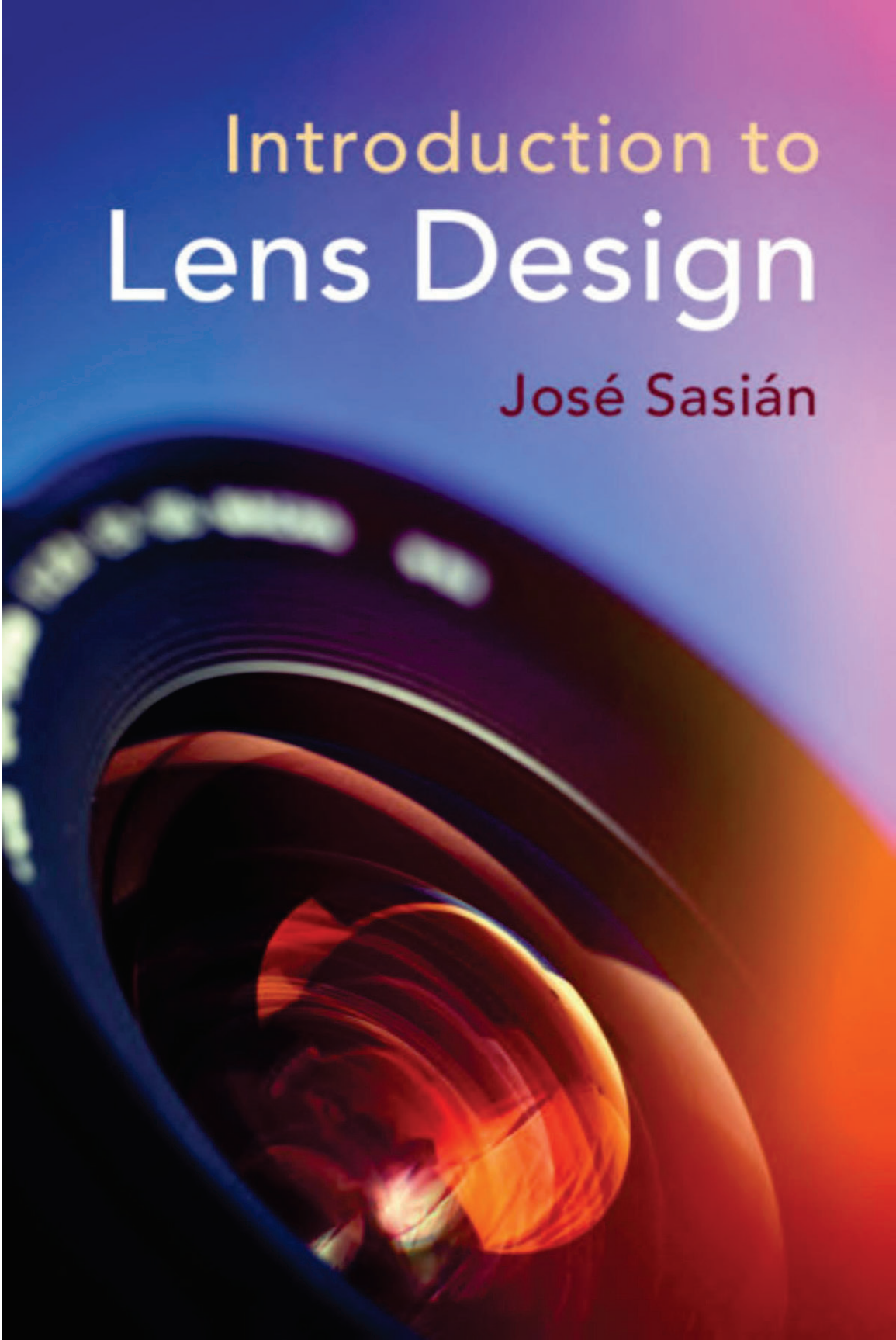


The background of the cover is an abstract, artistic representation of a lens or optical system. It features a series of concentric, curved lines that create a sense of depth and perspective, resembling the internal structure of a lens or a tunnel. The colors transition from a deep blue at the top to a vibrant orange and red at the bottom, with a purple hue in the middle. The overall effect is one of dynamic movement and optical complexity.

Introduction to Lens Design

José Sasián

Introduction to Lens Design

Optical lenses have many important applications, from telescopes and spectacles, to microscopes and lasers. This concise, introductory book provides an overview of the subtle art of lens design. It covers the fundamental optical theory, and the practical methods and tools employed in lens design, in a succinct and accessible manner. Topics covered include first-order optics, optical aberrations, achromatic doublets, optical relays, lens tolerances, designing with off-the-shelf lenses, miniature lenses, and zoom lenses. Covering all the key concepts of lens design, and providing suggestions for further reading at the end of each chapter, this book is an essential resource for graduate students working in optics and photonics, as well as for engineers and technicians working in the optics and imaging industries.

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Introduction to Lens Design

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6

Radiometry in a Lens System

Lens systems produce images by transferring radiant energy. At any plane transverse to the optical axis in an optical system there is a light distribution that may be subject to specifications. The light distribution is modeled with the laws of radiometry. To have a broader understanding about how optical systems work it is relevant to discuss how radiometric aspects impact the design of a lens system. In particular, we are concerned with the light distribution at the exit pupil and image planes. This chapter discusses basic and useful radiometric concepts in a lens system.

6.1 The Pinhole Camera

Let us consider a pinhole camera, as shown in Figure 6.1, where ds' is an element of area of the pinhole, and s' is the distance to the observation plane along the optical axis. At the observation plane there is an element of area, da' . Behind the pinhole aperture, which may be large in diameter, there is a Lambertian source of radiance, L_0' , in units of $\text{W/m}^2\text{-sr}$. The element of optical flux, $d\phi'^2$, in Watts, transferred to the element of area, da' , is:

$$d\phi'^2 = L_0' \frac{ds' da'}{s'^2} \cos^4(\theta'). \quad (6.1)$$

Then the element of irradiance, dI' , in W/m^2 , at any point in the observation plane is given by,

$$dI' = L_0' \frac{ds'}{s'^2} \cos^4(\theta'), \quad (6.2)$$

where θ' is the angle with respect to the optical axis of the line defined by ds' and da' .

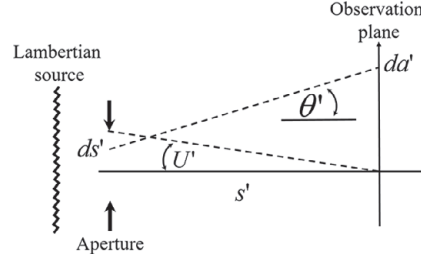


Figure 6.1 Geometry of a pinhole camera.

The irradiance in the optical axis, I_0' , is obtained by summing over all points of the pinhole aperture, which is assumed to be circular and to have a radius, a . This is given by,

$$\begin{aligned} I_0' &= 2\pi \frac{L_0'}{s'^2} \int_0^a r \left(\frac{r}{\sqrt{r^2 + s'^2}} \right)^4 dr = 2\pi L_0' \int_0^{U'} \sin(\zeta) \cos(\zeta) d\zeta \\ &= \pi \left(\frac{L_0'}{n'^2} \right) n'^2 \sin^2(U'). \end{aligned} \quad (6.3)$$

The quantity $\frac{L_0'}{n'^2} = cte$ is known as the radiance theorem, and remains invariant in an optical system that does not lose light; for example, by material absorption. Therefore, the on-axis irradiance, I_0' , is proportional to the square of the index of refraction of the observation media and proportional to the square of the sine of the semi-angle subtended by the pinhole aperture. The closer the observation point is to the pinhole camera, the larger the irradiance, I_0' , becomes.

For radiometric purposes and to first-order, light from the exit pupil of a lens system can be modeled like that of a pinhole camera. Then, the on-axis irradiance at the image plane of a lens system is given by,

$$I_0' = \pi \frac{L_0'}{n'^2} NA^2, \quad (6.4)$$

where $NA = n' \sin(U')$ is the numerical aperture of the lens system in image space.

Using the approximation,

$$\sin^2(U') \simeq \frac{1}{4(F/\#)^2}, \quad (6.5)$$

in the relationship for I_0' , allows one to conclude that the image brightness in the optical axis is inversely proportional to the square of the $F/\#$ at which an

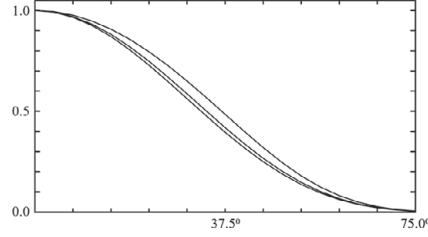


Figure 6.2 Relative illumination of pinhole cameras at $F/100$ (bottom curve), $F/2$ (intermediate curve), and at $F/1$ (top curve). The field of view is in degrees in the horizontal axis from 0° to 75° .

objective lens is working. Thus, to cast bright images, lenses with small $F/\#$ s are required.

6.2 Pinhole Camera Relative Illumination

If the aperture is small, i.e., a pinhole, and has an area ds' , then, for an off-axis observation point, the element of irradiance is given by,

$$dI' = \frac{L_0' ds'}{s'^2} \cos^4(\theta'). \quad (6.6)$$

The relative illumination at the observation plane is defined as the ratio of the off-axis irradiance to the on-axis irradiance, and is simply given by,

$$RI = \cos^4(\theta'). \quad (6.7)$$

This relation is known as the cosine⁴ law of illumination, and serves as a reference to describe the relative illumination of a lens system.

A pinhole can have different sizes and, in consideration of the distance to the observation plane, different $F/\#$'s can be defined. Figure 6.2 shows the relative illumination of pinhole cameras operating at $F/100$, $F/2$, and $F/1$. As can be seen, there is little difference in the relative illumination when above $F/2$.

The transverse position of the pinhole with respect to the optical axis is specified by $\vec{\rho}$, and the observation point is specified by $\vec{H}$. Then the relative illumination is given in terms of the first-order marginal, u' , and chief, $\bar{u}'$, ray slopes as,

$$\begin{aligned} RI(\vec{H}, \vec{\rho}) &= \cos^4(\theta') \\ &= 1 - 2u'^2(\vec{\rho} \cdot \vec{\rho}) - 4u'\bar{u}'(\vec{H} \cdot \vec{\rho}) - 2\bar{u}'^2(\vec{H} \cdot \vec{H}) + \dots \end{aligned} \quad (6.8)$$

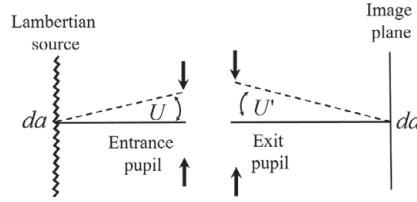


Figure 6.3 Representation of an optical system imaging a Lambertian source.

This relation serves as a reference to describe irradiance changes in an optical system.

6.3 Ratio of On-Axis Irradiance to Exitance in an Optical System

Let us consider now a lens system imaging a Lambertian source of radiance, L_0 . The aperture stop and the entrance pupil coincide in object space. If da is an element of area on the optical axis, then the element of optical flux emitted by the source is given by,

$$d\phi = 2\pi L_0 da \int_0^U \sin(\zeta) \cos(\zeta) d\zeta = \pi L_0 da \sin^2(U), \quad (6.9)$$

and an on-axis exitance in object space is given by,

$$I_0 = \pi L_0 \sin^2(U). \quad (6.10)$$

The angle, U , is the semi-angle subtended by the entrance pupil, as shown in Figure 6.3.

The ratio of the on-axis irradiance to exitance in the object and image planes is then,

$$\frac{I_0'}{I_0} = \frac{L_0'}{L_0} \frac{\sin^2(U')}{\sin^2(U)}. \quad (6.11)$$

Recalling the radiance theorem, which states that the ratio of radiances in different media is equal to the ratio of the square of the indices of refraction,

$$\frac{L_0'}{L_0} = \frac{n^2}{n'^2}, \quad (6.12)$$

and expressing the real ray angles, U and U' , as a function of the marginal and chief first-order ray slopes, allows us to write,

$$\frac{I_0'}{I_0} = \frac{n'^2}{n^2} \frac{\sin^2(U')}{\sin^2(U)} \simeq \frac{n'^2}{n^2} \frac{1+u^2}{u^2} \frac{(u' + \Delta u')^2}{1 + (u' + \Delta u')^2} \simeq \frac{1}{m^2} \left(1 + 2 \frac{W_{131}}{\mathcal{K}} \right), \quad (6.13)$$

where the transverse magnification is m , and where we have used the relationships $\frac{\Delta u'}{u'} = \frac{\overline{W}_{311}}{\mathcal{K}}$ and $\overline{W}_{311} = W_{131} + \frac{1}{2}\mathcal{K}\Delta(u^2)$. The increment in slope, $\Delta u'$, is included as the marginal real ray may not coincide with the first-order marginal ray in image space.

If the optical system obeys the sine condition, $u \sin(U') = u' \sin(U)$, there is no coma, $W_{131} = 0$, and the ratio of the on-axis irradiance to exitance is inversely proportional to the square of the transverse magnification. In the presence of coma aberration, W_{131} , or equivalently in the presence of pupil distortion, $\overline{W}_{311}$, the ratio of on-axis irradiance to exitance changes. Effectively, the exit pupil increases or decreases in size, changing U' , and then making the on-axis image point brighter or dimmer.

6.4 Lens System Relative Illumination

Let us consider an optical system where the aperture stop and the entrance pupil coincide. The exit pupil, being the image of the aperture stop in image space, can be distorted, not only for the on-axis beam but for off-axis beams. As the exit pupil distorts the image irradiance changes and, therefore, the relative illumination does too. To a second-order of approximation, the relative illumination, RI , of a system can be written as,

$$RI(\vec{H}) = 1 - \left(2\bar{u}'^2 - \frac{4}{\mathcal{K}} \overline{W}_{131} \right) (\vec{H} \cdot \vec{H}). \quad (6.14)$$

In the absence of pupil coma, $\overline{W}_{131}$, the relative illumination is that of a pinhole camera. In the presence of pupil coma, the exit pupil changes size and is anamorphically distorted. This distortion changes the convergence of the focusing beams and, thereby, the irradiance. Because pupil coma is related to image distortion, $\overline{W}_{131} = W_{311} + \frac{1}{2}\mathcal{K}\Delta(\bar{u}^2)$, the change in relative illumination can also be explained as a result of the decrease or increase of the imaging area for a given amount of optical flux.

If the stop is located at the exit pupil, then the relative illumination is, to second-order, that of a pinhole camera,

$$RI(\vec{H}) = 1 - 2\bar{u}'^2 (\vec{H} \cdot \vec{H}). \quad (6.15)$$

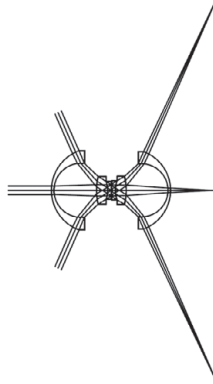


Figure 6.4 A wide angle objective lens following a $\cos^3$ illumination law.

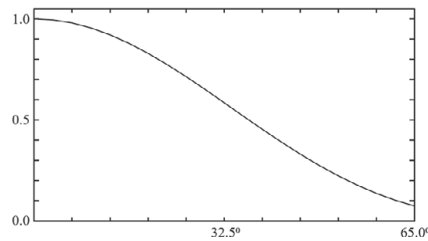


Figure 6.5 Relative illumination of the Roossinov lens. The field of view is in degrees in the horizontal axis from 0° to 65° .

Because pupil coma can affect the relative illumination, some wide-angle lenses take advantage to compensate for the $\cos^4$ law of illumination. The Roossinov lens of 1950 (US Patent 2,516,724), as shown in Figure 6.4, was designed as a nearly symmetrical lens where each half contributes a large amount of pupil coma which nearly cancels by symmetry. However, because the aperture stop is in the middle of the lens, only pupil coma that is contributed by the second half of the lens counts for the lens relative illumination; this pupil coma makes the lens to have a $\cos^3$ law of illumination, as shown in Figure 6.5.

The exit pupil as a function of the field of view is increasingly magnified and anamorphically distorted, as shown in Figure 6.6.

6.5 Light Vignetting

The aperture stop defines the relative aperture of the on-axis beam. It also defines the aperture for off-axis beams. However, there might be other

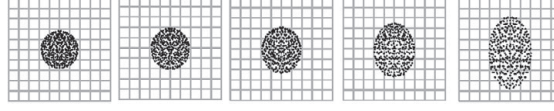


Figure 6.6 Spot diagrams showing the cross-section of different off-axis beams at the exit pupil plane. From left to right, on-axis to off-axis field positions.

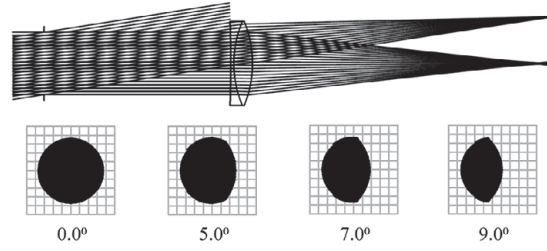


Figure 6.7 For off-axis beams the physical size of the lens aperture limits the amount of light. The loss of light increases as the field of view increases.

apertures in an optical system that limit the amount of light for off-axis beams. Rays of light can be obstructed, or clipped, for example by the physical size of a lens. This loss of light in an off-axis beam is known as light vignetting. Figure 6.7 shows how beam clipping can take place for off-axis beams. Because there is light loss, the relative illumination decreases.

However, in some lenses, light vignetting is introduced on purpose to suppress rays that have aberration beyond tolerances. Light vignetting has been used extensively in photographic lenses to reduce aberration at low $F/\#$ s and to reduce the overall lens diameter and weight.

6.6 Irradiance at the Exit Pupil Plane

It is of interest to describe the irradiance at the exit pupil plane of a lens system. We assume that the stop aperture coincides with the exit pupil, and that the object is a Lambertian source that is circular to maintain axial symmetry. The function,

$$\bar{I}(\vec{H}, \vec{\rho}) = \bar{I}_{000} + \bar{I}_{200}(\vec{\rho} \cdot \vec{\rho}) + \bar{I}_{111}(\vec{H} \cdot \vec{\rho}) + \bar{I}_{020}(\vec{H} \cdot \vec{H}) \dots, \quad (6.16)$$

provides the irradiance at the exit pupil as a function of the field, $\vec{H}$, and aperture, $\vec{\rho}$, vectors. Figure 6.8 shows graphically the second-order terms $\bar{I}_{200}(\vec{\rho} \cdot \vec{\rho})$, $\bar{I}_{111}(\vec{H} \cdot \vec{\rho})$, and $\bar{I}_{020}(\vec{H} \cdot \vec{H})$, which represent irradiance aberrations.

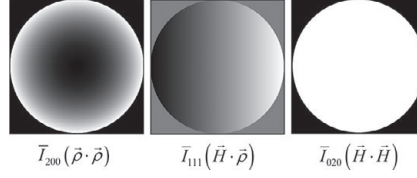


Figure 6.8 Second-order terms of irradiance changes at the exit pupil of a lens system.

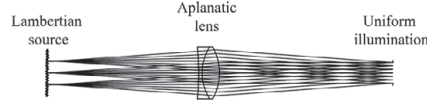


Figure 6.9 An aplanatic lens can be used to produce uniform illumination from a Lambertian source.

With $\bar{I}_{000} = 1$, the second-order coefficients in terms of image aberration coefficients are,

$$\bar{I}_{020}(\vec{H} \cdot \vec{H}) = \left[-2\bar{u}^2 - \frac{4}{\mathcal{K}} W_{311} \right] (\vec{H} \cdot \vec{H}), \quad (6.17)$$

$$\bar{I}_{111}(\vec{H} \cdot \vec{\rho}) = \left[-4\bar{u}'\bar{u} - \frac{4}{\mathcal{K}} W_{220} - \frac{6}{\mathcal{K}} W_{222} \right] (\vec{H} \cdot \vec{\rho}), \quad (6.18)$$

and

$$\bar{I}_{200}(\vec{\rho} \cdot \vec{\rho}) = \left[-2\bar{u}^2 - \frac{4}{\mathcal{K}} W_{131} \right] (\vec{\rho} \cdot \vec{\rho}). \quad (6.19)$$

If a lens system is free from coma aberration, $W_{131} = 0$, and collimates light, $\bar{u}' = 0$, then the illumination at the exit pupil would be uniform. This comes about because, for an axially symmetric object, the contributions of the term $\bar{I}_{111}(\vec{H} \cdot \vec{\rho})$ will cancel, i.e., $\bar{I}_{111}(\vec{H} \cdot \vec{\rho}) + \bar{I}_{111}(-\vec{H} \cdot \vec{\rho}) = 0$, the term $\bar{I}_{020}(\vec{H} \cdot \vec{H})$ does not depend on the aperture vector, $\vec{\rho}$, at the exit pupil, and the term $\bar{I}_{200}(\vec{\rho} \cdot \vec{\rho})$ will vanish.

An aplanatic lens, as shown in Figure 6.9, will produce uniform illumination at the exit pupil when the object is a Lambertian source. If the lens is aplanatic, but is imaging $\bar{u}' \neq 0$, like a microscope objective used in reverse, the irradiance at the exit pupil would follow a cosine⁴ law to second-order.

6.7 Optical Étendue

The optical flux, ϕ , transferred from a source of radiance, L_0 (here assumed uniform), and area, A_s , to an optical component or lens system is given by,

$$\phi = \frac{L_0}{n^2} \varepsilon, \quad (6.20)$$

where ε is the optical étendue associated with the optical component or system. Two first-order approximations to the étendue are $\varepsilon = \pi n^2 A_s \sin^2(U)$ and $\varepsilon = \pi^2 \mathcal{K}^2$. The étendue gives further meaning to the Lagrange invariant, $\mathcal{K}$, and it is a measure of the capacity of an optical component or lens system to transfer optical flux.

Every optical element or system has associated an étendue and, in order to avoid loss of light, the étendue of all the components of a system must be equal or greater than the étendue that defines the light gathering properties of the system. For example, the pixel size of a sensor often defines the $F/\#$ of an imaging system, and, given the sensor size area, the minimum étendue of the system is established. Then all components in the system must have at least the sensor's étendue to avoid light loss and fully illuminate the sensor.

As another example, consider a sharp imaging fisheye lens with a semi-field of view, θ , and with an entrance pupil radius, y_e . The étendue, ε , of the system in object space is $\varepsilon = \pi^2 y_e^2 \sin^2(\theta)$. Assuming telecentricity in image space, $\bar{u}' = 0$, and the radius of the image to be $\bar{Y}_i$, then we have that, whenever $\sin(U') \cong \frac{1}{2F/\#}$, the étendue in image space is $\varepsilon' = \frac{\pi^2 \bar{Y}_i^2 y_e^2}{f^2}$, where f is the focal length of the lens. If there is no light loss, we must have $\varepsilon' = \varepsilon$, which leads to the lens mapping relationship, $\bar{Y}_i = f \sin(\theta)$. This mapping differs from the ideal, $\bar{Y}_i = f \tan(\theta)$, mapping that many lenses produce.

Still another example of the use of étendue is in deriving the sine condition. The optical flux emitted from a Lambertian source is given by, $\phi = L_0 \pi A_s \sin^2(U)$. The flux forming a sharp image of the source is given by, $\phi' = \phi = L_0' \pi A_s' \sin^2(U')$. Using the radiance theorem, we obtain, $n^2 A_s \sin^2(U) = n'^2 A_s' \sin^2(U')$, which, after using, $n \bar{y}_o u = n' \bar{y}_i u'$, simplifies into the sine condition, $u' \sin(U) = u \sin(U')$. A discussion of the sine condition is given in Appendix 5.

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9

Image Evaluation

The quality of a lens system for a given application depends on several factors. Among them are the choice of lens form that best suits the application, the image quality provided by the lens, and how well the image quality can be maintained under actual lens fabrication and assembly errors, environmental conditions, and actual lens use. A starting point for a lens design is the lens specifications, which includes first-order requirements, packaging constraints, optical power-efficiency, and image quality requirements. Ultimately, lens cost is often a major design driver.

It is of critical importance that the application for the lens is well understood throughout, as much as possible, as the specifications for that lens follow from that understanding. In some cases, the specification list is wrong or incomplete, and this obviously impacts the lens design; it is often up to the engineer to discuss the specifications with the customer to correct any deficiency. The specifications may define the expected image quality in terms of image evaluation metrics that depend on the system application.

Once a lens form is chosen and the aberrations have been corrected, minimized, or balanced, it is necessary to evaluate how good or bad the image cast by that lens could be. This chapter discusses a variety of concepts and tools in image evaluation that are essential in lens design.

There are several topics that are related and that are of interest to understand and develop skill in their use as they relate to lens design. These topics can be treated from a geometrical optics point of view or from a physical optics point of view and are: image formation theory, image aberration theory, aberration evaluation, and image evaluation.

This chapter first discusses the image evaluation of a point object, and second the image evaluation of an extended object.

9.1 Image Evaluation of a Point Object

The concept of a point object, or light source, is non-physical, as it does not have an area. However, it is a useful concept, in that it defines where light rays or waves depart from. Consequently, and geometrically, the ideal image of a point object is a point image. The basic idea of image evaluation is that an image is composed of the images of a plurality of object points and, by assessing the individual point images, insight is obtained about the overall image quality of an extended object.

Observation of the images of point objects produced by aberration free lens systems with circular entrance pupils shows that the images are not pointy, but have a light distribution known as the Airy pattern, shown in Figure 9.1.

This pattern, $A(\rho)$, is described mathematically by the function,

$$A(\rho) = \left(\frac{2J_1(\pi\rho)}{(\pi\rho)} \right)^2, \quad (9.1)$$

where $J_1(\rho)$ is a first-order Bessel function of the first kind. The radial locations of the first three zeros in the Airy pattern take place at approximately $\rho = 1.22$, $\rho = 2.23$, and $\rho = 3.24$. The radius of the first zero as it relates to the wavelength of light and to the F -number is given by $\rho = 1.22\lambda F/\#$. Thus, taking the F -number of the eye to be $F/2.8$ and for a wavelength of $\lambda = 555$ nm, the radius of the first dark ring in the Airy pattern is $\rho = 1.89$ μm . Figure 9.2 shows a cross-section of the Airy pattern, and Figure 9.3 shows the encircled energy. About 83% of the energy is encircled within the first dark ring, and 91% within the second dark ring.

The Airy pattern occurs because of the wave nature of light and the phenomenon of diffraction. For many applications, the Airy pattern is considered as the ideal image of a point object that an aberration-free system can produce. In practice, one requirement for a system to produce an image that is substantially the Airy pattern is that the wavefront deformation across the exit pupil must be small in comparison to the wavelength of light.

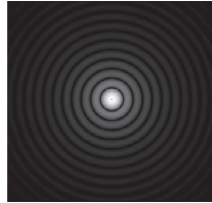


Figure 9.1 Enhanced Airy pattern produced by an aberration-free optical system.

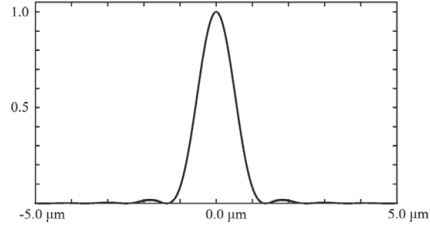


Figure 9.2 Normalized cross-section of the Airy pattern for $\lambda = 1 \mu\text{m}$ and $F/\# = 1$.

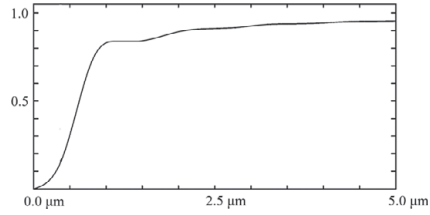


Figure 9.3 Normalized encircled energy in the Airy pattern as a function of the radial distance for $\lambda = 1 \mu\text{m}$ and $F/\# = 1$.

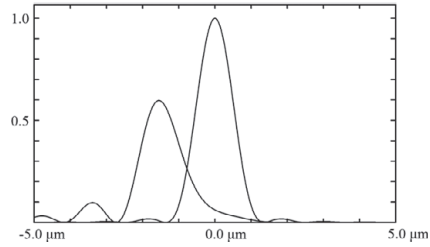


Figure 9.4 Cross-section of the diffraction pattern for the case of 1λ of coma aberration, with a peak of 0.6 (left), and no aberration with a peak of 1.0 (right) for $\lambda = 1 \mu\text{m}$ and $F/\# = 1$.

One major effect of aberration in the Airy pattern is to decrease the peak irradiance, as shown for the case of 1λ of coma aberration in Figure 9.4.

The ratio of the peak irradiance of the diffraction pattern produced by an aberrated beam to the peak irradiance of an un-aberrated beam was first estimated by Lord Rayleigh, but is known as the Strehl ratio. This ratio, R , was shown by A. Marechal to be related to the variance of the wavefront deformation, σ_w^2 , and can be approximated by the R. Shack relationship,

$$R = \exp \left(- \left(\frac{2\pi}{\lambda} \right)^2 \sigma_w^2 \right), \quad (9.2)$$

Table 9.1 *Primary aberration that produces a ratio of $R = 0.8$ at the ideal image plane, $\sigma_w \cong 0.07\lambda$*

$W_{020} = 0.247\lambda$	$W_{040} = 0.239\lambda$	$W_{131} = 0.604\lambda$	$W_{222} = 0.319\lambda$
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whenever the ratio satisfies $R \geq 0.1$. An optical system where the ratio is larger than 0.8 is considered to be very good, and is called diffraction limited. For this the standard deviation, σ_w , of the wavefront must be less than 0.07λ . Table 9.1 provides the amount of primary aberration that would produce a ratio of $R = 0.8$.

For the primary aberrations, the variance of the wavefront as a function of the field of view is given by,

$$\sigma_w^2 = \left(\frac{1}{12} \left(W_{020} + W_{040} + \left(W_{220} + \frac{1}{2} W_{222} \right) \vec{H} \cdot \vec{H} \right)^2 + \frac{1}{180} W_{040}^2 + \frac{1}{24} W_{222}^2 (\vec{H} \cdot \vec{H})^2 + \frac{1}{4} \left(\frac{2}{3} W_{131} |\vec{H}| + W_{311} (\vec{H} \cdot \vec{H}) |\vec{H}| \right)^2 + \frac{1}{72} W_{131}^2 \vec{H} \cdot \vec{H} \right). \quad (9.3)$$

The difference, $E = 1 - R$, can be considered the amount of energy that is taken out of the Airy disk and redistributed among the diffraction rings.

The Airy pattern is considered an ideal image and serves as a benchmark for comparison purposes. A useful analogy to keep in mind is that the diffraction pattern produced by a lens system, or light spot, is like the point of a pencil; the sharper it is, the finer the lines it can draw. Therefore, for actual systems it is important to evaluate the sharpness of the light spots they can produce. There are a variety of tools available to a lens designer to estimate how sharp the image of a point can be. For example, ray fans, wave fans, spot diagrams, RMS wavefront deformation plots, and encircled energy plots. These metrics do depend on the wavelength(s) the lens system is intended to work and, therefore, it is important to include in any analysis the spectral bandwidth of the system by weighting the contributions of a number of wavelengths.

9.1.1 Resolving Power and Depth of Focus

Two important attributes of an optical system are its resolving power and its depth of focus. Resolving power (RP) refers to the ability to separate two objects that are close to each other. Depth of focus (DOF) refers to the ability to maintain spot size as the observation plane is moved along the optical axis.

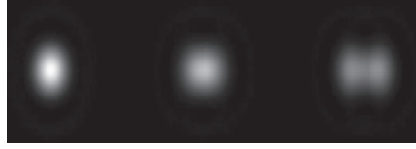


Figure 9.5 Images of two point objects. Left, not resolved; center, resolved according to Rayleigh's criterion; right, resolved.

For a system that is diffraction limited, these can be expressed in the terms of the wavelength, λ , and numerical aperture, NA , of the system as,

$$RP = k_1 \frac{\lambda}{NA}, \quad (9.4)$$

and

$$DOF = \pm k_2 \frac{n\lambda}{NA^2}, \quad (9.5)$$

where k_1 and k_2 are parameters that depend on the application, and n is the index of refraction in image space. As shown in Figure 9.5, Rayleigh's resolving power criterion refers to the ability to barely resolve the images of two point objects of equal intensity, and sets $k_1 = 0.61$. This criterion has the peak of the image of a first point falling on the first zero of the image of a second point. A defocus amount of $W_{020} = \lambda/4$ is considered tolerable and sets $k_2 = 0.5$. For a diffraction limited system working at $\lambda = 0.5 \mu\text{m}$, a rule of thumb is that the DOF is equal to the square of the $F/\#$ in micrometers. Thus, an $F/10$ system would have a DOF of $\pm 100 \mu\text{m}$. For systems that are not diffraction limited, both the resolving power and the depth of focus will depend on the application.

9.1.2 Spot Diagram

Spot diagrams are computed by plotting the intersection point of rays with the observation surface for a given field point, and, thus, they are a purely geometrical optics construct. This is shown in Figure 9.6 (left) for the case of five waves of coma aberration. Spot diagrams accompanied by a scale bar, or compared to the Airy disk diameter, provide a first estimate of the image size of a point. In addition, they can provide information about the symmetry of the aberration in a light beam. However, they do not convey how the energy is distributed spatially, unless the number of rays per bin is counted, and plotted as shown in Figure 9.6 (center). Spot diagrams can also be computed



Figure 9.6 Spot diagrams for five waves of coma aberration. Left, standard spot; center, binning and counting rays; right, diffraction calculated.

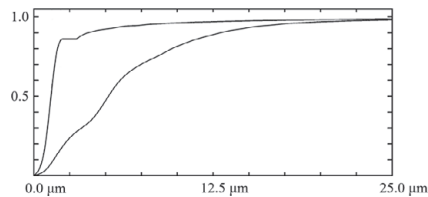


Figure 9.7 Encircled energy vs. radial distance for a diffraction limited system, left, and for the on-axis beam of the Petzval portrait objective, right.

using light diffraction calculations, and then a more accurate representation of the light distribution is obtained, as shown in Figure 9.6 (right). The rms spot size across the field of view is also a first estimate of the image quality of a system.

9.1.3 Encircled Energy

A plot of the encircled energy as a function of spot radius is an important tool to evaluate the quality of the image of a point object. Direct comparison with the encircled energy of a diffraction-limited beam provides objective information. Figure 9.7 shows the encircled energy for the on-axis beam of the Petzval portrait objective calculated for the F , d , and C wavelengths, and when the objective is scaled to a focal length of 100 mm. About 85% of the energy is encircled within a radius of 11.0 μm , while in the diffraction-limited beam 85% is encircled in a radius of about 3.0 μm .

The fact that the spot of light produced by the Petzval objective is much larger, about four-times, than the diffraction limited spot, does not mean that it is a poor lens. Considering that photographic film as a rough rule of thumb has a grain size of 25.0 μm , then the on-axis spot of the Petzval objective is not a limiting factor for image quality. Thus, the required image quality depends on application.

Some applications may require plots of ensquared energy, or energy enclosed in the meridional or sagittal directions, as a function of the field of

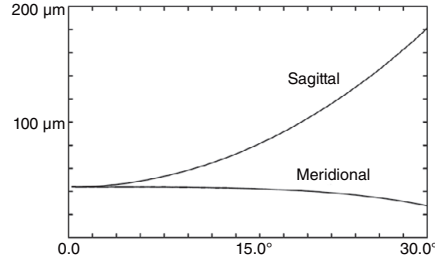


Figure 9.8 Plots of distance in the meridional or sagittal directions to enclose 80% of energy (geometrical) as a function of the field of view for the Wollaston landscape lens, $f' = 100$ mm.

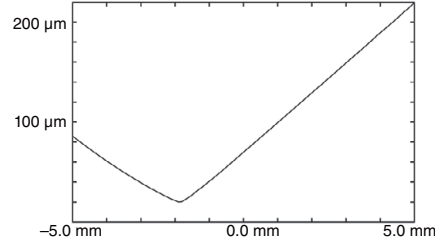


Figure 9.9 Radius that encircles 80% of the energy (geometrical) vs. axial position for the on-axis beam of the Wollaston landscape lens.

view, as shown in Figure 9.8 for the Wollaston landscape lens. Figure 9.9 also shows, for the Wollaston lens, the radius which encircles 80% of the energy (geometrical) for the on-axis beam as a function of the axial position. This provides information about the depth of focus.

9.2 Image Evaluation of an Extended Object

Application of linear shift invariant theory to model imaging by a lens system resulted in an insightful and powerful way to evaluate images of extended objects. In a linear shift invariant system (LSIS), the image, $i(x, y)$, is the convolution of the object, $o(x, y)$, with the system point spread function, $h(x, y)$, which is also called the impulse function. Mathematically this is written as,

$$i(x, y) = o(x, y) * h(x, y). \quad (9.6)$$

An object can be analyzed and described as a superposition of a plurality of spatial frequencies, $\exp\{-i2\pi(\xi x + \eta y)\}$, and likewise the image can



Figure 9.10 Spatial frequency pattern. Left, with full contrast; center and right, with reduced contrast; right, with contrast reversal.

also be analyzed and described by a plurality of spatial frequencies, $\exp\{-i2\pi(\xi x + \eta y)\}$. For example, an object represented by a square wave can be described by a superposition of cosine waves with different spatial frequencies, amplitudes, and phase shifts.

By taking the Fourier transform of the image, $i(x, y) = o(x, y) * h(x, y)$, we can write,

$$I(\xi, \eta) = O(\xi, \eta) \times H(\xi, \eta), \quad (9.7)$$

where $O(\xi, \eta)$ represents the spatial frequency spectrum of the object, $I(\xi, \eta)$ is the spatial frequency spectrum of the image, and $H(\xi, \eta)$ is the optical transfer function (OTF) of the system.

In general, the optical transfer function is complex, and can be written as,

$$H(\xi, \eta) = MTF(\xi, \eta) \times \exp(-i\phi(\xi, \eta)), \quad (9.8)$$

where $MTF(\xi, \eta)$ is its modulus, and $\phi(\xi, \eta)$ is the phase. The modulus, $MTF(\xi, \eta)$, is called the modulation transfer function, and provides the contrast with which a given spatial frequency is imaged. For a periodic function, the contrast is given by,

$$C(\xi, \eta) = MTF(\xi, \eta) = \frac{\text{MAX} - \text{MIN}}{\text{MAX} + \text{MIN}}, \quad (9.9)$$

where MAX and MIN are the maximum and the minimum irradiances. Figure 9.10 (left) shows one spatial frequency with which an object can be represented. As it is imaged by an optical system, the contrast can be reduced, as shown in Figure 9.10 (center and right). In addition, there can be a phase shift, i.e. a lateral displacement of the frequency pattern, that, when it is 180° , produces contrast reversal, as shown in Figure 9.10 (right). In contrast reversal, the peak irradiance displaces by half a period so that peaks and valleys swap location. Axially symmetric systems that are linear shift invariant can only produce phase shifts of 0° or 180° .

Under incoherent object illumination and for an aberration free system that has a circular entrance pupil, the point spread function, $h(x, y)$, is the Airy pattern. Its Fourier transform represents an ideal optical transfer function and, since for this case there is no phase shift, $\phi(\xi, \eta) = 0$, we have $H(\xi, \eta) = MTF(\xi, \eta)$ and,

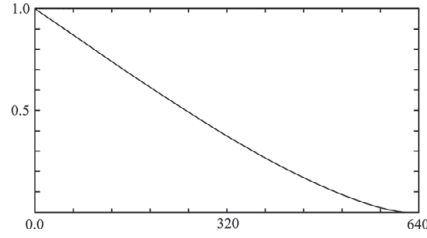


Figure 9.11 Modulation transfer function of an aberration-free system with circular entrance pupil working at $F/2.8$ and at $\lambda = 555$ nm. The vertical axis is the contrast, and the horizontal axis is the spatial frequency, $f = \sqrt{\xi^2 + \eta^2}$ in lp/mm.

$$MTF(\xi, \eta) = \frac{2}{\pi} (\psi - \cos(\psi) \sin(\psi)), \quad (9.10)$$

where

$$\psi = \arccos\left(\frac{\lambda \sqrt{\xi^2 + \eta^2}}{2NA}\right) = \arccos\left(\frac{f}{f_c}\right). \quad (9.11)$$

The MTF is a three-dimensional function as a function of the spatial frequencies, ξ and η . However, for an aberration-free system that is axially symmetric, the MTF is also axially symmetric, and a cross-section is sufficient to describe it, as shown in Figure 9.11. There is no modulation after the cut-off frequency, $f_c = 2NA/\lambda$; this is, the system cannot image spatial frequencies beyond this frequency, f_c . For a system in air working at $F/2.8$ and at $\lambda = 555$ nm, the cut-off frequency is $f_c \cong 643.5$ line-pairs per millimeter (lp/mm).

The MTF shows that only the zero spatial frequency is imaged with full contrast, and that the contrast progressively decreases as the spatial frequency increases. The MTF provides useful information about how an extended object is imaged. It tells what frequencies can be imaged and with what contrast. Clearly, this provides a different perspective about how to evaluate an image.

For off-axis field points, the MTF is no longer axially symmetric. Different cross-sections of the three-dimensional MTF provide the contrast of spatial frequencies that have different orientations with respect to the optical axis. MTF cross-sections in the meridional and sagittal directions are often plotted by lens design programs.

One can understand the OTF by imaging with a lens an object having a single spatial frequency, such as in Figure 9.10 (left), and measuring the contrast and phase shift in the image. By rotating the object and measuring contrast and phase shift, the OTF for that frequency can be determined. The

process can be repeated for many more spatial frequencies to determine the OTF of the lens more completely.

9.2.1 MTF Curves

Optical aberrations degrade the MTF, as shown in Figure 9.12, for example, for the case of defocus: $W_{020} = \lambda/4$, $W_{020} = \lambda/2$, and $W_{020} = \lambda$. For 1λ of defocus the phase (not shown) becomes negative after the modulation reaches zero and contrast reversal takes place. Defocus has a severe and adverse effect on the MTF, and about $W_{020} = \lambda/4$ can be tolerated in a diffraction-limited system.

The MTF is also plotted for the Wollaston landscape lens in Figure 9.13. The diffraction-limited MTF is plotted for reference, then the MTF for the on-axis field point, and finally two cross-sections in the meridional (tangential) and sagittal direction are plotted for the 30° field of view. One must keep in mind that the MTF is a three-dimensional function, and that plotting a few cross-sections of it may not be sufficient. However, it is standard practice in lens design to plot the S and T cross-sections. The Wollaston landscape lens is far from performing at the diffraction limit, but is still a useful imaging lens.

A healthy human eye is said to have a resolving power of about 1 arc-minute. But, in consideration of several degrading factors, an average

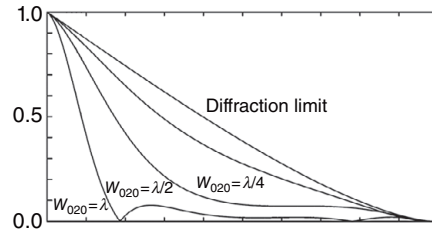


Figure 9.12 Diffraction-limited MTF and in the presence of defocus W_{020} .

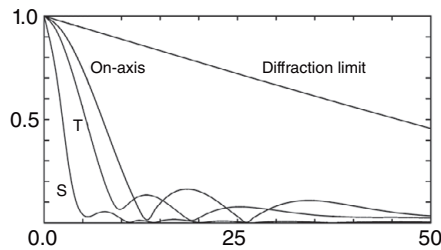


Figure 9.13 MTF for the Wollaston landscape lens.

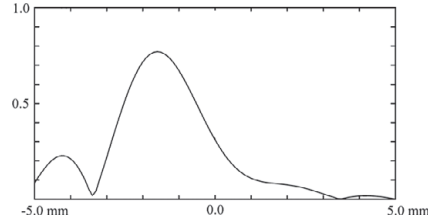


Figure 9.14 MTF for a spatial frequency of 10 lp/mm vs. axial position for the Wollaston landscape lens, $f' = 100$ mm. The zero position corresponds to the ideal image plane location, as calculated with first-order ray tracing.

resolving power can be set to 3 arc-minutes. Then, when looking at a photograph at a distance of 250 mm, the eye can resolve two spots that are separated by about 0.25 mm. This corresponds to an object spatial frequency of $f = 4$ lp/mm.

For 35 mm film ($24 \text{ mm} \times 36 \text{ mm}$) and assuming that the printed photograph would be 8-times larger ($192 \text{ mm} \times 288 \text{ mm}$), and viewed at a distance of 250 mm, the requirement for spatial frequency would be 4 lp/mm in the printed photograph and 32 lp/mm in the 35 mm film. Thus, a rule of thumb to specify a lens for 35 mm film is a contrast larger than 50% at 32 lp/mm. For high quality lenses the rule of thumb stiffens and becomes a contrast of 50% at 30 lp/mm and a contrast of 30% at 50 lp/mm. This is sometimes referred to as the 30/50 rule. However, the specification for MTF is highly dependent on application.

For the current CCD and CMOS image sensors with pixel sizes in the order of a few micrometers, the spatial frequency requirements are even higher. For example, to take advantage of a pixel size of $5 \text{ } \mu\text{m}$, the lens must be able to produce spatial frequencies in the order of 100 lp/mm.

The MTF for a given spatial frequency and for a given field can be plotted as a function of the axial position of the image plane. This is shown in Figure 9.14 for the Wollaston landscape lens for a spatial frequency of 10 lp/mm. Plots like this provide information about the best imaging plane and the depth of focus.

9.2.2 Image Simulation

The irradiance of an image can be calculated, and an image simulation can be produced in great detail under a number of variables, such as the illumination, the system aberrations, and the nature of the object to be imaged. An equation that describes the process of physical image formation was developed by H. H. Hopkins and is,

$$I(x, y) = \left(\frac{1}{f\lambda} \right)^2 \iint_{\infty} \sigma(x_0, y_0) |s(x, y) t(x, y) * * p s f(x, y)|^2 dx_0 dy_0, \quad (9.12)$$

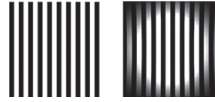


Figure 9.15 Left, bar target as an object. Right, image simulation using a Petzval lens. Note the decrease in relative illumination toward the image corners, and the pincushion distortion.

where $I(x, y)$ represents the irradiance of the image, $\sigma(x_0, y_0)$ represents the illumination, $s(x, y)$ is the optical field, $t(x, y)$ represents the object transmittance, and $psf(x, y)$ is the point spread function of the system. For incoherent illumination, the Hopkins equation simplifies to a convolution operation that can be carried with the fast Fourier transform. For partial coherent illumination, the equation is no longer a convolution, and the equation would take more time to be computed. Image simulation with the Hopkins equation can take significant time. However, it is used to calculate fine details in the image that an optical system could form.

As an example of image simulation, Figure 9.15 (left) shows a bar target as an object. Figure 9.15 (right) shows the simulated image that a Petzval portrait lens would produce. There is a decrease in relative illumination and in resolving power toward the corners of the image. Image simulations are helpful for a variety of purposes, such as checking image orientation, the overall appearance of the image, illumination, and resolving power.

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17

Miniature Lenses

There are a number of technological applications that use miniature lenses in which the lens diameter is a few millimeters, and typically smaller than 10 mm. For lens systems that employ such miniature lenses, several advantages result because of the scale. For a given lens form and, except for distortion, the aberrations scale down, while the wavelength of light remains the same. Given that lens volume is small, a wider possibility of lens materials becomes possible due to cost or material limitations. Lens weight is reduced, as well as dimensional changes due to temperature. Further, very thick lenses can be used in some applications. However, lens tolerances for lens thickness and decenter become tighter. Many microscope objectives, lenses for endoscopes, and lenses for mobile phones are in the category of miniature lenses. This chapter provides a discussion about lens design for mobile phones lenses.

17.1 Lens Specifications

Lenses for mobile phones, as illustrated in Figure 17.1, have been developed over the last two decades. Lens designs have evolved from having one or two lens elements, to three and four elements, to five-to-eight lens elements. Some of the lens elements have been made by glass molding, but currently they are made by plastic injection molding. To aid in the correction of aberration, highly aspheric surfaces are used. Table 17.1 provides typical lens specifications for field of view (FOV), focal length, F -number, total track length (TTL) from the vertex of the first surface of the lens to the image plane, chief ray angle of incidence at image sensor (CRA), relative illumination (RI), and number of lens elements.

Like any other photographic lens, the design of a mobile phone lens is driven by the light sensor; for example, a Charge Coupled Device (CCD) or

Table 17.1 *Typical mobile phone lens specifications*

Year	2006	2012	2018
Focal length	3–6 mm	3–5 mm	3–5 mm
FOV	66°	72°	78°
$F/\#$	2.8	2.2–2.4	2.0–1.4
TTL	<5.0 mm	<5.0 mm	<6.0 mm
Distortion	<1–2%	<1–2%	<1–2%
CRA	<24°	<30°	<33°
RI	>50%	>50%	>32%
# lens elements	3–5	4–6	5–8

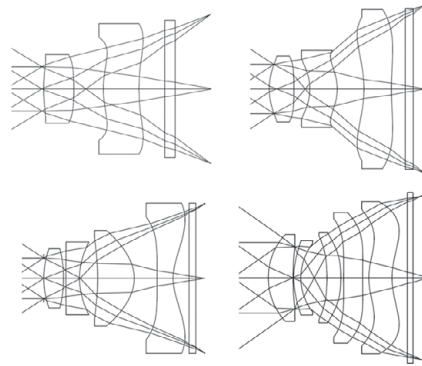


Figure 17.1 Mobile phone lens forms with two, three, four, and five lens elements. The plane parallel plate next to the image plane represents an infrared filter.

Complementary Metal Oxide Semiconductor (CMOS) image sensor. The diagonal of the active area of the sensor defines the minimum image circle diameter; some allowance in this diameter is given to allow for some lens/sensor decenter. There are many format sizes for an image sensor that are given in inches, such as $1/3''$. This format has a diagonal of 6 mm, a width of 4.8 mm, and a height of 3.6 mm. The inch designation of image sensors is an arcane reference to the vidicon tubes used in early video cameras. It is best to look up the actual digital image sensor dimensions for a given sensor specification.

The lens design is also driven by the specified total track length, which is often required to be less than 6 mm, so that the lens can be integrated in a thin mobile phone. Although most designs for mobile phone lenses are not telephotos, the ratio of the total track lens to the focal length, the telephoto ratio, is still used. A typical value for this ratio is 1.2.

Table 17.2 *Example of MTF specifications in fractions of the Nyquist frequency and in cycles/mm for an object at infinity*

	N_Q	$N_Q/2$	$N_Q/4$
MTF On-axis	>40%	>60%	>80%
MTF Off-axis @ 0.7 field	$S > 30\%$	$S > 50\%$	$S > 70\%$
	$T > 20\%$	$T > 40\%$	$T > 60\%$
MTF Off-axis @ 1.0 field	$S > 20\%$	$S > 30\%$	$S > 40\%$
	$T > 10\%$	$T > 20\%$	$T > 30\%$

Given the focal length and the image circle diameter, and assuming no distortion, the field of view can be calculated. Thus, for a focal length of 4.8 mm and an image circle diameter of 6 mm, the field of view is $\pm 32^\circ$.

Electronic sensors have pixels sensitive to the light incident on them. Typical pixel sizes for CMOS sensors vary in the range of 0.001 mm to 0.008 mm. A given spatial frequency can be recovered whenever it is sampled with twice that frequency. This sampling frequency is known as the Nyquist frequency, N_Q . An image sensor can sample an image at a spatial frequency of $1/(\text{pixel size})$. It follows that the sensor can recover a spatial frequency, $N_Q = 1/(2 \times \text{pixel size})$. If the pixel size is 1.5 μm , then $N_Q = 333$ cycles/mm (also line pairs per mm; lp/mm). The maximum frequency that a lens with a circular aperture can image is $N_C = 1/(\lambda F/\#)$. Using $\lambda = 0.5 \mu\text{m}$ and $F/2.2$ yields $N_C = 909$ cycles/mm. Thus, a diffraction limited lens at $F/2.2$ would not limit the sensor spatial frequency sampling. The image quality for a lens can then be specified in terms of the Nyquist frequency, for different fields and object distances, as shown in Table 17.2.

The design of a mobile phone lens also requires consideration of the spectral bandwidth to be used, for example, the visible spectrum from 400 nm to 700 nm. A filter is used in mobile phone lenses to suppress infrared radiation (IR). This filter is modeled as a parallel plate of BK7 glass as the last element of the lens system. The filter introduces spherical aberration that must be compensated by a lens element near the stop aperture. In addition, the spectral response of the sensor is used to weight the wavelengths used by the lens optimizer to reflect the sensor's sensitivity to wavelength.

A CMOS sensor includes an array of micro-lenses, also called lenslets, placed on top of the light sensitive pixels. As shown in Figure 17.2, the function of each micro-lens is that of a field lens that forms an image of the exit pupil of the mobile phone lens onto the surface of each light sensitive element of a pixel. In this way, light is redirected to the active areas in each pixel, as not all the area of a pixel is light sensitive. The array of micro-lenses

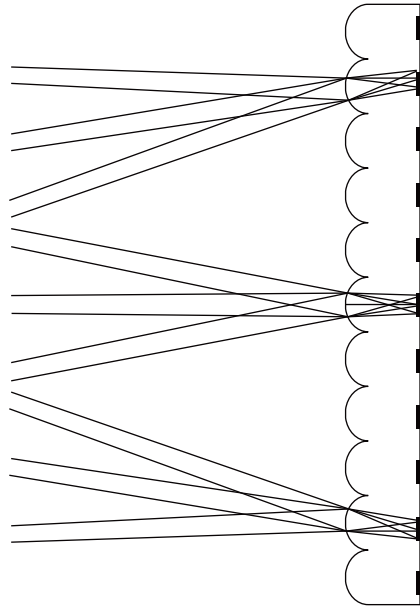


Figure 17.2 Model of array of micro-lenses to improve light coupling efficiency. The micro-lenses act as field lenses, forming a pupil at the CMOS light sensitive elements.

requires a specific chief ray angle (CRA) of incidence as a function of the ray height on the sensor. The specification can be simply to match a given CRA at the corner of the sensor, or to match within one or two degrees a table of angles of incidence as a function of the radial position on the sensor.

There must be enough back focal length (BFL) to allow placement of the image sensor and any clearances required by opto-mechanical considerations.

17.2 Lens Design Considerations

Mobile phone lenses have been evolutionary, in that every generation increased complexity from the previous one. The first-order layout in practice is usually taken from an existing lens design form. Given that lens element axial thickness can be substantial, thick lenses can be used, and more design forms are possible whenever the lens system has two or three lens elements, or when the total track length is not a concern. Many combinations of optical power for the lens elements are possible, for example PNP, PNP, NPP for a four-lens element system. The patent literature has hundreds of lens

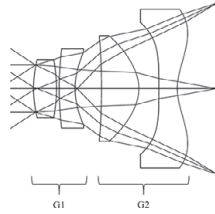


Figure 17.3 Four-lens element mobile phone lens. The first and second lens elements form the front group, and the third and fourth lens elements form the rear group.

design examples for mobile phone lenses and their forerunners, personal digital assistants.

Figure 17.3 shows a four-element mobile phone lens divided into two groups, G_1 and G_2 . The stop aperture is placed in front to reduce the chief ray angle of incidence at the image sensor, or slightly inside the lens to help provide some lens symmetry and to reduce odd aberrations. However, provided the relative illumination specification is met, the stop aperture can be placed in front, and some vignetting by an aperture inside the lens can be allowed. This vignetting has the effect of making the stop aperture appear inside the lens for off-axis field points. For stray light control, several apertures are usually specified within the mobile phone lens assembly.

Significant optical power of the lens system is in the front lens group, which must provide the majority of the correction for chromatic aberration, spherical aberration, and coma. The rear group corrects for Petzval field curvature, astigmatism, and distortion aberration. This group usually has highly aspheric surfaces. In particular, the last aspheric surface changes the optical power, from negative near the optical axis to positive near the edge of the lens. This positive power helps to decrease the CRA and introduces positive pupil coma, which helps to increase the relative illumination. Pupil spherical aberration is also present due to the positive power of the last lens near the edge.

The total track of the lens system has a strong impact on imaging quality. The smaller the telephoto ratio is, the more difficult it becomes to meet image quality requirements, in part because the lens is optically stressed and higher order aberrations become larger.

It is good practice not to specify unnecessary aspheric terms and to keep them at a minimum number. If many aspheric terms are used, the lens optimizer may create surface features in one lens that are canceled in another lens. Under lens decenter, the features can partially add rather than cancel, with the result that the as-built-lens becomes more sensitive to manufacturing

errors. Lens elements near the stop aperture may need only two or three aspheric terms, as their function is mainly to correct for spherical aberration.

As mentioned before, one problem with the small scale of miniature lenses is that lens element thickness and decenter errors can have a large impact by decreasing performance. For example, a thickness error of 0.1 mm can be tolerable in a 50 mm focal length lens, but not at all in a miniature lens with a focal length of 5 mm. Therefore, an important part of the lens design is to desensitize as much as possible a given design. The wavefront and distortion correction should not have rapid changes at the edge of the aperture or field, or slightly beyond the image circle, that could substantially decrease the performance of the as-built lens. Minimizing ray angle of incidence over all the lens surfaces helps to make the lens system less sensitive to errors.

The first and second lens elements in the front group usually have strong optical power to correct for chromatic aberration and provide most of the system's optical power. Any manufacturing or assembling errors in these lens elements can introduce significant aberration that would degrade the image sharpness. Field correcting elements in the rear group have a small beam footprint and a large chief ray height, and, under manufacturing errors, they can introduce asymmetrical image distortion.

During the early stages of the design of a mobile phone lens, and to explore possible lens forms, the surface conic constant and the fourth order aspheric coefficients can be released as variables for optimization. Once a promising design form is found, more aspheric terms can be added, provided they contribute to improve performance. It is important that, for optimization and evaluation, sufficient pupil and field sampling points be specified. Typically, the field is sampled with at least ten field points.

The optimization of a mobile phone lens can start with minimizing RMS spot size, then minimizing RMS wavefront error, and finally adjusting the lens to meet MTF requirements. If a given design form cannot meet image quality specifications, then a lens designer may consider changing lens materials or adding one more lens element. To add a lens element, a parallel plate of zero thickness is inserted in an air space, or a thick lens is split into two lenses with no air space between them. Fourth-order aspheric terms are included in the new surfaces, and the system is re-optimized, not necessarily allowing the radii of the new surfaces to vary. Then thickness is added to the new surfaces by small increments until a lens that is physically possible, this is with positive thicknesses and non-overlapping lenses, is obtained. As lens complexity is added, more design forms are possible, and lens image quality is expected to improve. The lens form of Figure 17.3 has been used as a starting point in many patented lenses to develop designs with more lenses. Parallel plates of

Table 17.3 *Example of a four-lens element design for a mobile phone lens.*
 $f' = 5.0 \text{ mm}$, $FOV = \pm 32^\circ$, $TTL/f = 1.4$, $F/2.8$, $CRA = 30.6^\circ$

Surface	Radius	Thickness	Plastic	K	A_4
STOP					
2	3.5432.72	0.87	E48R	-3.2457	
3	-4.57977	0.1		-11.7213	
4	126.2449	0.6	OKP4	0.0	
5	3.013807	0.8		-0.9977	-2.533×10^{-3}
6	-8.81693	1.61	E48R	0.0	4.161×10^{-3}
7	-1.61409	1.0		-2.2699	-0.0126713
8	72.73421	0.7	OKP4	0.0	-0.0100888
9	2.101288	0.7		-6.3020	-6.073×10^{-3}
10	Plano	0.3	BK7		
11	Plano	0.4			
Image					

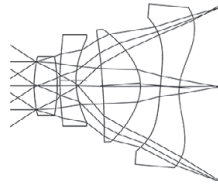


Figure 17.4 Example of a lens system with exaggerated lens element decenter and tilt.

glass have been added and then optimized to obtain five, six, and seven element designs.

Lens desensitizing to manufacturing errors is still an important requirement, so that the as-built lens meets fabrication yield specifications. Figure 17.4 shows exaggerated lens decenter and tilt, which can happen during lens assembly. Under lens element decenter and tilt, the axial symmetry is broken, and new aberration forms can appear. For example, in addition to uniform spherical aberration, uniform coma and uniform astigmatism can take place. In addition to linear coma, linear astigmatism and field tilt can also take place. Further, in addition to cubic distortion, asymmetric quadratic distortion can take place. Here uniform refers to the aberration being independent of the field of view, linear as dependent linearly with the field, and so forth. Appendix 4 provides a table of aberrations that can happen when the lens symmetry is reduced from axial to plane symmetry.

An example of a four-lens element design is given in Table 17.3 and shown in Figure 17.1 (bottom left). In this design, only conic constants and

fourth-order coefficients of asphericity were used. This is to emphasize that the use of many higher order aspheric coefficients, such as up to sixteenth order, may not be necessary as they can be redundant.

It is common to have errors in the aspheric coefficients due to truncating numbers, incorrect algebraic signs, neglecting numbers, or swapping coefficients. Thus, typically a lens, say from the patent literature, would need to be re-optimized to correct errors, to reflect actual indices of refraction, to adjust focal length, field of view, optical speed, MTF performance, and to reflect current manufacturing requirements such as minimum central and edge thicknesses.

17.3 Lens Manufacturing Considerations

Early lens systems used glass for the first lens element to provide a large v -number difference for the correction of chromatic aberration, and for making the lens system less sensitive to environmental changes such as temperature and humidity variations. However, with the development of plastic materials with low v -numbers, low birefringence, low water absorption, and the introduction of auto-focus, lens systems with all the lens elements made by plastic injection molding have been realized.

Under large volume production, lenses made by plastic injection are preferred to lenses made by glass molding, because of cost. Some advantages of plastic lens molding are the freedom to specify aspheric surfaces, and the choice to specify a lens flange to help precisely position a lens element with respect to other lens elements, thereby simplifying the lens system assembly.

For proper plastic flow and cooling, plastic lens manufacturers have some requirements for the aspect ratio of positive and negative lenses. Some guidelines are as follows: for positive lenses the ratio of lens central thickness to edge thickness should not be more than 3.2, and the edge thickness should not be less than 0.32 mm; for negative lenses the ratio of the maximum thickness to the central thickness should not be larger than 2.7, and the central thickness should not be less than 0.27 mm. These requirements are over the clear aperture of the lens and do not consider the lens flange; they are intended for the lens designer. Further, they also depend on the capabilities of the manufacturer. The ideal lens for injection molding approaches a lens with parallel surfaces so that plastic flow, cooling, and shrinkage are uniform. For miniature lenses, tolerances of 10 μm for thickness, decenter, total indicator runoff, and lens tilt are low; tolerances between 2 μm and 5 μm are medium and feasible to achieve in manufacturing, and tolerances between 0.5 μm and 2 μm are challenging to achieve, and are met for some miniature optics.

Table 17.4 *Properties of some plastics used in mobile phone lenses*

Code	n_d	v	γ	ρ
480R	1.525	55.95	$+1.44 \times 10^{-4}$	1.01
E48R	1.531	56.04	-2.62×10^{-4}	1.02
F52R	1.534	57.09	-2.21×10^{-4}	1.01
OKP4	1.607	26.90	-3.44×10^{-4}	1.20
OKP4HT	1.632	23.33	-2.72×10^{-4}	1.24

n_d , index of refraction; v , v -number; γ , opto-thermal coefficient; ρ , specific gravity.

The optical and mechanical properties of some plastics used in miniature lens systems are shown in Table 17.4. Some advantages of plastics over glass are that they are moldable at a lower temperature, have lower cost for large volumes, with lower weight, and aspheric and diffractive surfaces can be specified. Some disadvantages are the greater sensitivity to temperature changes, increased sensitivity to water absorption, internal light scattering or haze, reduced light transmission below 450 nm and above 1,000 nm, and a low resistance to abrasion.

Further Reading

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- Links to optical plastics vendors:
- <http://www.ogc.co.jp/e/products/fluorene/okp.html>
- <https://www.zeonex.com/Optics.aspx.html#glass-like>