

Joint Transmitter-Receiver Beamforming Over Space-Time Fading Channels

T. Zhang and A. Manikas

Communications and Signal Processing Group
Department of Electrical and Electronic Engineering
Imperial College London, Exhibition Road, London, SW7 2BT
Email: {a.manikas, tingting.zhang}@imperial.ac.uk

Abstract—The problem of joint transmitter and receiver beamforming in the downlink DS-CDMA over multipath fading channels is considered in this paper. The proposed investigation is based on the array manifold concept and thus, the space-time properties of the channel can be fully exploited, playing a crucial role in the operation of blind channel estimation and interference suppression techniques. The beamforming weights are designed to minimise the overall mean-squared-error (MSE) of the system. An iterative solution to the optimisation problem is firstly proposed under the system framework. A closed-form solution based on channel eigendecomposition is then proposed. The convergence of the iterative method to the closed-form solution and the equivalence of the two methods are verified through numerical simulations. The performance of the proposed approach is also supported by some illustrative examples.

NOTATIONS

A	Scalar
$\underline{A}$	Column Vector
$\mathbb{A}$	Matrix
$(\cdot)^T$	Transpose
$(\cdot)^H$	Conjugate transpose
$[\cdot]$	Round up to the nearest integer
$\otimes$	Kronecker product
$\text{diag}(\underline{A})$	Element-wise diagonalisation of vector $\underline{A}$
$\text{diag}(\mathbb{A})$	Column-wise diagonalisation of matrix $\mathbb{A}$
$\exp(\underline{A})$	Element-wise exponential of vector $\underline{A}$
$\underline{0}_N$	Zero vector of N elements
$\mathbb{I}_N$	Identity matrix of $N \times N$ dimension
$\underline{1}_N$	A vector of ones with length N

I. INTRODUCTION

The explosive demand for wireless internet and multimedia services requires more efficient techniques to be employed to enable high data rate downlink transmission in DS-CDMA systems. The use of beamforming is considered as an effective way to eliminate both multiple-access interference (MAI) and inter-symbol interference (ISI). While receiver beamforming has long been an active area of research, the physical limitation of a mobile station (MS), such as the space and battery power prohibits more advanced array processing at the MS. Transmit beamforming from the base station (BS) to MS serves as a powerful alternative for increasing downlink capacity and it is based on criteria such as maximum signal-to-noise ratio (MSNR) [1][2] and maximum pseudo signal-to-interference

plus noise ratio (PSINR) [2]. Currently more attention has been directed to joint transmitter-receiver beamforming that offers a performance advantage over transmitter-only, or receiver-only beamforming and a computational burden balanced over both ends of the transmission link. In [3], an iterative optimisation scheme that minimises the overall mean squared error (MSE) of the system is proposed over multipath channels. A generalized framework is constructed in [4] that embraces the most reasonable criteria of beamforming design. However, the geometry of the array is ignored in all the above mentioned approaches and this considerably limits the ability of antenna arrays in exploiting efficiently the inherent space-time structure of the multipath and multiple access channels. In [5], a mathematical model for a receiver-only beamforming system has been derived based on the spatio-temporal array manifold (STAR) vector. Utilising the geometrical information provided by the manifold vector, the proposed model offers a richer description of the space-time channel. The study presented in this paper will be based on the array manifold concept for designing a joint transmitter-receiver beamforming framework for “downlink” with significantly enhanced multiple-access interference suppression capabilities over multipath channels. Note that blind superresolution channel estimation techniques based on the STAR manifold concept have been developed [6] for uplink receiver beamforming systems; these techniques can be adapted for open loop downlink transmission - though this is not straightforward.

In this paper, the problem under consideration is joint transmitter-receiver beamforming in the downlink DS-CDMA system over multipath fading channels. In Section II, the antenna array system over space-time fading channels is modelled. Small aperture arrays with half wavelength spacing are considered in this paper due to the space limitation at the MS and, furthermore, due to their capability in combatting ambiguities and other negative side effects associated with large aperture arrays. In Section III, two approaches for joint beamforming are proposed under the MMSE criteria. The optimisation problem is first solved iteratively. Then a closed-form solution is derived based on the channel eigendecomposition. In Section IV numerical simulations are presented. The proposed approaches are also supported by a number of illustrative examples showing a considerable performance improvement of the proposed algorithm relative to other receiver/transmitter-only beamforming algorithms. The paper is concluded in Section V.

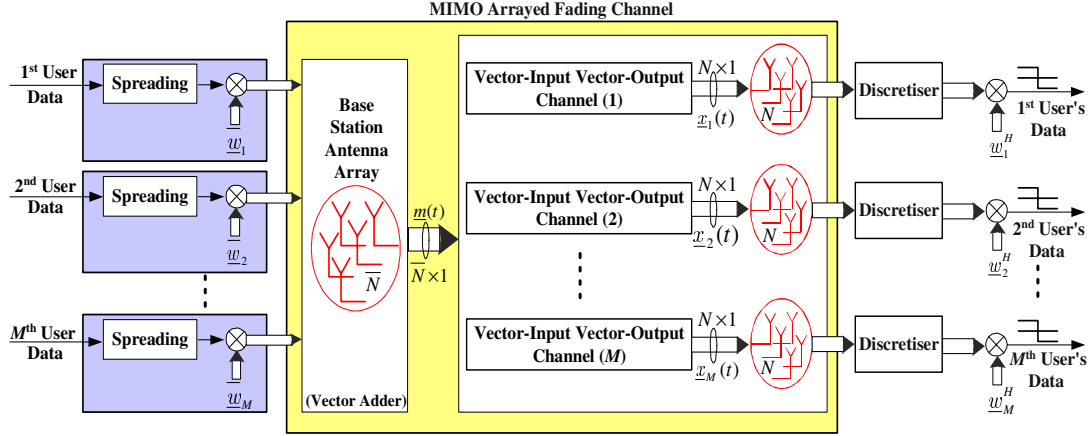


Fig. 1. System architecture with transmitter and receiver utilizing antenna-array. The “Vector-Input Vector-Output” channel model is shown in Fig. 2 in detail.

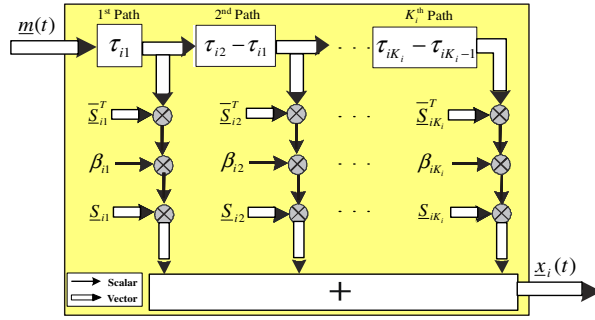


Fig. 2. “Vector-Input and Vector-Output” channel model between transmit antenna and receive antenna at the i^{th} user.

II. DOWNLINK SIGNAL MODELLING

Let us consider an M user binary phase-shift keying (BPSK) DS-CDMA system with $\bar{N}$ transmit and N receive antennas. Fig. 1 shows a typical arrayed system for joint transmitter-receiver beamforming. The i^{th} user's BPSK symbols $a_i[n], \forall n \in \mathcal{Z}$ are first spreaded with a unique spreading code $[\alpha_i[p] \in \pm 1, p = 0, \dots, \mathcal{N}_c - 1]$ of period $\mathcal{N}_c$ and a unity-norm beamforming weight vector $\underline{w}_i \in \mathbb{C}^{\bar{N} \times 1}$ is applied before transmission. The baseband transmitted signal is given by

$$\underline{m}(t) = \sum_{i=1}^M \sqrt{P_i} \underline{w}_i \sum_{n=-\infty}^{+\infty} a_i[n] \sum_{p=0}^{\mathcal{N}_c-1} \alpha_i[p] c(t - nT_{cs} - pT_c) \quad (1)$$

with $nT_{cs} + pT_c \leq t < nT_{cs} + (p+1)T_c$

where P_i is the transmission power of the i^{th} user; T_{cs} and T_c are respectively the data symbol period and chip period; and $c(t)$ denotes the unit amplitude chip pulse-shaping waveform of duration T_c .

Let us assume the transmitted signal arrives at the i^{th} user via K_i multipaths. Consider that the k^{th} path of the i^{th} user has

the direction-of-departure (DOD) $(\bar{\theta}_{ik}, \bar{\phi}_{ik})$ and the direction-of-arrival (DOA) (θ_{ik}, ϕ_{ik}) . Please note that a bar on the top of a symbol represents a transmitter's parameter. The symbols β_{ik} and τ_{ik} denote respectively the complex fading coefficient and path delay of the k^{th} path of the i^{th} user. The propagation channel can be modelled as a Vector-Input and Vector-Output (VIVO) channel model as shown in Fig. 2. Based on this model, the received baseband signal-vector of the i^{th} MS (user) can be written explicitly as

$$\underline{x}_i(t) = \sum_{k=1}^{K_i} \beta_{ik} \underline{S}_{ik} \bar{\underline{S}}_{ik}^T \underline{m}(t - \tau_{ik}) + \underline{n}_i(t) \quad (2)$$

where $\underline{n}_i(t)$ is the complex white Gaussian noise vector of covariance matrix $\sigma_i^2 \mathbb{I}_N$. The symbol $\underline{S}_{ik}$ represents receiver array manifold vector and is defined as

$$\underline{S}_{ik} \triangleq \underline{S}(\theta_{ik}, \phi_{ik}) = \exp(-j \cdot [\underline{r}_x, \underline{r}_y, \underline{r}_z] \cdot \underline{k}_{ik}) \quad (3)$$

where $[\underline{r}_x, \underline{r}_y, \underline{r}_z]$ denotes the Cartesian coordinate of the antenna-array elements and

$$\underline{k}_{ik} = \frac{2\pi F_c}{c} \cdot [\cos \theta_{ik} \cos \phi_{ik}, \sin \theta_{ik} \cos \phi_{ik}, \sin \phi_{ik}]^T$$

is the wavenumber vector pointing towards the direction (θ_{ik}, ϕ_{ik}) with F_c the carrier frequency, c the speed of light, and (θ_{ik}, ϕ_{ik}) the azimuth and elevation angles of the direction-of-arrival respectively. In a similar fashion is also defined the transmitter manifold vector $\bar{\underline{S}}_{ik} \triangleq \bar{\underline{S}}(\bar{\theta}_{ik}, \bar{\phi}_{ik})$. Without loss of generality, the elevation angle is assumed equal to zero in this paper.

The discretised baseband signal vector $\underline{x}[n]$ is formed by sampling $\underline{x}(t)$ with period $T_s = T_c$ which is then passed through a bank of N tapped-delay lines (TDL) (see [5] for more details). The multipath delay spread in this paper is assumed to lie within the range $[0, T_{cs})$, so a TDL of $2\mathcal{N}_c$ taps is capable of capturing the delay spread. To model the discretised signal $\underline{x}_i[n]$ at the output of the TDL, let us define the following

Spatio-Temporal ARray (STAR) manifold matrix

$$\mathbb{H}_{ik,j} = (\underline{S}_{ik} \bar{S}_{ik}^T) \otimes (\mathbb{J}^{l_{ik}} \underline{c}_j) \in \mathcal{C}^{2N\mathcal{N}_c \times \bar{N}} \quad (4)$$

associated with the signals from the j^{th} transmitter-beamformer (intended for any j^{th} user) and received by the i^{th} user via the k^{th} path; $l_{ik} = \lceil \tau_{ik}/T_c \rceil$ is the discretised multipath delay and the matrix $\mathbb{J}$ (or $\mathbb{J}^T$) is a $2N\mathcal{N}_c \times 2N\mathcal{N}_c$ time down-shift (or up-shift) operator matrix given as follows

$$\mathbb{J} = \begin{bmatrix} \underline{0}_{2N\mathcal{N}_c-1}^T & 0 \\ \mathbb{I}_{2N\mathcal{N}_c-1} & \underline{0}_{2N\mathcal{N}_c-1} \end{bmatrix} \quad (5)$$

and $\underline{c}_j$ is the j^{th} user's spreading code sequence padded with zeros of length N_c defined as

$$\underline{c}_j = [\alpha_j[0], \dots, \alpha_j[N_c - 1], \underline{0}_{N_c}^T]^T \quad (6)$$

By defining the vector $\underline{a}[n] = [a_1[n], \dots, a_M[n]]^T$ which denotes the data symbols of all users, the discretised received signal-vector $\underline{x}_i[n]$ of the i^{th} mobile can be modelled as

$$\underline{x}_i[n] = \underbrace{[\mathbb{H}_i^{\text{prev}}, \mathbb{H}_i, \mathbb{H}_i^{\text{next}}]}_{\triangleq \bar{\mathbb{G}}_i} (\mathbb{I}_3 \otimes \bar{\mathbb{W}}) \begin{bmatrix} \underline{a}[n-1] \\ \underline{a}[n] \\ \underline{a}[n+1] \end{bmatrix} + \underline{n}_i[n] \quad (7)$$

where $\underline{n}_i[n]$ represents the discretised noise vector. The composite channel matrix

$$\mathbb{H}_i = [\mathbb{H}_{i,1}, \mathbb{H}_{i,2}, \dots, \mathbb{H}_{i,M}] \in \mathcal{C}^{2N\mathcal{N}_c \times M\bar{N}}$$

contains the channel matrices of all users' signals that arrive at the i^{th} user with $\mathbb{H}_{i,j} \forall j = 1, \dots, M$ modelled as

$$\mathbb{H}_{i,j} = \sqrt{P_j} \sum_{k=1}^{K_i} \beta_{ik} \mathbb{H}_{ik,j} \in \mathcal{C}^{2N\mathcal{N}_c \times \bar{N}}$$

Furthermore, the two matrices associated with the previous and next symbols $\mathbb{H}_i^{\text{prev}}$ and $\mathbb{H}_i^{\text{next}}$, respectively, can be written as a function of $\mathbb{H}_i$ as follows:

$$\begin{aligned} \mathbb{H}_i^{\text{prev}} &= (\mathbb{I}_N \otimes (\mathbb{J}^T)^{N_c}) \mathbb{H}_i \\ \mathbb{H}_i^{\text{next}} &= (\mathbb{I}_N \otimes \mathbb{J}^{N_c}) \mathbb{H}_i \end{aligned}$$

Finally, in Eqn. (7), the matrix $\bar{\mathbb{W}}$ contains all the users' transmit beamforming vectors and is defined as follows

$$\bar{\mathbb{W}} = \text{diag}[\underline{w}_1, \underline{w}_2, \dots, \underline{w}_M] \in \mathcal{C}^{M\bar{N} \times M} \quad (8)$$

III. JOINT TRANSMITTER-RECEIVER BEAMFORMING

Joint transmitter-receiver beamforming is characterised by the interaction of the optimisation at both transmitter and receiver sides. The interaction procedure can be realised by an iterative transmitter and receiver optimisation approach in this paper. It is assumed that both the transmitter and receiver have perfect knowledge of the channel, though subspace based blind

method developed in [6] can be applied to retrieve channel state information. The beamforming weight matrix $\bar{\mathbb{W}}$ at the transmitter is fed to the receiver through a "feedback link" (close loop approach). With the channel and weight information available, the receiver beamforming weight is first adapted according to the MMSE criterion. By denoting the MMSE receiver weight vector of the i^{th} user (MS) as $\underline{w}_i$, the receiver weight vector can be constructed as

$$\underline{w}_i = [\bar{\mathbb{G}}_i \bar{\mathbb{G}}_i^H + \sigma_i^2 \mathbb{I}_{2N\mathcal{N}_c}]^{-1} \bar{\mathbb{G}}_i \underline{p}_{M+i} \quad (9)$$

where $\bar{\mathbb{G}}_i$ has been defined in Eqn. (7) and $\underline{p}_k$ is a column select vector defined as $\underline{p}_k = [0_{k-1}^T, 1, 0_{3M-k}^T]^T \in \mathcal{C}^{3M \times 1}$ which selects the k^{th} column of any pre-multiplied matrix. The output signal of the i^{th} user (MS) is given as

$$y_i[n] = \underline{w}_i^H \underline{x}_i[n]$$

After the weight vector at the receiver (MS) side has been constructed, each user's transmit beamforming vector $\underline{w}_i, \forall i$ is optimised at the BS based on the minimisation of the MSE of all users. That is

$$J = \min_{\underline{w}_1, \dots, \underline{w}_M} \mathcal{E} \left\{ \sum_{i=1}^M |a_i[n] - y_i[n]|^2 \right\} \quad (10)$$

It is important to point out that, as a consequence of applying the transmit beamforming weight $\underline{w}_i \forall i = 1, \dots, M$, the transmit power may increase. Thus, the transmit energy per bit interval per user is constrained as follows

$$\|\sqrt{P_i} \underline{w}_i\|^2 = P_i \quad \forall i = 1, \dots, M$$

which is equivalent to

$$\|\underline{w}_i\| = 1 \quad \forall i = 1, \dots, M$$

A. An iterative method based on Lagrange Multipliers

To solve the above constrained optimisation problem, the method of Lagrange multipliers is employed so as to convert the constrained minimisation problem into an unconstrained minimisation problem. Therefore, the augmented unconstrained cost function is

$$J = \min_{\underline{w}_i, \lambda_i, \forall i} \mathcal{E} \left\{ \sum_{i=1}^M |a_i[n] - \underline{w}_i^H \underline{x}_i[n]|^2 \right\} + \sum_{i=1}^M \lambda_i (\|\underline{w}_i\| - 1) \quad (11)$$

It can be proved (see Appendix) that, for given receivers' weight vector $\underline{w}_i$, the optimal transmitter associated with the i^{th} user is given as follows

$$\underline{w}_i = (\bar{\mathbb{G}}_i^H \bar{\mathbb{G}}_i + \mathbb{I}_{\bar{N}} \lambda_i)^{-1} \bar{\mathbb{H}}_{i,i}^H \underline{w}_i \quad (12)$$

where

$$\bar{\mathbb{G}}_i = (\mathbb{I}_3 \otimes \mathbb{W}^H) [\bar{\mathbb{H}}_i^{\text{prev}^T}, \bar{\mathbb{H}}_i^T, \bar{\mathbb{H}}_i^{\text{next}^T}]^T \in \mathcal{C}^{3M \times \bar{N}} \quad (13)$$

In Eqn. (13), the receivers' weight matrix $\mathbb{W}$ is defined as follows

$$\mathbb{W} = \text{diag}(\underline{w}_1, \underline{w}_2, \dots, \underline{w}_M) \in \mathcal{C}^{2N\mathcal{N}_c M \times M}$$

with $\underline{w}_i, \forall i = 1, \dots, M$ obtained using Eqn. (9). Furthermore, the matrix $\mathbb{H}_i = [\mathbb{H}_{1,i}^T, \mathbb{H}_{2,i}^T, \dots, \mathbb{H}_{M,i}^T]^T \in \mathcal{C}^{2N\mathcal{N}_c M \times \bar{N}}$ contains the channel matrices of the i^{th} user received by all M users in the system. Given both the transmitter and receiver beamforming vectors, it can be proven (see Appendix) that the Lagrange multiplier is given by

$$\lambda_i = \underline{w}_i^H \mathbb{H}_{i,i}^H \underline{w}_i - \underline{w}_i^H \mathbb{G}_i^H \mathbb{G}_i \underline{w}_i \quad (14)$$

From the above analysis, it can be seen that the transmitter beamformer and the receiver beamformer as well as the Lagrange multiplier are functions of each other, thus an iterative algorithm is suggested to obtain the solution. Thus, the proposed joint transmitter-receiver optimisation can be accomplished via the following steps.

- 1) The transmitter weight vectors $\underline{w}_i, \forall i$ are initialised as $\underline{w}_i = \frac{1}{\sqrt{N}} \mathbf{1}$.
- 2) The MMSE weight vector at each receiver $\underline{w}_i \forall i$ is calculated using Eqn. (9).
- 3) Given both the transmitter and receiver weight vectors, the corresponding Lagrange multiplier λ_i is calculated through Eqn. (14).
- 4) Using the receiver weight vector $\underline{w}_i$ of step 2 and the Lagrange multiplier λ_i obtained in Step 3, the transmitter weight $\underline{w}_i, \forall i$ can be updated via Eqn. (12) and is then normalised to have unity norm.
- 5) Repeat Steps 2-4 until a predefined convergence criterion is reached.

B. A closed-form solution based on Channel Eigendecomposition

The iterative algorithm that has been derived requires steps 2-4 to be repeated until convergence is reached. In this section, an approximate closed-form solution to the minimisation problem of Eqn. (10) is derived by performing the eigendecomposition of the channels. It can be proved that when users' spreading codes maintain good orthogonality, such as in a lightly or medium loaded CDMA system, the joint transmitter-receiver can be obtained in one step. Simulation results also show that the iterative approach converges to the closed-form solution.

From Eqn. (10), the MSE of all users can be derived as

$$\begin{aligned} \mathcal{E} \left\{ \sum_{i=1}^M |a_i[n] - \underline{w}_i^H \underline{x}_i[n]|^2 \right\} \\ = \sum_{i=1}^M \left\{ \underline{w}_i^H (\mathbb{G}_i \mathbb{G}_i^H + \sigma_i^2 \mathbb{I}_{2N\mathcal{N}_c}) \underline{w}_i + 1 - \underline{w}_i^H \mathbb{G}_i \underline{p}_{M+i} \right. \\ \left. - \underline{p}_{M+i}^H \mathbb{G}_i^H \underline{w}_i \right\} \end{aligned} \quad (15)$$

$$= \sum_{i=1}^M \underline{p}_{M+i}^H \left(\mathbb{I}_{3M} + \sigma_i^{-2} \mathbb{G}_i^H \mathbb{G}_i \right)^{-1} \underline{p}_{M+i} \quad (16)$$

Note that Eqn. (15) is obtained by substituting $\underline{x}_i[n]$ with Eqn. (7). Then substituting $\underline{w}_i$ with Eqn. (9) and using the matrix inversion lemma¹, Eqn. (16) can be obtained. Furthermore, if the columns of $\mathbb{H}_i^{\text{prev}}$, $\mathbb{H}_i$ and $\mathbb{H}_i^{\text{next}}$ in $\mathbb{G}_i^H$ are orthogonal to each other, then $\sigma_i^{-2} \mathbb{G}_i^H \mathbb{G}_i$ approximates a diagonal dominated matrix and the matrix inversion in Eqn. (16) can be simply realised by taking the reciprocal of the diagonal elements of $(\mathbb{I}_{3M} + \sigma_i^{-2} \mathbb{G}_i^H \mathbb{G}_i)$. Bearing in mind that $\underline{p}_{M+i}$ is the selecting operator, Eqn. (16) can be simplified as

$$\sum_{i=1}^M (1 + \underline{w}_i^H \mathbb{R}_{i,i} \underline{w}_i)^{-1} \quad (17)$$

where $\mathbb{R}_{i,i} = \sigma_i^{-2} \mathbb{H}_{i,i}^H \mathbb{H}_{i,i}$. Obviously, to minimise Eqn. (17) is equivalent to maximise $\underline{w}_i^H \mathbb{R}_{i,i} \underline{w}_i, \forall i = 1, \dots, M$. Based on the Rayleigh-Ritz Theorem (see Chapter 4 in [7]), the upper bound of $\underline{w}_i^H \mathbb{R}_{i,i} \underline{w}_i$ is achieved if $\underline{w}_i$ is the principal eigenvector of $\mathbb{R}_{i,i}$, i.e.

$$\underline{w}_i = \text{principal eigenvector of } \mathbb{R}_{i,i} \quad (18)$$

IV. NUMERICAL STUDIES

In this section, the performance of the proposed joint beamforming scheme is investigated through a number of numerical simulations. Consider that both the transmitter (or base station) and the receiver (or the mobile user) employ uniform linear arrays of half-wavelength spacing. The system is operating in the presence of $M = 3$ co-channel BPSK DS-CDMA users. Signals are transmitted to each user via 5 multipaths. The DOD and DOA of each multipath are uniformly distributed within $[0, 180^\circ]$ and TOA of each path is randomly selected from a uniform distribution between $[0, 30T_c)$. The path fading coefficients are complex Gaussian random variables and the multipath power sum for each user has unit magnitude. Each user employs a unique Gold sequence of length $\mathcal{N}_c = 31$ with rectangular chip pulse-shaping. The array is assumed to collect a block of 200 data symbols for processing at each time. The channel is assumed to be stationary during each data block and the channel state information is available to both transmitter and receiver.

Firstly, the MSE convergence curve of the proposed iterative method is studied for various SNR levels. The transmitter (BS) is equipped with an antenna array of 5 elements ($\bar{N} = 5$) while the receivers (MS) are equipped with 3-element arrays ($N = 3$). The transmitter beamforming weights are initialised as $\underline{w}_i = \frac{1}{\sqrt{N}} \mathbf{1}, \forall i$. Fig. 3 shows that the channel eigendecomposition method serves very well as a lower bound for the iterative method for a lightly loaded system. Both methods achieve the equivalent MSE level, however, it takes the iterative method a number of iterations to converge. The closed-form solution thus is more efficient to be employed at the transmitter.

¹Matrix Inversion Lemma: $(A + BCD)^{-1} = A^{-1} - A^{-1}B(DA^{-1}B + C^{-1})^{-1}DA^{-1}$. By substituting $A = \mathbb{I}_{3M}$, $B = \mathbb{G}_i^H$, $C = \sigma_i^{-2} \mathbb{I}_{2N\mathcal{N}_c}$ and $D = \mathbb{G}_i$, Eqn. (16) can be obtained.

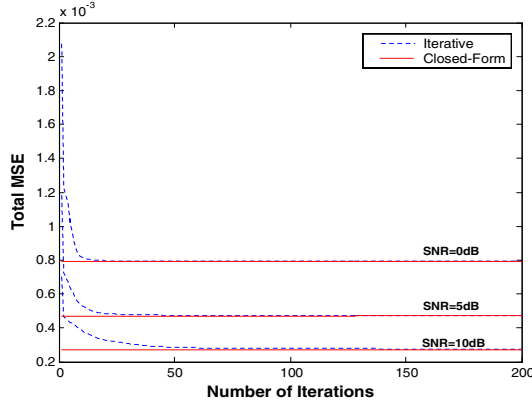


Fig. 3. Convergence study for the iterative method with input $E_b/N_0 = 0, 5, 10\text{dB}$. It is bounded by the closed-form solution.

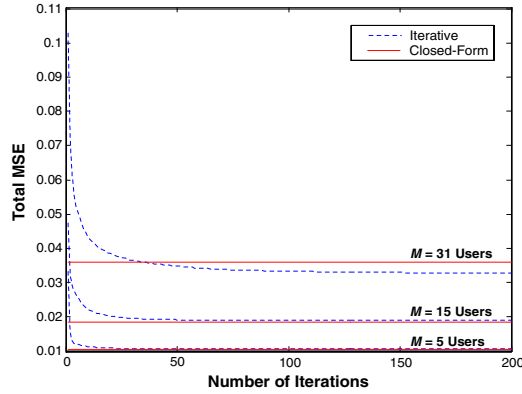


Fig. 4. Overall MSE performance of proposed methods in systems with varied number of users ($M = 5, 15, 31$).

Next, the CDMA system is extended to scenarios where the number of users is very large. Three scenarios are considered,

- $M = 5$ (lightly loaded system)
- $M = 15$ (medium loaded system)
- $M = 31$ (heavily loaded system)

and the overall MSE performance of the system is plotted in Fig. 4. It is observed that, for systems with $M = 5$ and $M = 15$, the convergence of the iterative method to the closed-form solution is very well maintained. In these cases, the closed-form approach instead of the iterative method can be used at the transmitter. As the number of users in the system increases, the orthogonality of users' spreading sequences degrades. A mismatch occurs when $M = 31$. In this case, the iterative method still converges to the global minimum of the cost function while the closed-form approach results in an MSE level slightly higher than the iterative method. Thus, in fully loaded systems, the iterative method is slightly better than the closed-form solution. Without loss of generality, in the following discussion, a system of 3 users will be considered and only the performance of closed-form solution will be plotted

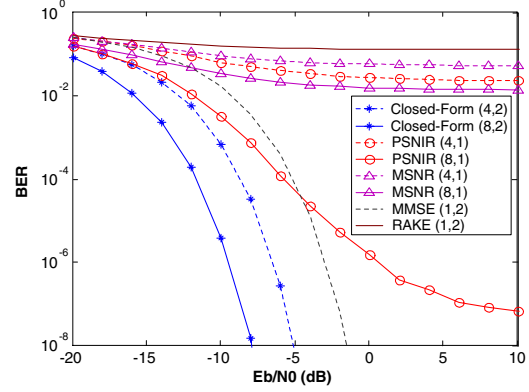


Fig. 5. BER performance of joint transmitter-receiver beamforming, transmitter-only beamforming and receiver-only beamforming as different number of transmitting antennas are employed ($\bar{N} = 1, 4, 8$).

and compared with other existing methods.

Fig. 5 and Fig. 6 show respectively the BER and SNIR performance of the proposed channel eigendecomposition scheme as the number of transmit antennas increases while the number of receive antennas is fixed. In particular, the two figures present the performance of the i^{th} user (taken as the desired user) as a function of E_b/N_0 . The near-far problem is considered by setting the signal to interference ratio -10dB (i.e. SIR = -10dB). For the proposed method, the number of receive antennas is kept constant at $N = 2$ while the number of transmit antenna elements is $\bar{N} = 4$ or 8 . The performance of two transmitter-only beamforming methods [1][2], namely MSNR and PSNIR (extended to the space-time channel model presented in this paper) are also present in Figs. 5 and 6. Note that these two methods are designed to maximise the following cost functions respectively,

$$\begin{cases} \max_{\bar{\mathbf{w}}_i} \text{SNIR}_i = \frac{\bar{\mathbf{w}}_i^H \mathbb{H}_{i,i}^H \mathbb{H}_{i,i} \bar{\mathbf{w}}_i}{\sigma_j^2} & : \text{MSNR} \\ \max_{\bar{\mathbf{w}}_i} \text{PSNIR}_i = \frac{\bar{\mathbf{w}}_i^H \mathbb{H}_{i,i}^H \mathbb{H}_{i,i} \bar{\mathbf{w}}_i}{\mathbb{G}_i^H \mathbb{G}_i - \bar{\mathbf{w}}_i^H \mathbb{H}_{i,i}^H \mathbb{H}_{i,i} \bar{\mathbf{w}}_i + \sigma_i^2} & : \text{PSNIR} \end{cases}$$

The transmit weight vector $\bar{\mathbf{w}}_i$ that maximises SNIR_i of the i^{th} user (MS) is equal to the principal eigenvector of $\mathbb{H}_{i,i}^H \mathbb{H}_{i,i}$. In the case of maximising PSNIR, $\bar{\mathbf{w}}_i$ is found to be the generalised eigenvector of $(\mathbb{H}_{i,i}^H \mathbb{H}_{i,i}, (\mathbb{G}_i^H \mathbb{G}_i - \bar{\mathbf{w}}_i^H \mathbb{H}_{i,i}^H \mathbb{H}_{i,i} \bar{\mathbf{w}}_i + \sigma_i^2 \mathbb{I}_{\bar{N}}))$ where $\mathbb{G}_i$ is defined in Eqn. (13).

The STAR-2D-RAKE receiver and STAR-MMSE receiver with $N = 2$ array elements employed at the MS are also evaluated and their performance is shown in Figs. 5 and 6. From these figures, it is clear that for transmitter-only beamforming schemes, the PSNIR outperforms MSNR (because of PSNIR's capability in suppressing co-channel interferences). For receiver-only beamforming schemes, the MMSE receiver performs much better than the RAKE receiver. It can be concluded that as the number of array elements increases at the transmitter, the BER and SNIR performances of the proposed method outperform all the other methods in the given scenario.

Next, let us consider the situation when the number of transmitter antenna elements $\bar{N}$ at the base station is kept constant

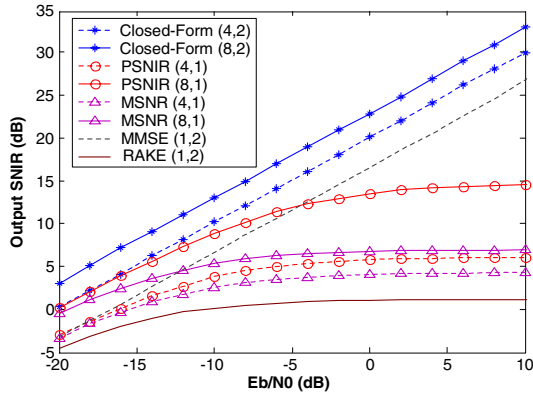


Fig. 6. Output SNIR performance of joint transmitter-receiver beamforming, transmitter-only beamforming and receiver-only beamforming as different number of transmitting antennas are employed ($\bar{N} = 1, 4, 8$).

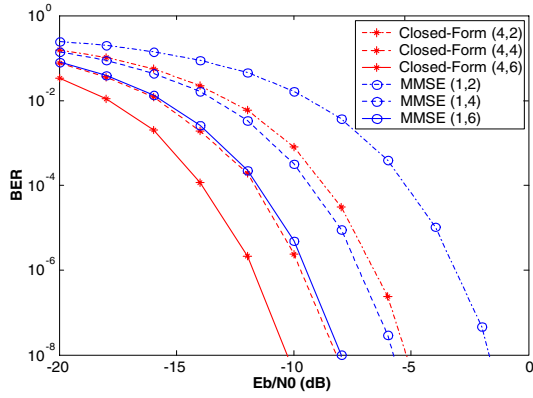


Fig. 7. BER performance of joint transmitter-receiver beamforming and receiver-only beamforming as the number of receiver elements are chosen as $N = 2, 4, 6$.

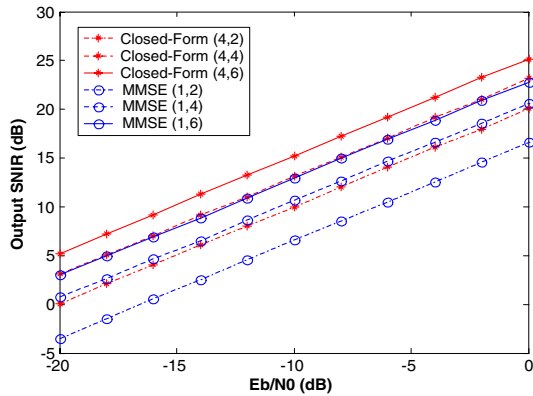


Fig. 8. Output SNIR performance of joint transmitter-receiver beamforming and receiver-only beamforming as the number of receiver elements are chosen as $N = 2, 4, 6$.

while the number of receiver antenna elements N at the MS receiver increases. It can be seen from Figs. 7 and 8 that the proposed joint optimisation method improves the BER and SNIR performance of the system. Note that the array set of $(\bar{N}, N) = (4, 4)$ has a comparable performance to the receiver-only MMSE beamformer with an array set of $(\bar{N}, N) = (1, 6)$. However, as there is a size restriction at the mobiles, by using transmit beamforming at the BS the number of receive antenna element at the MS can be reduced while a comparable performance can still be achieved.

V. APPENDIX

Derivation of $\bar{w}_1, \dots, \bar{w}_M$ and $\lambda_1, \lambda_2, \dots, \lambda_M$:

Based on the unconstrained cost function in Eqn. (11), the M weight vectors $\bar{w}_i, \forall i = 1, \dots, M$ are optimised individually by taking the partial derivative of J w.r.t. $\bar{w}_i, \forall i$. The gradient is readily obtained as

$$\frac{\partial J}{\partial \bar{w}_i} = 2\mathbf{G}_i^H \mathbf{G}_i \bar{w}_i + 2\lambda_i \bar{w}_i - 2\mathbf{H}_{i,i}^H \bar{w}_i$$

where $\mathbf{G}_i$ is defined as in Eqn. (13).

Letting the gradient equal to zero, the solution to the above equation is given by

$$\bar{w}_i = (\mathbf{G}_i^H \mathbf{G}_i + \mathbb{I}_N \lambda_i)^{-1} \mathbf{H}_{i,i}^H \bar{w}_i \quad (19)$$

The transmit weight $\bar{w}_i$ is then normalised in order to satisfy the constraint $\|\bar{w}_i\| = 1$. The Lagrange Multiplier λ_i can then be derived from Eqn. (19) as

$$\lambda_i \underbrace{\bar{w}_i^H \bar{w}_i}_{=1} = \bar{w}_i^H \mathbf{H}_{i,i}^H \bar{w}_i - \bar{w}_i^H \mathbf{G}_i^H \mathbf{G}_i \bar{w}_i \quad (20)$$

VI. CONCLUSION

In this paper, the problem of joint transmitter-receiver beamforming in a multipath fading environment has been studied. It has been shown that by utilizing the STAR manifold concept, the channel can be properly exploited in both space and time domain. Both an “iterative” and a “closed-form” solution to the optimisation problem have been provided. Numerical simulations have shown that the proposed approach is more powerful than some existing beamforming methods.

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