

INTERNATIONAL EDITION

Third Edition

MECHANICS OF MATERIALS

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lower portion is

$$dF_2 = \sigma_2 dA = -\frac{E_2 y}{\rho} dA \quad (4.26)$$

But, denoting by n the ratio E_2/E_1 of the two moduli of elasticity, we can express dF_2 as

$$dF_2 = -\frac{(nE_1)y}{\rho} dA = -\frac{E_1 y}{\rho} (n dA) \quad (4.27)$$

Comparing Eqs. (4.25) and (4.27), we note that the same force dF_2 would be exerted on an element of area $n dA$ of the first material. In other words, the resistance to bending of the bar would remain the same if both portions were made of the first material, provided that the width of each element of the lower portion were multiplied by the factor n . Note that this widening (if $n > 1$), or narrowing (if $n < 1$), must be effected in a direction parallel to the neutral axis of the section, since it is essential that the distance y of each element from the neutral axis remain the same. The new cross section obtained in this way is called the *transformed section* of the member (Fig. 4.25).

Since the transformed section represents the cross section of a member made of a *homogeneous material* with a modulus of elasticity E_1 , the method described in Sec. 4.4 can be used to determine the neutral axis of the section and the normal stress at various points of the section. The neutral axis will be drawn *through the centroid of the transformed section* (Fig. 4.26), and the stress σ_x at any point of the corresponding

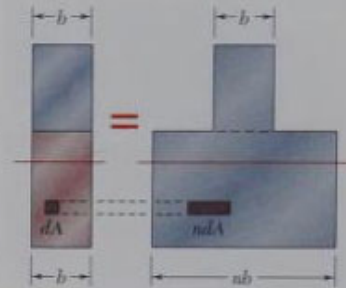


Fig. 4.25 Transformed section for composite bar.

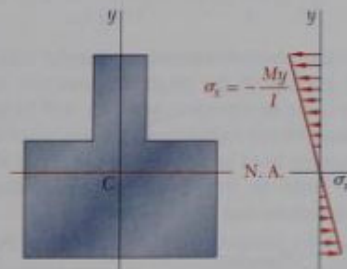


Fig. 4.26 Distribution of stresses in transformed section.

fictitious homogeneous member will be obtained from Eq. (4.16)

$$\sigma_x = -\frac{My}{I} \quad (4.16)$$

where y is the distance from the neutral surface, and I the *moment of inertia of the transformed section* with respect to its centroidal axis.

To obtain the stress σ_1 at a point located in the upper portion of the cross section of the original composite bar, we simply compute the stress σ_x at the corresponding point of the transformed section. However, to obtain the stress σ_2 at a point in the lower portion of the cross section, we must *multiply by n* the stress σ_x computed at the corresponding point of the transformed section. Indeed, as we saw earlier,