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# → U.S. PAT. 3,511,936, issued May 12, 1970 Performance of an Efficient Parallel Data Transmission System

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see also Chang &  
Gibbs, COM 8/68, p. 525  
and J.W. Smith,  
BSTJ 3/68 p. 273

REFERENCE: Saltzberg, B. R.: PERFORMANCE OF AN EFFICIENT PARALLEL DATA TRANSMISSION SYSTEM, Bell Telephone Laboratories, Inc., Holmdel, N. J. Rec'd 6/12/67. Paper 19TP67-972, approved by the IEEE Data Communication and Telegraph Systems Committee for publication without oral presentation. IEEE TRANS. ON COMMUNICATION TECHNOLOGY, 15-6, December 1967, pp. 805-811.

ABSTRACT: A parallel quadrature AM data transmission system may be implemented with a number of overlapping channels, each carrying a signaling rate  $b$ , spaced  $b/2$  apart in frequency. When a large number of channels are used, the system allows transmission speeds very close to the Nyquist rate, with little sensitivity to delay and amplitude distortion of the transmission medium. The receiver requires precise phasing of the demodulating carriers and sampling times in order to keep crosstalk between channels small. In the presence of delay and amplitude distortion, better results are obtained when half cosine roll-offs are used for shaping each channel than for full cosine roll-off. This transmission scheme appears to be a promising technique for achieving good performance at high information rates over bandlimited dispersive transmission media.

KEYWORDS: Amplitude Modulation, Audio Frequency, Crosstalk, Data Transmission Systems, Detection, Digital Signals, Distortion, Filters, Multiplex, Signal-to-Noise Ratio, Telephone Systems.

## INTRODUCTION

IN PARALLEL data transmission, an available frequency band is divided into several channels by independently modulating a number of carriers of different frequency. Since each channel occupies a relatively narrow frequency band, parallel transmission is effective in combatting the effects of amplitude and delay distortion and impulsive noise.

It is tempting to avoid spectral overlap of the channels in order to eliminate interchannel interference. However, this leads to inefficient use of the available spectrum. Higher overall signaling rates can be achieved if spectral overlap is permitted, and some orthogonality relationship is used to minimize the interference between adjacent channels. Some commercial data terminals use this approach.<sup>[1], [2]</sup> Recently, Chang has presented general conditions under which bandlimited channels, each carrying a signaling rate  $b$ , may be spaced  $b/2$  apart in frequency with no resultant intersymbol or interchannel interference.<sup>[3]</sup> Under these conditions, a total signaling rate very close to the Nyquist rate allowed by the available

bandwidth may be achieved through the use of a large number of channels. The spectral roll-off of each channel can be quite gentle, thus permitting easy filter design and rapidly decaying waveforms.

This paper considers the performance of a parallel data transmission system, which meets Chang's criteria, over a dispersive transmission medium.

### SYSTEM DESCRIPTION AND ANALYSIS

The amplitude spectrum of an efficient parallel data transmission signal is shown in Fig. 1. Each channel transmits a signaling rate  $b$ , and the channels are spaced  $b/2$  apart. The channels all have identical spectral shaping and are each symmetric about its center frequency. The roll-offs about the frequencies displaced  $b/2$  from the center frequency are the square root of Nyquist roll-offs.<sup>[4]</sup> This eliminates intersymbol interference in each channel. The shaping shown in Fig. 1 is a full cosine roll-off. Interchannel interference is eliminated when adjacent channels are in phase quadrature in addition to these requirements.

In an unpublished proposal by Becker, each channel is formed by vestigial sideband modulation with equal vestigial and data roll-offs. The carriers for adjacent channels are in quadrature. An equivalent system may be constructed using quadrature AM, which has identical performance in the presence of noise and amplitude and delay distortion.<sup>[6], [6], [7]</sup>

A block diagram of this efficient parallel quadrature AM system is shown in Fig. 2, where two adjacent channels are indicated. Each channel transmits two independent data streams, each of which suppressed-carrier amplitude modulates one of a pair of quadrature carriers whose frequency is equal to that of the center of the channel. Each data stream has a signaling rate  $b/2$ , and the timing of the two streams is staggered by  $1/b$ . Adjacent channels are staggered oppositely, so that the data streams which modulate the cosine carriers of the even numbered channels are in phase with the data streams that modulate the sine carriers of the odd numbered channels, and conversely.

All filters  $F(\omega)$  in the transmitter and receiver are identical and assumed to be real. The use of the same filtering at the transmitter and receiver assures optimum performance in the presence of white Gaussian noise. The filters are bandlimited to the signaling rate in order to eliminate the possibility of interference between any channels that are not immediately adjacent.

$$F(\omega) = 0, \quad |\omega| \geq \pi b. \quad (1)$$

In addition, the transmit and receive filters in tandem have a Nyquist roll-off.—

$$F^2(\omega) + F^2(\pi b - \omega) = 1, \quad 0 \leq \omega \leq \pi b/2. \quad (2)$$

The line signal is of the following form:

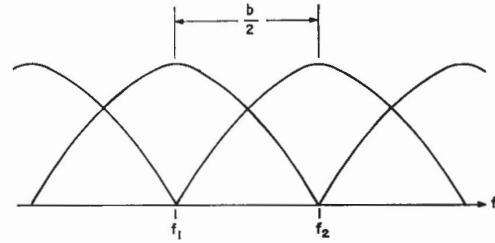


Fig. 1. Spectrum of an efficient parallel data transmission system, full cosine roll-off.

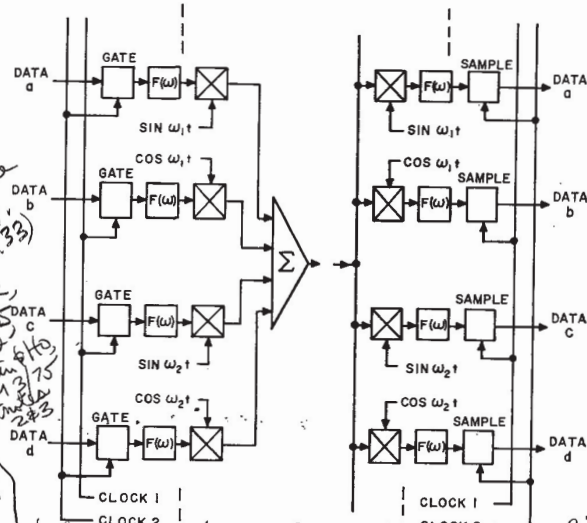


Fig. 2. Block diagram of parallel quadrature AM system.

$$s(t) = \sum_{m \text{ odd}} \sum_{n=-\infty}^{\infty} a_{nm} f(t - nT) \cos(\omega_0 + m\pi b)t + \sum_{m \text{ odd}} \sum_{n=-\infty}^{\infty} b_{nm} f(t - nT - T/2) \sin(\omega_0 + m\pi b)t + \sum_{m \text{ even}} \sum_{n=-\infty}^{\infty} a_{nm} f(t - nT - T/2) \cos(\omega_0 + m\pi b)t + \sum_{m \text{ even}} \sum_{n=-\infty}^{\infty} b_{nm} f(t - nT) \sin(\omega_0 + m\pi b)t \quad (3)$$

where

$$T = 2/b, \quad (4)$$

$f(t)$  is the inverse Fourier transform of  $F(\omega)$ , and  $a_{nm}$  and  $b_{nm}$  are the information-bearing random variables.

Because of the symmetry of the system, only the distortion in one subchannel due to the linear dispersive transmission medium need be considered. The results will be the same whether an even or odd numbered channel or the sine or cosine subchannel is chosen. The measure of distortion will be the maximum reduction in noise margin for all possible message sequences in all channels. This distortion measure is often referred to as the eye pattern closure. Since the system is linear and time-invariant, the distortion can be determined from the response to a single pulse transmitted on each subchannel.

The frequency response of the transmission medium is of the form

$$H(\omega) = A(\omega)\exp[j\varphi(\omega)] \quad (5)$$

and let any constant time delay be ignored. Without loss of generality, the distortion in the cosine subchannel of the  $k$ th channel will be considered, where  $k$  is odd and not one of the two end channels.

There are six possible components to the distortion in this subchannel. The intersymbol interference and the interference of the quadrature subchannel due to dispersive media have been well studied.<sup>[8]</sup> In addition, distortion results from crosstalk of the adjacent upper and lower channels. These components will be considered separately.

The spectrum of the received signal when a single pulse is transmitted in the subchannel of interest is

$$S_1(\omega) = (1/2)H(\omega)[F(\omega - \omega_k) + F(\omega + \omega_k)] \quad (6)$$

where

$$\omega_k = \omega_0 + k\pi b \quad (7)$$

is the center frequency of the channel.

This signal is multiplied by  $2 \cos \omega_k t$  and filtered by  $F(\omega)$ . The resulting spectrum is

$$R_1(\omega) = (1/2)F^2(\omega)[H(\omega + \omega_k) + H(\omega - \omega_k)]. \quad (8)$$

This signal is sampled at time  $t = t_0$  in order to recover the data. The distortion is defined as the ratio of the sum of the absolute values of the signal at all other sample times to the signal amplitude at the central sampling instant.

$$D_1 = \frac{\sum_{n \neq 0} |r_1(t_0 + nT)|}{r_1(t_0)} \quad (9)$$

where

$$r_1(t) = 1/2\pi \int_{-\infty}^{\infty} R_1(\omega)\exp(j\omega t)d\omega. \quad (10)$$

The received signal for a single pulse on the quadrature subchannel is

$$S_2(\omega) = (j/2)H(\omega)\{F(\omega + \omega_k)\exp[j(\omega + \omega_k)T/2] - F(\omega - \omega_k)\exp[j(\omega - \omega_k)T/2]\}. \quad (11)$$

The phase factor is due to the time displacement of the pulse by half a signal period. After demodulating with  $2 \cos \omega_k t$  and filtering, the spectrum of the signal presented to the sampler of the in-phase subchannel is

$$R_2(\omega) = (j/2)F^2(\omega)[H(\omega - \omega_k) - H(\omega + \omega_k)] \times \exp(j\omega T/2). \quad (12)$$

All samples of this interfering signal contribute to the distortion.

$$D_2 = \frac{\sum_{n \neq 0} |r_2(t_0 + nT)|}{r_1(t_0)} \quad (13)$$

*quadrature orthogonal if H is const.*

A single pulse on the in-phase channel of the  $(k + 1)$ th channel leads to a received signal

$$S_3(\omega) = (1/2)H(\omega)\{F(\omega - \omega_k - \pi b) \times \exp[j(\omega - \omega_k - \pi b)T/2] + F(\omega + \omega_k + \pi b) \times \exp[j(\omega + \omega_k + \pi b)T/2]\}. \quad (14)$$

Using (4),

$$S_3(\omega) = -(1/2)H(\omega)\{F(\omega - \omega_k - 2\pi/T) \times \exp[j(\omega - \omega_k)T/2] + F(\omega + \omega_k + 2\pi/T) \times \exp[j(\omega + \omega_k)T/2]\}. \quad (15)$$

When multiplied by  $2 \cos \omega_k t$  and filtered, this interfering component becomes

$$R_3(\omega) = -(1/2)F(\omega)[F(\omega - 2\pi/T)H(\omega + \omega_k) + F(\omega + 2\pi/T)H(\omega - \omega_k)]\exp(j\omega T/2). \quad (16)$$

Again, all sample values of this signal contribute to the distortion.

$$D_3 = \frac{\sum_{n=-\infty}^{\infty} |r_3(t_0 + nT)|}{r_1(t_0)}. \quad (17)$$

The received single pulse signal for the upper quadrature subchannel is

$$S_4(\omega) = (j/2)H(\omega)[F(\omega + \omega_k + 2\pi/T) - F(\omega - \omega_k - 2\pi/T)]. \quad (18)$$

As presented to the sampler under consideration, this signal is

$$R_4(\omega) = (j/2)F(\omega)[F(\omega + 2\pi/T)H(\omega - \omega_k) - F(\omega - 2\pi/T)H(\omega + \omega_k)] \quad (19)$$

and  $D_4$  is defined as in (12) and (16).

Similarly, the interference components from the  $(k - 1)$ th channel are

$$R_5(\omega) = -(1/2)F(\omega)[F(\omega + 2\pi/T)H(\omega + \omega_k) + F(\omega - 2\pi/T)H(\omega - \omega_k)]\exp(j\omega T/2) \quad (20)$$

$$R_6(\omega) = (j/2)F(\omega)[F(\omega - 2\pi/T)H(\omega - \omega_k) - F(\omega + 2\pi/T)H(\omega + \omega_k)] \quad (21)$$

due to the in-phase and quadrature subchannels, respectively.  $D_5$  and  $D_6$  are also defined as in (12) and (16).

The total distortion is the sum of these six components.

$$D = \sum_{m=1}^6 D_m. \quad (22)$$

It should be noted that  $D$  strongly depends on the choice of sampling time  $t_0$ . As a strict measure of the loss of noise margin, the distortion should account for any reduction in the value of  $r_1(t_0)$ . However, if  $t_0$  is properly chosen, this effect is slight compared with the amplitude of the interfering components. The distortion measure used here has the advantage of being applicable to multilevel

signaling with any size alphabet. In particular, for  $m$ -level signaling, errors will result without noise when

$$D > \frac{1}{2m-1}. \quad (23)$$

### RESULTS

#### No Impairment

When the transmission medium is distortionless,  $H(\omega) = 1$ , and there is no distortion in the signal presented to the sampler at time  $t_0 = 0$ . The spectrum of the signal of the subchannel of interest at this point is just  $F^2(\omega)$ . Since this spectrum satisfies (2),

$$r_1(nT) = 0, \quad n \neq 0. \quad (24)$$

There is no crosstalk at all from the quadrature subchannel. Equation (11) becomes

$$R_2(\omega) = 0 \quad (25)$$

when there is no distortion in the transmission medium.

Consider now the crosstalk from the upper in-phase subchannel. From (15)

$$R_3(\omega) = -(1/2)F(\omega)[F(\omega - 2\pi/T) + F(\omega + 2\pi/T)] \times \exp(j\omega T/2). \quad (26)$$

Since  $F(\omega)$  is even and satisfies (1),

$$r_3(t) = -1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \times \cos \omega(t + T/2) d\omega. \quad (27)$$

Substituting  $u = \omega - \pi/T$  into (26),

$$r_3(t) = 1/2\pi \int_{-\pi/T}^{\pi/T} F(u + \pi/T)F(u - \pi/T) \times \sin(ut - \pi t/T + uT/2) du. \quad (28)$$

At the sampling times,  $t_0 = nT$ ,

$$r_3(nT) = \frac{(-1)^n}{2\pi} \int_{-\pi/T}^{\pi/T} F(u + \pi/T)F(u - \pi/T) \times \sin u(nT + T/2) du. \quad (29)$$

Since the integrand of (28) is an odd function,  $r_3(nT) = 0$ . Thus there is no contribution to the distortion at the proper sampling instant. However, (28) is in general nonzero at times other than integral multiples of  $T$ .

The spectrum of the crosstalk signal due to the upper quadrature channel is

$$R_4(\omega) = (j/2)F(\omega)[F(\omega + 2\pi/T) - F(\omega - 2\pi/T)] \quad (30)$$

$$r_4(t) = 1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \sin \omega t d\omega \quad (31)$$

or, again substituting  $u = \omega - \pi/T$ ,

$$r_4(t) = 1/2\pi \int_{-\pi/T}^{\pi/T} F(u + \pi/T)F(u - \pi/T) \times \sin(u - \pi/T)t du. \quad (32)$$

It can readily be seen that the integrand of (32) is an odd function when  $t_0 = nT$ , so that  $r_4(nT) = 0$ . However, there is an addition to the signal of interest at other times.

Similar results are obtained for the crosstalk from the lower channel.

Figure 3 is an outline of the upper half of a binary eye pattern when a full cosine roll-off is used.

$$F(\omega) = \cos \omega T/4, \quad |\omega| < 2\pi/T. \quad (33)$$

The curves in Fig. 3 are plots of  $r_1$ ,  $r_1(1 + D)$ , and  $r_1(1 - D)$  versus  $t_0/T$ . It can be seen that the width of the open part of the eye is only about 24 percent, so that for binary transmission, errors will result without noise when the sampling time deviates from the proper instant by 12 percent. Multilevel systems can tolerate far less sampling time error. Accurate timing, in terms of fractional allowable error, is therefore essential in efficient parallel systems even when the transmission medium is distortionless.

#### Phase Offset

In the previous section it was assumed that the demodulating carrier had the correct phase. Degradation will result when a phase error exists. For the purposes of analysis, a phase error in the demodulating carrier is equivalent to a constant phase offset of the same magnitude and opposite sign in the transmission medium. The resultant distortion will be calculated when

$$H(\omega) = \exp[j\varphi_0 \text{sgn } \omega] \quad (34)$$

where  $\text{sgn } \omega = \omega/|\omega|$ .

Calculations similar to the previous ones lead to the results

$$r_1(t) = (1/\pi) \cos \varphi_0 \int_0^{2\pi/T} F^2(\omega) \cos \omega t d\omega \quad (35)$$

$$r_2(t - T/2) = (1/\pi) \sin \varphi_0 \int_0^{2\pi/T} F^2(\omega) \cos \omega t d\omega \quad (36)$$

$$r_3(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \times \cos(\omega t + \varphi_0) d\omega \quad (37)$$

$$r_4(t) = 1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \times \sin(\omega t + \varphi_0) d\omega \quad (38)$$

$$r_5(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \times \cos(\omega t - \varphi_0) d\omega \quad (39)$$

and

$$r_6(t) = -1/2\pi \int_0^{2\pi/T} F(\omega)F(2\pi/T - \omega) \sin(\omega t - \varphi_0) d\omega. \quad (40)$$

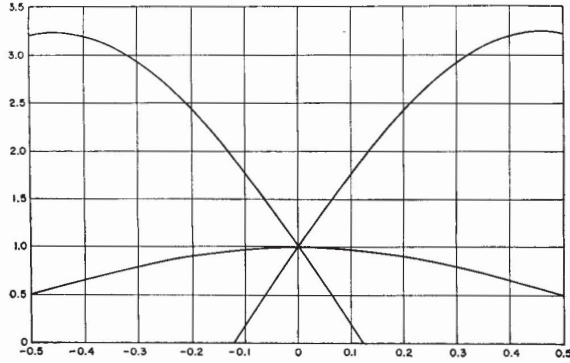


Fig. 3. Upper half of binary eye pattern outline for subchannel of parallel quadrature AM system, full cosine roll-off, no distortion.

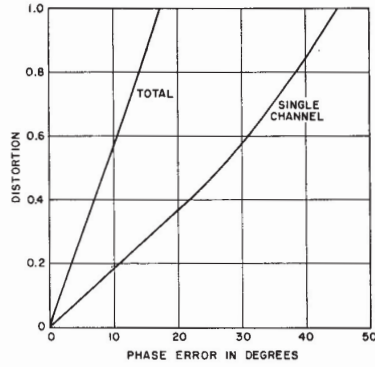


Fig. 4. Sensitivity to carrier phase offset, full cosine roll-off.

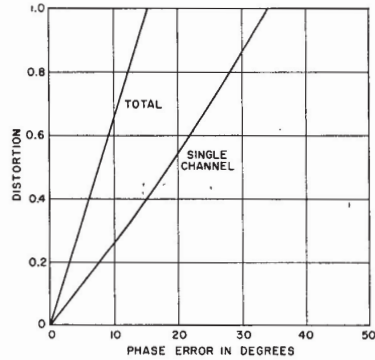


Fig. 5. Sensitivity to carrier phase offset, half cosine roll-off.

All components but the first contribute to the distortion. Figure 4 shows the distortion at the best sampling time  $t_0$ , versus  $\varphi_0$  for a system using full cosine roll-offs. The upper curve shows the total distortion, while the lower curve is a plot of  $D_1 + D_2$ , the distortion resulting when the upper and lower channels are removed. It is seen that the parallel system is approximately three times as sensitive as the single channel to carrier phase error.

Similar results are plotted in Fig. 5 for a system using half cosine roll-offs.



$$F(\omega) = \begin{cases} 1, & 0 < \omega < \pi/2T \\ \cos \frac{\omega T - \pi/2}{2}, & \pi/2T < \omega < 3\pi/2T \\ 0, & \omega > 3\pi/2T. \end{cases} \quad (41)$$

It can be seen that although the single channel is more sensitive to carrier phase error than when full roll-off is used, due to slower decay of the waveform, the sensitivity of the parallel system is only slightly increased since there is less crosstalk from adjacent channels.

#### Delay Distortion

If the parallel system includes many channels, then the bandwidth of any one channel is small. In this case it is usually possible to approximate the delay distortion of the transmission medium by a linear function of frequency over the frequency band of one channel. Since we are not concerned with flat delay, the delay distortion may be considered to be zero at the center of the band of interest. Then, if we are examining the  $k$ th channel, the delay distortion can be written as

$$L(\omega) = d(\omega - \omega_k)/2\pi \quad (42)$$

where  $d$  is the slope of the delay distortion in s/Hz. Then

$$\varphi(\omega) = \int L(\omega) d\omega \quad (43)$$

$$\varphi(\omega) = \varphi_0 + d(\omega - \omega_k)^2/4\pi \quad (44)$$

and

$$H(\omega) = \exp\{j[\varphi_0 + d(\omega - \omega_k)^2/4\pi] \operatorname{sgn}(\omega)\}. \quad (45)$$

After some manipulation we obtain

$$r_1(t) = 1/\pi \int_0^{2\pi/T} F^2(\omega) \cos(\varphi_0 + d\omega^2/4\pi) \cos \omega t d\omega \quad (46)$$

$$r_2(t - T/2) = 1/\pi \int_0^{2\pi/T} F^2(\omega) \sin(\varphi_0 + d\omega^2/4\pi) \times \cos \omega t d\omega \quad (47)$$

$$r_3(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times \cos(\omega t + \varphi_0 + d\omega^2/4\pi) d\omega \quad (48)$$

$$r_4(t) = 1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times \sin(\omega t + \varphi_0 + d\omega^2/4\pi) d\omega \quad (49)$$

$$r_5(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times \cos(\omega t - \varphi_0 - d\omega^2/4\pi) d\omega \quad (50)$$

and

$$r_6(t) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times \sin(\omega t - \varphi_0 - d\omega^2/4\pi) d\omega. \quad (51)$$

The distortion at the optimum sampling time is plotted in Figs. 6 and 7 for the full cosine roll-off and half cosine roll-off systems, respectively. In each case, curves are shown for the channel without crosstalk from adjacent channels, and for the parallel system both when  $\varphi_0$  is set equal to zero and when the optimum demodulating phase is used. The delay distortion is normalized to  $d/T^2$ . This normalized quantity is equal to the ratio of the difference in delay between adjacent carrier frequencies to the signal period  $T$ . It is also equal to the phase difference between adjacent carriers, multiplied by  $\pi$ . The curves are all approximately linear, at least in the low distortion region. The slopes  $k$  of these lines are given in parenthesis for each curve. The distortion can then be approximated by

$$D = kd/T^2. \quad (52)$$

Equation (52) illustrates the effectiveness of a parallel system in combatting delay distortion. If a given bandwidth is subdivided so as to transmit a given total signaling rate, then the signal period of each channel  $T$  is proportional to the number of channels. The distortion  $D$  is then inversely proportional to the square of the number of channels.

A comparison of Figs. 6 and 7 shows that although the full roll-off system is considerably less sensitive to delay distortion than the half roll-off system if there are no adjacent channels, the reverse is true for the efficient parallel system. In this case, the increased crosstalk from adjacent channels dominates the self-distortion to the extent that the total performance is definitely worse when full roll-offs are used.

The best demodulating phase is very near to that of the received signal at the carrier frequency when there are no interfering channels. However, Figs. 6 and 7 show that the performance of the parallel system can be substantially improved by altering the demodulating phase. The best phase is approximately equal to

$$-\varphi_0 = d/4\pi T^2. \quad (53)$$

When (53) holds then, from (44),

$$\varphi(\omega_k \pm \pi/T) = 0. \quad (54)$$

The arguments of (54) are the frequencies midway between channels, where the maxima of the crosstalk spectra occur. A near optimum strategy therefore consists of eliminating the phase error at the center of the crosstalk regions, thus reducing the crosstalk and accepting the increased self-distortion. Figure 8 illustrates the effect of demodulating phase on distortion for the example of full cosine roll-off and delay distortion  $d/T^2 = 0.4$ . It can be seen that although the self-distortion is near minimum when  $\varphi_0 = 0$ , the total distortion is close to its minimum when (53) is satisfied by a phase of  $18^\circ$ .

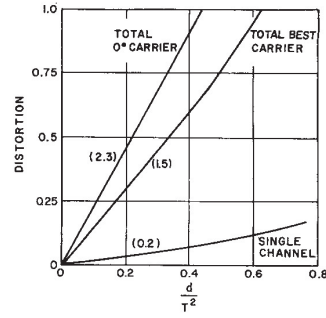


Fig. 6. Sensitivity to linear delay distortion, full cosine roll-off.

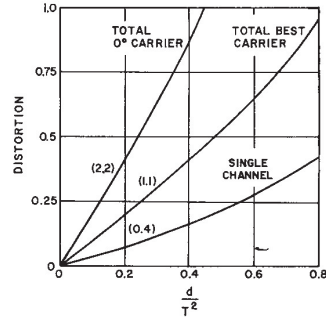


Fig. 7. Sensitivity to linear delay distortion, half cosine roll-off.

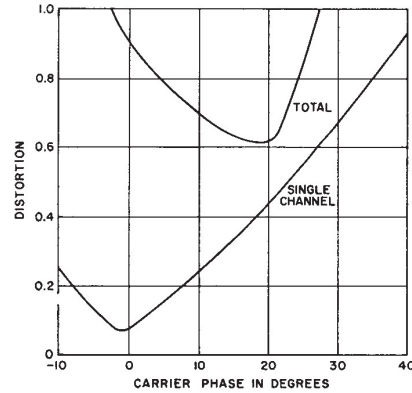


Fig. 8. Performance in the presence of linear delay distortion as a function of carrier phase, full cosine roll-off,  $d/T^2 = 0.4$ .

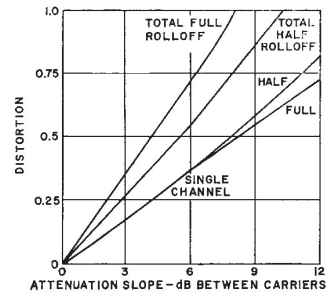


Fig. 9. Sensitivity to amplitude distortion.

### Amplitude Distortion

The system performance will be considered when the transmission medium has no delay distortion, but has an amplitude characteristic that is not constant with frequency. Then

$$r_1(t) = 1/2\pi \int_0^{2\pi/T} F^2(\omega) [A(\omega_k + \omega) + A(\omega_k - \omega)] \times \cos \omega t d\omega \quad (55)$$

$$r_2(t - T/2) = 1/2\pi \int_0^{2\pi/T} F^2(\omega) [A(\omega_k + \omega) - A(\omega_k - \omega)] \sin \omega t d\omega \quad (56)$$

$$r_3(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times A(\omega_k + \omega) \cos \omega t d\omega \quad (57)$$

$$r_4(t) = 1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times A(\omega_k + \omega) \sin \omega t d\omega \quad (58)$$

$$r_5(t - T/2) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times A(\omega_k - \omega) \cos \omega t d\omega \quad (59)$$

and

$$r_6(t) = -1/2\pi \int_0^{2\pi/T} F(\omega) F(2\pi/T - \omega) \times A(\omega_k - \omega) \sin \omega t d\omega. \quad (60)$$

Figure 9 contains plots of performance when the amplitude distortion is approximated by an exponential function of frequency over the frequency band of one channel. This type of distortion is commonly found in voice telephone circuits.<sup>[9]</sup> The abscissa is the variation in dB of the amplitude characteristic over a frequency interval  $b/2$ . The distortion now decreases only linearly with the number of channels.

The self-distortion is almost the same for full and half cosine roll-offs. As in the case of delay distortion, the performance of the total system is better when half roll-offs are used. The increase of distortion due to crosstalk is less for amplitude distortion than for delay distortion.

### CONCLUSION

Parallel quadrature AM transmission provides a method of transmitting digital data at speeds very close to the Nyquist rate of bandlimited channels without using sharp cutoff filters. In addition, the use of a large number of narrow channels is effective in combatting delay and amplitude distortion of the transmission medium.

The distortions calculated in this paper are applicable to channels in the interior of the frequency band. The two end channels will have considerably less distortion since crosstalk from only one other channel can exist. This effect is helpful in using a larger part of the frequency range of transmission media such as voice telephone facilities, in which the degradations increase gradually at the band edges.

In the presence of delay distortion, it is essential that the receiver separately adjusts the demodulating carrier phase and the sampling time for each channel. In addition, these adjustments must be fairly precise and free of jitter due to the high sensitivity of the system to these errors. An efficient parallel data system will therefore inevitably be a costly one to implement.

The strategy of designing an efficient parallel system should concentrate more on reducing crosstalk between adjacent channels than on perfecting the individual channels themselves, since the distortions due to crosstalk tend to dominate.

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