

Introduction

1.1 INTRODUCTION TO THE STUDY OF MECHANISMS

The study of mechanisms is very important. With the continuing advances made in the design of instruments, automatic controls, and automated equipment, the study of mechanisms takes on new significance. *Mechanisms* may be defined as that division of machine design which is concerned with the kinematic design of linkages, cams, gears, and gear trains. *Kinematic design* is design on the basis of motion requirements in contrast to design on the basis of strength requirements. An example of each of the mechanisms listed above will be given in order to present a comprehensive picture of the components to be studied.

A sketch of a linkage is shown in Fig. 1.1. This particular arrangement is known as the slider-crank mechanism. Link 1 is the frame and is stationary, link 2 is the crank, link 3 is the connecting rod, and link 4 is the slider. A common application of this linkage is in the internal-combustion engine where link 4

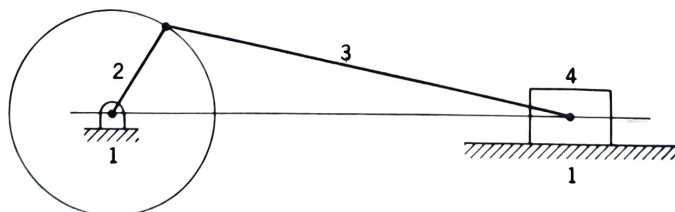


FIGURE 1.1 Slider-crank mechanism.

2 INTRODUCTION

becomes the piston (Fig. 1.2a). This figure also demonstrates how difficult it may be to discern the basic kinematic device when looking at a photograph or a drawing of a complete machine. Figure 1.2b shows the *kinematic diagram* of the slider-crank mechanism corresponding to the left-side crankshaft-connecting-rod-piston in the photograph of Fig. 1.2a. Such a kinematic diagram is much easier to work with and allows the designer to separate the kinematic considerations from the larger problem of machine design.

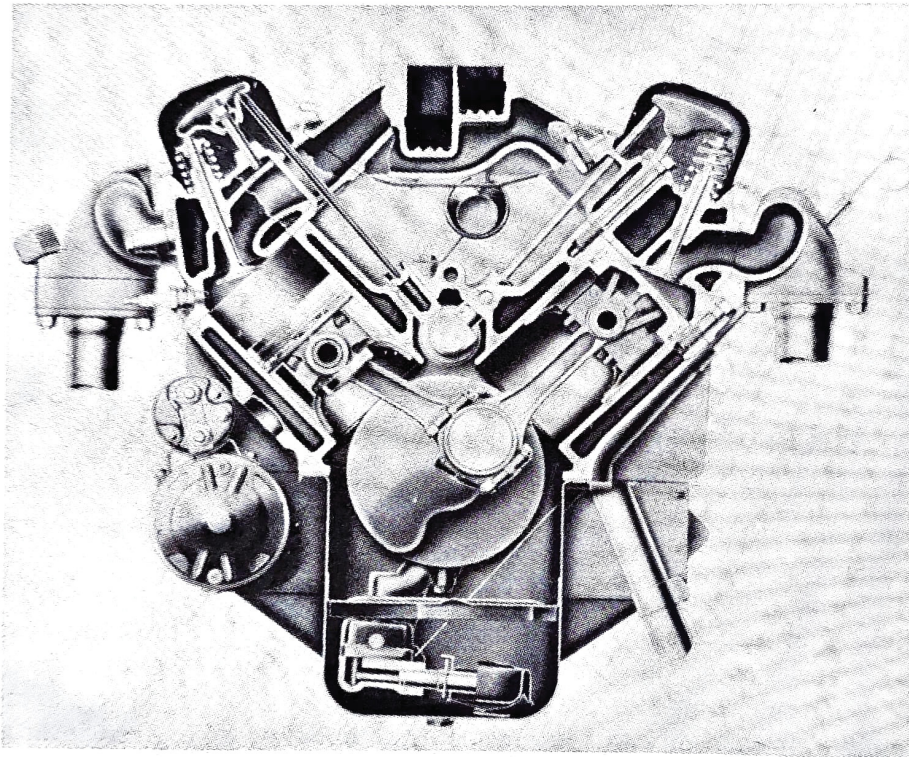


FIGURE 1.2a Chevrolet V-8 engine showing slider-crank mechanism. (General Motors Corporation).

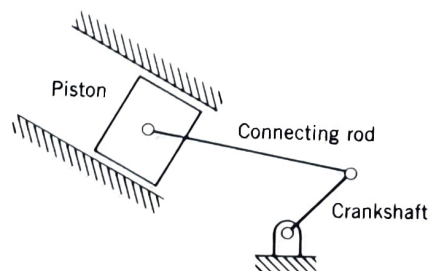


FIGURE 1.2b Kinematic diagram of engine mechanism.

Figure 1.3 shows the sketch of a cam and follower. The cam rotates at a constant angular velocity, and the follower moves up and down. On the upward motion the follower is driven by the cam, and on the return motion it is driven by the action of gravity or of a spring. Cams are used in many machines, but one of the most common is the automotive engine where two cams are used per cylinder to operate the intake and exhaust valves, also shown in Fig. 1.2a. A three-dimensional cam is shown in Fig. 1.4. In this cam, the motion of the follower depends not only upon the rotation of the cam but also upon the axial motion of the cam.

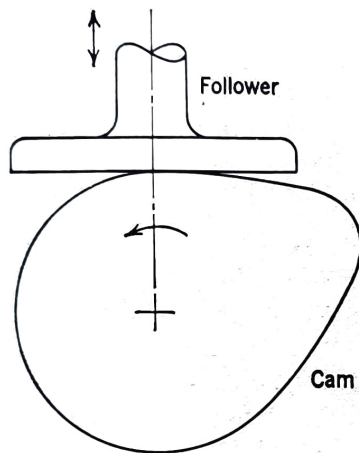


FIGURE 1.3
Two-dimensional cam.

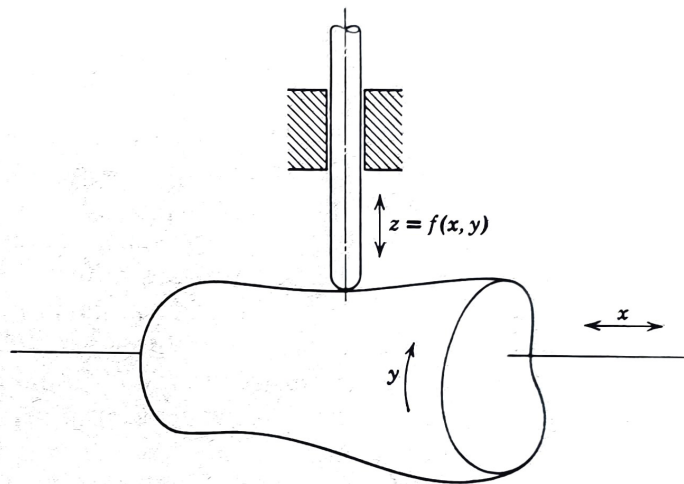


FIGURE 1.4 Three-dimensional cam.

Gears are used in many applications to transmit motion from one shaft to another with a constant angular velocity ratio. Figure 1.5 shows several commonly used gears.

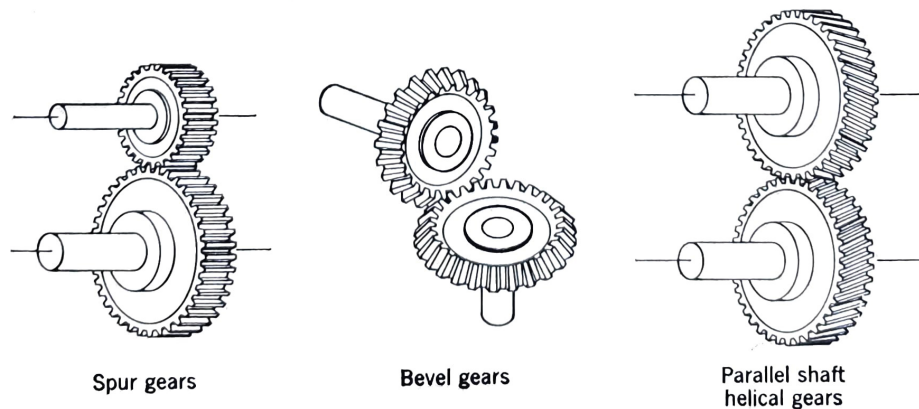


FIGURE 1.5 (continued next page)

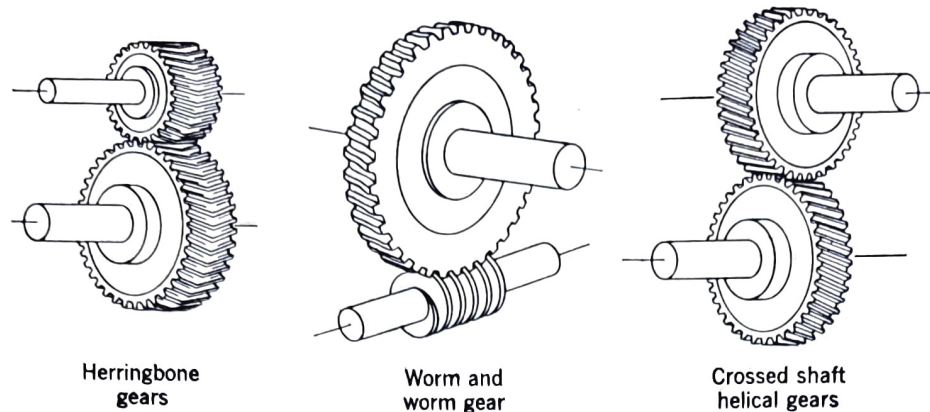


FIGURE 1.5 (continued)

In some cases, the desired reduction in angular velocity is too great to achieve using only two gears. When this occurs, several gears must be connected together to give what is known as a *gear train*. Figure 1.6 shows a gear train where the speed is stepped down in going from gear 1 to gear 2 and again in going from gear 3 to gear 4. Gear 1 is the driver, and gears 2 and 3 are mounted on the same shaft. In many gear trains, it is necessary to be able to shift gears in and out of mesh so as to obtain different combinations of speeds. A good example of this is the automobile transmission where three speeds forward and one in reverse are obtained by shifting two gears.

In devices such as instruments and automatic controls, obtaining the correct motion is all-important. The power transmitted by the elements may be so slight as to be negligible, which allows the components to be proportioned primarily on the basis of motion, strength being of secondary importance.

There are other machines, however, where the kinematic analysis is only one step in the design. After it has been determined how the various machine components will act to accomplish the desired motion, the forces acting upon

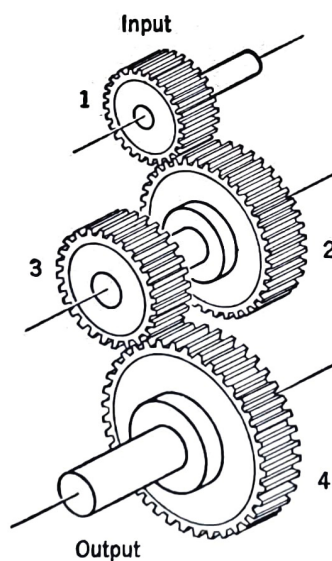


FIGURE 1.6 Gear train.

these parts must be analyzed. From this, the physical size of the parts may be determined. A machine tool is a good example; its strength and rigidity are more difficult to attain than the desired motions.

It is important at this time to define the terms used in the study of mechanisms. This is done in the following section.

1.2 MECHANISM, MACHINE

In the study of mechanisms the terms mechanism and machine will be used repeatedly. These are defined as follows:

A *mechanism* is a combination of rigid or resistant bodies so formed and connected that they move upon each other with definite relative motion. An example is the crank, connecting rod, and piston of an internal-combustion engine as shown diagrammatically in Fig. 1.2b.

A *machine* is a mechanism or collection of mechanisms which transmit force from the source of power to the resistance to be overcome. An example is the internal-combustion engine.

1.3 MOTION

In dealing with the study of mechanisms, it is necessary to define the various types of motion produced by these mechanisms.

Plane Motion

Translation

When a rigid body so moves that the position of each straight line of the body is parallel to all of its other positions, the body has motion of translation.

1. Rectilinear translation. All points of the body move in parallel straight line paths. When the body moves back and forth in this manner, it is said to reciprocate. This is illustrated in Fig. 1.7, where the slider 4 reciprocates between the limits B' and B'' .
2. Curvilinear translation. The paths of the points are identical curves parallel to a fixed plane. Figure 1.8 shows the mechanism that was used in connecting

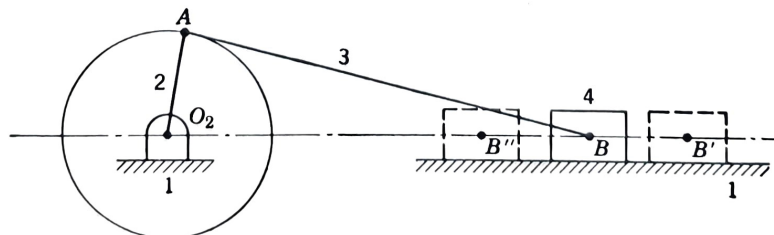


FIGURE 1.7

6 INTRODUCTION

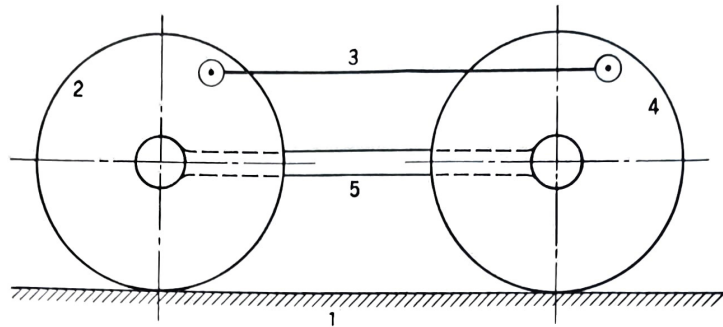


FIGURE 1.8

the drive wheels of the steam locomotive. In this mechanism, link 3 has curvilinear translation, and all points in the body trace out identical cycloids as wheels 2 and 4 roll along track 1. Link 5 moves with rectilinear translation.

Rotation

If each point of a rigid body having plane motion remains at a constant distance from a fixed axis that is perpendicular to the plane of motion, the body has motion of rotation. If the body rotates back and forth through a given angle, it is said to oscillate. This is shown in Fig. 1.9, where link 2 rotates and link 4 oscillates between the positions B' and B'' .

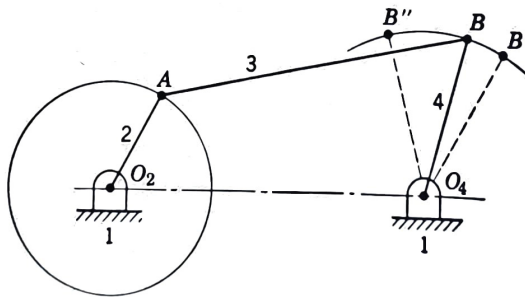


FIGURE 1.9 Four-bar linkage.

Rotation and Translation

Many bodies have motion which is a combination of rotation and translation. Link 3 in Fig. 1.7, links 2 and 4 in Fig. 1.8, and link 3 in Fig. 1.9 are examples of this type of motion.

Helical Motion

When a rigid body moves so that each point of the body has motion of rotation about a fixed axis and at the same time has translation parallel to the axis, the

body has helical motion. An example of helical motion is the motion of a nut as the nut is screwed onto a bolt.

Spherical Motion

When a rigid body moves so that each point of the body has motion about a fixed point while remaining at a constant distance from it, the body has spherical motion.

Spatial Motion

A body moving with rotation about three nonparallel axes and translation in three independent directions is said to be undergoing general spatial motion.

1.4 CYCLE, PERIOD, AND PHASE OF MOTION

When the parts of a mechanism have passed through all the possible positions they can assume after starting from some simultaneous set of relative positions and have returned to their original relative positions, they have completed a *cycle* of motion. The time required for a cycle of motion is the *period*. The simultaneous relative positions of a mechanism at a given instant during a cycle are a *phase*.

1.5 PAIRING ELEMENTS

The geometrical forms by which two members of a mechanism are joined together so that the relative motion between these two members is consistent are known as *pairing elements*. If the joint by which two members are connected has surface contact such as a pin joint, the connection is known as a *lower pair*. If the connection takes place at a point or along a line such as in a ball bearing or between two gear teeth in contact, it is known as a *higher pair*. A pair that permits only relative rotation is a *revolute*, or *turning pair*, and one that allows only sliding is a *sliding pair*. A turning pair can be either a lower or a higher pair depending upon whether a pin and bushing or a ball bearing is used for the connection. A sliding pair will be a lower pair as between a piston and cylinder wall.

1.6 LINK, CHAIN

A *link* is a rigid body having two or more pairing elements by means of which it may be connected to other bodies for purposes of transmitting force or motion. Generally, a link is a rigid member with provision at each end for connection to two other links. This may be extended, however, to include three, four, or even more connections. Figures 1.10a, b, and c show these arrangements. Perhaps the extreme case of a multiply connected link is the master rod in a nine-cylinder radial aircraft engine as seen in Fig. 1.10d.

A well-known example of a link with three connections is the bell crank, which can be arranged as shown in Fig. 1.11a or Fig. 1.11b. This link is generally

8 INTRODUCTION

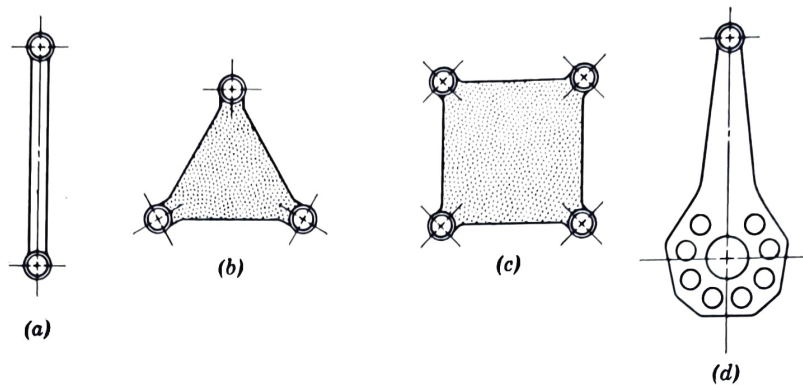


FIGURE 1.10

used for motion reduction and can be proportioned for a given ratio with a minimum of distortion of the required motion.

When a number of links is connected by means of pairs, the resulting system is a kinematic *chain*. If these links are connected in such a way that no motion is possible, a locked chain (structure) results. A constrained chain is obtained when the links are so connected that, no matter how many motion cycles are passed through, the relative motion will always be the same between the links.

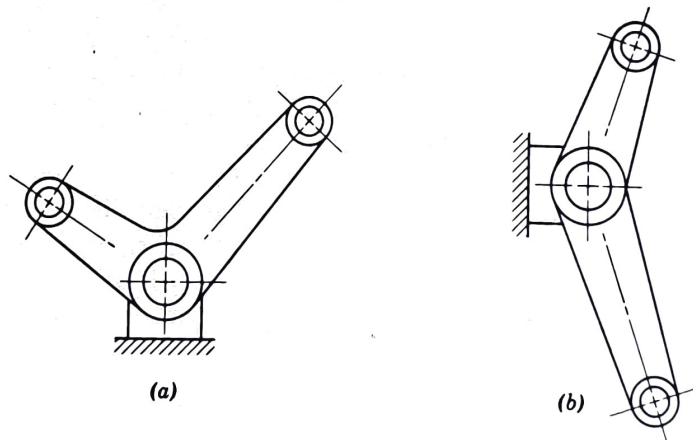


FIGURE 1.11

It is also possible to connect links so that an unconstrained chain results, which means that the motion pattern will vary from time to time depending on the amount of friction present in the joints. If one of the links of a constrained chain is made a fixed link, the result is a mechanism.

1.7 INVERSION

If in a mechanism, the link which was originally fixed is allowed to move and another link becomes fixed, the mechanism is said to be inverted. The inversion

of a mechanism does not change the motion of its links relative to each other but does change their absolute motions (relative to the ground).

1.8 TRANSMISSION OF MOTION

In the study of mechanisms, it is necessary to investigate the method in which motion may be transmitted from one member to another. Motion may be transmitted in three ways: (a) direct contact between two members such as between a cam and follower or between two gears, (b) through an intermediate link or connecting rod, and (c) by a flexible connector such as a belt or chain.

The angular velocity ratio is determined for the case of two members in direct contact. Figure 1.12 shows cam 2 and follower 3 in contact at point P . The cam has clockwise rotation, and the velocity of point P as a point on body 2 is represented by the vector $\mathbf{PM}_2$. The line NN' is normal to the two surfaces at point P and is known as the *common normal*, the *line of transmission*, or the *line of action*. The common tangent is represented by TT' . The vector $\mathbf{PM}_2$ is broken into two components, $\mathbf{Pn}$ along the common normal and $\mathbf{Pt}_2$ along the common tangent. Because of the fact that the cam and the follower are rigid members and must remain in contact, the normal component of the velocity of P as a point on body 3 must be equal to the normal component of P as a point on body 2. Therefore, knowing the direction of the velocity vector of P as a point on body 3 to be perpendicular to the radius O_3P and its normal component, it is possible to find the velocity $\mathbf{PM}_3$ as shown in the sketch. From this vector, the angular velocity of the follower may be determined from the relation $V = R\omega$, where V equals the linear velocity of a point moving along a path of radius R and ω equals the angular velocity of the radius R .

In direct-contact mechanisms, it is often necessary to determine the velocity of sliding. From the sketch this can be seen to be the vector difference between the tangential components of the velocities of the points of contact. This difference

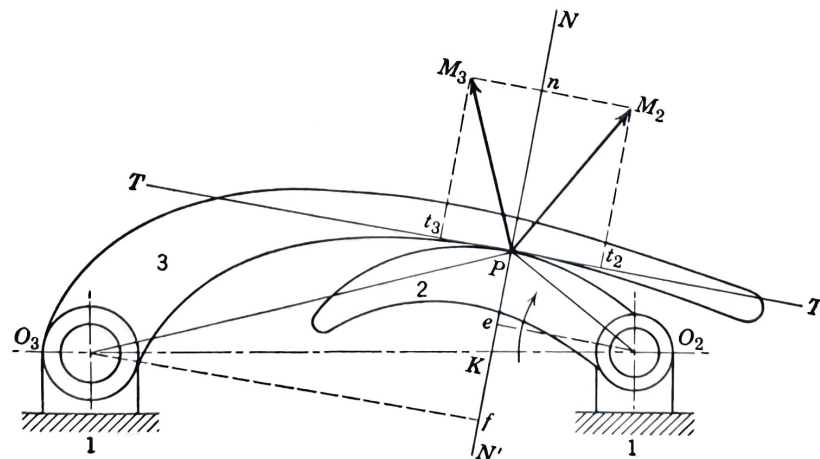


FIGURE 1.12

10 INTRODUCTION

is given by the distance $\overline{t_2 t_3}$ because the component $\mathbf{Pt}_3$ is opposite in direction to that of $\mathbf{Pt}_2$. If t_2 and t_3 fall on the same side of P , then the distance will subtract. If the contact point P should fall on the line of centers $\overline{O_2 O_3}$, then the vectors $\mathbf{PM}_2$ and $\mathbf{PM}_3$ will be equal and in the same direction. The tangential components must also be equal and in the same direction so that the velocity of sliding will be zero. The two members will then have pure rolling motion. Thus, it may be said that *the condition for pure rolling is that the point of contact shall lie on the line of centers.*

For the mechanism of Fig. 1.12, the motion between the cam and the follower will be a combination of rolling and sliding. Pure rolling can only take place where the point of contact P falls on the line of centers. However, contact at this point may not be possible because of the proportions of the mechanism. Pure sliding cannot occur between cam 2 and follower 3. For this to happen, a point on one link, within the limits of its travel, has to come in contact with all the successive points on the active surface of the other link.

It is possible to determine a relation so that the angular velocity ratio of two members in direct contact can be determined without going through the geometrical construction outlined above. From O_2 and O_3 drop perpendiculars upon the common normal striking it at e and f , respectively. The following relations will be seen to hold:

$$\omega_2 = \frac{PM_2}{O_2P} \quad \text{and} \quad \omega_3 = \frac{PM_3}{O_3P}$$

$$\frac{\omega_3}{\omega_2} = \frac{PM_3}{O_3P} \times \frac{O_2P}{PM_2}$$

From the fact that triangles PM_2n and O_2Pe are similar,

$$\frac{PM_2}{O_2P} = \frac{Pn}{O_2e}$$

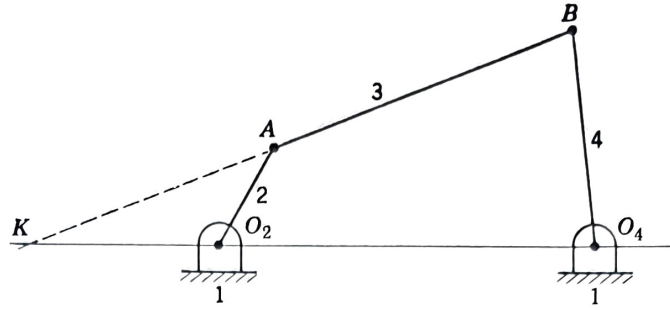
Also, PM_3n and O_3Pf are similar triangles; therefore,

$$\frac{PM_3}{O_3P} = \frac{Pn}{O_3f}$$

Therefore,

$$\frac{\omega_3}{\omega_2} = \frac{Pn}{O_3f} \times \frac{O_2e}{Pn} = \frac{O_2e}{O_3f}$$

With the common normal intersecting the line of centers at K , triangles O_2Ke


FIGURE 1.13

and O_3Kf are also similar; therefore,

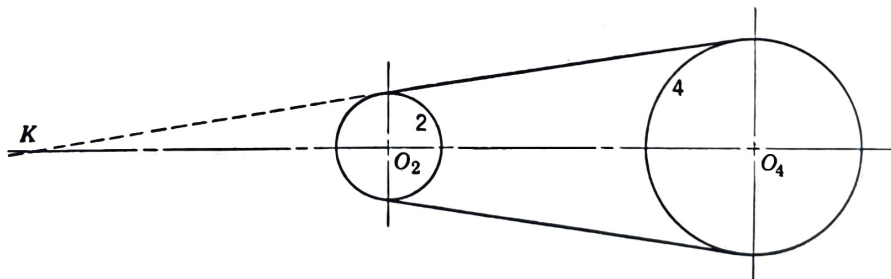
$$\frac{\omega_3}{\omega_2} = \frac{O_2e}{O_3f} = \frac{O_2K}{O_3K} \quad (1.1)$$

Therefore, for a pair of curved surfaces in direct contact, the angular velocities are inversely proportional to the segments into which the line of centers is cut by the common normal. From this it can be seen that *for constant angular velocity ratio the common normal must intersect the line of centers in a fixed point*.

It is also possible to derive the above relations for the transmission of motion through an intermediate link or connecting rod and for the transmission of motion through a flexible connector. Figures 1.13 and 1.14 show these two cases, respectively, where the angular velocity ratio is given by

$$\frac{\omega_4}{\omega_2} = \frac{O_2K}{O_4K} \quad (1.2)$$

In Fig. 1.14, the ratio ω_4/ω_2 is independent of the center distance O_2O_4 .


FIGURE 1.14

1.9 MOBILITY, OR NUMBER OF DEGREES OF FREEDOM

Mobility is one of the most fundamental concepts to the study of kinematics. By definition, the *mobility* of a mechanism is the number of degrees of freedom it possesses. An equivalent definition of mobility is the minimum number of in-

12 INTRODUCTION

dependent parameters required to specify the location of every link within a mechanism.

A single link constrained to move with planar motion, such as the one shown in Fig. 1.15a, possesses three degrees of freedom. The x - and y -coordinates of the point P along with the angle θ form an independent set of three parameters describing its location. Two unconnected planar links are shown in Fig. 1.15b. Since each link possesses three degrees of freedom, these two links possess a total of six degrees of freedom. If the two links are pinned together at a point by means of a revolute joint, as shown in Fig. 1.15c, the two-link system will possess only four degrees of freedom. Four independent parameters describing the location of the two links could, for example, be the x - and y -coordinates of the point P_1 , the angle θ_1 , and the angle θ_2 . There are many other parameters that could be used to specify the location of these links, but only four of these can be independent. Once the values of the independent parameters are specified, the position of every point in both links is determined.

In the simple example described above, connecting two planar links with a revolute joint had the effect of removing two degrees of freedom from the system. Stated in another way, a revolute joint permits a single degree of freedom (pure rotation) between the links it connects. Using this type of logic, it is possible to

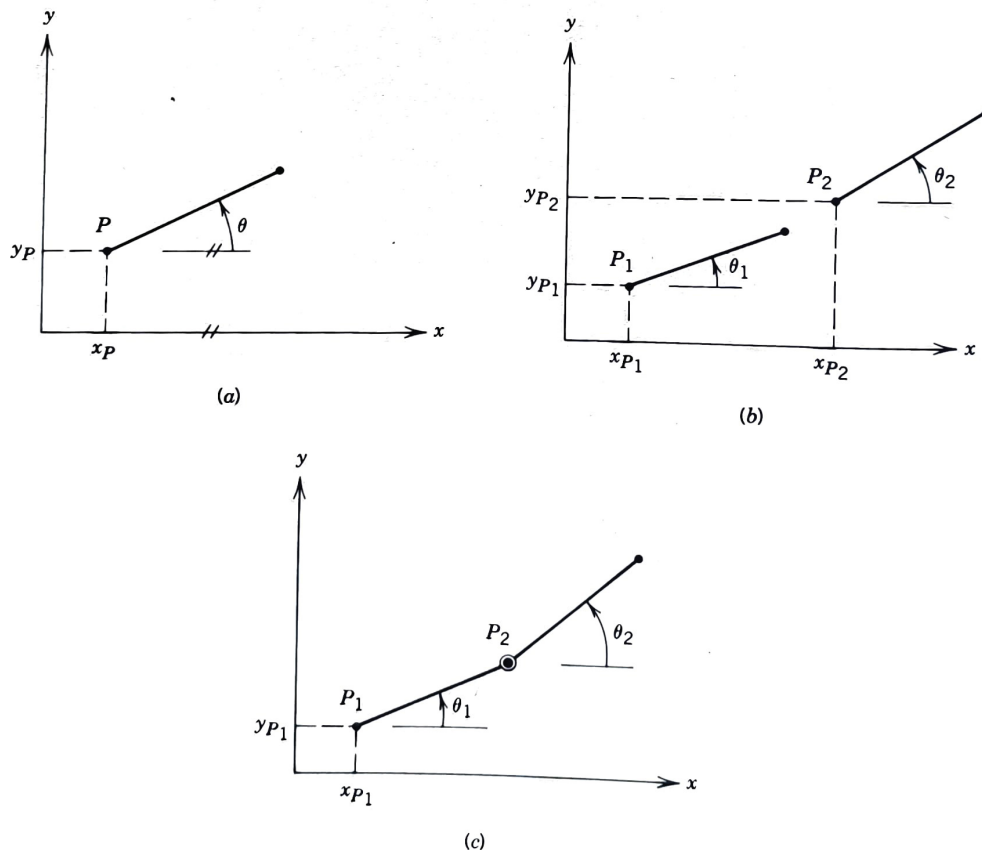


FIGURE 1.15

develop a general equation which will help predict the mobility of any planar mechanism.

For example, a planar mechanism having n links is to be designed. Before any connections are made, the system of n links will have a total of $3n$ degrees of freedom. Recognizing that one link of every mechanism will always be considered to be fixed to the ground removes three degrees of freedom. This leaves the system with a total of $3n - 3$, or $3(n - 1)$, degrees of freedom. Each one-degree-of-freedom joint removes two degrees of freedom from the system. Similarly, each two-degree-of-freedom joint removes one degree of freedom from the system. The total mobility of the system is given by Grubler's equation

$$M = 3(n - 1) - 2f_1 - f_2 \quad (1.3)$$

where

M = the mobility, or number of degrees of freedom

n = the total number of links, including the ground

f_1 = the number of one-degree-of-freedom joints

f_2 = the number of two-degree-of-freedom joints

Care must be used when applying this equation because there are a number of special mechanism geometries for which it will not work. Although no all-inclusive rule exists for predicting when the mobility equation may give an incorrect result, special cases often occur when several links of a mechanism are parallel. For example, applying Grubler's equation to the mechanism of Fig. 1.16 gives

$$M = 3(5 - 1) - 2(6) = 0$$

Nevertheless, this device can actually move as a result of its special geometry and is a mechanism with one degree of freedom. It must also be noted that a joint connecting k links at a single point must be counted as $k - 1$ joints. For example, a revolute joint connecting three links at a single point is counted as two joints. Only four types of joints are commonly found in planar mechanisms.

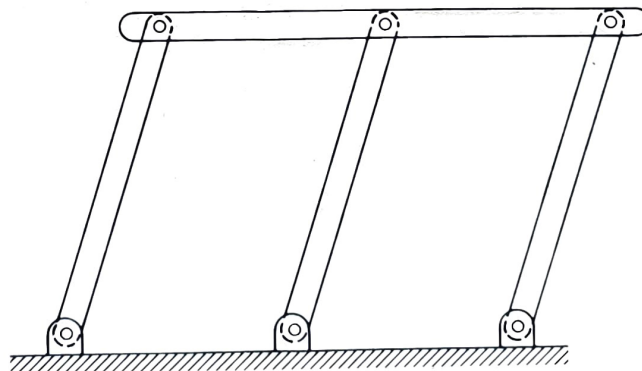


FIGURE 1.16

14 INTRODUCTION

These are the revolute, the prismatic, and the rolling contact joints (each having one degree of freedom), and the cam or gear joint (having two degrees of freedom). These joints are depicted in Fig. 1.17. The following definitions apply to the actual mobility of a device:

$M \geq 1$: the device is a mechanism with M degrees of freedom

$M = 0$: the device is a statically determinate structure

$M \leq -1$: the device is a statically indeterminate structure

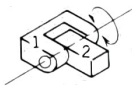
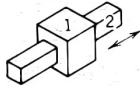
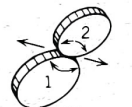
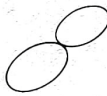
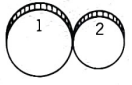
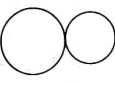
Joint Type (Symbol)	Physical Form	Schematic Representation	Degrees of Freedom
Revolute (R)			1 (Pure rotation)
Prismatic (P)			1 (Pure sliding)
Cam or gear			2 (Rolling and sliding)
Rolling contact			1 (Rolling without sliding)

FIGURE 1.17 Common types of joints found in planar mechanisms.

Example 1.1. Determine the mobility of the four-bar linkage of Fig. 1.18.

There are four links and four revolute joints, each having one degree of freedom. The mobility is given by

$$M = 3(4 - 1) - 2(4)$$

$$M = 1$$

So this is a one-degree-of-freedom mechanism.

Example 1.2. Determine the mobility of the device of Fig. 1.19.

There are four links connected by five single-degree-of-freedom joints (the joint connecting three links at a point counts twice). The mobility is given by

$$M = 3(4 - 1) - 2(5)$$

$$M = -1$$

This is a statically indeterminate structure.

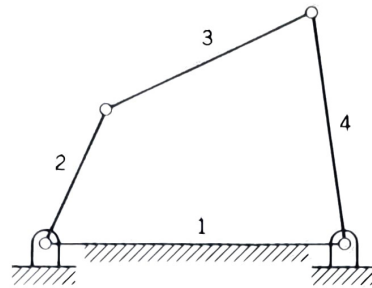


FIGURE 1.18

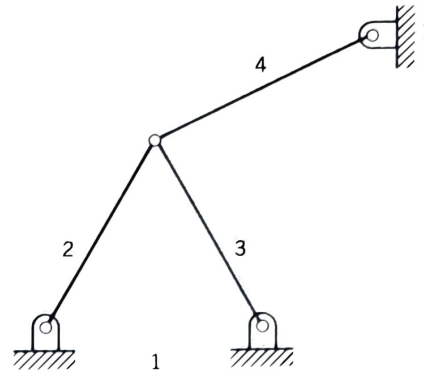


FIGURE 1.19

Example 1.3. Determine the mobility of the device of Fig. 1.20.

There are three links, two one-degree-of-freedom revolute joints and one two-degree-of-freedom higher-pair joint. In the higher-pair joint, the two contacting bodies may translate along the common tangent to the two surfaces or rotate about the contact point, thus giving two degrees of freedom. The mobility is given by

$$M = 3(3 - 1) - 2(2) - 1(1)$$

$$M = 1$$

This is a one-degree-of-freedom mechanism.

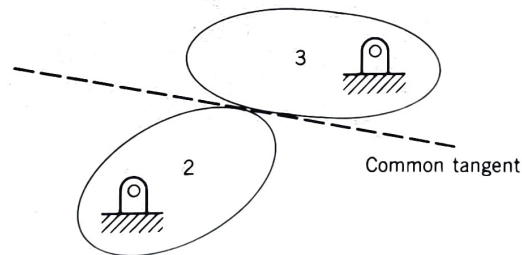


FIGURE 1.20

Problems

1.1. (a) If $\omega_2 = 20$ rad/min, calculate the angular velocity of link 3 for the two cases shown in Fig. 1.21. (b) Calculate the maximum angle and the minimum angle of the follower with the horizontal.

1.2. Lay out the mechanisms for Problem 1.1 to full scale and graphically determine the velocity of sliding between links 2 and 3. Use a velocity scale of 1 in. = 10 in./min.

1.3. If $\omega_2 = 20$ rad/min for the mechanism shown in Fig. 1.21, using graphical construction, determine the angular velocities of link 3 for one revolution of the cam in 60 increments starting from the position where $\omega_3 = 0$. Plot ω_3 versus cam angle θ letting the scale of ω_3 be 1 in. = 2.0 rad/min and the scale of θ be $\frac{1}{2}$ in. = 60.

16 INTRODUCTION

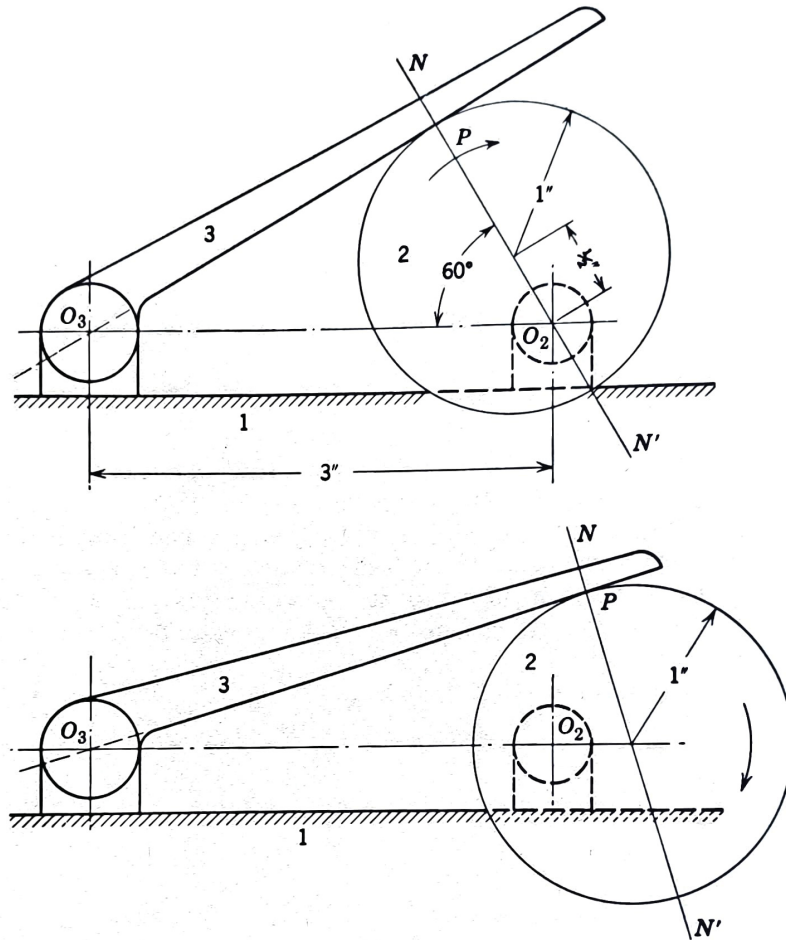
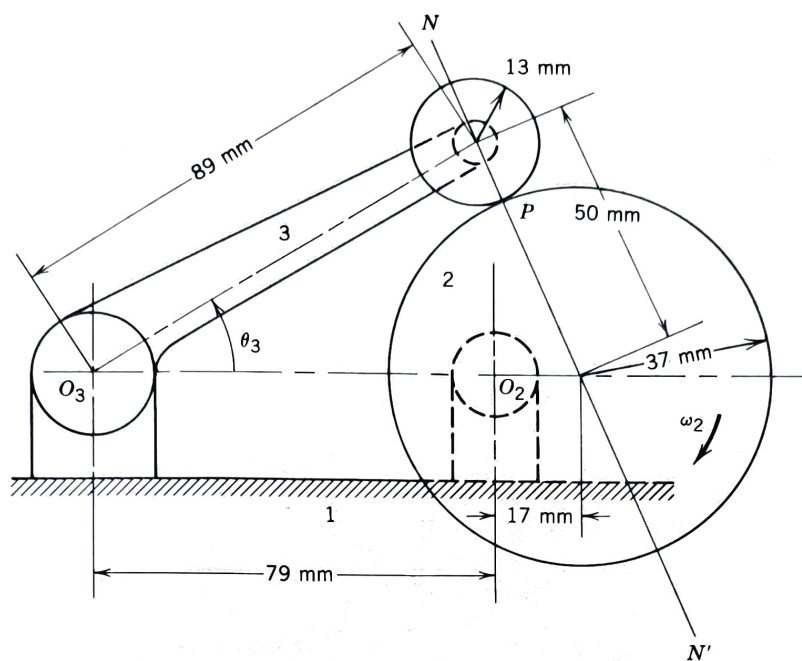
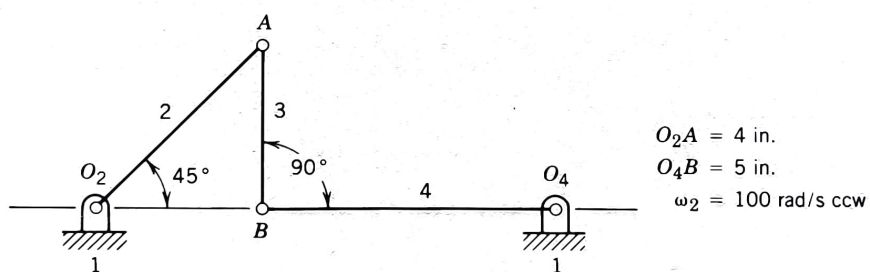
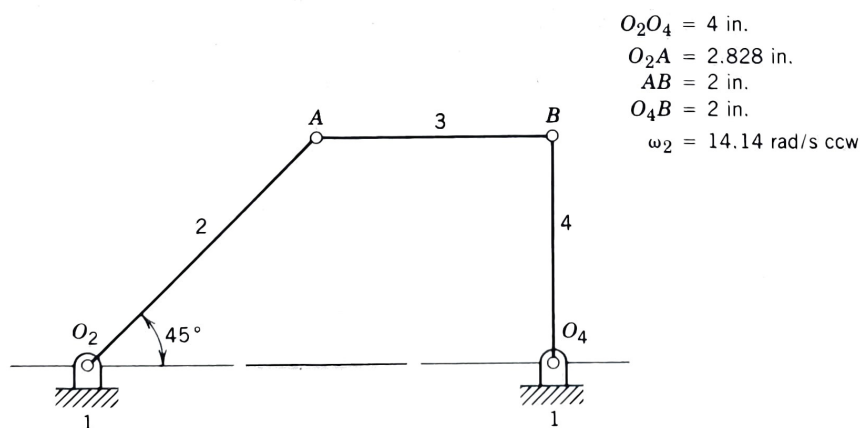


FIGURE 1.21

- 1.4.** (a) If $\omega_2 = 1800 \text{ rad/s}$, calculate the angular velocity of link 3 for the mechanism shown in Fig. 1.22. (b) Calculate the maximum angle and the minimum angle of the follower relative to the horizontal.
- 1.5.** For the linkage shown in Fig. 1.23, determine ω_4 and V_B .
- 1.6.** For the linkage shown in Fig. 1.24, determine V_B and ω_4 .
- 1.7.** Prove for the linkage shown in Fig. 1.13 that the angular velocities of the driven and driver links are inversely proportional to the segments into which the line of centers is cut by the line of transmission.
- 1.8.** Prove for the belt and pulleys shown in Fig. 1.14 that the angular velocities of the pulleys are inversely proportional to the segments into which the line of centers is cut by the line of transmission.
- 1.9.** In a linkage as shown in Fig. 1.13, the crank 2 is 19 mm long and rotates at a constant angular velocity of 15 rad/s. Link 3 is 38 mm long and link 4 is 25 mm long. The distance between centers O_2 and O_4 is 51 mm. Graphically determine the angular velocity of link


FIGURE 1.22

FIGURE 1.23

FIGURE 1.24

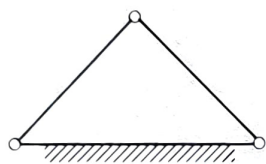


FIGURE 1.25

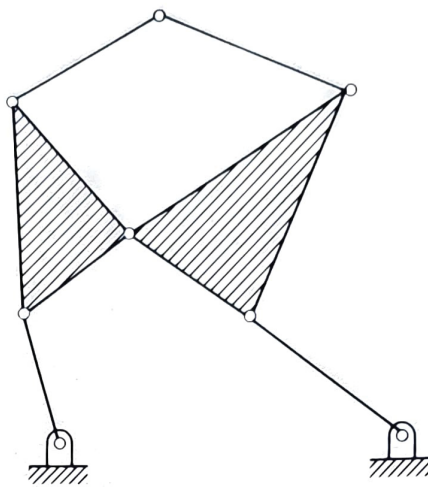


FIGURE 1.26

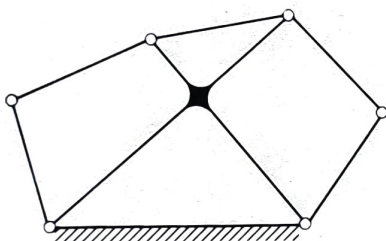


FIGURE 1.27

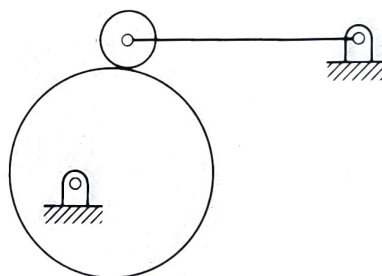


FIGURE 1.28

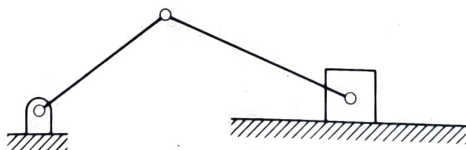


FIGURE 1.29

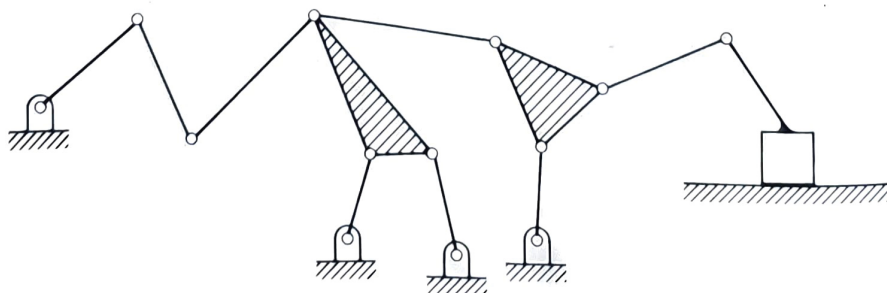
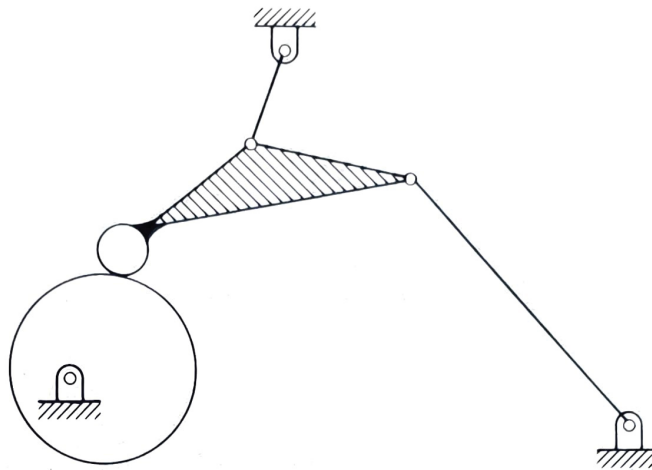
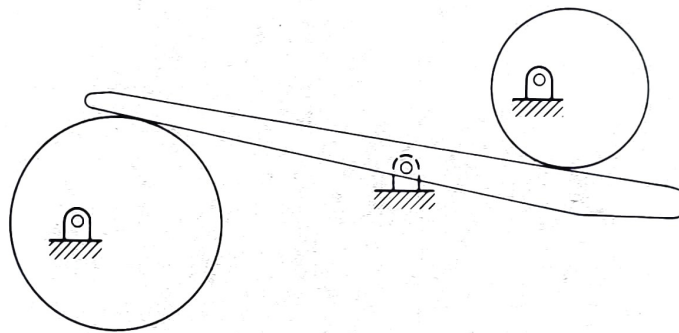


FIGURE 1.30


FIGURE 1.31

FIGURE 1.32

4 when link 2 is counterclockwise 45° from the horizontal. State whether or not ω_4 is constant.

1.10. A pulley of 100 mm diam drives one of 200 mm diam by means of a belt. If the angular velocity of the drive pulley is 65 rad/s and the center distance between pulleys is 400 mm, graphically determine the speed of the 200 mm pulley. Will its speed be constant?

1.11. Determine the mobility (number of degrees of freedom) of the devices shown in Figs. 1.25 through 1.32.