

Optimized additive mixing of colored light-emitting diode sources

Harald Ries

Philipps-Universität Marburg
Renthof 5
35032 Marburg, Germany

Ines Leike

Julius Muschaweck, MEMBER SPIE
OEC AG
Paul-Gerhardt-Allee 42
81245 München, Germany

Abstract. Given a set of available LEDs or other light sources with known specifications including spectrum, total luminous flux (in lumens), and efficacy (in lumens per watts), we show how to select that combination which yields light of the desired (photometric) color and, in addition, maximizes various objectives such as efficacy, luminous flux, and color rendering index. © 2004 Society of Photo-Optical Instrumentation Engineers. [DOI: 10.1117/1.1753273]

Subject terms: light-emitting diode; color mixing; optimization; color-rendering index; efficiency; efficacy; luminous flux.

Paper IE-OE12 received Jan. 9, 2004; revised manuscript received Mar. 1, 2004; accepted for publication Mar. 3, 2004. This paper is a revision of a paper presented at the SPIE conference on Design of Efficient Illumination Systems, Aug. 2003, San Diego, Calif. The paper presented there appears (unrefereed) in SPIE Proceedings Vol. 5186.

1 Introduction

LEDs are widely used in a variety of lighting applications. Because of advantages such as small size and long lifetime, more and more they are replacing conventional light sources. Recent improvements in both flux and flux density widen the range of applications. In most applications, color is an important issue. Often, white light is desired with an option for fine-tuning.

LED sources have a narrow spectral range as compared to the visible spectrum. This fact has physical reasons. Both for electroluminescence and for thermal sources, the spectral width is essentially determined by the temperature of the emitting material and increases with temperature.¹ While thermal light sources must operate at temperatures of the order of the temperature of the sun (6000 K), the semiconductor chip in an LED operates a little above room temperature. Due to these physical roots, the spectral width of LEDs is unlikely to change substantially with new materials. Thus, we have essentially two options to produce white light with LED sources. A straightforward, inherently lower efficiency option is to start with an appropriate (blue or UV) LED and then down convert some of the light to longer wavelengths with a suitable luminescent dye. This approach is used in the commercially available white LEDs. A higher efficiency and more flexible option is to mix the light of several different LEDs. In this paper, we discuss issues concerned with the latter approach.

A specific color can be achieved with a number of LEDs by adjusting the individual current and thus the total flux of each LED. Equivalently, if large fluxes requiring many LEDs of each type are desired, one could adjust the number of LEDs of each type rather than the individual current. This paper focuses on choosing the right relative proportions of various light sources such as the right number of LEDs of each type or the right individual currents. We do not discuss here various optical means to mix the light of these LEDs in the phase space of radiation.

In general, there are more different light sources than necessary to achieve a certain color. This enables us to simultaneously optimize for other objectives such as achieving the desired color with maximum flux, minimum price, maximum efficiency, or with maximum color rendering index. Many of these objectives are linear functions, rendering the optimization a linear programming approach.

Color rendering and efficacy issues of light mixed from LEDs was discussed by Ohno.² He showed that mixing narrow-band sources to achieve a given color leads to very different color rendering and efficacies, depending on the choice of the individual peak wavelengths of the sources. He did not address the issue of optimization to determine the “best” choice.

Optimal mixing of four or more LEDs to achieve a maximum of certain linear combinations of efficacy and color rendering indexes was investigated by Zukauskas et al.,^{3,4} using hypothetical Gaussian spectra with arbitrary peak wavelengths. Their work, based on an extensive search of possible wavelength combinations, answers important questions for LED die manufacturers. In contrast, the work presented here is based on available LEDs and also includes actual quantum conversion efficiencies, addressing questions that are important to luminaire manufacturers, who must combine available LEDs for color mixing.

In this paper, we use as an example the data of Luxeon® LEDs, including their spectral power distribution, luminous flux, and efficacies as given by data sheets.⁵ The spectral power distributions of these LEDs are shown in Fig. 1, other relevant parameters for the following calculations are listed in Table 1.

2 Human Color Perception

The human eye distinguishes color with three sensors that differ in their spectral sensitivity. To quantify color the Commission Internationale de L'Eclairage (CIE) standardized⁶ the spectral sensitivity of the three receptors

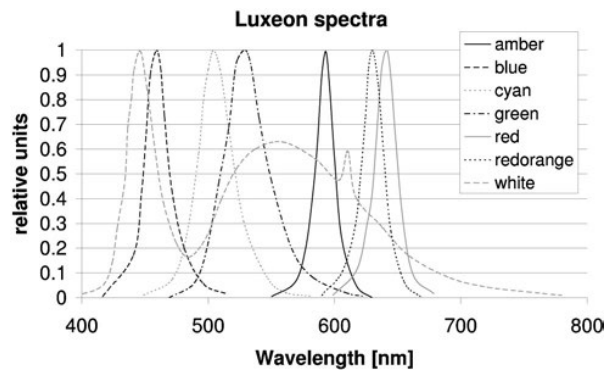


Fig. 1 Relative spectra of Luxeon® LEDs according to Ref. 5.

of a human observer $X(\lambda)$, $Y(\lambda)$, and $Z(\lambda)$.⁶ The green spectral sensitivity $Y(\lambda)$ is also known as the spectral sensitivity function for photopic vision. The spectral sensitivities are depicted in Fig. 2.

Every source of light has an associated characteristic spectrum of wavelengths $S(\lambda)$. Weighting this spectral power distribution with the tristimulus curves $X(\lambda)$ (black), $Y(\lambda)$ (gray), and $Z(\lambda)$ (light gray) yields three scalar values X , Y , and Z , the tristimulus values:

$$X = \int X(\lambda) S(\lambda) d\lambda,$$

$$Y = \int Y(\lambda) S(\lambda) d\lambda,$$

$$Z = \int Z(\lambda) S(\lambda) d\lambda. \quad (1)$$

The sum of these values is connected with the perception of brightness, while their ratio determines the perceived color. Accordingly, color can be represented as a point in a 2-D space.⁷ There are several equivalent representations. Figure 3 shows the color space according to the definition⁶ of the CIE from 1931. Here, the color coordinates x and y are defined as

Table 1 Parameters for Luxeon LEDs according to Ref. 5.

LED	Luminous Flux (lm)	Efficacy (lm/W)	CIE 1931 x, y Values
Blue	5	4.2	0.141, 0.043
Cyan	30	25.1	0.087, 0.579
Green	25	20.9	0.231, 0.703
Amber	36	34.9	0.579, 0.420
Red-orange	55	53.3	0.696, 0.304
Red	44	42.6	0.709, 0.291
White	18	15.0	0.327, 0.342

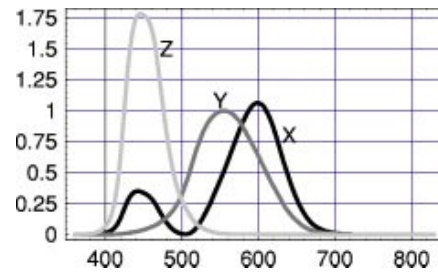


Fig. 2 Tristimulus curves, as defined by the CIE.

$$x = \frac{X}{X + Y + Z},$$

$$y = \frac{Y}{X + Y + Z}. \quad (2)$$

The values range from 0 and 1. The tristimulus curves overlap in such a way that not all combinations of coordinates x and y are physically possible. The range of all physically achievable colors is delimited by the horseshoe-shaped color diagram, as shown in Fig. 3. The color coordinates of the available Luxeon® LEDs are also indicated in Fig. 3. We also visualize the location of the color coordinates of Planckian emitters with temperature, as indicated in the plot.

3 Color Mixing

The CIE x - y color coordinates obtained by mixing the light from several LEDs with relative fluxes c_i are given by

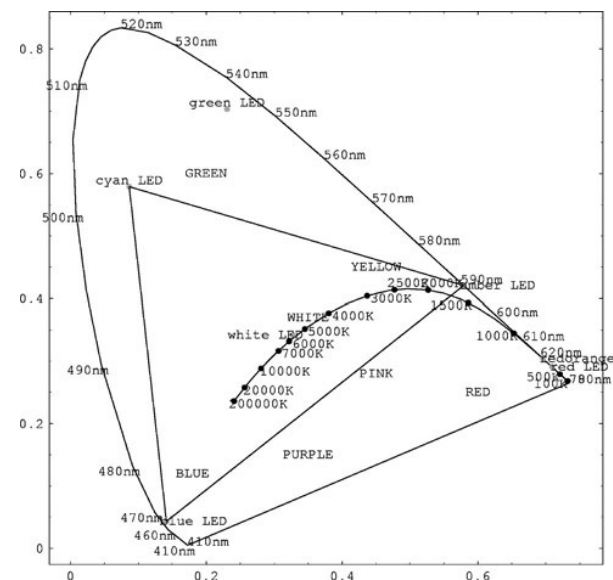


Fig. 3 CIE 1931 (x, y) diagram with color coordinates of Luxeon® LEDs.

$$x = \frac{\sum c_i X_i}{\sum c_i (X_i + Y_i + Z_i)},$$

$$y = \frac{\sum c_i Y_i}{\sum c_i (X_i + Y_i + Z_i)}, \quad (3)$$

where X_i , Y_i , and Z_i denote the respective tristimulus values of the individual LEDs driven at maximum current.

Achieving a desired color by mixing the light of given LEDs means adjusting the drive current of each LED independently relative to the maximum current specified by the manufacturer. The color that can be generated by combining elementary sources is a linear combination of the colors of the elementary sources with coefficients all ranging between 0 and 1. Thus, the range of colors that can be achieved from a given set of sources can be visualized as the convex hull of the color coordinates of the elementary sources. Consider, for example, the blue, cyan, and amber LEDs in Fig. 3. Adjusting their currents enables us to generate all colors within the indicated triangle, particularly white and colors close to white. On the other hand, many saturated colors and low-temperature Planckian emitter colors cannot be reached with this combination. The relative fluxes of these three LEDs are uniquely determined by the desired color.

With more than three LEDs of different colors, the relative fluxes necessary to achieve the desired color are no longer unique. In general, given N LEDs there are $N-3$ degrees of freedom to mix a given color. The remaining relative fluxes can be determined by solving Eqs. (3). These equations are linear in the weights c_i , and therefore have precisely one solution, which may or may not be within the physically feasible range. A solution outside this range indicates that the required color cannot be produced. These degrees of freedom offer the opportunity to achieve additional objectives. If these objectives are linear functions of the relative fluxes, the optimization turns out to be linear. Linear optimization is much less demanding than the general optimization task because any linear function always attains its extreme value on the boundary of its domain. Furthermore, linear functions cannot exhibit multiple local extremes. This enables efficient, fast, and robust numerical optimization for a large class of optimization problems⁸ of significant practical importance such as minimizing investment costs or, maximizing efficacy or luminous flux for a given color.

In contrast, nonlinear merit functions require the full machinery of global nonlinear optimization, as used, e.g., by Zukauskas et al.^{3,4} In these cases, the optimization is much more demanding. The objective function is, in general, a complex multidimensional landscape with several local optima, and the difficulty to find the global optimum increases rapidly with the dimensions of the parameter space,⁸ i.e., the number of LEDs. We also discuss one notable nonlinear application, which is maximizing the color-rendering index.

In the following we provide a selection of solved examples for both types of optimization that are relevant for practical purposes. In our calculations, we used the optimization algorithms of Mathematica 4 (Ref. 9). In the nonlin-

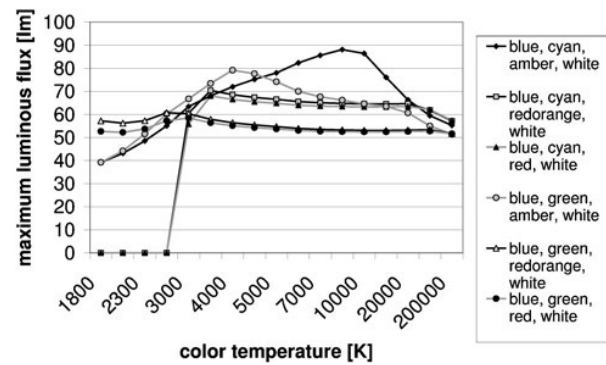


Fig. 4 Maximum luminous flux for different combinations of four LEDs as a function of color temperature.

ear case, we avoided being trapped in local minima by restarting the calculation with appropriately changed starting points.

3.1 Example: Maximize Luminous Flux for Given Colors

Luminous flux is an important specification issue for a large number of applications. The total flux from N LEDs is the sum of all fluxes of the individual sources. Therefore, maximizing total flux yields a linear optimization problem.

To obtain the colors of Planckian emitters with temperatures between 1800 K and 20,000 K by mixing three colored and one white Luxeon® LED, we require a blue LED; either a green or a cyan LED; and one of the red, red-orange, or amber LED. There is one degree of freedom left, which we used to maximize luminous flux for a given color.

The maximum luminous flux for each of the possible combinations of LEDs in dependence on color temperature is shown in Fig. 4. Zero luminous flux indicates that this color temperature cannot be reached with the given combination. The combinations of (blue, cyan, amber, white) and (blue, green, amber, white) enable us to achieve a particularly high luminous flux for most color temperatures.

3.2 Example: Maximize Efficacy for Given Colors

The efficacy of light sources K is defined as the ratio of luminous flux as perceived by the human eye and the total electrical power consumption of the source. The luminous flux is calculated by weighting the spectral distribution with the green spectral sensitivity $Y(\lambda)$ and multiplying by the theoretical maximum efficacy $K_m = 683 \text{ lm/W}$ for monochromatic light of 555 nm:

$$K = \frac{K_m \int S(\lambda) Y(\lambda) d\lambda}{q \int S(\lambda) d\lambda}, \quad (4)$$

where q is the efficiency of conversion from electrical power (in watts) to radiant flux (in watts). The efficacy of the combination of N LEDs is not *a priori* a linear combination of the efficacies of the individual LEDs. Nevertheless, one can formulate this task as a linear optimization with the additional linear constraint that the sum of all contributions, i.e., the total electrical power, is constant. This

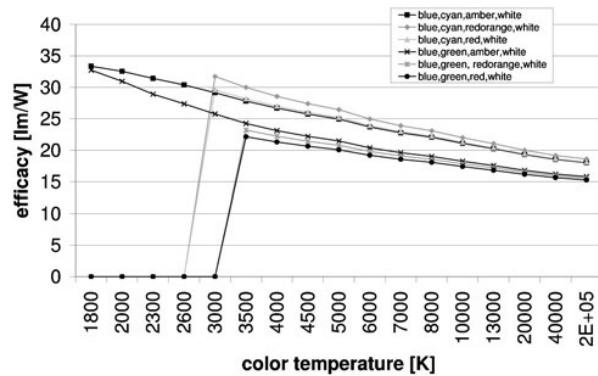


Fig. 5 Maximum efficacy for various combinations of LEDs as a function of color temperature.

does not reduce the number of degrees of freedom. Maximizing efficacy is equivalent to maximizing flux for fixed total input power. The result of the efficacy optimization is shown in Fig. 5. Since the efficacies of the individual LEDs differ widely, with approximately 5 lm/W for blue, 30 lm/W for cyan, 36 lm/W for amber, and 16 lm/W for white, the efficacy decreases from the red to the blue end of the Planckian locus. The impossibility to mix colors below 3000 K with some LED combinations is indicated as zero efficacy in the figure.

A combination of (blue, cyan, red-orange, and white) yields highest efficacy over the entire color temperature region above 3000 K. For this LED combination, the relative fluxes are shown in Fig. 6. Here, the fluxes are rescaled such that one of the LEDs is driven at maximum power.

The result for the relative fluxes shows that the cyan LED is driven at maximum power, whereas the white LED is not used at all. The reason is that the red-orange LED is much more efficient than the white LED and can replace the white LED in mixing the color.

3.3 Example: Maximize Color-Rendering Index

The color-rendering index (CRI) is a CIE standardized way to quantify the appearance of colors when illuminated with

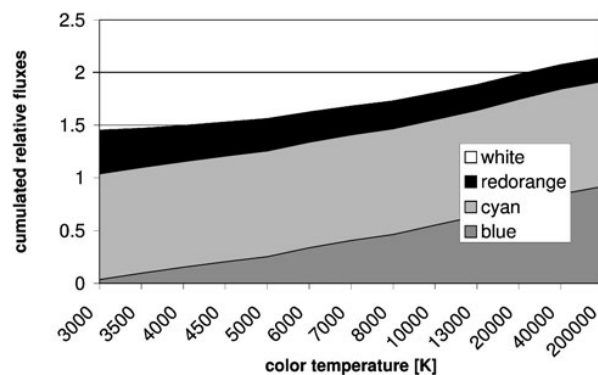


Fig. 6 Relative fluxes for optimal efficacy mixing of blue, cyan, red-orange, and white as a function of color temperature; the white LED was used in the computations, but is driven at zero power, since the red-orange LED is much more efficient and therefore replaces the white LED.

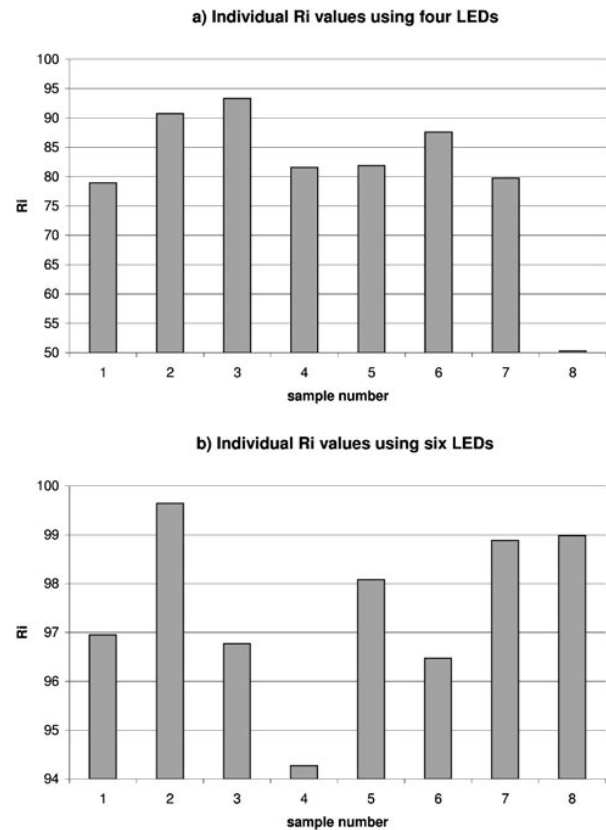


Fig. 7 Individual color rendering indices R_i for the eight CIE color samples.

a light source in comparison with the appearance when illuminated with a given reference source.¹⁰ To determine the general CRI, the CIE specifies eight color samples. The colors of these samples are evaluated while being illuminated by the light source to be assessed, and these colors are then compared with the corresponding colors using the reference source. The quantified, scaled color differences are then averaged and subtracted from 100 to obtain the CRI. Thus, the CRI will always be less than or equal to 100 and may even be negative. No actual experiment is required; the CRI can be computed using lamp spectra only. However, the notion of CRI is often used in misleading ways. It is important to realize that the CRI is not an absolute measure of “quality” of a lamp. On the contrary, the

Table 2 Relative fluxes of the LEDs for maximum CRI.

LED	Relative Contribution for Maximum CRI (Four LEDs)	Relative Contribution for Maximum CRI (Six LEDs)
Blue	0.0654	0.061
Cyan	0.0746	0.108
Green	(not needed)	0.005
Amber	0.0915	0.025
Red-orange	(not needed)	0.026
White	1.0	1.0

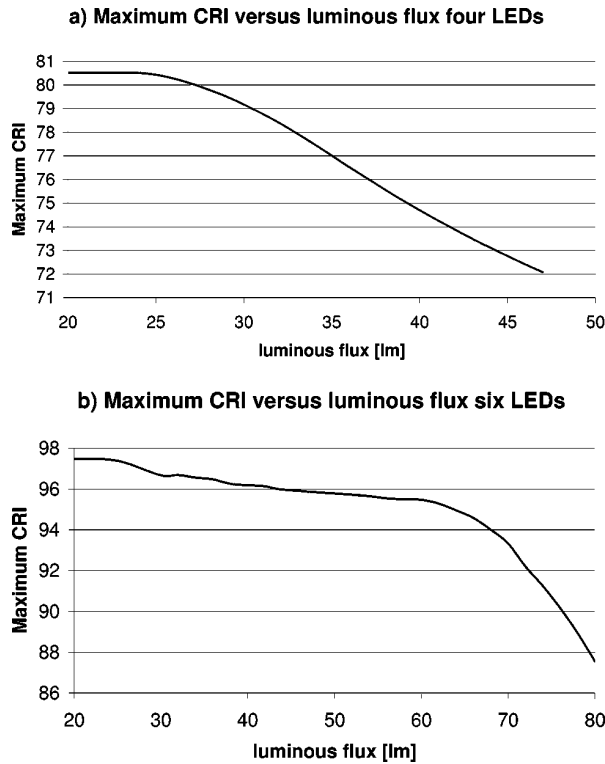


Fig. 8 Maximum CRI versus luminous flux for (a) a combination of four LEDs and (b) a combination of six LEDs.

CRI states in a quantitative way only how similar colors look under a certain lamp as compared to a reference lamp. Therefore, CRI values must always specify the reference lamp, according to the CIE standard. Also, in certain applications a lamp with a relatively low CRI may actually look “better” to a human observer. However, if stated correctly and used with caution, the CRI is generally accepted as a useful measure of lamp quality. A CRI between 90 and 100 is usually considered as excellent.

The calculation of CRI involves nonlinear operations. Optimization of the CRI is therefore inherently nonlinear—with all the problems of nonlinear optimization, such as finding appropriate starting points, avoiding local optima, problems with boundaries, etc. It turns out that the combination of blue, cyan, amber, and white yields the highest CRI for a 6000-K Planckian reference source. A CRI of 80.5 has been found in a global optimization for this combination. For six LEDs (blue, cyan, green, amber, red-orange, and white) a CRI of 97.5 is achieved. Figure 7 shows the contributions for the individual color samples. Note the different scales for four and six LEDs. Table 2 shows the relative fluxes of the LEDs.

3.3.1 Competing objectives

Optimization invariably is based on one scalar objective function. If one single quantity is optimized, the optical system may perform very poorly in other aspects. For example, for the best CRI values, efficacy is very low. The reason is that good CRI requires a broad source spectra, whereas efficacy is highest for monochromatic radiation with a wavelength of 555 nm. CRI and luminous flux val-

ues are also two competing properties. For the optimal CRI values of the combinations just considered, the luminous fluxes are rather low—about 25 lm for the four LED combination and 26 lm for the combination of six LEDs.

Of course, given a sufficient number of sources, optimization can include additional constraints beyond matching the desired color. In the following example, we calculated the best CRI under the assumption that the luminous flux has a specified value. Figure 8 shows the result of these optimization processes for four and six LEDs. Another approach would be to use a suitable scalar function of competing individual objectives as the combined objective to be optimized.

4 Conclusions

Only three LED sources are generally required to achieve a desired color by mixing their light. We showed how to take advantage of the large number of available sources and simultaneously optimize a number of additional objectives. Many of these, like total flux or efficacy, turn out to be linear functions of the LED weights, making optimization robust, efficient, and fast. However, the approach can be applied to nonlinear merit functions as well; we showed maximizing the CRI as an example. The mixing and optimization procedures discussed here are not restricted to LEDs. They can be used for all sources, provided that spectrum and flux are known.

References

1. H. Ries and A. J. McEvoy, “Chemical potential and temperature of light,” *J. Photochem. Photobiol., A* **59**, 11–18 (1991).
2. Y. Ohno, “Color issues of white LEDs,” OIDA Workshop Preliminary Report (Oct. 26–27, 2000).
3. A. Zukauskas et al., “Optimization of white polychromatic semiconductor lamps,” *Appl. Phys. Lett.* **80**, 234 (2002).
4. A. Zukauskas, F. Ivanauskas, R. Vaicekauskas, M. S. Shur, and R. Gaska, “Optimization of multichip white solid state lighting source with four or more LEDs,” *Proc. SPIE* **4445**, 148–155 (2001).
5. “Luxeon emitter technical data sheet DS25,” Lumileds Lighting LLC, San Jose, CA (Nov. 2002).
6. “Joint ISO/CIE standard 10527: colorimetric observers,” Commission Internationale de l’Eclairage, Vienna (1991).
7. D. Malacara, *Color Vision and Colorimetry: Theory and Applications*, SPIE Optical Engineering Press, Bellingham, WA (2002).
8. W. H. Press, B. P. Flannery, S. A. Teukolsky, and W. T. Vetterling, *Numerical Recipes*, 2nd ed., Cambridge University Press, New York (1992).
9. *Mathematica 4.0*, Wolfram Research Inc., Champaign, IL (1999); www.wolfram.com.
10. CIE publication 13.3-1995, Commission Internationale de l’Eclairage, Vienna (1995).



Harald Ries received his MSc degree in physics from Ludwig-Maximilians University, Munich, Germany, for research at the Max-Planck Institute of high energy physics and continued postgraduate work on thermodynamics of light and solar energy at Munich University, for which he received his PhD degree in physics in 1985. In 1999 Ries founded Optics & Energy Concepts (OEC), where he chairs the supervisory board of OEC AG and is chief scientist.

Since January 2000 he has also been an associate professor in the Physics Department of the Philipps University, Marburg, Germany, where he leads a research group on optics.



Ines Leike received her MSc degree in physics from the University of Leipzig, Germany, for research concerning heavy Higgs particles in 1987. For her postgraduate work on nonideal plasmas she received her PhD degree from the Humboldt University of Berlin in 1992. Between 1996 and 2000 she was with the German Aerospace Center (DLR) in the field of lidar physics and she joined Optics & Energy Concepts (OEC) in 2000.

mixed-use buildings. Between 1996 and 1999 he developed stationary high-temperature solar collectors with vacuum tubes, at first for the Bavarian Research Foundation and then in the United States in a collaboration project between Duke Energy and the University of Chicago. In 1999 he founded Optics & Energy Concepts (OEC) with Harald Ries and is the CEO.



Julius Muschaweck received his MSc degree for the analysis of a solar domestic hot water system from the Ludwig-Maximilians University, Munich, Germany, where he continued to work on several scientific projects, including the development of the European standard testing procedure for solar hot water systems. From 1992 to 1996, he was the managing partner of a real estate development company, where he was responsible for energy concepts, technical consulting, finance, and taxes for the construction of