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Theory and Problems of ELECTRIC CIRCUITS

Fourth Edition

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PREFACE

This book is designed for use as a textbook for a first course in circuit analysis or as a supplement to standard texts and can be used by electrical engineering students as well as other engineering and technology students. Emphasis is placed on the basic laws, theorems, and problem-solving techniques which are common to most courses.

The subject matter is divided into 17 chapters covering duly-recognized areas of theory and study. The chapters begin with statements of pertinent definitions, principles, and theorems together with illustrative examples. This is followed by sets of solved and supplementary problems. The problems cover a range of levels of difficulty. Some problems focus on fine points, which helps the student to better apply the basic principles correctly and confidently. The supplementary problems are generally more numerous and give the reader an opportunity to practice problem-solving skills. Answers are provided with each supplementary problem.

The book begins with fundamental definitions, circuit elements including dependent sources, circuit laws and theorems, and analysis techniques such as node voltage and mesh current methods. These theorems and methods are initially applied to DC-resistive circuits and then extended to RLC circuits by the use of impedance and complex frequency. Chapter 5 on amplifiers and op amp circuits is new. The op amp examples and problems are selected carefully to illustrate simple but practical cases which are of interest and importance in the student's future courses. The subject of waveforms and signals is also treated in a new chapter to increase the student's awareness of commonly used signal models.

Circuit behavior such as the steady state and transient response to steps, pulses, impulses, and exponential inputs is discussed for first-order circuits in Chapter 7 and then extended to circuits of higher order in Chapter 8, where the concept of complex frequency is introduced. Phasor analysis, sinusoidal steady state, power, power factor, and polyphase circuits are thoroughly covered. Network functions, frequency response, filters, series and parallel resonance, two-port networks, mutual inductance, and transformers are covered in detail. Application of Spice and PSpice in circuit analysis is introduced in Chapter 15. Circuit equations are solved using classical differential equations and the Laplace transform, which permits a convenient comparison. Fourier series and Fourier transforms and their use in circuit analysis are covered in Chapter 17. Finally, two appendixes provide a useful summary of the complex number system, and matrices and determinants.

This book is dedicated to our students from whom we have learned to teach well. To a large degree it is they who have made possible our satisfying and rewarding teaching careers. And finally, we wish to thank our wives, *Zahra Nahvi* and *Nina Edminister* for their continuing support, and for whom all these efforts were happily made.

MAHMOOD NAHVI
JOSEPH A. EDMINSTER

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Introduction

1.1 ELECTRICAL QUANTITIES AND SI UNITS

The International System of Units (SI) will be used throughout this book. Four basic quantities and their SI units are listed in Table 1-1. The other three basic quantities and corresponding SI units, not shown in the table, are temperature in degrees kelvin (K), amount of substance in moles (mol), and luminous intensity in candelas (cd).

All other units may be derived from the seven basic units. The electrical quantities and their symbols commonly used in electrical circuit analysis are listed in Table 1-2.

Two supplementary quantities are plane angle (also called phase angle in electric circuit analysis) and solid angle. Their corresponding SI units are the radian (rad) and steradian (sr).

Degrees are almost universally used for the phase angles in sinusoidal functions, for instance, $\sin(\omega t + 30^\circ)$. Since ωt is in radians, this is a case of mixed units.

The decimal multiples or submultiples of SI units should be used whenever possible. The symbols given in Table 1-3 are prefixed to the unit symbols of Tables 1-1 and 1-2. For example, mV is used for millivolt, 10^{-3} V, and MW for megawatt, 10^6 W.

1.2 FORCE, WORK, AND POWER

The derived units follow the mathematical expressions which relate the quantities. From “force equals mass times acceleration,” the *newton* (N) is defined as the unbalanced force that imparts an acceleration of 1 meter per second squared to a 1-kilogram mass. Thus, $1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2$.

Work results when a force acts over a distance. A *joule* of work is equivalent to a *newton-meter*: $1 \text{ J} = 1 \text{ N} \cdot \text{m}$. Work and energy have the same units.

Power is the rate at which work is done or the rate at which energy is changed from one form to another. The unit of power, the *watt* (W), is one joule per second (J/s).

Table 1-1

Quantity	Symbol	SI Unit	Abbreviation
length	L, l	meter	m
mass	M, m	kilogram	kg
time	T, t	second	s
current	I, i	ampere	A

Table 1-2

Quantity	Symbol	SI Unit	Abbreviation
electric charge	Q, q	coulomb	C
electric potential	V, v	volt	V
resistance	R	ohm	Ω
conductance	G	siemens	S
inductance	L	henry	H
capacitance	C	farad	F
frequency	f	hertz	Hz
force	F, f	newton	N
energy, work	W, w	joule	J
power	P, p	watt	W
magnetic flux	ϕ	weber	Wb
magnetic flux density	$\mathbf{B}$	tesla	T

EXAMPLE 1.1. In simple rectilinear motion a 10-kg mass is given a constant acceleration of 2.0 m/s^2 . (a) Find the acting force F . (b) If the body was at rest at $t = 0$, $x = 0$, find the position, kinetic energy, and power for $t = 4 \text{ s}$.

(a) $F = ma = (10 \text{ kg})(2.0 \text{ m/s}^2) = 20.0 \text{ kg} \cdot \text{m/s}^2 = 20.0 \text{ N}$

(b) At $t = 4 \text{ s}$, $x = \frac{1}{2}at^2 = \frac{1}{2}(2.0 \text{ m/s}^2)(4 \text{ s})^2 = 16.0 \text{ m}$

$$\text{KE} = Fx = (20.0 \text{ N})(16.0 \text{ m}) = 3200 \text{ N} \cdot \text{m} = 3.2 \text{ kJ}$$

$$P = \text{KE}/t = 3.2 \text{ kJ}/4 \text{ s} = 0.8 \text{ kJ/s} = 0.8 \text{ kW}$$

1.3 ELECTRIC CHARGE AND CURRENT

The unit of current, the *ampere* (A), is defined as the constant current in two parallel conductors of infinite length and negligible cross section, 1 meter apart in vacuum, which produces a force between the conductors of 2.0×10^{-7} newtons per meter length. A more useful concept, however, is that current results from charges in motion, and 1 ampere is equivalent to 1 coulomb of charge moving across a fixed surface in 1 second. Thus, in time-variable functions, $i(\text{A}) = dq/dt(\text{C/s})$. The derived unit of charge, the *coulomb* (C), is equivalent to an ampere-second.

The moving charges may be positive or negative. Positive ions, moving to the left in a liquid or plasma suggested in Fig. 1-1(a), produce a current i , also directed to the left. If these ions cross the plane surface S at the rate of *one coulomb per second*, then the resulting current is 1 ampere. Negative ions moving to the right as shown in Fig. 1-1(b) also produce a current directed to the left.

Table 1-3

Prefix	Factor	Symbol
pico	10^{-12}	p
nano	10^{-9}	n
micro	10^{-6}	μ
milli	10^{-3}	m
centi	10^{-2}	c
deci	10^{-1}	d
kilo	10^3	k
mega	10^6	M
giga	10^9	G
tera	10^{12}	T

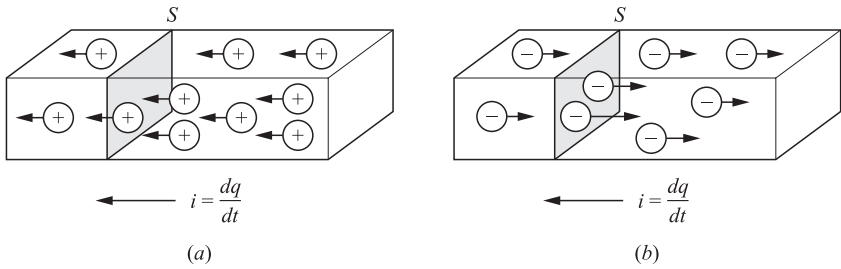


Fig. 1-1

Of more importance in electric circuit analysis is the current in metallic conductors which takes place through the motion of electrons that occupy the outermost shell of the atomic structure. In copper, for example, one electron in the outermost shell is only loosely bound to the central nucleus and moves freely from one atom to the next in the crystal structure. At normal temperatures there is constant, random motion of these electrons. A reasonably accurate picture of conduction in a copper conductor is that approximately 8.5×10^{28} *conduction* electrons per cubic meter are free to move. The electron charge is $-e = -1.602 \times 10^{-19}$ C, so that for a current of one ampere approximately 6.24×10^{18} electrons per second would have to pass a fixed cross section of the conductor.

EXAMPLE 1.2. A conductor has a constant current of five amperes. How many electrons pass a fixed point on the conductor in one minute?

$$5 \text{ A} = (5 \text{ C/s})(60 \text{ s/min}) = 300 \text{ C/min}$$

$$\frac{300 \text{ C/min}}{1.602 \times 10^{-19} \text{ C/electron}} = 1.87 \times 10^{21} \text{ electrons/min}$$

1.4 ELECTRIC POTENTIAL

An electric charge experiences a force in an electric field which, if unopposed, will accelerate the particle containing the charge. Of interest here is the work done to move the charge against the field as suggested in Fig. 1-2(a). Thus, if *1 joule* of work is required to move the charge *Q*, *1 coulomb* from position 0 to position 1, then position 1 is at a potential of *1 volt* with respect to position 0; $1 \text{ V} = 1 \text{ J/C}$. This electric potential is capable of doing work just as the mass in Fig. 1-2(b), which was raised against the gravitational force *g* to a height *h* above the ground plane. The potential energy *mgh* represents an ability to do work when the mass *m* is released. As the mass falls, it accelerates and this potential energy is converted to kinetic energy.

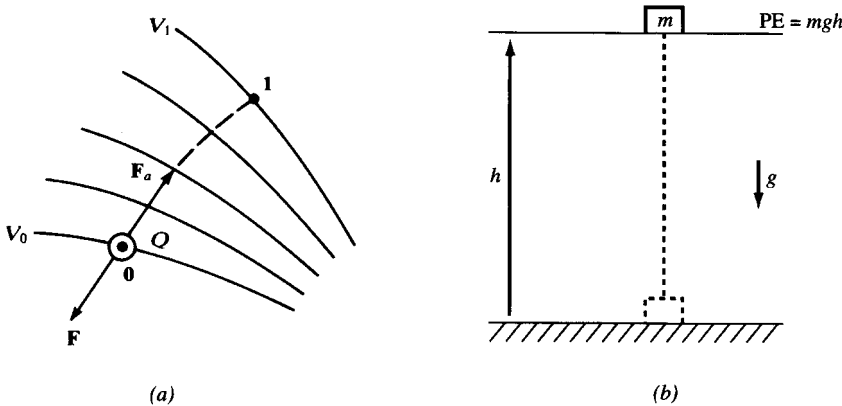


Fig. 1-2

EXAMPLE 1.3. In an electric circuit an energy of $9.25 \mu\text{J}$ is required to transport $0.5 \mu\text{C}$ from point a to point b . What electric potential difference exists between the two points?

$$1 \text{ volt} = 1 \text{ joule per coulomb} \quad V = \frac{9.25 \times 10^{-6} \text{ J}}{0.5 \times 10^{-6} \text{ C}} = 18.5 \text{ V}$$

1.5 ENERGY AND ELECTRICAL POWER

Electric energy in joules will be encountered in later chapters dealing with capacitance and inductance whose respective electric and magnetic fields are capable of storing energy. The rate, in *joules per second*, at which energy is transferred is electric power in *watts*. Furthermore, the product of voltage and current yields the electric power, $p = vi$; $1 \text{ W} = 1 \text{ V} \cdot 1 \text{ A}$. Also, $\text{V} \cdot \text{A} = (\text{J/C}) \cdot (\text{C/s}) = \text{J/s} = \text{W}$. In a more fundamental sense power is the time derivative $p = dw/dt$, so that instantaneous power p is generally a function of time. In the following chapters time average power P_{avg} and a root-mean-square (RMS) value for the case where voltage and current are sinusoidal will be developed.

EXAMPLE 1.4. A resistor has a potential difference of 50.0 V across its terminals and 120.0 C of charge per minute passes a fixed point. Under these conditions at what rate is electric energy converted to heat?

$$(120.0 \text{ C/min})/(60 \text{ s/min}) = 2.0 \text{ A} \quad P = (2.0 \text{ A})(50.0 \text{ V}) = 100.0 \text{ W}$$

Since $1 \text{ W} = 1 \text{ J/s}$, the rate of energy conversion is one hundred joules per second.

1.6 CONSTANT AND VARIABLE FUNCTIONS

To distinguish between constant and time-varying quantities, capital letters are employed for the constant quantity and lowercase for the variable quantity. For example, a constant current of 10 amperes is written $I = 10.0 \text{ A}$, while a 10-ampere time-variable current is written $i = 10.0 f(t) \text{ A}$. Examples of common functions in circuit analysis are the sinusoidal function $i = 10.0 \sin \omega t \text{ (A)}$ and the exponential function $v = 15.0 e^{-at} \text{ (V)}$.

Solved Problems

- 1.1** The force applied to an object moving in the x direction varies according to $F = 12/x^2 \text{ (N)}$.
 (a) Find the work done in the interval $1 \text{ m} \leq x \leq 3 \text{ m}$. (b) What constant force acting over the same interval would result in the same work?

$$(a) \quad dW = F dx \quad \text{so} \quad W = \int_1^3 \frac{12}{x^2} dx = 12 \left[\frac{-1}{x} \right]_1^3 = 8 \text{ J}$$

$$(b) \quad 8 \text{ J} = F_c(2 \text{ m}) \quad \text{or} \quad F_c = 4 \text{ N}$$

- 1.2** Electrical energy is converted to heat at the rate of 7.56 kJ/min in a resistor which has 270 C/min passing through. What is the voltage difference across the resistor terminals?

From $P = VI$,

$$V = \frac{P}{I} = \frac{7.56 \times 10^3 \text{ J/min}}{270 \text{ C/min}} = 28 \text{ J/C} = 28 \text{ V}$$

- 1.3** A certain circuit element has a current $i = 2.5 \sin \omega t$ (mA), where ω is the angular frequency in rad/s, and a voltage difference $v = 45 \sin \omega t$ (V) between terminals. Find the average power P_{avg} and the energy W_T transferred in one period of the sine function.

Energy is the time-integral of instantaneous power:

$$W_T = \int_0^{2\pi/\omega} vi \, dt = 112.5 \int_0^{2\pi/\omega} \sin^2 \omega t \, dt = \frac{112.5\pi}{\omega} \text{ (mJ)}$$

The average power is then

$$P_{\text{avg}} = \frac{W_T}{2\pi/\omega} = 56.25 \text{ mW}$$

Note that P_{avg} is independent of ω .

- 1.4** The unit of energy commonly used by electric utility companies is the kilowatt-hour (kWh).
(a) How many joules are in 1 kWh? (b) A color television set rated at 75 W is operated from 7:00 p.m. to 11:30 p.m. What total energy does this represent in kilowatt-hours and in megajoules?

- (a) $1 \text{ kWh} = (1000 \text{ J/s})(3600 \text{ s/h}) = 3.6 \text{ MJ}$
(b) $(75.0 \text{ W})(4.5 \text{ h}) = 337.5 \text{ Wh} = 0.3375 \text{ kWh}$
 $(0.3375 \text{ kWh})(3.6 \text{ MJ/kWh}) = 1.215 \text{ MJ}$

- 1.5** An AWG #12 copper wire, a size in common use in residential wiring, contains approximately 2.77×10^{23} free electrons per meter length, assuming one free conduction electron per atom. What percentage of these electrons will pass a fixed cross section if the conductor carries a constant current of 25.0 A?

$$\begin{aligned} \frac{25.0 \text{ C/s}}{1.602 \times 10^{-19} \text{ C/electron}} &= 1.56 \times 10^{20} \text{ electron/s} \\ (1.56 \times 10^{20} \text{ electrons/s})(60 \text{ s/min}) &= 9.36 \times 10^{21} \text{ electrons/min} \\ \frac{9.36 \times 10^{21}}{2.77 \times 10^{23}}(100) &= 3.38\% \end{aligned}$$

- 1.6** How many electrons pass a fixed point in a 100-watt light bulb in 1 hour if the applied constant voltage is 120 V?

$$\begin{aligned} 100 \text{ W} &= (120 \text{ V}) \times I(\text{A}) \quad I = 5/6 \text{ A} \\ \frac{(5/6 \text{ C/s})(3600 \text{ s/h})}{1.602 \times 10^{-19} \text{ C/electron}} &= 1.87 \times 10^{22} \text{ electrons per hour} \end{aligned}$$

- 1.7** A typical 12 V auto battery is rated according to *ampere-hours*. A 70-A · h battery, for example, at a discharge rate of 3.5 A has a life of 20 h. (a) Assuming the voltage remains constant, obtain the energy and power delivered in a complete discharge of the preceding battery. (b) Repeat for a discharge rate of 7.0 A.

- (a) $(3.5 \text{ A})(12 \text{ V}) = 42.0 \text{ W}$ (or J/s)
 $(42.0 \text{ J/s})(3600 \text{ s/h})(20 \text{ h}) = 3.02 \text{ MJ}$
(b) $(7.0 \text{ A})(12 \text{ V}) = 84.0 \text{ W}$
 $(84.0 \text{ J/s})(3600 \text{ s/h})(10 \text{ h}) = 3.02 \text{ MJ}$

The ampere-hour rating is a measure of the energy the battery stores; consequently, the energy transferred for total discharge is the same whether it is transferred in 10 hours or 20 hours. Since power is the rate of energy transfer, the power for a 10-hour discharge is twice that in a 20-hour discharge.

Supplementary Problems

- 1.8** Obtain the work and power associated with a force of 7.5×10^{-4} N acting over a distance of 2 meters in an elapsed time of 14 seconds. *Ans.* 1.5 mJ, 0.107 mW
- 1.9** Obtain the work and power required to move a 5.0-kg mass up a frictionless plane inclined at an angle of 30° with the horizontal for a distance of 2.0 m along the plane in a time of 3.5 s. *Ans.* 49.0 J, 14.0 W
- 1.10** Work equal to 136.0 joules is expended in moving 8.5×10^{18} electrons between two points in an electric circuit. What potential difference does this establish between the two points? *Ans.* 100 V
- 1.11** A pulse of electricity measures 305 V, 0.15 A, and lasts 500 μ s. What power and energy does this represent? *Ans.* 45.75 W, 22.9 mJ
- 1.12** A unit of power used for electric motors is the *horsepower* (hp), equal to 746 watts. How much energy does a 5-hp motor deliver in 2 hours? Express the answer in MJ. *Ans.* 26.9 MJ
- 1.13** For $t \geq 0$, $q = (4.0 \times 10^{-4})(1 - e^{-250t})$ (C). Obtain the current at $t = 3$ ms. *Ans.* 47.2 mA
- 1.14** A certain circuit element has the current and voltage
- $$i = 10e^{-5000t} \text{ (A)} \quad v = 50(1 - e^{-5000t}) \text{ (V)}$$
- Find the total energy transferred during $t \geq 0$. *Ans.* 50 mJ
- 1.15** The *capacitance* of a circuit element is defined as Q/V , where Q is the magnitude of charge stored in the element and V is the magnitude of the voltage difference across the element. The SI derived unit of capacitance is the *farad* (F). Express the farad in terms of the basic units.
Ans. $1 \text{ F} = 1 \text{ A}^2 \cdot \text{s}^4 / \text{kg} \cdot \text{m}^2$

Sinusoidal Steady-State Circuit Analysis

9.1 INTRODUCTION

This chapter will concentrate on the steady-state response of circuits driven by sinusoidal sources. The response will also be sinusoidal. For a linear circuit, the assumption of a *sinusoidal* source represents no real restriction, since a source that can be described by a periodic function can be replaced by an equivalent combination (Fourier series) of sinusoids. This matter will be treated in Chapter 17.

9.2 ELEMENT RESPONSES

The voltage-current relationships for the single elements R , L , and C were examined in Chapter 2 and summarized in Table 2-1. In this chapter, the functions of v and i will be sines or cosines with the argument ωt . ω is the angular frequency and has the unit rad/s. Also, $\omega = 2\pi f$, where f is the frequency with unit cycle/s, or more commonly hertz (Hz).

Consider an inductance L with $i = I \cos(\omega t + 45^\circ)$ A [see Fig. 9-1(a)]. The voltage is

$$v_L = L \frac{di}{dt} = \omega LI [-\sin(\omega t + 45^\circ)] = \omega LI \cos(\omega t + 135^\circ) \quad (\text{V})$$

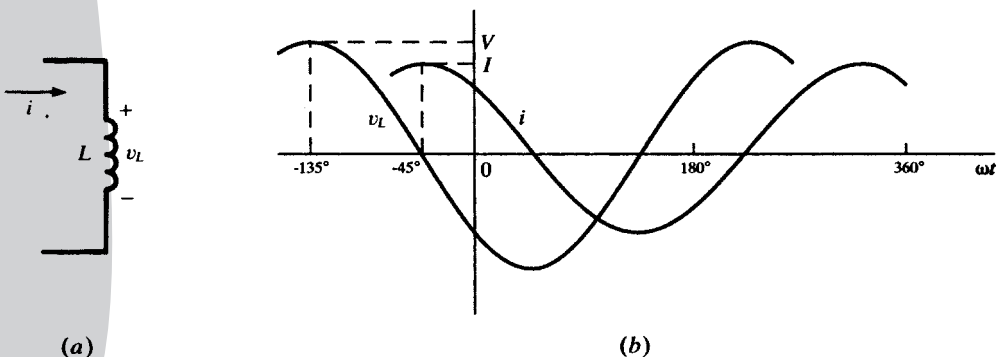


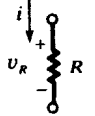
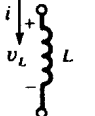
Fig. 9-1

A comparison of v_L and i shows that the current *lags* the voltage by 90° or $\pi/2$ rad. The functions are sketched in Fig. 9-1(b). Note that the current function i is to the right of v , and since the horizontal scale is ωt , events displaced to the right occur later in time. This illustrates that i *lags* v . The horizontal scale is in radians, but note that it is also marked in degrees (-135° , 180° , etc.). This is a case of mixed units just as with $\omega t + 45^\circ$. It is not mathematically correct but is the accepted practice in circuit analysis. The vertical scale indicates two different quantities, that is, v and i , so there should be two scales rather than one.

While examining this sketch, it is a good time to point out that a sinusoid is completely defined when its magnitude (V or I), frequency (ω or f), and phase (45° or 135°) are specified.

In Table 9-1 the responses of the three basic circuit elements are shown for applied current $i = I \cos \omega t$ and voltage $v = V \cos \omega t$. If sketches are made of these responses, they will show that for a resistance R , v and i are *in phase*. For an inductance L , i *lags* v by 90° or $\pi/2$ rad. And for a capacitance C , i *leads* v by 90° or $\pi/2$ rad.

Table 9-1

	$i = I \cos \omega t$	$v = V \cos \omega t$
	$v_r = RI \cos \omega t$	$i_R = \frac{V}{R} \cos \omega t$
	$v_L = \omega LI \cos(\omega t + 90^\circ)$	$i_L = \frac{V}{\omega L} \cos(\omega t - 90^\circ)$
	$v_C = \frac{I}{\omega C} \cos(\omega t - 90^\circ)$	$i_C = \omega CV \cos(\omega t + 90^\circ)$

EXAMPLE 9.1 The RL series circuit shown in Fig. 9-2 has a current $i = I \sin \omega t$. Obtain the voltage v across the two circuit elements and sketch v and i .

$$\begin{aligned} v_R &= RI \sin \omega t & v_L &= L \frac{di}{dt} = \omega LI \sin(\omega t + 90^\circ) \\ v &= v_R + v_L = RI \sin \omega t + \omega LI \sin(\omega t + 90^\circ) \end{aligned}$$

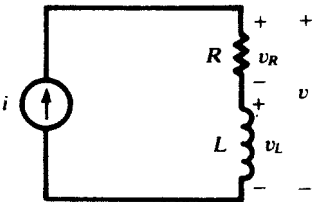


Fig. 9-2

Since the current is a sine function and

$$v = V \sin(\omega t + \theta) = V \sin \omega t \cos \theta + V \cos \omega t \sin \theta \tag{1}$$

we have from the above

$$v = RI \sin \omega t + \omega LI \sin \omega t \cos 90^\circ + \omega LI \cos \omega t \sin 90^\circ \tag{2}$$

Equating coefficients of like terms in (1) and (2),

$$V \sin \theta = \omega L I \quad \text{and} \quad V \cos \theta = RI$$

Then

$$v = I \sqrt{R^2 + (\omega L)^2} \sin[\omega t + \arctan(\omega L/R)]$$

and

$$V = I \sqrt{R^2 + (\omega L)^2} \quad \text{and} \quad \theta = \tan^{-1} \frac{\omega L}{R}$$

The functions i and v are sketched in Fig. 9-3. The phase angle θ , the angle by which i lags v , lies within the range $0^\circ \leq \theta \leq 90^\circ$, with the limiting values attained for $\omega L \ll R$ and $\omega L \gg R$, respectively. If the circuit had an applied voltage $v = V \sin \omega t$, the resulting current would be

$$i = \frac{V}{\sqrt{R^2 + (\omega L)^2}} \sin(\omega t - \theta)$$

where, as before, $\theta = \tan^{-1}(\omega L/R)$.

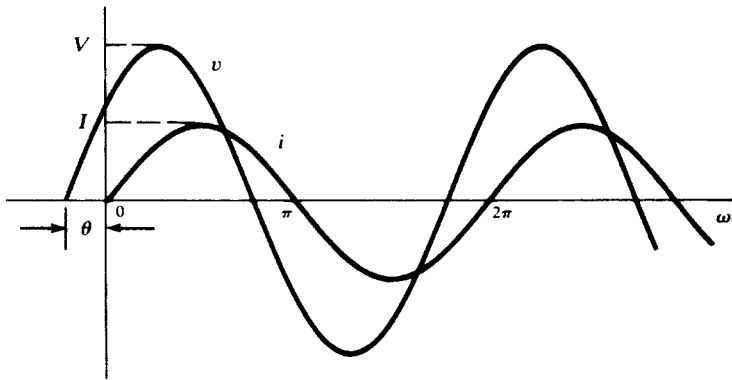


Fig. 9-3

EXAMPLE 9.2 If the current driving a series RC circuit is given by $i = I \sin \omega t$, obtain the total voltage across the two elements.

$$v_R = RI \sin \omega t \quad v_C = (1/\omega C) \sin(\omega t - 90^\circ)$$

$$v = v_R + v_C = V \sin(\omega t - \theta)$$

where

$$V = I \sqrt{R^2 + (1/\omega C)^2} \quad \text{and} \quad \theta = \tan^{-1}(1/\omega CR)$$

The negative phase angle shifts v to the right of the current i . Consequently i leads v for a series RC circuit. The phase angle is constrained to the range $0^\circ \leq \theta \leq 90^\circ$. For $(1/\omega C) \ll R$, the angle $\theta = 0^\circ$, and for $(1/\omega C) \gg R$, the angle $\theta = 90^\circ$. See Fig. 9-4.

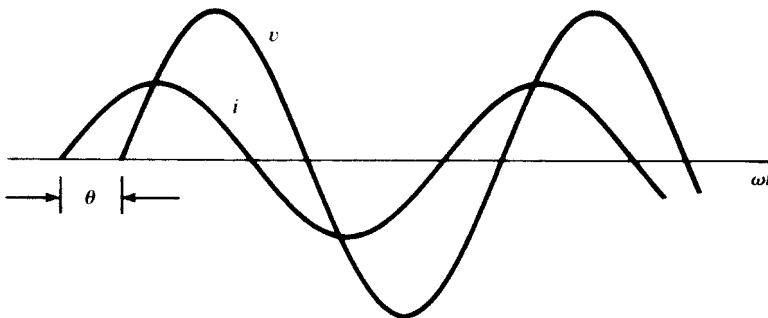


Fig. 9-4

9.3 PHASORS

A brief look at the voltage and current sinusoids in the preceding examples shows that the amplitudes and phase differences are the two principal concerns. A directed line segment, or *phasor*, such as that shown rotating in a counterclockwise direction at a constant angular velocity ω (rad/s) in Fig. 9-5, has a projection on the horizontal which is a cosine function. The length of the phasor or its magnitude is the amplitude or maximum value of the cosine function. The angle between two positions of the phasor is the phase difference between the corresponding points on the cosine function.

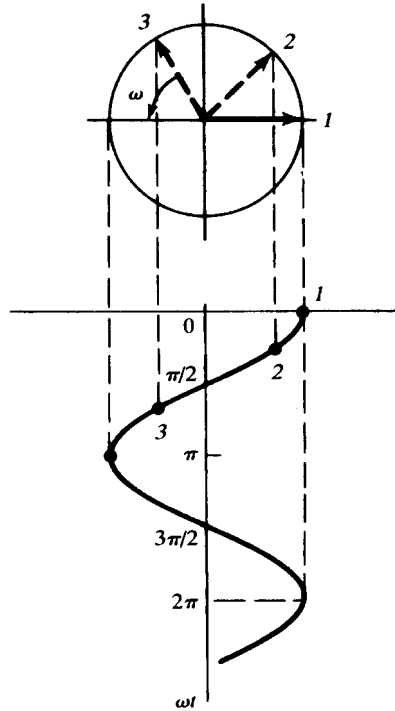


Fig. 9-5

Throughout this book *phasors will be defined from the cosine function*. If a voltage or current is expressed as a sine, it will be changed to a cosine by subtracting 90° from the phase.

Consider the examples shown in Table 9-2. Observe that the phasors, which are directed line segments and vectorial in nature, are indicated by boldface capitals, for example, **V**, **I**. The phase angle of the cosine function is the angle on the phasor. The phasor diagrams here and all that follow may be considered as a snapshot of the counterclockwise-rotating directed line segment taken at $t = 0$. The frequency f (Hz) and ω (rad/s) generally do not appear but they should be kept in mind, since they are implicit in any sinusoidal steady-state problem.

EXAMPLE 9.3 A series combination of $R = 10\ \Omega$ and $L = 20\text{ mH}$ has a current $i = 5.0 \cos(500t + 10^\circ)$ (A). Obtain the voltages v and **V**, the phasor current **I** and sketch the phasor diagram.

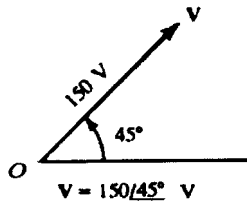
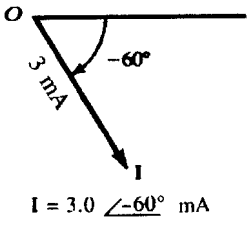
Using the methods of Example 9.1,

$$\begin{aligned} v_R &= 50.0 \cos(500t + 10^\circ) & v_L &= L \frac{di}{dt} = 50.0 \cos(500t + 100^\circ) \\ v &= v_R + v_L = 70.7 \cos(500t + 55^\circ) \quad (\text{V}) \end{aligned}$$

The corresponding phasors are

$$\mathbf{I} = 5.0 \angle 10^\circ \text{ A} \quad \text{and} \quad \mathbf{V} = 70.7 \angle 55^\circ \text{ V}$$

Table 9-2

Function	Phasor Representation
$v = 150 \cos(500t + 45^\circ) \text{ (V)}$	
$i = 3.0 \sin(2000t + 30^\circ) \text{ (mA)}$ $= 3.0 \cos(2000t - 60^\circ) \text{ (mA)}$	

The phase angle of 45° can be seen in the time-domain graphs of i and v shown in Fig. 9-6(a), and the phasor diagram with $\mathbf{I}$ and $\mathbf{V}$ shown in Fig. 9-6(b).

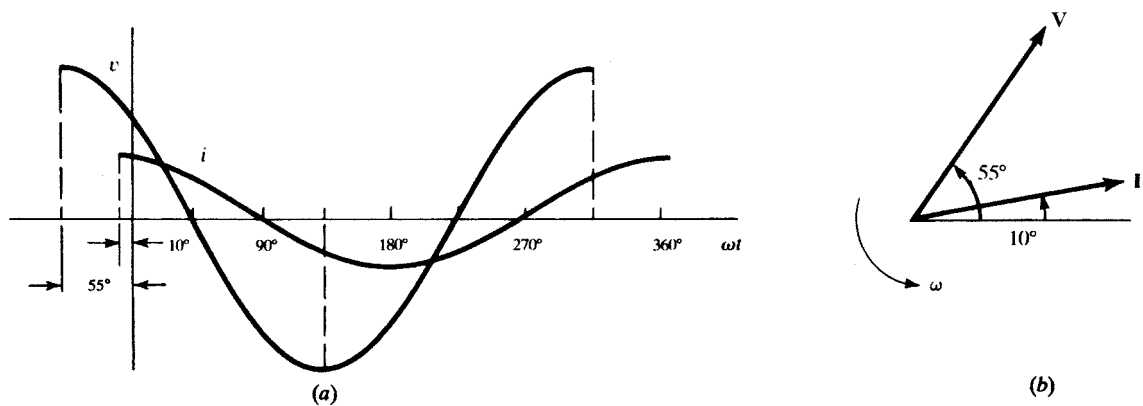


Fig. 9-6

Phasors can be treated as complex numbers. When the horizontal axis is identified as the real axis of a complex plane, the phasors become complex numbers and the usual rules apply. In view of *Euler’s identity*, there are three equivalent notations for a phasor.

polar form

rectangular form

exponential form

$$\mathbf{V} = V \angle \theta$$
$$\mathbf{V} = V(\cos \theta + j \sin \theta)$$
$$\mathbf{V} = V e^{j\theta}$$

The cosine expression may also be written as

$$v = V \cos(\omega t + \theta) = \text{Re}[V e^{j(\omega t + \theta)}] = \text{Re}[\mathbf{V} e^{j\omega t}]$$

The exponential form suggests how to treat the product and quotient of phasors. Since $(V_1 e^{j\theta_1})(V_2 e^{j\theta_2}) = V_1 V_2 e^{j(\theta_1 + \theta_2)}$,

$$(V_1 \angle \theta_1)(V_2 \angle \theta_2) = V_1 V_2 \angle \theta_1 + \theta_2$$

and, since $(V_1 e^{j\theta_1}) / (V_2 e^{j\theta_2}) = (V_1 / V_2) e^{j(\theta_1 - \theta_2)}$,

$$\frac{V_1 \angle \theta_1}{V_2 \angle \theta} = V_1 / V_2 \angle \theta_1 - \theta_2$$

The rectangular form is used in summing or subtracting phasors.

EXAMPLE 9.4 Given $\mathbf{V}_1 = 25.0 \angle 143.13^\circ$ and $\mathbf{V}_2 = 11.2 \angle 26.57^\circ$, find the ratio $\mathbf{V}_1 / \mathbf{V}_2$ and the sum $\mathbf{V}_1 + \mathbf{V}_2$.

$$\mathbf{V}_1 / \mathbf{V}_2 = \frac{25.0 \angle 143.13^\circ}{11.2 \angle 26.57^\circ} = 2.23 \angle 116.56^\circ = -1.00 + j1.99$$

$$\mathbf{V}_1 + \mathbf{V}_2 = (-20.0 + j15.0) + (10.0 + j5.0) = -10.0 + j20.0 = 23.36 \angle 116.57^\circ$$

9.4 IMPEDANCE AND ADMITTANCE

A sinusoidal voltage or current applied to a passive *RLC* circuit produces a sinusoidal response. With time functions, such as $v(t)$ and $i(t)$, the circuit is said to be in the *time domain*, Fig. 9-7(a); and when the circuit is analyzed using phasors, it is said to be in the *frequency domain*, Fig. 9-7(b). The voltage and current may be written, respectively,

$$v(t) = V \cos(\omega t + \theta) = \text{Re}[\mathbf{V} e^{j\omega t}] \quad \text{and} \quad \mathbf{V} = V \angle \theta$$

$$i(t) = I \cos(\omega t + \phi) = \text{Re}[\mathbf{I} e^{j\omega t}] \quad \text{and} \quad \mathbf{I} = I \angle \phi$$

The ratio of phasor voltage $\mathbf{V}$ to phasor current $\mathbf{I}$ is defined as *impedance* $\mathbf{Z}$, that is, $\mathbf{Z} = \mathbf{V} / \mathbf{I}$. The reciprocal of impedance is called *admittance* $\mathbf{Y}$, so that $\mathbf{Y} = 1 / \mathbf{Z}$ (S), where $1 \text{ S} = 1 \Omega^{-1} = 1 \text{ mho}$. $\mathbf{Y}$ and $\mathbf{Z}$ are complex numbers.

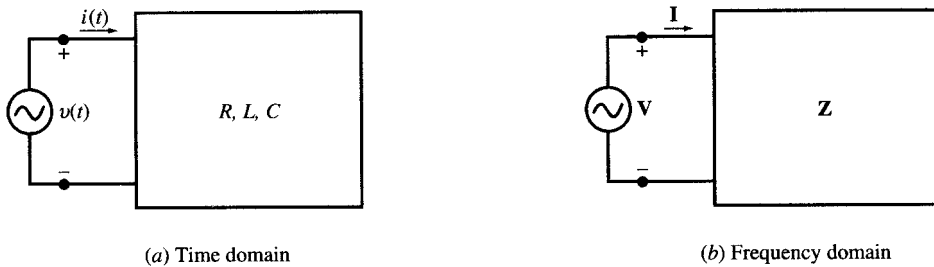


Fig. 9-7

When impedance is written in Cartesian form the real part is the resistance R and the imaginary part is the *reactance* X . The sign on the imaginary part may be positive or negative: When positive, X is called the *inductive reactance*, and when negative, X is called the *capacitive reactance*. When the admittance is written in Cartesian form, the real part is *admittance* G and the imaginary part is *susceptance* B . A positive sign on the susceptance indicates a *capacitive susceptance*, and a negative sign indicates an *inductive susceptance*. Thus,

$$\mathbf{Z} = R + jX_L \quad \text{and} \quad \mathbf{Z} = R - jX_C$$

$$\mathbf{Y} = G - jB_L \quad \text{and} \quad \mathbf{Y} = G + jB_C$$

The relationships between these terms follow from $\mathbf{Z} = 1 / \mathbf{Y}$. Then,

$$R = \frac{G}{G^2 + B^2} \quad \text{and} \quad X = \frac{-B}{G^2 + B^2}$$

$$G = \frac{R}{R^2 + X^2} \quad \text{and} \quad B = \frac{-X}{R^2 + X^2}$$

These expressions are not of much use in a problem where calculations can be carried out with the numerical values as in the following example.

EXAMPLE 9.5 The phasor voltage across the terminals of a network such as that shown in Fig. 9-7(b) is $100.0 \angle 45^\circ$ V and the resulting current is $5.0 \angle 15^\circ$ A. Find the equivalent impedance and admittance.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{100.0 \angle 45^\circ}{5.0 \angle 15^\circ} = 20.0 \angle 30^\circ = 17.32 + j10.0 \Omega$$

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = \frac{1}{\mathbf{Z}} = 0.05 \angle -30^\circ = (4.33 - j2.50) \times 10^{-2} \text{ S}$$

Thus, $R = 17.32 \Omega$, $X_L = 10.0 \Omega$, $G = 4.33 \times 10^{-2} \text{ S}$, and $B_L = 2.50 \times 10^{-2} \text{ S}$.

Combinations of Impedances

The relation $\mathbf{V} = \mathbf{IZ}$ (in the frequency domain) is formally identical to Ohm's law, $v = iR$, for a resistive network (in the time domain). Therefore, impedances combine exactly like resistances:

$$\text{impedances in series} \quad \mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 + \mathbf{Z}_2 + \cdots$$

$$\text{impedances in parallel} \quad \frac{1}{\mathbf{Z}_{\text{eq}}} = \frac{1}{\mathbf{Z}_1} + \frac{1}{\mathbf{Z}_2} + \cdots$$

In particular, for two parallel impedances, $\mathbf{Z}_{\text{eq}} = \mathbf{Z}_1 \mathbf{Z}_2 / (\mathbf{Z}_1 + \mathbf{Z}_2)$.

Impedance Diagram

In an *impedance diagram*, an impedance $\mathbf{Z}$ is represented by a point in the right half of the complex plane. Figure 9-8 shows two impedances; $\mathbf{Z}_1$, in the first quadrant, exhibits inductive reactance, while $\mathbf{Z}_2$, in the fourth quadrant, exhibits capacitive reactance. Their series equivalent, $\mathbf{Z}_1 + \mathbf{Z}_2$, is obtained by vector addition, as shown. Note that the "vectors" are shown without arrowheads, in order to distinguish these complex numbers from phasors.

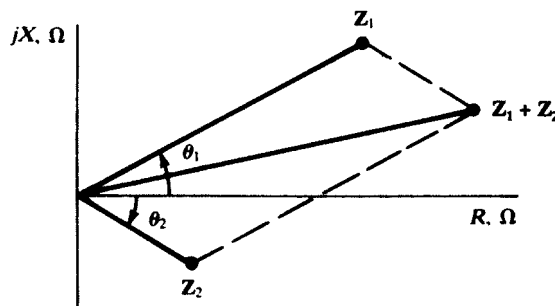


Fig. 9-8

Combinations of Admittances

Replacing $\mathbf{Z}$ by $1/\mathbf{Y}$ in the formulas above gives

$$\text{admittances in series} \quad \frac{1}{\mathbf{Y}_{\text{eq}}} = \frac{1}{\mathbf{Y}_1} + \frac{1}{\mathbf{Y}_2} + \cdots$$

$$\text{admittances in parallel} \quad \mathbf{Y}_{\text{eq}} = \mathbf{Y}_1 + \mathbf{Y}_2 + \cdots$$

Thus, series circuits are easiest treated in terms of impedance; parallel circuits, in terms of admittance.

Admittance Diagram

Figure 9-9, an *admittance diagram*, is analogous to Fig. 9-8 for impedance. Shown are an admittance $\mathbf{Y}_1$ having capacitive susceptance and an admittance $\mathbf{Y}_2$ having inductive susceptance, together with their vector sum, $\mathbf{Y}_1 + \mathbf{Y}_2$, which is the admittance of a parallel combination of $\mathbf{Y}_1$ and $\mathbf{Y}_2$.

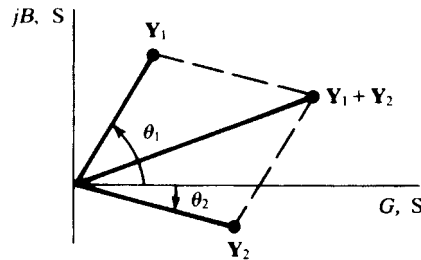


Fig. 9-9

9.5 VOLTAGE AND CURRENT DIVISION IN THE FREQUENCY DOMAIN

In view of the analogy between impedance in the frequency domain and resistance in the time domain, Sections 3.6 and 3.7 imply the following results.

(1) Impedances in series divide the total voltage in the ratio of the impedances:

$$\frac{\mathbf{V}_r}{\mathbf{V}_s} = \frac{\mathbf{Z}_r}{\mathbf{Z}_s} \quad \text{or} \quad \mathbf{V}_r = \frac{\mathbf{Z}_r}{\mathbf{Z}_{\text{eq}}} \mathbf{V}_T$$

See Fig. 9-10.

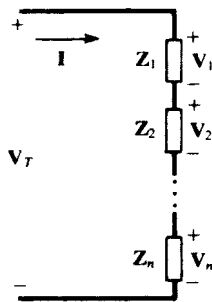


Fig. 9-10

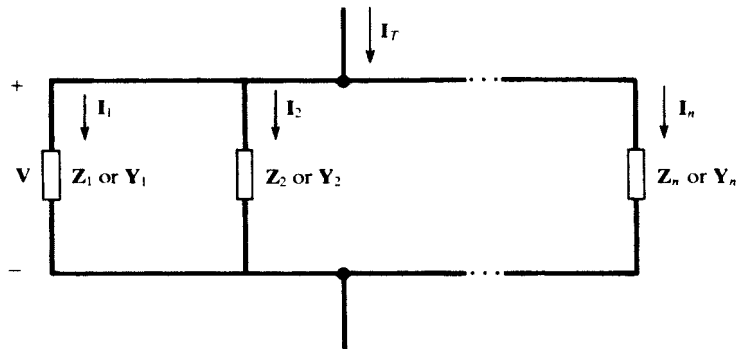


Fig. 9-11

(2) Impedances in parallel (admittances in series) divide the total current in the inverse ratio of the impedances (direct ratio of the admittances):

$$\frac{\mathbf{I}_r}{\mathbf{I}_s} = \frac{\mathbf{Z}_s}{\mathbf{Z}_r} = \frac{\mathbf{Y}_r}{\mathbf{Y}_s} \quad \text{or} \quad \mathbf{I}_r = \frac{\mathbf{Z}_{\text{eq}}}{\mathbf{Z}_r} \mathbf{I}_T = \frac{\mathbf{Y}_r}{\mathbf{Y}_{\text{eq}}} \mathbf{I}_T$$

See Fig. 9-11.

9.6 THE MESH CURRENT METHOD

Consider the frequency-domain network of Fig. 9-12. Applying KVL, as in Section 4.3, or simply by inspection, we find the matrix equation

$$\begin{bmatrix} \mathbf{Z}_{11} & \mathbf{Z}_{12} & \mathbf{Z}_{13} \\ \mathbf{Z}_{21} & \mathbf{Z}_{22} & \mathbf{Z}_{23} \\ \mathbf{Z}_{31} & \mathbf{Z}_{32} & \mathbf{Z}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix}$$

for the unknown mesh currents $\mathbf{I}_1, \mathbf{I}_2, \mathbf{I}_3$. Here, $\mathbf{Z}_{11} \equiv \mathbf{Z}_A + \mathbf{Z}_B$, the *self-impedance* of mesh 1, is the sum of all impedances through which $\mathbf{I}_1$ passes. Similarly, $\mathbf{Z}_{22} \equiv \mathbf{Z}_B + \mathbf{Z}_C + \mathbf{Z}_D$ and $\mathbf{Z}_{33} \equiv \mathbf{Z}_D + \mathbf{Z}_E$ are the self-impedances of meshes 2 and 3.

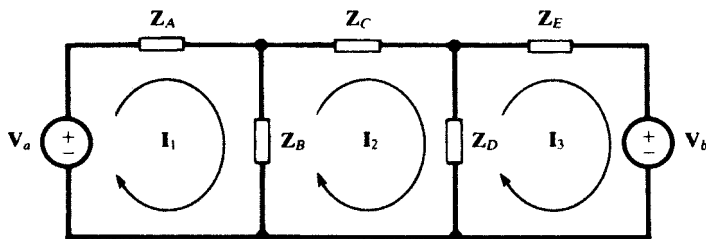


Fig. 9-12

The 1,2-element of the $\mathbf{Z}$ -matrix is defined as:

$$\mathbf{Z}_{12} \equiv \sum \pm (\text{impedance common to } \mathbf{I}_1 \text{ and } \mathbf{I}_2)$$

where a summand takes the plus sign if the two currents pass through the impedance in the same direction, and takes the minus sign in the opposite case. It follows that, invariably, $\mathbf{Z}_{12} = \mathbf{Z}_{21}$. In Fig. 9-12, $\mathbf{I}_1$ and $\mathbf{I}_2$ thread $\mathbf{Z}_B$ in opposite directions, whence

$$\mathbf{Z}_{12} = \mathbf{Z}_{21} = -\mathbf{Z}_B$$

Similarly,

$$\begin{aligned} \mathbf{Z}_{13} = \mathbf{Z}_{31} &\equiv \sum \pm (\text{impedance common to } \mathbf{I}_1 \text{ and } \mathbf{I}_3) = 0 \\ \mathbf{Z}_{23} = \mathbf{Z}_{32} &\equiv \sum \pm (\text{impedance common to } \mathbf{I}_2 \text{ and } \mathbf{I}_3) = -\mathbf{Z}_D \end{aligned}$$

The $\mathbf{Z}$ -matrix is symmetric.

In the $\mathbf{V}$ -column on the right-hand side of the equation, the entries $\mathbf{V}_k$ ($k = 1, 2, 3$) are defined exactly as in Section 4.3:

$$\mathbf{V}_k \equiv \sum \pm (\text{driving voltage in mesh } k)$$

where a summand takes the plus sign if the voltage drives in the direction of $\mathbf{I}_k$, and takes the minus sign in the opposite case. For the network of Fig. 9-12,

$$\mathbf{V}_1 = +\mathbf{V}_a \quad \mathbf{V}_2 = 0 \quad \mathbf{V}_3 = -\mathbf{V}_b$$

Instead of using the meshes, or “windows” of the (planar) network, it is sometimes expedient to choose an appropriate set of *loops*, each containing one or more meshes in its interior. It is easy to see that two loop currents might have the same direction in one impedance and opposite directions in another. Nevertheless, the preceding rules for writing the $\mathbf{Z}$ -matrix and the $\mathbf{V}$ -column have been formulated in such a way as to apply either to meshes or to loops. These rules are, of course, identical to those used in Section 4.3 to write the $\mathbf{R}$ -matrix and $\mathbf{V}$ -column.

EXAMPLE 9.6 Suppose that the phasor voltage across $\mathbf{Z}_B$, with polarity as indicated in Fig. 9-13 is sought. Choosing meshes as in Fig. 9-12 would entail solving for both $\mathbf{I}_1$ and $\mathbf{I}_2$, then obtaining the voltage as $\mathbf{V}_B = (\mathbf{I}_2 - \mathbf{I}_1)\mathbf{Z}_B$. In Fig. 9-13 three loops (two of which are meshes) are chosen so as to make $\mathbf{I}_1$ the only current in $\mathbf{Z}_B$. Furthermore, the direction of $\mathbf{I}_1$ is chosen such that $\mathbf{V}_B = \mathbf{I}_1\mathbf{Z}_B$. Setting up the matrix equation:

$$\begin{bmatrix} \mathbf{Z}_A + \mathbf{Z}_B & -\mathbf{Z}_A & 0 \\ -\mathbf{Z}_A & \mathbf{Z}_A + \mathbf{Z}_C + \mathbf{Z}_D & \mathbf{Z}_D \\ 0 & \mathbf{Z}_D & \mathbf{Z}_D + \mathbf{Z}_E \end{bmatrix} \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix} = \begin{bmatrix} -\mathbf{V}_a \\ \mathbf{V}_a \\ \mathbf{V}_b \end{bmatrix}$$

from which

$$\mathbf{V}_B = \mathbf{Z}_B \mathbf{I}_1 = \frac{\mathbf{Z}_B}{\Delta_z} \begin{vmatrix} -\mathbf{V}_a & -\mathbf{Z}_A & 0 \\ \mathbf{V}_a & \mathbf{Z}_A + \mathbf{Z}_B + \mathbf{Z}_C & \mathbf{Z}_D \\ \mathbf{V}_b & \mathbf{Z}_D & \mathbf{Z}_D + \mathbf{Z}_E \end{vmatrix}$$

where Δ_z is the determinant of the $\mathbf{Z}$ -matrix.

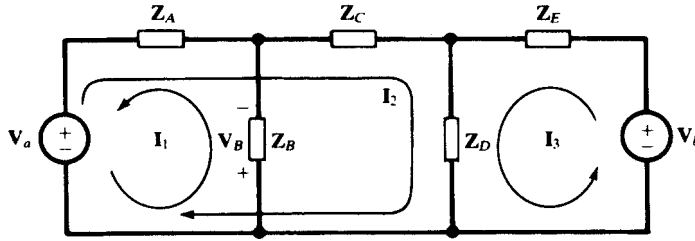


Fig. 9-13

Input and Transfer Impedances

The notions of input resistance (Section 4.5) and transfer resistance (Section 4.6) have their exact counterparts in the frequency domain. Thus, for the single-source network of Fig. 9-14, the *input impedance* is

$$\mathbf{Z}_{\text{input},r} \equiv \frac{\mathbf{V}_r}{\mathbf{I}_r} = \frac{\Delta_z}{\Delta_{rr}}$$

where Δ_{rr} is the cofactor of $\mathbf{Z}_{rr}$ in Δ_z ; and the *transfer impedance* between mesh (or loop) r and mesh (loop) s is

$$\mathbf{Z}_{\text{transfer},rs} \equiv \frac{\mathbf{V}_r}{\mathbf{I}_s} = \frac{\Delta_z}{\Delta_{rs}}$$

where Δ_{rs} is the cofactor of $\mathbf{Z}_{rs}$ in Δ_z .

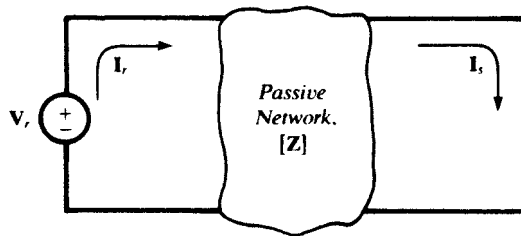


Fig. 9-14

As before, the superposition principle for an arbitrary n -mesh or n -loop network may be expressed as

$$\mathbf{I}_k = \frac{\mathbf{V}_1}{\mathbf{Z}_{\text{transfer},1k}} + \cdots + \frac{\mathbf{V}_{k-1}}{\mathbf{Z}_{\text{transfer},(k-1)k}} + \frac{\mathbf{V}_k}{\mathbf{Z}_{\text{input},k}} + \frac{\mathbf{V}_{k+1}}{\mathbf{Z}_{\text{transfer},(k+1)k}} + \cdots + \frac{\mathbf{V}_n}{\mathbf{Z}_{\text{transfer},nk}}$$

9.7 THE NODE VOLTAGE METHOD

The procedure is exactly as in Section 4.4, with admittances replacing reciprocal resistances. A frequency-domain network with n principal nodes, one of them designated as the reference node, requires $n - 1$ node voltage equations. Thus, for $n = 4$, the matrix equation would be

$$\begin{bmatrix} \mathbf{Y}_{11} & \mathbf{Y}_{12} & \mathbf{Y}_{13} \\ \mathbf{Y}_{21} & \mathbf{Y}_{22} & \mathbf{Y}_{23} \\ \mathbf{Y}_{31} & \mathbf{Y}_{32} & \mathbf{Y}_{33} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \\ \mathbf{V}_3 \end{bmatrix} = \begin{bmatrix} \mathbf{I}_1 \\ \mathbf{I}_2 \\ \mathbf{I}_3 \end{bmatrix}$$

in which the unknowns, $\mathbf{V}_1$, $\mathbf{V}_2$, and $\mathbf{V}_3$, are the voltages of principal nodes 1, 2, and 3 with respect to principal node 4, the reference node.

$\mathbf{Y}_{11}$ is the *self-admittance* of node 1, given by the sum of all admittances connected to node 1. Similarly, $\mathbf{Y}_{22}$ and $\mathbf{Y}_{33}$ are the self-admittances of nodes 2 and 3.

$\mathbf{Y}_{12}$, the *coupling admittance* between nodes 1 and 2, is given by *minus* the sum of all admittances connecting nodes 1 and 2. It follows that $\mathbf{Y}_{12} = \mathbf{Y}_{21}$. Similarly, for the other coupling admittances: $\mathbf{Y}_{13} = \mathbf{Y}_{31}$, $\mathbf{Y}_{23} = \mathbf{Y}_{32}$. The $\mathbf{Y}$ -matrix is therefore symmetric.

On the right-hand side of the equation, the $\mathbf{I}$ -column is formed just as in Section 4.4; i.e.,

$$\mathbf{I}_k = \sum (\text{current driving into node } k) \quad (k = 1, 2, 3)$$

in which a current driving *out of* node k is counted as negative.

Input and Transfer Admittances

The matrix equation of the node voltage method,

$$[\mathbf{Y}][\mathbf{V}] = [\mathbf{I}]$$

is identical in form to the matrix equation of the mesh current method,

$$[\mathbf{Z}][\mathbf{I}] = [\mathbf{V}]$$

Therefore, in theory at least, *input* and *transfer admittances* can be defined by analogy with input and transfer impedances:

$$\mathbf{Y}_{\text{input},r} \equiv \frac{\mathbf{I}_r}{\mathbf{V}_r} = \frac{\Delta_{\mathbf{Y}}}{\Delta_{rr}}$$

$$\mathbf{Y}_{\text{transfer},rs} \equiv \frac{\mathbf{I}_r}{\mathbf{V}_s} = \frac{\Delta_{\mathbf{Y}}}{\Delta_{rs}}$$

where now Δ_{rr} and Δ_{rs} are the cofactors of $\mathbf{Y}_{rr}$ and $\mathbf{Y}_{rs}$ in $\Delta_{\mathbf{Y}}$. In practice, these definitions are often of limited use. However, they are valuable in providing an expression of the superposition principle (for voltages);

$$\mathbf{V}_k = \frac{\mathbf{I}_1}{\mathbf{Y}_{\text{transfer},1k}} + \cdots + \frac{\mathbf{I}_{k-1}}{\mathbf{Y}_{\text{transfer},(k-1)k}} + \frac{\mathbf{I}_k}{\mathbf{Y}_{\text{input},k}} + \frac{\mathbf{I}_{k+1}}{\mathbf{Y}_{\text{transfer},(k+1)k}} + \cdots + \frac{\mathbf{I}_{n-1}}{\mathbf{Y}_{\text{transfer},(n-1)k}}$$

for $k = 1, 2, \dots, n - 1$. In words: the voltage at any principal node (relative to the reference node) is obtained by adding the voltages produced at that node by the various driving currents, these currents acting one at a time.

9.8 THÉVENIN'S AND NORTON'S THEOREMS

These theorems are exactly as given in Section 4.9, with the open-circuit voltage V' , short-circuit current I' , and representative resistance R' replaced by the open-circuit phasor voltage $\mathbf{V}'$, short-circuit phasor current $\mathbf{I}'$, and representative impedance $\mathbf{Z}'$. See Fig. 9-15.

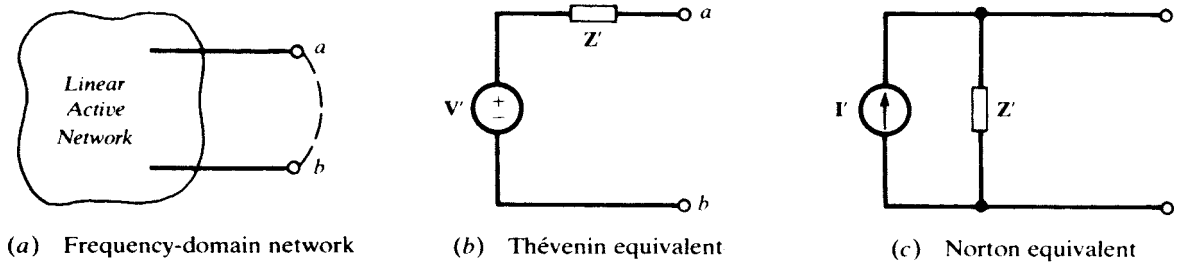


Fig. 9-15

9.9 SUPERPOSITION OF AC SOURCES

How do we apply superposition to circuits with more than one sinusoidal source? If all sources have the same frequency, superposition is applied in the phasor domain. Otherwise, the circuit is solved for each source, and time-domain responses are added.

EXAMPLE 9.7 A practical coil is connected in series between two voltage sources $v_1 = 5 \cos \omega_1 t$ and $v_2 = 10 \cos(\omega_2 t + 60^\circ)$ such that the sources share the same reference node. See Fig. 9-54. The voltage difference across the terminals of the coil is therefore $v_1 - v_2$. The coil is modeled by a 5-mH inductor in series with a 10- Ω resistor. Find the current $i(t)$ in the coil for (a) $\omega_1 = \omega_2 = 2000$ rad/s and (b) $\omega_1 = 2000$ rad/s, $\omega_2 = 2\omega_1$.

- (a) The impedance of the coil is $R + jL\omega = 10 + j10 = 10\sqrt{2} \angle 45^\circ \Omega$. The phasor voltage between its terminals is $\mathbf{V} = \mathbf{V}_1 - \mathbf{V}_2 = 5 - 10 \angle 60^\circ = -j5\sqrt{3}$ V. The current is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{-j5\sqrt{3}}{10\sqrt{2} \angle 45^\circ} \approx \frac{-j8.66}{14.14 \angle 45^\circ} = 0.61 \angle -135^\circ \text{ A}$$

$$i = 0.61 \cos(2000t - 135^\circ)$$

- (b) Because the coil has different impedances at $\omega_1 = 2000$ and $\omega_2 = 4000$ rad/s, the current may be represented in the time domain only. By applying superposition, we get $i = i_1 - i_2$, where i_1 and i_2 are currents due to v_1 and v_2 , respectively.

$$\mathbf{I}_1 = \frac{\mathbf{V}_1}{\mathbf{Z}_1} = \frac{5}{10 + j10} = 0.35 \angle -45^\circ \text{ A}, \quad i_1(t) = 0.35 \cos(2000t - 45^\circ)$$

$$\mathbf{I}_2 = \frac{\mathbf{V}_2}{\mathbf{Z}_2} = \frac{10 \angle 60^\circ}{10 + j20} = 0.45 \angle -3.4^\circ \text{ A}, \quad i_2(t) = 0.45 \cos(4000t - 3.4^\circ)$$

$$i = i_1 - i_2 = 0.35 \cos(2000t - 45^\circ) - 0.45 \cos(4000t - 3.4^\circ)$$

Solved Problems

- 9.1** A 10-mH inductor has current $i = 5.0 \cos 2000t$ (A). Obtain the voltage v_L .

From Table 9-1, $v_L = \omega LI \cos(\omega t + 90^\circ) = 100 \cos(2000t + 90^\circ)$ (V). As a sine function,

$$v_L = 100 \sin(2000t + 180^\circ) = -100 \sin 2000t \text{ (V)}$$

- 9.2** A series circuit, with $R = 10 \Omega$ and $L = 20$ mH, has current $i = 2.0 \sin 500t$ (A). Obtain total voltage v and the angle by which i lags v .

By the methods of Example 9.1,

$$\theta = \arctan \frac{500(20 \times 10^{-3})}{10} = 45^\circ$$

$$v = I\sqrt{R^2 + (\omega L)^2} \sin(\omega t + \theta) = 28.3 \sin(500t + 45^\circ) \quad (\text{V})$$

It is seen that i lags v by 45° .

- 9.3** Find the two elements in a series circuit, given that the current and total voltage are

$$i = 10 \cos(5000t - 23.13^\circ) \quad (\text{A}) \quad v = 50 \cos(5000t + 30^\circ) \quad (\text{V})$$

Since i lags v (by 53.13°), the elements are R and L . The ratio of $V_{\max}$ to $I_{\max}$ is $50/10$. Hence,

$$\frac{50}{10} = \sqrt{R^2 + (5000L)^2} \quad \text{and} \quad \tan 53.13^\circ = 1.33 = \frac{5000L}{R}$$

Solving, $R = 3.0 \, \Omega$, $L = 0.8 \, \text{mH}$.

- 9.4** A series circuit, with $R = 2.0 \, \Omega$ and $C = 200 \, \text{pF}$, has a sinusoidal applied voltage with a frequency of $99.47 \, \text{MHz}$. If the maximum voltage across the capacitance is $24 \, \text{V}$, what is the maximum voltage across the series combination?

$$\omega = 2\pi f = 6.25 \times 10^8 \, \text{rad/s}$$

From Table 9-1, $I_{\max} = \omega C V_{C,\max} = 3.0 \, \text{A}$. Then, by the methods of Example 9.2,

$$V_{\max} = I_{\max} \sqrt{R^2 + (1/\omega C)^2} = \sqrt{(6)^2 + (24)^2} = 24.74 \, \text{V}$$

- 9.5** The current in a series circuit of $R = 5 \, \Omega$ and $L = 30 \, \text{mH}$ lags the applied voltage by 80° . Determine the source frequency and the impedance $\mathbf{Z}$.

From the impedance diagram, Fig. 9-16,

$$5 + jX_L = \mathbf{Z} \angle 80^\circ \quad X_L = 5 \tan 80^\circ = 28.4 \, \Omega$$

Then $28.4 = \omega(30 \times 10^{-3})$, whence $\omega = 945.2 \, \text{rad/s}$ and $f = 150.4 \, \text{Hz}$.

$$\mathbf{Z} = 5 + j28.4 \, \Omega$$

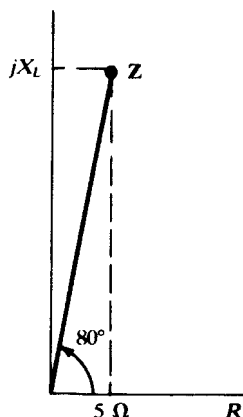


Fig. 9-16

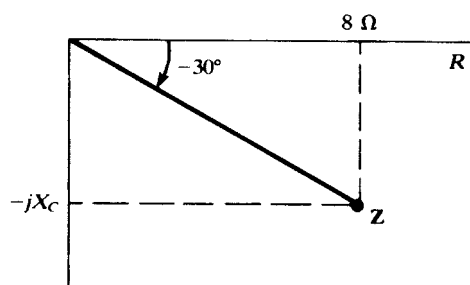


Fig. 9-17

- 9.6** At what frequency will the current lead the voltage by 30° in a series circuit with $R = 8\ \Omega$ and $C = 30\ \mu\text{F}$?

From the impedance diagram, Fig. 9-17,

$$8 - jX_C = Z \angle -30^\circ \quad -X_C = 8 \tan(-30^\circ) = -4.62\ \Omega$$

Then
$$4.62 = \frac{1}{2\pi f(30 \times 10^{-6})} \quad \text{or} \quad f = 1149\ \text{Hz}$$

- 9.7** A series RC circuit, with $R = 10\ \Omega$, has an impedance with an angle of -45° at $f_1 = 500\ \text{Hz}$. Find the frequency for which the magnitude of the impedance is (a) twice that at f_1 , (b) one-half that at f_1 .

From $10 - jX_C = Z_1 \angle -45^\circ$, $X_C = 10\ \Omega$ and $Z_1 = 14.14\ \Omega$.

(a) For twice the magnitude,

$$10 - jX_C = 28.28 \angle \theta \quad \text{or} \quad X_C = \sqrt{(28.28)^2 - (10)^2} = 26.45\ \Omega$$

Then, since X_C is inversely proportional to f ,

$$\frac{10}{26.45} = \frac{f_2}{500} \quad \text{or} \quad f_2 = 189\ \text{Hz}$$

(b) A magnitude $Z_3 = 7.07\ \Omega$ is impossible; the smallest magnitude possible is $Z = R = 10\ \Omega$.

- 9.8** A two-element series circuit has voltage $\mathbf{V} = 240 \angle 0^\circ\ \text{V}$ and current $\mathbf{I} = 50 \angle -60^\circ\ \text{A}$. Determine the current which results when the resistance is reduced to (a) 30 percent, (b) 60 percent, of its former value.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{240 \angle 0^\circ}{50 \angle -60^\circ} = 4.8 \angle 60^\circ = 2.40 + j4.16\ \Omega$$

(a) $30\% \times 2.40 = 0.72 \quad \mathbf{Z}_1 = 0.72 + j4.16 = 4.22 \angle 80.2^\circ\ \Omega$

$$\mathbf{I}_1 = \frac{240 \angle 0^\circ}{4.22 \angle 80.2^\circ} = 56.8 \angle -80.2^\circ\ \text{A}$$

(b) $60\% \times 2.40 = 1.44 \quad \mathbf{Z}_2 = 1.44 + j4.16 = 4.40 \angle 70.9^\circ\ \Omega$

$$\mathbf{I}_2 = \frac{240 \angle 0^\circ}{4.40 \angle 70.9^\circ} = 54.5 \angle -70.9^\circ\ \text{A}$$

- 9.9** For the circuit shown in Fig. 9-18, obtain $\mathbf{Z}_{\text{eq}}$ and compute $\mathbf{I}$.

For series impedances,

$$\mathbf{Z}_{\text{eq}} = 10 \angle 0^\circ + 4.47 \angle 63.4^\circ = 12.0 + j4.0 = 12.65 \angle 18.43^\circ\ \Omega$$

Then
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_{\text{eq}}} = \frac{100 \angle 0^\circ}{12.65 \angle 18.43^\circ} = 7.91 \angle -18.43^\circ\ \text{A}$$

- 9.10** Evaluate the impedance $\mathbf{Z}_1$ in the circuit of Fig. 9-19.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = 20 \angle 60^\circ = 10.0 + j17.3\ \Omega$$

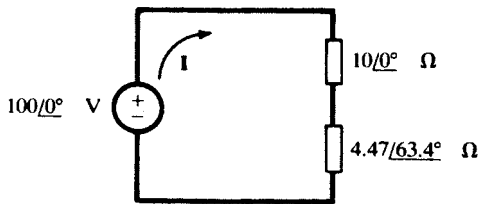


Fig. 9-18

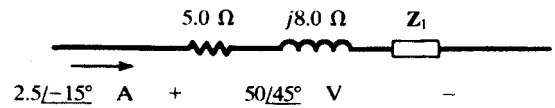


Fig. 9-19

Then, since impedances in series add,

$$5.0 + j8.0 + Z_1 = 10.0 + j17.3 \quad \text{or} \quad Z_1 = 5.0 + j9.3 \quad \Omega$$

9.11 Compute the equivalent impedance Z_{eq} and admittance Y_{eq} for the four-branch circuit shown in Fig. 9-20.

Using admittances,

$$Y_1 = \frac{1}{j5} = -j0.20 \text{ S} \qquad Y_3 = \frac{1}{15} = 0.067 \text{ S}$$

$$Y_2 = \frac{1}{5 + j8.66} = 0.05 - j0.087 \text{ S} \qquad Y_4 = \frac{1}{-j10} = j0.10 \text{ S}$$

Then

$$Y_{eq} = Y_1 + Y_2 + Y_3 + Y_4 = 0.117 - j0.187 = 0.221 \angle -58.0^\circ \text{ S}$$

and

$$Z_{eq} = \frac{1}{Y_{eq}} = 4.53 \angle 58.0^\circ \quad \Omega$$

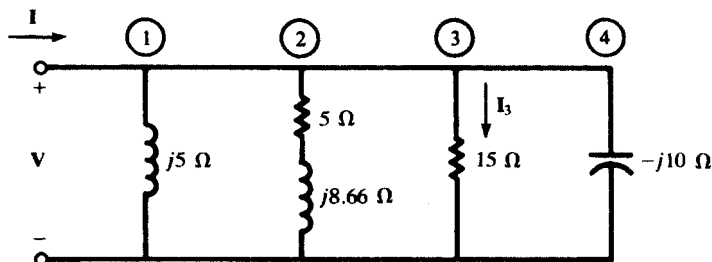


Fig. 9-20

9.12 The total current I entering the circuit shown in Fig. 9-20 is $33.0 \angle -13.0^\circ \text{ A}$. Obtain the branch current I_3 and the voltage V .

$$V = IZ_{eq} = (33.0 \angle -13.0^\circ)(4.53 \angle 58.0^\circ) = 149.5 \angle 45.0^\circ \text{ V}$$

$$I_3 = VY_3 = (149.5 \angle 45.0^\circ) \left(\frac{1}{15} \angle 0^\circ \right) = 9.97 \angle 45.0^\circ \text{ A}$$

9.13 Find Z_1 in the three-branch network of Fig. 9-21, if $I = 31.5 \angle 24.0^\circ \text{ A}$ for an applied voltage $V = 50.0 \angle 60.0^\circ \text{ V}$.

$$\mathbf{Y} = \frac{\mathbf{I}}{\mathbf{V}} = 0.630 \angle -36.0^\circ = 0.510 - j0.370 \text{ S}$$

Then

$$0.510 - j0.370 = \mathbf{Y}_1 + \frac{1}{10} + \frac{1}{4.0 + j3.0}$$

whence $\mathbf{Y}_1 = 0.354 \angle -45^\circ \text{ S}$ and $\mathbf{Z}_1 = 2.0 + j2.0 \Omega$.

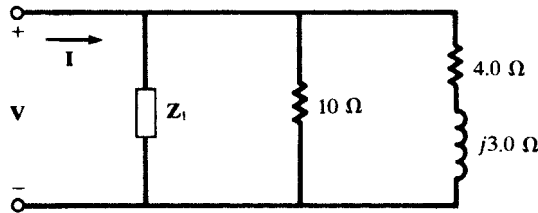


Fig. 9-21

- 9.14** The constants R and L of a coil can be obtained by connecting the coil in series with a known resistance and measuring the coil voltage V_x , the resistor voltage V_1 , and the total voltage V_T (Fig. 9-22). The frequency must also be known, but the phase angles of the voltages are not

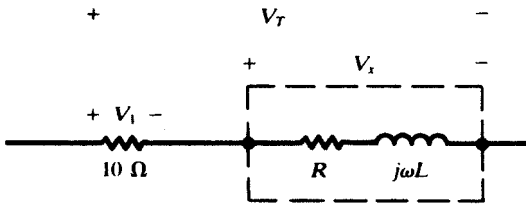


Fig. 9-22

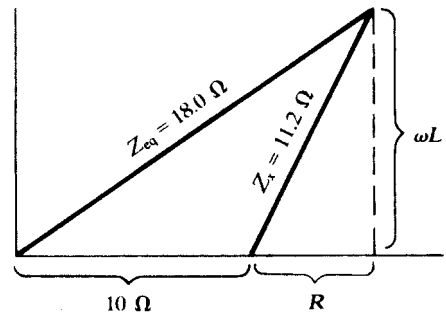


Fig. 9-23

known. Given that $f = 60 \text{ Hz}$, $V_1 = 20 \text{ V}$, $V_x = 22.4 \text{ V}$, and $V_T = 36.0 \text{ V}$, find R and L .

The measured voltages are effective values; but, as far as impedance calculations are concerned, it makes no difference whether effective or peak values are used.

The (effective) current is $I = V_1/10 = 2.0 \text{ A}$. Then

$$Z_x = \frac{22.4}{2.0} = 11.2 \Omega \quad Z_{eq} = \frac{36.0}{2.0} = 18.0 \Omega$$

From the impedance diagram, Fig. 9-23,

$$\begin{aligned} (18.0)^2 &= (10 + R)^2 + (\omega L)^2 \\ (11.2)^2 &= R^2 + (\omega L)^2 \end{aligned}$$

where $\omega = 2\pi 60 = 377 \text{ rad/s}$. Solving simultaneously,

$$R = 4.92 \Omega \quad L = 26.7 \text{ mH}$$

- 9.15** In the parallel circuit shown in Fig. 9-24, the effective values of the currents are: $I_x = 18.0 \text{ A}$, $I_1 = 15.0 \text{ A}$, $I_T = 30.0 \text{ A}$. Determine R and X_L .

The problem can be solved in a manner similar to that used in Problem 9.14 but with the admittance diagram.

The (effective) voltage is $V = I_1(4.0) = 60.0$ V. Then

$$Y_x = \frac{I_x}{V} = 0.300 \text{ S} \quad Y_{\text{eq}} = \frac{I_T}{V} = 0.500 \text{ S} \quad Y_1 = \frac{1}{4.0} = 0.250 \text{ S}$$

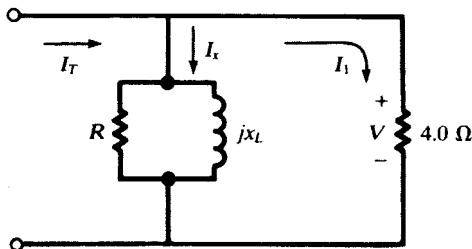


Fig. 9-24

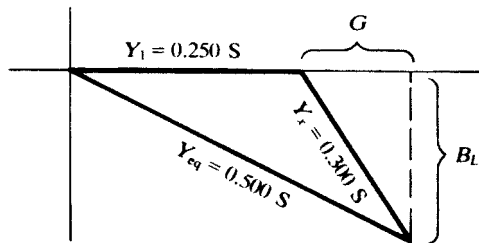


Fig. 9-25

From the admittance diagram, Fig. 9-25,

$$(0.500)^2 = (0.250 + G)^2 + B_L^2$$

$$(0.300)^2 = G^2 + B_L^2$$

which yield $G = 0.195$ S, $B_L = 0.228$ S. Then

$$R = \frac{1}{G} = 5.13 \, \Omega \quad \text{and} \quad jX_L = \frac{1}{-jB_L} = j4.39 \, \Omega$$

i.e., $X_L = 4.39 \, \Omega$.

9.16 Obtain the phasor voltage V_{AB} in the two-branch parallel circuit of Fig. 9-26.

By current-division methods, $I_1 = 4.64 \angle 120.1^\circ$ A and $I_2 = 17.4 \angle 30.1^\circ$ A. Either path AXB or path AYB may be considered. Choosing the former,

$$V_{AB} = V_{AX} + V_{XB} = I_1(20) - I_2(j6) = 92.8 \angle 120.1^\circ + 104.4 \angle -59.9^\circ = 11.6 \angle -59.9^\circ \text{ V}$$

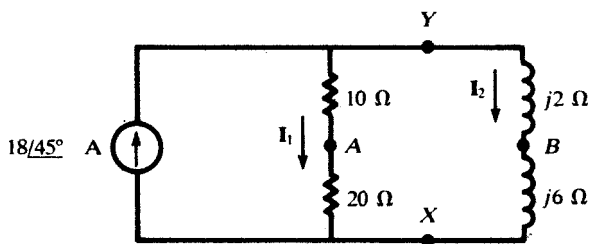


Fig. 9-26

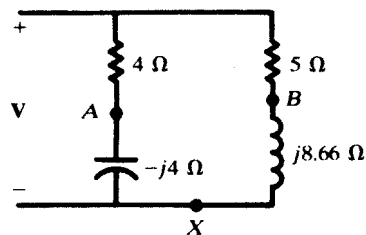


Fig. 9-27

9.17 In the parallel circuit shown in Fig. 9-27, $V_{AB} = 48.3 \angle 30^\circ$ V. Find the applied voltage V .

By voltage division in the two branches:

$$V_{AX} = \frac{-j4}{4 - j4} V = \frac{1}{1 + j} V \quad V_{BX} = \frac{j8.66}{5 + j8.66} V$$

and so

$$V_{AB} = V_{AX} - V_{BX} = \left(\frac{1}{1 + j} - \frac{j8.66}{5 + j8.66} \right) V = \frac{1}{-0.268 + j1} V$$

or

$$V = (-0.268 + j1)V_{AB} = (1.035 \angle 105^\circ)(48.3 \angle 30^\circ) = 50.0 \angle 135^\circ \text{ V}$$

9.18 Obtain the voltage V_x in the network of Fig. 9-28, using the mesh current method.

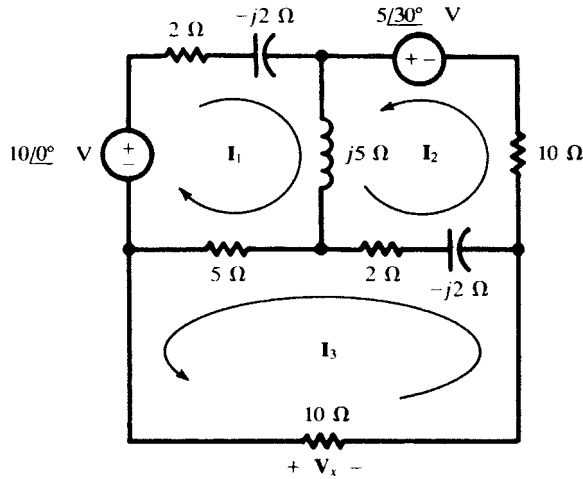


Fig. 9-28

One choice of mesh currents is shown on the circuit diagram, with I_3 passing through the $10\text{-}\Omega$ resistor in a direction such that $V_x = I_3(10)$ (V). The matrix equation can be written by inspection:

$$\begin{bmatrix} 7 + j3 & j5 & 5 \\ j5 & 12 + j3 & -(2 - j2) \\ 5 & -(2 - j2) & 17 - j2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 10 \angle 0^\circ \\ 5 \angle 30^\circ \\ 0 \end{bmatrix}$$

Solving by determinants,

$$I_3 = \frac{\begin{vmatrix} 7 + j3 & j5 & 10 \angle 0^\circ \\ j5 & 12 + j3 & 5 \angle 30^\circ \\ 5 & -2 + j2 & 0 \end{vmatrix}}{\begin{vmatrix} 7 + j3 & j5 & 5 \\ j5 & 12 + j3 & -2 + j2 \\ 5 & -2 + j2 & 17 - j2 \end{vmatrix}} = \frac{667.96 \angle -169.09^\circ}{1534.5 \angle 25.06^\circ} = 0.435 \angle -194.15^\circ \text{ A}$$

and $V_x = I_3(10) = 4.35 \angle -194.15^\circ$ V.

9.19 In the network of Fig. 9-29, determine the voltage V which results in a zero current through the $2 + j3 \text{ }\Omega$ impedance.

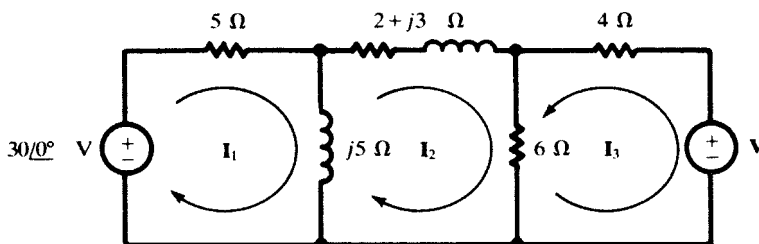


Fig. 9-29

Choosing mesh currents as shown on the circuit diagram,

$$\mathbf{I}_2 = \frac{1}{\Delta z} \begin{vmatrix} 5 + j5 & 30 \angle 0^\circ & 0 \\ -j5 & 0 & 6 \\ 0 & \mathbf{V} & 10 \end{vmatrix} = 0$$

Expanding the numerator determinant by cofactors of the second column,

$$-(30 \angle 0^\circ) \begin{vmatrix} -j5 & 6 \\ 0 & 10 \end{vmatrix} - \mathbf{V} \begin{vmatrix} 5 + j5 & 0 \\ -j5 & 6 \end{vmatrix} = 0 \quad \text{whence} \quad \mathbf{V} = 35.4 \angle 45.0^\circ \text{ V}$$

9.20 Solve Problem 9.19 by the node voltage method.

The network is redrawn in Fig. 9-30 with one end of the $2 + j3$ impedance as the reference node. By the rule of Section 9.7 the matrix equation is

$$\begin{bmatrix} \frac{1}{5} + \frac{1}{j5} + \frac{1}{2 + j3} & -\left(\frac{1}{5} + \frac{1}{j5}\right) \\ -\left(\frac{1}{5} + \frac{1}{j5}\right) & \frac{1}{5} + \frac{1}{j5} + \frac{1}{4} + \frac{1}{6} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{30 \angle 0^\circ}{5} \\ \frac{-30 \angle 0^\circ}{5} - \frac{\mathbf{V}}{4} \end{bmatrix}$$

For node voltage $\mathbf{V}_1$ to be zero, it is necessary that the numerator determinant in the solution for $\mathbf{V}_1$ vanish.

$$N_1 = \begin{vmatrix} \frac{30 \angle 0^\circ}{5} & -0.200 + j0.200 \\ \frac{-30 \angle 0^\circ}{5} - \frac{\mathbf{V}}{4} & 0.617 - j0.200 \end{vmatrix} = 0 \quad \text{from which} \quad \mathbf{V} = 35.4 \angle 45^\circ \text{ V}$$

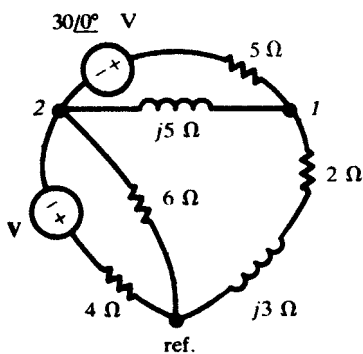


Fig. 9-30

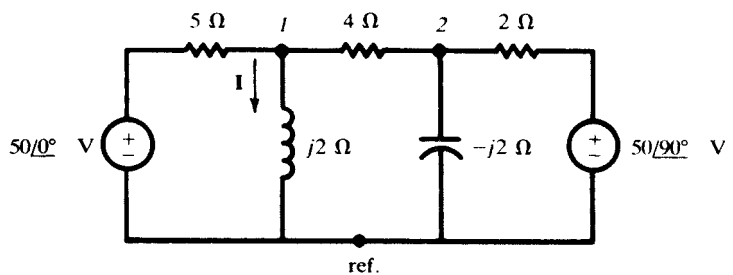


Fig. 9-31

9.21 Use the node voltage method to obtain the current $\mathbf{I}$ in the network of Fig. 9-31.

There are three principal nodes in the network. The reference and node 1 are selected so that the node 1 voltage is the voltage across the $j2\text{-}\Omega$ reactance.

$$\begin{bmatrix} \frac{1}{5} + \frac{1}{j2} + \frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{1}{4} + \frac{1}{-j2} + \frac{1}{2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1 \\ \mathbf{V}_2 \end{bmatrix} = \begin{bmatrix} \frac{50 \angle 0^\circ}{5} \\ \frac{50 \angle 90^\circ}{2} \end{bmatrix}$$

from which

$$\mathbf{V}_1 = \frac{\begin{vmatrix} 10 & -0.250 \\ j25 & 0.750 + j0.500 \end{vmatrix}}{\begin{vmatrix} 0.450 - j0.500 & -0.250 \\ -0.250 & 0.750 + j0.500 \end{vmatrix}} = \frac{13.52 \angle 56.31^\circ}{0.546 \angle -15.94^\circ} = 24.76 \angle 72.25^\circ \text{ V}$$

and

$$\mathbf{I} = \frac{24.76 \angle 72.25^\circ}{2 \angle 90^\circ} = 12.38 \angle -17.75^\circ \text{ A}$$

9.22 Find the input impedance at terminals ab for the network of Fig. 9-32.

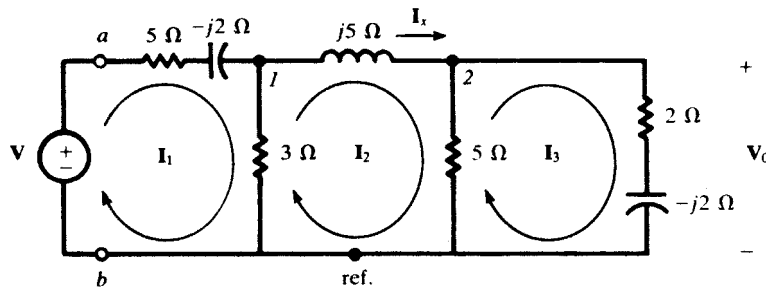


Fig. 9-32

With mesh current $\mathbf{I}_1$ selected as shown on the diagram.

$$\mathbf{Z}_{\text{input},1} = \frac{\Delta_z}{\Delta_{11}} = \frac{\begin{vmatrix} 8-j2 & -3 & 0 \\ -3 & 8+j5 & -5 \\ 0 & -5 & 7-j2 \end{vmatrix}}{\begin{vmatrix} 8+j5 & -5 \\ -5 & 7-j2 \end{vmatrix}} = \frac{315.5 \angle 16.19^\circ}{45.2 \angle 24.86^\circ} = 6.98 \angle -8.67^\circ \Omega$$

9.23 For the network in Fig. 9-32, obtain the current in the inductor, $\mathbf{I}_x$, by first obtaining the transfer impedance. Let $\mathbf{V} = 10 \angle 30^\circ \text{ V}$.

$$\mathbf{Z}_{\text{transfer},12} = \frac{\Delta_z}{\Delta_{12}} = \frac{315.5 \angle 16.19^\circ}{\begin{vmatrix} -3 & -5 \\ 0 & 7-j2 \end{vmatrix}} = 14.45 \angle 32.14^\circ \Omega$$

Then
$$\mathbf{I}_x = \mathbf{I}_2 = \frac{\mathbf{V}}{\mathbf{Z}_{\text{transfer},12}} = \frac{10 \angle 30^\circ}{14.45 \angle 32.14^\circ} = 0.692 \angle -2.14^\circ \text{ A}$$

9.24 For the network in Fig. 9-32, find the value of the source voltage $\mathbf{V}$ which results in $\mathbf{V}_0 = 5.0 \angle 0^\circ \text{ V}$.

The transfer impedance can be used to compute the current in the $2 - j2 \Omega$ impedance, from which $\mathbf{V}_0$ is readily obtained.

$$\mathbf{Z}_{\text{transfer},13} = \frac{\Delta_z}{\Delta_{13}} = \frac{315.5 \angle 16.19^\circ}{15 \angle 0^\circ} = 21.0 \angle 16.19^\circ \Omega$$

$$\mathbf{V}_0 = \mathbf{I}_3(2 - j2) = \frac{\mathbf{V}}{\mathbf{Z}_{\text{transfer},13}}(2 - j2) = \mathbf{V}(0.135 \angle -61.19^\circ)$$

Thus, if $V_0 = 5.0 \angle 0^\circ \text{ V}$,

$$V = \frac{5.0 \angle 0^\circ}{0.135 \angle -61.19^\circ} = 37.0 \angle 61.19^\circ \text{ V}$$

Alternate Method

The node voltage method may be used. V_0 is the node voltage V_2 for the selection of nodes indicated in Fig. 9-32.

$$V_0 = V_2 = \frac{\begin{vmatrix} \frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5} & \frac{V}{5-j2} \\ -\frac{1}{j5} & 0 \end{vmatrix}}{\begin{vmatrix} \frac{1}{5-j2} + \frac{1}{3} + \frac{1}{j5} & -\frac{1}{j5} \\ -\frac{1}{j5} & \frac{1}{j5} + \frac{1}{5} + \frac{1}{2-jj2} \end{vmatrix}} = V(0.134 \angle -61.15^\circ)$$

For $V_0 = 5.0 \angle 0^\circ \text{ V}$, $V = 37.3 \angle 61.15^\circ \text{ V}$, which agrees with the previous answer to within roundoff errors.

- 9.25** For the network shown in Fig. 9-33, obtain the input admittance and use it to compute node voltage V_1 .

$$Y_{\text{input},1} = \frac{\Delta_Y}{\Delta_{11}} = \frac{\begin{vmatrix} \frac{1}{10} + \frac{1}{j5} + \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} + \frac{1}{3+j4} + \frac{1}{-j10} \end{vmatrix}}{\frac{1}{2} + \frac{1}{3+j4} + \frac{1}{-j10}} = 0.311 \angle -49.97^\circ \text{ S}$$

$$V_1 = \frac{I_1}{Y_{\text{input},1}} = \frac{5.0 \angle 0^\circ}{0.311 \angle -49.97^\circ} = 16.1 \angle 49.97^\circ \text{ V}$$

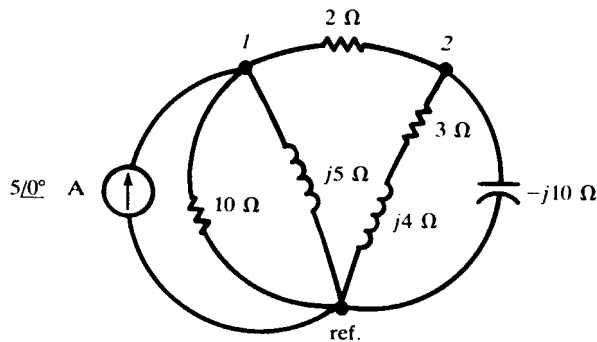


Fig. 9-33

- 9.26** For the network of Problem 9.25, compute the transfer admittance $Y_{\text{transfer},12}$ and use it to obtain node voltage V_2 .

$$Y_{\text{transfer},12} = \frac{\Delta_Y}{\Delta_{12}} = \frac{0.194 \angle -55.49^\circ}{-(-0.50)} = 0.388 \angle -55.49^\circ \text{ S}$$

$$V_2 = \frac{I_1}{Y_{\text{transfer},12}} = 12.9 \angle 55.49^\circ \text{ V}$$

9.27 Replace the active network in Fig. 9-34(a) at terminals ab with a Thévenin equivalent.

$$\mathbf{Z}' = j5 + \frac{5(3 + j4)}{5 + 3 + j4} = 2.50 + j6.25 \quad \Omega$$

The open-circuit voltage $\mathbf{V}'$ at terminals ab is the voltage across the $3 + j4 \Omega$ impedance:

$$\mathbf{V}' = \left(\frac{10 \angle 0^\circ}{8 + j4} \right) (3 + j4) = 5.59 \angle 26.56^\circ \quad \text{V}$$

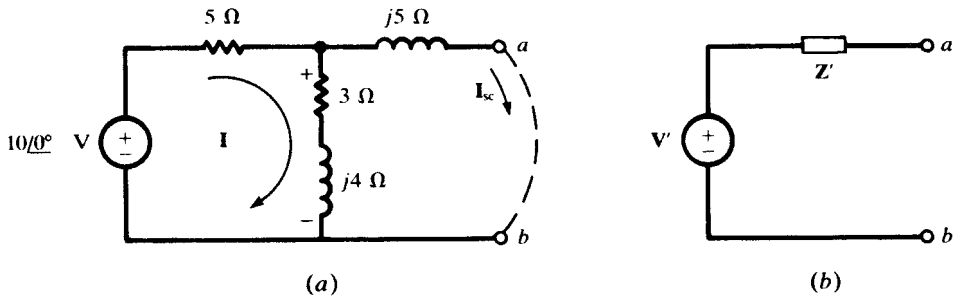


Fig. 9-34

9.28 For the network of Problem 9.27, obtain a Norton equivalent circuit (Fig. 9-35).

At terminals ab , $\mathbf{I}_{\text{sc}}$ is the Norton current $\mathbf{I}'$. By current division,

$$\mathbf{I}' = \frac{10 \angle 0^\circ}{5 + \frac{j5(3 + j4)}{3 + j9}} \left(\frac{3 + j4}{3 + j9} \right) = 0.830 \angle -41.63^\circ \quad \text{A}$$

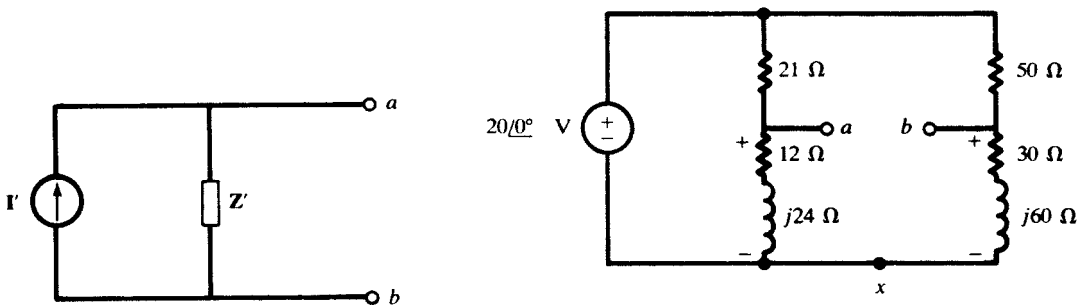


Fig. 9-36

The shunt impedance $\mathbf{Z}'$ is as found in Problem 9.27, $\mathbf{Z}' = 2.50 + j6.25 \Omega$.

9.29 Obtain the Thévenin equivalent for the bridge circuit of Fig. 9-36. Make $\mathbf{V}'$ the voltage of a with respect to b .

By voltage division in either branch,

$$\mathbf{V}_{ax} = \frac{12 + j24}{33 + j24} (20 \angle 0^\circ) \quad \mathbf{V}_{bx} = \frac{30 + j60}{80 + j60} (20 \angle 0^\circ)$$

Hence,
$$\mathbf{V}_{ab} = \mathbf{V}_{ax} - \mathbf{V}_{bx} = (20 \angle 0^\circ) \left(\frac{12 + j24}{33 + j24} - \frac{30 + j60}{80 + j60} \right) = 0.326 \angle 169.4^\circ \quad \mathbf{V} = \mathbf{V}'$$

Viewed from ab with the voltage source shorted out, the circuit is two parallel combinations in series, and so

$$\mathbf{Z}' = \frac{21(12 + j24)}{33 + j24} + \frac{50(30 + j60)}{80 - +j60} = 47.35 \angle 26.81^\circ \quad \Omega$$

- 9.30** Replace the network of Fig. 9-37 at terminals ab with a Norton equivalent and with a Thévenin equivalent.

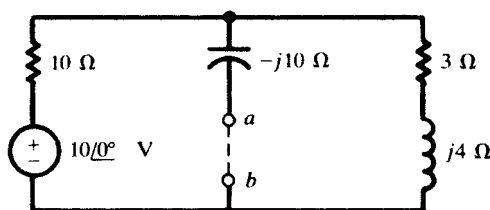


Fig. 9-37

By current division,

$$\mathbf{I}_{sc} = \mathbf{I}' = \left[\frac{10 \angle 0^\circ}{10 + \frac{(-j10)(3 + j4)}{3 - j6}} \right] \left(\frac{3 + j4}{3 - j6} \right) = 0.439 \angle 105.26^\circ \quad \text{A}$$

and by voltage division in the open circuit,

$$\mathbf{V}_{ab} = \mathbf{V}' = \frac{3 + j4}{13 + j4} (10 \angle 0^\circ) = 3.68 \angle 36.03^\circ \quad \text{V}$$

Then

$$\mathbf{Z}' = \frac{\mathbf{V}'}{\mathbf{I}'} = \frac{3.68 \angle 36.03^\circ}{0.439 \angle 105.26^\circ} = 8.37 \angle -69.23^\circ \quad \Omega$$

See Fig. 9-38.

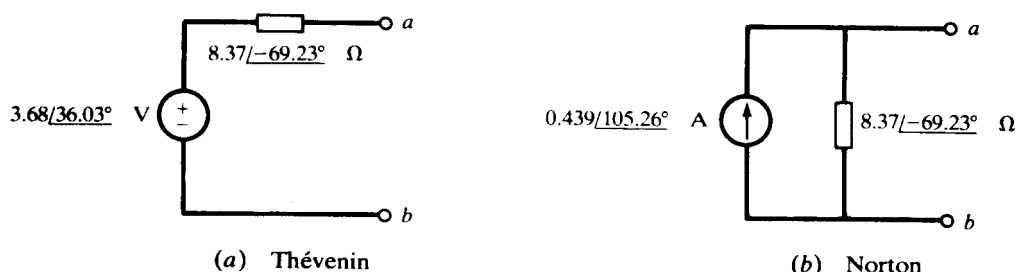


Fig. 9-38

Supplementary Problems

- 9.31** Two circuit elements in a series connection have current and total voltage

$$i = 13.42 \sin(500t - 53.4^\circ) \text{ (A)} \quad v = 150 \sin(500t + 10^\circ) \text{ (V)}$$

Identify the two elements. *Ans.* $R = 5 \Omega$, $L = 20 \text{ mH}$

- 9.32** Two circuit elements in a series connection have current and total voltage

$$i = 4.0 \cos(2000t + 13.2^\circ) \text{ (A)} \quad v = 200 \sin(2000t + 50.0^\circ) \text{ (V)}$$

Identify the two elements. *Ans.* $R = 30 \Omega$, $C = 12.5 \mu\text{F}$

- 9.33** A series RC circuit, with $R = 27.5 \Omega$ and $C = 66.7 \mu\text{F}$, has sinusoidal voltages and current, with angular frequency 1500 rad/s . Find the phase angle by which the current leads the voltage. *Ans.* 20°

- 9.34** A series RLC circuit, with $R = 15 \Omega$, $L = 80 \text{ mH}$, and $C = 30 \mu\text{F}$, has a sinusoidal current at angular frequency 500 rad/s . Determine the phase angle and whether the current leads or lags the total voltage. *Ans.* 60.6° , leads

- 9.35** A capacitance $C = 35 \mu\text{F}$ is in parallel with a certain element. Identify the element, given that the voltage and total current are

$$v = 150 \sin 3000t \text{ (V)} \quad i_T = 16.5 \sin(3000t + 72.4^\circ) \text{ (A)}$$

Ans. $R = 30.1 \Omega$

- 9.36** A two-element series circuit, with $R = 20 \Omega$ and $L = 20 \text{ mH}$, has an impedance $40.0 \angle \theta \Omega$. Determine the angle θ and the frequency. *Ans.* 60° , 276 Hz

- 9.37** Determine the impedance of the series RL circuit, with $R = 25 \Omega$ and $L = 10 \text{ mH}$, at (a) 100 Hz , (b) 500 Hz , (c) 1000 Hz . *Ans.* (a) $25.8 \angle 14.1^\circ \Omega$; (b) $40.1 \angle 51.5^\circ \Omega$; (c) $67.6 \angle 68.3^\circ \Omega$

- 9.38** Determine the circuit constants of a two-element series circuit if the applied voltage

$$v = 150 \sin(5000t + 45^\circ) \text{ (V)}$$

results in a current $i = 3.0 \sin(5000t - 15^\circ) \text{ (A)}$. *Ans.* 25Ω , 8.66 mH

- 9.39** A series circuit of $R = 10 \Omega$ and $C = 40 \mu\text{F}$ has an applied voltage $v = 500 \cos(2500t - 20^\circ) \text{ (V)}$. Find the resulting current i . *Ans.* $25\sqrt{2} \cos(2500t + 25^\circ) \text{ (A)}$

- 9.40** Three impedances are in series: $Z_1 = 3.0 \angle 45^\circ \Omega$, $Z_2 = 10\sqrt{2} \angle 45^\circ \Omega$, $Z_3 = 5.0 \angle -90^\circ \Omega$. Find the applied voltage $\mathbf{V}$, if the voltage across Z_1 is $27.0 \angle -10^\circ \text{ V}$. *Ans.* $126.5 \angle -24.6^\circ \text{ V}$

- 9.41** For the three-element series circuit in Fig. 9-39, (a) find the current $\mathbf{I}$; (b) find the voltage across each impedance and construct the voltage phasor diagram which shows that $\mathbf{V}_1 + \mathbf{V}_2 + \mathbf{V}_3 = 100 \angle 0^\circ \text{ V}$. *Ans.* (a) $6.28 \angle -9.17^\circ \text{ A}$; (b) see Fig. 9-40.

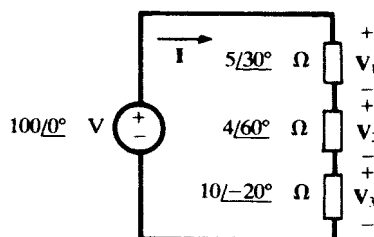


Fig. 9-39

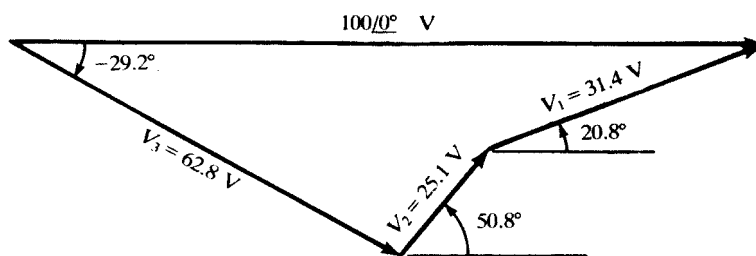


Fig. 9-40

- 9.42 Find $\mathbf{Z}$ in the parallel circuit of Fig. 9-41, if $\mathbf{V} = 50.0 \angle 30.0^\circ \text{ V}$ and $\mathbf{I} = 27.9 \angle 57.8^\circ \text{ A}$.

Ans. $5.0 \angle -30^\circ \Omega$

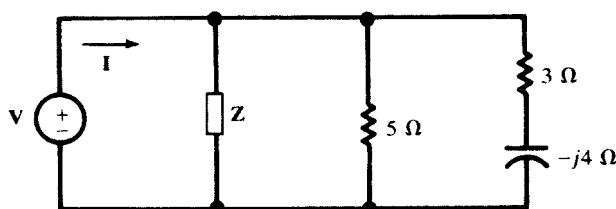


Fig. 9-41

- 9.43 Obtain the conductance and susceptance corresponding to a voltage $\mathbf{V} = 85.0 \angle 205^\circ \text{ V}$ and a resulting current $\mathbf{I} = 41.2 \angle -141.0^\circ \text{ A}$. Ans. $0.471 \text{ S}, 0.117 \text{ S (capacitive)}$

- 9.44 A practical coil contains resistance as well as inductance and can be represented by either a series or parallel circuit, as suggested in Fig. 9-42. Obtain R_p and L_p in terms of R_s and L_s .

Ans. $R_p = R_s + \frac{(\omega L_s)^2}{R_s}, L_p = L_s + \frac{R_s^2}{\omega^2 L_s}$

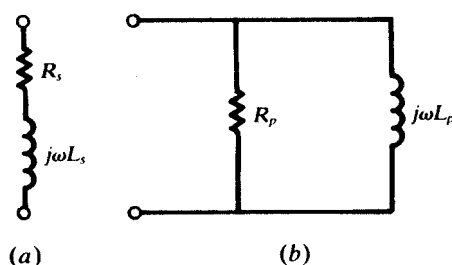


Fig. 9-42

- 9.45 In the network shown in Fig. 9-43 the 60-Hz current magnitudes are known to be: $I_T = 29.9 \text{ A}$, $I_1 = 22.3 \text{ A}$, and $I_2 = 8.0 \text{ A}$. Obtain the circuit constants R and L . Ans. $5.8 \Omega, 38.5 \text{ mH}$

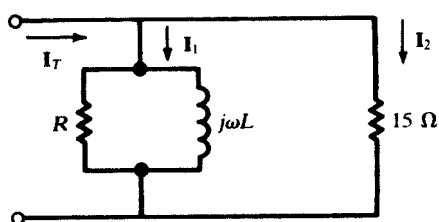


Fig. 9-43

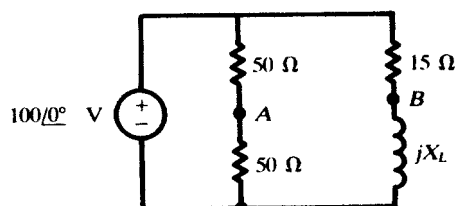


Fig. 9-44

- 9.46 Obtain the magnitude of the voltage V_{AB} in the two-branch parallel network of Fig. 9-44, if X_L is (a) $5\ \Omega$, (b) $15\ \Omega$, (c) $0\ \Omega$. *Ans.* 50 V , whatever X_L

- 9.47 In the network shown in Fig. 9-45, $V_{AB} = 36.1\ \angle 3.18^\circ\text{ V}$. Find the source voltage V . *Ans.* $75\ \angle -90^\circ\text{ V}$

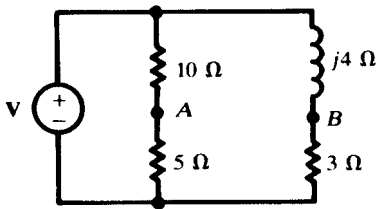


Fig. 9-45

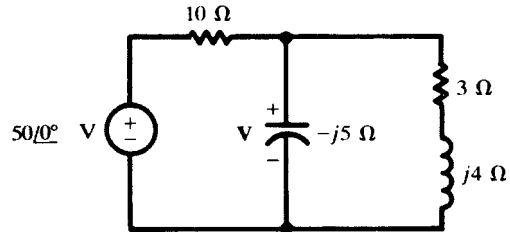


Fig. 9-46

- 9.48 For the network of Fig. 9-46 assign two different sets of mesh currents and show that for each, $\Delta_z = 55.9\ \angle -26.57^\circ\ \Omega^2$. For each choice, calculate the phasor voltage V . Obtain the phasor voltage across the $3 + j4\ \Omega$ impedance and compare with V . *Ans.* $V = V_{3+j4} = 22.36\ \angle -10.30^\circ\text{ V}$
- 9.49 For the network of Fig. 9-47, use the mesh current method to find the current in the $2 + j3\ \Omega$ impedance due to each of the sources V_1 and V_2 . *Ans.* $2.41\ \angle 6.45^\circ\text{ A}$, $1.36\ \angle 141.45^\circ\text{ A}$

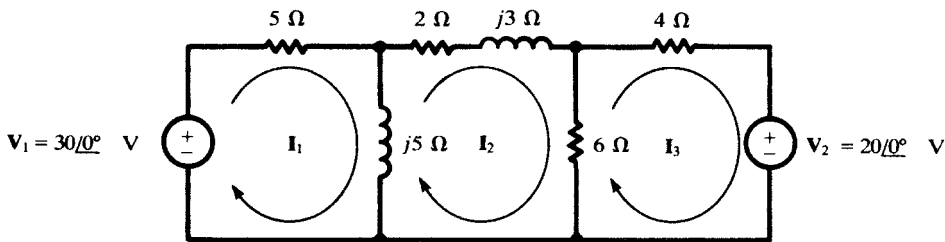


Fig. 9-47

- 9.50 In the network shown in Fig. 9-48, the two equal capacitances C and the shunting resistance R are adjusted until the detector current I_D is zero. Assuming a source angular frequency ω , determine the values of R_x and L_x . *Ans.* $R_x = 1/(\omega^2 C^2 R)$, $L_x = 1/(2\omega C)$

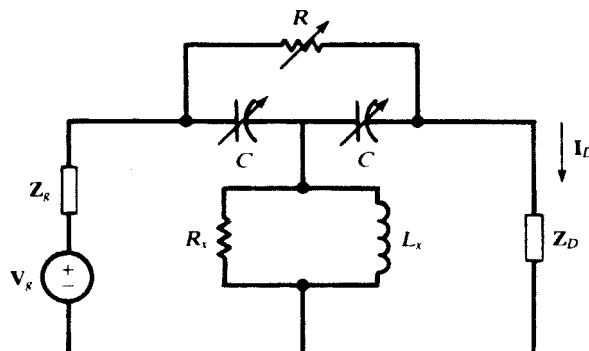


Fig. 9-48

- 9.51 For the network of Fig. 9-49, obtain the current ratio $\mathbf{I}_1/\mathbf{I}_3$. *Ans.* $3.3 \angle -90^\circ$

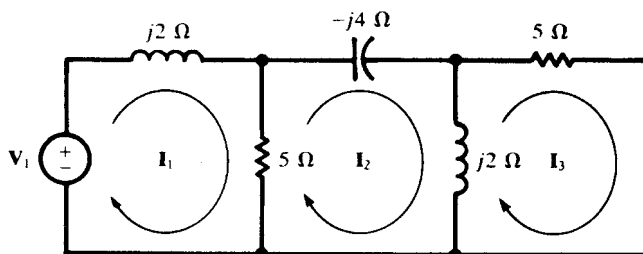


Fig. 9-49

- 9.52 For the network of Fig. 9-49, obtain $\mathbf{Z}_{\text{input},1}$ and $\mathbf{Z}_{\text{transfer},13}$. Show that $\mathbf{Z}_{\text{transfer},31} = \mathbf{Z}_{\text{transfer},13}$.
Ans. $1.31 \angle 21.8^\circ \Omega$, $4.31 \angle -68.2^\circ \Omega$

- 9.53 In the network of Fig. 9-50, obtain the voltage ratio $\mathbf{V}_1/\mathbf{V}_2$ by application of the node voltage method.

Ans. $\frac{\Delta_{11}}{\Delta_{12}} = 1.61 \angle -29.8^\circ$

- 9.54 For the network of Fig. 9-50, obtain the driving-point impedance $\mathbf{Z}_{\text{input},1}$. *Ans.* $5.59 \angle 17.35^\circ \Omega$

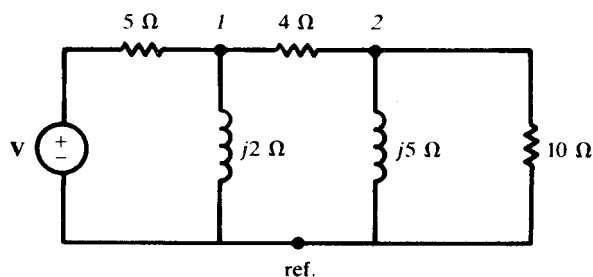


Fig. 9-50

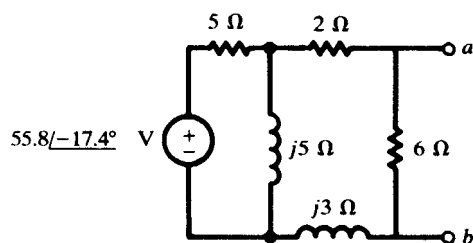


Fig. 9-51

- 9.55 Obtain the Thévenin and Norton equivalent circuits at terminals ab for the network of Fig. 9-51. Choose the polarity such that $\mathbf{V}' = \mathbf{V}_{ab}$. *Ans.* $\mathbf{V}' = 20.0 \angle 0^\circ \text{ V}$, $\mathbf{I}' = 5.56 \angle -23.06^\circ \text{ A}$, $\mathbf{Z}' = 3.60 \angle 23.06^\circ \Omega$

- 9.56 Obtain the Thévenin and Norton equivalent circuits at terminals ab for the network of Fig. 9-52.
Ans. $\mathbf{V}' = 11.5 \angle -95.8^\circ \text{ V}$, $\mathbf{I}' = 1.39 \angle -80.6^\circ \text{ A}$, $\mathbf{Z}' = 8.26 \angle -15.2^\circ \Omega$

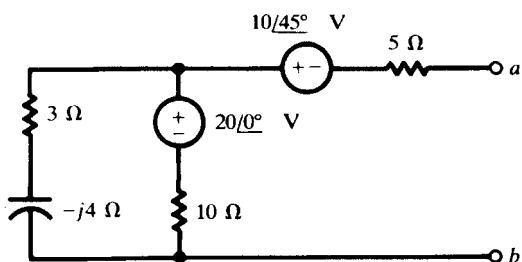


Fig. 9-52

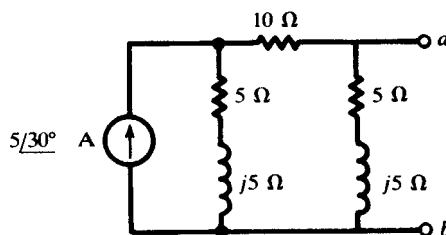


Fig. 9-53

- 9.57** Obtain the Thévenin and Norton equivalent circuits at terminals ab for the network of Fig. 9-53.

Ans. $V' = 11.18 \angle 93.43^\circ \text{ V}$, $I' = 2.24 \angle 56.56^\circ \text{ A}$, $Z' = 5.0 \angle 36.87^\circ \Omega$

- 9.58** In the circuit of Fig. 9-54, $v_1 = 10 \text{ V}$ and $v_2 = 5 \sin 2000t$. Find i .

Ans. $i = 1 - 0.35 \sin(2000t - 45^\circ)$

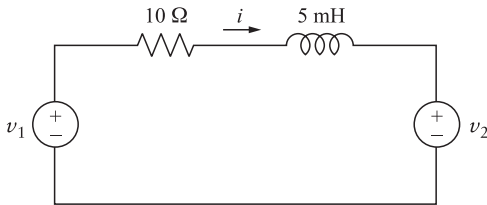


Fig. 9-54

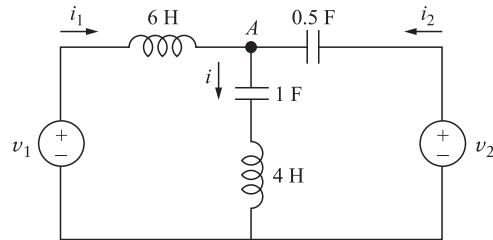


Fig. 9-55

- 9.59** In the circuit of Fig. 9-55, $v_1 = 6 \cos \omega t$ and $v_2 = \cos(\omega t + 60^\circ)$. Find v_A if $\omega = 2 \text{ rad/sec}$. *Hint:* Apply KCL at node A in the phasor domain. *Ans.* $v_A = 1.11 \sin 2t$

- 9.60** In the circuit of Problem 9.59 find phasor currents I_1 and I_2 drawn from the two sources. *Hint:* Apply phasor KVL to the loops on the left and right sides of the circuit.

Ans. $I_1 = 508 \angle -100.4^\circ$, $I_2 = 1057 \angle -145^\circ$, both in mA

- 9.61** Find v_A in the circuit of Problem 9.59 if $\omega = 0.5 \text{ rad/s}$. *Ans.* $V_a = 0$

- 9.62** In the circuit of Fig. 9-55, $v_1 = V_1 \cos(0.5t + \theta_1)$ and $v_2 = V_2 \cos(0.5t + \theta_2)$. Find the current through the 4 H inductor. *Ans.* $i = (V_2/4) \sin(0.5t + \theta_2) - (V_1/3) \sin(0.5t + \theta_1)$

- 9.63** In the circuit of Fig. 9-55, $v_1 = V_1 \cos(t + \theta_1)$ and $v_2 = V_2 \cos(t + \theta_2)$. Find v_A .

Ans. $v_A = \infty$, unless $V_1 = V_2 = 0$, in which case $v_A = 0$

- 9.64** In the circuit of Fig. 9-55, $v_1 = V_1 \cos(2t)$ and $v_2 = V_2 \cos(0.25t)$. Find v_A .

Ans. $v_A = -0.816V_1 \cos(2t) - 0.6V_2 \cos(0.25t)$