

Newtonian Physics

Benjamin Crowell



Book 1 in the **Light and Matter** series of introductory physics textbooks
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- 1 Newtonian Physics
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- 5 Optics
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To Paul Herrschaft and Rich Muller.



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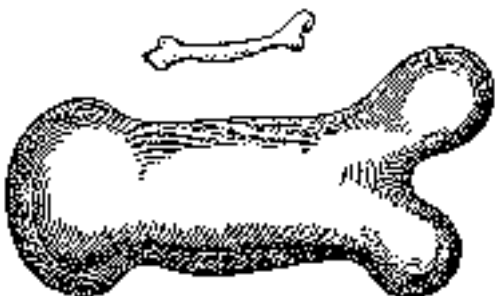


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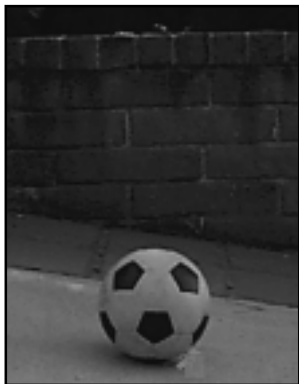
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2 Velocity and Relative Motion

2.1 Types of Motion



Rotation.



Simultaneous rotation and motion through space.



One person might say that the tipping chair was only rotating in a circle about its point of contact with the floor, but another could describe it as having both rotation and motion through space.

If you had to think consciously in order to move your body, you would be severely disabled. Even walking, which we consider to be no great feat, requires an intricate series of motions that your cerebrum would be utterly incapable of coordinating. The task of putting one foot in front of the other is controlled by the more primitive parts of your brain, the ones that have not changed much since the mammals and reptiles went their separate evolutionary ways. The thinking part of your brain limits itself to general directives such as “walk faster,” or “don’t step on her toes,” rather than micromanaging every contraction and relaxation of the hundred or so muscles of your hips, legs, and feet.

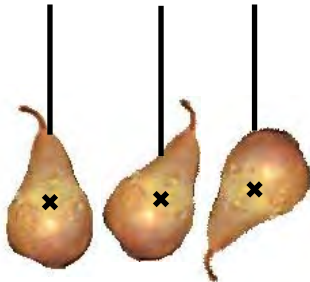
Physics is all about the conscious understanding of motion, but we’re obviously not immediately prepared to understand the most complicated types of motion. Instead, we’ll use the divide-and-conquer technique. We’ll first classify the various types of motion, and then begin our campaign with an attack on the simplest cases. To make it clear what we are and are not ready to consider, we need to examine and define carefully what types of motion can exist.

Rigid-body motion distinguished from motion that changes an object’s shape

Nobody, with the possible exception of Fred Astaire, can simply glide forward without bending their joints. Walking is thus an example in which there is both a general motion of the whole object and a change in the shape of the object. Another example is the motion of a jiggling water balloon as it flies through the air. We are not presently attempting a mathematical description of the way in which the shape of an object changes. Motion without a change in shape is called rigid-body motion. (The word “body” is often used in physics as a synonym for “object.”)

Center-of-mass motion as opposed to rotation

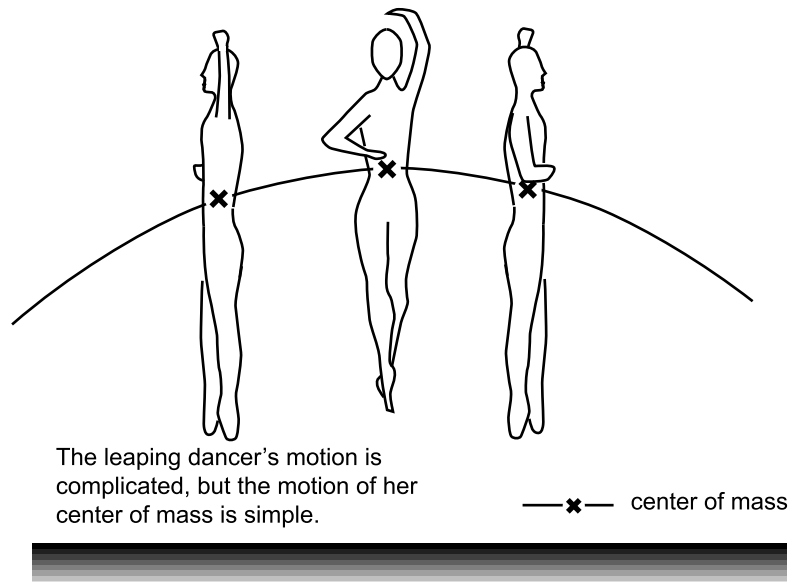
A ballerina leaps into the air and spins around once before landing. We feel intuitively that her rigid-body motion while her feet are off the ground consists of two kinds of motion going on simultaneously: a rotation and a motion of her body as a whole through space, along an arc. It is not immediately obvious, however, what is the most useful way to define the distinction between rotation and motion through space. Imagine that you attempt to balance a chair and it falls over. One person might say that the only motion was a rotation about the chair’s point of contact with the floor, but another might say that there was both rotation and motion down and to the side.



No matter what point you hang the pear from, the string lines up with the pear's center of mass. The center of mass can therefore be defined as the intersection of all the lines made by hanging the pear in this way. Note that the X in the figure should not be interpreted as implying that the center of mass is on the surface — it is actually inside the pear.

The motion of an object's center of mass is usually much simpler than the motion of any other point on it.

The same leaping dancer, viewed from above. Her center of mass traces a straight line, but a point away from her center of mass, such as her elbow, traces the much more complicated path shown by the dots.

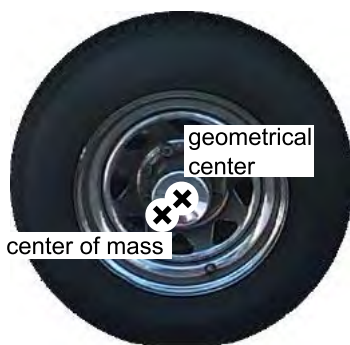


It turns out that there is one particularly natural and useful way to make a clear definition, but it requires a brief digression. Every object has a balance point, referred to in physics as the *center of mass*. For a two-dimensional object such as a cardboard cutout, the center of mass is the point at which you could hang the object from a string and make it balance. In the case of the ballerina (who is likely to be three-dimensional unless her diet is particularly severe), it might be a point either inside or outside her body, depending on how she holds her arms. Even if it is not practical to attach a string to the balance point itself, the center of mass can be defined as shown in the figure on the left.

Why is the center of mass concept relevant to the question of classifying rotational motion as opposed to motion through space? As illustrated in the figure above, it turns out that the motion of an object's center of mass is nearly always far simpler than the motion of any other part of the object. The ballerina's body is a large object with a complex shape. We might expect that her motion would be much more complicated than the motion of a small, simply-shaped object, say a marble, thrown up at the same angle as the angle at which she leapt. But it turns out that the motion of the ballerina's center of mass is exactly the same as the motion of the marble. That is, the motion of the center of mass is the same as the motion the ballerina would have if all her mass was concentrated at a point. By restricting our attention to the motion of the center of mass, we can therefore simplify things greatly.



We can now replace the ambiguous idea of “motion as a whole through space” with the more useful and better defined concept of “*center-of-mass motion*.” The motion of any rigid body can be cleanly split into rotation and center-of-mass motion. By this definition, the tipping chair does have both rotational and center-of-mass motion. Concentrating on the center of



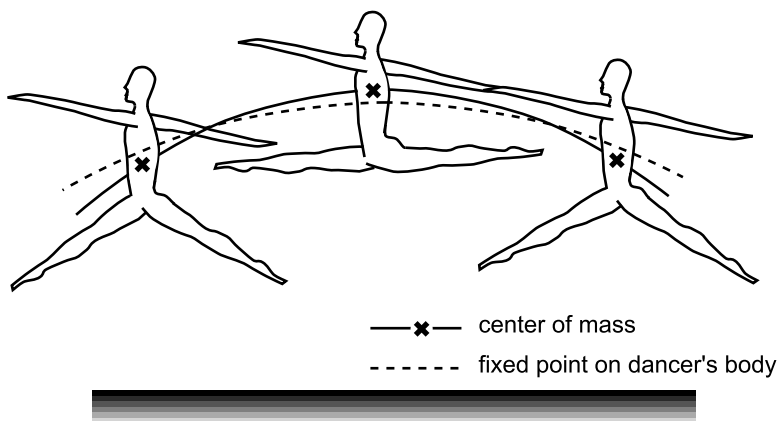
An improperly balanced wheel has a center of mass that is not at its geometric center. When you get a new tire, the mechanic clamps little weights to the rim to balance the wheel.

mass motion allows us to make a simplified *model* of the motion, as if a complicated object like a human body was just a marble or a point-like particle. Science really never deals with reality; it deals with models of reality.

Note that the word “center” in “center of mass” is not meant to imply that the center of mass must lie at the geometrical center of an object. A car wheel that has not been balanced properly has a center of mass that does not coincide with its geometrical center. An object such as the human body does not even have an obvious geometrical center.

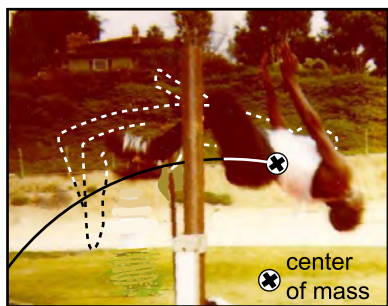
It can be helpful to think of the center of mass as the average location of all the mass in the object. With this interpretation, we can see for example that raising your arms above your head raises your center of mass, since the

A fixed point on the dancer's body follows a trajectory that is flatter than what we expect, creating an illusion of flight.



higher position of the arms' mass raises the average.

Ballerinas and professional basketball players can create an illusion of flying horizontally through the air because our brains intuitively expect them to have rigid-body motion, but the body does not stay rigid while executing a grand jete or a slam dunk. The legs are low at the beginning and end of the jump, but come up higher at the middle. Regardless of what the limbs do, the center of mass will follow the same arc, but the low position of the legs at the beginning and end means that the torso is higher compared to the center of mass, while in the middle of the jump it is lower compared to the center of mass. Our eye follows the motion of the torso and tries to interpret it as the center-of-mass motion of a rigid body. But since the torso follows a path that is flatter than we expect, this attempted interpretation fails, and we experience an illusion that the person is flying horizontally. Another interesting example from the sports world is the high jump, in which the jumper's curved body passes over the bar, but the center of mass passes under the bar! Here the jumper lowers his legs and upper body at the peak of the jump in order to bring his waist higher compared to the center of mass.



The high-jumper's body passes over the bar, but his center of mass passes under it.

Photo by Dunia Young.

Later in this course, we'll find that there are more fundamental reasons (based on Newton's laws of motion) why the center of mass behaves in such a simple way compared to the other parts of an object. We're also postponing any discussion of numerical methods for finding an object's center of mass. Until later in the course, we will only deal with the motion of objects'

centers of mass.

Center-of-mass motion in one dimension

In addition to restricting our study of motion to center-of-mass motion, we will begin by considering only cases in which the center of mass moves along a straight line. This will include cases such as objects falling straight down, or a car that speeds up and slows down but does not turn.

Note that even though we are not explicitly studying the more complex aspects of motion, we can still analyze the center-of-mass motion while ignoring other types of motion that might be occurring simultaneously. For instance, if a cat is falling out of a tree and is initially upside-down, it goes through a series of contortions that bring its feet under it. This is definitely not an example of rigid-body motion, but we can still analyze the motion of the cat's center of mass just as we would for a dropping rock.



Check

Consider a person running, a person pedaling on a bicycle, a person coasting on a bicycle, and a person coasting on ice skates. In which cases is the center-of-mass motion one-dimensional? Which cases are examples of rigid-body motion?

2.2 Describing Distance and Time

Center-of-mass motion in one dimension is particularly easy to deal with because all the information about it can be encapsulated in two variables: x , the position of the center of mass relative to the origin, and t , which measures a point in time. For instance, if someone supplied you with a sufficiently detailed table of x and t values, you would know pretty much all there was to know about the motion of the object's center of mass.

A point in time as opposed to duration

In ordinary speech, we use the word “time” in two different senses, which are to be distinguished in physics. It can be used, as in “a short time” or “our time here on earth,” to mean a length or duration of time, or it can be used to indicate a clock reading, as in “I didn’t know what time it was,” or “now’s the time.” In symbols, t is ordinarily used to mean a point in time, while Δt signifies an interval or duration in time. The capital Greek letter delta, Δ , means “the change in...,” i.e. a duration in time is the change or difference between one clock reading and another. The notation Δt does not signify the product of two numbers, Δ and t , but rather one single number, Δt . If a matinee begins at a point in time $t=1$ o’clock and ends at $t=3$ o’clock, the duration of the movie was the change in t ,

$$\Delta t = 3 \text{ hours} - 1 \text{ hour} = 2 \text{ hours} .$$

To avoid the use of negative numbers for Δt , we write the clock reading “after” to the left of the minus sign, and the clock reading “before” to the right of the minus sign. A more specific definition of the delta notation is therefore that delta stands for “after minus before.”

Even though our definition of the delta notation guarantees that Δt is positive, there is no reason why t can’t be negative. If t could not be negative, what would have happened one second before $t=0$? That doesn’t mean



Coasting on a bike and coasting on skates give one-dimensional center-of-mass motion, but running and pedaling require moving body parts up and down, which makes the center of mass move up and down. The only example of rigid-body motion is coasting on skates. (Coasting on a bike is not rigid-body motion, because the wheels twist.)

that time “goes backward” in the sense that adults can shrink into infants and retreat into the womb. It just means that we have to pick a reference point and call it $t=0$, and then times before that are represented by negative values of t .

Although a point in time can be thought of as a clock reading, it is usually a good idea to avoid doing computations with expressions such as “2:35” that are combinations of hours and minutes. Times can instead be expressed entirely in terms of a single unit, such as hours. Fractions of an hour can be represented by decimals rather than minutes, and similarly if a problem is being worked in terms of minutes, decimals can be used instead of seconds.

Self-Check



Of the following phrases, which refer to points in time, which refer to time intervals, and which refer to time in the abstract rather than as a measurable number?

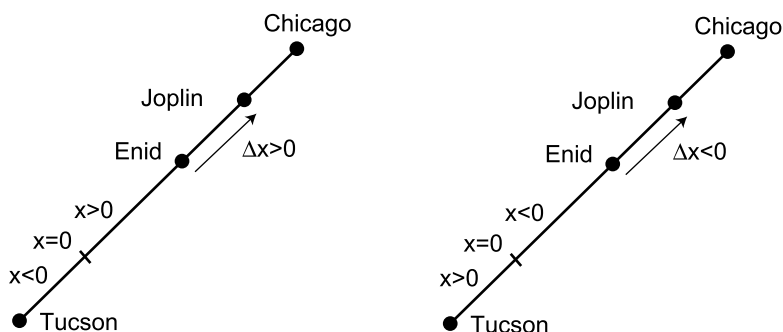
- (a) “The time has come.”
- (b) “Time waits for no man.”
- (c) “The whole time, he had spit on his chin.”

Position as opposed to change in position

As with time, a distinction should be made between a point in space, symbolized as a coordinate x , and a change in position, symbolized as Δx .

As with t , x can be negative. If a train is moving down the tracks, not only do you have the freedom to choose any point along the tracks and call it $x=0$, but it’s also up to you to decide which side of the $x=0$ point is positive x and which side is negative x .

Since we’ve defined the delta notation to mean “after minus before,” it is possible that Δx will be negative, unlike Δt which is guaranteed to be positive. Suppose we are describing the motion of a train on tracks linking Tucson and Chicago. As shown in the figure, it is entirely up to you to decide which way is positive.



Two equally valid ways of describing the motion of a train from Tucson to Chicago. In the first example, the train has a positive Δx as it goes from Enid to Joplin. In the second example, the same train going forward in the same direction has a negative Δx .



(a) a point in time; (b) time in the abstract sense; (c) a time interval

Note that in addition to x and Δx , there is a third quantity we could define, which would be like an odometer reading, or actual distance traveled. If you drive 10 miles, make a U-turn, and drive back 10 miles, then your Δx is zero, but your car's odometer reading has increased by 20 miles. However important the odometer reading is to car owners and used car dealers, it is not very important in physics, and there is not even a standard name or notation for it. The change in position, Δx , is more useful because it is so much easier to calculate: to compute Δx , we only need to know the beginning and ending positions of the object, not all the information about how it got from one position to the other.

Self-Check



A ball hits the floor, bounces to a height of one meter, falls, and hits the floor again. Is the Δx between the two impacts equal to zero, one, or two meters?

Frames of reference

The example above shows that there are two arbitrary choices you have to make in order to define a position variable, x . You have to decide where to put $x=0$, and also which direction will be positive. This is referred to as choosing a *coordinate system* or choosing a *frame of reference*. (The two terms are nearly synonymous, but the first focuses more on the actual x variable, while the second is more of a general way of referring to one's point of view.) As long as you are consistent, any frame is equally valid. You just don't want to change coordinate systems in the middle of a calculation.

Have you ever been sitting in a train in a station when suddenly you notice that the station is moving backward? Most people would describe the situation by saying that you just failed to notice that the train was moving — it only seemed like the station was moving. But this shows that there is yet a third arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange that anyone would bother with a coordinate system that was moving relative to the earth, but for instance the frame of reference moving along with a train might be far more convenient for describing things happening inside the train.



Zero, because the “after” and “before” values of x are the same.

2.3 Graphs of Motion; Velocity.

Motion with constant velocity

In example (a), an object is moving at constant speed in one direction. We can tell this because every two seconds, its position changes by five meters.

In algebra notation, we'd say that the graph of x vs. t shows the same change in position, $\Delta x = 5.0$ m, over each interval of $\Delta t = 2.0$ s. The object's velocity or speed is obtained by calculating $v = \Delta x / \Delta t = (5.0 \text{ m}) / (2.0 \text{ s}) = 2.5 \text{ m/s}$. In graphical terms, the velocity can be interpreted as the slope of the line. Since the graph is a straight line, it wouldn't have mattered if we'd taken a longer time interval and calculated $v = \Delta x / \Delta t = (10.0 \text{ m}) / (4.0 \text{ s})$. The answer would still have been the same, 2.5 m/s.

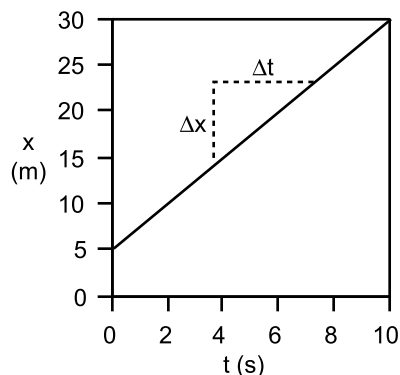
Note that when we divide a number that has units of meters by another number that has units of seconds, we get units of meters per second, which can be written m/s. This is another case where we treat units as if they were algebra symbols, even though they're not.

In example (b), the object is moving in the opposite direction: as time progresses, its x coordinate decreases. Recalling the definition of the Δ notation as "after minus before," we find that Δt is still positive, but Δx must be negative. The slope of the line is therefore negative, and we say that the object has a negative velocity, $v = \Delta x / \Delta t = (-5.0 \text{ m}) / (2.0 \text{ s}) = -2.5 \text{ m/s}$. We've already seen that the plus and minus signs of Δx values have the interpretation of telling us which direction the object moved. Since Δt is always positive, dividing by Δt doesn't change the plus or minus sign, and the plus and minus signs of velocities are to be interpreted in the same way. In graphical terms, a positive slope characterizes a line that goes up as we go to the right, and a negative slope tells us that the line went down as we went to the right.

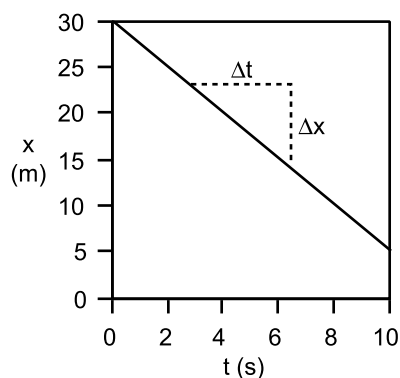
Motion with changing velocity

Now what about a graph like example (c)? This might be a graph of a car's motion as the driver cruises down the freeway, then slows down to look at a car crash by the side of the road, and then speeds up again, disappointed that there is nothing dramatic going on such as flames or babies trapped in their car seats. (Note that we are still talking about one-dimensional motion. Just because the graph is curvy doesn't mean that the car's path is curvy. The graph is not like a map, and the horizontal direction of the graph represents the passing of time, not distance.)

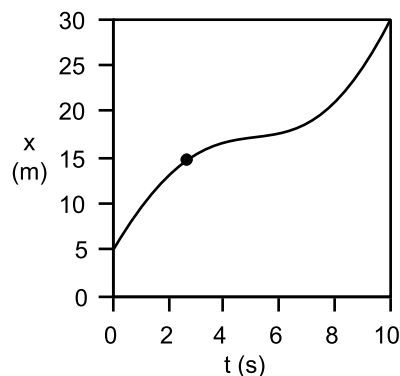
Example (c) is similar to example (a) in that the object moves a total of 25.0 m in a period of 10.0 s, but it is no longer true that it makes the same amount of progress every second. There is no way to characterize the entire graph by a certain velocity or slope, because the velocity is different at every moment. It would be incorrect to say that because the car covered 25.0 m in 10.0 s, its velocity was 2.5 m/s. It moved faster than that at the beginning and end, but slower in the middle. There may have been certain instants at which the car was indeed going 2.5 m/s, but the speedometer swept past that value without "sticking," just as it swung through various other values of speed. (I definitely want my next car to have a speedometer calibrated in m/s and showing both negative and positive values.)



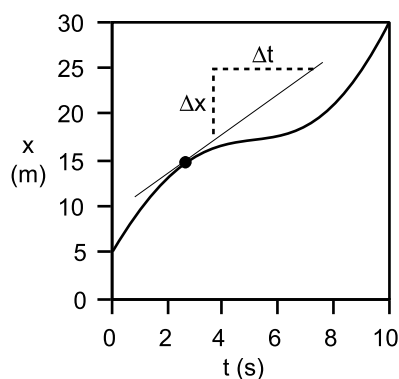
(a) Motion with constant velocity.



(b) Motion that decreases x is represented with negative values of Δx and v .



(c) Motion with changing velocity.



(d) The velocity at any given moment is defined as the slope of the tangent line through the relevant point on the graph.

definition of velocity

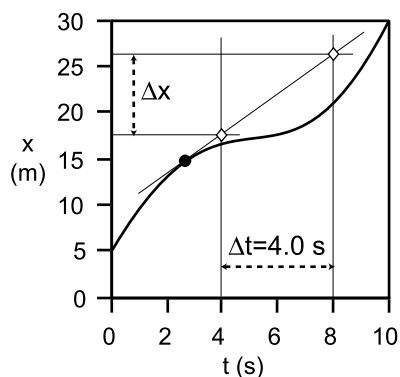
The velocity of an object at any given moment is the slope of the tangent line through the relevant point on its x - t graph.

One interpretation of this definition is that the velocity tells us how many meters the object would have traveled in one second, if it had continued moving at the same speed for at least one second. To some people the graphical nature of this definition seems “inaccurate” or “not mathematical.” The equation $v = \Delta x / \Delta t$ by itself, however, is only valid if the velocity is constant, and so cannot serve as a general definition.

Example

Question: What is the velocity at the point shown with a dot on the graph?

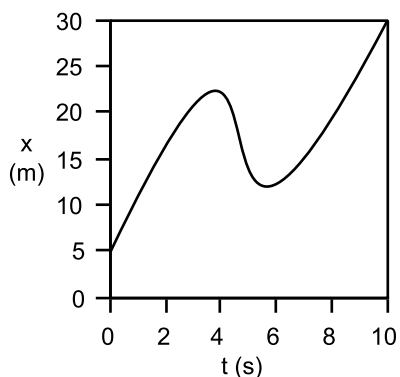
Solution: First we draw the tangent line through that point. To find the slope of the tangent line, we need to pick two points on it. Theoretically, the slope should come out the same regardless of which two points we picked, but in practical terms we’ll be able to measure more accurately if we pick two points fairly far apart, such as the two white diamonds. To save work, we pick points that are directly above labeled points on the t axis, so that $\Delta t = 4.0$ s is easy to read off. One diamond lines up with $x \approx 17.5$ m, the other with $x \approx 26.5$ m, so $\Delta x = 9.0$ m. The velocity is $\Delta x / \Delta t = 2.2$ m/s.



Example: finding the velocity at the point indicated with the dot.

Conventions about graphing

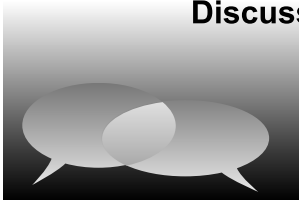
The placement of t on the horizontal axis and x on the upright axis may seem like an arbitrary convention, or may even have disturbed you, since your algebra teacher always told you that x goes on the horizontal axis and y goes on the upright axis. There is a reason for doing it this way, however. In example (e), we have an object that reverses its direction of motion twice. It can only be in one place at any given time, but there can be more than one time when it is at a given place. For instance, this object passed through $x = 17$ m on three separate occasions, but there is no way it could have been in more than one place at $t = 5.0$ s. Resurrecting some terminology you learned in your trigonometry course, we say that x is a function of t , but t is not a function of x . In situations such as this, there is a useful convention that the graph should be oriented so that any vertical line passes through the curve at only one point. Putting the x axis across the page and t upright would have violated this convention. To people who are used to interpreting graphs, a graph that violates this convention is as annoying as



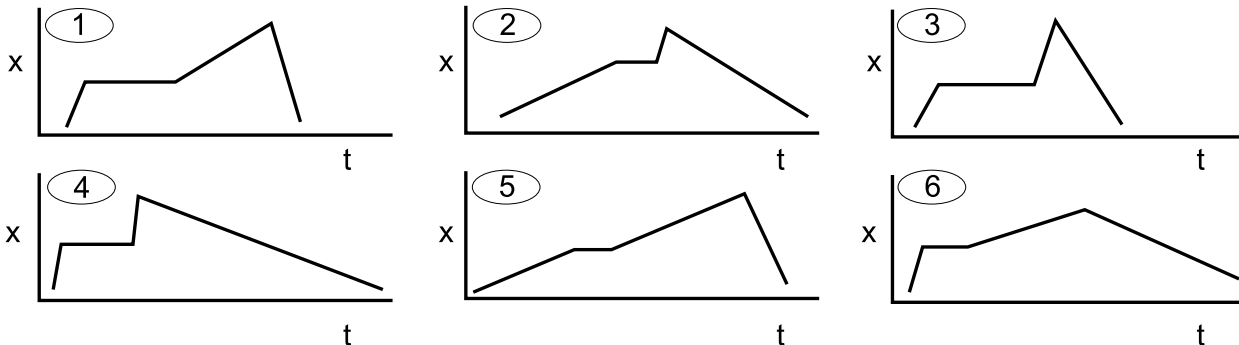
(e) Reversing the direction of motion.

fingernails scratching on a chalkboard. We say that this is a graph of “ x versus t .” If the axes were the other way around, it would be a graph of “ t versus x .” I remember the “versus” terminology by visualizing the labels on the x and t axes and remembering that when you read, you go from left to right and from top to bottom.

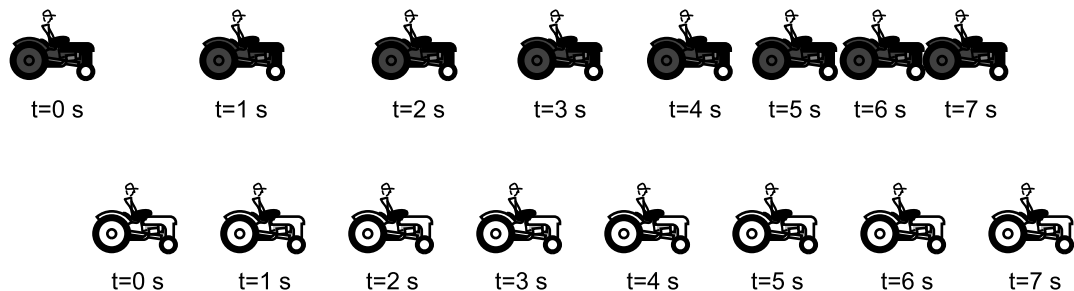
Discussion questions



A. An ant walks forward, pauses, then runs quickly ahead. It then suddenly reverses direction and walks slowly back the way it came. Which graph could represent its motion?



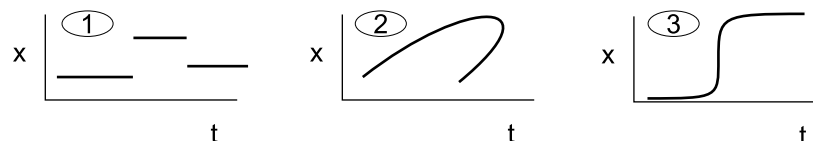
B. The figure shows a sequence of positions for two racing tractors. Compare the tractors’ velocities as the race progresses.

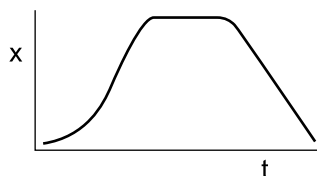


C. If an object had a straight-line motion graph with $\Delta x=0$ and $\Delta t \neq 0$, what would be true about its velocity? What would this look like on a graph? What about $\Delta t=0$ and $\Delta x \neq 0$?

D. If an object has a wavy motion graph like the one in example (e) on the previous page, which are the points at which the object reverses its direction? What is true about the object’s velocity at these points?

E. Discuss anything unusual about the following three graphs.





Discussion question G.

F. I have been using the term “velocity” and avoiding the more common English word “speed,” because some introductory physics texts define them to mean different things. They use the word “speed,” and the symbol “ s ” to mean the absolute value of the velocity, $s=|v|$. Although I have thrown in my lot with the minority of books that don’t emphasize this distinction in technical vocabulary, there are clearly two different concepts here. Can you think of an example of a graph of x vs. t in which the object has constant speed, but not constant velocity?

G. In the graph on the left, describe how the object’s velocity changes.

H. Two physicists duck out of a boring scientific conference to go get beer. On the way to the bar, they witness an accident in which a pedestrian is injured by a hit-and-run driver. A criminal trial results, and they must testify. In her testimony, Dr. Transverz Waive says, “The car was moving along pretty fast, I’d say the velocity was +40 mi/hr. They saw the old lady too late, and even though they slammed on the brakes they still hit her before they stopped. Then they made a U turn and headed off at a velocity of about -20 mi/hr, I’d say.” Dr. Longitud N.L. Vibrasheun says, “He was really going too fast, maybe his velocity was -35 or -40 mi/hr. After he hit Mrs. Hapless, he turned around and left at a velocity of, oh, I’d guess maybe +20 or +25 mi/hr.” Is their testimony contradictory? Explain.

2.4 The Principle of Inertia

Physical effects relate only to a change in velocity

Consider two statements that were at one time made with the utmost seriousness:

People like Galileo and Copernicus who say the earth is rotating must be crazy. We know the earth can't be moving. Why, if the earth was really turning once every day, then our whole city would have to be moving hundreds of leagues in an hour. That's impossible! Buildings would shake on their foundations. Gale-force winds would knock us over. Trees would fall down. The Mediterranean would come sweeping across the east coasts of Spain and Italy. And furthermore, what force would be making the world turn?

All this talk of passenger trains moving at forty miles an hour is sheer hogwash! At that speed, the air in a passenger compartment would all be forced against the back wall. People in the front of the car would suffocate, and people at the back would die because in such concentrated air, they wouldn't be able to expel a breath.

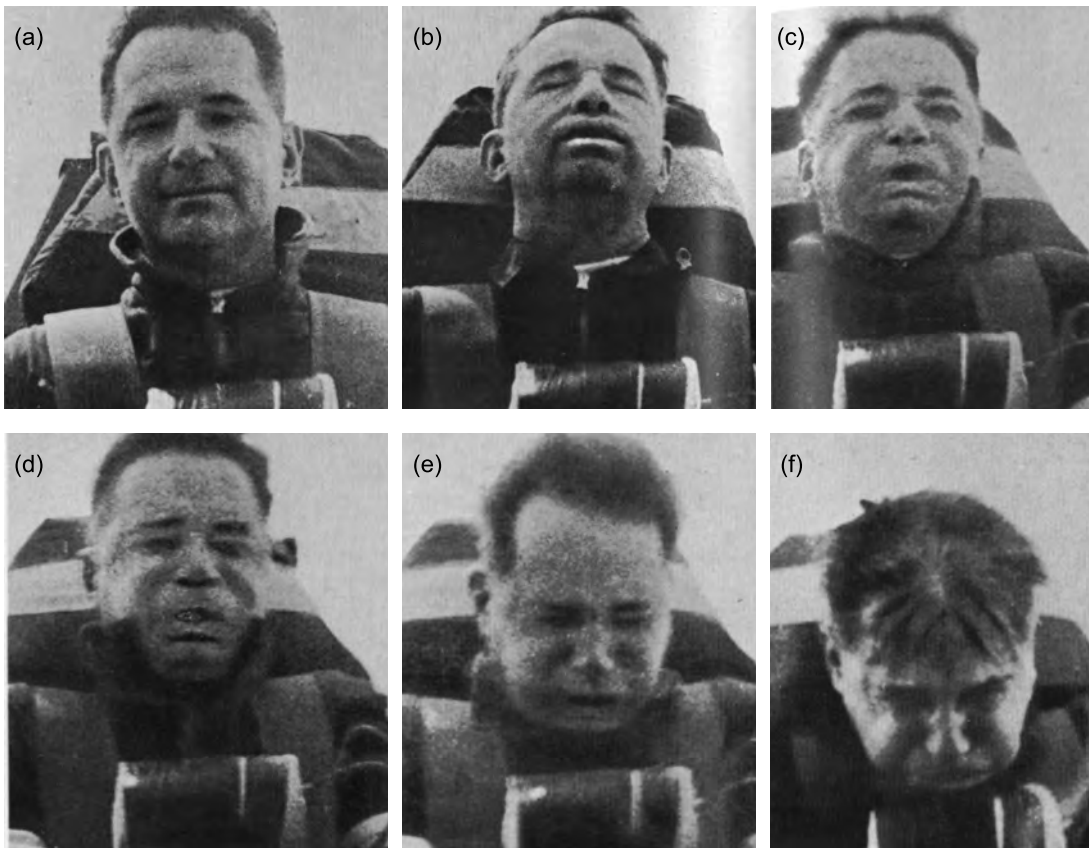
Some of the effects predicted in the first quote are clearly just based on a lack of experience with rapid motion that is smooth and free of vibration. But there is a deeper principle involved. In each case, the speaker is assuming that the mere fact of motion must have dramatic physical effects. More subtly, they also believe that a force is needed to keep an object in motion: the first person thinks a force would be needed to maintain the earth's rotation, and the second apparently thinks of the rear wall as pushing on the air to keep it moving.

Common modern knowledge and experience tell us that these people's predictions must have somehow been based on incorrect reasoning, but it is not immediately obvious where the fundamental flaw lies. It's one of those things a four-year-old could infuriate you by demanding a clear explanation of. One way of getting at the fundamental principle involved is to consider how the modern concept of the universe differs from the popular conception at the time of the Italian Renaissance. To us, the word "earth" implies a planet, one of the nine planets of our solar system, a small ball of rock and dirt that is of no significance to anyone in the universe except for members of our species, who happen to live on it. To Galileo's contemporaries, however, the earth was the biggest, most solid, most important thing in all of creation, not to be compared with the wandering lights in the sky known as planets. To us, the earth is just another object, and when we talk loosely about "how fast" an object such as a car "is going," we really mean the car-object's velocity relative to the earth-object.

Motion is relative

According to our modern world-view, it really isn't that reasonable to expect that a special force should be required to make the air in the train have a certain velocity relative to our planet. After all, the "moving" air in the "moving" train might just happen to have zero velocity relative to some other planet we don't even know about. Aristotle claimed that things "naturally" wanted to be at rest, lying on the surface of the earth. But experiment after experiment has shown that there is really nothing so

There is nothing special about motion or lack of motion relative to the planet earth.



This Air Force doctor volunteered to ride a rocket sled as a medical experiment. The obvious effects on his head and face are not because of the sled's speed but because of its rapid changes in speed: increasing in (b) and (c), and decreasing in (e) and (f). In (d) his speed is greatest, but because his speed is not increasing or decreasing very much at this moment, there is little effect on him.

special about being at rest relative to the earth. For instance, if a mattress falls out of the back of a truck on the freeway, the reason it rapidly comes to rest with respect to the planet is simply because of friction forces exerted by the asphalt, which happens to be attached to the planet.

Galileo's insights are summarized as follows:

The Principle of Inertia

No force is required to maintain motion with constant velocity in a straight line, and absolute motion does not cause any observable physical effects.

There are many examples of situations that seem to disprove the principle of inertia, but these all result from forgetting that friction is a force. For instance, it seems that a force is needed to keep a sailboat in motion. If the wind stops, the sailboat stops too. But the wind's force is not the only force on the boat; there is also a frictional force from the water. If the sailboat is cruising and the wind suddenly disappears, the backward frictional force still exists, and since it is no longer being counteracted by the wind's forward force, the boat stops. To disprove the principle of inertia, we would have to find an example where a moving object slowed down even though no forces whatsoever were acting on it.

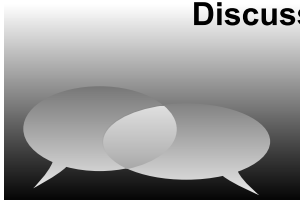
Self-Check



What is incorrect about the following supposed counterexamples to the principle of inertia?

- (1) When astronauts blast off in a rocket, their huge velocity does cause a physical effect on their bodies — they get pressed back into their seats, the flesh on their faces gets distorted, and they have a hard time lifting their arms.
- (2) When you're driving in a convertible with the top down, the wind in your face is an observable physical effect of your absolute motion.

Discussion questions



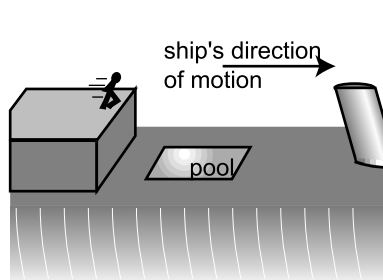
A. A passenger on a cruise ship finds, while the ship is docked, that he can leap off of the upper deck and just barely make it into the pool on the lower deck. If the ship leaves dock and is cruising rapidly, will this adrenaline junkie still be able to make it?

B. You are a passenger in the open basket hanging under a helium balloon. The balloon is being carried along by the wind at a constant velocity. If you are holding a flag in your hand, will the flag wave? If so, which way? [Based on a question from PSSC Physics.]

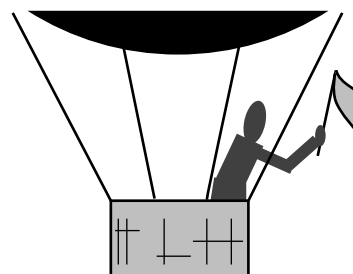
C. Aristotle stated that all objects naturally wanted to come to rest, with the unspoken implication that "rest" would be interpreted relative to the surface of the earth. Suppose we could transport Aristotle to the moon, put him in a space suit, and kick him out the door of the spaceship and into the lunar landscape. What would he expect his fate to be in this situation? If intelligent



Discussion question D.



Discussion question A.



Discussion question B.

creatures inhabited the moon, and one of them independently came up with the equivalent of Aristotelian physics, what would they think about objects coming to rest?

D. The bottle is sitting on a level table in a train's dining car, but the surface of the beer is tilted. What can you infer about the motion of the train?



(1) The effect only occurs during blastoff, when their velocity is changing. Once the rocket engines stop firing, their velocity stops changing, and they no longer feel any effect. (2) It is only an observable effect of your motion relative to the air.

2.5 Addition of Velocities

Addition of velocities to describe relative motion

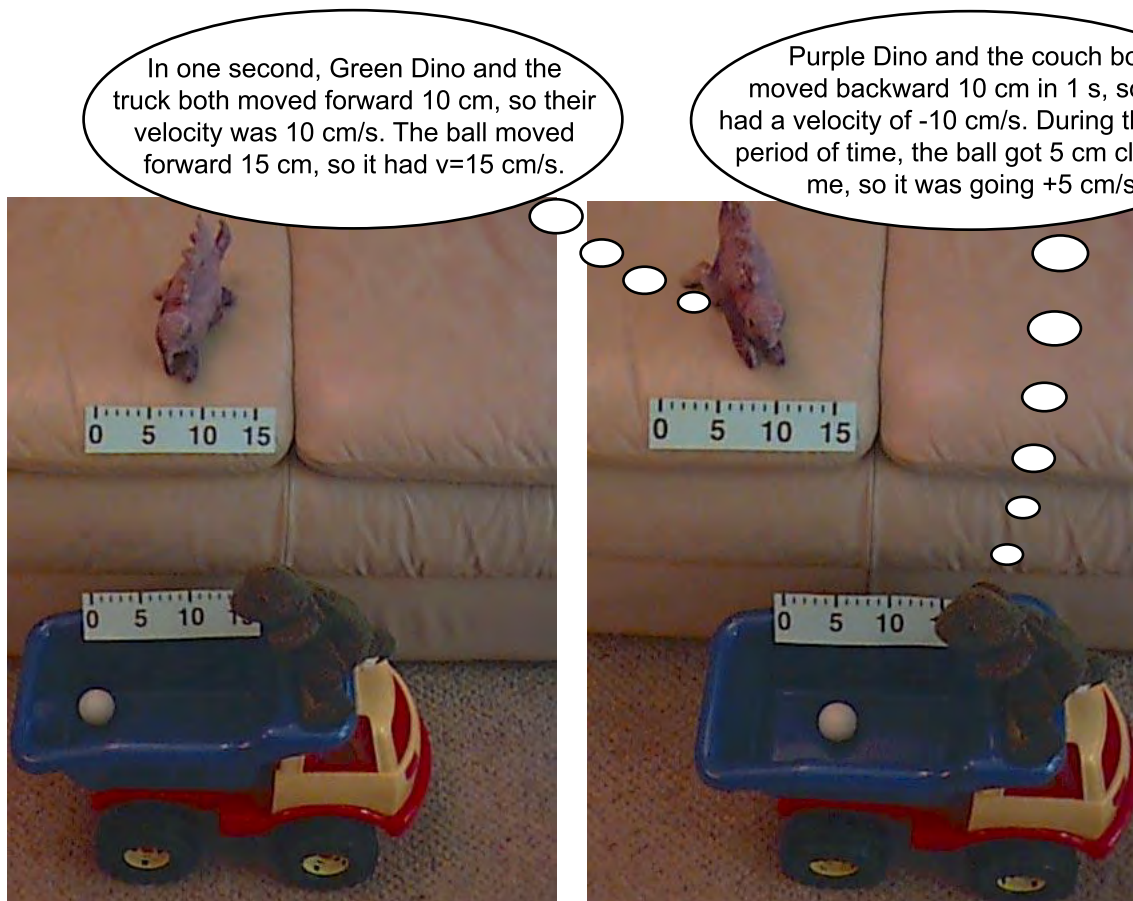
Since absolute motion cannot be unambiguously measured, the only way to describe motion unambiguously is to describe the motion of one object relative to another. Symbolically, we can write v_{PQ} for the velocity of object P relative to object Q.

*Relative velocities
add together.*

Velocities measured with respect to different reference points can be compared by addition. In the figure below, the ball's velocity relative to the couch equals the ball's velocity relative to the truck plus the truck's velocity relative to the couch:

$$\begin{aligned} v_{BC} &= v_{BT} + v_{TC} \\ &= 5 \text{ cm/s} + 10 \text{ cm/s} \end{aligned}$$

The same equation can be used for any combination of three objects, just by substituting the relevant subscripts for B, T, and C. Just remember to write the equation so that the velocities being added have the same subscript twice in a row. In this example, if you read off the subscripts going from left to right, you get BC...=...BTTC. The fact that the two "inside" subscripts on the right are the same means that the equation has been set up correctly. Notice how subscripts on the left look just like the subscripts on the right, but with the two T's eliminated.



These two highly competent physicists disagree on absolute velocities, but they would agree on relative velocities. Purple Dino considers the couch to be at rest, while Green Dino thinks of the truck as being at rest. They agree, however, that the truck's velocity relative to the couch is $v_{TC}=10$ cm/s, the ball's velocity relative to the truck is $v_{BT}=5$ cm/s, and the ball's velocity relative to the couch is $v_{BC}=v_{BT}+v_{TC}=15$ cm/s.

Negative velocities in relative motion

My discussion of how to interpret positive and negative signs of velocity may have left you wondering why we should bother. Why not just make velocity positive by definition? The original reason why negative numbers were invented was that bookkeepers decided it would be convenient to use the negative number concept for payments to distinguish them from receipts. It was just plain easier than writing receipts in black and payments in red ink. After adding up your month's positive receipts and negative payments, you either got a positive number, indicating profit, or a negative number, showing a loss. You could then show the that total with a high-tech "+" or "-" sign, instead of looking around for the appropriate bottle of ink.

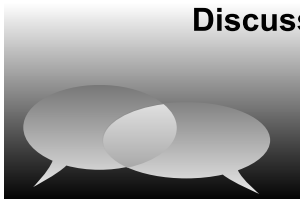
If you consistently label velocities as positive or negative depending on their directions, then adding velocities will also give signs that consistently relate to direction.

Nowadays we use positive and negative numbers for all kinds of things, but in every case the point is that it makes sense to add and subtract those things according to the rules you learned in grade school, such as "minus a minus makes a plus, why this is true we need not discuss." Adding velocities has the significance of comparing relative motion, and with this interpretation negative and positive velocities can be used within a consistent framework. For example, the truck's velocity relative to the couch equals the truck's velocity relative to the ball plus the ball's velocity relative to the couch:

$$\begin{aligned}v_{TC} &= v_{TB} + v_{BC} \\&= -5 \text{ cm/s} + 15 \text{ cm/s} \\&= 10 \text{ cm/s}\end{aligned}$$

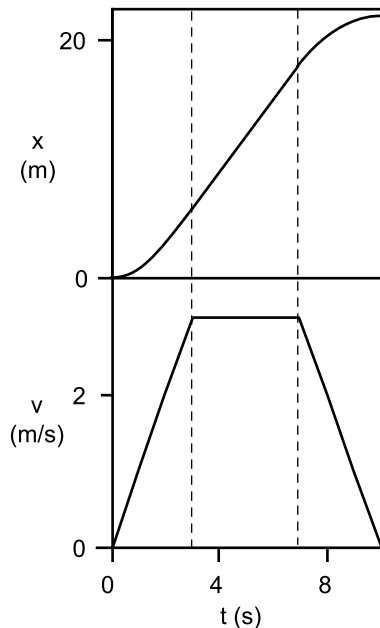
If we didn't have the technology of negative numbers, we would have had to remember a complicated set of rules for adding velocities: (1) if the two objects are both moving forward, you add, (2) if one is moving forward and one is moving backward, you subtract, but (3) if they're both moving backward, you add. What a pain that would have been.

Discussion questions



- A. Interpret the general rule $v_{AB} = -v_{BA}$ in words.
- B. If we have a specific situation where $v_{AB} + v_{BC} = 0$, what is going on?

2.6 Graphs of Velocity Versus Time



Since changes in velocity play such a prominent role in physics, we need a better way to look at changes in velocity than by laboriously drawing tangent lines on x -versus- t graphs. A good method is to draw a graph of velocity versus time. The examples on the left show the x - t and v - t graphs that might be produced by a car starting from a traffic light, speeding up, cruising for a while at constant speed, and finally slowing down for a stop sign. If you have an air freshener hanging from your rear-view mirror, then you will see an effect on the air freshener during the beginning and ending periods when the velocity is changing, but it will not be tilted during the period of constant velocity represented by the flat plateau in the middle of the v - t graph.

Students often mix up the things being represented on these two types of graphs. For instance, many students looking at the top graph say that the car is speeding up the whole time, since “the graph is becoming greater.” What is getting greater throughout the graph is x , not v .

Similarly, many students would look at the bottom graph and think it showed the car backing up, because “it’s going backwards at the end.” But what is decreasing at the end is v , not x . Having both the x - t and v - t graphs in front of you like this is often convenient, because one graph may be easier to interpret than the other for a particular purpose. Stacking them like this means that corresponding points on the two graphs’ time axes are lined up with each other vertically. However, one thing that is a little counterintuitive about the arrangement is that in a situation like this one involving a car, one is tempted to visualize the landscape stretching along the horizontal axis of one of the graphs. The horizontal axes, however, represent time, not position. The correct way to visualize the landscape is by mentally rotating the horizon 90 degrees counterclockwise and imagining it stretching along the upright axis of the x - t graph, which is the only axis that represents different positions in space.

2.7 ∫ Applications of Calculus

The integral symbol, $\int$, in the heading for this section indicates that it is meant to be read by students in calculus-based physics. Students in an algebra-based physics course should skip these sections. The calculus-related sections in this book are meant to be usable by students who are taking calculus concurrently, so at this early point in the physics course I do not assume you know any calculus yet. This section is therefore not much more than a quick preview of calculus, to help you relate what you’re learning in the two courses.

Newton was the first person to figure out the tangent-line definition of velocity for cases where the x - t graph is nonlinear. Before Newton, nobody had conceptualized the description of motion in terms of x - t and v - t graphs. In addition to the graphical techniques discussed in this chapter, Newton also invented a set of symbolic techniques called calculus. If you have an equation for x in terms of t , calculus allows you, for instance, to find an equation for v in terms of t . In calculus terms, we say that the function $v(t)$

is the derivative of the function $x(t)$. In other words, the derivative of a function is a new function that tells how rapidly the original function was changing. We now use neither Newton's name for his technique (he called it "the method of fluxions") nor his notation. The more commonly used notation is due to Newton's German contemporary Leibnitz, whom the English accused of plagiarizing the calculus from Newton. In the Leibnitz notation, we write

$$v = \frac{dx}{dt}$$

to indicate that the function $v(t)$ equals the slope of the tangent line of the graph of $x(t)$ at every time t . The Leibnitz notation is meant to evoke the delta notation, but with a very small time interval. Because the dx and dt are thought of as very small Δx 's and Δt 's, i.e. very small differences, the part of calculus that has to do with derivatives is called differential calculus.

Differential calculus consists of three things:

- The concept and definition of the derivative, which is covered in this book, but which will be discussed more formally in your math course.
- The Leibnitz notation described above, which you'll need to get more comfortable with in your math course.
- A set of rules for that allows you to find an equation for the derivative of a given function. For instance, if you happened to have a situation where the position of an object was given by the equation $x=2t^7$, you would be able to use those rules to find $dx/dt=14t^6$. This bag of tricks is covered in your math course.

Summary

Selected Vocabulary

center of mass the balance point of an object
velocity the rate of change of position; the slope of the tangent line on an x - t graph.

Notation

x a point in space
 t a point in time, a clock reading
 Δ “change in;” the value of a variable afterwards minus its value before
 Δx a distance, or more precisely a change in x , which may be less than the distance traveled; its plus or minus sign indicates direction
 Δt a duration of time
 v velocity
 v_{AB} the velocity of object A relative to object B

Standard Terminology Avoided in This Book

displacement a name for the symbol Δx .
speed the absolute value of the velocity, i.e. the velocity stripped of any information about its direction

Summary

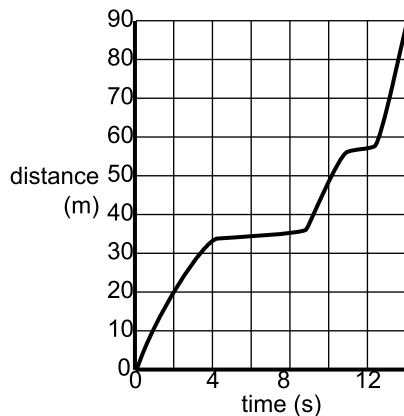
An object’s center of mass is the point at which it can be balanced. For the time being, we are studying the mathematical description only of the motion of an object’s center of mass in cases restricted to one dimension. The motion of an object’s center of mass is usually far simpler than the motion of any of its other parts.

It is important to distinguish location, x , from distance, Δx , and clock reading, t , from time interval Δt . When an object’s x - t graph is linear, we define its velocity as the slope of the line, $\Delta x/\Delta t$. When the graph is curved, we generalize the definition so that the velocity is the slope of the tangent line at a given point on the graph.

Galileo’s principle of inertia states that no force is required to maintain motion with constant velocity in a straight line, and absolute motion does not cause any observable physical effects. Things typically tend to reduce their velocity relative to the surface of our planet only because they are physically rubbing against the planet (or something attached to the planet), not because there is anything special about being at rest with respect to the earth’s surface. When it seems, for instance, that a force is required to keep a book sliding across a table, in fact the force is only serving to cancel the contrary force of friction.

Absolute motion is not a well-defined concept, and if two observers are not at rest relative to one another they will disagree about the absolute velocities of objects. They will, however, agree about relative velocities. If object A is in motion relative to object B, and B is in motion relative to C, then A’s velocity relative to C is given by $v_{AC} = v_{AB} + v_{BC}$. Positive and negative signs are used to indicate the direction of an object’s motion.

Homework Problems



Problem 1.

1 ✓. The graph shows the motion of a car stuck in stop-and-go freeway traffic. (a) If you only knew how far the car had gone during this entire time period, what would you think its velocity was? (b) What is the car's maximum velocity?

2. (a) Let θ be the latitude of a point on the Earth's surface. Derive an algebra equation for the distance, L , traveled by that point during one rotation of the Earth about its axis, i.e. over one day, expressed in terms of L , θ , and R , the radius of the earth. Check: Your equation should give $L=0$ for the North Pole.

(b✓) At what speed is Fullerton, at latitude $\theta=34^\circ$, moving with the rotation of the Earth about its axis? Give your answer in units of mi/h. [See the table in the back of the book for the relevant data.]

3 ★✓. A person is parachute jumping. During the time between when she leaps out of the plane and when she opens her chute, her altitude is given by the equation

$$y=(10000 \text{ m}) - (50 \text{ m/s})[t+(5.0 \text{ s})e^{-t/5.0 \text{ s}}] \text{ .}$$

Find her velocity at $t=7.0 \text{ s}$. (This can be done on a calculator, without knowing calculus.) Because of air resistance, her velocity does not increase at a steady rate as it would for an object falling in vacuum.

4 S. A light-year is a unit of distance used in astronomy, and defined as the distance light travels in one year. The speed of light is $3.0 \times 10^8 \text{ m/s}$. Find how many meters there are in one light-year, expressing your answer in scientific notation.

5 S. You're standing in a freight train, and have no way to see out. If you have to lean to stay on your feet, what, if anything, does that tell you about the train's velocity? Its acceleration? Explain.

6 ∫. A honeybee's position as a function of time is given by $x=10t-t^3$, where t is in seconds and x in meters. What is its velocity at $t=3.0 \text{ s}$?

S A solution is given in the back of the book.

✓ A computerized answer check is available.

★ A difficult problem.

∫ A problem that requires calculus.

Glossary

Acceleration. The rate of change of velocity; the slope of the tangent line on a v - t graph.

Attractive. Describes a force that tends to pull the two participating objects together. Cf. repulsive, oblique.

Center of mass. The balance point of an object.

Component. The part of a velocity, acceleration, or force that is along one particular coordinate axis.

Displacement. (avoided in this book) A name for the symbol Δx .

Fluid. A gas or a liquid.

Fluid friction. A friction force in which at least one of the object is a fluid (i.e. either a gas or a liquid).

Gravity. A general term for the phenomenon of attraction between things having mass. The attraction between our planet and a human-sized object causes the object to fall.

Inertial frame. A frame of reference that is not accelerating, one in which Newton's first law is true

Kinetic friction. A friction force between surfaces that are slipping past each other.

Light. Anything that can travel from one place to another through empty space and can influence matter, but is not affected by gravity.

Magnitude. The "amount" associated with a vector; the vector stripped of any information about its direction.

Mass. A numerical measure of how difficult it is to change an object's motion.

Matter. Anything that is affected by gravity.

Mks system. The use of metric units based on the meter, kilogram, and second. Example: meters per second is the mks unit of speed, not cm/s or km/hr.

Noninertial frame. An accelerating frame of reference, in which Newton's first law is violated

Nonuniform circular motion. Circular motion in which the magnitude of the velocity vector

changes

Normal force. The force that keeps two objects from occupying the same space.

Oblique. Describes a force that acts at some other angle, one that is not a direct repulsion or attraction. Cf. attractive, repulsive.

Operational definition. A definition that states what operations should be carried out to measure the thing being defined.

Parabola. The mathematical curve whose graph has y proportional to x^2 .

Radial. Parallel to the radius of a circle; the in-out direction. Cf. tangential.

Repulsive. Describes a force that tends to push the two participating objects apart. Cf. attractive, oblique.

Scalar. A quantity that has no direction in space, only an amount. Cf. vector.

Significant figures. Digits that contribute to the accuracy of a measurement.

Speed. (avoided in this book) The absolute value of or, in more than one dimension, the magnitude of the velocity, i.e. the velocity stripped of any information about its direction

Spring constant. The constant of proportionality between force and elongation of a spring or other object under strain.

Static friction. A friction force between surfaces that are not slipping past each other.

Système International.. Fancy name for the metric system.

Tangential. Tangent to a curve. In circular motion, used to mean tangent to the circle, perpendicular to the radial direction Cf. radial.

Uniform circular motion. Circular motion in which the magnitude of the velocity vector remains constant

Vector. A quantity that has both an amount (magnitude) and a direction in space. Cf. scalar.

Velocity. The rate of change of position; the slope of the tangent line on an x - t graph.

Weight. The force of gravity on an object, equal to mg .

Mathematical Review

Algebra

Quadratic equation:

The solutions of $ax^2 + bx + c = 0$

$$\text{are } x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Logarithms and exponentials:

$$\ln(ab) = \ln a + \ln b$$

$$e^{a+b} = e^a e^b$$

$$\ln e^x = e^{\ln x} = x$$

$$\ln(a^b) = b \ln a$$

Geometry, area, and volume

area of a triangle of base b and height $h = \frac{1}{2}bh$

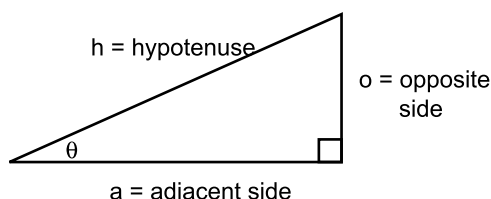
circumference of a circle of radius $r = 2\pi r$

area of a circle of radius $r = \pi r^2$

surface area of a sphere of radius $r = 4\pi r^2$

volume of a sphere of radius $r = \frac{4}{3}\pi r^3$

Trigonometry with a right triangle



Definitions of the sine, cosine, and tangent:

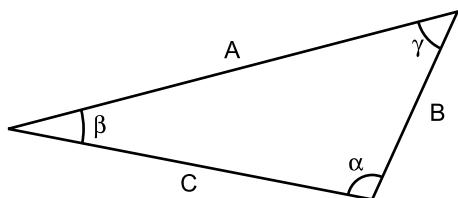
$$\sin \theta = \frac{o}{h}$$

$$\cos \theta = \frac{a}{h}$$

$$\tan \theta = \frac{o}{a}$$

Pythagorean theorem: $h^2 = a^2 + o^2$

Trigonometry with any triangle



Law of Sines:

$$\frac{\sin \alpha}{A} = \frac{\sin \beta}{B} = \frac{\sin \gamma}{C}$$

Law of Cosines:

$$C^2 = A^2 + B^2 - 2AB \cos \gamma$$

Properties of the derivative and integral (for students in calculus-based courses)

Let f and g be functions of x , and let c be a constant.

Linearity of the derivative:

$$\frac{d}{dx}(cf) = c \frac{df}{dx}$$

$$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$$

The chain rule:

$$\frac{d}{dx}f(g(x)) = f'(g(x))g'(x)$$

Derivatives of products and quotients:

$$\frac{d}{dx}(fg) = \frac{df}{dx}g + \frac{dg}{dx}f$$

$$\frac{d}{dx}\left(\frac{f}{g}\right) = \frac{f'g - fg'}{g^2}$$

Some derivatives:

$$\frac{d}{dx}x^m = mx^{m-1} \quad (\text{except for } m=0)$$

$$\frac{d}{dx}\sin x = \cos x$$

$$\frac{d}{dx}\cos x = -\sin x$$

$$\frac{d}{dx}e^x = e^x$$

$$\frac{d}{dx}\ln x = \frac{1}{x}$$

The fundamental theorem of calculus:

$$\int \frac{df}{dx} dx = f$$

Linearity of the integral:

$$\int cf(x) dx = c \int f(x) dx$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

Integration by parts:

$$\int f dg = fg - \int g df$$

Trig Tables

θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$	θ	$\sin \theta$	$\cos \theta$	$\tan \theta$
0°	0.000	1.000	0.000	30°	0.500	0.866	0.577	60°	0.866	0.500	1.732
1	0.017	1.000	0.017	31	0.515	0.857	0.601	61	0.875	0.485	1.804
2	0.035	0.999	0.035	32	0.530	0.848	0.625	62	0.883	0.469	1.881
3	0.052	0.999	0.052	33	0.545	0.839	0.649	63	0.891	0.454	1.963
4	0.070	0.998	0.070	34	0.559	0.829	0.675	64	0.899	0.438	2.050
5	0.087	0.996	0.087	35	0.574	0.819	0.700	65	0.906	0.423	2.145
6	0.105	0.995	0.105	36	0.588	0.809	0.727	66	0.914	0.407	2.246
7	0.122	0.993	0.123	37	0.602	0.799	0.754	67	0.921	0.391	2.356
8	0.139	0.990	0.141	38	0.616	0.788	0.781	68	0.927	0.375	2.475
9	0.156	0.988	0.158	39	0.629	0.777	0.810	69	0.934	0.358	2.605
10	0.174	0.985	0.176	40	0.643	0.766	0.839	70	0.940	0.342	2.747
11	0.191	0.982	0.194	41	0.656	0.755	0.869	71	0.946	0.326	2.904
12	0.208	0.978	0.213	42	0.669	0.743	0.900	72	0.951	0.309	3.078
13	0.225	0.974	0.231	43	0.682	0.731	0.933	73	0.956	0.292	3.271
14	0.242	0.970	0.249	44	0.695	0.719	0.966	74	0.961	0.276	3.487
15	0.259	0.966	0.268	45	0.707	0.707	1.000	75	0.966	0.259	3.732
16	0.276	0.961	0.287	46	0.719	0.695	1.036	76	0.970	0.242	4.011
17	0.292	0.956	0.306	47	0.731	0.682	1.072	77	0.974	0.225	4.331
18	0.309	0.951	0.325	48	0.743	0.669	1.111	78	0.978	0.208	4.705
19	0.326	0.946	0.344	49	0.755	0.656	1.150	79	0.982	0.191	5.145
20	0.342	0.940	0.364	50	0.766	0.643	1.192	80	0.985	0.174	5.671
21	0.358	0.934	0.384	51	0.777	0.629	1.235	81	0.988	0.156	6.314
22	0.375	0.927	0.404	52	0.788	0.616	1.280	82	0.990	0.139	7.115
23	0.391	0.921	0.424	53	0.799	0.602	1.327	83	0.993	0.122	8.144
24	0.407	0.914	0.445	54	0.809	0.588	1.376	84	0.995	0.105	9.514
25	0.423	0.906	0.466	55	0.819	0.574	1.428	85	0.996	0.087	11.430
26	0.438	0.899	0.488	56	0.829	0.559	1.483	86	0.998	0.070	14.301
27	0.454	0.891	0.510	57	0.839	0.545	1.540	87	0.999	0.052	19.081
28	0.469	0.883	0.532	58	0.848	0.530	1.600	88	0.999	0.035	28.636
29	0.485	0.875	0.554	59	0.857	0.515	1.664	89	1.000	0.017	57.290
								90	1.000	0.000	∞

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Moon: Loewy and Puiseux, 1894.

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Mars Climate Orbiter: NASA/JPL/Caltech. *Red blood cell*: C. Magowan et al.

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High jumper: Dunia Young. *Rocket sled*: U.S. Air Force.

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X-33 art: NASA. *Astronauts aboard Mir*: NASA. *Construction of the International Space Station*: NASA.

Gravity map: Data from US Navy Geosat and European Space Agency ERS-1 satellites, analyzed by David Sandwell and Walter Smith.

Chapter 4

Isaac Newton: Painting by Sir Godfrey Kneller, National Portrait Gallery, London.

Chapter 5

Space shuttle launch: NASA.

Chapter 6

The Ring Toss: Clarence White, ca. 1903.

Chapter 7

Aerial photo of Mondavi vineyards: NASA.

Chapter 8

Galloping horse: Eadweard Muybridge, 1878.

Useful Data

Metric Prefixes

M-	mega-	10^6
k-	kilo-	10^3
m-	milli-	10^{-3}
μ - (Greek mu)	micro-	10^{-6}
n-	nano-	10^{-9}

(Centi-, 10^{-2} , is used only in the centimeter.)

Notation and Units

quantity	unit	symbol
distance	meter, m	$x, \Delta x$
time	second, s	$t, \Delta t$
mass	kilogram, kg	m
area	m^2 (square meters)	A
volume	m^3 (cubic meters)	V
density	kg/m^3	ρ
force	Newton, 1 N=1 kgm/s ²	F
velocity	m/s	v
acceleration	m/s ²	a

symbol	meaning
$\propto$	is proportional to
$\approx$	is approximately equal to
$\sim$	on the order of

The Greek Alphabet

α	A	alpha	ν	N	nu
β	B	beta	ξ	Ξ	xi
γ	Γ	gamma	o	O	omicron
δ	Δ	delta	π	Π	pi
ϵ	E	epsilon	ρ	P	rho
ζ	Z	zeta	σ	Σ	sigma
η	H	eta	τ	T	tau
θ	Θ	theta	υ	Y	upsilon
ι	I	iota	ϕ	Φ	phi
κ	K	kappa	χ	X	chi
λ	Λ	lambda	ψ	Ψ	psi
μ	M	mu	ω	Ω	omega

Conversions

Conversions between SI and other units:

1 inch	=	2.54 cm (exactly)
1 mile	=	1.61 km
1 pound	=	4.45 N
(1 kg)(g)	=	2.2 lb
1 gallon	=	$3.78 \times 10^3 \text{ cm}^3$

Conversions between U.S. units:

1 foot	=	12 inches
1 yard	=	3 feet
1 mile	=	5280 ft

Earth, Moon, and Sun

body	mass (kg)	radius (km)	radius of orbit (km)
earth	5.97×10^{24}	6.4×10^3	1.49×10^8
moon	7.35×10^{22}	1.7×10^3	3.84×10^5
sun	1.99×10^{30}	7.0×10^5	

The radii and radii of orbits are average values. The moon orbits the earth and the earth orbits the sun.

Subatomic Particles

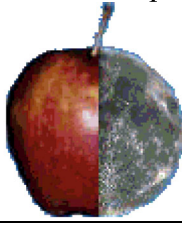
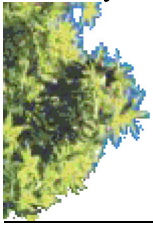
particle	mass (kg)	radius (m)
electron	9.109×10^{-31}	? – less than about 10^{-17}
proton	1.673×10^{-27}	about 1.1×10^{-15}
neutron	1.675×10^{-27}	about 1.1×10^{-15}

The radii of protons and neutrons can only be given approximately, since they have fuzzy surfaces. For comparison, a typical atom is about 10^{-9} m in radius.

Fundamental Constants

speed of light	$c = 3.00 \times 10^8 \text{ m/s}$
gravitational constant	$G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2 \cdot \text{kg}^{-2}$

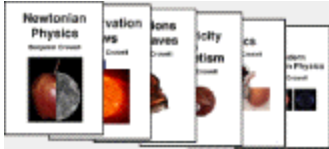
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Motion in One Dimension

I didn't learn until I was nearly through with college that I could understand a book much better if I mentally outlined it for myself before I actually began reading. It's a technique that warns my brain to get little cerebral file folders ready for the different topics I'm going to learn, and as I'm reading it allows me to say to myself, "Oh, the reason they're talking about this now is because they're preparing for this other thing that comes later," or "I don't need to sweat the details of this idea now, because they're going to explain it in more detail later on."

At this point, you're about to dive in to the main subjects of this book, which are force and motion. The concepts you're going to learn break down into the following three areas:

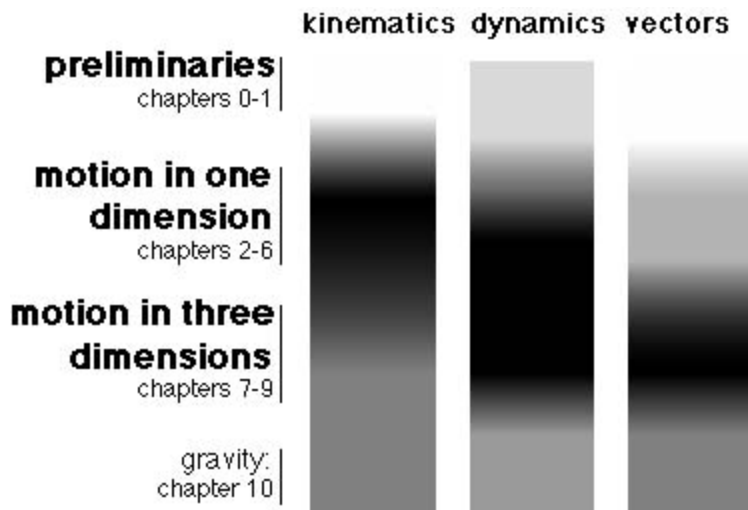
kinematics -- how to describe motion numerically

dynamics -- how force affects motion

vectors -- a mathematical way of handling the three-dimensional nature of force and motion

Roughly speaking, that's the order in which we'll cover these three areas, but the earlier chapters do contain quite a bit of preparation for the later topics. For instance, even before the present point in the book you've learned about the Newton, a unit of force. The discussion of force properly belongs to dynamics, which we aren't tackling head-on for a few more chapters, but I've found that when I teach kinematics it helps to be able to refer to forces now and then to show why it makes sense to define certain kinematical concepts. And although I don't explicitly introduce vectors until ch. 8, the groundwork is being laid for them in earlier chapters.

Here's a roadmap to the rest of the book:



2 Velocity and Relative Motion

2.1 Types of Motion

If you had to think consciously in order to move your body, you would be severely disabled. Even walking, which we consider to be no great feat, requires an intricate series of motions that your cerebrum would be utterly incapable of coordinating. The task of putting one foot in front of the other is controlled by the more primitive parts of your brain, the ones that have not changed much since the mammals and reptiles went their separate evolutionary ways. The thinking part of your brain limits itself to general directives such as "walk faster," or "don't step on her toes," rather than micromanaging every contraction and relaxation of the hundred or so muscles of your hips, legs, and feet.

Physics is all about the conscious understanding of motion, but we're obviously not immediately prepared to understand the most complicated types of motion. Instead, we'll use the

divide-and-conquer technique. We'll first classify the various types of motion, and then begin our campaign with an attack on the simplest cases. To make it clear what we are and are not ready to consider, we need to examine and define carefully what types of motion can exist.

Rigid-body motion distinguished from motion that changes an object's shape

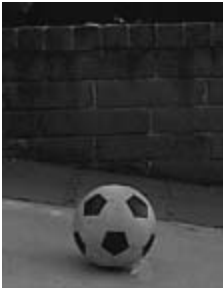
Nobody, with the possible exception of Fred Astaire, can simply glide forward without bending their joints. Walking is thus an example in which there is both a general motion of the whole object and a change in the shape of the object. Another example is the motion of a jiggling water balloon as it flies through the air. We are not presently attempting a mathematical description of the way in which the shape of an object changes. Motion without a change in shape is called rigid-body motion. (The word "body" is often used in physics as a synonym for "object.")

Center-of-mass motion as opposed to rotation

A ballerina leaps into the air and spins around once before landing. We feel intuitively that her rigid-body motion while her feet are off the ground consists of two kinds of motion going on simultaneously: a rotation and a motion of her body as a whole through space, along an arc. It is not immediately obvious, however, what is the most useful way to define the distinction between rotation and motion through space. Imagine that you attempt to balance a chair and it falls over. One person might say that the only motion was a rotation about the chair's point of contact with the floor, but another might say that there was both rotation and motion down and to the side.



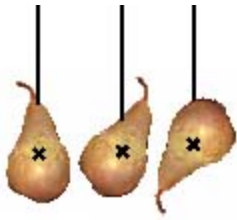
Rotation.



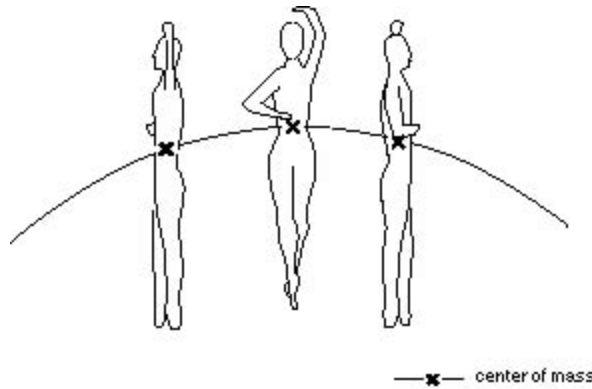
Simultaneous rotation and motion through space.



One person might say that the tipping chair was only rotating in a circle about its point of contact with the floor, but another could describe it as having both rotation and motion through space.



No matter what point you hang the pear from, the string lines up with the pear's center of mass. The center of mass can therefore be defined as the intersection of all the lines made by hanging the pear in this way. Note that the X in the figure should not be interpreted as implying that the center of mass is on the surface -- it is actually inside the pear.



The leaping dancer's motion is complicated, but the motion of her center of mass is simple.

It turns out that there is one particularly natural and useful way to make a clear definition, but it requires a brief digression. Every object has a balance point, referred to in physics as the *center of mass*. For a two-dimensional object such as a cardboard cutout, the center of mass is the point at which you could hang the object from a string and make it balance. In the case of the ballerina (who is likely to be three-dimensional unless her diet is particularly severe), it might be a point either inside or outside her body, depending on how she holds her arms. Even if it is not practical to attach a string to the balance point itself, the center of mass can be defined as shown in the figure on the left.

Why is the center of mass concept relevant to the question of classifying rotational motion as opposed to motion through space? As illustrated in the figure above, it turns out that the motion of an object's center of mass is nearly always far simpler than the motion of any other part of the object.

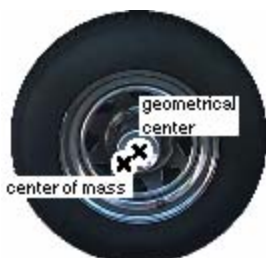
The ballerina's body is a large object with a complex shape. We might expect that her motion would be much more complicated than the motion of a small, simply-shaped object, say a marble, thrown up at the same angle as the angle at which she leapt. But it turns out that the motion of the ballerina's center of mass is exactly the same as the motion of the marble. That is, the motion of the center of mass is the same as the motion the ballerina would have if all her mass was concentrated at a point. By restricting our attention to the motion of the center of mass, we can therefore simplify things greatly.



The same leaping dancer, viewed from above. Her center of mass traces a straight line, but a point away from her center of mass, such as her elbow, traces the much more complicated path shown by the dots.

We can now replace the ambiguous idea of "motion as a whole through space" with the more useful and better defined concept of "*center-of-mass motion*." The motion of any rigid body can be cleanly split into rotation and center-of-mass motion. By this definition, the tipping chair does have both rotational and center-of-mass motion. Concentrating on the center of mass motion allows us to make a simplified *model* of the motion, as if a complicated object like a human body was just a marble or a point-like particle. Science really never deals with reality; it deals with models of reality.

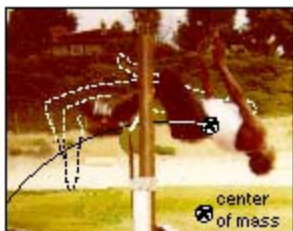
Note that the word "center" in "center of mass" is not meant to imply that the center of mass must lie at the geometrical center of an object. A car wheel that has not been balanced properly has a center of mass that does not coincide with its geometrical center. An object such as the human



An improperly balanced wheel has a center of mass that is

not at its geometric center. When you get a new tire, the mechanic clamps little weights to the rim to balance the wheel.

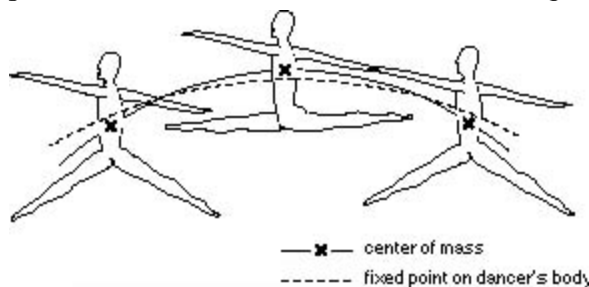
A fixed point on the dancer's body follows a trajectory that is flatter than what we expect, creating an illusion of flight.



The high-jumper's body passes over the bar, but his center of mass passes under it. Photo by Dunia Young.

body does not even have an obvious geometrical center.

It can be helpful to think of the center of mass as the average location of all the mass in the object. With this interpretation, we can see for example that raising your arms above your head raises your center of mass, since the higher position of the arms' mass raises the average.



Ballerinas and professional basketball players can create an illusion of flying horizontally through the air because our brains intuitively expect them to have rigid-body motion, but the body does not stay rigid while executing a grand jete or a slam dunk. The legs are low at the beginning and end of the jump, but come up higher at the middle. Regardless of what the limbs do, the center of mass will follow the same arc, but the low position of the legs at the beginning and end means that the torso is higher compared to the center of mass, while in the middle of the jump it is lower compared to the center of mass. Our eye follows the motion of the torso and tries to interpret it as the center-of-mass motion of a rigid body. But since the torso follows a path that is flatter than we expect, this attempted interpretation fails, and we experience an illusion that the person is flying horizontally. Another interesting example from the sports world is the high jump, in which the jumper's curved body passes over the bar, but the center of mass passes under the bar! Here the jumper lowers his legs and upper body at the peak of the jump in order to

bring his waist higher compared to the center of mass.

Later in this course, we'll find that there are more fundamental reasons (based on Newton's laws of motion) why the center of mass behaves in such a simple way compared to the other parts of an object. We're also postponing any discussion of numerical methods for finding an object's center of mass. Until later in the course, we will only deal with the motion of objects' centers of mass.

Center-of-mass motion in one dimension

In addition to restricting our study of motion to center-of-mass motion, we will begin by considering only cases in which the center of mass moves along a straight line. This will include cases such as objects falling straight down, or a car that speeds up and slows down but does not turn.

Note that even though we are not explicitly studying the more complex aspects of motion, we can still analyze the center-of-mass motion while ignoring other types of motion that might be occurring simultaneously. For instance, if a cat is falling out of a tree and is initially upside-down, it goes through a series of contortions that bring its feet under it. This is definitely not an example of rigid-body motion, but we can still analyze the motion of the cat's center of mass just as we would for a dropping rock.

Self-Check

Consider a person running, a person pedaling on a bicycle, a person coasting on a bicycle, and a person coasting on ice skates. In which cases is the center-of-mass motion one-dimensional? Which cases are examples of rigid-body motion?

2.2 Describing Distance and Time

Center-of-mass motion in one dimension is particularly easy to deal with because all the

information about it can be encapsulated in two variables: x , the position of the center of mass relative to the origin, and t , which measures a point in time. For instance, if someone supplied you with a sufficiently detailed table of x and t values, you would know pretty much all there was to know about the motion of the object's center of mass.

A point in time as opposed to duration

In ordinary speech, we use the word "time" in two different senses, which are to be distinguished in physics. It can be used, as in "a short time" or "our time here on earth," to mean a length or duration of time, or it can be used to indicate a clock reading, as in "I didn't know what time it was," or "now's the time." In symbols, t is ordinarily used to mean a point in time, while Δt signifies an interval or duration in time. The capital Greek letter delta, Δ , means "the change in...," i.e. a duration in time is the change or difference between one clock reading and another. The notation Δt does not signify the product of two numbers, Δ and t , but rather one single number, Δt . If a matinee begins at a point in time $t=1$ o'clock and ends at $t=3$ o'clock, the duration of the movie was the change in t ,

$$\Delta t = 3 \text{ hours} - 1 \text{ hour} = 2 \text{ hours} .$$

To avoid the use of negative numbers for Δt , we write the clock reading "after" to the left of the minus sign, and the clock reading "before" to the right of the minus sign. A more specific definition of the delta notation is therefore that delta stands for "after minus before."

Even though our definition of the delta notation guarantees that Δt is positive, there is no reason why t can't be negative. If t could not be negative, what would have happened one second before $t=0$? That doesn't mean that time "goes backward"

in the sense that adults can shrink into infants and retreat into the womb. It just means that we have to pick a reference point and call it $t=0$, and then times before that are represented by negative values of t .

Although a point in time can be thought of as a clock reading, it is usually a good idea to avoid doing computations with expressions such as "2:35" that are combinations of hours and minutes. Times can instead be expressed entirely in terms of a single unit, such as hours. Fractions of an hour can be represented by decimals rather than minutes, and similarly if a problem is being worked in terms of minutes, decimals can be used instead of seconds.

Self-Check

Of the following phrases, which refer to points in time, which refer to time intervals, and which refer to time in the abstract rather than as a measurable number?

- (a) "The time has come."
- (b) "Time waits for no man."
- (c) "The whole time, he had spit on his chin."

Position as opposed to change in position

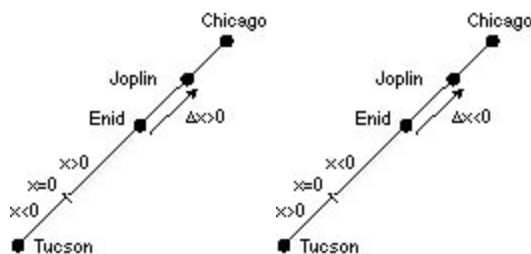
As with time, a distinction should be made between a point in space, symbolized as a coordinate x , and a change in position, symbolized as Δx .

As with t , x can be negative. If a train is moving down the tracks, not only do you have the freedom to choose any point along the tracks and call it $x=0$, but it's also up to you to decide which side of the $x=0$ point is positive x and which side is negative x .

Since we've defined the delta notation to mean "after minus before," it is possible that Δx will be negative, unlike Δt which is guaranteed to be

positive. Suppose we are describing the motion of a train on tracks linking Tucson and Chicago. As shown in the figure, it is entirely up to you to decide which way is positive.

Two equally valid ways of describing the motion of a train from Tucson to Chicago. In the first example, the train has a positive Δx as it goes from Enid to Joplin. In the second example, the same train going forward in the same direction has a negative Δx .



Note that in addition to x and Δx , there is a third quantity we could define, which would be like an odometer reading, or actual distance traveled. If you drive 10 miles, make a U-turn, and drive back 10 miles, then your Δx is zero, but your car's odometer reading has increased by 20 miles. However important the odometer reading is to car owners and used car dealers, it is not very important in physics, and there is not even a standard name or notation for it. The change in position, Δx , is more useful because it is so much easier to calculate: to compute Δx , we only need to know the beginning and ending positions of the object, not all the information about how it got from one position to the other.

Self-Check

A ball hits the floor, bounces to a height of one meter, falls, and hits the floor again. Is the Δx between the two impacts equal to zero, one, or two meters?

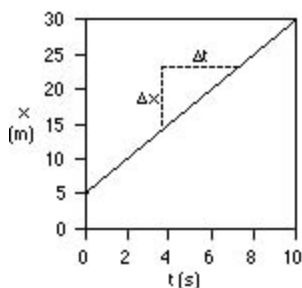
Frames of reference

The example above shows that there are two arbitrary choices you have to make in order to define a position variable, x . You have to decide where to put $x=0$, and also which direction will be

positive. This is referred to as choosing a *coordinate system* or choosing a *frame of reference*. (The two terms are nearly synonymous, but the first focuses more on the actual x variable, while the second is more of a general way of referring to one's point of view.) As long as you are consistent, any frame is equally valid. You just don't want to change coordinate systems in the middle of a calculation.

Have you ever been sitting in a train in a station when suddenly you notice that the station is moving backward? Most people would describe the situation by saying that you just failed to notice that the train was moving it only seemed like the station was moving. But this shows that there is yet a third arbitrary choice that goes into choosing a coordinate system: valid frames of reference can differ from each other by moving relative to one another. It might seem strange that anyone would bother with a coordinate system that was moving relative to the earth, but for instance the frame of reference moving along with a train might be far more convenient for describing things happening inside the train.

2.3 Graphs of Motion; Velocity.



(a) Motion with constant velocity.

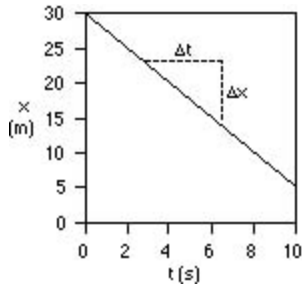
Motion with constant velocity

In example (a), an object is moving at constant speed in one direction. We can tell this because every two seconds, its position changes by five meters.

In algebra notation, we'd say that the graph of x vs. t shows the same change in position, $\Delta x = 5.0$ m, over each interval of $\Delta t = 2.0$ s. The object's velocity or speed is obtained by calculating $v = \Delta x / \Delta t = (5.0 \text{ m}) / (2.0 \text{ s}) = 2.5 \text{ m/s}$. In graphical terms, the velocity can be interpreted as the slope of the line. Since the graph is a straight line, it wouldn't have

mattered if we'd taken a longer time interval and calculated $v = \Delta x / \Delta t = (10.0 \text{ m}) / (4.0 \text{ s})$. The answer would still have been the same, 2.5 m/s.

Note that when we divide a number that has units of meters by another number that has units of seconds, we get units of meters per second, which can be written m/s. This is another case where we treat units as if they were algebra symbols, even though they're not.

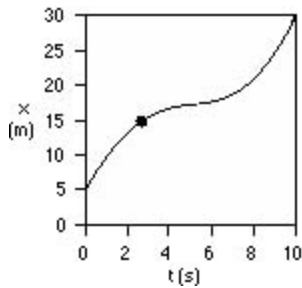


(b) Motion that decreases x is represented with negative values of Δx and v .

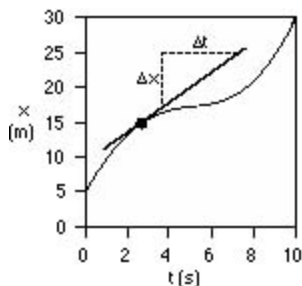
In example (b), the object is moving in the opposite direction: as time progresses, its x coordinate decreases. Recalling the definition of the Δ notation as "after minus before," we find that Δt is still positive, but Δx must be negative. The slope of the line is therefore negative, and we say that the object has a negative velocity, $v = \Delta x / \Delta t = (-5.0 \text{ m}) / (2.0 \text{ s}) = -2.5 \text{ m/s}$. We've already seen that the plus and minus signs of Δx values have the interpretation of telling us which direction the object moved. Since Δt is always positive, dividing by Δt doesn't change the plus or minus sign, and the plus and minus signs of velocities are to be interpreted in the same way. In graphical terms, a positive slope characterizes a line that goes up as we go to the right, and a negative slope tells us that the line went down as we went to the right.

Motion with changing velocity

Now what about a graph like example (c)? This might be a graph of a car's motion as the driver cruises down the freeway, then slows down to look at a car crash by the side of the road, and then speeds up again, disappointed that there is nothing dramatic going on such as flames or babies trapped in their car seats. (Note that we are still talking about one-dimensional motion. Just because the graph is curvy doesn't mean that the car's path is curvy. The graph is not like a map,



(c) Motion with changing velocity.



(d) The velocity at any given moment is defined as the slope of the tangent line through the relevant point on the graph.

and the horizontal direction of the graph represents the passing of time, not distance.)

Example (c) is similar to example (a) in that the object moves a total of 25.0 m in a period of 10.0 s, but it is no longer true that it makes the same amount of progress every second. There is no way to characterize the entire graph by a certain velocity or slope, because the velocity is different at every moment. It would be incorrect to say that because the car covered 25.0 m in 10.0 s, its velocity was 2.5 m/s. It moved faster than that at the beginning and end, but slower in the middle. There may have been certain instants at which the car was indeed going 2.5 m/s, but the speedometer swept past that value without "sticking," just as it swung through various other values of speed. (I definitely want my next car to have a speedometer calibrated in m/s and showing both negative and positive values.)

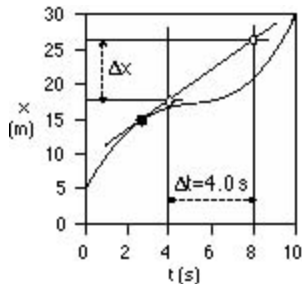
We assume that our speedometer tells us what is happening to the speed of our car at every instant, but how can we define speed mathematically in a case like this? We can't just define it as the slope of the curvy graph, because a curve doesn't have a single well-defined slope as does a line. A mathematical definition that corresponded to the speedometer reading would have to be one that attached a different velocity value to a single point on the curve, i.e. a single instant in time, rather than to the entire graph. If we wish to define the speed at one instant such as the one marked with a dot, the best way to proceed is illustrated in (d), where we have drawn the line through that point called the tangent line, the line that "hugs the curve." We can then adopt the following definition of velocity:

definition of velocity

The velocity of an object at any given moment

is the slope of the tangent line through the relevant point on its x - t graph.

One interpretation of this definition is that the velocity tells us how many meters the object would have traveled in one second, if it had continued moving at the same speed for at least one second. To some people the graphical nature of this definition seems "inaccurate" or "not mathematical." The equation $v = \Delta x / \Delta t$ by itself, however, is only valid if the velocity is constant, and so cannot serve as a general definition.



Example: finding the velocity at the point indicated with the dot.

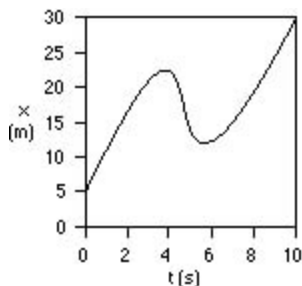
Example

Question: What is the velocity at the point shown with a dot on the graph?

Solution: First we draw the tangent line through that point. To find the slope of the tangent line, we need to pick two points on it. Theoretically, the slope should come out the same regardless of which two points we picked, but in practical terms we'll be able to measure more accurately if we pick two points fairly far apart, such as the two white diamonds. To save work, we pick points that are directly above labeled points on the t axis, so that $\Delta t = 4.0$ s is easy to read off. One diamond lines up with $x = 17.5$ m, the other with $x = 26.5$ m, so $\Delta x = 9.0$ m. The velocity is $\Delta x / \Delta t = 2.2$ m/s.

Conventions about graphing

The placement of t on the horizontal axis and x on the upright axis may seem like an arbitrary convention, or may even have disturbed you, since your algebra teacher always told you that x goes on the horizontal axis and y goes on the upright axis. There is a reason for doing it this way, however. In example (e), we have an object that reverses its direction of motion twice. It can only be in one place at any given time, but there can be more than one time when it is at a given place. For instance, this object passed through $x = 17$ m on three separate occasions, but there is no way it

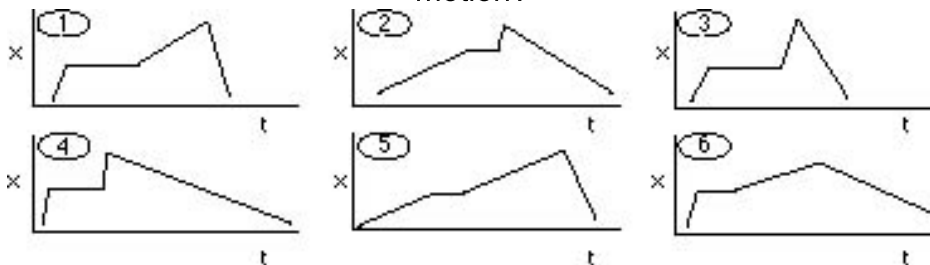


(e) Reversing the direction of motion.

could have been in more than one place at $t=5.0$ s. Resurrecting some terminology you learned in your trigonometry course, we say that x is a function of t , but t is not a function of x . In situations such as this, there is a useful convention that the graph should be oriented so that any vertical line passes through the curve at only one point. Putting the x axis across the page and t upright would have violated this convention. To people who are used to interpreting graphs, a graph that violates this convention is as annoying as fingernails scratching on a chalkboard. We say that this is a graph of " x versus t ." If the axes were the other way around, it would be a graph of " t versus x ." I remember the "versus" terminology by visualizing the labels on the x and t axes and remembering that when you read, you go from left to right and from top to bottom.

Discussion questions

A. An ant walks forward, pauses, then runs quickly ahead. It then suddenly reverses direction and walks slowly back the way it came. Which graph could represent its motion?



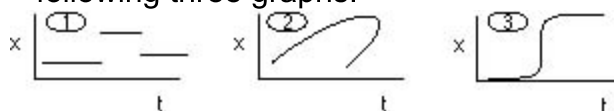
B. The figure shows a sequence of positions for two racing tractors. Compare the tractors' velocities as the race progresses.



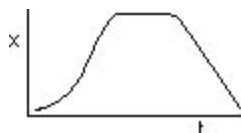
C. If an object had a straight-line motion graph with $\Delta x=0$ and $\Delta t \neq 0$, what would be true about its velocity? What would this look like on a graph? What about $\Delta t=0$ and $\Delta x \neq 0$?

D. If an object has a wavy motion graph like the one in example (e) on the previous page, which are the points at which the object reverses its direction? What is true about the object's velocity at these points?

E. Discuss anything unusual about the following three graphs.



F. I have been using the term "velocity" and avoiding the more common English word "speed," because some introductory physics texts define them to mean different things. They use the word "speed," and the symbol "s" to mean the absolute value of the velocity, $s = |v|$. Although I have thrown in my lot with the minority of books that don't emphasize this distinction in technical vocabulary, there are clearly two different concepts here. Can you think of an example of a graph of x vs. t in which the object has constant speed, but not constant velocity?



Discussion question
G.

G. In the graph on the left, describe how the object's velocity changes.

H. Two physicists duck out of a boring scientific conference to go get beer. On the way to the bar, they witness an accident in which a pedestrian is injured by a hit-and-run driver. A criminal trial results, and they must testify. In her testimony, Dr. Transverz Waive says, "The car was moving along pretty fast, I'd say the velocity was +40 mi/hr. They saw the old lady too late, and even though they slammed on the brakes they still hit her before they stopped. Then they made a U turn and headed off at a velocity of about -20 mi/hr, I'd say." Dr. Longitud N.L. Vibrasheun says, "He was really going too fast, maybe his velocity was -35 or -40 mi/hr. After he hit Mrs. Hapless, he turned around and left at a velocity of, oh, I'd guess maybe +20 or +25 mi/hr." Is their testimony contradictory? Explain.

I. The following group exercise involves Michael Johnson's world-record 200-meter sprint in the 1996 Olympics. The table gives the distance he has covered at various times. (The data are made up, except for his total time.) Each group is to find a value of $\Delta x / \Delta t$ between two specified instants, with the members of the group checking each other's answers. We will then compare

everyone's results and discuss how this relates to velocity.

	t (s)	x (m)
A	10.200	100.0000
B	10.210	100.0990
C	10.300	100.9912
D	11.200	110.0168
E	19.320	200.0000

group 1: Find $\Delta x/\Delta t$ using points A and B.

group 2: Find $\Delta x/\Delta t$ using points A and C.

group 3: Find $\Delta x/\Delta t$ using points A and D.

group 4: Find $\Delta x/\Delta t$ using points A and E.

2.4 The Principle of Inertia

Physical effects relate only to a change in velocity

Consider two statements that were at one time made with the utmost seriousness:

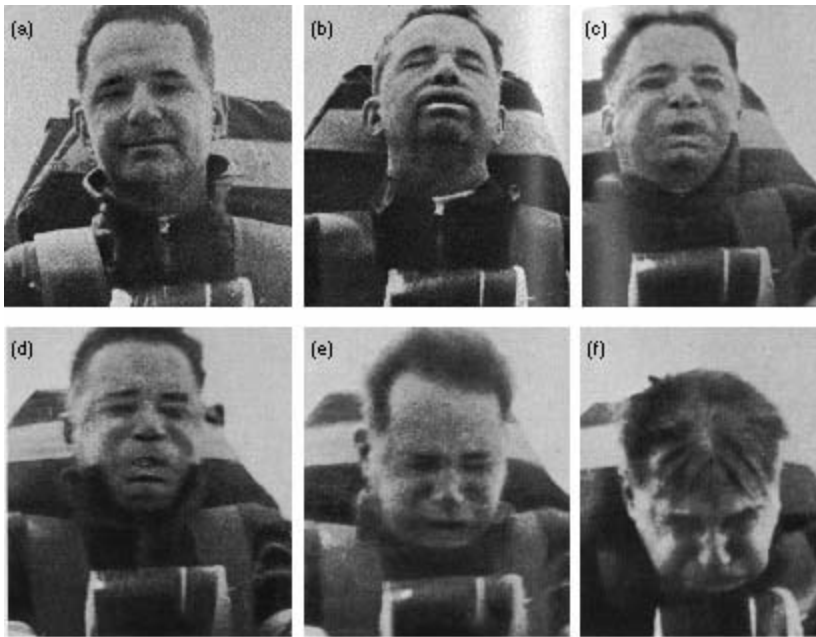
People like Galileo and Copernicus who say the earth is rotating must be crazy. We know the earth can't be moving. Why, if the earth was really turning once every day, then our whole city would have to be moving hundreds of leagues in an hour. That's impossible! Buildings would shake on their foundations. Gale-force winds would knock us over. Trees would fall down. The Mediterranean would come sweeping across the east coasts of Spain and Italy. And furthermore, what force would be making the world turn?

All this talk of passenger trains moving at forty miles an hour is sheer hogwash! At that speed, the air in a passenger compartment would all be forced against the back wall. People in the front of the car would suffocate, and people at the back would die because in such concentrated air, they wouldn't be able to expel a breath.

Some of the effects predicted in the first quote are clearly just based on a lack of experience with rapid motion that is smooth and free of vibration. But there is a deeper principle involved. In each case, the speaker is assuming that the mere fact of

motion must have dramatic physical effects. More subtly, they also believe that a force is needed to keep an object in motion: the first person thinks a force would be needed to maintain the earth's rotation, and the second apparently thinks of the rear wall as pushing on the air to keep it moving.

Common modern knowledge and experience tell us that these people's predictions must have somehow been based on incorrect reasoning, but it is not immediately obvious where the fundamental flaw lies. It's one of those things a four-year-old could infuriate you by demanding a clear explanation of. One way of getting at the fundamental principle involved is to consider how the modern concept of the universe differs from the popular conception at the time of the Italian Renaissance. To us, the word "earth" implies a planet, one of the nine planets of our solar system, a small ball of rock and dirt that is of no significance to anyone in the universe except for members of our species, who happen to live on it. To Galileo's contemporaries, however, the earth was the biggest, most solid, most important thing in all of creation, not to be compared with the wandering lights in the sky known as planets. To us, the earth is just another object, and when we talk loosely about "how fast" an object such as a car "is going," we really mean the car-object's velocity relative to the earth-object.



This Air Force doctor volunteered to ride a rocket sled as a medical experiment. The obvious effects on his head and face are not because of the sled's speed but because of its rapid changes in speed: increasing in (b) and (c), and decreasing in (e) and (f). In (d) his speed is greatest, but because his speed is not increasing or decreasing very much at this moment, there is little effect on him.

Motion is relative

According to our modern world-view, it really isn't that reasonable to expect that a special force should be required to make the air in the train have a certain velocity relative to our planet. After all, the "moving" air in the "moving" train might just happen to have zero velocity relative to some other planet we don't even know about. Aristotle claimed that things "naturally" wanted to be at rest, lying on the surface of the earth. But experiment after experiment has shown that there is really nothing so special about being at rest relative to the earth. For instance, if a mattress falls out of the back of a truck on the freeway, the reason it rapidly comes to rest with respect to the planet is simply because of friction forces exerted by the asphalt, which happens to be attached to the planet.

Galileo's insights are summarized as follows:

The Principle of Inertia

No force is required to maintain motion with

constant velocity in a straight line, and absolute motion does not cause any observable physical effects.

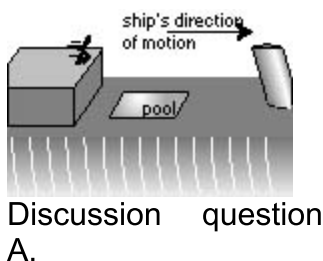
There are many examples of situations that seem to disprove the principle of inertia, but these all result from forgetting that friction is a force. For instance, it seems that a force is needed to keep a sailboat in motion. If the wind stops, the sailboat stops too. But the wind's force is not the only force on the boat; there is also a frictional force from the water. If the sailboat is cruising and the wind suddenly disappears, the backward frictional force still exists, and since it is no longer being counteracted by the wind's forward force, the boat stops. To disprove the principle of inertia, we would have to find an example where a moving object slowed down even though no forces whatsoever were acting on it.

Self-Check

What is incorrect about the following supposed counterexamples to the principle of inertia?

(1) When astronauts blast off in a rocket, their huge velocity does cause a physical effect on their bodies they get pressed back into their seats, the flesh on their faces gets distorted, and they have a hard time lifting their arms.

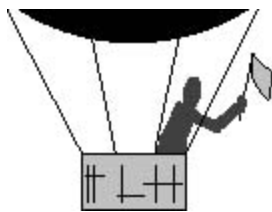
(2) When you're driving in a convertible with the top down, the wind in your face is an observable physical effect of your absolute motion.



Discussion questions

A. A passenger on a cruise ship finds, while the ship is docked, that he can leap off of the upper deck and just barely make it into the pool on the lower deck. If the ship leaves dock and is cruising rapidly, will this adrenaline junkie still be able to make it?

B. You are a passenger in the open basket hanging under a helium balloon. The balloon is being carried along by the wind at a constant velocity. If you are holding a flag in your hand, will the flag wave? If so, which



Discussion question
B.

way? [Based on a question from PSSC Physics.]

C. Aristotle stated that all objects naturally wanted to come to rest, with the unspoken implication that "rest" would be interpreted relative to the surface of the earth. Suppose we could transport Aristotle to the moon, put him in a space suit, and kick him out the door of the spaceship and into the lunar landscape. What would he expect his fate to be in this situation? If intelligent creatures inhabited the moon, and one of them independently came up with the equivalent of Aristotelian physics, what would they think about objects coming to rest?

2.5 Addition of Velocities

Addition of velocities to describe relative motion

Since absolute motion cannot be unambiguously measured, the only way to describe motion unambiguously is to describe the motion of one object relative to another. Symbolically, we can write v_{PQ} for the velocity of object P relative to object Q.

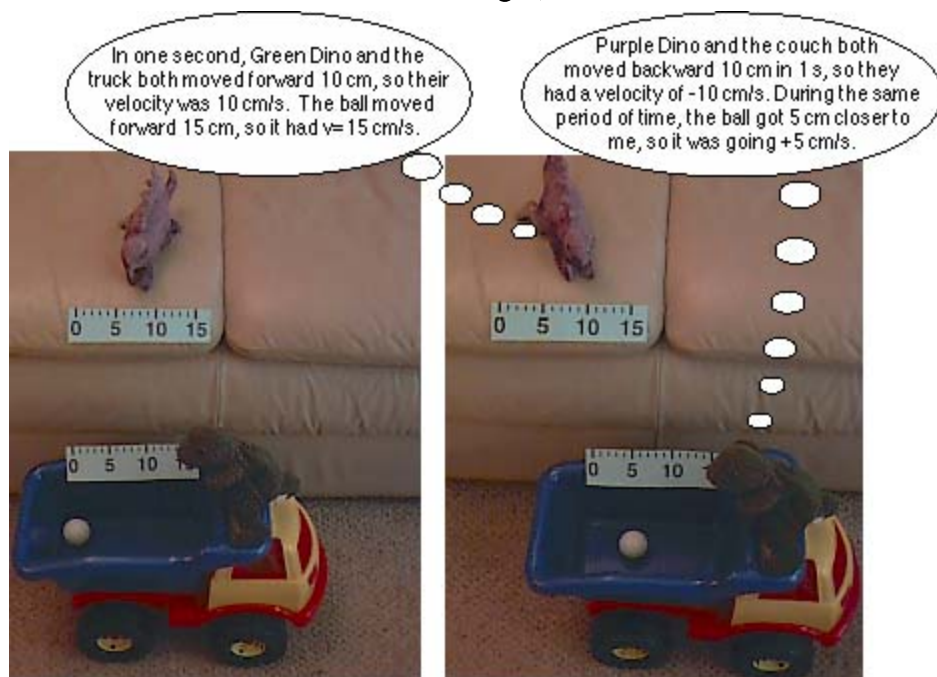
Velocities measured with respect to different reference points can be compared by addition. In the figure below, the ball's velocity relative to the couch equals the ball's velocity relative to the truck plus the truck's velocity relative to the couch:

$$v_{BC} = v_{BT} + v_{TC}$$

$$= 5 \text{ cm/s} + 10 \text{ cm/s}$$

The same equation can be used for any combination of three objects, just by substituting the relevant subscripts for B, T, and C. Just remember to write the equation so that the velocities being added have the same subscript twice in a row. In this example, if you read off the subscripts going from left to right, you get BC...=...BTTC. The fact that the two "inside" subscripts on the right are the same means that the

equation has been set up correctly. Notice how subscripts on the left look just like the subscripts on the right, but with the two T's eliminated.



These two highly competent physicists disagree on absolute velocities, but they would agree on relative velocities. Purple Dino considers the couch to be at rest, while Green Dino thinks of the truck as being at rest. They agree, however, that the truck's velocity relative to the couch is $v_{TC}=10$ cm/s, the ball's velocity relative to the truck is $v_{BT}=5$ cm/s, and the ball's velocity relative to the couch is $v_{BC}=v_{BT}+v_{TC}=15$ cm/s.

Negative velocities in relative motion

My discussion of how to interpret positive and negative signs of velocity may have left you wondering why we should bother. Why not just make velocity positive by definition? The original reason why negative numbers were invented was that bookkeepers decided it would be convenient to use the negative number concept for payments to distinguish them from receipts. It was just plain easier than writing receipts in black and payments in red ink. After adding up your month's positive receipts and negative payments, you either got a positive number, indicating profit, or a negative number, showing a loss. You could then show the that total with a high-tech "+" or "-" sign, instead of looking around for the appropriate bottle of ink.

Nowadays we use positive and negative numbers for all kinds of things, but in every case the point is that it makes sense to add and subtract those things according to the rules you learned in grade school, such as "minus a minus makes a plus, why this is true we need not discuss." Adding velocities has the significance of comparing relative motion, and with this interpretation negative and positive velocities can be used within a consistent framework. For example, the truck's velocity relative to the couch equals the truck's velocity relative to the ball plus the ball's velocity relative to the couch:

$$\begin{aligned} v_{TC} &= v_{TB} + v_{BC} \\ &= 5 \text{ cm/s} + 15 \text{ cm/s} \\ &= 10 \text{ cm/s} \end{aligned}$$

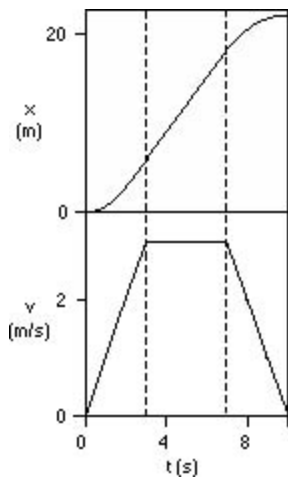
If we didn't have the technology of negative numbers, we would have had to remember a complicated set of rules for adding velocities: (1) if the two objects are both moving forward, you add, (2) if one is moving forward and one is moving backward, you subtract, but (3) if they're both moving backward, you add. What a pain that would have been.

Discussion questions

A. Interpret the general rule $v_{AB} = -v_{BA}$ in words.

B. If we have a specific situation where $v_{AB} + v_{BC} = 0$, what is going on?

2.6 Graphs of Velocity Versus Time



Since changes in velocity play such a prominent role in physics, we need a better way to look at changes in velocity than by laboriously drawing tangent lines on x -versus- t graphs. A good method is to draw a graph of velocity versus time. The examples on the left show the x - t and v - t graphs that might be produced by a car starting from a traffic light, speeding up, cruising for a while at constant speed, and finally slowing down for a stop sign. If you have an air freshener hanging from your rear-view mirror, then you will see an effect on the air freshener during the beginning and ending periods when the velocity is changing, but it will not be tilted during the period of constant velocity represented by the flat plateau in the middle of the v - t graph.

Students often mix up the things being represented on these two types of graphs. For instance, many students looking at the top graph say that the car is speeding up the whole time, since "the graph is becoming greater." What is getting greater throughout the graph is x , not v .

Similarly, many students would look at the bottom graph and think it showed the car backing up, because "it's going backwards at the end." But what is decreasing at the end is v , not x . Having both the x - t and v - t graphs in front of you like this is often convenient, because one graph may be easier to interpret than the other for a particular purpose. Stacking them like this means that corresponding points on the two graphs' time axes are lined up with each other vertically. However, one thing that is a little counterintuitive about the arrangement is that in a situation like this one involving a car, one is tempted to visualize the landscape stretching along the horizontal axis of one of the graphs. The horizontal axes, however, represent time, not position. The correct way to

visualize the landscape is by mentally rotating the horizon 90 degrees counterclockwise and imagining it stretching along the upright axis of the x - t graph, which is the only axis that represents different positions in space.

2.7 ∫ Applications of Calculus

Note: As of version 4.7, Netscape does not display integral symbols properly. If you're using Netscape, they will be displayed as a numerical code.

The integral symbol, $\int$, in the heading for this section indicates that it is meant to be read by students in calculus-based physics. Students in an algebra-based physics course should skip these sections. The calculus-related sections in this book are meant to be usable by students who are taking calculus concurrently, so at this early point in the physics course I do not assume you know any calculus yet. This section is therefore not much more than a quick preview of calculus, to help you relate what you're learning in the two courses.

Newton was the first person to figure out the tangent-line definition of velocity for cases where the x - t graph is nonlinear. Before Newton, nobody had conceptualized the description of motion in terms of x - t and v - t graphs. In addition to the graphical techniques discussed in this chapter, Newton also invented a set of symbolic techniques called calculus. If you have an equation for x in terms of t , calculus allows you, for instance, to find an equation for v in terms of t . In calculus terms, we say that the function $v(t)$ is the derivative of the function $x(t)$. In other words, the derivative of a function is a new function that tells how rapidly the original function was changing. We now use neither Newton's name for his technique (he called it "the method of fluxions") nor his notation. The more commonly used notation is due to Newton's German contemporary

Leibnitz, whom the English accused of plagiarizing the calculus from Newton. In the Leibnitz notation, we write

$$v = \frac{dx}{dt}$$

to indicate that the function $v(t)$ equals the slope of the tangent line of the graph of $x(t)$ at every time t .

The Leibnitz notation is meant to evoke the delta notation, but with a very small time interval.

Because the dx and dt are thought of as very small Δx 's and Δt 's, i.e. very small differences, the part of calculus that has to do with derivatives is called differential calculus.

Differential calculus consists of three things:

- The concept and definition of the derivative, which is covered in this book, but which will be discussed more formally in your math course.
- The Leibnitz notation described above, which you'll need to get more comfortable with in your math course.
- A set of rules for that allows you to find an equation for the derivative of a given function. For instance, if you happened to have a situation where the position of an object was given by the equation $x=2t^7$, you would be able to use those rules to find $dx/dt=14t^6$. This bag of tricks is covered in your math course.

Summary

Selected Vocabulary

center of mass the balance point of an object

velocity the rate of change of position; the slope of the tangent line on an x - t graph.

Notation

x a point in space

t a point in time, a clock reading

Δ "change in;" the value of a variable afterwards minus its value before

Δx	a distance, or more precisely a change in x , which may be less than the distance traveled; its plus or minus sign indicates direction
Δt	a duration of time
v	velocity
v_{AB}	the velocity of object A relative to object B

Standard Terminology Avoided in This Book

displacement	a name for the symbol Δx .
speed	the absolute value of the velocity, i.e. the velocity stripped of any information about its direction

Summary

An object's center of mass is the point at which it can be balanced. For the time being, we are studying the mathematical description only of the motion of an object's center of mass in cases restricted to one dimension. The motion of an object's center of mass is usually far simpler than the motion of any of its other parts.

It is important to distinguish location, x , from distance, Δx , and clock reading, t , from time interval Δt . When an object's x - t graph is linear, we define its velocity as the slope of the line, $\Delta x/\Delta t$. When the graph is curved, we generalize the definition so that the velocity is the slope of the tangent line at a given point on the graph.

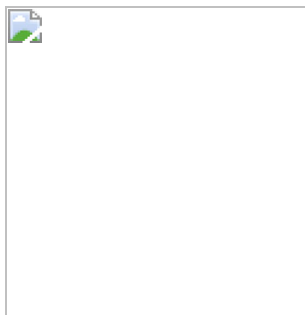
Galileo's principle of inertia states that no force is required to maintain motion with constant velocity in a straight line, and absolute motion does not cause any observable physical effects. Things typically tend to reduce their velocity relative to the surface of our planet only because they are physically rubbing against the planet (or something attached to the planet), not because there is anything special about being at rest with respect to the earth's surface. When it seems, for instance, that a force is required to keep a book sliding across a table, in fact the force is only serving to cancel the contrary force of friction.

Absolute motion is not a well-defined concept, and if two observers are not at rest relative to one another they will disagree about the absolute velocities of objects. They will, however, agree about relative velocities. If object A is in motion relative to object B, and B is in motion

relative to C, then A's velocity relative to C is given by $v_{AC} = v_{AB} + v_{BC}$.

Positive and negative signs are used to indicate the direction of an object's motion.

Homework Problems



Problem 1.

1. The graph shows the motion of a car stuck in stop-and-go freeway traffic. (a) If you only knew how far the car had gone during this entire time period, what would you think its velocity was? (b) What is the car's maximum velocity?

2. (a) Let θ be the latitude of a point on the Earth's surface. Derive an algebra equation for the distance, L , traveled by that point during one rotation of the Earth about its axis, i.e. over one day, expressed in terms of L , θ , and R , the radius of the earth. Check: Your equation should give $L=0$ for the North Pole.

(b3) At what speed is Fullerton, at latitude $\theta=34^\circ$, moving with the rotation of the Earth about its axis? Give your answer in units of mi/h. [See the inside front cover of the book for the relevant data.]

3. A person is parachute jumping. During the time between when she leaps out of the plane and when she opens her chute, her altitude is given by the equation

$$y = (10000 \text{ m}) - (50 \text{ m/s})[t + (5.0 \text{ s})e^{-t/5.0 \text{ s}}]$$

Find her velocity at $t=7.0 \text{ s}$. (This can be done on a calculator, without knowing calculus.) Because of air resistance, her velocity does not increase at a steady rate as it would for an object falling in vacuum.

4 S. A light-year is a unit of distance used in astronomy, and defined as the distance light travels in one year. The speed of light is $3.0 \times 10^8 \text{ m/s}$. Find how many meters there are in one light-year, expressing your answer in scientific notation.

5 S. You're standing in a freight train, and have no way to see out. If you have to lean to stay on your feet, what, if anything, does that tell you about the train's velocity? Its acceleration? Explain.

6 . A honeybee's position as a function of time is given by $x=10t-t^3$, where t is in seconds and x in meters. What is its velocity at $t=3.0$ s?

Key to the symbols used in the homework problems.

S A solution is given in the back of the book.

A difficult problem.

A computerized answer check is available.

A problem that requires calculus.