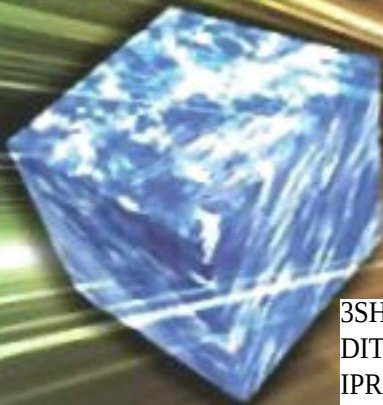


# Digital Color Imaging

## HANDBOOK



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Edited by  
**Gaurav Sharma**

# Digital Color Imaging

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# Digital Color Imaging

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Edited by  
**Gaurav Sharma**  
*Xerox Corporation*  
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# *Preface*

The field of color imaging deals with the capture, processing, communication, and reproduction of color images. The origins of color imaging can be traced back to prehistoric times when cave dwellers created the first color drawings depicting events in their lives, using primitive materials and techniques available to them. Since then, color images have played an important role in history, and color imaging has advanced hand in hand with progress in science and technology. In the past 10 to 15 years, this field, like many others, has been significantly transformed by the digital revolution.

Digital color imaging devices such as digital still and video cameras, color scanners, displays, printers, DVD players, and cable/satellite set-top boxes are now commonplace in both home and office environments. A vast majority of color imagery is now captured digitally. An even larger fraction is digital during some part of the image's life cycle, so it is subject to computer-based processing. Digital technology enables unprecedented functionality and flexibility in the capture, processing, exchange, and output of color images. A knowledge of color science, color systems, appropriate processing algorithms, and device characteristics is necessary to fully harness this functionality and flexibility. As a result, the field of digital color imaging is a highly interdisciplinary area involving elements of physics, visual science, chemistry, psychophysics, computational algorithms, systems engineering, and mathematical optimization. While excellent texts and reference material exist in each of these areas, it has hitherto been the responsibility of researchers in the color imaging field to cull out relevant information. The goal of this handbook is to present aspects of these diverse elements as they relate to digital color imaging in a single and concise compilation. It is my hope that the handbook's assimilation of these different aspects and perspectives will aid students who are starting out in this area, as well as practitioners and researchers with expertise in specific domains who seek a better understanding of the rest of the system.

Chapters 1 through 3 are intended to cover the basics of color vision, perception, and physics that underpin digital color imaging. The material in these chapters will serve as useful background for those who are new to this area and as a refresher and update for color engineers with significant experience in the field. The end-to-end aspects of control and management of color in digital imaging systems are addressed in Chapter 4. Chapter 5 is

concerned with device color characterization, whereby the responses of individual color imaging devices (e.g., digital cameras, scanners, color printers, and displays) are measured and suitably accounted for in the capture and output of color images.

Chapters 6 and 7 address the important subject of digital halftoning, which deals with the rendition of images on printers and display devices that are capable of only bilevel reproduction or, more generally, of a limited number of levels. Since the vast majority of printers used in the printing and publishing industries are halftone printers, this topic is of significant interest in color imaging. Chapter 8 describes the compression of color images, which is a prerequisite for efficient use of network bandwidth and storage resources. The chapter cannot, and is not intended to, span the vast field of image compression. Instead, it focuses on aspects of image compression that are specifically pertinent to color images, a topic that is often left unaddressed by a number of image compression techniques. Brief overviews of the widely used JPEG and the emerging JPEG2000 image compression standards are included in the chapter.

Chapter 9 discusses color quantization or palettization of color images for use in frame-buffer systems with limited memory. While typical desktop displays today are “full-color” and typically do not require palettization, the issue is regaining importance in smaller displays on hand-held mobile devices, which are much more limited. Chapter 10 discusses techniques for pictorial gamut mapping. These techniques address the fundamental trade-offs encountered when printing or displaying color images on common output devices that are capable of producing only a limited range of colors. Computationally efficient transforms for digital color imaging are discussed in Chapter 11. Finally, Chapter 12 covers color image processing in digital cameras, a topic that has assumed great importance with the explosion in the use of these devices for image capture.

Each chapter of the handbook is largely self-contained and can be read in isolation, provided the reader is generally familiar with the area. Cross-references among the chapters capture the important interrelationships in the information presented in the individual chapters. Chapter 1 also includes a broad overview of digital color imaging systems with references to, and connections between, the material in the other chapters, which may not be directly apparent. This is intended to facilitate the understanding of digital color imaging from a systems perspective, which is becoming increasingly important in today’s open, interconnected world. Additional material related to the book will be made available on the publisher’s web site [www.crcpress.com](http://www.crcpress.com). In particular, due to concerns of increased cost and the limitations of color accuracy in the printing process, a number of images that were originally in color have been included only as black-and-white figures in the book; full-color electronic versions of these figures are available online.

I would like to take this opportunity to thank all the authors for their excellent contributions. They have done an admirable job in writing for a

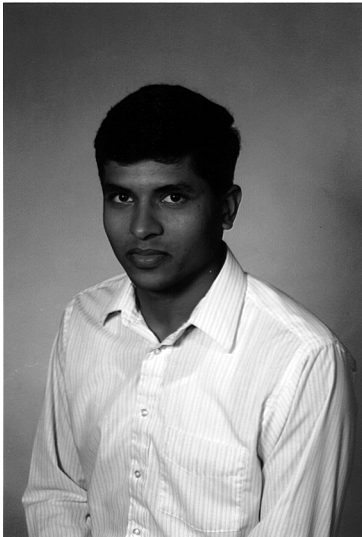
fairly wide audience while still communicating their individual research insights and accomplishments. The quality of the handbook can be directly attributed to their diligence.

I would also like to thank the outstanding staff at CRC press for their excellent support in the production and editing of this handbook. In particular, I would like to thank Nora Konopka for initiating this project, Helena Redshaw for urging me and the contributors to stay on schedule and for handling the submissions of all the materials, and Susan Fox for handling the copy editing and final production. Without their dedicated assistance, this project would have never been completed.

**Gaurav Sharma**  
*Xerox Corporation*  
*Webster, NY*  
*g.sharma@ieee.org*

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## About the Editor



**Gaurav Sharma** is a member of the research staff at Xerox Corporation's Solutions and Services Technology Center, where he currently leads a research project on color imaging. He is also involved in teaching in an adjunct capacity at the Electrical and Computer Engineering Departments at the Rochester Institute of Technology, Rochester, New York. He received a BE degree in electronics and communication engineering from University of Roorkee, India, in 1990; an ME degree in electrical communication engineering from the Indian Institute of Science, Bangalore, India, in 1992; and an MS degree in applied mathematics and a Ph.D. degree in electrical and computer engineering from North Carolina State University, Raleigh, in 1995 and 1996, respectively.

From August 1992 through August 1996, he was a research assistant at the Center for Advanced Computing and Communications in the Electrical and Computer Engineering Department at North Carolina State University. His research and graduate work during this period focused on metrics for the evaluation and design of color recording devices. Since August 1996, he has been with Xerox Corporation. His research interests include color science and imaging, image security and halftoning, signal restoration, and error correction coding. Dr. Sharma is a member of Sigma Xi, Phi Kappa Phi, and Pi Mu Epsilon and is the current vice president of the Rochester chapter of the IEEE Signal Processing Society. He has authored or co-authored more than 40 technical papers in the fields of color, digital imaging, and image processing. He holds four U.S. patents and has more than a dozen pending U.S. patent applications.

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# Contributors

**A. Ufuk Agar**

Hewlett-Packard Laboratories  
Color Imaging & Printing  
Technologies Department, HP  
Labs  
Palo Alto, California

**Jan P. Allebech**

Purdue University  
School of ECE  
West Lafayette, Indiana

**Raja Balasubramanian**

Xerox Webster Research Center  
Webster, New York

**Farhan A. Baqai**

Sony Corporation  
Media Processing Division  
San Jose, California

**Luc Brun**

Université de Reims Champagne  
Ardenne  
Reims, France

**Patrick Emmel**

Clariant  
Masterbatches Division  
Muttenz, Switzerland

**Mark D. Fairchild**

Rochester Institute of Technology  
Munsell Color Science Lab, Center  
for Imaging Science  
Rochester, New York

**Edward Giorgianni**

Eastman Kodak Company  
Imaging Research & Advanced  
Development Division  
Rochester, New York

**Charles Hains**

Xerox Corporation  
Webster, New York

**Garrett M. Johnson**

Rochester Institute of Technology  
Center for Imaging Science  
Rochester, New York

**R. Victor Klassen**

Xerox Corporation  
Webster, New York

**Keith Knox**

Xerox Corporation  
Xerox Digital Imaging Technology  
Center  
Webster, New York

**Thomas Madden**  
Eastman Kodak Company  
Imaging Research & Advanced  
Development Division  
Rochester, New York

**Jan Morovic**  
University of Derby  
Colour & Imaging Institute  
Kingsway, Derby, England

**Ken Parulski**  
Eastman Kodak Company  
Digital & Applied Imaging Division  
Rochester, New York

**Ricardo L. de Queiroz**  
Xerox Corporation  
Corporate Research & Technology  
Webster, New York

**Gaurav Sharma**  
Xerox Corporation  
Webster, New York

**Kevin E. Spaulding**  
Eastman Kodak Company  
Imaging Research & Advanced  
Development Division  
Rochester, New York

**Alain Trémeau**  
Université Jean Monnet  
de Saint-Etienne  
Saint-Etienne, France

**Shen-Ge Wang**  
Xerox Corporation  
Webster, New York

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# *Color fundamentals for digital imaging*

*Gaurav Sharma*  
*Xerox Corporation*

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## 1.1 *Introduction*

In our daily lives, color images surround us in print, television, computer displays, photographs, and movies. While these color images are taken for granted by a majority of readers and viewers, their production engages an entire industry of scientists, engineers, and practitioners. A knowledge of fundamental color principles is central to the work of this industry. The purpose of this chapter is to provide a concise introduction to some of these fundamentals of color science, colorimetry, color technology, and color systems. The presentation in the chapter is organized as a progressive introduction of principles from a logical rather than historical perspective. While suitable references and background material are included, the purpose is not to exhaustively document historical development of the principles or necessarily trace concepts to primary originators.

The perception of color is the result of interaction between a physical stimulus; receptors in the human eye that sense the stimulus; and the neural

system and the brain; which are responsible for communicating and interpreting the signals sensed by the eye. This clearly involves several physical, neural, and cognitive phenomena, which must be understood so as to comprehend color vision completely. While research continues in the integration of all these aspects of color, significant success has been achieved in understanding the physical and (to a lesser extent) neural phenomena involved in color sensation. The first part of this chapter attempts to summarize the current understanding in these areas with particular emphasis on the aspects that are of interest in color imaging applications.

The second part of the chapter is a brief overview of color recording and reproduction devices, their underlying physical principles, and color characteristics. Color measuring instrumentation, digital image recording devices such as scanners and digital color cameras, and color reproduction devices such as displays and printers are described. The spectral and color characteristics of images are also briefly discussed. The third part of the chapter describes the concepts of device-independent color and color management. The final section offers concluding remarks on the content covered elsewhere in the chapter.

Where appropriate, each section begins with a description of general principles and then briefly discusses their application in color imaging applications. Several of the topics covered here are discussed in significant detail in later chapters, but the material here provides a broad system-wide overview and indicates the connections and interrelations that may otherwise not be apparent.

## 1.2 *Physical stimuli for color*

The physical stimulus for color is electromagnetic radiation in the visible region of the spectrum, which is commonly referred to as *light*. In air or a vacuum, the visible region of the electromagnetic spectrum is typically specified by the wavelength region between  $\lambda_{min} = 360$  nm and  $\lambda_{max} = 830$  nm. Light stimulates retinal receptors in the eye, which ultimately causes the phenomenon of vision and the perception of color.

Our current understanding about the nature of light and color can be traced to the work of Sir Isaac Newton.<sup>215</sup> Newton's careful experiments<sup>215,216</sup> with sunlight and a prism helped dispel existing misconceptions and led to the realization that light can be decomposed into a spectrum of *monochromatic* components that cannot be further decomposed. Accordingly, light is characterized physically by its spectral composition. Typically, the characterization takes the form of a spectral power distribution (SPD), which characterizes light by the distribution of power (or energy per unit time) as a function of wavelength.<sup>†</sup>

<sup>†</sup> Note that the selection of wavelength rather than frequency or wave number for the specification of spectral power distribution of light is a rather arbitrary choice but has become a commonly accepted convention in the photometry, color measurement, and imaging communities.

Absolute spectral power distributions for light emitted or reflected off a surface are specified typically in radiometric units of Watts per steradian per square meter.<sup>253,335</sup> In practice, absolute SPDs are rarely (if ever) required for the purposes of color measurement and specification, and relative SPDs, where the scale/units are arbitrary, are commonly used. Figure 1.1 illustrates the relative SPDs of typical daylight, cool white fluorescent office lighting, and an incandescent lamp. The abscissa on the plot indicates the wavelength, and the ordinate indicates the relative density of light power. The mathematical interpretation of the spectral power distribution is as follows: if  $I(\lambda)$  denotes the spectral power distribution, the power in an infinitesimal interval  $d\lambda$  centered about  $\lambda_0$  is given by  $I(\lambda_0)d\lambda$ .

Light incident on the eye may originate in different ways. When viewing self-luminous objects, the light directly originates from the object being viewed. More commonly, the object being viewed is illuminated by an external light source, such as daylight outdoors, or light from a lamp/overhead fixture indoors. In such situations, the SPD of light entering the eye is the product of the SPD of the light source and the spectral reflectance of the object. If the SPD of the illuminating source is given by  $I(\lambda)$ , and the spectral reflectance of the object is  $r(\lambda)$ , the SPD of the reflected light is given by the product  $I(\lambda)r(\lambda)$ . A similar relation is applicable to objects such as slides that are viewed in transmission, where the spectral reflectance is replaced by the spectral transmittance  $t(\lambda)$ . It is worth noting that the above mathematical relation is based on an idealized model of illuminant-object interaction that does not account for several geometry/surface effects such as the combina-

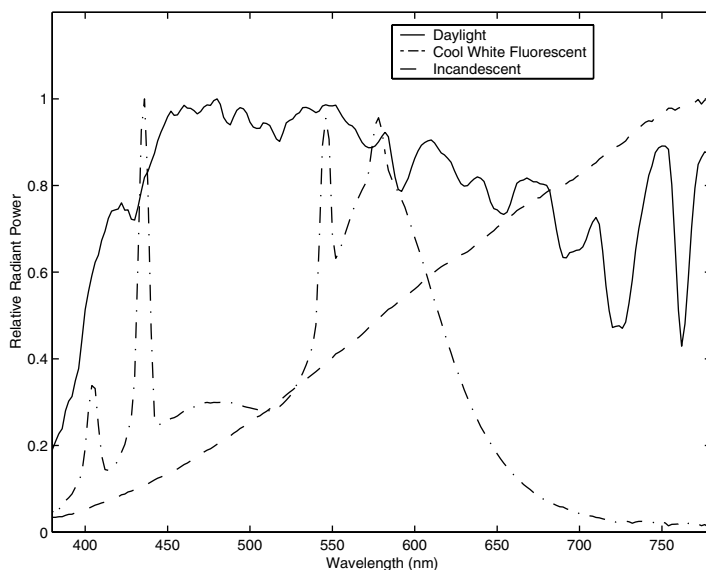


**Figure 1.1** Measured relative spectral power distributions (SPDs) for daylight, cool white fluorescent office lighting, and an incandescent lamp.

tion of specular and body reflectance components.<sup>189(pp. 43–45)</sup> The model is, however, reasonably accurate for most imaging situations if care is taken to measure using a light source and geometry similar to that used in final viewing. Figure 1.2 illustrates a set of spectral reflectances for five different objects. One can see that the spectral reflectances of objects can demonstrate significant wavelength selectivity in that they reflect light of certain wavelengths with significantly more strength than light of other wavelengths. This spectral selectivity is typically the main determinant of the color appearance of the object.

### 1.2.1 The stimulus error

In discussing objects, it is common to say that they possess certain colors. For instance, the sky may be described as blue, an apple as red, and grass as green. In actuality, however, there is no color without an observer; therefore, attributing a color to an object is not strictly accurate. The attribution of colors to objects/lights is a particular instance of what psychologists refer to as the *stimulus error*<sup>27,296</sup> wherein a sensation experienced by an observer is identified with the stimulus causing the sensation. Color scientists and researchers have been aware of the stimulus error that pervades our common usage of color terms. Newton himself demonstrated this awareness in his quote, “The rays, to speak properly, are not colored; in them there is nothing else than a certain power and disposition to stir up a sensation of this or that color.” Thus, speaking precisely, the light from the sky is not blue but evokes the sensation of blue when viewed by an observer.

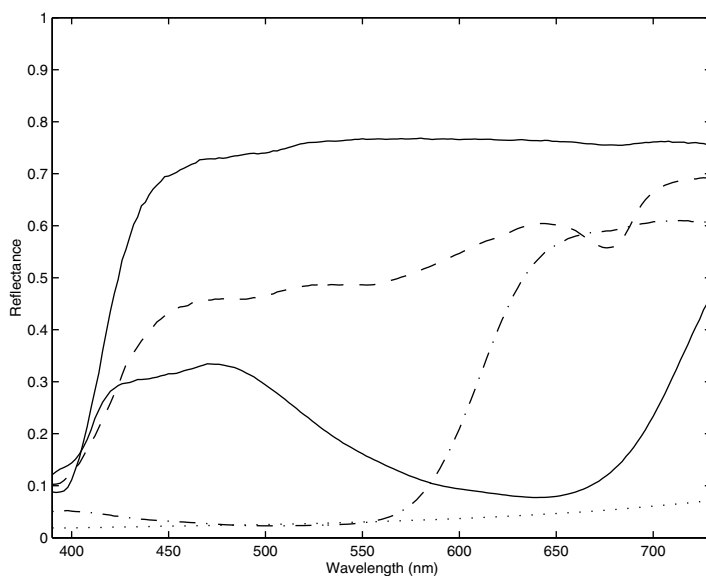


Figure 1.2 Measured spectral reflectance functions for five different natural objects.

As with several other sensations, the stimulus error is firmly ingrained in our usage of color terms, and one would have to go to great lengths and use awkward, pedantic language to avoid it entirely. Consequently, we continue to use color terms in commonly used contexts and talk, for instance, of cyan, magenta, and yellow colorants. It is, however, important to realize that an accurate interpretation of such statements requires a discounting of the stimulus error.

The stimulus error is often combined with other misuses of color terminology. For instance, one often hears the statement that a prism decomposes white light into its constituent colors. This statement is clearly inaccurate and unacceptable in technical usage. The proper statement would be that a prism decomposes light into its constituent spectral or wavelength components. Spectral power distributions of light, spectral reflectance functions, and spectral sensitivity functions are physical descriptions that are independent of observed sensation, and describing these in terms of color sensations is therefore incomplete and inaccurate. Errors of this type are therefore to be consciously avoided in technical descriptions of color.

### 1.3 Human color perception and trichromacy

Figure 1.3 shows a rough schematic of the human eye. The incident light is focused by the cornea and the eye's lens to form an image of the object being viewed onto the retina located at the back of the eyeball. The cornea provides most of the refraction needed to bring the light to a focus on the retina, and the primary purpose of the lens is to allow the eye to focus on objects at different viewing distances by changing the shape of the lens through the process of *accommodation*.<sup>153(p. 100)</sup> Photoreceptors within the retinal membrane are responsible for sensing the image and creating the neural signals that are responsible for the sense of sight. There are two kinds of photoreceptors: *rods* and *cones*. The rods are extremely sensitive to light and primarily useful for vision under very low light levels, termed as *scotopic* vision. In scotopic vision, only shades of gray can be perceived, and no color

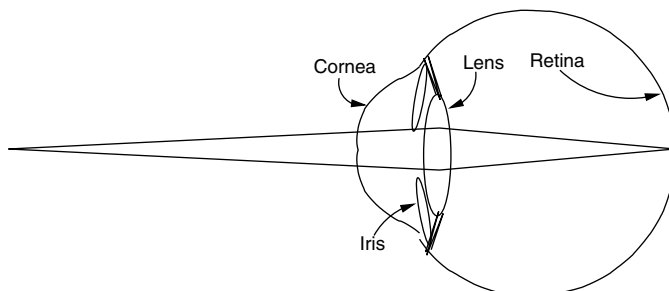


Figure 1.3 Schematic of the human eye.

is seen. This is the case, for instance, when objects are viewed under starlight. Under typical light levels used in imaging applications, the rods become saturated and do not contribute to vision; instead, the less-sensitive cones are active. The term *photopic* vision is used to describe this domain. There is a gradual change from photopic to scotopic vision as the illumination level is lowered, and in the intermediate *mesopic* form of vision both rods and cones are active. Typical light levels for these three domains of vision are listed in Section 1.5.1.

The cones are responsible for color vision. Observers with normal color vision<sup>†</sup> have three different types of cones, with photosensitive pigments that differ in their spectral absorption characteristics and, consequently, in their spectral sensitivities. The three types of cones are commonly called S, M, and L cones, which are abbreviated forms of short, medium, and long wavelength sensitive cones, respectively.<sup>‡</sup> Under a fixed set of viewing conditions, the response of these cones can be accurately modeled by a linear system defined by the spectral sensitivities of the cones. If the spectral distribution of light incident on the retina is given by  $f(\lambda)$ , where  $\lambda$  represents wavelength (we are ignoring any spatial variations in the light for the time being), the responses of the three cones can be modeled as a three vector with components given by

$$c_i = \int_{\lambda_{min}}^{\lambda_{max}} s_i(\lambda) f(\lambda) d\lambda \quad i = 1, 2, 3 \quad (1.1)$$

where  $s_i(\lambda)$  denotes the sensitivity of the  $i$ th type of cones, and  $\lambda_{min}, \lambda_{max}$  denote the interval of wavelengths outside of which all these sensitivities are zero. As indicated earlier, in air or vacuum, this visible region of the electromagnetic spectrum is specified by the wavelength region between  $\lambda_{min} = 360$  nm and  $\lambda_{max} = 830$  nm. Estimates of the effective sensitivities of the LMS cones (i.e., cone fundamentals<sup>256</sup>) are shown in [Figure 1.4](#).

Mathematically, the expressions in Equation 1.1 correspond to inner product operations<sup>96</sup> in the Hilbert space of square integrable functions  $\mathcal{L}^2([\lambda_{min}, \lambda_{max}])$ . Hence, the cone response mechanism corresponds to a projection of the spectrum onto the space spanned by three sensitivity functions  $\{s_i(\lambda)\}_{i=1}^3$ . This space is called the *human visual subspace* (HVSS).<sup>55,56,125,304,310</sup> The perception of color depends on further nonlinear processing of the retinal responses. However, to a first order of approximation, under similar conditions of adaptation, the sensation of color may be specified by the responses of the cones. This is the basis of all colorimetry and will be implicitly assumed throughout this section. A discussion of perceptual uniformity and appearance will be postponed until Sections 1.7 and 1.9.

† Around 8% of males and 0.5% of females are color deficient.

‡ Note that the common statement that the eye has three cones sensitive, respectively, to red, green, and blue light is not only inappropriate and erroneous for reasons described in Section 2.1, but also creates a circular definition.

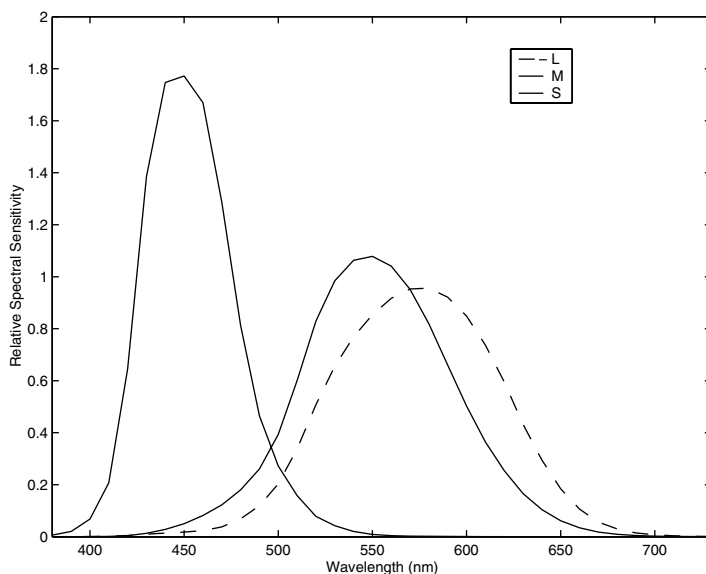


**Figure 1.4** Estimated effective sensitivities of the L, M, S cones (cone fundamentals).

For computation, the spectral quantities in Equation 1.1 may be replaced by their sampled counterparts to obtain summations as numerical approximations to the integrals. For most color spectra, a sampling rate of 10 nm provides sufficient accuracy but, in applications involving fluorescent lamps with sharp spectral peaks, a higher sampling rate or alternative approaches may be required.<sup>189,264,302,303</sup> If  $N$  uniformly spaced samples are used over the visible range  $[\lambda_{min}, \lambda_{max}]$ , Equation 1.1 can be written as

$$c_i = \sum_{i=0}^{N-1} s_i(\lambda_i) f(\lambda_i) \Delta\lambda = \mathbf{s}_i^T \mathbf{f} \quad i = 1, 2, 3 \quad (1.2)$$

In this equation,  $\{\lambda_i\}_{i=0}^{N-1}$  are the uniformly spaced wavelengths covering the visible region of the spectrum,  $\lambda_i = \lambda_0 + i\Delta\lambda$ , with  $\Delta\lambda$  as the wavelength sampling interval. The superscript  $T$  denotes the transpose operation,  $\mathbf{f} = [f(\lambda_0), f(\lambda_1), \dots, f(\lambda_{N-1})]^T$  is the  $N \times 1$  vector of samples of  $f(\lambda)$ , and  $\mathbf{s}_i = \Delta\lambda [s_i(\lambda_0), s_i(\lambda_1), \dots, s_i(\lambda_{N-1})]^T$  is the  $N \times 1$  vector of samples of  $s_i(\lambda)$  scaled by the sampling interval  $\Delta\lambda$ . Note that, for notational simplicity, we have absorbed the influence of the sampling interval as a scaling factor into the cone sensitivity vectors  $\{\mathbf{s}_i\}_{i=1}^3$ . Equation 1.2 can be compactly written using matrix-vector notation as

$$\mathbf{c} = \mathbf{S}^T \mathbf{f} \quad (1.3)$$

where  $\mathbf{c} = [c_1, c_2, c_3]^T$ ,  $\mathbf{S} = [s_{21}, s_{22}, s_{23}] =$  the  $N \times 3$  matrix with the cone sensitivity vectors as its columns. The HVSS then corresponds to the column space of  $\mathbf{S}$ .

In normal human observers, the spectral sensitivities of the three cones are linearly independent. Furthermore, the differences between the spectral sensitivities of color-normal observers are (relatively) small<sup>277(p.343),328,335</sup> and arise primarily due to the difference in the spectral transmittance of the eye's lens and the optical medium ahead of the retina.<sup>71,211,219,220,328</sup>

If a standardized set of cone responses is defined, color may be specified using the three-vector  $\mathbf{c}$  in Equation 1.3, known as a *tristimulus* vector. Just as several different coordinate systems may be used for specifying position in three-dimensional space, any nonsingular, well-defined linear transformation of the tristimulus vector  $\mathbf{c}$  can also serve the purpose of color specification. Because the cone responses are difficult to measure directly, but nonsingular linear transformations of the cone responses are readily determined through color-matching experiments, such a transformed coordinate system is used for the measurement and specification of color.

## 1.4 Color matching

Two spectra, represented by  $N$ -vectors  $\mathbf{f}$  and  $\mathbf{g}$ , produce the same cone responses and therefore represent the same color if

$$\mathbf{S}^T \mathbf{f} = \mathbf{S}^T \mathbf{g} \quad (1.4)$$

Because  $\mathbf{S}$  is an  $N \times 3$  matrix with  $N > 3$ , the above system of equations has multiple solutions. This implies that many different spectra match in color.

It is, in fact, possible to draw significantly stronger conclusions from Equations 1.3 and 1.4. One of the characteristics of color vision that can be deduced based on these equations is the phenomenon of *trichromacy*, which states that it is possible to produce a color match for a given stimulus (equivalently, identical cone responses under the same viewing conditions) by using only combinations of light from three light sources.<sup>105,200,201</sup> To establish this, consider three color *primaries*, i.e., three *colorimetrically independent* light sources  $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$ . The term *colorimetrically independent* will be used in this chapter to denote a collection of spectra such that the color of any one cannot be visually matched by any linear combination of the others. Mathematically, colorimetric independence of  $\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3$  is equivalent to the linear independence of the three-vectors  $\mathbf{S}^T \mathbf{p}_1, \mathbf{S}^T \mathbf{p}_2$ , and  $\mathbf{S}^T \mathbf{p}_3$ . Hence, if  $\mathbf{P} = [\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3]$ , the  $3 \times 3$  matrix  $\mathbf{S}^T \mathbf{P}$  is nonsingular.

For any visible spectrum  $\mathbf{f}$  the three-vector

$$\mathbf{a}(\mathbf{f}) \stackrel{\text{def}}{=} (\mathbf{S}^T \mathbf{P})^{-1} \mathbf{S}^T \mathbf{f}$$

satisfies the relation

$$\mathbf{S}^T \mathbf{f} = \mathbf{S}^T \mathbf{P} \mathbf{a}(\mathbf{f}) \quad (1.5)$$

which is the relation for a color match. Hence, for any visible spectrum  $\mathbf{f}$ , there exists a linear combination of the primaries,  $\mathbf{P} \mathbf{a}(\mathbf{f})$ , which matches the color of  $\mathbf{f}$ . This statement encapsulates the principle of trichromacy. It can be further seen that  $\mathbf{a}(\mathbf{f})$  specifies the unique linear combination of primaries that matches  $\mathbf{f}$  in color. This follows from the nonsingularity of  $\mathbf{S}^T \mathbf{P}$ , which ensures that if  $\mathbf{S}^T \mathbf{f} = \mathbf{S}^T \mathbf{P} \mathbf{v}_1 = \mathbf{S}^T \mathbf{P} \mathbf{v}_2$ , then  $\mathbf{v}_1 = \mathbf{v}_2$ . The elements of  $\mathbf{a}(\mathbf{f})$  represent the relative intensities or “strengths” of the primaries required to match the color of  $\mathbf{f}$ .

Some additional elaboration is necessary to establish the correspondence between the above mathematical argument and a physical experiment in which colors are matched using three primaries. In the mathematical computation, it is possible that the obtained vector of primary intensities,  $\mathbf{a}(\mathbf{f})$ , has negative components (in fact, it can be readily shown that, for any set of physical primaries, there exist visible spectra for which this happens). Because negative intensities of the primaries cannot be produced, the spectrum  $\mathbf{P} \mathbf{a}(\mathbf{f})$  is not realizable using the primaries. A physical realization corresponding to the equations is, however, still possible by rearranging the terms in Equation 1.5 and “subtracting” the primaries with negative intensities from  $\mathbf{f}$ . The double negation cancels out and corresponds to the addition of positive amounts of the appropriate primaries to  $\mathbf{f}$ .

The setup for a typical color-matching experiment is shown schematically in Figure 1.5. The observer views a small circular field that is split into two halves. The spectrum  $\mathbf{f}$  is displayed on one half of a visual field. On the other half of the visual field appears a linear combination of the primary sources. The observer attempts to visually match the input spectrum by adjusting the relative intensities of the primary sources. The vector  $\mathbf{a}(\mathbf{f})$  denotes the relative intensities of the three primaries when a match is obtained. Physically, it may be impossible to match the input spectrum by adjusting the intensities of the primaries. When this happens, the observer is allowed to move one or two of the primaries so that they illuminate the same field as input spectrum,  $\mathbf{f}$  (see Figure 1.6). As noted earlier, this procedure is mathematically equivalent to subtracting that amount of primary from the primary field; i.e., the strengths in  $\mathbf{a}(\mathbf{f})$  corresponding to the primaries that were moved are negative. As demonstrated in the last paragraph, all visible spectra can be matched using this method.

### 1.4.1 Color-matching functions

The linearity of color matching expressed in Equation 1.4 implies that, if the color tristimulus values for a basis set of spectra are known, the color values for all linear combinations of those spectra can be readily deduced. The unit intensity monochromatic spectra, given by  $\{\mathbf{e}_i\}_{i=1}^N$ , where  $\mathbf{e}_i$  is an  $N$ -vector having a one in the  $i$ th position and zeros elsewhere, form an orthonormal basis in terms of which all spectra can be expressed. Hence, the color match-

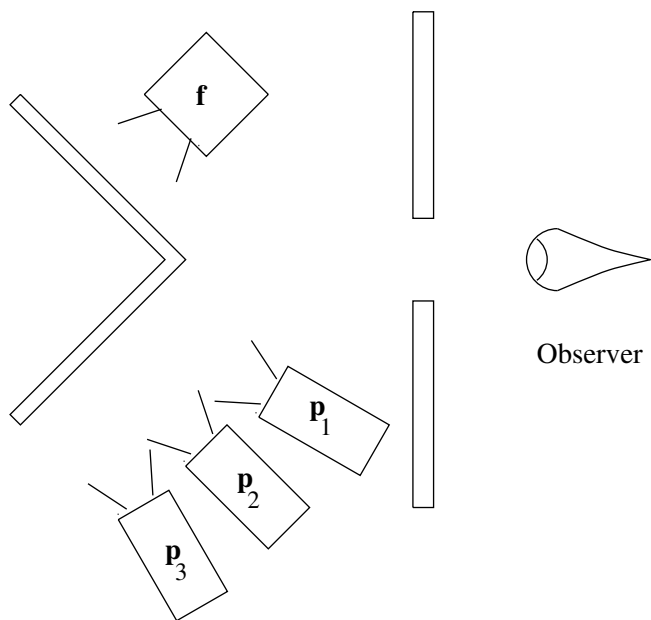


*Figure 1.5* Color matching experiment.

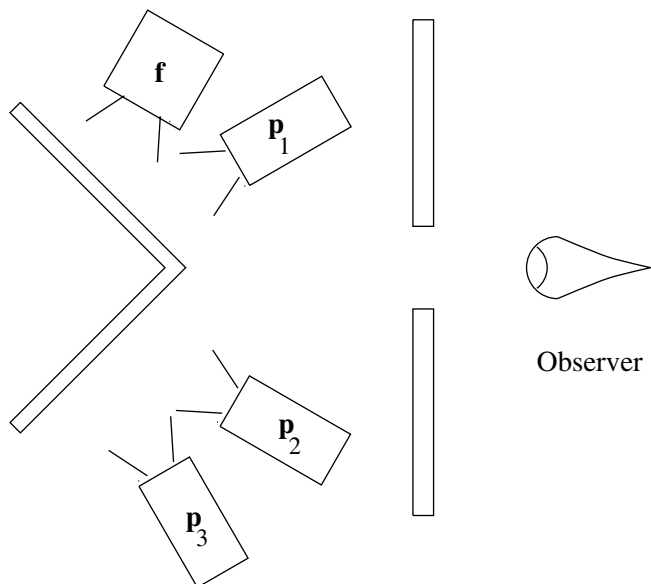


*Figure 1.6* Color matching experiment with negative value for primary  $p_1$ .

ing properties of all spectra (with respect to a given set of primaries) can be specified in terms of the color matching properties of these monochromatic spectra.

Consider the color matching experiment of the last section for the monochromatic spectra. Denoting the relative intensities of the three primaries required for matching  $\mathbf{e}_i$  by  $\mathbf{a}_i = \mathbf{a}(\mathbf{e}_i)$ , the matches for all the monochromatic spectra can be written as

$$\mathbf{S}^T \mathbf{e}_i = \mathbf{S}^T \mathbf{P} \mathbf{a}_i \quad i = 1, 2, \dots, N \quad (1.6)$$

Combining the results of all  $N$  monochromatic spectra, we get

$$\mathbf{S}^T \mathbf{I}_N = \mathbf{S}^T \mathbf{P} \mathbf{A}^T \quad (1.7)$$

where  $\mathbf{I}_N = [\mathbf{e}_1, \mathbf{e}_2, \dots, \mathbf{e}_N]$  is the  $N \times N$  identity matrix, and  $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \dots, \mathbf{a}_N]^T$  is the *color matching matrix* corresponding to the primaries  $\mathbf{P}$ .<sup>†</sup> The entries in the  $k$ th column of  $\mathbf{A}$  correspond to the relative amount of the  $k$ th primary required to match  $\{\mathbf{e}_i\}_{i=1}^N$ , respectively. The columns of  $\mathbf{A}$  are therefore referred to as the *color-matching functions* (CMFs) (associated with the primaries  $\mathbf{P}$ ).

Now, reconsider the matching of a general spectrum  $\mathbf{f} = [f_1, f_2, \dots, f_N]^T$  in a color matching experiment using the primaries  $\mathbf{P}$ . The stimulus can be decomposed in terms of the unit intensity monochromatic stimuli  $\{\mathbf{e}_i\}_{i=1}^N$  as

$$\mathbf{f} = \mathbf{I}_N \mathbf{f} = [\mathbf{e}_1 \mathbf{e}_2 \dots \mathbf{e}_N] [f_1, f_2, \dots, f_N]^T = \sum_{i=1}^N f_i \mathbf{e}_i \quad (1.8)$$

Recall, a linear combination of the primaries with relative intensities specified by the tristimulus vector  $\mathbf{a}_i$  matches the monochromatic spectrum  $\mathbf{e}_i$ . From the linearity of color matching and the above decomposition, it therefore follows that a linear combination of the primaries with relative intensities specified by the tristimulus vector

$$\sum_{i=1}^N f_i \mathbf{a}_i = \mathbf{A}^T \mathbf{f}$$

matches the spectrum  $\mathbf{f}$ . Thus, the tristimulus vector  $\mathbf{A}^T \mathbf{f}$  represents the relative intensities of the primaries  $\mathbf{P}$  that match the color of  $\mathbf{f}$ .

<sup>†</sup> In defining  $\mathbf{A}$  as the matrix whose  $i$ th row is  $\mathbf{a}_i^T$ , we breach the common convention used throughout the rest of the chapter according to which a bold lower case subscripted letter denotes a column of the matrix denoted by the corresponding bold upper case letter.

From Equation 1.7, it can be readily seen that the color-matching matrix

$$\mathbf{A} = \mathbf{S}(\mathbf{P}^T \mathbf{S})^{-1} \quad (1.9)$$

Hence, the CMFs are a nonsingular linear transformation of the sensitivities of the three cones in the eye. It also follows that the color of two spectra,  $\mathbf{f}$  and  $\mathbf{g}$ , matches if and only if  $\mathbf{A}^T \mathbf{f} = \mathbf{A}^T \mathbf{g}$ . As mentioned earlier, color of a visible spectrum,  $\mathbf{f}$ , may be specified in terms of the tristimulus values,  $\mathbf{A}^T \mathbf{f}$ , instead of  $\mathbf{S}^T \mathbf{f}$ . The fact that the color-matching matrix is readily determinable using the procedure outlined above makes such a scheme for specifying color considerably more attractive in comparison to one based on the actual cone sensitivities. Note also that the HVSS which was defined as the column space of  $\mathbf{S}$  can alternately be defined as the column space of  $\mathbf{A}$ . Using Equation 1.9, we see that

$$\mathbf{A}^T \mathbf{P} = (\mathbf{S}(\mathbf{P}^T \mathbf{S})^{-1})^T \mathbf{P} = (\mathbf{S}^T \mathbf{P})^{-1} \mathbf{S}^T \mathbf{P} = \mathbf{I}_3 \quad (1.10)$$

where  $\mathbf{I}_3$  is the  $3 \times 3$  identity matrix. Equation 1.10 can also be obtained by direct reasoning. Consider a color matching experiment in which the stimulus to be matched by a combination of the primaries is one of the primaries itself, say  $\mathbf{p}_1$ . The unique values of the relative intensities of the primaries required to match  $\mathbf{p}_1$  are  $\mathbf{A}^T \mathbf{p}_1$ . Because  $\mathbf{p}_1 = \mathbf{P}[100]^T$  clearly matches itself,  $\mathbf{A}^T \mathbf{p}_1 = [100]^T$ . Similar relations hold for  $\mathbf{p}_2$  and  $\mathbf{p}_3$ , and Equation 1.10 is obtained by concatenating the corresponding color match relations for all three primaries.

### 1.4.2 Metamerism and black space

As stated in Equation 1.4, two spectra represented by  $N$ -vectors  $\mathbf{f}$  and  $\mathbf{g}$  match in color if  $\mathbf{S}^T \mathbf{f} = \mathbf{S}^T \mathbf{g}$  (or  $\mathbf{A}^T \mathbf{f} = \mathbf{A}^T \mathbf{g}$ ). Because  $\mathbf{S}$  (or equivalently  $\mathbf{A}$ ) is an  $N \times 3$  matrix, with  $N > 3$ , it is clear that several different spectra appear to be the same color to the observer. Two distinct spectra that appear the same are called *metamers*, and such a color match is said to be a *metameric match* (as opposed to a spectral match). Figure 1.7 shows plots of two metameric SPDs. Note that the colorimetry corresponding to these distributions is identical, but the SPDs exhibit very significant differences. The specific SPDs plotted here correspond to the SPD for CIE standard illuminant D65 (see Section 1.5.2) and a metameric match obtained to the corresponding SPD using typical CRT primaries.

The vector space view of color matching outlined above was first presented in a cohesive mathematical framework by Cohen and Kaupauf.<sup>55–58</sup> Tutorial descriptions using current notation and terminology appear in References 125, 299, 300, and 304. This approach allows us to deduce a number of interesting and useful properties of color vision. One such property is the decomposition of the  $N$ -dimensional spectral space into the three-dimensional HVSS and the  $(N - 3)$ -dimensional *metameric black space*, which was first hypothesized by Wyszecki.<sup>332</sup> Mathematically, this result states that any visible spectrum,  $\mathbf{f}$ , can be written as

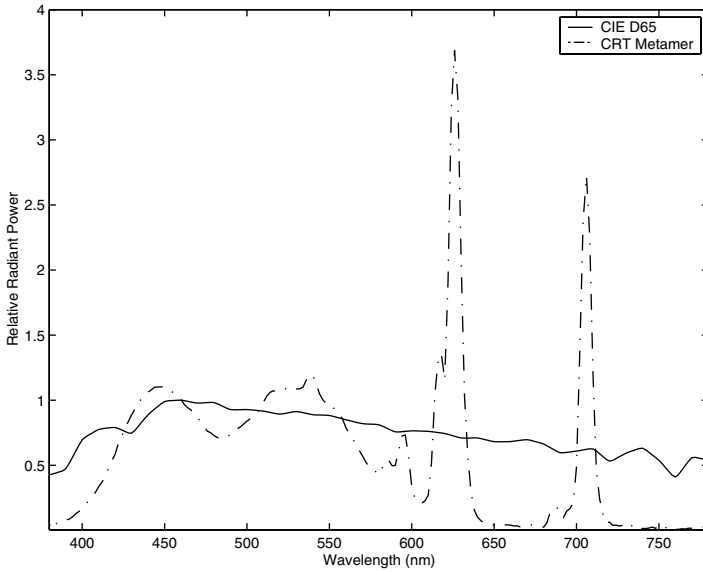


Figure 1.7 Example of a pair of metameretic radiances.

$$\mathbf{f} = \mathbf{P}_A \mathbf{f} + \mathbf{P}_A^\perp \mathbf{f} \quad (1.11)$$

where  $\mathbf{P}_A = \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T$  is the orthogonal projector onto the column space of  $\mathbf{A}$ , i.e., the HVSS, and

$$\mathbf{P}_A^\perp = (\mathbf{I}_N - \mathbf{P}_A)$$

is the orthogonal projector onto the black space, which is the orthogonal complement of the HVSS. The projection,  $\mathbf{P}_A \mathbf{f}$ , is called the *fundamental metamer* of  $\mathbf{f}$ , because all metamers of  $\mathbf{f}$  are given by

$$\left\{ \mathbf{P}_A \mathbf{f} + \mathbf{P}_A^\perp \mathbf{g} \mid \mathbf{g} \in \mathbb{R}^N \right\}$$

Spectra that match in color have identical projections onto the HVSS. Conversely, spectra having identical projections onto the HVSS match in color. For a given spectrum  $\mathbf{f}$ , the tristimulus value  $\mathbf{t} = \mathbf{A}^T \mathbf{f}$  and the corresponding CMFs  $\mathbf{A}$  can be used to compute the corresponding fundamental metamer as

$$\mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} \mathbf{t} = \mathbf{A}(\mathbf{A}^T \mathbf{A})^{-1} (\mathbf{A}^T \mathbf{f}) = \mathbf{P}_A \mathbf{f} \quad (1.12)$$

and, vice versa, the fundamental metamer  $\mathbf{P}_A \mathbf{f}$  and CMFs  $\mathbf{A}$  can be used to compute the tristimulus,

$$\mathbf{t} = \mathbf{A}^T \mathbf{P}_A \mathbf{f} = \mathbf{A}^T \mathbf{A} (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{f} = \mathbf{A}^T \mathbf{f} \quad (1.13)$$

Equation 1.13 also illustrates the fact that the tristimulus values for a spectrum and its fundamental metamer are equivalent. Thus, the fundamental metamer offers an alternate representation of exactly the same information that is contained in the tristimulus values. The representation is, however, an  $N$ -vector in a three-dimensional subspace of the  $N$ -dimensional spectral space and therefore quite powerful and useful in the comparison of colors and spectra.<sup>56</sup> Tristimulus values are not ideally suited for the same task because of the dimensional mismatch between three-dimensional tristimulus values and  $N$ -dimensional spectra.

Another direct consequence of the above description of color matching is the fact that the primaries in any color matching experiment are unique only up to metamers. Because metamers are visually identical, the CMFs are not changed if each of the three primaries are replaced by any of their metamers.

The physical realization of metamers imposes additional constraints over and above those predicated by the equations above. In particular, any physically realizable spectrum needs to be non-negative, and hence it is possible that the metamers described by the above mathematics may not be realizable. In cases where a realizable metamer exists, set theoretic approaches may be used to incorporate non-negativity and other constraints.<sup>261,299</sup>

## 1.5 Colorimetry

It was mentioned in Section 1.4.1 that the color of a visible spectrum  $\mathbf{f}$  can be specified in terms of the tristimulus values,  $\mathbf{A}^T \mathbf{f}$ , where  $\mathbf{A}$  is a matrix of CMFs. To have agreement between different measurements, it is necessary to define a standard set of CMFs with respect to which the tristimulus values are stated. A number of different standards have been defined for a variety of applications, and it is worth reviewing some of these standards and the historical reasons behind their development.

### 1.5.1 CIE standards

The Commission Internationale de l'Eclairage (International Commission on Illumination, CIE) is the primary organization responsible for standardization of color metrics and terminology. A colorimetry standard was first defined by the CIE in 1931 and continues to form the basis of modern colorimetry. The CIE 1931 recommendations define a standard colorimetric observer by providing two different but equivalent set of CMFs. The first set of CMFs are known as the CIE RGB CMFs,  $\bar{r}(\lambda)$ ,  $\bar{g}(\lambda)$ ,  $\bar{b}(\lambda)$ . These are

associated with monochromatic primaries at wavelengths of 700.0, 546.1, and 435.8 nm, respectively, with their radiant intensities adjusted so that the tristimulus values of the equi-energy spectrum are all equal.<sup>47</sup> The equi-energy spectrum is the one whose SPD is constant (as a function of wavelength). The CIE RGB CMFs are shown in [Figure 1.8](#).

The second set of CMFs, known as the CIE XYZ CMFs,  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$ , are shown in [Figure 1.9](#). They were recommended for reasons of more convenient application in colorimetry and are defined in terms of a linear transformation of the CIE RGB CMFs.<sup>150</sup> When these CMFs were first defined, calculations were typically performed on desk calculators, and the repetitive summing and differencing due to the negative lobes of the CIE RGB CMFs were prone to errors. Hence, the transformation from the CIE RGB CMFs to CIE XYZ CMFs was determined so as to avoid negative values at all wavelengths.<sup>177</sup> Because an infinite number of transformations can be defined to meet this non-negativity requirement, additional criteria were used in the choice of the CMFs.<sup>85,153(p. 531)</sup> Two of the important considerations were the choice of  $\bar{y}(\lambda)$  coincident with the *luminous efficiency function*<sup>335</sup> and the normalization of the three CMFs so as to yield equal tristimulus values for the equi-energy spectrum. The luminous efficiency function gives the relative sensitivity of the eye to the energy at each wavelength. From the discussion of Section 1.4, it is readily seen that CMFs that are non-negative for all wavelengths cannot be obtained with any physically realizable primaries. Hence, any set of primaries corresponding to the CIE XYZ CMFs is not physically realizable. [Table 1.1](#) provides a listing of the CIE XYZ color matching functions, sampled at 5-nm intervals in the range of 380 to 780 nm. Data used in this table are also available at the CIE web site.<sup>47</sup>

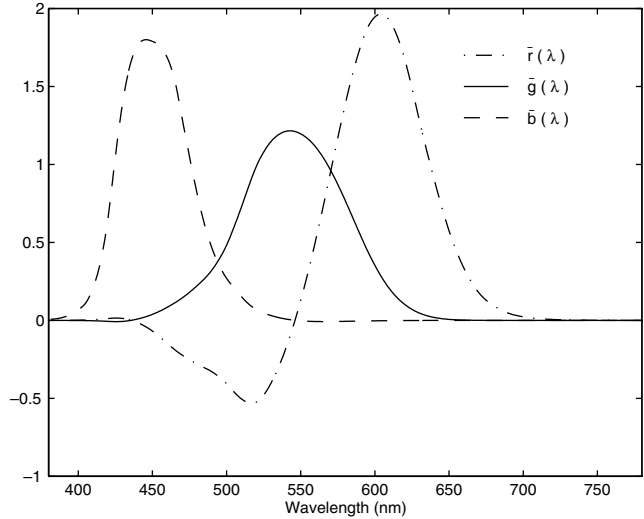


**Figure 1.8** CIE  $\bar{r}(\lambda)$ ,  $\bar{g}(\lambda)$ ,  $\bar{b}(\lambda)$  color matching functions.

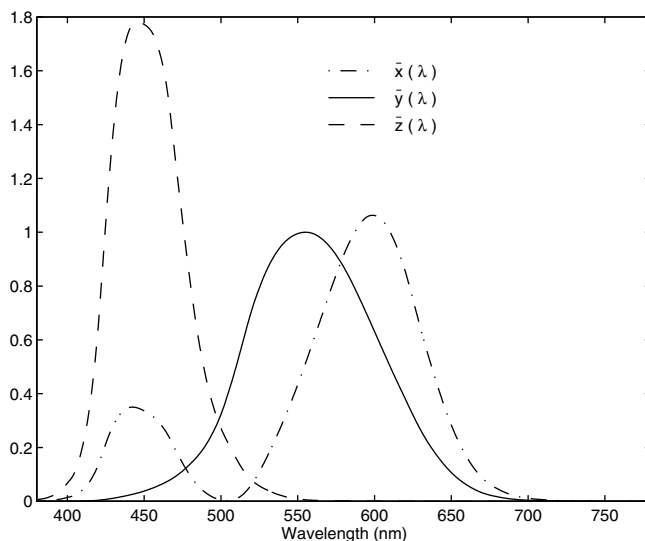


**Figure 1.9** CIE  $\bar{x}(\lambda)$ ,  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$  color matching functions.

The tristimulus values obtained with the CIE RGB CMFs are called the *CIE RGB tristimulus values*, and those obtained with the CIE XYZ CMFs are called the *CIE XYZ tristimulus values*. In most color imaging applications, and in color research, CIE XYZ values are used, and the CIE RGB tristimulus values are rarely used. The Y tristimulus value is usually called the *luminance* and correlates with the perceived brightness of the radiant spectrum. The luminance is described in units of candela per square meter ( $\text{cd}/\text{m}^2$ ). Typical ambient luminance levels under sunlight, indoor lighting, moonlight, and starlight are of the order of  $10^5$ ,  $10^2$ ,  $10^{-1}$ , and  $10^{-3}$   $\text{cd}/\text{m}^2$ , respectively. The scotopic, mesopic, and photopic domains of vision defined in Section 1.3 correspond roughly to luminance intervals 0.000001–0.034  $\text{cd}/\text{m}^2$ , 0.034–3.4  $\text{cd}/\text{m}^2$ , and over 3.4  $\text{cd}/\text{m}^2$ , respectively.

The two sets of CMFs described above are suitable for describing color-matching when the angular subtense of the matching fields at the eye is between one and four degrees.<sup>47,335(p. 131)</sup> When the inadequacy of these CMFs for matching fields with larger angular subtense became apparent, the CIE defined an alternate standard colorimetric observer in 1964 with different sets of CMFs.<sup>47</sup> Because imaging applications (unlike quality control applications in manufacturing) involve complex visual fields where the color-homogeneous areas have small angular subtense, the CIE 1964 ( $10^\circ$  observer) CMFs will not be discussed here.

### 1.5.2 Colorimetry for reflective objects

The discussion in the last section was based on the assumption that  $\mathbf{f}$  is the spectral radiance of the light incident on the eye. Reflective objects are

*Table 1.1* CIE XYZ CMFs for the 2° 1931 CIE Standard Observer and SPDs for Standard Illuminants D50, D65, and A Tabulated at 5-nm Wavelength Intervals (Except for the D50 SPD, these data are available from the CIE web site.<sup>47</sup>)

| Wavelength<br>$\lambda$ (nm) | $\bar{x}(\lambda)$ | $\bar{y}(\lambda)$ | $\bar{z}(\lambda)$ | $I_{D50}(\lambda)$ | $I_{D65}(\lambda)$ | $I_A(\lambda)$ |
|------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|----------------|
| 380                          | 0.0014             | 0.0000             | 0.0065             | 24.4875            | 49.97550           | 9.795100       |
| 385                          | 0.0022             | 0.0001             | 0.0105             | 27.1791            | 52.31180           | 10.899600      |
| 390                          | 0.0042             | 0.0001             | 0.0201             | 29.8706            | 54.64820           | 12.085300      |
| 395                          | 0.0076             | 0.0002             | 0.0362             | 39.5894            | 68.70150           | 13.354300      |
| 400                          | 0.0143             | 0.0004             | 0.0679             | 49.3081            | 82.75490           | 14.708000      |
| 405                          | 0.0232             | 0.0006             | 0.1102             | 52.9104            | 87.12040           | 16.148000      |
| 410                          | 0.0435             | 0.0012             | 0.2074             | 56.5128            | 91.48600           | 17.675300      |
| 415                          | 0.0776             | 0.0022             | 0.3713             | 58.2733            | 92.45890           | 19.290700      |
| 420                          | 0.1344             | 0.0040             | 0.6456             | 60.0338            | 93.43180           | 20.995000      |
| 425                          | 0.2148             | 0.0073             | 1.0391             | 58.9257            | 90.05700           | 22.788300      |
| 430                          | 0.2839             | 0.0116             | 1.3856             | 57.8175            | 86.68230           | 24.670900      |
| 435                          | 0.3285             | 0.0168             | 1.6230             | 66.3212            | 95.77360           | 26.642500      |
| 440                          | 0.3483             | 0.0230             | 1.7471             | 74.8249            | 104.86500          | 28.702700      |
| 445                          | 0.3481             | 0.0298             | 1.7826             | 81.0360            | 110.93600          | 30.850800      |
| 450                          | 0.3362             | 0.0380             | 1.7721             | 87.2472            | 117.00800          | 33.085900      |
| 455                          | 0.3187             | 0.0480             | 1.7441             | 88.9297            | 117.41000          | 35.406800      |
| 460                          | 0.2908             | 0.0600             | 1.6692             | 90.6122            | 117.81200          | 37.812100      |
| 465                          | 0.2511             | 0.0739             | 1.5281             | 90.9902            | 116.33600          | 40.300200      |
| 470                          | 0.1954             | 0.0910             | 1.2876             | 91.3681            | 114.86100          | 42.869300      |
| 475                          | 0.1421             | 0.1126             | 1.0419             | 93.2383            | 115.39200          | 45.517400      |
| 480                          | 0.0956             | 0.1390             | 0.8130             | 95.1085            | 115.92300          | 48.242300      |
| 485                          | 0.0580             | 0.1693             | 0.6162             | 93.5356            | 112.36700          | 51.041800      |
| 490                          | 0.0320             | 0.2080             | 0.4652             | 91.9627            | 108.81100          | 53.913200      |
| 495                          | 0.0147             | 0.2586             | 0.3533             | 93.8432            | 109.08200          | 56.853900      |
| 500                          | 0.0049             | 0.3230             | 0.2720             | 95.7237            | 109.35400          | 59.861100      |
| 505                          | 0.0024             | 0.4073             | 0.2123             | 96.1685            | 108.57800          | 62.932000      |
| 510                          | 0.0093             | 0.5030             | 0.1582             | 96.6133            | 107.80200          | 66.063500      |
| 515                          | 0.0291             | 0.6082             | 0.1117             | 96.8711            | 106.29600          | 69.252500      |
| 520                          | 0.0633             | 0.7100             | 0.0782             | 97.1290            | 104.79000          | 72.495900      |
| 525                          | 0.1096             | 0.7932             | 0.0573             | 99.6141            | 106.23900          | 75.790300      |
| 530                          | 0.1655             | 0.8620             | 0.0422             | 102.0991           | 107.68900          | 79.132600      |
| 535                          | 0.2257             | 0.9149             | 0.0298             | 101.4269           | 106.04700          | 82.519300      |
| 540                          | 0.2904             | 0.9540             | 0.0203             | 100.7547           | 104.40500          | 85.947000      |
| 545                          | 0.3597             | 0.9803             | 0.0134             | 101.5359           | 104.22500          | 89.412400      |
| 550                          | 0.4334             | 0.9950             | 0.0087             | 102.3170           | 104.04600          | 92.912000      |
| 555                          | 0.5121             | 1.0000             | 0.0057             | 101.1585           | 102.02300          | 96.442300      |
| 560                          | 0.5945             | 0.9950             | 0.0039             | 100.0000           | 100.00000          | 100.000000     |
| 565                          | 0.6784             | 0.9786             | 0.0027             | 98.8675            | 98.16710           | 103.582000     |
| 570                          | 0.7621             | 0.9520             | 0.0021             | 97.7350            | 96.33420           | 107.184000     |
| 575                          | 0.8425             | 0.9154             | 0.0018             | 98.3265            | 96.06110           | 110.803000     |
| 580                          | 0.9163             | 0.8700             | 0.0017             | 98.9180            | 95.78800           | 114.436000     |

*Table 1.1* CIE XYZ CMFs for the 2° 1931 CIE Standard Observer and SPDs for Standard Illuminants D50, D65, and A Tabulated at 5-nm Wavelength Intervals (Except for the D50 SPD, these data are available from the CIE web site.<sup>47</sup>)

| Wavelength<br>$\lambda$ (nm) | $\bar{x}(\lambda)$ | $\bar{y}(\lambda)$ | $\bar{z}(\lambda)$ | $I_{D50}(\lambda)$ | $I_{D65}(\lambda)$ | $I_A(\lambda)$ |
|------------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|----------------|
| 585                          | 0.9786             | 0.8163             | 0.0014             | 96.2084            | 92.23680           | 118.080000     |
| 590                          | 1.0263             | 0.7570             | 0.0011             | 93.4988            | 88.68560           | 121.731000     |
| 595                          | 1.0567             | 0.6949             | 0.0010             | 95.5933            | 89.34590           | 125.386000     |
| 600                          | 1.0622             | 0.6310             | 0.0008             | 97.6878            | 90.00620           | 129.043000     |
| 605                          | 1.0456             | 0.5668             | 0.0006             | 98.4784            | 89.80260           | 132.697000     |
| 610                          | 1.0026             | 0.5030             | 0.0003             | 99.2691            | 89.59910           | 136.346000     |
| 615                          | 0.9384             | 0.4412             | 0.0002             | 99.1553            | 88.64890           | 139.988000     |
| 620                          | 0.8544             | 0.3810             | 0.0002             | 99.0415            | 87.69870           | 143.618000     |
| 625                          | 0.7514             | 0.3210             | 0.0001             | 97.3817            | 85.49360           | 147.235000     |
| 630                          | 0.6424             | 0.2650             | 0.0000             | 95.7218            | 83.28860           | 150.836000     |
| 635                          | 0.5419             | 0.2170             | 0.0000             | 97.2895            | 83.49390           | 154.418000     |
| 640                          | 0.4479             | 0.1750             | 0.0000             | 98.8572            | 83.69920           | 157.979000     |
| 645                          | 0.3608             | 0.1382             | 0.0000             | 97.2622            | 81.86300           | 161.516000     |
| 650                          | 0.2835             | 0.1070             | 0.0000             | 95.6672            | 80.02680           | 165.028000     |
| 655                          | 0.2187             | 0.0816             | 0.0000             | 96.9285            | 80.12070           | 168.510000     |
| 660                          | 0.1649             | 0.0610             | 0.0000             | 98.1898            | 80.21460           | 171.963000     |
| 665                          | 0.1212             | 0.0446             | 0.0000             | 100.5966           | 81.24620           | 175.383000     |
| 670                          | 0.0874             | 0.0320             | 0.0000             | 103.0034           | 82.27780           | 178.769000     |
| 675                          | 0.0636             | 0.0232             | 0.0000             | 101.0682           | 80.28100           | 182.118000     |
| 680                          | 0.0468             | 0.0170             | 0.0000             | 99.1330            | 78.28420           | 185.429000     |
| 685                          | 0.0329             | 0.0119             | 0.0000             | 93.2570            | 74.00270           | 188.701000     |
| 690                          | 0.0227             | 0.0082             | 0.0000             | 87.3809            | 69.72130           | 191.931000     |
| 695                          | 0.0158             | 0.0057             | 0.0000             | 89.4922            | 70.66520           | 195.118000     |
| 700                          | 0.0114             | 0.0041             | 0.0000             | 91.6035            | 71.60910           | 198.261000     |
| 705                          | 0.0081             | 0.0029             | 0.0000             | 92.2460            | 72.97900           | 201.359000     |
| 710                          | 0.0058             | 0.0021             | 0.0000             | 92.8886            | 74.34900           | 204.409000     |
| 715                          | 0.0041             | 0.0015             | 0.0000             | 84.8715            | 67.97650           | 207.411000     |
| 720                          | 0.0029             | 0.0010             | 0.0000             | 76.8544            | 61.60400           | 210.365000     |
| 725                          | 0.0020             | 0.0007             | 0.0000             | 81.6828            | 65.74480           | 213.268000     |
| 730                          | 0.0014             | 0.0005             | 0.0000             | 86.5112            | 69.88560           | 216.120000     |
| 735                          | 0.0010             | 0.0004             | 0.0000             | 89.5455            | 72.48630           | 218.920000     |
| 740                          | 0.0007             | 0.0002             | 0.0000             | 92.5798            | 75.08700           | 221.667000     |
| 745                          | 0.0005             | 0.0002             | 0.0000             | 85.4048            | 69.33980           | 224.361000     |
| 750                          | 0.0003             | 0.0001             | 0.0000             | 78.2299            | 63.59270           | 227.000000     |
| 755                          | 0.0002             | 0.0001             | 0.0000             | 67.9609            | 55.00540           | 229.585000     |
| 760                          | 0.0002             | 0.0001             | 0.0000             | 57.6918            | 46.41820           | 232.115000     |
| 765                          | 0.0001             | 0.0000             | 0.0000             | 70.3074            | 56.61180           | 234.589000     |
| 770                          | 0.0001             | 0.0000             | 0.0000             | 82.9230            | 66.80540           | 237.008000     |
| 775                          | 0.0001             | 0.0000             | 0.0000             | 80.5985            | 65.09410           | 239.370000     |
| 780                          | 0.0000             | 0.0000             | 0.0000             | 78.2740            | 63.38280           | 241.675000     |

viewed under an illuminating light source and, accordingly, their colorimetry is specified under a suitable illuminant. For the purposes of defining colorimetry, a reflective object can be represented by the  $N$ -vector,  $\mathbf{r}$ , of samples of its spectral reflectance  $r(\lambda)$ . When the object is viewed under an illuminant with SPD  $l(\lambda)$ , represented in sampled form by the  $N$ -vector  $\mathbf{l}$ , the resulting SPD at the eye is the product  $l(\lambda)r(\lambda)$  of the illuminant SPD and the object reflectance, which can be represented in sampled form as the  $N$ -vector  $\mathbf{Lr}$ , where  $\mathbf{L}$  is the diagonal illuminant matrix with entries from  $\mathbf{l}$  along the diagonal. The CIE XYZ tristimulus values defining the color are therefore given by

$$\mathbf{t} = \mathbf{A}^T \mathbf{L} \mathbf{r} = \mathbf{A}_L^T \mathbf{r} \quad (1.14)$$

where  $\mathbf{A}$  is the matrix of CIE XYZ CMFs, and  $\mathbf{A}_L = \mathbf{L}\mathbf{A}$ . Color measurement for transmissive objects can be similarly defined in terms of their spectral transmittance. The color matching functions can be scaled by a common scale factor so that the Y stimulus value corresponds to the luminance in units of  $\text{cd}/\text{m}^2$ . However, as mentioned earlier, the absolute SPDs for the illuminant are rarely known or required in applications of colorimetry of reflective objects. In the colorimetry of reflective objects, it is therefore common to normalize the tristimulus values (or equivalently the CMFs) so that the Y coordinate is 100 for a *perfect reflector*, whose spectral reflectance is unity across all wavelengths. Because computation of CIE XYZ colorimetry is a basic step commonly employed in color imaging, it is useful to list this computation of CIE XYZ values explicitly:

$$\begin{aligned} X &= k \sum_{i=0}^{N-1} \bar{x}(\lambda_i) l(\lambda_i) r(\lambda_i) \\ Y &= k \sum_{i=0}^{N-1} \bar{y}(\lambda_i) l(\lambda_i) r(\lambda_i) \\ Z &= k \sum_{i=0}^{N-1} \bar{z}(\lambda_i) l(\lambda_i) r(\lambda_i) \end{aligned} \quad (1.15)$$

where  $\{\lambda_i\}_{i=0}^{N-1}$  are the uniformly spaced wavelengths covering the visible region of the spectrum  $\lambda_i = \lambda_0 + i\Delta\lambda$ , with  $\Delta\lambda$  as the wavelength sampling interval, and the normalization factor  $k$  given by

$$k = \frac{100}{\sum_{i=0}^{N-1} \bar{y}(\lambda_i) l(\lambda_i)} \quad (1.16)$$

In addition to the CMFs, the CIE has defined a number of standard illuminants for use in colorimetry of nonluminous reflecting objects. The relative SPDs of a number of these standard illuminants are shown in [Figure 1.10](#). The corresponding values are also tabulated in [Table 1.1](#). To represent different phases of daylight, a continuum of daylight illuminants has been defined<sup>47</sup> that are uniquely specified in terms of their *correlated color temperature* (CCT). Because the temperature of a blackbody radiator describes its complete spectral power distribution and thereby its color, it is commonly referred to as the color temperature of the blackbody. For an arbitrary illuminant, the CCT is defined as the color temperature of the blackbody radiator that is visually closest to the illuminant (in color).<sup>335</sup> The D65 and D50 illuminant spectra shown in [Figure 1.10](#) are two daylight illuminants commonly used in colorimetry and have CCTs of 6500 and 5000 K, respectively. The CIE illuminant A represents a blackbody radiator at a temperature of 2856 K and closely approximates the spectra of incandescent lamps. Sources with lower CCT tend to be more red, whereas those with higher temperatures are bluer. Illuminants with similar CCT are assumed to be similar with regard to their color rendering of illuminated objects. This is, however, true only for illuminants whose spectra closely resemble that of a blackbody radiator, and other spectra that have identical CCT can have very different distributions and color rendering properties.<sup>202</sup> An example of the problem with the use of CCT for specifying the color-rendering properties of an illuminant is shown in [Figure 1.11](#), where two synthesized illuminants are shown along with a reflectance spectrum measured from a cyan print sample. Though the illuminants have the same luminance and an identical CCT of 5000K, the color difference for the reflectance sample under the two illuminants is rather

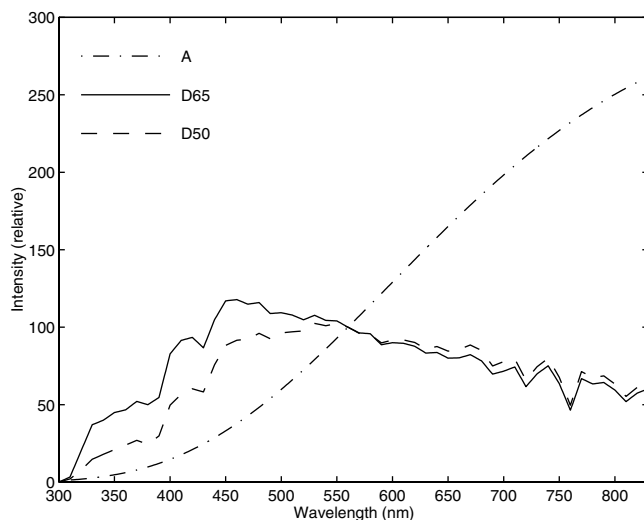


**Figure 1.10** CIE standard illuminants.

large, corresponding to 44.4  $\Delta E_{ab}^*$  units. (For the definition of  $\Delta E_{ab}^*$ , see Section 1.7.2.)

The definition of metameric matches and metamers can be extended to reflective objects. Two objects with (different) spectral reflectances  $\mathbf{r}_1$  and  $\mathbf{r}_2$  are said to be metamers (or in metameric match) under an illuminant with SPD  $\mathbf{l}$  if

$$\mathbf{A}_L^T \mathbf{r}_1 = \mathbf{A}_L^T \mathbf{r}_2 \quad (1.17)$$

In analogy with the HVSS, the column space of  $\mathbf{A}_L$  is defined as the *Human Visual Illuminant Subspace* (HVISS).<sup>310</sup> In a fashion similar to that described in Section 1.4.2 for spectral radiances, the space of reflectances may also be decomposed into two orthogonal components, one being the HVISS and the other a black reflectance space, representing the absence of a visual stimulus. Every reflectance spectrum can then be represented as the summation of two orthogonal components, one in the three-dimensional HVISS and the other in the black reflectance space. Reflective metamers under a specified viewing illuminant have identical HVISS components, and their differences therefore lie entirely in the black reflectance space.

Metamerism is both a boon and a curse in color applications. Most color output systems (such as CRTs and color photography) exploit metamerism to reproduce color. However, in the matching of reflective materials, a metameric match under one viewing illuminant is usually insufficient to establish a match under other viewing illuminants. A common manifestation of this phenomenon is the color match of (different) fabrics under one illumination and mismatch under another. This situation is referred to as *illuminant metamerism*. Figure 1.12 shows an example of illuminant metamerism. The plots in this figure show the spectral reflectances of four different metameric samples that have identical colorimetry under CIE illuminant D50 but exhibit significant differences under other illuminants such as cool white fluorescent or CIE illuminant A. The four reflectances used in this example are spectral reflectances obtained with different color reproduction processes, representing one each of a photographic, xerographic, inkjet, and lithographic process. Details on how these metameric spectra were obtained can be found in Reference 270.

### 1.5.3 Chromaticity coordinates and chromaticity diagrams

Because color is specified by tristimuli, different colors may be visualized as vectors in three-dimensional space. However, such a visualization is difficult to reproduce on two-dimensional media and therefore inconvenient. A useful two-dimensional representation of colors is obtained if tristimuli are normalized to lie in the *unit plane*, i.e., the plane over which the tristimulus values sum up to unity. Such a normalization is convenient, as it destroys only information about the “intensity” of the stimulus and preserves com-

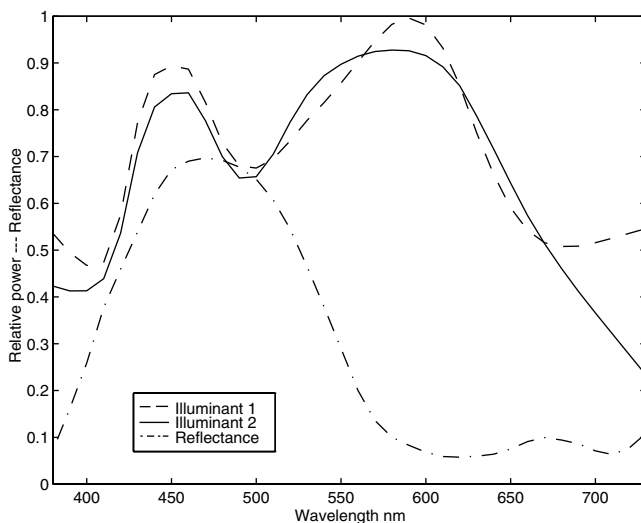


**Figure 1.11** Correlated color temperature (CCT) counter-example with two illuminants with CCT = 5000 K, and a spectral reflectance.

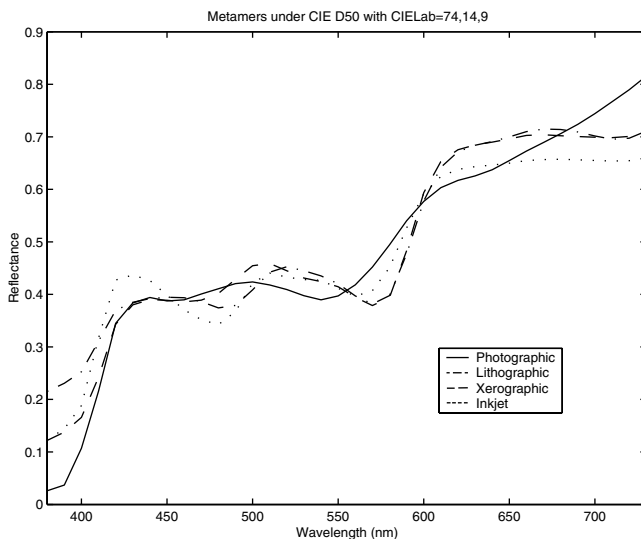


**Figure 1.12** Reflective metamers under CIE illuminant D50 corresponding to different color reproduction processes.

plete information about the direction. The coordinates of the normalized tristimulus vector are called *chromaticity coordinates*, and a plot of colors on the unit plane using these coordinates is called a *chromaticity diagram*. Because the three chromaticity coordinates sum up to unity, typical diagrams plot only two chromaticity coordinates along mutually perpendicular axes.

The most commonly used chromaticity diagram is the CIE xy chromaticity diagram. The CIE xyz chromaticity coordinates can be obtained from the X,Y, Z tristimulus values in CIE XYZ space as

$$\begin{aligned}x &= \frac{X}{X+Y+Z'} \\y &= \frac{Y}{X+Y+Z'} \\z &= \frac{Z}{X+Y+Z'}\end{aligned}\tag{1.18}$$

Figure 1.13 shows a plot of the curve corresponding to visible monochromatic spectra on the CIE xy chromaticity diagram. This shark-fin-shaped curve, along which the wavelength (in nm) is indicated, is called the *spectrum locus*. From the linear relation between radiance spectra and the tristimulus values, it can readily be seen that the chromaticity coordinates of any additive-combination of two spectra lie on the line segment joining their chromaticity coordinates.<sup>335</sup> From this observation, it follows that the region of chromaticities of all realizable spectral stimuli is the convex hull of the spectrum locus. In Figure 1.13, this region of physically realizable chromaticities is the region inside the closed curve formed by the spectrum locus and the broken line joining its two extremes, which is known as the *purple line*.

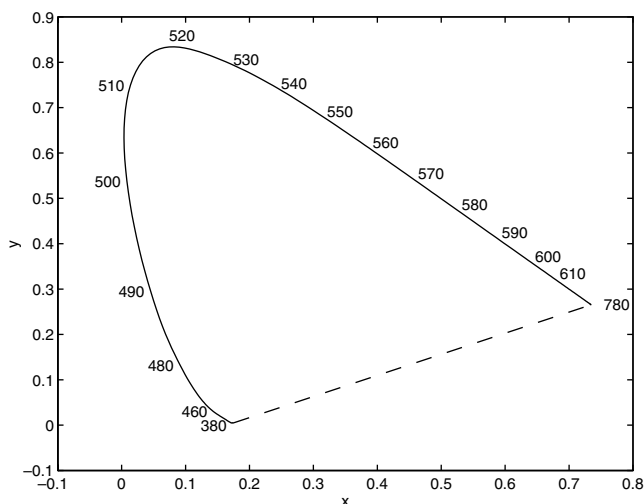


Figure 1.13 CIE xy chromaticity diagram.

### 1.5.4 Transformation of primaries: NTSC, SMPTE, and CCIR primaries

If a different set of primary sources,  $\mathbf{Q}$ , is used in the color matching experiment, a different set of CMFs,  $\mathbf{B}$ , is obtained. Because all CMFs are non-singular linear transformations of the human cone responses, the CMFs are related by a linear transformation. The relation between the two sets of CMFs can be obtained as follows. Using Equation 1.7 for the two sets of primaries and corresponding CMFs, both can be related to the eye's cone sensitivities and to each other as

$$\mathbf{S}^T = \mathbf{S}^T \mathbf{I}_N = \mathbf{S}^T \mathbf{P} \mathbf{A}^T = \mathbf{S}^T \mathbf{Q} \mathbf{B}^T \quad (1.19)$$

Post-multiplying both sides by  $(\mathbf{S}^T \mathbf{P})^{-1}$  we have

$$(\mathbf{S}^T \mathbf{P})^{-1} \mathbf{S}^T = \mathbf{A}^T = (\mathbf{S}^T \mathbf{P})^{-1} \mathbf{S}^T \mathbf{Q} \mathbf{B}^T \quad (1.20)$$

Substituting the relation from the left side equality above into the right side yields

$$\mathbf{A}^T = \mathbf{A}^T \mathbf{Q} \mathbf{B}^T \quad (1.21)$$

Note that the columns of the  $3 \times 3$  matrix  $\mathbf{A}^T \mathbf{Q}$  are the tristimulus values of the primaries  $\mathbf{Q}$  with respect to the primaries  $\mathbf{P}$ . From the colorimetric independence of the primaries  $\mathbf{Q}$ , it therefore follows that  $\mathbf{A}^T \mathbf{Q}$  is non-singular, and we have

$$\mathbf{B}^T = (\mathbf{A}^T \mathbf{Q})^{-1} \mathbf{A}^T \quad (1.22)$$

Note that the same transformation,  $(\mathbf{A}^T \mathbf{Q})^{-1}$ , is useful for the conversion of tristimuli in the primary system  $\mathbf{P}$  to tristimuli in the primary system  $\mathbf{Q}$ .

Color television was one of the first consumer products exploiting the phenomenon of trichromacy. The three light-emitting color phosphors in the television cathode ray tube (CRT) form the three primaries in this "color matching experiment." In the United States, the National Television Systems Committee (NTSC) recommendations for a receiver primary system based on three phosphor primaries were adopted by the Federal Communications Commission (FCC) in 1953 for use as a standard in color television (TV). The FCC standard specified the CIE  $xy$  chromaticity coordinates for the phosphors<sup>68</sup> as (0.670, 0.330) (red), (0.210, 0.710) (green), and (0.140, 0.080) (blue).<sup>221</sup> In addition, the tristimulus values (1, 1, 1) were assumed to correspond to a "white color" typically specified as the illuminant D65. The chromaticity coordinates along with the white balance condition define the CIE XYZ tristimuli of the NTSC primaries, which determine the relation of NTSC RGB tristimuli to CIE XYZ tristimuli as per Equation 1.22.

In the early color TV system, the signal-origination colorimetry was coupled with the colorimetry of displays, with the tacit assumption that processing at the receiver involves only decoding, and no color processing is performed. As display technology changed, manufacturers began using more efficient phosphors and incorporated some changes in the decoding as a compensation for the nonstandard phosphors.<sup>225</sup> Similar changes took place in the monitors used by broadcasters, but they were unaware of the compensating mechanisms in the consumer TV sets. As a result, there was considerable color variability in the broadcast TV system.<sup>68</sup> To overcome this problem, the chromaticities of a set of controlled phosphors was defined for use in broadcast monitors, and it now forms the Society of Motion Picture and Television Engineers (SMPTE) “C” phosphor specification.<sup>279,280</sup> Current commercial TV broadcasts in the U.S. are based on this specification.

With the development of newer display technologies that are not based on CRTs (see Section 1.11.1.5), it is now recognized that signal-origination colorimetry needs to be decoupled from the receiver colorimetry and that color correction at the receiver should compensate for the difference. However, for compatibility reasons and to minimize noise in transformations, it is still desirable to keep the reference primaries for broadcast colorimetry close to the phosphor primaries. Toward this end, the International Radio Consultative Committee (CCIR)<sup>140</sup> has defined a set of phosphor primaries by the chromaticity coordinates (0.640, 0.330) (red), (0.300, 0.600) (green), and (0.150, 0.060) (blue) for use in high-definition television (HDTV) systems.

Prior to transmission, tristimuli in SMPTE RGB and CCIR RGB spaces are nonlinearly compressed (by raising them to a power of 0.45) and encoded for reducing transmission bandwidth.<sup>39,140</sup> The reasons for these operations will be explained in Section 1.11.1.1. Note, however, that the encoding and nonlinear operations must be reversed before the signals can be converted to tristimuli spaces associated with other primaries. Transformations for the conversion of color tristimulus values between various systems can be found in References 234 (pp. 66–67), 142 (p. 71), and 231.

## 1.6 *Alternative color specification systems*

One of the limitations of the system of colorimetry outlined above is its non-intuitiveness and lack of clear relation to commonly understood color perception attributes such as hue, saturation, and lightness/brightness.<sup>†</sup> In describing perceived colors, most individuals resort to the use of color names such as white, black, red, green, yellow, blue, pink, etc. These terms, however, have no inherent ordering and are therefore limited in their utility unless they are conceptually organized into a *color order system*<sup>330</sup> based on perceptual principles.

<sup>†</sup> Readers are referred to [Chapter 2](#) of this handbook or to References 132, 133, 135, and 335 (p. 487) for definitions of hue, chroma, saturation, lightness, brightness, and other color appearance terminology. Common notions of these terms will, however, suffice for the purposes of this chapter.

An early *color order system* for the specification of color based on the perceptual dimensions of hue, lightness, and chroma was developed by Munsell as a teaching aid for art students.<sup>208</sup> In the Munsell Color System, the hue, lightness, and chroma dimensions are described by the Munsell hue, Munsell value, and Munsell chroma, respectively.<sup>25</sup> A semi-numerical specification of the color is obtained by classifying colors into ten principal hues, with ten sub-hues for each hue, and a value between zero and ten. For defined hue and value, a chroma specification was experimentally obtained by selecting samples (with colors of the corresponding hue and value) of increasing chroma with equal perceived differences between neighboring samples. The step sizes for the perceptually equally spaced samples were determined so as to be consistent across different hue and value coordinates. A physical embodiment in the form of a color atlas<sup>209</sup> is an integral part of the Munsell system. The *Munsell Book of Color*, as the atlas was called, contains reflective samples that (when viewed under daylight) are spaced apart in perceptually equal steps of these attributes.<sup>335</sup> Colors in the Munsell system are specified by the combination of the Munsell hue, Munsell value, and Munsell chroma classifications/numbers. The Munsell system has undergone significant extension and evolution and is still in use.<sup>209</sup> In addition to the Munsell system, several other *color order systems* are in existence. The predominant among these are the Swedish Natural Color System<sup>123,124,147,275,289</sup> and the Optical Society of America (OSA) Uniform Color Scales (OSA-UCS) system.<sup>1,186,187</sup>

In the printing industry, it is common to create desired colors by using specially formulated colorants or premixed inks. These are typically known as *spot* colors. The colors are often communicated and specified by using printed samples that are organized by colorant and given distinct designations. Designers may thus choose a color from the available samples and communicate the color to printers using its designation, which specifies which ink is to be used in the printing process. The Pantone Matching System<sup>224</sup> is the main example of such a colorant-based empirical color specification system. Clearly, such a system has several limitations, the primary one being the variation in the specified “color” with a change in viewing illumination. Nonetheless, the system is in widespread use in the design and printing industries and has been extended to additional applications beyond printing.

The color specification systems described above are convenient for the specification of colors of uniform regions with reasonable spatial extent, such as those encountered in paints, color plastics, and textiles. The systems are therefore commonly used in the textiles and coloring industries. The color order systems are also commonly used in color research because of their desirable perceptual attributes. The Pantone Matching System is also commonly used for the specification of color in document imaging applications, typically for regions of uniform color such as a background or a corporate logo. These systems, however, they are not suited for the specification of colors in images where the colors are spatially and typically continuously

varying. A numerical system for color such as the CIE system is thus a requirement for imaging applications. The color order systems, however, play a significant role in the development and enhancement of the CIE numerical system; in particular, in the determination of color spaces with meaningful correlates of perceptual attributes and uniformity with respect to perception.

## 1.7 Uniform color spaces and color differences

The standards for colorimetry defined in Section 1.5 provide a system for specifying color in terms of tristimulus values that can be used to represent colors unambiguously in a three-dimensional space. It is natural to consider the relation of the distance between colors in this three-dimensional space to the perceived difference between them. Before such a comparison can be made, it is necessary to have some means for quantifying perceived color differences. For widely different color stimuli, an observer's assessment of the magnitude of color difference is rather variable and subjective.<sup>335(p. 486)</sup> At the same time, there is little practical value in quantifying large differences in color, and most research has concentrated on quantifying small color differences. For this purpose, the notion of a *just noticeably difference* (JND) in stimuli has been used extensively as a unit by color scientists.

Several researchers have examined the distribution of just noticeably different colors in CIE xy chromaticity and CIE XYZ tristimuli spaces and have found that it varies widely over the color space.<sup>34,185,329,333,334</sup> Hence, the CIE XYZ space is perceptually nonuniform in that equal perceptual differences between colors do not correspond to equal distances in the tristimulus space. Because perceptual uniformity is an extremely desirable feature for defining tolerances in color reproduction systems, considerable research has been directed toward the development of *uniform color spaces*. Traditionally, the problem has been decomposed into two sub-problems: one of determining a uniform lightness scale and the other of determining a uniform chromaticity diagram for equi-lightness color stimuli. The two are then combined with suitable scaling factors for the chromaticity scale and the lightness scale to make their units correspond to the same factor of a JND.

The historical milestones in the search for uniform brightness and lightness scales are described in Wyszecki and Stiles.<sup>335(pp. 493–499)</sup> Typical experiments determine these scales either by a process of repeated bisection of the scale extremes or by moving up in increments of a JND. Details of these standard psychophysical techniques can be found, for instance, in Gesscheider.<sup>100</sup> A cube-root power law relation between brightness and luminance provides a satisfactory fit for most experimental data and therefore has the most widespread acceptance at present.<sup>335(p. 494)</sup>

The search for a uniform lightness scale was complemented by efforts toward determination of a *uniform chromaticity scale* for constant lightness. Two of these attempts are noteworthy. The first determined a linear transformation of the tristimulus space that yielded a chromaticity diagram with

just noticeably different colors being roughly equi-spaced.<sup>151,152</sup> This was the precursor of the CIE 1960  $u,v$  diagram.<sup>335(p. 503)</sup> The second was primarily motivated by the Munsell system and used a nonlinear transformation of the CIE XYZ tristimuli to obtain a *chromatic-value* diagram in which the distances of Munsell colors of equal lightness would be in proportion to their hue and chroma differences.<sup>4</sup> The form for the nonlinear transformation was based on a color vision model proposed earlier by Adams,<sup>3</sup> and the diagram is therefore referred to as Adams' chromatic-value diagram.

Based on the aforementioned research, the CIE has recommended two uniform color spaces for practical applications: the CIE 1976  $L^*u^*v^*$  (CIELUV) space and the CIE 1976  $L^*a^*b^*$  (CIELAB) space.<sup>47</sup> These spaces are defined in terms of transformations from CIE XYZ tristimuli into these spaces. Both spaces employ a common lightness scale,  $L^*$ , that depends only on the luminance value  $Y$ . The lightness scale is combined with different uniform chromaticity diagrams to obtain a three-dimensional uniform color space. For the CIELUV space, a later version of the CIE 1960  $u,v$  diagram is used, whereas CIELAB uses a modification of Adams' chromatic-value diagram.<sup>335(p. 503)</sup> In either case, the transformations include a normalization involving the tristimuli of a white stimulus, which provides a crude approximation to the eye's adaptation (see Section 1.9.1). Euclidean distances in either space provide a color-difference formula for evaluating color differences in perceptually relevant units. Both spaces also include correlates for the three perceptual attributes of lightness, chroma, and hue.

### 1.7.1 The CIE 1976 $L^*u^*v^*$ space

The  $L^*$ ,  $u^*$ ,  $v^*$  values corresponding to a stimulus with CIE XYZ tristimulus values  $X$ ,  $Y$ ,  $Z$  are given by<sup>47</sup>

$$L^* = 116f\left(\frac{Y}{Y_n}\right) - 16 \quad (1.23)$$

$$u^* = 13L^*(u' - u'_n) \quad (1.24)$$

$$v^* = 13L^*(v' - v'_n) \quad (1.25)$$

where

$$f(x) = \begin{cases} x^{\frac{1}{3}} & x > 0.008856 \\ 7.787x + \frac{16}{116} & x \leq 0.008856 \end{cases} \quad (1.26)$$

$$u' = \frac{4X}{X + 15Y + 3Z} \quad (1.27)$$

$$v' = \frac{9Y}{X + 15Y + 3Z} \quad (1.28)$$

$$u_n' = \frac{4X_n}{X_n + 15Y_n + 3Z_n} \quad (1.29)$$

$$v_n' = \frac{9Y_n}{X_n + 15Y_n + 3Z_n} \quad (1.30)$$

and  $X_n, Y_n, Z_n$  are the tristimuli of the white stimulus, which is typically the brightest stimulus in the field of view (see the next chapter for a more detailed discussion of adapting white, and also Section 1.9.1).

The Euclidean distance between two color stimuli in CIELUV space is denoted by  $\Delta E_{uv}^*$  (delta E-uv) and is a measure of the total color difference between them. On average, a  $\Delta E_{uv}^*$  value of around 2.9 corresponds to a JND.<sup>192</sup> As mentioned earlier, the value of  $L^*$  serves as a correlate of lightness. In the  $u^*v^*$  plane, the radial distance ( $\sqrt{(u^*)^2 + (v^*)^2}$ ) and angular position ( $\arctan\left(\frac{u^*}{v^*}\right)$ ) serve as correlates of chroma and hue, respectively.

### 1.7.2 The CIE 1976 $L^*a^*b^*$ space

The  $L^*$  coordinate of the CIELAB space is identical to the  $L^*$  coordinate for the CIELUV space, and the transformations for the  $a^*$  and  $b^*$  coordinates are given by

$$a^* = 500 \left( f\left(\frac{X}{X_n}\right) - f\left(\frac{Y}{Y_n}\right) \right) \quad (1.31)$$

$$b^* = 200 \left( f\left(\frac{Y}{Y_n}\right) - f\left(\frac{Z}{Z_n}\right) \right) \quad (1.32)$$

where  $f(\cdot), X_n, Y_n$ , and  $Z_n$  are as defined earlier.

Because CIELAB is used extensively in imaging, transforms to and from CIELAB to other color spaces are commonly employed. For this reason, it is useful to list the inverse of the above transform, which converts a CIELAB value to a corresponding set of CIE XYZ values as follows:

$$f_Y = \frac{L^* + 16}{116} \quad (1.33)$$

$$f_X = \frac{a^*}{500} + f_Y \quad (1.34)$$

$$f_Z = f_Y - \frac{b^*}{200} \quad (1.35)$$

$$X = X_n f^{-1}(f_X) \quad (1.36)$$

$$Y = Y_n f^{-1}(f_Y) \quad (1.37)$$

$$Z = Z_n f^{-1}(f_Z) \quad (1.38)$$

where  $f_Y, f_X$ , and  $f_Z$  are intermediate terms representing  $f(Y/Y_n), f(X/X_n)$ , and  $f(Z/Z_n)$ , respectively, and  $f^{-1}(\cdot)$  denotes the inverse of the function  $f(\cdot)$  in Equation 1.26, given by

$$f^{-1}(t) = \begin{cases} t^3 & t > 0.206893 \\ \frac{1}{7.787} \left( t - \frac{16}{116} \right) & 0 \leq t \leq 0.206893 \end{cases} \quad (1.39)$$

In CIELAB, the radial distance in the  $a^*b^*$  plane

$$C_{ab}^* = \sqrt{(a^*)^2 + (b^*)^2} \quad (1.40)$$

serves as a correlate or measure of perceived chroma. The angular position in the  $a^*b^*$  plane

$$h_{ab}^* = \arctan\left(\frac{a^*}{b^*}\right) \quad (1.41)$$

serves as a correlate of perceived hue. Euclidean distance between two color stimuli in CIELAB space is denoted by  $\Delta E_{ab}^*$  (delta E-ab). For a sample color with CIELAB values  $L_2^*, a_2^*, b_2^*$ , the color difference from a standard color with CIELAB values  $L_s^*, a_s^*, b_s^*$  is given by

$$\Delta E_{ab}^* = \sqrt{(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2} \quad (1.42)$$

where  $\Delta L^* = L_2^* - L_s^*$ ,  $\Delta a^* = a_2^* - a_s^*$ , and  $\Delta b^* = b_2^* - b_s^*$  are the distances of the sample from the standard along the  $L^*, a^*$ , and  $b^*$  axes, respectively. A  $\Delta E_{ab}^*$  value of around 2.3 corresponds to a JND.<sup>192</sup> This correlation

is, however, quite approximate, and there are significant variations in a visual JND over color space.

The CIELAB space also incorporates an opponent-color encoding (see Sections 1.9 and 1.9.2 for details). The opponent encoding and the  $L^*$ ,  $a^*$ , and  $b^*$  axes are illustrated in Figure 1.14. The  $a^*$  axis corresponds to red–green opponent hues, with distances along the positive  $a^*$  axis corresponding to a measure of redness and distances along the negative  $a^*$  axis corresponding to a measure of greenness. Likewise, the  $b^*$  axis corresponds to the yellow–blue opponent hues, with the distances along the positive  $b^*$  axis representing a measure of yellowness and distances along the negative  $b^*$  axis representing a measure of blueness. Thus, the CIELAB coordinates of a color can also provide a rough description for the color’s perceptual attributes. A color with CIELAB of 50, 0, 0 is a mid-gray color with no apparent hue — commonly referred to as a *neutral* or *achromatic* color. A CIELAB value of 50, 0, 70 represents a mid-lightness saturated yellow color, and a CIELAB value of 90, –10, –7 represents a pastel cyan (bluish-green) color.

### 1.7.3 Limitations of CIELAB and CIELUV spaces

The CIELAB color space is widely used in color imaging and printing industries, and CIELUV is commonly used in the display industry. Both spaces, however, have several limitations. As may be expected, the CIELUV and CIELAB color spaces are only approximately uniform and are often inadequate for specific applications. The uniformity of CIELAB and CIELUV is about the same, but the largest departures from uniformity occur in different

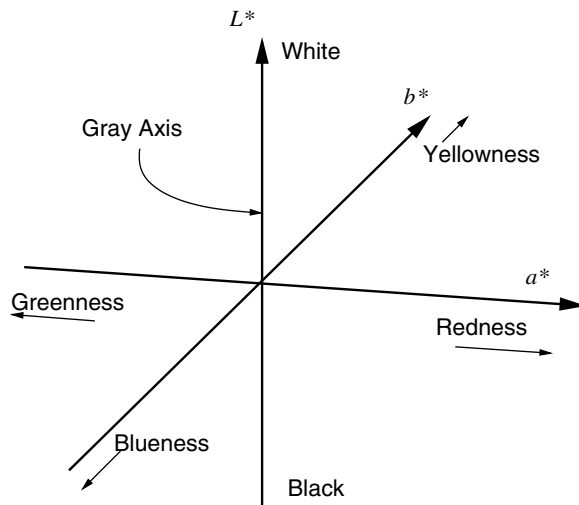


**Figure 1.14** Interpretation of CIELAB axes and opponent nature of the  $a^*$  and  $b^*$  axes.

regions of the color space.<sup>163,229,245</sup> Several other uniform color spaces and color difference formulae have been proposed since acceptance of the CIE standards. Because CIELAB has gained wide acceptance as a standard, most of the difference formulae attempt to use alternate (non-Euclidean) *distance measures*<sup>†</sup> in the CIELAB space. Some of these are discussed in the following section.

The CIELAB and CIELUV spaces are also deficient with regard to the correlates for hue. Figure 1.15 shows a plot of the  $a^*-b^*$  plane where the loci of points corresponding to a constant perceived hue have been plotted. (The tables used to generate this figure were provided by Braun<sup>33</sup> and are based on visual data of Hung et al.<sup>129</sup>) The center of the figure corresponds to the  $L^*$  axis. Since the angular position

$$\arctan\left(\frac{a^*}{b^*}\right)$$

in CIELAB is a correlate of hue, one expects colors of a single hue should correspond to planes in CIELAB emanating radially outward from the CIELAB axis. The loci of points corresponding to a constant perceived hue in Figure 1.15 should therefore correspond to lines going radially outward from the center of the figure. This is, however, not the case. Particularly, the

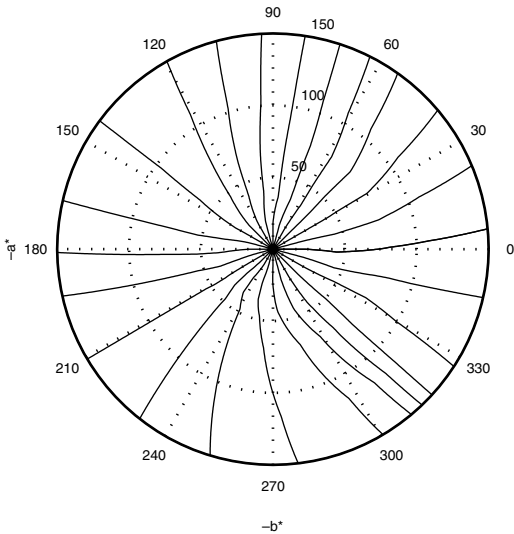


**Figure 1.15** Loci of points corresponding to constant perceived hue in  $a^*-b^*$  plane (interpolated data).

<sup>†</sup> Note that several of these distance measures are asymmetric and as such do not satisfy the mathematical requirements for a metric.<sup>96(p. 91)</sup>

loci of points corresponding to constant perceived hues in the blue regions (in the vicinity of the negative  $b^*$  axis) show a very high curvature, indicating that the CIELAB hue correlate in this region is in poor agreement with actual perceived hue.

In several color processing operations, it is desirable to preserve a specified perceptual color attribute. A common example of this is the process of gamut-mapping (described in significant detail in a subsequent chapter), where it is often desirable to modify colors while preserving hue. Early attempts at preserving hue in the process of gamut mapping were based on the CIELAB space, where the gamut mapping was decomposed into individual mappings in CIELAB *hue planes*, i.e., planes emanating outward from the  $L^*$  axis. While these mappings preserve the CIELAB correlate of hue, the actual perceived hue is often poorly preserved due to the discrepancy between CIELAB hue and perceived hue described above. Understandably, the most significant artifacts are seen in the blue region, and the most common instantiation is the shift from blue to purple in the mapping of colors in the vicinity of the CRT blue primary to a printer gamut. More recently, the problem has been addressed by either warping the CIELAB space to correct for the hue “nonlinearity” before the gamut mapping<sup>33</sup> or through the use of alternate spaces, other than CIELAB, that provide better correlates of hue. Details are included in the chapter on gamut mapping.

#### 1.7.4 *Alternative color difference formulae*

As indicated earlier, the Euclidean distance in CIELAB does not provide a very good agreement with the perceived magnitude of the color difference. Several alternative color difference formulae have therefore been proposed. Due to the widespread use of CIELAB, the most commonly used formulae are based on the CIELAB space and differ from  $\Delta E_{ab}^*$  in that the computation of the distance between colors is not the Euclidean distance. Instead, the color difference formulae commonly decompose the Euclidean distance in CIELAB into components corresponding to differences in lightness, hue, and chroma, and these components are weighted differently to define a new color difference. Prominent among this class of uniform color difference formulae are the CMC(l:c) distance function,<sup>52</sup> the BFD(l:c) function,<sup>182,183</sup> the CIE-94 color difference formula,<sup>49</sup> and the recently developed CIE-DE2000 color difference formula.<sup>179</sup> The CMC(l:c) and the BFD(l:c) formulae have a long history and have influenced the development of the newer color difference formulae. These two color difference formulae are compared to  $\Delta E_{ab}^*$  and a number of other uniform color spaces using perceptibility and acceptability criteria in Reference 192. In image processing applications involving color, the CIELAB and CIELUV spaces have been used extensively whereas, in industrial color control applications, the CMC formulae have found wider acceptance. In 1994, the CIE issued a new recommendation for the computation of color differences in CIELAB space that incorporates several of the robust and attractive features of the CMC(l:c) distance function.<sup>49</sup> Additional

features of the CMC and BFD color difference formulae have also been incorporated into CIE-DE2000.<sup>179</sup>

For the computation of the color difference of a sample color with CIELAB values  $L_2^*, a_2^*, b_2^*$  from a standard color with CIELAB values  $L_s^*, a_s^*, b_s^*$ , a generic class of formula representing several of these color difference formulae can be written as<sup>179</sup>

$$\Delta E = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^2 + R_T \phi(\Delta C^* \Delta H^*)} \quad (1.43)$$

where  $k_L, k_C, k_H$  = positive, real-valued scaling parameters chosen based on the application where the formula is used

$S_L, S_C, S_H$  = lightness-, chroma-, and hue-dependent scaling functions, respectively

$R_T$  = an additional scaling function that depends on chroma and hue

The terms  $\Delta L^*, \Delta C^*, \Delta H^*$  are referred to as the lightness, chroma, and hue differences, respectively. These are defined in terms of the standard and sample CIELAB values as<sup>†</sup>

$$\Delta L^* = L_2^* - L_s^* \quad (1.44)$$

$$\Delta C^* = C_{ab,2}^* - C_{ab,s}^* \quad (1.45)$$

$$\begin{aligned} \Delta H^* &= \sqrt{(\Delta E_{ab}^*)^2 - ((\Delta L^*)^2 + (\Delta C^*)^2)} \\ &= \sqrt{(\Delta a^*)^2 - (\Delta b^*)^2 + (\Delta C^*)^2} \\ &= \frac{2(a_2^* b_s^* - a_s^* b_2^*)}{\sqrt{C_{ab,2}^* C_{ab,s}^* + a_2^* a_s^* + b_2^* b_s^*}} \end{aligned} \quad (1.46)$$

The sample and standard chroma values  $C_{ab,2}^*$  and  $C_{ab,s}^*$ , respectively, are computed from the CIELAB coordinates as indicated in Equation 1.40, and the terms  $\Delta L^*, \Delta a^*, \Delta b^*$ , and  $\Delta E_{ab}^*$  are as defined in Equation 1.42. The function  $\phi(\cdot)$  is a function of the product of chroma and hue differences. Typically,  $k_L = k_C = k_H = 1$  are used as the default parameter values in imaging applications.

For the CMC and CIE-94 color difference formulae, the  $R_T$  term in Equation 1.43 is absent, simplifying these formulae to

† The CIE-DE2000 formula requires an additional chroma-dependent scaling of the  $a^*$  axis prior to computation of the lightness, chroma, and hue differences.

$$\Delta E = \sqrt{\left(\frac{\Delta L^*}{k_L S_L}\right)^2 + \left(\frac{\Delta C^*}{k_C S_C}\right)^2 + \left(\frac{\Delta H^*}{k_H S_H}\right)^2} \quad (1.47)$$

The weighting functions  $S_L$ ,  $S_C$ , and  $S_H$  for the CMC formula are computed from the CIELAB coordinates of the standard as<sup>2,52,205</sup>

$$S_L^{CMC} = \begin{cases} \frac{0.040975 L_{s}^*}{1 + 0.01765 L_{s}^*} & L^* \geq 16 \\ 0.511 & L^* < 16 \end{cases} \quad (1.48)$$

$$S_C^{CMC} = \frac{0.0638 C_{ab,s}^*}{1 + 0.0131 C_{ab,s}^*} + 0.638 \quad (1.49)$$

$$S_H^{CMC} = S_C^{CMC} (T^{CMC} F^{CMC} + 1 - F^{CMC}) \quad (1.50)$$

$$F^{CMC} = \sqrt{\frac{(C_{ab,s}^*)^4}{(C_{ab,s}^*)^4 + 1900}} \quad (1.51)$$

$$T^{CMC} = \begin{cases} 0.56 + |0.2 \cos(h_{ab,s}^* + 168^\circ)| & 164^\circ \leq h_{ab,s}^* \leq 345^\circ \\ 0.36 + |0.4 \cos(h_{ab,s}^* + 35^\circ)| & \text{otherwise} \end{cases} \quad (1.52)$$

where the CIELAB hue angle of the standard,  $h_{ab,s}^*$ , is computed in the interval  $[0, 360^\circ]$  according to the definition in Equation 1.41. The parameter  $k_H$  is unity for the CMC color difference formula, and the parameters  $k_L$  and  $k_C$  (defined by the user) weight the importance of lightness and chroma relative to hue. The resulting color difference computed using the above scaling functions in Equation 1.47 is referred to as the  $\Delta E_{CMC(k_L, k_C)}$  color difference. Note that the definitions of the lightness, hue, and chroma weighting functions in terms of the CIELAB coordinates of the standard make the CMC color formula asymmetric in that the distance between two samples is dependent on which one is chosen as the standard and which one as the sample.

The complexity of the CMC formula makes it difficult to understand intuitively based on equations alone. A graphical illustration of the CMC difference is therefore presented in Figure 1.16. The figure represents the CIELAB  $a^*-b^*$  plane, with the  $a^*$  and  $b^*$  axes as shown. The figure includes plots of several elliptical closed curves that represent the loci of points whose color difference with respect to a standard color at the “center” of the “ellipse” is one  $\Delta E_{CMC(1,1)}$  unit. The “center” representing the standard has been excluded from the figure for clarity. Note that the figure is representative of  $a^*-b^*$  planes at all values of  $L^*$ , because the  $S_H^{CMC}$  and  $S_C^{CMC}$  weighting

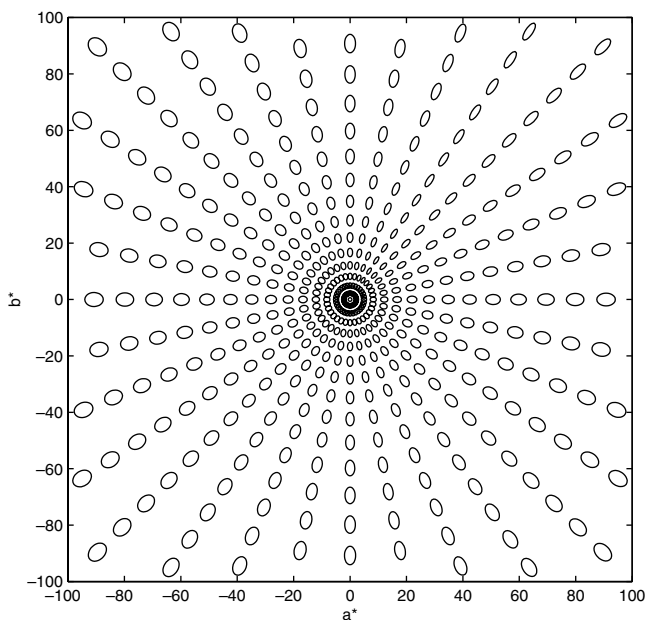


**Figure 1.16** “Ellipses” corresponding to a color difference of  $\Delta E_{CMC(1:1)} = 1.0$  in the  $a^*-b^*$  plane.

functions are independent of  $L^*$ . The figure, however, does not illustrate the effect of the weighting function  $S_L^{CMC}$ . The predominant trend seen in Figure 1.16 is the elongation of the “ellipses” as one goes radially outward from the center. A plot of similar “ellipses” for the  $\Delta E_{ab}^*$  color difference would result in uniform circles of radius 1 throughout the plane. Thus, if the CMC formula is considered accurate, i.e., in better agreement with perceived differences, it indicates that the  $\Delta E_{ab}^*$  color difference overemphasizes hue and chroma differences at higher chroma values. Visual color difference data verify that this is indeed the dominant deficiency in the uniformity of CIELAB.

While the CMC formula corrects for this dominant nonuniformity in CIELAB, because of its complexity and the number of terms, there are concerns about its statistical validity and generalization to data and situations beyond those for which it was originally derived.<sup>19</sup> In 1994, the CIE proposed a new color difference formula that incorporated only the more robust features of the CMC formula. The CIE-94 color difference formula<sup>49</sup> is obtained by using the following weighting functions in the difference equation of Equation 1.47:

$$S_C^{CIE94} = 1 + 0.045C_{ab}^* \quad (1.53)$$

$$S_H^{CIE94} = 1 + 0.015C_{ab}^* \quad (1.54)$$

The lightness weighting function  $S_L^{CIE94} = 1.0$ , and, in typical imaging applications, the parametric factors  $k_L$ ,  $k_C$ , and  $k_H$  are all chosen to be unity. Asymmetric and symmetric versions of the formula have been defined. For the asymmetric formula, the chroma in Equations 1.53 and 1.54 corresponds to the chroma of the standard color, i.e.,  $C_{ab}^* = C_{ab,s}^*$ . Just like the CMC formula, this implies that the distance between two samples is dependent on which one is chosen as the standard and which one as the sample. For the symmetric version of the formula, the chroma for the weighting functions in Equations 1.53 and 1.54 is defined as the geometric mean of the chromas of the sample and standard colors, i.e.,  $C_{ab}^* = \sqrt{(C_{ab,s}^* C_{ab,2}^*)}$ . This ensures that the distance between two samples is independent of which one is chosen as the standard. The color difference obtained using the above scaling functions in Equation 1.47 is referred to as the  $\Delta E_{94}^*$  color difference.

The CIE-94 color difference formula is significantly simpler than the CMC formula. From the scaling functions of Equations 1.53 and 1.54, it is clear that the CIE-94 color difference formula scales down hue and chroma differences for higher chroma colors in comparison to CIELAB and thus corrects for the predominant deficiency in CIELAB. Along the  $L^*$  axis, the scaling factors are all unity, which ensures that (asymmetric)  $\Delta E_{94}^*$  color differences about neutral colors that lie on the  $L^*$  axis are identical to the  $\Delta E_{ab}^*$  Euclidean color difference. For chromatic colors, the weighting factors in Equations 1.53 and 1.54 are greater than unity, and the  $\Delta E_{94}^*$  is therefore smaller than the  $\Delta E_{ab}^*$  Euclidean color difference. A graphical visualization of the  $\Delta E_{94}^*$  color difference formula is shown in Figure 1.17, where several elliptical closed curves are plotted in the  $a^*-b^*$  plane. These “ellipses” correspond to the loci of points whose color difference with respect to a standard color at the “center” of the “ellipse” is one  $\Delta E_{94}^*$  unit (asymmetric formula). Once again, the central points have been excluded for clarity of the figure. The figure demonstrates trends similar to Figure 1.16, with “ellipses” elongated along the radial direction as one goes radially outward from the  $L^*$  axis and increasing in size along both dimensions. Due to the lack of a hue-dependent weighting term in the  $\Delta E_{94}^*$  color difference formula, “ellipses” at the same radial distance from the origin in Figure 1.17 are congruent, unlike the corresponding “ellipses” in Figure 1.16. Through an integration of the chroma weighting function for the CIE-94 color difference formula, it is also possible to create an alternate color space in which the Euclidean distance is in very good agreement with the  $\Delta E_{94}^*$  color difference formula.<sup>244,295</sup> Such a space is useful for visualizing  $\Delta E_{94}^*$  color differences, as they correspond to the well-understood notion of Euclidean distance.

Recently, a new color difference formula that has been proposed for adoption as a CIE standard<sup>179</sup> is termed the CIE-DE2000 color difference formula, and the corresponding color difference is denoted as  $\Delta E_{00}$ . The CIE-DE2000 color difference formula incorporates a hue-dependent weighting function  $S_H$  similar to the CMC formula. It also includes an additional term in the color difference that depends on the hue and chroma difference product that is motivated by the BFD formula (the  $R_T$  term in Equation 1.43).

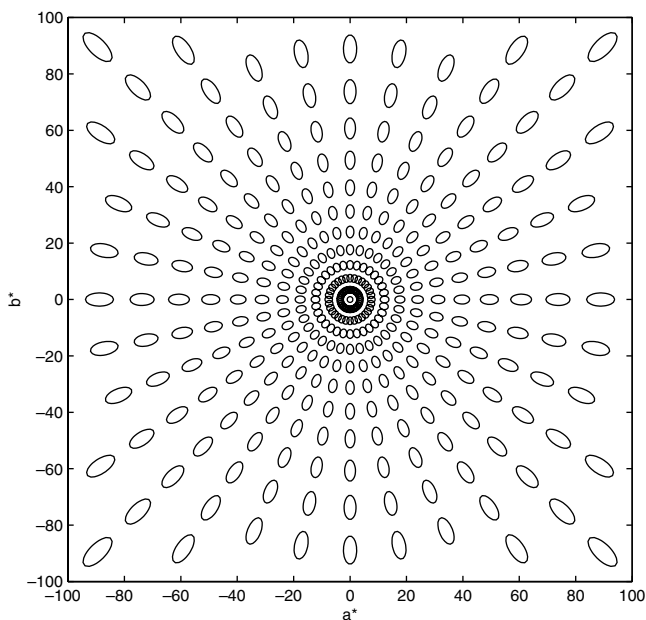


**Figure 1.17** “Ellipses” corresponding to a color difference of  $\Delta E_{94}^* = 1.0$  in the  $a^*-b^*$  plane.

The formula also includes a global rescaling of the  $a^*$  axis prior to computation of the hue and chroma. The CIEDE2000 color difference formula is designed to be symmetric by using averages of the standard and sample color values in the weighting functions. The resulting formula is fairly complex, and the reader is referred to Reference 179 for details. Figure 1.18 visually illustrates the behavior of the  $\Delta E_{00}$  color difference formula in the  $a^*-b^*$  plane. The closed curves plotted in this figure correspond to the loci of points whose color difference, with respect to a standard color located roughly in the center of the curve, is one  $\Delta E_{00}$  unit. The closed curves are well behaved in most regions and shaped like ellipses. In the blue region (around the negative  $b^*$  axis), however, the closed curves become non-convex and take on a very distorted shape. This is potentially problematic and probably does not agree with any psychophysical color difference data, most of which predict loci of visually equidistant color samples from a standard as convex almost elliptical closed curves. It would therefore be prudent to wait for a resolution of these issues before using the CIE-DE2000 color difference formula. In addition to the issue illustrated in Figure 1.18, additional concerns remain with regard to the CIE-DE2000.<sup>170,180</sup> Because the scaling functions have been derived from color difference datasets, there are questions about differences in the conditions under which the data were gathered and about the influence of the conditions on the scaling functions. In particular, some of the data might be influenced by lightness and chroma

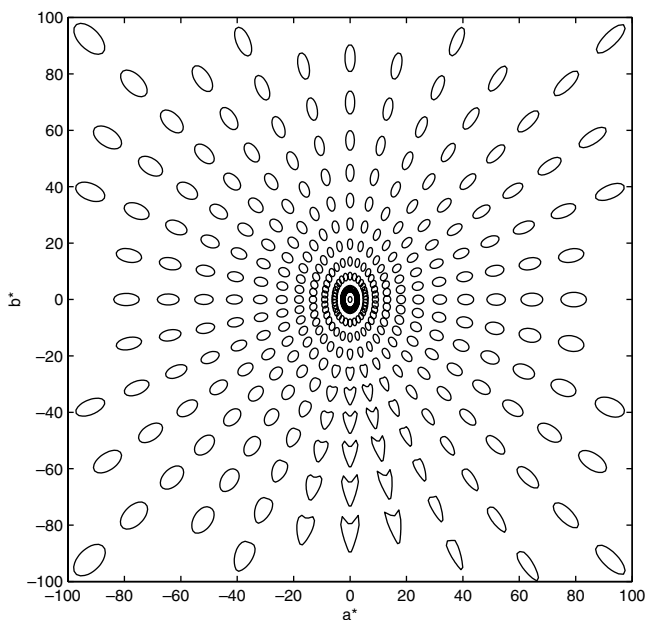


**Figure 1.18** “Ellipses” corresponding to a CIEDE2000 color difference of  $\Delta E_{00} = 1.0$  in the  $a^*-b^*$  plane.

crispening,<sup>†</sup> limiting the scenarios under which the formula is applicable.<sup>170</sup> In imaging applications, colors are typically surrounded by other similar colors, whereas visual datasets used for deriving the color difference formulae are based on comparisons of colors on a fixed background — normally mid-gray. For imaging applications, therefore, it is particularly important that the local lightness/chroma crispening in the experimental visual data should not influence the color difference formulae. In addition to the above concerns with color difference formulae, there are several fundamental questions associated with the CIELAB space itself.<sup>169</sup> Research on improved uniform color spaces and color difference formulae is therefore likely to continue.

## 1.8 Limitations of CIE colorimetry

CIE colorimetry as discussed above has several additional limitations beyond the deficiency in uniformity of the CIELAB and CIELUV color spaces that was already addressed above. First, variations among observers’ color matching characteristics are not comprehended by the CIE standard observer and can result in a mismatch in colors for an actual color-normal observer, even when a metameric match is predicted by the standard observer. This is termed *observer metamerism*. While the extent of variation cannot be inherently reduced, it is useful to quantify it, because it forms a basis for deciding

<sup>†</sup> A description of lightness crispening can be found in the next chapter.

what color tolerances are allowable and the extent to which illumination has to be controlled to avoid illuminant metamerism (discussed in Section 1.5.2). Toward this end, the CIE has developed an additional concept of a *standard deviate observer*,<sup>48</sup> which allows users to establish confidence limits representing observer variability. The standard deviate observer was derived based on analysis of the variation in the original color matching data used to derive the CIE 1964 10° observer.

There is also significant debate about the accuracy of the standard CIE 1931 CMFs, given that these were derived from experimental data gathered almost a century ago, using equipment limited by the technology available at that time. In particular, the data used to generate the 2° standard observer<sup>110,328</sup> cannot be entirely reconciled with the data gathered later and used to generate the 10° standard observer.<sup>284</sup> Research has indicated some small but systematic deviations from the original CMFs in the short-wavelength regions.<sup>78,278,285</sup> It has been conjectured that these deviations may partly explain some of the blue hue nonlinearity in CIELAB that was discussed earlier. The CIE is also involved in an effort to derive new cone fundamentals (CMFs) for the 2° standard observer based on the more recent 10° observer data.<sup>256,307</sup> There are also fundamental questions about the connection between photometry and colorimetry and whether it is indeed possible to define the photometric (luminance) response in terms of the colorimetric response as one of the CMFs. Colorimetry therefore continues to be an area of active research. In addition to issues with established standards for color-normal observers, significant work is also required to better understand the “color” vision characteristics of color-anomalous observers.

Despite the above limitations, it is worth noting that the CIE system for colorimetry is a significant advance over methods for color specification prevailing before its adoption. The CIE system of colorimetry provides a methodology for the quantitative specification of colors (or at least, color correspondences). In this regard, the perception of color is significantly advantaged over other human senses such as touch and smell for which no system for quantitative specification currently exists and none is under imminent development.

## 1.9 Psychophysics of color

The human visual system as a whole displays considerable adaptation. It is estimated that the total intensity range over which colors can be sensed is around 10<sup>8</sup>:1. While the cones themselves respond only over a 1000:1 intensity range, the vast total operating range is achieved by adjustment of their sensitivity to light as a function of the incident photon flux.<sup>320</sup> This adjustment is believed to be largely achieved through a feedback from the neuronal layers that provide temporal lowpass filtering and adjust the cones’ output as a function of average illumination. A small fraction of the adaptation corresponding to a factor of around 8:1 is the result of a 4:1 change in the diameter of the pupil that acts as the aperture of the eye.<sup>135(p. 23)</sup>

Another fascinating aspect of human vision is the invariance of object colors under lights with widely varying intensity levels and spectral distributions. Thus, objects are often recognized as having approximately the same color in phases of daylight having considerable difference in their spectral power distribution and also under artificial illumination. This phenomenon is called *color constancy*. The term *chromatic adaptation* is used to describe the changes in the visual system that relate to this and other psychophysical phenomena.

While colorimetry provides a representation of colors in terms of three independent variables, it was realized early on that humans perceive color as having four distinct *hues* corresponding to the perceptually unique sensations of red, green, yellow, and blue. Thus, while yellow can be produced by the additive combination of red and green, it is clearly perceived as being qualitatively different from each of the two components. Hering<sup>120</sup> had considerable success in explaining color perception in terms of an opponent-colors theory, which assumed the existence of neural signals of opposite kinds, with the red–green hues forming one opponent pair and the yellow–blue hues constituting the other. Such a theory also satisfactorily explains both the existence of some intermediate hues (such as red–yellow, yellow–green, green–blue, and blue–red) and the absence of other intermediate hues (such as reddish-greens and yellowish-blues).

Initially, the trichromatic theory and the opponent-colors theory were considered competitors for explaining color vision. However, neither one by itself was capable of giving satisfactory explanations of several important color vision phenomena. In more recent years, these competing theories have been combined in the form of *zone theories of color vision*, which assume that there are two separate but sequential zones in which these theories apply. Thus, in these theories, it is postulated that the retinal color-sensing mechanism is trichromatic, but an opponent-color encoding is employed in the neural pathways carrying the retinal responses to the brain. These theories of color vision have formed the basis of a number of color appearance models that attempt to explain psychophysical phenomena. Typically, in the interests of simplicity, these models follow the theories only approximately and involve empirically determined parameters. The simplicity, however, allows their practical use in color reproduction applications involving different media where a perceptual match is more desirable and relevant than a colorimetric match.

A somewhat different but widely publicized color vision theory was the *retinex* (from *retina* and *cortex*) theory of Edwin Land.<sup>174,173</sup> Through a series of experiments, Land demonstrated that integrated broadband *reflectances* in red, green, and blue channels show a much stronger correlation with perceived color than the actual spectral composition of radiant light incident at the eye, or corresponding integrated radiances. He further postulated that the human visual system is able to infer the broadband reflectances from a scene through a successive comparison of spatially neighboring areas, which offers an alternate spatial form of adaptation similar to chromatic adaptation.

The *retinex* computation is one of the few models of vision that attempts to comprehend spatial interactions in color vision. Recent years have seen a resurgence of interest in the retinex theory.<sup>203</sup> Computational versions of the theory have recently been used, with some success, in the enhancement of color images,<sup>145,146,239</sup> in illuminant estimation for digital camera images,<sup>97</sup> gamut mapping of high dynamic range images,<sup>281</sup> and other applications.

One may note here that some of the uniform color spaces include some aspects of color constancy and color appearance in their definitions. In particular, both the CIELAB and CIELUV spaces employ an opponent color encoding and use white-point normalizations that partly explain color constancy. However, the notion of a color appearance model is distinct from that of a uniform color space. Typical uniform color spaces are useful only for comparing stimuli under similar conditions of adaptation and can yield incorrect results if used for comparing stimuli under different adaptation conditions. The CIE does not recommend the use of CIELAB and CIELUV spaces in conditions where the illuminant is “too different from average daylight.”<sup>47</sup> Even under these restricted conditions, it is apparent from the discussion of Section 1.7.3 that, as appearance spaces, CIELAB and CIELUV are rather crude approximations. Color appearance models and spaces are the topic of the next chapter. A very brief outline is included here to provide a broad overview and establish the connections with the basic colorimetry and color difference formulae we have described here.

### 1.9.1 Chromatic adaptation and color constancy

Several mechanisms of chromatic adaptation have been proposed to explain the phenomenon of color constancy. Perhaps the most widely used of these in imaging applications is one proposed by von Kries.<sup>167</sup> He hypothesized that the chromatic adaptation is achieved through individual adaptive gain control on each of the three cone responses. Thus, instead of Equation 1.3, a more complete model represents the cone responses as

$$\mathbf{c}' = \mathbf{D}\mathbf{S}^T\mathbf{f} \quad (1.55)$$

where  $\mathbf{D}$  is a diagonal matrix corresponding to the gains of the three channels, and the other terms are as before. The gains of the three channels depend on the state of adaptation of the eye, which is determined by pre-exposed stimuli and the surround, but independent of the test stimulus  $\mathbf{f}$ . This is known as the *von Kries coefficient rule*.

The term *asymmetric matching* is used to describe matching of color stimuli under different adaptation conditions. Using the von Kries coefficient rule, two radiant spectra,  $\mathbf{f}_1$  and  $\mathbf{f}_2$ , viewed under adaptation conditions specified by the diagonal matrices  $\mathbf{D}_1$  and  $\mathbf{D}_2$ , respectively, will match if

$$\mathbf{D}_1\mathbf{S}^T\mathbf{f}_1 = \mathbf{D}_2\mathbf{S}^T\mathbf{f}_2 \quad (1.56)$$

Thus, under the von Kries coefficient rule, chromatic adaptation can be modeled as a diagonal transformation for tristimuli specified in terms of the eye's cone responses. Usually, tristimulus values are specified not relative to the cone responses themselves but to CMFs that are linear transformations of the cone responses. In this case, it can readily be seen<sup>335(p. 432)</sup> that the tristimuli of color stimuli that are in an asymmetric color match are related by a similarity transformation<sup>101</sup> of the diagonal matrix  $\mathbf{D}_1^{-1}\mathbf{D}_2$ .

A von Kries transformation is commonly used in color rendering applications because of its simplicity and is a part of several standards for device-independent color imaging.<sup>61,290</sup> Typically, the diagonal matrix  $\mathbf{D}_1^{-1}\mathbf{D}_2$  is determined by assuming that the cone responses on either side of Equation 1.56 are identical for white stimuli (usually a perfect reflector illuminated by the illuminant under consideration). The white-point normalization in CIELAB space was primarily motivated by such a model. Because the CIE XYZ CMFs are not *per se* the cone responses of the eye, the diagonal transformation representing the normalization is not a von Kries transformation and was chosen more for convenience than accuracy.<sup>79</sup>

In actual practice, the von Kries transformation can explain results obtained from psychophysical experiments only approximately.<sup>335(pp. 433–451)</sup> At the same time, the constancy of metameric matches under different adaptation conditions provides strong evidence for the fact that the cone response curves vary only in scale while preserving the same shape.<sup>131(p. 15)</sup> It therefore seems most likely that part of the adaptation lies in the nonlinear processing of the cone responses in the neural pathways leading to the brain.

A number of alternatives to the von Kries adaptation rule have been proposed to obtain better agreement with experimental observations. Most of these are nonlinear and use additional information that is often unavailable in imaging applications. Several of these are discussed in the next chapter, and additional information can be found in References 81 and 135 (pp. 81 and 217).

The phenomenon of color constancy suggests that the human visual system transforms recorded stimuli into representations of the scene reflectance that are (largely) independent of the viewing illuminant. Several researchers have investigated algorithms for estimating illuminant-independent descriptors of reflectance spectra from recorded tristimuli, which have come to be known as “computational color constancy algorithms.”<sup>88–90,93,195</sup> Several of these algorithms rely on low-dimensional linear models of object and illuminant spectra, which will be discussed briefly in Section 1.11.5.1. A discussion of how these algorithms relate to the von Kries transformation rule and to human color vision can also be found in References 89, 90, and 322.

### 1.9.2 Opponent processes theory and color appearance models

The modeling of chromatic adaptation is just one part of the overall goal of color appearance modeling. While color appearance models are empirically determined, they are usually based on physiological models of color vision.

Most modern color vision models are based on “wiring diagrams” of the type shown in [Figure 1.19](#). The front end of the model consists of L, M, and S (long, medium, and short wavelength sensitive) cones. The cone responses undergo nonlinear transformations and are combined into two opponent color chromatic channels (R-G and Y-B) and one achromatic channel (A). A positive signal in the R-G channel is an indication of redness, whereas a negative signal indicates greenness. Similarly, yellowness and blueness are opposed in the Y-B channel. The outputs of these channels combine to determine the perceptual attributes of hue, saturation, and brightness.

It is obvious that the above color-vision model is an oversimplification. Actual color appearance models are considerably more intricate and involve a much larger number of parameters, with mechanisms to account for spatial effects of surround and the adaptation of the cone responses, which was briefly discussed in the last section. Due to the immense practical importance of color appearance modeling to color reproduction systems, there has been considerable research in this area, and a standard color appearance model has been developed by the CIE.<sup>50,181</sup> Additional details of the state of the research in color appearance models can be found in the next chapter.

A common use of color appearance models in imaging applications is in reproducing images that are to be viewed under different viewing condi-

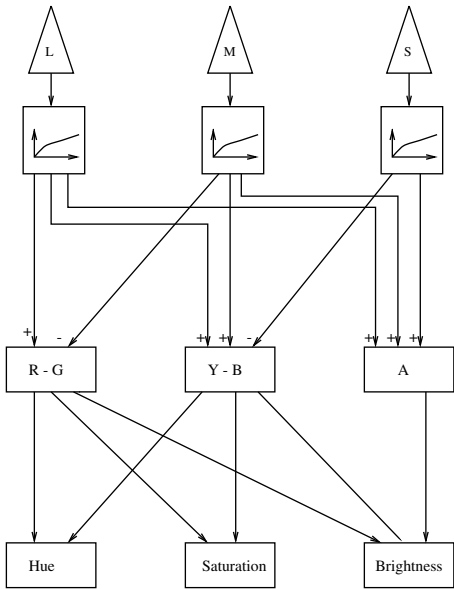


**Figure 1.19** Typical “wiring diagram” for human color vision models (adapted from Reference 246).

tions from the original. An example of this situation would be the reproduction of a bright daylight-lit outdoor scene in a photograph/printed image that is to be viewed indoors under significantly lower light levels and illumination with relative SPDs very different from daylight. Matching the colorimetry of the outdoor scene under the indoor illumination is neither feasible nor desirable due to the very different states of adaptation of the eye under the differing conditions.<sup>136</sup> Instead, it is desirable to match the appearance as closely as possible. This is facilitated by color appearance models.

Figure 1.20 illustrates the conceptual use of the color appearance model in situations similar to those outlined above. For given colorimetry under (specified) reference viewing conditions, a color appearance model predicts the colorimetry required under the (specified) test viewing conditions for producing the same color appearance. To take a concrete example, if you view a reflective print under one illuminant and want to create a print that, when viewed under a different illuminant, produces an identical appearance (in your mind), the color appearance model gives you the colorimetry that your desired print should have. Note that the appearance that you are trying to match is that of the “original” print under the original viewing conditions and not the appearance of the original print under the new viewing illuminant. This is a common source of confusion in the use of color appearance models and chromatic adaptation transforms: it is incorrectly assumed that these models predict the color appearance of a physical color sample as it

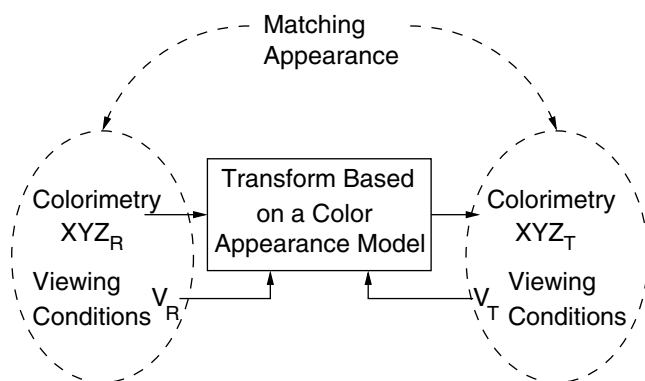


**Figure 1.20** Use of a color appearance model to predict colorimetry under test viewing conditions that will match appearance of reference colorimetry under corresponding viewing conditions.

is moved across viewing illuminants. A color appearance model is not intended to predict the color appearance of a physical object under a different “test” viewing illuminant from its colorimetry under a “reference” viewing illuminant. Clearly, if we have metameric samples that match under the “reference illuminant,” a color appearance model will indicate that they match under all test illuminants whereas, in practice, we know this is not true. Color appearance models only model the adaptation process in the eye — they cannot and do not model the physical changes in the viewing illuminant; these can be properly accounted for only by using spectral measurements using spectrophotometers and spectroradiometers.

Despite the statements in the above paragraphs, arguments from color constancy can be used to say that, for most objects, color appearance models would provide a good approximation to the colorimetry of the original object under the new illuminant. However, because color constancy holds only approximately, so does this argument, and it is important to understand that the failure of a color appearance model in predicting the change in appearance of a physical object when it is moved from one viewing illuminant to another is an instance of incorrect use of the model rather than a failure of the model.

## *1.10 Spatial characteristics of color vision*

The CIELAB color space and associated color difference formulae try to account for the nonlinearities in the color sensing process for the comparison of relatively large uniform color regions and do not account for any spatial interactions in the eye. Because the eye acts as a spatial lowpass filter that can average over high spatial-frequency differences in images, a point-by-point computation of color differences in CIELAB color space is not appropriate for the comparison of images. Though the eye exhibits lowpass characteristics for both luminance<sup>†</sup> and chrominance spatial information, the bandwidth for the chrominance channels is much lower than that of the luminance channels. This fact has been exploited in numerous applications, including color TV, where the transmission bandwidth allocated to the chrominance channels is significantly lower than that for the luminance channel.

Several researchers have modeled the spatial properties of the eye as a linear filter and studied the frequency response of the eye for luminance and chrominance spatial patterns.<sup>87,95,103,164,345</sup> However, a complete model for human vision that is perceptually uniform and incorporates the effects of the spatial interactions is yet to be developed. While sophisticated models that explain a large number of psychophysical effects have been developed for luminance information,<sup>66,178</sup> their extensions and the development of alternative models that are capable of handling color data are still an active

<sup>†</sup> Strictly speaking, the luminance response of the eye is believed to be the bandpass but, for all practical purposes, the low-frequency attenuation can be ignored.

area of research.<sup>331</sup> Several of the models in development are quite complicated and incorporate features designed to explain a wide variety of psychophysical effects.<sup>73,346</sup>

While sophisticated models are necessary in some cases,<sup>67</sup> in several imaging applications significant improvements can be made by incorporating relatively simple spatial models of human color vision. Computational simplicity is also a necessity in situations where the models are used in iterative loops for the optimization of image processing algorithms. As a consequence, several simplified models have been developed that model the spatial characteristics of color vision as simple luminance/lightness and chrominance contrast sensitivity functions. One of the popular models in this class of simplified models is the spatial extension of the CIELAB (S-CIELAB) model,<sup>345</sup> which will be used here for illustration of some of the features of these models.

The S-CIELAB model first transforms a color image into three opponent color image planes,  $O_1$ ,  $O_2$ , and  $O_3$ , corresponding to black–white (*luminance*), red–green, yellow–blue image components, respectively. These three color coordinates are defines as a linear transformation of the CIE XYZ tristimulus values as

$$\begin{aligned} O_1 &= 0.279X + 0.72Y - 0.107Z \\ O_2 &= -0.449X + 0.29Y - 0.077Z \\ O_3 &= 0.086X + 0.59Y - 0.501Z \end{aligned} \quad (1.57)$$

Each opponent-colors image is convolved with a kernel filtered by two-dimensional separable spatial kernels consisting of mixed Gaussians

$$f^j(x, y) = k^j \sum_i w_i^j E_i^j(x, y) \quad (1.58)$$

$$E_i^j(x, y) = k_i^j \exp \left( \frac{-(x^2 + y^2)}{(\sigma_i^j)^2} \right) \quad (1.59)$$

where  $f^1(x, y)$ ,  $f^2(x, y)$ , and  $f^3(x, y)$  are the kernels for  $O_1$ ,  $O_2$ , and  $O_3$ , respectively. Variables  $x$  and  $y$  denote the spatial dimensions, and the standard deviation  $\sigma_i^j$  determines the spatial spread of the corresponding Gaussian in the mixture. In the discrete implementation, the scale factor  $k_i^j$  is chosen so that  $E_i^j(x, y)$  sums to 1 over the spatial extent along  $x$  and  $y$ . The scale factors  $k^j$  are chosen so that, for each color plane, its two-dimensional kernel  $f^j(x, y)$  sums to one. The parameters  $(w_i^j, \sigma_i^j)$  for the three opponent-colors are listed in Table 1.2, where  $\sigma_i^j$  are in degrees of visual angle.

Table 1.2 Parameters for the s-CIELAB HVS Model

|                               | $w_i^j$                  | $\sigma_i^j$             |
|-------------------------------|--------------------------|--------------------------|
| Luminance $O_1$ ( $i = 3$ )   | 0.921<br>0.105<br>-0.108 | 0.0283<br>0.133<br>4.336 |
| Red-green $O_2$ ( $i = 2$ )   | 0.531<br>0.330           | 0.0392<br>0.494          |
| Blue-yellow $O_3$ ( $i = 2$ ) | 0.488<br>0.371           | 0.0536<br>0.386          |

In the S-CIELAB model, the opponent color images  $O_1$ ,  $O_2$ , and  $O_3$  are spatially filtered through the corresponding spatial kernels  $f^1(x,y)$ ,  $f^2(x,y)$ , and  $f^3(x,y)$  to obtain images  $O_1'$ ,  $O_2'$ , and  $O_3'$ , respectively.

$$O_j'(x, y) = (O_j(x, y) * f^j(x, y)) \quad j = 1, 2, 3 \quad (1.60)$$

where  $*$  denotes convolution. The filtered images  $O_1'$ ,  $O_2'$ , and  $O_3'$  are then transformed back to CIE XYZ coordinates using the inverse of the transformation in Equation 1.57. The CIE XYZ coordinates are then transformed into CIELAB using the transformation described earlier in Section 1.7.2. The resulting image is the S-CIELAB representation of the original image. To evaluate the differences between two images,<sup>†</sup> the images are converted into their S-CIELAB representations, and pixel-wise color differences are computed between them in the CIELAB space, producing a color difference image. Typically, the CIE  $\Delta E_{ab}^*$  color difference formula has been used, although the other difference formulae based on CIELAB described in Section 1.7.4 could also be used. The color difference image obtained from this process represents a spatial map of the visual difference or distortion between the images.<sup>342</sup> The distortion map may be used directly to determine regions where differences will be perceived, or the error over the image pixels may be averaged to determine a single measure of the perceived difference between the images.

The spatial filter kernels for S-CIELAB corresponding to the parameter values of Table 1.2 are shown in Figure 1.21a, b, and c for the black–white, red–green, and yellow–blue opponent channels, respectively. The shapes of the kernels clearly indicate the greater detail resolution for the black–white (*luminance*) channel in comparison to the red–green and yellow–blue chrominance channels, which is designed to mimic the behavior of the human visual system. Because the individual kernels sum to unity, filtering a large uniform region does not change its values, and the color difference computation

<sup>†</sup> It is assumed that the two images are registered well with each other. The S-CIELAB metric is not meaningful for misregistered images.

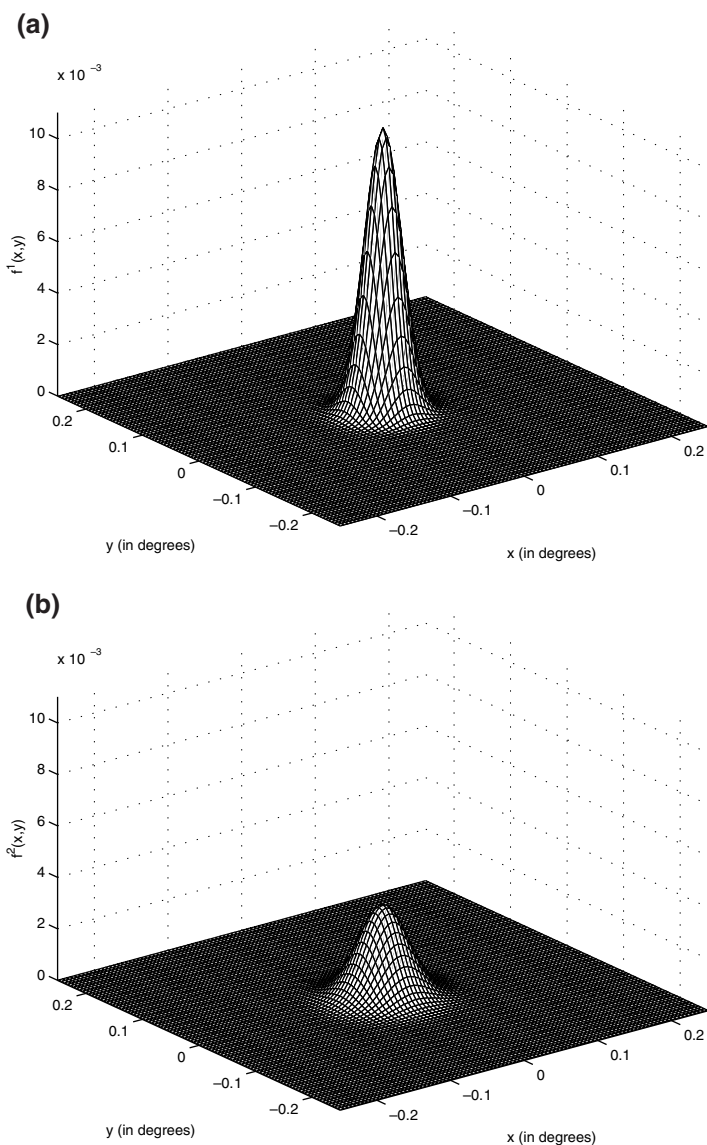


**Figure 1.21** Spatial convolution kernels for the S-CEILAB opponent color channels: (a)  $f^1(x,y)$ , black–white; and (b)  $f^2(x,y)$ , red–green (continues).

reduces to the direct computation of color difference between the original images in CIELAB. Thus, the S-CIELAB model agrees with CIELAB for large uniform regions, which is desirable, as CIELAB is already a widely used model for computing color differences between spatially uniform regions of differing color.

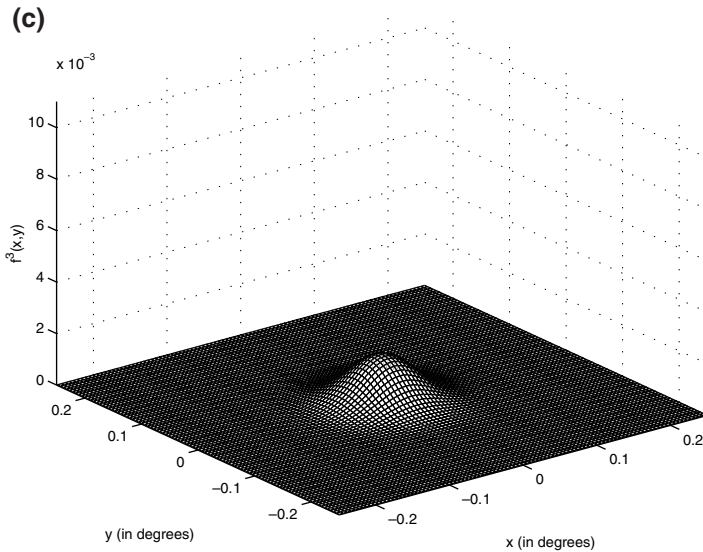


**Figure 1.21 (continued)** Spatial convolution kernels for the S-CEILAB opponent color channels: (c)  $f^\beta(x,y)$ , yellow–blue.

The S-CIELAB model is attractive because of its simplicity and close association to the well-accepted CIELAB standard. It has therefore been used recently in several different applications.<sup>10,342–344</sup> Several other models with similar characteristics have also been employed successfully in color imaging applications.<sup>87,95,103,164</sup> These models provide varying degrees of agreement with psychophysical data and few, if any, comprehensive objective tests of the different models exist. Research on improving the models and on the development of new models is also continuing.

## 1.11 Color image reproduction and recording devices

In the physical world, color images exist as spatially varying spectral radiance or reflectance distributions. To process these images digitally, the images must be sampled both spatially and spectrally. The issues involved in spatial sampling and reconstruction of images have been discussed at length in signal processing literature and will not be repeated here. The aspects of spectral sampling and color recording for images are also important, and these are addressed here. The reproduction of color images works in the opposite direction from the recording process. The physical realization of color images from recorded information requires synthesis of spatially varying spectral radiance or reflectance distributions. This section provides an overview of common color reproduction devices, the spectral characteristics of natural images and images from these reproduction devices, and methods for recording these images. Color reproduction devices are discussed first,

because color recording systems may also be used to record color reproductions and may exploit the characteristics of the reproduction device.

### 1.11.1 Color output systems

Nature provides a variety of mechanisms by which color may be produced. As many as 15 distinct physical mechanisms have been identified that are responsible for color in nature.<sup>210</sup> While only a fraction of these mechanisms are suitable for technological exploitation, there is still considerable diversity in available technologies and devices for displaying and printing color images.

Color output devices can be classified broadly into two main types: *additive* and *subtractive*. Additive color systems produce color on a dark background through the combination of differently colored lights, known as primaries. The qualifier additive is used to signify the fact that the final spectrum is the sum (or average) of the spectra of the individual lights, as was assumed in the discussion of color matching in Section 1.4. Typically, the additive primaries are red, green, and blue (RGB). The additive mixing of RGB primaries is illustrated in Figure 1.22. The combination of red and green forms yellow, of red and blue forms magenta, and of blue and green forms cyan. Combination of all three primaries at full intensities produces white. Intermediate colors are obtained by varying the individual primary intensities. Examples of additive color systems include color cathode ray tube (CRT) displays and projection video systems.

Color in subtractive systems is produced through a process of removing (subtracting) unwanted spectral components from “white” light. Typically,

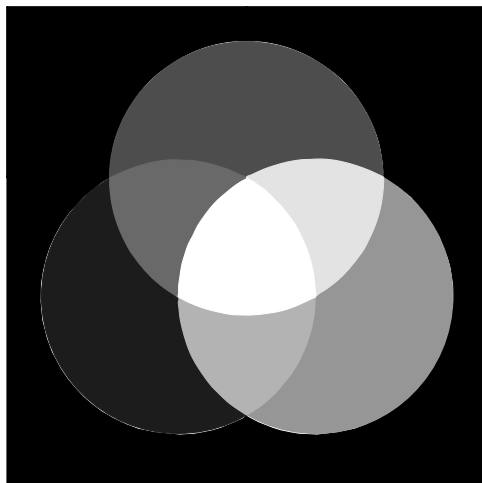


**Figure 1.22** (See color insert following page 430) Additive mixing of red, green, and blue primaries.

such systems produce color on transparent or reflective media, which are illuminated by “white” light having spectral components distributed over the visible spectrum. By overlaying these media with colorants that selectively absorb light of certain wavelengths while transmitting other wavelengths, different colors are produced. Typical subtractive systems are based on cyan, magenta, and yellow (CMY) colorants that absorb light in the red, green, and blue spectral regions, respectively. The red, green, and blue spectral regions are roughly defined as the intervals 600–700, 500–600, and 400–500 nm, respectively. Each colorant absorbs its complementary color and transmits the rest of the visible range of the spectrum. The process is illustrated in Figure 1.23. The individual CMY colorants eliminate RGB spectral regions, respectively. The combination (overlay) of cyan and magenta eliminates both red and green, producing blue; the combination of cyan and yellow eliminates red and blue, producing green; and the combination of magenta and yellow eliminates green and blue, producing red. The combination of the maximum amounts of all three produces black. Intermediate colors are produced by varying the colorant amounts. Dye sublimation printers, color photographic prints, color slides, and halftone color printers are representatives of the subtractive process. Halftone color printing, which is commonly used for lithographic/electro-photographic printing and in most desktop inkjet color printers, may also be viewed as a *hybrid* system,<sup>266</sup> since the colorants combine subtractively, but the perceived color is the average of the differently colored regions over a small area.

Any practical output system is capable of producing only a limited range of colors. The range of producible colors on a device is referred to as its *gamut*. Red, green, and blue primaries are chosen for additive systems,

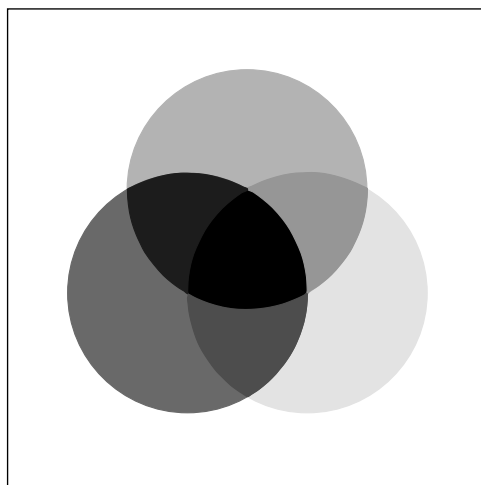


**Figure 1.23** (See color insert) Subtractive combinations of cyan, magenta, and yellow colorants.

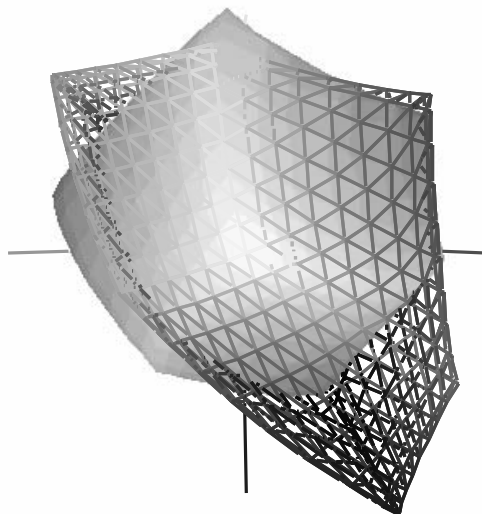
because they offer the largest possible gamut; for the same reason, subtractive systems use cyan (C), magenta (M), and yellow (Y) colorants. In subtractive systems, it is also common to use a fourth black (K) colorant that absorbs light (almost) uniformly over the visible spectrum. This improves the gamut by allowing darker colors to be produced and also allows better reproduction of achromatic (gray) colors and lower cost through the replacement of the more expensive CMY colorants with the K. An excellent description of additive and subtractive color reproduction and additional reasons behind the choice of primaries can be found in Reference 341, Chapter 3.

The gamut of a device can be represented by a three-dimensional solid in any color space such as CIEXYZ/CIELAB. Because two-dimensional representations are more convenient for display, it is common to utilize a CIE xy chromaticity diagram for this purpose. On the CIE xy chromaticity diagram, the gamut of an additive system appears as a convex polygon with the primaries representing the vertices. For the usual case of three red, green, and blue primaries, the gamut is a triangle on the CIE xy chromaticity diagram. Because most subtractive and hybrid systems are nonlinear, their gamuts have irregular shape and are not characterized by such elegant geometric constructs. The two-dimensional representation of the gamut on the CIE xy diagram presents only an incomplete (and difficult-to-interpret) picture, because it does not represent the full three-dimensional data. With the increase in computing speeds and advances in computer graphics, visualization techniques are now used to render three-dimensional views of the gamuts.<sup>154,251</sup> The ability to manipulate these views interactively is extremely useful in understanding the capabilities and limitations of the different color output devices. Three different views comparing the gamut of a CRT monitor and the gamut of a dye-sublimation continuous tone color printer are shown in [Figure 1.24](#), and identical views for an inkjet halftone color printer are shown in [Figure 1.25](#). In both cases, the wire frames represent the gamut of a CRT monitor, and the solids represent the gamuts of the printer.<sup>†</sup> These views demonstrate that the gamuts of these three output devices are fairly different, with several colors that can be produced on one device and not on another. Overall, the gamut of the monitor is the largest, followed by the gamut of the continuous tone printer and then by the inkjet halftone printer, which has a rather small gamut in comparison to the other devices. This mismatch in the gamut between the devices poses significant challenges in cross-media color reproduction and is discussed in some detail in [Chapter 10](#), on gamut mapping.

To discuss colorimetric reproduction on color output devices, it is useful to introduce some terminology. The term *control values* is used to denote signals that drive a device. The operation of the device can be represented as a multidimensional mapping from control values to colors specified in a device-independent color space. This mapping is referred to as the (device)

<sup>†</sup> The gamuts displayed here are computed assuming that no flare is present when viewing the CRT display. Flare, i.e., ambient external room light reflected by the CRT display screen, can cause a significant reduction in the effective gamut of a CRT display.<sup>263</sup>

(a)



(b)

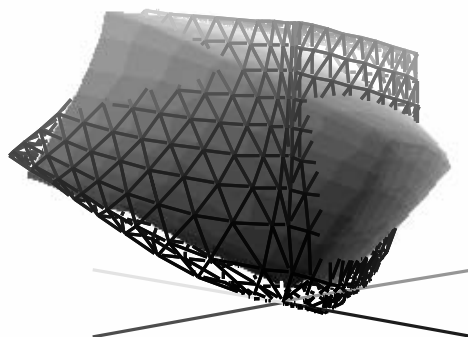


**Figure 1.24** (See color insert) Comparison of a CRT monitor gamut (shown as a wire frame) and a continuous-tone dye-sublimation printer gamut (shown as a solid) in CIELAB space: (a) top view along the  $L^*$  axis, and (b) a perspective projection (continues).

(c)

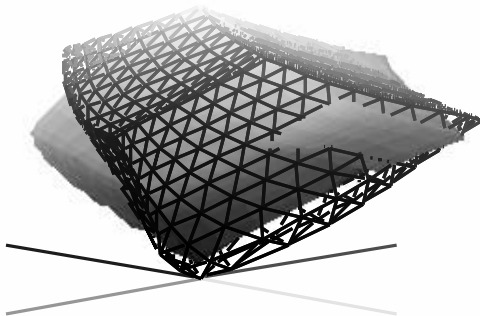


**Figure 1.24 (continued)** (See color insert) Comparison of a CRT monitor gamut (shown as a wire frame) and a continuous-tone dye-sublimation printer gamut (shown as a solid) in CIELAB space: (c) another perspective projection.

*forward device transform.* interactively specified colors in a device-independent color space must be mapped to device control values to obtain colorimetric output, it is necessary to determine the inverse of the multidimensional forward device transform. The determination of the forward device transform and the inverse transform necessary for mapping device independent colors to device control values is sometimes called *device profiling*.

If the device's operation can be accurately represented by a parametric model, the profiling is readily done by determining the model parameters from a few measurements. If no useful model exists, a purely empirical approach is necessary in which the forward device transform is directly measured over a grid of device control values. The inversion may be performed in a closed form if the characterization uses a device model that allows this. If an empirical approach is employed in characterization, or if the model used is noninvertible (often the case with nonlinear models), one has to resort to numerical methods in the inversion step.

#### 1.11.1.1 Cathode ray tubes

The most widely used display device for television and computer monitors is the color CRT. The CRT produces visible light by bombardment of a thin

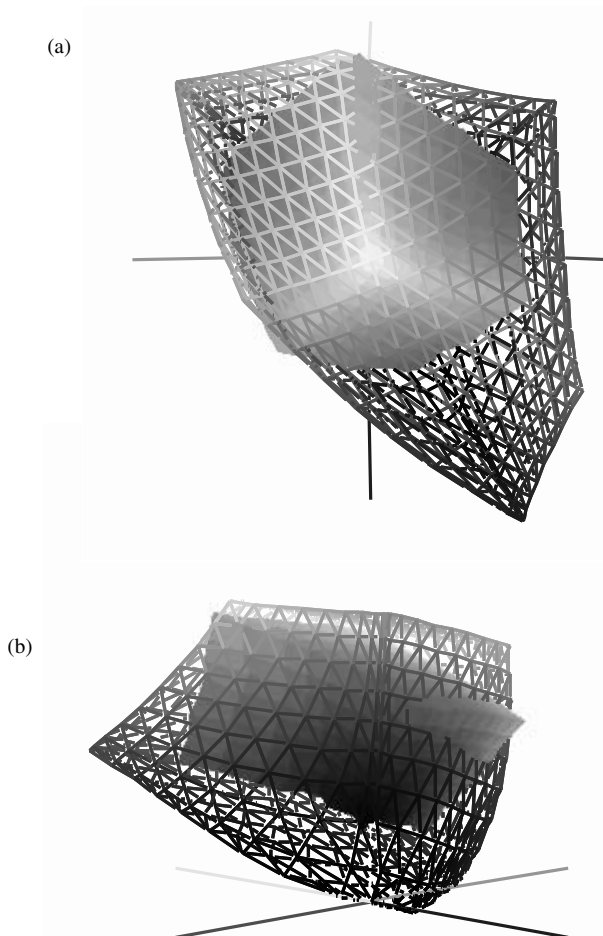


**Figure 1.25** (See color insert) Comparison of a CRT monitor gamut (shown as a wire frame) and an inkjet printer gamut (shown as a solid) in CIELAB space: (a) top view along the  $L^*$  axis and (b) a perspective projection (continues).

layer of phosphor material by an energetic beam of electrons. The electron beam causes the phosphor to fluoresce and emit light whose spectral characteristics are governed by the chemical nature of the phosphor. The most commonly used color CRT tubes are the shadow-mask type, in which a mosaic of red, green, and blue light-emitting phosphors on a screen is illuminated by three independent electron beams. The intensity of light emitted by the phosphors is governed by the velocity and number of electrons. The beam is scanned across the screen by electrostatic or electromagnetic deflection mechanisms. The number of electrons is modulated in synchronism with the scan to obtain spatial variations in the intensity of the light emitted by the three phosphors. At normal viewing distances, the light

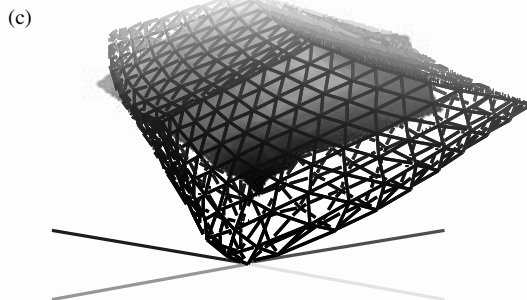


**Figure 1.25 (continued)** (See color insert) Comparison of a CRT monitor gamut (shown as a wire frame) and an inkjet printer gamut (shown as a solid) in CIELAB space: (c) another perspective projection.

from the mosaic is spatially averaged by the eye, and the CRT thus forms an additive color system.

There are several design choices in the manufacture of shadow mask CRTs. Other competing designs offer improved resolution by utilizing a layered phosphor instead of a mosaic. The reader is referred to References 37 and 323 for a description of the different technologies and involved trade-offs. A detailed description of physical principles involved in the operation of these devices is provided in Reference 272 (pp. 79–200).

Color in CRT displays is controlled through the application of different voltages to the red, green, and blue guns. For a complete colorimetric characterization of these devices, the CIE XYZ tristimulus values (or other tristimuli) must be specified as a spatially varying function of the voltages applied to the three guns. A brute-force approach to this problem, using a multidimensional lookup table, is not feasible because of the extremely large number of measurements required. Hence, simplifying assumptions should be made so as to make the problem tractable.

Assumptions of spatial uniformity, gun independence, and phosphor constancy are commonly made to simplify CRT colorimetry.<sup>29</sup> Spatial uniformity implies that the color characterization of the CRT does not vary with position. Gun independence refers to the assumption that the three phosphors and their driving mechanisms do not interact. This implies that the incident intensity at the eye when the guns are operated simultaneously is the sum of the intensities when the guns are operated individually. Phosphor constancy refers to the assumption that the relative spectral power distribution of light emitted by the phosphors does not change with driving voltage (i.e., at all driving voltages, the spectra emitted by a phosphor are scalar multiples of a single spectrum).

With the above three assumptions, the problem of characterizing the CRT reduces to a problem of relating the intensities of the individual red, green, and blue channels to their corresponding gun voltages. The problem

can be further simplified through the use of a parametric model for the operation of the individual guns. Typically, these models are based on the exponential relation between the beam current and grid voltage in vacuum tubes.<sup>43,175</sup> For each gun, the spectrum of emitted light in response to a control voltage,  $v$ , is modeled by an expression of the form<sup>23,222</sup>

$$\left( (1 - \beta) \frac{v}{v_m} + \beta \right)^\gamma h(\lambda) \quad (1.61)$$

where  $v_m$  = maximum value of the voltage  
 $h(\lambda)$  = emitted phosphor spectrum at the maximum voltage  
 $\beta$  = an offset  
 $\gamma$  = the exponential parameter

For appropriate setup of the monitor offset and brightness controls,<sup>23</sup> the offset term  $\beta = 0$  and the relation simplifies to  $(v/v_m)^\gamma h(\lambda)$ . The exponent,  $\gamma$ , is commonly referred to as the *monitor gamma* and is normally around 2.2 for most color monitors. Because the power-law parametric model is only approximate, several modifications of it have been used by researchers.<sup>21,23,64,99</sup> In practice, the addition of the offset term as illustrated in Equation 1.61 offers a significant improvement, because it accounts for deviations from the pure power-law behavior due to differences in the brightness control setting for the CRT.<sup>21,23,263</sup> Using the parametric models, CRT monitors can be readily characterized using a small number of measurements.

The CRT phosphors define a set of additive primaries. If the CMFs corresponding to these primaries are used in color specification, they can be directly used to drive the electron guns if the signals are precorrected for the power-law nonlinearity mentioned above. The transformation from CIE XYZ tristimulus values is given by a linear transformation, corresponding to a transformation from the CIE primaries to the phosphor primaries, followed by a one-dimensional transformation that is determined by the parametric model used to represent the operation of the individual electron guns.<sup>23</sup> Typically, this operation involves exponentiation to the power of  $1/\gamma$  and is known as *gamma correction*. As mentioned in Section 1.5.4, TV signals are normally gamma corrected before transmission. One may note here that quantization of gamma corrected signals results in wider quantization intervals at higher intensities where the sensitivity of the eye is also lower. Therefore, just like speech companding, gamma correction of color tristimuli prior to quantization in a digital system (or transmission in a limited bandwidth system) reduces the perceptibility of errors and contours in comparison to a scheme in which no gamma correction is used.<sup>130(p. 393),163,166,227,232</sup>

For colors that the phosphors are capable of producing, fairly good color reproduction can be obtained on a CRT using the models mentioned above. Using only a few measurements of the individual channel responses, the models provide a very good approximation to the actual display. Typical color errors<sup>23,263</sup> from the offset-gamma model described above are an aver-

age  $\Delta E_{ab}^*$  under 1 and a maximum  $\Delta E_{ab}^*$  around 2, and the corresponding numbers in  $\Delta E_{94}^*$  units are 0.5 and 1.6, respectively. In comparison to other color reproduction devices, these color errors are extremely small. CRTs have therefore been often used in color vision research where a very high color accuracy is required. In practical color imaging applications, the limitations in the CRT color gamut due to the phosphors used cause significant color errors for colors that lie beyond the gamut. This is one of the primary sources of color errors seen in broadcast TV.

The assumptions of gun independence and phosphor constancy have been tested by several researchers and found to hold reasonably well.<sup>21,29,64,65,263</sup> However, in most CRT monitors, for the same driving voltage, the light intensity is brightest at the center and falls off toward the edges. The change in luminance over the screen can be as high as 25%.<sup>196(p.104)</sup> Therefore, the assumption of spatial uniformity does not strictly hold. Because the eye's sensitivity itself is not uniform over the entire field of view, and because the eye adapts well to the smooth variation in intensity across the screen, the spatial nonuniformity of CRTs is not too noticeable.<sup>†</sup> An algorithm for correcting for spatial inhomogeneity is discussed in Reference 63.

#### 1.11.1.2 LCD displays

Liquid crystal display (LCD) flat panels are becoming increasingly common as computer color displays due to their compact size and low power consumption. These displays are now available at increasingly higher spatial resolutions and in larger screen sizes with image quality that meets or exceeds that of typical cathode-ray-tube (CRT) displays.<sup>327</sup> While the market for CRTs continues to grow at present, in the long run, flat panel displays are expected to replace CRTs as the primary computer displays.<sup>38,321</sup>

With widespread use, there is also an increased need for color management for LCD displays, which enables accurate control of color in displayed images. While the color characteristics of CRT displays and methods for their color calibration have been extensively studied and reported, the color characteristics of LCD displays and methods for calibration have come to the forefront only in the last few years and have received only limited attention in published literature. A recent comparison of the color characteristics and gamut considerations for LCD displays vs. CRTs can be found in Reference 263.

The most common LCD displays for computers are back-lit active matrix LCDs (AMLCDs) of the "twisted nematic" type.<sup>162(p. 72)</sup> These are manufactured by deposition and patterning of (active) pixel electronics on a glass substrate. Each pixel element consists of a pair of linear polarizers with liquid crystal (LC) material sandwiched in between. [Figure 1.26](#) graphically illustrates a pixel element. The two linear polarizers are orthogonally oriented;

<sup>†</sup> The imperceptibility of the slow lightness variation over a CRT screen can also be explained in terms of the bandpass nature of the eye's lightness/luminance spatial response, which severely attenuates the perceptual impact of this variation.

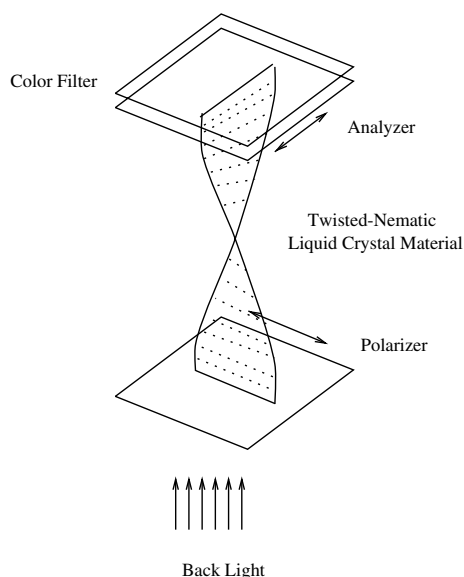


**Figure 1.26** Structure of an LCD display pixel.

light does not pass through the display except for actions of the LCs. The surfaces adjacent to the LC molecules are typically designed so that (in the absence of any electric field) the LC molecules align in a  $90^\circ$  twisted configuration, which rotates the plane of polarization of incident linearly polarized light by a  $90^\circ$  angle.<sup>291(pp. 429–430)</sup> The “input” polarizer on the backside polarizes the light coming from the lamp behind the display. This polarized light encounters the LC molecules, which rotate its plane of polarization by  $90^\circ$ , allowing it to pass through the output polarizer, resulting in an ON pixel. The pixel is turned OFF by the application of an electric field. Due to their *dielectric anisotropy*, the LC molecules tend to align with the electric field and move away from their twisted state. In a strong enough electric field, the molecules are almost completely aligned with the electric field. This causes the pixel to be turned off, as the LC molecules no longer produce the  $90^\circ$  rotation in the plane of polarization that is required for the output polarizer to transmit the light.

Color displays are produced by laying a mosaic of red, green, and blue colored filters on the substrate glass aligned with the pixel array. Quite often, the individual RGB pixels are rectangular and arranged so that three horizontally adjacent rectangular RGB pixels constitute a single square “color pixel” (which is set farther away from other “color pixels” in comparison to the spacing between the individual RGB pixels). The display thus appears to be composed of stripes of rectangular RGB pixels going vertically across the screen. The backlight is typically a fluorescent lamp with three prominent peaks in the red, green, and blue regions of the spectrum.

In most AMLCD color displays, the RGB pixels are driven and controlled independently. The emitted light is combined and averaged in the eye (just as for CRTs). Therefore, the three RGB channels combine through simple addition of light, and one can expect channel independence to hold for these displays. This allows a complete characterization of the display from a per-channel characterization. Furthermore, if the switching mechanism of the LCD pixel cell described above is spectrally nonselective (i.e., when a pixel is driven by applying a voltage the percent change in spectral transmittance is the same across all wavelengths), the channel-chromaticity-constancy assumption of also applies, further simplifying the characterization to a determination of the individual channel intensity or tone response curves. The tone-response curves for the individual channels then correspond to what is commonly referred to as the *electro-optic response*, in LCD terminology. The electro-optic response of an LCD pixel cell (for on-axis viewing) tends to be an S-shaped curve (like sigmoidal functions).<sup>162</sup> This response is quite different from the power-law relation for CRTs and cannot readily be modeled by the parametric power-law relation of Equation 1.61. Additional measurements are therefore typically required for the characterization of LCD displays and alternate parametric relations have also been proposed.<sup>172</sup>

Because LCD displays were first deployed in an environment dominated by CRTs, the color filters for these devices were designed to have the same chromaticities as the CRT primaries. This allows the devices to be directly driven by the same signals as those used for CRTs, with only one-dimensional compensations for the per-channel tone-response curves. Often, displays incorporate built-in, switchable per-channel compensation curves that could change the actual observed tone-response curves for the display. These compensation curves are also typically set up to effectively mimic a power-law relation<sup>84</sup> between the driving signals and the observed luminance.

Using the assumption of channel independence, LCD displays can be color profiled with good accuracy. Average and maximum color errors in  $\Delta E_{ab}^*$  units are around 2.4 and 4.5 with corresponding numbers in  $\Delta E_{94}^*$  units around 1 and 2, respectively.<sup>263</sup> This level of error is well below the acceptable range for most imaging applications, although use in color science experimentation would require greater accuracy.

A common problem with most AMLCD displays is one of limited viewing angle. Due to the birefringent nature of liquid crystal (LC) molecules, polarized light entering an LC material off axis is treated differently from polarized light entering along the optical axis ( $0^\circ$  angle of incidence wrt to the normal). The LC molecules are less effective in performing the proper rotation for polarized light that enters the LC material off axis. The electro-optic transfer function of LCDs therefore tends to be angle dependent.<sup>291(p. 430)</sup> In addition, the LCs are dispersive and operate differently on different wavelengths of light, especially when responding to off-axis incident light. Thus, as one moves off axis (either up or down or from side to side), significant variations in hue and contrast occur with the change in

viewing angle.<sup>91</sup> As one moves far enough off axis, the contrast is diminished to the point that it limits the effective viewing angle.

A change in viewing angle of 30° for an AMCLD display can result in average color shifts of over 5  $\Delta E_{ab}^*$  units.<sup>263</sup> Figure 1.27 shows color shifts observed in an experimental display for a change in viewing angle from 0 to 30° projected on the  $a^*-b^*$  plane. The lines in this plot represent the color shifts with the “\*” representing the color seen along a 30° viewing angle and the other end representing the color seen along a 0° viewing angle. As the viewing angle changes from 0 to 30°, the colors move toward the center of the  $a^*$  and  $b^*$  plane. Thus, the color shifts are such that colors tend to desaturate as the viewing angle increases. The predominant effect seen in off-axis viewing is a reduction in contrast and saturation. The large magnitude of the differences indicates that such an LCD display should be used only for a limited viewing angle if accurate color is desired.

Several innovative techniques have been developed to increase this useful viewing-angle range.<sup>119,15,161,230</sup> Recent commercial displays include alternative technologies such as *in plane switching* to provide an improved usable angle of viewing.<sup>197</sup>

### 1.11.1.3 Contone printers

Continuous *tone* printers are subtractive color devices. They use different concentrations of the cyan, magenta, and yellow colorants to control the

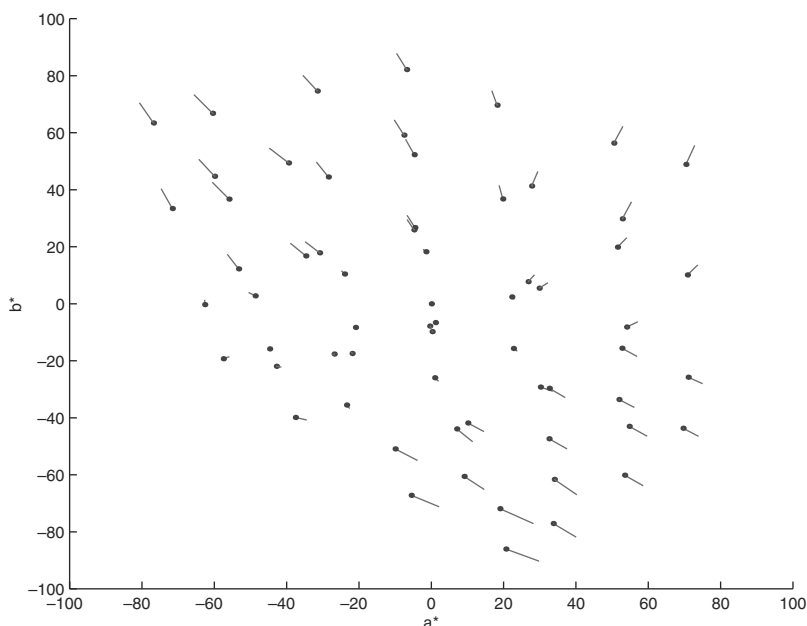


**Figure 1.27** Color shifts in  $a^*$  and  $b^*$  for the LCD display test patches for a change in viewing angle from 0 to 30°.

absorption in the red, green, and blue regions and thereby produce different colors. The term *continuous* comes from the fact that, at each spatial location, these printers vary the colorant concentrations or amounts over a continuous range (in contrast with halftone printers, which are discussed next).

Most subtractive color reproduction systems are inherently nonlinear and cannot be modeled as easily or accurately as additive systems. The subtractive principle is schematically shown in Figure 1.28 for a transmissive system. If the incident light spectrum is  $I(\lambda)$ , the spectrum of the light transmitted through the three layers is given by  $g(\lambda) = I(\lambda)t_1(\lambda)t_2(\lambda)t_3(\lambda)$ , where  $t_i(\lambda)$  is the spectral transmittance of the  $i$ th layer. If the colorants are transparent (i.e., do not scatter incident light), and their absorption coefficients are assumed to be proportional to their concentration (Bouguer–Beer law), it can be shown that<sup>108(Chap. 7)</sup> the *optical density* of the  $i$ th colorant layer, which is defined as the negation of the logarithm (base 10) of its transmittance, is given by

$$d_i(c_i; \lambda) = -\log_{10} t_i(c_i; \lambda) = c_i d_i(\lambda) \quad (1.62)$$

where  $t_i(c_i; \lambda)$  is the transmittance of the  $i$ th colorant layer,  $c_i$  is the concentration of the  $i$ th colorant (which varies between 0 and 1), and  $d_i(\lambda) = d_i(1; \lambda)$  is the density at maximum concentration.

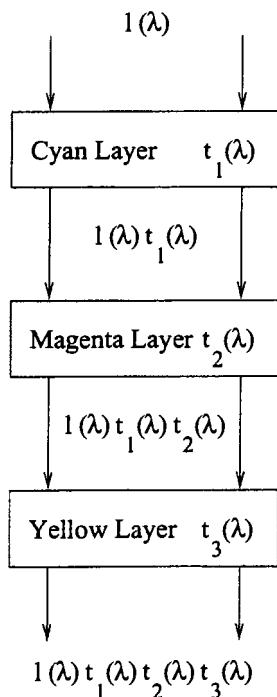


Figure 1.28 Subtractive color reproduction.

Using samples of the spectra involved, the spectrum of transmitted light can be represented as<sup>300</sup>

$$\mathbf{g} = \mathbf{L}[10^{-\mathbf{D}\mathbf{c}}] \quad (1.63)$$

where  $\mathbf{L}$  = a diagonal matrix representing an illuminant spectrum  
 $\mathbf{c}$  = the vector of colorant concentrations  
 $\mathbf{D} = [\mathbf{d}_1\mathbf{d}_2\mathbf{d}_3]$

The remaining boldface symbols represent sample vectors of the corresponding spectral functions, and the exponentiation is computed component-wise.

For prints produced on paper, the transmitted light is reflected by the paper surface and travels once again through the colorant layers. This process is readily incorporated in the model of Equation 1.63 as an additional diagonal matrix that represents the reflectance spectrum of the substrate and a doubling of the densities  $\{d_i(\lambda)\}_{i=1}^3$ . Technically, for diffuse illumination the Kubelka–Munk model (see Reference 108, Chapter 7) should be used with the scattering terms set to zero. The mathematical details are, however, unaffected by this technicality, except for a change in the scaling of densities. For simplicity, the substrate reflectance can be conceptually included in the illuminant matrix  $\mathbf{L}$ , and the same equations can be used for reflective media. While the assumption of transparent layers with no scattering and no interaction between layers is sometimes too simplistic (for instance, for halftone prints and/or pigmented colorants), it is also fairly accurate for a number of useful cases including typical photographic slides and (to a lesser degree) photographic prints, and it has been successfully used in these scenarios.<sup>261,262</sup>

An example of a photographic contone subtractive color reproduction process is illustrated in Figures 1.29 and 1.30. The measurements used in these figures correspond to a Kodak IT8 photographic target.<sup>217</sup> Figure 1.29 illustrates the reflectances for white paper and for the maximum density cyan, magenta, and yellow colorants on the target. Figure 1.30 shows the estimated spectral densities for cyan, magenta, and yellow colorants (at maximum concentrations) as determined from these measurements. In these plots, the spectral regions corresponding to a high density value correspond to spectral wavelengths where the colorants strongly absorb light and regions of 0 density correspond to regions where the colorants transmit all the light through with no absorption. Note that the colorants all have relatively wide absorption bands with significant overlap between the absorption regions of the different colorants.

Even the simplified model of Equation 1.63 cannot be used for a closed-form calibration of a subtractive system. Analytical models therefore often assume that the three dyes have nonoverlapping rectangular-shaped absorption regions that cover the visible region of the spectrum. This is known as the *block-dye* assumption. Using the block dye assumption, colorant concen-

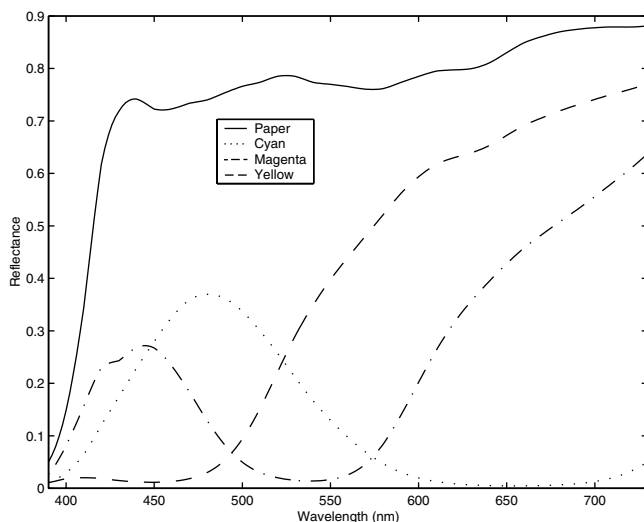


**Figure 1.29** Spectral reflectances for white paper, cyan, magenta, and yellow from the Kodak IT8 target.

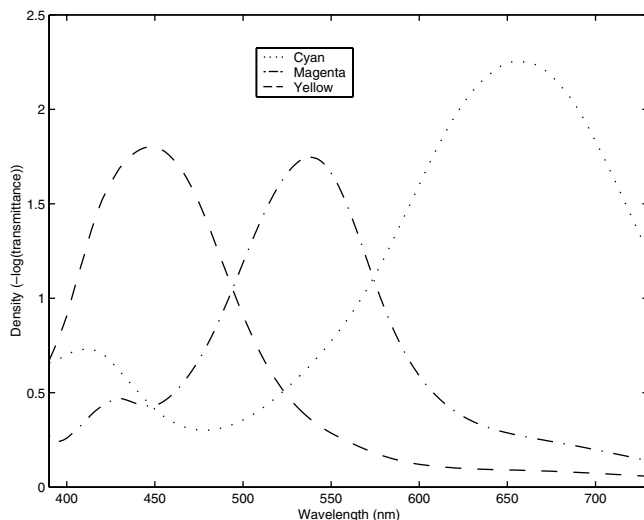


**Figure 1.30** Spectral densities for cyan, magenta, and yellow from the Kodak IT8 target.

trations required to produce a given CIE tristimulus can be determined in closed form.<sup>300</sup> Under the assumption of nonoverlapping spectral absorption bands for the colorants, the colorants' interactions are eliminated, and the system can be transformed into an equivalent additive system. Because real

colorants deviate from this ideal desired behavior, the absorptions for a colorant in the visible band but outside its primary absorption band are often referred to as *unwanted absorptions*. Because unwanted absorptions are a hard reality of actual colorants, the utility of the block-dye assumption is restricted to educational illustrations of the subtractive process.

Most contone printers available currently use thermal dye diffusion technology. The coloring dyes in such a system are transferred from a dispensing film into the reproduction medium by means of heat-induced diffusion. Often, a fourth black dye is used in addition to the cyan, magenta, and yellow dyes to achieve greater contrast and better reproduction of achromatic (gray) colors. A review of the physical processes involved in a thermal dye diffusion printer can be found in References 113 and 114. Note that conventional photography uses subtractive color reproduction; thus, continuous tone printing is possible using photochemical methods that mimic photography. For an overview of this method and for more details of the dye diffusion printing process, the reader is referred to References 72 and 148.

In practice, the models for subtractive printers described above are applicable to only a small set of printers. For several applications, the colorants are not completely transparent; therefore, the Kubelka–Munk theory,<sup>16,108,168</sup> which accounts for scattering of light by the colorants, is more appropriate than the Bouguer–Beer law. Because accurate determination of the model parameters is fairly complicated, and there are interactions between the media and the colorant layers that are not accounted for even in the Kubelka–Munk theory, often purely empirical techniques are used to profile color printers. The model in Equation 1.63, though somewhat restrictive, has been successfully used to model and profile a thermal dye diffusion printer<sup>17</sup> and has also proven very useful in analytical simulations of printers and in making design choices.<sup>139</sup>

Typical empirical approaches for color printer calibration begin by measuring the color of test prints corresponding to a uniform grid of control values. This provides a sampling of the mapping from device control values to a device-independent color space. A variety of interpolation-based techniques are then utilized to determine the required inverse transformation, typically in the form of a lookup table over a uniform grid in a color space.<sup>128,218,240</sup> Interesting alternate approaches have also utilized neural networks<sup>160</sup> and an iterated scheme that concentrates measurements in regions of greatest impact.<sup>40</sup>

#### 1.11.1.4 Halftone printers

Contone printers require reliable and accurate spatial control of colorant concentrations, which is difficult to achieve and control accurately. As a result, contone printers are rather expensive. Most desktop printers are therefore based on the simpler technique of halftoning, which has long been the color reproduction method of choice in commercial lithographic printing. Like CRT displays, halftoning exploits the spatial lowpass characteristics of the human

visual system. Color halftone images are produced by placing a large number of small, differently colored dots on paper. Due to the lowpass nature of the eye's spatial response, the effective spectrum seen by the eye is the average of the spectra over a small angular subtense. Different colors are produced by varying the relative areas of the differently colored dots. In contrast with contone printing, the concentration of a colorant within a dot is not varied, so halftone printers are considerably easier and less expensive to manufacture. Special processing of images is necessary to determine the dot patterns for the different colors prior to printing on a halftone printer. This processing is the subject of [Chapter 6](#), "Digital Color Halftones," and only the halftone printing mechanism will be briefly discussed in this chapter.

For reasons similar to those for contone processes, most three-ink halftone systems use cyan, magenta, and yellow colorants for printing the dots (see Reference 341, Chapter 3). Just as in contone printers, a fourth black colorant is often introduced to conserve the more expensive colorants, reduce ink usage, and produce denser blacks (see Reference 341, p. 282).

A model for halftone printers was first suggested by Neugebauer,<sup>213</sup> who observed that the overlap of CMY colorant halftone dots produces eight primary colors: the paper (no-ink), each of the three colorants (cyan, magenta and yellow), combinations of two colorants (red, green, blue), and combinations of all three (black). The observation generalizes from the three-colorant case to the case of four or any arbitrary number of colorants. The colorants combine subtractively over the regions in which they overlap and, in general, produce up to  $2^K$  different colors with  $K$  colorants. These distinct colors are called the *Neugebauer primaries*. Neugebauer proposed that the tristimulus values of the print can be expressed as the weighted average of the tristimuli of the Neugebauer primaries with the weights equal to their fractional area coverages. Due to the penetration and scattering of light in paper, known as the *Yule–Nielsen effect*<sup>†</sup> (or, alternatively, as the optical dot gain),<sup>51,340</sup> the simple Neugebauer model does not perform well in practice. As a result, several empirical modifications have been suggested for the model.<sup>255</sup> Recently, considerable success has been demonstrated in using a spectral version of the Neugebauer model with empirical corrections for the Yule–Nielsen effect,<sup>9,11,252,308,336</sup> in which the reflectance of a CMYK halftone print is modeled as

$$r(\lambda; \mathbf{w}) = \left( \sum_{i=1}^{16} w_i r_i^{1/n}(\lambda) \right)^n \quad (1.64)$$

where  $r_i(\lambda)$  and  $w_i$  are, respectively, the spectral reflectance and the fractional area of the  $i$ th primary (there are 16 primaries for a 4-colorant [CMYK] system); and  $n$  is the empirical Yule–Nielsen correction factor. As a further

<sup>†</sup> Note that in the printing of the original paper,<sup>340</sup> Nielsen's name was misspelled as "Nielsen." Both spellings have therefore been used in existing literature.

simplification, the reflectances of the Neugebauer primaries composed of overprints of more than one colorant may be expressed in terms of the transmittances of the different colorant layers as was done in the subtractive model of Equation 1.63. However, because this assumption of additivity of densities reduces accuracy, it is usually not invoked; instead, the primary reflectances  $\{r_i(\lambda)\}_{i=1}^{16}$  are directly measured from prints.

Normally, the fractional areas  $w_i$  are themselves related to the fractional areas  $c, m, y, k$  covered by the cyan, magenta, yellow, and black colorants, respectively. The relations determining the  $w_i$  from  $c, m, y, k$  depend on the method used to generate the halftone images. Equations for these relations were first derived by Demichel<sup>69</sup> by assuming independent random coverage for the colorants. Rotated halftone screens<sup>341</sup> that approximate this random independent coverage assumption are often used for printing, because they are robust to commonly occurring registration (alignment) errors between the different colorants images (*separations*). Alternative halftone screens have different characteristics and consequently different relationships between the fractional colorant coverages and the primary areas.<sup>8,243</sup>

Prior to the work of Neugebauer, halftone color reproduction was often confused with subtractive reproduction, and the Neugebauer model therefore offered very significant improvements.<sup>242</sup> However, the actual halftone process is considerably more complicated. As a result, several empirical modifications have been suggested for the model, such as the *Yule–Nielsen* parameter  $n$  and the spectral version of the model with the *Yule–Nielsen* modification. While these models have had some success, complete and accurate physical models for the color halftone printing process and the *Yule–Nielsen* effect continue to be elusive. Promising new directions are therefore a subject of continuing research.<sup>74,75,111</sup> Additional details on the research in this area can be found in [Chapter 3](#).

One obstacle in the direct use of Neugebauer models in characterizing a halftone printer is that the relationship between the control values and the printing area of the different colorants is usually not known *a priori*. Hence, an empirical component is normally required even for characterization schemes using a model. This empirical component is in the form of one-dimensional pretransformations of device control values, which often serve the additional purpose of increasing characterization accuracy along the *achromatic* or *neutral gray* axis, where the eye has significantly greater sensitivity.<sup>286</sup> Purely empirical schemes similar to those used for characterizing contone printers can also be used for halftone printers. The models mentioned above are nonlinear and nonseparable in the device control values and cannot be inverted analytically. Hence, for both model-based and empirical schemes, the inversion of the characterization needs to be performed numerically. In either case, the final mapping from required color tristimuli to device control values is realized as a multidimensional lookup table. The models, however, have an advantage over a purely empirical approach in that they offer a significant reduction in the number of measurements required. An interesting generalization of the Neugebauer model is dis-

cussed in References 8, 9, and 252, where the model is interpreted as interpolating between a number of end points specified by the primaries. Accuracy is improved by using local interpolation over smaller cells, which in turn implies more measurements. The generalization, known as the cellular Neugebauer model, thus offers a graceful trade-off between accuracy and the number of measurements required. Due to the large number of effects ignored by most models, they can offer only limited accuracy. Therefore, in graphics arts and printing industries, where there is greater emphasis on quality, measurement-intensive empirical schemes are often preferred.<sup>248</sup> A comparison of some model-based and measurement-based empirical schemes for electronic imaging applications can be found in Reference 157.

Halftone printers have been manufactured using very different technologies for printing dots on paper.<sup>148(p. 4–8)</sup> The most promising current technologies utilize inkjet, thermal transfer, and electrophotography to produce the halftone dots. Even a brief mention of the principles and technology of these devices is beyond the scope of this chapter, and the interested reader is referred to References 72, 148, 241, and 254 for details.

#### *1.11.1.5 Recent advances in color displays and printing*

The increasing use of portable computers and hand-held mobile devices has fostered considerable research in a variety of display technologies. While the LCD display is dominant in this class, a number of alternative technologies are actively being developed. These include plasma displays, electroluminescent displays, and displays based on organic light-emitting diodes (OLEDs). A description of the historical development, physical principles, and relative merits and demerits of some of these technologies can be found in References 37, 155, 199, and 272. Most of them are additive color systems similar to a CRT and use a mosaic of red, green, and blue “dots” to produce color, although some LCD devices are based on the subtractive principle<sup>155,321</sup> or on spectrally selective reflection.<sup>107</sup>

A majority of the color display devices mentioned so far rely on the spatial lowpass characteristics of the human eye to produce different colors using a mosaic of differently colored regions. An alternative system for producing color, known as *field sequential color* (FSC), is based on the temporal lowpass nature of the eye’s response. In an FSC system, red, green, and blue image frames are projected in rapid succession onto the viewing screen, and the temporal averaging in the observer’s eye produces the illusion of a single colored image. An FSC system was originally selected by the FCC for color TV transmission but, before it could be commercialized, it was replaced by the monochrome-compatible NTSC system in use today. The primary drawback in such a system was the high frame rate and bandwidth requirements.<sup>257(p. 218–219)</sup> Recently, there has been a resurgence of interest in FSC systems. An interesting example of a recent FSC system is Texas Instruments’ digital micromirror device (DMD)<sup>339</sup> that utilizes an array of deformable micromirrors. In the deformed state, each micromirror deflects light from an

illuminating lamp onto a single picture element (pixel) on the viewing screen. The duty cycles of the deformation of different mirrors are modulated to produce image intensity variations on the screen. Color is produced by placing a color filter wheel between the lamp and the micromirror device and synchronizing the red, green, and blue frames with the color wheel. Alternative configurations using three separate DMD devices or two devices in a five-primary projection system have also been reported.<sup>338</sup> From a color imaging standpoint, DMD displays are rather interesting, as they are almost linear and allow considerable flexibility in the choice of the primaries through the use of different color filters in the filter wheel.

There have also been significant new advances in color printing in recent years. Color halftone printers have continually improved in resolution, speed, and cost. Some devices now incorporate limited contone capability through a coarse variation in colorant concentrations/drop volumes. The gamut of printers has also been enlarged by using improved colorants or, more recently, by using more than three/four inks, which is referred to as high-fidelity (hi-fi) printing.<sup>26,104,223</sup> It is also common in inkjet printers to use dilute cyan and magenta colorants in addition to the regular cyan, magenta, and yellow colorants. This is particularly beneficial, because the lower density colorants produce more spatially uniform and less visible halftone structure in light regions such as Caucasian skin tones and pale blue skies, which are quite critical to the good reproduction of photographs.

The color profiling for several of the devices mentioned above is performed using model-based or empirical techniques already in use for existing devices. Other devices are still in their infancy, and little information if any is available on their color characterization and performance. As they find increased acceptance, there will no doubt be a greater demand for more accurate color characterization and for reasonable models of these devices. This, therefore, will be an active area of color imaging research in the future.

### *1.11.2 Image characteristics*

The spectral radiance or reflectance of an image carries the most complete color information. Most color spectra are fairly smooth, and a 10-nm wavelength sampling is normally sufficient for representing these spectra.<sup>303</sup> The spectra of fluorescent gas-discharge lamps have sharp spectral peaks and therefore require a higher sampling rate or alternative model-based approaches.<sup>189,264,302</sup> A 10-nm sampling of spectra in the 400- to 700-nm region provides  $N = 31$  samples<sup>†</sup> for each spectrum. Thus, color spectra lie in a 31-dimensional vector space. Color spectra are usually smooth, so they do not really exhibit 31 degrees of freedom. A principal-component analysis of common reflectance spectra reveals that a large fraction of the energy in

<sup>†</sup> The 400- to 700-nm interval is chosen for illustration and the exact interval is not central to the argument here. Typical color measurement instruments report spectra over this or a larger wavelength region.

these spectra is contained in a few principal components.<sup>149</sup> Several researchers have investigated this problem and have estimated that color reflectance spectra can be represented by using anywhere between 4 and 12 principal components, depending on the accuracy desired.<sup>54,194,314,319</sup>

The spectral reflectance characteristics discussed in the last paragraph covered almost the entire range of naturally occurring reflective objects. Because most color reproduction systems exploit trichromacy and have only three additive/subtractive primaries, their spectra often have even fewer degrees of freedom. Thus, the spectra from a CRT display lie in a three-dimensional space defined by the spectra of the phosphor primaries. The same is true of all additive systems that utilize three primaries. For subtractive and hybrid systems, although there often are only three primaries, they are not confined to a three-dimensional linear space, due to the nonlinearities in the subtractive process. Despite the nonlinearities, the presence of only three degrees of freedom in the color production process may often be exploited in imaging applications.<sup>270</sup>

### 1.11.3 Computer-generated imager

The previous section focused on natural image scenes and *hard-copy* prints of these images. In the area of multimedia applications, the computer is an alternative, rapidly growing source of color images. Computer-generated scenes and animations are now common in video games and movies.<sup>53,247</sup> These applications strive for realistic images and therefore have to model the interactions between lights and objects in the scene, using both geometric optics and colorimetry to produce proper color, shading, and texture for the surfaces involved.<sup>92(Chap. 14),93</sup> For these applications to be successful in multimedia applications, it is necessary that the color information be recorded in a form that permits accurate rendering of the image on multiple output devices. This is particularly relevant for recording on movie film, which has entirely different characteristics from the CRT.<sup>171</sup>

Color can also be used to effectively encode information for presentation and visualization. Bar graphs and pie charts used in presentations are simple examples of the use of colors to distinguish different regions. The capabilities of color encoding are also exploited more fully in applications that use color to meaningfully represent quantitative differences in the data. Examples of such applications include visualization of multispectral/stereo data using *pseudo-color* images, and the use of color to portray three-dimensional relationships in multidimensional databases. The proper use of color in these and other multimedia applications greatly enhances their effectiveness.<sup>92(p. 601),306</sup> Usually, the color images are designed for viewing on a CRT monitor and tend to exploit the full capabilities of the monitor. When printing these images, it is necessary to preserve their information and effectiveness. This is often a significant challenge, because the printer gamuts are significantly smaller than monitor gamuts. Methods of compensating for this type of gamut mismatch are discussed in [Chapter 10](#).

### 1.11.4 Color recording systems

Systems for recording color information include both color measurement instrumentation useful for large uniform regions and color image capture devices designed to capture spatially varying color information. Color measurement instruments are used primarily for determining the color characteristics of objects and of imaging input and output devices. They include both spectral measuring devices such as spectroradiometers/spectrophotometers and filter-based instruments that directly measure colorimetry. The correct color measurement instrument for a particular application depends on several factors including the cost of the instrument, the accuracy desired, the variety of viewing conditions for which a calibration is desired, and the number and type of devices that must be calibrated. Color image capture devices include conventional and digital color cameras, which record color information from a scene of real world objects, and color scanners, which record color information from a prior reproduction such as a printed color image, slide, or negative.

#### 1.11.4.1 Spectroradiometers and spectrophotometers

Sampling of the spectral distribution provides the most direct and complete technique for recording color information. A *spectroradiometer* is a device that measures the power of optical radiation as a function of wavelength. [Figure 1.31](#) shows the schematic cross section of a spectroradiometer. The light is collimated by the lens onto a dispersive element, which decomposes it into its spectrum. The spectrum is then sampled and recorded using single or multiple detectors. Typically, a diffraction grating is used as the dispersive element, because it provides an almost linear relationship between wavelength and displacement in the detector plane, as opposed to an optical prism, for which the correspondence is highly nonlinear. The linear relationship considerably simplifies calibration procedures.

Modern spectroradiometers use charge-coupled-device (CCD) arrays as the detectors because of their linear characteristics. A sampling of the spec-

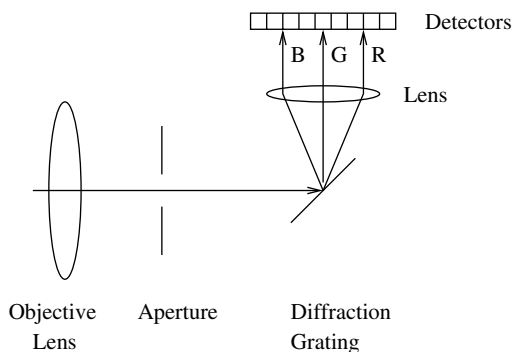


**Figure 1.31** Schematic cross section of a typical spectroradiometer.

trum is achieved automatically through the placement of physically distinct detectors in the measurement plane. Because the separation between the detectors need not correspond directly to a convenient wavelength spacing, the detector outputs are usually interpolated to obtain the final spectral samples. Even though the CCDs are almost linear in their response at a given wavelength, their spectral sensitivity is not uniform. Therefore, a gain compensation procedure is usually necessary to obtain calibrated output from the device.<sup>204(p. 338)</sup>

The range and the sampling interval of spectroradiometers vary according to their intended application. Spectroradiometers used for color typically report measurements over the range of 380–780 nm and are capable of a spectral resolution ranging from 1 to 10 nm. For most color work (including that using fluorescent lamps as sources), 2-nm sampling is sufficient.<sup>303</sup> However, because some light sources have monochromatic emission lines in their spectra, a deconvolution of the spectroradiometer measurements may sometimes be necessary to obtain greater accuracy.<sup>264</sup>

Spectroradiometers can be used to measure both self-luminous and reflective objects. For the measurement of reflectance spectra, a spectrally smooth (preferably, white) light source and a spectrally referenced sample (preferably, white) are required. Comparison measurements between the known sample and the sample under question are made under identical conditions, allowing the determination of the unknown sample's reflectance spectrum, from which the color descriptor under any viewing illuminant can be obtained. Normally, the exact spectrum of the illuminant used with the spectroradiometer in the reflectance measurement is immaterial, provided it has sufficient power over the spectral range of interest. However, for the measurement of fluorescent materials, the power in the ultraviolet regions is also important, and the illuminant must closely approximate the desired standard illuminant under which colors are to be computed.<sup>347</sup>

While spectroradiometers can measure both luminous and nonluminous objects, they are expensive, larger, and less portable than the other instruments, and they generally are more difficult to operate due to their multiple operating modes. In particular, it can be difficult to set up a spectroradiometer for measuring reflectance samples under controlled conditions. There are automated systems for performing measurements on multiple samples using an x-y stage and a single light source.

An interesting application of spectroradiometry that extends beyond the visible spectrum is in multispectral scanners carried by remote sensing satellites. These scanners disperse radiation into different spectral bands in much the same way as the spectroradiometers discussed above. Early cameras in these satellites used 5 to 12 spectral bands extending from the visible into the infrared region.<sup>36,287</sup> The Airborne Visible Infrared Imaging Spectrometer (AVIRIS),<sup>106</sup> which samples the range of 400 to 2500 nm at 10-nm resolution, is an example of more recent scanners that use a larger number of bands.

A *spectrophotometer* is a device for measuring the spectral reflectance of an object. Unlike a spectroradiometer, a spectrophotometer does not measure

self-luminous objects. Therefore, spectrophotometers are useful for the calibration of printers and scanners but not of CRT displays. Spectrophotometers have their own internal light source that illuminates the sample under measurement. There are many geometries for the sensor/illuminant combination, and each may be optimal for different applications. For color imaging applications, the sensor and illuminant in the device are often set up for a  $45^\circ/0^\circ$  condition as shown in [Figure 1.32](#). Some devices illuminate the sample diffusely and measure off axis, allowing an option to include or exclude the specular component from the sample. Some devices contain an optical grating, others contain a filter wheel, and yet others use spectrally different light sources to illuminate the sample.

Spectrophotometers measure reflectance as the ratio of two (uncalibrated) spectroradiometric measurements<sup>266, 347</sup> using the same principle that was described in the last section. This is shown schematically in [Figure 1.33](#). The light source is contained within the spectrophotometer and is used to illuminate both a standard sample with known reflectance,  $r_s(\lambda)$ , and the test object whose reflectance,  $r_o(\lambda)$ , is to be measured. If  $l(\lambda)$  denotes the SPD of the light source, and the device makes  $K$  spectral measurements at  $\Delta\lambda$  wavelength intervals in the region  $[\lambda_0, \lambda_0 + (K - 1)\Delta\lambda]$ , the reference measurements can be represented as

$$m_s(k) = d_k l(\lambda_0 + k\Delta\lambda) r_s(\lambda_0 + k\Delta\lambda), \quad 0 \leq k \leq (K - 1)$$

where  $d_k$  denotes the detector sensitivity at  $(\lambda_0 + k\Delta\lambda)$ . Similarly, the object measurements are given by

$$m_o(k) = d_k l(\lambda_0 + k\Delta\lambda) r_o(\lambda_0 + k\Delta\lambda), \quad 0 \leq k \leq (K - 1)$$

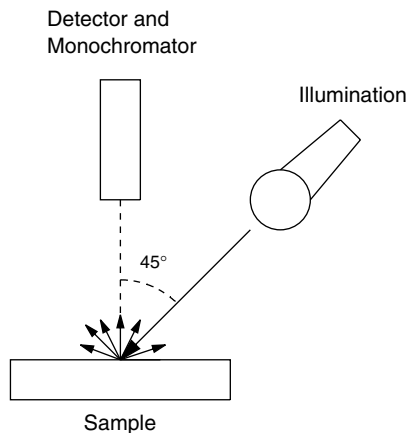


**Figure 1.32**  $45\text{--}0^\circ$  measurement geometry.

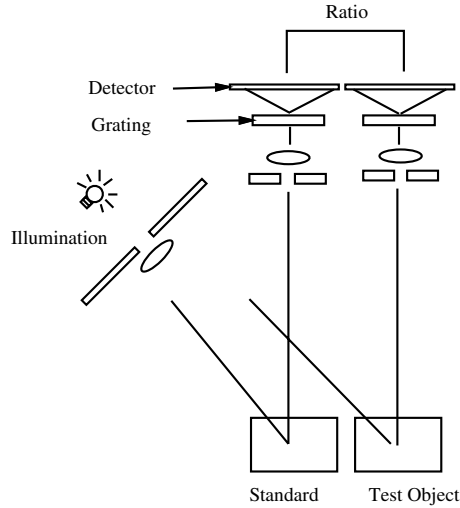


**Figure 1.33** Spectrophotometer measurement.

The object reflectance samples can therefore be determined as

$$r_o(\lambda_0 + k\Delta\lambda) = \frac{m_o(k)}{m_s(k)} r_s(\lambda_0 + k\Delta\lambda), \quad 0 \leq k \leq (K - 1)$$

Mathematically, it can be seen that the detector sensitivity,  $d_k$ , and the illuminant,  $l(\lambda)$ , cancel out and have no impact on the measurement. However, to obtain good performance in the presence of quantization and measurement noise and errors due to the limited dynamic range of the detectors, it is desirable that the product of these quantities be nearly constant as a function of wavelength. For similar reasons, it is desirable that the reflectance of the standard sample be close to unity at all wavelengths. To avoid unnecessary duplication of the optics and sensors, the measurements of the reference standard and the object are usually performed sequentially instead of using the parallel scheme shown in [Figure 1.33](#). In addition, for added convenience and to save time, typical measurement devices make one measurement of the standard that is stored and used for a number of successive object measurements.

Because most real-world reflectances are relatively smooth functions of wavelength<sup>303</sup> and have low dimensionality as discussed in Section 1.11.2, most spectrophotometers work with much larger sampling intervals than spectroradiometers, typically reporting reflectance at 5-, 10-, or 20-nm intervals. The built-in illumination in these devices is usually a filtered incandescent or xenon arc lamp whose spectrum is smooth (unlike fluorescent lamps) and therefore does not unduly amplify the measurement noise and quantization errors. Spectrophotometers used in color work usually sample the spectrum in the 380- to 780-nm range, although the lower-wavelength end

of the spectrum may be truncated or less accurate in some devices because of the lower energy in incandescent lamps at the ultraviolet end. Owing to the lower resolution requirement, and because of the less stringent calibration required (due to the normalization of illuminant and detector sensitivities), spectrophotometers are considerably less expensive than spectroradiometers and are also more stable over time.

The design of spectroradiometers and spectrophotometers should account for a large number of factors excluded from the simplistic description given above. Both devices suffer from systematic and nonsystematic deviations from the ideal behavior described above and must be accurately calibrated to known radiant and reflectance standards prior to use. In particular, stray light, detector nonlinearity, effects of polarization, variations in illumination and measurement geometry, and unaccounted fluorescence and thermochromism of samples are sources of systematic errors. Detector noise and quantum fluctuations in photon flux are examples of random errors encountered in measurements. The reader is referred to References 108 ([Chapter 8](#)) and 109 ([Chapter 9](#)) for a thorough, though somewhat dated, account of the systematic errors in these devices and their calibration procedures. A more current, though brief, overview is also provided in References 16 (pp. 74–86) and 135 ([Chapter 5](#)). Detector noise models for older instruments that used thermal detectors and vacuum tubes are described in Reference 35, and a more recent account of noise models for semiconductor detectors of radiation is provided in References 28, 70, 144, and 204. Some methods for accounting and correcting some of the systematic errors in spectrophotometers are discussed in Reference 24. The propagation of spectrophotometric errors in colorimetry has also been analyzed in Reference 83.

Color recording devices that attempt to sample spectral information suffer from a number of obvious drawbacks. First, because the total energy in the spectrum is split into a number of spectral samples, a sizable measurement aperture and/or long integration time will be required for reliable measurements of the spectra. In addition, the required optical components make some of the spectral devices rather expensive and therefore inappropriate for desktop use. Finally, measurement devices that exploit trichromacy are less accurate but can offer acceptable color performance and significant speed advantage at a fraction of the cost. Spectroradiometers and spectrophotometers are therefore used primarily for color calibration, where the larger aperture and longer measurement times are not prohibitive (in contrast with devices for recording complete spatially varying images).

#### 1.11.4.2 *Colorimeters and photometers*

As suggested by its name, a *colorimeter* measures color tristimuli and reports these as color values in CIE XYZ, CIELAB, or related color spaces. Some colorimeters have an internal light source for the measurement of color of reflective objects, whereas others measure only self-luminous or externally

illuminated objects. For some devices, tristimulus values for the sample under a few different illuminants are available. Most colorimeters are small hand-held devices with no moving parts and a single light detector. They achieve their spectral “separation” by way of color filters placed before the detector or with spectrally different light sources used for illumination.<sup>†</sup> To accurately report CIE colorimetry, colorimeters must record information from which the colorimetry can be derived. The sensitivities of these devices are therefore often designed to match a set of color-matching functions. It is common for filter-based colorimeters to use four independent filters, one each to approximate the  $\bar{y}(\lambda)$ ,  $\bar{z}(\lambda)$ , and two to approximate the two individual humps of the  $\bar{x}(\lambda)$  color matching functions.<sup>16(p. 86)</sup>

Colorimeters are less expensive than spectrophotometers and spectroradiometers, but they do not provide the detailed spectral information that allows the calibration of a printer for an arbitrary viewing illuminant. Those that measure self-luminous sources are used in the calibration of CRTs. *Photometers* are single-channel devices that provide a measurement of the luminance of a self-luminous or externally illuminated object. They are inexpensive and find use primarily in the calibration of CRTs when the chromaticity of the CRT phosphors is known.

#### 1.11.4.3 Photographic film-based recording schemes

Photographic film is not a digital recording device; however, a brief discussion of this medium is worthwhile, as it often forms the primary input to many digital color imaging systems. Film used for color photography records the color information in three spectral bands corresponding roughly to the red, green, and blue regions of the spectrum.

The image to be recorded is focused by a lens onto the film. The film contains three emulsion layers with silver halide crystals that act as the light sensors and sensitizing dyes that make the crystals in the three layers respond to different spectral regions. Typically, the top layer is blue sensitive; this is followed by a yellow filter and green and red sensitive layers, respectively. The yellow filter keeps blue light from getting to the lower layers that are also sensitive to blue light. Light in each of the three spectral bands initiates the formation of development centers in the corresponding film layer. When the film is chemically processed, the silver halide crystals at the development centers are converted into grains of silver, and unexposed crystals are removed. The number of grains of silver in a given layer at a particular location is determined by the incident light energy in the image in the corresponding spectral band at that location. Thus, the spatial distribution of silver grains in the three layers forms a record of the spatial distribution of blue, green, and red energy in the image.

The relation between the density of silver grains and the incident light spectrum is highly nonlinear. In addition, the formation of silver grains is not deterministic, and the randomness in grain formation contributes to

<sup>†</sup> The latter configuration is disadvantaged in the measurement of fluorescent samples.<sup>16(p. 75)</sup>

noise in the recording process, known as *film grain noise*. Film grain noise is often modeled as a Poisson or Gaussian random process<sup>143(pp. 619–622),258,259,294</sup> and constitutes multiplicative noise in the recorded image intensity.<sup>234(p. 342)</sup>

An image record in the form of three layers of silver grains is obviously of limited use. Therefore, further chemical processing of the film is necessary. For the purposes of this discussion, it suffices to note that this processing replaces the silver grains in the red, green, and blue layers with cyan, magenta, and yellow dyes in accordance with the principles of *subtractive color reproduction*, which were be discussed in Section 1.11.1.3. A more complete description for color photography can be found in Reference 130, and simplified mathematical models for the process are described in Reference 234 (pp. 335–339).

As an aside, one may note that, prior to the invention of spectrophotometers and spectroradiometers, two techniques were developed to record the spectral information of entire images on (monochromatic) film. In the micro-dispersion method of color photography, the light from each small region of image was split into its spectral components using dispersive elements, and the corresponding spectra (of rather small spatial extent) were recorded on film. The second method, known as Lippman photography, recorded the color information in the form of a standing wave pattern by using a mercury coating on the rear of the film as a mirror. Both methods required extremely fine-grain film to achieve the high resolution required and long exposure times to compensate for the low energy at each spectral wavelength. The reader is referred to Reference 130 for a slightly more detailed account of these techniques.

#### 1.11.4.4 *Digital color cameras and scanners*

Digital color cameras and color scanners are color recording devices that operate on similar principles, though their intended use is quite different and often poses different challenges. Both of these devices record color information by transmitting the image through a number of color filters having different spectral transmittances and sampling the resulting “colored” images using electronic sensors.

Digital color cameras are designed to capture color images of real-world objects in much the same way as conventional cameras, with the difference that the images are recorded electronically instead of using film. Because the scenes may involve moving objects, they typically have two-dimensional CCD arrays that capture the image in a single electronically controlled exposure. Different schemes may be used to achieve the spatial sampling and color filtering operations concurrently. One arrangement uses three CCD arrays with red, green, and blue color filters, respectively. In such an arrangement, precise mechanical and optical alignment is necessary to maintain correspondence between the images from the different channels. Often, the green channel is offset by one-half a pixel in the horizontal direction to increase bandwidth beyond that achievable by individual CCDs.<sup>118</sup> For econ-

omy and to avoid the problems of registering multiple images, another common arrangement uses a color filter mosaic that is overlaid on the CCD array during the semiconductor processing steps. Because the green region of the spectrum is perceptually more significant, such mosaics are laid out so as to have green, red, and blue recording pixels in the ratio 2:1:1 or 3:1:1.<sup>226</sup> Image restoration techniques are then used to reconstruct the full images for each of the channels.<sup>30,32,59,293,301</sup> Recently, a novel camera design has been developed that avoids the problems of both color filter arrays and of multiple images by using a single sensor and effectively layering the red, green, and blue filters by using the spectral selectivity of light penetration in silicon.<sup>94</sup> One of the aspects of color capture that is more challenging for color cameras than for scanners and colorimeters is the lack of control over scene illumination. While the eye adapts to the scene as described in Section 1.9.1, producing a visual appearance that is largely independent of the scene illumination, cameras do not incorporate these adapting mechanisms and consequently produce images with severe color casts/shifts. Algorithms for estimating the relevant scene illumination characteristics and correction of the casts are therefore necessary. Details on the image processing for digital color cameras can be found in [Chapter 12](#).

Scanners are usually designed for scanning images reproduced on paper or transparencies and include their own sources of illumination. Because the objects are stationary, these devices do not need to capture the entire image in a single exposure. Typical drum or flatbed moving stage scanners use a single sensor per channel which is scanned across the image to provide spatial sampling. The single sensor makes the characterization of the device easier and more precise and also allows the use of more expensive and accurate sensors. For desktop scanners, speed is of greater importance, and they usually employ an array of three linear CCD sensors with red, green, and blue color filters. The linear sensors extend across one dimension of the scanned image. This allows three filtered channels of the image along a line to be acquired simultaneously. To sample the entire image, the linear array is moved optically or mechanically across the other dimension of the image. In another variation of these devices, three different lamps are used in conjunction with a single linear CCD array to obtain a three-band image from three successive measurements.

Colorimeters, digital cameras, and scanners can be mathematically represented by very similar models. In the remainder of this section, a scanner will be used for illustration of such a model. However, the same discussion applies to colorimeters and cameras with trivial modifications that will be pointed out where required.

The schematic of a typical desktop color scanner is shown in [Figure 1.34](#). The scanner lamp illuminates the image, and the light reflected off a small area is imaged by the lens onto a beam splitter that splits the light into a number of channels with filters having different spectral transmittances (the typical case of three channels is shown in the figure). The filtered outputs are integrated over the electromagnetic spectrum by optical detectors to

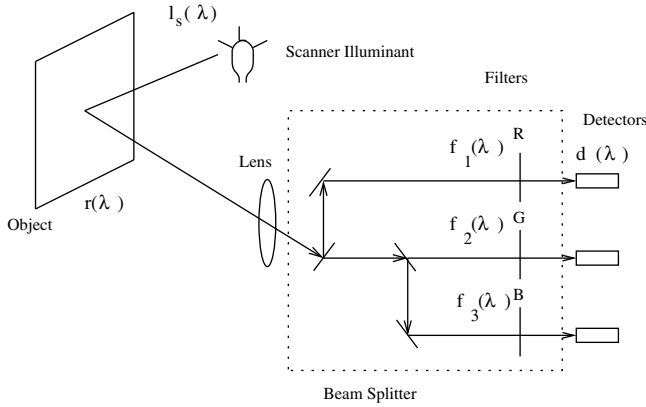


**Figure 1.34** Schematic of a color scanner.

obtain a scanner measurement vector. This process is repeated over the entire image to obtain a “color” representation of the image. In actual scanners, the scanner measurements of the small area corresponding to a sampling unit are influenced by the color of the surrounding areas.<sup>282</sup> Ideally, restoration schemes should be used to remove the blur from the recorded image. However, due to the computational requirements, this is rarely done, and this aspect of the problem will be ignored in the subsequent discussion.

For sensors commonly used in electronic scanners, the response at a single spatial location can be modeled in a manner similar to Equation 1.1 as

$$\begin{aligned}
 t_i^s &= \int_{-\infty}^{\infty} f_i(\lambda) d(\lambda) r(\lambda) I_s(\lambda) d\lambda + \varepsilon_i \\
 &= \int_{-\infty}^{\infty} m_i(\lambda) r(\lambda) I(\lambda) d\lambda + \varepsilon_i
 \end{aligned} \tag{1.65}$$

where

$K$  = number of scanner recording channels

$\{f_i(\lambda)\}_{i=1}^K$  = the spectral transmittances of the color filters

$d(\lambda)$  = sensitivity of the detector used in the measurements

$I_s(\lambda)$  = SPD of the illuminant

$r(\lambda)$  = spectral reflectance of the area being scanned

$\varepsilon_i$  = measurement noise

$m_i(\lambda) = f_i(\lambda) d(\lambda)$  is the product of filter transmittance and detector sensitivity

$t_i^s$  = the value obtained from the  $i$ th channel

In a manner analogous to Equation 1.3, Equations 1.65 may be replaced by their discrete approximations using matrix vector notation as

$$\mathbf{t}_s = \mathbf{M}^T \mathbf{L}_s \mathbf{r} + \boldsymbol{\varepsilon} \quad (1.66)$$

where  $\mathbf{t}_s$  = the  $K \times 1$  vector of scanner measurements  
 $\mathbf{r}$  = the  $N \times 1$  vector of reflectance samples  
 $\mathbf{L}_s$  = an  $N \times N$  diagonal matrix with samples of the radiant spectrum of the scanner illuminant along the diagonal  
 $\mathbf{M}$  = an  $N \times K$  matrix whose  $i$ th column,  $\mathbf{m}_i$ , is the vector of samples of the product of the  $i$ th filter transmittance and the detector sensitivity  
 $\boldsymbol{\varepsilon}$  = the  $K \times 1$  measurement noise vector

Note that, while these devices “sample” color spectra very coarsely, to assure that the above model is accurate it is necessary to meet sampling restrictions on the color spectra involved.<sup>303</sup> Due to their higher efficiency and lower heat dissipation, fluorescent lamps are often used in desktop scanners. Because their spectra have sharp spectral peaks, the sampling rate requirements (with uniform sampling) in the model of Equation 1.66 can be prohibitively high. A more efficient model for such a case is proposed in Reference 265, where a decomposition of the illuminant into the sum of a band-limited (smooth) component and impulses (monochromatic emission lines) is used to substantially reduce the dimensionality of the model while retaining the mathematical form of Equation 1.66.

For colorimeters and color cameras, the stimulus is normally a luminous object or an object illuminated by an illuminant external to the device. For these devices, the product,  $\mathbf{L}_s \mathbf{r}$  (or its equivalent), defines the spectral radiance whose color is to be recorded. From the model in Equation 1.66, it can be inferred that, in the absence of noise, exact CIE XYZ tristimulus values can be obtained from the data recorded by colorimeters and color cameras if there exists a transformation that transforms the sensor response matrix,  $\mathbf{M}$ , into the matrix of CIE XYZ color matching functions,  $\mathbf{A}$ .<sup>300</sup> This is equivalent to the requirement that the HVSS be contained in the *sensor visual space* defined as the column space of  $\mathbf{M}$ .<sup>310</sup> For devices using three channels, this reduces to the requirement that  $\mathbf{M}$  be a nonsingular linear transformation of  $\mathbf{A}$ . This fact has been known for some time and is referred to as the Luther–Ives condition.<sup>131,184</sup> Recent reiterations of this result can be found in References 102 and 127. A device that satisfies (generalizations of) the Luther–Ives condition will be said to be *colorimetric*.

For color scanners, the analysis is slightly more involved, because the illuminant used in the scanner is usually different from the illuminant under which the scanned object is viewed by an observer. Under these conditions, it can be shown that the CIE XYZ tristimulus values of the scanned object under the viewing illuminant can be determined exactly from the noiseless

scanner measurements if the human visual (viewing) illuminant space (HVISS) is contained in the *scanner visual space* (SVS) defined as the column space of  $\mathbf{L}_s\mathbf{M}$ . Because the spectra of fluorescent lamps used in most scanners is quite different from that of the daylight illuminants used in colorimetry, this condition is rarely met in practice. In addition, color tristimuli under multiple viewing illuminants often need to be estimated from a single scan of the image, and the above criterion would require an inordinately large number of detectors. In addition to the problems caused by fluorescent lamps, actual colorimeters, cameras, and scanners are subject to a wide variety of restrictions arising out of economic considerations and limitations of the processes and materials for manufacturing filters, optical components, and sensors. Techniques from signal processing are therefore useful for the evaluation and design of these devices.

It is the filters,  $\{f_i(\lambda)\}_{i=1}^3$ , over which the designer has the most control. A quality measure for evaluating single-color filters was first proposed by Neugebauer.<sup>214</sup> Recently, this was extended to provide a computationally simple measure of goodness for multiple filters in terms of the principal angles between the HVISS and the SVS.<sup>310</sup> The measure was used for the evaluation and design of color scanning filters.<sup>311,312</sup> The same measure was also successfully applied to the combinatorial problem of selecting an appropriate set of filters for a scanner from given off-the-shelf candidate filters.<sup>313</sup> A minimum-mean-squared-error approach, which requires more statistical information than purely subspace-based approaches, was introduced in Reference 316, where numerical approaches for minimizing errors in uniform color spaces were also considered. In Reference 317, noise was included in the analysis, and References 324–326 emphasize the reduction of perceived color errors in a hybrid device capable of measuring both reflective and emissive objects through the use of linearized versions of CIELAB space.<sup>27</sup> An alternate novel approach accounting for noise was proposed in Reference 76, where a filter-set was chosen from the multitude satisfying the Luther–Ives condition so as to minimize the perceptual impact of noise. In References 260 and 267, a unified treatment encompassing a number of these approaches is presented, and their performances are compared. Recently, the comprehensive figure of merit defined in References 260 and 267 has been extended through the inclusion of a signal dependent noise model and has been applied to the optimization of digital camera color sensitivities.<sup>235–238</sup>

It may be emphasized here that, for a rendition of the recorded reflectance scene under multiple viewing illuminants, more than three channels are usually necessary. A simple generalization of the Luther–Ives condition would require three  $K$  channels for  $K$  different viewing illuminants. In practice, however, between four and seven optimally designed spectral channels provide sufficient accuracy for common viewing illuminants.<sup>268,317</sup>

Note that, in recording color images digitally, both the spectrum and the spatial dimensions need to be “sampled.” The different quality measures mentioned above consider only the spectral sampling aspect of the above problem. These are therefore suitable for evaluating the color recording fidel-

ity for large patches of uniform color and do not represent the complete performance for the image recording system. This is particularly true in CFA camera devices where the spatial and spectral sampling are performed jointly.

In actual devices, it is possible to have systematic deviations from the linear model of Equation 1.66. Sources of error include fluorescence of samples in scanners, stray light, inclusion of ultraviolet and infrared radiation in the measurements (which is not accounted for if the visible region of the spectrum is used in the model), and limited dynamic range of detectors.<sup>86</sup> However, if proper precautions are taken, these errors are small and can be included in the noise process with minimal loss of functionality.<sup>86</sup>

It should also be noted here that the above discussion applies to a system for recording color where the input spectra are not constrained to lie in a restricted set. In recording color information from color reproductions that exploit trichromacy and utilize three primaries, the requirements for obtaining precise color information are much less stringent, and sensors with any three linearly independent channels typically suffice. A proof of this result for a system using three additive primaries (whose spectra vary only in amplitude and not in spectral shape) appears in Reference 125. An example of an application where this can be readily seen is the measurement of colors produced on a cathode ray tube (CRT).<sup>117</sup> Note, however, that the calibration of these noncolorimetric recording systems is highly dependent on the primaries used in creating the images. Thus, they yield large color errors with images that are not produced with the primaries used in calibration.

For subtractive color reproduction systems (described in Section 1.11.1.3) that use varying densities of cyan, magenta, and yellow dyes to reproduce colors, one can conclude that any three sensors from whose measurements the densities can be inferred will suffice. In fact, in such a restricted system, not only is it possible to obtain colorimetric information about the original, but it is also possible to reconstruct complete spectral information from the three-channel record.<sup>261,262,270</sup> The mathematical characterization of this requirement requires assumptions on the spectra of the dyes and models for the specific processes used, which are discussed in References 261 and 270. In practical systems, it is sufficient to have any three reasonably narrow color filters with peaks in the red, green, and blue regions.<sup>130(p. 247),249</sup> Because this is far less demanding as a design objective than the colorimetric criteria discussed above, and because a large fraction of input images to scanners are in the form of photographic prints that use subtractive reproduction, most present-day scanners are designed to satisfy this requirement. The cost levied by this design trade-off is greater user intervention, because distinct calibrations of the scanner are required for accurately scanning reproductions produced with different subtractive primaries.<sup>112</sup> With the explosion in the number of color reproduction systems, it is difficult to maintain a correspondingly large number of calibration profiles and the color accuracy of such scanners is likely to be further compromised.

If scanners are designed to be colorimetric, a linear transformation, independent of the scanned object characteristics, can be used to accurately

estimate the CIE XYZ tristimulus values from the scanner measurements. However, due to the nonlinear relationship between density and tristimuli, scanners designed to measure dye densities perform poorly with a linear transformation. A number of heuristic nonlinear calibration schemes have therefore been used in practice. Three-dimensional lookup tables,<sup>128</sup> least-squares polynomial regression,<sup>128,158</sup> and neural networks<sup>160</sup> are examples of these approaches. Note, however, that these approaches offer significant gains over a simple linear transformation only when the characterization is performed for a restricted class of inputs.<sup>112</sup>

### 1.11.5 *Multispectral recording and reproduction systems*

A *multispectral* image is an image in which each pixel has *multiple* channels that carry information about its *spectral* content. Multispectral images span the domain of images from conventional three-channel color images to hyperspectral imagery with hundreds of bands/channels used in remote sensing applications. Multispectral scanners often utilize narrowband spectral filters to record energy in different regions of the spectrum in a manner very similar to the color recording devices mentioned in Section 1.11.4.4. Traditionally, multiband sensors with more than three channels have been used in remote sensing applications. A major difference between these and the color recording devices arises from the fact that they are not attempting to capture information so as to satisfy a human observer. Therefore, these devices are not restricted to operating within the visible region of the electromagnetic spectrum and typically use infrared, visible, and microwave regions of the spectrum.<sup>36</sup> For the same reason, while dimensionality reduction of recorded data is often done while processing (see Section 1.11.5.1), there is no direct analog of trichromacy in remote sensing.

Traditionally, color imaging has worked with three channels, both for capture and reproduction.<sup>†</sup> Recent years have, however, seen a significant interest in multispectral image capture in the visible range of the spectrum and reproduction of multispectral images using objectives beyond those of simple color matching.

A variety of schemes have been proposed for multispectral image capture in the visible range. Devices in existence today are experimental, and most of these perform image capture using a single-channel camera in front of which are placed a number of spectrally selective filters that provide the separation of the image into multiple bands. Proposed filtering options include conventional absorption filters,<sup>297</sup> electro-optic tunable filters,<sup>115</sup> and interference filters<sup>121</sup> that offer narrow bandwidths. While absorption filters produce wider spectral bands than the electro-optic tunable filters and the interference filters, they also exhibit significantly less variation in their spectral transmittance with change in angular placement, which is often a source

<sup>†</sup> While the use of four colorants is common in color printing, and additional colorants have also been used in hi-fi color reproduction, the traditional approaches have chosen these in a constrained fashion, with essentially only three independent channels.<sup>270,271</sup>

of error in these systems. Because the number of channels is large, color-filter array type schemes that jointly sample the image spectrally and spatially are not common, though the use of filters in conjunction with trichromatic cameras has been investigated.<sup>138</sup> The large number of filters also implies a longer time for the scene capture and, consequently, the current devices are capable of capturing only static scenes with no motion, such as paintings or still-life scenes. For document imaging applications, the original to be captured is often a reproduction, and in these cases it is often feasible to recover spectral information from a simple three-channel record and realize the benefits of multispectral imaging.<sup>20,262,270</sup>

Multispectral capture offers several benefits over conventional three-channel capture. All of these advantages stem from the fact that the additional spectral data represent the information in the original scene more completely. The data can be used, for instance, to simulate changes of the SPD of the scene illuminant,<sup>116</sup> which may then be used for display or print renderings of the original as it would have appeared under a desired illuminant.<sup>†</sup> The use of multispectral data for this purpose is similar to the use of spectrophotometers in color characterization, where colorimetry is then calculated from the spectral measurements by specifying a desired illuminant.<sup>‡</sup> The additional information in the multispectral capture also enables analysis of characteristics of the original<sup>22,193</sup> that would not be captured by conventional color capture. This is particularly valuable in the digitization of art archives and provides a mechanism for preserving these digitally, allowing better restoration and the ability to share them as more realistic reproductions. Several academic and government institutions have, therefore, initiated research in this area.

Additional capabilities of captured multispectral data are realized in conjunction with multispectral output systems that use more than three channels for output and can incorporate additional criteria for reproduction. The additional capabilities enabled by multichannel output devices can be used to try to effect a spectral match between the reproduction and the original that was (multispectrally) recorded. Such a reproduction would eliminate illuminant metamerism and provide a color match between the original and the reproduction across any viewing illuminant. The complicated nature of color hardcopy reproduction and limited availability and capability of colorant materials make this ideal goal extremely difficult to realize.<sup>18,305</sup> Some preliminary experimental results on spectral hardcopy reproduction using six colorants have recently been reported.<sup>292</sup> The large

† As already noted in Section 1.9.2, a color appearance model is not designed to address this problem. However, from the arguments at the end of Section 1.9.2, it is also clear that the benefit of multispectral capture for this application can be evaluated by comparing its efficacy over the incorrect use of a color appearance model in this situation. Such an evaluation is offered in Reference 116 using CIELAB as a crude appearance model.

‡ Note also that this application suffers from the same pitfalls as the color measurements in that the effects of fluorescence, if any, can be comprehended only to the extent that the illuminant being simulated is similar to the illuminant under which the scene is captured.

dimensionality of the spectral space ensures that, for any reasonable number of colorants, most spectra are outside the spectral gamut of the reproduction system. Practical systems in the near future can only hope to provide approximate spectral matching at best. One limitation with this approach is that the differences between the spectra of the original and the reproduction might be visible under common lighting conditions, and, even under a single lighting condition, color differences could result that could be eliminated by colorimetric matching. Thus, it is beneficial to consider alternative reproduction objectives for multispectral output. One straightforward extension is to minimize color differences under multiple common illuminants. This addresses illuminant metamerism for the standard observer but does not address observer metamerism arising due to differences among observers. An alternative objective is to consider the color variations among observers and use the freedom offered by the additional channels to minimize the impact of observer metamerism.<sup>121</sup> This is particularly meaningful for additive multiprimary displays and has been recently applied in that context.<sup>121,337</sup>

In addition to their use in color imaging applications outlined above, multispectral data are also extremely useful as a research tool for the realistic simulation of conventional imaging systems.<sup>31,309</sup> Such simulation and analysis can help direct improved design of these recording systems. The encoding of multispectral images is thus far in experimental formats, but some recent work has proposed mechanisms for exchange and an encoding format that is compatible with colorimetry in that three of the channels are selected to correspond to colorimetry.<sup>165</sup>

#### 1.11.5.1 *Principal-component recording*

The color recording devices of Section 1.11.4.4 attempt to sample the spectra of images while preserving visual information. A recording of the spectrum itself provides greater information but is extremely slow and expensive. Because spectral information of reflective images is extremely useful for determining color under different illuminants, alternative schemes for recording their spectral information are of interest.

Note that, in the absence of noise, the scanned image in Equation 1.66 can be directly used to determine the projection of the image spectra onto the SVS. Hence, to obtain good reconstruction of reflectance spectra, the sensors can be chosen so that a large fraction of the energy in reflectance spectra lies in the SVS. In the absence of noise, the Karhunen–Loève (KL) transformation provides the mathematical solution to this problem in terms of the statistics of the ensemble of reflectance spectra.<sup>126</sup> The best spectral reconstruction of scanned spectra in a mean-squared error sense is obtained from a  $K$  channel scanner when the SVS corresponds to the span of the  $K$  *principal components* of the reflectance spectra, i.e., the eigenvectors associated with the  $K$  largest eigenvalues of the spectral reflectance correlation matrix.

The reflectance spectra of most naturally occurring objects are smooth functions of wavelength; the same is true of spectra produced using pho-

tography, printing, or paints. As a result, these spectra can be accurately represented by a few principal components. Various studies of reflectance spectra have estimated that between three and seven principal components (depending on application) provide satisfactory reconstruction of reflectance spectra for most color work.<sup>54,122,198,314,319</sup> Note that this offers a significant reduction in dimensionality in comparison with spectrophotometric measurements using uniform sampling.

Linear models for object reflectance spectra based on the principal-components idea have been used by many researchers for recovering illuminant and surface reflectance data from recorded images and for color correction applications.<sup>41,122,315,319</sup> Most of this research used KL transform on a spectrophotometrically recorded ensemble of reflectance spectra, and the problem of designing spectral recording devices based inherently on the principal-components approach has received little attention. There are, however, commercial color measuring devices that attempt to reconstruct spectral data from sensor measurements.<sup>42</sup> In addition, the principal-components approach has been used in analyzing multispectral satellite imagery, and the idea of a recorder based on principal components has also been suggested for acquiring satellite images.<sup>287(Chap. 7)</sup>

One may note here that some naturally occurring reflectance spectra do not adhere to the smoothness assumption. Examples of such spectra are colors produced due to multiple film interference in certain minerals and iridescent colors on some bird feathers and in shells containing calcium carbonate.<sup>210(p. 261),267</sup> A principal-components scheme leads to relatively large errors in such spectra. Hence, in imaging applications involving these objects, the principal-components approach would be inappropriate for approximation of their spectra.

### *1.11.6 Quantization and coding*

Color images recorded with the different input devices described in the last section need to be quantized for digital processing. Both scalar and vector quantization techniques can be used in the quantization of color data. For simplicity, most color devices do independent quantization of the RGB channels with 8 to 12 bits per channel, with either uniform quantizers or companded quantizers that perform a gamma correction before the quantization. As mentioned earlier, the gamma correction significantly reduces the perceptibility of quantization errors, particularly in the eight-bit devices. For computer color displays based on a frame buffer architecture,<sup>298</sup> often only 8, 12, or 16 bits of video memory are allocated to each pixel, thereby allowing simultaneous display of only  $2^8$ ,  $2^{12}$ , or  $2^{16}$  colors, respectively. Vector quantization techniques have therefore been used extensively for displaying images on these devices. [Chapter 9](#) surveys and discusses these techniques.

With the proliferation of digital color imagery, the problem of coding color images for transmission and storage has gained increased importance.

It was recognized early on that the highly correlated RGB spaces were not suitable for independent coding.<sup>233</sup> Consequently, most of the methods transform the data into a luminance channel and two chrominance channels that are then coded independently. A luminance-chrominance space also allows coding schemes to exploit the properties of human vision by allocating significantly fewer bits to the high-frequency chrominance components, which are perceptually less significant.

The most prevalent compression scheme at present is the JPEG standard for still images<sup>228</sup> and the MPEG standard for video data.<sup>207</sup> These standards are both based on the discrete-cosine transform (DCT).<sup>5</sup> While these standards do not explicitly specify the color spaces to be used, in current implementations, it is common to use the YCrCb space,<sup>39</sup> with the Cr and Cb components subsampled by a factor of two along both spatial dimensions.<sup>212</sup> The YCrCb color space is a luminance-chrominance color space, based on gamma-corrected RGB, that has been proposed for use as a standard in HDTV. The Y component is a luminance channel similar to  $L^*$ , and the Cr and Cb are opponent chrominance channels similar to  $a^*$  and  $b^*$ , respectively. The chapter on color image compression provides more details on the compression of color images and the new JPEG-2000 standard.

#### *1.11.7 Device color spaces*

The measurements from color recording devices and the control values for color output devices are the color representations of recorded images or the images to be reproduced, respectively. Hence, it is common to say that these values represent colors in the device's color space. Thus, typically there are RGB color spaces for scanners, cameras, and other input devices, and there are CMY/CMYK color spaces for color printers. Unlike the CIE standard color spaces discussed in Section 1.5.1, most of these color spaces are not standard and cannot be directly used for the meaningful archival/communication of image data. However, if these device spaces are related to the standard color spaces in a clear unambiguous way, these can also be potentially used for the exchange of color information. An advantage of such a scheme is that no transformations of the image data are required for display if the data are specified in the native color space of the device. Because a large fraction of the images on the World Wide Web are primarily targeted for display on CRT monitors that have very similar characteristics, a new standard color space, sRGB, has been proposed for use based on these characteristics.<sup>6</sup> The sRGB color space is basically a gamma-corrected tristimulus space that uses the CRT phosphors as primaries for determining the CMFs. In addition, the sRGB standard includes provisions for specifying the viewing conditions (white-point chromaticities, image surround, flare, etc.). The sRGB space is tied to the characteristics of common CRT displays and is therefore limited in some respects. In particular, there are colors that can be produced on common printers but lie outside the range of colors that are can be encoded in sRGB. These colors tend to be in the cyan and bright

yellow regions. Recently, extensions to the space to remedy some of its limitations with regard to gamut have been proposed.<sup>283</sup> However, as displays evolve to newer technologies, the original motivation for using the sRGB standard as a method for keeping data in a format that requires minimal transformation is also no longer valid.

Standardized CMYK spaces have also been defined in the graphic arts industry to allow color data to be supplied to printing press operators in a form that can be unambiguously interpreted (unlike device CMYK specific to a single press). The standards in this area vary by geography and include SWOP<sup>288</sup> (North America), Euroscale (Europe), and Japan Color (Japan). These standards often combine multiple independent specifications based on the printing conditions (e.g., for coated vs. uncoated substrates).

## 1.12 Color management and calibration

For proper color reproduction, the input and output devices involved must be calibrated. Historically, the systems used for color reproduction were calibrated<sup>†</sup> in a closed-loop configuration. As shown in Figure 1.35, in a closed-loop configuration, the complete system is calibrated from input through output. Thus, for color photography, the film sensitivities, dye absorptances, and developmental interactions were appropriately chosen so as to result in acceptable reproduction. In offset printing, the scanner was used to generate CMYK “separations” that were suitable for generating

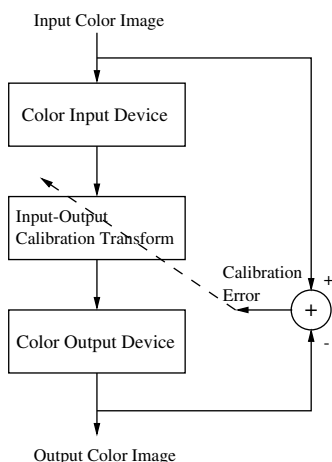


**Figure 1.35** Closed-loop system calibration.

<sup>†</sup> Note that, in Chapter 5, the terms *calibration* and *characterization* have specific meanings as per-channel corrections and the full three-dimensional color correction that apply to the “calibrated” device. In this chapter, we do not follow that convention strictly, because the terms *calibration* and *characterization* are in common use with overloaded meanings. The use of these terms in this chapter should be apparent from the context.

halftone prints. With the increased use of digital computers and the evolution of desktop printing, it became obvious that such an approach has severe limitations. In particular, as the number of devices increases, calibrations for each input–output device pair are difficult to construct and maintain. In addition, because the calibrated data in a closed-loop calibration scheme are specific to one output device, they are not suitable for archival purposes or exchange with devices outside the system.

With the growth of networking and increased exchange of color image data between geographically divided systems, it was recognized that several of these problems can be solved by calibrating each device to a standard device-independent (DVI) color space, which can then be used for the exchange of data between different devices and for archival uses. As shown in [Figure 1.36](#), in these systems, the data from an input device are converted to a device-independent color space and then transformed into the device-space of the target device for reproduction.

To enable proper management of color, several components are required. Color measurement instrumentation discussed earlier in Sections 1.11.4.1 and 1.11.4.2 is necessary for calibrating input and output systems. Standard formats for the storage and communication of these device calibrations are required so that different applications can make use of the calibrations. Also necessary are systems and algorithms that use the calibrations effectively to achieve desired results. These components are briefly outlined in the remainder of this section. As with other topics, several are discussed in detail in subsequent chapters.

### *1.12.1 Calibration and profiles*

Calibration of a color imaging device relates its input/output to DVI color values. For an input device, calibration provides a mapping from device measurement values (e.g., scanner RGB) to DVI color descriptors (e.g., CIE XYZ, CIELAB etc.), and, for an output device, the calibration process yields a mapping from DVI color descriptors to device control values (e.g., CMYK, monitor RGB) which produce those color descriptors.

#### *1.12.1.1 Input device calibration*

To calibrate a scanner, the first step is to select a collection of color patches that span the gamut of interest. Ideally, these colors should not be *metameric* for the scanner or the eye (under the illuminant for which the calibration is being produced). Metamerism is defined as the property in which different spectra map to the same values under a set of sensitivity functions. These patches are measured using a color measurement instrument, such as a spectrophotometer or a colorimeter, which will provide the device-independent color values  $\{\mathbf{t}_k\}_{k=1}^{M_q}$ , where  $M_q$  denotes the number of patches. Any of the (device-independent) color spaces such as CIE XYZ, CIELAB, etc. can be used for this purpose. The use of CIELAB is common, as this space includes

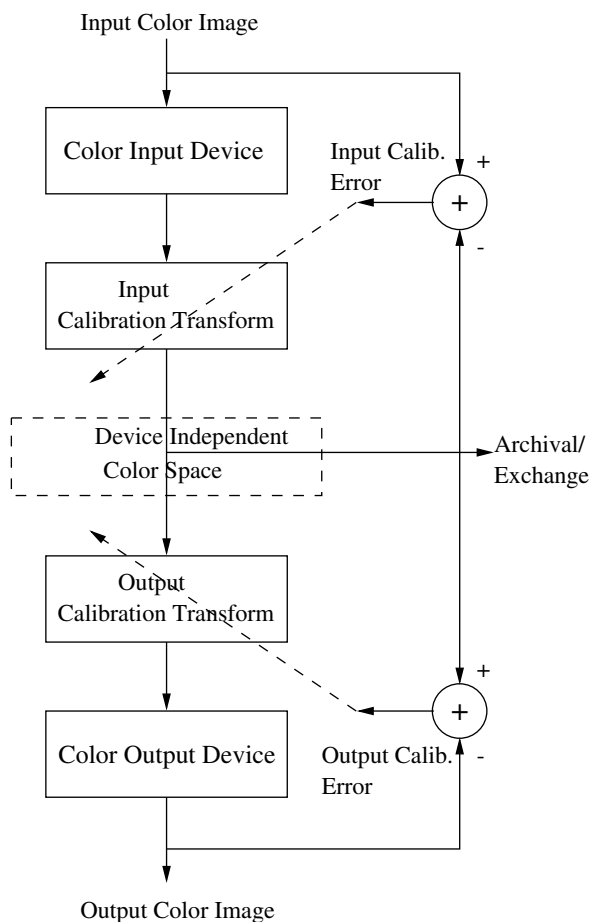


**Figure 1.36** Device-independent color calibration.

information on the viewing illuminant in the white point. The patches are also measured with the scanner to obtain the scanner measurements  $\{\mathbf{u}_k\}_{k=1}^{M_q}$ .

To determine the CIE values for an arbitrary measured patch, the collected data are used to construct an interpolating function that maps from the space of scanner measurement values to the chosen device-independent color space. This function,  $F(\cdot)$ , can then be used to relate any of the scanner RGB values to colorimetric XYZ values or LAB values, i.e.,  $F(\mathbf{u}) = \mathbf{t}$ . Normally, a parametric form is chosen for  $F(\cdot)$ , and the parameters are determined through regression. Several different schemes have been used for this pur-

pose, ranging from straightforward linear and polynomial regression to neural networks.<sup>159(Chap. 11)</sup> Usually,  $F(\cdot)$  is complicated and computationally expensive. For this reason,  $F(\cdot)$  is usually used to produce a finely sampled lookup table (LUT) from which  $F(\cdot)$  at arbitrary points is obtained by using simple interpolation schemes.<sup>159(Chap. 4)</sup>

Calibration for digital cameras and video cameras is usually done in a similar fashion by using a target of patches with known reflectances or color values. Because the eye is very sensitive to deviations from the neutral (achromatic) colors, sometimes an additional one-dimensional transform is included on each of the RGB channels so that the  $R = G = B$  line corresponds to neutral colors. This procedure is commonly referred to as *gray/white balancing*.

### 1.12.1.2 Output device calibration

For calibrating an output device, a transformation from DVI color values to the space of device control values is required. This requires a two-step procedure. In the first step, the printer characterization, which determines the forward transformation from printer control values to DVI color values, is determined. Then, this forward transform is used in the next step to determine the inverse mapping from DVI color values to device control values.

Because CRT monitors are represented well by the parametric models described in Section 1.11.1.1, the forward characterization of these devices is usually done by determining the model parameters from a few measurements. It can also be readily seen that, due to their additive nature, the inverse transformation from CIE XYZ (or other tristimulus) values to the CRT control voltages can be computed by means of a simple matrix transformation followed by a one-dimensional transform for the gamma correction. This scheme is used in all cases except those requiring the highest accuracy, for which LUT-based schemes may be used, and additional corrections may be made for the surface reflection (flare) from the monitor screen. In addition, it may be necessary to correct for the significant spatial nonuniformity over the CRT screen.<sup>63,266</sup>

For the forward characterization of printers, an empirical scheme similar to that described for scanner calibration is commonly used. By selecting a set of printer control values  $\{\mathbf{c}_k\}_{k=1}^{M_p}$  covering the range of allowable control values, measuring the corresponding DVI color values  $\{\mathbf{v}_k\}_{k=1}^{M_p}$ , and using some interpolation scheme, the forward mapping,  $\mathbf{v} = G(\mathbf{c})$  from control values to DVI color values is determined. For halftone printers, alternatively, the Neugebauer models mentioned in Section 1.11.1.4 have also been used. Because even the models are nonlinear and not readily invertible, for the inverse mapping  $G^{-1}(\cdot)$ , an interpolating function invariably is used to get a finely sampled LUT. Due to the four degrees of freedom in the control values of CMYK printers, there exist multiple control values that result in the same printed color. Because this poses a challenge in obtaining a smooth inverse mapping, the inverse often determines the

amounts of three virtual CMY colorants from which the CMYK control values are obtained by some functional relation. This process is viewed as the incorporation of black ink and removal of underlying CMY inks and is commonly referred to as *undercolor removal* (UCR). Typically, the UCR mapping from the virtual CMY values to actual CMYK control values is designed so as to better render achromatic colors, reduce total colorant amounts (for faster drying/better adhesion to paper), and (in some cases) reduce the use of the expensive CMY colorants.

### 1.12.1.3 Device profiles

To make the calibration transformations available to different applications that wish to use them, the calibration transformation for each device is stored in a device profile. In the early days of color management, different manufacturers used their own proprietary formats for the storage of these profiles, which were therefore useful only for applications from the same manufacturer. To realize the full benefits from the DVI calibration of devices, the desktop publishing industry is increasingly moving toward open systems. A standard format for the storage of device profiles has been defined and is being widely adopted.<sup>62</sup> This International Color Consortium (ICC) profile format specifies a wide variety of input and output device profiles suitable for efficiently representing the color calibration information. The use of a standardized format allows the profiles to be used by different applications from different manufacturers. Limitations of the profiles in some respects have resulted in some nonstandard implementations that have compromised compatibility, but these are being addressed by the ICC with input from member corporations.

## 1.12.2 Color management systems

A color management system (CMS) is responsible for interpreting the device profiles and performing the appropriate transformations to and from the device-independent space. The goal of a CMS is to provide predictable and consistent color without requiring specialized skills from the user. Thus, CMSs tie together device profiles with *color matching modules* that use these device profiles to transform device-dependent image data to DVI color spaces or to the device color spaces of target output devices on which the images are to be displayed. In addition, the CMS provides the user with flexibility to choose different *rendering intents* for different images. Thus, for instance, in reproducing a company logo, a perfect colorimetric match is usually desired; in producing bar graphs and pie charts for presentation, it is desirable that the colors be highly saturated for maximal impact; and in reproducing pictorial images, it is desirable that the closest perceptual match be obtained (which will depend on viewing conditions). Color management functions can be performed at several different phases of the imaging process — in the devices (e.g., Adobe's Postscript level 2/3 for

printers), in device drivers (e.g., Cannon Colorgear), applications (e.g., Adobe's Photoshop/Acrobat), or in the operating system (e.g., Apple's ColorSync/Microsoft Windows 2000/XP). A description of some of the practical aspects of color management in printing applications and a listing of vendors of color management software and solutions can be found in Reference 137. The notion of embedding color management in the operating system has the potential of making the process transparent to the end user. The system, however, also introduces limitations that can result in unexpected behavior when users are unaware of the color management or have incompatible components. As a result, even though several vendors of operating systems for desktop and workstation computers have incorporated CMSs into their products, these are not extensively utilized at present.

Ideally, with color management, one could accurately transfer color information from one medium (e.g., a CRT) to another (e.g., print). Unfortunately, this is an extremely difficult task for two reasons:

1. There are significant differences in gamuts of different devices (this was mentioned and demonstrated in Section 1.11.1).
2. The difference in typical viewing conditions for different media implies that a simple colorimetric match does not give an appearance match.

There is therefore significant interest in *gamut mapping* algorithms that map the colors in an image to suitable colors that can be reproduced on the target device. Methods that model the adaptations of the eye and allow the computation of appearance matched images under different viewing conditions are also an active area of research.

### 1.12.3 Gamut mapping

Gamut mapping is the process of mapping the displayable colors from one media to those of another media. As defined earlier, the ideal goal for color matching depends on the type of image and the intent of the user. The problem is probably most acute for pictorial images for which, ideally, one would like to map the colors so as to obtain the best possible appearance match between the images on the different media.<sup>206,286</sup> The strategy for gamut mapping could be either image dependent or image independent. Because image-dependent methods can use different strategies for different images, they often produce better results. However, they are also significantly slower than image-independent techniques, because they require a fresh computation for each image and are therefore seldom used in automatic gamut-mapping schemes. Several color applications, however, indicate out-of-gamut colors in an image via a key color or a flashing highlight (on a CRT). This allows the user to perform a transformation such that these colors are mapped satisfactorily. Such an approach, however, requires considerable operator skill to obtain satisfactory results.

The simplest technique of gamut mapping (particularly for CRT monitors) is clipping in the space of device control values. Because the space of control values is not a UCS, clipping of control values does not yield the closest printable color (as perceived by an observer). To remedy this problem, a simple extension would be to map out-of-gamut colors to the nearest in-gamut color in a UCS. While this approach offers significantly better results than the device space clipping,<sup>98</sup> it can often result in unacceptable hue shifts that are perceptually very objectionable. An additional limitation of this and the clipping approach is that smoothly varying regions beyond the device gamut can potentially be mapped to a single color, creating undesirable abrupt edges in the previously smooth regions and causing loss of significant information (such as shape from shading).

Another approach used for gamut mapping is a gamut compression algorithm that compresses all the colors in the image in a manner that reduces the colorimetric dynamic range in the image while ensuring that the colors can be reproduced. For example, one could move all the colors in the image toward one point, such as a mid-gray, until all the colors in the image are within the device gamut. Unlike the clipping approach, this method will retain some of the variation in smoothly varying image regions that are beyond the device gamut. In addition to UCSs, notions of hue, chroma, and saturation/value are extremely useful in gamut mapping research, as these can be directly related to viewers' objections to artifacts produced by gamut mapping algorithms and can be used in appropriately choosing the clipping and compression. In Section 1.7, it was pointed out that, in addition to being a UCS, the CIELAB space allows the computation of correlates of lightness, hue, and chroma. As a result, CIELAB has been used extensively in gamut mapping research. In the process, some limitations of CIELAB have also been discovered and have been addressed through empirical modifications. The "blue hue nonlinearity" was one such limitation that was discussed in Section 1.7.3.

The transformations to and from the perceptual spaces are usually nonlinear, and speed is often an issue.<sup>60</sup> For this reason, some CMSs may use a high-resolution LUT along with a linear interpolator to perform the transformation. Performing multiple transforms on the data can result in a loss of fidelity and introduce visually noticeable errors due to the accumulation of errors associated with finite precision arithmetic. Some CMSs can cache input and output transformations, concatenate them, and perform these in a single operation when the final image is desired. This not only saves time but also improves accuracy. Some of these powerful techniques are discussed in [Chapter 11](#).

#### *1.12.4 Appearance matching*

Section 1.9 briefly discussed the adaptation in the eye in response to change in viewing conditions. Since different color reproduction media have different viewing conditions, in reproducing images across these different media, an appearance match instead of a pure colorimetric match is usually desired.

The simplest instance of appearance matching is the *white-point* matching method based on the von Kries transformation as mentioned in Section 1.7. This transformation converts tristimuli into a space of cone responses and applies a diagonal correction matrix that equates the white points under the two viewing conditions.<sup>335(p. 432)</sup> White-point matching is often used in color imaging applications, and there is support for it in several color management applications and packages such as PostScript.

In addition to white-point adaptation, there are several well characterized psychophysical effects that change with change in viewing conditions. In particular, it is well documented that the apparent contrast (perceived intensity gradient) of an image decreases in a dark surround in comparison to a bright surround.<sup>134(pp. 49–57)</sup> Often, this change in contrast is modeled by relating the luminance to the perceived lightness as a power-law relation, similar to that for CIELAB in Equation 1.32, with the exponent increasing as the surround gets brighter.<sup>80,134(pp. 56–57)</sup> Thus, *gamma correction* has also been used extensively to compensate for these effects, in addition to correction for monitor nonlinearity.

There has been considerable research in defining color appearance models that account for chromatic adaptation, influence of surround, and other psychophysical phenomena that affect the perception of images.<sup>81</sup> These models hold tremendous potential for use in cross-media color reproduction. The CIE is involved in an ongoing attempt to define a standard appearance model for use in imaging applications. An interim version of the model is available as a standard.<sup>50</sup> The model has also been recently refined in References 82 and 176. More details on the status of color appearance research can be found in [Chapter 2](#), which is devoted to this topic.

## 1.13 Summary

This chapter provides an introduction to the fundamentals of color science and technology as applied to color digital imaging. It also attempts to provide a systems view of color imaging systems, where the interactions between image capture devices, image and color processing operations, image display and printing systems, inherent characteristics of natural and man-made images, and the human observer are outlined. Given the breadth of subject matter covered in the chapter, the presentation of several aspects is quite terse and abbreviated. A much more detailed view of several of the individual “components” within this system is presented in the remaining chapters of this book.

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chapter and I am grateful to them for their assistance. Finally, the insights (if any!) in this chapter reflect the knowledge I have gained from numerous colleagues, mentors, instructors, researchers, and authors who have helped shape my understanding of color science, color perception, and color imaging. I am grateful for having the opportunity to work in this wonderful field and to interact with you. Any errors/inaccuracies in the presentation can be attributed solely to my own lack of understanding.

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