

2. *Robichaux*

The Board states, “. . . Petitioner asserts that Robichaux and Scherer render obvious claim 1’s ‘high pass filter which receives as an input the difference signal in which low frequency components are absent.’” (Paper 12 at 32.) The Board further states, “Petitioner contends that a person of ordinary skill in the art would have had a reasonable expectation of success in implementing a combination in which ‘Robichaux’s torque signal TQ_ERR would be transmitted to Scherer’s transient control branch instead of Robichaux’s PID controller 220, and Scherer’s high-pass filter 4 would then output a filtered difference signal with low-frequency components removed.’” (*Id.* at 33.) The Board further states, “Patent Owner concludes that ‘[c]ombining these two prior art references, even in hindsight, would result in a desired engine torque signal which is high-pass filtered, and a ‘difference signal’ which is not high-pass filtered.’” (*Id.* at 35.) The Board yet further states that “. . . Patent Owner reiterates that neither Robichaux nor Scherer teaches high pass filtering a difference signal. . . . Patent Owner argues that if a person of ordinary skill in the art had considered combining the references’ teachings, ‘the [person of ordinary skill in the art] would have been taught to ‘high-pass filter’ an input to a comparator that outputs a ‘difference signal,’ rather than filtering the ‘difference signal.’” (*Id.* at 36.) The Board ultimately asserts that “[a]t this stage, Petitioner has shown sufficiently that a person of ordinary skill in the art would have had a reason

to add Scherer's transient control branch with its high-pass filter 4 to Robichaux's system." (*Id.* at 37.)

B. *Scherer and Robichaux In Combination Do Not Render Any of the Challenged Claims of the '565 Patent Obvious*

As an initial matter, Patent Owner strongly disagrees that "Patent Owner does not dispute Petitioner's contention that Robichaux indicates a need for transient torque control in lieu of PID controller 220 during transient conditions." (*Id.* at 37.) (Ex. 2004 at ¶ 32.) *Robichaux's* alleged indication of a "need for transient torque control in lieu of PID controller 220 during transient conditions" stems from the following single statement in *Robichaux*: "Robichaux's proportional-integral-derivative (PID) controller 220 processes the torque error signal during steady-state conditions. . . . Robichaux's PID controller 'is not active during certain transient torque limiting control modes.'" (Paper 12 at 33.) Patent Owner disagrees that a statement that the PID controller "is not active during certain transient torque limiting control modes" constitutes a statement of "a need for transient torque control in lieu of PID controller 220 during transient conditions." (Ex. 2004 at ¶ 32.)

To begin with, *Robichaux's* PID controller is only not active during "certain" transient torque limiting control modes, which implies that it is active during other than those "certain transient torque limiting control modes." (*Id.*) Second, a "transient torque limiting control mode" is a mode in which transient torque is

purposely limited as part and parcel of the basic design intent of *Robichaux*. (*Id.* at ¶ 33.) Rather than indicating “a need for transient torque control in lieu of PID controller 220 during transient conditions,” *Robichaux* discourages a POSA from and teaches away from doing so because *Robichaux* purposely causes the PID controller 220 to be not active during “*certain transient torque limiting control modes.*” (*Id.*) To extrapolate from that isolated teaching of *Robichaux* that rather than purposely not activate the PID controller during “certain transient torque limiting control modes” that *Robichaux* indicates “a need for transient torque control in lieu of PID controller 220 during transient conditions,” constitutes clear error. (*Id.* at ¶ 34.)

Additionally, Patent Owner disagrees that it only attacked *Robichaux* and *Scherer* individually, and did not fully consider the “combination of *Robichaux* and *Scherer*.” ***Scherer discourages and directly teaches away from using a high-pass filter in embodiments in which a comparator is used to produce a “difference signal” by subtracting an “estimated torque signal” from a “desired torque signal.”*** (*Id.* at ¶¶ 40-42; 62-65.) The Board and the Petitioner have “overlooked” or selectively ignored the *Scherer*’s explicit teaching of eliminating the high-pass filter altogether in embodiments in which a comparator is used to produce a “difference signal” by subtracting an “estimated torque signal” from a “desired torque signal.” (*Id.*) Certainly, a person of ordinary skill in the art would have been

taught by *Scherer* to high-pass filter the “desired torque signal” in embodiments in which a comparator is not used, and to eliminate the high-pass filter in embodiments in which the comparator is used. (Ex. 2004, 40-42; 62-65.)

Any attempt to reconstruct the claimed invention of the ’565 patent by combining the teachings of *Scherer* and *Robichaux* or any of the other prior art references relied upon by Petitioner (much less then modifying such combined teachings to supply a critical, patentability-imparting claim limitation that goes to the heart of the ’565 invention that is lacking in all references) constitutes clearly impermissible hindsight and is contrary to the fairly considered teachings of these prior art references. In fact, as has been clearly shown hereinabove, such attempts are based upon using the challenged claims as a blueprint or roadmap to pick and choose isolated teachings of these prior art references that support a particular thesis while ignoring and selectively discarding other teachings of these prior art references that undermine that particular thesis, which clearly constitutes the exercise of legally impermissible hindsight.

Both *Robichaux* and *Scherer* disclose an embodiment using a comparator to compare an estimated torque signal and a desired torque signal and to output a difference signal based upon this comparison. (Ex. 2004 at ¶ 63.) *Scherer* specifically eliminates the high-pass filter in this embodiment and clearly does not high-pass filter this “difference signal,” and *Robichaux* also does not disclose high-

pass filtering the difference signal output by the comparator. (*Id.* at ¶ 64.) The fact that none of the prior art references relied upon by Petitioner (and the Board) in two separate *inter partes* review trials regarding this same ('565) patent, despite at least two of these references disclosing a comparator that outputs a “difference signal” based upon a comparison of an estimated torque signal and a desired torque signal but not disclosing that this “difference signal” is high-pass filtered, is **a strong secondary indicium of non-obviousness** of the claimed invention as set forth in the challenged claims of the '565 patent.

Dr. McCarley, Petitioner’s expert, testified during his deposition in this matter as follows:

Q . . . What was the **starting point** for the creation of this declaration? How did you begin your work in any way, shape or form?

MR. CONKLIN: Objection, form.

A The complete process ***began with numerous discussions between myself and the attorneys for Sterne Kessler formulating – first of all understanding the issues, formulating a declaration plan, identifying potential prior art references, both patents and in common publications. Some written correspondence fleshing out what eventually became parts of the declaration***, usually my input to them, technical advice. And then the drafting of the declaration which began just a standard framework provided by the attorneys with the usual boilerplate [] in place for an IPR.

...

Q . . . And then conversely did they have any input on the technical sections?

A Yes.

Q How so?

A In our preliminary discussions, keeping in mind that both attorneys [VW's attorneys at Sterne Kessler] do have technical backgrounds, ***we [Dr. McCarley and VW's attorneys] sorted out a number of the interpretations of both '565 and the selected prior art patents collaboratively***, checking each other's understanding and interpretation of the words, which I'm sure you know can be sometimes a little bit obtuse in patent language. But the technical content underlying all of the arguments originates from either me or just common sense engineering knowledge.

Q Let me talk about the step of ***identifying the prior art***. Could you tell me how that took place?

A ***The staff at Sterne Kessler, who have expertise in searching and can do so more efficiently than I can, identified a number of patents, some of which I selected for use in this declaration with the approval of the attorneys.*** Is that adequate?

(Ex. 2003 at 10:6-14:4.)

Dr. McCarley further testified at his deposition as follows:

Q ... I want to try to understand to the extent you recall which of the references that are listed as exhibits in this IPR, which of those were the product of the Sterne Kessler search versus your own searching or gathering of prior [art references]? So do you have any recollection of which came from Sterne Kessler and which came from your own searching?

MR. CONKLIN: Objection, form.

A I believe that ***the four prior art patents cited in this declaration came from the Sterne Kessler research staff***. Not all at the same time. In particular the latter patent [later identified by Dr. McCarley as Robichaux (McCarley, 19:12-13)] was most definitely related to technical feedback that I provided when I was studying the first options.

...

Q ... ***do you know the identities of the folks at Sterne Kessler who did the prior art searching?***

A No. I assume that the attorneys that I worked with had something to do with it, but I know that they have a staff that is very skilled at this, and I have never inquired as to their names.

(*Id.* at 17:16-18:20.)

Dr. McCarley further testified at his deposition as follows:

Q ... And did you produce a draft from that framework?

A Yes.

Q And what happened after that with that draft?

A There was ***quite a bit of back and forth through many partially developed versions***. It was incremental. So this was the natural process of collaboratively creating a final document. But once again, all of the technical opinions in this document are either from me, strongly supported by me or simply common engineering knowledge.

Q When you say that one option of strongly supported by you, I take it that means it might have been a suggestion that came from the – one of the attorneys but you nevertheless supported it and then adopted it and put it in your report. Am I understanding you right?

A That would be fair.

(*Id.* at 21:21-22:17.)

Based upon the above and other deposition testimony of Dr. McCarley, it is plain to see that what the Petitioner's attorneys and expert have done, broadly speaking, is to collaboratively prepare expert declarations and copy and paste therefrom into their briefs in this proceeding, and without the expert even starting the obviousness analysis from the proper starting point, *i.e.*, stepping into the shoes of the hypothetical POSA working at the time of the invention and devising the prior art search strategy that the POSA would have devised without the benefit of the disclosure and claims of the subject patent.

Further, based upon the above and other deposition testimony of Dr. McCarley, and as will become more fully apparent hereinafter, it is plain to see that the Petitioner's expert in this case did not use the proper obviousness analytical

framework, and, as such, the Expert Declarations submitted by Petitioner in this proceeding are fundamentally flawed and lack probative value.

The case law and other authorities cited herein dictate that the *Graham v. John Deere* requirement of determining the scope and content of the prior art requires that the starting point of an obviousness analysis is a careful consideration of what prior art references a hypothetical person of ordinary skill in the art at the time the invention was made, without the benefit of the disclosure or claims of the subject patent, tasked with solving the problem and/or meeting the objectives that the inventor had in mind at the time of the invention, would have selected and combined during the normal course of research and development.

The scope and content of the prior art that is relevant to the obviousness analysis should be limited to the scope and content of the prior art that the hypothetical person of ordinary skill in the art attempting to solve the problem and/or meet the objectives that the inventor had in mind at the time of the invention would have found during the normal course of research and development. In cases in which the retained expert is handed or spoon fed the prior art references to consider by the petitioner's attorneys, such as the instant case, it becomes impossible for the Patent Owner's attorney to meaningfully and properly cross-examine the petitioner's expert about how he/she formulated the prior art search and what criteria he/she used in devising the search strategy and in ultimately selecting relevant prior art references,

and then, combining the teachings of those relevant prior art references that are selected and considered.

The overall approach and methodology of a law firm supplying the prior art references to a retained expert is fundamentally flawed and inherently unreliable, since the expert in this instance necessarily has an improper starting point. The proper starting point is for the expert to “***step back in time and into the shoes of the hypothetical person of ordinary skill in the art “when the invention was unknown and just before it was made”***” (MPEP §2142.7) and determine what prior art the POSA would have searched for and found in the normal course of research and development, assuming the POSA was attempting to solve the problem and/or meet the objectives the inventor had in mind at the time the invention was made. To reach a conclusion of obviousness requires more than a mere showing that the prior art includes separate references covering each separate limitation in a claim under examination. *Unigene Labs., Inc. v. Apotex, Inc.*, 655 F.3d 1352, 1360 (Fed. Cir. 2011). Rather, *obviousness requires the additional showing that a person of ordinary skill in the art at the time of the invention would have selected and combined those prior art elements in the normal course of research and development to yield the claimed invention. Id.*

“The ‘as a whole’ instruction in title 35 prevents evaluation of the invention part by part. . . . This form of hindsight reasoning, using the invention as a roadmap to find its prior art components, would discount the value of

combining various existing features or principles in a new way to achieve a new result - often the very definition of invention.”

Princeton Biochemicals, Inc. v. Coulter, Inc., 411 F.3d 1332, 1337 (Fed. Cir. 2005); *see also Allergan, Inc. v. Apotex, Inc.*, 754 F.3d 952 (Fed. Cir. 2014).

Based upon the foregoing, Patent Owner respectfully submits that Dr. McCarley’s expert declaration in this case is fundamentally flawed, inherently unreliable, and, ultimately, lacks probative value and should not be relied upon by the Board in finding that a claim limitation that is admittedly not disclosed in any of the prior art references.

VIII. CONCLUSION

Based upon all the above and foregoing, Patent Owner respectfully submits that all the challenged claims of the ’565 patent are patentable, and respectfully requests an ultimate finding to that effect.

October 29, 2020

Respectfully submitted,

/s/ Timothy Devlin

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CERTIFICATE OF WORD COUNT

Pursuant to 37 C.F.R. § 42.24(d), the undersigned certifies that there are 6,850 words in this paper, excluding any table of contents, table of authorities, mandatory notices under 37 C.F.R. § 42.8, certificate of word count, certificate of service, or appendix of exhibits. This certification relies on the word count of the word-processing system used to prepare this paper.

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CERTIFICATE OF SERVICE

I hereby certify that on October 29, 2020, I caused a true and correct copy of the foregoing materials to be served via the Patent Office electronic filing system, and electronic service via email to the following attorneys of record pursuant to Petitioner’s consent.

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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC
Patent Owner

Case IPR2020-00444
Patent 6,581,565 B2

**PETITIONER'S OBJECTIONS TO PATENT OWNER'S EVIDENCE
UNDER 37 C.F.R. § 42.64(b)(1)**

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U.S. Patent and Trademark Office
P.O. Box 1450
Alexandria, VA 22313-1450

Pursuant to 37 C.F.R. § 42.64(b)(1), Petitioner Volkswagen Group of America, Inc. (“VWGoA”) hereby objects under the Federal Rules of Evidence (“FRE”) to the admissibility of **Exhibit 2004**: Second Declaration of Dr. Russell Leonard, which Patent Owner Michigan Motor Technologies LLC filed with its October 29, 2020 Patent Owner’s Response.

VWGoA timely objects under 37 C.F.R. § 42.64(b)(1) within the allowed five business days of the October 29, 2020 service of the Exhibit 2004. VWGoA files and serves MMT with these objections to provide notice that VWGoA may move to exclude Exhibit 2004 under 37 C.F.R. § 42.64(c).

I. GROUNDS FOR OBJECTIONS

A. Witness’s Lack of Qualifications as an Expert

VWGoA objects to Exhibit 2004 in its entirety as improper expert testimony under FRE 702 and 703. The testimony contained in this exhibit is not provided by a witness who is sufficiently qualified as an expert by knowledge, skill, experience, training, or education in the subject matter at issue.

B. Testimony’s Lack of Reliability

VWGoA objects to Exhibit 2004, for example, paragraphs 21-78, as improper expert testimony under FRE 702 and 703. The testimony is based on insufficient facts or data, is not the product of reliable principles and methods, and

the expert has not reliably applied the appropriate principles and methods to the facts of the case.

II. CONCLUSION

To the extent that MMT fails to correct the defects associated with Exhibit 2004 in view of VWGoA's objections herein, VWGoA may file one or more motions to exclude Exhibit 2004 under 37 C.F.R. § 42.64(c).

Respectfully submitted,

STERNE, KESSLER, GOLDSTEIN & FOX P.L.L.C.

/MICHAEL D. SPECHT/

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Attorney for Petitioner
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CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that on November 5, 2020 the above-captioned **PETITIONER’S OBJECTIONS TO PATENT OWNER’S EVIDENCE UNDER 37 C.F.R. § 42.64(b)(1)** was served in its entirety via electronic mail upon the following counsel of record for the Patent Owner:

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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC
Patent Owner

Case IPR2020-00445
Patent 6,581,565 B2

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UNDER 37 C.F.R. § 42.64(b)(1)**

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II. CONCLUSION

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Respectfully submitted,

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CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that on November 5, 2020 the above-captioned **PETITIONER’S OBJECTIONS TO PATENT OWNER’S EVIDENCE UNDER 37 C.F.R. § 42.64(b)(1)** was served in its entirety via electronic mail upon the following counsel of record for the Patent Owner:

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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES, LLC,
Patent Owner

Case IPR2020-00444
Patent 6,581,565

**PETITIONER VOLKSWAGEN GROUP OF AMERICA, INC.'S
NOTICE OF REMOTE DEPOSITION**

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P.O. Box 1450
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Counsel for the parties have conferred and agreed that Petitioner will conduct cross-examination by deposition of the Patent Owner's witness at the date and time indicated below.

The cross-examination will be conducted remotely using video teleconferencing service(s) and/or telephony service(s) that provide remote access for authorized parties wishing to participate in the deposition via the Internet and/or telephonically.

Date/Time: Tuesday, January 5, 2021, 9:30 a.m. EST

Declarant's Name: Dr. Russell Leonard

Location: Dr. Leonard will be located at his home residence in Grand Rapids, MI. The other parties participating in the deposition, including the examining attorneys, the defending attorneys, and the court reporter, will be participating from geographic locations different than Dr. Leonard.

The cross-examination will address both the instant proceeding and proceeding IPR2020-00445, in which U.S. Patent No. 6,581,565 is also at issue.

The cross-examination will be recorded stenographically by a court reporter authorized by law to administer oaths. It will also be transcribed by real-time reporting software.

Given that strong and continuous Internet connectivity of a remote deposition is crucial, a test may be conducted with all parties prior to the deposition date. If connectivity or other technical issues arise during the test that cannot be corrected before the deposition date, then an alternative deposition date will be scheduled. If connectivity or other technical issues arise during the course of the deposition and are not corrected such that the deposition cannot be completed on the deposition date, an alternative deposition date will be used to complete the deposition.

Respectfully submitted,

STERNE, KESSLER, GOLDSTEIN & FOX P.L.L.C.

/Kyle E. Conklin/

Kyle E. Conklin
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CERTIFICATE OF SERVICE

The undersigned hereby certifies that the foregoing **PETITIONER VOLKSWAGEN GROUP OF AMERICA, INC.'S NOTICE OF REMOTE DEPOSITION** was served electronically via e-mail on December 18, 2020, in its entirety on the following counsel of record for Patent Owner:

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Respectfully submitted,

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**PETITIONER VOLKSWAGEN GROUP OF AMERICA, INC.'S
REPLY TO PATENT OWNER'S RESPONSE**

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PETITIONER’S UPDATED EXHIBIT LIST

Exhibit No.	Description
1001	U.S. Patent No. 6,581,565 B2 to Heslop <i>et al.</i> (“’565 Patent”)
1002	Prosecution File History of U.S. Patent No. 6,581,565 B2 (“’565 history”)
1003	Declaration of C. Arthur MacCarley, Ph.D., P.E.
1004	Curriculum Vitae of C. Arthur MacCarley, Ph.D., P.E.
1005	U.S. Patent No. 6,006,717 A to Suzuki <i>et al.</i> , titled “Direct-Injection Spark-Ignition Type Engine Control Apparatus,” issued December 28, 1999 (“Suzuki”)
1006	U.S. Patent No. 5,241,855 A to Cullen <i>et al.</i> , titled “Method and Apparatus for Inferring Engine Torque,” issued September 7, 1993 (“Cullen”)
1007	U.S. Patent No. 6,07,681 A to Scherer <i>et al.</i> , titled “Process and Apparatus for Adjusting the Torque of an Internal-Combustion Engine,” issued April 11, 2000 (“Scherer”)
1008	U.S. Patent No. 5,282,449 A to Takahashi <i>et al.</i> , titled “Method and System for Engine Control,” issued February 1, 1994 (“Takahashi”)
1009	“Bosch Fuel Injection Systems,” F. Aird, Berkeley Publishing Group, 2001, ISBN 1-55788-365-3 (“Aird”)
1010	U.S. Patent No. 6,209,519 B1 to Melchior <i>et al.</i> , titled “Method and Arrangement for Controlling the Quiet Running of an Internal Combustion Engine,” issued April 3, 2001 (“Melchior”)
1011	U.S. Patent No. 5,692,471 A to Zhang, titled “Method and Arrangement for Controlling a Vehicle,” issued December 2, 1997 (“Zhang”)
1012	“Bosch Automotive Handbook,” Robert Bosch GmbH, 2000, ISBN 0-7680-0669-4 (“Bosch”)
1013	<i>Intentionally Left Blank</i>
1014	John B. Heywood, Internal Combustion Engine Fundamentals, 1st Ed. McGraw-Hill, Inc., 1988. (“Heywood”)
1015	Ronald K. Jorgen, “Automotive Electronics Handbook,” McGraw-

Exhibit No.	Description
	Hill, Inc., ISBN0-07-033189-8, c.1994 (selected portions) (“Jurgen”)
1016	<i>Intentionally Left Blank</i>
1017	<i>Intentionally Left Blank</i>
1018	<i>Intentionally Left Blank</i>
1019	Understanding Automotive Electronics, Fifth Edition (1998), William B. Ribbens, Ph.D. (“Ribbens”)
1020	<i>Intentionally Left Blank</i>
1021	<i>Intentionally Left Blank</i>
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1029	Amended Complaint for Patent Infringement, <i>Michigan Motor Technologies LLC v. Volkswagen AG</i> , Case No. 2:19-cv-10485 (E.D. Mich.), filed April 30, 2019
1030	Transcript of the Deposition of Russell Leonard, Ph.D., January 5, 2021.
1031	Second Declaration of C. Arthur MacCarley, Ph.D., P.E.
1032	Ogata, Katsuhiko. Modern Control Engineering, 4th ed., Prentice Hall, 2002
1033	DAE Notes, RC High-Pass Filter Circuit, https://www.daenotes.com/electronics/digital-electronics/rc-high-pass-filter-circuit

MMT's Patent Owner Response ("POR") does not overcome the strong obviousness showing in Volkswagen Group of America's Petition. Indeed, the POR does little to advance arguments beyond those previously presented in the Patent Owner Preliminary Response ("POPR"), which the Board already correctly rejected in the Institution Decision.

The '565 patent broadly claims controlling engine torque output by fuel and spark adjustments that compensate for the difference between a demand torque and an estimated torque. To compensate for these differences between demand torque and estimated torque, the '565 patent claims the combination of basic components found in a typical engine controller—a demand torque controller, an estimator, a comparator, a high-pass filter, and a transient torque controller—that were all well known, as evidenced by Suzuki, Cullen, and Scherer. Indeed, Patent Owner's declarant, Dr. Leonard, acknowledged that all these recited components of the '565 patent's DISI engine controller (shown in Figure 3) were known in the prior art.

EX1030, 24:4-26:1.

In the POR, similar to the POPR, Patent Owner directs all its arguments to a single element—high-pass filtering a difference signal outputted from a comparator—and focuses on Suzuki and Scherer. However, as the Board correctly determined in the Institution Decision, the Petition provides a “reasoned explanation” for combining Suzuki and Scherer “that would high pass filter a

torque difference signal.” Decision on Institution, 37; Petition 37-38, 53-56; EX1003, ¶¶94-103. And the Board properly gauged that the POPR “does not explain ... a persuasive reason” why the applied references discourage the use of the alleged invention recited in the claims of the ’565 patent. DI, 38. The POR adds no significant new arguments beyond what was asserted—and rejected—in the POPR.

Accordingly, the Board should find all challenged claims unpatentable.

I. Patent Owner’s Arguments Do Not Rebut Petitioner’s Compelling Evidence Showing That the Prior Art Discloses Every Limitation of the Challenged Claims.

Patent Owner admits that the applied art discloses each component of the recited torque control device.¹ *See generally* POR; EX1030, 24:4-26:1, 27:4-28:11, 35:3-41:18. In fact, Patent Owner’s declarant admitted that a device having a torque demand controller generating a torque demand signal and a torque producer receiving the torque demand signal were well known before the ’565 patent. EX1030, 25:6-8, 27:8-11. Patent Owner’s declarant also admitted that using an estimator for outputting an estimated torque signal, a comparator for outputting a

¹ Patent Owner does also not contend that any elements recited in claims 2-6 are absent from the prior art. As the Board correctly found in its Institution Decision, the dependent claims are obvious in view of Petitioner’s evidence. Pet., 61-67, 70-87; DI, 46.

difference signal based on the estimated torque signal and the demand torque signal, and a high-pass filter for outputting a filtered signal to a transient torque controller were well known before the '565 patent. EX1030, 25:9-20, 27:12-28:1. Patent Owner's declarant further admitted that using a transient torque controller for outputting spark and fuel adjustment signals based on high-pass filtered signals was well known. *Id.*, 25:21-26:1, 28:2-11. Patent Owner's only dispute is that the applied art allegedly fails to disclose the recited arrangement of a high-pass filter receiving a difference signal outputted from a comparator. POR, 5; EX1030, 26:2-27:3.

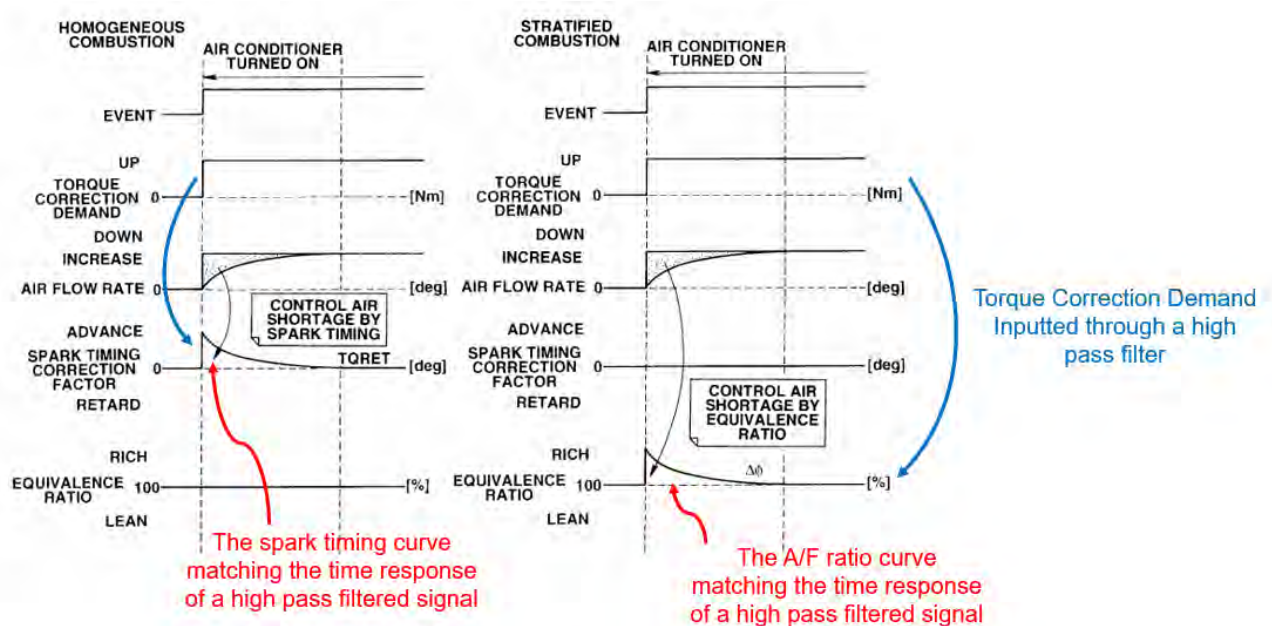
Petitioner's evidence convincingly shows why the combination of Suzuki, Cullen, and Scherer discloses all the limitations of independent claims 1 and 7, including the contested limitation "a high pass filter which receives as an input the difference signal." Pet., 28-61; DI, 36-38; EX1003, ¶¶85-153. And the POR does not rebut this compelling evidence. Instead, the POR (1) ignores Suzuki's implicit high-pass filtering teachings; (2) individually attacks the references and ignores what the Petition's prior art combination suggests as a whole; (3) applies unfounded and legally-flawed approaches for determining obviousness; and (4) presents baseless attacks against the credibility of Petitioner's expert witness Dr. MacCarley. EX1031, ¶¶32-34.

A. Suzuki alone suggests high-pass filtering a difference signal.

Patent Owner does not dispute that Suzuki discloses a comparator outputting a difference signal representing the difference between an estimated torque and a demand torque. *See generally* POR; EX1030, 37:14-39:1. Nor does Patent Owner dispute that Suzuki discloses a transient controller receiving the difference signal as an input and outputting fuel and spark adjustment signals. *See generally* POR; EX1030, 39:20-41:18. Instead, Patent Owner contends that Suzuki does not disclose a high-pass filter that receives the difference signal from the comparator because Suzuki does not expressly recite the term “high-pass filter” in the written specification. POR, 18-25. But “a reference must be considered not only for what it expressly teaches, but also for what it fairly suggests.” *In re Baird*, 16 F.3d 380, 383 (Fed. Cir. 1994) (quoting *In re Burckel*, 592 F.2d 1175, 1179 (C.C.P.A. 1979)).

As explained in the Petition and by Petitioner’s expert Dr. MacCarley, Figures 12-15 of Suzuki imply high-pass filtering a difference signal and then outputting the filtered signal to the homogeneous and stratified torque correction sections. Pet., 50–53; EX1003, ¶¶94, 124-127; EX1005, 7:51-64, 8:1-4, 8:51-60; EX2003, 10:2-11:9; EX1031, ¶¶35-45. Specifically, the torque correction factor signals—spark timing correction for homogeneous mode and fuel injection adjustment for stratified mode—illustrated in Suzuki’s Figures 12-15 match the

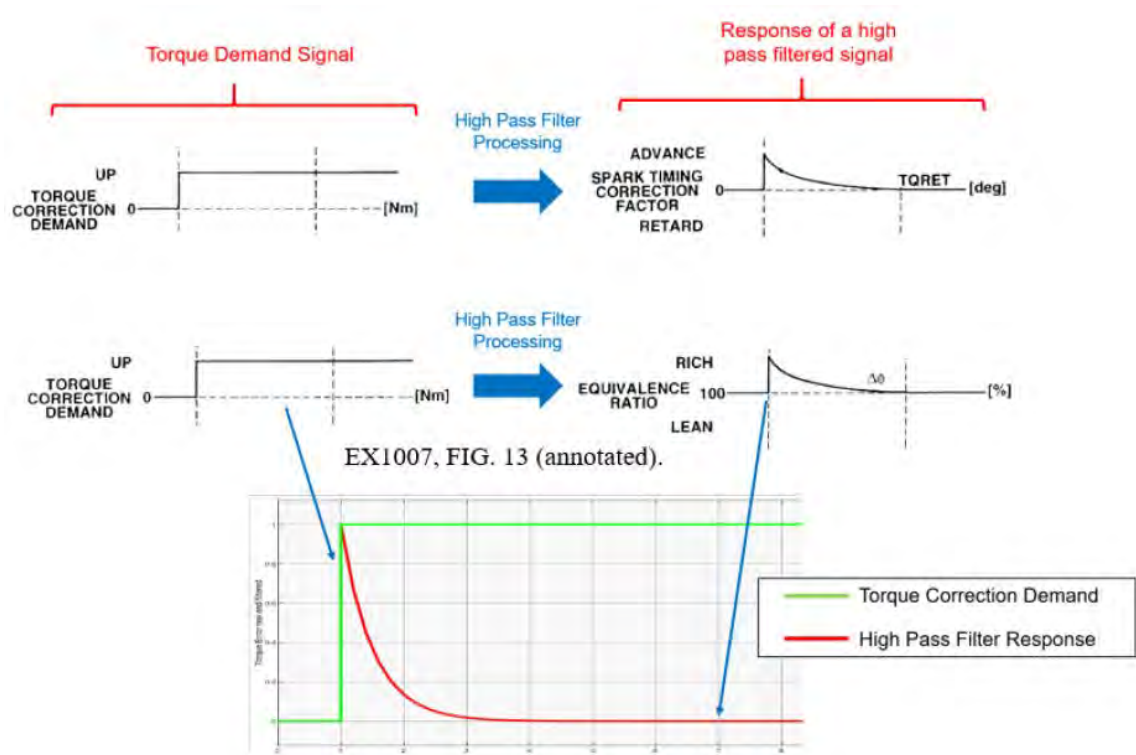
time response curve of a high-pass filter. Pet., 50–53; EX1003, ¶¶94, 124–127; EX1005, 7:51–63, 8:1–4, 8:51–60; EX1031, ¶¶35–45.



EX1003, ¶125 (annotating EX1005, FIG. 13).

Indeed, in his declaration, Dr. MacCarley provided a simulated correction signal (red) outputted from a high-pass filter in response to a step function (green), which matched Suzuki’s spark timing and equivalence ratio correction factor signals in response to step correction demands. EX1003, ¶126. Just as the high-pass filtered dynamic torque component operates in Scherer, Suzuki’s spark timing and equivalence ratio correction factors provide significant adjustments within a short time period and then disappear over time. EX1031, ¶41; EX1005, 8:51-60; EX1007, 4:46-52 (“**Due to the high pass characteristic of the high pass filter stage, ... the dynamic desired torque component (Md) will completely**

disappear.”)² And just as a high-pass filter typically offsets the phase lag of a controller’s response to a transient condition, the expedient adjustment shown by Suzuki’s spark timing and equivalence ratio curves offset the lag shown in the air flow rate curves in Figure 13 of Suzuki. EX1031, ¶¶40-41; EX1032, 622-623, FIG. 9-5, (“As seen from Figure 9-4, **the lead compensator is basically a high-pass filter**. ... The primary function of the lead compensator is to reshape the frequency-response curve ... to **offset the excessive phase lag** associated with components of the fixed system.”)



EX1003, FIG. A.

² Emphasis added unless noted.

Accordingly, a POSA would have understood that Suzuki implicitly discloses, or at least suggests, a high-pass filter receiving the difference signal in order correct the torque shortage via significant fuel and spark adjustments that disappear over time. EX1003, ¶¶94, 127; EX1031, ¶¶35-45.

Patent Owner does not refute Dr. MacCarley's compelling testimony showing that Suzuki implicitly discloses, or at least suggests, the use of a high-pass filter. EX1031, ¶¶42-45. Notably, neither Patent Owner nor Patent Owner's declarant Dr. Leonard addresses Dr. MacCarley's testimony specifically explaining how the Suzuki's torque correction factor signals illustrated in Figures 12-15 imply the use of a high-pass filter. POR, 18-25; EX2004, ¶¶26-36. Both Patent Owner and Patent Owner's declarant ignore the illustrated torque correction factor signals and, instead, paraphrase Suzuki's text that describes the vehicle operating conditions that trigger the torque demand shown in each of Suzuki's Figures 12-15. POR, 18-25; EX2004, ¶¶32-36. Patent Owner and its declarant do not provide any alternative explanation for how the torque correction factor signals are generated in Figures 12-15 of Suzuki. POR, 18-25; EX2004, ¶¶32-36. Patent Owner and its declarant also fail to explain how Suzuki's torque correction factor signals provide significant adjustments within a short time period and then disappear over time, without using a high-pass filter. POR, 18-25; EX2004, ¶¶32-36; EX1031, ¶42.

Instead, Patent Owner and its declarant ignore the illustrated correction factor signal waveform shapes. Both Patent Owner and its declarant summarily conclude these figures do not disclose high-pass filtering, without addressing the illustrated torque correction factor signal waveform shapes or providing any related technical analysis. POR, 18-25; EX2004, ¶¶26-36; EX1031, ¶43. In other words, Patent Owner's rebuttal is full of conclusory statements, lacking technical bases. This lack of technical explanation renders Patent Owner's rebuttal meritless. *See Wowza Media Sys., LLC v. Adobe Sys., Inc.*, IPR2013-00054, Paper 16, 4 (P.T.A.B. July 13, 2013) (“[T]he Declaration appeared, for the most part, simply to track and repeat the arguments for unpatentability presented in the Petition. ... Expert testimony that does not disclose the underlying facts or data on which the opinion is based is entitled to little or no weight.”).

Patent Owner further contends that if Suzuki supposedly used a high-pass filter, “Suzuki would only high pass filter the ‘torque correction demand signal.’” POR, 24. Patent Owner's assertion is wrong. EX1031, ¶44.

Suzuki discloses that the torque correction factor curves correspond to the adjustments made to spark timing during homogeneous mode and fuel injection during stratified mode **“to correct the torque shortage”** that is caused by the “delay in increasing the actual amount of air to the engine.” EX1005, 8:51-60. To correct for the torque shortage, Suzuki's torque correction factor signals would be

generated based on the difference between the demand torque and the estimated torque. That is, **Suzuki's torque correction factor signals would not be able to correct for torque shortage without first calculating the difference between the estimated and demand torques.** EX1031, ¶45. Thus, contrary to Patent Owner's baseless assertions, Suzuki's transient torque controller receives the high-pass filtered torque difference as an input before generating the torque correction factor signals. *Id.*

B. Patent Owner improperly attacks Scherer individually and ignores the Petition's combination of Suzuki and Scherer.

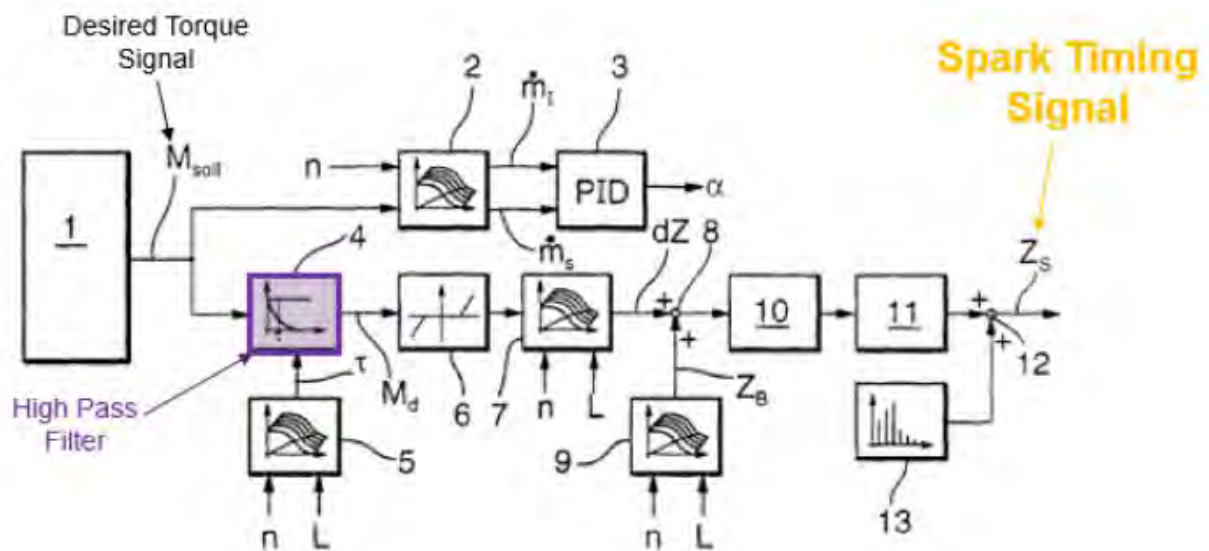
Even assuming that Suzuki does not disclose or suggest a high-pass filter, the Board's Institution Decision correctly found based on Petitioner's evidence that the combination of Suzuki and Scherer renders obvious a high-pass filter that receives a difference signal representing the difference between a torque demand and an estimated torque. DI, 36-38; Pet. 37-38, 53-56; EX1003, ¶¶94-103; EX1031, ¶46.

Scherer discloses a high-pass filter 4 that “filters the dynamic desired engine torque part (M_d) ... from the desired engine torque (M_{des}) ... and in turn provides the dynamic desired engine torque (M_d) to the device for the ignition point adjustment” in Figure 1 and the device for fuel adjustment in Figure 2. EX1007, 4:40-45; EX1031, ¶47. Scherer's ignition point adjustment system in Figure 1 and fuel adjustment system in Figure 2 are used “to adjust specifically that dynamic

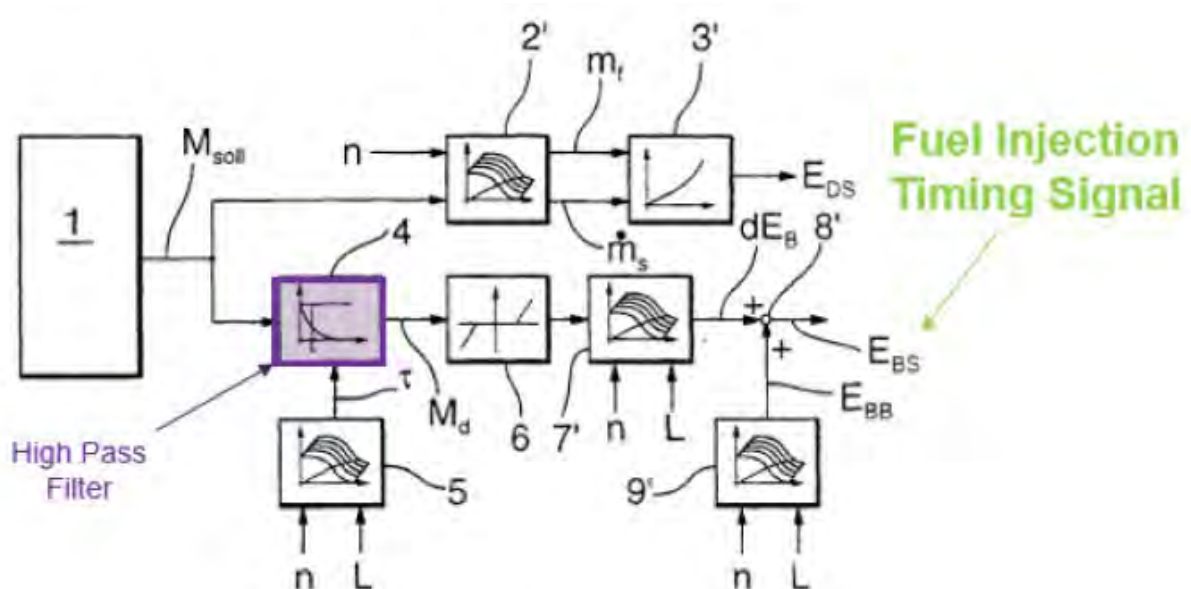
desired engine torque component which cannot be regulated by the air-flow rate-regulating branch.” EX1007, 3:59-31. As explained in the Petition and supported by Dr. MacCarley’s Declaration, a POSA would have understood that Scherer’s technique for adjusting ignition timing and fuel injection based on the high-pass filtered torque signal are analogous to Suzuki’s homogeneous and stratified torque correction sections. Pet., 36-37; EX1003, ¶98; EX1031, ¶47. Accordingly, it would have been obvious to add Scherer’s high-filter 4 **immediately upstream** of Suzuki’s homogeneous and stratified torque correction sections—Suzuki’s transient torque controller. EX1031, ¶47; EX1003, ¶100 (“Given that the homogeneous and stratified torque correction sections in Suzuki’s control unit 20 are intended to respond to dynamic changes in torque demand, **a POSA would have been motivated to use Scherer’s high-pass filter 4 to receive Suzuki’s torque difference signal and output the dynamic component of the torque difference signal to Suzuki’s homogeneous and stratified torque correction sections.**”). As the Board correctly observed in the Institution Decision, situating Scherer’s high-pass filter 4 immediately upstream of Suzuki’s homogeneous and stratified torque correction sections would result in the high-pass filter 4 receiving the difference signal outputted from Suzuki’s comparator. DI, 37; EX1031, ¶47.

Indeed, when looking to combine Scherer’s high-pass filter with Suzuki’s torque control unit, a POSA would have located the high-pass filter immediately

upstream of the Suzuki's homogeneous and stratified torque correction sections, as suggested by Scherer. EX1031, ¶48. As shown below in Figure 1 of Scherer, the high-pass filter 4 is situated immediately upstream of the provisions for spark adjustment so that the "ignition point adjusting branch is used only to adjust specifically that dynamic desired engine torque component." EX1007, 4:20-31; EX1031, ¶48. And as shown below in Figure 2 of Scherer, the high-pass filter 4 is situated immediately upstream of the provisions for fuel adjustment so that the fuel adjusting branch calculates "an injection start displacement (dE_B) corresponding to the dynamic desired engine torque component (M_d)." EX1007, 5:35-45; EX1031, ¶48. Because Suzuki's homogeneous and stratified torque correcting sections are directed to adjusting the dynamic torque component, a POSA would have been motivated to similarly locate Scherer's high-pass filter immediately upstream of Suzuki's homogeneous and stratified torque correcting sections to receive the dynamic torque component outputted from the high-pass filter. EX1005, 3:5-22; Pet. 36-39; EX1031, ¶48.



EX1007, FIG. 1 (annotated).



EX1007, FIG. 2 (annotated).

Moreover, a POSA would have understood that the high-pass filter is intended to be situated at the input side of the transient torque controller—ignition point or fuel injection branch—**regardless of receiving a torque demand or a torque difference signal**. EX1031, ¶49. First, as acknowledged by Patent Owner's

declarant, Scherer discloses that the desired torque signal outputted from the engine torque coordination unit 1 is merely an “example,” implying that other types of signals, such as a difference signal, may be inputted into the high-pass filter. EX1005, 3:60-63 (“[A]n engine torque coordination unit 1 provides a signal indicative, **for example**, of the desired engine torque (M_{des}) requested by the driver via the accelerator pedal.”); EX1030, 68:13-69:12. Second, Scherer discloses particular advantages associated with high-pass filter signal processing that are applicable to both a torque demand signal and a difference signal. EX1031, ¶49; EX1007, 4:45-52, 5:10-15. These advantages include (1) removing the low frequency component to permit a rapid response to correcting torque output and (2) reducing the torque output as the transient component decays to zero. EX1031, ¶49; EX1007, 4:45-52, 5:10-15; EX1030, 50:7-53:16. And a POSA would have understood that the difference signal in Suzuki is further processed before being inputted into the transient torque controller—Suzuki’s homogeneous and stratified torque correction sections. EX1031, ¶49. Indeed, as shown in FIG. 6, Suzuki translates the difference signal in step S35 into a corrective action (i.e., a torque correction factor), which is then inputted into the homogeneous and stratified torque correction sections. EX1031, ¶50. By situating Scherer’s high-pass filter immediately upstream of Suzuki’s torque correction sections, the high-pass filter would translate the difference signal into a correction action to be performed by the

transient controller—Suzuki’s torque correction sections. EX1031, ¶51. Such modification would allow Suzuki’s transient controller to keep the expedient “variation of the ignition point or of the injection start ... as small as possible.” EX1007, 2:54-62, 4:45-52; EX1031, ¶51.

Attempt to evade what is taught by the Suzuki-Scherer combination, Patent Owner improperly attacks Scherer in isolation and ignores the Petition’s combination. However, the Federal Circuit has consistently held that “[a] finding of obviousness... cannot be overcome by ‘by attacking references individually where the rejection is based upon the teachings of a combination of references.’” *See Bradium Techs. LLC v. Iancu*, 923 F.3d 1032, 1050 (Fed. Cir. 2019) (quoting *In re Merck & Co.*, 800 F.2d 1091, 1097 (Fed. Cir. 1986)). “[T]he test for obviousness is **what the combined teachings of the references** would have suggested to those having ordinary skill in the art.” *In re Mouttet*, 686 F.3d 1322, 1333 (Fed. Cir. 2012). Furthermore, “[a] claimed invention may be obvious even when the prior art does not teach each claim limitation, so long as the record contains some reason that would cause one of skill in the art to modify the prior art to obtain the claimed invention.” *Malico, Inc. v. Cooler Master USA Inc.*, 594 F. App’x 621, 626 (Fed. Cir. 2014); *see also Hartness Int’l, Inc. v. Simplimatic Eng’g Co.*, 819 F.2d 1100, 1108 (Fed. Cir. 1987) (“the inquiry is not whether each

element existed in the prior art, but whether the prior art made obvious the invention as a whole”).

Here, Patent Owner argues that “no prior art reference relied upon by the Petitioner or the Board discloses producing a ‘difference signal’ by comparing an ‘estimated torque signal’ and a ‘desired torque signal’ and then high-pass filtering such ‘difference signal.’” POR, 24. Pointing to the embodiments shown in Figures 1 and 2 of Scherer, Patent Owner argues that “only the desired torque signal is high-pass filtered and no comparator is used to compare an ‘estimated torque signal’ to the ‘desired torque signal.’” *Id.*, 26. That is, Patent Owner focuses solely on Scherer and ignores that Suzuki discloses outputting a difference signal to a transient torque controller, which, when combined with Scherer, situates a high-pass filter on the input side of the transient torque controller that render the high-pass filter of claims 1 and 7 obvious. EX1031, ¶¶52-53; EX1005, 7:35-44; EX1007, 4:20-23 (“In order to adjust the dynamic desired torque component which exceeds this quasi-static part, an ignition point adjusting branch, with **the input-side high-pass filter**, is provided in parallel to the air-flow-rate-regulating branch.”).

Accordingly, as explained in the Petition and supported by Dr. MacCarley’s declaration, a POSA would have understood that the combination of Suzuki and Scherer would use Scherer’s high-pass filter 4 to receive Suzuki’s torque

difference signal and output the dynamic component of the torque difference signal to Suzuki's homogeneous and stratified torque correction sections to improve the response time of Suzuki's controller. EX1003, ¶98; EX1031, ¶46-54.

C. A POSA would have been motivated to combine Suzuki and Scherer.

1. The Petition explained why a POSA would have combined Scherer and Suzuki.

The Board correctly determined “that Petitioner provides a reasoned explanation” why a skilled artisan would have combined Scherer's high-pass filter with Suzuki's torque controller. DI, 37; EX1031, ¶55. Combining Scherer's high-pass filter with Suzuki's torque controller would have yielded the predictable advantage of adjusting the torque output with a quicker response time during transient engine operations. Pet., 36-37; DI, 23-26; EX1003, ¶¶98-101; EX1031, ¶55; EX1007, 5:9-15 (“The embodiment of FIG. 1 therefore permits a comparatively **rapid engine torque adjustment** which can follow very fast desired engine torque changes”). The Petition further demonstrated the similarities between Suzuki and Scherer that would have encouraged a POSA to combine their teachings. Namely, Suzuki and Scherer are both in the field of engine control systems for monitoring torque output and controlling engine parameters in accordance with monitored torque output. Pet., 28-29; EX1003, ¶¶86, 101; EX1031, ¶55. And as the Petition explained, a POSA would have understood how

to combine Suzuki's and Scherer's teachings by simply modifying the software instructions stored in the microcomputer of Suzuki's control unit. Pet., 37-38; EX1003, ¶¶99; EX1031, ¶¶55; EX1030, 64:16-65:2. Indeed, supported by Dr. MacCarley's declaration, the Petition established that combining Scherer's high-pass filter 4 with Suzuki's control unit 20 would have merely been applying a known technique in a similar, known device in the same way to expedite the response time for correcting the transient component of the torque demand. Pet., 36 (citing *KSR Int'l Co. v. Teleflex Inc.*, 550 U.S. 398, 416 (2007)); EX1003, ¶¶98-103; EX1031, ¶¶55-56.

Accordingly, contrary to Patent Owner's assertions, *see, e.g.*, POR, 15, 24-25, 29-30, 34, the rationales to combine Suzuki and Scherer are based on the prior art disclosures themselves, not hindsight.

2. Scherer does not teach away from the claimed invention.

Patent Owner asserts that Scherer teaches away from being combined with Suzuki because the alternative embodiment shown in Figures 3 and 4 of Scherer uses a model-based estimated value stage, instead of a high-pass filter, to determine the dynamic desired engine torque component. POR, 25-29. Because Scherer's model-based estimated value stage employs an estimated determination unit 14 and a subtractor 15, rather than a high-pass filter, Patent Owner alleges that Scherer discourages a POSA from using a high-pass filter to receive a difference

signal outputted from a comparator. *Id.*, 27-29. Patent Owner’s reliance on Scherer’s alternative embodiment to support a teaching away argument is legally flawed and factually unsupported. EX1031, ¶¶57-58.

“A reference may be said to teach away when a person of ordinary skill, upon reading the reference, would be discouraged from following the path set out in the reference, or would be led in a direction divergent from the path that was taken by the applicant.” *Ricoh Co., Ltd. v. Quanta Comput. Inc.*, 550 F.3d 1325, 1332 (Fed. Cir. 2008) (quoting *In re Kahn*, 441 F.3d 977, 990 (Fed. Cir. 2006)). To teach away, a prior art reference must “criticize, discredit, or otherwise discourage the solution claimed.” *In re Fulton*, 391 F.3d 1195, 1201 (Fed. Cir. 2004). A reference does not teach away for merely expressing a general preference for an alternative invention. *See DePuy Spine, Inc. v. Medtronic Sofamor Danek, Inc.*, 567 F.3d 1314, 1327 (Fed. Cir. 2009). Moreover, “a prior art reference that does not specifically refer to one element of a combination does not, *per se*, teach away.” *Syntex (U.S.A.) LLC v. Apotex, Inc.*, 407 F.3d 1371, 1380 (Fed. Cir. 2005).

Nowhere does Scherer criticize, discredit, or discourage a POSA from using a high-pass filter that receives a difference signal. EX1031, ¶59. Patent Owner alleges that Scherer’s description of the embodiment shown in Figures 3 and 4 discourages a POSA from using of a high-pass filter with a comparator. POR, 27-29. This section of Scherer discloses “[t]his embodiment corresponds to that of

FIG. 1 except that, instead of the high pass filter stage, a model-based estimated value stage is provided to determine the dynamic desired engine torque component.” EX1007, 6:3-12; POR, 27. Scherer’s description of an alternative embodiment, however, would not have discouraged a POSA from using a high-pass filter that receives a difference signal. EX1031, ¶60. Rather, Scherer merely discloses two ways—a high-pass filter and a model-based estimation value stage—for determining a dynamic component of a torque signal. EX1031, ¶60. Scherer never discloses that the model-estimation (Figures 3 & 4) and the high-pass filter (Figures 1 & 2) are mutually exclusive. EX1031, ¶60. And Scherer does not discredit using both a high-pass filter and an estimation stage in a torque producer of an engine controller. EX1031, ¶60. Thus, Scherer’s two alternative embodiments—high-pass filtering in Figures 1 and 2 and the model estimation in Figures 3 and 4—are not a teaching away. EX1031, ¶57-61.

D. Patent Owner’s credibility attacks on Dr. MacCarley’s testimony are meritless.

In an attempt to distract the Board from the Petition’s merits, Patent Owner asserts that Dr. MacCarley’s testimony is inherently unreliable due to the mechanics of how his Declaration was drafted and certain observations concerning his cross-examination. POR, 30-35. These allegations against Dr. MacCarley’s testimony are unfounded.

1. Patent Owner mischaracterizes Dr. MacCarley's testimony.

Patent Owner's spurious attacks on the reliability of Dr. MacCarley's testimony are premised on mischaracterizations. Patent Owner contends that Dr. MacCarley "testified during his deposition... that his expert declaration in this case 'was a collaborative effort between us [between him and Petitioner's attorneys].'" POR, 30 (quoting EX2003, 8:12-9:7). Patent Owner further highlights that Dr. MacCarley acknowledged in his deposition that the prior art search was performed by Petitioner's counsel. POR, 31. From here, Patent Owner argues "based upon ... deposition testimony of Dr. MacCarley, ... it is plain to see that the Petitioner's expert in this case did not use the proper obviousness analytical framework, and, as such, the Expert Declaration in this Proceeding are [*sic*] fundamentally flawed and lack probative value." POR, 32.

Patent Owner selectively and deceptively removed relevant statements from the testimony quoted in its POR. When testifying about preparing the declaration, Dr. MacCarley concluded by stating "every statement in [the declaration] is, indeed, my opinion, and I take full responsibility for it." EX2003, 9:1-2. Furthermore, while Dr. MacCarley acknowledged that he did not conduct the prior art search, Dr. MacCarley noted that he provided input into the search and that "[u]ltimately, the selection of all of [Suzuki, Scherer, Takahashi, and Cullen] was

mine.” EX2003, 13:23-14:16, 15:14-16:4. Patent Owner removed and ignored these statements by Dr. MacCarley, which do not fit its false narrative.

Moreover, the line of questions eliciting this testimony is the exact type of questions the Board has said “generally are a waste of time” in *inter partes* proceedings. *Pevarello v. Lan*, Patent Interference No. 105,394, Paper 85, slip op., 21 (B.P.A.I. January 12, 2007). Indeed, the Board has held that when the manner of the expert’s testimony is in line with typical testimony before the Board, and the expert’s background, analysis, and supporting evidence are credible, an inquiry into “the mechanics of declaration preparation is ‘**a waste of time, both for the witness and the Board.**’” *Square, Inc. v. Think Computer Corp.*, CBM2014-00159, Paper 47, 26-27, (P.T.A.B. Nov. 27, 2015) (quoting *Pevarello*, Paper 85, slip op., at 19-21). Spending over forty years researching, teaching, and designing engine control systems, Dr. MacCarley has supported each of his positions set forth in his declaration with significant insight and secondary evidence, such as automotive handbooks, engineering textbooks, and relevant patents.

In sum, Patent Owner’s credibility attacks against Dr. MacCarley’s testimony are meritless and a waste of the Board’s time. The Board should disregard Patent Owner’s specious allegations.

2. Dr. MacCarley articulated and applied the appropriate test for obviousness.

Patent Owner further alleges that Dr. MacCarley failed to apply the proper framework for determining obviousness because he received prior art references from Petitioner's counsel. POR, 32-34. Particularly, Patent Owner argues that by being "spoon fed the prior art references to consider," Dr. MacCarley did not take the proper starting point when determining whether the claims of the '565 patent would have been obvious. *Id.*, 33-34. These arguments lack merit.

Dr. MacCarley properly considered the requirements for determining obviousness as it related to a POSA and the corresponding level of ordinary skill in the art. EX1003, ¶¶30-41. Indeed, during his deposition, Dr. MacCarley testified that the legal standards, set forth paragraphs 21-39 of his declaration, were the standards that he applied in conducting his analysis and formulating his opinions. EX2003, 21:17-22. He applied the proper standards in his analysis.

And Patent Owner's assertion that Dr. MacCarley's testimony is unreliable because he allegedly did not "determine what prior art the POSA would have searched for and found in the normal course of research and development," is legally flawed. It is well-settled law that there is "a presumption that **a person of ordinary skill in the art possesses knowledge of all prior art** references in the particular art." *Messerschmidt v. United States*, 29 Fed. Cl. 1, 27, *aff'd*, 14 F.3d 613 (Fed. Cir. 1993); *In re Winslow*, 365 F.2d 1017, 1020 (C.C.P.A. 1966) ("Section

103 requires us to presume full knowledge by the inventor of the prior art in the field of his endeavor.”). Since all applied references are analogous art in the field of engine control systems—a fact uncontested by Patent Owner—a POSA would have had knowledge of each of the applied prior art references: Suzuki, Cullen, Scherer, and Takahashi. Thus, Patent Owner’s argument contravenes established law.

Accordingly, Dr. MacCarley applied the proper framework in opining that the ’565 patent claims would have been obviousness to a POSA.

II. Conclusion

For the reasons above, claims 1-7 of U.S. Patent No. 6,581,565 would have been obvious and should be cancelled.

Respectfully submitted,

STERNE, KESSLER, GOLDSTEIN & FOX P.L.L.C.

/Michael D. Specht/

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Attorney for Petitioner
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Date: January 21, 2021

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**CERTIFICATE OF COMPLIANCE WITH TYPE-VOLUME LIMITATION,
TYPEFACE REQUIREMENTS, AND TYPE STYLE REQUIREMENTS**

1. This Petitioner's Reply to the Patent Owner's Response complies with the type-volume limitation of 5,600 words, comprising 4,561 words, excluding the parts exempted by 37 C.F.R. § 42.24(c).

2. This Petitioner's Reply to the Patent Owner's Response complies with the general format requirements of 37 C.F.R. § 42.6(a) and has been prepared using Microsoft® Word 2010 in 14-point Times New Roman.

Respectfully submitted,

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CERTIFICATION OF SERVICE (37 C.F.R. §§ 42.6(e))

The undersigned hereby certifies that on January 21, 2021 true and correct copies of the foregoing **PETITIONER'S REPLY TO PATENT OWNER'S RESPONSE** and **EXHIBITS 1030-1033** were served in their entireties on the following parties via email:

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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,

Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES, LLC,

Patent Owner

Case IPR2020-00445

Patent 6,581,565

Deposition of RUSSELL LEONARD, PH.D.

Conducted Virtually

Tuesday, January 5, 2021

9:31 a.m.

Job No.: 343665

Pages: 1-72

Reported by: Matthew Goldstein, RPR

VW EX1030
VW v. MMT
IPR2020-00444

1 Deposition of RUSSELL LEONARD, PH.D.,
2 conducted virtually:

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9 Pursuant to Notice, before Matthew Goldstein,
10 RPR, Notary Public in and for the State of
11 Maryland.

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C O N T E N T S		
EXAMINATION OF RUSSELL LEONARD, PH.D.		PAGE
By MR. CONKLIN		6
E X H I B I T S		
(Attached)		
LEONARD	DEPOSITION EXHIBIT	PAGE
Exhibit 1	Second Declaration of Dr. Russell Leonard	9
Exhibit 2	Declaration of C. Arthur MacCarley, Ph.D., P.E. in Support of Petition for Inter Partes Review of U.S. Patent No. 6,581,565	22
Exhibit 3	United States Patent 6,581,565	23
Exhibit 4	United States Patent 6,006,717	34
Exhibit 5	United States Patent 6,047,681	42
Exhibit 6	European Patent Application EP 1 076 165 A2	55

1 THE COURT REPORTER: Will counsel please
2 stipulate that in lieu of formally swearing in the
3 witness, the reporter will instead ask the witness
4 to acknowledge that their testimony will be true
5 under the penalties of perjury, that counsel will
6 not object to the admissibility of the transcript
7 based on proceeding in this way, and that the
8 witness has verified that he is in fact
9 Dr. Russell Leonard.

10 MR. CONKLIN: Counsel for petitioner
11 agrees to that stipulation.

12 MS. WILLIAMS: Counsel for patent owner
13 also agrees to the stipulation.

14 THE COURT REPORTER: Thank you, Counsel.
15 And, sir, do you hereby acknowledge that
16 your testimony will be true under the penalties of
17 perjury?

18 THE WITNESS: Are you speaking to me?

19 THE COURT REPORTER: Yes, sir.

20 THE WITNESS: Okay. Yes.

21 THE COURT REPORTER: Thank you.

22 And whenever you're ready, Counsel.

1 EXAMINATION BY COUNSEL FOR THE PETITIONER
2 BY MR. CONKLIN:

3 Q. Good morning, Dr. Leonard.

4 A. Good morning.

5 Q. You've been deposed a few times; is that
6 correct?

7 A. Yes.

8 Q. So you generally understand the process
9 of a deposition?

10 A. Yes.

11 Q. And it's important that today that we
12 try not to speak over each other over the video
13 conference software.

14 Do you understand that?

15 A. Yes.

16 Q. And you understand that it's important
17 that you provide verbal answers that can be
18 transcribed?

19 Do you understand that?

20 A. Yes.

21 Q. And I'm going to ask you numerous
22 questions today and if any of my questions are

1 unclear, will you please let me know that?

2 A. Yes.

3 Q. We will try to take periodic breaks
4 about every hour. If you need to take a break
5 before that, please let me know.

6 Do you understand that?

7 A. Yes.

8 Q. And do you have paper copies of the
9 documents that were sent to you?

10 A. Yes.

11 Q. And are those paper copies clean copies?

12 A. Yes.

13 Q. And there are no annotations or notes on
14 those copies?

15 A. No notes.

16 Q. And you understand that you're under
17 oath right now?

18 A. Yes.

19 Q. Dr. Leonard, are you -- other than this
20 Zoom interface, are you in communication with
21 counsel for MMT?

22 A. At this moment?

1 Q. Yes.

2 A. No.

3 Q. Dr. Leonard, do you understand why
4 you're here for this deposition right now?

5 A. I have my own understanding.

6 Q. Do you understand that you're here to
7 testify about your statements in two declarations
8 you provided in two separate IPR proceedings?

9 A. That is what I understand to be true.

10 Q. And specifically you're here to provide
11 testimony regarding U.S. patent 6,581,565; is that
12 correct?

13 A. That is correct.

14 Q. And you provided two declarations in IPR
15 2020-00444; correct?

16 A. Yes.

17 Q. And you also provided a declaration in
18 IPR 2020-00445; correct?

19 A. That is correct.

20 Q. And you understand that this deposition
21 will cover both of those proceedings?

22 A. I thought there was two depositions, but

1 if there's just going to be one, hopefully we do
2 them in some kind of -- you know, we separate them
3 somehow. I don't feel comfortable switching back
4 and forth between the two in flight.

5 Q. I will do my best to do that. I
6 will -- obviously you understand that there's some
7 references that are in both proceedings; correct?

8 A. Yeah, they are repetitive, correct.

9 (Leonard Deposition Exhibit 1 was marked
10 for identification and attached to the
11 transcript.)

12 BY MR. CONKLIN:

13 Q. Dr. Leonard, can you grab your copy of
14 Exhibit 2004 in IPR 2020-00444?

15 A. Can you just cut to the chase and tell
16 me generally what that one is called because that
17 would help me find it?

18 Q. That would be your second declaration in
19 the 444 IPR.

20 A. I have that.

21 Q. And is Exhibit 2004 a copy of your
22 declaration?

1 A. Yes.

2 Q. And did you sign that declaration?

3 A. Yes, I did.

4 Q. When is the last time you reviewed that
5 declaration?

6 A. This morning.

7 Q. I'm just going to do a little bit of
8 recordkeeping right here and run through the
9 documents, Dr. Leonard. So I apologize for the
10 list of documents that's going to be coming, but
11 it's going to help us in the future.

12 Do you have a copy of Exhibit 2004 of
13 the IPR 2020-00445 proceeding, that is your second
14 declaration?

15 A. Yeah, that's the one you just had me
16 get, is that not correct?

17 Q. I asked previously for the 00444
18 exhibit.

19 A. Oh, you're on the 445 now, is that what
20 you're doing?

21 Q. Yes. I would just like to introduce
22 these on the record.

1 A. All right. Give me a minute. I do have
2 that.

3 Q. And Exhibit 2004 of IPR 2020-00445 is a
4 copy of your declaration; correct?

5 A. Yes.

6 Q. And you signed that declaration?

7 A. Yes, I did.

8 Q. How did you prepare those declarations?

9 A. I reviewed the documents that I was told
10 to review and I wrote my truthful opinion.

11 Q. Did you type the declarations?

12 A. Yes.

13 Q. The entire declaration you typed?

14 A. There was some review back and forth,
15 but I typed my original thoughts.

16 Q. Do you recall what edits counsel for MMT
17 made?

18 A. Not exactly. They were -- there were
19 some edits.

20 Q. Are there any changes you would like to
21 make to either of those declarations?

22 A. Aside from a typo in one, I don't -- I

1 don't have any real factual information that I
2 would change.

3 Q. So as you're sitting here today, you do
4 not have any errors that you would like to correct
5 in these declarations?

6 A. No.

7 Q. When preparing these two declarations,
8 what documents did you review?

9 A. When I prepared the declaration for the
10 IPR 2020-00444, I reviewed the '565 patent, I
11 reviewed a reference called Suzuki, and I reviewed
12 a reference called Cullen, a reference called
13 Scherer, a reference called Takahashi. I reviewed
14 the petition for inter partes review, the
15 444 version. I reviewed the document of
16 institution from the board. I reviewed the
17 MacCarley declaration. There was a MacCarley
18 deposition transcript. There was a petitioner's
19 reply to patent owner's preliminary response.
20 There's a patent owner's response. I think I
21 covered them all or close to it.

22 Q. When preparing the declaration for the

1 IPR 2020-00445, what documents did you review?

2 A. It was a similar slate. I remember
3 reviewing a couple of MacCarley declarations, you
4 know, decision institute, those kind of things
5 similar to what I had said earlier. And then this
6 one in particular had some different references
7 and some similar references so Robichaux, Scherer,
8 Takahashi and Suzuki, and there's a few other
9 minor references.

10 Q. Did you review the Bosch Automotive
11 Handbook when preparing either declaration?

12 A. Yes.

13 Q. Did you review the Heywood textbook when
14 preparing either declaration?

15 A. Yes.

16 Q. Did you review the Jurgen textbook when
17 you prepared this declaration?

18 A. Yes.

19 Q. And did you review the Ribbens textbook
20 when you prepared this declaration?

21 A. Yes.

22 Q. Let's turn to your declaration in

1 IPR 2020-00444.

2 A. Which one, the first one?

3 Q. Exhibit 2004.

4 A. Okay.

5 Q. Can you look at paragraphs 19 and 20 of
6 that declaration?

7 A. Yes.

8 Q. Is this your understanding of
9 obviousness?

10 A. That's what I wrote, yes.

11 Q. Do you have any additional
12 understandings of obviousness other than these two
13 paragraphs?

14 A. I do. We've talked about it with MMT's
15 lawyers, but I didn't include that in this
16 declaration.

17 Q. And what is that additional
18 understanding that's not articulated here?

19 A. Well, I didn't put it in my declaration,
20 I'm not going to opine on that right now.

21 Q. So you don't have an additional
22 understanding of obviousness?

1 A. I do.

2 Q. Can you explain what that additional
3 understanding is?

4 A. I have to refer back to my declaration
5 here and, you know, my statement of obviousness is
6 right here.

7 Q. And when you reference right here, are
8 you referencing the paragraphs we were just
9 talking about in your second declaration?

10 A. 19 and 20.

11 Q. How long did it take you to prepare
12 these two declarations?

13 A. Including all the background reading and
14 all that, somewhere approximately 40 hours for
15 each one, maybe a little less on the second since
16 there's some duplicative references, but roughly.

17 Q. Can we turn to paragraph 14 of your
18 second declaration in IPR 2020-00444?

19 A. I'm there. Go ahead.

20 Q. Who provided you with this understanding
21 of the level of ordinary skill in the art?

22 A. It's one that I came up with and I ran

1 it by counsel from MMT and that's where it evolved
2 from.

3 Q. Did counsel from MMT provide input on
4 this definition?

5 A. Maybe only one word, but it was
6 99 percent me.

7 Q. What word would that have been?

8 A. I don't recall. I'd have to go back
9 through my -- if I even have saved it -- the
10 document versions, but I do recall that there was
11 one word that they -- that came up and I don't
12 remember if it was an addition or if it was just a
13 finessing to use a different -- a different word
14 to what I had, but it didn't change radically this
15 level of skill of ordinary art paragraph.

16 Q. You mentioned document versions, were
17 there multiple versions of your declaration?

18 A. As I write, it's not a linear process
19 necessarily, it's sort of a stepped process. I
20 write, I reflect, I read some more and then I
21 write again and I do that over and over again
22 until I get to a point where I'm satisfied with

1 it.

2 Q. Dr. Leonard, do you have a B.S. in
3 mechanical engineering?

4 A. No.

5 Q. Do you have a master's in mechanical
6 engineering?

7 A. No.

8 Q. Do you have a B.S. in electrical
9 engineering?

10 A. No.

11 Q. Do you have a master's in electrical
12 engineering?

13 A. No.

14 Q. Do you have a Ph.D. in mechanical
15 engineering?

16 A. No.

17 Q. Do you have a Ph.D. in electrical
18 engineering?

19 A. No.

20 Q. Dr. Leonard, what degrees do you have?

21 A. I have a Bachelor of Science in
22 secondary education that is a major with K

1 through 12 industrial engineering and technology
2 education with an automotive focus. I took every
3 automotive class that was available at the time.
4 I also worked as a mechanic during my undergrad.

5 I got a Master's of Education in the
6 structural systems design and adult education. At
7 that time, I was working as a systems engineer for
8 Hughes Aircraft and later -- or initially I was
9 working at Ford Motor Company teaching powertrain
10 controls to engineers. And when I finished --
11 toward the end of finishing that degree is when I
12 transitioned to being a systems engineer for
13 Hughes Aircraft.

14 I earned a doctor of philosophy in
15 higher educational leadership. I earned that
16 degree while being a professor in the College of
17 Engineering Technology at the School of Automotive
18 and Heavy Equipment. I instruct automotive
19 service and automotive engineering technology
20 courses.

21 And I've held numerous positions as an
22 engineer for Cadillac Motorcar Division, for

1 service operations at General Motors. I wrote
2 diagnostic software for the scan tool and that
3 interfaced with various electronic control
4 modules. And, again, I taught a powertrain
5 controls class for Ford Motor Company and that
6 delved into a lot of the microprocessor strategy
7 and things related to our subject matter here in
8 the '565 patent.

9 Q. What microprocessor strategies are
10 related to the '565 patent?

11 A. There's torque control, dealing with
12 spark air/fuel control and its relation to
13 controlling torque of an internal combustion
14 engine.

15 Q. And those strategies that you just
16 mentioned were known before the '565 patent?

17 A. What paragraph are you on?

18 Q. I was just asking about your answer you
19 just gave.

20 Was torque control known before the '565
21 patent?

22 A. Torque control certainly was known.

1 Q. Was torque control using spark and
2 air/fuel modifications known before the '565
3 patent?

4 A. Yes.

5 Q. Going back to the question of your
6 educational background, are your degrees in
7 education?

8 A. Yes.

9 Q. Dr. Leonard, what is a high-pass filter?

10 A. A high-pass filter is exactly what it
11 says, it allows high signals to go through and
12 blocks low signals in general.

13 Q. When you say "high signals," what is a
14 high signal?

15 A. A signal that's higher than a low
16 signal.

17 Q. In what way?

18 A. Well, you have a low signal, maybe it's
19 a low voltage, maybe it's low frequency, whatever
20 it is, it's a low signal, something that could be
21 characterized as a low signal. And a high signal
22 would be something that would be higher than that

1 that the high-pass filter allows through.

2 In this case, you could get into the
3 '565 patent, but what I say in my declaration is
4 that a high-pass filter receives the air signal
5 and blocks filters out with smaller differences
6 from the comparator 18 and only allows larger
7 differences to pass through to the transient
8 torque controller and that's in reference to the
9 '565 patent.

10 Q. Does a high-pass filter filter based on
11 frequency or magnitude?

12 A. It's magnitude. If I said frequency, I
13 didn't mean to say that in reference to this.

14 Q. Dr. Leonard, have you ever designed an
15 engine control system that uses a high-pass
16 filter?

17 A. No.

18 Q. Dr. Leonard, have you ever studied the
19 time response wave forms from a high-pass filter?

20 A. Yes, I reviewed those.

21 Q. Where?

22 A. In the information that I was asked to

1 review and also in other textbooks and in my
2 background as an engineer I've reviewed those.

3 Q. Can you explain what the time response
4 curve of a high-pass filter looks like using a
5 step input?

6 A. Well, generally if the step input is
7 high enough for long enough, then it would pass
8 through.

9 Q. And what does the outputted wave form
10 look like?

11 A. It can look like a curve.

12 Q. Does that curve decay over time?

13 A. It can.

14 (Leonard Deposition Exhibit 2 was marked
15 for identification and attached to the
16 transcript.)

17 BY MR. CONKLIN:

18 Q. Dr. Leonard, can you pull up
19 Exhibit 1003 of IPR 2020-00444?

20 A. Okay. I have it open.

21 Q. Okay. Can you go to page 68 of that
22 document?

1 A. I'm there.

2 Q. Actually, strike that question.

3 (Leonard Deposition Exhibit 3 was marked
4 for identification and attached to the
5 transcript.)

6 BY MR. CONKLIN:

7 Q. Dr. Leonard, can you please pull up
8 Exhibit 1001 of IPR 2020-00444?

9 A. That's the '565 patent; correct?

10 Q. Yes. That's correct.

11 A. Okay. I have it open.

12 Q. And can you take a look at the abstract
13 please?

14 A. The what?

15 Q. The abstract on the first page of the
16 '565 patent, please.

17 A. Yes, I'm there.

18 Q. And just take a moment to read that.

19 (Witness peruses the exhibit.)

20 THE WITNESS: Okay.

21 BY MR. CONKLIN:

22 Q. The '565 patent is directed to an engine

1 torque controller for spark ignition internal
2 combustion engines; correct?

3 A. Yes.

4 Q. And let's take a look at Claim 1 of the
5 '565 patent.

6 A. Yes.

7 Q. Claim 1 of the '565 patent recites a
8 torque demand controller; right?

9 A. Yes.

10 Q. Claim 1 recites a torque producer;
11 right?

12 A. Correct.

13 Q. The torque producer includes an
14 estimator; correct?

15 A. Yes.

16 Q. And that estimator outputs an estimated
17 torque signal; correct?

18 A. Yes.

19 Q. Claim 1 also recites a comparator;
20 correct?

21 A. Yes.

22 Q. Claim 1 further recites a high-pass

1 filter; correct?

2 A. Yes.

3 Q. And Claim 1 recites a transient torque
4 controller; right?

5 A. Yes.

6 Q. Torque demand controllers were known
7 before the '565 patent; right?

8 A. Yes.

9 Q. Torque producers were known before the
10 '565 patent; right?

11 A. Yes.

12 Q. Torque estimators were known before the
13 '565 patent; right?

14 A. Yes.

15 Q. Comparators were used in engine control
16 systems before the '565 patent; right?

17 A. Correct.

18 Q. High-pass filters were known before the
19 '565 patent; right?

20 A. Yes.

21 Q. And transient torque controllers were
22 known before the '565 patent; right?

1 A. Yes.

2 Q. Is it fair to say that you only dispute
3 that the petition has failed to show a high-pass
4 filter that receives as an input the difference
5 signal and outputs a filtered difference signal?

6 A. Repeat that question, please.

7 Q. Is it fair to say that you only dispute
8 that the petition has failed to show or render
9 obvious a high-pass filter that receives as an
10 input the difference signal and outputs a filtered
11 difference signal?

12 A. What I say generally is that a filtered
13 difference signal of desired engine torque and
14 estimated torque signals that comparator compares
15 those two and that difference is filtered through
16 a high-pass filter. And it's the order of
17 operations and the unique sequencing that the '565
18 patent discloses that is not found in any of the
19 prior art.

20 Q. So it's the combination of the recited
21 comparator and high-pass filter that is missing,
22 allegedly missing in the prior art?

1 A. Not only that, but the filtered
2 difference of desired engine torque and estimated
3 torque signals as well, you can't ignore that.

4 Q. Were torque demand controllers for
5 generating a torque demand signal known before the
6 '565 patent?

7 A. Say that again.

8 Q. Were torque demand controllers for
9 generating a torque demand signal known before the
10 '565 patent?

11 A. Yes.

12 Q. Were torque producers that receive the
13 torque demand signal known before the '565 patent?

14 A. Yes.

15 Q. Were comparators that compared the
16 estimated torque to a demanded torque to generate
17 a difference signal known before the '565 patent?

18 A. Yes. Can you repeat that question? I
19 just want to be sure, it's hard to hear.

20 Q. Were comparators that compared the
21 estimated torque to a demand torque to generate a
22 difference signal known before the '565 patent?

1 A. Yes.

2 Q. Were transient torque controllers that
3 receive a signal from a high-pass filter and
4 outputs a fuel or spark adjustment signal known
5 before the '565 patent?

6 A. Say that again.

7 Q. Were transient torque controllers that
8 receive a signal from a high-pass filter and
9 output fuel or spark adjustment signals known
10 before the '565 patent?

11 A. Yes.

12 Q. Were high-pass filters that filter out
13 low frequency components known before the '565
14 patent?

15 A. Yes.

16 Q. Dr. Leonard, what is a steady state
17 error?

18 A. Can you make your question more
19 specific?

20 Q. In paragraph 24 of your second
21 declaration in IPR 2020-00444, you mention steady
22 state error, what is a steady state error?

1 A. In the context of the '565 patent, in
2 referring to my declaration, torque demand signal
3 that represents a feedforward signal and that
4 torque -- let me see if I'm reading this right --
5 and the torque produced signal represents a
6 feedback signal. So you have feedforward and
7 feedback. These two signals are run through a
8 comparator that generates an error signal. The
9 error signal is generated by subtracting the
10 current torque estimate from the demand torque
11 signal. This error includes instances where
12 torque demand is less than torque produced and
13 instances where torque demand is greater than
14 torque produced.

15 Going on, the high-pass filter receives
16 the error signal and blocks, filters out smaller
17 differences from the comparator 18 and only allows
18 larger differences to pass through to the
19 transient torque controller. In general, a steady
20 state error in terms of operation of an internal
21 combustion engine is where there are little to no
22 transients.

1 Q. What kind of transients?

2 A. One example that I gave -- that I read
3 in the art was when air conditioner compressor
4 engages during operation of the engine, it
5 requires torque to run, it's going to use torque
6 from the engine through the accessory drive and if
7 nothing is done to counteract that, that's less
8 torque for doing other things that the engine must
9 do or is doing at that time.

10 Q. Dr. Leonard, in the last sentence of
11 paragraph 25, you mentioned hunting. What is
12 hunting?

13 A. So going back to my example of the air
14 compressor, air-conditioning compressor engaging
15 on an engine being driven by the accessory drive
16 belt using torque that the engine produces leaving
17 less torque for other operations, a situation
18 where the torque drops, the torque output of the
19 engine for use for motivating the vehicle down the
20 road let's say, it drops when that
21 air-conditioning compressor kicks on, now you have
22 a deficit or you have less torque available

1 because some of the torque is being used by that
2 air-conditioning compressor.

3 If you don't have a good system, you can
4 overshoot and add too much torque by changing some
5 parameters of the engine operation and then you
6 overshoot and then you got too much which you
7 would in response try to reduce torque. And if
8 your system is not as accurate like the '565
9 patent, you'll end up with too little torque again
10 and you'll go back and forth hunting for the right
11 amount of torque to run the engine and it will
12 swing depending upon how bad your system is
13 designed quite -- it could be noticeable and
14 objectionable to the customer.

15 Q. And in your opinion, when the air
16 conditioner compressor kicks on, is that a
17 transient condition?

18 A. As it cycles on and off, it would be
19 considered transient when it does those things.
20 When it cycles on, when it cycles off, that's a
21 change.

22 Q. How does a high-pass filter correct

1 steady state error?

2 A. So a high-pass filter, again, it will
3 filter out lower frequencies, things that are not
4 worthy of being responded to, and you'll get the
5 higher frequencies, which you'll respond to. So
6 when the air-conditioning compressor kicks on, you
7 have a need for more torque that will show up and
8 go through the high-pass filter, you'll adjust
9 accordingly until it kicks off and then you need
10 less torque.

11 Q. In that explanation, are you considering
12 cycling the air conditioner compressor on and off
13 to be a steady state issue?

14 A. It can be both. When it comes on, when
15 it goes off, it's transient. If it comes on and
16 stays on, it could be considered in a steady state
17 for that duration.

18 Q. And would a person of ordinary skill in
19 the art had under- -- would a person of ordinary
20 skill in the art understood that a high-pass
21 filter could be used in this manner?

22 A. Well, until the '565 patent came along,

1 people were attempting to do that, so, yes.

2 Q. This is a good point to take a break I
3 think. Can we go off the record?

4 (Recess from the record.)

5 BY MR. CONKLIN:

6 Q. Dr. Leonard, can you take a look at
7 paragraph 26 of Exhibit 2004 in IPR 2020-00444?

8 A. Yes. Go ahead.

9 Q. Suzuki discloses a torque control for a
10 direct injection spark ignition engine; correct?

11 A. Yes.

12 Q. And specifically Suzuki discloses a
13 method of torque control for a direct injection
14 spark ignition engine that operates in both
15 stratified and homogenous modes; correct?

16 A. Yes.

17 Q. And you agree that Suzuki provides a
18 simple solution to engine torque control; correct?

19 A. Yes. Simple in comparison to '565.

20 Q. And a POSA would have known how to
21 implement Suzuki's torque control system?

22 A. Following that teaching, yes.

1 Q. You would agree that Suzuki's controller
2 controls torque output using spark adjustments
3 during homogenous mode; right?

4 A. Correct.

5 Q. And Suzuki's torque controller controls
6 torque output using fuel adjustments during
7 stratified mode; correct?

8 A. Air/fuel ratio, correct.

9 (Leonard Deposition Exhibit 4 was marked
10 for identification and attached to the
11 transcript.)

12 BY MR. CONKLIN:

13 Q. Let's take a look at Figure 6 of the
14 Suzuki reference, Exhibit 1005 of IPR 2020-00444.

15 A. Which drawing are we looking at?

16 Q. Figure 6.

17 A. Okay. I'm there.

18 Q. Actually, before we get into Figure 6,
19 let's take a step back. Can you take a look at
20 Column 1, lines 4 through 6?

21 A. Column line -- or Column 1, lines 4
22 through 6?

1 Q. Yes.

2 A. Okay. I'm there.

3 Q. Suzuki is directed to a direct injection
4 spark ignition engine control apparatus for
5 correcting engine torque based on engine operating
6 conditions; right?

7 A. Correct.

8 Q. And can you take a look at Column 1,
9 lines 44 through 48?

10 A. Yes.

11 Q. Suzuki's system controls torque by
12 controlling at least the spark timing to correct
13 torque during homogenous combustion and by
14 controlling at least the air/fuel ratio to correct
15 torque during stratified combustion; correct?

16 A. Yes.

17 Q. Can we take a look at Column 4, lines 32
18 through 40?

19 A. Yes.

20 Q. Suzuki discloses using a control
21 unit 20; correct?

22 A. Yes.

1 Q. And this control unit 20 controls the
2 degree of opening of the electronic control
3 throttle valve 4; correct?

4 A. Yes.

5 Q. And control unit 20 controls the amount
6 of fuel injected to the engine by the fuel
7 injector 5; correct?

8 A. Correct.

9 Q. And control unit 20 controls the spark
10 timing of the spark plug 6; correct?

11 A. Yeah.

12 Q. A person of ordinary skill in the art
13 would have known how to implement control unit 20;
14 correct?

15 A. Yes.

16 Q. Let's turn to Figure 6 of Suzuki.

17 A. Okay.

18 Q. And for your reference, the
19 corresponding description starts on Column 7,
20 line 8.

21 A. Figure 6. Hold on a second. This is
22 Suzuki 105. Say that column again.

1 Q. Column 7, lines -- starting at line 8.

2 A. Okay.

3 Q. At step S31 in Figure 6 of Suzuki, the
4 controller reads a demand torque; right?

5 A. It says here that a target torque, TRQ,
6 calculated by torque demand control is retrieved.
7 It didn't sound exactly like what you were saying.
8 I'm reading out of the reference Column 7, 14, 15.

9 Q. Let's look at box S31 in Figure 6.

10 A. Okay.

11 Q. In Figure 6, at step S31, the target
12 torque correction demand is read; correct?

13 A. That's what it says there, yes.

14 Q. At step S33 in Figure 6 of Suzuki, the
15 controller estimates a torque output; correct?

16 A. Torque output air correction, correct.

17 Q. What does box S33 say?

18 A. It says "Estimate output torque during
19 air correction."

20 Q. So at step S33 the controller is
21 estimating output torque?

22 A. Yes.

1 Q. At step S34 of Figure 6 of Suzuki, the
2 controller calculates a torque shortage; correct?

3 A. Correct.

4 Q. And that calculation is target torque
5 minus estimated torque; correct?

6 A. Yes.

7 Q. The resulting signal of target torque
8 minus estimated torque is a difference signal;
9 correct?

10 A. You could call it that.

11 Q. What is a difference signal?

12 A. I will say yes, I don't want to explain
13 it. That's a difference signal, yes.

14 Q. Sorry, I had audio difficulties there,
15 let me read that answer real quick.

16 Dr. Leonard, I apologize, I'm going to
17 reask the question so I can hear your response.

18 What is a difference signal?

19 A. The difference between two items, two
20 signals in this case.

21 Q. And the calculated torque shortage in
22 S34, that is a difference signal; correct?

1 A. Yes.

2 Q. In step S35 in Figure 6 of Suzuki, the
3 controller calculates a torque correction factor
4 piper in accordance with the difference between
5 the estimated torque and the target torque;
6 correct?

7 A. Hold on. Say that question again.

8 Q. In step S35 in Figure 6 of Suzuki, the
9 controller calculates a torque correction factor
10 piper in accordance with the difference between
11 the estimated torque and the target torque;
12 correct?

13 A. S35 calculates the torque correction
14 factor piper in accordance with the torque
15 shortage.

16 Q. And the torque shortage is the
17 difference between target torque and the estimated
18 torque; correct?

19 A. Yes.

20 Q. In step S38 in Figure 6 of Suzuki, the
21 controller calculates a spark timing correction
22 factor TQRET based on the torque correction factor

1 during a homogenous mode; correct?

2 A. Correct.

3 Q. In step S40 in Figure 6 of Suzuki, the
4 controller -- strike that.

5 Dr. Leonard, referencing Column 7,
6 lines 65 through 67 of Suzuki.

7 A. Yes.

8 Q. Suzuki's engine controller proceeds to
9 make a fuel adjustment as shown in Figure 5 during
10 stratified mode; correct?

11 A. Say your question again.

12 Q. And I will revise it slightly.
13 Referencing lines 60 through 67 of Column 7,
14 Suzuki's controller proceeds to make a fuel
15 adjustment as shown in Figure 5 during the
16 stratified mode; correct?

17 A. Hold on a second because I've got
18 a -- I'm looking at a different spot now so let me
19 look at it.

20 (Witness peruses the exhibit.)

21 THE WITNESS: Given that I looked at all
22 this stuff now, go ahead with your revised

1 question again.

2 BY MR. CONKLIN:

3 Q. Referencing lines 60 through 67 of
4 Column 7, Suzuki's controller proceeds to make a
5 fuel adjustment as shown in Figure 5 during the
6 stratified mode; correct?

7 A. Yes.

8 Q. Dr. Leonard, please review the first
9 paragraph of Column 8.

10 A. Okay.

11 Q. Steps S31 to S37, S38, S39, S12 and S13,
12 perform a homogenous combustion torque correcting
13 function; correct?

14 A. Yes.

15 Q. Steps S31 to S37, S40 and S22 to S25
16 perform a stratified combustion torque correcting
17 function; correct?

18 A. Yes.

19 Q. Dr. Leonard, while we have been
20 referencing Exhibit 1005 of IPR 2020-00444, this
21 particular publication is the same publication as
22 Exhibit 1005 of IPR 2020-00445; right?

1 A. Say your question again, I'm -- I just
2 found what you were talking about, part of it,
3 what was the first part? Just go ahead and repeat
4 it all.

5 Q. We've been discussing Exhibit 1005 of
6 IPR 2020-00444, this particular document is the
7 same document as Exhibit 1005 of IPR 2020-00445;
8 right?

9 A. Yes.

10 (Leonard Deposition Exhibit 5 was marked
11 for identification and attached to the
12 transcript.)

13 BY MR. CONKLIN:

14 Q. Dr. Leonard, can you please grab
15 Exhibit 1007 of IPR 2020-00444?

16 A. 1007.

17 Q. This is the Scherer reference,
18 Dr. Leonard.

19 A. Oh, 1007. Okay. I have it.

20 Q. And Exhibit 1007 of IPR 2020-00445 is
21 the same document as Exhibit 1007 of IPR
22 2020-00444; right?

1 A. Yes.

2 Q. I'll refer to this document as the
3 Scherer reference.

4 Do you understand that?

5 A. Yes.

6 Q. Let's take a look at Figure 1 of
7 Scherer.

8 A. Okay.

9 Q. Figure 1 of Scherer illustrates an upper
10 control branch made of components 2 and 3;
11 correct?

12 A. Yes.

13 Q. Figure 1 of Scherer illustrates a lower
14 control branch made up of components --

15 (Technical interruption.)

16 Q. Let me start that over.

17 Figure 1 of Scherer illustrates a lower
18 control branch made up of components 4, 5, 6, 7,
19 8, 9, 10, 11, 12 and 13; correct?

20 A. Correct.

21 Q. Let's discuss the upper control branch.
22 Elements 2 and 3 collectively shown in the upper

1 branch adjust the throttle valve to control torque
2 output; correct?

3 A. Say that again.

4 Q. Elements 2 and 3 of the upper control
5 branch adjust the throttle valve to control torque
6 output; correct?

7 A. Yes.

8 Q. Element 3 is a PID controller; correct?

9 A. Yes.

10 Q. The upper branch in Figure 1 of Scherer
11 is directed to adjusting the steady state
12 component of torque output; correct?

13 A. Say your question again because I've got
14 an answer for you, I just want to make sure I'm
15 answering you correctly.

16 Q. Certainly.

17 The upper branch in Figure 1 of Scherer
18 is directed to adjusting the steady state
19 component of torque output; correct?

20 A. What they say in the reference is that
21 it's a quasi-static desired engine torque
22 correction.

1 Q. So the upper branch in Figure 1 of
2 Scherer is directed to adjusting the quasi-static
3 component of the torque output; correct?

4 A. That's what I understand from the
5 reference.

6 Q. Let's look at the lower branch in
7 Figure 1 of Scherer. Elements 4 through 13 shown
8 in the lower branch of the torque control device
9 adjust the ignition displacement for auto engines
10 and fuel timing -- strike that.

11 Elements 4 through 13 collectively shown
12 in the lower branch of the torque control device
13 adjust ignition displacement for auto engines;
14 correct?

15 A. Hang on a second, I'm looking at
16 the -- I'm looking at the wrong figure. Figure 1;
17 correct?

18 Q. That's right.

19 A. And you're saying 4 through 13?

20 Q. Yes.

21 A. Yeah, that's the branch that adjusts
22 ignition point according to the reference.

1 Q. And Element 4 is a high-pass filter;
2 correct?

3 A. Yes, that's what they say in the
4 reference.

5 Q. The lower branch shown in Figure 1 of
6 Scherer is directed to adjusting the transient
7 component of the torque output; right?

8 A. In the reference, they call it dynamic
9 desired torque component.

10 Q. Is dynamic desired torque component
11 analogous to a transient torque component?

12 A. Yes.

13 Q. Is it fair to say that the lower branch
14 of Scherer constitutes a transient torque control?

15 A. I would classify it as a dynamic torque
16 control by adjusting the ignition point branch.
17 In a rough linkage, yes.

18 Q. Earlier you mentioned Element 4 as a
19 high-pass filter; correct?

20 A. Yes.

21 Q. The high-pass filter 4 is on the input
22 side of this ignition adjustment branch; correct?

1 A. Yes.

2 Q. And downstream of high-pass filter 4
3 would be the components that control the ignition
4 adjustment; correct?

5 A. There's additional devices that modify
6 the control further downstream. You're talking
7 6 through 13.

8 Q. What is signal ZS in Figure 1, next to
9 Element 12?

10 A. It's an adjusting signal for the
11 ignition point adjustment.

12 Q. And the high-pass filter 4 is upstream
13 of this adjusting signal ZS; correct?

14 A. It's upstream of that output ZS.

15 Q. Let's turn to Figure 2 of Scherer.

16 A. Okay.

17 Q. Figure 2 of Scherer illustrates an upper
18 control branch made of components 2 prime and
19 3 prime; correct?

20 A. Correct.

21 Q. Figure 2 of Scherer illustrates a lower
22 control branch made of components 4 through 9;

1 correct?

2 A. 4 through 9 prime, correct.

3 Q. The lower control branch in Figure 2
4 includes a high-pass filter 4; correct?

5 A. Yes.

6 Q. And high-pass filter 4 is upstream of
7 the injection start adjusting signal EBS; correct?

8 A. Yes.

9 Q. And in Figure 2, the high-pass filter 4
10 is on the input side of the ignition adjustment
11 branch; correct?

12 A. Say again.

13 Q. The high-pass filter 4 is on the input
14 side of the lower ignition adjustment branch;
15 correct?

16 A. On which figure?

17 Q. Figure 2.

18 A. No. Incorrect.

19 Q. Why is -- why do you say high-pass
20 filter 4 is not on the input side of that lower
21 branch?

22 A. Which figure?

1 Q. Figure 2.

2 A. Figure 2.

3 Q. Sorry, I think I -- I'm misspeaking
4 here, Dr. Leonard, leading to the confusion.

5 In Figure 2, high-pass filter 4 is on
6 the input side of the fuel adjustment branch;
7 correct?

8 A. EBS is an injection start adjusting
9 signal for a compression ignition diesel engine
10 so, no.

11 Q. High-pass filter 4 in Figure 2 of
12 Scherer is on the input side of the lower control
13 branch; correct?

14 A. Figure 2?

15 Q. Yes.

16 A. Yes.

17 Q. Dr. Leonard, please take a look at the
18 paragraph starting on Column 1, line 7.

19 A. Column 1. All right. Line -- okay.

20 Q. Scherer is directed to processes and
21 apparatuses for adjusting the torque of an engine;
22 correct?

1 A. Torque of internal combustion engine,
2 correct.

3 Q. Dr. Leonard, please take a look at
4 line -- the paragraph starting at line 38 of
5 Column 2.

6 A. 38. Yes.

7 Q. One advantage of Scherer's system was to
8 provide a system that adjusts the engine torque to
9 a desired value with the least possible
10 expenditures in both steady state and transient
11 engine operations with a comparatively short
12 reaction time; correct?

13 A. That's what it says, correct.

14 Q. Referencing Figure 1 in the passage
15 starting at line 20 of Column 4, Scherer's system
16 reacts very rapidly to sudden changes of engine
17 torque; correct?

18 A. Okay. So we are referencing Figure 1
19 and this is Column 4, line 20, go ahead and say
20 your question.

21 Q. Scherer's lower ignition point
22 adjustment branch reacts very rapidly to sudden

1 changes of the engine torque; correct?

2 A. Yes, that's what it says.

3 Q. And that would be an advantage of the
4 system?

5 A. That is one of the purported advantages.

6 Q. Is there any reason why that's not an
7 advantage?

8 A. No.

9 Q. Scherer's lower ignition point
10 adjustment branch in Figure 1 also minimizes
11 exhaust admission and fuel consumption; right?

12 A. Yes.

13 Q. The high-pass filter 4 in Figure 1
14 filters the dynamic desired engine torque part
15 from the desired engine torque supplied to it and
16 in turn provides the dynamic desired engine torque
17 to the device for the ignition point adjustment
18 which follows; correct?

19 A. Go ahead and say your question again.

20 Q. And I was referencing lines 40 through
21 45 of Column 4.

22 A. Thank you.

1 Q. The high-pass filter 4 in Figure 1
2 filters the dynamic desired engine torque part
3 from the desired engine torque supplied to it and
4 in turn provides the dynamic desired engine torque
5 to the device for the ignition point adjustment
6 which follows; correct?

7 A. Yes, that's what it says, yep.

8 Q. And due to the high-pass characteristic
9 of the high-pass filter 4 in Figure 1 of Scherer,
10 the dynamic desired torque component will
11 completely disappear over time; correct?

12 A. Repeat your question again, I'm sorry.

13 Q. And due to the high-pass characteristic
14 of the high-pass filter 4, the dynamic desired
15 torque component will completely disappear over
16 time; correct?

17 A. That's qualified at a constant engine
18 load, yes, if you add that into it.

19 Q. Due to the high-pass characteristic of
20 the high-pass filter stage, after the decay of the
21 transients during a change of the engine torque
22 desired and at a constant engine load, the dynamic

1 desired torque component will completely disappear
2 over time; correct?

3 A. After decay, which is time, correct.

4 Q. And this results and shows a return of
5 the engine operating condition to the optimal
6 range for both consumption and admission; correct?

7 A. That's what they claim, yes, or not
8 claim, but that's what they state.

9 Q. And that would be an advantage of
10 Scherer's system; correct?

11 A. That's what they're claiming or
12 purporting as an advantage, yes.

13 Q. Is there any reason why that would not
14 be an advantage?

15 A. No, there isn't any reason why it would
16 not be an advantage.

17 Q. In Figure 1 of Scherer, the adjusting
18 signal ZS is for ignition point adjustment;
19 correct?

20 A. This question sounds familiar. The
21 output signal of the adder 12 is used as an
22 adjusting signal ZS for the ignition point

1 adjustment. That's Column 5, line 5, 6.

2 Q. Is that a "yes," the adjusting signal ZS
3 is for ignition point adjustment?

4 A. Yes.

5 Q. I'm referencing line 9 of Column 5. The
6 embodiment of Figure 1 permits a comparatively
7 rapid engine torque adjustment which can follow
8 very fast desired engine torque changes and after
9 the conclusion of a transient phase, changes back
10 to comparatively rapidly to an operation which is
11 optimal with respect to the exhaust admissions and
12 fuel consumption; right?

13 A. Yes.

14 Q. And this would be an advantage of
15 Scherer's system?

16 A. Yes.

17 Q. Figure 2 of Scherer shows a system that
18 is functionally analogous to Figure 1; correct?

19 A. Yes.

20 Q. I think this is a good point to take a
21 break. Can we go off the record?

22 (Recess from the record.)

1 (Leonard Deposition Exhibit 6 was marked
2 for identification and attached to the
3 transcript.)

4 BY MR. CONKLIN:

5 Q. Dr. Leonard, can you please take a look
6 at Exhibit 1020 of IPR 2020-00445, that is the
7 Robichaux reference.

8 A. Okay.

9 Q. Robichaux is directed to a system and
10 method for controlling engine torque; correct?

11 A. Yes.

12 Q. Now, Dr. Leonard, can you please turn to
13 Figure 3 of Robichaux?

14 A. Okay.

15 Q. Do you see Element 210 in the upper
16 left-hand corner of that figure?

17 A. Yes.

18 Q. Element 210 is a comparator; correct?

19 A. Okay. Say question again.

20 Q. Element 210 in Figure 3 of Robichaux,
21 that is a comparator; correct?

22 A. Yes.

1 Q. And comparator 210 outputs a signal
2 TQ_ERR; correct?

3 A. Yes.

4 Q. Signal TQ_ERR is a difference signal;
5 correct?

6 A. Yes.

7 Q. Element 220 is a controller; correct?

8 A. A PID controller.

9 Q. Is that, yes, Element 220 is a
10 controller?

11 A. It is a PID controller, Element 220.

12 Q. What is a PID controller?

13 A. Proportional integral derivative.

14 Q. The difference signal TQ_ERR is an input
15 to controller 220; correct?

16 A. A PID controller 220, correct.

17 Q. Moving upstream from comparator 210, do
18 you see the input signal TQ_ENG?

19 A. No. Where are you looking at that?

20 Q. In the far left of --

21 A. Oh, your faces were covering it up.
22 TQ_ENG.

1 Q. Signal TQ_ENG is a demand brake torque
2 signal; correct?

3 A. Desired engine brake torque is what it
4 says in this specification.

5 Q. So input signal TQ_ENG is a desired
6 engine brake torque; correct?

7 A. Correct.

8 Q. Turning to Element 216 in Figure 3.

9 A. Okay.

10 Q. Block 216 in Figure 3 estimates the
11 actual engine brake torque; correct?

12 A. That's what it says.

13 Q. In paragraph 30 of Robichaux, it
14 mentions Block 218. Is Block 218 Block 210 in
15 Figure 3?

16 A. I think we're looking at two different
17 things here. Where are you looking, what page?

18 Q. In paragraph 30 of Robichaux, it
19 mentions Block 218.

20 A. On page 5?

21 Q. Is the reference to Block 218 in
22 paragraph 30 --

1 A. So I'm on page 5 and then there's
2 paragraph 30.

3 Q. Correct.

4 A. Go ahead. And what was your question?

5 Q. In paragraph 30 of Robichaux, it
6 mentions Block 218, is the reference Block 218 in
7 paragraph 30 the same as the illustrated Block 210
8 in Figure 3?

9 A. Block 218, is that in Figure 3 that
10 you're asking about?

11 Q. Yes.

12 The comparator 210 --

13 A. Okay.

14 Q. -- in Figure 3.

15 A. Yes.

16 Q. Is the description in paragraph 30
17 referencing Block 218 a typo to comparator 210?

18 MS. WILLIAMS: Objection; form.

19 BY MR. CONKLIN:

20 Q. Strike that.

21 In paragraph 30 of Robichaux, Block 218
22 outputs an actual -- strike that.

1 Block 218 of Robichaux outputs a TQ
2 error signal based on a difference between TQ sum
3 and TQ net; correct?

4 A. Which block again?

5 Q. Referencing Block 218 in paragraph 30.

6 A. Where does that reference in the figure,
7 though, I don't see that?

8 Q. I'm asking you about paragraph 30 right
9 now.

10 A. Okay.

11 Q. Does Block 218 in paragraph 30 output a
12 TQ error based on a difference between TQ sum and
13 TQ net?

14 A. It appears that that's what's written in
15 the specification.

16 Q. Turning to Figure 3 and comparator 210.
17 Comparator 210 outputs a TQ_ERR signal based on a
18 difference between TQ sum and TQ net; correct?

19 A. If you're talking about Figure 3 -- and
20 this is in reference to Block 210; correct --

21 Q. Yes.

22 A. -- in Figure 3?

1 Q. Yes.

2 A. Go ahead and repeat that again because
3 this 218 through 10 thing is weird, it's
4 confusing.

5 Q. Comparator 210 outputs a TQ error signal
6 that is a difference between TQ sum and TQ net;
7 correct?

8 A. It's TQ_ERR. There is a -- so, yeah,
9 TQ_ERR, yes, as long as that's their output of
10 Block 210.

11 THE REMOTE TECHNICIAN: Counsel, I
12 believe we just lost someone, I'm not sure if they
13 were scheduled to drop out of the meeting.

14 MR. CONKLIN: Let's go off the record.

15 (Recess from the record.)

16 BY MR. CONKLIN:

17 Q. Dr. Leonard, what -- in Figure 3 of
18 Robichaux, what is Block 202?

19 A. Block 202 I don't see a definition, but
20 it does say it controls at least one operating
21 parameter of the engine based on requested engine
22 brake torque to deliver the desired engine brake

1 torque. They just call it Block 202.

2 Q. So Block 202 controls the engine torque?

3 A. It delivers desired engine brake torque.

4 Q. Block 202 is downstream of comparator
5 210 in Figure 3; correct?

6 A. Yes.

7 Q. Block 202 is downstream of controller
8 220 in Figure 3; correct?

9 A. PID controller, yes, 220.

10 Q. Turning to paragraph 35 of Robichaux.

11 A. On page 5?

12 Q. Yes.

13 A. Okay.

14 Q. Take a second to review that paragraph.

15 (Witness peruses the exhibit.)

16 BY MR. CONKLIN:

17 Q. In addition to controlling airflow,
18 Block 202 can be configured to control fuel
19 delivery or fuel flow to control the engine output
20 torque; correct?

21 A. Say your question again, I finally read
22 the paragraph.

1 Q. In addition to controlling airflow,
2 Block 202 can be configured to control fuel
3 delivery or fuel flow to control the engine output
4 torque; correct?

5 A. No -- wait a minute, hang on.

6 Q. Dr. Leonard, please read lines 38 and
7 39.

8 A. In the embodiment illustrated in
9 Figure 3, Section 202 uses airflow as the primary
10 control parameter. Of course fuel delivery or
11 fuel flow may also be controlled to control the
12 engine output torque.

13 Q. In addition to controlling airflow,
14 Robichaux discloses that fuel delivery or fuel
15 flow can also be controlled to adjust engine
16 torque; correct?

17 A. He says that in that paragraph, I don't
18 see it in the Section 202, but, you know, yeah,
19 you are correct, he says that in that paragraph
20 you're referencing, but there's so many errors in
21 this reference, I just don't trust it.

22 Q. What other errors are there,

1 Dr. Leonard?

2 A. Well, there's the errors and I don't
3 know what exactly they were trying to say with the
4 Block 218 and then the TQ_ERR, which also shows up
5 maybe possibly as TQ_ERROR, I'm not really sure.
6 It makes it a little confusing there. And so --

7 Q. Dr. Leonard, what is integrator windup?

8 A. Where are you referring to?

9 Q. Earlier you mentioned that Dr. MacCarley
10 had a declaration that you reviewed; correct?

11 A. Okay.

12 Q. And he references integrator windup,
13 what is your understanding of integrator windup?

14 A. Was that the first declaration or --

15 Q. Let me strike. I'll provide some
16 context there.

17 A. Please.

18 Q. What is your understanding of integrator
19 windup in relation to PID controllers?

20 A. Well, PID controller, the I stands for
21 integral.

22 Q. And what is integrator windup?

1 A. So that's a situation in a PID, feedback
2 controller where you got a large change in set
3 point and the integral term accumulates a
4 significant error during that rise and it
5 overshoots. It kind of goes back to that thing
6 about the hunting that we talked about earlier,
7 you know, you overshoot.

8 Q. Is hunting the same as integrator
9 windup?

10 A. It's analogous in -- it's analogous.

11 Q. Have you designed a PID controller
12 before?

13 A. No.

14 Q. Have you designed controllers before?

15 A. Yes.

16 Q. Would a person of ordinary skill in the
17 art understand how to program a controller?

18 A. Yes.

19 Q. Would it have been routine at the time
20 of the '565 patent to know how to program a
21 controller?

22 A. Yes.

1 Q. In your declaration, did you cite any
2 secondary references to support your opinion?

3 A. On 444 or 445?

4 Q. We'll take them individually. In your
5 second declaration, Exhibit 2005, of IPR
6 2020-00444, did you cite any secondary textbooks
7 or articles to support your opinion?

8 A. On -- this is -- this is document 2004,
9 I thought you said 2005; but, anyway, no, I did
10 not.

11 Q. And to clarify the record, Exhibit 2004
12 is your second declaration in the IPR 2020-00444?

13 A. That's correct.

14 Q. Why didn't you cite any additional
15 secondary textbooks or articles to support your
16 opinion?

17 A. Because I didn't need to. I didn't feel
18 like I needed to.

19 Q. In your second declaration in IPR
20 2020-00445, did you cite any secondary textbooks
21 or articles to support your opinion?

22 A. That is also document 2004; correct?

1 Q. Yes.

2 A. No.

3 Q. Let's go back to Exhibit 1005, Suzuki
4 reference.

5 A. Okay.

6 Q. And will you specifically look at
7 Column 8, lines 51 through 58?

8 A. Column 8, 51 through 58?

9 Q. Correct.

10 A. Okay.

11 Q. Figure 13 shows the response waveforms
12 associated when the air conditioner is turned on;
13 right?

14 A. Yes.

15 Q. And earlier you mentioned turning the
16 air conditioner on is an example of a transient
17 condition; correct?

18 A. Yeah, when it comes on and when it goes
19 off.

20 Q. And that would create a stepped input;
21 correct?

22 A. It would create a stepped need for

1 increased torque. So it would be a step -- the
2 torque would actually be lowered sort of, you
3 know, shorted, be less than what you had because
4 it's being absorbed by that accessory.

5 Q. In Suzuki, the spark timing is corrected
6 to correct torque shortage during homogenous mode;
7 correct?

8 A. Spark timing is corrected to correct the
9 torque shortage during homogenous combustion.

10 Q. Is that a "yes," spark timing is
11 corrected to correct torque shortage during
12 homogenous combustion?

13 A. Yes.

14 Q. Dr. Leonard, can you turn back to
15 Exhibit 1007, the Scherer reference?

16 A. All right.

17 Q. Please review Column 3, lines 59 through
18 62.

19 A. Yes.

20 Q. Scherer describes that the desired
21 engine torque signal MDES outputted from the
22 torque coordination unit 1 is merely an example;

1 right?

2 A. Say that again.

3 Q. Let me try to clarify that.

4 A. Hold on a second, I got to close a door
5 here to eliminate some background noise.

6 Go ahead.

7 Q. Scherer discloses that one example of a
8 signal indicative of the desired torque -- strike
9 that.

10 Do you see in line 62 the phrase, "for
11 example"?

12 A. Okay. Yes.

13 Q. What does for example mean to you?

14 A. It's an example.

15 Q. Does that mean there are other
16 possibilities?

17 A. I mean, that calls for speculation, but,
18 you know, they're saying for example, so that's
19 the example given.

20 Q. But it's not limiting it to the desired
21 engine torque MDES in this instance?

22 A. You know, I'm looking at this and it

1 says, "On the input side, an engine torque
2 coordination unit 1 provides a signal indicative,
3 for example, of desired engine torque M sub DES
4 requested by the driver via the accelerator
5 pedal." You know, if there's another embodiment,
6 can you direct me to that where --

7 Q. I'm just trying to get your
8 understanding of what this sentence means.

9 A. What it means is that the engine torque
10 coordination unit 1 provides a signal indicative
11 of the desired engine torque M sub DES requested
12 by the driver via the accelerator pedal.

13 Q. Let's go off the record.

14 (Recess from the record.)

15 BY MR. CONKLIN:

16 Q. Dr. Leonard, I have no further
17 questions.

18 A. Okay.

19 MS. WILLIAMS: Kyle, give me about
20 five minutes to see if I want to redirect.

21 MR. CONKLIN: Okay.

22 MS. WILLIAMS: All right.

1 (Recess from the record.)

2 MS. WILLIAMS: Patent owner has no
3 questions on redirect. We're good to go.

4 MR. CONKLIN: Thank you, Dr. Leonard,
5 for your time today.

6 THE WITNESS: You're welcome. Thank
7 you.

8 MR. CONKLIN: Off the record.

9 MS. WILLIAMS: Off the record.

10 (Off the record at 1:15 p.m.)

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1 CERTIFICATE OF SHORTHAND REPORTER - NOTARY PUBLIC

2 I, Matthew Goldstein, Reporter and Notary Public
3 within and for the state of Maryland do hereby
4 certify:

5 That RUSSELL LEONARD, Ph.D., the witness whose
6 deposition is hereinbefore set forth, was duly sworn
7 by me before the commencement of such deposition and
8 that such deposition was taken before me and is a
9 true record of the testimony given by such witness.

10 I further certify that the adverse party,
11 VOLKSWAGEN GROUP OF AMERICA, INC, was represented
12 by counsel at the deposition.

13 I further certify that the deposition of RUSSELL
14 LEONARD, Ph.D. occurred virtually on Tuesday,
15 January 5, 2021, commencing at 9:31 a.m. to 1:15
16 p.m.

17 I further certify that I am not related to any of
18 the parties to this action by blood or marriage, I
19 am not employed by or an attorney to any of the
20 parties to this action, and that I am in no way
21 interested, financially or otherwise, in the outcome
22 of this matter.

1 IN WITNESS WHEREOF, I have hereunto set my
2 hand this 15th of January, 2021.

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14 _____

15 Matthew Goldstein
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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner,

v.

MICHIGAN MOTOR TECHNOLOGIES, LLC
Patent Owner

Case IPR2020-00444
Patent 6,581,565

**DECLARATION OF C. ARTHUR MACCARLEY, PH.D.
IN SUPPORT OF PETITIONER'S REPLY**

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Patent Trial and Appeal Board
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I, C. Arthur MacCarley, Ph.D., P.E., declare as follows:

I. INTRODUCTION

1. I have been retained as an expert in this proceeding by counsel for Volkswagen Group of America, Inc. I previously submitted a declaration (EX1003) in support of the Petition for *inter partes* review of U.S. Patent No. 6,581,565 (“the ’565 patent”). This declaration is in support of Petitioner’s Reply to Patent Owner’s Response in the same proceeding, IPR2020-00444.

2. I understand that Patent Owner Michigan Motor Technologies LLC submitted a Patent Owner Response (Paper 16) (“POR”). I have been asked to provide my technical review, analysis, insights, and opinions regarding the POR.

3. My background and qualifications were provided in paragraphs 11-19 of my original declaration, Exhibit 1003, and my CV was submitted as Exhibit 1004. My statements in my original declaration regarding my review of the ’565 patent and related materials remain unchanged. In reaching my opinions, I carefully reviewed the POR, the Petition for *inter partes* review of the ’565 patent, my original declaration, and the materials reviewed as part of my original declaration. EX1003, ¶20 (listing references considered in my prior declaration). In addition, I reviewed and refer to the following materials:

Paper	Description
11	Institution Decision

Paper	Description
16	Patent Owner’s Response

Exhibit	Description
1030	Dr. Leonard’s Deposition Transcript
1032	Ogata, Katsuhiko. Modern Control Engineering, 4th ed., Prentice Hall, 2002
1033	DAE Notes, RC High-Pass Filter Circuit, https://www.daenotes.com/electronics/digital-electronics/rc-high-pass-filter-circuit
2004	Dr. Leonard’s Second Declaration

4. I have also reviewed all other materials cited herein.
5. My work on this case is being billed at a rate of \$300 per hour, with reimbursement for actual expenses. My compensation is not contingent upon the outcome of this *inter partes* review (“IPR”).

II. LEVEL OF ORDINARY SKILL IN THE ART

6. I incorporate by reference my understanding of the person of ordinary skill in the art (“POSA”) as set forth in paragraphs 40-41 of my previous declaration.
7. As stated in my prior declaration, I understand a person of ordinary skill in the art (“POSA”) in the context of the ’565 patent as of July 23, 2001 (the earliest possible priority date of the ’565 patent) would possess a Bachelor of Science degree in electrical, mechanical, automotive, or a similar field of

engineering; be knowledgeable of the principles and operation of internal combustion engines of that time period; and have the equivalent of two years' experience or hands-on education in the design or implementation of automotive or vehicular control systems or components. Experience could take the place of some formal training, as domain knowledge may be learned on the job. This description is approximate, and a higher level of education or skill might make up for less experience and vice versa.

III. UNDERSTANDING OF THE LAW

8. I incorporate by reference my understanding of the legal principles, including my discussion of claim construction and obviousness, as set forth in my previous declaration. EX1003, ¶¶21-39. I include here certain additional legal understandings.

9. As stated in my prior declaration, I have been told the following legal principles apply to analysis of patentability. I also have been told that, in an *inter partes* review proceeding, a patent claim may be deemed unpatentable if it is shown by a preponderance of the evidence that the claim is anticipated by one prior art reference under 35 U.S.C. § 102, or rendered obvious by one or more prior art references under 35 U.S.C. § 103.

10. I understand that documents and materials such as printed publications or patents that qualify as prior art can be used to render a patent claim

as anticipated or as obvious.

11. I understand that, once the claims of a patent have been properly construed, the second step in determining anticipation or obviousness of a patent claim requires a comparison of the properly construed claim language to the prior art on a limitation-by-limitation basis.

A. My Understanding of Claim Construction

12. I understand that during an *inter partes* review proceeding, claims are to be construed in light of the specification as would be read by a person of ordinary skill in the relevant art at the time of the purported priority date. I understand that claim terms are given their ordinary and customary meaning as would be understood by a person of ordinary skill in the relevant art in the context of the entire disclosure. A claim term, however, will not receive its ordinary meaning if the patentee acted as his own lexicographer and clearly set forth a definition of the claim term in the specification. In this case, the claim term will receive the definition set forth in the patent.

B. A Person of Ordinary Skill in the Art

13. I understand that a person of ordinary skill in the relevant art (herein “POSA”) is presumed to be aware of all pertinent art, thinks along conventional wisdom in the art, and is a person of ordinary creativity—not an automaton.

14. I have been asked to consider the level of ordinary skill in the field

that someone would have had at the time the alleged invention was made. In deciding the level of ordinary skill, I considered the following:

- the levels of education and experience of persons working in the field;
- the types of problems encountered in the field; and
- the sophistication of the technology.

15. I understand that the relevant time for considering whether a claim would have been obvious to a POSA is at the time of the alleged invention, which I have been told to assume for this case is July 23, 2001.

16. Regardless if I use “I” or “POSA” during my technical analysis below, all of my statements and opinions are to be understood to be based on how a POSA would have understood or read a document at the time of the alleged invention.

C. My Understanding of Obviousness

17. I understand that a patent claim is unpatentable if the claimed invention would have been obvious to a person of ordinary skill in the art at the time of the alleged invention. I understand that this means that even if all of the elements of the claim cannot be found in a single prior art reference that would anticipate the claim, the claim can still be unpatentable.

18. It is my understanding that, to obtain a patent, a claimed invention

must have been, as of its earliest possible priority date, non-obvious in view of the prior art in the field. I understand that a patent claim is obvious when the differences between the subject matter sought to be patented and the prior art are such that the subject matter as a whole would have been obvious to a person of ordinary skill in the art at the time the alleged invention was made. When considering the issues of obviousness, I have been told that I am to do the following:

- Determine the scope and content of the prior art;
- Ascertain the differences between the prior art and the claims at issue;
- Resolve the level of ordinary skill in the pertinent art; and
- Consider evidence of secondary indicia of non-obviousness (if available).

19. With respect to determining the proper scope of prior art to examine, I understand that in order for a prior art reference to be properly used in an obviousness ground under 35 U.S.C. § 103, the prior art reference must be analogous art to the claimed invention. I have been told that a reference is analogous art to the claimed invention only if: (1) the reference is from the same field of endeavor as the claimed invention; or (2) the reference is reasonably pertinent to the particular problem solved by the inventor.

20. I understand that factors relevant to determining the proper field of endeavor include the inventor's explanations of the subject matter (including the patent specification), as well as the claimed invention's structure and function. And I further understand that to be reasonably pertinent to the particular problem solved by the inventor, a prior art reference must logically commend itself to the inventor's attention in considering his or her problem.

21. I have been told that an analogous reference may be modified or combined with other analogous references or with the POSA's own knowledge if the person would have found the modification or combination obvious. I have also been told that a POSA is presumed to know all relevant, analogous prior art, and the obviousness analysis may take into account the inferences and creative steps that a POSA would employ.

22. In determining whether a prior art reference would have been combined with another prior art reference or other information known to a POSA, I have been told that the following principles may be considered:

- A combination of familiar elements according to known methods is likely to be obvious if it yields predictable results;
- The simple substitution of one known element for another is likely to be obvious if it yields predictable results;
- The use of a known technique to improve similar devices,

products, or methods in the same way is likely to be obvious if it yields predictable results;

- The application of a known technique to a prior art reference that is ready for improvement is likely to be obvious if it yields predictable results;
- Any need or problem known in the field and addressed by the reference can provide a reason for combining the elements in the manner claimed;
- A skilled artisan often will be able to fit the teachings of multiple references together like a puzzle; and
- The proper analysis of obviousness requires a determination of whether a POSA would have a “reasonable expectation of success”—not “absolute predictability” of success—in achieving the claimed invention by combining prior art references.

23. I understand that whether a prior art reference renders a claim unpatentable as obvious is determined from the perspective of a POSA. I have also been told that, while there is no requirement for the prior art to contain an express suggestion to combine known elements to achieve the claimed invention, a suggestion to combine known elements to achieve the claimed invention may come from the prior art as a whole or individually, as filtered through the knowledge of

one skilled in the art. In addition, I have been told the inferences and creative steps a person of ordinary skill in the art would employ are also relevant to the determination of obviousness.

24. I also understand that when a work is available in one field, design alternatives and other market forces can prompt variations of it, either in the same field or in another. I have also been told that if a POSA can implement a predictable variation and would see the benefit of doing so, that variation is likely to be obvious. I have been told that in many fields, there may be little discussion of obvious combinations, and in these fields market demand—not scientific literature—may drive design trends. I have been told that, when there is a design need or market pressure and there are a finite number of predictable solutions, a POSA has good reason to pursue those known options.

25. I have been told there is no rigid rule prescribing that a reference or combination of references must contain a “teaching, suggestion, or motivation” to combine references. But I also have been told that the “teaching, suggestion, or motivation” test can be a useful guide in establishing a rationale for combining elements of the prior art. I have been told that this test poses the question as to whether there is an express or implied teaching, suggestion, or motivation to combine prior art elements in a way that realizes the claimed invention and that it seeks to counter impermissible hindsight analysis.

26. As stated in my prior declaration, I understand that a patent claim can be considered obvious to a POSA at the time the application was filed and that the teachings of two or more prior art references may be combined if that combination would have been obvious to POSA. EX1003, ¶35.

27. I further understand that a reference must be considered not only for what it expressly teaches but also for what the fairly suggests to a POSA.

28. I further understand that the test for obviousness is what the combined teachings of the references would have suggested to a POSA. I also understand that a finding of obviousness cannot be overcome by attacking references individually where the obviousness rationale is based upon the teachings of a combination of references. And I further understand that a claimed invention may be obvious even when the prior art does not teach each claim limitation, so long as the record contains some reason that would cause a POSA to modify the prior art to obtain the claimed invention.

29. I further understand that whether a prior art reference teaches away from the claimed invention is a factor to be considered in determining obviousness. I also understand that it is improper to combine references that teach away from their combination. I further understand that teaching away means that a reference criticizes, discredits, or otherwise discourages the solution claimed or the manner of combination. And I understand that a reference does not teach away for merely

expressing a general preference for an alternative invention.

30. As stated in my prior declaration, I understand that the proper construction of any term is how a person of ordinary skill in the art would have understood the term based on the term's use in the claims, specification, and file history of the patent. EX1003, ¶25.

IV. OVERVIEW OF THE '565 PATENT

31. I incorporate by reference my discussion of the overview of the '565 Patent, including my discussion of the technology overview, the problem that the '565 patent alleges existed in the prior art, the alleged invention of the '565 patent, the prosecution history summary, and claim construction as set forth in my previous declaration. EX1003, ¶¶42-71.

V. THE PRIOR ART DISCLOSES EVERY LIMITATION OF THE CHALLENGED CLAIMS.

32. From my review of the POR, Dr. Leonard's declaration, and Dr. Leonard's transcript, Patent Owner's arguments are directed to a single limitation: "a high pass filter which receives as an input the difference signal." Patent Owner does also not contend that any elements recited in claims 2-6 are absent from the prior art.

33. I note that Patent Owner's declarant Dr. Leonard admitted that a device having a torque demand controller generating a torque demand signal and a torque producer receiving the torque demand signal were well known before the

time of the '565 patent. EX1030, 25:6-8, 27:8-11. Dr. Leonard also admitted that using an estimator for outputting an estimated torque signal, a comparator for outputting a difference signal based on the estimated torque signal and the demand torque signal, and a high-pass filter for outputting a filtered signal to a transient torque controller were well known before the time of the '565 patent. EX1030, 25:9-20, 27:12-28:1. Dr. Leonard further admitted that using a transient torque controller for outputting spark and fuel adjustment signals based on high-pass filtered signals was well known. EX1030, 25:21-26:1, 28:2-11. Patent Owner's only dispute is that the applied art fails to disclose the recited arrangement of a high-pass filter receiving a difference signal outputted from a comparator. POR, 9 EX1030, 26:2-27:3.

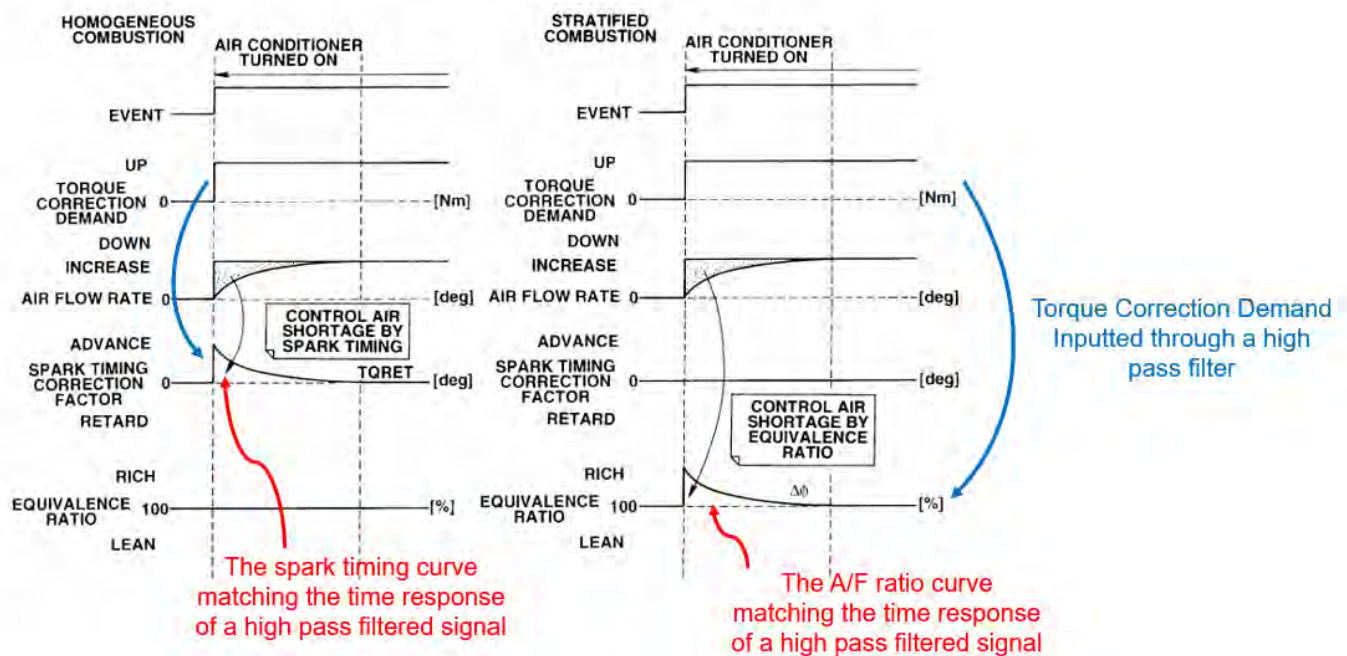
34. As explained in my prior declaration, the combination of Suzuki, Cullen, and Scherer discloses all the limitations of independent claims 1 and 7, including the contested limitation "a high pass filter which receives as an input the difference signal." DI, 36-38; EX1003, ¶¶85-153. In my opinion, the POR (1) ignores the Suzuki's implicit high-pass filtering teachings, (2) ignores what the combination of Suzuki and Scherer suggests as a whole; (3) applies a flawed argument that Scherer teaches away from Suzuki.

A. Suzuki alone implies high-pass filtering a difference signal.

35. From my review of the POR, Dr. Leonard's declaration, and Dr.

Leonard's transcript, Patent Owner does not dispute that Suzuki discloses a comparator outputting a difference signal representing the difference between an estimated torque and a demand torque. *See generally* POR; EX1030, 37:14-39:1. Nor does Patent Owner dispute that Suzuki discloses a transient controller receiving the difference signal as an input and outputting fuel and spark adjustment signals. *See generally* POR; EX1030, 39:20-41:18. Instead, Patent Owner contends that Suzuki does not disclose a high-pass filter that receives the difference signal from the comparator because Suzuki does not expressly disclose the term "high-pass filter" in the written specification. POR, 18-25.

36. Yet the function of a high-pass filter is clearly disclosed in Suzuki. As I explained in my first declaration, Figures 12-15 of Suzuki implicitly disclose a high-pass filter receiving a torque difference signal by showing its time response to a step change in the torque difference signal. The output of this filter is then passed to the homogeneous and stratified torque correction sections. EX1003, ¶¶94, 124-127; EX1005, 7:51-64, 8:1-4, 8:51-60; EX2003, 10:3-11:9. Specifically, the torque correction factor signals—spark timing correction for homogeneous mode and fuel injection adjustment for stratified mode, as illustrated in Suzuki's Figures 12-15, represent exactly the time response curve of a high-pass filter. EX1003, ¶¶94, 124-127; EX1005, 7:51-64, 8:1-4, 8:51-60.



EX1005, FIG. 13 (annotated).

37. Indeed, in my previous declaration, I provided a simulated correction signal (red) outputted from a high filter in response to a step function (green), which matched Suzuki's spark timing and equivalence ratio correction factor signals in response to step correction demands. EX1003, ¶126. While Suzuki does not explicitly refer to a high-pass filter, a POSA would have understood that Suzuki implicitly discloses a high-pass filter receiving the torque shortage signal to generate the torque correction curves for at least two reasons.

38. First, the need to use high-pass filters to supplement torque during a rapid throttle input transient, equivalent to an abrupt torque demand change, has been well known and used in most automotive fuel system well before the '565 patent. For example, high-pass filters have been implemented by an accelerator

pump on automotive carburetors well before the '565 patent. And high-pass filters have been implemented by equivalent transient fuel enhancement algorithms in the electronic control units of nearly all electronic fuel injection systems before the '565 patent. Thus, a POSA would have understood that Suzuki's torque correction signals implicitly disclose, or at the least suggest, the use of a high-pass filter processing the torque shortage signal to supplement torque during a transient condition.

39. Second, a "high pass filter" is known in the art by various nomenclatures and represented by many mathematical and graphical models, depending upon the application. For example, a first-order high-pass filter is most generally defined by its Laplace transfer function $H(s) = \frac{s+a}{s+b}$ including the special case $H(s) = \frac{s}{s+b}$ which is the filter referred to in the '565 patent. The transient response of this transfer function is shown below. EX1033, DAE Notes website, 2. Notably, the filtered output wave (labeled as (C) Output Wave) of the high-pass filter circuit matches the torque correction curves shown in Figure 13 of Suzuki.

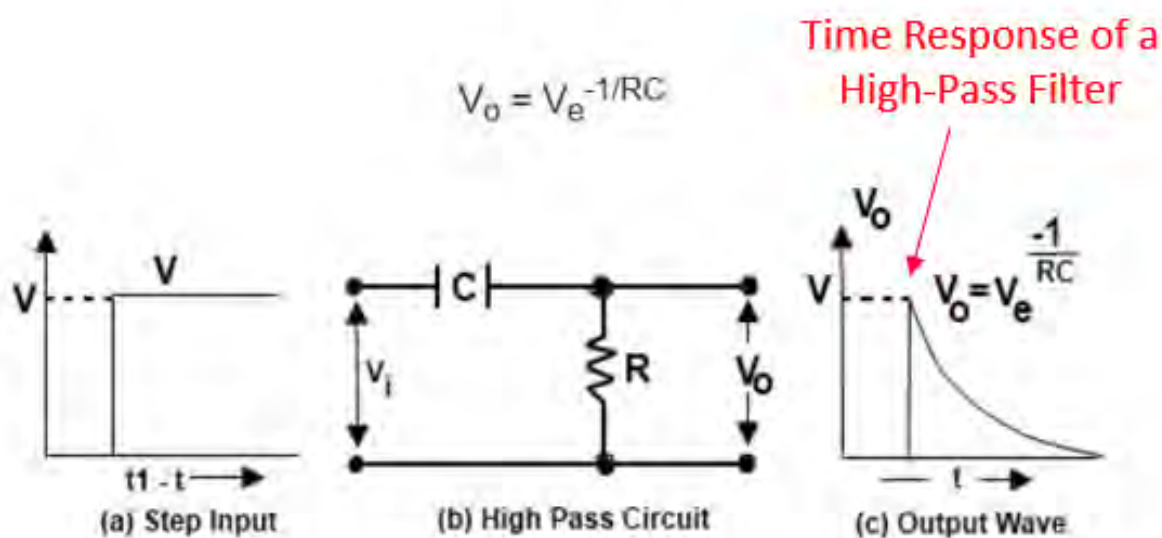


Figure 2: Step Voltage Input

EX1033, FIG. 2 (annotated).

40. Regarding the nomenclature, a high-pass filter is also commonly known as a low-frequency-block or DC-block filter (DC = direct current, or the steady state value of electrical current) in the field of signal processing. EX1033, 1. In control systems, high-pass filters were known before the '565 patent as a lead compensator, which are one of the most commonly-used controllers in its range of practical uses. EX1032, 622 (“As seen from Figure 9-4, **the lead compensator is basically a high-pass filter**. ... The high frequencies are passed, but low frequencies are attenuated”) (emphasis added). And as shown below, lead compensators (i.e., a high-pass filter) are commonly located downstream of a comparator junction of a feedback loop and upstream of another functional block for generating a control signal. EX1032, 622-623 (“The primary function of the

lead compensator is to reshape the frequency-response curve ... to **offset the excessive phase lag** associated with components of the fixed system.”) (emphasis added). Thus, a POSA would have understood that particular functionalities by a torque controller, such as the generation of the torque correction curves in Suzuki, implicitly discloses the use of a high-pass filter, even if Suzuki does not expressly disclose the term “high-pass filter” in the specification of the application.

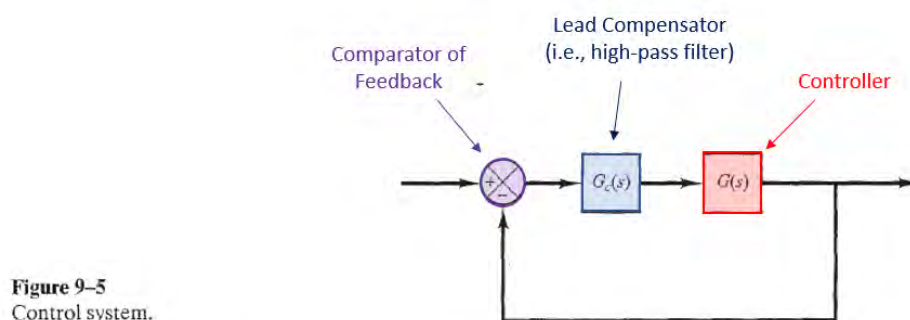


Figure 9-5
Control system.

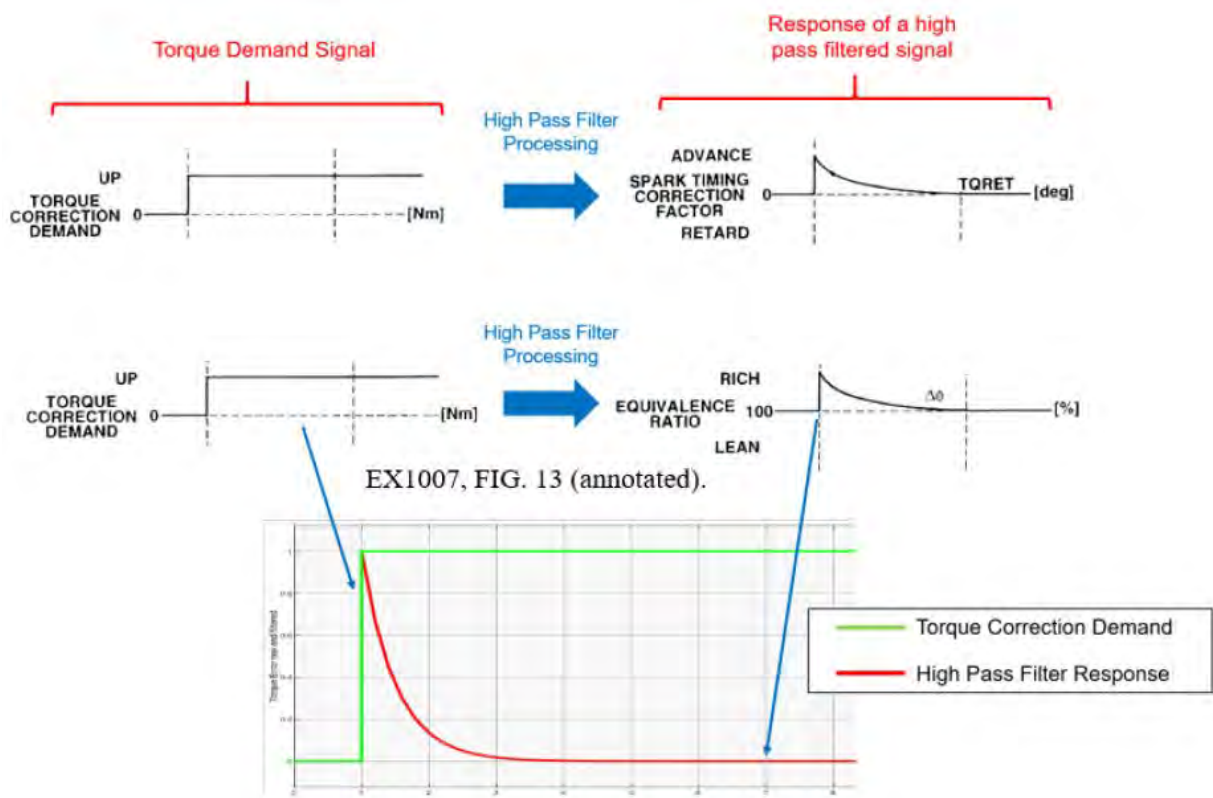
1. Assume the following lead compensator:

$$G_c(s) = K_c \alpha \frac{Ts + 1}{\alpha Ts + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \quad (0 < \alpha < 1).$$

EX1033, FIG. 9-5 (annotated).

41. As shown below in Figure 13 of Suzuki, the curves illustrated in Suzuki’s spark timing and equivalent ratio correction factor graphs implicitly disclose a filter processing the torque shortage (i.e., difference) signal. Indeed, like Scherer’s high-pass filtered dynamic torque component, Suzuki’s spark timing and equivalence ratio transient correction factors provide significant adjustments within a short time period and then disappear over time. EX1005, 8:51-61;

EX1007, 4:45-52 (“Due to the high pass characteristic of the high pass filter stage, ... the dynamic desired torque component (M_d) will completely disappear.”) (emphasis added). This expedient adjustment shown by the spark timing and equivalence ratio transient correction factor curves offset the lag shown in the air flow curves in Figure 13 of Suzuki. In my opinion, a POSA would have understood that Suzuki’s correction factor graphs shown in Figure 13 implicitly discloses a high-pass filter receiving the difference signal in order correct the torque shortage via significant fuel and spark adjustments that disappear over time. EX1003, ¶¶103, 127.



EX1003, FIG. A.

42. Notably, neither Patent Owner nor Patent Owner's expert Dr. Leonard addresses my testimony specifically explaining how the Suzuki's torque correction factor signals illustrated in Figures 12-15 imply the use of a high-pass filter. POR, 18-25; EX2004, ¶¶26-36. This omission is highly misleading. Both Patent Owner and Dr. Leonard ignore the illustrated torque correction factor signals and, instead, selectively paraphrase Suzuki's text that describes the vehicle operating conditions that trigger the torque demand shown in each of Suzuki's Figures 12-15. POR, 18-25; EX2004, ¶¶32-36. Patent Owner and Dr. Leonard do not provide any alternative explanation for how the torque correction factor signals are generated in Figures 12-15 of Suzuki. POR, 18-25; EX2004, ¶¶32-36. Patent Owner and Dr. Leonard also fail to explain how Suzuki's torque correction factor signals provide significant adjustments within a short time period and then disappear over time without using a high-pass filter. POR, 18-25; EX2004, ¶¶32-36.

43. Instead, Patent Owner and Dr. Leonard ignore the illustrated correction factor signal waveform shapes. Both Patent Owner and Dr. Leonard conclude these figures do not disclose high-pass filtering without addressing the illustrated torque correction factor signal waveform shapes or providing any related technical analysis. POR, 18-25; EX2004, ¶¶26-36. In my opinion, Patent Owner's rebuttal lacks technical bases and is full of incorrect, conclusory statements.

44. Patent Owner further contends that if Suzuki supposedly used a high-pass filter, “Suzuki would only high pass filter the ‘torque correction demand signal.’” POR, 24. Patent Owner’s assertion is technically wrong because Suzuki indeed filters the torque shortage signal to generate the torque correction factor curves as I explain below.

45. Suzuki discloses that the torque correction factor curves correspond to the adjustments made to spark timing during homogeneous mode and fuel injection during stratified mode **“to correct the torque shortage”** that is caused by the “delay in increasing the actual amount of air to the engine.” EX1005, 8:51-61. (emphasis added). To correct for the torque shortage, Suzuki’s torque correction factor signals are generated based on the difference between the demand torque and the estimated torque—a torque difference. That is, **Suzuki’s torque correction factor signals would not be able to correct for torque shortage without first calculating the difference between the estimated and demand torques.** Thus, in my opinion, Suzuki’s transient torque controller receives the high-pass filtered torque difference as an input before generating the torque correction factor signals.

B. The combination of Suzuki and Scherer renders obvious high-pass filtering a difference signal.

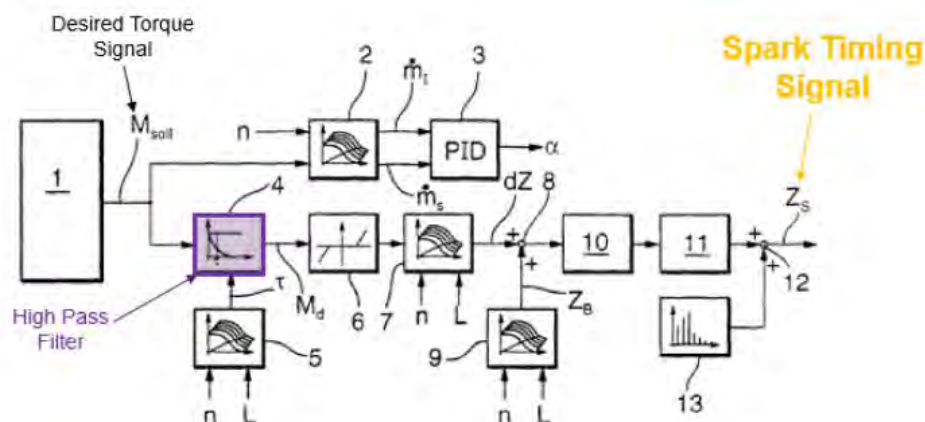
46. In my opinion, the Board’s Institution Decision correctly found based on Petitioner’s evidence that the combination of Suzuki and Scherer renders

obvious a high-pass filter that receives a difference signal representing the difference between a torque demand and an estimated torque. DI, 36-38; EX1003, ¶¶94-103.

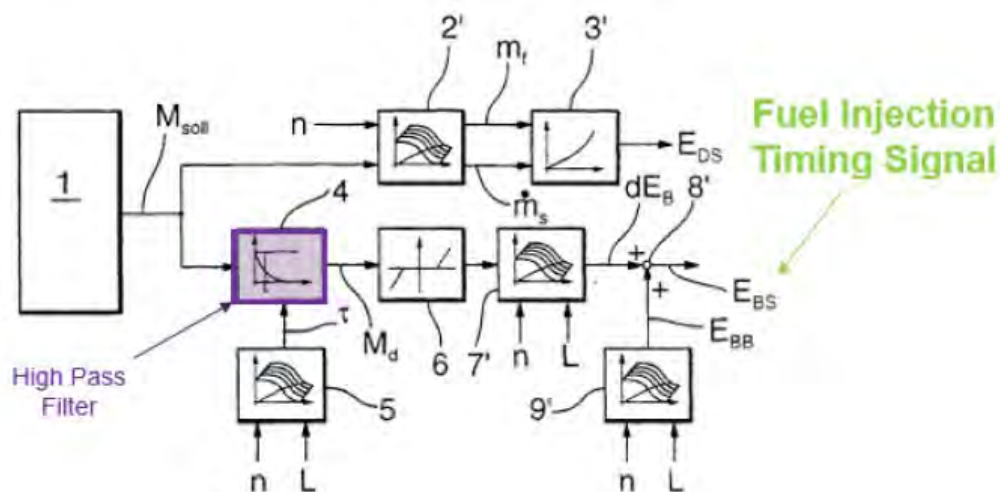
47. Scherer discloses a high-pass filter 4 that “filters the dynamic desired engine torque part (M_d) ... from the desired engine torque (M_{des}) ... and in turn provides the dynamic desired engine torque (M_d) to the device for the ignition point adjustment” in Figure 1 and the device for fuel adjustment in Figure 2. EX1007, 4:40-45. Scherer’s ignition point adjustment system in Figure 1 and fuel adjustment system in Figure 2 are used “to adjust specifically that dynamic desired engine torque component which cannot be regulated by the air-flow rate-regulating branch.” EX1007, 4:20-31. As explained in my previous declaration, a POSA would have understood that Scherer’s technique for adjusting ignition timing and fuel injection based on the high-pass filtered torque signal are analogous to Suzuki’s homogeneous and stratified torque correction sections. EX1003, ¶98. Accordingly, it would have been obvious to add Scherer’s high-filter 4 **immediately upstream** of Suzuki’s homogeneous and stratified torque correction sections—Suzuki’s transient torque controller. EX1003, ¶100 (“Given that the homogeneous and stratified torque correction sections in Suzuki’s control unit 20 are intended to respond to dynamic changes in torque demand, a POSA would have been motivated to use Scherer’s high-pass filter 4 to receive Suzuki’s

torque difference signal and output the dynamic component of the torque difference signal to Suzuki's homogeneous and stratified torque correction sections.") (emphasis added). As the Board correctly observed in the Institution Decision, situating Scherer's high-pass filter 4 immediately upstream of Suzuki's homogeneous and stratified torque correction sections would result in the high-pass filter 4 receiving the difference signal outputted from Suzuki's comparator. DI, 37.

48. In my opinion, when looking to combine Scherer's high-pass filter with Suzuki's torque control unit, a POSA would have located the high-pass filter immediately upstream of the Suzuki's homogeneous and stratified torque correction sections, as suggested by Scherer. As shown below in Figure 1 of Scherer, the high-pass filter 4 is situated immediately upstream of the provisions for spark adjustment so that the "ignition point adjusting branch is used only to adjust specifically that dynamic desired engine torque component." EX1007, 4:20-31. And as shown below in Figure 2 of Scherer, the high-pass filter 4 is situated immediately upstream of the provisions for fuel adjustment so that the fuel adjusting branch calculates "an injection start displacement (dE_B) corresponding to the dynamic desired engine torque component (M_d)." EX1007, 5:35-45.



EX1007, FIG. 1 (annotated).



EX1007, FIG. 2 (annotated).

Because Suzuki's homogeneous and stratified torque correcting sections are directed to adjusting the dynamic torque component, a POSA would have been motivated to similarly locate Scherer's high-pass filter immediately upstream of Suzuki's homogeneous and stratified torque correcting sections to receive the dynamic torque component outputted from the high-pass filter. EX1005, 3:5-22.

49. Moreover, in my opinion, a POSA would have understood that the

high-pass filter is intended to be situated at the input side of the transient torque controller—ignition point or fuel injection branch—**regardless of receiving a torque demand or a torque difference signal**. First, as acknowledged by Dr. Leonard, Scherer discloses that the desired torque signal outputted from the engine torque coordination unit 1 is merely an “example,” implying that other types of signals, such as a difference signal, may be inputted into the high-pass filter. EX1007, 3:60-63 (“[A]n engine torque coordination unit 1 provides a signal indicative, **for example**, of the desired engine torque (M_{des}) requested by the driver via the accelerator pedal.”) (emphasis added); EX1030, 68:13-69:12. Second, Scherer discloses particular advantages associated with high-pass filter signal processing that are applicable to both a torque demand signal and a difference signal. EX1007, 4:45-52, 5:10-15. These advantages include (1) removing the low frequency component to permit a rapid response to correcting torque output and (2) reducing the torque output as the transient component decays to zero. EX1007, 4:45-52, 5:10-15; EX1030, 50:7-53:16. Third, a POSA would have understood at the time of the ’565 patent that high-pass filters, such as a lead compensator, are commonly located downstream of a comparator junction of a feedback loop and upstream of another functional block, such as a transient controller in a control system, to offset lag during transient conditions. EX1032, 622-23, FIG. 9-5. Thus, a POSA would have understood that a torque difference signal, rather than the

Scherer's torque desired signal, can be inputted to a high-pass filter at an input side of a transient controller (i.e., Suzuki's homogeneous and stratified torque correcting sections) to expediently adjust torque output.

50. And a POSA would have understood that the transient difference signal in Suzuki is further processed to translate the difference signal into a corrective action to be performed by the transient torque controller—Suzuki's homogeneous and stratified torque correction sections. Indeed, as shown in Figure 6, Suzuki processes the difference signal in step S35 to determine a torque correction factor, which is then inputted into the homogeneous and stratified torque correction sections. In homogeneous-charge mode, the difference signal is translated to an adjustment of the ignition (spark) timing, which is the fastest possible way to adjust torque during homogeneous mode. In stratified-charge mode, the difference signal is translated to an adjustment of injected fuel quantity (i.e., to correct air-fuel ratio), which is the fastest torque-correcting action during stratified mode. A POSA would have known that one common technique for translating the difference signal to spark and fuel adjustments is using a high-pass filter.

51. By situating the high-pass filter immediately upstream of Suzuki's homogeneous and stratified torque correction sections, Scherer's high-pass filter would further process Suzuki's difference signal before being inputted into the

homogeneous and torque correction sections so that “variation of the ignition point or of the injection start is expediently kept as small as possible.” EX1007, 2:54-62, 4:45-52.

52. From my review of the POR and Dr. Leonard’s declaration, Patent Owner attacks Scherer in isolation and ignores what Suzuki and Scherer suggest as a whole to render obvious a high-pass filter receiving a difference signal.

53. Patent Owner argues that “no prior art reference relied upon by the Petitioner or the Board discloses producing a ‘difference signal’ by comparing an ‘estimated torque signal’ and a ‘desired torque signal’ and then high-pass filtering such ‘difference signal.’” POR, 24. Patent Owner points to the embodiments shown in Figures 1 and 2 of Scherer and argues that “only the desired torque signal is high-pass filtered and no comparator is used to compare an ‘estimated torque signal’ to the ‘desired torque signal.’” POR, 26. This is incorrect. Patent Owner’s argument focuses solely on Scherer and ignores the combined teachings of Suzuki outputting a difference signal to a transient torque controller and Scherer situating a high-pass filter on the input side of the transient torque controller, which renders the high-pass filter of claims 1 and 7 obvious. EX1005, 7:35-44; EX1007, 4:20-22 (“In order to adjust the dynamic desired torque component which exceeds this quasi-static part, an ignition point adjusting branch, with **the input-side high-pass filter**, is provided in parallel to the air-flow-rate-regulating branch.”) (emphasis

added).

54. Accordingly, as explained in my first declaration, aside from the fact that Suzuki already implicitly discloses a high-pass filter, a POSA would have understood to use Scherer's high-pass filter 4 to receive Suzuki's torque difference signal and output the dynamic (transient) component of the torque difference signal to Suzuki's homogeneous and stratified torque correction sections to improve the response time of Suzuki's controller. EX1003, ¶98.

C. A POSA would have been motivated to combine Suzuki and Scherer.

1. My first declaration explained why a POSA would have combined Scherer and Suzuki.

55. In my opinion, the Board correctly determined that "Petitioner provides a reasoned explanation" why a skilled artisan would have combined Scherer's high-pass filter with Suzuki's torque controller. DI, 37. Combining Scherer's high-pass filter with Suzuki's torque controller would have yielded the predictable advantage of adjusting the torque output with a quicker response time during transient engine operations. DI, 32, 36-37; EX1003, ¶¶98-101; EX1007, 5:10-15 ("The embodiment of FIG. 1 therefore permits a comparatively **rapid engine torque adjustment** which can follow very fast desired engine torque changes") (emphasis added). As I explained in my first declaration, the similarities between Suzuki and Scherer would have encouraged a POSA to combine their

teachings. Namely, Suzuki and Scherer are both in the field of engine control systems for monitoring torque output and controlling engine parameters in accordance with monitored torque output. EX1003, ¶¶86, 101. And as I explained in my first declaration, a POSA would have understood how to combine Suzuki's and Scherer's teachings by simply modifying the software instructions stored in the microcomputer of Suzuki's control unit. EX1003, ¶99. EX1030, 64:16-65:2. As I previously opined, combining Scherer's high-pass filter 4 with Suzuki's control unit 20 would have merely been applying a known technique in a similar, known device in the same way to expedite the response time for correcting the transient component of the torque demand. EX1003, ¶¶98-103.

56. Accordingly, contrary to Patent Owner's assertions (*see, e.g.*, POR, 15, 24-25, 29-30, 34), the motivation to combine was based on the prior art teachings themselves, not hindsight.

2. Scherer does not teach away from the claimed invention.

57. Patent Owner asserts that Scherer teaches away from being combined with Suzuki because the alternative embodiment shown in Figures 3 and 4 of Scherer uses a model-based estimated value stage, instead of a high-pass filter, to determine the dynamic desired engine torque component. POR, 25-29. Because Scherer's model-based estimated value stage employs an estimated determination unit 14 and a subtractor 15, rather than a high-pass filter, Patent Owner argues that

Scherer discourages a POSA from using a high-pass filter to receive a torque difference signal outputted from a comparator. POR, 27-29.

58. However, I disagree that Scherer's alternative embodiment shown in Figures 3 and 4 teaches away from being combined with Suzuki. Scherer's model-based estimated determination stage and subtractor are a programmable construct that can be implemented in software with other functionalities, such as a high-pass filter. The programming of the model-based estimator is left unspecified in Scherer to allow such generality in a given engine application. Thus, in my opinion, Scherer's alternative embodiment in Figures 3 and 4 invites the combination of components, such as a high-pass filter receiving a torque difference signal generated by a comparator.

59. Furthermore, nowhere does Scherer criticize, discredit, or discourage a POSA from using a high-pass filter that receives a torque difference signal. Patent Owner argues that Scherer's description of the embodiment shown in Figures 3 and 4 discourages a POSA from using of a high-pass filter following a comparator. POR, 27-29. This section of Scherer discloses "[t]his embodiment corresponds to that of FIG. 1 except that, instead of the high pass filter stage, a model-based estimated value stage is provided to determine the dynamic desired engine torque component." EX1007, 6:3-12; POR, 27.

60. Scherer's description of an alternative, more general embodiment

would not have discouraged a POSA from using a high-pass filter that receives a torque difference signal. Rather, Scherer merely discloses two ways—a high-pass filter and a more general, model-based estimation value stage—for determining a dynamic component of a torque signal. Scherer never discloses that the model-estimation (Figures 3 and 4) and the high-pass filter (Figures 1 and 2) are mutually exclusive. And Scherer does not discredit using both a high-pass filter and an estimation stage in a torque producer of an engine controller.

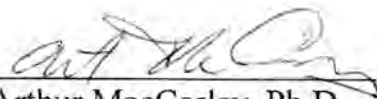
61. Thus, in my opinion, Scherer's two alternative embodiments—high-pass filtering in Figures 1 and 2 and the model estimation in Figures 3 and 4—are not a teaching away.

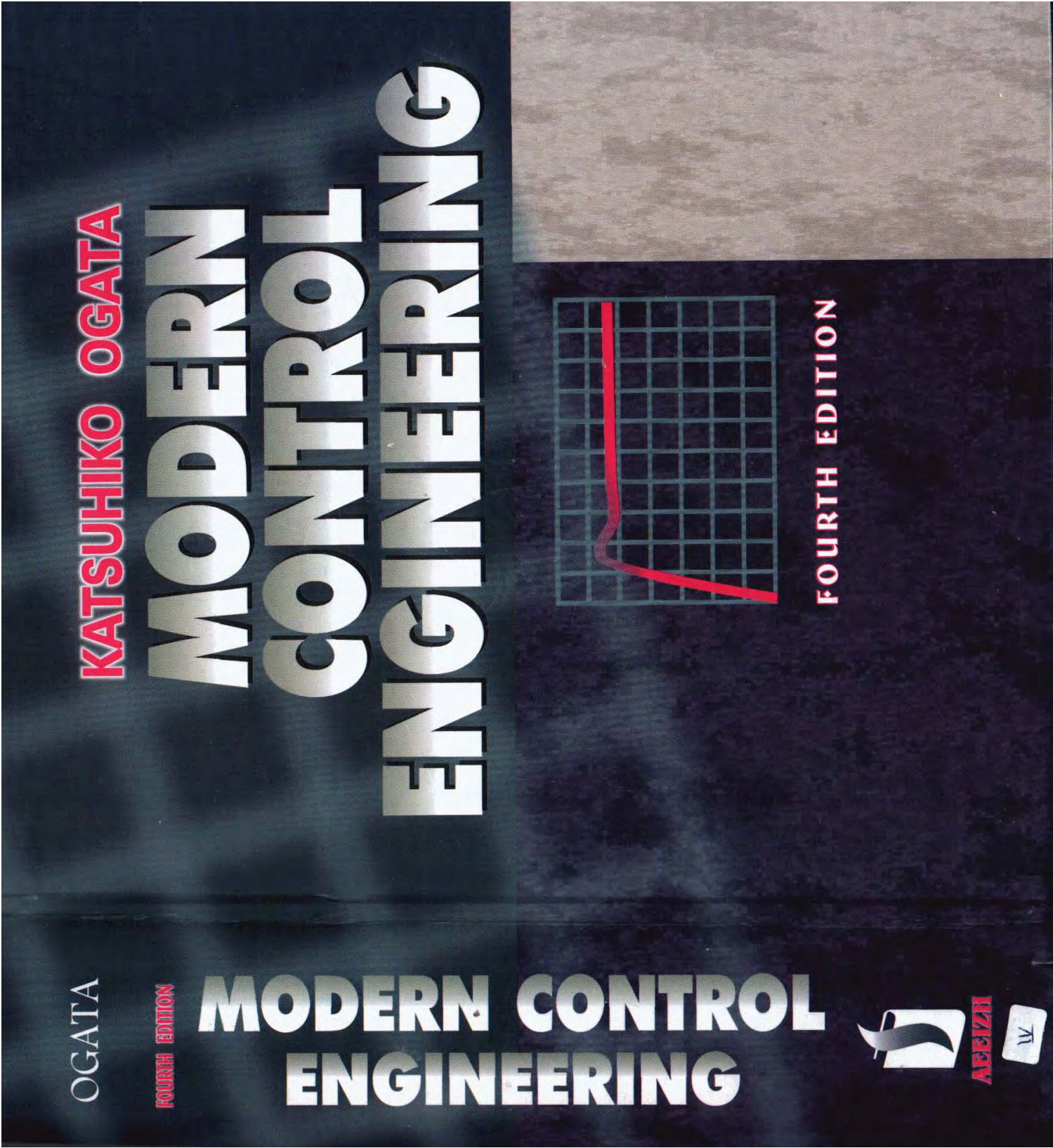
VI. CONCLUSION

In signing this Declaration, I recognize that this Declaration will be filed as evidence in an *inter partes* review before the Patent Trial and Appeal Board of the United States Patent and Trademark Office. I also recognize that I may be subject to cross-examination in the case and that cross-examination will take place within the United States. If cross-examination is required, I will appear for cross-examination within the United States during the time allotted.

I hereby declare that all statements made herein are made of my own knowledge and are true and that all statements made on information and belief are believed to be true; and further that these statements were made with the knowledge that willful false statements and the like so made are punishable by fine or imprisonment, or both, under Section 1001 of Title 18 of the United States Code.

Executed on this 21th day of January, 2021.


C. Arthur MacCarley, Ph.D.

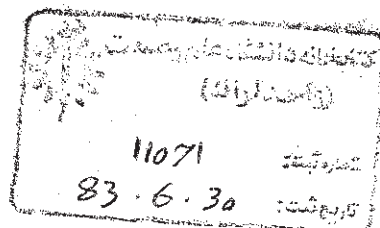


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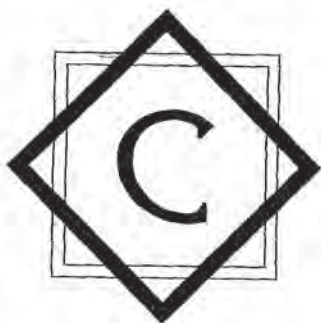
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Preface

This book presents a comprehensive treatment of the analysis and design of control systems. It is written at the level of the senior engineering (mechanical, electrical, aerospace, and chemical) student and is intended to be used as a text for the first course in control systems. The prerequisite on the part of the reader is that he or she has had introductory courses on differential equations, vector-matrix analysis, circuit analysis, and mechanics.

The main revision made in the fourth edition of the text is to present two-degrees-of-freedom control systems to design high performance control systems such that steady-state errors in following step, ramp, and acceleration inputs become zero. Also, newly presented is the computational (MATLAB) approach to determine the pole-zero locations of the controller to obtain the desired transient response characteristics such that the maximum overshoot and settling time in the step response be within the specified values. These subjects are discussed in Chapter 10. Also, Chapter 5 (primarily transient response analysis) and Chapter 12 (primarily pole placement and observer design) are expanded using MATLAB. Many new solved problems are added to these chapters so that the reader will have a good understanding of the MATLAB approach to the analysis and design of control systems. Throughout the book computational problems are solved with MATLAB.

This text is organized into 12 chapters. The outline of the book is as follows. Chapter 1 presents an introduction to control systems. Chapter 2 deals with Laplace transforms of commonly encountered time functions and some of the useful theorems on Laplace transforms. (If the students have an adequate background on Laplace transforms, this chapter may be skipped.) Chapter 3 treats mathematical modeling of dynamic systems

(mostly mechanical, electrical, and electronic systems) and develops transfer function models and state-space models. This chapter also introduces signal flow graphs. Discussions of a linearization technique for nonlinear mathematical models are included in this chapter.

Chapter 4 presents mathematical modeling of fluid systems (such as liquid-level systems, pneumatic systems, and hydraulic systems) and thermal systems. Chapter 5 treats transient response analyses of dynamic systems to step, ramp, and impulse inputs. MATLAB is extensively used for transient response analysis. Routh's stability criterion is presented in this chapter for the stability analysis of higher order systems. Steady-state error analysis of unity-feedback control systems is also presented in this chapter.

Chapter 6 treats the root-locus analysis of control systems. Plotting root loci with MATLAB is discussed in detail. In this chapter root-locus analyses of positive-feedback systems, conditionally stable systems, and systems with transport lag are included. Chapter 7 presents the design of lead, lag, and lag-lead compensators with the root-locus method. Both series and parallel compensation techniques are discussed.

Chapter 8 presents basic materials on frequency-response analysis. Bode diagrams, polar plots, the Nyquist stability criterion, and closed-loop frequency response are discussed including the MATLAB approach to obtain frequency response plots. Chapter 9 treats the design and compensation techniques using frequency-response methods. Specifically, the Bode diagram approach to the design of lead, lag, and lag-lead compensators is discussed in detail.

Chapter 10 first deals with the basic and modified PID controls and then presents computational (MATLAB) approach to obtain optimal choices of parameter values of controllers to satisfy requirements on step response characteristics. Next, it presents two-degrees-of-freedom control systems. The chapter concludes with the design of high performance control systems that will follow a step, ramp, or acceleration input without steady-state error. The zero-placement method is used to accomplish such performance.

Chapter 11 presents a basic analysis of control systems in state space. Concepts of controllability and observability are given here. This chapter discusses the transformation of system models (from transfer-function model to state-space model, and vice versa) with MATLAB. Chapter 12 begins with the pole placement design technique, followed by the design of state observers. Both full-order and minimum-order state observers are treated. Then, designs of type 1 servo systems are discussed in detail. Included in this chapter are the design of regulator systems with observers and design of control systems with observers. Finally, this chapter concludes with discussions of quadratic optimal regulator systems.

In this book, the basic concepts involved are emphasized and highly mathematical arguments are carefully avoided in the presentation of the materials. Mathematical proofs are provided when they contribute to the understanding of the subjects presented. All the material has been organized toward a gradual development of control theory.

Throughout the book, carefully chosen examples are presented at strategic points so that the reader will have a clear understanding of the subject matter discussed. In addition, a number of solved problems (A-problems) are provided at the end of each chapter, except Chapter 1. These solved problems constitute an integral part of the text. Therefore, it is suggested that the reader study all these problems carefully to obtain a

deeper understanding of the topics discussed. In addition, many problems (without solutions) of various degrees of difficulty are provided (B-problems). These problems may be used as homework or quiz purposes. An instructor using this text can obtain a complete solutions manual (for B-problems) from the publisher.

Most of the materials including solved and unsolved problems presented in this book have been class tested in senior level courses on control systems at the University of Minnesota.

If this book is used as a text for a quarter course (with 40 lecture hours), most of the materials in the first 10 chapters (except perhaps Chapter 4) may be covered. [The first nine chapters cover all basic materials of control systems normally required in a first course on control systems. Many students enjoy studying computational (MATLAB) approach to the design of control systems presented in Chapter 10. It is recommended that Chapter 10 be included in any control courses.] If this book is used as a text for a semester course (with 56 lecture hours), all or a good part of the book may be covered with flexibility in skipping certain subjects. Because of the abundance of solved problems (A-problems) that might answer many possible questions that the reader might have, this book can also serve as a self-study book for practicing engineers who wish to study basic control theory.

I would like to express my sincere appreciation to Professors Athimoottil V. Mathew (Rochester Institute of Technology), Richard Gordon (University of Mississippi), Guy Beale (George Mason University), and Donald T. Ward (Texas A & M University), who made valuable suggestions at the early stage of the revision process, and anonymous reviewers who made many constructive comments. Appreciation is also due to my former students, who solved many of the A-problems and B-problems included in this book.

Katsuhiko Ogata



Introduction to Control Systems

1-1 INTRODUCTION

Automatic control has played a vital role in the advance of engineering and science. In addition to its extreme importance in space-vehicle systems, missile-guidance systems, robotic systems, and the like, automatic control has become an important and integral part of modern manufacturing and industrial processes. For example, automatic control is essential in the numerical control of machine tools in the manufacturing industries, in the design of autopilot systems in the aerospace industries, and in the design of cars and trucks in the automobile industries. It is also essential in such industrial operations as controlling pressure, temperature, humidity, viscosity, and flow in the process industries.

Since advances in the theory and practice of automatic control provide the means for attaining optimal performance of dynamic systems, improving productivity, relieving the drudgery of many routine repetitive manual operations, and more, most engineers and scientists must now have a good understanding of this field.

Historical Review. The first significant work in automatic control was James Watt's centrifugal governor for the speed control of a steam engine in the eighteenth century. Other significant works in the early stages of development of control theory were due to Minorsky, Hazen, and Nyquist, among many others. In 1922, Minorsky worked on automatic controllers for steering ships and showed how stability could be determined from the differential equations describing the system. In 1932, Nyquist developed a relatively simple procedure for determining the stability of closed-loop systems on the

basis of open-loop response to steady-state sinusoidal inputs. In 1934, Hazen, who introduced the term servomechanisms for position control systems, discussed the design of relay servomechanisms capable of closely following a changing input.

During the decade of the 1940s, frequency-response methods (especially the Bode diagram methods due to Bode) made it possible for engineers to design linear closed-loop control systems that satisfied performance requirements. From the end of the 1940s to the early 1950s, the root-locus method due to Evans was fully developed.

The frequency-response and root-locus methods, which are the core of classical control theory, lead to systems that are stable and satisfy a set of more or less arbitrary performance requirements. Such systems are, in general, acceptable but not optimal in any meaningful sense. Since the late 1950s, the emphasis in control design problems has been shifted from the design of one of many systems that work to the design of one optimal system in some meaningful sense.

As modern plants with many inputs and outputs become more and more complex, the description of a modern control system requires a large number of equations. Classical control theory, which deals only with single-input-single-output systems, becomes powerless for multiple-input-multiple-output systems. Since about 1960, because the availability of digital computers made possible time-domain analysis of complex systems, modern control theory, based on time-domain analysis and synthesis using state variables, has been developed to cope with the increased complexity of modern plants and the stringent requirements on accuracy, weight, and cost in military, space, and industrial applications.

During the years from 1960 to 1980, optimal control of both deterministic and stochastic systems, as well as adaptive and learning control of complex systems, were fully investigated. From 1980 to the present, developments in modern control theory centered around robust control, H_∞ control, and associated topics.

Now that digital computers have become cheaper and more compact, they are used as integral parts of control systems. Recent applications of modern control theory include such nonengineering systems as biological, biomedical, economic, and socioeconomic systems.

Definitions. Before we can discuss control systems, some basic terminologies must be defined.

Controlled Variable and Manipulated Variable. The *controlled* variable is the quantity or condition that is measured and controlled. The *manipulated* variable is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally, the controlled variable is the output of the system. *Control* means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to correct or limit deviation of the measured value from a desired value.

In studying control engineering, we need to define additional terms that are necessary to describe control systems.

Plants. A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation. In this book, we shall call any physical object to be controlled (such as a mechanical device, a heating furnace, a chemical reactor, or a spacecraft) a plant.

Processes. The *Merriam-Webster Dictionary* defines a process to be a natural, progressively continuing operation or development marked by a series of gradual changes that succeed one another in a relatively fixed way and lead toward a particular result or end; or an artificial or voluntary, progressively continuing operation that consists of a series of controlled actions or movements systematically directed toward a particular result or end. In this book we shall call any operation to be controlled a *process*. Examples are chemical, economic, and biological processes.

Systems. A system is a combination of components that act together and perform a certain objective. A system is not limited to physical ones. The concept of the system can be applied to abstract, dynamic phenomena such as those encountered in economics. The word system should, therefore, be interpreted to imply physical, biological, economic, and the like, systems.

Disturbances. A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called *internal*, while an *external* disturbance is generated outside the system and is an input.

Feedback Control. Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and does so on the basis of this difference. Here only unpredictable disturbances are so specified, since predictable or known disturbances can always be compensated for within the system.

1-2 EXAMPLES OF CONTROL SYSTEMS

In this section we shall present several examples of control systems.

Speed Control System. The basic principle of a Watt's speed governor for an engine is illustrated in the schematic diagram of Figure 1-1. The amount of fuel admitted

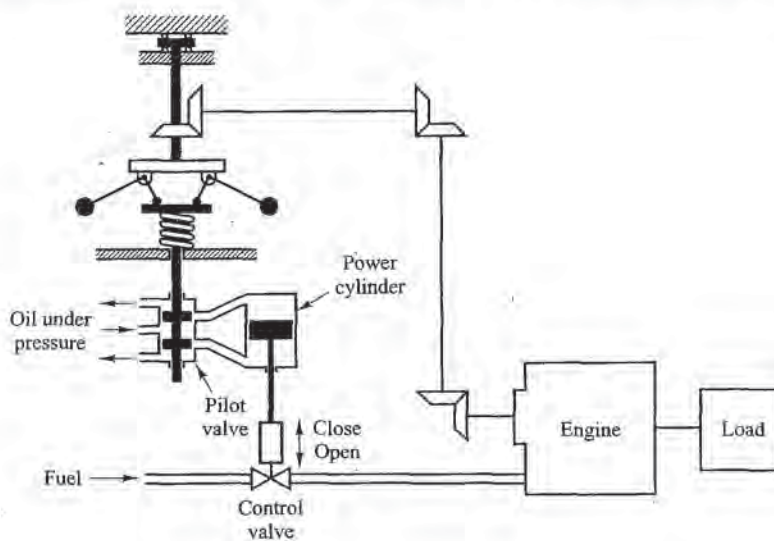
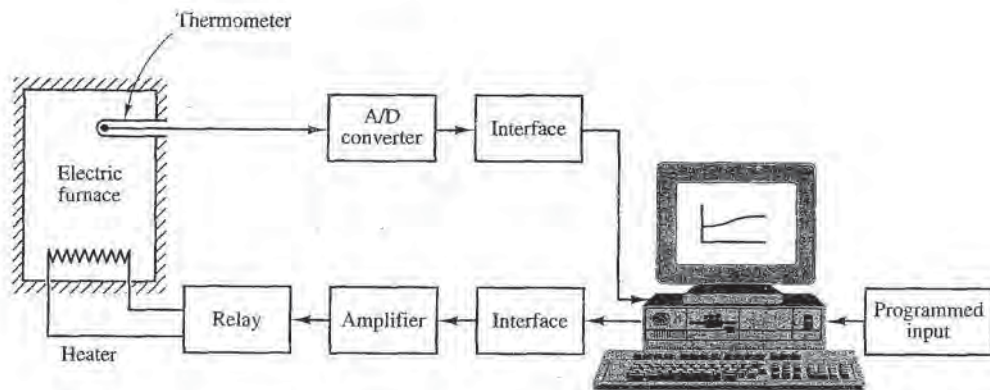


Figure 1-1
Speed control
system.

Figure 1-2
Temperature control
system.



to the engine is adjusted according to the difference between the desired and the actual engine speeds.

The sequence of actions may be stated as follows: The speed governor is adjusted such that, at the desired speed, no pressured oil will flow into either side of the power cylinder. If the actual speed drops below the desired value due to disturbance, then the decrease in the centrifugal force of the speed governor causes the control valve to move downward, supplying more fuel, and the speed of the engine increases until the desired value is reached. On the other hand, if the speed of the engine increases above the desired value, then the increase in the centrifugal force of the governor causes the control valve to move upward. This decreases the supply of fuel, and the speed of the engine decreases until the desired value is reached.

In this speed control system, the plant (controlled system) is the engine and the controlled variable is the speed of the engine. The difference between the desired speed and the actual speed is the error signal. The control signal (the amount of fuel) to be applied to the plant (engine) is the actuating signal. The external input to disturb the controlled variable is the disturbance. An unexpected change in the load is a disturbance.

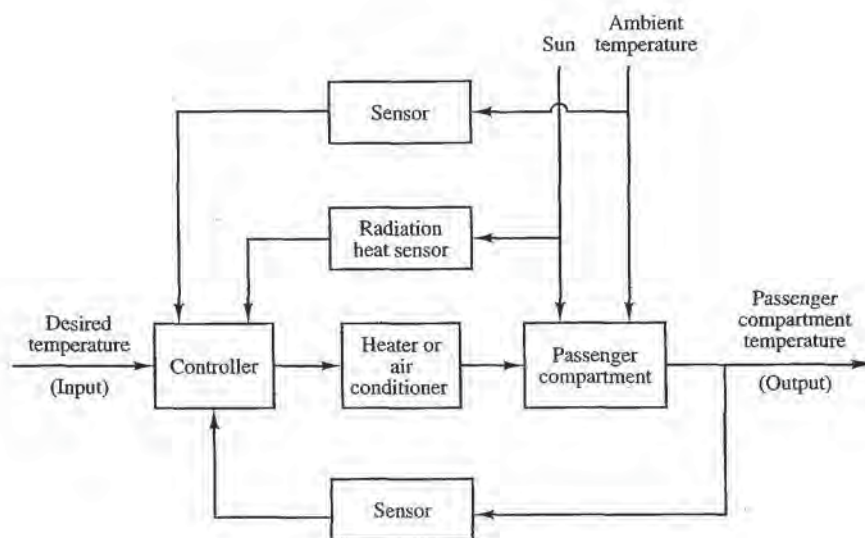
Temperature Control System. Figure 1-2 shows a schematic diagram of temperature control of an electric furnace. The temperature in the electric furnace is measured by a thermometer, which is an analog device. The analog temperature is converted to a digital temperature by an A/D converter. The digital temperature is fed to a controller through an interface. This digital temperature is compared with the programmed input temperature, and if there is any discrepancy (error), the controller sends out a signal to the heater, through an interface, amplifier, and relay, to bring the furnace temperature to a desired value.

EXAMPLE 1-1

Consider the temperature control of the passenger compartment of a car. The desired temperature (converted to a voltage) is the input to the controller. The actual temperature of the passenger compartment must be converted to a voltage through a sensor and fed back to the controller for comparison with the input.

Figure 1-3 is a functional block diagram of temperature control of the passenger compartment of a car. Note that the ambient temperature and radiation heat transfer from the sun, which are not constant while the car is driven, act as disturbances.

Figure 1-3
Temperature control
of passenger
compartment
of a car.



The temperature of the passenger compartment differs considerably depending on the place where it is measured. Instead of using multiple sensors for temperature measurement and averaging the measured values, it is economical to install a small suction blower at the place where passengers normally sense the temperature. The temperature of the air from the suction blower is an indication of the passenger compartment temperature and is considered the output of the system.

The controller receives the input signal, output signal, and signals from sensors from disturbance sources. The controller sends out an optimal control signal to the air conditioner or heater to control the amount of cooling air or warm air so that the passenger compartment temperature is about the desired temperature.

Business Systems. A business system may consist of many groups. Each task assigned to a group will represent a dynamic element of the system. Feedback methods of reporting the accomplishments of each group must be established in such a system for proper operation. The cross-coupling between functional groups must be made a minimum in order to reduce undesirable delay times in the system. The smaller this cross-coupling, the smoother the flow of work signals and materials will be.

A business system is a closed-loop system. A good design will reduce the managerial control required. Note that disturbances in this system are the lack of personnel or materials, interruption of communication, human errors, and the like.

The establishment of a well-founded estimating system based on statistics is mandatory to proper management. Note that it is a well-known fact that the performance of such a system can be improved by the use of lead time, or *anticipation*.

To apply control theory to improve the performance of such a system, we must represent the dynamic characteristic of the component groups of the system by a relatively simple set of equations.

Although it is certainly a difficult problem to derive mathematical representations of the component groups, the application of optimization techniques to business systems significantly improves the performance of the business system.

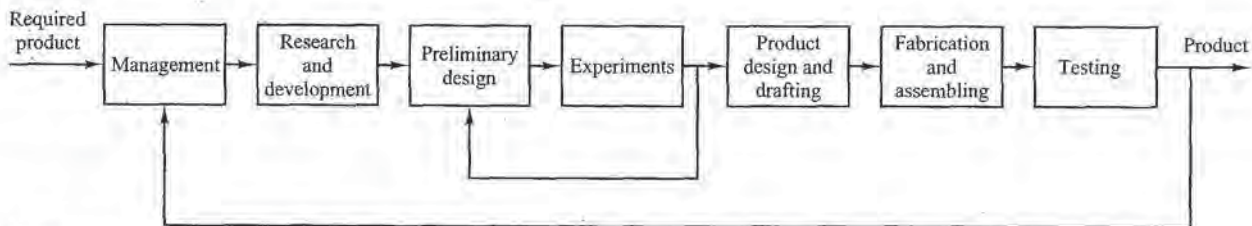


Figure 1-4
Block diagram of an engineering organizational system.

EXAMPLE 1-2 An engineering organizational system is composed of major groups such as management, research and development, preliminary design, experiments, product design and drafting, fabrication and assembling, and testing. These groups are interconnected to make up the whole operation. Such a system may be analyzed by reducing it to the most elementary set of components necessary that can provide the analytical detail required and by representing the dynamic characteristics of each component by a set of simple equations. (The dynamic performance of such a system may be determined from the relation between progressive accomplishment and time.) Draw a functional block diagram showing an engineering organizational system.

A functional block diagram can be drawn by using blocks to represent the functional activities and interconnecting signal lines to represent the information or product output of the system operation. A possible block diagram is shown in Figure 1-4.

1-3 CLOSED-LOOP CONTROL VERSUS OPEN-LOOP CONTROL

Feedback Control Systems. A system that maintains a prescribed relationship between the output and the reference input by comparing them and using the difference as a means of control is called a *feedback control system*. An example would be a room-temperature control system. By measuring the actual room temperature and comparing it with the reference temperature (desired temperature), the thermostat turns the heating or cooling equipment on or off in such a way as to ensure that the room temperature remains at a comfortable level regardless of outside conditions.

Feedback control systems are not limited to engineering but can be found in various nonengineering fields as well. The human body, for instance, is a highly advanced feedback control system. Both body temperature and blood pressure are kept constant by means of physiological feedback. In fact, feedback performs a vital function: It makes the human body relatively insensitive to external disturbances, thus enabling it to function properly in a changing environment.

Closed-Loop Control Systems. Feedback control systems are often referred to as *closed-loop control* systems. In practice, the terms feedback control and closed-loop control are used interchangeably. In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal (which may be the output signal itself or a function of the output signal and its derivatives and/or integrals), is fed to the controller so as to reduce the error and bring the output of the system to a desired value. The term closed-loop control always implies the use of feedback control action in order to reduce system error.

Open-Loop Control Systems. Those systems in which the output has no effect on the control action are called *open-loop control systems*. In other words, in an open-

loop control system the output is neither measured nor fed back for comparison with the input. One practical example is a washing machine. Soaking, washing, and rinsing in the washer operate on a time basis. The machine does not measure the output signal, that is, the cleanliness of the clothes.

In any open-loop control system the output is not compared with the reference input. Thus, to each reference input there corresponds a fixed operating condition; as a result, the accuracy of the system depends on calibration. In the presence of disturbances, an open-loop control system will not perform the desired task. Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances. Clearly, such systems are not feedback control systems. Note that any control system that operates on a time basis is open loop. For instance, traffic control by means of signals operated on a time basis is another example of open-loop control.

Closed-Loop versus Open-Loop Control Systems. An advantage of the closed-loop control system is the fact that the use of feedback makes the system response relatively insensitive to external disturbances and internal variations in system parameters. It is thus possible to use relatively inaccurate and inexpensive components to obtain the accurate control of a given plant, whereas doing so is impossible in the open-loop case.

From the point of view of stability, the open-loop control system is easier to build because system stability is not a major problem. On the other hand, stability is a major problem in the closed-loop control system, which may tend to overcorrect errors and thereby can cause oscillations of constant or changing amplitude.

It should be emphasized that for systems in which the inputs are known ahead of time and in which there are no disturbances it is advisable to use open-loop control. Closed-loop control systems have advantages only when unpredictable disturbances and/or unpredictable variations in system components are present. Note that the output power rating partially determines the cost, weight, and size of a control system. The number of components used in a closed-loop control system is more than that for a corresponding open-loop control system. Thus, the closed-loop control system is generally higher in cost and power. To decrease the required power of a system, open-loop control may be used where applicable. A proper combination of open-loop and closed-loop controls is usually less expensive and will give satisfactory overall system performance.

EXAMPLE 1-3 Most analyses and designs of control systems presented in this book are concerned with closed-loop control systems. Under certain circumstances (such as where no disturbances exist or the output is hard to measure) open-loop control systems may be desired. Therefore, it is worthwhile to summarize the advantages and disadvantages of using open-loop control systems.

The major advantages of open-loop control systems are as follows:

1. Simple construction and ease of maintenance.
2. Less expensive than a corresponding closed-loop system.
3. There is no stability problem.
4. Convenient when output is hard to measure or measuring the output precisely is economically not feasible. (For example, in the washer system, it would be quite expensive to provide a device to measure the quality of the washer's output, cleanliness of the clothes.)

The major disadvantages of open-loop control systems are as follows:

1. Disturbances and changes in calibration cause errors, and the output may be different from what is desired.
2. To maintain the required quality in the output, recalibration is necessary from time to time.

1-4 OUTLINE OF THE BOOK

We briefly describe here the organization and contents of the book.

Chapter 1 has given introductory materials on control systems. Chapter 2 presents basic Laplace transform theory necessary for understanding the control theory presented in this book. Chapter 3 deals with mathematical modeling of dynamic systems in terms of transfer functions and state-space equations. It discusses mathematical modeling of mechanical systems and electrical and electronic systems. This chapter also includes the signal flow graphs and linearization of nonlinear mathematical models. Chapter 4 treats mathematical modeling of liquid-level systems, pneumatic systems, hydraulic systems, and thermal systems. Chapter 5 treats transient-response analyses of first- and second-order systems as well as higher-order systems. Detailed discussions of transient-response analysis with MATLAB are presented. Routh's stability criterion and steady-state errors in unity-feedback control systems are also presented in this chapter.

Chapter 6 gives a root-locus analysis of control systems. General rules for constructing root loci are presented. Detailed discussions for plotting root loci with MATLAB are included. Chapter 7 deals with the design of control systems via the root-locus method. Specifically, root-locus approaches to the design of lead compensators, lag compensators, and lag-lead compensators are discussed in detail. Chapter 8 gives the frequency-response analysis of control systems. Bode diagrams, polar plots, Nyquist stability criterion, and closed-loop frequency response are discussed. Chapter 9 treats control systems design via the frequency-response approach. Here Bode diagrams are used to design lead compensators, lag compensators, and lag-lead compensators. Chapter 10 discusses the basic and modified PID controls. In this chapter two-degrees-of-freedom control systems are introduced. We design high-performance control systems using two-degrees-of-freedom configuration. MATLAB is extensively used in the design of such systems.

Chapter 11 presents basic materials for the state-space analysis of control systems. The solution of the time-invariant state equation is derived and concepts of controllability and observability are discussed. Chapter 12 treats the design of control systems in state space. This chapter begins with the pole-placement problems, followed by the design of state observers, and the design of regulator systems with observers and control systems with observers. Finally, quadratic optimal control is discussed.



The Laplace Transform*

2-1 INTRODUCTION

The Laplace transform method is an operational method that can be used advantageously for solving linear differential equations. By use of Laplace transforms, we can convert many common functions, such as sinusoidal functions, damped sinusoidal functions, and exponential functions, into algebraic functions of a complex variable s . Operations such as differentiation and integration can be replaced by algebraic operations in the complex plane. Thus, a linear differential equation can be transformed into an algebraic equation in a complex variable s . If the algebraic equation in s is solved for the dependent variable, then the solution of the differential equation (the inverse Laplace transform of the dependent variable) may be found by use of a Laplace transform table or by use of the partial-fraction expansion technique, which is presented in Section 2-5 and 2-6.

An advantage of the Laplace transform method is that it allows the use of graphical techniques for predicting the system performance without actually solving system differential equations. Another advantage of the Laplace transform method is that, when we solve the differential equation, both the transient component and steady-state component of the solution can be obtained simultaneously.

Outline of the Chapter. Section 2-1 presents introductory remarks. Section 2-2 briefly reviews complex variables and complex functions. Section 2-3 derives Laplace

*This chapter may be skipped if the student is already familiar with Laplace transforms.

transforms of time functions that are frequently used in control engineering. Section 2-4 presents useful theorems of Laplace transforms, and Section 2-5 treats the inverse Laplace transformation using the partial-fraction expansion of $B(s)/A(s)$, where $A(s)$ and $B(s)$ are polynomials in s . Section 2-6 presents computational methods with MATLAB to obtain the partial-fraction expansion of $B(s)/A(s)$, as well as the zeros and poles of $B(s)/A(s)$. Finally, Section 2-7 deals with solutions of linear time-invariant differential equations by the Laplace transform approach.

2-2 REVIEW OF COMPLEX VARIABLES AND COMPLEX FUNCTIONS

Before we present the Laplace transformation, we shall review the complex variable and complex function. We shall also review Euler's theorem, which relates the sinusoidal functions to exponential functions.

Complex Variable. A complex number has a real part and an imaginary part, both of which are constant. If the real part and/or imaginary part are variables, a complex quantity is called a *complex variable*. In the Laplace transformation we use the notation s as a complex variable; that is,

$$s = \sigma + j\omega$$

where σ is the real part and ω is the imaginary part.

Complex Function. A complex function $G(s)$, a function of s , has a real part and an imaginary part or

$$G(s) = G_x + jG_y$$

where G_x and G_y are real quantities. The magnitude of $G(s)$ is $\sqrt{G_x^2 + G_y^2}$, and the angle θ of $G(s)$ is $\tan^{-1}(G_y/G_x)$. The angle is measured counterclockwise from the positive real axis. The complex conjugate of $G(s)$ is $\bar{G}(s) = G_x - jG_y$.

Complex functions commonly encountered in linear control systems analysis are single-valued functions of s and are uniquely determined for a given value of s .

A complex function $G(s)$ is said to be *analytic* in a region if $G(s)$ and all its derivatives exist in that region. The derivative of an analytic function $G(s)$ is given by

$$\frac{d}{ds} G(s) = \lim_{\Delta s \rightarrow 0} \frac{G(s + \Delta s) - G(s)}{\Delta s} = \lim_{\Delta s \rightarrow 0} \frac{\Delta G}{\Delta s}$$

Since $\Delta s = \Delta\sigma + j\Delta\omega$, Δs can approach zero along an infinite number of different paths. It can be shown, but is stated without a proof here, that if the derivatives taken along two particular paths, that is, $\Delta s = \Delta\sigma$ and $\Delta s = j\Delta\omega$, are equal, then the derivative is unique for any other path $\Delta s = \Delta\sigma + j\Delta\omega$ and so the derivative exists.

For a particular path $\Delta s = \Delta\sigma$ (which means that the path is parallel to the real axis).

$$\frac{d}{ds} G(s) = \lim_{\Delta\sigma \rightarrow 0} \left(\frac{\Delta G_x}{\Delta\sigma} + j \frac{\Delta G_y}{\Delta\sigma} \right) = \frac{\partial G_x}{\partial \sigma} + j \frac{\partial G_y}{\partial \sigma}$$

For another particular path $\Delta s = j\Delta\omega$ (which means that the path is parallel to the imaginary axis).

$$\frac{d}{ds} G(s) = \lim_{j\Delta\omega \rightarrow 0} \left(\frac{\Delta G_x}{j\Delta\omega} + j \frac{\Delta G_y}{j\Delta\omega} \right) = -j \frac{\partial G_x}{\partial \omega} + \frac{\partial G_y}{\partial \omega}$$

If these two values of the derivative are equal,

$$\frac{\partial G_x}{\partial \sigma} + j \frac{\partial G_y}{\partial \sigma} = \frac{\partial G_y}{\partial \omega} - j \frac{\partial G_x}{\partial \omega}$$

or if the following two conditions

$$\frac{\partial G_x}{\partial \sigma} = \frac{\partial G_y}{\partial \omega} \quad \text{and} \quad \frac{\partial G_y}{\partial \sigma} = -\frac{\partial G_x}{\partial \omega}$$

are satisfied, then the derivative $dG(s)/ds$ is uniquely determined. These two conditions are known as the Cauchy–Riemann conditions. If these conditions are satisfied, the function $G(s)$ is analytic.

As an example, consider the following $G(s)$:

$$G(s) = \frac{1}{s+1}$$

Then

$$G(\sigma + j\omega) = \frac{1}{\sigma + j\omega + 1} = G_x + jG_y$$

where

$$G_x = \frac{\sigma + 1}{(\sigma + 1)^2 + \omega^2} \quad \text{and} \quad G_y = \frac{-\omega}{(\sigma + 1)^2 + \omega^2}$$

It can be seen that, except at $s = -1$ (that is, $\sigma = -1$, $\omega = 0$), $G(s)$ satisfies the Cauchy–Riemann conditions:

$$\begin{aligned} \frac{\partial G_x}{\partial \sigma} &= \frac{\partial G_y}{\partial \omega} = \frac{\omega^2 - (\sigma + 1)^2}{[(\sigma + 1)^2 + \omega^2]^2} \\ \frac{\partial G_y}{\partial \sigma} &= -\frac{\partial G_x}{\partial \omega} = \frac{2\omega(\sigma + 1)}{[(\sigma + 1)^2 + \omega^2]^2} \end{aligned}$$

Hence $G(s) = 1/(s+1)$ is analytic in the entire s plane except at $s = -1$. The derivative $dG(s)/ds$, except at $s = -1$, is found to be

$$\begin{aligned} \frac{d}{ds} G(s) &= \frac{\partial G_x}{\partial \sigma} + j \frac{\partial G_y}{\partial \sigma} = \frac{\partial G_y}{\partial \omega} - j \frac{\partial G_x}{\partial \omega} \\ &= -\frac{1}{(\sigma + j\omega + 1)^2} = -\frac{1}{(s+1)^2} \end{aligned}$$

Note that the derivative of an analytic function can be obtained simply by differentiating $G(s)$ with respect to s . In this example,

$$\frac{d}{ds} \left(\frac{1}{s+1} \right) = -\frac{1}{(s+1)^2}$$

Points in the s plane at which the function $G(s)$ is analytic are called *ordinary* points, while points in the s plane at which the function $G(s)$ is not analytic are called *singular* points. Singular points at which the function $G(s)$ or its derivatives approach infinity are called *poles*. Singular points at which the function $G(s)$ equals zero are called *zeros*.

If $G(s)$ approaches infinity as s approaches $-p$ and if the function

$$G(s)(s + p)^n, \quad \text{for } n = 1, 2, 3, \dots$$

has a finite, nonzero value at $s = -p$, then $s = -p$ is called a pole of order n . If $n = 1$, the pole is called a simple pole. If $n = 2, 3, \dots$, the pole is called a second-order pole, a third-order pole, and so on.

To illustrate, consider the complex function

$$G(s) = \frac{K(s + 2)(s + 10)}{s(s + 1)(s + 5)(s + 15)^2}$$

$G(s)$ has zeros at $s = -2, s = -10$, simple poles at $s = 0, s = -1, s = -5$, and a double pole (multiple pole of order 2) at $s = -15$. Note that $G(s)$ becomes zero at $s = \infty$. Since for large values of s

$$G(s) \doteq \frac{K}{s^3}$$

$G(s)$ possesses a triple zero (multiple zero of order 3) at $s = \infty$. If points at infinity are included, $G(s)$ has the same number of poles as zeros. To summarize, $G(s)$ has five zeros ($s = -2, s = -10, s = \infty, s = \infty, s = \infty$) and five poles ($s = 0, s = -1, s = -5, s = -15, s = -15$).

Euler's Theorem. The power series expansions of $\cos \theta$ and $\sin \theta$ are, respectively,

$$\cos \theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \frac{\theta^6}{6!} + \dots$$

$$\sin \theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \frac{\theta^7}{7!} + \dots$$

And so

$$\cos \theta + j \sin \theta = 1 + (j\theta) + \frac{(j\theta)^2}{2!} + \frac{(j\theta)^3}{3!} + \frac{(j\theta)^4}{4!} + \dots$$

Since

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

we see that

$$\cos \theta + j \sin \theta = e^{j\theta} \quad (2-1)$$

This is known as *Euler's theorem*.

Chapter 2 / The Laplace Transform

By using Euler's theorem, we can express sine and cosine in terms of an exponential function. Noting that $e^{-j\theta}$ is the complex conjugate of $e^{j\theta}$ and that

$$e^{j\theta} = \cos \theta + j \sin \theta$$

$$e^{-j\theta} = \cos \theta - j \sin \theta$$

we find, after adding or subtracting these two equations, that

$$\cos \theta = \frac{1}{2} (e^{j\theta} + e^{-j\theta}) \quad (2-2)$$

$$\sin \theta = \frac{1}{2j} (e^{j\theta} - e^{-j\theta}) \quad (2-3)$$

2-3 LAPLACE TRANSFORMATION

We shall first present a definition of the Laplace transformation and a brief discussion of the condition for the existence of the Laplace transform and then provide examples for illustrating the derivation of Laplace transforms of several common functions.

Let us define

$f(t)$ = a function of time t such that $f(t) = 0$ for $t < 0$

s = a complex variable

$\mathcal{L}$ = an operational symbol indicating that the quantity that it prefixes is to be transformed by the Laplace integral $\int_0^\infty e^{-st} dt$

$F(s)$ = Laplace transform of $f(t)$

Then the Laplace transform of $f(t)$ is given by

$$\mathcal{L}[f(t)] = F(s) = \int_0^\infty e^{-st} dt [f(t)] = \int_0^\infty f(t) e^{-st} dt$$

The reverse process of finding the time function $f(t)$ from the Laplace transform $F(s)$ is called the *inverse Laplace transformation*. The notation for the inverse Laplace transformation is $\mathcal{L}^{-1}$, and the inverse Laplace transform can be found from $F(s)$ by the following inversion integral:

$$\mathcal{L}^{-1}[F(s)] = f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s) e^{st} ds, \quad \text{for } t > 0 \quad (2-4)$$

where c , the abscissa of convergence, is a real constant and is chosen larger than the real parts of all singular points of $F(s)$. Thus, the path of integration is parallel to the $j\omega$ axis and is displaced by the amount c from it. This path of integration is to the right of all singular points.

Evaluating the inversion integral appears complicated. In practice, we seldom use this integral for finding $f(t)$. There are simpler methods for finding $f(t)$. We shall discuss such simpler methods in Sections 2-5 and 2-6.

It is noted that in this book the time function $f(t)$ is always assumed to be zero for negative time; that is,

$$f(t) = 0, \quad \text{for } t < 0$$

Existence of Laplace Transform. The Laplace transform of a function $f(t)$ exists if the Laplace integral converges. The integral will converge if $f(t)$ is sectionally continuous in every finite interval in the range $t > 0$ and if it is of exponential order as t approaches infinity. A function $f(t)$ is said to be of exponential order if a real, positive constant σ exists such that the function

$$e^{-\sigma t}|f(t)|$$

approaches zero as t approaches infinity. If the limit of the function $e^{-\sigma t}|f(t)|$ approaches zero for σ greater than σ_c and the limit approaches infinity for σ less than σ_c , the value σ_c is called the *abscissa of convergence*.

For the function $f(t) = Ae^{-\alpha t}$

$$\lim_{t \rightarrow \infty} e^{-\sigma t}|Ae^{-\alpha t}|$$

approaches zero if $\sigma > -\alpha$. The abscissa of convergence in this case is $\sigma_c = -\alpha$. The integral $\int_0^\infty f(t)e^{-st} dt$ converges only if σ , the real part of s , is greater than the abscissa of convergence σ_c . Thus the operator s must be chosen as a constant such that this integral converges.

In terms of the poles of the function $F(s)$, the abscissa of convergence σ_c corresponds to the real part of the pole located farthest to the right in the s plane. For example, for the following function $F(s)$,

$$F(s) = \frac{K(s+3)}{(s+1)(s+2)}$$

the abscissa of convergence σ_c is equal to -1 . It can be seen that for such functions as t , $\sin \omega t$, and $t \sin \omega t$ the abscissa of convergence is equal to zero. For functions like e^{-ct} , te^{-ct} , $e^{-ct} \sin \omega t$, and so on, the abscissa of convergence is equal $-c$. For functions that increase faster than the exponential function, however, it is impossible to find suitable values of the abscissa of convergence. Therefore, such functions as e^{t^2} and te^{t^2} do not possess Laplace transforms.

The reader should be cautioned that although e^{t^2} (for $0 \leq t \leq \infty$) does not possess a Laplace transform, the time function defined by

$$\begin{aligned} f(t) &= e^{t^2}, & \text{for } 0 \leq t \leq T < \infty \\ &= 0, & \text{for } t < 0, T < t \end{aligned}$$

does possess a Laplace transform since $f(t) = e^{t^2}$ for only a limited time interval $0 \leq t \leq T$ and not for $0 \leq t \leq \infty$. Such a signal can be physically generated. Note that the signals that we can physically generate always have corresponding Laplace transforms.

If a function $f(t)$ has a Laplace transform, then the Laplace transform of $Af(t)$, where A is a constant, is given by

$$\mathcal{L}[Af(t)] = A\mathcal{L}[f(t)]$$

This is obvious from the definition of the Laplace transform. Since Laplace transformation is a linear operation, if functions $f_1(t)$ and $f_2(t)$ have Laplace transforms, $F_1(s)$ and $F_2(s)$, respectively, then the Laplace transform of the function $\alpha f_1(t) + \beta f_2(t)$ is given by

$$\mathcal{L}[\alpha f_1(t) + \beta f_2(t)] = \alpha F_1(s) + \beta F_2(s)$$

In what follows, we derive Laplace transforms of a few commonly encountered functions.

Exponential Function. Consider the exponential function

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= Ae^{-\alpha t}, & \text{for } t \geq 0 \end{aligned}$$

where A and α are constants. The Laplace transform of this exponential function can be obtained as follows:

$$\mathcal{L}[Ae^{-\alpha t}] = \int_0^{\infty} Ae^{-\alpha t} e^{-st} dt = A \int_0^{\infty} e^{-(\alpha+s)t} dt = \frac{A}{s + \alpha}$$

It is seen that the exponential function produces a pole in the complex plane.

In deriving the Laplace transform of $f(t) = Ae^{-\alpha t}$, we required that the real part of s be greater than $-\alpha$ (the abscissa of convergence). A question may immediately arise as to whether or not the Laplace transform thus obtained is valid in the range where $\sigma < -\alpha$ in the s plane. To answer this question, we must resort to the theory of complex variables. In the theory of complex variables, there is a theorem known as the analytic extension theorem. It states that, if two analytic functions are equal for a finite length along any arc in a region in which both are analytic, then they are equal everywhere in the region. The arc of equality is usually the real axis or a portion of it. By using this theorem the form of $F(s)$ determined by an integration in which s is allowed to have any real positive value greater than the abscissa of convergence holds for any complex values of s at which $F(s)$ is analytic. Thus, although we require the real part of s to be greater than the abscissa of convergence to make the integral $\int_0^{\infty} f(t)e^{-st} dt$ absolutely convergent, once the Laplace transform $F(s)$ is obtained, $F(s)$ can be considered valid throughout the entire s plane except at the poles of $F(s)$.

Step Function. Consider the step function

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= A, & \text{for } t > 0 \end{aligned}$$

where A is a constant. Note that it is a special case of the exponential function $Ae^{-\alpha t}$, where $\alpha = 0$. The step function is undefined at $t = 0$. Its Laplace transform is given by

$$\mathcal{L}[A] = \int_0^{\infty} Ae^{-st} dt = \frac{A}{s}$$

In performing this integration, we assumed that the real part of s was greater than zero (the abscissa of convergence) and therefore that $\lim_{t \rightarrow \infty} e^{-st}$ was zero. As stated earlier, the Laplace transform thus obtained is valid in the entire s plane except at the pole $s = 0$.

The step function whose height is unity is called *unit-step* function. The unit-step function that occurs at $t = t_0$ is frequently written as $1(t - t_0)$. The step function of height A that occurs at $t = 0$ can then be written as $f(t) = A1(t)$. The Laplace transform of the unit-step function, which is defined by

$$\begin{aligned} 1(t) &= 0, & \text{for } t < 0 \\ &= 1, & \text{for } t > 0 \end{aligned}$$

is $1/s$, or

$$\mathcal{L}[1(t)] = \frac{1}{s}$$

Physically, a step function occurring at $t = 0$ corresponds to a constant signal suddenly applied to the system at time t equals zero.

Ramp Function. Consider the ramp function

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= At, & \text{for } t \geq 0 \end{aligned}$$

where A is a constant. The Laplace transform of this ramp function is obtained as

$$\begin{aligned} \mathcal{L}[At] &= \int_0^{\infty} Ate^{-st} dt = At \frac{e^{-st}}{-s} \Big|_0^{\infty} - \int_0^{\infty} \frac{Ae^{-st}}{-s} dt \\ &= \frac{A}{s} \int_0^{\infty} e^{-st} dt = \frac{A}{s^2} \end{aligned}$$

Sinusoidal Function. The Laplace transform of the sinusoidal function

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= A \sin \omega t, & \text{for } t \geq 0 \end{aligned}$$

where A and ω are constants, is obtained as follows. Referring to Equation (2-3), $\sin \omega t$ can be written

$$\sin \omega t = \frac{1}{2j} (e^{j\omega t} - e^{-j\omega t})$$

Hence

$$\begin{aligned} \mathcal{L}[A \sin \omega t] &= \frac{A}{2j} \int_0^{\infty} (e^{j\omega t} - e^{-j\omega t}) e^{-st} dt \\ &= \frac{A}{2j} \frac{1}{s - j\omega} - \frac{A}{2j} \frac{1}{s + j\omega} = \frac{A\omega}{s^2 + \omega^2} \end{aligned}$$

Similarly, the Laplace transform of $A \cos \omega t$ can be derived as follows:

$$\mathcal{L}[A \cos \omega t] = \frac{As}{s^2 + \omega^2}$$

Comments. The Laplace transform of any Laplace transformable function $f(t)$ can be found by multiplying $f(t)$ by e^{-st} and then integrating the product from $t = 0$ to $t = \infty$. Once we know the method of obtaining the Laplace transform, however, it is not necessary to derive the Laplace transform of $f(t)$ each time. Laplace transform tables can conveniently be used to find the transform of a given function $f(t)$. Table 2-1 shows Laplace transforms of time functions that will frequently appear in linear control systems analysis.

Table 2-1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab} \left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt}) \right]$	$\frac{1}{s(s+a)(s+b)}$

(continues on next page)

Table 2-1 (continued)

18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t \quad (0 < \zeta < 1)$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$ $(0 < \zeta < 1, \quad 0 < \phi < \pi/2)$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega} (\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

In the following discussion we present Laplace transforms of functions as well as theorems on the Laplace transformation that are useful in the study of linear control systems.

Translated Function. Let us obtain the Laplace transform of the translated function $f(t - \alpha)1(t - \alpha)$, where $\alpha \geq 0$. This function is zero for $t < \alpha$. The functions $f(t)1(t)$ and $f(t - \alpha)1(t - \alpha)$ are shown in Figure 2-1.

By definition, the Laplace transform of $f(t - \alpha)1(t - \alpha)$ is

$$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = \int_0^{\infty} f(t - \alpha)1(t - \alpha)e^{-st} dt$$

By changing the independent variable from t to τ , where $\tau = t - \alpha$, we obtain

$$\int_0^{\infty} f(t - \alpha)1(t - \alpha)e^{-st} dt = \int_{-\alpha}^{\infty} f(\tau)1(\tau)e^{-s(\tau+\alpha)} d\tau$$

Since in this book we always assume that $f(t) = 0$ for $t < 0$, $f(\tau)1(\tau) = 0$ for $\tau < 0$. Hence we can change the lower limit of integration from $-\alpha$ to 0. Thus

$$\begin{aligned} \int_{-\alpha}^{\infty} f(\tau)1(\tau)e^{-s(\tau+\alpha)} d\tau &= \int_0^{\infty} f(\tau)1(\tau)e^{-s(\tau+\alpha)} d\tau \\ &= \int_0^{\infty} f(\tau)e^{-s\tau}e^{-\alpha s} d\tau \\ &= e^{-\alpha s} \int_0^{\infty} f(\tau)e^{-s\tau} d\tau = e^{-\alpha s} F(s) \end{aligned}$$

where

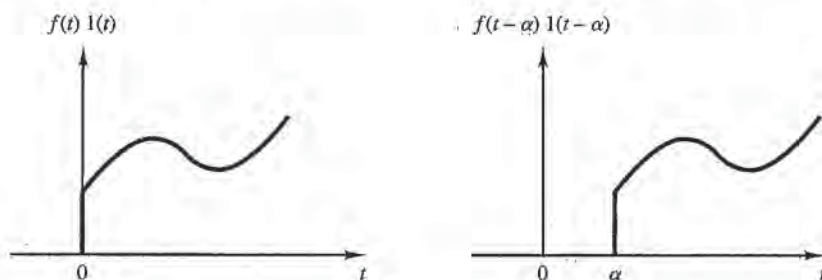
$$F(s) = \mathcal{L}[f(t)] = \int_0^{\infty} f(t)e^{-st} dt$$

And so

$$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s} F(s), \quad \text{for } \alpha \geq 0$$

This last equation states that the translation of the time function $f(t)1(t)$ by α (where $\alpha \geq 0$) corresponds to the multiplication of the transform $F(s)$ by $e^{-\alpha s}$.

Figure 2-1
Function $f(t)1(t)$
and translated
function
 $f(t - \alpha)1(t - \alpha)$.



Pulse Function. Consider the pulse function

$$f(t) = \frac{A}{t_0}, \quad \text{for } 0 < t < t_0$$

$$= 0, \quad \text{for } t < 0, t_0 < t$$

where A and t_0 are constants.

The pulse function here may be considered a step function of height A/t_0 that begins at $t = 0$ and that is superimposed by a negative step function of height A/t_0 beginning at $t = t_0$; that is,

$$f(t) = \frac{A}{t_0} 1(t) - \frac{A}{t_0} 1(t - t_0)$$

Then the Laplace transform of $f(t)$ is obtained as

$$\begin{aligned} \mathcal{L}[f(t)] &= \mathcal{L}\left[\frac{A}{t_0} 1(t)\right] - \mathcal{L}\left[\frac{A}{t_0} 1(t - t_0)\right] \\ &= \frac{A}{t_0 s} - \frac{A}{t_0 s} e^{-st_0} \\ &= \frac{A}{t_0 s} (1 - e^{-st_0}) \end{aligned} \quad (2-5)$$

Impulse Function. The impulse function is a special limiting case of the pulse function. Consider the impulse function

$$g(t) = \lim_{t_0 \rightarrow 0} \frac{A}{t_0}, \quad \text{for } 0 < t < t_0$$

$$= 0, \quad \text{for } t < 0, t_0 < t$$

Since the height of the impulse function is A/t_0 and the duration is t_0 , the area under the impulse is equal to A . As the duration t_0 approaches zero, the height A/t_0 approaches infinity, but the area under the impulse remains equal to A . Note that the magnitude of the impulse is measured by its area.

Referring to Equation (2-5), the Laplace transform of this impulse function is shown to be

$$\begin{aligned} \mathcal{L}[g(t)] &= \lim_{t_0 \rightarrow 0} \left[\frac{A}{t_0 s} (1 - e^{-st_0}) \right] \\ &= \lim_{t_0 \rightarrow 0} \frac{\frac{d}{dt_0} [A(1 - e^{-st_0})]}{\frac{d}{dt_0} (t_0 s)} = \frac{As}{s} = A \end{aligned}$$

Thus the Laplace transform of the impulse function is equal to the area under the impulse.

The impulse function whose area is equal to unity is called the *unit-impulse function* or the *Dirac delta function*. The unit-impulse function occurring at $t = t_0$ is usually denoted by $\delta(t - t_0)$. $\delta(t - t_0)$ satisfies the following:

$$\delta(t - t_0) = 0, \quad \text{for } t \neq t_0$$

$$\delta(t - t_0) = \infty, \quad \text{for } t = t_0$$

$$\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$$

It should be mentioned that an impulse that has an infinite magnitude and zero duration is mathematical fiction and does not occur in physical systems. If, however, the magnitude of a pulse input to a system is very large and its duration is very short compared to the system time constants, then we can approximate the pulse input by an impulse function. For instance, if a force or torque input $f(t)$ is applied to a system for a very short time duration, $0 < t < t_0$, where the magnitude of $f(t)$ is sufficiently large so that the integral $\int_0^{t_0} f(t) dt$ is not negligible, then this input can be considered an impulse input. (Note that when we describe the impulse input the area or magnitude of the impulse is most important, but the exact shape of the impulse is usually immaterial.) The impulse input supplies energy to the system in an infinitesimal time.

The concept of the impulse function is quite useful in differentiating discontinuous functions. The unit-impulse function $\delta(t - t_0)$ can be considered the derivative of the unit-step function $1(t - t_0)$ at the point of discontinuity $t = t_0$ or

$$\delta(t - t_0) = \frac{d}{dt} 1(t - t_0)$$

Conversely, if the unit-impulse function $\delta(t - t_0)$ is integrated, the result is the unit-step function $1(t - t_0)$. With the concept of the impulse function we can differentiate a function containing discontinuities, giving impulses, the magnitudes of which are equal to the magnitude of each corresponding discontinuity.

Multiplication of $f(t)$ by $e^{-\alpha t}$. If $f(t)$ is Laplace transformable, its Laplace transform being $F(s)$, then the Laplace transform of $e^{-\alpha t} f(t)$ is obtained as

$$\mathcal{L}[e^{-\alpha t} f(t)] = \int_0^{\infty} e^{-\alpha t} f(t) e^{-st} dt = F(s + \alpha) \quad (2-6)$$

We see that the multiplication of $f(t)$ by $e^{-\alpha t}$ has the effect of replacing s by $(s + \alpha)$ in the Laplace transform. Conversely, changing s to $(s + \alpha)$ is equivalent to multiplying $f(t)$ by $e^{-\alpha t}$. (Note that α may be real or complex.)

The relationship given by Equation (2-6) is useful in finding the Laplace transforms of such functions as $e^{-\alpha t} \sin \omega t$ and $e^{-\alpha t} \cos \omega t$. For instance, since

$$\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2 + \omega^2} = F(s), \quad \mathcal{L}[\cos \omega t] = \frac{s}{s^2 + \omega^2} = G(s)$$

it follows from Equation (2-6) that the Laplace transforms of $e^{-\alpha t} \sin \omega t$ and $e^{-\alpha t} \cos \omega t$ are given, respectively, by

$$\begin{aligned}\mathcal{L}[e^{-\alpha t} \sin \omega t] &= F(s + \alpha) = \frac{\omega}{(s + \alpha)^2 + \omega^2} \\ \mathcal{L}[e^{-\alpha t} \cos \omega t] &= G(s + \alpha) = \frac{s + \alpha}{(s + \alpha)^2 + \omega^2}\end{aligned}$$

Change of Time Scale. In analyzing physical systems, it is sometimes desirable to change the time scale or normalize a given time function. The result obtained in terms of normalized time is useful because it can be applied directly to different systems having similar mathematical equations.

If t is changed into t/α , where α is a positive constant, then the function $f(t)$ is changed into $f(t/\alpha)$. If we denote the Laplace transform of $f(t)$ by $F(s)$, then the Laplace transform of $f(t/\alpha)$ may be obtained as follows:

$$\mathcal{L}\left[f\left(\frac{t}{\alpha}\right)\right] = \int_0^{\infty} f\left(\frac{t}{\alpha}\right) e^{-st} dt$$

Letting $t/\alpha = t_1$ and $\alpha s = s_1$, we obtain

$$\begin{aligned}\mathcal{L}\left[f\left(\frac{t}{\alpha}\right)\right] &= \int_0^{\infty} f(t_1) e^{-s_1 t_1} d(\alpha t_1) \\ &= \alpha \int_0^{\infty} f(t_1) e^{-s_1 t_1} dt_1 \\ &= \alpha F(s_1)\end{aligned}$$

or

$$\mathcal{L}\left[f\left(\frac{t}{\alpha}\right)\right] = \alpha F(\alpha s)$$

As an example, consider $f(t) = e^{-t}$ and $f(t/5) = e^{-0.2t}$. We obtain

$$\mathcal{L}[f(t)] = \mathcal{L}[e^{-t}] = F(s) = \frac{1}{s + 1}$$

Hence

$$\mathcal{L}\left[f\left(\frac{t}{5}\right)\right] = \mathcal{L}[e^{-0.2t}] = 5F(5s) = \frac{5}{5s + 1}$$

This result can be verified easily by taking the Laplace transform of $e^{-0.2t}$ directly as follows:

$$\mathcal{L}[e^{-0.2t}] = \frac{1}{s + 0.2} = \frac{5}{5s + 1}$$

Comments on the Lower Limit of the Laplace Integral. In some cases, $f(t)$ possesses an impulse function at $t = 0$. Then the lower limit of the Laplace integral must

be clearly specified as to whether it is $0-$ or $0+$, since the Laplace transforms of $f(t)$ differ for these two lower limits. If such a distinction of the lower limit of the Laplace integral is necessary, we use the notations

$$\mathcal{L}_+[f(t)] = \int_{0+}^{\infty} f(t)e^{-st} dt$$

$$\mathcal{L}_-[f(t)] = \int_{0-}^{\infty} f(t)e^{-st} dt = \mathcal{L}_+[f(t)] + \int_{0-}^{0+} f(t)e^{-st} dt$$

If $f(t)$ involves an impulse function at $t = 0$, then

$$\mathcal{L}_+[f(t)] \neq \mathcal{L}_-[f(t)]$$

since

$$\int_{0-}^{0+} f(t)e^{-st} dt \neq 0$$

for such a case. Obviously, if $f(t)$ does not possess an impulse function at $t = 0$ (that is, if the function to be transformed is finite between $t = 0-$ and $t = 0+$), then

$$\mathcal{L}_+[f(t)] = \mathcal{L}_-[f(t)]$$

2-4 LAPLACE TRANSFORM THEOREMS

This section presents several theorems on Laplace transformation that are important in control engineering.

Real Differentiation Theorem. The Laplace transform of the derivative of a function $f(t)$ is given by

$$\mathcal{L}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0) \quad (2-7)$$

where $f(0)$ is the initial value of $f(t)$ evaluated at $t = 0$. [Here we assumed $f(0-) = f(0+) = f(0)$.]

For a given function $f(t)$, the values of $f(0+)$ and $f(0-)$ may be the same or different, as illustrated in Figure 2-2. The distinction between $f(0+)$ and $f(0-)$ is important

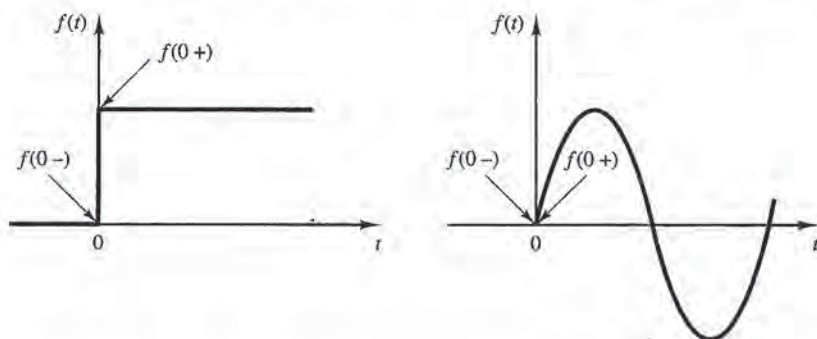


Figure 2-2
Step function and
sine function
indicating initial
values at $t = 0-$ and
 $t = 0+$.

when $f(t)$ has a discontinuity at $t = 0$ because in such a case $df(t)/dt$ will involve an impulse function at $t = 0$. If $f(0+) \neq f(0-)$, Equation (2-7) must be modified to

$$\mathcal{L}_+ \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0+)$$

$$\mathcal{L}_- \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0-)$$

To prove the real differentiation theorem, Equation (2-7), we proceed as follows. Integrating the Laplace integral by parts gives

$$\int_0^\infty f(t) e^{-st} dt = f(t) \frac{e^{-st}}{-s} \Big|_0^\infty - \int_0^\infty \left[\frac{d}{dt} f(t) \right] \frac{e^{-st}}{-s} dt$$

Hence

$$F(s) = \frac{f(0)}{s} + \frac{1}{s} \mathcal{L} \left[\frac{d}{dt} f(t) \right]$$

It follows that

$$\mathcal{L} \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0)$$

Similarly, we obtain the following relationship for the second derivative of $f(t)$:

$$\mathcal{L} \left[\frac{d^2}{dt^2} f(t) \right] = s^2 F(s) - sf(0) - \dot{f}(0)$$

where $\dot{f}(0)$ is the value of $df(t)/dt$ evaluated at $t = 0$. To derive this equation, define

$$\frac{d}{dt} f(t) = g(t)$$

Then

$$\begin{aligned} \mathcal{L} \left[\frac{d^2}{dt^2} f(t) \right] &= \mathcal{L} \left[\frac{d}{dt} g(t) \right] = s\mathcal{L}[g(t)] - g(0) \\ &= s\mathcal{L} \left[\frac{d}{dt} f(t) \right] - \dot{f}(0) \\ &= s^2 F(s) - sf(0) - \dot{f}(0) \end{aligned}$$

Similarly, for the n th derivative of $f(t)$, we obtain

$$\mathcal{L} \left[\frac{d^n}{dt^n} f(t) \right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} \dot{f}(0) - \dots - s^{(n-2)} \frac{d^{n-2}}{dt^{n-2}} f(0) - s^{(n-1)} \frac{d^{n-1}}{dt^{n-1}} f(0)$$

where $f(0), \dot{f}(0), \dots, \frac{d^{n-1}}{dt^{n-1}} f(0)$ represent the values of $f(t), df(t)/dt, \dots, d^{n-1}f(t)/dt^{n-1}$, respectively, evaluated at $t = 0$. If the distinction between $\mathcal{L}_+$ and $\mathcal{L}_-$ is necessary, we substitute $t = 0+$ or $t = 0-$ into $f(t), df(t)/dt, \dots, d^{n-1}f(t)/dt^{n-1}$, depending on whether we take $\mathcal{L}_+$ or $\mathcal{L}_-$.

Note that, in order for Laplace transforms of derivatives of $f(t)$ to exist, $d^n f(t)/dt^n$ ($n = 1, 2, 3, \dots$) must be Laplace transformable.

Note also that, if all the initial values of $f(t)$ and its derivatives are equal to zero, then the Laplace transform of the n th derivative of $f(t)$ is given by $s^n F(s)$.

EXAMPLE 2-1 Consider the cosine function:

$$\begin{aligned} g(t) &= 0, & \text{for } t < 0 \\ &= \cos \omega t, & \text{for } t \geq 0 \end{aligned}$$

The Laplace transform of this cosine function can be obtained directly as in the case of the sinusoidal function considered earlier. The use of the real differentiation theorem, however, will be demonstrated here by deriving the Laplace transform of the cosine function from the Laplace transform of the sine function. If we define

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= \sin \omega t, & \text{for } t \geq 0 \end{aligned}$$

then

$$\mathcal{L}[\sin \omega t] = F(s) = \frac{\omega}{s^2 + \omega^2}$$

The Laplace transform of the cosine function is obtained as

$$\begin{aligned} \mathcal{L}[\cos \omega t] &= \mathcal{L}\left[\frac{1}{\omega} \left(\frac{d}{dt} \sin \omega t\right)\right] = \frac{1}{\omega} [sF(s) - f(0)] \\ &= \frac{1}{\omega} \left[\frac{s\omega}{s^2 + \omega^2} - 0\right] = \frac{s}{s^2 + \omega^2} \end{aligned}$$

Final-Value Theorem. The final-value theorem relates the steady-state behavior of $f(t)$ to the behavior of $sF(s)$ in the neighborhood of $s = 0$. This theorem, however, applies if and only if $\lim_{t \rightarrow \infty} f(t)$ exists [which means that $f(t)$ settles down to a definite value for $t \rightarrow \infty$]. If all poles of $sF(s)$ lie in the left half s plane, $\lim_{t \rightarrow \infty} f(t)$ exists. But if $sF(s)$ has poles on the imaginary axis or in the right half s plane, $f(t)$ will contain oscillating or exponentially increasing time functions, respectively, and $\lim_{t \rightarrow \infty} f(t)$ will not exist. The final-value theorem does not apply to such cases. For instance, if $f(t)$ is the sinusoidal function $\sin \omega t$, $sF(s)$ has poles at $s = \pm j\omega$ and $\lim_{t \rightarrow \infty} f(t)$ does not exist. Therefore, this theorem is not applicable to such a function.

The final-value theorem may be stated as follows. If $f(t)$ and $df(t)/dt$ are Laplace transformable, if $F(s)$ is the Laplace transform of $f(t)$, and if $\lim_{t \rightarrow \infty} f(t)$ exists, then

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

To prove the theorem, we let s approach zero in the equation for the Laplace transform of the derivative of $f(t)$ or

$$\lim_{s \rightarrow 0} \int_0^{\infty} \left[\frac{d}{dt} f(t) \right] e^{-st} dt = \lim_{s \rightarrow 0} [sF(s) - f(0)]$$

Since $\lim_{s \rightarrow 0} e^{-st} = 1$, we obtain

$$\begin{aligned} \int_0^{\infty} \left[\frac{d}{dt} f(t) \right] dt &= f(t) \Big|_0^{\infty} = f(\infty) - f(0) \\ &= \lim_{s \rightarrow 0} sF(s) - f(0) \end{aligned}$$

from which

$$f(\infty) = \lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$$

The final-value theorem states that the steady-state behavior of $f(t)$ is the same as the behavior of $sF(s)$ in the neighborhood of $s = 0$. Thus, it is possible to obtain the value of $f(t)$ at $t = \infty$ directly from $F(s)$.

EXAMPLE 2-2 Given

$$\mathcal{L}[f(t)] = F(s) = \frac{1}{s(s+1)}$$

what is $\lim_{t \rightarrow \infty} f(t)$?

Since the pole of $sF(s) = 1/(s+1)$ lies in the left half s plane, $\lim_{t \rightarrow \infty} f(t)$ exists. So the final-value theorem is applicable in this case.

$$\lim_{t \rightarrow \infty} f(t) = f(\infty) = \lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} \frac{s}{s(s+1)} = \lim_{s \rightarrow 0} \frac{1}{s+1} = 1$$

In fact, this result can easily be verified, since

$$f(t) = 1 - e^{-t}, \quad \text{for } t \geq 0$$

Initial-Value Theorem. The initial-value theorem is the counterpart of the final-value theorem. By using this theorem, we are able to find the value of $f(t)$ at $t = 0+$ directly from the Laplace transform of $f(t)$. The initial-value theorem does not give the value of $f(t)$ at exactly $t = 0$ but at a time slightly greater than zero.

The initial-value theorem may be stated as follows: If $f(t)$ and $df(t)/dt$ are both Laplace transformable and if $\lim_{s \rightarrow \infty} sF(s)$ exists, then

$$f(0+) = \lim_{s \rightarrow \infty} sF(s)$$

To prove this theorem, we use the equation for the $\mathcal{L}_+$ transform of $df(t)/dt$:

$$\mathcal{L}_+ \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0+)$$

For the time interval $0+ \leq t \leq \infty$, as s approaches infinity, e^{-st} approaches zero. (Note that we must use $\mathcal{L}_+$ rather than $\mathcal{L}_-$ for this condition.) And so

$$\lim_{s \rightarrow \infty} \int_{0+}^{\infty} \left[\frac{d}{dt} f(t) \right] e^{-st} dt = \lim_{s \rightarrow \infty} [sF(s) - f(0+)] = 0$$

or

$$f(0+) = \lim_{s \rightarrow \infty} sF(s)$$

In applying the initial-value theorem, we are not limited as to the locations of the poles of $sF(s)$. Thus the initial-value theorem is valid for the sinusoidal function.

It should be noted that the initial-value theorem and the final-value theorem provide a convenient check on the solution, since they enable us to predict the system behavior in the time domain without actually transforming functions in s back to time functions.

Real-Integration Theorem. If $f(t)$ is of exponential order and $f(0-) = f(0+) = f(0)$, then the Laplace transform of $\int f(t) dt$ exists and is given by

$$\mathcal{L} \left[\int f(t) dt \right] = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s} \quad (2-8)$$

where $F(s) = \mathcal{L}[f(t)]$ and $f^{-1}(0) = \int f(t) dt$ evaluated at $t = 0$.

Note that if $f(t)$ involves an impulse function at $t = 0$, then $f^{-1}(0+) \neq f^{-1}(0-)$. So if $f(t)$ involves an impulse function at $t = 0$, we must modify Equation (2-8) as follows:

$$\begin{aligned}\mathcal{L}_+\left[\int f(t) dt\right] &= \frac{F(s)}{s} + \frac{f^{-1}(0+)}{s} \\ \mathcal{L}_-\left[\int f(t) dt\right] &= \frac{F(s)}{s} + \frac{f^{-1}(0-)}{s}\end{aligned}$$

The real-integration theorem given by Equation (2-8) can be proved in the following way. Integration by parts yields

$$\begin{aligned}\mathcal{L}\left[\int f(t) dt\right] &= \int_0^\infty \left[\int f(t) dt\right] e^{-st} dt \\ &= \left[\int f(t) dt\right] \frac{e^{-st}}{-s} \Big|_0^\infty - \int_0^\infty f(t) \frac{e^{-st}}{-s} dt \\ &= \frac{1}{s} \int f(t) dt \Big|_{t=0} + \frac{1}{s} \int_0^\infty f(t) e^{-st} dt \\ &= \frac{f^{-1}(0)}{s} + \frac{F(s)}{s}\end{aligned}$$

and the theorem is proved.

We see that integration in the time domain is converted into division in the s domain. If the initial value of the integral is zero, the Laplace transform of the integral of $f(t)$ is given by $F(s)/s$.

The preceding real-integration theorem given by Equation (2-8) can be modified slightly to deal with the definite integral of $f(t)$. If $f(t)$ is of exponential order, the Laplace transform of the definite integral $\int_0^t f(t) dt$ is given by

$$\mathcal{L}\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s} \quad (2-9)$$

where $F(s) = \mathcal{L}[f(t)]$. This is also referred to as the real-integration theorem. Note that if $f(t)$ involves an impulse function at $t = 0$ then $\int_{0+}^t f(t) dt \neq \int_{0-}^t f(t) dt$ and the following distinction must be observed:

$$\begin{aligned}\mathcal{L}_+\left[\int_{0+}^t f(t) dt\right] &= \frac{\mathcal{L}_+[f(t)]}{s} \\ \mathcal{L}_-\left[\int_{0-}^t f(t) dt\right] &= \frac{\mathcal{L}_-[f(t)]}{s}\end{aligned}$$

To prove Equation (2-9), first note that

$$\int_0^t f(t) dt = \int f(t) dt - f^{-1}(0)$$

where $f^{-1}(0)$ is equal to $\int f(t) dt$ evaluated at $t = 0$ and is a constant. Hence

$$\mathcal{L}\left[\int_0^t f(t) dt\right] = \mathcal{L}\left[\int f(t) dt\right] - \mathcal{L}[f^{-1}(0)]$$

Noting that $f^{-1}(0)$ is a constant so that

$$\mathcal{L}[f^{-1}(0)] = \frac{f^{-1}(0)}{s}$$

we obtain

$$\mathcal{L}\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s} + \frac{f^{-1}(0)}{s} - \frac{f^{-1}(0)}{s} = \frac{F(s)}{s}$$

Complex-Differentiation Theorem. If $f(t)$ is Laplace transformable, then, except at poles of $F(s)$,

$$\mathcal{L}[tf(t)] = -\frac{d}{ds} F(s)$$

where $F(s) = \mathcal{L}[f(t)]$. This is known as the complex-differentiation theorem. Also,

$$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2} F(s)$$

In general,

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), \quad \text{for } n = 1, 2, 3, \dots$$

To prove the complex-differentiation theorem, we proceed as follows:

$$\begin{aligned} \mathcal{L}[tf(t)] &= \int_0^\infty tf(t)e^{-st} dt = -\int_0^\infty f(t) \frac{d}{ds} (e^{-st}) dt \\ &= -\frac{d}{ds} \int_0^\infty f(t)e^{-st} dt = -\frac{d}{ds} F(s) \end{aligned}$$

Hence the theorem. Similarly, by defining $tf(t) = g(t)$, the result is

$$\begin{aligned} \mathcal{L}[t^2f(t)] &= \mathcal{L}[tg(t)] = -\frac{d}{ds} G(s) = -\frac{d}{ds} \left[-\frac{d}{ds} F(s) \right] \\ &= (-1)^2 \frac{d^2}{ds^2} F(s) = \frac{d^2}{ds^2} F(s) \end{aligned}$$

Repeating the same process, we obtain

$$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s), \quad \text{for } n = 1, 2, 3, \dots$$

Convolution Integral. Consider the Laplace transform of

$$\int_0^t f_1(t - \tau)f_2(\tau) d\tau$$

This integral is often written as

$$f_1(t) * f_2(t)$$

The mathematical operation $f_1(t) * f_2(t)$ is called *convolution*. Note that if we put $t - \tau = \xi$, then

$$\begin{aligned} \int_0^t f_1(t - \tau) f_2(\tau) d\tau &= - \int_t^0 f_1(\xi) f_2(t - \xi) d\xi \\ &= \int_0^t f_1(\tau) f_2(t - \tau) d\tau \end{aligned}$$

Hence

$$\begin{aligned} f_1(t) * f_2(t) &= \int_0^t f_1(t - \tau) f_2(\tau) d\tau \\ &= \int_0^t f_1(\tau) f_2(t - \tau) d\tau \\ &= f_2(t) * f_1(t) \end{aligned}$$

If $f_1(t)$ and $f_2(t)$ are piecewise continuous and of exponential order, then the Laplace transform of

$$\int_0^t f_1(t - \tau) f_2(\tau) d\tau$$

can be obtained as follows:

$$\mathcal{L} \left[\int_0^t f_1(t - \tau) f_2(\tau) d\tau \right] = F_1(s) F_2(s) \quad (2-10)$$

where

$$F_1(s) = \int_0^\infty f_1(t) e^{-st} dt = \mathcal{L}[f_1(t)]$$

$$F_2(s) = \int_0^\infty f_2(t) e^{-st} dt = \mathcal{L}[f_2(t)]$$

To prove Equation (2-10) note that $f_1(t - \tau)1(t - \tau) = 0$ for $\tau > t$. Hence

$$\int_0^t f_1(t - \tau) f_2(\tau) d\tau = \int_0^\infty f_1(t - \tau) 1(t - \tau) f_2(\tau) d\tau$$

Then

$$\begin{aligned} \mathcal{L} \left[\int_0^t f_1(t - \tau) f_2(\tau) d\tau \right] &= \mathcal{L} \left[\int_0^\infty f_1(t - \tau) 1(t - \tau) f_2(\tau) d\tau \right] \\ &= \int_0^\infty e^{-st} \left[\int_0^\infty f_1(t - \tau) 1(t - \tau) f_2(\tau) d\tau \right] dt \end{aligned}$$

Substituting $t - \tau = \lambda$ in this last equation and changing the order of integration, which is valid in this case because $f_1(t)$ and $f_2(t)$ are Laplace transformable, we obtain

$$\begin{aligned}\mathcal{L}\left[\int_0^t f_1(t-\tau)f_2(\tau) d\tau\right] &= \int_0^\infty f_1(t-\tau)1(t-\tau)e^{-st} dt \int_0^\infty f_2(\tau) d\tau \\ &= \int_0^\infty f_1(\lambda)e^{-s(\lambda+\tau)} d\lambda \int_0^\infty f_2(\tau) d\tau \\ &= \int_0^\infty f_1(\lambda)e^{-s\lambda} d\lambda \int_0^\infty f_2(\tau)e^{-s\tau} d\tau \\ &= F_1(s)F_2(s)\end{aligned}$$

This last equation gives the Laplace transform of the convolution integral. Conversely, if the Laplace transform of a function is given by a product of two Laplace transform functions, $F_1(s)F_2(s)$, then the corresponding time function (the inverse Laplace transform) is given by the convolution integral $f_1(t) * f_2(t)$.

Laplace Transform of Product of Two Time Functions. The Laplace transform of the product of two Laplace transformable functions $f(t)$ and $g(t)$ can be given by

$$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s-p) dp \quad (2-11)$$

To show this, we may proceed as follows: The Laplace transform of the product of $f(t)$ and $g(t)$ can be written as

$$\mathcal{L}[f(t)g(t)] = \int_0^\infty f(t)g(t)e^{-st} dt \quad (2-12)$$

Note that the inversion integral is

$$f(t) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(s)e^{st} ds, \quad \text{for } t > 0$$

where c is the abscissa of convergence for $F(s)$. Thus,

$$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_0^\infty \int_{c-j\infty}^{c+j\infty} F(p)e^{pt} dp g(t)e^{-st} dt$$

Because of the uniform convergence of the integrals considered, we may invert the order of integration:

$$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p) dp \int_0^\infty g(t)e^{-(s-p)t} dt$$

Noting that

$$\int_0^\infty g(t)e^{-(s-p)t} dt = G(s-p)$$

we obtain

$$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s-p) dp \quad (2-13)$$

Summary. Table 2-2 summarizes properties and theorems of the Laplace transforms. Most of them have been derived or proved in this section.

Table 2-2 Properties of Laplace Transforms

1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_{\pm}\left[\frac{d}{dt}f(t)\right] = sF(s) - f(0\pm)$
4	$\mathcal{L}_{\pm}\left[\frac{d^2}{dt^2}f(t)\right] = s^2F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathcal{L}_{\pm}\left[\frac{d^n}{dt^n}f(t)\right] = s^nF(s) - \sum_{k=1}^n s^{n-k}f^{(k-1)}(0\pm)$ where $f^{(k-1)}(t) = \frac{d^{k-1}}{dt^{k-1}}f(t)$
6	$\mathcal{L}_{\pm}\left[\int f(t) dt\right] = \frac{F(s)}{s} + \frac{1}{s}\left[\int f(t) dt\right]_{t=0\pm}$
7	$\mathcal{L}_{\pm}\left[\int \cdots \int f(t)(dt)^n\right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}}\left[\int \cdots \int f(t)(dt)^k\right]_{t=0\pm}$
8	$\mathcal{L}\left[\int_0^t f(t) dt\right] = \frac{F(s)}{s}$
9	$\int_0^{\infty} f(t) dt = \lim_{s \rightarrow 0} F(s) \quad \text{if } \int_0^{\infty} f(t) dt \text{ exists}$
10	$\mathcal{L}[e^{-\alpha t}f(t)] = F(s + a)$
11	$\mathcal{L}[f(t - \alpha)1(t - \alpha)] = e^{-\alpha s}F(s) \quad \alpha \geq 0$
12	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
13	$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2}F(s)$
14	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}F(s) \quad (n = 1, 2, 3, \dots)$
15	$\mathcal{L}\left[\frac{1}{t}f(t)\right] = \int_s^{\infty} F(s) ds \quad \text{if } \lim_{t \rightarrow 0} \frac{1}{t}f(t) \text{ exists}$
16	$\mathcal{L}\left[f\left(\frac{1}{a}\right)\right] = aF(as)$
17	$\mathcal{L}\left[\int_0^t f_1(t - \tau)f_2(\tau) d\tau\right] = F_1(s)F_2(s)$
18	$\mathcal{L}[f(t)g(t)] = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} F(p)G(s - p) dp$

2-5 INVERSE LAPLACE TRANSFORMATION

As noted earlier, the inverse Laplace transform can be obtained by use of the inversion integral given by Equation (2-4). However, the inversion integral is complicated and, therefore, its use is not recommended for finding inverse Laplace transforms of commonly encountered functions in control engineering.

A convenient method for obtaining inverse Laplace transforms is to use a table of Laplace transforms. In this case, the Laplace transform must be in a form immediately recognizable in such a table. Quite often the function in question may not appear in tables of Laplace transforms available to the engineer. If a particular transform $F(s)$ cannot be found in a table, then we may expand it into partial fractions and write $F(s)$ in terms of simple functions of s for which the inverse Laplace transforms are already known.

Note that these simpler methods for finding inverse Laplace transforms are based on the fact that the unique correspondence of a time function and its inverse Laplace transform holds for any continuous time function.

Partial-Fraction Expansion Method for Finding Inverse Laplace Transforms. For problems in control systems analysis, $F(s)$, the Laplace transform of $f(t)$, frequently occurs in the form

$$F(s) = \frac{B(s)}{A(s)}$$

where $A(s)$ and $B(s)$ are polynomials in s . In the expansion of $F(s) = B(s)/A(s)$ into a partial-fraction form, it is important that the highest power of s in $A(s)$ be greater than the highest power of s in $B(s)$. If such is not the case, the numerator $B(s)$ must be divided by the denominator $A(s)$ in order to produce a polynomial in s plus a remainder (a ratio of polynomials in s whose numerator is of lower degree than the denominator).

If $F(s)$ is broken up into components,

$$F(s) = F_1(s) + F_2(s) + \cdots + F_n(s)$$

and if the inverse Laplace transforms of $F_1(s)$, $F_2(s)$, $\dots$, $F_n(s)$ are readily available, then

$$\begin{aligned}\mathcal{L}^{-1}[F(s)] &= \mathcal{L}^{-1}[F_1(s)] + \mathcal{L}^{-1}[F_2(s)] + \cdots + \mathcal{L}^{-1}[F_n(s)] \\ &= f_1(t) + f_2(t) + \cdots + f_n(t)\end{aligned}$$

where $f_1(t)$, $f_2(t)$, $\dots$, $f_n(t)$ are the inverse Laplace transforms of $F_1(s)$, $F_2(s)$, $\dots$, $F_n(s)$, respectively. The inverse Laplace transform of $F(s)$ thus obtained is unique except possibly at points where the time function is discontinuous. Whenever the time function is continuous, the time function $f(t)$ and its Laplace transform $F(s)$ have a one-to-one correspondence.

The advantage of the partial-fraction expansion approach is that the individual terms of $F(s)$, resulting from the expansion into partial-fraction form, are very simple functions of s ; consequently, it is not necessary to refer to a Laplace transform table if we memorize several simple Laplace transform pairs. It should be noted, however, that in applying the partial-fraction expansion technique in the search for the inverse Laplace

transform of $F(s) = B(s)/A(s)$ the roots of the denominator polynomial $A(s)$ must be obtained in advance. That is, this method does not apply until the denominator polynomial has been factored.

Partial-Fraction Expansion when $F(s)$ Involves Distinct Poles Only. Consider $F(s)$ written in the factored form

$$F(s) = \frac{B(s)}{A(s)} = \frac{K(s + z_1)(s + z_2) \cdots (s + z_m)}{(s + p_1)(s + p_2) \cdots (s + p_n)}, \quad \text{for } m < n$$

where $p_1, p_2, \dots, p_n$ and $z_1, z_2, \dots, z_m$ are either real or complex quantities, but for each complex p_i or z_i there will occur the complex conjugate of p_i or z_i , respectively. If $F(s)$ involves distinct poles only, then it can be expanded into a sum of simple partial fractions as follows:

$$F(s) = \frac{B(s)}{A(s)} = \frac{a_1}{s + p_1} + \frac{a_2}{s + p_2} + \cdots + \frac{a_n}{s + p_n} \quad (2-14)$$

where a_k ($k = 1, 2, \dots, n$) are constants. The coefficient a_k is called the *residue* at the pole at $s = -p_k$. The value of a_k can be found by multiplying both sides of Equation (2-14) by $(s + p_k)$ and letting $s = -p_k$, which gives

$$\begin{aligned} \left[(s + p_k) \frac{B(s)}{A(s)} \right]_{s=-p_k} &= \left[\frac{a_1}{s + p_1} (s + p_k) + \frac{a_2}{s + p_2} (s + p_k) \right. \\ &\quad \left. + \cdots + \frac{a_k}{s + p_k} (s + p_k) + \cdots + \frac{a_n}{s + p_n} (s + p_k) \right]_{s=-p_k} \\ &= a_k \end{aligned}$$

We see that all the expanded terms drop out with the exception of a_k . Thus the residue a_k is found from

$$a_k = \left[(s + p_k) \frac{B(s)}{A(s)} \right]_{s=-p_k} \quad (2-15)$$

Note that, since $f(t)$ is a real function of time, if p_1 and p_2 are complex conjugates, then the residues a_1 and a_2 are also complex conjugates. Only one of the conjugates, a_1 or a_2 , needs to be evaluated because the other is known automatically.

Since

$$\mathcal{L}^{-1} \left[\frac{a_k}{s + p_k} \right] = a_k e^{-p_k t}$$

$f(t)$ is obtained as

$$f(t) = \mathcal{L}^{-1}[F(s)] = a_1 e^{-p_1 t} + a_2 e^{-p_2 t} + \cdots + a_n e^{-p_n t}, \quad \text{for } t \geq 0$$

EXAMPLE 2-3 Find the inverse Laplace transform of

$$F(s) = \frac{s + 3}{(s + 1)(s + 2)}$$

The partial-fraction expansion of $F(s)$ is

$$F(s) = \frac{s+3}{(s+1)(s+2)} = \frac{a_1}{s+1} + \frac{a_2}{s+2}$$

where a_1 and a_2 are found by using Equation (2-15):

$$a_1 = \left[(s+1) \frac{s+3}{(s+1)(s+2)} \right]_{s=-1} = \left[\frac{s+3}{s+2} \right]_{s=-1} = 2$$

$$a_2 = \left[(s+2) \frac{s+3}{(s+1)(s+2)} \right]_{s=-2} = \left[\frac{s+3}{s+1} \right]_{s=-2} = -1$$

Thus

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] \\ &= \mathcal{L}^{-1}\left[\frac{2}{s+1}\right] + \mathcal{L}^{-1}\left[\frac{-1}{s+2}\right] \\ &= 2e^{-t} - e^{-2t}, \quad \text{for } t \geq 0 \end{aligned}$$

EXAMPLE 2-4 Obtain the inverse Laplace transform of

$$G(s) = \frac{s^3 + 5s^2 + 9s + 7}{(s+1)(s+2)}$$

Here, since the degree of the numerator polynomial is higher than that of the denominator polynomial, we must divide the numerator by the denominator.

$$G(s) = s + 2 + \frac{s+3}{(s+1)(s+2)}$$

Note that the Laplace transform of the unit-impulse function $\delta(t)$ is 1 and that the Laplace transform of $d\delta(t)/dt$ is s . The third term on the right-hand side of this last equation is $F(s)$ in Example 2-3. So the inverse Laplace transform of $G(s)$ is given as

$$g(t) = \frac{d}{dt} \delta(t) + 2\delta(t) + 2e^{-t} - e^{-2t}, \quad \text{for } t \geq 0-$$

EXAMPLE 2-5 Find the inverse Laplace transform of

$$F(s) = \frac{2s+12}{s^2+2s+5}$$

Notice that the denominator polynomial can be factored as

$$s^2 + 2s + 5 = (s+1+j2)(s+1-j2)$$

If the function $F(s)$ involves a pair of complex-conjugate poles, it is convenient not to expand $F(s)$ into the usual partial fractions but to expand it into the sum of a damped sine and a damped cosine function.

Noting that $s^2 + 2s + 5 = (s+1)^2 + 2^2$ and referring to the Laplace transforms of $e^{-\alpha t} \sin \omega t$ and $e^{-\alpha t} \cos \omega t$, rewritten thus,

$$\begin{aligned} \mathcal{L}[e^{-\alpha t} \sin \omega t] &= \frac{\omega}{(s+\alpha)^2 + \omega^2} \\ \mathcal{L}[e^{-\alpha t} \cos \omega t] &= \frac{s+\alpha}{(s+\alpha)^2 + \omega^2} \end{aligned}$$

the given $F(s)$ can be written as a sum of a damped sine and a damped cosine function.

$$\begin{aligned} F(s) &= \frac{2s + 12}{s^2 + 2s + 5} = \frac{10 + 2(s + 1)}{(s + 1)^2 + 2^2} \\ &= 5 \frac{2}{(s + 1)^2 + 2^2} + 2 \frac{s + 1}{(s + 1)^2 + 2^2} \end{aligned}$$

It follows that

$$\begin{aligned} f(t) &= \mathcal{L}^{-1}[F(s)] \\ &= 5\mathcal{L}^{-1}\left[\frac{2}{(s + 1)^2 + 2^2}\right] + 2\mathcal{L}^{-1}\left[\frac{s + 1}{(s + 1)^2 + 2^2}\right] \\ &= 5e^{-t} \sin 2t + 2e^{-t} \cos 2t, \quad \text{for } t \geq 0 \end{aligned}$$

Partial-Fraction Expansion when $F(s)$ Involves Multiple Poles. Instead of discussing the general case, we shall use an example to show how to obtain the partial-fraction expansion of $F(s)$.

Consider the following $F(s)$:

$$F(s) = \frac{s^2 + 2s + 3}{(s + 1)^3}$$

The partial-fraction expansion of this $F(s)$ involves three terms,

$$F(s) = \frac{B(s)}{A(s)} = \frac{b_1}{s + 1} + \frac{b_2}{(s + 1)^2} + \frac{b_3}{(s + 1)^3}$$

where b_3 , b_2 , and b_1 are determined as follows. By multiplying both sides of this last equation by $(s + 1)^3$, we have

$$(s + 1)^3 \frac{B(s)}{A(s)} = b_1(s + 1)^2 + b_2(s + 1) + b_3 \quad (2-16)$$

Then letting $s = -1$, Equation (2-16) gives

$$\left[(s + 1)^3 \frac{B(s)}{A(s)} \right]_{s=-1} = b_3$$

Also, differentiation of both sides of Equation (2-16) with respect to s yields

$$\frac{d}{ds} \left[(s + 1)^3 \frac{B(s)}{A(s)} \right] = b_2 + 2b_1(s + 1) \quad (2-17)$$

If we let $s = -1$ in Equation (2-17), then

$$\frac{d}{ds} \left[(s + 1)^3 \frac{B(s)}{A(s)} \right]_{s=-1} = b_2$$

By differentiating both sides of Equation (2-17) with respect to s , the result is

$$\frac{d^2}{ds^2} \left[(s + 1)^3 \frac{B(s)}{A(s)} \right] = 2b_1$$

From the preceding analysis it can be seen that the values of b_3 , b_2 , and b_1 are found systematically as follows:

$$\begin{aligned}
 b_3 &= \left[(s+1)^3 \frac{B(s)}{A(s)} \right]_{s=-1} \\
 &= (s^2 + 2s + 3)_{s=-1} \\
 &= 2 \\
 b_2 &= \left\{ \frac{d}{ds} \left[(s+1)^3 \frac{B(s)}{A(s)} \right] \right\}_{s=-1} \\
 &= \left[\frac{d}{ds} (s^2 + 2s + 3) \right]_{s=-1} \\
 &= (2s + 2)_{s=-1} \\
 &= 0 \\
 b_1 &= \frac{1}{2!} \left\{ \frac{d^2}{ds^2} \left[(s+1)^3 \frac{B(s)}{A(s)} \right] \right\}_{s=-1} \\
 &= \frac{1}{2!} \left[\frac{d^2}{ds^2} (s^2 + 2s + 3) \right]_{s=-1} \\
 &= \frac{1}{2} (2) = 1
 \end{aligned}$$

We thus obtain

$$\begin{aligned}
 f(t) &= \mathcal{L}^{-1}[F(s)] \\
 &= \mathcal{L}^{-1} \left[\frac{1}{s+1} \right] + \mathcal{L}^{-1} \left[\frac{0}{(s+1)^2} \right] + \mathcal{L}^{-1} \left[\frac{2}{(s+1)^3} \right] \\
 &= e^{-t} + 0 + t^2 e^{-t} \\
 &= (1 + t^2) e^{-t}, \quad \text{for } t \geq 0
 \end{aligned}$$

Comments. For complicated functions with denominators involving higher-order polynomials, partial-fraction expansion may be quite time consuming. In such a case, use of MATLAB is recommended. (See Section 2-6.)

2-6 PARTIAL-FRACTION EXPANSION WITH MATLAB

MATLAB has a command to obtain the partial-fraction expansion of $B(s)/A(s)$. It also has a command to obtain the zeros and poles of $B(s)/A(s)$.

We shall first present the MATLAB approach to obtain the partial-fraction expansion of $B(s)/A(s)$. Then we discuss the MATLAB approach to obtain the zeros and poles of $B(s)/A(s)$.

Partial-Fraction Expansion with MATLAB. Consider the following function $B(s)/A(s)$:

$$\frac{B(s)}{A(s)} = \frac{\text{num}}{\text{den}} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_n}{s^n + a_1 s^{n-1} + \cdots + a_n}$$

where some of a_i and b_j may be zero. In MATLAB row vectors num and den specify the coefficients of the numerator and denominator of the transfer function. That is,

$$\begin{aligned} \text{num} &= [b_0 \ b_1 \ \dots \ b_n] \\ \text{den} &= [1 \ a_1 \ \dots \ a_n] \end{aligned}$$

The command

$$[r,p,k] = \text{residue}(\text{num},\text{den})$$

finds the residues (r), poles (p), and direct terms (k) of a partial-fraction expansion of the ratio of two polynomials $B(s)$ and $A(s)$.

The partial-fraction expansion of $B(s)/A(s)$ is given by

$$\frac{B(s)}{A(s)} = \frac{r(1)}{s - p(1)} + \frac{r(2)}{s - p(2)} + \dots + \frac{r(n)}{s - p(n)} + k(s) \quad (2-18)$$

Comparing Equations (2-14) and (2-18), we note that $p(1) = -p_1$, $p(2) = -p_2, \dots$, $p(n) = -p_n$; $r(1) = a_1$, $r(2) = a_2, \dots$, $r(n) = a_n$. [$k(s)$ is a direct term.]

EXAMPLE 2-6 Consider the following transfer function,

$$\frac{B(s)}{A(s)} = \frac{2s^3 + 5s^2 + 3s + 6}{s^3 + 6s^2 + 11s + 6}$$

For this function,

$$\begin{aligned} \text{num} &= [2 \ 5 \ 3 \ 6] \\ \text{den} &= [1 \ 6 \ 11 \ 6] \end{aligned}$$

The command

$$[r,p,k] = \text{residue}(\text{num},\text{den})$$

gives the following result:

$$\begin{aligned} [r,p,k] &= \text{residue}(\text{num},\text{den}) \\ r &= \\ &\quad -6.0000 \\ &\quad -4.0000 \\ &\quad 3.0000 \\ p &= \\ &\quad -3.0000 \\ &\quad -2.0000 \\ &\quad -1.0000 \\ k &= \\ &\quad 2 \end{aligned}$$

(Note that the residues are returned in column vector r , the pole locations in column vector p , and the direct term in row vector k .) This is the MATLAB representation of the following partial-fraction expansion of $B(s)/A(s)$:

$$\begin{aligned}\frac{B(s)}{A(s)} &= \frac{2s^3 + 5s^2 + 3s + 6}{s^3 + 6s^2 + 11s + 6} \\ &= \frac{-6}{s+3} + \frac{-4}{s+2} + \frac{3}{s+1} + 2\end{aligned}$$

The residue command can also be used to form the polynomials (numerator and denominator) from its partial-fraction expansion. That is, the command

$$[\text{num}, \text{den}] = \text{residue}(r, p, k)$$

where r , p , and k are as given in the previous MATLAB output, converts the partial-fraction expansion back to the polynomial ratio $B(s)/A(s)$, as follows:

```
[num,den] = residue(r,p,k);
printsys(num,den,'s')
num/den =
      2s^3 + 5s^2 + 3s + 6
      s^3 + 6s^2 + 11s + 6
```

The command

$$\text{printsys}(\text{num}, \text{den}, 's')$$

prints the num/den in terms of the ratio of polynomials in s .

Note that if $p(j) = p(j+1) = \dots = p(j+m-1)$ [that is, $p_j = p_{j+1} = \dots = p_{j+m-1}$], the pole $p(j)$ is a pole of multiplicity m . In such a case, the expansion includes terms of the form

$$\frac{r(j)}{s - p(j)} + \frac{r(j+1)}{[s - p(j)]^2} + \dots + \frac{r(j+m-1)}{[s - p(j)]^m}$$

For details, see Example 2-7.

EXAMPLE 2-7 Expand the following $B(s)/A(s)$ into partial-fractions with MATLAB.

$$\frac{B(s)}{A(s)} = \frac{s^2 + 2s + 3}{(s+1)^3} = \frac{s^2 + 2s + 3}{s^3 + 3s^2 + 3s + 1}$$

For this function, we have

$$\begin{aligned}\text{num} &= [0 \ 1 \ 2 \ 3] \\ \text{den} &= [1 \ 3 \ 3 \ 1]\end{aligned}$$

The command

$$[r, p, k] = \text{residue}(\text{num}, \text{den})$$

gives the result shown on the next page. It is the MATLAB representation of the following partial-fraction expansion of $B(s)/A(s)$:

$$\frac{B(s)}{A(s)} = \frac{1}{s+1} + \frac{0}{(s+1)^2} + \frac{2}{(s+1)^3}$$

```

num = [0 1 2 3];
den = [1 3 3 1];
[r,p,k] = residue(num,den)

r =

    1.0000
    0.0000
    2.0000

p =

   -1.0000
   -1.0000
   -1.0000

k =

    []

```

Note that the direct term k is zero.

To obtain the original function $B(s)/A(s)$ from r, p, and k, enter the following program to the computer:

```

[num,den] = residue(r,p,k);
printsys(num,den,'s')

```

Then the computer will show the num/den as follows:

$$\text{num/den} = \frac{s^2 + 2s + 3}{s^3 + 3s^2 + 3s + 1}$$

Finding Zeros and Poles of $B(s)/A(s)$ with MATLAB. MATLAB has a command

```
[z,p,K] = tf2zp(num,den)
```

to obtain the zeros, poles, and gain K of $B(s)/A(s)$.

Consider the system defined by

$$\frac{B(s)}{A(s)} = \frac{4s^2 + 16s + 12}{s^4 + 12s^3 + 44s^2 + 48s}$$

To obtain the zeros (z), poles (p), and gain (K), enter the following MATLAB program into the computer:

```

num = [0 0 4 16 12];
den = [1 12 44 48 0];
[z,p,K] = tf2zp(num,den)

```

Then the computer will produce the following output on the screen:

```

z =
    -3
    -1

p =
     0
   -6.0000
   -4.0000
   -2.0000

K =
     4

```

The zeros are at $s = -3$ and -1 . The poles are at $s = 0, -6, -4$, and -2 . The gain K is 4.
 If the zeros, poles, and gain K are given, then the following MATLAB program will yield the original num/den.

```

z = [-1; -3];
p = [0; -2; -4; -6];
K = 4;
[num,den] = zp2tf(z,p,K);
printsys(num,den,'s')

num/den =
      4s^2 + 16s + 12
-----
s^4 + 12s^3 + 44s^2 + 48s

```

2-7 SOLVING LINEAR, TIME-INVARIANT, DIFFERENTIAL EQUATIONS

In this section we are concerned with the use of the Laplace transform method in solving linear, time-invariant, differential equations.

The Laplace transform method yields the complete solution (complementary solution and particular solution) of linear, time-invariant, differential equations. Classical methods for finding the complete solution of a differential equation require the evaluation of the integration constants from the initial conditions. In the case of the Laplace transform method, however, this requirement is unnecessary because the initial conditions are automatically included in the Laplace transform of the differential equation.

If all initial conditions are zero, then the Laplace transform of the differential equation is obtained simply by replacing d/dt with s , d^2/dt^2 with s^2 , and so on.

In solving linear, time-invariant, differential equations by the Laplace transform method, two steps are involved.

1. By taking the Laplace transform of each term in the given differential equation, convert the differential equation into an algebraic equation in s and obtain the expression for the Laplace transform of the dependent variable by rearranging the algebraic equation.
2. The time solution of the differential equation is obtained by finding the inverse Laplace transform of the dependent variable.

In the following discussion, two examples are used to demonstrate the solution of linear, time-invariant, differential equations by the Laplace transform method.

EXAMPLE 2-8 Find the solution $x(t)$ of the differential equation

$$\ddot{x} + 3\dot{x} + 2x = 0, \quad x(0) = a, \quad \dot{x} = b$$

where a and b are constants.

By writing the Laplace transform of $x(t)$ as $X(s)$ or

$$\mathcal{L}[x(t)] = X(s)$$

we obtain

$$\mathcal{L}[\dot{x}] = sX(s) - x(0)$$

$$\mathcal{L}[\ddot{x}] = s^2X(s) - sx(0) - \dot{x}(0)$$

And so the given differential equation becomes

$$[s^2X(s) - sx(0) - \dot{x}(0)] + 3[sX(s) - x(0)] + 2X(s) = 0$$

By substituting the given initial conditions into this last equation, we obtain

$$[s^2X(s) - as - b] + 3[sX(s) - a] + 2X(s) = 0$$

or

$$(s^2 + 3s + 2)X(s) = as + b + 3a$$

Solving for $X(s)$, we have

$$X(s) = \frac{as + b + 3a}{s^2 + 3s + 2} = \frac{as + b + 3a}{(s + 1)(s + 2)} = \frac{2a + b}{s + 1} - \frac{a + b}{s + 2}$$

The inverse Laplace transform of $X(s)$ gives

$$\begin{aligned} x(t) &= \mathcal{L}^{-1}[X(s)] = \mathcal{L}^{-1}\left[\frac{2a + b}{s + 1}\right] - \mathcal{L}^{-1}\left[\frac{a + b}{s + 2}\right] \\ &= (2a + b)e^{-t} - (a + b)e^{-2t}, \quad \text{for } t \geq 0 \end{aligned}$$

which is the solution of the given differential equation. Notice that the initial conditions a and b appear in the solution. Thus $x(t)$ has no undetermined constants.

EXAMPLE 2-9 Find the solution $x(t)$ of the differential equation

$$\ddot{x} + 2\dot{x} + 5x = 3, \quad x(0) = 0, \quad \dot{x}(0) = 0$$

Noting that $\mathcal{L}[3] = 3/s$, $x(0) = 0$, and $\dot{x}(0) = 0$, the Laplace transform of the differential equation becomes

$$s^2X(s) + 2sX(s) + 5X(s) = \frac{3}{s}$$

Solving for $X(s)$, we find

$$\begin{aligned} X(s) &= \frac{3}{s(s^2 + 2s + 5)} = \frac{3}{5} \frac{1}{s} - \frac{3}{5} \frac{s + 2}{s^2 + 2s + 5} \\ &= \frac{3}{5} \frac{1}{s} - \frac{3}{10} \frac{2}{(s + 1)^2 + 2^2} - \frac{3}{5} \frac{s + 1}{(s + 1)^2 + 2^2} \end{aligned}$$

Hence the inverse Laplace transform becomes

$$\begin{aligned} x(t) &= \mathcal{L}^{-1}[X(s)] \\ &= \frac{3}{5} \mathcal{L}^{-1}\left[\frac{1}{s}\right] - \frac{3}{10} \mathcal{L}^{-1}\left[\frac{2}{(s + 1)^2 + 2^2}\right] - \frac{3}{5} \mathcal{L}^{-1}\left[\frac{s + 1}{(s + 1)^2 + 2^2}\right] \\ &= \frac{3}{5} - \frac{3}{10} e^{-t} \sin 2t - \frac{3}{5} e^{-t} \cos 2t, \quad \text{for } t \geq 0 \end{aligned}$$

which is the solution of the given differential equation.

EXAMPLE PROBLEMS AND SOLUTIONS

A-2-1. Find the poles of the following $F(s)$:

$$F(s) = \frac{1}{1 - e^{-s}}$$

Solution. The poles are found from

$$e^{-s} = 1$$

or

$$e^{-(\sigma + j\omega)} = e^{-\sigma}(\cos \omega - j \sin \omega) = 1$$

From this it follows that $\sigma = 0$, $\omega = \pm 2n\pi$ ($n = 0, 1, 2, \dots$). Thus, the poles are located at

$$s = \pm j2n\pi \quad (n = 0, 1, 2, \dots)$$

A-2-2. Find the Laplace transform of $f(t)$ defined by

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= te^{-3t}, & \text{for } t \geq 0 \end{aligned}$$

Solution. Since

$$\mathcal{L}[t] = G(s) = \frac{1}{s^2}$$

referring to Equation (2-6), we obtain

$$F(s) = \mathcal{L}[te^{-3t}] = G(s + 3) = \frac{1}{(s + 3)^2}$$

A-2-3. What is the Laplace transform of

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= \sin(\omega t + \theta), & \text{for } t \geq 0 \end{aligned}$$

where θ is a constant?

Solution. Noting that

$$\sin(\omega t + \theta) = \sin \omega t \cos \theta + \cos \omega t \sin \theta$$

we have

$$\begin{aligned}\mathcal{L}[\sin(\omega t + \theta)] &= \cos \theta \mathcal{L}[\sin \omega t] + \sin \theta \mathcal{L}[\cos \omega t] \\ &= \cos \theta \frac{\omega}{s^2 + \omega^2} + \sin \theta \frac{s}{s^2 + \omega^2} \\ &= \frac{\omega \cos \theta + s \sin \theta}{s^2 + \omega^2}\end{aligned}$$

A-2-4. Find the Laplace transform $F(s)$ of the function $f(t)$ shown in Figure 2-3, where $f(t) = 0$ for $t < 0$ and $2a \leq t$. Also find the limiting value of $F(s)$ as a approaches zero.

Solution. The function $f(t)$ can be written

$$f(t) = \frac{1}{a^2} 1(t) - \frac{2}{a^2} 1(t - a) + \frac{1}{a^2} 1(t - 2a)$$

Then

$$\begin{aligned}F(s) &= \mathcal{L}[f(t)] \\ &= \frac{1}{a^2} \mathcal{L}[1(t)] - \frac{2}{a^2} \mathcal{L}[1(t - a)] + \frac{1}{a^2} \mathcal{L}[1(t - 2a)] \\ &= \frac{1}{a^2} \frac{1}{s} - \frac{2}{a^2} \frac{1}{s} e^{-as} + \frac{1}{a^2} \frac{1}{s} e^{-2as} \\ &= \frac{1}{a^2 s} (1 - 2e^{-as} + e^{-2as})\end{aligned}$$

As a approaches zero, we have

$$\begin{aligned}\lim_{a \rightarrow 0} F(s) &= \lim_{a \rightarrow 0} \frac{1 - 2e^{-as} + e^{-2as}}{a^2 s} = \lim_{a \rightarrow 0} \frac{\frac{d}{da} (1 - 2e^{-as} + e^{-2as})}{\frac{d}{da} (a^2 s)} \\ &= \lim_{a \rightarrow 0} \frac{2se^{-as} - 2se^{-2as}}{2as} = \lim_{a \rightarrow 0} \frac{e^{-as} - e^{-2as}}{a} \\ &= \lim_{a \rightarrow 0} \frac{\frac{d}{da} (e^{-as} - e^{-2as})}{\frac{d}{da} (a)} = \lim_{a \rightarrow 0} \frac{-se^{-as} + 2se^{-2as}}{1} \\ &= -s + 2s = s\end{aligned}$$

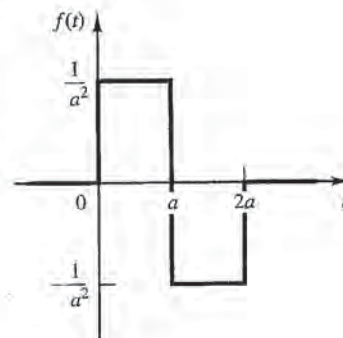


Figure 2-3
Function $f(t)$.

A-2-5. Find the initial value of $df(t)/dt$ when the Laplace transform of $f(t)$ is given by

$$F(s) = \mathcal{L}[f(t)] = \frac{2s + 1}{s^2 + s + 1}$$

Solution. Using the initial-value theorem,

$$\lim_{t \rightarrow 0^+} f(t) = f(0+) = \lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \frac{s(2s + 1)}{s^2 + s + 1} = 2$$

Since the $\mathcal{L}_+$ transform of $df(t)/dt = g(t)$ is given by

$$\begin{aligned} \mathcal{L}_+[g(t)] &= sF(s) - f(0+) \\ &= \frac{s(2s + 1)}{s^2 + s + 1} - 2 = \frac{-s - 2}{s^2 + s + 1} \end{aligned}$$

the initial value of $df(t)/dt$ is obtained as

$$\begin{aligned} \lim_{t \rightarrow 0^+} \frac{df(t)}{dt} &= g(0+) = \lim_{s \rightarrow \infty} s[sF(s) - f(0+)] \\ &= \lim_{s \rightarrow \infty} \frac{-s^2 - 2s}{s^2 + s + 1} = -1 \end{aligned}$$

A-2-6. The derivative of the unit-impulse function $\delta(t)$ is called a *unit-doublet* function. (Thus, the integral of the unit-doublet function is the unit-impulse function.) Mathematically, an example of the unit-doublet function, which is usually denoted by $u_2(t)$, may be given by

$$u_2(t) = \lim_{t_0 \rightarrow 0} \frac{1(t) - 2[1(t - t_0)] + 1(t - 2t_0)}{t_0^2}$$

Obtain the Laplace transform of $u_2(t)$.

Solution. The Laplace transform of $u_2(t)$ is given by

$$\begin{aligned} \mathcal{L}[u_2(t)] &= \lim_{t_0 \rightarrow 0} \frac{1}{t_0^2} \left(\frac{1}{s} - \frac{2}{s} e^{-t_0 s} + \frac{1}{s} e^{-2t_0 s} \right) \\ &= \lim_{t_0 \rightarrow 0} \frac{1}{t_0^2 s} \left[1 - 2 \left(1 - t_0 s + \frac{t_0^2 s^2}{2} + \cdots \right) + \left(1 - 2t_0 s + \frac{4t_0^2 s^2}{2} + \cdots \right) \right] \\ &= \lim_{t_0 \rightarrow 0} \frac{1}{t_0^2 s} [t_0^2 s^2 + (\text{higher-order terms in } t_0 s)] = s \end{aligned}$$

A-2-7. Find the Laplace transform of $f(t)$ defined by

$$\begin{aligned} f(t) &= 0, & \text{for } t < 0 \\ &= t^2 \sin \omega t, & \text{for } t \geq 0 \end{aligned}$$

Solution. Since

$$\mathcal{L}[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$$

applying the complex-differentiation theorem

$$\mathcal{L}[t^2 f(t)] = \frac{d^2}{ds^2} F(s)$$

to this problem, we have

$$\mathcal{L}[f(t)] = \mathcal{L}[t^2 \sin \omega t] = \frac{d^2}{ds^2} \left[\frac{\omega}{s^2 + \omega^2} \right] = \frac{-2\omega^3 + 6\omega s^2}{(s^2 + \omega^2)^3}$$

- A-2-8.** Prove that if $f(t)$ is of exponential order and if $\int_0^\infty f(t) dt$ exists [which means that $\int_0^\infty f(t) dt$ assumes a definite value] then

$$\int_0^\infty f(t) dt = \lim_{s \rightarrow 0} F(s)$$

where $F(s) = \mathcal{L}[f(t)]$.

Solution. Note that

$$\int_0^\infty f(t) dt = \lim_{t \rightarrow \infty} \int_0^t f(t) dt$$

Referring to Equation (2-9),

$$\mathcal{L} \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s}$$

Since $\int_0^\infty f(t) dt$ exists, by applying the final-value theorem to this case,

$$\lim_{t \rightarrow \infty} \int_0^t f(t) dt = \lim_{s \rightarrow 0} s \frac{F(s)}{s}$$

or

$$\int_0^\infty f(t) dt = \lim_{s \rightarrow 0} F(s)$$

- A-2-9.** Prove that if $f(t)$ is a periodic function with period T , then

$$\mathcal{L}[f(t)] = \frac{\int_0^T f(t) e^{-st} dt}{1 - e^{-Ts}}$$

Solution.

$$\mathcal{L}[f(t)] = \int_0^\infty f(t) e^{-st} dt = \sum_{n=0}^\infty \int_{nT}^{(n+1)T} f(t) e^{-st} dt$$

By changing the independent variable from t to τ , where $\tau = t - nT$,

$$\mathcal{L}[f(t)] = \sum_{n=0}^\infty e^{-nTs} \int_0^T f(\tau) e^{-s\tau} d\tau$$

where we used the fact that $f(\tau + nT) = f(\tau)$ because the function $f(t)$ is periodic with period T . Noting that

$$\begin{aligned} \sum_{n=0}^\infty e^{-nTs} &= 1 + e^{-Ts} + e^{-2Ts} + \dots \\ &= 1 + e^{-Ts}(1 + e^{-Ts} + e^{-2Ts} + \dots) \\ &= 1 + e^{-Ts} \left(\sum_{n=0}^\infty e^{-nTs} \right) \end{aligned}$$

we obtain

$$\sum_{n=0}^{\infty} e^{-nTs} = \frac{1}{1 - e^{-Ts}}$$

It follows that

$$\mathcal{L}[f(t)] = \frac{\int_0^T f(t)e^{-st} dt}{1 - e^{-Ts}}$$

A-2-10. What is the Laplace transform of the periodic function shown in Figure 2-4?

Solution. Note that

$$\begin{aligned} \int_0^T f(t)e^{-st} dt &= \int_0^{T/2} e^{-st} dt + \int_{T/2}^T (-1)e^{-st} dt \\ &= \left. \frac{e^{-st}}{-s} \right|_0^{T/2} - \left. \frac{e^{-st}}{-s} \right|_{T/2}^T \\ &= \frac{e^{-(1/2)Ts} - 1}{-s} + \frac{e^{-Ts} - e^{-(1/2)Ts}}{s} \\ &= \frac{1}{s} [e^{-Ts} - 2e^{-(1/2)Ts} + 1] \\ &= \frac{1}{s} [1 - e^{-(1/2)Ts}]^2 \end{aligned}$$

Referring to Problem A-2-9, we have

$$\begin{aligned} F(s) &= \frac{\int_0^T f(t)e^{-st} dt}{1 - e^{-Ts}} = \frac{(1/s)[1 - e^{-(1/2)Ts}]^2}{1 - e^{-Ts}} \\ &= \frac{1 - e^{-(1/2)Ts}}{s[1 + e^{-(1/2)Ts}]} = \frac{1}{s} \tanh \frac{Ts}{4} \end{aligned}$$

A-2-11. Find the inverse Laplace transform of $F(s)$, where

$$F(s) = \frac{1}{s(s^2 + 2s + 2)}$$

Solution. Since

$$s^2 + 2s + 2 = (s + 1 + j1)(s + 1 - j1)$$

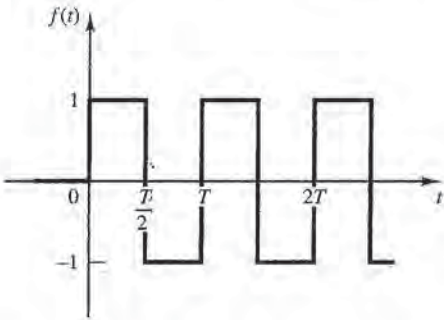


Figure 2-4
Periodic function
(square wave).

we notice that $F(s)$ involves a pair of complex-conjugate poles, and so we expand $F(s)$ into the form

$$F(s) = \frac{1}{s(s^2 + 2s + 2)} = \frac{a_1}{s} + \frac{a_2s + a_3}{s^2 + 2s + 2}$$

where a_1 , a_2 , and a_3 are determined from

$$1 = a_1(s^2 + 2s + 2) + (a_2s + a_3)s$$

By comparing coefficients of s^2 , s , and s^0 terms on both sides of this last equation, respectively, we obtain

$$a_1 + a_2 = 0, \quad 2a_1 + a_3 = 0, \quad 2a_1 = 1$$

from which

$$a_1 = \frac{1}{2}, \quad a_2 = -\frac{1}{2}, \quad a_3 = -1$$

Therefore,

$$\begin{aligned} F(s) &= \frac{1}{2} \frac{1}{s} - \frac{1}{2} \frac{s + 2}{s^2 + 2s + 2} \\ &= \frac{1}{2} \frac{1}{s} - \frac{1}{2} \frac{1}{(s + 1)^2 + 1^2} - \frac{1}{2} \frac{s + 1}{(s + 1)^2 + 1^2} \end{aligned}$$

The inverse Laplace transform of $F(s)$ gives

$$f(t) = \frac{1}{2} - \frac{1}{2} e^{-t} \sin t - \frac{1}{2} e^{-t} \cos t, \quad \text{for } t \geq 0$$

A-2-12. Obtain the inverse Laplace transform of

$$F(s) = \frac{5(s + 2)}{s^2(s + 1)(s + 3)}$$

Solution.

$$F(s) = \frac{5(s + 2)}{s^2(s + 1)(s + 3)} = \frac{b_1}{s} + \frac{b_2}{s^2} + \frac{a_1}{s + 1} + \frac{a_2}{s + 3}$$

where

$$\begin{aligned} a_1 &= \left. \frac{5(s + 2)}{s^2(s + 3)} \right|_{s=-1} = \frac{5}{2} \\ a_2 &= \left. \frac{5(s + 2)}{s^2(s + 1)} \right|_{s=-3} = \frac{5}{18} \\ b_2 &= \left. \frac{5(s + 2)}{(s + 1)(s + 3)} \right|_{s=0} = \frac{10}{3} \\ b_1 &= \frac{d}{ds} \left[\frac{5(s + 2)}{(s + 1)(s + 3)} \right]_{s=0} \\ &= \left. \frac{5(s + 1)(s + 3) - 5(s + 2)(2s + 4)}{(s + 1)^2(s + 3)^2} \right|_{s=0} = -\frac{25}{9} \end{aligned}$$

Thus

$$F(s) = -\frac{25}{9} \frac{1}{s} + \frac{10}{3} \frac{1}{s^2} + \frac{5}{2} \frac{1}{s+1} + \frac{5}{18} \frac{1}{s+3}$$

The inverse Laplace transform of $F(s)$ is

$$f(t) = -\frac{25}{9} + \frac{10}{3}t + \frac{5}{2}e^{-t} + \frac{5}{18}e^{-3t}, \quad \text{for } t \geq 0$$

A-2-13. Find the inverse Laplace transform of

$$F(s) = \frac{s^4 + 2s^3 + 3s^2 + 4s + 5}{s(s+1)}$$

Solution. Since the numerator polynomial is of higher degree than the denominator polynomial, by dividing the numerator by the denominator until the remainder is a fraction, we obtain

$$F(s) = s^2 + s + 2 + \frac{2s+5}{s(s+1)} = s^2 + s + 2 + \frac{a_1}{s} + \frac{a_2}{s+1}$$

where

$$a_1 = \left. \frac{2s+5}{s+1} \right|_{s=0} = 5$$

$$a_2 = \left. \frac{2s+5}{s} \right|_{s=-1} = -3$$

It follows that

$$F(s) = s^2 + s + 2 + \frac{5}{s} - \frac{3}{s+1}$$

The inverse Laplace transform of $F(s)$ is

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{d^2}{dt^2} \delta(t) + \frac{d}{dt} \delta(t) + 2\delta(t) + 5 - 3e^{-t}, \quad \text{for } t \geq 0$$

A-2-14. Derive the inverse Laplace transform of

$$F(s) = \frac{1}{s(s^2 + \omega^2)}$$

Solution.

$$F(s) = \frac{1}{s(s^2 + \omega^2)} = \frac{1}{\omega^2} \frac{1}{s} - \frac{1}{\omega^2} \frac{s}{s^2 + \omega^2}$$

Hence the inverse Laplace transform of $F(s)$ is obtained as

$$f(t) = \mathcal{L}^{-1}[F(s)] = \frac{1}{\omega^2} (1 - \cos \omega t), \quad \text{for } t \geq 0$$

A-2-15. Obtain the inverse Laplace transform of the following $F(s)$. [Use MATLAB to find the partial-fraction expansion of $F(s)$.]

$$F(s) = \frac{s^5 + 8s^4 + 23s^3 + 35s^2 + 28s + 3}{s^3 + 6s^2 + 8s}$$

Solution. The following MATLAB program will produce the partial-fraction expansion of $F(s)$:

```
num = [1 8 23 35 28 3];
den = [0 0 1 6 8 0];
[r,p,k] = residue(num,den)

r =
    0.3750
    0.2500
    0.3750

p =
    -4
    -2
     0

k =
     1  2  3
```

Note that $k = [1 \ 2 \ 3]$ means that $F(s)$ involves $s^2 + 2s + 3$ as shown below:

$$F(s) = s^2 + 2s + 3 + \frac{0.375}{s+4} + \frac{0.25}{s+2} + \frac{0.375}{s}$$

Hence, the inverse Laplace transform of $F(s)$ is given by

$$f(t) = \frac{d^2}{dt^2} \delta(t) + 2 \frac{d}{dt} \delta(t) + 3\delta(t) + 0.375e^{-4t} + 0.25e^{-2t} + 0.375, \quad \text{for } t \geq 0$$

A-2-16. Given the zero(s), pole(s), and gain K of $B(s)/A(s)$, obtain the function $B(s)/A(s)$ using MATLAB. Consider the three cases below.

- (1) There is no zero. Poles are at $-1 + 2j$ and $-1 - 2j$. $K = 10$.
- (2) A zero is at 0. Poles are at $-1 + 2j$ and $-1 - 2j$. $K = 10$.
- (3) A zero is at -1 . Poles are at $-2, -4$ and -8 . $K = 12$.

Solution. MATLAB programs to obtain $B(s)/A(s) = \text{num}/\text{den}$ for the three cases are shown below.

```
z = [];
p = [-1+2*j; -1-2*j];
K = 10;
[num,den] = zp2tf(z,p,K);
printsys(num,den)

num/den =
      10
-----
s^2 + 2s + 5
```

```
z = [0];
p = [-1+2*j; -1-2*j];
K = 10;
[num,den] = zp2tf(z,p,K);
printsys(num,den)

num/den =
      10s
-----
s^2 + 2s + 5
```

```
z = [-1];
p = [-2; -4; -8];
K = 12;
[num,den] = zp2tf(z,p,K);
printsys(num,den)

num/den =
      12s + 12
-----
s^3 + 14s^2 + 56s + 64
```

A-2-17. Solve the following differential equation:

$$\ddot{x} + 2\dot{x} + 10x = t^2, \quad x(0) = 0, \quad \dot{x}(0) = 0$$

Solution. Noting that the initial conditions are zeros, the Laplace transform of the equation becomes as follows:

$$s^2X(s) + 2sX(s) + 10X(s) = \frac{2}{s^3}$$

Hence

$$X(s) = \frac{2}{s^3(s^2 + 2s + 10)}$$

We need to find the partial-fraction expansion of $X(s)$. Since the denominator involves a triple pole, it is simpler to use MATLAB to obtain the partial-fraction expansion. The following MATLAB program may be used:

```
num = [0 0 0 0 0 2];
den = [1 2 10 0 0 0];
[r,p,k] = residue(num,den)
r =
    0.0060- 0.0087i
    0.0060+ 0.0087i
   -0.0120
   -0.0400
    0.2000
p =
   -1.0000+ 3.0000i
   -1.0000- 3.0000i
     0
     0
     0
k =
[]
```

From the MATLAB output, we find

$$X(s) = \frac{0.006 - 0.0087j}{s + 1 - 3j} + \frac{0.006 + 0.0087j}{s + 1 + 3j} + \frac{-0.012}{s} + \frac{-0.04}{s^2} + \frac{0.2}{s^3}$$

Combining the first two terms on the right-hand side of the equation, we get

$$X(s) = \frac{0.012(s + 1) + 0.0522}{(s + 1)^2 + 3^2} - \frac{0.012}{s} - \frac{0.04}{s^2} + \frac{0.2}{s^3}$$

The inverse Laplace transform of $X(s)$ gives

$$x(t) = 0.012e^{-t} \cos 3t + 0.0174e^{-t} \sin 3t - 0.012 - 0.04t + 0.1t^2, \quad \text{for } t \geq 0$$

PROBLEMS

B-2-1. Find the Laplace transforms of the following functions:

(a) $f_1(t) = 0,$ for $t < 0$
 $= e^{-0.4t} \cos 12t,$ for $t \geq 0$

(b) $f_2(t) = 0,$ for $t < 0$
 $= \sin\left(4t + \frac{\pi}{3}\right),$ for $t \geq 0$

B-2-2. Find the Laplace transforms of the following functions:

(a) $f_1(t) = 0,$ for $t < 0$
 $= 3 \sin(5t + 45^\circ),$ for $t \geq 0$

(b) $f_2(t) = 0,$ for $t < 0$
 $= 0.03(1 - \cos 2t),$ for $t \geq 0$

B-2-3. Obtain the Laplace transform of the function defined by

$$f(t) = 0, \quad \text{for } t < 0$$

$$= t^2 e^{-at}, \quad \text{for } t \geq 0$$

B-2-4. Obtain the Laplace transforms of the following functions:

(a) $f(t) = 0,$ for $t < 0$
 $= \sin \omega t \cdot \cos \omega t,$ for $t \geq 0$

(b) $f(t) = 0,$ for $t < 0$
 $= te^{-t} \sin 5t,$ for $t \geq 0$

B-2-5. Obtain the Laplace transform of the function defined by

$$f(t) = 0, \quad \text{for } t < 0$$

$$= \cos 2\omega t \cdot \cos 3\omega t, \quad \text{for } t \geq 0$$

B-2-6. What is the Laplace transform of the function $f(t)$ shown in Figure 2-5?

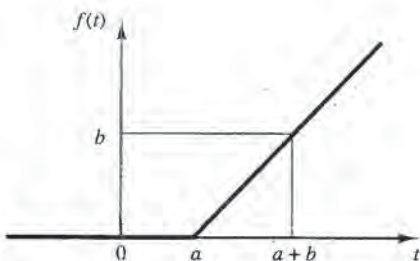


Figure 2-5
Function $f(t)$.

B-2-7. Obtain the Laplace transform of the function $f(t)$ shown in Figure 2-6.

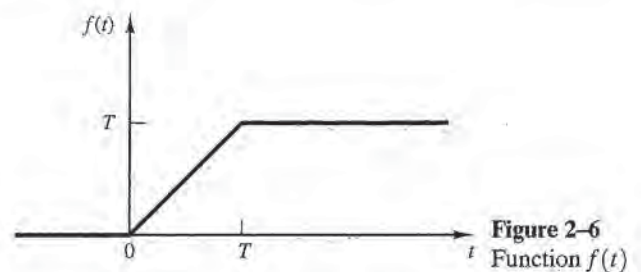


Figure 2-6
Function $f(t)$

B-2-8. Find the Laplace transform of the function $f(t)$ shown in Figure 2-7. Also, find the limiting value of $\mathcal{L}[f(t)]$ as a approaches zero.

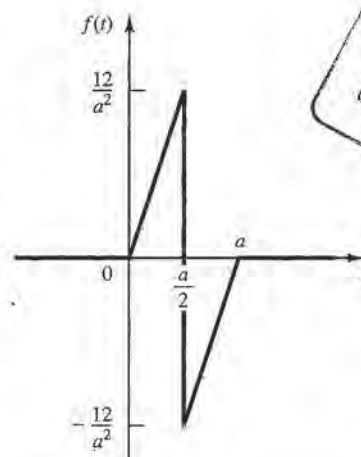


Figure 2-7
Function $f(t)$.

B-2-9. By applying the final-value theorem, find the final value of $f(t)$ whose Laplace transform is given by

$$F(s) = \frac{10}{s(s+1)}$$

Verify this result by taking the inverse Laplace transform of $F(s)$ and letting $t \rightarrow \infty$.

B-2-10. Given

$$F(s) = \frac{1}{(s+2)^2}$$

determine the values of $f(0+)$ and $\dot{f}(0+)$. (Use the initial-value theorem.)

B-2-11. Find the inverse Laplace transform of

$$F(s) = \frac{s+1}{s(s^2+s+1)}$$

B-2-12. Obtain the inverse Laplace transform of the following function:

$$F(s) = \frac{5e^{-s}}{s+1}$$

B-2-13. Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \frac{6s+3}{s^2}$

(b) $F_2(s) = \frac{5s+2}{(s+1)(s+2)^2}$

B-2-14. Find the inverse Laplace transforms of the following functions:

(a) $F_1(s) = \frac{1}{s^2(s^2+\omega^2)}$

(b) $F_2(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} \quad (0 < \zeta < 1)$

B-2-15. Obtain the partial-fraction expansion of the following function with MATLAB:

$$F(s) = \frac{10(s+2)(s+4)}{(s+1)(s+3)(s+5)^2}$$

Then, obtain the inverse Laplace transform of $F(s)$.

B-2-16. Consider the following function $F(s)$:

$$F(s) = \frac{s^4 + 5s^3 + 6s^2 + 9s + 30}{s^4 + 6s^3 + 21s^2 + 46s + 30}$$

Using MATLAB, obtain the partial-fraction expansion of $F(s)$. Then, obtain the inverse Laplace transform of $F(s)$.

B-2-17. A function $B(s)/A(s)$ consists of the following zeros, poles, and gain K :

zeros at $s = -1, s = -2$

poles at $s = 0, s = -4, s = -6$

gain $K = 5$

Obtain the expression for $B(s)/A(s) = \text{num/den}$ with MATLAB.

B-2-18. What is the solution of the following differential equation?

$$2\ddot{x} + 7\dot{x} + 3x = 0, \quad x(0) = 3, \quad \dot{x}(0) = 0$$

B-2-19. Solve the differential equation

$$\dot{x} + 2x = \delta(t), \quad x(0^-) = 0$$

B-2-20. Solve the following differential equation:

$$\ddot{x} + 2\zeta\omega_n\dot{x} + \omega_n^2x = 0, \quad x(0) = a, \quad \dot{x}(0) = b$$

where a and b are constants.

B-2-21. Obtain the solution of the differential equation

$$\dot{x} + ax = A \sin \omega t, \quad x(0) = b$$

B-2-22. Obtain the solution of the differential equation

$$\ddot{x} + 3\dot{x} + 6x = 0, \quad x(0) = 0, \quad \dot{x}(0) = 3$$

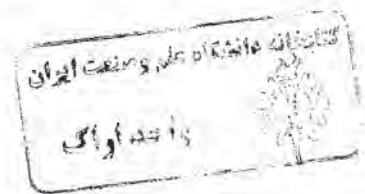
B-2-23. Solve the following differential equation:

$$\ddot{x} + 2\dot{x} + 10x = e^{-t}, \quad x(0) = 0, \quad \dot{x}(0) = 0$$

The forcing function e^{-t} is given at $t = 0$ when the system is at rest.



Mathematical Modeling of Dynamic Systems



3-1 INTRODUCTION

In studying control systems the reader must be able to model dynamic systems and analyze dynamic characteristics. A mathematical model of a dynamic system is defined as a set of equations that represents the dynamics of the system accurately or, at least, fairly well. Note that a mathematical model is not unique to a given system. A system may be represented in many different ways and, therefore, may have many mathematical models, depending on one's perspective.

The dynamics of many systems, whether they are mechanical, electrical, thermal, economic, biological, and so on, may be described in terms of differential equations. Such differential equations may be obtained by using physical laws governing a particular system, for example, Newton's laws for mechanical systems and Kirchhoff's laws for electrical systems. We must always keep in mind that deriving reasonable mathematical models is the most important part of the entire analysis of control systems.

Throughout this book we assume that the principle of causality applies to the systems considered. This means that the current output of the system (the output at time $t = 0$) depends on the past input (the input for $t < 0$) but does not depend on the future input (the input for $t > 0$).

Mathematical Models. Mathematical models may assume many different forms. Depending on the particular system and the particular circumstances, one mathematical model may be better suited than other models. For example, in optimal control problems, it is advantageous to use state-space representations. On the other hand, for the

transient-response or frequency-response analysis of single-input-single-output, linear, time-invariant systems, the transfer function representation may be more convenient than any other. Once a mathematical model of a system is obtained, various analytical and computer tools can be used for analysis and synthesis purposes.

Simplicity Versus Accuracy. In obtaining a mathematical model, we must make a compromise between the simplicity of the model and the accuracy of the results of the analysis. In deriving a reasonably simplified mathematical model, we frequently find it necessary to ignore certain inherent physical properties of the system. In particular, if a linear lumped-parameter mathematical model (that is, one employing ordinary differential equations) is desired, it is always necessary to ignore certain nonlinearities and distributed parameters that may be present in the physical system. If the effects that these ignored properties have on the response are small, good agreement will be obtained between the results of the analysis of a mathematical model and the results of the experimental study of the physical system.

In general, in solving a new problem, it is desirable to build a simplified model so that we can get a general feeling for the solution. A more complete mathematical model may then be built and used for a more accurate analysis.

We must be well aware of the fact that a linear lumped-parameter model, which may be valid in low-frequency operations, may not be valid at sufficiently high frequencies since the neglected property of distributed parameters may become an important factor in the dynamic behavior of the system. For example, the mass of a spring may be neglected in low-frequency operations, but it becomes an important property of the system at high frequencies. (For the case where a mathematical model involves considerable errors, robust control theory may be applied.)

Linear Systems. A system is called linear if the principle of superposition applies. The principle of superposition states that the response produced by the simultaneous application of two different forcing functions is the sum of the two individual responses. Hence, for the linear system, the response to several inputs can be calculated by treating one input at a time and adding the results. It is this principle that allows one to build up complicated solutions to the linear differential equation from simple solutions.

In an experimental investigation of a dynamic system, if cause and effect are proportional, thus implying that the principle of superposition holds, then the system can be considered linear.

Linear Time-Invariant Systems and Linear Time-Varying Systems. A differential equation is linear if the coefficients are constants or functions only of the independent variable. Dynamic systems that are composed of linear time-invariant lumped-parameter components may be described by linear time-invariant (constant-coefficient) differential equations. Such systems are called *linear time-invariant* (or *linear constant-coefficient*) systems. Systems that are represented by differential equations whose coefficients are functions of time are called *linear time-varying* systems. An example of a time-varying control system is a spacecraft control system. (The mass of a spacecraft changes due to fuel consumption.)

Outline of the Chapter. Section 3-1 has presented an introduction to the mathematical modeling of dynamic systems. Section 3-2 presents the transfer function and impulse-response function. Section 3-3 introduces automatic control systems and Section 3-4 discusses concepts of modeling in state space. Section 3-5 presents state-space representation of dynamic systems. Section 3-6 discusses transformation of mathematical models with MATLAB. Section 3-7 treats mathematical modeling of mechanical systems and discusses Newton's approach to modeling mechanical systems. Section 3-8 deals with mathematical modeling of electrical and electronic systems. Section 3-9 treats signal flow graphs and Mason's gain formula useful to control systems analysis. Finally, Section 3-10 discusses linearization of nonlinear mathematical models.

3-2 TRANSFER FUNCTION AND IMPULSE-RESPONSE FUNCTION

In control theory, functions called transfer functions are commonly used to characterize the input-output relationships of components or systems that can be described by linear, time-invariant, differential equations. We begin by defining the transfer function and follow with a derivation of the transfer function of a mechanical system. Then we discuss the impulse-response function.

Transfer Function. The *transfer function* of a linear, time-invariant, differential equation system is defined as the ratio of the Laplace transform of the output (response function) to the Laplace transform of the input (driving function) under the assumption that all initial conditions are zero.

Consider the linear time-invariant system defined by the following differential equation:

$$\begin{aligned} a_0 \overset{(n)}{y} + a_1 \overset{(n-1)}{y} + \cdots + a_{n-1} \dot{y} + a_n y \\ = b_0 \overset{(m)}{x} + b_1 \overset{(m-1)}{x} + \cdots + b_{m-1} \dot{x} + b_m x \quad (n \geq m) \end{aligned}$$

where y is the output of the system and x is the input. The transfer function of this system is the ratio of the Laplace transformed output to the Laplace transformed input when all initial conditions are zero, or

$$\begin{aligned} \text{Transfer function} = G(s) &= \frac{\mathcal{L}[\text{output}]}{\mathcal{L}[\text{input}]} \bigg|_{\text{zero initial conditions}} \\ &= \frac{Y(s)}{X(s)} = \frac{b_0 s^m + b_1 s^{m-1} + \cdots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n} \end{aligned}$$

By using the concept of transfer function, it is possible to represent system dynamics by algebraic equations in s . If the highest power of s in the denominator of the transfer function is equal to n , the system is called an *nth-order system*.

Comments on Transfer Function. The applicability of the concept of the transfer function is limited to linear, time-invariant, differential equation systems. The transfer function approach, however, is extensively used in the analysis and design of such systems. In what follows, we shall list important comments concerning the transfer function. (Note that in the list a system referred to is one described by a linear, time-invariant, differential equation.)

1. The transfer function of a system is a mathematical model in that it is an operational method of expressing the differential equation that relates the output variable to the input variable.
2. The transfer function is a property of a system itself, independent of the magnitude and nature of the input or driving function.
3. The transfer function includes the units necessary to relate the input to the output; however, it does not provide any information concerning the physical structure of the system. (The transfer functions of many physically different systems can be identical.)
4. If the transfer function of a system is known, the output or response can be studied for various forms of inputs with a view toward understanding the nature of the system.
5. If the transfer function of a system is unknown, it may be established experimentally by introducing known inputs and studying the output of the system. Once established, a transfer function gives a full description of the dynamic characteristics of the system, as distinct from its physical description.

EXAMPLE 3-1

Consider the satellite attitude control system shown in Figure 3-1. The diagram shows the control of only the yaw angle θ . (In the actual system there are controls about three axes.) Small jets apply reaction forces to rotate the satellite body into the desired attitude. The two skew symmetrically placed jets denoted by A or B operate in pairs. Assume that each jet thrust is $F/2$ and a torque $T = Fl$ is applied to the system. The jets are applied for a certain time duration and thus the torque can be written as $T(t)$. The moment of inertia about the axis of rotation at the center of mass is J .

Let us obtain the transfer function of this system by assuming that torque $T(t)$ is the input, and the angular displacement $\theta(t)$ of the satellite is the output. (We consider the motion only in the plane of the page.)

To derive the transfer function, we proceed according to the following steps.

1. Write the differential equation for the system.
2. Take the Laplace transform of the differential equation, assuming all initial conditions are zero.
3. Take the ratio of the output $\Theta(s)$ to the input $T(s)$. This ratio is the transfer function.

Applying Newton's second law to the present system and noting that there is no friction in the environment of the satellite, we have

$$J \frac{d^2\theta}{dt^2} = T$$

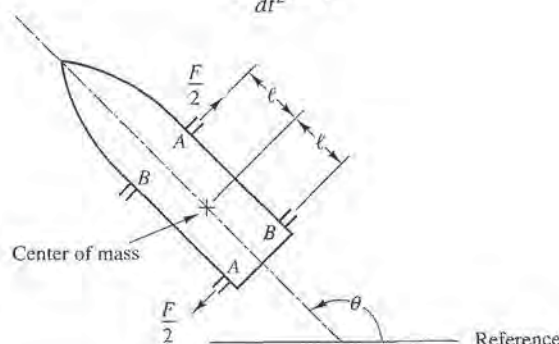


Figure 3-1
Schematic diagram
of a satellite attitude
control system.

Taking the Laplace transform of both sides of this last equation and assuming all initial conditions to be zero yields

$$Js^2\Theta(s) = T(s)$$

where $\Theta(s) = \mathcal{L}[\theta(t)] = T(s)$ and $T(s) = \mathcal{L}[T(t)]$. The transfer function of the system is thus obtained as

$$\text{Transfer function} = \frac{\Theta(s)}{T(s)} = \frac{1}{Js^2}$$

Convolution Integral. For a linear, time-invariant system the transfer function $G(s)$ is

$$G(s) = \frac{Y(s)}{X(s)}$$

where $X(s)$ is the Laplace transform of the input and $Y(s)$ is the Laplace transform of the output, where we assume that all initial conditions involved are zero. It follows that the output $Y(s)$ can be written as the product of $G(s)$ and $X(s)$, or

$$Y(s) = G(s)X(s) \quad (3-1)$$

Note that multiplication in the complex domain is equivalent to convolution in the time domain (see Section 2-4), so the inverse Laplace transform of Equation (3-1) is given by the following convolution integral:

$$\begin{aligned} y(t) &= \int_0^t x(\tau)g(t-\tau)d\tau \\ &= \int_0^t g(\tau)x(t-\tau)d\tau \end{aligned}$$

where both $g(t)$ and $x(t)$ are 0 for $t < 0$.

Impulse-Response Function. Consider the output (response) of a system to a unit-impulse input when the initial conditions are zero. Since the Laplace transform of the unit-impulse function is unity, the Laplace transform of the output of the system is

$$Y(s) = G(s) \quad (3-2)$$

The inverse Laplace transform of the output given by Equation (3-2) gives the impulse response of the system. The inverse Laplace transform of $G(s)$, or

$$\mathcal{L}^{-1}[G(s)] = g(t)$$

is called the impulse-response function. This function $g(t)$ is also called the weighting function of the system.

The impulse-response function $g(t)$ is thus the response of a linear system to a unit-impulse input when the initial conditions are zero. The Laplace transform of this function gives the transfer function. Therefore, the transfer function and impulse-response function of a linear, time-invariant system contain the same information about the system dynamics. It is hence possible to obtain complete information about the dynamic

characteristics of the system by exciting it with an impulse input and measuring the response. (In practice, a pulse input with a very short duration compared with the significant time constants of the system can be considered an impulse.)

3-3 AUTOMATIC CONTROL SYSTEMS

A control system may consist of a number of components. To show the functions performed by each component, in control engineering, we commonly use a diagram called the *block diagram*. This section first explains what a block diagram is. Next, it discusses introductory aspects of automatic control systems, including various control actions. Then, it presents a method for obtaining block diagrams for physical systems, and, finally, discusses techniques to simplify such diagrams.

Block Diagrams. A *block diagram* of a system is a pictorial representation of the functions performed by each component and of the flow of signals. Such a diagram depicts the interrelationships that exist among the various components. Differing from a purely abstract mathematical representation, a block diagram has the advantage of indicating more realistically the signal flows of the actual system.

In a block diagram all system variables are linked to each other through functional blocks. The *functional block* or simply *block* is a symbol for the mathematical operation on the input signal to the block that produces the output. The transfer functions of the components are usually entered in the corresponding blocks, which are connected by arrows to indicate the direction of the flow of signals. Note that the signal can pass only in the direction of the arrows. Thus a block diagram of a control system explicitly shows a unilateral property.

Figure 3-2 shows an element of the block diagram. The arrowhead pointing toward the block indicates the input, and the arrowhead leading away from the block represents the output. Such arrows are referred to as *signals*.

Note that the dimensions of the output signal from the block is the dimensions of the input signal multiplied by the dimensions of the transfer function in the block.

The advantages of the block diagram representation of a system lie in the fact that it is easy to form the overall block diagram for the entire system by merely connecting the blocks of the components according to the signal flow and that it is possible to evaluate the contribution of each component to the overall performance of the system.

In general, the functional operation of the system can be visualized more readily by examining the block diagram than by examining the physical system itself. A block diagram contains information concerning dynamic behavior, but it does not include any information on the physical construction of the system. Consequently, many dissimilar and unrelated systems can be represented by the same block diagram.

It should be noted that in a block diagram the main source of energy is not explicitly shown and that the block diagram of a given system is not unique. A number of different block diagrams can be drawn for a system, depending on the point of view of the analysis.

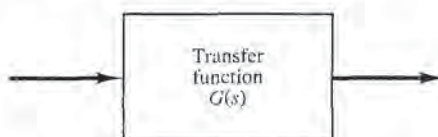


Figure 3-2
Element of a block diagram.

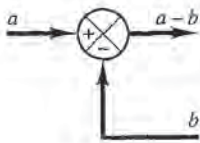


Figure 3-3
Summing point.

Summing Point. Referring to Figure 3-3, a circle with a cross is the symbol that indicates a summing operation. The plus or minus sign at each arrowhead indicates whether that signal is to be added or subtracted. It is important that the quantities being added or subtracted have the same dimensions and the same units.

Branch Point. A branch point is a point from which the signal from a block goes concurrently to other blocks or summing points.

Block Diagram of a Closed-Loop System. Figure 3-4 shows an example of a block diagram of a closed-loop system. The output $C(s)$ is fed back to the summing point, where it is compared with the reference input $R(s)$. The closed-loop nature of the system is clearly indicated by the figure. The output of the block, $C(s)$ in this case, is obtained by multiplying the transfer function $G(s)$ by the input to the block, $E(s)$. Any linear control system may be represented by a block diagram consisting of blocks, summing points, and branch points.

When the output is fed back to the summing point for comparison with the input, it is necessary to convert the form of the output signal to that of the input signal. For example, in a temperature-control system, the output signal is usually the controlled temperature. The output signal, which has the dimension of temperature, must be converted to a force or position or voltage before it can be compared with the input signal. This conversion is accomplished by the feedback element whose transfer function is $H(s)$, as shown in Figure 3-5. The role of the feedback element is to modify the output before it is compared with the input. (In most cases the feedback element is a sensor that measures the output of the plant. The output of the sensor is compared with the system input, and the actuating error signal is generated.) In the present example, the feedback signal that is fed back to the summing point for comparison with the input is $B(s) = H(s)C(s)$.

Figure 3-4
Block diagram of a closed-loop system.

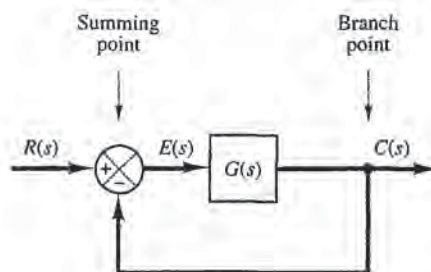
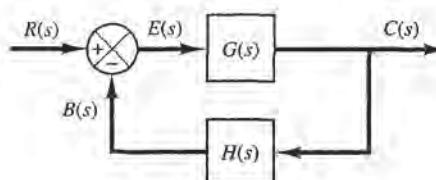


Figure 3-5
Closed-loop system.



Open-Loop Transfer Function and Feedforward Transfer Function. Referring to Figure 3-5, the ratio of the feedback signal $B(s)$ to the actuating error signal $E(s)$ is called the *open-loop transfer function*. That is,

$$\text{Open-loop transfer function} = \frac{B(s)}{E(s)} = G(s)H(s)$$

The ratio of the output $C(s)$ to the actuating error signal $E(s)$ is called the *feedforward transfer function*, so that

$$\text{Feedforward transfer function} = \frac{C(s)}{E(s)} = G(s)$$

If the feedback transfer function $H(s)$ is unity, then the open-loop transfer function and the feedforward transfer function are the same.

Closed-Loop Transfer Function. For the system shown in Figure 3-5, the output $C(s)$ and input $R(s)$ are related as follows: since

$$\begin{aligned} C(s) &= G(s)E(s) \\ E(s) &= R(s) - B(s) \\ &= R(s) - H(s)C(s) \end{aligned}$$

eliminating $E(s)$ from these equations gives

$$C(s) = G(s)[R(s) - H(s)C(s)]$$

or

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)} \quad (3-3)$$

The transfer function relating $C(s)$ to $R(s)$ is called the *closed-loop transfer function*. This transfer function relates the closed-loop system dynamics to the dynamics of the feedforward elements and feedback elements.

From Equation (3-3), $C(s)$ is given by

$$C(s) = \frac{G(s)}{1 + G(s)H(s)} R(s)$$

Thus the output of the closed-loop system clearly depends on both the closed-loop transfer function and the nature of the input.

Obtaining Cascaded, Parallel, and Feedback (Closed-Loop) Transfer Functions with MATLAB. In control systems analysis, we frequently need to calculate the cascaded transfer functions, parallel-connected transfer functions, and feedback-connected (closed-loop) transfer functions. MATLAB has convenient commands to obtain the cascaded, parallel, and feedback (closed-loop) transfer functions.

Suppose that there are two components $G_1(s)$ and $G_2(s)$ connected differently as shown in Figure 3-6, where

$$G_1(s) = \frac{\text{num1}}{\text{den1}}, \quad G_2(s) = \frac{\text{num2}}{\text{den2}}$$

To obtain the transfer functions of the cascaded system, parallel system, or feedback (closed-loop) system, the following commands may be used:

```
[num, den] = series(num1,den1,num2,den2)
[num, den] = parallel(num1,den1,num2,den2)
[num, den] = feedback(num1,den1,num2,den2)
```

As an example, consider the case where

$$G_1(s) = \frac{10}{s^2 + 2s + 10} = \frac{\text{num1}}{\text{den1}}, \quad G_2(s) = \frac{5}{s + 5} = \frac{\text{num2}}{\text{den2}}$$

MATLAB Program 3-1 gives $C(s)/R(s) = \text{num}/\text{den}$ for each arrangement of $G_1(s)$ and $G_2(s)$. Note that the command

```
printsys(num,den)
```

displays the num/den [that is, the transfer function $C(s)/R(s)$] of the system considered.

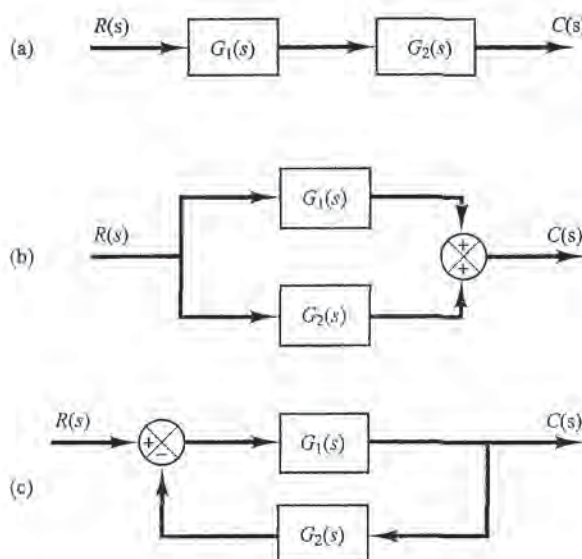


Figure 3-6
(a) Cascaded system;
(b) parallel system;
(c) feedback (closed-loop) system.

MATLAB Program 3-1

```
num1 = [0 0 10];
den1 = [1 2 10];
num2 = [0 5];
den2 = [1 5];
[num, den] = series(num1,den1,num2,den2);
printsys(num,den)

num/den =
      50
-----
s^3 + 7s^2 + 20s + 50

[num, den] = parallel(num1,den1,num2,den2);
printsys(num,den)

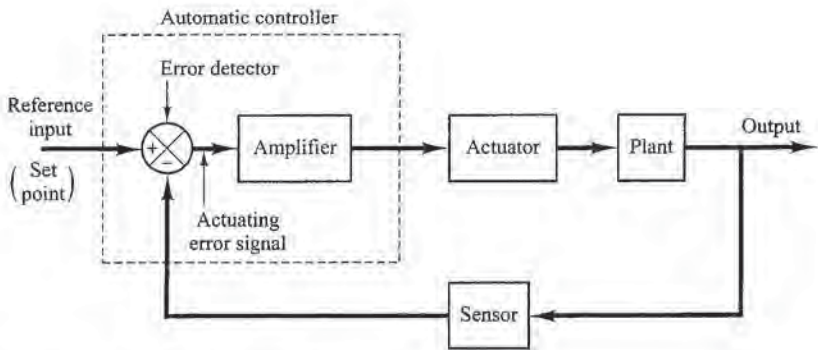
num/den =
      5s^2 + 20s + 100
-----
s^3 + 7s^2 + 20s + 50

[num, den] = feedback(num1,den1,num2,den2);
printsys(num,den)

num/den =
      10s + 50
-----
s^3 + 7s^2 + 20s + 100
```

Automatic Controllers. An automatic controller compares the actual value of the plant output with the reference input (desired value), determines the deviation, and produces a control signal that will reduce the deviation to zero or to a small value. The manner in which the automatic controller produces the control signal is called the *control action*. Figure 3-7 is a block diagram of an industrial control system, which

Figure 3-7
Block diagram of an industrial control system, which consists of an automatic controller, an actuator, a plant, and a sensor (measuring element).



consists of an automatic controller, an actuator, a plant, and a sensor (measuring element). The controller detects the actuating error signal, which is usually at a very low power level, and amplifies it to a sufficiently high level. The output of an automatic controller is fed to an actuator, such as an electric motor, a hydraulic motor, or a pneumatic motor or valve. (The actuator is a power device that produces the input to the plant according to the control signal so that the output signal will approach the reference input signal.)

The sensor or measuring element is a device that converts the output variable into another suitable variable, such as a displacement, pressure, or voltage, that can be used to compare the output to the reference input signal. This element is in the feedback path of the closed-loop system. The set point of the controller must be converted to a reference input with the same units as the feedback signal from the sensor or measuring element.

Classifications of Industrial Controllers. Industrial controllers may be classified according to their control actions as:

1. Two-position or on-off controllers
2. Proportional controllers
3. Integral controllers
4. Proportional-plus-integral controllers
5. Proportional-plus-derivative controllers
6. Proportional-plus-integral-plus-derivative controllers

Most industrial controllers use electricity or pressurized fluid such as oil or air as power sources. Consequently, controllers may also be classified according to the kind of power employed in the operation, such as pneumatic controllers, hydraulic controllers, or electronic controllers. What kind of controller to use must be decided based on the nature of the plant and the operating conditions, including such considerations as safety, cost, availability, reliability, accuracy, weight, and size.

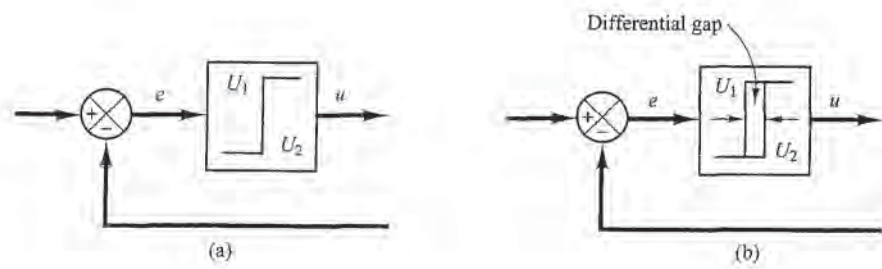
Two-Position or On-Off Control Action. In a two-position control system, the actuating element has only two fixed positions, which are, in many cases, simply on and off. Two-position or on-off control is relatively simple and inexpensive and, for this reason, is very widely used in both industrial and domestic control systems.

Let the output signal from the controller be $u(t)$ and the actuating error signal be $e(t)$. In two-position control, the signal $u(t)$ remains at either a maximum or minimum value, depending on whether the actuating error signal is positive or negative, so that

$$\begin{aligned} u(t) &= U_1, & \text{for } e(t) > 0 \\ &= U_2, & \text{for } e(t) < 0 \end{aligned}$$

where U_1 and U_2 are constants. The minimum value U_2 is usually either zero or $-U_1$. Two-position controllers are generally electrical devices, and an electric solenoid-operated valve is widely used in such controllers. Pneumatic proportional controllers with very high gains act as two-position controllers and are sometimes called pneumatic two-position controllers.

Figure 3-8
 (a) Block diagram of an on-off controller;
 (b) block diagram of an on-off controller with differential gap.



Figures 3-8 (a) and (b) show the block diagrams for two-position or on-off controllers. The range through which the actuating error signal must move before the switching occurs is called the *differential gap*. A differential gap is indicated in Figure 3-8(b). Such a differential gap causes the controller output $u(t)$ to maintain its present value until the actuating error signal has moved slightly beyond the zero value. In some cases, the differential gap is a result of unintentional friction and lost motion; however, quite often it is intentionally provided in order to prevent too frequent operation of the on-off mechanism.

Consider the liquid-level control system shown in Figure 3-9(a), where the electromagnetic valve shown in Figure 3-9(b) is used for controlling the inflow rate. This valve is either open or closed. With this two-position control, the water inflow rate is either a positive constant or zero. As shown in Figure 3-10, the output signal continuously moves between the two limits required to cause the actuating element to move from one fixed position to the other. Notice that the output curve follows one of two exponential curves, one corresponding to the filling curve and the other to the emptying curve. Such output oscillation between two limits is a typical response characteristic of a system under two-position control.

From Figure 3-10, we notice that the amplitude of the output oscillation can be reduced by decreasing the differential gap. The decrease in the differential gap, however, increases the number of on-off switchings per minute and reduces the useful life of the component. The magnitude of the differential gap must be determined from such considerations as the accuracy required and the life of the component.

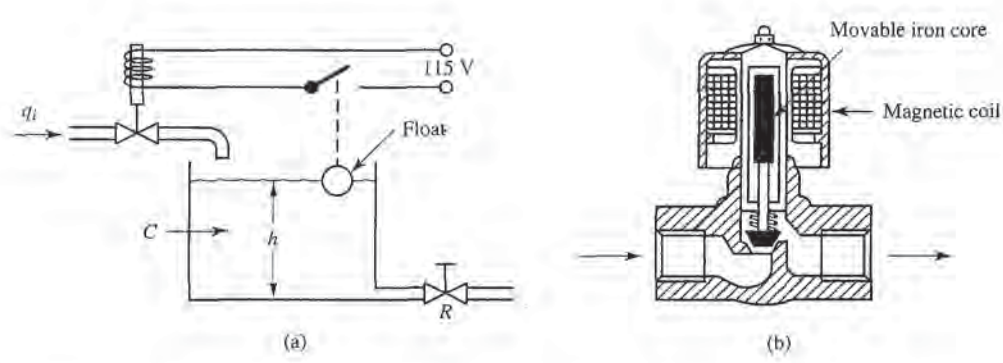
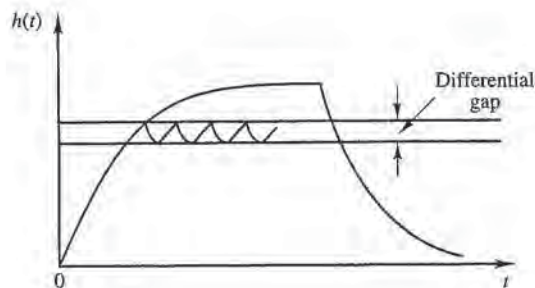


Figure 3-9
 (a) Liquid-level control system;
 (b) electromagnetic valve.

Figure 3-10
Level $h(t)$ versus t
curve for the system
shown in Figure 3-9(a).



Proportional Control Action. For a controller with proportional control action, the relationship between the output of the controller $u(t)$ and the actuating error signal $e(t)$ is

$$u(t) = K_p e(t)$$

or, in Laplace-transformed quantities,

$$\frac{U(s)}{E(s)} = K_p$$

where K_p is termed the proportional gain.

Whatever the actual mechanism may be and whatever the form of the operating power, the proportional controller is essentially an amplifier with an adjustable gain.

Integral Control Action. In a controller with integral control action, the value of the controller output $u(t)$ is changed at a rate proportional to the actuating error signal $e(t)$. That is,

$$\frac{du(t)}{dt} = K_i e(t)$$

or

$$u(t) = K_i \int_0^t e(t) dt$$

where K_i is an adjustable constant. The transfer function of the integral controller is

$$\frac{U(s)}{E(s)} = \frac{K_i}{s}$$

Proportional-Plus-Integral Control Action. The control action of a proportional-plus-integral controller is defined by

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t) dt$$

or the transfer function of the controller is

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} \right)$$

where T_i is called the *integral time*.

Proportional-Plus-Derivative Control Action. The control action of a proportional-plus-derivative controller is defined by

$$u(t) = K_p e(t) + K_p T_d \frac{de(t)}{dt}$$

and the transfer function is

$$\frac{U(s)}{E(s)} = K_p (1 + T_d s)$$

where T_d is called the *derivative time*.

Proportional-Plus-Integral-Plus-Derivative Control Action. The combination of proportional control action, integral control action, and derivative control action is termed proportional-plus-integral-plus-derivative control action. This combined action has the advantages of each of the three individual control actions. The equation of a controller with this combined action is given by

$$u(t) = K_p e(t) + \frac{K_p}{T_i} \int_0^t e(t) dt + K_p T_d \frac{de(t)}{dt}$$

or the transfer function is

$$\frac{U(s)}{E(s)} = K_p \left(1 + \frac{1}{T_i s} + T_d s \right)$$

where K_p is the proportional gain, T_i is the integral time, and T_d is the derivative time. The block diagram of a proportional-plus-integral-plus-derivative controller is shown in Figure 3-11.

Figure 3-11

Block diagram of a proportional-plus-integral-plus-derivative controller.

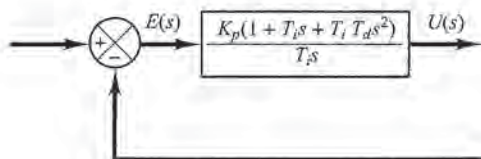
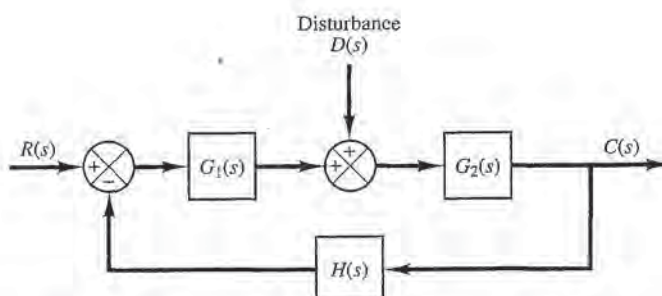


Figure 3-12
Closed-loop system
subjected to a
disturbance.



Closed-Loop System Subjected to a Disturbance. Figure 3-12 shows a closed-loop system subjected to a disturbance. When two inputs (the reference input and disturbance) are present in a linear system, each input can be treated independently of the other; and the outputs corresponding to each input alone can be added to give the complete output. The way each input is introduced into the system is shown at the summing point by either a plus or minus sign.

Consider the system shown in Figure 3-12. In examining the effect of the disturbance $D(s)$, we may assume that the reference input is zero; we may then calculate the response $C_D(s)$ to the disturbance only. This response can be found from

$$\frac{C_D(s)}{D(s)} = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

On the other hand, in considering the response to the reference input $R(s)$, we may assume that the disturbance is zero. Then the response $C_R(s)$ to the reference input $R(s)$ can be obtained from

$$\frac{C_R(s)}{R(s)} = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)}$$

The response to the simultaneous application of the reference input and disturbance can be obtained by adding the two individual responses. In other words, the response $C(s)$ due to the simultaneous application of the reference input $R(s)$ and disturbance $D(s)$ is given by

$$\begin{aligned} C(s) &= C_R(s) + C_D(s) \\ &= \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} [G_1(s)R(s) + D(s)] \end{aligned}$$

Consider now the case where $|G_1(s)H(s)| \gg 1$ and $|G_1(s)G_2(s)H(s)| \gg 1$. In this case, the closed-loop transfer function $C_D(s)/D(s)$ becomes almost zero, and the effect of the disturbance is suppressed. This is an advantage of the closed-loop system.

On the other hand, the closed-loop transfer function $C_R(s)/R(s)$ approaches $1/H(s)$ as the gain of $G_1(s)G_2(s)H(s)$ increases. This means that if $|G_1(s)G_2(s)H(s)| \gg 1$ then the closed-loop transfer function $C_R(s)/R(s)$ becomes independent of $G_1(s)$ and $G_2(s)$ and becomes inversely proportional to $H(s)$ so that the variations of $G_1(s)$ and $G_2(s)$ do not affect the closed-loop transfer function $C_R(s)/R(s)$. This is another advantage of the closed-loop system. It can easily be seen that any closed-loop system with unity feedback, $H(s) = 1$, tends to equalize the input and output.

Procedures for Drawing a Block Diagram. To draw a block diagram for a system, first write the equations that describe the dynamic behavior of each component. Then take the Laplace transforms of these equations, assuming zero initial conditions, and represent each Laplace-transformed equation individually in block form. Finally, assemble the elements into a complete block diagram.

As an example, consider the RC circuit shown in Figure 3-13(a). The equations for this circuit are

$$i = \frac{e_i - e_o}{R} \quad (3-4)$$

$$e_o = \frac{\int i dt}{C} \quad (3-5)$$

The Laplace transforms of Equations (3-4) and (3-5), with zero initial condition, become

$$I(s) = \frac{E_i(s) - E_o(s)}{R} \quad (3-6)$$

$$E_o(s) = \frac{I(s)}{Cs} \quad (3-7)$$

Equation (3-6) represents a summing operation, and the corresponding diagram is shown in Figure 3-13(b). Equation (3-7) represents the block as shown in Figure 3-13(c). Assembling these two elements, we obtain the overall block diagram for the system as shown in Figure 3-13(d).

Block Diagram Reduction. It is important to note that blocks can be connected in series only if the output of one block is not affected by the next following block. If there are any loading effects between the components, it is necessary to combine these components into a single block.

Any number of cascaded blocks representing nonloading components can be replaced by a single block, the transfer function of which is simply the product of the individual transfer functions.

A complicated block diagram involving many feedback loops can be simplified by a step-by-step rearrangement. Simplification of the block diagram by rearrangements

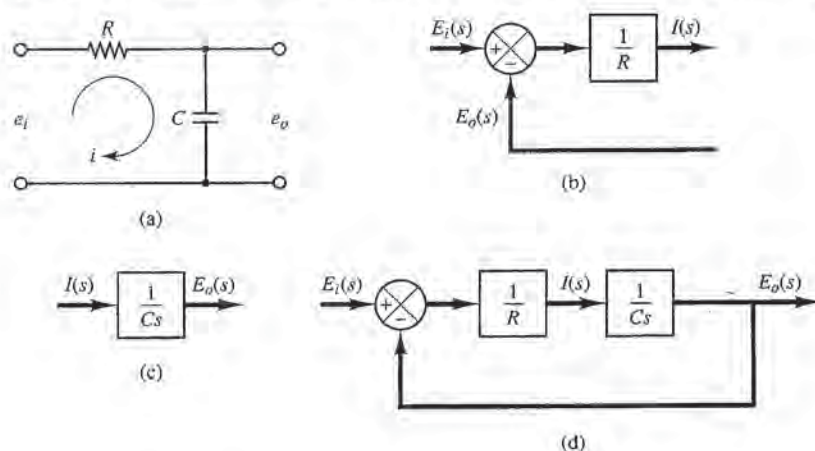


Figure 3-13
 (a) RC circuit;
 (b) block diagram
 representing
 Equation (3-6);
 (c) block diagram
 representing
 Equation (3-7);
 (d) block diagram of
 the RC circuit.

considerably reduces the labor needed for subsequent mathematical analysis. It should be noted, however, that as the block diagram is simplified the transfer functions in new blocks become more complex because new poles and new zeros are generated.

In simplifying a block diagram, remember the following.

1. The product of the transfer functions in the feedforward direction must remain the same.
2. The product of the transfer functions around the loop must remain the same.

EXAMPLE 3-2 Consider the system shown in Figure 3-14(a). Simplify this diagram.

By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1 , we obtain Figure 3-10(b). Eliminating the positive feedback loop,

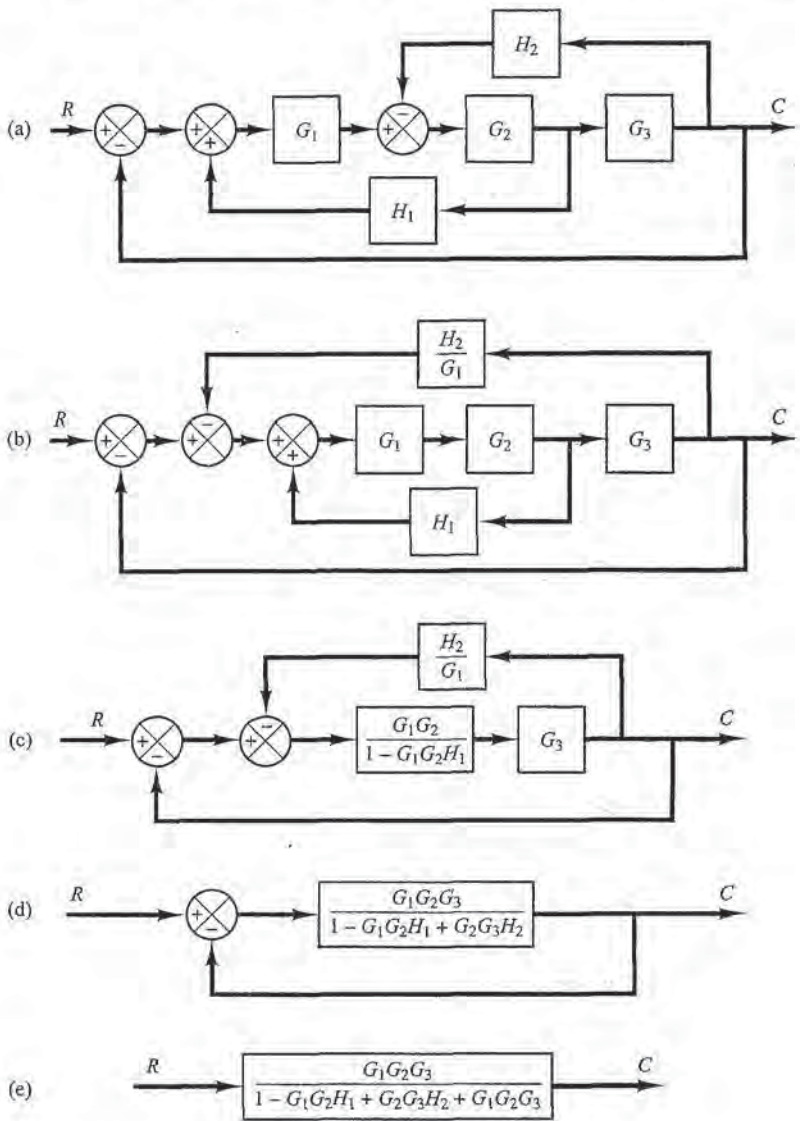


Figure 3-14
(a) Multiple-loop system;
(b)–(e) successive reductions of the block diagram shown in (a).

we have Figure 3-14(c). The elimination of the loop containing H_2/G_1 gives Figure 3-14(d). Finally, eliminating the feedback loop results in Figure 3-14(e).

Notice that the numerator of the closed-loop transfer function $C(s)/R(s)$ is the product of the transfer functions of the feedforward path. The denominator of $C(s)/R(s)$ is equal to

$$\begin{aligned} 1 - \sum (\text{product of the transfer functions around each loop}) \\ &= 1 - (G_1 G_2 H_1 - G_2 G_3 H_2 - G_1 G_2 G_3) \\ &= 1 - G_1 G_2 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 \end{aligned}$$

(The positive feedback loop yields a negative term in the denominator.)

3-4 MODELING IN STATE SPACE

In this section we shall present introductory material on state-space analysis of control systems.

Modern Control Theory. The modern trend in engineering systems is toward greater complexity, due mainly to the requirements of complex tasks and good accuracy. Complex systems may have multiple inputs and multiple outputs and may be time varying. Because of the necessity of meeting increasingly stringent requirements on the performance of control systems, the increase in system complexity, and easy access to large scale computers, modern control theory, which is a new approach to the analysis and design of complex control systems, has been developed since around 1960. This new approach is based on the concept of state. The concept of state by itself is not new since it has been in existence for a long time in the field of classical dynamics and other fields.

Modern Control Theory Versus Conventional Control Theory. Modern control theory is contrasted with conventional control theory in that the former is applicable to multiple-input-multiple-output systems, which may be linear or nonlinear, time invariant or time varying, while the latter is applicable only to linear time-invariant single-input-single-output systems. Also, modern control theory is essentially a time-domain approach, while conventional control theory is a complex frequency-domain approach. Before we proceed further, we must define state, state variables, state vector, and state space.

State. The state of a dynamic system is the smallest set of variables (called *state variables*) such that the knowledge of these variables at $t = t_0$, together with the knowledge of the input for $t \geq t_0$, completely determines the behavior of the system for any time $t \geq t_0$.

Note that the concept of state is by no means limited to physical systems. It is applicable to biological systems, economic systems, social systems, and others.

State Variables. The state variables of a dynamic system are the variables making up the smallest set of variables that determine the state of the dynamic system. If at

least n variables $x_1, x_2, \dots, x_n$ are needed to completely describe the behavior of a dynamic system (so that once the input is given for $t \geq t_0$ and the initial state at $t = t_0$ is specified, the future state of the system is completely determined), then such n variables are a set of state variables.

Note that state variables need not be physically measurable or observable quantities. Variables that do not represent physical quantities and those that are neither measurable nor observable can be chosen as state variables. Such freedom in choosing state variables is an advantage of the state-space methods. Practically, however, it is convenient to choose easily measurable quantities for the state variables, if this is possible at all, because optimal control laws will require the feedback of all state variables with suitable weighting.

State Vector. If n state variables are needed to completely describe the behavior of a given system, then these n state variables can be considered the n components of a vector $\mathbf{x}$. Such a vector is called a *state vector*. A state vector is thus a vector that determines uniquely the system state $\mathbf{x}(t)$ for any time $t \geq t_0$, once the state at $t = t_0$ is given and the input $u(t)$ for $t \geq t_0$ is specified.

State Space. The n -dimensional space whose coordinate axes consist of the x_1 axis, x_2 axis, $\dots$, x_n axis, where $x_1, x_2, \dots, x_n$ are state variables; is called a *state space*. Any state can be represented by a point in the state space.

State-Space Equations. In state-space analysis we are concerned with three types of variables that are involved in the modeling of dynamic systems: input variables, output variables, and state variables. As we shall see in Section 3-5, the state-space representation for a given system is not unique, except that the number of state variables is the same for any of the different state-space representations of the same system.

The dynamic system must involve elements that memorize the values of the input for $t \geq t_1$. Since integrators in a continuous-time control system serve as memory devices, the outputs of such integrators can be considered as the variables that define the internal state of the dynamic system. Thus the outputs of integrators serve as state variables. The number of state variables to completely define the dynamics of the system is equal to the number of integrators involved in the system.

Assume that a multiple-input-multiple-output system involves n integrators. Assume also that there are r inputs $u_1(t), u_2(t), \dots, u_r(t)$ and m outputs $y_1(t), y_2(t), \dots, y_m(t)$. Define n outputs of the integrators as state variables: $x_1(t), x_2(t), \dots, x_n(t)$. Then the system may be described by

$$\begin{aligned}\dot{x}_1(t) &= f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \dot{x}_2(t) &= f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ &\vdots \\ \dot{x}_n(t) &= f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t)\end{aligned}\tag{3-8}$$

The outputs $y_1(t), y_2(t), \dots, y_m(t)$ of the system may be given by

$$\begin{aligned} y_1(t) &= g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ y_2(t) &= g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ &\vdots \\ y_m(t) &= g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{aligned} \quad (3-9)$$

If we define

$$\begin{aligned} \mathbf{x}(t) &= \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix}, & \mathbf{f}(\mathbf{x}, \mathbf{u}, t) &= \begin{bmatrix} f_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ f_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ f_n(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix}, \\ \mathbf{y}(t) &= \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}, & \mathbf{g}(\mathbf{x}, \mathbf{u}, t) &= \begin{bmatrix} g_1(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ g_2(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \\ \vdots \\ g_m(x_1, x_2, \dots, x_n; u_1, u_2, \dots, u_r; t) \end{bmatrix}, & \mathbf{u}(t) &= \begin{bmatrix} u_1(t) \\ u_2(t) \\ \vdots \\ u_r(t) \end{bmatrix} \end{aligned}$$

then Equations (3-8) and (3-9) become

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}, \mathbf{u}, t) \quad (3-10)$$

$$\mathbf{y}(t) = \mathbf{g}(\mathbf{x}, \mathbf{u}, t) \quad (3-11)$$

where Equation (3-10) is the state equation and Equation (3-11) is the output equation. If vector functions $\mathbf{f}$ and/or $\mathbf{g}$ involve time t explicitly, then the system is called a time-varying system.

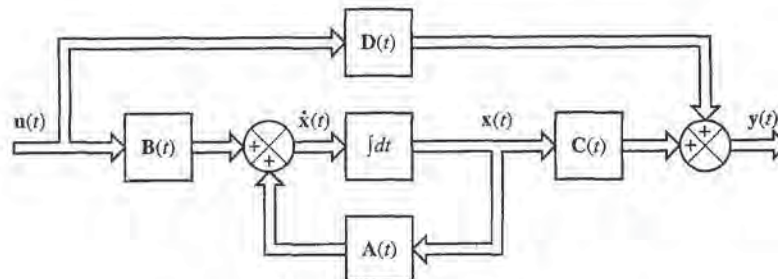
If Equations (3-10) and (3-11) are linearized about the operating state, then we have the following linearized state equation and output equation:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(t)\mathbf{x}(t) + \mathbf{B}(t)\mathbf{u}(t) \quad (3-12)$$

$$\mathbf{y}(t) = \mathbf{C}(t)\mathbf{x}(t) + \mathbf{D}(t)\mathbf{u}(t) \quad (3-13)$$

where $\mathbf{A}(t)$ is called the state matrix, $\mathbf{B}(t)$ the input matrix, $\mathbf{C}(t)$ the output matrix, and $\mathbf{D}(t)$ the direct transmission matrix. (Details of linearization of nonlinear systems about the operating state are discussed in Section 3-10.) A block diagram representation of Equations (3-12) and (3-13) is shown in Figure 3-15.

Figure 3-15
Block diagram of the linear, continuous-time control system represented in state space.



If vector functions $\mathbf{f}$ and $\mathbf{g}$ do not involve time t explicitly then the system is called a time-invariant system. In this case, Equations (3-12) and (3-13) can be simplified to

$$\dot{\mathbf{x}}(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{B}\mathbf{u}(t) \quad (3-14)$$

$$\dot{\mathbf{y}}(t) = \mathbf{C}\mathbf{x}(t) + \mathbf{D}\mathbf{u}(t) \quad (3-15)$$

Equation (3-14) is the state equation of the linear, time-invariant system.

Equation (3-15) is the output equation for the same system. In this book we shall be concerned mostly with systems described by Equations (3-14) and (3-15).

In what follows we shall present an example for deriving a state equation and output equation.

EXAMPLE 3-3

Consider the mechanical system shown in Figure 3-16. We assume that the system is linear. The external force $u(t)$ is the input to the system, and the displacement $y(t)$ of the mass is the output. The displacement $y(t)$ is measured from the equilibrium position in the absence of the external force. This system is a single-input-single-output system.

From the diagram, the system equation is

$$m\ddot{y} + b\dot{y} + ky = u \quad (3-16)$$

This system is of second order. This means that the system involves two integrators. Let us define state variables $x_1(t)$ and $x_2(t)$ as

$$x_1(t) = y(t)$$

$$x_2(t) = \dot{y}(t)$$

Then we obtain

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m}(-ky - b\dot{y}) + \frac{1}{m}u$$

or

$$\dot{x}_1 = x_2 \quad (3-17)$$

$$\dot{x}_2 = -\frac{k}{m}x_1 - \frac{b}{m}x_2 + \frac{1}{m}u \quad (3-18)$$

The output equation is

$$y = x_1 \quad (3-19)$$

In a vector-matrix form, Equations (3-17) and (3-18) can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u \quad (3-20)$$

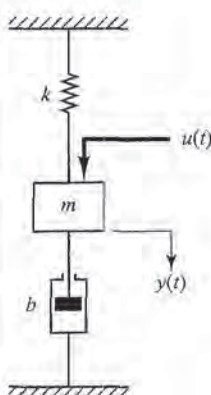


Figure 3-16
Mechanical system.

The output equation, Equation (3-19), can be written as

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (3-21)$$

Equation (3-20) is a state equation and Equation (3-21) is an output equation for the system. Equations (3-20) and (3-21) are in the standard form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ y &= \mathbf{C}\mathbf{x} + Du \end{aligned}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad D = 0$$

Figure 3-17 is a block diagram for the system. Notice that the outputs of the integrators are state variables.

Correlation Between Transfer Functions and State-Space Equations. In what follows we shall show how to derive the transfer function of a single-input-single-output system from the state-space equations.

Let us consider the system whose transfer function is given by

$$\frac{Y(s)}{U(s)} = G(s) \quad (3-22)$$

This system may be represented in state space by the following equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (3-23)$$

$$y = \mathbf{C}\mathbf{x} + Du \quad (3-24)$$

where $\mathbf{x}$ is the state vector, u is the input, and y is the output. The Laplace transforms of Equations (3-23) and (3-24) are given by

$$s\mathbf{X}(s) - \mathbf{x}(0) = \mathbf{A}\mathbf{X}(s) + \mathbf{B}U(s) \quad (3-25)$$

$$Y(s) = \mathbf{C}\mathbf{X}(s) + DU(s) \quad (3-26)$$

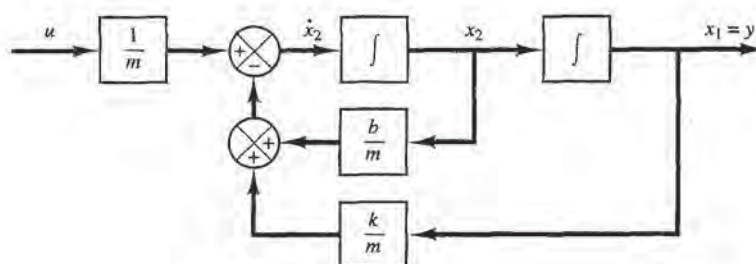


Figure 3-17
Block diagram of the
mechanical system
shown in Figure 3-16.

Since the transfer function was previously defined as the ratio of the Laplace transform of the output to the Laplace transform of the input when the initial conditions were zero, we set $\mathbf{x}(0)$ in Equation (3-25) to be zero. Then we have

$$s\mathbf{X}(s) - \mathbf{A}\mathbf{X}(s) = \mathbf{B}U(s)$$

or

$$(s\mathbf{I} - \mathbf{A})\mathbf{X}(s) = \mathbf{B}U(s)$$

By premultiplying $(s\mathbf{I} - \mathbf{A})^{-1}$ to both sides of this last equation, we obtain

$$\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}U(s) \quad (3-27)$$

By substituting Equation (3-27) into Equation (3-26), we get

$$Y(s) = [\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D]U(s) \quad (3-28)$$

Upon comparing Equation (3-28) with Equation (3-22), we see that

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D \quad (3-29)$$

This is the transfer-function expression of the system in terms of $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and D .

Note that the right-hand side of Equation (3-29) involves $(s\mathbf{I} - \mathbf{A})^{-1}$. Hence $G(s)$ can be written as

$$G(s) = \frac{Q(s)}{|s\mathbf{I} - \mathbf{A}|}$$

where $Q(s)$ is a polynomial in s . Therefore, $|s\mathbf{I} - \mathbf{A}|$ is equal to the characteristic polynomial of $G(s)$. In other words, the eigenvalues of $\mathbf{A}$ are identical to the poles of $G(s)$.

EXAMPLE 3-4 Consider again the mechanical system shown in Figure 3-16. State-space equations for the system are given by Equations (3-20) and (3-21). We shall obtain the transfer function for the system from the state-space equations.

By substituting $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, and D into Equation (3-29), we obtain

$$\begin{aligned} G(s) &= \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D \\ &= [1 \ 0] \left\{ \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \right\}^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} + 0 \\ &= [1 \ 0] \begin{bmatrix} s & -1 \\ \frac{k}{m} & s + \frac{b}{m} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \end{aligned}$$

Since

$$\begin{bmatrix} s & -1 \\ \frac{k}{m} & s + \frac{b}{m} \end{bmatrix}^{-1} = \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} \begin{bmatrix} s + \frac{b}{m} & 1 \\ -\frac{k}{m} & s \end{bmatrix}$$

we have

$$G(s) = [1 \quad 0] \frac{1}{s^2 + \frac{b}{m}s + \frac{k}{m}} \begin{bmatrix} s + \frac{b}{m} & 1 \\ -\frac{k}{m} & s \end{bmatrix} \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}$$

$$= \frac{1}{ms^2 + bs + k}$$

which is the transfer function of the system. The same transfer function can be obtained from Equation (3-16).

Transfer Matrix. Next, consider a multiple-input-multiple-output system. Assume that there are r inputs $u_1, u_2, \dots, u_r$, and m outputs $y_1, y_2, \dots, y_m$. Define

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}, \quad \mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_r \end{bmatrix}$$

The transfer matrix $\mathbf{G}(s)$ relates the output $\mathbf{Y}(s)$ to the input $\mathbf{U}(s)$, or

$$\mathbf{Y}(s) = \mathbf{G}(s)\mathbf{U}(s)$$

where $\mathbf{G}(s)$ is given by

$$\mathbf{G}(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + \mathbf{D}$$

[The derivation for this equation is the same as that for Equation (3-29).] Since the input vector $\mathbf{u}$ is r dimensional and the output vector $\mathbf{y}$ is m dimensional, the transfer matrix $\mathbf{G}(s)$ is an $m \times r$ matrix.

3-5 STATE-SPACE REPRESENTATION OF DYNAMIC SYSTEMS

A dynamic system consisting of a finite number of lumped elements may be described by ordinary differential equations in which time is the independent variable. By use of vector-matrix notation, an n th-order differential equation may be expressed by a first-order vector-matrix differential equation. If n elements of the vector are a set of state variables, then the vector-matrix differential equation is a *state* equation. In this section we shall present methods for obtaining state-space representations of continuous-time systems.

State-Space Representation of n th-Order Systems of Linear Differential Equations in which the Forcing Function Does Not Involve Derivative Terms. Consider the following n th-order system:

$$y^{(n)} + a_1 y^{(n-1)} + \dots + a_{n-1} \dot{y} + a_n y = u \quad (3-30)$$

Noting that the knowledge of $y(0), \dot{y}(0), \dots, y^{(n-1)}(0)$, together with the input $u(t)$ for $t \geq 0$, determines completely the future behavior of the system, we may take

$y(t), \dot{y}(t), \dots, y^{(n-1)}(t)$ as a set of n state variables. (Mathematically, such a choice of state variables is quite convenient. Practically, however, because higher-order derivative terms are inaccurate, due to the noise effects inherent in any practical situations, such a choice of the state variables may not be desirable.)

Let us define :

$$\begin{aligned} x_1 &= y \\ x_2 &= \dot{y} \\ &\vdots \\ x_n &= y^{(n-1)} \end{aligned}$$

Then Equation (3-30) can be written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\vdots \\ \dot{x}_{n-1} &= x_n \\ \dot{x}_n &= a_n x_1 - \dots - a_1 x_n + u \end{aligned}$$

or

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \quad (3-31)$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

The output can be given by

$$y = [1 \ 0 \ \cdots \ 0] \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

or

$$y = \mathbf{C}\mathbf{x} \quad (3-32)$$

where

$$\mathbf{C} = [1 \ 0 \ \cdots \ 0]$$

[Note that D in Equation (3-24) is zero.] The first-order differential equation, Equation (3-31), is the state equation, and the algebraic equation, Equation (3-32), is the output equation.

Note that the state-space representation for the transfer function system

$$\frac{Y(s)}{U(s)} = \frac{1}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

is given also by Equation (3-31) and (3-32).

State-Space Representation of n th Order Systems of Linear Differential Equations in which the Forcing Function Involves Derivative Terms. Consider the differential equation system that involves derivatives of the forcing function, such as

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} \dot{y} + a_n y = b_0 u^{(n)} + b_1 u^{(n-1)} + \cdots + b_{n-1} \dot{u} + b_n u \quad (3-33)$$

The main problem in defining the state variables for this case lies in the derivative terms. The state variables must be such that they will eliminate the derivatives of u in the state equation.

One way to obtain a state equation and output equation is to define the following n variables as a set of n state variables:

$$\begin{aligned} x_1 &= y - \beta_0 u \\ x_2 &= \dot{y} - \beta_0 \dot{u} - \beta_1 u = \dot{x}_1 - \beta_1 u \\ x_3 &= \ddot{y} - \beta_0 \ddot{u} - \beta_1 \dot{u} - \beta_2 u = \dot{x}_2 - \beta_2 u \\ &\vdots \\ x_n &= y^{(n-1)} - \beta_0 y^{(n-1)} - \beta_1 y^{(n-2)} - \cdots - \beta_{n-2} \dot{u} - \beta_{n-1} u = \dot{x}_{n-1} - \beta_{n-1} u \end{aligned} \quad (3-34)$$

where $\beta_0, \beta_1, \beta_2, \dots, \beta_n$ are determined from

$$\begin{aligned} \beta_0 &= b_0 \\ \beta_1 &= b_1 - a_1 \beta_0 \\ \beta_2 &= b_2 - a_1 \beta_1 - a_2 \beta_0 \\ \beta_3 &= b_3 - a_1 \beta_2 - a_2 \beta_1 - a_3 \beta_0 \\ &\vdots \\ \beta_n &= b_n - a_1 \beta_{n-1} - \cdots - a_{n-1} \beta_1 - a_n \beta_0 \end{aligned} \quad (3-35)$$

With this choice of state variables the existence and uniqueness of the solution of the state equation is guaranteed. (Note that this is not the only choice of a set of state variables.) With the present choice of state variables, we obtain

$$\begin{aligned} \dot{x}_1 &= x_2 + \beta_1 u \\ \dot{x}_2 &= x_3 + \beta_2 u \\ &\vdots \\ \dot{x}_{n-1} &= x_n + \beta_{n-1} u \\ \dot{x} &= -a_n x_1 - a_{n-1} x_2 - \cdots - a_1 x_n + \beta_n u \end{aligned} \tag{3-36}$$

[To derive Equation (3-36), see Problem A-3-6.] In terms of vector-matrix equations, Equation (3-36) and the output equation can be written as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \vdots \\ \dot{x}_{n-1} \\ \dot{x}_n \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix} + \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} + \beta_0 u$$

or

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} \tag{3-37}$$

$$y = \mathbf{Cx} + Du \tag{3-38}$$

where

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_{n-1} \\ \beta_n \end{bmatrix}, \quad \mathbf{C} = [1 \ 0 \ \cdots \ 0], \quad D = \beta_0 = b_0$$

In this state-space representation, matrices $\mathbf{A}$ and $\mathbf{C}$ are exactly the same as those for the system of Equation (3-30). The derivatives on the right-hand side of Equation (3-33) affect only the elements of the $\mathbf{B}$ matrix.

Note that the state-space representation for the transfer function

$$\frac{Y(s)}{U(s)} = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1} s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1} s + a_n}$$

is given also by Equations (3-37) and (3-38).

There are many ways to obtain state-space representations of systems. Some of them are presented in this chapter. Methods for obtaining canonical representations of systems in state space (such as controllable canonical form, observable canonical form, diagonal canonical form, and Jordan canonical form) are presented in Chapter 11.

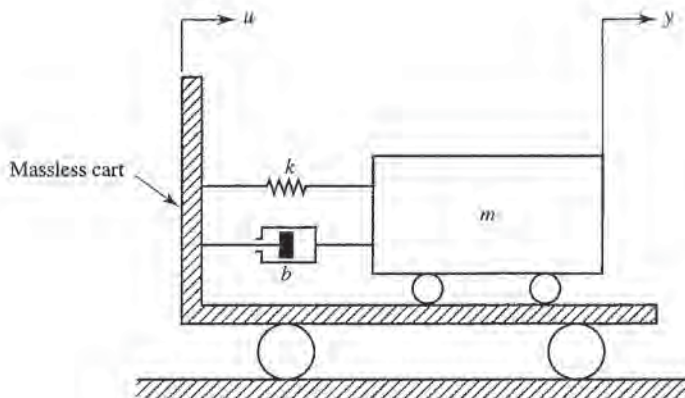
MATLAB can be used to obtain state-space representations of systems from transfer function representations, and vice versa. This subject is presented in Section 3-6.

EXAMPLE 3-5

Consider the spring-mass-dashpot system mounted on a massless cart as shown in Figure 3-18. A dashpot is a device that provides viscous friction, or damping. It consists of a piston and oil-filled cylinder. Any relative motion between the piston rod and the cylinder is resisted by the oil because the oil must flow around the piston (or through orifices provided in the piston) from one side of the piston to the other. The dashpot essentially absorbs energy. This absorbed energy is dissipated as heat, and the dashpot does not store any kinetic or potential energy. The dashpot is also called a *damper*.

Let us obtain mathematical models of this system by assuming that the cart is standing still for $t < 0$ and the spring-mass-dashpot system on the cart is also standing still for $t < 0$. In this system, $u(t)$ is the displacement of the cart and is the input to the system. At $t = 0$, the cart is moved at a constant speed, or $\dot{u} = \text{constant}$. The displacement $y(t)$ of the mass is the output. (The displacement is relative to the ground.) In this system, m denotes the mass, b denotes the viscous friction coefficient, and k denotes the spring constant. We assume that the friction force of the dashpot

Figure 3-18
Spring-mass-
dashpot system
mounted on a cart.



is proportional to $\dot{y} - \dot{u}$ and that the spring is a linear spring; that is, the spring force is proportional to $y - u$.

For translational systems, Newton's second law states that

$$ma = \sum F$$

where m is a mass, a is the acceleration of the mass, and $\sum F$ is the sum of the forces acting on the mass in the direction of the acceleration a . Applying Newton's second law to the present system and noting that the cart is massless, we obtain

$$m \frac{d^2 y}{dt^2} = -b \left(\frac{dy}{dt} - \frac{du}{dt} \right) - k(y - u)$$

or

$$m \frac{d^2 y}{dt^2} + b \frac{dy}{dt} + ky = b \frac{du}{dt} + ku$$

This equation represents a mathematical model of the system considered. Taking the Laplace transform of this last equation, assuming zero initial condition, gives

$$(ms^2 + bs + k)Y(s) = (bs + k)U(s)$$

Taking the ratio of $Y(s)$ to $U(s)$, we find the transfer function of the system to be

$$\text{Transfer function} = G(s) = \frac{Y(s)}{U(s)} = \frac{bs + k}{ms^2 + bs + k}$$

Such a transfer function representation of a mathematical model is used very frequently in control engineering.

Next we shall obtain a state-space model of this system. We shall first compare the differential equation for this system

$$\ddot{y} + \frac{b}{m} \dot{y} + \frac{k}{m} y = \frac{b}{m} \dot{u} + \frac{k}{m} u$$

with the standard form

$$\ddot{y} + a_1 \dot{y} + a_2 y = b_0 \ddot{u} + b_1 \dot{u} + b_2 u$$

and identify a_1 , a_2 , b_0 , b_1 , and b_2 as follows:

$$a_1 = \frac{b}{m}, \quad a_2 = \frac{k}{m}, \quad b_0 = 0, \quad b_1 = \frac{b}{m}, \quad b_2 = \frac{k}{m}$$

Referring to Equation (3-35), we have

$$\beta_0 = b_0 = 0$$

$$\beta_1 = b_1 - a_1 \beta_0 = \frac{b}{m}$$

$$\beta_2 = b_2 - a_1 \beta_1 - a_2 \beta_0 = \frac{k}{m} - \left(\frac{b}{m}\right)^2$$

Then, referring to Equation (3-34), define

$$x_1 = y - \beta_0 u = y$$

$$x_2 = \dot{x}_1 - \beta_1 u = \dot{x}_1 - \frac{b}{m} u$$

From Equation (3-36) we have

$$\dot{x}_1 = x_2 + \beta_1 u = x_2 + \frac{b}{m} u$$

$$\dot{x}_2 = -a_2 x_1 - a_1 x_2 + \beta_2 u = -\frac{k}{m} x_1 - \frac{b}{m} x_2 + \left[\frac{k}{m} - \left(\frac{b}{m}\right)^2\right] u$$

and the output equation becomes

$$y = x_1$$

or

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{b}{m} \\ \frac{k}{m} - \left(\frac{b}{m}\right)^2 \end{bmatrix} u \quad (3-39)$$

and

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (3-40)$$

Equations (3-39) and (3-40) give a state-space representation of the system. (Note that this is not the only state-space representation. There are infinitely many state-space representations for the system.)

3-6 TRANSFORMATION OF MATHEMATICAL MODELS WITH MATLAB

MATLAB is quite useful to transform the system model from transfer function to state space, and vice versa. We shall begin our discussion with transformation from transfer function to state space.

Let us write the closed-loop transfer function as

$$\frac{Y(s)}{U(s)} = \frac{\text{numerator polynomial in } s}{\text{denominator polynomial in } s} = \frac{\text{num}}{\text{den}}$$

Once we have this transfer-function expression, the MATLAB command

$$[A, B, C, D] = \text{tf2ss}(\text{num}, \text{den})$$

will give a state-space representation. It is important to note that the state-space representation for any system is not unique. There are many (infinitely many) state-space representations for the same system. The MATLAB command gives one possible such state-space representation.

Transformation From Transfer Function to State Space. Consider the transfer function system

$$\begin{aligned} \frac{Y(s)}{U(s)} &= \frac{s}{(s+10)(s^2+4s+16)} \\ &= \frac{s}{s^3+14s^2+56s+160} \end{aligned} \quad (3-41)$$

There are many (infinitely many) possible state-space representations for this system. One possible state-space representation is

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ -14 \end{bmatrix} u \\ y &= [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u \end{aligned}$$

Another possible state-space representation (among infinitely many alternatives) is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -14 & -56 & -160 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u \quad (3-42)$$

$$y = [0 \ 1 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u \quad (3-43)$$

MATLAB transforms the transfer function given by Equation (3-41) into the state-space representation given by Equations (3-42) and (3-43). For the example system considered here, MATLAB Program 3-2 will produce matrices **A**, **B**, **C**, and **D**.

MATLAB Program 3-2

```
num = [0 0 1 0];
den = [1 14 56 160];
[A,B,C,D] = tf2ss(num,den)

A =
    -14    -56   -160
         1         0         0
         0         1         0

B =
         1
         0
         0

C =
         0         1         0

D =
         0
```

Transformation From State Space to Transfer Function. To obtain the transfer function from state-space equations, use the following command:

$$[\text{num},\text{den}] = \text{ss2tf}(\text{A},\text{B},\text{C},\text{D},\text{iu})$$

iu must be specified for systems with more than one input. For example, if the system has three inputs (*u*₁, *u*₂, *u*₃), then *iu* must be either 1, 2, or 3, where 1 implies *u*₁, 2 implies *u*₂, and 3 implies *u*₃.

If the system has only one input, then either

$$[\text{num},\text{den}] = \text{ss2tf}(\text{A},\text{B},\text{C},\text{D})$$

or

$$[\text{num},\text{den}] = \text{ss2tf}(\text{A},\text{B},\text{C},\text{D},1)$$

may be used. For the case where the system has multiple inputs and multiple outputs, see Problem **A-3-13**.

EXAMPLE 3-6 Obtain the transfer function of the system defined by the following state-space equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -25 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 25 \\ -120 \end{bmatrix} u$$
$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

MATLAB Program 3-3 will produce the transfer function for the given system. The transfer function obtained is given by

$$\frac{Y(s)}{U(s)} = \frac{25s + 5}{s^3 + 5s^2 + 25s + 5}$$

MATLAB Program 3-3

```
A = [0 1 0; 0 0 1; -5 -25 -5];
B = [0; 25; -120];
C = [1 0 0];
D = [0];
[num,den] = ss2tf(A,B,C,D)

num =
    0    0.0000   25.0000   5.0000

den =
    1.0000   5.0000   25.0000   5.0000

% ***** The same result can be obtained by entering the following command: *****
[num,den] = ss2tf(A,B,C,D,1)

num =
    0    0.0000   25.0000   5.0000

den =
    1.0000   5.0000   25.0000   5.0000
```

3-7 MECHANICAL SYSTEMS

In this section we shall discuss mathematical modeling of mechanical systems. The fundamental law governing mechanical systems is Newton’s second law. It can be applied to any mechanical system. In this section we shall derive mathematical models of three mechanical systems. (Mathematical models of additional mechanical systems will be derived and analyzed throughout the remaining chapters.)

EXAMPLE 3-7 Obtain the transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$ of the mechanical system shown in Figure 3-19.

The equations of motion for the system shown in Figure 3-19 are

$$m_1 \ddot{x}_1 = -k_1 x_1 - k_2(x_1 - x_2) - b(\dot{x}_1 - \dot{x}_2) + u$$

$$m_2 \ddot{x}_2 = -k_3 x_2 - k_2(x_2 - x_1) - b(\dot{x}_2 - \dot{x}_1)$$

Simplifying, we obtain

$$m_1 \ddot{x}_1 + b\dot{x}_1 + (k_1 + k_2)x_1 = b\dot{x}_2 + k_2 x_2 + u$$

$$m_2 \ddot{x}_2 + b\dot{x}_2 + (k_2 + k_3)x_2 = b\dot{x}_1 + k_2 x_1$$

Taking the Laplace transforms of these two equations, assuming zero initial conditions, we obtain

$$[m_1 s^2 + bs + (k_1 + k_2)]X_1(s) = (bs + k_2)X_2(s) + U(s) \quad (3-44)$$

$$[m_2 s^2 + bs + (k_2 + k_3)]X_2(s) = (bs + k_2)X_1(s) \quad (3-45)$$

Solving Equation (3-45), for $X_2(s)$ and substituting it into Equation (3-44) and simplifying, we get

$$\begin{aligned} & [(m_1 s^2 + bs + k_1 + k_2)(m_2 s^2 + bs + k_2 + k_3) - (bs + k_2)^2]X_1(s) \\ & = (m_2 s^2 + bs + k_2 + k_3)U(s) \end{aligned}$$

from which we obtain

$$\frac{X_1(s)}{U(s)} = \frac{m_2 s^2 + bs + k_2 + k_3}{(m_1 s^2 + bs + k_1 + k_2)(m_2 s^2 + bs + k_2 + k_3) - (bs + k_2)^2} \quad (3-46)$$

From Equations (3-45) and (3-46) we have

$$\frac{X_2(s)}{U(s)} = \frac{bs + k_2}{(m_1 s^2 + bs + k_1 + k_2)(m_2 s^2 + bs + k_2 + k_3) - (bs + k_2)^2} \quad (3-47)$$

Equations (3-46) and (3-47) are the transfer functions $X_1(s)/U(s)$ and $X_2(s)/U(s)$, respectively.

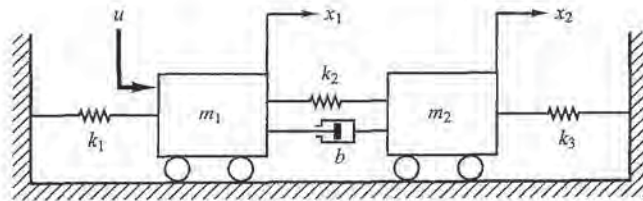
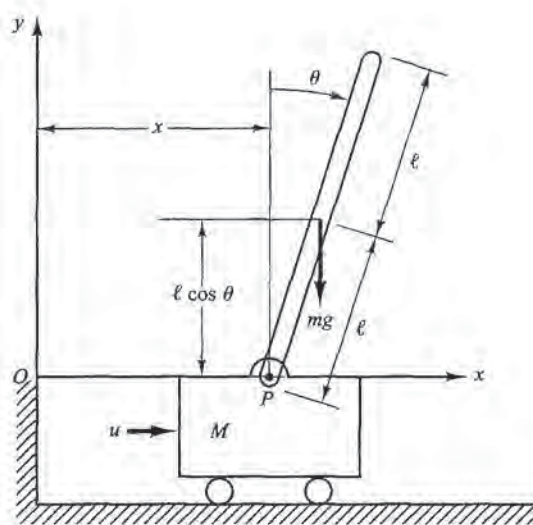
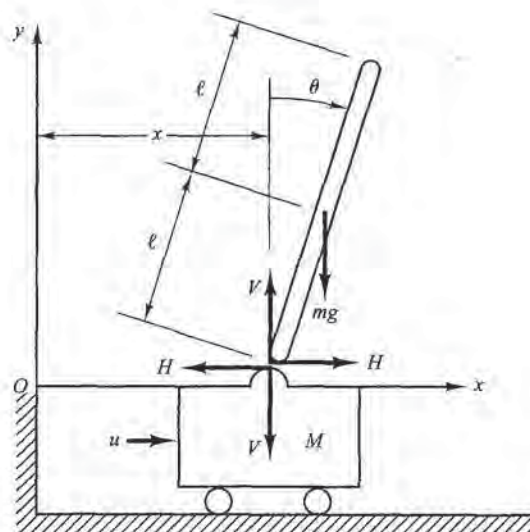


Figure 3-19
Mechanical system.

EXAMPLE 3-8 An inverted pendulum mounted on a motor-driven cart is shown in Figure 3-20(a). This is a model of the attitude control of a space booster on takeoff. (The objective of the attitude control problem is to keep the space booster in a vertical position.) The inverted pendulum is unstable in that it may fall over any time in any direction unless a suitable control force is applied. Here we consider



(a)



(b)

Figure 3-20
(a) Inverted
pendulum system;
(b) free-body
diagram.

only a two-dimensional problem in which the pendulum moves only in the plane of the page. The control force u is applied to the cart. Assume that the center of gravity of the pendulum rod is at its geometric center. Obtain a mathematical model for the system.

Define the angle of the rod from the vertical line as θ . Define also the (x, y) coordinates of the center of gravity of the pendulum rod as (x_G, y_G) . Then

$$x_G = x + l \sin \theta$$

$$y_G = l \cos \theta$$

To derive the equations of motion for the system, consider the free-body diagram shown in Figure 3-20(b). The rotational motion of the pendulum rod about its center of gravity can be described by

$$I\ddot{\theta} = Vl \sin \theta - Hl \cos \theta \quad (3-48)$$

where I is the moment of inertia of the rod about its center of gravity.

The horizontal motion of center of gravity of pendulum rod is given by

$$m \frac{d^2}{dt^2} (x + l \sin \theta) = H \quad (3-49)$$

The vertical motion of center of gravity of pendulum rod is

$$m \frac{d^2}{dt^2} (l \cos \theta) = V - mg \quad (3-50)$$

The horizontal motion of cart is described by

$$M \frac{d^2 x}{dt^2} = u - H \quad (3-51)$$

Since we must keep the inverted pendulum vertical, we can assume that $\theta(t)$ and $\dot{\theta}(t)$ are small quantities such that $\sin \theta \approx 0$, $\cos \theta = 1$, and $\theta\dot{\theta}^2 = 0$. Then, Equations (3-48) through (3-50) can be linearized. The linearized equations are

$$I\ddot{\theta} = Vl\theta - Hl \quad (3-52)$$

$$m(\ddot{x} + l\ddot{\theta}) = H \quad (3-53)$$

$$0 = V - mg \quad (3-54)$$

From Equations (3-51) and (3-53), we obtain

$$(M + m)\ddot{x} + ml\ddot{\theta} = u \quad (3-55)$$

From Equations (3-52), (3-53), and (3-54), we have

$$\begin{aligned} I\ddot{\theta} &= mgl\theta - Hl \\ &= mgl\theta - l(m\ddot{x} + ml\ddot{\theta}) \end{aligned}$$

or

$$(I + ml^2)\ddot{\theta} + ml\ddot{x} = mgl\theta \quad (3-56)$$

Equations (3-55) and (3-56) describe the motion of the inverted-pendulum-on-the-cart system. They constitute a mathematical model of the system.

EXAMPLE 3-9

Consider the inverted pendulum system shown in Figure 3-21. Since in this system the mass is concentrated at the top of the rod, the center of gravity is the center of the pendulum ball. For this case, the moment of inertia of the pendulum about its center of gravity is small, and we assume $I = 0$ in Equation (3-56). Then the mathematical model for this system becomes as follows:

$$(M + m)\ddot{x} + ml\ddot{\theta} = u \quad (3-57)$$

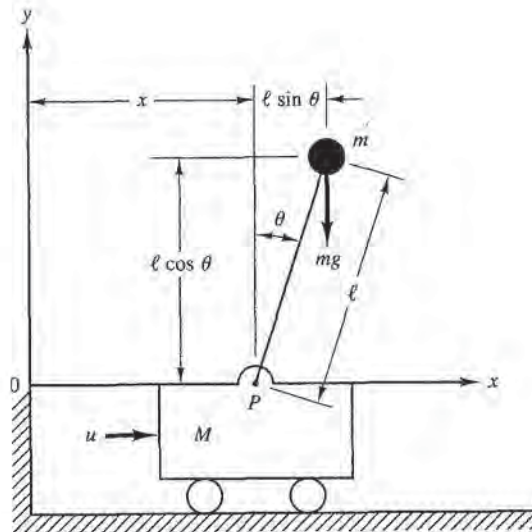
$$ml^2\ddot{\theta} + ml\ddot{x} = mgl\theta \quad (3-58)$$

Equations (3-57) and (3-58) can be modified to

$$Ml\ddot{\theta} = (M + m)g\theta - u \quad (3-59)$$

$$M\ddot{x} = u - mg\theta \quad (3-60)$$

Figure 3-21
Inverted-pendulum
system.



Equation (3-59) was obtained by eliminating $\dot{x}$ from Equations (3-57) and (3-58). Equation (3-60) was obtained by eliminating $\ddot{\theta}$ from Equations (3-57) and (3-58). From Equation (3-59) we obtain the plant transfer function to be

$$\begin{aligned}\frac{\Theta(s)}{-U(s)} &= \frac{1}{Mls^2 - (M + m)g} \\ &= \frac{1}{Ml\left(s + \sqrt{\frac{M + m}{Ml}}g\right)\left(s - \sqrt{\frac{M + m}{Ml}}g\right)}\end{aligned}$$

The inverted pendulum plant has one pole on the negative real axis [$s = -(\sqrt{M + m}/\sqrt{Ml})\sqrt{g}$] and another on the positive real axis [$s = (\sqrt{M + m}/\sqrt{Ml})\sqrt{g}$]. Hence, the plant is open-loop unstable.

Define state variables x_1 , x_2 , x_3 , and x_4 by

$$\begin{aligned}x_1 &= \theta \\ x_2 &= \dot{\theta} \\ x_3 &= x \\ x_4 &= \dot{x}\end{aligned}$$

Note that angle θ indicates the rotation of the pendulum rod about point P , and x is the location of the cart. If we consider θ and x as the outputs of the system, then

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \theta \\ x \end{bmatrix} = \begin{bmatrix} x_1 \\ x_3 \end{bmatrix}$$

(Notice that both θ and x are easily measurable quantities). Then, from the definition of the state variables and Equations (3-59) and (3-60), we obtain

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= \frac{M + m}{Ml}gx_1 - \frac{1}{Ml}u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= -\frac{m}{M}gx_1 + \frac{1}{M}u\end{aligned}$$

In terms of vector-matrix equations, we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{M+m}{Ml}g & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{m}{M}g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{1}{Ml} \\ 0 \\ \frac{1}{M} \end{bmatrix} u \quad (3-61)$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \quad (3-62)$$

Equations (3-61) and (3-62) give a state-space representation of the inverted pendulum system. (Note that state-space representation of the system is not unique. There are infinitely many such representations for this system.)

3-8 ELECTRICAL AND ELECTRONIC SYSTEMS

Basic laws governing electrical circuits are Kirchhoff's current law and voltage law. Kirchhoff's current law (node law) states that the algebraic sum of all currents entering and leaving a node is zero. (This law can also be stated as follows: The sum of currents entering a node is equal to the sum of currents leaving the same node.) Kirchhoff's voltage law (loop law) states that at any given instant the algebraic sum of the voltages around any loop in an electrical circuit is zero. (This law can also be stated as follows: The sum of the voltage drops is equal to the sum of the voltage rises around a loop.) A mathematical model of an electrical circuit can be obtained by applying one or both of Kirchhoff's laws to it.

This section first deals with simple electrical circuits and then treats mathematical modeling of operational amplifier systems.

LRC Circuit. Consider the electrical circuit shown in Figure 3-22. The circuit consists of an inductance L (henry), a resistance R (ohm), and a capacitance C (farad). Applying Kirchhoff's voltage law to the system, we obtain the following equations:

$$L \frac{di}{dt} + Ri + \frac{1}{C} \int i dt = e_i \quad (3-63)$$

$$\frac{1}{C} \int i dt = e_o \quad (3-64)$$

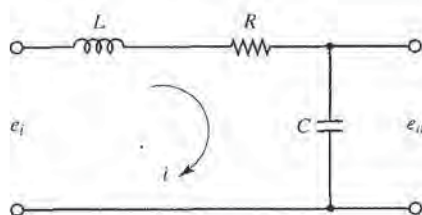


Figure 3-22
Electrical circuit.

Equations (3-63) and (3-64) give a mathematical model of the circuit.

A transfer function model of the circuit can also be obtained as follows: Taking the Laplace transforms of Equations (3-63) and (3-64), assuming zero initial conditions, we obtain

$$LsI(s) + RI(s) + \frac{1}{C} \frac{1}{s} I(s) = E_i(s)$$

$$\frac{1}{C} \frac{1}{s} I(s) = E_o(s)$$

If e_i is assumed to be the input and e_o the output, then the transfer function of this system is found to be

$$\frac{E_o(s)}{E_i(s)} = \frac{1}{LCs^2 + RCs + 1} \quad (3-65)$$

State-Space Representation. A state-space model of the system shown in Figure 3-22 may be obtained as follows: First, note that the differential equation for the system can be obtained from Equation (3-65) as

$$\ddot{e}_o + \frac{R}{L} \dot{e}_o + \frac{1}{LC} e_o = \frac{1}{LC} e_i$$

Then by defining state variables by

$$x_1 = e_o$$

$$x_2 = \dot{e}_o$$

and the input and output variables by

$$u = e_i$$

$$y = e_o = x_1$$

we obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{LC} & -\frac{R}{L} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{LC} \end{bmatrix} u$$

and

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

These two equations give a mathematical model of the system in state space.

Transfer Functions of Cascaded Elements. Many feedback systems have components that load each other. Consider the system shown in Figure 3-23. Assume that

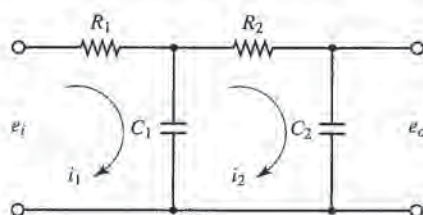


Figure 3-23
Electrical system.

e_i is the input and e_o is the output. The capacitances C_1 and C_2 are not charged initially. It will be shown that the second stage of the circuit (R_2C_2 portion) produces a loading effect on the first stage (R_1C_1 portion). The equations for this system are

$$\frac{1}{C_1} \int (i_1 - i_2) dt + R_1 i_1 = e_i \quad (3-66)$$

and

$$\frac{1}{C_1} \int (i_2 - i_1) dt + R_2 i_2 + \frac{1}{C_2} \int i_2 dt = 0 \quad (3-67)$$

$$\frac{1}{C_2} \int i_2 dt = e_o \quad (3-68)$$

Taking the Laplace transforms of Equations (3-66) through (3-68), respectively, using zero initial conditions, we obtain

$$\frac{1}{C_1 s} [I_1(s) - I_2(s)] + R_1 I_1(s) = E_i(s) \quad (3-69)$$

$$\frac{1}{C_1 s} [I_2(s) - I_1(s)] + R_2 I_2(s) + \frac{1}{C_2 s} I_2(s) = 0 \quad (3-70)$$

$$\frac{1}{C_2 s} I_2(s) = E_o(s) \quad (3-71)$$

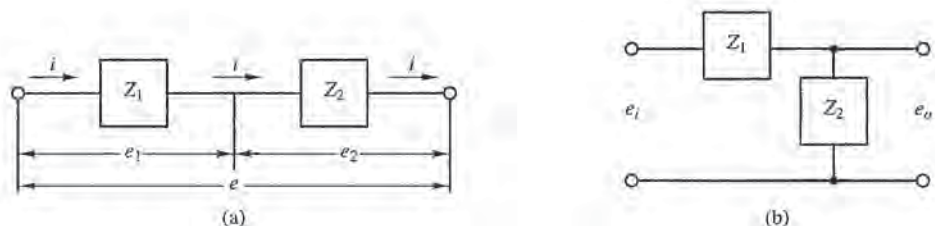
Eliminating $I_1(s)$ from Equations (3-69) and (3-70) and writing $E_i(s)$ in terms of $I_2(s)$, we find the transfer function between $E_o(s)$ and $E_i(s)$ to be

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \frac{1}{(R_1 C_1 s + 1)(R_2 C_2 s + 1) + R_1 C_2 s} \\ &= \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1} \end{aligned} \quad (3-72)$$

The term $R_1 C_2 s$ in the denominator of the transfer function represents the interaction of two simple RC circuits. Since $(R_1 C_1 + R_2 C_2 + R_1 C_2)^2 > 4 R_1 C_1 R_2 C_2$, the two roots of the denominator of Equation (3-72) are real.

The present analysis shows that, if two RC circuits are connected in cascade so that the output from the first circuit is the input to the second, the overall transfer function is not the product of $1/(R_1 C_1 s + 1)$ and $1/(R_2 C_2 s + 1)$. The reason for this is that, when we derive the transfer function for an isolated circuit, we implicitly assume that the output is unloaded. In other words, the load impedance is assumed to be infinite, which means that no power is being withdrawn at the output. When the second circuit is connected to the output of the first, however, a certain amount of power is withdrawn, and thus the assumption of no loading is violated. Therefore, if the transfer function of this system is obtained under the assumption of no loading, then it is not valid. The degree of the loading effect determines the amount of modification of the transfer function.

Figure 3-24
Electrical circuits.



Complex Impedances. In deriving transfer functions for electrical circuits, we frequently find it convenient to write the Laplace-transformed equations directly, without writing the differential equations. Consider the system shown in Figure 3-24(a). In this system, Z_1 and Z_2 represent complex impedances. The complex impedance $Z(s)$ of a two-terminal circuit is the ratio of $E(s)$, the Laplace transform of the voltage across the terminals, to $I(s)$, the Laplace transform of the current through the element, under the assumption that the initial conditions are zero, so that $Z(s) = E(s)/I(s)$. If the two-terminal element is a resistance R , capacitance C , or inductance L , then the complex impedance is given by R , $1/Cs$, or Ls , respectively. If complex impedances are connected in series, the total impedance is the sum of the individual complex impedances.

Remember that the impedance approach is valid only if the initial conditions involved are all zeros. Since the transfer function requires zero initial conditions, the impedance approach can be applied to obtain the transfer function of the electrical circuit. This approach greatly simplifies the derivation of transfer functions of electrical circuits.

Consider the circuit shown in Figure 3-24(b). Assume that the voltages e_i and e_o are the input and output of the circuit, respectively. Then the transfer function of this circuit is

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2(s)}{Z_1(s) + Z_2(s)}$$

For the system shown in Figure 3-22,

$$Z_1 = Ls + R, \quad Z_2 = \frac{1}{Cs}$$

Hence the transfer function $E_o(s)/E_i(s)$ can be found as follows:

$$\frac{E_o(s)}{E_i(s)} = \frac{\frac{1}{Cs}}{Ls + R + \frac{1}{Cs}} = \frac{1}{LCs^2 + RCs + 1}$$

which is, of course, identical to Equation (3-65).

EXAMPLE 3-10

Consider again the system shown in Figure 3-23. Obtain the transfer function $E_o(s)/E_i(s)$ by use of the complex impedance approach. (Capacitors C_1 and C_2 are not charged initially.)

The circuit shown in Figure 3-23 can be redrawn as that shown in Figure 3-25(a), which can be further modified to Figure 3-25(b).

In the system shown in Figure 3-25(b) the current I is divided into two currents I_1 and I_2 . Noting that

$$Z_2 I_1 = (Z_3 + Z_4) I_2, \quad I_1 + I_2 = I$$

we obtain

$$I_1 = \frac{Z_3 + Z_4}{Z_2 + Z_3 + Z_4} I, \quad I_2 = \frac{Z_2}{Z_2 + Z_3 + Z_4} I$$

Noting that

$$E_i(s) = Z_1 I + Z_2 I_1 = \left[Z_1 + \frac{Z_2(Z_3 + Z_4)}{Z_2 + Z_3 + Z_4} \right] I$$

$$E_o(s) = Z_4 I_2 = \frac{Z_2 Z_4}{Z_2 + Z_3 + Z_4} I$$

we obtain

$$\frac{E_o(s)}{E_i(s)} = \frac{Z_2 Z_4}{Z_1(Z_2 + Z_3 + Z_4) + Z_2(Z_3 + Z_4)}$$

Substituting $Z_1 = R_1$, $Z_2 = 1/(C_1 s)$, $Z_3 = R_2$, and $Z_4 = 1/(C_2 s)$ into this last equation, we get

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \frac{\frac{1}{C_1 s} \frac{1}{C_2 s}}{R_1 \left(\frac{1}{C_1 s} + R_2 + \frac{1}{C_2 s} \right) + \frac{1}{C_1 s} \left(R_2 + \frac{1}{C_2 s} \right)} \\ &= \frac{1}{R_1 C_1 R_2 C_2 s^2 + (R_1 C_1 + R_2 C_2 + R_1 C_2) s + 1} \end{aligned}$$

which is the same as that given by Equation (3-72).

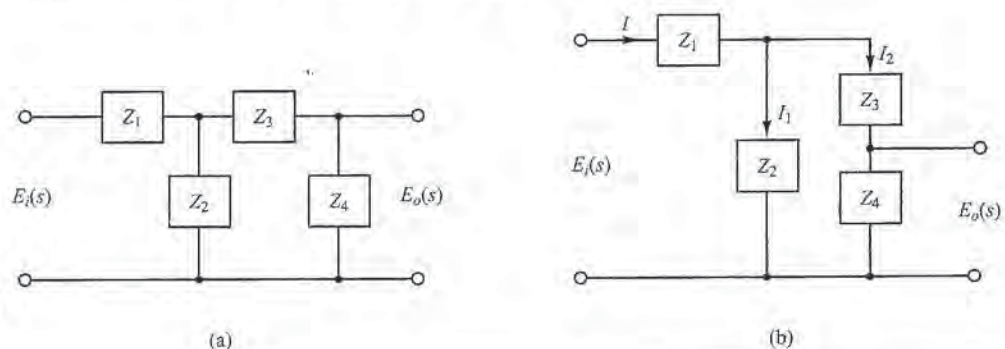


Figure 3-25
(a) The circuit of Figure 3-23 shown in terms of impedances; (b) equivalent circuit diagram.



Figure 3-26
(a) System consisting of two nonloading cascaded elements; (b) an equivalent system.

Transfer Functions of Nonloading Cascaded Elements. The transfer function of a system consisting of two nonloading cascaded elements can be obtained by eliminating the intermediate input and output. For example, consider the system shown in Figure 3-26(a). The transfer functions of the elements are

$$G_1(s) = \frac{X_2(s)}{X_1(s)} \quad \text{and} \quad G_2(s) = \frac{X_3(s)}{X_2(s)}$$

If the input impedance of the second element is infinite, the output of the first element is not affected by connecting it to the second element. Then the transfer function of the whole system becomes

$$G(s) = \frac{X_3(s)}{X_1(s)} = \frac{X_2(s)X_3(s)}{X_1(s)X_2(s)} = G_1(s)G_2(s)$$

The transfer function of the whole system is thus the product of the transfer functions of the individual elements. This is shown in Figure 3-26(b).

As an example, consider the system shown in Figure 3-27. The insertion of an isolating amplifier between the circuits to obtain nonloading characteristics is frequently used in combining circuits. Since amplifiers have very high input impedances, an isolation amplifier inserted between the two circuits justifies the nonloading assumption.

The two simple *RC* circuits, isolated by an amplifier as shown in Figure 3-27, have negligible loading effects, and the transfer function of the entire circuit equals the product of the individual transfer functions. Thus, in this case,

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \left(\frac{1}{R_1 C_1 s + 1} \right) (K) \left(\frac{1}{R_2 C_2 s + 1} \right) \\ &= \frac{K}{(R_1 C_1 s + 1)(R_2 C_2 s + 1)} \end{aligned}$$

Electronic Controllers. In what follows we shall discuss electronic controllers using operational amplifiers. We begin by deriving the transfer functions of simple operational-amplifier circuits. Then we derive the transfer functions of some of the operational-amplifier controllers. Finally, we give operational-amplifier controllers and their transfer functions in the form of a table.

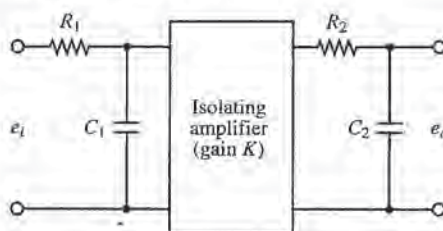
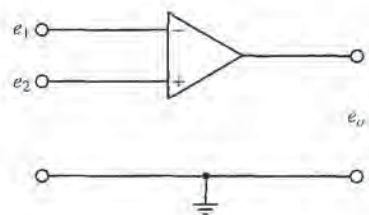


Figure 3-27
Electrical system.

Figure 3-28
Operational
amplifier.



Operational Amplifiers. Operational amplifiers, often called op amps, are frequently used to amplify signals in sensor circuits. Op amps are also frequently used in filters used for compensation purposes. Figure 3-28 shows an op amp. It is a common practice to choose the ground as 0 volt and measure the input voltages e_1 and e_2 relative to the ground. The input e_1 to the minus terminal of the amplifier is inverted, and the input e_2 to the plus terminal is not inverted. The total input to the amplifier thus becomes $e_2 - e_1$. Hence, for the circuit shown in Figure 3-28, we have

$$e_o = K(e_2 - e_1) = -K(e_1 - e_2)$$

where the inputs e_1 and e_2 may be dc or ac signals and K is the differential gain (voltage gain). The magnitude of K is approximately $10^5 \sim 10^6$ for dc signals and ac signals with frequencies less than approximately 10 Hz. (The differential gain K decreases with the signal frequency and becomes about unity for frequencies of 1 MHz $\sim$ 50 MHz.) Note that the op amp amplifies the difference in voltages e_1 and e_2 . Such an amplifier is commonly called a differential amplifier. Since the gain of the op amp is very high, it is necessary to have a negative feedback from the output to the input to make the amplifier stable. (The feedback is made from the output to the inverted input so that the feedback is a negative feedback.)

In the ideal op amp, no current flows into the input terminals, and the output voltage is not affected by the load connected to the output terminal. In other words, the input impedance is infinity and the output impedance is zero. In an actual op amp, a very small (almost negligible) current flows into an input terminal and the output cannot be loaded too much. In our analysis here, we make the assumption that the op amps are ideal.

Inverting Amplifier. Consider the operational amplifier circuit shown in Figure 3-29. Let us obtain the output voltage e_o .

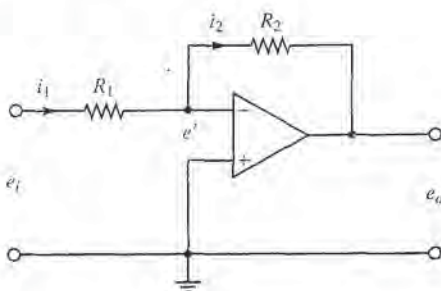


Figure 3-29
Inverting amplifier.

The equation for this circuit can be obtained as follows: Define

$$i_1 = \frac{e_i - e'}{R_1}, \quad i_2 = \frac{e' - e_o}{R_2}$$

Since only a negligible current flows into the amplifier, the current i_1 must be equal to current i_2 . Thus

$$\frac{e_i - e'}{R_1} = \frac{e' - e_o}{R_2}$$

Since $K(0 - e') = e_o$ and $K \gg 1$, e' must be almost zero, or $e' \doteq 0$. Hence we have

$$\frac{e_i}{R_1} = -\frac{e_o}{R_2}$$

or

$$e_o = -\frac{R_2}{R_1} e_i$$

Thus the circuit shown is an inverting amplifier. If $R_1 = R_2$, then the op-amp circuit shown acts as a sign inverter.

Noninverting Amplifier. Figure 3-30(a) shows a noninverting amplifier. A circuit equivalent to this one is shown in Figure 3-30(b). For the circuit of Figure 3-30(b), we have

$$e_o = K \left(e_i - \frac{R_1}{R_1 + R_2} e_o \right)$$

where K is the differential gain of the amplifier. From this last equation, we get

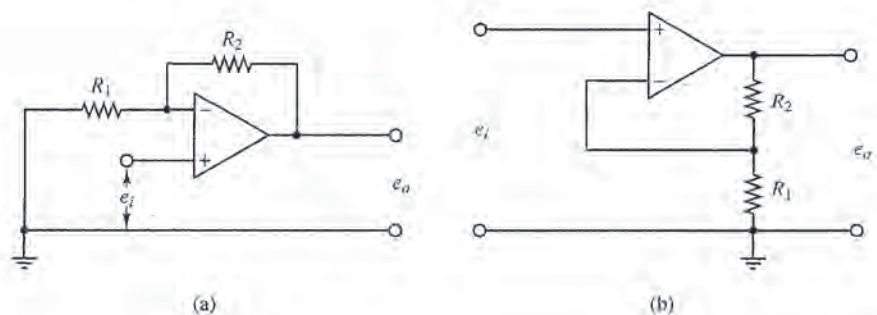
$$e_i = \left(\frac{R_1}{R_1 + R_2} + \frac{1}{K} \right) e_o$$

Since $K \gg 1$, if $R_1/(R_1 + R_2) \gg 1/K$, then

$$e_o = \left(1 + \frac{R_2}{R_1} \right) e_i$$

This equation gives the output voltage e_o . Since e_o and e_i have the same signs, the op-amp circuit shown in Figure 3-30(a) is noninverting.

Figure 3-30
(a) Noninverting operational amplifier;
(b) equivalent circuit.



EXAMPLE 3-11 Figure 3-31 shows an electrical circuit involving an operational amplifier. Obtain the output e_o .
Let us define

$$i_1 = \frac{e_i - e'}{R_1}, \quad i_2 = C \frac{d(e' - e_o)}{dt}, \quad i_3 = \frac{e' - e_o}{R_2}$$

Noting that the current flowing into the amplifier is negligible, we have

$$i_1 = i_2 + i_3$$

Hence

$$\frac{e_i - e'}{R_1} = C \frac{d(e' - e_o)}{dt} + \frac{e' - e_o}{R_2}$$

Since $e' \approx 0$, we have

$$\frac{e_i}{R_1} = -C \frac{de_o}{dt} - \frac{e_o}{R_2}$$

Taking the Laplace transform of this last equation, assuming the zero initial condition, we have

$$\frac{E_i(s)}{R_1} = -\frac{R_2 Cs + 1}{R_2} E_o(s)$$

which can be written as

$$\frac{E_o(s)}{E_i(s)} = -\frac{R_2}{R_1} \frac{1}{R_2 Cs + 1}$$

The op-amp circuit shown in Figure 3-31 is a first-order lag circuit. (Several other circuits involving op amps are shown in Table 3-1 together with their transfer functions.)

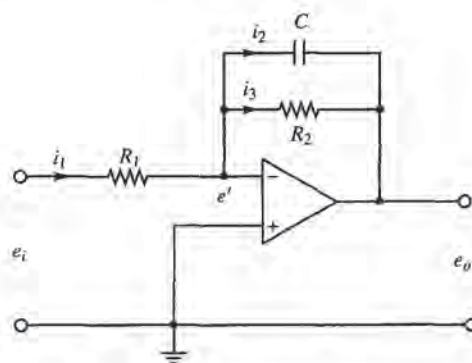
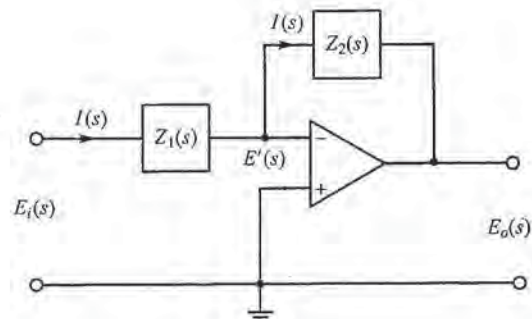


Figure 3-31
First-order lag circuit
using operational
amplifier.

Figure 3-32
Operational
amplifier circuit.



Impedance Approach to Obtaining Transfer Functions. Consider the op-amp circuit shown in Figure 3-32. Similar to the case of electrical circuits we discussed earlier, the impedance approach can be applied to op-amp circuits to obtain their transfer functions. For the circuit shown in Figure 3-32, we have

$$\frac{E_i(s) - E'(s)}{Z_1} = \frac{E'(s) - E_o(s)}{Z_2}$$

Since $E'(s) \doteq 0$, we have

$$\frac{E_o(s)}{E_i(s)} = - \frac{Z_2(s)}{Z_1(s)} \tag{3-73}$$

EXAMPLE 3-12 Referring to the op-amp circuit shown in Figure 3-31, obtain the transfer function $E_o(s)/E_i(s)$ by use of the impedance approach.

The complex impedances $Z_1(s)$ and $Z_2(s)$ for this circuit are

$$Z_1(s) = R_1 \quad \text{and} \quad Z_2(s) = \frac{1}{Cs + \frac{1}{R_2}} = \frac{R_2}{R_2Cs + 1}$$

The transfer function $E_o(s)/E_i(s)$ is, therefore, obtained as

$$\frac{E_o(s)}{E_i(s)} = - \frac{Z_2(s)}{Z_1(s)} = - \frac{R_2}{R_1} \frac{1}{R_2Cs + 1}$$

which is, of course, the same as that obtained in Example 3-11.

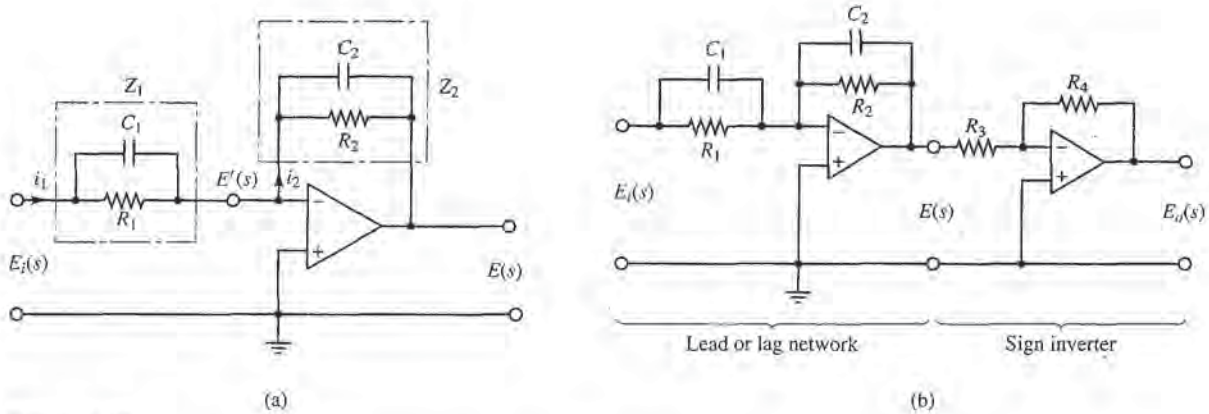


Figure 3-33
(a) Operational-amplifier circuit; (b) operational-amplifier circuit used as a lead or lag compensator.

Lead or Lag Networks Using Operational Amplifiers. Figure 3-33(a) shows an electronic circuit using an operational amplifier. The transfer function for this circuit can be obtained as follows: Define the input impedance and feedback impedance as Z_1 and Z_2 , respectively. Then

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1}, \quad Z_2 = \frac{R_2}{R_2 C_2 s + 1}$$

Hence, referring to Equation (3-73), we have

$$\frac{E(s)}{E_i(s)} = -\frac{Z_2}{Z_1} = -\frac{R_2}{R_1} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = -\frac{C_1}{C_2} \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}} \quad (3-74)$$

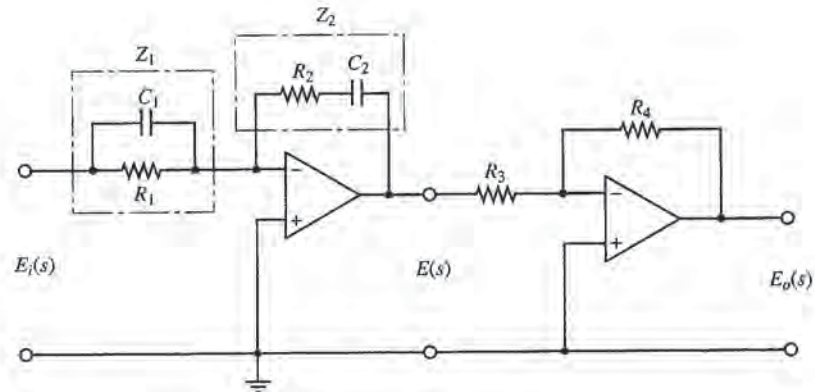
Notice that the transfer function in Equation (3-74) contains a minus sign. Thus, this circuit is sign inverting. If such a sign inversion is not convenient in the actual application, a sign inverter may be connected to either the input or the output of the circuit of Figure 3-33(a). An example is shown in Figure 3-33(b). The sign inverter has the transfer function of

$$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3}$$

The sign inverter has the gain of $-R_4/R_3$. Hence the network shown in Figure 3-33(b) has the following transfer function:

$$\begin{aligned} \frac{E_o(s)}{E_i(s)} &= \frac{R_2 R_4}{R_1 R_3} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1} = \frac{R_4 C_1}{R_3 C_2} \frac{s + \frac{1}{R_1 C_1}}{s + \frac{1}{R_2 C_2}} \\ &= K_c \alpha \frac{T s + 1}{\alpha T s + 1} = K_c \frac{s + \frac{1}{T}}{s + \frac{1}{\alpha T}} \end{aligned} \quad (3-75)$$

Figure 3-34
Electronic PID
controller.



where

$$T = R_1 C_1, \quad \alpha T = R_2 C_2, \quad K_c = \frac{R_4 C_1}{R_3 C_2}$$

Notice that

$$K_c \alpha = \frac{R_4 C_1}{R_3 C_2} \frac{R_2 C_2}{R_1 C_1} = \frac{R_2 R_4}{R_1 R_3}, \quad \alpha = \frac{R_2 C_2}{R_1 C_1}$$

This network has a dc gain of $K_c \alpha = R_2 R_4 / (R_1 R_3)$.

Note that this network is a lead network if $R_1 C_1 > R_2 C_2$, or $\alpha < 1$. It is a lag network if $R_1 C_1 < R_2 C_2$.

PID Controller Using Operational Amplifiers. Figure 3-34 shows an electronic proportional-plus-integral-plus-derivative controller (a PID controller) using operational amplifiers. The transfer function $E(s)/E_i(s)$ is given by

$$\frac{E(s)}{E_i(s)} = -\frac{Z_2}{Z_1}$$

where

$$Z_1 = \frac{R_1}{R_1 C_1 s + 1}, \quad Z_2 = \frac{R_2 C_2 s + 1}{C_2 s}$$

Thus

$$\frac{E(s)}{E_i(s)} = -\left(\frac{R_2 C_2 s + 1}{C_2 s}\right)\left(\frac{R_1 C_1 s + 1}{R_1}\right)$$

Noting that

$$\frac{E_o(s)}{E(s)} = -\frac{R_4}{R_3}$$

we have

$$\begin{aligned}
 \frac{E_o(s)}{E_i(s)} &= \frac{E_o(s)}{E(s)} \frac{E(s)}{E_i(s)} = \frac{R_4 R_2}{R_3 R_1} \frac{(R_1 C_1 s + 1)(R_2 C_2 s + 1)}{R_2 C_2 s} \\
 &= \frac{R_4 R_2}{R_3 R_1} \left(\frac{R_1 C_1 + R_2 C_2}{R_2 C_2} + \frac{1}{R_2 C_2 s} + R_1 C_1 s \right) \\
 &= \frac{R_4(R_1 C_1 + R_2 C_2)}{R_3 R_1 C_2} \left[1 + \frac{1}{(R_1 C_1 + R_2 C_2)s} + \frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2} s \right] \quad (3-76)
 \end{aligned}$$

Notice that the second operational-amplifier circuit acts as a sign inverter as well as a gain adjuster.

When a PID controller is expressed as

$$\frac{E_o(s)}{E_i(s)} = K_p \left(1 + \frac{T_i}{s} + T_d s \right)$$

K_p is called the proportional gain, T_i is called the integral time, and T_d is called the derivative time. From Equation (3-76) we obtain the proportional gain K_p , integral time T_i , and derivative time T_d to be

$$\begin{aligned}
 K_p &= \frac{R_4(R_1 C_1 + R_2 C_2)}{R_3 R_1 C_2} \\
 T_i &= \frac{1}{R_1 C_1 + R_2 C_2} \\
 T_d &= \frac{R_1 C_1 R_2 C_2}{R_1 C_1 + R_2 C_2}
 \end{aligned}$$

When a PID controller is expressed as

$$\frac{E_o(s)}{E_i(s)} = K_p + \frac{K_i}{s} + K_d s$$

K_p is called the proportional gain, K_i is called the integral gain, and K_d is called the derivative gain. For this controller

$$\begin{aligned}
 K_p &= \frac{R_4(R_1 C_1 + R_2 C_2)}{R_3 R_1 C_2} \\
 K_i &= \frac{R_4}{R_3 R_1 C_2} \\
 K_d &= \frac{R_4 R_2 C_1}{R_3}
 \end{aligned}$$

Table 3-1 shows a list of operational-amplifier circuits that may be used as controllers or compensators.

Table 3-1 Operational-Amplifier Circuits That May Be Used as Compensators

	Control Action	$G(s) = \frac{E_o(s)}{E_i(s)}$	Operational Amplifier Circuits
1	P	$\frac{R_4}{R_3} \frac{R_2}{R_1}$	
2	I	$\frac{R_4}{R_3} \frac{1}{R_1 C_2 s}$	
3	PD	$\frac{R_4}{R_3} \frac{R_2}{R_1} (R_1 C_1 s + 1)$	
4	PI	$\frac{R_4}{R_3} \frac{R_2}{R_1} \frac{R_2 C_2 s + 1}{R_2 C_2 s}$	
5	PID	$\frac{R_4}{R_3} \frac{R_2}{R_1} \frac{(R_1 C_1 s + 1) (R_2 C_2 s + 1)}{R_2 C_2 s}$	
6	Lead or lag	$\frac{R_4}{R_3} \frac{R_2}{R_1} \frac{R_1 C_1 s + 1}{R_2 C_2 s + 1}$	
7	Lag-lead	$\frac{R_6}{R_5} \frac{R_4}{R_3} \frac{[(R_1 + R_3) C_1 s + 1] (R_2 C_2 s + 1)}{(R_1 C_1 s + 1) [(R_2 + R_4) C_2 s + 1]}$	

3-9 SIGNAL FLOW GRAPHS

The block diagram is useful for graphically representing control system dynamics and is used extensively in the analysis and design of control systems. An alternate approach for graphically representing control system dynamics is the signal flow graph approach, due to S. J. Mason. It is noted that the signal flow graph approach and the block diagram approach yield the same information and one is in no sense superior to the other.

Signal Flow Graphs. A signal flow graph is a diagram that represents a set of simultaneous linear algebraic equations. When applying the signal flow graph method to analyses of control systems, we must first transform linear differential equations into algebraic equations in s .

A signal flow graph consists of a network in which nodes are connected by directed branches. Each node represents a system variable, and each branch connected between two nodes acts as a signal multiplier. Note that the signal flows in only one direction. The direction of signal flow is indicated by an arrow placed on the branch, and the multiplication factor is indicated along the branch. The signal flow graph depicts the flow of signals from one point of a system to another and gives the relationships among the signals.

As mentioned earlier, a signal flow graph contains essentially the same information as a block diagram. If a signal flow graph is used to represent a control system, then a gain formula, called Mason's gain formula, may be used to obtain the relationships among system variables without carrying out reduction of the graph.

Definitions. Before we discuss signal flow graphs, we must define certain terms.

Node. A node is a point representing a variable or signal.

Transmittance. The transmittance is a real gain or complex gain between two nodes. Such gains can be expressed in terms of the transfer function between two nodes.

Branch. A branch is a directed line segment joining two nodes. The gain of a branch is a transmittance.

Input node or source. An input node or source is a node that has only outgoing branches. This corresponds to an independent variable.

Output node or sink. An output node or sink is a node that has only incoming branches. This corresponds to a dependent variable.

Mixed node. A mixed node is a node that has both incoming and outgoing branches.

Path. A path is a traversal of connected branches in the direction of the branch arrows. If no node is crossed more than once, the path is open. If the path ends at the same node from which it began and does not cross any other node more than once, it is closed. If a path crosses some node more than once but ends at a different node from which it began, it is neither open nor closed.

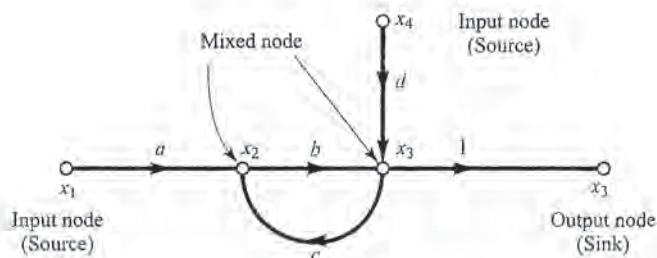
Loop. A loop is a closed path.

Loop gain. The loop gain is the product of the branch transmittances of a loop.

Nontouching loops. Loops are nontouching if they do not possess any common nodes.

Forward path. A forward path is a path from an input node (source) to an output node (sink) that does not cross any nodes more than once.

Figure 3–35
Signal flow graph.



Forward path gain. A forward path gain is the product of the branch transmittances of a forward path.

Figure 3–35 shows nodes and branches, together with transmittances.

Properties of Signal Flow Graphs. A few important properties of signal flow graphs are as follows:

1. A branch indicates the functional dependence of one signal on another. A signal passes through only in the direction specified by the arrow of the branch.
2. A node adds the signals of all incoming branches and transmits this sum to all outgoing branches.
3. A mixed node, which has both incoming and outgoing branches, may be treated as an output node (sink) by adding an outgoing branch of unity transmittance. (See Figure 3–35. Notice that a branch with unity transmittance is directed from x_3 to another node, also denoted by x_3 .) Note, however, that we cannot change a mixed node to a source by this method.
4. For a given system, a signal flow graph is not unique. Many different signal flow graphs can be drawn for a given system by writing the system equations differently.

Signal Flow Graph Algebra. A signal flow graph of a linear system can be drawn using the foregoing definitions. In doing so, we usually bring the input nodes (sources) to the left and the output nodes (sinks) to the right. The independent and dependent variables of the equations become the input nodes (sources) and output nodes (sinks), respectively. The branch transmittances can be obtained from the coefficients of the equations.

To determine the input-output relationship, we may use Mason's formula, which will be given later, or we may reduce the signal flow graph to a graph containing only input and output nodes. To accomplish this, we use the following rules:

1. The value of a node with one incoming branch, as shown in Figure 3–36(a), is $x_2 = ax_1$.
2. The total transmittance of cascaded branches is equal to the product of all the branch transmittances. Cascaded branches can thus be combined into a single branch by multiplying the transmittances, as shown in Figure 3–36(b).
3. Parallel branches may be combined by adding the transmittances, as shown in Figure 3–36(c).
4. A mixed node may be eliminated, as shown in Figure 3–36(d).
5. A loop may be eliminated, as shown in Figure 3–36(e). Note that

$$x_3 = bx_2, \quad x_2 = ax_1 + cx_3$$