Visionlearning®
your insight into science

Equations

Scientific Notation and Order of Magnitude

BY JANET SHIVER, PH.D., TERI WILLARD, ED.D.

Table of contents

Numbers and numeration systems
History of our numeration system
Scientific notation
How to write a number in scientific notation
Scientific notation and the decimal place
Orders of magnitude
Dividing numbers in scientific notation
Example: All creatures great and small
Conclusion

Terms you should know

base : a number raised to a power. For example, in the expression 10^3, 10 is the base.

exponent : the raised numeral written behind the base indicating the number of times the base is used as a factor. For example, the expression 10^3 means 10 x 10 x 10; the exponent of 10^3 is 3.

scientific notation : a method for expressing numbers using powers of 10. A number written in scientific notation has two factors, a number that is at least one, but less than ten, and a power of ten. For example, the age of the Earth can be expressed at 4.543×10^9.

order of magnitude : a classification that is a multiple of ten, expressed in scientific notation.

Measurement is the basis of science. Scientists measure and evaluate measurements to study and explain phenomena. Humans naturally want to compare the sizes of items. Say we wanted to compare how strong we are to how strong another animal is. We would see that the average human can comfortably lift one times her weight, while a gorilla can lift ten times his weight, an ant 100 times, and a dung beetle an astounding 1000 times its weight (that is a lot of dung!). To

Top

Takeda EX1070

make comparisons like these scientists often think in terms of orders of magnitude, so they might say that a dung beetle is 100 times as strong as a human.

Order of magnitude is a method for describing the size of measures in terms of powers of ten. An order of magnitude estimates the approximate value or size of something in base-ten units such as, “in the millions.” For example, our national debt is “in the ten-trillions” of dollars which means it can be anywhere from 10 trillion to 99 trillion. Order of magnitude is an especially important concept in science. It allows scientists to express a number in scientific notation and roughly determine how much larger or smaller one object is compared to another. To understand order of magnitude, we must first understand our base-ten system and scientific notation.

Numbers and numeration systems

A number is an abstract concept of an amount or measure, for example, two or three. A numeral, however, is a symbol representing a number, such as the symbol “2” or “3.” The set of symbols and rules that govern how numbers are represented is called a numeration system. Many early numeration systems were based upon the values that symbols represented, and to make larger numbers, symbols were written in order and their values were added together. For example, in the Egyptian numeration system, the symbol | represents the number one, and || represents “1 + 1,” or two. ||| represents three, and so on (see Figure 1 for an example). The problem with this system is that numbers became extremely long. This was partially addressed by giving new symbols to each new power of ten, for example, ten was denoted as ∩. But this didn’t completely solve the problem as 99 looks like this in Egyptian symbols:

∩∩∩∩∩∩∩∩∩|||||||||

Top

Figure 1: Numerical hieroglyphs at the Temple of Karnak in north Luxor, Egypt. image © Olaf Tausch

In contrast to the Egyptian system, our modern numeration system is called a place value system. Ten unique symbols are used to indicate amounts between zero and nine, and the position of a particular symbol represents the value of a particular power of ten. For example, the numeral “1” represents different values based on its position within the numeral. The numeral “11” represents the number eleven, not the number two as it would in the Egyptian system, because there is a “1” in the ten's place and a “1” in the one's place. So ten plus one equals eleven. One characteristic our system has in common with the Egyptian system is that we still add the values in each position to get our final number. Let’s take a look at the numeral 2,576. If we expand this numeral, we can show the value of each digit based on its position and how we would determine its value. The numeral 2,576 can be written in expanded notation as:

$$(2 \times 1000) + (5 \times 100) + (7 \times 10) + (6 \times 1)$$

Notice that each of the numerals is multiplied by a power of ten. For example, five is multiplied by the number 100. Since 100 is a multiple of ten, it can be written as 10 x 10 or 10^2. 1000 is also a multiple of ten and can be written as 10 x 10 x 10 or 10^3. Using these powers of ten, we can rewrite the expanded notation to highlight that feature:

$$(2 \times 10^3) + (5 \times 10^2) + (7 \times 10^1) + (6 \times 10^0)$$

Top

Writing it like this, you can see that the exponent denotes the position of the numeral. And the position of each numeral is important as it tells us the value, or power of ten, to be included in the total sum for the number. Being able to express a number using powers of ten is important to understanding order of magnitude, for example, take a look at the table below.

Table 1: Powers of ten.

Powers of ten	Numeral
10^5	100,000
10^4	10,000
10^3	1,000
10^2	100
10^1	10
10^0	1

Comprehension Checkpoint

In the number 50,607,920, the numeral "7" represents

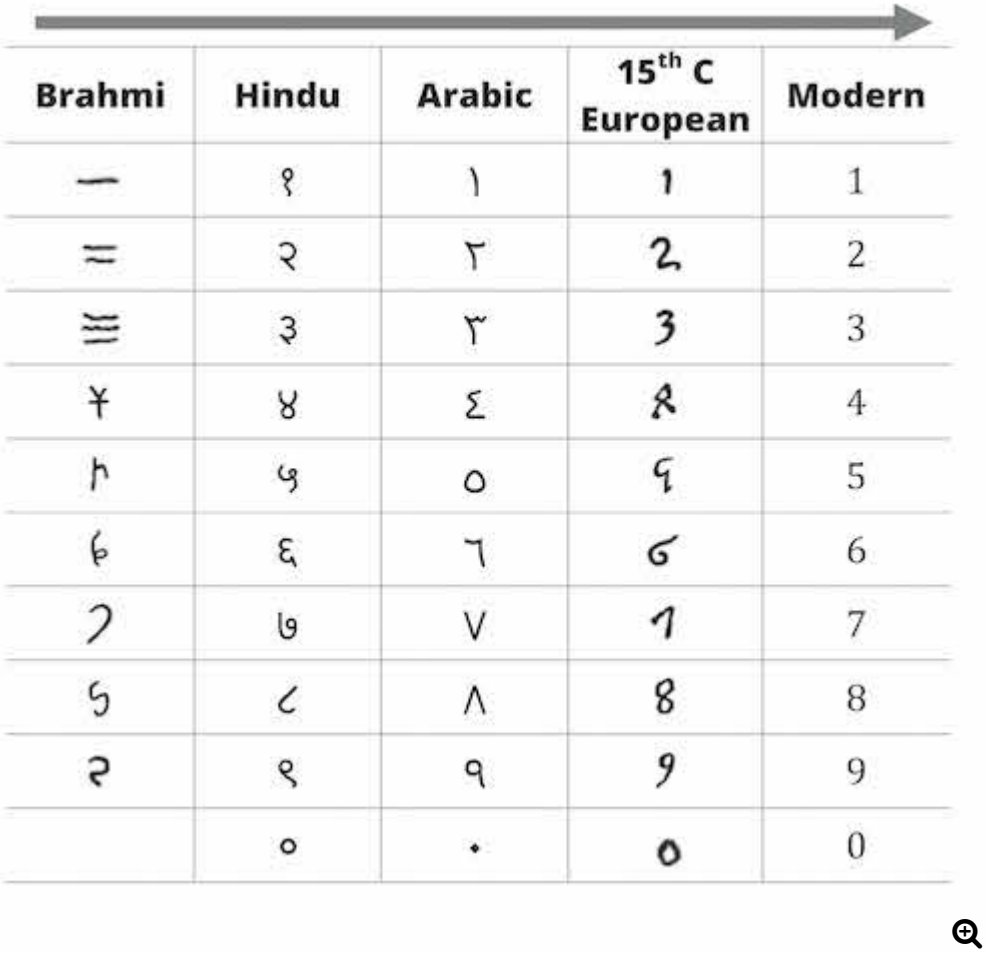
a. 7×10^3

b. 7×10^4

History of our numeration system

Where did our numeration system originate? Our base-ten numeration system began with symbols originating in India. These symbols traveled with Indian traders to the Arabic and Islamic people, and finally migrated to Europe. The symbols evolved over time as they were used by various civilizations. Abu'l Hasan Ahmad ibu Ibrahim al-Uqlidisi was the very first mathematician to use this type of system in a document dated 953. In this document, he discussed and wrote numbers in the base-ten place value form similar to what we use today. Because our system was influenced by developments of the Indians and Arabs, it is called the Hindu-Arabic system. (See Figure 2 for an example of the evolution of numerals.)

top



Brahmi	Hindu	Arabic	15th C European	Modern
—	१	١	1	1
=	२	٢	2	2
≡	३	٣	3	3
⊀	४	٤	4	4
⊦	५	٥	5	5
⊧	६	٦	6	6
⊃	७	٧	7	7
⊅	८	٨	8	8
⊇	९	٩	9	9
	०	٠	0	0

Figure 2: A selection of numerals from various cultures in history. You can see the progression of numerical symbols from Brahmi to Hindu to Arabic to 15th century European to our commonly recognized modern numbers. image © I.Taylor (Brahmi and 15th C European)

Scientific notation

While our modern numeration system makes it easier to write large numbers than the Egyptian system, writing very large and very small numbers can still be cumbersome and so scientists use scientific notation as a way to write very large or very small numbers in a much more concise way that makes working with these numbers easier. For example, the Earth has a surface area of 169,900,000 miles squared and the world's population is approximately 7,403,000,000. What if you needed to divide these two numbers to find out how many people per square mile were on the earth? These giant numbers can be cumbersome to divide. Instead, we could express them in scientific notation as 1.699×10^8 and 7.403×10^9. How did we convert these numbers and why is it easier to work with them in scientific notation? Let's look at where scientific notation came from towards understanding its use.

Top

Table 2: Standard versus scientific notation.

	Earth's Surface Area	World's Population	People per Square Mile
Standard number:	$169{,}900{,}000$	$7{,}403{,}000{,}000$	$\frac{7{,}403{,}000{,}000}{169{,}900{,}000}$
Scientific notation:	1.699×10^8	7.403×10^9	$\frac{7.403 \times 10^9}{1.699 \times 10^8}$

Scientific notation follows naturally from our base-ten system as a shorthand way to write very large or very small numbers. Scientific notation has two parts. Looking at an example, 1.6 x 10^8, we can see that the first part is a decimal number greater than or equal to one, but less than ten (in this case, 1.6). The second part is a multiple of ten expressed using an exponent (here, 10^8).

The notation we use today to denote an exponent was first used by Scottish mathematician, James Hume in 1636. However, he used Roman numerals for the exponents. Using Roman numerals as exponents became problematic since many of the exponents became very large so Hume's notation didn't last long. A year later in 1637, Rene Descartes became the first mathematician to use the Hindu-Arabic numerals of today as exponents. The exponent is used as a shorthand way to state how many times a number should be multiplied by itself, so 10^3 is equal to 10 x 10 x 10, and 2^4 is equal to 2 x 2 x 2 x 2.

In scientific notation we also use a decimal numeral. The Flemish mathematician Simon Stevin (Figure 3) first used a decimal point to represent a fraction with a denominator of ten in 1585. While decimals had been used by both the Arabs and Chinese long before this time, Stevin is credited with popularizing their use in Europe. An English translation of Stevin's work was published in 1608 and titled *Disme, The Arts of Tenths or Decimal Arithmetike*, and it inspired President Thomas Jefferson to propose a decimal-based currency for the United States. (For example, one tenth of a dollar is called a dime.)

Top

Figure 3: Statue of Simon Stevin (1548-1620) in Bruges. Stevin was a Flemish mathematician and engineer, credited with introducing decimal fractions.
image © Dennis Jarvis

While we can trace the history of the components (decimals and exponents) of scientific notation, it is difficult to determine who actually first used the term scientific notation. In fact, it wasn't until 1961 that the term could be found in a dictionary indicating that it was being widely used. Even though it is difficult to pinpoint the exact origins of the phrase, it is often thought to have begun with computer scientists. In the 1940's, Konrad Zuse introduced a concept that he referred to as the "floating point." Zuse's floating point was a method of representing any number as a decimal greater than or equal to one but less than ten, times a number raised to a power. This notation made it easier to represent and conduct calculations on large and small numbers in the binary code used by computers, even given their limited computing power at the time. Over the next two decades, the term scientific notation often referred to a number expressed as a decimal (described above) times any second number raised to a power. For example, 2.45×2^3 would have been described as scientific notation in the 1960s or earlier. Today, we only use the term scientific notation when the second number is the numeral 10 raised to a power, such as in 2.45×10^3.

Top

Comprehension Checkpoint

The expression 16.7 x 2^5 is an example of modern scientific notation.

a. True

b. False

How to write a number in scientific notation

Did you know that the Earth is 4,543,000,000 years old or that a carbon atom weighs just 0.00000000000000000000000199265 grams? Those are really large and small numbers! If you had to write them several times, or worse yet, use them to calculate something, you may have a difficult time keeping up with all of those zeros. To help simplify these numbers and make them easier to work with, we can express them using scientific notation.

Consider the age of the Earth, 4,543,000,000 years. To rewrite 4,543,000,000 years in scientific notation we must express it as a decimal between one and ten. To do that we divide it by 1,000,000,000 giving us 4.543 years. But the age of the earth is not 4.543 years. So to express this number correctly, we must show that it has to be multiplied by 1,000,000,000. So we can write this as 4.543 times 1,000,000,000. But this can be shortened further because 1,000,000,000 is equal to 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 x 10 which can be expressed as 10^9. So we can write the age of the earth in scientific notation as 4.543 x 10^9 years.

Table 3 shows how some small and large numbers are expressed in terms of powers of ten.

Table 3: Values for powers of 10.

Power of 10	Expanded Meaning	Equivalent Value
10^{-5}	$\frac{1}{10^5}$ or $\frac{1}{10\times10\times10\times10\times10} = \frac{1}{100000}$	0.00001
10^{-4}	$\frac{1}{10^4}$ or $\frac{1}{10\times10\times10\times10} = \frac{1}{10000}$	0.0001
10^{-3}	$\frac{1}{10^3}$ or $\frac{1}{10\times10\times10} = \frac{1}{1000}$	0.001
10^{-2}	$\frac{1}{10^2}$ or $\frac{1}{10\times10} = \frac{1}{100}$	0.01

Top

Power of 10	Expanded Meaning	Equivalent Value
10^{-1}	$\frac{1}{10^1}$	0.1
10^0	1	1
10^1	10	10
10^2	10 x 10	100
10^3	10 x 10 x 10	1,000
10^4	10 x 10 x 10 x 10	10,000
10^5	10 x 10 x 10 x 10 x 10	100,000

As you can see from the table, when the exponent on the number ten is a negative number, we can think of this as multiplying by 1/10. For example, 10^{-4} is the same as multiplying by the following:

$$\frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{10000}$$

This number can be written in different ways, such as:

$$\frac{1}{10000} = \frac{1}{10^4} = 10^{-4} = .0001$$

The takeaway is that negative exponents are used to express very small numbers in scientific notation.

Let's think about the weight of our carbon atom, 0.00000000000000000000000199265 grams. To express the weight of the carbon atom as a decimal number greater than or equal to one and less than ten, we would have to multiply by 100,000,000,000,000,000,000,000, or 10^{23}. This would give us a decimal number of 1.99265, but we then have to add in the factor to show how the number is truly expressed. To reverse this process and return the atom to its original weight we would have to divide by 10^{23}. So the original weight of a carbon atom can be written as:

Top

$$\frac{1.99265}{10^{23}}$$

This representation is very close to scientific notation, but scientific notation is always written using multiplication, not division. So we can rewrite it as:

$$1.99265 \times \frac{1}{10^{23}}$$

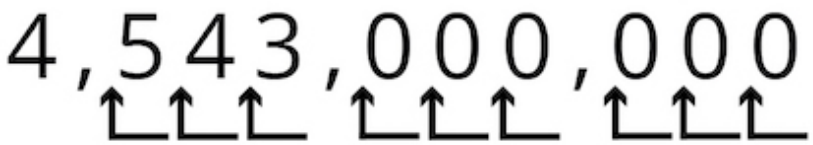
Finally, we express the power of ten with a negative exponent and place it in the numerator:

$$1.99265 \times 10^{-23} \text{ grams}$$

We now have the weight of an atom written correctly in scientific notation.

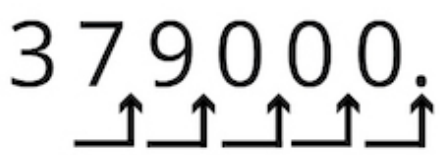
Scientific notation and the decimal place

A faster method for rewriting a number in scientific notation is to think of how many times the decimal point would have to be moved. For example, in our age of the Earth example, to express 4,543,000,000 as a number greater than one and less than ten we can simply move the decimal.

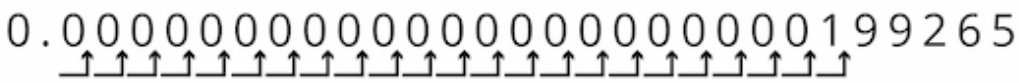


In this case, we moved our decimal nine places to the left to get the number 4.543. Each shift of the decimal represents division of the number by ten. To return the number back to its original form we must do the opposite, multiply by nine 10s. Therefore, to express the number in scientific notation, we would use a positive nine exponent, thus giving us the same answer as earlier: 4.543 x 10^9. To convert a number with a positive exponent back, you would move the decimal to the right the same number of places as indicated by the exponent. For example, 3.79 x 10^5:

Top



We can use a similar but opposite process to write very small numbers in scientific notation. Let's return to the carbon atom that weighed 0.00000000000000000000000199265 grams. To begin the process, we must move our decimal 23 times to the right.



Since we are moving to the right we are multiplying 0.00000000000000000000000199265 by 10 twenty-three times. To return the number back to its original form we must reverse the process and divide by 10 twenty-three times. Since multiplying by a negative exponent is the same as dividing a number, the original weight of a carbon atom can be written as 1.99265×10^{-23} grams.

Notice that when you moved the decimal to the left you made the number you started with smaller. To return it to its original size you had to multiply by a bunch of tens. This means that the exponent on the ten had to be positive. When you moved the decimal to the right, the opposite happened. You made the original number much bigger, so to return it to its original tiny size you had to divide by a bunch of tens, making the exponent on ten negative.

Comprehension Checkpoint

The number 0.0036 written in scientific notation is

a. 3.6×10^{3}

b. 3.6×10^{-3}

Orders of magnitude

Top

Scientists describe the magnitude or size of numbers using something called the order of magnitude. We can think of order of magnitude to mean the power of ten closest to a given quantity. For example, we often hear there are a billion people in China. This is not the exact number of people in China. It is just an order of magnitude approximation. Since 1,000,000,000 can be written as 10^9 we can say that the population of China has an order of magnitude of nine. We can find an order of magnitude by simply writing a number in scientific notation.

Dividing numbers in scientific notation

Scientists often use order of magnitude to compare the sizes or distances of items. Orders of magnitude give us a quick method for determining the relationship between two quantities. But to do this, we need to be able to divide numbers written in scientific notation. For example, the distance from the Earth to the Sun is 93,000,000 miles = 9.3 x 10^7 miles. However, the distance to the next nearest star, Proxima Centauri, is about 2.522 x 10^{13} miles. How many times greater is the distance from Proxima Centauri to Earth than the distance from the Earth to the Sun?

To solve this problem, we need to divide the two distances.

$$\frac{2.522 \times 10^{13} \text{ miles}}{9.3 \times 10^{7} \text{ miles}} = \frac{2.522}{9.3} \times \frac{10^{13}}{10^{7}} \approx 0.2712 \times 10^{6}$$

Notice that when you divide powers of ten, the net result is that the exponents get subtracted from one another:

$$10^{13} \div 10^{7} = 10^{13-7} = 10^{6}$$

Similarly, when you multiply powers of ten the net result is to add the exponents together. Now back to our example. Since the exponent on ten is six, it is tempting to say that the two numbers differ by a magnitude of six, but this is not correct. The convention is to report the answer in scientific notation. The answer 0.2712 x 10^6 is not in correct form, since 0.2712 is not expressed as a number at least one but less than ten. To convert this into scientific notation, you need to move the decimal one place to the right, which is the same as multiplying the number by ten. To compensate for this you would have to reduce the exponent by one digit, which is equivalent to dividing by ten, and arrive at the same answer 2.712 x 10^5. We can now correctly report that the two distances differ by a magnitude of five. Since 1 x 10^5 is equal to 10,000, we can say that Proxima Centauri is 10,000 times further from Earth than the Sun.

Top

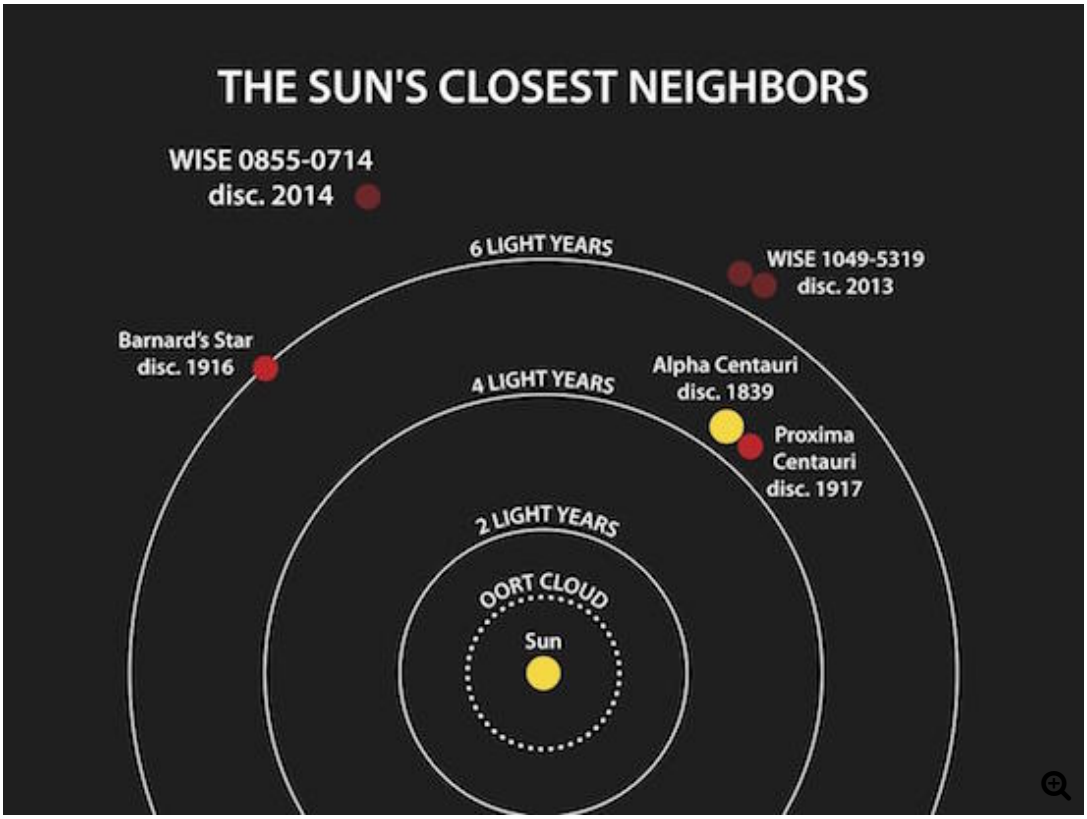


Figure 4: This diagram illustrates the locations of the star systems closest to the sun. Proxima Centauri is shown just after the 4 light year ring.
image © NASA/Penn State University

We see order of magnitude comparisons often arising in chemistry when discussing the concentrations of substances. For example, a scientist might say the concentration of arsenic in the hundreds of parts per million (ppm) range. This means, that for every one million "parts" of water you would find 0.01 parts of arsenic. For example, you could say that for every 1kg of water, you could find approximately 0.01mg of arsenic (1kg = 1,000,000mg). Let's write the comparison as a ratio.

$$\frac{0.01mg}{1kg} = \frac{0.01mg}{1000000mg}$$

Now let's express the numerator and denominator in scientific notation:

$$\frac{0.01mg}{1000000mg} = \frac{1 \times 10^{-2}mg}{1 \times 10^{6}mg}$$

Dividing we get:

$$\frac{1 \times 10^{-2}mg}{1 \times 10^{6}mg} = \frac{1}{1} \times \frac{10^{-2}}{10^{6}} = 1 \times 10^{-2-6} = 1 \times 10^{-8}$$

Top

We can now say that the amount of arsenic is eight orders of magnitude less than the amount of water.

Example: All creatures great and small

Let's return to the blue whale and plankton mentioned at the beginning of this module. The blue whale is the largest creature ever to inhabit the planet and yet it shares the ocean with one of the smallest, plankton, a creature that barely represents a tiny speck in the ocean. An average blue whale weighs approximately 190,000 kilograms while a single plankton weighs a mere 0.5 milligrams. To express these weights in the same units (if you need help with metric conversions, refer to our Metric System module) we can say that a Blue Whale weighs 190,000,000,000mg or 1.9×10^{11}mg and a Plankton weighs 0.5mg or 5×10^{-1}mg.

Comparing these two ocean dwellers we get:

$$\frac{1.9 \times 10^{11}}{5 \times 10^{-1}} = 0.38 \times (10^{11} \times 10^{1})$$

$$= 0.38 \times 10^{12}$$

$$= 3.8 \times 10^{-1} \times 10^{12}$$

$$= 3.8 \times 10^{11}$$

Thus the whale is 10^{11}, or 100,000,000,000 times larger than the plankton!

Comprehension Checkpoint

When you compare 4.52×10^{7} to 3.60×10^{-3},

a. 4.52×10^{7} is 10 orders of magnitude larger than 3.60×10^{-3}.

b. 4.52×10^{7} is 4 orders of magnitude larger than 3.60×10^{-3}.

Conclusion

Scientific notation and order of magnitude are fundamental concepts in all branches of science. Order of magnitude allows us to quickly estimate the size of something or the difference in

top

measure between two things by expressing it as a power of ten. With practice, expressing numbers in scientific notation provides a quick and easy way to compare, multiply and divide numbers. These concepts are especially useful when comparing very large and very small measurements such as the weight of an atom, the distance to a star, or the length of a tiny, microscopic ocean dweller.

Summary

The blue whale weighs approximately 190,000 kilograms, while a plankton weighs just 0.5 milligrams—a difference of 11 orders of magnitude. Scientific notation and order of magnitude are fundamental concepts in all branches of science. They are especially useful when expressing and comparing very large and very small measurements. This module traces the history of our base-ten numeration system and gives a step-by-step explanation of how to write numbers in scientific notation. Sample problems demonstrate how to divide numbers in scientific notation to determine orders of magnitude.

Key Concepts

- Scientists often deal with very large and very small measurements, and so they think in terms of order of magnitude to effectively express these measurements and differences between them.
- Orders of magnitude differences are embedded in our base-ten measurement system, where one order of magnitude represents a ten-fold difference.
- Scientific notation is used to make it easier to express extremely large or extremely small numbers, and is rooted in multiplying a number by some power of ten (10^x).
- Expressing numbers in scientific (base ten) notation can often make it easier to perform simple mathematical operations on that number.

Further Reading

The Metric System

Unit Conversion

References

Schombert, J. (n.d.). History of Astronomy. Retrieved from: http://abyss.uoregon.edu/~js/ast121/lectures/lec02.html

Top

Janet Shiver, Ph.D., Teri Willard, Ed.D. "Scientific Notation and Order of Magnitude" Visionlearning Vol. MAT-3 (7), 2016.

Top