

Theoretical and Measured Performance of a DFE Modem on a Fading Multipath Channel

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Abstract—A decision-feedback equalizer (DFE) is the basis of a recent development of a quadruple diversity troposcatter modem which can operate up to a data rate of 12.6 Mbit/s in a 99% bandwidth of 15 MHz. In this paper a theoretical approach is developed for the calculation of average bit error rate (ABER), including the effects of intersymbol interference due to multipath and the finiteness of the transversal filters used to realize the DFE. By omitting the intersymbol interference effect, the calculation provides a lower bound which can be used to assess the intersymbol interference penalty for a particular DFE structure. The paper includes calculations of a DFE configuration which has a three tap forward filter with tap spacing equal to one-half a symbol interval. Measured performance results from fading channel simulator tests of a three tap forward filter DFE are presented for data rates from 1.5 to 12.6 Mbit/s and for a wide range of multipath statistical conditions. The results for this DFE configuration show (1) excellent agreement between calculated and measured ABER, (2) a small intersymbol interference penalty when the 2σ multipath spread is less than approximately one-half the data symbol interval, and (3) successful operation at values of multipath spread up to twice the data symbol interval. In a sequel to this paper, the results of a field test of the DFE modem are presented. These live links test results are consistent with both the calculated and simulator measured data presented here.

I. INTRODUCTION

AN ADAPTIVE equalization technique has been successfully developed for use on tropospheric scatter channels at data rates up to 12.6 Mbit/s. A multichannel decision-feedback equalizer (DFE) provides the functions of diversity combining, elimination of intersymbol interference effects, coherent recombining of the multipath components, and removal of the carrier frequency Doppler component. These functions are accomplished in a time-varying environment with rms Doppler spreads up to 10 Hz and specular Doppler components such as airplane reflections up to 150 Hz. U.S. Army ECOM is responsible for the development of the DFE modem through a Sylvania/Signatron contract as part of the Megabit Digital Troposcatter Subsystem (MDTS).

A decision-feedback equalizer uses a linear equalizer or forward filter component to reduce the effects of future digit intersymbol interference, i.e., precursors. A backward filter which linearly operates on reconstructed data is used to eliminate past digit intersymbol interference, i.e., postcursors. The use of this forward and backward filter combination was suggested by Austin [1] who calculated optimum tap settings for

a tapped-delay line structure. Monsen [2, 3] established the optimum DFE structure under a minimum mean-square-error (MMSE) criterion and devised an implementation scheme which included a convergent adaptation procedure for a fading multipath channel. The optimum structure for the white noise channel consists of a forward filter, which is composed of a matched filter and an anti-causal tapped-delay-line filter with tap spacing equal to the symbol length, and a backward tapped-delay-line filter with symbol interval tap spacing. Adaptation can be accomplished in a band-limited channel by realizing the forward filter as a tapped-delay-line filter with Nyquist tap spacing and continuous updating of the forward and backward filter tap gains by an estimated gradient algorithm of the type developed by Widrow and Hoff [4]. This adaptation is optimum in a mean square error sense for the decision-feedback equalizer [2].

Because the backward filter cancels postcursors with a linear weighting of past decisions, errors in the decision process will tend to produce more errors and the possibility of error propagation exists. This phenomenon has been modeled as a Markov process and it has been shown both to have a small effect on a particular channel realization when the intersymbol interference is not excessive and a negligible effect on fading multipath channels [5]. This latter result for fading channels is due to:

- (1) the reduction of intersymbol interference by the diversity combining operation, and
- (2) the domination of the average bit error rate by noise effects rather than intersymbol interference.

The presence of independently fading multipath will result in both intersymbol interference and an inherent or implicit diversity gain potential. When the adaptive DFE was first formulated, its performance was shown [2] to be better in the presence of multipath than in its absence because the implicit diversity gain is larger than any intersymbol interference degradation. For the optimum DFE determined in [2], the average bit error rate performance is very close to the performance of the optimum matched filter receiver for single-pulse reception [5]. For a DFE with a finite number of taps, the intersymbol interference penalty, which is measured by the difference between MMSE reception of serially transmitted pulses and MMSE reception of a single pulse, has also been established [6] to be small in a fading channel application. Thus, the DFE represents an efficient, practical approach for the transmission of high speed data over slowly varying fading multipath channels.

The DFE model developed for fading channel application uses up to four forward filters to combine the received diver-

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sity signals. Each forward filter consists of a three tap transversal filter with tap spacing equal to one-half a symbol interval. The modulation format is 4 PSK, which gives a symbol interval equal to twice the bit period. The backward filter is a three tap transversal filter with symbol interval tap spacing. A detailed description of the DFE modem is given in [7].

The performance measure for a DFE modem in a fading channel application is the average bit error rate (ABER). The ABER is a function of the average received energy per bit $\bar{E}_b$, the received noise power in 1 Hz of bandwidth, N_0 , and the multipath spread, 2σ . In this paper a theoretical approach is developed for the calculation of ABER including the effects of intersymbol interference due to multipath and the finiteness of the transversal filter used to realize the forward filter component of the DFE. Computed ABER values are compared with measured performance results from fading channel simulator tests of a three tap forward filter DFE. The results for this DFE configuration show excellent agreement between calculated and measured ABER, a small intersymbol interference penalty when the multipath spread is less than approximately one-half the data symbol interval, and successful operation at values of 2σ up to twice the data symbol interval. In a sequel [11] to this paper, the results of a field test of the DFE modem are presented. These live link test results are consistent with both the calculated and simulator measured data presented here.

The remainder of this paper is organized as follows. Section II briefly describes the troposcatter channel model and the finite realization of the decision-feedback equalizer. A method of calculating the average bit error rate for the DFE on a fading multipath channel is outlined in Section III. The ABER calculated performance for a three tap forward filter DFE is given in Section IV along with the results of the fading channel simulator tests of the DFE modem.

II. COMMUNICATION SYSTEM DEFINITION

The communication system model under consideration is shown in Fig. 1. Complex notation is used to represent in-phase and quadrature components and explicit modulation/demodulation operations are not shown. The data to be transmitted (s_k) are selected from the set $(\pm 1 \pm j)$ for a quaternary phase-shift-keying system. The transmitted waveform is

$$s(t) = \sum_{k=-\infty}^{\infty} s_k f(t - kT) \quad (1)$$

where T is the interval between successive symbol transmission, i.e., symbol interval. The transmitter impulse response $f(t)$ is defined as a unit energy real function.

The fading multipath channel can be represented by an ensemble of zero-mean random functions with complex Gaussian statistics. Individual diversity channels are considered to be independent of each other and the ensemble is completely described by the second moment of the channel impulse response $h_i(t)$

$$\overline{h_i(t)h_j^*(\tau)} = \delta_{ij}\delta(t - \tau)\bar{E}_b p(t), \quad i, j = 1, 2, \dots, D \quad (2)$$

where δ_{ij} and $\delta(t)$ are the Kronecker delta and continuous delta function, respectively.

The unit area function $p(t)$ is the mean squared envelope of the impulse response or multipath profile. It represents the average power response at each delay value to impulse function excitation. Twice the rms width of this function is referred to as the multipath spread, 2σ . The received energy per bit per diversity channel is given by

$$\begin{aligned} & \int_{-\infty}^{\infty} dt \left| \int_{-\infty}^{\infty} f(t-u)h_i(u)h_i^*(u) du \right|^2 \\ &= \int_{-\infty}^{\infty} dt \int_{-\infty}^{\infty} f^2(t-u)\bar{E}_b p(u) du \\ &= \bar{E}_b \int_{-\infty}^{\infty} p(u) du = \bar{E}_b. \end{aligned}$$

The ensemble representation is used here because the analysis to follow assumes a slowly varying multipath channel which the equalizer can track. Hence the performance is determined by averaging the bit error probability for the MMSE equalizer over the channel ensemble. An overbar is used to denote channel ensemble averages. Brackets $\langle \rangle$ will be used to denote average over noise or source statistics for a particular channel realization. For the white Gaussian noise channels the additive noise terms $n_i(t)$ are zero mean and have second moments

$$\langle n_i(t)n_j^*(\tau) \rangle = N_0\delta_{ij}\delta(t - \tau), \quad i, j = 1, 2, \dots, D.$$

Each forward filter in a realizable DFE consists of a finite length tapped delay line filter with tap spacing of τ_s seconds and impulse response

$$w_i(t) = \sum_{k=-K_1}^{K_2} w_{ik}^* \delta(t - k\tau_s) \quad (3)$$

where the tap gain value is chosen with a complex conjugate for notational convenience later. The receiver filter has impulse response $f(-t)$ which matches the transmit pulse waveform. For this choice, in the absence of multipath, and after time synchronization, the optimum tap weights for each forward filter reduce to one tap on and the rest off. The output of the receiver filter is sampled at the symbol rate $1/T$ and held for a symbol interval. The backward filter correction sample is summed with the sampled receiver filter output to produce the detection variable z . The backward filter weights previous decisions $\hat{s}_{k-i}$ by the complex weight b_i to form the sample output

$$y = \sum_{i=1}^B b_i \hat{s}_{k-i} \quad (4)$$

at the decision time for the s_k symbol. The decision on s_k is denoted as $\hat{s}_k$. The decision process takes the form

$$\hat{s}_k = \text{sgn}(\text{Re}(z)) + j \text{sgn}(\text{Im}(z))$$

where sgn is the signum function.

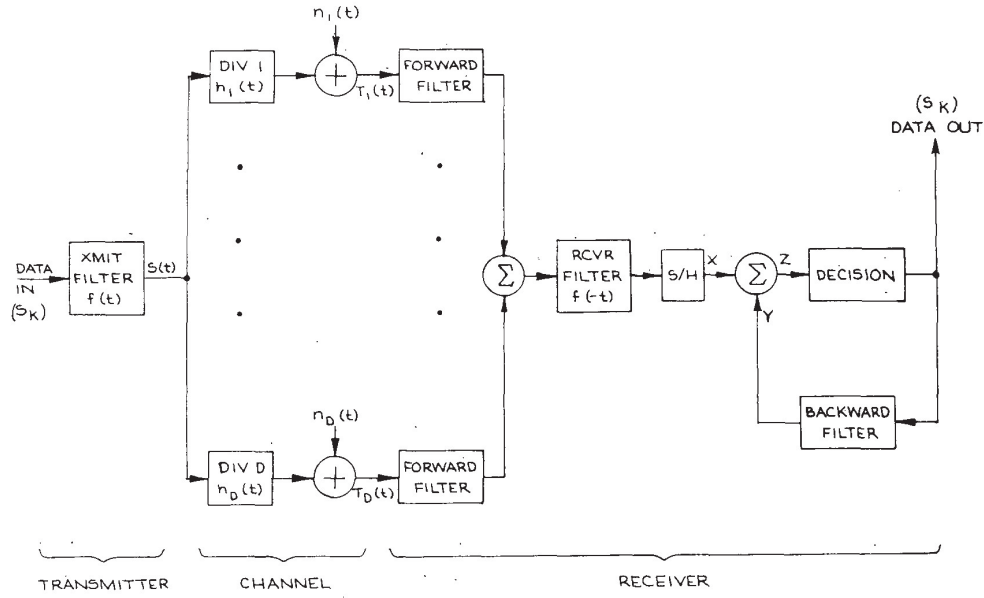


Fig. 1. Communication System Model.

The parameters of the DFE are the number of forward filter taps K , the forward filter tap spacing τ_s , and the number of backward filter taps, B . The optimum DFE requires $K = \infty = B$ and $\tau_s = 1/W$, where W is the two-sided bandwidth. A practical choice of parameters for troposcatter channels was determined from computer simulation of a fading channel equalizer application to be $K = 3 = B$ and $\tau_s = T/2$.

III. DFE PERFORMANCE ANALYSIS

Previous methods of determining average bit error rate (ABER) performance of practical equalizer structures have been restricted to Monte Carlo simulations [5, 6] using an ensemble of multipath channels. An analytic approach for calculating the ABER is complicated by the presence of an intersymbol interference term in the signal-to-noise ratio (SNR) expression for a particular channel realization. Because of this term, an average over the channel ensemble is a formidable task. On the other hand, omission of the intersymbol interference term leads to an ABER which for a fixed SNR monotonically decreases with increasing multipath spread. For any practical equalizer for a fixed SNR the ABER will initially decrease for increasing multipath spread, but then increase as the multipath exceeds the equalizer's capability to mitigate the intersymbol interference. Elimination of the intersymbol term provides a convenient lower bound which both shows the intersymbol interference penalty and is an accurate performance estimate for small multipath spreads. However, the lower bound is too loose for performance calculation when the rms multipath spread is on the order of the forward filter width. The analytic procedure to be developed allows accurate calculation of the lower bound for no intersymbol interference and by an approximation of the intersymbol interference effect provides a good performance estimate for the large multipath case.

For notational simplicity the analysis considers a nondiversity channel and extends the final result for higher order diversity systems. The received signal has the form

$$r(t) = \sum_{i=-\infty}^{\infty} s_i \int_{-\infty}^{\infty} h(u) f(t - iT - u) du + n(t). \quad (5)$$

We define the combined transmitter and receiver response as

$$g(t) = \int_{-\infty}^{\infty} f(t+u) f(u) du \quad (6)$$

and its convolution with the channel as

$$\sqrt{E_b} q(t) = \int_{-\infty}^{\infty} g(t-u) h(u) du. \quad (7)$$

For a sampling time t_0 , the detection variable z has the form

$$z = \sum_{k=-K_1}^{K_2} w_k^* \left[\sum_{i=-\infty}^{\infty} s_i \sqrt{E_b} q(t_0 - iT - k\tau_s) + v(t_0 - k\tau_s) \right]$$

where the $K = K_1 + K_2 + 1$ forward filter taps are apportioned between K_2 "past" taps and K_1 "future" taps and one center tap. The noise variate

$$v(t_0) = \int n(t_0 - u) f(u) du$$

is zero-mean Gaussian with second moment

$$\langle v(t_0 - k\tau) v^*(t_0 - l\tau) \rangle = N_0 g(k\tau - l\tau).$$

There exists an obvious representation in a $K = K_1 + K_2 + 1$ dimensional column vector form, viz.,

$$z = \mathbf{w}' \left(\sqrt{\bar{E}_b} \sum_{i=-\infty}^{\infty} s_i \mathbf{q}_i + \mathbf{v} \right) \quad (8)$$

where the prime refers to complex conjugate transpose. The vector $\mathbf{w}$ represents the forward filter complex tap gains, $\mathbf{q}_i$ represents the sampled continuous filter response for the i th transmitted symbol, and $\mathbf{v}$ is a zero-mean Gaussian noise vector process with positive definite covariance matrix $N_0 G_0$ with element values $N_0 g(k\tau_s - l\tau_s)$.

For QPSK, every other bit decision is made on the real part of z and there is quadrature symmetry and thus the real noise power affecting that decision is

$$\delta_0^2 = \langle \text{Re}(\mathbf{w}' \mathbf{v})^2 \rangle = \frac{N_0}{2} \mathbf{w}' G_0 \mathbf{w}. \quad (9)$$

If one assumes $s_k = 0, k \neq 0$, there is no intersymbol interference and the analysis for the ABER leads to an SNR expression for a particular channel realization which is a quadratic form of the type $\mathbf{q}_0' G_0^{-1} \mathbf{q}_0$, where $\mathbf{q}_0$ is a complex Gaussian vector with statistics determined by the channel and G_0 is a positive definite matrix. After a diagonalization procedure, the average of the bit error probability function of the SNR can be computed for both coherent and differentially coherent detection. This calculation leads to the lower bound expression. When the intersymbol interference is present, an effective signal-to-noise ratio can be defined which leads to a quadratic form, but both the vector and the matrix have random components. A method of averaging over a function of this quadratic form is not apparent. One course worth considering is to approximate the intersymbol interference effect in a manner which will modify the G_0 matrix but keep its deterministic nature. This can be accomplished if the following two approximations are made.

(1) Assume the interference symbols $s_i, i \neq 0$, are Gaussian distributed rather than complex binary.

(2) Approximate the matrix contribution to G_0 due to intersymbol interference by its mean.

This approach assumes that the intersymbol interference after equalization in a fading multipath channel can be well approximated by an equivalent non-fading additive Gaussian noise term. Since most of the bit errors are a result of fades in the desired signal, gross approximations in the interference characteristics do not significantly alter the ABER.

One can reasonably assume that the backward filter cancels the interference contribution due to past symbols; i.e., symbols which arrive before the symbol currently being decided on. If we arbitrarily take the symbol selected for decision as s_0 , the remaining interference is classified as future and is given by

$$\sqrt{\bar{E}_b} \sum_{i=1}^I s_i \mathbf{w}' \mathbf{q}_i \equiv \mathbf{w}' \mathbf{v}_1. \quad (10)$$

The sum can be truncated after a few terms, as the dot product will disappear for large i due to the finite duration of the pulse function $q(t)$. The first approximation is to take s_i as a zero mean complex Gaussian with mean magnitude squared value γ^2 . This quantity should be less than two, as approximating a unit magnitude binary variable by a unit variance Gaussian variable will certainly lead to pessimistic performance results due to the tails of the Gaussian distribution. A choice of $\gamma^2 = 1/2$ was found to provide excellent agreement between calculated and measured values.

Equation (8) can be rewritten as

$$z = \mathbf{w}' (\mathbf{q}_0 \sqrt{\bar{E}_b} s_0 + \mathbf{v} + \mathbf{v}_1) \quad (11)$$

and since the additive noise contribution from $\mathbf{v}$ is independent of the source digit contribution from $\mathbf{v}_1$, the effective noise power for the decision on the real part of z is

$$\begin{aligned} \delta^2 &= \langle [\text{Re}(\mathbf{w}'(\mathbf{v} + \mathbf{v}_1))]^2 \rangle \\ \delta^2 &= \frac{N_0}{2} \mathbf{w}' G_0 \mathbf{w} + \frac{1}{2} \mathbf{w}' \langle \mathbf{v}_1 \mathbf{v}_1' \rangle \mathbf{w} \equiv \frac{N_0}{2} \mathbf{w}' G \mathbf{w}. \end{aligned} \quad (12)$$

Performing the indicated averaging with respect to the source digits, one finds from the definition of $\mathbf{v}_1$ in (10) the covariance matrix

$$\langle \mathbf{v}_1 \mathbf{v}_1' \rangle = \bar{E}_b \gamma^2 \sum_{i=1}^I \mathbf{q}_i \mathbf{q}_i'. \quad (13)$$

The signal-to-noise ratio (SNR) affecting the decision on the real part of z is then

$$\rho \equiv \frac{\bar{E}_b}{2\delta^2} [\text{Re}(\mathbf{w}' \mathbf{q}_0)]^2 \quad (14)$$

which we seek to maximize as a function of the forward filter weight vector, $\mathbf{w}$. Consider the generalized dot product defined on the positive definite matrix G , i.e.,

$$(\mathbf{u}, \mathbf{v}) \equiv \mathbf{u}' G \mathbf{v}. \quad (15)$$

Equation (14) can be written as

$$\rho = \frac{\bar{E}_b}{N_0} \frac{[\text{Re}(\mathbf{w}, G^{-1} \mathbf{q}_0)]^2}{(\mathbf{w}, \mathbf{w})} \quad (16)$$

and by a generalization of the Schwarz Inequality we obtain

$$\rho \leq \frac{\bar{E}_b}{N_0} \frac{|\langle \mathbf{w}, G^{-1} \mathbf{q}_0 \rangle|^2}{(\mathbf{w}, \mathbf{w})} \leq \frac{\bar{E}_b}{N_0} (G^{-1} \mathbf{q}_0, G^{-1} \mathbf{q}_0) \quad (17)$$

with equality if and only if

$$\mathbf{w}_{\text{opt}} = G^{-1} \mathbf{q}_0. \quad (18)$$

Note (18) results in a pure real term in (16) to satisfy the $\text{Re}(\cdot)$ requirement. The maximum SNR is then

$$\rho_{\max} = \frac{\bar{E}_b}{N_0} \mathbf{q}_0' G^{-1} \mathbf{q}_0. \quad (19)$$

As a check on the normalization, if $\sigma = 0$ and $t_0 = 0$ we have from Eq. (7)

$$q(-k\tau) = g(-k\tau) = G_{0k} \quad (20)$$

and then

$$\rho_{\max} = \frac{\bar{E}_b}{N_0} g(0) = \frac{\bar{E}_b}{N_0} \quad (21)$$

as is expected in the absence of multipath.

We will now establish that the mean square error function

$$\langle \epsilon^2 \rangle = \frac{1}{2} \langle |z - ds|^2 \rangle \quad (22)$$

which is minimized by the decision-feedback equalizer, leads to the same set of forward filter weights as maximization of the SNR in (16) for the appropriate choice of scale factor d . After squaring and averaging, equation (22) becomes

$$\begin{aligned} \langle \epsilon^2 \rangle &= \bar{E}_b \mathbf{w}' \mathbf{q}_0 \mathbf{q}_0' \mathbf{w} - 2\sqrt{\bar{E}_b} d \operatorname{Re}(\mathbf{w}' \mathbf{q}_0) \\ &\quad + \frac{1}{2} N_0 \mathbf{w}' G \mathbf{w} + d^2 \\ &= |\sqrt{\bar{E}_b} \mathbf{w}' \mathbf{q}_0 - d|^2 + N_0 \mathbf{w}' G \mathbf{w}. \end{aligned} \quad (23)$$

Minimization of (23) is equivalent to the Lagrange multiplier problem

$$\min_{\mathbf{w}} [\mathbf{w}' G \mathbf{w} - 2\beta \mathbf{w}' \mathbf{q}_0] \quad (24)$$

which has solution

$$\mathbf{w} = \beta G^{-1} \mathbf{q}_0. \quad (25)$$

Since Eq. (18) and (25) are the same except for an unimportant scale factor, we have established that the maximum signal-to-noise ratio for a minimum mean-square error DFE and a particular channel realization is the quadratic form given by Eq. (19). The matrix G is defined from (12) and (13) as

$$G = G_0 + \frac{\bar{E}_b}{N_0} \gamma^2 \sum_{i=1}^I \mathbf{q}_i \mathbf{q}_i' \quad (26)$$

where G_0 is a function of the transmitter impulse response and is a deterministic matrix. The second term in (26) is a random matrix with statistics determined by the channel ensemble. Using G in this form for calculation of the ABER requires an average over a function of a quadratic form which has both random vectors and random matrix. The second approximation used to compute this average is to replace the random interference term by its mean, i.e.,

$$G \doteq G_0 + \frac{\bar{E}_b}{N_0} \gamma^2 \sum_{i=1}^I \overline{\mathbf{q}_i \mathbf{q}_i'}. \quad (27)$$

The matrix G is now a deterministic function of the transmitter and receiver filter characteristics and the channel statistics. The vector $\mathbf{q}_0$ is a random vector associated with the fading channel ensemble. The average bit error rate can now be computed by taking the average over the channel ensemble. The bit error probability for QPSK and a particular channel realization is from (11) and (19)

$$p_c = \frac{1}{2} \operatorname{pr} \{ \xi > 1 \} + \frac{1}{2} \operatorname{pr} \{ \xi < -1 \} \quad (28)$$

where ξ is a zero-mean Gaussian random variable with standard deviation equal to $\sqrt{1/2\rho_{\max}}$. The bit error probability is

$$\begin{aligned} p_c &= (2\pi)^{-1/2} \int_{2\rho_{\max}}^{\infty} \exp(-u^2/2) du \\ &= \frac{1}{2} \operatorname{erfc} \sqrt{\rho_{\max}}. \end{aligned} \quad (29)$$

The subscript c denotes that the detection process was coherent; i.e., the receiver knows the transmitted carrier phase. Differential detection of phase-shift-keyed signals yields a bit error probability [8]

$$p_d = \frac{1}{2} \exp(-\rho_{\max}). \quad (30)$$

The tap gains of an adaptive MMSE equalizer remove the phase and frequency difference between the transmitter and receiver carrier clocks to an ambiguity of 180° for binary transmission and 90° for quaternary transmission. This ambiguity is eliminated by differentially encoding the transmitted data. The performance of coherently detected and differentially encoded data signals for a fixed $\rho_{\max}$ is approximately double the error rate given by (29). When practical degradations are included in the analysis, the performance is closer to the differential detection expression (30). Thus the ensemble average of p_c is of interest as a fading channel performance bound and the ensemble average of p_d is of interest as an estimate of realizable modem performance.

Let $y(x)$ be the probability density function for $\rho_{\max}$ and $Y(s)$ its Laplace transform. We want to calculate the ensemble averages

$$\bar{p}_c = \int_0^\infty \frac{1}{2} \operatorname{erfc} \sqrt{x} y(x) dx \quad (31)$$

$$\bar{p}_d = \int_0^\infty \frac{1}{2} \exp(-x) y(x) dx = \frac{1}{2} Y(1). \quad (32)$$

The Laplace transform $Y(s)$ is easily obtained after a diagonalization of the quadratic form in Eq. (19).

$$\mathbf{x} = \frac{\bar{E}_b}{N_0} \mathbf{q}_0' G^{-1} \mathbf{q}_0 = \frac{\bar{E}_b}{N_0} = \alpha' \alpha \quad (33)$$

where α is a zero-mean complex Gaussian vector with a diagonal covariance matrix $\Gamma_{ij} = \lambda_i \delta_{ij}$. The diagonalization resulted from the transformation

$$\alpha = MG^{-1/2}q_0 \quad (34)$$

and M is the normalized orthogonal matrix for the matrix

$$G^{-1/2}q_0q_0'G^{-1/2} \quad (35)$$

i.e.,

$$G^{-1/2}q_0q_0'G^{-1/2}M = M\Gamma \quad (36)$$

$$M'M = I. \quad (37)$$

Thus the eigenvalues λ_i are also the eigenvalues of the symmetric matrix $G^{-1/2}q_0q_0'G^{-1/2}$ and the unsymmetric matrix $G^{-1}q_0q_0'$. Since the components of α are uncorrelated Gaussian, the probability density for $\rho_{\max}$ is the convolution of exponential densities which yields a Laplace transform in product form.

$$Y(s) = \prod_{k=-K_1}^{K_2} \left(1 + \frac{\bar{E}_b}{N_0} \lambda_k s\right)^{-1} \quad (38)$$

If there are D explicit diversity channels with identical statistics defined by (2), the Laplace transform for the SNR variate becomes

$$Y(s) = \prod_{k=-K_1}^{K_2} \left(1 + \frac{\bar{E}_b}{N_0} \lambda_k s\right)^{-D} \quad (39)$$

The error probability for DPSK is

$$\bar{p}_d = \frac{1}{2} Y(1) = \frac{1}{2} \prod_{k=-K_1}^{K_2} \left(1 + \frac{\bar{E}_b}{N_0} \lambda_k\right)^{-D} \quad (40)$$

For coherent detection it is necessary to find the coefficients A_{ik} in a partial fraction expansion of $Y(s)$, i.e.,

$$Y(s) = \sum_{i=1}^D \sum_{k=-K_1}^{K_2} A_{ik} \left(1 + \frac{\bar{E}_b}{N_0} \lambda_k s\right)^{-i} \quad (41)$$

A recursive method for finding the coefficients and the resulting error probability for coherent detection is detailed in [6] where the ABER for the infinite length optimum single pulse receiver was determined. The result is

$$\bar{p}_c = \sum_{i=1}^D \sum_{k=-K_1}^{K_2} A_{ik} p_i(\lambda_k) \quad (42)$$

where $p_i(\lambda_k)$ can be calculated in closed form as

$$p_i(\lambda_k) = \sqrt{\lambda_k} / 2\pi \sum_{m=0}^{i-1} G(m+0.5) / (1 + \lambda_k)^{m+0.5} m! \quad (43)$$

where $G(\cdot)$ is the gamma function.

The above form is computationally inconvenient when $p_i \ll 1$, but taking advantage of knowledge that

$$\lim_{i \rightarrow \infty} p_i(\lambda_k) = 0 \quad (44)$$

leads to a rapidly convergent series for $\lambda_k > 1$, i.e.,

$$p_i(\lambda_k) = \sqrt{\lambda_k} / 2\pi \sum_{m=k}^{\infty} G(m+0.5) / (1 + \lambda_k)^{m+0.5} m! \quad (45)$$

The lower bound for coherent detection calculated here differs from the infinite length receiver lower bound in [6] by the eigenvalue distribution. Clearly the infinite length receiver will be superior to any finite realization of an equalizer.

For a DPSK system or for the use of the DPSK expression to approximate the Gaussian noise nonfading performance of a practical modem, calculation of the average error probability does not require the determination of the eigenvalues. Note that (40) has the determinant form

$$\bar{p}_d = \frac{1}{2} \left(\det \left| I + \frac{\bar{E}_b}{N_0} G^{-1} C \right| \right)^{-D} \quad (46)$$

where the covariance matrix C is a function of the transmitter filter multipath profile, and the sampling time t_0 .

$$\begin{aligned} C_{kl} &= C_{kl}(t_0) = \overline{q(t_0 - k\tau_s)q^*(t_0 - l\tau_s)} \\ &= \int_{-\infty}^{\infty} g(t_0 - k\tau_s - u)g(t_0 - l\tau_s - u)p(u) du. \end{aligned} \quad (47)$$

The matrix G as defined in (27) has the form

$$G_{kl} = g(k\tau_s - l\tau_s) + \frac{\bar{E}_b}{N_0} \gamma^2 \sum_{i=1}^I C_{kl}(t_0 - iT). \quad (48)$$

The matrices (47) and (48) and the signal-to-noise ratio completely determine the ABER. For a coherent detection system the eigenvalues of $G^{-1}C$ can be found by a matrix iteration procedure [9] for unsymmetrical matrices. The eigenvalues represent the normalized mean power for each independent implicit diversity branch of which there are $K = K_1 + K_2 + 1$ branches. The sum of the eigenvalues is overbounded by unity. This is most easily shown through the physical consideration that the presence of multipath does not increase the mean received energy per bit. We have

$$\rho_{\max} = \frac{\bar{E}_b}{N_0} q_0' G^{-1} q_0 \quad (49)$$

and its ensemble mean

$$\begin{aligned} \bar{\rho}_{\max} &= \frac{E_b}{N_0} \overline{q_0' G^{-1} q_0} = \frac{E_b}{N_0} \text{Trace} \{G^{-1} \overline{q_0 q_0'}\} \\ &= \frac{\bar{E}_b}{N_0} \text{Trace} \{G^{-1} C\} = \frac{\bar{E}_b}{N_0} \sum_k \lambda_k. \end{aligned} \quad (50)$$

Since $\bar{\rho}_{\max} \leq \bar{E}_b / N_0$ from physical considerations, it follows that the eigenvalue sum is overbounded by unity. Thus one criterion in choosing the forward filter tap spacing and number of taps is that the eigenvalue sum be close to unity for the more extreme multipath situations.

IV. SIMULATOR TEST RESULTS AND CALCULATED PERFORMANCE

The DFE modem designed and developed in the MDTs program can accommodate up to four diversity channels with a forward filter structure of three taps per diversity channel. The taps are spaced at one-half the symbol interval. The modulation format used is QPSK with rectangular pulses filtered to a 99% bandwidth of 15 MHz at the highest data rate of 12.6 Mbit/s. The modem receiver uses coherent detection with a differential encoding format to resolve a 90° ambiguity. These parameters define the performance characteristic of the modem. A more detailed description of the physical and electrical characteristics of the MDTs modem is given in [7].

In this section, fading channel performance of the MDTs modem using a troposcatter channel simulator [10] is summarized and compared to the performance calculations derived from the analytic model developed in Section III. The troposcatter simulator consisted of a tapped delay line for each diversity with noise driven tap multipliers to represent the channel. Multipath widths of up to 10^{-6} s and rms fade rates from 0.1 to 10 Hz are within the simulator range. The simulator tap spacing is 66 ns. These measured and calculated results provide a baseline for evaluation of the performance results from live link testing during the early months of 1976. A description of the live link tests, a summary of the performance results and a comparison with calculated performance are included in the sequel [11] to this paper.

The performance of a single channel model receiver on a white Gaussian noise nonfading channel is shown in Figure 2. The degradation from ideal coherent detection is about 2 dB which is primarily due to the effect of differential encoding (approximately a factor of 2 in error rate), the adaptation self noise, quantization of the backward filter weights, and spectrum control. The expression for differential detection of phase-shift keyed signals (30), is seen to provide a convenient lower bound for the actual nonfading modem performance. Since most of the contribution to the average error probability occurs at instantaneous values of E_b/N_0 corresponding to the 10^{-1} to 10^{-2} region, it follows that the bound and measured fading performance would be close. As outlined in the previous section, use of the exponential performance characteristic has the advantage that the average error probability can be determined by calculating a determinant of rank equal to the number of forward filter taps rather than calculating the eigenvalues directly.

The fading performance of the DFE modem was measured at data rates of 1.5, 3.1, 6.3, 9.4, and 12.6 Mbit/s over a complete range of multipath profiles and channel fade rates. Performance characteristics can be summarized in terms of the ratio of twice the rms multipath spread σ to the QPSK symbol interval T . For a fixed $2\sigma/T$ and $\bar{E}_b/N_0$, performance was essentially independent of data rate. Performance for dual and quadruple diversity operation is also unaffected by choice of the rms Doppler spread* up to the maximum rate of 10 Hz.

The performance results calculated from the model devel-

*The rms Doppler spread is the standard deviation of the Doppler power spectrum. In the simulator it is established by the 3 dB bandwidth of a two-pole Butterworth filter used to filter the input white noise driving the tap gains.

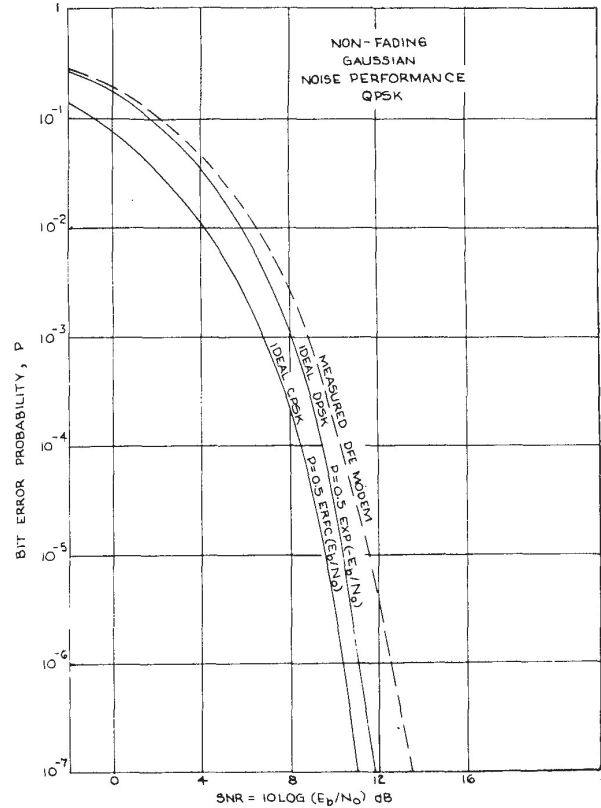


Fig. 2. Non-Fading Performance.

oped in Section III are presented in Fig. 3 for selected values of $2\sigma/T$. Two future bits of intersymbol interference were used in the calculation. Additional bits did not change the performance results for the range of multipath considered. A summary of the measured MDTs modem performance at all the data rates is given in Figures 4 through 8. The values of $2\sigma/T$ selected for the performance calculation agree with the 12.6 Mbit/s values in Fig. 4. The calculated and measured values compare well at all data rates except for extreme multipath conditions under dual diversity operation. As an example, Table 1 shows the calculated and measured $\bar{E}_b/N_0$ to achieve a 10^{-6} average error probability.

Except for the $2\sigma/T = 1.2$ /dual diversity example where extreme multipath conditions limited performance, the calculations are accurate to within 2 dB for dual diversity and they are accurate to within 1 dB for all $2\sigma/T$ values for quad diversity.

These performance curves show that the "optimum" value of $2\sigma/T$ is larger for larger diversity order as diversity helps reduce intersymbol interference. The performance improvement for small to moderate amounts of multipath spread is due to the use of the multipath as additional redundant channels to increase the diversity gain. The improvement relative to the $2\sigma/T = 0$ curve for no multipath is the implicit diversity gain. For larger values of multipath spread, the equalizer cannot use all the additional diversity paths offered by the multipath because of the multipath induced intersymbol interference. The penalty for removal of the intersymbol interference becomes significant beyond $2\sigma/T$ of 0.5, and thereafter sharply limits the implicit diversity gain. For a sufficiently

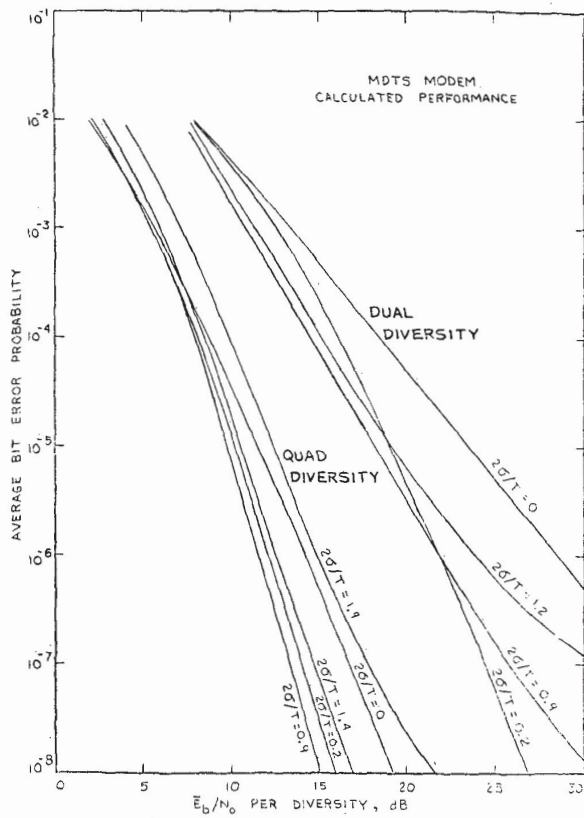


Fig. 3. Calculated Performance Results.

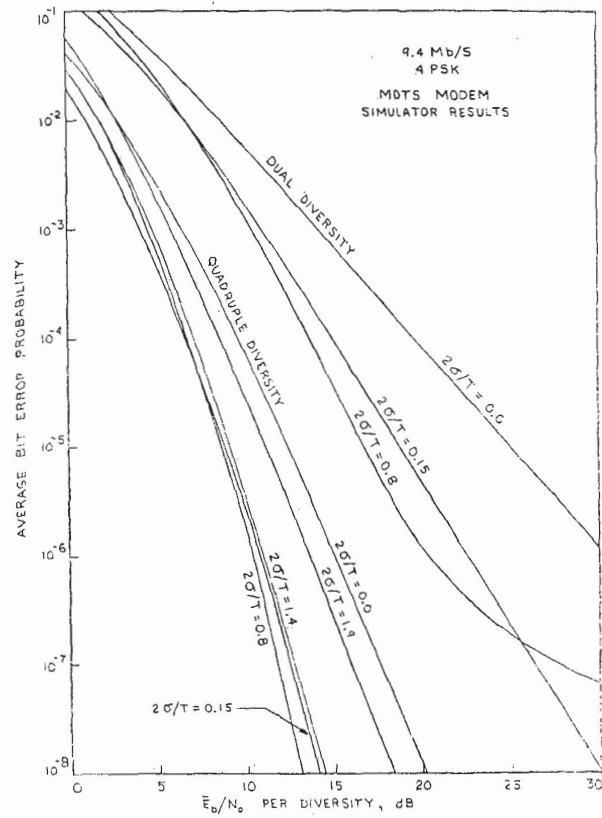


Fig. 5. Simulator Test Results, 9.4 Mbit/s.

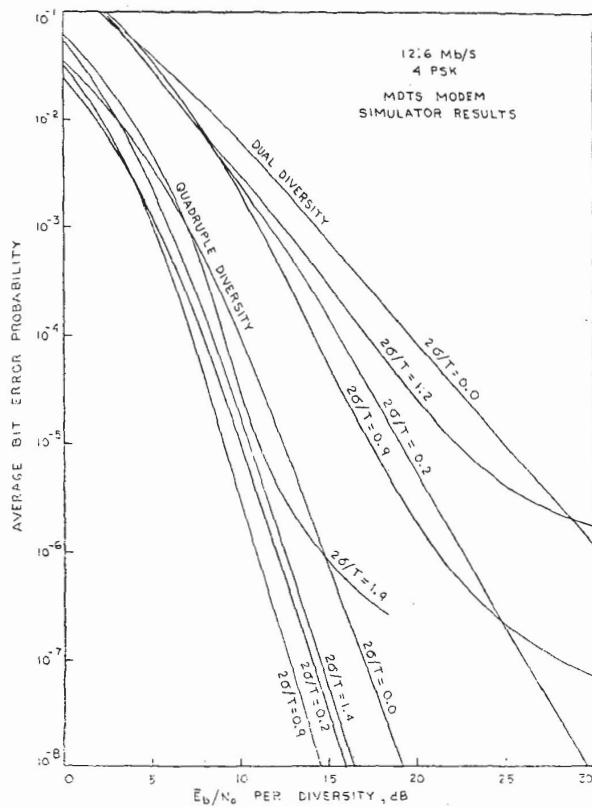


Fig. 4. Simulator Test Results, 12.6 Mbit/s.

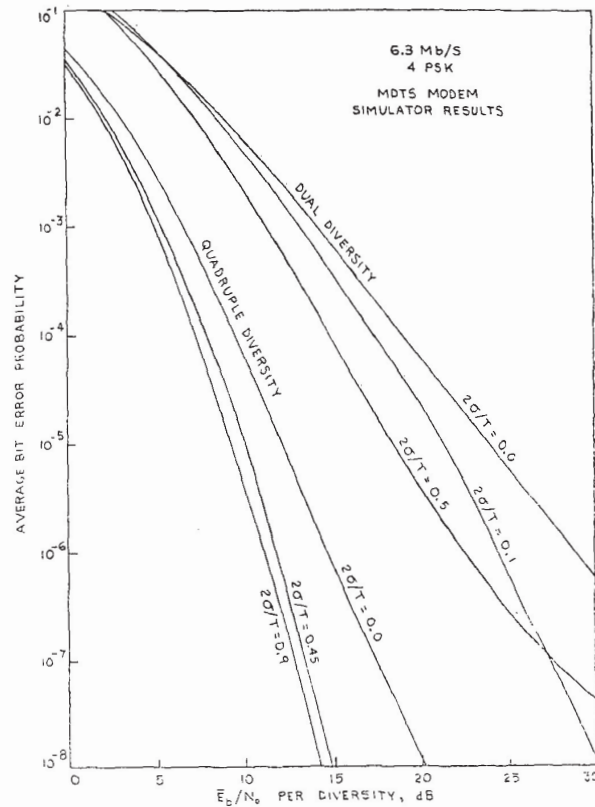


Fig. 6. Simulator Test Results, 6.3 Mbit/s.

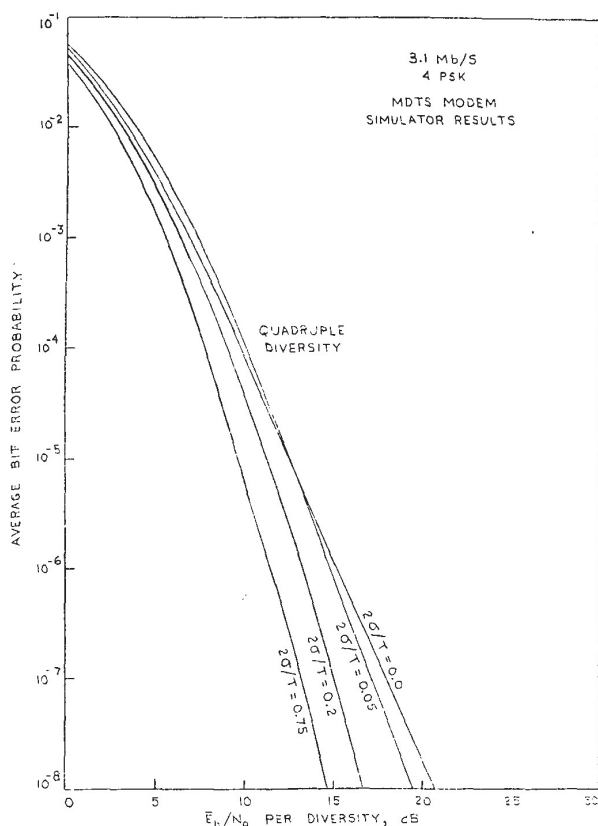


Fig. 7. Simulator Test Results, 3.1 Mbit/s.

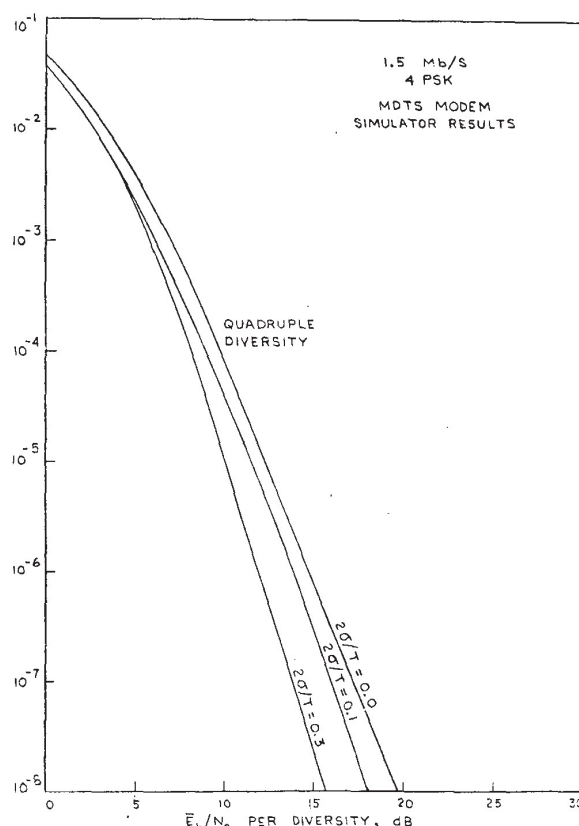


Fig. 8. Simulator Test Results, 1.5 Mbit/s.

TABLE 1
COMPARISON OF CALCULATED AND MEASURED
PERFORMANCE AT 12.6 MBIT/S

Diversity Order	$2\sigma/T$	Required for $\bar{P}=10^{-6}$	
		Measured $\bar{E}_b/N_0$ dB	Calc. $\bar{E}_b/N_0$ dB
2	0	30.0	28.5
2	0.2	22.5	22.0
2	0.9	21.0	21.7
2	1.2	>30	24.0
4	0	14.5	14.2
4	0.2	12.0	12.0
4	0.9	10.7	11.5
4	1.4	12.5	12.5
4	1.9	14.5	15.0

large multipath spread, an irreducible ABER will result as the equalizer cannot keep the data eye open under all possible multipath conditions. The use of more taps in the equalizer would, of course, increase the multipath handling capability. For data rates up to 6.3 Mbit/s and for most military troposcatter systems, the ratio $2\sigma/T$ will be less than 2.0. Operation at the highest data rate of 12.6 Mbit/s is expected to be limited to 150 to 200 mi troposcatter links in order to maintain a $2\sigma/T$ of less than 2.0.

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The development of a troposcatter modem capable of operation at rates eight times faster than heretofore achievable is an accomplishment requiring contributions from too many

people to mention all here. The author would like to thank and extend congratulations to all those individuals who participated in this effort. In particular, the unrelenting efforts of Mr. D. Kern, who directed the Sylvania design team and was responsible for the collection of the performance data presented here, should not go unmentioned. The contributions of Mr. P. Mahoney, who directed the SIGNATRON design team, to the area of high speed equalizer technology also deserves special note.

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Improvement of Transmission Characteristics of Superconductive Coaxial Cable: Spectral Analysis of Dimensional Irregularity of Raw Materials of Coaxial Cable

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Abstract—This paper deals with the improvement of the transmission characteristics of superconductive coaxial line (SCCX), and presents a quantitative relationship between the Fourier component of the dimensional irregularities in the raw material of SCCX, such as the thickness of the copper tape and the diameter of the insulator, and an abnormal increase of the transmission loss due to the periodic impedance irregularities. This paper also shows that even for the coaxial cable with very small attenuation constant (≈ 0.5 dB/km) and high operating frequency (up to several gigahertz), an approximate formula for loss peak due to the periodic impedance irregularities holds for the production length of cable. This is indicated by the comparison of an approximate formula with the result of digital computer simulation.

I. INTRODUCTION

THERE have been many attempts to apply superconductivity in communication technology since the use of superconductive coaxial cable for instrumentation in nuclear testing was first proposed in the United States in 1968.¹ And some theoretical and experimental investigations have also been made to explore the use of SCCX as an information transmitting medium.²⁻⁶

Some of the advantages of SCCX are as follows:

- a) high operating frequency and wide band (several gigahertz)
- b) small physical size
- c) low loss.

All these things, however, make it very difficult to produce SCCX with small impedance irregularities. Due to the cumulative effect of small periodic impedance irregularities, the frequency/attenuation curve exhibits sharp peaks at a series of harmonically related frequencies which cause signal distortion.⁹

These peaks have to be diminished and if possible removed. There are two factors which cause impedance irregularities:

- a) imperfections in the manufacturing processes
- b) periodic distortion of cable geometry due to the dimensional irregularities of the raw material.

By the dimensional irregularities of the raw material we mean the diameter deviations of the inner and outer conductors and the insulator. The diameter deviations of the outer conductor are caused by the thickness deviations of the copper tape used as the outer conductor. It is very difficult to distinguish attenuation peaks due to the first factor from those due to the second. In other words, it seems to be impossible to know the cause of these peaks exactly and to take the countermeasure that would remove these peaks. We are now in a position to solve the quantitative relationship between the attenuation dis-

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