

Figure 11.16 A sinusoidal wave traveling to the right. Any point on the wave moves a distance of one wavelength, λ, in a time equal to the period of the wave.

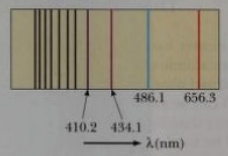
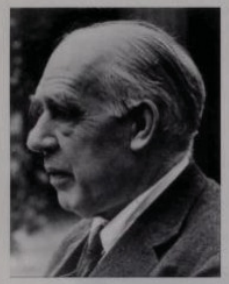


Figure 11.17 A series of spectral lines for atomic hydrogen. The prominent lines labeled are part of the Balmer series.



Niels Bohr (1885–1962)

A Danish physicist, Bohr was an active participant in the early development of quantum mechanics and provided much of its philosophical framework. During the 1920s and 1930s, Bohr headed the Institute for Advanced Studies in Copenhagen. This institute was a magnet for many of the world's best physicists and provided a forum for the exchange of ideas. When Bohr visited the United States in 1939 to attend a scientific conference, he brought news that the fission of uranium had been discovered by Hahn and Strassman in Berlin. The results were the foundations of the atomic bomb developed in the United States during World War II. Bohr was awarded the 1922 Nobel prize for his investigation of the structure of atoms and of the radiation emanating from them.

is depicted in Figure 11.16. The distance between two consecutive crests of the wave is called the **wavelength,** λ. As the wave travels to the right with speed v, any point on the wave travels a distance of one wavelength in a time interval of one period, T (the time for one cycle), so the wave speed is $v = \lambda/T$. The inverse of the period, $1/T$, is called the **frequency,** f, of the wave; it represents the number of cycles per second. Thus, the speed of the wave is often written as $v = \lambda f$. In this section, because we shall deal with electromagnetic waves—which travel at the speed of light, c—the appropriate relation is

$$c = \lambda f \qquad [11.13]$$

The emission spectrum of hydrogen shown in Figure 11.17 includes four prominent lines that occur at wavelengths of 656.3 nm, 486.1 nm, 434.1 nm, and 410.2 nm. In 1885, Johann Balmer (1825–1898) found that the wavelengths of these and less prominent lines can be described by this simple empirical equation:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{2^2} - \frac{1}{n^2}\right) \qquad \text{Balmer series} \qquad [11.14]$$

where n may have integral values of 3, 4, 5, . . . , and R_H is a constant, now called the **Rydberg constant.** If the wavelength is in meters, R_H has the value

$$R_H = 1.0973732 \times 10^7\ \text{m}^{-1}$$

The first line in the Balmer series, at 656.3 nm, corresponds to $n = 3$ in Equation 11.14; the line of 486.1 nm corresponds to $n = 4$; and so on.

In addition to emitting light at specific wavelengths, an element can also absorb light at specific wavelengths. The spectral lines corresponding to this process form what is known as an **absorption spectrum.** An absorption spectrum can be obtained by passing a continuous radiation spectrum (one containing all wavelengths) through a vapor of the element being analyzed. The absorption spectrum consists of a series of dark lines superimposed on the otherwise continuous spectrum.

At the beginning of the twentieth century, scientists were perplexed by the failure of classical physics to explain the characteristics of spectra. Why did atoms of a given element emit only certain wavelengths of radiation, so that the emission spectrum displayed discrete lines? Furthermore, why did the atoms absorb only those wavelengths that they emitted? In 1913 Bohr provided an explanation of

atomic spectra that included some features of the currently accepted theory. Using the simplest atom, hydrogen, Bohr described a model of what he thought must be the atom's structure. His model of the hydrogen atom contains some classical features as well as some revolutionary postulates that could not be justified within the framework of classical physics.[3] The basic assumptions of the Bohr theory as it applies to the hydrogen atom are as follows:

- *Assumptions of the Bohr theory*

1. The electron moves in circular orbits about the proton under the influence of the Coulomb force of attraction, as in Figure 11.18.
2. Only certain electron orbits are stable. These are orbits in which the hydrogen atom does not emit energy in the form of radiation. Hence, the total energy of the atom remains constant, and classical mechanics can be used to describe the electron's motion.
3. Radiation is emitted by the hydrogen atom when the electron "jumps" from a more energetic initial state to a lower state. The jump cannot be visualized or treated classically. In particular, the frequency, f, of the radiation emitted in the jump is related to the change in the atom's energy and **is independent of the frequency of the electron's orbital motion.** The frequency of the emitted radiation is found from

$$E_i - E_f = hf \qquad [11.15]$$

where E_i is the energy of the initial state, E_f is the energy of the final state, h is Planck's constant (see Section 10.9), and $E_i > E_f$.
4. The size of the allowed electron orbits is determined by a condition imposed on the electron's orbital angular momentum: the allowed orbits are those for which the electron's orbital angular momentum about the nucleus is an integral multiple of $\hbar = h/2\pi$.

$$m_e v r = n\hbar \qquad n = 1, 2, 3, \ldots \qquad [11.16]$$

Using these four assumptions, we can calculate the allowed energy levels and emission wavelengths of the hydrogen atom. The electrical potential energy of the system shown in Figure 11.18 is given by $U_e = -k_e e^2/r$, where k_e is the Coulomb constant, e is the charge on the electron, and r is the electron–proton separation. Thus, the total energy of the atom, which contains both kinetic and potential energy terms, is

$$E = K + U_e = \tfrac{1}{2}m_e v^2 - k_e\frac{e^2}{r} \qquad [11.17]$$

Applying Newton's second law to this system, we see that the Coulomb attractive force on the electron, $k_e e^2/r^2$ (Eq. 5.15), must equal the mass times the centripetal acceleration ($a = v^2/r$) of the electron:

$$\frac{k_e e^2}{r^2} = \frac{m_e v^2}{r}$$

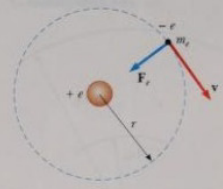


Figure 11.18 Diagram representing Bohr's model of the hydrogen atom in which the orbiting electron is allowed to be only in specific orbits of discrete radii.

[3]The Bohr model can be applied successfully to such hydrogen-like ions as singly ionized helium and doubly ionized lithium, but the theory does not properly describe the visible spectra of more complex atoms and ions.

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IPR2023-00783

From this expression, we find the kinetic energy to be

$$K = \tfrac{1}{2}m_e v^2 = \frac{k_e e^2}{2r} \qquad [11.18]$$

Substituting this value of K into Equation 11.17, we find that the total energy of the atom is

Total energy of the hydrogen atom

$$E = -\frac{k_e e^2}{2r} \qquad [11.19]$$

Note that the total energy is negative, indicating a bound electron–proton system. This means that energy in the amount of $k_e e^2/2r$ must be added to the atom just to remove the electron and make the total energy zero. An expression for r, the radius of the allowed orbits, can be obtained by eliminating v by substitution between Equations 11.16 and 11.18:

Radii of Bohr orbits in hydrogen

$$r_n = \frac{n^2\hbar^2}{m_e k_e e^2} \qquad n = 1, 2, 3, \ldots \qquad [11.20]$$

This result shows that the radii have discrete values, or are *quantized.*

The orbit for which $n = 1$ has the smallest radius; it is called the **Bohr radius,** a_0, and has the value

The Bohr radius

$$a_0 = \frac{\hbar^2}{m_e k_e e^2} = 0.0529 \text{ nm} \qquad [11.21]$$

The first three Bohr orbits are shown to scale in Figure 11.19.

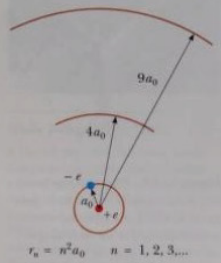


Figure 11.19 The first three Bohr orbits for hydrogen.

The quantization of the orbit radii immediately leads to energy quantization. This can be seen by substituting $r_n = n^2a_0$ into Equation 11.19. The allowed energy levels are found to be

$$E_n = -\frac{k_e e^2}{2a_0}\left(\frac{1}{n^2}\right) \qquad n = 1, 2, 3, \ldots \qquad [11.22]$$

Insertion of numerical values into Equation 11.22 gives

$$E_n = -\frac{13.6}{n^2} \text{ eV} \qquad n = 1, 2, 3, \ldots \qquad [11.23]$$

(Recall from Section 9.7 that 1 eV = 1.6×10^{-19} J.) The lowest stationary state corresponding to $n = 1$, called the **ground state,** has an energy of $E_1 = -13.6$ eV. The next state, the **first excited state,** has $n = 2$ and an energy of $E_2 = E_1/2^2 = -3.4$ eV. Figure 11.20 is an energy-level diagram showing the energies of these discrete energy states and the corresponding quantum numbers. The uppermost level, corresponding to $n = \infty$ (or $r = \infty$) and $E = 0$, represents the state for which the electron is removed from the atom. The minimum energy required to ionize the atom (that is, to completely remove an electron in the ground state from the proton's influence) is called the **ionization energy.** As can be seen from Figure 11.20, the ionization energy for hydrogen, based on Bohr's calculation, is 13.6 eV. This constituted another major achievement for the Bohr theory, because the ionization energy for hydrogen had already been measured to be precisely 13.6 eV.

Figure 11.20 also shows other spectral series (the Lyman series and the Paschen series) that were found after Balmer's discovery. These spectra obey other empirical formulas, which are reconciled with the Bohr model.

Equation 11.22, together with Bohr's third postulate, can be used to calculate the frequency of the radiation that is emitted when the electron jumps from an outer orbit to an inner orbit:

$$f = \frac{E_i - E_f}{h} = \frac{k_e e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad [11.24]$$

Because the quantity being measured is wavelength, it is convenient to convert frequency to wavelength, using $c = f\lambda$, to get

$$\frac{1}{\lambda} = \frac{f}{c} = \frac{k_e e^2}{2a_0 hc}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad [11.25]$$

The remarkable fact is that the *theoretical* expression, Equation 11.25, is identical to a generalized form of the empirical relations discovered by Balmer and others (see Eq. 11.14),

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad [11.26]$$

provided that the combination of constants $k_e e^2/2a_0 hc$ is equal to the experimentally determined Rydberg constant. After Bohr demonstrated the agreement of these two quantities to a precision of about 1%, it was soon recognized as the crowning achievement of his new theory of quantum mechanics. Furthermore, Bohr showed that all of the spectral series for hydrogen have a natural interpretation in his theory. Figure 11.20 shows these spectral series as transitions between energy levels.

Bohr immediately extended his model for hydrogen to other elements in which all but one electron had been removed. Ionized elements such as He^+, Li^{2+}, and Be^{3+} were suspected to exist in hot stellar atmospheres, where frequent atomic collisions occur with enough energy to completely remove one or more atomic electrons. Bohr showed that many mysterious lines observed in the Sun and several stars could not be due to hydrogen but were correctly predicted by his theory if attributed to singly ionized helium.

Although the Bohr model had some success in predicting the spectra of single valence electron atoms, it has some serious drawbacks. For example, it cannot account for the visible spectra of more complex atoms, and it is unable to predict many subtle spectral details of hydrogen and other simple atoms.

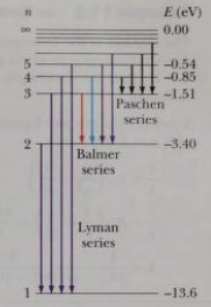


Figure 11.20 An energy level diagram for hydrogen. The discrete allowed energies are plotted on the vertical axis. Nothing is plotted on the horizontal axis, but the horizontal extent of the diagram is made large enough to show allowed transitions. Quantum numbers are given on the left.

CONCEPTUAL PROBLEM 4

Under normal experimental conditions, hydrogen atoms show more lines in emission than in absorption. Why?

CONCEPTUAL PROBLEM 5

Suppose the energy of the electron in a hydrogen atom is $-E$. What is the kinetic energy? What is the potential energy?

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

Example 11.5 An Electronic Transition in Hydrogen

The electron in the hydrogen atom makes a transition from the $n = 2$ energy state to the ground state (corresponding to $n = 1$). Find the wavelength and frequency of the emitted radiation.

Solution We can use Equation 11.26 directly to obtain λ, with $n_i = 2$ and $n_f = 1$:

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$\frac{1}{\lambda} = R_H\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R_H}{4}$$

$$\lambda = \frac{4}{3R_H} = \frac{4}{3(1.097 \times 10^{-7}\ \text{m}^{-1})}$$

$$= 1.215 \times 10^{-7}\ \text{m} = 121.5\ \text{nm} \quad \text{(ultraviolet)}$$

Because $c = f\lambda$, the frequency of the radiation is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{1.215 \times 10^{-7}\ \text{m}} = 2.47 \times 10^{15}\ \text{s}^{-1}$$

EXERCISE 8 What is the wavelength of the radiation emitted by hydrogen when the electron makes a transition from the $n = 3$ state to the $n = 1$ state?

Answer $\dfrac{9}{8R_H} = 102.6\ \text{nm}$

Bohr's Correspondence Principle

In our study of relativity in Chapter 9, we found that newtonian mechanics cannot be used to describe phenomena that occur at speeds approaching the speed of light. Newtonian mechanics is a special case of relativistic mechanics and is usable only when v is much less than c. In a similar way, **quantum mechanics is in agreement with classical physics where the energy differences between quantized levels vanish.** This principle, first set forth by Bohr, is called the **correspondence principle.**

For example, consider an electron orbiting the hydrogen atom with $n > 10\,000$. For such large values of n, the energy differences between adjacent levels approach zero and the levels are nearly continuous. As a consequence, the classical model is reasonably accurate in describing the system for large values of n. According to the classical picture, the frequency of the light emitted by the atom is equal to the frequency of revolution of the electron in its orbit about the nucleus. Calculations show that for $n > 10\,000$, this frequency differs from that predicted by quantum mechanics by less than 0.015%.

SUMMARY

Newton's universal law of gravity states that the gravitational force of attraction between any two particles of masses m_1 and m_2 separated by a distance r has the magnitude

$$F_g = G\frac{m_1 m_2}{r^2} \qquad [11.1]$$

where G is the gravitational constant, $6.672 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$.

Kepler's laws of planetary motion state the following:

1. Every planet moves in an elliptical orbit with the Sun at one of the focal points.
2. The radius vector drawn from the Sun to any planet sweeps out equal areas in equal time intervals.
3. The square of the orbital period of any planet is proportional to the cube of the semimajor axis for the elliptical orbit.

Kepler's second law is a consequence of the fact that the force of gravity is a *central force.* This implies that the angular momentum of the planet–Sun system is a constant of the motion.

Kepler's first and third laws are a consequence of the inverse-square nature of the universal law of gravity. Newton's second law, together with the force law given by Equation 11.1, verifies that the period, T, and radius, r, of the orbit of a planet about the Sun are related by

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 \qquad [11.4]$$

where M_s is the mass of the Sun. Most planets have nearly circular orbits about the Sun. For elliptical orbits, Equation 11.4 is valid if r is replaced by the semimajor axis, a.

If an isolated system consists of a particle of mass m moving with a speed of v in the vicinity of a massive body of mass M, the *total energy* of the system is constant and is

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r} \qquad [11.6]$$

If m moves in a circular orbit of radius r about M, where $M \gg m$, the total energy of the system is

$$E = -\frac{GMm}{2r} \qquad [11.9]$$

The total energy is negative for any bound system—that is, one in which the orbit is closed, such as a circular or an elliptical orbit.

The minimum speed an object must have to escape the gravitational field of a uniform sphere of mass M and radius R is

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \qquad [11.12]$$

The Bohr model of the atom is successful in describing the spectra of atomic hydrogen and hydrogen-like ions. One of the basic assumptions of the model is that the electron can exist only in discrete orbits such that the angular momentum mvr is an integral multiple of $h/2\pi = \hbar$. Assuming circular orbits and a simple coulombic attraction between the electron and proton, the energies of the quantum states for hydrogen are calculated to be

$$E_n = -\frac{k_e e^2}{2a_0}\left(\frac{1}{n^2}\right) \qquad [11.22]$$

where k_e is the Coulomb constant, e is electronic charge, $n = 1, 2, 3, \cdots$ is a positive integer called the **quantum number** of the state, and $a_0 = 0.0529$ nm is the **Bohr radius.**

If the electron in the hydrogen atom makes a transition from an orbit with a quantum number of n_i to one with a quantum number of n_f, where $n_f < n_i$, the frequency of the radiation emitted by the atom is

$$f = \frac{k_e e^2}{2a_0 h}\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \qquad [11.24]$$

Using $E = hf = hc/\lambda$, one can calculate the wavelengths of the radiation for various transitions in which there is a change in quantum number, $n_i \rightarrow n_f$. The calculated wavelengths are in excellent agreement with observed line spectra.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

CONCEPTUAL QUESTIONS

1. If the gravitational force on an object is directly proportional to its mass, why don't large masses fall with greater acceleration than small ones?
2. The gravitational force that the Sun exerts on the Moon is about twice as great as the gravitational force that the Earth exerts on the Moon. Why doesn't the Sun pull the Moon away from the Earth during a total eclipse of the Sun?
3. Explain why it takes more fuel for a spacecraft to travel from the Earth to the Moon than for the return trip. Estimate the difference.
4. Explain why there is no work done on a planet as it moves in a circular orbit around the Sun, even though a gravitational force is acting on the planet. What is the *net* work done on a planet during each revolution as it moves around the Sun in an elliptical orbit?
5. At what position in its elliptical orbit is the speed of a planet a maximum? At what position is the speed a minimum?
6. Why don't we put a communications satellite in orbit around the 45th parallel? Wouldn't this be more useful in the United States than one in orbit around the Equator?
7. In his 1798 experiment, Cavendish was said to have "weighed the Earth." Explain this statement.
8. If a hole could be dug to the center of the Earth, do you think that the force on a mass m would still obey Equation 11.1 there? What do you think the force on m would be at the center of the Earth?
9. The *Voyager* spacecraft was accelerated toward escape speed from the Sun by the gravitational force exerted on the spacecraft by Jupiter. How is this possible?
10. The *Apollo 13* spaceship developed trouble in the oxygen system about halfway to the Moon. Why did the mission continue on around the Moon and then return home rather than immediately turn back to Earth?
11. Discuss the similarities and differences between the classical description of planetary motion and the Bohr model of the hydrogen atom.
12. The Bohr theory of the hydrogen atom is based on several assumptions. Discuss those assumptions and their significance. Do any of them contradict classical physics?
13. Explain the significance behind the fact that the total energy of the atom in the Bohr model is negative.
14. Suppose that the electron in the hydrogen atom obeyed classical mechanics rather than quantum mechanics. Why should such a "hypothetical" atom emit a continuous spectrum rather than the observed line spectrum?

PROBLEMS

Section 11.1 Newton's Universal Law of Gravity Revisited

1. Which exerts a greater force of gravitational attraction on objects on the Earth: the Moon or the Sun? Calculate these forces on a 1.00-kg mass.
2. Two ocean liners, each with a mass of 40 000 metric tons, are moving on parallel courses, 100 m apart. What is the magnitude of the acceleration of one of the liners toward the other due to the mutual gravitational attraction? (Treat the ships as spheres.)
3. The gravitational field on the surface of the Moon is about one sixth that on the surface of the Earth. If the radius of the Moon is about one quarter that of the Earth, find the ratio of the average mass density of the Moon to the average mass density of the Earth.
4. A student proposes to measure the gravitational constant G by suspending two spherical masses from the ceiling of a tall cathedral and measuring the deflection of the cables from the vertical. Draw a free-body diagram for one of the masses. If two 100.0 kg masses are suspended at the end of 45.00 m long cables and the cables are attached to the ceiling 1.000 m apart, what is the separation of the masses?
5. On the way to the Moon the Apollo astronauts reach a point at which the Moon's gravitational pull becomes equal to that of Earth. Determine the distance of this point from the center of the Earth.

Section 11.2 Kepler's Laws

Section 11.3 The Universal Law of Gravity and the Motions of Planets

6. A satellite is in a circular orbit just above the surface of the Moon. (The radius of the Moon is 1738 km.) (a) What is the acceleration of the satellite? (b) What is the speed of the satellite? (c) What is the period of the satellite orbit?
7. The *Explorer VIII* satellite, placed into orbit November 3, 1960, to investigate the ionosphere, had the following orbit parameters: perigee 459 km and apogee 2289 km (both distances above the Earth's surface); period, 112.7 min. Find the ratio v_p/v_a.
8. Halley's Comet approaches the Sun to within 0.570 A.U., and its orbital period is 75.6 years. (A.U. is the abbreviation for astronomical unit, where 1 A.U. = 1.50×10^{11} m is the mean Earth–Sun distance.) How far from the Sun will Halley's Comet travel before it starts its return journey? (See Fig. P11.8.)

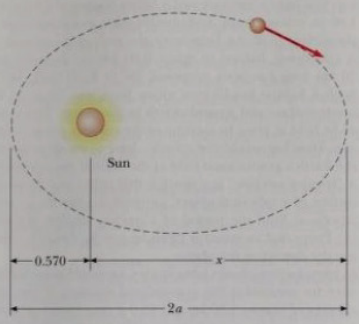


Figure P11.8

9. Io, a satellite of Jupiter, has an orbital period of 1.77 days and an orbital radius of 4.22×10^5 km. From these data, determine the mass of Jupiter.
10. Two planets X and Y travel counterclockwise in circular orbits about a star, as in Figure P11.10. The radii of their orbits is in the ratio 3:1. At some time, they are aligned as in Figure P11.10a, making a straight line with the star. Five years later, Planet X has rotated through 90° as in Figure P11.10b. Where is planet Y at this time?

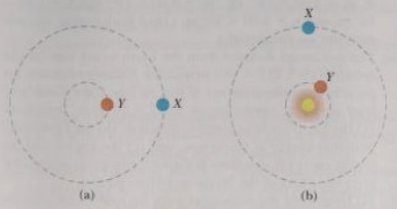


Figure P11.10

11. A synchronous satellite, which always remains above the same point on a planet's equator, is put in orbit around Jupiter to study the famous red spot. Jupiter rotates once every 9.84 h. Use the data of Table 11.2 to find the altitude of the satellite.

Section 11.4 Energy Considerations in Planetary and Satellite Motion

12. A satellite of mass 200 kg is placed in Earth orbit at a height of 200 km above the surface. (a) Assuming a circular orbit, how long does the satellite take to complete one orbit? (b) What is the satellite's speed? (c) What is the minimum energy necessary to place this satellite in orbit (assuming no air friction)?
13. A spaceship is fired from the Earth's surface with an initial speed 2.00×10^4 m/s. What will its speed be when it is very far from the Earth? (Neglect friction.)
14. A satellite of the Earth has a mass of 100 kg and is at an altitude of 2.00×10^6 m. (a) What is the potential energy of the satellite–Earth system? (b) What is the magnitude of the gravitational force exerted by the Earth on the satellite?
15. A 1000-kg satellite orbits the Earth at an altitude of 100 km. It is desired to increase the altitude of the circular orbit to 200 km. How much energy must be added to the system to effect this change in altitude?
16. A satellite of mass m orbits the Earth at an altitude h_1. It is desired to increase the altitude of the circular orbit to h_2. How much energy must be added to the system to effect this change in altitude?
17. A satellite moves in a circular orbit just above the surface of a planet. Show that the orbital speed v and escape speed of the satellite are related by the expression $v_{esc} = \sqrt{2}v$.
18. The planet Uranus has a mass about 14.0 times the Earth's mass, and its radius is equal to about 3.70 Earth radii. (a) By setting up ratios with the corresponding Earth values, find the acceleration due to gravity at the cloud tops of Uranus. (b) Ignoring the rotation of the planet, find the minimum escape speed from Uranus.
19. Determine the escape velocity for a rocket on the far side of Ganymede, the largest of Jupiter's moons. Ganymede's radius is 2.64×10^6 m, and its mass is 1.495×10^{23} kg. The mass of Jupiter is 1.90×10^{27} kg, and the distance between Jupiter and Ganymede is 1.071×10^9 m. Be sure to include the gravitational effect due to Jupiter, but you may ignore the motion of Jupiter and Ganymede as they revolve about their center of mass.
20. How much work is done by the Moon's gravitational field as a 1000 kg meteor comes in from outer space and impacts on the Moon's surface?
21. In Robert Heinlein's *The Moon Is a Harsh Mistress*, the colonial inhabitants of the Moon threaten to launch rocks down onto the Earth if they are not given independence (or at least representation). Assume that a rail gun can launch a rock at twice the speed that would be required to escape from an isolated stationary Moon. Calculate the speed of this rock as it enters the Earth's atmosphere.
22. A spacecraft in the shape of a long cylinder has a length of 100 m and its mass with occupants is 1000 kg. It has strayed in too close to a 1.0-m radius black hole having a mass 100 times that of the Sun (Fig. P11.22). If the nose of the space-

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IPR2023-00783

craft points toward the center of the black hole, and if distance between the nose of the spaceship and the black hole's center is 10.0 km, (a) determine the average acceleration of the spaceship. (b) What is the difference in the acceleration experienced by the occupants in the nose of the ship and those in the rear of the ship farthest from the black hole?



Figure P11.22

Section 11.5 Atomic Spectra and the Bohr Theory of Hydrogen

23. For a hydrogen atom in its ground state, use the Bohr model to compute (a) the orbital speed of the electron, (b) the kinetic energy of the electron, and (c) the electrical potential energy of the atom.
24. Radiation is emitted from a hydrogen atom, which undergoes a transition from the $n = 3$ state to the $n = 2$ state. Calculate (a) the energy, (b) the wavelength, and (c) the frequency of the emitted light.
25. A hydrogen atom is in its first excited state ($n = 2$). Using the Bohr theory of the atom, calculate (a) the radius of the orbit, (b) the linear momentum of the electron, (c) the angular momentum of the electron, and its (d) kinetic energy, (e) potential energy, and (f) total energy.
26. How much energy is required to ionize hydrogen (a) when it is in the ground state? (b) when it is in the state for which $n = 3$?
27. Show that the speed of the electron in the nth Bohr orbit in hydrogen is given by

$$v_n = \frac{k_e e^2}{n\hbar}$$

28. (a) Calculate the angular momentum of the Moon due to its orbital motion about the Earth. In your calculation, use 3.84×10^8 m as the average Earth–Moon distance and 2.36×10^6 s as the period of the Moon in its orbit. (b) Determine the corresponding quantum number. (c) By what fraction would the Earth–Moon distance have to be increased to increase the quantum number by 1?

Additional Problems

29. *Voyagers 1* and *2* surveyed the surface of Jupiter's moon Io and photographed active volcanoes spewing liquid sulfur to heights of 70 km above the surface of this moon. Estimate the speed with which the liquid sulfur left the volcano. Io's mass is 8.9×10^{22} kg and its radius is 1820 km.
30. You are an astronaut, and you observe a small planet to be spherical. After landing on the planet, you set off, walk straight ahead, and find yourself returning to your spacecraft from the opposite side after completing a lap of 25.0 km. You hold a hammer and a falcon feather at a height of 1.40 m, release them, and observe them to fall together to the surface in 29.2 s. Determine the mass of the planet.
31. A cylindrical habitat in space 6.00 km in diameter and 30 km long has been proposed (by G. K. O'Neill, 1974). Such a habitat would have cities, land, and lakes on the inside surface and air and clouds in the center. This would all be held in place by rotation of the cylinder about its long axis. How fast would the cylinder have to rotate to imitate the Earth's gravitational field at the walls of the cylinder?
32. A "treetop satellite" is a satellite that orbits just above the surface of a spherical object, assumed to offer no air resistance. (a) Find the period of a treetop satellite of Earth. (b) Prove that its speed is given by $v = R\sqrt{4\pi G\rho/3}$, where ρ is the density of the planet.
33. In introductory physics laboratories, a typical Cavendish balance for measuring the gravitational constant G uses lead spheres of masses 1.50 kg and 15.0 g the centers of which are separated by about 4.50 cm. Calculate the gravitational force between these spheres, treating each as a point mass located at the center of the sphere.
34. Show that the escape speed from the surface of a planet of uniform density is directly proportional to the radius of the planet.
35. Two hypothetical planets of masses m_1 and m_2 and radii r_1 and r_2, respectively, are at rest when they are an infinite distance apart. Because of their gravitational attraction, they head toward each other on a collision course. (a) When their center-to-center separation is d, find the speed of each planet and their *relative* velocity. (b) Find the kinetic energy of each planet *just* before they collide if $m_1 = 2.00 \times 10^{24}$ kg, $m_2 = 8.00 \times 10^{24}$ kg, $r_1 = 3.00 \times 10^6$ m, and $r_2 = 5.00 \times 10^6$ m. (*Hint:* Both energy and momentum are conserved.)
36. The maximum distance from the Earth to the Sun (at our aphelion) is 1.521×10^{11} m and the distance of closest approach (at perihelion) is 1.471×10^{11} m. If the Earth's orbital speed at perihelion is 3.027×10^4 m/s, determine (a) the Earth's orbital speed at aphelion, (b) the kinetic and potential energy at perihelion, and (c) the kinetic and potential energy at aphelion. Is the total energy constant? (Neglect the effect of the Moon and other planets.)
37. (a) Determine the amount of work (in joules) that must be done on a 100-kg payload to elevate it to a height of 1000 km above the Earth's surface. (b) Determine the amount of additional work that is required to put the payload into circular orbit at this elevation.
38. After a supernova explosion, a star may undergo a gravitational collapse to an extremely dense state known as a neutron star, in which all the electrons and protons are squeezed together to form neutrons. A neutron star having a mass about equal to that of the Sun would have a radius of about 10.0 km. Find (a) the free-fall acceleration at its surface, (b) the weight of a 70.0-kg person at its surface, and (c) the energy required to remove a neutron of mass 1.67×10^{-27} kg from its surface to infinity.
39. When it orbited the Moon, the *Apollo 11* spacecraft's mass was 9.979×10^3 kg, its period was 119 min, and its mean distance from the Moon's center was 1.849×10^6 m. Assuming its orbit to be circular and the Moon to be a uniform sphere, find (a) the mass of the Moon, (b) the orbital speed of the spacecraft, and (c) the minimum energy required for the craft to leave the orbit and escape the Moon's gravitational field.
40. Three point objects having masses m, $2m$, and $3m$, are fixed at the corners of a square of side length a such that the lighter object is at the upper left-hand corner, the heavier object is at the lower left-hand corner, and the remaining object is at the upper right-hand corner. Determine the magnitude and direction of the resulting gravitational acceleration at the center of the square.
41. X-ray pulses from Cygnus X-1, a celestial x-ray source, have been recorded during high-altitude rocket flights. The signals can be interpreted as originating when a blob of ionized matter orbits a black hole with a period of 5.00 ms. If the blob were in a circular orbit about a black hole the mass of which is $20.0M_{\text{Sun}}$, what is the orbital radius?
42. Studies of the relationship of the Sun to its galaxy—the Milky Way—have revealed that the Sun is located near the outer edge of the galactic disc, about 30 000 lightyears from the center. Furthermore, it has been found that the Sun has an orbital speed of approximately 250 km/s around the galactic center. (a) What is the period of the Sun's galactic motion? (b) What is the order of magnitude of the mass of the Milky Way galaxy? Supposing that the galaxy is made mostly of stars of which the Sun is typical, estimate the number of stars in the Milky Way.
43. Four possible transitions for a hydrogen atom are as follows:

(A) $n_i = 2$; $n_f = 5$ (B) $n_i = 5$; $n_f = 3$

(C) $n_i = 7$; $n_f = 4$ (D) $n_i = 4$; $n_f = 7$

(a) Which transition emits the shortest wavelength photon? (b) In which transition does the atom gain the most energy? (c) In which transition(s) does the atom lose energy?

44. *Vanguard I*, launched March 3, 1958, is the oldest human-made satellite still in orbit. Its initial orbit had an apogee of 3970 km and a perigee of 650 km. Its maximum speed was 8.23 km/s and it had a mass of 1.60 kg. (a) Determine the period of the orbit. (Use the semimajor axis.) (b) Determine the speeds at apogee and perigee. (c) Find the total energy of the satellite.
45. Two stars of masses M and m, separated by a distance d, revolve in circular orbits about their center of mass (Fig. P11.45). Show that each star has a period given by

$$T^2 = \frac{4\pi^2}{G(M+m)}d^3$$

(*Hint:* Apply Newton's second law to each star, and note that the center-of-mass condition requires that $Mr_2 = mr_1$, where $r_1 + r_2 = d$.)

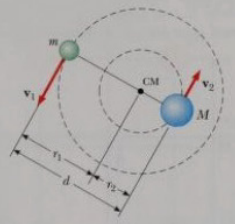


Figure P11.45

46. In 1978, astronomers at the U.S. Naval Observatory discovered that Pluto has a moon, called Charon, that eclipses the planet every 6.39 days. Given that the center-to-center separation between Pluto and Charon is 19 700 km, find the total mass ($M + m$) of the two bodies. (*Hint:* See Problem 45.)
47. Two hydrogen atoms collide head-on and end up with zero kinetic energy. Each then emits a 121.6-nm photon ($n = 2$ to $n = 1$ transition). At what speed were the atoms moving before the collision?
48. A rocket is given an initial speed of $v_0 = 2\sqrt{Rg}$ vertically upward at the surface of the Earth, which has radius R and surface free-fall acceleration g. The rocket motors are then cut off, and thereafter the rocket coasts under the action of gravitational forces only. (Ignore atmospheric friction and the Earth's rotation.) Derive an expression for the subsequent speed, v, as a function of the distance, r, from the center of the Earth in terms of g, R, and r.

Spreadsheet Problems

S1. Four point masses are fixed as shown in Figure PS11.1. The gravitational potential energy for a test particle of mass m moving along the x axis in the gravitational field of the fixed masses can be written as

$$U(x) = -\frac{GM_1 m}{r_1} - \frac{GM_2 m}{r_2} - \frac{GM_3 m}{r_3} - \frac{GM_4 m}{r_4}$$

where $r_2 = r_4 = [b^2 + (x + a)^2]^{1/2}$ and $r_2 = r_3 = [b^2 + (x - a)^2]^{1/2}$. Spreadsheet 11.1 calculates $U(x)$ and the force $F(x) = -dU(x)/dx$ exerted on the test particle. The quantities M_1, M_2, M_3, M_4, a, and b are input parameters.

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IPR2023-00783

The mass of the test particle is 1.00 kg. (This is the case of two particles on the y axis at $y = +1.00$ m and -1.00 m.) (a) Plot $U(x)$ and $F(x)$ versus x for $x = -4.00$ m to $+4.00$ m. (b) If the test particle has an energy of -7.50×10^{-11} J, describe the particle's motion. (c) How does the force on the particle vary as the particle moves along the x axis?

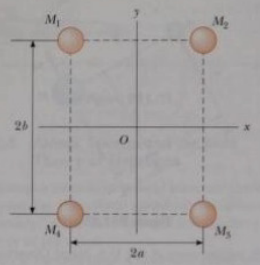


Figure PS11.1

S2. In Spreadsheet 11.1, let $a = 1.00$ m, $b = 0.50$ m, and $M_1 = M_2 = M_3 = M_4 = 1.00$ kg. The mass of the particle is 1.00 kg. (a) Plot $U(x)$ and $F(x)$ versus x. (b) If the test particle has an energy of -2.00×10^{-10} J, what motion is possible?

S3. If a projectile is fired straight up from the surface of the Earth with an initial speed v_0 that is less than the escape speed $v_{esc} = \sqrt{2gR_E} = 11.2$ km/s, the projectile will rise to a maximum height h above the Earth's surface and return. Neglecting air resistance and using conservation of energy, show that

$$h = \frac{R_E v_0^2}{2gR_E - v_0}$$

where R_E is the Earth's radius. Write a spreadsheet to calculate h for $v_0 = 1, 2, 3, \ldots, 11$ km/s. Plot h/R_E versus v_0. What happens as v_0 approaches the escape speed v_{esc}?

S4. The acceleration of an object moving in the gravitational field of the Earth is

$$\mathbf{a} = -GM_E \frac{\mathbf{r}}{r^3}$$

where $\mathbf{r}$ is the position vector directed from the center of the Earth to the object. Choosing the origin at the center of the Earth and assuming the object is moving in the xy plane, the acceleration has rectangular components

$$a_x = -\frac{GM_E x}{(x^2 + y^2)^{3/2}} \qquad a_y = -\frac{GM_E y}{(x^2 + y^2)^{3/2}}$$

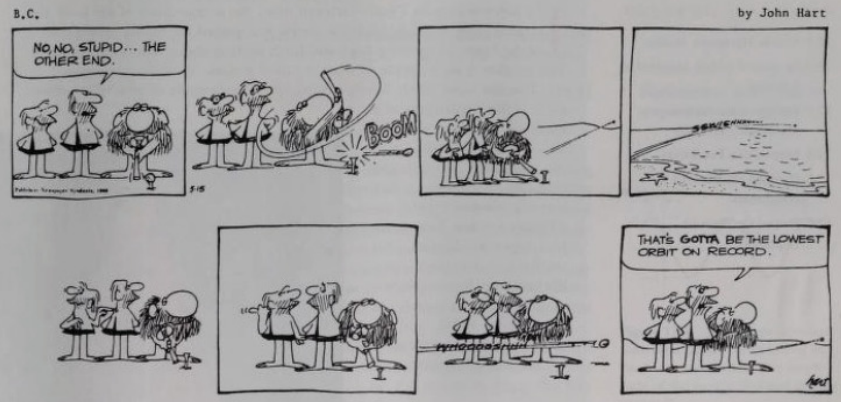
Write a spreadsheet (or computer program) to find the position of the object as a function of time. (Use the techniques developed in Spreadsheet 11.1 as a model for your own program. Assume that the initial position of the object is at $x = 0$ and $y = 2R_E$, where R_E is the radius of the Earth, and give the object an initial velocity of 5 km/s in the x direction. The time increment should be made as small as practical. Try 5 s. Plot the x and y coordinates of the object as functions of time. Does the object hit the Earth? Vary the initial velocity until a circular orbit is found.

S5. Modify your spreadsheet (or program) from Problem S4 to calculate the kinetic, potential, and total energies as functions of time. How do they vary with time for the various cases?

S6. Modify your spreadsheet (or program) from Problem S4 to calculate the angular momentum of the object. In rectangular coordinates, $L = m(xv_y - yv_x)$. How does the angular momentum vary with time? Kepler's second law of planetary motion implies that L remains constant. Is your calculated L constant?

ANSWERS TO CONCEPTUAL PROBLEMS

1. Kepler's second law is a consequence of conservation of angular momentum. Angular momentum is conserved as long as there is no torque, which is true for any central force, regardless of the particular dependence on separation. Thus, Kepler's second law would be unchanged. The particular mathematical form of Kepler's third law depends on the inverse square character of the force. If the force were an inverse cube, there would still be a proportional relationship between a power of the period and a power of the semimajor axis, but the powers would be different than in the existing law.
2. Air friction causes a decrease in the mechanical energy of the satellite–Earth system. This reduces the major axis of the orbit, bringing the satellite closer to the surface of the Earth. A satellite in a smaller orbit, however, must travel faster. Thus, the effect of the air friction is to speed up the satellite!
3. Kepler's third law (Eq. 11.4), which applies to all the planets, tells us that the period of a planet is proportional to $r^{3/2}$. Because Saturn and Jupiter are farther from the Sun than Earth, they have longer periods. The Sun's gravitational field (whose magnitude is GM_S/r^2) is much weaker at a distant Jovian planet. Thus, an outer planet experiences much smaller centripetal acceleration than Earth, and a correspondingly longer period.
4. In an emission spectrum, a gas is initially excited to higher quantum states. The emission spectrum is provided by subsequent downward transitions which can occur to all lower states. If the transition is not to the lowest state, the atom can make a subsequent transition from the intermediate state to the lowest state. For example, a transition can be made from $n = 10$ to $n = 5$, followed by a transition from $n = 5$ to $n = 1$. Thus, there is a rich mixture of spectral lines in the spectrum, corresponding to this large number of possible transitions. An absorption spectrum is normally performed with the gas at room temperature, at which almost all of the atoms are in the ground state. Thus, when radiation is absorbed, upward transitions involving *only the ground state* are possible. Thus, there are fewer transitions than for the emission spectrum and fewer lines.
5. In the discussion of the hydrogen atom, it can be seen that the kinetic energy is equal to half of the magnitude of the potential energy. Thus, we must have $K = E$ and $U_e = -2E$.



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IPR2023-00783

12

Oscillatory Motion

CHAPTER OUTLINE

12.1 Simple Harmonic Motion
12.2 Motion of a Mass Attached to a Spring
12.3 Energy of the Simple Harmonic Oscillator
12.4 Motion of a Pendulum
12.5 Damped Oscillations (Optional)
12.6 Forced Oscillations (Optional)

If a force acting on a body varies in time, the acceleration of the body also changes with time. If this force always acts toward the equilibrium position of the body, a repetitive back-and-forth motion about that position results. The motion is an example of what is called *periodic* or *oscillatory* motion.

You are most likely familiar with several examples of periodic motion, such as the oscillations of a mass on a spring, the motion of a pendulum, and the vibrations of a stringed musical instrument. Numerous systems exhibit oscillatory motion. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors; in alternating-current circuits, voltage, current, and electrical charge vary periodically with time.



Time exposure of a pendulum, which consists of a small sphere suspended by a light string. The time it takes the pendulum to undergo one complete oscillation is called the period of its motion, and the maximum displacement of the pendulum from the vertical position is called the *amplitude*. As we shall see in this chapter, the period of its motion for small amplitudes depends only on the length of the pendulum and the value of the free-fall acceleration. *(James Stevenson/SPL/Photo Researchers)*

Much of the material in this chapter deals with **simple harmonic motion.** This special kind of motion occurs when a restoring force acts on a body. In simple harmonic motion, an object oscillates between two spatial positions for an indefinite period of time. In real mechanical systems, retarding (frictional) forces are always present. Such forces reduce the mechanical energy of the system with time, and the oscillations are said to be *damped*. If an external driving force is applied, we call the motion a *forced oscillation*.

12.1 • SIMPLE HARMONIC MOTION

A particle moving along the x axis is said to exhibit **simple harmonic motion** when x, its displacement from equilibrium, varies in time according to the relationship

$$x = A\cos(\omega t + \phi) \qquad [12.1]$$

• *Displacement versus time for simple harmonic motion*

where A, ω, and ϕ are constants of the motion. In order to give physical significance to these constants, it is convenient to plot x as a function of t, as in Figure 12.1. First, we note that A, called the **amplitude** of the motion, is simply **the maximum displacement of the particle in either the positive or negative x direction.** The constant ω is called the **angular frequency** (defined in Eq. 12.4). The constant angle ϕ is called the **phase constant** (or phase angle) and, along with the amplitude A, is determined uniquely by the initial displacement and velocity of the particle. The constants ϕ and A tell us what the displacement and velocity were at time $t = 0$. The quantity $(\omega t + \phi)$ is called the **phase** of the motion and is useful for comparing the motions of two particles. Note that the function x is periodic and repeats itself each time ωt increases by 2π radians.

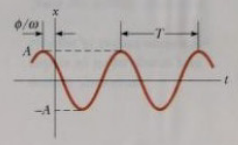


Figure 12.1 Displacement versus time for a particle undergoing simple harmonic motion. The amplitude of the motion is A and the period is T.

The **period,** T, of the motion is the time required for the particle to go through one full cycle of its motion. That is, the value of x at time t equals the value of x at time $t + T$. We can show that the period is given by $T = 2\pi/\omega$ by using the fact that the phase increases by 2π radians in a time of T:

$$\omega t + \phi + 2\pi = \omega(t + T) + \phi$$

Simplifying this expression, we see that $\omega T = 2\pi$, or

$$T = \frac{2\pi}{\omega} \qquad [12.2]$$

• *Period*

The inverse of the period is called the **frequency** of the motion, f. The frequency represents the **number of oscillations the particle makes per unit time:**

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad [12.3]$$

• *Frequency*

The units of f are cycles per second, or hertz (Hz). Rearranging Equation 12.3 gives

$$\omega = 2\pi f = \frac{2\pi}{T} \qquad [12.4]$$

• *Angular frequency*

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IPR2023-00783

The angular frequency ω has the units radians per second. We shall discuss the geometric significance of ω in Section 12.4.

We can obtain the velocity of a particle undergoing simple harmonic motion by differentiating Equation 12.1 with respect to time:

Velocity in simple harmonic motion

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \phi) \qquad [12.5]$$

The acceleration of the particle is dv/dt:

Acceleration in simple harmonic motion

$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi) \qquad [12.6]$$

Because $x = A\cos(\omega t + \phi)$, we can express Equation 12.6 in the form

$$a = -\omega^2 x \qquad [12.7]$$

From Equation 12.5 we see that, because the sine and cosine functions oscillate between ± 1, the extreme values of v are $\pm \omega A$. Likewise, Equation 12.6 tells us that the extreme values of the acceleration are $\pm \omega^2 A$. Therefore, the *maximum* values of the velocity and acceleration are

Maximum values of velocity and acceleration in simple harmonic motion

$$v_{max} = \omega A \qquad [12.8]$$

$$a_{max} = \omega^2 A \qquad [12.9]$$

Figure 12.2a plots displacement versus time for an arbitrary value of the phase constant. The velocity and acceleration-versus-time curves are illustrated in Figures 12.2b and 12.2c. They show that the phase of the velocity differs from the phase of the displacement by $\pi/2$ rad, or 90°. That is, when x is a maximum or a minimum,

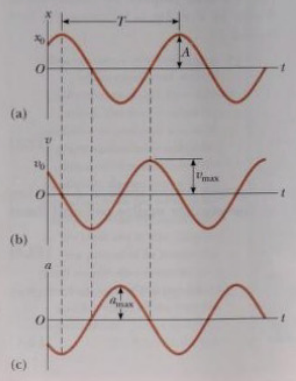


Figure 12.2 Graphical representation of simple harmonic motion: (a) displacement versus time, (b) velocity versus time, and (c) acceleration versus time. Note that at any specified time the velocity is 90° out of phase with the displacement and the acceleration is 180° out of phase with the displacement.

the velocity is zero. Likewise, when x is zero, the speed is a maximum. Furthermore, note that the phase of the acceleration differs from the phase of the displacement by π radians, or 180°. That is, when x is a maximum, a is a maximum in the opposite direction.

As we stated earlier, $x = A\cos(\omega t + \phi)$ is a general expression for the displacement of the particle from equilibrium, where the phase constant ϕ and the amplitude A must be chosen to meet the initial conditions of the motion. Suppose that the initial position, x_0, and initial velocity, v_0, of a single oscillator of known angular frequency are given; that is, at $t = 0$, $x = x_0$ and $v = v_0$. Under these conditions, Equations 12.1 and 12.5 give

$$x_0 = A\cos\phi \quad \text{and} \quad v_0 = -\omega A \sin\phi \qquad [12.10]$$

Dividing these two equations eliminates A, giving $\frac{v_0}{x_0} = -\omega\tan\phi$, or

$$\tan\phi = -\frac{v_0}{\omega x_0} \qquad [12.11]$$

Furthermore, if we take the sum $x_0^2 + (v_0/\omega)^2 = A^2\cos^2\phi + A^2\sin^2\phi$ (where we have used Eq. 12.10) and solve for A, we find that

The phase angle ϕ and amplitude A can be obtained from the initial conditions.

$$A = \sqrt{x_0^2 + \left(\frac{v_0}{\omega}\right)^2} \qquad [12.12]$$

Thus, we see that ϕ and A are known if x_0, ω, and v_0 are specified. We shall treat a few specific cases in the next section.

We conclude this section by pointing out the following important properties of a particle moving in simple harmonic motion:

Properties of simple harmonic motion

- The displacement, velocity, and acceleration all vary sinusoidally with time but are not in phase, as shown in Figure 12.2.
- The acceleration is proportional to the displacement, but in the opposite direction.
- The frequency and the period of motion are independent of the amplitude.

Example 12.1 An Oscillating Body

A body oscillates with simple harmonic motion along the x axis. Its displacement varies with time according to the equation

$$x = (4.00\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

where t is in seconds and the angles in the parentheses are in radians.

(a) Determine the amplitude, frequency, and period of the motion.

Solution By comparing this equation with the general equation for simple harmonic motion, $x = A\cos(\omega t + \phi)$, we see that $A = 4.00$ m and $\omega = \pi$ rad/s; therefore we find $f = \omega/2\pi = \pi/2\pi = 0.500\text{ s}^{-1}$ and $T = 1/f = 2.00$ s.

(b) Calculate the velocity and acceleration of the body at any time t.

Solution

$$v = \frac{dx}{dt} = -4.00\sin\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi\text{ m/s})\sin\left(\pi t + \frac{\pi}{4}\right)$$

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$$a = \frac{dv}{dt} = -4.00\pi \cos\left(\pi t + \frac{\pi}{4}\right)\frac{d}{dt}(\pi t)$$

$$= -(4.00\pi^2 \text{ m/s}^2)\cos\left(\pi t + \frac{\pi}{4}\right)$$

EXERCISE 1 Using the results to part (b), determine the position, velocity, and acceleration of the body at $t = 1.00$ s.
Answer $x = -2.83$ m; $v = 8.89$ m/s; $a = 27.9$ m/s^2

EXERCISE 2 Determine the maximum speed and the maximum acceleration of the body. Answer From the general expressions for v and a found in part (b), we see that the maximum values of the sine and cosine functions are unity. Therefore, v varies between $\pm 4.00\pi$ m/s, and a varies between $\pm 4.00\pi^2$ m/s^2. Thus, $v_{max} = 4.00\pi$ m/s and $a_{max} = 4.00\pi^2$ m/s^2.

EXERCISE 3 A particle moving with simple harmonic motion travels a total distance of 20.0 cm in each cycle of its motion, and its maximum acceleration is 50.0 m/s^2. Find (a) the angular frequency of the motion and (b) the maximum speed of the particle.
Answer (a) 31.6 rad/s (b) 1.58 m/s

12.2 • MOTION OF A MASS ATTACHED TO A SPRING

Consider a physical system consisting of a mass, m, attached to the end of a spring where the mass is free to move on a horizontal track (Fig. 12.3). We know from experience that such a system oscillates back and forth if disturbed from the equilibrium position $x = 0$, where the spring is unstretched. If the surface is frictionless, the mass can exhibit simple harmonic motion. One experimental arrangement that clearly demonstrates that such a system exhibits simple harmonic motion is illustrated in Figure 12.4. A mass oscillating vertically on a spring has a marking pen attached to it. While the mass is in motion, a sheet of paper is moved horizontally as shown, and the marking pen traces out a sinusoidal pattern. We can understand this qualitatively by first recalling that when the mass is displaced a small distance, x, from equilibrium, the spring exerts a force on it, given by **Hooke's law,**

$$F_s = -kx \qquad [12.13]$$

where k is the force constant of the spring. We call this a **linear restoring force** because it is linearly proportional to the displacement and always directed toward the equilibrium position *opposite* the displacement. That is, when the mass is displaced to the right in Figure 12.3, x is positive and the restoring force is to the left. When the mass is displaced to the left of $x = 0$, then x is negative and the restoring force is to the right.

If we apply Newton's second law to the motion of the mass in the x direction, we get

$$F_x = -kx = ma$$

$$a = -\frac{k}{m}x \qquad [12.14]$$

That is, just as we learned in Section 12.1, **the acceleration is proportional to the displacement of the mass from equilibrium and is in the opposite direction.** If the mass is displaced a maximum distance, $x = A$, at some initial time and released from rest, its *initial* acceleration is $-kA/m$ (that is, the acceleration has its extreme negative value). When the mass passes through the equilibrium position, $x = 0$ and its acceleration is zero. At this instant, its speed is a maximum. It then continues to travel to the left of equilibrium and finally reaches $x = -A$, at which time its acceleration is kA/m (maximum positive) and its speed is again zero. Thus, we see that the mass oscillates between the turning points $x = \pm A$. In one full cycle of its motion it travels a distance of $4A$.

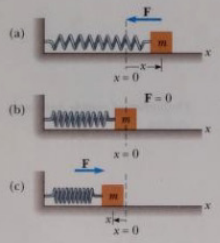


Figure 12.3 A mass attached to a spring on a frictionless track moves in simple harmonic motion. (a) When the mass is displaced to the right of equilibrium, the displacement is positive and the acceleration is negative. (b) At the equilibrium position, $x = 0$, the acceleration is zero but the speed is a maximum. (c) When the displacement is negative, the acceleration is positive.

Let us now describe this motion in a quantitative fashion. Recall that, by definition, $a = dv/dt = d^2x/dt^2$, and so we can express Equation 12.14 as

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \qquad [12.15]$$

If we denote the ratio k/m with the symbol ω^2,

$$\omega^2 = \frac{k}{m} \qquad [12.16]$$

then Equation 12.15 can be written in the form

$$\frac{d^2x}{dt^2} = -\omega^2 x \qquad [12.17]$$

What we now require is a solution to Equation 12.17—that is, a function $x(t)$ that satisfies this second-order differential equation. Because Equations 12.17 and 12.7 are equivalent, we see that the solution must be that of simple harmonic motion:

$$x(t) = A\cos(\omega t + \phi)$$

To see this explicitly, note that if $x = A\cos(\omega t + \phi)$, then

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \phi) = -\omega A \sin(\omega t + \phi)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \phi) = -\omega^2 A\cos(\omega t + \phi)$$

Comparing the expressions for x and d^2x/dt^2, we see that $d^2x/dt^2 = -\omega^2 x$ and Equation 12.17 is satisfied.

The following general statement can be made based on the foregoing discussion:

> Whenever the force acting on a particle is linearly proportional to the displacement and in the opposite direction, the particle exhibits simple harmonic motion.

We shall give additional physical examples in subsequent sections.

Because the period of simple harmonic motion is $T = 2\pi/\omega$ and the frequency is the inverse of the period, we can express the period and frequency of the motion for this mass–spring system as

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad [12.18]$$

• *Period of a mass–spring system*

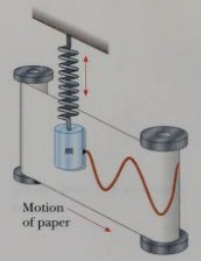


Figure 12.4 An experimental apparatus for demonstrating simple harmonic motion. A pen attached to the oscillating mass traces out a sine wave on the moving chart paper.

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IPR2023-00783

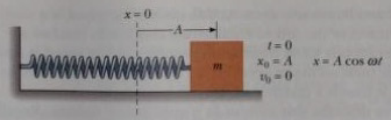


Figure 12.5 A mass–spring system that starts from rest at $x_0 = A$. In this case, $\phi = 0$, and so $x = A \cos \omega t$.

Frequency of a mass–spring system

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{\omega}{2\pi} \qquad [12.19]$$

That is, the period and frequency depend *only* on the mass and on the force constant of the spring. As we might expect, the frequency is larger for a stiffer spring (larger value of k) and decreases with increasing mass.

Special Case I In order to better understand the physical significance of our solution of the equation of motion, let us consider the following special case. Suppose we pull the mass from equilibrium by a distance of A and release it from rest in this stretched position, as in Figure 12.5. We must then require that our solution for $x(t)$ obey the initial conditions that at $t = 0$, $x_0 = A$ and $v_0 = 0$. These conditions are met if we choose $\phi = 0$, giving $x = A \cos \omega t$ as our solution. To check this solution, we note that it satisfies the condition that $x_0 = A$ at $t = 0$, because $\cos 0 = 1$. Thus, we see that A and ϕ contain the information on initial conditions.

Now let us investigate the behavior of the velocity and acceleration in this special case. Because $x = A \cos \omega t$, we have

$$v = \frac{dx}{dt} = -\omega A \sin \omega t \qquad \text{and} \qquad a = \frac{dv}{dt} = -\omega^2 A \cos \omega t$$

From the preceding velocity expression we see that at $t = 0$, $v_0 = 0$, as we require. The expression for the acceleration tells us that at $t = 0$, $a = -\omega^2 A$. Physically this makes sense, because the force on the mass is to the left when the displacement is positive. In fact, at this position $F_x = -kA$ (to the left), and the initial acceleration is $-kA/m$.

We could also use a more formal approach to show that $x = A \cos \omega t$ is the correct solution by using the relation $\tan \phi = -v_0/\omega x_0$ (Eq. 12.10). Because $v_0 = 0$ at $t = 0$, $\tan \phi = 0$ and so $\phi = 0$.

The displacement, velocity, and acceleration versus time are plotted in Figure 12.6 for this special case. Note that the acceleration reaches extreme values of $\pm \omega^2 A$ when the displacement has extreme values of $\mp A$. Furthermore, the velocity has extreme values of $\pm \omega A$, which both occur at $x = 0$. Hence, the quantitative solution agrees with our qualitative description of this system.

Special Case II Now suppose that the mass is given an initial velocity of $\mathbf{v}_0$ to the *right* at the unstretched position of the spring, so that at $t = 0$, $x_0 = 0$ and $v = v_0$ (Fig. 12.7). Our particular solution must now satisfy these initial conditions.

Applying Equation 12.11, $\tan \phi = -v_0/\omega x_0$, and the initial condition that $x_0 = 0$ at $t = 0$ gives $\tan \phi = -\infty$ or $\phi = -\pi/2$. Hence, the solution is given by $x = A \cos(\omega t - \pi/2)$, which can be written $x = A \sin \omega t$. Furthermore, from Equation 12.11 we see that $A = v_0/\omega$; therefore, we can express our solution as

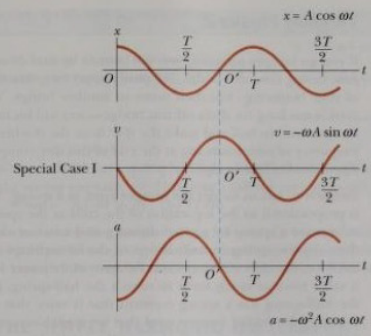


Figure 12.6 Displacement, velocity, and acceleration versus time for a particle undergoing simple harmonic motion under the initial conditions that at $t = 0$, $x = A$, and $v = 0$.

$$x = \frac{v_0}{\omega} \sin \omega t$$

The velocity and acceleration in this case are

$$v = \frac{dx}{dt} = v_0 \cos \omega t \qquad \text{and} \qquad a = \frac{dv}{dt} = -\omega v_0 \sin \omega t$$

This is consistent with the fact that the mass always has its maximum speed at $x = 0$, and the force and acceleration are zero at that position. The graphs of these functions versus time in Figure 12.6 correspond to the origin at O'. What would be the solution for x if the mass were initially moving to the left in Figure 12.7?

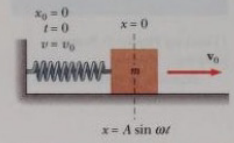


Figure 12.7 The mass–spring system starts its motion at the equilibrium position, $x = 0$ at $t = 0$. If its initial velocity is v_0 to the right, its x coordinate varies as $x = \frac{v_0}{\omega} \sin \omega t$.

Thinking Physics 1

We know that the period of oscillation of a mass hung on a spring is proportional to the square root of the mass. Thus, if we perform an experiment in which we hang a range of masses on the end of a spring and measure the period of oscillation, a graph of the square of the period against the mass will result in a straight line, as suggested in Figure TP12.1. But we find that the line does not go through the origin. Why not?

Reasoning The reason that the line does not go through the origin is that the spring itself has mass. Thus, the resistance to changes in motion of the system is a combination of the mass hung on the end of the spring and the mass of the oscillating coils of the spring. The entire mass of the spring is not oscillating, however. The bottom-most coil is oscillating over the same amplitude as the mass, and the top-most coil is not oscillating at all. For a cylindrical spring, energy arguments can be used to show that the effective additional mass representing the oscillations of the spring is one third of the mass of the spring. The square of the period is proportional to the total oscillating mass, but the graph in the diagram shows the square of the period against only the mass hung on the spring. A graph of period squared against total mass (mass hung on the spring plus the effective oscillating mass of the spring) would pass through the origin.

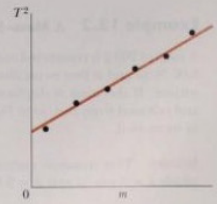


Figure TP12.1 (Thinking Physics 1)

Petitioner Intel Corp., Ex. 1038
IPR2023-00783



(Thinking Physics 2) Bungee jumping. *(Gamma)*

Thinking Physics 2

If you go bungee jumping, you will bounce up and down at the end of the cord after your daring dive off a bridge. Suppose you perform this dive and measure the frequency of your bouncing. You then move to another bridge. You discover that the bungee cord is too long for dives off this bridge—you will hit the ground. Thus, you fold the bungee cord in half and make the dive from the doubled bungee cord. How does the frequency of your bouncing at the end of this dive compare to the frequency after the dive from the first bridge?

Reasoning Let us model the bungee cord as a spring. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant that is twice that of the complete spring. Now consider the folded bungee cord that we model as two half-springs in parallel. Each half has a spring constant that is twice the original spring constant of the bungee cord. In addition, a given mass hanging on the folded bungee cord will experience two forces—one from each half-spring. As a result, the required force for a given extension will be four times as much as for the original bungee cord. The effective spring constant of the folded bungee cord is, therefore, four times as large as the original spring constant. Because the frequency of oscillation is proportional to the square root of the spring constant, your bouncing frequency on the folded cord will be twice that of the original cord.

CONCEPTUAL PROBLEM 1

Is a bouncing ball an example of simple harmonic motion? Is the daily movement of a student from home to school and back simple harmonic motion?

CONCEPTUAL PROBLEM 2

A mass is hung on a spring and the frequency of oscillation of the system, f, is measured. The mass, a second identical mass, and the spring are carried in the Space Shuttle to space. The two masses are attached to the ends of the spring and the system is taken out into space on a space walk. The spring is extended and the system is released to oscillate while floating in space. What is the frequency of oscillation for this system, in terms of f?

Example 12.2 A Mass–Spring System

A mass of 200 g is connected to a light spring of force constant 5.00 N/m and is free to oscillate on a horizontal, frictionless surface. If the mass is displaced 5.00 cm from equilibrium and released from rest, as in Figure 12.5, (a) find the period of its motion.

Solution This situation corresponds to Special Case I, in which $x = A\cos\omega t$ and $A = 5.00 \times 10^{-2}$ m. Therefore,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5.00\ \text{N/m}}{200 \times 10^{-3}\ \text{kg}}} = 5.00\ \text{rad/s}$$

and

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5.00} = 1.26\ \text{s}$$

(b) Determine the maximum speed and maximum acceleration of the mass.

Solution

$$v_{\max} = \omega A = (5.00\ \text{rad/s})(5.00 \times 10^{-2}\ \text{m}) = 0.250\ \text{m/s}$$

$$a_{\max} = \omega^2 A = (5.00\ \text{rad/s})^2(5.00 \times 10^{-2}\ \text{m}) = 1.25\ \text{m/s}^2$$

(c) Express the displacement, speed, and acceleration as functions of time.

Solution The expression $x = A\cos\omega t$ is our solution for Special Case I, and so we can use the results from (a), (b), and (c) to get

$$x = A\cos\omega t = (0.0500\ \text{m})\cos 5.00t$$

$$v = -\omega A\sin\omega t = -(0.250\ \text{m/s})\sin 5.00t$$

$$a = -\omega^2 A\cos\omega t = -(1.25\ \text{m/s}^2)\cos 5.00t$$

EXERCISE 4 (a) A 400-g mass is suspended from a spring hanging vertically, and the spring is found to stretch 8.0 cm. Find the spring constant. (b) How much will the spring stretch if the suspended mass is 575 g? Answer (a) 49 N/m (b) 12 cm

EXERCISE 5 A 3.0-kg mass is attached to a spring and pulled out horizontally to a maximum displacement from equilibrium of 0.50 m. What spring constant must the spring have if the mass is to achieve an acceleration equal to the free-fall acceleration? Answer 59 N/m

12.3 • ENERGY OF THE SIMPLE HARMONIC OSCILLATOR

Let us examine the mechanical energy of the mass–spring described in Figure 12.5. Because the surface is frictionless, we expect that the total mechanical energy is constant. We can use Equation 12.5 to express the kinetic energy as

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 A^2\sin^2(\omega t + \phi) \qquad [12.20]$$

• *Kinetic energy of a simple harmonic oscillator*

The elastic potential energy stored in the spring for any elongation x is $\frac{1}{2}kx^2$. Using Equation 12.1, we get

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2\cos^2(\omega t + \phi) \qquad [12.21]$$

• *Potential energy of a simple harmonic oscillator*

We see that K and U are always positive quantities. Because $\omega^2 = k/m$, we can express the *total energy* of the simple harmonic oscillator as

$$E = K + U = \tfrac{1}{2}kA^2[\sin^2(\omega t + \phi) + \cos^2(\omega t + \phi)]$$

But $\sin^2\theta + \cos^2\theta = 1$; therefore, this equation reduces to

$$E = \tfrac{1}{2}kA^2 \qquad [12.22]$$

• *Total energy of a simple harmonic oscillator*

That is, the energy of a simple harmonic oscillator is a constant of the motion and proportional to the square of the amplitude. In fact, the total mechanical energy is just equal to the maximum potential energy stored in the spring when $x = \pm A$. At these points, $v = 0$ and there is no kinetic energy. At the equilibrium position, $x = 0$ and $U = 0$, so that the total energy is all in the form of kinetic energy. That is, at $x = 0$, $E = \frac{1}{2}mv_{\max}^2 = \frac{1}{2}m\omega^2 A^2$.

Plots of the kinetic and potential energies versus time are shown in Figure 12.8a, where $\phi = 0$. In this situation, both K and U are always positive, and their sum at all times is a constant equal to $\frac{1}{2}kA^2$, the total energy of the system. The variations of K and U with displacement are plotted in Figure 12.8b. Energy is continuously being transferred between potential energy stored in the spring and the kinetic energy of the mass. Figure 12.9 illustrates the position, velocity, acceleration, kinetic energy, and potential energy of the mass–spring system for one full period of the motion. Most of the ideas discussed so far are incorporated in this important figure. We suggest that you study it carefully.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

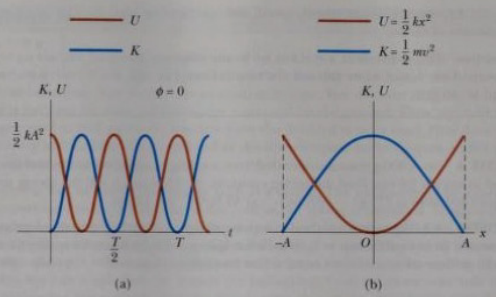


Figure 12.8 (a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with $\phi = 0$. (b) Kinetic energy and potential energy versus displacement for a simple harmonic oscillator. In either plot, note that $K + U =$ constant.

Finally, we can use energy to obtain the velocity for an arbitrary displacement, x, expressing the total energy at some arbitrary position as

$$E = K + U = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2$$

Velocity as a function of position for a simple harmonic oscillator

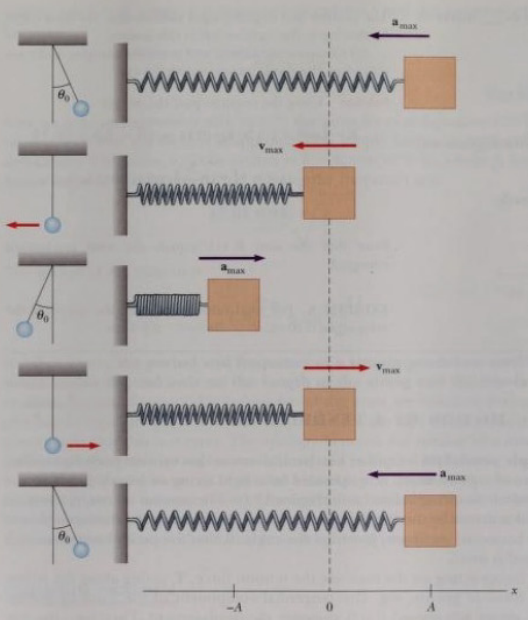
$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)} = \pm\omega\sqrt{A^2 - x^2} \qquad [12.23]$$

Again, this expression substantiates the fact that the speed is a maximum at $x = 0$ and zero at the turning points, $x = \pm A$.

Thinking Physics 3

A mass oscillating on the end of a horizontal spring slides back and forth over a frictionless surface. During one oscillation, you set an identical mass with Velcro (a sticky fabric) attached at the maximum displacement point. Just as the oscillating mass reaches its largest displacement and is momentarily at rest, it adheres to the new mass by means of the Velcro, and the two masses continue the oscillation together. Does the period of the oscillation change? Does the amplitude of oscillation change? Does the energy of the oscillation change?

Reasoning The period of oscillation *does* change, because the period depends on the mass that is oscillating. The amplitude does not change. Because the new mass was added while the original mass was at rest, the combined masses are at rest at this point, also, defining the amplitude as the same as in the original oscillation. The energy does not change, either. At the maximum displacement point, the energy is all potential energy stored in the spring, which depends only on the spring constant and the amplitude, not the mass. The increased mass will pass through the equilibrium point with less velocity than in the original oscillation, but with the same kinetic energy. Another approach is to think about how energy could be transferred into the oscillating system—no work was done (nor was there any other form of energy transfer), so the energy in the system cannot change.



t	x	v	a	K	U
0	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$
T/4	0	$-\omega A$	0	$\frac{1}{2}kA^2$	0
T/2	-A	0	$\omega^2 A$	0	$\frac{1}{2}kA^2$
3T/4	0	ωA	0	$\frac{1}{2}kA^2$	0
T	A	0	$-\omega^2 A$	0	$\frac{1}{2}kA^2$

Figure 12.9 Simple harmonic motion for a mass–spring system and its analogy to the motion of a simple pendulum. The parameters in the table at the right refer to the mass–spring system, assuming that at $t = 0$, $x = A$ so that $x = A\cos\omega t$ (Special Case I).

CONCEPTUAL PROBLEM 3

A mass–spring system undergoes simple harmonic motion with an amplitude A. Does the total energy change if the mass is doubled but the amplitude is not changed? Are the kinetic and potential energies at a given point in its motion affected by the change in mass? Explain.

Example 12.3 Oscillations on a Horizontal Surface

A 0.50-kg mass connected to a light spring of force constant 20.0 N/m oscillates on a horizontal, frictionless track. (a) Calculate the total energy of the system and the maximum speed of the mass if the amplitude of the motion is 3.0 cm.

Solution Using Equation 12.22, we get

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}\left(20.0\,\frac{\text{N}}{\text{m}}\right)(3.0 \times 10^{-2}\ \text{m})^2 = 9.0 \times 10^{-3}\ \text{J}$$

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

When the mass is at $x = 0$, $U = 0$ and $E = \frac{1}{2}mv_{max}^2$; therefore

$$\tfrac{1}{2}mv_{max}^2 = 9.0 \times 10^{-3}\ \text{J}$$

$$v_{max} = \sqrt{\frac{18.0 \times 10^{-3}\ \text{J}}{0.50\ \text{kg}}} = 0.19\ \text{m/s}$$

(b) What is the velocity of the mass when the displacement is equal to 2.0 cm?

Solution We can apply Equation 12.23 directly:

$$v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}$$

$$= \pm\sqrt{\frac{20.0}{0.50}(3.0^2 - 2.0^2) \times 10^{-4}}$$

$$= \pm 0.14\ \text{m/s}$$

The positive and negative signs indicate that the mass could be moving to the right or left at this instant.

(c) Compute the kinetic and potential energies of the system when the displacement equals 2.0 cm.

Solution Using the result to part (b), we get

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.50\ \text{kg})(0.14\ \text{m/s})^2 = 5.0 \times 10^{-3}\ \text{J}$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(20.0\ \frac{\text{N}}{\text{m}}\right)(2.0 \times 10^{-2}\ \text{m})^2$$

$$= 4.0 \times 10^{-3}\ \text{J}$$

Note that the sum $K + U$ equals the total mechanical energy, E.

EXERCISE 6 For what values of x does the speed of the mass equal 0.10 m/s? Answer ± 2.6 cm

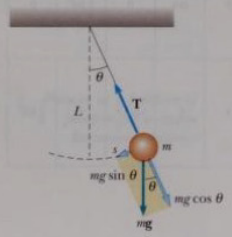


Figure 12.10 When θ is small, the simple pendulum oscillates in simple harmonic motion about the equilibrium position ($\theta = 0$). The restoring force is $mg \sin \theta$, the component of the force of gravity tangent to the circle.

12.4 • MOTION OF A PENDULUM

The **simple pendulum** is another mechanical system that exhibits periodic motion. It consists of a point mass, m, suspended by a light string of length L, where the upper end of the string is fixed as in Figure 12.10. The motion occurs in a vertical plane and is driven by the force of gravity. We shall show that the motion is that of a simple harmonic oscillator, provided the angle, θ, that the pendulum makes with the vertical is small.

The forces acting on the mass are the tension force, **T**, acting along the string, and the force of gravity, $m\mathbf{g}$. The tangential component of the force of gravity, $mg \sin \theta$, always acts toward $\theta = 0$, opposite the displacement. Therefore, the tangential force is a restoring force, and we can use Newton's second law to write the equation of motion in the tangential direction as

$$F_t = -mg \sin \theta = m\frac{d^2s}{dt^2}$$

where s is the displacement measured along the arc in Figure 12.10 and the minus sign indicates that F_t acts toward the equilibrium position. Because $s = L\theta$ and L is constant, this equation reduces to

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin \theta$$

The right side is proportional to $\sin \theta$ rather than to θ; hence, we conclude that the motion is not simple harmonic motion, because it is not of the form of Equation 12.17. However, if we assume that θ is *small*, we can use the approximation $\sin \theta \approx \theta$, where θ is measured in radians,[1] and the equation of motion becomes

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\theta \qquad [12.24]$$

• ***Equation of motion for the simple pendulum (small θ)***

Now we have an expression with exactly the same form as Equation 12.17, and so we conclude that the motion is approximately simple harmonic motion for small amplitudes. Therefore, θ can be written as $\theta = \theta_0 \cos(\omega t + \phi)$, where θ_0 is the **maximum angular displacement** and the angular frequency ω is

$$\omega = \sqrt{\frac{g}{L}} \qquad [12.25]$$

• ***Angular frequency of motion for a simple pendulum***

The period of the motion is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}} \qquad [12.26]$$

• ***Period of motion for a simple pendulum***

In other words, **the period and frequency of a simple pendulum oscillating at small angles depend only on the length of the string and the free-fall acceleration.** Because the period is *independent* of the mass, we conclude that *all* simple pendula of equal length at the same location oscillate with equal periods. Experiment shows that this is correct. The analogy between the motion of a simple pendulum and the mass–spring system is illustrated in Figure 12.9.

The simple pendulum can be used as a timekeeper. It is also a convenient device for making precise measurements of the free-fall acceleration. Such measurements are important, because variations in local values of $\mathbf{g}$ can provide information on the locations of oil and other valuable underground resources.

CONCEPTUAL PROBLEM 4

A simple pendulum is suspended from the ceiling of a stationary elevator, and the period is determined. Describe the changes, if any, in the period when the elevator (a) accelerates upward, (b) accelerates downward, and (c) moves with constant velocity.

CONCEPTUAL PROBLEM 5

Imagine that a pendulum is hanging from the ceiling of a car. As the car coasts freely down a hill, is the equilibrium position of the pendulum vertical? Does the period of oscillation change from that in a stationary car?

CONCEPTUAL PROBLEM 6

A grandfather clock depends on the period of a pendulum to keep correct time. Suppose a grandfather clock is calibrated correctly and then the temperature of the room in which it resides increases. Does the grandfather clock run slow, fast, or correctly? *Hint:* A metal expands when its temperature is raised.

[1] See Appendix A for more details on the small-angle approximation.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

Example 12.4 A Measure of Height

A man enters a tall tower, needing to know its height. He notes that a long pendulum extends from the ceiling almost to the floor and that its period is 12.0 s. How tall is the tower?

Solution If we use $T = 2\pi\sqrt{L/g}$ and solve for L, we get

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(12.0\ \text{s})^2}{4\pi^2} = 35.7\ \text{m}$$

EXERCISE 7 If the pendulum described in this example is taken to the moon, where the free-fall acceleration is 1.67 m/s², what is the period there? Answer 29.1 s

OPTIONAL

The Physical Pendulum

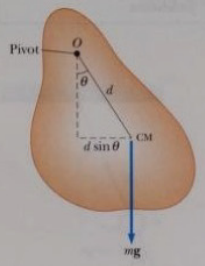


Figure 12.11 The physical pendulum consists of a rigid body pivoted at the point O, which is not at the center of mass. At equilibrium, the weight vector passes through O, corresponding to $\theta = 0$. The restoring torque about O when the system is displaced through an angle θ is $mgd \sin\theta$.

If a hanging object oscillates about a fixed axis that does not pass through its center of mass, and the object cannot be accurately approximated as a point mass, then it must be treated as a physical, or compound, pendulum. Consider a rigid body pivoted at a point O that is a distance of d from the center of mass (Fig. 12.11). The torque about O is provided by the force of gravity, and its magnitude is $mgd \sin\theta$. Using the fact that $\tau = I\alpha$, where I is the moment of inertia about the axis through O, we get

$$-mgd \sin\theta = I\frac{d^2\theta}{dt^2}$$

The minus sign on the left indicates that the torque about O tends to decrease θ. That is, the force of gravity produces a restoring torque.

If we again assume that θ is small, then the approximation $\sin\theta \approx \theta$ is valid and the equation of motion reduces to

$$\frac{d^2\theta}{dt^2} = -\left(\frac{mgd}{I}\right)\theta = -\omega^2\theta \qquad [12.27]$$

Note that the equation has the same form as Equation 12.17, and so the motion is approximately simple harmonic motion for small amplitudes. That is, the solution of Equation 12.27 is $\theta = \theta_0 \cos(\omega t + \phi)$, where θ_0 is the maximum angular displacement and

$$\omega = \sqrt{\frac{mgd}{I}}$$

The period is

Period of motion for a physical pendulum

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}} \qquad [12.28]$$

One can use this result to measure the moment of inertia of a planar rigid body. If the location of the center of mass and, hence, the distance d are known, the moment of inertia can be obtained through a measurement of the period. Finally, note that Equation 12.28 reduces to the period of a simple pendulum (Eq. 12.26) when $I = md^2$—that is, when all the mass is concentrated at the center of mass.

CONCEPTUAL PROBLEM 7

Two students are watching the swinging of a Foucault pendulum with a large bob in a museum. One student says, "I'm going to sneak past the fence and stick some chewing gum on the top of the pendulum bob, to change its frequency of oscillation." The other student says, "That won't change the frequency—the frequency of a pendulum is independent of mass." Which student is correct?

Example 12.5 A Swinging Rod

A uniform rod of mass M and length L is pivoted about one end and oscillates in a vertical plane (Fig. 12.12). Find the period of oscillation if the amplitude of the motion is small.

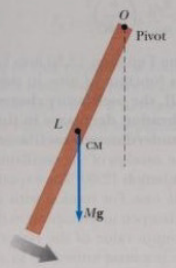


Figure 12.12 (Example 12.5) A rigid rod oscillating about a pivot through one end is a physical pendulum with $d = L/2$ and from Table 10.2, $I_0 = \frac{1}{3}ML^2$.

Solution The moment of inertia of a uniform rod about an axis through one end is $\frac{1}{3}ML^2$. The distance d from the pivot to the center of mass is $L/2$. Substituting these quantities into Equation 12.28 gives

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\frac{L}{2}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

Comment In one of the Moon landings, an astronaut walking on the Moon's surface had a belt hanging from his space suit, and the belt oscillated as a compound pendulum. A scientist on Earth observed this motion on TV and from it was able to estimate the free-fall acceleration on the Moon. How do you suppose this calculation was done?

EXERCISE 8 Calculate the period of a meter stick pivoted about one end and oscillating in a vertical plane as in Figure 12.12. Answer 1.64 s

EXERCISE 9 A simple pendulum has a length of 3.00 m. Determine the change in its period if it is taken from a point where $g = 9.80\ \text{m/s}^2$ to an elevation where the free-fall acceleration decreases to 9.79 m/s². Answer 1.78×10^{-3} s

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OPTIONAL

12.5 · DAMPED OSCILLATIONS

The oscillatory motions we have considered so far have occurred in the context of an ideal system—that is, one that oscillates indefinitely under the action of a linear restoring force. In realistic systems, dissipative forces, such as friction, are present and retard the motion of the system. As a consequence, the mechanical energy of the system diminishes in time, and the motion is said to be *damped*.

One common type of drag force, which we discussed in Chapter 5, is proportional to the velocity and acts in the direction opposite the motion. This type of drag is often observed when an object is oscillating in air, for instance. Because the drag force can be expressed as $\mathbf{R} = -b\mathbf{v}$, where b is a constant, and the restoring force exerted on the system is $-kx$, we can write Newton's second law as

$$\sum F_x = -kx - bv = ma_x \qquad [12.29]$$

$$-kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

The solution of this equation requires mathematics that may not yet be familiar to you, and so it will simply be stated without proof. When the parameters of the system are such that $b < \sqrt{4mk}$, the solution to Equation 12.29 is

$$x = Ae^{-(b/2m)t}\cos(\omega t + \phi) \qquad [12.30]$$

where the angular frequency of motion is

$$\omega = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2} \qquad [12.31]$$

This result can be verified by substituting Equation 12.30 into Equation 12.29. Figure 12.13a shows the displacement as a function of time in this case. We see that **when the drag force is relatively small, the oscillatory character of the motion is preserved but the amplitude of vibration decreases in time** and the motion ultimately ceases. This is known as an **underdamped oscillator.** The dashed blue lines in Figure 12.13a, which outline the *envelope* of the oscillatory curve, represent the exponential factor that appears in Equation 12.30. The exponential factor shows that *the amplitude decays exponentially with time.* For motion with a given spring constant and particle mass, the oscillations dampen more rapidly as the maximum value of the drag force approaches the maximum value of the restoring force. One example of a damped harmonic oscillator is a mass immersed in a fluid, as in Figure 12.13b.

It is convenient to express the angular frequency of vibration of a damped system (Eq. 12.31) in the form

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}$$

where $\omega_0 = \sqrt{k/m}$ represents the angular frequency of oscillation in the absence of a drag force (the undamped oscillator). In other words, when $b = 0$, the drag force is zero and the system oscillates with its natural frequency, ω_0. As the magnitude of the drag force increases, the oscillations dampen more rapidly. When b reaches a critical value, b_c, so that $b_c/2m = \omega_0$, the system does not oscillate and is said to be

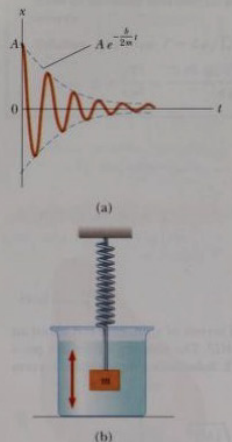


Figure 12.13 (a) Graph of the displacement versus time for a damped oscillator. Note the decrease in amplitude with time. (b) One example of a damped oscillator is a mass on a spring submersed in a liquid.

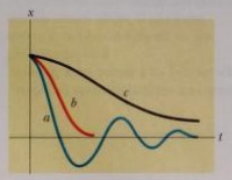


Figure 12.14 Plots of displacement versus time for (a) an underdamped oscillator, (b) a critically damped oscillator, and (c) an overdamped oscillator.

critically damped. In this case it returns to equilibrium in an exponential manner with time, as in Figure 12.14.

If the medium is highly viscous and the parameters meet the condition that $b/2m > \omega_0$, the system is **overdamped.** Again, the displaced system does not oscillate but simply returns to its equilibrium position. As the damping increases, the time it takes the displacement to reach equilibrium also increases, as indicated in Figure 12.14. In any case, when a drag force is present, the energy of the oscillator eventually falls to zero. The lost mechanical energy dissipates into thermal energy in the resistive medium.

12.6 · FORCED OSCILLATIONS

OPTIONAL

We have seen that the energy of a damped oscillator decreases in time as a result of the drag force. It is possible to compensate for this energy loss by applying an external force that does positive work on the system. At any instant, energy can be put into the system by an applied force that acts in the direction of motion of the oscillator. For example, a child on a swing can be kept in motion by appropriately timed "pushes." The amplitude of motion remains constant if the energy input per cycle of motion exactly equals the energy lost as a result of air drag.

A common example of a forced oscillator is a damped oscillator driven by an external force that varies periodically, such as $F = F_0 \cos \omega t$, where ω is the angular frequency of the force and F_0 is a constant. Adding this driving force to the left side of Equation 12.29 gives

$$F_0 \cos \omega t - b\frac{dx}{dt} - kx = m\frac{d^2x}{dt^2} \qquad [12.32]$$

Again, the solution of this equation is rather lengthy and will not be presented. However, after a sufficiently long period of time, when the energy input per cycle equals the energy lost per cycle, a steady-state condition is reached in which the oscillations proceed with constant amplitude. At this time, Equation 12.32 has the solution

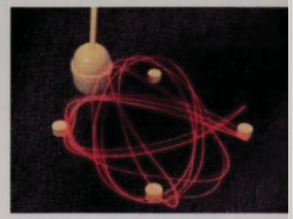
$$x = A\cos(\omega t + \phi) \qquad [12.33]$$

where

$$A = \frac{F_0/m}{\sqrt{(\omega^2 - \omega_0^2)^2 + \left(\frac{b\omega}{m}\right)^2}} \qquad [12.34]$$

and where $\omega_0 = \sqrt{k/m}$ is the angular frequency of the undamped oscillator ($b = 0$). From a physical point of view, one can argue that in a steady state the oscillator must have the same angular frequency as the driving force, and so the solution given by Equation 12.33 is expected.

Equation 12.34 shows that the amplitude of the forced oscillator is constant, because it is being driven by an external force. That is, the external agent provides the energy necessary to overcome the losses due to the drag force. Note that the mass oscillates at the angular frequency of the driving force, ω. For small damping, the amplitude becomes large when the frequency of the driving force is near the



This is a computer-generated simulation of the chaotic pattern of a driven pendulum. The nonlinearity in the motion is provided by the gravitational torque, which is proportional to the sine of the angle. An important research problem is to predict the conditions under which different physical systems exhibit chaotic behavior. *(© Yoav Levy/Phototake)*

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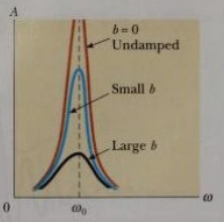


Figure 12.15 Graph of amplitude versus frequency for a damped oscillator when a periodic driving force is present. When the frequency of the driving force equals the natural frequency, ω_0, resonance occurs. Note that the shape of the resonance curve depends on the size of the damping coefficient, b.

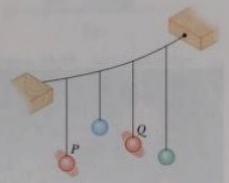


Figure 12.16 If pendulum P is set into oscillation, pendulum Q will eventually oscillate with the greatest amplitude because its length is equal to that of P and so they have the same natural frequency of vibration. The pendula are oscillating in a direction perpendicular to the plane formed by the stationary strings.

natural frequency of oscillation or when $\omega \approx \omega_0$. The dramatic increase in amplitude near the natural frequency is called **resonance,** and the angular frequency, ω_0, is called the **resonance frequency** of the system.

The reason for large-amplitude oscillations at the resonance frequency is that energy is being transferred to the system under the most favorable conditions. This can be better understood by taking the first time derivative of x, which gives an expression for the velocity of the oscillator. In doing so, one finds that v is proportional to $\sin(\omega t + \phi)$. When the applied force is in phase with $\mathbf{v}$, the rate at which work is done on the oscillator by the force $\mathbf{F}$ (in other words, the power) equals Fv. Because the quantity Fv is always positive when $\mathbf{F}$ and $\mathbf{v}$ are in phase, we conclude that **at resonance the applied force is in phase with the velocity, and the power transferred to the oscillator is a maximum.**

Figure 12.15 is a graph of amplitude as a function of frequency for the forced oscillator, with and without a resistive force. Note that the amplitude increases with decreasing damping ($b \to 0$) and that the resonance curve broadens as the damping increases. Under steady-state conditions, and at any driving frequency, the energy transferred into the system equals the energy lost because of the damping force; hence, the average total energy of the oscillator remains constant. In the absence of a damping force ($b = 0$), we see from Equation 12.34 that the steady-state amplitude approaches infinity as $\omega \to \omega_0$. In other words, if there are no losses in the system and we continue to drive an initially motionless oscillator with a sinusoidal force that is in phase with the velocity, the amplitude of motion will build up without limit (Fig. 12.15). This does not occur in practice because some damping is always present. That is, at resonance the amplitude is large but finite for small damping.

One experiment that demonstrates a resonance phenomenon is illustrated in Figure 12.16. Several pendula of different lengths are suspended from a stretched string. If one of them, such as P, is set swinging in the plane perpendicular to the plane of the string, the others will begin to oscillate, because they are coupled by the stretched string. Of those that are forced into oscillation by this coupling, pendulum Q, which is the same length as P (hence, the two pendula have the same natural frequency), will oscillate with the greatest amplitude.

Resonance appears in other areas of physics. For example, certain electrical circuits have natural (resonant) frequencies. A structure such as a bridge has natural frequencies and can be set into resonance by an appropriate driving force. A striking example of such a structural resonance occurred in 1940, when the Tacoma Narrows Bridge in Washington was destroyed by resonant vibrations (Fig. 12.17). The winds were not particularly strong on that occasion, but the bridge still collapsed because vortices (turbulences) generated by the wind blowing through the bridge occurred at a frequency that matched the natural frequency of the bridge.

Many other examples of resonant vibrations can be cited. A resonant vibration you may have experienced is the "singing" of telephone wires in the wind. Machines often break if one vibrating part is at resonance with some other moving part.

Thinking Physics 4

In an earthquake region, should damping in a building be large or small?

Reasoning The damping in a building should be large, so that the resonance curve is broad and flat. The building should respond to a wide range of earthquake frequencies, with a relatively small response. If the damping were small, so that the resonance curve were tall and skinny, the building would reject many earthquake frequencies but would have a disastrously large response to a narrow range of frequencies.



(a) (b)

Figure 12.17 (a) High winds set up vibrations in the bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. (b) Once established, this resonance condition led to the bridge's collapse. *(UPI/Bettmann Newsphotos)*

CONCEPTUAL PROBLEM 8

You stand on the end of a diving board and bounce to set it into oscillation. You find a maximum response, in terms of the amplitude of oscillation of the end of the board, when you bounce at frequency f. You now move to the middle of the board and repeat the experiment. Is the resonance frequency for forced oscillations at this point higher than f, lower than f, or the same as f? Why?

CONCEPTUAL PROBLEM 9

Some parachutes have holes in them to allow air to move smoothly through the chute. Without the holes, the air gathered under the chute as the parachutist falls is sometimes released from under the edges of the chute alternately and periodically from one side and then the other. Why might this periodic release of air cause a problem?

SUMMARY

The position of a simple harmonic oscillator varies periodically in time according to the relation

$$x = A\cos(\omega t + \phi) \qquad [12.1]$$

where A is the amplitude of the motion, ω is the angular frequency, and ϕ is the phase constant. The values of A and ϕ depend on the initial position and velocity of the oscillator.

The time for one complete oscillation is called the **period** of the motion, defined by

$$T = \frac{2\pi}{\omega} \qquad [12.2]$$

The inverse of the period is the **frequency** of the motion, which equals the number of oscillations per second.

The **velocity** and **acceleration** of a simple harmonic oscillator are

$$v = \frac{dx}{dt} = -\omega A\sin(\omega t + \phi) \qquad [12.5]$$

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$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \phi) \qquad [12.6]$$

Thus, the maximum velocity is ωA, and the maximum acceleration is $\omega^2 A$. The velocity is zero when the oscillator is at its turning points, $x = \pm A$, and the speed is a maximum at the equilibrium position, $x = 0$. The magnitude of the acceleration is a maximum at the turning points and is zero at the equilibrium position.

A mass–spring system moving on a frictionless track exhibits simple harmonic motion with the period

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \qquad [12.18]$$

where k is the force constant of the spring and m is the mass attached to the spring.

The kinetic energy and potential energy of a simple harmonic oscillator vary with time and are

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi) \qquad [12.20]$$

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \cos^2(\omega t + \phi) \qquad [12.21]$$

The **total energy** of a simple harmonic oscillator is a constant of the motion and is

$$E = \tfrac{1}{2}kA^2 \qquad [12.22]$$

The potential energy of a simple harmonic oscillator is a maximum when the particle is at its turning points (maximum displacement from equilibrium) and is zero at the equilibrium position. The kinetic energy is zero at the turning points and is a maximum at the equilibrium position.

A **simple pendulum** of length L exhibits simple harmonic motion for small angular displacements from the vertical, with a period of

$$T = 2\pi\sqrt{\frac{L}{g}} \qquad [12.26]$$

That is, the period is independent of the suspended mass.

A **physical pendulum** exhibits simple harmonic motion about a pivot that does not go through the center of mass. The period of this motion is

$$T = 2\pi\sqrt{\frac{I}{mgd}} \qquad [12.28]$$

where I is the moment of inertia about an axis through the pivot and d is the distance from the pivot to the center of mass.

Damped oscillations occur in a system in which a drag force opposes the linear restoring force. If such a system is set in motion and then left to itself, its mechanical energy decreases in time because of the presence of the nonconservative drag force. It is possible to compensate for this loss in energy by driving the system with an external periodic force that is in phase with the motion of the system. When the frequency of the driving force matches the natural frequency of the *undamped* oscillator that starts its motion from rest, energy is continuously transferred to the oscillator and its amplitude increases without limit.

CONCEPTUAL QUESTIONS

1. Does the acceleration of a simple harmonic oscillator remain constant during its motion? Is the acceleration ever zero? Explain.
2. Explain why the kinetic and potential energies of a mass–spring system can never be negative.
3. What is the total distance traveled by a body executing simple harmonic motion in a time equal to its period if its amplitude is A?
4. If a mass–spring system is hung vertically and set into oscillation, why does the motion eventually stop?
5. If a pendulum clock keeps perfect time at the base of a mountain, will it also keep perfect time when moved to the top of the mountain? Explain.
6. If a grandfather clock were running slow, how could we adjust the length of the pendulum to correct the time?
7. A pendulum bob is made from a ball filled with water. What happens to the frequency of vibration of this pendulum if the ball has a hole that allows water to slowly leak out?
8. A platoon of soldiers marches in step along a road. Why are they ordered to break step when crossing a bridge?
9. If the length of a simple pendulum is quadrupled, what happens to (a) its frequency and (b) its period?
10. The amplitude of a system moving in simple harmonic motion is doubled. Determine the change in (a) the total energy, (b) the maximum speed, (c) the maximum acceleration, and (d) the period.
11. A simple harmonic oscillator has a total energy of E. (a) Determine the kinetic and potential energies when the displacement equals one-half the amplitude. (b) For what value of the displacement does the kinetic energy equal the potential energy?
12. If the coordinate of a particle varies as $x = -A\cos\omega t$, what is the phase constant in Equation 12.1? At what position does the particle begin its motion?
13. Does the displacement of an oscillating particle between $t = 0$ and a later time t necessarily equal the position of the particle at time t? Explain.
14. Determine whether or not the following quantities can be in the same direction for a simple harmonic oscillator: (a) displacement and velocity, (b) velocity and acceleration, (c) displacement and acceleration.
15. Can the amplitude A and phase constant ϕ be determined for an oscillator if only the position is specified at $t = 0$? Explain.
16. A simple pendulum undergoes simple harmonic motion when θ is small. Is the motion periodic when θ is large? How does the period of motion change as θ increases?
17. Will damped oscillations occur for any values of b and k? Explain.
18. Is it possible to have damped oscillations when a system is at resonance? Explain.

PROBLEMS

Section 12.1 Simple Harmonic Motion

1. The displacement of a particle is given by the expression $x = (4.00\text{ m})\cos(3.00\pi t + \pi)$, where x is in meters and t is in seconds. Determine (a) the frequency and period of the motion, (b) the amplitude of the motion, (c) the phase constant, and (d) the displacement of the particle at $t = 0.250$ s.
2. A particle oscillates with simple harmonic motion so that its displacement varies according to the expression $x = (5.00\text{ cm})\cos(2t + \pi/6)$, where x is in centimeters and t is in seconds. At $t = 0$, find (a) the displacement of the particle, (b) its velocity, and (c) its acceleration. (d) Find the period and amplitude of the motion.
3. A piston in an automobile engine is in simple harmonic motion. If its amplitude of oscillation from the centerline is ± 5.00 cm and its mass is 2.00 kg, find the maximum velocity and acceleration of the piston when the auto engine is running at the rate of 3600 rev/min.
4. A 20.0-g particle moves in simple harmonic motion with a frequency of 3.00 oscillations/s and an amplitude of 5.00 cm. (a) Through what total distance does the particle move during one cycle of its motion? (b) What is its maximum speed? Where does this occur? (c) Find the maximum acceleration of the particle. Where in the motion does the maximum acceleration occur?
5. A particle moving along the x axis in simple harmonic motion starts from the origin at $t = 0$ and moves to the right. If the amplitude of its motion is 2.00 cm and the frequency is 1.50 Hz, (a) show that its displacement is given by the expression $x = (2.00\text{ cm})\sin(3.00\pi t)$. Determine (b) the maximum speed and the earliest time ($t > 0$) at which the particle has this speed, (c) the maximum acceleration and the earliest time ($t > 0$) at which the particle has this acceleration to the right, and (d) the total distance traveled between $t = 0$ and $t = 1.00$ s.
6. If the initial position, velocity, and acceleration of an object moving in simple harmonic motion are x_0, v_0, and a_0, and

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IPR2023-00783

if the angular frequency of oscillation is ω, (a) show that the position and velocity of the object for all time can be written as

$$x(t) = x_0 \cos \omega t + \left(\frac{v_0}{\omega}\right) \sin \omega t$$

$$v(t) = -x_0 \omega \sin \omega t + v_0 \cos \omega t$$

(b) If the amplitude of the motion is A, show that

$$v^2 - ax = v_0^2 - a_0 x_0 = A^2 \omega^2$$

Section 12.2 Motion of a Mass Attached to a Spring

(*Note:* Neglect the mass of the spring in all of these problems.)

7. An archer pulls her bow string back 0.400 m by exerting a force that increases uniformly from zero to 230 N. (a) What is the equivalent spring constant of the bow? (b) How much work is done in pulling the bow?
8. A 1.00-kg mass attached to a spring of force constant 25.0 N/m oscillates on a horizontal, frictionless track. At $t = 0$, the mass is released from rest at $x = -3.00$ cm. (That is, the spring is compressed by 3.00 cm.) Find (a) the period of its motion, (b) the maximum values of its speed and acceleration, and (c) the displacement, velocity, and acceleration as functions of time.
9. A 0.500-kg mass attached to a spring of force constant 8.00 N/m vibrates in simple harmonic motion with an amplitude of 10.0 cm. Calculate (a) the maximum value of its speed and acceleration, (b) the speed and acceleration when the mass is 6.00 cm from the equilibrium position, and (c) the time it takes the mass to move from $x = 0$ to $x = 8.00$ cm.
10. A block of unknown mass is attached to a spring of force constant 6.50 N/m and undergoes simple harmonic motion with an amplitude of 10.0 cm. When the mass is halfway between its equilibrium position and the endpoint, its speed is measured to be +30.0 cm/s. Calculate (a) the mass of the block, (b) the period of the motion, and (c) the maximum acceleration of the block.
11. A 1.00-kg mass is attached to a horizontal spring. The spring is initially stretched by 0.100 m and the mass is released from rest there. After 0.500 s, the speed of the mass is zero. What is the maximum speed of the mass?
12. A particle that hangs from a spring oscillates with an angular frequency of 2.00 rad/s. The spring is suspended from the ceiling of an elevator car and hangs motionless (relative to the elevator car) as the car descends at a constant speed of 1.50 m/s. The car then stops suddenly. (a) With what amplitude does the particle oscillate? (b) What is the equation of motion for the particle? (Choose the upward direction to be positive.)

Section 12.3 Energy of the Simple Harmonic Oscillator

(Neglect spring masses.)

13. An automobile having a mass of 1000 kg is driven into a brick wall in a safety test. The bumper behaves like a spring of constant 5.00×10^6 N/m and compresses 3.16 cm as the car is brought to rest. What was the speed of the car before impact, assuming no energy is lost during impact with the wall?
14. An ore car of mass 4000 kg starts from rest and rolls downhill from the mine on tracks. At the end of the tracks, 10.0 m lower in elevation, is a spring with force constant $k = 400\,000$ N/m. How much is the spring compressed in stopping the ore car? Ignore friction.
15. A 200-g mass is attached to a spring and executes simple harmonic motion with a period of 0.250 s. If the total energy of the system is 2.00 J, find (a) the force constant of the spring and (b) the amplitude of the motion.
16. A mass–spring system oscillates with an amplitude of 3.50 cm. If the force constant of the spring is 250 N/m and the mass is 0.500 kg, determine (a) the mechanical energy of the system, (b) the maximum speed of the mass, and (c) the maximum acceleration.
17. A 50.0-g mass connected to a spring of force constant 35.0 N/m oscillates on a horizontal, frictionless surface with an amplitude of 4.00 cm. Find (a) the total energy of the system and (b) the speed of the mass when the displacement is 1.00 cm. When the displacement is 3.00 cm, find (c) the kinetic energy and (d) the potential energy.
18. A 2.00-kg mass is attached to a spring and placed on a horizontal, smooth surface. A horizontal force of 20.0 N is required to hold the mass at rest when it is pulled 0.200 m from its equilibrium position (the origin of the x axis). The mass is now released from rest with an initial displacement of $x_0 = 0.200$ m, and it subsequently undergoes simple harmonic oscillations. Find (a) the force constant of the spring, (b) the frequency of the oscillations, and (c) the maximum speed of the mass. Where does this maximum speed occur? (d) Find the maximum acceleration of the mass. Where does it occur? (e) Find the total energy of the oscillating system. When the displacement equals one third the maximum value, find (f) the speed and (g) the acceleration.

Section 12.4 Motion of a Pendulum

19. A simple pendulum has a period of 2.50 s. (a) What is its length? (b) What would its period be on the Moon, where $g_M = 1.67$ m/s^2?
20. A "seconds" pendulum is one that moves through its equilibrium position once each second. (The period of the pendulum is 2.000 s.) The length of a seconds pendulum is 0.9927 m at Tokyo and 0.9942 m at Cambridge, England. What is the ratio of the free-fall accelerations at these two locations?
21. A simple pendulum has a mass of 0.250 kg and a length of 1.00 m. It is displaced through an angle of 15.0° and then released. What are (a) the maximum speed? (b) the maximum angular acceleration? (c) the maximum restoring force?
22. The angular displacement of a pendulum is represented by the equation $\theta = 0.320 \cos \omega t$, where θ is in radians and $\omega = 4.43$ rad/s. Determine the period and length of the pendulum.
23. A particle of mass m slides inside a frictionless hemispherical bowl of radius R. Show that if it starts from rest with a small displacement from equilibrium, the particle moves in simple harmonic motion with an angular frequency equal to that of a simple pendulum of length R. That is, $\omega = \sqrt{g/R}$.
24. A simple pendulum is 5.00 m long. (a) What is the period of simple harmonic motion for this pendulum if it is located in an elevator accelerating upward at 5.00 m/s^2? (b) What is its period if the elevator is accelerating downward at 5.00 m/s^2? (c) What is the period of simple harmonic motion for this pendulum if it is placed in a truck that is accelerating horizontally at 5.00 m/s^2?
25. A very light rod extends rigidly 0.500 m out from one end of a meter stick. The stick is suspended from a pivot at the far end of the rod and set into oscillation. (a) Determine the period of oscillation. (b) By what percentage does this differ from a 1.00–m-long simple pendulum?
26. Consider the physical pendulum of Figure 12.11. (a) If its moment of inertia about an axis passing through its center of mass and parallel to the axis passing through its pivot point is I_{CM}, show that its period is

$$T = 2\pi \sqrt{\frac{I_{CM} + md^2}{mgd}}$$

where d is the distance between the pivot point and center of mass. (b) Show that the period has a minimum value when d satisfies $md^2 = I_{CM}$.

Section 12.5 Damped Oscillations (Optional)

27. A pendulum of length 1.00 m is released from an initial angle of 15.0°. After 1000 s, its amplitude is reduced by friction to 5.50°. What is the value of $b/2m$?
28. Show that Equation 12.30 is a solution of Equation 12.29 provided that $b^2 < 4mk$.
29. Show that the time rate of change of mechanical energy for a damped, undriven oscillator is given by $dE/dt = -bv^2$ and hence is always negative. (*Hint:* Differentiate the expression for the mechanical energy of an oscillator, $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, and use Eq. 12.29.)

Section 12.6 Forced Oscillations (Optional)

30. Calculate the resonant frequencies of (a) a 3.00-kg mass attached to a spring of force constant 240 N/m and (b) a simple pendulum 1.50 m in length.
31. A 2.00-kg mass attached to a spring is driven by an external force $F = (3.00\ \text{N}) \cos(2\pi t)$. If the force constant of the spring is 20.0 N/m, determine (a) the period and (b) the amplitude of the motion. (*Hint:* Assume there is no damping—that is, $b = 0$, and use Eq. 12.34.)
32. Consider an *undamped*, forced oscillator ($b = 0$), and show that Equation 12.33 is a solution of Equation 12.32, with an amplitude given by Equation 12.34.

Additional Problems

33. A mass m is oscillating freely on a vertical spring (Fig. P12.33). When $m = 0.810$ kg, the period is 0.910 s. An unknown mass on the same spring has a period of 1.16 s. Determine (a) the spring constant k and (b) the unknown mass.



Figure P12.33

34. A mass m is oscillating freely on a vertical spring with a period T (Fig. P12.33). An unknown mass m' on the same spring oscillates with a period T'. Determine (a) the force constant k and (b) the unknown mass m'.
35. A pendulum of length L and mass M has a spring of force constant k connected to it at a distance h below its point of suspension (Fig. P12.35). Find the frequency of vibration of the system for small values of the amplitude (small θ). (Assume the vertical suspension of length L is rigid but neglect its mass.)
36. A horizontal plank of mass m and length L is pivoted at one end, and the opposite end is attached to a spring of force constant k (Fig. P12.36). The moment of inertia of the plank about the pivot is $\frac{1}{3}mL^2$. When the plank is displaced a small angle θ from its horizontal equilibrium position and released, show that it moves with simple harmonic motion with an angular frequency $\omega = \sqrt{3k/m}$.

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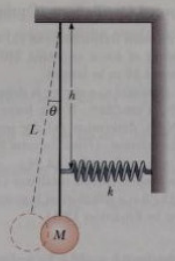


Figure P12.35

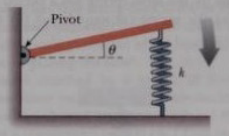


Figure P12.36

37. A large block P executes horizontal simple harmonic motion by sliding across a frictionless surface with a frequency $f = 1.50$ Hz. Block B rests on it, as shown in Figure P12.37, and the coefficient of static friction between the two is $\mu_s = 0.600$. What maximum amplitude of oscillation can the system have if the block is not to slip?

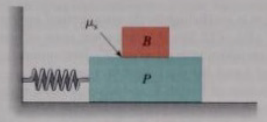


Figure P12.37

38. A 2.00-kg block hangs without vibrating at the end of a spring ($k = 500$ N/m) that is attached to the ceiling of an elevator car. The car is rising with an upward acceleration of $g/3$ when the acceleration suddenly ceases (at $t = 0$). (a) What is the angular frequency of oscillation of the block after the acceleration ceases? (b) By what amount is the spring stretched during the time that the elevator car is accelerating? (c) What are the amplitude of the oscillation and the initial phase angle observed by a rider in the car? Take the upward direction to be positive.

39. A simple pendulum having a length of 2.23 m and a mass of 6.74 kg is given an initial speed of 2.06 m/s at its equilibrium position. Assume it undergoes simple harmonic motion and determine its (a) period, (b) total energy, and (c) maximum angular displacement.

40. People who ride motorcycles and bicycles learn to look out for bumps in the road—and especially for washboarding, a condition of many equally spaced ridges or bumps a certain distance apart. What is so bad about washboarding? A motorcycle has several springs and shock absorbers in its suspension, but you can model it as a single spring supporting a mass. You can estimate the spring constant by thinking about how far the spring compresses when a big biker sits down on the seat. Compute, on the basis of reasonable estimates, the distance between the washboard bumps of which a motorcyclist must be particularly careful when he or she is traveling at highway speed.

41. A mass M is connected to a spring of mass m and oscillates in simple harmonic motion on a horizontal, frictionless track (Fig. P12.41). The force constant of the spring is k and the equilibrium length is ℓ. Find (a) the kinetic energy of the system when the mass has a speed of v and (b) the period of oscillation. [*Hint:* Assume that all portions of the spring oscillate in phase and that the velocity of a segment dx is proportional to the distance from the fixed end; that is, $v_x = (x/\ell)v$. Also, note that the mass of a segment of the spring is $dm = (m/\ell)dx$.]

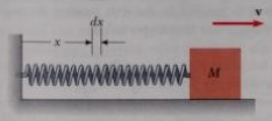


Figure P12.41

42. When a mass M, connected to the end of a spring of mass $m_s = 7.40$ g and force constant k is set into simple harmonic motion, the period of its motion is

$$T = 2\pi\sqrt{\frac{M + (m_s/3)}{k}}$$

A two-part experiment is conducted using various masses suspended vertically from the spring, as in Figure P12.33. (a) Displacements of 17.0, 29.3, 35.3, 41.3, 47.1, and 49.3 cm are measured for M values of 20.0, 40.0, 50.0, 60.0, 70.0, and 80.0 g, respectively. Construct a graph of Mg versus x, and perform a linear least-squares fit to the data. From the slope of your graph, determine a value for k for this spring. (b) The system is now set into simple harmonic motion, and periods are measured with a stopwatch. With $M = 80.0$ g, the total time for 10 oscillations is measured to be 13.41 s. The experiment is repeated with M values of 70.0, 60.0, 50.0, 40.0, and 20.0 g, with corresponding times for 10 oscillations of 12.52, 11.67, 10.67, 9.62, and 7.03 s. Obtain experimental values for T for each of these M values. Plot a graph of T^2 versus M and determine a value for k from the slope of the linear least-squares fit through the data points. Compare this value of k with that obtained in part (a). (c) Obtain a value for m_s from your graph and compare it with the given value of 7.40 g.

43. A mass m is connected to two rubber bands of length L, each under tension F, as in Figure P12.43. The mass is displaced vertically by a small distance, y. Assuming the tension does not change, show that (a) the restoring force is $-(2F/L)y$ and (b) the system exhibits simple harmonic motion with an angular frequency $\omega = \sqrt{2F/mL}$.



Figure P12.43

44. Consider a damped oscillator as illustrated in Figure 12.13. Assume the mass is 375 g, the spring constant is 100 N/m, and $b = 0.100$ kg/s. (a) How long does it take for the amplitude to drop to half its initial value? (b) How long does it take for the mechanical energy to drop to half its initial value? (c) Show that, in general, the fractional rate at which the amplitude decreases in a damped harmonic oscillator is one half the fractional rate at which the mechanical energy decreases.

45. A small, thin disk of radius r and mass m is attached rigidly to the face of a second thin disk of radius R and mass M as shown in Figure P12.45. The center of the small disk is located at the edge of the large disk. The large disk is mounted at its center on a frictionless axle. The assembly is rotated through a small angle θ from its equilibrium position and released. (a) Show that the speed of the center of the small disk as it passes through the equilibrium position is

$$v = 2\left[\frac{Rg(1 - \cos\theta)}{(M/m) + (r/R)^2 + 2}\right]^{1/2}$$

(b) Show that the period of the motion is

$$T = 2\pi\left[\frac{(M + 2m)R^2 + mr^2}{2mgR}\right]^{1/2}$$

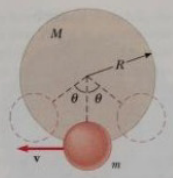


Figure P12.45

46. A mass m is connected to two springs of force constants k_1 and k_2, as in Figures P12.46a and P12.46b. In each case, the mass moves on a frictionless table and is displaced from equilibrium and released. Show that in each case the mass exhibits simple harmonic motion with periods

(a) $T = 2\pi\sqrt{\dfrac{m(k_1 + k_2)}{k_1 k_2}}$

(b) $T = 2\pi\sqrt{\dfrac{m}{k_1 + k_2}}$

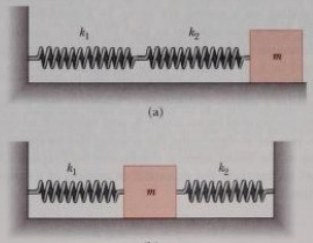


Figure P12.46

47. Imagine that a hole is drilled through the center of the Earth to its other side. It can be shown that an object of mass m at a distance r from the center of the Earth is pulled toward the center of the Earth only by the mass within the sphere of radius r (the reddish region in Fig. P12.47). Write Newton's law of gravitation for an object at the distance r from the center of the Earth, and show that the force on it is of Hooke's law form, $F = -kr$, where the effective force constant is $k = (4/3)\pi\rho Gm$, where ρ is the density of the Earth (assumed uniform) and G is the gravitational constant. Show that a sack of mail dropped into the hole will execute simple harmonic motion. When will it arrive at the other side of the Earth?

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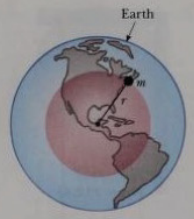


Figure P12.47

Spreadsheet Problems

S1. Use a spreadsheet to plot the position x of an object undergoing simple harmonic motion as a function of time. (a) Use $A = 2.00$ m, $\omega = 5.00$ rad/s, and $\phi = 0$. On the same graph, plot the function for $\phi = \pi/4$, $\pi/2$, and π. Explain how the different phase constants affect the position of the object. (b) Repeat part (a) with $\omega = 5.00$ rad/s. How does the change in ω affect the plots?

S2. The potential and kinetic energies of a particle of mass m oscillating on a spring are given by Equations 12.20 and 12.21. Use a spreadsheet to plot $U(t)$, $K(t)$, and their sum as functions of time on the same graph. The input variables should be k, A, m, and ϕ. What happens to the sum of $U(t)$ and $K(t)$ as the input variables are changed?

S3. Consider a simple pendulum of length L that is displaced from the vertical by an angle θ_0 and released. In this problem, you are to determine the angular displacement for any value of the initial angular displacement. Prepare a spreadsheet to integrate the equation of motion of a simple pendulum:

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta$$

Take the initial condition to be $\theta = \theta_0$ and $d\theta/dt = 0$ at $t = 0$. Choose values for θ_0 and L, and find the angular displacement θ as a function of time. Using the same values of θ_0, compare your results for θ with those obtained from $\theta(t) = \theta_0 \cos \omega_0 t$ where $\omega_0 = \sqrt{g/L}$. Repeat for different values of θ_0. Be sure to include large initial angular displacements. Does the period change when the initial angular displacement is changed? How do the periods for large values of θ_0 compare to those for small values of θ_0? (*Note:* Using the modified Euler's method to solve this differential equation you may find that the amplitude tends to increase with time. The fourth-order Runge-Kutta method discussed in the *Spreadsheet Investigations* Supplement would be a better choice to solve the differential equation. However, if you choose dt small enough, the solution using the modified Euler's method will still be good.)

S4. In the real world friction is always present. Modify your spreadsheet for Problem S3 to include the force of air resistance: $F_{air} = -bv$. In this situation, the equation of motion is

$$\frac{d^2\theta}{dt^2} = -\frac{g}{L}\sin\theta - \frac{b}{m}\frac{d\theta}{dt}$$

where m is the mass of the pendulum and b is the drag coefficient. Integrate this equation and calculate the sum of the kinetic and potential energies as functions of time using your calculated values of θ and $d\theta/dt$. Because energy is not constant, when in the pendulum's cycle is energy dissipated the fastest?

S5. The differential equation for the position x of a driven, damped harmonic oscillator of mass m is

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + m\omega_0^2 x = F_0 \cos \omega t$$

where ω_0 and b are constants, F_0 is the amplitude of the driving force, and ω is the driving angular frequency. For the case of a mass on a spring, $\omega_0 = \sqrt{k/m}$, where k is the force constant. Write a computer program or spreadsheet to integrate this equation of motion. Your input variables are the damping coefficient, b, the mass m, ω (the natural frequency of the system), and F_0. Choose the values for the initial velocity and position and calculate the position x as a function of time. Vary the driving angular frequency ω. Pick some values near ω_0. Does resonance occur? (*Note:* Your program may crash or blow up near resonance. The amplitudes of the motion may have large values when $\omega = \omega_0$. Also, the Euler method usually tends to overestimate the solution. Other methods such as the fourth-order Runge-Kutta method discussed in the *Spreadsheet Investigations* Supplement give a more accurate solution.)

ANSWERS TO CONCEPTUAL PROBLEMS

1. The bouncing ball is not an example of simple harmonic motion. The ball does not follow a sinusoidal function for its position as a function of time. The daily movement of a student is also not simple harmonic motion, since the student stays at a fixed position—school—for a long period of time. If this motion were sinusoidal, the student would move more and more slowly as she approached her desk, and, as soon as she sat down at the desk, she would start to move back toward home again!
2. When the spring with two masses is set into oscillation in space, the coil in the exact center of the spring does not move. Thus, we can imagine clamping the center coil in place without affecting the motion. If we do this, we have two separate oscillating systems, one on each side of the clamp. The half-spring on each side of the clamp has twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring. Thus, our clamped system of masses on two half-springs will vibrate with a frequency that is higher than f by a factor of $\sqrt{2}$.
3. No. Because $E = \frac{1}{2}kA^2$, changing the mass while keeping A constant has no effect on the total energy. The potential energy $U = \frac{1}{2}kx^2$ depends only on position, not mass. Because $K = E - U$, the kinetic energy will also depend on position and not mass.
4. If it accelerates upward, the effective "g" is greater than the free-fall acceleration, so the period decreases. If it accelerates downward, the effective "g" is less than the free-fall acceleration, so the period increases. If it moves with constant velocity, the period does not change. (If the pendulum is in free-fall, it does not oscillate.)
5. Since the car is accelerating downhill, the principle of equivalence tells us that there is an effective gravitational field component uphill. Combining this with the actual gravitational field, the net effective gravitational field is no longer vertical. Thus, the equilibrium position of the pendulum will not be vertical. In addition, the net effective gravitational field has changed in value from the actual field, so the period of the pendulum will change.
6. As the temperature increases, the length of the pendulum will increase, due to thermal expansion. With a longer length, the period of the pendulum will increase. Thus, it will take longer to execute each swing, so that each second according to the clock will take longer than an actual second. Thus, the clock will run *slow*.
7. The first student is correct. While changing the mass of a simple pendulum does not change the frequency, the Foucault pendulum is a *physical* pendulum—the bob has a size. Assuming that the cord is very light, when the gum is placed on top of the bob, the center of mass of the physical pendulum is shifted very slightly upward. This shortens the effective length of the pendulum, which alters the frequency.
8. The diving board is acting as a physical pendulum. With your mass at the end of the board, you and the board have the maximum possible moment of inertia. When you move to the center of the board, the moment of inertia of the system decreases, and the natural frequency of oscillation increases above f.
9. If the period of the alternating release of air matched the swinging period of the parachutist, there could be a resonance response. The swinging of the parachutist could become so energetic that the parachute could become fouled and its braking action could disappear.

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13

Wave Motion

CHAPTER OUTLINE

13.1 Three Wave Characteristics
13.2 Types of Waves
13.3 One-Dimensional Transverse Traveling Waves
13.4 Sinusoidal Traveling Waves
13.5 Superposition and Interference of Waves
13.6 The Speed of Transverse Waves on Strings
13.7 Reflection and Transmission of Waves
13.8 Energy Transmitted by Sinusoidal Waves on Strings
13.9 Sound Waves
13.10 The Doppler Effect

Most of us experienced waves as children, when we dropped pebbles into a pond. The disturbance created by a pebble manifests itself as ripple waves that move outward until they finally reach the shore. If you were to carefully examine the motion of a leaf floating near the point where the pebble entered the water, you would see that the leaf moves up and down and sideways about its original position, but does not undergo any net displacement away from or toward the source of the disturbance. The water wave (the disturbance) moves from one place to another, *yet the water is not carried with it.*

The world is full of other kinds of waves, including sound waves, waves on strings, earthquake waves, radio waves, and x-rays. Most waves can be placed in one of two categories. **Mechanical waves** are waves that disturb and propagate through a medium; sound waves, for which air is the medium, and earthquake waves, for which the Earth's crust is the medium, are two examples of mechanical waves. **Electromagnetic waves** are a special class of waves that do not require a medium in order to propagate; light waves and radio waves are two familiar examples. In this chapter we shall confine our attention to the study of mechanical waves, deferring our study of electromagnetic waves to Chapter 24.

The wave concept is abstract. When we observe a water wave, what we see is a rearrangement of the water's surface. Without the water, there would be no wave. In the case of this or any other mechanical wave, what we interpret as a wave corresponds to the disturbance of a medium. Therefore, we can consider a mechanical wave to be **the motion of a disturbance.** This motion is not to be confused with the motion of the particles that make up the medium. In general, we describe mechanical wave motion by specifying the positions of all particles of the disturbed medium as a function of time.



Large waves such as this travel ▶ great distances over the surface of the ocean, yet the water does not flow with the wave. The crests and troughs of the wave often form repetitive patterns. *(Superstock)*

13.1 · THREE WAVE CHARACTERISTICS

All mechanical waves require (1) some source of disturbance, (2) a medium that can be disturbed, and (3) some physical mechanism through which particles of the medium can influence each other. All waves carry energy and momentum, but the amount of energy transmitted through a medium and the mechanism responsible for the energy transport differ from case to case. For instance, the power of ocean waves during a storm is much greater than the power of sound waves generated by a musical instrument.

Three physical characteristics are important in describing waves: wavelength, frequency, and wave speed. **One wavelength is the minimum distance between any two points on a wave that behave identically**—for example, adjacent crests or adjacent troughs. Figure 13.1a shows displacement versus position for a sinusoidal wave at a specific time. The symbol λ (Greek lambda) is used to denote wavelength.

Most waves are periodic. **The frequency of such waves is the rate at which the disturbance repeats itself.** The period of the wave is the minimum time it takes the disturbance to repeat itself and is equal to the inverse of the frequency. Figure 13.1b shows displacement versus time for a sinusoidal wave at a fixed position, where the period is the time between identical displacements of a point on the wave.

Waves travel, or *propagate,* with a specific speed, which depends on the properties of the medium being disturbed. For instance, sound waves travel through air at 20°C with a speed of about 343 m/s, whereas the speed of sound in most solids is higher than 343 m/s.

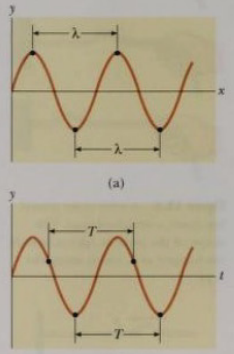


Figure 13.1 (a) The wavelength λ of a wave is the distance between adjacent crests or adjacent troughs. (b) The period T of a wave is the time it takes the wave to travel one wavelength.

Thinking Physics 1

Why is it important to be quiet in avalanche country? In the movie *On Her Majesty's Secret Service* (United Artists, 1969), the bad guys try to stop James Bond, who is escaping on skis, by firing a gun and causing an avalanche. Why did this happen?

Reasoning The essence of a wave is the propagation of a *disturbance* through a medium. An impulsive sound, like the gunshot in the James Bond movie, can cause an acoustical disturbance that propagates through the air and can impact a ledge of snow that is just ready to break free to begin an avalanche. Such a disastrous event occurred in 1916 during World War I, when Austrian soldiers in the Alps were smothered by an avalanche caused by cannon fire.

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13.2 · TYPES OF WAVES

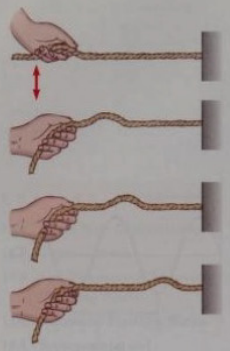


Figure 13.2 A wave pulse traveling down a stretched rope. The shape of the pulse is approximately unchanged as it travels along the rope.

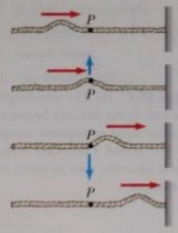


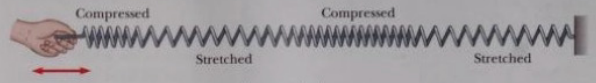
Figure 13.3 A pulse traveling on a stretched rope is a transverse wave. That is, any element *P* on the rope moves (blue arrows) in a direction perpendicular to the wave motion (red arrows).

One way to demonstrate wave motion is to flip the free end of a long rope that is under tension and has its opposite end fixed, as in Figure 13.2. In this manner, a single wave bump (or pulse) is formed and travels (to the right in Fig. 13.2) with a definite speed. This type of disturbance is called a **traveling wave,** and the rope is the medium through which it travels. Figure 13.2 represents four consecutive "snapshots" of the traveling wave. The shape of the wave pulse changes very little as it travels along the rope.[1]

Note that **as the wave pulse travels, each rope segment that is disturbed moves in a direction perpendicular to the wave motion.** Figure 13.3 illustrates this point for a particular segment, labeled *P*. Note that there is no motion of any part of the rope in the direction of the wave. A traveling wave such as this, in which the particles of the disturbed medium move perpendicularly to the wave velocity, is called a **transverse wave.**

In another class of waves, called **longitudinal waves,** the particles of the medium undergo displacements *parallel* to the direction of wave motion. Sound waves in air, for instance, are longitudinal. Their disturbance corresponds to a series of high- and low-pressure regions that may travel through air or through any material medium with a certain speed. A longitudinal pulse can be easily produced in a stretched spring, as in Figure 13.4. The free end is pumped back and forth along the length of the spring. This action produces compressed and stretched regions of the coil that travel along the spring, parallel to the wave motion.[2]

Some waves are neither transverse nor longitudinal but a combination of the two. Surface water waves are a good example. Figure 13.5 shows the motion of water particles at the surface as a wave moves to the right. Each particle moves in a circular path, and hence the disturbance has both transverse and longitudinal components. As the wave passes, water particles at the crests are moving in the direction of the wave, and particles at the troughs move in the opposite direction. Hence, no *net* displacement of water particles takes place. A cork bobbing on a pond surface as a wave passes exhibits this circular motion.



Longitudinal wave

Figure 13.4 A longitudinal pulse along a stretched spring. The displacement of the coils is in the direction of the wave motion. For the starting motion described in the text, the compressed region is followed by a stretched region.

[1]Strictly speaking, the pulse changes its shape and gradually spreads out during the motion. This effect is called *dispersion* and is common to many mechanical waves.

[2]In the case of longitudinal waves in a gas, each compressed area is a region of higher-than-average pressure and density, and each stretched region is a region of lower-than-average pressure and density.

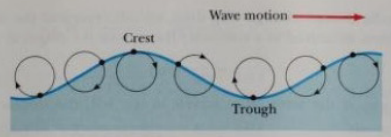


Figure 13.5 Wave motion on the surface of water. The molecules at the water's surface move in nearly circular paths. Each molecule is displaced horizontally and vertically from its equilibrium position, represented by circles.

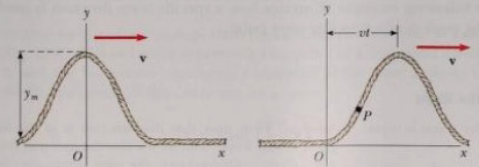
CONCEPTUAL PROBLEM 1

In a long line of people waiting to buy tickets at a movie theater, when the first person leaves, a pulse of motion occurs, as people step forward to fill in the gap. The gap moves through the line of people. What determines the speed of this pulse? Is it transverse or longitudinal? How about "the wave" at a baseball game, where people in the stands stand up and shout as the wave arrives at their location, and this pulse moves around the stadium—what determines the speed of this pulse? Is it transverse or longitudinal?

13.3 · ONE-DIMENSIONAL TRANSVERSE TRAVELING WAVES

So far we have provided only verbal and graphical descriptions of a traveling wave. Let us now develop a mathematical description of a one-dimensional traveling wave. Consider a wave pulse traveling to the right with constant speed, v, along a long, stretched string, as in Figure 13.6. The pulse moves along the x axis (the axis of the string), and the transverse (up-and-down) displacement of the string is measured with the coordinate y.

Figure 13.6a represents the shape and position of the pulse at time $t = 0$. At this time, the shape of the pulse, whatever it may be, can be represented as $y = f(x)$. That is, y is some definite function of x. The *maximum displacement* of the string, y_m, is called the **amplitude** of the wave. Because the speed of the wave pulse is v, the pulse travels to the right a distance of vt in the time t (Fig. 13.6b). If the shape



(a) Pulse at $t = 0$ (b) Pulse at time t

Figure 13.6 A one-dimensional wave pulse traveling to the right with a speed v. (a) At $t = 0$, the shape of the pulse is given by $y = f(x)$. (b) At some later time t, the shape remains unchanged and the vertical displacement of any point P of the medium is given by $y = f(x - vt)$.

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IPR2023-00783

of the wave pulse doesn't change with time, we can represent the displacement y for all later times, measured in a stationary frame with the origin at O, as

Pulse traveling to the right •

$$y = f(x - vt) \qquad [13.1]$$

In the same way, if the wave pulse travels to the left, the displacement of the string is

Pulse traveling to the left •

$$y = f(x + vt) \qquad [13.2]$$

The displacement y, sometimes called the **wave function,** depends on the two variables x and t. For this reason, it is often written $y(x, t)$, which is read "y as a function of x and t." It is important to understand the meaning of y.

Consider a point, P, on the string, identified by a particular value of its coordinates. As the wave pulse passes through P, the y coordinate of this point increases, reaches a maximum, and then decreases to zero. **The wave function $y(x, t)$ represents the y coordinate of any point P at any time t.** Furthermore, if t is fixed, then the wave function y as a function of x, sometimes called the *waveform*, defines a curve representing the actual shape of the pulse at that time. This is equivalent to a snapshot of the pulse at time t.

For a pulse that moves without changing its shape, the velocity of the pulse is the same as the velocity of any point along the pulse, such as the crest. To find the velocity of the pulse, we can calculate how far the crest moves horizontally in a short time and then divide that distance by the time interval. In order to follow the motion of the crest, some particular value, say x_0, must be substituted for $x - vt$ in Equation 13.1. (The value x_0 is called the *argument* of the function y.) Regardless of how x and t change individually, we must require that $x - vt = x_0$ in order to stay with the crest. This equation, therefore, represents the motion of the crest. At $t = 0$, the crest is at $x = x_0$; after an interval of dt, the crest is at $x = x_0 + v\,dt$. Therefore, in the time dt the crest has moved a distance of $dx = (x_0 + v\,dt) - x_0 = v\,dt$. Clearly, the wave speed, often called the **phase speed,** is

Phase speed •

$$v = \frac{dx}{dt} \qquad [13.3]$$

The *wave velocity*, or *phase velocity*, must not be confused with the *transverse velocity* of a particle in the medium (which is always perpendicular to the wave velocity). The phase velocity is the same at any point along the pulse.

The following example illustrates how a specific wave function is used to describe the motion of a traveling wave pulse.

Example 13.1 A Pulse Moving to the Right

A wave pulse moving to the right along the x axis is represented by the wave function

$$y(x, t) = \frac{2}{(x - 3.0t)^2 + 1}$$

where x and y are measured in centimeters and t is in seconds. Let us plot the waveform at $t = 0$, $t = 1.0$ s, and $t = 2.0$ s.

Solution First, note that this function is of the form $y = f(x - vt)$. By inspection, we see that the speed of the wave is $v = 3.0$ cm/s. Furthermore, the wave amplitude (the maximum value of y) is given by $A = 2.0$ cm. The location of the peak of the pulse occurs at the value of x for which the denominator is a minimum, that is, where $(x - 3.0t) = 0$. At times $t = 0$, $t = 1.0$ s, and $t = 2.0$ s, the wave function expressions are

$$y(x, 0) = \frac{2}{x^2 + 1} \quad \text{at } t = 0$$

$$y(x, 1.0) = \frac{2}{(x - 3.0)^2 + 1} \quad \text{at } t = 1.0 \text{ s}$$

$$y(x, 2.0) = \frac{2}{(x - 6.0)^2 + 1} \quad \text{at } t = 2.0 \text{ s}$$

We can now use these expressions to plot the wave function versus x at these times. For example, let us evaluate $y(x, 0)$ at $x = 0.50$ cm:

$$y(0.50, 0) = \frac{2}{(0.50)^2 + 1} = 1.6 \text{ cm}$$

Likewise, $y(1.0, 0) = 1.0$ cm, $y(2.0, 0) = 0.40$ cm, and so on. A continuation of this procedure for other values of x yields the waveform shown in Figure 13.7a. In a similar manner, one obtains the graphs of $y(x, 1.0)$ and $y(x, 2.0)$, shown in Figures 13.7b and 13.7c, respectively. These snapshots show that the wave pulse moves to the right without changing its shape and has a constant speed of 3.0 cm/s.

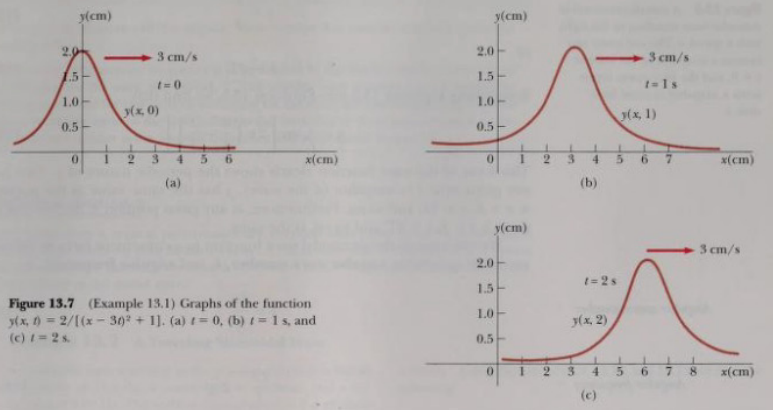


Figure 13.7 (Example 13.1) Graphs of the function $y(x, t) = 2/[(x - 3t)^2 + 1]$. (a) $t = 0$, (b) $t = 1$ s, and (c) $t = 2$ s.

13.4 • SINUSOIDAL TRAVELING WAVES

In this section we introduce an important periodic waveform known as a **sinusoidal wave** (Fig. 13.8). The red curve represents a snapshot of a sinusoidal traveling wave at $t = 0$, and the blue curve represents a snapshot of the wave at some later t. At $t = 0$, the displacement of the curve can be written

$$y = A \sin\left(\frac{2\pi}{\lambda} x\right) \qquad [13.4]$$

where the amplitude, A, as usual, represents the maximum value of the displacement, and λ is the wavelength as defined in Figure 13.1a. Thus, we see that the displacement repeats itself when x is increased by an integral multiple of λ. If the wave moves to the right with a phase speed of v, the wave function at some later t is

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IPR2023-00783

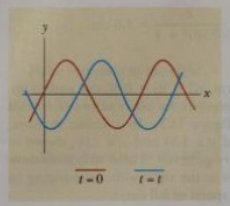


Figure 13.8 A one-dimensional sinusoidal wave traveling to the right with a speed v. The red curve represents a snapshot of the wave at $t = 0$, and the blue curve represents a snapshot at some later time t.

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] \qquad [13.5]$$

That is, the sinusoidal wave moves to the right a distance of vt in the time t, as in Figure 13.8. Note that the wave function has the form $f(x - vt)$ and represents a wave traveling to the right. If the wave were traveling to the left, the quantity $x - vt$ would be replaced by $x + vt$, just as in the case of the traveling pulse described by Equations 13.1 and 13.2.

Because the period T is the time it takes the wave to travel a distance of one wavelength, the phase speed, wavelength, and period are related by

$$v = \frac{\lambda}{T} \qquad [13.6]$$

or

$$\lambda = vT \qquad [13.7]$$

Substituting Equation 13.6 into Equation 13.5, we find that

$$y = A \sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right] \qquad [13.8]$$

This form of the wave function clearly shows the periodic nature of y. That is, at any given time t (a snapshot of the wave), y has the same value at the positions x, $x + \lambda$, $x + 2\lambda$, and so on. Furthermore, at any given position x, the value of y at times t, $t + T$, $t + 2T$, and so on, is the same.

We can express the sinusoidal wave function in a convenient form by defining two other quantities: **angular wave number,** k, and **angular frequency,** ω:

Angular wave number •

$$k \equiv \frac{2\pi}{\lambda} \qquad [13.9]$$

Angular frequency •

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \qquad [13.10]$$

Note that in Equation 13.10, we used the definition of frequency, $f = 1/T$. Using these definitions, we see that Equation 13.8 can be written in the more compact form

$$y = A \sin(kx - \omega t) \qquad [13.11]$$

We shall use this form most frequently.

Using Equations 13.9 and 13.10, we can express the phase speed, v, in the alternative forms

$$v = \frac{\omega}{k} \qquad [13.12]$$

Speed of a traveling sinusoidal wave •

$$v = f\lambda \qquad [13.13]$$

The wave function given by Equation 13.11 assumes that the displacement, y, is zero at $x = 0$ and $t = 0$. This need not be the case. If the transverse displacement is not zero at $x = 0$ and $t = 0$, we generally express the wave function in the form

$$y = A \sin(kx - \omega t - \phi) \qquad [13.14]$$

• *Wave function for a traveling sinusoidal wave*

where ϕ is called the **phase constant** and can be determined from the initial conditions.

Thinking Physics 2

The quantity that we call the angular wave number k is sometimes called *spatial frequency*. Why?

Reasoning The angular frequency ω is 2π divided by the *time* interval for one cycle of oscillation for the wave, the period, T. The angular wave number k is 2π divided by the *space* interval for one cycle of oscillation for the wave, the wavelength, λ (the spatial interval of one cycle of the wave). Due to the similarity in these parameters, it is reasonable to consider ω the *temporal* frequency and k the *spatial* frequency.

CONCEPTUAL PROBLEM 2

Sound waves from a musical performance are encoded onto radio waves for transmission from the radio studio to your home receiver. A radio wave with a wavelength of 3 meters carries a sound wave that has a wavelength of 3 meters in air. Which has the higher frequency, the radio wave or the sound wave?

Example 13.2 A Traveling Sinusoidal Wave

A sinusoidal wave traveling in the positive x direction has an amplitude of 15.0 cm, a wavelength of 40.0 cm, and a frequency of 8.00 Hz. The vertical displacement of the medium at $t = 0$ and $x = 0$ is also 15.0 cm, as shown in Figure 13.9. (a) Find the angular wave number, period, angular frequency, and speed of the wave.

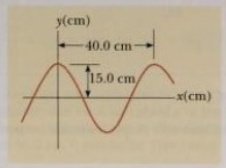


Figure 13.9 (Example 13.2) A sinusoidal wave of wavelength $\lambda = 40.0$ cm and amplitude $A = 15.0$ cm. The wave function can be written in the form $y = A\cos(kx - \omega t)$.

Solution Using Equations 13.9, 13.10, and 13.13, we find the following:

$$k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{40.0 \text{ cm}} = 0.157 \text{ rad/cm}$$

$$T = \frac{1}{f} = \frac{1}{8.00 \text{ s}^{-1}} = 0.125 \text{ s}$$

$$\omega = 2\pi f = 2\pi(8.00 \text{ s}^{-1}) = 50.3 \text{ rad/s}$$

$$v = f\lambda = (8.00 \text{ s}^{-1})(40.0 \text{ cm}) = 320 \text{ cm/s}$$

(b) Determine the phase constant ϕ, and write a general expression for the wave function.

Solution Because $A = 15.0$ cm and because it is given that $y = 15.0$ cm at $x = 0$ and $t = 0$, substitution into Equation 13.14 gives

$$15 = 15 \sin(-\phi) \quad \text{or} \quad \sin(-\phi) = 1$$

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Because $\sin(-\phi) = -\sin\phi$, we see that $\phi = -\pi/2$ rad (or $-90°$). Hence, the wave function is of the form

$$y = A\sin\left(kx - \omega t + \frac{\pi}{2}\right) = A\cos(kx - \omega t)$$

That the wave function must have this form can be seen by inspection, noting that the cosine argument is displaced by 90° from the sine function. Substituting the values for A, k, and ω into this expression gives

$$y = (15.0\text{ cm})\cos(0.157x - 50.3t)$$

Sinusoidal Waves on Strings

One method of producing a traveling sinusoidal wave on a very long string is shown in Figure 13.10. One end of the string is connected to a blade that is set in vibration. As the blade oscillates vertically with simple harmonic motion, a traveling wave moving to the right is set up on the string. Figure 13.10 represents snapshots of the wave at intervals of one quarter of a period. Note that **each particle of the string, such as *P*, oscillates vertically in the *y* direction with simple harmonic motion.** This must be the case because each particle follows the simple harmonic motion of the blade. Therefore, every segment of the string can be treated as a simple harmonic oscillator vibrating with a frequency equal to the frequency of vibration of the blade that drives the string.[3] Note that although each segment oscillates in the y direction, the wave (or disturbance) travels in the x direction with speed v.

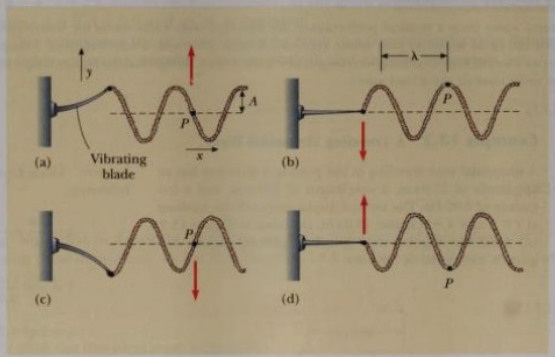


Figure 13.10 One method for producing a train of sinusoidal wave pulses on a continuous string. The left end of the string is connected to a blade that is set into vibration. Every segment of the string, such as the point *P*, oscillates with simple harmonic motion in the vertical direction.

[3]In this arrangement, we are assuming that the segment always oscillates in a vertical line. The tension in the string would vary if the segment were allowed to move sideways. Such a motion would make the analysis very complex.

Of course, this is the definition of a transverse wave. In this case, the energy carried by the traveling wave is supplied by the vibrating blade.

If the waveform at $t = 0$ is as described in Figure 13.10b, then the wave function can be written

$$y = A\sin(kx - \omega t)$$

We can use this expression to describe the motion of any point on the string. The point P (or any other point on the string) moves vertically, and so its x coordinate *remains constant.* Therefore, the **transverse speed,** v_y, of the point P (not to be confused with the wave speed, v) and its **transverse acceleration,** a_y, are

$$v_y = \left.\frac{dy}{dt}\right]_{x=\text{constant}} = \frac{\partial y}{\partial t} = -\omega A\cos(kx - \omega t) \qquad [13.15]$$

$$a_y = \left.\frac{dv_y}{dt}\right]_{x=\text{constant}} = \frac{\partial v_y}{\partial t} = -\omega^2 A\sin(kx - \omega t) \qquad [13.16]$$

The maximum values of these quantities are simply the absolute values of the coefficients of the cosine and sine functions:

$$(v_y)_{\max} = \omega A \qquad [13.17]$$

$$(a_y)_{\max} = \omega^2 A \qquad [13.18]$$

You should recognize that the transverse speed and transverse acceleration of any point on the string do not reach their maximum values simultaneously. In fact, the transverse speed reaches its maximum value (ωA) when the displacement $y = 0$, whereas the transverse acceleration reaches its maximum value ($\omega^2 A$) when $y = -A$. Finally, Equations 13.17 and 13.18 are identical to the corresponding equations for simple harmonic motion.

Example 13.3 A Sinusoidally Driven String

The string shown in Figure 13.10 is driven at a frequency of 5.00 Hz. The amplitude of the motion is 12.0 cm, and the wave speed is 20.0 m/s. Furthermore, the wave is such that $y = 0$ at $x = 0$ and $t = 0$. Determine the angular frequency and wave number for this wave, and write an expression for the wave function.

Solution Using Equations 13.10 and 13.12 gives

$$\omega = \frac{2\pi}{T} = 2\pi f = 2\pi(5.00\text{ Hz}) = 31.4\text{ rad/s}$$

$$k = \frac{\omega}{v} = \frac{31.4\text{ rad/s}}{20.0\text{ m/s}} = 1.57\text{ rad/m}$$

Because $A = 12.0\text{ cm} = 0.120\text{ m}$, we have

$$y = A\sin(kx - \omega t) = (0.120\text{ m})\sin(1.57x - 31.4t)$$

EXERCISE 1 Calculate the maximum values for the transverse speed and transverse acceleration of any point on the string. Answer 3.77 m/s; 118 m/s²

EXERCISE 2 When a particular wire is vibrating with a frequency of 4.00 Hz, a transverse wave of wavelength 60.0 cm is produced. Determine the speed of wave pulses along the wire. Answer 2.40 m/s

EXERCISE 3 For a certain transverse wave, the distance between two successive maxima is 1.2 m and eight maxima pass a given point along the direction of travel every 12 s. Calculate the wave speed. Answer 0.80 m/s

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IPR2023-00783

13.5 · SUPERPOSITION AND INTERFERENCE OF WAVES

Many interesting wave phenomena in nature cannot be described by a single moving pulse. Instead, one must analyze complex waveforms in terms of a combination of many traveling waves. To analyze such wave combinations, one can make use of the **superposition principle,** which states that

Linear waves obey the superposition principle.

> if two or more traveling waves are moving through a medium, the resultant wave function at any point is the sum of the wave functions of the individual waves.

This rather striking property is exhibited by many waves in nature. Waves that obey this principle are called *linear waves,* and they are generally characterized by small amplitudes. Waves that violate the superposition principle are called *nonlinear waves* and are often characterized by large amplitudes. In this book, we shall deal only with linear waves.

One consequence of the superposition principle is that **two traveling waves can pass through each other without being destroyed or even altered.** For instance, when two pebbles are thrown into a pond, the expanding circular surface waves do not destroy each other. In fact, the ripples pass through each other. The complex pattern that is observed can be viewed as two independent sets of expanding circles. Likewise, when sound waves from two sources move through air, they

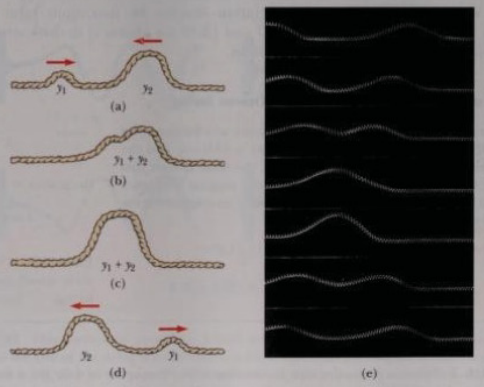


Figure 13.11 *(Left)* Two wave pulses traveling on a stretched string in opposite directions pass through each other. When the pulses overlap, as in (b) and (c), the net displacement of the string equals the sum of the displacements produced by each pulse. Because each pulse produces positive displacements of the string, we refer to their superposition as *constructive interference.* *(Right)* Photograph of superposition of two equal and symmetric pulses traveling in opposite directions on a stretched spring. *(Photo, Education Development Center, Newton, Mass.)*

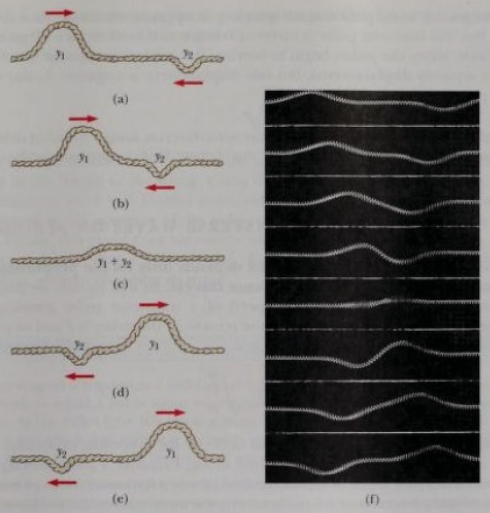


Figure 13.12 *(Left)* Two wave pulses traveling in opposite directions with displacements that are inverted relative to each other. When the two overlap as in (c), their displacements subtract from each other. *(Right)* Photograph of superposition of two symmetric pulses traveling in opposite directions, where one is inverted relative to the other. *(Photo, Education Development Center, Newton, Mass.)*

also can pass through each other. The sound one hears at a given point is the resultant of both disturbances.

A simple pictorial representation of the superposition principle is obtained by considering two pulses traveling in opposite directions on a stretched string, as in Figure 13.11. The wave function for the pulse moving to the right is y_1, and the wave function for the pulse moving to the left is y_2. The pulses have the same speed but different shapes. Each pulse is assumed to be symmetric (although this is not a necessary condition), and in both cases the vertical displacements of the string are taken to be positive. When the waves begin to overlap (Fig. 13.11b), the resulting complex waveform is given by $y_1 + y_2$. When the crests of the pulses exactly coincide (Fig. 13.11c), the resulting waveform, $y_1 + y_2$, is symmetric. The two pulses finally separate and continue moving in their original directions (Fig. 13.11d). Note that the final waveforms remain unchanged, as if the two pulses had never met! The combination of separate waves in the same region of space to produce a resultant wave is called **interference.**

For the two pulses shown in Figure 13.11, the vertical displacements caused by the pulses are in the same direction, and so the resultant waveform (when the pulses overlap) exhibits a displacement greater than those of the individual pulses. Now



Interference patterns produced by outward spreading waves from several drops of water falling into a pond. *(Martin Dohrn/SPL/Photo Researchers)*

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IPR2023-00783

consider two identical pulses, again traveling in opposite directions on a stretched string, but this time one pulse is inverted relative to the other, as in Figure 13.12. In this case, when the pulses begin to overlap, the resultant waveform is the sum of the two separate displacements, but one displacement is negative. Again, the two pulses pass through each other.

EXERCISE 4 Two waves are traveling in the same direction along a stretched string. Each has an amplitude of 4.0 cm and they are 90° out of phase. Find the amplitude of the resultant wave. Answer 5.7 cm

13.6 · THE SPEED OF TRANSVERSE WAVES ON STRINGS

For linear mechanical waves, **the speed depends only on the properties of the medium through which the disturbance travels.** In this section we shall focus our attention on determining the speed of a transverse pulse traveling on a stretched string. We shall show that if the tension in the string is F and its mass per unit length is μ, then the wave speed, v, is

Speed of a wave on a stretched string

$$v = \sqrt{\frac{F}{\mu}} \qquad [13.19]$$

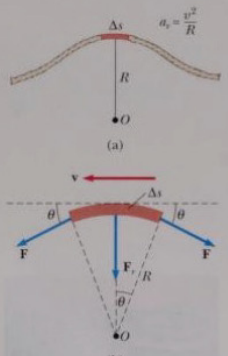


Figure 13.13 (a) To obtain the speed v of a wave on a stretched string, it is convenient to describe the motion of a small segment of the string in a moving frame of reference. (b) The net force on a small segment of length Δs is in the radial direction. The horizontal components of the tension force cancel.

Now let us use a mechanical analysis to derive the preceding expression for the speed of a pulse traveling on a stretched string. Consider a pulse moving to the right with a uniform speed of v, measured relative to a stationary frame of reference. Instead of using a stationary frame, however, it is more convenient to choose as our reference frame one that moves along with the pulse at the same speed, so that the pulse appears to be at rest in the frame, as in Figure 13.13a. This is permitted because Newton's laws are valid in either a stationary frame or one that moves with constant velocity. A small segment of the string, of length Δs, forms the approximate arc of a circle of radius R, as shown in Figure 13.13a and magnified in Figure 13.13b. In the pulse's frame of reference, the segment moves to the left with the speed v. This segment has a centripetal acceleration of v^2/R, which is supplied by the tension, F, in the string. The force **F** acts on each side of the segment, tangent to the arc, as in Figure 13.13b. The horizontal components of **F** cancel, and each vertical component $F \sin \theta$ acts radially inward toward the center of the arc. Hence, the total radial force is $2F \sin \theta$. Because the segment is small, θ is small, and we can use the small-angle approximation $\sin \theta \approx \theta$. Therefore, the total radial force can be expressed as

$$F_r = 2F \sin \theta \approx 2F\theta$$

The segment has the mass $m = \mu \, \Delta s$, where μ is the mass per unit length of the string. Because the segment forms part of a circle and subtends an angle of 2θ at the center, $\Delta s = R(2\theta)$, and hence

$$m = \mu \, \Delta s = 2\mu R\theta$$

If we apply Newton's second law to this segment, the radial component of motion gives

$$F_r = \frac{mv^2}{R} \quad \text{or} \quad 2F\theta = \frac{2\mu R\theta v^2}{R}$$

where F_r is the force that supplies the centripetal acceleration of the segment and maintains the curvature of this segment.

Solving for v gives

$$v = \sqrt{\frac{F}{\mu}}$$

Notice that this derivation is based on the assumption that the pulse height is small relative to the length of the string. Using this assumption, we were able to use the approximation $\sin \theta \approx \theta$. Furthermore, the model assumes that the tension, F, is not affected by the presence of the pulse, so that F is the same at all points on the string. Finally, this proof does *not* assume any particular shape for the pulse. Therefore, we conclude that a pulse of *any shape* will travel on the string with speed $v = \sqrt{F/\mu}$, without changing its shape.

Thinking Physics 3

A secret agent is trapped in a building on top of the elevator car at a lower floor. She attempts to signal a fellow agent on the roof by tapping a message on the elevator cable. As the pulses from this tapping move up the cable to the accomplice, does the speed with which they move stay the same, increase, or decrease? If the pulses are sent one second apart, are they received one second apart by her partner?

Reasoning The elevator cable can be modeled as a heavy string. The speed of waves on the cable is a function of the tension in the cable. As the waves move higher on the cable, they encounter increased tension, because each higher point on the cable must support the weight of all the cable below it (and the elevator). Thus, the speed of the pulses increases as they move higher on the cable. The frequency of the pulses will not be affected, because each pulse takes the same total time to reach the top—they will still arrive at the top of the cable at intervals of one second.

CONCEPTUAL PROBLEM 3

In mechanics, massless strings are often assumed. Why is this not a good assumption when discussing waves on strings?

Example 13.4 The Speed of a Pulse on a Cord

A uniform cord has a mass of 0.300 kg and a total length of 6.00 m. Tension is maintained in the cord by suspending a 2.00-kg mass from one end (Fig. 13.14). Find the speed of a pulse on this cord. Assume the tension is not affected by the mass of the cord.

Solution The tension F in the cord is equal to the weight of the suspended 2.00-kg mass:

$$F = mg = (2.00 \text{ kg})(9.80 \text{ m/s}^2) = 19.6 \text{ N}$$

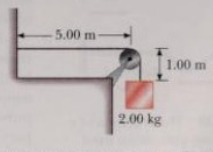


Figure 13.14 (Example 13.4) The tension F in the cord is maintained by the suspended mass. The wave speed is given by the expression $v = \sqrt{F/\mu}$.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

(This calculation of the tension neglects the small mass of the cord. Strictly speaking, the cord can never be exactly horizontal, and therefore the tension is not uniform.)

The mass per unit length μ is

$$\mu = \frac{m}{\ell} = \frac{0.300\ \text{kg}}{6.00\ \text{m}} = 0.0500\ \text{kg/m}$$

Therefore, the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6\ \text{N}}{0.0500\ \text{kg/m}}} = 19.8\ \text{m/s}$$

EXERCISE 5 Show that Equation 13.19 is dimensionally correct using the known dimensions of F and μ.

EXERCISE 6 Transverse waves travel with a speed of 20.0 m/s in a string under a tension of 6.00 N. What tension is required for a wave speed of 30.0 m/s in the same string?
Answer 13.5 N

13.7 · REFLECTION AND TRANSMISSION OF WAVES

Whenever a traveling wave pulse reaches a boundary, part or all of the pulse is *reflected*. Any part not reflected is said to be *transmitted* through the boundary. Consider a pulse traveling on a string that is fixed at one end (Fig. 13.15). When the pulse reaches the fixed boundary, it is reflected. Because the support attaching the string to the wall is rigid, none of the pulse is transmitted through the wall.

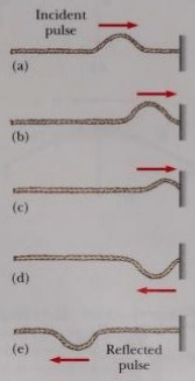


Figure 13.15 The reflection of a traveling wave pulse at the fixed end of a stretched string. The reflected pulse is inverted, but its shape remains the same.

Note that the reflected pulse has exactly the same amplitude as the incoming pulse but is inverted. The inversion can be explained as follows. When the pulse meets the end of the string that is fixed at the support, the string produces an upward force on the support. By Newton's third law, the support must exert an equal and opposite reaction force on the string. This downward force causes the pulse to invert on reflection.

Now consider another situation in which there is total reflection and zero transmission. In this case the pulse arrives at the end of a string that is free to move vertically, as in Figure 13.16. The tension at the free end is maintained by tying the string to a ring of negligible mass that is free to slide vertically on a frictionless post. Again, the pulse is reflected, but this time it is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward. In the process, the ring has upward momentum as it reaches the top of its motion and is then returned to its original position by the downward component of the tension force. This produces a reflected pulse that is not inverted, the amplitude of which is the same as that of the incoming pulse.

Finally, we may have a situation in which the boundary is intermediate between these two extreme cases; that is, it is neither completely rigid nor completely free. In this case, part of the wave is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string, as in Figure 13.17. When a pulse traveling on the light string reaches the boundary between the two strings, part of the pulse is reflected and inverted and part is transmitted to the heavier string. As one would expect, both the reflected pulse and the transmitted one have a smaller amplitude than the incident pulse. The inversion in the reflected wave is similar to the behavior of a pulse meeting a rigid boundary.

When a pulse traveling on a heavy string strikes the boundary of a lighter string, as in Figure 13.18, again part is reflected and part transmitted. This time, however, the reflected pulse is not inverted. In both the lighter-to-heavier case and the heavier-to-lighter case, the relative heights of the reflected and transmitted pulses depend on the relative densities of the two strings. In an extreme case, if the second string were infinitely more dense, it would behave like a rigid wall.

In the preceding section, we found that the speed of a wave on a string increases as the mass per unit length of the string decreases. In other words, a pulse travels more slowly on a heavy string than on a light string if both are under the same tension. The following general rules apply to reflected waves:

> When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is more massive than A), the reflected part of the pulse is inverted on reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is more massive than B), the reflected pulse is not inverted.

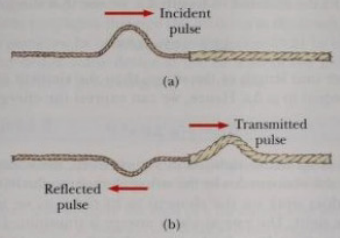


Figure 13.17 (a) A pulse traveling to the right on a light string attached to a heavier string. (b) Part of the incident pulse is reflected (and inverted), and part is transmitted to the heavier string. (Note that the change in pulse width is not shown.)

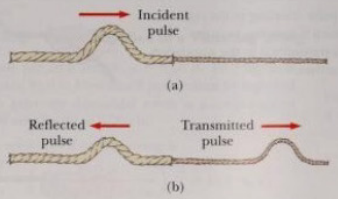


Figure 13.18 (a) A pulse traveling to the right on a heavy string attached to a lighter string. (b) The incident pulse is partially reflected and partially transmitted. In this case, the reflected pulse is not inverted. (Note that the change in pulse width is not shown.)

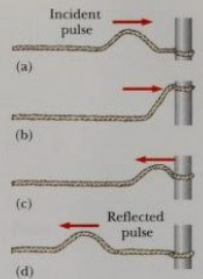


Figure 13.16 The reflection of a traveling wave pulse at the free end of a stretched string. In this case, the reflected pulse is not inverted.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

13.8 • ENERGY TRANSMITTED BY SINUSOIDAL WAVES ON STRINGS

As waves propagate through a medium, they transport energy. This is easily demonstrated by hanging a mass on a stretched string and then sending a pulse down the string, as in Figure 13.19. When the pulse meets the suspended mass, the mass is momentarily displaced, as in Figure 13.19b. In the process, energy is transferred to the mass, because work must be done in moving it upward.

This section examines the rate at which energy is transported along a string. We shall assume that this one-dimensional wave is sinusoidal when we calculate the power transferred. Later we shall extend the same ideas to three-dimensional waves.

Consider a sinusoidal wave traveling on a string (Fig. 13.20). The source of the energy is some external agent at the left end of the string, which does work in producing the oscillations. Let us focus our attention on an element of the string of length Δx and mass Δm. Each such segment moves vertically with simple harmonic motion. Furthermore, each segment has the same angular frequency, ω, and the same amplitude, A. As we found in Chapter 12, the total energy, E, associated with a particle moving with simple harmonic motion is $E = \frac{1}{2}kA^2 = \frac{1}{2}m\omega^2A^2$ (Eq. 12.22), where k is the effective force constant of the restoring force. If we apply this equation to the element of length Δx, we see that the total energy of this element is

$$\Delta E = \tfrac{1}{2}(\Delta m)\omega^2 A^2$$

If μ is the mass per unit length of the string, then the element of length Δx has a mass, Δm, that is equal to $\mu\,\Delta x$. Hence, we can express the energy as

$$\Delta E = \tfrac{1}{2}(\mu\,\Delta x)\omega^2 A^2 \qquad [13.20]$$

If the wave travels from left to right, as in Figure 13.20, the energy ΔE arises from the work done on the element Δm by the string element to the left of Δm. Similarly, the element Δm does work on the element to its right, so we see that energy is transmitted to the right. The rate at which energy is transmitted along the string, or the power (P), is dE/dt. If we let Δx approach 0, Equation 13.20 gives

$$P = \frac{dE}{dt} = \tfrac{1}{2}\left(\mu\,\frac{dx}{dt}\right)\omega^2 A^2$$

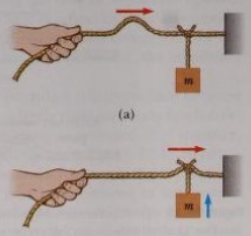


Figure 13.19 (a) A pulse traveling to the right on a stretched string on which a mass has been suspended. (b) Energy is transmitted to the suspended mass when the pulse arrives.

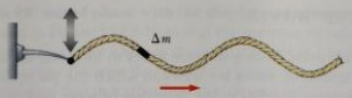


Figure 13.20 A sinusoidal wave traveling along the x axis on a stretched string. Every segment moves vertically, and each segment has the same total energy. The power transmitted by the wave equals the energy contained in one wavelength divided by the period of the wave.

Because dx/dt is equal to the wave speed, v, we have

$$P = \tfrac{1}{2}\mu\omega^2 A^2 v \qquad [13.21] \quad \bullet \textit{ Power}$$

This shows that the power transmitted by a sinusoidal wave on a string is proportional to (1) the wave speed, (2) the square of the frequency, and (3) the square of the amplitude. In fact, *all* sinusoidal waves have the following general property: **The power transmitted by any sinusoidal wave is proportional to the square of the angular frequency and to the square of the amplitude.**

Thus, we see that a wave traveling through a medium corresponds to energy transport through the medium, with no net transfer of matter. An oscillating source provides the energy and produces a harmonic disturbance of the medium. The disturbance is able to propagate through the medium as the result of the interaction between adjacent particles. In order to verify Equation 13.21 by direct experiment, one would have to design some device at the far end of the string to extract the energy of the wave without producing any reflections.

Thinking Physics 4

We have shown that waves carry energy—not only waves on strings, but all waves. Devise a demonstration that shows that waves transfer momentum as well as energy.

Reasoning Imagine that we cover the ends of a cardboard tube with rubber sheets, secured with rubber bands, to form a crude drum with two heads. We now lay the tube horizontally on the table and suspend a Ping-Pong ball from a string so that the Ping-Pong ball is just touching one of the rubber sheets. If we now thump the other rubber sheet with a finger, the sound will travel down the tube, strike the first rubber sheet, and the Ping-Pong ball will bounce away from the rubber sheet. This demonstrates that the sound wave is carrying momentum to the Ping-Pong ball.

Example 13.5 Power Supplied to a Vibrating String

A string of mass per unit length $\mu = 5.00 \times 10^{-2}$ kg/m is under a tension of 80.0 N. How much power must be supplied to the rope to generate sinusoidal waves at a frequency of 60.0 Hz and an amplitude of 6.00 cm?

Solution The wave speed on the string is

$$v = \sqrt{\frac{F}{\mu}} = \left(\frac{80.0\text{ N}}{5.00 \times 10^{-2}\text{ kg/m}}\right)^{1/2} = 40.0\text{ m/s}$$

Because $f = 60.0$ Hz, the angular frequency ω of the sinusoidal waves on the string has the value

$$\omega = 2\pi f = 2\pi(60.0\text{ Hz}) = 377\text{ s}^{-1}$$

Using these values in Equation 13.21 for the power, with $A = 6.00 \times 10^{-2}$ m, gives

$$\begin{aligned}\text{Power} &= \tfrac{1}{2}\mu\omega^2 A^2 v \\ &= \tfrac{1}{2}(5.00 \times 10^{-2}\text{ kg/m})(377\text{ s}^{-1})^2 \\ &\quad \times (6.00 \times 10^{-2}\text{ m})^2(40.0\text{ m/s}) \\ &= 512\text{ W}\end{aligned}$$

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

13.9 • SOUND WAVES

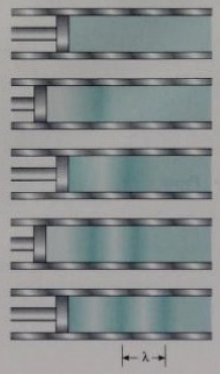


Figure 13.21 A sinusoidal longitudinal wave propagating down a tube filled with a compressible gas. The source of the wave is a vibrating piston at the left. The high- and low-pressure regions are dark and light, respectively.

Let us turn our attention from transverse waves to longitudinal ones. As stated in Section 13.2, longitudinal waves are waves in which the particles of the medium undergo displacements parallel to the direction of wave motion. Sound waves in air are the most important example of longitudinal waves. They can travel through any material medium, their speed depending on the properties of that medium.

The displacements accompanying a sound wave in air are longitudinal displacements of individual molecules from their equilibrium positions. Such displacements result if the source of the waves, such as the diaphragm of a loudspeaker, oscillates with simple harmonic motion. For instance, you can produce a one-dimensional harmonic sound wave in a long, narrow tube containing a gas by means of a vibrating piston at one end, as in Figure 13.21. The darker regions in the figure represent regions in which the gas is compressed and, consequently, the density and pressure are *above* their equilibrium values. Such a compressed layer of gas, called a **condensation,** is formed when the piston is being pushed into the tube. The condensation moves down the tube as a pulse, continuously compressing the layers in front of it. When the piston is withdrawn from the tube, the gas in front of it expands and consequently the pressure and density in this region fall below their equilibrium values. These low-pressure regions, called **rarefactions,** are represented by the lighter areas in Figure 13.21. The rarefactions also propagate along the tube, following the condensations. Both regions move with a speed equal to the speed of sound in that medium. The speed of sound waves in air at 20°C is about 343 m/s.

As the piston oscillates back and forth in a sinusoidal fashion, regions of condensation and rarefaction are continuously set up. The distance between two successive condensations (or two successive rarefactions) equals the wavelength, λ. As these regions travel down the tube, any small volume of the medium moves with simple harmonic motion parallel to the direction of the wave (in other words, longitudinally). If $s(x, t)$ is the displacement of a small volume element measured from its equilibrium position, we can express this displacement function as

$$s(x, t) = s_{max} \cos(kx - \omega t) \qquad [13.22]$$

where s_{max} is the **maximum displacement from equilibrium** (the displacement amplitude), k is the angular wave number, and ω is the angular frequency of the piston. Note that the displacement of the volume element is along x, the direction of motion of the sound wave, which of course means we are describing a longitudinal wave. The variation in the pressure of the gas, ΔP, measured from its equilibrium value is also sinusoidal; it is given by

$$\Delta P = \Delta P_{max} \sin(kx - \omega t) \qquad [13.23]$$

The **pressure amplitude,** ΔP_{max}, is the **maximum change in pressure from the equilibrium value.** It is proportional to the displacement amplitude, s_{max}:

Pressure amplitude •

$$\Delta P_{max} = \rho v \omega s_{max} \qquad [13.24]$$

where ρ is the density of the medium, v is the wave speed, and ωs_{max} is the maximum longitudinal speed of the medium in front of the piston.

Thus, we see that a sound wave may be considered as either a displacement wave or a pressure wave. A comparison of Equations 13.22 and 13.23 shows that **the pressure wave is 90° out of phase with the displacement wave.** Graphs of these functions are shown in Figure 13.22. Note that the pressure variation is a maximum when the displacement is zero, whereas the displacement is a maximum when the pressure variation is zero. Because the pressure is proportional to the density of the medium, the expression that describes how that density varies from its equilibrium value is similar to Equation 13.23.

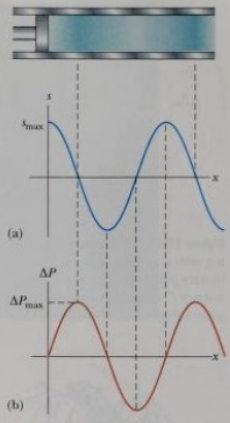


Figure 13.22 (a) Displacement amplitude versus position and (b) pressure amplitude versus position for a sinusoidal longitudinal wave. The displacement wave is 90° out of phase with the pressure wave.

Thinking Physics 5

Garth Brooks had a hit song with "The Thunder Rolls" (Caged Panther Music, Inc., 1990). Why *does* thunder produce an extended "rolling" sound? And how does lightning produce thunder in the first place?

Reasoning Let us assume we are at ground level and neglect ground reflections. When lightning strikes, a channel of ionized air carries a very large electric current from a cloud to the ground. This results in a very rapid temperature increase of this channel of air as the current flows through it. The temperature increase causes a sudden expansion of the air. This expansion is so sudden and so intense that a tremendous disturbance is produced in the air—thunder. The thunder rolls due to the fact that the lightning channel is a long extended source—the entire length of the channel produces the sound at essentially the same instant of time. Sound produced at the bottom of the channel reaches you first, if you are on the ground, because that is the point closest to you, and then sounds from progressively higher portions of the channel reach you. If the lightning channel were a perfectly straight line, the resulting sound might be a steady roar, but the zig-zagged shape of the path results in the rolling variation in loudness, with sound from some portions of the channel arriving at your location simultaneously. This results in periods of loud sounds, interspersed with other instants when the net sound reaching you is low in intensity.

13.10 • THE DOPPLER EFFECT

When a vehicle sounds its horn as it travels along a highway, the frequency of the sound you hear is higher as the vehicle approaches you than it is as the vehicle moves away from you. This is one example of the **Doppler effect,** named after the Austrian physicist Christian Johann Doppler (1803–1853).

In general, a Doppler effect is experienced whenever there is relative motion between the source of sound and the observer. When the source and observer are moving toward each other, the observer hears a frequency that is higher than the true frequency of the source. When the source and observer are moving away from each other, the observer hears a frequency that is lower than the true frequency of the source.[4]

[4]Although the Doppler effect is most commonly experienced with sound waves, it is a phenomenon common to all harmonic waves. For example, a shift in frequencies of light waves (electromagnetic waves) is produced by the relative motion of source and observer. The relativistic Doppler effect demands that there can be no distinction between observer moving toward source and source moving toward observer. In 1842 Doppler first reported the frequency shift in connection with light emitted by stars revolving about each other in double-star systems.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

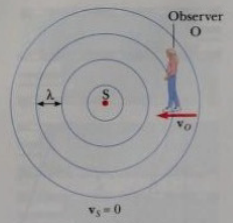


Figure 13.23 An observer O moving with a speed v_O toward a stationary point source S hears a frequency f' that is greater than the source frequency.



"I love hearing that lonesome wail of the train whistle as the magnitude of the frequency of the wave changes due to the Doppler effect."

The Doppler effect for electromagnetic waves is used in police radar systems to measure the speeds of motor vehicles. Likewise, astronomers use the effect to determine the relative motions of stars, galaxies, and other celestial objects.

First, let us consider the case where the observer O is moving with a speed of v_O and the sound source S is stationary, as in Figure 13.23. For simplicity, we shall assume that the air is stationary and that the observer moves directly toward the source (considered as a point source). We shall take the frequency of the source to be f, the wavelength to be λ, and the speed of sound to be v. A stationary observer would detect f vibrations each second, where $f = v/\lambda$. (That is, when the source and observer are both at rest, the observed frequency must equal the true frequency of the source.) However, the observer moving toward the source with the speed v_O receives an additional number of vibrations per second, equal to v_O/λ. Hence, the frequency the observer hears, f', is *increased:* $f' = v/\lambda + v_O/\lambda$. Because $\lambda = v/f$, we can express f' as

$$f' = f\left(\frac{v + v_O}{v}\right) \qquad \text{(Observer moving toward source)} \qquad \textbf{[13.25]}$$

Now consider the situation in which the source moves with a speed of v_S relative to the medium, and the observer is at rest. If the source moves directly toward observer A in Figure 13.24a, the wave fronts seen by the observer are closer to each other than they would be if the source were at rest. As a result, the wavelength λ' measured by observer A is shorter than the true wavelength λ of the source. During each vibration, with a duration of T (the period), the source moves a distance of $v_S T = v_S/f$. Therefore, the wavelength is *shortened* by this amount, and the observed wavelength has the shorter value $\lambda' = \lambda - v_S/f$. Because $\lambda = v/f$, the frequency heard by observer A is

$$f' = \frac{v}{\lambda'} = f\left(\frac{v}{v - v_S}\right) \qquad \text{(source moving toward observer)} \qquad \textbf{[13.26]}$$

That is, the frequency is *increased* when the source moves toward the observer. In a similar manner, if the source moves away from observer B at rest, the sign of v_S is reversed in Equation 13.26.

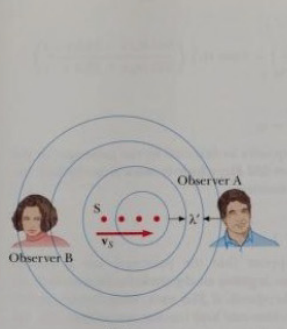
Finally, if both the source and the observer are in motion, one finds the following general formula for the observed frequency:

Frequency heard with observer and source in motion

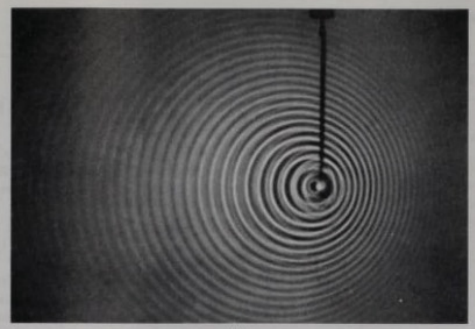
$$f' = f\left(\frac{v \pm v_O}{v \mp v_S}\right) \qquad \textbf{[13.27]}$$

In this expression, the upper signs ($+ v_O$ and $- v_S$) refer to motion of the one *toward* the other, and the lower signs ($- v_O$ and $+ v_S$) refer to motion of one *away from* the other.

When working with any Doppler effect problem, a convenient rule to remember concerning signs is the following: The word *toward* is associated with an *increase* in the observed frequency. The words *away from* are associated with a *decrease* in the observed frequency.



(a)



(b)

Figure 13.24 (a) A source S moving with a speed v_S toward a stationary observer A and away from a stationary observer B. Observer A hears an increased frequency, and observer B hears a decreased frequency. (b) The Doppler effect in water observed in a ripple tank. *(Courtesy Educational Development Center, Newton, Mass.)*

Thinking Physics 6

Suppose you place your stereo speakers far apart and run past them from right to left, or left to right. If you run rapidly enough and have excellent pitch discrimination, you may notice that the music that is playing seems to be out of tune when you are between the speakers. Why?

Reasoning When you are between the speakers, you are running away from one of them and toward the other. Thus, there is a Doppler shift downward for the sound from the speaker behind you and a Doppler shift upward for the sound from the speaker ahead of you. As a result, the sound from the two speakers will not be in tune. A calculation shows that a world-class sprint runner could run fast enough to generate about a semitone difference in the sound from the two speakers.

CONCEPTUAL PROBLEM 4

Suppose the wind blows. Does this cause a Doppler effect for an observer listening to a sound moving through the air? Is it like a moving source or a moving observer?

CONCEPTUAL PROBLEM 5

You are driving toward a cliff and you honk your horn. Is there a Doppler shift of the sound when you hear the echo? Is it like a moving source or moving observer? What if the reflection occurs not from a cliff but from the forward edge of a huge alien spacecraft which is moving toward you as you drive?

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IPR2023-00783

Example 13.6 The Noisy Siren

An ambulance travels down a highway at a speed of 33.5 m/s (75 mi/h). Its siren emits sound at a frequency of 400 Hz. What is the frequency heard by a passenger in a car traveling at 24.6 m/s (55 mi/h) in the opposite direction as the car approaches the ambulance and as the car moves away from the ambulance?

Solution Let us take the velocity of sound in air to be $v = 343$ m/s. We can use Equation 13.27 in both cases. As the ambulance and car approach each other, the observed apparent frequency is

$$f' = f\left(\frac{v + v_O}{v - v_S}\right) = (400 \text{ Hz})\left(\frac{343 \text{ m/s} + 24.6 \text{ m/s}}{343 \text{ m/s} - 33.5 \text{ m/s}}\right)$$

$$= 475 \text{ Hz}$$

Likewise, as they recede from each other, a passenger in the car hears a frequency

$$f' = f\left(\frac{v - v_O}{v + v_S}\right) = (400 \text{ Hz})\left(\frac{343 \text{ m/s} - 24.6 \text{ m/s}}{343 \text{ m/s} + 33.5 \text{ m/s}}\right)$$

$$= 338 \text{ Hz}$$

The *change* in frequency as detected by the passenger in the car is 475 − 338 = 137 Hz, which is more than 30% of the actual frequency emitted.

EXERCISE 7 Suppose that the passenger car is parked on the side of the highway as the ambulance travels down the highway at the speed of 33.5 m/s. What frequency will the passenger in the car hear as the ambulance (a) approaches the parked car and (b) recedes from the parked car? Answer (a) 443 Hz (b) 364 Hz

EXERCISE 8 A band is playing on a moving truck. The band strikes the note middle C (262 Hz), but it is heard by spectators ahead of the truck as C# (277 Hz). How fast is the truck moving? Answer 18.6 m/s

SUMMARY

A **transverse wave** is a wave in which the particles of the medium move in a direction perpendicular to the direction of the wave velocity. An example is a wave on a stretched string.

Longitudinal waves are waves in which the particles of the medium move parallel to the direction of the wave speed. Sound waves in air are longitudinal.

Any one-dimensional wave traveling with a speed of v in the positive x direction can be represented by a wave function of the form $y = f(x - vt)$. Likewise, the wave function for a wave traveling in the negative x direction has the form $y = f(x + vt)$. The shape of the wave at any instant (a snapshot of the wave) is obtained by holding t constant.

The **superposition principle** says that when two or more waves move through a medium, the resultant wave function equals the algebraic sum of the individual wave functions. Waves that obey this principle are said to be *linear*. When two waves combine in space, they interfere to produce a resultant wave.

The wave function for a one-dimensional harmonic wave traveling to the right can be expressed as

$$y = A \sin\left[\frac{2\pi}{\lambda}(x - vt)\right] = A \sin(kx - \omega t) \qquad [13.6, 13.12]$$

where A is the amplitude, λ is the wavelength, k is the angular wave number, and ω is the angular frequency. If T is the period and f is the frequency, then v, k, and ω can be written

$$v = \frac{\lambda}{T} = f\lambda \qquad [13.7, 13.14]$$

$$k \equiv \frac{2\pi}{\lambda} \qquad [13.10]$$

$$\omega \equiv \frac{2\pi}{T} = 2\pi f \qquad [13.11]$$

The speed of a wave traveling on a stretched string of mass per unit length μ and tension F is

$$v = \sqrt{\frac{F}{\mu}} \qquad [13.19]$$

When a pulse traveling on a string meets a fixed end, the pulse is reflected and inverted. If the pulse reaches a free end, it is reflected but not inverted.

The **power** transmitted by a harmonic wave on a stretched string is

$$P = \tfrac{1}{2}\mu\omega^2 A^2 v \qquad [13.21]$$

The change in frequency heard by an observer whenever there is relative motion between a wave source and the observer is called the **Doppler effect.** When the source and observer are moving toward each other, the observer hears a frequency that is higher than the true frequency of the source. When the source and observer are moving away from each other, the observer hears a frequency that is lower than the true frequency of the source.

CONCEPTUAL QUESTIONS

1. How would you set up a longitudinal wave in a stretched spring? Would it be possible to set up a transverse wave in a spring?
2. By what factor would you have to increase the tension in a taut string in order to double the wave speed?
3. When traveling on a taut string, does a wave pulse always invert on reflection? Explain.
4. Does the vertical speed of a segment of a horizontal taut string through which a wave is traveling depend on the wave speed?
5. A vibrating source generates a sinusoidal wave on a string under constant tension. If the power delivered to the string is doubled, by what factor does the amplitude change? Does the wave speed change under these circumstances?
6. Consider a wave traveling on a taut rope. What is the difference, if any, between the speed of the wave and the speed of a small section of the rope?
7. What happens to the wavelength of a wave on a string when the frequency is doubled? What happens to its speed? Assume the tension in the string remains the same.
8. When all the strings on a guitar are stretched to the same tension, will the speed of a wave along the more massive bass strings be faster or slower than the speed of a wave on the lighter strings?
9. If you stretch a rubber hose and pluck it, you can observe a pulse traveling up and down the hose. What happens to the speed if you stretch the hose tighter? If you fill the hose with water?
10. In a longitudinal wave in a spring, the coils move back and forth in the direction of wave motion. Does the speed of the wave depend on the maximum speed of each coil?
11. When two waves interfere, can the amplitude of the resultant wave be larger than either of the two original waves? Under what conditions?
12. A solid can transport both longitudinal waves and transverse waves, but a fluid can transport only longitudinal waves. Why?
13. In an earthquake both *S* (transverse) and *P* (longitudinal) waves are sent out. The *S* waves travel through the Earth more slowly than the *P* waves (4.5 km/s versus 7.8 km/s). By detecting the time of arrival of the waves, how can one determine how far away the epicenter of the quake was? How many detection centers are necessary to pinpoint the location of the epicenter?

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IPR2023-00783

PROBLEMS

Section 13.3 One-Dimensional Transverse Traveling Waves

1. At $t = 0$, a transverse wave pulse in a wire is described by the function

$$y = \frac{6}{x^2 + 3}$$

where x and y are in meters. Write the function $y(x, t)$ that describes this wave if it is traveling in the positive x direction with a speed of 4.50 m/s.

2. Ocean waves with a crest-to-crest distance of 10.0 m can be described by

$$y(x, t) = (0.800 \text{ m})\sin[0.628(x - vt)]$$

where $v = 1.20$ m/s. (a) Sketch $y(x, t)$ at $t = 0$. (b) Sketch $y(x, t)$ at $t = 2.00$ s. Note how the entire waveform has shifted 2.40 m in the positive x direction in this time interval.

3. A wave moving along the x axis is described by

$$y(x, t) = 5.0e^{-(x+5.0t)^2}$$

where x is in meters and t is in seconds. Determine (a) the direction of the wave motion and (b) the speed of the wave.

4. Two wave pulses A and B are moving in opposite directions along a taut string with a speed of 2.00 cm/s. The amplitude of A is twice the amplitude of B. The pulses are shown in Figure P13.4 at $t = 0$. Sketch the shape of the string at $t = 1$, 1.5, 2, 2.5, and 3 s.

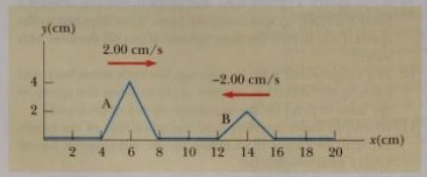


Figure P13.4

Section 13.4 Sinusoidal Traveling Waves

5. A sinusoidal wave is traveling along a rope. The oscillator that generates the wave completes 40.0 vibrations in 30.0 s. Also, a given peak travels 425 cm along the rope in 10.0 s. What is the wavelength?

6. A sinusoidal wave of wavelength 2.00 m and amplitude 0.100 m travels with a speed of 1.00 m/s on a string. Initially, the left end of the string is at the origin and the wave moves from left to right. Find (a) the frequency and angular frequency, (b) the angular wave number, and (c) the wave function for this wave. Determine the equation of motion for (d) the left end of the string, and (e) the point on the string at $x = 1.50$ m to the right of the left end. (f) What is the maximum speed of any point on the string?

7. (a) Write the expression for the function $y(x, t)$ for a sinusoidal wave traveling along a rope in the *negative* x direction with the following characteristics: $A = 8.00$ cm, $\lambda = 80.0$ cm, $f = 3.00$ Hz, and $y(0, 0) = 0$. (b) Write the expression for y as a function of x and t for the wave in part (a) assuming that $y(10 \text{ cm}, 0) = 0$.

8. A transverse wave on a string is described by

$$y = (0.120 \text{ m})\sin \pi(x/8 + 4t)$$

(a) Determine the transverse speed and acceleration of the string at $t = 0.200$ s for the point on the string located at $x = 1.60$ m. (b) What are the wavelength, period, and speed of propagation of this wave?

9. A wave is described by $y = (2.00 \text{ cm})\sin(kx - \omega t)$, where $k = 2.11$ rad/m, $\omega = 3.62$ rad/s, x is in meters, and t is in seconds. Determine the amplitude, wavelength, frequency, and speed of the wave.

10. A sinusoidal wave traveling in the $-x$ direction (to the left) has an amplitude of 20.0 cm, a wavelength of 35.0 cm, and a frequency of 12.0 Hz. The displacement of the wave at $t = 0$, $x = 0$ is $y = -3.00$ cm and has a positive velocity here. (a) Sketch the wave at $t = 0$. (b) Find the angular wave number, period, angular frequency, and phase velocity of the wave. (c) Write an expression for the wave function $y(x, t)$.

11. A transverse sinusoidal wave on a string has a period $T = 25.0$ ms and travels in the negative x direction with a speed of 30.0 m/s. At $t = 0$, a particle on the string at $x = 0$ has a displacement of 2.00 cm and is moving in the negative y direction with a speed of 2.00 m/s. (a) What is the amplitude of the wave? (b) What is the initial phase angle? (c) What is the maximum transverse speed of the string? (d) Write the wave function for the wave.

Section 13.5 Superposition and Interference of Waves

12. Two pulses traveling on the same string are described by

$$y_1 = \frac{5}{(3x - 4t)^2 + 2} \quad \text{and} \quad y_2 = \frac{-5}{(3x + 4t - 6)^2 + 2}$$

(a) In which direction does each pulse travel? (b) At what time do the two cancel? (c) At what point do the two waves always cancel?

13. Two sinusoidal waves in a string are defined by the functions

$$y_1 = (2.00 \text{ cm})\sin(20.0x - 30.0t)$$

$$y_2 = (2.00 \text{ cm})\sin(25.0x - 40.0t)$$

where y and x are in centimeters and t is in seconds. (a) What is the phase difference between these two waves at the point $x = 5.00$ cm at $t = 2.00$ s? (b) What is the positive x value closest to the origin for which the two phases differ by π at $t = 2.00$ s? (This is where the two waves add to zero.)

Section 13.6 The Speed of Transverse Waves on Strings

14. A piano string of mass-per-length 5.00×10^{-3} kg/m is under a tension of 1350 N. Find the speed with which a wave travels on this string.

15. A phone cord is 4.00 m long. The cord has a mass of 0.200 kg. If a transverse wave pulse travels from the receiver to the phone box in 0.100 s, what is the tension in the cord?

16. A 30.0-m steel wire and a 20.0-m copper wire, both with 1.00-mm diameters, are connected end to end and stretched to a tension of 150 N. How long does it take a transverse wave to travel the entire length of the two wires?

17. Transverse pulses travel with a speed of 200 m/s along a taut copper wire the diameter of which is 1.50 mm. What is the tension in the wire? (The density of copper is 8.92 g/cm^3.)

18. A light string of mass-per-unit length 8.00 g/m has its ends tied to two walls separated by a distance equal to three fourths the length of the string (Fig. P13.18). A mass m is suspended from the center of the string, putting a tension in the string. (a) Find an expression for the transverse wave speed in the string as a function of the hanging mass.

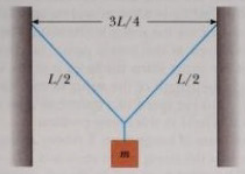


Figure P13.18

(b) How much mass should be suspended from the string to have a wave speed of 60.0 m/s?

19. An astronaut on the Moon wishes to measure the local value of the free-fall acceleration g_{Moon} by timing pulses traveling down a wire that has a large mass suspended from it. Assume a wire of mass 4.00 g is 1.60 m long and has a 3.00 kg mass suspended from it. A pulse requires 36.1 ms to traverse the length of the wire. Calculate g_{Moon} from these data. (You may neglect the mass of the wire when calculating the tension in it.)

Section 13.8 Energy Transmitted by Sinusoidal Waves on Strings

20. A taut rope has a mass of 0.180 kg and a length of 3.60 m. What power must be supplied to the rope in order to generate sinusoidal waves having an amplitude of 0.100 m and a wavelength of 0.500 m and traveling with a speed of 30.0 m/s?

21. Sinusoidal waves 5.00 cm in amplitude are to be transmitted along a string of linear density 4.00×10^{-2} kg/m. If the maximum power delivered by the source is 300 W and the string is under a tension of 100 N, what is the highest vibrational frequency at which the source can operate?

22. It is found that a 6.00-m segment of a long string contains four complete waves and has a mass of 180 g. The string is vibrating sinusoidally with a frequency of 50.0 Hz and a peak-to-valley displacement of 15.0 cm. (*Peak-to-valley* means the vertical distance from the farthest positive displacement to the farthest negative displacement.) (a) Write down the function that describes this wave traveling in the positive x direction. (b) Determine the power being supplied to the string.

23. A horizontal string can transmit a maximum power of P_0 (without breaking) if a wave with amplitude A and angular frequency ω is traveling along it. In order to increase this maximum power, a student folds the string and uses this double string as a transmitter. Determine the maximum power that can be transmitted along the double string, assuming that the tension remains the same.

24. A two-dimensional water wave spreads in circular wave fronts. Show that the amplitude A at a distance r from the initial disturbance is proportional to $1/\sqrt{r}$. (*Hint:* Consider the energy concentrated in the outward-moving ripple.)

Section 13.9 Sound Waves

(In this section, use the following values as needed unless otherwise specified: the equilibrium density of air, $\rho = 1.20$ kg/m^3; the speed of sound in air, $v = 343$ m/s. Also, pressure variations, ΔP, are measured relative to atmospheric pressure.)

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IPR2023-00783

25. A flowerpot is knocked off a balcony 20.0 m above the sidewalk heading for a man of height 1.75 m standing below. How high from the ground can the flowerpot be after which it would be too late for a shouted warning to reach the man in time? Assume that the man below requires 0.300 s to respond to the warning.
26. A rescue plane flies horizontally at a constant speed searching for a disabled boat. When the plane is directly above the boat, the boat's crew blows a loud horn. By the time the plane's sound detector perceives the horn's sound, the plane has traveled a distance equal to one-half its altitude above the ocean. If it takes the sound 2.00 s to reach the plane, determine (a) the speed of the plane, and (b) its altitude. Take the speed of sound to be 343 m/s.
27. A sound wave in a cylinder is described by Equations 13.22 through 13.24. Show that $\Delta P = \pm \rho v \omega \sqrt{s_{max}^2 - s^2}$.
28. Calculate the pressure amplitude of a 2.00-kHz sound wave in air if the displacement amplitude is 2.00×10^{-8} m.
29. An experimenter wishes to generate in air a sound wave that has a displacement amplitude of 5.50×10^{-6} m. The pressure amplitude is to be limited to 8.40×10^{-1} N/m^2. What is the minimum wavelength the sound wave can have?
30. A sound wave in air has a pressure amplitude of 4.00 N/m^2 and a frequency of 5.00 kHz. Take $\Delta P = 0$ at the point $x = 0$ when $t = 0$. (a) What is ΔP at $x = 0$ when $t = 2.00 \times 10^{-4}$ s, and (b) what is ΔP at $x = 0.0200$ m when $t = 0$?

Section 13.10 The Doppler Effect

31. A commuter train passes a passenger platform at a constant speed of 40.0 m/s. The train horn is sounded at its characteristic frequency of 320 Hz. (a) What total change in frequency is detected by a person on the platform as the train goes from approaching to receding? (b) What wavelength is detected by a person on the platform as the train approaches?
32. A driver traveling northbound on a highway is driving at a speed of 25.0 m/s. A police car driving southbound at a speed of 40.0 m/s approaches with its siren sounding at a base frequency of 2500 Hz. (a) What frequency is observed by the driver as the police car approaches? (b) What frequency is detected by the driver after the police car passes him? (c) Repeat parts (a) and (b) for the case when the police car is traveling northbound.
33. Standing at a crosswalk, you hear a frequency of 560 Hz from the siren of an approaching police car. After the police car passes, the observed frequency of the siren is 480 Hz. Determine the car's speed from these observations.
34. A tuning fork vibrating at 512 Hz falls from rest and accelerates at 9.80 m/s^2. How far below the point of release is the tuning fork when waves of frequency 485 Hz reach the release point? Take the speed of sound in air to be 340 m/s.
35. A block with a speaker bolted to it is connected to a spring having spring constant $k = 20.0$ N/m as in Figure P13.35. The total mass of the block and speaker is 5.00 kg, and the amplitude of this unit's motion is 0.500 m. If the speaker emits sound waves of frequency 440 Hz, determine the range in frequencies heard by the person to the right of the speaker.



Figure P13.35

Additional Problems

36. Unoccupied by spectators, a large set of football bleachers has solid seats and risers. You stand on the field in front of it and sharply clap two wooden boards together once. You hear from the bleachers a sound with definite pitch, which may remind you of a short toot on a trumpet. Account for this sound. Compute order-of-magnitude estimates for its frequency, wavelength, and duration on the basis of data you specify.
37. A 2.00-kg block hangs from a rubber string, being supported so that the string is not stretched. The unstretched length of the string is 0.500 m and its mass is 5.00 g. The spring constant for the string is 100 N/m. The block is released and stops at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the string in this "stretched" position? (c) Find the speed of a transverse wave in the string if the block is held in this lowest position.
38. A block of mass M hangs from a rubber string, being supported so that the string is not stretched. The unstretched length of the string is L_0 and its mass is m, much less than M. The spring constant for the string is k. The block is released and stops at the lowest point. (a) Determine the tension in the string when the block is at this lowest point. (b) What is the length of the string in this "stretched" position? (c) Find the speed of a transverse wave in the string if the block is held in this lowest position.
39. The wave function for a traveling wave on a taut string is (in SI units)

$$y(x, t) = (0.350 \text{ m}) \sin(10\pi t - 3\pi x + \pi/4)$$

(a) What is the speed and direction of travel of the wave? (b) What is the vertical displacement of the string at $t = 0$, $x = 0.100$ m? (c) What are the wavelength and frequency of the wave? (d) What is the maximum magnitude of the transverse speed of the string?
40. A stone is dropped into a deep canyon and is heard to strike the bottom 10.2 s after release. The speed of sound waves in air is 343 m/s. How deep is the canyon? What would be the percentage error in the depth if the time required for the sound to reach the canyon rim were ignored?
41. The ocean floor is underlain by a layer of basalt that constitutes the crust, or uppermost layer of the Earth, in this region. Below this crust is found denser peridotite rock, which forms the Earth's mantle. The boundary between these two layers is called the *Mohorovičić discontinuity* ("Moho" for short). If an explosive charge is set off at the surface of the basalt, it generates a seismic wave that is reflected back at the Moho. If the speed of this wave in basalt is 6.50 km/s, and the two-way travel time is 1.85 s, what is the thickness of this oceanic crust?
42. A worker strikes a steel pipeline with a hammer, generating both longitudinal and transverse waves. Reflected waves return 2.40 s apart. How far away is the reflection point? (For steel, $v_{long} = 6.20$ km/s and $v_{trans} = 3.20$ km/s.)
43. A rope of total mass m and length L is suspended vertically. Show that a transverse wave pulse will travel the length of the rope in a time $t = 2\sqrt{L/g}$. (*Hint:* First find an expression for the wave speed at any point a distance x from the lower end by considering the tension in the rope as resulting from the weight of the segment below that point.)
44. A train whistle ($f = 400$ Hz) sounds higher or lower in pitch depending on whether it approaches or recedes. (a) Prove that the difference in frequency between the approaching and receding train whistle is

$$\Delta f = \frac{2f\left(\frac{u}{v}\right)}{1 - \frac{u^2}{v^2}} \qquad \begin{array}{l} u = \text{speed of train} \\ v = \text{speed of sound} \end{array}$$

(b) Calculate this difference for a train moving at a speed of 130 km/h. Take the speed of sound in air to be 340 m/s.

45. In order to be able to determine her speed, a skydiver carries a tone generator. A friend on the ground at the landing site has equipment for receiving and analyzing sound waves. While the skydiver is falling at terminal speed, her tone generator emits a steady tone of 1800 Hz. (Assume that the air is calm and that the sound speed is 343 m/s, independent of altitude.) (a) If her friend on the ground (directly beneath the skydiver) receives waves of frequency 2150 Hz, what is the skydiver's speed of descent? (b) If the skydiver were also carrying sound-receiving equipment sensitive enough to detect waves reflected from the ground, what frequency would she receive?
46. A wave pulse traveling along a string of linear mass density is described by the relationship

$$y = [A_0 e^{-bx}] \sin(kx - \omega t)$$

where the factors in brackets before the sine are said to be the amplitude. (a) What is the power $P(x)$ carried by this wave at a point x? (b) What is the power $P(0)$ carried by this wave at the origin? (c) Compute the ratio $P(x)/P(0)$.
47. A bat, moving at 5.00 m/s, is chasing a flying insect. If the bat emits a 40.0-kHz chirp and receives back an echo at 40.4 kHz, at what speed is the insect moving toward or away from the bat? (Take the speed of sound in air to be $v = 340$ m/s.)
48. An earthquake on the ocean floor in the Gulf of Alaska induces a *tsunami* (sometimes called a *tidal wave*) that reaches Hilo, Hawaii, 4450 km distant, in a time of 9 h 30 min. Tsunamis have enormous wavelengths (100 km to 200 km), and for such waves the propagation speed is $v \approx \sqrt{gd}$, where d is the average depth of the water. From the information given, find the average wave speed and the average ocean depth between Alaska and Hawaii. (This method was used in 1856 to estimate the average depth of the Pacific Ocean long before soundings were made to give a direct determination.)

Spreadsheet Problem

S1. Two transverse wave pulses traveling in opposite directions along the x axis are represented by the following wave functions:

$$y_1(x, t) = \frac{6}{(x - 3t)^2} \qquad y_2(x, t) = -\frac{3}{(x + 3t)^2}$$

where x and y are measured in centimeters and t is in seconds. Write a spreadsheet or program to add the two pulses and obtain the shape of the composite waveform $y_{tot} = y_1 + y_2$ as a function of time. Plot y_{tot} versus x. Make separate plots for $t = 0$, 0.5, 1, 1.5, 2, 2.5, and 3.0 s.

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IPR2023-00783

ANSWERS TO CONCEPTUAL PROBLEMS

1. A pulse in a long line of people is longitudinal, since the movement of people is parallel to the direction of propagation of the pulse. The speed is determined by the reaction time of the people and the speed with which they can move, once a space opens up. There is also a psychological factor, in that people will not want to fill a space that opens up in front of them too quickly, so as not to intimidate the person in front of them. The "wave" at a stadium is transverse, since the fans stand up vertically, as the wave sweeps past them horizontally. The speed of this pulse depends on the limits of the fans' abilities to rise and sit rapidly, and on psychological factors associated with the anticipation of seeing the pulse approach the observer's location.
2. Both the radio wave and the sound wave have the same wavelength. The speed of propagation for electromagnetic waves, however, is much higher than the speed of sound, so the frequency of the radio wave is much higher than that of the sound wave. Audible sound frequencies range from about 15 Hz to 15 000 Hz, while radio frequencies are in kilohertz for AM radio and megahertz for FM.
3. A wave on a massless string would have an infinite speed of propagation because its linear mass density is zero.
4. Wind can change a Doppler shift. Both v_O and v_S in our equation must be interpreted as speeds of observer and source relative to air. If source and observer are moving relative to each other, the observer will hear one shifted frequency in still air and a different shifted frequency if wind is blowing. However, if source and observer are at rest relative to each other, there is no shift when wind blows past.
5. The echo *is* Doppler shifted, and the shift is like both a moving source and a moving observer. The sound which leaves your horn in the forward direction is Doppler shifted to a higher frequency, because it is coming from a moving source. As the sound reflects back and comes toward you, you are a moving observer, so there is a second Doppler shift to an even higher frequency. If the sound reflects from the spacecraft moving toward you, there is another moving source shift to an even higher frequency. The reflecting surface of the spacecraft acts as a moving source.

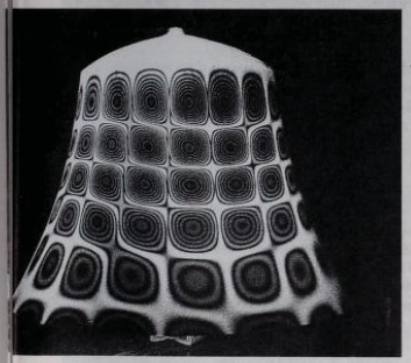
14

Superposition and Standing Waves

An important aspect of waves is their combined effect when two or more travel in the same medium. For instance, what happens to a string when a wave traveling toward a fixed end is reflected back on itself?

In a linear medium—that is, one in which the restoring force of the medium is proportional to the displacement of the medium—one can apply the **principle of superposition** to obtain the resultant disturbance. This principle can be applied to many types of waves under certain conditions, including waves on strings, sound waves, surface water waves, and electromagnetic waves. The superposition principle states that the net displacement of any part of the disturbed medium equals the vector sum of the displacements caused by the individual waves. In Chapter 13, the term **interference** was used to describe the effect produced by combining two waves that are moving simultaneously through a medium.

This chapter is concerned with the superposition principle as it applies to sinusoidal waves. If the sinusoidal waves that com-



◀ **Photograph of standing waves on a vibrating handbell. The photograph was taken using a technique called *time-average holographic interferometry*. Such a hologram consists of millions of superimposed holograms, but this hologram emphasizes the two positions of maximum deflection of the vibrating bell. The pattern recorded here occurred at a frequency of 2684 Hz, but other patterns are observed at lower and higher frequencies.** *(Courtesy of Professor Thomas D. Rossing, Northern Illinois University)*

CHAPTER OUTLINE

14.1 Superposition and Interference of Sinusoidal Waves
14.2 Standing Waves
14.3 Natural Frequencies in a Stretched String
14.4 Standing Waves in Air Columns
14.5 Beats: Interference in Time (Optional)
14.6 Complex Waves (Optional)

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IPR2023-00783

bine in a given medium have the same frequency and wavelength, one finds that a stationary pattern, called a **standing wave,** can be produced at certain frequencies under certain circumstances. For example, a stretched string fixed at both ends has a discrete set of standing wave patterns, called **modes of vibration,** that depend on the tension and the mass-per-unit-length of the string. These modes of vibration are found in the strings of stringed musical instruments. Other musical instruments, such as the organ and flute, make use of the natural frequencies of sound waves in hollow pipes. Such frequencies depend on the length and shape of the pipe and on whether the ends are open or closed.

In this chapter we also consider the superposition and interference of waves with different frequencies and wavelengths. When two sound waves with nearly the same frequency interfere, one hears variations in loudness called *beats.* The beat frequency corresponds to the rate of alternation between constructive and destructive interference. Finally, we describe how any complex periodic waveform can, in general, be described by a sum of sine and cosine functions.

14.1 • SUPERPOSITION AND INTERFERENCE OF SINUSOIDAL WAVES

Let us apply the superposition principle to two sinusoidal waves traveling in the same direction in a medium. If the two waves are traveling to the right and have the same frequency, wavelength, and amplitude but differ in phase, we can express their individual wave functions as

$$y_1 = A\sin(kx - \omega t) \qquad \text{and} \qquad y_2 = A\sin(kx - \omega t - \phi)$$

Hence, the resultant wave function, y, is

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t - \phi)]$$

In order to simplify this expression, it is convenient to use the trigonometric identity

$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

If we let $a = kx - \omega t$ and $b = kx - \omega t - \phi$, the resultant wave, y, reduces to

Resultant of two traveling sinusoidal waves •

$$y = \left(2A\cos\frac{\phi}{2}\right)\sin\left(kx - \omega t - \frac{\phi}{2}\right) \qquad [14.1]$$

This result has several important features. The resultant wave function, y, is also harmonic and has the *same* frequency and wavelength as the individual waves. The amplitude of the resultant wave is $2A\cos(\phi/2)$ and the phase is $\phi/2$. If the phase constant, ϕ, equals 0, then $\cos(\phi/2) = \cos 0 = 1$ and the amplitude of the resultant wave is $2A$. In other words, the amplitude of the resultant wave is twice the amplitude of either individual wave. In this case, the waves are said to be everywhere *in phase* and to thus **interfere constructively.** That is, the crests and troughs of the individual waves occur at the same positions, as is shown by the blue lines in Figure 14.1a. In general, constructive interference occurs when $\cos(\phi/2) = \pm 1$, or when $\phi = 0, 2\pi, 4\pi, \ldots$. However, if ϕ is equal to π radians or to any *odd* multiple of π then $\cos(\phi/2) = \cos(\pi/2) = 0$ and the resultant wave has *zero* amplitude everywhere. In this case, the two waves **interfere destructively.** That is, the crest of one

Constructive interference •

Destructive interference •

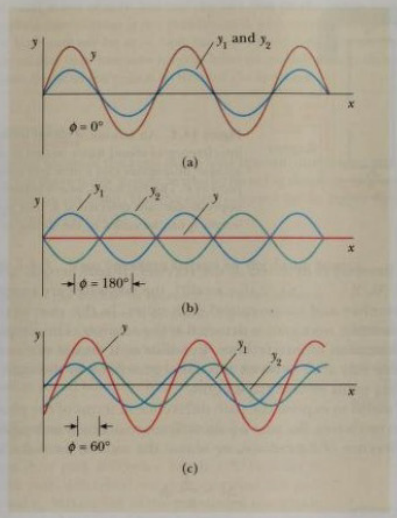


Figure 14.1 The superposition of two waves with amplitudes y_1 and y_2, where $y_1 = y_2$. (a) When the two waves are in phase, the result is constructive interference. (b) When the two waves are 180° out of phase, the result is destructive interference. (c) When the phase angle lies in the range $0 < \phi < 180°$, the resultant y falls somewhere between that shown in (a) and that shown in (b).

wave coincides with the trough of the second (Fig. 14.1b) and their displacements cancel at every point. Finally, when the phase constant has an arbitrary value between 0 and π, as in Figure 14.1c, the resultant wave has an amplitude the value of which is somewhere between 0 and $2A$.

Interference of Waves

One simple device for demonstrating interference of sound waves is illustrated in Figure 14.2. Sound from speaker S is sent into a tube at P, at which point there is a T-shaped junction. Half the sound power travels in one direction, and half in the opposite direction. Thus, the sound waves that reach receiver R at the other side can travel along either of two paths. The total distance from speaker to receiver is called the **path length,** r. The length of the lower path is fixed at r_1. The upper path length, r_2, can be varied by sliding the U-shaped tube (similar to that on a slide trombone). When the difference in the path lengths, $\Delta r = |r_2 - r_1|$, is either zero or some integral multiple of the wavelength λ, the two waves reaching the receiver are in phase and interfere constructively, as in Figure 14.1a. In this case, a maximum

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

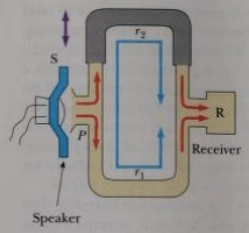


Figure 14.2 An acoustical system for demonstrating interference of sound waves. Sound from the speaker propagates into a tube and splits into two parts at *P*. The two waves, which superimpose at the opposite side, are detected at R. The upper path length r_2 can be varied by the sliding section.

in the sound intensity is detected at the receiver. If path length r_2 is adjusted so that Δr is $\lambda/2, 3\lambda/2, \ldots n\lambda/2$ (for n odd), the two waves are exactly 180° out of phase at the receiver and hence cancel each other. In this case of completely destructive interference, no sound is detected at the receiver. This simple experiment is a striking illustration of interference. In addition, it demonstrates the fact that a phase difference may arise between two waves generated by the same source when they travel along paths of unequal lengths.

It is often useful to express the path difference in terms of the phase difference, ϕ, between the two waves. Because a path difference of one wavelength corresponds to a phase difference of 2π radians, we obtain the ratio $\lambda/2\pi = \Delta r/\phi$, or

Relationship between path difference and phase angle •

$$\Delta r = \frac{\lambda}{2\pi}\phi \qquad [14.2]$$

Nature provides many other examples of interference phenomena. Later in the book we shall describe several interesting interference effects involving light waves.

Thinking Physics 1

If stereo speakers are connected to the amplifier "out of phase," one speaker is moving outward when the other is moving inward. This results in a weakness in the bass notes, which can be corrected by reversing the wires on one of the speaker connections. Why is there only an effect on the bass notes in this case and not the treble notes?

Reasoning Imagine that you are sitting in front of the speakers, midway between them. Then, the sound from each speaker travels the same distance to you, and there is no phase difference in the sound that is due to a path difference. Because the speakers are connected out of phase, the sound waves are half a wavelength out of phase on leaving the speaker and, consequently, on arriving at your ear. As a result, there will be cancellation of the sound for all frequencies in the ideal case of a zero-size head located exactly on the midpoint between the speakers. If the ideal head were moved off the centerline, there will be an additional phase difference introduced by the path-length difference for the sound from the two speakers. In the case of low frequency, long wavelength bass notes, the path-length differences will be a small fraction of a wavelength, so there will still be significant cancellation. For the high-frequency, short-wavelength treble notes, a small movement of the ideal head will result in a much larger fraction of a wavelength in path length difference or even multiple wavelengths. Thus, the treble notes could be in phase with this head movement. If we now add the fact that the head is not of zero size and the fact that there are two ears, we can see that the complete cancellation is not possible, and, with even small movements of the head, one or both ears will be at or near maxima for the treble notes. The size of the head is much smaller than bass wavelengths, however, so there is significant weakening of the bass over much of the region in front of the speakers.

CONCEPTUAL PROBLEM 1

As oppositely moving pulses of the same shape (one upward, one downward) on a string pass through each other, there is one instant at which the string shows no displacement from the equilibrium position at any point. Has the energy carried by the pulses disappeared at this instant in time? If not, where is it?

Example 14.1 Two Speakers Driven by the Same Source

A pair of speakers placed 3.00 m apart are driven by the same oscillator (Fig. 14.3). A listener is originally at point *O*, which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point *P*, which is a perpendicular distance 0.350 m from *O* before reaching the *first minimum* in sound intensity. What is the frequency of the oscillator?

Solution The first minimum occurs when the two waves reaching the listener at *P* are 180° out of phase—in other words, when their path difference equals $\lambda/2$. In order to calculate the path difference, we must first find the path lengths r_1 and r_2. Making use of the two shaded triangles in Figure 14.3, we find the path lengths to be

$$r_1 = \sqrt{(8.00\text{ m})^2 + (1.15\text{ m})^2} = 8.08\text{ m}$$

$$r_2 = \sqrt{(8.00\text{ m})^2 + (1.85\text{ m})^2} = 8.21\text{ m}$$

Hence, the path difference is $r_2 - r_1 = 0.13$ m. Because we require that this path difference be equal to $\lambda/2$ for the first minimum, we find that $\lambda = 0.26$ m.

To obtain the oscillator frequency, we can use $v = \lambda f$, where v is the speed of sound in air, 343 m/s:

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.26\text{ m}} = 1.3\text{ kHz}$$

EXERCISE 1 If the oscillator frequency is adjusted such that the listener hears the first minimum at a distance of 0.75 m from *O*, what is the new frequency? Answer 0.63 kHz

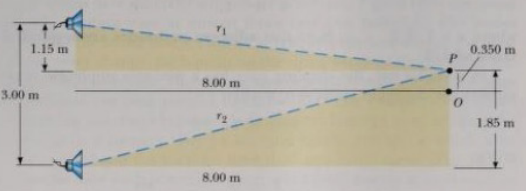


Figure 14.3 (Example 14.1)

14.2 • STANDING WAVES

If a stretched string is clamped at both ends, waves traveling in both directions are reflected from the ends. The incident and reflected waves combine according to the superposition principle.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

Consider two sinusoidal waves, in the same medium, that have the same amplitude, frequency, and wavelength but are traveling in *opposite* directions. If we neglect any phase angle ϕ for simplicity, their wave functions can be written

$$y_1 = A\sin(kx - \omega t) \qquad \text{and} \qquad y_2 = A\sin(kx + \omega t)$$

where y_1 represents a wave traveling to the right and y_2 represents a wave traveling to the left. Adding these two functions gives the resultant wave function, y:

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f$, as usual. Using the trigonometric identity $\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$, this reduces to

Wave function for a standing wave

$$y = (2A\sin kx)\cos \omega t \qquad [14.3]$$

This expression represents the wave function of a **standing wave.** From this result, we see that a standing wave has an angular frequency of ω and an amplitude of $2A\sin kx$ (the quantity in the parentheses of Eq. 14.3). That is, every particle of the string vibrates in simple harmonic motion with the same frequency. However, the amplitude of motion of a given particle depends on x. This is in contrast to the situation involving a traveling sinusoidal wave, in which all particles oscillate with the same amplitude as well as the same frequency.

Because the amplitude of the standing wave at any value of x is equal to $2A\sin kx$, we see that the *maximum* amplitude has the value $2A$. This occurs when the coordinate x satisfies the condition $\sin kx = 1$, or when

$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

Because $k = 2\pi/\lambda$, the positions of maximum amplitude, called **antinodes,** are

Position of antinodes

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \qquad [14.4]$$

where $n = 1, 3, 5, \ldots$. Note that **adjacent antinodes are separated by a distance of $\lambda/2$.**

In a similar way, the standing wave has a *minimum* amplitude of zero when x satisfies the condition $\sin kx = 0$ or when

$$kx = \pi, 2\pi, 3\pi, \ldots$$

giving

Position of nodes

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \qquad [14.5]$$

where $n = 1, 2, 3, \ldots$. **These points of zero amplitude, called nodes, are also spaced by $\lambda/2$.** The distance between a node and an adjacent antinode is $\lambda/4$.

The standing wave patterns produced at various times by two waves traveling in opposite directions are depicted graphically in Figure 14.4. The upper part of each figure represents the individual traveling waves, and the lower part represents the standing wave patterns. The nodes of the standing wave are labeled *N*, and the antinodes are labeled *A*. At $t = 0$ (Fig. 14.4a), the two waves are spatially identical,

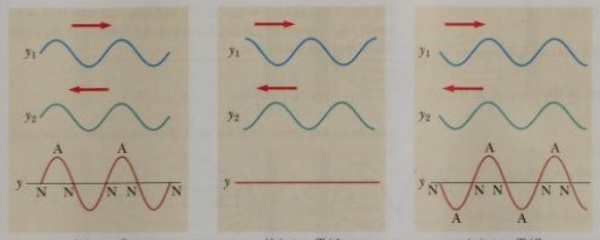


Figure 14.4 Standing wave patterns at various times produced by two waves of equal amplitude traveling in opposite directions. For the resultant wave y, the nodes (N) are points of zero displacement and the antinodes (A) are points of maximum displacement.

giving a standing wave of maximum amplitude, $2A$. One quarter of a period later, at $t = T/4$ (Fig. 14.4b), the individual waves have moved one quarter of a wavelength (one to the right and the other to the left). At this time, the individual displacements are equal and opposite for all values of x, and hence the resultant wave has zero displacement everywhere. At $t = T/2$ (Fig. 14.4c), the individual waves are again identical spatially, producing a standing wave pattern that is inverted relative to the $t = 0$ pattern.

It is instructive to describe the energy associated with the motion of a standing wave. To illustrate this point, consider a standing wave formed on a stretched string that is fixed at each end, as in Figure 14.5. Except for the nodes, which are stationary, all points on the string oscillate vertically with the same frequency. Furthermore, different points have different amplitudes of motion. Figure 14.5 represents snapshots of the standing wave at various times over one half-cycle. The nodes represent points on the string that are always at rest, and the locations of the maxima and minima never change. Each point on the string executes simple harmonic motion in the vertical direction. That is, one can view the standing wave as a large number of oscillators vibrating parallel to each other. The energy of the vibrating string continually alternates between elastic potential energy, at which time the string is momentarily stationary (Fig. 14.5a), and kinetic energy, at which time the string is horizontal and the particles have their maximum speed (Fig. 14.5c). The string particles have both potential energy and kinetic energy at intermediate times (Figs. 14.5b and 14.5d).

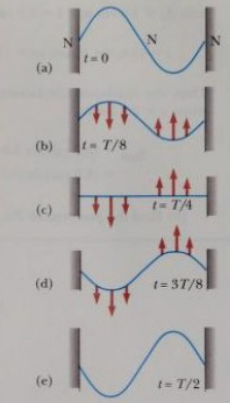


Figure 14.5 A standing wave pattern in a taut string showing snapshots during one half-cycle. (a) At $t = 0$, the string is momentarily at rest, and so $K = 0$ and all of the energy is potential energy U associated with the vertical displacements of the string segments. (b) At $t = T/8$, the string is in motion, and the energy is half kinetic and half potential. (c) At $t = T/4$, the string is horizontal (undeformed) and, therefore, $U = 0$; all of the energy is kinetic. The motion continues as indicated in (d) and (e), and ultimately the initial configuration in part (a) is repeated.

Example 14.2 Formation of a Standing Wave

Two waves traveling in opposite directions produce a standing wave. The individual wave functions are

$$y_1 = (4.0\text{ cm})\sin(3.0x - 2.0t)$$
$$y_2 = (4.0\text{ cm})\sin(3.0x + 2.0t)$$

where x and y are in centimeters. (a) Find the maximum displacement of the motion at $x = 2.3$ cm.

Solution When the two waves are summed, the result is a standing wave the function of which is given by Equation 14.3,

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

with $A_0 = 4.0$ cm and $k = 3.0$ rad/cm:

$$y = (2A_0 \sin kx)\cos \omega t = [(8.0 \text{ cm})\sin 3.0x]\cos \omega t$$

Thus, the maximum displacement of the motion at the position $x = 2.3$ cm is

$$y_{max} = (8.0 \text{ cm})\sin 3.0x]_{x=2.3}$$
$$= (8.0 \text{ cm})\sin(6.9 \text{ rad}) = 4.6 \text{ cm}$$

(b) Find the positions of the nodes and antinodes.

Solution Because $k = 2\pi/\lambda = 3$ rad/cm, we see that $\lambda = 2\pi/3$ cm. Therefore, from Equation 14.4 we find that the antinodes are located at

$$x = n\left(\frac{\pi}{6}\right) \text{ cm} \qquad (n = 1, 3, 5, \ldots)$$

and from Equation 14.5 we find that the nodes are located at

$$x = n\frac{\lambda}{2} = n\left(\frac{\pi}{3}\right) \text{ cm} \qquad (n = 1, 2, 3, \ldots)$$

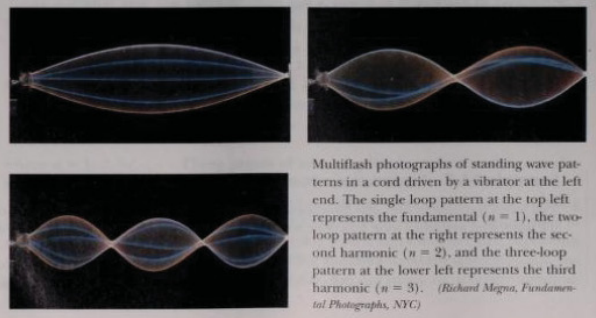
14.3 • NATURAL FREQUENCIES IN A STRETCHED STRING

Consider a string that has a length of L and is fixed at both ends, as in Figure 14.6. Standing waves are set up in the string by a continuous superposition of waves incident on and reflected from the ends. The string has a number of natural patterns of vibration, called **normal modes.** Each of these modes has a characteristic frequency; the frequencies are easily calculated.

First, note that the ends of the string must be nodes, because these points are fixed. If the string is displaced at its midpoint and released, all modes that do not have a node at the center point are excited. Consider the normal mode in which the center of the string is an antinode (Fig. 14.6b). For this normal mode, the length of the string equals $\lambda/2$ (the distance between nodes):

$$L = \lambda_1/2 \qquad \text{or} \qquad \lambda_1 = 2L$$

The next normal mode, of wavelength λ_2 (Fig. 14.6c), occurs when the length of the string equals one wavelength, that is, when $\lambda_2 = L$. The third normal mode (Fig. 14.6d) corresponds to the case in which the length equals $3\lambda/2$; therefore, $\lambda_3 = 2L/3$. In general, the wavelengths of the various normal modes can be conveniently expressed as



Multiflash photographs of standing wave patterns in a cord driven by a vibrator at the left end. The single loop pattern at the top left represents the fundamental ($n = 1$), the two-loop pattern at the right represents the second harmonic ($n = 2$), and the three-loop pattern at the lower left represents the third harmonic ($n = 3$). *(Richard Megna, Fundamental Photographs, NYC)*

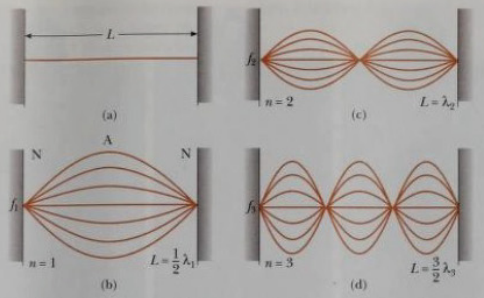


Figure 14.6 (a) A string of length L fixed at both ends. The normal modes of vibration form a harmonic series: (b) the fundamental frequency, or first harmonic; (c) the second harmonic; and (d) the third harmonic.

$$\lambda_n = \frac{2L}{n} \qquad (n = 1, 2, 3, \ldots) \qquad [14.6]$$

• *Wavelengths of normal modes*

where the index n refers to the nth mode of vibration. The natural frequencies associated with these modes are obtained from the relationship $f = v/\lambda$, where **the wave speed, v, is the same for all frequencies.** Using Equation 14.6, we find that the frequencies of the normal modes are

$$f_n = \frac{v}{\lambda_n} = \frac{n}{2L}v \qquad (n = 1, 2, 3, \ldots) \qquad [14.7]$$

• *Frequencies of normal modes as functions of wave speed and length of string*

Because $v = \sqrt{F/\mu}$ (Eq. 13.19), where F is the tension in the string and μ is its mass per unit length, we can express the natural frequencies of a stretched string (sometimes called *harmonics*) as

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (n = 1, 2, 3, \ldots) \qquad [14.8]$$

• *Frequencies of normal modes as functions of string tension and linear mass density*

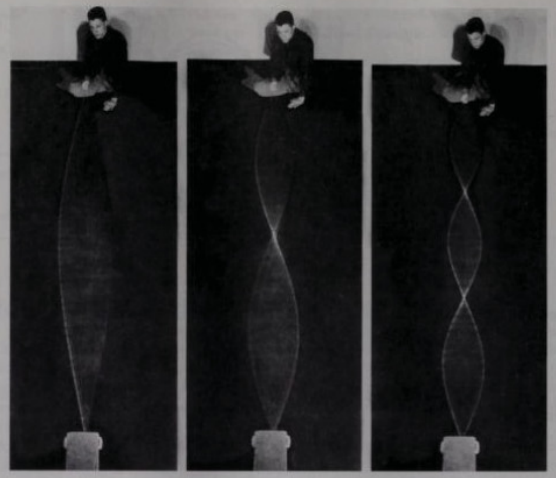
The lowest frequency, corresponding to $n = 1$, is called the **fundamental frequency,** f_1, and is

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{\mu}} \qquad [14.9]$$

• *Fundamental frequency of a taut string*

It is clear that the frequencies of the remaining modes are integral multiples of the fundamental frequency—that is, $2f_1$, $3f_1$, $4f_1$, and so on. These higher natural frequencies, together with the fundamental frequency, form a **harmonic series.** The fundamental, f_1, is the first harmonic; the frequency $f_2 = 2f_1$ is the second harmonic; the frequency f_n is the nth harmonic.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783



Photographs of standing waves. As one end of the rope is moved from side to side with increasing frequency, patterns with more and more loops are formed; only certain definite frequencies will produce fixed patterns. *(Photos, Education Development Center, Newton, Mass.)*

We can obtain the foregoing results in an alternative manner. Because we require that the string be fixed at $x = 0$ and $x = L$, the wave function $y(x, t)$ given by Equation 14.3 must be *zero* at these points for *all* times. That is, the boundary conditions require that $y(0, t) = 0$ and $y(L, t) = 0$ for all values of t. Because $y = (2A \sin kx)\cos \omega t$, the first condition, $y(0, t) = 0$, is automatically satisfied because $\sin kx = 0$ at $x = 0$. To meet the second condition, $y(L, t) = 0$, we require that $\sin kL = 0$. This condition is satisfied when the angle kL equals an integral multiple of π (180°). Therefore, the allowed values of k are[1]

$$k_n L = n\pi \qquad (n = 1, 2, 3, \ldots) \qquad \textbf{[14.10]}$$

Because $k_n = 2\pi/\lambda_n$, we find that

$$\left(\frac{2\pi}{\lambda_n}\right) L = n\pi \qquad \text{or} \qquad \lambda_n = \frac{2L}{n}$$

which is identical to Equation 14.6.

When a stretched string is distorted to a shape that corresponds to any one of its harmonics, after being released it will vibrate at the frequency of that harmonic. However, if the string is plucked or bowed, the resulting vibration will include frequencies of various harmonics, including the fundamental. In effect, the string

[1]We exclude $n = 0$, because this corresponds to the trivial case in which no wave exists ($k = 0$).

"selects" the normal-mode frequencies when disturbed by a nonharmonic disturbance (for example, a finger plucking it).

Figure 14.7 shows a stretched string vibrating with its first and second harmonics simultaneously. In this figure, the combined vibration is the superposition of the two vibrations shown in Figures 14.6b and 14.6c. The larger loop corresponds to the fundamental frequency of vibration, f_1, and the smaller loops correspond to the second harmonic, f_2. In general, the resulting motion, or displacement, of the string can be described by a superposition of the various harmonic wave functions, with different frequencies and amplitudes. Hence, the sound that one hears corresponds to a complex waveform associated with these modes of vibration. (We shall return to this point in Section 14.6.)

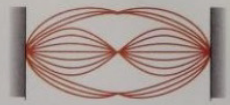


Figure 14.7 Multiple exposures of a string vibrating simultaneously in its first and second harmonics.

The frequency of a stringed instrument can be changed either by varying the strings' tension, F, or by changing their length, L. For example, the tension in the strings of guitars and violins is adjusted by a screw mechanism or by turning pegs on the neck of the instrument. As the tension increases, the frequencies of the normal modes increase according to Equation 14.8. Once the instrument is "tuned," the player varies the frequency by moving his or her fingers along the neck, thereby changing the length of the vibrating portion of the string. As this length is reduced, the frequency increases, because the normal-mode frequencies are inversely proportional to (vibrating) string length.

CONCEPTUAL PROBLEM 2

Guitarists sometimes play a "harmonic," by lightly touching a string at the exact center and plucking the string. The result is a clear note one octave higher than the fundamental of the string, even though the string is not pressed to the fingerboard. Why does this happen?

CONCEPTUAL PROBLEM 3

An archer shoots an arrow from a bow. Does the string of the bow exhibit standing waves after the arrow leaves? If so, and if the bow is perfectly symmetric so that the arrow leaves from the center of the string, what harmonics are excited?

Example 14.3 Give Me a C Note

A middle C string of the C-major scale on a piano has a fundamental frequency of 262 Hz, and the A note has a fundamental frequency of 440 Hz. (a) Calculate the frequencies of the next two harmonics of the C string.

Solution Because $f_1 = 262$ Hz, we can use Equations 14.8 and 14.9 to find the frequencies f_2 and f_3:

$$f_2 = 2f_1 = 524 \text{ Hz}$$

$$f_3 = 3f_1 = 786 \text{ Hz}$$

(b) If the strings for the A and C notes are assumed to have the same mass per unit length and the same length, determine the ratio of tensions in the two strings.

Solution Using Equation 14.8 for the two strings vibrating at their fundamental frequencies gives

$$f_{1A} = \frac{1}{2L}\sqrt{F_A/\mu} \qquad \text{and} \qquad f_{1C} = \frac{1}{2L}\sqrt{F_C/\mu}$$

$$f_{1A}/f_{1C} = \sqrt{F_A/F_C}$$

$$F_A/F_C = (f_{1A}/f_{1C})^2 = (440/262)^2 = 2.82$$

(c) In a real piano, the assumption we made in (b) is only half true. The string densities are equal, but the A string is 64% as long as the C string. What is the ratio of their tensions?

$$f_{1A}/f_{1C} = (L_C/L_A)\sqrt{F_A/F_C} = (100/64)\sqrt{F_A/F_C}$$

$$F_A/F_C = (0.64)^2(440/262)^2 = 1.16$$

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

EXERCISE 2 A stretched string is 160 cm long and has a linear density of 0.0150 g/cm. What tension in the string will result in a second harmonic of 460 Hz? Answer 813 N

EXERCISE 3 A string of linear density 1.0×10^{-3} kg/m and length 3.0 m is stretched between two points. One end is vibrated transversely at 200 Hz. What tension in the string will establish a standing-wave pattern with three loops along the string's length?
Answer 160 N

14.4 • STANDING WAVES IN AIR COLUMNS

Standing longitudinal waves can be set up in a tube of air, such as an organ pipe, as the result of interference between longitudinal waves traveling in opposite directions. The phase relationship between the incident wave and the wave reflected from one end depends on whether that end is open or closed. This is analogous to the phase relationships between incident and reflected transverse waves at the ends of a string. **The closed end of an air column is a displacement node,** just as the fixed end of a vibrating string is a displacement node. As a result, the reflected wave at a closed end of a tube of air is 180° out of phase with the incident wave. Furthermore, because the pressure wave is 90° out of phase with the displacement wave (Section 13.9), **the closed end of an air column corresponds to a pressure antinode** (that is, a point of maximum pressure variation).

If the end of an air column is open to the atmosphere, the air molecules have complete freedom of motion. The wave reflected from an open end is nearly in phase with the incident wave when the tube's diameter is small relative to the wavelength of the sound. As a consequence, **the open end of an air column is approximately a displacement antinode and a pressure node.**

Strictly speaking, the open end of an air column is not exactly an antinode. A condensation does not reach full expansion until it passes somewhat beyond an open end. For a thin-walled tube of circular cross-section, this correction that must be added for each open end is about $0.6R$, where R is the tube's radius. Hence, the effective length of the tube is somewhat greater than the true length, L.

The first three modes of vibration of a pipe that is open at both ends are shown in Figure 14.8a. Note that in this figure, the standing longitudinal waves are drawn as transverse waves because it is difficult to draw longitudinal displacements. When air is directed into the pipe from the left, longitudinal standing waves are formed and the pipe resonates at its natural frequencies. All modes of vibration are excited simultaneously (although not with the same amplitude). Note that the ends are displacement antinodes (approximately). In the fundamental mode, the wavelength is twice the length of the pipe, and hence the frequency of the fundamental, f_1, is $v/2L$. In a similar way, the frequencies of the higher harmonics are $2f_1$, $3f_1$, ... Thus, in a pipe that is open at both ends, the natural frequencies of vibration form a harmonic series; that is, the higher harmonics are integral multiples of the fundamental frequency. Because all harmonics are present, we can express the natural frequencies of vibration as

Natural frequencies of a pipe open at both ends •

$$f_n = n\frac{v}{2L} \qquad (n = 1, 2, 3, \ldots) \qquad [14.11]$$

where v is the speed of sound in air.

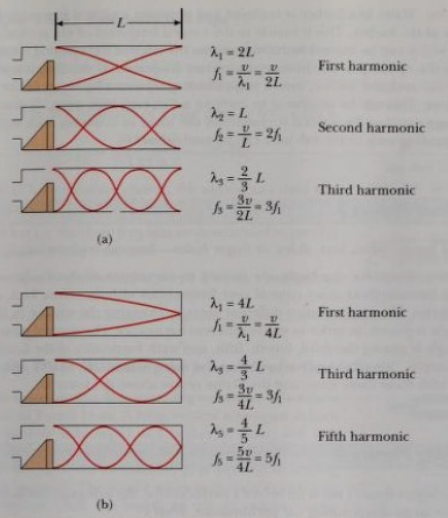


Figure 14.8 (a) Standing longitudinal waves in an organ pipe open at both ends. The natural frequencies that form a harmonic series are f_1, $2f_1$, $3f_1$, ... (b) Standing longitudinal waves in an organ pipe closed at one end. Only the *odd* harmonics are present, and so the natural frequencies are f_1, $3f_1$, $5f_1$, ...

If a pipe is closed at one end and open at the other, the closed end is a displacement node (Fig. 14.8b). In this case, the wavelength for the fundamental mode is four times the length of the tube. Hence, the fundamental, f_1, is equal to $v/4L$, and the frequencies of the higher harmonics are equal to $3f_1$, $5f_1$, ... That is, **in a pipe that is closed at one end, only odd harmonics are present,** and these are

$$f_n = n\frac{v}{4L} \qquad (n = 1, 3, 5, \ldots) \qquad [14.12]$$

• *Natural frequencies of a pipe closed at one end and open at the other*

Standing waves in air columns are the primary sources of the sounds produced by wind instruments.

Thinking Physics 2

Passing ocean waves sometimes cause the water in a harbor to undergo very large oscillations, called *seiches*. Why would this happen?

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

Reasoning Water in a harbor is enclosed and possesses a natural frequency based on the size of the harbor. This is similar to the natural frequency of the enclosed air in a bottle, which can be excited by blowing across the edge of the opening. Ocean waves pass by the opening of the harbor at a certain frequency. If this frequency matches that of the enclosed harbor, then a large standing wave can be set up in the water by resonance. This can be simulated by carrying a fish tank with water. If your walking frequency matches the natural frequency of the water as it sloshes back and forth, a large standing wave in the fish tank can be established.

Thinking Physics 3

A bugle has no valves, keys, slides, or finger holes—how can it play a song?

Reasoning Songs for the bugle are limited to harmonics of the fundamental frequency, because there is no control over frequencies without valves, keys, slides, or finger holes. The player obtains different notes by changing the tension in the lips as the bugle is played, in order to excite different harmonics. The normal playing range of a bugle is among the third, fourth, fifth, and sixth harmonics of the fundamental. For example, "Reveille" is played with just the three notes G, C, and E, and "Taps" is played with these three notes and the G one octave above the lower G.

Thinking Physics 4

If an orchestra doesn't warm up before a performance, the strings go flat and the wind instruments go sharp during the performance. Why?

Reasoning Without warming up, all the instruments will be at room temperature at the beginning of the concert. As the wind instruments are played, they fill with warm air from the player's exhalation. The increase in temperature of the air in the instrument causes an increase in the speed of sound, which raises the resonance frequencies of the air columns. As a result, the instruments go sharp. The strings on the stringed instruments also increase in temperature due to the friction of rubbing with the bow. This results in thermal expansion, which causes a decrease in the tension in the strings. With a decrease in tension, the wave speed on the strings drops, and the fundamental frequencies decrease. Thus, the stringed instruments go flat.

CONCEPTUAL PROBLEM 4

In Balboa Park in San Diego, California, there is an outdoor organ. Does the fundamental frequency of a particular pipe on this organ change on hot and cold days? How about on days with high and low atmospheric pressure?

CONCEPTUAL PROBLEM 5

If you have a series of identical glass bottles, with varying amounts of water in them, you can play musical notes by either striking the bottles with a spoon, or blowing across the open tops of the bottles. When hitting the bottles, the frequency of the note decreases as the water level rises. When blowing on the bottles, the frequency of the note increases as the water level rises. Why is the behavior of the frequency different in these two cases?

Example 14.4 Resonance in a Pipe

A pipe has a length of 1.23 m. (a) Determine the frequencies of the first three harmonics if the pipe is open at each end. Take $v = 343$ m/s as the speed of sound in air.

Solution The first harmonic of a pipe open at both ends is

$$f_1 = \frac{v}{2L} = \frac{343 \text{ m/s}}{2(1.23 \text{ m})} = 139 \text{ Hz}$$

Because all harmonics are present, the second and third harmonics are $f_2 = 2f_1 = 278$ Hz and $f_3 = 3f_1 = 417$ Hz.

(b) What are the three frequencies determined in part (a) if the pipe is closed at one end?

Solution The fundamental frequency of a pipe closed at one end is

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(1.23 \text{ m})} = 69.7 \text{ Hz}$$

In this case, only odd harmonics are present, and so the next two resonances have frequencies $f_3 = 3f_1 = 209$ Hz and $f_5 = 5f_1 = 349$ Hz.

(c) For the pipe open at both ends, how many harmonics are present in the normal human hearing range (20 to 20 000 Hz)?

Solution Because all harmonics are present, $f_n = nf_1$. For $f_n = 20\,000$ Hz, we have $n = 20\,000/139 = 144$, so that 144 harmonics are present in the audible range. Actually, only the first few harmonics have sufficient amplitude to be heard.

Example 14.5 Measuring the Frequency of a Tuning Fork

A simple apparatus for demonstrating resonance in a tube is described in Figure 14.9a. A long, vertical tube open at both ends is partially submerged in a beaker of water, and a vibrating tuning fork of unknown frequency is placed near the top. The length of the air column, L, is adjusted by moving the tube vertically. The sound waves generated by the fork are reinforced when the length of the air column corresponds to one of the resonant frequencies of the tube.

For a certain tube, the smallest value of L for which a peak occurs in the sound intensity is 9.00 cm. From this measurement, determine the frequency of the tuning fork and the value of L for the next two resonant modes.

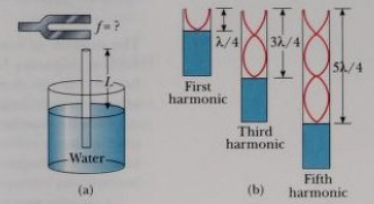


Figure 14.9 (Example 14.5) (a) Apparatus for demonstrating the resonance of sound waves in a tube closed at one end. The length L of the air column is varied by moving the tube vertically while it is partially submerged in water. (b) The first three normal modes of the system shown in part (a).

Reasoning Although the tube is open at both ends to allow the water in, the water surface acts like a wall at one end of the length L. Therefore this setup represents a pipe closed at one end and the fundamental has a frequency of $v/4L$ (Fig. 14.9b).

Solution Taking $v = 343$ m/s for the speed of sound in air and $L = 0.0900$ m, we get

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.0900 \text{ m})} = 953 \text{ Hz}$$

From this information about the fundamental mode, we see that the wavelength is $\lambda = 4L = 0.360$ m. Because the frequency of the source is constant, the next two resonance modes (Fig. 14.9b) correspond to lengths of $3\lambda/4 = 0.270$ m and $5\lambda/4 = 0.450$ m.

EXERCISE 4 The longest pipe on a certain organ is 4.88 m. What is the fundamental frequency (at 0.0°C) if the nondriven end of the pipe is (a) closed and (b) open?
Answer (a) 17.0 Hz (b) 34.0 Hz

EXERCISE 5 An air column 2.00 m in length is open at both ends. The frequency of a certain harmonic is 410 Hz, and the frequency of the next higher harmonic is 492 Hz. Determine the speed of sound in the air column. Answer 328 m/s

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

OPTIONAL

14.5 • BEATS: INTERFERENCE IN TIME

The interference phenomena with which we have been dealing so far involve the superposition of two or more waves with the same frequency, traveling in opposite directions. Because the resultant waveform in this case depends on the coordinates of the disturbed medium, we can refer to the phenomenon as *spatial interference.* Standing waves in strings and pipes are common examples of spatial interference.

We now consider another type of interference effect, one that results from the superposition of two waves with slightly *different frequencies.* In this case, when the two waves are observed at a given point, they are periodically in and out of phase. That is, there occurs a temporal alternation between constructive and destructive interference. We refer to this phenomenon as **interference in time** or **temporal interference.**

For example, if two tuning forks of slightly different frequencies are struck, one hears a sound of pulsating intensity, called a **beat.**

Definition of beats •

> A **beat** is the periodic variation in intensity at a given point due to the superposition of two waves with slightly different frequencies.

The number of beats one hears per second, or **beat frequency,** equals the difference in frequency between the two sources. The maximum beat frequency that the human ear can detect is about 20 beats/s. When the beat frequency exceeds this value, it blends indistinguishably with the compound sounds producing the beats.

One can use beats to tune a stringed instrument, such as a piano, by beating a note against a reference tone of known frequency. The string can then be adjusted to equal the frequency of the reference by tightening or loosening it until the beats become too infrequent to notice.

Consider two waves with equal amplitudes, traveling through a medium with slightly different frequencies, f_1 and f_2. We can represent the displacement that each wave would produce at a point as

$$y_1 = A \cos 2\pi f_1 t \qquad \text{and} \qquad y_2 = A \cos 2\pi f_2 t$$

Using the superposition principle, we find that the resultant displacement at that point is given by

$$y = y_1 + y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

It is convenient to write this in a form that uses the trigonometric identity

$$\cos a + \cos b = 2 \cos\left(\frac{a-b}{2}\right) \cos\left(\frac{a+b}{2}\right)$$

Letting $a = 2\pi f_1 t$ and $b = 2\pi f_2 t$, we find that

Resultant of two waves of different frequencies but equal amplitude •

$$y = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \cos 2\pi \left(\frac{f_1 + f_2}{2}\right) t \qquad [14.13]$$

Graphs demonstrating the individual waveforms as well as the resultant wave are shown in Figure 14.10. From the factors in Equation 14.13, we see that the resultant

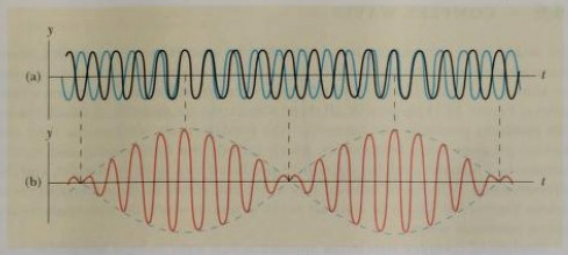


Figure 14.10 Beats are formed by the combination of two waves of slightly different frequencies traveling in the same direction. (a) The individual waves. (b) The combined wave has an amplitude (broken line) that oscillates in time.

vibration at a point has an effective frequency equal to the average frequency, $(f_1 + f_2)/2$, and an amplitude of

$$A = 2A \cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t \qquad [14.14]$$

That is, the *amplitude varies in time* with a frequency of $(f_1 - f_2)/2$. When f_1 is close to f_2, this amplitude variation is slow, as illustrated by the envelope (broken line) of the resultant waveform in Figure 14.10b.

Note that a beat, or a maximum in amplitude, will be detected whenever

$$\cos 2\pi \left(\frac{f_1 - f_2}{2}\right) t = \pm 1$$

That is, there are *two* maxima in each cycle. Because the amplitude varies with frequency as $(f_1 - f_2)/2$, the number of beats per second, or the beat frequency, f_b, is twice this value:

$$f_b = f_1 - f_2 \qquad [14.15]$$

• *Beat frequency*

For instance, if two tuning forks vibrate individually at frequencies of 438 Hz and 442 Hz, respectively, the resultant sound wave of the combination has a frequency of 440 Hz (the musical note A) and a beat frequency of 4 Hz. That is, the listener hears the 440-Hz sound wave go through an intensity maximum four times every second.

CONCEPTUAL PROBLEM 6

You have a standard tuning fork whose frequency you know and a second tuning fork whose frequency you don't know. When you play them together, you hear a beat frequency of 4 Hz. You know that the frequency of the mystery tuning fork is different from that of the standard fork by 4 Hz, but you can't tell whether it is higher or lower. What could you do to determine this? (*Hint:* You are chewing gum at the time that you perform these measurements.)

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

OPTIONAL

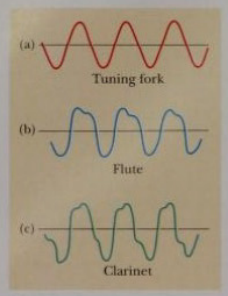


Figure 14.11 Waveform produced by (a) a tuning fork, (b) a flute, and (c) a clarinet, each at approximately the same frequency. *(Adapted from C. A. Culver, Musical Acoustics, 4th ed., New York, McGraw-Hill, 1956, p. 128.)*

14.6 • COMPLEX WAVES

The sound wave patterns produced by most instruments are very complex. Some characteristic waveforms produced by a tuning fork, a harmonic flute, and a clarinet are shown in Figure 14.11. Although each instrument has its own characteristic pattern, Figure 14.11 shows that all three waveforms are periodic. A struck tuning fork produces primarily one harmonic (the fundamental), whereas the flute and clarinet produce many frequencies, which include the fundamental and various harmonics. Thus, the complex waveforms produced by a violin or clarinet, and the corresponding richness of musical tones, are the result of the superposition of various harmonics. This is in contrast to the drum, in which the overtones do not form a harmonic series.

Analysis of complex waveforms appears at first sight to be a formidable task. However, if the waveform is periodic, it can be represented with arbitrary precision by the combination of a sufficiently large number of sinusoidal waves that form a harmonic series. In fact, one can represent any periodic function or any function over a finite interval as a series of sine and cosine terms by using a mathematical technique based on *Fourier's theorem.* The corresponding sum of terms that represents the periodic waveform is called a **Fourier series.**

Let $y(t)$ be any function that is periodic in time, with a period of T, so that $y(t + T) = y(t)$. **Fourier's theorem** states that this function can be written

$$y(t) = \sum_n (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t) \qquad [14.16]$$

where the lowest frequency is $f_1 = 1/T$.

The higher frequencies are integral multiples of the fundamental, and so $f_n = nf_1$. The coefficients A_n and B_n represent the amplitudes of the various waves. The amplitude of the nth harmonic is proportional to $\sqrt{A_n^2 + B_n^2}$, and its intensity is proportional to $A_n^2 + B_n^2$.

Figure 14.12 represents a harmonic analysis of the waveforms shown in Figure 14.11. Note the variation of relative intensity with harmonic content for the flute and clarinet. In general, any musical sound contains components that are members of a harmonic set with varying relative intensities.

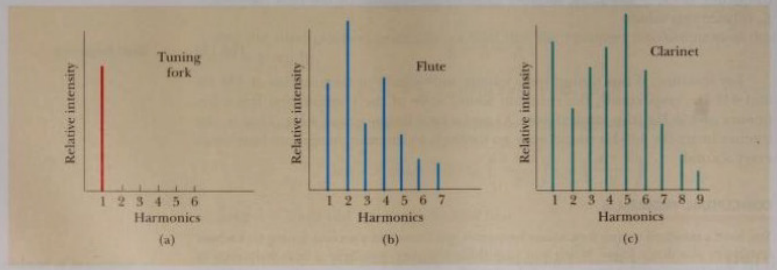


Figure 14.12 Harmonics of the waveforms shown in Figure 14.11. Note the variations in intensity of the various harmonics. *(Adapted from C. A. Culver, Musical Acoustics, 4th ed., New York, McGraw-Hill, 1956.)*

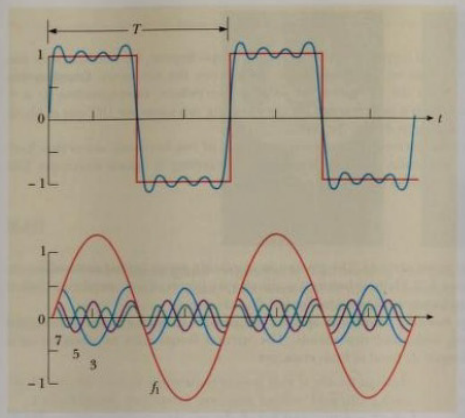


Figure 14.13 Harmonic synthesis of a square wave, which can be represented by the sum of odd harmonics of the fundamental. *(From M. L. Warren, Introductory Physics, New York, W. H. Freeman, 1979, p. 178; by permission of the publisher.)*

As an example of Fourier synthesis, consider the periodic square wave shown in Figure 14.13. The square wave is synthesized by a series of *odd* harmonics of the fundamental. The series contains only sine functions (that is, $B_n = 0$ for all n). Only the first four odd harmonics and their respective amplitudes are shown. One obtains a better fit to the true waveform by adding more harmonics.

Using modern technology, one can produce a variety of musical tones by mixing harmonics with varying amplitudes. Electronic music synthesizers that do this are now widely used.

Thinking Physics 5

A professor performs a demonstration in which he breathes helium and then speaks with a comical voice. One student explains, "The velocity of sound in helium is higher than in air, so the fundamental frequency of the standing waves in the mouth is increased." Another student says, "No, the fundamental frequency is determined by the vocal folds and cannot be changed. Only the *quality* of the voice has changed." Which student is correct?

Reasoning The second student is correct. The fundamental frequency of the complex tone from the voice is determined by the vibration of the vocal folds and is not changed by substituting a different gas in the mouth. The introduction of the helium into the mouth results in higher harmonics being excited more than in the normal voice, but the fundamental frequency of the voice is the same—only the quality has changed. The unusual inclusion of the higher frequency harmonics results in a common description of this effect as a "high-pitched" voice, but this is incorrect.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

SUMMARY

When two waves with equal amplitudes and frequencies superimpose, the resultant wave has an amplitude that depends on the phase angle, ϕ, between the two waves. **Constructive interference** occurs when the two waves are *in phase* everywhere, corresponding to $\phi = 0, 2\pi, 4\pi, \ldots$. **Destructive interference** occurs when the two waves are 180° out of phase everywhere, corresponding to $\phi = \pi, 3\pi, 5\pi, \ldots$.

Standing waves are formed from the superposition of two harmonic waves that have the same frequency, amplitude, and wavelength but are traveling in *opposite* directions. The resultant standing wave is described by the wave function

$$y = (2A \sin kx)\cos \omega t \qquad [14.3]$$

Hence, its amplitude varies as $\sin kx$. The maximum amplitude points (called **antinodes**) are separated by a distance $\lambda/2$. Halfway between antinodes are points of zero amplitude (called **nodes**). The distance between adjacent nodes is $\lambda/2$.

One can set up standing waves with specific frequencies in such systems as stretched strings, hollow pipes, rods, and drumheads. The natural frequencies of vibration of a stretched string of length, L, fixed at both ends, are

$$f_n = \frac{n}{2L}\sqrt{\frac{F}{\mu}} \qquad (n = 1, 2, 3, \ldots) \qquad [14.8]$$

where F is the tension in the string and μ is its mass-per-unit length. The natural frequencies of vibration form a **harmonic series**—that is, $f_1, 2f_1, 3f_1, \ldots$.

The standing wave patterns for longitudinal waves in a hollow pipe depend on whether the ends of the pipe are open or closed. If the pipe is open at both ends, the natural frequencies of vibration form a harmonic series. If one end is closed, only odd harmonics of the fundamental are present.

The phenomenon of **beats** occurs as a result of the superposition of two traveling waves of slightly different frequencies. For sound waves at a given point, one hears an alternation in sound intensity with time. Thus, beats correspond to *interference as time passes.*

Any periodic waveform can be represented by the combination of the sinusoidal waves that form a harmonic series. The process is called *Fourier synthesis* and is based on *Fourier's theorem.*

CONCEPTUAL QUESTIONS

1. When two waves interfere constructively or destructively, is there any gain or loss in energy? Explain.
2. Does the phenomenon of wave interference apply only to sinusoidal waves?
3. Some singers claim to be able to shatter a wine glass by maintaining a certain vocal pitch over a period of several seconds (Fig. Q14.3). What mechanism causes the glass to break? (The glass must be very clean in order to break.)
4. What limits the amplitude of motion of a real vibrating system that is driven at one of its natural frequencies?
5. Explain why all harmonics are present in an organ pipe open at both ends, but only the odd harmonics are present in a pipe closed at one end.
6. An airplane mechanic notices that the sound from a twin-engine aircraft rapidly varies in loudness when both engines are running. What could be causing this variation from loud to soft?
7. Why does a vibrating guitar string sound louder when placed on the instrument than it would if allowed to vibrate in the air while off the instrument?



Figure Q14.3 (Question 3) *(Left)* Standing wave pattern in a vibrating wine glass. The wine glass will shatter if the amplitude of vibration becomes too large. *(Courtesy of Professor Thomas D. Rossing, Northern Illinois University)* *(Right)* A wine glass shattered by the amplified sound of a human voice. *(© Ben Rose 1992/The IMAGE Bank)*

8. When the base of a vibrating tuning fork is placed against a chalkboard, the sound becomes louder. How does this affect the length of time for which the fork vibrates? Does this agree with conservation of energy?
9. To keep animals away from their cars, some people mount short, thin pipes on the fenders. The pipes give out a high-pitched wail when the cars are moving. How do they create the sound?
10. If you wet your fingers and lightly run them around the rim of a fine wine glass, a high-pitched sound is heard. Why? How could you produce various musical notes with a set of wine glasses?
11. When a bell is rung, standing waves are set up around the bell's circumference. What boundary conditions must be satisfied by the resonant wavelengths? How does a crack in the bell, such as in the Liberty Bell, affect the satisfying of the boundary conditions and the sound emanating from the bell?
12. Explain why your voice seems to sound better than usual when you sing in the shower.
13. What is the purpose of the slide on a trombone or the valves on a trumpet?
14. Despite a reasonably steady hand, a certain person often spills his coffee when carrying it to his seat. Discuss resonance as a possible cause of this difficulty and devise a means for solving the problem.

PROBLEMS

Section 14.1 Superposition and Interference of Sinusoidal Waves

1. Two harmonic waves are described by

$$y_1 = (5.00\ \text{m})\ \sin[\pi(4.00x - 1200t)]$$

$$y_2 = (5.00\ \text{m})\ \sin[\pi(4.00x - 1200t - 0.250)]$$

where x, y_1, and y_2 are in meters and t is in seconds. (a) What is the amplitude of the resultant wave? (b) What is the frequency of the resultant wave?

2. A sinusoidal wave is described by

$$y_1 = (0.0800\ \text{m})\ \sin[2\pi(0.100x - 80.0t)]$$

where y_1 and x are in meters and t is in seconds. Write an expression for a wave that has the same frequency, amplitude, and wavelength as y_1 but when added to y_1 gives a resultant with an amplitude of $8\sqrt{3}$ cm.

3. Two waves are traveling in the same direction along a stretched string. Each has an amplitude of 4.00 cm and they are 90.0° out of phase. Find the amplitude of the resultant wave.
4. Two identical sinusoidal waves with wavelengths of 3.00 m travel in the same direction at a speed of 2.00 m/s. The second wave originates from the same point as the first but at a later time. Determine the minimum possible time interval between the starting moments of the two waves if the amplitude of the resultant wave is the same as that of the two initial waves.
5. Two speakers are driven by a common oscillator at 800 Hz and face each other at a distance of 1.25 m. Locate the points along a line joining the two speakers at which relative minima would be expected. (Use $v = 343$ m/s.)
6. A tuning fork generates sound waves with a frequency of 246 Hz. The waves travel in opposite directions along a hall-

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

way, are reflected by walls, and return. What is the phase difference between the reflected waves when they meet? The corridor is 47.0 m long and the tuning fork is located 14.0 m from one end. The speed of sound in air is 343 m/s.

Section 14.2 Standing Waves

7. Use the trigonometric identity

$$\sin(a \pm b) = \sin a \cos b \pm \cos a \sin b$$

to show that the resultant of two wave functions each of amplitude A, angular frequency ω, and propagation number k and traveling in opposite directions can be written

$$y = (2A \sin kx) \cos \omega t$$

For simplicity, set all phase angles ϕ to zero.

8. Two sinusoidal waves traveling in opposite directions interfere to produce a standing wave described by

$$y = (1.50 \text{ m}) \sin(0.400x) \cos(200t)$$

where x is in meters and t is in seconds. Determine the wavelength, frequency, and speed of the interfering waves.

9. Two waves in a long string are given by

$$y_1 = (0.0150 \text{ m}) \cos\left(\frac{x}{2} - 40t\right)$$

$$y_2 = (0.0150 \text{ m}) \cos\left(\frac{x}{2} + 40t\right)$$

where the y's and x are in meters and t is in seconds. (a) Determine the positions of the nodes of the resulting standing wave. (b) What is the maximum displacement at the position $x = 0.400$ m?

10. Two waves that set up a standing wave in a long string are given by

$$y_1 = A \sin(kx - \omega t + \phi)$$

$$y_2 = A \sin(kx + \omega t)$$

Show (a) that the addition of the arbitrary phase angle will change only the position of the nodes and (b) that the distance between nodes remains constant.

Section 14.3 Natural Frequencies in a Stretched String

11. A student wants to establish a standing wave on a wire that is 1.80 m long and clamped at both ends. The wave speed is 540 m/s. What is the minimum frequency the student should apply to set up standing waves?

12. A standing wave is established in a 120–cm-long string fixed at both ends. The string vibrates in four segments when driven at 120 Hz. (a) Determine the wavelength. (b) What is the fundamental frequency?

13. A cello A-string vibrates in its fundamental mode with a frequency of 220 vibrations/s. The vibrating segment is 70.0 cm long and has a mass of 1.20 g. (a) Find the tension in the string. (b) Determine the frequency of the harmonic that causes the string to vibrate in three segments.

14. A string of length L, mass-per-unit length μ, and tension F is vibrating at its fundamental frequency. What effect will the following have on the fundamental frequency? (a) The length of the string is doubled, with all other factors held constant. (b) The mass per unit length is doubled, with all other factors held constant. (c) The tension is doubled, with all other factors held constant.

15. A 60.0-cm guitar string under a tension of 50.0 N has a mass per unit length of 0.100 g/cm. What is the highest resonant frequency that can be heard by a person capable of hearing frequencies up to 20 000 Hz?

16. A stretched wire vibrates in its fundamental mode at a frequency of 400 vibrations/s. What would be the fundamental frequency if the wire were half as long, with twice the diameter and with four times the tension?

17. A 2.00–m-long wire having a mass of 0.100 kg is fixed at both ends. The tension in the wire is maintained at 20.0 N. What are the frequencies of the first three allowed modes of vibration? If a node is observed at a point 0.400 m from one end, in what mode and with what frequency is it vibrating?

18. A violin string has a length of 0.350 m and is tuned to concert G, having a frequency $f_G = 392$ Hz. Where must the violinist place her finger to play concert A, having a frequency $f_A = 440$ Hz? If this position is to remain correct to one half the width of a finger (i.e., to within 0.600 cm), the string tension cannot be allowed to slip by more than what fraction?

19. Find the fundamental frequency and the next three frequencies that could cause a standing wave pattern on a string that is 30.0 m long, has a mass per unit length 9.00×10^{-3} kg/m, and is stretched to a tension of 20.0 N.

20. Standing-wave vibrations are set up in a crystal goblet with two nodes and two antinodes equally spaced around the 20.0-cm circumference of its rim. If transverse waves move around the glass at 900 m/s, an opera singer would have to produce a high harmonic with what frequency in order to shatter the glass with a resonant vibration?

Section 14.4 Standing Waves in Air Columns

(In this section, unless otherwise indicated, assume that the speed of sound in air is 343 m/s.)

21. Calculate the length for a pipe that has a fundamental frequency of 240 Hz if the pipe is (a) closed at one end and (b) open at both ends.

22. A glass tube (open at both ends) of length L is positioned near an audio speaker of frequency $f = 0.680$ kHz. For what values of L will the tube resonate with the speaker?

23. The overall length of a piccolo is 32.0 cm. The resonating air column vibrates as a pipe open at both ends. (a) Find the frequency of the lowest note a piccolo can play, assuming the speed of sound in air is 340 m/s. (b) Opening holes in the side effectively shortens the length of the resonant column. If the highest note a piccolo can sound is 4000 Hz, find the distance between adjacent nodes for this mode of vibration.

24. The fundamental frequency of an open organ pipe corresponds to middle C (261.6 Hz on the chromatic musical scale). The third resonance of a closed organ pipe has the same frequency. What are the lengths of the two pipes?

25. A tuning fork the frequency of which is f is used to set up a resonance condition in a pipe. Write an expression for the length that will cause the pipe to resonate in its nth mode if it is (a) open at both ends; (b) closed at one end. (Assume that the speed of sound is v.)

26. Do not stick anything into your ear! Estimate the length of your ear canal, from its opening at the external ear in to the eardrum. If you regard the canal as a tube that is open at one end and closed at the other, at approximately what fundamental frequency would you expect your hearing to be most sensitive? Explain why you can hear especially soft sounds just around this frequency.

27. A shower stall measures 86.0 cm × 86.0 cm × 210 cm. When you sing in the shower, which frequencies will sound the richest (resonate), assuming the shower acts as a pipe closed at both ends (nodes at opposite sides)? Assume also that the human voice ranges from 130 Hz to 2000 Hz (not necessarily one person's voice, however). Let the speed of sound in the hot shower stall be 355 m/s.

28. An open pipe 0.400 m in length is placed vertically in a cylindrical bucket, nearly touching the bottom of the bucket, which has area 0.100 m^2. Water is slowly poured into the bucket until a sounding tuning fork of frequency 440 Hz, held over the pipe, produces resonance. Find the mass of water in the bucket at this moment.

29. Water is pumped into a long cylinder at a rate of 18.0 cm^3/s. The radius of the cylinder is 4.00 cm, and at the open top of the cylinder there is a tuning fork vibrating with a frequency of 200 Hz. As the water rises, how much time elapses between successive resonances?

30. Water is pumped into a long cylinder at a volume flow rate R. The radius of the cylinder is r (cm), and at the open top of the cylinder there is a tuning fork vibrating with a frequency f. As the water rises, how much time elapses between successive resonances?

31. A piece of metal pipe is just the right length so that when it is cut into two pieces, their lowest resonance frequencies are 256 Hz for one and 440 Hz for the other. (a) What resonant frequency would have been produced by the original length of pipe, and (b) how long was the original piece?

32. A piece of cardboard tubing, closed at one end, is just the right length so that when it is cut into two (unequal) pieces, their lowest resonant frequencies are 256 Hz for the piece with the closed end and 440 Hz for the piece with both ends open. (a) What resonant frequency would have been produced by the original cardboard tubing, and (b) how long was the original piece?

Section 14.5 Beats: Interference in Time (Optional)

33. In certain ranges of a piano keyboard, more than one string is tuned to the same note to provide extra loudness. For example, the note at 110 Hz has two strings at this pitch. If one string slips from its normal tension of 600 N to 540 N, what beat frequency will be heard when the two strings are struck simultaneously?

34. A student holds a tuning fork oscillating at 256 Hz. He walks toward a wall at a constant speed of 1.33 m/s. (a) What beat frequency does he observe between the tuning fork and its echo? (b) How fast must he walk away from the wall to observe a beat frequency of 5.00 Hz?

35. A flute is designed so that it plays a frequency of 261.6 Hz, middle C, when all the holes are covered and the temperature is 20.0°C. (a) Consider the flute as a pipe that is open at both ends; find the length of the flute, assuming that middle C is the fundamental. (b) A second player, nearby in a colder room, also attempts to play middle C on an identical flute. A beat frequency of 3.00 Hz is heard. What is the temperature of the room? The speed of sound in air is described by

$$v = (331 \text{ m/s}) \sqrt{1 + \frac{T}{273^\circ}}$$

where T is the Celsius temperature.

Additional Problems

36. Two loudspeakers are placed on a wall, 2.00 m apart. A listener stands directly in front of one of the speakers, 3.00 m from the wall. The speakers are being driven by a single oscillator at a frequency of 300 Hz. (a) What is the phase difference between the two waves when they reach the observer? (b) What is the frequency closest to 300 Hz to which the oscillator may be adjusted such that the observer will hear minimal sound?

37. On a marimba, the wooden bar that sounds a tone when struck vibrates as a transverse standing wave with three antinodes and two nodes. The lowest frequency note is 87.0 Hz, produced by a bar 40.0 cm long. (a) Find the speed of transverse waves on the bar. (b) The loudness and duration of the emitted sound are enhanced by a resonant pipe suspended vertically below the center of the bar. If the pipe is open at the top end only and the speed of sound in air is 340 m/s, what is the length of the pipe required to resonate with the bar in part (a)?

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IPR2023-00783



(Problem 37) Marimba players in Mexico City. *(Murray Greenberg)*

38. A speaker at the front of a room and an identical speaker at the rear of the room are being driven by the same oscillator at 456 Hz. A student walks at a uniform rate of 1.50 m/s along the length of the room. How many beats does the student hear per second?
39. Two train whistles have identical frequencies of 180 Hz. When one train is at rest in the station sounding its whistle, a beat frequency of 2.00 Hz is heard from a moving train. What two possible speeds and directions can the moving train have?
40. Jane waits on a railroad platform, while two trains approach from the same direction at equal speeds of 8.00 m/s. Both trains are blowing their whistles (which have the same frequency), and one train is some distance behind the other. After the first train passes Jane, but before the second train passes her, she hears beats of frequency 4.00 Hz. What is the frequency of the train whistles?
41. A string (mass = 4.80 g, length = 2.00 m, and tension = 48.0 N), fixed at both ends, vibrates in its second ($n = 2$) natural mode. What is the wavelength in air of the sound generated by this vibrating string?
42. A pipe that is open at both ends has a fundamental frequency of 300 Hz when the speed of sound in air is 333 m/s. (a) What is the length of the pipe? (b) What is the frequency of the second harmonic when the temperature of the air is increased so that the speed of sound in the pipe is 344 m/s?
43. A string with a mass of 8.00 g and a length of 5.00 m has one end attached to a wall; the other end is draped over a pulley and attached to a hanging mass of 4.00 kg. If the string is plucked, what is the fundamental frequency of vibration?
44. In a major chord on the physical-pitch musical scale, the frequencies are in the ratios 4:5:6:8. A set of pipes, closed at one end, are to be cut so that when sounded in their fundamental mode, they will sound out a major chord. (a) What is the ratio of the lengths of the pipes? (b) What length pipes are needed if the lowest frequency of the chord is 256 Hz? (c) What are the frequencies of this chord?
45. Two wires are welded together. The wires are of the same material, but one is twice the diameter of the other one. They are subjected to a tension of 4.60 N. The thin wire has a length of 40.0 cm and a linear mass density of 2.00 g/m. The combination is fixed at both ends and vibrated in such a way that two antinodes are present with the node between them being right at the weld. (a) What is the frequency of vibration? (b) How long is the thick wire?
46. Two identical strings, each fixed at both ends, are arranged near each other. If string *A* starts oscillating in its fundamental mode, it is observed that string *B* will begin vibrating in its third ($n = 3$) natural mode. Determine the ratio of the tension of string *B* to the tension of string *A*.
47. A standing wave is set up in a string of variable length and tension by a vibrator of variable frequency. When the vibrator has a frequency f, using a string of length L and tension F, there are n antinodes set up in the string. (a) If the length of the string is doubled, by what factor should the frequency be changed to get the same number of antinodes? (b) If the frequency and length are held constant, what tension will produce $n + 1$ antinodes? (c) If the frequency is tripled and the length halved, by what factor should the tension be changed to get twice as many antinodes?
48. Radar detects the speed of a car, using the Doppler shift of microwaves that are reflected off the moving car, by beating the received wave with the transmitted wave and measuring the difference. The Doppler shift for microwaves is

$$f = f_0 \sqrt{\frac{c + v}{c - v}}$$

where f_0 is the transmitted frequency, c is the speed of light (3×10^8 m/s), and v is the relative speed of the two objects. (a) Show that the wave that reflects back to the source has a frequency

$$f = f_0 \frac{(c + v)}{(c - v)}$$

(b) Show that the expression for the beat frequency of the microwaves may be written as $f_b = 2v/\lambda$. (Because the beat frequency is much smaller than the transmitted frequency, use the approximation $f + f_0 = 2f_0$.) (c) What beat frequency is measured for a speed of 30.0 m/s (67 mph) if the microwaves have a frequency of 10.0 GHz? (1 GHz = 10^9 Hz.) (d) If the beat frequency measurement is accurate to ± 5 Hz, how accurate is the velocity measurement?
49. If two adjacent natural frequencies of an organ pipe are determined to be 0.550 kHz and 0.650 kHz, calculate the fundamental frequency and length of this pipe. (Use $v = 340$ m/s.)

50. A 0.0100 kg and 2.00-m-long wire is fixed at both ends and vibrates in its simplest mode under tension 200 N. When a tuning fork is placed near the wire, a beat frequency of 5.00 Hz is heard. (a) What is the frequency (frequencies) of the tuning fork? (b) What should the tension in the wire be to make the beats disappear?
51. The wave function for a standing wave is given in Equation 14.3 as $y = 2A \sin kx \cos \omega t$. (a) Rewrite this wave function in terms of the wavelength λ and wavespeed v of the wave. (b) Write the wave function of the simplest standing-wave vibration of a stretched string of length L. (c) Write the wave function for the second harmonic. (d) Generalize these results and write the wave function for the nth resonance vibration.

Spreadsheet Problems

S1. Spreadsheet 14.1 adds two traveling waves at some fixed time t. The resultant wave function is

$$y = y_1 + y_2 = A_1 \sin(k_1 x - \omega_1 t + \phi_1) + A_2 \sin(k_2 x - \omega_2 t + \phi_2)$$

Add two waves traveling in opposite directions having the same wavelengths, the same phases, and the same speeds. (a) Use $A_1 = A_2 = 0.10$ m, $\omega_1 = -\omega_2 = 3.0$ rad/s, $\phi_1 = \phi_2 = 0$, and $k_1 = k_2 = 2.0$ rad/m. View the associated graph. Choose different values of t to see the time evolution of the resultant wave function. Do you get standing waves? (b) Repeat part (a) using $A_1 = 0.10$ m, $A_2 = 0.20$ m.

S2. Use Spreadsheet 14.1 to add two traveling waves that differ only in phase. (a) Choose $A_1 = A_2 = 0.10$ m, $\omega_1 = \omega_2 = 2.5$ rad/s, $k_1 = k_2 = 1.0$ rad/m, $\phi_1 = 0$, and $\phi_2 = 0$, $\pi/8$, $\pi/4$, $\pi/2$, and π. View the associated graphs. For which values of ϕ_2 do you get constructive interference? Destructive interference? (b) Repeat part (a) using $A_2 = 0.20$ m.

S3. Use Spreadsheet 14.1 to add two traveling waves with different wavelengths. (a) Choose $A_1 = A_2 = 0.10$ m, $\omega_1 = \omega_2 = 3.0$ rad/s, $k_1 = 2.0$ rad/m, $k_2 = 1.0$ rad/m, and $\phi_1 = \phi_2 = 0$. View the associated graph. (b) Repeat part (a) with $k_2 = 3, 4, 5, 6, 7, 8, 9$, and 10. Examine the graph for each case. Explain the appearance of the graphs.

S4. Three waves with the same frequency, wavelength, and amplitude are traveling in the same direction. Each differs in phase such that $(\phi_2 - \phi_1) = (\phi_3 - \phi_2) = \Delta\phi$. Modify Spreadsheet 14.1 to add the three waves at some fixed time t. Calculate the resultant wave function at $t = 0.0$, 1.0 s, and 2.0 s. Use $A_1 = A_2 = A_3 = 0.050$ m, $\omega_1 = \omega_2 = \omega_3 = 2.5$ rad/s, and $k_1 = k_2 = k_3 = 1.0$ rad/m. Choose $\Delta\phi = \pi/6$. View the associated graph. Repeat for different values of $\Delta\phi$.

S5. Write a spreadsheet or computer program to add two traveling waves of different frequencies:

$$y_1 = y_0 \cos 2\pi f_1 t \qquad y_2 = y_0 \cos 2\pi f_2 t$$

If the two frequencies are close to each other, beats will be produced. Does your program show the beats? From your numerical results, how does the beat frequency relate to the frequencies of the two waves?

S6. The Fourier theorem states that any periodic wave of frequency f, no matter how complicated, can be expressed as a sum of even or odd harmonic functions. That is,

$$y(t) = \sum_{n=0}^{\infty} A_n \sin(n\omega t + \phi_n)$$

where $\omega = 2\pi f$ is the fundamental angular frequency. (a) Use $A_1 = 1$, $A_2 = 1/2$, $A_3 = 1/3$, and so on. All the ϕ_n's are zero. (b) Use $A_1 = 1$, $A_3 = 1/3$, $A_5 = 1/5$, and so on. All the even A_n's and all the ϕ_n's are zero. Write a spreadsheet or computer program that will calculate this sum. You should design your spreadsheet or program so that the number of terms in the sum can be easily specified. Start with just the first term ($n = 1$). Then rerun the calculation with more terms in the sum ($n = 2, 3, 4, \ldots 20$). Plot the sum so that one or two periods are displayed. Note how the waveform builds up as more terms are added.

ANSWERS TO CONCEPTUAL PROBLEMS

1. At the instant at which there is no displacement of the string, the string is still moving. Thus, the energy is stored at that instant completely as kinetic energy of the string.
2. At the center of the string, there is a node for the second harmonic, as well as for every even-numbered harmonic. By placing the finger at the center and plucking, the guitarist is eliminating any harmonic that does not have a node at that point, which is all of the odd harmonics. The even harmonics can vibrate relatively freely with the finger at the center because they exhibit no displacement at that point. The result is a sound with a mixture of frequencies that are integer multiples of the second harmonic, which is one octave higher than the fundamental.
3. The bow string is pulled away from equilibrium and released, similar to the way that a guitar string is pulled and released when it is plucked. Thus, standing waves will be excited in the bow string. If the arrow leaves from the exact center of the string, then a series of odd harmonics will be excited. Even harmonics will not be excited because they have a node at the point where the string exhibits its maximum displacement.
4. A change in temperature will result in a change in the speed

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IPR2023-00783

of sound and, therefore, in the fundamental frequency. The speed of sound for audible frequencies in the open atmosphere is only a function of temperature, and does not depend on the pressure. Thus, there will be no effect on the fundamental frequency of variations in atmospheric pressure.

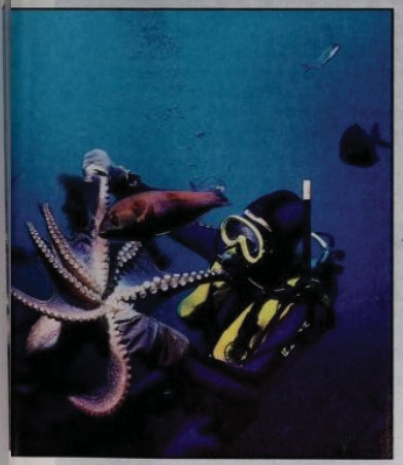
5. When the bottles are struck, standing wave vibrations are established in the glass material of the bottles. The frequencies of these vibrations are determined by the tension in the glass and the mass loading of the glass material. As the water level rises, there is more mass loading, since the glass is in contact with the water. This increased mass loading decreases the frequency. On the other hand, blowing into a bottle establishes a standing wave vibration in the air cavity above the water. As the water level rises, the length of this cavity decreases, and the frequency rises.
6. If you take the gum out of your mouth and stick it to one of the tines of the mystery tuning fork, it will lower its frequency, due to the mass loading of the gum. Now, when the two forks are played together, the beat frequency will be different. If the beat frequency is lower, then the forks are closer together in frequency, so the mystery fork must be higher in frequency than the standard. If the beat frequency is higher, the frequencies are farther apart, and the mystery fork must have had a lower frequency. This argument assumes that the piece of gum is small enough that the frequency shift for the fork is less than 4 Hz.

15

Fluid Mechanics

Matter is normally classified as being in one of three states: solid, liquid, or gaseous. Everyday experience tells us that a solid has a definite volume and shape. A brick maintains its familiar shape and size day in and day out. We also know that a liquid has a definite volume but no definite shape. Finally, an unconfined gas has neither definite volume nor definite shape. These definitions help us to picture the states of matter, but they are somewhat artificial. For example, asphalt and plastics are normally considered solids, but over long periods of time they tend to flow like liquids. Likewise, most substances can be a solid, liquid, or gas (or combinations of these), depending on the temperature and pressure. In general, the time it takes a particular substance to change its shape in response to an external force determines whether we treat the substance as a solid, liquid, or gas.

A **fluid** is a collection of molecules that are randomly arranged and held together by weak cohesive forces and forces exerted by the walls of a container. Both liquids and gases are fluids. In our treatment of the mechanics of fluids, we shall see that no new physical principles are needed to



◀ **A scuba diver plays with an octopus in Poor Knights Island, New Zealand. As the diver descends to greater depths, the water pressure increases above atmospheric pressure, and the internal pressure of the body also increases accordingly to maintain equilibrium. Scuba divers have been able to swim at depths of more than 1000 ft.** *(Darryl Torckler/Tony Stone Images)*

CHAPTER OUTLINE

15.1 Pressure
15.2 Variation of Pressure with Depth
15.3 Pressure Measurements
15.4 Buoyant Forces and Archimedes' Principle
15.5 Fluid Dynamics
15.6 Streamlines and the Equation of Continuity
15.7 Bernoulli's Principle
15.8 Other Applications of Bernoulli's Principle (Optional)

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

explain such effects as the buoyant force on a submerged object and the dynamic lift on an airplane wing. First, we consider a fluid at rest and derive an expression for the pressure exerted by the fluid as a function of its density and depth. We then treat fluids in motion, an area of study called fluid dynamics. A fluid in motion can be described by a model in which certain simplifying assumptions are made. We use this model to analyze some situations of practical importance. An underlying principle known as the **Bernoulli principle** enables us to determine relationships between the pressure, density, and velocity at every point in a fluid.

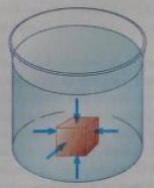


Figure 15.1 The force of the fluid on a submerged object at any point is perpendicular to the surface of the object. The force of the fluid on the walls of the container is perpendicular to the walls at all points.

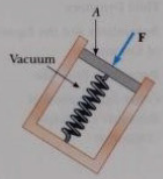


Figure 15.2 A simple device for measuring pressure in a fluid.

15.1 • PRESSURE

The study of fluid mechanics involves the density of a substance, defined as its mass per unit volume. For this reason, Table 15.1 lists the densities of various substances. These values vary slightly with temperature, because the volume of a substance is temperature dependent (as we shall see in Chapter 16). Note that under standard conditions (0°C and atmospheric pressure) the densities of gases are about 1/1000 the densities of solids and liquids. This difference implies that the average molecular spacing in a gas under these conditions is about ten times greater in each dimension than in a solid or liquid.

Fluids do not sustain shearing stresses, and thus the only stress that can exist on an object submerged in a fluid is one that tends to compress the object. The force exerted by the fluid on the object is always perpendicular to the surfaces of the object, as shown in Figure 15.1.

The pressure at a specific point in a fluid can be measured with the device pictured in Figure 15.2. The device consists of an evacuated cylinder enclosing a light piston connected to a spring. As the device is submerged in a fluid, the fluid presses down on the top of the piston and compresses the spring until the inward force of the fluid is balanced by the outward force of the spring. The fluid pressure can be measured directly if the spring is calibrated in advance. This is accomplished by applying a known force to the spring to compress it a given distance.

TABLE 15.1 Densities of Some Common Substances

Substance	ρ (kg/m^3)[a]	Substance	ρ (kg/m^3)[a]
Ice	0.917×10^3	Water	1.00×10^3
Aluminum	2.70×10^3	Sea water	1.03×10^3
Iron	7.86×10^3	Ethyl alcohol	0.806×10^3
Copper	8.92×10^3	Benzene	0.879×10^3
Silver	10.5×10^3	Mercury	13.6×10^3
Lead	11.3×10^3	Air	1.29
Gold	19.3×10^3	Oxygen	1.43
Platinum	21.4×10^3	Hydrogen	8.99×10^{-2}
Glycerine	1.26×10^3	Helium	1.79×10^{-1}

[a] All values are at standard atmospheric pressure and temperature (STP)—that is, atmospheric pressure and 0°C. To convert to grams per cubic centimeter, multiply by 10^{-3}.

If F is the magnitude of the normal force on the piston and A is the surface area of the piston, then the pressure, P, of the fluid at the level to which the device has been submerged is defined as the ratio of force to area:

$$P \equiv \frac{F}{A} \qquad [15.1]$$

• *Definition of pressure*

To define the pressure at a specific point, consider a fluid acting on the device shown in Figure 15.2. If the normal force exerted by the fluid is F over a surface element of area δA that contains the point in question, then the pressure at that point is

$$P = \lim_{\delta A \to 0} \frac{F}{\delta A} = \frac{dF}{dA} \qquad [15.2]$$

As we see in the next section, the pressure in a fluid varies with depth. Therefore, to get the total force on a flat wall of a container, we have to integrate Equation 15.2 over the surface.

Because pressure is force per unit area, it has units of N/m^2 in the SI system. Another name for the SI unit of pressure is **pascal** (Pa).

$$1 \text{ Pa} \equiv 1 \text{ N/m}^2 \qquad [15.3]$$

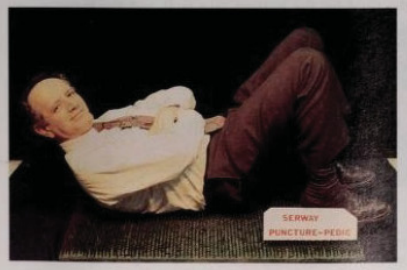


Snowshoes prevent the person from sinking into the soft snow because the person's weight is spread over a larger area, which reduces the pressure on the snow's surface. *(Earl Young/FPG)*

Thinking Physics 1

The daring physics professor, after a long lecture, stretches out for a nap on a bed of nails, as in the photograph. How is this possible?

Reasoning If you try to support your entire weight on a single nail, the pressure on your body is your weight divided by the very small area of the nail. This pressure is sufficiently large to penetrate the skin. However, if you distribute your weight over several hundred nails, as the professor is doing, the pressure is considerably reduced because the area that supports your weight is the total area of all nails in contact with your body. (Note that lying on a bed of nails is much more comfortable than sitting on a bed of nails. Standing on the bed of nails without shoes is not recommended.)



(Thinking Physics 1) *(Jim Lehman)*

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IPR2023-00783

Thinking Physics 2

Suction cups can be used to hold objects onto surfaces. Why don't astronauts use suction cups to hold onto the outside surface of the space shuttle?

Reasoning The action of a suction cup depends on the fact that air is pushed out from under the cup when it is pressed against a surface. When released, it tends to spring back a bit, causing the trapped gas under the cup to expand and have a reduced pressure. Thus, the difference between the atmospheric pressure on the outside of the cup and the reduced pressure inside provides a net force pushing the cup against the surface. For astronauts in orbit around the Earth, there is no air outside the surface of the spacecraft. Thus, if a suction cup were to be pressed against the outside surface of the spacecraft, the pressure differential could not occur.

CONCEPTUAL PROBLEM 1

A woman wearing high-heeled shoes is invited into a home in which the kitchen has vinyl floor covering. Why should the homeowner be concerned?

EXERCISE 1 Estimate the density of the *nucleus* of an atom. What does this result suggest concerning the structure of matter? (Use the fact that the mass of a proton is equal to 1.67×10^{-27} kg and its radius is approximately 10^{-15} m.) Answer 4.0×10^{17} kg/m^3; matter is mostly empty space.

15.2 • VARIATION OF PRESSURE WITH DEPTH

As divers know well, the pressure in the sea or a lake increases as they dive to greater depths. Likewise, atmospheric pressure decreases with increasing altitude. For this reason, aircraft flying at high altitudes must have pressurized cabins.

We now show how the pressure in a liquid increases linearly with depth. Consider a liquid of density ρ at rest and open to the atmosphere, as in Figure 15.3. Let us select a sample of the liquid contained within an imaginary cylinder of cross-sectional area A extending from the surface of the liquid to a depth h. The pressure exerted by the fluid on the bottom face is P, and the pressure on the top face of the cylinder is atmospheric pressure, P_0. Therefore, the upward force exerted by the liquid on the bottom of the cylinder is PA, and the downward force exerted by the atmosphere on the top is P_0A. Because the mass of liquid in the cylinder is $\rho V = \rho Ah$, the weight of the fluid in the cylinder is $w = \rho gV = \rho gAh$. Because the cylinder is in equilibrium, the upward force at the bottom must be greater than the downward force at the top of the sample to support its weight:

$$PA - P_0A = \rho gAh$$

or

$$P = P_0 + \rho gh \qquad [15.4]$$

Variation of pressure with depth •

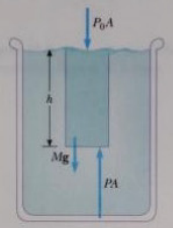


Figure 15.3 The variation of pressure with depth in a fluid. The net force on the volume of water within the darker region must be zero.

That is, the absolute pressure P at a depth h below the surface of a liquid open to the atmosphere is *greater* than atmospheric pressure by an amount ρgh.

In our calculations and end-of-chapter problems, we usually take atmospheric pressure to be

$$P_0 = 1.00 \text{ atm} \approx 1.013 \times 10^5 \text{ Pa}$$

Equation 15.4 verifies that the pressure is the same at all points having the same depth, independent of the shape of the container.

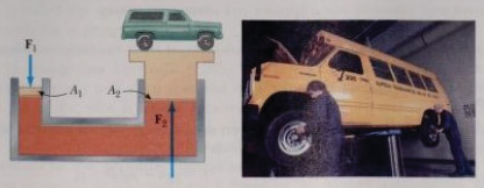
In view of the fact that the pressure in a liquid depends only on depth, any increase in pressure at the surface must be transmitted to every point in the fluid. This was first recognized by the French scientist Blaise Pascal (1623–1662) and is called **Pascal's law: A change in the pressure applied to an enclosed liquid is transmitted undiminished to every point of the liquid and to the walls of the container.** • *Pascal's law*

An important application of Pascal's law is the hydraulic press, illustrated by Figure 15.4. A force F_1 is applied to a small piston of area A_1. The pressure is transmitted through a liquid to a larger piston of area A_2. Because the pressure is the same on both sides, we see that $P = F_1/A_1 = F_2/A_2$. Therefore, the force F_2 is larger than F_1 by the multiplying factor A_2/A_1. Hydraulic brakes, car lifts, hydraulic jacks, and forklifts all make use of this principle.

Thinking Physics 3

Blood pressure is normally measured with the cuff of the sphygmomanometer around the arm. Suppose that the blood pressure were measured with the cuff around the calf of the leg of a standing person. Would the reading of the blood pressure be the same here as it was for the arm?

Reasoning The blood pressure measured at the calf would be larger than that measured at the arm. If we imagine the vascular system of the body to be a vessel containing a liquid (the blood), the pressure in the liquid will increase with depth. The blood at the calf is deeper in the liquid than that at the arm and is at a higher pressure.



(a) (b)

Figure 15.4 (a) Diagram of a hydraulic press. Because the increase in pressure is the same at the left and right sides, a small force $\mathbf{F}_1$ at the left produces a much larger force $\mathbf{F}_2$ at the right. (b) A bus under repair is supported by a hydraulic lift in a garage. *(Superstock)*

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IPR2023-00783



(Conceptual Problem 2) *(Henry Leap)*

CONCEPTUAL PROBLEM 2

A typical silo on a farm has many bands wrapped around its perimeter as shown in the photograph. Why is the spacing between successive bands smaller at the lower portions of the silo?

Example 15.1 The Car Lift

In a car lift used in a service station, compressed air exerts a force on a small piston of circular cross-section having a radius of 5.00 cm. This pressure is transmitted by a liquid to a second piston of radius 15.0 cm. What force must the compressed air exert in order to lift a car weighing 13 300 N? What air pressure will produce this force?

Solution Because the pressure exerted by the compressed air is transmitted undiminished throughout the fluid, we have

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \frac{\pi(5.00 \times 10^{-2}\ \text{m})^2}{\pi(15.0 \times 10^{-2}\ \text{m})^2}(1.33 \times 10^4\ \text{N})$$
$$= 1.48 \times 10^3\ \text{N}$$

The air pressure that will produce this force is

$$P = \frac{F_1}{A_1} = \frac{1.48 \times 10^3\ \text{N}}{\pi(5.00 \times 10^{-2}\ \text{m})^2} = 1.88 \times 10^5\ \text{Pa}$$

This pressure is approximately twice atmospheric pressure.

The input work (the work done by $\mathbf{F}_1$) is equal to the output work (the work done by $\mathbf{F}_2$), so that energy is conserved.

Example 15.2 The Force on a Dam

Water is filled to a height H behind a dam of width w (Fig. 15.5). Determine the resultant force on the dam.

Reasoning We cannot calculate the force on the dam by simply multiplying the area times the pressure, because the pressure varies with depth. The problem can be solved by finding the force dF on a narrow horizontal strip at depth h and then integrating the expression to find the total force on the dam.

Solution The pressure at the depth h beneath the surface at the shaded portion is

$$P = \rho gh = \rho g(H - y)$$

(We have left out atmospheric pressure because it acts on both sides of the dam.) Using Equation 15.2, we find the force on the shaded strip of area $dA = w\,dy$ to be

$$dF = P\,dA = \rho g(H - y)w\,dy$$

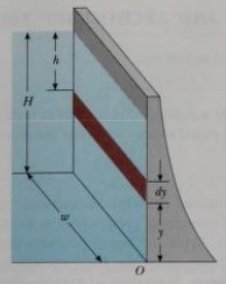


Figure 15.5 (Example 15.2) The total force on a dam must be obtained from the expression $F = \int P\,dA$, where dA is the area of the dark strip.

Therefore, the total force on the dam is

$$F = \int P\,dA = \int_0^H \rho g(H - y)w\,dy = \tfrac{1}{2}\rho g w H^2$$

Note that because the pressure increases with depth, the dam is designed so that its thickness increases with depth, as in Figure 15.5.

EXERCISE 2 Use the fact that the pressure increases linearly with depth to find the average pressure on the dam. Answer $\frac{1}{2}\rho gH$

EXERCISE 3 What is the hydrostatic force on the back of Grand Coulee Dam if the water in the reservoir is 150 m deep and the width of the dam is 1200 m? Answer 1.32×10^{11} N

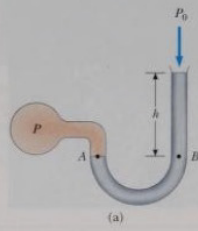
EXERCISE 4 In some places, the Greenland ice sheet is 1.0 km thick. Estimate the pressure on the ground underneath the ice ($\rho_{ice} = 920\ \text{kg/m}^3$). Answer 9×10^6 Pa

15.3 · PRESSURE MEASUREMENTS

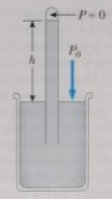
One simple device for measuring pressure is the open-tube manometer illustrated in Figure 15.6a. One end of a U-shaped tube containing a liquid is open to the atmosphere, and the other end is connected to a system of unknown pressure P. The difference in pressure $P - P_0$ is equal to ρgh. Therefore, we see that $P = P_0 + \rho gh$. The pressure P is called the **absolute pressure,** and the difference $P - P_0$ is called the **gauge pressure.** For example, the pressure you measure in your bicycle tire is gauge pressure.

Another instrument used to measure pressure is the common barometer, invented by Evangelista Torricelli (1608–1647). A long tube closed at one end is filled with mercury and then inverted into a dish of mercury (Fig. 15.6b). The closed end of the tube is nearly a vacuum, so its pressure can be taken as zero. Therefore, it follows that $P_0 = \rho_{Hg}gh$, where ρ_{Hg} is the density of the mercury and h is the height of the mercury column. One atmosphere ($P_0 = 1$ atm) of pressure is defined to be the pressure equivalent of a column of mercury that is exactly 0.7600 m in height at 0°C, with $g = 9.80665\ \text{m/s}^2$. At this temperature, mercury has a density of $13.595 \times 10^3\ \text{kg/s}^3$; therefore

$$P_0 = \rho_{Hg}gh = (13.595 \times 10^3\ \text{kg/m}^3)(9.80665\ \text{m/s}^2)(0.7600\ \text{m})$$
$$= 1.013 \times 10^5\ \text{Pa}$$



(a)



(b)

Figure 15.6 Two devices for measuring pressure: (a) an open-tube manometer and (b) a mercury barometer.

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IPR2023-00783

15.4 · BUOYANT FORCES AND ARCHIMEDES' PRINCIPLE

Archimedes' principle •

Archimedes' principle can be stated as follows:

> Any body completely or partially submerged in a fluid is buoyed up by a force the magnitude of which is equal to the weight of the fluid displaced by the body.

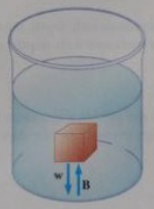


Figure 15.7 The external forces on the cube of water are the force of gravity **w** and the buoyant force **B**. Under equilibrium conditions, $B = w = mg$.



Archimedes (287–212 B.C.)

A Greek mathematician, physicist, and engineer, Archimedes was perhaps the greatest scientist of antiquity. According to legend, Archimedes was asked by King Hieron to determine whether the king's crown was made of pure gold or had been alloyed with some other metal. The task was to be performed without damaging the crown. Archimedes presumably arrived at a solution while taking a bath, noting a partial loss of weight after submerging his arms and legs in the water. As the story goes, he was so excited about his great discovery that he ran through the streets of Syracuse naked shouting, "Eureka!" which is Greek for "I have found it."

Everyone has experienced Archimedes' principle. Recall that it is relatively easy to lift someone if the person is in a swimming pool, whereas lifting that same individual on dry land is much harder. Evidently, water provides partial support to any object placed in it. The upward force that the fluid exerts on an object submerged in it is called the **buoyant force.** According to Archimedes' principle, **the magnitude of the buoyant force always equals the weight of the fluid displaced by the object.**

Archimedes' principle can be verified in the following manner. Suppose we focus our attention on the indicated cube of fluid in the container of Figure 15.7. This cube of fluid is in equilibrium under the action of the forces on it. One of these forces is the force of gravity. What cancels this downward force? Apparently, the rest of the fluid inside the container is holding it in equilibrium. Thus, the magnitude of the buoyant force is exactly equal to the weight of the fluid inside the cube:

$$B = w$$

Now, imagine that the cube of fluid is replaced by a cube of steel of the same dimensions. What is the buoyant force on the steel? The fluid surrounding a cube behaves in the same way whether a cube of fluid or a cube of steel is being buoyed up; therefore, **the buoyant force acting on the steel is the same as the buoyant force acting on a cube of fluid of the same dimensions.** This result applies for a submerged object of any shape, size, or density.

Let us show explicitly that the magnitude of the buoyant force is equal to the weight of the displaced fluid. The pressure at the bottom of the cube in Figure 15.7 is greater than the pressure at the top by an amount $\rho_f gh$, where ρ_f is the density of the fluid and h is the height of the cube. The pressure difference, ΔP, is equal to the buoyant force per unit area—that is, $\Delta P = B/A$. Thus, we see that $B = (\Delta P)A = (\rho_f gh)A = \rho_f gV$, where V is the volume of the cube. Because the mass of the fluid in the cube is $M = \rho_f V$, we see that

$$B = w = \rho_f Vg = Mg \qquad [15.5]$$

where w is the weight of the displaced fluid.

Before proceeding with a few examples, it is instructive to compare the forces acting on a totally submerged object with those acting on a floating object.

Case I: A Totally Submerged Object When an object is totally submerged in a fluid of density ρ_f, the magnitude of the upward buoyant force is $B = \rho_f V_0 g$, where V_0 is the volume of the object. If the object has a density ρ_0, its weight is $w = Mg = \rho_0 V_0 g$, and the net force on it is $B - w = (\rho_f - \rho_0)V_0 g$. Hence, if the density of the object is less than the density of the fluid, as in Figure 15.8a, the unsupported object will accelerate upward. If the density of the object is greater than the density of the fluid as in Figure 15.8b, the unsupported object will sink.

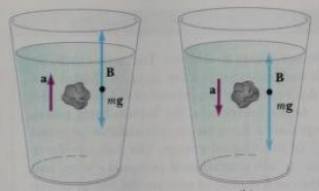


Figure 15.8 (a) A totally submerged object that is less dense than the fluid in which it is submerged will experience a net upward force. (b) A totally submerged object that is denser than the fluid sinks.

Case II: A Floating Object Now consider an object in static equilibrium floating on a fluid—that is, an object that is only partially submerged. In this case, the upward buoyant force is balanced by the downward force of gravity exerted on the object. If V is the volume of the fluid displaced by the object (which corresponds to that volume of the object beneath the fluid level), then the buoyant force has a magnitude $B = \rho_f Vg$. Because the weight of the object is $w = Mg = \rho_0 V_0 g$, and $w = B$, we see that $\rho_f Vg = \rho_0 V_0 g$, or

$$\frac{\rho_0}{\rho_f} = \frac{V}{V_0} \qquad [15.6]$$

Under normal conditions, the average density of a fish is slightly greater than the density of water. This being the case, a fish would sink if it did not have some mechanism for adjusting its density. The fish accomplishes this by internally regulating the size of its swim bladder. In this manner, fish are able to swim to various depths.



Hot-air balloons over Albuquerque, New Mexico. Because hot air is less dense than cold air, there is a net upward buoyant force on the ballons. *(William Moriarity/Rainbow)*

Thinking Physics 4

Suppose an office party is taking place on the top floor of a tall building. Carrying an iced soft drink, you step on the elevator, which begins to accelerate downward. What happens to the ice in the drink? Does it rise farther out of the liquid? Sink deeper into the liquid? Or is it unaffected by the motion?

Reasoning The level of the ice in the liquid is unaffected by the motion. The acceleration of the elevator is equivalent to a change in the gravitational field, according to the principle of equivalence. If the elevator accelerates downward, one might be tempted to say that the effect is the same as if gravity decreases—the weight of the ice cube decreases, causing it to float higher in the liquid. Recall, however, that the magnitude of the buoyant force is equal to the weight of the liquid displaced by the ice cube. The weight of the liquid also decreases with the effectively decreased gravity. With both the weight of the ice cube and the buoyant force decreasing by the same factor, the level of the ice cube in the liquid is unaffected.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

Thinking Physics 5

A florist delivery person is delivering a flower basket to a home. The basket includes an attached helium-filled balloon, which suddenly comes loose from the basket and begins to accelerate upward toward the sky. Startled by the release of the balloon, the delivery person drops the flower basket. As the basket falls, the basket–Earth system experiences an increase in kinetic energy and a decrease in gravitational potential energy, consistent with the conservation of mechanical energy. The balloon–Earth system, however, experiences an increase in *both* gravitational potential energy and kinetic energy. Is this inconsistent with the conservation of mechanical energy principle? If not, from where is the extra energy coming?

Reasoning In the case of the system of the flower basket and the Earth, a good approximation to the motion of the basket can be made by ignoring the effects of the air. Thus, the conservation of mechanical energy principle is obeyed, with the exception of a small amount of transformation to internal energy of the air and the basket due to air friction. For the balloon–Earth system, we cannot ignore the effects of the air—it is the buoyant force of the air that causes the balloon to rise.

CONCEPTUAL PROBLEM 3

Atmospheric pressure varies from day to day. Does a ship float higher in the water on a high pressure day compared to a low pressure day?

CONCEPTUAL PROBLEM 4

Suppose a damaged ship just barely floats in the ocean after a hole in its hull has been plugged. It is pulled toward shore and into a river, heading toward a dry dock for repair. As it is pulled up the river, it sinks. Why?

CONCEPTUAL PROBLEM 5

A pound of styrofoam and a pound of lead have the same weight. If they are placed on an equal arm balance, will it balance?

CONCEPTUAL PROBLEM 6

A person in a boat floating in a small pond throws an anchor overboard. Does the level of the pond rise, fall, or remain the same?

Example 15.3 A Submerged Object

A piece of aluminum is suspended from a string and then completely immersed in a container of water (Fig. 15.9). The mass of the aluminum is 1.0 kg, and its density is 2.7×10^3 kg/m^3. Calculate the tension in the string before and after the aluminum is immersed.

Solution When the aluminum is suspended in air, as in Figure 15.9a, the tension in the string, T_1 (the reading on the scale), is equal to the weight, Mg, of the aluminum, assuming that the buoyant force of air can be neglected:

$$T_1 = Mg = (1.0 \text{ kg})(9.80 \text{ m/s}^2) = 9.8 \text{ N}$$

When immersed in water, the aluminum experiences an upward buoyant force **B**, as in Figure 15.9b, which reduces the tension in the string. Because the system is in equilibrium,

$$T_2 + B - Mg = 0$$

$$T_2 = Mg - B = 9.8 \text{ N} - B$$

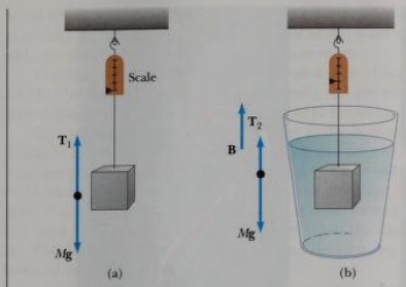


Figure 15.9 (Example 15.3) (a) When the aluminum is suspended in air, the scale reads the weight, Mg (neglecting the buoyancy of air). (b) When the aluminum is immersed in water, the buoyant force **B** reduces the scale reading to $T_2 = Mg - B$.

In order to calculate B, we must first calculate the volume of the aluminum:

$$V_{Al} = \frac{M}{\rho_{Al}} = \frac{1.0 \text{ kg}}{2.7 \times 10^3 \text{ kg/m}^3} = 3.7 \times 10^{-4} \text{ m}^3$$

Because the buoyant force equals the weight of the water displaced, we have

$$B = M_w g = \rho_w V_{Al} g$$
$$= (1.0 \times 10^3 \text{ kg/m}^3)(3.7 \times 10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2)$$
$$= 3.6 \text{ N}$$

Therefore,

$$T_2 = 9.8 \text{ N} - B = 9.8 \text{ N} - 3.6 \text{ N} = 6.2 \text{ N}$$

EXERCISE 5 What is the true weight of one cubic meter of a light plastic that has a specific gravity of 0.15? Note the true weight of an object is its weight in vacuum.
Answer 1.47 kN

15.5 · FLUID DYNAMICS

Thus far, our study of fluids has been restricted to fluids at rest. We now turn our attention to fluid dynamics—that is, fluids in motion. Instead of trying to study the motion of each particle of the fluid as a function of time, we describe the properties of the fluid at each point as a function of time.

Flow Characteristics

When fluid is in motion, its flow can be characterized as being one of two main types. The flow is said to be **steady** or **laminar** if each particle of the fluid follows a smooth path, so that the paths of different particles never cross each other, as in Figure 15.10. Thus, in steady flow, the velocity of the fluid at any point remains constant in time.

Above a certain critical speed, fluid flow becomes **turbulent.** Turbulent flow is an irregular flow characterized by small whirlpool-like regions, as in Figure 15.11. As an example, the flow of water in a stream becomes turbulent in regions in which rocks and other obstructions are encountered, often forming white-water rapids.

The term **viscosity** is commonly used in fluid flow to characterize the degree of internal friction in the fluid. This internal friction or viscous force is associated with the resistance of two adjacent layers of the fluid against moving relative to each other. Because of viscosity, part of the kinetic energy of a fluid is converted to thermal energy. This is similar to the mechanism by which an object sliding on a rough horizontal surface loses kinetic energy.

Petitioner Intel Corp., Ex. 1038
IPR2023-00783



Figure 15.10 An illustration of streamline flow around an automobile in a test wind tunnel. The streamlines in the airflow are made visible by smoke particles. *(Andy Sacks/Tony Stone Images)*



Figure 15.11 Turbulent flow: The tip of a rotating blade (the dark region at the top) forms a vortex in air that is being heated by an alcohol lamp (the wick is at the bottom). Note the air turbulence on both sides of the rotating blade. *(Kim Vandiver and Harold Edgerton, Palm Press, Inc.)*

Because the motion of a real fluid is complex and not yet fully understood, we make some simplifying assumptions in our approach. As we shall see, many features of real fluids in motion can be understood by considering the behavior of an ideal fluid. In our model, we make the following four assumptions:

Properties of an ideal fluid •

1. **Nonviscous fluid.** In a nonviscous fluid, internal friction is neglected. An object moving through the fluid experiences no viscous force.
2. **Incompressible fluid.** The density of an incompressible fluid is assumed to remain constant because it does not change due to a change in pressure on the fluid.
3. **Steady flow.** In steady flow, we assume that the velocity of the fluid at each point remains constant in time.
4. **Irrotational flow.** Fluid flow is irrotational if there is no angular momentum of the fluid about any point. If a small wheel placed anywhere in the fluid does not rotate about the wheel's center of mass, the flow is irrotational. (If the wheel were to rotate, as it would if turbulence were present, the flow would be rotational.)

The first two assumptions are properties of our ideal fluid. The last two are descriptions of the way that the fluid flows.



Figure 15.12 This diagram represents a set of streamlines (blue lines). A particle at P follows one of these streamlines, and its velocity is tangent to the streamline at each point along its path.

15.6 • STREAMLINES AND THE EQUATION OF CONTINUITY

The path taken by a fluid particle under steady flow is called a **streamline.** The velocity of the fluid particle is always tangent to the streamline, as shown in Figure 15.12. No two streamlines can cross each other, for if they did, a fluid particle could move either way at the crossover point and then the flow would not be steady. A set of streamlines, as shown in Figure 15.12, forms what is called a *tube of flow.* Note that fluid particles cannot flow into or out of the sides of this tube, because if they did, the streamlines would be crossing each other.

Consider an ideal fluid flowing through a pipe of nonuniform size, as in Figure 15.13. The particles in the fluid move along the streamlines in steady flow. At all points the velocity of any particle is tangent to the streamline along which it moves.

In a small time interval, Δt, the fluid at the bottom end of the pipe moves a distance $\Delta x_1 = v_1 \Delta t$. If A_1 is the cross-sectional area in this region, then the mass contained in the shaded region is $\Delta m_1 = \rho A_1 \Delta x_1 = \rho A_1 v_1 \Delta t$. In a similar way, the fluid that moves through the upper end of the pipe in the time Δt has a mass $\Delta m_2 = \rho A_2 v_2 \Delta t$. However, because *mass is conserved* and because the flow is steady, the mass that crosses A_1 in a time Δt must equal the mass that crosses A_2 in a time Δt. That is, $\Delta m_1 = \Delta m_2$, or $\rho A_1 v_1 = \rho A_2 v_2$. Because the density is common to both sides of this expression, we get

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad [15.7]$$

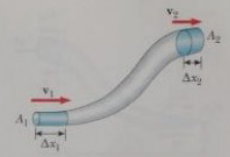


Figure 15.13 A fluid moving with streamline flow through a pipe of varying cross-sectional area. The volume of fluid flowing through A_1 in a time interval Δt must equal the volume flowing through A_2 in the same time interval. Therefore, $A_1 v_1 = A_2 v_2$.

• *Equation of continuity*

This expression, called the **equation of continuity,** says that **the product of the area and the fluid speed at all points along the pipe is a constant.** Therefore, the speed is high where the tube is constricted and low where the tube is wide. The product Av, which has the dimensions of volume/time, is called the **volume flow rate.** The condition Av = constant is equivalent to the fact that **the amount of fluid that enters one end of the tube in a given time interval equals the amount leaving in the same time interval, assuming no leaks.**

Example 15.4 Filling a Water Bucket

A water hose 2.00 cm in diameter is used to fill a 20.0-liter bucket. If it takes 1.00 min to fill the bucket, what is the speed v at which the water leaves the hose? (1 L = 10^3 cm^3)

Solution The cross-sectional area of the hose is

$$A = \pi r^2 = \pi \frac{d^2}{4} = \pi\left(\frac{2.00^2}{4}\right)\text{cm}^2 = \pi\ \text{cm}^2$$

According to the data given, the flow rate is equal to 20.0 liters/min. Equating this to the product Av gives

$$Av = 20.0\ \frac{\text{L}}{\text{min}} = \frac{20.0 \times 10^3\ \text{cm}^3}{60.0\ \text{s}}$$

$$v = \frac{20.0 \times 10^3\ \text{cm}^3}{(\pi\ \text{cm}^2)(60.0\ \text{s})} = 106\ \text{cm/s}$$

EXERCISE 6 If the diameter of the hose is reduced to 1.00 cm, what will the speed of the water be as it leaves the hose, assuming the same flow rate? Answer 424 cm/s

15.7 • BERNOULLI'S PRINCIPLE

As a fluid moves through a pipe of varying cross-section and elevation, the pressure changes along the pipe. In 1738 the Swiss physicist Daniel Bernoulli first derived an expression that relates the pressure to fluid speed and elevation.

Consider the flow of an ideal fluid through a nonuniform pipe in a time, Δt, as illustrated in Figure 15.14. The force on the lower end of the fluid is $P_1 A_1$, where P_1 is the pressure in section 1. The work done by this force is $W_1 = F_1 \Delta x_1 = P_1 A_1 \Delta x_1 = P_1 \Delta V$, where ΔV is the volume of section 1. In a similar manner, the

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IPR2023-00783

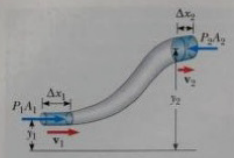


Figure 15.14 A fluid flowing through a constricted pipe with streamline flow. The fluid in the section of length Δx_1 moves to the section of length Δx_2. The volumes of fluid in the two sections are equal.

work done on the fluid at the upper end in the time Δt is $W_2 = -P_2A_2\,\Delta x_2 = -P_2\,\Delta V$. (The volume that passes through section 1 in a time Δt equals the volume that passes through section 2 in the same time interval.) This work is negative because the fluid force opposes the displacement. Thus the net work done by these forces in the time Δt is

$$W_F = (P_1 - P_2)\Delta V$$

If a fluid element of mass Δm enters the tube with speed v_1 and leaves with speed v_2, then the change in its kinetic energy is

$$\Delta K = \tfrac{1}{2}(\Delta m)v_2^2 - \tfrac{1}{2}(\Delta m)v_1^2$$

As the fluid element rises a vertical distance $y_2 - y_1$, the work done by gravity is negative and given by

$$W_g = -\Delta mg(y_2 - y_1)$$

Because the net work done on the fluid is $W_F + W_g$, we can apply the work-kinetic energy theorem to this volume of fluid to give

$$(P_1 - P_2)\Delta V - \Delta mg(y_2 - y_1) = \tfrac{1}{2}(\Delta m)v_2^2 - \tfrac{1}{2}(\Delta m)v_1^2$$

If we divide each term by ΔV, noting that $\rho = \Delta m/\Delta V$, and rearrange terms, the expression reduces to

$$P_1 + \tfrac{1}{2}\rho v_1^2 + \rho g y_1 = P_2 + \tfrac{1}{2}\rho v_2^2 + \rho g y_2 \qquad [15.8]$$

This is **Bernoulli's equation** as applied to an ideal fluid. It is often expressed as

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad [15.9]$$

Bernoulli's equation says that the sum of the pressure, (P), the kinetic energy per unit volume ($\frac{1}{2}\rho v^2$), and gravitational potential energy per unit volume ($\rho g y$) has the same value at all points along a streamline.

Note the Bernoulli's equation is *not* a sum of energy density terms because P is pressure, not energy density. When the fluid is at rest, $v_1 = v_2 = 0$ and Equation 15.8 becomes

$$P_1 - P_2 = \rho g(y_2 - y_1) = \rho g h$$

which agrees with Equation 15.4.



Daniel Bernoulli (1700–1782)

Bernoulli was a Swiss physicist and mathematician who made important discoveries in hydrodynamics. His most famous work, *Hydrodynamica*, published in 1738, is both a theoretical and a practical study of equilibrium, pressure, and velocity of fluids. In this publication, Bernoulli also attempted the first explanation of the behavior of gases with changing pressure and temperature; this was the beginning of the kinetic theory of gases. *(Corbis-Bettmann)*

Thinking Physics 6

In tornadoes, it sometimes occurs that the windows of a house explode outward. Sometimes, the roof of a house is lifted off the house. What causes these events to happen? How does opening the windows in a tornado (which may seem counterintuitive!) help in theory?

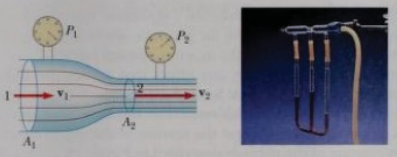
Reasoning The wind speed in a tornado is very high. According to Bernoulli's principle, such high wind speeds will result in low pressures. When these winds pass by a window, the pressure outside the window is reduced well below that of the pressure on the inside surface, which is due to the still air inside the house. As a result, the net force outward can burst the window. When these high-speed winds pass over the roof of a house, the pressure above the roof is much lower than the pressure below the roof, again from the still air inside the house. The pressure difference can be enough to lift the roof off the house. By opening the windows, the pressure differential between the outside and inside air can be reduced by allowing a flow of air through the open windows. This will reduce the possibility of exploding windows or a flying roof.

CONCEPTUAL PROBLEM 7

When you are driving a small car on the freeway and a truck passes you at a high speed, you feel pulled *toward* the truck. Why?

Example 15.5 The Venturi Tube

The horizontal constricted pipe illustrated in Figure 15.15, known as a *Venturi tube*, can be used to measure flow speeds such as air speed in flight. Let us determine the flow speed at point 2 if the pressure difference $P_1 - P_2$ is known.



(a) (b)

Figure 15.15 (a) (Example 15.5) The pressure P_1 is greater than the pressure P_2, because $v_1 < v_2$. This device can be used to measure the speed of fluid flow. (b) A Venturi tube. *(Courtesy of Central Scientific Co.)*

Solution Because the pipe is horizontal, $y_1 = y_2$ and Equation 15.8 applied to points 1 and 2 gives

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

From the equation of continuity (Eq. 15.7), we see that $A_1v_1 = A_2v_2$, or

$$v_1 = \frac{A_2}{A_1}v_2$$

Substituting this expression into the previous equation gives

$$P_1 + \tfrac{1}{2}\rho\left(\frac{A_2}{A_1}\right)^2 v_2^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

$$v_2 = A_1\sqrt{\frac{2(P_1 - P_2)}{\rho(A_1^2 - A_2^2)}}$$

We can also obtain an expression for v_1 using this result and the continuity equation. Note that because $A_2 < A_1$, it follows that $P_1 > P_2$. In other words, the pressure is reduced in the constricted part of the pipe. This result is somewhat analogous to the following situation: Consider a very crowded room in which people are squeezed together. As soon as a door is opened and people begin to exit, the squeezing (pressure) is least near the door where the motion (flow) is greatest.

EXERCISE 7 The Garfield Thomas water tunnel at Pennsylvania State University has a circular cross-section that constricts from a diameter of 3.6 m to the test section, which is 1.2 m in diameter. If the speed of flow is 3.0 m/s in the larger-diameter pipe, determine the speed of flow in the test section. Answer 27 m/s

15.8 · OTHER APPLICATIONS OF BERNOULLI'S PRINCIPLE

OPTIONAL

Consider the streamlines that flow around an airplane wing as shown in Figure 15.16. Let us assume that the airstream approaches the wing horizontally from the right with a velocity $\mathbf{v}_1$. The tilt of the wing causes the airstream to be deflected downward with a velocity $\mathbf{v}_2$. Because the airstream is deflected by the wing, the wing must exert a force on the airstream. According to Newton's third law, the airstream must exert an equal and opposite force $\mathbf{F}$ on the wing. This force has a

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IPR2023-00783

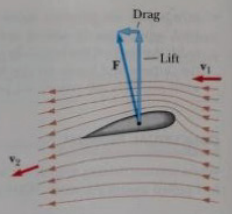


Figure 15.16 Streamline flow around a moving airplane wing. The air approaching from the right with a velocity $\mathbf{v}_1$ is deflected downward by the wing, leaving the trailing edge of the wing with a velocity $\mathbf{v}_2$. Because the airstream is deflected, it exerts a force $\mathbf{F}$ on the wing that has a vertical and horizontal component.



Figure 15.17 A stream of air passing over a tube dipped into a liquid will cause the liquid to rise in the tube as shown.

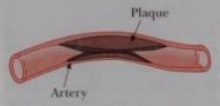


Figure 15.18 Blood must travel faster than normal through a constricted artery.

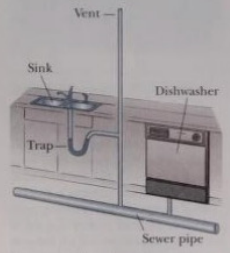


Figure 15.19 (Thinking Physics 7)

vertical component called the **lift** (or aerodynamic lift) and a horizontal component called **drag.** The lift depends on several factors, such as the speed of the airplane, the area of the wing, its curvature, and the angle between the wing and the horizontal. As this angle increases, turbulent flow can set in above the wing to reduce the lift.

The lift on the wing is consistent with Bernoulli's equation. The speed of the airstream is greater above the wing, hence the air pressure above the wing is less than the pressure below the wing, which results in a net upward force.

In general, an object experiences lift by any effect that causes the fluid to change its direction as it flows past the object. Some factors that influence the lift are the shape of the object, its orientation with respect to the fluid flow, spinning motion (for example, a spinning baseball), and the texture of the object's surface.

A number of devices operate in the manner described in Figure 15.17. A stream of air passing over an open tube reduces the pressure above the tube. This reduction in pressure causes the liquid to rise into the air stream. The liquid is then dispersed into a fine spray of droplets. You might recognize that this so-called atomizer is used in perfume bottles and paint sprayers.

Bernoulli's equation explains one symptom called vascular flutter in a person with advanced arteriosclerosis. The artery is constricted as a result of an accumulation of plaque on its inner walls (Fig. 15.18). If the blood speed is sufficiently high in the constricted region, the artery may collapse under external pressure, causing a momentary interruption in blood flow. At this point, Bernoulli's equation does not apply, and the vessel reopens under arterial pressure. As the blood rushes through the constricted artery, the internal pressure drops and again the artery closes. Such variations in blood flow can be heard with a stethoscope. If the plaque becomes dislodged and ends up in a smaller vessel that delivers blood to the heart, the person can suffer a heart attack.

Thinking Physics 7

Consider the portion of a home plumbing system shown in Figure 15.19. The water trap in the pipe below the sink captures a plug of water that prevents sewer gas from finding its way from the sewer pipe, up the sink drain, and into the home. Suppose the dishwasher is draining, so that water is moving to the left in the sewer pipe. What is the purpose of the vent, which is open to the air above the roof of the house? In which direction is air moving at the opening of the vent, upward or downward?

Reasoning Let us imagine that the vent is not present, so that the drain pipe for the sink is simply connected through the trap to the sewer pipe. As water from the dishwasher moves to the left in the sewer pipe, the pressure in the sewer pipe is reduced below atmospheric pressure, according to Bernoulli's principle. The pressure at the drain in the sink is still at atmospheric pressure. Thus, this pressure differential can push the plug of water in the water trap of the sink down the drain pipe into the sewer pipe, removing it as a barrier to sewer gas. With the addition of the vent to the roof, the reduced pressure of the dishwasher water will result in air entering the vent pipe at the roof. This will keep the pressure in the vent pipe and the right-hand side of the sink drain pipe at a pressure close to atmospheric, so that the plug of water in the water trap will remain.

SUMMARY

The **pressure,** P, in a fluid is the force per unit area that the fluid exerts on any surface:

$$P \equiv \frac{F}{A} \qquad [15.1]$$

In the SI system, pressure has units of N/m^2, and $1\ N/m^2 = 1$ pascal (Pa).

The pressure in a fluid varies with depth h according to the expression

$$P = P_0 + \rho g h \qquad [15.4]$$

where P_0 is atmospheric pressure ($= 1.013 \times 10^5\ N/m^2$) and ρ is the density of the fluid, assumed uniform.

Pascal's law states that when pressure is applied to an enclosed fluid, the pressure is transmitted undiminished to every point in the fluid and to every point on the walls of the container.

When an object is partially or fully submerged in a fluid, the fluid exerts an upward force on the object called the **buoyant force.** According to **Archimedes' principle,** the buoyant force is equal to the weight of the fluid displaced by the body.

Various aspects of fluid dynamics can be understood by assuming that the fluid is nonviscous and incompressible and that the fluid motion is a steady flow with no turbulence.

Using these assumptions, two important results regarding fluid flowing through a pipe of nonuniform size can be obtained:

- The flow rate through the pipe is a constant, which is equivalent to stating that the product of the cross-sectional area, A, and the speed, v, at any point is a constant. This gives the **equation of continuity:**

$$A_1 v_1 = A_2 v_2 = \text{constant} \qquad [15.7]$$

- The sum of the pressure, kinetic energy per unit volume, and gravitational potential energy per unit volume has the same value at all points along a streamline. This corresponds to **Bernoulli's equation:**

$$P + \tfrac{1}{2}\rho v^2 + \rho g y = \text{constant} \qquad [15.9]$$

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

CONCEPTUAL QUESTIONS

1. Figure Q15.1 shows aerial views from directly above two dams. Both dams are equally long (the vertical dimension in the diagram) and equally deep (into the page in the diagram). The dam on the left holds back a very large lake, and the dam on the right holds back a narrow river. Which dam has to be built more strongly?

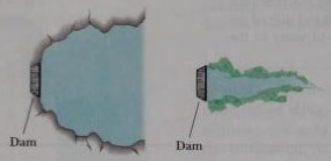


Figure Q15.1 (Conceptual Question 1)

2. Some physics students attach a long tube to the opening of a balloon. Leaving the balloon on the ground, the other end of the tube is hoisted to the roof of a multistory campus building. Students at the top of the building start pouring water into the tube. The students on the ground watch the balloon fill with water. On the roof, the students are surprised to see that the tube never seems to fill up—more and more water can continue to be poured into the tube. On the ground, the balloon bursts and drenches the students there. Why did the balloon-tube system never fill up, and why did the balloon burst?
3. A barge is carrying a load of gravel along a river. It approaches a low bridge, and the captain realizes that the top of the pile of gravel is not going to make it under the bridge. The captain orders the crew to quickly shovel gravel from the pile into the water. Is this a good decision?
4. Two drinking glasses having equal weights but different shapes and different cross-sectional areas are filled to the same level with water. According to the expression $P = P_0 + \rho g h$, the pressure is the same at the bottom of both glasses. In view of this, why does one weigh more than the other?
5. Pascal used a barometer with water as the working fluid. Why is it impractical to use water for a typical barometer?
6. A helium-filled balloon rises until its density becomes the same as that of the air. If a sealed submarine begins to sink, will it go all the way to the bottom of the ocean or will it stop when its density becomes the same as that of the surrounding water?
7. A fish rests on the bottom of a bucket of water while the bucket is being weighed. When the fish begins to swim around, does the weight change?
8. Lead has a greater density than iron, and both are denser than water. Is the buoyant force on a lead object greater than, less than, or equal to the buoyant force on an iron object of the same volume?
9. An ice cube is placed in a glass of water. What happens to the level of the water as the ice melts?
10. The water supply for a city is often provided from reservoirs built on high ground. Water flows from the reservoir, through pipes, and into your home when you turn the tap on your faucet. Why is the water flow more rapid out of a faucet on the first floor of a building than in an apartment on a higher floor?
11. If air from a hair dryer is blown over the top of a Ping-Pong ball, the ball can be suspended in air. Explain.
12. When ski-jumpers are airborne, why do they bend their bodies forward and keep their hands at their sides?
13. Explain why a sealed bottle partially filled with a liquid can float.
14. When is the buoyant force on a swimmer greater: after exhaling or after inhaling?
15. A piece of unpainted wood is partially submerged in a container filled with water. If the container is sealed and pressurized above atmospheric pressure, does the wood rise, fall, or remain at the same level? (*Hint:* Wood is porous.)
16. Because atmospheric pressure is about 10^5 N/m^2 and the area of a person's chest is about 0.13 m^2, the force of the atmosphere on one's chest is approximately 13 000 N. In view of this enormous force, why don't our bodies collapse?
17. If you release a ball while inside a freely falling elevator, the ball remains in front of you rather than falling to the floor, because the ball, the elevator, and you all experience the same downward acceleration, **g**. What happens if you repeat this experiment with a helium-filled balloon? (This one is tricky.)
18. Two identical ships set out to sea. One is loaded with a cargo of Styrofoam, and the other is empty. Which ship is more submerged?
19. A small piece of steel is tied to a block of wood. When the wood is placed in a tub of water with the steel on top, half of the block is submerged. If the block is inverted so that the steel is under water, does the amount of the block submerged increase, decrease, or remain the same? What happens to the water level in the tub when the block is inverted?
20. Prairie dogs ventilate their burrows by building a mound over one entrance, which is open to a stream of air. A second entrance at ground level is open to almost stagnant air. How does this construction create an air flow through the burrow?
21. An unopened can of diet cola floats when placed in a tank of water, whereas a can of regular cola of the same brand sinks in the tank. What do you suppose could explain this behavior?

PROBLEMS

Section 15.1 Pressure

1. A 50.0-kg woman balances on one heel of a high-heel shoe. If the heel is circular with radius 0.500 cm, what pressure does she exert on the floor?
2. A king orders a gold crown having a mass of 0.500 kg. When it arrives from the metalsmith, the volume of the crown is found to be 185 cm^3. Is the crown made of solid gold?
3. What is the total mass of the earth's atmosphere? (The radius of the Earth is 6.37×10^6 m, and atmospheric pressure at the surface is 1.013×10^5 N/m^2.)

Section 15.2 Variation of Pressure with Depth

4. Determine the absolute pressure at the bottom of a lake that is 30.0 m deep.
5. The spring of the pressure gauge shown in Figure 15.2 has a force constant of 1000 N/m, and the piston has a diameter of 2.00 cm. Find the depth in water for which the spring compresses by 0.500 cm.
6. The small piston of a hydraulic lift has a cross-sectional area of 3.00 cm^2, and the large piston has an area of 200 cm^2 (Fig. 15.4). What force must be applied to the small piston to raise a load of 15.0 kN? (In service stations this is usually accomplished with compressed air.)
7. What must be the contact area between a suction cup (completely exhausted) and a ceiling in order to support the weight of an 80.0-kg student?
8. A swimming pool has dimensions 30.0 m × 10.0 m and a flat bottom. When the pool is filled to a depth of 2.00 m with fresh water, what is the total force due to the water on the bottom? On each end? On each side?
9. Normal atmospheric pressure is 1.013×10^5 Pa. The approach of a storm causes the height of a mercury barometer to drop by 20.0 mm from the normal height. What is the atmospheric pressure? (The density of mercury is 13.59 g/cm^3.)
10. Mercury is poured into a U-shaped tube, as in Figure P15.10a. The left arm of the tube has a cross-sectional area of $A_1 = 10.0$ cm^2, and the right arm has a cross-sectional area of $A_2 = 5.00$ cm^2. One hundred grams of water are then poured into the right arm, as in Figure P15.10b. (a) Determine the length of the water column in the right arm of the U-shaped tube. (b) Given that the density of mercury is 13.6 g/cm^3, what distance, h, does the mercury rise in the left arm?
11. Blaise Pascal duplicated Torricelli's barometer using (as a Frenchman would) a red Bordeaux wine as the working liquid (Fig. P15.11). The density of the wine he used was 984 kg/m^3. What was the height h of the wine column for normal atmospheric pressure? Would you expect the vacuum above the column to be as good as for mercury?

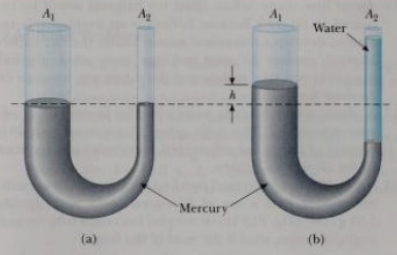


Figure P15.10

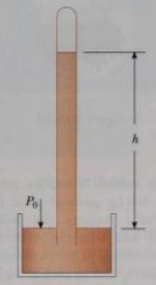


Figure P15.11

Section 15.4 Buoyant Forces and Archimedes' Principle

12. A Styrofoam slab has a thickness of 10.0 cm and a density of 300 kg/m^3. What is the area of the slab if it floats just awash in fresh water when a 75.0-kg swimmer is aboard?
13. A Styrofoam slab has thickness h and density ρ_s. What is the area of the slab if it floats with its top surface just awash in fresh water when a swimmer of mass m is on top?
14. A balloon is filled with 400 m^3 of helium. How big a payload can the balloon lift? (The density of air is 1.29 kg/m^3; the density of helium is 0.180 kg/m^3.)
15. A cube of wood 20.0 cm on a side and having a density of 650 kg/m^3 floats on water. (a) What is the distance from the horizontal top of the cube to the water level? (b) How

Petitioner Intel Corp., Ex. 1038
IPR2023-00783

much lead weight has to be placed on top of the cube so that its top is just level with the water?

16. About how many helium-filled toy balloons would be required to lift you? Because helium is an irreplaceable resource, develop a theoretical answer rather than an experimental answer. In your solution state what physical quantities you take as data and the values you measure or estimate for them.

17. A plastic sphere floats in water with 50.0 percent of its volume submerged. This same sphere floats in oil with 40.0 percent of its volume submerged. Determine the densities of the oil and the sphere.

18. A frog in a hemispherical pod finds that he just floats without sinking into a sea of blue-green ooze with density 1.35 g/cm^3 (Fig. P15.18). If the pod has radius 6.00 cm and negligible mass, what is the mass of the frog?



Figure P15.18

19. How many cubic meters of helium are required to lift a balloon with a 400-kg payload to a height of 8000 m? ($\rho_{He} = 0.18$ kg/m^3.) Assume the balloon maintains a constant volume and that the density of air decreases with altitude z according to the expression $\rho_{air} = \rho_0 e^{-z/8000}$, where z is in meters, and ρ_0 (= 1.25 kg/m^3) is the density of air at sea level.

20. A 10.0-kg block of metal measuring 15.0 cm × 10.0 cm × 10.0 cm is suspended from a scale and immersed in water, as in Figure 15.9b. The 15.0-cm dimension is vertical, and the top of the block is 5.00 cm from the surface of the water. (a) What are the forces on the top and bottom of the block? (Take $P_0 = 1.0130 \times 10^5$ N/m^2.) (b) What is the reading of the spring scale? (c) Show that the buoyant force equals the difference between the forces at the top and bottom of the block.

Section 15.5 Fluid Dynamics

Section 15.6 Streamlines and the Equation of Continuity

Section 15.7 Bernoulli's Principle

21. The volume flow rate of water through a horizontal pipe is 2.00 m^3/min. Determine the speed of flow at a point at which the diameter of the pipe is (a) 10.0 cm, (b) 5.00 cm.

22. The legendary Dutch boy who saved Holland by placing his finger in the hole of a dike had a finger 1.20 cm in diameter. Assuming the hole was 2.00-m below the surface of the sea (density 1030 kg/m^3), (a) what was the force on his finger? (b) If he removed his finger from the hole, how long would it take the released water to fill 1 acre of land to a depth of 1 foot, assuming the hole remained constant in size? (A typical U.S. family of four uses 1 acre-foot of water, 1234 m^3, in 1 year.)

23. A large storage tank, open at the top and filled with water, develops a small hole in its side at a point 16.0 m below the water level. If the rate of flow from the leak is 2.50×10^{-3} m^3/min, determine (a) the speed at which the water leaves the hole and (b) the diameter of the hole.

24. A horizontal pipe 10.0 cm in diameter has a smooth reduction to a pipe 5.00 cm in diameter. If the pressure of the water in the larger pipe is 8.00×10^4 Pa and the pressure in the smaller pipe is 6.00×10^4 Pa, at what rate does water flow through the pipes?

25. Water is pumped from the Colorado River up to Grand Canyon Village through a 15.0-cm diameter pipe. The river is at 564 m elevation and the village is at 2096 m. (a) What is the minimum pressure the water must be pumped at to arrive at the village? (b) If 4500 m^3 are pumped per day, what is the speed of the water in the pipe? (c) What additional pressure is necessary to deliver this flow? *Note:* You may assume that the acceleration due to gravity and the density of air are constant over this range of elevations.

26. Old Faithful geyser in Yellowstone Park erupts at approximately 1-hour intervals, and the height of the fountain reaches 40.0 m. (a) With what speed does the water leave the ground? (b) What is the pressure (above atmospheric) in the heated underground chamber?

Section 15.8 Other Applications of Bernoulli's Principle (Optional)

27. An airplane has a mass of 1.60×10^4 kg, and each wing has an area of 40.0 m^2. During level flight, the pressure on the lower wing surface is 7.00×10^4 Pa. Determine the pressure on the upper wing surface.

28. A Venturi tube may be used as a fluid flow meter (Fig. 15.15). If the difference in pressure $P_1 - P_2 = 21.0$ kPa, find the fluid flow rate in m^3/s given that the radius of the outlet tube is 1.00 cm, the radius of the inlet tube is 2.00 cm, and the fluid is gasoline ($\rho = 700$ kg/m^3).

29. A Pitot tube can be used to determine the velocity of air flow by measuring the difference between the total pressure and the static pressure (Fig. P15.29). If the fluid in the tube is mercury, density $\rho_{Hg} = 13\,600$ kg/m^3, and $\Delta h = 5.00$ cm, find the speed of air flow. (Assume that the air is stagnant at point A and take $\rho_{air} = 1.25$ kg/m^3.)

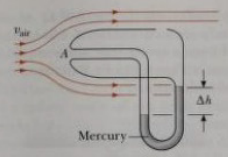


Figure P15.29

30. A siphon is used to drain water from a tank, as indicated in Figure P15.30. The siphon has a uniform diameter. Assume steady flow. (a) If the distance $h = 1.00$ m, find the speed of outflow at the end of the siphon. (b) What is the limitation on the height of the top of the siphon above the water surface? (In order to have a continuous flow of liquid, the pressure in the liquid cannot go below the vapor pressure of the liquid.)

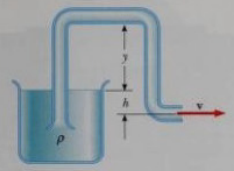


Figure P15.30

31. A large storage tank is filled to a height h_0. The tank is punctured at a height h from the bottom of the tank (Fig. P15.31). (a) Prove that the speed at which the water comes out is $\sqrt{2g(h_0 - h)}$ if the flow is steady and frictionless. (b) How far from the tank will the stream land?

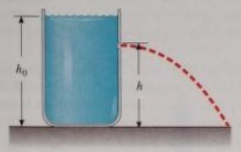


Figure P15.31

32. A hole is punched at height h in the side of a container of height h_0, full of water as in Figure P15.31. If the water is to shoot as far as possible horizontally, (a) how far from the bottom of the container should the hole be punched? (b) Neglecting friction losses, how far (initially) from the side of the container will the water land?

Additional Problems

33. A Ping-Pong ball has a diameter of 3.80 cm and average density of 0.0840 g/cm^3. What force would be required to hold it completely submerged under water?

34. Figure P15.34 shows a water tank with a valve at the bottom. If this valve is opened, what is the maximum height attained by the water stream coming out of the right side of the tank? Assume that $h = 10.0$ m, $L = 2.00$ m, and $\theta = 30.0°$ and that the cross-sectional area at A is very large compared with that at B.

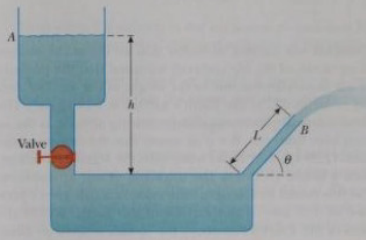


Figure P15.34

35. A helium-filled balloon is tied to a 2.00-m-long, 0.0500-kg uniform string. The balloon is spherical with a radius of 0.400 m. When released, it lifts a length, h, of string and then remains in equilibrium, as in Figure P15.35. Determine the value of h. The envelope of the balloon has mass 0.250 kg.

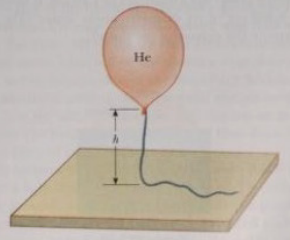


Figure P15.35

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