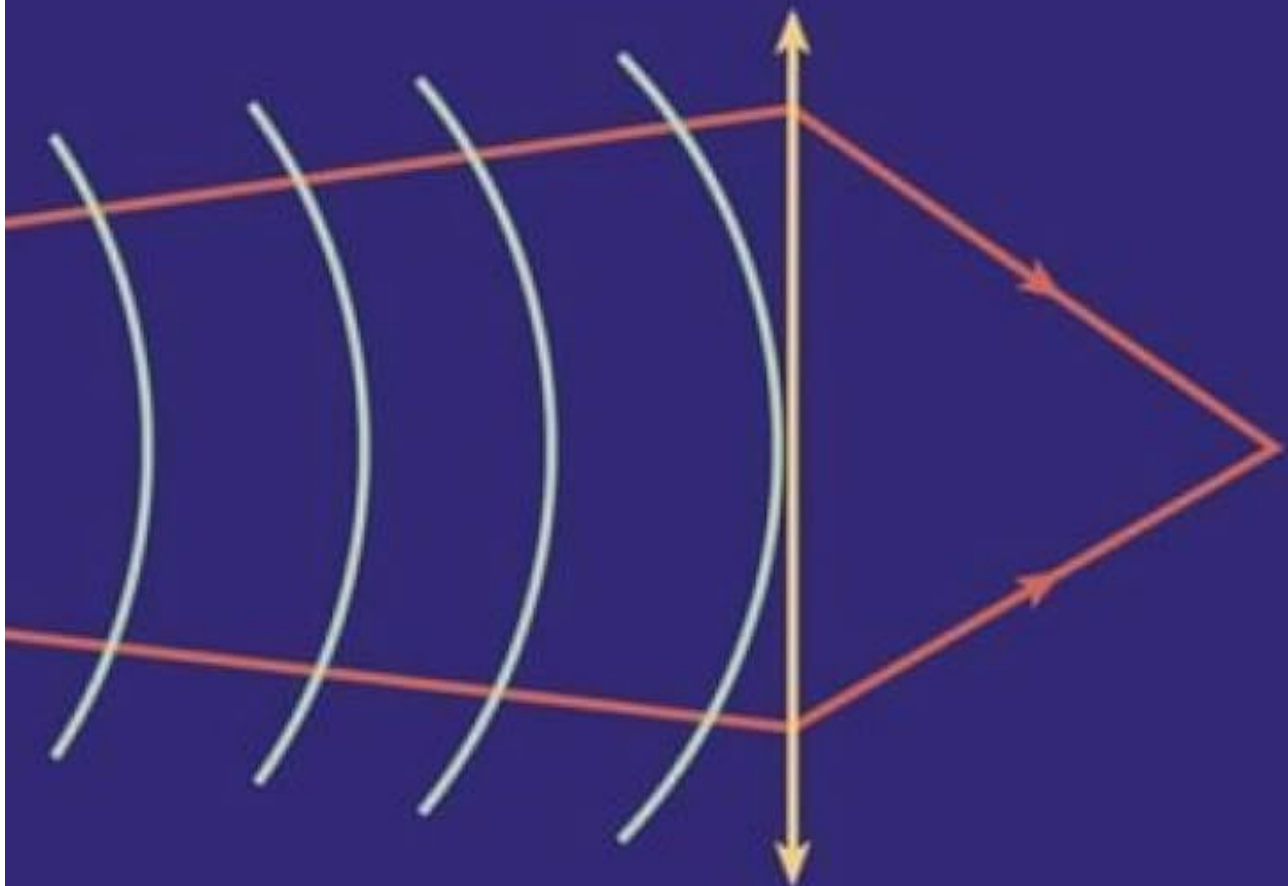


Michael P. Keating

Geometric, Physical, and Visual Optics



Second Edition



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CHAPTER 17

Angular Magnification

17.1 RELATIVE SIZE MAGNIFICATION

The foveal area of the human retina has the highest density of cones, and is the area that provides the best resolution of detail. The peripheral retina has a smaller density of cones, and provides much poorer resolution of detail. If a person's foveal area is impaired, the person will have decreased visual acuity even when the image is clear.

Increasing the size of the object can help a person with an impaired foveal area. This results in an increase in retinal image size, and enables the retina to obtain more information about the object's detail. For example, consider a person who holds a regular print book at 40 cm and has a retinal image size that is clear but too small for the detail to be resolved. The same person can perhaps hold a large print book at 40 cm and have a clear retinal image that is large enough for the detail to be resolved.

In the above situation, each book is held at 40 cm, each image is in focus on the retina, and the lateral magnification m of the observer's eye remains exactly the same when the books are switched. However, we are not so much interested in the lateral magnification as we are in the increase in the retinal image size. The ratio of the retinal image sizes is generically referred to as the *angular magnification* M . As shown below, the reason for the name is that the ratio of the retinal image sizes is equal to the ratio of the nodal angles subtended by the objects.

Figure 17.1 shows a thin lens in air eye model. The retina (screen) is a distance v_e from the lens. Figure 17.1a shows an object that has a nodal angle of w_1 , and results in a retinal image size of I_1 . Figure 17.1b shows a larger object at the same distance from the eye. The larger object has a nodal angle of w_2 and results in a retinal image size of I_2 . From Figure 17.1a,

$$\tan w_1 = \frac{I_1}{v_e}$$

From Figure 17.1b,

$$\tan w_2 = \frac{I_2}{v_e}$$

By definition,

$$M = \frac{I_2}{I_1} \quad (17.1)$$

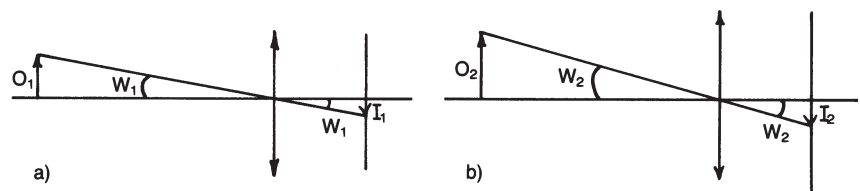
Since v_e is unchanged, it follows from the above three equations that

$$M = \frac{\tan w_2}{\tan w_1} \quad (17.2)$$

In paraxial cases, the tangent can be replaced by the angle, so

$$M = \frac{w_2}{w_1} \quad (17.3)$$

Figure 17.1 Relative size magnification.



Equation 17.3 is the mathematical statement that M is equal to the ratio of the subtended nodal angles.

EXAMPLE 17.1

Represent an unaccommodated emmetropic eye by a +60 D thin lens located 16.67 mm in front of a screen (the retina). For a set of letters located 40 cm from the lens, the ocular accommodative demand is 2.5 D and the lateral magnification is 16.67 mm/(−40 cm), which equals −0.042. What is the increase in retinal image size when viewing a 20/80 letter, which subtends an angle of 20', as compared to viewing a 20/20 letter, which subtends an angle of 5'? What is the change in the lateral magnification?

From Eqs. 17.1 and 17.3,

$$M = \frac{20'}{5'} = 4\times$$

The retinal image size increases by a factor of 4. Since the object and image distances are unchanged, the lateral magnification is unchanged.

Angular magnification is conventionally labeled with an $\times$, although the $\times$ is a dimensionless unit. There are other ways of increasing an observer's retinal image size without physically changing the object size, and these are considered in the next sections. The angular magnification due to an object size change is sometimes called *relative size magnification*.

17.2 ANGULAR MAGNIFICATION WITHOUT A PHYSICAL SIZE CHANGE

A distant hot air balloon appears as a tiny shape in the sky. Even a visually normal person cannot make out the colorful pattern on the balloon because the retinal image of the balloon is too small. As the balloon drifts closer, the observer's retinal image of the balloon grows in size until the pattern is resolvable.

Alternatively, an observer can make out the pattern on the distant balloon by looking through a telescope. The telescope optically magnifies the image of the balloon, and the pattern then becomes resolvable. Similarly, when a near object is too small for resolution of the fine detail, a magnifying lens or a microscope can be used to increase one's retinal image size.

Thus, without changing the physical size of the object, people can increase their retinal image sizes either by bringing the object closer or by using a magnifying system, such as a converging lens, a telescope, or a microscope. Just as in relative size magnification, the increase is quantified by comparing the magnified retinal image size to the original unmagnified retinal image size. Again, the magnification of interest is not lateral magnification, which is a comparison of an image size to its conjugate object size, but rather is a comparison of one image size to another image size. As above, *angular magnification* is the generic name for the ratio of one retinal image size to another.

Figure 17.2 shows the comparison for an object at two different distances. The person's eye is represented by the thin lens and screen. The object has size O and is initially at position 1, which is a distance of u_1 from the eye. The conjugate image size, determined by the nodal ray, is I_e . When the object is moved in to position 2 (which is a distance u_2 from the eye) and the person accommodates appropriately, then the retinal image is clear and the size is I_m . The angular magnification M is specified by

$$M = \frac{I_m}{I_e} \quad (17.4)$$

The angles w_e and w_m subtended by the images at the nodal point of the lens are given by

$$\tan w_m = I_m/v_e$$

and

$$\tan w_e = I_e/v_e$$

Figure 17.2 Relative distance magnification.

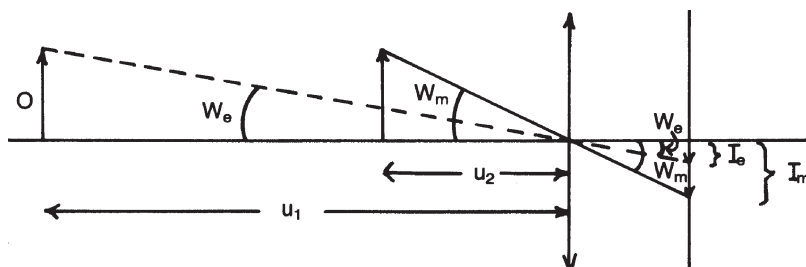
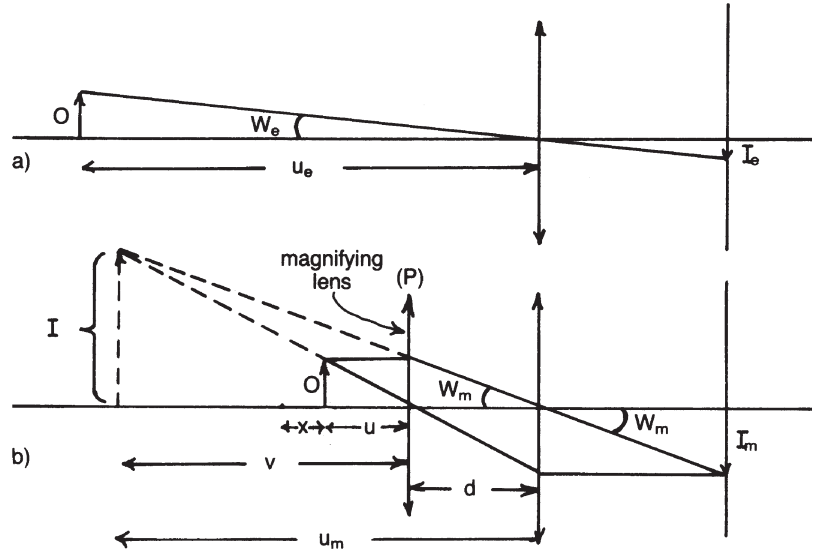


Figure 17.3 a, Unaided eye; b, eye aided by simple magnifier.



When accommodation occurs, the power of the lens changes but the lens–screen distance v_e remains the same. Then from the above three equations,

$$M = \frac{I_m}{I_e} = \frac{\tan w_m}{\tan w_e} \quad (17.5)$$

In the small angle approximation, which is accurate in the paraxial region, the tangents can be replaced by the angles and then

$$M = \frac{w_m}{w_e} \quad (17.6)$$

Equation 17.6 shows that the ratio of retinal image sizes is equal to the ratio of the nodal angles. In effect, one can increase (magnify) the retinal image size by increasing (magnifying) the angle of the nodal ray incident on the eye.

Figure 17.3 shows the situation for a person viewing a near object directly, and then viewing the near object through the magnifying lens. The retinal image size and the nodal angles are labeled the same as in Figure 17.2. Equations 17.4 through 17.6 again give the angular magnification.

It is important to note that angular magnification is always a comparison of one situation versus another. To emphasize this comparison, consider the relation between angular magnification and lateral magnification for the case in which the physical size of the object is unchanged. For the situation shown in Figure 17.3a, the lateral magnification m_1 for the direct viewing is defined as

$$m_1 = I_e/O$$

while the lateral magnification m_2 for the total magnifying lens–eye system (Figure 17.3b) is defined as

$$m_2 = I_m/O$$

Since the object size O is unchanged in the two different situations, one can use the above two equations together with Eq. 17.4 to obtain

$$M = \frac{m_2}{m_1} \quad (17.7)$$

Equation 17.7 shows that the angular magnification is equal to the ratio of the lateral magnifications, provided the object size remains unchanged.

In particular, the angular magnification M can be large even when the lateral magnification m_2 is small. In order for this to happen, m_1 has to be much smaller than m_2 . Similarly, the angular magnification M can be small even when m_2 is large. This occurs whenever m_1 is much larger than m_2 . Consequently, when dealing with angular magnification, it is important to keep the initial viewing conditions in mind.

(As an analogy, consider Pat and Mike, who have just returned from a casino each with \$3000. Are they happy or unhappy about their winnings? The answer depends on the initial amount that each went to the casino with. Pat went to the casino with \$65, while Mike went to the casino with \$9000. Even though both left the casino with \$3000, Pat is happy but Mike is not.)

17.3 RELATIVE DISTANCE MAGNIFICATION

As stated in Section 17.2, the retinal image size of an object can be increased by bringing the object closer to

the eye. The angular magnification that results in this case is sometimes referred to as *relative distance magnification*.

In Figure 17.2, the object is originally at a distance u_1 from the eye, and then is moved in to a distance u_2 from the eye. In each case, the angle subtended by the object at the nodal point is equal to the angle subtended by the image at the nodal point. From the triangles involving the object,

$$\tan w_e = O/(-u_1)$$

and

$$\tan w_m = O/(-u_2)$$

Then from Eq. 17.5, the angular magnification equals the ratio of the object distances, or

$$M = \frac{(u_1)}{(u_2)} \quad (17.8)$$

Again, the angular magnification M is dimensionless but is conventionally labeled with an $\times$. For example, an angular magnification of $9\times$ indicates a ninefold increase in the retinal image size (or in the angle of the eye's nodal ray). Note that as the object is moved closer to the eye, the person's accommodative demand increases. The person's near point is the closest the object can get and still be seen clearly.

EXAMPLE 17.2

G.O. Student has her notes propped on her desk 100 cm from her eyes, and is having trouble reading the fine print. She then moves the notes in to 20 cm from her eyes. What angular magnification does G.O. achieve? (i.e., What is the increase in G.O.'s retinal image size?)

From Eq. 17.8,

$$M = \frac{(-100 \text{ cm})}{(-20 \text{ cm})} = +5\times$$

G.O.'s retinal image increases by a factor of 5 when the notes are moved in.

EXAMPLE 17.3

G.O.'s homework solutions are 25 cm in front of her eyes. What angular magnification does G.O. achieve when she brings the homework solutions in to 20 cm in front of her eyes?

From Eq. 17.8,

$$M = \frac{(-25 \text{ cm})}{(-20 \text{ cm})} = +1.25\times$$

In each of the above two examples, the object ended up 20 cm from the eye, but the angular magnification

(or increase in retinal image size) is different because the starting conditions are different.

As a final note, it should be obvious that if the object is moved away from the eye instead of closer, the retinal image size will decrease and M will be less than one. For example, when the object at 20 cm is moved to 25 cm from the eye, the angular magnification due to the distance change is $0.80\times$. M less than one indicates a decrease in the retinal image size.

17.4 PLUS LENS AS A SIMPLE MAGNIFIER: INTUITIVE METHOD

When an object is placed inside the primary focal point of a plus (converging) lens, the conjugate image is virtual, erect, and larger. Thus, one can use a plus lens as a simple magnifying lens as shown in Figure 17.3.

In order to determine the angular magnification, we must first establish the initial conditions. Assume that initially the person directly views the object at a distance u_e in front of the eye (Figure 17.3a). The angle subtended by the object at the eye's nodal point is equal to the angle subtended by the retinal image at the nodal point, and both are labeled w . Then,

$$\tan w_e = O/(-u_e)$$

Figure 17.3b shows the person viewing the object through the magnifying lens. Note that the actual object to eye distance is different from u_e . The object is a distance of u in front of the magnifying lens, and the lens is a distance d in front of the eye. The virtual image (size I) is a distance of v from the magnifier and u_m from the eye. From the nodal ray for the eye,

$$\tan w_m = I/(-u_m)$$

From Eq. (17.5), the angular magnification M is then

$$M = \frac{I(u_e)}{O(u_m)}$$

I/O is the lateral magnification of the magnifier. Further, the ratio of the eye's object distances is just the angular magnification due to the distance changes (i.e. the relative distance magnification). Therefore, the total angular magnification M is equal to the product of the lateral magnification m_{lens} of the magnifying lens and the relative distance magnification M_{dis} or

$$M = m_{\text{lens}}M_{\text{dis}} \quad (17.9)$$

EXAMPLE 17.4

Ranger Rick, an emmetrope, is examining a small insect 20 cm in front of his eye. Rick then