

Summation Notation

Often mathematical formulae require the addition of many variables. Summation or sigma notation is a convenient and simple form of shorthand used to give a concise expression for a sum of the values of a variable.

Let $x_1, x_2, x_3, \dots, x_n$ denote a set of n numbers. x_1 is the first number in the set. x_i represents the i th number in the set.

Summation notation involves:

The summation sign

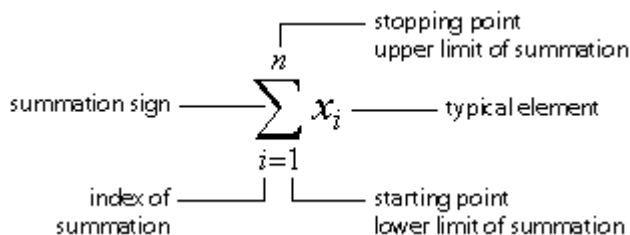
This appears as the symbol, Σ , which is the Greek upper case letter, S. The summation sign, Σ , instructs us to sum the elements of a sequence. A typical element of the sequence which is being summed appears to the right of the summation sign.

The variable of summation, i.e. the variable which is being summed

The variable of summation is represented by an index which is placed beneath the summation sign. The index is often represented by i . (Other common possibilities for representation of the index are j and t .) The index appears as the expression $i = 1$. The index assumes values starting with the value on the right hand side of the equation and ending with the value above the summation sign.

The starting point for the summation or the lower limit of the summation

The stopping point for the summation or the upper limit of summation



Some typical examples of summation

$\sum_{i=1}^n x_i$ This expression means sum the values of x , starting at x_1 and ending with x_n .

$$\sum_{i=1}^n x_i = x_1 + x_2 + x_3 + \dots + x_n$$

$\sum_{i=1}^{10} x_i$ This expression means sum the values of x , starting at x_1 and ending with x_{10} .

$$\sum_{i=1}^{10} x_i = x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

$\sum_{i=3}^{10} x_i$ This expression means sum the values of x , starting at x_3 and ending with x_{10} .

$$\sum_{i=3}^{10} x_i = x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 + x_{10}$$

$\sum x$ The limits of summation are often understood to mean $i = 1$ through n . Then the notation below and above the summation sign is omitted. Therefore this expression means sum the values of x , starting at x_1 and ending with x_n .

$$\sum x = x_1 + x_2 + x_3 + \cdots + x_n$$

$\sum_{i=1}^n x_i^2$ This expression means sum the squared values of x , starting at x_1 and ending with x_n .

$$\sum_{i=1}^n x_i^2 = x_1^2 + x_2^2 + x_3^2 + \cdots + x_n^2$$

Arithmetic operations may be performed on variables within the summation. For example:

$\left(\sum_{i=1}^n x_i \right)^2$ This expression means sum the values of x , starting at x_1 and ending with x_n and then square the sum.

$$\left(\sum_{i=1}^n x_i \right)^2 = (x_1 + x_2 + x_3 + \cdots + x_n)^2$$

Arithmetic operations may be performed on expressions containing more than one variable. For example:

$\sum_{i=1}^n x_i y_i$ This expression means form the product of x multiplied by y , starting at x_1 and y_1 and ending with x_n and y_n and then sum the products.

$$\sum_{i=1}^n x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + \cdots + x_n y_n$$

$\sum_{i=1}^n c = nc$ In this expression c is a constant, i.e. an element which does not involve the variable of summation and the sum involves n elements.

Problems

Data

i	x_i
1	1
2	2
3	3
4	4

1. Find $\sum_{i=1}^4 x_i$

$$\sum_{i=1}^4 x_i = x_1 + x_2 + x_3 + x_4 = 1 + 2 + 3 + 4 = 10$$

2. Find $\sum_{i=1}^4 x_i^2$

$$\sum_{i=1}^4 x_i^2 = x_1^2 + x_2^2 + x_3^2 + x_4^2 = 1^2 + 2^2 + 3^2 + 4^2 = 1 + 4 + 9 + 16 = 30$$

Data

i	x_i
1	-1
2	3
3	7

and c which is a constant = 11

3. Find $\sum_{i=1}^3 x_i^2$

$$\sum_{i=1}^3 x_i^2 = x_1^2 + x_2^2 + x_3^2 = (-1)^2 + 3^2 + 7^2 = 1 + 9 + 49 = 59$$

4. Find $\left(\sum_{i=1}^3 x_i\right)^2$

$$\left(\sum_{i=1}^3 x_i\right)^2 = (x_1 + x_2 + x_3)^2 = (-1 + 3 + 7)^2 = (9)^2 = 81$$

5. Find $\sum_{i=1}^3 c$

$$\sum_{i=1}^3 c = 11 + 11 + 11 = 33$$

Data

i	x_i	y_i
1	10	0
2	8	3

3	6	6
4	4	9
5	2	12

6. Find $\sum_{i=1}^5 x_i$

$$\sum_{i=1}^5 x_i = x_1 + x_2 + x_3 + x_4 + x_5 = 10 + 8 + 6 + 4 + 2 = 30$$

7. Find $\sum_{i=1}^5 y_i$

$$\sum_{i=1}^5 y_i = y_1 + y_2 + y_3 + y_4 + y_5 = 0 + 3 + 6 + 9 + 12 = 30$$

8. Find $\left(\sum_{i=1}^5 y_i \right)^2$

$$\left(\sum_{i=1}^5 y_i \right)^2 = (y_1 + y_2 + y_3 + y_4 + y_5)^2 = (0 + 3 + 6 + 9 + 12)^2 = (30)^2 = 900$$

9. Find $\sum_{i=1}^5 x_i y_i$

$$\sum_{i=1}^5 x_i y_i = x_1 y_1 + x_2 y_2 + x_3 y_3 + x_4 y_4 + x_5 y_5 = 0 + 24 + 36 + 36 + 24 = 120$$

[\[Index\]](#)