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SCHAUM'S OUTLINE OF

Theory and Problems of

ELECTRIC CIRCUITS

Fourth Edition

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CHAPTER 9

Sinusoidal Steady-State Circuit Analysis

9.1 INTRODUCTION

This chapter will concentrate on the steady-state response of circuits driven by sinusoidal sources. The response will also be sinusoidal. For a linear circuit, the assumption of a *sinusoidal* source represents no real restriction, since a source that can be described by a periodic function can be replaced by an equivalent combination (Fourier series) of sinusoids. This matter will be treated in Chapter 17.

9.2 ELEMENT RESPONSES

The voltage-current relationships for the single elements R, L, and C were examined in Chapter 2 and summarized in Table 2-1. In this chapter, the functions of v and i will be sines or cosines with the argument ωt. ω is the angular frequency and has the unit rad/s. Also, $\omega = 2\pi f$, where f is the frequency with unit cycle/s, or more commonly hertz (Hz).

Consider an inductance L with $i = I\cos(\omega t + 45°)$ A [see Fig. 9-1(a)]. The voltage is

$$v_L = L\frac{di}{dt} = \omega LI[-\sin(\omega t + 45°)] = \omega LI\cos(\omega t + 135°) \quad \text{(V)}$$

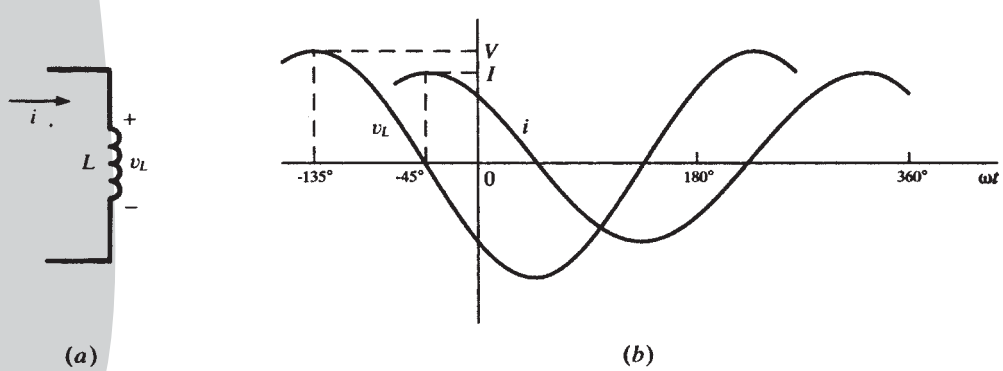

Fig. 9-1

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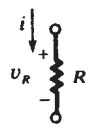
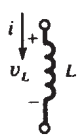
A comparison of v_L and i shows that the current *lags* the voltage by 90° or $\pi/2$ rad. The functions are sketched in Fig. 9-1(*b*). Note that the current function i is to the right of v, and since the horizontal scale is ωt, events displaced to the right occur later in time. This illustrates that *i lags v*. The horizontal scale is in radians, but note that it is also marked in degrees (−135°, 180°, etc.). This is a case of mixed units just as with $\omega t + 45°$. It is not mathematically correct but is the accepted practice in circuit analysis. The vertical scale indicates two different quantities, that is, v and i, so there should be two scales rather than one.

While examining this sketch, it is a good time to point out that a sinusoid is completely defined when its magnitude (V or I), frequency (ω or f), and phase (45° or 135°) are specified.

In Table 9-1 the responses of the three basic circuit elements are shown for applied current $i = I\cos\omega t$ and voltage $v = V\cos\omega t$. If sketches are made of these responses, they will show that for a resistance R, *v and i are in phase*. For an inductance L, *i lags v* by 90° or $\pi/2$ rad. And for a capacitance C, *i leads v* by 90° or $\pi/2$ rad.

Table 9-1

	$i = I\cos\omega t$	$v = V\cos\omega t$
	$v_r = RI\cos\omega t$	$i_R = \dfrac{V}{R}\cos\omega t$
	$v_L = \omega LI\cos(\omega t + 90°)$	$i_L = \dfrac{V}{\omega L}\cos(\omega t - 90°)$
	$v_C = \dfrac{I}{\omega C}\cos(\omega t - 90°)$	$i_C = \omega CV\cos(\omega t + 90°)$

EXAMPLE 9.1 The RL series circuit shown in Fig. 9-2 has a current $i = I\sin\omega t$. Obtain the voltage v across the two circuit elements and sketch v and i.

$$v_R = RI\sin\omega t \qquad v_L = L\frac{di}{dt} = \omega LI\sin(\omega t + 90°)$$

$$v = v_R + v_L = RI\sin\omega t + \omega LI\sin(\omega t + 90°)$$

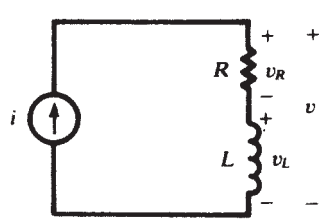


Fig. 9-2

Since the current is a sine function and

$$v = V\sin(\omega t + \theta) = V\sin\omega t\cos\theta + V\cos\omega t\sin\theta \tag{1}$$

we have from the above

$$v = RI\sin\omega t + \omega LI\sin\omega t\cos 90° + \omega LI\cos\omega t\sin 90° \tag{2}$$

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Equating coefficients of like terms in (*1*) and (*2*),

$$V \sin\theta = \omega L I \qquad \text{and} \qquad V \cos\theta = RI$$

Then
$$v = I\sqrt{R^2 + (\omega L)^2}\,\sin[\omega t + \arctan(\omega L/R)]$$

and
$$V = I\sqrt{R^2 + (\omega L)^2} \qquad \text{and} \qquad \theta = \tan^{-1}\frac{\omega L}{R}$$

The functions i and v are sketched in Fig. 9-3. The phase angle θ, the angle by which i lags v, lies within the range $0^\circ \le \theta \le 90^\circ$, with the limiting values attained for $\omega L \ll R$ and $\omega L \gg R$, respectively. If the circuit had an applied voltage $v = V \sin\omega t$, the resulting current would be

$$i = \frac{V}{\sqrt{R^2 + (\omega L)^2}}\sin(\omega t - \theta)$$

where, as before, $\theta = \tan^{-1}(\omega L/R)$.

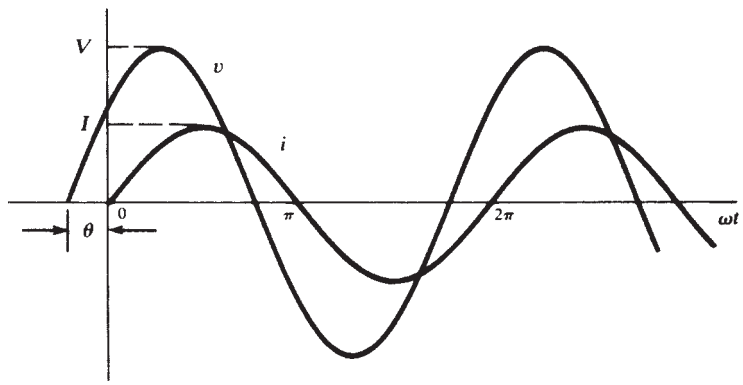

Fig. 9-3

EXAMPLE 9.2 If the current driving a series RC circuit is given by $i = I \sin\omega t$, obtain the total voltage across the two elements.

$$v_R = RI\sin\omega t \qquad v_C = (1/\omega C)\sin(\omega t - 90^\circ)$$
$$v = v_R + v_C = V\sin(\omega t - \theta)$$

where
$$V = I\sqrt{R^2 + (1/\omega C)^2} \qquad \text{and} \qquad \theta = \tan^{-1}(1/\omega CR)$$

The negative phase angle shifts v to the right of the current i. Consequently i *leads* v for a series RC circuit. The phase angle is constrained to the range $0^\circ \le \theta \le 90^\circ$. For $(1/\omega C) \ll R$, the angle $\theta = 0^\circ$, and for $(1/\omega C) \gg R$, the angle $\theta = 90^\circ$. See Fig. 9-4.

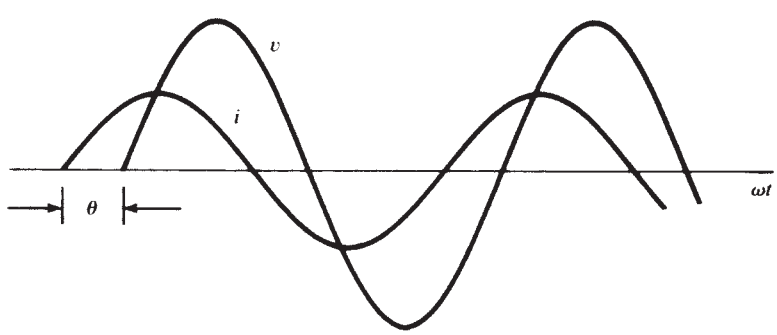

Fig. 9-4

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9.3 PHASORS

A brief look at the voltage and current sinusoids in the preceding examples shows that the amplitudes and phase differences are the two principal concerns. A directed line segment, or *phasor*, such as that shown rotating in a counterclockwise direction at a constant angular velocity ω (rad/s) in Fig. 9-5, has a projection on the horizontal which is a cosine function. The length of the phasor or its magnitude is the amplitude or maximum value of the cosine function. The angle between two positions of the phasor is the phase difference between the corresponding points on the cosine function.

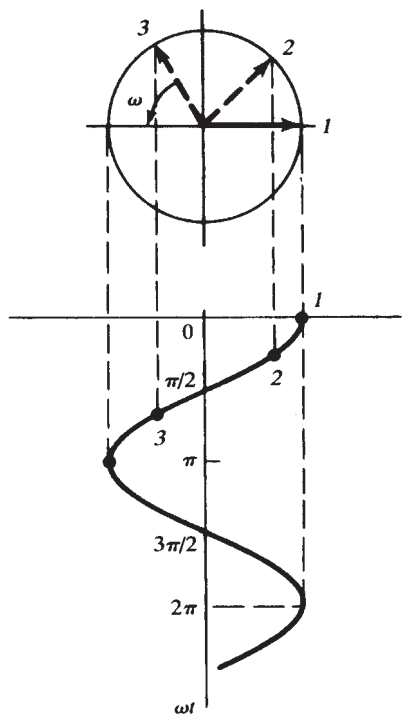

Fig. 9-5

Throughout this book *phasors will be defined from the cosine function*. If a voltage or current is expressed as a sine, it will be changed to a cosine by subtracting 90° from the phase.

Consider the examples shown in Table 9-2. Observe that the phasors, which are directed line segments and vectorial in nature, are indicated by boldface capitals, for example, **V**, **I**. The phase angle of the cosine function is the angle on the phasor. The phasor diagrams here and all that follow may be considered as a snapshot of the counterclockwise-rotating directed line segment taken at $t = 0$. The frequency f (Hz) and ω (rad/s) generally do not appear but they should be kept in mind, since they are implicit in any sinusoidal steady-state problem.

EXAMPLE 9.3 A series combination of $R = 10\,\Omega$ and $L = 20\,\text{mH}$ has a current $i = 5.0\cos(500t + 10°)$ (A). Obtain the voltages v and **V**, the phasor current **I** and sketch the phasor diagram.

Using the methods of Example 9.1,

$$v_R = 50.0\cos(500t + 10°) \qquad v_L = L\frac{di}{dt} = 50.0\cos(500t + 100°)$$

$$v = v_R + v_L = 70.7\cos(500t + 55°) \quad \text{(V)}$$

The corresponding phasors are

$$\mathbf{I} = 5.0\ \underline{/10°}\ \text{A} \qquad \text{and} \qquad \mathbf{V} = 70.7\ \underline{/55°}\ \text{V}$$

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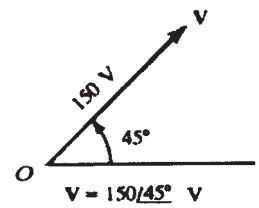
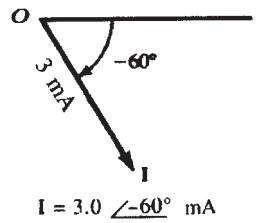
Table 9-2

Function	Phasor Representation
$v = 150\cos(500t + 45°)$ (V)	
$i = 3.0\sin(2000t + 30°)$ (mA) $= 3.0\cos(2000t - 60°)$ (mA)	

The phase angle of 45° can be seen in the time-domain graphs of i and v shown in Fig. 9-6(a), and the phasor diagram with **I** and **V** shown in Fig. 9-6(b).

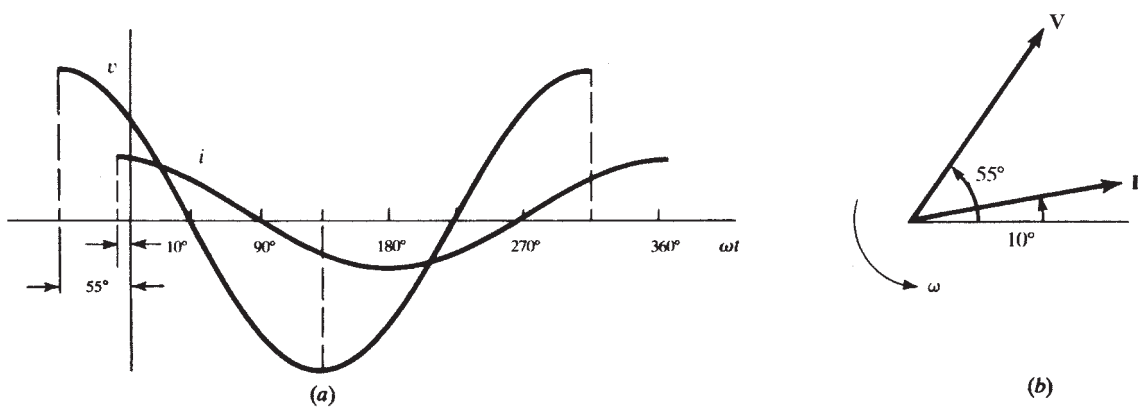


Fig. 9-6

Phasors can be treated as complex numbers. When the horizontal axis is identified as the real axis of a complex plane, the phasors become complex numbers and the usual rules apply. In view of *Euler's identity*, there are three equivalent notations for a phasor.

$$\text{polar form} \qquad \mathbf{V} = V\ \underline{/\theta}$$
$$\text{rectangular form} \qquad \mathbf{V} = V(\cos\theta + j\sin\theta)$$
$$\text{exponential form} \qquad \mathbf{V} = Ve^{j\theta}$$

The cosine expression may also be written as

$$v = V\cos(\omega t + \theta) = \mathrm{Re}\,[Ve^{j(\omega t+\theta)}] = \mathrm{Re}\,[\mathbf{V}e^{j\omega t}]$$

The exponential form suggests how to treat the product and quotient of phasors. Since $(V_1e^{j\theta_1})(V_2e^{j\theta_2}) + V_1V_2e^{j(\theta_1+\theta_2)}$,

$$(V_1\ \underline{/\theta_1})(V_2\ \underline{/\theta_2}) = V_1V_2\ \underline{/\theta_1 + \theta_2}$$

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and, since $(V_1 e^{j\theta_1})/(V_2 e^{j\theta_2}) = (V_1/V_2)e^{j(\theta_1-\theta_2)}$,

$$\frac{V_1 \ \underline{/\theta_1}}{V_2 \ \underline{/\theta}} = V_1/V_2 \ \underline{/\theta_1 - \theta_2}$$

The rectangular form is used in summing or subtracting phasors.

EXAMPLE 9.4 Given $\mathbf{V}_1 = 25.0 \ \underline{/143.13^\circ}$ and $\mathbf{V}_2 = 11.2 \ \underline{/26.57^\circ}$, find the ratio $\mathbf{V}_1/\mathbf{V}_2$ and the sum $\mathbf{V}_1 + \mathbf{V}_2$.

$$\mathbf{V}_1/\mathbf{V}_2 = \frac{25.0 \ \underline{/143.13^\circ}}{11.2 \ \underline{/26.57^\circ}} = 2.23 \ \underline{/116.56^\circ} = -1.00 + j1.99$$

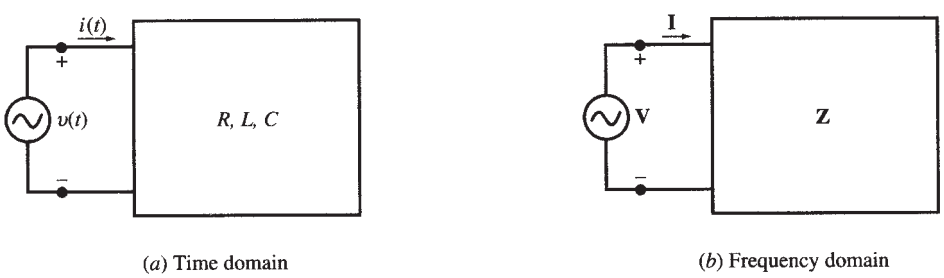
$$\mathbf{V}_1 + \mathbf{V}_2 = (-20.0 + j15.0) + (10.0 + j5.0) = -10.0 + j20.0 = 23.36 \ \underline{/116.57^\circ}$$

9.4 IMPEDANCE AND ADMITTANCE

A sinusoidal voltage or current applied to a passive *RLC* circuit produces a sinusoidal response. With time functions, such as $v(t)$ and $i(t)$, the circuit is said to be in the *time domain*, Fig. 9-7(*a*); and when the circuit is analyzed using phasors, it is said to be in the *frequency domain*, Fig. 9-7(*b*). The voltage and current may be written, respectively,

$$v(t) = V\cos(\omega t + \theta) = \mathrm{Re}[\mathbf{V}e^{j\omega t}] \qquad \text{and} \qquad \mathbf{V} = V \ \underline{/\theta}$$
$$i(t) = I\cos(\omega t + \phi) = \mathrm{Re}[\mathbf{I}e^{j\omega t}] \qquad \text{and} \qquad \mathbf{I} = I \ \underline{/\phi}$$

The ratio of phasor voltage $\mathbf{V}$ to phasor current $\mathbf{I}$ is defined as *impedance* $\mathbf{Z}$, that is, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$. The reciprocal of impedance is called *admittance* $\mathbf{Y}$, so that $\mathbf{Y} = 1/\mathbf{Z}$ (S), where $1\ \mathrm{S} = 1\ \Omega^{-1} = 1$ mho. $\mathbf{Y}$ and $\mathbf{Z}$ are complex numbers.



(*a*) Time domain (*b*) Frequency domain

Fig. 9-7

When impedance is written in Cartesian form the real part is the resistance R and the imaginary part is the *reactance* X. The sign on the imaginary part may be positive or negative: When positive, X is called the *inductive reactance*, and when negative, X is called the *capacitive reactance*. When the admittance is written in Cartesian form, the real part is *admittance* G and the imaginary part is *susceptance* B. A positive sign on the susceptance indicates a *capacitive susceptance*, and a negative sign indicates an *inductive susceptance*. Thus,

$$\mathbf{Z} = R + jX_L \qquad \text{and} \qquad \mathbf{Z} = R - jX_C$$
$$\mathbf{Y} = G - jB_L \qquad \text{and} \qquad \mathbf{Y} = G + jB_C$$

The relationships between these terms follow from $\mathbf{Z} = 1/\mathbf{Y}$. Then,

$$R = \frac{G}{G^2 + B^2} \qquad \text{and} \qquad X = \frac{-B}{G^2 + B^2}$$
$$G = \frac{R}{R^2 + X^2} \qquad \text{and} \qquad B = \frac{-X}{R^2 + X^2}$$

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These expressions are not of much use in a problem where calculations can be carried out with the numerical values as in the following example.

EXAMPLE 9.5 The phasor voltage across the terminals of a network such as that shown in Fig. 9-7(*b*) is $100.0\ \underline{/45^\circ}$ V and the resulting current is $5.0\ \underline{/15^\circ}$ A. Find the equivalent impedance and admittance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} \frac{100.0\ \underline{/45^\circ}}{5.0\ \underline{/15^\circ}} = 20.0\ \underline{/30^\circ} = 17.32 + j10.0\,\Omega$$

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}} = 0.05\ \underline{/-30} = (4.33 - j2.50) \times 10^{-2}\ \mathrm{S}$$

Thus, $R = 17.32\,\Omega$, $X_L = 10.0\,\Omega$, $G = 4.33 \times 10^{-2}$ S, and $B_L = 2.50 \times 10^{-2}$ S.

Combinations of Impedances

The relation $\mathbf{V} = \mathbf{IZ}$ (in the frequency domain) is formally identical to Ohm's law, $v = iR$, for a resistive network (in the time domain). Therefore, impedances combine exactly like resistances:

$$\text{impedances in series} \qquad \mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots$$

$$\text{impedances in parallel} \qquad \frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots$$

In particular, for two parallel impedances, $\mathbf{Z}_{\text{eq}} = \mathbf{Z}_1\mathbf{Z}_2/(\mathbf{Z}_1 + \mathbf{Z}_2)$.

Impedance Diagram

In an *impedance diagram*, an impedance $\mathbf{Z}$ is represented by a point in the right half of the complex plane. Figure 9-8 shows two impedances; $\mathbf{Z}_1$, in the first quadrant, exhibits inductive reactance, while $\mathbf{Z}_2$, in the fourth quadrant, exhibits capacitive reactance. Their series equivalent, $\mathbf{Z}_1 + \mathbf{Z}_2$, is obtained by vector addition, as shown. Note that the "vectors" are shown without arrowheads, in order to distinguish these complex numbers from phasors.

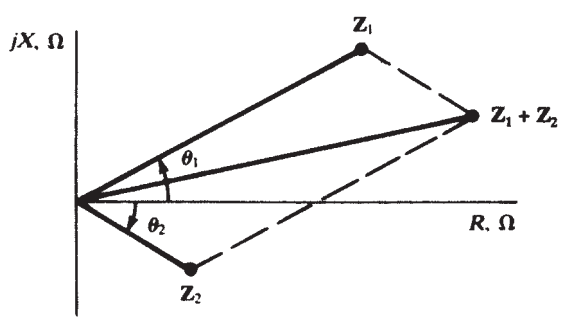

Fig. 9-8

Combinations of Admittances

Replacing $\mathbf{Z}$ by $1/\mathbf{Y}$ in the formulas above gives

$$\text{admittances in series} \qquad \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1} + \frac{1}{\mathbf{Y}_2} + \cdots$$

$$\text{admittances in parallel} \qquad \mathbf{Y}_{\text{eq}} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots$$

Thus, series circuits are easiest treated in terms of impedance; parallel circuits, in terms of admittance.

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Admittance Diagram

Figure 9-9, an *admittance diagram*, is analogous to Fig. 9-8 for impedance. Shown are an admittance $\mathbf{Y}_1$ having capacitive susceptance and an admittance $\mathbf{Y}_2$ having inductive susceptance, together with their vector sum, $\mathbf{Y}_1 + \mathbf{Y}_2$, which is the admittance of a parallel combination of $\mathbf{Y}_1$ and $\mathbf{Y}_2$.

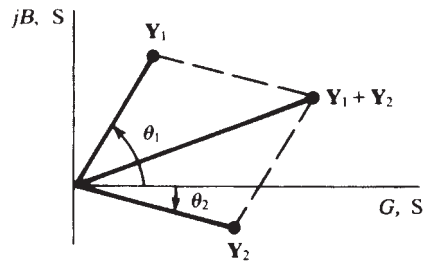

Fig. 9-9

9.5 VOLTAGE AND CURRENT DIVISION IN THE FREQUENCY DOMAIN

In view of the analogy between impedance in the frequency domain and resistance in the time domain, Sections 3.6 and 3.7 imply the following results.

(1) Impedances in series divide the total voltage in the ratio of the impedances:

$$\frac{\mathbf{V}_r}{\mathbf{V}_s} = \frac{\mathbf{Z}_r}{\mathbf{Z}_s} \qquad \text{or} \qquad \mathbf{V}_r = \frac{\mathbf{Z}_r}{\mathbf{Z}_{\text{eq}}}\mathbf{V}_T$$

See Fig. 9-10.

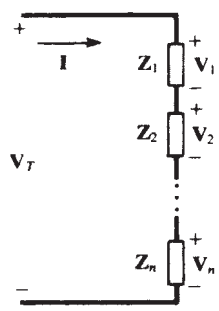

Fig. 9-10

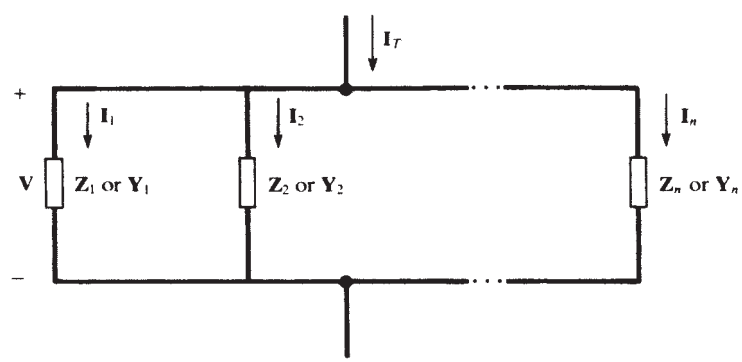

Fig. 9-11

(2) Impedances in parallel (admittances in series) divide the total current in the inverse ratio of the impedances (direct ratio of the admittances):

$$\frac{\mathbf{I}_r}{\mathbf{I}_s} = \frac{\mathbf{Z}_s}{\mathbf{Z}_r} = \frac{\mathbf{Y}_r}{\mathbf{Y}_s} \qquad \text{or} \qquad \mathbf{I}_r = \frac{\mathbf{Z}_{\text{eq}}}{\mathbf{Z}_r}\mathbf{I}_T = \frac{\mathbf{Y}_r}{\mathbf{Y}_{\text{eq}}}\mathbf{I}_T$$

See Fig. 9-11.

9.6 THE MESH CURRENT METHOD

Consider the frequency-domain network of Fig. 9-12. Applying KVL, as in Section 4.3, or simply by inspection, we find the matrix equation

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