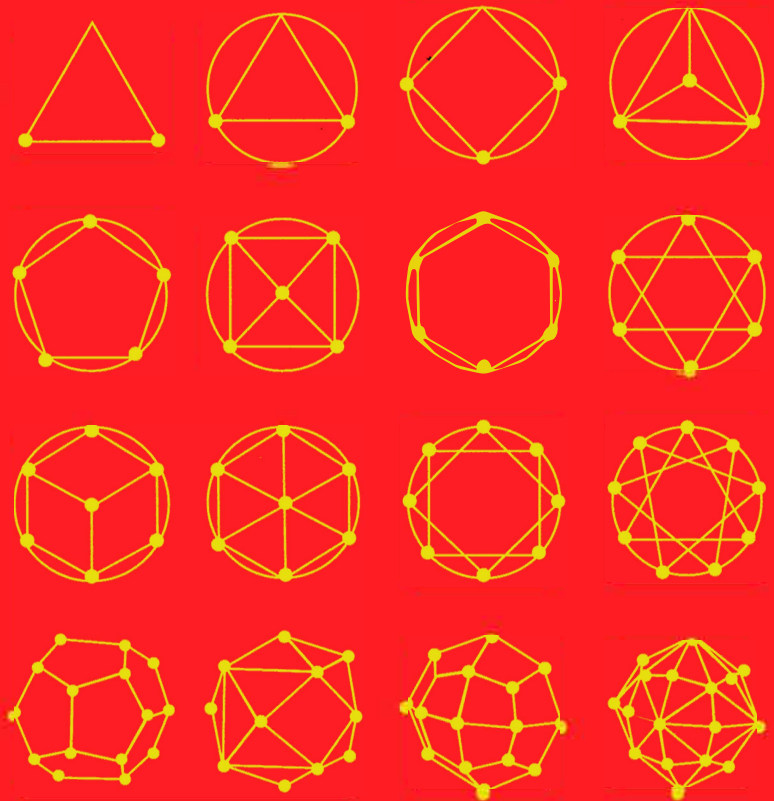




SIMPLY

MATHS



Facet Technologies, LLC
Exhibit 2055
LifeScan v. Facet
IPR2023-00712



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First published in Great Britain in 2022 by
Dorling Kindersley Limited
DK, One Embassy Gardens, 8 Viaduct Gardens,
London, SW11 7BV

The authorized representative in the EEA is
Dorling Kindersley Verlag GmbH, Arnulfstr. 124,
80636 Munich, Germany.

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10 9 8 7 6 5 4 3 2 1
001-325017-Feb/2022

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A CIP catalogue record for this book
is available from the British Library.
ISBN: 978-0-2415-1568-6

Printed and bound in China

For the curious
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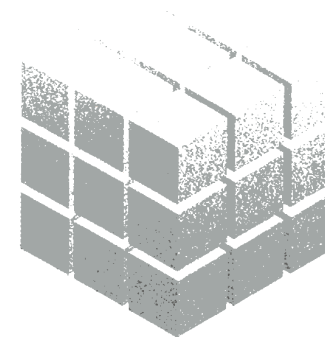
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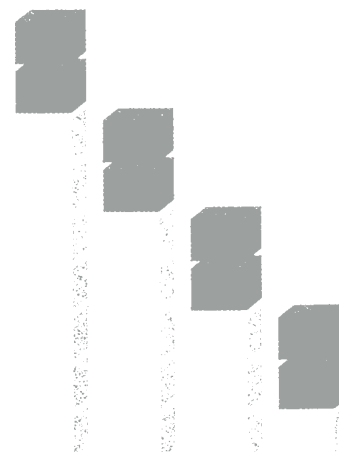
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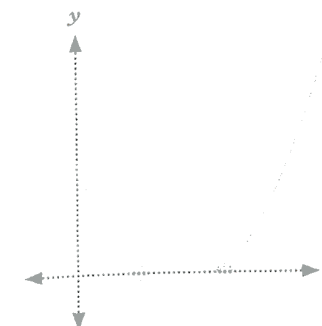
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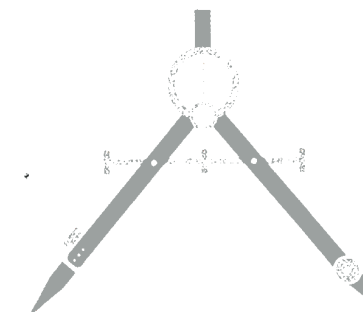
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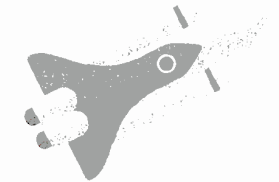
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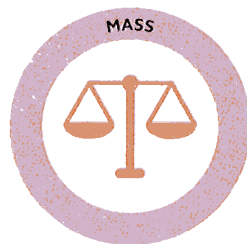


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M E A S U R E

Throughout history, humans have made sense of the physical world by describing it in terms of quantities such as length, mass, and time. Ancient civilizations developed many ways of measuring these quantities, culminating in the standard units we are familiar with today, such as metres, kilograms, and seconds. The modern world possesses instruments that can measure these quantities to an extremely high level of accuracy. Standardization creates a common language, while accuracy gives confidence that outcomes are reliable and predictable.

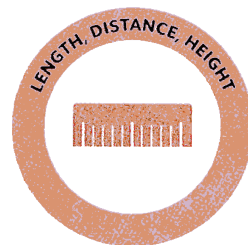


MASS

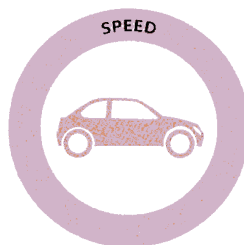
A range of measures
Different units of measurement are used to measure different things: from the speed of a moving object to the loudness of a sound. The latter is measured in decibels (db).



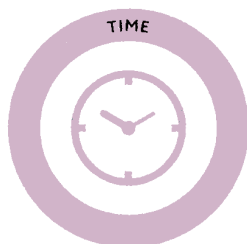
LOUDNESS



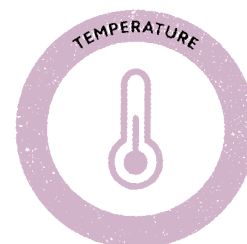
LENGTH, DISTANCE, HEIGHT



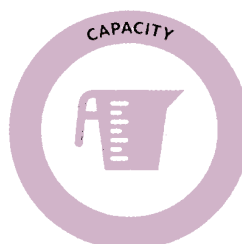
SPEED



TIME



TEMPERATURE



CAPACITY

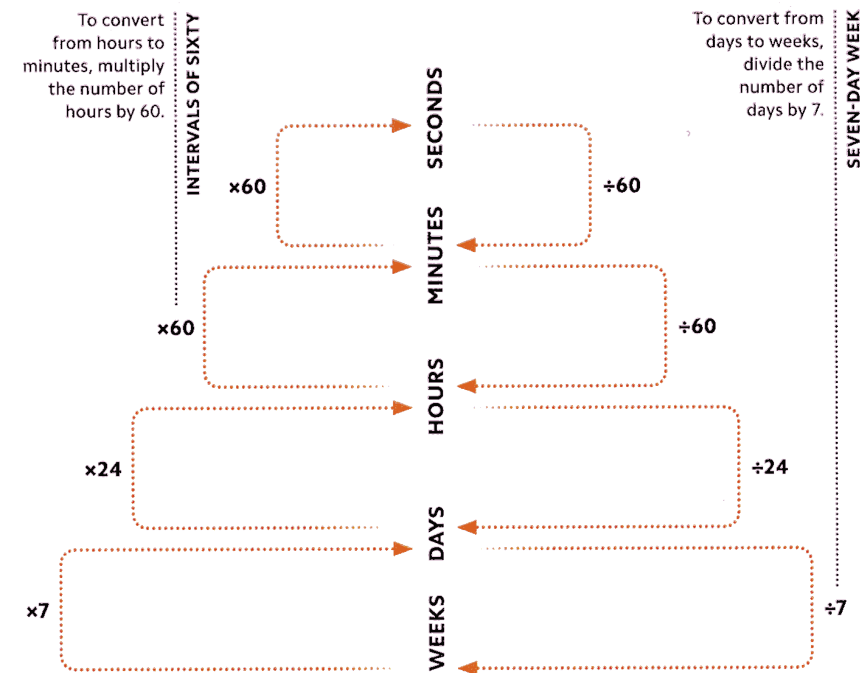
MADE TO MEASURE

A measurement is an attempt to quantify the size of something. There are two main systems of measurement: metric and imperial. They use different units of measurement. For example, in metric, distances are measured in lengths such as millimetres (mm), centimetres (cm), and kilometres (km). In imperial, inches (in), feet (ft), and miles are preferred. A measurement is only as accurate as the tool used to make it. Because of this, upper and lower bounds are useful for describing accuracy (see p.109).

A MATTER OF TIME

Time can be described as the duration of an event or the intervals between events. Standard units of time include seconds, minutes, hours, days, weeks, and years. Early civilizations derived some of these measurements from observing the Sun's movement in the sky.

- This is related to the rotation and orbit of Earth, which takes 24 hours (one day) to rotate on its axis and about 365 days (one year) to orbit the Sun.

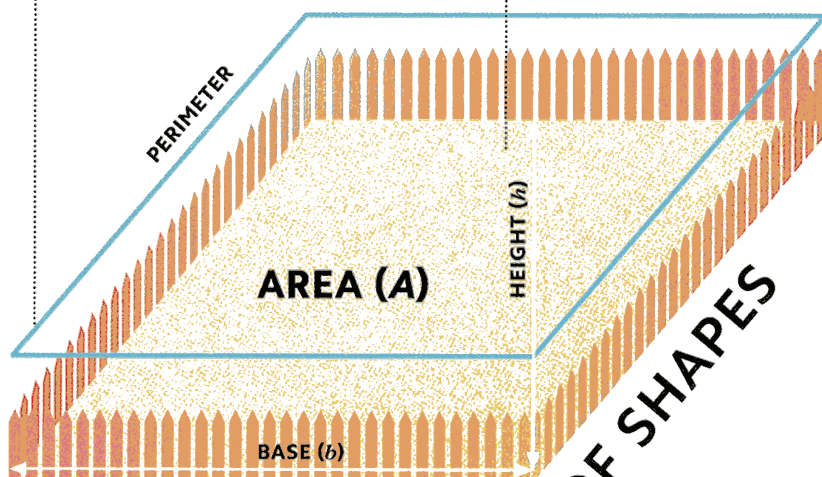


SHAPE'S EDGE

A parallelogram is made up of two pairs of identical, opposite sides. Adding the length of the base and an adjacent side and doubling the result will give the perimeter.

CALCULATING AREA

The formula to find the area of a parallelogram is $\text{Area} = \text{base} \times \text{height}$ ($A = b \times h$). For example, with a base of 5 m and a height of 3 m, the total area would be 15 m².



THE AREA

Area is the size of a two-dimensional surface. It is measured in square units of length such as square centimetres (cm²) and square metres (m²). Some shapes have a specific formula for calculating their area. A closed shape's perimeter is the sum of the lengths of its sides. Here, the garden is a parallelogram, the size of its surface is its area, and the sum of the garden's edges (the fence) is its perimeter.

TALKING IN CIRCLES

A circle is a round shape that consists of a single closed line made up of points a fixed distance from the circle's centre. A circle's circumference (perimeter) divided by its diameter gives π (pi), which is 3.1416 (to 4 decimal places). As this is true of all circles, π is useful for finding a circle's area or radius when one or the other is unknown. Circles also have unlimited lines of symmetry, and the smallest ratio of perimeter to area of any two-dimensional shape.

CIRCUMFERENCE

The total length of the circle's outer edge.

DIAMETER

A line that cuts a circle exactly in half.

RADIUS

A line from the centre to the outer edge. Two radii form a sector between them.

SEGMENT

Anatomy of a circle

A circle has many different parts formed by straight lines inside and outside of the circle. Each part has its own specific name.

CHORD

A straight line linking two points on the circumference. It divides a circle into two segments.

SECTOR

AREA

TANGENT

A line that passes a circle, touching it at one point.

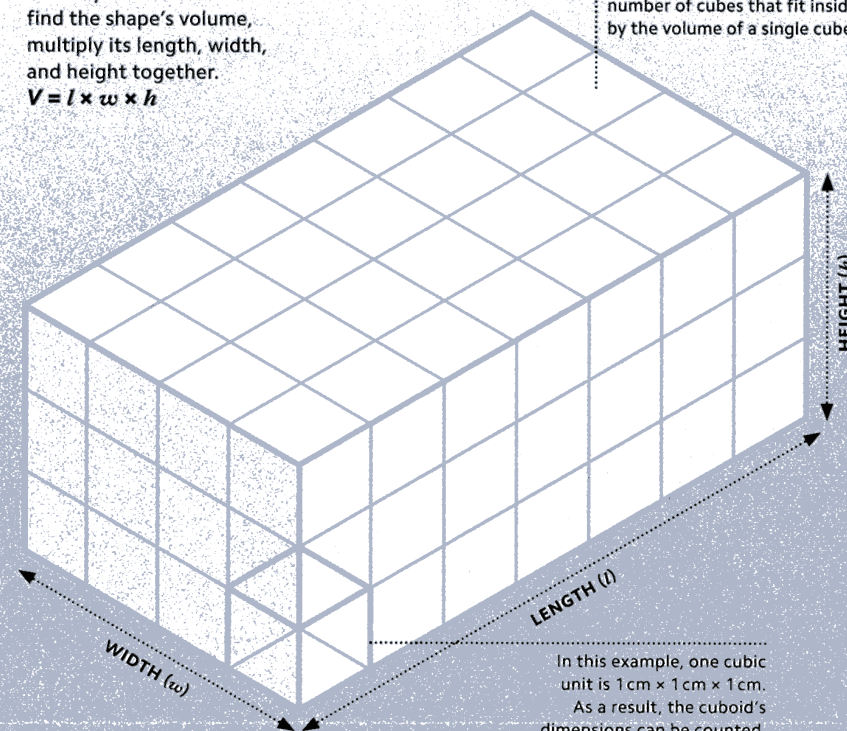
SPACE FILLER

The volume of an object is the amount of three-dimensional space it occupies. The volume of a solid, liquid, or gas is measured in cubic units. An object's volume can also be helpful for calculating its mass. Simple shapes often have a formula that can be used to find their volume. The volume contained within a hollow container is called its capacity.

Cuboid

The cubic units in a cuboid are easy to visualize. To find the shape's volume, multiply its length, width, and height together.

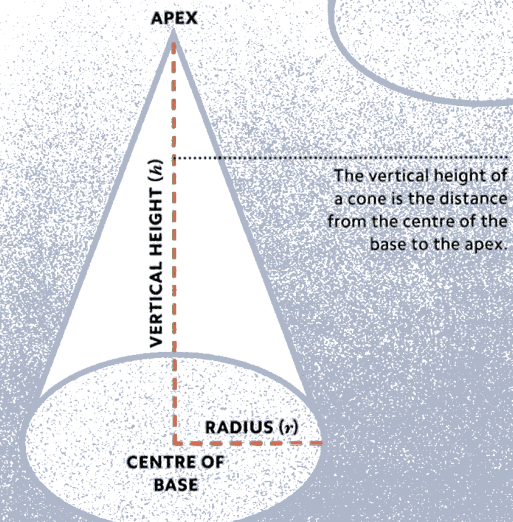
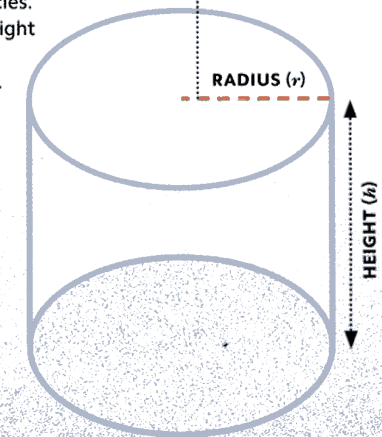
$$V = l \times w \times h$$



Cylinder

A cylinder is made up of two identical and parallel circles connected by a curved surface. Its height is the distance between the circles. A cylinder's radius and height must be known before its volume can be calculated.

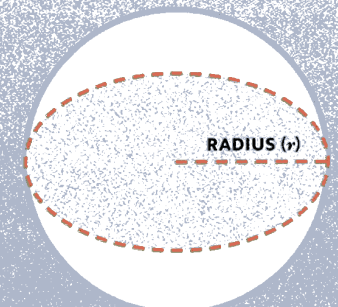
$$V = \pi \times r^2 \times h$$



Cone

A cone has a circular base connected to an apex (point) by a curved surface. To find a cone's volume, its radius and vertical height must be known.

$$V = \frac{1}{3} \times \pi \times r^2 \times h$$



Sphere

Every point on the surface of a sphere is the same distance from its centre. To calculate a sphere's volume, the only measurement required is the sphere's radius.

$$V = \frac{4}{3} \times \pi \times r^3$$

CYLINDER'S HEIGHT

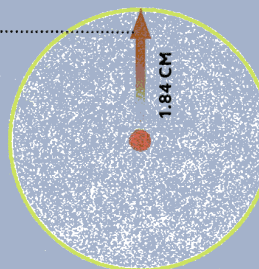
The height of the cylinder in its original state is now the width of this rectangle.

RECTANGULAR

A cylinder's middle section, unrolled, becomes a rectangle.

RADIUS

The circle has the same radius as the cylinder.

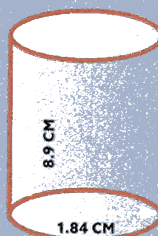


A NET RESULT

The total area of a three-dimensional object's surface is called its surface area. Knowing an object's surface area has many real-life uses, such as allowing us to find the exact amount of paint needed to cover the object.

The surface area of a building or living creature also determines how much heat is lost through its surface. It can be helpful to visualize a shape's surface area as if the shape has been flattened or opened out – this is called its net. Some three-dimensional shapes have their own formula for calculating their surface area.

8.9 CM



ORIGINAL FORM

A cylinder is made up of two identical circles connected by a curved surface.

Opened out flat

This is the net of a cylinder. The cylinder's surface area is the sum of the area of the rectangle and the circles.

A DEGREE OF ACCURACY

Defining the bounds

These scales measure an object's weight to the nearest 10 g. As a result, the exact weight of this apple could be anything from 75–85 g. This means the lower bound is 75 g and the upper bound is 85 g.

TO THE NEAREST 10 GRAMS

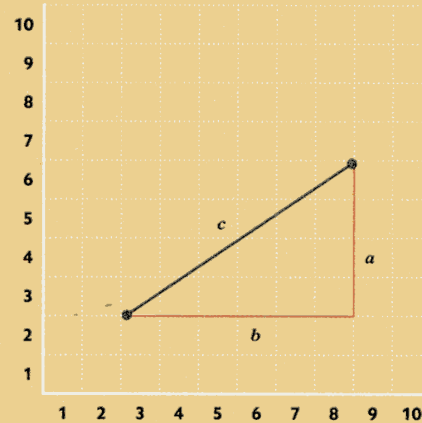
80g

No measurement can be exact. As a result, there are a number of ways to describe its accuracy. This includes to the nearest amount (such as the nearest 10 g), to the nearest percentage (such as the nearest 5%), or to a certain number of significant figures (e.g. 1.43 km, to 2 significant figures, is 1.4 km). The smallest amount that would round to the value of a measurement is called the lower bound, and the top limit is called the upper bound.

RIGHT-ANGLED TRIANGLES

Pythagoras' theorem derives its name from Pythagoras, the 6th-century-BCE mathematician and philosopher to whom it is traditionally attributed. This theorem allows the length of one side of a right-angled triangle to be found if the lengths of the other two sides are known. In algebra, this is phrased as $a^2 + b^2 = c^2$, where c is the hypotenuse (the side opposite the right angle) and a and b are the other sides.

"Reason is immortal, all else mortal."
Pythagoras



Finding the length

Create a right-angled triangle and write down Pythagoras' Theorem:

$$a^2 + b^2 = c^2$$

Measure lines a and b and input them into the formula:

$$4^2 + 6^2 = c^2$$

Calculate the square numbers:

$$16 + 36 = c^2$$

This gives the length of line c squared:

$$52 = c^2$$

Swap the order, and square root both sides:

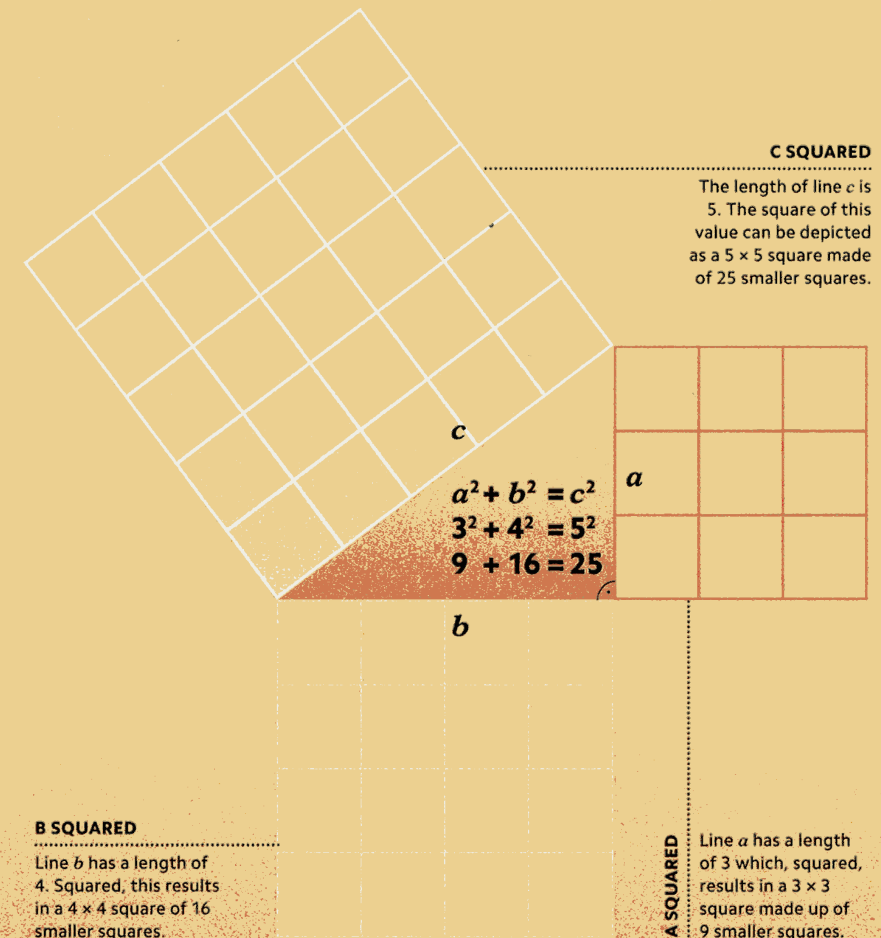
$$\sqrt{c^2} = \sqrt{52}$$

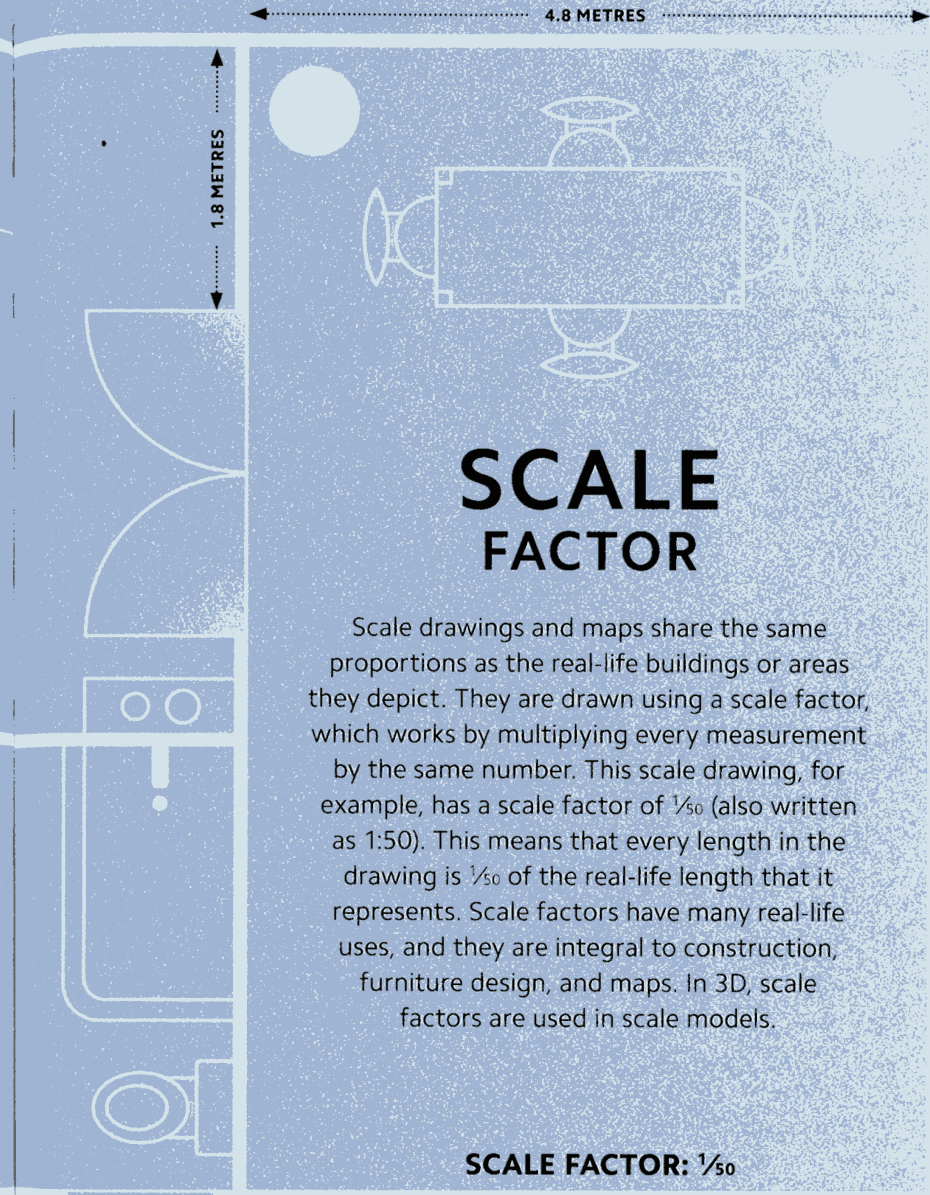
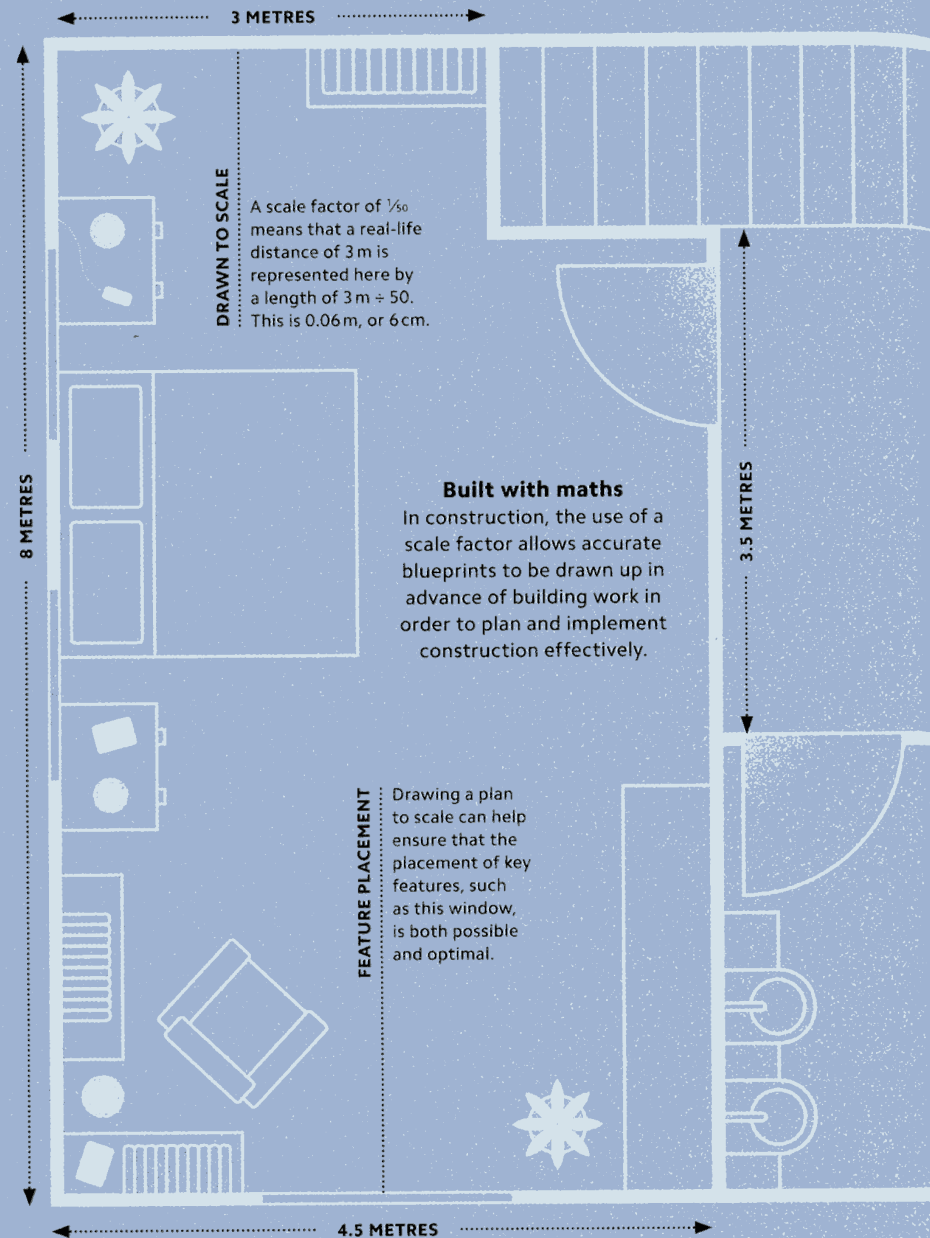
This gives the length of line c :

$$c = 7.2 \text{ (to 2 significant figures)}$$

Demonstrating Pythagoras

To demonstrate the theorem, draw the squared sides of the triangle as blocks of squares. Then count the number of individual squares within each block. The number of squares in a^2 and b^2 combined is equal to c^2 .





SCALE FACTOR

Scale drawings and maps share the same proportions as the real-life buildings or areas they depict. They are drawn using a scale factor, which works by multiplying every measurement by the same number. This scale drawing, for example, has a scale factor of $\frac{1}{50}$ (also written as 1:50). This means that every length in the drawing is $\frac{1}{50}$ of the real-life length that it represents. Scale factors have many real-life uses, and they are integral to construction, furniture design, and maps. In 3D, scale factors are used in scale models.



This doll is the same shape as the dolls to its side, but it is a different size. This means they are similar.

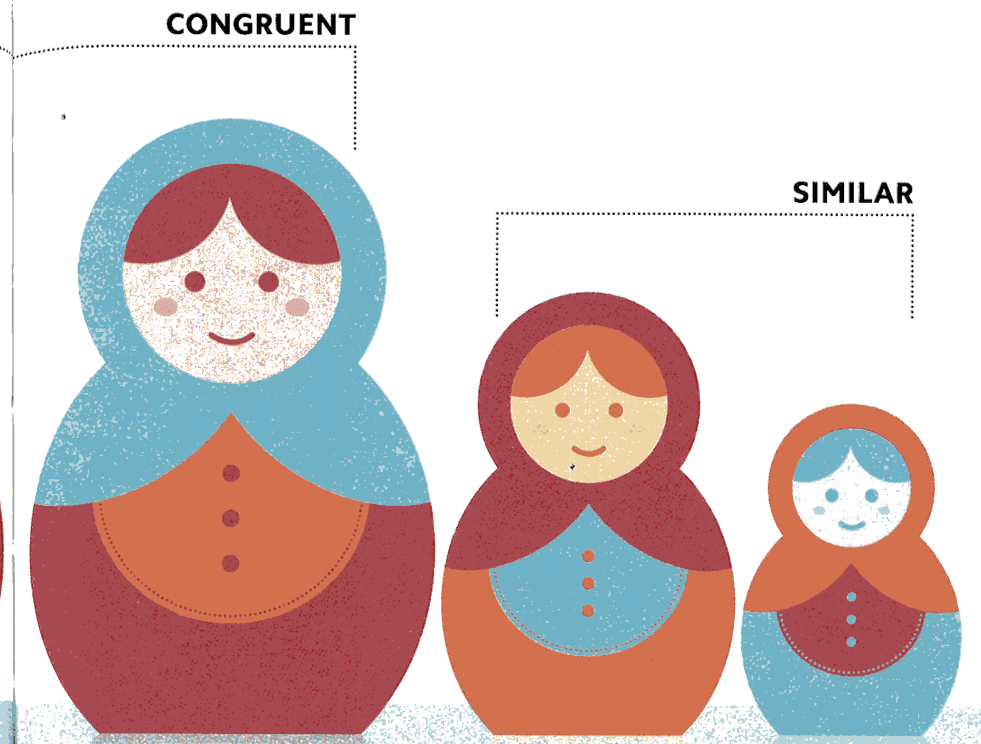
SIZE DIFFERENCE

These dolls are identical in size and shape and are therefore congruent.

IDENTICAL DOLLS

SIMILAR

In geometry, two shapes are referred to as similar if they share the same proportions. For example, if a shape's sides are double the length of another shape's sides, and both shapes have matching angles, they are similar. Furthermore, if two shapes have matching lengths in addition to matching angles they are referred to as congruent. The orientation or position of the shapes does not impact on whether they are similar or congruent. Congruence and similarity can also apply to three-dimensional objects such as dolls (see above).



CONGRUENT

SIMILAR

OR THE SAME?

Congruent or similar
Although these dolls share the same shape because their proportions are the same, some are larger than others. All of the dolls are similar, and the three pairs of identical size are congruent pairs.

Two congruent shapes will always share the same side lengths and angles.

UNKNOWN ANGLE

To find this angle, the side across from it (the tree's shadow) is labelled the opposite, while the tree's height is the adjacent.

Pythagoras' theorem (see pp.110–11) can be used to calculate this length as 11.0 m (to 3 significant figures).

UNKNOWN LENGTH

ADJACENT (6 M)

“Trigonometry ...
was not the work
of any one man
or nation.”

Carl Benjamin Boyer

Using a scientific calculator, divide the opposite by the adjacent. Using this number, press the inverse tangent button ($\tan^{-1}$) and enter it. This gives the angle as 56.9° (to 3 significant figures).

KNOWN LENGTHS

90°

CALCULATING WITH TRIANGLES

Trigonometry is based on the relationships between the side lengths and angles of triangles. In right-angled triangles, the longest side is the hypotenuse. The other two sides, relative to a specific angle, are called the opposite and the adjacent. The three main ratios in trigonometry are sine (sin), cosine (cos), and tangent (tan). The sine is the opposite divided by the hypotenuse, the cosine is the adjacent divided by the hypotenuse, and the tangent is the opposite divided by the adjacent. Because right-angled triangles with the same angles are similar (see pp.114–15) these ratios are always the same.

HYPOTENUSE

Choosing an angle and a ratio

Start by choosing an angle to find – in this case, the top left angle. This angle's opposite and adjacent lengths are both known. Because the tangent is derived from dividing the opposite side length by the adjacent side length, the tangent can be used to find the angle.

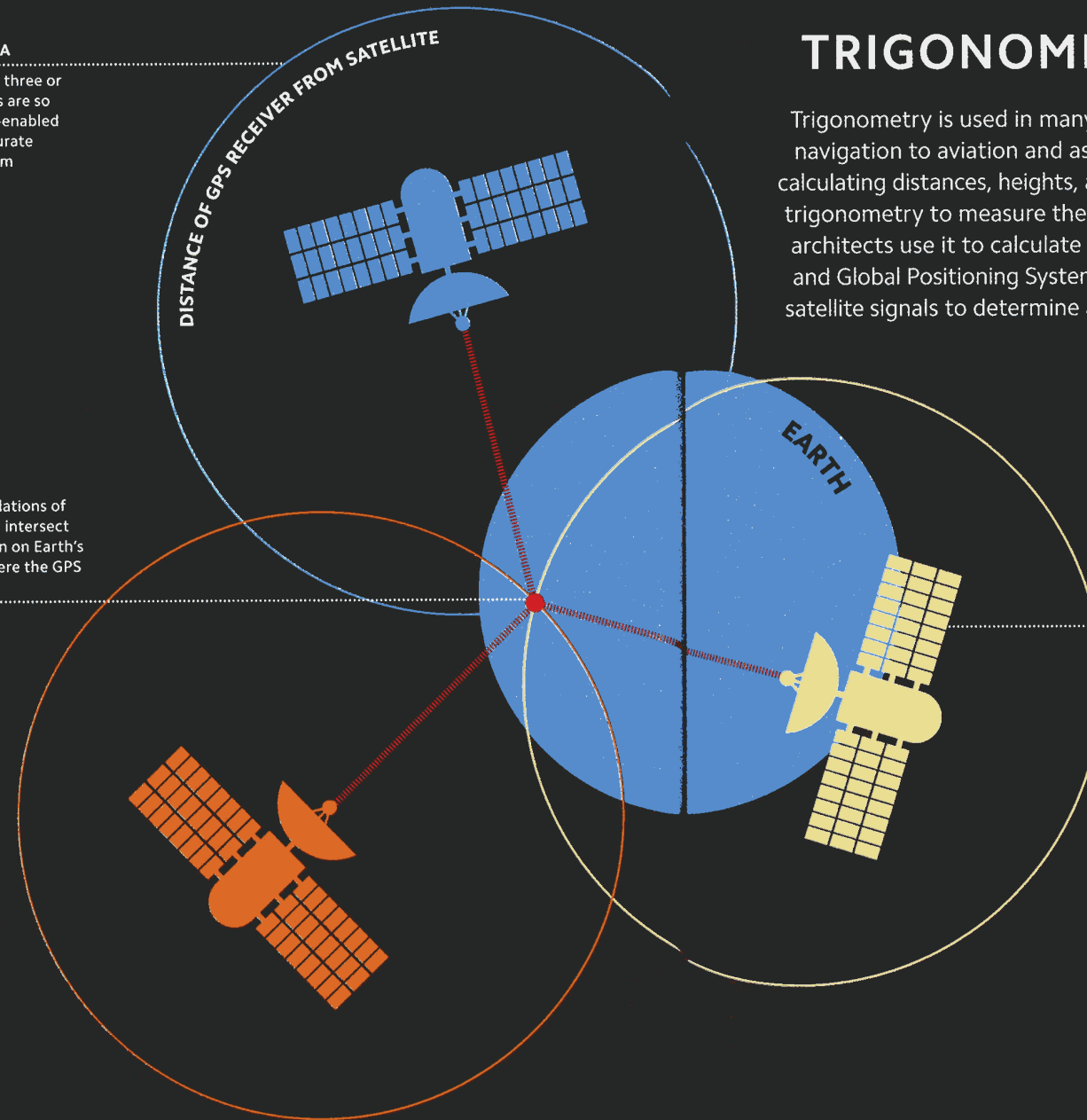
OPPOSITE (9.2 M)

NEAR-EXACT DATA

The calculations of three or more GPS satellites are so precise that a GPS-enabled smartphone is accurate to within around 5 m under open sky.

The distance calculations of the three satellites intersect at an exact location on Earth's surface. This is where the GPS receiver is located.

GPS RECEIVER



TRIGONOMETRY IN ACTION

Trigonometry is used in many different fields, from construction and navigation to aviation and astronomy. This is because it is useful for calculating distances, heights, and angles. For example, astronomers use trigonometry to measure the distance between stars in the night sky, architects use it to calculate the dimensions of buildings in planning, and Global Positioning Systems (GPS) rely on trigonometry applied to satellite signals to determine a GPS receiver's exact location on Earth.

SATELLITE

GPS satellites equipped with atomic clocks emit highly precise time stamps so that their location can be pinpointed at all times.

How does GPS work?

A GPS satellite works by giving its position in space and its distance from a GPS receiver on Earth. At least three satellites must do this at once for the intersection of their distance calculations to occur at a single point on Earth's surface. This process is known as trilateration.

OBLIQUE TRIANGLES

An oblique triangle is a triangle without a right angle. The sine and cosine rules are used to find an unknown side or unknown angle of an oblique triangle. They can only be used when some of the triangle's sides or angles are already known. Which rule is used is based on which values are known. When inputting known values into either sine or cosine rule, each angle (for example, A) is paired with its opposite side (a). Sine and cosine are abbreviated on a calculator keypad as sin and cos.

a

Variations of the rules
There are two versions of the sine rule: one for finding a side (see below) and one for finding an angle. Similarly there are two versions of the cosine rule: one for finding a side and another for finding an angle (see below).

SINE RULE
(UNKNOWN SIDE VERSION)

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

COSINE RULE
(UNKNOWN ANGLE VERSION)

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

C

b

A

B

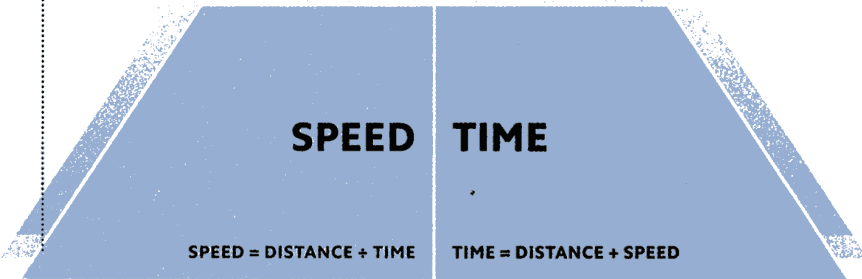
c

SPEED
The speed of a moving object can be found by dividing the amount of time it has been moving by the distance it has travelled.

DISTANCE
The distance an object has travelled can be found by multiplying its speed by the amount of time it has been moving.

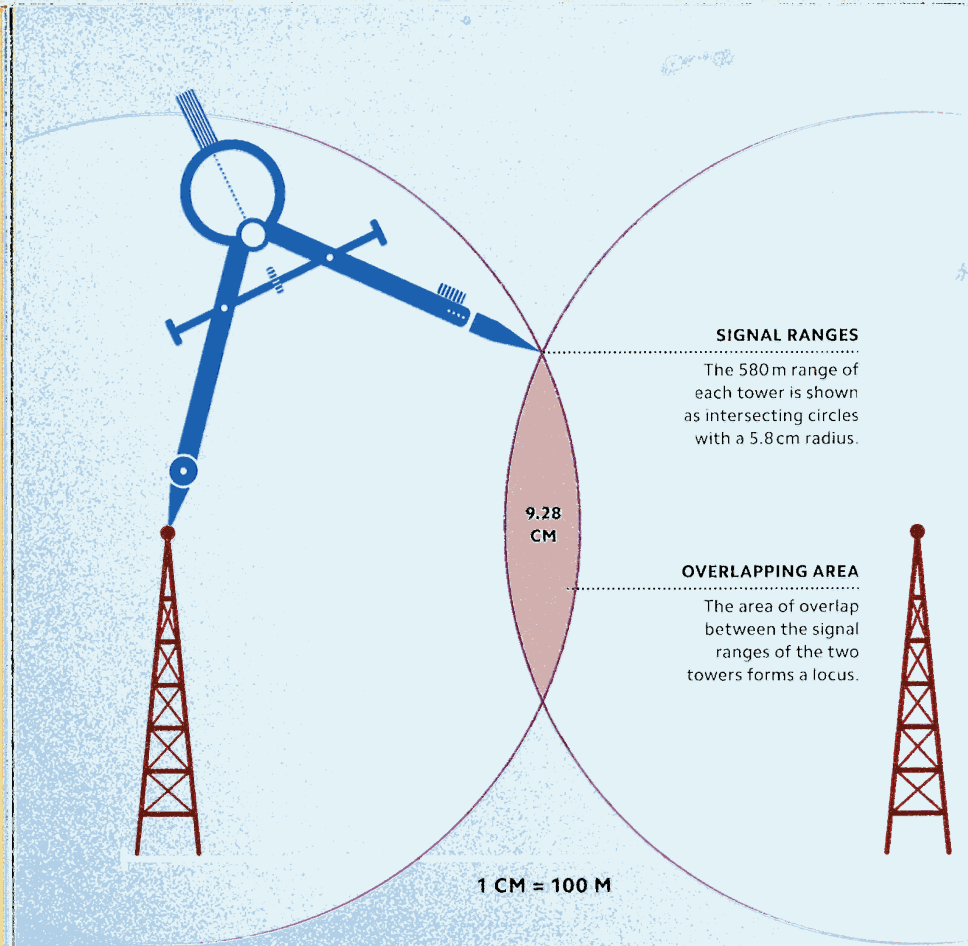
DISTANCE

DISTANCE = SPEED × TIME



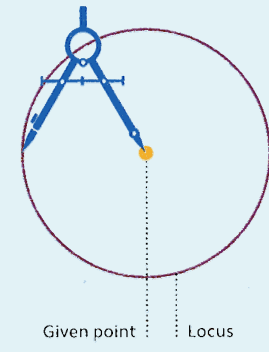
COMBINING UNITS

A compound measure uses two or more different units. For example, the speed of a moving object is often measured in kilometres per hour (kph) or miles per hour (mph). These compound measures account for both the amount of distance travelled and the amount of time that passes. Other compound measures include pressure, force, density, and the area and volume of objects (see pp.106–107).

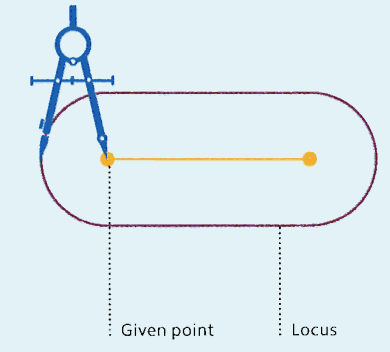


SETS OF POINTS

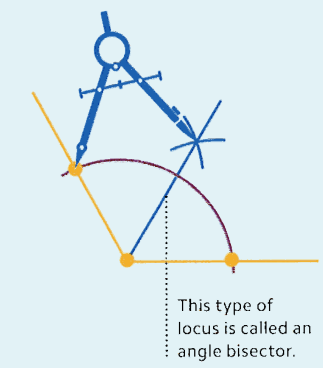
A construction is an accurate drawing made with a pair of compasses and a straight edge (usually a ruler). This could be a drawing of a locus (plural: loci) – a set of points which meet one or several conditions. One example of a locus is a circle (see opposite). Loci can be in 1D, 2D, or 3D. Loci can also be combined to form a new locus, such as in the simplified scale drawing of two transmission towers above. It shows a locus formed by the area of overlap between two loci (circles).



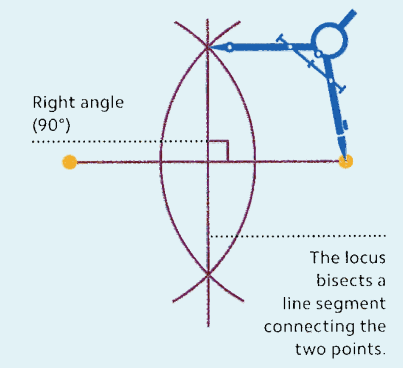
Set distance from a point
Place a pair of compasses at a given point and draw a full circle. This locus meets the condition of its points all being situated the same distance from its centre.



Set distance from a line segment
This locus consists of points a fixed distance from a line segment. Draw the semicircular ends with a pair of compasses, and then the straight sides with a ruler.



Equidistant from line segments
This locus is a straight line made up of a set of points that meet the condition of lying equidistant (the same distance) from the two intersecting yellow line segments.



Equidistant from two points
Drawing a locus equidistant from two points forms a perpendicular bisector: a straight line that passes (at 90°) through the midpoint of a line segment joining the two points.

STARTING POINT



INTENDED ROUTE

In a still body of water, the boat's route would be a vector of dimensions 30 across and 0 up or down.

INTENDED DIRECTION OF TRAVEL

DIRECTION OF CURRENT



INTENDED DESTINATION



MAGNITUDE AND DIRECTION

ACTUAL DIRECTION OF TRAVEL

HYPOTENUSE

With diagonal vectors, the vector forms the longest side of a right-angled triangle.

The vector moves 30 units to the right.

$\begin{pmatrix} 30 \\ -20 \end{pmatrix}$

The vector moves 20 units downwards.

VERTICAL DIRECTION

END POINT



A vector is a distance in a certain direction. Every vector has a size and a direction. In diagrams this is drawn as a line with an arrow on it. The length of the line indicates the size of the vector while the arrow signifies the direction. A vector is often expressed by giving the horizontal units (for example, metres) over the vertical units (see opposite). If the horizontal units are positive, the direction of the vector is right, and if negative, left. Similarly, if the vertical units are positive, the vector moves up, and if negative, down.

A diagonal journey

A boat sets out to sail to the opposite side of a 30m-wide river. En route, the river's current pushes the boat 20m downriver. The boat's path is a vector with dimensions 30 across and 20 down.

WHAT IS THE MATRIX?

A matrix (plural: matrices) is a rectangular grid of numbers or variables enclosed by square or curved brackets. Matrices record information in rows and columns that can be extended to hold huge amounts of data. They allow a computer to automate calculations and have applications in areas such as physics, computer science, and cryptography. Matrices can be added, subtracted, or multiplied.

They are also used to transform two-dimensional shapes (see opposite) and three-dimensional shapes.

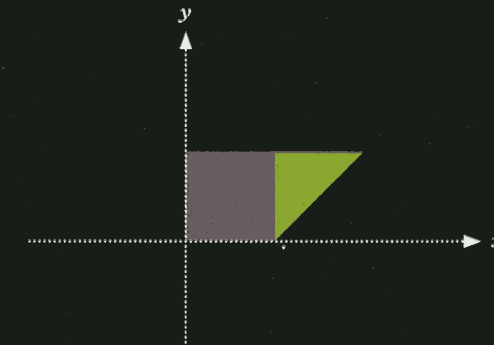
$$\begin{bmatrix} 3 & 8 \\ 4 & 6 \end{bmatrix} + \begin{bmatrix} 4 & 0 \\ 1 & -9 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5 & -3 \end{bmatrix}$$

$3 + 4 = 7$

Manipulating matrices

To add or subtract matrices, each element in the first matrix is added to or subtracted from the corresponding element in the second matrix. To multiply matrices, each row in the first matrix is multiplied by each column in the second matrix.

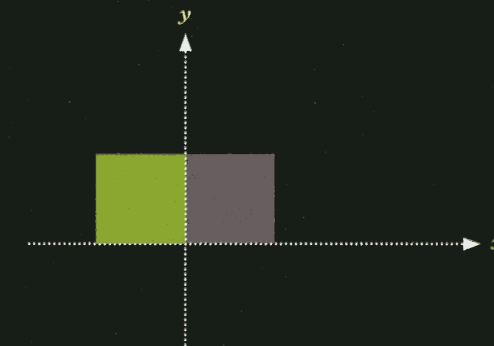
The first verified usage of matrices occurred in ancient China in the 2nd century BCE.



Horizontal shear

This matrix applies a horizontal shear to the original square (purple), slanting it to the right.

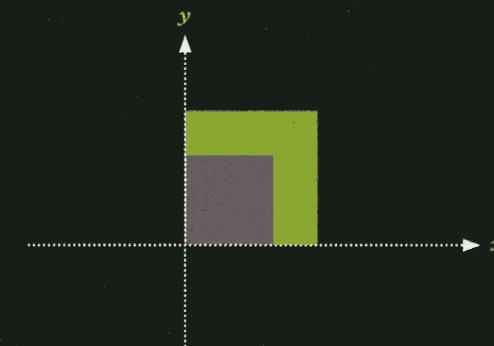
$$\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$



Reflection

This matrix reflects the original square (purple) across a line of symmetry at the y-axis.

$$\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$



Enlargement

This matrix enlarges the original square (purple) by factor 1.5.

$$\begin{bmatrix} 1.5 & 0 \\ 0 & 1.5 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix}$$