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Optical performance of axial gradient and aspheric surface lenses: study and analysis

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ABSTRACT

Using the caustic merit function for plane waves incident upon a singlet lens, a comparison between the optical performance of the lens using an axial gradient index with spherical surfaces or aspheric surfaces with constant index material has been investigated. Results indicate that the use of an inhomogeneous medium for the lens material is more effective in the controlling aberrations than the use of aspheric surfaces for a similar lens.

1. INTRODUCTION

In lens design, the aberrations of an optical system can be controlled by using of aspheric surfaces¹ or using an inhomogeneous medium² between the surfaces of the optical system. Sands³ has shown that the contributions of an axial gradient index (GRIN) to the third order aberrations of an optical system are equivalent to those of aspheric surfaces. However, it has been expected that inhomogeneous media would be more effective in some cases for controlling aberrations than the use of aspheric surfaces when higher order aberrations are considered. Attempts⁴ have been made to produce axial GRIN lens designs which have a comparable optical performance with an aspheric surfaces lens. These efforts have been hampered by the lack of optical design software which is capable of calculating the fifth and higher order aberration coefficients for the GRIN lens. As a result, the lens designer has not been able to study the effect of using GRIN materials on aberrations correction and balancing.

In this paper, a comparison between the effectiveness of axial GRIN materials and aspheric surfaces for controlling aberrations has been studied for the case of plane waves incident upon a singlet lens. Using the caustic merit function (CMF),⁵ two lens designs, which have the same back focal length as that of an initial lens, are produced. The first design is an axial GRIN lens with spherical surfaces, and the second design is an aspheric surfaces lens with constant index media. The optical performance of each design has been evaluated as a function of the lens parameters to arrive at an optimum design that has a minimum value of the average value of the CMF.

Starting with the parameters of the initial lens, the refractive index of the lens material is varied linearly along the optic axis of the lens while the other parameters (the radii of curvature of the surfaces and the back focal length) are kept constant. Then, values of the CMF are averaged over the entire entrance pupil and the average CMF is evaluated as a function of the GRIN parameter to reach the optimum axial GRIN design. Similarly, the optimum design of an aspheric lens has been obtained by variation of the aspherical deformation coefficients⁶ of the spherical surfaces of the initial lens while the other parameters (refractive index, thickness, and the back focal length) are kept constant. In a computational program, all the deformation coefficients⁶ of both surfaces of the lens are varied until a minimum value of the average CMF is achieved. This study has not addressed the issue of whether these designs represent global minima of the CMF and other optical merit functions.

The optical performance of the initial lens (homogeneous lens), the axial GRIN lens, the aspheric surfaces lens, and the axial GRIN lens given in Ref. [4] has been evaluated and compared for a wide range of field angles, using the average CMF and the RMS⁷ of blur circle radius. Also, for these lenses, the ray aberrations are evaluated and compared over the range of the entrance pupil radius by using the CMF.

2. THEORETICAL ANALYSIS

In general, tracing a single ray through an axial GRIN lens or an aspheric lens can be used to derive a formula for evaluating the irradiance or energy flux density^{8,9} along the path of this ray. Then, this formula can be employed to evaluate the caustic surface configurations at the focal plane of such a lens. The mathematical procedures for the derivation of the flux flow formula and the CMF are summarized below.

2.1. Ray Trace Procedures

2.1.1. Axial GRIN Lens

Consider a ray incident at the boundary surface, S_1 , of an axial GRIN lens (see Fig. 1), which has a refractive index profile as

$$n(z) = n_0(1 - \alpha z), \quad (1)$$

where n_0 is the refractive index at the vertex of the first surface of the lens, α is the gradient index parameter, and z -axis is the optic axis of the lens. Then, the solution of the ray equation¹⁰ in the axial GRIN medium gives the ray path which is expressed as

$$x = x_0 + p_0 \int_{z_0}^z \frac{dz}{l}, \quad (2)$$

$$y = y_0 + q_0 \int_{z_0}^z \frac{dz}{l}, \quad (3)$$

where p_0 and q_0 are the optical direction cosines of the ray as it leaves the surface S_1 and (x_0, y_0, z_0) are the initial coordinates of the incident point. Since n is a function of z alone, the optical direction cosines of the refracted ray through the axial GRIN medium, p and q , are invariant along the path of this ray. That is, $p = p_0$ and $q = q_0$. Also, the optical direction of this ray in the z direction, l , can be expressed as

$$l = (n^2(z) - p^2 - q^2)^{1/2} = (n^2(z) - p_0^2 - q_0^2)^{1/2}. \quad (4)$$

Combining Eqs. 1-4 and carrying out the integration gives

$$x = x_0 + (p_0/n_0\alpha) \ln Q, \quad (5)$$

$$y = y_0 + (q_0/n_0\alpha) \ln Q, \quad (6)$$

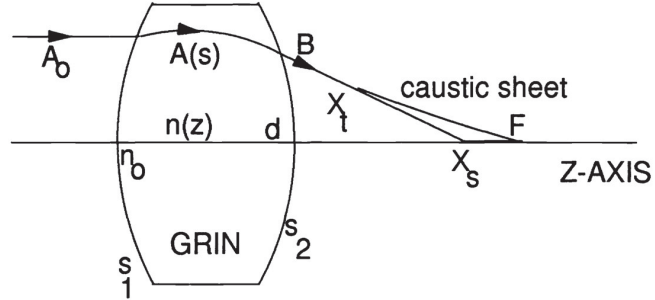


Figure 1: Schematic diagram for a ray trace in an axial GRIN lens.

where

$$\begin{aligned} Q &= (2\sqrt{z^2 + bz + c + 2z + b}) / (2\sqrt{z_0^2 + bz_0 + c + 2z_0 + b}), \\ c &= (n_0^2 - p_0^2 - q_0^2) / (n_0 \alpha)^2, \\ b &= -2/\alpha. \end{aligned}$$

The intersection point of the ray with the second surface of the lens, S_2 , and the optical direction cosines of this ray, $\vec{A}(s)$, just before this boundary, are specified by the use of a numerical ray trace technique.¹¹ Then, the optical directional cosines of the emergent ray, as it leaves the second surface are specified by the use of the laws of refraction.¹²

2.1.2. Aspheric Surface Lens

Consider a rotationally symmetric aspheric surface of an equation which is expressed as¹³

$$\vec{X}(s, \phi) = \vec{i} \cos \phi + \vec{j} \sin \phi + \vec{k} Z(s), \quad (7)$$

where

$$Z(s) = \frac{cs^2}{1 + \sqrt{1 - (cs)^2}} + e s^4 + f s^6 + g s^8 + h s^{10} + O(s^{12}),$$

$s^2 = x^2 + y^2$, and c is the vertex radius of curvature, ϕ is the polar coordinate angle measured from the positive x -axis, and the numerical coefficients (e , f , g , and h) are the aspherical deformation constants, and the term $O(s^{12})$ stands for the rest of the terms in the series which are neglected in these numerical calculations.

In this study, only meridional rays are considered. Then, following the mathematical description given in Ref. [13], (see Eqs. 6-20, Chap. 4), the incident ray is brought fairly close to the point where the ray actually crosses

the aspheric surface S_1 (see Fig. 4-2, Ref. [13]). Next, one considers the refraction process at this point and uses Eqs. 21-39 in Ref. [13] to compute the optical direction cosines of the refracted ray as it leaves the S_1 toward the second surface S_2 . Similarly, the same procedures are repeated to determine the intersection point of the ray with S_2 and the optical direction cosines of the emergent ray from this surface.

2.2 Caustic Merit Function

The configurations of the caustic surfaces⁸ in the image plane of a singlet lens can be specified throughout the use of the flux flow equation⁹ which represents the illuminance associated with each traced ray through such a lens. For an axial GRIN lens and an aspheric surfaces lens, the mathematical descriptions for the derivation of the CMF are summarized below

2.2.1. Axial GRIN Lens

The input to the flux density formula (see Eq. 28 in Ref. [9]) is obtained from the initial direction of the incident rays, the equations of the lens surfaces, the equations of the geodesic in the axial GRIN lens, and the intersection points of the traced rays with the surfaces of the lens (see Fig. 1). Since the caustic surfaces represents the loci of singularities of the flux density,¹⁴ then equating the denominator of Eq. 28 in Ref. [9] to zero gives the distances along the emergent ray, r_t and r_s , from the intersection point of this ray with S_2 to the caustic points, $\vec{X}_t$ and $\vec{X}_s$, in the tangential and sagittal planes respectively (see Fig. 1):

$$\vec{X}_t = \vec{X}(2) + r_t \vec{B}, \quad (8)$$

$$\vec{X}_s = \vec{X}(2) + r_s \vec{B}, \quad (9)$$

where $\vec{B}$ is a unit vector along the emergent ray and $\vec{X}(2)$ is the position vector of the intersection point of this ray with the second surface of the lens. Following the notation given in Ref. [5], the CMF is expressed as

$$CMF = |\vec{X}_t - \vec{X}_f| + |\vec{X}_s - \vec{X}_f|, \quad (10)$$

where $\vec{X}_f$ is the position vector of the paraxial image.

2.2.2. Aspheric Surfaces Lens

To derive the CMF for a meridional ray which is traced through an aspheric surfaces lens, the mathematical description given in Ref. [15] has been used to specify the values of r_t and r_s . Then, the use of Eqs. 8-10 specifies the value of CMF.

3. Results and Analysis

The parameters of both optimized designs, the axial GRIN and the aspheric surfaces lens, and a comparison between the optical performance of these designs with that of the initial lens are presented below.

3.1. Optimized Axial GRIN Lens

A singlet spherical surface lens of a constant refractive index is considered as a trial lens. The parameters of this lens are: surface curvatures 2.126026 and 21.45789 cm⁻¹, thickness 0.6 cm, and a refractive index 1.877091. Then, the refractive index is varied linearly along the optical axis (see Eq. 1) while the back focal length, f_b , is kept constant. For an axial GRIN lens, α can be expressed in terms of the other parameters of the lens as follows

$$f_b = f(1 - (n_0 - 1)c_1 d^*), \quad (11)$$

where

$$\begin{aligned} 1/f &= c_1(n_0 - 1) - c_2(n_d - 1) + c_1 c_2(n_0 - 1)(n_d - 1)d^*, \\ d^* &= \ln(1 - \alpha d)/(\alpha n_0), \\ n_d &= n_0(1 - \alpha d), \end{aligned}$$

c_1 and c_2 are the surface curvatures of the lens and d its thickness. Then, expanding $\ln(1 - \alpha d)$ to the third order¹⁶ and rearranging the terms of Eq. 11 gives a quadratic equation of α as

$$\alpha^2(f_b \gamma_3) + \alpha(f_b \gamma_2 - \mu d^2/2) + (f_b \gamma_1 + \mu d - 1) = 0, \quad (12)$$

where

$$\begin{aligned} \gamma_3 &= -d^3 c_1 c_2 (n_0 - 1)/2, \\ \gamma_2 &= -n_0 c_2 d + c_1 c_2 (n_0^2 - 1) d^2 / (2n_0), \\ \gamma_1 &= (n_0 - 1)(c_1 + c_2 + c_1 c_2 (n_0 + 1) d / n_0), \\ \mu &= (n_0 - 1) c_1 / n_0. \end{aligned}$$

In the optimization program, the value of d is varied and Eq. 12 is solved for the corresponding value of α . Then, the corresponding values of CMF are averaged over the entire area of the entrance pupil of radius 1.3 cm. For plane waves incident on the lens, the average CMF is evaluated as a function of α (see Fig. 2). Results show that the average CMF has a minimum value for $\alpha = 0.10887$ cm⁻¹ and $d = 0.5408$ cm. This value of α is nearly equal to that value given in Ref. [4].

3.2. Optimized Aspheric Surfaces Design

In this study, the effect of the deformation of the lens surface on the caustic merit function has been investigated for the plane waves incident upon the lens. The average CMF is evaluated as a function of the first deformation coefficient, e , of the surface S_1 while the other deformation coefficients of both lens surfaces are equated to zero (see Fig. 3). Referring to this figure, the average CMF has a minimum value for $e = -0.0075$. To obtain the final optimized design, all the deformation coefficients of both lens surfaces have been varied at the same time in the computational program. The parameters of the optimized aspheric surfaces design are given in Table 1, where the reader should be reminded that extensive efforts have not been put forth in this study to establish whether these optimized lenses represent systems with a global minima for the CMF and other optical merit functions.

3.3. Comparison of Optical Performance of Optimized Designs

To compare the optical performance of these lenses, the values of CMF are averaged over the entire area of the entrance pupil. For plane waves incident upon the following lenses: the initial, the optimized axial GRIN, the

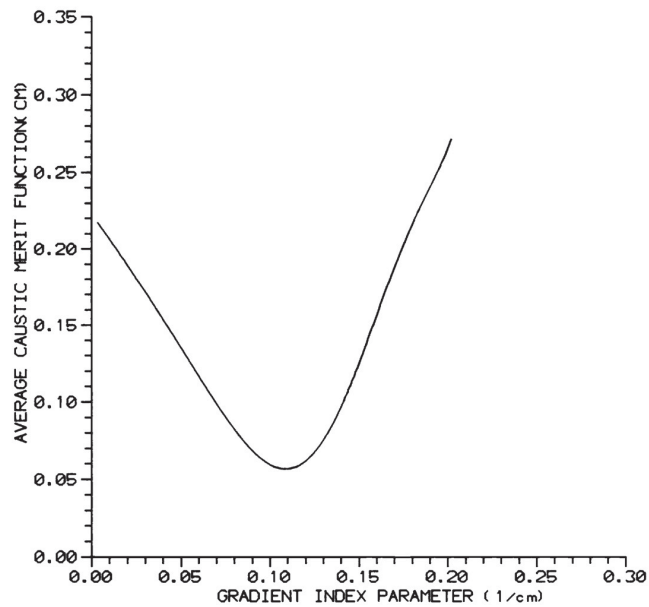


Figure 2: Average CMF as a function of the GRIN parameter.

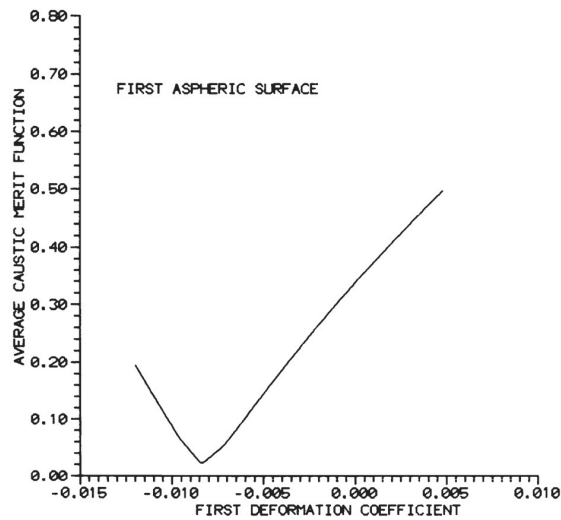


Figure 3: Average CMF as a function of the first deformation coefficient of the first surface on the lens.

Surface	1	2
c	2.126026	21.457898
e	-0.0075	-0.0030
f	-0.00016	0.0022
g	-0.00030	-0.0004
h	-0.0016	-0.0001
d	0.6	2.302234
n	1.877091	1.00

Table 1: Parameters of optimized aspheric surfaces lens.

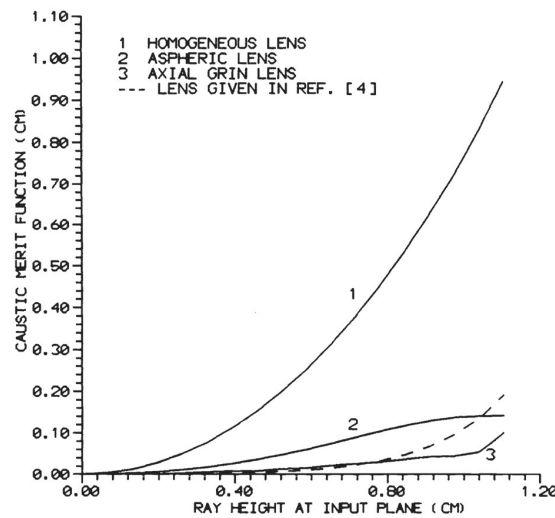


Figure 4: Comparison of the CMF as a function of the ray height at the entrance pupil.

optimized aspheric surfaces, and the axial GRIN lens given in Ref. [4], the values of average CMF are 0.44337, 0.06201, 0.07942, and 0.08914, respectively. This shows that the optimized axial GRIN has the smallest value of the average CMF when compared to other lenses in the group. Figure 4 presents the values of CMF as a function of the ray height at the entrance pupil for the group of lenses. Compared to the other lenses in the group, these results show that the axial GRIN lens has the smallest values of the CMF over the range of the entrance pupil radius. Figure 5 displays the values of the average CMF as a function of field angles. Through the range of field angle 0 - 20°, these results show that the optimized axial GRIN lens has much smaller values of average CMF than the optimized aspheric lens or initial lens and has similar performance as the GRIN lens reported in Ref. [13].

To investigate the ray distribution at the focal plane of the lens, the values of RMS blur circle radius are compared as a function of the field angles in Fig. 6. Results show that the use of axial GRIN optimization technique has reduced the values of RMS of the initial lens by 50% and 23% for the incident angles 0 and 20°, respectively, while the use of aspheric surfaces optimization technique has reduced the values of RMS by 23.7% and 12.8% for the same field angles. Figure 6 also shows that the values of RMS are very similar for the optimized axial GRIN lens and the GRIN lens given in Ref. [4].

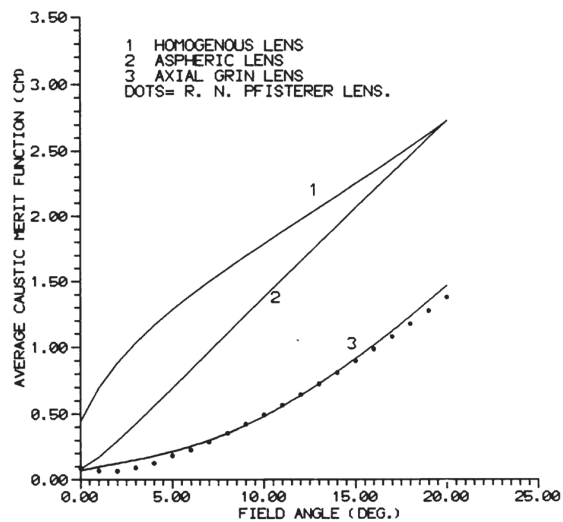


Figure 5: Comparison of average CMF as a function of field angles.

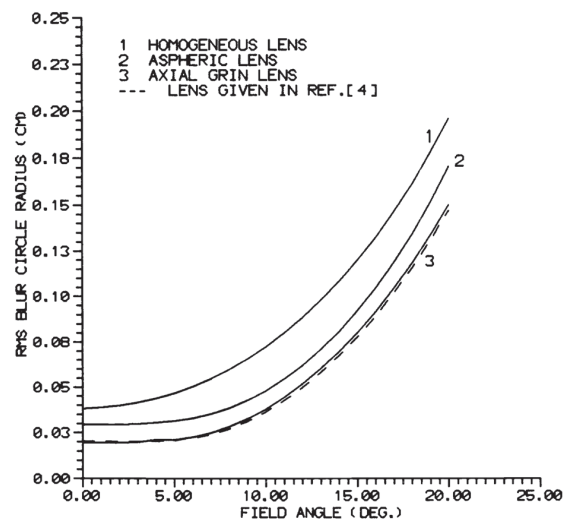


Figure 6: Comparison of RMS blur circle radius as a function of field angles.

4. CONCLUSION

The results of this study indicate that the caustic surfaces and the associated CMF, which represents spread of the image of the optical system from the ideal focal point of the system when all orders of geometrical aberrations are considered, are useful methods for optimizing the imaging performance of a GRIN lens with spherical surfaces and a constant index lens with aspherical surfaces. Further, this study suggest that an axial GRIN lens provides better optical performance than an aspherical lens with constant index media.

5. REFERENCES

1. G. G. Slyusarev, *Aberration and Optical Design Theory* (Adam Hilger Ltd, Bristol, England, 1984), Chap. 9.
2. D. T. Moore, "Design of singlet with continuously varying indices of refractions," J. Opt. Soc. Am. **61**, 886 (1971).
3. P. J. Sands, "Third order aberrations of inhomogeneous lenses," J. Opt. Soc. Am. **60.11**, 1436-1443 (1976).
4. R. N. Pfisterer, "Design of a 35-mm photographic objective using axial GRIN materials," Proc. SPIE **2000**, 359-365 (1993).
5. A. M. Kassim, D. L. Shealy and D. G. Burkhard, "Caustic merit function for optical design," Appl. Opt. **28.3**, 601-606 (1989).
6. A. Nussbaum and R. A. Phillips, *Contemporary Optics for Scientists and Engineers* (Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1976), Chap. 3.
7. T. H. Jamieson, *Optimization Techniques in Lens Design* (American Elsevier Publishing Company, New York, 1971), pp. 16-20.
8. D. G. Burkhard and D. L. Shealy, "Equation for the intensity of acoustic rays deflected by an object in a variable velocity medium," J. Acoust. Soc. Am. **56.5**, 1327-1335 (1974).
9. D. G. Burkhard, "Illuminance and caustic surfaces in multi-interface variable -index media. 1: Theory," Appl. Opt. **19.21**, 3682-3693 (1980).
10. E. W. Marchand, *Gradient Index Optics* (Academic, New York, 1978), Chap. 4.
11. A. Sharma, D. V. Kumar, and A. K. Ghatak, "Tracing rays through gradient index medium: a new method," Appl. Opt. **21.6**, 984-987 (1982).
12. W. T. Welford, *Aberrations of Optical Systems* (Adam Hilger Ltd, Bristol, England, 1986), p. 12.
13. Ref. [6], pp. 96-104.
14. D. G. Burkhard, "Illuminance and caustic surfaces in multi-surface variable -index media. 2: Application to radial dependent variable-index singlet," Appl. Opt. **19.21**, 3694-3703 (1980).
15. D. G. Burkhard and D. L. Shealy, "Simplified formula for the illuminance in an optical system," Appl. Opt. **20.5**, 897-909 (1981).
16. M. R. Spiegel, *Mathematical Handbook of Formula and Tables* (McGraw-Hill Company, New York, 1968), p. 111.