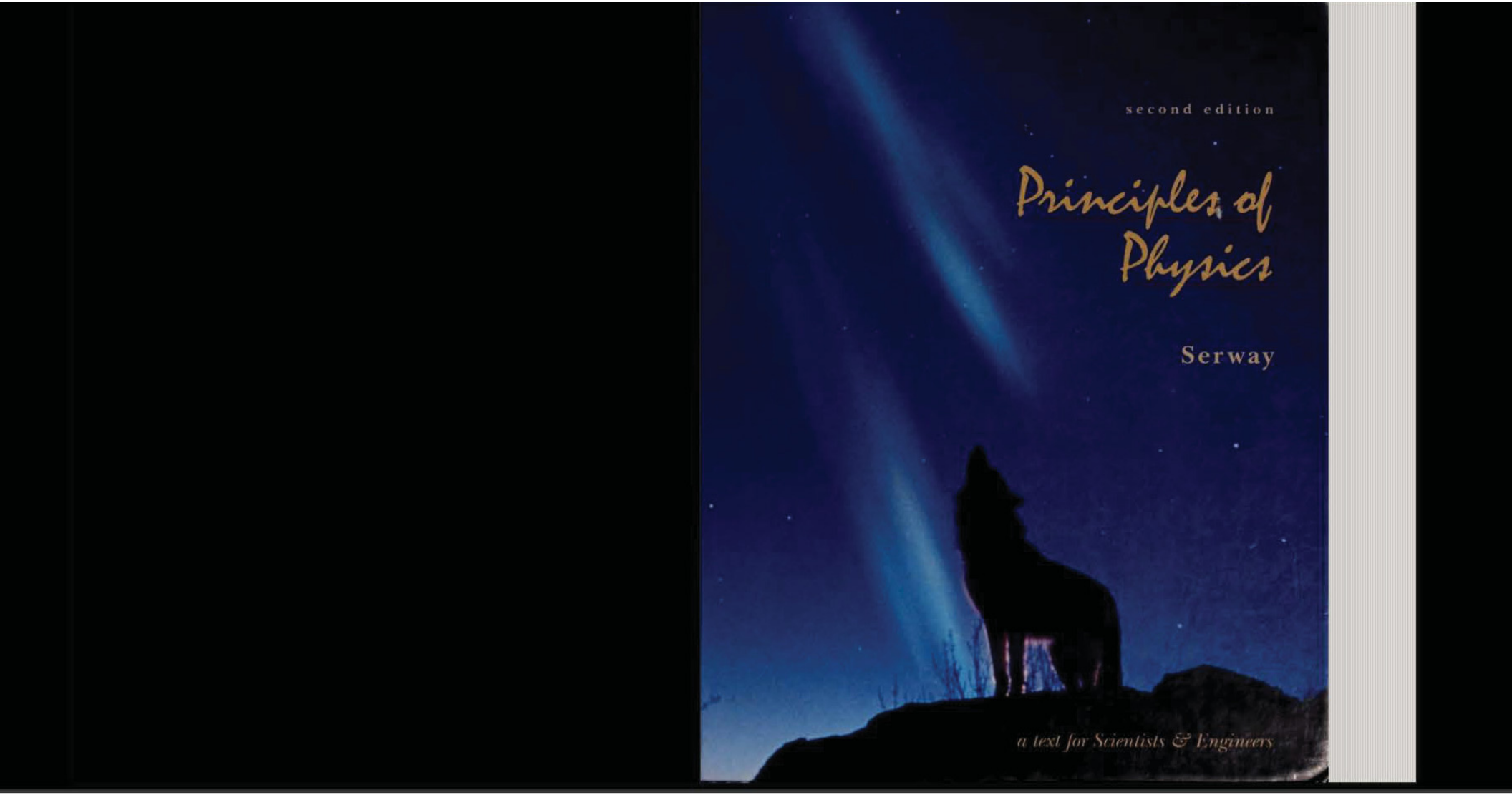


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PEDAGOGICAL USE OF COLOR

Mechanics

Displacement and position vectors

Velocity vectors (**v**)
Velocity component vectors

Force vectors (**F**)
Force component vectors

Acceleration vectors (**a**)
Acceleration component vectors

Torque ($\boldsymbol{\tau}$) and angular momentum (**L**) vectors

Linear or rotational motion directions

Springs

Pulleys

Electricity and Magnetism

Electric fields

Magnetic fields

Positive charges

Negative charges

Resistors

Batteries and other dc power supplies

Switches

Capacitors

Inductors (coils)

Voltmeters

Ammeters

Galvanometers

ac generators

Ground symbol

Light and Optics

Light rays

Lenses and prisms

Mirrors

Objects

Images

Some Fundamental Constants[a]

Quantity	Symbol	Value[b]
Atomic mass unit	u	$1.660\,540\,2(10) \times 10^{-27}$ kg 931.494 32(28) MeV/c^2
Avogadro's number	N_A	$6.022\,136\,7(36) \times 10^{23}$ $(\text{mol})^{-1}$
Bohr magneton	$\mu_B = \frac{e\hbar}{2m_e}$	$9.274\,015\,4(31) \times 10^{-24}$ J/T
Bohr radius	$a_0 = \frac{\hbar^2}{m_e e^2 k_e}$	$0.529\,177\,249(24) \times 10^{-10}$ m
Boltzmann's constant	$k_B = R/N_A$	$1.380\,658(12) \times 10^{-23}$ J/K
Compton wavelength	$\lambda_C = \frac{h}{m_e c}$	$2.426\,310\,58(22) \times 10^{-12}$ m
Deuteron mass	m_d	$3.343\,586\,0(20) \times 10^{-27}$ kg 2.013 553 214(24) u
Electron mass	m_e	$9.109\,389\,7(54) \times 10^{-31}$ kg $5.485\,799\,03(13) \times 10^{-4}$ u 0.510 999 06(15) MeV/c^2
Electron-volt	eV	$1.602\,177\,33(49) \times 10^{-19}$ J
Elementary charge	e	$1.602\,177\,33(49) \times 10^{-19}$ C
Gas constant	R	8.314 510(70) J/K·mol
Gravitational constant	G	$6.672\,59(85) \times 10^{-11}$ N·m^2/kg^2
Hydrogen ground state	$E_0 = \frac{m_e e^4 k_e^2}{2\hbar^2} = \frac{e^2 k_e}{2a_0}$	13.605 698(40) eV
Josephson frequency-voltage ratio	$2e/h$	$4.835\,976\,7(14) \times 10^{14}$ Hz/V
Magnetic flux quantum	$\Phi_0 = \frac{h}{2e}$	$2.067\,834\,61(61) \times 10^{-15}$ Wb
Neutron mass	m_n	$1.674\,928\,6(10) \times 10^{-27}$ kg 1.008 664 904(14) u 939.565 63(28) MeV/c^2
Nuclear magneton	$\mu_n = \frac{e\hbar}{2m_p}$	$5.050\,786\,6(17) \times 10^{-27}$ J/T
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ N/A^2 (exact)
Permittivity of free space	$\epsilon_0 = 1/\mu_0 c^2$	$8.854\,187\,817 \times 10^{-12}$ C^2/N·m^2 (exact)
Planck's constant	h $\hbar = h/2\pi$	$6.626\,075(40) \times 10^{-34}$ J·s $1.054\,572\,66(63) \times 10^{-34}$ J·s
Proton mass	m_p	$1.672\,623(10) \times 10^{-27}$ kg 1.007 276 470(12) u 938.272 3(28) MeV/c^2
Quantized Hall resistance	h/e^2	25812.805 6(12) Ω
Rydberg constant	R_H	$1.097\,373\,153\,4(13) \times 10^7$ m^{-1}
Speed of light in vacuum	c	$2.997\,924\,58 \times 10^8$ m/s (exact)

[a] These constants are the values recommended in 1986 by CODATA, based on a least-squares adjustment of data from different measurements. For a more complete list, see Cohen, E. Richard, and Barry N. Taylor, *Rev. Mod. Phys.* **59**:1121, 1987.

[b] The numbers in parentheses for the values below represent the uncertainties in the last two digits.

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Principles of Physics

SECOND EDITION

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Principles *of* Physics

SECOND EDITION

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James Madison University

with contributions by

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Preface

Principles of Physics is designed for a one-year introductory calculus-based physics course for engineering and science students. This second edition contains many new pedagogical features—most notably, an increased emphasis on physical concepts through the use of conceptual examples and conceptual problems. Based on comments from users of the first edition and reviewers' suggestions, a major effort was made to improve organization, clarity of presentation, precision of language, and accuracy throughout.

This project was conceived because of the well-known problem we continue to wrestle with in teaching the introductory calculus-based physics course. The course content (and hence the size of textbooks) continues to grow, while the number of contact hours with students has either dropped or remained unchanged. Furthermore, traditional one-year courses cover little if any 20th-century physics.



Bruce Byers, FPG

In preparing this book, I was motivated by the spreading interest in reforming this course, primarily through efforts of the Introductory University Physics Project (IUPP) sponsored by the American Association of Physics Teachers and American Institute of Physics. The primary goals and guidelines of this project are to:

- reduce course content following the "less may be more" theme
- incorporate contemporary physics naturally into the course
- organize the course in the context of one or more "story lines"
- treat all student constituents equitably

Recognizing a need for a textbook that could meet these guidelines several years ago, I studied the various proposed IUPP models and the many reports from IUPP committees. Eventually, I became actively involved in the review and planning of one specific model, initially developed at the U.S. Air Force Academy, entitled "A Particles Approach to Introductory Physics." I spent part of the summer of 1990 at the Academy working with Colonel

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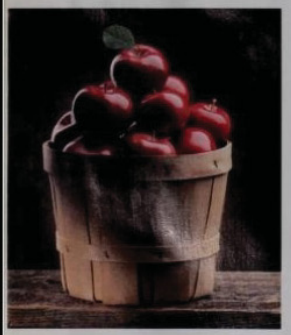
James Head and Lt. Col. Rolf Enger, the primary authors of the IUPP model, and other members of that department. This most useful collaboration was the starting point of this project.

In my opinion, the IUPP model developed at the U.S. Air Force Academy, and modified by a team of people over the last few years, was an excellent choice for designing a reformed curriculum for several reasons:

- It is an evolutionary approach (rather than a revolutionary approach), which should meet the current demands of the physics community.
- It deletes many topics in classical physics (such as alternating current circuits and optical instruments), and places less emphasis on rigid body motion, optics, and thermodynamics.
- Some topics in 20th century physics are introduced early in the textbook such as special relativity, energy and momentum quantization, and the Bohr model of the hydrogen atom.
- A deliberate attempt is made at showing the unity of physics.

CHANGES TO THE SECOND EDITION

A number of changes and improvements have been made in the second edition of this text. Many changes are in response to comments and suggestions provided by reviewers of the manuscript and instructors using the first edition. The following represent the major changes in the second edition:



Superstock

Organization The organization of the textbook is slightly different from that of the first edition. Chapter 6 in the first edition has been deleted, and the sections dealing with the fundamental forces in nature and the gravitational field are now incorporated in Chapter 5. The material on oscillatory motion and waves (Chapters 21, 22, and 23 in the first edition) is now covered earlier (Chapters 12, 13, and 14 in the second edition), following mechanics. The two chapters in the first edition covering interference and diffraction of light waves (Chapters 27 and 28) have been combined and condensed into one chapter entitled Wave Optics (Chapter 27). The section on polarization of light waves is now treated earlier in Chapter 24 entitled Electromagnetic Waves. Finally, by popular demand, Chapter 15 entitled Fluid Mechanics is new to this edition.

Content In this second edition, a concerted effort was made to place more emphasis on critical thinking and teaching physical concepts. This was accomplished by the addition of approximately 150 conceptual examples, called **Thinking Physics,** and over 200 **Conceptual Problems.** Most of the **Thinking Physics** examples, which include reasoning statements, provide students with a means of reviewing the concepts presented in that section. Some examples demonstrate the connection between the content of that chapter and other scientific disciplines. These examples also serve as models for students to respond to the **Conceptual Problems** posed in the text and end-of-chapter conceptual questions. The answers to all **Conceptual Problems** are contained at the end of each chapter.

Problems and Conceptual Questions A substantial revision of the end-of-chapter problems and conceptual questions was made in this second edition. Most of the new problems that have been added are intermediate in level, and many of them are problems that require students to make order-of-magnitude calculations or estimates. All problems have been carefully edited and reworded where necessary. Solutions to approximately 25 percent of the end-of-chapter problems and selected conceptual questions are included in the Study Guide and Student Solutions Manual. These problems and questions are identified by boxes around their numbers.

Significant Figures Significant figures in both worked examples and end-of-chapter problems have been handled with care. Most numerical examples and problems are worked out to three significant figures.

TEXTBOOK FEATURES

The textbook includes many features which are intended to enhance its usefulness to both the student and instructor:

Style I have attempted to write the book in a style that is clear, logical, and succinct. At the same time, my relaxed writing style is meant to make the reading more enjoyable and less intimidating.

Previews Most chapters begin with a chapter preview, which includes a brief discussion of chapter objectives and content. This feature enables the student to better understand how the topics covered in that chapter fit into the overall structure and objectives of the course.

Mathematical Level Calculus is introduced gradually, where it is needed, keeping in mind that many students often take a course in calculus concurrently. Most steps are shown when basic equations are developed, and reference is made to mathematical appendices provided at the end of the text. Vector products are covered where they are needed in physical applications.

Conceptual Examples and Problems As mentioned earlier, one of the objectives of this edition is to place more emphasis on critical thinking and conceptual understanding. To meet this objective, I have included many **Thinking Physics** examples and **Conceptual Problems** throughout each chapter. The reasoning or responses to all examples immediately follow the example statement, while the answers to all conceptual problems are included at the end of each chapter. Ideally, the student will use these features to better understand physical concepts before being presented with quantitative examples and working homework problems.

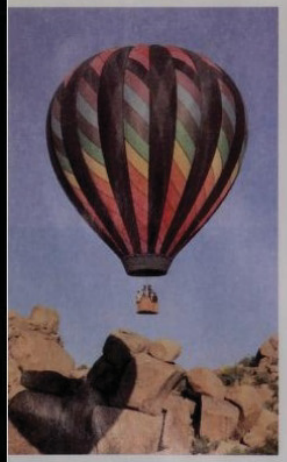
Worked Examples Every chapter includes several worked examples of varying difficulty which are intended to reinforce conceptual understanding and to serve as models for solving end-of-chapter problems.

Exercises Many examples are followed immediately by exercises with answers. Those exercises contained in an example box represent extensions of the worked examples. Other related exercises with answers are included at the end of many

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sections. All exercises are intended to encourage students to practice problem solving for immediate reinforcement, and to test the student's understanding of concepts and problem-solving skills. Students who work through these exercises on a regular basis should find the end-of-chapter problems less intimidating.

Important Statements and Equations Most important statements and definitions are set in **boldface** type for added emphasis and ease of review. Important equations are highlighted with a tan screen for review or reference.



Russell Schlepman

Illustrations The text material, worked examples, and end-of-chapter questions and problems are accompanied by numerous figures, photographs, and tables. Full color is used to add clarity to the figures and to make the visual presentations as realistic as possible. Three-dimensional effects are produced with the use of color airbrushed areas, where appropriate. Vectors are color-coded, and curves in *xy*-plots are drawn in color. Color photographs have been carefully selected, and their accompanying captions have been written to serve as an added instructional tool.

Margin Notes Margin notes are used to help students locate and review important statements, concepts, and equations.

Units The international system of units (SI) is used throughout the text. The British engineering system of units is used only to a limited extent in chapters on mechanics, heat, and thermodynamics.

Problem-Solving Strategies and Hints I have included general strategies and hints for solving the types of problems featured in both the worked examples and in the end-of-chapter problems. This feature will help students identify important steps and understand the logic for solving specific classes of problems.

Summaries Each chapter contains a summary which reviews the important concepts and equations discussed in that chapter.

Conceptual Questions A list of conceptual questions is provided at the end of each chapter. These questions, which require verbal or written responses, provide the student with a means of self-testing the concepts presented in the chapter. Others could serve as a basis for initiating classroom discussions. Answers to selected questions, indicated by boxes around their numbers, are included in the Study Guide and Student Solutions Manual.

End-of-Chapter Problems An extensive set of problems is included at the end of each chapter. Answers to odd-numbered problems are given at the end of the book; these pages have colored edges for ease of location. For the convenience of both the student and the instructor, about two thirds of the problems are keyed to specific sections of the chapter. The remaining problems, labeled "Additional Problems," are not keyed to specific sections. There are three levels of problems according to their level of difficulty. Straightforward problems are numbered in black, intermediate level problems are numbered in blue, and the most challenging problems are numbered in magenta.

Spreadsheet Problems Most chapters include several spreadsheet problems following the end-of-chapter problem sets. Spreadsheet modeling of physical phenomena enables the student to obtain graphical representations of physical phenomena and perform numerical analyses without the burden of having to learn a high-level computer language. Spreadsheets are particularly useful in exploratory investigations; "what if" questions can be addressed easily and depicted graphically.

The level of difficulty in the spreadsheet problems, as with all end-of-chapter problems, is indicated by the color of the problem number. For the most straightforward problems (black), a disk with spreadsheet problems is provided. The student must enter the pertinent data, vary the parameters, and interpret the results. Intermediate level problems (blue) usually require students to modify an existing template to perform the required analysis. The more challenging problems (magenta) require students to develop their own spreadsheet templates. Brief instructions on using the templates are provided in Appendix E.

Appendices and Endpapers Several appendices are provided at the end of the textbook, including the new appendix with instructions for problem-solving with spreadsheets. Most of the appendix material represents a review of mathematical techniques used in the text, including scientific notation, algebra, geometry, trigonometry, differential calculus, and integral calculus. Reference to these appendices is made throughout the text. Most mathematical sections include worked examples and exercises with answers. In addition to the mathematical reviews, the appendices contain tables of physical data, conversion factors, atomic masses, the SI units of physical quantities, and the periodic chart. The endpapers contain other useful information including the color code used in the text, lists of fundamental constants and physical data, conversions, and the Greek alphabet.

SAUNDERS CORE CONCEPTS IN PHYSICS CD-ROM

Saunders Core Concepts in Physics CD-ROM applies the power of multimedia to the calculus-based course, offering unparalleled full-motion animation and video, engaging interactive graphics, clear and concise text, and guiding narration. *Saunders Core Concepts in Physics* focuses on those concepts students typically find most difficult in the course, drawing from mechanics, thermodynamics, electromagnetism, and optics. The CD-ROM also presents step-by-step explorations of essential mathematics, problem-solving strategies, and animations of problems to promote conceptual understanding and sharpen problem-solving skills. Problems on this CD-ROM are taken primarily from Serway's *Principles of Physics*, 2/e. Available for Macintosh and Windows.

STUDENT ANCILLARIES

Study Guide and Student Solutions Manual by John R. Gordon, Ralph McGrew, Steve Van Wyk, and Ray Serway. The manual features detailed solutions to 25 percent of the end-of-chapter problems and selected conceptual questions from the text. These are indicated in the text with boxed numbers. The manual also features a skills section, important notes from key sections of the text, and a list of important equations and concepts.



Courtesy of NASA

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Spreadsheet Templates The Spreadsheet Template Disk contains spreadsheet files designed to be used with the end-of-chapter problems entitled Spreadsheet Problems. The files have been developed in Microsoft Excel 5.0 for the Macintosh, and in several formats for Windows and DOS users (see Appendix E). These can be used with most spreadsheet programs including all the recent versions of Lotus 1-2-3, Excel for Windows, Quattro Pro, and f(g) Scholar. Over 30 templates are provided on the disk.

Spreadsheet Investigations in Physics by Lawrence B. Golden and James R. Klein. This workbook with the accompanying disk illustrates how spreadsheets can be used for solving many physics problems. The workbook is divided into two parts. The first part consists of spreadsheet tutorials, while the second part is a short introduction to numerical methods. The tutorials include basic spreadsheet techniques and emphasize navigating the spreadsheet, entering data, constructing formulas, and graphing. The numerical methods include differentiation, integration, interpolation, and the solution of differential equations. Many examples and exercises are provided. Step-by-step instructions are given for constructing numerical models of selected physics problems. The exercises and examples used to illustrate the numerical methods are chosen from introductory physics and mathematics.

Pocket Guide by V. Gordon Lind. This 5" by 7" notebook is a section-by-section capsule of the textbook that provides a handy guide for looking up important concepts, formulas, and problem-solving hints.

Mathematical Methods for Introductory Physics with Calculus by Ronald C. Davidson. This brief book is designed for students who find themselves unable to keep pace in their physics class because of a lack of familiarity with the necessary mathematical tools. *Mathematical Methods* provides an overview of those mathematical topics that are needed in an introductory-level physics course through the use of many worked examples and exercises.

So You Want to Take Physics: A Preparation for Scientists and Engineers by Rodney Cole. This text is useful to those students who need additional preparation before or during a course in physics. The book includes typical problems with worked solutions, and a review of techniques in mathematics and physics. The friendly, straightforward style makes it easier to understand how mathematics is used in the context of physics.

Life Science Applications for Physics compiled by Jerry Faughn. This supplement provides examples, readings, and problems from the biological sciences as they relate to physics. Topics include "Friction in Human Joints," "Physics of the Human Circulatory System," "Physics of the Nervous System," and "Ultrasound and Its Applications." This supplement is useful in those courses having a significant number of pre-med students.

Physics Laboratory Manual by David Loyd supplements the learning of basic physical principles while introducing laboratory procedures and equipment. Each chapter of the laboratory manual includes a pre-laboratory assignment, a list of objectives, an equipment list, the theory behind the experiment, the experimental procedure to be followed, calculations, graphs, and questions. In addition, laboratory report templates are provided for each experiment so the student can record data, calculations, and experimental results.

INSTRUCTOR ANCILLARIES

Instructor's Solutions Manual by Steve Van Wyk, Ralph McGrew, and Ray Serway. This manual contains solutions to all of the problems in the text and provides answers to even-numbered problems. All solutions have been carefully reviewed for accuracy.

Printed Test Bank by Myron Schneiderwent. This test bank offers over 1500 multiple choice and conceptually oriented critical thinking questions.

Computerized Test Bank Available for the IBM PC (DOS and Windows) and Macintosh computers, this computerized version of the printed test bank also contains over 1500 questions. The test bank enables the instructor to create unique tests and permits the editing of questions as well as addition of new questions. The program gives answers to all problems and prints each answer on a separate grading key.

Overhead Transparency Acetates A set of 175 transparencies with approximately 300 full-color figures from the text.

Instructor's Manual to Accompany Physics Laboratory Manual by David Loyd. Each chapter contains a discussion of the experiment, teaching hints, answers to selected questions, and a post-laboratory quiz with short answer and essay questions. Also included is a list of suppliers of scientific equipment and a summary of all the equipment needed for the laboratory experiments in the manual.

Physics Demonstrations Videotape by J. C. Sprott. This video features two hours of demonstrations divided into 12 primary topics. Each topic contains between four and nine demonstrations for a total of 70 physics demonstrations.



Guy Sauvage, Photo Researchers, Inc.

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TEACHING OPTIONS

Although many topics found in traditional textbooks have been omitted from this textbook, instructors may find that the current text still contains more material than can be covered in a two-semester sequence. For this reason, I would like to offer the following suggestions. If you wish to place more emphasis on contemporary topics in physics, you should consider omitting parts or all of Chapters 15, 16, 17, 18, 24, 25, and 26. On the other hand, if you wish to follow a more traditional approach which places more emphasis on classical physics, you could omit Chapters 9, 11, 28, 29, 30, and 31. Either approach can be used without any loss in continuity. Other teaching options would fall somewhere in between these two extremes by omitting the sections labeled "Optional."

ACKNOWLEDGMENTS

As any author well knows, a project of this magnitude would be an impossible task without the assistance of many talented and dedicated individuals whose comments, criticisms, and suggestions are so important in developing a finished product. In preparing the second edition of this textbook, I have been guided by the expertise of many people who reviewed part or all of the manuscript. Special thanks go to John Jewett, Jr., Ron Bieniek, and Ralph McGrew who provided in-depth reviews of parts or all of the manuscript at various stages and offered many suggestions for its improvement. Laurent Hodges wrote several new end-of-chapter problems. I am most grateful to the following reviewers of the manuscript for their valuable feedback:

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Some end-of-chapter problems in Chapters 3, 6, and 8 of this second edition came from *University Physics*, 2nd edition by Alvin Hudson and Rex Nelson (Saunders College Publishing, 1990) or from *University Physics*, 2nd edition by George Arfken, David Griffing, Donald Kelly and Joseph Priest (Harcourt Brace Jovanovich, 1989). I thank the authors of these problems.

I am indebted to Colonel James Head and Lt. Col. Rolf Enger of the U.S. Air Force Academy for inspiring me to begin work on this project while I was at the Academy during the summer of 1990, and for their warm hospitality during my visit. They and their colleagues designed the original IUPP model upon which this textbook is based. Special thanks go to Ralph McGrew for assembling the problem sets, writing new problems, and checking solutions to all problems. I appreciate the tedious work of Ralph McGrew and Steve Van Wyk in writing the Instructor's Solutions Manual, and I thank Gloria Langer for her fine work in preparing this manual. I am grateful to John R. Gordon, Ralph McGrew, and Steve Van Wyk who co-authored the Study Guide and Student Solutions Manual, and to Linda Miller and Michael Rudmin for their assistance in its preparation. Dena Digilio-Betz was very helpful in locating many of the photographs used in this text. During the development of this text, I benefited from many useful discussions with my colleagues and other physics instructors including Robert Bauman, Don Chodrow, Jerry Faughn, John R. Gordon, Kevin Giovanetti, Dick Jacobs, Clem Moses, Dorn Peterson, Joseph Rudmin, and Gerald Taylor. I owe a debt of gratitude to Irene Nunes for clarifying much of the material in the first edition, and for eliminating many inconsistencies, redundancies, and other nightmares that authors must face. Special thanks and recognition go to the professional staff at Saunders College Publishing for their careful attention to detail and for their dedication to producing first quality textbooks, especially Susan Dust Pashos, Beth Ahrens, Alexandra Buczek, Sally Kusch, Carol Bleistine, and Angus McDonald. I sincerely appreciate the wisdom and enthusiasm of my publisher and good friend John Vondeling, who continues to publish high quality instructional products for science education.

Finally, I thank my entire family for their endless love, understanding, and continued encouragement, especially my beautiful and devoted wife Elizabeth, who is always at my side to share my joys and sorrows.

Ray Serway
Harrisonburg, VA
JULY 1997



Michel Gouverneur, Photo News, Gamma Sport

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To the Student

I feel it is appropriate to offer some words of advice which should be of benefit to you, the student. Before doing so, I will assume that you have read the Preface, which describes the various features of the text that will help you through the course.

HOW TO STUDY

Very often instructors are asked "How should I study physics and prepare for examinations?" There is no simple answer to this question, but I would like to offer some suggestions based on my own experiences in learning and teaching over the years.

First and foremost, maintain a positive attitude towards the subject matter, keeping in mind that physics is the most fundamental of all natural sciences. Other science courses that follow will use the same physical principles, so it is important that you understand and be able to apply the various concepts and theories discussed in the text.



Robin Smith/Tony Stone Images

CONCEPTS AND PRINCIPLES

It is essential that you understand the basic concepts and principles *before* attempting to solve assigned problems. This is best accomplished through a careful reading of the textbook before attending your lecture on that material. In the process, it is useful to jot down certain points which are not clear to you. Take careful notes in class, and then ask questions pertaining to those ideas that require clarification. Keep in mind that few people are able to absorb the full meaning of scientific material after one reading. Several readings of the text and notes may be necessary. Your lectures and laboratory work should supplement the text and clarify some of the more difficult material. You should reduce memorization of material to a minimum. Memorizing passages from a text, equations, and derivations does not necessarily mean you understand the material. Your understanding of the material will be enhanced through a combination of efficient study habits, discussions with other students and instructors,

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and your ability to solve the problems in the text. Ask questions whenever you feel it is necessary.

STUDY SCHEDULE

It is important to set up a regular study schedule, preferably on a daily basis. Make sure to read the syllabus for the course and adhere to the schedule set by your instructor. The lectures will be much more meaningful if you read the corresponding textual material before attending the lecture. As a general rule, you should devote about two hours of study time for every hour in class. If you are having trouble with the course, seek the advice of the instructor or students who have taken the course. You may find it necessary to seek further instruction from experienced students. Very often, instructors will offer review sessions in addition to regular class periods. It is important that you avoid the practice of delaying study until a day or two before an exam. More often than not, this will lead to disastrous results. Rather than an all-night study session, it is better to briefly review the basic concepts and equations, followed by a good night's rest. If you feel in need of additional help in understanding the concepts, preparing for exams, or in problem-solving, we suggest that you acquire a copy of the Study Guide/Student Solutions Manual which accompanies the text and should be available at your college bookstore.

USE THE FEATURES

You should make full use of the various features of the text discussed in the Preface. For example, marginal notes are useful for locating and describing important equations and concepts, while important statements and definitions are highlighted in **boldface** type or with a blue box. Many useful tables are contained in appendices, but most are incorporated in the text where they are used most often. Appendix B is a convenient review of mathematical techniques. Answers to odd-numbered problems are provided at the end of the book. Exercises (with answers), which follow some worked examples, represent extensions of those examples, and in most cases you are expected to perform a simple calculation. Their purpose is to test your problem-solving skills as you read through the text. Problem-Solving Strategies and Hints are included in selected chapters throughout the text to give you additional information to help you solve problems. An overview of the entire text is given in the table of contents, while the index will enable you to locate specific material quickly. Footnotes are sometimes used to supplement the discussion or to cite other references on the subject. Many chapters include problems that require the use of programmable calculators, computers, or simulation software. Consult your instructor; it may be possible to obtain some of the relevant software from the Saunders physics Web site. Spreadsheet problems can be solved either analytically or with the use of the *Spreadsheet Investigations* supplement. These are intended for those courses that place some emphasis on numerical methods. You may want to develop appropriate programs for some of these problems even if they are not assigned by your instructor.



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After reading a chapter, you should be able to define any new quantities introduced in that chapter, and discuss the principles and assumptions that were used to arrive at certain key relations. The chapter summaries and the review sections of the Study Guide/Student Solutions Manual should help you in this regard. In some cases, it will be necessary to refer to the index of the text to locate certain topics. You should be able to correctly associate with each physical quantity a symbol used to represent that quantity and the unit in which the quantity is specified. Furthermore, you should be able to express each important relation in a concise and accurate prose statement.

THE IMPORTANCE OF PROBLEM SOLVING

R.P. Feynman, Nobel laureate in physics, once said, "You do not know anything until you have practiced." In keeping with this statement, I strongly advise you to develop the skills necessary to solve a wide range of problems. Your ability to solve problems will be one of the main tests of your knowledge of physics; therefore, you should try to solve as many problems as possible. It is essential that you understand basic concepts and principles before attempting to solve problems. It is good practice to try to find alternate solutions to the same problem. For example, problems in mechanics can be solved using Newton's laws, but very often an alternative method using energy considerations is more direct. You should not deceive yourself into thinking you understand the problem after seeing its solution in class. You must be able to solve the problem and similar problems on your own.

The method of solving problems should be carefully planned. A systematic plan is especially important when a problem involves several concepts. First, read the problem several times until you are confident you understand what is being asked. Look for any key words that will help you interpret the problem, and perhaps allow you to make certain assumptions. Your ability to interpret the question properly is an integral part of problem solving. You should acquire the habit of writing down the information given in a problem, and decide what quantities need to be found. You might want to construct a table listing quantities given and quantities to be found. This procedure is sometimes used in the worked examples of the text. After you have decided on the method you feel is appropriate for the situation, proceed with your solution. General problem-solving strategies of this type are included in the text and are highlighted by a tan screen.

I often find that students fail to recognize the limitations of certain formulas or physical laws in a particular situation. It is very important that you understand and remember the assumptions which underlie a particular theory or formalism. For example, certain equations in kinematics apply only to a particle moving with constant acceleration. These equations are not valid for situations in which the acceleration is not constant, such as the motion of an object connected to a spring, or the motion of an object through a fluid.

GENERAL PROBLEM-SOLVING STRATEGY

Most courses in general physics require the student to learn the skills of problem solving, and examinations are largely composed of problems that test such skills. This brief section describes some useful ideas which will enable you to increase your

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accuracy in solving problems, enhance your understanding of physical concepts, eliminate initial panic or lack of direction in aproaching a problem, and organize your work. One way to help accomplish these goals is to adopt a problem-solving strategy. Many chapters in this text will include a section labeled "Problem-Solving Strategies and Hints" which should help you through the "rough spots."

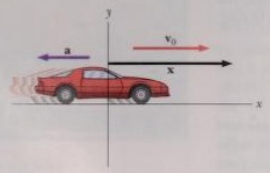
In developing problem-solving strategies, five basic steps are commonly used, and are exemplified in the sample Example below.

1. Draw a suitable diagram with appropriate labels and coordinate axes if needed.
2. As you examine what is being asked in the problem, identify the basic physical principle (or principles) that are involved, listing the knowns and unknowns.
3. Select a basic relationship or derive an equation that can be used to find the unknown, and solve the equation for the unknown symbolically.
4. Substitute the given values along with the appropriate units into the equation.
5. Obtain a numerical value for the unknown. The problem is verified and receives a check mark if the following questions can be properly answered: Do the units match? Is the answer reasonable? Is the plus or minus sign proper or meaningful?

One of the purposes of this strategy is to promote accuracy. Properly drawn diagrams can eliminate many sign errors. Diagrams also help to isolate the physical principles of the problem. Symbolic solutions and carefully labeled knowns and unknowns will help eliminate other careless errors. The use of symbolic solutions should help you think in terms of the physics of the problem. A check of units at the end of the problem can indicate a possible algebraic error. The physical layout and organization of your problem will make the final product more understandable and easier to follow. Once you have developed an organized system for examining problems and extracting relevant information, you will become a more confident problem solver.

Example

A person driving in a car at a speed of 20 m/s applies the brakes and stops in a distance of 100 m. What was the acceleration of the car?



Given:

$x_0 = 0$ m

$x = 100$ m

$v_0 = 20$ m/s

$v = 0$ m/s

$a = ?$

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$a = \frac{v^2 - v_0^2}{2(x - x_0)}$$

$$a = \frac{(0\ \text{m/s})^2 - (20\ \text{m/s})^2}{2(100\ \text{m})} = -2.0\ \text{m/s}^2$$

Check units for consistency:

$$\frac{\text{m}^2/\text{s}^2}{\text{m}} = \frac{\text{m}}{\text{s}^2}$$

EXPERIMENTS

Physics is a science based upon experimental observations. In view of this fact, I recommend that you try to supplement the text through various type of "hands-on" experiments, either at home or in the laboratory. These can be used to test ideas and models discussed in class or in the text. For example, the common "Slinky" toy is excellent for studying traveling waves; a ball swinging on the end of a long string can be used to investigate pendulum motion; various masses attached to the end of a vertical spring or rubber band can be used to determine their elastic nature; an old pair of Polaroid sunglasses and some discarded lenses and magnifying glass are the components of various experiments in optics; you can get an approximate measure of the acceleration of gravity simply by dropping a ball from a known height and measuring the time of its fall with a stopwatch. The list is endless. When physical models are not available, be imaginative and try to develop models of your own.



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THINKING PHYSICS AND CONCEPTUAL PROBLEMS

These new features are intended to develop your conceptual understanding of physics. You should be able to answer all **Conceptual Problems** based on previous content in the chapter. (Answers are provided at the chapter's end, but please try to answer them on your own first.) **Thinking Physics** examples provide the model for the form your answers should take. The answers ("Reasoning") for **Thinking Physics** are provided immediately for you because these examples often extend and further develop the chapter content. They may, for instance, illuminate the connection between physics and other scientific disciplines.

AN INVITATION TO PHYSICS

It is my sincere hope that you too will find physics an exciting and enjoyable experience, and that you will profit from this experience, regardless of your chosen profession. Welcome to the exciting world of physics.

The scientist does not study nature because it is useful; he studies it because he delights in it, and he delights in it because it is beautiful. If nature were not beautiful, it would not be worth knowing, and if nature were not worth knowing, life would not be worth living.

HENRI POINCARÉ

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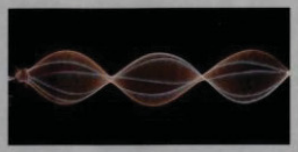


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An Invitation to Physics

Physics, the most fundamental physical science, is concerned with the basic principles of the Universe. It is the foundation on which the other physical sciences—astronomy, chemistry, and geology—are based. The beauty of physics lies in the simplicity of its fundamental theories and in the manner in which a small number of basic concepts, equations, and assumptions can alter and expand our view of the world.

All physical phenomena in our world can be described in terms of one or more of the following theories:

- Classical mechanics, which is concerned with the motion of objects moving at speeds that are low compared to the speed of light;
- Relativity, which is a theory describing particles moving at any speed, even those whose speeds approach the speed of light;
- Thermodynamics, which deals with heat, temperature, and the behavior of large numbers of particles;
- Electromagnetism, which involves the theory of electricity, magnetism, and electromagnetic fields;
- Quantum mechanics, a theory dealing with the behavior of submicroscopic particles as well as the macroscopic world.



◀ **White light directed through a triangular glass prism creates "Pheaenomena of Colours" on the manuscript of Isaac Newton's *Opticks*.** *(© Erich Lessing, Magnum Photos)*

When a discrepancy between theory and experiment arises in any of the theories listed above, the theories must be modified and experiments must be performed to test the predictions of the modified theories. Many times a theory is satisfactory under limited conditions; a more general theory might be satisfactory without such limi-

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tations. A classic example is Newtonian mechanics, which accurately describes the motions of objects at low speeds but does not apply to objects moving at speeds comparable to the speed of light. The special theory of relativity developed by Albert Einstein (1879–1955) successfully predicts the motions of objects at speeds approaching the speed of light and hence is a more general theory of motion.

Classical physics, developed prior to 1900, includes the theories, concepts, laws, and experiments in classical mechanics, thermodynamics, and electromagnetism. For example, Galileo Galilei (1564–1642) made significant contributions to classical mechanics through his work on the laws of motion with constant acceleration. In the same era, Johannes Kepler (1571–1630) used astronomical observations to develop empirical laws for the motions of planetary bodies.

The most important contributions to classical mechanics were provided by Isaac Newton (1642–1727), who developed classical mechanics as a systematic theory and was one of the originators of the calculus as a mathematical tool. Although major developments in classical physics continued in the 18th century, thermodynamics and electromagnetism were not developed until the latter part of the 19th century, principally because the apparatus for controlled experiments was either too crude or unavailable. Although many electric and magnetic phenomena had been studied earlier, it was the work of James Clerk Maxwell (1831–1879) that provided a unified theory of electromagnetism. In this book we shall discuss the various disciplines of classical physics in separate sections; however, we will see that the disciplines of mechanics and electromagnetism are basic to all the branches of classical and modern physics.

A major revolution in physics, usually referred to as *modern physics*, began near the end of the 19th century. Modern physics developed in response to the discovery that many physical phenomena could not be explained by classical physics. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Einstein's theory of relativity completely revolutionized the traditional concepts of space, time, and energy. Einstein's theory correctly described the motion of objects moving at speeds comparable to the speed of light. Newton's laws fail to make correct predictions for objects moving at such high speeds. The theory of relativity also assumes that the speed of light is the upper limit of the speed of an object or signal and shows that mass and energy are related. Quantum mechanics was formulated by a number of distinguished scientists to provide descriptions of physical phenomena at the atomic level.

Scientists continually work at improving our understanding of fundamental laws, and new discoveries are made every day. In many research areas there is a great deal of overlap among physics, chemistry, and biology. The many technological advances in recent times are the result of the efforts of many scientists, engineers, and technicians. Some of the most notable recent developments are (1) unmanned space missions and manned moon landings, (2) microcircuitry and high-speed computers, and (3) sophisticated imaging techniques used in scientific research and medicine. The impacts of such developments and discoveries on society have indeed been great, and it is very likely that future discoveries and developments will be exciting, challenging, and of great benefit to humanity.

1

Introduction and Vectors

The goal of physics is to provide a quantitative understanding of certain basic phenomena that occur in our Universe. Physics is a science based on experimental observations and mathematical analyses. The main objective behind such experiments and analyses is to develop theories that explain the phenomenon being studied and to relate those theories to other established theories. Fortunately, it is possible to explain the behavior of various physical systems using relatively few fundamental laws. Analytical procedures require the expression of those laws in the language of mathematics, the tool that provides a bridge between theory and experiment. In this chapter we shall discuss a few mathematical concepts and techniques that will be used throughout the book.

Because later chapters will be concerned with the laws of physics, it is necessary to provide clear definitions of the basic quantities involved in these laws. For example, such physical quantities as force, velocity, volume, and acceleration can be described in terms of more fundamental quantities. In the next several chapters we will encounter three of the latter: **length** (L), **time** (T), and **mass** (M). Later in the book we shall add units for two other standard quantities to our list, one for temperature (the kelvin), the other for electric current (the ampere). In our study of mechanics, however, we shall be concerned only with the units of mass, length, and time.

Some physical quantities require the measurement of more than one attribute. For example, physical quantities that have both numerical and direc-



◀ **This photograph was taken in Kinsley, Kansas, located midway between New York City and San Francisco.** *(Mike & Carol Werner/Comstock)*

CHAPTER OUTLINE

1.1 Standards of Length, Mass, and Time
1.2 Density and Atomic Mass
1.3 Dimensional Analysis
1.4 Conversion of Units
1.5 Order-of-Magnitude Calculations
1.6 Significant Figures
1.7 Coordinate Systems and Frames of Reference
1.8 Problem-Solving Strategy
1.9 Vectors and Scalars
1.10 Some Properties of Vectors
1.11 Components of a Vector and Unit Vectors

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tional properties—such as force, velocity, and acceleration—are represented by vectors. A large part of this chapter is concerned with vector algebra and the general properties of vectors. The addition and subtraction of vectors will be discussed, together with some common applications to physical situations. Because vectors will be used throughout this book, it is imperative that you understand both their graphical and their algebraic properties.

1.1 • STANDARDS OF LENGTH, MASS, AND TIME

If we make a measurement of a certain quantity and wish to describe it to someone, a unit for the quantity must be defined. For example, it would be meaningless for a visitor from another planet to talk to us about a length of 8 "glitches" if we did not know the meaning of the unit "glitch." However, if someone familiar with our system of measurement and weights reports that a wall is 2.0 meters high and our unit of length is defined to be 1.0 meter, we then know that the height of the wall is twice our fundamental unit of length. Likewise, if we are told that a person has a mass of 75 kilograms and our unit of mass is defined as 1.0 kilogram, then that person has a mass 75 times larger than our fundamental unit of mass. In 1960 an international committee agreed on a system of definitions and standards to describe fundamental physical quantities. It is called the **SI system** (Système International) of units. Its units of length, mass, and time are the meter, kilogram, and second, respectively.

Length

In A.D. 1120 the king of England decreed that the standard of length in his country would be the yard and that the yard would be precisely equal to the distance from the tip of his nose to the end of his outstretched arm. In a similar way, the original standard for the foot adopted by the French was the length of the royal foot of King Louis XIV. This standard prevailed until 1799, when the legal standard of length in France became the meter, defined as one ten-millionth of the distance from the Equator to the North Pole.

Many other systems have been developed in addition to those just discussed, but the French system has prevailed in most countries and in scientific circles everywhere. As recently as 1960, the length of the meter was still defined as the distance between two lines on a specific bar of platinum-iridium alloy stored under controlled conditions. This standard was abandoned for several reasons, a principal one being that the limited accuracy with which the separation between the lines can be determined does not meet the current requirements of science and technology. Until recently, the meter was defined as 1 650 763.73 wavelengths of orange-red light emitted from a krypton-86 lamp. However, in October 1983 the **meter was redefined to be the distance traveled by light in a vacuum during a time of 1/299 792 458 s.** In effect, this latest definition establishes that the speed of light in a vacuum is 299 792 458 m/s.

Definition of the meter •

Mass

Definition of the kilogram •

The SI unit of mass, **the kilogram, is defined as the mass of a specific platinum-iridium alloy cylinder kept at the International Bureau of Weights and Measures at Sèvres, France.** At this point, we should add a word of caution. Most beginning students of physics tend to confuse the physical quantities called *weight* and *mass*. For the present we shall not discuss the distinction between them; they will be clearly defined in later chapters. For now you should note that they are distinctly different quantities.

TABLE 1.1 Approximate Values of Some Measured Lengths

	Length (m)
Distance from Earth to most remote quasar known	1.4×10^{26}
Distance from Earth to most remote normal galaxies known	4×10^{25}
Distance from Earth to nearest large galaxy (M 31 in Andromeda)	2×10^{22}
Distance from Sun to nearest star (Proxima Centauri)	4×10^{16}
One lightyear	9.46×10^{15}
Mean orbit radius of Earth	1.5×10^{11}
Mean distance from Earth to Moon	3.8×10^{8}
Distance from Equator to North Pole	1×10^{7}
Mean radius of Earth	6.4×10^{6}
Typical altitude of orbiting Earth satellite	2×10^{5}
Length of a football field	9.1×10^{1}
Length of a housefly	5×10^{-3}
Size of smallest dust particles	1×10^{-4}
Size of cells of most living organisms	1×10^{-5}
Diameter of a hydrogen atom	1×10^{-10}
Diameter of a uranium nucleus	1.4×10^{-14}
Diameter of a proton	1×10^{-15}

Time

Before 1960 the standard of time was defined in terms of the average length of a solar day in the year 1900. (A solar day is the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day.) The basic unit of time, the second, was defined to be $(1/60)(1/60)(1/24) = 1/86\,400$ of the average solar day. In 1967 the second was redefined to take advantage of the great precision obtainable with a device known as an atomic clock, which uses the characteristic frequency of the cesium-133 atom as the "reference clock." **The second is now defined as 9 192 631 770 times the period of oscillation of radiation from the cesium atom.**

Approximate Values for Length, Mass, and Time

Approximate values of various lengths, masses, and time intervals are presented in Tables 1.1, 1.2, and 1.3, respectively. Note the wide range of values for these quantities.[1] You should study the tables and get a feel for what is meant by a mass of 100 kilograms, for example, or by a time interval of 3.2×10^{7} seconds. Systems of units commonly used are the SI system in which the units of length, mass, and

TABLE 1.2 Masses of Various Bodies (approximate values)

	Mass (kg)
Universe	10^{52}
Milky Way galaxy	10^{42}
Sun	2×10^{30}
Earth	6×10^{24}
Moon	7×10^{22}
Shark	3×10^{2}
Human	7×10^{1}
Frog	1×10^{-1}
Mosquito	1×10^{-5}
Bacterium	1×10^{-15}
Hydrogen atom	1.67×10^{-27}
Electron	9.11×10^{-31}

[1] If you are unfamiliar with the use of powers of ten (scientific notation), you should review Appendix B.1.

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TABLE 1.3 Approximate Values of Some Time Intervals

	Interval (s)
Age of the Universe	5×10^{17}
Age of the Earth	1.3×10^{17}
Time since the fall of the Roman Empire	5×10^{12}
Average age of a college student	6.3×10^{8}
One year	3.2×10^{7}
One day (time for one revolution of Earth around its axis)	8.6×10^{4}
Time between normal heartbeats	8×10^{-1}
Period[a] of audible sound waves	1×10^{-3}
Period of typical radio waves	1×10^{-6}
Period of vibration of an atom in a solid	1×10^{-13}
Period of visible light waves	2×10^{-15}
Duration of a nuclear collision	1×10^{-22}
Time for light to cross a proton	3.3×10^{-24}

[a] A period is defined as the time interval required for one complete vibration.

TABLE 1.4 Some Prefixes for Powers of Ten

Power	Prefix	Abbreviation
10^{-18}	atto	a
10^{-15}	femto	f
10^{-12}	pico	p
10^{-9}	nano	n
10^{-6}	micro	μ
10^{-3}	milli	m
10^{-2}	centi	c
10^{-1}	deci	d
10^{3}	kilo	k
10^{6}	mega	M
10^{9}	giga	G
10^{12}	tera	T
10^{15}	peta	P
10^{18}	exa	E

time are the meter (m), kilogram (kg), and second (s), respectively; the cgs or Gaussian system, in which the units of length, mass, and time are the centimeter (cm), gram (g), and second (s), respectively; and the British engineering system (sometimes called the *conventional system*), in which the units of length, mass, and time are the foot (ft), slug, and second, respectively. Throughout most of this book we shall use SI units, because they are almost universally accepted in science and industry. We will make limited use of conventional units in the study of classical mechanics.

Some of the most frequently used prefixes for the powers of ten and their abbreviations are listed in Table 1.4. For example, 10^{-3} m is equivalent to 1 millimeter (mm), and 10^{3} m is 1 kilometer (km). Likewise, 1 kg is 10^{3} g, and 1 megavolt (MV) is 10^{6} volts.

1.2 · DENSITY AND ATOMIC MASS

Density ·

The **density** ρ (Greek letter rho) of any substance is defined as its *mass per unit volume* (a table of the letters in the Greek alphabet is provided at the back of the book):

$$\rho \equiv \frac{m}{V} \qquad [1.1]$$

For example, aluminum has a density of $2.70 \times 10^{3}\ \text{kg/m}^3$ and lead has a density of $11.3 \times 10^{3}\ \text{kg/m}^3$. A list of densities for various substances is given in Table 1.5.

Atomic mass ·

The difference in density between aluminum and lead is due in part to their different *atomic masses;* the atomic mass of lead is 207 and that of aluminum is 27. However, the ratio of atomic masses, $207/27 = 7.67$, does not correspond to the ratio of densities, $11.3/2.70 = 4.19$. The discrepancy is due to the difference in

atomic spacings and atomic arrangements in the crystal structures of the two types of elements.

All ordinary matter consists of atoms, and each atom is made up of electrons and a nucleus. Practically all of the mass of an atom is contained in the nucleus, which consists of protons and neutrons. Thus, we can understand why the atomic masses of the elements differ. The mass of a nucleus is measured relative to the mass of an atom of carbon-12 (this isotope of carbon has six protons and six neutrons).

The mass of ^{12}C is defined to be exactly 12 atomic mass units (u), where $1\ \text{u} = 1.6605402 \times 10^{-27}$ kg. In these units, the proton and neutron have masses of about 1 u. To be more precise,

$$\text{Mass of proton} = 1.007\ 276\ \text{u}$$

$$\text{Mass of neutron} = 1.008\ 664\ \text{u}$$

The mass of the nucleus of ^{27}Al is approximately 27 u. In fact, a more precise measurement shows that the nuclear mass is always slightly *less* than the combined mass of the protons and neutrons making up the nucleus. The processes of nuclear fission and nuclear fusion are based on this mass difference.

One **mole** (mol) of any element (or compound) consists of Avogadro's number, N_A, of molecules of the substance. Avogadro's number, $N_A = 6.02 \times 10^{23}$, was defined so that 1 mole of carbon-12 atoms would have a mass of exactly 12 g. A mole of one element differs in mass from a mole of another. For example, 1 mole of aluminum has a mass of 27 g, and 1 mole of lead has a mass of 207 g. But 1 mole of aluminum contains the same number of atoms as 1 mole of lead, because there are 6.02×10^{23} atoms in 1 mole of *any* element. The mass of one molecule (or atom, if the substance is an element) is then given by

$$m = \frac{\text{molar mass}}{N_A} \qquad [1.2]$$

For example, the mass of an aluminum atom is

$$m = \frac{27\ \text{g/mol}}{6.02 \times 10^{23}\ \text{atoms/mol}} = 4.5 \times 10^{-23}\ \text{g/atom}$$

Note that 1 u is equal to N_A^{-1} g.

TABLE 1.5 Densities of Various Substances

Substance	Density ρ(kg/m^3)
Platinum	21.45×10^{3}
Gold	19.3×10^{3}
Uranium	18.7×10^{3}
Lead	11.3×10^{3}
Copper	8.93×10^{3}
Iron	7.86×10^{3}
Aluminum	2.70×10^{3}
Magnesium	1.75×10^{3}
Water	1.00×10^{3}
Air at atmospheric pressure	0.0012×10^{3}

Example 1.1 How Many Atoms in the Cube?

A solid cube of aluminum (density 2.7 g/cm^3) has a volume of 0.20 cm^3. How many aluminum atoms are contained in the cube?

Solution Because density equals mass per unit volume, the mass of the cube is

$$M = \rho V = (2.7\ \text{g/cm}^3)(0.20\ \text{cm}^3) = 0.54\ \text{g}$$

To find the number of atoms, N, we can set up a proportion using the fact that 1 mole of aluminum (27 g) contains 6.02×10^{23} atoms:

$$\frac{6.02 \times 10^{23}\ \text{atoms}}{27\ \text{g}} = \frac{N}{0.54\ \text{g}}$$

$$N = \frac{(0.54\ \text{g})(6.02 \times 10^{23}\ \text{atoms})}{27\ \text{g}} = 1.2 \times 10^{22}\ \text{atoms}$$

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1.3 · DIMENSIONAL ANALYSIS

The word *dimension* has a special meaning in physics. It usually denotes the physical nature of a quantity. Whether a distance is measured in units of feet or meters or furlongs, it is a distance. We say its dimension is *length*.

The symbols that will be used in this section to specify length, mass, and time are L, M, and T, respectively. We will often use square brackets [ ] to denote the dimensions of a physical quantity. For example, in this notation the dimensions of velocity, v, are written $[v] = L/T$, and the dimensions of area, A, are $[A] = L^2$. The dimensions of area, volume, velocity, and acceleration are listed in Table 1.6, along with their units in the three common systems of measurement. The dimensions of other quantities, such as force and energy, will be described as they are introduced in the book.

In many situations, you may be faced with having to derive or check a specific formula. Although you may have forgotten the details of the derivation, there is a useful and powerful procedure called *dimensional analysis* that can be used as a consistency check, to assist in the derivation, or to check your final expression. This procedure should always be used and should help minimize the rote memorization of equations. Dimensional analysis makes use of the fact that **dimensions can be treated as algebraic quantities.** That is, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. By following these simple rules, you can use dimensional analysis to help determine whether or not an expression has the correct form, because the relationship can be correct only if the dimensions on the two sides of the equation are the same.

To illustrate this procedure, suppose you wish to derive a formula for the distance x traveled by a car in a time t if the car starts from rest and moves with constant acceleration a. In Chapter 2 we shall find that the correct expression for this special case is $x = \frac{1}{2}at^2$. Let us check the validity of this expression from a dimensional analysis approach.

The quantity x on the left side has the dimension of length. In order for the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length. We can perform a dimensional check by substituting the basic dimensions for acceleration, L/T^2, and time, T, into the equation. That is, the dimensional form of the equation $x = \frac{1}{2}at^2$ can be written as

$$L = \frac{L}{T^2} \cdot T^2 = L$$

The units of time cancel as shown, leaving the unit of length.

TABLE 1.6 Dimensions of Area, Volume, Velocity, and Acceleration

System	Area (L^2)	Volume (L^3)	Velocity (L/T)	Acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
cgs	cm^2	cm^3	cm/s	cm/s^2
British engineering	ft^2	ft^3	ft/s	ft/s^2

Example 1.2 Analysis of an Equation

Show that the expression $v = v_0 + at$ is dimensionally correct, where v and v_0 represent velocities, a is acceleration, and t is a time interval.

Solution Because

$$[v] = [v_0] = \frac{L}{T}$$

and the dimensions of acceleration are L/T^2, the dimensions of at are

$$[at] = \frac{L}{T^2} \cdot T = \frac{L}{T}$$

and the expression is dimensionally correct. However, if the expression were given as $v = v_0 + at^2$, it would be dimensionally *incorrect*. Try it and see!

1.4 · CONVERSION OF UNITS

Sometimes it is necessary to convert units from one system to another. Conversion factors between SI and conventional units of length are as follows:

1 mile = 1609 m = 1.609 km 1 ft = 0.3048 m = 30.48 cm

1 m = 39.37 in. = 3.281 ft 1 in. = 0.0254 m = 2.54 cm

A more complete list of conversion factors can be found in Appendix A.

Units can be treated as algebraic quantities that can cancel each other. For example, suppose we wish to convert 15.0 in. to centimeters. Because 1 in. = 2.54 cm (exactly), we find that

$$15.0 \text{ in.} = (15.0 \text{ in.}) \left(2.54 \frac{\text{cm}}{\text{in.}}\right) = 38.1 \text{ cm}$$





(*Left*) Conversion of miles to kilometers. (*Right*) This modern car speedometer gives speed readings in both miles per hour and kilometers per hour. You should confirm the conversion between the two units for a few readings on the dial. *(Paul Silverman, Fundamental Photographs)*

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Example 1.3 **The Density of a Cube**

The mass of a solid cube is 856 g, and each edge has a length of 5.35 cm. Determine the density ρ of the cube in SI units.

Solution Because $1\ \text{g} = 10^{-3}\ \text{kg}$ and $1\ \text{cm} = 10^{-2}\ \text{m}$, the mass, m, and volume, V, in SI units are given by

$$m = 856\ \text{g} \times 10^{-3}\ \text{kg/g} = 0.856\ \text{kg}$$

$$V = L^3 = (5.35\ \text{cm} \times 10^{-2}\ \text{m/cm})^3$$
$$= (5.35)^3 \times 10^{-6}\ \text{m}^3 = 1.53 \times 10^{-4}\ \text{m}^3$$

Therefore

$$\rho = \frac{m}{V} = \frac{0.856\ \text{kg}}{1.53 \times 10^{-4}\ \text{m}^3} = 5.59 \times 10^3\ \text{kg/m}^3$$

1.5 · ORDER-OF-MAGNITUDE CALCULATIONS

It is often useful to compute an approximate answer to a given physical problem even in instances in which little information is available. This answer can then be used to determine whether or not a more precise calculation is necessary. Such an approximation is usually based on certain assumptions, which must be modified if greater precision is needed. Thus, we will sometimes refer to an *order of magnitude* of a certain quantity as the power of ten of the number that describes that quantity. Usually, when an order-of-magnitude calculation is made, the results are reliable to within a factor of 10. If a quantity increases in value by three orders of magnitude, this means that its value increases by a factor of $10^3 = 1000$. We use the symbol $\sim$ for *is on the order of*. Thus,

$$0.0086 \sim 10^{-2} \qquad 0.0031 \sim 10^{-3} \qquad 700 \sim 10^3$$

The spirit of attempting order-of-magnitude calculations, sometimes referred to as '*guesstimates*' or '*ball-park figures*' is captured by the following quotation: "Make an estimate before every calculation, try a simple physical argument . . . before every derivation, guess the answer to every puzzle. Courage: No one else needs to know what the guess is."[2]

Example 1.4 **The Number of Atoms in a Solid**

Estimate the number of atoms in 1 cm^3 of a solid.

Solution From Table 1.1 we note that the diameter of an atom is about 10^{-10} m. Thus, if in our model we assume that the atoms in the solid are solid spheres of this diameter, then the volume of each sphere is about $10^{-30}\ \text{m}^3$ (more precisely, volume $= 4\pi r^3/3 = \pi d^3/6$, where $r = d/2$). Therefore, because $1\ \text{cm}^3 = 10^{-6}\ \text{m}^3$, the number of atoms in the solid is on the order of $10^{-6}/10^{-30} = 10^{24}$ atoms.

A more precise calculation would require knowledge of the density of the solid and the mass of each atom. However, our estimate agrees with the more precise calculation to within a factor of ten.

[2] E. Taylor and J. A. Wheeler, *Spacetime Physics*, San Francisco, W. H. Freeman, 1966, p. 60.

Example 1.5 **How Much Gas Do We Use?**

Estimate the number of gallons of gasoline used by all cars in the United States each year.

Solution Because there are about 240 million people in the United States, an estimate of the number of cars in the country is 60 million (assuming one car and four people per family). We also estimate that the average distance traveled per year is 10 000 miles. If we assume gasoline consumption of 0.05 gal/mi, each car uses about 500 gal/year. Multiplying this by the total number of cars in the United States gives an estimated total consumption of 3×10^{10} gal, which corresponds to a yearly consumer expenditure of more than $30 billion! This is probably a low estimate because we have not accounted for commercial consumption.

1.6 · SIGNIFICANT FIGURES

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainty. The value of the uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experimenter, and the number of measurements performed.

Suppose that in a laboratory experiment we are asked to measure the area of a rectangular plate using a meter stick as a measuring instrument. Let us assume that the accuracy to which we can measure a particular dimension of the plate is ± 0.1 cm. If the length of the plate is measured to be 16.3 cm, we can claim only that its length lies somewhere between 16.2 cm and 16.4 cm. In this case, we say that the measured value has three significant figures. Likewise, if its width is measured to be 4.5 cm, the actual value lies between 4.4 cm and 4.6 cm. This measured value has only two significant figures. Note that the significant figures include the first reliable digit. Thus, we could write the measured values as 16.3 ± 0.1 cm and 4.5 ± 0.1 cm.

Suppose now that we would like to find the area of the plate by multiplying the two measured values. If we were to claim that the area is $(16.3\ \text{cm})(4.5\ \text{cm}) = 73.35\ \text{cm}^2$, our answer would be unjustifiable because it contains four significant figures, which is greater than the number of significant figures in either of the measured lengths. A good rule of thumb in determining the number of significant figures that can be claimed is as follows:

> When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the *least* accurate of the quantities being multiplied, where *least accurate* means *having the lowest number of significant figures*. The same rule applies to division.

Applying this rule to the multiplication example given previously, we see that the answer for the area can have only two significant figures because the length of 4.5 cm has only two significant figures. Thus, all we can claim is that the area is 73 cm^2, realizing that the value can range between $(16.2\ \text{cm})(4.4\ \text{cm}) = 71\ \text{cm}^2$ and $(16.4\ \text{cm})(4.6\ \text{cm}) = 75\ \text{cm}^2$.

Zeros may or may not be significant figures. Those used to position the decimal point in such numbers as 0.03 and 0.0075 are not significant. Thus there are one and two significant figures, respectively, in these two values. When the positioning of zeros comes after other digits, however, there is the possibility of misinterpre-

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tation. For example, suppose the mass of an object is given as 1500 g. This value is ambiguous because we do not know whether the last two zeros are being used to locate the decimal point or whether they represent significant figures in the measurement. In order to remove this ambiguity, it is common to use scientific notation to indicate the number of significant figures. In this case, we would express the mass as 1.5×10^3 g if there are two significant figures in the measured value, 1.50×10^3 g if there are three significant figures, and 1.500×10^3 g if there are four. Likewise, 0.000 15 should be expressed in scientific notation as 1.5×10^{-4} if it has two significant figures or as 1.50×10^{-4} if it has three significant figures. The three zeros between the decimal point and the digit 1 in the number 0.000 15 are not counted as significant figures, because they are present only to locate the decimal point. In general, a **significant figure** is a reliably known digit (other than a zero used to locate the decimal point).

For addition and subtraction, the number of decimal places must be considered when you are determining how many significant figures to report.

> When numbers are **added** or **subtracted,** the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum.

For example, if we wish to compute 123 + 5.35, the answer is 128 and not 128.35. If we compute the sum 1.0001 + 0.0003 = 1.0004, the result has the correct number of decimal places; consequently, it has five significant figures even though one of the terms in the sum, 0.0003, has only one significant figure. Likewise, if we perform the subtraction 1.002 − 0.998 = 0.004, the result has only one significant figure even though one term has four significant figures and the other has three. In this book, **most of the numerical examples and end-of-chapter problems will yield answers having either two or three significant figures.**

Example 1.6 Installing a Carpet

A carpet is to be installed in a room, the length of which is measured to be 12.71 m (four significant figures) and the width of which is measured to be 3.46 m (three significant figures). Find the area of the room.

Solution If you multiply 12.71 m by 3.46 m on your calculator, you will get an answer of 43.9766 m^2. How many of these numbers should you claim? Our rule of thumb for multiplication tells us that you can claim only the number of significant figures in the least accurate of the quantities being measured. In this example, we have only three significant figures in our least accurate measurement, so we should express our final answer as 44.0 m^2. Note that in the answer given, we used a general rule for rounding off numbers, which states that the last digit retained is to be increased by 1 if the first digit dropped was equal to 5 or greater. If the last digit is 5, the result should be rounded to the nearest even number. (This helps avoid accumulation of errors.)

1.7 · COORDINATE SYSTEMS AND FRAMES OF REFERENCE

Many aspects of physics deal in some way or another with locations in space. For example, the mathematical description of the motion of an object requires a method for describing the position of the object. Thus, it is fitting that we first discuss how to describe the position of a point in space. It is done by means of coordinates. A point on a line can be located with one coordinate; a point in a plane is located with two coordinates; and three coordinates are required to locate a point in space.

A coordinate system used to specify locations in space consists of:

- A fixed reference point O, called the *origin*,
- A set of specified axes or directions with an appropriate scale and labels on the axes,
- Instructions that tell us how to label a point in space relative to the origin and axes.

One convenient coordinate system that we will use frequently is the *cartesian coordinate system*, sometimes called the *rectangular coordinate system*. Such a system in two dimensions is illustrated in Figure 1.1. An arbitrary point in this system is labeled with the coordinates (x, y). Positive x is taken to the right of the origin, and positive y is upward from the origin. Negative x is to the left of the origin, and negative y is downward from the origin. For example, the point P, which has coordinates (5, 3), may be reached by going first 5 meters to the right of the origin and then 3 meters above the origin. In the same way, the point Q has coordinates (−3, 4), which correspond to going 3 meters to the left of the origin and 4 meters above the origin.

Sometimes it is more convenient to represent a point in a plane by its *plane polar coordinates* (r, θ), as in Figure 1.2a. In this coordinate system, r is the length of the line from the origin to the point, and θ is the angle between that line and a fixed axis, usually the positive x axis, with θ measured counterclockwise. From the right triangle in Figure 1.2b, we find $\sin\theta = y/r$ and $\cos\theta = x/r$. (A review of trigonometric functions is given in Appendix B.4.) Therefore, starting with plane polar coordinates, one can obtain the cartesian coordinates through the equations

$$x = r\cos\theta \qquad [1.3]$$

$$y = r\sin\theta \qquad [1.4]$$

Furthermore, it follows that

$$\tan\theta = \frac{y}{x} \qquad [1.5]$$

and

$$r = \sqrt{x^2 + y^2} \qquad [1.6]$$

You should note that these expressions relating the coordinates (x, y) to the coordinates (r, θ) apply only when θ is defined as in Figure 1.2a, in situations in which positive θ is an angle measured *counterclockwise* from the positive x axis. Other choices are made in navigation and astronomy. If the reference axis for the polar angle θ is chosen to be other than the positive x axis, or the sense of increasing θ is chosen differently, then the corresponding expressions relating the two sets of coordinates will change.

1.8 · PROBLEM-SOLVING STRATEGY

Most courses in general physics require the student to learn the skills of problem solving, and examinations usually include problems that test such skills. This brief section describes some useful suggestions that will enable you to increase your ac-

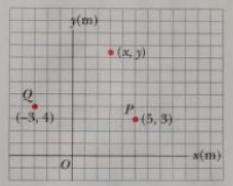
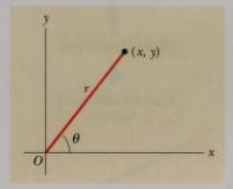
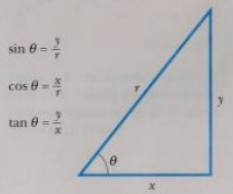


Figure 1.1 Designation of points in a cartesian coordinate system. Every point is labeled with coordinates (x, y).



(a)



(b)

Figure 1.2 (a) The plane polar coordinates of a point are represented by the distance r and the angle θ. (b) The right triangle used to relate (x, y) to (r, θ).

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curacy in solving problems, enhance your understanding of physical concepts, eliminate initial panic or lack of direction in approaching a problem, and organize your work. One way to help accomplish these goals is to adopt a problem-solving strategy. Many chapters in this book will include a section labeled "Problem-Solving Strategy," which should help you through the rough spots.

The following steps are commonly used to develop a problem-solving strategy:

A MENU FOR PROBLEM SOLVING

Read Problem → Draw Diagram → Identify Data → Choose Equation(s) → Solve Equation(s) → Evaluate and Check Answer

1. Read the problem carefully at least twice. Be sure you understand the nature of the problem before proceeding further.
2. Draw a suitable diagram with appropriate labels and coordinate axes if needed.
3. Imagine a movie, running in your mind, of what happens in the problem.
4. As you examine what is being asked in the problem, identify the basic physical principle or principles that are involved, listing the knowns and unknowns.
5. Select a basic relationship or derive an equation that can be used to find the unknown and symbolically solve the equation for the unknown.
6. Substitute the given values, along with the appropriate units, into the equation.
7. Obtain a numerical value with units for the unknown. You can have confidence in your result if the following questions can be properly answered: Do the units match? Is the answer reasonable? Is the plus or minus sign proper or meaningful?

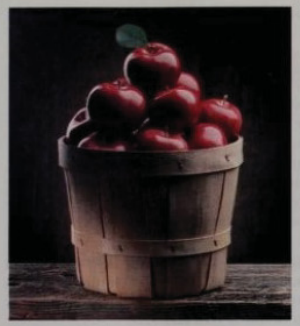
One of the purposes of this strategy is to promote accuracy. Properly drawn diagrams can eliminate many sign errors. Diagrams also help to isolate the physical principles of the problem. Symbolic solutions and carefully labeled knowns and unknowns will help eliminate other careless errors. The use of symbolic solutions should help you think in terms of the physics of the problem. A check of units at the end of the problem can indicate a possible algebraic error. The physical layout and organization of your problem will make the final product more understandable and easier to follow. Once you have developed an organized system for examining problems and extracting relevant information, you will become a more confident problem solver.

1.9 · VECTORS AND SCALARS

Each of the physical quantities that we shall encounter in this book can be placed in one of two categories: It is either a scalar or a vector. A scalar is a quantity that is completely specified by a positive or negative number with appropriate units:

> A **scalar** quantity has a positive or negative value and no direction. However, a **vector** is a physical quantity that must be specified by both magnitude and direction.

The number of apples in a basket is an example of a scalar quantity. If you are told there are 38 apples in the basket, this completes the required information; no specification of direction is required. Other examples of scalars are temperature,





(*Left*) The number of apples in the basket is one example of a scalar quantity. Can you think of other examples? (*Superstock*) (*Right*) Jennifer pointing in the right direction tells us to travel 5 blocks to the north to reach the courthouse. A vector is a physical quantity that must be specified by both magnitude and direction. *(Photo by Raymond A. Serway)*

volume, mass, and time intervals. The rules of ordinary arithmetic are used to manipulate scalar quantities.

Force is an example of a vector quantity. To completely describe the force on an object, we must specify both the direction of the applied force, the magnitude of the force, and the location of the force. When the motion (velocity) of an object is described, we must specify both the speed and the direction of its motion.

Another simple example of a vector quantity is the **displacement** of a particle, defined as its *change in position*. Suppose the particle moves from some point O to a point P along a straight path, as in Figure 1.3. We represent this displacement by drawing an arrow from O to P, where the tip of the arrow represents the direction of the displacement and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from O to P, such as the broken line in Figure 1.3, its displacement is still OP. The vector displacement along any indirect path from O to P is defined as being equivalent to the displacement represented by the direct path from O to P. The magnitude of the displacement is the shortest distance between the end points. Thus, **the displacement of a particle is completely known if its initial and final coordinates are known.** The path need not be specified. In other words, **the displacement is independent of the path** if the end points of the path are fixed.

It is important to note that the distance traveled by a particle is distinctly different from its displacement. The **distance** traveled (a scalar quantity) is the length of the path, which in general can be much greater than the magnitude of the displacement (see Fig. 1.3).

If the particle moves along the x axis from position x_i to position x_f, as in Figure 1.4, its displacement is given by $x_f - x_i$. (The indices i and f refer to the initial and final values.) We use the Greek letter delta (Δ) to denote the *change* in a quantity.

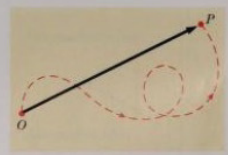


Figure 1.3 As a particle moves from O to P along the broken line, its displacement vector is the arrow drawn from O to P.

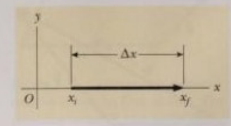


Figure 1.4 A particle moving along the x axis from x_i to x_f undergoes a displacement $\Delta x = x_f - x_i$.

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Therefore, we define the change in the position of the particle (the displacement) as

Definition of displacement along a line •

$$\Delta x \equiv x_f - x_i \qquad [1.7]$$

From this definition we see that Δx is positive if x_f is greater than x_i and negative if x_f is less than x_i. For example, if a particle changes its position from $x_i = -3$ units to $x_f = 5$ units, its displacement is 8 units.

Many physical quantities in addition to displacement are vectors. They include velocity, acceleration, force, and momentum, all of which will be defined in later chapters. In this book we will use boldface letters, such as **A**, to represent arbitrary vectors. Another common method of notating vectors with which you should be familiar is to use an arrow over the letter: $\vec{A}$.

The magnitude of the vector **A** is written A or, alternatively, $|\mathbf{A}|$. The magnitude of a vector is always positive and has physical units, such as meters for displacement or meters per second for velocity, as discussed earlier. Vectors combine according to special rules, which will be discussed in Sections 1.10 and 1.11.

Thinking Physics 1

Consider your commute to work or school in the morning. Which is larger, the distance you traveled or the magnitude of the displacement vector?

Reasoning The distance traveled, unless you have a very unusual commute, *must* be larger than the magnitude of the displacement vector. The distance includes all of the twists and turns that you made in following the roads from home to work or school. However, the magnitude of the displacement vector is the length of a straight line from your home to work or school. This is often described informally as "the distance as the crow flies." The only way that the distance could be the same as the magnitude of the displacement vector is if your commute is a perfect straight line, which is highly unlikely! The distance could *never* be less than the magnitude of the displacement vector, because the shortest distance between two points is a straight line.

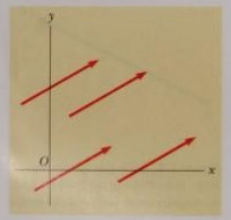


Figure 1.5 Four representations of the same vector.

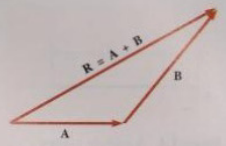


Figure 1.6 When vector **A** is added to vector **B**, the resultant **R** runs from the tail of **A** to the tip of **B**.

1.10 • SOME PROPERTIES OF VECTORS

Equality of Two Vectors Two vectors **A** and **B** are defined to be equal if they have the same magnitude and the same direction. That is, $\mathbf{A} = \mathbf{B}$ only if $A = B$ *and* **A** and **B** act in parallel. For example, all the vectors in Figure 1.5 are equal even though they have different starting points. This property allows us to translate a vector parallel to itself in a diagram without affecting the vector.

Addition When two or more vectors are added together, they must *all* have the same units. For example, it would be meaningless to add a velocity vector to a displacement vector, because they are different physical quantities. Scalars obey the same rule. For example, it would be meaningless to add time intervals and temperatures.

The rules for vector sums are conveniently described by geometric methods. To add vector **B** to vector **A**, first draw vector **A**, with its magnitude represented by a convenient scale, on graph paper and then draw vector **B** to the same scale with its tail starting from the tip of **A**, as in Figure 1.6. The *resultant vector* $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is the vector drawn from the tail of **A** to the tip of **B**. This is known as the *triangle method of addition*. An alternative graphical procedure for adding two vectors, known as the **parallelogram rule of addition,** is shown in Figure 1.7a. In this construction, the tails of the two vectors **A** and **B** are together, and the resultant vector **R** is the diagonal of a parallelogram formed with **A** and **B** as its sides.

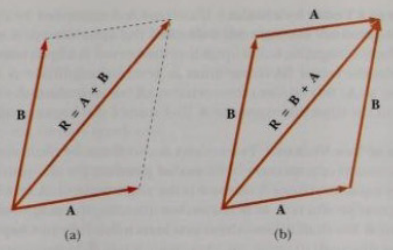


Figure 1.7 (a) In this construction, the resultant **R** is the diagonal of a parallelogram with sides **A** and **B**. (b) This construction shows that $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.

When two vectors are added, **the sum is independent of the order of the addition.** This can be seen from the geometric construction in Figure 1.7b and is known as the **commutative law of addition:**

$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A} \qquad [1.8]$$

• *Commutative law*

If three or more vectors are added, **their sum is independent of the way in which they are grouped.** A geometric proof of this for three vectors is given in Figure 1.8. This is called the **associative law of addition:**

$$\mathbf{A} + (\mathbf{B} + \mathbf{C}) = (\mathbf{A} + \mathbf{B}) + \mathbf{C} \qquad [1.9]$$

• *Associative law*

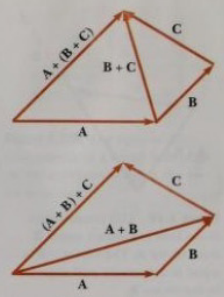


Figure 1.8 Geometric constructions for verifying the associative law of addition.

Geometric constructions can also be used to add more than three vectors. This is shown in Figure 1.9 for the case of four vectors. The resultant vector sum $\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C} + \mathbf{D}$ is **the vector that completes the polygon.** In other words, **R is the vector drawn from the tail of the first vector to the tip of the last vector.** Again, the order of the summation is unimportant.

Thus we conclude that **a vector is a quantity that has both magnitude and direction and also obeys the laws of vector addition,** as described in Figures 1.6 through 1.9.

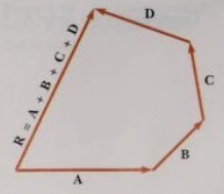


Figure 1.9 Geometric construction for summing four vectors. The resultant vector **R** completes the polygon.

Negative of a Vector The negative of the vector **A** is defined as the vector that, when added to **A**, gives zero for the vector sum. That is, $\mathbf{A} + (-\mathbf{A}) = 0$. The vectors **A** and $-\mathbf{A}$ have the same magnitude but opposite directions.

Subtraction of Vectors The operation of vector subtraction makes use of the definition of the negative of a vector. We define the operation $\mathbf{A} - \mathbf{B}$ as vector $-\mathbf{B}$ added to vector **A**:

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) \qquad [1.10]$$

The geometric construction for subtracting two vectors is shown in Figure 1.10.

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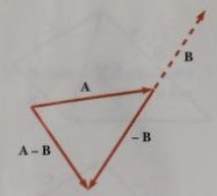


Figure 1.10 This construction shows how to subtract vector **B** from vector **A**. The vector $-\mathbf{B}$ is equal in magnitude and opposite to the vector **B**.

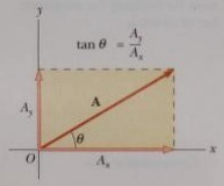


Figure 1.11 Any vector **A** lying in the *xy* plane can be represented by its components A_x and A_y.

Multiplication of a Vector by a Scalar If a vector **A** is multiplied by a positive scalar quantity m, the product $m\mathbf{A}$ is a vector that has the same direction as **A** and magnitude mA. If m is a negative scalar quantity, the vector $m\mathbf{A}$ is directed opposite to **A**. For example, the vector $5\mathbf{A}$ is five times as great in magnitude as **A** and has the same direction as **A**. By contrast, the vector $-\frac{1}{3}\mathbf{A}$ has one third the magnitude of **A** and points in the direction opposite **A** (because of the negative sign).

Multiplication of Two Vectors Two vectors **A** and **B** can be multiplied to produce either a scalar or a vector quantity. The **scalar product** (or dot product) $\mathbf{A} \cdot \mathbf{B}$ is a scalar quantity equal to $AB \cos \theta$, where θ is the angle between **A** and **B**. The **vector product** (or cross product) $\mathbf{A} \times \mathbf{B}$ is a vector quantity, the magnitude of which is equal to $AB \sin \theta$. We shall discuss these products more fully in Chapters 6 and 10, where they are first used.

Each chapter of this textbook contains a number of exercises. The purpose of such exercises is to test your understanding of the material just discussed by asking you to do a calculation or answer some question related to a worked example. Here is your first exercise.

EXERCISE 1 The magnitudes of two vectors **A** and **B** are $A = 8$ units and $B = 3$ units. Find the largest and smallest possible values for the magnitude of the resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$. Answer The largest value of R is 11 units, corresponding to the case when **A** and **B** point in the same direction. The smallest value of R is 5 units, corresponding to the case when **A** is directed opposite to **B**.

1.11 · COMPONENTS OF A VECTOR AND UNIT VECTORS

The geometric method of adding vectors is not the recommended procedure in situations in which great precision is required or in three-dimensional problems. In this section we describe a method of adding vectors that makes use of the *projections* of a vector along the axes of a rectangular coordinate system.

Components of the vector **A** ·

Consider a vector **A** lying in the *xy* plane and making an arbitrary angle θ with the positive *x* axis, as in Figure 1.11. The vector **A** can be represented by its **rectangular components**, A_x and A_y. The component A_x represents the projection of **A** along the *x* axis, whereas A_y represents the projection of **A** along the *y* axis. The components of a vector, which are scalar quantities, can be positive or negative. For example, in Figure 1.11, A_x and A_y are both positive.

From Figure 1.11 and the definition of the sine and cosine of an angle, we see that $\cos \theta = A_x/A$ and $\sin \theta = A_y/A$. Hence, the components of **A** are given by

$$A_x = A \cos \theta \quad \text{and} \quad A_y = A \sin \theta \qquad [1.11]$$

It is important to note that when using these component equations, θ must be measured counterclockwise from the positive *x* axis. These components form two sides of a right triangle, the hypotenuse of which has a magnitude A. Thus, it follows that the magnitude of **A** and its direction are related to its components through the expressions

Magnitude of **A** ·

$$A = \sqrt{A_x^2 + A_y^2} \qquad [1.12]$$

Direction of **A** ·

$$\tan \theta = \frac{A_y}{A_x} \qquad [1.13]$$

To solve for θ, we can write $\theta = \tan^{-1}(A_y/A_x)$, which is read "$\theta$ equals the angle whose tangent is the ratio A_y/A_x." **Note that the signs of the components A_x and A_y depend on the angle θ.** For example, if $\theta = 120°$, A_x is negative and A_y is positive. By contrast, if $\theta = 225°$, both A_x and A_y are negative. If both components are negative, the angle measured from the positive *x* axis will be the angle you determine on your calculator plus 180°. Figure 1.12 summarizes the signs of the components when **A** lies in the various quadrants.

If you choose reference axes or an angle other than those shown in Figure 1.11, the components of the vector must be modified accordingly. In many applications it is more convenient to express the components of a vector in a coordinate system having axes that are not horizontal and vertical but that are still perpendicular to each other. Suppose a vector **B** makes an angle θ' with the x' axis defined in Figure 1.13. The components of **B** along these axes are given by $B_{x'} = B \cos \theta'$ and $B_{y'} = B \sin \theta'$, as in Equation 1.9. The magnitude and direction of **B** are obtained from expressions equivalent to Equations 1.10 and 1.11. Thus, we can express the components of a vector in *any* coordinate system that is convenient for a particular situation.

The components of a vector differ when viewed from different coordinate systems. Furthermore, the components of a vector can change with respect to a fixed coordinate system if the vector changes in magnitude, orientation, or both.

Vector quantities are often expressed in terms of unit vectors. **A unit vector is a dimensionless vector 1 unit in length used to specify a given direction.** Unit vectors have no other physical significance. They are used simply as a convenience in describing a direction in space. We will use the symbols **i**, **j**, and **k** to represent unit vectors pointing in the *x*, *y*, and *z* directions, respectively. Thus, the unit vectors **i**, **j**, and **k** form a set of mutually perpendicular vectors as shown in Figure 1.14a, where the magnitude of the unit vectors equals unity. That is, $|\mathbf{i}| = |\mathbf{j}| = |\mathbf{k}| = 1$.

Consider a vector **A** lying in the *xy* plane, as in Figure 1.14b. The product of the component A_x and the unit vector **i** is the vector $A_x\mathbf{i}$ parallel to the *x* axis with magnitude A_x. Likewise, $A_y\mathbf{j}$ is a vector of magnitude A_y parallel to the *y* axis. When

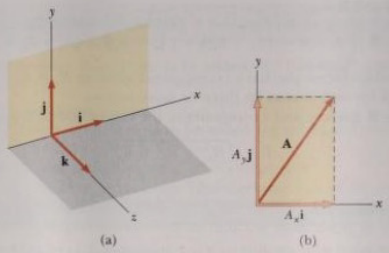


Figure 1.14 (a) The unit vectors **i**, **j**, and **k** are directed along the *x*, *y*, and *z* axes, respectively. (b) A vector **A** lying in the *xy* plane has component vectors $A_x\mathbf{i}$ and $A_y\mathbf{j}$ where A_x and A_y are the components of **A**.

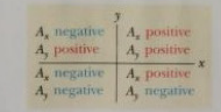


Figure 1.12 The signs of the components of a vector **A** depend on the quadrant in which the vector is located.

· *Unit vectors*

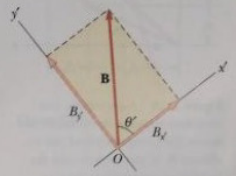


Figure 1.13 The components of vector **B** in a coordinate system that is tilted.

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using the unit form of a vector, we are simply multiplying a vector (the unit vector) by a scalar. Thus, the unit-vector notation for the vector **A** is written

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} \qquad [1.14]$$

The vectors $A_x\mathbf{i}$ and $A_y\mathbf{j}$ are the component vectors of **A**. These should not be confused with A_x and A_y, which we shall always refer to as the components of **A**.

Now suppose we wish to add vector **B** to vector **A**, where **B** has components B_x and B_y. The procedure for performing this sum is simply to add the x and y components separately. The resultant vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$ is therefore

$$\mathbf{R} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} \qquad [1.15]$$

Thus, the components of the resultant vector are given by

$$R_x = A_x + B_x$$
$$R_y = A_y + B_y \qquad [1.16]$$

The magnitude of **R** and the angle it makes with the x axis can then be obtained from its components using the relationships

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x + B_x)^2 + (A_y + B_y)^2} \qquad [1.17]$$

$$\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x} \qquad [1.18]$$

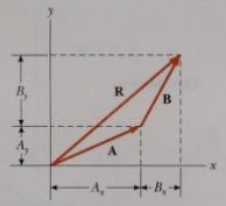


Figure 1.15 A geometric construction showing the relation between the components of the resultant **R** of two vectors and the individual component vectors.

The procedure just described for adding two vectors **A** and **B** using the component method can be checked using a geometric construction, as in Figure 1.15. Again you must take note of the *signs* of the components when using either the algebraic or the geometric method.

The extension of these methods to three-dimensional vectors is straightforward. If **A** and **B** both have x, y, and z components, we express them in the form

$$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k} \quad \text{and} \quad \mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$$

The sum of **A** and **B** is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (A_x + B_x)\mathbf{i} + (A_y + B_y)\mathbf{j} + (A_z + B_z)\mathbf{k} \qquad [1.19]$$

Thus, the resultant vector also has a z component, given by $R_z = A_z + B_z$. The same procedure can be used to sum up three or more vectors.

If a vector **R** has x, y, and z components, the magnitude of the vector is

$$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$$

Thinking Physics 2

You may have asked someone directions to a destination in a city and have been told something like, "Walk 3 blocks east and then 5 blocks south." If so, are you experienced with vector components?

Reasoning Yes, you are! Although you may not have thought of vector component language when you heard these directions, this is exactly what the directions represent. The perpendicular streets of the city reflect an x–y coordinate system—we can assign the x axis to the east–west streets and the y axis to the north–south streets. Thus, the comment of the person giving you directions can be translated as, "Undergo a displacement vector that has an x component of +3 blocks and a y component of −5 blocks." You would arrive at the same destination by undergoing the y component first, followed by the x component, demonstrating the commutative law of addition.

Many sections of this book will include conceptual problems that are nonnumerical like the Thinking Physics example above. The answers to all conceptual problems are found at the end of each chapter. Here is your first set of conceptual problems.

CONCEPTUAL PROBLEM 1

If one component of a vector is not zero, can its magnitude be zero? Explain.

CONCEPTUAL PROBLEM 2

If $\mathbf{A} + \mathbf{B} = 0$, what can you say about the components of the two vectors?

CONCEPTUAL PROBLEM 3

Figure CP1.3 shows two vectors lying in the xy-plane. Determine the signs of (a) the x components of **A** and **B**, (b) the y components of **A** and **B**, and (c) the x and y components of $\mathbf{A} + \mathbf{B}$.

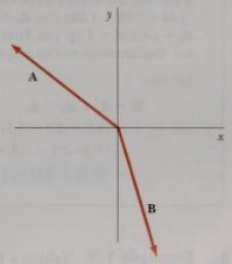


Figure CP1.3

CONCEPTUAL PROBLEM 4

Can the component of a vector ever be equal to the magnitude of the vector?

PROBLEM-SOLVING STRATEGY • Adding Vectors, Using Components

When two or more vectors are to be added, the following steps are recommended:

1. Select a coordinate system.
2. Draw a sketch of the vectors to be added (or subtracted) and label each vector.
3. Find the x and y components of all vectors.
4. Find the resultant components (the algebraic sum of the components) in both the x and y directions.
5. Use the Pythagorean theorem to find the magnitude of the resultant vector.
6. Use a suitable trigonometric function to find the angle the resultant vector makes with the x axis.

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Example 1.7 The Sum of Two Vectors

Find the sum of two vectors **A** and **B** lying in the *xy* plane and given by

$$\mathbf{A} = 2.0\mathbf{i} + 2.0\mathbf{j} \quad \text{and} \quad \mathbf{B} = 2.0\mathbf{i} - 4.0\mathbf{j}$$

Solution Note that $A_x = 2.0$, $A_y = 2.0$, $B_x = 2.0$, and $B_y = -4.0$. Therefore, the resultant vector **R** is

$$\mathbf{R} = \mathbf{A} + \mathbf{B} = (2.0 + 2.0)\mathbf{i} + (2.0 - 4.0)\mathbf{j} = 4.0\mathbf{i} - 2.0\mathbf{j}$$

or

$$R_x = 4.0, \qquad R_y = -2.0$$

The magnitude of **R** is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{20} = 4.5$$

EXERCISE 2 Find the angle θ that the resultant vector **R** makes with the positive *x* axis. Answer 330°

Example 1.8 The Resultant Displacement

A particle undergoes three consecutive displacements: $\mathbf{d}_1 = (1.5\mathbf{i} + 3.0\mathbf{j} - 1.2\mathbf{k})$ cm, $\mathbf{d}_2 = (2.3\mathbf{i} - 1.4\mathbf{j} - 3.6\mathbf{k})$ cm, and $\mathbf{d}_3 = (-1.3\mathbf{i} + 1.5\mathbf{j})$ cm. Find the components of the resultant displacement and its magnitude.

Solution

$$\begin{aligned}\mathbf{R} &= \mathbf{d}_1 + \mathbf{d}_2 + \mathbf{d}_3 \\ &= (1.5 + 2.3 - 1.3)\mathbf{i} + (3.0 - 1.4 + 1.5)\mathbf{j} \\ &\quad + (-1.2 - 3.6 + 0)\mathbf{k} \\ &= (2.5\mathbf{i} + 3.1\mathbf{j} - 4.8\mathbf{k})\text{ cm}\end{aligned}$$

That is, the resultant displacement has components $R_x = 2.5$ cm, $R_y = 3.1$ cm, and $R_z = -4.8$ cm. Its magnitude is

$$\begin{aligned}R &= \sqrt{R_x^2 + R_y^2 + R_z^2} \\ &= \sqrt{(2.5\text{ cm})^2 + (3.1\text{ cm})^2 + (-4.8\text{ cm})^2} = 6.2\text{ cm}\end{aligned}$$

Example 1.9 Taking a Hike

A hiker begins a trip by first walking 25.0 km due southeast from her base camp. On the second day she walks 40.0 km in a direction 60.0° north of east, at which point she discovers a forest ranger's tower.

(a) Determine the components of the hiker's displacements in the first and second days.

Solution If we denote the displacement vectors on the first and second days by **A** and **B**, respectively, and use the camp as the origin of coordinates, we get the vectors shown in Figure 1.16. Displacement **A** has a magnitude of 25.0 km and is 45.0° southeast. Its components are

$$A_x = A\cos(-45.0°) = (25.0\text{ km})(0.707) = 17.7\text{ km}$$

$$A_y = A\sin(-45.0°) = -(25.0\text{ km})(0.707) = -17.7\text{ km}$$

The negative value of A_y indicates that the y coordinate decreased in this displacement. The signs of A_x and A_y are also evident from Figure 1.16.

The second displacement, **B**, has a magnitude of 40.0 km and is 60.0° north of east. Its components are

$$B_x = B\cos 60.0° = (40.0\text{ km})(0.500) = 20.0\text{ km}$$

$$B_y = B\sin 60.0° = (40.0\text{ km})(0.866) = 34.6\text{ km}$$

(b) Determine the components of the hiker's total displacement for the trip.

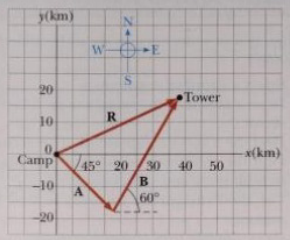


Figure 1.16 The total displacement of the hiker is the vector $\mathbf{R} = \mathbf{A} + \mathbf{B}$.

Solution The resultant displacement vector for the trip, $\mathbf{R} = \mathbf{A} + \mathbf{B}$, has components

$$R_x = A_x + B_x = 17.7\text{ km} + 20.0\text{ km} = 37.7\text{ km}$$

$$R_y = A_y + B_y = 17.7\text{ km} + 34.6\text{ km} = 16.9\text{ km}$$

In unit-vector form, we can write the total displacement as $\mathbf{R} = (37.7\mathbf{i} + 16.9\mathbf{j})$ km.

EXERCISE 3 Determine the magnitude and direction of the total displacement. Answer 41.3 km, 24.1° north of east from the base camp

SUMMARY

Mechanical quantities can be expressed in terms of three fundamental quantities, *length*, *mass*, and *time*, which in the SI system have the units *meters* (m), *kilograms* (kg), and *seconds* (s), respectively. It is often useful to use the *method of dimensional analysis* to check equations and to assist in deriving expressions.

The density of a substance is defined as its mass per unit volume. Different substances have different densities mainly because of differences in their atomic masses and atomic arrangements.

The number of molecules in 1 mole of any element or compound, called **Avogadro's number** (N_A), is 6.02×10^{23}.

Vectors are quantities that have both magnitude and direction and obey the vector law of addition. **Scalars** are quantities that have only magnitude.

Two vectors **A** and **B** can be added using either the triangle method or the parallelogram rule. In the triangle method (Fig. 1.6), the vector $\mathbf{C} = \mathbf{A} + \mathbf{B}$ runs from the tail of **A** to the tip of **B**. In the parallelogram method (Fig. 1.7a), **C** is the diagonal of a parallelogram having **A** and **B** as its sides.

The *x* component of the vector **A**, A_x, is equal to its projection along the *x* axis of a coordinate system as in Figure 1.11, where $A_x = A\cos\theta$ and θ is the angle **A** makes with the *x* axis. Likewise, the y component of **A**, A_y, is its projection along the y axis, where $A_y = A\sin\theta$. The resultant of two or more vectors can be found by resolving all vectors into their *x* and y components, adding their resultant *x* and y components, and then using the Pythagorean theorem to find the magnitude of the resultant vector. The angle that the resultant vector makes with the *x* axis can be found by the use of a suitable trigonometric function.

If a vector **A** has an *x* component equal to A_x and a y component equal to A_y, the vector can be expressed in unit-vector form as $\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j}$. In this notation, **i** is a unit vector in the positive *x* direction and **j** is a unit vector in the positive y direction. Because **i** and **j** are unit vectors, $|\mathbf{i}| = |\mathbf{j}| = 1$.

CONCEPTUAL QUESTIONS

1. Suppose that the three fundamental standards of the metric system were length, *density*, and time rather than length, *mass*, and time. The standard of density in this system is to be defined as that of water. What considerations about water would need to be addressed to make sure that the standard unit of density were to be as accurate as possible?
2. What types of natural phenomena could serve as alternative time standards?
3. The height of a horse is sometimes given in units of "hands." Why is this a poor standard of length?
4. Express the following quantities using the prefixes given in Table 1.4: (a) 3×10^{-4} m, (b) 5×10^{-5} s, (c) 72×10^2 g.
5. As one moves upward in atomic number in the periodic table, the atomic masses of the elements increases. Because the atoms are becoming more and more massive, why doesn't the *density* of elemental materials increase in the same way?
6. Suppose that two quantities *A* and *B* have different dimensions. Determine which of the following arithmetic opera-

Boxed numbers indicate questions that have answers available in the Student Solutions Manual and Study Guide.

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tions *could* be physically meaningful: (a) $A + B$, (b) A/B, (c) $B - A$, (d) AB.

7. What accuracy is implied in an order-of-magnitude calculation?
8. Apply an order-of-magnitude calculation to an everyday situation you might encounter. For example, how far do you walk or drive each day?
9. Which of the following are vectors and which are not: force, temperature, the volume of water in a can, the ratings of a TV show, the height of a building, the velocity of a sports car, the age of the Universe?
10. A vector **A** lies in the *xy* plane. For what orientations of **A** will both of its rectangular components be negative? For what orientations will its components have opposite signs?
11. A book is moved once around the perimeter of a tabletop with the dimensions 1.0 m × 2.0 m. If the book ends up at its initial position, what is its displacement? What is the distance traveled?
12. While traveling along a straight interstate highway you notice that the mile marker reads 260. You travel until you reach the 150-mile marker and then retrace your path to the 175-mile marker. What is the magnitude of your resultant displacement from the 260-mile marker?
13. If the component of vector **A** along the direction of vector **B** is zero, what can you conclude about the two vectors?
14. If **A** = **B**, what can you conclude about the components of **A** and **B**?
15. Is it possible to add a vector quantity to a scalar quantity? Explain.
16. A roller coaster travels 135 ft at an angle of 40.0° above the horizontal. How far does it move horizontally and vertically?
17. The resolution of vectors into components is equivalent to replacing the original vector with the sum of two vectors, whose sum is the same as the original vector. There are an infinite number of pairs of vectors that will satisfy this condition; we choose that pair with one vector parallel to the *x* axis and the second parallel to the *y* axis. What difficulties would be introduced by defining components relative to axes that are not perpendicular—for example, the *x* axis and a *y* axis oriented at 45° to the *x* axis?

PROBLEMS

Section 1.2 Density and Atomic Mass

1. Calculate the mass of an atom of (a) helium, (b) iron, (c) lead. Give your answers in atomic mass units and in grams. The atomic masses of the atoms given are 4.00, 55.9, and 207, respectively.
2. The standard kilogram is a platinum-iridium cylinder 39.0 mm in height and 39.0 mm in diameter. What is the density of the material?
3. One cubic meter (1.00 m^3) of aluminum has a mass of 2.70×10^3 kg, and one cubic meter of iron has a mass of 7.86×10^3 kg. Find the radius of a solid aluminum sphere that will balance a solid iron sphere of radius 2.00 cm on an equal-arm balance.
4. Let ρ_{Al} represent the density of aluminum and ρ_{Fe} that of iron. Find the radius of a solid aluminum sphere that balances a solid iron sphere of radius r_{Fe} on an equal-arm balance.
5. The mass of the planet Saturn is 5.64×10^{26} kg, and its radius is 6.00×10^7 m. (a) Calculate its density. (b) If Saturn were placed in a large enough ocean of water, would it float? Explain.

Section 1.3 Dimensional Analysis

6. The radius *r* of a circle inscribed in any triangle whose sides are *a*, *b*, and *c* is given by $r = [(s - a)(s - b)(s - c)/s]^{1/2}$, where *s* is an abbreviation for $(a + b + c)/2$. Check this formula for dimensional consistency.
7. The consumption of natural gas by a company satisfies the empirical equation $V = 1.50t + 0.00800t^2$, where *V* is the volume in millions of cubic feet and *t* the time in months. Express this equation in units of cubic feet and seconds. Put the proper units on the coefficients. Assume that a month is 30.0 days.
8. Newton's law of universal gravitation is given by

$$F = G\frac{Mm}{r^2}$$

Here *F* is the force of gravity, *M* and *m* are masses, and *r* is a length. Force has the units kg·m/s^2. What are the SI units of the proportionality constant *G*?

9. The period *T* of a simple pendulum (the time for one complete oscillation) is measured in time units and is given by

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

where ℓ is the length of the pendulum and *g* is the acceleration due to gravity in units of length divided by the square of time. Show that this equation is dimensionally consistent.

Boxed numbers indicate problems that have full solutions available in the Student Solutions Manual and Study Guide. Cyan numbers indicate intermediate level problems and magenta numbers indicate more challenging problems.

Section 1.4 Conversion of Units

10. An auditorium measures 40.0 m × 20.0 m × 12.0 m. The density of air is 1.20 kg/m^3. What are (a) the volume of the room in cubic feet and (b) the weight of air in the room in pounds?
11. An astronomical unit (AU) is defined as the average distance between the Earth and Sun. (a) How many astronomical units are there in one lightyear? (b) Determine the distance from Earth to the Andromeda galaxy in astronomical units.
12. Assume that it takes 7.00 minutes to fill a 30.0-gal gasoline tank. (a) Calculate the rate at which the tank is filled in gallons per second. (b) Calculate the rate at which the tank is filled in cubic meters per second. (c) Determine the time, in hours, required to fill a 1-cubic-meter volume at the same rate. (1 U.S. gal = 231 $in.^3$)
13. One gallon of paint (volume = 3.78×10^{-3} m^3) covers an area of 25.0 m^2. What is the thickness of the paint on the wall?
14. You can obtain a rough estimate of the size of a molecule by the following simple experiment. Let a droplet of oil spread out on a smooth water surface. The resulting oil slick will be approximately 1 molecule thick. If an oil droplet of mass 9.00×10^{-7} kg and density 918 kg/m^3 spreads out into a circle of radius 41.8 cm on the water surface, calculate the diameter of an oil molecule.
15. (a) Find a conversion factor to convert from miles per hour to kilometers per hour. (b) Until recently, federal law mandated that highway speeds would be 55 mi/h. Use the conversion factor of part (a) to find the speed in kilometers per hour. (c) The maximum highway speed has been raised to 65 mi/h in some places. In kilometers per hour, how much increase is this over the 55-mi/h limit?
16. (a) How many seconds are there in a year? (b) If one micrometeorite (a sphere with a diameter of 10^{-6} m) struck each square meter of the Moon each second, how many years would it take to cover the Moon to a depth of 1.00 m? (*Hint:* Consider a cubic box on the Moon 1.00 m on a side and find how long it would take to fill the box.)
17. The mass of the Sun is about 1.99×10^{30} kg, and the mass of a hydrogen atom, of which the Sun is mostly composed, is 1.67×10^{-27} kg. How many atoms are there in the Sun?

Section 1.5 Order-of-Magnitude Calculations

18. Compute the order of magnitude of the mass of (a) a bathtub filled with water, and (b) a bathtub filled with pennies. In your solution list the quantities you estimate and the value you estimate for each.
19. Estimate the number of piano tuners living in New York City. This problem was posed by the physicist Enrico Fermi, who was well known for making order-of-magnitude calculations.
20. Soft drinks are commonly sold in aluminum containers. Estimate the number of such containers thrown away or recycled each year by U.S. consumers. Approximately how many tons of aluminum does this represent?

Section 1.6 Significant Figures

21. Carry out the following arithmetic operations: (a) the sum of the numbers 756, 37.2, 0.83, and 2.5; (b) the product 3.2 × 3.563; (c) the product $5.6 \times \pi$.
22. How many significant figures are there in (a) 78.9 ± 0.2, (b) 3.788×10^9, (c) 2.46×10^{-6}, (d) 0.0053?
23. The radius of a circle is measured to be 10.5 ± 0.2 m. Calculate the (a) area and (b) circumference of the circle and give the uncertainty in each value.
24. The *radius* of a solid sphere is measured by a student to be (6.50 ± 0.20) cm, and its mass is measured to be (1.85 ± 0.02) kg. Determine the density of the sphere, in kilograms per cubic meter, and the uncertainty in the density.

Section 1.7 Coordinate Systems and Frames of Reference

25. The polar coordinates of a point are $r = 5.50$ m and $\theta = 240.0°$. What are the cartesian coordinates of this point?
26. A point in the *xy* plane has cartesian coordinates (−3.00, 5.00) m. What are the polar coordinates of this point?
27. Two points in a plane have polar coordinates (2.50 m, 30.0°) and (3.80 m, 120.0°). Determine (a) the cartesian coordinates of these points and (b) the distance between them.
28. If the polar coordinates of the point (*x*, *y*) are (*r*, θ), determine the polar coordinates for the points: (a) (−*x*, *y*), (b) (−2*x*, −2*y*), and (c) (3*x*, −3*y*).

Section 1.9 Vectors and Scalars

Section 1.10 Some Properties of Vectors

29. A person walks along a circular path of radius 5.00 m, around one half of the circle. (a) Find the magnitude of the displacement vector. (b) How far did the person walk? (c) What is the magnitude of the displacement if the circle is completed?
30. A jogger runs 100 m due west, then changes direction for the second leg of the run. At the end of the run, she is 175 m away from the starting point at an angle of 15.0° north of west. What were the direction and magnitude of her second displacement? Use a graph to solve this problem.
31. Each of the displacement vectors **A** and **B** shown in Figure P1.31 on page 26 has a magnitude of 3.0 m. Graphically find (a) **A** + **B**, (b) **A** − **B**, (c) **B** − **A**, (d) **A** − 2**B**.
32. Indiana Jones is trapped in a maze. To find his way out, he walks 10.0 m, makes a 90.0° right turn, walks 5.00 m, makes

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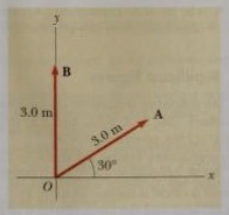


Figure P1.31

another 90.0° right turn, and walks 7.00 m. What is his displacement from his initial position?

Section 1.11 Components of a Vector and Unit Vectors

33. A vector has an x component of −25.0 units and a y component of 40.0 units. Find the magnitude and direction of this vector.
34. Find the horizontal and vertical components of the 100-m displacement of a superhero who flies from the top of a tall building along the path shown in Figure P1.34.

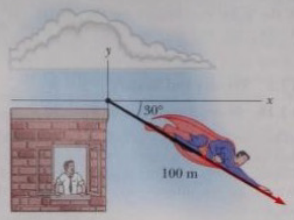


Figure P1.34

35. Instructions for finding a buried treasure include the following: Go 75 paces at 240°, turn to 135° and walk 125 paces, then travel 100 paces at 160°. Determine the resultant displacement from the starting point.
36. Given the vectors $\mathbf{A} = 2.00\mathbf{i} + 6.00\mathbf{j}$ and $\mathbf{B} = 3.00\mathbf{i} - 2.00\mathbf{j}$, (a) sketch the vector sum $\mathbf{C} = \mathbf{A} + \mathbf{B}$ and the vector subtraction $\mathbf{D} = \mathbf{A} - \mathbf{B}$. (b) Find analytical solutions for $\mathbf{C}$ and $\mathbf{D}$ first in terms of unit vectors and then in terms of polar coordinates, with angles measured with respect to the positive x axis.
37. Three vectors are oriented as shown in Figure P1.37, where $|\mathbf{A}| = 20.0$, $|\mathbf{B}| = 40.0$, and $|\mathbf{C}| = 30.0$ units. Find (a) the x and y components of the resultant vector and (b) the magnitude and direction of the resultant vector.

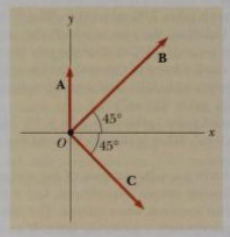


Figure P1.37

38. A novice golfer on the green takes three strokes to sink the ball. The successive displacements are 4.00 m due north, 2.00 m northeast, and 1.00 m 30.0° west of south. Starting at the same initial point, an expert golfer might make the hole in what single vector displacement?
39. A person going for a walk follows the path shown in Figure P1.39. The total trip consists of four straight-line paths. At the end of the walk, what is the person's resultant displacement, measured from the starting point?

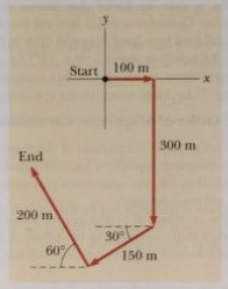


Figure P1.39

40. A vector is given by $\mathbf{R} = 2.00\mathbf{i} + 1.00\mathbf{j} + 3.00\mathbf{k}$. Find (a) the magnitudes of the x, y, and z components, (b) the magnitude of $\mathbf{R}$, and (c) the angles between $\mathbf{R}$ and the x, y, and z axes.
41. A particle undergoes two displacements. The first has a magnitude of 150 cm and makes an angle of 120.0° with the positive x axis. The *resultant* displacement has a magnitude of 140 cm and is directed at an angle of 35.0° to the positive x axis. Find the magnitude and direction of the second displacement.
42. A jet airliner moving initially at 300 mph due east enters a region where the wind is blowing at 100 mph in a direction 30.0° north of east. Determine the new speed and direction of the aircraft.

Additional Problems

43. A useful fact is that there are about $\pi \times 10^7$ s in one year. Use a calculator to find the percentage error in this approximation. *Note:*

$$\text{Percentage error} = \frac{|\text{assumed value} - \text{true value}|}{\text{true value}} \times 100\%$$

44. The eye of a hurricane passes over Grand Bahama Island. It is moving in a direction 60.0° north of west with a speed of 41.0 km/h. Three hours later the course of the hurricane suddenly shifts due north, and its speed slows to 25.0 km/h. How far is the hurricane from Grand Bahama 4.50 hours after it passes over the island?
45. Assume that there are 50 million passenger cars in the United States and that the average fuel consumption is 20 mi/gal of gasoline. If the average distance traveled by each car is 10 000 miles/year, how much gasoline would be saved per year if average fuel consumption could be increased to 25 mi/gal?
46. The basic function of the carburetor of an automobile is to atomize the gasoline and mix it with air to promote rapid combustion. As an example, assume that 30.0 cm^3 of gasoline is atomized into N spherical droplets, each with a radius of 2.00×10^{-5} m. What is the total surface area of these N spherical droplets?
47. Assume that 70 percent of the Earth's surface is covered with water at an average depth of 1 mile, and **estimate** the mass of the water on Earth in kilograms.
48. The data in the following table represent measurements of the masses and dimensions of solid cylinders of aluminum, copper, brass, tin, and iron. Use these data to calculate the densities of the substances. Compare your results for aluminum, copper, and iron with those given in Table 1.5.

Substance	Mass (g)	Diameter (cm)	Length (cm)
Aluminum	51.5	2.52	3.75
Copper	56.3	1.23	5.06
Brass	94.4	1.54	5.69
Tin	69.1	1.75	3.74
Iron	216.1	1.89	9.77

49. An air-traffic controller notices two aircraft on his radar screen. The first is at altitude 800 m, horizontal distance 19.2 km, and 25.0° south of west. The second aircraft is at altitude 1100 m, horizontal distance 17.6 km, and 20.0° south of west. What is the distance between the two aircraft? (Place the x axis west, the y axis south, and the z axis vertical.)
50. Two people pull on a stubborn mule, as shown by the helicopter view in Figure P1.50. Find (a) the single force that is equivalent to the two forces shown, and (b) the force that a third person would have to exert on the mule to make the net force equal to zero.

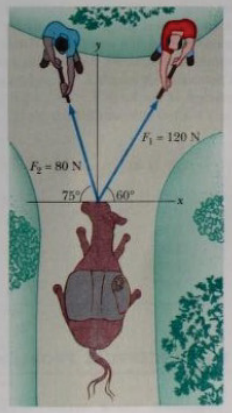


Figure P1.50

51. The distance from the Sun to the nearest star is 4×10^{16} m. The Milky Way galaxy is roughly a disk of radius $\sim 10^{21}$ m and thickness $\sim 10^{19}$ m. Find the **order of magnitude** of the number of stars in the Milky Way. Assume the

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distance between the Sun and the nearest neighbor is typical.

52. A rectangular parallelepiped has dimensions a, b, and c, as in Figure P1.52. (a) Obtain a vector expression for the face diagonal vector $\mathbf{R}_1$. What is the magnitude of this vector? (b) Obtain a vector expression for the body diagonal vector $\mathbf{R}_2$. Note that $\mathbf{R}_1$, $c\mathbf{k}$, and $\mathbf{R}_2$ form a right triangle and prove that the magnitude of $\mathbf{R}_2$ is $\sqrt{a^2 + b^2 + c^2}$.

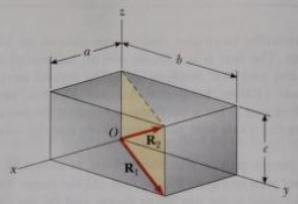


Figure P1.52

Spreadsheet Problems

S1. Spreadsheet programs are useful in examining data graphically. Most spreadsheet programs can fit the best straight line (a regression line) to a set of data. The following table gives experimental results for the measurement of the period T of a pendulum of length L. These data are consistent with an equation of the form $T = CL^n$, where C and n are constants and n is not necessarily an integer.

Length L(m)	Period T(s)
0.25	1.00
0.50	1.40
0.75	1.75
1.00	2.00
1.50	2.50
2.00	2.80

S1. (a) Use the least-squares or the regression-line procedure of your spreadsheet program to find the best fit to the data. Because $T = CL^n$, we see that

$$\log T = \log C + n \log L$$

First calculate a column of log T values and another of log L as the independent variable and log T as the dependent variable. The least-squares fit finds the slope n and the intercept log C. (b) Which data points deviate most from a straight line plot of T versus L^n? (c) Is the experimental value of n found in part (a) consistent with a dimensional analysis of $T = CL^n$?

S2. The period T and orbit radius R for the motions of four moons of Jupiter are

	Io	Europa	Ganymede	Callisto
T (days)	1.77	3.55	7.16	16.69
R (km)	422 000	671 000	1 070 000	1 880 000

(a) These data can be fitted by the formula $T = CR^n$. Follow the procedures in Problem S1a to find C and n. (b) A fifth satellite, Amalthea, has a period of 0.50 days. Use $T = CR^n$ to find the radius for its orbit.

S3. Use Spreadsheet 1.1 to find the total displacement corresponding to the sum of the three displacement vectors: $\mathbf{A} = 3.0\mathbf{i} + 4.0\mathbf{j}$, $\mathbf{B} = -2.3\mathbf{i} - 7.8\mathbf{j}$, and $\mathbf{C} = 5.0\mathbf{i} - 2.0\mathbf{j}$, where the units are in meters. (a) Give your answer in component form. (b) Give your answer in polar form. (c) Find a fourth displacement that returns you to the origin.

ANSWERS TO CONCEPTUAL PROBLEMS

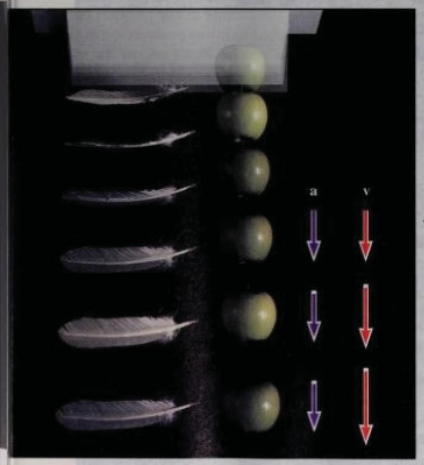
1. No. The magnitude of a vector $\mathbf{A}$ is equal to $\sqrt{A_x^2 + A_y^2 + A_z^2}$. Therefore, if any component is nonzero, $\mathbf{A}$ cannot be zero.
2. $\mathbf{A} = -\mathbf{B}$, therefore the components of the two vectors must have opposite signs and equal magnitudes.
3. (a) A_x is negative, B_x is positive; (b) A_y is positive, B_y is negative; (c) both components are negative.
4. It is possible to have a situation in which one component of a vector is equal to the magnitude of the vector, as long as the other component is *zero*. For example, a velocity vector lying directly along the east direction has an eastward component that is equal to the magnitude of the velocity vector.

2

Motion in One Dimension

Dynamics is the study of the motions of objects and the relationships of those motions to such physical concepts as force and mass. Before beginning our study of dynamics, however, it is convenient to describe motion using the concepts of space and time, without regard to the causes of the motion; this is a portion of mechanics called *kinematics*. In this chapter we shall consider motion along a straight line—that is, one-dimensional motion. Starting with the concept of displacement, discussed in Chapter 1, we shall define velocity and acceleration. Using these concepts, we shall proceed to study the motion of objects undergoing constant acceleration. In Chapter 3 we shall extend our discussion to two-dimensional motion.

From everyday experience we recognize that motion represents continuous change in the position of an object. The movement of an object through space may be accompanied by the rotation or vibration of the object. Such motions can be quite complex. However, it is sometimes possible to simplify matters by temporarily neglecting the internal motions of the moving object. In many situations, an object can be



◀ **An apple and a feather, released from rest in a 4-ft vacuum chamber, fall at the same rate, regardless of their masses. Neglecting air resistance, all objects fall to Earth with the same acceleration, $a = g$, of magnitude 9.80 m/s² as indicated by the violet arrows in this multiflash photograph. The velocity, v, of the two objects increases linearly with time, as indicated by the series of red arrows.**

CHAPTER OUTLINE

2.1 Average Velocity
2.2 Instantaneous Velocity
2.3 Acceleration
2.4 Motion Diagrams
2.5 One-Dimensional Motion with Constant Acceleration
2.6 Freely Falling Objects

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treated as a *particle* if the only motion being considered is translation through space.

Although an idealized particle is a mathematical point with no size, we can sometimes perform useful calculations by representing macroscopic objects as particles. For example, if we wish to describe the motion of the Earth around the Sun, we can approximate the Earth by treating it as a particle and thereby attain reasonable accuracy in predicting the Earth's orbit. This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and Sun. By contrast, we could not use a particle description to explain the internal structure of the Earth or such phenomena as tides, earthquakes, and volcanic activity. On a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles. However, the particle description of the gas molecules is generally inadequate for understanding those properties of the gas that depend on the internal motions (vibrations) and rotations of the gas molecules.

2.1 • AVERAGE VELOCITY

The motion of a particle is completely known if the position of the particle in space is known at all times. Consider a particle moving along the x axis from point P to point Q. Let its position at point P be x_i at some time t_i, and let its position at point Q be x_f at time t_f. At times other than t_i and t_f, the position of the particle between these two points may vary, as in Figure 2.1. Such a plot is often called a *position–time graph*. In the time interval $\Delta t = t_f - t_i$, the displacement of the particle is $\Delta x = x_f - x_i$. (Recall that displacement is defined as the change in the position of the particle, which is equal to its final position value minus its initial position value.) Using unit-vector notation, the **displacement vector** can be expressed as $\Delta \mathbf{x} \equiv (x_f - x_i)\mathbf{i}$.

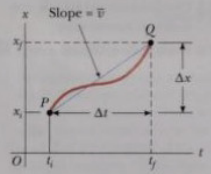


Figure 2.1 Position–time graph for a particle moving along the x axis. The average velocity $\bar{v}_x$ in the interval $\Delta t = t_f - t_i$ is obtained from the slope of the straight line connecting the points P and Q.

Definition of average velocity •

The x component of the **average velocity** of the particle, $\bar{\mathbf{v}}_x$, is defined as the ratio of its displacement vector, $\Delta \mathbf{x}$, and the time interval, Δt:

$$\bar{\mathbf{v}}_x \equiv \frac{\Delta \mathbf{x}}{\Delta t} = \frac{(x_f - x_i)\mathbf{i}}{t_f - t_i} \qquad [2.1]$$

From this definition we see that the average velocity has the dimensions of length divided by time (L/T)—m/s in SI units and ft/s in conventional units. The average velocity is *independent* of the path taken between the points P and Q. This is true because the average velocity is proportional to the displacement, $\Delta \mathbf{x}$, which in turn depends only on the initial and final coordinates of the particle. It therefore follows that if a particle starts at some point and returns to the same point via any path, its average velocity for this trip is zero, because its displacement along such a path is zero. Displacement should not be confused with the distance traveled, because the distance traveled for any motion is clearly nonzero. Thus, average velocity gives us no details of the motion between points P and Q. (How we evaluate the velocity at some instant in time is discussed in the next section.) Finally, note that the average velocity in one dimension can be positive or negative, depending on the sign of the displacement vector. (The time interval, Δt, is always positive.) If the x coordinate of the particle increases in time (that is, if $x_f > x_i$), then $\Delta \mathbf{x}$ is positive and $\bar{\mathbf{v}}_x$ is positive. This corresponds to an average velocity in the positive x direction. By contrast, if the coordinate decreases in time ($x_f < x_i$), $\Delta \mathbf{x}$ is negative; hence $\bar{\mathbf{v}}_x$ is negative. This corresponds to an average velocity in the negative x direction.

The average velocity can also be interpreted geometrically. A straight line drawn between the points P and Q in Figure 2.1 forms the hypotenuse of a right triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$. Therefore, we see that **the average velocity of the particle during the time interval t_i to t_f is equal to the slope of the straight line joining the initial and final points on the position–time graph**. (The word *slope* will often be used in reference to the graphs of physical data. Regardless of what data are plotted, the word *slope* will represent the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis.)

Example 2.1 Calculate the Average Velocity

A particle moving along the x axis is located at $x_i = 12$ m at $t_i = 1$ s and at $x_f = 4$ m at $t_f = 3$ s. Find its displacement and average velocity during this time interval.

Solution The displacement is

$$\Delta \mathbf{x} = (x_f - x_i)\mathbf{i} = (4\text{ m} - 12\text{ m})\mathbf{i} = -8\mathbf{i}\text{ m}$$

The average velocity is

$$\bar{\mathbf{v}}_x = \frac{\Delta \mathbf{x}}{\Delta t} = \frac{(x_f - x_i)\mathbf{i}}{t_f - t_i} = \frac{(4\text{ m} - 12\text{ m})\mathbf{i}}{3\text{ s} - 1\text{ s}} = -4\mathbf{i}\text{ m/s}$$

Because the displacement is negative for this time interval, we conclude that the particle has moved to the left, toward decreasing values of x.

EXERCISE 1 A jogger runs in a straight line, with an average speed of 5.00 m/s for 4.00 min, and then with an average speed of 4.00 m/s for 3.00 min. (a) What is her total displacement? (b) What is her average speed during this time?
Answer (a) 1920 m (b) $\frac{32}{7}$ m/s

2.2 • INSTANTANEOUS VELOCITY

We would like to be able to define the velocity of a particle at a particular instant of time, rather than just during a finite interval of time. The velocity of a particle at any instant of time, or at some point on a position–time graph, is called the **instantaneous velocity.** This concept is especially important when the average velocity is *not constant* through different time intervals.

Consider the motion of a particle between the two points P and Q on the position–time graph shown in Figure 2.2. As the point Q is brought closer and closer to the point P, the time intervals (Δt_1, Δt_2, Δt_3, . . .) get progressively smaller. The average velocity for each time interval is given by the slope of the appropriate dotted line in Figure 2.2. As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the line tangent to the curve at the point P. **The slope of the line tangent to the curve at P gives the instantaneous velocity at the time t_i.** In other words,

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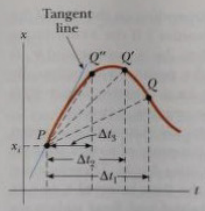


Figure 2.2 Position–time graph for a particle moving along the x axis. As the time intervals, starting at t_i, get smaller and smaller, the average velocity for one interval approaches the slope of the line tangent at P. The instantaneous velocity at P is obtained from the slope of the blue tangent line at the time t_i.

the instantaneous velocity, $\mathbf{v}_x$, equals the limiting value of the ratio $\Delta\mathbf{x}/\Delta t$ as Δt approaches zero:[1]

The instantaneous velocity vector is the derivative of the displacement vector with respect to time.

$$\mathbf{v}_x \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} \qquad [2.2]$$

In the calculus notation, this limit is called the *derivative* of x with respect to t, written $d\mathbf{x}/dt$:

$$\mathbf{v}_x \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{d\mathbf{x}}{dt} \qquad [2.3]$$

The instantaneous velocity can be positive, negative, or zero. When the slope of the position–time graph is positive, such as at the point P in Figure 2.3, v is positive. At point R, v is negative, because the slope is negative. Finally, the instantaneous velocity is zero at the peak Q (the turning point), where the slope is zero. **From here on, we shall usually use the word *velocity* to designate instantaneous velocity.**

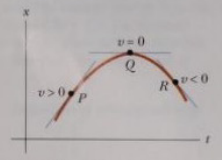


Figure 2.3 In the position–time graph shown, the velocity is positive at P, where the slope of the tangent line is positive; the velocity is zero at Q, where the slope of the tangent line is zero; and the velocity is negative at R, where the slope of the tangent line is negative.

The **instantaneous speed** of a particle is defined as the magnitude of the instantaneous velocity vector. Hence, by definition, *speed* can never be negative.

Those of you familiar with the calculus should recognize that there are specific rules for taking the derivatives of functions. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly.

Suppose x is proportional to some power of t, such as

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is

$$\frac{dx}{dt} = nAt^{n-1}$$

For example, if $x = 5t^3$, we see that $dx/dt = 3(5)t^{3-1} = 15t^2$.

[1] *Note:* The magnitude of the displacement, Δx, also approaches zero as Δt approaches zero. However, as Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the *true* slope of the line tangent to the x versus t curve.

Thinking Physics 1

Consider the following motions of an object in one dimension: (a) A ball is thrown directly upward, rises to a highest point, and falls back into the thrower's hand. (b) A race car starts from rest and speeds up to 100 m/s. (c) A Voyager spacecraft drifts through space at constant velocity. Are there any points in the motion of these particles at which the average velocity (over the entire interval) and the instantaneous velocity (at an instant of time within the interval) are the same? If so, identify the point(s).

Reasoning (a) The average velocity for the thrown ball is zero—the ball returns to the starting point at the end of the time interval. There is one point at which the instantaneous velocity is zero—at the top of the motion. (b) The average velocity for the motion of the race car cannot be evaluated unambiguously with the information given, but it must be some value between 0 and 100 m/s. Because the car will have every instantaneous velocity between 0 and 100 m/s at some time during the interval, there must be some instant at which the instantaneous velocity is equal to the average velocity. (c) Because the instantaneous velocity of the spacecraft is constant, its instantaneous velocity over *any* time interval and its average velocity at *any* time are the same.

Thinking Physics 2

Does an automobile speedometer measure average or instantaneous speed?

Reasoning Based on the fact that changes in the speed of the automobile are reflected in changes in the speedometer reading, we might be tempted to say that a speedometer measures instantaneous speed. We would be wrong, however. The reading on a speedometer is related to the rotation of the wheels. Often, a rotating magnet, whose rotation is related to that of the wheels, is used in a speedometer. The reading is based on the *time interval* between rotations of the magnet. Thus, the reading is an average velocity over this time interval. This time interval is generally very short, so that the average velocity is close to the instantaneous velocity. As long as the velocity changes are gradual, this approximation is very good. If the automobile brakes to a quick stop, however, the speedometer may not be able to keep up with the rapid changes in speed, and the measured average speed over the time interval may be very different than the instantaneous speed at the end of the interval.

CONCEPTUAL PROBLEM 1

Under what conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed?

Example 2.2 Average and Instantaneous Velocity

A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in meters and t is in seconds. The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, stops instantaneously at $t = 1$ s, and then heads back in the positive x direction for $t > 1$ s.

(a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution In the first time interval we set $t_i = 0$ and $t_f = 1$ s. Because $x = -4t + 2t^2$, we get for the first displacement

$$\Delta\mathbf{x}_{01} = (x_f - x_i)\mathbf{i} = [-4(1) + 2(1)^2]\mathbf{i} - [-4(0) + 2(0)^2]\mathbf{i}$$
$$= -2\mathbf{i}\ \text{m}$$

Likewise, in the second time interval we can set $t_i = 1$ s and $t_f = 3$ s. Therefore, the displacement in this interval is

$$\Delta\mathbf{x}_{13} = (x_f - x_i)\mathbf{i} = [-4(3) + 2(3)^2]\mathbf{i} - [-4(1) + 2(1)^2]\mathbf{i}$$
$$= 8\mathbf{i}\ \text{m}$$

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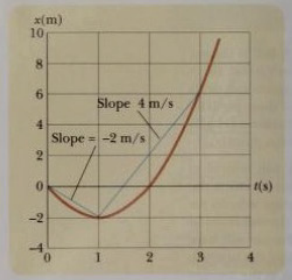


Figure 2.4 (Example 2.2) Position–time graph for a particle having an x coordinate that varies in time according to $x = -4t + 2t^2$.

Note that these displacements can also be read directly from the position–time graph (Fig. 2.4).

(b) Calculate the average velocity in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution In the first time interval, $\Delta t = t_f - t_i = 1$ s. Therefore, using Equation 2.1 and the results from (a) gives

$$\bar{\mathbf{v}}_x = \frac{\Delta \mathbf{x}_{01}}{\Delta t} = \frac{-2\mathbf{i} \text{ m}}{1 \text{ s}} = -2\mathbf{i} \text{ m/s}$$

Likewise, in the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{\mathbf{v}}_x = \frac{\Delta \mathbf{x}_{13}}{\Delta t} = \frac{8\mathbf{i} \text{ m}}{2 \text{ s}} = 4\mathbf{i} \text{ m/s}$$

These values (the coefficients of $\mathbf{i}$) agree with the slopes of the lines joining these points in Figure 2.4.

(c) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution We can find the instantaneous velocity at any time t by taking the first derivative of x with respect to t:

$$v = \frac{dx}{dt} = \frac{d}{dt}(-4t + 2t^2) = -4 + 4t$$

Thus, at $t = 2.5$ s, we find that $v = -4 + 4(2.5) = 6$ m/s.

We can also obtain this result by measuring the slope of the position–time graph at $t = 2.5$ s. (You should show that the velocity is $-4\mathbf{i}$ m/s at $t = 0$ and zero at $t = 1$ s.) Do you see any symmetry in the motion? For example, does the speed ever repeat itself?

Example 2.3 The Limiting Process

The position of a particle moving along the x axis varies in time according to the expression $x = (3 \text{ m/s}^2)\, t^2$, where x is in meters and t is in seconds. Find the velocity in terms of t at any time.

Solution The position–time graph for this motion is shown in Figure 2.5. We can compute the velocity at any time t by using the definition of the instantaneous velocity. If the initial coordinate of the particle at time t is $x_i = 3t^2$, then the coordinate at a later time, $t + \Delta t$, is

$$\begin{aligned} x_f &= 3(t + \Delta t)^2 = 3[t^2 + 2t\,\Delta t + (\Delta t)^2] \\ &= 3t^2 + 6t\,\Delta t + 3(\Delta t)^2 \end{aligned}$$

Therefore, the displacement in the time interval Δt is

$$\begin{aligned} \Delta \mathbf{x} &= (x_f - x_i)\mathbf{i} = [3t^2 + 6t\,\Delta t + 3(\Delta t)^2 - 3t^2]\mathbf{i} \\ &= (6t\,\Delta t + 3(\Delta t)^2)\mathbf{i} \end{aligned}$$

The average velocity in this time interval is

$$\bar{\mathbf{v}}_x = \frac{\Delta \mathbf{x}}{\Delta t} = (6t + 3\,\Delta t)\mathbf{i}$$

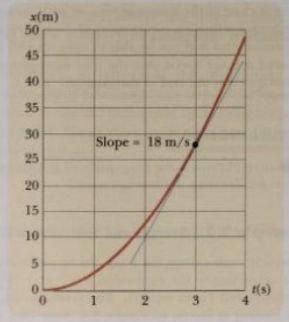


Figure 2.5 (Example 2.3) Position–time graph for a particle having an x coordinate that varies in time according to $x = 3t^2$. Note that the instantaneous velocity at $t = 3.0$ s is obtained from the slope of the blue line tangent to the curve at this point.

To find the instantaneous velocity, we take the limit of this expression as Δt approaches zero. In doing so, we see that the term $3\,\Delta t$ goes to zero; therefore,

$$\mathbf{v}_x = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = 6t\mathbf{i} \text{ m/s}$$

Notice that this expression gives us the velocity at *any* general time t. It tells us that $\mathbf{v}_x$ is increasing linearly in time. It is then a straightforward matter to find the velocity at some specific time from the expression $\mathbf{v}_x = 6t\mathbf{i}$. For example, at $t = 3.0$ s, the velocity is $\mathbf{v}_x = 6(3)\mathbf{i} = 18\mathbf{i}$ m/s. Again, this can be checked from the slope of the graph at $t = 3.0$ s.

We can also find v by taking the first derivative of x with respect to time. In this example, $x = 3t^2$, and we see that $v_x = dx/dt = 6t$, in agreement with our result of taking the limit explicitly.

2.3 • ACCELERATION

When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the speed of a car increases when you step on the gas. The car slows down when you apply the brakes, and it changes direction when you turn the wheel. However, we need a more precise definition of acceleration.

Suppose a particle moving along the x axis has a velocity $\mathbf{v}_{xi}$ at time t_i and a velocity $\mathbf{v}_{xf}$ at time t_f.

The **average acceleration** of a particle in the time interval $\Delta t = t_f - t_i$ is defined as the ratio $\Delta \mathbf{v}_x / \Delta t$, where $\Delta \mathbf{v}_x = \mathbf{v}_{xf} - \mathbf{v}_{xi}$ is the *change* in velocity in this time interval:

$$\bar{\mathbf{a}}_x \equiv \frac{\mathbf{v}_{xf} - \mathbf{v}_{xi}}{t_f - t_i} = \frac{\Delta \mathbf{v}_x}{\Delta t} \qquad [2.4]$$

• *Definition of average acceleration*

Acceleration is a vector quantity having dimensions of length divided by (time)^2, or L/T^2. Some of the common units of acceleration are meters per second per second (m/s^2) and feet per second per second (ft/s^2).

In some situations, the value of the average acceleration may be different for different time intervals. It is therefore useful to define the **instantaneous acceleration** as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we take the limit of the ratio $\Delta \mathbf{v}_x / \Delta t$ as Δt approaches zero, we get the **instantaneous acceleration:**

$$\mathbf{a}_x \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}_x}{\Delta t} = \frac{d\mathbf{v}_x}{dt} \qquad [2.5]$$

• *The instantaneous acceleration is the derivative of the velocity with respect to time.*

That is, **the instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity–time graph.** One can interpret the derivative of the velocity with respect to time as the *time rate of change of velocity*. Note that if $\mathbf{a}_x$ is positive, the acceleration is in the positive x direction, whereas negative $\mathbf{a}_x$ implies acceleration in the negative x direction. This does not mean that the particle is *moving* in the x direction. From now on we shall use the term *acceleration* to mean instantaneous acceleration.

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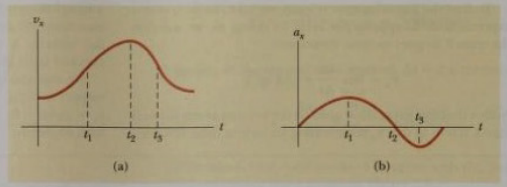


Figure 2.6 The instantaneous acceleration can be obtained from the velocity–time graph (a). At each instant, the acceleration in the a_x versus t graph (b) equals the slope of the line tangent to the v versus t curve.

Because $\mathbf{v}_x = d\mathbf{x}/dt$, the acceleration can also be written as

$$\mathbf{a}_x = \frac{d\mathbf{v}_x}{dt} = \frac{d}{dt}\left(\frac{d\mathbf{x}}{dt}\right) = \frac{d^2\mathbf{x}}{dt^2} \qquad [2.6]$$

That is, the acceleration equals the *second derivative* of the displacement with respect to time.

Figure 2.6 shows how the acceleration–time curve can be derived from the velocity–time curve. In these sketches, the acceleration at any time is simply the slope of the velocity–time graph at that time. Positive values of the acceleration correspond to those points at which the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time t_1, when the slope of the velocity–time graph is a maximum. The acceleration then goes to zero at time t_2, when the velocity is a maximum (that is, when the velocity is momentarily not changing and the slope of the v versus t graph is zero). Finally, the acceleration is negative when the velocity in the positive x direction is decreasing in time.

Acceleration can mean speeding up, slowing down, or changing direction.

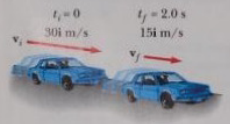


Figure 2.7 The velocity of the car decreases from 30**i** m/s to 15**i** m/s in a time interval of 2.0 s.

As an example of the computation of acceleration, consider the car pictured in Figure 2.7. In this case, the velocity of the car has changed from an initial value of 30**i** m/s to a final value of 15**i** m/s in a time interval of 2.0 s. The average acceleration during this time interval is

$$\bar{\mathbf{a}}_x = \frac{15\mathbf{i}\ \text{m/s} - 30\mathbf{i}\ \text{m/s}}{2.0\ \text{s}} = -7.5\mathbf{i}\ \text{m/s}^2$$

The minus sign in this example indicates that the acceleration vector is in the negative x direction (to the left). For the case of motion in a straight line, the direction of the velocity of an object and the direction of its acceleration are related as follows. **When the object's velocity and acceleration are in the same direction, the object is speeding up in that direction.** However, **when the object's velocity and acceleration are in opposite directions, the speed of the object decreases in time.**

CONCEPTUAL PROBLEM 2

If a car is traveling eastward, can its acceleration be westward? Explain.

CONCEPTUAL PROBLEM 3

Can a particle with constant acceleration ever stop and stay stopped?

Example 2.4 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $\mathbf{v}_x = (40 - 5t^2)\mathbf{i}$ m/s, where t is in seconds.

(a) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution The velocity–time graph for this function is given in Figure 2.8. The velocities at $t_i = 0$ and $t_f = 2.0$ s are found by substituting these values of t into the expression given for the velocity:

$$\mathbf{v}_{xi} = (40 - 5t_i^2)\mathbf{i}\ \text{m/s} = [40 - 5(0)^2]\mathbf{i}\ \text{m/s} = 40\mathbf{i}\ \text{m/s}$$
$$\mathbf{v}_{xf} = (40 - 5t_f^2)\mathbf{i}\ \text{m/s} = [40 - 5(2)^2]\mathbf{i}\ \text{m/s} = 20\mathbf{i}\ \text{m/s}$$

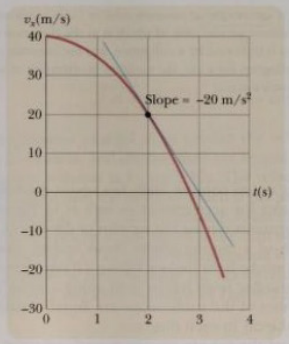


Figure 2.8 (Example 2.4) The velocity–time graph for a particle moving along the x axis according to the relation $v_x = (40 - 5t^2)$ m/s. Note that the acceleration at $t = 2.0$ s is obtained from the slope of the blue tangent line at that time.

Therefore, the average acceleration in the specified time interval, $\Delta t = t_f - t_i = 2.0$ s, is

$$\bar{\mathbf{a}}_x = \frac{\mathbf{v}_{xf} - \mathbf{v}_{xi}}{t_f - t_i} = \frac{(20 - 40)\mathbf{i}\ \text{m/s}}{(2 - 0)\text{s}} = -10\mathbf{i}\ \text{m/s}^2$$

The negative sign is consistent with the fact that the slope of the line joining the initial and final points on the velocity–time graph is negative.

(b) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at time t is $\mathbf{v}_{xi} = (40 - 5t^2)\mathbf{i}$ m/s, and the velocity at time $t + \Delta t$ is

$$\mathbf{v}_{xf} = 40\mathbf{i} - 5(t + \Delta t)^2\mathbf{i} = [40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2]\mathbf{i}$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta\mathbf{v}_x = \mathbf{v}_{xf} - \mathbf{v}_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\mathbf{i}\ \text{m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero, we get the acceleration at *any* time t:

$$\mathbf{a}_x = \lim_{\Delta t \to 0}\frac{\Delta\mathbf{v}_x}{\Delta t} = \lim_{\Delta t \to 0}(-10t - 5\,\Delta t)\mathbf{i} = -10t\mathbf{i}\ \text{m/s}^2$$

Therefore, at $t = 2.0$ s we find that

$$\mathbf{a}_x = (-10)(2)\mathbf{i}\ \text{m/s}^2 = -20\mathbf{i}\ \text{m/s}^2$$

This result can also be obtained by measuring the slope of the velocity–time graph at $t = 2.0$ s. Note that the acceleration is not constant in this example. Situations involving constant acceleration will be treated in Section 2.5.

EXERCISE 2 When struck by a club, a golf ball initially at rest acquires a speed of 31.0 m/s. If the ball is in contact with the club for 1.17 ms, what is the magnitude of the average acceleration of the ball? Answer 26 500 m/s²

2.4 · MOTION DIAGRAMS

The concepts of velocity and acceleration are often confused with each other, but in fact they are quite different quantities. It is instructive to make use of motion diagrams to describe the velocity and acceleration vectors as time progresses while

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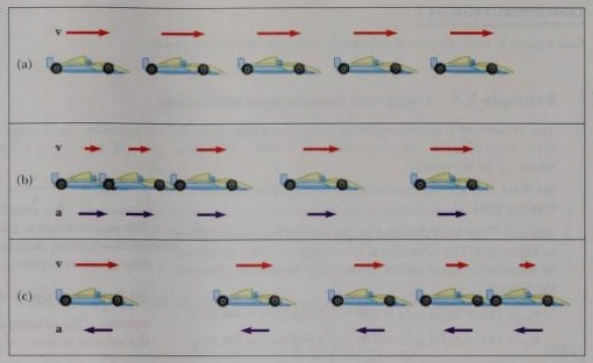


Figure 2.9 (a) Motion diagram for a car moving at constant velocity (zero acceleration). (b) Motion diagram for a car, the constant acceleration of which is in the direction of its velocity. The velocity vector at each instant is indicated by a red arrow, and the constant acceleration vector by a violet arrow. (c) Motion diagram for a car, the constant acceleration of which is in the direction *opposite* the velocity at each instant.

an object is in motion. In order not to confuse these two vector quantities, we use red for velocity vectors and violet for acceleration vectors as in Figure 2.9. The vectors are sketched at several instants during the motion of the object.

A stroboscopic photograph of a moving object shows several images of the object, each image taken as the strobe light flashes. Figure 2.9 represents three sets of strobe photographs of cars moving from left to right along a straight roadway. The time intervals between flashes of the stroboscope are equal in each diagram. Let us describe the motion of the car in each diagram.

In Figure 2.9a, the images of the car are equally spaced—the car moves the same distance in each time interval. Thus, the car moves with *constant positive velocity* and has zero acceleration.

In Figure 2.9b, the images of the car become farther apart as time progresses. In this case, the velocity vector increases in time because the car's displacement between adjacent positions increases as time progresses. Thus, the car is moving with a *positive velocity* and a *positive acceleration*.

In Figure 2.9c, the car slows as it moves to the right because its displacement between adjacent positions decreases as time progresses. In this case, the car moves initially to the right with a constant negative acceleration. The velocity vector decreases in time and eventually reaches zero. (This type of motion is exhibited by a car that skids to a stop after applying its brakes.) From this diagram we see that the acceleration and velocity vectors are *not* in the same direction.

2.5 • ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

If the acceleration of a particle varies in time, the motion can be complex and difficult to analyze. A common and simple type of one-dimensional motion occurs when the acceleration is constant or uniform. In this case the average acceleration over any time interval equals the instantaneous acceleration at any instant of time within the interval. As a consequence, the velocity increases or decreases at the same rate throughout the motion.

If we replace $\bar{\mathbf{a}}_x$ with $\mathbf{a}_x$ in Equation 2.4, we find that

$$\mathbf{a}_x = \frac{\mathbf{v}_{xf} - \mathbf{v}_{xi}}{t_f - t_i}$$

Because we are dealing with motion in one dimension, we shall drop the boldface notation for vectors and use a minus sign when the vector is in the negative x direction. For convenience, let $t_i = 0$ and t_f be any arbitrary time t. Also, let $v_i = v_{x0}$ (the initial velocity at $t = 0$) and $v_f = v_x$ (the velocity at any arbitrary time t). With this notation, we can express the acceleration as

$$a_x = \frac{v_x - v_{x0}}{t}$$

or

$$v_x = v_{x0} + a_x t \quad \text{(for constant } a_x\text{)} \qquad [2.7]$$

• *Velocity as a function of time*

This expression enables us to predict the velocity at *any* time t if the initial velocity, acceleration, and elapsed time are known. A graph of position versus time for this motion is shown in Figure 2.10a. The velocity–time graph shown in Figure 2.10b is a straight line, the slope of which is the acceleration, a_x. This is consistent with the fact that $a_x = dv_x/dt$ is a constant. From this graph and from Equation 2.7, we see that the velocity at any time t is the sum of the initial velocity, v_{x0}, and the change in velocity, $a_x t$. The graph of acceleration versus time (Fig. 2.10c) is a straight line with a slope of zero, because the acceleration is constant. Note that if the acceleration were negative, the slope of Figure 2.10b would be negative.

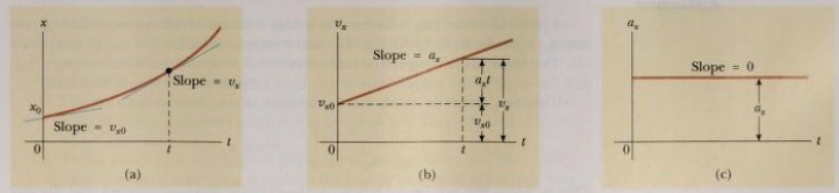


Figure 2.10 A particle moving along the x axis with constant acceleration a_x: (a) the position–time graph, (b) the velocity–time graph, and (c) the acceleration–time graph.

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Because the velocity varies linearly with time according to Equation 2.7, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity, v_{x0}, and the final velocity, v_x.

$$\bar{v}_x = \frac{v_{x0} + v_x}{2} \quad \text{(for constant } a_x\text{)} \qquad [2.8]$$

Note that this expression is useful only when the acceleration is constant—that is, when the velocity varies linearly with time.

We can now use Equations 2.1 and 2.8 to obtain the displacement as a function of time. Again we choose $t_i = 0$, at which time the initial position is $x_i = x_0$. This gives

$$\Delta x = \bar{v}_x \Delta t = \left(\frac{v_{x0} + v_x}{2}\right) t$$

$$x - x_0 = \frac{1}{2}(v_{x0} + v_x)t \quad \text{(for constant } a_x\text{)} \qquad [2.9]$$

We can obtain another useful expression for the displacement by substituting Equation 2.7 into Equation 2.9:

$$x - x_0 = \tfrac{1}{2}(v_{x0} + v_{x0} + a_x t)t$$

Displacement as a function of time

$$x - x_0 = v_{x0}t + \tfrac{1}{2}a_x t^2 \quad \text{(for constant } a_x\text{)} \qquad [2.10]$$

The validity of this expression can be checked by differentiating it with respect to time, to give

$$v_x = \frac{dx}{dt} = \frac{d}{dt}(x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2) = v_{x0} + a_x t$$

Finally, we can obtain an expression that does not contain the time by substituting the value of t from Equation 2.7 into Equation 2.9. This gives

$$x - x_0 = \tfrac{1}{2}(v_{x0} + v_x)\left(\frac{v_x - v_{x0}}{a_x}\right) = \frac{v_x^2 - v_{x0}^2}{2a_x}$$

Velocity as a function of displacement

$$v_x^2 = v_{x0}^2 + 2a_x(x - x_0) \quad \text{(for constant } a_x\text{)} \qquad [2.11]$$

A position–time graph for motion under constant acceleration, assuming positive a_x, is shown in Figure 2.10a. The curve representing Equation 2.10 is a parabola. The slope of the tangent to this curve at $t = 0$ equals the initial velocity, v_{x0}, and the slope of the tangent line at any time t equals the velocity at that time.

If motion occurs in which the acceleration is *zero*, then we see that

$$\left.\begin{aligned} v_x &= v_{x0} \\ x - x_0 &= v_x t \end{aligned}\right\} \quad \text{when } a_x = 0$$

That is, when the acceleration is zero, the velocity remains constant and the displacement changes linearly with time.

Equations 2.7 through 2.11 **may be used to solve any problem in one-dimensional motion with constant acceleration.** Keep in mind that these relationships were derived from the definitions of velocity and acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant. It is often convenient to choose the initial position of the particle as the origin of the motion, so that $x_0 = 0$ at $t = 0$. In such a case, the displacement is simply x.

TABLE 2.1 Kinematic Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v_x = v_{x0} + a_x t$	Velocity as a function of time
$x - x_0 = \frac{1}{2}(v_x + v_{x0})t$	Displacement as a function of velocity and time
$x - x_0 = v_{x0}t + \frac{1}{2}a_x t^2$	Displacement as a function of time
$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the position of the particle is x_0 and its velocity is v_{x0}.

The four kinematic equations that are used most often are listed in Table 2.1. The choice of which kinematic equation or equations you should use in a given situation depends on what is known beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns, such as the displacement and velocity at some instant. For example, suppose the initial velocity, v_{x0}, and acceleration, a_x, are given. You can then find (1) the velocity after a time t has elapsed using $v_x = v_{x0} + a_x t$, and (2) the displacement after a time t has elapsed using $x - x_0 = v_{x0}t + \frac{1}{2}a_x t^2$. You should recognize that the quantities that vary during the motion are velocity, displacement, and time.

You will get a great deal of practice in the use of these equations by solving exercises and problems. You will discover again and again that there is more than one method for obtaining a solution.

PROBLEM-SOLVING STRATEGY • Accelerated Motion

The following procedure is recommended for solving problems that involve accelerated motion:

1. Make sure all the units in the problem are consistent. That is, if distances are measured in meters, be sure that velocities have units of meters per second and accelerations have units of meters per second per second.
2. Choose a coordinate system.
3. Choose an instant to call the *initial point* and another the *final point*. Make a list of all the quantities given in the problem and a separate list of those to be determined.
4. Think about what is going on physically in the problem and then select from the list of kinematic equations the one or ones that will enable you to determine the unknowns.
5. Construct an appropriate motion diagram and check to see if your answers are consistent with the diagram.

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Example 2.5 Watch Out for the Speed Limit

A car traveling at a constant speed of 30.0 m/s (≈67 mi/h) passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets off in chase with a constant acceleration of 3.00 m/s². How long does it take the trooper to overtake the speeding car?

Reasoning To solve this problem algebraically, we write an expression for the position of each vehicle as a function of time. It is convenient to choose the origin at the position of the billboard and take $t = 0$ as the time the trooper begins moving. At that instant, the speeding car has already traveled a distance of 30.0 m, because it travels at a constant speed of 30.0 m/s. Thus, the initial position of the speeding car is $x_0 = 30.0$ m.

Solution Because the car moves with constant speed, its acceleration is zero, and applying Equation 2.10 gives

$$x_C = 30.0 \text{ m} + (30.0 \text{ m/s})t$$

Note that at $t = 0$, this expression does give the car's correct initial position, $x_C = x_0 = 30.0$ m.

For the trooper, who starts from the origin at $t = 0$, we have $x_0 = 0$, $v_{x0} = 0$, and $a_x = 3.00$ m/s². Hence, the position of the trooper as a function of time is

$$x_T = \tfrac{1}{2}a_x t^2 = \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2$$

The trooper overtakes the car at the instant that $x_T = x_C$, or

$$\tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = 30.0 \text{ m} + (30.0 \text{ m/s})t$$

This gives the quadratic equation

$$1.50t^2 - 30.0t - 30.0 = 0$$

the positive solution of which is $t = 21.0$ s. Note that in this time interval, the trooper has traveled a distance of about 660 m.

EXERCISE 3 This problem can also be easily solved graphically. On the *same* graph, plot the position versus time for each vehicle, and from the intersection of the two curves determine the time at which the trooper overtakes the speeding car.

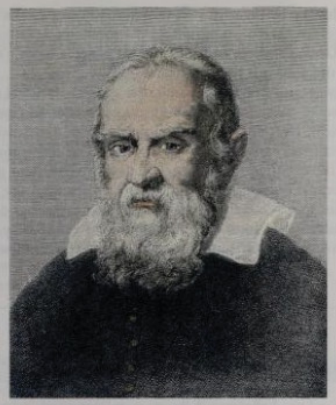
EXERCISE 4 A hockey puck sliding on a frozen lake comes to rest after traveling 200 m. Its initial speed was 3.00 m/s. (a) What was its acceleration if it is assumed to have been constant? (b) How long was it in motion? (c) What was its speed after traveling 150 m?

Answer (a) -2.25×10^{-2} m/s² (b) 133 s (c) 1.50 m/s

2.6 · FREELY FALLING OBJECTS

It is well known that all objects, when dropped, fall toward the Earth with nearly constant acceleration in the absence of air resistance. There is a legend that Galileo Galilei first discovered this fact by observing that two different weights dropped simultaneously from the Tower of Pisa hit the ground at approximately the same time. Although there is some doubt that this particular experiment was carried out, it is well established that Galileo did perform many systematic experiments on objects moving on inclined planes. Through careful measurements of distances and time intervals, he was able to show that the displacement of an object starting from rest is proportional to the square of the time the object is in motion. This observation is consistent with one of the kinematic equations we derived for motion under constant acceleration (Eq. 2.10). Galileo's achievements in the science of mechanics paved the way for Newton in his development of the laws of motion.

You might want to try the following experiment. Drop a coin and a crumpled-up piece of paper simultaneously from the same height. In the absence of air resistance, the two experience the same motion and hit the floor at the same time. In a real (nonideal) experiment, air resistance cannot be neglected. In the idealized case, however, where air resistance *is* neglected, such motion is referred to as *free fall*. If this same experiment could be conducted in a good vacuum, where air friction is truly negligible, the paper and coin would fall with the same acceleration, regardless of the shape or weight of the paper. This point is illustrated convincingly on page 29, in the photograph of the apple and feather falling in a vacuum. On August 2, 1971, such an experiment was conducted on the Moon by astronaut David Scott. He simultaneously released a geologist's hammer and a falcon's feather, and in unison they fell to the lunar surface. This demonstration surely would have pleased Galileo!



Galileo Galilei (1564–1642).

We shall denote the free-fall acceleration with the symbol **g**. The magnitude of **g** decreases with increasing altitude. Furthermore, there are slight variations in the magnitude of **g** with latitude. However, at the surface of the Earth the magnitude of **g** is approximately 9.80 m/s², or 980 cm/s², or 32 ft/s². Unless stated otherwise, we shall use the value 9.80 m/s² when doing calculations. Furthermore, we shall assume that the vector **g** is directed downward toward the center of the Earth.

When we use the expression *freely falling object*, we do not necessarily mean an object dropped from rest.



> A freely falling object is an object moving freely under the influence of gravity only, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling freely once they are released.

It is important to emphasize that any freely falling object experiences an acceleration directed downward, as shown in the multiflash photograph of a falling billiard

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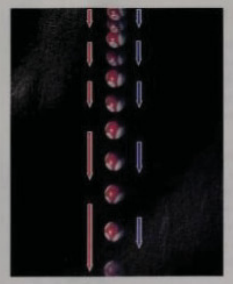


Figure 2.11 A multiflash photograph of a falling billiard ball. As the ball falls, the spacing between successive images increases, indicating that the ball accelerates downward. The motion diagram shows that the ball's velocity (red arrows) increases with time, whereas its acceleration (violet arrows) remains constant. *(© Richard Megna 1990, Fundamental Photographs)*

ball (Fig. 2.11). This is true regardless of the initial motion of the object. An object thrown upward (or downward) experiences the same acceleration as an object released from rest. **Once they are in free fall, all objects have a downward acceleration equal to the free-fall acceleration.**

If we neglect air resistance and assume that the gravitational acceleration does not vary with altitude, then a freely falling object moves in one dimension with constant acceleration. Therefore, the equations developed in Section 2.5 for objects moving with constant acceleration can be applied. The only modification that we need to make in these equations for freely falling objects is to note that the motion is in the vertical direction (the y direction) rather than the horizontal direction (the x direction) and that the acceleration is downward and has a magnitude of 9.80 m/s^2. Thus, for a freely falling object we commonly take $a_y = -g = -9.80$ m/s^2, where the minus sign means that the acceleration of the object is downward.

Thinking Physics 3

A skydiver jumps out of a helicopter. A few seconds later, another skydiver jumps out, so that they both fall along the same vertical line. Ignore air resistance, so that both skydivers fall with the same acceleration. Does the vertical distance between them stay the same? Does the difference in their velocities stay the same? If they were connected by a long bungee cord, would the tension in the cord become greater, less, or stay the same?

Reasoning At any given instant of time, the velocities of the jumpers are definitely different, because one had a head start. In a time interval after this instant, however, each jumper increases his or her velocity by the same amount, because they have the same acceleration. Thus, the difference in velocities remains the same. The first jumper will always be moving with a higher velocity than the second. Thus, in a given time interval, the first jumper will cover more distance than the second. Thus, the separation distance between them increases. As a result, once the distance between the divers reaches the value at which the bungee cord becomes straight, the tension in the bungee cord will increase as the jumpers move apart. Of course, the bungee cord will pull downward on the second jumper and upward on the first, resulting in a force in addition to gravity, which will affect their motion. If the bungee cord is robust enough, this will result in the separation distance of the divers becoming smaller as the cord pulls them together.

CONCEPTUAL PROBLEM 4

A child throws a ball into the air with some initial velocity. Another child drops a toy at the same instant. Compare the accelerations of the two objects while they are in the air.

CONCEPTUAL PROBLEM 5

A ball is thrown upward. While the ball is in the air, (a) what happens to its velocity? (b) Does its acceleration increase, decrease, or remain constant?

Example 2.6 Try to Catch the Dollar

Emily challenges David to catch a dollar bill as follows. She holds the bill vertically, as in Figure 2.12, with the center of the bill between David's index finger and thumb. David must catch the bill after Emily releases it without moving his hand downward. Who would you bet on?

Reasoning Place your bets on Emily. There is a time delay between the instant Emily releases the bill and the time David reacts and closes his fingers. The reaction time of most people is at best about 0.2 s. Because the bill is in free fall and undergoes a downward acceleration of 9.80 m/s^2, in 0.2 s it falls a distance of $\frac{1}{2}gt^2 \cong 0.2$ m = 20 cm. This distance is about twice the distance between the center of the bill and its top edge ($\cong$ 8 cm). Thus, David will be unsuccessful. You might want to try this on one of your friends.

Figure 2.12 (Example 2.6)



Example 2.7 Not a Bad Throw for a Rookie!

A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as in Figure 2.13. Determine (a) the time needed for the stone to reach its maximum height, (b) the maximum height, (c) the time needed for the stone to return to the level of the thrower, (d) the velocity of the stone at this instant, and (e) the velocity and position of the stone at t = 5.00 s.

Solution (a) To find the time necessary to reach the maximum height, use Equation 2.7, $v_y = v_{y0} + a_y t$, noting that $v_y = 0$ at maximum height:

$$20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t_1 = 0$$

$$t_1 = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = 2.04 \text{ s}$$

(b) This value of time can be substituted into Equation 2.10, $y = v_{y0}t + \frac{1}{2}a_y t^2$, to give the maximum height measured from the position of the thrower:

$$y_{max} = (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= 20.4 \text{ m}$$

(c) When the stone is back at the height of the thrower, the y coordinate is zero. From the expression $y = v_{y0}t + \frac{1}{2}a_y t^2$ (Eq. 2.10), with $y = 0$, we obtain the expression

$$20.0t - 4.90t^2 = 0$$

This is a quadratic equation and has two solutions for t. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

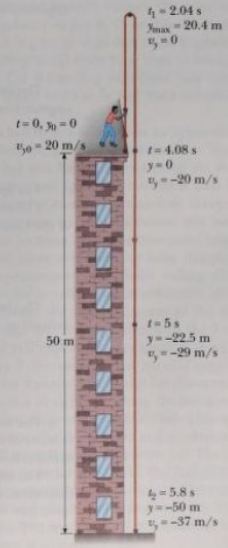


Figure 2.13 (Example 2.7) Position and velocity versus time for a freely falling particle initially thrown upward with a velocity v_{y0} = 20 m/s. (Not drawn to scale.)

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One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution—the one we are after—is $t = 4.08$ s.

(d) The value for t found in (c) can be inserted into $v_y = v_{y0} + a_y t$ (Eq. 2.7) to give

$$v_y = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s}) = -20.0 \text{ m/s}$$

Note that the velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction. This indicates that the motion is symmetric.

(e) From $v_y = v_{y0} + a_y t$ (Eq. 2.7), the velocity after 5.00 s is

$$v_y = 20 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s}) = -29.0 \text{ m/s}$$

We can use $y = v_{y0} t + \frac{1}{2} a_y t^2$ (Eq. 2.10) to find the position of the particle at $t = 5.00$ s:

$$y = (20.0 \text{ m/s})(5.00 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s})^2$$
$$= -22.5 \text{ m}$$

EXERCISE 5 Find (a) the velocity of the stone just before it hits the ground and (b) the total time the stone is in the air.
Answer (a) −37.1 m/s (b) 5.83 s

SUMMARY

The **average velocity** of a particle during some time interval is equal to the ratio of the displacement vector, $\Delta\mathbf{x}$, and the time interval, Δt:

$$\bar{\mathbf{v}}_x = \frac{\Delta \mathbf{x}}{\Delta t} \qquad [2.1]$$

The **instantaneous velocity** of a particle is defined as the limit of the ratio $\Delta\mathbf{x}/\Delta t$ as Δt approaches zero:

$$\mathbf{v}_x = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{x}}{\Delta t} = \frac{d\mathbf{x}}{dt} \qquad [2.3]$$

The **speed** of a particle is defined as the magnitude of the instantaneous velocity vector.

The **average acceleration** of a particle during some time interval is defined as the ratio of the change in its velocity, $\Delta\mathbf{v}_x$, and the time interval, Δt:

$$\bar{\mathbf{a}}_x = \frac{\Delta \mathbf{v}_x}{\Delta t} \qquad [2.4]$$

The **instantaneous acceleration** is equal to the limit of the ratio $\Delta\mathbf{v}_x/\Delta t$ as $\Delta t \to 0$. By definition, this equals the derivative of $\mathbf{v}_x$ with respect to t or the time rate of change of the velocity:

$$\mathbf{a}_x = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}_x}{\Delta t} = \frac{d\mathbf{v}_x}{dt} \qquad [2.5]$$

The slope of the tangent to the x versus t curve at any instant gives the instantaneous velocity of the particle. The slope of the tangent to the v versus t curve gives the instantaneous acceleration of the particle.

The **equations of kinematics** for a particle moving along the x axis with uniform acceleration a (constant in magnitude and direction) are

$$\left.\begin{aligned} v_x &= v_{x0} + a_x t && [2.7] \\ x - x_0 &= \tfrac{1}{2}(v_{x0} + v_x) t && [2.9] \\ x - x_0 &= v_{x0} t + \tfrac{1}{2} a_x t^2 && [2.10] \\ v_x^2 &= v_{x0}^2 + 2a_x(x - x_0) && [2.11] \end{aligned}\right\} \quad \text{(constant } a_x \text{ only)}$$

A body falling freely experiences an acceleration directed toward the center of the Earth. If air friction is neglected, and if the altitude of the motion is small compared with the Earth's radius, then one can assume that the free-fall acceleration, $\mathbf{g}$, is constant over the range of motion, where g is equal to 9.80 m/s², or 32 ft/s². Assuming y to be positive upward, the acceleration is given by $-g$, and the equations of kinematics for a body in free fall are the same as those already given, with the substitutions $x \to y$ and $a_y \to -g$.

CONCEPTUAL QUESTIONS

1. Are there any conditions under which the average velocity of a particle moving in one dimension is larger than the average speed?
2. The average velocity for a particle moving in one dimension has a positive value. Is it possible for the instantaneous velocity of the particle to have been negative at any time in the interval? Suppose the particle started at the origin, $x = 0$. If the average velocity is positive, could the particle ever have been in the $-x$ region?
3. If the average velocity of an object is zero in some time interval, what can you say about the displacement of the object for that interval?
4. Can the instantaneous velocity of an object ever be greater in magnitude than the average velocity? Can it ever be less?
5. Consider the following combinations of signs and values for velocity and acceleration of a particle with respect to a one-dimensional x axis:

	Velocity	Acceleration
a.	Positive	Positive
b.	Positive	Negative
c.	Positive	Zero
d.	Negative	Positive
e.	Negative	Negative
f.	Negative	Zero
g.	Zero	Positive
h.	Zero	Negative

Describe what a particle is doing in each case, and give a real-life example for an automobile on an east–west one-dimensional axis, with east considered the positive direction.

6. Can the equations of kinematics (Eqs. 2.7–2.11) be used in a situation in which the acceleration varies in time? Can they be used when the acceleration is zero?
7. A student at the top of a building of height h throws one ball upward with an initial speed v_0 and then throws a second ball downward with the same initial speed. How do the final velocities of the balls compare when they reach the ground?
8. Two cars are moving in the same direction in parallel lanes along a highway. At some instant, the velocity of car A exceeds the velocity of car B. Does this mean that the acceleration of A is greater than that of B? Explain.
9. Car A traveling south from New York to Miami has a speed of 25 m/s. Car B traveling west from New York to Chicago also has a speed of 25 m/s. Are their velocities equal? Explain.
10. You drop a ball from a window on an upper floor of a building. It strikes the ground with velocity v. You now repeat the drop, but you have a friend down on the street who throws another ball upward at velocity v. Your friend throws the ball upward at exactly the same time that you drop yours from the window. At some location, the balls pass each other. Is this location *at* the halfway point between window and ground, *above* this point, or *below* this point?
11. Galileo experimented with balls rolling down inclined planes in order to reduce the acceleration along the plane and thus reduce the rate of descent of the balls. Suppose the angle that the incline makes with the horizontal is θ. How would you expect the acceleration along the plane to decrease as θ decreases? What specific trigonometric dependence on θ would you expect for the acceleration?
12. A ball rolls in a straight line along the horizontal direction. Using motion diagrams (or multiflash photographs) as in Figure 2.9, describe the velocity and acceleration of the ball for each of the following situations: (a) The ball moves to the right at a constant speed. (b) The ball moves from right to left and continually slows down. (c) The ball moves from right to left and continually speeds up. (d) The ball moves to the right, first speeding up at a constant rate and then slowing down at a constant rate.

Boxed numbers indicate questions that have answers available in the Student Solutions Manual and Study Guide.

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Figure Q2.14 See Question 14.

13. A rapidly growing plant doubles in height each week. At the end of the 25th day, the plant reaches the height of the building. At what time was the plant one fourth the height of the building?
14. A pebble is dropped into a water well, and the splash is heard 16 s later, as illustrated in the cartoon strip. What is the *approximate* distance from the rim of the well to the water's surface?
15. The motion of the Earth's crustal plates is described by a model referred to as *plate tectonic motion*. Measurements indicate that coastal portions of southern California have northward plate tectonic motions of 2.5 cm per year. Estimate the time it would take for this motion to carry southern California to Alaska.
16. A heavy object falls from a height h under the influence of gravity. It is released at $t = 0$ and strikes the ground at time t. Ignore air resistance. (a) When the object is at height $0.5\,h$, is the time earlier than $0.5\,t$, later than $0.5\,t$, or equal to $0.5\,t$? (b) When the time is $0.5\,t$, is the height of the object greater than $0.5\,h$, less than $0.5\,h$, or equal to $0.5\,h$?

PROBLEMS

Section 2.1 Average Velocity

1. The position of a pinewood derby car was observed at various times; the results are summarized in the table. Find the average velocity of the car for (a) the first second, (b) the last 3 seconds, and (c) the entire period of observation.

x (m)	0	2.3	9.2	20.7	36.8	57.5
t (s)	0	1.0	2.0	3.0	4.0	5.0

2. A motorist drives north for 35.0 minutes at 85.0 km/h and then stops for 15.0 minutes. He then continues north, traveling 130 km in 2.00 h. (a) What is his total displacement? (b) What is his average velocity?
3. The displacement versus time for a certain particle moving along the x axis is shown in Figure P2.3. Find the average velocity in the time intervals (a) 0 to 2 s, (b) 0 to 4 s, (c) 2 s to 4 s, (d) 4 s to 7 s, (e) 0 to 8 s.
4. A person walks first at a constant speed of 5.00 m/s along a straight line from point A to point B and then back along the line from B to A at a constant speed of 3.00 m/s. (a) What is her average speed over the entire trip? (b) Her average velocity over the entire trip?
5. A person first walks at a constant speed v_1 along a straight line from A to B and then back along the line from B to A at a constant speed v_2. (a) What is her average speed over the entire trip? (b) Her average velocity over the entire trip?

Boxed numbers indicate problems that have full solutions available in the Student Solutions Manual and Study Guide. Cyan numbers indicate intermediate level problems and magenta numbers indicate more challenging problems.

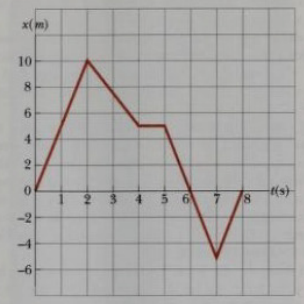


Figure P2.3

Section 2.2 Instantaneous Velocity

6. At $t = 1.00$ s, a particle moving with constant velocity is located at $x = -3.00$ m, and at $t = 6.00$ s the particle is located at $x = 5.00$ m. (a) From this information, plot the position as a function of time. (b) Determine the velocity of the particle from the slope of this graph.
7. A position–time graph for a particle moving along the x axis is shown in Figure P2.7. (a) Find the average velocity in the time interval $t = 1.5$ s to $t = 4.0$ s. (b) Determine the instantaneous velocity at $t = 2.0$ s by measuring the slope of the tangent line shown in the graph. (c) At what value of t is the velocity zero?

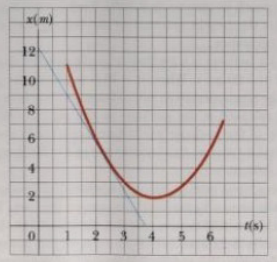


Figure P2.7

8. (a) Use the data in Problem 1 to construct a smooth graph of position versus time. (b) By constructing tangents to the $x(t)$ curve, find the instantaneous velocity of the car at several instants. (c) Plot the instantaneous velocity versus time and, from this, determine the average acceleration of the car. (d) What was the initial velocity of the car?
9. Find the instantaneous velocity of the particle described in Figure P2.3 at the following times: (a) $t = 1.0$ s, (b) $t = 3.0$ s, (c) $t = 4.5$ s, and (d) $t = 7.5$ s.

Section 2.3 Acceleration

10. A particle is moving with a velocity $\mathbf{v}_0 = 60.0\hat{\mathbf{i}}$ m/s at $t = 0$. Between $t = 0$ and $t = 15.0$ s the velocity decreases uniformly to zero. What was the acceleration during this 15.0-s interval? What is the significance of the sign of your answer?
11. A 50-g superball traveling at 25.0 m/s bounces off a brick wall and rebounds at 22.0 m/s. A high-speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval? (*Note:* $1\text{ ms} = 10^{-3}$ s.)
12. A velocity–time graph for an object moving along the x axis is shown in Figure P2.12. (a) Plot a graph of the acceleration versus time. (b) Determine the average acceleration of the object in the time intervals $t = 5$ s to $t = 15$ s and $t = 0$ to $t = 20$ s.

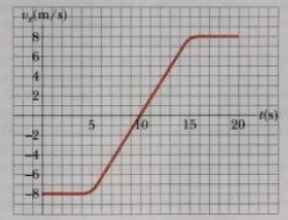


Figure P2.12

13. A particle moves along the x axis according to the equation $x = 2.0 + 3.0t - t^2$, where x is in meters and t is in seconds. At $t = 3.00$ s, find (a) the position of the particle, (b) its velocity, and (c) its acceleration.
14. When struck by a club, a golf ball initially at rest acquires a speed of 31.0 m/s. If the ball is in contact with the club for 1.17 ms, what is the magnitude of the average acceleration of the ball?

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15. Figure P2.15 shows a graph of v versus t for the motion of a motorcyclist as he starts from rest and moves along the road in a straight line. (a) Find the average acceleration for the time interval $t_0 = 0$ to $t_1 = 6.0$ s. (b) Estimate the time at which the acceleration has its greatest positive value and the value of the acceleration at that instant. (c) When is the acceleration zero? (d) Estimate the maximum negative value of the acceleration and the time at which it occurs.

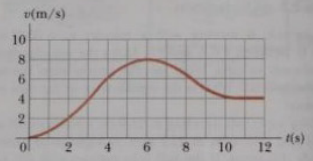


Figure P2.15

Section 2.4 Motion Diagrams

Section 2.5 One-Dimensional Motion with Constant Acceleration

Section 2.6 Freely Falling Objects

16. A truck travels 40.0 m in 8.50 s while uniformly slowing down to a final speed of 2.80 m/s. Find (a) its original speed and (b) its acceleration.
17. A body moving with uniform acceleration has a velocity of 12.0 cm/s in the positive x direction when its x coordinate is 3.00 cm. If its x coordinate 2.00 s later is -5.00 cm, what is the magnitude of its acceleration?
18. The new BMW M3 can accelerate from 0 to 60.0 mi/h in 5.00 s. (a) What is this acceleration in m/s²? (b) How long does it take this car to go from 60.0 mi/h to 130 mi/h?
19. The minimum distance required to stop a car moving at 35.0 mi/h is 40.0 ft. What is the minimum stopping distance for the same car moving at 70.0 mi/h, assuming the same rate of acceleration?
20. A drag racer starts her car from rest and accelerates at 10.0 m/s² for the entire distance of 400 m ($\frac{1}{4}$ mile). (a) How long did it take the race car to travel this distance? (b) What is the speed of the race car at the end of the run?
21. A jet plane lands with a speed of 100 m/s and can accelerate at a maximum rate of -5.00 m/s² as it comes to rest. (a) From the instant the plane touches the runway, what is the minimum time needed before it can come to rest? (b) Can this plane land on a small tropical island airport where the runway is 0.800 km long?
22. A locomotive slows from a speed of 26.0 m/s to 0 in 18.0 s. What distance does it travel?
23. The driver of a car slams on the brakes when he sees a tree blocking the road. The car slows uniformly with acceleration -5.60 m/s² for 4.20 s, making straight skid marks 62.4 m long. With what speed does the car then strike the tree?
24. *Help! One of our equations is missing!* We describe motion under constant acceleration with the parameters v_x, v_{xo}, a_x, t, and $x - x_0$. Of the equations listed in Table 2.1, the first does not involve $x - x_0$; the second does not contain a_x; the third omits v_x; and the last does not contain t. To complete the set, there should be an equation not involving v_{xo}. Derive this equation from the others. Use it to solve Problem 23 in one step.
25. Until recently, the world's land-speed record was held by Col. John P. Stapp, USAF. On March 19, 1954, he rode a rocket-propelled sled that moved down a track at 632 mi/h. He and the sled were safely brought to rest in 1.40 s. Determine (a) the negative acceleration he experienced and (b) the distance he traveled during this negative acceleration.
26. An electron in a cathode-ray-tube (CRT) accelerates from 2.00×10^4 m/s to 6.00×10^6 m/s over 1.50 cm. (a) How long does the electron take to travel this 1.50 cm? (b) What is its acceleration?
27. A student throws a set of keys vertically upward to her sorority sister, who is in a window 4.00 m above. The keys are caught 1.50 s later by the sister's outstretched hand. (a) With what initial velocity were the keys thrown? (b) What was the velocity of the keys just before they were caught?
28. A ball is thrown directly downward, with an initial speed of 8.00 m/s, from a height of 30.0 m. After what interval does the ball strike the ground?
29. A baseball is hit so that it travels straight upward after being struck by the bat. A fan observes that it takes 3.00 s for the ball to reach its maximum height. Find (a) its initial velocity and (b) the height reached by the ball. Ignore the effects of air resistance.
30. A hot-air balloon is traveling vertically upward at a constant speed of 5.00 m/s. When it is 21.0 m above the ground, a package is released from the balloon. (a) For how long after being released is the package in the air? (b) What is the velocity of the package just before impact with the ground? (c) Repeat (a) and (b) for the case of the balloon descending at 5.00 m/s.
31. A daring stunt woman sitting on a tree limb wishes to drop vertically onto a horse galloping under the tree. The speed of the horse is 10.0 m/s, and the distance from the limb to the saddle is 3.00 m. (a) What must be the horizontal distance between the saddle and limb when the woman makes her move? (b) How long is she in the air?



Figure P2.30 Hot air balloon over Carefree, Arizona. *(Russell Schlepman)*

32. A ball is thrown vertically upward from the ground with an initial speed of 15.0 m/s. (a) How long does it take the ball to reach its maximum altitude? (b) Find its maximum altitude. (c) Determine the velocity and acceleration of the ball at $t = 2.00$ s.
33. A ball accelerates at 0.500 m/s² while moving down an inclined plane 9.00 m long. When it reaches the bottom, the ball rolls up another plane, where, after moving 15.0 m, it comes to rest. (a) What is the speed of the ball at the bottom of the first plane? (b) How long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball's speed 8.00 m along the second plane?
34. The height of a helicopter above the ground is given by $h = 3.00t^3$, where h is in meters and t is in seconds. After 2.00 s, the helicopter releases a small mailbag. How long after its release does the mailbag reach the ground?
35. Speedy Sue driving at 30.0 m/s enters a one-lane tunnel. She then observes a slow-moving van 155 m ahead traveling at 5.00 m/s. Sue applies her brakes but can decelerate only at 2.00 m/s² because the road is wet. Will there be a collision? If yes, determine how far into the tunnel and at what time the collision occurs. If no, determine the distance of closest approach between Sue's car and the van.

Additional Problems

36. A motorist is traveling at 18.0 m/s when he sees a deer in the road 38.0 m ahead. (a) If the maximum negative acceleration of the vehicle is -4.50 m/s², what is the maximum reaction time, Δt, of the motorist that will allow him to avoid hitting the deer? (b) If his reaction time is 0.300 s, how fast will he be traveling when he hits the deer?
37. Another scheme to catch the roadrunner has failed and a safe falls from rest from the top of a 25.0-m high cliff toward Wile E. Coyote, who is standing at the base. Wile first notices the safe after it has fallen 15.0 m. How long does he have to get out of the way?
38. A rocket is fired vertically upward with an initial velocity of 80.0 m/s. It accelerates upward at 4.00 m/s² until it reaches an altitude of 1000 m. At that point, its engines fail and the rocket goes into free flight, with an acceleration of -9.80 m/s². (a) How long is the rocket in motion? (b) What is its maximum altitude? (c) What is its velocity just before it collides with the Earth? (*Hint:* Consider the motion while the engine is operating separate from the free-fall motion.)
39. In a 100-m race, Maggie and Judy cross the finish line in a dead heat, both taking 10.2 s. They both accelerated uniformly, and Maggie took 2.00 s and Judy 3.00 s to attain maximum speed, which they maintained for the rest of the race. (a) What was the acceleration of each sprinter? (b) What were their respective maximum speeds? (c) Which sprinter was ahead at the 6.00-s mark and by how much?
40. The position of a softball tossed vertically upward is described by the equation $y = 7.00t - 4.90t^2$, where y is in meters and t in seconds. Find (a) the initial speed v_0 at $t_0 = 0$, (b) the velocity at $t = 1.26$ s, and (c) the acceleration of the ball.
41. A hard rubber ball, released at chest height, falls to the pavement and bounces back to nearly the same height. When it is in contact with the pavement, the lower side of the ball is temporarily flattened. Before this dent in the ball pops out, suppose that its maximum depth is about 1 centimeter. Make an order-of-magnitude estimate of the maximum acceleration of the ball. State your assumptions, the quantities you estimate, and the values you estimate for them.
42. A teenager has a car that accelerates at 3.00 m/s² and decelerates at -4.50 m/s². On a trip to the store, he accelerates from rest to 12.0 m/s, drives at a constant speed for 5.00 s, and then comes to a momentary stop at the corner. He then accelerates to 18.0 m/s, drives at a constant speed for 20.0 s, decelerates for 2.67 s; he continues for 4.00 s at this speed and then comes to a stop. (a) How long does the trip take? (b) How far has he traveled? (c) What is his average speed for the trip? (d) How long would it take to walk to the store and back if he walks at 1.50 m/s?

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43. An inquisitive physics student and mountain climber climbs a 50.0-m cliff that overhangs a calm pool of water. He throws two stones vertically downward, 1.00 s apart, and observes that they cause a single splash. The first stone has an initial speed of 2.00 m/s. (a) How long after release of the first stone do the two stones hit the water? (b) What initial velocity must the second stone have if they are to hit simultaneously? (c) What is the speed of each stone at the instant the two hit the water?

44. In the first hour of travel, a train moves with a speed v, in the next half hour it has a speed $3v$, in the next 90.0 min it travels with a speed $v/2$, and in the final 2.00 h it travels with a speed $v/3$. (a) Plot the speed–time graph for this trip. (b) How far does the train travel in this trip? (c) What is the average speed of the train over the entire trip?

45. A rock is dropped from rest into a well. (a) If the sound of the splash is heard 2.40 s later, how far below the top of the well is the surface of the water? The speed of sound in air (for the ambient temperature) is 336 m/s. (b) If the travel time for the sound is neglected, what percentage error is introduced when the depth of the well is calculated?

46. A car and train move together along parallel paths at 25.0 m/s. The car then undergoes a uniform acceleration of -2.50 m/s^2 because of a red light and comes to rest. It remains at rest for 45.0 s, then accelerates back to a speed of 25.0 m/s at a rate of 2.50 m/s^2. How far behind the train is the car when it reaches the speed of 25.0 m/s, assuming that the speed of the train has remained 25.0 m/s?

47. Kathy Kool buys a sports car that can accelerate at the rate of 4.90 m/s^2. She decides to test the car by dragging with another speedster, Stan Speedy. Both start from rest, but experienced Stan leaves the starting line 1.00 s before Kathy. If Stan moves with a constant acceleration of 3.50 m/s^2 and Kathy maintains an acceleration of 4.90 m/s^2, find (a) the time it takes Kathy to overtake Stan, (b) the distance she travels before she catches him, and (c) the speeds of both cars at the instant she overtakes him.

48. In a 100-m linear accelerator, an electron is accelerated to 1.00% the speed of light in 40.0 m before it coasts 60.0 m to a target. (a) What is the electron's acceleration during the first 40.0 m? (b) How long does the total flight take?

49. In order to protect his food from hungry bears, a Boy Scout raises his food pack, of mass m, with a rope that is thrown over a tree limb at height h above his hands. He walks away from the vertical rope with constant velocity v_0, holding the free end of the rope in his hands (Fig. P2.49). (a) Show that the velocity v of the food pack is $x(x^2 + h^2)^{-1/2}v_0$ where x is the distance he has walked away from the vertical rope. (b) Show that the acceleration a of the food pack is $h^2(x^2 + h^2)^{-3/2}v_0^2$. (c) What values do the acceleration and velocity v have shortly after the Boy Scout leaves the vertical rope? (d) What values do the velocity and acceleration approach as the distance x continues to increase?

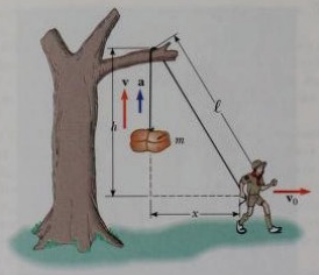


Figure P2.49

50. In Problem 49, let the height, h, equal 6.00 m and the speed, v_0, equal 2.00 m/s. Assume that the food pack starts from rest. (a) Tabulate and graph the speed–time graph. (b) Tabulate and graph the acceleration–time graph. (Let the range of time be from 0 to 5.00 s and the time intervals be 0.50 s.)

51. Two objects, A and B, are connected by a rigid rod that has a length L. The objects slide along perpendicular guide rails, as shown in Figure P2.51. If A slides to the left with a constant speed v, find the velocity of B when $\alpha = 60.0°$.

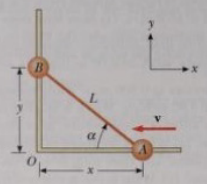


Figure P2.51

Spreadsheet Problems

S1. Use Spreadsheet 2.1 to plot position and velocity as functions of time for one object traveling at constant velocity and another traveling with constant acceleration. Choose a variety of velocity and acceleration values, and view the graphs. Be sure to investigate zero and negative values. *Note:* There are two graphs associated with Spreadsheet 2.1, one for position versus time and one for velocity versus time.

S2. Spreadsheet 2.2 models the sport of drag racing. Two dragsters can have different accelerations $A1$ and $A2$ as well as different maximum speeds $V1$ and $V2$. Both cars start from rest at the same starting position. However, a time delay t' in starting times may be introduced if the cars are quite different. Enter the following data:

	Acceleration (ft/s^2)	Maximum Speed (ft/s)	Delay Time t' (s)
Car 1	5	300	—
Car 2	6	250	0.5

(a) Which car wins a 1/4-mile race? (b) If the acceleration of Car 1 is increased to 5.2 ft/s^2, which car now wins?

S3. Modify Spreadsheet 2.1 to solve this problem. The police have set up a "speed trap" on the highway. From a police car hidden behind a billboard, an officer with a radar gun measures a motorist's speed to be 35.0 m/s. Three seconds later, she alerts her partner, who is in another police car 100 m down the road. The second police car starts from rest accelerating at 2.00 m/s^2 in pursuit of the speeder 2.00 s after receiving the alert. (a) How much time elapses before the speeder is overtaken? (b) What is the police car's speed when it overtakes the speeder? (c) How far does the second police car travel before overtaking the speeder?

ANSWERS TO CONCEPTUAL PROBLEMS

1. If the particle moves along a line without changing direction, the displacement and distance over any time interval will be the same. As a result, the magnitude of the average velocity and the average speed will be the same. If the particle reverses direction, however, then the displacement will be less than the distance. In turn, the magnitude of the average velocity will be smaller than the average speed.
2. Yes. This occurs when a car is slowing down, so that the direction of its acceleration is opposite to its direction of motion.
3. For an accelerating particle to stop at all, the velocity and acceleration must have opposite signs, so that the speed is decreasing. Given that this is the case, the particle will eventually come to rest. If the acceleration remains constant, however, the particle must begin to move again, opposite to the direction of its original motion. If the particle comes to rest and then stays at rest, the acceleration has disappeared at the moment the motion stops. This is the case for a braking car—the acceleration is negative and goes to zero as the car comes to rest.
4. Once the objects leave the hand, both are freely falling objects, and both experience the same downward acceleration equal to the free-fall acceleration of magnitude 9.80 m/s^2.
5. (a) As it travels upward, its speed decreases by 9.80 m/s during each second of its motion. When it reaches the peak of its motion, its speed becomes zero. As the ball moves downward, its speed increases by 9.80 m/s each second. (b) The acceleration of the ball remains constant while it is in the air. Its magnitude of its acceleration is the free-fall acceleration, $g = 9.80$ m/s^2.

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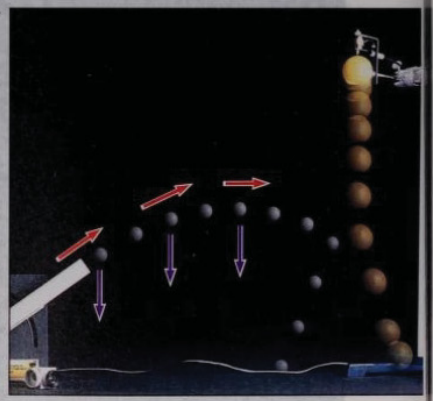
3

Motion in Two Dimensions

CHAPTER OUTLINE

3.1 The Displacement, Velocity, and Acceleration Vectors
3.2 Two-Dimensional Motion with Constant Acceleration
3.3 Projectile Motion
3.4 Uniform Circular Motion
3.5 Tangential and Radial Acceleration

In this chapter we deal with the kinematics of a particle moving in a plane, which is two-dimensional motion. Some common examples of motion in a plane are the motion of projectiles and satellites and the motion of charged particles in uniform electric fields. We begin by showing that displacement, velocity, and acceleration are vector quantities. As in the case of one-dimensional motion, we derive the kinematic equations for two-dimensional motion from the fundamental definitions of displacement,



A multiflash photograph of a popular lecture demonstration in which a projectile is fired at a target that is being held by a magnet in the device at the top right of the photograph. The conditions of the experiment are that the gun is aimed at the target and the projectile leaves the gun at the instant the target is released from rest. Under these conditions the projectile will hit the target, independent of the initial speed of the projectile. The reason is that they both experience the same downward acceleration, and hence the velocities of the projectile and target change by the same amount in the same time interval. Note that the velocity of the projectile (red arrows) changes in direction and magnitude, whereas its downward acceleration (violet arrows) remains constant. *(Central Scientific Company)*

velocity, and acceleration. As special cases of motion in two dimensions, we then treat constant-acceleration motion in a plane and uniform circular motion.

3.1 • THE DISPLACEMENT, VELOCITY, AND ACCELERATION VECTORS

In Chapter 2 we found that the motion of a particle moving along a straight line is completely known if its coordinate is known as a function of time. Now let us extend this idea to motion in the xy plane. We begin by describing the position of a particle with a *position vector* $\mathbf{r}$, drawn from the origin of some reference frame to the particle located in the xy plane, as in Figure 3.1. At time t_i, the particle is at the point P, and at some later time, t_f, the particle is at Q, where the indices i and f refer to initial and final values. As the particle moves from P to Q in the time interval $\Delta t = t_f - t_i$, the position vector changes from $\mathbf{r}_i$ to $\mathbf{r}_f$. As we learned in Chapter 2, the displacement of a particle is the difference between its final position and initial position. Therefore, the **displacement vector** for the particle of Figure 3.1 equals the difference between its final position vector and its initial position vector:

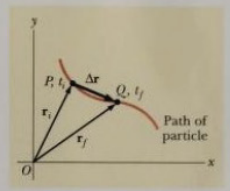


Figure 3.1 A particle moving in the xy plane is located with the position vector $\mathbf{r}$ drawn from the origin to the particle. The displacement of the particle as it moves from P to Q in the time interval $\Delta t = t_f - t_i$ is equal to the vector $\Delta\mathbf{r} = \mathbf{r}_f - \mathbf{r}_i$.

$$\Delta\mathbf{r} \equiv \mathbf{r}_f - \mathbf{r}_i \qquad [3.1]$$

• *Definition of the displacement vector*

The direction of $\Delta\mathbf{r}$ is indicated in Figure 3.1. As we see from the figure, the magnitude of the displacement vector is *less* than the distance traveled along the curved path.

We define the **average velocity** of the particle during the time interval Δt as the ratio of the displacement to that time interval:

$$\bar{\mathbf{v}} \equiv \frac{\Delta\mathbf{r}}{\Delta t} \qquad [3.2]$$

• *Definition of average velocity*

Because displacement is a vector quantity and the time interval is a scalar quantity, we conclude that the average velocity is a *vector* quantity directed along $\Delta\mathbf{r}$. Note that the average velocity between points P and Q is *independent of the path* between the two points. This is because the average velocity is proportional to the displacement, which in turn depends only on the initial and final position vectors and not on the path taken between those two points. As we did with one-dimensional motion, we conclude that if a particle starts its motion at some point and returns to this point via any path, its average velocity is zero for this trip, because its displacement is zero. Consider again the motion of a particle between two points in the xy plane, as in Figure 3.2. As the time intervals over which we observe the motion become smaller and smaller, the direction of the displacement approaches that of the line tangent to the path at the point P.

The **instantaneous velocity, v**, is defined as the limit of the average velocity, $\Delta\mathbf{r}/\Delta t$, as Δt approaches zero:

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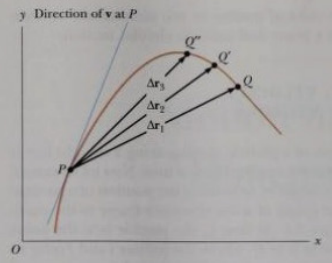


Figure 3.2 As a particle moves between two points, its average velocity is in the direction of the displacement vector $\Delta\mathbf{r}$. As the end point of the path is moved from Q to Q' to Q'', the respective displacements and corresponding time intervals become smaller and smaller. In the limit that the end point approaches P, Δt approaches zero and the direction of $\Delta\mathbf{r}$ approaches that of the line tangent to the curve at P. By definition, the instantaneous velocity at P is in the direction of this tangent line.

Definition of instantaneous velocity

$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt} \qquad [3.3]$$

That is, the instantaneous velocity equals the derivative of the position vector with respect to time. The direction of the instantaneous velocity vector at any point in a particle's path is along a line that is tangent to the path at that point and in the direction of motion; this is illustrated in Figure 3.3. The magnitude of the instantaneous velocity vector is called the *speed*.

As a particle moves from one point to another along some path, as in Figure 3.3, its instantaneous velocity vector changes from $\mathbf{v}_i$ at time t_i to $\mathbf{v}_f$ at time t_f.

The **average acceleration** of a particle as it moves from P to Q is defined as the ratio of the change in the instantaneous velocity vector, $\Delta\mathbf{v}$, to the elapsed time, Δt:

Definition of average acceleration

$$\bar{\mathbf{a}} \equiv \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad [3.4]$$

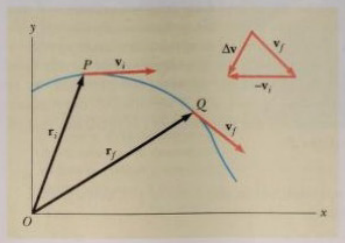


Figure 3.3 The average acceleration vector $\bar{\mathbf{a}}$ for a particle moving from P to Q is in the direction of the change in velocity, $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$.

Because the average acceleration is the ratio of a vector quantity, $\Delta\mathbf{v}$, and a scalar quantity, Δt, we conclude that $\bar{\mathbf{a}}$ is a vector quantity directed along $\Delta\mathbf{v}$. As is indicated in Figure 3.3, the direction of $\Delta\mathbf{v}$ is found by adding the vector $-\mathbf{v}_i$ (the negative of $\mathbf{v}_i$) to the vector $\mathbf{v}_f$, because by definition $\Delta\mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$.

The **instantaneous acceleration, a,** is defined as the limiting value of the ratio $\Delta\mathbf{v}/\Delta t$ as Δt approaches zero:

Definition of instantaneous acceleration

$$\mathbf{a} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt} \qquad [3.5]$$

In other words, the instantaneous acceleration equals the derivative of the velocity vector with respect to time.

It is important to recognize that various changes can cause a particle to accelerate. First, the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion. Second, only the direction of the velocity vector may change with time as its magnitude (speed) remains constant, as in curved-path (two-dimensional) motion. Third, both the magnitude and the direction of the velocity vector may change.

Thinking Physics 1

The gas pedal in an automobile is called the *accelerator*. Are there any other automobile controls that could also be considered "accelerators"?

Reasoning The gas pedal is called the accelerator because the cultural usage of the word *acceleration* refers to an *increase in speed*. The scientific definition, however, is that acceleration occurs *whenever the velocity changes* in any way. Thus, the *brake pedal* can also be considered an accelerator, because it causes the car to slow down. The *steering wheel* is also an accelerator, because it changes the direction of the velocity vector.

CONCEPTUAL PROBLEM 1

(a) Can an object accelerate if its speed is constant? (b) Can an object accelerate if its velocity is constant?

CONCEPTUAL PROBLEM 2

Can a particle have a constant velocity and varying speed? Can a particle have a constant speed and a varying velocity? Give examples of a positive answer and explain a negative answer.

3.2 • TWO-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

Let us consider two-dimensional motion during which the acceleration remains constant. That is, we assume that the magnitude and direction of the acceleration remain unchanged during the motion.

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The motion of a particle can be determined by its position vector $\mathbf{r}$. The position vector for a particle moving in the xy plane can be written

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} \qquad [3.6]$$

where x, y, and $\mathbf{r}$ change with time as the particle moves. If the position vector is known, the velocity of the particle can be obtained from Equations 3.3 and 3.6, which give

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} \qquad [3.7]$$

Because $\mathbf{a}$ is assumed constant, its components a_x and a_y are also constants. Therefore, we can apply the equations of kinematics to the x and y components of the velocity vector. Substituting $v_x = v_{x0} + a_x t$ and $v_y = v_{y0} + a_y t$ into Equation 3.7 gives

$$\begin{aligned}\mathbf{v} &= (v_{x0} + a_x t)\mathbf{i} + (v_{y0} + a_y t)\mathbf{j} \\ &= (v_{x0}\mathbf{i} + v_{y0}\mathbf{j}) + (a_x\mathbf{i} + a_y\mathbf{j})t\end{aligned}$$

Velocity vector as a function of time

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \qquad [3.8]$$

This result states that the velocity of a particle at some time t equals the vector sum of its initial velocity, $\mathbf{v}_0$, and the additional velocity $\mathbf{a}t$ acquired in the time t as a result of its constant acceleration.

In a similar way, from Equation 2.9 we know that the x and y coordinates of a particle moving with constant acceleration are

$$x = x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2 \quad \text{and} \quad y = y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2$$

Substituting these expressions into Equation 3.6 gives

$$\begin{aligned}\mathbf{r} &= (x_0 + v_{x0}t + \tfrac{1}{2}a_x t^2)\mathbf{i} + (y_0 + v_{y0}t + \tfrac{1}{2}a_y t^2)\mathbf{j} \\ &= (x_0\mathbf{i} + y_0\mathbf{j}) + (v_{x0}\mathbf{i} + v_{y0}\mathbf{j})t + \tfrac{1}{2}(a_x\mathbf{i} + a_y\mathbf{j})t^2\end{aligned}$$

Position vector as a function of time

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \qquad [3.9]$$

This equation implies that the displacement vector $\mathbf{r} - \mathbf{r}_0$ is the vector sum of a displacement $\mathbf{v}_0 t$, arising from the initial velocity of the particle, and a displacement $\frac{1}{2}\mathbf{a}t^2$, resulting from the uniform acceleration of the particle. Graphical representations of Equations 3.8 and 3.9 are shown in Figure 3.4a and 3.4b. For simplicity in drawing the figure, we have taken $\mathbf{r}_0 = 0$ in Figure 3.4b. That is, we assume that the particle is at the origin at $t = 0$. Note from Figure 3.4b that $\mathbf{r}$ is generally not along the direction of $\mathbf{v}_0$ or $\mathbf{a}$, because the relationship between these quantities is a vector expression. For the same reason, from Figure 3.4a we see that $\mathbf{v}$ is generally not along the direction of $\mathbf{v}_0$ or $\mathbf{a}$. Finally, if we compare the two figures, we see that $\mathbf{v}$ and $\mathbf{r}$ are not in the same direction.

Because Equations 3.8 and 3.9 are *vector* expressions, we may write their x and y component forms with $\mathbf{r}_0 = 0$:

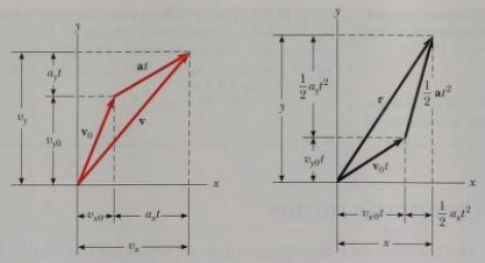


Figure 3.4 Vector representations and rectangular components of (a) the velocity and (b) the displacement of a particle moving with a uniform acceleration $\mathbf{a}$.

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \qquad \begin{cases} v_x = v_{x0} + a_x t \\ v_y = v_{y0} + a_y t \end{cases}$$

$$\mathbf{r} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \qquad \begin{cases} x = v_{x0}t + \frac{1}{2}a_x t^2 \\ y = v_{y0}t + \frac{1}{2}a_y t^2 \end{cases}$$

These components are illustrated in Figure 3.4. In other words, two-dimensional motion having constant acceleration is equivalent to two independent motions in the x and y directions having constant accelerations a_x and a_y.

Example 3.1 Motion in a Plane

A particle starts from the origin at $t = 0$ with an initial velocity having an x component of 20 m/s and a y component of -15 m/s. The particle moves in the xy plane with an x component of acceleration only, given by $a_x = 4.0$ m/s^2. (a) Determine the components of velocity as a function of time and the total velocity vector at any time.

Solution With $v_{x0} = 20$ m/s and $a_x = 4.0$ m/s^2, the equations of kinematics give

$$v_x = v_{x0} + a_x t = (20 + 4.0t) \text{ m/s}$$

Also, with $v_{y0} = -15$ m/s and $a_y = 0$,

$$v_y = v_{y0} = -15 \text{ m/s}$$

Therefore, using these results and noting that the velocity vector $\mathbf{v}$ has two components, we get

$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j} = [(20 + 4.0t)\mathbf{i} - 15\mathbf{j}] \text{ m/s}$$

We could also obtain this result using Equation 3.8 directly, noting that $\mathbf{a} = 4.0\mathbf{i}$ m/s^2 and $\mathbf{v}_0 = (20\mathbf{i} - 15\mathbf{j})$ m/s. Try it!

(b) Calculate the velocity and speed of the particle at $t = 5.0$ s.

Solution With $t = 5.0$ s, the result from (a) gives

$$\mathbf{v} = [(20 + 4(5.0))\mathbf{i} - 15\mathbf{j}] \text{ m/s} = (40\mathbf{i} - 15\mathbf{j}) \text{ m/s}$$

That is, at $t = 5.0$ s, $v_x = 40$ m/s, and $v_y = -15$ m/s. Knowing these two components for this two-dimensional motion, we know the numerical value of the velocity vector. To determine the angle θ that $\mathbf{v}$ makes with the x axis, we use the fact that $\tan\theta = v_y/v_x$, or

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-15 \text{ m/s}}{40 \text{ m/s}}\right) = -21^\circ$$

The speed is the magnitude of $\mathbf{v}$:

$$v = |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{(40)^2 + (-15)^2} \text{ m/s} = 43 \text{ m/s}$$

(*Note*: If you calculate v_0 from the x and y components of $\mathbf{v}_0$, you will find that $v > v_0$. Why?)

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EXERCISE 1 Determine the x and y coordinates of the particle at any time t and the displacement vector at this time. Answer $x = (20t + 2.0t^2)$ m; $y = (-15t)$ m; $\mathbf{r} = [(20t + 2.0t^2)\mathbf{i} - 15t\mathbf{j}]$ m

EXERCISE 2 A particle starts from rest at $t = 0$ at the origin and moves in the xy plane with a constant acceleration of $\mathbf{a} = (2.0\mathbf{i} + 4.0\mathbf{j})$ m/s^2. After a time t has elapsed, determine (a) the x and y components of velocity, (b) the coordinates of the particle, and (c) the speed of the particle. Answer (a) $v_x = 2t$ m/s, $v_y = 4t$ m/s (b) $x = t^2$ m, $y = 2t^2$ m (c) $4.47t$ m/s

3.3 • PROJECTILE MOTION

Assumptions of projectile motion •

Anyone who has observed a baseball in motion (or any object thrown into the air) has observed projectile motion. The ball moves in a curved path when thrown at some angle with respect to the Earth's surface. This very common form of motion is surprisingly simple to analyze if the following two assumptions are made: (1) the free-fall acceleration, g, is constant over the range of motion and is directed downward,[1] and (2) the effect of air resistance is negligible.[2] With these assumptions, we find that the path of a projectile, which we call its *trajectory*, is *always* a parabola. **We use these assumptions throughout this chapter.**

If we choose our reference frame such that the y direction is vertical and positive upward, then $a_y = -g$ (as in one-dimensional free fall) and $a_x = 0$ (because air resistance is neglected). Furthermore, let us assume that at $t = 0$, the projectile leaves the origin ($x_0 = y_0 = 0$) with speed v_0, as in Figure 3.5. If the vector $\mathbf{v}_0$ makes an angle θ_0 with the horizontal, where θ_0 is the angle at which the projectile leaves the origin as in Figure 3.5, then from the definitions of the cosine and sine functions we have

$$\cos\theta_0 = \frac{v_{x0}}{v_0} \quad \text{and} \quad \sin\theta_0 = \frac{v_{y0}}{v_0}$$

Therefore, the initial x and y components of velocity are

$$v_{x0} = v_0\cos\theta_0 \quad \text{and} \quad v_{y0} = v_0\sin\theta_0$$

Substituting these expressions into Equations 3.8 and 3.9 with $a_x = 0$ and $a_y = -g$ gives the velocity components and coordinates for the projectile at any time t:

Horizontal velocity component •
$$v_x = v_{x0} = v_0\cos\theta_0 = \text{constant} \qquad [3.10]$$

Vertical velocity component •
$$v_y = v_{y0} - gt = v_0\sin\theta_0 - gt \qquad [3.11]$$

Horizontal position component •
$$x = v_{x0}t = (v_0\cos\theta_0)t \qquad [3.12]$$

Vertical position component •
$$y = v_{y0}t - \tfrac{1}{2}gt^2 = (v_0\sin\theta_0)t - \tfrac{1}{2}gt^2 \qquad [3.13]$$

[1] This approximation is reasonable as long as the range of motion is small compared with the radius of the Earth (6.4×10^6 m). In effect, this approximation is equivalent to assuming that the earth is flat within the range of motion considered.

[2] This approximation is generally *not* justified, especially at high velocities. In addition, the spin of a projectile, such as a baseball, can give rise to some very interesting effects associated with aerodynamic forces (for example, a curveball thrown by a pitcher).

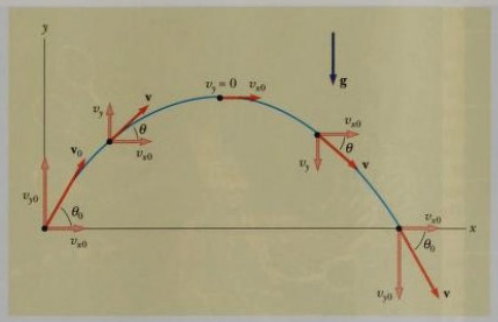


Figure 3.5 The parabolic path of a projectile that leaves the origin with a velocity $\mathbf{v}_0$. The velocity vector $\mathbf{v}$ changes with time in both magnitude and direction. The change in the velocity vector is the result of acceleration in the negative y direction. The x component of velocity remains constant in time because there is no acceleration along the horizontal direction. Also, the y component of velocity is zero at the peak of the path.

From Equation 3.10 we see that v_x remains constant in time and is equal to v_{x0}; there is no horizontal component of acceleration. For the y motion note that v_y and y are similar to Equations 2.12 and 2.14 for freely falling bodies. In fact, *all* of the equations of kinematics developed in Chapter 2 are applicable to projectile motion.

If we solve for t in Equation 3.12 and substitute this expression for t into Equation 3.13, we find that

$$y = (\tan\theta_0)x - \left(\frac{g}{2v_0^2\cos^2\theta_0}\right)x^2 \qquad [3.14]$$

which is valid for the angles in the range $0 < \theta_0 < \pi/2$. This expression is of the form $y = ax - bx^2$, which is the equation of a parabola that passes through the origin. Thus, we have proved that the trajectory of a projectile is a parabola. Note that the trajectory is *completely* specified if v_0 and θ_0 are known.

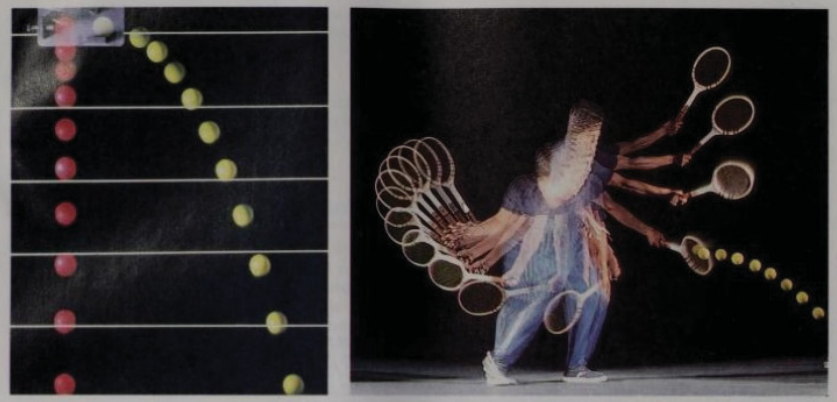
One can obtain the speed, v, of the projectile as a function of time by noting that Equations 3.10 and 3.11 give the x and y components of velocity at any instant. Therefore, by definition, because v is equal to the magnitude of $\mathbf{v}$,

$$v = \sqrt{v_x^2 + v_y^2} \qquad [3.15]$$ • *Speed*

Also, because the velocity vector is tangent to the path at any instant, as shown in Figure 3.5, the angle θ that v makes with the horizontal can be obtained from v_x and v_y through the expression

$$\tan\theta = \frac{v_y}{v_x} \qquad [3.16]$$

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(Left) This multiflash photograph of two balls released simultaneously illustrates both free fall (red ball) and projectile motion (yellow ball). The yellow ball was projected horizontally, whereas the red ball was released from rest. Can you explain why both balls reach the floor simultaneously? *(© Richard Megna 1990, Fundamental Photographs)* *(Right)* Multiflash exposure of a tennis player executing a backhand swing. Note that the ball follows a parabolic path characteristic of a projectile. Such photographs can be used to study the quality of sports equipment and the performance of an athlete. *(© Zimmerman, FPG International)*

The vector expression for the position vector of the projectile as a function of time follows directly from Equation 3.9, with $\mathbf{a} = \mathbf{g}$:

$$\mathbf{r} = \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{g}t^2$$

This expression gives the same information as the combination of Equations 3.12 and 3.13 and is plotted in Figure 3.6. Note that this expression for $\mathbf{r}$ is consistent with Equation 3.13, because the expression for $\mathbf{r}$ is a vector equation and $\mathbf{a} = \mathbf{g} = -g\mathbf{j}$ when the upward direction is taken to be positive.

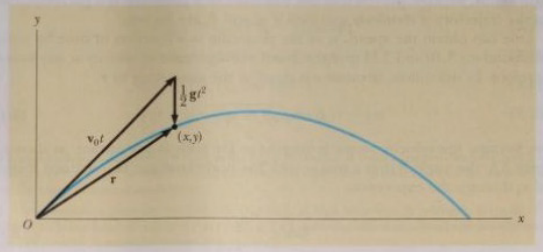


Figure 3.6 The displacement vector $\mathbf{r}$ of a projectile, the initial velocity of which is $\mathbf{v}_0$ at the origin. The vector $\mathbf{v}_0 t$ would be the displacement of the projectile if gravity were absent, and the vector $\frac{1}{2}\mathbf{g}t^2$ is its vertical displacement due to its downward gravitational acceleration.

It is interesting to note that the motion of a particle can be considered the superposition of the term $\mathbf{v}_0 t$, which would be the displacement if no acceleration were present, and the term $\frac{1}{2}\mathbf{g}t^2$, which arises from the acceleration due to gravity. In other words, if there were no gravitational acceleration, the particle would continue to move along a straight path in the direction of $\mathbf{v}_0$. Therefore, the vertical distance $\frac{1}{2}\mathbf{g}t^2$ through which the particle "falls" off the straight-path line is the same distance a freely falling body would fall during the same time interval. **We conclude that projectile motion is the superposition of two motions: (1) constant velocity motion in the initial direction and (2) the motion of a particle freely falling in the vertical direction under constant acceleration.**

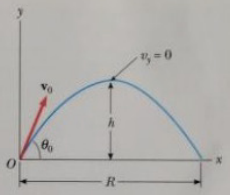


Figure 3.7 A projectile fired from the origin at $t = 0$ with an initial velocity $\mathbf{v}_0$. The maximum height of the projectile is h, and its horizontal range is R. At the peak of the trajectory, the y component of its velocity is zero and the projectile has coordinates $(\frac{R}{2}, h)$.

Horizontal Range and Maximum Height of a Projectile Let us assume that a projectile is fired from the origin at $t = 0$ with a positive v_y component, as in Figure 3.7. There are two special points that are interesting to analyze: the peak that has Cartesian coordinates $(R/2, h)$ and the point having coordinates $(R, 0)$. The distance R is called the *horizontal range* of the projectile and h is its *maximum height*. Let us find h and R in terms of v_0, θ_0, and g.

We can determine h by noting that at the peak $v_y = 0$. Therefore, Equation 3.11 can be used to determine the time t_1 it takes to reach the peak:

$$t_1 = \frac{v_0 \sin \theta_0}{g}$$

Substituting this expression for t_1 into Equation 3.13 and replacing y with h gives h in terms of v_0 and θ_0:

$$h = (v_0 \sin \theta_0)\frac{v_0 \sin \theta_0}{g} - \tfrac{1}{2}g\left(\frac{v_0 \sin \theta_0}{g}\right)^2$$

$$h = \frac{{v_0}^2 \sin^2 \theta_0}{2g} \qquad [3.17]$$

The range, R, is the horizontal distance traveled in twice the time it takes to reach the peak—that is, in a time $2t_1$. (This can be seen by setting $y = 0$ in Equation 3.13 and solving the quadratic for t. One solution of this quadratic is $t = 0$, and the other is $t = 2t_1$.) Using Equation 3.12 and noting that $x = R$ at $t = 2t_1$, we find that

$$R = (v_0 \cos \theta_0)2t_1 = (v_0 \cos \theta_0)\frac{2v_0 \sin \theta_0}{g} = \frac{2{v_0}^2 \sin \theta_0 \cos \theta_0}{g}$$

Because $\sin 2\theta = 2 \sin \theta \cos \theta$, R can be written in the most compact form

$$R = \frac{{v_0}^2 \sin 2\theta_0}{g} \qquad [3.18]$$

Keep in mind that Equations 3.17 and 3.18 are useful for calculating h and R only if v_0 and θ_0 are known and only for a symmetric path, as shown in Figure 3.7 (which means that only $\mathbf{v}_0$ has to be specified). The general expressions given by Equations 3.10 through 3.13 are the *more important* results, because they give the coordinates and velocity components of the projectile at *any* time t.

You should note that the maximum value of R from Equation 3.18 is given by $R_{max} = {v_0}^2/g$. This result follows from the fact that the maximum value of $\sin 2\theta_0$

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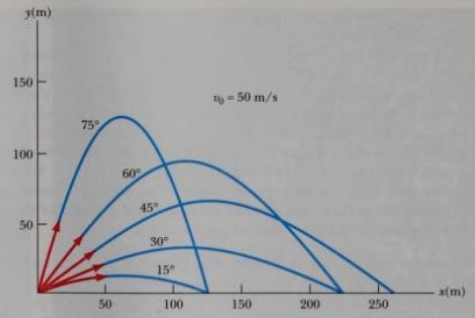


Figure 3.8 A projectile fired from the origin with an initial speed of 50 m/s at various angles of projection. Note that complementary values of θ_0 will result in the same value of x (range of the projectile).

is unity, which occurs when $2\theta_0 = 90°$. Therefore, we see that R is a maximum when $\theta_0 = 45°$, as you would expect if air resistance is neglected.

Figure 3.8 illustrates various trajectories for a projectile of a given initial speed. As you can see, the range is a maximum for $\theta_0 = 45°$. In addition, for any θ_0 other than 45°, a point with coordinates $(R, 0)$ can be reached by using either one of two complementary values of θ_0, such as 75° and 15°. Of course, the maximum height and the time of flight will be different for these two values of θ_0.

Before we look at some numerical examples dealing with projectile motion, let us pause to summarize what we have learned so far about this kind of motion:

- Provided air resistance is negligible, the horizontal component of velocity, v_x, remains constant because there is no horizontal component of acceleration.
- The vertical component of acceleration is equal to the free-fall acceleration, **g**.
- The vertical component of velocity, v_y, and the displacement in the y direction are identical to those of a freely falling body.
- Projectile motion can be described as a superposition of the two motions in the x and y directions.

PROBLEM-SOLVING STRATEGY • Projectile Motion

We suggest that you use the following approach to solving projectile motion problems:

1. Select a coordinate system.
2. Resolve the initial velocity vector into x and y components.
3. Treat the horizontal motion and the vertical motion independently.

4. Follow the techniques for solving problems with constant velocity to analyze the horizontal motion of the projectile.
5. Follow the techniques for solving problems with constant acceleration to analyze the vertical motion of the projectile.

Thinking Physics 2

A home run is hit in a baseball game. The ball is hit from home plate into the centerfield stands along a long, parabolic path. What is the acceleration of the ball (a) while it is rising, (b) at the highest point of the trajectory, and (c) while it is descending after reaching the highest point? Ignore air resistance.

Reasoning The answers to all three parts are the same—the acceleration is that due to gravity, 9.8 m/s^2, because the force of gravity is pulling downward on the ball during the entire motion. During the rising part of the trajectory, the downward acceleration results in the decreasing positive values of the vertical component of the velocity of the ball. During the falling part of the trajectory, the downward acceleration results in the increasing negative values of the vertical component of the velocity. Many people have trouble with the topmost point, mistakenly thinking that the acceleration of the ball at the highest point is zero. This interpretation arises from a confusion between zero vertical velocity and zero acceleration. Even though the ball has momentarily come to rest at the highest point (vertically; it is still moving horizontally), it is still accelerating, because the force of gravity is still acting. If the ball were to come to rest and experience zero acceleration, then the velocity would not change—the ball would remain suspended at the highest point! We don't see that happening, because the acceleration is *not* equal to zero.

CONCEPTUAL PROBLEM 3

Suppose that as you are running at constant speed, you wish to throw a ball such that you will catch it as it comes back down. How should you throw the ball?

CONCEPTUAL PROBLEM 4

As a projectile moves in its parabolic path, is there any point along its path at which the velocity and acceleration vectors are (a) perpendicular to each other? (b) parallel to each other?

Example 3.2 That's Quite an Arm

A stone is thrown from the top of a building upward at an angle of 30.0° to the horizontal and with an initial speed of 20.0 m/s, as in Figure 3.9. If the height of the building is 45.0 m, (a) how long is the stone "in flight"?

Solution The initial x and y components of the velocity are

$$v_{x0} = v_0 \cos\theta_0 = (20.0 \text{ m/s})(\cos 30.0°) = 17.3 \text{ m/s}$$

$$v_{y0} = v_0 \sin\theta_0 = (20.0 \text{ m/s})(\sin 30.0°) = 10.0 \text{ m/s}$$

To find t, we can use $y = v_{y0}t - \frac{1}{2}gt^2$ (Eq. 3.13) with $y = -45.0$ m and $v_{y0} = 10.0$ m/s (we have chosen the top of the building as the origin, as in Fig. 3.9):

$$-45.0 \text{ m} = (10.0 \text{ m/s})t - \tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

Solving the quadratic equation for t gives, for the positive root, $t = 4.22$ s. Does the negative root have any physical

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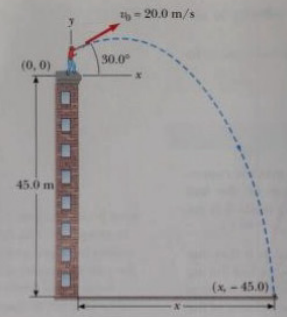


Figure 3.9 (Example 3.2)

meaning? (Can you think of another way of finding t from the information given?)

(b) What is the speed of the stone just before it strikes the ground?

Solution The y component of the velocity just before the stone strikes the ground can be obtained using the equation $v_y = v_{y0} - gt$ (Eq. 3.11) with $t = 4.22$ s:

$$v_y = 10.0 \text{ m/s} - (9.80 \text{ m/s}^2)(4.22 \text{ s}) = -31.4 \text{ m/s}$$

Because $v_x = v_{x0} = 17.3$ m/s, the required speed is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(17.3)^2 + (-31.4)^2} \text{ m/s} = 35.9 \text{ m/s}$$

EXERCISE 3 Where does the stone strike the ground? Answer 73.0 m from the base of the building.

Example 3.3 The Stranded Explorers

An Alaskan rescue plane drops a package of emergency rations to a stranded party of explorers, as shown in Figure 3.10. If the plane is traveling horizontally at 40.0 m/s at a height of 100 m above the ground, where does the package strike the ground relative to the point at which it is released?

Reasoning and Solution The coordinate system for this problem is selected as shown in Figure 3.10, with the positive x direction to the right and the positive y direction upward.

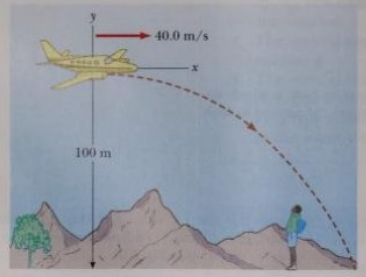


Figure 3.10 (Example 3.3) According to a ground observer, a package released from the rescue plane travels along the path shown. How does the path followed by the package appear to an observer on the plane (assumed to be moving at constant speed)?

Consider first the horizontal motion of the package. The only equation available to us is $x = v_{x0}t$ (Eq. 3.12).

The initial x component of the package velocity is the same as that of the plane when the package is released, 40.0 m/s. Thus, we have

$$x = (40.0 \text{ m/s})t$$

If we know t, the length of time the package is in the air, we can determine x, the distance traveled by the package in the horizontal direction. To find t, we move to the equations for the vertical motion of the package. We know that at the instant the package hits the ground, its y coordinate is -100 m. We also know that the initial component of velocity of the package in the vertical direction, v_{y0}, is zero because the package was released with only a horizontal component of velocity. From Equation 3.13, we have

$$y = -\tfrac{1}{2}gt^2$$

$$-100 \text{ m} = -\tfrac{1}{2}(9.80 \text{ m/s}^2)t^2$$

$$t^2 = 20.4 \text{ s}^2$$

$$t = 4.51 \text{ s}$$

The value for the time of flight substituted into the equation for the x coordinate gives

$$x = (40.0 \text{ m/s})(4.51 \text{ s}) = 180 \text{ m}$$

The package hits the ground not directly under the drop point but 180 m to the right of that point.

EXERCISE 4 What are the horizontal and vertical components of the velocity of the package just before it hits the ground? Answer $v_x = 40.0$ m/s; $v_y = -44.1$ m/s

EXERCISE 5 It has been said that in his youth George Washington threw a silver dollar across a river. Assuming that the river was 75 m wide, (a) what *minimum initial* speed was necessary to get the coin across the river, and (b) how long was the coin in flight? Answer (a) 27.1 m/s (b) 3.91 s

3.4 • UNIFORM CIRCULAR MOTION

Figure 3.11a shows an object moving in a circular path with *constant linear speed* v. Such motion is called **uniform circular motion.** It is often surprising to students to find that **even though the object moves at a constant speed, it still has an acceleration.** To see why, consider the defining equation for average acceleration, $\bar{\mathbf{a}} = \Delta\mathbf{v}/\Delta t$ (Eq. 3.4).

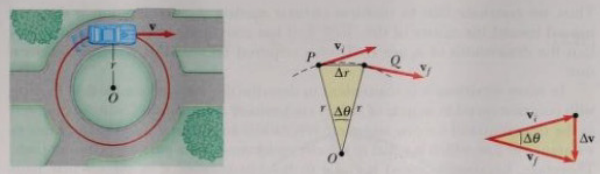
Note that the acceleration depends on *the change in the velocity vector.* Because velocity is a vector quantity, there are two ways in which an acceleration can be produced, as mentioned in Section 3.1: by a change in the *magnitude* of the velocity and by a change in the *direction* of the velocity. It is the latter situation that is occurring for an object moving with constant speed in a circular path. The velocity vector is always tangent to the path of the object and perpendicular to the radius r of the circular path. We now show that the acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. An acceleration of this nature is called a *centripetal acceleration* (center seeking) and its magnitude is

$$a_r = \frac{v^2}{r} \qquad [3.19]$$

• *Magnitude of centripetal acceleration*

where r is the radius of the circle.

To derive Equation 3.19, consider Figure 3.11b. Here an object is seen first at point P and then at point Q. The particle is at P at time t_i, and its velocity at that time is $\mathbf{v}_i$; it is at Q at some later time t_f, and its velocity at that time is $\mathbf{v}_f$. Let us



(a) (b) (c)

Figure 3.11 (a) An object moving along a circular path at constant speed experiences uniform circular motion. (b) As the particle moves from P to Q, the direction of its velocity vector changes from $\mathbf{v}_i$ to $\mathbf{v}_f$. (c) The construction for determining the direction of the change in velocity $\Delta\mathbf{v}$ that is toward the center of the circle.

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also assume here that $\mathbf{v}_i$ and $\mathbf{v}_f$ differ only in direction; their magnitudes are the same (that is, $v_i = v_f = v$). In order to calculate the acceleration of the particle, let us begin with the defining equation for average acceleration (Eq. 3.4):

$$\bar{\mathbf{a}} = \frac{\mathbf{v}_f - \mathbf{v}_i}{t_f - t_i} = \frac{\Delta \mathbf{v}}{\Delta t}$$

This equation indicates that we must vectorially subtract $\mathbf{v}_i$ from $\mathbf{v}_f$, where $\Delta \mathbf{v} = \mathbf{v}_f - \mathbf{v}_i$ is the change in the velocity. Because $\mathbf{v}_i + \Delta \mathbf{v} = \mathbf{v}_f$, the vector $\Delta \mathbf{v}$ can be found using the vector triangle in Figure 3.11c. When Δt is very small, Δr and $\Delta \theta$ are also very small. In this case, $\mathbf{v}_f$ is almost parallel to $\mathbf{v}_i$ and the vector $\Delta \mathbf{v}$ is approximately perpendicular to them, pointing toward the center of the circle.

Now consider the triangle in Figure 3.11b, which has sides Δr and r. This triangle and the one in Figure 3.11c that has sides Δv and v are similar. (Two triangles are *similar* if the angle between any two sides is the same for both triangles and if the ratio of lengths of these sides is the same.) This enables us to write a relationship between the lengths of the sides:

$$\frac{\Delta v}{v} = \frac{\Delta r}{r}$$

This equation can be solved for Δv and the expression obtained from it can be substituted into $\bar{a} = \Delta v / \Delta t$ (Eq. 3.4) to give

$$\bar{a} = \frac{v \, \Delta r}{r \, \Delta t}$$

Now imagine that points P and Q in Figure 3.11b come extremely close together. In this case $\Delta \mathbf{v}$ would point toward the center of the circular path, and because the acceleration is in the direction of $\Delta \mathbf{v}$, it too is toward the center. Furthermore, as P and Q approach each other, Δt approaches zero, and the ratio $\Delta r / \Delta t$ approaches the speed v. Hence, in the limit $\Delta t \to 0$, the magnitude of the acceleration is

$$a_r = \frac{v^2}{r}$$

Thus, we conclude that in uniform circular motion the acceleration is directed inward toward the center of the circle and has magnitude v^2/r. You should show that the dimensions of a_r are $\mathrm{L/T^2}$—as required because this is a true acceleration.

In many situations it is convenient to describe the motion of a particle moving with constant speed in a circle of radius r in terms of the **period,** T, which is defined as the time required for one complete revolution. In the time T the particle moves a distance of $2\pi r$, which is equal to the circumference of the particle's circular path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or $v = 2\pi r / T$, it follows that

$$T = \frac{2\pi r}{v} \qquad [3.20]$$

Thinking Physics 3

An airplane travels from Los Angeles to Sydney, Australia. For 15 hours after cruising altitude is reached, the instruments on the plane indicate that the ground speed holds rock-steady at 700 km/hr and that the heading of the airplane does not change. Is the velocity of the airplane constant?

Reasoning The velocity is not constant, due to the curvature of the Earth. Even though the speed does not change and the heading is always toward Sydney (is this actually true?), the airplane is traveling around a significant portion of the circumference of the Earth. Thus, the direction of the velocity vector does change. We could extend this by imagining that the airplane passes over Sydney and continues (assuming it has enough fuel!) around the Earth until it arrives at Los Angeles again. It is impossible for an airplane to have a constant velocity (relative to the Universe, not to the Earth's surface) and return to its starting point.

EXERCISE 6 Find the acceleration of a particle moving with a constant speed of 8.0 m/s in a circle 2.0 m in radius. Answer 32 m/s² toward the center of the circle

3.5 • TANGENTIAL AND RADIAL ACCELERATION

Let us consider the motion of a particle along a curved path where the velocity changes both in direction and in magnitude, as described in Figure 3.12. In this situation, the velocity vector is always tangent to the path; however, the acceleration vector $\mathbf{a}$ is at some angle to the path.

As the particle moves along the curved path in Figure 3.12, we see that the direction of the total acceleration vector, $\mathbf{a}$, changes from point to point. This vector can be resolved into two components: a radial component, a_r, and a tangential component, a_t. The *total* acceleration vector, $\mathbf{a}$, can be written as the vector sum of the component vectors:

$$\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t \qquad [3.21]$$ • *Total acceleration*

The tangential acceleration arises from the change in the speed of the particle, and the projection of the tangential acceleration along the direction of the velocity is

$$a_t = \frac{d\,|\mathbf{v}|}{dt} \qquad [3.22]$$ • *Tangential acceleration*

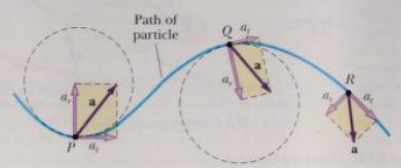


Figure 3.12 The motion of a particle along an arbitrary curved path lying in the xy plane. If the velocity vector $\mathbf{v}$ (always tangent to the path) changes in direction and magnitude, the acceleration $\mathbf{a}$ has a tangential component a_t and a radial component a_r.

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The radial acceleration is due to the change in direction of the velocity vector and has an absolute magnitude given by

Radial or centripetal acceleration

$$a_r = \frac{v^2}{r} \qquad [3.23]$$

where r is the radius of curvature of the path at the point in question. Because $\mathbf{a}_r$ and $\mathbf{a}_t$ are perpendicular component vectors of $\mathbf{a}$, it follows that $a = \sqrt{a_r^2 + a_t^2}$. As in the case of uniform circular motion, $\mathbf{a}_r$ always points toward the center of curvature, as shown in Figure 3.12. Also, at a given speed, a_r is large when the radius of curvature is small (as at points P and Q in Fig. 3.12) and small when r is large (such as at point R). The direction of $\mathbf{a}_t$ is either in the same direction as $\mathbf{v}$ (if v is increasing) or opposite $\mathbf{v}$ (if v is decreasing).

Note that in the case of uniform circular motion, where v is constant, $a_t = 0$ and the acceleration is always radial, as we described in Section 3.4. (*Note:* Eq. 3.23 is identical to Eq. 3.19.) In other words, uniform circular motion is a special case of motion along a curved path. Furthermore, if the direction of $\mathbf{v}$ does not change, then there is no radial acceleration and the motion is one-dimensional ($a_r = 0$, but a_t may not be zero).

Thinking Physics 4

In uniform circular motion, the acceleration vector is always perpendicular to the velocity vector. (a) Suppose a particle is moving at uniform speed around an *elliptical* path. What is the relationship between the acceleration and velocity vectors in this case? (b) Now, let the elliptical path be that of Halley's Comet in orbit around the Sun. This is *not* a case of uniform speed—the comet moves slowly when it is far from the Sun and rapidly when it is close. Are the acceleration and velocity vectors perpendicular in this case?

Reasoning (a) We consider first the case of the particle moving at constant speed around the elliptical path (Fig. 3.13a). Because the speed is constant, there must be no component of the acceleration tangent to the path. Thus, the acceleration must be

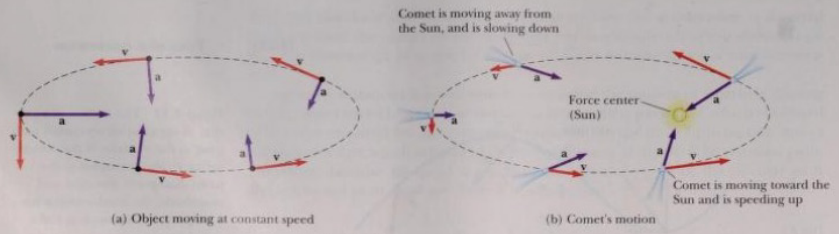


Figure 3.13 (Thinking Physics 4)

perpendicular to the path and, therefore, always perpendicular to the velocity vector. The magnitude of the acceleration vector is not constant—it will be largest at the ends of the ellipse, because the velocity is changing direction the most rapidly there. Figure 3.13a shows the velocity and acceleration vectors at several locations around the path.

(b) In the case of Halley's Comet, or any other object in an elliptical orbit around a gravitational-force center, the direction of the acceleration is the same as the direction of the gravitational force on the object, which is toward the *focus* of the ellipse, where the gravitational force center lies (Fig. 3.13b). Thus, the acceleration and velocity vectors are not perpendicular, except for the points at the ends of the ellipse. As the comet moves away from the Sun, the acceleration has a component opposite to the velocity. As it moves toward the Sun, the acceleration has a component parallel to the velocity. This results in the slowing down of the comet as it moves away from the Sun and its speeding up as it moves toward the Sun. Notice in Figure 3.13b that the velocity vectors have large magnitudes when the comet is near the Sun and small magnitudes when it is far away.

Example 3.4 The Swinging Ball

A ball tied to the end of a string 0.50 m in length swings in a vertical circle under the influence of gravity, as in Figure 3.14. When the string makes an angle of $\theta = 20°$ with the vertical, the ball has a speed of 1.5 m/s. (a) Find the magnitude of the radial component of acceleration at this instant.

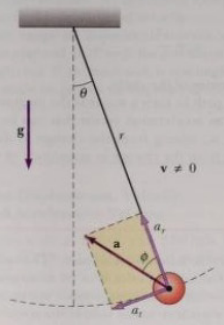


Figure 3.14 (Example 3.4) Motion of a ball suspended by a string of length r. The ball swings with nonuniform circular motion in a vertical plane, and its acceleration $\mathbf{a}$ has a radial component a_r and a tangential component a_t.

Solution Because $v = 1.5$ m/s and $r = 0.50$ m, we find that

$$a_r = \frac{v^2}{r} = \frac{(1.5 \text{ m/s})^2}{0.50 \text{ m}} = 4.5 \text{ m/s}^2$$

(b) When the ball is at an angle θ to the vertical, it has a tangential acceleration of magnitude $g \sin \theta$ (the component of $\mathbf{g}$ tangent to the circle). Therefore, at $\theta = 20°$, $a_t = g \sin 20° = 3.4$ m/s^2. Find the magnitude and direction of the *total* acceleration at $\theta = 20°$.

Solution Because $\mathbf{a} = \mathbf{a}_r + \mathbf{a}_t$, the magnitude of $\mathbf{a}$ at $\theta = 20°$ is

$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(4.5)^2 + (3.4)^2} \text{ m/s}^2 = 5.6 \text{ m/s}^2$$

If ϕ is the angle between $\mathbf{a}$ and the string, then

$$\phi = \tan^{-1}\frac{a_t}{a_r} = \tan^{-1}\left(\frac{3.4 \text{ m/s}^2}{4.5 \text{ m/s}^2}\right) = 36°$$

Note that all of the vectors—$\mathbf{a}$, $\mathbf{a}_r$, and $\mathbf{a}_t$—change in direction *and* magnitude as the ball swings through the circle. When the ball is at its lowest elevation ($\theta = 0$), $a_t = 0$, because there is no tangential component of $\mathbf{g}$ at this angle, and a_r is a *maximum* because v is a maximum. If the ball has enough speed to reach its highest position ($\theta = 180°$), a_t is again zero, but a_r is a minimum because v is now a minimum. Finally, in the two horizontal positions ($\theta = 90°$ and 270°), $|\mathbf{a}_t| = g$ and a_r has a value between its minimum and maximum values.

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SUMMARY

If a particle moves with *constant* acceleration $\mathbf{a}$ and has velocity $\mathbf{v}_0$ and position $\mathbf{r}_0$ at $t = 0$, its velocity and position vectors at some later time t are

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \quad [3.8]$$

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2 \quad [3.9]$$

For two-dimensional motion in the xy plane under constant acceleration, these vector expressions are equivalent to two component expressions, one for the motion along x and one for the motion along y.

Projectile motion is two-dimensional motion under constant acceleration, where $a_x = 0$ and $a_y = -g$. In this case, if $x_0 = y_0 = 0$, the components of Equations 3.8 and 3.9 reduce to

$$v_x = v_{x0} = \text{constant} \quad [3.10]$$

$$v_y = v_{y0} - gt \quad [3.11]$$

$$x = v_{x0}t \quad [3.12]$$

$$y = v_{y0}t - \tfrac{1}{2}gt^2 \quad [3.13]$$

where $v_{x0} = v_0 \cos\theta_0$, $v_{y0} = v_0 \sin\theta_0$, v_0 is the initial speed of the projectile, and θ_0 is the angle $\mathbf{v}_0$ makes with the positive x axis. Note that these expressions give the velocity components (and hence the velocity vector) and the coordinates (and hence the position vector) at *any* time t that the projectile is in motion.

It is useful to think of projectile motion as the superposition of two motions: (1) uniform motion in the x direction and (2) motion in the vertical direction subject to a constant downward acceleration of magnitude $g = 9.80\ \text{m/s}^2$.

A particle moving in a circle of radius r with constant speed v undergoes a centripetal (or radial) acceleration, $\mathbf{a}_r$, because the direction of $\mathbf{v}$ changes in time. The magnitude of $\mathbf{a}_r$ is

$$a_r = \frac{v^2}{r} \quad [3.19]$$

and its direction is always toward the center of the circle.

If a particle moves along a curved path in such a way that the magnitude and direction of $\mathbf{v}$ change in time, the particle has an acceleration vector that can be described by two components: (1) a radial component, a_r, arising from the change in direction of $\mathbf{v}$, and (2) a tangential component, a_t, arising from the change in magnitude of $\mathbf{v}$.

CONCEPTUAL QUESTIONS

1. If you know the position vectors of a particle at two points along its path and also know the time it took to get from one point to the other, can you determine the particle's instantaneous velocity? Its average velocity? Explain.
2. Explain whether or not the following particles have an acceleration: (a) a particle moving in a straight line with constant speed and (b) a particle moving around a curve with constant speed.
3. Correct the following statement: "The racing car rounds the turn at a constant velocity of 90 miles per hour."
4. A spacecraft drifts through space at a constant velocity. Suddenly a gas leak in the side of the spacecraft causes a constant acceleration of the spacecraft in a direction perpendicular to the initial velocity. The orientation of the spacecraft does not change, so that the acceleration remains perpendicular to the original direction of the velocity. What is the shape of the path followed by the spacecraft in this situation?
5. A ball is projected horizontally from the top of a building. One second later another ball is projected horizontally from the same point with the same velocity. At what point in the motion will the balls be closest to each other? Will the first ball always be traveling faster than the second ball? What will be the time difference between when the balls hit the ground? Can the horizontal projection velocity of the second ball be changed so that the balls arrive at the ground at the same time?
6. At the end of a pendulum's arc, its velocity is zero. Is its acceleration also zero at that point?
7. If a rock is dropped from the top of a sailboat's mast, will it hit the deck at the same point whether the boat is at rest or in motion at constant velocity?
8. Two projectiles are thrown with the same magnitude of initial velocity, one at an angle θ with respect to the level ground and the other at angle $90^\circ - \theta$. Both projectiles will strike the ground at the same distance from the projection point. Will both projectiles be in the air for the same time interval?
9. A projectile is fired at some angle to the horizontal with some initial speed v_0, and air resistance is neglected. Is the projectile a freely falling body? What is its acceleration in the vertical direction? What is its acceleration in the horizontal direction?
10. State which of the following quantities, if any, remain constant as a projectile moves through its parabolic trajectory: (a) speed, (b) acceleration, (c) horizontal component of velocity, (d) vertical component of velocity.
11. The maximum range of a projectile occurs when it is launched at an angle of 45.0° with the horizontal, if air resistance is neglected. If air resistance is not neglected, will the optimum angle be greater or less than 45.0°? Explain.
12. An object moves in a circular path with constant speed v. (a) Is the velocity of the object constant? (b) Is its acceleration constant? Explain.
13. A projectile is fired on the Earth with some initial velocity. Another projectile is fired on the Moon with the *same* initial velocity. Neglecting air resistance, which projectile has the greater range? Which reaches the greater altitude? (Note that the free-fall acceleration on the Moon is about $1.6\ \text{m/s}^2$.)
14. A coin on a table is given an initial horizontal velocity such that it ultimately leaves the end of the table and hits the floor. At the instant the coin leaves the end of the table, a ball is released from the same height and falls to the floor. Explain why the two objects hit the floor simultaneously, even though the coin has an initial velocity.
15. Describe how a driver can steer a car traveling at constant speed so that (a) the acceleration is zero or (b) the magnitude of the acceleration remains constant.
16. An ice skater is executing a figure eight, consisting of two equal, tangent circular paths. Throughout the first loop she increases her speed uniformly, and during the second loop she moves at a constant speed. Make a sketch of her acceleration vector at several points along the path of motion.
17. Construct motion diagrams showing the velocity and acceleration of a projectile at several points along its path if (a) the projectile is fired horizontally and (a) the projectile is fired at an angle θ with the horizontal.
18. A baseball is thrown such that its initial x and y components of velocity are known. Neglecting air resistance, describe how you would calculate, at the instant the ball reaches the top of its trajectory, (a) its coordinates, (b) its velocity, and (c) its acceleration. How would these results change if air resistance were taken into account?

Boxed numbers indicate questions that have answers available in the Student Solutions Manual and Study Guide.

PROBLEMS

Section 3.1 The Displacement, Velocity, and Acceleration Vectors

1. A motorist drives south at 20.0 m/s for 3.00 min, then turns west and travels at 25.0 m/s for 2.00 min, and finally travels northwest at 30.0 m/s for 1.00 min. For this 6.00-min trip, find (a) the net vector displacement of the motorist, (b) the motorist's average speed, and (c) the average velocity of the motorist.
2. Suppose that the position-vector function for a particle is given as $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, with $x(t) = at + b$ and $y(t) = ct^2 + d$, where $a = 1.00\ \text{m/s}$, $b = 1.00\ \text{m}$, $c = 0.125\ \text{m/s}^2$, and $d = 1.00\ \text{m}$. (a) Calculate the average velocity during the time interval $t = 2.00\ \text{s}$ to $t = 4.00\ \text{s}$. (b) Determine the velocity and speed at $t = 2.00\ \text{s}$.
3. A golf ball is hit off a tee at the edge of a cliff. Its x and y coordinates versus time are given by the following expressions:

$$x = (18.0\ \text{m/s})t \qquad y = (4.00\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$$

(a) Write a vector expression for the position $\mathbf{r}$ versus time t, using the unit vectors $\mathbf{i}$ and $\mathbf{j}$. By using derivatives, repeat

Boxed numbers indicate problems that have full solutions available in the Student Solutions Manual and Study Guide. Cyan numbers indicate intermediate level problems and magenta numbers indicate more challenging problems.

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for (b) the velocity vector $\mathbf{v}$ versus time and (c) the acceleration vector $\mathbf{a}$ versus time. (d) Find the x and y coordinates of the golf ball at $t = 3.00$ s. Using the unit vectors $\mathbf{i}$ and $\mathbf{j}$, write expressions for (e) the velocity $\mathbf{v}$ and (f) the acceleration $\mathbf{a}$ at the instant $t = 3.00$ s.

4. The coordinates of an object moving in the xy plane vary with time according to the equations $x = -(5.00 \text{ m}) \sin(t)$ and $y = (4.00 \text{ m}) - (5.00 \text{ m}) \cos(t)$, where t is in seconds. (a) Determine the components of velocity and components of acceleration at $t = 0$ s. (b) Write expressions for the position vector, the velocity vector, and the acceleration vector at any time $t > 0$. (c) Describe the path of the object in an xy plot.

Section 3.2 Two-Dimensional Motion with Constant Acceleration

5. At $t = 0$, a particle moving in the xy plane with constant acceleration has a velocity of $\mathbf{v}_0 = (3.00\mathbf{i} - 2.00\mathbf{j})$ m/s at the origin. At $t = 3.00$ s, the particle's velocity is $\mathbf{v} = (9.00\mathbf{i} + 7.00\mathbf{j})$ m/s. Find (a) the acceleration of the particle and (b) its coordinates at any time t.

6. The vector position of a particle varies in time according to the expression $\mathbf{r} = (3.00\mathbf{i} - 6.00t^2\mathbf{j})$ m. (a) Find expressions for the velocity and acceleration as functions of time. (b) Determine the particle's position and velocity at $t = 1.00$ s.

7. A fish swimming in a horizontal plane has velocity $\mathbf{v}_0 = (4.00\mathbf{i} + 1.00\mathbf{j})$ m/s when its displacement from a certain rock is $\mathbf{r}_0 = (10.0\mathbf{i} - 4.00\mathbf{j})$ m. After the fish swims with constant acceleration for 20.0 s, its velocity is $\mathbf{v} = (20.0\mathbf{i} - 5.00\mathbf{j})$ m/s. (a) What are the components of the acceleration? (b) What is the direction of the acceleration with respect to unit vector $\mathbf{i}$? (c) Where is the fish at $t = 25.0$ s and in what direction is it moving?

8. A particle initially located at the origin has an acceleration of $\mathbf{a} = 3.00\mathbf{j}$ m/s^2 and an initial velocity of $\mathbf{v}_0 = 5.00\mathbf{i}$ m/s. Find (a) the vector position and velocity at any time t and (b) the coordinates and speed of the particle at $t = 2.00$ s.

Section 3.3 Projectile Motion

(Neglect air resistance in all problems. Use $g = 9.80$ m/s^2.)

9. In a local bar, a customer slides an empty beer mug on the counter for a refill. The bartender is momentarily distracted and does not see the mug, which slides off the counter and strikes the floor 1.40 m from the base of the counter. If the height of the counter is 0.860 m, (a) with what velocity did the mug leave the counter, and (b) what was the direction of the mug's velocity just before it hit the floor?

10. A student decides to measure the muzzle velocity of a pellet from his gun. He points the gun horizontally. He places a target on a vertical wall a distance x away from the gun. The pellet hits the target a vertical distance y below the gun. (a) Show that the position of the pellet when traveling through the air is given by $y = Ax^2$, where A is a constant. (b) Express the constant A in terms of the initial velocity and the free-fall acceleration. (c) If $x = 3.00$ m and $y = 0.210$ m, what is the initial speed of the pellet?

11. A golfer wants to drive a golf ball a distance of 310 yards (283 m). If the four-wood launches the ball at 15.0° above the horizontal, what must be the initial speed of the ball to achieve the required distance?

12. One strategy in a snowball fight is to throw a snowball at a high angle over level ground. While your opponent is watching the first one, you throw a second snowball at a low angle timed to arrive before or at the same time as the first one. Assume both snowballs are thrown with a speed of 25.0 m/s. The first one is thrown at an angle of 70.0° with respect to the horizontal. (a) At what angle should the second snowball be thrown to arrive at the same point as the first? (b) How many seconds later should the second snowball be thrown after the first to arrive at the same time?

13. A tennis player standing 12.6 m from the net hits the ball at 3.00° above the horizontal. To clear the net, the ball must rise at least 0.330 m. If the ball just clears the net at the apex of its trajectory, how fast was the ball moving when it left the racket?

14. An artillery shell is fired with an initial velocity of 300 m/s at 55.0° above the horizontal. It explodes on a mountainside 42.0 s after firing. What are the x and y coordinates of the shell where it explodes relative to its firing point?

15. An astronaut on a strange planet finds that she can jump a *maximum* horizontal distance of 15.0 m if her initial speed is 3.00 m/s. What is the free-fall acceleration on the planet?

16. A ball is tossed from an upper-story window. The ball is given an initial velocity of 8.00 m/s at an angle of 20.0° below the horizontal. It strikes the ground 3.00 s later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown. (c) How long does it take the ball to reach a point 10.0 m below the level of launching?

17. A ball is thrown horizontally from the top of a building 35.0 m high. The ball strikes the ground at a point 80.0 m from the base of the building. Find (a) the time the ball is in flight, (b) its initial velocity, and (c) the x and y components of velocity just before the ball strikes the ground.

18. A cannon with a muzzle speed of 1000 m/s is used to destroy a target on a mountain top. The target is 2000 m from the cannon horizontally and 800 m above the ground. At what angle, relative to the ground, should the cannon be fired?

19. A placekicker must kick a football from a point 36.0 m (about 40 yards) from the goal and the ball must clear the crossbar, which is 3.05 m high. The ball leaves the ground when kicked with a speed of 20.0 m/s at an angle of 53.0° to the horizontal. (a) By how much does the ball clear or fall short of clearing the crossbar? (b) Does the ball approach the crossbar while still rising or while falling?

20. A fireman, 50.0 m away from a burning building, directs a stream of water from a fire hose at an angle of 30.0° above the horizontal. If the speed of the stream is 40.0 m/s, at what height will the stream of water strike the building?

21. The speed of a projectile when it reaches its maximum height is one half of its speed when it is at half its maximum height. What is the initial projection angle of the projectile?

22. A soccer player kicks a rock horizontally off a 40.0-m–high cliff into a pool of water. If the player hears the sound of the splash 3.00 s later, what was the initial speed given to the rock? Assume the speed of sound in air to be 343 m/s.

23. During the 1968 Olympics in Mexico City, Bob Beamon executed a record long jump. The horizontal distance he achieved was 8.90 m. His center of gravity started at an elevation of 1.00 m, reached a maximum height of 1.90 m, and finished at 0.150 m. From these data determine (a) his time of flight, (b) his horizontal and vertical velocity components at the takeoff time, and (c) his takeoff angle.

Section 3.4 Uniform Circular Motion

24. The orbit of the Moon around the Earth is approximately circular, with a mean radius of 3.84×10^8 m. It takes 27.3 days for the Moon to complete one revolution around the Earth. Find (a) the mean orbital speed of the Moon and (b) its centripetal acceleration.

25. An athlete rotates a 1.00-kg discus along a circular path of radius 1.06 m (Figure P3.25). The maximum speed of the discus is 20.0 m/s. Determine the magnitude of the maximum radial acceleration of the discus.

26. A tire 0.500 m in radius rotates at a constant rate of 200 rev/min. Find the speed and magnitude of acceleration of a small stone lodged in the outer edge of the tread of the tire.

27. Young David who slew Goliath experimented with slings before tackling the giant. He found that he could revolve a sling of length 0.600 m at the rate of 8.00 rev/s. If he increased the length to 0.900 m, he could revolve the sling only 6.00 times per second. (a) Which rate of rotation gives the greater linear speed? (b) What is the centripetal acceleration at 8.00 rev/s? (c) What is the centripetal acceleration at 6.00 rev/s?

28. An astronaut standing on the Moon fires a gun so that the bullet leaves the barrel initially moving in a horizontal direction. (a) What must be the muzzle speed of the bullet so that it travels completely around the Moon and returns to its original location? (b) How long is the bullet in flight? Assume that the free-fall acceleration on the Moon is one sixth that on the Earth.

29. If the rotation of the Earth increased so that the centripetal acceleration was equal to the gravitational acceleration at the Equator, (a) what would be the tangential speed of a person standing at the Equator, and (b) how long would a day be?

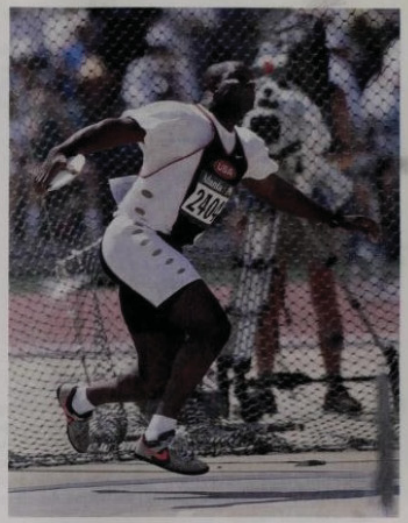


Figure P3.25

Section 3.5 Tangential and Radial Acceleration

30. An automobile, the speed of which is increasing at a rate of 0.600 m/s^2, travels along a circular road of radius 20.0 m. When the instantaneous speed of the automobile is 4.00 m/s, find (a) the tangential acceleration component, (b) the centripetal acceleration component, and (c) the magnitude and direction of the total acceleration.

31. A train slows down as it rounds a sharp horizontal turn, slowing from 90.0 km/h to 50.0 km/h in the 15.0 s that it takes to round the bend. The radius of the curve is 150 m. Compute the acceleration at the moment the train speed reaches 50.0 km/h, assuming it continues to slow down at this time.

32. A point on a rotating turntable 20.0 cm from the center accelerates from rest to a final speed of 0.700 m/s in 1.75 s. At $t = 1.25$ s, find the magnitude and direction of

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(a) the centripetal acceleration, (b) the tangential acceleration, and (c) the total acceleration of the point.

33. Figure P3.33 represents the total acceleration of a particle moving clockwise in a circle of radius 2.50 m at a given instant of time. At this instant, find (a) the centripetal acceleration, (b) the speed of the particle, and (c) its tangential acceleration.

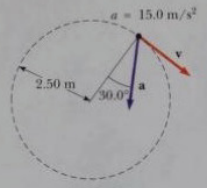


Figure P3.33

34. A ball swings in a vertical circle at the end of a 1.50-m long rope. When it is 36.9° past the lowest point on its way up, the ball's total acceleration is $(-22.5\mathbf{i} + 20.2\mathbf{j})$ m/s². At that instant, (a) sketch a vector diagram showing the components of its acceleration, (b) determine the magnitude of its centripetal acceleration, and (c) determine the speed and velocity of the ball.

Additional Problems

35. At $t = 0$ a particle leaves the origin with a velocity of 6.00 m/s in the positive y direction. Its acceleration is given by $\mathbf{a} = (2.00\mathbf{i} - 3.00\mathbf{j})$ m/s². When the particle reaches its *maximum* y coordinate, its y component of velocity is zero. At this instant, find (a) the velocity of the particle and (b) its x and y coordinates.

36. A ball on the end of a string is whirled around in a horizontal circle of radius 0.300 m. The plane of the circle is 1.20 m above the ground. The string breaks and the ball lands 2.00 m (horizontally) away from the point on the ground directly beneath the ball's location when the string breaks. Find the centripetal acceleration of the ball during its circular motion.

37. A home run is hit in such a way that the baseball just clears a wall 21.0 m high, located 130 m from home plate. The ball is hit at an angle of 35.0° to the horizontal, and air resistance is negligible. Find (a) the initial speed of the ball, (b) the time it takes the ball to reach the wall, and (c) the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1.00 m above the ground.)

38. The astronaut orbiting the Earth in the photograph is preparing to dock with a spinning Westar VII satellite. The satellite is in a circular orbit 600 km above the Earth's surface, where the free-fall acceleration is 8.21 m/s². The radius of the Earth is 6400 km. Determine the speed of the satellite and the time required to complete one orbit around the Earth.



Figure P3.38 *(Courtesy of NASA)*

39. A particle has velocity components

$$v_x = +4.00 \text{ m/s} \qquad v_y = -(6.00 \text{ m/s}^2)t + 4.00 \text{ m/s}$$

Calculate the speed of the particle and the direction $\theta = \tan^{-1}(v_y/v_x)$ of the velocity vector at $t = 2.00$ s.

40. When baseball outfielders throw the ball in, they usually allow it to take one bounce on the theory that the ball arrives sooner this way. Suppose that after the bounce the ball rebounds at the same angle θ as before, as in Figure P3.40 but loses half its speed. (a) Assuming the ball is always thrown with the same initial speed, at what angle θ should the ball be thrown in order to go the same distance D with one bounce (blue path) as one thrown upward at 45.0° with no bounce (green path)? (b) Determine the ratio of the times for the one-bounce and no-bounce throws.

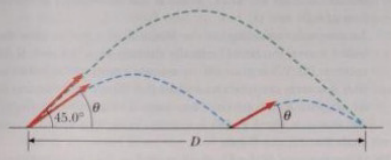


Figure P3.40

41. A girl can throw a ball a maximum horizontal distance of 40.0 m on a level field. How far can she throw the same ball vertically upward? Assume that her muscles give the ball the same speed in each case.

42. A girl can throw a ball a maximum horizontal distance of R on a level field. How far can she throw the same ball vertically upward? Assume that her muscles give the ball the same speed in each case.

43. A bomber is flown horizontally, with a ground speed of 275 m/s, at an altitude of 3000 m over level terrain. Neglect the effects of air resistance. (a) How far from the point that is vertically under the point of release will a bomb hit the ground? (b) If the plane maintains its original course and speed, where will it be when the bomb hits the ground? (c) For the preceding conditions, at what angle from the vertical at the point of release must the telescopic bomb sight be set so that the bomb will hit the target seen in the sight at the time of release?

44. A football is thrown toward a receiver with an initial speed of 20.0 m/s, at an angle of 30.0° above the horizontal. At that instant, the receiver is 20.0 m from the quarterback. In what direction and with what constant speed should the receiver run in order to catch the football at the level at which it was thrown?

45. A hawk is flying horizontally at 10.0 m/s in a straight line, 200 m above the ground. A mouse it has been carrying is released from its grasp. The hawk continues on its path at the same speed for 2.00 s before attempting to retrieve its prey. To accomplish the retrieval, it dives in a straight line at constant speed and recaptures the mouse 3.00 m above the ground. (a) Assuming no air resistance, find the diving speed of the hawk. (b) What angle did the hawk make with the horizontal during its descent? (c) For how long did the mouse "enjoy" free fall?

46. A rocket is launched at an angle of 53.0° to the horizontal with an initial speed of 100 m/s. For 3.00 s it moves along its initial line of motion with an acceleration of 30.0 m/s². Then its engines fail and the rocket proceeds to move in free fall. Find (a) the maximum altitude reached by the rocket, (b) its total time of flight, and (c) its horizontal range.

47. A car is parked on a steep incline overlooking the ocean, where the incline makes an angle of 37.0° with the horizontal. The negligent driver leaves the car in neutral, and the parking brakes are defective. The car rolls from rest down the incline with a constant acceleration of 4.00 m/s², traveling 50.0 m to the edge of a cliff. The cliff is 30.0 m above the ocean. Find (a) the speed of the car when it reaches the edge of the cliff and the time it takes to get there, (b) the velocity of the car when it lands in the ocean, (c) the total time the car is in motion, and (d) the position of the car when it lands in the ocean, relative to the base of the cliff.

48. The determined coyote is out once more to try to capture the elusive roadrunner. The coyote wears a pair of Acme jet-powered roller skates, which provide a constant horizontal acceleration of 15.0 m/s², as pictured in Figure P3.48. The coyote starts off at rest 70.0 m from the edge of a cliff at the instant the roadrunner zips past him in the direction of the cliff. (a) If the roadrunner moves with constant speed, determine the minimum speed he must have in order to reach the cliff before the coyote. (b) If the cliff is 100 m above the base of a canyon, determine where the coyote lands in the canyon (assume his skates are still in operation when he is in flight). (c) Determine the coyote's velocity components just before he lands in the canyon. (As usual, the roadrunner saves himself by making a sudden turn at the cliff.)

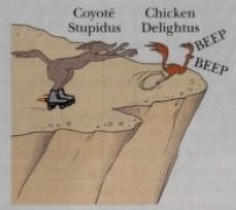


Figure P3.48

49. A skier leaves the ramp of a ski jump with a velocity of 10.0 m/s, 15.0° above the horizontal, as in Figure P3.49. The slope is inclined at 50.0°, and air resistance is negligi-

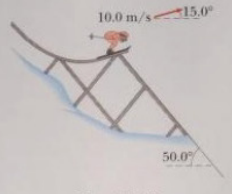


Figure P3.49

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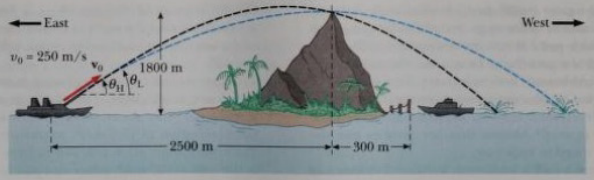


Figure P3.51

ble. Find (a) the distance from the ramp to where the jumper lands and (b) the velocity components just before the landing. (How do you think the results might be affected if air resistance were included? Note that jumpers lean forward in the shape of an airfoil, with their hands at their sides, to increase their distance. Why does this work?)

50. Describe what you might do to give your hand a large acceleration without hurting yourself or striking your hand against anything. Compute an order-of-magnitude estimate of this acceleration, stating the quantities you measure or estimate and their values.

51. An enemy ship is on the east side of a mountain island, as shown in Figure P3.51. The enemy ship can maneuver to within 2500 m of the 1800-m–high mountain peak and can shoot projectiles with an initial speed of 250 m/s. If the western shoreline is horizontally 300 m from the peak, what are the distances from the western shore at which a ship can be safe from the bombardment of the enemy ship?

Spreadsheet Problems

S1. The punter for the St. Louis Rams needs to kick the football 40 yd to 45 yd (36.6 m to 41.1 m) to keep the opposing team inside the 5-yd line. Use Spreadsheet 3.1 to estimate the angle at which he should kick the ball. *Note:* He must keep the ball in the air as long as possible in order to help prevent a runback (in other words, he wants a long "hang time"). The initial speed of the ball should be in the range of 20 m/s to 30 m/s.

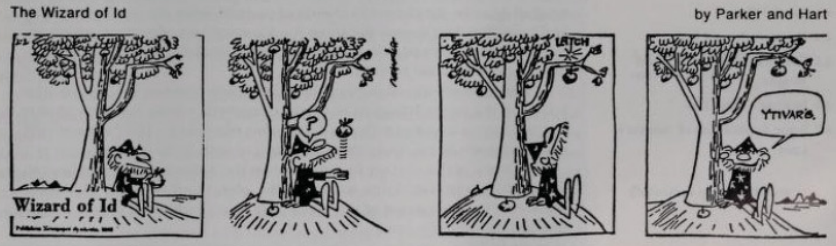
S2. Modify Spreadsheet 3.1 by adding two columns and then calculate the maximum height reached by the football in Problem S1 and the total time it is in the air.

S3. The x and y coordinates of a projectile are given by Equations 3.12 and 3.13. Using these equations, set up a spreadsheet to calculate the x and y coordinates of a projectile and the components of its velocity v_x and v_y as functions of time. The initial speed and initial angle of the projectile should be input parameters. Use the graph function of the spreadsheet to plot the x and y coordinates as functions of time. Also graph v_x and v_y as functions of time. Use this spreadsheet to solve the following problem.

The New York Giants are tied with the Chicago Bears with only a few seconds left in the game. The Giants have the football and call the placekicker into the game. He must kick the ball 52 yd (47.5 m) for a field goal. If the crossbar of the goal is 10 ft (3.05 m) high, at what angle and speed should he kick the ball to score the winning points? Is there only one solution to this problem? Solutions can be found by varying the parameters and studying the graph of y versus x.

ANSWERS TO CONCEPTUAL PROBLEMS

1. (a) Yes. Although its speed may be constant, the direction of its motion (that is, the direction of **v**) may change, causing an acceleration. For example, an object moving in a circle with constant speed has an acceleration directed toward the center of the circle. (b) No. An object that moves with constant velocity has zero acceleration. Constant velocity means that both the direction and magnitude of **v** remain constant.
2. A constant velocity is interpreted as constant in both magnitude and direction. Thus, a constant velocity clearly implies a nonvarying speed. On the other hand, a constant speed indicates that only the magnitude of the velocity vector is constant—the direction can change. Thus, as long as a particle covers the same distance in equal time intervals, the speed is constant, and its direction can change continuously, giving a varying velocity. A familiar example is a particle moving in a circular path at constant speed, such that the velocity is constantly changing direction.
3. You should simply throw it straight up in the air. Because the ball was moving along with you, it would follow a parabolic trajectory with a horizontal component of velocity that is the same as yours.
4. (a) At the top of its flight, **v** is horizontal and **a** is vertical. This is the only point at which the velocity and acceleration vectors are perpendicular. (b) If the object is thrown straight up or down, then **v** and **a** will be parallel throughout the downward motion. Otherwise, the velocity and acceleration vectors are never parallel.



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4

The Laws of Motion

CHAPTER OUTLINE

4.1 The Concept of Force
4.2 Newton's First Law and Inertial Frames
4.3 Inertial Mass
4.4 Newton's Second Law
4.5 The Gravitational Force and Weight
4.6 Newton's Third Law
4.7 Some Applications of Newton's Laws

In the preceding two chapters on kinematics, we described the motions of particles based on the definitions of displacement, velocity, and acceleration. However, we would like to be able to answer specific questions related to the causes of motion, such as, "What mechanism causes motion?" and, "Why do some objects accelerate at higher rates than others?" In this chapter we shall describe the change in motion of particles using the concepts of force and mass. We then discuss the three fundamental laws of motion, which are based on experimental observations and were formulated about three centuries ago by Sir Isaac Newton.

Classical mechanics describes the relationship between the motion of a body and the forces acting on it. Classical mechanics deals only with objects that are large compared with the dimensions of atoms ($\sim 10^{-10}$ m) and that move at speeds much slower than the speed of light (3.00×10^8 m/s).

We learn in this chapter how to describe the acceleration of an object in terms of the resultant force acting on the object and its mass. This force represents the interaction of the object with its environment. Mass is a measure of the object's tendency to resist an acceleration when a force acts on it.

We also discuss *force laws*, which describe how to



Calf-roping, as shown in this photograph taken in Steamboat, Colorado, is a standard rodeo event. The external forces exerted on the horse are the force of friction between the horse and ground, the downward force of gravity, the force exerted by the rope attached to the calf, the downward force exerted by the cowboy on the horse, and the upward force of the ground. Can you identify the forces acting on the calf? *(Fourbyfive, Inc.)*

calculate the force on an object if its environment is known. We shall see that, although the force laws are simple in form, they successfully explain a wide variety of phenomena and experimental observations. They, together with the laws of motion, are the foundations of classical mechanics.

4.1 • THE CONCEPT OF FORCE

Everyone has a basic understanding of the concept of force as a result of everyday experiences. When you push or pull an object, you exert a force on it. You exert a force when you throw or kick a ball. In these examples, the word *force* is associated with the result of muscular activity and with some change in the state of motion of an object. Forces do not always cause an object to move, however. For example, as you sit reading this book, the force of gravity acts on your body, and yet you remain stationary. Likewise, you can push on a block of stone and yet fail to move it.

What force (if any) causes a distant star to drift freely through space? Newton answered such questions by stating that the change in velocity of an object is caused by unbalanced forces. Therefore, if an object moves with uniform motion (constant velocity), no force is required to maintain the motion. Because only a force can cause a change in velocity, we can think of force as that which causes a body to accelerate.

• ***A body accelerates due to an external force.***

Now consider a situation in which several forces act simultaneously on an object. In this case, the object accelerates only if the **net force** acting on it is not equal to zero. We shall often refer to the net force as the *resultant force* or as the *unbalanced force*. **If the net force is zero, the acceleration is zero and the velocity of the object remains constant.** That is, if the net force acting on the object is zero, either the object will be at rest or it will move with constant velocity. **When a body has constant velocity or is at rest, it is said to be in equilibrium.**

• ***Definition of equilibrium***

This chapter is concerned with the relation between the force on an object and the acceleration of that object. If you pull on a spring, as in Figure 4.1a, the spring stretches. If the spring is calibrated, the distance it stretches can be used to measure the strength of the force. If a child pulls hard enough on a cart to overcome friction, as in Figure 4.1b, the cart moves. When a football is kicked, as in Figure 4.1c, it is both deformed and set in motion. These are all examples of a class of forces called *contact forces*. That is, they represent the result of physical contact between two objects.

Another class of forces, which do not involve physical contact between two objects but act through empty space, are known as *field forces*. The force of gravitational attraction between two objects is an example of this class of force, illustrated in Figure 4.1d. This gravitational force keeps objects bound to the Earth and gives rise to what we commonly call the *weight* of an object. The planets of our solar system are bound under the action of gravitational forces. Another common example of a field force is the electric force that one electric charge exerts on another electric charge, as in Figure 4.1e. These charges might be an electron and proton forming a hydrogen atom. A third example of a field force is the force that a bar magnet exerts on a piece of iron, as shown in Figure 4.1f. The forces holding an atomic nucleus together are also field forces but are usually very short-range. They are the dominating interaction for particle separations of the order of 10^{-15} m.

Early scientists, including Newton, were uneasy with the concept of a force acting between two disconnected objects. To overcome this conceptual problem,



A football is set in motion as a result of the contact force **F** on it due to the kicker's foot. The ball is distorted in the short time it is in contact with the foot. *(Ralph Cowan, Tony Stone Worldwide)*

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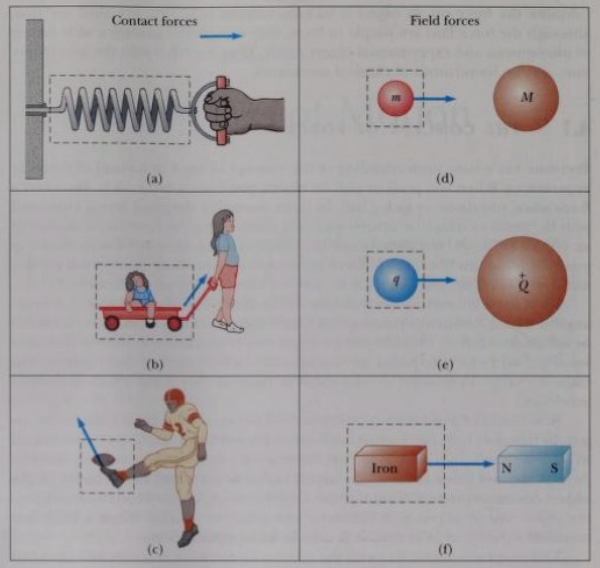


Figure 4.1 Some examples of forces applied to various objects. In each case a force is exerted on the particle or object within the boxed area. The environment external to the boxed area provides this force.

Michael Faraday (1791–1867) introduced the concept of a *field*. According to this approach, when a mass m_1 is placed at some point P near a mass m_2, one can say that m_1 interacts with m_2 by virtue of the gravitational field that exists at P. The field at P is created by mass m_2. Likewise, a field exists at the position of m_2 created by m_1. In fact, all objects create a gravitational field in the space around them.

The distinction between contact forces and field forces is not as sharp, however, as the previous section might lead you to believe. At the atomic level, all the forces we classify as contact forces turn out to be due to repulsive electrical (field) forces of the type illustrated in Figure 4.1e. Nevertheless, in developing models for macroscopic phenomena, it is convenient to use both classifications of forces. However, the only known *fundamental* forces in nature are all field forces: (1) gravitational attractions between objects, (2) electromagnetic forces between charges, (3) strong nuclear forces between subatomic particles, and (4) weak nuclear forces that arise in certain radioactive decay processes. In classical physics we are concerned only with gravitational and electromagnetic forces.

Fundamental forces in nature

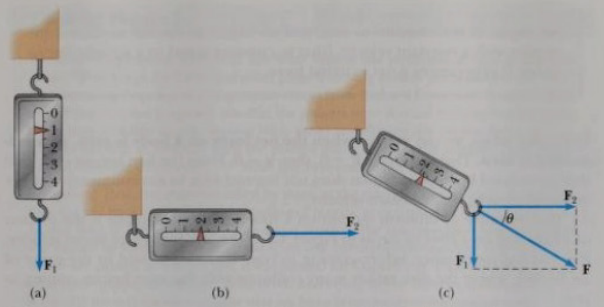


Figure 4.2 The vector nature of a force is tested with a spring scale. (a) The downward vertical force $\mathbf{F}_1$ elongates the spring 1 unit. (b) The horizontal force $\mathbf{F}_2$ elongates the spring 2 units. (c) The combination of $\mathbf{F}_1$ and $\mathbf{F}_2$ elongates the spring $\sqrt{1^2 + 2^2} = \sqrt{5}$ units.

It is sometimes convenient to use the deformation of a spring to measure force. Suppose a force is applied vertically to a spring that has a fixed upper end, as in Figure 4.2a. We can calibrate the spring by defining the unit force, $\mathbf{F}_1$, as the force that produces an elongation of 1.00 cm. If a force $\mathbf{F}_2$, applied horizontally as in Figure 4.2b, produces an elongation of 2.00 cm, the magnitude of $\mathbf{F}_2$ is 2 units. If the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are applied simultaneously, as in Figure 4.2c, the elongation of the spring is $\sqrt{5} = 2.24$ cm. The single force $\mathbf{F}$ that would produce this same elongation is the vector sum of $\mathbf{F}_1$ and $\mathbf{F}_2$, as described in Figure 4.2c. That is, $|\mathbf{F}| = \sqrt{F_1^2 + F_2^2} = \sqrt{5}$ units, and its direction is $\theta = \arctan(-0.500) = -26.6°$. **Because forces are vectors, you must use the rules of vector addition to get the resultant force on a body.**

Springs that elongate in proportion to an applied force are said to obey **Hooke's law.** Such springs can be constructed and calibrated to measure unknown forces.

4.2 · NEWTON'S FIRST LAW AND INERTIAL FRAMES

Before about 1600, scientists believed that the natural state of matter was the state of rest. Galileo was the first to take a different approach to motion and the natural state of matter. He devised thought experiments, such as an object moving on a frictionless surface, and concluded that it is not the nature of an object to stop once it is set in motion; rather, it is an object's nature to resist changes in its motion. In his words, "Any velocity, once imparted to a moving body, will be rigidly maintained as long as the external causes of retardation are removed."

This new approach to motion was later formalized by Newton in a statement that has come to be known as **Newton's first law of motion:**

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A statement of Newton's first law

An object at rest remains at rest, and an object in motion continues in motion with a constant velocity (that is, constant speed in a straight line), unless it experiences a net external force.

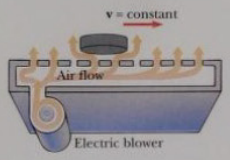


Figure 4.3 A disk moving on a layer of air is an example of uniform motion—that is, motion in which the acceleration is zero.

In simpler terms, we can say that **when the net force on a body is zero, its acceleration is zero.** That is, when $\Sigma \mathbf{F} = 0$, then $\mathbf{a} = 0$. From the first law, we conclude that an isolated body (a body that does not interact with its environment) is either at rest or moving with constant velocity.

One example of uniform motion on a nearly frictionless plane is the motion of a light disk on a layer of air, as in Figure 4.3. If the disk is given an initial velocity, it coasts a great distance before coming to rest. This idea is used in the game of air hockey, where the disk makes many collisions with the walls before coming to rest.

As a second example, consider a spaceship traveling in space, far removed from any planets or other matter. The spaceship requires some propulsion system to change its velocity. However, if the propulsion system is turned off when the spaceship reaches a velocity $\mathbf{v}$, the spaceship "coasts" in space with that velocity, and the astronauts get a free ride (that is, no propulsion system is required to keep them moving at the velocity $\mathbf{v}$).

Inertial Frames

Inertial frame

Newton's first law, sometimes called the *law of inertia*, defines a special set of reference frames called inertial frames. **An inertial frame of reference is one in which Newton's first law is valid.** Any reference frame that moves with constant velocity with respect to an inertial frame is itself an inertial frame. A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame. The Earth is not an inertial frame because of its orbital motion around the Sun and rotational motion around its own axis. As the Earth travels in its nearly circular orbit around the Sun, it experiences a centripetal acceleration of about $4.4 \times 10^{-3}\ \text{m/s}^2$ toward the Sun. In addition, because the Earth rotates around its own axis once every 24 h, a point on the Equator experiences an additional centripetal acceleration of $3.37 \times 10^{-2}\ \text{m/s}^2$ toward the center of the Earth. However, these accelerations are small compared with g and can often be neglected. In most situations **we shall assume that a frame on or near the Earth's surface is an inertial frame.**

Thus, if an object is moving with constant velocity, an observer in one inertial frame (say, one at rest with respect to the object) will claim that the acceleration and the resultant force on the object are zero. An observer in *any other* inertial frame will also find that $\mathbf{a} = 0$ and $\mathbf{F} = 0$ for the object. According to the first law, a body at rest and one moving with constant velocity are equivalent. Unless stated otherwise, we shall usually write the laws of motion with respect to an observer "at rest" in an inertial frame.

Thinking Physics 1

When the Copernican Theory of the Solar System was proposed, a natural question arose: "What keeps the Earth and other planets moving in their paths around the Sun?" An interesting response to this question comes from Richard Feynman[1]: "In those days, one of the theories proposed was that the planets went around because behind them there were invisible angels, beating their wings and driving the planets forward. . . . It turns out that in order to keep the planets going around, the invisible angels must fly in a different direction. . . ." What did Feynman mean by this?

Reasoning First, the question asked by those at the time of Copernicus indicates that they did not have a proper understanding of inertia, as described by Newton's first law. At that time in history, before Galileo and Newton, the interpretation was that *motion* was caused by force. This is different from our current understanding that *changes in motion* are caused by force. Thus, it was natural for Copernicus' contemporaries to ask what force propelled a planet in its orbit. According to our current understanding, it is equally natural for us to realize that no force is necessary—the motion simply continues due to inertia. Thus, in Feynman's imagery, we do not need the angels pushing the planet *from behind*. We do need the angels to push *inward*, however, to provide the centripetal acceleration needed to change its direction into a circle. Of course, the angels are not real, from a scientific point of view—they are a metaphor for the *gravitational force*.

CONCEPTUAL PROBLEM 1

Is it possible to have motion in the absence of a force?

CONCEPTUAL PROBLEM 2

If a single force acts on an object, is there an acceleration? If an object experiences an acceleration, is a force acting on it? If an object experiences no acceleration, is there no force acting?

4.3 • INERTIAL MASS

Inertia

If you attempt to change the velocity of an object, the object resists this change. **Inertia** is solely a property of an individual object; it is a measure of the response of an object to an external force. For instance, consider two large, solid cylinders of equal size, one balsa wood and the other steel. If you were to push the cylinders along a horizontal, rough surface, the force required to give the steel cylinder some acceleration would be larger than the force needed to give the balsa wood cylinder the same acceleration. Therefore, we say that the steel cylinder has more inertia than the balsa wood cylinder.

Mass is used to measure inertia, and the SI unit of mass is the kilogram. The greater the mass of a body, the less it accelerates (changes its state of motion) under the action of an applied force.

A quantitative measurement of mass can be made by comparing the accelera-

[1]R. P. Feynman, R. B. Leighton, and M. Sands, *The Feynman Lectures on Physics*, Vol. 1, Reading, Massachusetts, 1963, Addison-Wesley Publishing Co., p. 7-2.

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