838	-22.838	-22.838	-22.838	-6.530
45	-44.052	-22.851	-22.851	-22.851	-22.851	-22.849	-22.847	-6.521
46	-43.976	-23.394	-23.394	-23.392	-23.391	-23.391	-19.819	-6.612
47	-43.880	-23.657	-23.657	-23.657	-23.657	-23.656	-17.939	-6.749
48	-43.769	-23.993	-23.993	-23.992	-23.992	-23.992	-12.510	-6.124
49	-43.747	-23.932	-23.932	-23.932	-23.932	-23.932	-15.409	-7.000
50	-43.690	-24.010	-24.010	-24.010	-24.010	-24.010	-14.480	-7.110
51	-43.085	-24.022	-24.021	-24.021	-24.021	-24.021	-14.359	-7.122
52	-43.628	-24.091	-24.091	-24.091	-24.091	-24.091	-13.389	-7.240
53	-43.575	-24.146	-24.148	-24.147	-24.146	-24.147	-12.479	-7.354
54	-43.526	-24.191	-24.191	-24.191	-24.191	-24.191	-11.630	-7.457
55	-43.480	-24.223	-24.225	-24.227	-24.223	-24.227	-7.537	-7.562
56	-43.435	-24.253	-24.251	-24.252	-24.253	-24.252	-8.269	-8.268
57	-43.393	-24.270	-24.269	-24.269	-24.269	-24.269	-8.182	-8.170
58	-43.323	-24.795	-24.795	-24.795	-24.795	-21.912	-8.434	-8.432
59	-43.224	-25.171	-25.171	-25.170	-25.170	-19.536	-9.211	-9.200
60	-43.143	-25.366	-25.366	-25.366	-25.366	-17.669	-10.066	-10.066
61	-43.056	-25.545	-25.545	-25.545	-25.545	-11.874	-11.891	-11.892
62	-43.009	-25.568	-25.568	-25.568	-25.569	-11.884	-11.875	-11.875
63	-42.959	-25.586	-25.586	-25.586	-25.586	-11.885	-11.885	-11.885
64	-42.907	-25.604	-25.604	-25.604	-25.604	-11.890	-11.889	-11.890
65	-42.832	-26.175	-26.175	-26.175	-23.810	-11.947	-11.947	-11.947
66	-42.761	-26.407	-26.407	-26.407	-23.008	-12.221	-12.221	-12.221
67	-42.685	-26.623	-26.623	-26.623	-22.127	-12.457	-12.457	-12.457
68	-42.617	-26.762	-26.762	-26.762	-21.525	-12.600	-12.600	-12.600
69	-42.547	-26.909	-26.909	-26.909	-20.691	-12.903	-12.905	-12.910
70	-42.530	-26.930	-26.930	-26.930	-20.620	-12.920	-12.920	-12.920
71	-42.477	-27.050	-27.050	-27.050	-19.462	-13.823	-13.823	-13.822
72	-42.415	-27.121	-27.121	-27.121	-18.901	-14.000	-14.000	-13.989
73	-42.351	-27.211	-27.212	-27.211	-15.640	-15.640	-15.640	-15.640
74	-42.293	-27.236	-27.236	-27.236	-15.640	-15.639	-15.639	-15.639
75	-42.232	-27.259	-27.259	-27.259	-15.648	-15.648	-15.648	-15.648
76	-42.169	-27.280	-27.279	-27.279	-15.663	-15.663	-15.663	-15.663
77	-42.094	-27.964	-27.964	-25.850	-15.745	-15.745	-15.745	-15.745
78	-42.024	-28.222	-28.222	-25.276	-15.825	-15.826	-15.826	-15.825
79	-41.959	-28.366	-28.366	-24.979	-15.860	-15.858	-15.860	-15.860
80	-41.887	-28.620	-28.620	-24.173	-16.615	-16.615	-16.616	-16.615

Table 71

FIG. 95

	Label							
Design #	8	9	10	11	12	13	14	15
41	-3.639	-3.628	-3.647	-3.629	-3.641	-3.634	-3.627	-3.653
42	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609	-4.609
43	-6.370	-6.373	-6.369	-6.369	-6.369	-6.370	-6.368	-6.370
44	-6.527	-6.527	-6.527	-6.527	-6.527	-6.527	-6.524	-6.524
45	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521	-6.521
46	-6.612	-6.612	-6.612	-6.612	-6.612	-6.610	-6.610	-6.610
47	-6.749	-6.749	-6.749	-6.748	-6.748	-6.747	-6.746	-6.744
48	-8.124	-8.124	-8.124	-8.124	-8.121	-8.121	-8.121	6.031
49	-7.000	-7.000	-7.000	-7.000	-7.000	-7.000	-7.000	-6.998
50	-7.110	-7.110	-7.110	-7.110	-7.100	-7.100	-7.100	-7.100
51	-7.122	-7.122	-7.122	-7.122	-7.122	-7.122	-7.116	-7.114
52	-7.240	-7.239	-7.239	-7.239	-7.239	-7.234	-7.234	-7.233
53	-7.312	-7.406	-7.344	-7.380	-7.341	-7.346	-7.335	-7.327
54	-7.457	-7.457	-7.457	-7.457	-7.457	-7.457	-7.450	-7.451
55	-7.675	-7.559	-7.013	-7.420	-10.845	-7.658	-7.783	-7.752
56	-8.208	-8.208	-8.209	-8.209	-8.206	-8.172	-8.158	-4.855
57	-8.166	-8.166	-8.166	-8.166	-8.166	-8.166	-8.166	-5.244
58	-8.432	-8.432	-8.432	-8.432	-8.432	-8.432	-8.432	0.000
59	-9.200	-9.200	-9.200	-9.200	-9.200	-9.200	0.505	0.519
60	-10.066	-10.066	-10.063	-10.057	-10.057	-0.011	-0.010	0.005
61	-11.893	-11.892	-11.892	-11.892	-0.308	-0.308	-0.308	-0.310
62	-11.874	-11.874	-11.874	-11.870	-2.509	0.345	0.345	0.345
63	-11.884	-11.885	-11.885	-11.885	-0.377	-0.376	-0.376	-0.371
64	-11.890	-11.890	-11.890	-11.890	-0.365	-0.365	-0.365	-0.365
65	-11.947	-11.947	-11.947	-11.946	-0.003	0.000	0.000	0.000
66	-12.221	-12.221	-12.221	-10.474	-0.012	-0.012	-0.012	-0.012
67	-12.456	-12.456	-12.456	-9.936	0.000	0.000	0.000	0.000
68	-12.600	-12.600	-12.600	-8.687	-0.002	-0.001	-0.001	0.000
69	-12.901	-12.897	-12.902	-3.081	-3.046	-3.034	-3.074	-3.000
70	-12.910	-12.910	-12.910	-3.110	-3.080	-3.020	-3.000	-2.950
71	-13.822	-13.823	-5.103	-5.055	-5.055	-5.054	-5.054	2.768
72	-13.987	-13.987	-4.360	-4.360	-4.360	-4.360	-4.360	-4.360
73	-15.640	-9.495	-4.617	-4.616	-4.616	-4.617	-4.616	-4.614
74	-15.640	-9.565	-4.609	-4.609	-4.609	-4.609	-4.609	-4.608
75	-15.648	-8.562	-4.613	-4.613	-4.613	-4.613	-4.613	-4.613
76	-15.663	-9.604	-4.627	-4.626	-4.626	-4.626	-4.626	-4.626
77	-15.745	-8.854	-4.726	-4.724	-4.724	-4.724	-4.724	-4.724
78	-15.825	-6.846	-5.638	-5.642	-5.644	-5.641	-5.659	0.068
79	-15.859	-5.640	-5.840	-5.840	-5.840	-5.840	-5.839	-0.179
80	-12.278	-6.699	-6.692	-6.691	-6.694	-6.690	0.001	0.002

Table 72

FIG. 96

	Label							
Design #	16	17	18	19	20	21	22	23
41	-3.686	-3.668	6.587	6.464	6.471	6.465	6.464	6.507
42	-4.609	6.228	6.226	6.229	6.229	6.229	6.246	6.269
43	5.753	5.801	6.614	6.773	6.758	6.762	6.762	6.750
44	6.524	6.524	6.524	6.524	6.528	6.528	6.528	6.528
45	6.519	6.519	6.519	6.519	6.523	6.523	6.523	6.523
46	6.611	6.611	6.611	6.611	6.611	6.611	6.611	6.611
47	6.741	6.746	6.746	6.749	6.749	6.749	6.749	6.749
48	6.031	6.031	6.031	6.032	6.032	6.032	6.033	6.035
49	7.000	7.000	7.000	7.000	7.000	7.000	7.000	7.000
50	7.109	7.109	7.109	7.109	7.110	7.110	7.110	7.110
51	7.120	7.120	7.120	7.121	7.121	7.121	7.121	7.121
52	7.237	7.237	7.237	7.237	7.237	7.238	7.238	7.238
53	7.399	7.365	7.267	7.327	7.392	7.354	7.264	7.414
54	7.444	7.457	7.457	7.457	7.457	7.457	7.457	7.467
55	7.469	7.260	7.080	8.013	10.246	9.165	7.233	8.013
56	4.856	8.207	8.190	8.190	8.202	8.210	8.186	8.196
57	5.244	8.168	8.168	8.168	8.168	8.168	8.168	8.168
58	0.000	8.432	8.432	8.432	8.432	8.432	8.432	8.432
59	0.524	0.524	3.854	9.711	9.711	9.711	9.711	9.711
60	0.005	0.005	0.005	10.061	10.061	10.061	10.064	10.064
61	-0.310	-0.311	-0.310	2.201	11.885	11.885	11.886	11.885
62	0.345	0.345	0.345	0.384	11.884	11.884	11.885	11.885
63	-0.353	-0.353	-0.354	2.624	11.874	11.874	11.874	11.874
64	-0.365	-0.365	-0.340	2.595	11.879	11.879	11.879	11.879
65	0.000	0.000	0.000	0.062	11.947	11.947	11.947	11.947
66	-0.012	-0.012	-0.012	-0.012	11.972	11.973	11.975	11.975
67	-0.001	0.000	0.000	0.002	9.336	12.455	12.456	12.456
68	-0.001	-0.001	0.002	0.004	8.664	12.601	12.601	12.601
69	2.963	3.151	3.052	3.030	3.041	12.913	12.904	12.886
70	2.900	2.900	3.040	3.090	3.240	12.910	12.910	12.910
71	2.768	2.768	2.769	2.767	2.768	9.507	13.611	13.611
72	4.357	4.357	4.362	4.362	4.362	4.362	13.979	13.996
73	4.613	4.617	4.617	4.617	4.616	4.617	9.495	15.639
74	4.607	4.609	4.610	4.609	4.608	4.609	9.565	15.639
75	4.613	4.613	4.613	4.613	4.613	4.614	9.562	15.647
76	4.626	4.626	4.626	4.626	4.626	4.626	9.504	15.663
77	4.723	4.724	4.724	4.724	4.725	4.725	8.864	15.745
78	0.039	4.532	5.978	6.085	6.071	6.159	6.139	15.825
79	0.179	5.839	5.840	5.839	5.840	5.841	5.841	15.859
80	0.002	-0.004	6.686	6.685	6.689	6.695	6.709	12.278

Table 73

FIG. 97

	Label							
Design #	24	25	26	27	28	29	30	31
41	15.111	22.519	22.500	22.519	22.521	22.511	22.503	44.275
42	12.543	22.685	22.685	22.686	22.686	22.688	22.688	44.193
43	8.754	22.824	22.822	22.823	22.823	22.824	22.823	44.113
44	6.535	22.837	22.838	22.838	22.838	22.838	22.838	44.062
45	6.523	22.846	22.849	22.849	22.851	22.851	22.851	44.051
46	6.611	19.820	23.392	23.392	23.392	23.392	23.392	43.976
47	6.749	17.938	23.656	23.656	23.656	23.657	23.657	43.890
48	6.044	19.120	23.583	23.583	23.583	23.583	23.583	43.859
49	7.000	15.409	23.932	23.932	23.932	23.932	23.932	43.747
50	7.120	14.480	24.010	24.010	24.010	24.010	24.010	43.690
51	7.121	14.399	24.022	24.021	24.021	24.021	24.021	43.686
52	7.236	13.389	24.091	24.091	24.091	24.091	24.091	43.628
53	12.477	7.364	24.146	24.146	24.147	24.147	24.148	43.575
54	7.457	11.630	24.191	24.191	24.191	24.191	24.191	43.526
55	7.057	7.253	24.230	24.230	24.227	24.226	24.227	43.479
56	8.190	8.214	24.253	24.252	24.252	24.252	24.252	43.435
57	8.188	8.188	24.269	24.269	24.269	24.270	24.270	43.393
58	8.432	8.433	21.911	24.795	24.795	24.795	24.795	43.323
59	9.711	9.711	19.089	25.210	25.210	25.210	25.210	43.219
60	10.064	10.064	17.669	25.366	25.366	25.366	25.366	43.143
61	11.883	11.882	11.878	25.546	25.546	25.546	25.546	43.058
62	11.885	11.885	11.885	25.567	25.567	25.566	25.566	43.010
63	11.874	11.874	11.874	25.588	25.588	25.588	25.588	42.959
64	11.879	11.879	11.879	25.606	25.606	25.606	25.606	42.907
65	11.947	11.947	11.947	23.810	26.175	26.175	26.175	42.832
66	11.975	11.975	11.977	23.203	26.366	26.366	26.366	42.764
67	12.456	12.456	12.456	22.128	26.622	26.622	26.622	42.685
68	12.600	12.601	12.600	21.527	26.762	26.762	26.762	42.617
69	12.895	12.899	12.910	20.892	26.909	26.910	26.909	42.547
70	12.920	12.920	12.920	20.620	26.930	26.930	26.930	42.530
71	13.812	13.813	13.812	19.786	27.028	27.028	27.028	42.479
72	13.996	13.996	13.996	18.901	27.121	27.121	27.121	42.416
73	15.640	15.640	15.640	15.640	27.211	27.211	27.212	42.351
74	15.639	15.641	15.639	15.640	27.236	27.236	27.236	42.293
75	15.647	15.648	15.648	15.648	27.259	27.259	27.259	42.232
76	15.663	15.663	15.663	15.663	27.279	27.279	27.279	42.169
77	15.740	15.745	15.745	15.745	25.850	27.964	27.964	42.094
78	15.825	15.824	15.825	15.826	25.276	28.221	28.221	42.024
79	15.859	15.859	15.859	15.860	24.979	28.366	28.366	41.959
80	16.616	16.615	16.615	16.615	24.173	28.620	28.620	41.887

Table 74

FIG. 98

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-41.821	-28.769	-28.769	-23.684	-16.845	-16.844	-16.842	-16.844
82	-41.760	-28.848	-28.848	-23.491	-16.906	-16.905	-16.905	-16.904
83	-41.699	-28.961	-28.960	-23.010	-17.629	-17.630	-17.631	-14.794
84	-41.639	-29.050	-29.050	-22.576	-17.973	-17.973	-17.974	-13.458
85	-41.579	-29.119	-29.119	-22.235	-18.205	-18.206	-18.207	-10.709
86	-41.521	-29.202	-29.202	-19.869	-20.914	-19.870	-15.504	-11.049
87	-41.459	-29.236	-29.236	-20.250	-20.250	-20.250	-11.981	-15.369
88	-41.470	-29.230	-29.230	-20.250	-20.250	-20.250	-15.360	-11.090
89	-41.393	-29.263	-29.263	-20.266	-20.261	-20.262	-15.380	-11.085
90	-41.326	-29.287	-29.297	-20.278	-20.278	-20.277	-15.368	-11.099
91	-41.255	-29.310	-29.310	-20.298	-20.298	-20.299	-15.342	-11.114
92	-41.186	-29.946	-28.676	-20.382	-20.382	-20.382	-14.423	-12.070
93	-41.122	-30.267	-28.368	-20.437	-20.437	-20.437	-11.520	-13.125
94	-41.060	-30.466	-28.172	-20.467	-20.467	-20.467	-13.048	-13.053
95	-41.000	-30.689	-27.922	-20.987	-20.987	-19.413	-13.248	-13.248
96	-40.942	-30.884	-27.690	-21.251	-21.251	-18.827	-13.445	-13.445
97	-40.887	-31.034	-27.516	-21.408	-21.408	-18.464	-13.580	-13.581
98	-40.834	-31.159	-27.377	-21.533	-21.533	-18.136	-13.737	-13.737
99	-40.781	-31.262	-27.270	-21.623	-21.623	-17.864	-14.270	-14.266
100	-40.731	-31.355	-27.173	-21.712	-21.712	-17.499	-14.653	-14.653
101	-40.680	-31.436	-27.086	-21.782	-21.782	-17.099	-15.016	-15.016
102	-40.629	-31.502	-27.043	-21.827	-21.827	-16.360	-16.383	-14.324
103	-40.578	-31.565	-26.989	-22.249	-21.434	-16.400	-16.400	-14.293
104	-40.531	-31.636	-26.895	-22.684	-20.972	-16.569	-16.575	-13.874
105	-40.520	-31.656	-26.880	-22.740	-20.910	-16.600	-16.600	-13.820
106	-40.482	-31.696	-26.823	-22.812	-20.724	-16.706	-16.696	-13.471
107	-40.433	-31.749	-26.769	-23.062	-20.561	-16.800	-16.805	-12.532
108	-40.382	-31.796	-26.724	-23.204	-20.335	-17.642	-15.980	-12.703
109	-40.330	-31.839	-26.687	-23.329	-20.047	-18.214	-14.661	-14.680
110	-40.275	-31.876	-26.675	-23.385	-19.954	-18.338	-14.629	-14.608
111	-40.217	-31.906	-26.677	-23.414	-19.937	-18.371	-14.621	-14.621
112	-40.157	-31.935	-26.689	-23.430	-19.948	-18.379	-14.631	-14.631
113	-40.096	-31.960	-26.708	-23.437	-19.986	-18.357	-14.674	-14.638
114	-40.032	-31.984	-26.733	-23.437	-20.035	-18.325	-14.686	-14.686
115	-39.966	-32.007	-26.761	-23.435	-20.089	-18.291	-14.717	-14.717
116	-39.899	-32.028	-26.791	-23.436	-20.126	-18.282	-14.727	-14.734
117	-39.830	-32.048	-26.825	-23.430	-20.191	-18.225	-15.093	-14.397
118	-39.760	-32.067	-26.861	-23.423	-20.255	-18.167	-15.279	-14.232
119	-39.689	-32.086	-26.898	-23.419	-20.312	-18.119	-15.410	-14.123
120	-39.616	-32.103	-26.934	-23.418	-20.360	-18.084	-15.506	-14.041

Table 75

FIG. 99

	Label							
Design #	8	9	10	11	12	13	14	15
81	-10.064	-4.016	-7.749	-7.790	-7.745	-7.783	0.710	0.600
82	-8.314	-8.314	-8.314	-8.314	-8.314	-2.668	0.000	0.000
83	-8.424	-8.424	-8.426	-8.424	-8.425	-2.035	-0.010	-0.010
84	-9.139	-9.131	-9.146	-9.133	-5.714	-1.273	-1.253	-1.255
85	-10.706	-10.711	-10.709	-7.012	-2.943	-2.947	-2.946	-2.946
86	-10.979	-11.029	-11.133	-2.350	-3.921	-3.938	-4.265	-4.066
87	-11.077	-11.070	-11.079	-4.614	-3.298	-3.771	-3.824	-3.316
88	-11.060	-11.070	-11.070	-4.470	-3.650	-3.740	-3.630	-2.900
89	-11.087	-11.083	-11.081	-4.419	-4.105	-3.440	-3.137	-3.531
90	-11.105	-11.095	-11.095	-3.757	-3.409	-3.572	-3.471	-4.449
91	-11.115	-11.118	-11.122	-3.938	-3.922	-3.779	-3.779	-3.260
92	-12.114	-12.134	-7.676	-5.315	-5.314	-5.318	-0.787	-0.790
93	-13.127	-13.129	-5.948	-5.949	-5.948	-5.948	-0.345	-0.320
94	-13.052	-11.799	-5.914	-5.916	-5.916	-5.916	-1.319	0.426
95	-13.248	-11.225	-5.983	-5.983	-5.982	-5.983	-0.233	-0.148
96	-13.448	-10.590	-6.363	-6.363	-6.363	-5.062	-0.001	-0.001
97	-13.580	-6.575	-10.100	-6.574	-4.830	-6.577	0.002	0.000
98	-13.731	-8.188	-8.183	-8.195	-5.469	-2.226	-2.219	-2.231
99	-12.752	-8.255	-8.253	-8.253	-5.304	-2.266	-2.266	-2.266
100	-12.010	-8.425	-8.425	-8.425	-4.723	-2.483	-2.393	-2.393
101	-10.050	-10.047	-10.050	-5.349	-5.160	-5.018	-1.627	-0.546
102	-10.084	-10.073	-10.052	-4.892	-5.291	-5.290	-1.643	-0.211
103	-10.075	-10.051	-10.079	-5.181	-5.085	-5.214	-1.847	-0.622
104	-10.625	-10.633	-9.069	-5.233	-5.235	-5.234	-1.098	-1.100
105	-10.670	-10.640	-9.040	-5.270	-5.230	-5.180	-1.880	-0.020
106	-10.976	-10.978	-8.219	-6.210	-5.838	-3.878	-1.657	-1.383
107	-12.532	-10.913	-7.356	-7.977	-5.724	-2.519	-2.613	-1.587
108	-12.793	-9.852	-7.796	-7.617	-4.400	-4.277	-2.310	-0.268
109	-11.046	-10.598	-8.374	-6.228	-5.911	-2.886	-2.891	-0.761
110	-11.630	-9.544	-9.075	-6.528	-4.747	-4.441	-1.922	2.042
111	-11.639	-9.258	-9.417	-6.176	-5.514	-3.923	-1.828	-1.313
112	-11.636	-9.367	-9.387	-5.875	-5.869	-3.813	-1.898	-1.040
113	-11.521	-9.916	-8.643	-5.892	-5.987	-3.727	-1.210	-1.838
114	-11.349	-10.251	-8.607	-5.991	-5.990	-3.645	-1.592	-1.552
115	-10.977	-10.743	-8.406	-6.066	-6.061	-3.492	-1.877	-1.380
116	-10.870	-10.866	-8.386	-6.096	-6.093	-2.911	-2.905	-0.561
117	-10.896	-10.898	-8.337	-6.306	-5.924	-2.887	-2.924	-0.568
118	-10.944	-10.948	-8.124	-6.860	-5.436	-3.346	-2.594	-0.489
119	-10.991	-10.989	-7.689	-7.441	-5.086	-3.647	-2.493	-0.229
120	-11.297	-10.718	-7.592	-7.592	-6.023	-3.732	-2.449	-0.229

Table 76

FIG. 100

	Label							
Design #	16	17	18	19	20	21	22	23
81	0.645	0.697	0.666	8.351	8.351	8.355	8.360	8.348
82	0.000	0.001	2.667	8.314	8.314	8.314	8.314	8.313
83	-0.009	-0.010	2.080	8.423	8.423	8.423	8.423	8.422
84	-1.110	2.734	2.911	4.729	9.212	9.223	9.217	9.219
85	2.943	2.949	2.944	2.948	7.012	10.711	10.708	10.709
86	3.314	3.315	3.187	3.641	5.117	11.020	11.009	11.046
87	4.672	3.541	3.560	3.537	3.314	11.074	11.063	11.073
88	2.870	3.480	3.890	4.110	4.250	11.070	11.070	11.080
89	3.414	3.445	3.385	3.791	4.620	11.084	11.081	11.085
90	3.381	3.458	3.883	4.403	3.534	11.099	11.096	11.098
91	3.241	3.862	3.858	3.855	3.859	11.118	11.118	11.119
92	-0.794	2.871	5.049	5.049	5.062	5.046	12.965	12.058
93	-0.342	1.023	5.944	5.943	5.943	5.941	11.537	13.123
94	0.432	0.428	5.927	5.927	5.928	5.928	11.755	13.064
95	-0.134	0.518	5.981	5.982	5.982	5.982	11.228	13.247
96	0.001	0.001	5.059	6.361	6.366	6.366	10.589	13.446
97	-0.002	0.000	4.630	6.569	6.570	6.586	10.100	13.580
98	2.266	2.211	2.210	5.488	8.196	8.184	8.177	13.741
99	2.266	2.266	2.266	5.305	8.254	8.254	8.254	12.753
100	2.412	2.423	2.433	4.725	8.424	8.424	8.427	12.010
101	0.292	1.989	4.607	5.235	5.585	10.016	10.058	10.061
102	0.199	1.862	4.857	5.294	5.316	10.059	10.067	10.062
103	0.027	1.840	5.156	5.161	5.163	10.081	10.077	10.076
104	0.507	1.769	5.187	5.185	5.194	9.676	9.674	11.026
105	0.020	1.690	5.210	5.230	5.240	9.040	10.650	10.690
106	1.491	1.542	3.981	6.008	6.031	8.239	11.040	10.907
107	1.618	2.523	2.564	5.792	7.592	7.091	10.313	12.546
108	0.281	2.326	4.252	4.414	7.640	7.778	9.850	12.681
109	1.329	1.678	3.802	5.983	5.951	10.149	11.333	8.689
110	-0.656	0.583	4.770	4.332	6.906	9.687	8.910	11.611
111	1.225	1.746	3.838	5.773	5.960	9.394	9.297	11.633
112	1.917	1.027	3.804	5.827	5.920	9.356	9.346	11.637
113	1.236	1.801	3.741	5.929	5.922	8.864	9.904	11.524
114	1.564	1.580	3.645	6.016	5.964	8.808	10.249	11.350
115	1.608	1.607	3.569	5.819	6.299	8.379	10.867	10.860
116	0.588	2.629	3.173	6.054	6.117	8.394	10.870	10.863
117	0.566	2.894	2.920	5.893	6.340	8.330	10.898	10.898
118	0.521	2.446	3.520	5.319	8.968	8.070	10.908	10.993
119	0.235	2.468	3.679	5.067	7.472	7.673	10.990	10.989
120	0.226	2.480	3.747	5.033	7.568	7.612	10.729	11.285

Table 77

FIG. 101

	Label							
	24	25	26	27	28	29	30	31
Design #								
81	16.864	16.864	16.864	16.864	23.655	28.775	28.775	41.621
82	16.904	16.904	16.906	16.906	23.491	28.848	28.848	41.760
83	14.802	17.628	17.628	17.628	23.011	28.960	28.960	41.699
84	13.339	17.986	17.988	17.988	22.564	29.051	29.051	41.639
85	10.707	18.206	18.205	18.205	22.235	29.119	29.119	41.579
86	11.056	15.550	20.962	19.857	19.813	29.201	29.201	41.521
87	11.074	15.370	20.252	20.248	20.251	29.236	29.236	41.459
88	11.090	15.380	20.250	20.250	20.250	29.230	29.230	41.470
89	11.082	15.382	20.261	20.261	20.260	29.263	29.263	41.393
90	11.101	15.368	20.277	20.278	20.278	29.287	29.287	41.326
91	11.113	15.342	20.298	20.298	20.299	29.310	29.310	41.256
92	12.054	14.489	20.380	20.379	20.380	28.082	29.941	41.186
93	13.123	13.122	20.437	20.437	20.437	28.357	30.267	41.122
94	13.060	13.066	20.466	20.466	20.467	28.173	30.485	41.060
95	13.247	13.247	19.415	20.987	20.986	27.922	30.688	41.000
96	13.445	13.446	18.826	21.251	21.251	27.690	30.884	40.942
97	13.580	13.580	18.464	21.408	21.408	27.518	31.034	40.887
98	13.732	13.732	18.136	21.534	21.533	27.377	31.159	40.834
99	14.267	14.269	17.864	21.623	21.623	27.270	31.262	40.781
100	14.653	14.653	17.499	21.712	21.712	27.173	31.355	40.731
101	15.019	15.019	17.095	21.783	21.782	27.095	31.436	40.680
102	14.325	16.399	16.342	21.827	21.827	27.043	31.502	40.629
103	14.295	16.400	16.400	21.434	22.249	26.989	31.565	40.578
104	13.899	16.562	16.562	20.987	22.674	26.897	31.635	40.530
105	13.820	16.600	16.600	20.910	22.749	26.880	31.650	40.520
106	13.473	16.699	16.702	20.724	22.912	26.823	31.696	40.482
107	12.517	16.811	16.794	20.560	23.062	26.769	31.749	40.433
108	12.727	15.977	17.644	20.334	23.205	26.724	31.795	40.382
109	14.629	14.686	18.227	20.039	23.331	26.686	31.839	40.330
110	14.610	14.629	18.338	19.954	23.385	26.675	31.875	40.275
111	14.628	14.617	18.369	19.939	23.413	26.678	31.908	40.217
112	14.629	14.632	18.379	19.947	23.430	26.689	31.935	40.158
113	14.656	14.654	18.387	19.986	23.437	26.708	31.960	40.096
114	14.686	14.686	18.325	20.035	23.437	26.733	31.984	40.032
115	14.716	14.719	18.290	20.089	23.435	26.761	32.007	39.966
116	14.734	14.728	18.280	20.126	23.436	26.791	32.028	39.899
117	14.392	15.097	18.224	20.191	23.429	26.825	32.048	39.830
118	14.220	15.290	18.164	20.259	23.423	26.861	32.067	39.760
119	14.124	15.410	18.119	20.312	23.419	26.896	32.086	39.689
120	14.045	15.504	18.085	20.360	23.418	26.934	32.103	39.616

Table 7N

FIG. 102

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-39.542	-32.120	-26.971	-23.419	-20.406	-18.051	-15.604	-13.944
122	-39.520	-32.120	-26.980	-23.420	-20.410	-18.050	-15.620	-13.930
123	-39.467	-32.136	-27.007	-23.423	-20.446	-18.027	-15.684	-13.860
124	-39.390	-32.151	-27.044	-23.431	-20.479	-18.015	-15.738	-13.811
125	-39.312	-32.165	-27.080	-23.442	-20.509	-18.011	-15.782	-13.773
126	-39.233	-32.177	-27.115	-23.457	-20.535	-18.015	-15.810	-13.761
127	-39.152	-32.188	-27.151	-23.475	-20.558	-18.026	-15.831	-13.764
128	-39.070	-32.198	-27.186	-23.496	-20.579	-18.041	-15.845	-13.776
129	-38.988	-32.206	-27.221	-23.519	-20.600	-18.059	-15.859	-13.792
130	-38.901	-32.212	-27.256	-23.544	-20.620	-18.079	-15.871	-13.810
131	-38.814	-32.217	-27.292	-23.572	-20.641	-18.100	-15.884	-13.828
132	-38.725	-32.219	-27.326	-23.602	-20.662	-18.122	-15.897	-13.847
133	-38.635	-32.220	-27.361	-23.633	-20.684	-18.145	-15.912	-13.865
134	-38.544	-32.219	-27.396	-23.666	-20.706	-18.168	-15.928	-13.883
135	-38.451	-32.216	-27.430	-23.700	-20.731	-18.192	-15.945	-13.900
136	-38.357	-32.212	-27.464	-23.736	-20.756	-18.218	-15.964	-13.918
137	-38.261	-32.205	-27.497	-23.773	-20.783	-18.241	-15.984	-13.936
138	-38.164	-32.196	-27.529	-23.812	-20.812	-18.266	-16.005	-13.955
139	-38.150	-32.190	-27.530	-23.820	-20.820	-18.270	-16.010	-13.960
140	-38.065	-32.185	-27.561	-23.851	-20.843	-18.292	-16.028	-13.974
141	-37.966	-32.172	-27.592	-23.892	-20.876	-18.319	-16.051	-13.993
142	-37.863	-32.157	-27.621	-23.933	-20.910	-18.348	-16.076	-14.014
143	-37.760	-32.140	-27.650	-23.975	-20.946	-18.377	-16.102	-14.036
144	-37.656	-32.120	-27.677	-24.017	-20.984	-18.408	-16.129	-14.058
145	-37.550	-32.099	-27.702	-24.060	-21.024	-18.441	-16.157	-14.082
146	-37.443	-32.075	-27.727	-24.104	-21.066	-18.475	-16.186	-14.107
147	-37.334	-32.049	-27.749	-24.147	-21.110	-18.511	-16.217	-14.133
148	-37.224	-32.021	-27.770	-24.190	-21.155	-18.549	-16.249	-14.161
149	-37.112	-31.990	-27.789	-24.233	-21.201	-18.589	-16.283	-14.190
150	-37.000	-31.957	-27.806	-24.276	-21.249	-18.631	-16.318	-14.220
151	-36.886	-31.923	-27.822	-24.318	-21.298	-18.676	-16.356	-14.252
152	-36.770	-31.885	-27.835	-24.359	-21.348	-18.722	-16.395	-14.286
153	-36.654	-31.846	-27.846	-24.399	-21.399	-18.771	-16.437	-14.321
154	-36.536	-31.805	-27.855	-24.438	-21.450	-18.821	-16.481	-14.359
155	-36.417	-31.761	-27.862	-24.476	-21.502	-18.873	-16.527	-14.398
156	-36.360	-31.740	-27.860	-24.490	-21.530	-18.900	-16.550	-14.420
157	-36.296	-31.715	-27.867	-24.513	-21.554	-18.927	-16.576	-14.440
158	-36.175	-31.667	-27.869	-24.548	-21.605	-18.983	-16.627	-14.484
159	-36.052	-31.617	-27.869	-24.581	-21.657	-19.039	-16.681	-14.531
160	-35.928	-31.565	-27.867	-24.613	-21.708	-19.097	-16.737	-14.581

Table 79

FIG. 103

	Label							
Design #	8	9	10	11	12	13	14	15
121	-11.514	-10.628	-7.664	-7.612	-4.476	-4.476	-1.815	-1.123
122	-11.550	-10.490	-7.630	-7.450	-4.490	-4.480	-1.730	-1.220
123	-11.692	-10.348	-8.087	-7.207	-4.776	-4.237	-1.868	-1.313
124	-11.793	-10.248	-8.253	-7.086	-4.948	-4.085	-1.837	-1.141
125	-11.881	-10.145	-8.428	-6.866	-5.234	-3.784	-2.185	-0.797
126	-11.924	-10.105	-8.499	-6.786	-5.342	-3.663	-2.325	-0.650
127	-11.946	-10.098	-8.526	-6.766	-5.376	-3.631	-2.364	-0.611
128	-11.955	-10.110	-8.527	-6.781	-5.387	-3.652	-2.346	-0.634
129	-11.980	-10.127	-8.523	-6.804	-5.361	-3.681	-2.320	-0.665
130	-11.984	-10.147	-8.517	-6.828	-5.335	-3.709	-2.295	-0.694
131	-11.970	-10.165	-8.515	-6.848	-5.324	-3.731	-2.277	-0.716
132	-11.977	-10.182	-8.515	-6.865	-5.319	-3.748	-2.265	-0.731
133	-11.985	-10.198	-8.518	-6.879	-5.316	-3.760	-2.259	-0.741
134	-11.996	-10.212	-8.524	-6.890	-5.317	-3.768	-2.257	-0.746
135	-12.008	-10.225	-8.531	-6.900	-5.321	-3.775	-2.257	-0.750
136	-12.021	-10.238	-8.540	-6.909	-5.326	-3.780	-2.259	-0.752
137	-12.036	-10.251	-8.550	-6.917	-5.332	-3.785	-2.261	-0.753
138	-12.051	-10.264	-8.561	-6.926	-5.339	-3.790	-2.263	-0.754
139	-12.050	-10.270	-8.566	-6.930	-5.340	-3.790	-2.260	-0.750
140	-12.067	-10.278	-8.572	-6.935	-5.346	-3.795	-2.266	-0.754
141	-12.085	-10.292	-8.584	-6.944	-5.353	-3.799	-2.270	-0.755
142	-12.103	-10.307	-8.597	-6.954	-5.361	-3.805	-2.273	-0.756
143	-12.121	-10.322	-8.610	-6.964	-5.369	-3.810	-2.276	-0.757
144	-12.141	-10.338	-8.623	-6.975	-5.377	-3.816	-2.280	-0.758
145	-12.161	-10.355	-8.637	-6.986	-5.386	-3.822	-2.284	-0.760
146	-12.182	-10.373	-8.652	-6.998	-5.395	-3.829	-2.287	-0.761
147	-12.205	-10.392	-8.668	-7.011	-5.404	-3.835	-2.291	-0.762
148	-12.228	-10.411	-8.684	-7.024	-5.414	-3.842	-2.296	-0.764
149	-12.252	-10.432	-8.701	-7.037	-5.425	-3.850	-2.300	-0.765
150	-12.278	-10.454	-8.719	-7.052	-5.436	-3.858	-2.305	-0.767
151	-12.305	-10.477	-8.738	-7.067	-5.447	-3.866	-2.310	-0.768
152	-12.334	-10.501	-8.757	-7.083	-5.460	-3.874	-2.315	-0.770
153	-12.364	-10.526	-8.778	-7.099	-5.472	-3.883	-2.320	-0.772
154	-12.396	-10.553	-8.800	-7.117	-5.486	-3.893	-2.326	-0.774
155	-12.429	-10.581	-8.823	-7.136	-5.500	-3.903	-2.332	-0.776
156	-12.450	-10.599	-8.839	-7.149	-5.510	-3.910	-2.336	-0.778
157	-12.465	-10.611	-8.848	-7.155	-5.515	-3.914	-2.338	-0.778
158	-12.503	-10.642	-8.874	-7.176	-5.531	-3.925	-2.345	-0.780
159	-12.542	-10.676	-8.902	-7.199	-5.548	-3.937	-2.352	-0.782
160	-12.585	-10.711	-8.931	-7.222	-5.567	-3.950	-2.360	-0.785

Table 80

FIG. 104

	Label							
	16	17	18	19	20	21	22	23
Design #								
121	1.123	1.815	4.476	4.476	7.632	7.643	10.529	11.513
122	1.210	1.740	4.470	4.490	7.490	7.760	10.500	11.540
123	1.486	1.475	4.503	4.505	7.296	8.017	10.382	11.668
124	1.115	1.879	4.030	5.005	7.029	8.282	10.233	11.802
125	0.797	2.183	3.787	5.231	6.868	8.426	10.146	11.880
126	0.650	2.325	3.663	5.342	6.766	8.500	10.105	11.924
127	0.611	2.364	3.631	5.376	6.766	8.526	10.098	11.945
128	0.634	2.346	3.652	5.367	6.781	8.528	10.110	11.955
129	0.665	2.320	3.681	5.351	6.804	8.523	10.127	11.960
130	0.694	2.295	3.709	5.335	6.828	8.517	10.147	11.964
131	0.716	2.277	3.731	5.324	6.848	8.515	10.165	11.970
132	0.731	2.265	3.749	5.318	6.865	8.515	10.182	11.977
133	0.741	2.259	3.760	5.316	6.879	8.518	10.198	11.985
134	0.746	2.257	3.768	5.317	6.890	8.524	10.212	11.996
135	0.750	2.257	3.775	5.321	6.900	8.531	10.225	12.008
136	0.752	2.258	3.780	5.326	6.909	8.540	10.238	12.021
137	0.753	2.261	3.785	5.332	6.917	8.550	10.251	12.036
138	0.754	2.263	3.790	5.339	6.926	8.561	10.264	12.051
139	0.750	2.260	3.790	5.340	6.930	8.560	10.270	12.060
140	0.754	2.266	3.795	5.346	6.935	8.572	10.278	12.067
141	0.755	2.270	3.799	5.353	6.944	8.584	10.292	12.085
142	0.756	2.273	3.805	5.361	6.954	8.597	10.307	12.103
143	0.757	2.276	3.810	5.369	6.964	8.610	10.322	12.121
144	0.758	2.280	3.816	5.377	6.975	8.623	10.336	12.141
145	0.760	2.284	3.822	5.386	6.986	8.637	10.355	12.161
146	0.761	2.287	3.829	5.395	6.998	8.652	10.373	12.182
147	0.762	2.291	3.835	5.404	7.011	8.668	10.392	12.205
148	0.764	2.296	3.842	5.414	7.024	8.684	10.411	12.228
149	0.765	2.300	3.850	5.425	7.037	8.701	10.432	12.252
150	0.767	2.305	3.858	5.436	7.052	8.719	10.454	12.278
151	0.768	2.310	3.866	5.447	7.067	8.738	10.477	12.305
152	0.770	2.316	3.874	5.460	7.083	8.757	10.501	12.334
153	0.772	2.320	3.883	5.472	7.099	8.778	10.526	12.364
154	0.774	2.326	3.893	5.486	7.117	8.800	10.553	12.396
155	0.776	2.332	3.903	5.500	7.136	8.823	10.581	12.429
156	0.780	2.330	3.910	5.510	7.140	8.830	10.590	12.450
157	0.778	2.338	3.914	5.516	7.155	8.848	10.611	12.465
158	0.780	2.345	3.926	5.531	7.176	8.874	10.642	12.503
159	0.782	2.352	3.937	5.548	7.199	8.902	10.676	12.542
160	0.785	2.360	3.950	5.567	7.222	8.931	10.711	12.585

Table 81

FIG. 105

	Label							
	24	25	26	27	28	29	30	31
Design #								
121	13.944	15.604	18.051	20.406	23.419	26.971	32.120	39.542
122	13.930	15.620	18.050	20.410	23.420	26.980	32.120	39.530
123	13.870	15.679	18.028	20.445	23.423	27.007	32.136	39.467
124	13.807	15.740	18.014	20.479	23.431	27.044	32.151	39.390
125	13.774	15.782	18.011	20.509	23.442	27.079	32.165	39.312
126	13.761	15.810	18.015	20.535	23.457	27.115	32.177	39.253
127	13.764	15.831	18.026	20.558	23.475	27.151	32.188	39.152
128	13.776	15.846	18.041	20.580	23.496	27.186	32.198	39.070
129	13.792	15.859	18.059	20.600	23.519	27.221	32.208	38.986
130	13.810	15.871	18.079	20.620	23.544	27.256	32.212	38.901
131	13.828	15.884	18.100	20.641	23.572	27.292	32.217	38.814
132	13.847	15.897	18.122	20.662	23.602	27.326	32.219	38.725
133	13.865	15.912	18.145	20.684	23.633	27.361	32.220	38.635
134	13.883	15.928	18.168	20.706	23.668	27.396	32.219	38.544
135	13.900	15.945	18.192	20.731	23.700	27.430	32.216	38.451
136	13.918	15.964	18.216	20.758	23.736	27.464	32.212	38.357
137	13.936	15.984	18.241	20.783	23.773	27.497	32.205	38.261
138	13.955	16.005	18.266	20.812	23.812	27.529	32.196	38.164
139	13.960	16.010	18.270	20.820	23.820	27.530	32.190	38.150
140	13.974	16.028	18.292	20.843	23.851	27.561	32.185	38.085
141	13.993	16.051	18.319	20.876	23.892	27.592	32.172	37.985
142	14.014	16.076	18.348	20.910	23.933	27.621	32.157	37.883
143	14.036	16.102	18.377	20.946	23.975	27.650	32.140	37.780
144	14.058	16.129	18.408	20.984	24.017	27.677	32.120	37.656
145	14.082	16.157	18.441	21.024	24.060	27.702	32.099	37.550
146	14.107	16.186	18.475	21.066	24.104	27.727	32.075	37.443
147	14.133	16.217	18.511	21.110	24.147	27.749	32.049	37.334
148	14.161	16.249	18.549	21.156	24.190	27.770	32.021	37.224
149	14.190	16.283	18.589	21.201	24.233	27.789	31.990	37.112
150	14.220	16.318	18.631	21.249	24.276	27.806	31.957	37.000
151	14.252	16.356	18.676	21.298	24.318	27.822	31.923	36.886
152	14.286	16.395	18.722	21.348	24.359	27.835	31.885	36.770
153	14.321	16.437	18.771	21.399	24.399	27.846	31.846	36.654
154	14.359	16.481	18.821	21.450	24.438	27.855	31.805	36.536
155	14.398	16.527	18.873	21.502	24.476	27.862	31.761	36.416
156	14.420	16.550	18.900	21.530	24.490	27.860	31.740	36.360
157	14.440	16.576	18.927	21.554	24.513	27.867	31.715	36.296
158	14.484	16.627	18.983	21.605	24.548	27.869	31.667	36.175
159	14.531	16.681	19.039	21.657	24.581	27.869	31.617	36.052
160	14.581	16.737	19.097	21.708	24.613	27.867	31.565	35.926

Table 82

FIG. 106

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-35.603	-31.511	-27.869	-24.843	-21.758	-19.156	-16.795	-14.633
162	-35.676	-31.455	-27.856	-24.671	-21.806	-19.215	-16.854	-14.688
163	-35.651	-31.397	-27.847	-24.697	-21.856	-19.275	-16.916	-14.745
164	-35.424	-31.337	-27.836	-24.721	-21.903	-19.335	-16.979	-14.805
165	-35.296	-31.275	-27.823	-24.743	-21.949	-19.394	-17.043	-14.867
166	-35.167	-31.212	-27.808	-24.763	-21.994	-19.453	-17.108	-14.932
167	-35.038	-31.147	-27.790	-24.781	-22.037	-19.512	-17.174	-14.998
168	-34.909	-31.081	-27.771	-24.797	-22.075	-19.569	-17.240	-15.065
169	-34.779	-31.014	-27.750	-24.811	-22.117	-19.626	-17.306	-15.134
170	-34.650	-30.946	-27.728	-24.823	-22.155	-19.681	-17.372	-15.203
171	-34.522	-30.877	-27.704	-24.833	-22.191	-19.735	-17.436	-15.273
172	-34.394	-30.808	-27.679	-24.842	-22.226	-19.786	-17.501	-15.342
173	-34.268	-30.739	-27.653	-24.850	-22.259	-19.839	-17.563	-15.411
174	-34.150	-30.670	-27.630	-24.860	-22.290	-19.890	-17.620	-15.480
175	-34.144	-30.671	-27.627	-24.856	-22.290	-19.886	-17.625	-15.479
176	-34.022	-30.603	-27.601	-24.862	-22.320	-19.936	-17.685	-15.546
177	-33.902	-30.537	-27.574	-24.866	-22.348	-19.982	-17.743	-15.611
178	-33.785	-30.472	-27.548	-24.870	-22.375	-20.026	-17.799	-15.673
179	-33.672	-30.409	-27.522	-24.874	-22.401	-20.069	-17.853	-15.734
180	-33.562	-30.347	-27.497	-24.877	-22.426	-20.110	-17.905	-15.793
181	-33.456	-30.288	-27.474	-24.880	-22.450	-20.149	-17.954	-15.849
182	-33.353	-30.231	-27.450	-24.883	-22.473	-20.187	-18.002	-15.903
183	-33.254	-30.176	-27.428	-24.886	-22.495	-20.223	-18.048	-15.954
184	-33.160	-30.124	-27.407	-24.889	-22.516	-20.257	-18.091	-16.003
185	-33.068	-30.073	-27.387	-24.892	-22.536	-20.290	-18.133	-16.050
186	-32.981	-30.026	-27.368	-24.895	-22.556	-20.321	-18.172	-16.094
187	-32.898	-29.980	-27.350	-24.898	-22.574	-20.351	-18.210	-16.136
188	-32.817	-29.936	-27.333	-24.901	-22.592	-20.380	-18.246	-16.176
189	-32.740	-29.894	-27.317	-24.904	-22.609	-20.407	-18.280	-16.214
190	-32.666	-29.854	-27.301	-24.906	-22.626	-20.434	-18.313	-16.250
191	-32.595	-29.816	-27.287	-24.910	-22.642	-20.459	-18.344	-16.285
192	-32.527	-29.780	-27.273	-24.913	-22.657	-20.483	-18.374	-16.318
193	-32.464	-29.746	-27.260	-24.915	-22.671	-20.505	-18.401	-16.348
194	-32.400	-29.712	-27.248	-24.918	-22.686	-20.528	-18.429	-16.379
195	-32.342	-29.681	-27.236	-24.921	-22.699	-20.548	-18.454	-16.406
196	-32.285	-29.652	-27.225	-24.924	-22.712	-20.568	-18.478	-16.433
197	-32.230	-29.623	-27.215	-24.927	-22.724	-20.587	-18.502	-16.459
198	-32.178	-29.596	-27.204	-24.929	-22.736	-20.605	-18.524	-16.483
199	-32.127	-29.568	-27.194	-24.932	-22.747	-20.623	-18.546	-16.507
200	-32.075	-29.542	-27.185	-24.935	-22.759	-20.640	-18.567	-16.530

Table 83

FIG. 107

	Label							
Design #	8	9	10	11	12	13	14	15
161	-12.630	-10.749	-8.963	-7.247	-5.586	-3.963	-2.365	-0.788
162	-12.678	-10.790	-8.996	-7.274	-5.606	-3.978	-2.376	-0.790
163	-12.728	-10.833	-9.032	-7.303	-5.628	-3.993	-2.386	-0.794
164	-12.782	-10.879	-9.070	-7.334	-5.652	-4.010	-2.396	-0.797
165	-12.838	-10.927	-9.111	-7.367	-5.677	-4.028	-2.406	-0.800
166	-12.897	-10.979	-9.154	-7.402	-5.704	-4.047	-2.418	-0.804
167	-12.959	-11.034	-9.201	-7.440	-5.734	-4.068	-2.430	-0.808
168	-13.023	-11.091	-9.250	-7.480	-5.765	-4.090	-2.443	-0.813
169	-13.089	-11.151	-9.302	-7.523	-5.798	-4.114	-2.456	-0.817
170	-13.157	-11.214	-9.356	-7.568	-5.834	-4.140	-2.473	-0.823
171	-13.226	-11.278	-9.413	-7.616	-5.871	-4.167	-2.489	-0.828
172	-13.295	-11.344	-9.472	-7.666	-5.911	-4.195	-2.507	-0.834
173	-13.365	-11.410	-9.532	-7.717	-5.952	-4.225	-2.525	-0.840
174	-13.430	-11.470	-9.590	-7.770	-5.990	-4.250	-2.540	-0.850
175	-13.435	-11.477	-9.693	-7.769	-5.994	-4.256	-2.543	-0.846
176	-13.503	-11.544	-9.654	-7.822	-6.037	-4.287	-2.562	-0.852
177	-13.570	-11.609	-9.714	-7.875	-6.080	-4.319	-2.582	-0.859
178	-13.636	-11.673	-9.774	-7.927	-6.122	-4.350	-2.601	-0.865
179	-13.699	-11.736	-9.832	-7.978	-6.164	-4.381	-2.620	-0.872
180	-13.760	-11.798	-9.888	-8.027	-6.204	-4.411	-2.638	-0.878
181	-13.819	-11.853	-9.942	-8.076	-6.243	-4.440	-2.656	-0.884
182	-13.875	-11.909	-9.993	-8.120	-6.281	-4.468	-2.673	-0.890
183	-13.929	-11.961	-10.043	-8.164	-6.317	-4.495	-2.690	-0.895
184	-13.980	-12.011	-10.089	-8.205	-6.351	-4.520	-2.705	-0.901
185	-14.028	-12.059	-10.134	-8.244	-6.383	-4.544	-2.720	-0.906
186	-14.074	-12.104	-10.176	-8.281	-6.413	-4.566	-2.734	-0.910
187	-14.118	-12.147	-10.216	-8.316	-6.442	-4.588	-2.747	-0.915
188	-14.160	-12.188	-10.253	-8.349	-6.469	-4.608	-2.760	-0.919
189	-14.199	-12.226	-10.289	-8.381	-6.495	-4.627	-2.771	-0.923
190	-14.237	-12.263	-10.323	-8.411	-6.520	-4.645	-2.783	-0.927
191	-14.272	-12.298	-10.355	-8.439	-6.543	-4.662	-2.793	-0.930
192	-14.306	-12.331	-10.386	-8.466	-6.565	-4.679	-2.803	-0.934
193	-14.337	-12.361	-10.414	-8.490	-6.585	-4.694	-2.812	-0.937
194	-14.369	-12.392	-10.442	-8.515	-6.605	-4.709	-2.821	-0.940
195	-14.397	-12.419	-10.467	-8.537	-6.623	-4.722	-2.830	-0.943
196	-14.424	-12.446	-10.492	-8.558	-6.640	-4.735	-2.837	-0.945
197	-14.451	-12.472	-10.516	-8.579	-6.658	-4.747	-2.845	-0.948
198	-14.476	-12.496	-10.538	-8.598	-6.673	-4.759	-2.852	-0.950
199	-14.500	-12.519	-10.560	-8.618	-6.689	-4.771	-2.860	-0.953
200	-14.524	-12.542	-10.581	-8.636	-6.704	-4.782	-2.866	-0.955

Table 84

FIG. 108

	Label							
	16	17	18	19	20	21	22	23
Design #								
161	0.788	2.368	3.963	5.589	7.247	8.963	10.749	12.630
162	0.790	2.376	3.978	5.606	7.274	8.998	10.790	12.678
163	0.794	2.388	3.993	5.628	7.303	9.032	10.833	12.728
164	0.797	2.398	4.010	5.652	7.334	9.070	10.879	12.782
165	0.800	2.406	4.028	5.677	7.367	9.111	10.927	12.836
166	0.804	2.418	4.047	5.704	7.402	9.154	10.979	12.897
167	0.808	2.430	4.066	5.734	7.440	9.201	11.034	12.959
168	0.813	2.443	4.090	5.765	7.480	9.250	11.091	13.023
169	0.817	2.458	4.114	5.798	7.523	9.302	11.151	13.089
170	0.823	2.473	4.140	5.834	7.568	9.356	11.214	13.157
171	0.828	2.489	4.167	5.871	7.616	9.413	11.278	13.226
172	0.834	2.506	4.195	5.911	7.666	9.472	11.344	13.295
173	0.840	2.525	4.225	5.952	7.717	9.532	11.410	13.365
174	0.850	2.540	4.250	5.990	7.770	9.590	11.470	13.430
175	0.846	2.543	4.256	5.994	7.769	9.593	11.477	13.436
176	0.853	2.562	4.287	6.037	7.822	9.654	11.544	13.503
177	0.859	2.582	4.319	6.080	7.875	9.714	11.609	13.570
178	0.865	2.601	4.350	6.122	7.927	9.774	11.673	13.636
179	0.872	2.620	4.381	6.164	7.978	9.832	11.735	13.699
180	0.878	2.638	4.411	6.204	8.027	9.888	11.796	13.760
181	0.884	2.656	4.440	6.244	8.075	9.942	11.853	13.819
182	0.890	2.673	4.468	6.281	8.120	9.993	11.909	13.875
183	0.896	2.690	4.495	6.317	8.164	10.043	11.961	13.929
184	0.901	2.705	4.520	6.351	8.205	10.089	12.011	13.980
185	0.906	2.720	4.544	6.383	8.244	10.134	12.059	14.028
186	0.910	2.734	4.566	6.413	8.281	10.176	12.105	14.074
187	0.915	2.747	4.586	6.442	8.316	10.216	12.147	14.118
188	0.919	2.760	4.608	6.469	8.349	10.253	12.188	14.160
189	0.923	2.771	4.627	6.495	8.381	10.289	12.226	14.199
190	0.927	2.783	4.645	6.520	8.411	10.323	12.263	14.237
191	0.930	2.793	4.662	6.543	8.439	10.355	12.298	14.272
192	0.934	2.803	4.679	6.565	8.466	10.386	12.331	14.306
193	0.937	2.812	4.694	6.585	8.490	10.414	12.361	14.337
194	0.940	2.821	4.709	6.605	8.515	10.442	12.392	14.369
195	0.943	2.830	4.722	6.623	8.537	10.467	12.419	14.397
196	0.945	2.837	4.735	6.640	8.558	10.492	12.446	14.424
197	0.948	2.845	4.747	6.658	8.579	10.516	12.472	14.451
198	0.950	2.852	4.759	6.673	8.598	10.538	12.496	14.476
199	0.953	2.860	4.771	6.689	8.618	10.560	12.519	14.500
200	0.955	2.866	4.782	6.704	8.636	10.580	12.542	14.524

Table 85

FIG. 109

	Label							
	24	25	26	27	28	29	30	31
Design #								
161	14.633	16.795	19.150	21.756	24.643	27.863	31.511	35.603
162	14.688	16.854	19.215	21.808	24.671	27.856	31.455	35.678
163	14.745	16.916	19.275	21.856	24.697	27.847	31.397	35.551
164	14.805	16.979	19.335	21.903	24.721	27.836	31.337	35.424
165	14.867	17.043	19.394	21.949	24.743	27.823	31.275	35.296
166	14.932	17.108	19.453	21.994	24.763	27.808	31.212	35.167
167	14.998	17.174	19.512	22.037	24.781	27.790	31.147	35.038
168	15.065	17.240	19.569	22.078	24.797	27.771	31.081	34.909
169	15.134	17.306	19.626	22.117	24.811	27.750	31.014	34.779
170	15.203	17.372	19.681	22.155	24.823	27.728	30.946	34.650
171	15.273	17.436	19.735	22.191	24.833	27.704	30.877	34.522
172	15.342	17.501	19.788	22.226	24.842	27.679	30.808	34.394
173	15.411	17.563	19.839	22.259	24.850	27.653	30.739	34.268
174	15.480	17.626	19.890	22.290	24.860	27.630	30.670	34.150
175	15.479	17.625	19.888	22.290	24.856	27.627	30.671	34.144
176	15.546	17.685	19.936	22.320	24.862	27.601	30.603	34.022
177	15.611	17.743	19.982	22.348	24.866	27.574	30.537	33.902
178	15.673	17.799	20.026	22.375	24.870	27.548	30.472	33.785
179	15.734	17.853	20.069	22.401	24.874	27.522	30.409	33.672
180	15.793	17.905	20.110	22.426	24.877	27.497	30.347	33.562
181	15.849	17.954	20.149	22.450	24.880	27.473	30.288	33.455
182	15.903	18.002	20.187	22.473	24.883	27.450	30.231	33.353
183	15.954	18.048	20.223	22.495	24.886	27.428	30.176	33.254
184	16.003	18.091	20.257	22.516	24.889	27.407	30.124	33.160
185	16.050	18.133	20.290	22.536	24.892	27.387	30.073	33.068
186	16.094	18.172	20.321	22.556	24.895	27.368	30.025	32.981
187	16.136	18.210	20.351	22.574	24.898	27.350	29.980	32.898
188	16.176	18.246	20.380	22.592	24.901	27.333	29.936	32.817
189	16.214	18.280	20.407	22.609	24.904	27.317	29.894	32.740
190	16.250	18.313	20.434	22.626	24.906	27.301	29.854	32.666
191	16.285	18.344	20.459	22.642	24.910	27.287	29.818	32.595
192	16.318	18.374	20.483	22.657	24.913	27.273	29.780	32.527
193	16.348	18.401	20.505	22.671	24.915	27.260	29.748	32.464
194	16.379	18.429	20.528	22.686	24.918	27.248	29.712	32.400
195	16.406	18.454	20.548	22.699	24.921	27.236	29.681	32.342
196	16.433	18.478	20.568	22.712	24.924	27.225	29.652	32.285
197	16.459	18.502	20.587	22.724	24.927	27.215	29.623	32.230
198	16.483	18.524	20.605	22.736	24.929	27.204	29.595	32.178
199	16.507	18.546	20.623	22.747	24.932	27.194	29.568	32.127
200	16.530	18.567	20.640	22.759	24.935	27.185	29.542	32.075

Table 86

FIG. 110

	Label							
	0	1	2	3	4	5	6	7
Design #								
201	-32.030	-29.518	-27.177	-24.937	-22.770	-20.656	-18.586	-16.551
202	-31.991	-29.497	-27.159	-24.939	-22.778	-20.670	-18.603	-16.570
203	-31.943	-29.472	-27.160	-24.941	-22.788	-20.686	-18.623	-16.592
204	-31.897	-29.450	-27.153	-24.944	-22.799	-20.701	-18.641	-16.611
205	-31.863	-29.431	-27.146	-24.946	-22.808	-20.714	-18.656	-16.627
206	-31.831	-29.416	-27.141	-24.948	-22.814	-20.724	-18.668	-16.641
207	-31.803	-29.401	-27.135	-24.950	-22.821	-20.734	-18.681	-16.654
208	-31.800	-29.402	-27.142	-24.960	-22.834	-20.749	-18.697	-16.670
209	-31.732	-29.365	-27.122	-24.963	-22.838	-20.757	-18.709	-16.686
210	-31.670	-29.332	-27.107	-24.961	-22.843	-20.771	-18.727	-16.706
211	-31.644	-29.307	-27.086	-24.937	-22.836	-20.774	-18.735	-16.718
212	-31.640	-29.319	-27.107	-24.959	-22.856	-20.786	-18.746	-16.727

Table #7

FIG. 111

	Label							
Design #	8	9	10	11	12	13	14	15
201	-14.545	-12.593	-10.599	-8.652	-6.717	-4.791	-2.872	-0.957
202	-14.564	-12.581	-10.617	-8.667	-6.730	-4.801	-2.878	-0.959
203	-14.587	-12.603	-10.637	-8.684	-6.743	-4.811	-2.884	-0.961
204	-14.608	-12.622	-10.655	-8.701	-6.757	-4.821	-2.891	-0.963
205	-14.622	-12.637	-10.668	-8.712	-6.766	-4.827	-2.894	-0.964
206	-14.637	-12.652	-10.681	-8.723	-6.775	-4.834	-2.899	-0.966
207	-14.650	-12.664	-10.692	-8.733	-6.784	-4.841	-2.902	-0.967
208	-14.664	-12.676	-10.701	-8.737	-6.783	-4.835	-2.893	-0.954
209	-14.683	-12.696	-10.722	-8.759	-6.804	-4.858	-2.912	-0.970
210	-14.704	-12.716	-10.741	-8.776	-6.819	-4.868	-2.922	-0.978
211	-14.717	-12.731	-10.757	-8.793	-6.836	-4.884	-2.936	-0.990
212	-14.724	-12.735	-10.758	-8.790	-6.830	-4.875	-2.923	-0.974

Table #8

FIG. 112

	Label							
Design #	16	17	18	19	20	21	22	23
201	0.957	2.872	4.791	6.717	8.652	10.599	12.563	14.545
202	0.959	2.878	4.801	6.730	8.667	10.617	12.581	14.564
203	0.961	2.884	4.811	6.743	8.684	10.637	12.603	14.587
204	0.963	2.891	4.821	6.757	8.701	10.655	12.622	14.606
205	0.964	2.894	4.827	6.766	8.712	10.668	12.637	14.622
206	0.966	2.899	4.834	6.775	8.723	10.681	12.652	14.637
207	0.967	2.902	4.841	6.784	8.733	10.692	12.664	14.650
208	0.964	2.922	4.862	6.807	8.755	10.718	12.689	14.674
209	0.970	2.912	4.859	6.804	8.759	10.722	12.698	14.683
210	0.966	2.910	4.857	6.808	8.765	10.729	12.704	14.692
211	0.956	2.904	4.855	6.809	8.769	10.736	12.712	14.701
212	0.974	2.923	4.875	6.830	8.790	10.755	12.735	14.724

Table 69

FIG. 113

	Label							
	24	25	26	27	28	29	30	31
Design #								
201	16.551	18.586	20.650	22.770	24.937	27.177	29.518	32.030
202	16.570	18.603	20.670	22.778	24.939	27.169	29.497	31.991
203	16.592	18.623	20.686	22.788	24.941	27.160	29.472	31.943
204	16.611	18.641	20.701	22.799	24.945	27.153	29.450	31.897
205	16.627	18.656	20.714	22.808	24.946	27.146	29.431	31.863
206	16.641	18.668	20.724	22.814	24.948	27.141	29.416	31.831
207	16.654	18.681	20.734	22.821	24.950	27.135	29.401	31.803
208	16.676	18.699	20.746	22.825	24.943	27.113	29.357	31.723
209	16.686	18.709	20.757	22.836	24.953	27.122	29.365	31.732
210	16.697	18.720	20.768	22.845	24.959	27.124	29.362	31.726
211	16.706	18.729	20.775	22.850	24.962	27.127	29.366	31.729
212	16.727	18.746	20.788	22.856	24.959	27.107	29.318	31.639

Table 98

FIG. 114

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
1	-5	0.97	0.74	0.63	0.53	0.41	0
2	-4.8	1.02	0.78	0.67	0.57	0.43	0
3	-4.6	0.86	0.73	0.62	0.53	0.45	0
4	-4.4	0.76	0.65	0.55	0.47	0.4	0
5	-4.2	1.03	0.88	0.75	0.63	0.54	0
6	-4	1	0.85	0.72	0.61	0.52	0
7	-3.8	1.16	0.9	0.77	0.65	0.56	0
8	-3.6	1.14	0.97	0.83	0.7	0.6	0
9	-3.4	1.19	1.01	0.86	0.73	0.62	0
10	-3.2	1.41	1.2	1.02	0.87	0.66	0
11	-3	1.33	1.13	0.96	0.81	0.62	0
12	-2.8	1.49	1.14	0.97	0.82	0.63	0
13	-2.6	1.43	1.22	1.03	0.88	0.75	0
14	-2.4	1.74	1.48	1.26	1.07	0.91	0
15	-2.2	1.44	1.23	1.04	0.89	0.75	0
16	-2	1.45	1.23	1.05	0.89	0.76	0
17	-1.8	1.81	1.54	1.31	1.11	0.85	0
18	-1.6	1.87	1.59	1.35	1.15	0.88	0
19	-1.4	1.82	1.55	1.32	1.12	0.96	0
20	-1.2	2.01	1.7	1.45	1.23	1.05	0
21	-1	2.01	1.71	1.45	1.23	0.94	0
22	-0.8	2.03	1.73	1.47	1.25	1.06	0
23	-0.6	2.04	1.73	1.47	1.25	1.06	0
24	-0.4	2.27	1.93	1.64	1.39	1.18	0
25	-0.2	2.26	1.93	1.64	1.39	1.06	0
26	0	2.27	1.93	1.64	1.39	1.18	0
27	0	2.27	1.93	1.64	1.39	1.18	0
28	0.2	2.28	1.95	1.66	1.41	1.2	0
29	0.4	2.83	2.4	2.04	1.74	1.48	0
30	0.6	2.55	2.16	1.84	1.56	1.33	0
31	0.8	2.54	2.16	1.84	1.56	1.19	0
32	1	3.26	2.77	2.35	2	1.53	0
33	1.2	3.29	2.79	2.37	2.02	1.72	0
34	1.4	3.3	2.8	2.38	2.02	1.55	0
35	1.6	2.97	2.53	2.15	1.83	1.56	0
36	1.8	3.31	2.82	2.39	2.03	1.73	0
37	2	3.69	3.13	2.66	2.26	1.73	0
38	2.2	3.68	2.83	2.4	2.04	1.74	0
39	2.4	3.84	3.26	2.77	2.36	1.8	0
40	2.6	3.89	3.31	2.81	2.39	1.83	0

Table 91

FIG. 115

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
41	2.8	3.51	2.99	2.53	2.15	1.65	0
42	3	3.49	2.97	2.52	2.16	1.54	0
43	3.2	3.86	3.28	2.79	2.37	1.93	0
44	3.4	3.48	2.96	2.51	2.14	1.63	0
45	3.6	3.48	2.96	2.51	2.14	1.63	0
46	3.8	3.48	2.96	2.51	2.14	1.64	0
47	4	3.63	3.08	2.62	2.23	1.7	0
48	4.2	3.67	3.12	2.65	2.26	1.72	0
49	4.4	3.5	2.98	2.53	2.15	1.66	0
50	4.6	3.45	2.94	2.5	2.12	1.62	0
51	4.78	3.72	3.16	2.68	2.28	1.75	0
52	4.8	3.64	3.1	2.63	2.24	1.71	0
53	5	3.42	2.9	2.47	2.1	1.6	0
54	5.2	2.82	2.4	2.04	1.73	1.47	0
55	5.4	2.27	1.93	1.64	1.4	1.19	0
56	5.6	1.77	1.5	1.28	1.08	0.92	0
57	5.8	2.71	2.31	1.96	1.67	1.42	0
58	6	2.49	2.07	1.76	1.49	1.27	0
59	6.2	3.53	3	2.55	2.17	1.66	0
60	6.4	3.58	3.04	2.58	2.2	1.68	0
61	6.6	3.8	3.23	2.74	2.33	1.78	0
62	6.8	3.87	3.29	2.8	2.38	1.64	0
63	7	3.49	2.97	2.52	2.14	1.82	0
64	7.2	3.5	2.97	2.53	2.15	1.83	0
65	7.4	3.16	2.68	2.28	1.94	1.65	0
66	7.6	3.69	3.14	2.67	2.27	1.73	0
67	7.8	3.33	2.83	2.41	2.04	1.74	0
68	8	3.75	3.19	2.71	2.3	1.76	0
69	8.2	3.77	3.2	2.72	2.32	1.59	0
70	8.4	3.55	3.02	2.57	2.18	1.85	0
71	8.46	3.56	3.02	2.57	2.18	1.86	0
72	8.6	3.54	3.01	2.56	2.17	1.85	0
73	8.8	3.54	3.01	2.56	2.17	1.86	0
74	9	3.72	3.16	2.68	2.28	1.75	0
75	9.2	3.35	2.85	2.42	2.06	1.75	0
76	9.4	3.36	2.85	2.43	2.06	1.76	0
77	9.6	3.36	2.86	2.43	2.06	1.76	0
78	9.8	3.55	3.01	2.56	2.18	1.67	0
79	10	3.56	3.03	2.57	2.19	1.57	0
80	10.2	3.57	3.04	2.58	2.19	1.68	0

Table 92

FIG. 116

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
81	10.4	2.77	2.35	2	1.7	1.44	0
82	10.6	2.83	2.41	2.05	1.74	1.48	0
83	10.8	2.87	2.44	2.07	1.76	1.5	0
84	11	2.76	2.35	1.99	1.7	1.44	0
85	11.2	2.17	1.84	1.57	1.33	1.13	0
86	11.4	2.35	2	1.7	1.44	1.23	0
87	11.6	2.22	1.89	1.6	1.36	1.16	0
88	11.8	2.34	1.99	1.69	1.43	1.22	0
89	11.83	2.33	1.98	1.69	1.43	1.22	0
90	12	2.34	1.99	1.69	1.44	1.22	0
91	12.2	2.34	1.99	1.69	1.44	1.22	0
92	12.4	2.43	2.06	1.76	1.49	1.27	0
93	12.6	2.25	1.92	1.63	1.38	1.18	0
94	12.8	2.26	1.92	1.63	1.39	1.19	0
95	13	2.26	1.92	1.63	1.39	1.18	0
96	13.2	1.96	1.67	1.42	1.21	1.03	0
97	13.4	1.81	1.54	1.31	1.11	0.94	0
98	13.6	1.83	1.56	1.32	1.12	0.95	0
99	13.8	1.64	1.39	1.18	1	0.86	0
100	14	1.39	1.18	1.01	0.86	0.73	0
101	14.2	1.42	1.21	1.03	0.87	0.74	0
102	14.4	1.44	1.23	1.04	0.89	0.75	0
103	14.6	1.44	1.22	1.04	0.88	0.75	0
104	14.8	1.45	1.23	1.04	0.89	0.75	0
105	15	1.31	1.12	0.95	0.81	0.69	0
106	15.05	1.29	1.09	0.93	0.79	0.67	0
107	15.2	0.93	0.79	0.67	0.57	0.48	0
108	15.4	0.98	0.83	0.71	0.6	0.51	0
109	15.6	0.79	0.67	0.57	0.48	0.41	0
110	15.8	0.84	0.72	0.61	0.52	0.44	0
111	16	0.84	0.71	0.61	0.52	0.44	0
112	16.2	0.86	0.72	0.61	0.52	0.44	0
113	16.4	0.85	0.72	0.61	0.52	0.44	0
114	16.6	0.86	0.72	0.61	0.52	0.44	0
115	16.8	0.85	0.72	0.61	0.52	0.44	0
116	17	0.85	0.72	0.62	0.52	0.45	0
117	17.2	0.85	0.72	0.62	0.52	0.44	0
118	17.4	0.74	0.63	0.54	0.45	0.39	0
119	17.6	0.75	0.64	0.54	0.46	0.39	0
120	17.8	0.75	0.67	0.57	0.48	0.41	0

Table 93

FIG. 117

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
121	18	0.7	0.6	0.51	0.43	0.37	0
122	18.2	0.74	0.63	0.54	0.46	0.39	0
123	18.23	0.67	0.57	0.49	0.41	0.35	0
124	18.4	0.69	0.59	0.5	0.43	0.36	0
125	18.6	0.72	0.61	0.52	0.44	0.37	0
126	18.8	0.75	0.64	0.54	0.46	0.39	0
127	19	0.79	0.67	0.57	0.48	0.41	0
128	19.2	0.82	0.7	0.6	0.51	0.43	0
129	19.4	0.86	0.73	0.62	0.53	0.45	0
130	19.6	0.89	0.76	0.65	0.55	0.47	0
131	19.8	0.93	0.79	0.67	0.57	0.48	0
132	20	0.96	0.82	0.7	0.59	0.5	0
133	20.2	1	0.85	0.72	0.61	0.52	0
134	20.4	1.03	0.89	0.75	0.63	0.54	0
135	20.6	1.07	0.91	0.77	0.65	0.56	0
136	20.8	0.99	0.84	0.71	0.61	0.52	0
137	21	1.14	0.97	0.82	0.7	0.52	0
138	21.2	1.07	0.91	0.77	0.65	0.56	0
139	21.4	1	0.85	0.72	0.61	0.52	0
140	21.42	1	0.85	0.73	0.62	0.52	0
141	21.6	1.04	0.88	0.75	0.64	0.49	0
142	21.8	0.97	0.83	0.7	0.6	0.51	0
143	22	1.01	0.86	0.73	0.62	0.47	0
144	22.2	0.94	0.8	0.68	0.58	0.44	0
145	22.4	0.87	0.74	0.63	0.54	0.41	0
146	22.6	0.9	0.77	0.65	0.55	0.42	0
147	22.8	0.84	0.71	0.61	0.52	0.44	0
148	23	0.78	0.66	0.56	0.48	0.41	0
149	23.2	0.8	0.68	0.58	0.49	0.38	0
150	23.4	0.74	0.63	0.54	0.46	0.35	0
151	23.6	0.76	0.65	0.55	0.47	0.36	0
152	23.8	0.7	0.6	0.51	0.43	0.37	0
153	24	0.65	0.55	0.47	0.4	0.34	0
154	24.2	0.66	0.56	0.48	0.41	0.31	0
155	24.4	0.61	0.52	0.44	0.38	0.29	0
156	24.6	0.56	0.48	0.41	0.35	0.26	0
157	24.69	0.57	0.48	0.41	0.35	0.3	0
158	24.8	0.58	0.49	0.42	0.36	0.27	0
159	25	0.54	0.45	0.39	0.33	0.28	0
160	25.2	0.49	0.42	0.36	0.3	0.26	0

Table 94

FIG. 118

Design #	SNPs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
161	25.4	0.51	0.43	0.37	0.31	0.26	0
162	25.6	0.47	0.4	0.34	0.29	0.24	0
163	25.8	0.46	0.41	0.35	0.29	0.22	0
164	26	0.44	0.37	0.32	0.27	0.23	0
165	26.2	0.41	0.34	0.29	0.25	0.21	0
166	26.4	0.41	0.35	0.3	0.25	0.21	0
167	26.6	0.4	0.34	0.29	0.24	0.19	0
168	26.8	0.35	0.3	0.25	0.21	0.18	0
169	27	0.34	0.29	0.24	0.21	0.18	0
170	27.2	0.32	0.28	0.23	0.2	0.17	0
171	27.4	0.31	0.27	0.23	0.19	0.16	0
172	27.6	0.27	0.23	0.2	0.17	0.14	0
173	27.8	0.26	0.22	0.19	0.16	0.14	0
174	28	0.25	0.21	0.18	0.15	0.13	0
175	28.19	0.24	0.2	0.17	0.15	0.13	0
176	28.2	0.24	0.2	0.17	0.15	0.13	0
177	28.4	0.23	0.2	0.17	0.14	0.12	0
178	28.6	0.22	0.19	0.16	0.14	0.12	0
179	28.8	0.21	0.18	0.15	0.13	0.11	0
180	29	0.2	0.17	0.15	0.12	0.1	0
181	29.2	0.17	0.15	0.13	0.11	0.09	0
182	29.4	0.17	0.14	0.12	0.1	0.09	0
183	29.6	0.16	0.14	0.12	0.1	0.08	0
184	29.8	0.15	0.13	0.11	0.09	0.08	0
185	30	0.15	0.12	0.11	0.09	0.08	0
186	30.2	0.14	0.12	0.1	0.09	0.07	0
187	30.4	0.13	0.11	0.1	0.08	0.07	0
188	30.6	0.13	0.11	0.09	0.08	0.07	0
189	30.8	0.12	0.1	0.09	0.07	0.06	0
190	31	0.12	0.1	0.08	0.07	0.05	0
191	31.2	0.1	0.08	0.07	0.06	0.05	0
192	31.4	0.1	0.08	0.07	0.06	0.05	0
193	31.6	0.09	0.08	0.07	0.06	0.05	0
194	31.8	0.09	0.07	0.06	0.05	0.05	0
195	32	0.08	0.07	0.06	0.05	0.04	0
196	32.2	0.08	0.07	0.06	0.05	0.04	0
197	32.4	0.08	0.07	0.06	0.05	0.04	0
198	32.6	0.07	0.06	0.05	0.04	0.04	0
199	32.8	0.07	0.06	0.05	0.04	0.04	0
200	33	0.07	0.06	0.05	0.04	0.03	0

Table 95

FIG. 119

Design #	SNRs	5.00%	15.00%	30.00%	45.00%	60.00%	100.00%
201	33.2	0.06	0.05	0.05	0.04	0.03	0
202	33.4	0.06	0.05	0.04	0.04	0.03	0
203	33.6	0.06	0.05	0.04	0.04	0.03	0
204	33.8	0.06	0.05	0.04	0.03	0.03	0
205	34	0.05	0.04	0.04	0.03	0.03	0
206	34.2	0.05	0.04	0.04	0.03	0.03	0
207	34.4	0.05	0.04	0.04	0.03	0.03	0
208	34.6	0.05	0.04	0.03	0.03	0.02	0
209	34.8	0.05	0.04	0.03	0.03	0.02	0
210	35	0.04	0.04	0.03	0.03	0.02	0
211	35.2	0.04	0.04	0.03	0.03	0.02	0
212	35.4	0.04	0.03	0.03	0.02	0.02	0
213	35.6	0.04	0.03	0.03	0.02	0.02	0

Table 96

FIG. 120

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
1	-5.0000	0.1977	0.1571	0.0407	25.9021
2	-4.8000	0.2058	0.1637	0.0420	25.6676
3	-4.6000	0.2141	0.1707	0.0434	25.4273
4	-4.4000	0.2227	0.1779	0.0448	25.1813
5	-4.2000	0.2316	0.1854	0.0462	24.9291
6	-4.0000	0.2407	0.1931	0.0476	24.6715
7	-3.8000	0.2502	0.2011	0.0491	24.4072
8	-3.6000	0.2600	0.2094	0.0505	24.1364
9	-3.4000	0.2700	0.2180	0.0520	23.8588
10	-3.2000	0.2804	0.2269	0.0535	23.5744
11	-3.0000	0.2910	0.2361	0.0550	23.2828
12	-2.8000	0.3020	0.2456	0.0564	22.9836
13	-2.6000	0.3133	0.2554	0.0579	22.6764
14	-2.4000	0.3248	0.2655	0.0594	22.3611
15	-2.2000	0.3367	0.2759	0.0608	22.0374
16	-2.0000	0.3489	0.2867	0.0622	21.7045
17	-1.8000	0.3613	0.2977	0.0636	21.3620
18	-1.6000	0.3741	0.3092	0.0650	21.0097
19	-1.4000	0.3872	0.3208	0.0663	20.6467
20	-1.2000	0.4005	0.3330	0.0675	20.2727
21	-1.0000	0.4141	0.3454	0.0687	19.8868
22	-0.8000	0.4280	0.3582	0.0698	19.4885
23	-0.6000	0.4421	0.3713	0.0708	19.0766
24	-0.4000	0.4565	0.3847	0.0718	18.6515
25	-0.2000	0.4711	0.3985	0.0726	18.2112
26	0.0000	0.4859	0.4127	0.0733	17.7550
27	0.1871	0.5	0.4262	0.0738	17.3143
28	0.2000	0.5010	0.4272	0.0738	17.2830
29	0.4000	0.5162	0.4420	0.0742	16.7936
30	0.6000	0.5316	0.4571	0.0744	16.2853
31	0.8000	0.5471	0.4726	0.0745	15.7577
32	1.0000	0.5628	0.4885	0.0743	15.2096
33	1.2000	0.5786	0.5047	0.0739	14.6402
34	1.4000	0.5944	0.5212	0.0732	14.0480
35	1.6000	0.6104	0.5380	0.0724	13.4304
36	1.8000	0.6270	0.5552	0.0718	12.9362
37	2.0000	0.6442	0.5727	0.0715	12.4812
38	2.2000	0.6619	0.5905	0.0714	12.0896
39	2.4000	0.6802	0.6087	0.0716	11.7577
40	2.6000	0.6991	0.6271	0.0720	11.4812

Table 97

FIG. 121

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
41	2.8000	0.7186	0.6459	0.0727	11.2561
42	3.0000	0.7385	0.6650	0.0737	11.0780
43	3.2000	0.7593	0.6844	0.0749	10.9425
44	3.4000	0.7804	0.7041	0.0764	10.8448
45	3.6000	0.8021	0.7241	0.0781	10.7804
46	3.8000	0.8243	0.7443	0.0800	10.7445
47	4.0000	0.8470	0.7649	0.0821	10.7322
48	4.2000	0.8702	0.7858	0.0844	10.7388
49	4.4000	0.8938	0.8070	0.0868	10.7590
50	4.6000	0.9178	0.8284	0.0894	10.7862
51	4.8000	0.9421	0.8501	0.0920	10.8216
52	5.0000	0.9668	0.8721	0.0947	10.8545
53	5.2000	0.9917	0.8944	0.0973	10.8828
54	5.2657	1.0000	0.9018	0.0982	10.8905
55	5.4000	1.0169	0.9170	0.1000	10.9024
56	5.6000	1.0423	0.9398	0.1025	10.9096
57	5.8000	1.0679	0.9628	0.1050	10.9012
58	6.0000	1.0934	0.9862	0.1072	10.8741
59	6.2000	1.1191	1.0098	0.1093	10.6262
60	6.4000	1.1448	1.0336	0.1112	10.7558
61	6.6000	1.1704	1.0577	0.1126	10.8614
62	6.8000	1.1960	1.0820	0.1141	10.5429
63	7.0000	1.2210	1.1065	0.1145	10.3491
64	7.2000	1.2477	1.1312	0.1164	10.2925
65	7.4000	1.2747	1.1562	0.1185	10.2474
66	7.6000	1.3026	1.1814	0.1212	10.2619
67	7.8000	1.3306	1.2067	0.1239	10.2665
68	8.0000	1.3587	1.2323	0.1264	10.2601
69	8.2000	1.3869	1.2580	0.1288	10.2420
70	8.4000	1.4150	1.2839	0.1311	10.2121
71	8.6000	1.4432	1.3100	0.1332	10.1702
72	8.8000	1.4714	1.3362	0.1352	10.1178
73	8.9903	1.4984	1.3613	0.1371	10.0694
74	9.0000	1.4999	1.3626	0.1373	10.0734
75	9.2000	1.5300	1.3892	0.1409	10.1412
76	9.4000	1.5600	1.4159	0.1442	10.1840
77	9.6000	1.5899	1.4427	0.1472	10.2025
78	9.8000	1.6195	1.4697	0.1499	10.1975
79	10.0000	1.6491	1.4968	0.1523	10.1750
80	10.2000	1.6786	1.5240	0.1546	10.2052

Table 98

FIG. 122

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
81	10.4000	1.7104	1.5514	0.1590	10.2461
82	10.6000	1.7413	1.5790	0.1623	10.2771
83	10.8000	1.7721	1.6067	0.1654	10.2967
84	11.0000	1.8037	1.6345	0.1692	10.3523
85	11.2000	1.8362	1.6625	0.1737	10.4487
86	11.4000	1.8686	1.6906	0.1779	10.5249
87	11.6000	1.9009	1.7189	0.1819	10.5824
88	11.8000	1.9330	1.7474	0.1856	10.6235
89	12.0000	1.9652	1.7760	0.1892	10.8515
90	12.2000	1.9974	1.8046	0.1926	10.5705
91	12.2081	1.9987	1.8060	0.1927	10.6711
92	12.4000	2.0297	1.8338	0.1959	10.6843
93	12.6000	2.0621	1.8629	0.1993	10.6962
94	12.8000	2.0947	1.8921	0.2026	10.7072
95	13.0000	2.1275	1.9215	0.2059	10.7170
96	13.2000	2.1604	1.9511	0.2093	10.7247
97	13.4000	2.1934	1.9808	0.2125	10.7283
98	13.6000	2.2264	2.0107	0.2157	10.7258
99	13.8000	2.2594	2.0407	0.2187	10.7156
100	14.0000	2.2924	2.0709	0.2215	10.6966
101	14.2000	2.3253	2.1011	0.2242	10.6682
102	14.4000	2.3581	2.1315	0.2266	10.6302
103	14.6000	2.3908	2.1620	0.2288	10.5831
104	14.8000	2.4234	2.1926	0.2308	10.5273
105	15.0000	2.4559	2.2233	0.2326	10.4639
106	15.2000	2.4884	2.2541	0.2343	10.3941
107	15.2711	2.4999	2.2651	0.2348	10.3676
108	15.4000	2.5207	2.2850	0.2357	10.3170
109	15.6000	2.5530	2.3160	0.2370	10.2322
110	15.8000	2.5851	2.3471	0.2380	10.1392
111	16.0000	2.6171	2.3783	0.2387	10.0379
112	16.2000	2.6489	2.4097	0.2392	9.9289
113	16.4000	2.6806	2.4411	0.2395	9.8122
114	16.6000	2.7122	2.4727	0.2396	9.6892
115	16.8000	2.7438	2.5043	0.2394	9.5605
116	17.0000	2.7756	2.5382	0.2394	9.4386
117	17.2000	2.8075	2.5682	0.2393	9.3185
118	17.4000	2.8392	2.6003	0.2389	9.1891
119	17.6000	2.8709	2.6326	0.2383	9.0518
120	17.8000	2.9025	2.6651	0.2374	8.9076

Table 99

FIG. 123

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
121	18.0000	2.9341	2.6977	0.2364	8.7621
122	18.2000	2.9057	2.7306	0.2351	8.6105
123	18.4000	2.9073	2.7637	0.2336	8.4532
124	18.4172	3.0000	2.7665	0.2335	8.4394
125	18.6000	3.0288	2.7969	0.2319	8.2922
126	18.8000	3.0605	2.8304	0.2301	8.1301
127	19.0000	3.0923	2.8640	0.2282	7.9695
128	19.2000	3.1243	2.8979	0.2264	7.8128
129	19.4000	3.1566	2.9320	0.2246	7.6606
130	19.6000	3.1891	2.9662	0.2228	7.5114
131	19.8000	3.2216	3.0007	0.2209	7.3626
132	20.0000	3.2543	3.0353	0.2189	7.2125
133	20.2000	3.2870	3.0701	0.2169	7.0633
134	20.4000	3.3197	3.1051	0.2146	6.9116
135	20.6000	3.3523	3.1402	0.2121	6.7543
136	20.8000	3.3847	3.1754	0.2093	6.5913
137	21.0000	3.4170	3.2108	0.2062	6.4231
138	21.2000	3.4491	3.2462	0.2029	6.2502
139	21.4000	3.4811	3.2817	0.1993	6.0734
140	21.5191	3.5000	3.3029	0.1971	5.9667
141	21.6000	3.5128	3.3173	0.1955	5.8936
142	21.8000	3.5445	3.3530	0.1915	5.7115
143	22.0000	3.5760	3.3887	0.1873	5.5275
144	22.2000	3.6073	3.4244	0.1829	5.3421
145	22.4000	3.6385	3.4601	0.1784	5.1557
146	22.6000	3.6695	3.4958	0.1737	4.9688
147	22.8000	3.7003	3.5314	0.1689	4.7819
148	23.0000	3.7310	3.5670	0.1639	4.5957
149	23.2000	3.7615	3.6026	0.1589	4.4107
150	23.4000	3.7919	3.6381	0.1538	4.2274
151	23.6000	3.8221	3.6734	0.1486	4.0462
152	23.8000	3.8522	3.7087	0.1434	3.8675
153	24.0000	3.8821	3.7439	0.1382	3.6919
154	24.2000	3.9119	3.7789	0.1330	3.5202
155	24.4000	3.9417	3.8138	0.1279	3.3529
156	24.6000	3.9714	3.8485	0.1228	3.1909
157	24.7934	4.0000	3.8820	0.1180	3.0401
158	24.8000	4.0010	3.8831	0.1179	3.0351
159	25.0000	4.0306	3.9175	0.1131	2.8869
160	25.2000	4.0603	3.9517	0.1086	2.7487

Table 100

FIG. 124

Design #	SNR	Opt. Cap	Std. Cap	Gain [bits]	Gain %
161	25.4000	4.0904	3.9858	0.1046	2.6251
162	25.6000	4.1208	4.0196	0.1012	2.5185
163	25.8000	4.1514	4.0532	0.0982	2.4225
164	26.0000	4.1820	4.0867	0.0953	2.3319
165	26.2000	4.2124	4.1198	0.0926	2.2443
166	26.4000	4.2426	4.1529	0.0896	2.1560
167	26.6000	4.2726	4.1857	0.0867	2.0723
168	26.8000	4.3021	4.2183	0.0838	1.9866
169	27.0000	4.3315	4.2507	0.0808	1.9006
170	27.2000	4.3605	4.2828	0.0777	1.8141
171	27.4000	4.3892	4.3146	0.0745	1.7273
172	27.6000	4.4175	4.3462	0.0713	1.6404
173	27.8000	4.4454	4.3774	0.0680	1.5535
174	28.0000	4.4730	4.4083	0.0647	1.4670
175	28.1991	4.5000	4.4387	0.0613	1.3817
176	28.2000	4.5001	4.4388	0.0613	1.3813
177	28.4000	4.5268	4.4689	0.0580	1.2968
178	28.6000	4.5531	4.4985	0.0546	1.2139
179	28.8000	4.5789	4.5278	0.0513	1.1330
180	29.0000	4.6042	4.5561	0.0480	1.0544
181	29.2000	4.6289	4.5840	0.0449	0.9784
182	29.4000	4.6530	4.6112	0.0417	0.9053
183	29.6000	4.6765	4.6377	0.0387	0.8352
184	29.8000	4.6993	4.6635	0.0358	0.7682
185	30.0000	4.7214	4.6883	0.0330	0.7046

Table 18J

FIG. 125

	Label							
	0	1	2	3	4	5	6	7
Design #								
1	-18.468	-18.468	-18.468	-18.468	-18.466	-18.466	-18.468	-18.468
2	-18.487	-18.487	-18.464	-18.464	-18.463	-18.463	-18.464	-18.463
3	-18.502	-18.486	-18.486	-18.486	-18.486	-18.465	-18.486	-18.486
4	-18.502	-18.480	-18.480	-18.480	-18.460	-18.460	-18.463	-18.460
5	-18.513	-18.513	-18.513	-18.513	-18.443	-18.464	-18.513	-18.468
6	-18.472	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
7	-18.519	-18.483	-18.483	-18.483	-18.468	-18.466	-18.483	-18.474
8	-18.478	-18.478	-18.472	-18.472	-18.472	-18.472	-18.472	-18.472
9	-18.494	-18.466	-18.464	-18.465	-18.464	-18.464	-18.464	-18.464
10	-18.477	-18.477	-18.475	-18.475	-18.474	-18.474	-18.475	-18.475
11	-18.489	-18.478	-18.478	-18.478	-18.475	-18.475	-18.478	-18.478
12	-18.509	-18.479	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465
13	-18.540	-18.529	-18.451	-18.527	-18.451	-18.451	-18.451	-18.451
14	-18.529	-18.518	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
15	-18.506	-18.505	-18.494	-18.505	-18.444	-18.480	-18.494	-18.480
16	-18.529	-18.504	-18.461	-18.461	-18.459	-18.461	-18.461	-18.461
17	-18.471	-18.470	-18.470	-18.470	-18.470	-18.470	-18.470	-18.470
18	-18.506	-18.505	-18.468	-18.468	-18.465	-18.468	-18.468	-18.468
19	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476	-18.476
20	-18.482	-18.482	-18.482	-18.482	-18.454	-18.482	-18.482	-18.482
21	-18.474	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466	-18.466
22	-18.478	-18.474	-18.474	-18.474	-18.464	-18.474	-18.474	-18.474
23	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524	-18.524
24	-18.483	-18.479	-18.479	-18.479	-18.461	-18.468	-18.479	-18.479
25	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496	-18.496
26	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467	-18.467
27	-18.474	-18.471	-18.471	-18.471	-18.471	-18.471	-18.471	-18.471
28	-18.538	-18.537	-18.537	-18.537	-18.446	-18.446	-18.446	-18.446
29	-18.499	-18.499	-18.467	-18.467	-18.466	-18.466	-18.466	-18.466
30	-18.495	-18.495	-18.495	-18.495	-18.453	-18.459	-18.486	-18.460
31	-18.494	-18.494	-18.485	-18.485	-18.485	-18.485	-18.485	-18.485
32	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528	-18.528
33	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488	-18.488
34	-18.536	-18.535	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464
35	-20.584	-20.684	-20.584	-20.584	-20.584	-20.684	-20.584	-20.584
36	-21.574	-21.574	-21.574	-21.574	-21.573	-21.573	-21.574	-21.573
37	-22.222	-22.222	-22.222	-22.222	-22.220	-22.222	-22.222	-22.222
38	-22.705	-22.705	-22.704	-22.705	-22.703	-22.703	-22.703	-22.703
39	-23.088	-23.087	-23.087	-23.087	-23.087	-23.087	-23.087	-23.087
40	-23.404	-23.404	-23.404	-23.404	-23.402	-23.404	-23.404	-23.404

Table 102

FIG. 126

	Label							
	8	9	10	11	12	13	14	15
Design #								
1	-18.462	-18.463	-18.466	-18.463	-18.466	-18.466	-18.466	-18.466
2	-18.462	-18.462	-18.463	-18.463	-18.463	-18.463	-18.463	-18.463
3	-18.445	-18.455	-18.466	-18.462	-18.466	-18.466	-18.466	-18.466
4	-18.459	-18.459	-18.459	-18.459	-18.460	-18.459	-18.459	-18.459
5	-18.436	-18.438	-18.438	-18.438	-18.442	-18.442	-18.442	-18.442
6	-18.460	-18.466	-18.466	-18.466	-18.460	-18.466	-18.466	-18.466
7	-18.449	-18.450	-18.450	-18.450	-18.466	-18.450	-18.450	-18.450
8	-18.391	-18.447	-18.472	-18.472	-18.472	-18.472	-18.472	-18.472
9	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464	-18.464
10	-18.444	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
11	-18.424	-18.431	-18.456	-18.450	-18.472	-18.469	-18.458	-18.468
12	-18.428	-18.463	-18.463	-18.463	-18.465	-18.465	-18.465	-18.465
13	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451	-18.451
14	-18.453	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
15	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444	-18.444
16	-18.458	-18.458	-18.456	-18.458	-18.459	-18.459	-18.458	-18.458
17	-18.415	-18.463	-18.470	-18.469	-18.470	-18.470	-18.470	-18.470
18	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456	-18.456
19	-18.455	-18.455	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457
20	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454	-18.454
21	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465	-18.465
22	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459	-18.459
23	-18.391	-18.391	-18.391	-18.391	-18.456	-18.456	-18.394	-18.394
24	-18.452	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457	-18.457
25	-18.398	-18.418	-18.420	-18.420	-18.495	-18.495	-18.422	-18.420
26	-18.465	-18.465	-18.465	-18.465	-18.467	-18.465	-18.465	-18.465
27	-18.459	-18.459	-18.459	-18.459	-18.465	-18.463	-18.462	-18.462
28	-18.408	-18.445	-18.445	-18.445	-18.445	-18.445	-18.445	-18.445
29	-18.456	-18.456	-18.457	-18.456	-18.461	-18.461	-18.458	-18.459
30	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453	-18.453
31	-18.440	-18.440	-18.440	-18.440	-18.484	-18.440	-18.440	-18.440
32	-18.389	-18.389	-18.390	-18.390	-18.419	-18.419	-18.419	-18.419
33	-18.428	-18.428	-18.430	-18.429	-18.486	-18.456	-18.449	-18.449
34	-18.450	-18.450	-18.450	-18.450	-18.452	-18.450	-18.450	-18.450
35	-16.071	-16.073	-16.073	-16.073	-16.073	-16.073	-16.073	-16.073
36	-14.716	-14.716	-14.716	-14.716	-14.722	-14.722	-14.717	-14.717
37	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720	-13.720
38	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905	-12.905
39	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206	-12.206
40	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589	-11.589

Table 103

FIG. 127

	Label							
	16	17	18	19	20	21	22	23
Design #								
1	18.471	18.471	18.466	18.466	18.468	18.466	18.468	18.468
2	18.479	18.469	18.469	18.469	18.468	18.468	18.469	18.468
3	18.502	18.502	18.497	18.502	18.481	18.497	18.497	18.497
4	18.502	18.502	18.483	18.483	18.483	18.483	18.483	18.483
5	18.481	18.481	18.481	18.481	18.481	18.481	18.481	18.481
6	18.471	18.471	18.471	18.471	18.471	18.471	18.471	18.471
7	18.509	18.468	18.468	18.468	18.468	18.468	18.468	18.468
8	18.476	18.467	18.467	18.467	18.466	18.466	18.467	18.466
9	18.496	18.496	18.496	18.496	18.496	18.496	18.496	18.496
10	18.541	18.542	18.478	18.479	18.452	18.452	18.452	18.452
11	18.489	18.468	18.467	18.467	18.465	18.465	18.467	18.465
12	18.509	18.484	18.484	18.484	18.466	18.466	18.466	18.466
13	18.540	18.469	18.469	18.469	18.469	18.469	18.469	18.469
14	18.528	18.528	18.528	18.528	18.476	18.476	18.528	18.476
15	18.505	18.464	18.464	18.464	18.464	18.464	18.464	18.464
16	18.528	18.471	18.466	18.469	18.460	18.462	18.465	18.462
17	18.543	18.544	18.456	18.456	18.455	18.455	18.455	18.455
18	18.505	18.505	18.505	18.505	18.440	18.505	18.505	18.505
19	18.541	18.542	18.471	18.471	18.453	18.453	18.453	18.453
20	18.474	18.474	18.474	18.474	18.468	18.468	18.474	18.468
21	18.477	18.477	18.467	18.474	18.466	18.466	18.467	18.466
22	18.478	18.478	18.478	18.478	18.478	18.478	18.478	18.478
23	18.524	18.491	18.491	18.491	18.491	18.491	18.491	18.491
24	18.484	18.484	18.484	18.484	18.484	18.484	18.484	18.484
25	18.483	18.483	18.474	18.474	18.474	18.474	18.474	18.474
26	18.468	18.468	18.468	18.468	18.468	18.468	18.468	18.468
27	18.470	18.468	18.468	18.468	18.466	18.468	18.468	18.468
28	18.519	18.481	18.481	18.481	18.481	18.481	18.481	18.481
29	18.500	18.474	18.474	18.474	18.474	18.474	18.474	18.474
30	18.495	18.495	18.495	18.495	18.495	18.495	18.495	18.495
31	18.483	18.481	18.481	18.481	18.481	18.481	18.481	18.481
32	18.528	18.520	18.499	18.499	18.473	18.477	18.499	18.499
33	18.488	18.485	18.485	18.485	18.485	18.485	18.485	18.485
34	18.534	18.488	18.488	18.488	18.487	18.487	18.487	18.487
35	20.584	20.583	20.583	20.583	20.583	20.583	20.583	20.583
36	21.574	21.572	21.571	21.571	21.571	21.571	21.571	21.571
37	22.218	22.218	22.218	22.218	22.208	22.209	22.218	22.217
38	22.705	22.705	22.705	22.705	22.702	22.702	22.705	22.704
39	23.088	23.088	23.088	23.088	23.088	23.088	23.088	23.088
40	23.404	23.404	23.404	23.404	23.401	23.401	23.401	23.401

Table 184

FIG. 128

	Label							
	24	25	26	27	28	29	30	31
Design #								
1	18.465	18.465	18.465	18.465	18.466	18.466	18.466	18.466
2	18.452	18.463	18.463	18.463	18.468	18.464	18.463	18.463
3	18.434	18.434	18.436	18.435	18.437	18.437	18.437	18.437
4	18.410	18.410	18.445	18.410	18.483	18.483	18.445	18.472
5	18.309	18.456	18.460	18.460	18.481	18.481	18.481	18.481
6	18.438	18.445	18.466	18.453	18.471	18.471	18.471	18.471
7	18.433	18.451	18.462	18.462	18.468	18.466	18.463	18.456
8	18.464	18.464	18.464	18.464	18.465	18.465	18.464	18.464
9	18.408	18.408	18.420	18.420	18.496	18.497	18.420	18.420
10	18.452	18.452	18.452	18.452	18.452	18.452	18.452	18.452
11	18.461	18.464	18.464	18.464	18.465	18.464	18.464	18.464
12	18.453	18.453	18.455	18.453	18.455	18.455	18.455	18.456
13	18.448	18.448	18.448	18.448	18.469	18.448	18.448	18.448
14	18.404	18.417	18.418	18.418	18.476	18.419	18.418	18.418
15	18.457	18.462	18.464	18.462	18.464	18.464	18.464	18.464
16	18.458	18.460	18.460	18.460	18.460	18.460	18.460	18.460
17	18.455	18.455	18.455	18.455	18.455	18.455	18.455	18.455
18	18.421	18.432	18.437	18.437	18.439	18.439	18.439	18.439
19	18.452	18.452	18.452	18.452	18.453	18.453	18.452	18.452
20	18.455	18.455	18.455	18.455	18.467	18.467	18.466	18.467
21	18.459	18.460	18.461	18.460	18.466	18.466	18.462	18.466
22	18.426	18.432	18.432	18.432	18.478	18.478	18.478	18.478
23	18.428	18.428	18.428	18.428	18.490	18.440	18.428	18.428
24	18.416	18.418	18.429	18.419	18.484	18.463	18.472	18.473
25	18.456	18.456	18.456	18.456	18.456	18.456	18.456	18.456
26	18.465	18.465	18.465	18.465	18.465	18.465	18.465	18.465
27	18.464	18.464	18.464	18.464	18.464	18.464	18.464	18.464
28	18.437	18.437	18.453	18.437	18.453	18.453	18.453	18.453
29	18.453	18.453	18.453	18.453	18.473	18.453	18.453	18.453
30	18.427	18.438	18.438	18.438	18.445	18.438	18.438	18.438
31	18.451	18.451	18.451	18.451	18.455	18.451	18.451	18.451
32	18.428	18.429	18.429	18.428	18.451	18.437	18.429	18.434
33	18.447	18.447	18.447	18.447	18.448	18.448	18.447	18.447
34	18.439	18.439	18.439	18.439	18.440	18.439	18.439	18.439
35	16.071	16.071	16.072	16.071	16.086	16.072	16.072	16.072
36	14.720	14.720	14.720	14.720	14.720	14.720	14.720	14.720
37	13.722	13.722	13.728	13.726	13.731	13.728	13.728	13.728
38	12.905	12.905	12.905	12.905	12.905	12.905	12.905	12.905
39	12.205	12.205	12.205	12.205	12.205	12.205	12.205	12.205
40	11.590	11.590	11.590	11.590	11.590	11.590	11.590	11.590

Table 105

FIG. 129

	Label							
	0	1	2	3	4	5	6	7
Design #								
41	-23.669	-23.669	-23.669	-23.669	-23.662	-23.662	-23.665	-23.665
42	-23.892	-23.892	-23.892	-23.892	-23.891	-23.891	-23.892	-23.891
43	-24.086	-24.086	-24.086	-24.086	-24.085	-24.085	-24.086	-24.086
44	-24.254	-24.254	-24.253	-24.254	-24.245	-24.250	-24.253	-24.253
45	-24.397	-24.397	-24.397	-24.397	-24.395	-24.395	-24.397	-24.397
46	-24.525	-24.524	-24.524	-24.524	-24.524	-24.524	-24.524	-24.524
47	-24.635	-24.635	-24.635	-24.635	-24.633	-24.634	-24.635	-24.635
48	-24.738	-24.738	-24.730	-24.738	-24.727	-24.729	-24.730	-24.730
49	-24.821	-24.821	-24.816	-24.816	-24.816	-24.816	-24.816	-24.816
50	-24.897	-24.897	-24.892	-24.897	-24.890	-24.891	-24.892	-24.892
51	-24.963	-24.960	-24.960	-24.960	-24.955	-24.960	-24.960	-24.960
52	-25.024	-25.023	-25.015	-25.015	-25.015	-25.015	-25.015	-25.015
53	-25.074	-25.074	-25.074	-25.074	-25.038	-25.041	-25.061	-25.061
54	-25.083	-25.082	-25.082	-25.082	-25.078	-25.078	-25.078	-25.078
55	-25.109	-25.109	-25.108	-25.108	-25.108	-25.108	-25.108	-25.108
56	-25.148	-25.147	-25.143	-25.144	-25.143	-25.143	-25.143	-25.143
57	-25.176	-25.174	-25.174	-25.174	-25.173	-25.173	-25.173	-25.173
58	-25.202	-25.202	-25.202	-25.202	-25.189	-25.192	-25.197	-25.195
59	-25.220	-25.220	-25.215	-25.215	-25.214	-25.214	-25.215	-25.214
60	-25.238	-25.236	-25.236	-25.236	-25.228	-25.228	-25.236	-25.228
61	-25.243	-25.243	-25.243	-25.243	-25.222	-25.222	-25.233	-25.236
62	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243	-25.243
63	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792	-24.792
64	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737	-24.737
65	-38.698	-26.007	-23.569	-23.569	-21.371	-21.371	-21.371	-21.371
66	-39.014	-25.904	-23.497	-23.497	-21.322	-21.322	-21.322	-21.322
67	-39.276	-25.788	-23.433	-23.433	-21.281	-21.281	-21.281	-21.281
68	-39.422	-25.843	-23.419	-23.419	-21.222	-21.222	-21.222	-21.222
69	-39.538	-25.889	-23.430	-23.430	-21.156	-21.156	-21.156	-21.156
70	-39.606	-26.034	-23.434	-23.434	-21.077	-21.077	-21.077	-21.077
71	-39.643	-26.177	-23.477	-23.477	-20.995	-20.995	-20.995	-20.995
72	-39.643	-26.413	-23.533	-23.533	-20.878	-20.878	-20.878	-20.878
73	-31.746	-31.746	-31.746	-31.746	-14.945	-14.945	-14.945	-14.945
74	-31.751	-31.751	-31.734	-31.746	-14.948	-14.948	-14.952	-14.949
75	-31.689	-31.689	-31.688	-31.688	-15.023	-15.023	-15.024	-15.024
76	-31.638	-31.638	-31.622	-31.622	-15.108	-15.108	-15.110	-15.110
77	-31.574	-31.574	-31.564	-31.570	-15.198	-15.198	-15.201	-15.200
78	-31.554	-31.547	-31.456	-31.456	-15.296	-15.297	-15.310	-15.310
79	-38.426	-31.170	-28.865	-28.865	-14.205	-14.568	-15.742	-15.742
80	-38.361	-30.759	-28.491	-28.491	-14.105	-14.576	-15.661	-15.661

Table 106

FIG. 130

	Label							
Design #	8	9	10	11	12	13	14	15
41	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041	-11.041
42	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545	-10.545
43	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096	-10.096
44	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688	-9.688
45	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317	-9.317
46	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977	-8.977
47	-8.668	-8.668	-8.668	-8.668	-8.669	-8.668	-8.668	-8.668
48	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385	-8.385
49	-8.128	-8.128	-8.128	-8.128	-8.129	-8.129	-8.128	-8.129
50	-7.892	-7.892	-7.892	-7.892	-7.897	-7.896	-7.896	-7.896
51	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685	-7.685
52	-7.496	-7.496	-7.496	-7.496	-7.497	-7.497	-7.496	-7.496
53	-7.337	-7.337	-7.337	-7.337	-7.346	-7.337	-7.337	-7.337
54	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280	-7.280
55	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183	-7.183
56	-7.054	-7.054	-7.057	-7.056	-7.058	-7.058	-7.057	-7.057
57	-6.949	-6.949	-6.950	-6.950	-6.951	-6.951	-6.951	-6.951
58	-6.860	-6.860	-6.860	-6.860	-6.869	-6.866	-6.860	-6.866
59	-6.789	-6.794	-6.794	-6.794	-6.797	-6.797	-6.797	-6.797
60	-6.739	-6.739	-6.739	-6.739	-6.742	-6.742	-6.741	-6.741
61	-6.715	-6.715	-6.715	-6.715	-6.718	-6.717	-6.715	-6.715
62	-6.696	-6.696	-6.696	-6.696	-6.698	-6.698	-6.698	-6.698
63	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815	-6.815
64	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812	-6.812
65	-5.930	-6.192	-6.424	-6.424	-6.878	-6.878	-6.878	-6.878
66	-5.657	-6.089	-6.361	-6.361	-6.860	-6.860	-6.860	-6.860
67	-5.810	-6.038	-6.309	-6.309	-6.851	-6.851	-6.851	-6.851
68	-5.761	-5.999	-6.255	-6.255	-6.862	-6.862	-6.862	-6.862
69	-5.707	-5.913	-6.203	-6.203	-6.880	-6.880	-6.880	-6.880
70	-5.674	-5.865	-6.164	-6.164	-6.913	-6.913	-6.913	-6.913
71	-5.641	-5.816	-6.119	-6.119	-6.957	-6.957	-6.957	-6.957
72	-5.601	-5.760	-6.071	-6.071	-7.015	-7.015	-7.015	-7.015
73	31.747	31.747	31.746	31.746	-11.518	-11.518	-11.518	-11.518
74	31.742	31.742	31.742	31.742	-11.521	-11.521	-11.521	-11.521
75	31.700	31.700	31.674	31.677	-11.571	-11.571	-11.571	-11.571
76	31.637	31.638	31.625	31.631	-11.616	-11.616	-11.616	-11.616
77	31.574	31.574	31.584	31.570	-11.658	-11.658	-11.658	-11.658
78	31.553	31.553	31.441	31.484	-11.695	-11.695	-11.695	-11.695
79	31.181	31.181	31.181	31.181	-11.375	-11.375	-11.375	-11.375
80	30.361	30.758	28.491	28.491	-11.302	-11.302	-11.302	-11.302

Table 167

FIG. 131

	Label							
	16	17	18	19	20	21	22	23
Design #								
41	23.671	23.671	23.668	23.671	23.657	23.663	23.667	23.666
42	23.892	23.892	23.892	23.892	23.891	23.891	23.891	23.891
43	24.086	24.086	24.084	24.086	24.080	24.080	24.080	24.080
44	24.254	24.254	24.254	24.254	24.249	24.249	24.249	24.249
45	24.397	24.397	24.397	24.397	24.397	24.397	24.397	24.397
46	24.526	24.526	24.526	24.526	24.516	24.517	24.526	24.523
47	24.635	24.635	24.635	24.635	24.635	24.635	24.635	24.635
48	24.735	24.735	24.731	24.735	24.731	24.731	24.731	24.731
49	24.821	24.821	24.821	24.821	24.818	24.819	24.820	24.820
50	24.896	24.895	24.895	24.895	24.891	24.891	24.894	24.891
51	24.963	24.961	24.961	24.961	24.948	24.951	24.960	24.957
52	25.021	25.020	25.013	25.013	25.012	25.012	25.012	25.012
53	25.074	25.074	25.062	25.062	25.061	25.061	25.061	25.061
54	25.081	25.081	25.081	25.081	25.079	25.079	25.079	25.079
55	25.109	25.109	25.106	25.106	25.106	25.106	25.106	25.106
56	25.148	25.148	25.143	25.148	25.138	25.138	25.142	25.140
57	25.176	25.176	25.176	25.176	25.167	25.167	25.173	25.170
58	25.203	25.203	25.203	25.203	25.185	25.185	25.203	25.194
59	25.219	25.219	25.219	25.219	25.204	25.209	25.214	25.214
60	25.230	25.230	25.226	25.230	25.225	25.225	25.225	25.225
61	25.240	25.240	25.240	25.240	25.240	25.240	25.240	25.240
62	25.243	25.243	25.243	25.243	25.228	25.237	25.243	25.243
63	38.522	26.837	24.259	24.259	21.835	21.835	21.835	21.835
64	38.430	26.613	24.103	24.103	21.747	21.747	21.747	21.747
65	38.697	25.964	23.561	23.561	21.380	21.380	21.380	21.380
66	39.013	25.893	23.500	23.500	21.319	21.319	21.319	21.319
67	39.272	25.861	23.457	23.457	21.263	21.263	21.263	21.263
68	39.422	25.861	23.430	23.430	21.215	21.215	21.215	21.215
69	39.560	25.899	23.429	23.429	21.154	21.154	21.154	21.154
70	39.625	26.010	23.440	23.440	21.079	21.079	21.079	21.078
71	39.618	26.207	23.493	23.493	20.983	20.983	20.983	20.983
72	39.638	26.474	23.540	23.540	20.865	20.865	20.865	20.865
73	11.518	11.518	11.518	11.518	0.398	0.398	0.398	0.398
74	11.522	11.522	11.522	11.522	0.402	0.402	0.403	0.403
75	11.574	11.574	11.574	11.574	0.462	0.462	0.462	0.462
76	11.619	11.619	11.619	11.619	0.528	0.528	0.528	0.528
77	11.658	11.658	11.658	11.658	0.595	0.595	0.595	0.595
78	11.692	11.692	11.692	11.692	0.665	0.665	0.665	0.665
79	11.652	11.652	11.652	11.652	0.746	0.760	0.845	0.845
80	11.303	11.303	11.303	11.303	0.716	0.717	0.717	0.717

Table 108

FIG. 132

	Label							
	24	25	26	27	28	29	30	31
Design #								
41	11.041	11.041	11.041	11.041	11.041	11.041	11.041	11.041
42	10.545	10.545	10.545	10.545	10.545	10.545	10.545	10.545
43	10.098	10.099	10.099	10.099	10.099	10.099	10.099	10.099
44	9.685	9.688	9.686	9.685	9.686	9.686	9.688	9.686
45	9.316	9.316	9.316	9.316	9.316	9.316	9.316	9.316
46	8.978	8.978	8.978	8.978	8.978	8.978	8.978	8.978
47	8.666	8.667	8.667	8.667	8.667	8.667	8.667	8.667
48	8.385	8.385	8.385	8.385	8.385	8.385	8.385	8.385
49	8.125	8.125	8.125	8.125	8.126	8.126	8.125	8.125
50	7.893	7.894	7.894	7.894	7.894	7.894	7.894	7.894
51	7.685	7.686	7.686	7.686	7.686	7.687	7.686	7.686
52	7.496	7.498	7.496	7.499	7.501	7.499	7.499	7.499
53	7.332	7.335	7.337	7.337	7.337	7.337	7.337	7.337
54	7.280	7.280	7.280	7.280	7.280	7.280	7.280	7.280
55	7.183	7.184	7.184	7.184	7.186	7.186	7.185	7.185
56	7.054	7.058	7.058	7.058	7.059	7.059	7.059	7.059
57	6.950	6.950	6.952	6.952	6.952	6.952	6.952	6.952
58	6.863	6.863	6.863	6.863	6.864	6.863	6.863	6.863
59	6.791	6.791	6.791	6.791	6.810	6.799	6.798	6.798
60	6.745	6.745	6.745	6.745	6.746	6.746	6.745	6.745
61	6.709	6.709	6.710	6.710	6.711	6.711	6.711	6.711
62	6.693	6.700	6.700	6.700	6.702	6.702	6.700	6.700
63	5.960	5.923	6.256	6.259	6.886	6.886	6.886	6.886
64	5.576	5.836	6.181	6.181	6.844	6.844	6.844	6.844
65	5.939	6.170	6.431	6.431	6.876	6.878	6.876	6.876
66	5.863	6.093	6.361	6.361	6.862	6.862	6.862	6.852
67	5.794	6.017	6.297	6.297	6.856	6.856	6.856	6.856
68	5.748	5.965	6.249	6.249	6.865	6.865	6.865	6.865
69	5.706	5.911	6.201	6.201	6.880	6.880	6.880	6.880
70	5.673	5.865	6.161	6.161	6.912	6.912	6.912	6.912
71	5.638	5.814	6.118	6.118	6.962	6.962	6.962	6.962
72	5.598	5.755	6.066	6.066	7.014	7.014	7.014	7.014
73	14.945	14.945	14.945	14.945	-0.398	-0.398	-0.398	-0.398
74	14.947	14.947	14.947	14.947	-0.397	-0.397	-0.397	-0.397
75	15.017	15.019	15.031	15.029	-0.465	-0.465	-0.466	-0.466
76	15.103	15.103	15.107	15.104	-0.528	-0.528	-0.529	-0.528
77	15.199	15.199	15.203	15.201	-0.597	-0.597	-0.597	-0.597
78	15.293	15.299	15.322	15.314	-0.667	-0.667	-0.668	-0.667
79	15.258	15.258	15.258	15.258	-0.593	-0.593	-0.593	-0.593
80	14.104	14.575	15.660	15.660	-0.716	-0.717	-0.717	-0.717

Table 169

FIG. 133

	Label							
	0	1	2	3	4	5	6	7
Design #								
81	-38.770	-30.565	-28.305	-28.305	-14.112	-14.591	-15.728	-15.728
82	-39.045	-30.428	-28.150	-28.150	-14.151	-14.629	-15.813	-15.813
83	-39.237	-30.332	-28.008	-28.008	-14.213	-14.681	-15.922	-15.922
84	-42.842	-27.881	42.842	-25.702	29.062	-24.594	31.203	-24.594
85	-42.855	-27.882	42.853	-25.687	29.031	-24.553	31.121	-24.553
86	-42.464	-27.897	42.842	-25.674	29.001	-24.507	31.047	-24.507
87	-42.259	-27.918	42.803	-25.658	28.972	-24.450	30.979	-24.450
88	-42.044	-27.950	42.732	-25.640	28.944	-24.384	30.918	-24.384
89	-41.819	-27.996	42.639	-25.619	28.920	-24.309	30.866	-24.309
90	-41.584	-28.049	42.521	-25.594	28.899	-24.226	30.820	-24.226
91	-41.354	-28.060	42.517	-25.592	28.899	-24.218	30.819	-24.218
92	-41.313	-28.130	42.382	-25.565	28.883	-24.131	30.781	-24.131
93	-41.061	-28.204	42.235	-25.546	28.871	-24.041	30.748	-24.041
94	-40.815	-28.281	42.094	-25.529	28.867	-23.954	30.725	-23.954
95	-40.588	-28.384	41.955	-25.523	28.866	-23.867	30.704	-23.867
96	-40.382	-28.507	41.826	-25.531	28.868	-23.781	30.688	-23.781
97	-40.205	-28.658	41.701	-25.556	28.869	-23.693	30.674	-23.693
98	-40.054	-28.809	41.580	-25.607	28.868	-23.609	30.656	-23.609
99	-39.931	-28.990	41.461	-25.677	28.857	-23.523	30.638	-23.523
100	-39.837	-29.167	41.338	-25.777	28.842	-23.437	30.617	-23.437
101	-39.769	-29.347	41.210	-25.909	28.820	-23.348	30.594	-23.348
102	-39.729	-29.519	41.072	-26.082	28.790	-23.251	30.569	-23.251
103	-39.717	-29.676	40.928	-26.350	28.749	-23.144	30.538	-23.144
104	-39.740	-29.838	40.770	-26.653	28.695	-23.030	30.504	-23.030
105	-39.801	-30.016	40.599	-27.009	28.627	-22.860	30.464	-22.947
106	-39.861	-30.186	40.421	-27.323	28.552	-22.705	30.427	-22.896
107	-39.873	-30.238	40.366	-27.420	28.529	-22.658	30.421	-22.880
108	-39.692	-30.332	40.263	-27.572	28.464	-22.588	30.408	-22.856
109	-39.907	-30.471	40.107	-27.763	28.409	-22.501	30.409	-22.830
110	-39.901	-30.592	39.964	-27.911	28.334	-22.431	30.432	-22.810
111	-39.885	-30.712	39.825	-28.026	28.255	-22.368	30.473	-22.799
112	-39.845	-30.817	39.722	-28.095	28.173	-22.304	30.566	-22.787
113	-39.795	-30.919	39.633	-28.136	28.094	-22.240	30.678	-22.778
114	-39.727	-31.016	39.585	-28.134	28.013	-22.184	30.850	-22.763
115	-39.632	-31.098	39.589	-28.108	27.930	-22.083	31.071	-22.749
116	-40.170	-32.458	-24.267	-26.000	-15.469	-15.469	-18.490	-17.922
117	-40.113	-32.436	-24.253	-26.017	-15.428	-15.428	-18.613	-18.023
118	-40.042	-32.414	-24.246	-26.045	-15.378	-15.378	-18.740	-18.141
119	-39.959	-32.389	-24.245	-26.093	-15.323	-15.323	-18.892	-18.271
120	-39.866	-32.361	-24.251	-26.129	-15.266	-15.266	-19.035	-18.407

Table 118

FIG. 134

	Label							
	8	9	10	11	12	13	14	15
Design #								
81	38.770	30.564	28.304	28.304	-11.260	-11.260	-11.260	-11.260
82	39.045	30.428	28.150	28.150	-11.220	-11.220	-11.220	-11.220
83	39.236	30.328	28.011	28.011	-11.174	-11.174	-11.174	-11.174
84	-8.991	-12.436	-8.991	-12.636	-8.991	-13.018	-8.991	-13.018
85	-8.995	-12.533	-8.995	-12.741	-8.995	-13.146	-8.995	-13.146
86	-8.991	-12.638	-8.991	-12.852	-8.991	-13.286	-8.991	-13.286
87	-8.970	-12.766	-8.970	-12.985	-8.970	-13.450	-8.970	-13.450
88	-8.931	-12.915	-8.931	-13.140	-8.931	-13.637	-8.931	-13.637
89	-8.879	-13.083	-8.879	-13.313	-8.879	-13.842	-8.879	-13.842
90	-8.810	-13.271	-8.810	-13.505	-8.810	-14.064	-8.810	-14.064
91	-8.807	-13.276	-8.807	-13.512	-8.807	-14.073	-8.807	-14.073
92	-8.729	-13.463	-8.729	-13.703	-8.729	-14.290	-8.729	-14.290
93	-8.644	-13.658	-8.644	-13.896	-8.644	-14.508	-8.644	-14.508
94	-8.563	-13.841	-8.563	-14.081	-8.563	-14.707	-8.563	-14.707
95	-8.488	-13.996	-8.488	-14.234	-8.488	-14.882	-8.488	-14.882
96	-8.419	-14.121	-8.419	-14.357	-8.419	-15.032	-8.419	-15.032
97	-8.355	-14.215	-8.355	-14.447	-8.355	-15.161	-8.355	-15.161
98	-8.299	-14.280	-8.299	-14.513	-8.299	-15.273	-8.299	-15.273
99	-8.244	-14.335	-8.244	-14.548	-8.244	-15.366	-8.244	-15.366
100	-8.199	-14.363	-8.199	-14.561	-8.185	-15.447	-8.190	-15.447
101	-8.165	-14.372	-8.165	-14.551	-8.123	-15.519	-8.124	-15.519
102	-8.133	-14.380	-8.132	-14.514	-8.059	-15.585	-8.059	-15.585
103	-8.103	-14.325	-8.101	-14.448	-7.991	-15.648	-7.991	-15.648
104	-8.070	-14.265	-8.067	-14.358	-7.916	-15.702	-7.916	-15.702
105	-8.033	-14.180	-8.031	-14.237	-7.833	-15.746	-7.833	-15.746
106	-8.001	-14.102	-7.999	-14.128	-7.755	-15.782	-7.755	-15.782
107	-7.994	-14.077	-7.990	-14.094	-7.730	-15.794	-7.730	-15.794
108	-7.980	-14.038	-7.978	-14.041	-7.687	-15.814	-7.687	-15.814
109	-7.966	-13.981	-7.963	-13.981	-7.628	-15.842	-7.628	-15.842
110	-7.959	-13.935	-7.957	-13.935	-7.578	-15.871	-7.578	-15.871
111	-7.958	-13.895	-7.957	-13.895	-7.531	-15.897	-7.531	-15.892
112	-7.967	-13.861	-7.967	-13.861	-7.491	-15.947	-7.495	-15.905
113	-7.981	-13.829	-7.981	-13.829	-7.455	-15.990	-7.459	-15.920
114	-8.002	-13.795	-8.002	-13.795	-7.423	-16.030	-7.431	-15.932
115	-8.027	-13.760	-8.027	-13.760	-7.397	-16.076	-7.403	-15.949
116	40.170	32.459	24.267	26.000	-9.724	-9.724	-8.849	-8.881
117	40.113	32.438	24.253	26.016	-9.774	-9.774	-8.798	-8.827
118	40.041	32.414	24.246	26.044	-9.829	-9.829	-8.735	-8.760
119	39.958	32.398	24.245	26.081	-9.889	-9.889	-8.662	-8.682
120	39.865	32.360	24.250	26.128	-9.954	-9.954	-8.580	-8.596

Table 111

FIG. 135

	Label							
Design #	16	17	18	19	20	21	22	23
81	11.260	11.260	11.260	11.260	0.782	0.782	0.782	0.782
82	11.219	11.219	11.219	11.219	0.855	0.855	0.855	0.855
83	11.176	11.176	11.176	11.176	0.939	0.939	0.939	0.939
84	14.917	12.616	14.917	12.616	18.770	12.616	18.440	12.616
85	15.029	12.559	15.029	12.559	18.943	12.559	18.588	12.559
86	15.158	12.498	15.158	12.498	19.110	12.498	18.752	12.498
87	15.323	12.421	15.323	12.421	19.290	12.421	18.924	12.421
88	15.528	12.328	15.528	12.328	19.470	12.328	19.102	12.328
89	15.765	12.227	15.765	12.227	19.641	12.227	19.272	12.227
90	16.040	12.111	16.040	12.111	19.803	12.111	19.434	12.111
91	16.060	12.107	16.060	12.107	19.810	12.107	19.440	12.107
92	16.334	11.986	16.334	11.986	19.950	11.986	19.582	11.988
93	16.630	11.865	16.630	11.865	20.081	11.865	19.714	11.865
94	16.906	11.750	16.906	11.750	20.189	11.750	19.819	11.750
95	17.152	11.664	17.152	11.667	20.285	11.630	19.911	11.630
96	17.361	11.607	17.361	11.606	20.372	11.494	19.990	11.494
97	17.532	11.562	17.532	11.562	20.447	11.378	20.055	11.385
98	17.667	11.531	17.667	11.531	20.512	11.289	20.106	11.300
99	17.779	11.505	17.779	11.505	20.569	11.210	20.149	11.229
100	17.870	11.489	17.870	11.489	20.621	11.149	20.185	11.172
101	17.942	11.482	17.942	11.482	20.671	11.099	20.215	11.126
102	18.000	11.481	18.000	11.481	20.721	11.059	20.244	11.089
103	18.041	11.488	18.041	11.488	20.769	11.027	20.268	11.061
104	18.070	11.499	18.070	11.499	20.816	11.000	20.288	11.037
105	18.084	11.513	18.084	11.513	20.863	10.976	26.303	11.017
106	18.089	11.536	18.089	11.536	20.917	10.958	20.324	11.000
107	18.091	11.544	18.091	11.544	20.940	10.950	20.331	10.993
108	18.090	11.562	18.090	11.562	20.986	10.937	20.352	10.981
109	18.081	11.593	18.081	11.593	21.073	10.915	20.385	10.961
110	18.062	11.627	18.062	11.627	21.179	10.887	20.428	10.936
111	18.031	11.666	18.031	11.666	21.309	10.864	20.480	10.904
112	17.983	11.700	17.983	11.700	21.446	10.807	20.532	10.857
113	17.921	11.735	17.921	11.735	21.603	10.747	20.598	10.796
114	17.840	11.763	17.840	11.763	21.778	10.668	20.661	10.715
115	17.747	11.779	17.747	11.779	21.949	10.577	20.706	10.623
116	9.724	9.724	8.649	8.880	2.894	2.894	3.101	3.098
117	9.774	9.774	8.798	8.827	2.866	2.866	3.125	3.126
118	9.829	9.829	8.738	8.760	2.831	2.831	3.159	3.157
119	9.889	9.889	8.662	8.683	2.787	2.787	3.194	3.192
120	9.954	9.954	8.581	8.596	2.735	2.735	3.234	3.232

Table 112

FIG. 136

	Label							
	24	25	26	27	28	29	30	31
Design #								
81	14.114	14.594	15.729	15.729	-0.783	-0.783	-0.783	-0.783
82	14.152	14.629	15.814	15.814	-0.856	-0.856	-0.856	-0.856
83	14.215	14.684	15.916	15.916	-0.939	-0.939	-0.939	-0.939
84	0.734	2.397	0.734	2.397	0.504	2.397	0.515	2.397
85	0.628	2.487	0.628	2.487	0.412	2.487	0.424	2.487
86	0.514	2.585	0.514	2.585	0.316	2.585	0.329	2.585
87	0.375	2.700	0.375	2.700	0.203	2.700	0.215	2.700
88	0.211	2.836	0.211	2.836	0.069	2.836	0.082	2.836
89	0.029	2.988	0.029	2.988	-0.079	2.988	-0.067	2.986
90	-0.178	3.161	-0.178	3.161	-0.250	3.161	-0.239	3.161
91	-0.186	3.167	-0.186	3.167	-0.257	3.167	-0.246	3.167
92	-0.395	3.346	-0.395	3.346	-0.435	3.346	-0.424	3.346
93	-0.612	3.534	-0.612	3.534	-0.625	3.534	-0.614	3.534
94	-0.808	3.712	-0.808	3.712	-0.808	3.712	-0.808	3.712
95	-0.978	3.870	-0.978	3.870	-0.978	3.870	-0.978	3.870
96	-1.124	4.007	-1.124	4.007	-1.124	4.007	-1.124	4.007
97	-1.244	4.122	-1.244	4.122	-1.244	4.122	-1.244	4.122
98	-1.336	4.218	-1.336	4.218	-1.336	4.218	-1.336	4.218
99	-1.409	4.297	-1.409	4.297	-1.409	4.297	-1.409	4.297
100	-1.464	4.365	-1.464	4.365	-1.464	4.365	-1.464	4.365
101	-1.501	4.420	-1.501	4.420	-1.501	4.420	-1.501	4.420
102	-1.524	4.465	-1.524	4.465	-1.524	4.465	-1.524	4.465
103	-1.533	4.505	-1.533	4.505	-1.533	4.505	-1.533	4.505
104	-1.530	4.540	-1.530	4.540	-1.530	4.540	-1.530	4.540
105	-1.514	4.570	-1.514	4.571	-1.514	4.571	-1.514	4.571
106	-1.495	4.593	-1.495	4.593	-1.495	4.609	-1.495	4.609
107	-1.490	4.596	-1.490	4.596	-1.490	4.619	-1.490	4.618
108	-1.480	4.604	-1.480	4.604	-1.480	4.639	-1.480	4.638
109	-1.466	4.612	-1.466	4.612	-1.466	4.669	-1.466	4.666
110	-1.456	4.614	-1.456	4.614	-1.456	4.693	-1.456	4.691
111	-1.435	4.619	-1.436	4.619	-1.456	4.719	-1.456	4.716
112	-1.421	4.592	-1.422	4.592	-1.468	4.737	-1.468	4.733
113	-1.407	4.566	-1.408	4.566	-1.484	4.761	-1.484	4.748
114	-1.401	4.525	-1.401	4.525	-1.511	4.757	-1.510	4.754
115	-1.397	4.473	-1.397	4.473	-1.545	4.759	-1.545	4.753
116	15.469	15.469	18.491	17.923	-2.894	-2.894	-3.101	-3.099
117	15.428	15.428	18.613	18.023	-2.866	-2.866	-3.126	-3.126
118	15.378	15.378	18.749	18.141	-2.831	-2.831	-3.159	-3.157
119	15.324	15.324	18.891	18.272	-2.787	-2.787	-3.194	-3.192
120	15.266	15.266	19.034	18.406	-2.735	-2.735	-3.233	-3.232

Table 113

FIG. 137

	Label							
	0	1	2	3	4	5	6	7
Design #								
121	-39.738	-32.309	-24.163	-26.205	39.739	32.310	-19.642	-19.232
122	-39.649	-32.294	-24.211	-26.262	39.652	32.295	-19.652	-19.205
123	-39.536	-32.264	-24.242	-26.313	39.558	32.276	-19.673	-19.189
124	-39.535	-32.267	-24.247	-26.323	39.535	32.267	-19.676	-19.188
125	-39.425	-32.234	-24.266	-26.372	39.425	32.235	-19.705	-19.182
126	-39.280	-32.184	-24.279	-26.420	39.280	32.185	-19.748	-19.190
127	-39.115	-32.122	-24.285	-26.466	39.114	32.120	-19.802	-19.206
128	-38.948	-32.053	-24.274	-26.504	38.949	32.054	-19.853	-19.220
129	-38.784	-31.991	-24.260	-26.553	38.784	31.991	-19.906	-19.238
130	-38.640	-31.943	-24.236	-26.616	38.640	31.943	-19.954	-19.226
131	-38.523	-31.917	-24.206	-26.700	38.522	31.915	-19.999	-19.205
132	-38.424	-31.907	-24.169	-26.796	38.425	31.908	-20.038	-19.169
133	-38.432	-32.010	38.436	-27.388	-17.451	-21.330	-17.003	-22.982
134	-38.350	-32.013	38.356	-27.381	-17.487	-21.326	-16.995	-23.037
135	38.274	-32.018	38.274	-27.382	-17.524	-21.318	-16.982	-23.104
136	-38.195	-32.021	38.202	-27.387	-17.562	-21.304	-16.961	-23.182
137	-38.121	-32.028	38.125	-27.401	-17.603	-21.286	-16.930	-23.274
138	-38.052	-32.040	38.050	-27.427	-17.643	-21.261	-16.888	-23.380
139	-37.977	-32.047	37.977	-27.455	-17.685	-21.234	-16.841	-23.485
140	-37.932	-32.052	37.936	-27.475	-17.710	-21.218	-16.811	-23.549
141	-37.904	-32.056	37.909	-27.490	-17.728	-21.207	-16.788	-23.592
142	-37.835	-32.069	37.833	-27.533	-17.771	-21.185	-16.735	-23.696
143	-37.760	-32.075	37.765	-27.574	-17.819	-21.165	-16.683	-23.789
144	-37.689	-32.084	37.687	-27.619	-17.873	-21.153	-16.633	-23.875
145	-37.610	-32.087	37.614	-27.661	-17.935	-21.147	-16.586	-23.953
146	-37.532	-32.090	37.534	-27.705	-18.009	-21.151	-16.540	-24.027
147	-37.462	-32.088	37.462	-27.746	-18.093	-21.162	-16.497	-24.094
148	-37.365	-32.082	37.369	-27.784	-18.187	-21.184	-16.457	-24.159
149	-37.280	-32.074	37.279	-27.821	-18.284	-21.215	-16.423	-24.220
150	-37.189	-32.062	37.189	-27.855	-18.380	-21.253	-16.398	-24.278
151	-37.094	-32.045	37.094	-27.885	-18.474	-21.299	-16.383	-24.334
152	-36.994	-32.023	36.996	-27.912	-18.564	-21.350	-16.383	-24.386
153	-36.891	-31.997	36.891	-27.935	-18.649	-21.405	-16.394	-24.439
154	-36.781	-31.964	36.781	-27.953	-18.731	-21.462	-16.421	-24.487
155	-36.668	-31.927	36.666	-27.965	-18.808	-21.519	-16.460	-24.531
156	-36.536	-31.876	36.541	-27.969	-18.887	-21.579	-16.517	-24.571
157	-36.412	-31.824	36.419	-27.969	-18.956	-21.631	-16.575	-24.606
158	-36.408	-31.822	36.408	-27.968	-18.959	-21.633	-16.577	-24.606
159	-36.256	-31.750	36.259	-27.953	-19.032	-21.685	-16.653	-24.631
160	-36.085	-31.658	36.082	-27.921	-19.106	-21.732	-16.740	-24.645

Table 114

FIG. 138

	Label							
Design #	8	9	10	11	12	13	14	15
121	-6.470	-6.470	-11.613	-11.613	-5.566	-5.582	-13.954	-14.014
122	-6.536	-6.536	-11.556	-11.556	-5.489	-5.486	-14.052	-14.129
123	-6.623	-6.623	-11.505	-11.505	-5.407	-5.404	-14.171	-14.262
124	-6.629	-6.629	-11.499	-11.499	-5.397	-5.393	-14.181	-14.273
125	-6.729	-6.729	-11.456	-11.456	-5.311	-5.308	-14.303	-14.405
126	-6.870	-6.870	-11.425	-11.425	-5.212	-5.210	-14.452	-14.563
127	-7.045	-7.045	-11.413	-11.413	-5.115	-5.114	-14.605	-14.723
128	-7.239	-7.239	-11.420	-11.420	-5.034	-5.033	-14.742	-14.868
129	-7.422	-7.422	-11.438	-11.438	-4.968	-4.967	-14.857	-14.991
130	-7.572	-7.572	-11.454	-11.454	-4.920	-4.919	-14.940	-15.088
131	-7.681	-7.681	-11.460	-11.460	-4.891	-4.890	-14.989	-15.157
132	-7.759	-7.759	-11.460	-11.460	-4.850	-4.550	-15.014	-15.205
133	-3.256	-6.122	-3.256	-6.122	-13.075	-9.682	-13.146	-9.682
134	-3.230	-6.176	-3.230	-6.176	-13.104	-9.682	-13.191	-9.682
135	-3.208	-6.216	-3.208	-6.216	-13.123	-9.681	-13.228	-9.681
136	-3.192	-6.246	-3.192	-6.246	-13.132	-9.682	-13.259	-9.677
137	-3.178	-6.265	-3.178	-6.265	-13.130	-9.680	-13.283	-9.668
138	-3.163	-6.275	-3.163	-6.275	-13.115	-9.673	-13.302	-9.654
139	-3.154	-6.283	-3.154	-6.283	-13.099	-9.670	-13.318	-9.640
140	-3.149	-6.285	-3.149	-6.286	-13.083	-9.667	-13.328	-9.630
141	-3.145	-6.286	-3.145	-6.286	-13.070	-9.664	-13.331	-9.622
142	-3.135	-6.282	-3.135	-6.286	-13.038	-9.659	-13.344	-9.602
143	-3.129	-6.279	-3.129	-6.287	-13.003	-9.657	-13.359	-9.583
144	-3.122	-6.273	-3.122	-6.287	-12.964	-9.656	-13.376	-9.561
145	-3.118	-6.266	-3.117	-6.287	-12.921	-9.658	-13.397	-9.538
146	-3.113	-6.255	-3.109	-6.285	-12.871	-9.660	-13.423	-9.510
147	-3.109	-6.242	-3.103	-6.284	-12.815	-9.664	-13.452	-9.477
148	-3.106	-6.224	-3.094	-6.282	-12.752	-9.670	-13.489	-9.438
149	-3.102	-6.203	-3.085	-6.280	-12.685	-9.677	-13.531	-9.391
150	-3.099	-6.176	-3.073	-6.278	-12.618	-9.687	-13.582	-9.338
151	-3.096	-6.143	-3.058	-6.278	-12.552	-9.705	-13.645	-9.277
152	-3.095	-6.103	-3.040	-6.280	-12.490	-9.726	-13.724	-9.209
153	-3.094	-6.054	-3.018	-6.284	-12.433	-9.755	-13.815	-9.132
154	-3.096	-5.995	-2.986	-6.293	-12.389	-9.805	-13.919	-9.050
155	-3.103	-5.925	-2.947	-6.310	-12.359	-9.868	-14.026	-8.968
156	-3.119	-5.848	-2.899	-6.346	-12.357	-9.959	-14.140	-8.897
157	-3.148	-5.767	-2.841	-6.400	-12.374	-10.067	-14.245	-8.842
158	-3.148	-5.764	-2.839	-6.402	-12.375	-10.071	-14.248	-8.841
159	-3.206	-5.686	-2.765	-6.502	-12.429	-10.216	-14.364	-8.819
160	-3.322	-5.636	-2.677	-6.667	-12.520	-10.390	-14.483	-8.853

Table 115

FIG. 139

	Label							
	16	17	18	19	20	21	22	23
Design #								
121	19.643	19.233	13.955	14.016	24.164	26.206	11.618	11.618
122	19.652	19.205	14.052	14.129	24.210	26.261	11.560	11.560
123	19.674	19.187	14.170	14.260	24.246	26.321	11.504	11.504
124	19.676	19.189	14.182	14.275	24.247	26.323	11.502	11.502
125	19.706	19.184	14.305	14.408	24.268	26.372	11.458	11.458
126	19.750	19.190	14.454	14.564	24.281	26.422	11.426	11.426
127	19.800	19.206	14.605	14.722	24.283	26.463	11.416	11.416
128	19.854	19.222	14.744	14.868	24.276	26.504	11.421	11.421
129	19.905	19.230	14.857	14.992	24.259	26.553	11.438	11.438
130	19.956	19.227	14.941	15.089	24.237	26.615	11.455	11.455
131	19.998	19.205	14.990	15.157	24.205	26.698	11.461	11.461
132	20.038	19.170	15.015	15.206	24.170	26.797	11.460	11.460
133	27.384	22.988	32.015	21.350	13.255	16.943	13.194	17.422
134	27.387	23.044	32.018	21.341	13.270	16.949	13.193	17.467
135	27.386	23.111	32.020	21.328	13.286	16.947	13.190	17.512
136	27.393	23.191	32.027	21.311	13.300	16.932	13.180	17.556
137	27.406	23.282	32.032	21.289	13.313	16.907	13.166	17.599
138	27.426	23.382	32.038	21.262	13.322	16.870	13.144	17.640
139	27.467	23.490	32.048	21.235	13.332	16.826	13.117	17.684
140	27.479	23.565	32.055	21.219	13.337	16.796	13.098	17.709
141	27.495	23.598	32.061	21.208	13.340	16.775	13.084	17.726
142	27.533	23.697	32.067	21.184	13.350	16.725	13.060	17.770
143	27.577	23.791	32.079	21.165	13.362	16.675	13.011	17.817
144	27.619	23.876	32.084	21.152	13.376	16.627	12.971	17.872
145	27.664	23.955	32.090	21.147	13.398	16.581	12.925	17.934
146	27.704	24.026	32.090	21.149	13.422	16.537	12.876	18.006
147	27.745	24.093	32.088	21.161	13.452	16.495	12.819	18.091
148	27.784	24.156	32.083	21.182	13.489	16.455	12.756	18.184
149	27.821	24.219	32.074	21.214	13.531	16.422	12.688	18.282
150	27.855	24.278	32.062	21.253	13.582	16.397	12.620	18.378
151	27.885	24.334	32.045	21.299	13.646	16.384	12.553	18.474
152	27.912	24.388	32.024	21.350	13.723	16.382	12.491	18.563
153	27.935	24.439	31.997	21.405	13.816	16.395	12.434	18.649
154	27.953	24.486	31.965	21.463	13.920	16.422	12.389	18.732
155	27.966	24.532	31.926	21.521	14.028	16.462	12.360	18.810
156	27.971	24.572	31.879	21.578	14.139	16.516	12.356	18.886
157	27.969	24.605	31.824	21.631	14.243	16.575	12.373	18.956
158	27.968	24.606	31.822	21.633	14.246	16.577	12.375	18.959
159	27.953	24.631	31.751	21.684	14.363	16.651	12.427	19.031
160	27.919	24.643	31.656	21.730	14.488	16.739	12.520	19.105

Table 116

FIG. 140

	Label							
	24	25	26	27	28	29	30	31
Design #								
121	0.241	0.241	5.559	5.557	-0.243	-0.243	6.468	6.468
122	0.299	0.299	5.486	5.482	-0.300	-0.300	6.535	6.535
123	0.369	0.369	5.397	5.395	-0.376	-0.376	6.618	6.618
124	0.379	0.379	5.393	5.389	-0.380	-0.380	6.629	6.629
125	0.462	0.462	5.306	5.303	-0.465	-0.465	6.729	6.729
126	0.579	0.579	5.208	5.207	-0.581	-0.581	6.870	6.870
127	0.724	0.724	5.115	5.113	-0.723	-0.723	7.046	7.046
128	0.881	0.881	5.031	5.030	-0.883	-0.883	7.239	7.239
129	1.033	1.033	4.967	4.968	-1.033	-1.034	7.422	7.422
130	1.159	1.159	4.918	4.917	-1.160	-1.160	7.572	7.572
131	1.255	1.255	4.881	4.880	-1.255	-1.255	7.681	7.681
132	1.326	1.326	4.849	4.849	-1.329	-1.328	7.759	7.759
133	0.172	2.924	0.172	2.924	9.485	6.389	9.485	6.389
134	0.192	2.974	0.192	2.974	9.533	6.378	9.533	6.378
135	0.102	3.014	0.102	3.014	9.568	6.370	9.569	6.370
136	0.077	3.042	0.077	3.042	9.589	6.361	9.596	6.361
137	0.059	3.064	0.059	3.064	9.602	6.353	9.615	6.353
138	0.047	3.080	0.047	3.080	9.605	6.345	9.627	6.345
139	0.035	3.089	0.035	3.089	9.603	6.335	9.634	6.335
140	0.029	3.093	0.029	3.093	9.598	6.329	9.636	6.329
141	0.026	3.094	0.026	3.094	9.593	6.324	9.637	6.323
142	0.021	3.100	0.021	3.100	9.582	6.317	9.641	6.314
143	0.015	3.101	0.015	3.101	9.566	6.309	9.642	6.301
144	0.012	3.103	0.012	3.103	9.559	6.304	9.646	6.291
145	0.008	3.101	0.008	3.102	9.528	6.298	9.650	6.278
146	0.007	3.100	0.007	3.103	9.505	6.295	9.656	6.266
147	0.005	3.095	0.004	3.102	9.474	6.291	9.662	6.250
148	0.005	3.089	0.002	3.100	9.436	6.289	9.668	6.230
149	0.006	3.081	-0.001	3.098	9.390	6.284	9.676	6.207
150	0.007	3.069	-0.004	3.095	9.336	6.281	9.687	6.179
151	0.010	3.055	-0.008	3.094	9.276	6.279	9.703	6.145
152	0.015	3.038	-0.013	3.093	9.207	6.280	9.726	6.104
153	0.022	3.014	-0.021	3.093	9.130	6.284	9.759	6.054
154	0.033	2.984	-0.033	3.094	9.048	6.293	9.805	5.994
155	0.048	2.945	-0.049	3.101	8.966	6.310	9.868	5.924
156	0.072	2.899	-0.071	3.119	8.896	6.345	9.958	5.849
157	0.106	2.841	-0.105	3.146	8.842	6.400	10.069	5.766
158	0.107	2.839	-0.106	3.148	8.840	6.402	10.071	5.765
159	0.163	2.767	-0.161	3.205	8.819	6.501	10.213	5.686
160	0.257	2.680	-0.254	3.324	8.864	6.668	10.390	5.639

Table 117

FIG. 141

	Label							
	0	1	2	3	4	5	6	7
Design #								
161	-35.860	-31.521	35.880	-27.852	-19.181	-21.766	-16.846	-24.633
162	-35.666	-31.406	35.563	-27.759	-19.248	-21.799	-16.934	-24.628
163	-35.507	-31.322	35.508	-27.769	-19.301	-21.835	-17.000	-24.637
164	-35.373	-31.256	35.373	-27.752	-19.353	-21.874	-17.058	-24.654
165	-35.247	-31.195	35.249	-27.736	-19.403	-21.913	-17.108	-24.672
166	-35.132	-31.143	35.131	-27.730	-19.453	-21.954	-17.158	-24.695
167	-35.013	-31.087	35.014	-27.719	-19.503	-21.994	-17.210	-24.716
168	-34.901	-31.036	34.902	-27.711	-19.552	-22.034	-17.260	-24.737
169	-34.790	-30.986	34.787	-27.702	-19.603	-22.074	-17.313	-24.758
170	-34.677	-30.930	34.676	-27.689	-19.652	-22.112	-17.366	-24.776
171	-34.564	-30.875	34.560	-27.677	-19.703	-22.150	-17.421	-24.793
172	-34.446	-30.817	34.449	-27.666	-19.751	-22.185	-17.476	-24.807
173	-34.335	-30.759	34.335	-27.643	-19.800	-22.220	-17.531	-24.820
174	-34.219	-30.700	34.221	-27.624	-19.848	-22.254	-17.587	-24.832
175	-34.107	-30.641	-24.643	-27.805	34.107	30.641	-22.286	-19.896
176	-34.106	-30.640	-24.843	-27.604	34.106	30.640	-22.287	-19.896
177	-33.992	-30.579	-24.851	-27.583	33.993	30.580	-22.316	-19.941
178	-33.880	-30.519	-24.859	-27.561	33.881	30.520	-22.346	-19.986
179	-33.770	-30.459	-24.864	-27.538	33.770	30.459	-22.373	-20.029
180	-33.661	-30.399	-24.870	-27.515	33.661	30.399	-22.400	-20.071
181	-33.554	-30.341	-24.875	-27.493	33.554	30.341	-22.426	-20.111
182	-33.452	-30.284	-24.879	-27.470	33.451	30.284	-22.449	-20.149
183	-33.348	-30.227	-24.881	-27.447	33.350	30.228	-22.472	-20.187
184	-33.253	-30.175	-24.886	-27.427	33.252	30.175	-22.495	-20.223
185	-33.159	-30.123	-24.888	-27.406	33.159	30.123	-22.515	-20.257

Table 118

FIG. 142

	Label							
	8	9	10	11	12	13	14	15
Design #								
161	-3.564	-5.671	-2.596	-6.941	-12.071	-10.618	-14.638	-8.955
162	-3.776	-5.733	-2.552	-7.160	-12.793	-10.792	-14.757	-9.108
163	-3.899	-5.774	-2.533	-7.290	-12.873	-10.904	-14.841	-9.183
164	-3.977	-5.800	-2.521	-7.375	-12.931	-10.962	-14.907	-9.233
165	-4.034	-5.822	-2.516	-7.440	-12.982	-11.045	-14.964	-9.274
166	-4.073	-5.837	-2.510	-7.487	-13.025	-11.095	-15.016	-9.306
167	-4.107	-5.856	-2.511	-7.531	-13.070	-11.143	-15.068	-9.339
168	-4.133	-5.872	-2.511	-7.566	-13.112	-11.187	-15.118	-9.369
169	-4.156	-5.888	-2.512	-7.600	-13.157	-11.229	-15.170	-9.401
170	-4.177	-5.908	-2.518	-7.632	-13.203	-11.272	-15.222	-9.433
171	-4.196	-5.929	-2.524	-7.666	-13.253	-11.318	-15.277	-9.470
172	-4.220	-5.953	-2.533	-7.701	-13.305	-11.366	-15.333	-9.509
173	-4.242	-5.979	-2.543	-7.737	-13.359	-11.416	-15.390	-9.551
174	-4.265	-6.008	-2.555	-7.776	-13.416	-11.468	-15.449	-9.596
175	-7.818	-9.641	-13.473	-11.523	-6.035	-4.291	-15.508	-17.643
176	-7.819	-9.642	-13.474	-11.524	-6.035	-4.291	-15.509	-17.644
177	-7.860	-9.691	-13.532	-11.579	-6.070	-4.315	-15.568	-17.698
178	-7.903	-9.742	-13.592	-11.636	-6.104	-4.339	-15.626	-17.752
179	-7.947	-9.794	-13.651	-11.692	-6.140	-4.365	-15.685	-17.805
180	-7.992	-9.846	-13.711	-11.749	-6.177	-4.392	-15.743	-17.858
181	-8.037	-9.897	-13.768	-11.804	-6.213	-4.418	-15.798	-17.908
182	-8.081	-9.947	-13.823	-11.859	-6.249	-4.445	-15.852	-17.956
183	-8.128	-9.999	-13.879	-11.914	-6.286	-4.472	-15.906	-18.004
184	-8.165	-10.044	-13.930	-11.963	-6.318	-4.496	-15.955	-18.049
185	-8.206	-10.091	-13.980	-12.013	-6.352	-4.521	-16.003	-18.091

Table 119

FIG. 143

	Label							
	16	17	18	19	20	21	22	23
Design #								
161	27.655	24.636	31.523	21.769	14.639	16.849	12.670	19.184
162	27.698	24.627	31.405	21.799	14.757	16.934	12.794	19.246
163	27.769	24.697	31.322	21.836	14.841	17.000	12.873	19.302
164	27.751	24.664	31.255	21.874	14.906	17.056	12.931	19.353
165	27.740	24.675	31.197	21.914	14.964	17.108	12.981	19.404
166	27.729	24.694	31.142	21.953	15.016	17.157	13.025	19.452
167	27.721	24.717	31.089	21.995	15.068	17.209	13.069	19.503
168	27.711	24.737	31.037	22.034	15.116	17.260	13.112	19.553
169	27.699	24.755	30.982	22.072	15.170	17.312	13.168	19.601
170	27.689	24.775	30.930	22.111	15.221	17.365	13.203	19.652
171	27.673	24.791	30.872	22.148	15.277	17.421	13.254	19.702
172	27.660	24.808	30.818	22.186	15.333	17.476	13.305	19.752
173	27.644	24.822	30.760	22.221	15.390	17.531	13.359	19.800
174	27.625	24.833	30.701	22.254	15.449	17.587	13.415	19.848
175	22.286	19.895	15.508	17.643	24.843	27.605	13.473	11.523
176	22.286	19.895	15.508	17.643	24.843	27.604	13.473	11.523
177	22.317	19.942	15.568	17.698	24.852	27.584	13.532	11.578
178	22.346	19.986	15.627	17.753	24.859	27.562	13.592	11.635
179	22.374	20.029	15.685	17.808	24.864	27.539	13.652	11.692
180	22.400	20.071	15.742	17.858	24.869	27.515	13.710	11.749
181	22.425	20.111	15.798	17.908	24.874	27.493	13.768	11.805
182	22.450	20.150	15.852	17.957	24.879	27.471	13.824	11.859
183	22.473	20.188	15.906	18.004	24.882	27.448	13.879	11.913
184	22.495	20.223	15.955	18.048	24.885	27.427	13.930	11.964
185	22.516	20.257	16.004	18.092	24.889	27.407	13.981	12.013

Table 120

FIG. 144

	Label							
	24	25	26	27	28	29	30	31
Design #								
161	0.424	2.591	-0.428	3.561	8.963	6.939	10.617	5.667
162	0.573	2.552	-0.572	3.776	9.108	7.160	10.793	5.734
163	0.655	2.532	-0.655	3.899	9.183	7.290	10.904	5.774
164	0.707	2.522	-0.706	3.977	9.233	7.376	10.982	5.801
165	0.740	2.513	-0.742	4.032	9.272	7.438	11.044	5.820
166	0.767	2.511	-0.766	4.073	9.306	7.485	11.095	5.838
167	0.784	2.509	-0.786	4.106	9.338	7.530	11.143	5.855
168	0.799	2.510	-0.799	4.133	9.369	7.565	11.186	5.871
169	0.812	2.516	-0.809	4.159	9.403	7.602	11.231	5.891
170	0.820	2.519	-0.819	4.178	9.434	7.633	11.272	5.908
171	0.829	2.527	-0.826	4.201	9.472	7.668	11.320	5.931
172	0.833	2.533	-0.834	4.220	9.509	7.701	11.366	5.953
173	0.839	2.542	-0.841	4.241	9.550	7.737	11.416	5.979
174	0.845	2.554	-0.846	4.264	9.595	7.776	11.468	6.007
175	0.855	2.563	6.035	4.291	-0.866	-2.563	7.818	9.641
176	0.858	2.564	6.036	4.292	-0.865	-2.562	7.819	9.641
177	0.859	2.577	6.069	4.314	-0.860	-2.575	7.859	9.691
178	0.864	2.593	6.104	4.339	-0.865	-2.594	7.902	9.742
179	0.869	2.610	6.140	4.365	-0.870	-2.610	7.947	9.794
180	0.875	2.627	6.177	4.392	-0.874	-2.626	7.993	9.846
181	0.880	2.643	6.213	4.419	-0.880	-2.642	8.037	9.898
182	0.885	2.658	6.248	4.444	-0.886	-2.660	8.080	9.947
183	0.890	2.675	6.285	4.471	-0.891	-2.676	8.125	9.998
184	0.896	2.691	6.319	4.496	-0.896	-2.691	8.166	10.046
185	0.900	2.705	6.352	4.520	-0.902	-2.707	8.206	10.091

Table 121

FIG. 145

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
63	7	2.79	2.51	2.26	2.03	1.46	0
64	7.2	2.88	2.59	2.33	2.1	1.38	0
65	7.4	2.76	2.47	2.23	2	1.31	0
66	7.6	2.74	2.47	2.22	2	1.31	0
67	7.8	2.74	2.47	2.22	2	1.31	0
68	8	2.72	2.45	2.2	1.96	1.3	0
69	8.2	2.7	2.43	2.18	1.96	1.29	0
70	8.4	2.66	2.39	2.15	1.94	1.27	0
71	8.6	2.62	2.36	2.12	1.91	1.26	0
72	8.8	2.56	2.3	2.07	1.87	1.22	0
73	8.99	2.59	1.28	1.15	1.04	0.93	0
74	9	2.58	1.28	1.15	1.04	0.93	0
75	9.2	2.57	1.25	1.13	1.02	0.91	0
76	9.4	2.54	1.23	1.1	0.99	0.89	0
77	9.6	2.51	1.2	1.08	0.97	0.87	0
78	9.8	2.48	1.18	1.06	0.96	0.86	0
79	10	2.39	2.15	1.93	1.74	1.14	0
80	10.2	2.13	1.92	1.73	1.56	1.13	0
81	10.4	2.31	2.09	1.87	1.69	1.11	0
82	10.6	2.26	2.03	1.83	1.64	1.06	0
83	10.8	2.2	1.98	1.78	1.6	1.17	0
84	11	2.19	1.97	1.77	1.59	1.06	0
85	11.2	2.15	1.93	1.74	1.56	1.03	0
86	11.4	2.1	1.89	1.7	1.53	1.12	0
87	11.6	2.05	1.84	1.66	1.49	1.09	0
88	11.8	2.2	1.98	1.78	1.61	1.05	0
89	12	2.13	1.91	1.72	1.55	1.02	0
90	12.2	2.04	1.84	1.65	1.49	1.09	0
91	12.21	2.04	1.83	1.65	1.49	1.08	0
92	12.4	2.17	1.96	1.76	1.58	1.04	0
93	12.6	2.12	1.91	1.72	1.39	1.02	0
94	12.8	2.07	1.87	1.68	1.51	0.99	0
95	13	2.02	1.82	1.64	1.47	0.97	0
96	13.2	1.96	1.76	1.59	1.43	1.04	0
97	13.4	1.89	1.7	1.53	1.38	1.01	0
98	13.6	1.83	1.65	1.48	1.33	0.97	0
99	13.8	1.76	1.58	1.42	1.28	0.93	0
100	14	1.89	1.7	1.53	1.38	1	0
101	14.2	1.85	1.66	1.5	1.35	0.68	0
102	14.4	1.81	1.63	1.47	1.32	0.96	0

Table 122

FIG. 146

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
103	14.6	1.77	1.59	1.43	1.29	0.94	0
104	14.8	1.72	1.56	1.39	1.25	0.91	0
105	15	1.68	1.67	1.5	1.35	0.85	0
106	15.2	1.81	1.63	1.47	1.32	0.87	0
107	15.27	1.8	1.62	1.46	1.32	0.86	0
108	15.4	1.79	1.61	1.46	1.17	0.86	0
109	15.6	1.68	1.43	1.28	1.15	0.84	0
110	15.8	1.72	1.55	1.4	1.25	0.82	0
111	16	1.54	1.38	1.25	1.12	0.82	0
112	16.2	1.53	1.37	1.24	1.11	0.81	0
113	16.4	1.51	1.36	1.22	1.1	0.8	0
114	16.6	1.66	1.49	1.34	1.09	0.79	0
115	16.8	1.62	1.46	1.31	1.18	0.77	0
116	17	1.57	1.41	1.27	1.14	0.75	0
117	17.2	1.55	1.39	1.25	1.02	0.74	0
118	17.4	1.52	1.37	1.23	1	0.73	0
119	17.6	1.34	1.21	1.09	0.98	0.71	0
120	17.8	1.46	1.31	1.18	1.06	0.7	0
121	18	1.27	1.14	1.03	0.92	0.67	0
122	18.2	1.36	1.22	1.1	0.99	0.66	0
123	18.4	1.31	1.18	1.06	0.95	0.63	0
124	18.42	1.3	1.17	1.05	0.95	0.62	0
125	18.6	1.25	1.12	1.01	0.91	0.66	0
126	18.8	1.3	1.17	1.05	0.95	0.56	0
127	19	1.2	1.08	0.97	0.88	0.57	0
128	19.2	1.18	1.06	0.96	0.86	0.57	0
129	19.4	1.1	0.99	0.89	0.81	0.59	0
130	19.6	1.14	1.02	0.92	0.83	0.54	0
131	19.8	1.16	1.04	0.93	0.84	0.55	0
132	20	1.04	0.94	0.84	0.76	0.55	0
133	20.2	1.03	0.93	0.84	0.75	0.49	0
134	20.4	1.06	0.94	0.85	0.69	0.5	0
135	20.6	1.05	0.95	0.86	0.69	0.5	0
136	20.8	0.95	0.86	0.77	0.69	0.51	0
137	21	0.95	0.86	0.77	0.7	0.46	0
138	21.2	0.95	0.86	0.77	0.63	0.46	0
139	21.4	0.96	0.86	0.77	0.62	0.46	0
140	21.52	0.86	0.77	0.69	0.62	0.45	0
141	21.6	0.85	0.77	0.68	0.62	0.45	0
142	21.8	0.85	0.76	0.68	0.62	0.45	0
143	22	0.78	0.71	0.64	0.57	0.42	0

Table 123

FIG. 147

Design #	SNRs	5.00%	40.00%	50.00%	60.00%	70.00%	100.00%
144	22.2	0.81	0.73	0.66	0.53	0.39	0
145	22.4	0.75	0.68	0.61	0.55	0.36	0
146	22.6	0.78	0.7	0.63	0.57	0.37	0
147	22.8	0.72	0.65	0.59	0.53	0.35	0
148	23	0.67	0.6	0.54	0.48	0.36	0
149	23.2	0.69	0.62	0.56	0.5	0.33	0
150	23.4	0.64	0.57	0.52	0.47	0.34	0
151	23.6	0.66	0.59	0.53	0.43	0.31	0
152	23.8	0.61	0.55	0.49	0.44	0.29	0
153	24	0.56	0.5	0.45	0.41	0.3	0
154	24.2	0.57	0.52	0.46	0.42	0.27	0
155	24.4	0.53	0.48	0.43	0.39	0.29	0
156	24.6	0.54	0.49	0.44	0.33	0.26	0
157	24.79	0.6	0.45	0.4	0.36	0.27	0
158	24.8	0.5	0.45	0.41	0.36	0.24	0
159	25	0.46	0.42	0.38	0.34	0.25	0
160	25.2	0.48	0.43	0.39	0.35	0.25	0
161	25.4	0.44	0.4	0.36	0.32	0.23	0
162	25.6	0.45	0.41	0.37	0.33	0.22	0
163	25.8	0.42	0.38	0.34	0.3	0.22	0
164	26	0.38	0.34	0.31	0.28	0.2	0
165	26.2	0.38	0.34	0.31	0.28	0.2	0
166	26.4	0.37	0.33	0.3	0.27	0.2	0
167	26.6	0.36	0.32	0.29	0.26	0.19	0
168	26.8	0.35	0.32	0.28	0.25	0.18	0
169	27	0.34	0.31	0.28	0.25	0.18	0
170	27.2	0.3	0.27	0.24	0.22	0.16	0
171	27.4	0.29	0.26	0.23	0.21	0.15	0
172	27.6	0.28	0.25	0.23	0.2	0.15	0
173	27.8	0.27	0.24	0.22	0.2	0.14	0
174	28	0.26	0.23	0.21	0.19	0.14	0
175	28.2	0.26	0.23	0.2	0.18	0.13	0
176	28.2	0.25	0.23	0.2	0.18	0.13	0
177	28.4	0.24	0.22	0.2	0.18	0.13	0
178	28.6	0.21	0.19	0.17	0.15	0.11	0
179	28.8	0.22	0.2	0.18	0.16	0.12	0
180	29	0.19	0.17	0.16	0.14	0.1	0
181	29.2	0.18	0.17	0.15	0.13	0.1	0
182	29.4	0.2	0.18	0.16	0.13	0.09	0
183	29.6	0.17	0.15	0.14	0.12	0.09	0
184	29.8	0.16	0.15	0.13	0.12	0.09	0
185	30	0.15	0.14	0.12	0.11	0.08	0

Table 124

FIG. 148

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US09/69681

A. CLASSIFICATION OF SUBJECT MATTER
IPC(8) - H04B 7/185 (2010.01)
USPC - 455/12.1
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC(8): H04B 7/185 (2010.01)
USPC: 455/12.1

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched
IPC(8): H04B 7/185 (2010.01) (keyword limited; terms below)
USPC: 455/7, 12.1 (keyword limited; terms below)

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
PubWEST(PGPB, USPT, EPAB, JPAB); Google; Search Terms: signal constellation diagram point transmitter transceiver receiver channel communication network coder coding encoder encoding bit byte mapping mapper maps map modulat signal transmission symbol bits demodulat de-modulat estimat predict likelihood likely probable PAM pulse amplitude modulat

C. DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X	US 2004/0258177 A1 (SHEN et al.) 23 December 2004 (23.12.2004), entire document, especially abstract, para. [0009], [0016], [0019], [0056], [0061], [0065], [0075], [0149], [0155], [0157], [0174], [0318], and [0320].	1-3, 15-18, 32-34, 47
Y		4-15, 19-28, 30-31, 35-46
Y	US 2007/0280147 A1 (CATREUX-ERCEG et al.) 06 December 2007 (06.12.2007), entire document, especially para. [0004], [0012], [0014], [0030], [0040], [0051], [0062], and [0064]-[0065].	4-15, 19-28, 30-31, 35-46
A	US 2007/0054614 A1 (WALKER et al.) 08 March 2007 (08.03.2007), entire document	1-28, 30-47
A	US 2005/0286409 A1 (YOON et al.) 29 December 2005 (29.12.2005), entire document	1-28, 30-47

☐ Further documents are listed in the continuation of Box C ☐

* Special categories of cited documents:
"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier application or patent but published on or after the international filing date
"L" document which may throw doubts on priority claim(s) or which is cited to establish the publication date of another citation or other special reason (as specified)
"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed
"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
12 April 2010 (12.04.2010)	03 MAY 2010
Name and mailing address of the ISA/US Mail Stop PCT, Attn: ISA/US, Commissioner for Patents P.O. Box 1450, Alexandria, Virginia 22313-1450 Facsimile No. 571-273-3201	Authorized officer: Lee W. Young PCT Helpdesk: 571-272-4300 PCT OSP: 571-272-7774

Form PCT/ISA/210 (second sheet) (July 2009)

Espacenet

Bibliographic data: JP2010538502 (A) — 2010-12-09

DESIGN METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS

Inventor(s):

Applicant(s):

Classification: - **international:** *H04L27/34*
- **cooperative:** H04B15/00; H04L1/0003; H04L1/0009; H04L27/3405; H04L27/3483; H04W24/02

Application number: JP20100511344 20080605 Global Dossier

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Abstract not available for JP2010538502 (A)
Abstract of corresponding document: WO2008151308 (A1)

Communication systems are described that use geometrically shaped constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. In several embodiments, the geometrically shaped is optimized based upon a capacity measure such as parallel decoding capacity or joint capacity. In many embodiments, a capacity optimized geometrically shaped constellation can be used to replace a conventional constellation as part of a firmware upgrade to transmitters and receivers within a communication system. In a number of embodiments, the geometrically shaped constellation is optimized for an Additive White Gaussian Noise channel or a fading channel. In numerous embodiments, the communication uses adaptive rate encoding and the location of points within the

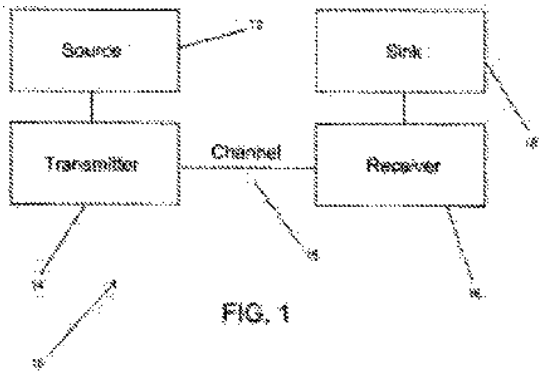


FIG. 1

geometrically shaped constellation changes as the code rate changes. One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes dmin.

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(71)出願人　509333531
パーソウム　マジュド　エフ.
アメリカ合衆国　カリフォルニア州　アーバイン　ドゥランゾ　アイル　３７４
(71)出願人　509333542
ジョーンズ　クリストファー　リチャード
アメリカ合衆国　カリフォルニア州　パシフィック　パリサデス　カレ　パトリシア　１５６３
(74)代理人　100106002
弁理士　正林　真之
(74)代理人　100120891
弁理士　林　一好

最終頁に続く

(54)【発明の名称】容量を最適化したコンステレーションで信号を送信する設計方法装置及び方法及び装置

(57)【要約】
同様のＳＮＲバンドの中で動作する従来のコンステレーションと比較して、増加した容量を有する幾何学的形状のコンステレーションを用いる通信システムについて説明する。いくつかの実施の形態では、幾何学的形状を、並列復号容量又は共通容量などの容量測定に基づいて最適化する。多くの実施の形態では、容量を最適化した幾何学的形状のコンステレーションを用いて、通信システムの内部の送信機と受信機のファームウェアアップデートの一部として従来のコンステレーションを交換することができる。いくつかの実施の形態では、幾何学的形状のコンステレーションを、加法性ホワイトガウスノイズチャネル又はフェージングチャネルについて最適化する。複数の実施の形態では、通信は適合型レート符号化を使用し、幾何学的形状のコンステレーションの内部のポイントの位置は、符号レートが変化するとともに変化する。本発明の一実施の形態は、通信チャネルを介して信号を受信機に送信するように構成された送信機を含み、送信機が、ユーザビットを受信して、拡張された出力符号化ビットレートで符号化ビットを出力するように構成

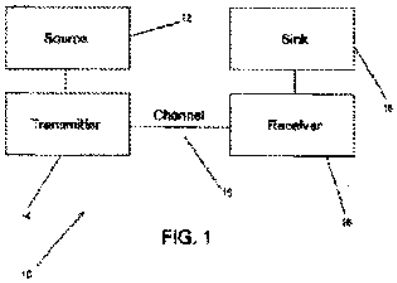


FIG. 1

【特許請求の範囲】
【請求項1】
通信チャネルを介して信号を受信機に送信するように構成された送信機を含むデジタル通信システムであって、
前記送信機が、
ユーザビットを受信して、拡張された出力符号化ビットレートで符号化ビットを出力するように構成された符号器と、
符号化ビットを記号コンステレーションの記号にマッピングするように構成されたマッパーと、
前記マッパーによって生成された記号を用いて前記通信チャネルを介して送信するための信号を生成するように構成された変調器とを有し、
前記受信機が、
前記通信チャネルを介して受信した信号を復調するように構成された復調器と、
復調された信号から尤度を推定するように構成されたデマッパーと、
前記デマッパーによって生成された尤度から復号されたビットを推定するように構成された復号器と、を有し、
前記記号コンステレーションが、d_{min}を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションである、デジタル通信システム。
【請求項2】
前記幾何学的に離間した記号コンステレーションは、追加的な制限に従って容量最適化されている、請求項１記載の通信システム。
【請求項3】
幾何学的形状の記号コンステレーションは、並列復号（ｐａｒａｌｌｅｌ　ｄｅｃｏｄｅ）を用いて容量について最適化されている、請求項１記載の通信システム。
【請求項4】
幾何学的形状の記号コンステレーションは、共通容量（ｊｏｉｎｔ　ｃａｐａｃｉｔｙ）を用いて容量について最適化されている、請求項１記載の通信システム。
【請求項5】
前記通信チャネルはＡＷＧＮチャネルである、請求項１記載の通信システム。
【請求項6】
前記通信チャネルはフェージングチャネルである、請求項１記載の通信システム。
【請求項7】
前記幾何学的に離間した記号コンステレーションのコンステレーションポイントは、ユーザレートに応じて可変である、請求項１記載の通信システム。
【請求項8】
通信チャネルを介してユーザ情報のビットを送信する方法であって、該方法は、
符号体系を用いて前記ユーザ情報のビットを符号化するステップと、
前記ユーザ情報の符号化ビットを記号コンステレーションにマッピングするステップであって、
前記記号コンステレーションは、d_{min}を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションであるステップと、
変調方式に従って、記号を変調するステップと、
前記通信チャネルを介して変調された信号を送信するステップと、
変調された信号を受信するステップと、
前記変調方式に従って前記変調された信号を復調するステップと、
尤度を生成するために前記幾何学的形状の信号コンステレーションを用いて復調された信号をデマッピングするステップと、

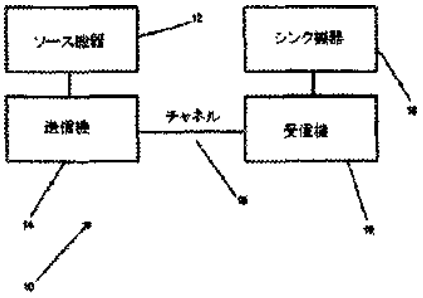
復号されたビットの推定値を取得するために前記尤度を復号するステップと、を含む方法。
【請求項９】
通信システムで使用される幾何学的形状の記号コンステレーションのポイントを選択する方法であって、該方法は、
適当なコンステレーションのサイズと、次元当たりの所望の容量を選択するステップと、
システムが動作する可能性のある初期ＳＮＲを推定するステップと、
d_{min}を最大化するコンステレーションに対して前記コンステレーションのＳＮＲ性能に所定の向上が達成されるまで、容量測定値を最大化するように前記コンステレーションのポイントの位置を反復的に最適化するステップと、を含む方法。
【請求項１０】
処理によって得られる通信システムで使用される幾何学的形状の記号コンステレーションであって、前記処理は、
適当なコンステレーションのサイズと、次元当たりの所望の容量を選択するステップと、
システムが動作する可能性のある初期ＳＮＲを推定するステップと、
d_{min}を最大化するコンステレーションに対して前記コンステレーションのＳＮＲ性能に所定の向上が達成されるまで、容量測定値を最大化するように前記コンステレーションのポイントの位置を反復的に最適化するステップと、を含む幾何学的形状の記号コンステレーション。
【請求項１１】
処理によって得られる通信システムで使用される幾何学的形状の記号コンステレーションであって、前記処理は、
適当なコンステレーションのサイズと、所望のＳＮＲを選択するステップと、
前記コンステレーションの容量測定値を最大化するように前記コンステレーションのポイントの位置を最適化するステップと、を含む幾何学的形状の記号コンステレーション。
【請求項１２】
情報を送信する方法であって、該方法は、
幾何学的（ｇｅｏｍｔｅｒｉｃａｌｌｙ）形状の記号コンステレーションを用いてチャネルを介して情報を送信するステップと、
前記幾何学的形状の記号コンステレーションの中のポイントの位置を変更して、目標ユーザデータレートを変更するステップと、を含む方法。
【発明の詳細な説明】
【技術分野】
【０００１】
<連邦政府による資金提供を受けた研究開発の記載>
本発明は一般的には、帯域幅、及び／又は電力効率の良いデジタル信号システムに関し、より詳細には、容量を増加させた不等間隔のコンステレーションの使用に関する。
【背景技術】
【０００２】
本発明は、ＮＡＳＡにより認可された契約第ＮＡＳ７－０３００１号の下で政府の支援を受けてなされた。政府は本発明に対し一定の権利を有する。
【０００３】
用語「コンステレーション」とは、一般的なデジタル通信システムによって送信することが可能な記号を説明するために用いる。受信機は、受信した信号をコンステレーションにマッピングすることによって送信された記号の検出を試みる。コンステレーションのポイントの間の最小距離（d_{min}）は、信号対ノイズ比（ＳＮＲ）が最大のコンステレーションの容量を示す。したがって、多くの通信システムで使用されるコンステレーションは、d_{min}を最大化するように設計される。コンステレーションの次元（ｄｉｍｅｎｓｉｏｎ）の数が増加すると、一次元当たりの一定のコンステレーションエネルギーに対す

る最小距離が増加する。
　したがって、優良な最小距離特性を有する複数の多次元のコンステレーションが設計されてきた。
【0004】
　通信システムはシャノン限界（Ｓｈａｎｎｏｎ　ｌｉｍｉｔ）として知られる、理論的な最大容量を有する。多くの通信システムは、符号を用いて通信チャネルの容量を増加させようとしている。ターボ符号及びＬＤＰＣ符号などの符号化技術を用いて顕著な符号化利得が達成されてきた。符号化技術を用いて達成できる符号化利得は、通信システムのコンステレーションによって制限されている。シャノン限界は、コンステレーションの中心部の記号をコンステレーションの端部の記号よりもより頻繁に送信する、無限のコンステレーションであるガウス分布として知られる、理論的なコンステレーションに基づいて考えることができる。
　実際のコンステレーションは有限であり、記号を同様の確率で送信するため、ガウス型の容量と比較して少ない容量を有する。コンステレーションの容量は、そのコンステレーションを用いた符号化を行った場合に得ることができる利得の限界を表すと考えられる。
【図面の簡単な説明】
【0015】
【図1】本発明の実施の形態に係る通信システムの概念図である。
【図2】本発明の実施の形態に係る送信機の概念図である。
【図3】本発明の実施の形態に係る受信機の概念図である。
【図4a】チャネルの並列復号容量の概念図である。
【図4b】チャネルの共通容量の概念図である。
【図5】本発明の実施の形態に係る、固定符号レートと変調方式を有する通信システムで使用するための、容量について最適化したコンステレーションを取得するための処理を示すフローチャートである。
【図6a】従来のＰＡＭ－２，４，８，１６，３２のガウス容量とＰＤ容量との比較を示すグラフである。
【図6b】従来のＰＡＭ－２，４，８，１６，３２のガウス容量と共通容量との比較を示すグラフである。
【図7】従来のＰＡＭ－２，４，８，１６，３２コンステレーションのＰＤ容量及び共通容量についてガウス容量に対するＳＮＲギャップを示すグラフである。
【図8a】従来と最適化されたＰＡＭ－２，４，８，１６，３２コンステレーションのＰＤ容量のガウス容量に対するＳＮＲギャップの比較を示すグラフである。
【図8b】従来と最適化されたＰＡＭ－２，４，８，１６，３２コンステレーションの共通容量のガウス容量に対するＳＮＲギャップの比較を示すグラフである。
【図9】複数の異なる長さのＬＤＰＣ符号を用いたシミュレーションで、従来とＰＤ容量を最適化したＰＡＭ－３２コンステレーションのフレームエラーレート性能を示すグラフである。
【図10a】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図10b】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図10c】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図10d】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

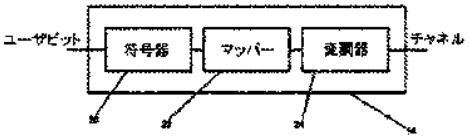
【図11a】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションの設計テーブルである。
【図11b】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションの設計テーブルである。
【図12a】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図12b】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図12c】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図12d】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図13a】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションの設計テーブルである。
【図13b】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションの設計テーブルである。
【図14a】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図14b】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図14c】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図14d】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図15a】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションの設計テーブルである。
【図15b】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションの設計テーブルである。
【図16a】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図16b】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図16c】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図16d】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図17a】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションの設計テーブルである。

【図17b】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションの設計テーブルである。
【図18】従来と容量最適化ＰＳＫコンステレーションのガウス容量に対するＳＮＲギャップを示すグラフである。
【図19】ＰＤ容量最適化ＰＳＫ－３２コンステレーションのコンステレーションポイントの位置を示すグラフである。
【図20】本発明の実施の形態に係る、異なるＳＮＲにおける、ＰＤ容量を最適化した一連のＰＳＫ－３２コンステレーションを示す図である。
【図21】本発明の実施の形態に係る、二つのＰＤ最適化ＰＡＭ－８コンステレーションの直交デカルト積（ｏｒｔｈｏｇｏｎａｌ　Ｃａｒｔｅｓｉａｎ　ｐｒｏｄｕｃｔ）から構築されるＱＡＭ－６４を示す図である。
【図22a】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図22b】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図23a】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図23b】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図24a】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図24b】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

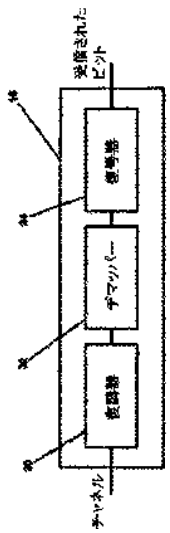
【図１】



【図２】



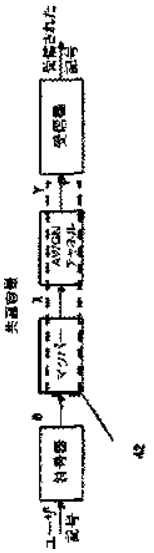
【図３】



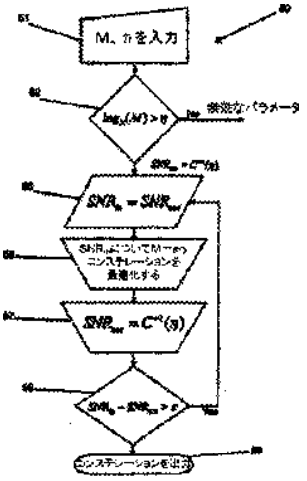
【図4a】



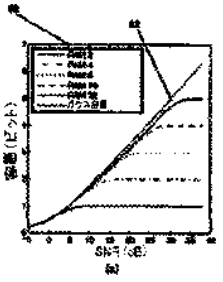
【図4b】



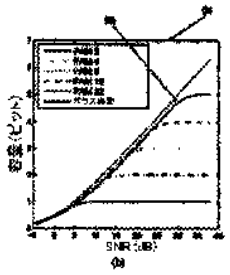
【図５】



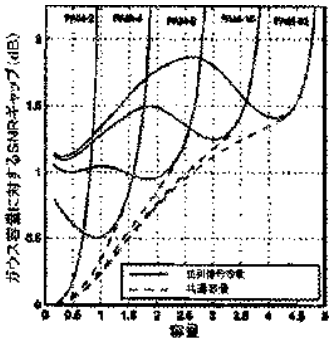
【図6a】



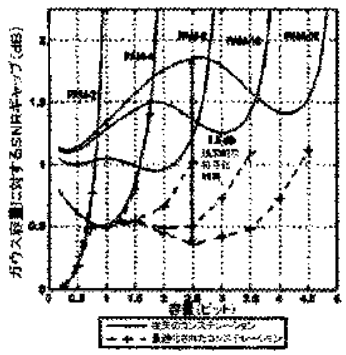
【図６ｂ】



【図７】



【図8a】



【図8b】

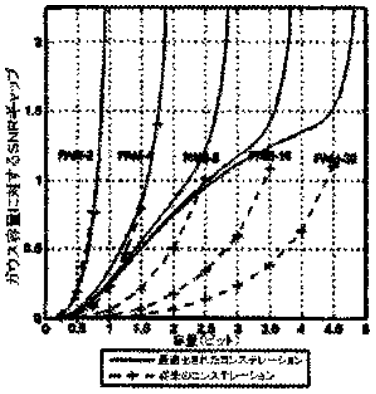
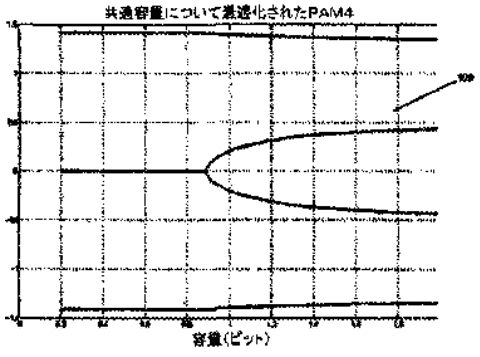


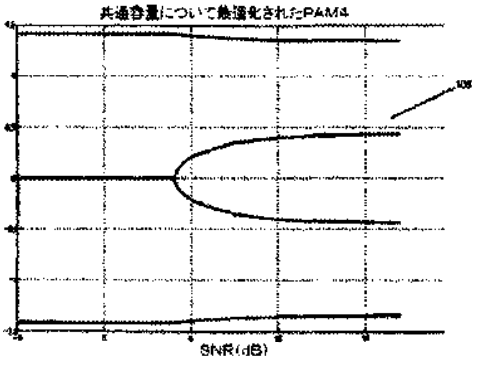
FIG. 9

【図9】

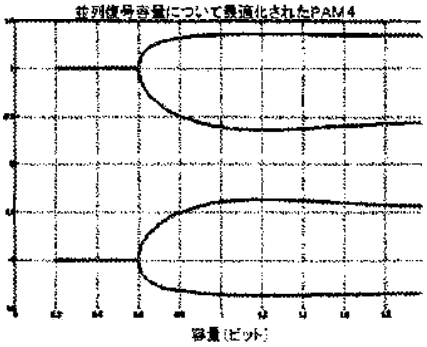
【図10a】



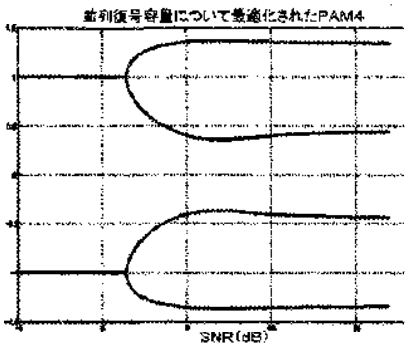
【図10b】



【図10c】



【図10d】



【図11a】

異なるレートで共通容量について最適化されたPAM－4コンステレーション

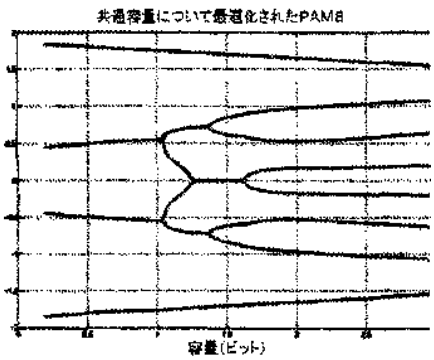
(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.03	2.71	5.00	7.15	9.24
a_0	-1.41	-1.41	-1.40	-1.37	-1.36
a_1	0.00	0.00	-0.20	-0.33	-0.39
a_2	0.00	0.00	0.20	0.33	0.39
a_3	1.41	1.41	1.40	1.37	1.36

【図11b】

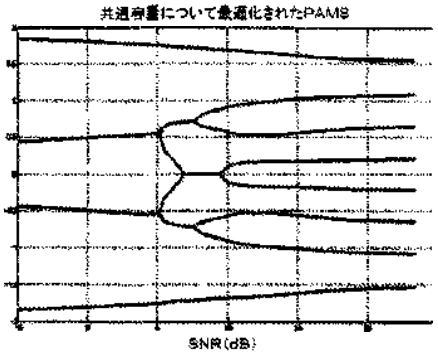
異なるレートで並列復号容量について最適化されたPAM－4コンステレーション

(bps)	0.50	0.75	1.00	1.25	1.50
(SNR)	0.19	3.11	5.56	7.22	9.25
a_0	-1.00	-1.30	-1.36	-1.37	-1.36
a_1	-1.00	-0.56	-0.39	-0.33	-0.39
a_2	1.00	1.30	1.36	0.33	1.36
a_3	1.00	0.56	0.39	1.37	0.39

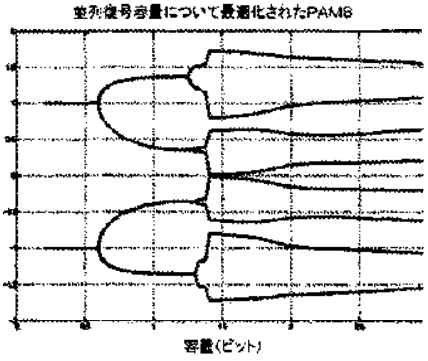
【図12a】



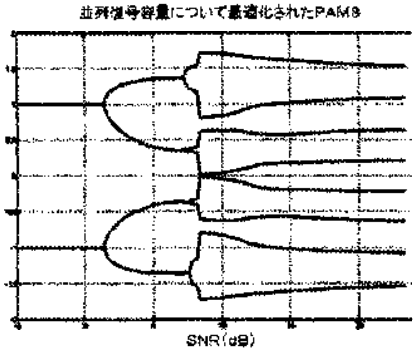
【図12b】



【図12c】



【図12d】



【図13a】

異なるレートで共通容量について最適化されたPAM－8コンステレーション

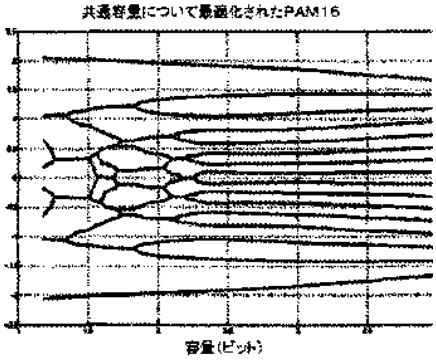
(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.00	4.82	8.66	12.26	15.93
a_0	-1.81	-1.76	-1.70	-1.66	-1.60
a_1	-0.50	-0.55	-0.84	-0.97	-1.03
a_2	-0.50	-0.55	-0.63	-0.53	-0.58
a_3	-0.50	-0.55	-0.00	-0.17	-0.19
a_4	0.50	0.55	0.00	0.17	0.19
a_5	0.50	0.55	0.63	0.53	0.58
a_6	0.50	0.55	0.84	0.97	1.03
a_7	1.81	1.76	1.70	1.66	1.60

【図13b】

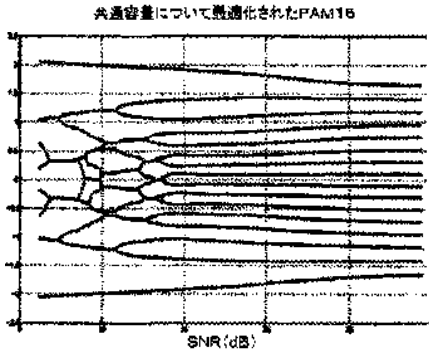
異なるレートで並列復号容量について最適化されたPAM－8コンステレーション

(bps)	0.5	1.0	1.5	2.0	2.5
(SNR)	0.19	5.27	9.00	12.42	15.93
a_0	-1.00	-1.36	-1.72	-1.64	-1.60
a_1	-1.00	-1.36	-0.81	-0.97	-1.03
a_2	-1.00	-0.39	1.72	1.64	-0.19
a_3	-1.00	-0.39	-0.62	-0.58	-0.58
a_4	1.00	1.36	0.62	0.58	1.60
a_5	1.00	1.36	0.02	0.15	1.03
a_6	1.00	0.39	0.81	0.97	0.19
a_7	1.00	0.39	-0.02	-0.15	0.58

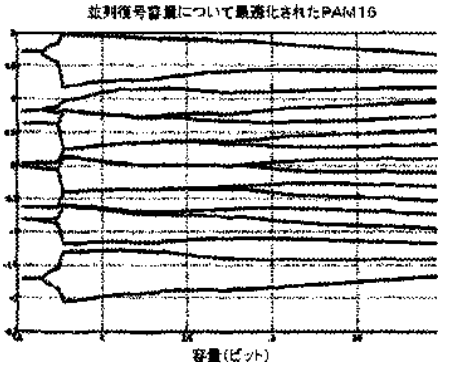
【図14a】



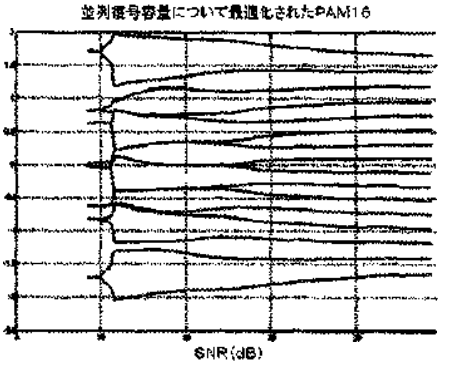
【図14b】



【図14c】



【図14d】



【図15a】

異なるレートで共通容量について最適化されたPAM－16コンステレーション

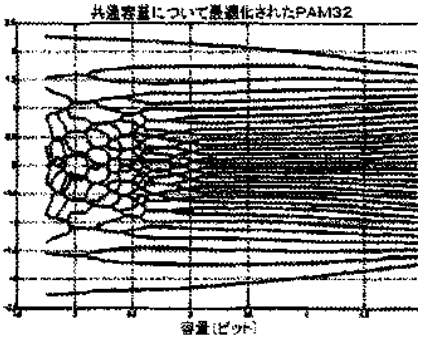
(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	8.52	11.04	13.25	15.68	22.12
a_0	-2.02	-1.99	-1.91	-1.89	-1.79
a_1	-1.19	-1.23	-1.40	-1.42	-1.42
a_2	-1.16	-1.16	-1.05	-1.15	-1.15
a_3	-0.98	-0.69	-0.82	-0.84	-0.90
a_4	-0.15	-0.69	-0.68	-0.62	-0.68
a_5	-0.24	-0.40	-0.43	-0.42	-0.47
a_6	-0.24	-0.17	-0.24	-0.26	-0.28
a_7	-0.24	-0.17	-0.09	-0.08	-0.09
a_8	0.24	0.17	0.09	0.02	0.09
a_9	0.24	0.17	0.24	0.26	0.28
a_{10}	0.24	0.40	0.43	0.42	0.47
a_{11}	0.24	0.69	0.68	0.62	0.68
a_{12}	0.98	0.69	0.82	0.84	0.90
a_{13}	1.16	1.16	1.05	1.15	1.15
a_{14}	1.16	1.23	1.40	1.42	1.42
a_{15}	2.02	1.99	1.91	1.89	1.79

【図15b】

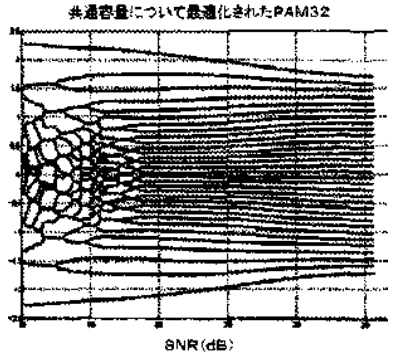
異なるレートで並列復号容量について最適化されたPAM－16コンステレーション

(bps)	1.5	2.0	2.5	3.0	3.5
(SNR)	9.80	12.35	15.42	18.72	21.19
a_0	-1.72	-1.98	-1.85	-1.84	-1.75
a_1	-1.72	-1.29	-1.35	-1.42	-1.42
a_2	-0.81	1.94	1.89	1.84	1.75
a_3	-0.81	-1.19	-1.14	-1.17	-1.15
a_4	1.72	-0.98	-0.25	-0.40	-0.47
a_5	1.72	-0.65	-0.70	-0.65	-0.68
a_6	-0.62	-0.98	-0.34	-0.29	-0.28
a_7	-0.62	-0.65	-0.16	-0.02	-0.09
a_8	0.62	1.89	1.15	1.11	1.15
a_9	0.62	0.75	0.76	0.84	0.90
a_{10}	0.62	1.26	1.35	1.42	1.42
a_{11}	0.62	0.75	0.70	0.63	0.68
a_{12}	-0.81	0.66	0.60	0.05	0.09
a_{13}	0.81	0.29	0.34	0.29	0.28
a_{14}	-0.62	0.06	0.60	-0.05	-0.09
a_{15}	-0.62	0.29	0.35	0.40	0.47

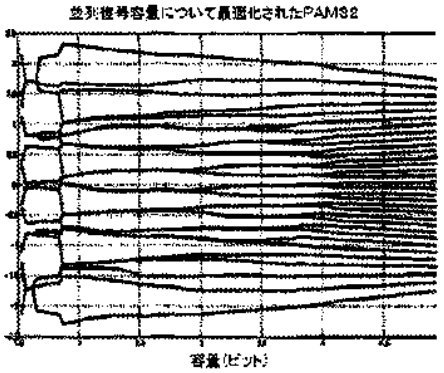
【図16a】



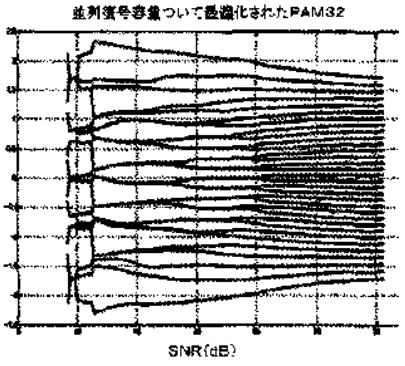
【図16b】



【図16c】



【図16d】



【図17a】

異なるレートで共通容量について最適化されたPAM－32コンステレーション

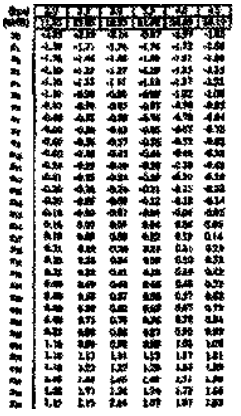


FIG. 20

【図20】

【図21】

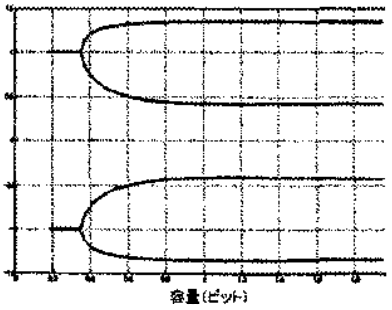
【図19】

【図18】

【図17】

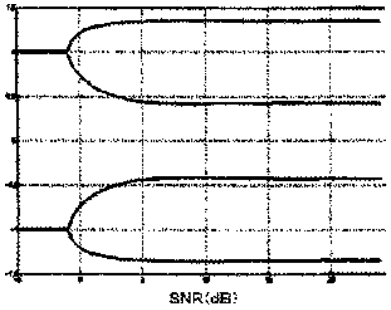
【図22a】

フェージングチャネルにわたって並列復号容量について符号化されたPAM4



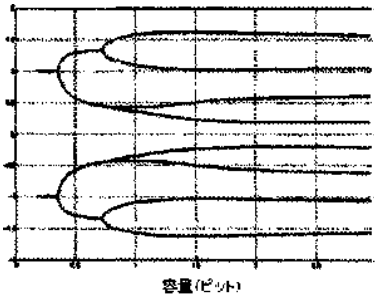
【図22b】

フェージングチャネルにわたって並列復号容量について符号化されたPAM4



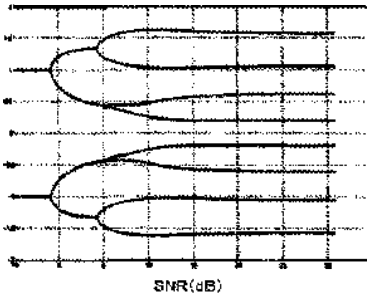
【図23a】

フェージングチャネルにわたって並列復号容量について符号化されたPAM8



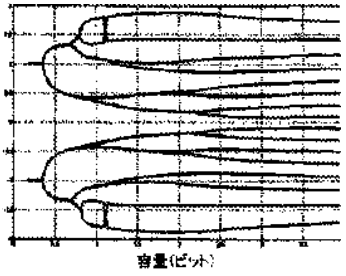
【図23b】

フェージングチャネルにわたって並列復号容量について符号化されたPAM8



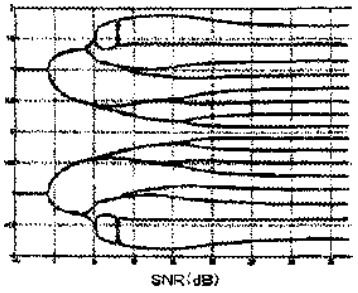
【図24a】

フェージングチャネルにわたって並列復号容量について符号化されたPAM16



【図24b】

フェージングチャネルにわたって並列復号容量について符号化されたPAM16



【手続補正書】
【提出日】平成21年6月5日(2009.6.5)
【手続補正1】
【補正対象書類名】特許請求の範囲
【補正対象項目名】全文
【補正方法】変更
【補正の内容】
【特許請求の範囲】
【請求項1】
通信チャネルを介して信号を受信機に送信するように構成された送信機を含むデジタル通信システムであって、
前記送信機が、
ユーザビットを受信して、拡張された出力符号化ビットレートで符号化ビットを出力するように構成された符号器と、
符号化ビットを記号コンステレーションの記号にマッピングするように構成されたマッパーと、
前記マッパーによって生成された記号を用いて前記通信チャネルを介して送信するための信号を生成するように構成された変調器とを有し、
前記受信機が、
前記通信チャネルを介して受信した信号を復調するように構成された復調器と、
復調された信号から尤度を推定するように構成されたデマッパーと、
前記デマッパーによって生成された尤度から復号されたビットを推定するように構成された復号器と、を有し、
前記記号コンステレーションが、$d_{m,n}$を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションである、デジタル通信システム。
【請求項2】
前記幾何学的に離間した記号コンステレーションは、追加的な制限に従って容量最適化されている、請求項１記載の通信システム。
【請求項3】
幾何学的形状の記号コンステレーションは、並列復号（ｐａｒａｌｌｅｌ　ｄｅｃｏｄｅ）を用いて容量について最適化されている、請求項１記載の通信システム。
【請求項4】
幾何学的形状の記号コンステレーションは、共通容量（ｊｏｉｎｔ　ｃａｐａｃｉｔｙ）を用いて容量について最適化されている、請求項１記載の通信システム。
【請求項5】
前記通信チャネルはＡＷＧＮチャネルである、請求項１記載の通信システム。
【請求項6】
前記通信チャネルはフェージングチャネルである、請求項１記載の通信システム
【請求項7】
前記幾何学的に離間した記号コンステレーションのコンステレーションポイントは、ユーザレートに応じて可変である、請求項１記載の通信システム。
【請求項8】
通信チャネルを介してユーザ情報のビットを送信機と受信機の間で送信する方法であって、該方法は、
前記送信機の中の符号器を用いて、符号体系に従って前記ユーザ情報のビットを符号化するステップと、
前記送信機の中のマッパーを用いて、前記ユーザ情報の符号化ビットを記号コンステレーションにマッピングするステップであって、
前記記号コンステレーションは、$d_{m,n}$を最大化する信号コンステレーションと比較

して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションであるステップと、

前記送信機の中の変調器を用いて、変調方式に従って、記号を変調するステップと、

前記通信チャネルを介して変調された信号を前記送信機を用いて送信するステップと、

変調された信号を受信機で受信するステップと、

前記受信機の中の復調器を用いて、前記変調方式に従って前記変調された信号を復調するステップと、

前記受信機の中のデマッパーを用いて、尤度を生成するために幾何学的形状の信号コンステレーションを用いて復調された信号をデマッピングするステップと、

前記受信機を用いて、復号されたビットの推定値を取得するために前記尤度を復号するステップと、を含む方法。

【請求項9】

コンピューティング装置を用いて、通信システムにおいて送信機のマッパーと受信機のデマッパーで使用するために、幾何学的形状の記号コンステレーションのポイントを選択する方法であって、該方法は、

前記コンピューティング装置のメモリに、選択されたコンステレーションのサイズを記憶するステップと、

前記コンピューティング装置を用いて、記憶したコンステレーションのサイズを用いて容量測定値を最大化するように前記コンステレーションのポイントの位置を反復的に最適化するステップと、を含み、

前記反復的に最適化するステップを所定の終了条件が満たされるまで実行して、結果として、d_{min}を最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する幾何学的に離間した記号コンステレーションとする、方法。

【請求項10】

容量最適化された幾何学的形状の記号コンステレーションを用いて、変調のためにユーザ情報の符号化ビットを記号にマッピングするように構成されたマッパーを含む送信機であって、

前記幾何学的形状の記号コンステレーションは、処理によって得られ、前記処理は、

コンピューティング装置のメモリに、選択されたコンステレーションのサイズと所望のＳＮＲを記憶するステップと、

前記コンピューティング装置を用いて、記憶した前記コンステレーションのサイズと前記所望のＳＮＲを用いて前記コンステレーションの容量測定値を最大化するように前記コンステレーションのポイントの位置を最適化するステップであって、最適化された前記コンステレーションのポイントは幾何学的に離間し、d_{min}を最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する、ステップと、

最適化によってコンピュータから得られた前記コンステレーションのポイントを用いて、前記マッパーを構成設定するステップと、を含む方法。

【請求項11】

通信チャネルを介してユーザ情報のビットを送信機と受信機の間で送信する方法であって、該方法は、

初期目標ユーザデータレートに到達するように、固定数のコンステレーションポイントを有する記号コンステレーションの内部のポイントの位置を決定するステップと、

前記記号コンステレーションを用いて前記送信機と前記受信機とを構成設定するステップと、

前記送信機を用いて、

符号体系に従ってユーザ情報のビットを符号化するステップと、

前記ユーザ情報の符号化ビットを、前記記号コンステレーションにマッピングするステップと、

変調方式に従って記号を変調するステップと、

前記通信チャネルを介して変調された信号を送信機に送信するステップと、

前記受信機を用いて，

前記送信機から送信される変調された信号を受信するステップと、

前記変調方式に前記変調された信号を復調するステップと，

尤度を生成するために信号コンステレーションを用いて復調された信号をデマッピングするステップと、

復号されたビットの推定値を取得するために前記尤度を復号するステップと、

前記記号コンステレーションの内部の固定数のポイントの位置を変更して目標ユーザデータレートを変更するステップと、

前記ユーザ情報のビットの送信時に、変更された記号コンステレーションを用いて前記送信機と前記受信機とを構成設定するステップと、を含む方法．

【請求項12】

容量最適化された幾何学的形状のコンステレーションを用いて、変調された信号から尤度を生成するように構成されたデマッパーを有する受信機であって、

前記幾何学的形状のコンステレーションは、処理によって得られ、前記処理は、

コンピューティング装置のメモリに、選択されたコンステレーションのサイズと所望のＳＮＲを記憶するステップと、

前記コンピューティング装置を用いて、記憶した前記コンステレーションのサイズと前記所望のＳＮＲを用いて前記コンステレーションの容量測定値を最大化するように前記コンステレーションのポイントの位置を最適化するステップであって、最適化された前記コンステレーションのポイントは幾何学的に離間し、d_{min}を最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する、ステップと、

最適化によってコンピュータから得られた前記コンステレーションのポイントを用いて、前記デマッパーを構成設定するステップと、を含む受信機．

【請求項13】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項８又は９に記載の方法．

【請求項14】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項８又は９に記載の方法．

【請求項15】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項１０に記載の送信機．

【請求項16】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項１０に記載の送信機．

【請求項17】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項１２に記載の受信機．

【請求項18】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項１２に記載の受信機．

【請求項19】

前記幾何学的に離間した記号コンステレーションのコンステレーションポイントは、ユーザレートに応じて可変である、請求項１記載の通信システム．

【請求項20】

前記送信機が、前記通信システムのユーザビットレートを変更するために、符号レートを変更するように構成され、

前記送信機が、前記マッパーにより使用される前記記号コンステレーションのコンステレーションポイントを前記符号器の前記符号レートに基づいて移動するように構成され、

前記受信機が、符号レートの変更に関する情報を前記送信機から受信するように構成さ

れ、
　前記受信機が、前記ユーザレートの変更の通知の受信に応じて、前記デマッパーによって使用されるコンステレーションと前記復号器とを変更するように構成される、請求項１９記載の通信システム。
【請求項21】
　前記受信機が、前記通信システムの前記コンステレーションと前記符号レートの変更の受信を応答確認するように構成され、
　前記送信機は、前記受信機からの応答確認を受信すると、新しい符号レートと記号コンステレーションを用いて送信を開始するように構成される、請求項２０記載の通信システム。
【請求項22】
　前記送信機と前記受信機との間にはバックチャネル通信リンクが存在し、
　前記受信機は、受信機性能を監視し、観察された受信機性能に応じて、符号レートの変更を開始するように構成され、
　前記受信機は、前記バックチャネル通信リンクを介して前記送信機にユーザビットレートの変更を送信するように構成され、
　前記マッパーと前記デマッパーは、幾何学的形状のコンステレーションの前記コンステレーションポイントを、前記ユーザビットレートの変更に基づいて移動するように構成されている、請求項２０記載の通信システム。

INTERNATIONAL SEARCH REPORT

International application No.
PCT/US2008/065894

A. CLASSIFICATION OF SUBJECT MATTER
IPC(8) - H04J11/00 (2008.04)
USPC - 370/207
According to International Patent Classification (IPC) or to both national classification and IPC

B. FIELDS SEARCHED

Minimum documentation searched (classification system followed by classification symbols)
IPC(8) - H04J 11/00 (2008.04)
USPC - 370/207

Documentation searched other than minimum documentation to the extent that such documents are included in the fields searched

Electronic data base consulted during the international search (name of data base and, where practicable, search terms used)
PatBase, Google Patents

C DOCUMENTS CONSIDERED TO BE RELEVANT

Category*	Citation of document, with indication, where appropriate, of the relevant passages	Relevant to claim No.
X ---- Y	US 2004/0022179 A1 (GIANNAKIS et al) 05 February 2004 (05.02.2004) entire document	9-11 ---- 1-8, 12
Y	US 2007/0116161 A1 (TOKORO et al) 24 May 2007 (24.05.2007) entire document	1-8, 12
Y	US 6,603,801 B1 (ANDREN et al) 05 August 2003 (05.08.2003) entire document	4
Y	US 2006/0156843 A1 (GLASS et al) 18 July 2006 (13.07.2006) entire document	3

☐ Further documents are listed in the continuation of Box C.　☐

* Special categories of cited documents:
"A" document defining the general state of the art which is not considered to be of particular relevance
"E" earlier application or patent but published on or after the international filing date
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"O" document referring to an oral disclosure, use, exhibition or other means
"P" document published prior to the international filing date but later than the priority date claimed
"T" later document published after the international filing date or priority date and not in conflict with the application but cited to understand the principle or theory underlying the invention
"X" document of particular relevance; the claimed invention cannot be considered novel or cannot be considered to involve an inventive step when the document is taken alone
"Y" document of particular relevance; the claimed invention cannot be considered to involve an inventive step when the document is combined with one or more other such documents, such combination being obvious to a person skilled in the art
"&" document member of the same patent family

Date of the actual completion of the international search	Date of mailing of the international search report
03 October 2008	22 OCT 2008
Name and mailing address of the ISA/US Mail Stop PCT, Attn: ISA/US, Commissioner for Patents P.O. Box 1450, Alexandria, Virginia 22313-1450 Facsimile No. 571-273-3201	Authorized officer: Blaine R. Copenheaver PCT Helpdesk: 571-272-4300 PCT OSP: 571-272-7774

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(72)発明者　パーソウム　マジェド　エフ.
アメリカ合衆国　カリフォルニア州　アーバイン　ドゥランゾ　アイル　３７４
(72)発明者　ジョーンズ　クリストファー　リチャード
アメリカ合衆国　カリフォルニア州　パシフィック　パリサデス　カレ　パトリシア　１５６３
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【要約の続き】
された符号器と、符号化ビットを記号コンステレーションの記号にマッピングするように構成されたマッパーと、マッパーによって生成された記号を用いて通信チャネルを介して送信するための信号を生成するように構成された変調器とを有する。さらに、受信機が、通信チャネルを介して受信した信号を復調するように構成された復調器と、復調された信号から尤度を推定するように構成されたデマッパーと、デマッパーによって生成された尤度から復号されたビットを推定するように構成された復号器とを有する。さらに、記号コンステレーションが、d_{min}を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションである。
【選択図】図１

Espacenet

Bibliographic data: JP2010538502 (A) — 2010-12-09

DESIGN METHODOLOGY AND METHOD AND APPARATUS FOR SIGNALING WITH CAPACITY OPTIMIZED CONSTELLATIONS

Inventor(s):

Applicant(s):

Classification: - **international:** *H04L27/34*
- **cooperative:** H04B15/00; H04L1/0003; H04L1/0009; H04L27/3405; H04L27/3483; H04W24/02

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Abstract not available for JP2010538502 (A)
Abstract of corresponding document: WO2008151308 (A1)

Communication systems are described that use geometrically shaped constellations that have increased capacity compared to conventional constellations operating within a similar SNR band. In several embodiments, the geometrically shaped is optimized based upon a capacity measure such as parallel decoding capacity or joint capacity. In many embodiments, a capacity optimized geometrically shaped constellation can be used to replace a conventional constellation as part of a firmware upgrade to transmitters and receivers within a communication system. In a number of

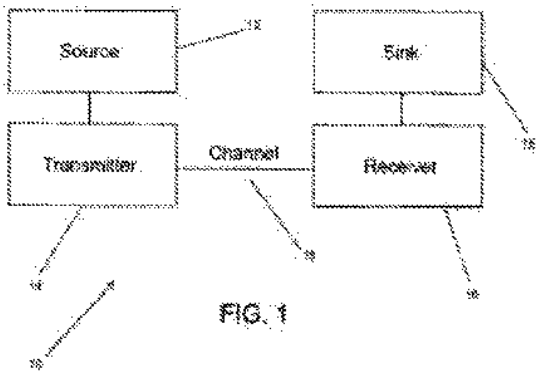


FIG. 1

embodiments, the geometrically shaped constellation is optimized for an Additive White Gaussian Noise channel or a fading channel. In numerous embodiments, the communication uses adaptive rate encoding and the location of points within the geometrically shaped constellation changes as the code rate changes. One embodiment of the invention includes a transmitter configured to transmit signals to a receiver via a communication channel, wherein the transmitter, includes a coder configured to receive user bits and output encoded bits at an expanded output encoded bit rate, a mapper configured to map encoded bits to symbols in a symbol constellation, a modulator configured to generate a signal for transmission via the communication channel using symbols generated by the mapper. In addition, the receiver includes a demodulator configured to demodulate the received signal via the communication channel, a demapper configured to estimate likelihoods from the demodulated signal, a decoder that is configured to estimate decoded bits from the likelihoods generated by the demapper. Furthermore, the symbol constellation is a capacity optimized geometrically spaced symbol constellation that provides a given capacity at a reduced signal-to-noise ratio compared to a signal constellation that maximizes dmin.

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(73)特許権者　509333531
パーソウム　マジュド　エフ.
アメリカ合衆国　カリフォルニア州　アーバイン　ドゥランゾ　アイル　３７４
(73)特許権者　509333542
ジョーンズ　クリストファー　リチャード
アメリカ合衆国　カリフォルニア州　パシフィック　パリサデス　カレ　パトリシア　１５６３
(74)代理人　100106002
弁理士　正林　真之
(74)代理人　100120891
弁理士　林　一好

最終頁に続く

(54)【発明の名称】容量を最適化したコンステレーションで信号を送信する設計方法論及び方法及び装置

(57)【特許請求の範囲】
【請求項１】
通信チャネルを介して信号を受信機に送信するように構成された送信機を含むデジタル通信システムであって、
前記送信機が、
ユーザビットを受信して、拡張された出力符号化ビットレートで符号化ビットを出力するように構成された符号器と、
符号化ビットを記号コンステレーションの記号にマッピングするように構成されたマッパーと、
前記マッパーによって生成された記号を用いて前記通信チャネルを介して送信するための信号を生成するように構成された変調器とを有し、
前記受信機が、
前記通信チャネルを介して受信した信号を復調するように構成された復調器と、
復調された信号から尤度を推定するように構成されたデマッパーと、
前記デマッパーによって生成された尤度から復号されたビットを推定するように構成された復号器と、を有し、
前記記号コンステレーションに含まれるポイント（信号点）の位置は、コンステレーションの容量測定値が最大となるように最適化され、かつ、これらのポイント（信号点）は当該記号コンステレーション上で幾何学的に離間し、そしてこの最適化された記号コンステレーションは、最小ポイント間距離ｄｍｉｎが最大化された従来の信号コンステレーシ

ョンと比較して、より低い信号対ノイズ比で所与の容量を提供するものであることを特徴とする、デジタル通信システム。

【請求項２】

前記幾何学的に離間した記号コンステレーションは、追加的な制限に従って容量最適化されている、請求項１記載の通信システム。

【請求項３】

幾何学的形状の記号コンステレーションは、並列復号（ｐａｒａｌｌｅｌ　ｄｅｃｏｄｅ）を用いて容量について最適化されている、請求項１記載の通信システム。

【請求項４】

幾何学的形状の記号コンステレーションは、共通容量（ｊｏｉｎｔ　ｃａｐａｃｉｔｙ）を用いて容量について最適化されている、請求項１記載の通信システム。

【請求項５】

前記通信チャネルはＡＷＧＮチャネルである、請求項１記載の通信システム。

【請求項６】

前記通信チャネルはフェージングチャネルである、請求項１記載の通信システム。

【請求項７】

前記幾何学的に離間した記号コンステレーションのコンステレーションポイントは、ユーザレートに応じて可変である、請求項１記載の通信システム。

【請求項８】

通信チャネルを介してユーザ情報のビットを送信機と受信機の間で送信する方法であって、該方法は、

前記送信機の中の符号器を用いて、符号体系に従って前記ユーザ情報のビットを符号化するステップと、

前記送信機の中のマッパーを用いて、前記ユーザ情報の符号化ビットを記号コンステレーションにマッピングするステップであって、

前記記号コンステレーションに含まれるポイント（信号点）の位置は、コンステレーションの容量測定値が最大となるように最適化され、かつ、これらのポイント（信号点）は当該記号コンステレーション上で幾何学的に離間し、そしてこの最適化された記号コンステレーションは、最小ポイント間距離ｄｍｉｎが最大化された従来の信号コンステレーションと比較して、より低い信号対ノイズ比で所与の容量を提供するステップと、

前記送信機の中の変調器を用いて、変調方式に従って、記号を変調するステップと、

前記通信チャネルを介して変調された信号を前記送信機を用いて送信するステップと、

変調された信号を受信機で受信するステップと、

前記受信機の中の復調器を用いて、前記変調方式に従って前記変調された信号を復調するステップと、

前記受信機の中のデマッパーを用いて、尤度を生成するために幾何学的形状の信号コンステレーションを用いて復調された信号をデマッピングするステップと、

前記受信機を用いて、復号されたビットの推定値を取得するために前記尤度を復号するステップと、を含む方法。

【請求項９】

コンピューティング装置を用いて、通信システムにおいて送信機のマッパーと受信機のデマッパーで使用するために、幾何学的形状の記号コンステレーションのポイントを選択する方法であって、該方法は、

前記コンピューティング装置のメモリに、選択されたコンステレーションのサイズを記憶するステップと、

前記コンピューティング装置を用いて、記憶したコンステレーションのサイズを用いて容量測定値を最大化するように前記コンステレーションのポイントの位置を反復的に最適化するステップと、を含み、

前記反復的に最適化するステップによる結果として、ｄｍｉｎを最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する幾何学的に離

間した記号コンステレーションとする、方法。
【請求項１０】
容量最適化された幾何学的形状の記号コンステレーションを用いて、変調のためにユーザ情報の符号化ビットを記号にマッピングするように構成されたマッパーを含む送信機であって、
前記幾何学的形状の記号コンステレーションは、処理によって得られ、前記処理は、
コンピューティング装置のメモリに、選択されたコンステレーションのサイズと所望のＳＮＲを記憶するステップと、
前記コンピューティング装置を用いて、記憶した前記コンステレーションのサイズと前記所望のＳＮＲを用いて前記コンステレーションの容量測定値を最大化するように前記コンステレーションのポイントの位置を最適化するステップであって、最適化された前記コンステレーションのポイントは幾何学的に離間し、ｄｍｉｎを最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する、ステップと、
最適化によってコンピュータから得られた前記コンステレーションのポイントを用いて、前記マッパーを構成設定するステップと、を含む方法。
【請求項１１】
通信チャネルを介してユーザ情報のビットを送信機と受信機の間で送信する方法であって、該方法は、
初期目標ユーザデータレートに到達するように、固定数のコンステレーションポイントを有する記号コンステレーションの内部のポイントの位置を決定し、前記記号コンステレーションのポイントの位置が、前記固定数のポイント及び初期目標ユーザデータレートに基づいてコンステレーションの容量測定値を最大化するように最適化され、前記最適化されたコンステレーションの位置は幾何学的に離間し、ｄｍｉｎを最大化する信号コンステレーションと比較して低減された信号対ノイズ比で所与の容量を提供するステップと、
前記記号コンステレーションを用いて前記送信機と前記受信機とを構成設定するステップと、
前記送信機を用いて、
符号体系に従ってユーザ情報のビットを符号化するステップと、
前記ユーザ情報の符号化ビットを、前記記号コンステレーションにマッピングするステップと、
変調方式に従って記号を変調するステップと、
前記通信チャネルを介して変調された信号を送信機に送信するステップと、
前記受信機を用いて、
前記送信機から送信される変調された信号を受信するステップと、
前記変調方式に前記変調された信号を復調するステップと、
尤度を生成するために信号コンステレーションを用いて復調された信号をデマッピングするステップと、
復号されたビットの推定値を取得するために前記尤度を復号するステップと、
前記記号コンステレーションの内部の固定数のポイントの位置を変更して目標ユーザデータレートを変更し、前記記号コンステレーションのポイントの位置が、前記固定数のポイント及び前記目標ユーザデータレートに基づいて前記コンステレーションの容量測定値を最大化するように最適化され、前記最適化されたコンステレーションの位置は幾何学的に離間し、ｄｍｉｎを最大化する信号コンステレーションと比較して低減された信号対ノイズ比で所与の容量を提供するステップと、
前記ユーザ情報のビットの送信時に、変更された記号コンステレーションを用いて前記送信機と前記受信機とを構成設定するステップと、を含む方法。
【請求項１２】
容量最適化された幾何学的形状のコンステレーションを用いて、変調された信号から尤度を生成するように構成されたデマッパーを有する受信機であって、
前記幾何学的形状のコンステレーションは、処理によって得られ、前記処理は、

コンピューティング装置のメモリに、選択されたコンステレーションのサイズと所望のＳＮＲを記憶するステップと、

前記コンピューティング装置を用いて、記憶した前記コンステレーションのサイズと前記所望のＳＮＲを用いて前記コンステレーションの容量測定値を最大化するように前記コンステレーションのポイントの位置を最適化するステップであって、最適化された前記コンステレーションのポイントは幾何学的に離間し、ｄｍｉｎを最大化する信号コンステレーションと比較して削減された信号対ノイズ比で所与の容量を提供する、ステップと、

最適化によってコンピュータから得られた前記コンステレーションのポイントを用いて、前記デマッパーを構成設定するステップと、を含む受信機。

【請求項１３】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項８又は９に記載の方法。

【請求項１４】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項８又は９に記載の方法。

【請求項１５】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項１０に記載の送信機。

【請求項１６】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項１０に記載の送信機。

【請求項１７】

前記幾何学的形状のコンステレーションは、並列復号容量を用いて容量について最適化されている、請求項１２に記載の受信機。

【請求項１８】

前記幾何学的形状のコンステレーションは、共通容量を用いて容量について最適化されている、請求項１２に記載の受信機。

【請求項１９】

前記幾何学的に離間した記号コンステレーションのコンステレーションポイントは、ユーザレートに応じて可変である、請求項１記載の通信システム。

【請求項２０】

前記送信機が、前記通信システムのユーザビットレートを変更するために、符号レートを変更するように構成され、

前記送信機が、前記マッパーにより使用される前記記号コンステレーションのコンステレーションポイントを前記符号器の前記符号レートに基づいて移動するように構成され、

前記受信機が、符号レートの変更に関する情報を前記送信機から受信するように構成され、

前記受信機が、前記ユーザレートの変更の通知の受信に応じて、前記デマッパーによって使用されるコンステレーションと前記復号器とを変更するように構成される、請求項１９記載の通信システム。

【請求項２１】

前記受信機が、前記通信システムの前記コンステレーションと前記符号レートの変更の受信を応答確認するように構成され、

前記送信機は、前記受信機からの応答確認を受信すると、新しい符号レートと記号コンステレーションを用いて送信を開始するように構成される、請求項２０記載の通信システム。

【請求項２２】

前記送信機と前記受信機との間にはバックチャネル通信リンクが存在し、

前記受信機は、受信機性能を監視し、観察された受信機性能に応じて、符号レートの変更を開始するように構成され、

前記受信機は、前記バックチャネル通信リンクを介して前記送信機にユーザビットレートの変更を送信するように構成され、
前記マッパーと前記デマッパーは、幾何学的形状のコンステレーションの前記コンステレーションポイントを、前記ユーザビットレートの変更に基づいて移動するように構成されている、請求項２０記載の通信システム。

【発明の詳細な説明】
【技術分野】
【０００１】
＜連邦政府による資金提供を受けた研究開発の記載＞
本発明は一般的には、帯域幅、及び／又は電力効率の良いデジタル信号システムに関し、より詳細には、容量を増加させた不等間隔のコンステレーションの使用に関する。
【背景技術】
【０００２】
本発明は、ＮＡＳＡにより認可された契約第ＮＡＳ７－０３００１号の下で政府の支援を受けてなされた。政府は本発明に対し一定の権利を有する。
【０００３】
用語「コンステレーション」とは、一般的なデジタル通信システムによって送信することが可能な記号を説明するために用いる。受信機は、受信した信号をコンステレーションにマッピングすることによって送信された記号の検出を試みる。コンステレーションのポイントの間の最小距離（d_{min}）は、信号対ノイズ比（ＳＮＲ）が最大のコンステレーションの容量を示す。したがって、多くの通信システムで使用されるコンステレーションは、d_{min}を最大化するように設計される。コンステレーションの次元（ｄｉｍｅｎｓｉｏｎ）の数が増加すると、一次元当たりの一定のコンステレーションエネルギーに対する最小距離が増加する。
したがって、優良な最小距離特性を有する複数の多次元のコンステレーションが設計されてきた。
【０００４】
通信システムはシャノン限界（Ｓｈａｎｎｏｎ　ｌｉｍｉｔ）として知られる、理論的な最大容量を有する。多くの通信システムは、符号を用いて通信チャネルの容量を増加させようとしている。ターボ符号及びＬＤＰＣ符号などの符号化技術を用いて顕著な符号化利得が達成されてきた。符号化技術を用いて達成できる符号化利得は、通信システムのコンステレーションによって制限されている。シャノン限界は、コンステレーションの中心部の記号をコンステレーションの端部の記号よりもより頻繁に送信する、無限のコンステレーションであるガウス分布として知られる、理論的なコンステレーションに基づいて考えることができる。
実際のコンステレーションは有限であり、記号を同様の確率で送信するため、ガウス型の容量と比較して少ない容量を有する。コンステレーションの容量は、そのコンステレーションを用いた符号化を行った場合に得ることができる利得の限界を表すと考えられる。
【発明の概要】
【発明が解決しようとする課題】
【０００５】
従来から不等間隔のコンステレーションを発展させようという試行がなされてきた。例えば、非符号化システムのエラーレートを最小化させるために最適化された不等間隔のコンステレーションを用いるシステムが提案されている。他の提案されたシステムは、ガウス分布を模擬するために、蓋然性は等しいが、不等間隔の記号を用いたコンステレーションを使用する。
【０００６】
他のアプローチは、コンステレーションの次元を増加させる、又は、以前に送信した記号を考慮して新たに送信する記号を選択する。しかしながら、これらのコンステレーショ

ンは、以前として最短距離の基準に基づいて設計されていた。

【課題を解決するための手段】

【０００７】

送信機と受信機の間の制限付き容量が、最初にシャノン（Ｓｈａｎｎｏｎ　１９４８を参照）によって開示されたガウスチャネル容量限界に近づくように変調を行うシステムと方法について説明する。従来の通信システムは、シャノン・ガウス容量に対して大きなギャップを残す変調を採用している。本発明の変調はこのギャップを削減し、いくつかのケースにおいて、このギャップをほぼ解消する。本発明は、特に、いくつかの変調ポイントを他のものより頻繁に送信する傾向に設計された符号化メカニズムを必要とせずに、ビット又は記号のマッパー及びデマッパーのそれぞれの入力と出力の間の容量を最大化するように、ポイント（１以上の次元空間）を配置する方法を提供する。本発明を実際に適用すると、システムが、より少ない電力で所与のレートでデータを送信する、又は、同一の電力量でより高いレートでデータを送信することができる。

【０００８】

本発明の一実施の形態は、通信チャネルを介して信号を受信機に送信するように構成された送信機を含み、送信機が、ユーザビットを受信して、拡張された出力符号化ビットレートで符号化ビットを出力するように構成された符号器と、符号化ビットを記号コンステレーションの記号にマッピングするように構成されたマッパーと、マッパーによって生成された記号を用いて通信チャネルを介して送信するための信号を生成するように構成された変調器とを有する。さらに、受信機が、通信チャネルを介して受信した信号を復調するように構成された復調器と、復調された信号から尤度を推定するように構成されたデマッパーと、デマッパーによって生成された尤度から復号されたビットを推定するように構成された復号器と、を有する。さらに、記号コンステレーションが、d_{min}を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションである。

【０００９】

さらなる本発明の実施の形態は、符号体系を用いてユーザ情報のビットを符号化するステップと、ユーザ情報の符号化ビットを記号コンステレーションにマッピングするステップであって、記号コンステレーションは、d_{min}を最大化する信号コンステレーションと比較して低減された信号対ノイズ比で、所与の容量を提供する、容量最適化された幾何学的に離間した記号コンステレーションであるステップと、変調方式に従って、記号を変調するステップと、通信チャネルを介して変調された信号を送信するステップと、変調された信号を受信するステップと、上記変調方式に従って変調された信号を復調するステップと、尤度を生成するために幾何学的形状の信号コンステレーションを用いて復調された信号をデマッピングするステップと、復号されたビットの推定値を取得するために尤度を復号するステップとを含む。

【００１０】

本発明のさらなる実施の形態は、適当なコンステレーションのサイズと、次元当たりの所望の容量を選択するステップと、システムが動作する可能性のある初期ＳＮＲを推定するステップと、d_{min}を最大化するコンステレーションに対してコンステレーションのＳＮＲ性能に所定の向上が達成されるまで、容量測定値を最大化するようにコンステレーションのポイントの位置を反復的に最適化してステップと、を含む。

【００１１】

本発明のさらなる実施の形態は、適当なコンステレーションのサイズと、次元当たりの所望の容量を選択するステップと、システムが動作する可能性のある初期ＳＮＲを推定するステップと、d_{min}を最大化するコンステレーションに対してコンステレーションのＳＮＲ性能に所定の向上が達成されるまで、容量測定値を最大化するようにコンステレーションのポイントの位置を反復的に最適化してステップと、を含む。

【００１２】

本発明のさらなる実施の形態は、適当なコンステレーションのサイズと、所望のＳＮＲ

を選択するステップと、コンステレーションの容量測定値を最大化するようにコンステレーションのポイントの位置を最適化するステップと、を含む。

【００１３】

本発明のさらなる実施の形態は、所与のコンステレーションのサイズの平方根であるコンステレーションのサイズを有する幾何学的形状のＰＡＭコンステレーションを取得するステップであって、幾何学的形状のＰＡＭコンステレーションは、d_{min}を最大化するコンステレーションよりも大きな容量を有するステップと、幾何学的形状のＰＡＭコンステレーションを用いて直交化されたＰＡＭコンステレーションを生成するステップと、幾何学的形状のＰＡＭコンステレーションを直交化されたＰＡＭコンステレーションとを組み合わせて、幾何学的形状のＱＡＭコンステレーションを生成するステップとを含む。

【００１４】

本発明のさらなる実施の形態は、幾何学的形状の記号コンステレーションを用いてチャネルを介して情報を送信するステップと、幾何学的形状の記号コンステレーションの中のポイントの位置を変更して、目標ユーザデータレートを変更するステップと、を含む。

【図面の簡単な説明】

【００１５】

【図１】本発明の実施の形態に係る通信システムの概念図である。

【図２】本発明の実施の形態に係る送信機の概念図である。

【図３】本発明の実施の形態に係る受信機の概念図である。

【図４ａ】チャネルの並列復号容量の概念図である。

【図４ｂ】チャネルの共通容量の概念図である。

【図５】本発明の実施の形態に係る、固定符号レートと変調方式を有する通信システムで使用するための、容量について最適化したコンステレーションを取得するための処理を示すフローチャートである。

【図６ａ】従来のＰＡＭ－２，４，８，１６，３２のガウス容量とＰＤ容量との比較を示すグラフである。

【図６ｂ】従来のＰＡＭ－２，４，８，１６，３２のガウス容量と共通容量との比較を示すグラフである。

【図７】従来のＰＡＭ－２，４，８，１６，３２コンステレーションのＰＤ容量及び共通容量についてガウス容量に対するＳＮＲギャップを示すグラフである。

【図８ａ】従来と最適化されたＰＡＭ－２，４，８，１６，３２コンステレーションのＰＤ容量のガウス容量に対するＳＮＲギャップの比較を示すグラフである。

【図８ｂ】従来と最適化されたＰＡＭ－２，４，８，１６，３２コンステレーションの共通容量のガウス容量に対するＳＮＲギャップの比較を示すグラフである。

【図９】複数の異なる長さのＬＤＰＣ符号を用いたシミュレーションで、従来とＰＤ容量を最適化したＰＡＭ－３２コンステレーションのフレームエラーレート性能を示すグラフである。

【図１０ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

【図１０ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

【図１０ｃ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

【図１０ｄ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。

【図１１ａ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰ

ＡＭ－４コンステレーションの設計テーブルである。
【図１１ｂ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４コンステレーションの設計テーブルである。
【図１２ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１２ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１２ｃ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１２ｄ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１３ａ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションの設計テーブルである。
【図１３ｂ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－８コンステレーションの設計テーブルである。
【図１４ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１４ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１４ｃ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１４ｄ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１５ａ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－１６コンステレーションの設計テーブルである。
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【図１６ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１６ｃ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１６ｄ】ユーザビットレート／次元に対する、及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図１７ａ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰＡＭ－３２コンステレーションの設計テーブルである。
【図１７ｂ】本発明の実施の形態に係る、ＰＤ容量及び共通容量について最適化されたＰ

ＡＭ－３２コンステレーションの設計テーブルである。
【図１８】従来と容量最適化ＰＳＫコンステレーションのガウス容量に対するＳＮＲギャップを示すグラフである。
【図１９】ＰＤ容量最適化ＰＳＫ－３２コンステレーションのコンステレーションポイントの位置を示すグラフである。
【図２０】本発明の実施の形態に係る、異なるＳＮＲにおける、ＰＤ容量を最適化した一連のＰＳＫ－３２コンステレーションを示す図である。
【図２１】本発明の実施の形態に係る、二つのＰＤ最適化ＰＡＭ－８コンステレーションの直交デカルト積（ｏｒｔｈｏｇｏｎａｌ　Ｃａｒｔｅｓｉａｎ　ｐｒｏｄｕｃｔ）から構築されるＱＡＭ－６４を示す図である。
【図２２ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図２２ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－４コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図２３ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図２３ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－８コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図２４ａ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【図２４ｂ】ユーザビットレート／次元に対する、及びＳＮＲに対する、フェージングチャネルにわたって、ＰＤ容量について最適化されたＰＡＭ－１６コンステレーションのコンステレーションポイントの位置を示す軌跡図である。
【発明を実施するための形態】
【００１６】
図面を参照して、不等間隔の（すなわち、「幾何学的」形状の）ポイントを有する、本発明の実施の形態に係る通信システムについて説明する。いくつかの実施の形態において、幾何学的形状のポイントの位置は、d_{min}を最大にするコンステレーションによって必要とされるＳＮＲ（信号対ノイズ比）と比較して、削減されたＳＮＲにおいて所与の容量測定値（ｃａｐａｃｉｔｙ　ｍｅａｓｕｒｅ）を提供するように設計されている。多くの実施の形態では、コンステレーションは、所定の範囲のチャネルＳＮＲ（信号対ノイズ比）で増加した容量を提供するように選択される。コンステレーションポイントの位置を選択する際に使用することができる容量測定値は、これに限定されないが、ＰＤ（ｐａｒａｌｌｅｌ　ｄｅｃｏｄｅ）容量及び共通容量（ｊｏｉｎｔ　ｃａｐａｃｉｔｙ）を含む。
【００１７】
多くの実施の形態において、通信システムは、これに限定されないが、ＬＤＰＣ符号及びターボ符号を含む、容量に接近する符号を利用している。以下でさらに説明するが、容量に接近するチャネル符号を利用する通信システムのコンステレーションポイントの直接的な最適化は、最適化されるコンステレーションポイントに関わるＳＮＲに応じて様々なコンステレーションを生成することができる。したがって、同一のコンステレーションが、全ての符号化レートにわたって適用される同一の符号化利得を実現する可能性は低く、このため、同一のコンステレーションが、全てのレートにわたって最善の性能を実現可能とすることはない。多くの例で、一つの符号レートのコンステレーションは、他の符号レートでは実現できない利得を実現することができる。本発明の実施の形態による、容量を

最適化したコンステレーションを選択して、特定の符号化レートに基づく符号化利得を増加させる処理について説明する。複数の実施の形態において通信システムは、チャネルの状態、符号レートの変更、及び／又は、目標ユーザビットレートの変更に応じて、コンステレーションのポイントの位置を調整することが可能である。

＜通信システム＞

【００１８】

本発明の実施の形態に係る通信システムを図１に示す。通信システム１０は、ユーザビットを送信機１４に提供するソース機器（ｓｏｕｒｃｅ）１２を含む。送信機は、所定の変調方式を用いて、受信機１６にチャネルを介して記号を送信する。受信機は、変調方式の情報を用いて、送信機から受信した信号を復号する。復号されたビットは、受信機と接続したシンク機器（ｓｉｎｋ　ｄｅｖｉｃｅ）に提供される。

【００１９】

本実施の形態に係る送信機を図２に示す。送信機１４は、ソース機器からユーザビットを受信して、所定の符号体系に従ってビットに符号化する符号器２０を含む。複数の実施の形態では、ターボ符号又はＬＤＰＣ符号などの容量アプローチ符号を使用する。他の実施の形態では、他の符号体系を用いて、通信システムの内部の符号化利得を提供することができる。マッパー（ｍａｐｐｅｒ）２２が符号器に接続されている。マッパーは、符号器により出力されたビットを、マッパーの内部に保存された、幾何分布された信号コンステレーションの中の記号にマッピングする。マッパーは、記号を変調器２４に提供し、変調器２４は、チャネルを介して送信するために記号を変調する。

【００２０】

本実施の形態に係る受信機を図３に示す。受信機１６は、チャネルを介して受信した信号を復調して、記号又は同様のビットを得る復調器３０を含む。デマッパーは、送信機が用いた幾何学的形状の記号コンステレーションの情報を用いてこれらの尤度を決定する。デマッパー３２は、これらの尤度を復号器３４に送出し、復号器３４は、符号化されたビットストリームを復号して、受信されたビットのシーケンスをシンクに送出する。

＜幾何学的形状のコンステレーション＞

【００２１】

本発明の実施の形態に係る送信機と受信機は、幾何学的形状の記号コンステレーションを用いる。いくつかの実施の形態では、コンステレーションの容量を最適化する幾何学的形状の記号コンステレーションを使用する。本発明の実施の形態に従って、使用することができる様々な幾何学的形状の記号コンステレーション、幾何学的形状の記号コンステレーションを導く技術について、以下、説明する。

＜幾何学的形状のコンステレーションの選択＞

【００２２】

本発明の実施の形態に係る通信システムで使用する幾何学的形状のコンステレーションは、符号化レートを固定とするか否かを含む、様々な要因に応じて選択することができる。多くの実施の形態において、幾何学的形状のコンステレーションを用いて、通信システムの内部の送信機のマッパーと受信機のデマッパーにおける従来のコンステレーション（すなわち、d_{min}について最大化したコンステレーション）を置きかえることができる。通信システムのアップグレードは、コンステレーションの選択を含み、多くの場合、アップグレードは、単純なファームウェアのアップグレードによって遂成できる。他の実施の形態では、特定の性能要件に適合するために、幾何学的形状のコンステレーションを符号レートと併せて選択する。性能要件は、例えば、指定されたビットレート、最大送信電力などの要素を含むことができる。既存の通信システムをアップグレードする場合、及び新しい通信システムを設計する場合の幾何学的なコンステレーションの選択処理について、以下説明する。

＜既存の通信システムのアップグレード＞

【００２３】

d_{min}について最大化したコンステレーションの容量より大きな容量を提供する幾何

学的形状のコンステレーションを、本発明の実施の形態に係る通信システムにおいて従来のコンステレーションに代えて用いることができる。多くの場合、幾何学的形状のコンステレーションに交換することは、通信システムの内部の送信機及び受信機のファームウェア又はソフトウェアのアップグレードによって実現することができる。幾何学的形状のコンステレーションの全てが、d_{min}について最大化したコンステレーションの容量より大きな容量を有するとは限らない。d_{min}について最大化したコンステレーションの容量より大きな容量を有する幾何学的形状のコンステレーションを選択するための一つのアプローチは、所与のＳＮＲに対するコンステレーションの容量の測定値に対して、コンステレーションの形状を最適化することである。最適化処理で用いることができる容量測定値は、これに限定されないが、共通容量（ｊｏｉｎｔ　ｃａｐａｃｉｔｙ）又は並列復号（ＰＤ：　ｐａｒａｌｌｅｌ　ｄｅｃｏｄｉｎｇ）容量を含むことができる。

＜共通容量及び並列復号容量＞

【００２４】

コンステレーションは、コンステレーションポイントの総数Ｍと実次元の数Ｎｄｉｍによってパラメータ化することができる。復号器とコンステレーションデマッパーとの間に確率伝搬反復がないシステムでは、コンステレーションデマッパーはチャネルの一部として考えることができる。ＰＤ容量を決定する目的のため、チャネルの一部として想定できる通信システムの部分の概念図を図４ａに示す。チャネルの一部として想定される通信システムの部分をゴーストライン４０で示す。このようにして定義されるチャネルの容量が、並列復号（ＰＤ容量）であり、以下の式で与えられる。

【数１】

$$C_{PD} = \sum_{i=0}^{i-1} I(X_i;Y)$$

ここで、Ｘｉは、送信された記号のＩビットのうちのｉ番目のビットであり、Ｙは、受信された記号であり、Ｉ（Ａ；Ｂ）は、ランダム変数ＡとＢの間の相互情報を示す。

【００２５】

他の方法で表現すると、チャネルのＰＤ容量は、送信機の符号器（ＬＤＰＣエンコーダなど）の出力ビットと、受信機のデマッパーで算出された尤度との間の相互情報としてとらえることができる。ＰＤ容量は、コンステレーションの中のポイントの配置と、ラベリング割り当てとの両方に影響される。

【００２６】

デマッパーと復号器の間の確率伝搬反復が存在すると、デマッパーは、もはやチャネルの一部としてとらえることができなくなり、コンステレーションの共通容量が，システム性能において知りうる限り最もタイトな限界となる。コンステレーションの共通容量を決定する目的のため、チャネルの一部として想定される通信システムの部分の概念図を図４ｂに示す。チャネルの一部として想定される通信システムの部分をゴーストライン４２で示す。チャネルの共通容量は以下の式で与えられる。

$C_{JOINT} = I(X;Y)$

【００２７】

共通容量は、リンクの送信側のマッパーの入力と、チャネル（例えば、ＡＷＧＮチャネルとフェージングチャネルを含む）の出力との間で実現可能な容量を表す。実際のシステムは、復号の前にチャネル観測を「デマップ」する必要がある。一般的に、このステップは、容量の損失の原因となる。事実、$C_G > C_{JOINT} > C_{PD}$又は$C_G = C_{JOINT} > C_{PD}$であることを検証することができる。すなわち、C_{JOINT}は、C_{PD}によって達成可能な容量の上限となる。本発明の方法は、所与の通信システム容量の実際の限界がC_{JOINT}及びC_{PD}によって制限されるという事実を考慮することに動機付けされている。本発明のいくつかの実施の形態では、これらの測定値を最大化する幾何学的形

状のコンステレーションを選択する。
＜最適な容量を有するコンステレーションの選択＞
【００２８】
本発明の実施の形態の幾何学的形状のコンステレーションは、容量測定値を最適化するように設計することができる。容量測定値はＰＤ容量又は共通容量を含むが、これに限定されない。本発明の実施の形態に係る、固定符号レートを有する通信システムで使用するための、幾何学的形状のコンステレーションのポイント、及び潜在的なラベリングを選択する処理について、図５に示す。処理５０は、適当なコンステレーションのサイズＭ、及び次元ηごとの所望の容量の選択（５１）から開始する。図示の実施の形態では、この処理は、コンステレーションのサイズが所望の容量をサポートできることを確実にするためのチェック（５２）を含む。コンステレーションのサイズが、所望の容量をサポートできる場合、処理は、指定された容量についてＭ－ａｒｙコンステレーションを反復して最適化する。最適なコンステレーションは、通信システムが動作するＳＮＲに依存するため、指定された容量のコンステレーションの最適化は、しばしば反復的な処理を含む。必要な容量を与える最適なコンステレーションのＳＮＲは、先験的に知られてはいない。本発明の説明において、ＳＮＲは、次元当たりの平均ノイズエネルギーに対する次元当たりの平均コンステレーションとの比として定義する。ほどんどの場合、容量は、次元当たりの記号当たりの目標ユーザビットレートに設定することができる。いくつかの場合、いくつかの実施上のマージンを目標ユーザビットレートの上端に追加することによって、必要なユーザレートをより低いレートで提供することができる実際のシステムが実現可能となる。このマージンは符号に依存する。以下の手順を用いて、ユーザレートの上端にあるマージンを含む目標容量を決定することができる。第１に、符号（例えば、ＬＤＰＣ又はターボ）を、従来の等間隔のコンステレーションと併せてシミュレーションする。第２に、このシミュレーションの結果から、必要とされるエラーレート時の動作の実際のＳＮＲを見つける。第３に、そのＳＮＲにおける従来のコンステレーションの容量を算出する。最後に、その容量について幾何学的形状のコンステレーションを最適化する。
【００２９】
図示の実施の形態では、反復最適化のループは、システムが動作しがちなＳＮＲの初期推定値（すなわち、SNR_{in}）の選択を含む。いくつかの実施の形態では、初期推定値は、従来のコンステレーションを使用するために必要とされるＳＮＲである。他の実施の形態では、他の技術を用いて初期ＳＮＲを選択することができる。次に、初期SNR_{in}推定値で選択された容量測定値を最大化するようにコンステレーションを最適化することによって、Ｍ－ａｒｙコンステレーションを取得する（５６）。所与のＳＮＲ推定値に対する最適化されたコンステレーションを取得するための様々な技術について以下、説明する。
【００３０】
最適化されたＭ－ａｒｙコンステレーションが次元ηごとに所望の容量を提供するＳＮＲ（SNR_{out}）を決定する（５７）。SNR_{out}とSNR_{in}が収束したか否かを判定する（５８）。図示の実施の形態では、収束は、SNR_{out}がSNR_{in}と等しくなることによって示される。いくつかの実施の形態では、収束は、SNR_{out}とSNR_{in}との間の差違が所定の閾値を下回るかに基づいて判定することができる。SNR_{out}とSNR_{in}とが収束していない場合、処理は、新しいSNR_{in}としてSNR_{out}をもう一度反復選択をする（５５）。SNR_{out}とSNR_{in}とが収束した場合、コンステレーションの容量測定値は最適化されている。低いＳＮＲで容量最適化したコンステレーションは、d_{min}を最大化するコンステレーションよりも著しく優れた性能向上（必要される最小のＳＮＲに削減して測定される）を達成することができる幾何学的形状のコンステレーションである。以下、より詳細に説明する。
【００３１】
図５に示す処理は、所与のＳＮＲに対するＭ－ａｒｙコンステレーションのＰＤ容量又は共通容量を最大化することができる。図５に示す処理は、容量について最適化したＭ－

ａｒｙコンステレーションを選択することを示しているが、同様の処理を用いて、所与のＳＮＲにおけるガウス容量とのＳＮＲギャップが、従来のコンステレーションのＳＮＲギャップよりも低い所定のマージン、例えば、０．５ｄｂである、Ｍ－ａｒｙコンステレーションの生成時に終了するようにしてもよい。あるいは、本発明の実施の形態によると、従来のコンステレーションの容量よりも大きな容量を有するＭ－ａｒｙコンステレーションを識別する、その他の処理を用いることができる。本発明の実施の形態に係る幾何学的形状のコンステレーションは、通信システムが動作するＳＮＲの範囲において、最適な容量を有していなくとも、d_{min}を最大にするコンステレーションの容量よりも大きな容量を達成することができる。

【００３２】

私たちは、共通容量を最大にするように設計されたコンステレーションは、ＧＦ（ｑ）上の記号による符号、又は多段階復号法による符号に特に好適であることがあることに留意する。反対に、ＰＤ容量について最適化するコンステレーションは、ＧＦ（２）上の記号を有する符号のより一般的なケースにより好適である。

＜所与のＳＮＲにおけるＭ－ＡＲＹコンステレーションの容量の最適化＞

【００３３】

容量を最適化したコンステレーションを取得するための処理は、しばしば、所与のＳＮＲにおけるＭ－ＡＲＹコンステレーションのポイントについて最適な位置を決定する処理を含む。図５に示す最適化処理５６などの最適化処理は、一般的に、制限無し又は制限付き非線形最適化を含む。最大化される可能な目的関数は共通又はＰＤ容量関数である。これらの関数は、これに限定されないが、ＡＷＧＮ（加法性ホワイトガウスノイズ）チャネル又はレイリーフェージングチャネルを含むチャネルを目的とすることができる。最適化処理は、その記号ラベリングによって識別されるコンステレーションポイントごとの位置を付与する。目的が共通容量である場合、ポイント位置の群が変更されない限りにおいて、ビットラベリングを変更しても共通容量は変更されない。この意味において、ポイントビットラベリングは無関係となる。

【００３４】

最適化処理は、一般的に、所与のＳＮＲにおいて最大のＰＤ容量又は共通容量を与えるコンステレーションを見つける。最適化処理自体は、とりわけ、所与のＳＮＲにおいて最高の容量を与えるいくつかのコンステレーションを考慮し、当該コンステレーションを選択する、反復数値処理をしばしば含む。他の実施の形態では、必要なＰＤ容量又は共通容量を与える最小のＳＮＲを要求するコンステレーションを見つけることもできる。これは、図５に示す最適化処理を反復して実行することを必要とする。

【００３５】

コンステレーションポイントの位置の最適化制限は、これに限定されないが、ポイント位置の上限と下限、結果として生じるコンステレーションのピーク対平均電力、及び結果として生じるゼロ平均を含むことができる。大域的に最適なコンステレーションがゼロ平均（直流成分を含まない）を有することは、容易に示すことができる。ゼロ平均制限を明示的に含むことは、最適化ルーチンのより迅速な収束を助ける。ポイント位置とラベリングの全ての組み合わせを全数探索（ｅｘｈａｕｓｔｉｖｅ　ｓｅａｒｃｈ）することが可能な場合を除き、解がほぼ大域的に最適である場合が必ずしも常ではない。全数探索が可能な場合、非線形オプティマイザにより提供される解は事実、大域的に最適となる。

【００３６】

上述の処理は、固定符号レート及び変調方式を有する通信システムの用途において、従来の容量と比較して増加した容量を有する幾何学的形状のコンステレーションを得ることができる手法の例を提供する。d_{min}を最大化する従来のコンステレーションと比較して、容量について最適化されたコンステレーションを用いて達成可能な実際のメリットについて以下、説明する。

＜最適化されて幾何学的に離間したコンステレーションによって達成可能なメリット＞

【００３７】

任意の通信方法によって達成可能な、究極的な理論的容量をガウス容量C_Gと想定して、以下の式で定義する。

【数２】

$$C_G = \frac{1}{2}log_2(1+SNR)$$

【００３８】

ここで、信号対ノイズ（ＳＮＲ）を予測ノイズ電力に対する予測信号電力との比率とする。コンステレーションの容量とC_Gとの間に残存するギャップは、所与のコンステレーションの設計の品質の目安と考えることができる。

【００３９】

理論的に最適な符号器と組み合わせた従来の変調方式の間の容量のギャップについて、図６ａ及び図６ｂを参照して観察する。図６ａは、d_{min}を最大化する従来のＰＡＭ－２、４、８、１６、及び３２のコンステレーションのガウス容量とＰＤ容量との間の比較を示すグラフ６０を含み、ガウス容量と様々なＰＡＭコンステレーションのＰＤ容量の間にギャップ６２が存在している。図６ｂは、d_{min}を最大化する従来のＰＡＭ－２、４、８、１６、及び３２のコンステレーションのガウス容量と共通容量との間の比較を示すグラフ６４を含み、ガウス容量と様々なＰＡＭコンステレーションの共通容量の間にギャップ６６が存在している。これらの容量のギャップが表す範囲は、従来のＰＡＭコンステレーションが究極的な理論的容量、すなわちガウス容量を取得するには少なすぎる。

【００４０】

ガウス容量に近いポイントにおいて、図６ａに示す曲線と図６ｂに示す曲線の間の差をより見やすくするため、コンステレーションごとに、異なる容量の値についてガウス容量に対するＳＮＲギャップを図７に示す。興味深いことに、図７のグラフ７０から（共通容量とは異なり）、同一のＳＮＲにおいて、ＰＤ容量は、必ずしもコンステレーションポイントの数とともに増加しないことがわかる。以下でさらに論じるが、これは、ＰＤ容量について最適化されたＰＡＭコンステレーションの場合には当てはまらない。

【００４１】

図８ａ及び図８ｂに、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４，８，１６、及び３２のコンステレーションの性能の概要を示す（ＢＰＳＫは、全ての符号レートにおいて最適なＰＡＭ－２コンステレーションであることに留意されたい）。コンステレーションは、次元（すなわち、符号レート）ごとに異なる目標ユーザビットについて、ＰＤ容量及び共通容量について最適化される。最適化されたコンステレーションは、次元当たりの目標ユーザビットに応じて異なり、また、ＰＤ容量を最大化するように設定されているか、共通容量を最大化するように設計されているか否かに応じても異なる。ＰＤを最適化したＰＡＭコンステレーションは全て、グレーラベリングを用いてラベルされているが、常にバイナリ反映型（ｂｉｎａｒｙ　ｒｅｆｌｅｃｔｉｖｅ）グレーラベリングであるわけではない。数直線上のいかなる場所にコンステレーションポイントを設けてもよいという自由を与えたとしても、全てのグレーラベルが、可能な最大のＰＤ容量を達成するとは限らないことに留意されたい。図８ａに、ＰＤ容量について最適化されたコンステレーションごとのＳＮＲギャップを示す。図８ｂに、共通容量について最適化されたコンステレーションごとのガウス容量に対するＳＮＲギャップを示す。グラフの「＋」は各々、異なるコンステレーションを示すことを強調したい。

【００４２】

図８ａを参照すると、ＰＤ容量について符号化されたコンステレーションを用いて達成された符号化利得を、ＰＡＭ－３２について２．５ビットの次元当たりにユーザビットレートでＳＮＲギャップを比較して評価することができる。記号当たりに５ビットを送信するシステムでは、２．５ビットのユーザビットレート／次元（次元当たりのユーザビットレート）が符号レートの１／２を構成する。その符号レートでは、ＰＤ容量について最適化されたコンステレーションは、従来のＰＡＭ－３２コンステレーションと比較すると、

略１．５ｄＢの符号化利得をさらに提供する。

【００４３】

ＰＤ容量について最適化されたコンステレーションを用いて達成することができるＳＮＲ利得は、シミュレーションによって検証することができる。従来のＰＡＭ－３２コンステレーション、及びＰＤ容量について最適化されたＰＡＭ－３２コンステレーションと併せて１／２ＬＤＰＣ符号のレート（ｒａｔｅ　１／２　ＬＤＰＣ　ｃｏｄｅ）を用いて実行したシミュレーションの結果を図９に示す。グラフ９０は、ＳＭＲに関する異なるコンステレーションのフレームエラーレート性能のグラフを含み、異なる長さの符号（すなわち、ｋ＝４，０９６及びｋ＝１６，３８４）を用いている。使用する符号とは無関係に、ＰＤ容量について最適化されたコンステレーションは、略１．３ｄＢの利得を達成しており、これは図８ａから予想される利得に極めて近い。

＜容量最適化ＰＡＭコンステレーション＞

【００４４】

上述の概要の処理を用いた場合、ＳＮＲに対するＰＡＭコンステレーションの内部のポイントの位置を示す、容量について最適化されたＰＡＭコンステレーションの軌跡図を生成することができる。次元当たりの様々な一般的なユーザビットレートにおける、ＰＤ容量及び共通容量について最適化されたＰＡＭ－４、８、１６、及び３２コンステレーションの軌跡図と対応する設計テーブルを、図１０ａ～図１７ｂに示す。軌跡図と設計テーブルは、低スペクトル効率から高スペクトル効率の範囲に対応する低ＳＮＲから高ＳＮＲまでのＰＡＭ－４、８、１６、及び３２コンステレーションのポイントの位置と、ラベリングを示す。

【００４５】

図１０ａにおいて、軌跡図１００は、達成される容量に対してグラフ化した、共通容量について最適化されたＰＡＭ－４コンステレーションのポイントの位置を表す。ＳＮＲに対してグラフ化した、共通容量について最適化されたＰＡＭ－４コンステレーションのポイントの位置を表す同様の軌跡図１０５を図１０ｂに示す。図１０ｃにおいて、ＰＤ容量について最適化されたＰＡＭ－４のポイントの位置は、達成可能な容量についてグラフ化されており、図１０ｄにおいて、ＰＤ容量について最適化されたＰＡＭ－４のポイントの位置は、ＳＮＲについてグラフ化されており、低ＳＮＲにおいて、ＰＤ容量について最適化されたＰＡＭ－４コンステレーションが有する特異ポイントは２つだけであるが、共通容量について最適化されたコンステレーションは３つである。ＳＮＲが増加すると、最適化ごとに実際、４つの特異ポイントを提供する。この現象は、図１０ａｂと図１０ｃｄの縦のスライスをテーブルに抽出した、対象となるＰＡＭ－４コンステレーションのいくつかの設計を説明する図１１ａ及び図１１ｂで明確に説明される。選択されたＳＮＲスライスは、容量＝｛０．５，０．７５，１．０，１．２５，１．５｝ビット／記号（ｂｐｓ）を達成する設計を表す。ＰＡＭ－４が最大で$\log_2(4)=2$ｂｐｓを実現できるとした場合、これらの設計ポイントはそれぞれ、符号レートＲ＝｛１／４，３／８，１／２．５／８，３／４｝の情報を有するシステムを表す。

【００４６】

図１２ａ、１２ｂ及び図１２ｃ、１２ｄは、達成可能な容量及びＳＮＲに対する、ＰＤ容量及び共通容量について最適化したＰＡＭ－８コンステレーションのポイントの軌跡図を表す。図１３ａ及び図１３ｂは、達成可能な容量η＝｛０．５，１．０，１．５，２．０，２．５｝ｂｐｓに対応するＳＮＲにおける、これらのグラフからのスライスを示す。これらのスライスの各々は、符号レート$R=\eta \text{bps}/\log_2(8)$を有するシステムに対応するため、Ｒ＝｛１／６，１／３，１／２，２／３，５／６｝となる。これらのテーブルのコンステレーションの相対的な性能の一例として、ＳＮＲ＝９．００ｄＢ又は１．５ｂｐｓについて最適化した、ＰＤ容量最適化ＰＡＭ－８コンステレーションを示す図１３ｂを考慮する。次に、図８ａに示したグラフを検討すると、最適化されたコンステレーションの究極的なガウス容量（ＣＧ）に対するギャップは、略０．５ｄＢであることがわかる。同一のスペクトル効率における、従来のＰＡＭ－８コンステレーションのギャッ

プは略１．０ｄＢとなる。最適化されたコンステレーションの利点は、同一のレートについて０．５ｄＢ（この場合、Ｒ＝１／２）である。この利得は、通信システムにおいてマッパーとデマッパーを変更するだけで、全ての他のブロックを同一としたままで得ることができる。

【００４７】

ＰＡＭ－１６ＰＤ容量及び共通容量最適化コンステレーションの軌跡図と設計テーブルを提供する同様の情報を、図１４ａ、１４ｂ、１４ｃ、１４ｄと図１５ａ、１５ｂに表す。同様に、図１６ａ、１６ｂ、１６ｃ、１６ｄ、１７ａ、１７ｂに、ＰＡＭ－３２ＰＤ容量及び共通容量最適化コンステレーションの軌跡図と設計テーブルを示す。

＜容量最適化ＰＳＫコンステレーション＞

【００４８】

従来のＰＳＫ（ｐｈａｓｅ　ｓｈｉｆｔ　ｋｅｙｅｄ）コンステレーションは既に極めて最適である。これは、図１８に示す、従来のＰＳＫと、容量を最適化したＰＳＫコンステレーションのＳＮＲギャップを比較したグラフ１８０から見てとることができる。このグラフは、ＰＤ容量とガウス容量との間のギャップを、従来のＰＳＫ－４、８、１６、３２と、ＰＤ容量最適化ＰＳＫ－４、８、１６、３２についてグラフ化している。

【００４９】

ＳＮＲにわたるＰＤ最適化ＰＳＫ－３２ポイントの軌跡図を図１９に示す。実際に全てのＰＳＫがスペクトル効率η＜５又はη＝５を有する点に特徴がある。これは図２０から見てとることができる。低いＳＮＲ（０．４ｄＢ）では、最適化されたＰＳＫ－３２設計は、従来のＰＳＫ－４と同様であり、ＳＮＲ＝８．４ｄＢでは、最適化されたＰＳＫ－３２は、従来のＰＳＫ－８と同様であり、ＳＮＲ＝１４．８ｄＢでは、最適化されたＰＳＫ－３２は、従来のＰＳＫ－１６と同様であり、最後に、２０．４ｄＢを上回るＳＮＲでは、最適化されたＰＳＫ－３２は、従来のＰＳＫ－３２と同様であることに留意されたい。従来のＰＳＫコンステレーションと比較すると、最適化されたＰＳＫ－３２が優れたＰＤ容量を提供するこれらの離散的なポイントの間（例えば、ＳＮＲ＝２と４、５ｄＢ）にＳＮＲがある。

【００５０】

図１８の最適化されたＰＳＫ－３２のガウス容量曲線に対するギャップと併せた、図１９におけるＰＤを最適化されたＰＳＫ３２のポイントの軌跡は、潜在的な設計方法論を示唆していることに留意されたい。具体的には、設計者は、コンステレーションポイントを図１９の軌跡から一つの選択するように制御を行ったただ一つの可調整パラメータと併せて、最適化されたＰＳＫ－３２を用いるだけで、従来のＰＳＫ－４、８、１６によって実現可能な性能と同等以上の性能を達成することができた。かかるアプローチは、そのレートを、例えば、レートの４／５に変化させて、４ビット／記号の全体のスペクトル効率（符号プラス最適化されたＰＳＫ－３２変調）を達成し、１／５まで下げて１ビット／記号の全体のスペクトル効率を達成する、高いレート適応型チャネル符号と結合してもよい。かかる適応型変調と符号化システムは、基本的に、図１８の右端の曲線によって表される最適な連続体で実行することができる。

＜適応型レート設計＞

【００５１】

上述の例では、ＰＳＫ－３２のスペクトル適応型の使用について説明した。これと類似の技術を、送信機と受信機の間のリンクにわたって、他の容量最適化されたコンステレーションに適用することができる。例えば、システムがサービス品質を実現する場合、要求に応じてスペクトル効率の増減を送信機に指示することができる。本発明によると、エンドユーザのレート目標に適合する符号レートの選択と併せて、目標とするスペクトル効率に正確に設計された容量最適化されたコンステレーションを送信マッパーにロードすることができる。かかる変調／符号レートの変更が発生すると、メッセージは受信機に伝播されて、受信機は、変更を見込んで、新しい送信側の構成に合うように、デマッパー／復号器の構成を選択することができる。