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OUTLINE OF**

Theory and Problems of ELECTRIC CIRCUITS

Fourth Edition

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PREFACE

This book is designed for use as a textbook for a first course in circuit analysis or as a supplement to standard texts and can be used by electrical engineering students as well as other engineering and technology students. Emphasis is placed on the basic laws, theorems, and problem-solving techniques which are common to most courses.

The subject matter is divided into 17 chapters covering duly-recognized areas of theory and study. The chapters begin with statements of pertinent definitions, principles, and theorems together with illustrative examples. This is followed by sets of solved and supplementary problems. The problems cover a range of levels of difficulty. Some problems focus on fine points, which helps the student to better apply the basic principles correctly and confidently. The supplementary problems are generally more numerous and give the reader an opportunity to practice problem-solving skills. Answers are provided with each supplementary problem.

The book begins with fundamental definitions, circuit elements including dependent sources, circuit laws and theorems, and analysis techniques such as node voltage and mesh current methods. These theorems and methods are initially applied to DC-resistive circuits and then extended to RLC circuits by the use of impedance and complex frequency. Chapter 5 on amplifiers and op amp circuits is new. The op amp examples and problems are selected carefully to illustrate simple but practical cases which are of interest and importance in the student's future courses. The subject of waveforms and signals is also treated in a new chapter to increase the student's awareness of commonly used signal models.

Circuit behavior such as the steady state and transient response to steps, pulses, impulses, and exponential inputs is discussed for first-order circuits in Chapter 7 and then extended to circuits of higher order in Chapter 8, where the concept of complex frequency is introduced. Phasor analysis, sinusoidal steady state, power, power factor, and polyphase circuits are thoroughly covered. Network functions, frequency response, filters, series and parallel resonance, two-port networks, mutual inductance, and transformers are covered in detail. Application of Spice and PSpice in circuit analysis is introduced in Chapter 15. Circuit equations are solved using classical differential equations and the Laplace transform, which permits a convenient comparison. Fourier series and Fourier transforms and their use in circuit analysis are covered in Chapter 17. Finally, two appendixes provide a useful summary of the complex number system, and matrices and determinants.

This book is dedicated to our students from whom we have learned to teach well. To a large degree it is they who have made possible our satisfying and rewarding teaching careers. And finally, we wish to thank our wives, *Zahra Nahvi* and *Nina Edminister* for their continuing support, and for whom all these efforts were happily made.

MAHMOOD NAHVI
JOSEPH A. EDMINSTER

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EXAMPLE 1.3. In an electric circuit an energy of $9.25 \mu\text{J}$ is required to transport $0.5 \mu\text{C}$ from point a to point b . What electric potential difference exists between the two points?

$$1 \text{ volt} = 1 \text{ joule per coulomb} \quad V = \frac{9.25 \times 10^{-6} \text{ J}}{0.5 \times 10^{-6} \text{ C}} = 18.5 \text{ V}$$

1.5 ENERGY AND ELECTRICAL POWER

Electric energy in joules will be encountered in later chapters dealing with capacitance and inductance whose respective electric and magnetic fields are capable of storing energy. The rate, in *joules per second*, at which energy is transferred is electric power in *watts*. Furthermore, the product of voltage and current yields the electric power, $p = vi$; $1 \text{ W} = 1 \text{ V} \cdot 1 \text{ A}$. Also, $\text{V} \cdot \text{A} = (\text{J/C}) \cdot (\text{C/s}) = \text{J/s} = \text{W}$. In a more fundamental sense power is the time derivative $p = dw/dt$, so that instantaneous power p is generally a function of time. In the following chapters time average power P_{avg} and a root-mean-square (RMS) value for the case where voltage and current are sinusoidal will be developed.

EXAMPLE 1.4. A resistor has a potential difference of 50.0 V across its terminals and 120.0 C of charge per minute passes a fixed point. Under these conditions at what rate is electric energy converted to heat?

$$(120.0 \text{ C/min})/(60 \text{ s/min}) = 2.0 \text{ A} \quad P = (2.0 \text{ A})(50.0 \text{ V}) = 100.0 \text{ W}$$

Since $1 \text{ W} = 1 \text{ J/s}$, the rate of energy conversion is one hundred joules per second.

1.6 CONSTANT AND VARIABLE FUNCTIONS

To distinguish between constant and time-varying quantities, capital letters are employed for the constant quantity and lowercase for the variable quantity. For example, a constant current of 10 amperes is written $I = 10.0 \text{ A}$, while a 10-ampere time-variable current is written $i = 10.0 f(t) \text{ A}$. Examples of common functions in circuit analysis are the sinusoidal function $i = 10.0 \sin \omega t \text{ (A)}$ and the exponential function $v = 15.0 e^{-at} \text{ (V)}$.

Solved Problems

1.1 The force applied to an object moving in the x direction varies according to $F = 12/x^2 \text{ (N)}$.
(a) Find the work done in the interval $1 \text{ m} \leq x \leq 3 \text{ m}$. (b) What constant force acting over the same interval would result in the same work?

$$(a) \quad dW = F dx \quad \text{so} \quad W = \int_1^3 \frac{12}{x^2} dx = 12 \left[\frac{-1}{x} \right]_1^3 = 8 \text{ J}$$

$$(b) \quad 8 \text{ J} = F_c(2 \text{ m}) \quad \text{or} \quad F_c = 4 \text{ N}$$

1.2 Electrical energy is converted to heat at the rate of 7.56 kJ/min in a resistor which has 270 C/min passing through. What is the voltage difference across the resistor terminals?

From $P = VI$,

$$V = \frac{P}{I} = \frac{7.56 \times 10^3 \text{ J/min}}{270 \text{ C/min}} = 28 \text{ J/C} = 28 \text{ V}$$

and, since $(V_1 e^{j\theta_1}) / (V_2 e^{j\theta_2}) = (V_1 / V_2) e^{j(\theta_1 - \theta_2)}$,

$$\frac{V_1 \angle \theta_1}{V_2 \angle \theta} = V_1 / V_2 \angle \theta_1 - \theta_2$$

The rectangular form is used in summing or subtracting phasors.

EXAMPLE 9.4 Given $\mathbf{V}_1 = 25.0 \angle 143.13^\circ$ and $\mathbf{V}_2 = 11.2 \angle 26.57^\circ$, find the ratio $\mathbf{V}_1 / \mathbf{V}_2$ and the sum $\mathbf{V}_1 + \mathbf{V}_2$.

$$\mathbf{V}_1 / \mathbf{V}_2 = \frac{25.0 \angle 143.13^\circ}{11.2 \angle 26.57^\circ} = 2.23 \angle 116.56^\circ = -1.00 + j1.99$$

$$\mathbf{V}_1 + \mathbf{V}_2 = (-20.0 + j15.0) + (10.0 + j5.0) = -10.0 + j20.0 = 23.36 \angle 116.57^\circ$$

9.4 IMPEDANCE AND ADMITTANCE

A sinusoidal voltage or current applied to a passive *RLC* circuit produces a sinusoidal response. With time functions, such as $v(t)$ and $i(t)$, the circuit is said to be in the *time domain*, Fig. 9-7(a); and when the circuit is analyzed using phasors, it is said to be in the *frequency domain*, Fig. 9-7(b). The voltage and current may be written, respectively,

$$v(t) = V \cos(\omega t + \theta) = \text{Re}[\mathbf{V} e^{j\omega t}] \quad \text{and} \quad \mathbf{V} = V \angle \theta$$

$$i(t) = I \cos(\omega t + \phi) = \text{Re}[\mathbf{I} e^{j\omega t}] \quad \text{and} \quad \mathbf{I} = I \angle \phi$$

The ratio of phasor voltage $\mathbf{V}$ to phasor current $\mathbf{I}$ is defined as *impedance* $\mathbf{Z}$, that is, $\mathbf{Z} = \mathbf{V} / \mathbf{I}$. The reciprocal of impedance is called *admittance* $\mathbf{Y}$, so that $\mathbf{Y} = 1 / \mathbf{Z}$ (S), where $1 \text{ S} = 1 \Omega^{-1} = 1 \text{ mho}$. $\mathbf{Y}$ and $\mathbf{Z}$ are complex numbers.

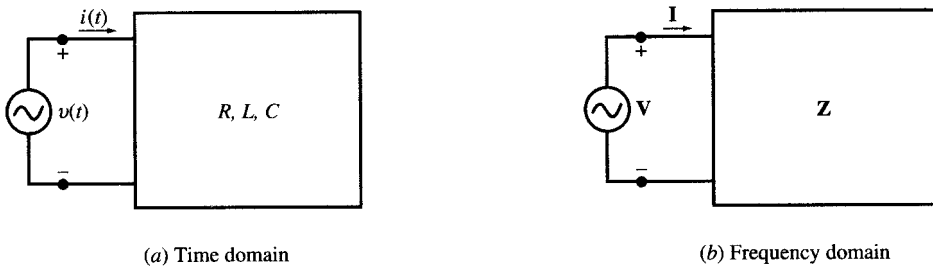


Fig. 9-7

When impedance is written in Cartesian form the real part is the resistance R and the imaginary part is the *reactance* X . The sign on the imaginary part may be positive or negative: When positive, X is called the *inductive reactance*, and when negative, X is called the *capacitive reactance*. When the admittance is written in Cartesian form, the real part is *admittance* G and the imaginary part is *susceptance* B . A positive sign on the susceptance indicates a *capacitive susceptance*, and a negative sign indicates an *inductive susceptance*. Thus,

$$\mathbf{Z} = R + jX_L \quad \text{and} \quad \mathbf{Z} = R - jX_C$$

$$\mathbf{Y} = G - jB_L \quad \text{and} \quad \mathbf{Y} = G + jB_C$$

The relationships between these terms follow from $\mathbf{Z} = 1 / \mathbf{Y}$. Then,

$$R = \frac{G}{G^2 + B^2} \quad \text{and} \quad X = \frac{-B}{G^2 + B^2}$$

$$G = \frac{R}{R^2 + X^2} \quad \text{and} \quad B = \frac{-X}{R^2 + X^2}$$

These expressions are not of much use in a problem where calculations can be carried out with the numerical values as in the following example.

EXAMPLE 9.5 The phasor voltage across the terminals of a network such as that shown in Fig. 9-7(b) is $100.0 \angle 45^\circ$ V and the resulting current is $5.0 \angle 15^\circ$ A. Find the equivalent impedance and admittance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{100.0 \angle 45^\circ}{5.0 \angle 15^\circ} = 20.0 \angle 30^\circ = 17.32 + j10.0 \Omega$$

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}} = 0.05 \angle -30^\circ = (4.33 - j2.50) \times 10^{-2} \text{ S}$$

Thus, $R = 17.32 \Omega$, $X_L = 10.0 \Omega$, $G = 4.33 \times 10^{-2} \text{ S}$, and $B_L = 2.50 \times 10^{-2} \text{ S}$.

Combinations of Impedances

The relation $\mathbf{V} = \mathbf{IZ}$ (in the frequency domain) is formally identical to Ohm's law, $v = iR$, for a resistive network (in the time domain). Therefore, impedances combine exactly like resistances:

$$\text{impedances in series} \quad \mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots$$

$$\text{impedances in parallel} \quad \frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots$$

In particular, for two parallel impedances, $\mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)$.

Impedance Diagram

In an *impedance diagram*, an impedance $\mathbf{Z}$ is represented by a point in the right half of the complex plane. Figure 9-8 shows two impedances; $\mathbf{Z}_1$, in the first quadrant, exhibits inductive reactance, while $\mathbf{Z}_2$, in the fourth quadrant, exhibits capacitive reactance. Their series equivalent, $\mathbf{Z}_1 + \mathbf{Z}_2$, is obtained by vector addition, as shown. Note that the "vectors" are shown without arrowheads, in order to distinguish these complex numbers from phasors.

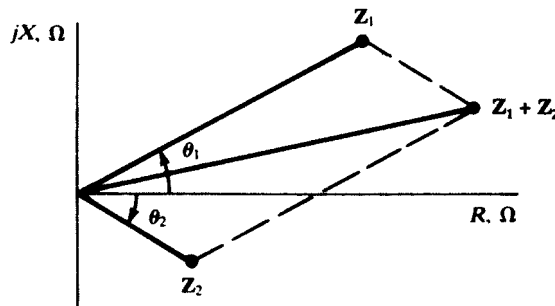


Fig. 9-8

Combinations of Admittances

Replacing $\mathbf{Z}$ by $1/\mathbf{Y}$ in the formulas above gives

$$\text{admittances in series} \quad \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1} + \frac{1}{\mathbf{Y}_2} + \cdots$$

$$\text{admittances in parallel} \quad \mathbf{Y}_{\text{eq}} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots$$

Thus, series circuits are easiest treated in terms of impedance; parallel circuits, in terms of admittance.

Admittance Diagram

Figure 9-9, an *admittance diagram*, is analogous to Fig. 9-8 for impedance. Shown are an admittance $\mathbf{Y}_1$ having capacitive susceptance and an admittance $\mathbf{Y}_2$ having inductive susceptance, together with their vector sum, $\mathbf{Y}_1 + \mathbf{Y}_2$, which is the admittance of a parallel combination of $\mathbf{Y}_1$ and $\mathbf{Y}_2$.

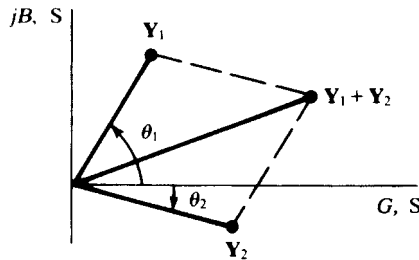


Fig. 9-9

9.5 VOLTAGE AND CURRENT DIVISION IN THE FREQUENCY DOMAIN

In view of the analogy between impedance in the frequency domain and resistance in the time domain, Sections 3.6 and 3.7 imply the following results.

(1) Impedances in series divide the total voltage in the ratio of the impedances:

$$\frac{\mathbf{V}_r}{\mathbf{V}_s} = \frac{\mathbf{Z}_r}{\mathbf{Z}_s} \quad \text{or} \quad \mathbf{V}_r = \frac{\mathbf{Z}_r}{\mathbf{Z}_{\text{eq}}} \mathbf{V}_T$$

See Fig. 9-10.

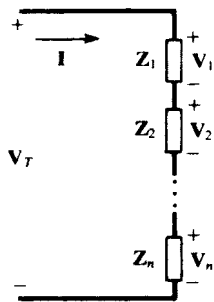


Fig. 9-10

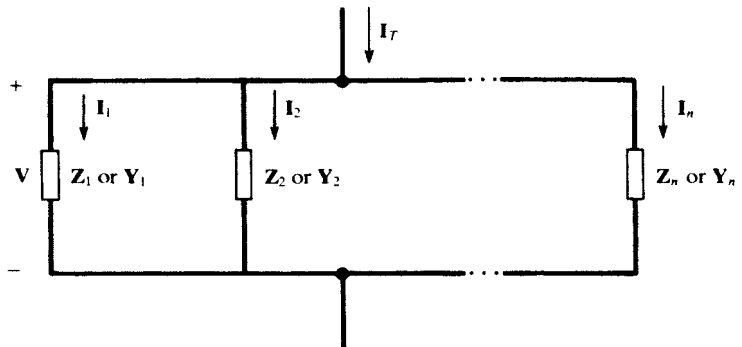


Fig. 9-11

(2) Impedances in parallel (admittances in series) divide the total current in the inverse ratio of the impedances (direct ratio of the admittances):

$$\frac{\mathbf{I}_r}{\mathbf{I}_s} = \frac{\mathbf{Z}_s}{\mathbf{Z}_r} = \frac{\mathbf{Y}_r}{\mathbf{Y}_s} \quad \text{or} \quad \mathbf{I}_r = \frac{\mathbf{Z}_{\text{eq}}}{\mathbf{Z}_r} \mathbf{I}_T = \frac{\mathbf{Y}_r}{\mathbf{Y}_{\text{eq}}} \mathbf{I}_T$$

See Fig. 9-11.

9.6 THE MESH CURRENT METHOD

Consider the frequency-domain network of Fig. 9-12. Applying KVL, as in Section 4.3, or simply by inspection, we find the matrix equation