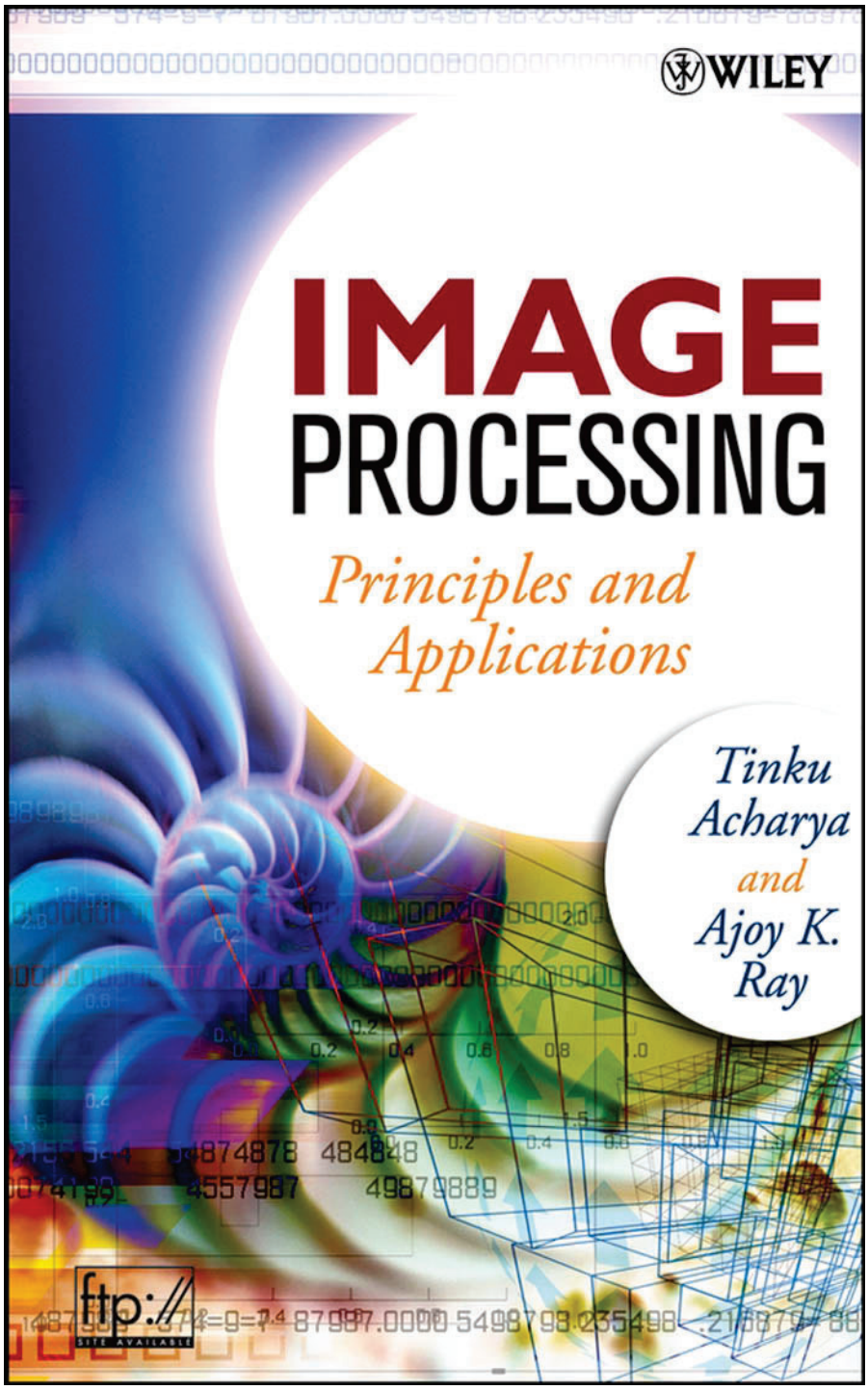

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Image Processing

Principles and Applications

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as the ideal image with additive noise, which is modelled as a random field. Sampling of such a two-dimensional random field model of images has been discussed in [5].

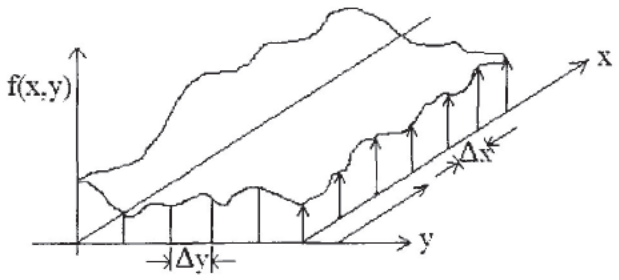

Fig. 2.4 Two-dimensional sampling array.

Let $f(x,y)$ be a continuous-valued intensity image and let $s(x,y)$ be a two-dimensional sampling function of the form

$$s(x,y) = \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} \delta(x - j\Delta x, y - k\Delta y.)$$

The two-dimensional sampling function is an infinite array of dirac delta functions as shown in Figure 2.4. The sampling function, also known as a comb function, is arranged in a regular grid of spacing Δx and Δy along X- and Y axes respectively. The sampled image may be represented as

$$\begin{aligned} f_s(x,y) &= f(x,y)s(x,y) \\ &= \sum_{j=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} f(j\Delta x, k\Delta y) \cdot \delta(x - j\Delta x, y - k\Delta y) \end{aligned}$$

The sampled image $f_s(x,y)$ is an array of image intensity values at the sample points $(j\Delta x, k\Delta y)$ in a regular two-dimensional grid. Images may be sampled using rectangular and hexagonal lattice structures as shown in Figure 2.5. One of the important questions is how small Δx and Δy should be, so that we will be able to reconstruct the original image from the sampled image. The answer to this question lies in the Nyquist theorem, which states that a time varying signal should be sampled at a frequency which is at least twice of the maximum frequency component present in the signal. Comprehensive discussions may be found in [1, 2, 4, 6].

2.3.1 Image Sampling

A static image is a two-dimensional spatially varying signal. The sampling period, according to Nyquist criterion, should be smaller than or at the most

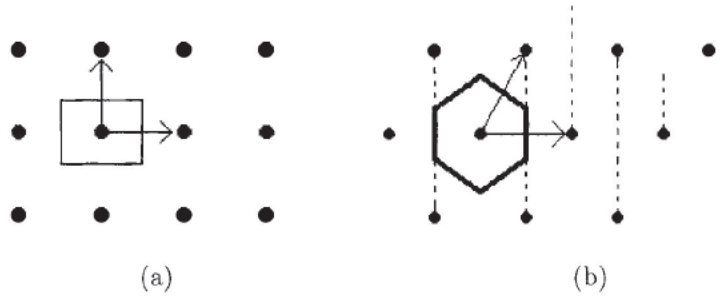

Fig. 2.5 (a) Rectangular and (b) hexagonal lattice structure of the sampling grid.

equal to half of the period of the finest detail present within an image. This implies that the sampling frequency along x axis $w_{xs} \geq 2w_x^L$ and along y axis $w_{ys} \geq 2w_y^L$, where w_x^L and w_y^L are the limiting factors of sampling along x and y directions. Since we have chosen sampling of Δx along X-axis and Δy along Y-axis, $\Delta x \leq \frac{\pi}{w_x^L}$ and $\Delta y \leq \frac{\pi}{w_y^L}$. The values of Δx and Δy should be chosen in such a way that the image is sampled at Nyquist frequency. If Δx and Δy values are smaller, the image is called oversampled, while if we choose large values of Δx and Δy the image will be undersampled. If the image is oversampled or exactly sampled, it is possible to reconstruct the bandlimited image. If the image is undersampled, then there will be spectral overlapping, which results in *aliasing effect*. We have shown images sampled at different spatial resolutions in Figure 2.6 to demonstrate that the aliasing effect increases as the sampling resolution decreases.

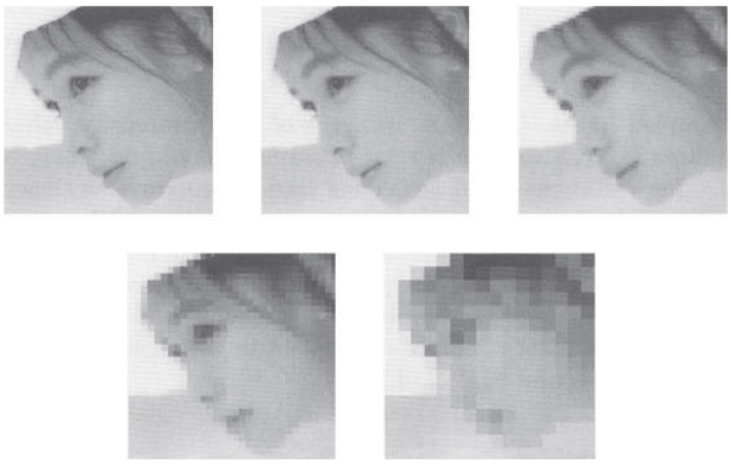

Fig. 2.6 Images sampled at 256 × 256, 128 × 128 , 64 × 64, 32 × 32, and 16 × 16 rectangular sampling grids.

If the original image is bandlimited in Fourier domain, and if the sampling is performed at the Nyquist rate, then it is possible to reconstruct the original image using appropriate sample interpolation. This property is valid in both the cases (i.e., for deterministic and random image fields). The theory of sampling in a lattice of two or more dimensions has been well documented in [7]. The aliasing errors caused by undersampling of the image has been discussed in [6]. The aliasing error can be reduced substantially by using a presampling filter. Thus by choosing a filter which attenuates the high spatial frequencies, the errors get reduced. However, if there is any attenuation in the pass band of the reconstruction filter, it results in a loss of resolution of the sampled image [2]. Reports on the results of sampling errors using Fourier and optimal sampling have been presented in [8].

2.3.2 Image Quantization

Conversion of the sampled analog pixel intensities to discrete valued integer numbers is the process of *quantization*. Quantization involves assigning a single value to each sample in such a way that the image reconstructed from the quantized sample values are of good quality and the error introduced because of quantization is small. The dynamic range of values that the samples of the image can assume is divided into a finite number of intervals, and each interval is assigned a single level. Early work on quantization may be found in [9]–[11].

Some of the interesting questions are as follows:

- How many quantized levels are sufficient to represent each sample?
- How do we choose the quantization levels?

As the number of quantization levels increases, obviously the quantized image will approximate the original continuous-valued image in a better way with less quantization error. When the quantization levels are chosen equally spaced at equal interval, it is known as uniform quantization. When the sample intensity values are equally likely to occur at different intervals, uniform quantization is always preferred. In many situations, however, the image samples assume values in a small range quite frequently and other values infrequently. In such a situation, it is preferable to use nonuniform quantization. The quantization in such cases should be such that they will be finely spaced in the small regions in which the sample values occur frequently, and coarsely spaced in other regions. The uniform and nonuniform quantization levels are shown in Figures 2.7(a) and 2.7(b) respectively. The process of nonuniform quantization is implemented by the process of companding, in which each sample is first processed by a nonlinear compressor, then quantized uniformly and finally again processed by an expander before reconstruction of the original image [11].