

SCHAUM'S
ouTlines

ELECTRIC CIRCUITS

Fourth Edition

MAHMOOD NAHVI JOSEPH A. EDMINISTER

Covers all course fundamentals —
supplements any class text

Hundreds of fully worked problems

Hundreds of review questions

The ideal guide for
independent study

The perfect aid for better
grades

**MORE THAN
30 MILLION
SCHAUM'S
OUTLINES
SOLD**

**THE ORIGINAL AND MOST POPULAR
COLLEGE COURSE SERIES AROUND THE WORLD**

www.EngineeringEBooksPdf.com

IPR2023-00697
Theta EX2011

**SCHAUM'S
OUTLINE OF**

Theory and Problems of ELECTRIC CIRCUITS

Fourth Edition

MAHMOOD NAHVI, Ph.D.

*Professor of Electrical Engineering
California Polytechnic State University*

JOSEPH A. EDMINISTER

*Professor Emeritus of Electrical Engineering
The University of Akron*

Schaum's Outline Series

McGRAW-HILL

*New York Chicago San Francisco Lisbon
London Madrid Mexico City Milan New Dehli
San Juan Seoul Singapore Sydney Toronto*

www.EngineeringEBooksPdf.com

**IPR2023-00697
Theta EX2011**

McGraw-Hill

A Division of The McGraw-Hill Companies



Copyright © 2003, 1997, 1986, 1965] by The McGraw-Hill Companies, Inc. All rights reserved. Manufactured in the United States of America. Except as permitted under the United States Copyright Act of 1976, no part of this publication may be reproduced or distributed in any form or by any means, or stored in a database or retrieval system, without the prior written permission of the publisher.

0-07-142582-9

The material in this eBook also appears in the print version of this title: 0-07-139307-2.

All trademarks are trademarks of their respective owners. Rather than put a trademark symbol after every occurrence of a trademarked name, we use names in an editorial fashion only, and to the benefit of the trademark owner, with no intention of infringement of the trademark. Where such designations appear in this book, they have been printed with initial caps.

McGraw-Hill eBooks are available at special quantity discounts to use as premiums and sales promotions, or for use in corporate training programs. For more information, please contact George Hoare, Special Sales, at george_hoare@mcgraw-hill.com or (212) 904-4069.

TERMS OF USE

This is a copyrighted work and The McGraw-Hill Companies, Inc. ("McGraw-Hill") and its licensors reserve all rights in and to the work. Use of this work is subject to these terms. Except as permitted under the Copyright Act of 1976 and the right to store and retrieve one copy of the work, you may not decompile, disassemble, reverse engineer, reproduce, modify, create derivative works based upon, transmit, distribute, disseminate, sell, publish or sublicense the work or any part of it without McGraw-Hill's prior consent. You may use the work for your own noncommercial and personal use; any other use of the work is strictly prohibited. Your right to use the work may be terminated if you fail to comply with these terms.

THE WORK IS PROVIDED "AS IS". MCGRAW-HILL AND ITS LICENSORS MAKE NO GUARANTEES OR WARRANTIES AS TO THE ACCURACY, ADEQUACY OR COMPLETENESS OF OR RESULTS TO BE OBTAINED FROM USING THE WORK, INCLUDING ANY INFORMATION THAT CAN BE ACCESSED THROUGH THE WORK VIA HYPERLINK OR OTHERWISE, AND EXPRESSLY DISCLAIM ANY WARRANTY, EXPRESS OR IMPLIED, INCLUDING BUT NOT LIMITED TO IMPLIED WARRANTIES OF MERCHANTABILITY OR FITNESS FOR A PARTICULAR PURPOSE. McGraw-Hill and its licensors do not warrant or guarantee that the functions contained in the work will meet your requirements or that its operation will be uninterrupted or error free. Neither McGraw-Hill nor its licensors shall be liable to you or anyone else for any inaccuracy, error or omission, regardless of cause, in the work or for any damages resulting therefrom. McGraw-Hill has no responsibility for the content of any information accessed through the work. Under no circumstances shall McGraw-Hill and/or its licensors be liable for any indirect, incidental, special, punitive, consequential or similar damages that result from the use of or inability to use the work, even if any of them has been advised of the possibility of such damages. This limitation of liability shall apply to any claim or cause whatsoever whether such claim or cause arises in contract, tort or otherwise.

DOI: 10.1036/0071425829

Sinusoidal Steady-State Circuit Analysis

9.1 INTRODUCTION

This chapter will concentrate on the steady-state response of circuits driven by sinusoidal sources. The response will also be sinusoidal. For a linear circuit, the assumption of a *sinusoidal* source represents no real restriction, since a source that can be described by a periodic function can be replaced by an equivalent combination (Fourier series) of sinusoids. This matter will be treated in Chapter 17.

9.2 ELEMENT RESPONSES

The voltage-current relationships for the single elements R , L , and C were examined in Chapter 2 and summarized in Table 2-1. In this chapter, the functions of v and i will be sines or cosines with the argument ωt . ω is the angular frequency and has the unit rad/s. Also, $\omega = 2\pi f$, where f is the frequency with unit cycle/s, or more commonly hertz (Hz).

Consider an inductance L with $i = I \cos(\omega t + 45^\circ)$ A [see Fig. 9-1(a)]. The voltage is

$$v_L = L \frac{di}{dt} = \omega LI [-\sin(\omega t + 45^\circ)] = \omega LI \cos(\omega t + 135^\circ) \quad (\text{V})$$

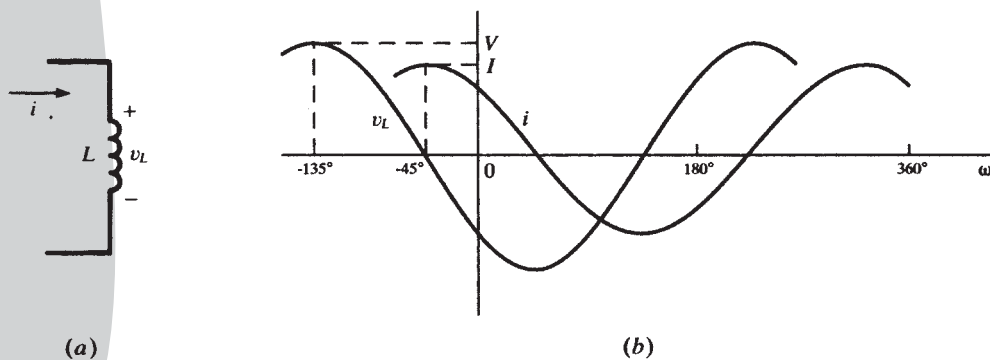


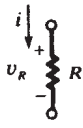
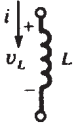
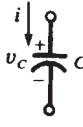
Fig. 9-1

A comparison of v_L and i shows that the current *lags* the voltage by 90° or $\pi/2$ rad. The functions are sketched in Fig. 9-1(b). Note that the current function i is to the right of v , and since the horizontal scale is ωt , events displaced to the right occur later in time. This illustrates that i *lags* v . The horizontal scale is in radians, but note that it is also marked in degrees (-135° , 180° , etc.). This is a case of mixed units just as with $\omega t + 45^\circ$. It is not mathematically correct but is the accepted practice in circuit analysis. The vertical scale indicates two different quantities, that is, v and i , so there should be two scales rather than one.

While examining this sketch, it is a good time to point out that a sinusoid is completely defined when its magnitude (V or I), frequency (ω or f), and phase (45° or 135°) are specified.

In Table 9-1 the responses of the three basic circuit elements are shown for applied current $i = I \cos \omega t$ and voltage $v = V \cos \omega t$. If sketches are made of these responses, they will show that for a resistance R , v and i are *in phase*. For an inductance L , i *lags* v by 90° or $\pi/2$ rad. And for a capacitance C , i *leads* v by 90° or $\pi/2$ rad.

Table 9-1

	$i = I \cos \omega t$	$v = V \cos \omega t$
	$v_R = RI \cos \omega t$	$i_R = \frac{V}{R} \cos \omega t$
	$v_L = \omega LI \cos(\omega t + 90^\circ)$	$i_L = \frac{V}{\omega L} \cos(\omega t - 90^\circ)$
	$v_C = \frac{I}{\omega C} \cos(\omega t - 90^\circ)$	$i_C = \omega CV \cos(\omega t + 90^\circ)$

EXAMPLE 9.1 The RL series circuit shown in Fig. 9-2 has a current $i = I \sin \omega t$. Obtain the voltage v across the two circuit elements and sketch v and i .

$$v_R = RI \sin \omega t \quad v_L = L \frac{di}{dt} = \omega LI \sin(\omega t + 90^\circ)$$

$$v = v_R + v_L = RI \sin \omega t + \omega LI \sin(\omega t + 90^\circ)$$

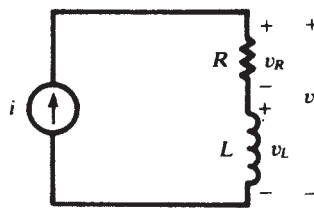


Fig. 9-2

Since the current is a sine function and

$$v = V \sin(\omega t + \theta) = V \sin \omega t \cos \theta + V \cos \omega t \sin \theta \quad (1)$$

we have from the above

$$v = RI \sin \omega t + \omega LI \sin \omega t \cos 90^\circ + \omega LI \cos \omega t \sin 90^\circ \quad (2)$$

Equating coefficients of like terms in (1) and (2),

$$V \sin \theta = \omega L I \quad \text{and} \quad V \cos \theta = R I$$

Then

$$v = I \sqrt{R^2 + (\omega L)^2} \sin [\omega t + \arctan (\omega L / R)]$$

and

$$V = I \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{\omega L}{R}$$

The functions i and v are sketched in Fig. 9-3. The phase angle θ , the angle by which i lags v , lies within the range $0^\circ \leq \theta \leq 90^\circ$, with the limiting values attained for $\omega L \ll R$ and $\omega L \gg R$, respectively. If the circuit had an applied voltage $v = V \sin \omega t$, the resulting current would be

$$i = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \sin (\omega t - \theta)$$

where, as before, $\theta = \tan^{-1} (\omega L / R)$.

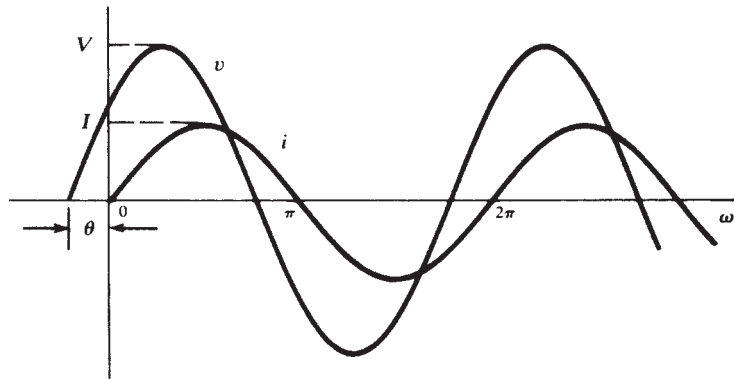


Fig. 9-3

EXAMPLE 9.2 If the current driving a series RC circuit is given by $i = I \sin \omega t$, obtain the total voltage across the two elements.

$$v_R = R I \sin \omega t \quad v_C = (1/\omega C) \sin (\omega t - 90^\circ)$$

$$v = v_R + v_C = V \sin (\omega t - \theta)$$

where

$$V = I \sqrt{R^2 + (1/\omega C)^2} \quad \text{and} \quad \theta = \tan^{-1} (1/\omega C R)$$

The negative phase angle shifts v to the right of the current i . Consequently i leads v for a series RC circuit. The phase angle is constrained to the range $0^\circ \leq \theta \leq 90^\circ$. For $(1/\omega C) \ll R$, the angle $\theta = 0^\circ$, and for $(1/\omega C) \gg R$, the angle $\theta = 90^\circ$. See Fig. 9-4.

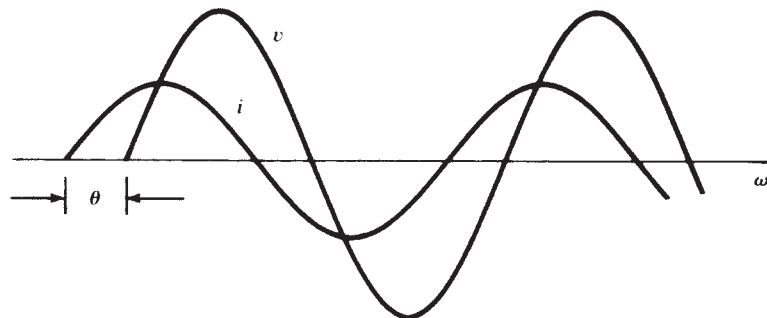


Fig. 9-4

9.3 PHASORS

A brief look at the voltage and current sinusoids in the preceding examples shows that the amplitudes and phase differences are the two principal concerns. A directed line segment, or *phasor*, such as that shown rotating in a counterclockwise direction at a constant angular velocity ω (rad/s) in Fig. 9-5, has a projection on the horizontal which is a cosine function. The length of the phasor or its magnitude is the amplitude or maximum value of the cosine function. The angle between two positions of the phasor is the phase difference between the corresponding points on the cosine function.

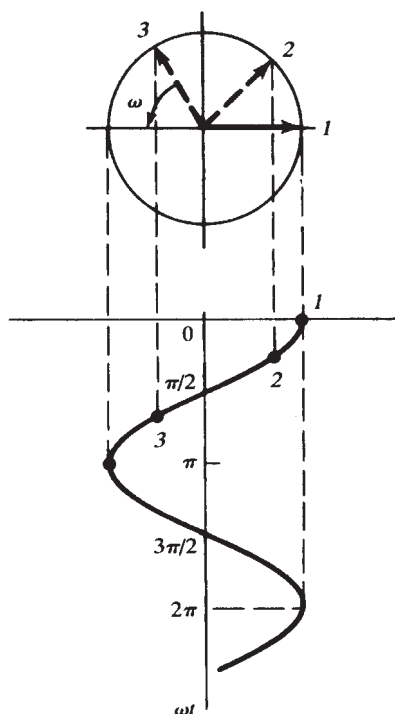


Fig. 9-5

Throughout this book *phasors will be defined from the cosine function*. If a voltage or current is expressed as a sine, it will be changed to a cosine by subtracting 90° from the phase.

Consider the examples shown in Table 9-2. Observe that the phasors, which are directed line segments and vectorial in nature, are indicated by boldface capitals, for example, $\mathbf{V}$, $\mathbf{I}$. The phase angle of the cosine function is the angle on the phasor. The phasor diagrams here and all that follow may be considered as a snapshot of the counterclockwise-rotating directed line segment taken at $t = 0$. The frequency f (Hz) and ω (rad/s) generally do not appear but they should be kept in mind, since they are implicit in any sinusoidal steady-state problem.

EXAMPLE 9.3 A series combination of $R = 10\ \Omega$ and $L = 20\text{ mH}$ has a current $i = 5.0 \cos(500t + 10^\circ)$ (A). Obtain the voltages v and $\mathbf{V}$, the phasor current $\mathbf{I}$ and sketch the phasor diagram.

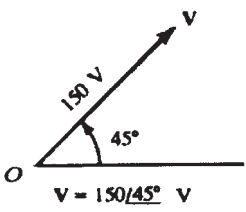
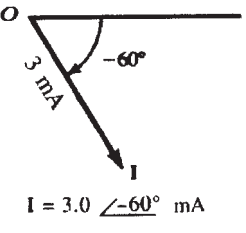
Using the methods of Example 9.1,

$$\begin{aligned} v_R &= 50.0 \cos(500t + 10^\circ) & v_L &= L \frac{di}{dt} = 50.0 \cos(500t + 100^\circ) \\ v &= v_R + v_L = 70.7 \cos(500t + 55^\circ) \quad (\text{V}) \end{aligned}$$

The corresponding phasors are

$$\mathbf{I} = 5.0 \angle 10^\circ \text{ A} \quad \text{and} \quad \mathbf{V} = 70.7 \angle 55^\circ \text{ V}$$

Table 9-2

Function	Phasor Representation
$v = 150 \cos(500t + 45^\circ) \text{ (V)}$	
$i = 3.0 \sin(2000t + 30^\circ) \text{ (mA)}$ $= 3.0 \cos(2000t - 60^\circ) \text{ (mA)}$	

The phase angle of 45° can be seen in the time-domain graphs of i and v shown in Fig. 9-6(a), and the phasor diagram with $\mathbf{I}$ and $\mathbf{V}$ shown in Fig. 9-6(b).

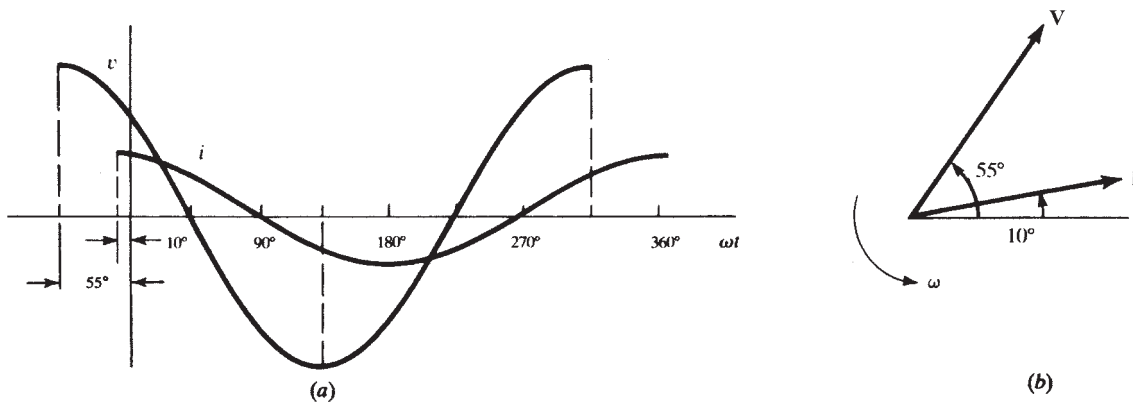


Fig. 9-6

Phasors can be treated as complex numbers. When the horizontal axis is identified as the real axis of a complex plane, the phasors become complex numbers and the usual rules apply. In view of *Euler's identity*, there are three equivalent notations for a phasor.

$$\begin{array}{ll}
 \text{polar form} & \mathbf{V} = V \angle \theta \\
 \text{rectangular form} & \mathbf{V} = V(\cos \theta + j \sin \theta) \\
 \text{exponential form} & \mathbf{V} = V e^{j\theta}
 \end{array}$$

The cosine expression may also be written as

$$v = V \cos(\omega t + \theta) = \operatorname{Re}[V e^{j(\omega t + \theta)}] = \operatorname{Re}[\mathbf{V} e^{j\omega t}]$$

The exponential form suggests how to treat the product and quotient of phasors. Since $(V_1 e^{j\theta_1})(V_2 e^{j\theta_2}) = V_1 V_2 e^{j(\theta_1 + \theta_2)}$,

$$(V_1 \angle \theta_1)(V_2 \angle \theta_2) = V_1 V_2 \angle \theta_1 + \theta_2$$

and, since $(V_1 e^{j\theta_1})/(V_2 e^{j\theta_2}) = (V_1/V_2)e^{j(\theta_1-\theta_2)}$,

$$\frac{V_1 \angle \theta_1}{V_2 \angle \theta_2} = V_1/V_2 \angle \theta_1 - \theta_2$$

The rectangular form is used in summing or subtracting phasors.

EXAMPLE 9.4 Given $\mathbf{V}_1 = 25.0 \angle 143.13^\circ$ and $\mathbf{V}_2 = 11.2 \angle 26.57^\circ$, find the ratio $\mathbf{V}_1/\mathbf{V}_2$ and the sum $\mathbf{V}_1 + \mathbf{V}_2$.

$$\mathbf{V}_1/\mathbf{V}_2 = \frac{25.0 \angle 143.13^\circ}{11.2 \angle 26.57^\circ} = 2.23 \angle 116.56^\circ = -1.00 + j1.99$$

$$\mathbf{V}_1 + \mathbf{V}_2 = (-20.0 + j15.0) + (10.0 + j5.0) = -10.0 + j20.0 = 23.36 \angle 116.57^\circ$$

9.4 IMPEDANCE AND ADMITTANCE

A sinusoidal voltage or current applied to a passive *RLC* circuit produces a sinusoidal response. With time functions, such as $v(t)$ and $i(t)$, the circuit is said to be in the *time domain*, Fig. 9-7(a); and when the circuit is analyzed using phasors, it is said to be in the *frequency domain*, Fig. 9-7(b). The voltage and current may be written, respectively,

$$v(t) = V \cos(\omega t + \theta) = \text{Re}[\mathbf{V}e^{j\omega t}] \quad \text{and} \quad \mathbf{V} = V \angle \theta$$

$$i(t) = I \cos(\omega t + \phi) = \text{Re}[\mathbf{I}e^{j\omega t}] \quad \text{and} \quad \mathbf{I} = I \angle \phi$$

The ratio of phasor voltage $\mathbf{V}$ to phasor current $\mathbf{I}$ is defined as *impedance* $\mathbf{Z}$, that is, $\mathbf{Z} = \mathbf{V}/\mathbf{I}$. The reciprocal of impedance is called *admittance* $\mathbf{Y}$, so that $\mathbf{Y} = 1/\mathbf{Z}$ (S), where $1 \text{ S} = 1 \Omega^{-1} = 1 \text{ mho}$. $\mathbf{Y}$ and $\mathbf{Z}$ are complex numbers.

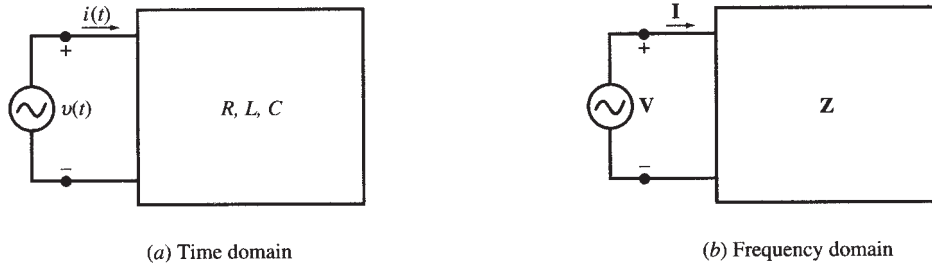


Fig. 9-7

When impedance is written in Cartesian form the real part is the resistance R and the imaginary part is the *reactance* X . The sign on the imaginary part may be positive or negative: When positive, X is called the *inductive reactance*, and when negative, X is called the *capacitive reactance*. When the admittance is written in Cartesian form, the real part is *admittance* G and the imaginary part is *susceptance* B . A positive sign on the susceptance indicates a *capacitive susceptance*, and a negative sign indicates an *inductive susceptance*. Thus,

$$\mathbf{Z} = R + jX_L \quad \text{and} \quad \mathbf{Z} = R - jX_C$$

$$\mathbf{Y} = G - jB_L \quad \text{and} \quad \mathbf{Y} = G + jB_C$$

The relationships between these terms follow from $\mathbf{Z} = 1/\mathbf{Y}$. Then,

$$R = \frac{G}{G^2 + B^2} \quad \text{and} \quad X = \frac{-B}{G^2 + B^2}$$

$$G = \frac{R}{R^2 + X^2} \quad \text{and} \quad B = \frac{-X}{R^2 + X^2}$$

These expressions are not of much use in a problem where calculations can be carried out with the numerical values as in the following example.

EXAMPLE 9.5 The phasor voltage across the terminals of a network such as that shown in Fig. 9-7(b) is $100.0 \angle 45^\circ$ V and the resulting current is $5.0 \angle 15^\circ$ A. Find the equivalent impedance and admittance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{100.0 \angle 45^\circ}{5.0 \angle 15^\circ} = 20.0 \angle 30^\circ = 17.32 + j10.0 \, \Omega$$

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}} = 0.05 \angle -30^\circ = (4.33 - j2.50) \times 10^{-2} \text{ S}$$

Thus, $R = 17.32 \, \Omega$, $X_L = 10.0 \, \Omega$, $G = 4.33 \times 10^{-2} \text{ S}$, and $B_L = 2.50 \times 10^{-2} \text{ S}$.

Combinations of Impedances

The relation $\mathbf{V} = \mathbf{IZ}$ (in the frequency domain) is formally identical to Ohm's law, $v = iR$, for a resistive network (in the time domain). Therefore, impedances combine exactly like resistances:

$$\text{impedances in series} \quad \mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots$$

$$\text{impedances in parallel} \quad \frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots$$

In particular, for two parallel impedances, $\mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)$.

Impedance Diagram

In an *impedance diagram*, an impedance $\mathbf{Z}$ is represented by a point in the right half of the complex plane. Figure 9-8 shows two impedances; $\mathbf{Z}_1$, in the first quadrant, exhibits inductive reactance, while $\mathbf{Z}_2$, in the fourth quadrant, exhibits capacitive reactance. Their series equivalent, $\mathbf{Z}_1 + \mathbf{Z}_2$, is obtained by vector addition, as shown. Note that the “vectors” are shown without arrowheads, in order to distinguish these complex numbers from phasors.

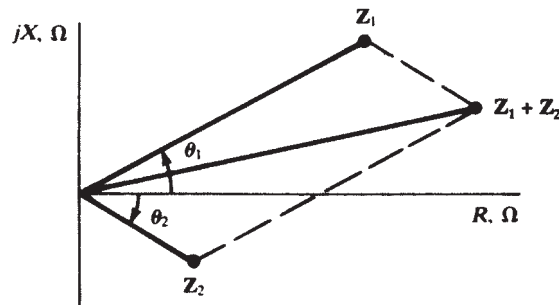


Fig. 9-8

Combinations of Admittances

Replacing $\mathbf{Z}$ by $1/\mathbf{Y}$ in the formulas above gives

$$\text{admittances in series} \quad \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1} + \frac{1}{\mathbf{Y}_2} + \cdots$$

$$\text{admittances in parallel} \quad \mathbf{Y}_{\text{eq}} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots$$

Thus, series circuits are easiest treated in terms of impedance; parallel circuits, in terms of admittance.

Admittance Diagram

Figure 9-9, an *admittance diagram*, is analogous to Fig. 9-8 for impedance. Shown are an admittance Y_1 having capacitive susceptance and an admittance Y_2 having inductive susceptance, together with their vector sum, $Y_1 + Y_2$, which is the admittance of a parallel combination of Y_1 and Y_2 .

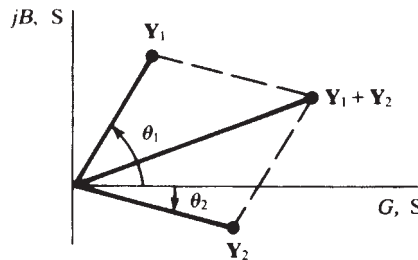


Fig. 9-9

9.5 VOLTAGE AND CURRENT DIVISION IN THE FREQUENCY DOMAIN

In view of the analogy between impedance in the frequency domain and resistance in the time domain, Sections 3.6 and 3.7 imply the following results.

(1) Impedances in series divide the total voltage in the ratio of the impedances:

$$\frac{V_r}{V_s} = \frac{Z_r}{Z_s} \quad \text{or} \quad V_r = \frac{Z_r}{Z_{eq}} V_T$$

See Fig. 9-10.

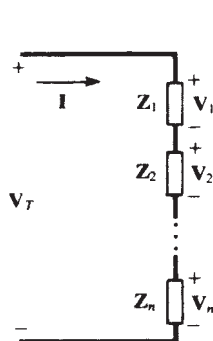


Fig. 9-10

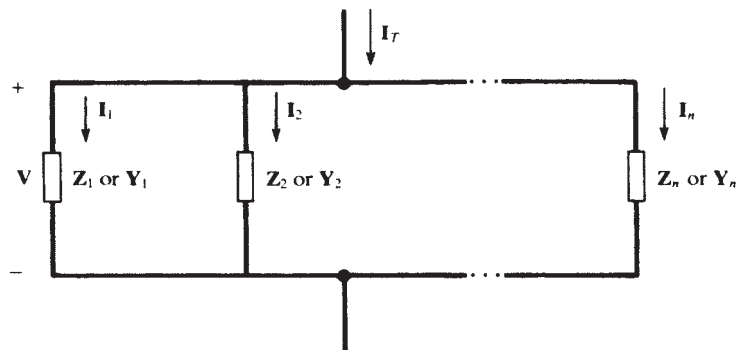


Fig. 9-11

(2) Impedances in parallel (admittances in series) divide the total current in the inverse ratio of the impedances (direct ratio of the admittances):

$$\frac{I_r}{I_s} = \frac{Z_s}{Z_r} = \frac{Y_r}{Y_s} \quad \text{or} \quad I_r = \frac{Z_{eq}}{Z_r} I_T = \frac{Y_r}{Y_{eq}} I_T$$

See Fig. 9-11.

9.6 THE MESH CURRENT METHOD

Consider the frequency-domain network of Fig. 9-12. Applying KVL, as in Section 4.3, or simply by inspection, we find the matrix equation