

EXHIBIT 1021

REGULARITY-PRESERVING IMAGE INTERPOLATION

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ABSTRACT

Common image interpolation methods assume that the underlying signal is continuous and may require that it possess one or more continuous derivatives. These assumptions are not generally true of natural images, most of which have instantaneous luminance transitions at the boundaries between objects. Continuity requirements on the interpolating function produce interpolated images with oversmoothed edges. To avoid this effect, a wavelet-based interpolation method that imposes no continuity constraints is introduced. The algorithm estimates the regularity of edges by measuring the decay of wavelet transform coefficients across scales and attempts to preserve the underlying regularity by extrapolating a new subband to be used in image resynthesis. The algorithm produces noticeably sharper edges than traditional techniques and exhibits an average PSNR improvement of 2.5dB over bilinear and bicubic techniques.

1. INTRODUCTION

Traditional image interpolation methods rely on assumptions about the nature of the image being interpolated which are often false in practice. Linear interpolation imposes continuity, bicubic interpolation assumes continuity of both the signal and its first derivative, and splines have continuous higher-order derivatives. Since natural images tend to consist of smooth sections joined by abrupt discontinuities at object boundaries, the assumptions implicitly underlying these interpolation algorithms are unrealistic. In practice, assuming higher-order continuity than is warranted by the original image causes the edges and textures to be smoothed and blurred. This change in visual smoothness is immediately noticeable and perceptually disturbing.

The wavelet transform, however, provides a means by which the local smoothness of a signal may be quantified; the mathematical smoothness (or *regularity*) is bounded by the rate of decay of its wavelet transform coefficients across scales. The algorithm proposed in this paper create new wavelet subbands by extrapolating the local coefficient decay. These new, fine-scale

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subbands are used together with the original wavelet subbands to synthesize an image of twice the original size. Extrapolation of the coefficient decay preserves the local regularity of the original image, thus avoiding the oversmoothing problems associated with traditional interpolation methods.

The structure of this paper is as follows. Section 2 introduces the concept of Hölder regularity and discusses its relationship with the wavelet transform. The correlation between wavelet subbands is then used to motivate the regularity-preserving interpolation algorithm of Section 3. Section 4 examines the results of this interpolation on natural images.

2. REGULARITY AND WAVELETS

A function $f : \mathbb{R} \rightarrow \mathbb{R}$ has Hölder regularity (or Hölder exponent) $\alpha = n + r$ with $n \in \mathbb{N}$ and $0 \leq r < 1$ if there exists a constant $K < \infty$ such that

$$|f^n(y) - f^n(x)| \leq K|y - x|^r \quad \forall x, y \in \mathbb{R}.$$

The Hölder regularity essentially indicates the number of continuous derivatives that a function possesses. Functions with a large Hölder exponent are both mathematically and visually smooth, while small Hölder exponents are associated with relatively rougher functions [1]. The wavelet transform provides a means for measuring the smoothness of a function in practice.

2.1. Measuring Hölder Regularity

The *undecimated dyadic wavelet transform* is a discretized version of the continuous-time wavelet transform [2]. It is computed by the filter bank shown in Figure 1, which projects a signal $f(x)$ onto a set of translations and dilations of a *mother wavelet* $\psi(x)$:

$$\psi_{k,l}(x) = \psi(2^k x - l),$$

where the *scale* k and the *offset* l are integers.

A signal has Hölder exponent r if there exists a finite constant K such that the wavelet transform coefficients $w_{k,l} = \langle f, \psi_{k,l} \rangle$ satisfy

$$|w_{k,l}| \leq C 2^{-k(r+\frac{1}{2})} \quad (1)$$

for all scales $k \in \mathbb{R}^+$ and all translations $l \in \mathbb{R}$ [3]. This theorem characterizes the regularity of a function

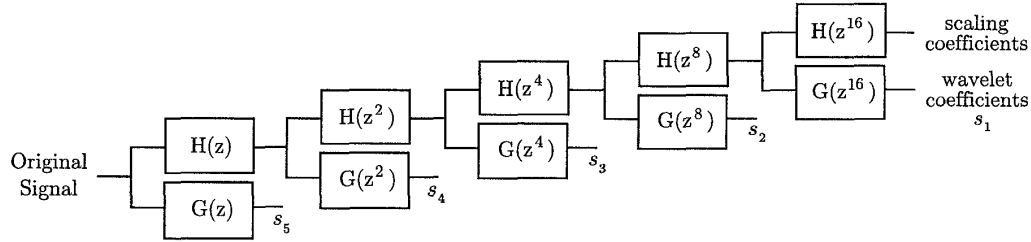


FIGURE 1: Undecimated Dyadic Wavelet Transform Filter Bank

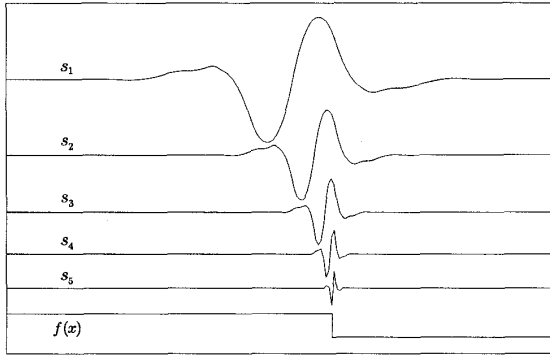


FIGURE 2: Wavelet Decomposition of a Step Edge $f(x)$

by the decay of the magnitude of its wavelet transform coefficients across scales. As such, it can be used to explain the similarities between scales that appear in multiresolution image processing.

2.2. Correlation Between Scales

Early research in multiresolution image processing [4] indicated the presence of similarly-shaped but differently-sized features at several scales simultaneously, as illustrated in Figure 2. Smoother functions exhibit more similarity between scales.

These similarities can be mathematically quantified by using the decay theorem of Equation (1). It can be shown that the correlation between the wavelet subbands at scale 2^m and 2^n is bounded by

$$|\text{Corr}(s_m, \tilde{s}_n)| \leq K 2^{-(m+n)(\alpha+\frac{1}{2})}. \quad (2)$$

This inequality indicates that the similarity between scales decreases exponentially as the regularity of the function analyzed increases, an observation that is corroborated by experimental data and the work of other authors [4, 5]. The similarity of features at different scales can be used to synthesize a new, fine-scale subband to be used in interpolation.

3. IMAGE INTERPOLATION

Traditional interpolation methods operate in the time domain by first interleaving the known samples with zeros and then lowpass filtering the interleaved signal to fill in the missing samples. In contrast, the regularity-preserving interpolation method synthesizes a new wavelet subband based on the decay of the known wavelet transform coefficients.

The original image can be considered to be the low-pass output of a wavelet analysis stage, as shown in Figure 3. The original image can therefore be input to a single wavelet synthesis stage along with the corresponding high frequency subbands to produce an image interpolated by a factor of two in both directions. Creation of these high-frequency subbands is therefore required for this interpolation strategy.

After an initial, non-separable edge detection, the unknown high-frequency subbands are created separately by a two-step process. First, edges with significant correlation across scales in each row are identified. The rate of decay of the wavelet coefficients near these edges is then extrapolated to approximate the high-frequency subband required to resynthesize a row of twice the original size. The same procedure is then applied to each column of the row-interpolated image. The entire system is illustrated in Figure 3.

3.1. Edge Localization

Suppose that the undecimated wavelet decomposition of each 1D signal produces wavelet subbands $s_1, s_2, \dots, s_L$ in order from coarsest scale to finest scale (*cf.* Figure 2). To create the new subband s_{L+1} , strong luminance changes are first identified by an edge detection algorithm similar to that in [5]. These edges are then selectively eliminated from consideration based on the decay of their wavelet coefficients across scales. Experimental observations indicate that strong edges achieve near-equality in (1) and (2). This observation leads to an efficient method for distinguishing edges in textured areas from strong edges at object boundaries based on

correlation between scales [6].

If a series of correlated features converges to a candidate edge in spatial location as the scale increases from s_1 to s_L and the rate of decay of the feature magnitudes approximates the bound in (1), the edge is retained for further processing. Edges for which no convergent sequence of correlated features exist are rejected from consideration, since they are most likely edges occurring in textured areas. For example, the series of similarly-shaped features in each subband in Figure 2 corresponds to the step in $f(x)$.

3.2. Coefficient Synthesis

For those edges which are retained, the feature in subband s_{L-1} is used as a template for creating a feature in the new subband s_{L+1} . Subband s_{L-1} is used because the features in the finest scale subband s_L tend to be dominated by noise for typical signals; subband s_L is the output of the highest-frequency filter in the analysis filter bank and therefore contains most of the high-frequency noise in the image. Subband s_{L-1} is also preferable to coarser scale subbands because the features are sufficiently localized that they correspond to a single edge. Features in subbands s_1 through s_{L-2} have broad spatial support and often correspond to several distinct edges.

The feature in s_{L-1} is scaled by a factor of $1/4$ and copied into the new subband s_{L+1} using a cubic spline approximation. The location of the scaled feature in the new subband is determined by extrapolating the locations of similar features in all the coarser subbands. The magnitude of the new feature is chosen to satisfy the bound in (1), where the rate of magnitude decay is determined by a linear least-squares fit to the logarithms of the coefficient magnitudes. The new subband is then input to a one-stage wavelet analysis bank along with the original signal, yielding an interpolated signal of twice the size.

4. EXPERIMENTAL RESULTS

The regularity-preserving interpolation algorithm maintains sharp edges without smoothing, unlike bilinear and bicubic interpolation. Figure 4 illustrates the visual differences between these interpolation methods for a subsection of the *Lena* image. The wavelet-based interpolation method preserves the strong edges of the original image as well as the texture in *Lena*'s hat better than the other methods. Similar results were obtained for other strong edges in *Lena* and for other images in the USC image database.

In addition to the striking visual difference, there is also a significant PSNR difference between the meth-

ods, as shown in Table 1. These PSNR values were computed by interpolating an original image subsampled by two and comparing the result to the original full-sized image. The greatest PSNR improvement was 4.2dB for *Lena*, an image with many strong edges relative to the number of textures. The improvement on *Mandrill*, however, was a negligible 0.182dB because of the relative abundance of textures, which, unlike edges, do not improve significantly with wavelet interpolation. The average improvement was approximately 2.5dB.

5. CONCLUSION

The wavelet-based interpolation method described in this paper avoids problems caused by traditional interpolation methods such as bilinear and bicubic interpolation. Rather than making assumptions about the continuity of the underlying signal and its derivatives, the wavelet-based method computes a measure of local regularity and preserves this local regularity in the interpolated image. This regularity-based interpolation results in both perceptual and quantifiable improvement in the interpolated image. The algorithm provides the most improvement over traditional methods on images with strong, well defined edges that separate smooth, high-regularity regions.

6. REFERENCES

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Image	Bilinear	Bicubic	Wavelet
<i>Lena</i>	27.3	27.5	31.7
<i>Couple</i>	26.8	27.0	29.6
<i>Mandrill</i>	23.4	23.6	23.8
<i>Peppers</i>	28.3	28.5	31.2

Table 1: PSNRs for Image Interpolation Methods

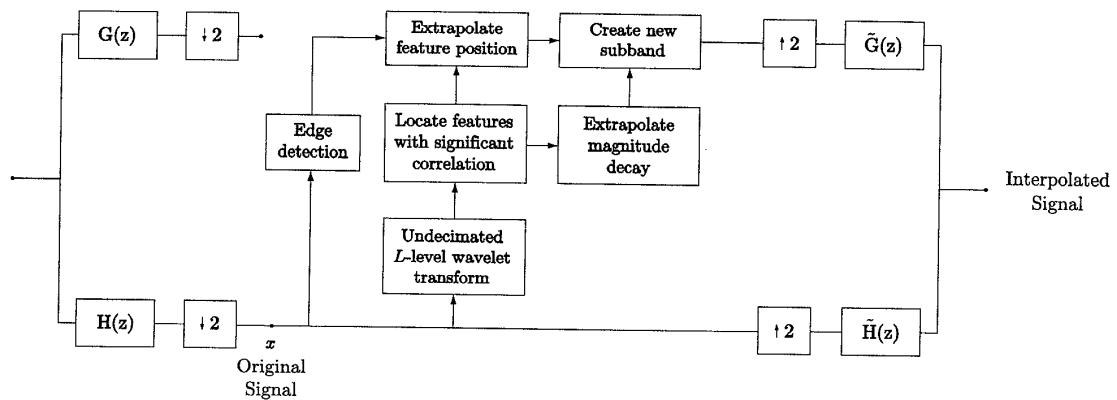


FIGURE 3: Block Diagram of Interpolation System



FIGURE 4: Interpolation Examples