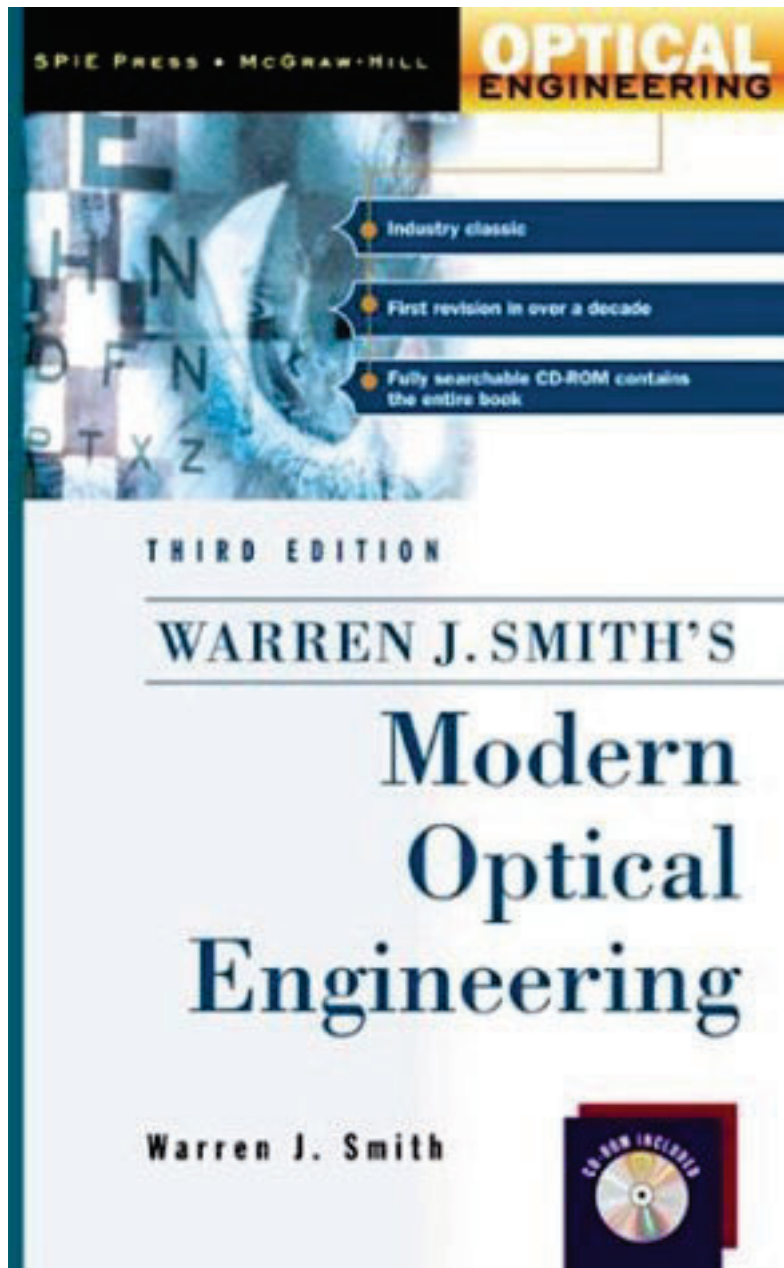




Modern Optical Engineering

The Design of Optical Systems

Warren J. Smith

*Chief Scientist, Kaiser Electro-Optics Inc.
Carlsbad, California
and Consultant in Optics and Design*

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Image Evaluation

11.1 Introduction

In the preceding chapter we discussed the means by which ray paths are traced through an optical system and how the numerical values of the image aberrations may be determined. In this chapter, we will consider the interpretation of the results of such optical computations. The basic question to which we address ourselves is: “What effect does a given amount of aberration have on the performance of the optical system?”

We have seen that raytracing yields an incomplete picture of the image-forming characteristics of a system, since the image formed by a “perfect” lens or mirror is not the geometric point that raytracing might lead us to expect, but a finite-sized diffraction pattern—the Airy disk and the surrounding rings. For small departures from perfection (i.e., aberrations which cause a deformation of the wave front amounting to less than one or two wavelengths) it is thus appropriate to consider the manner in which an aberration affects the distribution of energy in the diffraction pattern. For larger amounts of aberration, however, the illumination distribution as described by raytracing can yield a quite adequate representation of the performance of the system. Thus, it is convenient to divide our considerations into (1) the effects of small amounts of aberration, which we treat in terms of the wave nature of light, and (2) the effects of large amounts of aberration, which may be treated geometrically.

11.2 Optical Path Difference: Focus Shift

We will begin our discussion of small amounts of aberration by determining the optical path difference (OPD) or wave-front deformation introduced by a longitudinal shift of the reference point. Figure 11.1 shows a spherical wave front (solid line) emerging from the pupil of a “perfect” optical system with a focus at point F . We wish to determine the OPD with respect to a reference point at R , which is some arbitrary distance δ from F . If we construct a reference sphere (dashed), centered on R , which coincides with the wave front at the axis, then the OPD for a given zone (of radius Y) is the distance* from the reference sphere to the wave front measured along the radius of the reference sphere, as indicated in Fig. 11.1.

From the figure we can see that, for modest amounts of OPD, the path difference is equal to the radius of the reference sphere ($l + \delta$) minus the radius of the wave front (l) all less $\delta \cos U$.

$$\begin{aligned}\frac{\text{OPD}}{n} &= (l + \delta - \delta \cos U - l) \\ &= \delta (1 - \cos U)\end{aligned}$$

To an approximation sufficient for our purposes, we can make the substitution

$$\cos U \approx 1 - \frac{1}{2} \sin^2 U$$

and the optical path difference resulting from a shift of the reference point by an amount δ is given by

$$\text{OPD} = \frac{1}{2} n \delta \sin^2 U \quad (11.1)$$

A longitudinal shift of the reference point is equivalent to defocusing the system; by use of Rayleigh’s quarter-wave criterion we can establish a rough allowance for the tolerable depth of focus. Setting the OPD equal to a quarter wavelength of light and solving for the permissible focus shift,

$$\text{Depth of focus } \delta = \pm \frac{\lambda}{2n \sin^2 U_m} = 2\lambda (f/\#)^2 \quad (11.2a)$$

where λ is the wavelength of light, n is the index of the final medium, and U_m is the final slope of the marginal ray through the system. Note that U_m is used because the maximum amount of OPD occurs at the edge of the wave front. We can convert this to transverse measure by

*Times the index n of the final medium, if the final medium is not air.

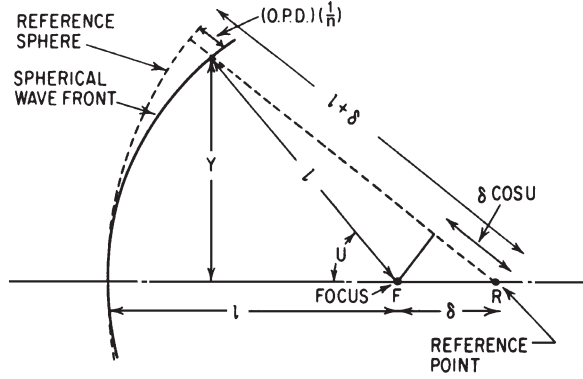


Figure 11.1 The optical path difference (OPD) introduced by a small longitudinal displacement (δ) of the reference point is equal to the index (n) times [the radius of the reference sphere ($l + \delta$) minus the radius of the wave front (l) minus $\delta \cos U$].

multiplying by the ray slope; using $\sin U_m$ as a close enough approximation for the slope, we get

Transverse $\lambda/4$ defocus,

$$H' = \frac{0.5 \lambda}{n \sin U_m} = \frac{0.5 \lambda}{\text{NA}} = \lambda (f/\#) \quad (11.2b)$$

where $\text{NA} = n \sin U_m$ and $(f/\#) = f\text{-number}$.

11.3 Optical Path Difference: Spherical Aberration

We begin by determining the OPD with respect to a reference sphere centered at the paraxial focus. In Fig. 11.2, the deformed wave front is shown as a solid line and the ray (normal to the wave front) from zone Y intersects the axis at point M . The reference sphere, centered at P , is shown dashed, and the OPD is, as before, the radial distance between the two surfaces times the index. Since the wave front is shown lagging behind the reference sphere, the sign of the OPD is shown negative, to be consistent.

The ray is normal to the wave front and the radius is normal to the reference sphere; thus the angle α between the surface normals is also the angle between the surfaces, and, as indicated in the lower sketch, the change in OPD corresponding to a small change in height dY is given by the relation

$$\alpha = \frac{(-d\text{OPD})}{n dY}$$

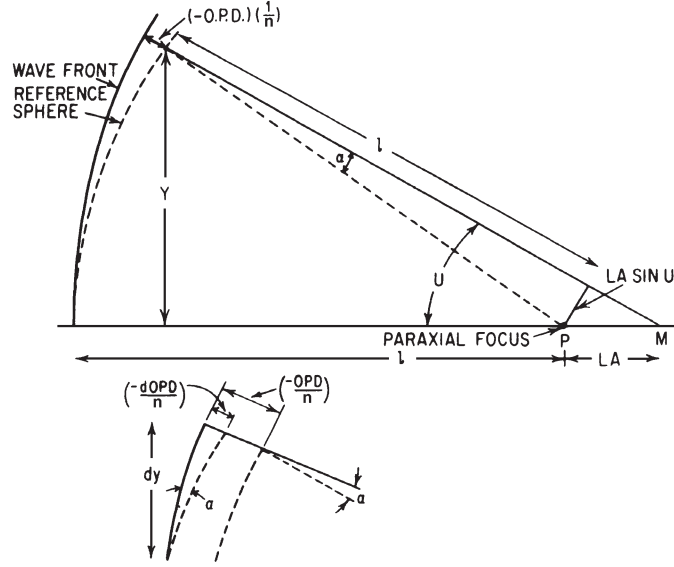


Figure 11.2 The OPD (with reference to the paraxial focal point) produced by spherical aberration. The small diagram indicates the relationship $\alpha = (-1/n) d\text{OPD}/dy$. In the upper sketch, it is apparent that $\alpha = \text{LA} \sin U/l$.

But the angular aberration α is also related to the spherical aberration by

$$\alpha = \frac{(\text{LA}) \sin U}{l} = \frac{(\text{LA}) Y}{l^2}$$

Combining and solving for $d\text{OPD}$ we get

$$d\text{OPD} = \frac{-Y n (\text{LA}) dY}{l^2}$$

Now the longitudinal spherical aberration is a function of Y and can be represented by the series

$$\text{LA} = aY^2 + bY^4 + cY^6 + \dots \quad (11.3)$$

Making this substitution and integrating

$$\begin{aligned} \text{OPD} &= - \int_0^Y \frac{nY}{l^2} (aY^2 + bY^4 + cY^6 + \dots) dY \\ &= - \frac{n}{l^2} \left(\frac{aY^4}{4} + \frac{bY^6}{6} + \frac{cY^8}{8} + \dots \right) \Big|_0^Y \end{aligned}$$

$$\begin{aligned}
&= \frac{-nY^2}{2l^2} \left(\frac{aY^2}{2} + \frac{bY^4}{3} + \frac{cY^6}{4} + \cdots \right) \\
&= -\frac{1}{2} n \sin^2 U \left(\frac{aY^2}{2} + \frac{bY^4}{3} + \frac{cY^6}{4} + \cdots \right) \quad (11.4)
\end{aligned}$$

Now Eq. 11.4 is the OPD with respect to the paraxial focus of the system. It is reasonable to expect that a more desirable reference point than the paraxial focus exists. Thus, by combining Eqs. 11.1 and 11.4, we get

$$\text{OPD} = \frac{1}{2} n \sin^2 U \left[\delta - \left(\frac{aY^2}{2} + \frac{bY^4}{3} + \frac{cY^6}{4} + \cdots \right) \right] \quad (11.5)$$

which is the OPD with respect to an axial point a distance δ from the paraxial focus.

Third-order spherical aberration. In many optical systems, the spherical aberration is almost entirely third-order; this is true for almost all systems composed of simple positive elements, and very nearly true for many other systems. Under such circumstances, Eq. 11.3 reduces to.

$$\text{LA} = aY^2 \quad (11.6)$$

and Eq. 11.5 reduces to

$$\text{OPD} = \frac{1}{2} n \sin^2 U [\delta - \frac{1}{2} aY^2] \quad (11.7)$$

Now at the edge of the aperture $Y = Y_m$ and $\text{LA} = \text{LA}_m$; substituting these values into Eq. 11.6, we find that (for third-order spherical)

$$a = \frac{\text{LA}_m}{Y_m^2}$$

and that

$$\text{OPD} = \frac{1}{2} n \sin^2 U \left[\delta - \frac{1}{2} \text{LA}_m \left(\frac{Y}{Y_m} \right)^2 \right] \quad (11.8)$$

To determine the value of δ which will result in the smallest amount of OPD, we can try several values of δ in Eq. 11.8 and plot the OPD for each as a function of Y . This has been done for shifts of $\delta = 0$, $\frac{1}{2}\text{LA}_m$, and LA_m ; the results are plotted in Fig. 11.3. It is apparent that the smallest departure from the spherical reference surface occurs when the OPD is zero at the margin. The corresponding shift of the reference point is $\text{LA}_m/2$. Therefore, from the standpoint of wave-front aberration, the best focus is midway between the marginal and paraxial focal points.

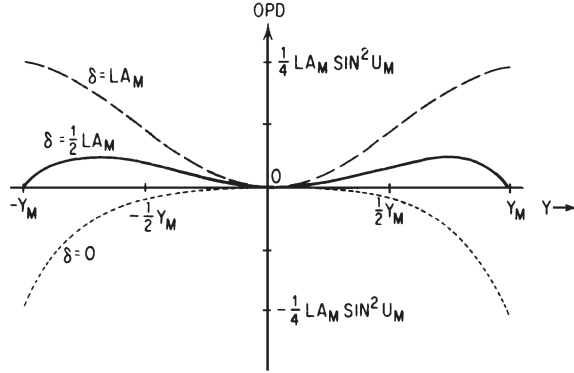


Figure 11.3 The OPD of a system with third-order spherical aberration, plotted as a function of Y for three positions of the reference point.

If we now substitute $\delta = LA_m/2$ into Eq. 11.8, we find (by differentiating with respect to Y and setting the result equal to zero) that the maximum OPD occurs at $Y = Y_m \sqrt{0.5} = 0.707 Y_m$ and is given by

$$\text{OPD} = \frac{LA_m}{16} n \sin^2 U_m$$

This is one-quarter of the OPD at the paraxial focus.

Applying Rayleigh's criterion by setting the OPD equal to one-quarter wavelength, we find the amount of marginal spherical aberration corresponding to this OPD is

$$LA_m = \frac{4\lambda}{n \sin^2 U_m} = 16\lambda (f/\#)^2 \quad (11.9a)$$

Again, making an approximate conversion to transverse aberration by multiplying by $\sin U_m$, we get

$$\text{TA}_m = \frac{4\lambda}{n \sin U_m} = \frac{4\lambda}{\text{NA}} = 8\lambda (f/\#) \quad (11.9b)$$

Fifth-order spherical aberration. When the spherical aberration consists of third and fifth order (and this includes the vast majority of all optical systems), we can write.

$$LA = aY^2 + bY^4$$

Substituting $LA = LA_m$ at $Y = Y_m$ and $LA = LA_z$ at $Y = 0.707 Y_m$, we find that the constants a and b are related to the marginal and zonal spherical by the following expressions:

$$\begin{aligned}
 LA_m &= aY_m^2 + bY_m^4 \\
 LA_z &= \frac{aY_m^2}{2} + \frac{bY_m^4}{4} \\
 a &= \frac{4LA_z - LA_m}{Y_m^2} \\
 b &= \frac{2LA_m - 4LA_z}{Y_m^4}
 \end{aligned}$$

The OPD is represented by truncating Eq. 11.5

$$OPD = \frac{1}{2} n \sin^2 U \left(\delta - \frac{aY^2}{2} - \frac{bY^4}{3} \right)$$

and the graph of OPD versus Y is a curve of the type shown in the upper plot of Fig. 11.4. The exact shape of the curve is, of course, dependent on the values of a , b , and δ .

The best focus occurs when

$$\delta = \frac{-3a^2}{16b} = \frac{-3(4LA_z - LA_m)^2}{32(LA_m - 2LA_z)} = \frac{3}{4} LA_{\max} \quad (11.10)$$

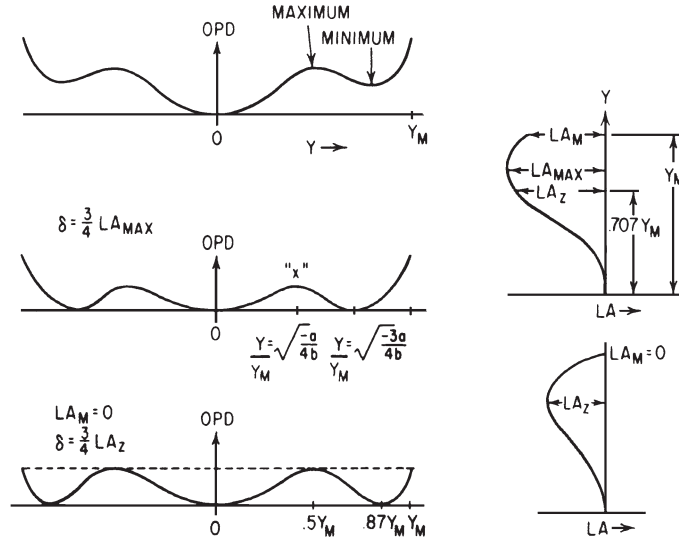


Figure 11.4 OPD vs. Y in the presence of third- and fifth-order aberration. Upper: OPD is a sixth-order function of Y , its shape depending on the aberration coefficients, a and b , and the position of the reference point (δ). Middle: OPD vs. Y when $\delta = (\frac{3}{4})LA_{\max}$. Lower: OPD is minimized when $LA_m = 0$ and $\delta = (\frac{3}{4})LA_z$.

since at this point the OPD is zero for three values of Y as shown in the middle plot of Fig. 11.4. At this focus, the OPD at the margin is

$$\text{OPD}_m = \frac{1}{2} n \sin^2 U_m \left(\frac{-3a^2}{16b} - \frac{aY_m^2}{2} - \frac{bY_m^4}{3} \right) \quad (11.11)$$

and at the maximum (point x), which occurs at

$$Y = Y_m \sqrt{-\frac{a}{4b}}$$

the OPD is given by

$$\text{OPD}_x = \frac{na^3 \sin^2 U_m}{96b^2 Y_m^2} \quad (11.12)$$

If the marginal spherical aberration of the system is corrected (so that $\text{LA}_m = 0$) then the values of OPD at the margin and at point X are equal, as indicated in the lower plot of Fig. 11.4. This is the condition for minimum OPD in the presence of fifth-order spherical. Then the shift of the reference point is given by

$$\delta = \frac{3}{4} \text{LA}_z$$

indicating that the best focus is three-fourths of the way from the paraxial focus to the zonal focus. The residual OPD is given by

$$\text{OPD}_m = \text{OPD}_x = \frac{n\text{LA}_z \sin^2 U_m}{24} \quad (11.13)$$

This is one-eighth of the OPD at the paraxial focus. Equating this to one-quarter wavelength, we find that the Rayleigh criterion allows a residual zonal spherical of

$$\text{LA}_z = \frac{6\lambda}{n \sin^2 U_m} \quad (11.14a)$$

To make an approximate conversion to transverse aberration, we multiply by $\sin U_z$, which is approximately equal to $0.7 \sin U_m$, and we get

$$\text{TA}_z = \frac{4.2\lambda}{n \sin U_m} = \frac{4.2\lambda}{\text{NA}} \quad (11.14b)$$

The Wave Aberration Polynomial

Equations 3.1 and 3.2 presented a power series expansion which expressed the transverse ray aberration as a function of h , s , and θ (see Fig. 3.1 for the meaning of these terms.) A similar expression can be derived for the wave-front aberration, or OPD.

$$\begin{aligned}
\text{OPD} = & A'_1 s^2 + A'_2 s h \cos \theta \\
& + B'_1 s^4 + B'_2 s^3 h \cos \theta + B'_3 s^2 h^2 \cos^2 \theta + B'_4 s^2 h^2 + B'_5 s h^3 \cos \theta \\
& + C'_1 s^6 + C'_2 s^5 h \cos \theta + C'_4 s^4 h^2 + C'_5 s^4 h^2 \cos^2 \theta + C'_7 s^3 h^3 \cos \theta \\
& + C'_8 s^3 h^3 \cos^3 \theta + C'_{10} s^2 h^4 + C'_{11} s^2 h^4 \cos^2 \theta + C'_{12} s h^5 \cos \theta \\
& + D'_1 s^8 + \dots
\end{aligned}$$

Note that although the constants here correspond to those in Eqs. 3.1 and 3.2, they are not numerically the same. However, the expressions are related by

$$y' = \text{TA}_y = \frac{l}{n} \frac{\partial \text{OPD}}{\partial y} \quad \text{and} \quad x' = \text{TA}_x = \frac{l}{n} \frac{\partial \text{OPD}}{\partial x}$$

where l is the pupil-to-image distance and n is the image space index. Note that the exponent of the semiaperture term s is larger by one in the wave-front expression than in the ray-intercept equations. This equation allows us to determine the shape of the wave front for any combination of aberrations.

11.4 Aberration Tolerances

The preceding sections form a basis for the establishment of what are usually referred to as *aberration tolerances*. We should note, however, that the use of the word “tolerance” in this connection does not carry the same go, no-go connotation that it does in matters mechanical, where parts may suddenly cease to fit or function when tolerances are exceeded. *Any* amount of aberration degrades the image; a larger amount simply degrades it more. Thus, it might be more accurate to call this section “Aberration Allowances.”

The Rayleigh criterion, or limit, allows not more than one-quarter wavelength of OPD over the wave front with respect to a reference sphere about a selected image point in order that the image may be “sensibly” perfect. For convenience, we will use the term one Rayleigh limit to mean an OPD of one-quarter wavelength. We have previously noted that the image formed by a perfect lens is a diffraction pattern which contains 84 percent of its energy in a central disk, the remaining 16 percent being distributed in the rings of the pattern. When the OPD is less than several Rayleigh limits, the size of the central disk is basically unchanged, but a noticeable shift of energy from the central disk to the rings takes place.

RMS OPD

The preceding discussions have measured the OPD in terms of its maximum departure from the reference sphere. This is often referred

to as peak-to-peak or peak-to-valley (P-V) OPD. It correlates well to image quality when the shape of the wave front is relatively smooth. However, it is inadequate if the wave front is abruptly irregular. In such circumstances the RMS OPD is a better measure of the effect of the wave-front deformation. RMS stands for “root mean square,” and is the square root of the average (or mean) of the squares of all the OPD values sampled over the full aperture of the system. Consider, for example, an otherwise perfect optical system with a bump on one surface. If the bump covers only a very small area, its effect on the image will be correspondingly small, even if the P-V OPD of the bump in the wave front is quite large. In this sort of case the RMS OPD would be very small and would represent the effect of the bump on the image much more accurately than the P-V OPD would. The relationship between RMS OPD and P-V OPD for the case of the very smooth wave-front deformation caused by defocusing is

$$\text{RMS OPD} = \frac{\text{P-V OPD}}{3.5}$$

For a less smooth wave-front deformation the denominator in this expression will be larger; this is especially true for deformations caused by high-order aberrations or by fabrication errors. Most workers assume a denominator of 4 or 5 in the above expression when dealing with random errors. Thus the Rayleigh quarter-wave criterion corresponds to an RMS OPD of a fourteenth- or a twentieth-wave. The fact that a twentieth-wave sounds much more impressive than a quarter-wave may have contributed to the popularity of RMS OPD among suppliers of optical systems.

Strehl Ratio

The *Strehl ratio* is the illumination at the center of the Airy disk for an aberrated system expressed as a fraction of the corresponding illumination for a perfect system, as shown in Fig. 11.5. It is a good measure of image quality when the optical system is well corrected. A Strehl ratio of 80 percent corresponds to a quarter-wave P-V OPD (exactly for defocus, approximately for most aberrations.) For modest amounts of OPD, the relationship between the Strehl ratio and the RMS OPD is well approximated by

$$\text{Strehl ratio} = e^{-(2\pi\omega)^2}$$

where ω is the RMS OPD in waves.

For various amounts of OPD, the several measures of image quality are related as indicated in the following table. It assumes that the OPD is due to defocusing. The P-V OPD is given in both Rayleigh lim-

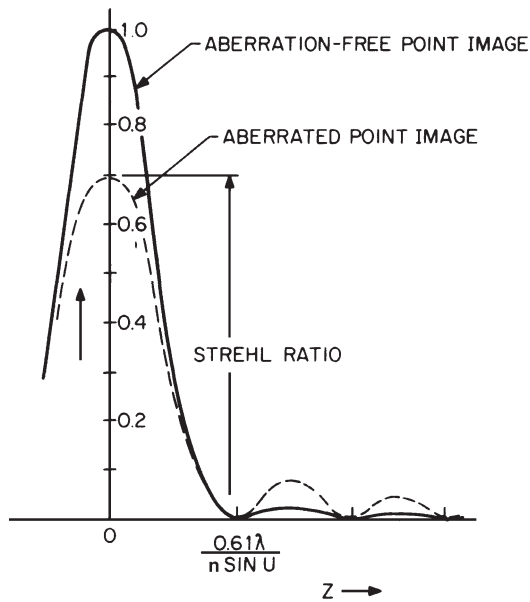


Figure 11.5

Relation of Image Quality Measures to OPD

P-V OPD	RMS OPD	Strehl ratio	% energy in	
			Airy disk	Rings
0.0	0.0	1.00	84	16
0.25RL = $\lambda/16$	0.018λ	0.99	83	17
0.5RL = $\lambda/8$	0.036λ	0.95	80	20
1.0RL = $\lambda/4$	0.07λ	0.80	68	32
2.0RL = $\lambda/2$	0.14λ	0.4*	40	60
3.0RL = 0.75λ	0.21λ	0.1*	20	80
4.0RL = λ	0.29λ	0.0*	10	90

*The smaller values of the Strehl ratio do not correlate well with image quality.

its (RL) and wavelengths. The *Marechal criterion* for image quality is a Strehl ratio of 0.80, which corresponds to the Rayleigh limit for defocusing but is otherwise more general than the quarter-wave limit.

Thus it is apparent that an amount of aberration corresponding to one Rayleigh limit does cause a small but appreciable change in the characteristics of the image. For most systems, however, one may assume that, if the aberrations are reduced to the Rayleigh limit, the performance will be first class and that it will take a determined investigator a considerable amount of effort to detect the resultant difference in a performance. An occasional system does require correction to a fraction of the

Rayleigh limit. Microscopes and telescopes are usually corrected to meet or better the Rayleigh criterion, on the axis at least; photographic lenses approach this level of correction only infrequently.

The following tabulation indicates the amount of aberration corresponding to one Rayleigh limit ($OPD = \lambda/4$) when the reference point is chosen to minimize the P-V OPD.

Out of Focus

Longitudinal:

$$\Delta l' = \frac{\lambda}{2n \sin^2 U_m} \quad (11.15)$$

Transverse:

$$H' = \frac{0.5\lambda}{NA}$$

Third-Order Marginal Spherical

Longitudinal:

$$LA_m = \frac{4\lambda}{n \sin^2 U_m} \quad (11.16)$$

Transverse:

$$TA_m = \frac{4\lambda}{NA}$$

Zonal Residual Spherical ($LA_m = 0$)

Longitudinal:

$$LA_z = \frac{6\lambda}{n \sin^2 U_m} \quad (11.17)$$

Transverse:

$$TA_z = \frac{4.2\lambda}{NA}$$

Tangential Coma

$$Coma_T = \frac{1.5\lambda}{NA} \quad (11.18)$$

Chromatic aberration

Axial color:

$$LAch = L'_F - L'_C = \frac{\lambda}{n \sin^2 U_m} \quad (11.19)$$

$$TAch = \frac{\lambda}{NA}$$

Lateral color:

$$TchA = H'_F - H'_C = \frac{0.5\lambda}{NA} \quad (11.20)$$

The symbols are λ , the wavelength of light; n , the index of the medium in which the image is formed; U_m , the slope angle of the marginal axial ray at the axial image; H , the image height; $NA = n \sin U_m$, the numerical aperture.

The allowance for longitudinal color is derived from the out-of-focus allowance; if the reference point is midway between the long- and short-wavelength focal points, it is apparent that they may be separated by twice the out-of-focus allowance before the Rayleigh limit is exceeded. For the chromatic aberrations these amounts are less significant in terms of their effect on the image quality (e.g., MTF) than are the quarter-wave amounts of the monochromatic aberrations. This is because only the extreme wavelengths (e.g., C and F) are a quarter-wave off the nominal wave front; all the other wavelengths are at less than a quarter-wave. Since for most systems the spectral response is at least somewhat peaked up for the central wavelengths, this means that for chromatic aberrations in amounts corresponding to Eqs. 11.19 and 11.20, more than half of the effective illumination has less than an eighth-wave OPD. Thus for ordinary chromatic one can assume that 1.8 to 2.5 (depending on whether the system spectral response is flat or peaked) times the amounts indicated above will produce about the same effect on the image as the quarter-wave amounts for the monochromatic aberrations. If the chromatic is in the form of secondary spectrum, factors of 2.5 to 4.5 are appropriate. Note that the human visual response is quite peaked and factors approaching the larger ones above are suitable for visual systems.

The allowance for coma is frequently exceeded, since it is extremely difficult to correct a system to this level of quality over an appreciable field. The out-of-focus allowance is, of course, applicable to curvature of field, and values of z_s and z_t (x_s and x_t) should (ideally, at least) be less than twice this amount. However, it is a rare system that can be corrected to this level, and most optical systems which cover an extended field exceed this allowance many times over.

Example A

For a visual optical system with a relative aperture of $f/5$, $\sin U_m = 0.10$ and $\lambda = 0.55 \mu\text{m} = 0.00055 \text{ mm}$. The aberration allowances corresponding to one-quarter wave OPD are thus given by:

$$\text{Out of focus} = \pm \frac{0.00055}{2 (0.1)^2} = \pm 0.0275 \text{ mm}$$

$$\text{Marginal spherical} = \pm \frac{4 (0.00055)}{(0.1)^2} = \pm 0.22 \text{ mm}$$

$$\text{Zonal spherical} = \pm \frac{6 (0.00055)}{(0.1)^2} = \pm 0.33 \text{ mm (LA}_m = 0)$$

$$\text{Tangential coma} = \pm \frac{1.5 (0.00055)}{0.1} = \pm 0.00825 \text{ mm}$$

$$\text{Axial chromatic} = \pm \frac{0.00055}{(0.1)^2} = \pm 0.055 \text{ mm (= } \pm 0.13 \text{ mm realistically)}$$

11.5 Image Energy Distribution (Geometric)

When the aberrations exceed the Rayleigh limit by several times, diffraction effects become relatively insignificant, and the results of geometric raytracing may be used to predict the appearance of a point image with a fair degree of accuracy. This can be done by dividing the entrance pupil of the optical system into a large number of equal areas and tracing a ray from the object point through the center of each of the small areas. The intersection of each ray with the selected image plane is plotted, and since each ray represents the same fraction of the total energy in the image, the density of the points in the plot is a measure of the power density (irradiance, illuminance) in the image. Obviously the more rays that are traced, the more accurate the representation of the geometrical image becomes. A ray intercept plot of this type is called a *spot diagram*. Figure 11.6 indicates several methods of placing the rays in the entrance pupil and shows an example of a spot diagram. The rectangular ray placement is the most used, being the easiest to do and also having utility in OPD and MTF calculations.

The preparation of a spot diagram obviously entails a great amount of raytracing. As pointed out in Section 10.4, the rays on each side of the meridional plane are mirror images of each other; this reduces the necessary raytracing by 50 percent. The number of rays to be traced can be reduced markedly by an interpolation process. To produce a spot diagram which faithfully reproduces the image, several hundred ray intersections are required. However, if 20 or 30 rays are traced, it is possible to fit an interpolation equation to their intercept coordinates so that the required (larger) number of points can be computed from the equation. Equations such as Eqs. 3.1 and 3.2 are suitable for this purpose. However, the high computation speed now available in most desktop computers makes this unnecessary, and most spot diagrams are made by simply tracing several hundred rays through the system.

For an accurate analysis, the effects of wavelength on the energy distribution must also be included. This is accomplished by tracing additional rays at different wavelengths; the variation of system

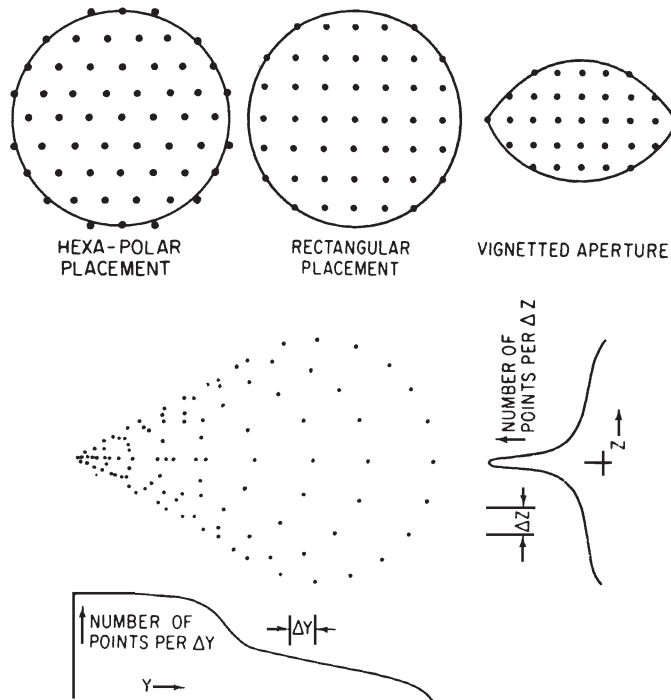


Figure 11.6 The upper sketches show the placement of rays in the entrance pupil so that each ray “represents” an equal area. Shown below are a spot diagram (for a system with pure coma) and the line spread functions (below and to the right) obtained by counting the number of points between parallel lines separated by a small distance, ΔY or ΔZ .

sensitivity with wavelength may be taken into account by tracing fewer rays in the less-sensitive wavelengths or by an appropriate weighting scheme. For devices with appreciable fields of view, spot diagrams must also be prepared for several obliquities.

Focusing must also be taken into account. Since it is difficult to predict in advance the exact position of the plane of best focus, spot diagrams are often prepared for several positions of the image plane and the best is selected. One way of accomplishing this is to hold the final ray data (intercepts and directions) in the computer memory and to calculate a new set of intercepts for each focus shift.

11.6 Spread Functions—Point and Line

The image of a point (whether the data are derived from a spot diagram or from an exact diffraction calculation) can be considered from a three-dimensional point of view to be a sort of illumination

mountain, as sketched in Fig. 11.7. The *point spread function* can be described two dimensionally by a series of cross sections through the three-dimensional solid. The solid corresponding to a line image is also shown in Fig. 11.7. The cross section of the line solid is called the *line spread function* and can be obtained by integrating the point solid along sections parallel to the direction of the line, since the line image is simply the summation of an infinite number of point images along its length. The lower part of Fig. 11.6 shows a spot diagram for a system with pure third-order coma and the line spread functions derived from it.

A *knife-edge trace* is a plot of the energy which passes a knife edge versus the position of the knife edge as the knife is scanned laterally through the image of a point. The slope, or derivative, of the knife-edge scan is equal to the value of the line spread function. This relationship is often used to measure the line spread function in order to measure the MTF (see Sec. 11.8).

11.7 Geometric Spot Size Due to Spherical Aberration

Third-order spherical aberration

The meridional spread of an image can, of course, be read directly from a ray intercept curve (see Fig. 3.24, for example). For points on the axis, the image blur is symmetrical and it is possible to obtain simple expressions for the size of the blur spot.

Figure 11.8 shows the ray paths near the image plane of a system afflicted with third-order spherical aberration. It is apparent that the minimum diameter blur spot for this system occurs at a point between the marginal focus and the paraxial focus. This point is three-quarters

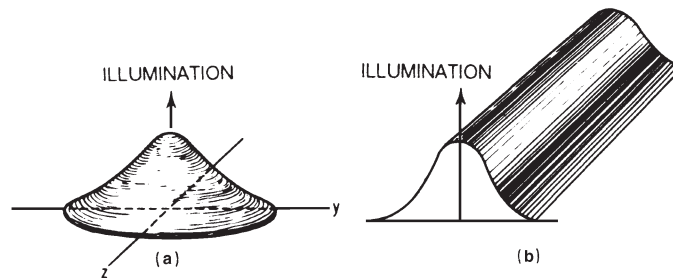


Figure 11.7 The energy distribution in the image of a point (a) and a line (b). The line image (b) is generated by summing an infinite number of point images (a) along its length. The line spread function is the cross section of (b).

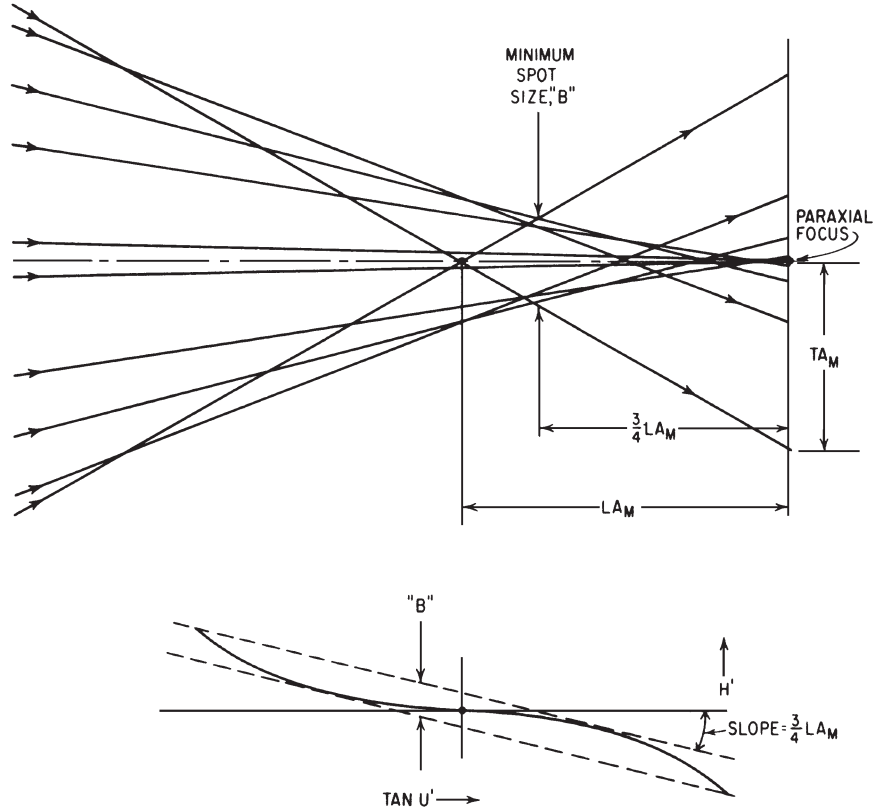


Figure 11.8 The upper figure shows the ray paths near the focus of a system with third-order spherical aberration. The smallest blur spot occurs at $0.75LA_m$ from the paraxial focus. The lower figure is a ray intercept curve (H' vs. $\tan U'$) for the same case; the slope of the dashed lines ($dH'/d \tan U'$) equals $0.75LA_m$ and their separation indicates the diameter of the blur spot.

of the way from the paraxial focus to the marginal focus, and the diameter of the spot at this point is given by:

$$\begin{aligned} B &= \frac{1}{2}LA_m \tan U_m \\ &= \frac{1}{2}TA_m \end{aligned} \quad (11.21)$$

Fifth-order spherical aberration

When the spherical aberration consists of both third and fifth orders, the situation is more complex. From a geometric standpoint, the minimum spot size can be shown to occur when the marginal spherical is equal to two-thirds of the (0.707) zonal spherical, or

$$LA_z = 1.5LA_m$$

and $LA = \text{zero}$ at $y = 1.12Y_m$. For most systems, this means that both LA_m and LA_z are undercorrected when the minimum geometric spot size is desired.

Then the “best” focus occurs at

$$\delta = 1.25LA_m = 0.83LA_z$$

and the size of the blur spot is

$$\begin{aligned} B &= \frac{1}{2}LA_m \tan U_m \\ &= \frac{1}{3}LA_z \tan U_m \end{aligned} \quad (11.22)$$

However, if the marginal spherical is corrected to zero, then the “best” geometric focus is at

$$\delta = 0.42LA_z$$

and for small values of U , the minimum blur spot size is

$$B = 0.84LA_z \tan U_m \quad (11.23)$$

The “best” focus positions described above are not necessarily those one would select visually, and the reader may have noticed that they differ from those selected on the basis of OPD in Sec. 11.3. Figure 11.9 shows a ray intercept curve for fifth-order spherical with the marginal spherical corrected to zero. The slope of the two solid lines indicates the amount of focus shift required to minimize the blur spot. (Remember that the slope $\Delta H/\Delta \tan U$ is equivalent to a focus shift, and that the vertical separation of the lines indicates the size of the blur.) However, the dashed pair of lines (which enclose the ray intercepts from about 80 percent of the aperture) indicate a focus position at which there is a much higher concentration of light within a much smaller spot, and this is usually the preferred focus, even though the *total* spread of the image is greater by a factor of almost 2.

The concept of minimum blur size is little used in optical systems for visual or photographic work, since the minimum geometric blur position is seldom, if ever, chosen as the focus. However, in systems which use photodetectors, one frequently wishes to determine the smallest detector that will collect all the energy in the image. Under such circumstances, the blur spot sizes given by Eqs. 11.21, 11.22, and 11.23 are extremely useful; in Chap. 13, a number of very convenient equations are presented which make use of this concept to predict the performance of several simple optical systems which are frequently used in conjunction with photodetectors. The geometric spot minimum is

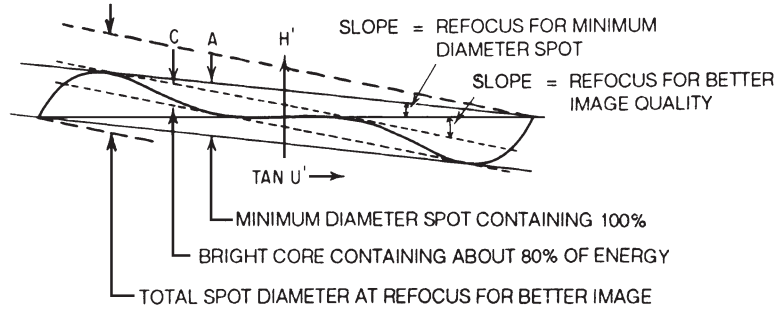


Figure 11.9 The image blur spot size for third- and fifth-order spherical aberration, balanced for $LA_m = 0$, illustrating the effects of various focus settings.

often a consideration when a system's performance is well below that of a "diffraction-limited" system.

Example B

A visual system, working at $f/5$ ($\sin U_m = 0.1$), which has an under-corrected third-order longitudinal spherical aberration of 0.22 mm, will have its minimum diameter blur spot $0.75 \times 0.22 = 0.165$ mm ahead of the paraxial focus, and by Eq. 11.21 the size of this blur spot will be equal to

$$B = \frac{1}{2} \times 0.22 \times 0.1005 = 0.011 \text{ mm}$$

It is interesting to note that on the basis of the OPD analysis, the best focus should occur $0.5 \times 0.22 = 0.11$ mm ahead of the paraxial focus and that the diameter of the central disk of the Airy pattern is equal to

$$\frac{1.22\lambda}{n \sin U} = \frac{1.22 (0.00055)}{0.1} = 0.0066 \text{ mm}$$

This central disk should contain about 68 percent of the energy in the image, since a marginal spherical of 0.22 mm is equal to just one Rayleigh limit (as shown in Example A).

If an $f/5$ system has third- and fifth-order spherical with a corrected marginal and a zonal residual of 0.33 mm (again in longitudinal measure), the smallest geometric spot size would be found at about $0.42 \times 0.33 = 0.14$ mm from the paraxial focus and the spot size would be

$$B = 0.84 \times 0.33 \times 0.1005 = 0.028 \text{ mm}$$

Here the comparison with the OPD analysis is less fortuitous. The zonal spherical of 0.33 mm is again equivalent to one Rayleigh limit; we would expect the central disk of the diffraction pattern to be 0.0066

mm as above, and the best focus to be about $0.75 \times 0.33 = 0.25$ mm from the paraxial focus. The agreement with geometry is somewhat better if we use the focus indicated by the dashed lines of Fig. 11.9; the position of “best focus” is almost exactly the same as the OPD best focus and the diameter of the intense center spot of the geometric pattern is to the order of 0.01 mm.

11.8 The Modulation Transfer Function

A type of target commonly used to test the performance of an optical system consists of a series of alternating light and dark bars of equal width, as indicated in Fig. 11.10a. Several sets of patterns of different spacings are usually imaged by the system under test and the finest set in which the line structure can be discerned is considered to be the limit of resolution of the system, which is expressed as a certain number of lines per millimeter.* When a pattern of this sort is imaged by an optical system, each geometric line (i.e., of infinitesimal width) in the object is imaged as a blurred line, whose cross section is the line spread function. Figure 11.10b indicates a cross section of the brightness of the bar object, and Fig. 11.10c shows how the image spread function “rounds off” the “corners” of the image. In Fig. 11.10d, the effect of the image blur on progressively finer patterns is indicated. It is apparent that when the illumination contrast in the image is less than the smallest amount that the system (e.g., the eye, film, or photodetector) can detect, the pattern can no longer be “resolved.”

If we express the contrast in the image as a “modulation,” given by the equation

$$\text{Modulation} = \frac{\text{max.} - \text{min.}}{\text{max.} + \text{min.}}$$

(where max. and min. are the image illumination levels as indicated in Fig. 11.10d), we can plot the modulation as a function of the number of lines per millimeter in the image, as indicated in Fig. 11.11a. The intersection of the modulation function line with a line representing the smallest amount of modulation which the system sensor can detect will give the limiting resolution of the system. The curve indicating the smallest amount of modulation detectable by a system or sensor (i.e., the threshold) is often called an AIM curve, where the initials stand for the aerial image modulation required to produce a response in the sys-

*Note that in optical work the convention is to consider a “line” to consist of one light bar and one dark bar, i.e., one cycle. In television parlance, both light and dark lines are counted. Thus, 10 “optical” lines indicate 10 light and 10 dark lines, whereas 10 “television” lines indicate 5 light and 5 dark lines. To avoid confusion, “optical” lines are frequently referred to as line pairs, e.g., 10 line pairs per millimeter.

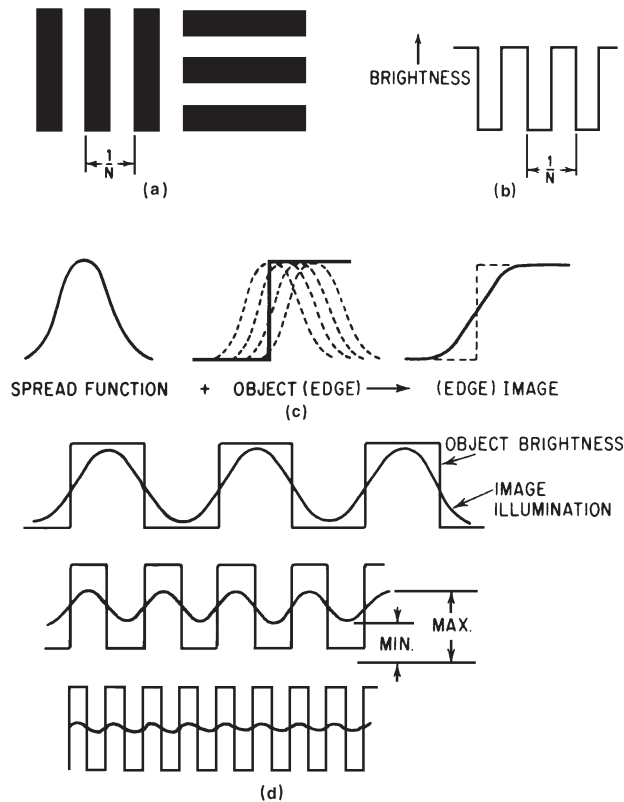


Figure 11.10 The imagery of a bar target. (a) A typical bar target used in testing optical systems consists of alternating light and dark bars. If the pattern has a frequency of N lines per millimeter, then it has a period of $1/N$ millimeters, as indicated. (b) A plot of the brightness of (a) is a square wave. (c) When an image is formed, each point is imaged as a blur, with an illumination distribution described by the spread function. The image then consists of the summation of all the spread functions. (d) As the test pattern is made finer, the contrast between the light and dark areas of the image is reduced.

tem or sensor. The response characteristics of the eye, films, image tubes, CCDs, etc., are appropriately described by an AIM curve. Note that the modulation threshold usually rises with spatial frequency, although there are exceptions. Figure 5.4 is effectively an AIM curve for the eye; note that at very low angular frequencies the contrast threshold of the eye rises (for physiologic reasons).

It should be apparent that the limiting resolution does not fully describe the performance of the system. Figure 11.11b shows two modulation plots with the same limiting resolution, but with quite different performances. The plot with the greater modulation at the lower frequencies is obviously superior, since it will produce crisper, more

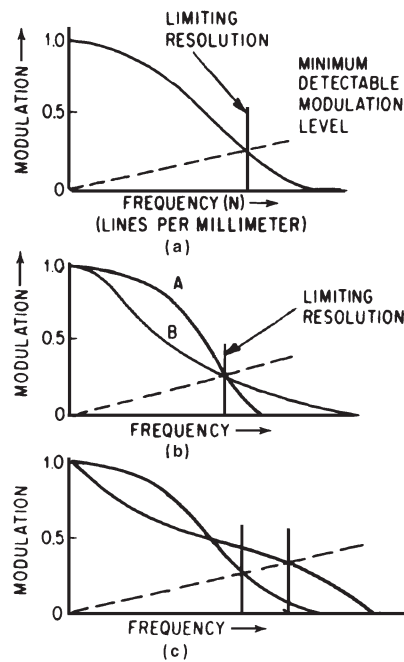


Figure 11.11 (a) The image modulation can be plotted as a function of the frequency of the test pattern. When the modulation drops below the minimum that can be detected, the target is not resolved. (b) The system represented by (a) will produce a superior image, although both (a) and (b) have the same limiting resolution.

contrasty images. Unfortunately, the type of choice one is usually faced with in deciding between two systems is less obvious. Consider Fig. 11.11c, where one system shows high limiting resolution and the other shows high contrast at low target frequencies. In cases of this type, the decision must be based on the relative importance of contrast versus resolution in the function of the system.*

The preceding discussion has been based on patterns whose brightness distribution is a “square wave” (Fig. 11.10b) and whose image illumination distribution is distorted or “rounded off” by characteristics of the optical system, as indicated in Fig. 11.10d. However, if the object pattern brightness distribution is in the form of a sine wave, the distribution in the image is also described by a sine wave, regardless of the shape of the spread function. This fact has led to the widespread use of the modulation transfer function to describe the performance of a lens system. The modulation transfer function is the ratio of the

*The *Strehl definition* is the ratio of the light intensity at the peak of the diffraction pattern of an aberrated image to that at the peak of an aberration-free image, and is one of the many criteria that have been proposed for image evaluation. It can be computed by calculating the volume under the (three-dimensional) modulation transfer function and dividing by the volume under the curve for an aberration-free lens (Sec. 11.10). A similar criterion for quick general evaluation of image quality is the normalized area under the modulation transfer curve.

modulation in the image to that in the object as a function of the frequency (cycles per unit of length) of the sine-wave pattern.

$$\text{MTF}(v) = \frac{M_i}{M_o}$$

A plot of MTF against frequency v is thus an almost universally applicable measure of the performance of an image-forming system and has been applied not only to lenses but to films, phosphors, image tubes, the eye, and even to complete systems such as camera-carrying aircraft.

One particular advantage of the MTF is that it can be cascaded by simply multiplying the MTFs of two or more components to obtain the MTF of the combination. For example, if a camera lens with an MTF of 0.5 at 20 cycles per millimeter is used with a film with an MTF of 0.7 at this frequency, the combination will have an MTF of $0.5 \times 0.7 = 0.35$. If the object to be photographed with this camera has a contrast (modulation) of 0.1, then the image modulation is $0.1 \times 0.35 = 0.035$, close to the limit of visual detection.

One should note, however, that MTFs do not cascade between optical components which are directly coherently “connected,” i.e., lenses which are not separated by a diffuser of some sort. This is because the aberrations of one component may compensate for the aberrations in another, and thus produce an image quality for the combination which is superior to that of either component. Any “corrected” optical system illustrates this point.

In the past, the MTF has been referred to as *frequency response*, *sine wave response*, or *contrast transfer function*.

If we assume an object consisting of alternating light and dark bands, the brightness (luminance, radiance) of which varies according to a cosine (or sine) function, as indicated by the upper part of Fig. 11.12, the distribution of brightness can be expressed mathematically as

$$G(x) = b_0 + b_1 \cos(2\pi vx) \quad (11.24)$$

where v is the frequency of the brightness variation in cycles per unit length, $(b_0 + b_1)$ is the maximum brightness, $(b_0 - b_1)$ is the minimum brightness, and x is the spatial coordinate perpendicular to the bands. The modulation of this pattern is then

$$M_0 = \frac{(b_0 + b_1) - (b_0 - b_1)}{(b_0 + b_1) + (b_0 - b_1)} = \frac{b_1}{b_0} \quad (11.25)$$

When this line pattern is imaged by an optical system, each point in the object will be imaged as a blur. The energy distribution within this blur will depend on the relative aperture of the system and the aberrations present. Since we are dealing with a linear object, the image of

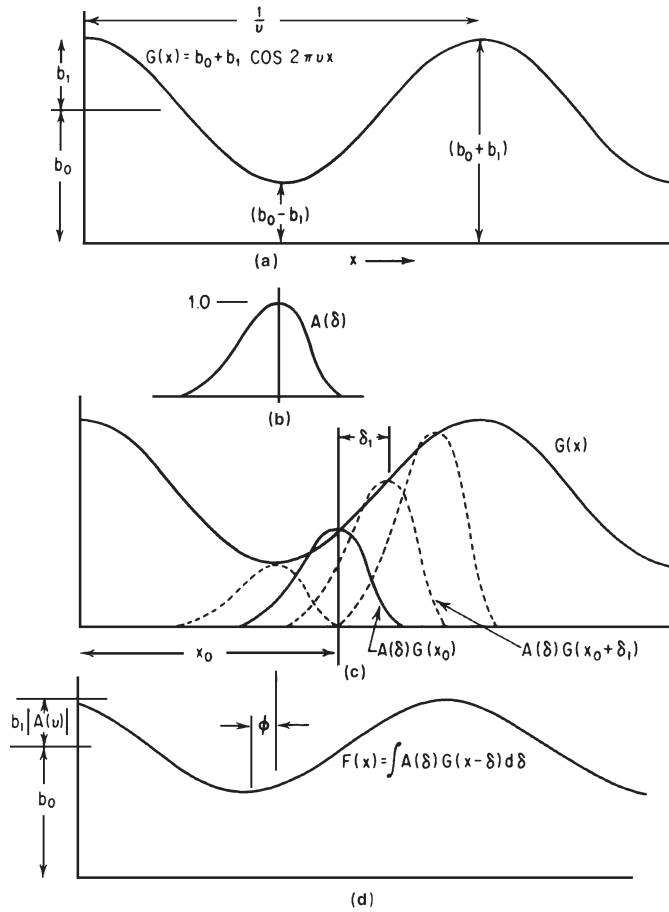


Figure 11.12 Convolution of the object brightness distribution function $G(x)$ with the line spread function $A(\delta)$. (a) The object function, $G(x) = b_0 + b_1 \cos(2\pi vx)$, plotted against x . (b) The line spread function $A(\delta)$. Note the asymmetry. (c) Illustrating the manner in which $G(x)$ is modified by $A(\delta)$. A point (or more accurately, a line element) at x_0 is imaged by the system as $G(x_0)$ times $A(\delta)$. Similarly at $x_0 + \delta_1$, the image of the line element is described by $A(\delta)G(x_0 + \delta_1)$. Thus the image function at a given x has a value equal to the summation of the contributions from all the points whose spread-out images reach x . (d) The image function $F(x) = \int A(\delta)G(x - \delta) d\delta$ has been shifted by ϕ and has a modulation $M_i = M_0|A(v)|$.

each line element can be described by the line spread function (Sec. 11.5, Fig. 11.7) indicated in Fig. 11.12 as $A(\delta)$. We now assume (for convenience) that the dimensions x and $(1/v)$ in Eq. 11.24 are the corresponding dimensions in the image. It is apparent that the image energy distribution at a position x is the summation of the product of $G(x)$ and $A(\delta)$ and can be expressed as

$$F(x) = \int A(\delta) G(x-\delta) d\delta \quad (11.26)$$

Combining Eqs. 11.24 and 11.26, we get

$$F(x) = b_0 \int A(\delta) d\delta + b_1 \int A(\delta) \cos [2\pi v (x-\delta)] d\delta \quad (11.27)$$

After normalizing by dividing by $\int A(\delta) d\delta$, Eq. 11.27 can be transformed to

$$\begin{aligned} F(x) &= b_0 + b_1 |A(v)| \cos (2\pi vx - \phi) \\ &= b_0 + b_1 A_c(v) \cos (2\pi vx) + b_1 A_s(v) \sin (2\pi vx) \end{aligned} \quad (11.28)$$

where

$$|A(v)| = [A_c^2(v) + A_s^2(v)]^{1/2} \quad (11.29)$$

and

$$A_c(v) = \frac{\int A(\delta) \cos (2\pi v \delta) d\delta}{\int A(\delta) d\delta} \quad (11.30)$$

$$A_s(v) = \frac{\int A(\delta) \sin (2\pi v \delta) d\delta}{\int A(\delta) d\delta} \quad (11.31)$$

$$\cos \phi = \frac{A_c(v)}{|A(v)|} \quad (11.32)$$

$$\tan \phi = \frac{A_s(v)}{A_c(v)} \quad (11.33)$$

Note that the resulting image energy distribution $F(x)$ is still modulated by a cosine function of the same frequency v , demonstrating that a cosine distribution object is always imaged as a cosine distribution image. If the line spread function $A(\delta)$ is asymmetrical, a phase shift ϕ is introduced. This is a lateral shift of the location of the image (at this frequency).

The modulation in the image is given by

$$M_i = \frac{b_1}{b_0} |A(v)| = M_0 |A(v)| \quad (11.34)$$

and $|A(v)|$ is the modulation transfer function.

$$\text{MTF}(v) = |A(v)| = \frac{M_i}{M_0}$$

The *optical transfer function* (OTF) is the complex function which describes this process. It is a function of the spatial frequency v of the sine-wave pattern. The real part of the OTF is the *modulation transfer function* (MTF) and the imaginary part is the *phase transfer function* (PTF). If the PTF is linear with frequency, it is, of course, just a simple lateral displacement of the image (as, for example, distortion), but if it is nonlinear, it can have an effect on the image quality. A phase shift of 180° is a reversal of contrast, in that the image pattern is light where it should be dark, and vice versa. See Fig. 15.24 for example.

11.9 Computation of the Modulation Transfer Function

We will illustrate the computation of the MTF by a grossly simplified and abbreviated example. In actual practice, a much larger number of points must be used in the computation if accurate results are to be obtained.

We assume that a spot diagram has been prepared from raytrace data as shown in Fig. 11.13a. The line spread function is determined by integrating the spot diagram in one direction; in practice, one assumes an increment Δx and counts all the spots between the lines bounding the increment. A normalized plot of N_x against x then represents the line spread function $A(x)$. (Note that the point spread function could be derived from a diffraction calculation if the diffraction MTF were desired.)

Since real spread functions are rarely (if ever) represented by ordinary analytic functions, we cannot use Eqs. 11.30 and 11.31 in their integral form. A close approximation (which lends itself nicely to electronic computer usage) is given by the equivalent summation equations,

$$A_c(v) = \frac{\sum A(x) \cos(2\pi vx) \Delta x}{\sum A(x) \Delta x} \quad (11.35)$$

$$A_s(v) = \frac{\sum A(x) \sin(2\pi vx) \Delta x}{\sum A(x) \Delta x} \quad (11.36)$$

As a numerical example, we will determine the value of the MTF for a frequency of $v = 0.1$, i.e., one-tenth cycle per unit length. The values of $A(x)$, the line spread function, which we will use are given in line 2 of Fig. 11.14 for various values of x . Line 4 of the table gives the values of $2\pi vx$ for these same values of x , and Lines 5 and 6 give the values of $\cos(2\pi vx)$ and $\sin(2\pi vx)$ for each point.

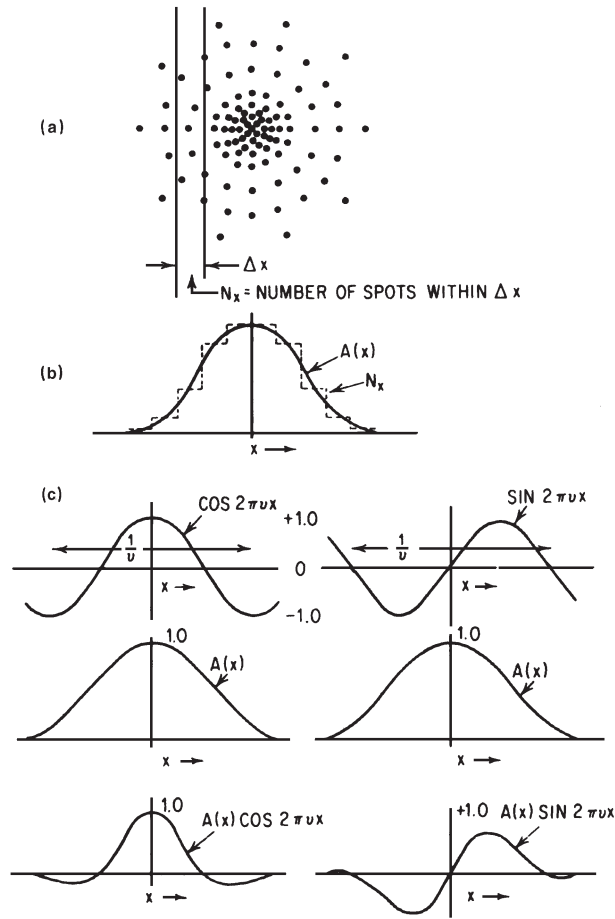


Figure 11.13 The calculation of the modulation transfer factor for a given frequency, v . (a) The spot diagram is summed in one direction by counting the number of spots (ray intersections) in each increment, Δx . (b) The number of spots is plotted against x to get the line spread function $A(x)$. $A(x)$ is usually normalized to peak at unity. (c) $A(x) \cos(2\pi vx)$ and $A(x) \sin(2\pi vx)$ are generated by point-for-point multiplication of $A(x)$ by the trigonometric functions. Then $\int A(x) \cos(2\pi vx) dx$ and $\int A(x) \sin(2\pi vx) dx$ are the areas under their respective curves (remembering that area below the x -axis is negative). Similarly $\int A(x) dx$ is the area under the curve of $A(x)$ vs. x . These values are used in Eqs. 11.29 through 11.34 to get the MTF and phase shift ϕ for the frequency v .

(1) $x (\Delta x = 1.0)$	- 4.5	- 3.5	- 2.5	- 1.5	- 0.5	+ 0.5	+ 1.5	+ 2.5	+ 3.5	+ 4.5
(2) $A(x)$	0.05	0.2	0.5	0.8	1.0	1.0	0.8	0.5	0.2	0.05
$\Sigma A(x) \Delta x = + 5.10$										
(3) vx	- 0.45	- 0.35	- 0.25	- 0.15	- 0.05	+ 0.05	+ 0.15	+ 0.25	+ 0.35	+ 0.45
(4) $2\pi vx$	- 0.9 π (- 162°)	- 0.7 π (- 126°)	- 0.5 π (- 90°)	- 0.3 π (- 54°)	- 0.1 π (- 18°)	+ 0.1 π (+ 18°)	+ 0.3 π (+ 54°)	+ 0.5 π (+ 90°)	+ 0.7 π (+ 126°)	+ 0.9 π (+ 162°)
(5) $\cos (2\pi vx)$	- 0.95106	- 0.58779	0	+ 0.58779	+ 0.95106	+ 0.95106	+ 0.58779	0.0	- 0.58779	- 0.95106
(6) $\sin (2\pi vx)$	- 0.30902	- 0.80902	- 1.0	- 0.80902	- 0.30902	+ 0.30902	+ 0.80902	+ 1.0	+ 0.80902	+ 0.30902
(7) $A(x) \cos (2\pi vx)$	- 0.04755	- 0.11756	0.0	+ 0.47023	+ 0.95106	+ 0.95106	+ 0.47023	0.0	- 0.11756	- 0.04755
$\Sigma A(x) \cos (2\pi vx) \Delta x = + 2.51236$										
(8) $A(x) \sin (2\pi vx)$	- 0.01545	- 0.16180	- 0.5	- 0.64722	- 0.30902	+ 0.30902	+ 0.64722	+ 0.5	+ 0.16180	+ 0.01545
$\Sigma A(x) \sin (2\pi vx) \Delta x = 0.0$										

Figure 11.14 Numerical computation of the modulation transfer function for a frequency of $v = 0.1$ cycles per millimeter, from the line spread function (x) given in line 2.

Now Lines 7 and 8 give $A(x) \cos(2\pi vx)$ and $A(x) \sin(2\pi vx)$, respectively. Since Δx is equal to 1.0 in this example, we can obtain the required summations for Eqs. 11.35 and 11.36 by summing across lines 2, 7, and 8, giving us

$$\sum A(x) \Delta x = +5.10$$

$$\sum A(x) \cos(2\pi vx) \Delta x = +2.51236$$

$$\sum A(x) \sin(2\pi vx) \Delta x = 0.0$$

Note that the last value is a foregone conclusion when $A(x)$ is a symmetrical function of x , since the positive and negative values of the sine function on either side of $x = 0$ cause one side to cancel the other when summed. Thus, when $A(x)$ is symmetrical, the labor of the calculation can be reduced by a factor of 4, since only one-half of the cosine function needs to be evaluated.

Inserting the above values into Eqs. 11.35 and 11.36, we find that

$$A_c(0.1) = \frac{2.51236}{5.1} = +0.4926$$

$$A_s(0.1) = \frac{0.0}{5.1} = 0.0$$

and that by Eqs. 11.29 and 11.33

$$\text{MTF}(0.1) = |A(0.1)| = (0.493^2 + 0^2)^{1/2} = 0.493$$

$$\tan \phi = \frac{0}{+2.512} = 0.0$$

Thus, for a frequency $v = 0.1$ cycles per unit length, we find a modulation transfer factor of 49 percent. This calculation can be repeated for several values of v , and a plot of the MTF against frequency, similar in appearance to those of Fig. 11.11, can be prepared from the results. As mentioned above, a much smaller value of Δx must be used if accurate results are to be obtained.

Square-wave vs. sine-wave targets

Once the MTF has been determined (plotted) for a range of frequencies, it is possible to determine an analogous function for the modulation transfer of a square wave pattern, i.e., a bar target of the type shown in Fig. 11.10. This is done by resolving the square wave into its Fourier components and taking the sine wave response to each component. Thus, for a given frequency v , the square wave modulation transfer $S(v)$ is given by the following equation [in which $\text{MTF}(v)$ is written $M(v)$ for clarity].

$$S(v) = \frac{4}{\pi} \left[M(v) - \frac{M(3v)}{3} + \frac{M(5v)}{5} - \frac{M(7v)}{7} + \dots \right] \quad (11.37a)$$

The inverse of this function is

$$M(v) = \frac{\pi}{4} \left[S(v) + \frac{S(3v)}{3} - \frac{S(5v)}{5} + \frac{S(7v)}{7} - \dots \right] \quad (11.37b)$$

Practicle resolution considerations

A rough indication of the practical meaning of resolution can be gained from the following, which lists the resolution required to photograph printed or typewritten copy.

Excellent reproduction (reproduces serifs, etc.) requires 8 resolution line pairs per the height of a lowercase letter e.

Legible (easily) reproduction requires 5 line pairs per letter height.

Decipherable (e, c, o partly closed) requires 3 line pairs per letter height.

Point sizes of type (where P is the point size) are

Height of an upper case letter = $0.22P$ mm = $0.0085P$ in

Height of a lower case letter = $0.15P$ mm = $0.006P$ in

The correlation between resolution in cycles per minimum dimension (height, length of military targets) and certain functions (often referred to as *Johnson's law*) is

Detect	1.0 line pairs per dimension
Orient	1.4 line pairs per dimension
Aim	2.5 line pairs per dimension
Recognize	4.0 line pairs per dimension
Identify	6–8 line pairs per dimension
Recognize with 50% accuracy	7.5 line pairs per height
Recognize with 90% accuracy	12. line pairs per height

11.10 Special Modulation Transfer Functions: Diffraction-Limited Systems

Section 11.9 discussed MTF in geometric terms; the spot-diagram techniques set forth there are applicable only when the aberrations are large. When they are small, the interactions between the diffraction

effects of the system aperture and the aberrations become very complex. If there are no aberrations present, the MTF of a system is related to the size of the diffraction pattern (which is a function of the numerical aperture of the system and the wavelength of the light used). For a “perfect” optical system, the MTF is

$$\text{MTF}(v) = \frac{2}{\pi} (\phi - \cos \phi \sin \phi) \quad (11.38)$$

where

$$\phi = \cos^{-1} \left(\frac{\lambda v}{2\text{NA}} \right)$$

and v is the frequency in cycles per millimeter, λ is the wavelength in millimeters, NA is the numerical aperture ($n' \sin U'$), and $\cos^{-1}(x)$ means the angle whose cosine is x .*

It is apparent that $\text{MTF}(v)$ is equal to zero when ϕ is zero; thus, the “limiting resolution” for an aberration-free system, often called the *cutoff frequency*, is

$$v_0 = \frac{2\text{NA}}{\lambda} = \frac{1}{\lambda (f/\#)} \quad (11.39)$$

where λ is in millimeters, $f/\#$ is the relative aperture of the system, and v_0 is in cycles per millimeter. Notice that an optical system is a low-pass filter which cannot transmit information at a higher spatial frequency than the cutoff frequency v_0 .

For an afocal system (or one with the image at infinity), the cutoff frequency is given by

$$v_0 = D/\lambda \text{ cycles/radian}$$

A plot of Eq. 11.38 is shown in Fig. 11.15; the frequency scale is in terms of v_0 , the limiting frequency given by Eq. 11.39. It should be noted that for *ordinary* systems, this level of performance cannot be exceeded. A geometric MTF curve derived from the *raytrace* data (and neglecting diffraction) of a well-corrected lens will sometimes exceed the values of Fig. 11.15; such results are, of course, incorrect and derive from the fact that the light ray concept only partially describes

*Equation 11.38 applies to uniformly illuminated and transmitting circular apertures. For apertures of *any* other shape, the diffraction MTF is equal to the (normalized) area common to the aperture and the aperture displaced. Equation 11.38 is thus the (normalized) area common to two circles of radius R , as their centers are separated by an amount equal to $2vR/v_0$. For a rectangular aperture the plot of MTF would thus be a straight line. The cutoff frequency v_0 is computed from Eq. 11.39 in each case using the aperture size (i.e., the $f/\#$ or NA) in the direction of the resolution.

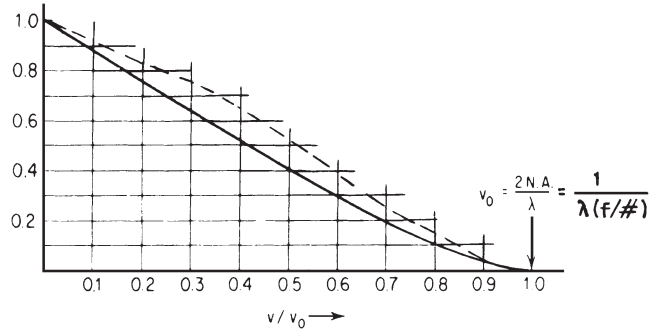


Figure 11.15 The modulation transfer function of an aberration-free system (solid line). Note that frequency is expressed as a fraction of the cutoff frequency. The dashed line is the modulation factor for a square wave (bar) target. Both curves are based on diffraction effects and assume a system with a uniformly transmitting circular aperture.

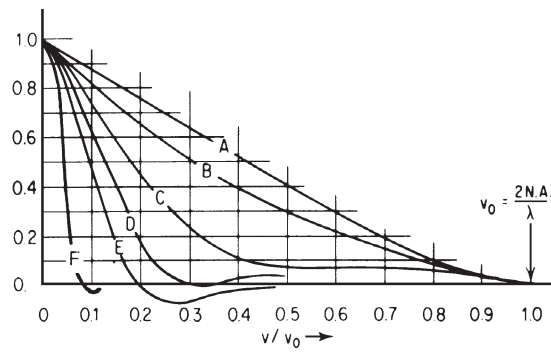


Figure 11.16 The effect of defocusing on the modulation transfer function of an aberration-free system.

- | | | |
|--------------|----------------------------|--------------------|
| (a) In focus | | OPD = 0.0 |
| (b) Defocus | $= \lambda/(2n \sin^2 U)$ | OPD = $\lambda/4$ |
| (c) Defocus | $= \lambda/(n \sin^2 U)$ | OPD = $\lambda/2$ |
| (d) Defocus | $= 3\lambda/(2n \sin^2 U)$ | OPD = $3\lambda/4$ |
| (e) Defocus | $= 2\lambda/(n \sin^2 U)$ | OPD = λ |
| (f) Defocus | $= 4\lambda/(n \sin^2 U)$ | OPD = 2λ |

(Curves are based on diffraction effects—not on a geometric calculation.)

the behavior of electromagnetic radiation. Note also that aberrations always reduce the MTF.

The effects of small amounts of defocusing on the diffraction-limited MTF are shown in Fig. 11.16. Note that curve B corresponds to the depth of focus allowed by one Rayleigh limit as discussed in Secs. 11.2 and 11.4. The small effect produced by an OPD of one-quarter wavelength indicates the astuteness of Rayleigh's selection of this amount as one which would not "sensibly" affect the image quality.

By way of comparison, Fig. 11.17 shows the MTF plots which would be obtained by geometrical calculations of a perfect system defocused by the same amounts. The agreement between Fig. 11.16, whose curves are derived from wave-front analysis, and Fig. 11.17 is poor for small amounts of OPD. However, when the defocusing is sufficient to introduce an OPD of one wavelength or more, the agreement becomes much better. Note that all the curves of Fig. 11.17 are of the same family and that one can be derived from another by a simple ratioing of the frequency scale. These curves are representations of

$$\text{MTF}(v) = \frac{2J_1(\pi Bv)}{\pi Bv} \approx \frac{J_1(2\pi\delta NA v)}{\pi\delta NA v} \quad (11.40)$$

where $J_1(\)$ indicates the first-order Bessel function,* B is the diameter of the blur spot produced by defocusing, δ is the longitudinal defocusing, NA is the numerical aperture, and v is the frequency in cycles per unit length.

Note that in Figs. 11.16 and 11.17, some of the curves show a negative value for the MTF. This indicates that the phase shift in the image (ϕ in Eq. 11.33) is 180° and that the image is light where it should be dark and vice versa. This is known as spurious resolution (since a line pattern can be seen, but it is not a true image of the object) and is a phenomenon which is frequently observed in defocused, well-corrected lenses or in lenses whose defocused image of a point is a nearly uniformly illuminated circular blur. See Fig. 15.24 for example.

In Fig. 11.18, the effects of third-order spherical aberration on MTF are shown. Note once again that the effect of an amount of aberration corresponding to the Rayleigh limit ($\text{OPD} = \lambda/4$) is quite modest. The situation here is quite similar to the defocusing case, in that MTF curves based on geometrical calculations are in poor agreement with Fig. 11.18 where the aberration is small, but in quite reasonable

$$*J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{n+2k}}{2^{n+2k} k!(n+k)!} \quad J_1(x) = \frac{x}{2} - \frac{(x/2)^3}{1^2 2} + \frac{(x/2)^5}{1^2 2^2 3} - \dots$$

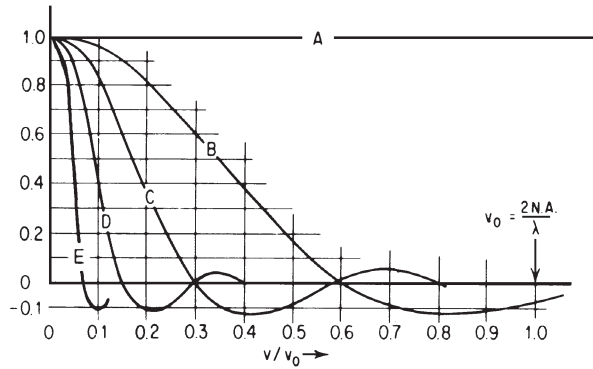


Figure 11.17 The effect of defocusing on the geometrically calculated modulation transfer function of an aberration-free system.

- | | |
|---------------------------------------|-------------------|
| (a) In focus | OPD = 0.0 |
| (b) Defocus = $\lambda/(2n \sin^2 U)$ | OPD = $\lambda/4$ |
| (c) Defocus = $\lambda/(n \sin^2 U)$ | OPD = $\lambda/2$ |
| (d) Defocus = $2\lambda/(n \sin^2 U)$ | OPD = λ |
| (e) Defocus = $4\lambda/(n \sin^2 U)$ | OPD = 2λ |

These geometrically derived plots are in poor agreement with the exact diffraction plots of Fig. 11.16 when the defocusing is small. The agreement at OPD = λ (curve D above, curve E in Fig. 11.16 is fair; the match at OPD = 2λ is quite good).

agreement where the aberration is to the order of one or two wavelengths of OPD.

Figure 11.19 shows the effect of a central obstruction in the aperture of a diffraction-limited system. Note that the introduction of a disk into the aperture* drops the response at low frequencies but raises it slightly at high frequencies (although it cannot change v_0 , the cutoff frequency). Thus, a system of this type tends to show greatly reduced contrast on coarse targets and a somewhat higher limit of resolution (when used with a system which requires a modulation of more than zero to detect resolution). This is the result of shifting light from the airy disk to the rings of the diffraction pattern.

MTF with coherent and semi-coherent illumination

All the preceding discussions (except that dealing with a central obstruction of the aperture) have assumed a uniformly illuminated

*Apodization is the use of a variable transmission filter or coating at the aperture to modify the diffraction pattern. Coatings which reduce the transmission at the center of the aperture tend to "favor" the response at high frequencies; coatings which reduce transmission at the edge of the aperture tend to favor the lower frequencies.

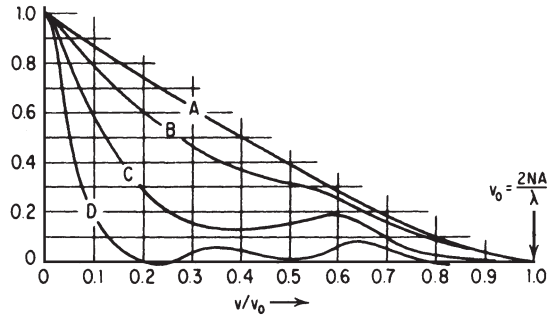


Figure 11.18 The effect of third-order spherical aberration on the modulation transfer function.

- (a) $LA_m = 0.0$ $OPD = 0$
 (b) $LA_m = 4\lambda/(n \sin^2 U)$ $OPD = \lambda/4$
 (c) $LA_m = 8\lambda/(n \sin^2 U)$ $OPD = \lambda/2$
 (d) $LA_m = 16\lambda/(n \sin^2 U)$ $OPD = \lambda$

These curves are based on diffraction wave-front computations for an image plane midway between the marginal and paraxial foci.

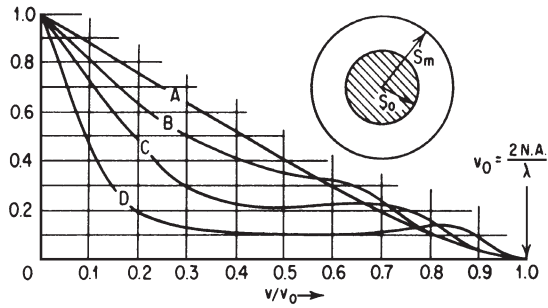


Figure 11.19 The effect of a central obscuration on the modulation transfer function of an aberration-free system.

- (a) $s_0/s_m = 0.0$
 (b) $s_0/s_m = 0.25$
 (c) $s_0/s_m = 0.5$
 (d) $s_0/s_m = 0.75$

and uniformly transmitting aperture. When the illumination system is arranged so that only the central part of the aperture is illuminated (and this can be done with Koehler illumination if a projection condenser images the source at a size smaller than the pupil of the projection lens), then the MTF plot is modified in a way which is nearly the reverse of that shown in Fig. 11.19.

Fourier theory tells us that we can consider the brightness distribution of an object to be the sum of many sinusoidal brightness distributions of differing frequencies, intensities, and orientations. To simplify matters, let us assume that we are projecting the image of a simple sinusoidal grating. Remembering that a sinusoidal grating has only the first diffraction order, consider the system shown in Fig. 11.20. If the illumination is *coherent* (i.e., collimated), the light from a point in the grating will be diffracted into the first order, as illustrated in Fig. 11.20a. If the angle of diffraction is less than that of the numerical aperture (NA) of the projection lens, the full power will be projected into the image. But if the grating frequency is high enough (so that $v \geq \text{NA}/\lambda$), the diffracted ray will pass outside the lens aperture, and no light corresponding to this frequency will make it into the image. The result of this situation is an MTF plot as shown in Fig. 11.20c, with 100 percent MTF for spatial frequencies of NA/λ or less and zero MTF for all higher frequencies. Note that NA/λ is just half the cutoff frequency ($v_0 = 2\text{NA}/\lambda$), as given in Eq. 11.39 for the incoherent illumination case.

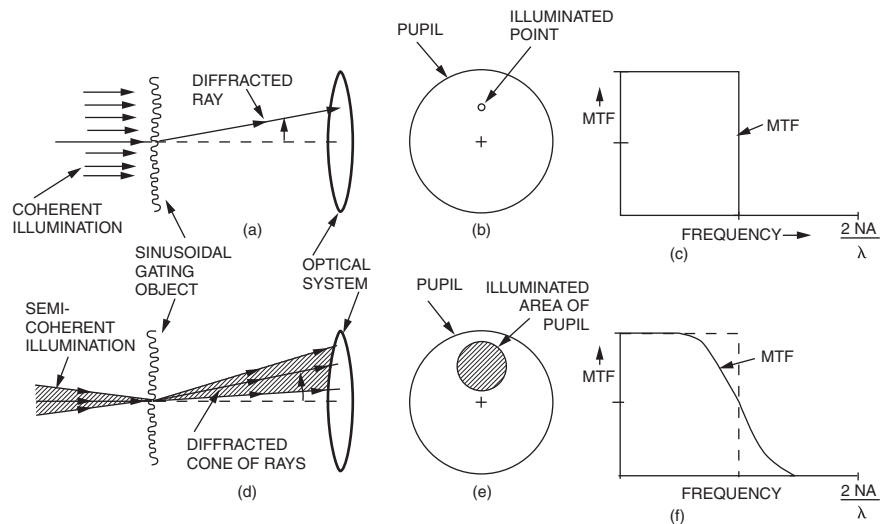


Figure 11.20 (a–c) The MTF with coherent illumination. (d–f) The MTF with semicoherent illumination (which partially fills the pupil).

If the illumination is *semicoherent*, the projection lens pupil will be partially filled, as indicated in Fig. 11.20d, e. As we consider an increasing grating frequency, the location of the illuminated area in the pupil will move toward the edge. However, at the edge of the pupil, the cutoff is gradual rather than abrupt, as in the coherent case described above, and we get an MTF plot of the sort shown in Fig. 11.20f.

Figure 11.21 shows the effect on the MTF for several values of the illumination system NA, expressed as a fraction of the lens NA. These partial coherence effects are useful in both microlithography and microscopy. Note that decentering or tilting the illuminating beam can be used to get directional effects and that ring illumination can emphasize a particular frequency.

As mentioned previously, MTFs have been applied to image-receiving systems which are not imaging systems. Figure 11.22 shows the MTF curves for a number of photographic emulsions. Since the MTF of a film is computed on the basis of equivalent relative exposures derived from density measurements on films exposed to sinusoidal test patterns, it is possible to have a film MTF greater than unity. This results from the chemical effects of development of the film on adjacent areas and will be noticed at the low-frequency end of the curves in Fig. 11.22. An AIM curve, as described in Section 11.8, can also be used to represent the response characteristics of nonimaging devices and sensors such as films.

11.11 Radial Energy Distribution

The data of a point spread function or a spot diagram can be presented in the form of a *radial energy distribution plot*. If the blur spot is

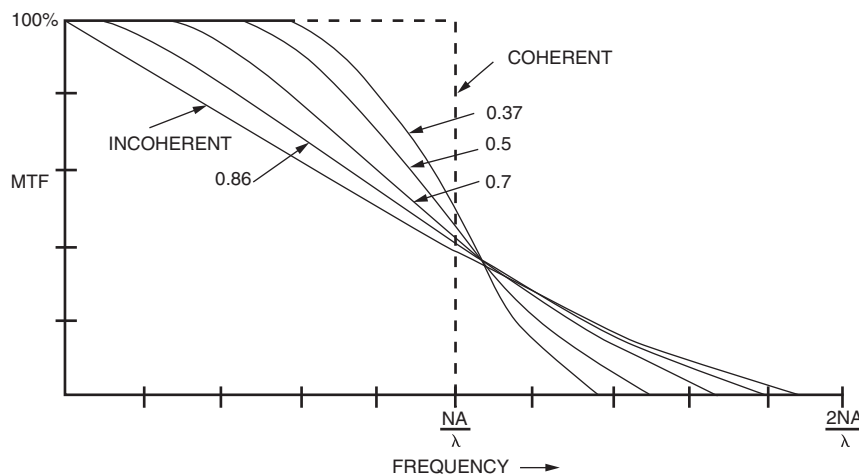


Figure 11.21 MTF vs. frequency for a partially filled pupil (semicoherent illumination). Numbers are the ratio of illuminating system NA to optical system NA.

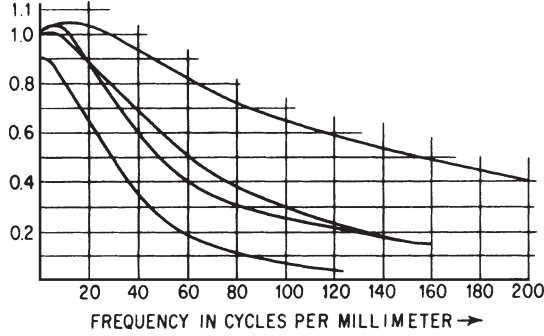


Figure 11.22 Modulation transfer functions of several photographic emulsions.

symmetrical, it is apparent that a small circular aperture centered in the image would pass a portion of the total energy and block the rest. A larger aperture would pass a greater portion of the energy and so on. A graph of the encircled fraction of the energy plotted against the radius (semidiameter) of the aperture is called the radial energy distribution curve.

A radial energy distribution curve, such as the example shown in Fig. 11.23, can be used to compute the MTF for an optical system by means of the summation equation

$$\text{MTF}(v) = \sum_{i=1}^{i=m} \Delta E_i J_0[2\pi v \bar{R}_i]$$

where v is the frequency in cycles per unit length, ΔE_i is the difference ($E_i - E_{i-1}$) between two values of E , the fractional energy, $\bar{R}_i$ is the average $\frac{1}{2}(R_i + R_{i-1})$ of the corresponding values of the radius and $J_0(\quad)$ indicates the zero-order Bessel function.*

Although this radial energy distribution relationship is (strictly speaking) valid only for point images which have rotational symmetry, i.e., for images on the optical axis, it can be used to predict approximate averaged resolution for off-axis points. This procedure, while it cannot yield separate radial and tangential values for resolution, does serve to give the designer a rough idea of the state of correction of the system.

* $J_0(x) = 1 - \left(\frac{x}{2}\right)^2 + \frac{\left(\frac{x}{2}\right)^4}{1^2 2^2} - \frac{\left(\frac{x}{2}\right)^6}{1^2 2^2 3^2} + \cdots$

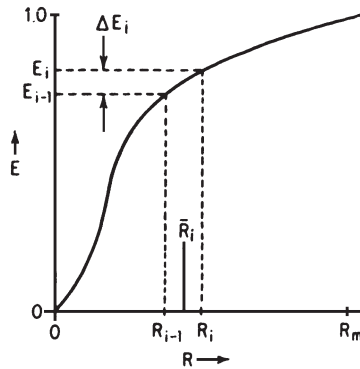


Figure 11.23 Radial energy distribution plot. The curve indicates the fraction E of the total energy in an image pattern which falls within a circle of radius R . Thus all the energy is encompassed by a circle of radius R_m ; E_i of the energy by a circle of radius R_i .

11.12 Point Spread Functions for the Primary Aberrations

The figures of this section illustrate the effects of the primary aberrations on the point spread function (PSF) of an optical system. Figures 11.24 through 11.28 each show four point spread functions, the first for a peak-to-valley OPD (wave-front deformation) of an eighth-wave, the second for a quarter-wave (which is the Rayleigh criterion), the third for a half-wave, and the fourth for a full wavelength of OPD. The caption for each figure also gives the rms (root mean square) OPD and the Strehl ratio for each PSF (see Sec. 11.4 and Fig. 11.5).

Figure 11.24 shows the effect of simple defocusing on the PSF. Note that for defocusing, the Rayleigh criterion (which is the OPD equal to a quarter-wave) is identical to the Marechal criterion (Strehl ratio equal to 0.80). In Fig. 11.25, which shows the effect of simple third-order spherical aberration, the PSF for an eighth-wave is almost identical to that for defocusing, and the quarter-wave PSF is very similar. But when we compare the half- and full-wave plots, the differences are quite apparent, despite the fact that the effects on the MTF and resolution are still comparable.

The coma PSF in Fig. 11.26, however, is noticeably different even at an OPD of an eighth-wave, where the unsymmetrical rings in the diffraction pattern are already apparent. At one wave of OPD the PSF is clearly showing the “comma-” or “comet-shaped” figure that one gets from a geometric optics spot diagram (see Fig. 11.6 for example).

Figure 11.27 may be a bit surprising to some readers. Most discussions of astigmatism (including that in Sec. 3.2 of this text) which are based on geometric optics indicate that the blur spot between the sagittal and tangential focal lines is an ellipse or a circle, depending on where the image is examined. However, in the PSF for either half-

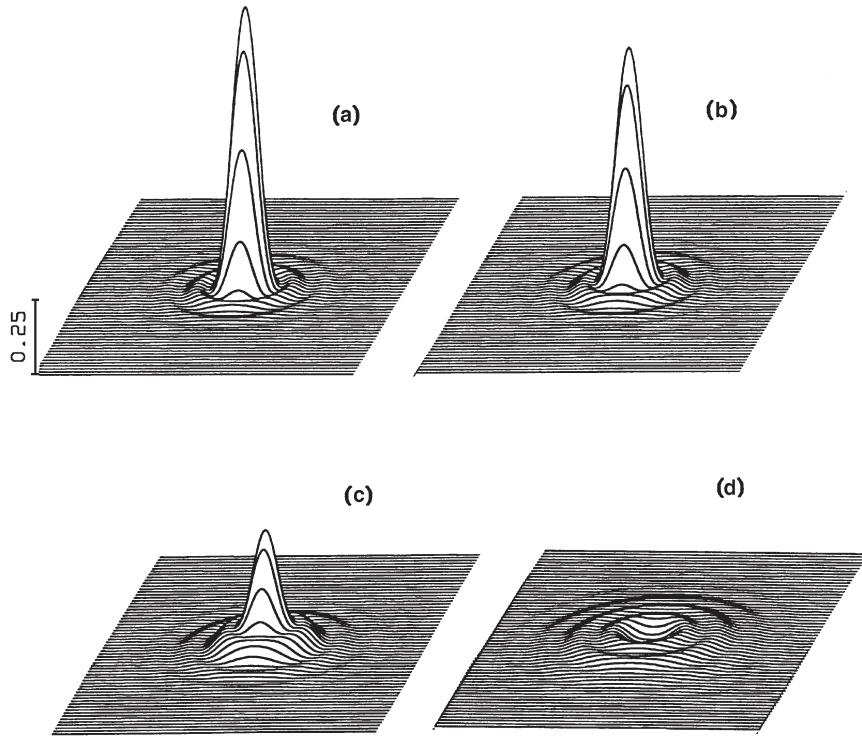


Figure 11.24 Point spread functions for different amounts of defocus. (a) 0.125 wave (P-V); 0.037 wave rms; 0.95 Strehl. (b) 0.25 wave (P-V); 0.074 wave rms; 0.80 Strehl. (c) 0.50 wave (P-V); 0.148 wave rms; 0.39 Strehl. (d) 1.00 wave (P-V); 0.297 wave rms; 0.00 Strehl.

or full-wave OPD we can easily see that the blur spot is not circular but has a definite four-sided aspect. Anyone who has microscopically examined the image of a point source formed by a lens with astigmatism as its major aberration will have observed this (and probably has wondered where the square-shaped image blur came from). It may help to understand this phenomenon to realize that the two focal lines are effectively acting as apertures, and the diffraction effect of this is to introduce the cross-shaped illumination distribution.

The most customary balance between third-order and fifth-order spherical aberration is with the aberration of the marginal ray corrected to zero. This state of correction produces the least peak-to-valley OPD (as demonstrated in Sec. 11.3). In Fig. 11.28, the eighth-wave PSF is not very different from that for pure third-order spherical, or even defocus, but at a quarter-wave one begins to notice that the rings are more pronounced than they are in Figs. 11.24 and 11.25. This effect

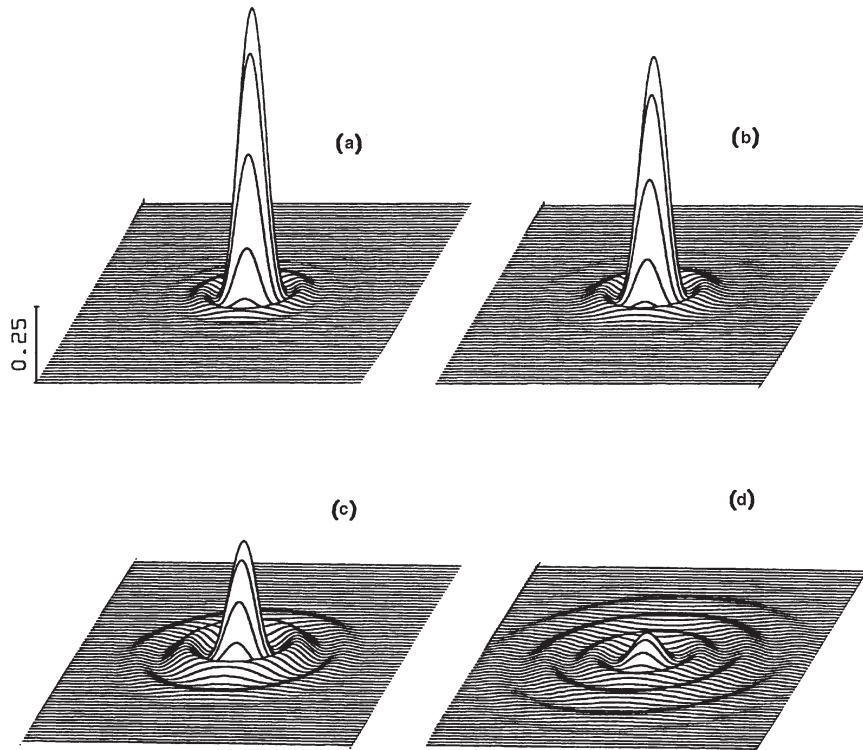


Figure 11.25 Point spread functions for different amounts of third-order spherical aberration. (a) 0.125 wave (P-V); 0.040 wave rms; 0.94 Strehl. (b) 0.25 wave (P-V); 0.080 wave rms; 0.78 Strehl. (c) 0.50 wave (P-V); 0.159 wave rms; 0.37 Strehl. (d) 1.00 wave (P-V); 0.318 wave rms; 0.08 Strehl. *Note:* Reference sphere centered at $0.5LA_m$ (midway between marginal and paraxial foci).

is quite noticeable in a star test, where heavy rings in the diffraction pattern are an indication of a zonal spherical residual.

The final figure in this series, Fig. 11.29, compares the PSF for the various aberrations, each of which is set at a value which equals the Marechal criterion (a Strehl ratio of 80 percent). There are differences apparent if one looks very closely at the defocus and spherical plots, and there are obvious differences for astigmatism and coma. However, the net effect on the image quality is surprisingly similar. This is, of course, the reason that the Rayleigh criterion of a quarter-wave (peak-to-valley) OPD and the Marechal criterion of 0.80 Strehl are so widely accepted by lens designers as a standard of image quality.

Note: These figures were prepared by applying an optical software program to systems which were set up to show only the particular aberration under consideration. A paraboloidal reflector was the obvious choice for the defocusing PSF because its axial image is total-

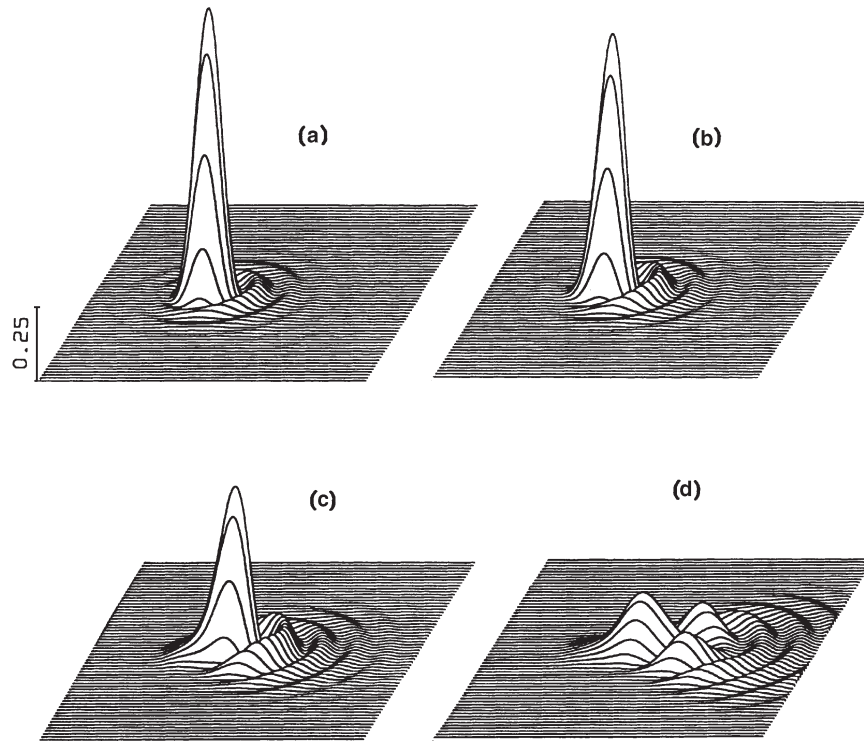


Figure 11.26 Point spread functions for different amounts of third-order coma. (a) 0.125 wave (P-V); 0.031 wave rms; 0.96 Strehl. (b) 0.25 wave (P-V); 0.061 wave rms; 0.86 Strehl. (c) 0.50 wave (P-V); 0.123 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.25 wave rms; 0.18 Strehl. *Note:* P-V OPD reference sphere centered at 0.25Coma_T from chief ray intersection point. rms OPD reference sphere centered at 0.226Coma_T from chief ray intersection point.

ly aberration-free. The spherical aberration plots were created by deforming the paraboloid with a fourth-order deformation term for the third-order spherical plot and fourth- and sixth-order deformations for the third- and fifth-order spherical plot. The coma PSF was calculated using a paraboloidal reflector with its aperture stop at the focal plane (which eliminates astigmatism), as in Fig. 13.37a. The image was put on a curved surface which approximated a sphere of radius equal to the focal length of the reflector and which was centered at the center of curvature of the paraboloid. The astigmatism PSF was produced by introducing an additional cylindrical parabolic reflector.

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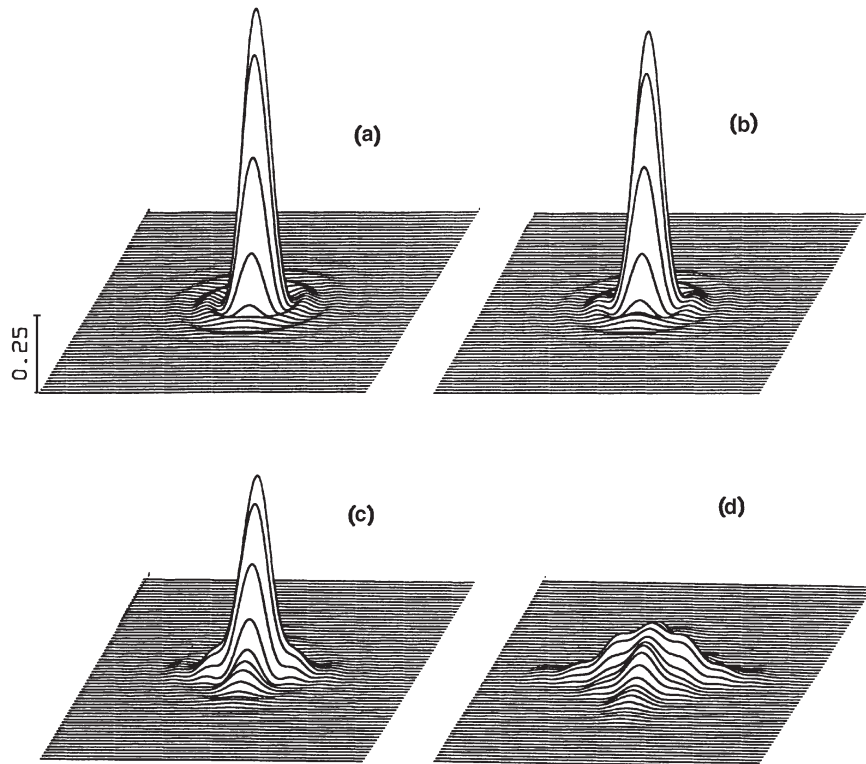


Figure 11.27 Point spread functions for different amounts of astigmatism. (a) 0.125 wave (P-V); 0.026 wave rms; 0.97 Strehl. (b) 0.25 wave (P-V); 0.052 wave rms; 0.90 Strehl. (c) 0.50 wave (P-V); 0.104 wave rms; 0.65 Strehl. (d) 1.00 wave (P-V); 0.207 wave rms; 0.18 Strehl. *Note:* Reference sphere centered midway between sagittal and tangential foci.

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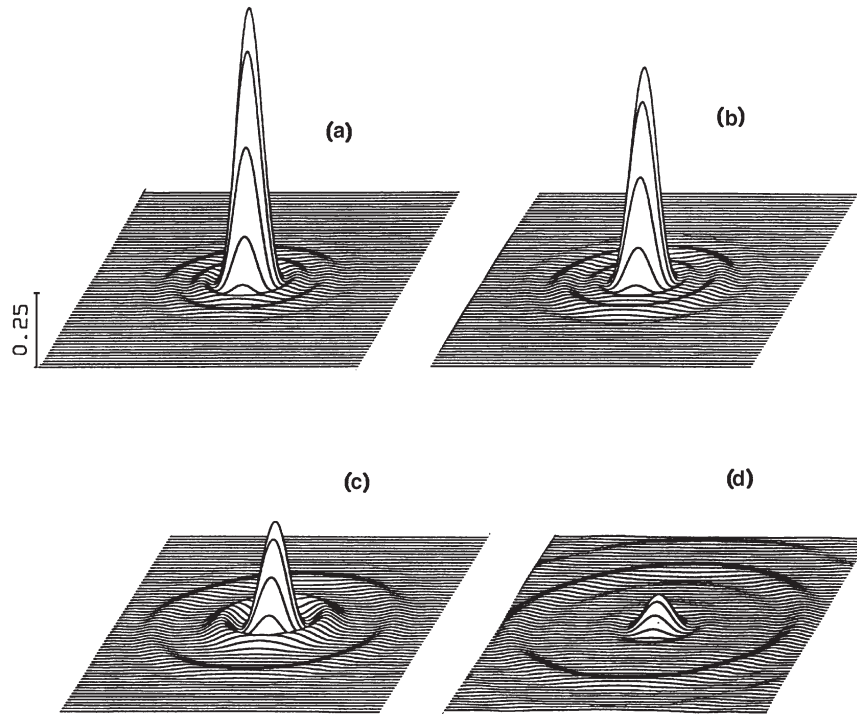


Figure 11.28 Point spread functions for different amounts of zonal spherical aberration (third- and fifth-order spherical balanced so that marginal spherical equals zero). (a) 0.125 wave (P-V); 0.042 wave rms; 0.93 Strehl. (b) 0.25 wave (P-V); 0.085 wave rms; 0.75 Strehl. (c) 0.50 wave (P-V); 0.208 wave rms; 0.35 Strehl. (d) 1.00 wave (P-V); 0.403 wave rms; 0.09 Strehl. *Note:* Reference sphere centered at $0.75LA_z$ for P-V and at $0.8LA_z$ for rms.

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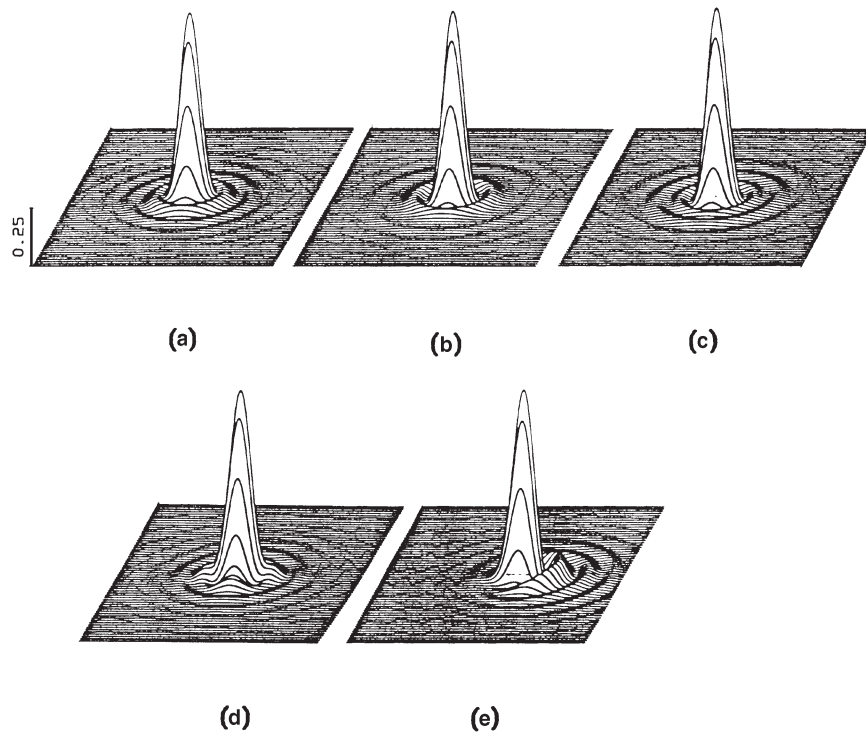


Figure 11.29 Point spread functions for five different aberrations, each with a Strehl ratio of 0.80 (the Marechal criterion). In each case the center of the reference sphere is located to minimize the rms OPD, which is 0.075 wave for all five aberrations. (a) Defocus: 0.25 wave (P-V). (b) Third-order spherical: 0.235 wave (P-V). (c) Balanced third- and fifth-order spherical: 0.221 wave (P-V). (d) Astigmatism: 0.359 wave (P-V). (e) Coma: 0.305 wave (P-V).

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Exercises

1 The longitudinal spherical aberration of a spherical reflector is equal to $y^2/8f$ (to a third-order approximation). What is the maximum diameter a 36-in focal-length spherical reflector may have without exceeding an OPD of one-quarter wavelength for visual light, $\lambda = 20 \times 10^{-6}$ in? (use Eg. 11.16)

answer: 4.7 in

2 The third-order sagittal coma of a parabolic reflector is given by $-y^2\theta/4f$, where θ is the half-field angle in radians. What is the maximum diameter a

36-in focal-length spherical reflector cover without exceeding the Rayleigh limit? ($\lambda = 2 \times 10^{-5}$ in)

answer: ± 0.0041 radians (0.47° total field)

3 An $f/5$ system is defocused by 0.05 mm. What is the modulation transfer factor for a “sine wave” target with a spatial frequency (at the image) of 120 cycles per millimeter? Use Fig. 11.16, Eq. 11.2, and assume $\lambda = 0.5 \mu\text{m}$.

ANSWER: 0.23