

TOWNSEND

CUDDELEY'S
YEAR BOOK
SECOND EDITION

TJ
184
D78
1991

ZF, Ex. 1017

DUDLEY'S GEAR HANDBOOK

Dennis P. Townsend Editor in Chief

Lewis Research Center, NASA

Second Edition

McGRAW-HILL, INC.

New York St. Louis San Francisco Auckland Bogotá
Caracas Lisbon London Madrid Mexico Milan
Montreal New Delhi Paris San Juan São Paulo
Singapore Sydney Tokyo Toronto

Library of Congress Cataloging-in-Publication Data

Dudley's gear handbook.—2nd ed. / Dennis P. Townsend, editor.
p. cm.

Rev. ed. of: Gear handbook. 1st ed. 1962.

Includes index.

ISBN 0-07-017903-4

I. Gearing—Handbooks, manuals, etc. I. Dudley, Darle W.
II. Townsend, Dennis P. III. Gear handbook.

TJ184.D83 1992

621.8'3—dc20

91-1967

CIP

Copyright © 1991, 1962 by McGraw-Hill, Inc. All rights reserved. Printed in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a data base or retrieval system, without the prior written permission of the publisher.

1 2 3 4 5 6 7 8 9 0 DOC/DOC 9 7 6 5 4 3 2 1

ISBN 0-07-017903-4

The sponsoring editor for this book was Robert W. Hauserman, the editing supervisor was Fred Dahl, and the production supervisor was Suzanne W. Babeuf. It was set in Times Roman by ComCom, Inc.

Printed and bound by R. R. Donnelley & Sons Company.

Information contained in this work has been obtained by McGraw-Hill, Inc., from sources believed to be reliable. However, neither McGraw-Hill nor its authors guarantees the accuracy or completeness of any information published herein, and neither McGraw-Hill nor its authors shall be responsible for any errors, omissions, or damages arising out of use of this information. This work is published with the understanding that McGraw-Hill and its authors are supplying information but are not attempting to render engineering or other professional services. If such services are required, the assistance of an appropriate professional should be sought.



engr
TJ
184
D78
1991

APG3651

CONTENTS

Contributors xi

Preface xiii

Chapter 1. Theory of Gearing and Application

1.1

- 1.1 Gear Ratio, Pitch Circles, Centroides, Pitch Cylinders, Pitch Cones, Axodes, and Pitch Surfaces / 1.1
- 1.2 Parametric Representation of Curves and Surfaces / 1.26
- 1.3 Envelope of a Family of Surfaces (Curves), Basic Law of Meshing / Contact Lines on Mating Surfaces / 1.30
- 1.4 Basic Kinematic Relations / 1.32
- 1.5 Conditions of Nonundercutting / 1.33
- 1.6 Envelope of Contact Lines on Generating Surface Σ_1 / 1.35
- 1.7 Limit Contact Normal / 1.35
- 1.8 Relations between Principal Curvatures and Directions for Mutually Enveloped Surfaces / 1.37
- 1.9 Contact Ellipse and Bearing Contact / 1.38
- 1.10 Modification of Gear Tooth Surfaces / 1.39
- References / 1.42

Chapter 2. Gear Types and Nomenclature

2.1

- 2.1 Types of Gears / 2.1
- 2.2 Nomenclature of Gears / 2.30
- References / 2.49

Chapter 3. Gear Arrangements

3.1

- 3.1 Possibilities in Gear Arrangements / 3.1
- 3.2 How the Arrangement May Drastically Affect Ratio, Power, Efficiency, and Gearbox Volume / 3.3
- 3.3 Parallel Axis Arrangements / 3.6
- 3.4 Parallel Axis Epicyclic Gearing / 3.14
- 3.5 Right-Angle Gear Arrangements / 3.24
- 3.6 Differential Gearing / 3.26
- 3.7 Speed-Changing Transmissions Using Shifting / 3.28
- 3.8 Speed-Changing Transmissions Using Clutches and Brakes / 3.30
- 3.9 High Ratio Arrangements / 3.31
- References / 3.34

Chapter 4. Gear Tooth Design**4.1**

- 4.1 Basic Requirements of Gear Teeth / 4.1
- 4.2 Standard Systems of Gear Tooth Proportions / 4.28
- 4.3 General Equations Relating to Center Distance / 4.72
- 4.4 Elements of Center Distance / 4.83
- References / 4.94

Chapter 5. Special Calculations of Spur and Helical Gears**5.1**

- 5.1 Notation / 5.1
- 5.2 Fundamental Gear Equation / 5.3
- 5.3 Involute Function and Roll Angle / 5.3
- 5.4 Line of Action and Contact Ratio / 5.4
- 5.5 Tooth Thickness Calculation / 5.6
- 5.6 Hertzian Stress Along Contact Path / 5.7
- 5.7 Backlash and Center Distance / 5.8
- 5.8 Tooth Thickness from Pin or Ball Measurement / 5.9
- 5.9 Interference Limit / 5.13
- 5.10 High Contact Ratio Gear / 5.15
- 5.11 Speed Ratio of Epicyclic Gears / 5.16
- 5.12 Compound Epicyclic Gear Index / 5.16
- 5.13 Nonstandard-Addendum Gear / 5.16
- References / 5.19

Chapter 6. Gear Tooth Calculations**6.1**

- 6.1 Calculating Involute Tooth Profile Coordinates / 6.1
- 6.2 Calculating Gear Tooth Fillet Coordinates / 6.3
- 6.3 Calculating the Inverse of an Involute / 6.6
- 6.4 Calculating Addendum Modification / 6.8
- 6.5 Derivation of AGMA Hertzian Stress Equation / 6.15
- 6.6 Gear Bending Stress Analysis / 6.19
- 6.7 Interference of Internal Gears / 6.22
- References / 6.27

Chapter 7. Gear Tolerances**7.1**

- 7.1 Terminology and Definitions Pertaining to Gear Tolerances / 7.1
- 7.2 Introduction of Gear Tolerances / 7.5
- 7.3 Tolerance Effects / 7.8
- 7.4 Types of Tolerances / 7.9
- 7.5 Factors Affecting Choice of Tolerance / 7.11
- 7.6 Drawings and Tolerances for Various Gear Types / 7.12
- 7.7 Tolerance Standards / 7.15
- 7.8 Special Considerations for Tolerance Values / 7.27
- 7.9 Assembly and Application Considerations / 7.41
- References / 7.44

Chapter 8. Gear Materials

- 8.1 Ferrous Gear Materials / 8.1
- 8.2 Gear Materials / 8.2
- 8.3 Carbon and Alloy Steels / 8.13
- 8.4 Nonferrous Gear Material / 8.28
- References / 8.41

Chapter 9. Gear Drawings

9.1

- 9.1 General Comments Concerning Preparation of Gear Drawings / 9.1
- 9.2 Gear Drawing Practice / 9.2
- 9.3 Gear Drawing Specifics / 9.6
- 9.4 Preparation of Detail Manufacturing Drawings Using Computer-Aided Design and Computer-Aided Drafting Equipment (CAD) / 9.22
- References / 9.28

Chapter 10. Gear Reactions and Mountings

10.1

- 10.1 Mechanics of Gear Reactions / 10.1
- 10.2 Basic Gear Reactions, Bearing Loads, and Mounting Types / 10.4
- 10.3 Basic Mounting Arrangements and Recommendations / 10.9
- 10.4 Bearing Load Calculations for Spur Gears / 10.12
- 10.5 Bearing Load Calculations for Helicals / 10.18
- 10.6 Mounting Practice for Bevel and Hypoid Gears / 10.23
- 10.7 Calculation of Bevel and Hypoid Bearing Loads / 10.34
- 10.8 Bearing Load Calculations for Worms / 10.46
- 10.9 Bearing Load Calculations for Spiroids / 10.52
- 10.10 Bearing Load Calculations for Other Gear Types / 10.54
- 10.11 Design of the Body of the Gear / 10.55
- References / 10.57

Chapter 11. Load Rating of Gears

11.1

- 11.1 Considerations / 11.1
- 11.2 Main Nomenclature / 11.2
- 11.3 Coplanar Gears (Involute Parallel Gears and Bevel Gears) / 11.4
- 11.4 Coplanar Gears: Simplified Estimates and Design Criteria / 11.8
- 11.5 Coplanar Gears: Detailed Analysis, Conventional Fatigue Limits, and Service Factors / 11.13
- 11.6 RH—Conventional Fatigue Limit of Factor K / 11.21
- 11.7 RF—Conventional Fatigue Limit of Factors U_L / 11.40
- 11.8 Coplanar Gears: Detailed Life Curves and Yielding / 11.47
- 11.9 Coplanar Gears: Prevention of Tooth Wear and Scoring / 11.54
- 11.10 Crossed Helical Gears / 11.55
- 11.11 Hypoid Gears / 11.56
- 11.12 Worm Gearing / 11.57
- 11.13 State of the Standards for Load Rating of Gears / 11.59
- References / 11.59

Chapter 12. Loaded Gears in Action**12.1**

- 12.1 Efficiency of Gears / 12.1
- 12.2 Sliding Velocity / 12.2
- 12.3 Efficiency of Coplanar Gears (Parallel Axes and Intersecting Axes) / 12.4
- 12.4 Efficiency of Nonplanar Gears (Nonintersecting and Nonparallel Axes) / 12.11
- 12.5 Gear Trains / 12.17
- 12.6 Gear Performance / 12.36
- 12.7 Factors That Affect the Performance of Gear Units / 12.37
- 12.8 Gear Failures / 12.40
- 12.9 Performance Testing of Gears / 12.60
- References / 12.69

Chapter 13. Gear Vibration**13.1**

- 13.1 Fundamentals of Vibration / 13.1
- 13.2 Measurement of Vibration / 13.3
- 13.3 Some Examples of Vibration in Geared Units / 13.7
- 13.4 Approximate Vibration Limits / 13.10
- 13.5 Control of Vibration in Manufacturing Gears and in the Field / 13.16
- 13.6 Vibration Analysis Techniques / 13.19
- References / 13.20

Chapter 14. Gear Noise**14.1**

- 14.1 Gear Noise: The General Problem / 14.2
- 14.2 Measurements and Standards / 14.7
- 14.3 Power Considerations / 14.10
- 14.4 Transmission Error in Gears / 14.11
- 14.5 Gear Noise Reduction at the Design Stage / 14.19
- 14.6 Gear Noise Modeling / 14.24
- 14.7 Gear Rattle / 14.25
- 14.8 Gear Noise Troubleshooting / 14.26
- 14.9 Closing Comments / 14.28
- References / 14.29

Chapter 15. Gear Lubrication**15.1**

- 15.1 Power Losses / 15.2
- 15.2 Lubrication and Cooling Methods / 15.4
- 15.3 Lubrication of Loaded Tank Flank and Surface Distress / 15.8
- 15.4 Estimation of the Influence of Lubrication on Surface Distress / 15.16
- 15.5 Lubricant and Additive Effects on Gear Fatigue Life / 15.31
- References / 15.31

Chapter 16. Gear Cutting**16.1**

- 16.1 Gear Milling / 16.1
- 16.2 Gear Hobbing / 16.6
- 16.3 Gear Shaping / 16.26
- 16.4 Gears Cut by Shear Speed / 16.38
- 16.5 Gear Broaching / 16.45

Chapter 17. Gears Made by Dies **17.1**

- 17.1 Die Processes / 17.1
- 17.2 Injection Molding / 17.2
- 17.3 Die-Cast Gears / 17.6
- 17.4 Powdered Metal Gears / 17.7
- 17.5 Forged Gears / 17.12
- 17.6 Cold Drawing and Extrusion / 17.14
- 17.7 Gear Teeth and Worm Thread Rolling / 17.16
- 17.8 Gear Stamping / 17.17
- References / 17.21

Chapter 18. Gear Finishing by Shaving, Rolling, and Honing **18.1**

- 18.1 The Rotary Gear Shaving Process / 18.1
- 18.2 Basic Principles / 18.4
- 18.3 The Shaving Cutter / 18.12
- 18.4 Shaving Machines / 18.14
- 18.5 Gear Roll-Finishing / 18.18
- 18.6 Rotary Gear Honing / 18.27

Chapter 19. Grinding of Spur and Helical Gears **19.1**

- 19.1 Reasons for Grinding / 19.1
- 19.2 Grinding Process Mechanics and Process Parameters / 19.2
- 19.3 Abrasives / 19.4
- 19.4 Grinding Wheels / 19.7
- 19.5 Grinding Processes / 19.10
- 19.6 Cycle Time Estimates / 19.19
- References / 19.21

Chapter 20. Bevel and Hypoid Gear Manufacturing **20.1**

- 20.1 The Basic Process / 20.1
- 20.2 The Basic Generator / 20.2
- 20.3 The Basic Nongenerator / 20.7
- 20.4 Gear Blank Requirements Prior to Tooth Processing / 20.9
- 20.5 Bevel and Hypoid Gear Manufacturing Procedures / 20.11
- 20.6 Bevel Gear Work-Holding Equipment / 20.12
- 20.7 Straight Bevel Gear Machines / 20.12
- 20.8 Face-Mill and Face-Hob Types of Cutting Machines and Grinders—Spiral Bevel, Zerol Bevel, and Hypoid Gears / 20.20
- 20.9 Spiral Bevel and Hypoid Gear Lapping / 20.40
- 20.10 Tooling System / 20.45
- References / 20.49

Chapter 21. High Ratio Right-Angle Gearing Manufacturing **21.1**

- 21.1 The Rearrangement of Material / 21.2
- 21.2 The Thread-Milling Process / 21.2

- 21.3 Thread-Hobbing Process / 21.4
- 21.4 Determination of Chip Load / 21.4
- 21.5 Speeds and Feeds / 21.5
- 21.6 Calculate Thread-Milling Time: Single Cutters / 21.6
- 21.7 Calculating Hobbing Time for Worms / 21.7
- 21.8 Hobbing Worm Gears / 21.7
- 21.9 Infeed Hobbing of Worm Gears / 21.10
- 21.10 Tangential Hobbing of Worm Gears / 21.12
- 21.11 Fly Cutting / 21.13
- 21.12 Feeds for Use in Cutting Worm Gears / 21.14
- 21.13 Time Required to Cut Worm Gears / 21.15
- 21.14 Double-Enveloping Worms and Worm Gears / 21.15
- 21.15 Cutting Time for Double-Enveloping Gear Sets / 21.18
- 21.16 Worm-Thread Grinding / 21.18
- 21.17 Worm Rolling / 21.21
- 21.18 Caburizing and Optional Finishing / 21.23
- 21.19 Powdered Metal Compacts / 21.23
- References / 21.30

Chapter 22. Gear Cutting Tools

22.1

-
- 22.1 Hobs for Spur and Helical Gears / 22.1
 - 22.2 Gear Shaper Cutters / 22.11
 - 22.3 Gear Milling Cutters / 22.21
 - 22.4 Shaving Cutters / 22.25
 - 22.5 Gear Tooth Broaches / 22.33
 - 22.6 Hard Gear Finishing / 22.41
 - References / 22.44

Chapter 23. Gear Inspection Devices and Procedures

23.1

-
- 23.1 Analytical Gear Checking / 23.2
 - 23.2 Functional Gear Checking / 23.6
 - 23.3 Checking Spur and Helical Gears / 23.13
 - 23.4 Bevel and Hypoid Gear Inspection / 23.23
 - 23.5 Worm Gear Inspection / 23.38
 - 23.6 Checking for Gear Wear / 23.42
 - 23.7 Checking Extra Large and Extra Small Gears / 23.45
 - References / 23.51

Chapter 24. Numerical Data Tables

24.1

-
- 24.1 Gear Measuring Tables / 24.1
 - 24.2 Trigonometric Functions / 24.20
 - 24.3 Involute Functions / 24.20
 - 24.4 Arc and Chord Data / 24.27
 - 24.5 Hardness Testing Data / 24.27
 - References / 24.30

Index follows Chapter 24

CHAPTER 2

GEAR TYPES AND NOMENCLATURE

Raymond J. Drago

*Associate Technical Fellow, Boeing Helicopters
Philadelphia, Pennsylvania*

Chief Engineer

*Drive Systems Technology, Inc.
Glen Mills, Pennsylvania*

Before we can discuss the different types of gears, we would do well to define just what we mean by a "gear." Perhaps the most succinct definition I have ever heard was provided by a loading dock worker who asked where he should deliver the "wheels with notches in them." In more technically correct terms, a *gear* is a toothed wheel that is usually, but not necessarily, round. The teeth may have any of an almost infinite variety of profiles. The purpose of gearing is to transmit motion and/or power from one shaft to another. This motion transfer may or may not be uniform, and may also be accompanied by changes in direction, speed, and shaft torque.

Unfortunately, even among gear specialists there is some ambiguity in the terms used to describe gears and gear-related parameters. In our treatment herein, to the extent possible, we follow the conventions recommended by the American Gear Manufacturers Association.¹

2.1 TYPES OF GEARS

The first part of this chapter covers the types of gears in common use. The basic nomenclature and formulas for each type of gear are covered in the second part of this chapter.

The text material for each type of gear gives some comments on where the gears are used, how the gears may be made, the possible efficiency, and occasional comments on lubrication of the gears. These comments should all be considered introductory. Much more specific information is given in the following chapters of this book. Also,

the references given in this book show the reader where more comprehensive data can be obtained.

2.1.1 Classifications

In general, gears may be divided into several broad classifications based on the arrangement of the axes of the gear pair. The most general type of gearing consists of a gear pair whose axes are neither perpendicular nor parallel and do not intersect (that is, they do not lie in the same plane). All other types are special cases of this basic form. In the ensuing sections we discuss most of the major gear types, generally in order of increasing complexity. We limit our discussion to specific gear types and do not include arrangements of these types.

Our discussions consider, qualitatively, the various aspects of many different types of gears. In order to provide an easy basis for comparing these many and varied gear types, Table 2.1 provides a broad-based comparison. It is by no means an exhaustive comparison, but it serves to provide a reasonable basis for preliminary design. In using this table, the reader must keep clearly in mind the fact that the data are "typical" or "nominal." Surely, any experienced gear technologist can point out specific cases that vary substantially in virtually every category.

Within each type of gear classification there exist many variations in actual tooth form. The involute or involute-based form is, at least for parallel axis gears, the most common tooth form; however, many other special forms exist. We do not discuss tooth forms, per se, in this section.

2.1.2 Parallel Axis Gears

The simplest types of gears are those that connect parallel shafts. They are generally relatively easy to manufacture and are capable of transmitting large amounts of power with high efficiency. Parallel axis gears transmit power with greater efficiency than any other type or form of gearing.

Spur Gears. The spur gear has teeth on the outside of a cylinder and the teeth are parallel to the axis of the cylinder. This simple type of gear is the most common type. Its volume of usage is the largest of all types.

The shape of the tooth is that of an involute form. There are, however, some notable exceptions. Precision mechanical clocks very often use cycloidal teeth since they have lower separating loads and generally operate more smoothly than involute gears and have less tendency to bind. The cycloidal form is not used for power gearing because such gears are difficult to manufacture, sensitive to small changes in center distance, and not as strong or as durable as their involute brothers.

Figure 2.1 shows a close-up view of the teeth on a set of spur gears having about a 5:1 ratio. The teeth are 20° involute tooth form. The pinion is made with more than a standard addendum and the gear addendum is shorter than standard. The whole depth is standard for high-strength gears. Note the large radius of curvature in the root fillet region. This reduces the bending stress due to a lower stress concentration factor. Also note that the pinion teeth look very sturdy. This is an effect of the long- and short-addendum design just mentioned. (Details of specific design strategy are covered

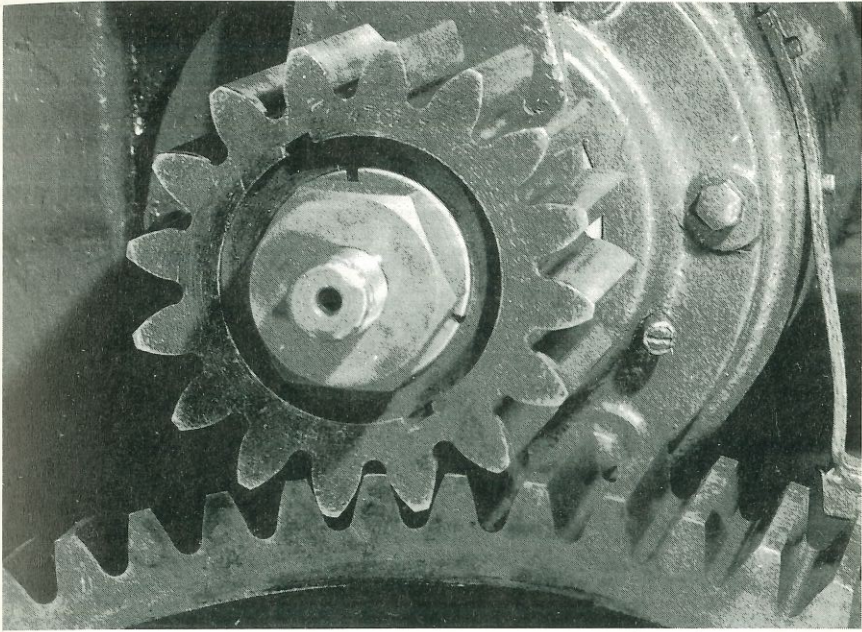


FIGURE 2.1 16-tooth spur pinion driving a gear with about 75 teeth. This is a special design of 20° involute teeth for high load-carrying capacity.

in later chapters of this book.)* Figure 2.2 shows a set of spur gears in an accessory gear box.

The most common pressure angles used for spur gears are $14\frac{1}{2}$, 20, and 25°. In general, the $14\frac{1}{2}$ ° pressure angle is not used for new designs (and has, in fact, been withdrawn as an AGMA standard tooth form); however, it is used for special designs and for some replacement gears. Lower pressure angles have the advantage of smoother and quieter tooth action because of the large profile contact ratio. In addition, lower loads are imposed on the support bearings because of a decreased radial load component; however, the tangential load component remains unchanged with pressure angle. The problem of undercutting associated with small numbers of pinion teeth is more severe with the lower pressure angle. Lower pressure angle gears also have lower bending strength and surface durability ratings and operate with higher sliding velocities (which contributes to their relatively poor scoring and wear performance characteristics) than their higher pressure angle counterparts.

Higher pressure angles have the advantage of better load-carrying capacity, with respect to both strength and durability, and lower sliding velocities (thus better scoring and wear performance characteristics). In some cases, very high pressure angles, 28, 30, and, in a few cases, as high as 45°, are employed in some special slow-speed gears for very high load capacity where noise is not the predominant consideration.

A minimum of equipment is required to produce this type of gear; thus it is usually the least expensive of all forms of gearing. While the most common tooth form for spur

*Drago³ and Dudley⁴ (specific books on gear design) give design strategy details beyond the scope of this book.

TABLE 2.1 General Comparison of Gear Types

Type of gear	Approx. range of efficiencies	Max.* widths generally used†	Type of load imposed on support bearings	Nominal range of reduction ratio‡	Nominal max. pitch line velocity, fpm		Member of pair	Methods of manufacture	Methods of refining
					Aircraft and high precision	Commercial			
Parallel Axes									
External spur gears	97-99.5	$F = d$	Radial	1:1-5:1	20,000	4,000	Both	Hob, shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis only), hone
Internal spur gears	97-99.5	Dictated by mating gear	Radial	1:5-7:1	20,000	4,000		Shape, mill, broach, stamp, sinter	Grind, shave, lap (crossed axis), hone
External helical gears	97-99.5	$F = d$	Radial and thrust	1:1-10:1	40,000	4,000	Both	Hob, shape, mill	Grind, shave, lap, hone
Internal helical gears	97-99.5	Dictated by mating gear	Radial and thrust	1:5:1-10:1	20,000	4,000		Shape, mill	Shave, grind, lap, hone
Internal herringbone or double helical	97-99.5	Dictated by mating gear	Radial	2:1-20:1	20,000	4,000		Shape	Lap, shave, hone
External herringbone or double-helical gear	97-99.5	$F = 2d$	Radial	1:1-20:1	40,000	4,000	Both	Hob, shape, mill	Grind, shave, lap, hone
Intersecting Axes									
Straight bevel gear	97-99.5	$\frac{1}{2}$ cone distance	Radial and thrust	1:1-8:1	10,000	1,000	Both	Generating, forming	Grind
Zerol bevel gear	97-99.5	28 % of cone distance	Radial and thrust	1:1-8:1	10,000	1,000	Both Gear	Generating Forming	Grind
Spiral bevel gear	97-99.5	$\frac{1}{2}$ cone distance	Radial and thrust	1:1-8:1	25,000	4,000	Both Gear	Generating Forming	Grind, lap
Face gear	95-99.5	From $0.2d$ at a low ratio to d at a high ratio	Radial and thrust	3:1-8:1	5,000	4,000	Pinion Gear	Same as external spur Shape	Lap, grind, shave, hone Lap
Beveloid	95-99.5	$\frac{5}{p_n}$	Radial and thrust	1:1-8:1	5,000	4,000	Both	Hob, generating	Lap, grind

Nonintersecting Nonparallel Axes

Crossed helical	50-95	$F_1 = 4p_n \sin \psi_1$ $F_2 = 4p_n \sin \psi_2$	Radial and thrust	1:1-100:1	10,000	4,000	Both	Hob, shape, mill	Grind, lap, shave
Cylindrical worm	50-90	$F_w = 5p_n \cos \lambda$ $F_g = 0.67d$	Radial and thrust	3:1-100:1	10,000	5,000	Pinion Gear	Mill, hob Hob	Grind Lap
Double-enveloping worm	50-98	$F_w = 0.9D$ $F_g = 0.9d$	Radial and thrust	3:1-100:1	10,000	4,000	Worm Wheel	Shape, hob Hob, mill	Lap, grind
Hypoid	90-98	$F_g = \frac{1}{3}$ cone distance	Radial and thrust	1:1-10:1	10,000	4,000	Both Gear	Generating Forming	Grind, lap
High-reduction hypoid	50-90	$F_g = 0.15D$	Radial and thrust	10:1-50:1	10,000	4,000	Pinion Gear	Generating Forming	Grind, lap Grind, lap
Spiroid	50-97	$F_p = 0.24D$ $F_g = 0.14D$	Radial and thrust	9:1-100:1	10,000	6,000	Pinion Gear	Mill, hob Hob, mold	Grind, chase
Planoid	90-98	$F_g = \frac{1}{3}$ cone distance	Radial and thrust	1.5:1-10:1	10,000	4,000	Pinion Gear	Hob Mill, broach	Lap, grind Grind
Helicon	50-98	$F_p = 0.21D$ $F_g = 0.12D$	Radial and thrust	3:1-100:1	10,000	6,000	Pinion Gear	Mill, hob Hob, mold	Grind, chase
Face gear	95-99.5	From 0.2D at a low ratio to D at a high ratio $\frac{5}{p_n}$	Radial and thrust	3:1-8:1	10,000	4,000	Pinion	Same as external spur	Same as external spur
Beveloid	50-95		Radial and thrust	1:1-100:1	10,000	4,000	Both Gear	Shape Generation, hob	Lap Grind, lap

* Face width given is only a "nominal" maximum. Consult other parts of the Gear Handbook for detail limitations on face width.

† d = pinion pitch diameter

D = gear pitch diameter

λ = lead angle

ψ = helix angle

p_n = normal circular pitch

†† The gear types showing an upper ratio of 100:1 can be built to 300:1 or higher. The ratio of 100:1 is shown as a normal maximum limit.

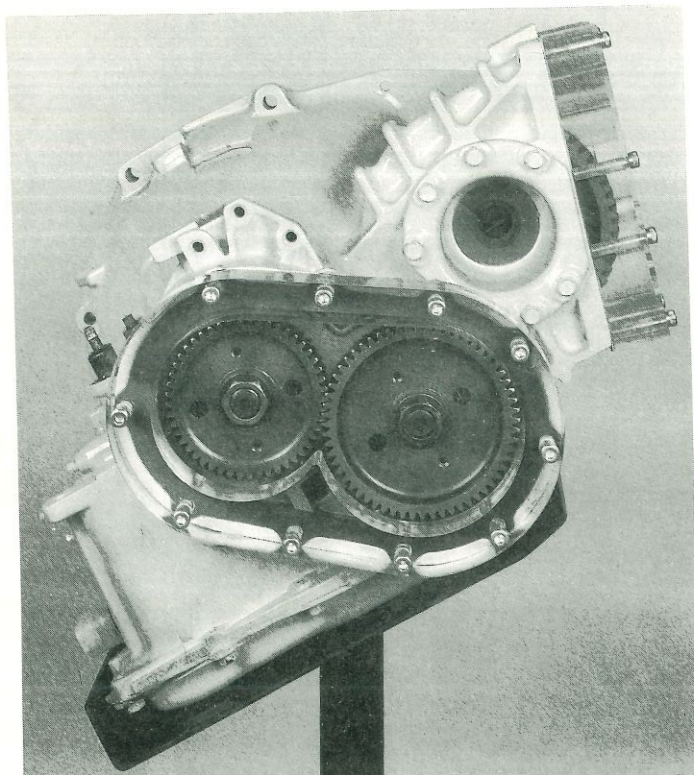


FIGURE 2.2 With the cover removed, a set of high-speed spur gears can be seen in an accessory box. Courtesy of the Gleason Works, Rochester, N.Y.

gears is the involute, other tooth forms are possible as long as they provide conjugate motion.

Helical Gears. When gear teeth are cut on a spiral that wraps around a cylinder, they are designated as helical. Helical teeth enter the meshing zone progressively and, therefore, have a smoother action than spur gear teeth and tend to be quieter. In addition, the load transmitted may be somewhat larger, or the life of the gears may be greater for the same loading, than with an equivalent pair of spur gears. Conversely, in some cases smaller-size helical gears (compared with spur gears) may be used to transmit the same loading. Helical gears produce an end thrust along the axis of the shafts in addition to separating and tangential (driving) loads of spur gears. Where suitable means can be provided to take this thrust, such as thrust collars or ball or tapered roller bearings, it is no great disadvantage. The efficiency of a helical gear set, which is dependent on the total normal tooth load (as well as the sliding velocity and friction coefficient, etc.), will usually be slightly lower than for an equivalent spur gear set.

Conceptually, helical gears may be thought of as stepped spur gears in which the size of the step becomes infinitely smaller. For external parallel axis helical gears to mesh, they must have the same helix angle but be of different hand. Just the opposite is true for an internal helical mesh; that is, the external pinion and the internal gear must have the same hand of helix. For crossed axis helical gears (discussed later in this section), the helix angles on the pinion and gear may or may not be of the same hand, depending on the direction of rotation desired and the magnitude of their individual helix angles.

Involute profiles are usually employed for helical gears and the same comments made earlier about spur gears hold true for helicals.

In order to provide significant improvement in noise over spur gears, the face overlap must be at least unity. That is, one end of the gear face must be advanced at least one circular pitch from the other. If the face overlap (also called face contact ratio) is less than unity, the helical gear, for most analytical purposes (that is, stress calculations) is treated as if it were a spur gear with no advantage taken of the fact that it is helical.

Helix angles from only a few degrees up to about 45° are practical. As the helix angle increases from zero, in general, the noise level is reduced and the load capacity is increased. At angles much above 15 to 20° , however, the tooth bending capacity generally begins to drop off. This is due to the fact that the transverse tooth thickness decreases rapidly.

Double-helical or herringbone gears are frequently used to obtain the noise benefits of single-helical gears without the disadvantage of thrust loading. The common types of helical gears are shown in Fig. 2.3, while Fig. 2.4 shows a typical single-helical set. The terms *double helical* and *herringbone* are sometimes used interchangeably. Actu-

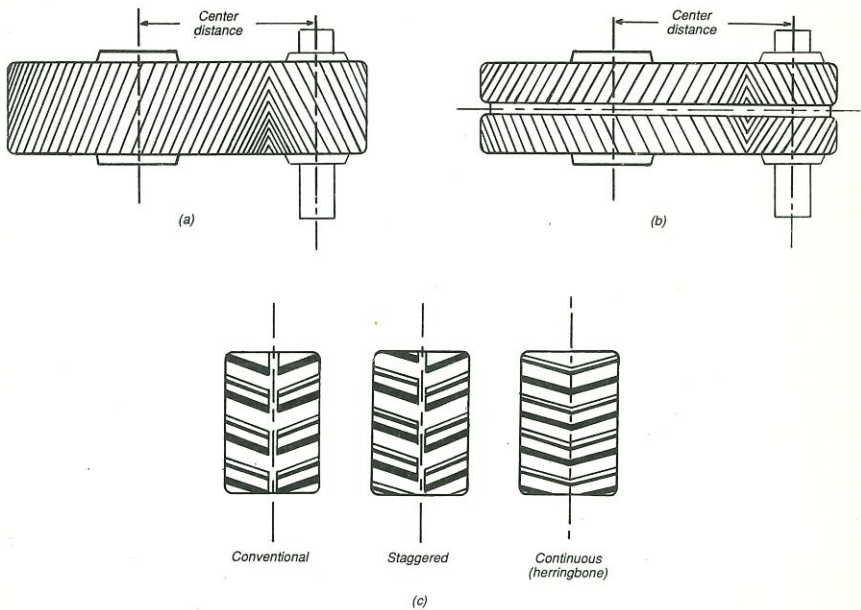


FIGURE 2.3 Types of helical-gear teeth. (a) Single helical; (b) double helical; (c) types of double helical.

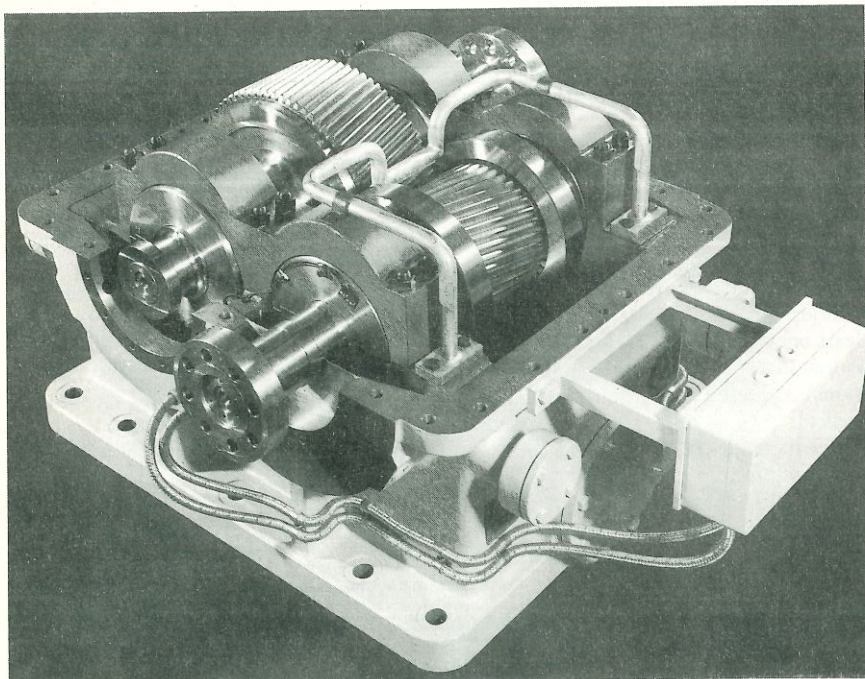


FIGURE 2.4 Single-helical-gear set designed for high speed and high horsepower. Courtesy of Maag Gear Wheel Company, Zurich, Switzerland.

ally, herringbone is more correctly used to describe double-helical teeth that are cut continuously into a solid blank, as shown in Fig. 2.3c. Double-helical gears may be cut integral with the blank, either in line or staggered, or two separate parts may be assembled to form the double.

While double-helical or herringbone gears do eliminate the net thrust load on the shaft, it is important to note that the two halves of the gear must internally react to the full thrust load. This being the case, the design of the gear blank for one-piece gears, or the construction of the assembly, must be carefully evaluated to insure that it is strong enough to take the thrust loads and to provide enough rigidity so that the deflections are not excessive.

The efficiency of helical gears is quite good though not quite equal to that of simple spurs. This is due to the fact that all other things being equal, the normal (or total) tooth load on a helical gear is higher than that on a spur for an equivalent tangential load.

Internal Gears. The internal gear has teeth on the inside of a cylinder. The teeth may be made either spur or helical. The teeth of an involute form internal gear have a concave shape rather than a convex shape. Internal gears are generally more efficient since the sliding velocity along the profile is lower than for an equivalent external set. Because of the concave nature of the internal tooth profile, its base is thicker than an

equivalent external gear tooth (either spur or helical). The tooth strength of an internal gear is greater than that of an equivalent external gear.

The internal gear has other advantages. It operates at a closer center distance with its mating pinion than do external gears of the same size. This permits a more compact design. The internal gear eliminates the use of an idler gear when it is necessary to have two parallel shafts rotate in the same direction. The internal gear forms its own guard over the meshing gear teeth. This is highly desirable for preventing accidents in some kinds of machines.

Internal gears cannot be used where the number of teeth in the pinion is almost the same as that of the gear. When this occurs, the tips of the pinion teeth interfere with the tips of the gear teeth. While a good guide is to maintain a ratio of 2:1 between the number of teeth on the internal gear and its mating pinion, lower ratios may be practical in some cases, particularly if the tooth utilizes shifted involute design characteristics. For full-depth 20°-pressure-angle teeth, for example, the internal gear pitch diameter must be at least one and one-half times that of the external gear; smaller ratios require considerable modification of the tooth shape to prevent interference as the teeth enter and leave the mating gear. Internal gears also have the disadvantage that few machine tools can produce them.

It may be more difficult to support a pinion in mesh with an internal gear because the pinion must usually be overhung mounted. Internal gears tend to find their greatest application in various forms of epicyclic gear systems and in large slewing-type drives, generally with integral bearings. In such configurations, the internal gear is frequently much larger than its mating external pinion, and thus it is usually possible to build a carrier structure inside the internal gear so that the pinion can be straddle mounted.

A typical internal gear arrangement is shown in Fig. 2.5, while Fig. 2.6 shows a large internal spur gear. In the case of an internal helical gear, its hand of helix must be the same as its mating pinion; that is, both the external helical pinion and its mating helical internal gear must be either right- or left-handed.

2.1.3 Nonparallel, Coplanar Gears (Intersecting Axes)

While the involute is the tooth form of almost universal choice for parallel axis gears, most gears that operate on nonparallel, coplanar axes do not employ involute profiles. A wide variety of gear types fall into this category. The most obvious use for gears of

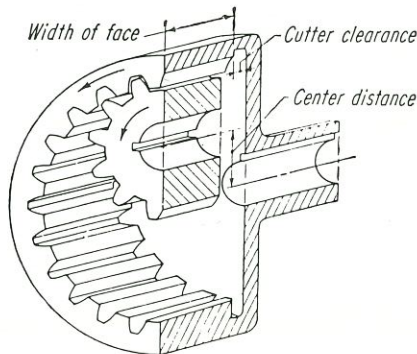


FIGURE 2.5 Typical internal-gear arrangement.

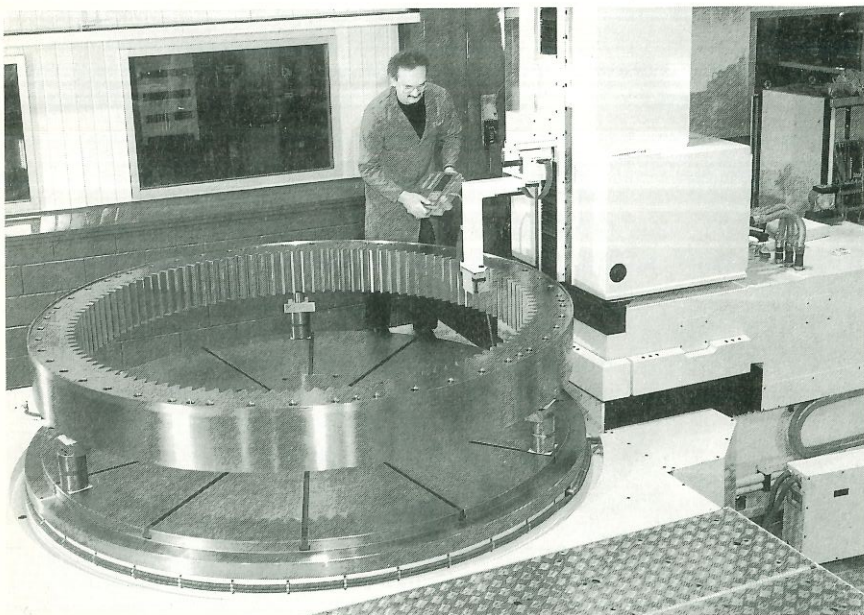


FIGURE 2.6 90-inch-diameter-internal-ring gear for an epicyclic speed reducer being measured on a 200-inch-diameter Maag gear checker. (Courtesy of Philadelphia Gear Corp., King of Prussia, Pa.)

this type is the redirection of power flow around a corner, such as might be required, for example, when connecting a horizontally mounted turbine engine to the vertically mounted rotor shaft on a helicopter. In fact, while power flow is a frequent reason for choosing one of these gear types, other features unique to a particular type of gear, such as the ability of straight bevel gears to be used in a differential arrangement for a rear-wheel-drive automobile, might well be the determining factor in making the decision to use one of these gear types. Descriptions of the major, and some interesting minor, ones follow.

Bevel Gears. There are four basic types of bevel gears: straight, Zerol,* spiral, and skew tooth. In addition, there are three different manufacturing methods (face milling, face hobbing, and tapered hobbing) that are employed to produce true curved tooth bevel gears, each of which produces a different tooth geometry. In our discussion herein we address the overall generic attributes of the different types of bevel gears without addressing the manufacturing variations.

All bevel gears impose both thrust and radial loads in addition to the transmitted tangential loads on their support bearings. The thrust is a result of the tapered nature of the gear blank, regardless of the tooth form employed (that is, even straight bevel gears produce an axial thrust). The diametral pitch for all bevel gears is conventionally measured at the heel of the tooth, while the pressure and spiral angles are conventionally measured at the mean section (tooth midface). In all cases, the pitch apex of the pinion and that of its mating gear must intersect at the point at which the axes of the

*Zerol is a registered trademark of the Gleason Works, Rochester, New York.

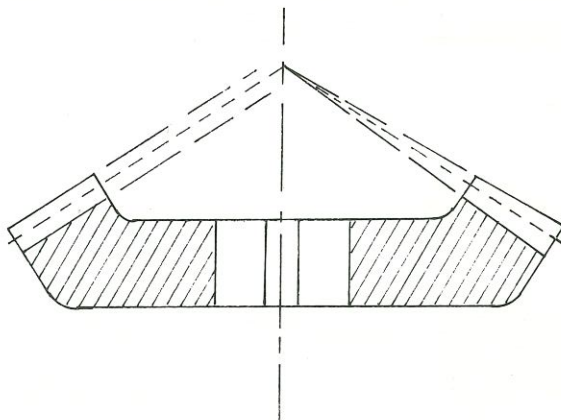


FIGURE 2.7 Basic bevel-gear configuration. Teeth may be tapered in depth or they may be made with a parallel depth, depending on manufacturing system used.

pinion and gear shafts intersect. Figure 2.7 shows the basic bevel gear configuration that is common to all types of bevel gears. By convention, the direction of rotation of a bevel gear is specified as clockwise or counterclockwise as viewed from the heel end of the gear looking toward the pitch apex.

Straight. The simplest form of the bevel gear is the straight bevel gear. Except for the thrust load produced by the blank taper, straight bevel gear teeth are a three-dimensional analogy to spur gear teeth. One very common use for straight bevel gears is in a bevel gear differential. Extensions of the straight teeth intersect at the axis of the gear. The tooth profile in a section normal to the tooth approximates that of an involute spur gear having a number of teeth equal to the actual number of teeth in the bevel gear divided by the cosine of the pitch angle (equivalent number of teeth). The teeth are always tapered in thickness and may have either constant or tapering height. The outer or heel part of the tooth is larger than the inner part called the toe. See Fig. 2.8. Although straight bevel gears have instantaneous line contact, they are often manufactured to have localized contact to permit more tolerance in mounting. One method of achieving this type of contact is the Coniflex* system, employed by Gleason Works.

Zerol. An improvement over straight bevel gears in terms of contact conditions, noise level, and power capacity is the Zerol bevel gear (Fig. 2.9). This gear is similar to a straight bevel except that the teeth are curved along their axis; however, the mean spiral angle is zero,[†] thus the bearing reaction loads (particularly the thrust loads) are the same as for straight bevels. Zerol gears are manufactured with the same type of cutters and on the same machines as spiral bevel gears. This is economically important as it eliminates the need for more than one type of bevel gear cutting equipment.

*Coniflex is a registered trademark of the Gleason Works, Rochester, New York.

[†]Actually, the term *Zerol* applies to any "spiral" bevel gear with a spiral angle of 10° or less, but most often Zerol bevels have a zero mean spiral angle.

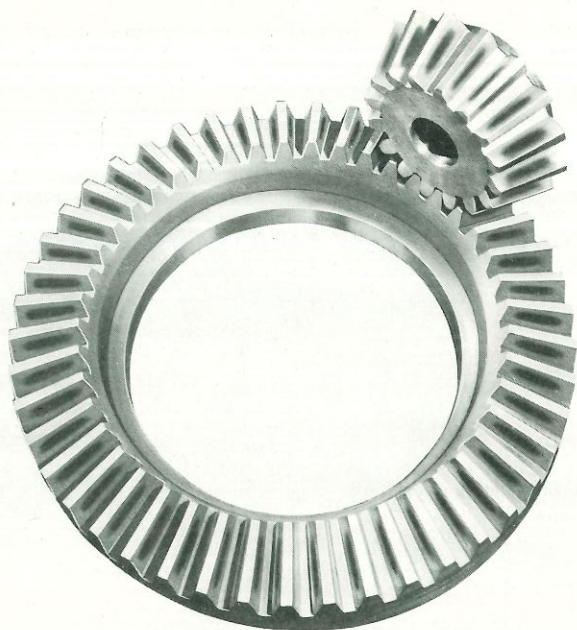


FIGURE 2.8 Straight-bevel gears. (Courtesy of the Gleason Works, Rochester, N.Y.)

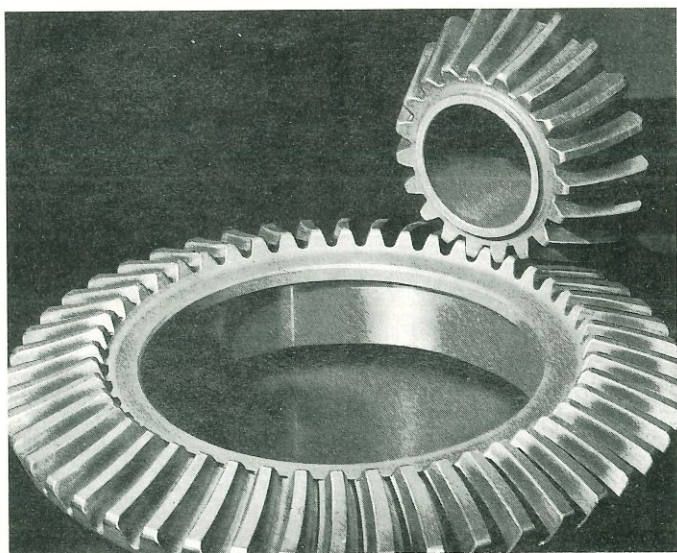


FIGURE 2.9 Zerol-bevel gears. (Courtesy of the Gleason Works, Rochester, N.Y.)

Localized tooth bearing of Zerol gears is accomplished by lengthwise mismatch (face curvature mismatch) and profile modification. The face width of a Zerol bevel gear is restricted to specific values due to cutter limitations. Zerol bevel gears are very frequently used in turbine engine, helicopter, and other high-speed devices such as accessory drives.

Zerol bevel gears may be compared to double helical or herringbone parallel axis gear teeth in that they have no more thrust load than their straight counterparts but provide advantages related to the improved contact ratio due to their lengthwise curvature. A pressure angle of 20° is the most common; however, Zerols may be manufactured with $14\frac{1}{2}$, $22\frac{1}{2}$, or 25° as well. Because Zerols are not generally used in very-high-load applications, the use of high pressure angles, which tend to reduce contact ratio and thus increase noise, should be carefully considered.

Spiral. The most complex form of bevel gear is the spiral bevel. It is commonly used in applications that require high load capacity at higher operating speeds than are typically possible with either straight or Zerol bevel gears. The relationship between spiral and straight bevel gears is comparable to that between helical and spur gears. By convention, the hand of spiral is defined as either left- or right-hand as determined by viewing the gear from the apex end looking toward the heel end.

The teeth of spiral bevel gears are curved and oblique (Fig. 2.10), and as a result they have a considerable amount of overlap. This insures more than one tooth in contact at all times and results in gradual engagement and disengagement with continuous multiple-tooth contact. This results in higher overall contact ratios than are possible with either straight or Zerol bevel gears. Because of this improved contact ratio and the resultant load sharing, spiral bevels have better load-carrying capacity and run more smoothly and quietly than either straight or Zerol bevel gears of the same size. The improved load capacity of spiral bevels also allows them to be smaller in size for a given load capacity than an equivalent straight or Zerol bevel gear.

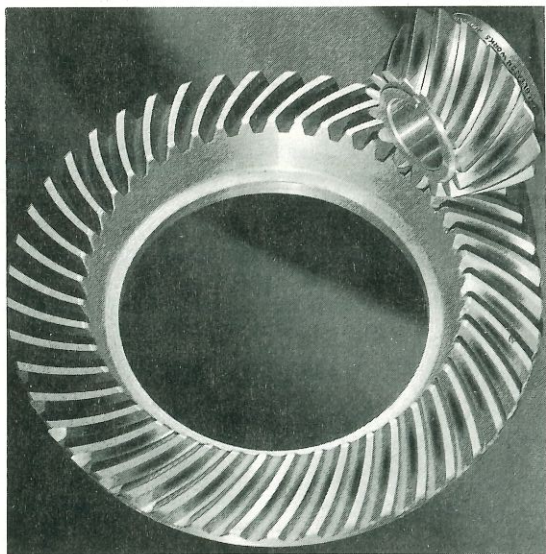


FIGURE 2.10 Spiral-bevel gears. (Courtesy of the Gleason Works, Rochester, N.Y.)

Spiral bevel gears impose significantly more thrust load on their support bearings than do either Zerol or straight bevel gears. While plain thrust bearings have been used successfully, a rolling element thrust bearing is usually required with spiral bevel gears. By comparison, plain bearings are quite often used with straight or Zerol bevel gears. The more common pressure angles for spiral bevel gears are 16 and 20°, with the latter now being almost standard. The most common spiral angle in use is 35°. In more highly loaded applications, particularly those that operate at high speeds as well, such as aircraft or helicopter transmissions, a higher pressure angle (typically 20½ to 25°) and lower spiral angles (typically 15 to 25°) are more often used.

The profile contact ratio on all bevel gears is usually substantially lower than equivalent spur gears; thus they tend to be somewhat noisier. In addition, due to their higher total load for a given tangential load and the higher sliding that accompanies the localized contact, they are less efficient than either spur or helical gears. Still, the efficiency of a well made set of bevel gears is generally better than 98 percent, and that of hardened and ground sets is usually above 99.2 percent.

Spiral bevel gears can be made by any of three generic methods. Each of these three methods produces a unique lengthwise tooth curvature and profile. In general, while gears within one system may be interchangeable, and gear sets made by each system are setwise interchangeable, pinions and gears are not interchangeable between systems, despite the fact that the basic configuration (that is, diametral pitch, spiral angle, pressure angle, etc.) may be identical.

1. Circular lengthwise tooth curvature: For this system, developed in the United States, face-milling cutters with multiple blades are used. While several methods of implementing this process exist, they are all somewhat similar in their basic motions. The blank executes a rolling motion relative to the cutter, which simulates one tooth of the imaginary crown gear. This is repeated for every tooth space, the cutter being withdrawn and turned back into starting position every time while the job is indexed for the next gap. This process is known as single indexing.

All tooth spaces have first to undergo a roughing operation and are finished by a subsequent operation. In general, the root lines of the teeth are not parallel to the pitch line, but set at a specific root angle. The cutters must be set parallel to the root lines while producing the gears; that is, they must be tilted relative to the pitch line. This causes the normal pressure angle to change, so that the pressure angles of the cutters must be corrected accordingly. This correction depends on the root angle and on the spiral angle and is different for either tooth side. In operation, the concave tooth surfaces of one member are always in mesh with the convex tooth side of the other. This system is used by Gleason machines and is the only one for which grinding machines have been developed.

2. Involute lengthwise tooth curvature: This system was originally developed in Germany. The tooth is a tapered hob, usually single-thread, with a constant pitch determined in accordance with the normal diametral pitch. The machining process is by continuous indexing: Tool and job rotate continuously and uniformly, the relationship between the two rotary motions being established by change gears as a function of the number of teeth being cut. The job is adjusted with its cone surface tangential to the surface of the imaginary flat gear.

This system is utilized by some Klingelnberg machines but is rapidly being replaced by the more versatile face-milling and face-hobbing processes.

3. Epicycloidal lengthwise tooth curvature: This system, originally evolved in Italy and England but still not free from some shortcomings, has been further developed in Switzerland and Germany. The machines available for producing this type of spiral bevel gear, also known as cyclo-palloid tooth form bevels, are similar to the one for the

production of circular tooth length curves in that the tool is also a face-type milling cutter with inserted blades, which can be of the inside-cutting, outside-cutting, and gashing type, alternatively. The indexing is now, however, not intermittent but continuous. The cutter and the blank rotate continuously and uniformly, so that not only is one tooth space generated at a time, but all tooth spaces consecutively. Accordingly, the blades are not evenly distributed over the periphery of the cutter, but combined in groups (that is, the cutter has multiple starts). One group generally comprises one roughing, one outside-cutting, and one inside-cutting blade, and allows simultaneous machining of the tooth bottom and both tooth sides. The number of blade groups depends on the size of the cutter and usually varies from 3 to 11. This basic system is utilized by both Oerlikon and Klingelnberg, with some variation on each.

Skew. The final type of bevel gear is the skew tooth. This is similar to a spiral bevel gear and is quite often also referred to (incorrectly) as a spiral bevel gear. Actually, the skew tooth has no lengthwise curvature; rather the teeth are simply cut straight but at an angle to the shaft centerline (as shown in Fig. 2.11) so that some face overlap may occur. This provides an improvement in load capacity (Fig. 2.12) when compared with a straight bevel gear, but not to the level possible with a true spiral (curved tooth) bevel gear. Skew tooth gears are used primarily in large (over 30-in. gear pitch diameter) sizes only. They are produced on planing generator machines.

Until the late 1980s, because of limitations in the size of the face-milling machines available, most "spiral" bevel gears over 30 to 40 in. in diameter were planed; thus they were actually skew tooth, not true spiral bevel gears. Since pinion and gear must be conjugate, the pinions that mate with these large gears must also be planed. Relatively recent advances in spiral bevel gear manufacturing have led to the development of large machines of both the face-milling and face-hobbing type that can cut true spiral bevel gear teeth on gears over 100 in. in pitch diameter.

Face Gears (On-Center). A face gear set is actually composed of a spur or helical pinion that is in mesh with a "face" gear. The pinions are really not any different from their parallel axis counterparts except for the fact that they are in mesh with a face gear. Face gears have teeth cut into the blank such that the axis of the teeth lie in a plane

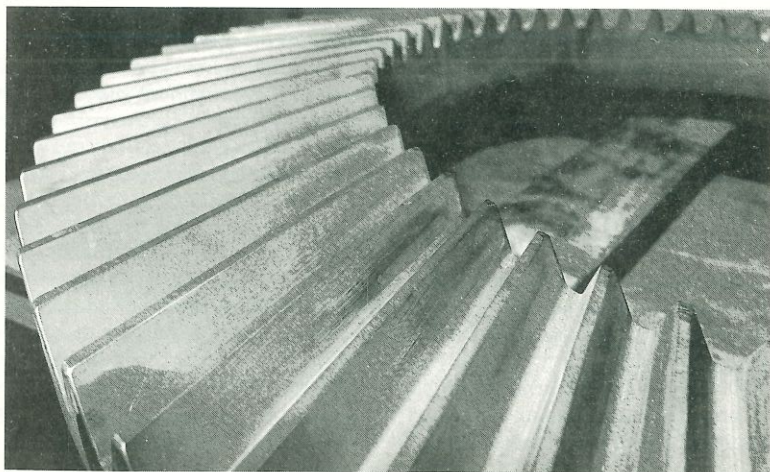


FIGURE 2.11 Skew-bevel gear.

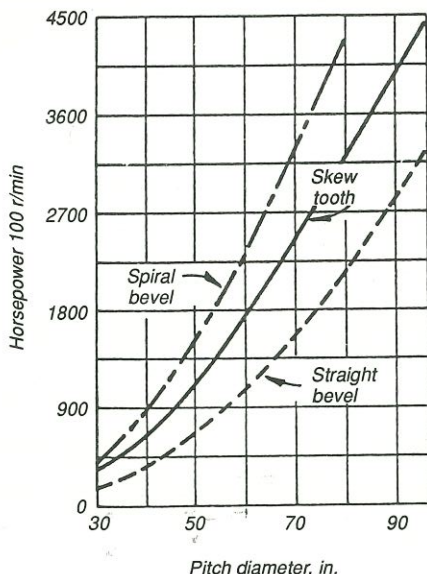


FIGURE 2.12 Comparison of large-straight skew and spiral-bevel-gear capacities. (Data supplied by Philadelphia Gear Corp., King of Prussia, Pa.)

that is perpendicular to the shaft axis. The mating pinion is either a spur or a helical gear. The pinion and gear axes are coplanar for on-center face gears. The pinion and face gear axes most often form a 90° shaft angle; however, small deviations from 90° are possible. In operation, this type of gear is similar to an equivalent set of straight bevel gears.

The face gear tooth changes shape from one end of the tooth to the other. The load capacity of face gears, compared with that of bevel gears, is rather small; thus they are used mostly for motion transmission rather than as power gears. Face gears are, relative to bevel gears, easy to make and somewhat less expensive as well. A typical arrangement is shown in Fig. 2.13. Toys and games make extensive use of face gears.

Conical Involute Gearing. Conical involute gearing is a completely generalized form of involute gear. It is an involute gear (as shown in Fig. 2.14) with tapered tooth thickness, root, and outside diameter. Commonly known as Beveloid* gears, they are useful primarily for precision instrument drives where the combination of high precision and limited load-carrying ability fits the application. Like other involute gears, Beveloids are relatively insensitive to positional errors although shaft angles must be quite accurate. They will mate conjugately with all other involute gears (spurs, worms, helicals, and racks). Like crossed axis helicals (see later discussion), however, the area

*Beveloid is a registered trademark of the Invincible Gear Co., Livonia, Michigan.

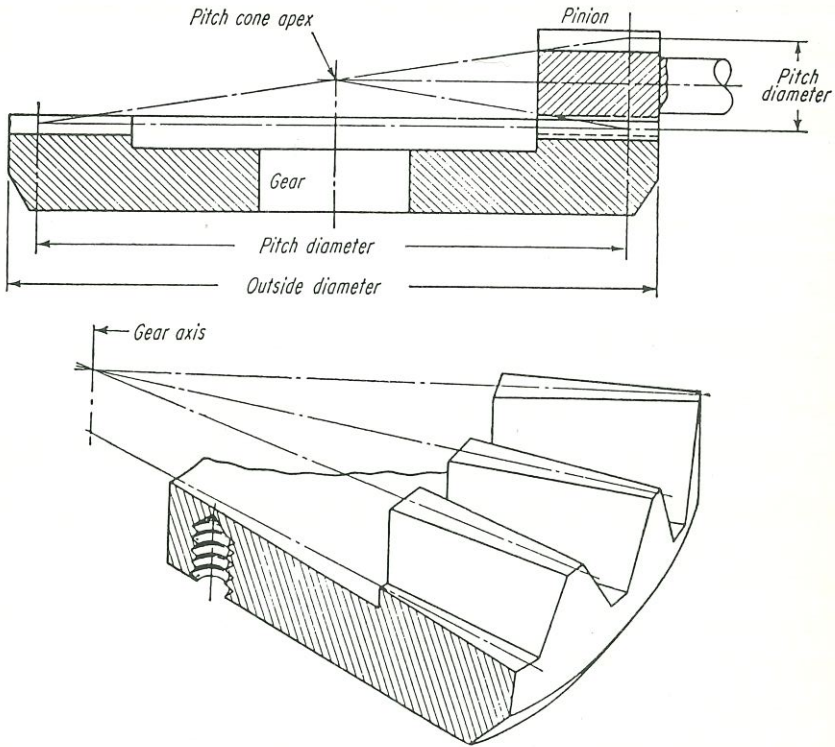


FIGURE 2.13 Face-gear arrangement.

of contact between these gears and Beveloids is limited; thus load capacity is low. For small tooth numbers and large cone angles, undercutting is quite severe.

This type of gear is not in widespread use at this time, but may be used to advantage in some circumstances, especially for precision motion-transmission applications in which the gear shafts must be located at varying orientations in space.

2.1.4 Nonparallel, Noncoplanar Gears (Nonintersecting Axes)

Gears in this classification are generally the most complex, both in terms of geom-

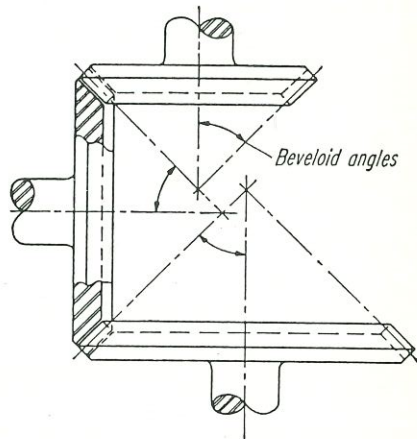


FIGURE 2.14 Beveloid-gear arrangement.

etry and manufacture. The simpler types, discussed first in the following discussion, are, however, quite easy to manufacture and are reasonably inexpensive, but they do not carry large loads. The more complex types are generally more expensive but provide better load capacity and other features that make them especially suited for a wide variety of special applications.

Crossed Axis Helicals. Crossed helical gears are satisfactory for the normal range of ratios used for single reduction helical gears. They provide both speed reduction and extreme versatility of shaft positioning at a relatively low initial cost. At higher ratios, or for anything above moderate loads, however, worm, Spiroid, or Helicon gears are generally preferable. (See Fig. 2.15 for typical crossed axis helical gears.)

Crossed axis helical gears are usually cut in the same manner as conventional helical gears using identical tooling, since they are, until mounted on their crossed axes, actually nothing more than conventional helical gears! Since crossed helical gears have a great deal of sliding action, special attention must be given to the selection of gear materials and their lubricants to reduce friction to a minimum and eliminate any possibility of seizing between mating gears. Experience has indicated that iron on iron functions satisfactorily. However, a hardened steel pinion driving a bronze gear will also work quite well.

Figure 2.16 shows some of the special relations that must exist for this type of gearing to function properly. It is interesting to note that, while mating parallel axis external helical gears must have opposite hands of helix, crossed axis helicals can have

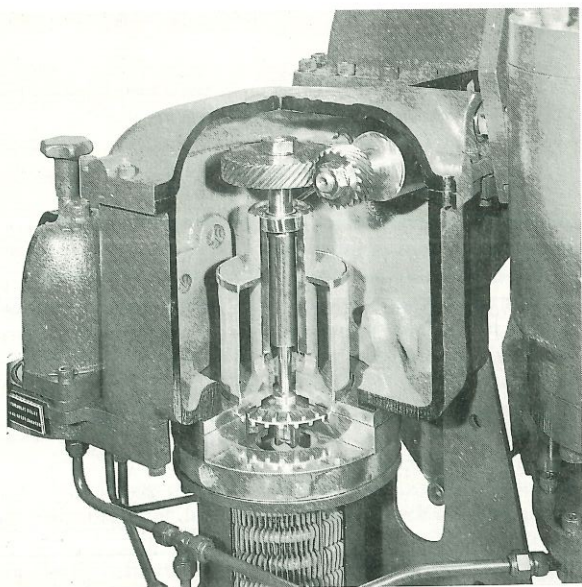


FIGURE 2.15 Crossed-axis-helical-gear set, driving a pump and governor for a steam turbine application. (Courtesy of the General Electric Co., Lynn, Mass.)

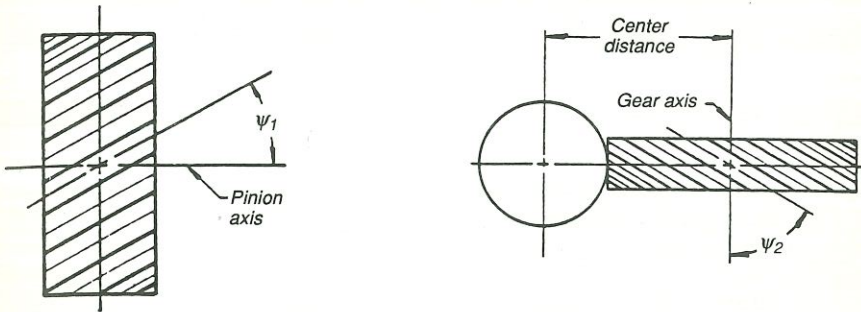


FIGURE 2.16 Crossed-axis-helical-gear arrangement.

either the same or opposite hands of helix, depending on the shaft angle and the relative directions of rotation of the driver and driven gears. Additionally, the reduction ratio between the pinion and the gear is a function of their relative numbers of teeth but not directly of their pitch diameters. This provides a great deal of flexibility in choosing the ratio, center distance, and diameters of the gears.

Finally, because the relative contact between the mating gears is the theoretical intersection of two cylinders, the load capacity of the gears is not directly influenced by the face width; that is, once the face width is extended such that it covers the full cylindrical intersection, extending either or both face widths farther is of no practical value since the extra face width will not be in contact.

Cylindrical Worm Gearing. The most basic form of worm gearing is a straight cylindrical worm in mesh with a simple helical gear. In reality, this is really an extreme case of crossed axis helical gearing. Such gears can provide considerably higher reduction ratios than simple crossed axis gear sets, but their load capacity is low, and the wear rate is high. For light loads, however, this configuration can be an economical alternative.

Single-Enveloping Worm Gearing. Better load capacity can be achieved if the simple helical gear is modified such that it is throated to allow the worm to fit down farther into the gear to achieve greater tooth contact area and thus smoother operation and improved load capacity. This common worm gear set is often referred to as single-enveloping since the gear envelops the worm but the worm remains straight. While many variations of this configuration are used, in the most common case the worm is essentially straight-sided while the gear or wheel is generally involute in form. The contact point is theoretically a line varying in length up to full face width of the gear with different tooth designs. Under load, this line becomes a thin elliptical band of contact. See Fig. 2.17 for a two-stage worm gear drive.

Basic arrangement views are shown in Fig. 2.18. The hand of helix for both members is the same. While the worm is generally made of steel, the wheel is usually made of either cast iron or, more frequently, one of several types of bronze. The worm teeth may be through hardened, but they are often, especially in higher-load and higher-speed applications, case hardened and ground. In some cases, however, to improve accuracy and efficiency, the worm teeth are ground even though they are not hardened after cutting. The wheel, due to the high sliding conditions that exist in this type of gearing, is usually made of cast iron or bronze, as noted in the foregoing; however, in some high-temperature environments it is necessary to make the wheel from steel as well.

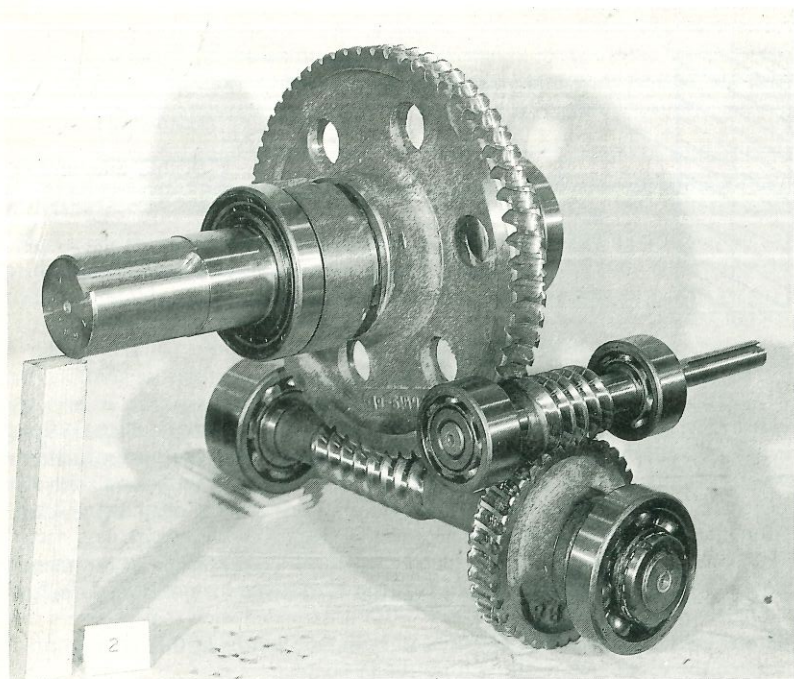


FIGURE 2.17 Two stages of cylindrical-worm gears. (Courtesy of Hamilton Gear and Machine Co., Ltd., Hamilton, Canada)

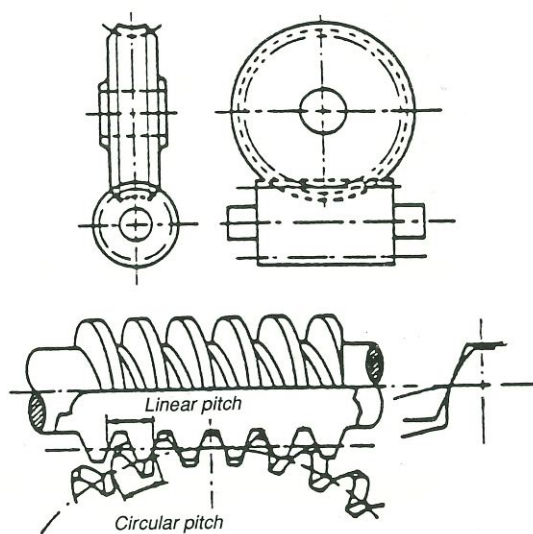


FIGURE 2.18 Schematic of cylindrical-worm gears.

By virtue of their inherent high contact ratio, the mechanical power rating of worm gears is quite high; however, in practice, their actual continuous-duty rating is substantially lower. This is due to high heat generation that can raise the lubricant temperature to unacceptable levels when the box is operated continuously. Fan-cooled worm boxes are quite common, and higher-power-capacity worm housings are almost always finned to aid heat dissipation. This large difference between the thermal and mechanical ratings of the typical worm gear drive gives rise to a peculiar property associated with worm drives: that is, their apparent ability to sustain relatively high short-term overloads without experiencing any damage. In reality, worm gears do not have a particularly good overload capability; rather, their thermal limitations cause them to be operated at loads below their mechanical limits. When operated for short periods of time at "overload," they are actually operating above their continuous-duty thermal limits but below their mechanical (stress-related) ratings; thus, since it takes an appreciable period of time for the temperature to rise, they sustain these short-term "overloads" quite well.

The advent of synthetic fluids has been a boon for worm drives of all types for two reasons. First, synthetic fluids have the ability to operate at higher bulk temperatures than the compounded mineral-based fluids that are commonly used for worm gears. Second, the friction coefficient associated with the use of synthetic fluids tends to be somewhat lower than that associated with compounded worm gear oils; thus less heating is produced. These factors combine to decrease the margin between the thermal and mechanical limits of newer work gear sets designed and rated to run with synthetic fluids; thus their apparent overload capability is reduced.

Worm gear efficiency is quite dependent on operating speed. The same set may show an efficiency of, say, 75 percent at a low speed and 85 percent at a higher speed. Ratio, material, accuracy, and geometric design all affect worm gear efficiency. Typical efficiencies run from 35 to 90 percent, with higher or lower values occurring in special cases.

A worm set can be used where irreversibility is desired, since, if the lead angle is less than the friction angle, the wheel cannot drive the worm. Usually, worms with lead angles less than 5° are self-locking. Care should be exercised when designing self-locking worms, since this feature is a static one. Vibration can cause the set to slip under dynamic conditions (that is, during a cutoff of power under load, the wheel, due to the inertia of the driving load, may overdrive the worm for a considerable time). Similarly, a worm set that, when stationary, cannot be driven through the gear shaft, may well begin to rotate if the unit is subjected to vibrations. The property of self-locking or irreversibility is better thought of as anti-back-driving rather than positive irreversibility. In critical applications, a brake on the worm should also be provided to insure that the system will positively not back-drive.

When the gears are assembled into their housing, the position of the gear along its axis must be adjusted such that an acceptable contact pattern is obtained on the bench. This contact pattern should favor the leaving side of the mesh so that a wedge is formed at the entering side of the mesh. This wedge at the entering side will cause lubricant to be drawn into the contact zone and thus minimize wear.

Double-Enveloping Worm Gearing. The capacity of a single-enveloping worm gear set as described in the foregoing discussion is improved by allowing the gear or wheel to envelop the worm. A further improvement in capacity may be achieved by allowing the worm to envelop the wheel as well. Such drives are known as double-enveloping. Double-enveloping worm gear drives, by getting more teeth into contact, tend to provide higher load capacity than do cylindrical or single-enveloping worm sets. This

is accomplished by changing the shape of the worm (as shown in Fig. 2.19) from a cylinder to an hourglass.

All of the comments made earlier pertaining to cylindrical worms also apply to double-enveloping worms. Because of the shape of the worm, clearly shown in Fig. 2.20, this type of worm gearing is more expensive to produce; but where weight or size are considerations, the cost differential is relatively small.

The forms of the worm and the gear in the double-enveloping drive design are regenerative; that is, the worm and gear tend to reproduce each other in use. This is true of no other worm and gear design. This condition aids proper break-in and contact pattern development. Since the worm and wheel envelop each other, assembly is not as straightforward with double-enveloping gear sets as it is with a single-enveloping set. In general, the worm and wheel must be assembled obliquely, and thus provisions must be made in the design of the housing to accommodate this requirement. This can often be accomplished by splitting the housing such that the worm is one part and the wheel is the other half. In addition, the position of the worm along its axis and the position of the gear along its axis must be adjusted simultaneously such that an acceptable pattern is obtained. This bench patterning procedure is in contrast to the simpler procedure required of a single-enveloping set in which only the position of the gear along its axis need be adjusted at assembly to obtain an acceptable contact pattern at assembly.

Hypoid Gears. Hypoid gears (Fig. 2.21) resemble spiral bevel gears except that the teeth are asymmetrical; that is, the pressure angle on each side of the tooth is different. Many of the same machines used to manufacture spiral bevel gears can also be used to manufacture hypoids.

The pitch surfaces of hypoid gears are hyperboloids of revolution. The teeth in mesh have line contact; however, under load, these lines spread to become elliptical regions of contact inclined across the face width of the teeth. One condition that must exist if a hypoid gear set is to have conjugate action is that the normal pitch of both members must be the same. The number of teeth in a gear and pinion are not, however, directly proportional to the ratio of their pitch diameters. This makes it possible to make large pinions while minimizing the size of the driven gear. This is one of the most attractive features of hypoid gearing. High reduction ratios with small offsets must usually utilize an overhung-mounted pinion. Frequently, however, the pinion must be straddle mounted (Fig. 2.22) if sufficient offset exists. Aside from space considerations, the tradeoff usually centers on efficiency, since efficiency generally decreases with increasing offset.

In operation, hypoid gears are usually smoother and quieter than spiral bevel gears due to their inherent higher total contact ratio. However, as in all cases of nonintersecting gear sets, high sliding takes place across the face of the teeth. The efficiency of hypoid gears is thus much less than that of a similar set of spiral bevel gears, typically 90 to 95 percent as compared with over 99 percent for many spiral bevel gears. Hypoids do, however, generally have greater tolerance to shock loading and can frequently be used at much higher single-stage ratios than spiral bevel gears.

Spiroid and Helicon Gearing.* Like most other skew axis concepts, Spiroid and Helicon are primarily screw action gears while, by comparison, spur, helical, and bevel gears are primarily rolling action gears.

*Spiroid and Helicon are registered trademarks of Illinois Tool Works, Chicago, Illinois.

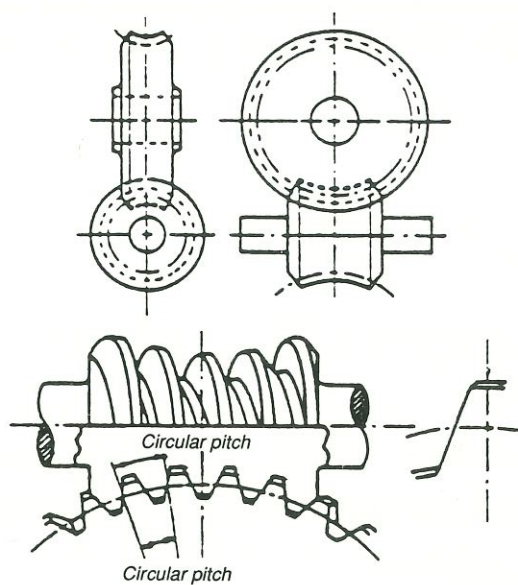


FIGURE 2.19 Schematic of double-enveloping-worm gears.

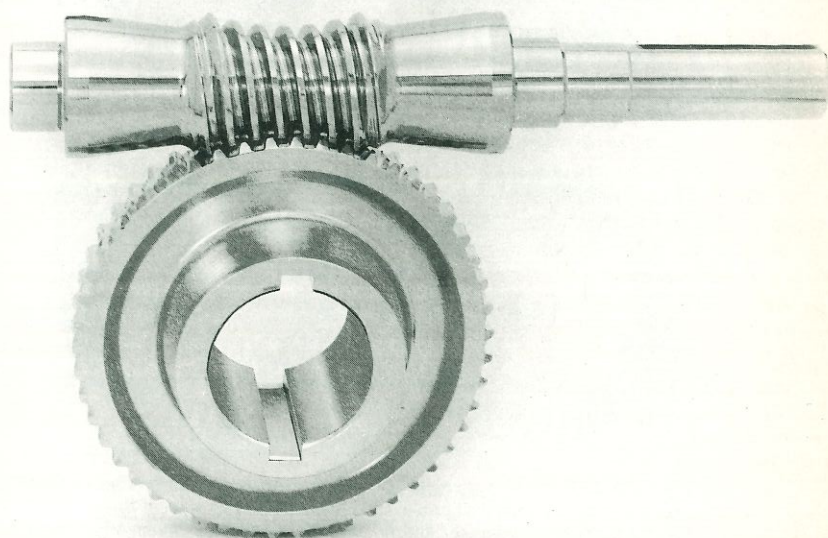


FIGURE 2.20 Double-enveloping-worm-gear set.

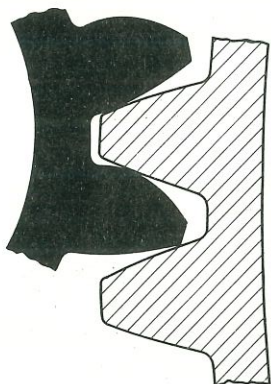
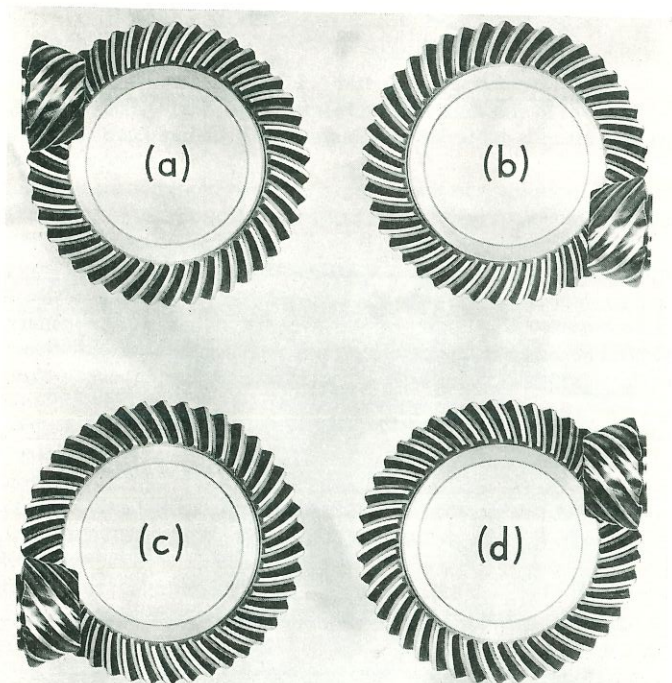


FIGURE 2.21 Hypoid-gear data. (1) Hypoid tooth profile showing unequal pressure angles and unequal profile curvatures on the two sides of the tooth. (2) Hypoid gears and pinions (a) and (b) are referenced to having an offset below center, while those in (c) and (d) have an offset above center. In determining the direction of offset, it is customary to look at the gear with the pinion at the right. For below-center offset, the pinion has a left-hand spiral, and for above-center offset the pinion has a right-hand spiral.

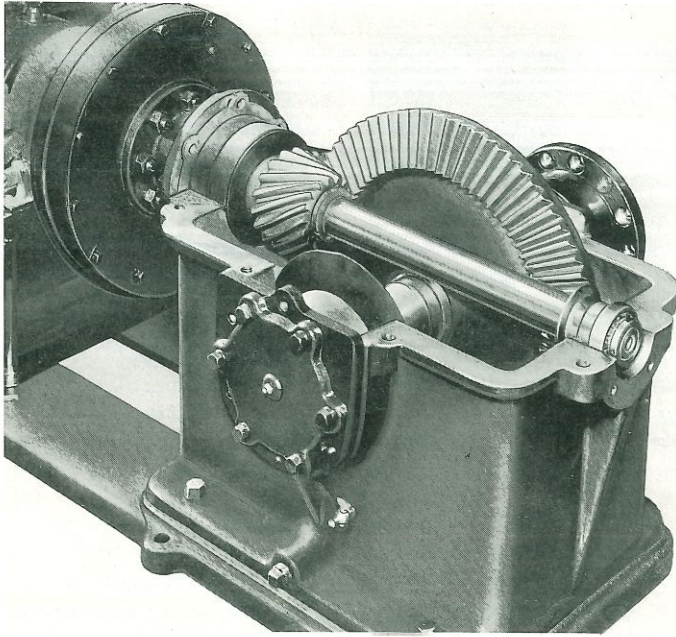


FIGURE 2.22 Hypoid-gear set. Both pinion and gear are straddle mounted. (*Courtesy of the Gleason Works, Rochester, N.Y.*)

The gear member of either a Spiroid (Fig. 2.23), or Helicon (Fig. 2.24), set resembles a high-pitch-angle bevel gear, while the pinions are more akin to worms. Both forms have more teeth in contact than an equivalent size worm set. The primary difference between Spiroid and Helicon gears is their minimum ratio capabilities [about 10:1 for Spiroid and 4:1 for Helicon (although lower ratios are practical if powdered metal fabrication is employed)] and the range of acceptable offsets. Spiroid pinions (Fig. 2.23) are typically tapered by about 5 or 10° on a side, while Helicon pinions (Fig. 2.24) are cylindrical. Pinions of both types, used in lower ratio designs (that is, less than 30:1), typically utilize multiple threads so that the number of teeth in the gear may be maintained at least at 30.

This type of gearing fits in between bevel and worm gears (Fig. 2.25) in terms of ratio, and generally provides better performance than do worms. They can, like worms, be designed to be anti-back-driving, but, again like worms, they cannot be designed to be entirely self-locking under all operating conditions, particularly where significant vibrations are present. They can also incorporate accurate control of backlash since the contact pattern is largely controlled by adjusting the pinion position, while the backlash is largely controlled by adjusting the position of the gear along its axis at assembly. These features make them a good choice for positioning drives such as radar or other antenna. These gear types are relatively insensitive to small position shifts; thus, in precision position-control applications, they are usually shimmed into intimate double flank contact to eliminate backlash.

The contact conditions are line, again like worm and hypoid gears, but the contact line is essentially radial; thus, the flow of oil into the contact zone is good, and efficiency

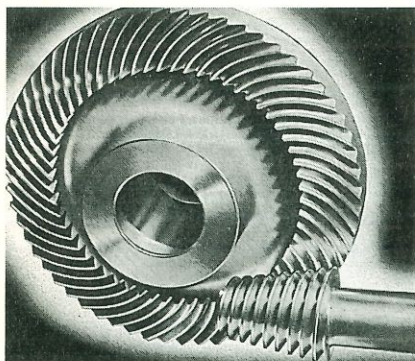


FIGURE 2.23 Spiroid-gear set. (Courtesy of Illinois Tool Works, Spiroid Division, Chicago, Ill.)

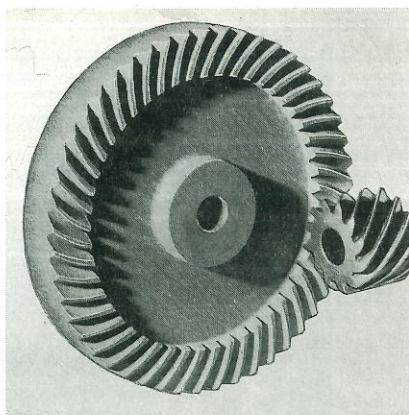


FIGURE 2.24 Helicon-gear set. (Courtesy of Illinois Tool Works, Spiroid Division, Chicago, Ill.)

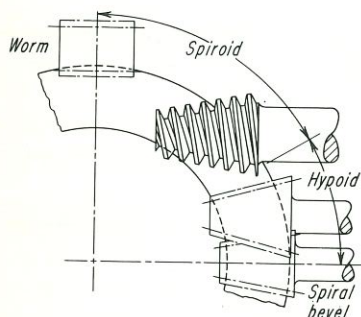


FIGURE 2.25 Schematic comparisons of worm gears, Spiroid gears, Helicon gears, hypoid gears, and spiral-bevel gears.

is improved over worms—usually about as good as hypoids. As with worms, the efficiency of a Spiroid or Helicon set varies with speed, but not to the large extent that occurs with worms.

One feature of Spiroids and Helicons that makes them both quiet and capable of great positional accuracy is the relatively high number of teeth in contact. The multiplicity of teeth in contact (for a typical design perhaps 10 percent of the gear teeth are in theoretical contact) provides an error averaging function so that individual tooth errors are not significant in terms of gear position.

Although generally less efficient than spur or helical gears, Spiroid or Helicon gears are usually superior to worm drives particularly at ratios less than 40:1, when compared on the basis of constant pinion size.

The offset required for a typical Spiroid gear set is about one-third the gear diameter, but small variations can also be accommodated. Similarly, the shaft angle is usually limited to 80 to 100°, with 90° being the standard in the vast majority of cases. In some special applications, shaft angles as low as 70° or as high as 120° are possible, but each case is a special design.

In general, the gears may be cut on standard hobbing machines. For a given configuration, a Spiroid gear set will have a greater load capacity than a Helicon set, but the Helicon position is somewhat easier to manufacture due to the cylindrical nature of its pinion as compared with the tapered shape of the Spiroid pinion.

Face Gears (Off-Center). If the pinion of a face gear set is offset such that its axis and that of the gear do not intersect, it is termed *off-center*. The discussion provided earlier

for on-center face gears applies equally well here. Such gear sets are not ordinarily used to transmit significant amounts of power; rather they are generally used in motion-transmission applications where uniformity of motion is not a critical factor.

2.1.5 Special Gear Types

Our discussion to this point would lead the reader to believe that all gears are in the form of surfaces of revolution (that is, cylinders and cones). Such is not the case. There are many applications in which gears are used to transmit motions that are intentionally nonuniform. There are few areas in which man's mechanical ingenuity can be employed more effectively than in the design of special-motion gears. The variety is almost endless, limited only by application and imagination. In order to provide some insight into the subject, we discuss some representative cases.

Square or Rectangular Gears. Consider the case in which a constant input speed is to be converted into a varying output speed. Gears that are square or rectangular, depending on the actual output required, can be used to produce such motion (as shown in Fig. 2.26). While the gears are in the position shown, the greatest radius of the driver mates with the smallest radius of the driven gear; the speed of the driven gear is, therefore, then at its maximum. As the gears revolve in the directions indicated by the arrows, the radius of the driver gradually decreases, and that of the driven gear gradually increases, until the points b and b' are in contact. The speed of the driven gear, therefore, gradually decreases during this eighth of a revolution. From the moment of contact between the points b and b' , the reverse action takes place, and the speed of the driven gear gradually increases until the points c and c' are in contact. Thus, during the entire revolution, the driven gear continues to alternate from a gradually decreasing to a gradually increasing speed, and vice versa.

For rectangular gears to work properly together, it is necessary first that the pitch peripheries of the two gears be equal in length, and, second, that the sum of the radii of each pair of points (points that come into contact with each other) on the two pitch peripheries be equal to the distance between the centers of the gears.

Triangular Gears. Figure 2.27 represents a pair of triangular gears, the object of which is to obtain an alternating, varying speed from the uniformly rotating driver, as in rectangular gears. Triangular gears give fewer changes of speed per revolution than do rectangular gears. In Fig. 2.27, the speed of the driven gear is at its minimum when

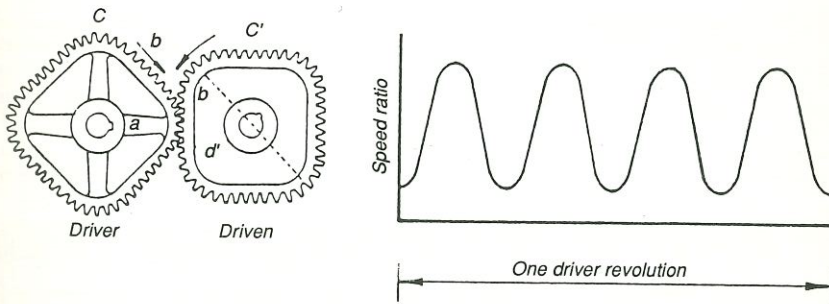


FIGURE 2.26 Square-gear characteristics.

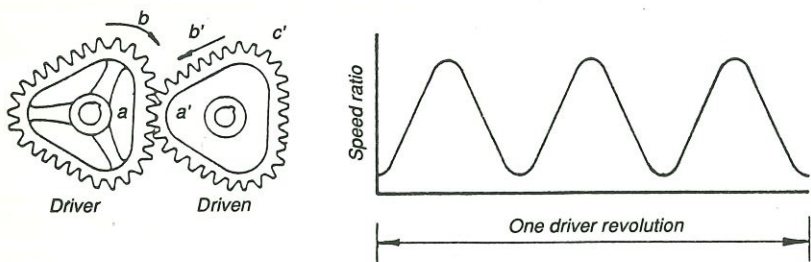


FIGURE 2.27 Triangular-gear characteristics.

the gears are in the positions shown in the figure. Since, from the positions, the radius of the driver gradually increases, and that of the driven gear decreases, as far as the points b and b' , the speed of the driven gear will gradually increase until the points b and b' are in contact—or for one-sixth of an entire revolution. The reverse action will then take place until the points c and c' are in contact, and so on. Thus, while in rectangular gears each gradually increasing or decreasing period takes place during one-eighth of a revolution, in triangular gears each of these periods occupies one-sixth of a revolution; that is, in rectangular gears there are eight alternatively increasing and decreasing periods in one entire revolution of the driven gear, and in triangular gearing there are but six.

Elliptical Gears. With elliptical gears (Fig. 2.28), we have still another means of obtaining the same result, with the difference that in elliptical gears each period of gradually increasing and decreasing speed takes place during one-fourth of a revolution. In other words, there are but four periods of increasing and decreasing speed during an entire revolution of the driver.

Scroll Gears. Figure 2.29 shows a pair of scroll gears. From the positions shown in the figure (in which the greatest radius of the driver meshes with the smallest radius of the driven gear), as the gears revolve in the directions indicated by the arrows, the radius of the driver gradually and uniformly decreases, while that of the driven gear gradually and uniformly increases. The speed of the driven gear is, therefore, at its

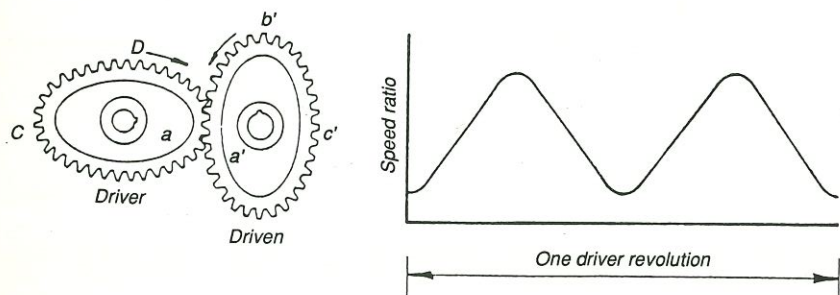


FIGURE 2.28 Elliptical-gear characteristics.

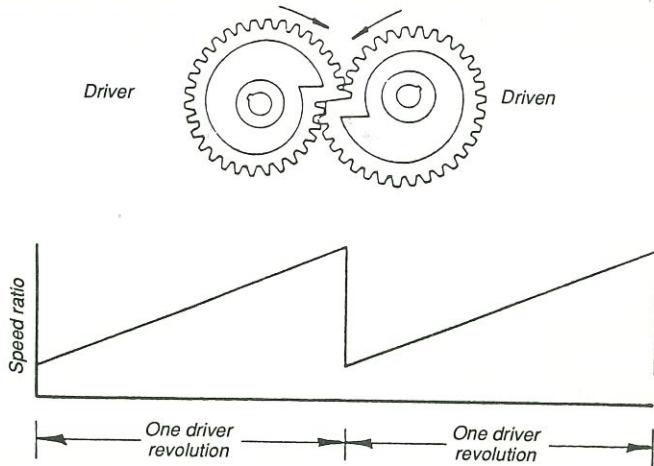


FIGURE 2.29 Scroll-gear characteristics.

maximum when the gears are in the positions shown, and gradually and uniformly decreases during the entire revolution. The moment before the positions shown in the figure are reached, the smallest radius of the driver gears with the greatest radius of the driven gear, the speed of the latter is at its minimum and suddenly (as the gears assume the positions in the figure) changes to its maximum.

Multiple-Sector Gears. The mechanism represented in Fig. 2.30 is a pair of multiple-sector gears, and the object is to obtain a series of discrete, different uniform speeds. In the figure, as long as the arcs ab and $a'b'$ are in mesh, the speed of the driven gear is the same. When the arcs cd and $c'd'$ come into mesh, the speed of the driven gear becomes slower, but remains the same throughout the meshing of these two arcs. Similarly, when the arcs ef and $e'f'$ come into mesh, the speed of the driven gear becomes still slower, but remains uniform during the meshing of these arcs. Thus, during each revolution, the driven gear has three periods of uniform speed, each differing from the others. For sector gears to work properly together, it is necessary that the arcs that mesh together be equal in length ($ab = a'b'$, $cd = c'd'$, etc.) and that the sum of the arc lengths on one gear be equal to the sum of the arc lengths on the other ($ab + cd + ef = a'b' + c'd' + e'f'$). Also, the sum of the radii of each two arcs together must be equal to the distance between the centers of the gears. Sector gears are somewhat difficult to construct because considerable care must be taken to insure that no two sectors of the driver gear mesh at the same time with the driven gear. To illustrate, suppose that the arcs ab and $a'b'$ mesh together at the same time as do the arcs ef and $e'f'$; that is, that the last few teeth of ab mesh with the driven gear at the moment when the first few teeth of ef do the same. The driven gear will then strive to drive the driven gear at its maximum and minimum speeds at the same time, an attempt that must obviously result in a fracture. In the figure, the arc cf ceases to mesh with the driven gear at the moment when the arc ab begins to gear. Thus, each arc of the driver must escape engagement just in time for its successor to begin engagement,

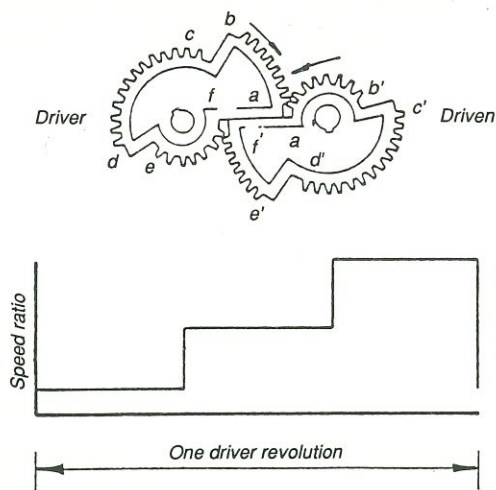


FIGURE 2.30 Multiple-sector-gear characteristics.

and yet leave between these events no appreciable interval to disturb the uniformity of motion.

In actuality, a discontinuous condition will exist for all sector-type gears at the time when different sectors exchange control of the speed of the driven member. These seemingly instantaneous periods of acceleration and deceleration limit the application of such gears to very low speeds lest the acceleration forces become exceedingly large.

2.2 NOMENCLATURE OF GEARS*

In this part of Chap. 2 the technical nomenclature of the various types of gears is given. Table 2.2 shows metric system symbols and English system symbols. The following tables (due to space limitations) show nomenclature tables in English symbols only.

2.2.1 Spur Gear Nomenclature and Basic Formulas

Figure 2.31 shows tooth elements that apply directly to spur gears. The same elements apply to other gear types.

Table 2.3 shows spur gear formulas. Note that the value of π may need to be used as 3.14159265 when very precise calculations are needed. Also, the determination of tooth thickness has complications of what backlash is needed and effects of long or short addendum. (These matters are covered later in this book.)

*Most of the material for this part of Chap. 2 was furnished by Darle W. Dudley, President of the Dudley Engineering Co., Inc., San Diego, California.

TABLE 2.2 Gear Terms, Symbols, and Units Used in the Calculation of Gear Dimensional Data

Term	Metric		English	
	Symbol	Units	Symbol	Units
Number of teeth, pinion	z_1	—	N_P or n	—
Number of teeth, gear	z_2	—	N_G or N	—
Number of threads, worm	z_1	—	N_W	—
Number of crown teeth	z	—	N_c	—
Tooth ratio	u	—	m_G	—
Addendum, pinion	h_{a1}	mm	a_P	in.
Addendum, gear	h_{a2}	mm	a_G	in.
Addendum, chordal	h_a	mm	a_c	in.
Rise of arc	—	mm	—	in.
Dedendum	h_f	mm	b	in.
Working depth	h'	mm	h_k	in.
Whole depth	h	mm	h_t	in.
Clearance	c	mm	c	in.
Tooth thickness	s	mm	t	in.
Arc tooth thickness, pinion	s_1	mm	t_P	in.
Arc tooth thickness, gear	s_2	mm	t_G	in.
Tooth thickness, chordal	$\bar{s}$	mm	t_c	in.
Backlash, transverse	j	mm	B	in.
Backlash, normal	j_n	mm	B_n	in.
Pitch diameter, pinion	d_{p1}	mm	d	in.
Pitch diameter, gear	d_{p2}	mm	D	in.
Pitch diameter, cutter	d_{p0}	mm	d_c	in.
Base diameter, pinion	d_{b1}	mm	d_b	in.
Base diameter, gear	d_{b2}	mm	D_b	in.
Outside diameter, pinion	d_{a1}	mm	d_o	in.
Outside diameter, gear	d_{a2}	mm	D_o	in.
Inside diameter, face gear	d_{i2}	mm	D_i	in.
Root diameter, pinion or worm	d_{f1}	mm	d_R	in.
Root diameter, gear	d_{f2}	mm	D_R	in.
Form diameter	d'_f	mm	d_f	in.
Limit diameter	d_ℓ	mm	d_ℓ	in.
Excess involute allowance	Δd_ℓ	mm	Δd_ℓ	in.
Ratio of diameters	ϵ	—	m	—
Center distance	a	mm	C	in.
Face width	b	mm	F	in.
Net face width	b'	mm	F_e	in.
Module, transverse	m or m_t	mm	—	—
Module, normal	m_n	mm	—	—
Diametral pitch, transverse	—	—	P_d or P_t	in. ⁻¹
Diametral pitch, normal	—	—	P_n	in. ⁻¹
Circular pitch	p	mm	p	in.
Circular pitch, transverse	p_t	mm	p_t	in.
Circular pitch, normal	p_n	mm	p_n	in.
Base pitch	p_b	mm	p_b	in.
Axial pitch	p_x	mm	p_x	in.
Lead (length)	p_z	mm	L	in.

TABLE 2.2 Gear Terms, Symbols, and Units (Continued)

Term	Metric		English	
	Symbol	Units	Symbol	Units
Pressure angle	α or α_t	deg	ϕ or ϕ_t	deg
Pressure angle, normal	α_n	deg	ϕ_n	deg
Pressure angle, axial	α_x	deg	ϕ_x	deg
Pressure angle of cutter	α_0	deg	ϕ_c	deg
Helix angle	β	deg	ψ	deg
Lead angle	γ	deg	λ	deg
Shaft angle	Σ	deg	Σ	deg
Roll angle	θ_r	deg	ϵ_r	deg
Pitch angle, pinion	δ'_1	deg	γ	deg
Pitch angle, gear	δ'_2	deg	Γ	deg
Pi	π	—	π	—
Contact ratio	ϵ_a	—	m_p	—
Zone of action	g_a	mm	Z	in.
Edge radius, tool	r_{a0}	mm	r_T	in.
Radius of curvature, root fillet	ρ_f	mm	ρ_f	in.
Circular thickness factor	k	—	k	—
Cone distance	R	mm	A	in.
Outer cone distance	R_a	mm	A_o	in.
Mean cone distance	R_m	mm	A_m	in.
Inner cone distance	R_f	mm	A_i	in.

NOTE: Abbreviations for units: mm = millimeters, in. = inches, deg = degrees.

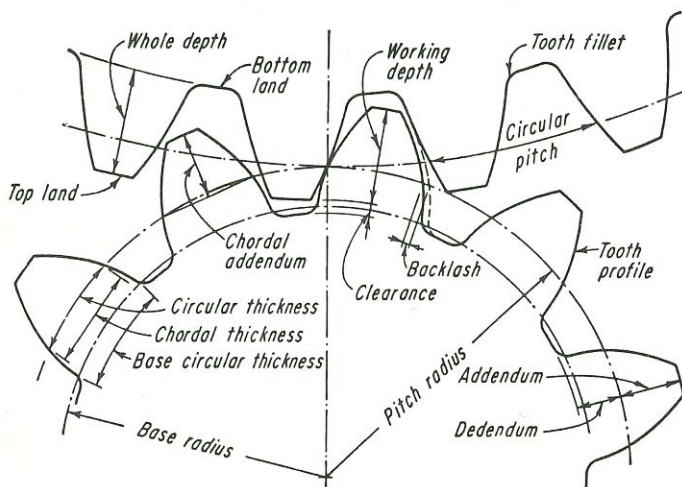

FIGURE 2.31 Spur-gear nomenclature.

TABLE 2.3 Spur Gear Formulas

To find	Having	Formula
Diametral pitch.....	Number of teeth and pitch diameter	$P = \frac{N}{D}$
Diametral pitch.....	Circular pitch	$P = \frac{3.1416^*}{p}$
Pitch diameter.....	Number of teeth and diametral pitch	$D = \frac{N}{P}$
Outside diameter.....	Pitch diameter and addendum	$D_o = D + 2a$
Root diameter.....	Outside diameter and whole depth	$D_R = D_o - 2h_t$
Root diameter.....	Pitch diameter and dedendum	$D_R = D - 2b$
Number of teeth.....	Pitch diameter and diametral pitch	$N = D \times P$
Base-circle diameter.....	Pitch diameter and pressure angle	$D_b = D \times \cos \phi$
Circular pitch.....	Pitch diameter and number of teeth	$p = \frac{3.1416D}{N}$
Circular pitch.....	Diametral pitch	$p = \frac{3.1416}{P}$
Center distance.....	Number of gear teeth and number of pinion teeth and diametral pitch	$C = \frac{N_g + N_P}{2P} = \frac{D_g + D_P}{2}$
Approximate thickness of tooth..	Diametral pitch	$t_t = \frac{1.5708}{P}$ (see design formulas, Chap. 5)

*Exact value of π is 3.14159265.

2.2.2 Helical Gear Nomenclature and Basic Formulas

Helical gear teeth spiral around a "base" cylinder. The angle of this spiral with respect to the axis is the helix angle. The advance of the spiral in going 360° around the base cylinder is a fixed quantity called the *lead*.

The spur gear tooth is parallel to the axis. This means that a spur gear can be thought of as a special case of a helical gear in which the numerical value of the lead is infinity!

Figure 2.32 shows the nomenclature of helical gear teeth. View *a* shows a section through the teeth that is perpendicular to the gear axis. This is known as the *transverse section*. View *b* shows how a "normal" section can be taken through a tooth perpendicular to a point on the pitch helix.

Both the transverse section and the normal section data must be used in helical gear calculations. Table 2.4 shows the formulas needed to calculate data for both of these sections.

A *rack* is a gear part in which the number of teeth in a 360° circle is infinite. Circular gears, of course, become straight when the radius of curvature is infinite.

Involute teeth become straight-sided for the rack tooth form. Figure 2.33 shows rack tooth parts for spur, helical, and straight bevel teeth. The helical rack shows very clearly the difference between the normal plane and the front plane (which corresponds to the transverse plane of circular gears).

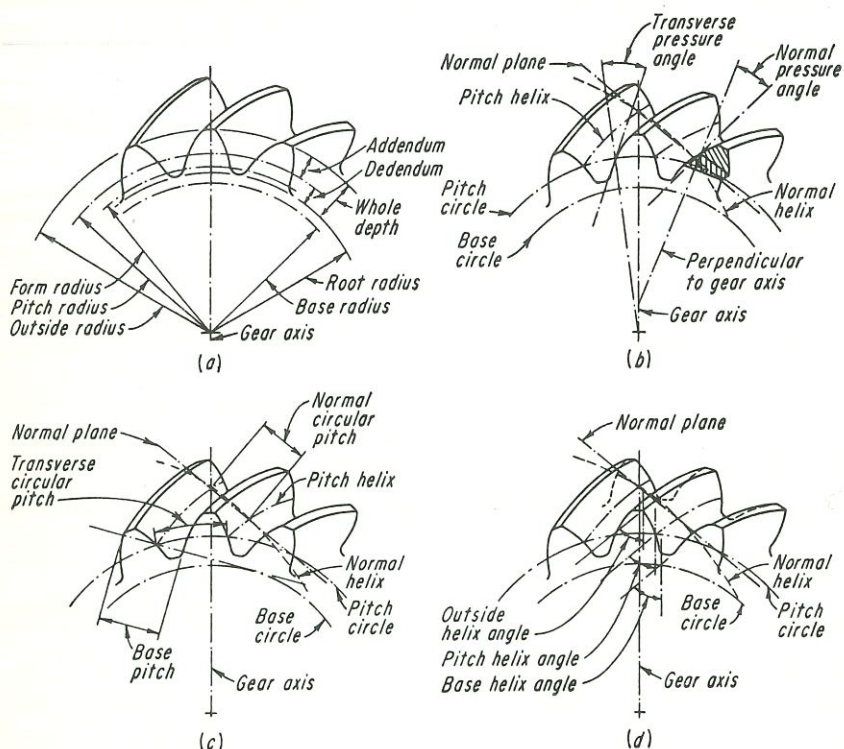


FIGURE 2.32 Helical-gear nomenclature.

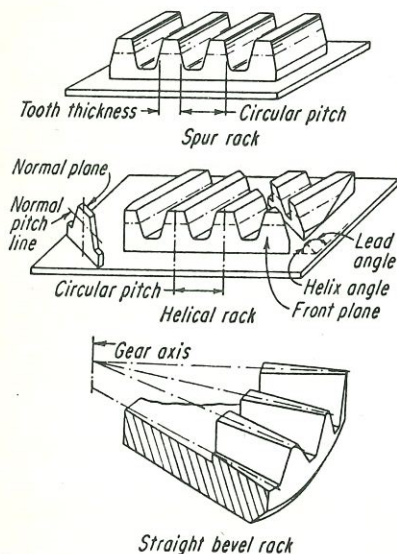


FIGURE 2.33 Spur-, helical-, and bevel-gear racks.

2.2.3 Internal Gear Nomenclature and Formulas

The internal gear has teeth on the *inside* of a ring rather than on the outside. An internal gear must mesh with a pinion having external teeth. (Two internal gears cannot mesh with each other.)

The nomenclature for the transverse section of an internal gear drive is shown in Fig. 2.34. Note that the internal tooth tends to have concave curves rather than convex curves. Also note that the tips of the teeth tend almost to hit each other as they enter and leave the meshing zone.

Formulas for internal gear dimensions are given in Table 2.5. Note that some of these formulas are similar to formulas in Table 2.3 except for a minus sign.

TABLE 2.4 Helical Gear Formulas

To find	Having	Formula
Normal diametral pitch.....	Number of teeth, pitch diameter, and helix angle	$P_n = \frac{N}{D \cos \psi}$
Normal diametral pitch.....	Normal circular pitch	$P_n = \frac{3.1416^*}{p_n}$
Normal diametral pitch.....	Transverse diametral pitch and helix angle	$P_n = \frac{P}{\cos \psi}$
Normal circular pitch.....	Normal diametral pitch	$p_n = \frac{3.1416}{P_n}$
Normal circular pitch.....	Transverse circular pitch	$p_n = p \cos \psi$
Pitch diameter.....	Number of teeth, normal diametral pitch, and helix angle	$D = \frac{N}{P_n \cos \psi}$
Center distance.....	Pinion and gear pitch diameter	$C = \frac{D_P + D_G}{2}$
Outside diameter.....	Pitch diameter and addendum	$D_o = D + 2a$
Approximate normal tooth thickness..	Normal diametral pitch	$t_n = \frac{1.571}{P_n}$ (see design formulas, Chap. 5)
Transverse tooth thickness.....	Normal tooth thickness and helix angle	$t_t = \frac{t_n}{\cos \psi}$
Normal pressure angle.....	Transverse pressure angle and helix angle	$\tan \phi_n = \tan \phi \cos \psi$

*Exact value of π is 3.14159265.

TABLE 2.5 Internal Spur Gear Formulas

To find	Having	Formula
Pitch diameter: Of gear.....	Number of teeth in gear and diametral pitch	$D = \frac{N_G}{P}$
Of pinion.....	Number of teeth in pinion and diametral pitch	$d = \frac{N_P}{P}$
Internal diameter (gear).....	Pitch diameter and addendum	$D_i = D - 2a$
Center distance.....	Number of teeth in gear and pinion; diametral pitch	$C = \frac{N_G - N_P}{2P}$
	Pitch diameter of gear and pinion	$C = \frac{D - d}{2}$

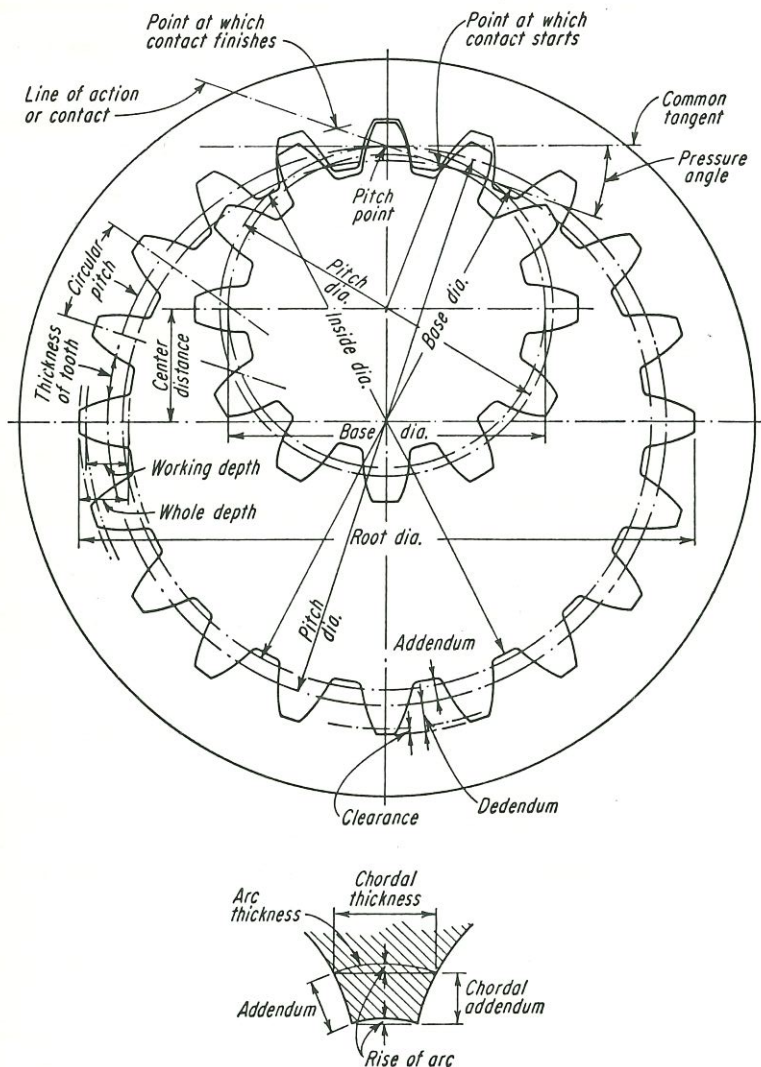


FIGURE 2.34 Internal-gear and pinion nomenclature.

Internal gears may be either spur or helical. If the teeth are helical, data for the teeth in the normal section may be determined by formulas in Table 2.4.

2.2.4 Crossed Helical Gear Nomenclature and Formulas

Since the axes of crossed helical gears are not parallel to each other and do not intersect, the two meshing gears tend to have different helix angles. (If the numerical amount of

each helix angle is the same, then the two gears cannot have opposite helix angles—because this would make the axes parallel instead of crossed.) Figure 2.35 shows standard nomenclature for the crossed axis situation with helical gears. The basic formulas are given in Table 2.6.

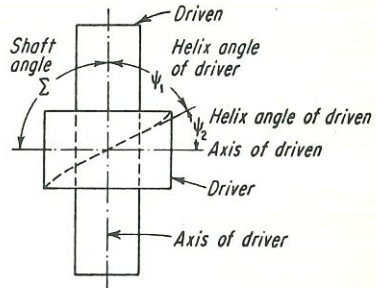


FIGURE 2.35 Crossed-helical-shaft angle and helix.

2.2.5 Bevel Gear Nomenclature and Formulas

Section 2.1.3 gave general information on gear types in the bevel gear family of gears. Nomenclature and formulas will now be given for three of these types that are in rather common usage.

Straight. Figure 2.36 shows section through of a pair of straight bevel gears in mesh. Note how the teeth are on the outside of cones. Dimensions are measured to a “crown point” that exists in space but may not exist in metal! (Usually this sharp corner is rounded off in the process of actual manufacture.)

The formulas for the straight bevel are given in Table 2.7. By tradition, the pitch diameters are taken at the large end of the tooth. This end also goes by names such as the *back face* or the *heel* end. (The other end of the tooth is termed the *front face* or *toe* end.)

Spiral. The spiral bevel gear has a curved shape (lengthwise). This curved shape is positioned at an angle to a pitch cone element. The angle is specified at the center of the face width (not the large end of the tooth where the pitch diameter is specified). Figure 2.37 shows the spiral angle.

The formulas for conventional spiral bevel gears are given in Table 2.8. Spiral bevel gears are often made to pressure angles other than 20° . They can also be made for shaft angles other than 90° . Beyond this, there is more than one system for the design and manufacture of bevel gears. Chapter 20 gives considerably more bevel gear data. The

TABLE 2.6 Crossed Helical Gear (Nonparallel Shaft, Helicals)

To find	Having	Formula
Shaft angle.....	Helix angle of pinion and helix angle of gear	$\Sigma = \psi_P + \psi_G$
Pitch diameter of pinion....	Number of teeth in pinion, normal diametral pitch, and helix angle of pinion	$D_P = \frac{N_P}{P_n \times \cos \psi_P}$
Pitch diameter of gear.....	Number of teeth in gear, normal diametral pitch and helix angle of gear	$D_G = \frac{N_G}{P_n \times \cos \psi_G}$
Center distance.....	Pitch diameter of pinion, and pitch diameter of gear	$C = \frac{D_P + D_G}{2}$
Speed ratio.....	Number of teeth in gear and number of teeth in pinion	$m_G = \frac{N_G}{N_P}$

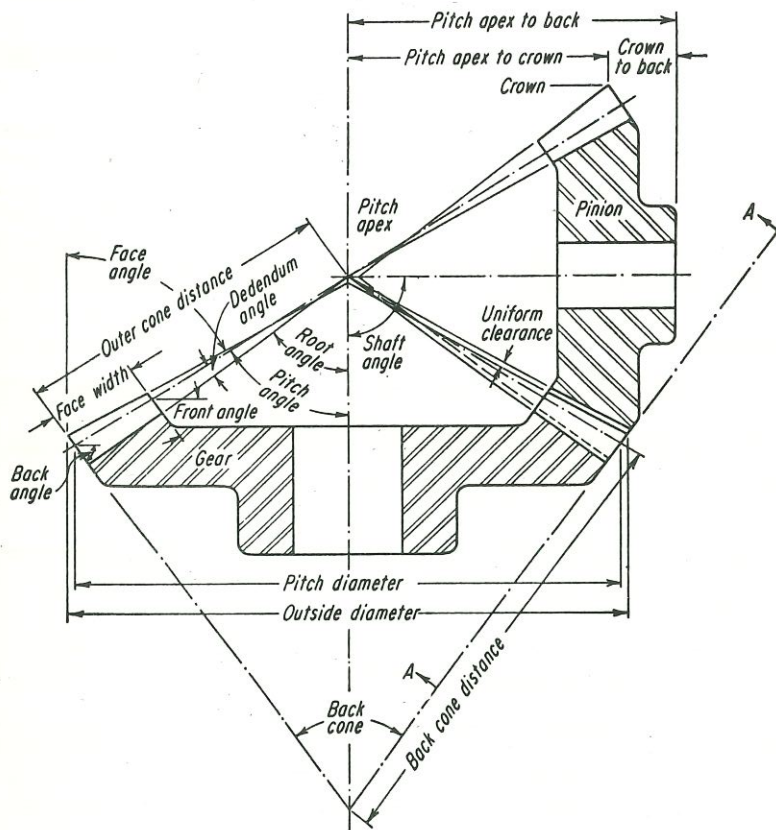


FIGURE 2.36 Bevel-gear nomenclature.

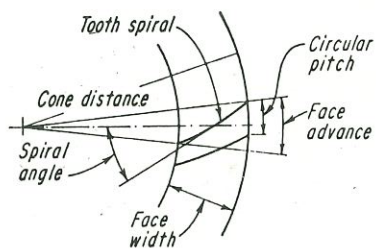


FIGURE 2.37 Spiral-bevel-gear nomenclature.

TABLE 2.7 Straight Bevel Gear Formulas (20° pressure angle, 90° shaft angle)

To find	Having	Formula
Pitch diameter of pinion.....	Number of pinion teeth and diametral pitch	$d = \frac{n}{P_d}$
Pitch diameter of gear.....	Number of gear teeth and diametral pitch	$D = \frac{N}{P_d}$
Pitch angle of pinion.....	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1} \frac{n}{N}$
Pitch angle of gear.....	Pitch angle of pinion	$\Gamma = 90^\circ - \gamma$
Outer cone distance of pinion and gear	Gear pitch diameter and pitch angle of gear	$A_o = \frac{D}{2 \sin \Gamma}$
Circular pitch of pinion and gear	Diametral pitch	$p = \frac{3.1416^*}{P_d}$
Dedendum angle of pinion....	Dedendum of pinion and outer cone distance	$\delta_P = \tan^{-1} \frac{b_{oP}}{A_o}$
Dedendum angle of gear.....	Dedendum of gear and outer cone distance	$\delta_G = \tan^{-1} \frac{b_{oG}}{A_o}$
Face angle of pinion blank....	Pinion pitch angle and dedendum angle of gear	$\gamma_o = \gamma + \delta_G$
Face angle of gear blank.....	Gear pitch angle and dedendum angle of gear	$\Gamma_o = \Gamma + \delta_P$
Root angle of pinion.....	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_P$
Root angle of gear.....	Pitch angle of gear and dedendum angle of gear	$\Gamma_R = \Gamma - \delta_G$
Outside diameter of pinion....	Pinion pitch diameter, pinion addendum and pitch angle of pinion	$d_o = d + 2a_{oP} \cos \gamma$
Outside diameter of gear.....	Pitch diameter of gear, addendum, and pitch angle of gear	$D_o = D + 2a_{oG} \cos \Gamma$
Pitch apex to crown of pinion..	Pitch diameter of gear, addendum, and pitch angle of pinion	$x_o = \frac{D}{2} - a_{oP} \sin \gamma$
Pitch apex to crown of gear....	Pitch diameter of pinion, addendum, and pitch angle of gear	$X_o = \frac{d}{2} - a_{oG} \sin \Gamma$
Circular thickness of pinion...	Circular pitch and gear circular tooth thickness	$t = p - T$
Chordal thickness of pinion....	Circular tooth thickness, pitch diameter of pinion, and backlash	$t_c = t - \frac{t^3}{6d^2} - \frac{B}{2}$
Chordal thickness of gear.....	Gear circular tooth thickness, pitch diameter of gear, and backlash	$T_c = T - \frac{T^3}{6D^2} - \frac{B}{2}$
Chordal addendum of pinion...	Addendum angle, circular tooth thickness, pitch diameter, and pitch angle of pinion	$a_{cP} = a_{oP} + \frac{t^2 \cos \gamma}{4d}$
Chordal addendum of gear.....	Addendum angle, circular tooth thickness, pitch angle, and pitch diameter of gear	$a_{cG} = a_{oG} + \frac{T^2 \cos \Gamma}{4D}$
Tooth angle of pinion.....	Outer cone distance, tooth thickness, dedendum of pinion, and pressure angle	$\frac{3,438}{A_o} \left(\frac{t}{2} + b_{oP} \tan \phi \right) \text{ minutes}$

TABLE 2.7 (Continued)

To find	Having	Formula
Tooth angle of gear.....	Outer cone distance, circular tooth thickness, dedendum of gear, and pressure angle	$\frac{3,438}{A_o} \left(\frac{T}{2} + b_{oG} \tan \phi \right)$ minutes

NOTE: $\tan^{-1}$ means "the angle whose tangent is."

*Exact value of π is 3.14159265.

references at the end of this chapter give further information—beyond the scope of this book.

Zerol. The Zerol bevel gear tooth has lengthwise curvature like the spiral bevel gear tooth. At the center of the face width, though, the spiral angle is 0° .

Figure 2.38 shows how the hand of a Zerol bevel is designated. Note that a right-hand spiral bevel pinion has to have left-hand spiral bevel gear to mesh with it, and vice versa. This same rule applies to spiral bevel gears.

Table 2.9 shows formulas for Zerol bevel gears. Both Zerol and spiral bevel gears have the pitch diameters specified at the large end of the tooth.

2.2.6 Worm Gear Nomenclature and Formulas

When the word *worm* is used in connection with gear types, the implication is that there is some "enveloping." A single-enveloping set of worm gears has a cylindrical worm

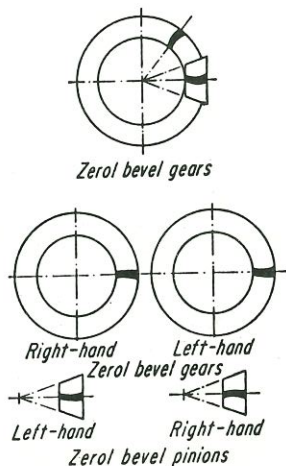


FIGURE 2.38 Zerol-bevel-gear nomenclature.

TABLE 2.8 Spiral Bevel Gear Formulas (20° pressure angle, 90° shaft angle)

To find	Having	Formula
Pitch diameter of pinion.....	Number of pinion teeth and diametral pitch	$d = \frac{n}{P_d}$
Pitch diameter of gear.....	Number of gear teeth and diametral pitch	$D = \frac{N}{P_d}$
Pitch angle of pinion.....	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1} \frac{n}{N}$
Pitch angle of gear.....	Pitch angle of pinion	$\Gamma = 90^\circ - \gamma$
Outer cone distance of pinion and gear....	Pitch diameter of gear and pitch angle of gear	$A_o = \frac{D}{2 \sin \Gamma}$
Circular pitch of pinion and gear.....	Diametral pitch	$p = \frac{3.1416^*}{P_d}$
Dedendum angle of pinion.....	Dedendum of pinion and outer cone distance	$\delta_P = \tan^{-1} \frac{b_{oP}}{A_o}$
Dedendum angle of gear.....	Dedendum of gear and outer cone distance	$\delta_G = \tan^{-1} \frac{b_{oG}}{A_o}$
Face angle of pinion blank.....	Pitch angle of pinion and dedendum angle of gear	$\gamma_o = \gamma + \delta_G$
Face angle of gear blank.....	Pitch angle of gear and dedendum angle of gear	$\Gamma_o = \Gamma + \delta_P$
Root angle of pinion.....	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_P$
Root angle of gear.....	Pitch angle of gear and dedendum angle of gear	$\Gamma_R = \Gamma - \delta_G$
Outside diameter of pinion.....	Pitch diameter, addendum, and pitch angle of pinion	$d_o = d + 2a_{oP} \cos \gamma$
Outside diameter of gear.....	Pitch diameter, pitch angle, and addendum of gear	$D_o = D + 2a_{oG} \cos \Gamma$
Pitch apex to crown of pinion.....	Pitch diameter of gear, pitch angle, and addendum of pinion	$x_o = \frac{D}{2} - a_{oP} \sin \gamma$
Pitch apex to crown of gear.....	Pitch diameter of pinion, pitch angle, and addendum of gear	$X_o = \frac{d}{2} - a_{oG} \sin \Gamma$
Circular thickness of pinion.....	Circular pitch of pinion and circular thickness of gear	$t = p - T$

*Exact value of π is 3.14159265.

TABLE 2.9 Zerol Bevel Gear Formulas (20° pressure angle, 90° shaft angle)

To find	Having	Formula
Pitch diameter of pinion.....	Number of pinion teeth and diametral pitch	$d = \frac{n}{P_d}$
Pitch diameter of gear.....	Number of gear teeth and diametral pitch	$D = \frac{N}{P_d}$
Pitch angle of pinion.....	Number of pinion teeth and number of gear teeth	$\gamma = \tan^{-1} \frac{n}{N}$
Pitch angle of gear.....	Pitch angle of pinion	$\Gamma = 90^\circ - \gamma$
Outer cone distance of pinion and gear	Pitch diameter of gear and pitch angle of gear	$A_o = \frac{D}{2 \sin \Gamma}$
Circular pitch of pinion and gear....	Diametral pitch	$p = \frac{3.1416^*}{P_d}$
Face angle of pinion blank.....	Pitch angle of pinion and dedendum angle of gear	$\gamma_o = \gamma + \delta_g$
Face angle of gear blank.....	Pitch angle of gear and dedendum angle of pinion	$\Gamma_o = \Gamma + \delta_p$
Root angle of pinion.....	Pitch angle of pinion and dedendum angle of pinion	$\gamma_R = \gamma - \delta_p$
Root angle of gear.....	Pitch angle of gear and dedendum angle of gear	$\Gamma_o = \Gamma - \delta_g$
Outside diameter of pinion.....	Pitch diameter, pitch angle, and addendum of pinion	$d_o = d + 2a_p \cos \gamma$
Outside diameter of gear.....	Pitch diameter, pitch angle, and addendum of gear	$D_o = D + 2a_g \cos \Gamma$
Pitch apex to crown of pinion.....	Pitch diameter of gear, pitch angle, and addendum of pinion	$x_o = \frac{D}{2} - a_p \sin \gamma$
Pitch apex to crown of gear.....	Pitch diameter of pinion, pitch angle, and addendum of gear	$X_o = \frac{d}{2} - a_g \sin \Gamma$
Circular thickness of pinion.....	Circular pitch of pinion and circular thickness of gear	$t = p - T$
Circular thickness of gear.....	Circular pitch, pressure angle, and addendum of pinion and gear	$T = \frac{p}{2} - (a_p - a_g) \tan \phi$

NOTE: $\tan^{-1}$ means "the angle whose tangent is."

*Exact value of π is 3.14159265.

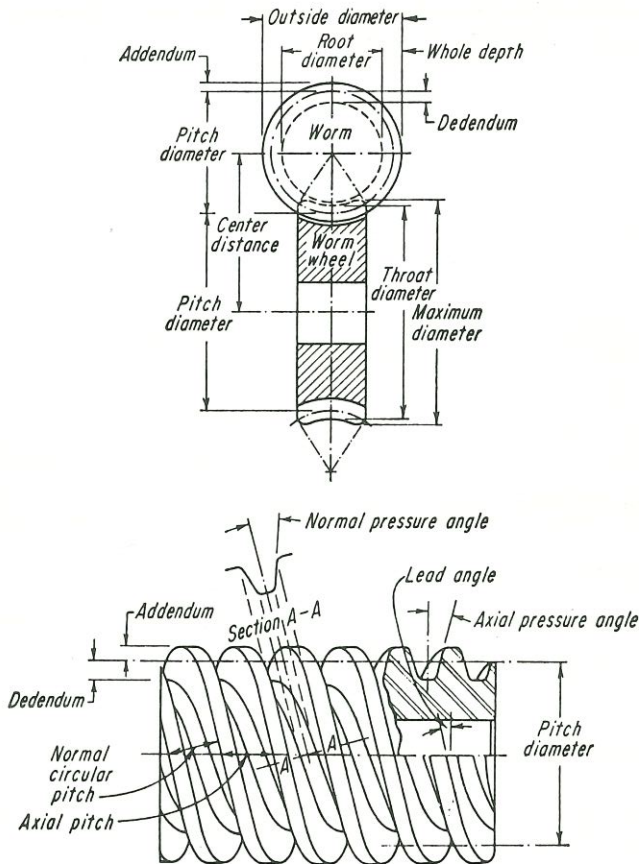


FIGURE 2.39 Cylindrical-worm-gear nomenclature.

in mesh with a gear that is throated to tend to wrap around the worm. Figure 2.39 shows a typical arrangement.

When both the worm and the gear wrap around each other, the combination is designated as double-enveloping. Figure 2.40 shows a typical arrangement of this kind.

Cylindrical Worm Gears. The worm thread (or tooth) may be dimensioned in both a normal section and an axial section. The transverse section is normally not used. Figure 2.39 shows the nomenclature that is used.

The special formulas for worm gearing are given in Table 2.10. The pitch diameter of the throated gear changes as you go across the face width. Standard practice is to take the pitch diameter at the center of the throat. Note the details of how this is done in Fig. 2.39.

Double-Enveloping Worm Gears. The typical style of design and the normal nomenclature for this kind of gearing is shown in Fig. 2.40. The pitch diameters are

TABLE 2.10 Worm Gear Formulas

To find	Having	Formula
Worm:		
Lead.....	Number of threads in worm and axial pitch	$L = N_W \times p_x$
Pitch diameter.....	Center distance	$d = \frac{C^{0.875}}{1.7} \text{ to } \frac{C^{0.875}}{3}$
Root diameter.....	Outside diameter of pinion and whole depth of tooth	$d_R = d_o - 2h_t$
Outside diameter.....	Pitch diameter and addendum	$d_o = d + 2a$
Minimum face.....	Throat diameter, pitch diameter, and addendum	$f = 2 \sqrt{\left(\frac{D_t}{2}\right)^2 - \left(\frac{D}{2} - a\right)^2}$
Lead angle.....	Lead and pitch diameter	$\tan \lambda = \frac{L}{3.1416 \times d}$
Normal pitch.....	Axial pitch and lead angle	$p_n = p_x \times \cos \lambda$
Gear:		
Nominal pitch diameter....	Number of teeth in gear and axial pitch	$D = \frac{N_G \times p_x}{3.1416^*}$
Throat diameter.....	Nominal pitch diameter and addendum	$D_t = D + 2a$
Effective face.....	Pitch diameter and working depth	$F_e = \sqrt{(d + h_k)^2 - d^2}$
Center distance.....	Pitch diameter of gear and pitch diameter of worm	$C = \frac{D + d}{2}$

*Exact value of π is 3.14159265.

specified at the center of the throat on the worm and the center of the throat on the gear.

The formulas for double-enveloping worm gears are given in Table 2.11.

2.2.7 Face Gears

The gear member of a face gear set has teeth cut on the end of a cylindrical-shaped blank. Figure 2.41 shows a typical face gear arrangement and the nomenclature used.

The formulas for an on-center 90°-shaft-angle set of face gears are given in Table 2.12. Additional design data (for a standard design) are given in Tables 2.13 and 2.14.

There are other possible face gear designs with shaft angles that are not 90° or conditions of not being on-center. These can be thought of as special designs beyond the scope of this book.

TABLE 2.11 Double-Enveloping Worm Gear Formulas

To find	Having	Formula
Worm root diameter*.....	Pitch diameter of pinion and dedendum of gear; center distance	$d_R = d - 2b_g$ $d_R = \frac{C^{0.875}}{3}$ (approx.)
Worm pitch diameter*.....	Center distance	$d = \frac{C^{0.875}}{2.2}$ (approx.) $D = 2C - d$
Gear pitch diameter.....	Center distance and worm pitch diameter	
Axial circular pitch.....	Gear pitch diameter and number of teeth in gear	$p_z = \frac{\pi D}{N_g}$
Normal circular pitch.....	Axial circular pitch and pitch cone angle of pinion	$p_n = p_z \cos \lambda$
Whole depth of tooth*.....	Normal circular pitch	$h_t = \frac{p_n}{2}$
Working depth of tooth*.....	Whole depth of tooth	$h_k = 0.9h_t$
Dedendum*.....	Working depth of tooth	$b_g = 0.611h_k$
Normal pressure angle*.....		$\phi_n = 20^\circ$
Axial pressure angle.....	Normal pressure angle and lead angle, deg	$\phi_x = \tan^{-1} \frac{\tan \phi_n}{\cos \lambda_c}$
Lead angle at center of worm.....	Pitch diameter, gear ratio, and worm pitch diameter	$\lambda_c = \tan^{-1} \frac{D}{m_g d}$
Lead angle average*.....	Pitch diameter, gear ratio, and worm pitch diameter	$\lambda = \tan^{-1} \frac{0.87D}{m_g d}$

*Proportions given in these formulas represent recommendations for Cone-drive gears. Cone-drive is a registered trademark of Cone-Drive Textron, Traverse City, Michigan.

2.2.8 Spiroid Gear Nomenclature and Formulas

There is a family of gears that is generally described by the name *Spiroid*. The members of this family are Spiroid and Helicon. The gear member of each of these kinds of gear sets has teeth cut on the end of a blank. The pinion member does not mesh on center but instead meshes with a considerable offset from the central position. Figure 2.42 shows a standard Spiroid arrangement. The dimension A is the offset.

The Spiroid pinion is tapered. The teeth spiral around the pinion somewhat like threads on a worm, but they are on the surface of a cone rather than on a cylinder. The Helicon pinion differs from the Spiroid pinion by being cylindrical. This makes it quite comparable to a cylindrical worm.

Table 2.16 gives some formulas for Spiroid gear sets. Other dimensions needed to make a standard design are given as part of Fig. 2.42. Specific data for Helicon gear sets are not given. The Spiroid Division of the Illinois Tool Works in Chicago, Illinois should be consulted for additional data.

2.2.9 Beveloid Gears

This special type of gear is patented by the Vinco Corporation. The Vinco Corporation went out of business about twenty years ago, but this type of gear is still in use. The

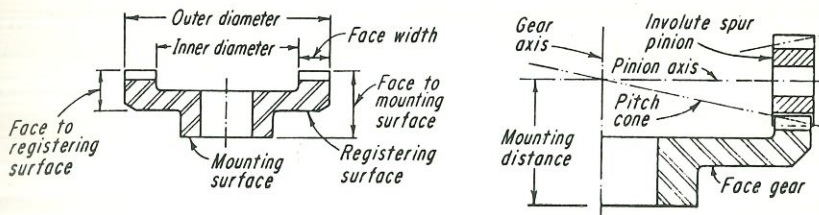


FIGURE 2.41 On-center-face-gear nomenclature.

TABLE 2.12 Face Gears—On Center 90° Shaft Angle*

To find	Having	Formula
Gear ratio.....	Number of pinion and gear teeth	$m_G = \frac{N_G}{N_P}$ (from 1.5 to 12.5)
Inside diameter.....	Diametral pitch, number of gear and pinion teeth, pressure angle	$D_{iG} = \frac{1}{P} \sqrt{8N_P + N_G \cos^2 \phi}$
Outside diameter (pointed teeth).	Diametral pitch, number of gear teeth, pressure angle, and tangency pressure angle	$D_{oG} = \frac{N_G \cos \phi \sec \phi_o}{P}$ (see Table 2.13)
Max. face width.....	Face-gear inside and outside diameters	$F_G = \frac{D_o - D_i}{2}$
Face to pinion axis.....	Pinion outside diameter and working depth of tooth	Face to pinion axis = $\frac{d_o}{2} - h_k$
Face to mounting surface.....	Mounting distance and face to pinion axis	Face to mounting surface = mounting distance - face to pinion axis
Min. numbers of teeth.....		See Table 2.14

*Based upon AGMA standard 203.03, "Fine-Pitch On-Center Face Gears for 20-degree Involute Spur Pinions."

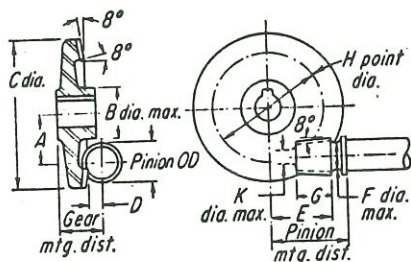
TABLE 2.13 Tangency Pressure Angle at Face Gear Outside Diameter

N_P	ϕ_o	$\sec \phi_o$	N_P	ϕ_o	$\sec \phi_o$	N_P	ϕ_o	$\sec \phi_o$
12	40.765	1.32031	25	34.596	1.21481	38	32.837	1.19016
13	39.669	1.29913	26	34.429	1.21237	39	32.730	1.18874
14	38.692	1.28120	27	34.268	1.21005	40	32.627	1.18736
15	37.815	1.26583	28	34.113	1.20783	41	32.525	1.18602
16	37.022	1.25249	29	33.965	1.20572	42	32.427	1.18473
17	36.300	1.24081	30	33.821	1.20369	43	32.332	1.18352
18	36.012	1.23626	31	33.683	1.20176	44	32.240	1.18228
19	35.778	1.23261	32	33.550	1.19990	45	32.150	1.18111
20	35.557	1.22919	33	33.421	1.19812	46	32.061	1.17996
21	35.346	1.22598	34	33.297	1.19640	47	31.976	1.17887
22	35.145	1.22295	35	33.176	1.19475	48	31.893	1.17781
23	34.954	1.22009	36	33.059	1.19317	49	31.810	1.17675
24	34.771	1.21738	37	32.946	1.19164	50	31.734	1.17578

TABLE 2.14 Minimum Numbers of Teeth in Pinion and Face Gear

Diametral pitch range	Min. numbers of teeth		Diametral pitch range	Min. numbers of teeth	
	Pinion	Gear		Pinion	Gear
20-48	12	18	73-76	19	29
49-52	13	20	77-80	20	30
53-56	14	21	81-84	21	32
57-60	15	23	85-88	22	33
61-64	16	24	89-92	23	35
65-68	17	26	93-96	24	36
69-72	18	27	97-100	25	38

TABLE 2.15 Spiroid Gear Dimensions



A	B	C	D	E	F	G	H	K
0.500	0.625	1.500	0.129	0.596	0.2969	0.365	1.101	0.2031
0.750	1.000	2.250	0.176	0.894	0.4219	0.548	1.051	0.2812
1.000	1.375	3.000	0.248	1.192	0.5938	0.731	2.202	0.3750
1.250	1.625	3.750	0.295	1.490	0.7031	0.914	2.752	0.4531
1.500	2.000	4.500	0.338	1.788	0.8281	1.096	3.303	0.5312
1.875	2.625	5.625	0.402	2.236	1.0156	1.370	4.129	0.6406
2.250	3.187	6.750	0.461	2.683	1.1562	1.644	4.954	0.7500
2.750	4.000	8.250	0.536	3.279	1.3594	2.010	6.055	0.8750
3.250	4.750	9.750	0.608	3.875	1.5625	2.375	7.156	1.0156
3.750	5.625	11.250	0.677	4.471	1.7656	2.741	8.257	1.1406
4.375	6.750	13.125	0.757	5.216	2.0000	3.197	9.633	1.2969
5.125	8.000	15.375	0.851	6.111	2.2812	3.745	11.285	1.4844

TABLE 2.16 Spiroid Gear Formulas

To find	Having	Formula
Ratio.....	Number of teeth in gear, number of threads in pinion	$m_G = \frac{N_G}{N_P}$
Pinion spiral angle.....	Theoretical lead, pinion OD cone angle, pinion pitch radius	$\tan \alpha_P = \frac{L \sec \tau}{2\pi r}$
Gear spiral angle.....	Ratio, pinion pitch radius, gear pitch radius, pinion spiral angle	$\sin \alpha_G = m_G \left(\frac{r}{R_G} \right) \sin \alpha_P$
Pinion pitch point.....	Length of pinion primary pitch cone	$= \frac{1}{3}$ axial length primary pitch cone from small end

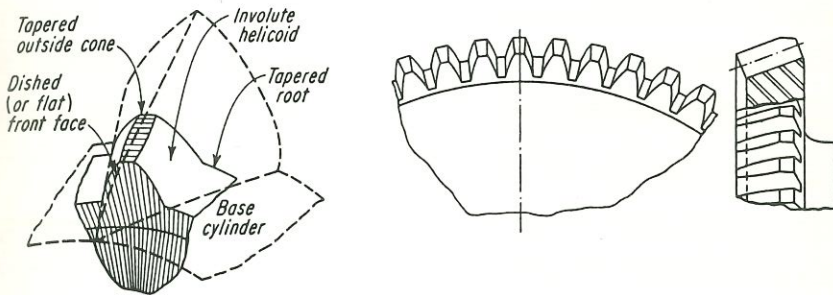


FIGURE 2.42 Beveloid-tooth and gear nomenclature.

Invincible Gear Company in Livonia, Michigan now holds the patent rights on this gear type and still manufactures Beveloid gears.

Figure 2.42 shows a Beveloid gear and its features. Beveloid gears can be engaged with spur gears, helical gears, cylindrical worms, racks, and other Beveloid gears. They can be used with intersecting, parallel, or skew shafts. They are relatively insensitive to mounting errors.

REFERENCES

1. ANSI/AGMA 1012-F90, "Gear Nomenclature, Definition of Terms with Symbols," American Gear Manufacturers Association, April 1990.
2. AGMA 904-B89, "Metric Usage," American Gear Manufacturers Association, May 1989.
3. Drago, R. J., *Fundamentals of Gear Design*, Butterworth, Stoneham, Massachusetts, 1988.
4. Dudley, D. W., *Handbook of Practical Gear Design*, McGraw-Hill, New York, 1984.
5. Klingelberg Söhne, The Klingelberg Spiral Bevel Gear Systems, Remscheid, Germany.
6. Machine Tool Works, Oerlikon-Bührle Ltd., Spiralomatic, Spiral Bevel Gear Cutting Machines, Zurich, Switzerland.
7. Smith, L. J., "The Involute Helicoid and The Universal Gear," American Gear Manufacturers Association, Fall Technical Meeting, 1989.

CHAPTER 3

GEAR ARRANGEMENTS

Paul F. Fox

*Product Engineer,
Auburn Gear
Auburn, Indiana*

Gears may be arranged in a great variety of ways. In complex pieces of machinery, the problem of fitting a gear arrangement into the available space is one of the major gear design problems. Even for relatively simple designs, the task of designing an optimum arrangement is very important. In these situations, factors such as the cost and manufacturability of a design must be optimized.

This chapter discusses various gear arrangements that can be found in industry today. It should be understood that there are an infinite number of arrangement possibilities for complex machinery drives. As a result, the arrangements shown in this chapter represent just a few possibilities for designs rather than a complete listing of all combinations.

Chapter 3 can be best understood if the reader is already familiar with the theory of gearing given in Chap. 1 and the kinds of gear teeth given in Chap. 2.

3.1 POSSIBILITIES IN GEAR ARRANGEMENTS

In general terms, a gear drive implies one toothed member that engages another toothed member and transmits motion and/or torque between the two. The second toothed member may be externally toothed or internally toothed, or it may be a section of a rack (a rack is a gear of infinite pitch diameter). A spur or helical pinion can be used to mesh with any of these three gear types. Many of the other gear types cannot mesh with the gear member if it is anything other than externally toothed. In some cases it is the geometry of the gear part that is limiting. In most cases, though, the gear part might be made in theory, but the available machinery may not have the capability to make the part.

Some types of gear teeth are interchangeable and some are not. In an interchangeable gear system, toothed gear wheels a , b , c , and d might be made. These gears may mesh in sequence with a meshing with b , b meshing with c , and c

meshing with d . If these gears were interchangeable, they could be taken apart and reassembled and run properly with a meshing with c or b meshing with d . Generally speaking, involute profile gears have the property of interchangeability, and other types of gears do not. Spur and helical gears are normally made with involute teeth and are interchangeable. An exception to this is the single-enveloping worm gear, where interchangeability does not exist even when the worm member is made to an involute profile.

In some gear types, it is geometrically impractical to make the pinion member with as little as one tooth. A spur pinion with only one tooth would not be capable of continuous, or conjugate, gear action. On the other hand, a worm with one thread is very practical and is often used. Some gear types can be made with the pinion member having as few as five teeth. Most gear types can have pinions with over 16 teeth. A single-enveloping worm, however, is impractical to manufacture at 16 threads, and there is seldom a good reason to use this many threads on a worm.

Table 3.1 shows in tabular form the meshing possibilities of all the major types of gear teeth. This table is based on normal practices and should not be considered an absolute rule. For instance, an internal bevel gear is theoretically possible and has been used in a few special cases. However, normal bevel gear machines will not make such a gear.

TABLE 3.1 Gear Meshing Possibilities

Type of Gear Teeth	Pinion and Gear	Pinion and Rack	Pinion and Internal Gear	Interchangeability	One Tooth Pinion	Pinion of 5 Teeth	Pinion of 16 or More Teeth
Spur	Yes	Yes	Yes	Yes	No	No*	Yes
Helical	Yes	Yes	Yes	No	No*	No*	Yes
Straight bevel	Yes	No*	No	No*	No	No*	Yes
Zerol bevel	Yes	No	No	No	No	No*	Yes
Spiral bevel	Yes	No	No	No	No*	No*	Yes
Hypoid	Yes	No	No	No	Yes	Yes	Yes
Face gear	Yes	No	No	No	No	No*	Yes
Crossed helical	Yes	Yes	No	Yes	Yes	Yes	Yes
Single-enveloping worm	Yes	No*	No*	No	Yes	Yes	No*
Double-enveloping worm	Yes	No	No	No	Yes	Yes	No*
Beveloid	Yes	Yes	No	Yes	No	No*	Yes
Spiroid	Yes	No	No	No	Yes	Yes	No*
Planoid	Yes	No	No	No	No*	Yes	Yes
Helicon	Yes	No	No	No	Yes	Yes	Yes

*Items with an asterisk indicate that it is mechanically possible to have this design but that the design is not normally used in the gear trade.

Trademarks: Zerol, registered trademark of Gleason Works, Rochester, N.Y.; Beveloid, registered trademark of Vinco Corp., Detroit, Mich.; Spiroid, Planoid, Helicon, registered trademarks of Illinois Tool Works, Chicago, Ill.

3.2 HOW THE ARRANGEMENT MAY DRASTICALLY AFFECT RATIO, POWER, EFFICIENCY, AND GEARBOX VOLUME

The arrangement of the gear drive can play a very important part in determining how easy or hard it is to achieve things like high ratio, high power capacity, high efficiency, or a very compact unit.

In ordinary spur or helical design practice, it is reasonable to handle ratios from 1:1 to about 8:1 in a single reduction. Ratios of even 10:1 are possible. Suppose a ratio of 120:1 was needed. This would require a triple-reduction unit of spur or helical gears. It would also take a triple-reduction bevel gear unit. A triple-reduction unit has three sets of pinions and gears, or a total of six toothed parts. With a worm gear set, it is possible to handle this much ratio in just a single reduction (only two toothed parts).

For each toothed part in a gear set, a shaft and generally two bearings are required. Where large ratios are needed, the choice of gear arrangement is often determined by finding the arrangement with the least number of parts that will do the job adequately.

Table 3.2 gives some general information about how different ratios can be achieved and the number of gear parts required.

The power-transmitting capacity of different gear arrangements is quite variable. For instance, there has never been a worm gear set that would handle 3750 kW (5000 hp) as a continuous-duty rating. On the other hand, many helical gear sets are in regular service carrying more than 7500 kW (10,000 hp). Defining precisely the upper limit of power for a particular gear arrangement is about as difficult as defining the altitude at

TABLE 3.2 How to Obtain Ratios

Kind of Arrangement	Minimum Number of Toothed Parts	Ratio Range		
		5:1	50:1	100:1
Single reduction:				
Spur	2	Yes	No	No
Helical	2	Yes	No	No
Bevel	2	Yes	No	No
Hypoid	2	Yes	Yes	Yes
Face	2	Yes	No	No
Worm	2	Yes	Yes	Yes
Spiroid	2	No	Yes	Yes
Planoid	2	Yes	No	No
Simple planetary	3	Yes	No	No
Fixed differential	5	No	Yes	Yes
Planocentric	2	No	Yes	Yes
Harmonic drive	2	No	Yes	Yes

which the earth's atmosphere ceases. The upper limit of a gear's capacity is determined by such factors as:

1. How large a gear can be made by available gear manufacturing equipment?
2. What is the upper limit on tooth surface loading and pitch line velocity for the best material available?
3. Are the job requirements of ratio, input speed, life expectancy, and lubrication favorable for a maximum power level?

Table 3.3 shows some nominal maximum power capacities for different gear arrangements. These should be used only as a general guide. In all cases there are a few gear sets in use (or that could be put in use) at considerably higher power ratings. Conversely, the job requirements of many applications make it completely impractical to achieve the ratings shown for the specific application. No power limits are shown for the fixed differential, the planocentric, or the Harmonic Drive. These kinds of units have been used mostly at low power levels (less than 40 kW), but they can probably be extended up into fairly high power ranges.

For specific data on load rating for a given gear design, Chap. 11 should be consulted.

TABLE 3.3 General Survey of Power and Efficiency

Kind of Arrangement	Nominal Maximum kW (hp)	Typical Efficiency, %		
		5:1 Ratio	50:1 Ratio	100:1 Ratio
Single reduction:				
Spur	2,240 (3,000)	98		
Helical	22,400 (30,000)	98		
Straight bevel	370 (500)	98		
Zerol bevel	745 (1,000)	98		
Spiral bevel	3,730 (5,000)	98		
Hypoid	745 (1,000)	95	80	60
Crossed helical	75 (100)	95	80	60
Cylindrical worm	560 (750)	95	80	60
Double-enveloping worm	745 (1,000)	95	80	60
Spiroid	370 (500)	95	80	60
Planoid	745 (1,000)	95		
Helicon	75 (100)	95	80	60
Double reduction:				
Spur	2,240 (3,000)	97	96	94
Helical	22,400 (30,000)	97	96	94
Spiral bevel	3,730 (5,000)	97	96	
Simple planetary	7,460 (10,000)	97		
Fixed differential			80	60
Planocentric			90	85
Harmonic drive			90	85

The efficiency that can be achieved in a gear set is quite variable. The kind of oil, the amount of pitch line velocity, and the kind of bearings enter quite directly into the efficiency. In a spur gear set, the best designs made under favorable conditions will achieve as much as 99-percent efficiency. Some spur sets under poor conditions may not achieve even 90-percent efficiency.

Table 3.3 shows the nominal order-of-magnitude efficiencies for different gear arrangements. These represent general trade experience with good designs but are not the best possible efficiency that can be achieved with an optimum design and a favorable application.

Table 3.3 also shows that the efficiency drops off rather quickly as the ratio increases. Generally this is caused by the fact that the sliding velocity increases as the ratio increases. A worm, for instance, at high ratio has a very large amount of sliding.

The fixed differential gear is capable of high ratio, but it too has a drop off in efficiency. This is caused by the fact that a large amount of tooth meshing occurs at high torque (see Sec. 3.6).

Suppose a 100:1 ratio is needed. So far as arrangement goes, the double-reduction approach using either spur, helical, or bevel gears is the easiest way to achieve high efficiency. With a favorable application and a good design, an efficiency of 96 percent can be achieved with helical gears. It is nearly impossible to obtain this kind of efficiency with any of the single-reduction gears that can reach a 100:1 ratio.

Chapter 12 covers the general subject of gear efficiency and shows curves for many gear types. This chapter should be consulted for more detailed information on gear efficiency.

The volume or weight of a gearbox is also considerably affected by the arrangement. The weight of a gearbox for a given job may vary as much as 50:1! For instance, a single-reduction helical gear unit made of low-hardness material in industrial practice might weigh 4500 kg (10,000 lb). A planetary gear set made of fully hardened gears for an aircraft application might handle the same power level, for the same design life, with the same degree of reliability as the industrial gear, and weigh only 90 kg (200 lb).

In general terms, a 50:1 spread in gear weight can be broken down as follows:

- Weight ratio for low tooth loads on low-hardness material vs. high loads on high-hardness material—7:1
- Weight ratio for heavy construction using thick walls and solid shafts vs. thin walls and hollow shafts—3:1
- Weight ratio of best arrangement to save weight vs. normal arrangement—2½:1

The opportunity to save weight in gear designs depends somewhat on the ratio required. For instance, a worm gear set is not particularly lightweight at a low ratio of 5:1. At a high ratio of 50:1, the worm gear becomes rather light because it can still do the job with just two toothed parts, whereas a spur or helical set would take at least twice as many parts to do the job.

Table 3.4 shows a general picture of what arrangements might be small for different gear ratios. In using this table, it should be kept in mind that intensity of tooth loading is often more important than arrangement in reducing weight. For instance, a case-hardened gear set used in a nominal arrangement might result in a lighter unit than a set of low-hardness cut gears used in the most weight-saving arrangement.

In using Table 3.4 it should also be kept in mind that the desired power rating must be favorable for the design. Some designs cannot handle high torque. These designs are small in weight only in comparison with other designs when their required torque levels are well within their capability.

TABLE 3.4 Gearbox Relative Size and Weight

Kind of Arrangement	Ratio Range			
	5:1	20:1	50:1	100:1
Single reduction:				
Spur, helical, bevel	Small			
Worm		Small	Small	Small
Hypoid	Small	Small	Small	Small
Spiroid		Small	Small	Small
Planoid	Small			
Double reduction:				
Single power path, helical gears		Medium size		
Multiple power path, helical gears		Small	Very small	
Epicyclic gears:				
Simple planetary	Very small			
Compound planetary		Very small		
Double-reduction planetary		Very small	Very small	
Fixed differential		Small	Very small	Very small

Generally speaking, the volume of the gearbox is proportional to the weight—provided that the type of construction is the same. Aircraft gearboxes with magnesium casings will have more volume for their weight than industrial boxes with cast-iron casings.

3.3 PARALLEL AXIS ARRANGEMENTS

Perhaps the most common gear arrangements are those consisting of a pinion and gear meshing on parallel axes. The teeth for these kinds of arrangements may be either spur, helical, or herringbone. (Chapter 2 defines the kinds of gear teeth and defines and explains gear nomenclature.)

Figure 3.1 shows the basic forms of the parallel axis arrangements discussed in this section.

3.3.1 Simple Gear Train

A single pinion and gear is an example of a simple gear train. The ratio for such a unit is figured quite simply:

$$u = \frac{z_2}{z_1} \quad (3.1)$$

where z_2 is the number of gear teeth, and z_1 is the number of pinion teeth.

A simple gear train is one that consists of an input pinion and an output gear with

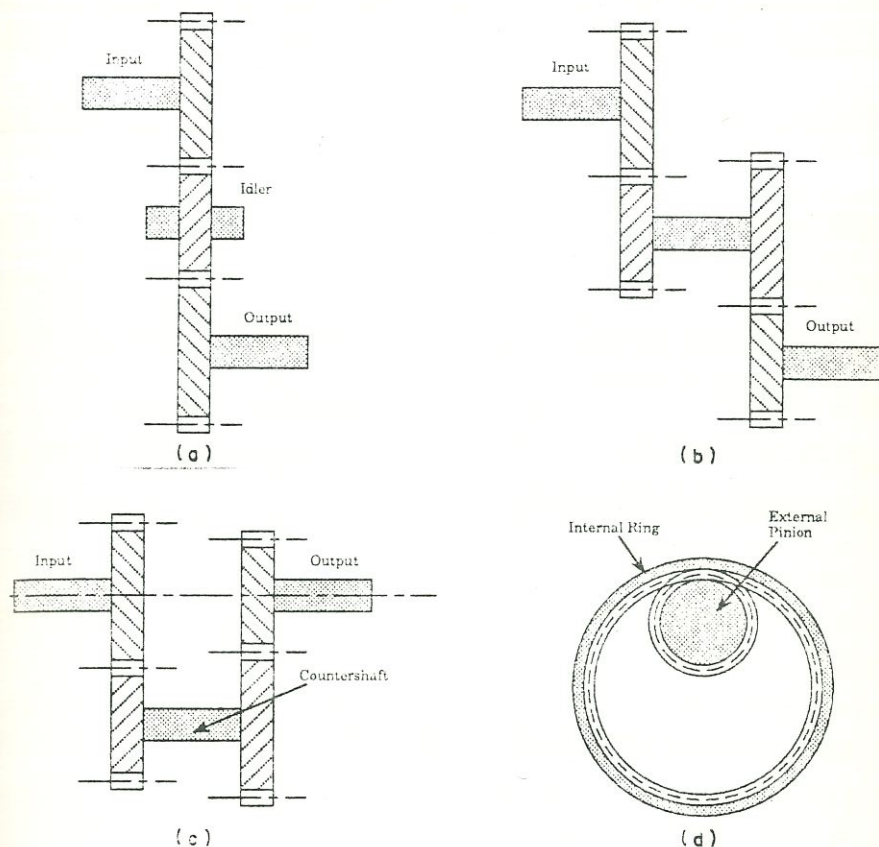


FIGURE 3.1 Parallel axis gear arrangements. (a) Simple gear train; (b) compound gear train; (c) reverted gear train; (d) internal gear train

any number of idlers in between. With a simple gear train the following relationships hold true:

1. Regardless of the number of idlers, the speed ratio is always the number of teeth in the output gear divided by the number of teeth in the input pinion.
2. An idler does not change the ratio of the gear train, but it does change the direction of rotation of the output gear.
3. The ratio of the output torque to the input torque is equal to the inverse of the speed ratio.

Figure 3.2 shows a simple gear train with four idlers. The ratio for this arrangement is calculated simply by dividing the number of teeth in the gear at the lower left corner of the picture by the number of teeth in the pinion at the upper right corner.

Figure 3.3 shows a helical pinion meshing with a gear of infinite pitch diameter, also known as a rack.

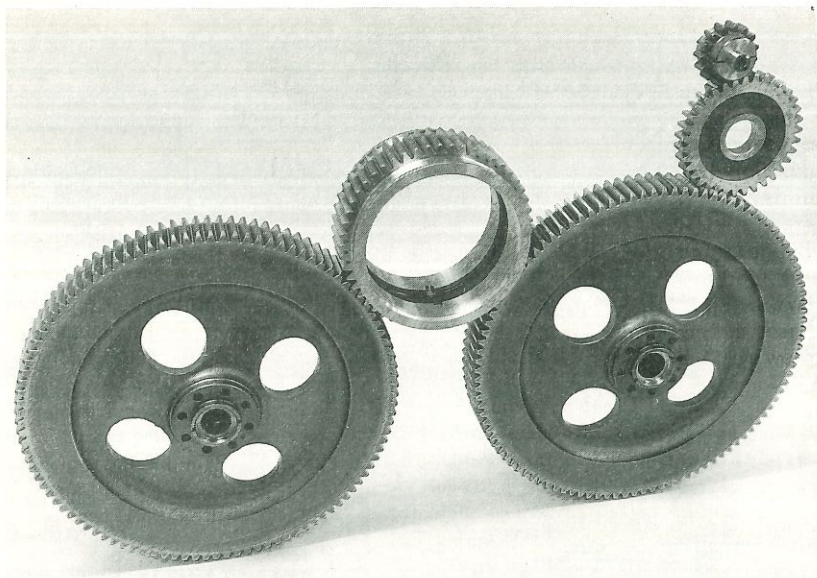


FIGURE 3.2 Simple gear train with three idlers

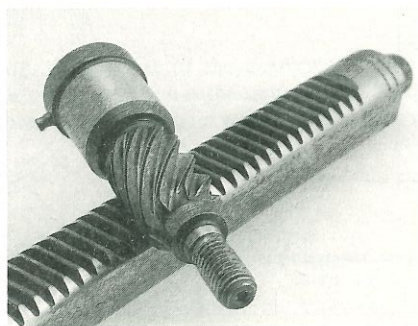


FIGURE 3.3 Helical pinion and rack

3.3.2 Compound Gear Train

A compound gear train is considered to be a set of simple gear trains mounted in series with each other. The ratio for this type of gear train is calculated by:

$$u = u_1 u_2 u_3 \text{ (etc.)}$$

or

$$u = \frac{z_{21}}{z_{11}} \frac{z_{22}}{z_{12}} \frac{z_{23}}{z_{13}} \text{ (etc.)} \quad (3.2)$$

where u_1 = ratio of the first reduction

u_2 = ratio of the second reduction

- z_{11} = number of pinion teeth, first reduction
- z_{12} = number of pinion teeth, second reduction
- z_{21} = number of gear teeth, first reduction
- z_{22} = number of gear teeth, second reduction

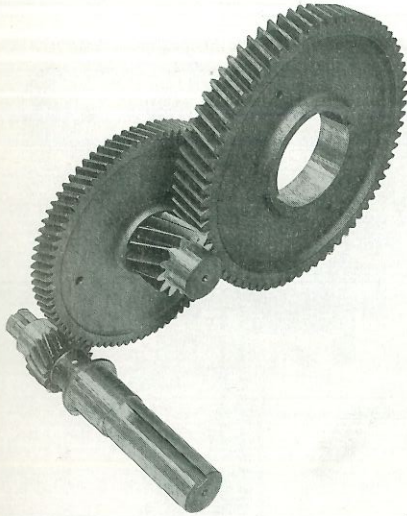


FIGURE 3.4 Compound helical gear train

A wide variety of arrangements are possible with parallel axes using simple and compound gear trains. In addition to compound reductions, there may be multiple power inputs or multiple power outputs. There may also be multiple power paths through the unit. This would be analogous to parallel circuits in electrical wiring.

Figure 3.4 shows a compound gear train using helical gears.

Figure 3.5 shows a three-speed power shift transmission with multiple compound reductions.

3.3.3 Articulated Gear Unit

In a double-reduction design, the first-reduction gear may be solidly mounted on the second-reduction pinion shaft with only two bearings supporting the assembly. Or the pinion and gear may each be mounted on a pair of bearings, and the two connected by a flexible quill shaft. The latter arrangement is called an *articulated design*. The articulated design is quite popular for the higher-speed, higher-horsepower gear units. It has advantages in that it provides flexibility between a first-reduction gear and a second reduction, which reduces dynamic tooth loading; and in the fact that both the gear and pinion have their own separate pair of bearings, making it possible to provide better control on the contact across the face width at each mesh.

Figure 3.6 shows a schematic arrangement for an articulated MDT unit. Further details of an articulated design are shown in Fig. 3.7. Figure 3.8 shows an MDT unit of articulated design that was used as a main drive on a super aircraft carrier.

3.3.4 Reverted Gear Train (Countershaft)

A special form of compound gear train is the reverted, or countershaft, arrangement. These gear trains enable multiple speed ratios to be achieved with a minimum number of shafts and bearings.

The significant characteristics of a reverted gear train are that the gear pairs have a common center distance with the input and output shafts being on the same center line, and both the input and output shafts rotate in the same direction. The ratio calculation for these arrangements is the same as for compound gear trains [Eq. (3.2)].

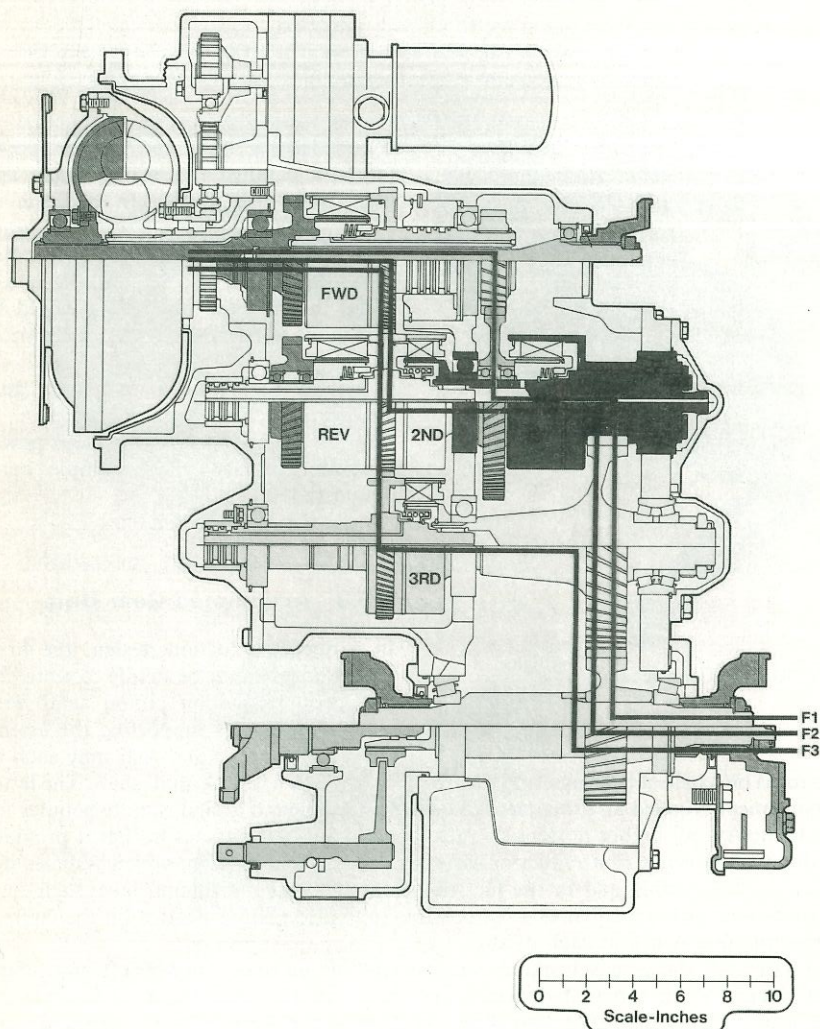


FIGURE 3.5 Three-speed power shift transmission with multiple compound reductions

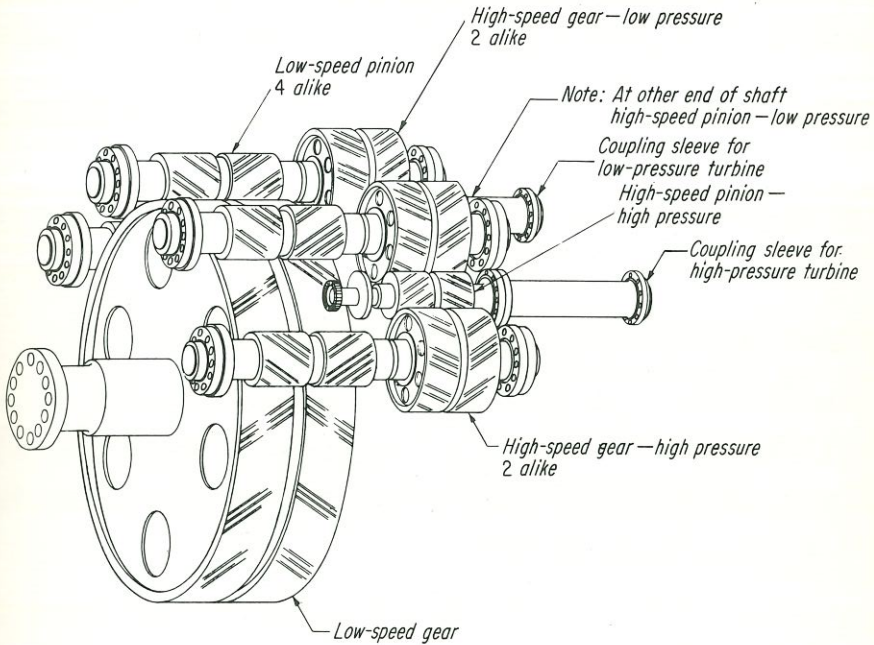


FIGURE 3.6 Schematic arrangement of articulated MDT unit

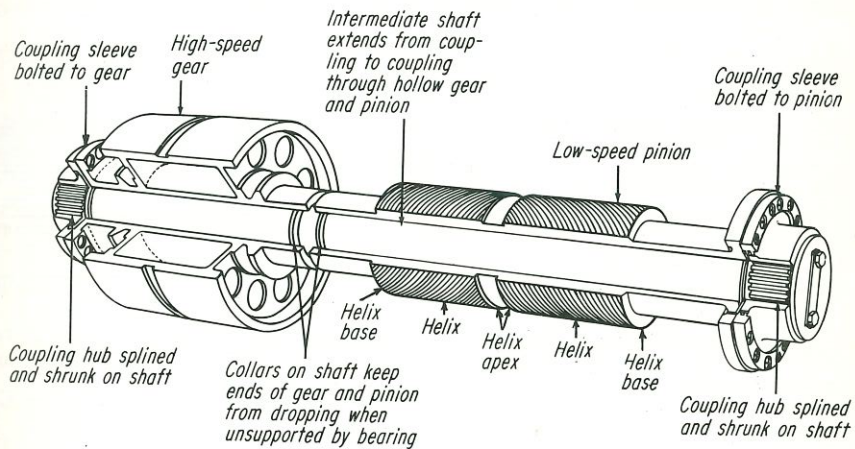


FIGURE 3.7 Detail of articulated design of high-speed and low-speed pinion. Note intermediate shaft passing through hollow pinion and gear, and the fact that both pinion and gear have their own separate bearings

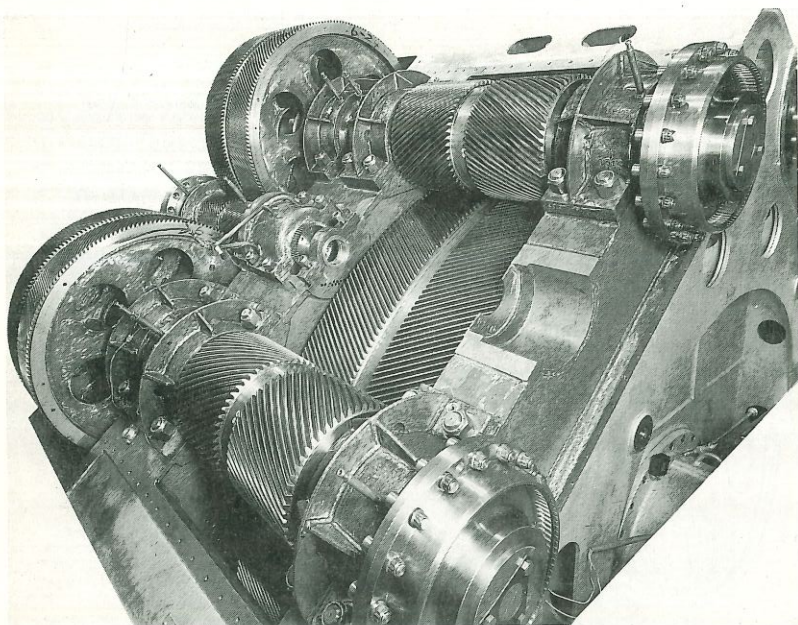


FIGURE 3.8 Side view of MDT unit of type used on super aircraft carrier *Saratoga*. Note articulated design. Rating in excess of 50,000 hp. (Courtesy of U.S. Navy and General Electric Co.)

Figure 3.9 shows a six-speed transmission of the type used in trucks. Note the two countershafts in this design.

3.3.5 Internal Gear Train

The internal gear train is a special type of simple gear train. In an internal arrangement, the speed ratio is calculated in a manner similar to that for a simple gear train [Eq. (3.1)], with the exception that the rotation of the output member is in the same direction as the input.

With an internal gear train, a larger ratio reduction is possible when compared with an external mesh design.

However, because manufacturing difficulties are greater in producing internal gears than for external gears of similar size, internal mesh arrangements are seldom used for simple ratio reductions.

3.3.6 Parallel Axis Gears Combined with Right-Angle Gears

In many cases taking a basic parallel-axis gear unit and adding a first or last reduction of right-angle gearing works well. Doing so permits the drive to turn a right-angle corner, and it may make the drive more compact and result in the use of fewer bearings and other parts.

Figure 3.10 shows a multiple-reduction unit with a combination of spur and helical gear reductions and a bevel reduction.

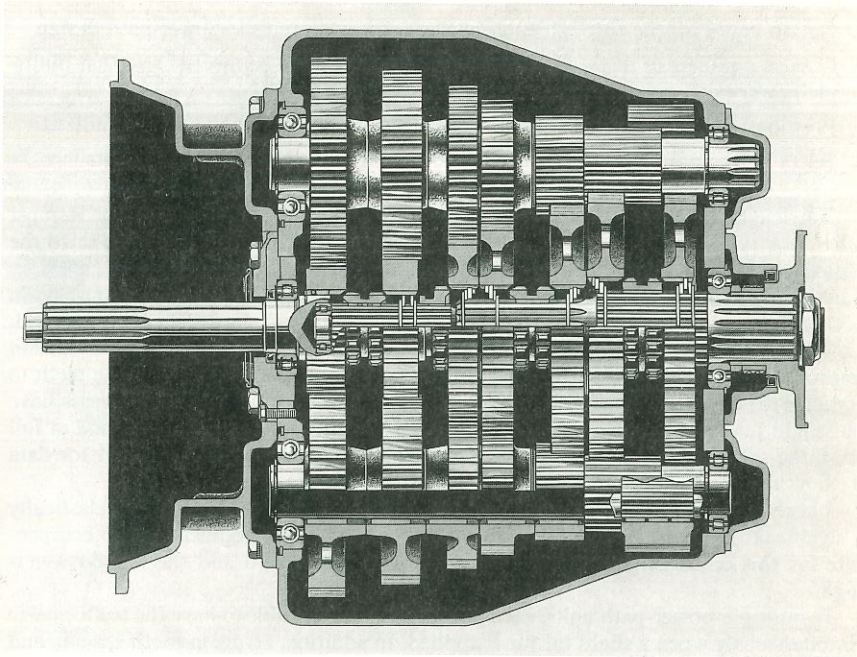


FIGURE 3.9 Six-speed truck transmission

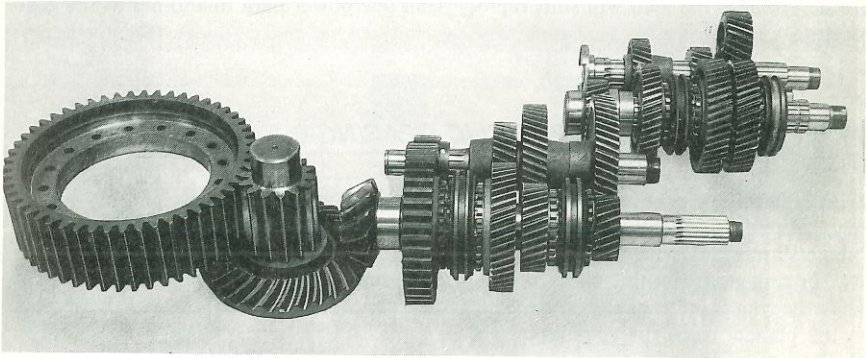


FIGURE 3.10 Double-reduction differential drive and transmission gearing

3.3.7 Design Problems with Parallel Axis Gears

There are several problems in parallel axis gear design that should be addressed if a design is to be successful. In picking an arrangement, designers should be alert to the design problems and satisfy themselves that they can design the arrangement selected so as to obtain a practical design to manufacture and a reliable design in the field.

Some of the problems that may show up are how to:

1. Obtain uniform tooth loading across the face width
2. Compensate for excessive windup of long, skinny pinions or long shafts

3. Obtain equal torque through each power path in a multiple-power-path design
4. Choose numbers of teeth, gear locations, and gear-shaft connections so that multiple-power-path gearing will assemble
5. Position idlers so that the resultant bearing loads on the shafts are minimized
6. Select helix angles and hands-on helical gearing to minimize axial thrusts

The length-to-diameter ratios of helical and spur gears can be fairly large. In most other gear types, the pinion does not have a face width more than about one-third the pinion pitch diameter. In spur gearing the L/D ratio may be as great as 1, while in double helical gearing it may approach 2. Needless to say, high accuracy is needed in boring the casings and machining the teeth when a relatively large face width is used. Low-hardness gearing of about 200 BHN exhibits some tendency to wear in and improve contact. As much as 0.001 in. may be worn off the high ends of the teeth to equalize the contact without too serious a result. Medium- to high-hardness gears have but slight tendency to wear in. If the contact across the face width is not good at full load, the teeth may break before any equalizing wear occurs. (See Chap. 11 for data on how to handle misalignment when load-rating gears.)

Large pinions with an L/D ratio of 1 or more will, under load, twist elastically a significant amount. It may be necessary to provide helix modifications to compensate for this on large gears where the material is fairly hard and the horsepower is high.

In multiple-power-path units, each power path must be able to have the teeth engage simultaneously when a slight torque is applied. In addition, errors in tooth spacing and runout must be slight enough in the gears so that the flexibility of the connecting shafts can keep a uniform load on each path. If the connections are too stiff or the teeth not precise enough, the load will shift rapidly from one power path to another as the gear rotates.

3.4 PARALLEL AXIS EPICYCLIC GEARING

A large family of gear arrangements goes under the name of "epicyclic," or "planetary," gearing. Generally speaking, the epicyclic train has a central "sun" gear, several "planets" meshing with the sun and spaced uniformly around the sun, and an "annulus," or ring gear, meshing with the planets. The sun and planets are externally toothed while the ring is internally toothed. The name *epicyclic* comes from the fact that points on the planets trace epicycloidal curves in space.

The name *planetary* gear is often used interchangeably with epicyclic to denote the whole family of gears. In a stricter sense, the planetary is but one type of epicyclic gear arrangement.

There are a large number of possible epicyclic arrangements. These may be divided into three general groups:

1. Simple epicyclic trains
2. Compound epicyclic trains
3. Coupled epicyclic trains

Figure 3.11 shows schematic arrangements of the two most common types of epicyclic gear trains, the simple epicyclic train, and the compound epicyclic train.

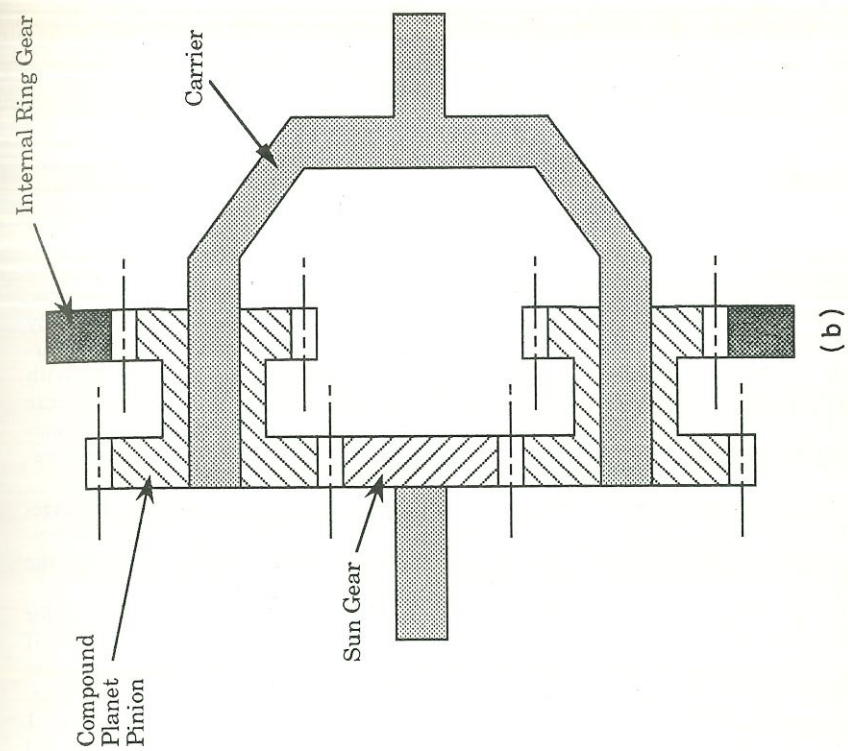
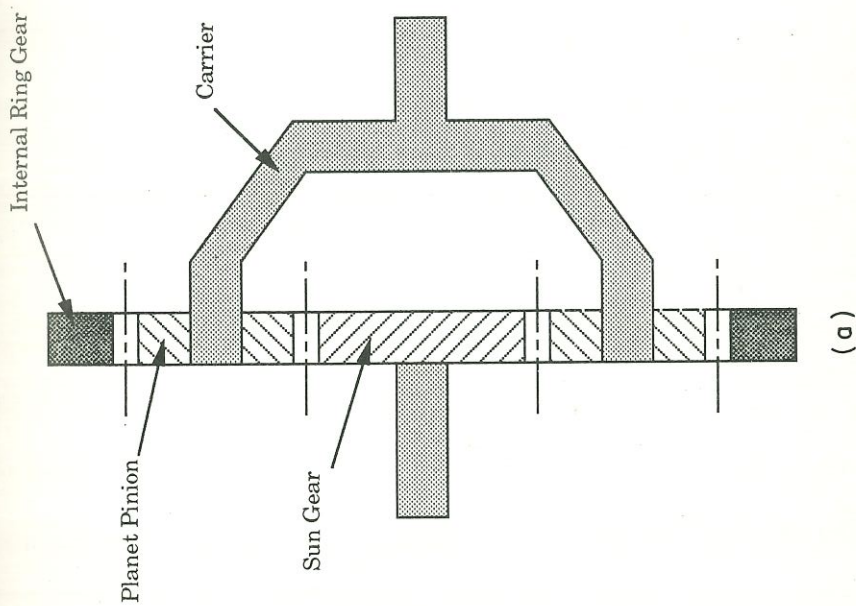


FIGURE 3.11 Epicyclic gear trains

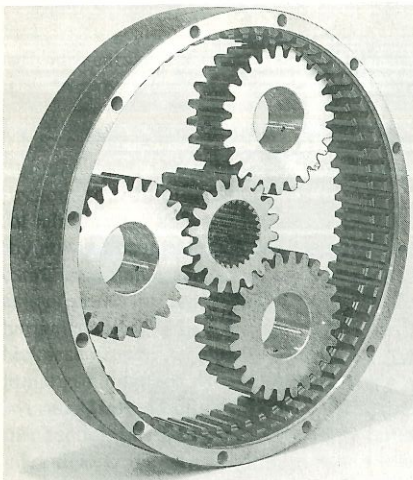


FIGURE 3.12 Simple epicyclic gear arrangement

3.4.1 Simple Epicyclic Trains

The simple epicyclic train consists only of the elements of sun, planet, and ring gear. The arrangement is often called a "simple" epicyclic gear or "simple" planetary gear to distinguish it from the more complicated arrangements.

Figure 3.12 shows a simple epicyclic gear arrangement. The figure shows only three planets. The number of planets varies considerably in actual practice. With some ratios it has been possible to squeeze in as many as twenty planets. Sometimes as few as one is used for light load applications.

Figure 3.13 shows schematic arrangements of the three types of simple epicyclic gears. These are the planetary, the star, and the solar arrangements.

Table 3.5 shows the overall ratio for each of the three simple epicyclic trains. If one of these units were driven backward, the ratio would be the reciprocal of the ratio calculated from the table. A range of ratios is shown for each kind of epicyclic gear. Note that only the solar will result in small ratio reductions. The ratio range of 1.7:1 to 2:1 is a range that is difficult to handle with simple epicyclic gearing. The star will work in this range, but the design is impractical in most cases.

Epicyclic gears will not assemble unless the tooth numbers are properly chosen. In

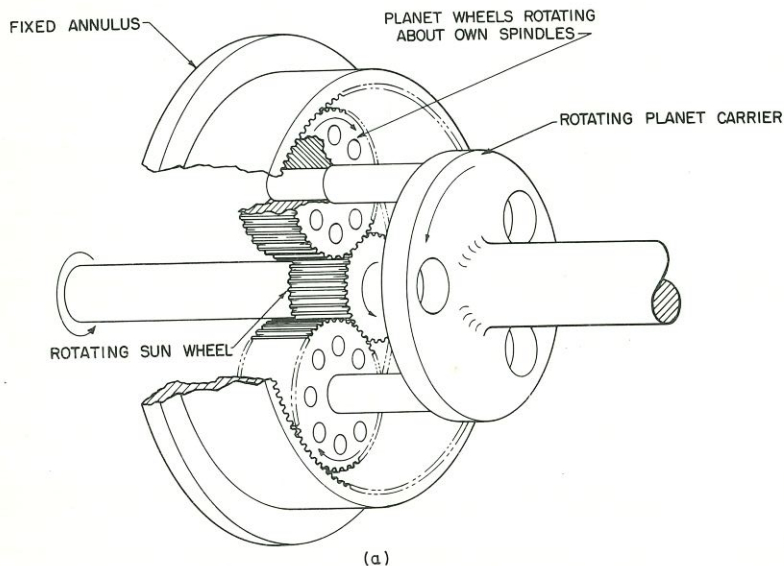


FIGURE 3.13 Simple epicyclic gears. (a) Planetary gear. (b) Star gear. (c) Solar gear

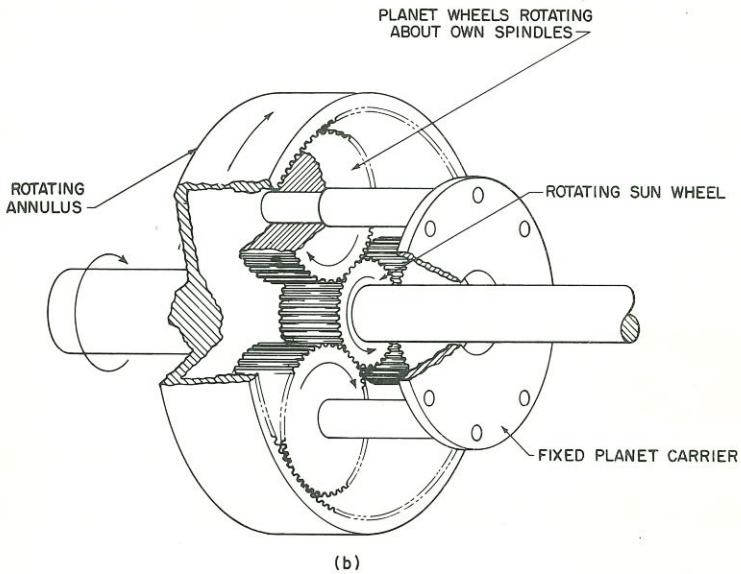
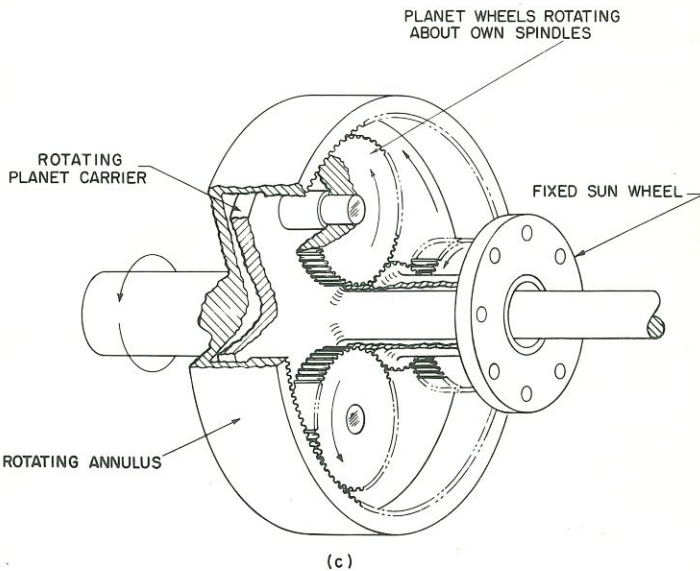


FIGURE 3.13 (Continued)

order to insure equal spacing of the planets around the sun, the following relationship must be met:

$$\frac{z_3 + z_2}{\text{number of planets}} = I \quad (3.3)$$

TABLE 3.5 Simple Epicyclic Gear Data

Kind of Arrangement	Fixed Member	Input Member	Output Member	Overall Ratio	Range of Ratios Normally Used
Planetary	Ring	Sun	Cage	$\frac{z_3}{z_2} + 1$	3:1–12:1
Star	Cage	Sun	Ring	$\frac{z_3}{z_2}$	2:1–11:1
Solar	Sun	Ring	Cage	$\frac{z_2}{z_3} + 1$	1.2:1–1.7:1

z_1 = number of planet teeth

z_2 = number of sun teeth

z_3 = number of ring teeth

where z_2 is the number of sun teeth and z_3 is the number of ring teeth, then I must equal an integer.

As an example, suppose you had a simple planetary arrangement with the following numbers of teeth: $z_2 = 21$, $z_1 = 15$, and $z_3 = 51$. Using Eq. (3.2), it can be determined that 3, 4, 6, or 8 planets are just a few of the possible numbers that can be equally spaced in this design. As a second example, let us look at a slightly different arrangement: $z_2 = 21$, $z_1 = 16$, and $z_3 = 53$. With this arrangement, it is possible to have only two equally spaced planets around the sun, which in most power transmission drives would be extremely impractical.

In cases where a specific planetary ratio is desired, but equal spacing of a given number of planets is impossible, the following equation can be used:

$$\frac{360}{z_3 + z_2} = x \quad (3.4)$$

where x equals the angular spacing increment.

Using the example of the 21-tooth sun and 53-tooth ring, the angular spacing increment would be 4.8649° . As a result, the angular spacing between planets divided by 4.8649° must be an integer. Therefore, a four-planet design could be assembled with planets spaced 87.5676° , 92.4324° , 87.5676° , and 92.4324° apart.

3.4.2 Compound Epicyclic Trains

The compound epicyclic train has two planet members attached together on a common shaft. Figure 3.11 shows a schematic arrangement of a compound planetary gear.

Table 3.6 shows the basic data for compound epicyclic gears. Note that the compound planetary and the compound star will handle much higher ratios than the simple epicyclic types.

Like the simple epicyclic gear arrangement, certain criteria must be met in order for the arrangement to assemble:

$$d_{p_3} = d_{p_3} + d_{p_{11}} + d_{p_{12}} \quad (3.5)$$

Transmissions:

- automatic, 3.10, 3.24, 3.32
- gear shift, 3.10, 3.13, 3.31
- reverse gear units, 3.26

Transmission error, 1.40-1.42, 14.11-14.19

accuracy effects:

- eccentricity, 14.18
- other, 14.19
- tooth profile, 14.18

bevel and hypoid, 14.17, 14.18

effect of face contact ratio, 14.24

effect of misalignment, 14.25

effect of profile modification, 14.13-14.15

effect of tip relief, 14.25

factors contributing to, 14.11

high contact ratio gears, 14.15, 14.16

loaded, 14.11, 14.13

measurement, 14.12

spur gears, 14.11

Transverse shaving, 22.26, 22.27

Triangle gears, 2.27

Trigonometric functions, 24.23

Undercut:

- design table, 4.33
- general discussion, 4.16, 4.17
- in spiral bevel gears, 4.57
- in straight bevel gears, 4.56

Undercutting, 5.13-5.15

Underpass shaving, 22.26, 22.27

Unitool method, 20.33

V and H check, 23.32-23.36

Velocity limits, 13.11, 13.16

Velocity plots, 13.9

Velocity polygon:

- crossed helical, 1.20,
- hypoid, 1.25
- worm, 1.22

Velocity transducers, 13.6

Versacut process, 20.34

Vibration of gears, 12.38, 13.1-13.20

approximate vibration limits, 13.10-13.15

acceleration limits, 13.12, 13.17

displacements limits, 13.13, 13.15

general vibration tendencies, 13.13, 13.14

velocity limits, 13.11, 13.16

vibration trade standard, 13.14

control of vibration, 13.15

field tests, 13.18

testing gear units, 13.16

tests of power packages, 13.18

Vibration of gears (Cont.):

example, 13.7-13.10

oscilloscope patterns, 13.10

spectrum analysis, 13.8

sun gear vibration, 13.10

velocity plot, 13.9

fundamentals, 13.1-13.3

measurements, 13.3-13.7

displacement probes, 13.5

measurement problems, 13.6

proximity probes, 13.5, 13.13

sensing devices, 13.4-13.6

velocity transducer, 13.6

vibration analysis techniques, 13.19

table of analysis methods, 13.20

Waterfall plots, 14.26, 14.27

Wear, 11.54, 12.41-12.44, 15.13, 23.42-23.44

from inadequate lubrication, 15.15

Weight of gears, relative, 3.5

Windage, 15.3

Windage losses, 12.24-12.28

Winter and Michaelis, 15.24

Wire size:

selection, 24.2

table, 24.2

Workholding devices, 20.13

Worm gears:

blank proportions, 10.49

calculation of bearing loads, 10.46-10.49

calculation procedure, 4.60-4.63

cutter diameter, 4.63

cutting, double-enveloping, 21.15

cylindrical, design, 4.60-4.63

description, 2.19-2.22

drafting practice, 9.18, 9.21

efficiency, 12.11-12.13

estimating power capacity, 11.58

formulas, 2.44, 2.46

hobbing, 21.4-21.13

chip load, 21.4, 21.5

fly cutting, 20.13

general information, 21.4-21.6

infed method, 21.10-21.12

production time, 21.15, 21.18, 21.20

speed and feed, 21.5, 21.14

tangential method, 21.12, 21.13

inspection, contact check, 23.38

of double-enveloping gears, 23.40-23.42

worm checking, 23.40

Worm gears (*Cont.*):

- lead angle, 4.62
- nomenclature, 2.43, 2.45
- normal pitches, 4.62
- operating pitch cylinder, 1.21
- powder metal compacts, 21.23–21.30
- rating, 11.57, 11.58
- time required, 21.29
- tolerances, 7.24, 7.25
- tooth forms, 4.59

Worms:

- carburizing, 21.23
- cutter for, 21.12, 21.13
- cutting throated worm, 21.16–21.18
- definition, 2.40, 2.43
- grinding threads, 21.18–21.20
- hobbing threads, 21.3–21.6

Worms (*Cont.*):

- inspection of, 23.38–23.42
- milling threads, 21.2, 21.3
- optional finishing, 21.23
- rolled threads, 17.16, 17.17
- rolling, 21.21, 21.22
- sample drawing, 9.20, 9.21
- tolerances, 7.14

Yielding, 11.51, 11.52

Zerol bevel gears:

- description, 2.11, 2.13
- formulas, 2.42
- nomenclature, 2.40

Zinc-base die castings, 8.33, 8.34

Zinc die castings, 8.33, 8.34