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Standardized Polymer Durometry

ABSTRACT: Polymer durometry is the measurement of the hardness of plastic and rubber materials using an indenter. Many different hardness scales and indenter geometries have been standardized for durometry. Indentation into a solid is governed by linear elastic mechanics, and this paper relates the Young's modulus to standardized hardness scale reading. We nondimensionalized these indentation mechanics, to uncover a normalized Young's modulus, Y , and the dimensionless hardness, $\mathcal{H}$, and then, a new dimensionless indenter stiffness that we call *mechanical indentability*, M_i . We use M_i for classifying standardized hardness scales. Our results allow practitioners to determine the Young's modulus from measured hardness. Further, our results can be used to convert between hardness scales, not only between hardness scales of the same indenter geometry, but also between those of differing geometry.

KEYWORDS: durometer, durometry, hardness, indentation, Young's modulus

Introduction

A durometer³ transduces the indentation of a standardized nondestructive indenting device to a standardized hardness scale reading. Hardness is thus a relative measure, as a material will register a different value of hardness on one scale than another. However, linear elasticity can be used to deduce the Young's modulus from this relative measure. In this paper, we introduce the nondimensionalized Young's modulus, Y , and hardness, $\mathcal{H}$ to reveal a new dimensionless indenter stiffness, which we call *mechanical indentability*, M_i . In practice, each durometer scale is unreliable both above and below certain scale readings. In this paper, we explain these bounds for reliability in terms of the sensitivity of the Young's modulus to the measured hardness. This durometer sensitivity is a highly nonlinear function of hardness. Our work allows Young's modulus to be deduced from measured hardness and also provides a pathway to convert between hardness scales. Worked examples are given for both.

Some solve the mechanics of linear elastic indentation for different indenter shapes [1–7]. Others develop empirical relations that relate Young's modulus to indentation [5,8–14]. Still others empirically relate Young's modulus to scale hardness [5,11,12,15] and fit power laws to these measurements. For inelastic indentation, Johnson [16] examined the role of plasticity and others, viscoelasticity [17,18]. Table 1 summarizes this literature chronologically. Headings for columns 1–7 refer to different indenter tip shapes. Columns 1 and 3 in Table 1 distinguish between flat tips (right circular cylinders) and truncated cones. The literature in column 3 accounts for deep indentation where material contacts the conical indenter surface, just beyond the truncated flat tip. The literature in column 6 accounts for a conical indenter of finite, hemispherical sharpness. Further to Table 1, only one paper reports measured val-

ues of Young's moduli versus hardness [5] and these being for rubber using a hemispherical indenter.

Our work applies well to standardized hardness scales [19–28], and is equally useful to those developing their own specialized hardness scales [14,29,30].

Table 2 lists our dimensional variables and Table 3 our dimensionless ones.

Durometer Mechanics

A durometer consists of a protruding indenter attached to a spring that is preloaded in compression. Figure 1 shows the inside of a Shore scale D durometer. Analog durometers transduce the indenter displacement to a standardized scale. The more the indenter depresses the spring, the harder the material and thus higher the reading. Figures 2 and 3 illustrate the working principle, which most durometer designs follow [31–40]. The material force on the indenter, F , is given by

$$F = F_0 + ks \quad (1)$$

where:

F_0 = spring preload,

k = spring constant, and

s = standardized hardness scale reading, having dimensions called *degrees of hardness*.⁴ The initial indenter protrusion is given by

$$p_0 = c + d \quad (2)$$

where:

c = initial spring compression (under preload F_0), and

d = final indentation. Now

$$c = \sigma s \quad (3)$$

where:

σ = spring sensitivity having dimensions of *displacement per degree of hardness*.

By substituting c , the initial spring compression in Eq 3, into Eq 2, the final indentation d is given by

⁴Also called *shore units* or simply *durometer units*.

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³Also called *hardness tester*, sometimes *sclerometer* or *indentometer*, formerly *hardometer*, and originally *plastometer* [40].

TABLE 1—Literature on the relation between young's modulus and hardness.

	Flat	Hemisphere	Truncated Cone	Cone	Nondimensionalizing	Scale Conversion	References
Timoskenko (1939)	●	●					[1]
Larrick (1940)			●				[8]
Harding and Sneddon (1945)	●	●		●			[2]
Sneddon (1946)	●						[3]
Sneddon (1948)				●			[4]
Gent (1958)	●	●					[5]
Yeh and Livingston (1961)	●	●					[10]
Sneddon (1965)	●	●		●			[6]
Johnson (1970)	●	●		●			[16]
Briscoe and Sebastian (1993)	●	●	●	●			[12]
Briscoe et al. (1994)	●	●		●			[7]
Busfield and Thomas (1999)		●					[13]
Boyce et al. (2003)			●	●			[15]
Fraczek et al. (2005)				●			[14]
This paper	●	●		●	●	●	

TABLE 2—Dimensional variables.

Indentation	d	L	≥ 0
Applied force	F	ML/t^2	≥ 0
Young's modulus	E	M/Lt^2	> 0
Indenter radius	r	L	> 0
Initial spring compression	c	L	≥ 0
Initial indenter protrusion	p_0	L	> 0
Scale reading	s	DH	≥ 0
Scale reading range	s_r	DH	> 0
Scale sensitivity (displacement/DH)	$\sigma \equiv p_0/s_r$	L/DH	> 0
Spring constant (force/DH)	k	ML/t^2DH	> 0
Initial compressive spring force	F_0	ML/t^2	≥ 0
Young's modulus lower bound for durometer scale	$E_0 \equiv 3F_0/8rp_0$	M/Lt^2	≥ 0
Force per unit unstrained area	f	ML/t^2	≥ 0
Shear modulus	G	M/Lt^2	> 0

Note: M =mass; L =length; t =time; DH=degrees of hardness.

$$d = p_0 - \sigma s \quad (4)$$

Nondimensionalizing Indentation

To deepen our understanding of hardness measurement we render the linear elastic mechanics of indentation dimensionless for each popular standardized indenter tip shape: The cone, the hemisphere, and the truncated cone. We then succeed in uncovering a general framework for interpreting hardness measurements in terms of the Young's modulus.

Conical

We begin by nondimensionalizing the linear elastic mechanics for the indentation d caused by a force F on a conical indenter [2]

$$d = \sqrt{\frac{\pi F(1 - \nu^2)}{2E \tan \alpha}} \quad (5)$$

where:

α =one-half of the included cone angle,
 E =Young's modulus, and

ν =Poisson's ratio.⁵

We then eliminate the force F in Eq 5 using Eq 1 and eliminate the final indentation d in Eq 5 using Eq 4. Next, we nondimensionalize and normalize the hardness s to $\mathcal{H}$, and the Young's modulus E to Y (see Table 3)

$$Y \equiv \frac{2rp_0}{F_0} \frac{E}{(1 - \nu^2)} \quad (6)$$

We then define Y_c as the *dimensionless* conical Young's modulus, which is our dimensionless Young's modulus Y over our conical tip radius r . Our dimensionless hardness $\mathcal{H}$ is the product of the scale sensitivity with the scale reading divided by the initial indenter protrusion (see Table 3) which simply for ASTM D2240 [27] is $\mathcal{H} = 100 s$. We thus discover a dimensionless group proportional to an indenter stiffness, which we call mechanical indentability, Mi . By mechanical indentability, we mean the mechanical device's ability to indent. The resulting dimensionless relation between Young's modulus and measured hardness for conical indentation is

$$Y_c = \frac{(1 + Mi\mathcal{H})}{(1 - \mathcal{H})^2} \quad (7)$$

⁵Eq 5 matches Eq 3.1.13 of Harding and Sneddon [2], and the middle (unnumbered) equation on p. 505 of Sneddon [4], but differs by a typo from Eq 6.5 in Sneddon [6].

TABLE 3—Dimensionless groups.

Poisson's ratio	ν
Extension ratio	λ
Indenter half-angle	α
Mechanical indentability	$Mi \equiv kp_0/\sigma F_0$
Hemispherical tip radius	$\rho = 3\sqrt{r/p_0}/2$
Conical tip radius	$\varsigma \equiv \pi r/p_0 \tan \alpha$
Hardness	$\mathcal{H} \equiv \sigma s/p_0$
Dimensionless Young's modulus ⁶	$Y = 2rp_0E/F_0(1-\nu^2) = 8rp_0E/3F_0$
Hemispherical Young's modulus	$Y_h = Y/\rho$
Conical Young's modulus	$Y_c = Y/\varsigma$

⁶Since $\nu \approx 0.5$ for rubbers and plastics, most polymer engineers will prefer to use the approximation $Y \approx (8rp_0/3F_0)E$.

Figure 4 shows the relation between the logarithm of the conical Young's modulus Y_c and the hardness $\mathcal{H}$ for the hardness scales now standardized by ASTM D2240-05 as scales B [$Mi = 13.64$] or M [$Mi = 1.36$].

Round

Similarly, beginning with the linear elastic mechanics for the indentation d caused by a force F on a hemispherical indenter [1] yields

$$d^{3/2} = \frac{3F}{4r^{1/2}} \frac{(1-\nu^2)}{E} \quad (8)$$

which matches Eq 1 in Gent [5] when $\nu = 1/2$. Nondimensionalizing then yields the relation between dimensionless Young's modulus and measured hardness

$$Y_h = \frac{(1 + Mi\mathcal{H})}{(1 - \mathcal{H})^{3/2}} \quad (9)$$

where we have defined Y_h as the hemispherical Young's modulus, which is our dimensionless Young's modulus Y divided by our hemispherical tip radius ρ . We thus rediscover the dimensionless



FIG. 1—Shore quadrant scale D durometer.

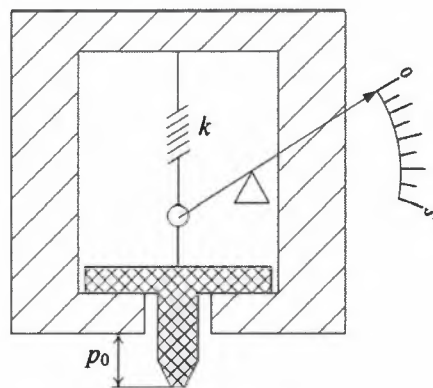


FIG. 2—Basic durometer at rest.

mechanical indentability, Mi . The conical and hemispherical dimensionless Eqs 7 and 9 share the same form, differing only by their geometric shape factors and the denominator power. Figure 5 shows the relation between the logarithm of the hemispherical Young's modulus Y_h and the hardness $\mathcal{H}$ now standardized in ASTM D2240-05 as scales O [$Mi = 13.64$], OO [$Mi = 4.47$], OOO [$Mi = 4.47$], OOO-S [$Mi = 10.57$], or E [$Mi = 13.64$] and ASTM F1957-99 scale CF [$Mi = 80.36$].

Flat

Also similarly, beginning with the linear elastic mechanics for the indentation d caused by a force F on a flat tip indenter [1] yields

$$d = \frac{F(1-\nu^2)}{2Er} \quad (10)$$

which for $\nu = 1/2$ matches Eq 5 in Gent [5]. Nondimensionalizing then yields the relation between dimensionless Young's modulus and measured hardness

$$Y = \frac{(1 + Mi\mathcal{H})}{(1 - \mathcal{H})} \quad (11)$$

We again uncover the same dimensionless mechanical indentability, Mi . Equations 7, 9, and 11 are the main results of this paper. Equation 11 shares the same form with Eqs 7 and 9, these differing

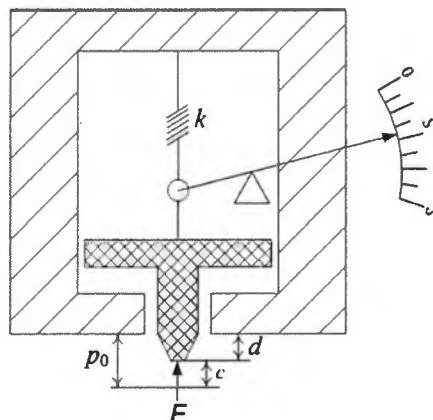


FIG. 3—Basic durometer during indentation.

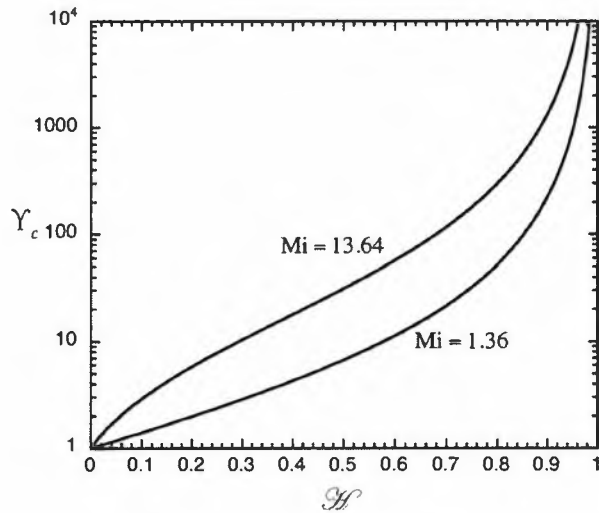


FIG. 4—Mechanical indentability and the relation between dimensionless conical Young's modulus and dimensionless hardness (Eq 7). Used to deduce Young's modulus from hardness measured with ASTM D2240-05 scales B [$M_i = 13.64$] or M [$M_i = 1.36$].

only by their geometric shape factors and the denominator powers. Figure 6 shows the relation between the logarithm of the Young's modulus Y and the hardness $\mathcal{H}$ for the scale now standardized in ASTM D2240-05 as scale A [$M_i = 13.64$].

Comparing Eqs 5, 8, and 10, we see that for each of these indenter shapes, indentation depends on the same ratio $((1-\nu^2)/E)$. Hence, deducing E from measured hardness requires us to know ν . Since $\nu \approx 1/2$ for rubbers and plastics, most polymer engineers will prefer to use the approximation (see Table 3)

$$Y \approx \frac{8rp_0}{3F_0} E \quad (12)$$

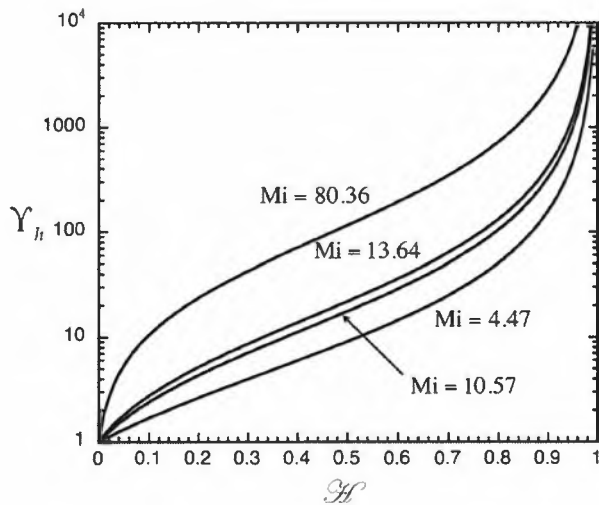


FIG. 5—Mechanical indentability and the relation between dimensionless hemispherical Young's modulus and dimensionless hardness. Used to deduce Young's modulus from hardness measured with ASTM D2240-05 scales O [$M_i = 13.64$], OO [$M_i = 4.47$], OOO [$M_i = 4.47$], OOO-S [$M_i = 10.57$], or E [$M_i = 13.64$] and ASTM F1957-99 scale CF [$M_i = 80.36$].

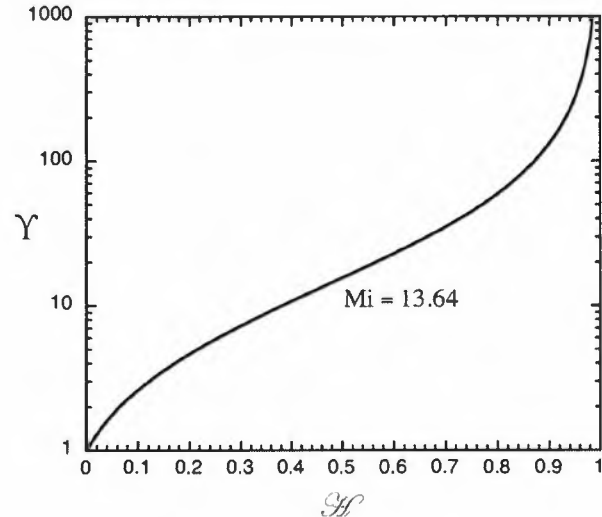


FIG. 6—Mechanical indentability and the relation between dimensionless Young's modulus and dimensionless hardness. Used to deduce Young's modulus from hardness measured with ASTM D2240-05 scale A [$M_i = 13.64$].

Dimensionless Durometer Bounds

The dimensionless Young's modulus Y is bounded on the low end and is unbounded on the high end

$$1 \leq Y \leq +\infty \quad (13)$$

and the hardness $\mathcal{H}$ is bounded on both ends

$$0 \leq \mathcal{H} \leq 1 \quad (14)$$

Equation 11 implies the nonzero intercept for the dimensionless Young's modulus Y

$$Y|_{\mathcal{H}=0} = 1 \quad (15)$$

Hence, for any given durometer scale, there is a lower bound to the Young's modulus E_0 for which measured hardness can be detected due to a spring preload. Specifically, values of Y subceeding unity are thus off-scale

$$Y < 1 \quad (16)$$

or in dimensional terms

$$E < E_0 \quad (17)$$

Therefore, a standardized hardness scale is characterized by the lower bound of the Young's modulus E_0 . Although a standardized hardness scale is unbounded, the upper range of the Young's modulus is very sensitive to changes in measured hardness. This provokes our following study of indenter nonlinearity.

Indenter Nonlinearity

We use the differences in nonlinearity to compare the sensitivities of the standardized hardness scales to one another. To examine the nonlinearity between the indenter types, we expand our dimensionless durometer Eqs 7, 9, and 11 at midscale. Expanding Eq 11 at midscale gives

$$Y = 1 + (1 + Mi) \left\{ 1 + 4 \left(\mathcal{H} - \frac{1}{2} \right) + 8 \left(\mathcal{H} - \frac{1}{2} \right)^2 + \dots \right\} \quad (18)$$

Equation 9 yields

$$Y_h = \sqrt{2} \left\{ 1 + (1 + Mi) + (5Mi + 6) \left(\mathcal{H} - \frac{1}{2} \right) + \left(\frac{27Mi + 30}{2} \right) \left(\mathcal{H} - \frac{1}{2} \right)^2 + \dots \right\} \quad (19)$$

and Eq 7

$$Y_c = 2 \left\{ 1 + (1 + Mi) + (6Mi + 8) \left(\mathcal{H} - \frac{1}{2} \right) + (20Mi + 24) \left(\mathcal{H} - \frac{1}{2} \right)^2 + \dots \right\} \quad (20)$$

For flat tip indentors, the ratio of the coefficient of the first non-linear term to that of the linear one is universally $n_f = 2$. For hemispherical and conical indentors, these same ratios are more complicated, namely

$$n_h = \frac{27Mi + 30}{2(5Mi + 6)} \quad (21)$$

and

$$n_c = \frac{2(5Mi + 6)}{3Mi + 4} \quad (22)$$

and combining

$$0 < Mi \leq \infty \quad (23)$$

with Eqs 21 and 22 yields

$$\frac{5}{4} < \frac{n_h}{n_f} \leq \frac{27}{20} \quad (24)$$

Equation 24 compares nonlinearity of the hemispherical indenter to the flat one. In other words durometers employing hemispherical indentors are always more highly nonlinear devices than those with flat tips.

Similarly

$$\frac{3}{2} < \frac{n_c}{n_f} \leq \frac{5}{3} \quad (25)$$

from which we can more generally conclude that, at midscale, the relationship $Y(\mathcal{H}, Mi)$ for durometers employing conical indentors, is always more highly nonlinear than for durometers with hemispherical indentors, which is already more highly nonlinear than for flat ones.

Error in Measured Hardness

Variation in hardness measurement can arise either from imperfections in the durometer or in the durometer technique, or sample variability or material heterogeneity. By material heterogeneity we mean the variation in Young's modulus from place to place on or near the sample surface. If the imprecision in the measured hardness is known, we can then use Eq 7, Eq 9, or Eq 11 to calculate the confidence interval for the Young's modulus deduced therefrom. Since Eq 7, Eq 9, or Eq 11 is nonlinear, such confidence intervals

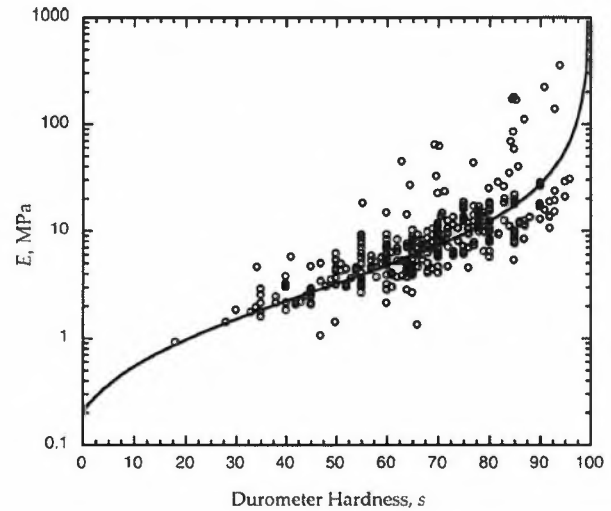


FIG. 7—Comparison between theory (Eqs 7, 9, and 11) (solid line) and experimental (open circles) data for commercially available rubber. Darker regions are more densely populated open circles.

do not straddle the deduced Young's modulus evenly. On the other hand, if the heterogeneity in the Young's modulus at or near the surface is known, Eq 7, Eq 9 or Eq 11 can also be used to calculate the range in the corresponding hardness. Since Eq 7, Eq 9, or Eq 11 is nonlinear, this range will not evenly straddle the mean hardness.

For a durometer's usable range, standards often recommend [27]

$$\frac{1}{5} \leq \mathcal{H} \leq \frac{9}{10} \quad (26)$$

but Eqs 7, 9, and 11 provide an objective basis for defining this range.

Good technique in durometry requires the presser foot to be flush to the surface. Failing to hold the durometer flush to the surface results in lower measured hardness and error which Mohamed and Aggag analyze [41]. A poor technique to avoid is pressing the durometer onto the test surface too quickly so as to drop it onto the surface. This poor handling can impair the durometer calibration, and calibration procedures have been standardized [27,42]. A test bar or a rubber reference block can also be used to make sure the durometer is in working condition [43].

Durometer Hardness to Young's Modulus

To test Eqs 7, 9, and 11 we mine experimental data from a large commercial repository. Specifically, using MatWeb [44], durometer hardness and secant modulus [45] measurements for flat, conical and hemispherical indentors, respectively scales A, D, and O on commercially available rubbers. Since Eqs 7, 9, and 11 are for Young's modulus, classical rubber elasticity theory [46] was used to convert the secant moduli to Young's moduli (see Appendix). A total of 468 different commercially available rubbers (including filled and unfilled) are plotted as open circles in Fig. 7, so darker regions are more densely populated. The results show that Eqs 7, 9, and 11 are mostly accurate with some deviations. Filled rubbers will exhibit $\nu \ll 1/2$ and will deviate from Eqs 7, 9, and 11. Commercial software for these conversions to Young's modulus is now available [47].

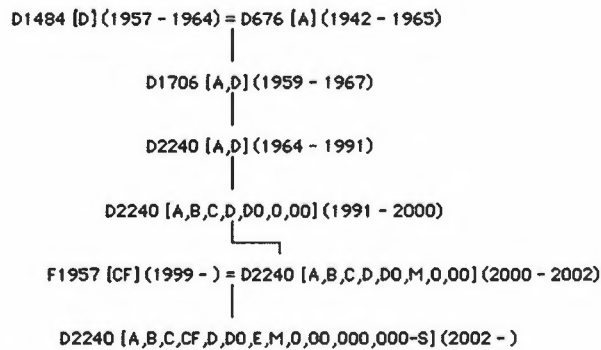


FIG. 8—ASTM International D2240 genealogy.

Conversion

The number of durometer scales has grown and conversion between scales is often necessary. Also, to intelligently conceive new scales, we need to relate them to existing ones. By trial, industry develops new durometer designs, and the standards eventually standardized some of these.

Figure 8 shows the genealogy of ASTM D2240-05 that begins with just the A scale and multiplies into today's 12 scales. There is significant overlap of Young's modulus between these 12 durometers scales, Eqs 7, 9, and 11 can be used to minimize such overlap when developing new scales.

In some older standards [48], durometry books [31], and in industry conversion nomograms are commonly encountered. These are flawed and typically supplied with a disclaimer "for comparison only." They do not state their basis and their end points are often wrong. Some align the zeros, all misalign the hundreds, and still others combine both of these flaws. We use Eqs 7, 9, and 11 to create nomograms based on linear elastic indentation mechanics. Figures 9 and 10 are such nomograms using scale A as the base scale and using linearly spaced Young's modulus for conversion below the preload of scale A. Figure 9 is sorted by midscale modulus, and Fig. 10, by minimum modulus. At midscale, standardized

scale 000 is used for softest materials while scale D, for the hardest.

By definition, all scales must line up at 100 since this represents infinite modulus for all scales, but not all scales should align at 0 because of differing preloads. The preload determines the minimum modulus the material must have to displace the indenter at all. When using nomograms, one proceeds carefully near 0 and 100. Specifically, near 100, durometers are extremely nonlinear and a precise reading is difficult. For 0 readings on preloaded durometers, the converted hardness is only an upper bound.

Nomograms provide no information about Young's modulus. Figure 11 converts hardness to Young's modulus and still can be used to convert between standardized scales. To convert, read up from the measured hardness to the measured scale. Then move horizontally at constant Young's modulus to the desired scale. From Fig. 11 one can see that ASTM scales have *crossover points*. Most of the crossover points occur near the lower and higher ends of the scales, so practitioners are cautioned to avoid these.

Discussion

Equations 7, 9, and 11 are the main results of this paper. Equation 11 provides a path for estimating Young's modulus from any hardness measured using a flat tip indenter, Eq 9 provides such a path for the hemispherical indenter, and Eq 7, for the conical indenter. Equations 7, 9, and 11 provide the pathways for interconverting between scales employing either the same indenter shapes, or even between those employing different ones. Though Eqs 7, 9, and 11 apply to single indentation durometry, they do not apply to *differential durometry*, where standardized tests [19,20,26] record the difference between two indentations such as Rockwell hardness, for example.

Following our dimensionless approach to durometry, and using the solution of an arbitrary indenter tip shape in Sneddon [6], it may be possible to obtain dimensionless equations for any number of novel indenter shapes. Then using our analysis of the lower bound of the Young's modulus, and the nonlinearity of the indenter

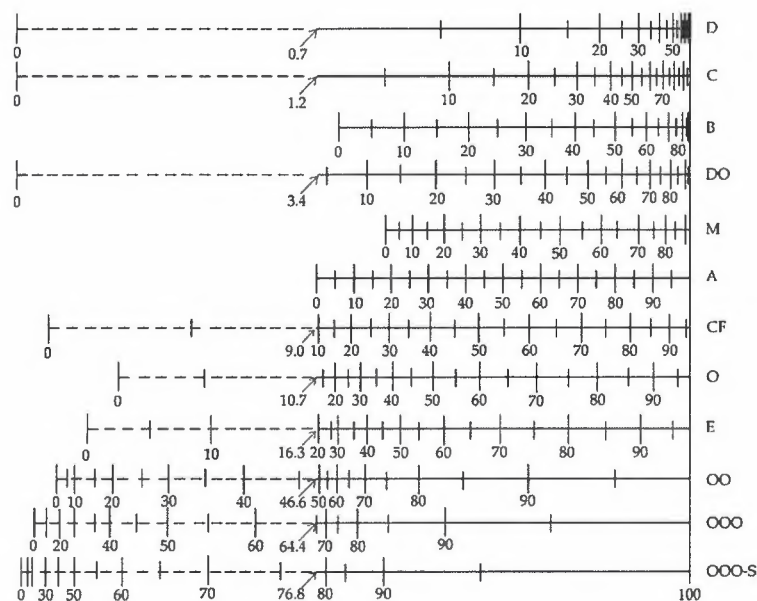


FIG. 9—Midscale sorted nomogram for ASTM D2240-05 durometer scale conversion based on linear elastic indentation mechanics.

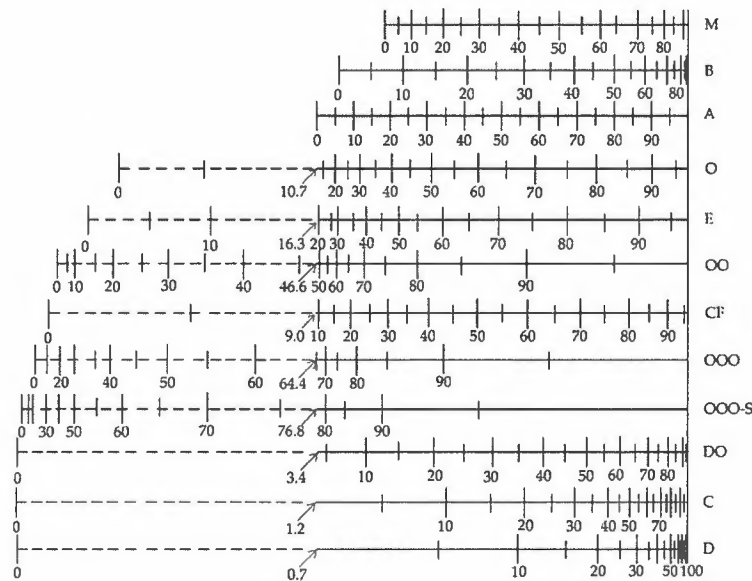


FIG. 10—Minimum modulus sorted nomogram for ASTM D2240-05 durometer scale conversion based on linear elastic indentation mechanics.

mechanics, one can better design durometers and standardized hardness scales for special applications.

Table 4 lists durometer specifications for some of the most popular standardized hardness scales [27,28].

Worked Examples

An engineer is interested in knowing the Young's modulus for the rubber roll on her roll coating machine. She made scale A standardized hardness measurements on this roll and obtained a mean scale reading of $s=90$ degrees of hardness, or $\mathcal{H}=0.90$. For scale A from Table 4, she finds the mechanical indentability, $M_i=13.64$. Using Eq 11, she deduces a dimensionless Young's modulus, $Y=133$, and then confirms this using Fig. 6. Finding scale A properties in Table 4, she finds the indenter radius of $r=0.0395$ cm, the initial spring force of $F_0=0.55$ N, and the initial protrusion of $p_0=0.25$ cm. Solving for the Young's modulus, E , in the definition of Y (from Table 3) yields

$$E = \frac{3F_0Y}{8p_0r} = \frac{3(0.55 \text{ N})(133)}{8(0.0025 \text{ m})(0.000395 \text{ m})} = 27.8 \text{ MPa} \quad (27)$$

To compare with an older quality control chart, this engineer needs scale D measurements. She is thus now interested in converting her measured scale A to D. Using Fig. 11 we see $E=27.8$ MPa corresponds to the scale D reading of $s=36.9$ degrees of hardness.

Conclusions

Nondimensionalizing the indenter mechanics uncovered a new dimensionless indenter stiffness which we call mechanical indentability, M_i , which can be used for classifying hardness scales. The major results of this paper are Eqs 7, 9, and 11 that, respectively, relate the Young's modulus to measured scale hardness for flat, hemispherical, and conical indenter geometries. From these equations we have produced three figures that show the effect of the

mechanical indentability, M_i , on these relations between Young's modulus and measured scale hardness. Our results can also be used to convert between different hardness scales, not only between hardness scales of the same indenter geometry, but also between those differing in geometry.

Some believe that there is no correspondence between s and E . For instance, the ASTM standard for durometry states that "No simple relationship exists between indentation hardness determined by this test method and any fundamental property of the material tested" [27] but Eqs 7, 9, and 11 contradict this advice. We do know that Eqs 7, 9, and 11 may not apply when the tested surface is oily or dirty. Also, for heterogeneous samples where the Young's modulus near the tested surface differs from the modulus elsewhere, then moduli deduced from Eq 7, Eq 9, or Eq 11 will differ from those measured on a tensile tester. In fact, combining durometry with layer removal, one can use Eq 7, Eq 9, or Eq 11 to get Young's modulus profiles through spatially heterogeneous materials.

We also know that Eqs 7, 9, and 11 are only accurate for indentations that are linear elastic or nearly so. Johnson [16] has explored

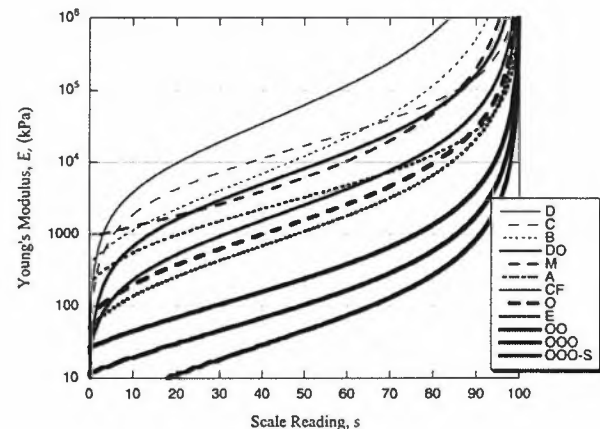


FIG. 11—Conversion chart for Young's modulus and between scales.

TABLE 4—Dimensional and dimensionless values for standardized hardness scales.

		Shore ^a	A ^b	B ^b	C ^b	D ^b
r	cm	0.0395	0.0395	0.0670	0.0395	0.0670
p_0	cm	0.254	0.250	0.250	0.250	0.250
s_r	DH	100	100	100	100	100
σ	cm/DH	2.54×10^{-3}	2.50×10^{-3}	2.50×10^{-3}	2.50×10^{-3}	2.50×10^{-3}
k	N/DH	0.0751	0.0750	0.0750	0.4445	0.4445
F_0	N	0.549	0.550	0.550	0	0
α	Degrees	...	...	15	...	15
ρ		...	...	...	...	...
ς		...	...	3.14	...	3.14
E_0	kPa	205	209	387	0	0
Mi		13.68	13.64	13.64	∞	∞
		O ^b	DO ^b	OO ^b	M ^b	
r	cm	0.119	0.119	0.119	0.0394	
p_0	cm	0.250	0.250	0.250	0.125	
s_r	DH	100	100	100	100	
σ	cm/DH	2.50×10^{-3}	2.50×10^{-3}	2.50×10^{-3}	1.25×10^{-3}	
k	N/DH	0.0750	0.4445	0.00908	0.00440	
F_0	N	0.55	0	0.203	0.324	
α	Degrees	...	...	...	15	
ρ		1.03	1.03	1.03	...	
ς		...	...	...	3.69	
E_0	kPa	71.7	0	26.5	912	
Mi		13.64	∞	4.47	1.36	
		E ^b	OOO ^b	OOO-S ^b	CF ^c	
r	cm	0.25	0.635	1.07	0.635	
p_0	cm	0.250	0.250	0.50	0.50	
s_r	DH	100	100	100	100	
σ	cm/DH	2.50×10^{-3}	2.50×10^{-3}	5.00×10^{-3}	5.00×10^{-3}	
k	N/DH	0.0750	0.00908	0.01765	0.8829	
F_0	N	0.55	0.203	0.167	1.0987	
α	Degrees	...	...	...	...	
ρ		1.50	2.39	2.19	1.69	
ς		...	...	...	...	
E_0	kPa	49.5	11.5	2.57	21.9	
Mi		13.64	4.47	10.57	80.36	

^aSource: Gent [5].^bSource: ASTM D2240-05 [27].^cSource: ASTM F1957-99 [28].

material plasticity, how it affects durometry, and specifically, how it delimits linear elasticity in indentation. In practice, engineers need only carefully inspect the indented surfaces following durometry for permanent deformation. The more severe such marks, the less accurate Eq 7, Eq 9, or Eq 11 will be.

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Appendix

Using classical rubber elasticity theory [46] to convert secant modulus at unit strain to Young's modulus, begin with the classic extension force relation

$$f = G \left(\lambda - \frac{1}{\lambda^2} \right) \quad (28)$$

where:

f =force per unit unstrained area,
 G =shear modulus, and
 λ =extension ratio.

Using the following relation from linear elastic mechanics to relate shear modulus, G , to Young's modulus, E , and using the good approximation for simple rubbers that the Poisson ratio, $\nu \approx 0.5$:

$$G = \frac{E}{2(1 + \nu)} \approx \frac{E}{3} \quad (29)$$

Substituting Eq 29 into Eq 28 and since at unit strain $\lambda = 2$

$$f = \frac{E}{3} \left(\lambda - \frac{1}{\lambda^2} \right) = \frac{7E}{12} \quad (30)$$

Hence, the universal ratio of the secant modulus at unit strain to the Young's modulus for a simple rubber is $7/12$.

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