$$\left.\begin{aligned}
&10\text{ cfm:}\quad H_5 = (1.0)(0.0032) = 0.0032\text{ in H}_2\text{O} = 0.0081\text{ cm H}_2\text{O}\\
&20\text{ cfm:}\quad H_5 = \left(\frac{20}{10}\right)^2(0.0032) = 0.0128\text{ in H}_2\text{O} = 0.0325\text{ cm H}_2\text{O}\\
&30\text{ cfm:}\quad H_5 = \left(\frac{30}{10}\right)^2(0.0032) = 0.0288\text{ in H}_2\text{O} = 0.0731\text{ cm H}_2\text{O}
\end{aligned}\right\}\quad(6.30)$$

6 *Exhaust from PCB Duct and Chassis.* The velocity of the air at the PCB exit is the same as the velocity at point 5. However, at the exit, one full velocity head is lost, as shown by Eq. 6.24. The losses for each flow rate are as follows:

$$\left.\begin{aligned}
&10\text{ cfm:}\quad H_6 = (1.0)(0.0032) = 0.0032\text{ in H}_2\text{O} = 0.0081\text{ cm H}_2\text{O}\\
&20\text{ cfm:}\quad H_6 = \left(\frac{20}{10}\right)^2(0.0032) = 0.0128\text{ in H}_2\text{O} = 0.0325\text{ cm H}_2\text{O}\\
&30\text{ cfm:}\quad H_6 = \left(\frac{30}{10}\right)^2(0.0032) = 0.0288\text{ in H}_2\text{O} = 0.0731\text{ cm H}_2\text{O}
\end{aligned}\right\}\quad(6.31)$$

The exhaust losses are relatively small, so that very little energy is wasted at the exit.

The static pressure loss at each flow is the sum of the individual pressure losses, which are shown in Tables 6.1 and 6.1A.

Table 6.1 is used to plot the chassis air flow impedance curve, which is shown in Figure 6.20. The static pressure curve for two different vane axial flow fans are superimposed on the same curve, using the manufacturers data. Both fans have 400 cycle motors in similar 2 in diameter housings.

**Table 6.1 Static Pressure Loss for Different Air Flow Rates through Electronic Box in English Units**

| Station Point | Pressure Loss (in $H_2O$) | | | |
|---|---|---|---|---|
| | 10 cfm | 20 cfm | 30 cfm | |
| 1 | 0.0386 | 0.1544 | 0.3470 | |
| 2 | 0.0340 | 0.1360 | 0.3060 | |
| 3 | 0.0287 | 0.1150 | 0.2590 | |
| 4 | 0.0406 | 0.1620 | 0.3650 | |
| 5 | 0.0032 | 0.0128 | 0.0288 | |
| 6 | 0.0032 | 0.0128 | 0.0288 | ← Exhaust |
| Total in $H_2O$ | 0.1483 | 0.5930 | 1.3346 | |

SEC et al. v. MRI
SEC Exhibit 1027.0201
IPR2023-00255

**Table 6.1A Static Pressure Loss for Different Air Flow Rates through Electronic Box in Metric Units**

| Station Point | Pressure Loss (cm $H_2O$) $4719\ \frac{cm^3}{sec}$ | $9438\ \frac{cm^3}{sec}$ | $14{,}157\ \frac{cm^3}{sec}$ | |
|---|---|---|---|---|
| 1 | 0.0980 | 0.3920 | 0.8820 | |
| 2 | 0.0860 | 0.3450 | 0.7770 | |
| 3 | 0.0729 | 0.2920 | 0.6570 | |
| 4 | 0.1030 | 0.4110 | 0.9270 | |
| 5 | 0.0081 | 0.0325 | 0.0731 | |
| 6 | 0.0081 | 0.0325 | 0.0731 | ← Exhaust |
| Total cm $H_2O$ | 0.3761 | 1.5050 | 3.3892 | |

Fan A is a 3 phase 15,500 rpm, 25 watt motor. Fan B is a single phase 11,000 rpm 18 watt motor which speeds up at high altitudes [44].

If fan A is used, the air flow rate through the chassis will be about 30 cfm with a constant cfm flow rate at all altitudes. If fan B is used, the sea level air flow rate will be 23 cfm. Fan B, however, is a variable speed fan which increases in speed as the altitude is increased (Rotron trade name is Altivar). This type of fan is usually very desirable for cooling electronic boxes at high altitudes where the air density is reduced [44].

The air weight flow through the electronic box can be determined from the average air density ($\rho$) in the box, using Eq. 6.32, which is obtained from the gas laws [45–47].

$$\rho = \frac{P}{RT} \tag{6.32}$$

where $P = 14.7\ \text{lb/in}^2$ sea level pressure (English units)
$P = 1034.4\ \text{g/cm}^2$ sea level pressure (metric units)
$R = 53.3\ \text{ft/°R}$ gas constant for air (English units)
$R = 2924\ \text{cm/°K}$ gas constant for air (metric units)

$$t_{av} = \frac{131 + 160}{2} = 145.5°\text{F} = 63°\text{C (average temperature)}$$

$$T = 460 + 145.5 = 605.5°\text{R (absolute air temperature)}$$

$$\rho = \frac{(14.7\ \text{lb/in}^2)(144\ \text{in}^2/\text{ft}^2)}{(53.3\ \text{ft/°R})(605.5°\text{R})} = 0.0656\ \frac{\text{lb}}{\text{ft}^3}\ \text{(English units)} \tag{6.33}$$

The weight flow is determined from Eq. 6.34.

$$W = \rho F \tag{6.34}$$

SEC et al. v. MRI
SEC Exhibit 1027.0202
IPR2023-00255

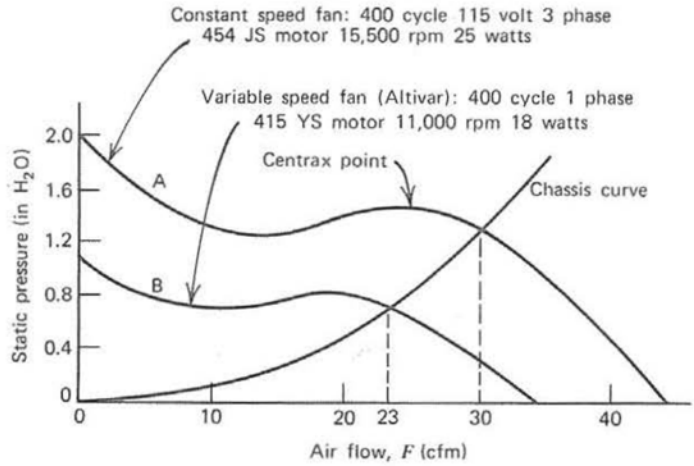



**Figure 6.20** Chassis air flow impedance curve superimposed on two different vane axial fan curves with different speeds (both fans have same 2 in diameter outer housing).

Using fan A, where the air flow $F = 30$ cfm, the weight flow is

$$W = \left(0.0656 \frac{\text{lb}}{\text{ft}^3}\right)\left(30 \frac{\text{ft}^3}{\text{min}}\right) = 1.97 \frac{\text{lb}}{\text{min}} \tag{6.35}$$

Using fan B, where the air flow is 23 cfm, the weight flow is:

$$W = \left(0.0656 \frac{\text{lb}}{\text{ft}^3}\right)\left(23 \frac{\text{ft}^3}{\text{min}}\right) = 1.51 \frac{\text{lb}}{\text{min}} \tag{6.36}$$

Equation 6.8 showed that the minimum required cooling air flow rate is 1.35 lb/min. Since fans A and B can both supply more than the minimum required flow rate, either fan will be acceptable.

## 6.8 HOLLOW CORE PCBs

Cooling air is often contaminated with moisture, which can bridge printed circuits and electrical connector contacts. This can result in a high resistance path across two conductors, which can affect the electrical operation of high impedance circuits. Many of the new specifications for advanced electronic systems will, therefore, not permit cooling air to come into direct contact with the electronic components or circuits. Under these circumstances, the fan cooling air is usually ducted through the walls of the chassis, which have been made into heat exchangers. Plug-in types of printed circuit boards (PCBs) with metal cores of strips, similar to those shown in Figures 4.1 and 4.2, and then used to conduct away the heat.

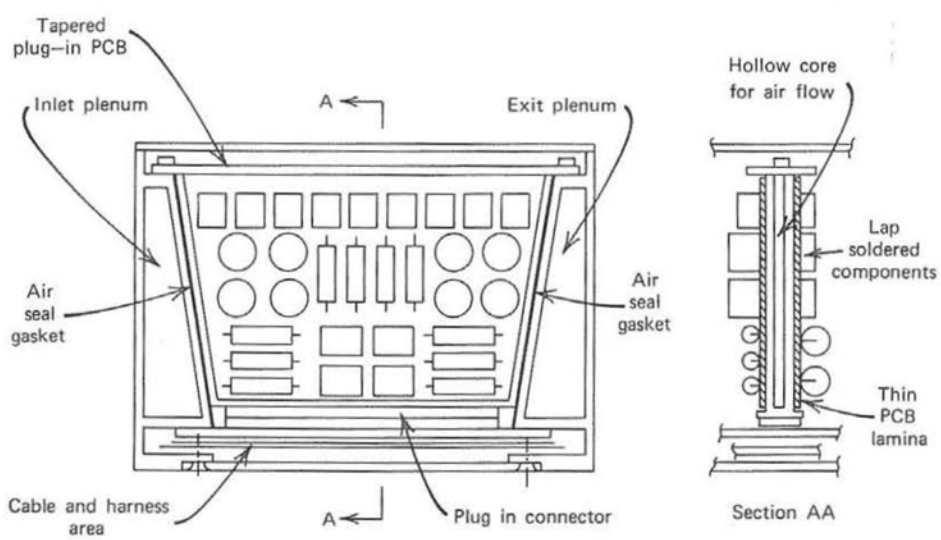
SEC et al. v. MRI
SEC Exhibit 1027.0203
IPR2023-00255

The length of the heat flow path from the PCB components to the cooling air in the chassis walls, as shown in Figure 4.2, is quite long. This may result in a large temperature rise. If a hollow core can be provided for the cooling air between each circuit board, with the components on the outside, the length of the heat flow path from the component to the cooling air will be sharply reduced. This will reduce the temperature rise to the components, so that they will run cooler and last longer.

Electrical lead wires must not extend into the cooling air ducts in the hollow cores, so that components must be lap soldered to the circuit boards. Existing flow soldering and wave soldering equipment, normally used for high volume production of printed circuits, could not be used to assemble lap soldered components.

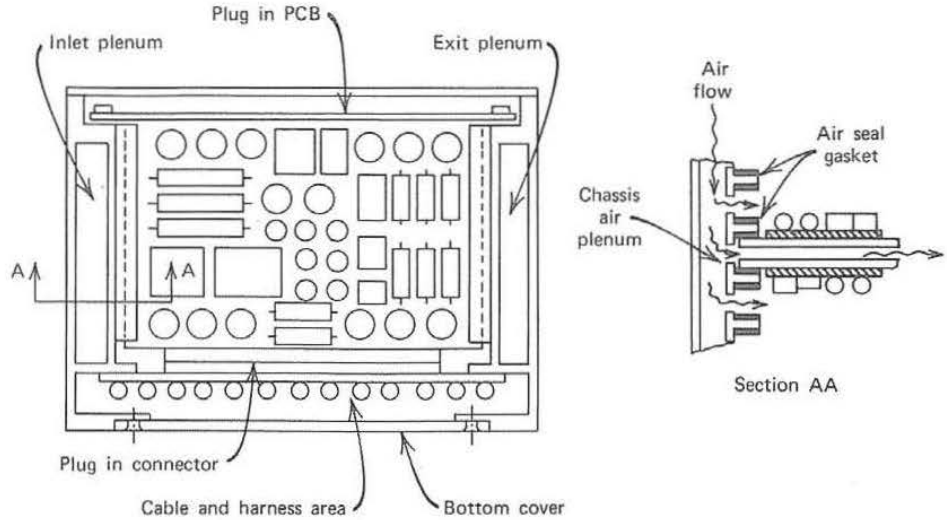
The most difficult task associated with plug-in hollow core PCBs is the air seal. A low resistance air flow path must be provided to the PCB, with a seal that can be engaged and disengaged many times with very little air leakage.

Figure 6.21*a*, *b*, and *c* shows some different types of air seals which have been used successfully for plug-in types of hollow PCBs. Figure 6.21*a* relies on tapered PCB sides to provide a seal with a soft gasket interface. Figure 6.21*b* uses an overlap interface, with a gasket that seals the outer surfaces at the extreme edges of the PCB. In the foregoing two designs, the inlet cooling air plenum and the exhaust air plenum are part of the chassis. In Figure 6.21*c*, the inlet cooling air slot and the exhaust air slot are part of the PCB itself. The PCBs are stacked together and clamped using four axial clamp bars, located at the four corners. This stacking arrangement forms a

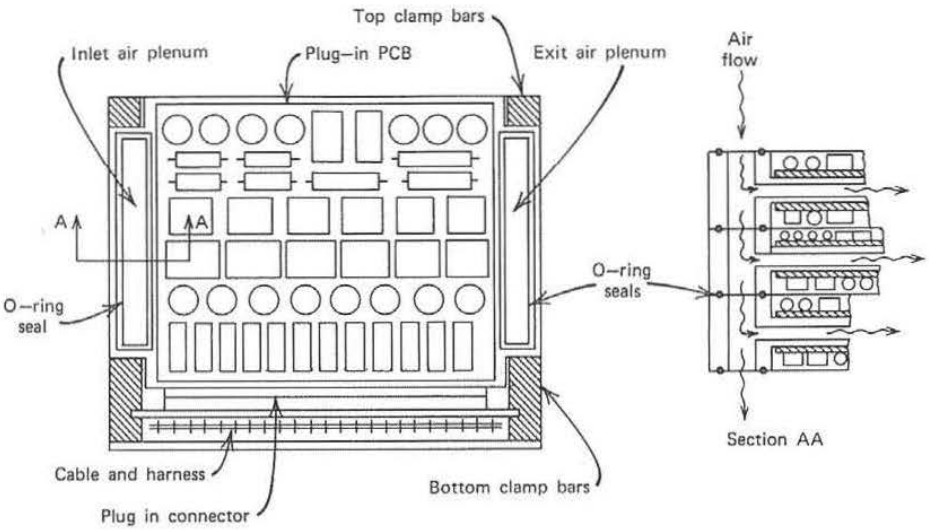



(a)

**Figure 6.21** Hollow core air cooled circuit boards.




(b)




(c)

**Figure 6.21** (Continued)

cooling air plenum out of the slots on each side of the PCB. An O-ring seal is used around the perimeter of each side slot to reduce the air leakage [48].

Equation 6.9 can still be used to determine the forced convection coefficient $h_c$ for the hollow core PCB. If a slot width of 0.050 in (0.127 cm) is used for the hollow core PCB, the average forced convection coefficient shown in Eq. 6.15 can be doubled to about 12 Btu/hr ft$^2$ °F.

Extreme care must be exercised if the hollow core PCBs will be operating in a vibration environment. The thin epoxy fiberglass circuit boards will

have a low resonant frequency, which will result in large vibration displacement amplitudes and rapid fatigue failures [1].

To reduce the failure rate and to provide a long fatigue life, stiffening ribs should be provided in the hollow core air space between the circuit boards. If more stiffening is required, external stiffening ribs can be added to the outside surface of the circuit boards between the electronic components.

## 6.9 COOLING AIR FANS FOR ELECTRONIC EQUIPMENT

Many different types of fans are available for cooling electronic equipment. These can generally be broken down into four major types: propeller, tube axial, vane axial, and centrifugal blowers. These fans can be single phase, three phase, 60 cycles, 400 cycles, 800 cycle ac/dc, constant speed, variable speed, and flow rates from 1 cfm to several thousand cfm [44].

Propeller fans can be obtained in a large variety of sizes, with different blade shapes and arrangements. Large propeller fans may be belt driven from an electric motor, while small fans may have the propeller fan attached directly to the electric motor, as shown in Figure 6.22. This type of fan can normally provide a static pressure of about 0.30 in (0.76 cm) of water at 0 cfm, or no delivery. Propeller fans are well suited for flushing operations through large racks or cabinets, which require large volumes or air at relatively low static pressures.

Tube axial fans have their motors mounted in the fan hub. This results in a fan with an axial depth that is much shorter than a propeller fan but which has a greater static pressure capability than a propeller fan of the same diameter. This type of fan, which is shown in Figure 6.23, is normally used where low noise, long life, and low cost are important. Typical applications are in areas that might use propeller fans, but where the depth is the limiting factor and the air flow requirements exceed those of a flushing operation [44].

Figure 6.24 shows a small vane axial fan which operates at a high rotational speed and provides static pressures of more than 4 in (10.16 cm) of water at zero delivery. Fans of this type are usually used for cooling electronic assemblies with high heat densities, which must operate in severe thermal and vibration environments. They generally operate at a high frequency of 400 Hz, and are available with variable speed [44] motors, which

**Figure 6.22** Typical small propeller fan.

SEC et al. v. MRI
SEC Exhibit 1027.0206
IPR2023-00255

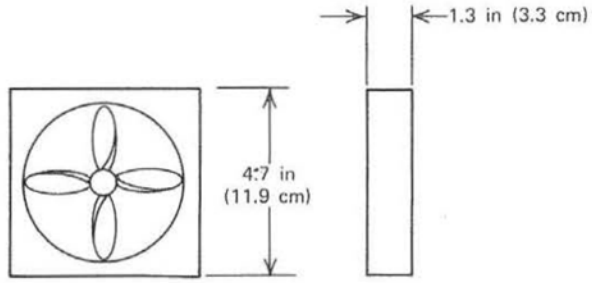



**Figure 6.23** Typical flat tube axial fan that has a rated life of more than 10 years of continuous duty at 20°C.

speed up at high altitudes to provide extra cooling with low density air. These high speed fans generate a high level of acoustic noise, so that mufflers are recommended for fans that will be used in areas adjacent to working personnel.

The pressure flow characteristics of these axial flow fans, with high blade angles, produces a region of discontinuity which corresponds to a stalling condition on the blade airfoil. It is generally not advisable to operate fans in this area (which is sometimes called the centrax point) or at lower flow volumes, as shown in Figure 6.25. Operation should take place in the area to the right of the centrax point.

When the system operates to the left of the centrax point, pressure surges can occur. For example, the static pressure at points A and B in Figure 6.25 are the same, but the cfm flow conditions are not. Therefore, the flow conditions through the box can switch back and forth between points A and B. If the higher flow at point B is required, the reduced flow at point A may produce overheating in the electronics.

If the box impedance curve falls to the left of the centrax point, it is usually possible to bleed air from the fan with shims or holes. This will move the box impedance curve to the right of the centrax point.

The fan will then be delivering an excess cfm flow rate, so that some air will be thrown away. Since the fan will be operating to the right side of the

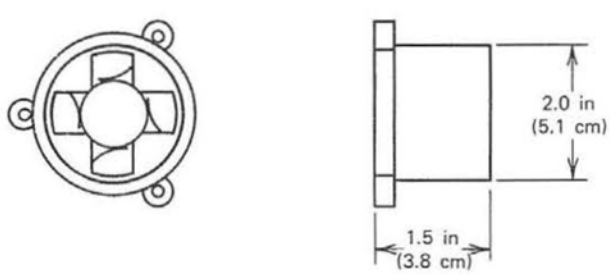



**Figure 6.24** High speed, small size vane axial fan.

SEC et al. v. MRI
SEC Exhibit 1027.0207
IPR2023-00255

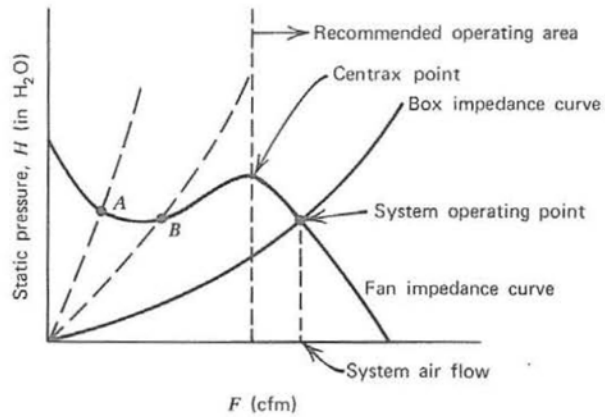



**Figure 6.25** Recommended operating area for axial flow fans.

centrax point, its operation will be stable and not be subjected to pressure surges. Tests should be run to ensure that the proper amount of air is still flowing through the box for cooling.

High static pressures and high air flow rates can be obtained with low rotational speeds using centrifugal blowers (sometimes called squirrel cage blowers). Three types of blades are commonly used: backward curved, radial or paddle blade, and forward curved. A typical centrifugal blower is shown in Figure 6.26.

The highest pressures for a given propeller diameter and speed are usually developed by the forward curved fans. However, a relatively small increase in volume flow may cause a considerable increase in the power required.

Backward curved fans made with air foil section blades have produced efficiencies well above 80%. These fans are often referred to as nonoverloading fans, because they work well over a large range of volume flow.

## 6.10 AIR FILTERS

Electronic equipment may be required to operate around heavy machinery or in dusty areas where the air can pick up oil, lint, and other impurities.

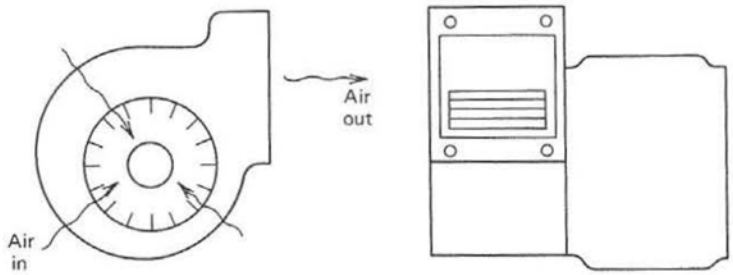



**Figure 6.26** Centrifugal (squirrel cage) fan.

SEC et al. v. MRI
SEC Exhibit 1027.0208
IPR2023-00255

If these contaminates are permitted to enter the cooling air passages, they might plug up the openings and reduce the air flow, causing overheating of the electronics. To prevent this from happening, air filters are often used.

The pressure drop across an air filter can be very large if the air velocity through the filter is too high. To keep the pressure drop to a minimum, the air velocity through the filter should be less than about 250 ft/min (127 cm/sec). A lower air velocity will reduce the pressure drop through the filter even more, but will require a large filter surface area. This is desirable if there is sufficient room available for the larger filter.

Some filter manufacturers recommend a minimum air flow velocity of about 200 ft/min (102 cm/sec) through their filters. This velocity is supposed to make the suspended particles strike the filter with sufficient force to capture the particle. A lower velocity through their filter might permit some particles to change their direction so that they pass through larger openings in a filter that is not too dense.

## 6.11 CUTOFF SWITCHES

When cooling air is required to provide a safe operating temperature for electronic equipment, some means should be provided to turn the equipment off when the cooling air flow is interrupted, to prevent damage to the equipment. The required air flow may not be supplied because of a fan failure, power failure, clogged ducts, or clogged filters.

An air flow interlock switch is convenient for turning off the power in electronic systems that are not required to operate in severe vibration, shock, or acceleration environments. This device has an air flow sensor that extends into the air stream. If the air flow falls below a certain critical level, the sensor shuts off the main power to prevent the electronics from overheating.

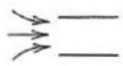
Electronic equipment that must operate in severe vibration, shock, or acceleration environments can use miniature thermal cutoff switches. These switches should be mounted adjacent to the hottest components to prevent them from overheating. They can be purchased to open at virtually any temperature, and the cutoff point is accurate to about ±2°C.

## 6.12 STATIC PRESSURE LOSS TABLES AND CHARTS

Static pressure losses for various flow relations and geometry changes are shown in terms of the velocity head for different structures and conditions in Tables 6.2, 6.3, and 6.4 [40, 41, 69].

SEC et al. v. MRI
SEC Exhibit 1027.0209
IPR2023-00255

**Table 6.2 Entry Losses for Different Shapes**

| Type of Opening | Description | Number of Velocity Heads ($H_v$) Lost |
|---|---|---|
|  | Plain duct end | 0.93 |
| | Flanged duct end | 0.49 |
| | Well–rounded entry | 0.04 |
| | Protruding entry | 2.70 |

## 6.13 HIGH ALTITUDE CONDITIONS

Electronic equipment is often required to operate at high altitudes, without conditioned cooling air. For fan cooled systems, the ambient air within the electronic equipment bay is normally used as the air source.

At high altitudes, the air density is very low. The weight flow of cooling air may be so sharply reduced that the equipment can be damaged as a result of overheating. If the system is protected with a thermal switch, it will simply shut the equipment off.

Many types of fans are available for cooling electronic systems. Some fans will speed up as the air density is reduced, so that they will deliver more cfm and more weight flow at the higher altitudes [44, 49].

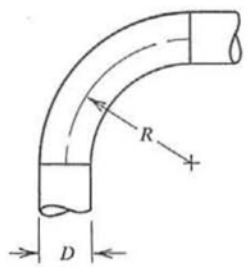
Constant speed fans, either single phase or three phase, will not change their speeds very much. They will run at approximately the same speed, and deliver approximately the same cfm, regardless of the altitude. The weight flow the fan delivers will change as the density or altitude changes. A constant speed fan will, therefore, deliver a different weight flow for different altitudes. As the weight flow through the electronic box changes, the temperatures within the box will also change.

As the altitude (or pressure) changes, the cfm flow through the box will remain the same if a constant speed fan is used. When the altitude is increased, the air density is decreased and therefore the weight flow is decreased. There is less resistance to the flow through the box, so the box flow impedance curve goes down, as shown in Figure 6.27 [42, 44, 49].

With the reduced density at high altitudes, the ability of the fan to draw against a high static pressure is reduced. The static pressure curve for the

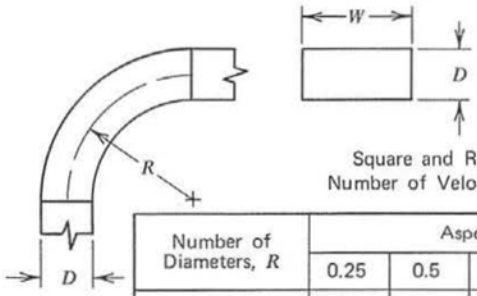
SEC et al. v. MRI
SEC Exhibit 1027.0210
IPR2023-00255

**Table 6.3 Turn Losses in Elbows.**




Round Elbows

| Number of Diameters, $R$ | Number of Velocity Heads ($H_v$) Lost |
|---|---|
| 2.75 | 0.20 |
| 2.50 | 0.22 |
| 2.25 | 0.24 |
| 2.00 | 0.27 |
| 1.75 | 0.32 |
| 1.50 | 0.39 |
| 1.25 | 0.55 |



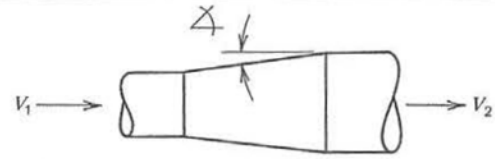

Square and Rectangular Elbows
Number of Velocity Heads ($H_v$) Lost

| Number of Diameters, $R$ | Aspect Ratio, $W/D$ | | | | | |
|---|---|---|---|---|---|---|
| | 0.25 | 0.5 | 1.0 | 2.0 | 3.0 | 4.0 |
| 0.0 (Mitre) | 1.50 | 1.32 | 1.15 | 1.04 | 0.92 | 0.86 |
| 0.5 | 1.36 | 1.21 | 1.05 | 0.95 | 0.84 | 0.79 |
| 1.0 | 0.45 | 0.28 | 0.21 | 0.21 | 0.20 | 0.19 |
| 1.5 | 0.28 | 0.18 | 0.13 | 0.13 | 0.12 | 0.12 |
| 2.0 | 0.24 | 0.15 | 0.11 | 0.11 | 0.10 | 0.10 |
| 3.0 | 0.24 | 0.15 | 0.11 | 0.11 | 0.10 | 0.10 |

fan therefore goes down, as shown in Figure 6.27. The rate of reduction for the box impedance curve is the same as the rate of reduction for the fan static pressure curve, so the two curves cross at the same cfm flow point. As a result, the cfm flow through the box remains approximately constant as the altitude changes.

The static pressure drop through the box at the altitude condition can be determined from the static pressure drop through the box at sea level

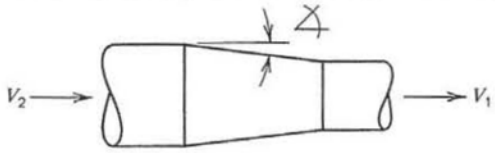
**Table 6.4 Expansion and Contraction Losses.**




$$H_L = a\left(\frac{V_1 - V_2}{4005}\right)^2 \qquad V = \text{velocity} = \frac{ft}{\text{min}}$$

Expansion Loss

| Taper Angle (deg) | Loss Constant $a$ |
|---|---|
| $3\frac{1}{2}$ | 0.22 |
| 5 | 0.28 |
| 10 | 0.44 |
| 15 | 0.58 |
| 20 | 0.72 |
| 25 | 0.87 |
| 30 | 1.00 |
| Over 30 | 1.00 |




Contraction Loss

| Taper Angle (deg) | Loss Constant $a$ |
|---|---|
| 5 | 0.05 |
| 10 | 0.06 |
| 15 | 0.08 |
| 20 | 0.10 |
| 25 | 0.11 |
| 30 | 0.13 |
| 45 | 0.20 |
| 60 | 0.30 |

conditions if the altitude and sea level densities are known. The relation is linear as shown in Eq. 6.37 [42, 44, 49].

$$\Delta P_{\text{alt}} = \Delta P_{\text{sl}}\left(\frac{\rho_{\text{alt}}}{\rho_{\text{sl}}}\right) \tag{6.37}$$

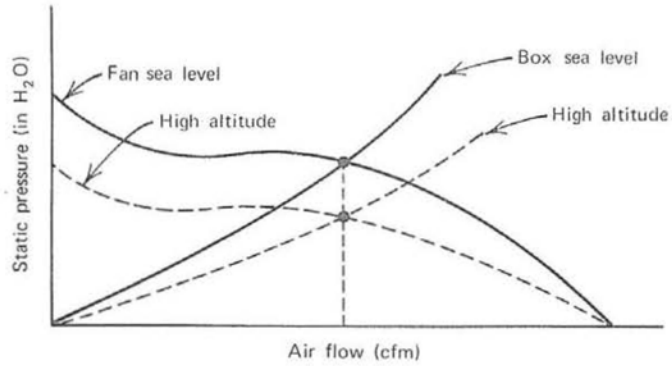



**Figure 6.27** Effects of density (or altitude) on fan performance.

## 6.14 SAMPLE PROBLEM—FAN COOLED BOX AT 30,000 FEET

Determine if the fan cooled box shown in Figure 6.13 is capable of operating at an altitude of 30,000 ft, using a constant speed fan delivering 30 cfm, as shown in Figure 6.20. The total power dissipation of the box is 165 watts, or 563 Btu/hr. This includes the power dissipation of 25 watts for the fan. The ambient air temperature in the equipment bay is expected to be 104°F (40.0°C) according to MIL-E-5400, the electronic equipment specification for class 1X equipment.

### *SOLUTION*

The average density of the cooling air passing through the box is not known because the exit air temperature is not known. Therefore, assume a temperature rise and determine the average air density. Compute the temperature rise based upon the power dissipation and compare it with the assumed value. Correct it if necessary, and repeat the process until reasonably good correlation is obtained. Start by assuming a temperature rise of 66°F (36.6°C). The average air temperature is then

$$t_{av} = \frac{t_{in} + t_{out}}{2} = \frac{104 + 170}{2} = 137°\text{F}$$

The air pressure at 30,000 ft is obtained from Table 6.5 as 629.6 lb/ft$^2$. Now the air density is obtained from Eq. 6.32.

$$\rho = \frac{P}{RT} = \frac{629.6}{(53.3)(460 + 137)} = 0.0198 \ \frac{\text{lb}}{\text{ft}^3} \tag{6.38}$$

SEC et al. v. MRI
SEC Exhibit 1027.0213
IPR2023-00255

**Table 6.5 Changes of Temperature and Pressure with Altitude [37, 38]**

| Temperature and Pressure as Functions of Geometric Altitude: English Units | | |
|---|---|---|
| Altitude (ft) | Temperature (°R) | Pressure ($P$, lbf ft$^{-2}$) |
| −16,500 | 577.58 | 3.6588 + 3 |
| −15,000 | 572.22 | 3.5462 |
| −10,000 | 554.37 | 3.0020 |
| − 5,000 | 536.52 | 2.5277 |
| 0 | 518.69 | 2.1162 |
| 5,000 | 500.86 | 1.7609 |
| 10,000 | 483.04 | 1.4556 |
| 15,000 | 465.23 | 1.1948 |
| 20,000 | 447.43 | 9.7327 + 2 |
| 25,000 | 429.64 | 7.8633 |
| 30,000 | 411.86 | 6.2966 + 2 |
| 35,000 | 394.08 | 4.9934 |
| 40,000 | 389.99 | 3.9312 |
| 45,000 | 389.99 | 3.0945 |
| 50,000 | 389.99 | 2.4361 |
| 55,000 | 389.99 | 1.9180 |
| 60,000 | 389.99 | 1.5103 |
| 65,000 | 389.99 | 1.1893 |
| 70,000 | 389.99 | 9.3672 + 1 |
| 75,000 | 389.99 | 7.3784 |
| 80,000 | 389.99 | 5.8125 + 1 |
| 85,000 | 394.32 | 4.5827 |
| 90,000 | 402.48 | 3.6292 |
| 95,000 | 410.64 | 2.8878 |
| 100,000 | 418.79 | 2.3085 |
| 105,000 | 426.94 | 1.8536 |
| 110,000 | 435.09 | 1.4947 |
| 115,000 | 443.23 | 1.2102 |
| 120,000 | 451.37 | 9.8372 + 0 |
| 125,000 | 459.50 | 8.0267 |
| 130,000 | 467.63 | 6.5735 + 0 |
| 135,000 | 475.76 | 5.4025 |
| 140,000 | 483.88 | 4.4552 |
| 145,000 | 492.00 | 3.6862 |
| 150,000 | 500.11 | 3.0597 |
| 155,000 | 508.22 | 2.5475 |
| 160,000 | 508.79 | 2.1247 |
| 165,000 | 508.79 | 1.7723 |
| 170,000 | 508.79 | 1.4784 |
| 175,000 | 508.79 | 1.2334 |
| 180,000 | 497.49 | 1.0272 + 0 |



SEC et al. v. MRI
SEC Exhibit 1027.0214
IPR2023-00255

Table 6.5 Continued

| Temperature and Pressure as Functions of Geometric Altitude: English Units | | |
|---|---|---|
| Altitude (ft) | Temperature (°R) | Pressure ($P$, lbf ft$^{-2}$) |
| 185,000 | 485.36 | 8.5160 − 1 |
| 190,000 | 473.24 | 7.0278 |
| 195,000 | 461.12 | 5.7713 |
| 200,000 | 449.00 | 4.7151 |
| 210,000 | 424.79 | 3.0955 |
| 220,000 | 400.60 | 1.9835 |
| 230,000 | 376.44 | 1.2368 |
| 240,000 | 352.30 | 7.4774 − 2 |
| 250,000 | 328.20 | 4.3640 |

The average air weight flow through the box is determined from Eq. 6.34.

$$W = \rho F = \left(0.0198 \frac{\text{lb}}{\text{ft}^3}\right)\left(30 \frac{\text{ft}^3}{\text{min}}\right) = 0.594 \frac{\text{lb}}{\text{min}} \tag{6.39}$$

The temperature rise of the cooling air as it passes through the box is determined from Eq. 6.7.

$$\Delta t_A = \frac{Q}{WC_p} = \frac{563 \text{ Btu/hr}}{(0.594 \text{ lb/min})(60 \text{ min/hr})(0.24 \text{ Btu/lb °F})} = 65.8°\text{F} \tag{6.40}$$

The assumed temperature rise of 66°F agrees well with the calculated value of 65.8°F, so that the assumed value is accurate.

The temperature rise across the convection air film to the surface of the electronic components for the altitude condition is still determined from Eq. 5.7, using Eqs. 6.9 through 6.18 for the required information.

Substitute Eq. 6.39 into Eq. 6.12.

$$G = \frac{W}{A} = \frac{(0.594 \text{ lb/min})(60 \text{ min/hr})}{(0.00625 \text{ ft}^2)(7 \text{ ducts})} = 814 \frac{\text{lb}}{\text{hr ft}^2} \tag{6.41}$$

Substitute into Eq. 6.11 for the Reynolds number:

$$N_R = \frac{(814 \text{ lb/hr ft}^2)(0.0165 \text{ ft})}{0.050 \text{ lb/ft hr}} = 269 \text{ (dimensionless)} \tag{6.42}$$

SEC et al. v. MRI
SEC Exhibit 1027.0215
IPR2023-00255

Substitute into Eq. 6.10 for the $J$ factor:

$$J = \frac{6}{(N_R)^{0.98}} = \frac{6}{(269)^{0.98}} = 0.0249 \tag{6.43}$$

Substitute into Eq. 6.9 for the forced air convection coefficient at 30,000 ft (physical properties of air are obtained from Figure 6.28 for an average air temperature of 137°F).

$$h_c = \frac{(0.0249)(0.24)(814)}{[(0.24)(0.05)/0.017]^{0.666}} = 6.13 \frac{\text{Btu}}{\text{hr ft}^2\ °\text{F}} \tag{6.44}$$

Notice that although there has been a relatively large reduction in the Reynolds number, the forced convection coefficient at the altitude condition, Eq. 6.44, has not changed much from the forced convection coefficient at sea level, as shown in Eq. 6.15. This is quite typical for laminar flow conditions. The forced convection coefficient there depends much more on the hydraulic diameter than on the Reynolds number. A smaller hydraulic diameter increases the value of the coefficient (see Figure 6.38).

The temperature rise across the convection film to the components is determined from Eq. 5.7.

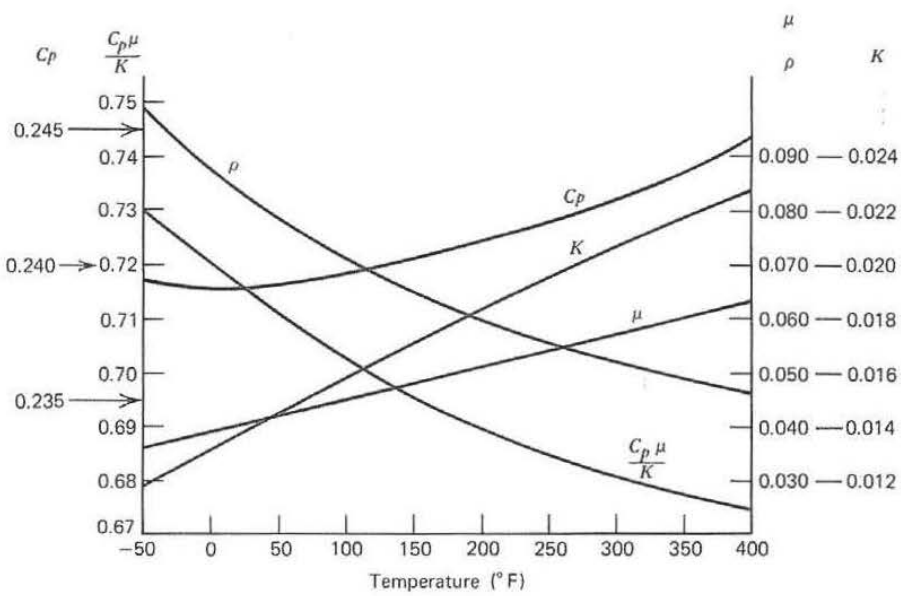



**Figure 6.28** Physical properties of dry air at 1 atm (English units).

*Given* $Q = 20$ watts $= 68.3$ Btu/hr (PCB heat)

$h_c = 6.13$ Btu/hr ft$^2$ °F (ref. Eq. 6.44)

$$A = \frac{(1.3)(8)(9)}{144} = 0.65 \text{ ft}^2 \text{ [effective area 1 PCB (including back side)]}$$

$$\Delta t_B = \frac{Q}{h_c A} = \frac{68.3}{(6.13)(0.65)} = 17.1°\text{F} \tag{6.45}$$

The component hot spot temperature is determined from Eq. 6.17.

*Given* $t_{in} = 104°$F

$\Delta t_A = 65.8°$F (ref. Eq. 6.40)

$\Delta t_B = 17.1°$F (ref. Eq. 6.45)

$$t_S = 104 + 65.8 + 17.1 = 186.9°\text{F} \tag{6.46}$$

Since this temperature is less than 212°F (100°C), the design is satisfactory for the altitude condition.

If a slightly cooler operating box is desired, the cooling air fan can be used as an exhaust fan. The 25 watts of heat from the fan will not be added to the inlet cooling air. This will reduce the cooling air temperature about 10°F and will also reduce the component hot spot temperature about 10°F, as shown in Eq. 6.47.

$$\Delta T = \frac{Q}{WC_p} = \frac{(25 \text{ watts})(3.413 \text{ Btu/hr/watt})}{(0.594 \text{ lb/min})(60 \text{ min/hr})(0.24 \text{ Btu/lb °F})} = 10°\text{F} \tag{6.47}$$

The pressure drop through the box at the 30,000 ft altitude condition can be determined from Eq. 6.37.

*Given* $\Delta P_{sl} = 1.30$ in $H_2O$ (ref. Figure 6.20 sea level for constant speed fan)

$\rho_{alt} = 0.0198$ lb/ft$^3$ 30,000 ft (ref. Eq. 6.38)

$\rho_{sl} = 0.0704$ lb/ft$^3$ sea level and 104°F

$$\Delta P_{alt} = 1.30\left(\frac{0.0198}{0.0704}\right) = 0.366 \text{ in } H_2O \tag{6.48}$$

## 6.15 OTHER CONVECTION COEFFICIENTS

Convection coefficients can become quite high for turbulent air flow conditions through ducts. These ducts may be air passages that are formed

SEC et al. v. MRI
SEC Exhibit 1027.0217
IPR2023-00255

between plug-in PCBs or ducts that deliver air between two electronic boxes. The convection coefficient is shown in Eq. 6.49.

$$h = 0.0198\left(\frac{K}{D}\right)(N_R)^{0.8} \tag{6.49}$$

Equation 6.49 can be used to find the forced convection coefficient for a pressurized enclosed duct. For example, consider a flat rectangular duct with a cross section $a = 3.0$ in, $b = 0.50$ in, and many feet long. Air at 20 psig and 150°F flows at 200 cfm. Find the convection heat transfer coefficient. The physical properties of air are needed. These are shown in Figure 6.28.

$$D = \frac{2ab}{a+b} \text{ (hydraulic diameter)}$$

$$D = \frac{2(3.0)(0.50)}{3.0 + 0.50} = 0.857 \text{ in} = 0.0714 \text{ ft}$$

$$\rho = \frac{P}{RT} \text{ (air density; ref. Eq. 6.32)}$$

$$\rho = \frac{(20.0 + 14.7)(144 \text{ in}^2/\text{ft}^2)}{(53.3)(460 + 150)°\text{R}} = 0.154 \frac{\text{lb}}{\text{ft}^3}$$

$$A = \frac{(3.0)(0.5) \text{ in}^2}{144 \text{ in}^2/\text{ft}^2} = 0.0104 \text{ ft}^2 \text{ (cross section area)}$$

$$v = \frac{\text{cfm}}{A} = \frac{(200 \text{ ft}^3/\text{min})(60 \text{ min/hr})}{0.0104 \text{ ft}^2} = 1.15 \times 10^6 \frac{\text{ft}}{\text{hr}} \text{ (velocity)}$$

$\mu = 0.055$ lb/ft hr (viscosity at 34.7 psia, 150°F; ref. Figure 6.28)

$K = 0.0175$ Btu/hr ft °F (conductivity at 150°F; ref. Figure 6.28)

The Reynolds number is

$$N_R = \frac{VD\rho}{\mu} = \frac{(1.15 \times 10^6)(0.0714)(0.154)}{0.055} = 230{,}000 \text{ (dimensionless)}$$

Since $N_R$ is well above 2300, the flow is obviously turbulent. Substitute into Eq. 6.49.

$$h = (0.0198)\left(\frac{0.0175}{0.0714}\right)(230{,}000)^{0.8} = 94.5 \frac{\text{Btu}}{\text{hr ft}^2\ °\text{F}} \tag{6.50}$$

SEC et al. v. MRI
SEC Exhibit 1027.0218
IPR2023-00255

For the flow of gasses parallel to plane surfaces, the general heat transfer convection coefficient can be determined from Eq. 6.51.

$$h = 0.055\left(\frac{K}{L}\right)(N_R)^{0.75} \tag{6.51}$$

The length of the surface is shown by $L$. This is to be limited to 2 ft, even if the length is greater than 2 ft.

When the air is blowing across a cylinder or wire, the convection coefficient can take the form shown in Eq. 6.52 [7, 8].

$$h = b\left(\frac{K}{D}\right)(N_R)^{m} \tag{6.52}$$

The constants $b$ and $m$, which are a function of the Reynolds number, are shown in Table 6.6 for different conditions [7, 8].

The characteristic dimension $D$ to be used in the equation for the square and diamond cross section is that of a circular cross section that has the same perimeter dimension.

For air flow over spheres, the convection heat transfer coefficient can be determined from Eq. 6.53.

$$h = 0.33\left(\frac{K}{D}\right)(N_R)^{0.6} \tag{6.53}$$

## 6.16 SAMPLE PROBLEM—COOLING A TO-5 TRANSISTOR

A 2N2905 transistor (TO-5 case size) dissipates 0.25 watt under steady state power conditions in a 160°F (71°C) ambient environment. The transistor is

**Table 6.6 Values to Be Used with Eq. 6.52.**

| Shape | Reynolds Number | $b$ | $m$ |
|---|---|---|---|
| | 0.4–4.0<br>4.0–40<br>40–4,000<br>4,000–40,000<br>40,000–400,000 | 0.891<br>0.821<br>0.615<br>0.174<br>0.0239 | 0.330<br>0.385<br>0.466<br>0.618<br>0.805 |
| | 5,000–10,000 | 0.0921 | 0.675 |
| | 5,000–100,000 | 0.222 | 0.585 |

SEC et al. v. MRI
SEC Exhibit 1027.0219
IPR2023-00255

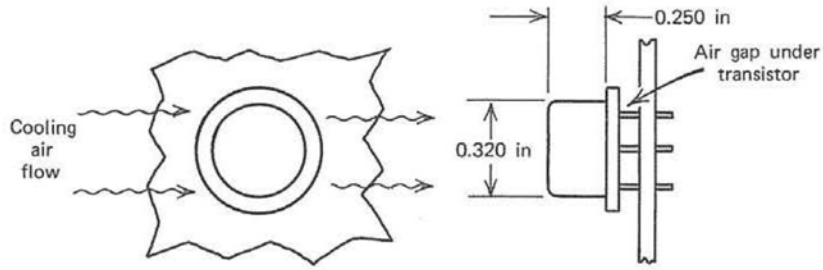



**Figure 6.29** Cooling air flow over a transistor.

mounted above the circuit board as shown in Figure 6.29. Determine the case temperature when the air velocity over the transistor is 250 ft/min.

***SOLUTION***

The Reynolds number ($N_R$) must be computed first, using the following information. (Physical properties of air are determined from Figure 6.28.)

$$V = 250\,\text{ft/min} \times 60\,\text{min/hr} = 15{,}000\,\text{ft/hr (air velocity)}$$

$$D = \frac{0.320\,\text{in}}{12\,\text{in/ft}} = 0.0267\,\text{ft (hydraulic diameter)}$$

$$\rho = \frac{P}{RT} = \frac{(14.7\,\text{lb/in}^2)(144\,\text{in}^2/\text{ft}^2)}{(53.3\,\text{ft/°R})(460 + 160)\,\text{°R}} = 0.064\,\frac{\text{lb}}{\text{ft}^3}\,\text{(air density)}$$

$$\mu = 0.050\,\text{lb/ft hr (viscosity at 160°F)}$$

$$K = 0.017\,\text{Btu/hr ft °F (thermal conductivity at 160°F)}$$

Substitute into Eq. 6.11 for the Reynolds number:

$$N_R = \frac{(15000)(0.0267)(0.064)}{0.050} = 512\,\text{(dimensionless)} \tag{6.54}$$

The Reynolds number is between 40 and 4000, so that the values shown in Table 6.6 are $b = 0.615$ and $m = 0.466$. Substituting these values into Eq. 6.52 results in Eq. 6.55.

$$h = 0.615\left(\frac{K}{D}\right)(N_R)^{0.466} \tag{6.55}$$

$$h = 0.615\left(\frac{0.017}{0.0267}\right)(512)^{0.466} = 7.17\,\frac{\text{Btu}}{\text{hr ft}^2\,\text{°F}} \tag{6.56}$$

SEC et al. v. MRI
SEC Exhibit 1027.0220
IPR2023-00255

The temperature rise across the boundary layer, from the ambient air to the transistor surface, is determined from Eq. 5.7.

$$\Delta t = \frac{Q}{hA} \text{ (ref. Eq. 5.7)}$$

*Given* $Q = 0.25 \text{ watt} \times 3.413 \text{ Btu/hr/watt} = 0.85 \text{ Btu/hr}$

$h = 7.16 \text{ Btu/hr ft}^2 \text{ °F (ref. Eq. 6.56)}$

$A$ = surface area of transistor

$$= \frac{2(\pi/4)(0.32)^2 + \pi(0.32)(0.25) \text{ in}^2}{144 \text{ in}^2/\text{ft}^2} = 0.00286 \text{ ft}^2$$

$$\Delta t = \frac{0.85 \text{ Btu/hr}}{(7.16 \text{ Btu/hr ft}^2 \text{ °F})(0.00286 \text{ ft}^2)} = 41.5\text{°F} \tag{6.57}$$

The case temperature of the transistor becomes

$$t_{\text{case}} = 160 + 41.5 = 201.5\text{°F} \tag{6.58}$$

The junction temperature of the transistor is determined using the thermal impedance from the junction to the case ($\Theta_{jc}$) which is usually obtained from the component manufacturer. When the value of $\Theta_{jc}$ = 59.4°F/watt (33°C/watt) the temperature rise from the case to the junction is:

$$\Delta t_{jc} = 59.4 \frac{\text{°F}}{\text{watt}} \times 0.25 \text{ watt} = 14.8\text{°F} \tag{6.59}$$

The junction temperature of the transistor can then be obtained from Eq. 6.60.

$$t_j = t_{\text{case}} + \Delta t_{jc} \tag{6.60}$$

$$t_j = 201.5 + 14.8 = 216.3\text{°F} \tag{6.61}$$

## 6.17 CONDITIONED COOLING AIR FROM AN EXTERNAL SOURCE

Conditioned cooling air is often supplied to the electronic equipment from an external source, such as the compressor section of an aircraft engine. For example, in the F-16 airplane, the electronic equipment cooling air is obtained from the seventeenth stage of the engine compressor. The air temperature and pressure are both very high, and the air may have a high moisture content. This air is therefore throttled, cooled, and dried before it is passed through the electronic equipment.

SEC et al. v. MRI
SEC Exhibit 1027.0221
IPR2023-00255

The amount of conditioned air required to cool an electronic system will vary with the temperature of the cooling air. Less air is required when the cooling air temperature is low, and more air is required when the cooling air temperature is high. Extensive test data and analysis have shown that an efficient thermal design can be achieved if the temperature of the cooling air is not allowed to exceed 160°F (71°C) as it exits from the electronic chassis. This condition should result in a maximum component surface temperature of about 212°F (100°C) when the component mounting techniques shown in Chapter 4 are used.

Equation 6.7 can be used to establish the cooling air flow curve for an electronic box when the power dissipation of the box is known. The technique is demonstrated with a sample problem.

## 6.18 SAMPLE PROBLEM—GENERATING A COOLING AIR FLOW CURVE

An electronic box, with a power dissipation of 330 watts, has conditioned cooling air supplied by the airplane engine. Generate the flow curve required to maintain a box cooling air exit temperature of 160°F (71°C) when the cooling air inlet temperature varies from −65°F (−54°C) to +131°F (+55°C). The conversion tables in Section 1.11 are used to convert to English and metric units.

*SOLUTION*

$$Q = 330 \text{ watts} = 1126.3 \text{ Btu/hr} = 78.87 \text{ cal/sec (heat dissipation)}$$

$$C_p = 0.24 \text{ Btu/lb °F} = 0.24 \text{ cal/g °C (specific heat)}$$

$$\Delta t = 160°\text{F} - t_{\text{in}} \text{ (English) } (71°\text{C} - t_{\text{in}} \text{ metric})$$

Start with an air inlet temperature of −65°F for English units, which corresponds to −54°C for metric units.

$$\Delta t = 160°\text{F} - (-65)°\text{F} = 225°\text{F (English units)}$$

$$\Delta t = 71°\text{C} - (-54)°\text{C} = 125°\text{C (metric units)}$$

Substituting into Eq. 6.7, the cooling air weight flow is determined using English units.

$$W = \frac{1126.3 \text{ Btu/hr}}{(225°\text{F})(0.24 \text{ Btu/lb °F})(60 \text{ min/hr})} = 0.347 \frac{\text{lb}}{\text{min}} \tag{6.62}$$

SEC et al. v. MRI
SEC Exhibit 1027.0222
IPR2023-00255

Using Eq. 6.7, the weight flow for metric units is

$$W = \frac{78.87 \text{ cal/sec}}{(125°\text{C})(0.24 \text{ cal/g °C})} = 2.63 \frac{\text{g}}{\text{sec}} \tag{6.62a}$$

At an air inlet temperature of 80°F (26.6°C)

$$\Delta t = 160°\text{F} - 80°\text{F} = 80°\text{F (English units)}$$

$$\Delta t = 71°\text{C} - 26.6°\text{C} = 44.4°\text{C (metric units)}$$

Substituting into Eq. 6.7 for the weight flow, we obtain

$$W = \frac{1126.3 \text{ Btu/hr}}{(80°\text{F})(0.24)(60 \text{ min/hr})} = 0.978 \frac{\text{lb}}{\text{min}} \text{ (English units)} \tag{6.63}$$

$$W = \frac{78.87 \text{ cal/sec}}{(44.4°\text{C})(0.24)} = 7.40 \frac{\text{g}}{\text{sec}} \text{ (metric units)} \tag{6.63a}$$

At an air inlet temperature of 131°F (55°C):

$$\Delta t = 160°\text{F} - 131°\text{F} = 29°\text{F (English units)}$$

$$\Delta t = 71°\text{C} - 55°\text{C} = 16°\text{C (metric units)}$$

Substituting into Eq. 6.7 yields

$$W = \frac{1126.3 \text{ Btu/hr}}{(29°\text{F})(0.24)(60 \text{ min/hr})} = 2.70 \frac{\text{lb}}{\text{min}} \text{ (English units)} \tag{6.64}$$

$$W = \frac{78.87 \text{ cal/sec}}{(16°\text{C})(0.24)} = 20.54 \frac{\text{g}}{\text{sec}} \text{ (metric units)} \tag{6.64a}$$

When the cooling air flow rate is plotted against the cooling air temperature, the cooling air flow curve will be produced, as shown in Figure 6.30.

## 6.19 STATIC PRESSURE LOSSES FOR VARIOUS ALTITUDE CONDITIONS

Electronic systems are often required to operate at different altitude conditions. As the altitude varies, it is desirable to maintain a constant cooling air weight flow rate, similar to the curve shown in Figure 6.30. At high altitudes the air density is lower, so that the cooling air flow velocity must be increased to provide the same air flow weight. As the air velocity increases, however, the static pressure drop through the electronic system

SEC et al. v. MRI
SEC Exhibit 1027.0223
IPR2023-00255

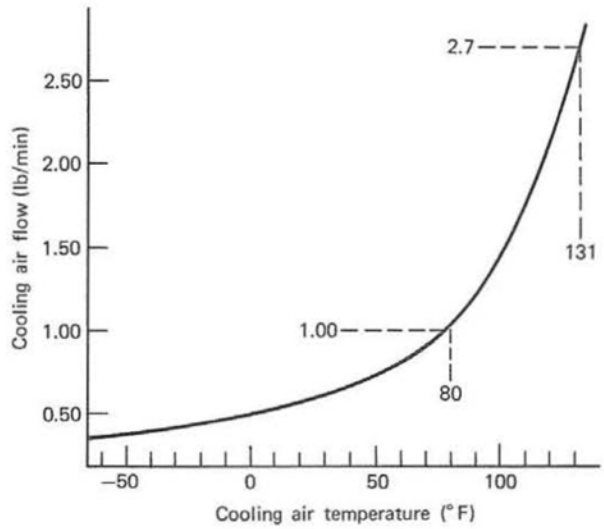



**Figure 6.30** Typical cooling air weight flow curve.

also increases. It is therefore important to know how these pressure losses will change with the altitude, to ensure an adequate flow of cooling air to the electronics under all conditions.

The complexity of the static pressure drop relations for various altitude conditions can be simplified by considering the density of the cooling air at the altitude condition and comparing it with the density of air at some standard condition. The standard selected is usually based on a temperature of 59°F (15°C) and a pressure of 14.7 lb/in² (1034.4 g/cm²), where the air density is 0.0765 lb/ft³ (0.00122 g/cm³).

It is convenient to use a density ratio that is called sigma and is normally shown by the Greek letter sigma ($\sigma$), which is defined as follows:

$$\sigma = \frac{\text{air density at any condition}}{\text{air density at standard conditions}} = \frac{\rho}{\rho_{\text{std}}}$$

$$\sigma = \frac{\rho}{0.0765\ \text{lb/ft}^3}\ \text{(for English units)} \tag{6.65}$$

$$\sigma = \frac{\rho}{0.00122\ \text{g/cm}^3}\ \text{(for metric units)} \tag{6.65a}$$

It is desirable to work with the average density of the cooling air for high altitude conditions. At high altitudes, the outside air pressure and density are very low, so that the cooling air velocity must be very high to provide a reasonable weight flow. High air velocities mean high flow losses, so that a high pressure must be used to push the cooling air through the box. Under these circumstances, the inlet air density can be many times greater than

the outlet air density. If these density changes are ignored at high altitude conditions, it can lead to large pressure drop errors. The average air density within the box is therefore used, to improve the accuracy of the calculations.

The density ratio ($\sigma$) can be written in another way by using the density relation obtained from the gas law relation shown in Eq. 6.32. Then substituting into Eq. 6.65, we obtain

$$\sigma = \frac{P_1/R_1T_1}{P_2/R_2T_2} = \frac{\dfrac{P_1}{(53.3\ \text{ft/°R})\,T_1}}{\dfrac{14.7\ \text{lb/in}^2}{(53.3\ \text{ft/°R})(460+59)\,\text{°R}}} = 35.3\,\frac{P_1}{T_1}\ \text{(English units)}$$

$$\sigma = \frac{\dfrac{P_1}{(2924\ \text{cm/°K})\,T_1}}{\dfrac{1034.4\ \text{g/cm}^2}{(2924\ \text{cm/°K})(273+15)\,\text{°K}}} = 0.279\,\frac{P_1}{T_1}\ \text{(metric units)}$$

The inlet and the outlet air pressures should be averaged to obtain the average value for the electronic box in English units and metric units. The resulting equation is convenient for working with electronic boxes at high altitude conditions.

$$\sigma = \frac{35.3}{2}\left(\frac{P_{\text{in}}}{T_{\text{in}}} + \frac{P_{\text{out}}}{T_{\text{out}}}\right)\ \text{(English units)} \tag{6.66}$$

where $P$ = pressure = lb/in²
$T$ = absolute temperature (460 + °F) = °R
$P_{\text{in}} = P_{\text{out}} + \Delta P$ = lb/in² (inlet pressure)
$P_{\text{out}}$ = outside pressure = lb/in²

$$\sigma = \frac{0.279}{2}\left(\frac{P_{\text{in}}}{T_{\text{in}}} + \frac{P_{\text{out}}}{T_{\text{out}}}\right)\ \text{(metric units)} \tag{6.66a}$$

where $P$ = pressure = g/cm²
$T$ = absolute temperature (273 + °C) = °K
$P_{\text{in}} = P_{\text{out}} + \Delta P$ = g/cm² (inlet pressure)
$P_{\text{out}}$ = outside pressure = g/cm²

The value of $\sigma$ is usually multiplied by the static pressure drop $\Delta P$ and plotted against the cooling air weight flow on a log-log plot. This results in a straight line, as shown in Figure 6.31.

For fully turbulent flow conditions, the slope of the $\sigma\,\Delta P$ line on a log-log plot is 2.0. For laminar flow conditions, the slope will usually vary from about 1.2 to about 1.7, depending upon the severity of the restrictions in the cooling air flow path.

SEC et al. v. MRI
SEC Exhibit 1027.0225
IPR2023-00255

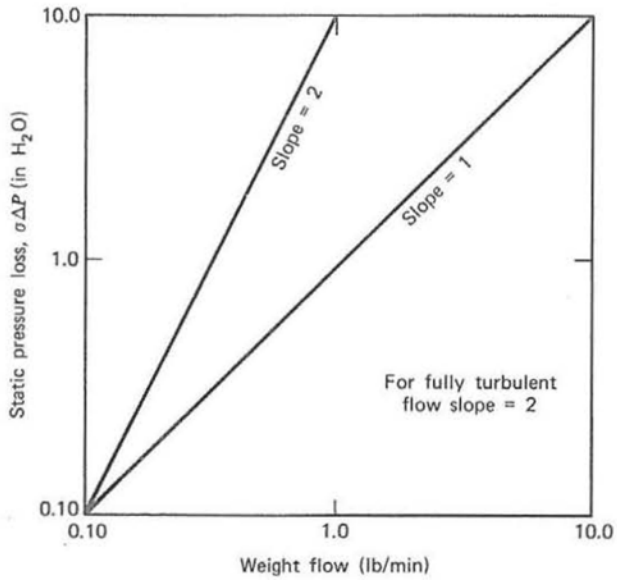



**Figure 6.31** Typical $\sigma\ \Delta P$ pressure loss curves for electronic boxes.

Once the $\sigma\ \Delta\ P$ curve has been established for a particular electronic box, it is very easy to determine the pressure drop through the box for any altitude and for any cooling air flow condition.

## 6.20 SAMPLE PROBLEM—STATIC PRESSURE DROP AT 65,000 FEET

An electronic box with a power dissipation of 330 watts has conditioned cooling air supplied (by the airplane turbine) with a weight flow rate as shown in Figure 6.30. This flow rate will provide a cooling air exit temperature of 160°F (71°C) for any inlet air temperature shown in the curve. Test data show that the static pressure drop through the box is 2.0 in of water (5.08 cm of water) when the air inlet temperature is 80°F (26.6°C) and the cooling air flow rate is 1.0 lb/min (7.56 g/sec).

1 Determine the static pressure drop through the box at an altitude of 65,000 ft (19,812 m) with an air flow rate of 1.0 lb/min at 80°F. The outside air pressure at this altitude is 0.83 lb/in² (58.4 g/cm²).
2 Determine the static pressure drop through the box at sea level and at an altitude of 65,000 ft when the air inlet temperature is 131°F (55°C) and the cooling air flow rate is 2.7 lb/min (20.43 g/sec).

SEC et al. v. MRI
SEC Exhibit 1027.0226
IPR2023-00255

*SOLUTION*

For the solution of part 1 of the problem, the value of $\sigma$ is first computed for the sea level condition. Equations 6.65 and 6.65a will give about the same results as Eqs. 6.66 and 6.66a because the 2.0 in pressure drop only represents a $2 \times 0.036 = 0.072$ lb/in² pressure drop. This is small compared to the atmospheric pressure of 14.7 lb/in², so that $P_{in}$ is virtually the same as $P_{out}$ in Eq. 6.66.

The average air density for the 80°F (26.6°C) sea level condition is computed from Eq. 6.32, using English units first. The average temperature in the box must be used.

$$t_{av} = \frac{t_{in} + t_{out}}{2} = \frac{80 + 160}{2} = 120°\text{F (English units)}$$

$$t_{av} = \frac{26.6 + 71}{2} = 48.8°\text{C (metric units)}$$

In English units:

$$\rho = \frac{(14.7\ \text{lb/in}^2)(144\ \text{in}^2/\text{ft}^2)}{(53.3\ \text{ft/°R})(460 + 120)°\text{R}} = 0.0685\ \frac{\text{lb}}{\text{ft}^3} \tag{6.67}$$

In metric units:

$$\rho = \frac{1034.4\ \text{g/cm}^2}{(2924\ \text{cm/°K})(273 + 48.8)°\text{K}} = 0.001099\ \frac{\text{g}}{\text{cm}^3} \tag{6.67a}$$

Substitute into Eq. 6.65 for the sea level value of $\sigma$ using English units.

$$\sigma = \frac{\rho}{0.0765} = \frac{0.0685}{0.0765} = 0.895\ \text{(dimensionless)} \tag{6.68}$$

Substitute into Eq. 6.65a for the sea level value of $\sigma$ using metric units.

$$\sigma = \frac{\rho}{0.001224} = \frac{0.001099}{0.001224} = 0.897\ \text{(dimensionless)} \tag{6.68a}$$

The value of $\sigma$ is multiplied by the static pressure drop ($\Delta P$) head of water, shown previously as 2.0 in $H_2O$ (5.08 cm $H_2O$), to obtain the $\sigma\,\Delta P$ values of static pressure drop:

$$\sigma\,\Delta P = (0{,}895)(2.0\ \text{in H}_2\text{O}) = 1.79\ \text{in H}_2\text{O (English units)} \tag{6.69}$$

$$\sigma\,\Delta P = (0.897)(5.08\ \text{cm H}_2\text{O}) = 4.55\ \text{cm H}_2\text{O (metric units)} \tag{6.69a}$$

SEC et al. v. MRI
SEC Exhibit 1027.0227
IPR2023-00255

The values shown above are valid only for a cooling air flow rate of 1.0 lb/min (7.56 g/sec) and an air inlet temperature of 80°F (26.6°C). However, this $\sigma \Delta P$ value is exactly the same *at any altitude* condition, *as long as the air flow rate and temperature stay the same*.

The static pressure drop at an altitude of 65,000 ft, with the 1.0 lb/min flow rate, can be determined if the value of $\sigma$ can be obtained for the high altitude condition. Using Eqs. 6.66 and 6.66a, the following information is required.

| *Item* | *English Units* | *Metric Units* |
|---|---|---|
| $P_{out}$ | 0.83 lb/in² | 58.4 g/cm² |
| $T_{out}$ | 460 + 160 = 620°R | 273 + 71 = 344°K |
| $T_{in}$ | 460 + 80 = 540°R | 273 + 26.6 = 299.6°K |
| $P_{in}$ | 0.83 lb/in² + $\Delta P$ | 58.4 g/cm² + $\Delta P$ |

The static pressure drop $\Delta P$ through the electronic box is not known for the high altitude condition, but its value is required. Therefore, a value is assumed and the calculation is made. The assumed value is then compared with the calculated value. If they are not close, another guess is made and the calculation is repeated. Close agreement can usually be obtained with two or three iterations. Considering English units first, assume a static pressure drop, $\Delta P$, of 22 in $H_2O$.

$$\Delta P = 22 \text{ in } H_2O \times 0.036 \frac{\text{lb/in}^2}{\text{in } H_2O} = 0.792 \text{ lb/in}^2$$

$$P_{in} = 0.83 \text{ lb/in}^2 + 0.792 \text{ lb/in}^2 = 1.622 \text{ lb/in}^2$$

Substitute into Eq. 6.66 for the $\sigma$ value at 65,000 ft with a flow rate of 1.0 lb/min at 80°F.

$$\sigma_{alt} = \frac{35.3}{2}\left(\frac{1.622}{540} + \frac{0.83}{620}\right) = 0.0766 \tag{6.70}$$

For metric units, assume a static pressure drop $\Delta P$ of 56 cm $H_2O$.

$$\Delta P = 56 \text{ cm } H_2O = 56 \text{ g/cm}^2$$

$$P_{in} = 58.4 \text{ g/cm}^2 + 56 \text{ g/cm}^2 = 114.4 \text{ g/cm}^2$$

Substitute into Eq. 6.66a for the $\sigma$ value at 19,812 m with a flow rate of 7.56 g/sec at 26.6°C.

$$\sigma_{alt} = \frac{0.279}{2}\left(\frac{114.4}{299.6} + \frac{58.4}{344}\right) = 0.0769 \tag{6.70a}$$

SEC et al. v. MRI
SEC Exhibit 1027.0228
IPR2023-00255

The static pressure drop at the altitude condition is determined from Eq. 6.71.

$$\Delta P_{\text{alt}} = \frac{\sigma \Delta P}{\sigma_{\text{alt}}} \tag{6.71}$$

For English units, substitute Eqs. 6.69 and 6.70 into Eq. 6.71.

$$\Delta P_{\text{alt}} = \frac{1.79}{0.0766} = 23.4 \text{ in } H_2O \tag{6.72}$$

For metric units, substitute Eqs. 6.69a and 6.70a into Eq. 6.71.

$$\Delta P_{\text{alt}} = \frac{4.556}{0.0769} = 59.2 \text{ cm } H_2O \tag{6.72a}$$

A pressure drop of 22 in $H_2O$ (and 56 cm $H_2O$) was assumed for the box at an altitude of 65,000 ft (19,812 m), with a flow rate of 1.0 lb/min (7.56 g/sec) at 80°F (26.6°C). The calculated value turned out to be 23.4 in $H_2O$ (and 59.2 cm $H_2O$) on the first try. A second iteration would show that the correct value is closer to 23 in $H_2O$ (58.4 cm $H_2O$).

For the solution to part 2 of the problem, the slope of the $\sigma \Delta P$ curve on a log-log plot similar to Figure 6.31 must be known. For fully turbulent flow systems the $\sigma \Delta P$ slope is 2.0. This is the theoretical maximum that can be obtained with an air flow system. In actual systems, however, this slope is seldom obtained. The actual slope for a typical box will generally have a range from about 1.2 for a box with very few widely spaced PCBs to about 1.8 for a box with heat exchanger side walls containing 20 wavy fins per inch.

When the electronic box has a $\sigma \Delta P$ slope of 1.4, for example, the pressure drop at any cooling air weight flow condition can be determined from the general definition of the slope, as shown in Figure 6.32 and Eq. 6.73.

$$\text{Slope } S = \frac{\Delta Y}{\Delta X} = \frac{\log Y_2 - \log Y_1}{\log X_2 - \log X_1} \tag{6.73}$$

Common logarithms to the base 10 or natural logarithms to the base $e$ can be used for Eq. 6.73. Considering common logarithms to the base 10 with English units, the $\sigma \Delta P$ static pressure drop through the box for a flow rate of 2.7 lb/min is computed.

$$1.4 = \frac{\log \sigma \Delta P_2 - \log 1.79}{\log 2.7 - \log 1.0} = \frac{\log \sigma \Delta P_2 - 0.2528}{0.4313 - 0}$$

$$\log \sigma \Delta P_2 = 1.4(0.4313) + 0.2528 = 0.8566$$

SEC et al. v. MRI
SEC Exhibit 1027.0229
IPR2023-00255

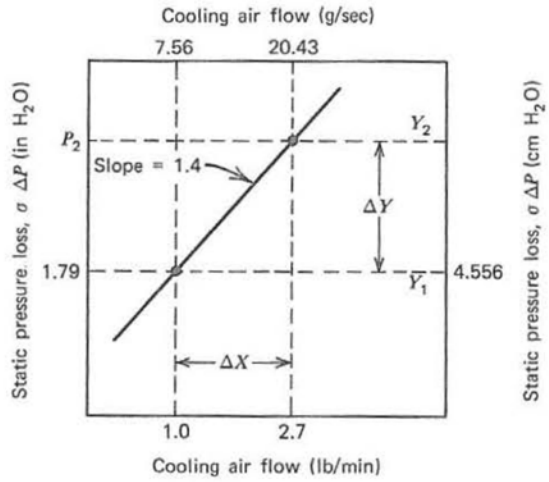



**Figure 6.32** $\sigma\ \Delta P$ cooling curve with a slope of 1.4.

Take the antilog of both sides:

$$\sigma\ \Delta P_2 = 7.188 \text{ in } H_2O \tag{6.74}$$

Considering common logarithms to the base 10 with metric units, the $\sigma\ \Delta P$ static pressure drop through the box for a flow rate of 20.43 g/sec is computed.

$$1.4 = \frac{\log \sigma\ \Delta P_2 - \log 4.556}{\log 20.43 - \log 7.56} = \frac{\log \sigma\ \Delta P_2 - 0.6586}{1.3102 - 0.8785}$$

$$\log \sigma\ \Delta P_2 = 1.4\ (0.4317) + 0.6586 = 1.2629$$

Take the antilog of both sides:

$$\sigma\ \Delta P_2 = 18.32 \text{ cm } H_2O \tag{6.74a}$$

Before the static pressure drop through the box at sea level conditions, with a flow rate of 2.7 lb/min, can be computed from the $\sigma\ \Delta P$ values, the average air temperature and average air density must be computed. In English units:

$$t_{av} = \frac{t_{in} + t_{out}}{2} = \frac{131 + 160}{2} = 145.5°F$$

$$\rho = \frac{(14.7)(144) \text{ lb/ft}^2}{(53.3 \text{ ft/°R})(460 + 145.5)\text{°R}} = 0.0656 \frac{\text{lb}}{\text{ft}^3} \tag{6.75}$$

SEC et al. v. MRI
SEC Exhibit 1027.0230
IPR2023-00255

In metric units:

$$t_{av} = \frac{55 + 71}{2} = 63°C$$

$$\rho = \frac{1034.4 \text{ g/cm}^2}{(2924 \text{ cm/°K})(273 + 63)°\text{K}} = 0.001053 \frac{\text{g}}{\text{cm}^3} \tag{6.75a}$$

Substitute Eq. 6.75 into Eq. 6.65 for English units:

$$\sigma = \frac{\rho}{0.0765} = \frac{0.0656}{0.0765} = 0.857 \text{ (dimensionless)} \tag{6.76}$$

Substitute Eq. 6.75a into Eq. 6.65a for metric units:

$$\sigma = \frac{\rho}{0.001224} = \frac{0.00105}{0.001224} = 0.857 \text{ (dimensionless)} \tag{6.76a}$$

The sea level static pressure drop through the box with a flow rate of 2.7 lb/min (20.43 g/sec) is now computed from Eqs. 6.74 and 6.75 in English units.

$$\Delta P_{sl} = \frac{\sigma \, \Delta P_2}{\sigma} = \frac{7.188}{0.857} = 8.39 \text{ in } H_2O \tag{6.77}$$

In metric units, with Eqs. 6.74a and 6.75a:

$$\Delta P_{sl} = \frac{\sigma \, \Delta P_2}{\sigma} = \frac{18.32}{0.857} = 21.38 \text{ cm } H_2O \tag{6.77a}$$

The static pressure drop through the box at an altitude of 65,000 ft, with a flow rate of 2.7 lb/min, can be calculated as before. Assume a pressure drop of 62 in $H_2O$ to start.

$$\Delta P = 62 \text{ in } H_2O \times 0.036 \text{ (lb/in}^2\text{)/in } H_2O = 2.232 \text{ lb/in}^2$$

$$P_{in} = 0.83 \text{ lb/in}^2 + 2.232 \text{ lb/in}^2 = 3.062 \text{ lb/in}^2$$

$$P_{out} = 0.83 \text{ lb/in}^2$$

$$T_{in} = 460 + 131 = 591°R$$

$$T_{out} = 460 + 160 = 620°R$$

$$\sigma_{alt} = \frac{35.3}{2}\left(\frac{3.062}{591} + \frac{0.83}{620}\right) = 0.115$$

The $\sigma \, \Delta P$ value at 65,000 ft must be the same as the value shown by Eq.

SEC et al. v. MRI
SEC Exhibit 1027.0231
IPR2023-00255

6.74 for the 2.7 lb/min flow rate. The static pressure drop through the box at 65,000 ft is, therefore,

$$\Delta P_{\text{alt}} = \frac{\sigma \, \Delta P}{\sigma_{\text{alt}}} = \frac{7.188}{0.115} = 62.5 \text{ in } H_2O \tag{6.78}$$

A static pressure drop of 62 in $H_2O$ was assumed for the 65,000 ft altitude condition for a flow rate of 2.7 lb/min. The calculated value is 62.5 in $H_2O$, which is close enough for these calculations and shows that the assumed value is accurate.

The force that pushes the cooling air through the electronic box is the pressure differential across the box. As this pressure difference increases, the velocity of the cooling air through the box will increase until the critical velocity is reached. Once the critical velocity is reached, any further increase in the pressure difference across the box will not produce any further increase in cooling air velocity.

Critical flow velocity will be reached when the ratio of the downstream pressure (or outside pressure) to the upstream pressure (or inside pressure) falls below the critical pressure ratio ($P_{\text{cr}}$) of 0.53, as shown below.

$$P_{\text{cr}} = \frac{P_{\text{out}}}{P_{\text{in}}} = 0.53 \text{ (critical pressure ratio)}$$

In the previous sample problem, this pressure ratio ($R_p$) at the 65,000 ft altitude is:

$$R_p = \frac{P_{\text{out}}}{P_{\text{in}}} = \frac{0.83}{3.062} = 0.27$$

Since this value is less than 0.53, it shows that the critical velocity has been reached. Therefore, the 2.7 lb/min cooling air flow rate at the 65,000 ft altitude condition can never be obtained.

## 6.21 TOTAL PRESSURE DROP FOR VARIOUS ALTITUDE CONDITIONS

Section 6.3 and Figures 6.7 through 6.9 show the basic relations between static pressures and velocity pressures for electronic boxes. Equation 6.3 shows that when the cooling air is forced through an electronic system, the total pressure is simply the sum of the velocity pressure and the static pressure.

$$H_t = H_v + H_s \qquad \text{(ref. Eq. 6.3)}$$

SEC et al. v. MRI
SEC Exhibit 1027.0232
IPR2023-00255

Cooling requirements for electronic equipment will sometimes specify the allowable pressure drop through the system in terms of the *static* pressure and sometimes in terms of the *total* pressure. Since the two pressures are different, it is necessary to understand the method for converting from one to another.

The static pressure drop through an electronic box is usually determined first. Section 6.4 shows how the static pressure losses are related to the velocity of the cooling air as it is being forced through the box.

After the static pressure losses have been determined for a box, it is necessary to compute the velocity pressure (or velocity head) for that box if the total pressure (or total head $H_t$) losses are required.

The velocity head $H_v$ should be based upon the velocity of the cooling air in the most restricted flow section through the box, where the air velocity is the highest. These values can be computed from Eq. 6.2 or 6.4 for English units and Eq. 6.2a for metric units.

## 6.22 SAMPLE PROBLEM—TOTAL PRESSURE LOSS THROUGH AN ELECTRONIC BOX

An air flow analysis of an electronic box shows that the static pressure drop ($H_s$) through the box is 1.75 in of water. The most severe flow restriction through the box is at the inlet, where the cooling air velocity is 2200 ft/min. Determine the total pressure drop through the box.

*SOLUTION*

The velocity head at the air inlet is determined from Eq. 6.4.

$$H_v = \left(\frac{V}{4005}\right)^2 = \left(\frac{2200}{4005}\right)^2 = 0.302 \text{ in } H_2O \tag{6.79}$$

The total pressure drop through the box is determined from Eq. 6.3.

$$H_t = H_v + H_s = 0.302 + 1.75 = 2.052 \text{ in } H_2O \tag{6.80}$$

## 6.23 FINNED COLD PLATES AND HEAT EXCHANGERS

Cold plates and heat exchangers are often fabricated of aluminum, with many thin plate fins spaced close together. When the fins are spaced close together, there is a large increase in the forced convection heat transfer coefficient ($h$) and a large increase in the surface area ($A$) available for heat transfer. The product of $h_c \times A$ therefore increases very rapidly, which

SEC et al. v. MRI
SEC Exhibit 1027.0233
IPR2023-00255

improves the thermal efficiency of the design. This results in a lower component hot spot temperature and improves the reliability of the electronic equipment [50].

There is a practical limit to the number of thin plate fins that can be used. In a dip brazed box, the maximum number of straight fins is about 22 fins per inch. More fins will often result in plugged passages because it is difficult to clean out the dip brazing salts, which may become trapped between the fins. If eutectic bonding or fluxless brazing is used, as many as 30 fins per inch can be used without plugging passages. Increasing the number of fins, however, will increase the pressure drop through the box, which can sharply reduce the cooling air flow rate and may result in overheating of the electronics.

Extensive test data [3] and analysis have shown that the forced convection heat transfer coefficient for multiple plate fin types of cold plates and heat exchangers can be determined with Eq. 6.9 [15, 43].

$$h_c = JC_pG\left(\frac{C_p\mu}{K}\right)^{-2/3} \quad \text{(ref. Eq. 6.9)}$$

The heat transfer coefficient $J$ can be determined from the Reynolds number ($N_R$). For laminar flow conditions, with $N_R$ between about 400 and 1500, the approximate $J$ value for finned cold plates and heat exchangers is shown in Eq. 6.81, compiled from data given in [43].

$$J = \frac{0.72}{(N_R)^{0.7}} \tag{6.81}$$

The typical cross section through a multiple fin cold plate or heat exchangers is shown in Figure 6.33.

The cold plate or heat exchanger structure is often used for the side walls of electronic boxes. Printed circuit boards (PCBs) are then supported between the side walls. Copper or aluminum sheets are often laminated to the PCBs to provide a good thermal conduction heat flow path along the PCBs, as shown in Figure 4.1. The heat can then flow along the PCBs to the air cooled side walls of the chassis, which contain multiple fin heat exchangers, as shown in Figure 4.2.

Since the cooling medium used is air, it is important to bring the cooling air as close to the electronic components as possible. This will reduce the temperature rise and improve the thermal efficiency.

In many new packaging applications the cooling air is ducted directly to the circuit boards, which have hollow cores, as shown in Figure 6.21. For improved heat transfer efficiency, multiple fin heat exchangers are often added to the hollow core PCBs. The convection heat transfer coefficient, shown by Eqs. 6.9 and 6.81, can be used to determine the thermal charac-

SEC et al. v. MRI
SEC Exhibit 1027.0234
IPR2023-00255

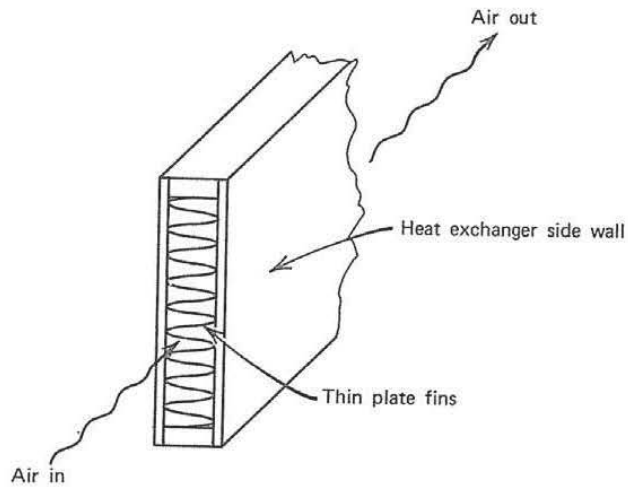



**Figure 6.33** Typical cold plate or heat exchanger.

teristics of systems which have hollow core air passages through the center of the PCBs.

## 6.24 PRESSURE LOSSES IN MULTIPLE FIN HEAT EXCHANGERS

Multiple fin heat exchangers and cold plates require a substantial amount of energy to overcome the friction that resists the flow of cooling air through these units. When the fin density is high or when the surface finish on the fins is very rough, even more energy is required to force the air through the long, narrow tubes that are formed by the heat exchanger fins.

It is convenient to express the friction losses in terms of the height of a column of water $H_L$ in inches (or centimeters) because the magnitudes of the numbers expressed in pounds per square inch (or grams per square centimeter) are so small.

Friction losses increase very rapidly as the velocity of the cooling air flow increases. However, the losses appear to be linear with respect to the length of the flow path through the heat exchanger.

The static head loss due to friction resulting from the flow through the multiple fin heat exchanger tubes only is shown in Eq. 6.82. Losses due to entrance resistance, turns, expansions, and contractions are not included in this equation. These other losses must be evaluated separately using techniques similar to those shown in Section 6.7, and then added to Eq. 6.82 to obtain the full static head loss for the electronic box. This is known as the Darcy Equation, and it is valid for English or metric units as long as a

SEC et al. v. MRI
SEC Exhibit 1027.0235
IPR2023-00255

consistent set of units is used. This equation can be used for laminar and turbulent flow conditions [3, 15, 17, 43, 51].

$$H_L = 4f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right) \tag{6.82}$$

where $V$ = flow velocity = ft/sec or cm/sec
$L$ = length of air flow path = in, ft, cm
$D$ = hydraulic diameter of one duct = in, ft, cm
$g$ = acceleration of gravity
$g$ = 32.2 ft/sec² = 386 in/sec² = 980 cm/sec²
$f$ = friction factors, dimensionless
$f = 16/N_R$ = Fanning friction factor for laminar flow in round tubes (ref. Table 6.7)*

Equation 6.82 will result in the static pressure loss in terms of the height of a column of air. Sometimes this equation is written with the density ($\rho$) included, as shown in Eq. 6.83. When the density is included, it changes the units of the losses from a height dimension to a pressure dimension, lb/in² or lb/ft² for English units and g/cm² for metric units.

$$\Delta P_L = 4f\left(\frac{L}{D}\right)\left(\frac{\rho V^2}{2g}\right) \tag{6.83}$$

Sometimes it is more convenient to obtain the static head loss in terms of $\sigma\,\Delta P$ directly in inches of water, as shown by Eq. 6.84. English units must be used with the constant of 0.226; then the weight flow $W$ = lb/min and the cross section area $A$ = in².

$$\sigma\,\Delta P = 0.226(4f)\left(\frac{L}{D}\right)\left(\frac{W}{A}\right)^2 \tag{6.84}$$

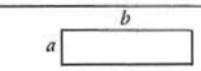
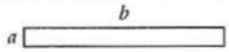
where $\sigma\,\Delta P$ = static pressure loss = in $H_2O$
$f$ = Fanning friction factor = dimensionless (ref. Table 6.7)
$L$ = length of heat exchanger = in
$D$ = hydraulic diameter, one duct = in
$W$ = cooling air weight flow, one duct = lb/min
$A$ = cross section flow area, one duct = in²

The friction factor $f$ is usually determined from test data for laminar and turbulent flow conditions and for different surface roughness. In the laminar flow region, for smooth wall surfaces, the friction factor can generally be

* *Do not become confused with the Hagen–Poiseulle equation for laminar flow in round tubes, shown in Eqs. 10.3 and 10.4. In the Hagen–Poiseulle equation, the laminar flow friction factor* $f = 64/N_R$.

SEC et al. v. MRI
SEC Exhibit 1027.0236
IPR2023-00255

**Table 6.7 Laminar Flow Friction Factors ($f$) for Fully Developed Velocity and Temperature Profiles in Smooth Pipes with Different Cross Sections.**

| Geometry | Friction Factor |
|---|---|
| | $f = \frac{13.3}{N_R}$ |
| $\frac{b}{a} = 1$ | $f = \frac{14.2}{N_R}$ |
| | $f = \frac{16}{N_R}$ |
|  $\frac{b}{a} = 4$ | $f = \frac{18.3}{N_R}$ |
|  $\frac{b}{a} = 8$ | $f = \frac{20.5}{N_R}$ |
| $\frac{b}{a} = \infty$ | $f = \frac{24}{N_R}$ |

related directly to the Reynolds number ($N_R$). Table 6.7 shows some typical friction factors for different cross sections that have smooth walls [43]. From Kays & London, Compact Heat Exchangers, published by McGraw-Hill Book Co.

## 6.25 FIN EFFICIENCY FACTOR

Fins are used extensively with forced convection cooling systems to increase the amount of heat that is removed. The fins extend into the air stream to expose a large amount of surface area, which improves the heat transfer. One of the factors involved in the transfer is the temperature difference between the cooling air and the surface of the fin. More heat can be transferred when there is a large temperature difference.

Sometimes the fin is relatively long, so that there is a temperature gradient along the fin. The base of the fin may be substantially hotter than the tip of the fin, as shown in Figure 6.34.

The tip of the fin should be nearly as hot as the base of the fin, in order to transfer the maximum amount of heat. This requires a large fin thickness ($\delta$), a short length ($d$), and a high thermal conductivity ($K$), to reduce the thermal gradient along the fin [53].

To compensate for the difference in the amount of heat that is transferred from a fin with a nonuniform temperature, a fin efficiency factor ($\eta$) is used. This fin efficiency is defined as the ratio of heat that is actually transferred from the fin to the heat that would be transferred at the maximum uniform

SEC et al. v. MRI
SEC Exhibit 1027.0237
IPR2023-00255

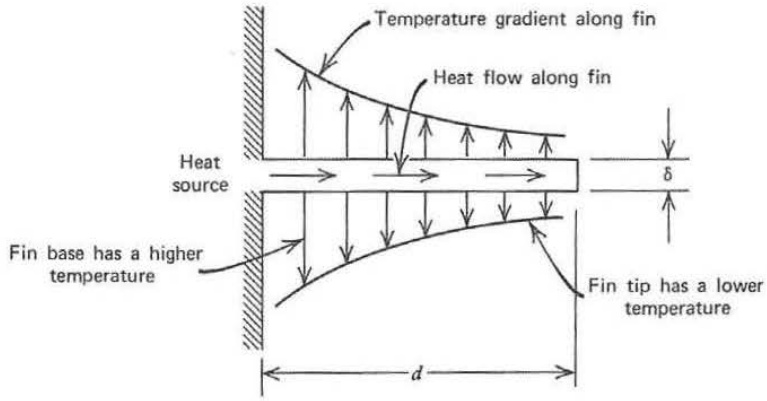



**Figure 6.34** Heat flow from a cooling fin.

temperature condition, where the tip of the fin is at the same temperature as the base. The fin efficiency factor is shown in Eq. 5.21 [3, 15, 17].

$$\eta = \frac{\tanh md}{md} \qquad \text{(ref. Eq. 5.21)}$$

where $\tanh$ = hyperbolic tangent

$m = \sqrt{2h_c/K\delta}$
$h_c$ = convection coefficient
$K$ = fin thermal conductivity
$\delta$ = fin thickness
$d$ = effective height of fin from base

Sometimes fins extend between two surfaces, which are both heat sources. When this condition exists, the height of the fin from the base is half of the total height, as shown in Figure 6.35.

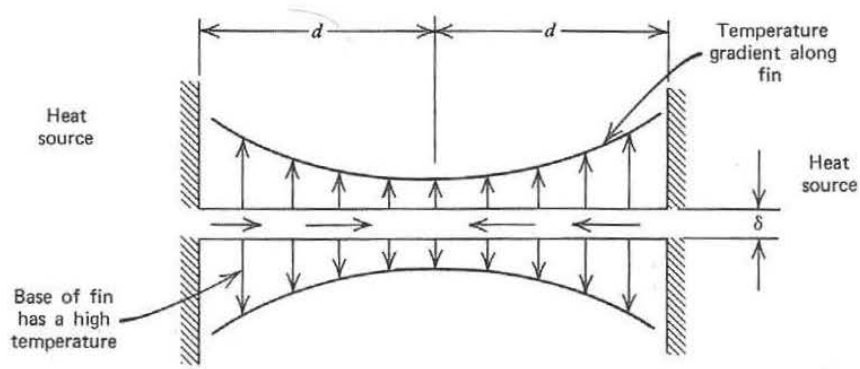



**Figure 6.35** Cooling fin with two heat sources on opposite ends.

Figure 6.36 shows an electronic box with multiple fin air cooled heat exchangers in the side panels.

The fin efficiency for a single fin, as shown in Figure 6.35, can be determined from Figure 5.7 when the values of $m$ and $d$ are known.

The use of the heat exchanger convection coefficient equation, pressure drop equations, and fin efficiency equation can be demonstrated with the use of a sample problem.

## 6.26 SAMPLE PROBLEM—HOLLOW CORE PCB WITH A FINNED HEAT EXCHANGER

An electronic system has many discrete, hybrid, and LSI components that will dissipate a large amount of heat. The size of the electronic box will permit the use of several 4.0 in (10.16 cm) × 6.2 in (15.75 cm) plug-in PCBs. The system must be capable of operating in a class 1X environment, using forced convection cooling at sea level conditions, where the surrounding ambient air temperature is 131°F (55°C). A maximum steady state power

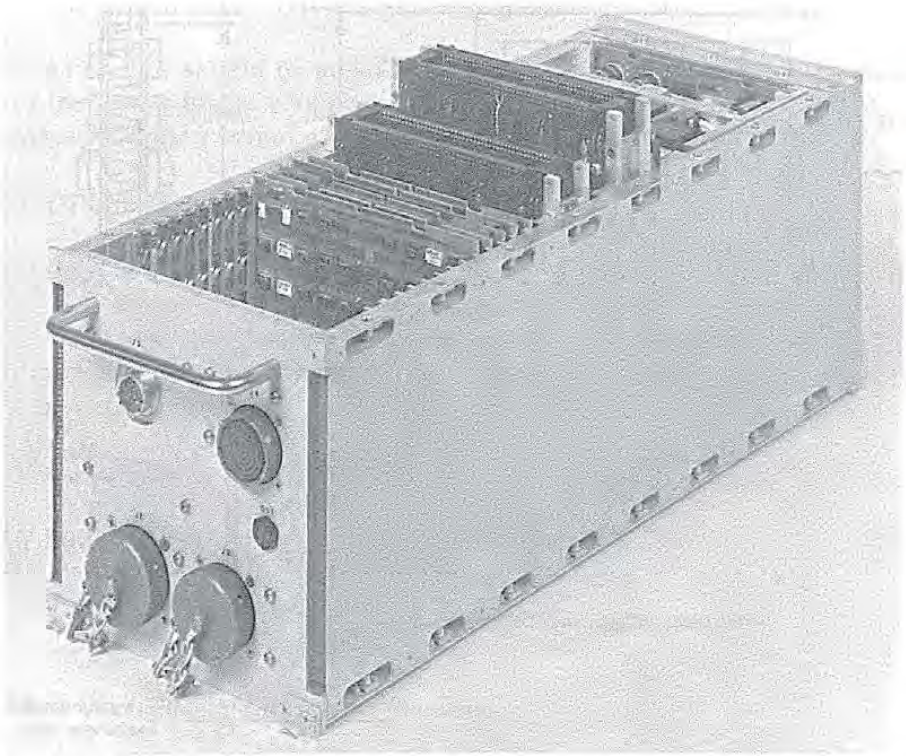


**Figure 6.36** Electronic box with plug-in modules that are cooled by conduction of heat to air cooled epoxy bonded heat exchangers in the side walls. (Courtesy Litton Stystems, Inc.)

SEC et al. v. MRI
SEC Exhibit 1027.0239
IPR2023-00255

dissipation of 50 watts is expected for each PCB under certain conditions, with a relatively uniform heat distribution across the component mounting surface. The maximum allowable component mounting surface temperature is 212°F (100°C). Because of the high power dissipation, a hollow core construction is proposed for the PCBs.

Determine the following:

1. The minimum cooling air weight flow required through each PCB.
2. The heat transfer convection coefficient for a multiple fin heat exchanger at the center of the PCB.
3. The maximum component case or surface mounting temperature.
4. The static pressure drop through the PCB heat exchanger.

A sketch of the proposed PCB is shown in Figure 6.37.

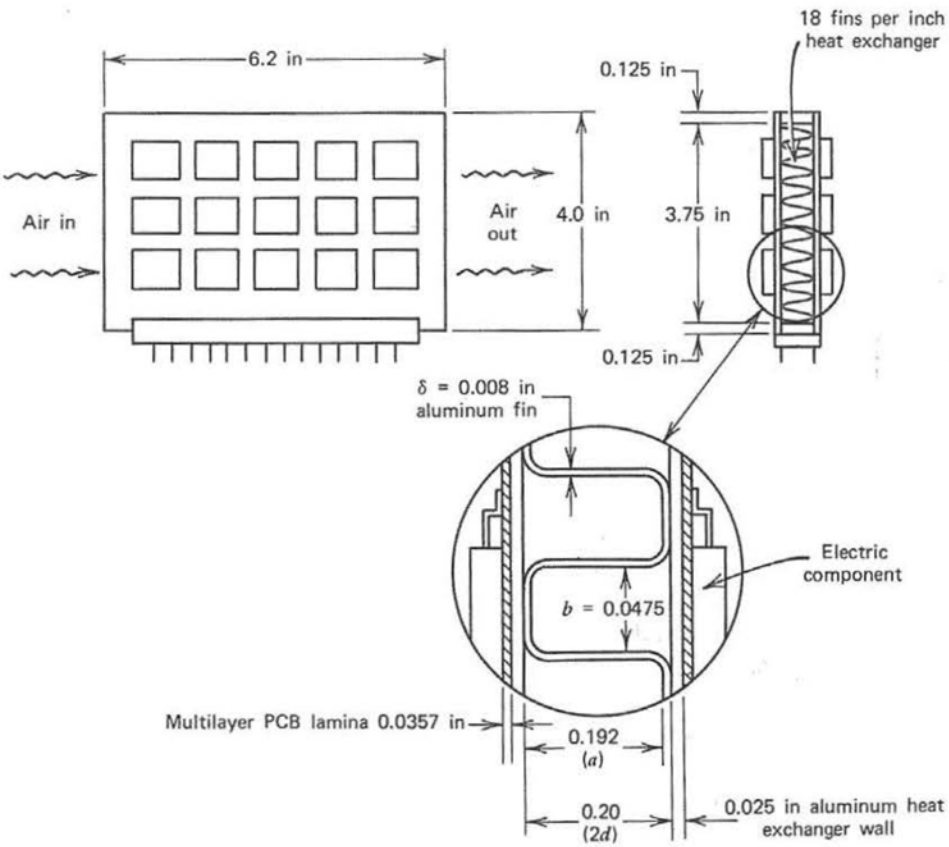



**Figure 6.37** Hollow core PCB with a dip brazed aluminum heat exchanger at the center and electronic components mounted on both sides.

SEC et al. v. MRI
SEC Exhibit 1027.0240
IPR2023-00255

*SOLUTION FOR PART 1*

An efficient cooling system can be designed by providing an exit air temperature of 160°F (71.1°C). The information required is as follows (see Section 6.17):

$$Q = 50 \text{ watts} = 170.6 \text{ Btu/hr} = 11.95 \text{ cal/sec (power dissipation)}$$

$$C_p = 0.24 \text{ Btu/lb °F} = 0.24 \text{ cal/g °C (specific heat)}$$

$$\Delta t = 160°\text{F} - 131°\text{F} = 29°\text{F} \quad \text{or} \quad 71.1°\text{C} - 55°\text{C} = 16.1°\text{C} \tag{6.85}$$

Substitute into Eq. 6.7 to determine the minimum required cooling air weight flow through each PCB in English units.

$$W_{\min} = \frac{170.6 \text{ Btu/hr}}{(29°\text{F})(0.24 \text{ Btu/lb °F})(60 \text{ min/hr})} = 0.408 \frac{\text{lb}}{\text{min}} \tag{6.86}$$

In metric units:

$$W_{\min} = \frac{11.95 \text{ cal/sec}}{(16.1°\text{C})(0.24 \text{ cal/g °C})} = 3.09 \frac{\text{g}}{\text{sec}} \tag{6.86a}$$

An extra 10% should be added to the foregoing cooling air flow rates to cover the possibility of cooling air leakage. A good seal is often difficult to maintain for a long period of time in a severe service environment.

*SOLUTION FOR PART 2*

The forced convection coefficient for the PCB heat exchanger shown in Figure 6.37 is determined from Eq. 6.9, and the $J$ factor is determined from Eq. 6.81. The Reynolds number ($N_R$) is also required, and it can be obtained with the use of Eq. 6.11. But first, the cooling air flow through one air duct must be determined.

$$\text{number of ducts} = 18 \text{ fins/in} \times 3.75 \text{ inch} = 67.5 \text{ ducts}$$

Air flow through one duct, in English units:

$$W = \frac{0.408 \text{ lb/min}}{67.5 \text{ ducts}} = 0.00604 \frac{\text{lb}}{\text{min}} \text{ per duct}$$

Air flow through one duct, in metric units:

$$W = \frac{3.09 \text{ g/sec}}{67.5 \text{ ducts}} = 0.0458 \frac{\text{g}}{\text{sec}} \text{ per duct}$$

SEC et al. v. MRI
SEC Exhibit 1027.0241
IPR2023-00255

Hydraulic diameter of one duct ($a = 0.192$ in, $b = 0.0475$ in):

$$D = \frac{4 \times \text{area}}{\text{perimeter}} = \frac{4ab}{2a + 2b} = \frac{2ab}{a + b} \tag{6.87}$$

$$D = \frac{2(0.192)(0.0475)}{0.192 + 0.0475} = 0.0761 \text{ in} = 0.00635 \text{ ft} = 0.193 \text{ cm}$$

Cross section area of one duct:

$$A = ab = (0.192)(0.0475) = 0.00912 \text{ in}^2 = 0.0000633 \text{ ft}^2 = 0.0588 \text{ cm}^2$$

The weight velocity flow through one duct is determined from Eq. 6.12.

$$G = \frac{0.00604 \text{ lb/min} \times 60 \text{ min/hr}}{0.0000633 \text{ ft}^2} = 5725 \frac{\text{lb}}{\text{hr ft}^2} \text{ (English units)}$$

$$G = \frac{0.0458 \text{ g/sec}}{0.0588 \text{ cm}^2} = 0.779 \frac{\text{g}}{\text{sec cm}^2} \text{ (metric units)}$$

The average cooling air temperature is:

$$t_{av} = \frac{t_{in} + t_{out}}{2} = \frac{131°\text{F} + 160°\text{F}}{2} = 145.5°\text{F} = 63.0°\text{C}$$

The physical properties of air (ref. Figure 6.28; conversion between English units and metric units is obtained from Section 1.11) [7, 8, 37, 38, 52].

$$K = 0.017 \text{ Btu/hr ft °F} = 0.0000702 \text{ cal/sec cm °C (conductivity)}$$

$$C_p = 0.24 \text{ Btu/lb °F} = 0.24 \text{ cal/g °C (specific heat)}$$

$$\mu = 0.050 \text{ lb/ft hr} = 0.000206 \text{ g/cm sec (viscosity)}$$

$$\rho = 0.0656 \text{ lb/ft}^3 = 0.00105 \text{ g/cm}^3 \text{ (density)}$$

Substitute into Eq. 6.11 for the Reynolds number. In English units:

$$N_R = \frac{(5725 \text{ lb/hr ft}^2)(0.00635 \text{ ft})}{0.050 \text{ lb/ft hr}} = 727 \text{ (dimensionless)} \tag{6.88}$$

In metric units:

$$N_R = \frac{(0.779 \text{ g/sec cm}^2)(0.193 \text{ cm})}{0.000206 \text{ g/cm sec}} = 729 \text{ (dimensionless)} \tag{6.88a}$$

Substitute into Eq. 6.81 for the Colburn $J$ factor.

SEC et al. v. MRI
SEC Exhibit 1027.0242
IPR2023-00255

$$J = \frac{0.72}{(N_R)^{0.7}} = \frac{0.72}{(727)^{0.7}} = 0.00715 \text{ (dimensionless)}$$

Substitute into Eq. 6.9 for the heat exchanger forced convection coefficient, in English units.

$$h_c = \frac{(0.00715)(0.24)(5725)}{[(0.24)(0.050)/0.017]^{0.666}} = 12.4 \frac{\text{Btu}}{\text{hr ft}^2\ ^\circ\text{F}} \tag{6.89}$$

In metric units:

$$h_c = \frac{(0.00715)(0.24)(0.779)}{[(0.24)(0.000206)/0.0000702]^{0.666}} = 0.00168 \frac{\text{cal}}{\text{sec cm}^2\ ^\circ\text{C}} \tag{6.89a}$$

The forced convection coefficient will be slightly higher at the cooling air entrance section of the heat exchanger because of the extra turbulence that is developed. This small increase is conservatively ignored here.

The fin efficiency factor shown in Eq. 5.21 must be considered because there will be a temperature gradient along the heat exchanger fins. Since there are electronic components mounted on both sides of the PCB, there will be a heat input to the heat exchanger fins from two different surfaces, as shown in Figure 6.35.

*Given* $\delta = 0.008$ in $= 0.000666$ ft $= 0.0203$ cm (fin thickness)
$K = 99$ Btu/hr ft °F $= 0.409$ cal/sec cm °C 6061 (aluminum fin)
$d = 0.20$ in/2 $= 0.10$ in $= 0.00833$ ft $= 0.254$ cm (note, heat in from both sides)
$h_c = 12.4$ Btu/hr ft² °F $= 0.00168$ cal/sec cm² °C (convection coeff.)

Substitute into Eq. 5.21 for English units, with dimensions in feet.

$$m = \sqrt{\frac{2(12.4)}{99(0.000666)}} = 19.4$$

$$md = 19.4(0.00833) = 0.1616$$

In metric units:

$$m = \sqrt{\frac{2(0.00168)}{(0.409)(0.0203)}} = 0.636$$

$$md = (0.636)(0.254) = 0.1615$$

Substitute into Eq. 5.21 for fin efficiency.

SEC et al. v. MRI
SEC Exhibit 1027.0243
IPR2023-00255

$$\eta = \frac{\tanh 0.1615}{0.1615} = \frac{0.16011}{0.1615} = 99.1\%$$

The corrected value of the forced convection coefficient in English units is

$$h_c = 12.4(0.991) = 12.3 \frac{\text{Btu}}{\text{hr ft}^2\ ^\circ\text{F}} \tag{6.90}$$

In metric units:

$$h_c = 0.00168(0.991) = 0.00166 \frac{\text{cal}}{\text{sec cm}^2\ ^\circ\text{C}} \tag{6.90a}$$

The value of the forced convection coefficient ($h_c$) is sharply influenced by the hydraulic diameter in the laminar flow regions. With low values of cooling air weightflow velocities, the convection coefficient for a given hydraulic diameter does not show much change. As the hydraulic diameter decreases, the forced convection coefficient increases rapidly, as shown in Figure 6.38. As the fluid flow increases and the turbulent region is approached, the convection coefficient increases very rapidly [70].

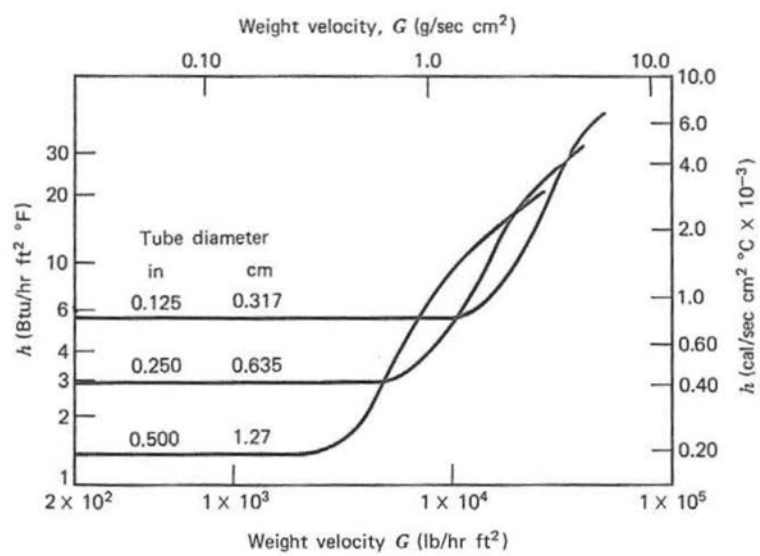



**Figure 6.38** Variation of forced convection coefficient with hydraulic diameter and flow rate. (Ref: General Electric Heat Transfer Data Book 1975)

*SOLUTION FOR PART 3*

The maximum component case temperatures will occur on the components that are mounted closest to the cooling air exhaust end of the heat exchanger.

A large number of interconnections are required for the circuits, so multilayer circuit laminars will be required. Interconnections between layers are made with plated throughholes, which are filled with solder or epoxy to improve the mechanical strength of the interconnection.

The thickness of the multilayer circuit lamina from the component to the heat exchanger will be about 0.0357 in (0.0907 cm). This thickness must be closely controlled because the component heat will flow through the multilayer circuit lamina to the air cooled heat exchanger at the center of the PCB. A thicker circuit lamina will have a higher thermal resistance, which will increase the temperature rise across the lamina.

Plated throughholes will tend to reduce the thermal resistance through the circuit lamina. An enlarged view through the circuit lamina, showing the heat flow path, is shown in Figure 6.39.

Some of the component heat may be lost by external convection and radiation, but this is conservatively ignored because the PCBs face each other and are close enough to restrict natural convection. It is therefore assumed that all the heat is transferred from the components to the cooling air by conduction through the PCB.

The heat flow path from the component surface (or case) to the cooling

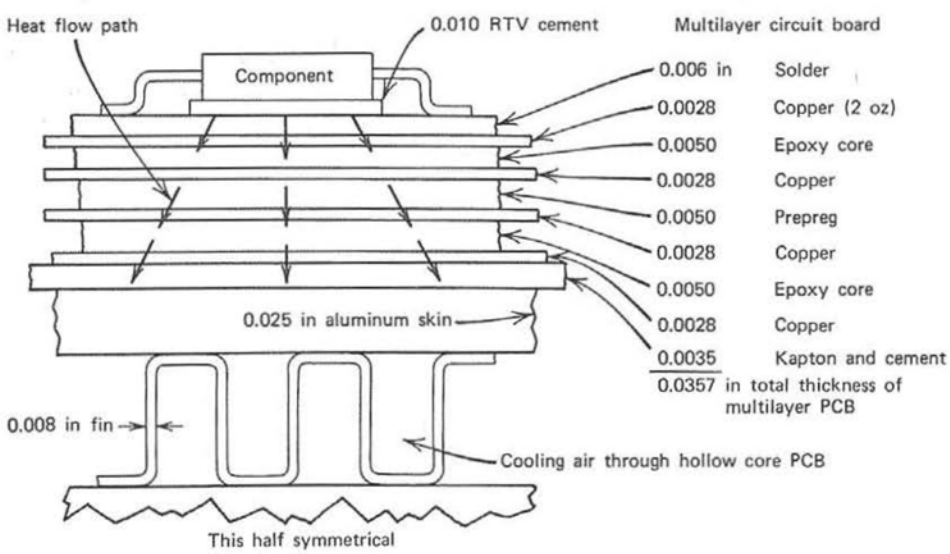



**Figure 6.39** Enlarged section through a hollow PCB with a heat exchanger core and multilayer circuit laminas on both sides.

SEC et al. v. MRI
SEC Exhibit 1027.0245
IPR2023-00255

air in the hollow core heat exchanger is evaluated by determining the temperature rise along four segments of the heat flow path as follows:

$\Delta t_1$ = temperature rise across component interface, from case to circuit lamina
$\Delta t_2$ = temperature rise through the multilayer circuit lamina to the aluminum heat exchanger skin
$\Delta t_3$ = temperature rise across the convection air film, from the aluminum heat exchanger surface to the center of cooling air stream
$\Delta t_4$ = temperature rise of the cooling air stream from the entrance of the heat exchanger to the exit

Each section along the heat flow path is examined in more detail.

$\Delta t_1$: temperature rise across component interface from case to circuit lamina

The electronic types being examined are hybrid and large scale integrated (LSI) components. These usually come in relatively large case sizes, approximately 1.0 in (2.54 cm) × 1.0 in (2.54 cm). These components will dissipate a large amount of heat, since the total power dissipation on the PCB is 50 watts. When the power density on such a component is more than about 4 watts/in$^2$, a good thermally conductive cement should be used to prevent an excessive temperature rise across the component interface as shown in Figure 6.40. Room temperature vulcanizing (RTV) cement is often used to permit the component to be removed if it has to be replaced. Epoxy cements are available with higher thermal conductivities, but components bonded with this material are very difficult to replace without damaging the PCB.

Test data show that some heat is conducted from the body of the hybrid component to the electrical lead wires. This heat flows along the lead wires to the circuit lamina solder pads, then through the circuit lamina into the heat exchanger.

The heat loss through the lead wires is usually ignored because the strain relief produces a long lead length with a high resistance, and there is another

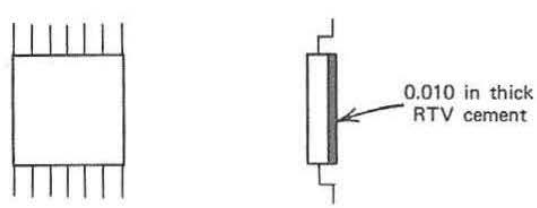



**Figure 6.40** Hybrid component fastened to PCB with RTV cement.

SEC et al. v. MRI
SEC Exhibit 1027.0246
IPR2023-00255

high resistance through the small cross section area through the circuit lamina.

Considering a maximum power dissipation of 4 watts for one hybrid, the temperature rise across an RTV interface can be determined from Eq. 3.1. See Section 1.11 for conversions between English units and metric units.

*Given* $Q = 4.0$ watts $= 13.65$ Btu/hr $= 0.956$ cal/sec (heat flow)
$L = 0.010$ in $= 0.000833$ ft $= 0.0254$ cm (RTV thickness)
$K = 0.22$ Btu/hr ft °F $= 0.000908$ cal/sec cm °C (RTV conductivity)

$$A = \frac{(1.0)(1.0)\ \text{in}^2}{144\ \text{in}^2/\text{ft}^2} = 0.00694\ \text{ft}^2 = 6.45\ \text{cm}^2 \text{ (RTV area)}$$

In English units, with dimensions in feet:

$$\Delta t_1 = \frac{(13.65)(0.000833)}{(0.22)(0.00694)} = 7.4°\text{F} \tag{6.91}$$

In metric units:

$$\Delta t_1 = \frac{(0.956)(0.0254)}{(0.000908)(6.45)} = 4.1°\text{C} \tag{6.91a}$$

$\Delta t_2$: temperature rise through the multilayer circuit lamina to the aluminum heat exchanger skin

The multilayer circuit lamina consists of 0.005 in thick epoxy fiberglass sheets, with etched copper circuits on each side. Two of these 0.005 in sheets are bonded together with a prepreg epoxy fiberglass cement using high temperatures and pressures to force the prepreg to flow between the copper runs, eliminating any air bubbles. Holes for interconnecting circuits are drilled through the bonded assembly, which is then plated with a thin copper layer. The plating forms an electrically conducting tube through the drilled holes and also builds up the thickness of the copper runs on both sides of the bonded lamina assembly. The original copper thickness on the lamina is usually 0.0014 in or about 1 ounce. The plating process usually adds about another ounce, which brings the total thickness of the copper runs on the outer surfaces to 0.0028 in.

The copper-plated throughholes (PTH) tend to increase the effective thermal conductivity through the epoxy fiberglass laminas, because copper has a very high thermal conductivity. Typical values are about 20%, depending upon the number of PTHs used.

In addition, the copper circuit layers within the bonded lamina will tend to spread the heat flowing through the lamina if there are many copper circuit runs. This tends to increase the effective cross section area along

SEC et al. v. MRI
SEC Exhibit 1027.0247
IPR2023-00255

the heat flow path through the circuit lamina about 15%, depending upon the number of copper circuit runs.

The temperature rise through the circuit lamina is also determined from Eq. 3.1.

*Given* $Q = 4$ watts $= 13.65$ Btu/hr $= 0.956$ cal/sec (heat)
$L = 0.0357$ in $= 0.00297$ ft $= 0.0907$ cm (thickness of circuit lamina)
$K = 0.20$ Btu/hr ft °F $= 0.000826$ cal/sec cm °C (conductivity with no PTHs)
$K = 0.24$ Btu/hr ft °F $= 0.000991$ cal/sec cm °c (conductivity with PTHs)
$A = (1.0)(1.0) = 1.0$ in$^2$ $= 0.00694$ ft$^2$ $= 6.452$ cm$^2$ (area with no heat spreading)
$A = 1.15$ in$^2$ $= 0.00798$ ft$^2$ $= 7.419$ cm$^2$ (area with heat spreading)

Conservatively ignore the improved conductivity of the PTHs and the spreading effects of the copper runs. In English units:

$$\Delta t_2 = \frac{(13.65)(0.00297)}{(0.20)(0.00604)} = 29.2°\text{F} \tag{6.92}$$

In metric units:

$$\Delta t_2 = \frac{(0.956)(0.0907)}{(0.000826)(6.452)} = 16.3°\text{C} \tag{6.92a}$$

$\Delta t_3$: temperature rise across the convection air film from the aluminum heat exchanger surface to the center of the cooling air stream

Equation 5.7 is used to determine this temperature rise. However, first the surface area of the heat exchanger must be calculated. If the total power dissipation is considered, the total heat exchanger surface area must be used. It is convenient to consider the solid height of the fins to compute the side wall area, as shown in Figure 6.41.

$$\text{fin solid height} = (18 \text{ fins/in})(3.75 \text{ in})(0.008 \text{ in}) = 0.54 \text{ inch}$$

$$\text{fin area} = (67.5 \text{ fins})(0.192 \text{ in})(6.2 \text{ in})(2 \text{ surfaces})$$

$$\text{fin area} = 160.7 \text{ in}^2 = 1.116 \text{ ft}^2 = 1036.8 \text{ cm}^2$$

$$\text{sidewall area} = (3.21 \text{ in})(6.2 \text{ in})(2 \text{ surfaces})$$

$$\text{sidewall area} = 39.8 \text{ in}^2 = 0.276 \text{ ft}^2 = 256.8 \text{ cm}^2$$

$$\text{total surface area } A = 1.392 \text{ ft}^2 = 1293.6 \text{ cm}^2$$

SEC et al. v. MRI
SEC Exhibit 1027.0248
IPR2023-00255

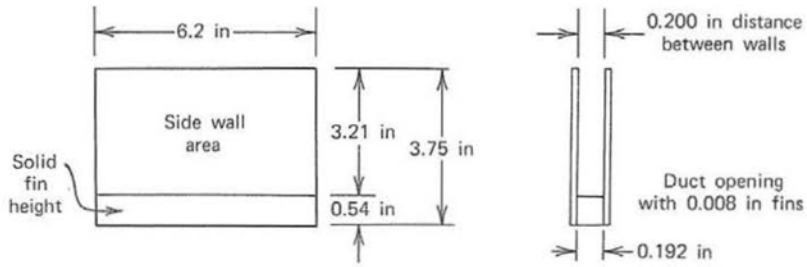



**Figure 6.41** Heat exchanger fins shown as a solid height.

Heat:

$$Q = 50 \text{ watts} = 170.65 \frac{\text{Btu}}{\text{hr}} = 11.95 \frac{\text{cal}}{\text{sec}} \text{ (total power)}$$

$$h_c = 12.3 \frac{\text{Btu}}{\text{hr ft}^2\,°\text{F}} = 0.00166 \frac{\text{cal}}{\text{sec cm}^2\,°\text{C}} \text{ (Ref. Eqs. 6.90 and 6.90a)}$$

For English units, with dimensions in feet:

$$\Delta t_3 = \frac{Q}{h_c A} = \frac{170.65}{(12.3)(1.392)} = 10.0°\text{F} \tag{6.93}$$

In metric units:

$$\Delta t_3 = \frac{11.95}{(0.00166)(1293.6)} = 5.6°\text{C} \tag{6.93a}$$

$\Delta t_4$: temperature rise of the cooling air from the entrance of the heat exchanger to the exit

As the cooling air flows through the PCB heat exchanger, it will pick up the heat dissipated from the electronic components. The temperature rise of the cooling air is determined by considering the total PCB power dissipation with Eq. 6.7.

*Given* $Q = 50$ watts $= 170.65$ Btu/hr $= 11.95$ cal/sec (total power)
$W = 0.408$ lb/min $= 3.09$ g/sec (ref. Eqs. 6.86 and 6.86a)
$C_p = 0.24$ Btu/lb °F $= 0.24$ cal/g °C (specific heat)

In English units:

$$\Delta t_4 = \frac{Q}{WC_p} = \frac{170.65 \text{ Btu/hr}}{(0.408 \text{ lb/min})(60 \text{ min/hr})(0.24 \text{ Btu/lb °F})} = 29.0°\text{F} \tag{6.94}$$

SEC et al. v. MRI
SEC Exhibit 1027.0249
IPR2023-00255

In metric units:

$$\Delta t_4 = \frac{11.95 \text{ cal/sec}}{(3.09 \text{ g/sec})(0.24 \text{ cal/g °C})} = 16.1°\text{C} \tag{6.94a}$$

The maximum component case temperature is obtained by adding all of the temperature rises $\Delta t_1$ through $\Delta t_4$ to the inlet air temperature of 131°F (55°C).

$$t_{\text{case}} = t_{\text{in}} + \Delta t_1 + \Delta t_2 + \Delta t_3 + \Delta t_4 \tag{6.95}$$

For English units, substitute Eqs. 6.91 through 6.94 into Eq. 6.95.

$$t_{\text{case}} = 131 + 7.4 + 29.2 + 10.0 + 29.0 = 206.6°\text{F} \tag{6.96}$$

For metric units, substitute Eqs. 6.91a through 6.94a into Eq. 6.95.

$$t_{\text{case}} = 55 + 4.1 + 16.3 + 5.6 + 16.1 = 97.1°\text{C} \tag{6.96a}$$

*SOLUTION PART 4*

The static pressure drop through the PCB heat exchanger only is desired from Eq. 6.82, in terms of the height of a column of water.

The friction factor is obtained from Table 6.7, considering the cross section of one tube of the heat exchanger to be a rectangular duct with an aspect ratio of 4. Using the Reynolds number computed in Eqs. 6.88 and 6.88a, the friction in English units and metric units is shown in Eq. 6.97.

$$f = \frac{18.3}{N_R} = \frac{18.3}{727} = 0.0252 \text{ (dimensionless)} \tag{6.97}$$

The cooling air velocity ($V$) through the heat exchanger is shown in Eq. 6.98. It is obtained from the physical properties of the air, which were previously determined for the Reynolds number shown in Eqs. 6.88 and 6.88a. The area ($A$) of one duct was shown in Eq. 6.87.

$$V = \frac{W}{\rho A} \tag{6.98}$$

In English units:

$$V = \frac{0.00604 \text{ lb/min}}{(0.0656 \text{ lb/ft}^3)(0.0000633 \text{ ft}^2)(60 \text{ sec/min})}$$

$$= 24.2 \frac{\text{ft}}{\text{sec}} = 290.4 \frac{\text{in}}{\text{sec}} \tag{6.99}$$

SEC et al. v. MRI
SEC Exhibit 1027.0250
IPR2023-00255