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A simple 180° field of view F-theta all-sky camera

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ABSTRACT

All-sky cameras with field of view (FOV) of 180° are used in several astronomical and terrestrial applications. In its simplest form, such a camera consists of a simple photographic camera which photographs the image of the sky reflected in a spherical mirror. This design is attractive due to its comparatively low cost as the spherical mirror has very relaxed tolerances on surface shape and accuracy. However, the image such a camera produces suffers from angular distortion near the edge of the image. If instead a spherical mirror an aspherical one is used, it is possible to eliminate angular distortion and to obtain constant angular magnification over the whole 180° FOV. The derivation of the differential equation of the required aspherical surface is outlined and the results of numerical integration of the equation are described.

1. ALL-SKY CAMERA

The name "all-sky camera" originates in astronomical applications of special cameras capable of photographing the whole visible sky at once. Such cameras have field of view (FOV) of 180° and produce circular images on the film or plate. They are used for meteor photography^{1,2}, atmospheric phenomena photography and probably for several other specialized applications. Specialized 180° FOV lenses, the so called fish-eye lenses, are obtainable. They are usually expensive, big and vulnerable due to very large, exposed, front lens surface. Because of that, old and simple versions of such cameras are still attractive, at least for amateurs. In its simplest form, such simple camera is composed of an ordinary small format photographic camera and a spherical mirror^{4,5}. Camera looks into the image of the sky produced by the mirror at greatly reduced angular scale, thus making it possible to photograph the whole visible sky with ordinary camera optics. The camera is held above the mirror by a tripod or similar construction. Such a simple camera system can cover very large FOV, even greater than 180° , if needed. The spherical mirror need not be very accurate one, and even such imperfect mirrors as Christmas-tree decorations and car hubs were successfully used^{3,5}. A concave mirror can also be used, but in this case the image of the sky is in front of the mirror, thus requiring

larger separation between the mirror and the camera.

In usual case of objects at infinity, the mirror forms their image in its focal plane, and the camera looks at this image. This means that the camera must be focused to that, relatively close, image. And, as the image surface is curved, the camera lens must have adequate depth of the focus for the whole image, from center to the edge. This implies that larger separations between camera and mirror are more favorable. Closing the lens can improve the depth of the focus, but this is not a popular measure in low light level applications, as is case in astronomy. And last, but not least, the angular size of the camera image (the camera is in front of the mirror!) is reduced as the camera is moved farther away from the mirror. Images produced by such a camera always show a silhouette of the camera and camera support in the image, but, as the silhouette occupies relatively small area of the image, it is usually not a problem in most applications.

Design of such a camera is well described in ref. 4, and I will quote only a few simple formulas, derived from paraaxial equations. If we have a camera with given lens and image size, we can compute angular size of the largest circular image that fits into this image size. In our case it is wise to keep the image about 20% smaller, to compensate for focusing the lens. There is no need to calculate the effect of focusing exactly, as in the final stage the size of the image is adjusted by changing the separation between the camera and the mirror a little. The linear image sizes used for calculations in this article are shown on the Fig. 1.

The angular FOV of the camera alone (COV) is given by the well-known formula

$$\text{COV} = 2 * \arctan(r/f) \quad (1)$$

where r is useful semi-diameter of the image circle, as defined on the Fig. 1., and f is the focal length of the camera lens. As the image of the sky, which covers the whole 180° must fit inside the camera FOV, we calculate the necessary angular magnification (actually reduction) which convex mirror must produce:

$$M' = \text{COV}/180 \quad (2)$$

Now, by using paraaxial image formulas, the relation between the separation of the camera and the mirror (d) and the radius of curvature of the convex mirror (R) is found to be

$$R = 2*d/(M-1) \quad (3)$$

with

$$M = 1/M' \quad (4)$$

The sketch of such a camera is shown on the Fig. 2.

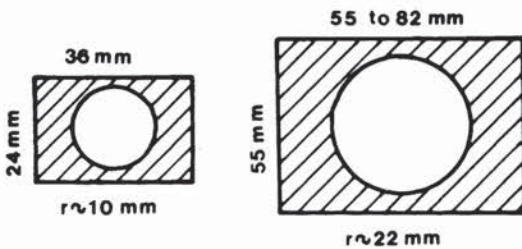


Figure 1: image produced by the all-sky camera is inscribed in the available image area. Examples are given for the two most commonly used film formats, the 24x36 mm (Leica) and the 55x55-55x82 mm (6x9) film formats.

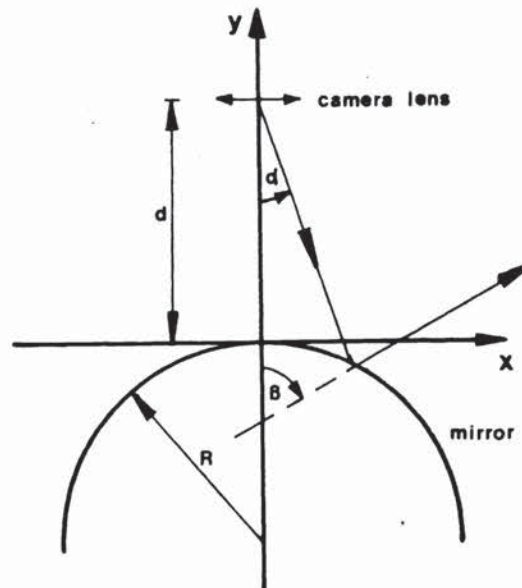


Figure 2: geometry of a simple all-sky camera. Note that the ray paths are reversed, thus simplifying analysis of the system. The camera lens is placed at T.

In paraaxial approximation such a camera has constant angular magnification M , so that

$$\beta = M \cdot \alpha \quad (5)$$

In reality, this simple relation is not valid for larger angles, and the camera suffers from angular distortion in outer parts of the image, as is illustrated on the Fig. 3. Camera for which eq. 5. holds for all image angles, is sometimes called "F-theta" camera.

2. F-THETA ALL SKY CAMERA WITH ASPHERICAL MIRROR

We have seen that spherical mirror produces reduction in angular magnification when image angles approach 90° . By using an aspherical mirror it is possible to keep angular magnification constant, even for very large image angles, well over 90° . I will outline the derivation of the equation of the required aspherical surface. The geometry of the problem is shown on the Fig. 4.

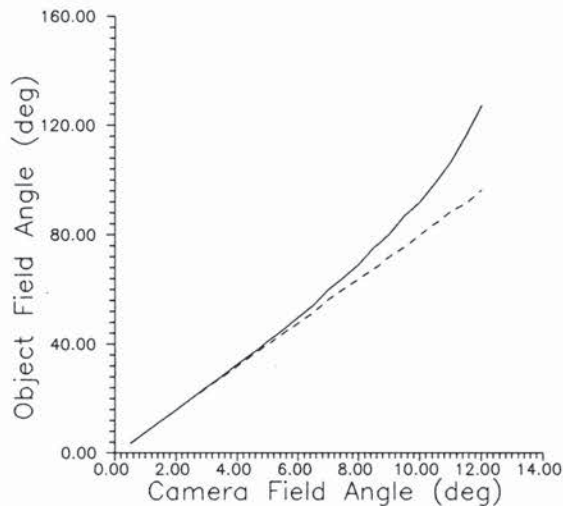


Figure 3: The angular distortion (solid line) of the simple all-sky camera using a convex spherical mirror with radius of curvature $R=0,286$ m. The 35 mm camera with a 50 mm lens is placed 1 m above the mirror. The paraaxial equation 5. with $M=8$ is drawn dotted.

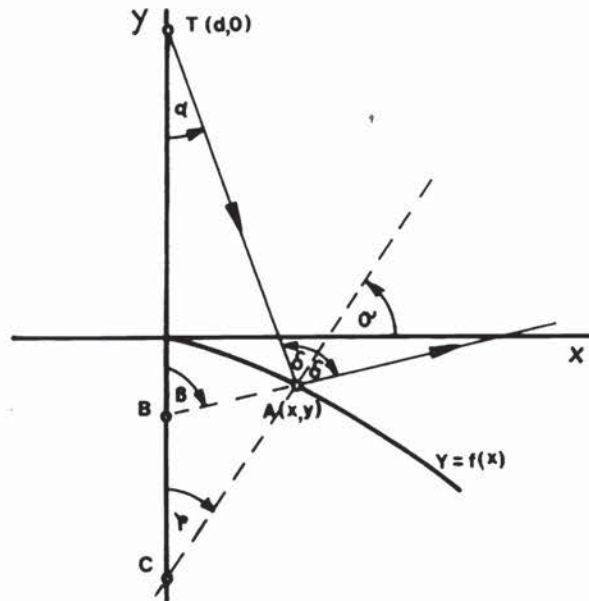


Figure 4: geometry of aspherical mirror used in an all-sky camera. For ease in analysis the ray paths are reversed, and we trace rays from the camera lens, situated at point T, to the mirror and, after the reflection, into the infinity.

The light ray goes from the point T situated on the y-axis at height d above the mirror vertex, and travels toward the mirror at angle α to the y-axis. It is reflected from the mirror at point A(x,y) toward the sky, at angle β to the y axis. Both angles are defined to be positive as indicated on the picture. The mirror surface is defined by the equation of the generating curve $y=f(x)$, and the actual mirror surface will be generated by the rotation of this curve around the y-axis. The ray path is reversed to make the derivation of the generating curve easier.

From the triangle ABT is easily found that

$$\beta = 2\delta - \alpha \quad (6)$$

and from the triangle ACT

$$\delta = \alpha + \phi \quad (7)$$

The slope of the normal to the curve $y=f(x)$ at point A can be found by using analytical geometry to be

$$\tan(\sigma) = -1 / (d(f(x))/dx)_A \quad (8)$$

From the triangle COD, $\phi = 90^\circ - \sigma$ and

$$\tan(\phi) = 1/\tan(\sigma) = -(d(f(x))/dx) \quad (9)$$

To find the coordinates (x,y) of the point A, we note that it is common to the ray and to the mirror surface. By combining equations of the ray and of the surface we find that

$$f(x) = -x/\tan(\alpha) + d \quad (10)$$

If we want to have a constant angular magnification, then for all angles α right to the edge of the camera FOV it must be

$$\beta = M * \alpha \quad (11)$$

By combining eq. 6, 7, 9 and 11 it is found that

$$d(f(x))/dx = -\tan(\alpha * (M-1)/2) \quad (12)$$

α can be eliminated by using eq. 10 to obtain the differential equation for the function f(x):

$$d(f(x))/dx = -\tan(N * \arctan(x/(d-f(x)))) \quad (13)$$

$$N = (M-1)/2$$

subject to the boundary condition $y=0$ for $x=0$ (see Fig. 4.). It is also obvious that the curve $f(x)$ is symmetrical with respect to the y-axis.

For very small angles we can approximate tangent and inverse tangent functions with the angles themselves. Also, in such a case y is much smaller than d , so it can be neglected in the denominator at the right side of eq. 13. The result of these approximations is the equation

$$f(x) = -Nx^2/2 \quad (14)$$

which is a parabola. If we calculate the radius of the curvature of this parabola at its vertex we get

$$R_0 = 2*d/(M-1) \quad (15)$$

which is exactly what we have found from simple paraaxial considerations (see eq. 3).

It should be said here that the distance from the image center to the image point at an angle from the axis is actually $f * \tan(\alpha)$ and not $f * \alpha$, so the distances measured on the film will not be strictly proportional to the field angle in object space. However, if we use normal, or even better, a telephoto camera lens, the difference is small. For instance, for 35 mm camera with a normal 50 mm lens the error at the edge of the field is about 1%, and for the 135 mm telephoto it is under 0.2%. And, as the lens itself can have residual distortion of the same order of magnitude, there is no

point in insisting on strict linear proportionality between the object field angle and the distance of object image from the center of the image.

The raytracing of mirrors discussed in this article was done with a simple, custom written computer program which was able to use tabulated curve coordinates obtained from numerical integration of eq. 13. This has resulted in slight waviness of some of the curves on raytrace graphs. This is caused by numerical inaccuracies in calculated slopes of mirror surface elements and not by the mirror surface itself.

3. NUMERICAL INTEGRATION OF THE SURFACE EQUATION

The equation 13. was numerically integrated for several magnifications, M , chosen to describe the most commonly used film sizes and lens focal lengths. The magnifications and lens parameters are listed in Table 1.

Table 1: lens focal lengths and magnifications (M) used in numerical integrations of eq. 13. Field semi-angles are calculated from the eq. 1. with r defined on the Fig. 1.

lens F.L.	FOV (semi-angle)	M
24x36 mm film size ($r=10$ mm)		
35 mm	16°	6
50 mm	11°	8
100 mm	5.7°	16
135 mm	4.2°	22
6x9 film size ($r=22$ mm)		
80 mm	16°	6
180 mm	7°	13

The distance between the camera and the mirror was normalized to $d=1$, and the numerical integration was performed only for positive x , as the mirror surface is axially symmetric. The necessary size of the mirror (maximal x) for field semi-angle of 90° was determined by raytracing. The results are tabulated in the Table 2. at the end of this paper.

The mirror shape for the case $M=8$ is shown on the Fig. 5, compared to the sphere with radius of curvature calculated from eq. 3, and shapes and sizes for various magnifications are compared on the Fig. 6. The effect of changing the distance d between the camera and the mirror is illustrated on the Fig. 7. As it can be seen, the

main effect of changing the distance between the camera and the mirror is change in the image size and magnification M , but the linearity of the combination is not impaired much. This means that the distance between the camera and the mirror is not critical, and that, what is more important, the mirror will work well with all lenses of reasonably similar FOV and thus of similar focal lengths (recall that M is determined by the FOV of the camera lens).

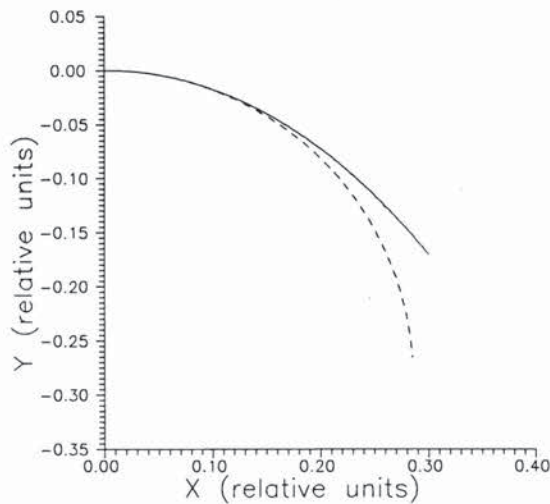


Figure 5: The shape of the aspherical mirror for $M=8$ (solid line) compared to the spherical mirror (dotted line) which produces the same paraaxial angular magnification.

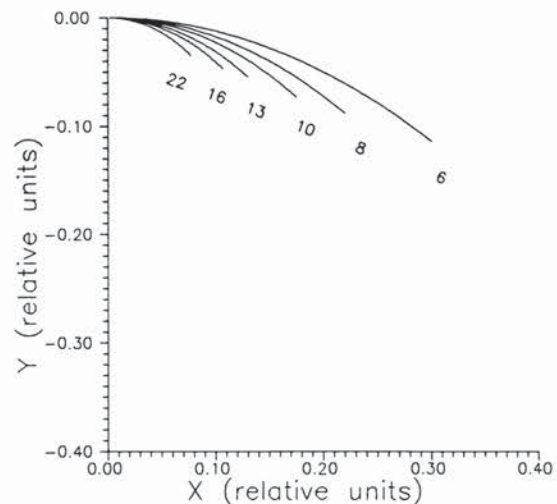


Figure 6: shapes and sizes of aspherical mirrors for various M and FOV of 180° . Mirrors for all magnifications, indicated under the corresponding curve, are truncated at the points which correspond to the semi-field of view of 90° .

The approximate parabolic equation is actually very close to the real surface shape as is illustrated on the Fig. 8. for the magnification $M=8$, and parabolic mirror can be used for field angles up to around 60° if we can accept a small level of angular distortion. The results of raytracing the parabolic mirror from Fig. 8. are shown on the Fig. 9.

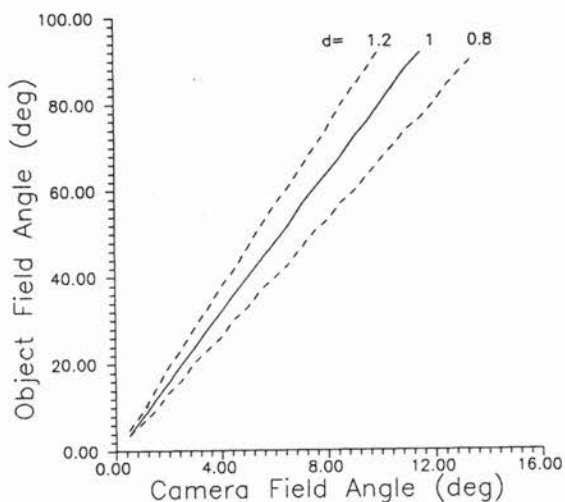


Figure 7: effect of changing the separation between the camera and the aspherical mirror. The mirror is calculated for $M = 8$ and $d=1$.

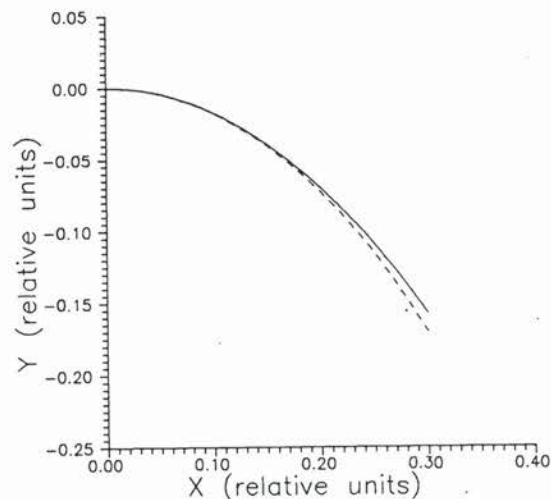


Figure 8: the parabolic shape (solid line) as approximation to the calculated aspherical shape (dashed) for $M=8$.

4. CONCLUSION

We see that it is possible to make a very wide angle camera with constant angular magnification right to the edge of the field of view by using an aspherical convex mirror. The ultimate field angle can be much greater than 180° , as it is limited only by the angular size of the mirror image itself, which is roughly equal to the circular field of view of the camera alone. By using large angular magnification of the mirror-camera system, this image can be quite small and total FOV of more than 340° can be obtained. An example of such a mirror is shown on the Fig. 10. The price paid for this is large distance between the camera and the mirror as large M implies use of long focal length camera lenses, to keep camera FOV small.

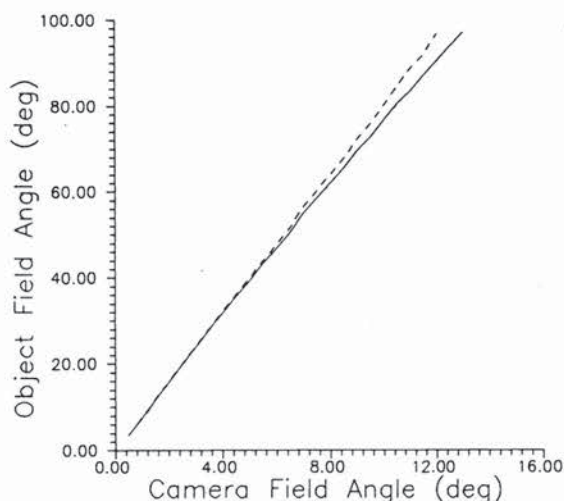


Figure 9: results of raytracing the parabolical mirror (solid) from fig 8. compared to the exact solution of eq. 13. (dashed).

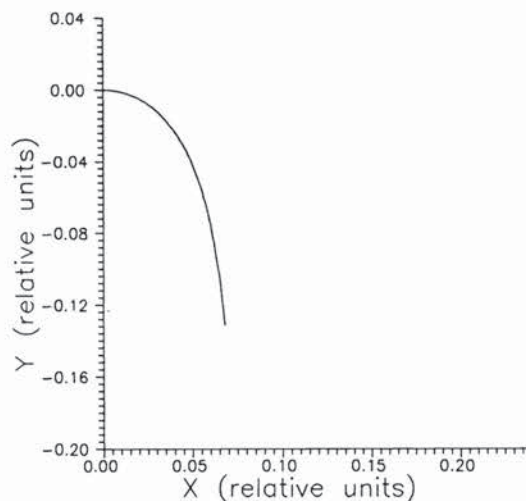


Figure 10: example of extremely wide field mirror with $M=50$ and FOV of 340° . The camera is placed at $y=1$.

Also, it should be kept in mind that camera itself will be imaged in the center of the image together with its support system, so the center of the field will be screened by the camera. And last, but not least, no attempt was made to analyze other image aberrations, as no adequate raytracing program was obtainable to me.

At the moment I can not tell if such large angular fields will ever be needed, but the possibility is here. On the other side, possibility to have a simple 180° FOV system is quite appealing for astronomical and meteorological applications. If the aspherical mirror is made in series it can be very cheap, as it can be produced in many simple ways, for instance as a polished sheet metal pressing (stainless steel or aluminum) or aluminized plastic. Such a mirror will be light and transportable and could be brought almost everywhere where the normal camera itself can be brought.

In my opinion such a mirror will be of interest to amateur astronomers, who make important contributions in those fields of astronomy where a simple instrumentation is sufficient, and where many observers are required, as is the case in the mentioned photographic meteor patrols etc. Also, all who love nature could carry such a mirror with them to photograph rare atmospheric phenomena, if they got a chance to do so. And they are going almost everywhere and at anytime.

Table 2 (next two pages): numerically calculated mirror shapes for several magnifications, M . The last tabulated points corresponds to the field semi-angle of 90° .

x	y(M=6)	y(M=8)	y(M=10)
0	0	0	0
0.0050	-0.000034	-0.000048	-0.000062
0.0100	-0.000131	-0.000184	-0.000236
0.0150	-0.000291	-0.000407	-0.000523
0.0200	-0.000513	-0.000718	-0.000923
0.0250	-0.000797	-0.001116	-0.001436
0.0300	-0.001144	-0.001603	-0.002063
0.0350	-0.001554	-0.002177	-0.002803
0.0400	-0.002026	-0.002840	-0.003657
0.0450	-0.002561	-0.003591	-0.004627
0.0500	-0.003158	-0.004430	-0.005711
0.0550	-0.003819	-0.005358	-0.006912
0.0600	-0.004542	-0.006376	-0.008230
0.0650	-0.005328	-0.007483	-0.009665
0.0700	-0.006176	-0.008679	-0.011219
0.0750	-0.007088	-0.009966	-0.012892
0.0800	-0.008063	-0.011343	-0.014686
0.0850	-0.009101	-0.012811	-0.016602
0.0900	-0.010202	-0.014370	-0.018641
0.0950	-0.011366	-0.016021	-0.020803
0.1000	-0.012594	-0.017764	-0.023091
0.1050	-0.013885	-0.019600	-0.025506
0.1100	-0.015239	-0.021529	-0.028050
0.1150	-0.016657	-0.023551	-0.030723
0.1200	-0.018139	-0.025668	-0.033529
0.1250	-0.019684	-0.027880	-0.036467
0.1300	-0.021293	-0.030188	-0.039540
0.1350	-0.022966	-0.032591	-0.042751
0.1400	-0.024704	-0.035091	-0.046100
0.1450	-0.026505	-0.037689	-0.049590
0.1500	-0.028370	-0.040385	-0.053224
0.1550	-0.030300	-0.043179	-0.057002
0.1600	-0.032294	-0.046073	-0.060927
0.1650	-0.034353	-0.049067	-0.065003
0.1700	-0.036476	-0.052162	-0.069230
0.1750	-0.038664	-0.055358	-0.073611
0.1800	-0.040917	-0.058657	
0.1850	-0.043235	-0.062059	
0.1900	-0.045618	-0.065565	
0.1950	-0.048066	-0.069176	
0.2000	-0.050579	-0.072892	
0.2050	-0.053157	-0.076714	
0.2100	-0.055801	-0.080643	
0.2150	-0.058510	-0.084680	
0.2200	-0.061285	-0.088826	
0.2250	-0.064125		
0.2300	-0.067031		
0.2350	-0.070003		
0.2400	-0.073040		
0.2450	-0.076144		
0.2500	-0.079314		
0.2550	-0.082549		
0.2600	-0.085851		
0.2650	-0.089218		
0.2700	-0.092652		
0.2750	-0.096152		
0.2800	-0.099718		
0.2850	-0.103350		
0.2900	-0.107049		
0.2950	-0.110813		
0.3000	-0.114644		

x	y(M=13)	y(M=16)	y(M=22)
0	0	0	0
0.0025	-0.000021	-0.000026	-0.000036
0.0050	-0.000079	-0.000098	-0.000138
0.0075	-0.000174	-0.000218	-0.000305
0.0100	-0.000308	-0.000385	-0.000539
0.0125	-0.000478	-0.000598	-0.000839
0.0150	-0.000687	-0.000859	-0.001205
0.0175	-0.000933	-0.001168	-0.001639
0.0200	-0.001217	-0.001523	-0.002140
0.0225	-0.001539	-0.001927	-0.002709
0.0250	-0.001899	-0.002378	-0.003347
0.0275	-0.002297	-0.002878	-0.004055
0.0300	-0.002733	-0.003426	-0.004833
0.0325	-0.003208	-0.004023	-0.005683
0.0350	-0.003721	-0.004669	-0.006605
0.0375	-0.004273	-0.005364	-0.007601
0.0400	-0.004864	-0.006110	-0.008672
0.0425	-0.005494	-0.006906	-0.009819
0.0450	-0.006163	-0.007752	-0.011045
0.0475	-0.006872	-0.008650	-0.012350
0.0500	-0.007621	-0.009599	-0.013736
0.0525	-0.008409	-0.010601	-0.015205
0.0550	-0.009238	-0.011656	-0.016760
0.0575	-0.010107	-0.012765	-0.018402
0.0600	-0.011018	-0.013927	-0.020134
0.0625	-0.011969	-0.015145	-0.021959
0.0650	-0.012962	-0.016418	-0.023879
0.0675	-0.013997	-0.017748	-0.025897
0.0700	-0.015074	-0.019134	-0.028017
0.0725	-0.016193	-0.020579	-0.030241
0.0750	-0.017355	-0.022083	-0.032574
0.0775	-0.018561	-0.023647	-0.035020
0.0800	-0.019810	-0.025272	
0.0825	-0.021104	-0.026959	
0.0850	-0.022442	-0.028709	
0.0875	-0.023825	-0.030524	
0.0900	-0.025254	-0.032404	
0.0925	-0.026729	-0.034350	
0.0950	-0.028251	-0.036364	
0.0975	-0.029819	-0.038448	
0.1000	-0.031436	-0.040603	
0.1025	-0.033100	-0.042830	
0.1050	-0.034813	-0.045130	
0.1075	-0.036576	-0.047506	
0.1100	-0.038389		
0.1125	-0.040253		
0.1150	-0.042168		
0.1175	-0.044135		
0.1200	-0.046155		
0.1225	-0.048229		
0.1250	-0.050356		
0.1275	-0.052539		
0.1300	-0.054778		

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