THE THEORY AND PRACTICE OF MODEM DESIGN

John A. C. Bingham

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Palo Alto, California



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Chapter 1

Introduction

1.1 THE ARRANGEMENT OF THIS BOOK

The order in which the separate parts of modem design are treated in this book is as logical as we could manage, but this book cannot be used strictly sequentially; much jumping forward and backward will be needed. For example, marketing is dealt with first in Chapter 2, but what you should design is obviously very dependent on whether, and how well, you can design it, and, just as obviously, on what people want (or can be persuaded to want) to buy; marketing factors must be considered at many stages of the design. Similarly, although transmitters are dealt with first, many decisions on their design must anticipate the problems of receivers: carrier and timing recovery, equalizer training, and, perhaps, echo cancellation.

The rest of this introductory chapter contains definitions of the terms and symbols that are used throughout this book, a brief discussion of modem standards and of the organizations that develop them, a fairly detailed description of those features of the telephone network that are important for data transmission, and a brief description of the principal ways in which the other media differ from it.

Chapter 2, contributed by Ken Krechmer, a modem marketing specialist, deals with the interaction of technology, applications, and standards in the creation of new modem products.

Chapters 3 through 11 deal with most of the components of a traditional modem as defined in Fig. 1.1. There is one omission; scramblers and descramblers should be there, but there was no logical chapter to put them in; they are discussed in Chapter 12, which is something of a catchall.

Figure 1.1 Components of a "full," a "traditional," and a narrowly defined modem.

2

Chapter 3 is entitled "Baseband Transmission," but the material in it is of fundamental importance for bandpass transmission as well. All the modulation methods considered in this book are linear, and it will be shown that, with the use of such methods, all passband channels have equivalent low-pass (or baseband) transfer functions and impulse responses.

Chapter 12 deals with the buffering, error correction, and scrambling that are needed to enable a synchronous modem to deal efficiently with asynchronous, character-formatted data. It then describes those functions that are needed to convert a private-line voice-band modem into one suitable for use on the Public Switched Telephone Network (PSTN).

Appendix I describes and lists some computer programs useful for both simulation and design; particularly important is a program for filter approximation that allows the design of all the filters in a modem to be treated as one comprehensive task. Appendix II contains some practical hints for diagnosing problems during the development stage.

The list of references at the end is dishearteningly long, but they serve three purposes—to acknowledge most (I hope) of the major contributions to the subject in the past twenty years, to trace and correlate previous developments of theories and methods, and to show where to find much of the detailed theory that there was no room for in the book. The references are cited by authors' initials rather than by the more usual anonymous numbers, in the hope that this will make them easier to recognize and then remember.

1.2 DEFINITIONS

1.2.1 Modem

The word "modem" is a concatenation of **mod**ulator and **dem**odulator, but there is a wide range of opinion as to what constitutes modulation and demodulation, and whether, indeed, a modem should comprise more than just a *mod* and a *dem*. The functions needed for transmission and reception of data are shown in block diagram form in Fig. 1.1, with narrow, traditional and broad interpretations of what constitutes a modem.

There are some conceptual advantages to be gained from requiring that the modulator be memoryless [Ma5]. This leads to a definition of a modem that is logically consistent but too narrow for practical use.

The traditional definition of a *mod* is "a device that accepts serial binary pulses from a data source and modulates some property (amplitude, frequency, or phase) of an analog signal in order to create a signal suitable for transmission in an analog medium." As shown in Fig. 1.1, this device comprises encoder, modulator, and filters. If the input data cannot be guaranteed to be sufficiently random for satisfactory operation of a sophisticated receiver, a scrambler will also be needed. It should be noted that this scrambler is for randomizing, and not for encryption or secrecy; these latter functions are the responsibility of the

data source. A *dem* is similarly defined as ... receiving device. Most of this book is concerned with modems defined in this way.

In the late 1960s AT&T coined the term "data set" to mean the combination of a modem with all the control functions needed to operate on the switched telephone network. This was a useful distinction, but it did not catch on; "modem" is now often broadened to include all these functions.

In most standards, and discussions of interfaces a modem is called *Data-Communication Equipment* (DCE) and the data source and destination—whether they are video terminals, computers, multiplexers, or other devices—are called *Data-Terminal Equipment* (DTE).

1.2.2 Two- and Four-Wire Lines

A four-wire (4W) line is a pair of two-wire (2W) lines, one for transmitting and one for receiving, in which the signals in the two directions are to be kept totally separate. However, perfect separation can be maintained only if the four-wire configuration is sustained from transmitter to receiver. If the lines are combined in a 4W/2W combining network (often called a *hybrid* or a *hybrid transformer*) at any point in the signal path, impedance mismatches will cause reflections and interference between the two signals; this is discussed in more detail in Section 11.1

The terms "two-wire" and "four-wire" ("2W" and "4W") will be used in this book even if the medium contains no wire, and a "line" or channel will be considered four-wire, as far as the modem is concerned, if complete separation of transmit and receive signals is effected *outside* the modem.

1.2.3 Private and Switched Lines

Private, leased, or *dedicated* lines (usually four-wire) are for the exclusive use of "leased-line" modems—either a pair (in a simple point-to-point connection) or several (on a multidrop network for a polling or a contention system). If the medium is the telephone network, their transmission characteristics are usually guaranteed to meet certain specifications (see Section 1.4), but if the link includes any radio transmission, the quality of it may be as variable as that of a nondedicated (i.e., switched) line.

Dial-up modems can establish a point-to-point connection on the PSTN by any combination of manual or automatic dialing† or answering. The quality of the circuit is not guaranteed, but all phone companies establish objectives. The links established are almost always two-wire because four-wire dialing is tedious and expensive.

† In a fo...

1.2.4 Simplex, Half-Duplex, and Full-Duplex

These three terms are used to describe a modem, or a transmission channel, or a combination of the two, and care must be taken to describe any particular arrangement precisely. Full-duplex modems will not work on half-duplex channels, and half-duplex modems can sometimes work in a full-duplex mode!

"Simplex" means capable of passing signals in one direction only. A remote modem for a telemetering system might be simplex; a two-wire line with a conventional unidirectional amplifier is simplex.

"Half-duplex" means capable of passing signals in either direction, but not in both simultaneously. A telephone channel via a satellite often includes a device that suppresses echoes with very long delays by allowing transmission in only one direction at a time; this renders the channel half-duplex. Echo suppressors are slowly being replaced by echo cancelers, which are theoretically full-duplex devices; these are discussed more fully in Section 1.4.

Any modem that does not share circuitry between transmitter and receiver can transmit and receive simultaneously if a four-wire line is used. However, such operation is usually taken for granted, and to qualify as full-duplex, a modem must be able to operate thus on a two-wire line.

When a modem is connected to a two-wire line, its output impedance cannot be matched exactly to the input impedance of the line, and some part of its transmitted signal (usually badly distorted) will always be reflected back. For this reason half-duplex receivers are disabled (received data is clamped) when their local transmitter is operative.

Full-duplex operation on a two-wire line requires the ability to separate a receive signal from the reflection of the transmitted signal. This is accomplished by either FDM in which the signals in the two directions occupy different frequency bands and are separated by filtering, or by Echo Canceling (EC) in which a locally synthesized replica of the reflected transmitted signal is subtracted from the composite received signal. These two methods are described in more detail in Chapter 11.

The implication of the term "full-duplex" is usually that the modem can transmit and receive simultaneously at "full" speed. Modems that provide a low-speed reverse channel are sometimes called "split-speed" or "asymmetric" modems; since a 4800/4800 bit/s modem is full-duplex and a 4800/0 is half-duplex, a 4800 bit/s modem with a 300 bit/s reverse channel might be pedantically called a 17/32 duplex!

1.2.5 Synchronous and Asynchronous Data

"Synchronous" means that the data is always accompanied by a clock signal; data changes on one edge of the clock (the conventions are discussed further in Chapter 12) and should be sampled by the receiving device on the other edge. Synchronous data is almost always grouped in blocks, and it is the responsibility

of the data source to assemble those blocks ... extra bits needed for error detecting and/or correcting according to one of many different protocols (BISYNC, SDLC, HDLC, etc.). The data source and destination expect the modem to be transparent to this type of data; conversely, the modem can ignore the blocking of the data. Discussion of these protocols is beyond the scope of this book.

Asynchronous data is not accompanied by any clock, and the transmitting and receiving modem know only the nominal data rate. To prevent slipping of the data relative to the modems' clocks, this data is always grouped in very short blocks (characters) with framing bits (start and stop bits); the most common code used for this is the seven-bit *American Standard Code for Information Interchange* (ASCII) with even parity; this is a subset of International Alphabet No. 5 that is defined in the CCITT recommendation V.3.

All the encoding and modulation methods considered in this book are synchronous, so if a modem is to be able to accept asynchronous character-formatted data, its transmitter must include a means for synchronizing this data; ways of doing this are discussed in Section 12.1. The most common method generates *character-formatted synchronous* data; this sounds like an oxymoron, but merely means that the synchronous data stream retains a character format as far as possible by including start and stop bits of precise duration.

1.2.6 Symbol Rates and "Bauds"

In most modems the input bits are accepted in groups of from one to six bits, and each group is considered as one *symbol;* the symbol rate is therefore an integer dividend of the bit rate. The symbol rate can be measured in bauds,† but this is a much misunderstood and misused term. Since we need the "b" subscript for the bit rate, we will eschew bauds altogether and use f_s for the symbol rate in units of symbols per second.

1.2.7 Symbols and Terminology

The symbols shown in Table 1.1 will be used consistently throughout the book; others will be introduced when needed and may be consistent only within a chapter. As far as possible, initials of the quantities represented are used in order to serve as mnemonics. Some general rules should be borne in mind:

1. The symbols a and b are reserved for binary data; a usually for primary data and b, for the results of some coding operation.
2. The symbol c is used for the tap coefficients of an equalizer.

† A baud can be best defined by the analogy baud : hertz :: symbol : cycle. The frequently used expression "baud rate" is tautological.

TABLE 1.1 Symbols Used Throughout the Book

$A(f)$	The amplitude part of $S(f)$
f_b	Bit rate [bit/s (bits per second)]
f_c	Carrier frequency (Hz)
f_N	Nyquist frequency, $= 1/2T$ (Hz)
f_s	Symbol rate [s/s (symbols per second)]
$h(t)$	The impulse response of an end-to-end channel, or any shortened version thereof
$H(z)$	The z transform of $h(t)$, $= h_1 + h_2 z^{-1} + \cdots + h_M z^{-M+1}$
$Q(x)$	The most convenient function for calculating error probabilities, $= \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(\frac{-u^2}{2}\right) du = \frac{\text{erfc}(x/2)}{2}$
$R(z)$	The autocorrelation function of $H(z)$, $= \sum_{m=-M}^{M} r_m z^{-m}$
$S(f)$	The complex frequency spectrum of $s(t)$, $= A(f) \exp j[2\pi f + \theta(f)]$
$S(f,\tau)$	The spectrum of $s(t)$ sampled at $t = nT + \tau$; band-limited to $-f_s < f < f_s$
$\theta(f)$	The phase part of $S(f)$
T	The symbol period, $= 1/f_s$

3. The symbol f is either frequency (usually subscripted), or a time domain transfer function; F is a frequency domain transfer function.
4. The symbols i to n follow the FORTRAN convention and are variable integers. Uppercase letters are fixed integers (e.g., m usually varies from 1 to M).
5. The symbols u to y (and occasionally, under duress, z) are continuously variable signals, progressing as consistently as possible from baseband transmit, through passband transmit and receive, to baseband receive.
6. The symbol z is the variable of the z transform, and z^{-1} is the delay operator; D, the delay operator of coding theory, is not used.

1.3 MODEM STANDARDS AND RECOMMENDATIONS†

Communication between dissimilar entities—persons, organizations, or equipment—is greatly hampered without a complete definition of the interface. The creation of such a mutually established interface is the purpose of any communications standard. Currently there is no complete definition of all the forms of communication that can occur at any but the simplest interface. Nevertheless, standards organizations act as forums for the development of interface standards, and the marketplace acts to test the acceptability of such standards. This combined process has produced sufficient standardization and wide enough acceptance of related assumptions to allow dissimilar manufacturers' communications products to interwork with increasing ease.

† This section was written by Ken Krechmer and John Bingham.

popularity as quad

4.4.1 Baseband Equivalent of a Single-Sideband Signal

With an SSB signal we are only interested in the real part of the imp response (the quadrature channel contains redundant information); there the third and fourth terms in the integrand of (4.8) can be discarded. Furt more, there is only one sideband to contribute to the integral, so that for case, for example, of an upper sideband:

$$h_p(t) = G(f)\,|C(f_c + f)|\cos(2\pi ft + \phi(f_c + f) + \theta_c)\,df$$

Again, it is important to note the role of the phase of the demodulat carrier; if there were no distortion in the channel, but θ_c were $\pi/2$, then h_p would be the Hilbert transform of the desired response. In the QAM case, adaptive equalizer would be easily able to perform the derotation, but for SSB signal the equalizer is only one-dimensional, and implementation of t necessary transfer function (equivalent to a further Hilbert transform) may difficult. If there is distortion the effect is not so dramatic, but θ_c must still chosen carefully; criteria for this choice will be discussed in Chapter 6.

4.5 ORTHOGONAL MODULATION OF MULTIPLE CARRIERS

If a single multilevel (many bits per symbol) signal is transmitted through distorted channel, some form of adaptive equalization will be required in th receiver. If a linear equalizer is used, noise enhancement will occur and th SNR of the signal delivered to the detector will be reduced.

Some early modems [DH&M and F&L1] used the technique of dividing th band into many subbands and transmitting the high-speed serial data stream a a multiplexed set of low-speed data streams modulated onto a set of carrie evenly spaced throughout the band. Holsinger [Ho1] proved that the metho results in the lowest possible error rate if the channel has amplitude distortio and a linear equalizer is used in the receiver. Chang [Ch1] showed how th carrier phases and data timing of the separate channels can be arranged t maintain a flat power spectrum of the composite signal and to permit separa tion of the overlapped channels in the receiver without interchannel interfe ence; Saltzberg [Sa2] developed the special case where each subchannel use SQAM, and considered the effects of channel distortion on such a modulate signal.

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Since the development of powerful DSP capabilities Orthogonally Multiplexed QAM (OMQAM) has been used by Gandalf, Telebit (see [Ba] and [TC]), and NEC for modems that transfer between 9.6 and 19.2 kbit/s in a telephone voice channel, but it has never been what one might call "mainstream." This is because it requires complicated, and usually proprietary, DSP techniques in the receiver, and because the trade-offs of improved performance (if any) against complexity of implementation have been difficult to assess. We will attempt that assessment here.

Performance with Gaussian Noise. If a distorted channel is divided into many subchannels, the variation of attenuation and delay across each subband is greatly reduced. Therefore, the impulse responses of these subchannels will be much less dispersed, and adaptive equalization may not be needed. Consequently, it is often claimed that the method has the advantage that the noise is not enhanced and the SNR is not reduced. However, a type of piecewise amplitude equalization may be performed by the separate AGCs in each subband. They make the average attenuation of all the subchannels appear to be equal† and, in the process, amplify the noise in the bands that had been attenuated. We will examine this effect carefully in the next two sections.

We shall find that a system with a predetermined bit loading of the bands—a nonadaptive system—may perform worse than a conventional single-band system and that the full potential of this modulation method is realized only if the transmitter can be configured to "match" the channel. This requires measurements of the SNRs in the receiver and feedback to the transmitter. Therefore, even though the other sections of this chapter are concerned only with transmitters, for this method we will have to consider also those parts of the receiver that affect the transmitter.

Performance with Impulse Noise. We shall find that a multichannel modem is more sensitive to narrow-band spikes of noise‡ than is a single-channel modem, which averages the noise across the whole band. Conversely, it is *less* sensitive to narrow time spikes (impulses) of noise than is a single-channel modem because it averages them over a long symbol period.

Complexity of Implementation. If the number of bands is very large (the Telebit modem can use as many as 512), each band will be very narrow and essentially undistorted; adaptive equalization will not be needed. If the number of bands is only moderate (the NEC modem uses 14), some simple equalization will be needed for the highest data speeds.

However, the fact that no equalizer—or only a very simple one—is needed does not necessarily mean that the implementation is simple; the multichannel

† Similarly, if separate timing recovery is performed for each subband, this is a type of piecewise delay equilization because it makes the average delay of all the subchannels appear to be equal.
‡ Assuming that it has not avoided them by adaptively configuring the bands.

filters or the DFT needed in the receiver to separate the subchannels may be
complex as a single-channel adaptive equalizer.

4.5.1 Ideal Performance of a Fixed System with Distortion and Noise

This method is very different from single-channel modulation, so, in orde
generate motivation for the study of it—the "why" before the "how"—
must first calculate its theoretically ideal performance in the presence of dis
tion and noise.

The ideal performance can be achieved only if (1) the number of band
large enough that the effects of *intrachannel* distortion can be ignored and
the reciever is able to completely separate the signals in the different bands so
to eliminate *interchannel* interference (ICI). However, calculation of the er
rate for a system with so many bands would be very tedious, so we will do it o
for the 14-band system described by Hirosaki [Hi . .]—ignoring, at leas
first, the need for an adaptive equalizer [Hi1].

The symbol rate is 200 s/s, and the total number of bits per symbol B_{tot}, is
so that the aggregate bit rate is 19.2 kbit/s. Twelve of the fourteen QAM tra
mitters used in the subbands encode seven bits per symbol (i.e., $B_i = 7$
$i = 2-13$) and the two in the bands at the edges of the 400 – 3200 Hz over
band encode just six ($B_1 = B_{14} = 6$).

If the channel is a so-called worst-case 3002 line as shown in Fig. 4.23,
amplitude response can be approximated by 14 flat segments centered ab
equally spaced carrier frequencies as shown ($f_{c,i} = 300 + 200i$ for $i = 1$-
with ideal filtering each subchannel may be considered to be undistorted
attenuated by an amount A_i. If the power transmitted in each band is norm
ized to unity, the total received power is