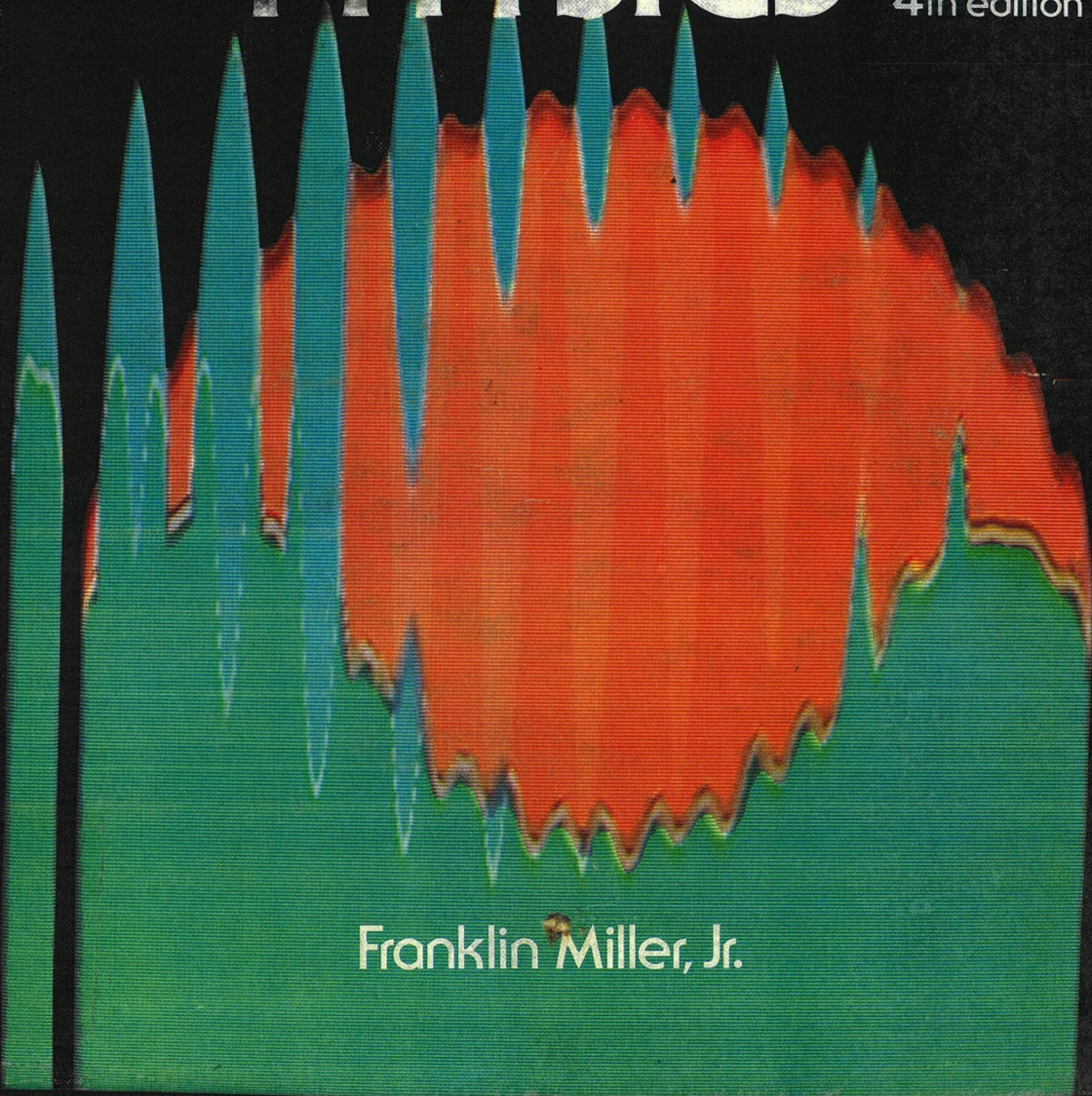


COLLEGE PHYSICS

4th edition



Franklin Miller, Jr.

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Kenyon College



HARCOURT BRACE JOVANOVIICH, INC.

New York Chicago San Francisco Atlanta

Cover photo: Manfred Kage from Peter Arnold

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Library of Congress Catalog Card Number: 76-47086

ISBN: 0-15-511734-3

Printed in the United States of America

Picture credits appear on page 817.

PROBLEMS

- 99 Solve for u and v : $3u + v = 7$; $2u - v = 3$.
 100 Solve for p and q : $q = p + 2$; $p + q = 8$.
 101 Solve for A and B : $A - 4B = -6$; $2B + A = 18$.
 102 Solve for x and y : $10x + 2y = 11$; $20(x - y) = -8$.

Find the real solutions, if any, of the following quadratic equations:

- 103 $2t^2 - 3t + 1 = 0$
 104 $v^2 + 5v + 6 = 0$
 105 $x^2 - 10 = 3x$
 106 $y^2 + 1 = -y$
 107 $3 + z^2 + 2z = 2$
 108 $4\theta^4 - 5\theta^2 + 1 = 0$ (Hint: Let θ^2 be the variable; call it x if you wish. Solve for θ^2 by the quadratic formula, and then find θ .)

B-4 Logarithms

The logarithm of a number is the power to which a base must be raised to obtain the number. Ordinary or "common" logarithms are to base 10:

$$\text{If } y = 10^x, \text{ then } x = \log y$$

For example, $\log 10^5 = 5.000$; $\log 2 = 0.3010$. The base of natural logarithms is e , a number whose value is 2.71828 . . . :

$$\text{If } y = e^x, \text{ then } x = \ln y$$

For example, $\ln 10 = 2.3026$; $\ln 1000 = 6.9078$. Logarithms to base e are written as "ln" to distinguish them from logarithms to base 10, which are written as "log." For any number N , the two logarithms are related as follows:

$$\ln N = 2.3026 \log N$$

There are several definitions of e , given in any calculus book. One interesting and useful property is related to the slope of a graph. If $y = e^x$, then the slope, given by dy/dx , is also e^x . This is not true for any other base; the slope of $y = 10^x$ is *not* 10^x .

What follows applies to logarithms to any base.

Multiplication.

$$\text{If } N = AB, \text{ then } \log N = \log A + \log B$$

Division.

$$\text{If } N = A/B, \text{ then } \log N = \log A - \log B$$

Exponentiation.

$$\text{If } N = A^n, \text{ then } \log N = n \log A$$

Some special cases of this last rule are: $\log 1 = 0$; $\log(10^x) = x$; $\ln(e^x) = x$.

Logarithms are no longer used for routine numerical calculations (although many pocket calculators perform exponentiation by internally calculating and using logarithms to base 2). In our study of physics, logarithms are used to interpret graphs of experimental data. For example, the activity (in counts/min) of a radioactive sample might be decreasing according to an exponential equation:

$$N = N_0 e^{-\lambda t}$$

where N_0 is the initial activity and t is the elapsed time. Taking logarithms of both sides of this equation gives

$$\begin{aligned} \ln N &= \ln N_0 + \ln(e^{-\lambda t}) \\ \ln N &= \ln N_0 - \lambda t \\ 2.30 \ln N &= 2.30 \ln N_0 - 2.30 \lambda t \\ \log N &= \log N_0 - 2.30 \lambda t \end{aligned}$$

Thus a graph of either $\ln N$ or $\log N$ is a straight line. From the intercept the value of N_0 can be found, and from the slope the value of λ can be found.

PROBLEMS ON LOGARITHMS

- 109 From the value of $\log 2$ given above, calculate $\ln 2$.
 110 Starting with the value of $\log 100$, calculate the value of $\ln 100$ and verify that $\ln 100 = \ln 1000 - \ln 10$.
 111 Calculate the value of $\ln [(2e^5)^3]$.
 112 If $p = 4e^{5x}$ and $q = 2e^{-x}$, calculate $\ln(p/q)$ if $x = 3$.
 113 If you have a calculator with exponentiation capability, find the value of y if $\log y = 2.4350$.