

PRINCIPLES OF PHYSICS SERIES

ELECTRICITY AND MAGNETISM

by

FRANCIS WESTON SEARS

Department of Physics

Massachusetts Institute of Technology



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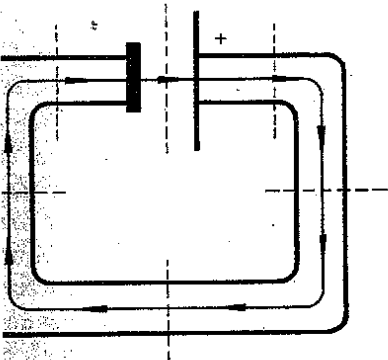
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ends of a wire are connected to different potentials, such as the EMF of a cell. If there is a current in the wire but the potential difference remains constant in time. Consider any short element of the wire bounded by two transverse planes as shown shaded in Fig. 4-2. If electrons in the wire are moving from left to right, negative charge is entering the element at its left face and an equal quantity of charge is leaving from its right face, and hence the potential difference across the element must be the same and therefore must be the same as the flow of electrons is like the

generator to which its ends are connected in a closed circuit. An EMF source and a generator by which is normally at the higher potential in a closed circuit. The light line in the diagram shows the direction of the conventional current in the opposite direction. The same direction as the conventional current in the opposite direction.



4-3. A closed circuit consisting of an electrolytic cell and a wire.

4-4]

ELECTRICAL CONDUCTIVITY

point of the circuit, the current immediately ceases. The circuit is then said to be *open*.

Notice carefully that the direction of the conventional current is "from plus to minus" in the external circuit only. Within the cell, the direction is from minus to plus.

4-4 Electrical conductivity. In order to maintain a current in a conductor there must be an electric field or a potential gradient within it. Conducting materials differ from one another, however, in the magnitude of the current density established by a given electric field. Fig. 4-4 represents a portion of a conductor of cross section A within which there is a (conventional) current i and a current density $J = \frac{i}{A}$. The elec-

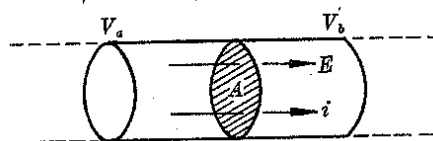


FIG. 4-4. Current within a portion of a conductor.

tric intensity at the shaded section is E . The ratio of the current density to the electric intensity, or the current density per unit electric intensity, is called the *electrical conductivity* of the material and is represented by σ .

$$\sigma = \frac{J}{E}, \quad \text{or} \quad J = \sigma E \quad (4-6)$$

The greater the conductivity, the greater the current density for a given electric intensity. The conductivity of a given material varies with temperature, and to a small degree with other physical conditions which will be mentioned later. For many materials, notably the metals, the conductivity is independent of the current density. On the other hand, there are substances for which the conductivity varies markedly with current density.

Current density is a vector, in the same direction as the field. The vector form of Eq. (4-6) is therefore

$$\vec{J} = \sigma \vec{E}$$

Eq. (4-6) may be put in the following form. If we replace E by $-\frac{dV}{dx}$, and J by $\frac{i}{A}$, we obtain

$$\begin{aligned} \frac{i}{A} &= \sigma \left(-\frac{dV}{dx} \right) \\ i &= -\sigma A \frac{dV}{dx} \end{aligned} \quad (4-7)$$

It is interesting to note that this equation is exactly analogous to the equation of steady-state heat conduction,

$$H = -KA \frac{dt}{dx}.$$

The electrical current i corresponds to the heat current H , the electrical conductivity σ to the thermal conductivity K , and the potential gradient $\frac{dV}{dx}$ to the temperature gradient $\frac{dt}{dx}$.

The relation between the conduction of electricity and the conduction of heat is more than a mere mathematical analogy. The free electrons which are the carriers of charge in electrical conduction also play an important role in the conduction of heat, and hence a correlation can be expected between electrical and thermal conductivities. It is a familiar fact that good electrical conductors, such as the metals, are good conductors of heat, while poor electrical conductors are also poor conductors of heat.

Theoretical reasoning which we cannot go into here predicts the following relation, known as the *Wiedemann-Franz law*, between the thermal and electrical conductivities of metals:

$$\frac{K}{\sigma} = 3 \left(\frac{k}{e} \right)^2 T$$

where k is the Boltzmann constant, e the electronic charge, and T the Kelvin temperature.

4-5 Resistance and resistivity. Ohm's law. While the relation $J = \sigma E$ is the fundamental equation of electrical conduction, it is more convenient in practice to work with currents and potential differences than with current densities and electric intensities. For this purpose we start with the equivalent form

$$i = -\sigma A \frac{dV}{dx}, \quad (4-7)$$

and consider a conductor of length L and constant cross section A (Fig. 4-5) in which there is a current i . Let V_a and V_b be the potentials at its terminals. We shall restrict the discussion to materials for which σ is independent of J and shall assume the temperature (and any other factors affecting σ) to be the same at all points of the conductor. Under these

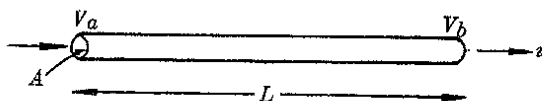


FIG. 4-5. Conductor of length L and constant cross section.

conditions i , σ , and A are all constant. Take the X -axis along the

$$\begin{aligned} i dx &= \\ i \int_0^L dx &= \\ iL &= \\ i &= \end{aligned}$$

This is the desired relation between the potential difference between its terminals and the conductance of the wire. The current for a given potential difference is then determined. In practice, the reciprocal of G is almost universally used. Resistance R (or r). Evidently, for a homogeneous material and for which σ is constant,

$$R =$$

The conducting properties of a material are often expressed in terms of the reciprocal of the conductance, called the resistance, and denoted by R .

In terms of resistivity Eq. (4-9) becomes

$$R = \frac{\rho L}{A}$$

and in terms of resistance Eq. (4-8) becomes

$$i = \frac{V_a - V_b}{R}$$

or

$$V_{ab} = iR$$

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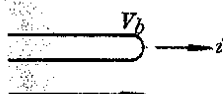
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and constant cross section.

conditions i , σ , and A are all constants and Eq. (4-7) can readily be inte-
grated. Take the X -axis along the wire with origin at end a . Then

$$\begin{aligned} i dx &= -\sigma A dV \\ i \int_0^L dx &= -\sigma A \int_{V_a}^{V_b} dV \\ iL &= \sigma A (V_a - V_b) \\ i &= \frac{\sigma A}{L} (V_a - V_b). \end{aligned} \quad (4-8)$$

This is the desired relation between the current in the conductor and
the potential difference between its terminals. The factor $\frac{\sigma A}{L}$ is called
the *conductance* of the wire. The greater the conductance, the greater
the current for a given potential difference.

In practice, the reciprocal of the conductance, called the *resistance*,
is almost universally used. Resistance is represented by the letter R
(or r). Evidently, for a homogeneous conductor of constant cross section
and for which σ is constant,

$$R = \frac{L}{\sigma A}. \quad (4-9)$$

The conducting properties of materials are commonly tabulated in
terms of the reciprocal of the conductivity, called the *resistivity* and de-
noted by ρ .

$$\rho = \frac{1}{\sigma}.$$

In terms of resistivity Eq. (4-9) becomes

$$R = \frac{\rho L}{A}, \quad (4-10)$$

and in terms of resistance Eq. (4-8) becomes

$$i = \frac{V_a - V_b}{R} = \frac{V_{ab}}{R}$$

or

$$V_{ab} = Ri. \quad (4-11)$$