This is the MATLAB representation of the following state-space equations:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5.0325 & -25.1026 & -5.008 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 25.04 \quad 5.008] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

A–11–6. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

where $\mathbf{x}$ = state vector (n-vector)
$\mathbf{u}$ = control vector (r-vector)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times r$ constant matrix

Obtain the response of the system to each of the following inputs:

(**a**) The r components of $\mathbf{u}$ are impulse functions of various magnitudes.

(**b**) The r components of $\mathbf{u}$ are step functions of various magnitudes.

(**c**) The r components of $\mathbf{u}$ are ramp functions of various magnitudes.

Solution.

(**a**) *Impulse response:* Referring to Equation (11–43), the solution to the given state equation is

$$\mathbf{x}(t) = e^{\mathbf{A}(t-t_0)}\mathbf{x}(t_0) + \int_{t_0}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)\,d\tau$$

Substituting $t_0 = 0-$ into this solution, we obtain

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0-) + \int_{0-}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{u}(\tau)\,d\tau$$

Let us write the impulse input $\mathbf{u}(t)$ as

$$\mathbf{u}(t) = \delta(t)\mathbf{w}$$

where $\mathbf{w}$ is a vector whose components are the magnitudes of r impulse functions applied at $t = 0$. The solution of the state equation when the impulse input $\delta(t)\mathbf{w}$ is given at $t = 0$ is

$$\begin{aligned} \mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0-) + \int_{0-}^{t} e^{\mathbf{A}(t-\tau)}\mathbf{B}\delta(\tau)\mathbf{w}\,d\tau \\ &= e^{\mathbf{A}t}\mathbf{x}(0-) + e^{\mathbf{A}t}\mathbf{B}\mathbf{w} \end{aligned} \tag{11–95}$$

(**b**) *Step response:* Let us write the step input $\mathbf{u}(t)$ as

$$\mathbf{u}(t) = \mathbf{k}$$

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10601

where $\mathbf{k}$ is a vector whose components are the magnitudes of r step functions applied at $t = 0$. The solution to the step input at $t = 0$ is given by

$$\begin{aligned}\mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\mathbf{k}\,d\tau \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left[\int_0^t \left(\mathbf{I} - \mathbf{A}\tau + \frac{\mathbf{A}^2\tau^2}{2!} - \cdots\right) d\tau\right]\mathbf{B}\mathbf{k} \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left(\mathbf{I}t - \frac{\mathbf{A}t^2}{2!} + \frac{\mathbf{A}^2t^3}{3!} - \cdots\right)\mathbf{B}\mathbf{k}\end{aligned}$$

If $\mathbf{A}$ is nonsingular, then this last equation can be simplified to give

$$\begin{aligned}\mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left[-(\mathbf{A}^{-1})(e^{-\mathbf{A}t} - \mathbf{I})\right]\mathbf{B}\mathbf{k} \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I})\mathbf{B}\mathbf{k}\end{aligned} \tag{11–96}$$

(c) *Ramp response:* Let us write the ramp input $\mathbf{u}(t)$ as

$$\mathbf{u}(t) = t\mathbf{v}$$

where $\mathbf{v}$ is a vector whose components are magnitudes of ramp functions applied at $t = 0$. The solution to the ramp input $t\mathbf{v}$ given at $t = 0$ is

$$\begin{aligned}\mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0) + \int_0^t e^{\mathbf{A}(t-\tau)}\mathbf{B}\tau\mathbf{v}\,d\tau \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\int_0^t e^{-\mathbf{A}\tau}\tau\,d\tau\mathbf{B}\mathbf{v} \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + e^{\mathbf{A}t}\left(\frac{\mathbf{I}}{2}t^2 - \frac{2\mathbf{A}}{3!}t^3 + \frac{3\mathbf{A}^2}{4!}t^4 - \frac{4\mathbf{A}^3}{5!}t^5 + \cdots\right)\mathbf{B}\mathbf{v}\end{aligned}$$

If $\mathbf{A}$ is nonsingular, then this last equation can be simplified to give

$$\begin{aligned}\mathbf{x}(t) &= e^{\mathbf{A}t}\mathbf{x}(0) + (\mathbf{A}^{-2})(e^{\mathbf{A}t} - \mathbf{I} - \mathbf{A}t)\mathbf{B}\mathbf{v} \\ &= e^{\mathbf{A}t}\mathbf{x}(0) + \left[\mathbf{A}^{-2}(e^{\mathbf{A}t} - \mathbf{I}) - \mathbf{A}^{-1}t\right]\mathbf{B}\mathbf{v}\end{aligned} \tag{11–97}$$

A–11–7. Obtain the response $y(t)$ of the following system:

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_1\end{bmatrix} = \begin{bmatrix}-1 & -0.5 \\ 1 & 0\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0.5 \\ 0\end{bmatrix}u, \qquad \begin{bmatrix}x_1(0) \\ x_2(0)\end{bmatrix} = \begin{bmatrix}0 \\ 0\end{bmatrix}$$

$$y = [1 \quad 0]\begin{bmatrix}x_1 \\ x_2\end{bmatrix}$$

where $u(t)$ is the unit-step input occurring at $t = 0$, or

$$u(t) = 1(t)$$

Solution. For this system

$$\mathbf{A} = \begin{bmatrix}-1 & -0.5 \\ 1 & 0\end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix}0.5 \\ 0\end{bmatrix}$$

The state transition matrix $\Phi(t) = e^{\mathbf{A}t}$ can be obtained as follows:

$$\Phi(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$$

0817

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10602

Since

$$(s\mathbf{I} - \mathbf{A})^{-1} = \begin{bmatrix} s+1 & 0.5 \\ -1 & s \end{bmatrix}^{-1} = \frac{1}{s^2 + s + 0.5}\begin{bmatrix} s & -0.5 \\ 1 & s+1 \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{s + 0.5 - 0.5}{(s+0.5)^2 + 0.5^2} & \dfrac{-0.5}{(s+0.5)^2 + 0.5^2} \\ \dfrac{1}{(s+0.5)^2 + 0.5^2} & \dfrac{s + 0.5 + 0.5}{(s+0.5)^2 + 0.5^2} \end{bmatrix}$$

we have

$$\mathbf{\Phi}(t) = e^{\mathbf{A}t} = \mathcal{L}^{-1}[(s\mathbf{I} - \mathbf{A})^{-1}]$$

$$= \begin{bmatrix} e^{-0.5t}(\cos 0.5t - \sin 0.5t) & -e^{-0.5t}\sin 0.5t \\ 2e^{-0.5t}\sin 0.5t & e^{-0.5t}(\cos 0.5t + \sin 0.5t) \end{bmatrix}$$

Since $\mathbf{x}(0) = 0$, referring to Equation (11–96), we have

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0) + \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I})\mathbf{B}k$$

$$= \mathbf{A}^{-1}(e^{\mathbf{A}t} - \mathbf{I})\mathbf{B}$$

$$= \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}\begin{bmatrix} 0.5e^{-0.5t}(\cos 0.5t - \sin 0.5t) - 0.5 \\ e^{-0.5t}\sin 0.5t \end{bmatrix}$$

$$= \begin{bmatrix} e^{-0.5t}\sin 0.5t \\ -e^{-0.5t}(\cos 0.5t + \sin 0.5t) + 1 \end{bmatrix}$$

Hence, the output $y(t)$ can be given by

$$y(t) = [1 \quad 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 = e^{-0.5t}\sin 0.5t$$

A–11–8. The Cayley–Hamilton theorem states that every $n \times n$ matrix $\mathbf{A}$ satisfies its own characteristic equation. The characteristic equation is not, however, necessarily the scalar equation of least degree that $\mathbf{A}$ satisfies. The least-degree polynomial having $\mathbf{A}$ as a root is called the *minimal polynomial*. That is, the minimal polynomial of an $n \times n$ matrix $\mathbf{A}$ is defined as the polynomial $\phi(\lambda)$ of least degree,

$$\phi(\lambda) = \lambda^m + a_1\lambda^{m-1} + \cdots + a_{m-1}\lambda + a_m, \qquad m \leq n$$

such that $\phi(\mathbf{A}) = \mathbf{0}$, or

$$\phi(\mathbf{A}) = \mathbf{A}^m + a_1\mathbf{A}^{m-1} + \cdots + a_{m-1}\mathbf{A} + a_m\mathbf{I} = \mathbf{0}$$

The minimal polynomial plays an important role in the computation of polynomials in an $n \times n$ matrix.

Let us suppose that $d(\lambda)$, a polynomial in λ, is the greatest common divisor of all the elements of $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$. Show that, if the coefficient of the highest-degree term in λ of $d(\lambda)$ is chosen as 1, then the minimal polynomial $\phi(\lambda)$ is given by

$$\phi(\lambda) = \left|\frac{\lambda\mathbf{I} - \mathbf{A}}{d(\lambda)}\right|$$

Solution. By assumption, the greatest common divisor of the matrix $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$ is $d(\lambda)$. Therefore,

$$\text{adj}(\lambda\mathbf{I} - \mathbf{A}) = d(\lambda)\mathbf{B}(\lambda)$$

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10603

where the greatest common divisor of the n^2 elements (which are functions of λ) of $\mathbf{B}(\lambda)$ is unity. Since

$$(\lambda\mathbf{I} - \mathbf{A})\,\text{adj}(\lambda\mathbf{I} - \mathbf{A}) = |\lambda\mathbf{I} - \mathbf{A}|\mathbf{I}$$

we obtain

$$d(\lambda)(\lambda\mathbf{I} - \mathbf{A})\mathbf{B}(\lambda) = |\lambda\mathbf{I} - \mathbf{A}|\mathbf{I} \tag{11–98}$$

from which we find that $|\lambda\mathbf{I} - \mathbf{A}|$ is divisible by $d(\lambda)$. Let us put

$$|\lambda\mathbf{I} - \mathbf{A}| = d(\lambda)\psi(\lambda) \tag{11–99}$$

Because the coefficient of the highest-degree term in λ of $d(\lambda)$ has been chosen to be 1, the coefficient of the highest-degree term in λ of $\psi(\lambda)$ is also 1. From Equations (11–98) and (11–99), we have

$$(\lambda\mathbf{I} - \mathbf{A})\mathbf{B}(\lambda) = \psi(\lambda)\mathbf{I}$$

Hence,

$$\psi(\mathbf{A}) = \mathbf{0}$$

Note that $\psi(\lambda)$ can be written as

$$\psi(\lambda) = g(\lambda)\phi(\lambda) + \alpha(\lambda)$$

where $\alpha(\lambda)$ is of lower degree than $\phi(\lambda)$. Since $\psi(\mathbf{A}) = \mathbf{0}$ and $\phi(\mathbf{A}) = \mathbf{0}$, we must have $\alpha(\mathbf{A}) = \mathbf{0}$. Also, since $\phi(\lambda)$ is the minimal polynomial, $\alpha(\lambda)$ must be identically zero, or

$$\psi(\lambda) = g(\lambda)\phi(\lambda)$$

Note that because $\phi(\mathbf{A}) = \mathbf{0}$, we can write

$$\phi(\lambda)\mathbf{I} = (\lambda\mathbf{I} - \mathbf{A})\mathbf{C}(\lambda)$$

Hence,

$$\psi(\lambda)\mathbf{I} = g(\lambda)\phi(\lambda)\mathbf{I} = g(\lambda)(\lambda\mathbf{I} - \mathbf{A})\mathbf{C}(\lambda)$$

and we obtain

$$\mathbf{B}(\lambda) = g(\lambda)\mathbf{C}(\lambda)$$

Note that the greatest common divisor of the n^2 elements of $\mathbf{B}(\lambda)$ is unity. Hence,

$$g(\lambda) = 1$$

Therefore,

$$\psi(\lambda) = \phi(\lambda)$$

Then, from this last equation and Equation (11–99), we obtain

$$\phi(\lambda) = \frac{|\lambda\mathbf{I} - \mathbf{A}|}{d(\lambda)}$$

A–11–9. If an $n \times n$ matrix $\mathbf{A}$ has n distinct eigenvalues, then the minimal polynomial of $\mathbf{A}$ is identical to the characteristic polynomial. Also, if the multiple eigenvalues of $\mathbf{A}$ are linked in a Jordan chain, the minimal polynomial and the characteristic polynomial are identical. If, however, the multiple eigenvalues of $\mathbf{A}$ are not linked in a Jordan chain, the minimal polynomial is of lower degree than the characteristic polynomial.

0819

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10604

Using the following matrices **A** and **B** as examples, verify the foregoing statements about the minimal polynomial when multiple eigenvalues are involved:

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

Solution. First, consider the matrix **A**. The characteristic polynomial is given by

$$|\lambda\mathbf{I} - \mathbf{A}| = \begin{vmatrix} \lambda - 2 & -1 & -4 \\ 0 & \lambda - 2 & 0 \\ 0 & -3 & \lambda - 1 \end{vmatrix} = (\lambda - 2)^2(\lambda - 1)$$

Thus, the eigenvalues of **A** are 2, 2, and 1. It can be shown that the Jordan canonical form of **A** is

$$\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and the multiple eigenvalues are linked in the Jordan chain as shown.

To determine the minimal polynomial, let us first obtain $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$. It is given by

$$\text{adj}(\lambda\mathbf{I} - \mathbf{A}) = \begin{bmatrix} (\lambda - 2)(\lambda - 1) & (\lambda + 11) & 4(\lambda - 2) \\ 0 & (\lambda - 2)(\lambda - 1) & 0 \\ 0 & 3(\lambda - 2) & (\lambda - 2)^2 \end{bmatrix}$$

Notice that there is no common divisor of all the elements of $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$. Hence, $d(\lambda) = 1$. Thus, the minimal polynomial $\phi(\lambda)$ is identical to the characteristic polynomial, or

$$\begin{aligned} \phi(\lambda) &= |\lambda\mathbf{I} - \mathbf{A}| = (\lambda - 2)^2(\lambda - 1) \\ &= \lambda^3 - 5\lambda^2 + 8\lambda - 4 \end{aligned}$$

A simple calculation proves that

$$\mathbf{A}^3 - 5\mathbf{A}^2 + 8\mathbf{A} - 4\mathbf{I} = \mathbf{0}$$

but

$$\mathbf{A}^2 - 3\mathbf{A} + 2\mathbf{I} \neq \mathbf{0}$$

Thus, we have shown that the minimal polynomial and the characteristic polynomial of this matrix **A** are the same.

Next, consider the matrix **B**. The characteristic polynomial is given by

$$|\lambda\mathbf{I} - \mathbf{B}| = \begin{vmatrix} \lambda - 2 & 0 & 0 \\ 0 & \lambda - 2 & 0 \\ 0 & -3 & \lambda - 1 \end{vmatrix} = (\lambda - 2)^2(\lambda - 1)$$

A simple computation reveals that matrix **B** has three eigenvectors, and the Jordan canonical form of **B** is given by

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10605

Thus, the multiple eigenvalues are not linked. To obtain the minimal polynomial, we first compute $\text{adj}(\lambda\mathbf{I} - \mathbf{B})$:

$$\text{adj}(\lambda\mathbf{I} - \mathbf{B}) = \begin{bmatrix} (\lambda - 2)(\lambda - 1) & 0 & 0 \\ 0 & (\lambda - 2)(\lambda - 1) & 0 \\ 0 & 3(\lambda - 2) & (\lambda - 2)^2 \end{bmatrix}$$

from which it is evident that

$$d(\lambda) = \lambda - 2$$

Hence,

$$\phi(\lambda) = \frac{|\lambda\mathbf{I} - \mathbf{B}|}{d(\lambda)} = \frac{(\lambda - 2)^2(\lambda - 1)}{\lambda - 2} = \lambda^2 - 3\lambda + 2$$

As a check, let us compute $\phi(\mathbf{B})$:

$$\phi(\mathbf{B}) = \mathbf{B}^2 - 3\mathbf{B} + 2\mathbf{I} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 9 & 1 \end{bmatrix} - 3\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

For the given matrix $\mathbf{B}$, the degree of the minimal polynomial is lower by 1 than that of the characteristic polynomial. As shown here, if the multiple eigenvalues of an $n \times n$ matrix are not linked in a Jordan chain, the minimal polynomial is of lower degree than the characteristic polynomial.

A–11–10. Show that by use of the minimal polynomial, the inverse of a nonsingular matrix $\mathbf{A}$ can be expressed as a polynomial in $\mathbf{A}$ with scalar coefficients as follows:

$$\mathbf{A}^{-1} = -\frac{1}{a_m}\left(\mathbf{A}^{m-1} + a_1\mathbf{A}^{m-2} + \cdots + a_{m-2}\mathbf{A} + a_{m-1}\mathbf{I}\right) \tag{11–100}$$

where $a_1, a_2, \ldots, a_m$ are coefficients of the minimal polynomial

$$\phi(\lambda) = \lambda^m + a_1\lambda^{m-1} + \cdots + a_{m-1}\lambda + a_m$$

Then obtain the inverse of the following matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & -2 \\ 1 & 0 & -3 \end{bmatrix}$$

Solution. For a nonsingular matrix $\mathbf{A}$, its minimal polynomial $\phi(\mathbf{A})$ can be written as

$$\phi(\mathbf{A}) = \mathbf{A}^m + a_1\mathbf{A}^{m-1} + \cdots + a_{m-1}\mathbf{A} + a_m\mathbf{I} = \mathbf{0}$$

where $a_m \neq 0$. Hence,

$$\mathbf{I} = -\frac{1}{a_m}\left(\mathbf{A}^m + a_1\mathbf{A}^{m-1} + \cdots + a_{m-2}\mathbf{A}^2 + a_{m-1}\mathbf{A}\right)$$

Premultiplying by $\mathbf{A}^{-1}$, we obtain

$$\mathbf{A}^{-1} = -\frac{1}{a_m}\left(\mathbf{A}^{m-1} + a_1\mathbf{A}^{m-2} + \cdots + a_{m-2}\mathbf{A} + a_{m-1}\mathbf{I}\right)$$

which is Equation (11–100).

0821

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10606

For the given matrix $\mathbf{A}$, $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$ can be given as

$$\text{adj}(\lambda\mathbf{I} - \mathbf{A}) = \begin{bmatrix} \lambda^2 + 4\lambda + 3 & 2\lambda + 6 & -4 \\ 3\lambda + 7 & \lambda^2 + 2\lambda - 3 & -2\lambda + 2 \\ \lambda + 1 & 2 & \lambda^2 - 7 \end{bmatrix}$$

Clearly, there is no common divisor $d(\lambda)$ of all elements of $\text{adj}(\lambda\mathbf{I} - \mathbf{A})$. Hence, $d(\lambda) = 1$. Consequently, the minimal polynomial $\phi(\lambda)$ is given by

$$\phi(\lambda) = \frac{|\lambda\mathbf{I} - \mathbf{A}|}{d(\lambda)} = |\lambda\mathbf{I} - \mathbf{A}|$$

Thus, the minimal polynomial $\phi(\lambda)$ is the same as the characteristic polynomial.

Since the characteristic polynomial is

$$|\lambda\mathbf{I} - \mathbf{A}| = \lambda^3 + 3\lambda^2 - 7\lambda - 17$$

we obtain

$$\phi(\lambda) = \lambda^3 + 3\lambda^2 - 7\lambda - 17$$

By identifying the coefficients a_i of the minimal polynomial (which is the same as the characteristic polynomial in this case), we have

$$a_1 = 3, \qquad a_2 = -7, \qquad a_3 = -17$$

The inverse of $\mathbf{A}$ can then be obtained from Equation (11–100) as follows:

$$\begin{aligned} \mathbf{A}^{-1} &= -\frac{1}{a_3}(\mathbf{A}^2 + a_1\mathbf{A} + a_2\mathbf{I}) = \frac{1}{17}(\mathbf{A}^2 + 3\mathbf{A} - 7\mathbf{I}) \\ &= \frac{1}{17}\left\{\begin{bmatrix} 7 & 0 & -4 \\ -2 & 7 & 8 \\ -2 & 2 & 9 \end{bmatrix} + 3\begin{bmatrix} 1 & 2 & 0 \\ 3 & -1 & -2 \\ 1 & 0 & -3 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right\} \\ &= \frac{1}{17}\begin{bmatrix} 3 & 6 & -4 \\ 7 & -3 & 2 \\ 1 & 2 & -7 \end{bmatrix} \\ &= \begin{bmatrix} \frac{3}{17} & \frac{6}{17} & -\frac{4}{17} \\ \frac{7}{17} & -\frac{3}{17} & \frac{2}{17} \\ \frac{1}{17} & \frac{2}{17} & -\frac{7}{17} \end{bmatrix} \end{aligned}$$

A–11–11. Show that if matrix $\mathbf{A}$ can be diagonalized, then

$$e^{\mathbf{A}t} = \mathbf{P}e^{\mathbf{D}t}\mathbf{P}^{-1}$$

where $\mathbf{P}$ is a diagonalizing transformation matrix that transforms $\mathbf{A}$ into a diagonal matrix, or $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix.

Show also that if matrix $\mathbf{A}$ can be transformed into a Jordan canonical form, then

$$e^{\mathbf{A}t} = \mathbf{S}e^{\mathbf{J}t}\mathbf{S}^{-1}$$

where $\mathbf{S}$ is a transformation matrix that transforms $\mathbf{A}$ into a Jordan canonical form $\mathbf{J}$, or $\mathbf{S}^{-1}\mathbf{A}\mathbf{S} = \mathbf{J}$, where $\mathbf{J}$ is in a Jordan canonical form.

Solution. Consider the state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

0822

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10607

If a square matrix can be diagonalized, then a diagonalizing matrix (transformation matrix) exists and it can be obtained by a standard method. Let $\mathbf{P}$ be a diagonalizing matrix for $\mathbf{A}$. Let us define

$$\mathbf{x} = \mathbf{P}\hat{\mathbf{x}}$$

Then

$$\dot{\hat{\mathbf{x}}} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P}\hat{\mathbf{x}} = \mathbf{D}\hat{\mathbf{x}}$$

where $\mathbf{D}$ is a diagonal matrix. The solution of this last equation is

$$\hat{\mathbf{x}}(t) = e^{\mathbf{D}t}\hat{\mathbf{x}}(0)$$

Hence,

$$\mathbf{x}(t) = \mathbf{P}\hat{\mathbf{x}}(t) = \mathbf{P}e^{\mathbf{D}t}\mathbf{P}^{-1}\mathbf{x}(0)$$

Noting that $\mathbf{x}(t)$ can also be given by the equation

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

we obtain $e^{\mathbf{A}t} = \mathbf{P}e^{\mathbf{D}t}\mathbf{P}^{-1}$, or

$$e^{\mathbf{A}t} = \mathbf{P}e^{\mathbf{D}t}\mathbf{P}^{-1} = \mathbf{P}\begin{bmatrix} e^{\lambda_1 t} & & & 0 \\ & e^{\lambda_2 t} & & \\ & & \ddots & \\ 0 & & & e^{\lambda_n t} \end{bmatrix}\mathbf{P}^{-1} \tag{11–101}$$

Next, we shall consider the case where matrix $\mathbf{A}$ may be transformed into a Jordan canonical form. Consider again the state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

First obtain a transformation matrix $\mathbf{S}$ that will transform matrix $\mathbf{A}$ into a Jordan canonical form so that

$$\mathbf{S}^{-1}\mathbf{A}\mathbf{S} = \mathbf{J}$$

where $\mathbf{J}$ is a matrix in a Jordan canonical form. Now define

$$\mathbf{x} = \mathbf{S}\hat{\mathbf{x}}$$

Then

$$\dot{\hat{\mathbf{x}}} = \mathbf{S}^{-1}\mathbf{A}\mathbf{S}\hat{\mathbf{x}} = \mathbf{J}\hat{\mathbf{x}}$$

The solution of this last equation is

$$\hat{\mathbf{x}}(t) = e^{\mathbf{J}t}\hat{\mathbf{x}}(0)$$

Hence,

$$\mathbf{x}(t) = \mathbf{S}\hat{\mathbf{x}}(t) = \mathbf{S}e^{\mathbf{J}t}\mathbf{S}^{-1}\mathbf{x}(0)$$

Since the solution $\mathbf{x}(t)$ can also be given by the equation

$$\mathbf{x}(t) = e^{\mathbf{A}t}\mathbf{x}(0)$$

we obtain

$$e^{\mathbf{A}t} = \mathbf{S}e^{\mathbf{J}t}\mathbf{S}^{-1}$$

0823

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10608

Note that $e^{\mathbf{J}t}$ is a triangular matrix [which means that the elements below (or above, as the case may be) the principal diagonal line are zeros] whose elements are $e^{\lambda t}, te^{\lambda t}, \frac{1}{2}t^2e^{\lambda t}$, and so forth. For example, if matrix $\mathbf{J}$ has the following Jordan canonical form:

$$\mathbf{J} = \begin{bmatrix} \lambda_1 & 1 & 0 \\ 0 & \lambda_1 & 1 \\ 0 & 0 & \lambda_1 \end{bmatrix}$$

then

$$e^{\mathbf{J}t} = \begin{bmatrix} e^{\lambda_1 t} & te^{\lambda_1 t} & \frac{1}{2}t^2e^{\lambda_1 t} \\ 0 & e^{\lambda_1 t} & te^{\lambda_1 t} \\ 0 & 0 & e^{\lambda_1 t} \end{bmatrix}$$

Similarly, if

$$\mathbf{J} = \left[\begin{array}{ccc:ccc:c} \lambda_1 & 1 & 0 & & & 0 \\ 0 & \lambda_1 & 1 & & & \\ 0 & 0 & \lambda_1 & & & \\ \hdashline & & & \lambda_4 & 1 & \\ & & & 0 & \lambda_4 & \\ \hdashline & & & & & \lambda_6 \\ 0 & & & & & \lambda_7 \end{array}\right]$$

then

$$e^{\mathbf{J}t} = \left[\begin{array}{ccc:ccc:c} e^{\lambda_1 t} & te^{\lambda_1 t} & \frac{1}{2}t^2e^{\lambda_1 t} & & & 0 \\ 0 & e^{\lambda_1 t} & te^{\lambda_1 t} & & & \\ 0 & 0 & e^{\lambda_1 t} & & & \\ \hdashline & & & e^{\lambda_4 t} & te^{\lambda_4 t} & \\ & & & 0 & e^{\lambda_4 t} & \\ \hdashline & & & & & e^{\lambda_6 t} \quad 0 \\ 0 & & & & & 0 \quad e^{\lambda_7 t} \end{array}\right]$$

A–11–12. Consider the following polynomial in λ of degree $m - 1$, where we assume $\lambda_1, \lambda_2, \ldots, \lambda_m$ to be distinct:

$$p_k(\lambda) = \frac{(\lambda - \lambda_1)\cdots(\lambda - \lambda_{k-1})(\lambda - \lambda_{k+1})\cdots(\lambda - \lambda_m)}{(\lambda_k - \lambda_1)\cdots(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})\cdots(\lambda_k - \lambda_m)}$$

where $k = 1, 2, \ldots, m$. Notice that

$$p_k(\lambda_i) = \begin{cases} 1, & \text{if } i = k \\ 0, & \text{if } i \neq k \end{cases}$$

Then the polynomial $f(\lambda)$ of degree $m - 1$,

$$\begin{aligned} f(\lambda) &= \sum_{k=1}^{m} f(\lambda_k)p_k(\lambda) \\ &= \sum_{k=1}^{m} f(\lambda_k)\frac{(\lambda - \lambda_1)\cdots(\lambda - \lambda_{k-1})(\lambda - \lambda_{k+1})\cdots(\lambda - \lambda_m)}{(\lambda_k - \lambda_1)\cdots(\lambda_k - \lambda_{k-1})(\lambda_k - \lambda_{k+1})\cdots(\lambda_k - \lambda_m)} \end{aligned}$$

takes on the values $f(\lambda_k)$ at the points λ_k. This last equation is commonly called *Lagrange's interpolation formula.* The polynomial $f(\lambda)$ of degree $m - 1$ is determined from m independent

0824

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10609

data $f(\lambda_1)$, $f(\lambda_2)$, ..., $f(\lambda_m)$. That is, the polynomial $f(\lambda)$ passes through m points $f(\lambda_1), f(\lambda_2), \ldots, f(\lambda_m)$. Since $f(\lambda)$ is a polynomial of degree $m-1$, it is uniquely determined. Any other representations of the polynomial of degree $m-1$ can be reduced to the Lagrange polynomial $f(\lambda)$.

Assuming that the eigenvalues of an $n \times n$ matrix $\mathbf{A}$ are distinct, substitute $\mathbf{A}$ for λ in the polynomial $p_k(\lambda)$. Then we get

$$p_k(\mathbf{A}) = \frac{(\mathbf{A}-\lambda_1\mathbf{I})\cdots(\mathbf{A}-\lambda_{k-1}\mathbf{I})(\mathbf{A}-\lambda_{k+1}\mathbf{I})\cdots(\mathbf{A}-\lambda_m\mathbf{I})}{(\lambda_k-\lambda_1)\cdots(\lambda_k-\lambda_{k-1})(\lambda_k-\lambda_{k+1})\cdots(\lambda_k-\lambda_m)}$$

Notice that $p_k(\mathbf{A})$ is a polynomial in $\mathbf{A}$ of degree $m-1$. Notice also that

$$p_k(\lambda_i\mathbf{I}) = \begin{cases} \mathbf{I}, & \text{if } i = k \\ \mathbf{0}, & \text{if } i \neq k \end{cases}$$

Now define

$$\begin{aligned} f(\mathbf{A}) &= \sum_{k=1}^{m} f(\lambda_k)p_k(\mathbf{A}) \\ &= \sum_{k=1}^{m} f(\lambda_k)\frac{(\mathbf{A}-\lambda_1\mathbf{I})\cdots(\mathbf{A}-\lambda_{k-1}\mathbf{I})(\mathbf{A}-\lambda_{k+1}\mathbf{I})\cdots(\mathbf{A}-\lambda_m\mathbf{I})}{(\lambda_k-\lambda_1)\cdots(\lambda_k-\lambda_{k-1})(\lambda_k-\lambda_{k+1})\cdots(\lambda_k-\lambda_m)} \end{aligned} \tag{11–102}$$

Equation (11–102) is known as Sylvester's interpolation formula. Equation (11–102) is equivalent to the following equation:

$$\begin{vmatrix} 1 & 1 & \cdots & 1 & \mathbf{I} \\ \lambda_1 & \lambda_2 & \cdots & \lambda_m & \mathbf{A} \\ \lambda_1^2 & \lambda_2^2 & \cdots & \lambda_m^2 & \mathbf{A}^2 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \lambda_1^{m-1} & \lambda_2^{m-1} & \cdots & \lambda_m^{m-1} & \mathbf{A}^{m-1} \\ f(\lambda_1) & f(\lambda_2) & \cdots & f(\lambda_m) & f(\mathbf{A}) \end{vmatrix} = \mathbf{0} \tag{11–103}$$

Equations (11–102) and (11–103) are frequently used for evaluating functions $f(\mathbf{A})$ of matrix $\mathbf{A}$, for example, $(\lambda\mathbf{I}-\mathbf{A})^{-1}$, $e^{\mathbf{A}t}$, and so forth. Note that Equation (11–103) can also be written as

$$\begin{vmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^{m-1} & f(\lambda_1) \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^{m-1} & f(\lambda_2) \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 1 & \lambda_m & \lambda_m^2 & \cdots & \lambda_m^{m-1} & f(\lambda_m) \\ \mathbf{I} & \mathbf{A} & \mathbf{A}^2 & \cdots & \mathbf{A}^{m-1} & f(\mathbf{A}) \end{vmatrix} = \mathbf{0} \tag{11–104}$$

Show that Equations (11–102) and (11–103) are equivalent. To simplify the arguments, assume that $m = 4$.

0825

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10610

Solution. Equation (11–103), where $m = 4$, can be expanded as follows:

$$\Delta = \begin{vmatrix} 1 & 1 & 1 & 1 & \mathbf{I} \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 & \mathbf{A} \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \mathbf{A}^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & \lambda_4^3 & \mathbf{A}^3 \\ f(\lambda_1) & f(\lambda_2) & f(\lambda_3) & f(\lambda_4) & f(\mathbf{A}) \end{vmatrix}$$

$$= f(\mathbf{A}) \begin{vmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & \lambda_4^3 \end{vmatrix} - f(\lambda_4) \begin{vmatrix} 1 & 1 & 1 & \mathbf{I} \\ \lambda_1 & \lambda_2 & \lambda_3 & \mathbf{A} \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \mathbf{A}^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & \mathbf{A}^3 \end{vmatrix}$$

$$+ f(\lambda_3) \begin{vmatrix} 1 & 1 & 1 & \mathbf{I} \\ \lambda_1 & \lambda_2 & \lambda_4 & \mathbf{A} \\ \lambda_1^2 & \lambda_2^2 & \lambda_4^2 & \mathbf{A}^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_4^3 & \mathbf{A}^3 \end{vmatrix} - f(\lambda_2) \begin{vmatrix} 1 & 1 & 1 & \mathbf{I} \\ \lambda_1 & \lambda_3 & \lambda_4 & \mathbf{A} \\ \lambda_1^2 & \lambda_3^2 & \lambda_4^2 & \mathbf{A}^2 \\ \lambda_1^3 & \lambda_3^3 & \lambda_4^3 & \mathbf{A}^3 \end{vmatrix}$$

$$+ f(\lambda_1) \begin{vmatrix} 1 & 1 & 1 & \mathbf{I} \\ \lambda_2 & \lambda_3 & \lambda_4 & \mathbf{A} \\ \lambda_2^2 & \lambda_3^2 & \lambda_4^2 & \mathbf{A}^2 \\ \lambda_2^3 & \lambda_3^3 & \lambda_4^3 & \mathbf{A}^3 \end{vmatrix}$$

Since

$$\begin{vmatrix} 1 & 1 & 1 & 1 \\ \lambda_1 & \lambda_2 & \lambda_3 & \lambda_4 \\ \lambda_1^2 & \lambda_2^2 & \lambda_3^2 & \lambda_4^2 \\ \lambda_1^3 & \lambda_2^3 & \lambda_3^3 & \lambda_4^3 \end{vmatrix} = (\lambda_4 - \lambda_3)(\lambda_4 - \lambda_2)(\lambda_4 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)$$

and

$$\begin{vmatrix} 1 & 1 & 1 & \mathbf{I} \\ \lambda_i & \lambda_j & \lambda_k & \mathbf{A} \\ \lambda_i^2 & \lambda_j^2 & \lambda_k^2 & \mathbf{A}^2 \\ \lambda_i^3 & \lambda_j^3 & \lambda_k^3 & \mathbf{A}^3 \end{vmatrix} = (\mathbf{A} - \lambda_k \mathbf{I})(\mathbf{A} - \lambda_j \mathbf{I})(\mathbf{A} - \lambda_i \mathbf{I})(\lambda_k - \lambda_j)(\lambda_k - \lambda_i)(\lambda_j - \lambda_i)$$

we obtain

$$\begin{aligned} \Delta &= f(\mathbf{A})[(\lambda_4 - \lambda_3)(\lambda_4 - \lambda_2)(\lambda_4 - \lambda_1)(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)] \\ &\quad - f(\lambda_4)[(\mathbf{A} - \lambda_3\mathbf{I})(\mathbf{A} - \lambda_2\mathbf{I})(\mathbf{A} - \lambda_1\mathbf{I})(\lambda_3 - \lambda_2)(\lambda_3 - \lambda_1)(\lambda_2 - \lambda_1)] \\ &\quad + f(\lambda_3)[(\mathbf{A} - \lambda_4\mathbf{I})(\mathbf{A} - \lambda_2\mathbf{I})(\mathbf{A} - \lambda_1\mathbf{I})(\lambda_4 - \lambda_2)(\lambda_4 - \lambda_1)(\lambda_2 - \lambda_1)] \\ &\quad - f(\lambda_2)[(\mathbf{A} - \lambda_4\mathbf{I})(\mathbf{A} - \lambda_3\mathbf{I})(\mathbf{A} - \lambda_1\mathbf{I})(\lambda_4 - \lambda_3)(\lambda_4 - \lambda_1)(\lambda_3 - \lambda_1)] \\ &\quad + f(\lambda_1)[(\mathbf{A} - \lambda_4\mathbf{I})(\mathbf{A} - \lambda_3\mathbf{I})(\mathbf{A} - \lambda_2\mathbf{I})(\lambda_4 - \lambda_3)(\lambda_4 - \lambda_2)(\lambda_3 - \lambda_2)] \\ &= \mathbf{0} \end{aligned}$$

0826

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10611

Solving this last equation for $f(\mathbf{A})$, we obtain

$$f(\mathbf{A}) = f(\lambda_1)\frac{(\mathbf{A}-\lambda_2\mathbf{I})(\mathbf{A}-\lambda_3\mathbf{I})(\mathbf{A}-\lambda_4\mathbf{I})}{(\lambda_1-\lambda_2)(\lambda_1-\lambda_3)(\lambda_1-\lambda_4)} + f(\lambda_2)\frac{(\mathbf{A}-\lambda_1\mathbf{I})(\mathbf{A}-\lambda_3\mathbf{I})(\mathbf{A}-\lambda_4\mathbf{I})}{(\lambda_2-\lambda_1)(\lambda_2-\lambda_3)(\lambda_2-\lambda_4)}$$
$$+ f(\lambda_3)\frac{(\mathbf{A}-\lambda_1\mathbf{I})(\mathbf{A}-\lambda_2\mathbf{I})(\mathbf{A}-\lambda_4\mathbf{I})}{(\lambda_3-\lambda_1)(\lambda_3-\lambda_2)(\lambda_3-\lambda_4)} + f(\lambda_4)\frac{(\mathbf{A}-\lambda_1\mathbf{I})(\mathbf{A}-\lambda_2\mathbf{I})(\mathbf{A}-\lambda_3\mathbf{I})}{(\lambda_4-\lambda_1)(\lambda_4-\lambda_2)(\lambda_4-\lambda_3)}$$
$$= \sum_{k=1}^{m} f(\lambda_k)\frac{(\mathbf{A}-\lambda_1\mathbf{I})\cdots(\mathbf{A}-\lambda_{k-1}\mathbf{I})(\mathbf{A}-\lambda_{k+1}\mathbf{I})\cdots(\mathbf{A}-\lambda_m\mathbf{I})}{(\lambda_k-\lambda_1)\cdots(\lambda_k-\lambda_{k-1})(\lambda_k-\lambda_{k+1})\cdots(\lambda_k-\lambda_m)}$$

where $m = 4$. Thus, we have shown the equivalence of Equations (11–102) and (11–103). Although we assumed $m = 4$, the entire argument can be extended to an arbitrary positive integer m. (For the case when the matrix A involves multiple eigenvalues, refer to Problem **A–11–13**.)

A–11–13. Consider Sylvester's interpolation formula in the form given by Equation (11–104):

$$\begin{vmatrix} 1 & \lambda_1 & \lambda_1^2 & \cdots & \lambda_1^{m-1} & f(\lambda_1) \\ 1 & \lambda_2 & \lambda_2^2 & \cdots & \lambda_2^{m-1} & f(\lambda_2) \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 1 & \lambda_m & \lambda_m^2 & \cdots & \lambda_m^{m-1} & f(\lambda_m) \\ \mathbf{I} & \mathbf{A} & \mathbf{A}^2 & \cdots & \mathbf{A}^{m-1} & f(\mathbf{A}) \end{vmatrix} = \mathbf{0}$$

This formula for the determination of $f(\mathbf{A})$ applies to the case where the minimal polynomial of $\mathbf{A}$ involves only distinct roots.

Suppose that the minimal polynomial of $\mathbf{A}$ involves multiple roots. Then the rows in the determinant that correspond to the multiple roots become identical, and therefore modification of the determinant in Equation (11–104) becomes necessary.

Modify the form of Sylvester's interpolation formula given by Equation (11–104) when the minimal polynomial of $\mathbf{A}$ involves multiple roots. In deriving a modified determinant equation, assume that there are three equal roots $(\lambda_1 = \lambda_2 = \lambda_3)$ in the minimal polynomial of $\mathbf{A}$ and that there are other roots $(\lambda_4, \lambda_5, \ldots, \lambda_m)$ that are distinct.

Solution. Since the minimal polynomial of $\mathbf{A}$ involves three equal roots, the minimal polynomial $\phi(\lambda)$ can be written as

$$\phi(\lambda) = \lambda^m + a_1\lambda^{m-1} + \cdots + a_{m-1}\lambda + a_m$$
$$= (\lambda-\lambda_1)^3(\lambda-\lambda_4)(\lambda-\lambda_5)\cdots(\lambda-\lambda_m)$$

An arbitrary function $f(\mathbf{A})$ of an $n \times n$ matrix $\mathbf{A}$ can be written as

$$f(\mathbf{A}) = g(\mathbf{A})\phi(\mathbf{A}) + \alpha(\mathbf{A})$$

where the minimal polynomial $\phi(\mathbf{A})$ is of degree m and $\alpha(\mathbf{A})$ is a polynomial in $\mathbf{A}$ of degree $m-1$ or less. Hence we have

$$f(\lambda) = g(\lambda)\phi(\lambda) + \alpha(\lambda)$$

where $\alpha(\lambda)$ is a polynomial in λ of degree $m-1$ or less, which can thus be written as

$$\alpha(\lambda) = \alpha_0 + \alpha_1\lambda + \alpha_2\lambda^2 + \cdots + \alpha_{m-1}\lambda^{m-1} \qquad (11\text{–}105)$$

0827

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10612

In the present case we have

$$\begin{aligned} f(\lambda) &= g(\lambda)\phi(\lambda) + \alpha(\lambda) \\ &= g(\lambda)\left[(\lambda - \lambda_1)^3(\lambda - \lambda_4)\cdots(\lambda - \lambda_m)\right] + \alpha(\lambda) \end{aligned} \tag{11–106}$$

By substituting $\lambda_1, \lambda_4, \ldots, \lambda_m$ for λ in Equation (11–106), we obtain the following $m - 2$ equations:

$$\begin{aligned} f(\lambda_1) &= \alpha(\lambda_1) \\ f(\lambda_4) &= \alpha(\lambda_4) \\ &\vdots \\ f(\lambda_m) &= \alpha(\lambda_m) \end{aligned} \tag{11–107}$$

By differentiating Equation (11–106) with respect to λ, we obtain

$$\frac{d}{d\lambda} f(\lambda) = (\lambda - \lambda_1)^2 h(\lambda) + \frac{d}{d\lambda}\alpha(\lambda) \tag{11–108}$$

where

$$(\lambda - \lambda_1)^2 h(\lambda) = \frac{d}{d\lambda}\left[g(\lambda)(\lambda - \lambda_1)^3(\lambda - \lambda_4)\cdots(\lambda - \lambda_m)\right]$$

Substitution of λ_1 for λ in Equation (11–108) gives

$$\left.\frac{d}{d\lambda} f(\lambda)\right|_{\lambda=\lambda_1} = f'(\lambda_1) = \left.\frac{d}{d\lambda}\alpha(\lambda)\right|_{\lambda=\lambda_1}$$

Referring to Equation (11–105), this last equation becomes

$$f'(\lambda_1) = \alpha_1 + 2\alpha_2\lambda_1 + \cdots + (m-1)\alpha_{m-1}\lambda_1^{m-2} \tag{11–109}$$

Similarly, differentiating Equation (11–106) twice with respect to λ and substituting λ_1 for λ, we obtain

$$\left.\frac{d^2}{d^2\lambda} f(\lambda)\right|_{\lambda=\lambda_1} = f''(\lambda_1) = \left.\frac{d^2}{d\lambda^2}\alpha(\lambda)\right|_{\lambda=\lambda_1}$$

This last equation can be written as

$$f''(\lambda_1) = 2\alpha_2 + 6\alpha_3\lambda_1 + \cdots + (m-1)(m-2)\alpha_{m-1}\lambda_1^{m-3} \tag{11–110}$$

Rewriting Equations (11–110), (11–109), and (11–107), we get

$$\begin{aligned} \alpha_2 + 3\alpha_3\lambda_1 + \cdots + \frac{(m-1)(m-2)}{2}\alpha_{m-1}\lambda_1^{m-3} &= \frac{f''(\lambda_1)}{2} \\ \alpha_1 + 2\alpha_2\lambda_1 + \cdots + (m-1)\alpha_{m-1}\lambda_1^{m-2} &= f'(\lambda_1) \\ \alpha_0 + \alpha_1\lambda_1 + \alpha_2\lambda_1^2 + \cdots + \alpha_{m-1}\lambda_1^{m-1} &= f(\lambda_1) \\ \alpha_0 + \alpha_1\lambda_4 + \alpha_2\lambda_4^2 + \cdots + \alpha_{m-1}\lambda_4^{m-1} &= f(\lambda_4) \\ &\vdots \\ \alpha_0 + \alpha_1\lambda_m + \alpha_2\lambda_m^2 + \cdots + \alpha_{m-1}\lambda_m^{m-1} &= f(\lambda_m) \end{aligned} \tag{11–111}$$

0828

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10613

These m simultaneous equations determine the α_k values (where $k = 0, 1, 2, \ldots, m-1$). Noting that $\phi(\mathbf{A}) = \mathbf{0}$ because it is a minimal polynomial, we have $f(\mathbf{A})$ as follows:

$$f(\mathbf{A}) = g(\mathbf{A})\phi(\mathbf{A}) + \alpha(\mathbf{A}) = \alpha(\mathbf{A})$$

Hence, referring to Equation (11–105), we have

$$f(\mathbf{A}) = \alpha(\mathbf{A}) = \alpha_0\mathbf{I} + \alpha_1\mathbf{A} + \alpha_2\mathbf{A}^2 + \cdots + \alpha_{m-1}\mathbf{A}^{m-1} \tag{11–112}$$

where the α_k values are given in terms of $f(\lambda_1), f'(\lambda_1), f''(\lambda_1), f(\lambda_4), f(\lambda_5), \ldots, f(\lambda_m)$. In terms of the determinant equation, $f(\mathbf{A})$ can be obtained by solving the following equation:

$$\begin{vmatrix}
0 & 0 & 1 & 3\lambda_1 & \cdots & \dfrac{(m-1)(m-2)}{2}\lambda_1^{m-3} & \dfrac{f''(\lambda_1)}{2} \\
0 & 1 & 2\lambda_1 & 3\lambda_1^2 & \cdots & (m-1)\lambda_1^{m-2} & f'(\lambda_1) \\
1 & \lambda_1 & \lambda_1^2 & \lambda_1^3 & \cdots & \lambda_1^{m-1} & f(\lambda_1) \\
1 & \lambda_4 & \lambda_4^2 & \lambda_4^3 & \cdots & \lambda_4^{m-1} & f(\lambda_4) \\
\cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\
\cdot & \cdot & \cdot & \cdot & & \cdot & \cdot \\
1 & \lambda_m & \lambda_m^2 & \lambda_m^3 & \cdots & \lambda_m^{m-1} & f(\lambda_m) \\
\mathbf{I} & \mathbf{A} & \mathbf{A}^2 & \mathbf{A}^3 & \cdots & \mathbf{A}^{m-1} & f(\mathbf{A})
\end{vmatrix} = \mathbf{0} \tag{11–113}$$

Equation (11–113) shows the desired modification in the form of the determinant. This equation gives the form of Sylvester's interpolation formula when the minimal polynomial of $\mathbf{A}$ involves three equal roots. (The necessary modification of the form of the determinant for other cases will be apparent.)

A–11–14. Using Sylvester's interpolation formula, compute $e^{\mathbf{A}t}$, where

$$\mathbf{A} = \begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$

Solution. Referring to Problem **A–11–9**, the characteristic polynomial and the minimal polynomial are the same for this $\mathbf{A}$. The minimal polynomial (characteristic polynomial) is given by

$$\phi(\lambda) = (\lambda - 2)^2(\lambda - 1)$$

Note that $\lambda_1 = \lambda_2 = 2$ and $\lambda_3 = 1$. Referring to Equation (11–112) and noting that $f(\mathbf{A})$ in this problem is $e^{\mathbf{A}t}$, we have

$$e^{\mathbf{A}t} = \alpha_0(t)\mathbf{I} + \alpha_1(t)\mathbf{A} + \alpha_2(t)\mathbf{A}^2$$

where $\alpha_0(t), \alpha_1(t)$, and $\alpha_2(t)$ are determined from the equations

$$\begin{aligned}
\alpha_1(t) + 2\alpha_2(t)\lambda_1 &= te^{\lambda_1 t} \\
\alpha_0(t) + \alpha_1(t)\lambda_1 + \alpha_2(t)\lambda_1^2 &= e^{\lambda_1 t} \\
\alpha_0(t) + \alpha_1(t)\lambda_3 + \alpha_2(t)\lambda_3^2 &= e^{\lambda_3 t}
\end{aligned}$$

0829

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10614

Substituting $\lambda_1 = 2$, and $\lambda_3 = 1$ into these three equations gives

$$\alpha_1(t) + 4\alpha_2(t) = te^{2t}$$
$$\alpha_0(t) + 2\alpha_1(t) + 4\alpha_2(t) = e^{2t}$$
$$\alpha_0(t) + \alpha_1(t) + \alpha_2(t) = e^{t}$$

Solving for $\alpha_0(t)$, $\alpha_1(t)$, and $\alpha_2(t)$, we obtain

$$\alpha_0(t) = 4e^{t} - 3e^{2t} + 2te^{2t}$$
$$\alpha_1(t) = -4e^{t} + 4e^{2t} - 3te^{2t}$$
$$\alpha_2(t) = e^{t} - e^{2t} + te^{2t}$$

Hence,

$$e^{\mathbf{A}t} = \left(4e^{t} - 3e^{2t} + 2te^{2t}\right)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \left(-4e^{t} + 4e^{2t} - 3te^{2t}\right)\begin{bmatrix} 2 & 1 & 4 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix}$$
$$+ \left(e^{t} - e^{2t} + te^{2t}\right)\begin{bmatrix} 4 & 16 & 12 \\ 0 & 4 & 0 \\ 0 & 9 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} e^{2t} & 12e^{t} - 12e^{2t} + 13te^{2t} & -4e^{t} + 4e^{2t} \\ 0 & e^{2t} & 0 \\ 0 & -3e^{t} + 3e^{2t} & e^{t} \end{bmatrix}$$

A–11–15. Consider an $n \times n$ matrix $\mathbf{A}$. Show that

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\sum_{j=0}^{m-1} s^j \sum_{i=1+j}^{m} \alpha_{m-i}\mathbf{A}^{i-j-1}}{\sum_{i=0}^{m} \alpha_{m-i}s^i}$$

where the α_i's are coefficients of the minimal polynomial of $\mathbf{A}$:

$$\alpha_0\mathbf{A}^m + \alpha_1\mathbf{A}^{m-1} + \cdots + \alpha_{m-1}\mathbf{A} + \alpha_m\mathbf{I} = \mathbf{0}$$

where $\alpha_0 = 1$ and m is the degree of minimal polynomial ($m \leq n$).

Solution. Let us put

$$\mathbf{P} = (s\mathbf{I} - \mathbf{A})^{-1}$$

Then

$$s\mathbf{P} = \mathbf{AP} + \mathbf{I}$$

By premultiplying both sides of this equation by $(s\mathbf{I} + \mathbf{A})$, we obtain

$$s^2\mathbf{P} = \mathbf{A}^2\mathbf{P} + \mathbf{A} + s\mathbf{I}$$

Similarly, by premultiplying both sides of this last equation by $(s\mathbf{I} + \mathbf{A})$, we obtain

$$s^3\mathbf{P} = \mathbf{A}^3\mathbf{P} + \mathbf{A}^2 + s\mathbf{A} + s^2\mathbf{I}$$

0830

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10615

By repeating this process, we obtain the following set of equations:

$$\mathbf{P} = \mathbf{P}$$
$$s\mathbf{P} = \mathbf{A}\mathbf{P} + \mathbf{I}$$
$$s^2\mathbf{P} = \mathbf{A}^2\mathbf{P} + \mathbf{A} + s\mathbf{I}$$
$$s^3\mathbf{P} = \mathbf{A}^3\mathbf{P} + \mathbf{A}^2 + s\mathbf{A} + s^2\mathbf{I}$$
$$\vdots$$
$$s^m\mathbf{P} = \mathbf{A}^m\mathbf{P} + \mathbf{A}^{m-1} + s\mathbf{A}^{m-2} + \cdots + s^{m-2}\mathbf{A} + s^{m-1}\mathbf{I}$$

where m is the degree of the minimal polynomial of $\mathbf{A}$. Then, by multiplying the $s^i\mathbf{P}$'s by α_{m-i} (where $i = 0, 1, 2, \ldots, m$) in the preceding $m + 1$ equations in the order given and adding the product together, we get

$$\begin{aligned}\alpha_m\mathbf{P} + \alpha_{m-1}s\mathbf{P} + \alpha_{m-2}s^2\mathbf{P} + \cdots + \alpha_0 s^m\mathbf{P} \\ = \sum_{i=0}^{m}\alpha_{m-i}\mathbf{A}^i\mathbf{P} + \sum_{i=1}^{m}\alpha_{m-i}\mathbf{A}^{i-1} + s\sum_{i=2}^{m}\alpha_{m-i}\mathbf{A}^{i-2} + \cdots \\ + s^{m-2}\sum_{i=m-1}^{m}\alpha_{m-i}\mathbf{A}^{i-m+1} + s^{m-1}\alpha_0\mathbf{I}\end{aligned} \tag{11–114}$$

Noting that

$$\sum_{i=0}^{m}\alpha_{m-i}\mathbf{A}^i\mathbf{P} = (\alpha_0\mathbf{A}^m + \alpha_1\mathbf{A}^{m-1} + \cdots + \alpha_{m-1}\mathbf{A} + \alpha_m\mathbf{I})\mathbf{P} = \mathbf{0}$$

we can simplify Equation (11–114) as follows:

$$\sum_{i=0}^{m}\alpha_{m-i}s^i\mathbf{P} = \sum_{j=0}^{m-1}s^j\sum_{i=1+j}^{m}\alpha_{m-i}\mathbf{A}^{i-j-1}$$

Therefore,

$$(s\mathbf{I} - \mathbf{A})^{-1} = \mathbf{P} = \frac{\sum_{j=0}^{m-1}s^j\sum_{i=1+j}^{m}\alpha_{m-i}\mathbf{A}^{i-j-1}}{\sum_{i=0}^{m}\alpha_{m-i}s^i}$$

If the minimal polynomial and characteristic polynomial of $\mathbf{A}$ are identical, then $m = n$. If $m = n$, then this last equation becomes

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{\sum_{j=0}^{n-1}s^j\sum_{i=1+j}^{n}\alpha_{n-i}\mathbf{A}^{i-j-1}}{|s\mathbf{I} - \mathbf{A}|} \tag{11–115}$$

where

$$|s\mathbf{I} - \mathbf{A}| = \sum_{i=0}^{n}\alpha_{n-i}s^i, \qquad \alpha_0 = 1$$

A–11–16. Show that the system described by

0831

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10616

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{11–116}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} \tag{11–117}$$

where $\mathbf{x}$ = state vector (n-vector)

$\mathbf{u}$ = control vector (r-vector)

$\mathbf{y}$ = output vector (m-vector) $(m \leq n)$

$\mathbf{A} = n \times n$ matrix

$\mathbf{B} = n \times r$ matrix

$\mathbf{C} = m \times n$ matrix

is completely output controllable if and only if the composite $m \times nr$ matrix $\mathbf{P}$, where

$$\mathbf{P} = [\mathbf{CB} \mid \mathbf{CAB} \mid \mathbf{CA}^2\mathbf{B} \mid \cdots \mid \mathbf{CA}^{n-1}\mathbf{B}]$$

is of rank m. (Notice that complete state controllability is neither necessary nor sufficient for complete output controllability.)

Solution. Suppose that the system is output controllable and the output $\mathbf{y}(t)$ starting from any $\mathbf{y}(0)$, the initial output, can be transferred to the origin of the output space in a finite time interval $0 \leq t \leq T$. That is,

$$\mathbf{y}(T) = \mathbf{C}\mathbf{x}(T) = \mathbf{0} \tag{11–118}$$

Since the solution of Equation (11–116) is

$$\mathbf{x}(t) = e^{\mathbf{A}t}\left[\mathbf{x}(0) + \int_0^t e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)\,d\tau\right]$$

at $t = T$, we have

$$\mathbf{x}(T) = e^{\mathbf{A}T}\left[\mathbf{x}(0) + \int_0^T e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)\,d\tau\right] \tag{11–119}$$

Substituting Equation (11–119) into Equation (11–118), we obtain

$$\begin{aligned}\mathbf{y}(T) &= \mathbf{C}\mathbf{x}(T) \\ &= \mathbf{C}e^{\mathbf{A}T}\left[\mathbf{x}(0) + \int_0^T e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)\,d\tau\right] = \mathbf{0}\end{aligned} \tag{11–120}$$

On the other hand, $\mathbf{y}(0) = \mathbf{C}\mathbf{x}(0)$. Notice that the complete output controllability means that the vector $\mathbf{C}\mathbf{x}(0)$ spans the m-dimensional output space. Since $e^{\mathbf{A}T}$ is nonsingular, if $\mathbf{C}\mathbf{x}(0)$ spans the m-dimensional output space, so does $\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0)$, and vice versa. From Equation (11–120) we obtain

$$\begin{aligned}\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0) &= -\mathbf{C}e^{\mathbf{A}T}\int_0^T e^{-\mathbf{A}\tau}\mathbf{B}\mathbf{u}(\tau)\,d\tau \\ &= -\mathbf{C}\int_0^T e^{\mathbf{A}\tau}\mathbf{B}\mathbf{u}(T-\tau)\,d\tau\end{aligned}$$

Note that $\int_0^T e^{\mathbf{A}\tau}\mathbf{B}\mathbf{u}(T-\tau)\,d\tau$ can be expressed as the sum of $\mathbf{A}^i\mathbf{B}_j$; that is,

$$\int_0^T e^{\mathbf{A}\tau}\mathbf{B}\mathbf{u}(T-\tau)\,d\tau = \sum_{i=0}^{n-1}\sum_{j=1}^{r}\gamma_{ij}\mathbf{A}^i\mathbf{B}_j$$

0832

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10617

where

$$\gamma_{ij} = \int_0^T \alpha_i(\tau) u_j(T - \tau)\, d\tau = \text{scalar}$$

and $\alpha_i(\tau)$ satisfies

$$e^{\mathbf{A}\tau} = \sum_{i=0}^{p-1} \alpha_i(\tau)\mathbf{A}^i \qquad (p\text{: degree of the minimal polynomial of } \mathbf{A})$$

and $\mathbf{B}_j$ is the jth column of $\mathbf{B}$. Therefore, we can write $\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0)$ as

$$\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0) = -\sum_{i=0}^{p-1}\sum_{j=1}^{r} \gamma_{ij}\mathbf{C}\mathbf{A}^i\mathbf{B}_j$$

From this last equation, we see that $\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0)$ is a linear combination of $\mathbf{C}\mathbf{A}^i\mathbf{B}_j (i = 0, 1, 2, \ldots, p - 1; j = 1, 2, \ldots, r)$. Note that if the rank of $\mathbf{Q}$, where

$$\mathbf{Q} = [\mathbf{C}\mathbf{B} \mid \mathbf{C}\mathbf{A}\mathbf{B} \mid \mathbf{C}\mathbf{A}^2\mathbf{B} \mid \cdots \mid \mathbf{C}\mathbf{A}^{p-1}\mathbf{B}] \qquad (p \le n)$$

is m, then so is the rank of $\mathbf{P}$, and vice versa. [This is obvious if $p = n$. If $p < n$, then the $\mathbf{C}\mathbf{A}^h\mathbf{B}_j$ (where $p \le h \le n - 1$) are linearly dependent on $\mathbf{C}\mathbf{B}_j, \mathbf{C}\mathbf{A}\mathbf{B}_j, \ldots, \mathbf{C}\mathbf{A}^{p-1}\mathbf{B}_j$. Hence, the rank of $\mathbf{P}$ is equal to that of $\mathbf{Q}$.] If the rank of $\mathbf{P}$ is m, then $\mathbf{C}e^{\mathbf{A}T}\mathbf{x}(0)$ spans the m-dimensional output space. This means that if the rank of $\mathbf{P}$ is m, then $\mathbf{C}\mathbf{x}(0)$ also spans the m-dimensional output space and the system is completely output controllable.

Conversely, suppose that the system is completely output controllable, but the rank of $\mathbf{P}$ is k, where $k < m$. Then the set of all initial outputs that can be transferred to the origin is of k-dimensional space. Hence, the dimension of this set is less than m. This contradicts the assumption that the system is completely output controllable. This completes the proof.

Note that it can be immediately proved that, in the system of Equations (11–116) and (11–117), complete state controllability on $0 \le t \le T$ implies complete output controllability on $0 \le t \le T$ if and only if m rows of $\mathbf{C}$ are linearly independent.

A–11–17. Discuss the state controllability of the following system:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -3 & 1 \\ -2 & 1.5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 4 \end{bmatrix} u \tag{11–121}$$

Solution. For this system,

$$\mathbf{A} = \begin{bmatrix} -3 & 1 \\ -2 & 1.5 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

Since

$$\mathbf{A}\mathbf{B} = \begin{bmatrix} -3 & 1 \\ -2 & 1.5 \end{bmatrix}\begin{bmatrix} 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

we see that vectors $\mathbf{B}$ and $\mathbf{A}\mathbf{B}$ are not linearly independent and the rank of the matrix $[\mathbf{B} \mid \mathbf{A}\mathbf{B}]$ is 1. Therefore, the system is not completely state controllable. In fact, elimination of x_2 from Equation (11–121), or the following two simultaneous equations,

$$\dot{x}_1 = -3x_1 + x_2 + u$$
$$\dot{x}_2 = -2x_1 + 1.5x_2 + 4u$$

yields

$$\ddot{x}_1 + 1.5\dot{x}_1 - 2.5x_1 = \dot{u} + 2.5u$$

0833

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10618

or, in the form of a transfer function,

$$\frac{X_1(s)}{U(s)} = \frac{s + 2.5}{(s + 2.5)(s - 1)}$$

Notice that cancellation of the factor $(s + 2.5)$ occurs in the numerator and denominator of the transfer function. Because of this cancellation, this system is not completely state controllable. This is an unstable system. Remember that stability and controllability are quite different things. There are many systems that are unstable, but are completely state controllable.

A–11–18. A state-space representation of a system in the controllable canonical form is given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.4 & -1.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \tag{11–122}$$

$$y = [0.8 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11–123}$$

The same system may be represented by the following state-space equation, which is in the observable canonical form:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -0.4 \\ 1 & -1.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.8 \\ 1 \end{bmatrix} u \tag{11–124}$$

$$y = [0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \tag{11–125}$$

Show that the state-space representation given by Equations (11–122) and (11–123) gives a system that is state controllable, but not observable. Show, on the other hand, that the state-space representation defined by Equations (11–124) and (11–125) gives a system that is not completely state controllable, but is observable. Explain what causes the apparent difference in the controllability and observability of the same system.

Solution. Consider the system defined by Equations (11–122) and (11–123). The rank of the controllability matrix

$$[\mathbf{B} \mid \mathbf{AB}] = \begin{bmatrix} 0 & 1 \\ 1 & -1.3 \end{bmatrix}$$

is 2. Hence, the system is completely state controllable. The rank of the observability matrix

$$[\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^*] = \begin{bmatrix} 0.8 & -0.4 \\ 1 & -0.5 \end{bmatrix}$$

is 1. Hence the system is not observable.

Next consider the system defined by Equations (11–124) and (11–125). The rank of the controllability matrix

$$[\mathbf{B} \mid \mathbf{AB}] = \begin{bmatrix} 0.8 & -0.4 \\ 1 & -0.5 \end{bmatrix}$$

is 1. Hence, the system is not completely state controllable. The rank of the observability matrix

$$[\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^*] = \begin{bmatrix} 0 & 1 \\ 1 & -1.3 \end{bmatrix}$$

is 2. Hence, the system is observable.

0834

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10619

The apparent difference in the controllability and observability of the same system is caused by the fact that the original system has a pole-zero cancellation in the transfer function. Referring to Equation (3–29), for $D = 0$ we have

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

If we use Equations (11–122) and (11–123), then

$$\begin{aligned} G(s) &= \begin{bmatrix} 0.8 & 1 \end{bmatrix} \begin{bmatrix} s & -1 \\ 0.4 & s + 1.3 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{1}{s^2 + 1.3s + 0.4} \begin{bmatrix} 0.8 & 1 \end{bmatrix} \begin{bmatrix} s + 1.3 & 1 \\ -0.4 & s \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= \frac{s + 0.8}{(s + 0.8)(s + 0.5)} \end{aligned}$$

[Note that the same transfer function can be obtained by using Equations (11–124) and (11–125).] Clearly, cancellation occurs in this transfer function.

If a pole-zero cancellation occurs in the transfer function, then the controllability and observability vary, depending on how the state variables are chosen. Remember that, to be completely state controllable and observable, the transfer function must not have any pole-zero cancellations.

A–11–19. Prove that the system defined by

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} \\ \mathbf{y} &= \mathbf{C}\mathbf{x} \end{aligned}$$

where $\mathbf{x}$ = state vector (n-vector)
$\mathbf{y}$ = output vector (m-vector) $\quad (m \leq n)$
$\mathbf{A} = n \times n$ matrix
$\mathbf{C} = m \times n$ matrix

is completely observable if and only if the composite $mn \times n$ matrix $\mathbf{P}$, where

$$\mathbf{P} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix}$$

is of rank n.

Solution. We shall first obtain the necessary condition. Suppose that

$$\text{rank } \mathbf{P} < n$$

Then there exists $\mathbf{x}(0)$ such that

$$\mathbf{P}\mathbf{x}(0) = \mathbf{0}$$

or

$$\mathbf{P}\mathbf{x}(0) = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \vdots \\ \mathbf{CA}^{n-1} \end{bmatrix} \mathbf{x}(0) = \begin{bmatrix} \mathbf{C}\mathbf{x}(0) \\ \mathbf{CA}\mathbf{x}(0) \\ \vdots \\ \mathbf{CA}^{n-1}\mathbf{x}(0) \end{bmatrix} = \mathbf{0}$$

0835

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10620

Hence, we obtain, for a certain $\mathbf{x}(0)$,

$$\mathbf{C}\mathbf{A}^i\mathbf{x}(0) = \mathbf{0}, \qquad \text{for } i = 0, 1, 2, \ldots, n-1$$

Notice that from Equation (11–48) or (11–50), we have

$$e^{\mathbf{A}t} = \alpha_0(t)\mathbf{I} + \alpha_1(t)\mathbf{A} + \alpha_2(t)\mathbf{A}^2 + \cdots + \alpha_{m-1}(t)\mathbf{A}^{m-1}$$

where $m (m \leq n)$ is the degree of the minimal polynomial for $\mathbf{A}$. Hence, for a certain $\mathbf{x}(0)$, we have

$$\mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0) = \mathbf{C}[\alpha_0(t)\mathbf{I} + \alpha_1(t)\mathbf{A} + \alpha_2(t)\mathbf{A}^2 + \cdots + \alpha_{m-1}(t)\mathbf{A}^{m-1}]\mathbf{x}(0) = \mathbf{0}$$

Consequently, for a certain $\mathbf{x}(0)$,

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t) = \mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0) = \mathbf{0}$$

which implies that, for a certain $\mathbf{x}(0)$, $\mathbf{x}(0)$ cannot be determined from $\mathbf{y}(t)$. Therefore, the rank of matrix $\mathbf{P}$ must be equal to n.

Next we shall obtain the sufficient condition. Suppose that rank $\mathbf{P} = n$. Since

$$\mathbf{y}(t) = \mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0)$$

by premultiplying both sides of this last equation by $e^{\mathbf{A}^*t}\mathbf{C}^*$, we get

$$e^{\mathbf{A}^*t}\mathbf{C}^*\mathbf{y}(t) = e^{\mathbf{A}^*t}\mathbf{C}^*\mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0)$$

If we integrate this last equation from 0 to t, we obtain

$$\int_0^t e^{\mathbf{A}^*t}\mathbf{C}^*\mathbf{y}(t)\,dt = \int_0^t e^{\mathbf{A}^*t}\mathbf{C}^*\mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0)\,dt \tag{11–126}$$

Notice that the left-hand side of this equation is a known quantity. Define

$$\mathbf{Q}(t) = \int_0^t e^{\mathbf{A}^*t}\mathbf{C}^*\mathbf{y}(t)\,dt = \text{known quantity} \tag{11–127}$$

Then, from Equations (11–126) and (11–127), we have

$$\mathbf{Q}(t) = \mathbf{W}(t)\mathbf{x}(0) \tag{11–128}$$

where

$$\mathbf{W}(t) = \int_0^t e^{\mathbf{A}^*\tau}\mathbf{C}^*\mathbf{C}e^{\mathbf{A}\tau}\,d\tau$$

It can be established that $\mathbf{W}(t)$ is a nonsingular matrix as follows: If $|\mathbf{W}(t)|$ were equal to 0, then

$$\mathbf{x}^*\mathbf{W}(t_1)\mathbf{x} = \int_0^{t_1} \|\mathbf{C}e^{\mathbf{A}t}\mathbf{x}\|^2\,dt = 0$$

which means that

$$\mathbf{C}e^{\mathbf{A}t}\mathbf{x} = \mathbf{0}, \qquad \text{for } 0 \leq t \leq t_1$$

which implies that rank $\mathbf{P} < n$. Therefore, $|\mathbf{W}(t)| \neq 0$, or $\mathbf{W}(t)$ is nonsingular. Then, from Equation (11–128), we obtain

$$\mathbf{x}(0) = [\mathbf{W}(t)]^{-1}\mathbf{Q}(t) \tag{11–129}$$

and $\mathbf{x}(0)$ can be determined from Equation (11–129).

Hence, we have proved that $\mathbf{x}(0)$ can be determined from $\mathbf{y}(t)$ if and only if rank $\mathbf{P} = n$. Note that $\mathbf{x}(0)$ and $\mathbf{y}(t)$ are related by

$$\mathbf{y}(t) = \mathbf{C}e^{\mathbf{A}t}\mathbf{x}(0) = \alpha_0(t)\mathbf{C}\mathbf{x}(0) + \alpha_1(t)\mathbf{C}\mathbf{A}\mathbf{x}(0) + \cdots + \alpha_{n-1}(t)\mathbf{C}\mathbf{A}^{n-1}\mathbf{x}(0)$$

0836

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10621

PROBLEMS

B–11–1. Consider the following transfer function system:

$$\frac{Y(s)}{U(s)} = \frac{s + 6}{s^2 + 5s + 6}$$

Obtain the state-space representation of this system in (a) controllable canonical form and (b) observable canonical form.

B–11–2. Consider the following system:

$$\dddot{y} + 6\ddot{y} + 11\dot{y} + 6y = 6u$$

Obtain a state-space representation of this system in a diagonal canonical form.

B–11–3. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 \\ -4 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1]$$

Transform the system equations into the controllable canonical form.

B–11–4. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1 \quad 0]$$

Obtain the transfer function $Y(s)/U(s)$.

B–11–5. Consider the following matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Obtain the eigenvalues λ_1, λ_2, λ_3, and λ_4 of the matrix **A**. Then obtain a transformation matrix **P** such that

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \text{diag}(\lambda_1, \lambda_2, \lambda_3, \lambda_4)$$

B–11–6. Consider the following matrix **A**:

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

Compute $e^{\mathbf{A}t}$ by three methods.

B–11–7. Given the system equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

find the solution in terms of the initial conditions $x_1(0)$, $x_2(0)$, and $x_3(0)$.

B–11–8. Find $x_1(t)$ and $x_2(t)$ of the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

where the initial conditions are

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

B–11–9. Consider the following state equation and output equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -6 & 1 & 0 \\ -11 & 0 & 1 \\ -6 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Show that the state equation can be transformed into the following form by use of a proper transformation matrix:

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -6 \\ 1 & 0 & -11 \\ 0 & 1 & -6 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

Then obtain the output y in terms of z_1, z_2, and z_3.

B–11–10. Obtain a state-space representation of the following system with MATLAB:

$$\frac{Y(s)}{U(s)} = \frac{10.4s^2 + 47s + 160}{s^3 + 14s^2 + 56s + 160}$$

0837

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10622

B–11–11. Obtain a transfer function representation of the following system with MATLAB:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$y = [0 \quad 0 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B–11–12. Obtain a transfer function representation of the following system with MATLAB:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B–11–13. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & -2 & -2 \\ 0 & -1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Is the system completely state controllable and completely observable?

B–11–14. Consider the system given by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Is the system completely state controllable and completely observable? Is the system completely output controllable?

B–11–15. Is the following system completely state controllable and completely observable?

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [20 \quad 9 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B–11–16. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [c_1 \quad c_2 \quad c_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Except for an obvious choice of $c_1 = c_2 = c_3 = 0$, find an example of a set of c_1, c_2, c_3 that will make the system unobservable.

B–11–17. Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 3 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The output is given by

$$y = [1 \quad 1 \quad 1] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(a) Show that the system is not completely observable.

(b) Show that the system is completely observable if the output is given by

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

0838

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10623



Design of Control Systems in State Space

12–1 INTRODUCTION

This chapter discusses state-space design methods based on the pole-placement method and the quadratic optimal regulator method. The pole-placement method is somewhat similar to the root-locus method in that we place closed-loop poles at desired locations. The basic difference is that in the root-locus design we place only the dominant closed-loop poles at the desired locations, while in the pole-placement design we place all closed-loop poles at desired locations.

We begin by presenting the basic materials on pole placement in regulator systems. We then discuss the design of state observers, followed by the design of regulator systems and control systems using the pole-placement-with-state-observer approach. Finally, we present the quadratic optimal regulator systems.

Outline of the Chapter. Section 12–1 has presented introductory material. Section 12–2 discusses the pole-placement approach to the design of control systems. We begin with the derivation of the necessary and sufficient conditions for arbitrary pole placement. Then we derive equations for the state feedback gain matrix **K** for pole placement. Section 12–3 presents the solution of the pole-placement problem with MATLAB. Section 12–4 discusses the design of servo systems using the pole-placement approach. Section 12–5 presents state observers. We discuss both full-order and minimum-order state observers. Also, transfer functions of observer controllers are derived. Section 12–6 presents the design of regulator systems with observers. Section 12–7 treats the design of control systems with observers. Finally, Section 12–8 discusses quadratic optimal

0839

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10624

regulator systems. Note that the state feedback gain matrix **K** can be obtained by both the pole-placement method and the quadratic optimal control method.

12–2 POLE PLACEMENT

In this section we shall present a design method commonly called the *pole-placement* or *pole-assignment technique.* We assume that all state variables are measurable and are available for feedback. It will be shown that if the system considered is completely state controllable, then poles of the closed-loop system may be placed at any desired locations by means of state feedback through an appropriate state feedback gain matrix.

The present design technique begins with a determination of the desired closed-loop poles based on the transient-response and/or frequency-response requirements, such as speed, damping ratio, or bandwidth, as well as steady-state requirements.

Let us assume that we decide that the desired closed-loop poles are to be at $s = \mu_1$, $s = \mu_2, \ldots, s = \mu_n$. By choosing an appropriate gain matrix for state feedback, it is possible to force the system to have closed-loop poles at the desired locations, provided that the original system is completely state controllable.

In this chapter we limit our discussions to single-input–single-output systems. That is, we assume the control signal $u(t)$ and output signal $y(t)$ to be scalars. In the derivation in this section we assume that the reference input $r(t)$ is zero. [In Section 12–7 we discuss the case where the reference input $r(t)$ is nonzero.]

In what follows we shall prove that a necessary and sufficient condition that the closed-loop poles can be placed at any arbitrary locations in the s plane is that the system be completely state controllable. Then we shall discuss methods for determining the required state feedback gain matrix.

It is noted that when the control signal is a vector quantity, the mathematical aspects of the pole-placement scheme become complicated. We shall not discuss such a case in this book. (When the control signal is a vector quantity, the state feedback gain matrix is not unique. It is possible to choose freely more than n parameters; that is, in addition to being able to place n closed-loop poles properly, we have the freedom to satisfy some or all of the other requirements, if any, of the closed-loop system.)

Design by Pole Placement. In the conventional approach to the design of a single-input–single-output control system, we design a controller (compensator) such that the dominant closed-loop poles have a desired damping ratio ζ and an undamped natural frequency ω_n. In this approach, the order of the system may be raised by 1 or 2 unless pole–zero cancellation takes place. Note that in this approach we assume the effects on the responses of nondominant closed-loop poles to be negligible.

Different from specifying only dominant closed-loop poles (the conventional design approach), the present pole-placement approach specifies all closed-loop poles. (There is a cost associated with placing all closed-loop poles, however, because placing all closed-loop poles requires successful measurements of all state variables or else requires the inclusion of a state observer in the system.) There is also a requirement on the part of the system for the closed-loop poles to be placed at arbitrarily chosen locations. The requirement is that the system be completely state controllable. We shall prove this fact in this section.

0840

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10625

Consider a control system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–1}$$
$$y = \mathbf{C}\mathbf{x} + Du$$

where $\mathbf{x}$ = state vector (n-vector)
y = output signal (scalar)
u = control signal (scalar)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times 1$ constant matrix
$\mathbf{C}$ = $1 \times n$ constant matrix
D = constant (scalar)

We shall choose the control signal to be

$$u = -\mathbf{K}\mathbf{x} \tag{12–2}$$

This means that the control signal u is determined by an instantaneous state. Such a scheme is called state feedback. The $1 \times n$ matrix $\mathbf{K}$ is called the state feedback gain matrix. We assume that all state variables are available for feedback. In the following analysis we assume that u is unconstrained. A block diagram for this system is shown in Figure 12–1.

This closed-loop system has no input. Its objective is to maintain the zero output. Because of the disturbances that may be present, the output will deviate from zero. The nonzero output will be returned to the zero reference input because of the state feedback scheme of the system. Such a system where the reference input is always zero is called a regulator system. (Note that if the reference input to the system is always a nonzero constant, the system is also called a regulator system.)

Substituting Equation (12–2) into Equation (12–1) gives

$$\dot{\mathbf{x}}(t) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(t)$$

The solution of this equation is given by

$$\mathbf{x}(t) = e^{(\mathbf{A}-\mathbf{B}\mathbf{K})t}\mathbf{x}(0) \tag{12–3}$$

where $\mathbf{x}(0)$ is the initial state caused by external disturbances. The stability and transient-response characteristics are determined by the eigenvalues of matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$. If matrix

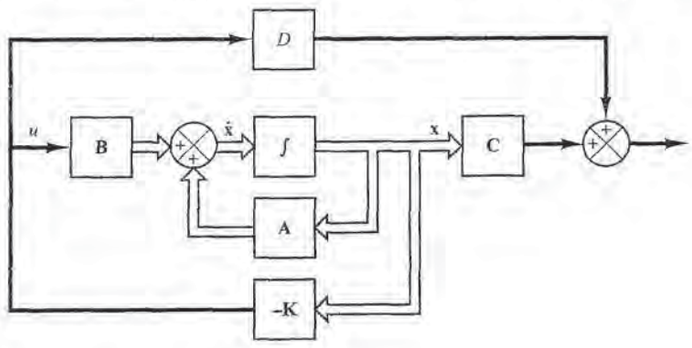


Figure 12–1
Closed-loop control system with $u = -\mathbf{K}\mathbf{x}$.

0841

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10626

$\mathbf{K}$ is chosen properly, the matrix $\mathbf{A} - \mathbf{BK}$ can be made an asymptotically stable matrix, and for all $\mathbf{x}(0) \neq \mathbf{0}$, it is possible to make $\mathbf{x}(t)$ approach $\mathbf{0}$ as t approaches infinity. The eigenvalues of matrix $\mathbf{A} - \mathbf{BK}$ are called the regulator poles. If these regulator poles are placed in the left-half s plane, then $\mathbf{x}(t)$ approaches $\mathbf{0}$ as t approaches infinity. The problem of placing the regulator poles (closed-loop poles) at the desired location is called a pole-placement problem.

In what follows, we shall prove that arbitrary pole placement for a given system is possible if and only if the system is completely state controllable.

Necessary and Sufficient Condition for Arbitrary Pole Placement We shall now prove that a necessary and sufficient condition for arbitrary pole placement is that the system be completely state controllable. We shall first derive the necessary condition. We begin by proving that if the system is not completely state controllable, then there are eigenvalues of matrix $\mathbf{A} - \mathbf{BK}$ that cannot be controlled by state feedback.

Suppose that the system of Equation (12–1) is not completely state controllable. Then the rank of the controllability matrix is less than n, or

$$\text{rank}\left[\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}\right] = q < n$$

This means that there are q linearly independent column vectors in the controllability matrix. Let us define such q linearly independent column vectors as $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_q$. Also, let us choose $n - q$ additional n-vectors $\mathbf{v}_{q+1}, \mathbf{v}_{q+2}, \ldots, \mathbf{v}_n$ such that

$$\mathbf{P} = \left[\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \mathbf{v}_{q+2} \mid \cdots \mid \mathbf{v}_n\right]$$

is of rank n. Then it can be shown that

$$\hat{\mathbf{A}} = \mathbf{P}^{-1}\mathbf{AP} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right], \qquad \hat{\mathbf{B}} = \mathbf{P}^{-1}\mathbf{B} = \left[\begin{array}{c} \mathbf{B}_{11} \\ \hline \mathbf{0} \end{array}\right]$$

(See Problem **A–12–1** for the derivation of these equations.) Now define

$$\hat{\mathbf{K}} = \mathbf{KP} = \left[\mathbf{k}_1 \mid \mathbf{k}_2\right]$$

Then we have

$$\begin{aligned} |s\mathbf{I} - \mathbf{A} + \mathbf{BK}| &= |\mathbf{P}^{-1}(s\mathbf{I} - \mathbf{A} + \mathbf{BK})\mathbf{P}| \\ &= |s\mathbf{I} - \mathbf{P}^{-1}\mathbf{AP} + \mathbf{P}^{-1}\mathbf{BKP}| \\ &= |s\mathbf{I} - \hat{\mathbf{A}} + \hat{\mathbf{B}}\hat{\mathbf{K}}| \\ &= \left|s\mathbf{I} - \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right] + \left[\begin{array}{c} \mathbf{B}_{11} \\ \hline \mathbf{0} \end{array}\right]\left[\mathbf{k}_1 \mid \mathbf{k}_2\right]\right| \\ &= \left|\begin{array}{cc} s\mathbf{I}_q - \mathbf{A}_{11} + \mathbf{B}_{11}\mathbf{k}_1 & -\mathbf{A}_{12} + \mathbf{B}_{11}\mathbf{k}_2 \\ \mathbf{0} & s\mathbf{I}_{n-q} - \mathbf{A}_{22} \end{array}\right| \\ &= |s\mathbf{I}_q - \mathbf{A}_{11} + \mathbf{B}_{11}\mathbf{k}_1| \cdot |s\mathbf{I}_{n-q} - \mathbf{A}_{22}| = 0 \end{aligned}$$

where $\mathbf{I}_q$ is a q-dimensional identity matrix and $\mathbf{I}_{n-q}$ is an $(n - q)$-dimensional identity matrix.

0842

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10627

Notice that the eigenvalues of $\mathbf{A}_{22}$ do not depend on $\mathbf{K}$. Thus, if the system is not completely state controllable, then there are eigenvalues of matrix $\mathbf{A}$ that cannot be arbitrarily placed. Therefore, to place the eigenvalues of matrix $\mathbf{A} - \mathbf{BK}$ arbitrarily, the system must be completely state controllable (necessary condition).

Next we shall prove a sufficient condition: that is, if the system is completely state controllable, then all eigenvalues of matrix $\mathbf{A}$ can be arbitrarily placed.

In proving a sufficient condition, it is convenient to transform the state equation given by Equation (12–1) into the controllable canonical form.

Define a transformation matrix $\mathbf{T}$ by

$$\mathbf{T} = \mathbf{MW} \tag{12–4}$$

where $\mathbf{M}$ is the controllability matrix

$$\mathbf{M} = \begin{bmatrix} \mathbf{B} & \vdots & \mathbf{AB} & \vdots & \cdots & \vdots & \mathbf{A}^{n-1}\mathbf{B} \end{bmatrix} \tag{12–5}$$

and

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{12–6}$$

where the a_i's are coefficients of the characteristic polynomial

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

Define a new state vector $\hat{\mathbf{x}}$ by

$$\mathbf{x} = \mathbf{T}\hat{\mathbf{x}}$$

If the rank of the controllability matrix $\mathbf{M}$ is n (meaning that the system is completely state controllable), then the inverse of matrix $\mathbf{T}$ exists, and Equation (12–1) can be modified to

$$\dot{\hat{\mathbf{x}}} = \mathbf{T}^{-1}\mathbf{AT}\hat{x} + \mathbf{T}^{-1}\mathbf{B}u \tag{12–7}$$

where

$$\mathbf{T}^{-1}\mathbf{AT} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \tag{12–8}$$

0843

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10628

$$\mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \tag{12–9}$$

[See Problems **A–12–2** and **A–12–3** for the derivation of Equations (12–8) and (12–9).] Equation (12–7) is in the controllable canonical form. Thus, given a state equation, Equation (12–1), it can be transformed into the controllable canonical form if the system is completely state controllable and if we transform the state vector $\mathbf{x}$ into state vector $\hat{\mathbf{x}}$ by use of the transformation matrix $\mathbf{T}$ given by Equation (12–4).

Let us choose a set of the desired eigenvalues as $\mu_1, \mu_2, \ldots, \mu_n$. Then the desired characteristic equation becomes

$$(s - \mu_1)(s - \mu_2)\cdots(s - \mu_n) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n = 0 \tag{12–10}$$

Let us write

$$\mathbf{KT} = \begin{bmatrix} \delta_n & \delta_{n-1} & \cdots & \delta_1 \end{bmatrix} \tag{12–11}$$

When $u = -\mathbf{KT}\hat{\mathbf{x}}$ is used to control the system given by Equation (12–7), the system equation becomes

$$\dot{\hat{\mathbf{x}}} = \mathbf{T}^{-1}\mathbf{AT}\hat{\mathbf{x}} - \mathbf{T}^{-1}\mathbf{BKT}\hat{\mathbf{x}}$$

The characteristic equation is

$$|s\mathbf{I} - \mathbf{T}^{-1}\mathbf{AT} + \mathbf{T}^{-1}\mathbf{BKT}| = 0$$

This characteristic equation is the same as the characteristic equation for the system, defined by Equation (12–1), when $u = -\mathbf{Kx}$ is used as the control signal. This can be seen as follows: Since

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu} = (\mathbf{A} - \mathbf{BK})\mathbf{x}$$

the characteristic equation for this system is

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}| = |\mathbf{T}^{-1}(s\mathbf{I} - \mathbf{A} + \mathbf{BK})\mathbf{T}| = |s\mathbf{I} - \mathbf{T}^{-1}\mathbf{AT} + \mathbf{T}^{-1}\mathbf{BKT}| = 0$$

0844

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10629

Now let us simplify the characteristic equation of the system in the controllable canonical form. Referring to Equations (12–8), (12–9), and (12–11), we have

$$
\begin{aligned}
&|s\mathbf{I} - \mathbf{T}^{-1}\mathbf{A}\mathbf{T} + \mathbf{T}^{-1}\mathbf{B}\mathbf{K}\mathbf{T}| \\
&= \left| s\mathbf{I} - \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & \cdots & -a_1 \end{bmatrix} + \begin{bmatrix} 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} \delta_n & \delta_{n-1} & \cdots & \delta_1 \end{bmatrix} \right| \\
&= \begin{vmatrix} s & -1 & \cdots & 0 \\ 0 & s & \cdots & 0 \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ a_n + \delta_n & a_{n-1} + \delta_{n-1} & \cdots & s + a_1 + \delta_1 \end{vmatrix} \\
&= s^n + (a_1 + \delta_1)s^{n-1} + \cdots + (a_{n-1} + \delta_{n-1})s + (a_n + \delta_n) = 0
\end{aligned} \tag{12–12}
$$

This is the characteristic equation for the system with state feedback. Therefore, it must be equal to Equation (12–10), the desired characteristic equation. By equating the coefficients of like powers of s, we get

$$
\begin{aligned}
a_1 + \delta_1 &= \alpha_1 \\
a_2 + \delta_2 &= \alpha_2 \\
&\cdot \\
&\cdot \\
&\cdot \\
a_n + \delta_n &= \alpha_n
\end{aligned}
$$

Solving the preceding equations for the δ_i's and substituting them into Equation (12–11), we obtain

$$
\begin{aligned}
\mathbf{K} &= \begin{bmatrix} \delta_n & \delta_{n-1} & \cdots & \delta_1 \end{bmatrix} \mathbf{T}^{-1} \\
&= \begin{bmatrix} \alpha_n - a_n & \vdots & \alpha_{n-1} - a_{n-1} & \vdots & \cdots & \vdots & \alpha_2 - a_2 & \vdots & \alpha_1 - a_1 \end{bmatrix} \mathbf{T}^{-1}
\end{aligned} \tag{12–13}
$$

Thus, if the system is completely state controllable, all eigenvalues can be arbitrarily placed by choosing matrix $\mathbf{K}$ according to Equation (12–13) (sufficient condition).

We have thus proved that a necessary and sufficient condition for arbitrary pole placement is that the system be completely state controllable.

It is noted that if the system is not completely state controllable, but is stabilizable, then it is possible to make the entire system stable by placing the closed-loop poles at desired locations for q controllable modes. The remaining $n - q$ uncontrollable modes are stable. So the entire system can be made stable.

0845

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10630

Determination of Matrix K Using Transformation Matrix T. Suppose that the system is defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

and the control signal is given by

$$u = -\mathbf{K}\mathbf{x}$$

The feedback gain matrix $\mathbf{K}$ that forces the eigenvalues of $\mathbf{A} - \mathbf{BK}$ to be $\mu_1, \mu_2, \ldots, \mu_n$ (desired values) can be determined by the following steps (if μ_i is a complex eigenvalue, then its conjugate must also be an eigenvalue of $\mathbf{A} - \mathbf{BK}$):

Step 1: Check the controllability condition for the system. If the system is completely state controllable, then use the following steps:

Step 2: From the characteristic polynomial for matrix $\mathbf{A}$, that is,

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

determine the values of $a_1, a_2, \ldots, a_n$.

Step 3: Determine the transformation matrix $\mathbf{T}$ that transforms the system state equation into the controllable canonical form. (If the given system equation is already in the controllable canonical form, then $\mathbf{T} = \mathbf{I}$.) It is not necessary to write the state equation in the controllable canonical form. All we need here is to find the matrix $\mathbf{T}$. The transformation matrix $\mathbf{T}$ is given by Equation (12–4), or

$$\mathbf{T} = \mathbf{MW}$$

where $\mathbf{M}$ is given by Equation (12–5) and $\mathbf{W}$ is given by Equation (12–6).

Step 4: Using the desired eigenvalues (desired closed-loop poles), write the desired characteristic polynomial:

$$(s - \mu_1)(s - \mu_2)\cdots(s - \mu_n) = s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n$$

and determine the values of $\alpha_1, \alpha_2, \ldots, \alpha_n$.

Step 5: The required state feedback gain matrix $\mathbf{K}$ can be determined from Equation (12–13), rewritten thus:

$$\mathbf{K} = [\alpha_n - a_n \mid \alpha_{n-1} - a_{n-1} \mid \cdots \mid \alpha_2 - a_2 \mid \alpha_1 - a_1]\mathbf{T}^{-1}$$

Determination of Matrix K Using Direct Substitution Method. If the system is of low order ($n \leq 3$), direct substitution of matrix $\mathbf{K}$ into the desired characteristic polynomial may be simpler. For example, if $n = 3$, then write the state feedback gain matrix $\mathbf{K}$ as

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3]$$

Substitute this $\mathbf{K}$ matrix into the desired characteristic polynomial $|s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$ and equate it to $(s - \mu_1)(s - \mu_2)(s - \mu_3)$, or

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}| = (s - \mu_1)(s - \mu_2)(s - \mu_3)$$

0846

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10631

Since both sides of this characteristic equation are polynomials in s, by equating the coefficients of the like powers of s on both sides, it is possible to determine the values of k_1, k_2, and k_3. This approach is convenient if $n = 2$ or 3. (For $n = 4, 5, 6, \ldots$, this approach may become very tedious.)

Note that if the system is not completely controllable, matrix $\mathbf{K}$ cannot be determined. (No solution exists.)

Determination of Matrix K Using Ackermann's Formula. There is a well-known formula, known as Ackermann's formula, for the determination of the state feedback gain matrix $\mathbf{K}$. We shall present this formula in what follows.

Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where we use the state feedback control $u = -\mathbf{K}\mathbf{x}$. We assume that the system is completely state controllable. We also assume that the desired closed-loop poles are at $s = \mu_1, s = \mu_2, \ldots, s = \mu_n$.

Use of the state feedback control

$$u = -\mathbf{K}\mathbf{x}$$

modifies the system equation to

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} \tag{12–14}$$

Let us define

$$\tilde{\mathbf{A}} = \mathbf{A} - \mathbf{B}\mathbf{K}$$

The desired characteristic equation is

$$\begin{aligned} |s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}| = |s\mathbf{I} - \tilde{\mathbf{A}}| &= (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n) \\ &= s^n + \alpha_1 s^{n-1} + \cdots + \alpha_{n-1}s + \alpha_n = 0 \end{aligned}$$

Since the Cayley–Hamilton theorem states that $\tilde{\mathbf{A}}$ satisfies its own characteristic equation, we have

$$\phi(\tilde{\mathbf{A}}) = \tilde{\mathbf{A}}^n + \alpha_1 \tilde{\mathbf{A}}^{n-1} + \cdots + \alpha_{n-1}\tilde{\mathbf{A}} + \alpha_n \mathbf{I} = \mathbf{0} \tag{12–15}$$

We shall utilize Equation (12–15) to derive Ackermann's formula. To simplify the derivation, we consider the case where $n = 3$. (For any other positive integer n, the following derivation can be easily extended.)

Consider the following identities:

$$\begin{aligned} \mathbf{I} &= \mathbf{I} \\ \tilde{\mathbf{A}} &= \mathbf{A} - \mathbf{B}\mathbf{K} \\ \tilde{\mathbf{A}}^2 &= (\mathbf{A} - \mathbf{B}\mathbf{K})^2 = \mathbf{A}^2 - \mathbf{A}\mathbf{B}\mathbf{K} - \mathbf{B}\mathbf{K}\tilde{\mathbf{A}} \\ \tilde{\mathbf{A}}^3 &= (\mathbf{A} - \mathbf{B}\mathbf{K})^3 = \mathbf{A}^3 - \mathbf{A}^2\mathbf{B}\mathbf{K} - \mathbf{A}\mathbf{B}\mathbf{K}\tilde{\mathbf{A}} - \mathbf{B}\mathbf{K}\tilde{\mathbf{A}}^2 \end{aligned}$$

0847

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10632

Multiplying the preceding equations in order by α_3, α_2, α_1, and α_0 (where $\alpha_0 = 1$), respectively, and adding the results, we obtain

$$\begin{aligned}
&\alpha_3\mathbf{I} + \alpha_2\tilde{\mathbf{A}} + \alpha_1\tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3 \\
&= \alpha_3\mathbf{I} + \alpha_2(\mathbf{A} - \mathbf{BK}) + \alpha_1(\mathbf{A}^2 - \mathbf{ABK} - \mathbf{BK}\tilde{\mathbf{A}}) + \mathbf{A}^3 - \mathbf{A}^2\mathbf{BK} \\
&\quad - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2 \\
&= \alpha_3\mathbf{I} + \alpha_2\mathbf{A} + \alpha_1\mathbf{A}^2 + \mathbf{A}^3 - \alpha_2\mathbf{BK} - \alpha_1\mathbf{ABK} - \alpha_1\mathbf{BK}\tilde{\mathbf{A}} - \mathbf{A}^2\mathbf{BK} \\
&\quad - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2
\end{aligned} \tag{12–16}$$

Referring to Equation (12–15), we have

$$\alpha_3\mathbf{I} + \alpha_2\tilde{\mathbf{A}} + \alpha_1\tilde{\mathbf{A}}^2 + \tilde{\mathbf{A}}^3 = \phi(\tilde{\mathbf{A}}) = \mathbf{0}$$

Also, we have

$$\alpha_3\mathbf{I} + \alpha_2\mathbf{A} + \alpha_1\mathbf{A}^2 + \mathbf{A}^3 = \phi(\mathbf{A}) \neq \mathbf{0}$$

Substituting the last two equations into Equation (12–16), we have

$$\phi(\tilde{\mathbf{A}}) = \phi(\mathbf{A}) - \alpha_2\mathbf{BK} - \alpha_1\mathbf{BK}\tilde{\mathbf{A}} - \mathbf{BK}\tilde{\mathbf{A}}^2 - \alpha_1\mathbf{ABK} - \mathbf{ABK}\tilde{\mathbf{A}} - \mathbf{A}^2\mathbf{BK}$$

Since $\phi(\tilde{\mathbf{A}}) = \mathbf{0}$, we obtain

$$\begin{aligned}
\phi(\mathbf{A}) &= \mathbf{B}(\alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2) + \mathbf{AB}(\alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}}) + \mathbf{A}^2\mathbf{BK} \\
&= \begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}\end{bmatrix}\begin{bmatrix}\alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K}\end{bmatrix}
\end{aligned} \tag{12–17}$$

Since the system is completely state controllable, the inverse of the controllability matrix

$$\begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}\end{bmatrix}$$

exists. Premultiplying both sides of Equation (12–17) by the inverse of the controllability matrix, we obtain

$$\begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}\end{bmatrix}^{-1}\phi(\mathbf{A}) = \begin{bmatrix}\alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K}\end{bmatrix}$$

Premultiplying both sides of this last equation by $[0 \quad 0 \quad 1]$, we obtain

$$[0 \quad 0 \quad 1]\begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}\end{bmatrix}^{-1}\phi(\mathbf{A}) = [0 \quad 0 \quad 1]\begin{bmatrix}\alpha_2\mathbf{K} + \alpha_1\mathbf{K}\tilde{\mathbf{A}} + \mathbf{K}\tilde{\mathbf{A}}^2 \\ \alpha_1\mathbf{K} + \mathbf{K}\tilde{\mathbf{A}} \\ \mathbf{K}\end{bmatrix} = \mathbf{K}$$

which can be rewritten as

$$\mathbf{K} = [0 \quad 0 \quad 1]\begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}\end{bmatrix}^{-1}\phi(\mathbf{A})$$

This last equation gives the required state feedback gain matrix $\mathbf{K}$.

For an arbitrary positive integer n, we have

$$\mathbf{K} = [0 \quad 0 \quad \cdots \quad 0 \quad 1]\begin{bmatrix}\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}\end{bmatrix}^{-1}\phi(\mathbf{A}) \tag{12–18}$$

0848

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10633

Equation (12–18) is known as Ackermann's formula for the determination of the state feedback gain matrix **K**.

Regulator Systems and Control Systems. Systems that include controllers can be divided into two categories: regulator systems (where the reference input is constant, including zero) and control systems (where the reference input is time varying). In what follows we shall consider regulator systems. Control systems will be treated in Section 12–7.

Choosing the Locations of Desired Closed-Loop Poles. The first step in the pole-placement design approach is to choose the locations of the desired closed-loop poles. The most frequently used approach is to choose such poles based on experience in the root-locus design, placing a dominant pair of closed-loop poles and choosing other poles so that they are far to the left of the dominant closed-loop poles.

Note that if we place the dominant closed-loop poles far from the $j\omega$ axis, so that the system response becomes very fast, the signals in the system become very large, with the result that the system may become nonlinear. This should be avoided.

Another approach is based on the quadratic optimal control approach. This approach will determine the desired closed-loop poles such that it balances between the acceptable response and the amount of control energy required. (See Section 12–8.) Note that requiring a high-speed response implies requiring large amounts of control energy. Also, in general, increasing the speed of response requires a larger, heavier actuator, which will cost more.

EXAMPLE 12–1 Consider the regulator system shown in Figure 12–2. The plant is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The system uses the state feedback control $\mathbf{u} = -\mathbf{K}\mathbf{x}$. Let us choose the desired closed-loop poles at

$$s = -2 + j4, \qquad s = -2 - j4, \qquad s = -10$$

(We make such a choice because we know from experience that such a set of closed-loop poles will result in a reasonable or acceptable transient response.) Determine the state feedback gain matrix **K**.

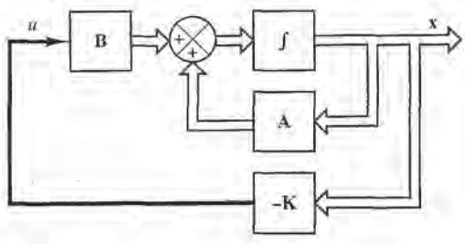


Figure 12–2
Regulator system.

0849

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10634

First, we need to check the controllability matrix of the system. Since the controllability matrix $\mathbf{M}$ is given by

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

we find that $|\mathbf{M}| = -1$, and therefore, rank $\mathbf{M} = 3$. Thus, the system is completely state controllable and arbitrary pole placement is possible.

Next, we shall solve this problem. We shall demonstrate each of the three methods presented in this chapter.

Method 1: The first method is to use Equation (12–13). The characteristic equation for the system is

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}| &= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 & 5 & s+6 \end{vmatrix} \\ &= s^3 + 6s^2 + 5s + 1 \\ &= s^3 + a_1 s^2 + a_2 s + a_3 = 0 \end{aligned}$$

Hence,

$$a_1 = 6, \qquad a_2 = 5, \qquad a_3 = 1$$

The desired characteristic equation is

$$\begin{aligned} (s + 2 - j4)(s + 2 + j4)(s + 10) &= s^3 + 14s^2 + 60s + 200 \\ &= s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3 = 0 \end{aligned}$$

Hence,

$$\alpha_1 = 14, \qquad \alpha_2 = 60, \qquad \alpha_3 = 200$$

Referring to Equation (12–13), we have

$$\mathbf{K} = [\alpha_3 - a_3 \mid \alpha_2 - a_2 \mid \alpha_1 - a_1]\mathbf{T}^{-1}$$

where $\mathbf{T} = \mathbf{I}$ for this problem because the given state equation is in the controllable canonical form. Then we have

$$\begin{aligned} \mathbf{K} &= [200 - 1 \mid 60 - 5 \mid 14 - 6] \\ &= [199 \quad 55 \quad 8] \end{aligned}$$

Method 2: By defining the desired state feedback gain matrix $\mathbf{K}$ as

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3]$$

and equating $|s\mathbf{I} - \mathbf{A} + \mathbf{BK}|$ with the desired characteristic equation, we obtain

$$\begin{aligned} |s\mathbf{I} - \mathbf{A} + \mathbf{BK}| &= \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [k_1 \quad k_2 \quad k_3] \right| \\ &= \begin{vmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 1 + k_1 & 5 + k_2 & s + 6 + k_3 \end{vmatrix} \\ &= s^3 + (6 + k_3)s^2 + (5 + k_2)s + 1 + k_1 \\ &= s^3 + 14s^2 + 60s + 200 \end{aligned}$$

0850

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10635

Thus,

$$6 + k_3 = 14, \qquad 5 + k_2 = 60, \qquad 1 + k_1 = 200$$

from which we obtain

$$k_1 = 199, \qquad k_2 = 55, \qquad k_3 = 8$$

or

$$\mathbf{K} = [199 \quad 55 \quad 8]$$

Method 3: The third method is to use Ackermann's formula. Referring to Equation (12–18), we have

$$\mathbf{K} = [0 \quad 0 \quad 1][\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}]^{-1}\phi(\mathbf{A})$$

Since

$$\begin{aligned}\phi(\mathbf{A}) &= \mathbf{A}^3 + 14\mathbf{A}^2 + 60\mathbf{A} + 200\mathbf{I} \\ &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}^3 + 14\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}^2 \\ &\quad + 60\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix} + 200\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix}\end{aligned}$$

and

$$[\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}$$

we obtain

$$\begin{aligned}\mathbf{K} &= [0 \quad 0 \quad 1]\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -6 \\ 1 & -6 & 31 \end{bmatrix}^{-1}\begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix} \\ &= [0 \quad 0 \quad 1]\begin{bmatrix} 5 & 6 & 1 \\ 6 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 199 & 55 & 8 \\ -8 & 159 & 7 \\ -7 & -43 & 117 \end{bmatrix} \\ &= [199 \quad 55 \quad 8]\end{aligned}$$

As a matter of course, the feedback gain matrix **K** obtained by the three methods are the same. With this state feedback, the closed-loop poles are placed at $s = -2 \pm j4$ and $s = -10$, as desired.

It is noted that if the order n of the system were 4 or higher, methods 1 and 3 are recommended, since all matrix computations can be carried out by a computer. If method 2 is used, hand computations become necessary because a computer may not handle the characteristic equation with unknown parameters $k_1, k_2, \ldots, k_n$.

0851

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10636

Comments. It is important to note that matrix **K** is not unique for a given system, but depends on the desired closed-loop pole locations (which determine the speed and damping of the response) selected. Note that the selection of the desired closed-loop poles or the desired characteristic equation is a compromise between the rapidity of the response of the error vector and the sensitivity to disturbances and measurement noises. That is, if we increase the speed of error response, then the adverse effects of disturbances and measurement noises generally increase. If the system is of second order, then the system dynamics (response characteristics) can be precisely correlated to the location of the desired closed-loop poles and the zero(s) of the plant. For higher-order systems, the location of the closed-loop poles and the system dynamics (response characteristics) are not easily correlated. Hence, in determining the state feedback gain matrix **K** for a given system, it is desirable to examine by computer simulations the response characteristics of the system for several different matrices **K** (based on several different desired characteristic equations) and to choose the one that gives the best overall system performance.

12–3 SOLVING POLE-PLACEMENT PROBLEMS WITH MATLAB

Pole-placement problems can be solved easily with MATLAB. MATLAB has two commands—acker and place—for the computation of feedback-gain matrix **K**. The command acker is based on Ackermann's formula. This command applies to single-input systems only. The desired closed-loop poles can include multiple poles (poles located at the same place).

If the system involves multiple inputs, for a specified set of closed-loop poles the state-feedback gain matrix **K** is not unique and we have an additional freedom (or freedoms) to choose **K**. There are many approaches to constructively utilize this additional freedom (or freedoms) to determine **K**. One common use is to maximize the stability margin. The pole placement based on this approach is called the robust pole placement. The MATLAB command for the robust pole placement is place.

Although the command place can be used for both single-input and multiple-input systems, this command requires that the multiplicity of poles in the desired closed-loop poles be no greater than the rank of **B**. That is, if matrix **B** is an $n \times 1$ matrix, the command place requires that there be no multiple poles in the set of desired closed-loop poles.

For single-input systems, the commands acker and place yield the same **K**. (But for multiple-input systems, one must use the command place instead of acker.)

It is noted that when the single-input system is barely controllable, some computational problem may occur if the command acker is used. In such a case the use of the place command is preferred, provided that no multiple poles are involved in the desired set of closed-loop poles.

To use the command acker or place, we first enter the following matrices in the program:

$$\mathbf{A} \text{ matrix}, \qquad \mathbf{B} \text{ matrix}, \qquad \mathbf{J} \text{ matrix}$$

where **J** matrix is the matrix consisting of the desired closed-loop poles such that

$$\mathbf{J} = [\mu_1 \quad \mu_2 \quad \cdots \quad \mu_n]$$

0852

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10637

Then we enter

```
K = acker(A,B,J)
```

or

```
K = place(A,B,J)
```

It is noted that the command eig (A-B*K) may be used to verify that K thus obtained gives the desired eigenvalues.

EXAMPLE 12–2 Consider the same system as treated in Example 12–1. The system equation is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

By using state feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to have the closed-loop poles at $s = \mu_i$ $(i = 1, 2, 3)$, where

$$\mu_1 = -2 + j4, \qquad \mu_2 = -2 - j4, \qquad \mu_3 = -10$$

Determine the state feedback-gain matrix **K** with MATLAB.

MATLAB programs that generate matrix **K** are shown in MATLAB Programs 12–1 and 12–2. MATLAB Program 12–1 uses command acker and MATLAB Program 12–2 uses command place.

MATLAB Program 12–1

```
A = [0 1 0;0 0 1;-1 -5 -6];
B = [0;0;1];
J = [-2+j*4 -2-j*4 -10];
K = acker(A,B,J)

K =

   199   55   8
```

MATLAB Program 12–2

```
A = [0 1 0;0 0 1;-1 -5 -6];
B = [0;0;1];
J = [-2+j*4 -2-j*4 -10];
K = place(A,B,J)
  place: ndigits = 15

K =

  199.0000   55.0000   8.0000
```


0853

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10638

EXAMPLE 12–3 Consider the same system as discussed in Example 12–1. It is desired that this regulator system have closed-loop poles at

$$s = -2 + j4, \qquad s = -2 - j4, \qquad s = -10$$

The necessary state feedback gain matrix $\mathbf{K}$ was obtained in Example 12–1 as follows:

$$\mathbf{K} = [199 \quad 55 \quad 8]$$

Using MATLAB, obtain the response of the system to the following initial condition:

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

Response to Initial Condition: To obtain the response to the given initial condition $\mathbf{x}(0)$, we substitute $u = -\mathbf{K}\mathbf{x}$ into the plant equation to get

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}, \qquad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

To plot the response curves (x_1 versus t, x_2 versus t, and x_3 versus t), we may use the command initial. We first define the state-space equations for the system as follows:

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{I}\mathbf{u}$$
$$\mathbf{y} = \mathbf{I}\mathbf{x} + \mathbf{I}\mathbf{u}$$

where we included $\mathbf{u}$ (a three-dimensional input vector). This $\mathbf{u}$ vector is considered $\mathbf{0}$ in the computation of the response to the initial condition. Then we define

```
sys = ss(A - BK, eye(3), eye(3), eye(3))
```

and use the initial command as follows:

```
x = initial(sys, [1;0;0],t)
```

where t is the time duration we want to use, such as

```
t = 0:0.01:4;
```

Then obtain x1, x2, and x3 as follows:

```
x1 = [1  0  0]*x';
x2 = [0  1  0]*x';
x3 = [0  0  1]*x';
```

and use the plot command. This program is shown in MATLAB Program 12–3. The resulting response curves are shown in Figure 12–3.

0854

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10639

MATLAB Program 12–3

```
% Response to initial condition:

A = [0  1  0;0  0  1;-1  -5  -6];
B = [0;0;1];
K = [199  55  8];
sys = ss(A-B*K, eye(3), eye(3), eye(3));
t = 0:0.01:4;
x = initial(sys,[1;0;0],t);
x1 = [1  0  0]*x';
x2 = [0  1  0]*x';
x3 = [0  0  1]*x';

subplot(3,1,1); plot(t,x1), grid
title('Response to Initial Condition')
ylabel('state variable x1')

subplot(3,1,2); plot(t,x2),grid
ylabel('state variable x2')

subplot(3,1,3); plot(t,x3),grid
xlabel('t (sec)')
ylabel('state variable x3')
```

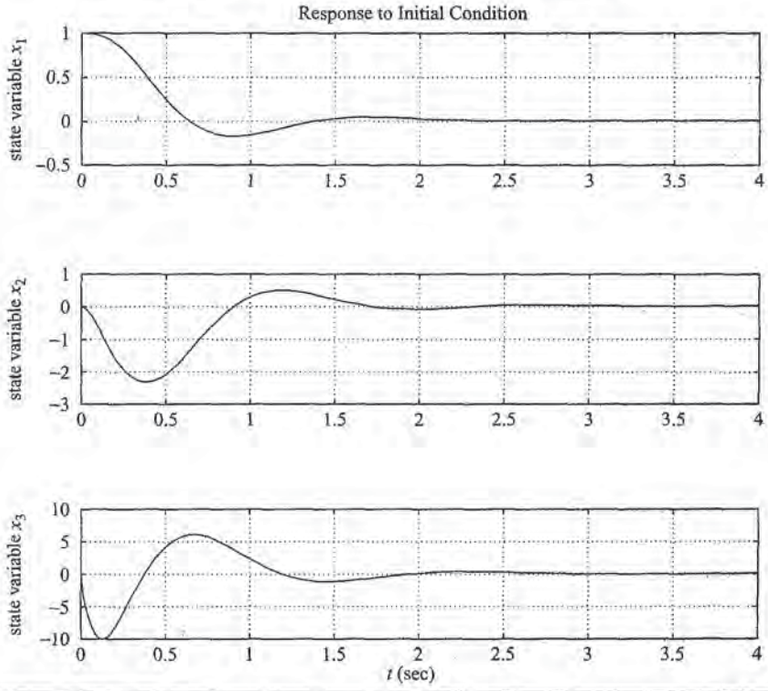


Figure 12–3
Response to initial condition.

0855

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10640

12–4 DESIGN OF SERVO SYSTEMS

In this section we shall discuss the pole-placement approach to the design of type 1 servo systems. Here we shall limit our systems each to have a scalar control signal u and a scalar output y.

In what follows we shall first discuss a problem of designing a type 1 servo system when the plant involves an integrator. Then we shall discuss the design of a type 1 servo system when the plant has no integrator.

Design of Type 1 Servo System when the Plant Has An Integrator. Assume that the plant is defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–19}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–20}$$

where $\mathbf{x}$ = state vector for the plant (n-vector)
u = control signal (scalar)
y = output signal (scalar)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times 1$ constant matrix
$\mathbf{C}$ = $1 \times n$ constant matrix

As stated earlier, we assume that both the control signal u and the output signal y are scalars. By a proper choice of a set of state variables, it is possible to choose the output to be equal to one of the state variables. (See the method presented in Chapter 3 for obtaining a state-space representation of the transfer function system in which the output y becomes equal to x_1.)

Figure 12–4 shows a general configuration of the type 1 servo system when the plant has an integrator. Here we assumed that $y = x_1$. In the present analysis we assume that the reference input r is a step function. In this system we use the following state-feedback control scheme:

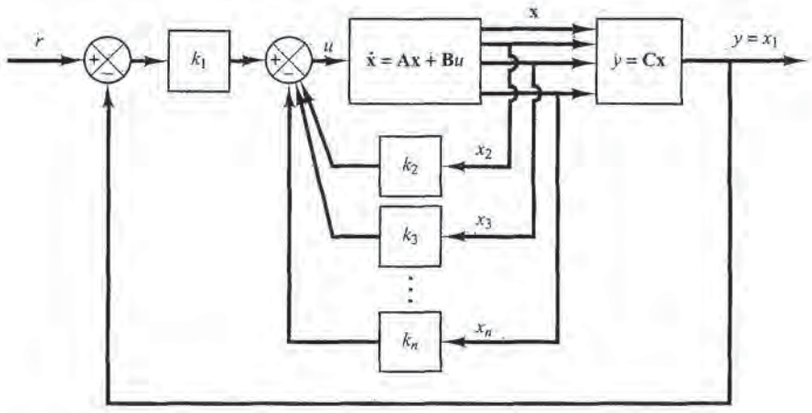


Figure 12–4
Type 1 servo system when the plant has an integrator.

0856

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10641

$$u = -\begin{bmatrix} 0 & k_2 & k_3 & \cdots & k_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \cdot \\ \cdot \\ \cdot \\ x_n \end{bmatrix} + k_1(r - x_1)$$

$$= -\mathbf{K}\mathbf{x} + k_1 r \tag{12–21}$$

where

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & \cdots & k_n \end{bmatrix}$$

Assume that the reference input (step function) is applied at $t = 0$. Then, for $t > 0$, the system dynamics can be described by Equations (12–19) and (12–21), or

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}k_1 r \tag{12–22}$$

We shall design the type 1 servo system such that the closed-loop poles are located at desired positions. The designed system will be an asymptotically stable system, $y(\infty)$ will approach the constant value r, and $u(\infty)$ will approach zero. (r is a step input.)

Notice that at steady state we have

$$\dot{\mathbf{x}}(\infty) = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(\infty) + \mathbf{B}k_1 r(\infty) \tag{12–23}$$

Noting that $r(t)$ is a step input, we have $r(\infty) = r(t) = r(\text{constant})$ for $t > 0$. By subtracting Equation (12–23) from Equation (12–22), we obtain

$$\dot{\mathbf{x}}(t) - \dot{\mathbf{x}}(\infty) = (\mathbf{A} - \mathbf{B}\mathbf{K})[\mathbf{x}(t) - \mathbf{x}(\infty)] \tag{12–24}$$

Define

$$\mathbf{x}(t) - \mathbf{x}(\infty) = \mathbf{e}(t)$$

Then Equation (12–24) becomes

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{e} \tag{12–25}$$

Equation (12–25) describes the error dynamics.

The design of the type 1 servo system here is converted to the design of an asymptotically stable regulator system such that $\mathbf{e}(t)$ approaches zero, given any initial condition $\mathbf{e}(0)$. If the system defined by Equation (12–19) is completely state controllable, then, by specifying the desired eigenvalues $\mu_1, \mu_2, \ldots, \mu_n$ for the matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$, matrix $\mathbf{K}$ can be determined by the pole-placement technique presented in Section 12–2.

The steady-state values of $\mathbf{x}(t)$ and $u(t)$ can be found as follows: At steady state ($t = \infty$), we have, from Equation (12–22),

$$\dot{\mathbf{x}}(\infty) = \mathbf{0} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}(\infty) + \mathbf{B}k_1 r$$

Since the desired eigenvalues of $\mathbf{A} - \mathbf{B}\mathbf{K}$ are all in the left-half s plane, the inverse of matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$ exists. Consequently, $\mathbf{x}(\infty)$ can be determined as

$$\mathbf{x}(\infty) = -(\mathbf{A} - \mathbf{B}\mathbf{K})^{-1}\mathbf{B}k_1 r$$

0857

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10642

Also, $u(\infty)$ can be obtained as

$$u(\infty) = -\mathbf{K}\mathbf{x}(\infty) + k_1 r = 0$$

(See Example 12–4 to verify this last equation.)

EXAMPLE 12–4 Design a type 1 servo system when the plant transfer function has an integrator. Assume that the plant transfer function is given by

$$\frac{Y(s)}{U(s)} = \frac{1}{s(s+1)(s+2)}$$

The desired closed-loop poles are $s = -2 \pm j2\sqrt{3}$ and $s = -10$. Assume that the system configuration is the same as that shown in Figure 12–4 and the reference input r is a step function. Obtain the unit-step response of the designed system.

Define state variables x_1, x_2, and x_3 as follows:

$$x_1 = y$$

$$x_2 = \dot{x}_1$$

$$x_3 = \dot{x}_2$$

Then the state-space representation of the system becomes

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–26}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–27}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Referring to Figure 12–4 and noting that $n = 3$, the control signal u is given by

$$u = -(k_2 x_2 + k_3 x_3) + k_1(r - x_1) = -\mathbf{K}\mathbf{x} + k_1 r \tag{12–28}$$

where

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3]$$

The state-feedback gain matrix $\mathbf{K}$ can be obtained easily with MATLAB. See MATLAB Program 12–4.

MATLAB Program 12–4

```
A = [0 1 0;0 0 1;0 -2 -3];
B = [0;0;1];
J = [-2+j*2*sqrt(3) -2-j*2*sqrt(3) -10];
K = acker(A,B,J)

K =

  160.0000   54.0000   11.0000
```


0858

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10643

The state feedback gain matrix **K** is thus

$$K = [160 \quad 54 \quad 11]$$

Unit-Step Response of the Designed System: The unit-step response of the designed system can be obtained as follows:

Since

$$\mathbf{A} - \mathbf{BK} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} [160 \quad 54 \quad 11] = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{bmatrix}$$

from Equation (12–22) the state equation for the designed system is

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -160 & -56 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 160 \end{bmatrix} r \tag{12–29}$$

and the output equation is

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \tag{12–30}$$

Solving Equations (12–29) and (12–30) for $y(t)$ when r is a unit-step function gives the unit-step response curve $y(t)$ versus t. MATLAB Program 12–5 yields the unit-step response. The resulting unit-step response curve is shown in Figure 12–5.

MATLAB Program 12–5

```
% ---------- Unit-step response ----------
% ***** Enter the state matrix, control matrix, output matrix,
% and direct transmission matrix of the designed system *****
AA = [0  1  0;0  0  1;-160  -56  -14];
BB = [0;0;160];
CC = [1  0  0];
DD = [0];
% ***** Enter step command and plot command *****
t = 0:0.01:5;
y = step(AA,BB,CC,DD,1,t);
plot(t,y)
grid
title('Unit-Step Response')
xlabel('t Sec')
ylabel('Output y')
```


0859

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10644

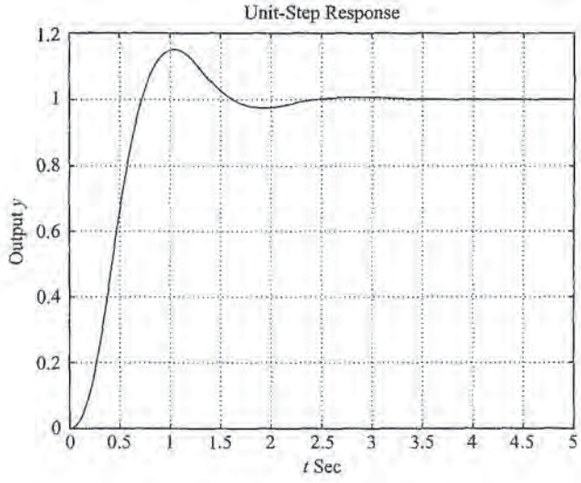


Figure 12–5
Unit-step response curve $y(t)$ versus t for the system designed in Example 12–4.

Note that since

$$u(\infty) = -\mathbf{K}\mathbf{x}(\infty) + k_1 r(\infty) = -\mathbf{K}\mathbf{x}(\infty) + k_1 r$$

we have

$$u(\infty) = -[160 \quad 54 \quad 11]\begin{bmatrix} x_1(\infty) \\ x_2(\infty) \\ x_3(\infty) \end{bmatrix} + 160r$$

$$= -[160 \quad 54 \quad 11]\begin{bmatrix} r \\ 0 \\ 0 \end{bmatrix} + 160r = 0$$

At steady state the control signal u becomes zero.

Design of Type 1 Servo System when the Plant Has No Integrator. If the plant has no integrator (type 0 plant), the basic principle of the design of a type 1 servo system is to insert an integrator in the feedforward path between the error comparator and the plant, as shown in Figure 12–6. (The block diagram of Figure 12–6 is a basic form of the type 1 servo system where the plant has no integrator.) From the diagram, we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–31}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–32}$$

$$u = -\mathbf{K}\mathbf{x} + k_I\xi \tag{12–33}$$

$$\dot{\xi} = r - y = r - \mathbf{C}\mathbf{x} \tag{12–34}$$

where $\mathbf{x}$ = state vector of the plant (n-vector)

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10645

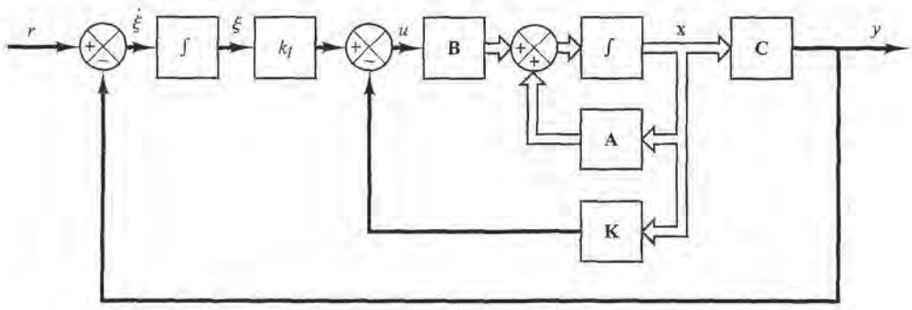


Figure 12–6
Type 1 servo system.

u = control signal (scalar)
y = output signal (scalar)
ξ = output of the integrator (state variable of the system, scalar)
r = reference input signal (step function, scalar)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times 1$ constant matrix
$\mathbf{C}$ = $1 \times n$ constant matrix

We assume that the plant given by Equation (12–31) is completely state controllable. The transfer function of the plant can be given by

$$G_p(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

To avoid the possibility of the inserted integrator being canceled by the zero at the origin of the plant, we assume that $G_p(s)$ has no zero at the origin.

Assume that the reference input (step function) is applied at $t = 0$. Then, for $t > 0$, the system dynamics can be described by an equation that is a combination of Equations (12–31) and (12–34):

$$\begin{bmatrix} \dot{\mathbf{x}}(t) \\ \dot{\xi}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) \\ \xi(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u(t) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r(t) \qquad (12\text{–}35)$$

We shall design an asymptotically stable system such that $\mathbf{x}(\infty)$, $\xi(\infty)$, and $u(\infty)$ approach constant values, respectively. Then, at steady state, $\dot{\xi}(t) = 0$, and we get $y(\infty) = r$.

Notice that at steady state we have

$$\begin{bmatrix} \dot{\mathbf{x}}(\infty) \\ \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(\infty) \\ \xi(\infty) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u(\infty) + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r(\infty) \qquad (12\text{–}36)$$

Noting that $r(t)$ is a step input, we have $r(\infty) = r(t) = r$ (constant) for $t > 0$. By subtracting Equation (12–36) from Equation (12–35), we obtain

$$\begin{bmatrix} \dot{\mathbf{x}}(t) - \dot{\mathbf{x}}(\infty) \\ \dot{\xi}(t) - \dot{\xi}(\infty) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}(t) - \mathbf{x}(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} [u(t) - u(\infty)] \qquad (12\text{–}37)$$

0861

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10646

Define

$$\mathbf{x}(t) - \mathbf{x}(\infty) = \mathbf{x}_e(t)$$
$$\xi(t) - \xi(\infty) = \xi_e(t)$$
$$u(t) - u(\infty) = u_e(t)$$

Then Equation (12–37) can be written as

$$\begin{bmatrix} \dot{\mathbf{x}}_e(t) \\ \dot{\xi}_e(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_e(t) \\ \xi_e(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u_e(t) \tag{12–38}$$

where

$$u_e(t) = -\mathbf{K}\mathbf{x}_e(t) + k_I \xi_e(t) \tag{12–39}$$

Define a new $(n + 1)$th-order error vector $\mathbf{e}(t)$ by

$$\mathbf{e}(t) = \begin{bmatrix} \mathbf{x}_e(t) \\ \xi_e(t) \end{bmatrix} = (n + 1)\text{-vector}$$

Then Equation (12–38) becomes

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{e} + \hat{\mathbf{B}}u_e \tag{12–40}$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix}, \qquad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix}$$

and Equation (12–39) becomes

$$u_e = -\hat{\mathbf{K}}\mathbf{e} \tag{12–41}$$

where

$$\hat{\mathbf{K}} = [\mathbf{K} \mid -k_I]$$

The state error equation can be obtained by substituting Equation (12–41) into Equation (12–40):

$$\dot{\mathbf{e}} = (\hat{\mathbf{A}} - \hat{\mathbf{B}}\hat{\mathbf{K}})\mathbf{e} \tag{12–42}$$

If the desired eigenvalues of matrix $\hat{\mathbf{A}} - \hat{\mathbf{B}}\hat{\mathbf{K}}$ (that is, the desired closed-loop poles) are specified as $\mu_1, \mu_2, \ldots, \mu_{n+1}$, then the state-feedback gain matrix $\mathbf{K}$ and the integral gain constant k_I can be determined by the pole-placement technique pesented in Section 12–2, provided that the system defined by Equation (12–40) is completely state controllable. Note that if the matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}$$

has rank $n + 1$, then the system defined by Equation (12–40) is completely state controllable. (See Problem **A–12–12**.)

0862

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10647

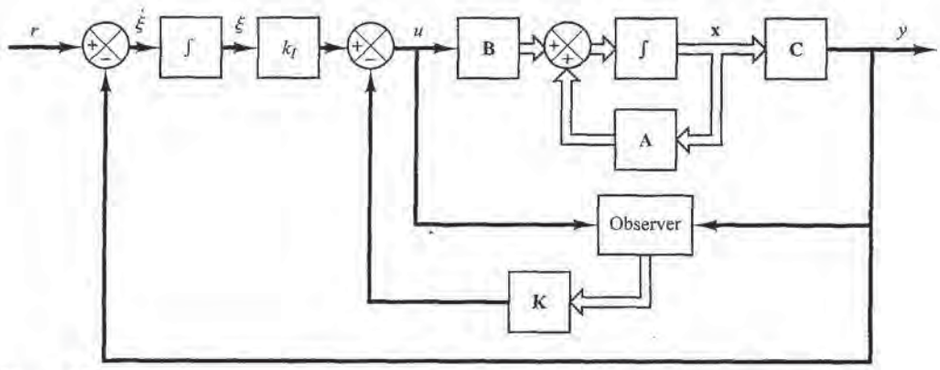


Figure 12–7
Type 1 servo system with state observer.

As is usually the case, not all state variables can be directly measurable. If this is the case, we need to use a state observer. Figure 12–7 shows a block diagram of a type 1 servo system with a state observer. [In the figure, each block with an integral symbol represents an integrator $(1/s)$.] Detailed discussions of state observers are given in Section 12–5.

EXAMPLE 12–5 Consider the inverted-pendulum control system shown in Figure 12–8. In this example, we are concerned only with the motion of the pendulum and motion of the cart in the plane of the page.

It is desired to keep the inverted pendulum upright as much as possible and yet control the position of the cart, for instance, move the cart in a step fashion. To control the position of the cart, we need to build a type 1 servo system. The inverted-pendulum system mounted on a cart does not have an integrator. Therefore, we feed the position signal y (which indicates the position of the cart) back to the input and insert an integrator in the feedforward path, as shown in Figure

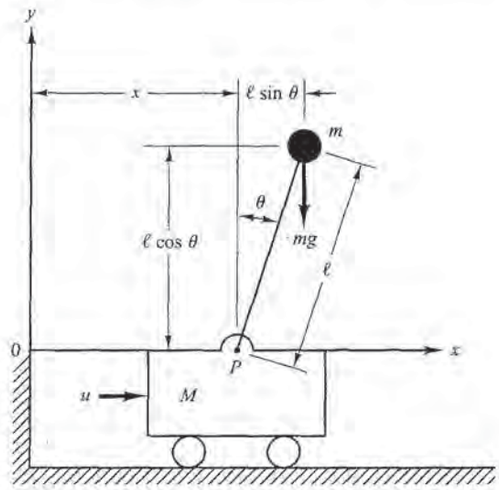


Figure 12–8
Inverted-pendulum control system.

0863

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10648

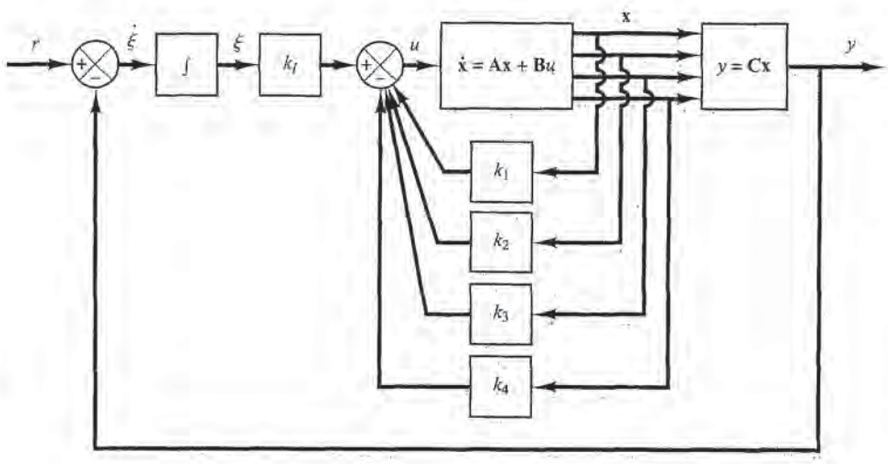


Figure 12–9
Inverted-pendulum control system. (Type 1 servo system when the plant has no integrator.)

12–9. We assume that the pendulum angle θ and the angular velocity $\dot{\theta}$ are small, so that $\sin\theta \doteq \theta$, $\cos\theta \doteq 1$, and $\theta\dot{\theta}^2 \doteq 0$. We also assume that the numerical values for M, m, and l are given as

$$M = 2 \text{ kg}, \qquad m = 0.1 \text{ kg}, \qquad l = 0.5 \text{ m}$$

Referring to Equations (3–59) and (3–60), the equations for the inverted-pendulum control system are

$$Ml\ddot{\theta} = (M + m)g\theta - u \tag{12–43}$$

$$M\ddot{x} = u - mg\theta \tag{12–44}$$

When the given numerical values are substituted, Equations (12–43) and (12–44) become

$$\ddot{\theta} = 20.601\theta - u \tag{12–45}$$

$$\ddot{x} = 0.5u - 0.4905\theta \tag{12–46}$$

Let us define the state variables x_1, x_2, x_3, and x_4 as

$$x_1 = \theta$$

$$x_2 = \dot{\theta}$$

$$x_3 = x$$

$$x_4 = \dot{x}$$

Then, referring to Equations (12–45) and (12–46) and Figure 12–9 and considering the cart position x as the output of the system, we obtain the equations for the system as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–47}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–48}$$

$$u = -\mathbf{K}\mathbf{x} + k_I\xi \tag{12–49}$$

$$\dot{\xi} = r - y = r - \mathbf{C}\mathbf{x} \tag{12–50}$$

0864

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10649

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, \quad \mathbf{C} = [0 \quad 0 \quad 1 \quad 0]$$

For the type 1 servo system, we have the state error equation as given by Equation (12–40):

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{e} + \hat{\mathbf{B}}u_e \tag{12–51}$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 20.601 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.4905 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

and the control signal is given by Equation (12–41):

$$u_e = -\hat{\mathbf{K}}\mathbf{e}$$

where

$$\hat{\mathbf{K}} = [\mathbf{K} \mid -k_I] = [k_1 \quad k_2 \quad k_3 \quad k_4 \mid -k_I]$$

To obtain a reasonable speed and damping in the response of the designed system (for example, the settling time of approximately 4 ~ 5 sec and the maximum overshoot of 15% ~ 16% in the step response of the cart), let us choose the desired closed-loop poles at $s = \mu_i$ ($i = 1, 2, 3, 4, 5$), where

$$\mu_1 = -1 + j\sqrt{3}, \quad \mu_2 = -1 - j\sqrt{3}, \quad \mu_3 = -5, \quad \mu_4 = -5, \quad \mu_5 = -5$$

We shall determine the necessary state-feedback gain matrix by the use of MATLAB.

Before we proceed further, we must examine the rank of matrix $\mathbf{P}$, where

$$\mathbf{P} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}$$

Matrix $\mathbf{P}$ is given by

$$\mathbf{P} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 20.601 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ -0.4905 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix} \tag{12–52}$$

The rank of this matrix can be found to be 5. Therefore, the system defined by Equation (12–51) is completely state controllable, and arbitrary pole placement is possible. MATLAB Program 12–6 produces the state feedback gain matrix $\hat{\mathbf{K}}$.

0865

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10650

MATLAB Program 12–6

```
A = [0 1 0 0; 20.601 0 0 0; 0 0 0 1; -0.4905 0 0 0];
B = [0;-1;0;0.5];
C = [0 0 1 0];
Ahat = [A zeros(4,1); -C 0];
Bhat = [B;0];
J = [-1+j*sqrt(3) -1-j*sqrt(3) -5 -5 -5];
Khat = acker(Ahat,Bhat,J)
Khat =
 -157.6336 -35.3733 -56.0652 -36.7466 50.9684
```

Thus, we get

$$\mathbf{K} = [k_1 \quad k_2 \quad k_3 \quad k_4] = [-157.6336 \quad -35.3733 \quad -56.0652 \quad -36.7466]$$

and

$$k_I = -50.9684$$

Unit Step-Response Characteristics of the Designed System. Once we determine the feedback gain matrix $\mathbf{K}$ and the integral gain constant k_1, the step response in the cart position can be obtained by solving the following equation, which is obtained by substituting Equation (12–49) into Equation (12–35):

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{B}k_I \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \xi \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r \tag{12–53}$$

The output of the system is $x_3(t)$, or

$$y = [0 \quad 0 \quad 1 \quad 0 \quad 0] \begin{bmatrix} \mathbf{x} \\ \xi \end{bmatrix} + [0]r \tag{12–54}$$

Define the state matrix, control matrix, output matrix, and direct transmission matrix of the system given by Equations (12–53) and (12–54) as AA, BB, CC, and DD, respectively. MATLAB Program 12–7 may be used to obtain the step-response curves of the designed system. Notice that, to obtain the unit-step response, we entered the command

```
[y,x,t] = step(AA,BB,CC,DD,1,t)
```

Figure 12–10 shows curves x_1 versus t, x_2 versus t, x_3 (= output y) versus t, x_4 versus t, and x_5 (= ξ) versus t. Notice that $y(t)$ [= $x_3(t)$] has approximately 15% overshoot and the settling time is approximately 4.5 sec. $\xi(t)$ [= $x_5(t)$] approaches 1.1. This result can be derived as follows: Since

$$\dot{\mathbf{x}}(\infty) = \mathbf{0} = \mathbf{A}\mathbf{x}(\infty) + \mathbf{B}u(\infty)$$

or

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ r \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix} u(\infty)$$

0866

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10651

MATLAB Program 12–7

```
%**** The following program is to obtain step response
% of the inverted-pendulum system just designed *****

A = [0  1  0  0;20.601  0  0  0;0  0  0  1;-0.4905  0  0  0];
B = [0;-1;0;0.5];
C = [0  0  1  0]
D = [0];
K = [-157.6336  -35.3733  -56.0652  -36.7466];
KI = -50.9684;
AA = [A - B*K  B*KI;-C  0];
BB = [0;0;0;0;1];
CC = [C  0];
DD = [0];

%***** To obtain response curves x1 versus t, x2 versus t,
% x3 versus t, x4 versus t, and x5 versus t, separately, enter
% the following command *****

t = 0:0,02:6;
[y,x,t] = step(AA,BB,CC,DD,1,t);

x1 = [1  0  0  0  0]*x';
x2 = [0  1  0  0  0]*x';
x3 = [0  0  1  0  0]*x';
x4 = [0  0  0  1  0]*x';
x5 = [0  0  0  0  1]*x';

subplot(3,2,1); plot(t,x1); grid
title('x1 versus t')
xlabel('t Sec'); ylabel('x1')

subplot(3,2,2); plot(t,x2); grid
title('x2 versus t')
xlabel('t Sec'); ylabel('x2')

subplot(3,2,3); plot(t,x3); grid
title('x3 versus t')
xlabel('t Sec'); ylabel('x3')

subplot(3,2,4); plot(t,x4); grid
title('x4 versus t')
xlabel('t Sec'); ylabel('x4')

subplot(3,2,5); plot(t,x5); grid
title('x5 versus t')
xlabel('t Sec'); ylabel('x5')
```


0867

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10652

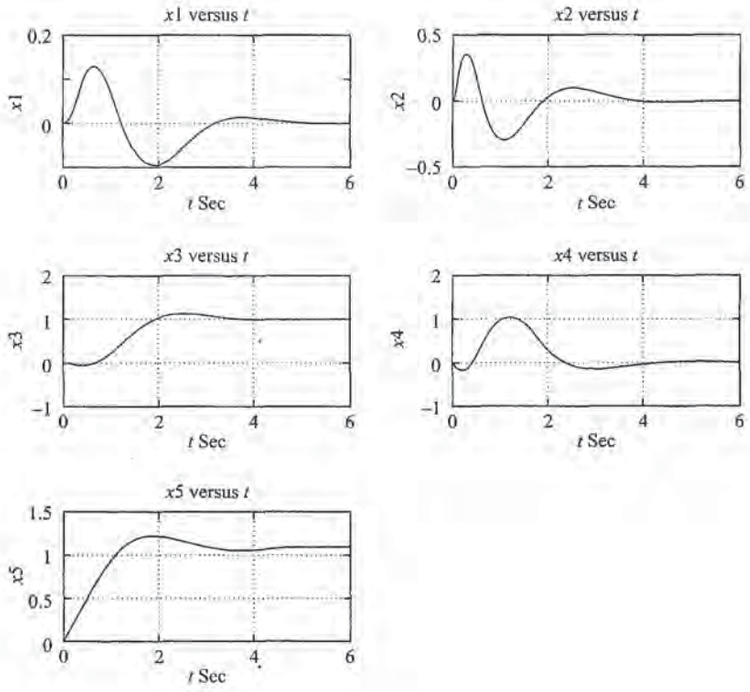


Figure 12–10
Curves x_1 versus t, x_2 versus t, x_3 (= output y) versus t, x_4 versus t, and $x_5 (= \xi)$ versus t.

we get

$$u(\infty) = 0$$

Since $u(\infty) = 0$, we have, from Equation (12–33),

$$u(\infty) = 0 = -\mathbf{K}\mathbf{x}(\infty) + k_1\xi(\infty)$$

and so

$$\xi(\infty) = \frac{1}{k_1}\left[\mathbf{K}\mathbf{x}(\infty)\right] = \frac{1}{k_1}k_3x_3(\infty) = \frac{-56.0652}{-50.9684}r = 1.1r$$

Hence, for $r = 1$, we have

$$\xi(\infty) = 1.1$$

It is noted that, as in any design problem, if the speed and damping are not quite satisfactory, then we must modify the desired characteristic equation and determine a new matrix $\hat{\mathbf{K}}$. Computer simulations must be repeated until a satisfactory result is obtained.

12–5 STATE OBSERVERS

In the pole-placement approach to the design of control systems, we assumed that all state variables are available for feedback. In practice, however, not all state variables are available for feedback. Then we need to estimate unavailable state variables.

0868

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10653

Estimation of unmeasurable state variables is commonly called *observation*. A device (or a computer program) that estimates or observes the state variables is called a *state observer*, or simply an *observer*. If the state observer observes all state variables of the system, regardless of whether some state variables are available for direct measurement, it is called a *full-order state observer*. There are times when this will not be necessary, when we will need observation of only the unmeasurable state variables, but not of those that are directly measurable as well. For example, since the output variables are observable and they are linearly related to the state variables, we need not observe all state variables, but observe only $n - m$ state variables, where n is the dimension of the state vector and m is the dimension of the output vector.

An observer that estimates fewer than n state variables, where n is the dimension of the state vector, is called a *reduced-order state observer* or, simply, a *reduced-order observer*. If the order of the reduced-order state observer is the minimum possible, the observer is called a *minimum-order state observer* or *minimum-order observer*. In this section, we shall discuss both the full-order state observer and the minimum-order state observer.

State Observer. A state observer estimates the state variables based on the measurements of the output and control variables. Here the concept of observability discussed in Section 11–7 plays an important role. As we shall see later, state observers can be designed if and only if the observability condition is satisfied.

In the following discussions of state observers, we shall use the notation $\tilde{\mathbf{x}}$ to designate the observed state vector. In many practical cases, the observed state vector $\tilde{\mathbf{x}}$ is used in the state feedback to generate the desired control vector.

Consider the plant defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–55}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–56}$$

The observer is a subsystem to reconstruct the state vector of the plant. The mathematical model of the observer is basically the same as that of the plant, except that we include an additional term that includes the estimation error to compensate for inaccuracies in matrices $\mathbf{A}$ and $\mathbf{B}$ and the lack of the initial error. The estimation error or observation error is the difference between the measured output and the estimated output. The initial error is the difference between the initial state and the initial estimated state. Thus, we define the mathematical model of the observer to be

$$\begin{aligned}\dot{\tilde{\mathbf{x}}} &= \mathbf{A}\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e(y - \mathbf{C}\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{K}_e\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e y\end{aligned} \tag{12–57}$$

where $\tilde{\mathbf{x}}$ is the estimated state and $\mathbf{C}\tilde{\mathbf{x}}$ is the estimated output. The inputs to the observer are the output y and the control input u. Matrix $\mathbf{K}_e$, which is called the observer gain matrix, is a weighting matrix to the correction term involving the difference between the measured output y and the estimated output $\mathbf{C}\tilde{\mathbf{x}}$. This term continuously corrects the model output and improves the performance of the observer. Figure 12–11 shows the block diagram of the system and the full-order state observer.

0869

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10654

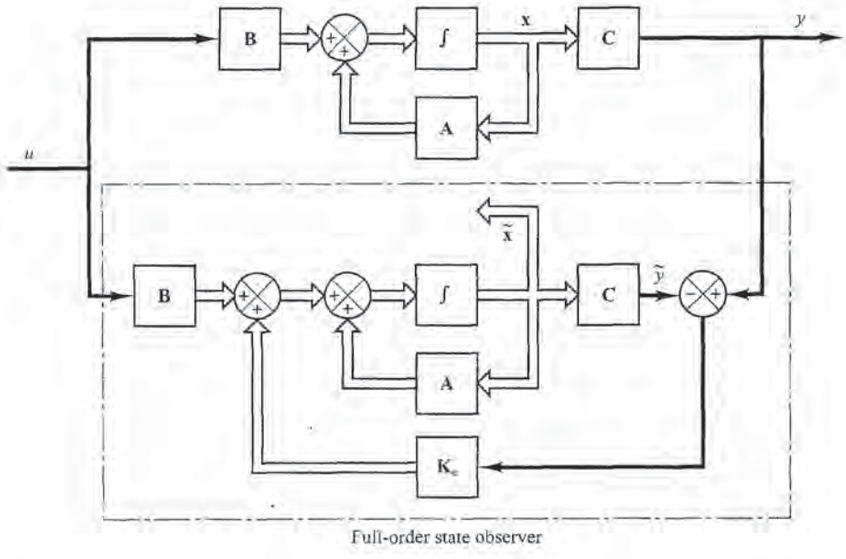


Figure 12–11
Block diagram of system and full-order state observer, when input u and output y are scalars.

Full-Order State Observer. The order of the state observer that will be discussed here is the same as that of the plant. Assume that the plant is defined by Equations (12–55) and (12–56) and the observer model is defined by Equation (12–57).

To obtain the observer error equation, let us subtract Equation (12–57) from Equation (12–55):

$$\begin{aligned}\dot{\mathbf{x}} - \dot{\tilde{\mathbf{x}}} &= \mathbf{A}\mathbf{x} - \mathbf{A}\tilde{\mathbf{x}} - \mathbf{K}_e(\mathbf{C}\mathbf{x} - \mathbf{C}\tilde{\mathbf{x}}) \\ &= (\mathbf{A} - \mathbf{K}_e\mathbf{C})(\mathbf{x} - \tilde{\mathbf{x}})\end{aligned} \tag{12–58}$$

Define the difference between $\mathbf{x}$ and $\tilde{\mathbf{x}}$ as the error vector $\mathbf{e}$, or

$$\mathbf{e} = \mathbf{x} - \tilde{\mathbf{x}}$$

Then Equation (12–58) becomes

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\mathbf{e} \tag{12–59}$$

From Equation (12–59), we see that the dynamic behavior of the error vector is determined by the eigenvalues of matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$. If matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ is a stable matrix, the error vector will converge to zero for any initial error vector $\mathbf{e}(0)$. That is $\tilde{\mathbf{x}}(t)$ will converge to $\mathbf{x}(t)$ regardless of the values of $\mathbf{x}(0)$ and $\tilde{\mathbf{x}}(0)$. If the eigenvalues of matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ are chosen in such a way that the dynamic behavior of the error vector is asymptotically stable and is adequately fast, then any error vector will tend to zero (the origin) with an adequate speed.

If the plant is completely observable, then it can be proved that it is possible to choose matrix $\mathbf{K}_e$ such that $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ has arbitrarily desired eigenvalues. That is, the observer gain matrix $\mathbf{K}_e$ can be determined to yield the desired matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$. We shall discuss this matter in what follows.

0870

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10655

Dual Problem. The problem of designing a full-order observer becomes that of determining the observer gain matrix $\mathbf{K}_e$ such that the error dynamics defined by Equation (12–59) are asymptotically stable with sufficient speed of response. (The asymptotic stability and the speed of response of the error dynamics are determined by the eigenvalues of matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$.) Hence, the design of the full-order observer becomes that of determining an appropriate $\mathbf{K}_e$ such that $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ has desired eigenvalues. Thus, the problem here becomes the same as the pole-placement problem we discussed in Section 12–2. In fact, the two problems are mathematically the same. This property is called duality.

Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

In designing the full-order state observer, we may solve the dual problem, that is, solve the pole-placement problem for the dual system

$$\dot{\mathbf{z}} = \mathbf{A}^*\mathbf{z} + \mathbf{C}^*v$$

$$n = \mathbf{B}^*\mathbf{z}$$

assuming the control signal v to be

$$v = -\mathbf{K}\mathbf{z}$$

If the dual system is completely state controllable, then the state feedback gain matrix $\mathbf{K}$ can be determined such that matrix $\mathbf{A}^* - \mathbf{C}^*\mathbf{K}$ will yield a set of the desired eigenvalues.

If $\mu_1, \mu_2, \cdots, \mu_n$ are the desired eigenvalues of the state observer matrix, then by taking the same μ_i's as the desired eigenvalues of the state-feedback gain matrix of the dual system, we obtain

$$|s\mathbf{I} - (\mathbf{A}^* - \mathbf{C}^*\mathbf{K})| = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n)$$

Noting that the eigenvalues of $\mathbf{A}^* - \mathbf{C}^*\mathbf{K}$ and those of $\mathbf{A} - \mathbf{K}^*\mathbf{C}$ are the same, we have

$$|s\mathbf{I} - (\mathbf{A}^* - \mathbf{C}^*\mathbf{K})| = |s\mathbf{I} - (\mathbf{A} - \mathbf{K}^*\mathbf{C})|$$

Comparing the characteristic polynomial $|s\mathbf{I} - (\mathbf{A} - \mathbf{K}^*\mathbf{C})|$ and the characteristic polynomial $|s\mathbf{I} - (\mathbf{A} - \mathbf{K}_e\mathbf{C})|$ for the observer system [refer to Equation (12–57)], we find that $\mathbf{K}_e$ and $\mathbf{K}^*$ are related by

$$\mathbf{K}_e = \mathbf{K}^*$$

Thus, using the matrix $\mathbf{K}$ determined by the pole-placement approach in the dual system, the observer gain matrix $\mathbf{K}_e$ for the original system can be determined by using the relationship $\mathbf{K}_e = \mathbf{K}^*$. (See Problem **A–12–10** for the details.)

Necessary and Sufficient Condition for State Observation. As discussed, a necessary and sufficient condition for the determination of the observer gain matrix $\mathbf{K}_e$ for the desired eigenvalues of $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ is that the dual of the original system

$$\dot{\mathbf{z}} = \mathbf{A}^*\mathbf{z} + \mathbf{C}^*v$$

0871

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10656

be completely state controllable. The complete state controllability condition for this dual system is that the rank of

$$[\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

be n. This is the condition for complete observability of the original system defined by Equations (12–55) and (12–56). This means that a necessary and sufficient condition for the observation of the state of the system defined by Equations (12–55) and (12–56) is that the system be completely observable.

Once we select the desired eigenvalues (or desired characteristic equation), the full-order state observer can be designed, provided the plant is completely observable. The desired eigenvalues of the characteristic equation should be chosen so that the state observer responds at least two to five times faster than the closed-loop system considered. As stated earlier, the equation for the full-order state observer is

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e y \tag{12–60}$$

It is noted that thus far we have assumed the matrices **A**, **B**, and **C** in the observer to be exactly the same as those of the physical plant. If there are discrepancies in **A**, **B**, and **C** in the observer and in the physical plant, the dynamics of the observer error are no longer governed by Equation (12–59). This means that the error may not approach zero as expected. Therefore, we need to choose $\mathbf{K}_e$ so that the observer is stable and the error remains acceptably small in the presence of small modeling errors.

Transformation Approach to Obtain State Observer Gain Matrix $\mathbf{K}_e$. By following the same approach as we used in deriving the equation for the state feedback gain matrix **K**, we can obtain the following equation:

$$\mathbf{K}_e = \mathbf{Q}\begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_1 - a_1 \end{bmatrix} = (\mathbf{W}\mathbf{N}^*)^{-1}\begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_1 - a_1 \end{bmatrix} \tag{12–61}$$

where $\mathbf{K}_e$ is an $n \times 1$ matrix,

$$\mathbf{Q} = (\mathbf{W}\mathbf{N}^*)^{-1}$$

and

$$\mathbf{N} = [\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

[Refer to Problem **A–12–10** for the derivation of Equation (12–61).]

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10657

Direct Substitution Approach to Obtain State Observer Gain Matrix $\mathbf{K}_e$. Similar to the case of pole placement, if the system is of low order, then direct substitution of matrix $\mathbf{K}_e$ into the desired characteristic polynomial may be simpler. For example, if $\mathbf{x}$ is a 3-vector, then write the observer gain matrix $\mathbf{K}_e$ as

$$\mathbf{K}_e = \begin{bmatrix} k_{e1} \\ k_{e2} \\ k_{e3} \end{bmatrix}$$

Substitute this $\mathbf{K}_e$ matrix into the desired characteristic polynomial:

$$|s\mathbf{I} - (\mathbf{A} - \mathbf{K}_e\mathbf{C})| = (s - \mu_1)(s - \mu_2)(s - \mu_3)$$

By equating the coefficients of the like powers of s on both sides of this last equation, we can determine the values of k_{e1}, k_{e2}, and k_{e3}. This approach is convenient if $n = 1$, 2, or 3, where n is the dimension of the state vector $\mathbf{x}$. (Although this approach can be used when $n = 4, 5, 6, \ldots$, the computations involved may become very tedious.)

Another approach to the determination of the state observer gain matrix $\mathbf{K}_e$ is to use Ackermann's formula. This approach is presented in the following.

Ackermann's Formula. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–62}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–63}$$

In Section 12–2 we derived Ackermann's formula for pole placement for the system defined by Equation (12–62). The result was given by Equation (12–18), rewritten thus:

$$\mathbf{K} = [0 \quad 0 \cdots 0 \quad 1][\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}]^{-1}\phi(\mathbf{A})$$

For the dual of the system defined by Equations (12–62) and (12–63),

$$\dot{\mathbf{z}} = \mathbf{A}^*\mathbf{z} + \mathbf{C}^*v$$

$$n = \mathbf{B}^*\mathbf{z}$$

the preceding Ackermann's formula for pole placement is modified to

$$\mathbf{K} = [0 \quad 0 \cdots 0 \quad 1][\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]^{-1}\phi(\mathbf{A}^*) \tag{12–64}$$

As stated earlier, the state observer gain matrix $\mathbf{K}_e$ is given by $\mathbf{K}^*$, where $\mathbf{K}$ is given by Equation (12–64). Thus,

$$\mathbf{K}_e = \mathbf{K}^* = \phi(\mathbf{A}^*)^* \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{CA}^{n-2} \\ \mathbf{CA}^{n-1} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \tag{12–65}$$

0873

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10658

where $\phi(s)$ is the desired characteristic polynomial for the state observer, or

$$\phi(s) = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_n)$$

where $\mu_1, \mu_2, \ldots, \mu_n$ are the desired eigenvalues. Equation (12–65) is called Ackermann's formula for the determination of the observer gain matrix $\mathbf{K}_e$.

Comments on Selecting the Best $\mathbf{K}_e$. Referring to Figure 12–11, notice that the feedback signal through the observer gain matrix $\mathbf{K}_e$ serves as a correction signal to the plant model to account for the unknowns in the plant. If significant unknowns are involved, the feedback signal through the matrix $\mathbf{K}_e$ should be relatively large. However, if the output signal is contaminated significantly by disturbances and measurement noises, then the output y is not reliable and the feedback signal through the matrix $\mathbf{K}_e$ should be relatively small. In determining the matrix $\mathbf{K}_e$, we should carefully examine the effects of disturbances and noises involved in the output y.

Remember that the observer gain matrix $\mathbf{K}_e$ depends on the desired characteristic equation

$$(s - \mu_1)(s - \mu_2)\cdots(s - \mu_n) = 0$$

The choice of a set of $\mu_1, \mu_2, \ldots, \mu_n$ is, in many instances, not unique. As a general rule, however, the observer poles must be two to five times faster than the controller poles to make sure the observation error (estimation error) converges to zero quickly. This means that the observer estimation error decays two to five times faster than does the state vector $\mathbf{x}$. Such faster decay of the observer error compared with the desired dynamics makes the controller poles dominate the system response.

It is important to note that if sensor noise is considerable, we may choose the observer poles to be slower than two times the controller poles, so that the bandwidth of the system will become lower and smooth the noise. In this case the system response will be strongly influenced by the observer poles. If the observer poles are located to the right of the controller poles in the left-half s plane, the system response will be dominated by the observer poles rather than by the control poles.

In the design of the state observer, it is desirable to determine several observer gain matrices $\mathbf{K}_e$ based on several different desired characteristic equations. For each of the several different matrices $\mathbf{K}_e$, simulation tests must be run to evaluate the resulting system performance. Then we select the best $\mathbf{K}_e$ from the viewpoint of overall system performance. In many practical cases, the selection of the best matrix $\mathbf{K}_e$ boils down to a compromise between speedy response and sensitivity to disturbances and noises.

EXAMPLE 12–6 Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = [0 \quad 1]$$

We use the observed state feedback such that

$$u = -\mathbf{K}\hat{\mathbf{x}}$$

0874

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10659

Design a full-order state observer, assuming that the system configuration is identical to that shown in Figure 12–11. Assume that the desired eigenvalues of the observer matrix are

$$\mu_1 = -10, \qquad \mu_2 = -10$$

The design of the state observer reduces to the determination of an appropriate observer gain matrix $\mathbf{K}_e$.

Let us examine the observability matrix. The rank of

$$[\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^*] = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

is 2. Hence, the system is completely observable and the determination of the desired observer gain matrix is possible. We shall solve this problem by three methods.

Method 1: We shall determine the observer gain matrix by use of Equation (12–61). The given system is already in the observable canonical form. Hence, the transformation matrix $\mathbf{Q} = (\mathbf{WN}^*)^{-1}$ is $\mathbf{I}$. Since the characteristic equation of the given system is

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & -20.6 \\ -1 & s \end{vmatrix} = s^2 - 20.6 = s^2 + a_1 s + a_2 = 0$$

we have

$$a_1 = 0, \qquad a_2 = -20.6$$

The desired characteristic equation is

$$(s + 10)^2 = s^2 + 20s + 100 = s^2 + \alpha_1 s + \alpha_2 = 0$$

Hence,

$$\alpha_1 = 20, \qquad \alpha_2 = 100$$

Then the observer gain matrix $\mathbf{K}_e$ can be obtained from Equation (12–61) as follows:

$$\mathbf{K}_e = (\mathbf{WN}^*)^{-1}\begin{bmatrix} \alpha_2 - a_2 \\ \alpha_1 - a_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 100 + 20.6 \\ 20 - 0 \end{bmatrix} = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

Method 2: Referring to Equation (12–59):

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\mathbf{e}$$

the characteristic equation for the observer becomes

$$|s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C}| = 0$$

Define

$$\mathbf{K}_e = \begin{bmatrix} k_{e1} \\ k_{e2} \end{bmatrix}$$

Then the characteristic equation becomes

$$\left|\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} + \begin{bmatrix} k_{e1} \\ k_{e2} \end{bmatrix}[0 \quad 1]\right| = \begin{vmatrix} s & -20.6 + k_{e1} \\ -1 & s + k_{e2} \end{vmatrix}$$

$$= s^2 + k_{e2}s - 20.6 + k_{e1} = 0 \tag{12–66}$$

0875

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10660

Since the desired characteristic equation is

$$s^2 + 20s + 100 = 0$$

by comparing Equation (12–66) with this last equation, we obtain

$$k_{e1} = 120.6, \qquad k_{e2} = 20$$

or

$$\mathbf{K}_e = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

Method 3: We shall use Ackermann's formula given by Equation (12–65):

$$\mathbf{K}_e = \phi(\mathbf{A}) \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where

$$\phi(s) = (s - \mu_1)(s - \mu_2) = s^2 + 20s + 100$$

Thus,

$$\phi(\mathbf{A}) = \mathbf{A}^2 + 20\mathbf{A} + 100\mathbf{I}$$

and

$$\mathbf{K}_e = (\mathbf{A}^2 + 20\mathbf{A} + 100\mathbf{I}) \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 120.6 & 412 \\ 20 & 120.6 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 120.6 \\ 20 \end{bmatrix}$$

As a matter of course, we get the same $\mathbf{K}_e$ regardless of the method employed.

The equation for the full-order state observer is given by Equation (12–57),

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e y$$

or

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & -100 \\ 1 & -20 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 120.6 \\ 20 \end{bmatrix} y$$

Finally, it is noted that, similar to the case of pole placement, if the system order n is 4 or higher, methods 1 and 3 are preferred, because all matrix computations can be carried out by a computer, while method 2 always requires hand computation of the characteristic equation involving unknown parameters $k_{e1}, k_{e2}, \ldots, k_{en}$.

Effects of the Addition of the Observer on a Closed-Loop System. In the pole-placement design process, we assumed that the actual state $\mathbf{x}(t)$ was available for feedback. In practice, however, the actual state $\mathbf{x}(t)$ may not be measurable, so we will need to design an observer and use the observed state $\tilde{\mathbf{x}}(t)$ for feedback as shown in Figure 12–12. The design process, therefore, becomes a two-stage process, the first stage being the determination of the feedback gain matrix $\mathbf{K}$ to yield the desired characteristic equation and the second stage being the determination of the observer gain matrix $\mathbf{K}_e$ to yield the desired observer characteristic equation.

Let us now investigate the effects of the use of the observed state $\tilde{\mathbf{x}}(t)$, rather than the actual state $\mathbf{x}(t)$, on the characteristic equation of a closed-loop control system.

0876

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10661

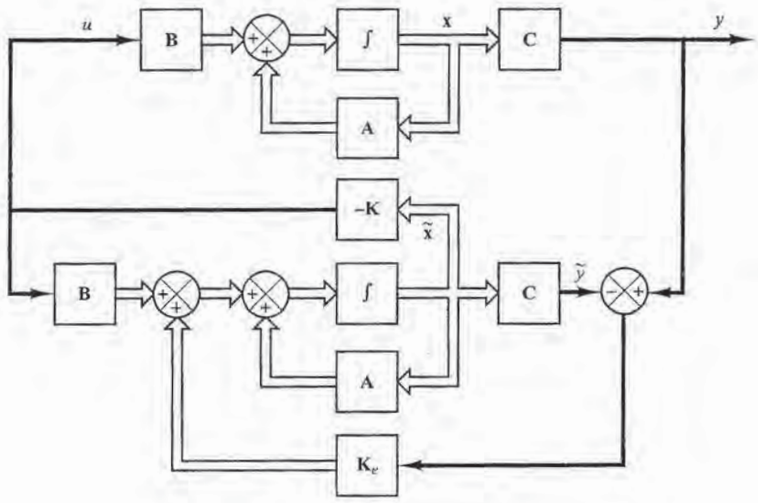


Figure 12–12
Observed-state feedback control system.

Consider the completely state controllable and completely observable system defined by the equations

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

For the state-feedback control based on the observed state $\tilde{\mathbf{x}}$,

$$u = -\mathbf{K}\tilde{\mathbf{x}}$$

With this control, the state equation becomes

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\tilde{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \tag{12–67}$$

The difference between the actual state $\mathbf{x}(t)$ and the observed state $\tilde{\mathbf{x}}(t)$ has been defined as the error $\mathbf{e}(t)$:

$$\mathbf{e}(t) = \mathbf{x}(t) - \tilde{\mathbf{x}}(t)$$

Substitution of the error vector $\mathbf{e}(t)$ into Equation (12–67) gives

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\mathbf{K}\mathbf{e} \tag{12–68}$$

Note that the observer error equation was given by Equation (12–59), repeated here:

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\mathbf{e} \tag{12–69}$$

Combining Equations (12–68) and (12–69), we obtain

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{K} \\ \mathbf{0} & \mathbf{A} - \mathbf{K}_e\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} \tag{12–70}$$

0877

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10662

Equation (12–70) describes the dynamics of the observed-state feedback control system. The characteristic equation for the system is

$$\begin{vmatrix} s\mathbf{I} - \mathbf{A} + \mathbf{BK} & -\mathbf{BK} \\ \mathbf{0} & s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} \end{vmatrix} = 0$$

or

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}||s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C}| = 0$$

Notice that the closed-loop poles of the observed-state feedback control system consist of the poles due to the pole-placement design alone and the poles due to the observer design alone. This means that the pole-placement design and the observer design are independent of each other. They can be designed separately and combined to form the observed-state feedback control system. Note that, if the order of the plant is n, then the observer is also of nth order (if the full-order state observer is used), and the resulting characteristic equation for the entire closed-loop system becomes of order $2n$.

Transfer Function of the Observer-Based Controller. Consider the plant defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$$

$$y = \mathbf{Cx}$$

Assume that the plant is completely observable. Assume that we use observed-state feedback control $u = -\mathbf{K}\tilde{\mathbf{x}}$. Then, the equations for the observer are given by

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C} - \mathbf{BK})\tilde{\mathbf{x}} + \mathbf{K}_e y \tag{12–71}$$

$$u = -\mathbf{K}\tilde{\mathbf{x}} \tag{12–72}$$

where Equation (12–71) is obtained by substituting $u = -\mathbf{K}\tilde{\mathbf{x}}$ into Equation (12–57).

By taking the Laplace transform of Equation (12–71), assuming a zero initial condition, and solving for $\tilde{\mathbf{X}}(s)$, we obtain

$$\tilde{\mathbf{X}}(s) = (s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{BK})^{-1}\mathbf{K}_e Y(s)$$

By substituting this $\tilde{\mathbf{X}}(s)$ into the Laplace transform of Equation (12–72), we obtain

$$U(s) = -\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{BK})^{-1}\mathbf{K}_e Y(s) \tag{12–73}$$

Then the transfer function $U(s)/Y(s)$ can be obtained as

$$\frac{U(s)}{Y(s)} = -\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{BK})^{-1}\mathbf{K}_e$$

Figure 12–13 shows the block diagram representation for the system. Notice that the transfer function

$$\mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{BK})^{-1}\mathbf{K}_e$$

acts as a controller for the system. Hence, we call the transfer function

$$\frac{U(s)}{-Y(s)} = \frac{\text{num}}{\text{den}} = \mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{BK})^{-1}\mathbf{K}_e \tag{12–74}$$

0878

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10663

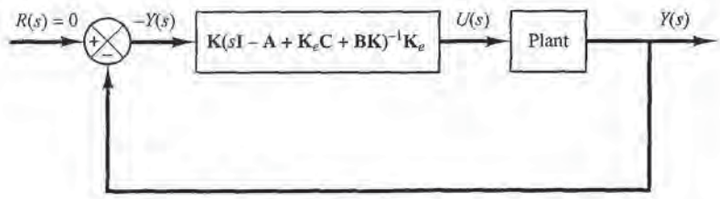


Figure 12–13
Block diagram representation of system with a controller-observer.

the observer-based controller transfer function or, simply, the observer-controller transfer function.

Note that the observer-controller matrix

$$\mathbf{A} - \mathbf{K}_e\mathbf{C} - \mathbf{BK}$$

may or may not be stable, although $\mathbf{A} - \mathbf{BK}$ and $\mathbf{A} - \mathbf{K}_e\mathbf{C}$ are chosen to be stable. In fact, in some cases the matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C} - \mathbf{BK}$ may be poorly stable or even unstable.

EXAMPLE 12–7 Consider the design of a regulator system for the following plant:

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u \tag{12–75}$$

$$y = \mathbf{Cx} \tag{12–76}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 0]$$

Suppose that we use the pole-placement approach to the design of the system and that the desired closed-loop poles for this system are at $s = \mu_i$ $(i = 1, 2)$, where $\mu_1 = -1.8 + j2.4$ and $\mu_2 = -1.8 - j2.4$. The state-feedback gain matrix $\mathbf{K}$ for this case can be obtained as follows:

$$\mathbf{K} = [29.6 \quad 3.6]$$

Using this state-feedback gain matrix $\mathbf{K}$, the control signal u is given by

$$u = -\mathbf{Kx} = -[29.6 \quad 3.6]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Suppose that we use the observed-state feedback control instead of the actual-state feedback control, or

$$u = -\mathbf{K}\tilde{\mathbf{x}} = -[29.6 \quad 3.6]\begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

where we choose the observer poles to be at

$$s = -8, \qquad s = -8$$

Obtain the observer gain matrix $\mathbf{K}_e$ and draw a block diagram for the observed-state feedback control system. Then obtain the transfer function $U(s)/[-Y(s)]$ for the observer controller, and draw another block diagram with the observer controller as a series controller in the feedforward path. Finally, obtain the response of the system to the following initial condition:

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad \mathbf{e}(0) = \mathbf{x}(0) - \tilde{\mathbf{x}}(0) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

0879

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10664

For the system defined by Equation (12–75), the characteristic polynomial is

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & -1 \\ -20.6 & s \end{vmatrix} = s^2 - 20.6 = s^2 + a_1 s + a_2$$

Thus,

$$a_1 = 0, \qquad a_2 = -20.6$$

The desired characteristic polynomial for the observer is

$$(s - \mu_1)(s - \mu_2) = (s + 8)(s + 8) = s^2 + 16s + 64$$
$$= s^2 + \alpha_1 s + \alpha_2$$

Hence,

$$\alpha_1 = 16, \qquad \alpha_2 = 64$$

For the determination of the observer gain matrix, we use Equation (12–61), or

$$\mathbf{K}_e = (\mathbf{W}\mathbf{N}^*)^{-1} \begin{bmatrix} \alpha_2 - a_2 \\ \alpha_1 - a_1 \end{bmatrix}$$

where

$$\mathbf{N} = [\mathbf{C}^* \;\vdots\; \mathbf{A}^*\mathbf{C}^*] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\mathbf{W} = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Hence,

$$\mathbf{K}_e = \left\{ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right\}^{-1} \begin{bmatrix} 64 + 20.6 \\ 16 - 0 \end{bmatrix}$$
$$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 84.6 \\ 16 \end{bmatrix} = \begin{bmatrix} 16 \\ 84.6 \end{bmatrix} \tag{12–77}$$

Equation (12–77) gives the observer gain matrix $\mathbf{K}_e$. The observer equation is given by Equation (12–60):

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e y \tag{12–78}$$

Since

$$u = -\mathbf{K}\tilde{\mathbf{x}}$$

Equation (12–78) becomes

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C} - \mathbf{B}\mathbf{K})\tilde{\mathbf{x}} + \mathbf{K}_e y$$

or

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} - \begin{bmatrix} 16 \\ 84.6 \end{bmatrix} [1 \quad 0] - \begin{bmatrix} 0 \\ 1 \end{bmatrix} [29.6 \quad 3.6] \right\} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 16 \\ 84.6 \end{bmatrix} y$$
$$= \begin{bmatrix} -16 & 1 \\ -93.6 & -3.6 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 16 \\ 84.6 \end{bmatrix} y$$

The block diagram of the system with observed-state feedback is shown in Figure 12–14(a).

0880

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10665

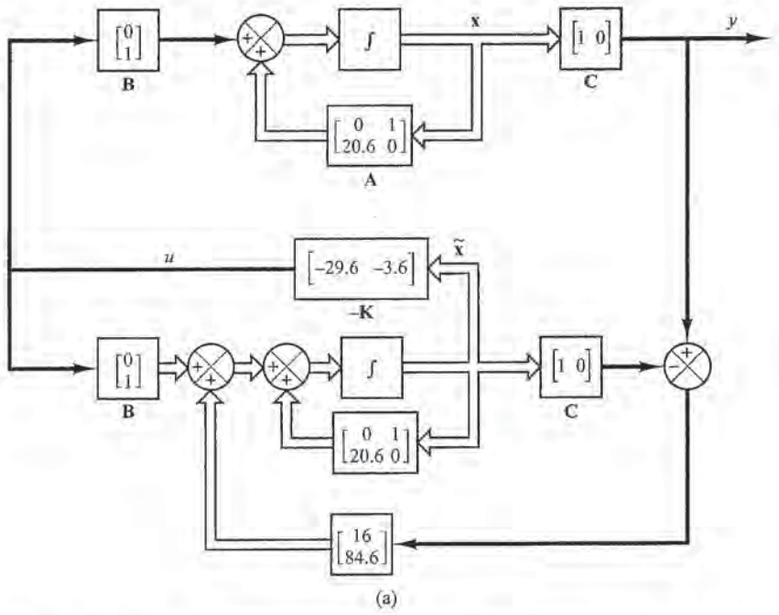


Figure 12–14
(a) Block diagram of system with observed-state feedback; (b) block diagram of transfer function system.

Referring to Equation (12–74), the transfer function of the observer-controller is

$$\frac{U(s)}{-Y(s)} = \mathbf{K}(s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C} + \mathbf{B}\mathbf{K})^{-1}\mathbf{K}_e$$

$$= [29.6 \quad 3.6]\begin{bmatrix} s + 16 & -1 \\ 93.6 & s + 3.6 \end{bmatrix}^{-1}\begin{bmatrix} 16 \\ 84.6 \end{bmatrix}$$

$$= \frac{778.16s + 3690.72}{s^2 + 19.6s + 151.2}$$

The same transfer function can be obtained with MATLAB. For example, MATLAB Program 12–8 produces the transfer function of the observer controller when the observer is of full order. Figure 12–14(b) shows a block diagram of the system.

0881

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10666

MATLAB Program 12–8

```
% Obtaining transfer function of observer controller --- full-order observer
A = [0  1;20.6  0];
B = [0;1];
C = [1  0];
K = [29.6  3.6];
Ke = [16;84.6];
AA = A-Ke*C-B*K;
BB = Ke;
CC = K;
DD = 0;
[num,den] = ss2tf(AA,BB,CC,DD)

num =

  1.0e+003*

    0   0.7782   3.6907

den =

  1.0000  19.6000  151.2000
```

The dynamics of the observed-state feedback control system just designed can be described by the following equations: For the plant,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 20.6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For the observer,

$$\begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{x}}_2 \end{bmatrix} = \begin{bmatrix} -16 & 1 \\ -93.6 & -3.6 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 16 \\ 84.6 \end{bmatrix} y$$

$$u = -[29.6 \quad 3.6] \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix}$$

The system, as a whole, is of fourth order. The characteristic equation for the system is

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}||s\mathbf{I} - \mathbf{A} + \mathbf{K}_e\mathbf{C}| = (s^2 + 3.6s + 9)(s^2 + 16s + 64)$$
$$= s^4 + 19.6s^3 + 130.6s^2 + 374.4s + 576 = 0$$

The characteristic equation can also be obtained from the block diagram for the system shown in Figure 12–14(b). Since the closed-loop transfer function is

$$\frac{Y(s)}{U(s)} = \frac{778.16s + 3690.72}{(s^2 + 19.6s + 151.2)(s^2 - 20.6) + 778.16s + 3690.72}$$

0882

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10667

the characteristic equation is

$$(s^2 + 19.6s + 151.2)(s^2 - 20.6) + 778.16s + 3690.72$$
$$= s^4 + 19.6s^3 + 130.6s^2 + 374.4s + 576 = 0$$

As a matter of course, the characteristic equation is the same for the system in state-space representation and in transfer-function representation.

Finally, we shall obtain the response of the system to the following initial condition:

$$x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \qquad e(0) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix}$$

Referring to Equation (12–70), the response to the initial condition can be determined from

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{K}_e\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix}, \qquad \begin{bmatrix} \mathbf{x}(0) \\ \mathbf{e}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

A MATLAB Program to obtain the response is shown in MATLAB Program 12–9. The resulting response curves are shown in Figure 12–15.

MATLAB Program 12–9

```
A = [0  1; 20.6  0];
B = [0;1];
C = [1  0];
K = [29.6  3.6];
Ke = [16; 84.6];
sys = ss([A-B*K  B*K; zeros(2,2)  A-Ke*C],eye(4),eye(4),eye(4));
t = 0:0.01:4;
z = initial(sys,[1;0;0.5;0],t);
x1 = [1  0  0  0]*z';
x2 = [0  1  0  0]*z';
e1 = [0  0  1  0]*z';
e2 = [0  0  0  1]*z';

subplot(2,2,1); plot(t,x1 ),grid
title('Response to Initial Condition')
ylabel('state variable x1')

subplot(2,2,2); plot(t,x2),grid
title('Response to Initial Condition')
ylabel('state variable x2')

subplot(2,2,3); plot(t,e1),grid
xlabel('t (sec)'), ylabel('error state variable e1')

subplot(2,2,4); plot(t,e2),grid
xlabel('t (sec)'), ylabel('error state variable e2')
```


0883

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10668

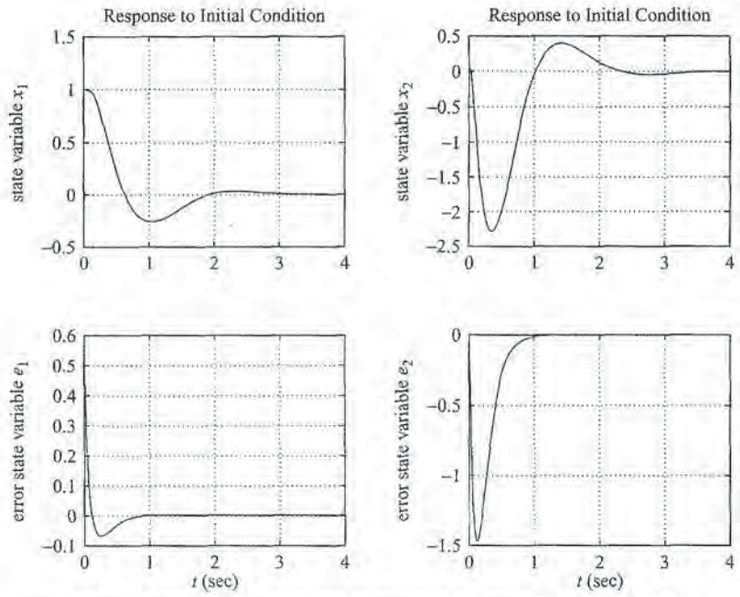


Figure 12–15
Response curves to initial condition.

Minimum-Order Observer. The observers discussed thus far are designed to reconstruct all the state variables. In practice, some of the state variables may be accurately measured. Such accurately measurable state variables need not be estimated.

Suppose that the state vector $\mathbf{x}$ is an n-vector and the output vector $\mathbf{y}$ is an m-vector that can be measured. Since m output variables are linear combinations of the state variables, m state variables need not be estimated. We need to estimate only $n - m$ state variables. Then the reduced-order observer becomes an $(n - m)$th-order observer. Such an $(n - m)$th-order observer is the minimum-order observer. Figure 12–16 shows the block diagram of a system with a minimum-order observer.

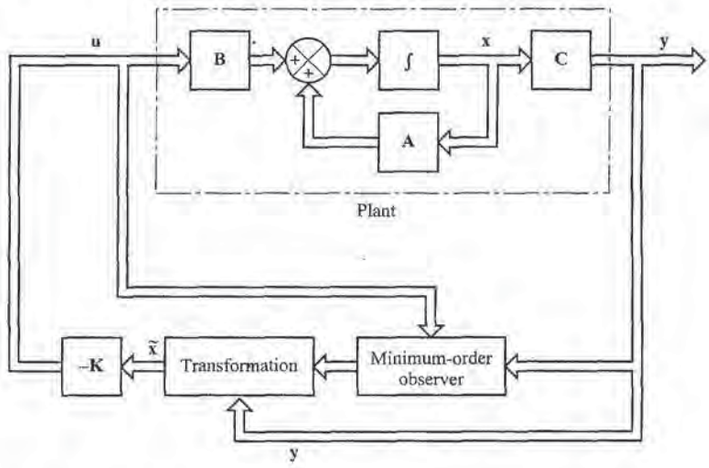


Figure 12–16
Observed-state feedback control system with a minimum-order observer.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10669

It is important to note, however, that if the measurement of output variables involves significant noises and is relatively inaccurate, then the use of the full-order observer may result in a better system performance.

To present the basic idea of the minimum-order observer, without undue mathematical complications, we shall present the case where the output is a scalar (that is, $m = 1$) and derive the state equation for the minimum-order observer. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–79}$$

$$y = \mathbf{C}\mathbf{x} \tag{12–80}$$

where the state vector $\mathbf{x}$ can be partitioned into two parts x_a (a scalar) and $\mathbf{x}_b$ [an $(n - 1)$-vector]. Here the state variable x_a is equal to the output y and thus can be directly measured, and $\mathbf{x}_b$ is the unmeasurable portion of the state vector. Then the partitioned state and output equations become

$$\begin{bmatrix} \dot{x}_a \\ \dot{\mathbf{x}}_b \end{bmatrix} = \left[\begin{array}{c|c} A_{aa} & \mathbf{A}_{ab} \\ \hline \mathbf{A}_{ba} & \mathbf{A}_{bb} \end{array}\right] \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix} + \begin{bmatrix} B_a \\ \mathbf{B}_b \end{bmatrix} u \tag{12–81}$$

$$y = [1 \;\vdots\; \mathbf{0}] \begin{bmatrix} x_a \\ \mathbf{x}_b \end{bmatrix} \tag{12–82}$$

where A_{aa} = scalar
$\mathbf{A}_{ab} = 1 \times (n - 1)$ matrix
$\mathbf{A}_{ba} = (n - 1) \times 1$ matrix
$\mathbf{A}_{bb} = (n - 1) \times (n - 1)$ matrix
B_a = scalar
$\mathbf{B}_b = (n - 1) \times 1$ matrix

From Equation (12–81), the equation for the measured portion of the state becomes

$$\dot{x}_a = A_{aa}x_a + \mathbf{A}_{ab}\mathbf{x}_b + B_a u$$

or

$$\dot{x}_a - A_{aa}x_a - B_a u = \mathbf{A}_{ab}\mathbf{x}_b \tag{12–83}$$

The terms on the left-hand side of Equation (12–83) can be measured. Equation (12–83) acts as the output equation. In designing the minimum-order observer, we consider the left-hand side of Equation (12–83) to be known quantities. Thus, Equation (12–83) relates the measurable quantities and unmeasurable quantities of the state.

From Equation (12–81), the equation for the unmeasured portion of the state becomes

$$\dot{\mathbf{x}}_b = \mathbf{A}_{ba}x_a + \mathbf{A}_{bb}\mathbf{x}_b + \mathbf{B}_b u \tag{12–84}$$

Noting that terms $\mathbf{A}_{ba}x_a$ and $\mathbf{B}_b u$ are known quantities, Equation (12–84) describes the dynamics of the unmeasured portion of the state.

0885

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10670

In what follows we shall present a method for designing a minimum-order observer. The design procedure can be simplified if we utilize the design technique developed for the full-order state observer.

Let us compare the state equation for the full-order observer with that for the minimum-order observer. The state equation for the full-order observer is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

and the "state equation" for the minimum-order observer is

$$\dot{\mathbf{x}}_b = \mathbf{A}_{bb}\mathbf{x}_b + \mathbf{A}_{ba}x_a + \mathbf{B}_b u$$

The output equation for the full-order observer is

$$y = \mathbf{C}\mathbf{x}$$

and the "output equation" for the minimum-order observer is

$$\dot{x}_a - A_{aa}x_a - B_a u = \mathbf{A}_{ab}\mathbf{x}_b$$

The design of the minimum-order observer can be carried out as follows: First, note that the observer equation for the full-order observer was given by Equation (12–57), which we repeat here:

$$\dot{\tilde{\mathbf{x}}} = (\mathbf{A} - \mathbf{K}_e\mathbf{C})\tilde{\mathbf{x}} + \mathbf{B}u + \mathbf{K}_e y \tag{12–85}$$

Then, making the substitutions of Table 12–1 into Equation (12–85), we obtain

$$\dot{\tilde{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\mathbf{x}}_b + \mathbf{A}_{ba}x_a + \mathbf{B}_b u + \mathbf{K}_e(\dot{x}_a - A_{aa}x_a - B_a u) \tag{12–86}$$

where the state observer gain matrix $\mathbf{K}_e$ is an $(n-1) \times 1$ matrix. In Equation (12–86), notice that in order to estimate $\tilde{\mathbf{x}}_b$, we need the derivative of x_a. This presents a difficulty, because differentiation amplifies noise. If $x_a\,(= y)$ is noisy, the use of $\dot{x}_a$ is unacceptable. To avoid this difficulty, we eliminate $\dot{x}_a$ in the following way. First rewrite Equation (12–86) as

Table 12–1 List of Necessary Substitutions for Writing the Observer Equation for the Minimum-Order State Observer

Full-Order State Observer	Minimum-Order State Observer
$\tilde{\mathbf{x}}$	$\tilde{\mathbf{x}}_b$
$\mathbf{A}$	$\mathbf{A}_{bb}$
$\mathbf{B}u$	$\mathbf{A}_{ba}x_a + \mathbf{B}_b u$
y	$\dot{x}_a - A_{aa}x_a - B_a u$
$\mathbf{C}$	$\mathbf{A}_{ab}$
$\mathbf{K}_e$ $(n \times 1$ matrix)	$\mathbf{K}_e$ $[(n-1) \times 1$ matrix]

0886

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10671

$$\dot{\tilde{\mathbf{x}}}_b - \mathbf{K}_e\dot{x}_a = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\mathbf{x}}_b + (\mathbf{A}_{ba} - \mathbf{K}_e A_{aa})y + (\mathbf{B}_b - \mathbf{K}_e B_a)u$$
$$= (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})(\tilde{\mathbf{x}}_b - \mathbf{K}_e y) + [(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}]y + (\mathbf{B}_b - \mathbf{K}_e B_a)u \tag{12–87}$$

Define

$$\mathbf{x}_b - \mathbf{K}_e y = \mathbf{x}_b - \mathbf{K}_e x_a = \boldsymbol{\eta}$$

and

$$\tilde{\mathbf{x}}_b - \mathbf{K}_e y = \tilde{\mathbf{x}}_b - \mathbf{K}_e x_a = \tilde{\boldsymbol{\eta}} \tag{12–88}$$

Then Equation (12–87) becomes

$$\dot{\tilde{\boldsymbol{\eta}}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\boldsymbol{\eta}} + [(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}]y + (\mathbf{B}_b - \mathbf{K}_e B_a)u \tag{12–89}$$

Define

$$\hat{\mathbf{A}} = \mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab}$$
$$\hat{\mathbf{B}} = \hat{\mathbf{A}}\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}$$
$$\hat{\mathbf{F}} = \mathbf{B}_b - \mathbf{K}_e B_a$$

Then Equation (12–89) becomes

$$\dot{\tilde{\boldsymbol{\eta}}} = \hat{\mathbf{A}}\tilde{\boldsymbol{\eta}} + \hat{\mathbf{B}}y + \hat{\mathbf{F}}u \tag{12–90}$$

Equation (12–90) and Equation (12–88) together define the minimum-order observer.

Since

$$y = [1 \mid \mathbf{0}]\begin{bmatrix} x_a \\ \hline \mathbf{x}_b \end{bmatrix}$$

$$\tilde{\mathbf{x}} = \begin{bmatrix} x_a \\ \hline \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} y \\ \hline \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \hline \mathbf{I}_{n-1} \end{bmatrix}[\tilde{\mathbf{x}}_b - \mathbf{K}_e y] + \begin{bmatrix} 1 \\ \hline \mathbf{K}_e \end{bmatrix} y$$

where $\mathbf{0}$ is a row vector consisting of $(n-1)$ zeros, if we define

$$\hat{\mathbf{C}} = \begin{bmatrix} \mathbf{0} \\ \hline \mathbf{I}_{n-1} \end{bmatrix}, \qquad \hat{\mathbf{D}} = \begin{bmatrix} 1 \\ \hline \mathbf{K}_e \end{bmatrix}$$

then we can write $\tilde{\mathbf{x}}$ in terms of $\tilde{\boldsymbol{\eta}}$ and y as follows:

$$\tilde{\mathbf{x}} = \hat{\mathbf{C}}\tilde{\boldsymbol{\eta}} + \hat{\mathbf{D}}y \tag{12–91}$$

This equation gives the transformation from $\tilde{\boldsymbol{\eta}}$ to $\tilde{\mathbf{x}}$.

Figure 12–17 shows the block diagram of the observed-state feedback control system with the minimum-order observer, based on Equations (12–79), (12–80), (12–90), (12–91) and $u = -\mathbf{K}\tilde{\mathbf{x}}$.

Next we shall derive the observer error equation. Using Equation (12–83), Equation (12–86) can be modified to

$$\dot{\tilde{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\mathbf{x}}_b + \mathbf{A}_{ba}x_a + \mathbf{B}_b u + \mathbf{K}_e\mathbf{A}_{ab}\mathbf{x}_b \tag{12–92}$$

0887

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10672

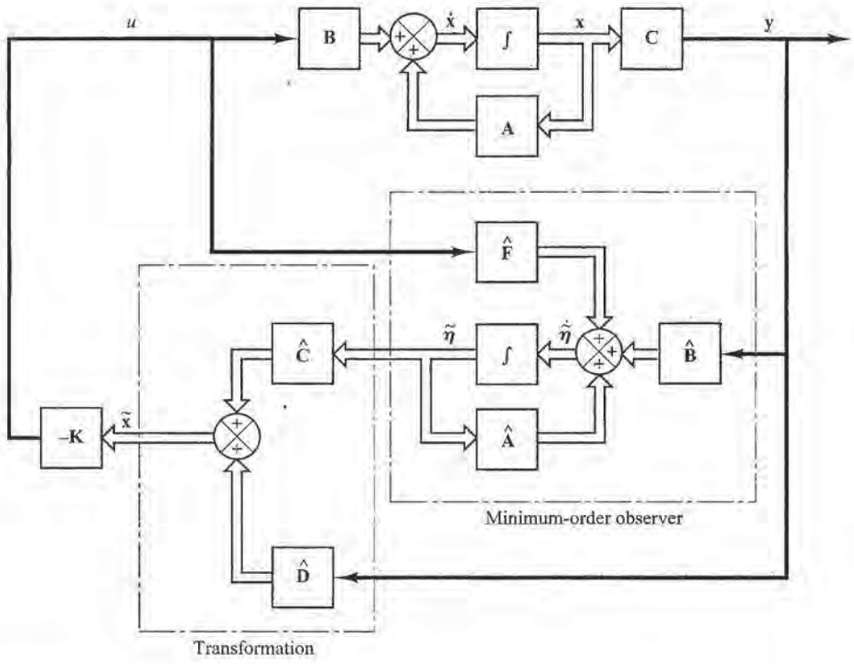


Figure 12–17
System with observed-state feedback, where the observer is the minimum-order observer.

By subtracting Equation (12–92) from Equation (12–84), we obtain

$$\dot{\mathbf{x}}_b - \dot{\tilde{\mathbf{x}}}_b = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})(\mathbf{x}_b - \tilde{\mathbf{x}}_b) \tag{12–93}$$

Define

$$\mathbf{e} = \mathbf{x}_b - \tilde{\mathbf{x}}_b = \boldsymbol{\eta} - \tilde{\boldsymbol{\eta}}$$

Then Equation (12–93) becomes

$$\dot{\mathbf{e}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{e} \tag{12–94}$$

This is the error equation for the minimum-order observer. Note that $\mathbf{e}$ is an $(n-1)$-vector.

The error dynamics can be chosen as desired by following the technique developed for the full-order observer, provided that the rank of matrix

$$\begin{bmatrix} \mathbf{A}_{ab} \\ \mathbf{A}_{ab}\mathbf{A}_{bb} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{A}_{ab}\mathbf{A}_{bb}^{n-2} \end{bmatrix}$$

is $n-1$. (This is the complete observability condition applicable to the minimum-order observer.)

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10673

The characteristic equation for the minimum-order observer is obtained from Equation (12–94) as follows:

$$|s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab}| = (s - \mu_1)(s - \mu_2)\cdots(s - \mu_{n-1})$$
$$= s^{n-1} + \hat{\alpha}_1 s^{n-2} + \cdots + \hat{\alpha}_{n-2}s + \hat{\alpha}_{n-1} = 0 \qquad (12\text{–}95)$$

where $\mu_1, \mu_2, \ldots, \mu_{n-1}$ are desired eigenvalues for the minimum-order observer. The observer gain matrix $\mathbf{K}_e$ can be determined by first choosing the desired eigenvalues for the minimum-order observer [that is, by placing the roots of the characteristic equation, Equation (12–95), at the desired locations] and then using the procedure developed for the full-order observer with appropriate modifications. For example, if the formula for determining matrix $\mathbf{K}_e$ given by Equation (12–61) is to be used, it should be modified to

$$\mathbf{K}_e = \hat{\mathbf{Q}}\begin{bmatrix} \hat{\alpha}_{n-1} - \hat{a}_{n-1} \\ \hat{\alpha}_{n-2} - \hat{a}_{n-2} \\ \cdot \\ \cdot \\ \cdot \\ \hat{\alpha}_1 - \hat{a}_1 \end{bmatrix} = (\hat{\mathbf{W}}\hat{\mathbf{N}}^*)^{-1}\begin{bmatrix} \hat{\alpha}_{n-1} - \hat{a}_{n-1} \\ \hat{\alpha}_{n-2} - \hat{a}_{n-2} \\ \cdot \\ \cdot \\ \cdot \\ \hat{\alpha}_1 - \hat{a}_1 \end{bmatrix} \qquad (12\text{–}96)$$

where $\mathbf{K}_e$ is an $(n - 1) \times 1$ matrix and

$$\hat{\mathbf{N}} = [\mathbf{A}_{ab}^* \mid \mathbf{A}_{bb}^*\mathbf{A}_{ab}^* \mid \cdots \mid (\mathbf{A}_{bb}^*)^{n-2}\mathbf{A}_{ab}^*] = (n - 1) \times (n - 1) \text{ matrix}$$

$$\hat{\mathbf{W}} = \begin{bmatrix} \hat{a}_{n-2} & \hat{a}_{n-3} & \cdots & \hat{a}_1 & 1 \\ \hat{a}_{n-3} & \hat{a}_{n-4} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \hat{a}_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix} = (n - 1) \times (n - 1) \text{ matrix}$$

Note that $\hat{a}_1, \hat{a}_2, \ldots, \hat{a}_{n-2}$ are coefficients in the characteristic equation for the state equation

$$|s\mathbf{I} - \mathbf{A}_{bb}| = s^{n-1} + \hat{a}_1 s^{n-2} + \cdots + \hat{a}_{n-2}s + \hat{a}_{n-1} = 0$$

Also, if Ackermann's formula given by Equation (12–65) is to be used, then it should be modified to

$$\mathbf{K}_e = \phi(\mathbf{A}_{bb})\begin{bmatrix} \mathbf{A}_{ab} \\ \mathbf{A}_{ab}\mathbf{A}_{bb} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{A}_{ab}\mathbf{A}_{bb}^{n-3} \\ \mathbf{A}_{ab}\mathbf{A}_{bb}^{n-2} \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix} \qquad (12\text{–}97)$$

0889

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10674

where

$$\phi(\mathbf{A}_{bb}) = \mathbf{A}_{bb}^{n-1} + \hat{\alpha}_1 \mathbf{A}_{bb}^{n-2} + \cdots + \hat{\alpha}_{n-2}\mathbf{A}_{bb} + \hat{\alpha}_{n-1}\mathbf{I}$$

Observed-State Feedback Control System with Minimum-Order Observer. For the case of the observed-state feedback control system with full-order state observer, we have shown that the closed-loop poles of the observed-state feedback control system consist of the poles due to the pole-placement design alone, plus the poles due to the observer design alone. Hence, the pole-placement design and the full-order observer design are independent of each other.

For the observed-state feedback control system with minimum-order observer, the same conclusion applies. The system characteristic equation can be derived as

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}||s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab}| = 0 \qquad (12\text{–}98)$$

(See Problem **A–12–11** for the details.) The closed-loop poles of the observed-state feedback control system with a minimum-order observer comprise the closed-loop poles due to pole placement [the eigenvalues of matrix $(\mathbf{A} - \mathbf{BK})$] and the closed-loop poles due to the minimum-order observer [the eigenvalues of matrix $(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})$]. Therefore, the pole-placement design and the design of the minimum-order observer are independent of each other.

Determining Observer Gain Matrix K_e with MATLAB. Because of the duality of pole-placement and observer design, the same algorithm can be applied to both the pole-placement problem and the observer-design problem. Thus, the commands acker and place can be used to determine the observer gain matrix $\mathbf{K}_e$.

The closed-loop poles of the observer are the eigenvalues of matrix $\mathbf{A} - \mathbf{K}_e\mathbf{C}$. The closed-loop poles of the pole-placement are the eigenvalues of matrix $\mathbf{A} - \mathbf{BK}$.

Referring to the duality problem between the pole-placement problem and observer-design problem, we can determine $\mathbf{K}_e$ by considering the pole-placement problem for the dual system. That is, we determine $\mathbf{K}_e$ by placing the eigenvalues of $\mathbf{A}^* - \mathbf{C}^*\mathbf{K}_e$ at the desired place. Since $\mathbf{K}_e = \mathbf{K}^*$, for the full-order observer we use the command

```
Ke = acker(A',C',L)'
```

where L is the vector of the desired eigenvalues for the observer. Similarly, for the full-order observer, we may use

```
Ke = place(A',C',L)'
```

provided L does not include multiple poles. [In the above commands, prime (') indicates the transpose.] For the minimum-order (or reduced-order) observers, use the following commands:

```
Ke = acker(Abb',Aab',L)'
```

or

```
Ke = place(Abb',Aab',L)'
```


0890

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10675

EXAMPLE 12–8 Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Let us assume that we want to place the closed-loop poles at

$$s_1 = -2 + j2\sqrt{3}, \quad s_2 = -2 - j2\sqrt{3}, \quad s_3 = -6$$

Then the necessary state-feedback gain matrix $\mathbf{K}$ can be obtained as follows:

$$\mathbf{K} = [90 \quad 29 \quad 4]$$

(See MATLAB Program 12–10 for a MATLAB computation of this matrix $\mathbf{K}$.)

Next, let us assume that the output y can be measured accurately so that state variable x_1 (which is equal to y) need not be estimated. Let us design a minimum-order observer. (The minimum-order observer is of second order.) Assume that we choose the desired observer poles to be at

$$s = -10, \quad s = -10$$

Referring to Equation (12–95), the characteristic equation for the minimum-order observer is

$$\begin{aligned} |s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab}| &= (s - \mu_1)(s - \mu_2) \\ &= (s + 10)(s + 10) \\ &= s^2 + 20s + 100 = 0 \end{aligned}$$

In what follows, we shall use Ackermann's formula given by Equation (12–97).

$$\mathbf{K}_e = \phi(\mathbf{A}_{bb}) \begin{bmatrix} \mathbf{A}_{ab} \\ \cdots\cdots \\ \mathbf{A}_{ab}\mathbf{A}_{bb} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{12–99}$$

where

$$\phi(\mathbf{A}_{bb}) = \mathbf{A}_{bb}^2 + \hat{\alpha}_1\mathbf{A}_{bb} + \hat{\alpha}_2\mathbf{I} = \mathbf{A}_{bb}^2 + 20\mathbf{A}_{bb} + 100\mathbf{I}$$

Since

$$\tilde{\mathbf{x}} = \begin{bmatrix} x_a \\ \cdots \\ \mathbf{x}_b \end{bmatrix} = \begin{bmatrix} x_1 \\ \cdots \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}, \quad \mathbf{A} = \left[\begin{array}{c|cc} 0 & 1 & 0 \\ \hline 0 & 0 & 1 \\ -6 & -11 & -6 \end{array}\right], \quad \mathbf{B} = \begin{bmatrix} 0 \\ \cdots \\ 0 \\ 1 \end{bmatrix}$$

we have

$$A_{aa} = 0, \quad \mathbf{A}_{ab} = [1 \quad 0], \quad \mathbf{A}_{ba} = \begin{bmatrix} 0 \\ -6 \end{bmatrix}$$
$$\mathbf{A}_{bb} = \begin{bmatrix} 0 & 1 \\ -11 & -6 \end{bmatrix}, \quad B_a = 0, \quad \mathbf{B}_b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Equation (12–99) now becomes

0891

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10676

$$\mathbf{K}_e = \left\{\begin{bmatrix} 0 & 1 \\ -11 & -6 \end{bmatrix}^2 + 20\begin{bmatrix} 0 & 1 \\ -11 & -6 \end{bmatrix} + 100\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right\}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 89 & 14 \\ -154 & 5 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 5 \end{bmatrix}$$

(A MATLAB computation of this $\mathbf{K}_e$ is given in MATLAB Program 12–10.)

MATLAB Program 12–10

```
A = [0  1  0;0  0  1;-6  -11  -6];
B = [0;0;1];
J = [-2+j*2*sqrt(3)  -2-j*2*sqrt(3)  -6];
K = acker(A,B,J)

K =

    90.0000   29.0000    4.0000

Abb = [0  1;-11  -6];
Aab = [1  0];
L = [-10  -10];
Ke = acker(Abb',Aab',L)'

Ke =

    14
     5
```

Referring to Equations (12–88) and (12–89), the equation for the minimum-order observer can be given by

$$\dot{\tilde{\boldsymbol{\eta}}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\boldsymbol{\eta}} + [(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e A_{aa}]y + (\mathbf{B}_b - \mathbf{K}_e B_a)u \qquad (12\text{–}100)$$

where

$$\tilde{\boldsymbol{\eta}} = \tilde{\mathbf{x}}_b - \mathbf{K}_e y = \tilde{\mathbf{x}}_b - \mathbf{K}_e x_1$$

Noting that

$$\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab} = \begin{bmatrix} 0 & 1 \\ -11 & -6 \end{bmatrix} - \begin{bmatrix} 14 \\ 5 \end{bmatrix}[1 \quad 0] = \begin{bmatrix} -14 & 1 \\ -16 & -6 \end{bmatrix}$$

the equation for the minimum-order observer, Equation (12–100), becomes

$$\begin{bmatrix} \dot{\tilde{\eta}}_2 \\ \dot{\tilde{\eta}}_3 \end{bmatrix} = \begin{bmatrix} -14 & 1 \\ -16 & -6 \end{bmatrix}\begin{bmatrix} \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{bmatrix} + \left\{\begin{bmatrix} -14 & 1 \\ -16 & -6 \end{bmatrix}\begin{bmatrix} 14 \\ 5 \end{bmatrix} + \begin{bmatrix} 0 \\ -6 \end{bmatrix} - \begin{bmatrix} 14 \\ 5 \end{bmatrix}0\right\}y + \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix} - \begin{bmatrix} 14 \\ 5 \end{bmatrix}0\right\}u$$

or

$$\begin{bmatrix} \dot{\tilde{\eta}}_2 \\ \dot{\tilde{\eta}}_3 \end{bmatrix} = \begin{bmatrix} -14 & 1 \\ -16 & -6 \end{bmatrix}\begin{bmatrix} \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{bmatrix} + \begin{bmatrix} -191 \\ -260 \end{bmatrix}y + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$$

0892

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10677

where

$$\begin{bmatrix} \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{bmatrix} = \begin{bmatrix} \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} - \mathbf{K}_e y$$

or

$$\begin{bmatrix} \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix} = \begin{bmatrix} \tilde{\eta}_2 \\ \tilde{\eta}_3 \end{bmatrix} + \mathbf{K}_e x_1$$

If the observed-state feedback is used, then the control signal u becomes

$$u = -\mathbf{K}\tilde{\mathbf{x}} = -\mathbf{K}\begin{bmatrix} x_1 \\ \tilde{x}_2 \\ \tilde{x}_3 \end{bmatrix}$$

where $\mathbf{K}$ is the state feedback gain matrix. Figure 12–18 is a block diagram showing the configuration of the system with observed-state feedback, where the observer is the minimum-order observer.

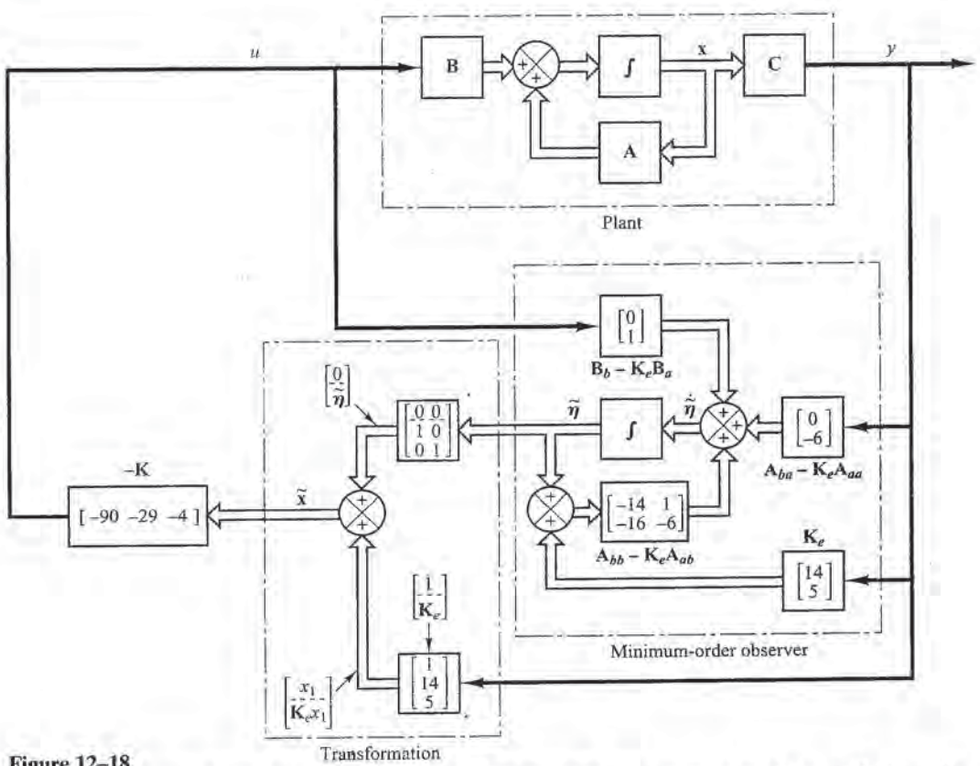


Figure 12–18
System with observed state feedback, where the observer is the minimum-order observer designed in Example 12–8.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10678

Transfer Function of Minimum-Order Observer-Based Controller. In the minimum-order observer equation given by Equation (12–89):

$$\dot{\tilde{\boldsymbol{\eta}}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\tilde{\boldsymbol{\eta}} + [(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_e\mathbf{A}_{aa}]y + (\mathbf{B}_b - \mathbf{K}_eB_a)u$$

define, similar to the case of the derivation of Equation (12–90),

$$\hat{\mathbf{A}} = \mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab}$$
$$\hat{\mathbf{B}} = \hat{\mathbf{A}}\mathbf{K}_e + \mathbf{A}_{ba} - \mathbf{K}_eA_{aa}$$
$$\hat{\mathbf{F}} = \mathbf{B}_b - \mathbf{K}_eB_a$$

Then, the following three equations define the minimum-order oberver:

$$\dot{\tilde{\boldsymbol{\eta}}} = \hat{\mathbf{A}}\tilde{\boldsymbol{\eta}} + \hat{\mathbf{B}}y + \hat{\mathbf{F}}u \tag{12–101}$$
$$\tilde{\boldsymbol{\eta}} = \tilde{\mathbf{x}}_b - \mathbf{K}_ey \tag{12–102}$$
$$u = -\mathbf{K}\tilde{\mathbf{x}} \tag{12–103}$$

Since Equation (12–103) can be rewritten as

$$u = -\mathbf{K}\tilde{\mathbf{x}} = -\begin{bmatrix} K_a & \mathbf{K}_b \end{bmatrix}\begin{bmatrix} y \\ \tilde{\mathbf{x}}_b \end{bmatrix} = -K_ay - \mathbf{K}_b\tilde{\mathbf{x}}_b$$
$$= -\mathbf{K}_b\tilde{\boldsymbol{\eta}} - (K_a + \mathbf{K}_b\mathbf{K}_e)y \tag{12–104}$$

by substituting Equation (12–104) into Equation (12–101), we obtain

$$\dot{\tilde{\boldsymbol{\eta}}} = \hat{\mathbf{A}}\tilde{\boldsymbol{\eta}} + \hat{\mathbf{B}}y + \hat{\mathbf{F}}[-\mathbf{K}_b\tilde{\boldsymbol{\eta}} - (K_a + \mathbf{K}_b\mathbf{K}_e)y]$$
$$= (\hat{\mathbf{A}} - \hat{\mathbf{F}}\mathbf{K}_b)\tilde{\boldsymbol{\eta}} + [\hat{\mathbf{B}} - \hat{\mathbf{F}}(K_a + \mathbf{K}_b\mathbf{K}_e)]y \tag{12–105}$$

Define

$$\tilde{\mathbf{A}} = \hat{\mathbf{A}} - \hat{\mathbf{F}}\mathbf{K}_b$$
$$\tilde{\mathbf{B}} = \hat{\mathbf{B}} - \hat{\mathbf{F}}(K_a + \mathbf{K}_b\mathbf{K}_e)$$
$$\tilde{\mathbf{C}} = -\mathbf{K}_b$$
$$\tilde{D} = -(K_a + \mathbf{K}_b\mathbf{K}_e)$$

Then Equations (12–105) and (12–104) can be written as

$$\dot{\tilde{\boldsymbol{\eta}}} = \tilde{\mathbf{A}}\tilde{\boldsymbol{\eta}} + \tilde{\mathbf{B}}y \tag{12–106}$$
$$u = \tilde{\mathbf{C}}\tilde{\boldsymbol{\eta}} + \tilde{D}y \tag{12–107}$$

Equations (12–106) and (12–107) define the minimum-order observer-based controller. By considering u as the output and $-y$ as the input, $U(s)$ can be written as

$$U(s) = [\tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}} + \tilde{D}]Y(s)$$
$$= -[\tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}} + \tilde{D}][-Y(s)]$$

Since the input to the observer controller is $-Y(s)$, rather than $Y(s)$, the transfer function of the observer controller is

$$\frac{U(s)}{-Y(s)} = \frac{\text{num}}{\text{den}} = -[\tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}} + \tilde{D}] \tag{12–108}$$

This transfer function can be easily obtained by using the following MATLAB statement:

```
[num,den] = ss2tf(Atilde, Btilde, -Ctilde, -Dtilde)
```
(12–109)

0894

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10679

12–6 DESIGN OF REGULATOR SYSTEMS WITH OBSERVERS

In this section we shall consider a problem of designing regulator systems by using the pole-placement-with-observer approach.

Consider the regulator system shown in Figure 12–19. (The reference input is zero.) The plant transfer function is

$$G(s) = \frac{10(s + 2)}{s(s + 4)(s + 6)}$$

Using the pole-placement approach, design a controller such that when the system is subjected to the following initial condition:

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{e}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $\mathbf{x}$ is the state vector for the plant and $\mathbf{e}$ is the observer error vector, the maximum undershoot of $y(t)$ is 25 to 35% and the settling time is about 4 sec. Assume that we use the minimum-order observer. (We assume that only the output y is measurable.)

We shall use the following design procedure:

1. Derive a state-space model of the plant.
2. Choose the desired closed-loop poles for pole placement. Choose the desired observer poles.
3. Determine the state feedback gain matrix $\mathbf{K}$ and the observer gain matrix $\mathbf{K}_e$.
4. Using the gain matrices $\mathbf{K}$ and $\mathbf{K}_e$ obtained in step 3, derive the transfer function of the observer controller. If it is a stable controller, check the response to the given initial condition. If the response is not acceptable, adjust the closed-loop pole location and/or observer pole location until an acceptable response is obtained.

Design step 1: We shall derive the state-space representation of the plant. Since the plant transfer function is

$$\frac{Y(s)}{U(s)} = \frac{10(s + 2)}{s(s + 4)(s + 6)}$$

the corresponding differential equation is

$$\dddot{y} + 10\ddot{y} + 24\dot{y} = 10\dot{u} + 20u$$

Referring to Section 3-5, let us define the state variables x_1, x_2, and x_3 as follows:

$$x_1 = y - \beta_0 u$$
$$x_2 = \dot{x}_1 - \beta_1 u$$
$$x_3 = \dot{x}_2 - \beta_2 u$$

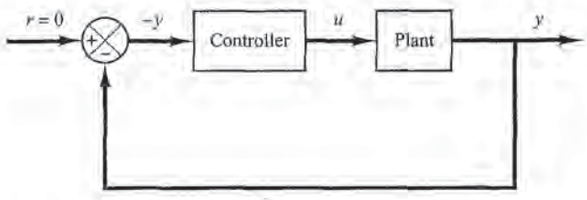


Figure 12–19
Regulator system.

0895

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10680

where

$$\beta_0 = 0$$
$$\beta_1 = 0$$
$$\beta_2 = 10$$
$$\beta_3 = -80$$

[See Equation (3-35) for the calculation of β's.] Then the state-space equation and output equation can be obtained as

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -24 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \\ -80 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$

Design step 2: As the first trial, let us choose the desired closed-loop poles at

$$s = -1 + j2, \qquad s = -1 - j2, \qquad s = -5$$

and choose the desired observer poles at

$$s = -10, \qquad s = -10$$

Design step 3: We shall use MATLAB to compute the state feedback gain matrix **K** and the observer gain matrix $\mathbf{K}_e$. MATLAB Program 12–11 produces matrices **K** and $\mathbf{K}_e$. In the program, matrices **J** and **L** represent the desired closed-loop poles for pole placement and the desired poles for the observer, respectively. The matrices **K** and $\mathbf{K}_e$ are obtained as

MATLAB Program 12–11

```
% Obtaining the state feedback gain matrix K

A = [0  1  0;0  0  1;0  -24  -10];
B = [0;10;-80];
C = [1  0  0];
J = [-1+j*2  -1-j*2  -5];
K = acker(A,B,J)

K =

   1.2500   1.2500   0.19375

% Obtaining the observer gain matrix Ke

Aaa = 0; Aab = [1  0]; Aba = [0;0]; Abb = [0  1;-24  -10];Ba = 0; Bb = [10;-80];
L = [-10  -10];
Ke = acker(Abb',Aab',L)'

Ke =

   10
  -24
```


0896

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10681

$$\mathbf{K} = [1.25 \quad 1.25 \quad 0.19375]$$

$$\mathbf{K}_e = \begin{bmatrix} 10 \\ -24 \end{bmatrix}$$

Design step 4: We shall determine the transfer function of the observer controller. Referring to Equation (12–108), the transfer function of the observer controller can be given by

$$G_c(s) = \frac{U(s)}{-Y(s)} = \frac{\text{num}}{\text{den}} = -[\tilde{\mathbf{C}}(s\mathbf{I} - \tilde{\mathbf{A}})^{-1}\tilde{\mathbf{B}} + \tilde{D}]$$

We shall use MATLAB to calculate the transfer function of the observer controller. MATLAB Program 12–12 produces this transfer function. The result is

$$G_c(s) = \frac{9.1s^2 + 73.5s + 125}{s^2 + 17s - 30}$$

$$= \frac{9.1(s + 5.6425)(s + 2.4344)}{(s + 18.6119)(s - 1.6119)}$$

Define the system with this observer controller as System 1. Figure 12–20 shows the block diagram of System 1.

MATLAB Program 12–12

```
% Determination of transfer function of observer controller

A = [0  1  0;0  0  1;0  -24  -10];
B = [0;10;-80];
Aaa = 0; Aab = [1  0]; Aba = [0;0]; Abb = [0  1;-24  -10];
Ba = 0; Bb = [10;-80];
Ka = 1.25; Kb = [1.25    0.19375];
Ke = [10;-24];
Ahat = Abb - Ke*Aab;
Bhat = Ahat*Ke + Aba - Ke*Aaa;
Fhat = Bb - Ke*Ba;
Atilde = Ahat - Fhat*Kb;
Btilde = Bhat - Fhat*(Ka + Kb*Ke);
Ctilde = -Kb;
Dtilde = -(Ka + Kb*Ke);
[num,den] = ss2tf(Atilde, Btilde, -Ctilde, -Dtilde)

num =
   9.1000  73.5000  125.0000

den =

   1.0000  17.0000  -30.0000
```


0897

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10682

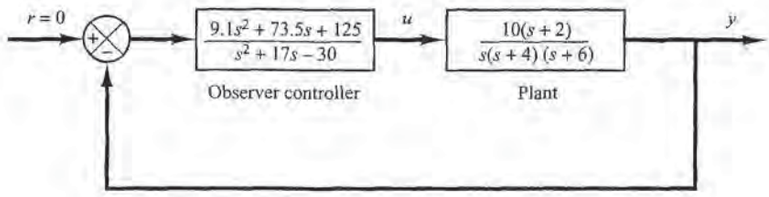


Figure 12–20
Block diagram of System 1.

The observer controller has a pole in the right-half s plane ($s = 1.6119$). The existence of an open-loop right-half s plane pole in the observer controller means that the system is open-loop unstable, although the closed-loop system is stable. The latter can be seen from the characteristic equation for the system:

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}| \cdot |s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab}|$$
$$= s^5 + 27s^4 + 255s^3 + 1025s^2 + 2000s + 2500$$
$$= (s + 1 + j2)(s + 1 - j2)(s + 5)(s + 10)(s + 10) = 0$$

(See MATLAB Program 12–13 for the calculation of the characteristic equation.)

A disadvantage of using an unstable controller is that the system becomes unstable if the dc gain of the system becomes small. Such a control system is neither desirable nor acceptable. Hence, to get a satisfactory system, we need to modify the closed-loop pole location and/or observer pole location.

MATLAB Program 12–13

```
% Obtaining the characteristic equation

[num1,den1] = ss2tf(A-B*K,eye(3),eye(3),eye(3),1);
[num2,den2] = ss2tf(Abb-Ke*Aab,eye(2),eye(2),eye(2),1);
charact_eq = conv(den1,den2)

charact_eq =

   1.0e+003*

   0.0010   0.0270   0.2550   1.0250   2.0000   2.5000
```

Second trial: Let us keep the desired closed-loop poles for pole placement as before, but modify the observer pole locations as follows:

$$s = -4.5, \qquad s = -4.5$$

Thus,

$$\mathbf{L} = \begin{bmatrix} -4.5 & -4.5 \end{bmatrix}$$

Using MATLAB, we find the new $\mathbf{K}_e$ to be

$$\mathbf{K}_e = \begin{bmatrix} -1 \\ 6.25 \end{bmatrix}$$

0898

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10683

Next, we shall obtain the transfer function of the observer controller. MATLAB Program 12–14 produces this transfer function as follows:

$$G_c(s) = \frac{1.2109s^2 + 11.2125s + 25.3125}{s^2 + 6s + 2.1406}$$
$$= \frac{1.2109(s + 5.3582)(s + 3.9012)}{(s + 5.619)(s + 0.381)}$$

MATLAB Program 12–14

```
% Determination of transfer function of observer controller.
A = [0  1  0;0  0  1;0  -24  -10];
B = [0;10;-80];
Aaa = 0; Aab = [1  0]; Aba = [0;0]; Abb = [0  1;-24  -10];
Ba = 0; Bb = [10;-80];
Ka = 1.25; Kb = [1.25  0.19375];
Ke = [-1;6.25];
Ahat = Abb - Ke*Aab;
Bhat = Ahat*Ke + Aba - Ke*Aaa;
Fhat = Bb - Ke*Ba;
Atilde = Ahat - Fhat*Kb;
Btilde = Bhat - Fhat*(Ka + Kb*Ke);
Ctilde = -Kb;
Dtilde = -(Ka + Kb*Ke);
[num,den] = ss2tf(Atilde,Btilde,-Ctilde,-Dtilde)

num =

   1.2109  11.2125  25.3125

den =

   1.0000  6.0000  2.1406
```

Notice that this is a stable controller. Define the system with this observer controller as System 2. We shall proceed to obtain the response of System 2 to the given initial condition:

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad \mathbf{e}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

By substituting $u = -\mathbf{K}\tilde{\mathbf{x}}$ into the state-space equation for the plant, we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\tilde{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\begin{bmatrix} x_a \\ \tilde{\mathbf{x}}_b \end{bmatrix} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\begin{bmatrix} x_a \\ \mathbf{x}_b - \mathbf{e} \end{bmatrix}$$
$$= \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\left\{\mathbf{x} - \begin{bmatrix} 0 \\ \mathbf{e} \end{bmatrix}\right\} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x} + \mathbf{B}\begin{bmatrix} K_a & \mathbf{K}_b \end{bmatrix}\begin{bmatrix} 0 \\ \mathbf{e} \end{bmatrix} \qquad (12\text{–}110)$$

0899

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10684

The error equation for the minimum-order observer is

$$\dot{\mathbf{e}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{e} \tag{12–111}$$

By combining Equations (12–110) and (12–111), we get

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK}_b \\ \mathbf{0} & \mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix}$$

with the initial condition

$$\begin{bmatrix} \mathbf{x}(0) \\ \mathbf{e}(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

MATLAB Program 12–15 produces the response to the given initial condition. The response curves are shown in Figure 12–21. They seem to be acceptable.

MATLAB Program 12–15

```
% Response to initial condition.

A = [0  1  0;0  0  1;0  -24  -10];
B = [0;10;-80];
K = [1.25  1.25  0.19375];
Kb = [1.25  0.19375];
Ke = [-1;6.25];
Aab = [1  0]; Abb = [0  1;-24  -10];
AA = [A-B*K  B*Kb; zeros(2,3)  Abb-Ke*Aab];
sys = ss(AA,eye(5),eye(5),eye(5));
t = 0:0.01:8;
x = initial(sys,[1;0;0;1;0],t);
x1 = [1  0  0  0  0]*x';
x2 = [0  1  0  0  0]*x';
x3 = [0  0  1  0  0]*x';
e1 = [0  0  0  1  0]*x';
e2 = [0  0  0  0  1]*x';

subplot(3,2,1); plot(t,x1); grid
xlabel ('t (sec)'); ylabel('x1')

subplot(3,2,2); plot(t,x2); grid
xlabel ('t (sec)'); ylabel('x2')

subplot(3,2,3); plot(t,x3); grid
xlabel ('t (sec)'); ylabel('x3')

subplot(3,2,4); plot(t,e1); grid
xlabel('t (sec)'); ylabel('e1')

subplot(3,2,5); plot(t,e2); grid
xlabel('t (sec)'); ylabel('e2')
```


0900

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10685

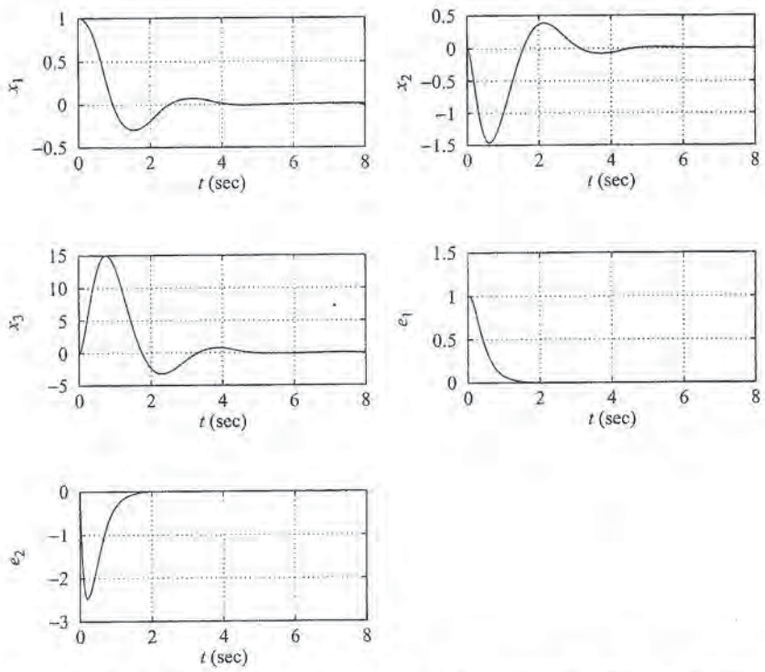


Figure 12–21
Response to the given initial condition; $x_1(0) = 1$, $x_2(0) = 0$, $x_3(0) = 0$, $e_1(0) = 1$, $e_2(0) = 0$.

Next, we shall check the frequency-response characteristics. The bode diagram of the open-loop system just designed is shown in Figure 12–22. The phase margin is about 40° and the gain margin is $+\infty$ dB. The Bode diagram of the closed-loop system is shown in Figure 12–23. The bandwidth of the system is approximately 3.8 rad/sec.

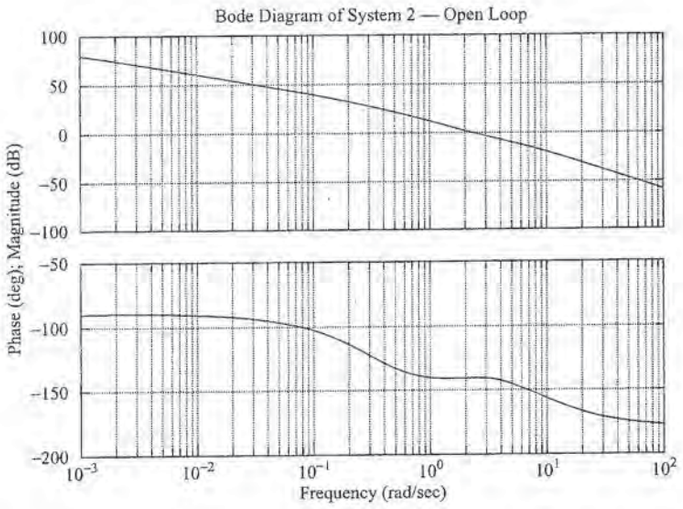


Figure 12–22
Bode diagram for the open-loop transfer function of System 2.

0901

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10686

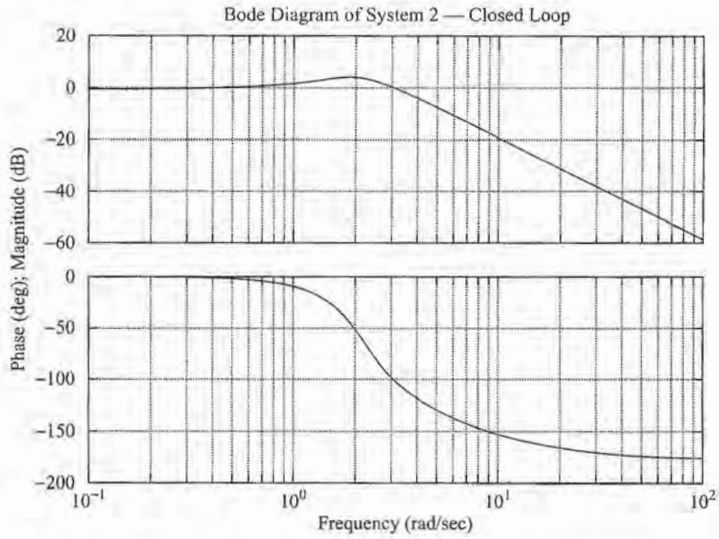


Figure 12–23
Bode diagram for the closed-loop transfer function of System 2.

Finally, we shall compare the root-locus plots of the first system with $L = [-10 \quad -10]$ and the second system with $L = [-4.5 \quad -4.5]$. The plot for the first system given in Figure12–24(a) shows that the system is unstable for small dc gain and becomes stable for large dc gain. The plot for the second system given in Figure 12–24(b), on the other hand, shows that the system is stable for any positive dc gain.

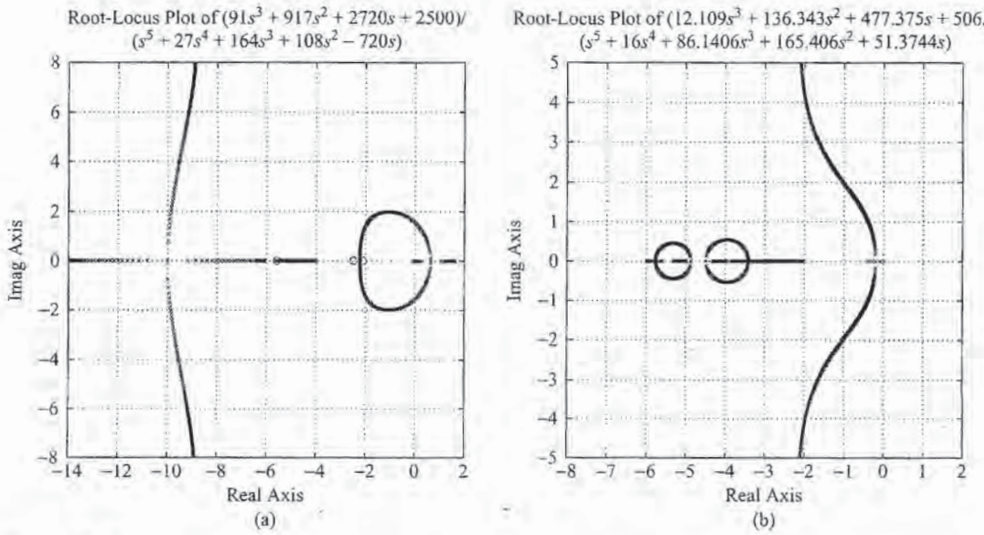


Figure 12–24
(a) Root-locus plot of the system with observer poles at $s = -10$ and $s = -10$; (b) root-locus plot of the system with observer poles at $s = -4.5$ and $s = -4.5$.

0902

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10687

Comments

1. In designing regulator systems, note that if the dominant controller poles are placed far to the left from the $j\omega$ axis, the elements of the state feedback gain matrix $\mathbf{K}$ will become large. Large gain values will make the actuator output become large, so that saturation may take place. Then the designed system will not behave as designed.
2. Also, by placing the observer poles far to the left of the $j\omega$ axis, the observer controller becomes unstable, although the closed-loop system is stable. An unstable observer controller is not acceptable.
3. If the observer controller becomes unstable, move the observer poles to the right in the left-half s plane until the observer controller becomes stable. Also, the desired closed-loop pole locations may need to be modified.
4. Note that if the observer poles are placed far to the left of the $j\omega$ axis, the bandwidth of the observer will increase and will cause noise problems. If there is a serious noise problem, the observer poles should not be placed too far to the left of the $j\omega$ axis. The general requirement is that the bandwidth should be sufficiently low that the sensor noise will not become a problem.
   The bandwidth of the system with the minimum-order observer is higher than that of the system with the full-order observer, provided that the multiple observer poles are placed at the same place for both observers. If the sensor noise is a serious problem, use of a full-order observer is recomnended.

12–7 DESIGN OF CONTROL SYSTEMS WITH OBSERVERS

In Section 12–6 we discussed the design of regulator systems with observers. (The systems did not have reference or command inputs.) In this section we consider the design of control systems with observers when the systems have reference inputs or command inputs. The output of the control system must follow the input that is time varying. In following the command input, the system must exhibit satisfactory performance (a reasonable rise time, overshoot, settling time, and so on).

In this section we consider control systems that are designed by use of the pole-placement-with-observer approach. Specifically, we consider control systems using observer controllers. In Section 12–6 we discussed regulator systems, whose block diagram is shown in Figure 12–25. This system has no reference input, or $r = 0$. When the system has a reference input, several different block diagram configurations are conceivable, each having an observer controller. Two of these configurations are shown in Figures 12–26 (a) and (b); we shall consider them in this section.

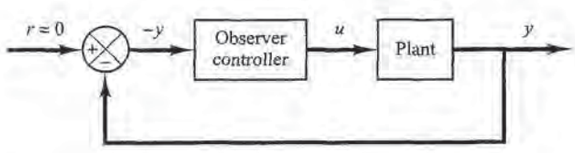
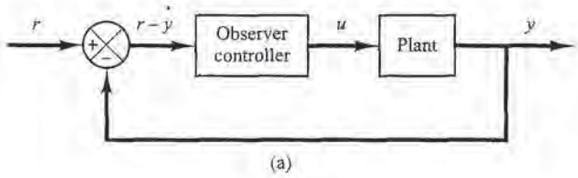


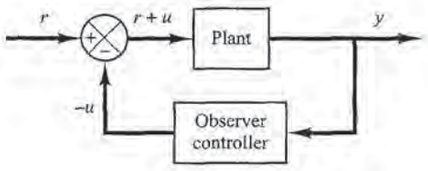
Figure 12–25
Regulator system.

0903

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10688



(a)



(b)

Figure 12–26
(a) Control system with observer controller in the feedforward path; (b) Control system with observer controller in the feedback path.

Configuration 1: Consider the system shown in Figure 12–27. In this system the reference input is simply added at the summing point. We would like to design the observer controller such that in the unit-step response the maximum overshoot is less than 30% and the settling time is about 5 sec.

In what follows we first design a regulator system. Then, using the observer controller designed, we simply add the reference input r at the summing point.

Before we design the observer controller, we need to obtain a state-space representation of the plant. Since

$$\frac{Y(s)}{U(s)} = \frac{1}{s(s^2 + 1)}$$

we obtain

$$\dddot{y} + \dot{y} = u$$

By choosing the state variables as

$$x_1 = y$$
$$x_2 = \dot{y}$$
$$x_3 = \ddot{y}$$

we get

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

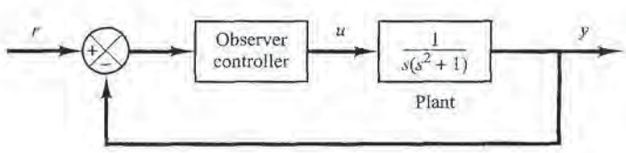


Figure 12–27
Control system with observer controller in the feedforward path.

0904

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10689

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 0 \quad 0]$$

Next, we choose the desired closed-loop poles for pole placement at

$$s = -1 + j, \qquad s = -1 - j, \qquad s = -8$$

and the desired observer poles at

$$s = -4, \qquad s = -4$$

The state feedback gain matrix $\mathbf{K}$ and the observer gain matrix $\mathbf{K}_e$ can be obtained as follows:

$$\mathbf{K} = [16 \quad 17 \quad 10]$$

$$\mathbf{K}_e = \begin{bmatrix} 8 \\ 15 \end{bmatrix}$$

See MATLAB Program 12–16.

The transfer function of the observer controller is obtained by use of MATLAB Program 12–17. The result is

$$G_c(s) = \frac{302s^2 + 303s + 256}{s^2 + 18s + 113} = \frac{302(s + 0.5017 + j0.772)(s + 0.5017 - j0.772)}{(s + 9 + j5.6569)(s + 9 - j5.6569)}$$

MATLAB Program 12–16

```
A = [0 1 0;0 0 1;0 -1 0];
B = [0;0;1];
J = [-1+j -1-j -8];
K = acker(A,B,J)

K =

   16 17 10

Aab = [1 0];
Abb = [0 1;-1 0];
L = [-4 -4];
Ke = acker(Abb',Aab',L)'

Ke =
   8
   15
```


0905

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10690

MATLAB Program 12–17

```
% Determination of transfer function of observer controller

A = [0  1  0;0  0  1;0  -1  0];
B = [0;0;1];
Aaa = 0; Aab = [1  0]; Aba = [0;0]; Abb = [0  1;-1  0];
Ba = 0; Bb = [0;1];
Ka = 16; Kb=[17  10];
Ke = [8;15];
Ahat = Abb - Ke*Aab;
Bhat = Ahat*Ke + Aba - Ke*Aaa;
Fhat = Bb - Ke*Ba;
Atilde = Ahat - Fhat*Kb;
Btilde = Bhat - Fhat*(Ka + Kb*Ke);
Ctilde = -Kb;
Dtilde = -(Ka + Kb*Ke);
[num,den] = ss2tf(Atilde,Btilde,-Ctilde,-Dtilde)

num =

   302.0000  303.0000  256.0000
den =

   1  18  113
```

Figure 12–28 shows the block diagram of the regulator system just designed. Figure 12–29 shows the block diagram of a possible configuration of the control system based on the regulator system shown in Figure 12–28. The unit-step response curve for this control system is shown in Figure 12–30. The maximum overshoot is about 28% and the settling time is about 4.5 sec. Thus, the designed system satisfies the design requirements.

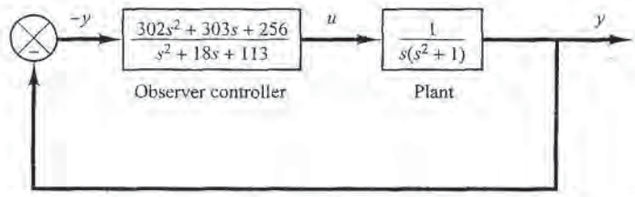


Figure 12–28
Regulator system with observer controller.

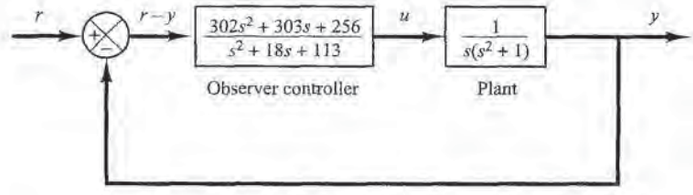


Figure 12–29
Control system with observer controller in the feedforward path.

0906

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10691

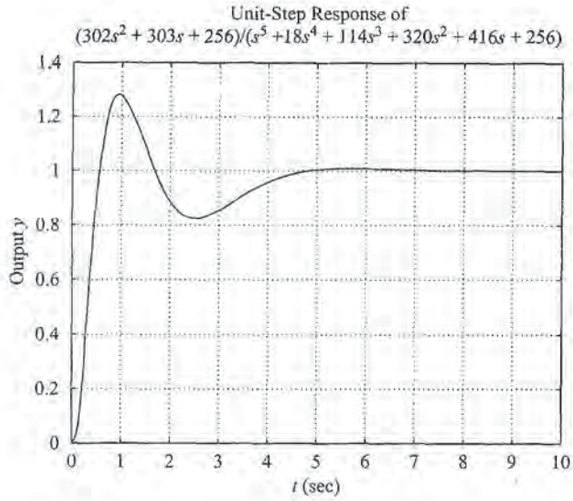


Figure 12–30
Unit-step response of the control system shown in Figure 12–29.

Configuration 2: A different configuration of the control system is shown in Figure 12–31. The observer controller is placed in the feedback path. The input r is introduced into the closed-loop system through the box with gain N. From this block diagram, the closed-loop transfer function is obtained as

$$\frac{Y(s)}{R(s)} = \frac{N(s^2 + 18s + 113)}{s(s^2 + 1)(s^2 + 18s + 113) + 302s^2 + 303s + 256}$$

We determine the value of constant N such that for a unit-step input r, the output y is unity as t approaches infinity. Thus we choose

$$N = \frac{256}{113} = 2.2655$$

The unit-step response of the system is shown in Figure 12–32. Notice that the maximum overshoot is very small, approximately 4%. The settling time is about 5 sec.

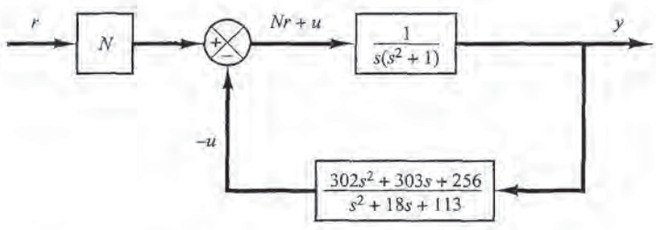


Figure 12–31
Control system with observer controller in the feedback path.

0907

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10692

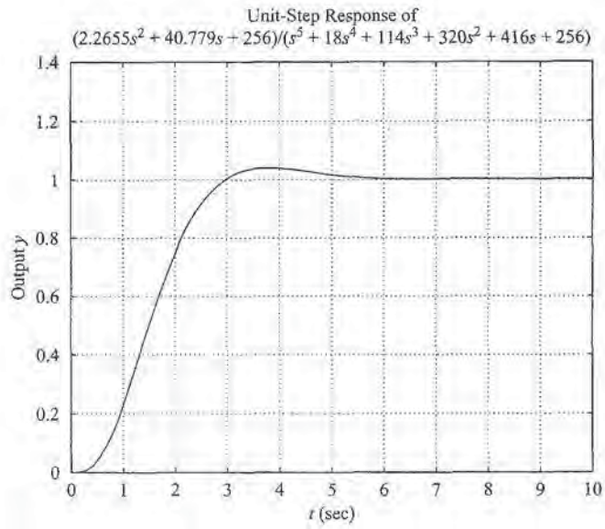


Figure 12–32
The unit-step response of the system shown in Figure 12–31. (The closed-loop poles for pole placement are at $s = -1 \pm j$, $s = -8$. The observer poles are at $s = -4$, $s = -4$.)

Comments. We considered two possible configurations for the closed-loop control systems using observer controllers. As stated earlier, other configurations are possible.

The first configuration, which places the observer controller in the feedforward path, generally gives a fairly large overshoot. The second configuration, which places the observer controller in the feedback path, gives a smaller overshoot. This response curve is quite similar to that of the system designed by the pole-placement approach without using the observer controller. See the unit-step response curve of the system, shown in Figure 12–33, designed by the pole-placement approach without observer. Here the desired closed-loop poles used are

$$s = -1 + j, \qquad s = -1 - j, \qquad s = -8$$

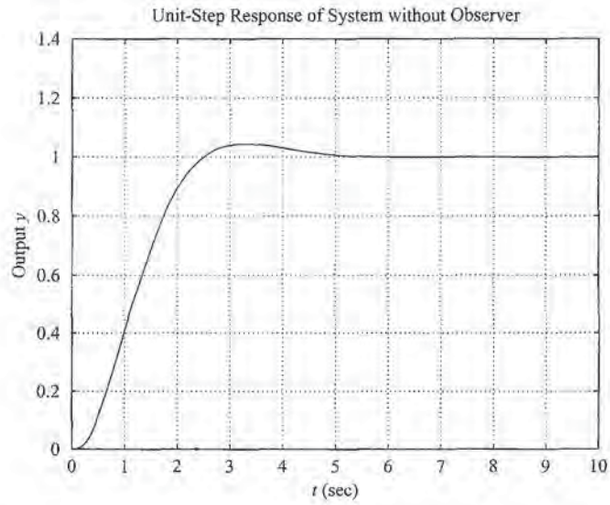


Figure 12–33
The unit-step response of the control system designed by the pole placement approach without observer. (The closed-loop poles are at $s = -1 \pm j$, $s = -8$.)

0908

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10693

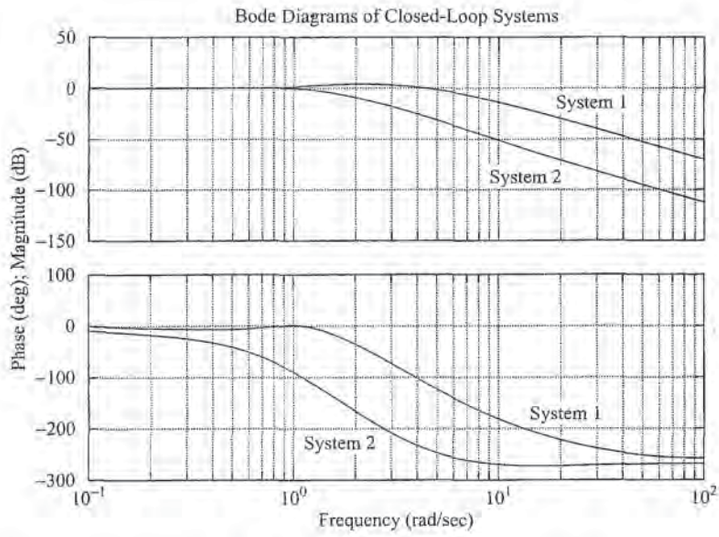


Figure 12–34
Bode diagrams of closed-loop system 1 (shown in Figure 12–29) and closed-loop system 2 (shown in Figure 12–31).

Note that, in these two systems, the rise time and settling time are determined primarily by the desired closed-loop poles for pole placement. (See Figures 12–32 and 12–33.)

The Bode diagrams of closed-loop system 1 (shown in Figure 12–29) and closed-loop system 2 (shown in Figure 12–31) are shown in Figure 12–34. From this figure, we find that the bandwidth of system 1 is 5 rad/sec and that of system 2 is 1.3 rad/sec.

Summary of State-Space Design Method

1. The state-space design method based on the pole-placement-combined-with-observer approach is very powerful. It is a time-domain method. The desired closed-loop poles can be arbitrarily placed, provided the plant is completely state controllable.
2. If not all state variables can be measured, an observer must be incorporated to estimate the unmeasurable state variables.
3. In designing a system using the pole-placement approach, several different sets of desired closed-loop poles need be considered, the response characteristics compared, and the best one chosen.
4. The bandwidth of the observer controller is generally large, because we choose observer poles far to the left in the s plane. A large bandwidth passes high-frequency noises and causes the noise problem.
5. Adding an observer to the system generally reduces the stability margin. In some cases, an observer controller may have zero(s) in the right-half s plane, which means that the controller may be stable but of nonminimum phase. In other cases, the controller may have pole(s) in the right-half s plane—that is, the controller is unstable. Then the designed system may become conditionally stable.
6. When the system is designed by the pole-placement-with-observer approach, it is advisable to check the stability margins (phase margin and gain margin), using a

0909

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10694

frequency-response method. If the system designed has poor stability margins, it is possible that the designed system may become unstable if the mathematical model involves uncertainties.

7. Note that for nth-order systems, classical design methods (root-locus and frequency-response methods) yield low-order compensators (first or second order). Since the observer-based controllers are nth-order (or mth-order if the minimum-order observer is used) for an nth-order system, the designed system will become $2n$th order [or $(n + m)$th order]. Since lower-order compensators are cheaper than higher-order ones, the designer should first apply classical methods and, if no suitable compensators can be determined, then try the pole-placement-with-observer design approach presented in this chapter.

12–8 QUADRATIC OPTIMAL REGULATOR SYSTEMS

An advantage of the quadratic optimal control method over the pole-placement method is that the former provides a systematic way of computing the state feedback control gain matrix.

Quadratic Optimal Regulator Problems. We shall now consider the optimal regulator problem that, given the system equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{12–112}$$

determines the matrix $\mathbf{K}$ of the optimal control vector

$$\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t) \tag{12–113}$$

so as to minimize the performance index

$$J = \int_0^\infty (\mathbf{x}^*\mathbf{Q}\mathbf{x} + \mathbf{u}^*\mathbf{R}\mathbf{u})\, dt \tag{12–114}$$

where $\mathbf{Q}$ is a positive-definite (or positive-semidefinite) Hermitian or real symmetric matrix and $\mathbf{R}$ is a positive-definite Hermitian or real symmetric matrix. Note that the second term on the right-hand side of Equation (12–114) accounts for the expenditure of the energy of the control signals. The matrices $\mathbf{Q}$ and $\mathbf{R}$ determine the relative importance of the error and the expenditure of this energy. In this problem, we assume that the control vector $\mathbf{u}(t)$ is unconstrained.

As will be seen later, the linear control law given by Equation (12–113) is the optimal control law. Therefore, if the unknown elements of the matrix $\mathbf{K}$ are determined so as to minimize the performance index, then $\mathbf{u}(t) = -\mathbf{K}\mathbf{x}(t)$ is optimal for any initial state $\mathbf{x}(0)$. The block diagram showing the optimal configuration is shown in Figure 12–35.

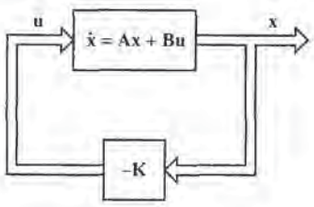


Figure 12–35
Optimal regulator system.

0910

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10695

Now let us solve the optimization problem. Substituting Equation (12–113) into Equation (12–112), we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}$$

In the following derivations, we assume that the matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$ is stable, or that the eigenvalues of $\mathbf{A} - \mathbf{B}\mathbf{K}$ have negative real parts.

Substituting Equation (12-113) into Equation (12-114) yields

$$J = \int_0^\infty (\mathbf{x}^*\mathbf{Q}\mathbf{x} + \mathbf{x}^*\mathbf{K}^*\mathbf{R}\mathbf{K}\mathbf{x})\, dt$$

$$= \int_0^\infty \mathbf{x}^*(\mathbf{Q} + \mathbf{K}^*\mathbf{R}\mathbf{K})\mathbf{x}\, dt$$

Let us set

$$\mathbf{x}^*(\mathbf{Q} + \mathbf{K}^*\mathbf{R}\mathbf{K})\mathbf{x} = -\frac{d}{dt}(\mathbf{x}^*\mathbf{P}\mathbf{x})$$

where $\mathbf{P}$ is a positive-definite Hermitian or real symmetric matrix. Then we obtain

$$\mathbf{x}^*(\mathbf{Q} + \mathbf{K}^*\mathbf{R}\mathbf{K})\mathbf{x} = -\dot{\mathbf{x}}^*\mathbf{P}\mathbf{x} - \mathbf{x}^*\mathbf{P}\dot{\mathbf{x}} = -\mathbf{x}^*\left[(\mathbf{A} - \mathbf{B}\mathbf{K})^*\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K})\right]\mathbf{x}$$

Comparing both sides of this last equation and noting that this equation must hold true for any $\mathbf{x}$, we require that

$$(\mathbf{A} - \mathbf{B}\mathbf{K})^*\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{B}\mathbf{K}) = -(\mathbf{Q} + \mathbf{K}^*\mathbf{R}\mathbf{K}) \tag{12–115}$$

It can be proved that if $\mathbf{A} - \mathbf{B}\mathbf{K}$ is a stable matrix, there exists a positive-definite matrix $\mathbf{P}$ that satisfies Equation (12–115). (See Problem **A–12–15**.)

Hence our procedure is to determine the elements of $\mathbf{P}$ from Equation (12–115) and see if it is positive definite. (Note that more than one matrix $\mathbf{P}$ may satisfy this equation. If the system is stable, there always exists one positive-definite matrix $\mathbf{P}$ to satisfy this equation. This means that, if we solve this equation and find one positive-definite matrix $\mathbf{P}$, the system is stable. Other $\mathbf{P}$ matrices that satisfy this equation are not positive definite and must be discarded.)

The performance index J can be evaluated as

$$J = \int_0^\infty \mathbf{x}^*(\mathbf{Q} + \mathbf{K}^*\mathbf{R}\mathbf{K})\mathbf{x}\, dt = -\mathbf{x}^*\mathbf{P}\mathbf{x}\Big|_0^\infty = -\mathbf{x}^*(\infty)\mathbf{P}\mathbf{x}(\infty) + \mathbf{x}^*(0)\mathbf{P}\mathbf{x}(0)$$

Since all eigenvalues of $\mathbf{A} - \mathbf{B}\mathbf{K}$ are assumed to have negative real parts, we have $\mathbf{x}(\infty) \to \mathbf{0}$. Therefore, we obtain

$$J = \mathbf{x}^*(0)\mathbf{P}\mathbf{x}(0) \tag{12–116}$$

Thus, the performance index J can be obtained in terms of the initial condition $\mathbf{x}(0)$ and $\mathbf{P}$.

To obtain the solution to the quadratic optimal control problem, we proceed as follows: Since $\mathbf{R}$ has been assumed to be a positive-definite Hermitian or real symmetric matrix, we can write

$$\mathbf{R} = \mathbf{T}^*\mathbf{T}$$

0911

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10696

where $\mathbf{T}$ is a nonsingular matrix. Then Equation (12-115) can be written as

$$(\mathbf{A}^* - \mathbf{K}^*\mathbf{B}^*)\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{BK}) + \mathbf{Q} + \mathbf{K}^*\mathbf{T}^*\mathbf{TK} = \mathbf{0}$$

which can be rewritten as

$$\mathbf{A}^*\mathbf{P} + \mathbf{PA} + [\mathbf{TK} - (\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P}]^*[\mathbf{TK} - (\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P}] - \mathbf{PBR}^{-1}\mathbf{B}^*\mathbf{P} + \mathbf{Q} = \mathbf{0}$$

The minimization of J with respect to $\mathbf{K}$ requires the minimization of

$$\mathbf{x}^*[\mathbf{TK} - (\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P}]^*[\mathbf{TK} - (\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P}]\mathbf{x}$$

with respect to $\mathbf{K}$. (See Problem **A–12–16.**) Since this last expression is nonnegative, the minimum occurs when it is zero, or when

$$\mathbf{TK} = (\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P}$$

Hence,

$$\mathbf{K} = \mathbf{T}^{-1}(\mathbf{T}^*)^{-1}\mathbf{B}^*\mathbf{P} = \mathbf{R}^{-1}\mathbf{B}^*\mathbf{P} \tag{12–117}$$

Equation (12–117) gives the optimal matrix $\mathbf{K}$. Thus, the optimal control law to the quadratic optimal control problem when the performance index is given by Equation (12–114) is linear and is given by

$$\mathbf{u}(t) = -\mathbf{Kx}(t) = -\mathbf{R}^{-1}\mathbf{B}^*\mathbf{Px}(t)$$

The matrix $\mathbf{P}$ in Equation (12–117) must satisfy Equation (12–115) or the following reduced equation:

$$\mathbf{A}^*\mathbf{P} + \mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}^*\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{12–118}$$

Equation (12–118) is called the reduced-matrix Riccati equation. The design steps may be stated as follows:

1. Solve Equation (12–118), the reduced-matrix Riccati equation, for the matrix $\mathbf{P}$. [If a positive-definite matrix $\mathbf{P}$ exists (certain systems may not have a positive-definite matrix $\mathbf{P}$), the system is stable, or matrix $\mathbf{A} - \mathbf{BK}$ is stable.]
2. Substitute this matrix $\mathbf{P}$ into Equation (12–117). The resulting matrix $\mathbf{K}$ is the optimal matrix.

A design example based on this approach is given in Example 12–9. Note that if the matrix $\mathbf{A} - \mathbf{BK}$ is stable, the present method always gives the correct result.

Finally, note that if the performance index is given in terms of the output vector rather than the state vector, that is,

$$J = \int_0^\infty (\mathbf{y}^*\mathbf{Qy} + \mathbf{u}^*\mathbf{Ru})\,dt$$

then the index can be modified by using the output equation

$$\mathbf{y} = \mathbf{Cx}$$

to

$$J = \int_0^\infty (\mathbf{x}^*\mathbf{C}^*\mathbf{QCx} + \mathbf{u}^*\mathbf{Ru})\,dt \tag{12–119}$$

and the design steps presented in this section can be applied to obtain the optimal matrix $\mathbf{K}$.

0912

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10697

EXAMPLE 12–9 Consider the system shown in Figure 12–36. Assuming the control signal to be

$$u(t) = -\mathbf{K}\mathbf{x}(t)$$

determine the optimal feedback gain matrix $\mathbf{K}$ such that the following performance index is minimized:

$$J = \int_0^\infty (\mathbf{x}^T\mathbf{Q}\mathbf{x} + u^2)\,dt$$

where

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} \qquad (\mu \geq 0)$$

From Figure 12–36, we find that the state equation for the plant is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

We shall demonstrate the use of the reduced-matrix Riccati equation in the design of the optimal control system. Let us solve Equation (12-118), rewritten as

$$\mathbf{A}^*\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^*\mathbf{P} + \mathbf{Q} = \mathbf{0}$$

Noting that matrix $\mathbf{A}$ is real and matrix $\mathbf{Q}$ is real symmetric, we see that matrix $\mathbf{P}$ is a real symmetric matrix. Hence, this last equation can be written as

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}[1][0 \quad 1]\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

This equation can be simplified to

$$\begin{bmatrix} 0 & 0 \\ p_{11} & p_{12} \end{bmatrix} + \begin{bmatrix} 0 & p_{11} \\ 0 & p_{12} \end{bmatrix} - \begin{bmatrix} p_{12}^2 & p_{12}p_{22} \\ p_{12}p_{22} & p_{22}^2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

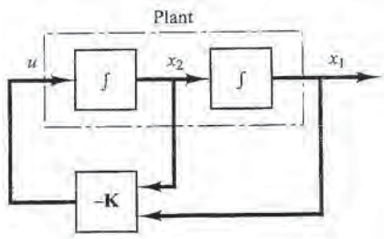


Figure 12–36
Control system.

0913

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10698

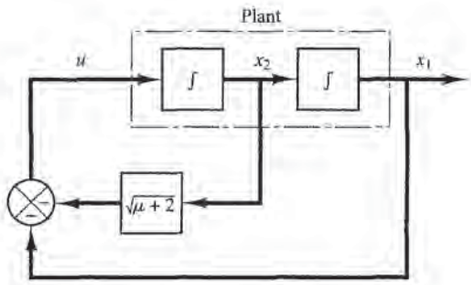


Figure 12–37
Optimal control of the plant shown in Figure 12–36.

from which we obtain the following three equations:

$$1 - p_{12}^2 = 0$$
$$p_{11} - p_{12}p_{22} = 0$$
$$\mu + 2p_{12} - p_{22}^2 = 0$$

Solving these three simultaneous equations for p_{11}, p_{12}, and p_{22}, requiring $\mathbf{P}$ to be positive definite, we obtain

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} \sqrt{\mu + 2} & 1 \\ 1 & \sqrt{\mu + 2} \end{bmatrix}$$

Referring to Equation (12–117), the optimal feedback gain matrix $\mathbf{K}$ is obtained as

$$\begin{aligned} \mathbf{K} &= \mathbf{R}^{-1}\mathbf{B}^*\mathbf{P} \\ &= [1][0 \quad 1]\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} \\ &= [p_{12} \quad p_{22}] \\ &= [1 \quad \sqrt{\mu + 2}] \end{aligned}$$

Thus, the optimal control signal is

$$u = -\mathbf{K}\mathbf{x} = -x_1 - \sqrt{\mu + 2}\, x_2 \tag{12–120}$$

Note that the control law given by Equation (12–120) yields an optimal result for any initial state under the given performance index. Figure 12–37 is the block diagram for this system.

Since the characteristic equation is

$$|s\mathbf{I} - \mathbf{A} + \mathbf{B}\mathbf{K}| = s^2 + \sqrt{\mu + 2}\, s + 1 = 0$$

if $\mu = 1$, the two closed-loop poles are located at

$$s = -0.866 + j, \qquad s = -0.866 - j$$

These correspond to the desired closed-loop poles when $\mu = 1$.

Solving Quadratic Optimal Regulator Problems with MATLAB. In MATLAB, the command

```
lqr(A,B,Q,R)
```


PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10699

solves the continuous-time, linear, quadratic regulator problem and the associated Riccati equation. This command calculates the optimal feedback gain matrix **K** such that the feedback control law

$$u = -\mathbf{K}\mathbf{x}$$

minimizes the performance index

$$J = \int_0^\infty (\mathbf{x}^*\mathbf{Q}\mathbf{x} + \mathbf{u}^*\mathbf{R}\mathbf{u})\, dt$$

subject to the constraint equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

Another command

```
[K,P,E] = lqr(A,B,Q,R)
```

returns the gain matrix **K**, eigenvalue vector **E**, and matrix **P**, the unique positive-definite solution to the associated matrix Riccati equation:

$$\mathbf{P}\mathbf{A} + \mathbf{A}^*\mathbf{P} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}^*\mathbf{P} + \mathbf{Q} = \mathbf{0}$$

If matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$ is a stable matrix, such a positive-definite solution **P** always exists. The eigenvalue vector **E** gives the closed-loop poles of $\mathbf{A} - \mathbf{B}\mathbf{K}$.

It is important to note that for certain systems matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$ cannot be made a stable matrix, whatever **K** is chosen. In such a case, there does not exist a positive-definite matrix **P** for the matrix Riccati equation. For such a case, the commands

```
K = lqr(A,B,Q,R)
[K,P,E] = lqr(A,B,Q,R)
```

do not give the solution. See MATLAB Program 12–18.

EXAMPLE 12–10 Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Show that the system cannot be stabilized by the state-feedback control scheme

$$u = -\mathbf{K}\mathbf{x}$$

whatever matrix **K** is chosen. (Notice that this system is not state controllable.)
Define

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

Then

$$\mathbf{A} - \mathbf{B}\mathbf{K} = \begin{bmatrix} -1 & 1 \\ 0 & 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$
$$= \begin{bmatrix} -1 - k_1 & 1 - k_2 \\ 0 & 2 \end{bmatrix}$$

0915

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10700

Hence, the characteristic equation becomes

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}| = \begin{vmatrix} s + 1 + k_1 & -1 + k_2 \\ 0 & s - 2 \end{vmatrix}$$
$$= (s + 1 + k_1)(s - 2) = 0$$

The closed-loop poles are located at

$$s = -1 - k_1, \qquad s = 2$$

Since the pole at $s = 2$ is in the right-half s plane, the system is unstable whatever **K** matrix is chosen. Hence, quadratic optimal control techniques cannot be applied to this system.

Let us assume that matrices **Q** and **R** in the quadratic performance index are given by

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = [1]$$

and that we write MATLAB Program 12–18. The resulting MATLAB solution is

```
K = [NaN  NaN]
```

(NaN means 'not a number.') Whenever the solution to a quadratic optimal control problem does not exist, MATLAB tells us that matrix **K** consists of NaN.

MATLAB Program 12–18

```
% --------- Design of quadratic optimal regulator system ---------
A = [-1 1;0  2];
B = [1;0];
Q = [1  0;0  1];
R = [1];
K = lqr(A,B,Q,R)
Warning: Matrix is singular to working precision.
K =
   NaN  NaN
% ***** If we enter the command [K,P,E] = lqr(A,B,Q,R), then *****
[K,P,E] = lqr(A,B,Q,R)
Warning: Matrix is singular to working precision.
K =
   NaN  NaN
P =
   -Inf  -Inf
   -Inf  -Inf
E =
   -2.0000
   -1.4142
```


PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10701

EXAMPLE 12–11 Consider the system described by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

The performance index J is given by

$$J = \int_0^\infty (\mathbf{x}'\mathbf{Q}\mathbf{x} + u'Ru)\,dt$$

where

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad R = [1]$$

Assume that the following control u is used.

$$u = -\mathbf{K}\mathbf{x}$$

Determine the optimal feedback gain matrix $\mathbf{K}$.

The optimal feedback gain matrix $\mathbf{K}$ can be obtained by solving the following Riccati equation for a positive-definite matrix $\mathbf{P}$:

$$\mathbf{A}'\mathbf{P} + \mathbf{P}\mathbf{A} - \mathbf{P}\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\mathbf{P} + \mathbf{Q} = \mathbf{0}$$

The result is

$$\mathbf{P} = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

Substituting this $\mathbf{P}$ matrix into the following equation gives the optimal $\mathbf{K}$ matrix:

$$\mathbf{K} = R^{-1}\mathbf{B}'\mathbf{P}$$

$$= [1][0 \quad 1]\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = [1 \quad 1]$$

Thus, the optimal control signal is given by

$$u = -\mathbf{K}\mathbf{x} = -x_1 - x_2$$

MATLAB 12–19 also yields the solution to this problem.

MATLAB Program 12–19

```
% ---------- Design of quadratic optimal regulator system ----------
A = [0  1;0  -1];
B = [0;1];
Q = [1  0; 0  1];
R = [1];
K = lqr(A,B,Q,R)
K =
   1.0000   1.0000
```


0917

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10702

EXAMPLE 12–12 Consider the system given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -35 & -27 & -9 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The performance index J is given by

$$J = \int_0^\infty (\mathbf{x}'\mathbf{Q}\mathbf{x} + u'Ru)\,dt$$

where

$$\mathbf{Q} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad R = [1]$$

Obtain the positive-definite solution matrix $\mathbf{P}$ of the Riccati equation, the optimal feedback gain matrix $\mathbf{K}$, and the eigenvalues of matrix $\mathbf{A} - \mathbf{B}\mathbf{K}$.

MATLAB Program 12–20 will solve this problem.

MATLAB Program 12–20

```
% ---------- Design of quadratic optimal regulator system ----------
A = [0 1 0;0 0 1;-35 -27 -9];
B = [0;0;1];
Q = [1 0 0;0 1 0;0 0 1];
R = [1];
[K,P,E] = lqr(A,B,Q,R)

K =

    0.0143    0.1107    0.0676

P =

    4.2625    2.4957    0.0143
    2.4957    2.8150    0.1107
    0.0143    0.1107    0.0676

E =

   -5.0958
   -1.9859 + 1.7110i
   -1.9859 - 1.7110i
```


0918

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10703

Next, let us obtain the response $\mathbf{x}$ of the regulator system to the initial condition $\mathbf{x}(0)$, where

$$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

With state feedback $u = -\mathbf{K}\mathbf{x}$, the state equation for the system becomes

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x}$$

Then the system, or sys, can be given by

```
sys = ss(A-B*K, eye(3), eye(3), eye(3))
```

MATLAB Program 12–21 produces the response to the given initial condition. The response curves are shown in Figure 12–38.

MATLAB Program 12–21

```
% Response to initial condition.
A = [0  1  0;0  0  1;-35  -27  -9];
B = [0;0;1];
K = [0.0143  0.1107  0.0676];
sys = ss(A-B*K, eye(3),eye(3),eye(3));
t = 0:0.01:8;
x = initial(sys,[1;0;0],t);
x1 = [1  0  0]*x';
x2 = [0  1  0]*x';
X3 = [0  0  1]*x';

subplot(2,2,1); plot(t,x1); grid
xlabel('t (sec)'); ylabel('x1')

subplot(2,2,2); plot(t,x2); grid
xlabel('t (sec)'); ylabel('x2)

subplot(2,2,3); plot(t,x3); grid
xlabel('t (sec)'); ylabel('x3')
```

EXAMPLE 12–13 Consider the system shown in Figure 12–39. The plant is defined by the following state-space equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x} + Du$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0], \quad D = [0]$$

The control signal u is given by

$$u = k_1(r - x_1) - (k_2x_2 + k_3x_3) = k_1r - (k_1x_1 + k_2x_2 + k_3x_3)$$

0919

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10704

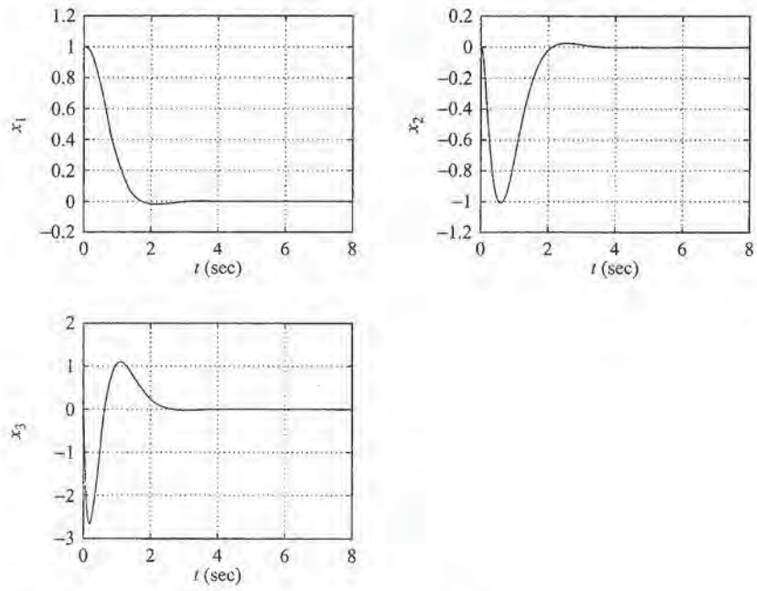


Figure 12–38
Response curves to initial condition.

In determining an optimal control law, we assume that the input is zero, or $r = 0$.

Let us determine the state-feedback gain matrix $\mathbf{K}$, where

$$\mathbf{K} = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

such that the following performance index is minimized:

$$J = \int_0^\infty (\mathbf{x}'\mathbf{Q}\mathbf{x} + u'Ru)\, dt$$

where

$$\mathbf{Q} = \begin{bmatrix} q_{11} & 0 & 0 \\ 0 & q_{22} & 0 \\ 0 & 0 & q_{33} \end{bmatrix}, \qquad R = 1, \qquad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \\ \ddot{y} \end{bmatrix}$$

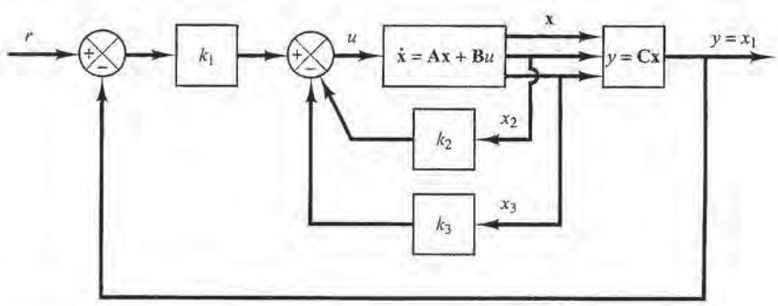


Figure 12–39
Control system.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10705

To get a fast response, q_{11} must be sufficiently large compared with q_{22}, q_{33}, and R. In this problem, we choose

$$q_{11} = 100, \qquad q_{22} = q_{33} = 1, \qquad R = 0.01$$

To solve this problem with MATLAB, we use the command

```
K = lqr(A,B,Q,R)
```

MATLAB Program 12–22 yields the solution to this problem.

MATLAB Program 12–22

```
% ---------- Design of quadratic optimal control system ----------
A = [0  1  0;0  0  1;0  -2  -3];
B = [0;0;1];
Q = [100  0  0;0  1  0;0  0  1];
R = [0.01];
K = lqr(A,B,Q,R)
K =
   100.0000   53.1200   11.6711
```

Next we shall investigate the step-response characteristics of the designed system using the matrix **K** thus determined. The state equation for the designed system is

$$\begin{aligned}\dot{\mathbf{x}} &= \mathbf{A}\mathbf{x} + \mathbf{B}u \\ &= \mathbf{A}\mathbf{x} + \mathbf{B}(-\mathbf{K}\mathbf{x} + k_1 r) \\ &= (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}k_1 r\end{aligned}$$

and the output equation is

$$y = \mathbf{C}\mathbf{x} = [1 \quad 0 \quad 0]\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

To obtain the unit-step response, use the following command:

```
[y,x,t] = step(AA,BB,CC,DD)
```

where

$$AA = \mathbf{A} - \mathbf{B}\mathbf{K}, \qquad BB = \mathbf{B}k_1, \qquad CC = \mathbf{C}, \qquad DD = D$$

MATLAB Program 12–23 produces the unit-step response of the designed system. Figure 12–40 shows the response curves x_1, x_2, and x_3 versus t on one diagram.

0921

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10706

MATLAB Program 12–23

```
% ---------- Unit-step response of designed system ----------

A = [0  1  0;0  0  1;0  -2  -3];
B = [0;0;1]
C = [1  0  0];
D = [0];
K = [100.0000  53.1200  11.6711];
k1 = K(1); k2 = K(2); k3 = K(3);

% ***** Define the state matrix, control matrix, output matrix,
% and direct transmission matrix of the designed systems as AA,
% BB, CC, and DD *****

AA = A - B*K;
BB = B*k1;
CC = C;
DD = D;
t = 0:0.01:8;
[y,x,t] = step (AA,BB,CC,DD,1,t);

plot(t,x)
grid
title('Response Curves x1, x2, x3, versus t')
xlabel('t Sec')
ylabel('x1,x2,x3')
text(2.6,1.35,'x1')
text(1.2,1.5,'x2')
text(0.6,3.5,'x3')
```

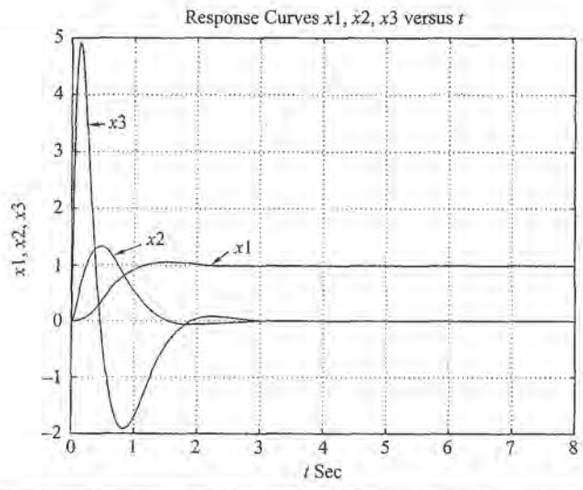


Figure 12–40
Response curves x_1 versus t, x_2 versus t, and x_3 versus t.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10707

Concluding Comments

1. Given any initial state $\mathbf{x}(t_0)$, the optimal regulator problem is to find an allowable control vector $\mathbf{u}(t)$ that transfers the state to the desired region of the state space and for which the performance index is minimized. For the existence of an optimal control vector $\mathbf{u}(t)$, the system must be completely state controllable.
2. The system that minimizes (or maximizes, as the case may be) the selected performance index is, by definition, optimal. Although the controller may have nothing to do with "optimality" in many practical applications, the important point is that the design based on the quadratic performance index yields a stable control system.
3. The characteristic of an optimal control law based on a quadratic performance index is that it is a linear function of the state variables, which implies that we need to feed back all state variables. This requires that all such variables be available for feedback. If not all state variables are available for feedback, then we need to employ a state observer to estimate unmeasurable state variables and use the estimated values to generate optimal control signals.

   Note that the closed-loop poles of the system designed by the use of the quadratic optimal regulator approach can be found from

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}| = 0$$

   Since these closed-loop poles correspond to the desired closed-loop poles in the pole-placement approach, the transfer functions of the observer controllers can be obtained from either Equation (12–74) if the observer is of full-order type or Equation (12–108) if the observer is of minimum-order type.
4. When the optimal control system is designed in the time domain, it is desirable to investigate the frequency-response characteristics to compensate for noise effects. The system frequency-response characteristics must be such that the system attenuates highly in the frequency range where noise and resonance of components are expected. (To compensate for noise effects, we must in some cases either modify the optimal configuration and accept suboptimal performance or modify the performance index.)
5. If the upper limit of integration in the performance index J given by Equation (12-114) is finite, then it can be shown that the optimal control vector is still a linear function of the state variables, but with time-varying coefficients. (Therefore, the determination of the optimal control vector involves that of optimal time-varying matrices.)

EXAMPLE PROBLEMS AND SOLUTIONS

A–12–1. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$$

Suppose that this system is not completely state controllable. Then the rank of the controllability matrix is less than n, or

$$\text{rank}\,[\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}] = q < n \qquad (12\text{–}121)$$

0923

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10708

This means that there are q linearly independent column vectors in the controllability matrix. Let us define such q linearly independent column vectors as $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_q$. Also, let us choose $n - q$ additional n-vectors $\mathbf{v}_{q+1}, \mathbf{v}_{q+2}, \ldots, \mathbf{v}_n$ such that

$$\mathbf{P} = [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \mathbf{v}_{q+2} \mid \cdots \mid \mathbf{v}_n]$$

is of rank n. By using matrix $\mathbf{P}$ as the transformation matrix, define

$$\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \hat{\mathbf{A}}, \qquad \mathbf{P}^{-1}\mathbf{B} = \hat{\mathbf{B}}$$

Show that $\hat{\mathbf{A}}$ can be given by

$$\hat{\mathbf{A}} = \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right]$$

where $\mathbf{A}_{11}$ is a $q \times q$ matrix, $\mathbf{A}_{12}$ is a $q \times (n - q)$ matrix, $\mathbf{A}_{22}$ is an $(n - q) \times (n - q)$ matrix, and $\mathbf{0}$ is an $(n - q) \times q$ matrix. Show also that matrix $\hat{\mathbf{B}}$ can be given by

$$\hat{\mathbf{B}} = \left[\begin{array}{c} \mathbf{B}_{11} \\ \hline \mathbf{0} \end{array}\right]$$

where $\mathbf{B}_{11}$ is a $q \times 1$ matrix and $\mathbf{0}$ is an $(n - q) \times 1$ matrix.

Solution. Notice that

$$\mathbf{A}\mathbf{P} = \mathbf{P}\hat{\mathbf{A}}$$

or

$$[\mathbf{A}\mathbf{f}_1 \mid \mathbf{A}\mathbf{f}_2 \mid \cdots \mid \mathbf{A}\mathbf{f}_q \mid \mathbf{A}\mathbf{v}_{q+1} \mid \cdots \mid \mathbf{A}\mathbf{v}_n]$$
$$= [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \cdots \mid \mathbf{v}_n]\hat{\mathbf{A}} \tag{12–122}$$

Also,

$$\mathbf{B} = \mathbf{P}\hat{\mathbf{B}} \tag{12–123}$$

Since we have q linearly independent column vectors $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_q$, we can use the Cayley–Hamilton theorem to express vectors $\mathbf{A}\mathbf{f}_1, \mathbf{A}\mathbf{f}_2, \ldots, \mathbf{A}\mathbf{f}_q$ in terms of these q vectors. That is,

$$\mathbf{A}\mathbf{f}_1 = a_{11}\mathbf{f}_1 + a_{21}\mathbf{f}_2 + \cdots + a_{q1}\mathbf{f}_q$$
$$\mathbf{A}\mathbf{f}_2 = a_{12}\mathbf{f}_1 + a_{22}\mathbf{f}_2 + \cdots + a_{q2}\mathbf{f}_q$$
$$\vdots$$
$$\mathbf{A}\mathbf{f}_q = a_{1q}\mathbf{f}_1 + a_{2q}\mathbf{f}_2 + \cdots + a_{qq}\mathbf{f}_q$$

0924

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10709

Hence, Equation (12–122) may be written as follows:

$$[\mathbf{Af}_1 \mid \mathbf{Af}_2 \mid \cdots \mid \mathbf{Af}_q \mid \mathbf{Av}_{q+1} \mid \cdots \mid \mathbf{Av}_n]$$

$$= [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \cdots \mid \mathbf{v}_n] \left[\begin{array}{ccc|ccc} a_{11} & \cdots & a_{1q} & a_{1q+1} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2q} & a_{2q+1} & \cdots & a_{2n} \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ a_{q1} & \cdots & a_{qq} & a_{qq+1} & \cdots & a_{qn} \\ \hline 0 & \cdots & 0 & a_{q+1q+1} & \cdots & a_{q+1n} \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ \cdot & & \cdot & \cdot & & \cdot \\ 0 & \cdots & 0 & a_{nq+1} & \cdots & a_{nn} \end{array}\right]$$

Define

$$\begin{bmatrix} a_{11} & \cdots & a_{1q} \\ a_{21} & \cdots & a_{2q} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{q1} & \cdots & a_{qq} \end{bmatrix} = \mathbf{A}_{11}$$

$$\begin{bmatrix} a_{1q+1} & \cdots & a_{1n} \\ a_{2q+1} & \cdots & a_{2n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{qq+1} & \cdots & a_{qn} \end{bmatrix} = \mathbf{A}_{12}$$

$$\begin{bmatrix} 0 & \cdots & 0 \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ 0 & \cdots & 0 \end{bmatrix} = \mathbf{A}_{21} = (n - q) \times q \text{ zero matrix}$$

$$\begin{bmatrix} a_{q+1q+1} & \cdots & a_{q+1n} \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ a_{nq+1} & \cdots & a_{nn} \end{bmatrix} = \mathbf{A}_{22}$$

Then Equation (12–122) can be written as

$$[\mathbf{Af}_1 \mid \mathbf{Af}_2 \mid \cdots \mid \mathbf{Af}_q \mid \mathbf{Av}_{q+1} \mid \cdots \mid \mathbf{Av}_n]$$

$$= [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \cdots \mid \mathbf{v}_n] \left[\begin{array}{c|c} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{array}\right]$$

0925

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10710

Thus,

$$\mathbf{AP} = \mathbf{P}\begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}$$

Hence,

$$\mathbf{P}^{-1}\mathbf{AP} = \hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \hline \mathbf{0} & \mathbf{A}_{22} \end{bmatrix}$$

Next, referring to Equation (12–123), we have

$$\mathbf{B} = [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \cdots \mid \mathbf{v}_n]\hat{\mathbf{B}} \tag{12–124}$$

Referring to Equation (12–121), notice that vector $\mathbf{B}$ can be written in terms of q linearly independent column vectors $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_q$. Thus, we have

$$\mathbf{B} = b_{11}\mathbf{f}_1 + b_{21}\mathbf{f}_2 + \cdots + b_{q1}\mathbf{f}_q$$

Consequently, Equation (12–124) may be written as follows:

$$b_{11}\mathbf{f}_1 + b_{21}\mathbf{f}_2 + \cdots + b_{q1}\mathbf{f}_q = [\mathbf{f}_1 \mid \mathbf{f}_2 \mid \cdots \mid \mathbf{f}_q \mid \mathbf{v}_{q+1} \mid \cdots \mid \mathbf{v}_n] \begin{bmatrix} b_{11} \\ b_{21} \\ \cdot \\ \cdot \\ \cdot \\ b_{q1} \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Thus,

$$\hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B}_{11} \\ \hline \mathbf{0} \end{bmatrix}$$

where

$$\mathbf{B}_{11} = \begin{bmatrix} b_{11} \\ b_{21} \\ \cdot \\ \cdot \\ \cdot \\ b_{q1} \end{bmatrix}$$

A–12–2. Consider a completely state controllable system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

Define the controllability matrix as $\mathbf{M}$:

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}]$$

0926

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10711

Show that

$$\mathbf{M}^{-1}\mathbf{A}\mathbf{M} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & \cdots & 0 & -a_{n-2} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix}$$

where $a_1, a_2, \ldots, a_n$ are the coefficients of the characteristic polynomial

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

Solution. Let us consider the case where $n = 3$. We shall show that

$$\mathbf{A}\mathbf{M} = \mathbf{M}\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \tag{12–125}$$

The left-hand side of Equation (12–125) is

$$\mathbf{A}\mathbf{M} = \mathbf{A}[\mathbf{B} \mid \mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B}] = [\mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B} \mid \mathbf{A}^3\mathbf{B}]$$

The right-hand side of Equation (12–125) is

$$[\mathbf{B} \mid \mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B}]\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} = [\mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B} \mid -a_3\mathbf{B} - a_2\mathbf{A}\mathbf{B} - a_1\mathbf{A}^2\mathbf{B}] \tag{12–126}$$

The Cayley–Hamilton theorem states that matrix $\mathbf{A}$ satisfies its own characteristic equation or, in the case of $n = 3$,

$$\mathbf{A}^3 + a_1\mathbf{A}^2 + a_2\mathbf{A} + a_3\mathbf{I} = \mathbf{0} \tag{12–127}$$

Using Equation (12–127), the third column of the right-hand side of Equation (12–126) becomes

$$-a_3\mathbf{B} - a_2\mathbf{A}\mathbf{B} - a_1\mathbf{A}^2\mathbf{B} = (-a_3\mathbf{I} - a_2\mathbf{A} - a_1\mathbf{A}^2)\mathbf{B} = \mathbf{A}^3\mathbf{B}$$

Thus, Equation (12–126), the right-hand side of Equation (12–125), becomes

$$[\mathbf{B} \mid \mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B}]\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} = [\mathbf{A}\mathbf{B} \mid \mathbf{A}^2\mathbf{B} \mid \mathbf{A}^3\mathbf{B}]$$

Hence, the left-hand side and the right-hand side of Equation (12–125) are the same. We have thus shown that Equation (12–125) is true. Consequently,

$$\mathbf{M}^{-1}\mathbf{A}\mathbf{M} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}$$

The preceding derivation can be easily extended to the general case of any positive integer n.

A–12–3. Consider a completely state controllable system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

Define

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{A}\mathbf{B} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}]$$

0927

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10712

and

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

where the a_i's are coefficients of the characteristic polynomial

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

Define also

$$\mathbf{T} = \mathbf{MW}$$

Show that

$$\mathbf{T}^{-1}\mathbf{AT} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix}, \quad \mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \cdot \\ \cdot \\ \cdot \\ 0 \\ 1 \end{bmatrix}$$

Solution. Let us consider the case where $n = 3$. We shall show that

$$\mathbf{T}^{-1}\mathbf{AT} = (\mathbf{MW})^{-1}\mathbf{A}(\mathbf{MW}) = \mathbf{W}^{-1}(\mathbf{M}^{-1}\mathbf{AM})\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \tag{12–128}$$

Referring to Problem **A–12–2**, we have

$$\mathbf{M}^{-1}\mathbf{AM} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}$$

Hence, Equation (12–128) can be rewritten as

$$\mathbf{W}^{-1}\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\mathbf{W} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$$

Therefore, we need to show that

$$\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\mathbf{W} = \mathbf{W}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \tag{12–129}$$

The left-hand side of Equation (12–129) is

$$\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -a_3 & 0 & 0 \\ 0 & a_1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

0928

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10713

The right-hand side of Equation (12–129) is

$$\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} = \begin{bmatrix} -a_3 & 0 & 0 \\ 0 & a_1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Clearly, Equation (12–129) holds true. Thus, we have shown that

$$\mathbf{T}^{-1}\mathbf{A}\mathbf{T} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$$

Next, we shall show that

$$\mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \tag{12–130}$$

Note that Equation (12–130) can be written as

$$\mathbf{B} = \mathbf{T}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \mathbf{M}\mathbf{W}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

Noting that

$$\mathbf{T}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = [\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}]\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = [\mathbf{B} \mid \mathbf{AB} \mid \mathbf{A}^2\mathbf{B}]\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \mathbf{B}$$

we have

$$\mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The derivation shown here can be easily extended to the general case of any positive integer n.

A–12–4. Consider the state equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -4 & -3 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

The rank of the controllability matrix $\mathbf{M}$,

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{AB}] = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}$$

is 2. Thus, the system is completely state controllable. Transform the given state equation into the controllable canonical form.

Solution. Since

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s-1 & -1 \\ 4 & s+3 \end{vmatrix} = (s-1)(s+3) + 4$$
$$= s^2 + 2s + 1 = s^2 + a_1 s + a_2$$

0929

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10714

we have

$$a_1 = 2, \qquad a_2 = 1$$

Define

$$\mathbf{T} = \mathbf{MW}$$

where

$$\mathbf{M} = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}, \qquad \mathbf{W} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

Then

$$\mathbf{T} = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix}$$

and

$$\mathbf{T}^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0.5 \end{bmatrix}$$

Define

$$\mathbf{x} = \mathbf{T}\hat{\mathbf{x}}$$

Then the state equation becomes

$$\dot{\hat{\mathbf{x}}} = \mathbf{T}^{-1}\mathbf{AT}\hat{\mathbf{x}} + \mathbf{T}^{-1}\mathbf{B}u$$

Since

$$\mathbf{T}^{-1}\mathbf{AT} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ -4 & -3 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ -2 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}$$

and

$$\mathbf{T}^{-1}\mathbf{B} = \begin{bmatrix} 0.5 & 0 \\ 0.5 & 0.5 \end{bmatrix}\begin{bmatrix} 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

we have

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}u$$

which is in the controllable canonical form.

A–12–5. Consider a system defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$$

$$y = \mathbf{Cx}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 0]$$

0930

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10715

The characteristic equation of the system is

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & -1 \\ 2 & s+3 \end{vmatrix} = s^2 + 3s + 2 = (s+1)(s+2) = 0$$

The eigenvalues of matrix **A** are -1 and -2.

It is desired to have eigenvalues at -3 and -5 by using a state-feedback control $u = -\mathbf{K}\mathbf{x}$. Determine the necessary feedback gain matrix **K** and the control signal u.

Solution. The given system is completely state controllable, since the rank of

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{AB}] = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}$$

is 2. Hence, arbitrary pole placement is possible.

Since the characteristic equation of the original system is

$$s^2 + 3s + 2 = s^2 + a_1 s + a_2 = 0$$

we have

$$a_1 = 3, \qquad a_2 = 2$$

The desired characteristic equation is

$$(s+3)(s+5) = s^2 + 8s + 15 = s^2 + \alpha_1 s + \alpha_2 = 0$$

Hence,

$$\alpha_1 = 8, \qquad \alpha_2 = 15$$

It is important to point out that the original state equation is not in the controllable canonical form, because matrix **B** is not

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Hence, the transformation matrix **T** must be determined.

$$\mathbf{T} = \mathbf{MW} = [\mathbf{B} \mid \mathbf{AB}]\begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 2 \\ 2 & -6 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

Hence,

$$\mathbf{T}^{-1} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

Referring to Equation (12–13), the necessary feedback gain matrix is given by

$$\mathbf{K} = [\alpha_2 - a_2 \mid \alpha_1 - a_1]\mathbf{T}^{-1}$$

$$= [15 - 2 \mid 8 - 3]\begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix} = [6.5 \quad 2.5]$$

Thus, the control signal u becomes

$$u = -\mathbf{K}\mathbf{x} = -[6.5 \quad 2.5]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

0931

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10716

A–12–6. A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

Define state variables as

$$x_1 = y$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_2$$

By use of the state-feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to place the closed-loop poles at

$$s = -2 + j2\sqrt{3}, \quad s = -2 - j2\sqrt{3}, \quad s = -10$$

Obtain the necessary state-feedback gain matrix **K** with MATLAB.

Solution. The state-space equations for the system become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 0u$$

Hence,

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 10 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \qquad D = [0]$$

(Note that, for the pole placement, matrices **C** and D do not affect the state-feedback gain matrix **K**.)

Two MATLAB programs for obtaining state-feedback gain matrix **K** are given in MATLAB Programs 12–24 and 12–25.

MATLAB Program 12–24

```
A = [0  1  0;0  0  1;-6  -11  -6];
B = [0;0;10];
J = [-2+j*2*sqrt(3)  -2-j*2*sqrt(3)  -10];
K = acker(A,B,J)

K =

   15.4000    4.5000    0.8000
```


0932

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10717

MATLAB Program 12–25

```
A = [0  1  0;0  0  1;  -6  -11  -6];
B = [0;0;10];
J = [-2+j*2*sqrt(3)  -2-j*2*Sqrt(3)  -10];
K = place(A,B,J)
place: ndigits= 15

K =

   15.4000    4.5000    0.8000
```

A–12–7. Consider a completely observable system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

$$y = \mathbf{C}\mathbf{x}$$

Define the observability matrix as $\mathbf{N}$:

$$\mathbf{N} = [\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^* \mid \cdots \mid (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

Show that

$$\mathbf{N}^*\mathbf{A}(\mathbf{N}^*)^{-1} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{bmatrix} \tag{12–131}$$

where $a_1, a_2, \ldots, a_n$ are the coefficients of the characteristic polynomial

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

Solution. Let us consider the case where $n = 3$. Then Equation (12–131) can be written as

$$\mathbf{N}^*\mathbf{A}(\mathbf{N}^*)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} \tag{12–132}$$

Equation (12–132) may be rewritten as

$$\mathbf{N}^*\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\mathbf{N}^* \tag{12–133}$$

We shall show that Equation (12–133) holds true. The left-hand side of Equation (12–133) is

$$\mathbf{N}^*\mathbf{A} = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}\mathbf{A} = \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ \mathbf{CA}^3 \end{bmatrix} \tag{12–134}$$

0933

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10718

The right-hand side of Equation (12–133) is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\mathbf{N}^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{CA} \\ \mathbf{CA}^2 \\ -a_3\mathbf{C} - a_2\mathbf{CA} - a_1\mathbf{CA}^2 \end{bmatrix} \tag{12–135}$$

The Cayley–Hamilton theorem states that matrix **A** satisfies its own characteristic equation, or

$$\mathbf{A}^3 + a_1\mathbf{A}^2 + a_2\mathbf{A} + a_3\mathbf{I} = \mathbf{0}$$

Hence,

$$-a_1\mathbf{CA}^2 - a_2\mathbf{CA} - a_3\mathbf{C} = \mathbf{CA}^3$$

Thus, the right-hand side of Equation (12–135) becomes the same as the right-hand side of Equation (12–134). Consequently,

$$\mathbf{N}^*\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\mathbf{N}^*$$

which is Equation (12–133). This last equation can be modified to

$$\mathbf{N}^*\mathbf{A}(\mathbf{N}^*)^{-1} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$$

The derivation presented here can be extended to the general case of any positive integer n.

A–12–8. Consider a completely observable system defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u \tag{12–136}$$

$$y = \mathbf{Cx} + Du \tag{12–137}$$

Define

$$\mathbf{N} = [\mathbf{C}^* \;\vdots\; \mathbf{A}^*\mathbf{C}^* \;\vdots\; \cdots \;\vdots\; (\mathbf{A}^*)^{n-1}\mathbf{C}^*]$$

and

$$\mathbf{W} = \begin{bmatrix} a_{n-1} & a_{n-2} & \cdots & a_1 & 1 \\ a_{n-2} & a_{n-3} & \cdots & 1 & 0 \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ a_1 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

where the a's are coefficients of the characteristic polynomial

$$|s\mathbf{I} - \mathbf{A}| = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

Define also

$$\mathbf{Q} = (\mathbf{WN}^*)^{-1}$$

0934

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10719

Show that

$$\mathbf{Q}^{-1}\mathbf{A}\mathbf{Q} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & \cdots & 0 & -a_{n-2} \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ \cdot & \cdot & & \cdot & \cdot \\ 0 & 0 & \cdots & 1 & -a_1 \end{bmatrix}$$

$$\mathbf{C}\mathbf{Q} = [0 \quad 0 \quad \cdots \quad 0 \quad 1]$$

$$\mathbf{Q}^{-1}\mathbf{B} = \begin{bmatrix} b_n - a_n b_0 \\ b_{n-1} - a_{n-1} b_0 \\ \cdot \\ \cdot \\ \cdot \\ b_1 - a_1 b_0 \end{bmatrix}$$

where the b_k's ($k = 0, 1, 2, \ldots, n$) are those coefficients appearing in the numerator of the transfer function when $\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D$ is written as follows:

$$\mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} + D = \frac{b_0 s^n + b_1 s^{n-1} + \cdots + b_{n-1}s + b_n}{s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n}$$

where $D = b_0$.

Solution. Let us consider the case where $n = 3$. We shall show that

$$\mathbf{Q}^{-1}\mathbf{A}\mathbf{Q} = (\mathbf{W}\mathbf{N}^*)\mathbf{A}(\mathbf{W}\mathbf{N}^*)^{-1} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix} \tag{12–138}$$

Note that, by referring to Problem **A–12–7**, we have

$$(\mathbf{W}\mathbf{N}^*)\mathbf{A}(\mathbf{W}\mathbf{N}^*)^{-1} = \mathbf{W}[\mathbf{N}^*\mathbf{A}(\mathbf{N}^*)^{-1}]\mathbf{W}^{-1} = \mathbf{W}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\mathbf{W}^{-1}$$

Hence, we need to show that

$$\mathbf{W}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}\mathbf{W}^{-1} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}$$

or

$$\mathbf{W}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\mathbf{W} \tag{12–139}$$

0935

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10720

The left-hand side of Equation (12–139) is

$$\mathbf{W}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix} = \begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{bmatrix}$$

$$= \begin{bmatrix} -a_3 & 0 & 0 \\ 0 & a_1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

The right-hand side of Equation (12–139) is

$$\begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\mathbf{W} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_3 & 0 & 0 \\ 0 & a_1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Thus, we see that Equation (12–139) holds true. Hence, we have proved Equation (12–138).

Next we shall show that

$$\mathbf{CQ} = [0 \quad 0 \quad 1]$$

or

$$\mathbf{C}(\mathbf{WN}^*)^{-1} = [0 \quad 0 \quad 1]$$

Notice that

$$[0 \quad 0 \quad 1](\mathbf{WN}^*) = [0 \quad 0 \quad 1]\begin{bmatrix} a_2 & a_1 & 1 \\ a_1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix}$$

$$= [1 \quad 0 \quad 0]\begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \mathbf{C}$$

Hence, we have shown that

$$[0 \quad 0 \quad 1] = \mathbf{C}(\mathbf{WN}^*)^{-1} = \mathbf{CQ}$$

Next define

$$\mathbf{x} = \mathbf{Q}\hat{\mathbf{x}}$$

Then Equation (12–136) becomes

$$\dot{\hat{\mathbf{x}}} = \mathbf{Q}^{-1}\mathbf{AQ}\hat{\mathbf{x}} + \mathbf{Q}^{-1}\mathbf{B}u \tag{12–140}$$

and Equation (12–137) becomes

$$y = \mathbf{CQ}\hat{\mathbf{x}} + Du \tag{12–141}$$

For the case of $n = 3$, Equation (12–140) becomes

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \\ \dot{\hat{x}}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -a_3 \\ 1 & 0 & -a_2 \\ 0 & 1 & -a_1 \end{bmatrix}\begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \hat{x}_3 \end{bmatrix} + \begin{bmatrix} \gamma_3 \\ \gamma_2 \\ \gamma_1 \end{bmatrix}u$$

0936

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10721

where

$$\begin{bmatrix} \gamma_3 \\ \gamma_2 \\ \gamma_1 \end{bmatrix} = \mathbf{Q}^{-1}\mathbf{B}$$

The transfer function $G(s)$ for the system defined by Equations (12-140) and (12-141) is

$$G(s) = \mathbf{CQ}(s\mathbf{I} - \mathbf{Q}^{-1}\mathbf{AQ})^{-1}\mathbf{Q}^{-1}\mathbf{B} + D$$

Noting that

$$\mathbf{CQ} = [0 \quad 0 \quad 1]$$

we have

$$G(s) = [0 \quad 0 \quad 1]\begin{bmatrix} s & 0 & a_3 \\ -1 & s & a_2 \\ 0 & -1 & s + a_1 \end{bmatrix}^{-1}\begin{bmatrix} \gamma_3 \\ \gamma_2 \\ \gamma_1 \end{bmatrix} + D$$

Note that $D = b_0$. Since

$$\begin{bmatrix} s & 0 & a_3 \\ -1 & s & a_2 \\ 0 & -1 & s + a_1 \end{bmatrix}^{-1} = \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}\begin{bmatrix} s^2 + a_1 s + a_2 & -a_3 & -a_3 s \\ s + a_1 & s^2 + a_1 s & -a_2 s - a_3 \\ 1 & s & s^2 \end{bmatrix}$$

we have

$$\begin{aligned} G(s) &= \frac{1}{s^3 + a_1 s^2 + a_2 s + a_3}[1 \quad s \quad s^2]\begin{bmatrix} \gamma_3 \\ \gamma_2 \\ \gamma_1 \end{bmatrix} + D \\ &= \frac{\gamma_1 s^2 + \gamma_2 s + \gamma_3}{s^3 + a_1 s^2 + a_2 s + a_3} + b_0 \\ &= \frac{b_0 s^3 + (\gamma_1 + a_1 b_0)s^2 + (\gamma_2 + a_2 b_0)s + \gamma_3 + a_3 b_0}{s^3 + a_1 s^2 + a_2 s + a_3} \\ &= \frac{b_0 s^3 + b_1 s^2 + b_2 s + b_3}{s^3 + a_1 s^2 + a_2 s + a_3} \end{aligned}$$

Hence,

$$\gamma_1 = b_1 - a_1 b_0, \qquad \gamma_2 = b_2 - a_2 b_0, \qquad \gamma_3 = b_3 - a_3 b_0$$

Thus, we have shown that

$$\mathbf{Q}^{-1}\mathbf{B} = \begin{bmatrix} \gamma_3 \\ \gamma_2 \\ \gamma_1 \end{bmatrix} = \begin{bmatrix} b_3 - a_3 b_0 \\ b_2 - a_2 b_0 \\ b_1 - a_1 b_0 \end{bmatrix}$$

Note that what we have derived here can be easily extended to the case when n is any positive integer.

A–12–9. Consider a system defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u$$

$$y = \mathbf{Cx}$$

0937

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10722

where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 \\ -4 & -3 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \qquad \mathbf{C} = [1 \quad 1]$$

The rank of the observability matrix $\mathbf{N}$,

$$\mathbf{N} = [\mathbf{C}^* \mid \mathbf{A}^*\mathbf{C}^*] = \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}$$

is 2. Hence, the system is completely observable. Transform the system equations into the observable canonical form.

Solution. Since

$$|s\mathbf{I} - \mathbf{A}| = s^2 + 2s + 1 = s^2 + a_1 s + a_2$$

we have

$$a_1 = 2, \qquad a_2 = 1$$

Define

$$\mathbf{Q} = (\mathbf{W}\mathbf{N}^*)^{-1}$$

where

$$\mathbf{N} = \begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}, \qquad \mathbf{W} = \begin{bmatrix} a_1 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$$

Then

$$\mathbf{Q} = \left\{ \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -3 & -2 \end{bmatrix} \right\}^{-1} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

and

$$\mathbf{Q}^{-1} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix}$$

Define

$$\mathbf{x} = \mathbf{Q}\hat{\mathbf{x}}$$

Then the state equation becomes

$$\dot{\hat{\mathbf{x}}} = \mathbf{Q}^{-1}\mathbf{A}\mathbf{Q}\hat{\mathbf{x}} + \mathbf{Q}^{-1}\mathbf{B}u$$

or

$$\begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ -4 & -3 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$

$$= \begin{bmatrix} 0 & -1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u \qquad (12\text{–}142)$$

The output equation becomes

$$y = \mathbf{C}\mathbf{Q}\hat{\mathbf{x}}$$

0938

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10723

or

$$y = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \tag{12–143}$$

Equations (12–142) and (12–143) are in the observable canonical form.

A–12–10. For the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

consider the problem of designing a state observer such that the desired eigenvalues for the observer gain matrix are $\mu_1, \mu_2, \cdots, \mu_n$.

Show that the observer gain matrix given by Equation (12–61), rewritten as

$$\mathbf{K}_e = (\mathbf{W}\mathbf{N}^*)^{-1} \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \cdot \\ \cdot \\ \cdot \\ \alpha_1 - a_1 \end{bmatrix} \tag{12–144}$$

can be obtained from Equation (12–13) by considering the dual problem. That is, the matrix $\mathbf{K}_e$ can be determined by considering the pole-placement problem for the dual system, obtaining the state-feedback gain matrix $\mathbf{K}$, and taking its conjugate transpose, or $\mathbf{K}_e = \mathbf{K}^*$.

Solution. The dual of the given system is

$$\dot{\mathbf{z}} = \mathbf{A}^*\mathbf{z} + \mathbf{C}^*v \tag{12–145}$$

$$n = \mathbf{B}^*\mathbf{z}$$

Using the state-feedback control

$$v = -\mathbf{K}\mathbf{z}$$

Equation (12–145) becomes

$$\dot{\mathbf{z}} = (\mathbf{A}^* - \mathbf{C}^*\mathbf{K})\mathbf{z}$$

Equation (12–13), which is rewritten here, is

$$\mathbf{K} = \begin{bmatrix} \alpha_n - a_n & \vdots & \alpha_{n-1} - a_{n-1} & \vdots & \cdots & \vdots & \alpha_2 - a_2 & \vdots & \alpha_1 - a_1 \end{bmatrix} \mathbf{T}^{-1} \tag{12–146}$$

where

$$\mathbf{T} = \mathbf{M}\mathbf{W} = \begin{bmatrix} \mathbf{C}^* & \vdots & \mathbf{A}^*\mathbf{C}^* & \vdots & \cdots & \vdots & (\mathbf{A}^*)^{n-1}\mathbf{C}^* \end{bmatrix} \mathbf{W}$$

For the original system, the observability matrix is

$$\begin{bmatrix} \mathbf{C}^* & \vdots & \mathbf{A}^*\mathbf{C}^* & \vdots & \cdots & \vdots & (\mathbf{A}^*)^{n-1}\mathbf{C}^* \end{bmatrix} = \mathbf{N}$$

Hence, matrix $\mathbf{T}$ can also be written as

$$\mathbf{T} = \mathbf{N}\mathbf{W}$$

Since $\mathbf{W} = \mathbf{W}^*$, we have

$$\mathbf{T}^* = \mathbf{W}^*\mathbf{N}^* = \mathbf{W}\mathbf{N}^*$$

0939

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10724

and

$$(\mathbf{T}^*)^{-1} = (\mathbf{WN}^*)^{-1}$$

Taking the conjugate transpose of both sides of Equation (12–146), we have

$$\mathbf{K}^* = (\mathbf{T}^{-1})^* \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \vdots \\ \alpha_1 - a_1 \end{bmatrix} = (\mathbf{T}^*)^{-1} \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \vdots \\ \alpha_1 - a_1 \end{bmatrix} = (\mathbf{WN}^*)^{-1} \begin{bmatrix} \alpha_n - a_n \\ \alpha_{n-1} - a_{n-1} \\ \vdots \\ \alpha_1 - a_1 \end{bmatrix}$$

Since $\mathbf{K}_e = \mathbf{K}^*$, this last equation is the same as Equation (12–144). Thus, we obtained Equation (12–144) by considering the dual problem.

A–12–11. Consider an observed-state feedback control system with a minimum-order observer described by the following equations:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–147}$$

$$y = \mathbf{C}\mathbf{x}$$

$$u = -\mathbf{K}\tilde{x} \tag{12–148}$$

where

$$\mathbf{x} = \begin{bmatrix} x_a \\ \hdashline \mathbf{x}_b \end{bmatrix}, \qquad \tilde{\mathbf{x}} = \begin{bmatrix} x_a \\ \hdashline \tilde{\mathbf{x}}_b \end{bmatrix}$$

(x_a is the state variable that can be directly measured, and $\tilde{\mathbf{x}}_b$ corresponds to the observed state variables.)

Show that the closed-loop poles of the system comprise the closed-loop poles due to pole placement [the eigenvalues of matrix $(\mathbf{A} - \mathbf{BK})$] and the closed-loop poles due to the minimum-order observer [the eigenvalues of matrix $(\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})$]

Solution. The error equation for the minimum-order observer may be derived as given by Equation (12–94), rewritten thus:

$$\dot{\mathbf{e}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{e} \tag{12–149}$$

where

$$\mathbf{e} = \mathbf{x}_b - \tilde{\mathbf{x}}_b$$

From Equation (12–147) and (12-148), we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{BK}\tilde{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{BK}\begin{bmatrix} x_a \\ \hdashline \tilde{\mathbf{x}}_b \end{bmatrix} = \mathbf{A}\mathbf{x} - \mathbf{BK}\begin{bmatrix} x_a \\ \hdashline \mathbf{x}_b - \mathbf{e} \end{bmatrix}$$

$$= \mathbf{A}\mathbf{x} - \mathbf{BK}\left\{\mathbf{x} - \begin{bmatrix} 0 \\ \hdashline \mathbf{e} \end{bmatrix}\right\} = (\mathbf{A} - \mathbf{BK})\mathbf{x} + \mathbf{BK}\begin{bmatrix} 0 \\ \hdashline \mathbf{e} \end{bmatrix} \tag{12–150}$$

Combining Equations (12-149) and (12-150) and writing

$$\mathbf{K} = [K_a \mid \mathbf{K}_b]$$

we obtain

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK}_b \\ \mathbf{0} & \mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix} \tag{12–151}$$

0940

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10725

Equation (12–151) describes the dynamics of the observed-state feedback control system with a minimum-order observer. The characteristic equation for this system is

$$\begin{vmatrix} s\mathbf{I} - \mathbf{A} + \mathbf{BK} & -\mathbf{BK}_b \\ \mathbf{0} & s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab} \end{vmatrix} = 0$$

or

$$|s\mathbf{I} - \mathbf{A} + \mathbf{BK}||s\mathbf{I} - \mathbf{A}_{bb} + \mathbf{K}_e\mathbf{A}_{ab}| = 0$$

The closed-loop poles of the observed-state feedback control system with a minimum-order observer consist of the closed-loop poles due to pole placement and the closed-loop poles due to the minimum-order observer. (Therefore, the pole-placement design and the design of the minimum-order observer are independent of each other.)

A–12–12. Consider a completely state controllable system defined by

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{B}u \tag{12–152}$$
$$y = \mathbf{Cx}$$

where $\mathbf{x}$ = state vector (n-vector)
u = control signal (scalar)
y = output signal (scalar)
$\mathbf{A}$ = $n \times n$ constant matrix
$\mathbf{B}$ = $n \times 1$ constant matrix
$\mathbf{C}$ = $1 \times n$ constant matrix

Suppose that the rank of the following $(n + 1) \times (n + 1)$ matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}$$

is $n + 1$. Show that the system defined by

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{e} + \hat{\mathbf{B}}u_e \tag{12–153}$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix}, \qquad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ \mathbf{0} \end{bmatrix}, \qquad u_e = u(t) - u(\infty)$$

is completely state controllable.

Solution. Define

$$\mathbf{M} = [\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}]$$

Because the system given by Equation (12–152) is completely state controllable, the rank of matrix $\mathbf{M}$ is n. Then the rank of

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}$$

is $n + 1$. Consider the following equation:

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} \mathbf{AM} & \mathbf{B} \\ -\mathbf{CM} & 0 \end{bmatrix} \tag{12–154}$$

Since matrix

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ -\mathbf{C} & 0 \end{bmatrix}$$

0941

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10726

is of rank $n + 1$, the left-hand side of Equation (12–154) is of rank $n + 1$. Therefore, the right-hand side of Equation (12–154) is also of rank $n + 1$. Since

$$\begin{bmatrix} \mathbf{AM} & \mathbf{B} \\ -\mathbf{CM} & 0 \end{bmatrix} = \begin{bmatrix} \mathbf{A}[\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}] & \mathbf{B} \\ -\mathbf{C}[\mathbf{B} \mid \mathbf{AB} \mid \cdots \mid \mathbf{A}^{n-1}\mathbf{B}] & 0 \end{bmatrix}$$
$$= \begin{bmatrix} \mathbf{AB} \mid \mathbf{A}^2\mathbf{B} \mid \cdots \mid \mathbf{A}^n\mathbf{B} \mid \mathbf{B} \\ -\mathbf{CB} \mid -\mathbf{CAB} \mid \cdots \mid -\mathbf{CA}^{n-1}\mathbf{B} \mid 0 \end{bmatrix}$$
$$= [\hat{\mathbf{A}}\hat{\mathbf{B}} \mid \hat{\mathbf{A}}^2\hat{\mathbf{B}} \mid \cdots \mid \hat{\mathbf{A}}^n\hat{\mathbf{B}} \mid \hat{\mathbf{B}}]$$

we find that the rank of

$$[\hat{\mathbf{B}} \mid \hat{\mathbf{A}}\hat{\mathbf{B}} \mid \hat{\mathbf{A}}^2\hat{\mathbf{B}} \mid \cdots \mid \hat{\mathbf{A}}^n\hat{\mathbf{B}}]$$

is $n + 1$. Thus, the system defined by Equation (12–153) is completely state controllable.

A–12–13. Consider the system shown in Figure 12–41. Using the pole-placement-with-observer approach, design a regulator system such that the system will maintain the zero position ($y_1 = 0$ and $y_2 = 0$) in the presence of disturbances. Choose the desired closed-loop poles for the pole placement part to be

$$s = -2 + j2\sqrt{3}, \quad s = -2 - j2\sqrt{3}, \quad s = -10, \quad s = -10$$

and the desired poles for the minimum-order observer to be

$$s = -15, \quad s = -16$$

First, determine the state feedback gain matrix $\mathbf{K}$ and observer gain matrix $\mathbf{K}_e$. Then, obtain the response of the system to an arbitrary initial condition—for example,

$$y_1(0) = 0.1, \quad y_2(0) = 0, \quad \dot{y}_1(0) = 0, \quad \dot{y}_2(0) = 0$$
$$e_1(0) = 0.1, \quad e_2(0) = 0.05$$

where e_1 and e_2 are defined by

$$e_1 = y_1 - \tilde{y}_1$$
$$e_2 = y_2 - \tilde{y}_2$$

Assume that $m_1 = 1$ kg, $m_2 = 2$ kg, $k = 36$ N/m, and $b = 0.6$ N-s/m.

Solution. The equations for the system are

$$m_1\ddot{y}_1 = k(y_2 - y_1) + b(\dot{y}_2 - \dot{y}_1) + u$$
$$m_2\ddot{y}_2 = k(y_1 - y_2) + b(\dot{y}_1 - \dot{y}_2)$$

By substituting the given numerical values for m_1, m_2, k, and b and simplifying, we obtain

$$\ddot{y}_1 = -36y_1 + 36y_2 - 0.6\dot{y}_1 + 0.6\dot{y}_2 + u$$
$$\ddot{y}_2 = 18y_1 - 18y_2 + 0.3\dot{y}_1 - 0.3\dot{y}_2$$

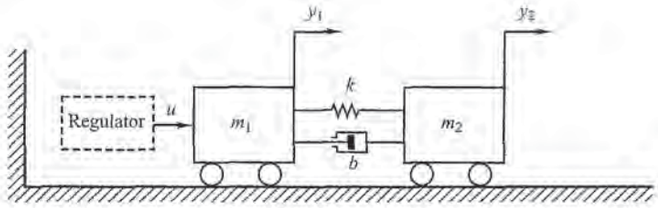


Figure 12–41
Mechanical system.

0942

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10727

Let us choose the state variables as follows:

$$x_1 = y_1$$
$$x_2 = y_2$$
$$x_3 = \dot{y}_1$$
$$x_4 = \dot{y}_2$$

Then, the state-space equations become

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -36 & 36 & -0.6 & 0.6 \\ 18 & -18 & 0.3 & -0.3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} u$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Define

$$\mathbf{A} = \left[\begin{array}{cc:cc} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hdashline -36 & 36 & -0.6 & 0.6 \\ 18 & -18 & 0.3 & -0.3 \end{array}\right] = \begin{bmatrix} \mathbf{A}_{aa} & \mathbf{A}_{ab} \\ \hdashline \mathbf{A}_{ba} & \mathbf{A}_{bb} \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ \hdashline 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{B}_a \\ \hdashline \mathbf{B}_b \end{bmatrix}$$

The state feedback gain matrix $\mathbf{K}$ and observer gain matrix $\mathbf{K}_e$ can be obtained easily by use of MATLAB as follows:

$$\mathbf{K} = [130.4444 \quad -41.5556 \quad 23.1000 \quad 15.4185]$$

$$\mathbf{K}_e = \begin{bmatrix} 14.4 & 0.6 \\ 0.3 & 15.7 \end{bmatrix}$$

(See MATLAB Program 12–26.)

MATLAB Program 12–26

```
A = [0 0 1 0;0 0 0 1;-36 36 -0.6 0.6;18 -18 0.3 -0.3];
B = [0;0;1;0];
J = [-2+j*2*sqrt(3) -2-j*2*sqrt(3) -10 -10];
K = acker(A,B,J)
K =
   130.4444   -41.5556   23.1000   15.4185
Aab = [1 0;0 1];
Abb = [-0.6 0.6;0.3 -0.3];
L = [-15 -16];
Ke = place(Abb',Aab',L)'
place: ndigits= 15
Ke =
   14.4000    0.6000
    0.3000   15.7000
```


0943

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10728

Response to Initial Condition: Next, we obtain the response of the designed system to the given initial condition. Since

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$u = -\mathbf{K}\tilde{\mathbf{x}}$$
$$\tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_a \\ \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{y} \\ \tilde{\mathbf{x}}_b \end{bmatrix}$$

we have

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\tilde{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}}) \tag{12–155}$$

Note that

$$\mathbf{x} - \tilde{\mathbf{x}} = \begin{bmatrix} \mathbf{x}_a \\ \mathbf{x}_b \end{bmatrix} - \begin{bmatrix} \mathbf{x}_a \\ \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{x}_b - \tilde{\mathbf{x}}_b \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{e} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}\mathbf{e} = \mathbf{F}\mathbf{e}$$

where

$$\mathbf{F} = \begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}$$

Then, Equation (12–155) can be written as

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\mathbf{K}\mathbf{F}\mathbf{e} \tag{12–156}$$

Since, from Equation (12–94), we have

$$\dot{\mathbf{e}} = (\mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab})\mathbf{e} \tag{12–157}$$

by combining Equations (12-156) and (12-157) into one equation, we have

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}\mathbf{K}\mathbf{F} \\ \mathbf{0} & \mathbf{A}_{bb} - \mathbf{K}_e\mathbf{A}_{ab} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix}$$

The state matrix here is a 6×6 matrix. The response of the system to the given initial condition can be obtained easily with MATLAB. (See MATLAB Program 12–27.) The resulting response curves are shown in Figure 12–42. The response curves seem to be acceptable.

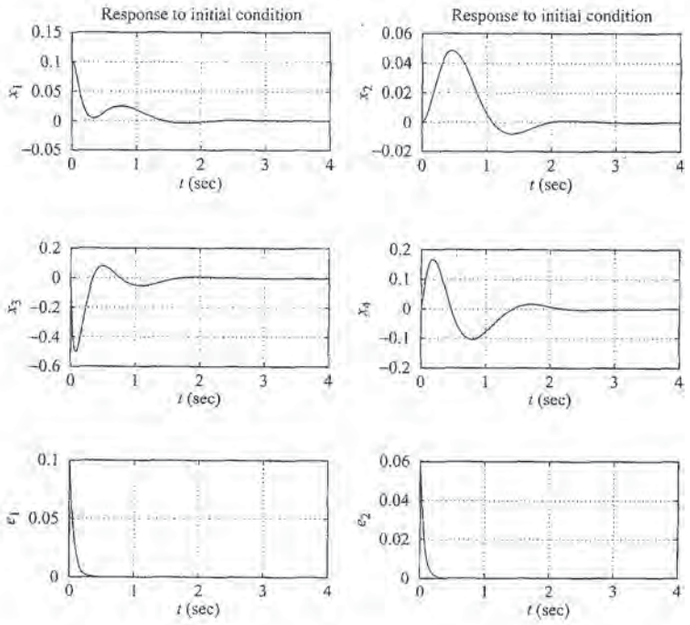


Figure 12–42
Response curves to initial condition.

0944

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10729

MATLAB Program 12–27

```
% Response to initial condition

A = [0  0  1  0;0  0  0  1;-36  36  -0.6  0.6;18  -18  0.3  -0.3];
B = [0;0;1;0];
K = [130.4444  -41.5556  23.1000  15.4185];
Ke = [14.4  0.6;0.3  15.7];
F = [0  0;0  0;1  0;0  1];
Aab = [1  0;0  1];
Abb = [-0.6  0.6;0.3  -0.3];
AA = [A-B*K  B*K*F; zeros(2,4)  Abb-Ke*Aab];
sys = ss(AA,eye(6),eye(6),eye(6));
t = 0:0.01:4;
y = initial(sys,[0.1;0;0;0;0.1;0.05],t);
x1 = [1  0  0  0  0  0]*y';
x2 = [0  1  0  0  0  0]*y';
x3 = [0  0  1  0  0  0]*y';
x4 = [0  0  0  1  0  0]*y';
e1 = [0  0  0  0  1  0]*y';
e2 = [0  0  0  0  0  1]*y';

subplot(3,2,1); plot(t,x1); grid; title('Response to initial condition'),
xlabel('t (sec)'); ylabel('x1')
subplot(3,2,2); plot(t,x2); grid; title('Response to initial condition'),
xlabel('t (sec)'); ylabel('x2')
subplot(3,2,3); plot(t,x3); grid; xlabel('t (sec)'); ylabel('x3')
subplot(3,2,4); plot(t,x4); grid; xlabel('t (sec)'); ylabel('x4')
subplot(3,2,5); plot(t,e1); grid; xlabel('t (sec)');ylabel('e1')
subplot(3,2,6); plot(t,e2); grid; xlabel('t (sec)'); ylabel('e2')
```

A–12–14. Consider the system shown in Figure 12–43. Design both the full-order and minimum-order observers for the plant. Assume that the desired closed-loop poles for the pole placement part are located at

$$s = -2 + j2\sqrt{3}, \qquad s = -2 - j2\sqrt{3}$$

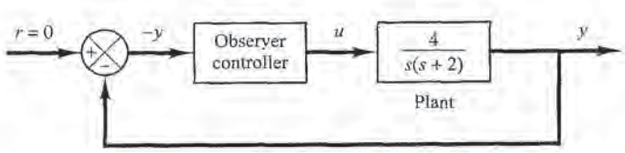


Figure 12–43
Regulator system.

0945

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10730

Assume also that the desired observer poles are located at

(a) $s = -8, s = -8$ for the full-order observer

(b) $s = -8$ for the minimum-order observer

Compare the responses to the initial conditions specified below:

(a) for the full-order observer:

$$x_1(0) = 1, \quad x_2(0) = 0, \quad e_1(0) = 1, \quad e_2(0) = 0$$

(b) for the minimum-order observer:

$$x_1(0) = 1, \quad x_2(0) = 0, \quad e_1(0) = 1$$

Also, compare the bandwidths of both systems.

Solution. We first determine the state-space representation of the system. By defining state variables x_1 and x_2 as

$$x_1 = y$$
$$x_2 = \dot{y}$$

we obtain

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 4 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For the pole-placement part, we determine the state feedback gain matrix $\mathbf{K}$. Using MATLAB, we find $\mathbf{K}$ to be

$$\mathbf{K} = [4 \quad 0.5]$$

(See MATLAB Program 12–28.)

Next, we determine the observer gain matrix $\mathbf{K}_e$ for the full-order observer. Using MATLAB, we find $\mathbf{K}_e$ to be

$$\mathbf{K}_e = \begin{bmatrix} 14 \\ 36 \end{bmatrix}$$

(See MATLAB Program 12–28.)

MATLAB Program 12–28

```
% Obtaining matrices K and Ke.

A = [0  1;0  -2];
B = [0;4];
C = [1  0];
J = [-2+j*2*sqrt(3)  -2-j*2*sqrt(3)];
L = [-8  -8];
K = acker(A,B,J)

K =

   4.0000  0.5000

Ke = acker(A',C',L)'

Ke =

   14
   36
```


0946

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10731

Now we find the response of this system to the given initial condition. Referring to Equation (12–70), we have

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\mathbf{e}} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{BK} & \mathbf{BK} \\ \mathbf{0} & \mathbf{A} - \mathbf{K}_e\mathbf{C} \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \mathbf{e} \end{bmatrix}$$

This equation defines the dynamics of the designed system using the full-order observer. MATLAB Program 12-29 produces the response to the given initial condition. The resulting response curves are shown in Figure 12–44.

MATLAB Program 12–29

```
% Response to initial condition ---- full-order observer

A = [0  1;0  -2];
B = [0;4];
C = [1  0];
K = [4  0.5];
Ke = [14;36];
AA = [A-B*K  B*K; zeros(2,2)  A-Ke*C];
sys = ss(AA, eye(4), eye(4), eye(4));
t = 0:0.01:8;
x = initial(sys, [1;0;1;0],t);
x1 = [1  0  0  0]*x';
x2 = [0  1  0  0]*x';
e1 = [0  0  1  0]*x';
e2 = [0  0  0  1]*x';

subplot(2,2,1); plot(t,x1); grid
xlabel('t (sec)'); ylabel('x1')

subplot(2,2,2); plot(t,x2); grid
xlabel('t (sec)'); ylabel('x2')

subplot(2,2,3); plot(t,e1); grid
xlabel('t (sec)'); ylabel('e1')

subplot(2,2,4); plot(t,e2); grid
xlabel('t (sec)'); ylabel('e2')
```

To obtain the transfer function of the observer controller, we use MATLAB. MATLAB Program 12–30 produces this transfer function. The result is

$$\frac{\text{num}}{\text{den}} = \frac{74s + 256}{s^2 + 18s + 108} = \frac{74(s + 3.4595)}{(s + 9 + j5.1962)(s + 9 - j5.1962)}$$

Next, we obtain the observer gain matrix K_e for the minimum-order observer. MATLAB Program 12–31 produces K_e. The result is

$$K_e = 6$$

0947

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10732

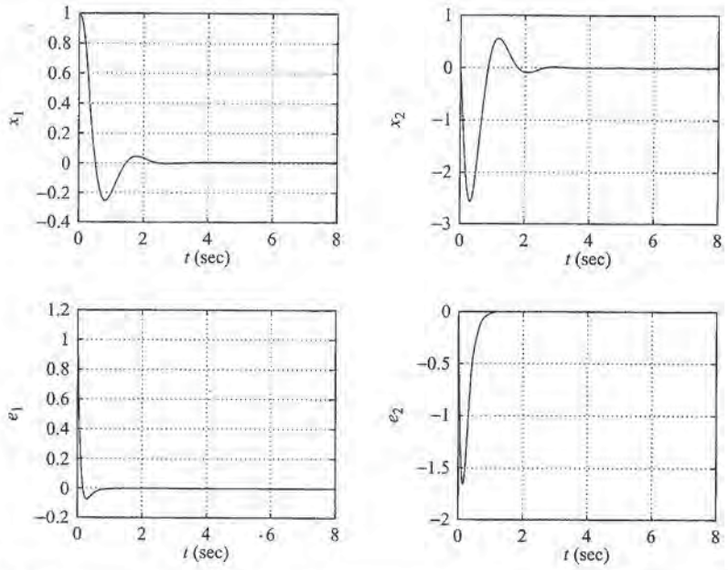


Figure 12–44
Response curves to initial condition.

MATLAB Program 12–30

```
% Determination of transfer function of observer controller ---- full-order observer
A = [0  1;0  -2];
B = [0;4];
C = [1  0];
K = [4  0.5];
Ke = [14;36];
[num,den] = ss2tf(A-Ke*C-B*K, Ke,K,0)
num =
     0   74.0000   256.0000
den =
     1   18   108
```

MATLAB Program 12–31

```
% Obtaining Ke ---- minimum-order observer
Aab = [1];
Abb = [-2];
LL = [-8];
Ke = acker(Abb',Aab',LL)'
Ke =
     6
```


0948

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10733

The response of the system with minimum-order observer to the initial condition can be obtained as follows: By substituting $u = -\mathbf{K}\tilde{\mathbf{x}}$ into the plant equation given by Equation (12–79) we find

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\tilde{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x} + \mathbf{B}\mathbf{K}(\mathbf{x} - \tilde{\mathbf{x}})$$

$$= (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}\begin{bmatrix} K_a & K_b \end{bmatrix}\begin{bmatrix} 0 \\ e \end{bmatrix}$$

or

$$\dot{\mathbf{x}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{x} + \mathbf{B}K_b e$$

The error equation is

$$\dot{e} = (A_{bb} - K_e A_{ab})e$$

Hence the system dynamics are defined by

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}K_b \\ 0 & A_{bb} - K_e A_{ab} \end{bmatrix}\begin{bmatrix} \mathbf{x} \\ e \end{bmatrix}$$

Based on this last equation, MATLAB Program 12–32 produces the response to the given initial condition. The resulting response curves are shown in Figure 12–45.

MATLAB Program 12–32

```
% Response to intial condition ---- minimum-order observer

A = [0  1;0  -2];
B = [0;4];
K = [4  0.5];
Kb = 0.5;
Ke = 6;
Aab = 1; Abb = -2;
AA = [A-B*K  B*Kb; zeros(1,2)  Abb-Ke*Aab];
sys = ss(AA,eye(3),eye(3),eye(3));
t = 0:0.01:8;
x = initial(sys,[1;0;1],t);
x1 = [1  0  0]*x';
x2 = [0  1  0]*x';
e = [0  0  1]*x';

subplot(2,2,1); plot(t,x1); grid
xlabel('t (sec)'); ylabel('x1')

subplot(2,2,2); plot(t,x2); grid
xlabel('t (sec)'); ylabel('x2')

subplot(2,2,3); plot(t,e); grid
xlabel('t (sec)'); ylabel('e')
```


0949

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10734

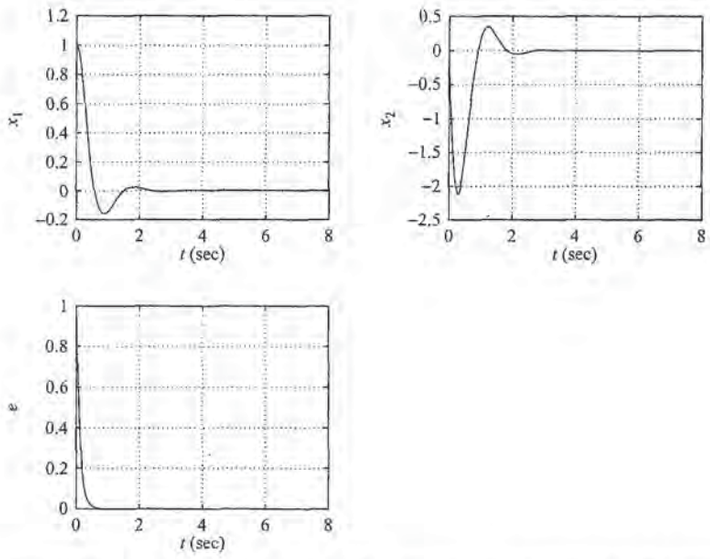


Figure 12–45
Response curves to initial condition.

The transfer function of the observer controller, when the system uses the minimum-order observer, can be obtained by use of MATLAB Program 12–33. The result is

$$\frac{\text{num}}{\text{den}} = \frac{7s + 32}{s + 10} = \frac{7(s + 4.5714)}{s + 10}$$

MATLAB Program 12–33

```
% Determination of transfer function of observer controller ---- minimum-order observer
A = [0  1;0  -2];
B = [0;4];
Aaa = 0; Aab = 1; Aba = 0; Abb = -2;
Ba = 0; Bb = 4;
Ka = 4; Kb = 0.5;
Ke = 6;
Ahat = Abb - Ke*Aab;
Bhat = Ahat*Ke + Aba - Ke*Aaa;
Fhat = Bb - Ke*Ba;
Atilde = Ahat - Fhat*Kb;
Btilde = Bhat - Fhat*(Ka + Kb*Ke);
Ctilde = -Kb;
Dtilde = -(Ka + Kb*Ke);
[num,den] = ss2tf(Atilde, Btilde, -Ctilde, -Dtilde)
num =
   7  32
den =
   1  10
```


0950

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10735

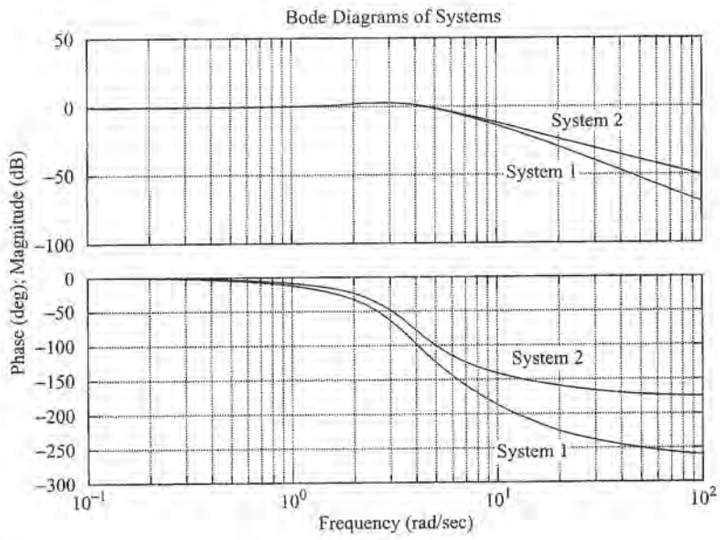


Figure 12–46
Bode diagrams of System 1 (system with full-order observer) and System 2 (system with minimum-order observer).
System 1 = $(296s + 1024)/(s^4 + 20s^3 + 144s^2 + 512s + 1024)$
System 2 = $(28s + 128)/(s^3 + 12s^2 + 48s + 128)$

The observer controller is clearly a lead compensator.

The Bode diagrams of System 1 (closed-loop system with full-order observer) and of System 2 (closed-loop system with minimum-order observer) are shown in Figure 12–46. Clearly, the bandwidth of System 2 is wider than that of System 1. System 1 has a better high-frequency noise-rejection characteristic than System 2.

A–12–15. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where $\mathbf{x}$ is a state vector (n-vector) and $\mathbf{A}$ is an $n \times n$ constant matrix. We assume that $\mathbf{A}$ is nonsingular. Prove that if the equilibrium state $\mathbf{x} = \mathbf{0}$ of the system is asymptotically stable (that is, if $\mathbf{A}$ is a stable matrix), then there exists a positive-definite Hermitian matrix $\mathbf{P}$ such that

$$\mathbf{A}^*\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}$$

where $\mathbf{Q}$ is a positive-definite Hermitian matrix.

Solution. The matrix differential equation.

$$\dot{\mathbf{X}} = \mathbf{A}^*\mathbf{X} + \mathbf{X}\mathbf{A}, \qquad \mathbf{X}(0) = \mathbf{Q}$$

has the solution

$$\mathbf{X} = e^{\mathbf{A}^* t}\mathbf{Q}e^{\mathbf{A}t}$$

Integrating both sides of this matrix differential equation from $t = 0$ to $t = \infty$, we obtain

$$\mathbf{X}(\infty) - \mathbf{X}(0) = \mathbf{A}^*\left(\int_0^\infty \mathbf{X}\,dt\right) + \left(\int_0^\infty \mathbf{X}\,dt\right)\mathbf{A}$$

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10736

Noting that $\mathbf{A}$ is a stable matrix and, therefore, $\mathbf{X}(\infty) = \mathbf{0}$, we obtain

$$-\mathbf{X}(0) = -\mathbf{Q} = \mathbf{A}^*\left(\int_0^\infty \mathbf{X}\,dt\right) + \left(\int_0^\infty \mathbf{X}\,dt\right)\mathbf{A}$$

Let us put

$$\mathbf{P} = \int_0^\infty \mathbf{X}\,dt = \int_0^\infty e^{\mathbf{A}^* t}\mathbf{Q}e^{\mathbf{A}t}\,dt$$

Note that the elements of $e^{\mathbf{A}t}$ are finite sums of terms like $e^{\lambda_i t}, te^{\lambda_i t}, \ldots, t^{m_i-1}e^{\lambda_i t}$, where the λ_i are the eigenvalues of $\mathbf{A}$ and m_i is the multiplicity of λ_i. Since the λ_i possess negative real parts,

$$\int_0^\infty e^{\mathbf{A}^* t}\mathbf{Q}e^{\mathbf{A}t}\,dt$$

exists. Note that

$$\mathbf{P}^* = \int_0^\infty e^{\mathbf{A}^* t}\mathbf{Q}e^{\mathbf{A}t}\,dt = \mathbf{P}$$

Thus $\mathbf{P}$ is Hermitian (or symmetric if $\mathbf{P}$ is a real matrix). We have thus shown that for a stable $\mathbf{A}$ and for a positive-definite Hermitian matrix $\mathbf{Q}$, there exists a Hermitian matrix $\mathbf{P}$ such that $\mathbf{A}^*\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}$. We now need to prove that $\mathbf{P}$ is positive definite. Consider the following Hermitian form:

$$\begin{aligned}\mathbf{x}^*\mathbf{P}\mathbf{x} &= \mathbf{x}^*\int_0^\infty e^{\mathbf{A}^* t}\mathbf{Q}e^{\mathbf{A}t}\,dt\,\mathbf{x}\\ &= \int_0^\infty \left(e^{\mathbf{A}t}\mathbf{x}\right)^*\mathbf{Q}\left(e^{\mathbf{A}t}\mathbf{x}\right)dt > 0, \qquad \text{for } \mathbf{x} \neq \mathbf{0}\\ &= 0, \qquad \text{for } \mathbf{x} = \mathbf{0}\end{aligned}$$

Hence, $\mathbf{P}$ is positive definite. This completes the proof.

A–12–16. Consider the following scalar system:

$$\dot{x} = ax + bu \tag{12–158}$$

where $a < 0$ and the performance index is given by

$$J = \int_0^\infty (qx^2 + ru^2)\,dt \tag{12–159}$$

where $q > 0$ and $r > 0$. The optimal control law that will minimize the performance index J can be given by

$$u = -Kx \tag{12–160}$$

Substituting Equation (12–160) into Equation (12–158) gives

$$\dot{x} = (a - bK)x$$

0952

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10737

Also, substituting Equation (12–160) into Equation (12–159) gives

$$J = \int_0^\infty (q + rK^2)x^2\,dt$$

Let us set

$$(q + rK^2)x^2 = -\frac{d}{dt}(px^2)$$

or

$$(q + rK^2)x^2 = -2px\dot{x} = -2p(a - bK)x^2$$

which can be simplified to

$$[q + rK^2 + 2p(a - bK)]x^2 = 0$$

For this last equation to hold true for any $x(t)$, we require

$$q + rK^2 + 2p(a - bK) = 0 \tag{12–161}$$

Referring to Problem **A–12–15**, if $a - bK$ is stable (meaning that it is negative), then, for a given $q + rK^2$, there exists a positive p such that

$$(a - bK)p + p(a - bK) = -q - rK^2$$

which is the same as Equation (12–161). Hence, there exists a p that satisfies Equation (12–161).

Show that the optimal control law can be given by

$$u = -Kx = -\frac{pb}{r}x$$

and p can be determined as a positive root of the following equation:

$$q + 2ap - \frac{p^2b^2}{r} = 0 \tag{12–162}$$

Solution. For a stable system, we have $x(\infty) = 0$. Hence, the performance index can be evaluated as follows:

$$J = \int_0^\infty (q + rK^2)x^2dt = -\int_0^\infty \frac{d}{dt}(px^2)\,dt$$
$$= -[px^2(\infty) - px^2(0)] = px^2(0)$$

To minimize the value of J [*for a given* $x(0)$] *with respect to* K, we set

$$\frac{\partial p}{\partial K} = 0 \tag{12–163}$$

where, referring to Equation (12–161),

$$p = -\frac{q + rK^2}{2(a - bK)} \tag{12–164}$$

0953

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10738

Thus,

$$\frac{\partial p}{\partial K} = -\frac{2rK(a - bK) - (q + rK^2)(-b)}{2(a - bK)^2} = 0$$

which yields

$$2rK(a - bK) + b(q + rK^2) = 0$$

Hence, we have

$$\frac{q + rK^2}{2(a - bK)} = -\frac{rK}{b} \tag{12–165}$$

From Equations (12–164) and (12–165), we obtain

$$p = \frac{rK}{b}$$

or

$$K = \frac{pb}{r} \tag{12–166}$$

By substituting Equation (12–166) into Equation (12–161), we obtain

$$q + 2pa - \frac{p^2b^2}{r} = 0 \tag{12–167}$$

which is Equation (12–162). The value of p can be determined as a positive root of the quadratic equation given by Equation (12–167).

The same results can be obtained in a different way. First note that Equation (12–161) can be modified as follows:

$$q + 2pa + \left(\sqrt{r}K - \frac{pb}{\sqrt{r}}\right)^2 - \frac{p^2b^2}{r} = 0 \tag{12–168}$$

Then, considering this last equation as a function of K, the minimum of the left-hand side of this last equation with respect to K occurs when

$$\sqrt{r}K - \frac{pb}{\sqrt{r}} = 0$$

or

$$K = \frac{pb}{r} \tag{12–169}$$

which is Equation (12–166). Thus, the minimization of J with respect to K is the same as the minimization of the left-hand side of Equation (12–161) with respect to K. By substituting Equation (12–169) into Equation (12–168), we obtain

$$q + 2pa - \frac{p^2b^2}{r} = 0$$

which is Equation (12–162).

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10739

A–12–17. Consider the control system described by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u \tag{12–170}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

Assuming the linear control law

$$u = -\mathbf{K}\mathbf{x} = -k_1 x_1 - k_2 x_2 \tag{12–171}$$

determine the constants k_1 and k_2 so that the following performance index is minimized:

$$J = \int_0^\infty \mathbf{x}^T \mathbf{x}\, dt$$

Consider only the case where the initial condition is

$$\mathbf{x}(0) = \begin{bmatrix} c \\ 0 \end{bmatrix}$$

Choose the undamped natural frequency to be 2 rad/sec.

Solution. Substituting Equation (12–171) into Equation (12–170), we obtain

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{K}\mathbf{x}$$

or

$$\begin{aligned} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-k_1 x_1 - k_2 x_2] \\ &= \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \end{aligned} \tag{12–172}$$

Thus,

$$\mathbf{A} - \mathbf{B}\mathbf{K} = \begin{bmatrix} 0 & 1 \\ -k_1 & -k_2 \end{bmatrix}$$

Elimination of x_2 from Equation (12–172) yields

$$\ddot{x}_1 + k_2 \dot{x}_1 + k_1 x_1 = 0$$

Since the undamped natural frequency is specified as 2 rad/sec, we obtain

$$k_1 = 4$$

Therefore,

$$\mathbf{A} - \mathbf{B}\mathbf{K} = \begin{bmatrix} 0 & 1 \\ -4 & -k_2 \end{bmatrix}$$

$\mathbf{A} - \mathbf{B}\mathbf{K}$ is a stable matrix if $k_2 > 0$. Our problem now is to determine the value of k_2 so that the performance index

$$J = \int_0^\infty \mathbf{x}^T \mathbf{x}\, dt = \mathbf{x}^T(0)\mathbf{P}(0)\mathbf{x}(0)$$

0955

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10740

is minimized, where the matrix $\mathbf{P}$ is determined from Equation (12–115), rewritten

$$(\mathbf{A} - \mathbf{BK})^*\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{BK}) = -(\mathbf{Q} + \mathbf{K}^*\mathbf{RK})$$

Since in this system $\mathbf{Q} = \mathbf{I}$ and $\mathbf{R} = \mathbf{0}$, this last equation can be simplified to

$$(\mathbf{A} - \mathbf{BK})^*\mathbf{P} + \mathbf{P}(\mathbf{A} - \mathbf{BK}) = -\mathbf{I} \tag{12–173}$$

Since the system involves only real vectors and real matrices, $\mathbf{P}$ becomes a real symmetric matrix. Then Equation (12–173) can be written as

$$\begin{bmatrix} 0 & -4 \\ 1 & -k_2 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} + \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -4 & -k_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

Solving for the matrix $\mathbf{P}$, we obtain

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix} = \begin{bmatrix} \dfrac{5}{2k_2} + \dfrac{k_2}{8} & \dfrac{1}{8} \\ \dfrac{1}{8} & \dfrac{5}{8k_2} \end{bmatrix}$$

The performance index is then

$$\begin{aligned} J &= \mathbf{x}^T(0)\mathbf{P}\mathbf{x}(0) \\ &= \begin{bmatrix} c & 0 \end{bmatrix}\begin{bmatrix} p_{11} & p_{12} \\ p_{12} & p_{22} \end{bmatrix}\begin{bmatrix} c \\ 0 \end{bmatrix} = p_{11}c^2 \\ &= \left(\frac{5}{2k_2} + \frac{k_2}{8}\right)c^2 \end{aligned} \tag{12–174}$$

To minimize J, we differentiate J with respect to k_2 and set $\partial J/\partial k_2$ equal to zero as follows:

$$\frac{\partial J}{\partial k_2} = \left(\frac{-5}{2k_2^2} + \frac{1}{8}\right)c^2 = 0$$

Hence,

$$k_2 = \sqrt{20}$$

With this value of k_2, we have $\partial^2 J/\partial k_2^2 > 0$. Thus, the minimum value of J is obtained by substituting $k_2 = \sqrt{20}$ into Equation (12–174), or

$$J_{\min} = \frac{\sqrt{5}}{2}c^2$$

The designed system has the control law

$$u = -4x_1 - \sqrt{20}x_2$$

The designed system is optimal in that it results in a minimum value for the performance index J under the assumed initial condition.

0956

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10741

A–12–18. Consider the same inverted-pendulum system as discussed in Example 12–5. The system is shown in Figure 12–8, where $M = 2$ kg, $m = 0.1$ kg, and $l = 0.5$ m. The block diagram for the system is shown in Figure 12–9. The system equations are given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

$$y = \mathbf{C}\mathbf{x}$$

$$u = -\mathbf{K}\mathbf{x} + k_I\xi$$

$$\dot{\xi} = r - y = r - \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, \quad \mathbf{C} = [0 \quad 0 \quad 1 \quad 0]$$

Referring to Equation (12–51), the error equation for the system is given by

$$\dot{\mathbf{e}} = \hat{\mathbf{A}}\mathbf{e} + \hat{\mathbf{B}}u_e$$

where

$$\hat{\mathbf{A}} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 20.601 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ -0.4905 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 \end{bmatrix}, \quad \hat{\mathbf{B}} = \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \\ 0 \end{bmatrix}$$

and the control signal is given by Equation (12–41):

$$u_e = -\hat{\mathbf{K}}\mathbf{e}$$

where

$$\hat{\mathbf{K}} = [\mathbf{K} \mid -k_I] = [k_1 \quad k_2 \quad k_3 \quad k_4 \mid -k_I]$$

$$\mathbf{e} = \begin{bmatrix} \mathbf{x}_e \\ \xi_e \end{bmatrix} = \begin{bmatrix} \mathbf{x}(t) - \mathbf{x}(\infty) \\ \xi(t) - \xi(\infty) \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

0957

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10742

Using MATLAB, determine the state feedback gain matrix $\hat{\mathbf{K}}$ such that the following performance index J is minimized:

$$J = \int_0^\infty (\mathbf{e}^*\mathbf{Q}\mathbf{e} + u^*Ru)\,dt$$

where

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 0.01$$

Obtain the unit-step response of the system designed.

Solution. A MATLAB program to determine $\hat{\mathbf{K}}$ is given in MATLAB Program 12–34. The result is

$$\hat{\mathbf{K}} = [-188.0799 \quad -37.0738 \quad -26.6767 \quad -30.5824 \quad 10.0000]$$

Hence

$$k_1 = -188.0799, \qquad k_2 = -37.0738, \qquad k_3 = -26.6767, \qquad k_4 = -30.5824, \qquad k_I = -10.0000$$

MATLAB Program 12–34

```
% Design of quadratic optimal control system
A = [0 1 0 0;20.601 0 0 0;0 0 0 1;-0.4905 0 0 0];
B = [0;-1;0;0.5];
C = [0 0 1 0];
D = [0];
Ahat = [A zeros(4,1);-C 0];
Bhat = [B;0];
Q = [100 0 0 0 0;0 1 0 0 0;0 0 1 0 0;0 0 0 1 0;0 0 0 0 1];
R = [0.01];
Khat = lqr(Ahat,Bhat,Q,R)
Khat =
   -188.0799 -37.0738 -26.6767 -30.5824 10.0000
```

Unit-Step Response Once we have determined the feedback gain matrix $\mathbf{K}$ and the integral gain constant k_I, we can determine the unit-step response of the designed system. The system equation is

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \xi \end{bmatrix} + \begin{bmatrix} \mathbf{B} \\ 0 \end{bmatrix} u + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r \tag{12–175}$$

0958

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10743

[Refer to Equation (12–35).] Since

$$u = -\mathbf{K}\mathbf{x} + k_I\xi$$

Equation (12–175) can be written as follows:

$$\begin{bmatrix} \dot{\mathbf{x}} \\ \dot{\xi} \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}\mathbf{K} & \mathbf{B}k_I \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ \xi \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r \tag{12–176}$$

MATLAB Program 12–35 gives the unit-step response of the system given by equation (12–176). The resulting response curves are presented in Figure 12–47. It shows response curves $\theta\,[= x_1(t)]$ versus t, $\dot{\theta}\,[= x_2(t)]$ versus t, $y\,[= x_3(t)]$ versus t, $\dot{y}\,[= x_4(t)]$ versus t, and $\xi\,[= x_5(t)]$ versus t, where the input $r(t)$ to the cart is a unit-step function $[r(t) = 1\ \text{m}]$. All initial conditions are set equal to zero. Figure 12–48 is an enlarged version of the cart position $y\,[= x_3(t)]$ versus t. The cart moves backward a very small amount for the first 0.6 sec or so. (Notice that the cart velocity is

MATLAB Program 12–35

```
% Unit-step response

A = [0  1  0  0;20.601  0  0  0;0  0  0  1;-0.4905  0  0  0];
B = [0;-1;0;0.5];
C = [0  0  1  0];
D = [0];
K = [-188.0799  -37.0738  -26.6767  -30.5824];
kI = -10.0000;
AA = [A-B*K  B*kI; -C  0];
BB = [0;0;0;0;1];
CC= [C  0];
DD = D;
t = 0:0.01:10;
[y,x,t] = step(AA,BB,CC,DD,1,t);
x1 = [1  0  0  0  0]*x';
x2 = [0  1  0  0  0]*x';
x3 = [0  0  1  0  0]*x';
x4 = [0  0  0  1  0]*x';
x5 = [0  0  0  0  1]*x';

subplot(3,2,1); plot(t,x1); grid;
xlabel('t (sec)'); ylabel('x1')

subplot(3,2,2); plot(t,x2); grid;
xlabel('t (sec)'); ylabel('x2')

subplot(3,2,3); plot(t,x3); grid;
xlabel('t (sec)'); ylabel('x3')

subplot(3,2,4); plot(t,x4); grid;
xlabel('t (sec)'); ylabel('x4')

subplot(3,2,5); plot(t,x5); grid;
xlabel('t (sec)'); ylabel('x5')
```


0959

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10744

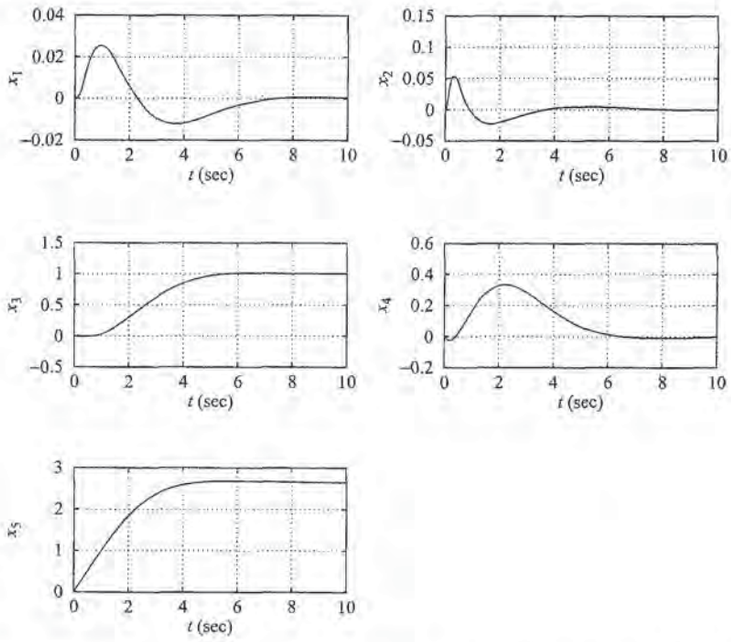


Figure 12–47
Response curves to a unit-step input.

negative for the first 0.4 sec.) This is due to the fact that the inverted-pendulum-on-the-cart system is a nonminimum-phase system.

Comparing the step-response characteristics of this system with those of Example 12-5, we notice that the response of the present system is less oscillatory and exhibits less maximum overshoot in the position response (x_3 versus t). The system designed by use of the quadratic optimal regulator approach generally gives such characteristics—less oscillatory and well damped.

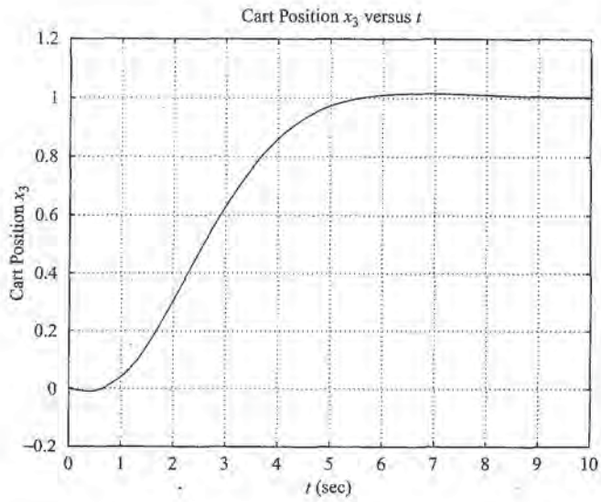


Figure 12–48
Cart position versus t curve.

0960

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10745

PROBLEMS

B–12–1. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1 \quad 0]$$

Transform the system equations into (a) controllable canonical form and (b) observable canonical form.

B–12–2. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 0 & 1 \\ 1 & -2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 1 \quad 1]$$

Transform the system equations into the observable canonical form.

B–12–3. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -5 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

By using the state-feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to have the closed-loop poles at $s = -2 \pm j4, s = -10$. Determine the state-feedback gain matrix $\mathbf{K}$.

B–12–4. Solve Problem B–12–3 with MATLAB.

B–12–5. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u$$

Show that this system cannot be stabilized by the state-feedback control $u = -\mathbf{K}\mathbf{x}$, whatever matrix $\mathbf{K}$ is chosen.

B–12–6. A regulator system has a plant

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

Define state variables as

$$x_1 = y$$
$$x_2 = \dot{x}_1$$
$$x_3 = \dot{x}_2$$

By use of the state-feedback control $u = -\mathbf{K}\mathbf{x}$, it is desired to place the closed-loop poles at

$$s = -2 + j2\sqrt{3}, \quad s = -2 - j2\sqrt{3}, \quad s = -10$$

Determine the necessary state-feedback gain matrix $\mathbf{K}$.

B–12–7. Solve Problem B–12–6 with MATLAB.

B–12–8. Consider the type 1 servo system shown in Figure 12–49. Matrices $\mathbf{A}$, $\mathbf{B}$, and $\mathbf{C}$ in Figure 12–49 are given by

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -5 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Determine the feedback gain constants k_1, k_2, and k_3 such that the closed-loop poles are located at

$$s = -2 + j4, \quad s = -2 - j4, \quad s = -10$$

Obtain the unit-step response and plot the output $y(t)$-versus-t curve.

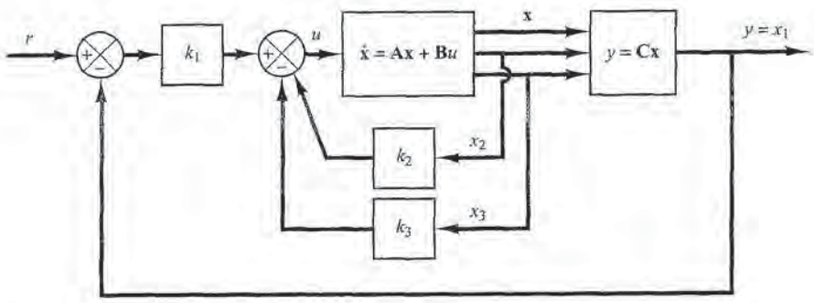


Figure 12–49
Type 1 servo system.

0961

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10746

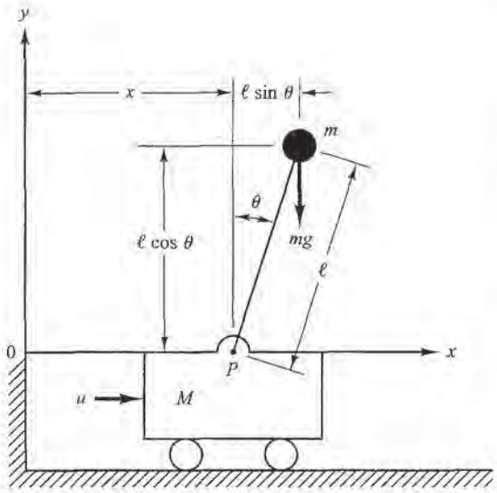


Figure 12–50
Inverted-pendulum system.

B–12–9. Consider the inverted-pendulum system shown in Figure 12–50. Assume that

$$M = 2\text{ kg}, \quad m = 0.5\text{ kg}, \quad l = 1\text{ m}$$

Define state variables as

$$x_1 = \theta, \quad x_2 = \dot{\theta}, \quad x_3 = x, \quad x_4 = \dot{x}$$

and output variables as

$$y_1 = \theta = x_1, \quad y_2 = x = x_3$$

Derive the state-space equations for this system.

It is desired to have closed-loop poles at

$$s = -4 + j4, \quad s = -4 - j4, \quad s = -20, \quad s = -20$$

Determine the state-feedback gain matrix **K**.

Using the state-feedback gain matrix **K** thus determined, examine the performance of the system by computer simulation. Write a MATLAB program to obtain the response of the system to an arbitrary initial condition. Obtain the response curves $x_1(t)$ versus t, $x_2(t)$ versus t, $x_3(t)$ versus t, and $x_4(t)$ versus t for the following set of initial condition:

$$x_1(0) = 0, \quad x_2(0) = 0, \quad x_3(0) = 0, \quad x_4(0) = 1\text{ m/s}$$

B–12–10. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0]$$

Design a full-order state observer. The desired observer poles are $s = -5$ and $s = -5$.

B–12–11. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -5 & -6 & 0 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Design a full-order state observer, assuming that the desired poles for the observer are located at

$$s = -10, \quad s = -10, \quad s = -15$$

B–12–12. Consider the system defined by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1.244 & 0.3956 & -3.145 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1.244 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Given the set of desired poles for the observer to be

$$s = -5 + j5\sqrt{3}, \quad s = -5 - j5\sqrt{3}, \quad s = -10$$

design a full-order observer.

B–12–13. Consider the double integrator system defined by

$$\ddot{y} = u$$

If we choose the state variables as

$$x_1 = y$$
$$x_2 = \dot{y}$$

then the state-space representation for the system becomes as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

0962

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10747

It is desired to design a regulator for this system. Using the pole-placement-with-observer approach, design an observer controller.

Choose the desired closed-loop poles for the pole-placement part to be

$$s = -0.7071 + j0.7071, \qquad s = -0.7071 - j0.7071$$

and assuming that we use a minimum-order observer, choose the desired observer pole at

$$s = -5$$

B–12–14. Consider the system

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$
$$y = \mathbf{C}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -11 & -6 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{C} = [1 \quad 0 \quad 0]$$

Design a regulator system by the pole-placement-with-observer approach. Assume that the desired closed-loop poles for pole placement are located at

$$s = -1 + j, \qquad s = -1 - j, \qquad s = -5$$

The desired observer poles are located at

$$s = -6, \qquad s = -6, \qquad s = -6$$

Also, obtain the transfer function of the observer controller.

B–12–15. Using the pole-placement-with-observer approach, design observer controllers (one with a full-order observer and the other with a minimum-order observer) for the system shown in Figure 12–51. The desired closed-loop poles for the pole-placement part are

$$s = -1 + j2, \qquad s = -1 - j2, \qquad s = -5$$

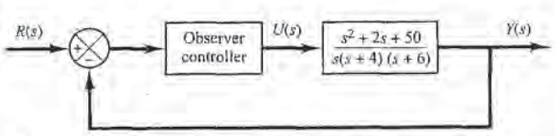


Figure 12–51
Control system with observer controller in the feedforward path.

The desired observer poles are

$s = -10, \quad s = -10, \quad s = -10$ for the full-order observer

$s = -10, \quad s = -10$ for the minimum-order observer.

Compare the unit-step responses of the designed systems. Compare also the bandwidths of both systems.

B–12–16. Using the pole-placement-with-observer approach, design the control systems shown in Figures 12–52(a) and (b). Assume that the desired closed-loop poles for the pole placement are located at

$$s = -2 + j2, \qquad s = -2 - j2$$

and the desired observer poles are located at

$$s = -8, \qquad s = -8$$

Obtain the transfer function of the observer controller. Compare the unit-step responses of both systems. [In System (b), determine the constant N so that the steady-state output $y(\infty)$ is unity when the input is a unit-step input.]

B–12–17. Consider the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x}$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -2 & -a \end{bmatrix}$$

a = adjustable parameter > 0

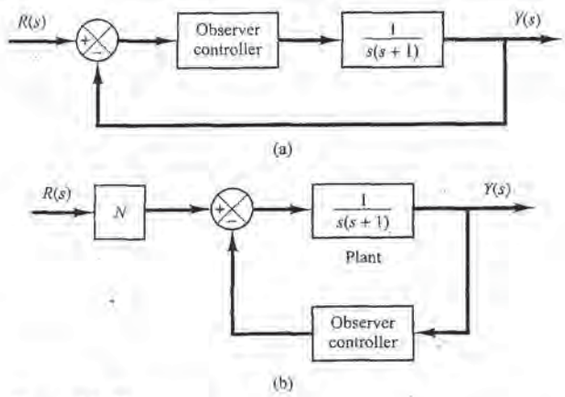


Figure 12–52
Control systems with observer controller: (a) observer controller in the feedforward path; (b) observer controller in the feedback path.

0963

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10748

Determine the value of the parameter a so as to minimize the following performance index:

$$J = \int_0^\infty \mathbf{x}^T \mathbf{x}\, dt$$

Assume that the initial state $\mathbf{x}(0)$ is given by

$$\mathbf{x}(0) = \begin{bmatrix} c_1 \\ 0 \\ 0 \end{bmatrix}$$

B–12–18. Consider the system shown in Figure 12–53. Determine the value of the gain K so that the damping ratio ζ of the closed-loop system is equal to 0.5. Then determine also the undamped natural frequency ω_n of the closed-loop system. Assuming that $e(0) = 1$ and $\dot{e}(0) = 0$, evaluate

$$\int_0^\infty e^2(t)\, dt$$

B–12–19. Determine the optimal control signal u for the system defined by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \qquad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

such that the following performance index is minimized:

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{x} + u^2)\, dt$$

B–12–20. Consider the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

It is desired to find the optimal control signal u such that the performance index

$$J = \int_0^\infty (\mathbf{x}^T \mathbf{Q}\mathbf{x} + u^2)\, dt, \qquad \mathbf{Q} = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}$$

is minimized. Determine the optimal signal $u(t)$.

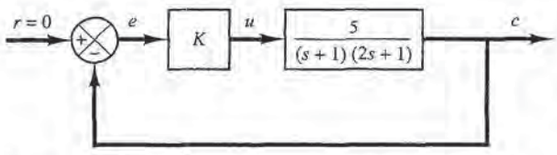


Figure 12–53
Control system.

B–12–21. Consider the inverted-pendulum system shown in Figure 12–50. It is desired to design a regulator system that will maintain the inverted pendulum in a vertical position in the presence of disturbances in terms of angle θ and/or angular velocity $\dot{\theta}$. The regulator system is required to return the cart to its reference position at the end of each control process. (There is no reference input to the cart.)

The state-space equation for the system is given by

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 20.601 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -0.4905 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0.5 \end{bmatrix}, \qquad \mathbf{x} = \begin{bmatrix} \theta \\ \dot{\theta} \\ x \\ \dot{x} \end{bmatrix}$$

We shall use the state-feedback control scheme

$$u = -\mathbf{K}\mathbf{x}$$

Using MATLAB, determine the state-feedback gain matrix $\mathbf{K} = [k_1 \;\; k_2 \;\; k_3 \;\; k_4]$ such that the following performance index J is minimized:

$$J = \int_0^\infty (\mathbf{x}^* \mathbf{Q}\mathbf{x} + u^* R u)\, dt$$

where

$$\mathbf{Q} = \begin{bmatrix} 100 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad R = 1$$

Then obtain the system response to the following initial condition:

$$\begin{bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \\ x_4(0) \end{bmatrix} = \begin{bmatrix} 0.1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Plot response curves θ versus t, $\dot{\theta}$ versus t, x versus t, and $\dot{x}$ versus t.

0964

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10749



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0967

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0968

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10753



Index

A

Abscissa of convergence, 13–14
Ackermann's formula:
  for observer gain matrix, 860–61
  for pole placement, 834–36
Actuator, 63
Air heating system, 209
Aircraft elevator control system, 216
Analytic extension theorem, 15
Analytic function, 10
Angle:
  of arrival, 354
  of departure, 348, 354
Angle condition, 339
Asymptotes:
  Bode diagram, 500–05
  root loci, 342–43, 352–53, 402–05
Attenuation, 225
Attitude-rate control system, 474
Automatic control system, 58
Automatic controller, 62
Automobile suspension system, 131–32
Auxiliary polynomial, 279

B

Back emf, 140
  constant, 140
Bandwidth, 572, 574
Basic control actions:
  integral, 65
  on-off, 63–64
  proportional, 65
  proportional-plus-derivative, 66
  proportional-plus-integral, 66
  proportional-plus-integral-plus-derivative, 66
  two-position, 63–64
Bleed-type relay, 163
Block, 58
Block diagram, 58
  reduction, 68
Bode diagram, 497
  error in asymptotic expression of, 502
  of first-order factors, 500–01, 503
  general procedure for plotting, 507
  plotting with MATLAB, 516–21
  of quadratic factors, 504–06
  of system defined in state space, 522–23
Branch, 104
  point, 59
Break frequency, 500
Breakaway point, 343–45, 353
Break-in point, 344–45, 349, 353
Bridged-T networks, 648
Business system, 5

0969

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10754

C

Canonical forms:
  controllable, 753
  diagonal, 754
  Jordan, 755, 757
  observable, 754
Capacitance:
  of pressure system, 159, 161
  of thermal system, 189
  of water tank, 155
Cauchy–Riemann conditions, 11
Cauchy's theorem, 594
Cayley–Hamilton theorem, 772–73, 805
Characteristic equation, 756
Characteristic polynomial, 75
Characteristic roots, 756
Circular root locus, 350–51
Classification of control systems, 288
Closed-loop control system, 6
Closed-loop frequency response, 575–76
Closed-loop transfer function, 60
Cofactor, 109
Command compensation, 733
Compensation:
  feedback, 417
  parallel, 417
  series, 417
Compensator:
  lag, 431, 631–32
  lag–lead, 441, 443, 639–41
  lead, 421–22, 623–24
Complete observability, 787–88
  conditions for, 788–89
  in the *s* plane, 788
Complete output controllablility, 819
Complete state controllability, 780–81, 782–85
  in the *s* plane, 784–85
Complex-conjugate poles:
  cancellation of undesirable, 647–48
Complex-differentiation theorem, 28
Complex function, 10
Complex impedence, 93
Complex variable, 10
Conditional stability, 379, 638–39
Conditionally stable system, 378–79, 410, 553–54
Conduction heat transfer, 189
Conformal mapping, 560–62
Conical water tank system, 211
Constant-gain loci, 368–69
Constant-magnitude loci (M circles), 576–77
Constant phase-angle loci (N circles), 578–79
Control actions, 62
Controllability, 779
  matrix, 781, 830
  output, 785
Controllable canonical form, 753, 794
Controlled variable, 2
Controller, 63
Controller tuning, 682
Convection heat transfer, 189
Conventional control theory, 70
Convolution, 29
  integral, 28, 30, 57
Corner frequency, 500–01
Critically damped system, 227
Cutoff frequency, 572
Cutoff rate, 573

D

Damped natural frequency, 226
Damper, 80, 184
Damping ratio, 225
  lines of constant, 365
Dashpot, 80, 184–185
Dead space, 112
Dead time, 379, 511, 586
  approximation of, 383–84
Decade, 499
Decibel, 497
Delay time, 230
Derivative control action, 170, 285
Derivative gain, 102
Derivative time, 66, 102
Detectability, 792
Determinant of graph, 109
Diagonal canonical form, 754, 798
Diagonalization of $n \times n$ matrix, 756
Differential amplifier, 96
Differential gap, 64
Differentiating system, 294
Dirac delta function, 21
Direct transmission matrix, 72
Disturbance, 3
Dominant closed-loop poles, 242
Duality, 858

E

e^{At}:
  computation of, 774–77
Eigenvalue, 756
  invariance of, 759
Electronic PID controller, 101
Engine-speed control system, 144
Engineering organizational system, 6
Equivalent moment of inertia , 297
Equivalent viscous-friction coefficient, 297

0970

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10755

Error equation:
for minimum-order observer, 875
Error vector, 857
Estimated error, 856
output, 856
state, 856
Euler's theorem, 12
Evans, W.R., 337
Exponential order, 14
Exponential response curve, 222

F

Feedback compensation, 417
Feedback control, 3, 6
Feedback control system, 6
Feedback element, 59
Feedback gain matrix:
determination of, 833–35
Feedforward compensation, 417, 421–29
Feedforward transfer function, 60
Final value theorem, 25
First-order system, 221–24
unit-impulse response of, 223
unit-ramp response of, 222–23
unit-step response of, 221–22
Flapper, 162
valve, 215
Forward path, 104
gain, 105
Free-body diagram, 87–88
Frequency response, 492
correlation between step response and, 569–72
lag compensation based on, 631–32
lag–lead compensation based on, 641
lead compensation based on, 622–24
Full-order state observer, 856–57
Functional block, 58

G

Gain adjustment, 583
Gain crossover frequency, 562
Gain margin, 562–64
Gas constant:
for air, 197
universal, 160
Gear train, 296
system, 296

H

High-pass filter, 622
Higher-order systems:
transient response of, 240
Hurwitz determinants, 316
Hurwitz stability criterion, 316–17, 319–22
equivalence of Routh's stability criterion and, 319–21
Hydraulic controller:
integral, 181
jet-pipe, 202
proportional, 183
proportional-plus-derivative, 186
proportional-plus-integral, 185
proportional-plus-integral-plus-derivative, 187
Hydraulic servo system, 207
Hydraulic servomotor, 176–177, 182
characteristic curves of, 180
Hydraulic system, 175–76
advantages and disadvantages of, 176

I

Ideal gas law, 160
Impedance:
approach to obtain transfer function, 93–94
Impulse function, 20
Impulse response, 254, 256
function, 57
Industrial controllers, 63
Initial condition:
response to, 263–70
Initial value theorem, 26
Input matrix, 72
Input node, 104
Integral control, 283
Integral control action, 65, 281
Integral controller, 63
Integral gain, 102
Integral time, 66, 102
Inverse Laplace transform, 13
partial-fraction expansion method for obtaining, 32–36
Inverse Laplace transformation, 13
Inverse polar plot, 557–58
Inversion integral, 13
Inverted-pendulum system, 86–90
Inverted-pendulum control system, 850–55
Inverting amplifier, 96–97
I-PD control, 702, 733
I-PD-controlled system, 702

J

Jet-pipe controller, 203
Jordan blocks, 783
Jordan canonical form, 755, 757, 774, 783, 790, 801–02

K

Kalman, R.E., 779, 791
Kirchhoff's current law, 90
Kirchhoff's loop law, 90
Kirchhoff's node law, 90
Kirchhoff's voltage law, 90

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10756

L

Lag compensation, 631–32
Lag compensator, 418, 429, 620, 630–32, 638, 645–46
  Bode diagram of, 631
  design by frequency-response method, 631–32
  design by root-locus method, 429–32
  polar plot of, 631
Lag network, 100–01, 418, 422, 495–96
Lag–lead compensation, 620, 639–41
Lag–lead compensator:
  Bode diagram of, 640
  design by frequency-response method, 641–44
  design by root-locus method, 441–43
  electronic, 440
  polar plot of, 640
Lag–lead network, 418, 459
  electronic, 440
  mechanical, 457–58
Lagrange polynomial, 812
Lagrange's interpolation formula, 811
Laminar-flow resistance, 153
Laplace integral:
  lower limit of, 22
Laplace transform, 13
  change of time scale, 22
  existance of, 14
  of exponential function, 15
  of impulse function, 20
  of periodic function, 45–46
  of product of two time functions, 30
  of pulse function, 20
  of ramp function, 16
  of sinusoidal function, 16
  of step function, 15
  of translated function, 19
Laplace transform method:
  solving differential equations by, 40–42
Laplace transforms:
  properties of, 31
  table of, 17–18
Lead compensation, 620, 645
Lead compensator, 418, 421, 478, 621–22
  Bode diagram of, 622
  design by frequency-response method, 622–24
  design by root-locus method, 422–23
  polar plot of, 621
Lead network, 100–01, 418, 422, 458, 495–96
  electronic, 421
  mechanical, 456–57
Linear system, 54
  constant coefficient, 54
Linear time-invariant system, 54
Linear time-varying system, 54
Linear transformation, 759
Linearization:
  of hydraulic servo system, 177–180
  of nonlinear systems, 112–14
Liquid-level control system, 153–58
Liquid-level systems, 153, 155–56, 192–96
Logarithmic decrement, 301
Logarithmic plot, 497
Log-magnitude versus phase plot, 539
Loop, 104
  gain, 104

M

M circles, 576
Magnitude condition, 339
Manipulated variable, 2
Mapping theorem, 544–45, 594
Mason, S.J., 104
Mason's gain formula, 104, 109–11
Mathematical model, 53–54
MATLAB:
  obtaining maximum overshoot with, 252–53, 307
  obtaining peak time with, 252–53, 307
  obtaining response to initial condition, 263–68
  partial-fraction expansion with, 36–39
  plotting Bode diagram with, 516–23
  plotting root loci with, 358–68
  writing text in diagrams, 247–49, 652
MATLAB commands:
  [A,B,C,D] = tf2ss(num,den), 83, 760
  abs, 651
  acker, 839–40, 877
  angle, 651
  bode 516
  bode(A,B,C,D), 516, 522
  bode(A,B,C,D,iu), 522
  bode(sys), 516
  bode(A,B,C,D,w), 516
  bode(num,den), 516
  bode(num,den,w), 516
  c = step(num,den,t), 249
  for loop, 308, 314, 693
  [Gm,pm,wcp,wcg,] = margin(sys),567
  gtext ('text'), 249
  impulse(A,B,C,D), 254
  impulse(num, den),254
  initial(A,B,C,D,[initial condition],t), 268
  K = lqr(A,B,Q,R), 902
  [K,P,E] = lqr(A,B,Q,R), 902
  [K,r] = rlocfind(num,den), 369
  logspace(d1,d2), 516

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10757

MATLAB commands *(Cont.)*
logspace(d1,d2,n), 516–517
lqr(A,B,Q,R), 901
lsim(A,B,C,D,u,t), 260
lsim(num,den,r,t), 260
magdB = 20*log10(mag), 516, 519
[mag,phase,w] = bode(A,B,C,D), 516
[mag,phase,w] = bode(A,B,C,D,iu,w), 516
[mag,phase,w] = bode(A,B,C,D,w), 516
[mag,phase,w] = bode(num,den), 516
[mag,phase,w] = bode(num,den,w), 516, 519, 574
[mag,phase,w] = bode(sys), 516
[mag,phase,w] = bode(sys,w), 574
mesh, 250–52, 314
[Mp,k] = max(mag), 574
NaN, 903
[num,den] = feedback(num1,den1, num2,den2), 61
[num,den] = parallel(num1,den1, num2,den2), 61
[num,den] = residue(r,p,k), 38
[num,den] = series(num1,den1, num2,den2), 61
[num, den] = sst2tf(A,B,C,D), 84, 364, 761
[num,den] = ss2tf(A,B,C,D,iu), 84,761
[NUM,den] = ss2tf(A,B,C,D,iu), 128, 763
nyquist(A,B,C,D), 532, 536, 538
nyquist(A,B,C,D,iu), 537
nyquist(A,B,C,D,iu,w), 532, 537
nyquist(A,B,C,D,w), 532
nyquist(num,den), 532
nyquist(num, den,w), 532
nyquist(sys), 532
place, 839–40, 877
polar, 651–52
printsys(num,den), 61, 249
printsys(num,den,'s'), 38, 249
r = abs(z), 651
[r,K] = rlocus(A,B,C,D), 359
[r,K] = rlocus(A,B,C,D,K), 359
[r,K] = rlocus(num,den), 358
[r,K] = rlocus(num,den,K), 358
[r,K] = rlocus(sys), 359
[r,p,K]= residue(num,den), 37
[re,im,w] = nyquist(A,B,C,D), 532
[re,im,w] = nyquist(A,B,C,D,iu,w), 532
[re,im,w] = nyquist(A,B,C,D,w), 532
[re,im,w] = nyquist(num,den), 532
[re,im,w] = nyquist(num,den,w), 532, 600
[re,im,w] = nyquist(sys), 532
residue, 37
resonant_frequency = w(k), 574
resonant_peak = 20*log10(Mp), 574
rlocfind, 369–70
rlocus(A,B,C,D), 358
rlocus(A,B,C,D,K), 358
rlocus(num,den), 358
rlocus(num,den,K), 358
semilogx, 519–20
sgrid, 365
step(A,B,C,D), 244, 246
step(num,den), 244
step(num,den,t), 244
step(sys), 244
sys = ss(A,B,C,D), 244
sys = tf(num,den), 244
text, 247
theta = angle(z), 651
y = lsim(A,B,C,D,u,t), 260
y = lsim(num,den,r,t), 260
[y, x, t] = impulse(A,B,C,D), 254
[y, x, t] = impulse(A,B,C,D,iu), 254
[y, x, t] = impulse(A,B,C,D,iu,t), 254
[y, x, t] = impulse(num,den), 254
[y, x, t] = impulse(num,den,t), 254
[y, x, t] = step(A,B,C,D,iu), 244
[y, x, t] = step(A,B,C,D,iu, t), 244
[y, x, t] = step(num,den,t), 244, 249
z = re+j*im, 651
Matrix exponential, 765, 773
closed solution for, 768
computation of, 773–77
Matrix Riccati equation, 902
Maximum overshoot:
in unit-impulse response, 239
in unit-step response, 230, 232
versus ζ curve, 234
Maximum percent overshoot, 230
Maximum phase lead angle, 622
Measuring element, 63
Mechanical vibratory system, 271
computer simulations of, 274–75
Mercury thermometer system, 210
Minimal polynomial, 773, 805, 808
Minimum-order observer, 871–77
based controller, 881
Minimum-order state observer, 856
Minimum-phase system, 371, 509–10
Minimum-phase transfer function, 509, 584
Mixed node, 104
Modern control theory, 70
Modified Nyquist path, 559
Modified *s*-plane contour, 559
Motor torque constant, 139

0973

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10758

Motorcycle suspension system, 132
Multiple-loop system, 554

N

N circles, 578
Newton's second law, 81
Nichols chart, 581
Nichols plots, 539
Node, 104
Nonbleed-type relay, 163
Nonhomogeneous state equation:
  solution of, 770–71
Noninverting amplifier, 97
Nonlinear mathematical models:
  linear approximation of, 112–14
Nonlinear system, 112
Nonminimum-phase systems, 371, 509–11
Nonminimum-phase transfer function, 509
Nontouching loop, 104
Nonuniqueness:
  of a set of state variables, 759–60
Nozzle-flapper amplifier, 162
Number-decibel conversion line, 498
Nyquist, H., 1, 542
Nyquist path, 545
Nyquist plot, 523
  of positive-feedback system, 603–05
  of system defined in state space, 536–39
Nyquist stability analysis, 550–61
Nyquist stability criterion, 540, 546–47, 550

O

Observability, 779, 786
  complete, 780–85
  matrix, 787
Observable canonical form, 754, 796
Observation, 856
Observed-state feedback control system, 865
Observer, 856
  mathematical model of, 856
Observer-based controller:
  transfer function of, 865–66
Observer-controller matrix, 866
Observer-controller transfer function, 866
Observer error equation, 857
Observer gain matrix, 856–57
Octave, 499
Offset, 322
On-off control action, 63–64
One-degree-of-freedom control system, 703
op amps, 96
Open-loop control system, 6
  advantages of, 7
  disadvantages of, 8
Open-loop frequency response curve:
  reshaping of, 620
Open-loop transfer function, 60
Operational amplifier, 96
Operational amplifier circuits:
  for lead or lag compensator, 100–01
  table of, 103
Optimal regulator problem, 910
Ordinary point, 12
Output controllability, 785, 819
Output equation, 72
Output matrix, 72
Output node, 104
Overdamped system, 228–29
Overlapped spool valve, 201
Overlapped valve, 182

P

Pade approximation, 384
  second-order, 410
Parallel compensation, 417
  technique, 451–52
Partial-fraction expansion, 32–36
  with MATLAB, 36–40
Path, 104
PD controller, 725
Peak time, 230, 232
Performance index, 897
Performance specifications, 416
Periodic function, 45–46
Phase crossover frequency, 564
Phase margin, 539, 562–63
  versus ζ curve, 570
PI controller, 724
PID control schemes, 681
PID control system, 701
  basic, 700
  with input filter, 732
  two-degrees-of-freedom, 703
PID controller, 725
  modified, 726
  using operational amplifiers, 101–02
PI-D control, 700
Pilot valve, 176, 182
PI-PD control, 703
PID-PD control, 703
Plant, 2
Pneumatic actuating valve, 169–70
Pneumatic controllers, 162–75
Pneumatic nozzle-flapper amplifier, 162
Pneumatic on-off controller, 167
Pneumatic pressure system, 197–99
Pneumatic proportional controller, 164–68
  force-balance type, 167–68
  force-distance type, 164–65
Pneumatic proportional-plus-derivative control action, 170–72

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10759

Pneumatic proportional-plus-integral control action, 172–74
Pneumatic proportional-plus-integral-plus-derivative control action, 174–75
Pneumatic relay, 163
  bleed type, 163
  nonbleed type, 163
  reverse acting, 164
Pneumatic systems, 158–61
Pneumatic two-position controller, 167
Polar plot, 523–27
Pole:
  of order n, 12
  simple, 12
Pole placement, 827–39
  design of system by, 827–39
  technique, 827
Pole assignment technique, 827
Pole-placement problem, 829
  solving with MATLAB, 839–40
Polytropic exponent, 160
Positive-feedback system, 373, 376–77
  Nyquist plot for, 603–05
  root loci for, 373–77
Positional servo system, 139–41
Pressure system, 159–61
Primary branch, 382
Principle of causality, 53
Principle of duality, 791
Principle of superposition, 54
Process, 3
Proportional control, 282, 286
  of system with inertia load, 283–84
Proportional control action, 65
Proportional gain, 65, 102
Proportional-plus-derivative control:
  of second-order system, 287–88
  of system with inertia load, 286–87
Proportional-plus-derivative control action, 66, 286–87
Proportional-plus-integral control action, 66
Proportional-plus-integral-plus-derivative control action, 66
Pulse function, 20

Q

Quadratic factor, 504
  log-magnitude curves of, 505
  phase-angle curves of, 505
Quadratic optimal control problem:
  optimal control law to, 899
Quadratic optimal regulator system, 897–99

R

Ramp response, 257
  MATLAB approach to obtain, 257–60
Real differentiation theorem, 23–24
Real-integration theorem, 26–27
Reduced characteristic equation, 356
Reduced-matrix Riccati equation, 899
Reduced-order observer, 856
Reduced-order state observer, 856
Regulator poles, 829
Regulator system with observer, 882–90
Relative stability, 220
Residue, 33
Residue theorem, 595
Resistance:
  laminar-flow, 153
  liquid flow, 153
  of pressure system, 159
  of thermal system, 189
  turbulent-flow, 154
Resonant frequency, 506, 526, 568, 574
Resonant peak:
  versus ζ curve, 507
Resonant peak magnitude, 507, 568
Response:
  to arbitrary input, 260
  to initial condition, 263–70
  to torque disturbance, 283–85
Reverse-acting relay, 163–64
Riccati equation, 899
Rise time, 230–31
  obtaining with MATLAB, 252–53, 307
Robust pole placement, 839
Root loci:
  general rules for constructing, 351–55
  for positive-feedback system, 373
  for system with transport lag, 379–83
Root locus, 339
  method, 337–38
Routh's stability criterion, 275–81

S

Satellite attitude control system, 56
Schwarz matrix, 335
Second-order system, 224
  impulse response of, 238–39
  standard form of, 226
  step response of, 225–29
  transient-response specification of, 229–35
  unit-step response curves of, 229
Sensor, 63
Series compensation, 417
Servo system, 224
  design of type 1, 843–50

0975

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10760

Servo system, (*Cont.*)
with velocity feedback, 235
Servomechanism, 2
Set-point kick, 700
Settling time, 230, 232–33
obtaining with MATLAB, 252–53, 307
versus ζ curve, 234
Sign inverter, 97
Signal flow graph, 104
algebra, 105
Simple pole, 12
Singular points, 12
Sink 104
Sinusoidal signal generator, 584
Sinusoidal transfer function, 495
Smith, Otto, J. M., 411
Smith predictor, 411–13
Source, 104
Space vehicle control system, 478
Speed control system, 204–05, 324
Spool valve:
linealized mathematical model of, 179
Spring-loaded pendulum system, 148–49
Square-law nonlinearity, 112
S-shaped curve, 683
Stability analysis, 550–60
in the complex plane, 242–43
Stabilizability, 792
Stack controller, 167
Standard second-order system, 249
State, 70
State controllability:
complete, 780–85
State equation, 72
solution of homogeneous, 764, 767
solution of nonhomogeneous, 770–71
Laplace transform solution of, 767–68, 771–72
State feedback, 828
State-feedback gain matrix, 828
MATLAB approach to determine, 839–40
State matrix, 72
State observation:
necessary and sufficient conditions for, 858–59
State observer, 855–57
design with MATLAB, 877
State observer gain matrix:
Ackermann's formula to obtain, 834–36
direct substitution approach to obtain, 860
transformation approach to obtain, 859
State space, 71
State-space equation, 71–73
correlation between transfer function and, 74–75
solution of, 764–72
State-space representation:
in canonical forms, 753–55
of *n*th order system, 76–80
State-transition matrix, 768
properties of, 769
State variable, 70–71
State vector, 71
Static acceleration error constant, 291, 515
determination of, 515–16
Static position error constant, 289, 513
Static velocity error constant, 290, 514
determination of, 514–15
Steady-state error:
in terms of gain K, 293
Steady-state response, 220
Step function, 15
Step response, 803–04
of second-order system, 225–29
Summing point, 59
Suspension system:
automobile, 131
motorcycle, 132
Sylvester's interpolation formula, 812–16
System, 3
type 0, 288, 293, 513, 529, 586
type 1, 288, 293, 514, 529, 586
type 2, 288, 293, 515, 529, 586
System response to initial condition:
MATLAB approach to obtain, 263–70

T

Tachometer, 236
feedback, 452
Taylor series expansion, 112–14
Temperature control systems, 4
Thermal capacitance, 189
Thermal resistance, 189
Thermal systems, 188–91
Thermometer system, 210–11
Three-degrees-of-freedom system, 749
Three-dimensional plot 250
of unit-step response curves, 252, 315
Traffic control system, 7
Transfer function:
of cascaded elements, 91
of cascaded systems, 61
closed-loop, 60
of closed-loop system, 61
of dead time, 379
experimental determination of, 587

0976

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10761

Transfer function, (*Cont.*)
expression in terms of **A**, **B**, **C**, and *D*, 75
of feedback system, 61
feedforward, 60
of minimum-order observer-based controller, 881
of nonloading cascaded elements, 95
observer-controller, 866, 884–86
open-loop, 60
of parallel systems, 61
sinusoidal, 495
of transport lag, 379
Transfer matrix, 76, 245, 590
Transformation:
from state space to transfer function, 761
from transfer function to state space, 760
Transient response, 220
analysis with MATLAB, 243–70
of higher-order system, 240
specifications, 229–30
Translated function, 19
Transmittance, 104
Transport lag, 379, 511, 586–87
approximation of, 383
phase angle characteristics of, 511
Tuning rules, 681
Turbulent-flow resistance, 154
Two-degrees-of-freedom control system, 704–06, 709, 719, 740–41, 749–51
Two-position control action, 63–64
Type 0 system, 288, 293, 586
log-magnitude curve for, 513
polar plot of, 529
Type 1 servo system:
design of, 843–55
pole-placement design of, 843–50
Type 1 system, 288, 293, 586
log-magnitude curve for, 514
polar plot of, 529
Type 2 system, 288, 293, 586
log-magnitude curve for, 515
polar plot of, 529

U

Uncontrollable system, 785
Undamped natural frequency, 225
Underdamped system, 226–27
Underlapped spool valve, 201
Underlapped valve, 177, 182
Unit-doublet function, 44
Unit-impulse function, 21
Unit-impulse response:
of first-order system, 223
of second-order system, 238–39
Unit-impulse response curves:
a family of, 238
obtained by use of MATLAB, 254–56
Unit-ramp response:
of first-order system, 222–23
of second-order system, 257–260
of system defined in state space, 258–60
Unit-step function, 15
Unit-step response:
of first-order system, 221
of second-order system, 225–29
Unit-step response curves:
of second-order system, 225–35
Universal gas constant, 160

V

Valve:
overlapped, 182
underlapped, 182
zero-lapped, 182
Valve coefficient, 179
Vectors:
linear dependence of, 779
linear independence of, 778
Velocity error, 290
Velocity feedback, 236, 452, 646

W

Watt's speed governor, 3
Weighting function, 57

Z

Zero, 12
of order m, 12
Zero-lapped valve, 182
Zero placement, 705, 707, 722
approach to improve response characteristics, 705
Ziegler–Nichols tuning rules, 682–83
first method, 683–84
second method, 684–86

0977

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10762

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10763

UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner

Case IPR2020-00445
Patent 6,581,565

PETITIONER VOLKSWAGEN GROUP OF AMERICA, INC.'S REPLY TO PATENT OWNER'S RESPONSE

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Patent Trial and Appeal Board
U.S. Patent & Trademark Office
P.O. Box 1450
Alexandria, VA 22313-1450

TABLE OF CONTENTS

I. PATENT OWNER'S ARGUMENTS DO NOT REBUT PETITIONER'S COMPELLING EVIDENCE SHOWING THAT THE PRIOR ART DISCLOSES EVERY LIMITATION OF THE CHALLENGED CLAIMS. 2

A. Patent Owner improperly attacks Robichaux and Scherer individually and ignores the Petition's combination of Robichaux and Scherer. 4

B. Robichaux and Scherer do not teach away from the claimed invention. 12

1. Deactivation of Robichaux's PID controller does not teach away from the claimed invention.....13

2. Scherer's alternative embodiments do not teach away from the claimed invention. 14

C. Patent Owner's credibility attacks on Dr. MacCarley's testimony are meritless..... 16

1. Patent Owner mischaracterizes Dr. MacCarley's testimony. ...17

2. Dr. MacCarley articulated and applied the appropriate test for obviousness. 19

II. CONCLUSION..... 20

PETITIONER'S UPDATED EXHIBIT LIST

Exhibit No.	Description
1001	U.S. Patent No. 6,581,565 B2 to Heslop *et al*. ("'565 Patent")
1002	Prosecution File History of U.S. Patent No. 6,581,565 B2 ("'565 history")
1003	Declaration of C. Arthur MacCarley, Ph.D., P.E.
1004	Curriculum Vitae of C. Arthur MacCarley, Ph.D., P.E.
1005	U.S. Patent No. 6,006,717 A to Suzuki *et al.*, titled "Direct-Injection Spark-Ignition Type Engine Control Apparatus," issued December 28, 1999 ("Suzuki")
1006	U.S. Patent No. 5,241,855 A to Cullen *et al.*, titled "Method and Apparatus for Inferring Engine Torque," issued September 7, 1993 ("Cullen")
1007	U.S. Patent No. 6,07,681 A to Scherer *et al.*, titled "Process and Apparatus for Adjusting the Torque of an Internal-Combustion Engine," issued April 11, 2000 ("Scherer")
1008	U.S. Patent No. 5,282,449 A to Takahashi *et al.*, titled "Method and System for Engine Control," issued February 1, 1994 ("Takahashi")
1009	"Bosch Fuel Injection Systems," F. Aird, Berkeley Publishing Group, 2001, ISBN 1-55788-365-3 ("Aird")
1010	U.S. Patent No. 6,209,519 B1 to Melchior *et al.*, titled "Method and Arrangement for Controlling the Quiet Running of an Internal Combustion Engine," issued April 3, 2001 ("Melchior")
1011	U.S. Patent No. 5,692,471 A to Zhang, titled "Method and Arrangement for Controlling a Vehicle," issued December 2, 1997 ("Zhang")
1012	"Bosch Automotive Handbook," Robert Bosch GmbH, 2000, ISBN 0-7680-0669-4 ("Bosch")
1013	*Intentionally Left Blank*
1014	John B. Heywood, Internal Combustion Engine Fundamentals, 1st Ed. McGraw-Hill, Inc., 1988. ("Heywood")
1015	Ronald K. Jurgen, "Automotive Electronics Handbook," McGraw-

Exhibit No.	Description
	Hill, Inc., ISBN0-07-033189-8, c.1994 (selected portions) ("Jurgen")
1016	*Intentionally Left Blank*
1017	*Intentionally Left Blank*
1018	*Intentionally Left Blank*
1019	Understanding Automotive Electronics, Fifth Edition (1998), William B. Ribbens, Ph.D. ("Ribbens")
1020	*Intentionally Left Blank*
1021	*Intentionally Left Blank*
1022	*Intentionally Left Blank*
1023	*Intentionally Left Blank*
1024	*Intentionally Left Blank*
1025	*Intentionally Left Blank*
1026	*Intentionally Left Blank*
1027	*Intentionally Left Blank*
1028	*Intentionally Left Blank*
1029	Amended Complaint for Patent Infringement, *Michigan Motor Technologies LLC v. Volkswagen AG*, Case No. 2:19-cv-10485 (E.D. Mich.), filed April 30, 2019
1030	Transcript of the Deposition of Russell Leonard, Ph.D., January 5, 2021.
1031	Second Declaration of C. Arthur MacCarley, Ph.D., P.E.
1032	Ogata, Katsuhiko. Modern Control Engineering, 4th ed., Prentice Hall, 2002
1033	*Intentionally Left Blank*

MMT's Patent Owner Response ("POR") does not overcome the strong obviousness showing in Volkswagen Group of America's Petition. Indeed, the POR does little to advance arguments beyond those previously presented in the Patent Owner Preliminary Response ("POPR"), which the Board already correctly rejected in the Institution Decision.

The '565 patent broadly claims controlling engine torque output by fuel and spark adjustments that compensate for the difference between a demand torque and an estimated torque. To compensate for these differences between demand torque and estimated torque, the '565 patent claims the combination of basic components found in a typical engine controller—a demand torque controller, an estimator, a comparator, a high-pass filter, and a transient torque controller—that were all well known, as evidenced by Robichaux and Scherer. Indeed, Patent Owner's declarant, Dr. Leonard, acknowledged that all these recited components of the '565 patent's DISI engine controller (shown in Figure 3) were known in the prior art. EX1030, 24:4-26:1.

In the POR, similar to the POPR, Patent Owner directs all its arguments to a single element—high-pass filtering a difference signal outputted from a comparator—and focuses on Robichaux and Scherer. However, as the Board correctly determined in the Institution Decision, Petitioner has shown "sufficiently that a person of ordinary skill in the art would have had reason to modify

Robichaux's system to send torque error … from block 218 to high pass filter 4 of Scherer's transient control branch ….” Decision on Institution, 37; Petition, 32-34, 47-50; EX1003, ¶¶81-92, 108-114. The Board also properly determined that the POPR “does not demonstrate that the references criticize, discredit, or otherwise discourage use of the invention recited in the claims of the ’565 patent.” DI, 38. The POR adds no significant new arguments beyond what was asserted—and rejected—in the POPR.

Accordingly, the Board should find all challenged claims unpatentable.

I. Patent Owner's Arguments Do Not Rebut Petitioner's Compelling Evidence Showing That the Prior Art Discloses Every Limitation of the Challenged Claims.

Patent Owner admits that the applied prior art discloses each component of the recited torque control device.[1] *See generally* POR; EX1030, 24:4-26:1, 27:4-28:11, 35:3-41:18. In fact, Patent Owner's declarant admitted that a device having a torque demand controller generating a torque demand signal and a torque producer receiving the torque demand signal were well known before the ’565 patent. EX1030, 25:6-8, 27:8-11. Patent Owner's declarant also admitted that using

[1] Patent Owner does also not contend that any elements recited in claims 2-6 are absent from the prior art. As the Board correctly found in its Institution Decision, the dependent claims are obvious in view of Petitioner's evidence. Pet., 61-67, 70-87; DI, 46, 55.

an estimator for outputting an estimated torque signal, a comparator for outputting a difference signal based on the estimated torque signal and the demand torque signal, and a high-pass filter for outputting a filtered signal to a transient torque controller were well known before the ’565 patent. EX1030, 25:9-20, 27:12-28:1. Patent Owner’s declarant further admitted that using a transient torque controller for outputting spark and fuel adjustment signals based on high-pass filtered signals was well known. EX1030, 25:21-26:1, 28:2-11. Patent Owner’s only dispute is that the applied prior art allegedly fails to disclose the recited arrangement of a high-pass filter receiving a difference signal outputted from a comparator. POR, 4-5; EX1030, 26:2-27:3.

Petitioner’s evidence convincingly shows why the combination of Robichaux and Scherer discloses all the limitations of independent claims 1 and 7, including the contested limitation “a high pass filter which receives as an input the difference signal.” Pet., 26-57; DI, 36-38; EX1003, ¶¶81-129. And the POR does not rebut the Petition’s evidence. Instead, the POR (1) individually attacks the references and ignores what the Petition’s cited art combination suggests as a whole; (2) applies unfounded and legally-flawed approaches for determining obviousness; and (3) presents baseless attacks against the credibility of Petitioner’s expert witness Dr. MacCarley. EX1031, ¶¶32-34.

A. Patent Owner improperly attacks Robichaux and Scherer individually and ignores the Petition's combination of Robichaux and Scherer.

The Board's Institution Decision correctly found based on Petitioner's evidence that the combination of Robichaux and Scherer render obvious a high-pass filter that receives a difference signal representing the difference between a torque demand and an estimated torque. DI, 36-38; EX1031, ¶¶35-48.

Patent Owner admits that Robichaux discloses a comparator that generates a "torque error" signal by subtracting an estimated torque from a demand torque. Pet., 45-46 (citing EX1003, ¶¶105–107; EX1020, ¶¶0029–0030, FIG. 3); POR, 10, 22; EX1030, 59:1-60:10. Robichaux further discloses outputting the torque error signal to a PID controller 220 that is suspended during abrupt transient conditions. Pet., 47 (citing EX1020, ¶0032; EX1003, ¶¶85-87). Suspending the PID controller 220 during trainset conditions, Robichaux invited combinations with references disclosing transient torque control, as explained in the Petition and Dr. MacCarley's declaration. Pet., 47 (citing EX1003, ¶¶87, 109); EX1031, ¶36.

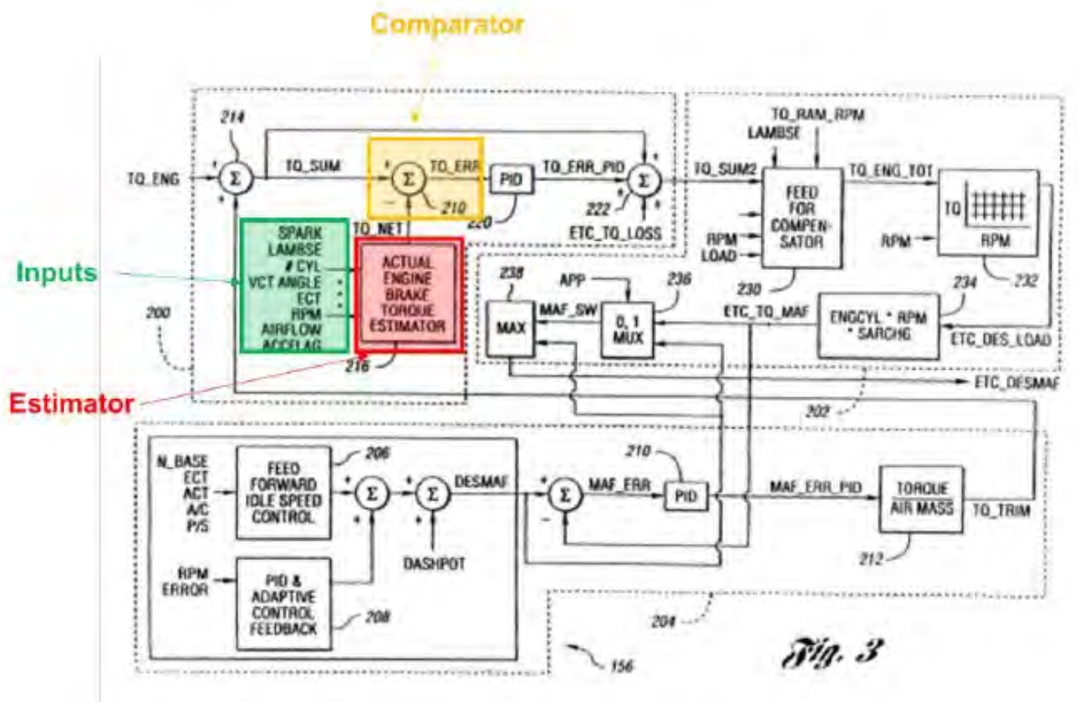


Fig. 3

EX1020, FIG. 3 (annotated).

Scherer discloses a high-pass filter 4 that "filters the dynamic desired engine torque part (Md) … from the desired engine torque (Mdes) … and in turn provides the dynamic desired engine torque (Md) to the device for the ignition point adjustment" in Figure 1 and the device for fuel adjustment in Figure 2. EX1007, 4:40-45; EX1031, ¶37. Scherer's ignition point adjustment system in Figure 1 and fuel adjustment system in Figure 2 are used "to adjust specifically that dynamic desired engine torque component which cannot be regulated by the air-flow-rate-regulating branch." EX1007, 4:2-31.

And as explained in the Petition, supported by Dr. MacCarley's declaration, and as the Board correctly determined, a POSA would have been motivated to

combine Scherer's transient control branch, including Scherer's high-pass filter 4, with Robichaux's controller so that Robichaux's controller can correct torque output during transient conditions. DI, 37; Pet., 30–36 (citing EX1003, ¶¶81–92; EX1007, 1:7–11, 2:51-59; 4:20–52, 4:60-67; 5:10–55, FIGS. 1, 2; EX1019, 138; EX1020 ¶¶0001, 0016–0018, 0023, 0025, 0027, 0032–0033, 0048, FIGS. 2, 3); EX1031, ¶38. Particularly, Dr. MacCarley explains that Robichaux's PID controller is subject to the well-known problem of "integrator wind-up," and that Robichaux recognized this problem and suspends the PID controller during transient conditions to prevent interference from other forms of control.[2] EX1003, ¶¶85-87; EX1031, ¶39; EX1020, ¶0032. As such, a POSA would have sought out other non-PID control, such as Scherer's transient control branch, to adjust torque output during transient conditions. EX1003, ¶¶85-87; EX1031, ¶39.

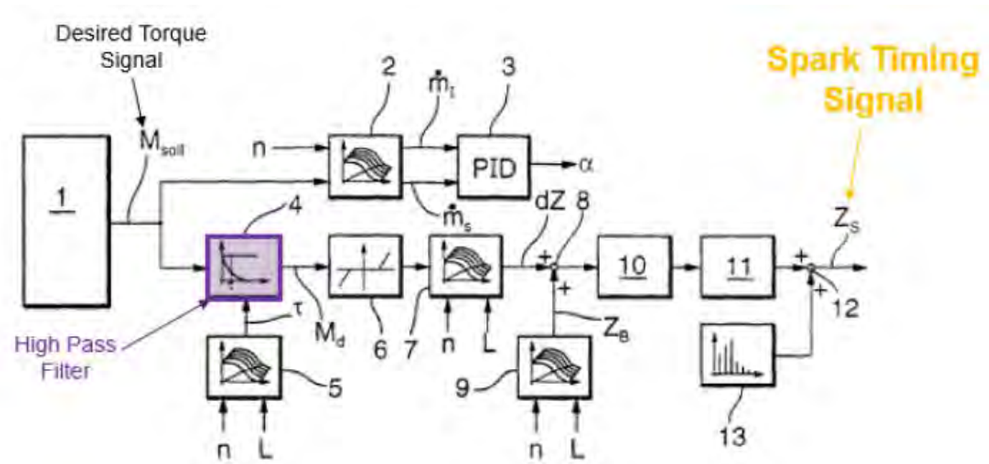
Indeed, combining Scherer's transient control branch with Robichaux's torque controller would have yielded the predictable advantage of permitting rapid torque adjustment during transient engine operations. Pet., 30-36, 47-50; DI, 37; EX1003, ¶¶82-92; EX1031, ¶40; EX1007, 5:9-14 ("The embodiment of FIG. 1

[2] Notably, neither Patent Owner nor Patent Owner's declarant Dr. Leonard addresses Dr. MacCarley's testimony about the known problem of integrator wind-up and how this would have motivated a POSA to include non-PID control architectures for controlling torque during transient conditions.

therefore permits a comparatively **rapid engine torque adjustment** which can follow very fast desired engine torque changes[.]"). The Petition further demonstrated similarities between Robichaux and Scherer that would have encouraged a POSA to combine their teachings. Namely, Robichaux and Scherer are both in the field of engine control systems for controlling engine parameters in accordance with monitored torque output. Pet., 30; EX1003, ¶82; EX1031, ¶40. And as explained in the Petition, a POSA would have understood how to combine Robichaux's and Scherer's teachings by simply modifying the software instructions stored in the microcomputer of Robichaux's controller. Pet., 34-36; EX1003, ¶91; EX1031, ¶40; EX1030, 64:16-65:2. Indeed, supported by Dr. MacCarley's declaration, the Petition established that combining Scherer's transient control branch with Robichaux's controller would have merely been applying a known technique in a similar, known device in the same way to expedite the response time for correcting the transient component of the torque demand. Pet., 34 (citing *KSR Int'l Co. v. Teleflex Inc.,* 550 U.S. 398, 416 (2007)); EX1003, ¶¶82-92; EX1031, ¶40.

And as the Board correctly found in the Institution Decision, combining Scherer's transient torque control branch in lieu of Robichaux's PID controller 220 would result in the high-pass filter 4 receiving the difference signal outputted from Robichaux's comparator, thereby allowing Robichaux's controller to precisely

control torque output in *both* steady-state and transient conditions. DI, 37; EX1031, ¶41. When looking to combine Scherer's transient torque control branch with Robichaux's torque control unit, a POSA would have configured Scherer's transient control branch to be parallel to Robichaux's PID controller 220 such that the Robichaux's torque error signal (TQ_ERROR) is inputted into Scherer's high pass filter during transient conditions. EX1031, ¶41; EX1003, ¶113. As shown below in Figure 1 Scherer, the transient control branch is configured parallel to a steady-state PID controller 3, which is analogous to Robichaux's PID controller 220. EX1031, ¶41.



EX1007, FIG. 1 (annotated).

Accordingly, when combining Scherer's teachings with Robichaux's teachings, a POSA would have configured Scherer's transient torque control branch to be parallel with Robichaux's PID controller 220 and to receive

Robichaux's torque error signal (TQ_ERROR) during transient conditions. EX1031, ¶42.

Moreover, a POSA would have understood that Scherer's high-pass filter is positioned at the input side of the transient torque controller. EX1031, ¶43. First, as acknowledged by Patent Owner's declarant, Scherer discloses that the desired torque signal outputted from the engine torque coordination unit 1 is an "example," implying that other types of signals, such as a difference signal, may be inputted into the high-pass filter. EX1007, 3:60-63 ("[A]n engine torque coordination unit 1 provides a signal indicative, **for example**, of the desired engine torque (Mdes) requested by the driver via the accelerator pedal.")[3]; EX1030, 68:13-69:12. Second, Scherer discloses particular advantages associated with high-pass filter signal processing that are applicable to both a torque demand signal and a difference signal. Pet., 33-36 (citing EX1007, 2:51-54, 4:20-52, 5:10-55); EX1031, ¶43; EX1007, 4:45-52, 5:9-14. These advantages include (1) removing the low frequency component to permit a rapid response to correcting torque output and (2) reducing the torque output as the transient component decays to zero. Pet., 33-36 (citing EX1007, 2:51-54, 4:20-52, 5:16-55); EX1031, ¶43; EX1007, 4:45-52, 5:9-14. And a POSA would have understood that Robichaux's difference signal would have been further processed to translate the difference signal into a correction

[3] Emphasis added unless noted.

action to be performed by a transient torque controller. EX1031, ¶44. Indeed, as shown in Figure 3, Robichaux uses the PID controller 220 to further process the torque error signal before generating the feedback signal for throttle valve control. EX1020, ¶¶0034-35; EX1031, ¶44. In view of this arrangement shown in Figure 3 of Robichaux, a POSA would have understood that the torque error signal needs to be further processed before making adjustments to the spark timing and fuel injection. EX1031, ¶44.

Accordingly, it would have been obvious to use Scherer's high-filter 4 to receive Robichaux's torque error signal (a difference signal). EX1031, ¶45; EX1003, ¶¶112-114.

Attempting to evade what is taught by the Robichaux-Scherer combination, Patent Owner attacks Scherer in isolation and ignores the Petition's combination. EX1031, ¶¶46-47. However, the Federal Circuit has consistently held that "[a] finding of obviousness… cannot be overcome by 'by attacking references individually where the rejection is based upon the teachings of a combination of references.'" *See Bradium Techs. LLC v. Iancu*, 923 F.3d 1032, 1050 (Fed. Cir. 2019); *In re Merck & Co.*, 800 F.2d 1091, 1097 (Fed. Cir. 1986)). "[T]he test for obviousness is **what the combined teachings of the references** would have suggested to those having ordinary skill in the art." *In re Mouttet*, 686 F.3d 1322, 1333 (Fed. Cir. 2012). Furthermore, "[a] claimed invention may be obvious even

when the prior art does not teach each claim limitation, so long as the record contains some reason that would cause one of skill in the art to modify the prior art to obtain the claimed invention." *Malico, Inc. v. Cooler Master USA Inc*., 594 F. App'x 621, 626 (Fed. Cir. 2014); *see also Hartness Int'l, Inc. v. Simplimatic Eng'g Co*., 819 F.2d 1100, 1108 (Fed. Cir. 1987) ("the inquiry is not whether each element existed in the prior art, but whether the prior art made obvious the invention as a whole[.]"). But this is precisely what the Patent Owner does here by arguing that "no prior art reference relied upon by the Petitioner or the Board discloses producing a 'difference signal' by comparing an 'estimated torque signal' and a 'desired torque signal' and then high-pass filtering such 'difference signal.'" POR, 15; EX1031, ¶¶46-47.

Pointing to the embodiments shown in Figures 1 and 2 of Scherer, Patent Owner argues that "only the dynamic component of the desired engine torque is high-pass filtered and no comparator is used to compare an 'estimated torque signal' to the 'desired torque signal.'" POR, 17. That is, Patent Owner ignores that Robichaux discloses outputting a difference signal to a PID controller 220 suspended during transient conditions, and that Scherer discloses situating a high-pass filter at an input side of a transient control branch, which when combined with Robichaux receives the difference signal during transient mode. Pet., 47-50 (citing EX1003, ¶¶85-90, 109-114; EX1020, ¶¶0030-0032; EX1007, 2:51-54, 4:20-5:9,

5:16-55); EX1031, ¶47; EX1020, ¶0032 (The PID controller 220 "is preferably **deactivated during these transient modes**"); EX1007, 4:20-24 ("In order to adjust the dynamic desired torque component which exceeds this quasi-static part, an ignition point adjusting branch, with the **input-side high-pass filter**, is provided **in parallel** to the air-flow-rate-regulating branch.")

Accordingly, as explained in the Petition and supported by Dr. MacCarley's declaration, a POSA would have been motivated to combine Scherer's transient control branch and its input high-pass filter with Robichaux's controller to receive Robichaux's torque error signal during transient mode "so that the transient torque control branch can respond to the transient circumstance in lieu of Robichaux's PID controller 220." DI, 37; *see also* EX1003, ¶81-92; EX1031, ¶35-48.

B. Robichaux and Scherer do not teach away from the claimed invention.

Patent Owner asserts that Robichaux and Scherer teach away from each other because (1) Robichaux's controller purposely limits control of transient torque output by deactivating the PID controller 220 during transient conditions, and (2) Scherer's alternative embodiment shown in Figures 3 and 4 uses a model-based estimated value stage, instead of a high-pass filter, to determine the dynamic desired engine torque component. POR, 12-23.

Patent Owner's assertions are premised on mischaracterizations of the applied prior art, technically unsupported conclusions, and legally-flawed approaches for determining obviousness. EX1031, ¶¶49-50.

1. Deactivation of Robichaux's PID controller does not teach away from the claimed invention.

Patent Owner asserts that Robichaux discourages a POSA from correcting torque output during transient conditions because Robichaux's controller suspends the PID controller during certain transient torque limiting control modes. POR, 21. But Patent Owner's argument against Robichaux is premised on the mischaracterization that Robichaux criticizes any attempt by an engine controller to correct transient torque output. EX1031, ¶51.

First, Patent Owner's assertion that Robichaux discourages any attempt to correct transient torque output simply cannot be true. Otherwise, Robichaux's engine controller would not provide adequate fuel and spark adjustments to correct torque output during transient conditions, such as shifting a gear or controlling idle engine speed. EX1031, ¶52.

Second, Patent Owner ignores the fact that Robichaux recognizes that other forms of controls are needed during transient conditions to adjust spark timing and fuel injection. EX1031, ¶53; *see* EX1020, ¶0032. Particularly, Robichaux discloses suspending the PID controller 220 "to prevent interference between the control parameters for air and fuel flow and the transient torque control parameters such as

spark retard or injector/cylinder cutout." EX1020, ¶0032; EX1003, ¶86. As explained in Dr. MacCarley's declaration, Robichaux recognizes that the integrator function of the PID typically overshoots the correction of torque output during transient conditions, and therefore, suspends the PID controller during transient conditions. EX1020, ¶0032; EX1003, ¶86. Accordingly, Robichaux invites other techniques, such as Scherer's transient control branch, that are more suitable for correcting torque output during transient conditions to be used in lieu of the PID controller. EX1031, ¶53.

Thus, the Board should reject Patent Owner's mischaracterization of Robichaux and its flawed legal argument that Robichaux teaches away from transient torque control. EX1031, ¶54.

2. Scherer's alternative embodiments do not teach away from the claimed invention.

Patent Owner asserts that Scherer discourages a POSA from using a high-pass filter with a difference signal because Scherer's alternative embodiment shown in Figures 3 and 4 uses a model estimation stage, rather than a high-pass filter, to determine a dynamic component of a desired torque signal. POR, 16-18, 21-22. But Scherer's disclosure of alternative embodiments—a high-pass filter and a model estimation stage—for determining a dynamic component does not mean Scherer teaches away from using a high-pass filter to receive a difference signal. EX1031, ¶55.

"A reference may be said to teach away when a person of ordinary skill, upon reading the reference, would be discouraged from following the path set out in the reference, or would be led in a direction divergent from the path that was taken by the applicant." *Ricoh Co., Ltd. v. Quanta Comput. Inc*., 550 F.3d 1325, 1332 (Fed. Cir. 2008) (quoting *In re Kahn*, 441 F.3d 977, 990 (Fed. Cir. 2006)). To teach away, a prior art reference must "criticize, discredit, or otherwise discourage the solution claimed." *In re Fulton*, 391 F.3d 1195, 1201 (Fed. Cir. 2004).

However, a reference does not teach away for merely expressing a general preference for an alternative invention. *See DePuy Spine, Inc. v. Medtronic Sofamor Danek, Inc*., 567 F.3d 1314, 1327 (Fed. Cir. 2009). Moreover, "a prior art reference that does not specifically refer to one element of a combination does not, *per se*, teach away." *Syntex (U.S.A.) LLC v. Apotex, Inc*., 407 F.3d 1371, 1380 (Fed. Cir. 2005).

Nowhere does Scherer criticize, discredit, or discourage a skilled artisan from using a high-filter to receive a difference signal. EX1031, ¶56. Patent Owner alleges that Scherer's description of the embodiment shown in Figure 3 discourages a POSA from using of a high-pass filter with a comparator. POR, 17-18, 21. This section of Scherer discloses "[t]his embodiment corresponds to that of FIG. 1 except that, instead of the high pass filter stage, a model-based estimated

value stage is provided to determine the dynamic desired engine torque component." EX1005, 6:3-12; POR, 17-18, 21.

Here, Scherer's description of the alternative embodiment, however, would not have discouraged a person of ordinary skill in the art from using a high-pass filter that receives a difference signal. EX1031, ¶57. Rather, Scherer merely discloses two ways—a high-pass filter and a model-based estimation value stage—for determining a dynamic component of a torque signal. EX1031, ¶57. Scherer never discloses that the model-estimation (Figures 3 & 4) stage and the high-pass filter (Figures 1 & 2) are mutually exclusive. EX1031, ¶57. And Scherer does not discredit using both a high-pass filter and an estimation stage in a torque producer of an engine controller. EX1031, ¶57. Thus, Scherer's two alternative embodiments—high-pass filter shown in Figures 1 and 2 and the model estimation shown in Figures 3 and 4—are not a teaching away. EX1031, ¶58.

C. Patent Owner's credibility attacks on Dr. MacCarley's testimony are meritless.

In an attempt to distract the Board from the Petition's merits, Patent Owner asserts additionally that Dr. MacCarley's testimony is inherently unreliable due to the mechanics of how his Declaration was drafted and certain observations concerning his cross-examination. POR, 23-28. These allegations against Dr. MacCarley's testimony are unfounded.

1. Patent Owner mischaracterizes Dr. MacCarley's testimony.

Patent Owner's spurious attacks on the reliability of Dr. MacCarley's testimony are premised on mischaracterizations. Patent Owner contends that Dr. MacCarley testified during his deposition that the starting point for creating the declaration was holding numerous discussions with Petitioner's counsel. POR, 23-24 (quoting EX2003, 10:6-14:4). Patent Owner further highlights that Dr. MacCarley acknowledged in his deposition that the prior art search was performed by Petitioner's counsel. POR, 24 (quoting EX2003: 17:16-18:20). From here, Patent Owner argues "[b]ased upon … deposition testimony of Dr. MacCarley, … it is plain to see that what the Petitioner's attorneys and expert have done … is to collaboratively prepare expert declarations … without the expert even starting the obviousness analysis from the proper starting point." POR, 25.

Patent Owner selectively and deceptively removed relevant statements from the testimony quoted in its POR. When testifying about the preparation of the declaration, Dr. MacCarley concluded by stating "the technical content underlying all of the arguments originates from either me or just common sense engineering knowledge." EX2003, 13:16-19. Furthermore, while Dr. MacCarley acknowledged that he did not conduct the search, Dr. MacCarley noted that he provided input into the search and that he selected the applied references—Robichaux, Scherer,

Takahashi, and Suzuki. *Id*., 13:23-14:4, 16:2-11. Patent Owner removed and ignored the statements by Dr. MacCarley that do not fit its false narrative.

The line of questions eliciting this testimony is the exact type of questions the Board has said "generally are a waste of time" in the context of interference proceedings. *Pevarello v. Lan*, Patent Interference No. 105,394, Paper 85, slip op., 21 (B.P.A.I. Jan. 12, 2007). Furthermore, the Board has held that when the manner of the expert's testimony is in line with typical testimony before the Board, and the expert's background, analysis, and supporting evidence are credible, an inquiry into "the mechanics of declaration preparation is '**a waste of time, both for the witness and the Board**.'" *Square, Inc. v. Think Comput. Corp., No*. CBM2014-00159, Paper 47, 26-27, (P.T.A.B. Nov. 27, 2015) (quoting *Pevarello*, Paper 85, slip op. at 19–21.) Spending over forty years researching, teaching, and designing engine control systems, Dr. MacCarley has supported each of his positions set forth in his declaration with significant insight and secondary evidence, such as automotive handbooks, engineering textbooks, and relevant patents.

In sum, Patent Owner's credibility attacks against Dr. MacCarley's testimony are meritless and a waste of the Board's time. The Board should disregard Patent Owner's specious allegations.

2. Dr. MacCarley articulated and applied the appropriate test for obviousness.

Patent Owner further alleges that Dr. MacCarley failed to apply the proper framework for determining obviousness because he received prior art references from Petitioner's counsel. POR, 25-28. Particularly, Patent Owner argues that by being "spoon fed the prior art references to consider," Dr. MacCarley did not take the proper starting point when determining whether the claims of the '565 patent would have been obvious. *Id*., 26-27. These arguments lack merit.

Dr. MacCarley properly considered the requirements for determining obviousness as it related to a POSA and the corresponding level of ordinary skill in the art. EX1003, ¶¶30-41. Indeed, during his deposition, Dr. MacCarley testified that he formulated his opinion based on the framework described in the legal standards section set forth in paragraphs 21-39 of his declaration. EX2003, 21:7-23. He applied the proper standards in his analysis.

And Patent Owner's assertion that Dr. MacCarley's testimony is unreliable because he allegedly did not "determine what prior art the POSA would have searched for and found in the normal course of research and development," is legally flawed. POR, 27. It is well-settled law that there is "a presumption that **a person of ordinary skill in the art possesses knowledge of all prior art** references in the particular art." *Messerschmidt v. United States*, 29 Fed. Cl. 1, 27, aff'd, 14 F.3d 613 (Fed. Cir. 1993); *In re Winslow*, 365 F.2d 1017, 1020 (C.C.P.A.

1966) ("Section 103 requires us to presume full knowledge by the inventor of the prior art in the field of his endeavor."). Since all applied references are analogous are in the field of engine control systems—a fact uncontested by Patent Owner—a POSA would have had knowledge of each of the applied prior art references: Robichaux, Scherer, Takahashi, and Suzuki. Thus, Patent Owner's argument contravenes established law.

Accordingly, Dr. MacCarley did not misapply the proper framework in opining that the '565 patent claims would have been obviousness to a POSA.

II. Conclusion

For the reasons above, claims 1-7 of U.S. Patent No. 6,581,565 would have been obvious and should be cancelled.

Respectfully submitted,

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CERTIFICATE OF COMPLIANCE WITH TYPE-VOLUME LIMITATION, TYPEFACE REQUIREMENTS, AND TYPE STYLE REQUIREMENTS

1. This Petitioner's Reply to the Patent Owner's Response complies with the type-volume limitation of 5,600 words, comprising 4,009 words, excluding the parts exempted by 37 C.F.R. § 42.24(c).

2. This Petitioner's Reply to the Patent Owner's Response complies with the general format requirements of 37 C.F.R. § 42.6(a) and has been prepared using Microsoft® Word 2010 in 14-point Times New Roman.

Respectfully submitted,

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CERTIFICATION OF SERVICE (37 C.F.R. §§ 42.6(e))

The undersigned hereby certifies that on January 21, 2021 true and correct copies of the foregoing **PETITIONER'S REPLY TO PATENT OWNER'S RESPONSE** and **EXHIBITS 1030-1032** were served in their entireties on the following parties via email:

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16159321

UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner

Case IPR2020-00444
U.S. Patent No. 6,581,565

JOINT MOTION TO TERMINATE *INTER PARTES* REVIEW

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United States Patent and Trademark Office
P.O. Box 1450
Alexandria, VA 22313-1450

TABLE OF CONTENTS

I. STATEMENT OF RELIEF REQUESTED 1
II. STATEMENT OF FACTS 1
III. ARGUMENT 2
A. The Board should terminate this IPR in its entirety. 2
B. Written Agreement 3
IV. CONCLUSION 4

PETITIONER'S UPDATED EXHIBIT LIST

Exhibit No.	Description
1001	U.S. Patent No. 6,581,565 B2 to Heslop *et al*. ("'565 Patent")
1002	Prosecution File History of U.S. Patent No. 6,581,565 B2 ("'565 history")
1003	Declaration of C. Arthur MacCarley, Ph.D., P.E.
1004	Curriculum Vitae of C. Arthur MacCarley, Ph.D., P.E.
1005	U.S. Patent No. 6,006,717 A to Suzuki *et al.*, titled "Direct-Injection Spark-Ignition Type Engine Control Apparatus," issued December 28, 1999 ("Suzuki")
1006	U.S. Patent No. 5,241,855 A to Cullen *et al.*, titled "Method and Apparatus for Inferring Engine Torque," issued September 7, 1993 ("Cullen")
1007	U.S. Patent No. 6,07,681 A to Scherer *et al.*, titled "Process and Apparatus for Adjusting the Torque of an Internal-Combustion Engine," issued April 11, 2000 ("Scherer")
1008	U.S. Patent No. 5,282,449 A to Takahashi *et al.*, titled "Method and System for Engine Control," issued February 1, 1994 ("Takahashi")
1009	"Bosch Fuel Injection Systems," F. Aird, Berkeley Publishing Group, 2001, ISBN 1-55788-365-3 ("Aird")
1010	U.S. Patent No. 6,209,519 B1 to Melchior *et al.*, titled "Method and Arrangement for Controlling the Quiet Running of an Internal Combustion Engine," issued April 3, 2001 ("Melchior")
1011	U.S. Patent No. 5,692,471 A to Zhang, titled "Method and Arrangement for Controlling a Vehicle," issued December 2, 1997 ("Zhang")
1012	"Bosch Automotive Handbook," Robert Bosch GmbH, 2000, ISBN 0-7680-0669-4 ("Bosch")
1013	*Intentionally Left Blank*
1014	John B. Heywood, Internal Combustion Engine Fundamentals, 1st Ed. McGraw-Hill, Inc., 1988. ("Heywood")

Exhibit No.	Description
1015	Ronald K. Jurgen, "Automotive Electronics Handbook," McGraw-Hill, Inc., ISBN0-07-033189-8, c.1994 (selected portions) ("Jurgen")
1016-1018	*Intentionally Left Blank*
1019	Understanding Automotive Electronics, Fifth Edition (1998), William B. Ribbens, Ph.D. ("Ribbens")
1020-1028	*Intentionally Left Blank*
1029	Amended Complaint for Patent Infringement, *Michigan Motor Technologies LLC v. Volkswagen AG*, Case No. 2:19-cv-10485 (E.D. Mich.), filed April 30, 2019
1030	Transcript of the Deposition of Russell Leonard, Ph.D., January 5, 2021.
1031	Second Declaration of C. Arthur MacCarley, Ph.D., P.E.
1032	Ogata, Katsuhiko. Modern Control Engineering, 4th ed., Prentice Hall, 2002
1033	DAE Notes, RC High-Pass Filter Circuit, https://www.daenotes.com/electronics/digital-electronics/rc-high-pass-filter-circuit
1034-1099	*Intentionally Left Blank*
1100	Agreement (CONFIDENTIAL)

I. STATEMENT OF RELIEF REQUESTED

Petitioner Volkswagen Group of America, Inc. ("VWGoA" or "Petitioner") and Patent Owner Michigan Motor Technologies LLC ("MMT" or "Patent Owner") have entered into a confidential agreement that resolves all underlying disputes between Petitioner and Patent Owner with respect to U.S. Patent No. 6,581,565 ("the '565 patent"). A copy of the parties' agreement is being filed with this motion as Exhibit 1100 (the "Agreement"). The parties are concurrently filing a separate request that the Agreement be treated as business confidential information and be kept separate from the files of the involved patent, pursuant to 37 C.F.R. § 42.74(c).

Accordingly, the parties jointly move the Board to terminate this *inter partes* review (IPR) proceeding in its entirety, without rendering a final written decision. The Board authorized the parties to file this joint motion to terminate in an email of March 17, 2021.

II. STATEMENT OF FACTS

On March 16, 2021, Petitioner and Patent Owner entered into a confidential Agreement. *See* EX1100 (Confidential). Under the terms of the Agreement, Petitioner and Patent Owner agreed to jointly seek termination of this proceeding.

III. ARGUMENT

37 C.F.R. § 42.72 provides that "[t]he Board may terminate a trial without rendering a final written decision, where appropriate, including where the trial is consolidated with another proceeding or pursuant to a joint request under 35 U.S.C. 317(a)." Termination of the present *inter partes* review proceeding in its entirety is in the interest of justice.

A. The Board should terminate this IPR in its entirety.

Public policy favors terminating the present *inter partes* review proceeding. The federal courts have expressed a strong interest in encouraging settlement in litigation. *See, e.g., Bergh v. Dept. of Trans.*, 794 F.2d 1575, 1577 (Fed. Cir. 1986) ("The law favors settlement of cases."), *cert. denied*, 479 U.S. 950 (1986). The Board's Trial Practice Guide stresses that "[t]here are strong public policy reasons to favor settlement between the parties to a proceeding." Office Patent Trial Practice Guide, 77 Fed. Reg. 48,756, 46,768 (Aug. 14, 2012).

Additionally, termination of the proceeding at this stage, in view of the Agreement, is appropriate. The USPTO can conserve significant administrative and judicial resources by terminating the proceeding now, removing the need for the Board to render a final written decision. The Board has terminated entire proceedings based on joint motions to terminate, even after the merits had been fully briefed and oral argument had occurred. *See Apex Med. Corp. v. Resmed Ltd.*,

IPR2013-00512, Paper 39, at 2-3 (P.T.A.B. Sept. 12, 2014) (granting joint motion to terminate after the parties had fully briefed the matter because the Board "has not yet decided the merits of this proceeding, and the record is not yet closed."); *Lam Research Corp. v. Flamm*, IPR2015-01764, Paper 27, at 5 (P.T.A.B. Dec. 15, 2016) (granting motion to terminate after all briefing and oral argument when statutory deadline for rendering final written decision is within two months of motion).

Finally, because the Board has not decided the merits of the proceeding, the expected normal course is to terminate the proceeding. Office Patent Trial Practice Guide, 77 Fed. Reg. at 48,768 ("The Board expects that a proceeding will terminate after the filing of a settlement agreement, unless the Board has already decided the merits of the proceeding.").

B. Written Agreement

The parties represent that their entire agreement in connection with the termination of this proceeding is embodied in the Agreement, which has been made in writing. There are no collateral agreements or understandings made in connection with, or in contemplation of, the termination of this *inter partes* review. The Agreement has been filed for access by the "Parties and Board Only" due to the highly sensitive business confidential information it contains. The parties desire that the Agreement be maintained as business confidential information under 35

U.S.C. § 317(b) and 37 C.F.R. § 42.74(c), and a separate joint request for such is being filed concurrently.

IV. CONCLUSION

Petitioner and Patent Owner respectfully request that the Board grant the parties' Joint Motion to Terminate this proceeding in its entirety.

Respectfully submitted,

/Jason A. Fitzsimmons/

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CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that true and correct copies of the foregoing **JOINT MOTION TO TERMINATE *INTER PARTES* REVIEW** and **Exhibit 1100** were served electronically via e-mail on March 19, 2021, on the following counsel for Patent Owner:

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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner

Case IPR2020-00444
U.S. Patent No. 6,581,565

JOINT REQUEST TO FILE AGREEMENT AS BUSINESS CONFIDENTIAL INFORMATION PURSUANT TO 35 U.S.C. § 317(b)

Mail Stop "PATENT BOARD"
Patent Trial and Appeal Board
United States Patent and Trademark Office
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Alexandria, VA 22313-1450

Pursuant to 35 U.S.C. § 317(b) and 37 C.F.R. § 42.74(c), Petitioner Volkswagen Group of America, Inc. ("VWGoA" or "Petitioner") and Patent Owner Michigan Motor Technologies LLC ("MMT" or "Patent Owner") hereby jointly request that a true copy of their agreement, filed concurrently herewith as Exhibit 1100 be treated as business confidential information and be kept separate from the file of this *inter partes* review (IPR) proceeding. Concurrently with the filing of this Request, Petitioner and Patent Owner are filing a Joint Motion to Terminate this *inter partes* review due to the agreement between the parties.

The parties consider the agreement to contain highly sensitive business confidential information that would substantially harm their business interests if publicly disclosed. As such, the parties hereby jointly request that the agreement be kept as a separate paper to be made available only under the provisions of 35 U.S.C § 317(b) and 37 C.F.R. § 42.74(c). The agreement has been filed for access by the "Parties and Board Only." The parties further jointly request that the Board order that in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the agreement, that any such written request be served upon the parties on the day the written request is provided to the Board.

Respectfully submitted,

/Jason A. Fitzsimmons/

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Date: March 19, 2021

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Respectfully submitted,

/s/ Timothy Devlin
Timothy Devlin (Reg. 41,706)
DEVLIN LAW FIRM LLC
Attorney for Patent Owner

Date: March 18, 2021

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CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that true and correct copies of the foregoing **JOINT REQUEST TO FILE AGREEMENT AS BUSINESS CONFIDENTIAL INFORMATION UNDER 35 U.S.C. § 317(b)** was served electronically via e-mail on March 19, 2021, on the following counsel for Patent Owner:

Timothy Devlin (Lead Counsel)
Robyn T. Williams (Backup Counsel)
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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner

Case IPR2020-00445
U.S. Patent No. 6,581,565

JOINT MOTION TO TERMINATE *INTER PARTES* REVIEW

Mail Stop "PATENT BOARD"
Patent Trial and Appeal Board
United States Patent and Trademark Office
P.O. Box 1450
Alexandria, VA 22313-1450

TABLE OF CONTENTS

I. STATEMENT OF RELIEF REQUESTED 1
II. STATEMENT OF FACTS 1
III. ARGUMENT 2
A. The Board should terminate this IPR in its entirety. 2
B. Written Agreement 3
IV. CONCLUSION 4

PETITIONER'S UPDATED EXHIBIT LIST

Exhibit No.	Description
1001	U.S. Patent No. 6,581,565 B2 to Heslop *et al*. ("'565 Patent")
1002	Prosecution File History of U.S. Patent No. 6,581,565 B2 ("'565 history")
1003	Declaration of C. Arthur MacCarley, Ph.D., P.E.
1004	Curriculum Vitae of C. Arthur MacCarley, Ph.D., P.E.
1005	U.S. Patent No. 6,006,717 A to Suzuki *et al.*, titled "Direct-Injection Spark-Ignition Type Engine Control Apparatus," issued December 28, 1999 ("Suzuki")
1006	U.S. Patent No. 5,241,855 A to Cullen *et al.*, titled "Method and Apparatus for Inferring Engine Torque," issued September 7, 1993 ("Cullen")
1007	U.S. Patent No. 6,07,681 A to Scherer *et al.*, titled "Process and Apparatus for Adjusting the Torque of an Internal-Combustion Engine," issued April 11, 2000 ("Scherer")
1008	U.S. Patent No. 5,282,449 A to Takahashi *et al.*, titled "Method and System for Engine Control," issued February 1, 1994 ("Takahashi")
1009	"Bosch Fuel Injection Systems," F. Aird, Berkeley Publishing Group, 2001, ISBN 1-55788-365-3 ("Aird")
1010	U.S. Patent No. 6,209,519 B1 to Melchior *et al.*, titled "Method and Arrangement for Controlling the Quiet Running of an Internal Combustion Engine," issued April 3, 2001 ("Melchior")
1011	U.S. Patent No. 5,692,471 A to Zhang, titled "Method and Arrangement for Controlling a Vehicle," issued December 2, 1997 ("Zhang")
1012	"Bosch Automotive Handbook," Robert Bosch GmbH, 2000, ISBN 0-7680-0669-4 ("Bosch")
1013	*Intentionally Left Blank*
1014	John B. Heywood, Internal Combustion Engine Fundamentals, 1st Ed. McGraw-Hill, Inc., 1988. ("Heywood")
1015	Ronald K. Jurgen, "Automotive Electronics Handbook," McGraw-

Exhibit No.	Description
	Hill, Inc., ISBN0-07-033189-8, c.1994 (selected portions) ("Jurgen")
1016-1018	*Intentionally Left Blank*
1019	Understanding Automotive Electronics, Fifth Edition (1998), William B. Ribbens, Ph.D. ("Ribbens")
1020-1028	*Intentionally Left Blank*
1029	Amended Complaint for Patent Infringement, *Michigan Motor Technologies LLC v. Volkswagen AG*, Case No. 2:19-cv-10485 (E.D. Mich.), filed April 30, 2019
1030	Transcript of the Deposition of Russell Leonard, Ph.D., January 5, 2021.
1031	Second Declaration of C. Arthur MacCarley, Ph.D., P.E.
1032	Ogata, Katsuhiko. Modern Control Engineering, 4th ed., Prentice Hall, 2002
1033-1099	*Intentionally Left Blank*
1100	Agreement (CONFIDENTIAL)

I. STATEMENT OF RELIEF REQUESTED

Petitioner Volkswagen Group of America, Inc. ("VWGoA" or "Petitioner") and Patent Owner Michigan Motor Technologies LLC ("MMT" or "Patent Owner") have entered into a confidential agreement that resolves all underlying disputes between Petitioner and Patent Owner with respect to U.S. Patent No. 6,581,565 ("the '565 patent"). A copy of the parties' agreement is being filed with this motion as Exhibit 1100 (the "Agreement"). The parties are concurrently filing a separate request that the Agreement be treated as business confidential information and be kept separate from the files of the involved patent, pursuant to 37 C.F.R. § 42.74(c).

Accordingly, the parties jointly move the Board to terminate this *inter partes* review (IPR) proceeding in its entirety, without rendering a final written decision. The Board authorized the parties to file this joint motion to terminate in an email of March 17, 2021.

II. STATEMENT OF FACTS

On March 16, 2021, Petitioner and Patent Owner entered into a confidential Agreement. *See* EX1100 (Confidential). Under the terms of the Agreement, Petitioner and Patent Owner agreed to jointly seek termination of this proceeding.

III. ARGUMENT

37 C.F.R. § 42.72 provides that "[t]he Board may terminate a trial without rendering a final written decision, where appropriate, including where the trial is consolidated with another proceeding or pursuant to a joint request under 35 U.S.C. 317(a)." Termination of the present *inter partes* review proceeding in its entirety is in the interest of justice.

A. The Board should terminate this IPR in its entirety.

Public policy favors terminating the present *inter partes* review proceeding. The federal courts have expressed a strong interest in encouraging settlement in litigation. *See, e.g., Bergh v. Dept. of Trans.*, 794 F.2d 1575, 1577 (Fed. Cir. 1986) ("The law favors settlement of cases."), *cert. denied*, 479 U.S. 950 (1986). The Board's Trial Practice Guide stresses that "[t]here are strong public policy reasons to favor settlement between the parties to a proceeding." Office Patent Trial Practice Guide, 77 Fed. Reg. 48,756, 46,768 (Aug. 14, 2012).

Additionally, termination of the proceeding at this stage, in view of the Agreement, is appropriate. The USPTO can conserve significant administrative and judicial resources by terminating the proceeding now, removing the need for the Board to render a final written decision. The Board has terminated entire proceedings based on joint motions to terminate, even after the merits had been fully briefed and oral argument had occurred. *See Apex Med. Corp. v. Resmed Ltd.*,

IPR2013-00512, Paper 39, at 2-3 (P.T.A.B. Sept. 12, 2014) (granting joint motion to terminate after the parties had fully briefed the matter because the Board "has not yet decided the merits of this proceeding, and the record is not yet closed."); *Lam Research Corp. v. Flamm*, IPR2015-01764, Paper 27, at 5 (P.T.A.B. Dec. 15, 2016) (granting motion to terminate after all briefing and oral argument when statutory deadline for rendering final written decision is within two months of motion).

Finally, because the Board has not decided the merits of the proceeding, the expected normal course is to terminate the proceeding. Office Patent Trial Practice Guide, 77 Fed. Reg. at 48,768 ("The Board expects that a proceeding will terminate after the filing of a settlement agreement, unless the Board has already decided the merits of the proceeding.").

B. Written Agreement

The parties represent that their entire agreement in connection with the termination of this proceeding is embodied in the Agreement, which has been made in writing. There are no collateral agreements or understandings made in connection with, or in contemplation of, the termination of this *inter partes* review. The Agreement has been filed for access by the "Parties and Board Only" due to the highly sensitive business confidential information it contains. The parties desire that the Agreement be maintained as business confidential information under 35

U.S.C. § 317(b) and 37 C.F.R. § 42.74(c), and a separate joint request for such is being filed concurrently.

IV. CONCLUSION

Petitioner and Patent Owner respectfully request that the Board grant the parties' Joint Motion to Terminate this proceeding in its entirety.

Respectfully submitted,

/Jason A. Fitzsimmons/

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CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that true and correct copies of the foregoing **JOINT MOTION TO TERMINATE *INTER PARTES* REVIEW** and **Exhibit 1100** were served electronically via e-mail on March 19, 2021 on the following counsel for Patent Owner:

Timothy Devlin (Lead Counsel)
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UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner

Case IPR2020-00445
U.S. Patent No. 6,581,565

JOINT REQUEST TO FILE AGREEMENT AS BUSINESS CONFIDENTIAL INFORMATION PURSUANT TO 35 U.S.C. § 317(b)

Mail Stop "PATENT BOARD"
Patent Trial and Appeal Board
United States Patent and Trademark Office
P.O. Box 1450
Alexandria, VA 22313-1450

Pursuant to 35 U.S.C. § 317(b) and 37 C.F.R. § 42.74(c), Petitioner Volkswagen Group of America, Inc. ("VWGoA" or "Petitioner") and Patent Owner Michigan Motor Technologies LLC ("MMT" or "Patent Owner") hereby jointly request that a true copy of their agreement, filed concurrently herewith as Exhibit 1100 be treated as business confidential information and be kept separate from the file of this *inter partes* review (IPR) proceeding. Concurrently with the filing of this Request, Petitioner and Patent Owner are filing a Joint Motion to Terminate this *inter partes* review due to the agreement between the parties.

The parties consider the agreement to contain highly sensitive business confidential information that would substantially harm their business interests if publicly disclosed. As such, the parties hereby jointly request that the agreement be kept as a separate paper to be made available only under the provisions of 35 U.S.C § 317(b) and 37 C.F.R. § 42.74(c). The agreement has been filed for access by the "Parties and Board Only." The parties further jointly request that the Board order that in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the agreement, that any such written request be served upon the parties on the day the written request is provided to the Board.

Respectfully submitted,

/Jason A. Fitzsimmons/

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Date: March 19, 2021

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Respectfully submitted,

/s/ Timothy Devlin
Timothy Devlin (Reg. 41,706)
DEVLIN LAW FIRM LLC
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Date: March 18, 2021

1526 Gilpin Ave.
Wilmington, DE 19806
(302) 449-9010

CERTIFICATE OF SERVICE (37 C.F.R. § 42.6(e))

The undersigned hereby certifies that true and correct copies of the foregoing **JOINT REQUEST TO FILE AGREEMENT AS BUSINESS CONFIDENTIAL INFORMATION UNDER 35 U.S.C. § 317(b)** was served electronically via e-mail on March 19, 2021, on the following counsel for Patent Owner:

Timothy Devlin (Lead Counsel)
Robyn T. Williams (Backup Counsel)
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Respectfully submitted,

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Paper 22
Date: April 20, 2021

UNITED STATES PATENT AND TRADEMARK OFFICE

BEFORE THE PATENT TRIAL AND APPEAL BOARD

VOLKSWAGEN GROUP OF AMERICA, INC.,
Petitioner,

v.

MICHIGAN MOTOR TECHNOLOGIES LLC,
Patent Owner.

IPR2020-00444
Patent 6,581,565 B2

Before NEIL T. POWELL, SCOTT B. HOWARD, and SEAN P. O'HANLON, *Administrative Patent Judges*.

POWELL, *Administrative Patent Judge*.

TERMINATION
Due to Settlement After Institution of Trial
35 U.S.C. § 317; 37 C.F.R. § 42.74

On March 19, 2021, with Board authorization, Petitioner and Patent Owner filed a joint motion to dismiss the petition and terminate the above-captioned proceeding. Paper 20. Along with the motion, the parties filed a copy of a Confidential Agreement (Ex. 1100), as well as a joint request to treat the Confidential Agreement as business confidential information (Paper 21). *See* 37 C.F.R. § 42.74(c) (a party to a settlement may request that the settlement agreement be treated as business confidential and be kept separate from the patent file).

Under 35 U.S.C. § 317(a), "[a]n inter partes review instituted under this chapter shall be terminated with respect to any petitioner upon the joint request of the petitioner and the patent owner, unless the Office has decided the merits of the proceeding before the request for termination is filed." It is also provided in 35 U.S.C. § 317(a) that if no petitioner remains, the Office may terminate the *inter partes* review.

In this case, the Board issued a decision to institute *inter partes* review. Paper 12. However, the Board has not yet decided the merits of the proceeding or entered a final written decision. In the joint motion, the parties state that they have settled their dispute and they jointly request termination of this *inter partes* review. Paper 20, 1. Accordingly, in the circumstances present here, we determine that it is appropriate to terminate the proceeding. 37 C.F.R. § 42.71(a). This Order does not constitute a final written decision pursuant to 35 U.S.C. § 318(a).

The parties also ask that the Confidential Agreement be treated as business confidential information and be kept separate from the file of the challenged patents. Paper 21. We determine that it is appropriate to treat the Confidential Agreement as business confidential information, and to

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10817

keep that document separate from the files of the challenged patents, pursuant to 37 C.F.R. § 42.74(c).

The parties further ask the Board to order that, "in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the [Confidential Agreement], that any such written request be served upon the parties on the day the written request is provided to the Board." Paper 21, 1. The statutory and regulatory provisions governing access to such settlement agreements stand on their own, and therefore, we deny the parties' request here.

For the foregoing reasons, it is:

ORDERED that the joint motion is *granted* and this proceeding is terminated with respect to Petitioner and Patent Owner;

FURTHER ORDERED that the joint request that the Confidential Agreement (Ex. 1100) be treated as business confidential information, to be kept separate from the patent file, is *granted* in this proceeding; and

FURTHER ORDERED that the joint request that the Board order that, in the event a person or entity makes a written request for access to the Confidential Agreement, that any such written request be served upon the parties on the day the written request is provided to the Board, is *denied* in this proceeding.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10818

IPR2020-00444
Patent 6,581,565 B2

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PATENT OWNER:

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PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10819

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Paper 22
Date: April 20, 2021

UNITED STATES PATENT AND TRADEMARK OFFICE

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POWELL, *Administrative Patent Judge*.

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PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10821

keep that document separate from the files of the challenged patents, pursuant to 37 C.F.R. § 42.74(c).

The parties further ask the Board to order that, "in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the [Confidential Agreement], that any such written request be served upon the parties on the day the written request is provided to the Board." Paper 21, 1. The statutory and regulatory provisions governing access to such settlement agreements stand on their own, and therefore, we deny the parties' request here.

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PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10822

IPR2020-00444
Patent 6,581,565 B2

PETITIONER:

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Paper 22
Date: April 20, 2021

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The parties also ask that the Confidential Agreement be treated as business confidential information and be kept separate from the file of the challenged patents. Paper 21. We determine that it is appropriate to treat the Confidential Agreement as business confidential information, and to

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10825

keep that document separate from the files of the challenged patents, pursuant to 37 C.F.R. § 42.74(c).

The parties further ask the Board to order that, "in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the [Confidential Agreement], that any such written request be served upon the parties on the day the written request is provided to the Board." Paper 21, 1. The statutory and regulatory provisions governing access to such settlement agreements stand on their own, and therefore, we deny the parties' request here.

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ORDERED that the joint motion is *granted* and this proceeding is terminated with respect to Petitioner and Patent Owner;

FURTHER ORDERED that the joint request that the Confidential Agreement (Ex. 1100) be treated as business confidential information, to be kept separate from the patent file, is *granted* in this proceeding; and

FURTHER ORDERED that the joint request that the Board order that, in the event a person or entity makes a written request for access to the Confidential Agreement, that any such written request be served upon the parties on the day the written request is provided to the Board, is *denied* in this proceeding.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10826

IPR2020-00445
Patent 6,581,565 B2

PETITIONER:

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PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10827

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Paper 22
Date: April 20, 2021

UNITED STATES PATENT AND TRADEMARK OFFICE

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35 U.S.C. § 317; 37 C.F.R. § 42.74

On March 19, 2021, with Board authorization, Petitioner and Patent Owner filed a joint motion to dismiss the petition and terminate the above-captioned proceeding. Paper 20. Along with the motion, the parties filed a copy of a Confidential Agreement (Ex. 1100), as well as a joint request to treat the Confidential Agreement as business confidential information (Paper 21). *See* 37 C.F.R. § 42.74(c) (a party to a settlement may request that the settlement agreement be treated as business confidential and be kept separate from the patent file).

Under 35 U.S.C. § 317(a), "[a]n inter partes review instituted under this chapter shall be terminated with respect to any petitioner upon the joint request of the petitioner and the patent owner, unless the Office has decided the merits of the proceeding before the request for termination is filed." It is also provided in 35 U.S.C. § 317(a) that if no petitioner remains, the Office may terminate the *inter partes* review.

In this case, the Board issued a decision to institute *inter partes* review. Paper 12. However, the Board has not yet decided the merits of the proceeding or entered a final written decision. In the joint motion, the parties state that they have settled their dispute and they jointly request termination of this *inter partes* review. Paper 20, 1. Accordingly, in the circumstances present here, we determine that it is appropriate to terminate the proceeding. 37 C.F.R. § 42.71(a). This Order does not constitute a final written decision pursuant to 35 U.S.C. § 318(a).

The parties also ask that the Confidential Agreement be treated as business confidential information and be kept separate from the file of the challenged patents. Paper 21. We determine that it is appropriate to treat the Confidential Agreement as business confidential information, and to

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10829

keep that document separate from the files of the challenged patents, pursuant to 37 C.F.R. § 42.74(c).

The parties further ask the Board to order that, "in the event a person or entity makes a written request, as stated in 37 C.F.R. § 42.74(c)(1)-(2), for access to the [Confidential Agreement], that any such written request be served upon the parties on the day the written request is provided to the Board." Paper 21, 1. The statutory and regulatory provisions governing access to such settlement agreements stand on their own, and therefore, we deny the parties' request here.

For the foregoing reasons, it is:

ORDERED that the joint motion is *granted* and this proceeding is terminated with respect to Petitioner and Patent Owner;

FURTHER ORDERED that the joint request that the Confidential Agreement (Ex. 1100) be treated as business confidential information, to be kept separate from the patent file, is *granted* in this proceeding; and

FURTHER ORDERED that the joint request that the Board order that, in the event a person or entity makes a written request for access to the Confidential Agreement, that any such written request be served upon the parties on the day the written request is provided to the Board, is *denied* in this proceeding.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10830

IPR2020-00445
Patent 6,581,565 B2

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AO 120 (Rev. 08/10)

TO:	REPORT ON THE FILING OR DETERMINATION OF AN ACTION REGARDING A PATENT OR TRADEMARK
Mail Stop 8 Director of the U.S. Patent and Trademark Office P.O. Box 1450 Alexandria, VA 22313-1450	

In Compliance with 35 U.S.C. § 290 and/or 15 U.S.C. § 1116 you are hereby advised that a court action has been filed in the U.S. District Court For the Eastern District of Texas on the following

☐ Trademarks or ☑ Patents. (☐ the patent action involves 35 U.S.C. § 292.):

DOCKET NO.	DATE FILED	U.S. DISTRICT COURT
4:21-cv-512	7/2/2021	For the Eastern District of Texas

PLAINTIFF	DEFENDANT
MICHIGAN MOTOR TECHNOLOGIES LLC	TATA MOTORS LTD., and JAGUAR LAND ROVER NORTH AMERICA, LLC

	PATENT OR TRADEMARK NO.	DATE OF PATENT OR TRADEMARK	HOLDER OF PATENT OR TRADEMARK
1	U.S. 6,557,540 B1	5/6/2003	Michigan Motor Technologies LLC
2	U.S. 6,581,565 B2	6/24/2003	Michigan Motor Technologies LLC
3	U.S. 6,558,260 B1	7/8/2003	Michigan Motor Technologies LLC
4	U.S. 6,581,574 B1	6/24/2003	Michigan Motor Technologies LLC
5	U.S. 6,736,122 B2	5/18/2004	Michigan Motor Technologies LLC
6	U.S. 8,909,482 B2	12/19/2014	Michigan Motor Technologies LLC

In the above—entitled case, the following patent(s)/ trademark(s) have been included:

DATE INCLUDED	INCLUDED BY ☐ Amendment ☐ Answer ☐ Cross Bill ☐ Other Pleading	
PATENT OR TRADEMARK NO.	DATE OF PATENT OR TRADEMARK	HOLDER OF PATENT OR TRADEMARK
1		
2		
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In the above—entitled case, the following decision has been rendered or judgement issued:

DECISION/JUDGEMENT

CLERK	(BY) DEPUTY CLERK	DATE

Copy 1—Upon initiation of action, mail this copy to Director Copy 3—Upon termination of action, mail this copy to Director
Copy 2—Upon filing document adding patent(s), mail this copy to Director Copy 4—Case file copy

AO 120 (Rev. 08/10)

TO:	Mail Stop 8 Director of the U.S. Patent and Trademark Office P.O. Box 1450 Alexandria, VA 22313-1450	REPORT ON THE FILING OR DETERMINATION OF AN ACTION REGARDING A PATENT OR TRADEMARK

In Compliance with 35 U.S.C. § 290 and/or 15 U.S.C. § 1116 you are hereby advised that a court action has been filed in the U.S. District Court Northern District of Illinois on the following

☑ Trademarks or ☐ Patents. (☐ the patent action involves 35 U.S.C. § 292.):

DOCKET NO.	DATE FILED	U.S. DISTRICT COURT
22cv3804	7/22/2022	Northern District of Illinois
PLAINTIFF Michigan Motor Technologies, LLC		DEFENDANT Bayerische Motoren Werke AG et al

PATENT OR TRADEMARK NO.	DATE OF PATENT OR TRADEMARK	HOLDER OF PATENT OR TRADEMARK
1 See Attachments		
2		
3		
4		
5		

In the above—entitled case, the following patent(s)/ trademark(s) have been included:

DATE INCLUDED	INCLUDED BY ☐ Amendment ☐ Answer ☐ Cross Bill ☐ Other Pleading	
PATENT OR TRADEMARK NO.	DATE OF PATENT OR TRADEMARK	HOLDER OF PATENT OR TRADEMARK
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In the above—entitled case, the following decision has been rendered or judgement issued:

DECISION/JUDGEMENT

CLERK	(BY) DEPUTY CLERK	DATE
Thomas G. Bruton	J. Hollimon	7/26/2022

Copy 1—Upon initiation of action, mail this copy to Director Copy 3—Upon termination of action, mail this copy to Director
Copy 2—Upon filing document adding patent(s), mail this copy to Director Copy 4—Case file copy

EXHIBIT 1

US006557540B1

(12) **United States Patent**
Mianzo et al.

(10) **Patent No.: US 6,557,540 B1**
(45) **Date of Patent: May 6, 2003**

(54) **METHOD OF CALCULATING A VALVE TIMING COMMAND FOR AN ENGINE**

(75) Inventors: **Lawrence A. Mianzo**, Plymouth, MI (US); **Brett D. Collins**, Ypsilanti, MI (US); **Ibrahim Haskara**, Brownstown, MI (US)

(73) Assignee: **Visteon Global Technologies, Inc.**, Dearborn, MI (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **10/014,286**

(22) Filed: **Dec. 11, 2001**

(51) **Int. Cl.**[7] **F01L 1/34**
(52) **U.S. Cl.** **123/678**; 123/90.15; 701/103
(58) **Field of Search** 123/678, 90.15, 123/90.19; 701/103

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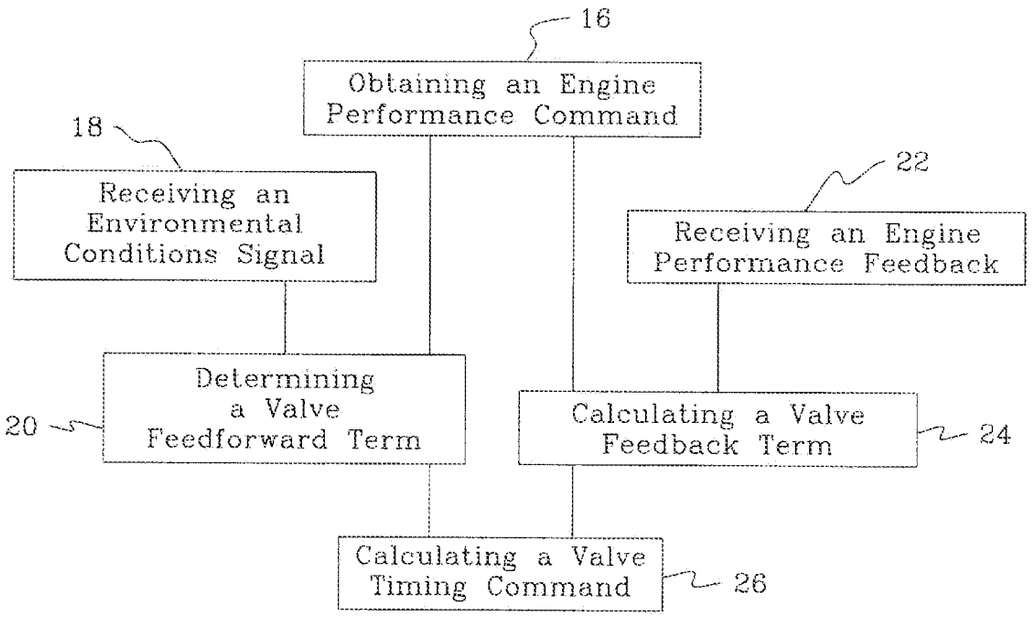
* cited by examiner

Primary Examiner—Erick Solis
(74) *Attorney, Agent, or Firm*—Brinks Hofer Gilson & Lione

(57) **ABSTRACT**

A method to calculate valve timing commands for an engine with variable valve timing is hereby disclosed. The method includes determining a valve feedforward term based on an engine performance command and an environmental conditions signal, calculating a valve feedback term based on the engine performance command and an engine performance feedback, and calculating a valve timing command based on the valve feedforward term and the valve feedback term.

16 Claims, 3 Drawing Sheets



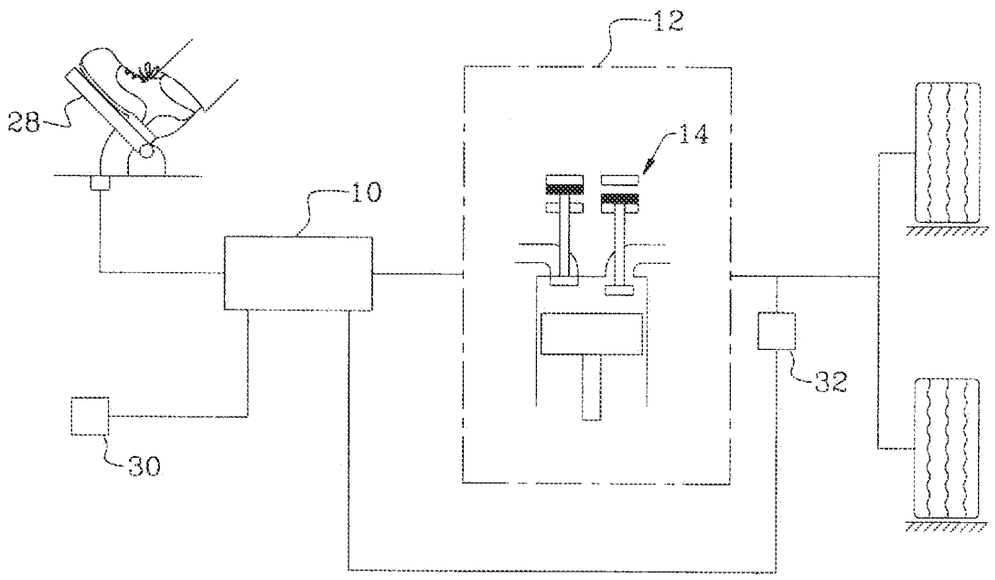


FIGURE 1

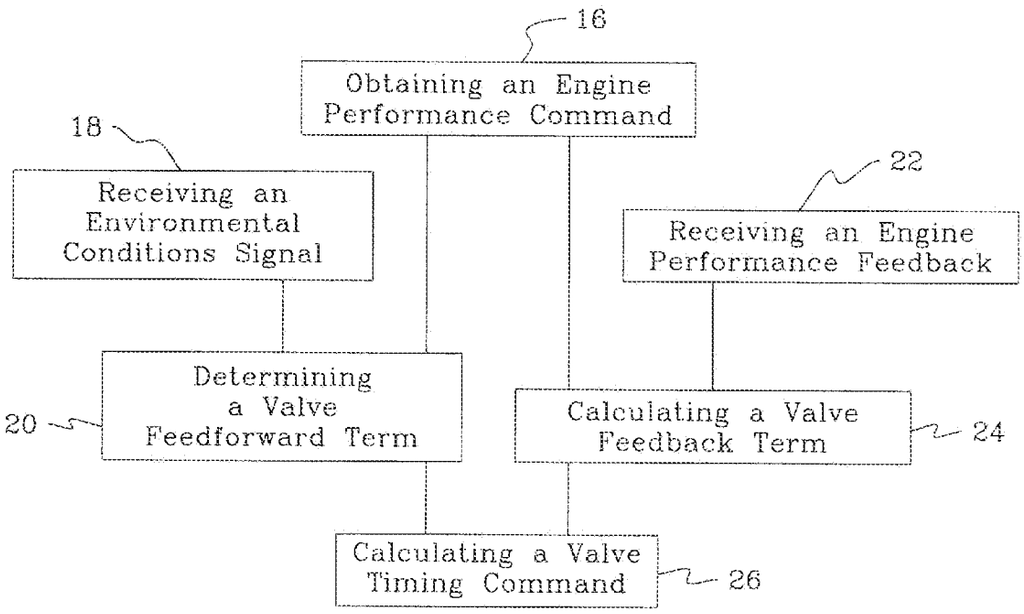

16
Obtaining an Engine Performance Command
18
Receiving an Environmental Conditions Signal
22
Receiving an Engine Performance Feedback
Determining a Valve Feedforward Term
20
Calculating a Valve Feedback Term
24
Calculating a Valve Timing Command
26

FIGURE 2

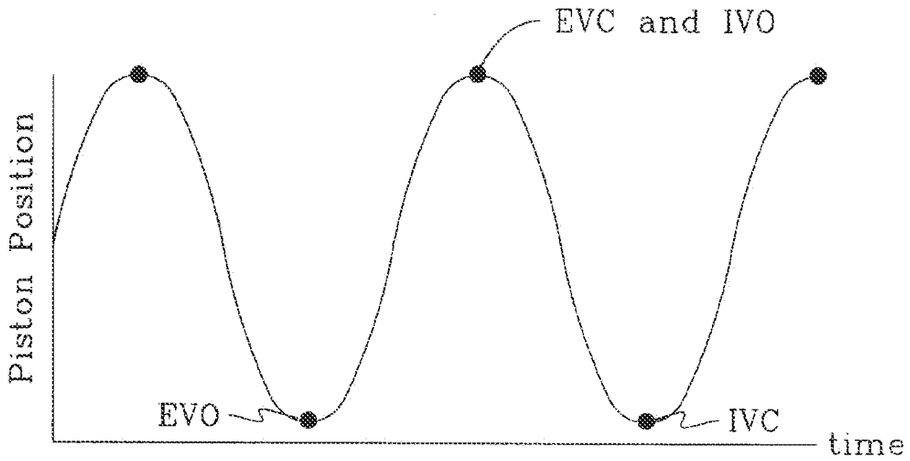


FIGURE 3A

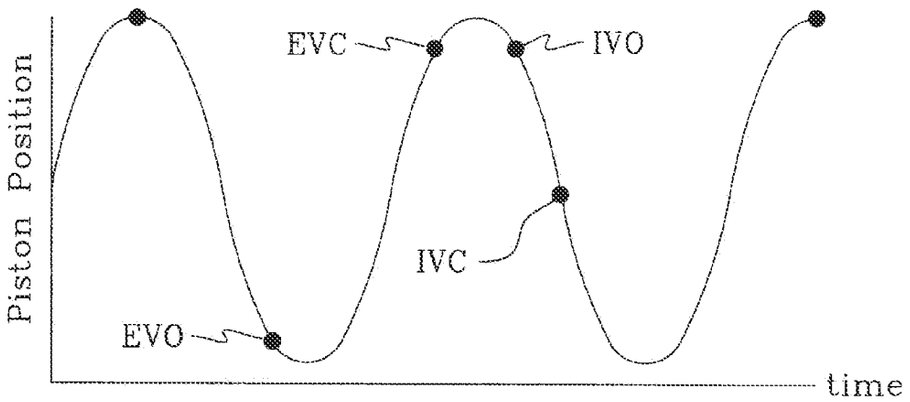


FIGURE 3B

METHOD OF CALCULATING A VALVE TIMING COMMAND FOR AN ENGINE

TECHNICAL FIELD

This invention relates generally to calculating commands for an engine and, more specifically, to calculating a valve timing command for an engine with variable timing valve actuators.

BACKGROUND

In gasoline engines of most vehicles, each cylinder of the engine cycles through four unique stages. In the first stage, an inlet valve opens and a piston draws air and fuel through the inlet valve and into the cylinder. The inlet valve closes and the piston reverses direction in the second stage to compress the air and fuel mixture. In the third stage, a spark combusts the mixture, which drives the piston (and powers the vehicle). An exhaust valve opens and the piston once again reverses direction, in the fourth stage, to push the combusted mixture through the exhaust valve and out of the cylinder.

The controlling of the inlet valve and the exhaust valve of each cylinder is a difficult task. The engine speed, which can exceed 6,000 rpm in most vehicles, dictates that the opening and closing of the inlet valve and the exhaust valve must be able to occur up to 50 times per second. In conventional engines, cams driven by the engine actuate the inlet valve and the exhaust valve. Modern research, however, has shown that fuel efficiency and power output of the engine may be optimized with an adjustment of the valve timing for a particular load on the engine. Some variable valve timing engines have been proposed, but the theoretical fuel efficiency and output power of these engines have not yet been reached.

BRIEF DESCRIPTION OF THE FIGURES

FIG. **1** is schematic of a vehicle having an engine controlled with the method of the preferred embodiment;

FIG. **2** is a flowchart of the method of the preferred embodiment; and

FIG. **3** is two timing charts of the valves of an engine.

DESCRIPTION OF THE PREFERRED METHODS

The following description of the two preferred method of the invention is not intended to limit the invention to these preferred methods, but rather to enable any person skilled in the art of variable valve timing control to make and use this invention.

As shown in FIG. **1**, the preferred method of the invention has been specifically created to be performed by a control unit **10** to calculate valve timing commands for an engine **12** with electromagnetic valve actuators **14**. The method, however, may be performed by any suitable device to calculate valve timing commands for any suitable engine with any suitable variable timing valves.

As shown in FIG. **2**, the preferred method of the invention has six principle actions: obtaining an engine performance command **16**; receiving an environmental conditions signal **18**; determining a valve feedforward term **20**; receiving an engine performance feedback **22**; calculating a valve feedback term **24**; and calculating a valve timing command **26** based on the valve feedforward term and the valve feedback term. The method performed by the control unit may, of course, include other suitable actions before, during, or after these principle actions.

The action of obtaining an engine performance command **16** preferably includes receiving a vehicle performance command from a driver. Preferably, the vehicle performance command is received from the foot of a driver with the use of a conventional pedal **28**, as shown in FIG. **1**. Alternatively, the vehicle performance command could be received from the driver with the use of any suitable device. The action of obtaining an engine performance command also preferably includes deriving the engine performance command from the vehicle performance command. The engine performance command is preferably based on the vehicle performance command, but may alternatively be based on additional suitable factors, such as a traction control signal or a cruise control signal. The engine performance command is preferably a desired engine torque and, for this reason, the engine performance command may be thought of as an engine torque command. The engine performance command, however, may alternatively be another suitable variable, such as a desired engine acceleration.

The action of receiving an environmental conditions signal **18** preferably includes receiving an environmental conditions signal from an environmental sensor **30** in the vehicle. The environmental sensor **30** preferably senses the ambient temperature outside the vehicle and communicates this data to the control unit **10**, which uses the data to determine the valve feedforward term. Other information, such as the ambient pressure, may be useful in the determination of the valve feedforward term. For this reason, the environmental sensor **30** may alternatively sense other suitable information. The environmental sensor **30** is preferably a conventional environmental sensor, but may alternatively be any suitable device.

Similarly, the action of receiving an engine performance feedback **22** preferably includes receiving the engine performance feedback from an engine sensor **32** in the vehicle. The engine sensor **32** preferably senses the engine speed and communicates this data to the control unit **10**, which uses the data to determine the valve feedforward term and the valve feedback term. Other engine measurables, such as engine torque data, may be useful in the determination of the terms. For this reason, the engine sensor **32** may alternatively sense other suitable information. The engine sensor **32** is preferably a conventional engine sensor, but may alternatively be any suitable sensor.

The action of determining a valve feedforward term **20** preferably includes determining the valve feedforward term based on the engine torque command, the ambient temperature data, and the engine speed data. The determination, however, may be based on other suitable factors, such as engine torque data, air-fuel ratio data, engine combustion stability data, or ambient pressure data. The control unit **10** preferably includes a look-up table, which has been optimized for fuel efficiency, power output, and engine emissions based on various engine torque commands, various ambient temperature data, and various engine speed data (or engine torque data). The control unit **10** may alternatively be programmed to perform a real-time optimization of the fuel efficiency, power output, and engine emission (or any other suitable measurement) based on the engine torque command, the ambient temperature data, and the engine speed data (or any other suitable commands and measurables).

The action of calculating the valve feedback term **24** preferably includes comparing the engine performance com-

mand and the engine performance feedback. By the definition of the term, the valve feedback term functions to compare the input with the output and to calculate a correction term based on the difference, if any. The comparison and the calculation are preferably accomplished by the control unit **10**, but may alternatively be accomplished by a suitable separate device.

As shown in FIG. **3**A, the position of the piston in the cylinder can be traced as a sinusoidal wave over a time period. The events of the opening of the exhaust valve ("EVO"), the closing of the exhaust valve ("EVC"), the opening of the inlet valve ("IVO"), and the closing of the inlet valve ("IVC") can be placed on this sinusoidal wave. As shown in FIG. **3**B, the events of the EVO, the EVC, the IVO, and the IVC may be shifted within the time period (note that the shift in the EVC and the IVO preferably mirror each another, but may alternatively be separately controlled). The adjustment of the timing of the EVO, the EVC/IVO, and the IVC when used partially, separately, or together may provide the desired fuel efficiency, power output, and emissions from the engine **12** of the vehicle. The action of calculating a valve timing command **26** preferably includes calculating an EVO command, an EVC/IVO command, and an IVC command. The control unit **10**, of course, may alternatively include other suitable parameters for the control of the variable timing valves.

The second preferred method of the invention includes the principle actions of the first preferred method and the additional principle action of receiving fuel conversion data and engine emissions data. These preferably include receiving the fuel conversion data and engine emissions data from suitable emission sensors (not shown) in the cylinder or the exhaust port of the engine **12**. The emission sensors preferably sense the amount of NO_x in the exhaust and communicates this data to the control unit **10**. The control unit **10** preferably uses this information to modify the output value from the look-up table, but may alternatively use this information to continually adjust the values in the look-up table. The control unit **10** may alternatively use this information as another factor in the determination of the valve feedback term. The emission sensors are preferably conventional emission sensors, but may alternatively be any suitable sensor.

As any person skilled in the art of variable valve engine control will recognize from the previous detailed description and from the figures and claims, modifications and changes can be made to the preferred methods without departing from the scope of this invention defined in the following claims.

We claim:

1. A method for calculating a valve timing command for an engine of a vehicle, comprising:

obtaining an engine performance command;

receiving an environmental conditions signal;

determining a valve feedforward term based on the engine performance command and the environmental conditions signal;

receiving an engine performance feedback;

calculating a valve feedback term based on the engine performance command and the engine performance feedback; and

calculating a valve timing command based on the valve feedforward term and the valve feedback term.

2. The method of claim **1** wherein said obtaining an engine performance command includes receiving a vehicle performance command from a driver of the vehicle and deriving the engine performance command based on the vehicle performance command.

3. The method of claim **1** wherein said receiving an environmental conditions signal includes receiving ambient temperature data.

4. The method of claim **3** wherein said determining a valve feedforward term includes determining the valve feedforward term based on the ambient temperature data.

5. The method of claim **4** wherein said determining a valve feedforward term includes determining the valve feedforward term based on the engine performance feedback.

6. The method of claim **5** wherein said determining a valve feedforward term includes referencing a look-up table with the engine performance command, the ambient temperature, and the engine performance feedback.

7. The method of claim **1** wherein said receiving an engine performance feedback includes receiving engine speed data.

8. The method of claim **1** wherein said obtaining an engine performance command includes obtaining an engine torque command; and wherein receiving an engine performance feedback includes receiving engine torque data.

9. The method of claim **8** wherein calculating a valve feedback term includes calculating a valve feedback term based on the engine torque command and the engine torque data.

10. The method of claim **1** wherein said calculating a valve timing command includes calculating an EVO command, an EVC/IVO command, and an IVC command.

11. The method of claim **1** further comprising receiving fuel conversion data.

12. The method of claim **11** wherein said determining a valve feedforward term includes determining a valve feedforward term based on the fuel conversion data.

13. The method of claim **11** wherein said calculating a valve feedback term includes calculating a valve feedback term based on the fuel conversion data.

14. The method of claim **1** further comprising receiving engine emissions data.

15. The method of claim **14** wherein said determining a valve feedforward term includes determining a valve feedforward term based on the engine emissions data.

16. The method of claim **11** wherein said calculating a valve feedback term includes calculating a valve feedback term based on the engine emissions data.

* * * * *

EXHIBIT 3

US006581565B2

(12) **United States Patent**
Heslop et al.

(10) **Patent No.: US 6,581,565 B2**
(45) **Date of Patent: Jun. 24, 2003**

(54) **ENGINE TORQUE CONTROLLER**

(75) Inventors: **Garon Nigel Heslop**, Billericay (GB); **Jon Dixon**, Maldon (GB); **Richard Stephen Blachford**, Cambridge (GB)

(73) Assignee: **Visteon Global Technologies, Inc.**, Dearborn, MI (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **10/191,641**

(22) Filed: **Jul. 9, 2002**

(65) **Prior Publication Data**

US 2003/0015169 A1 Jan. 23, 2003

(30) **Foreign Application Priority Data**

Jul. 23, 2001 (EP) 01306301

(51) **Int. Cl.**[7] **F02D 41/14**; F02B 17/00

(52) **U.S. Cl.** **123/295**; 123/406.23; 123/406.47; 123/478

(58) **Field of Search** 123/295, 350, 123/352, 399, 406.23, 406.47, 406.55, 478, 480, 488

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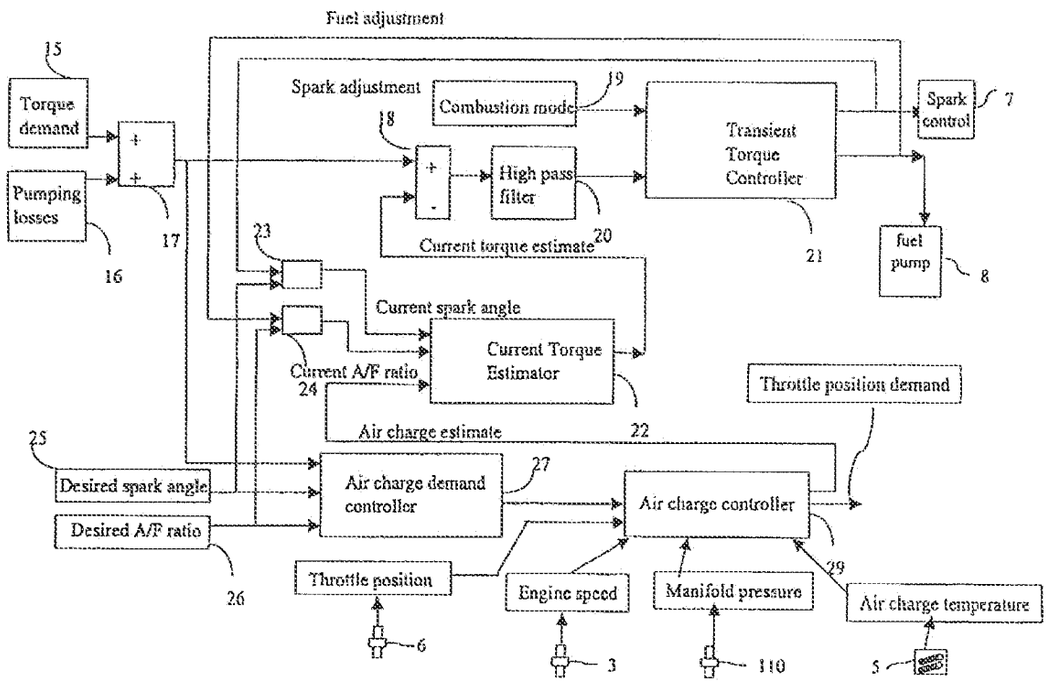
* cited by examiner

Primary Examiner—Tony M. Argenbright
(74) *Attorney, Agent, or Firm*—Brinks Hofer Gilson & Lione

(57) **ABSTRACT**

This invention relates to an engine torque controller for spark ignition internal combustion engines and more specifically for direct injection engines. The invention provides a torque controller and a method of controlling torque for an engine in which torque is controlled in dependence upon a filtered difference signal where the filtered difference signal is the difference between a desired torque signal and a signal representing an estimate of the current torque.

7 Claims, 3 Drawing Sheets



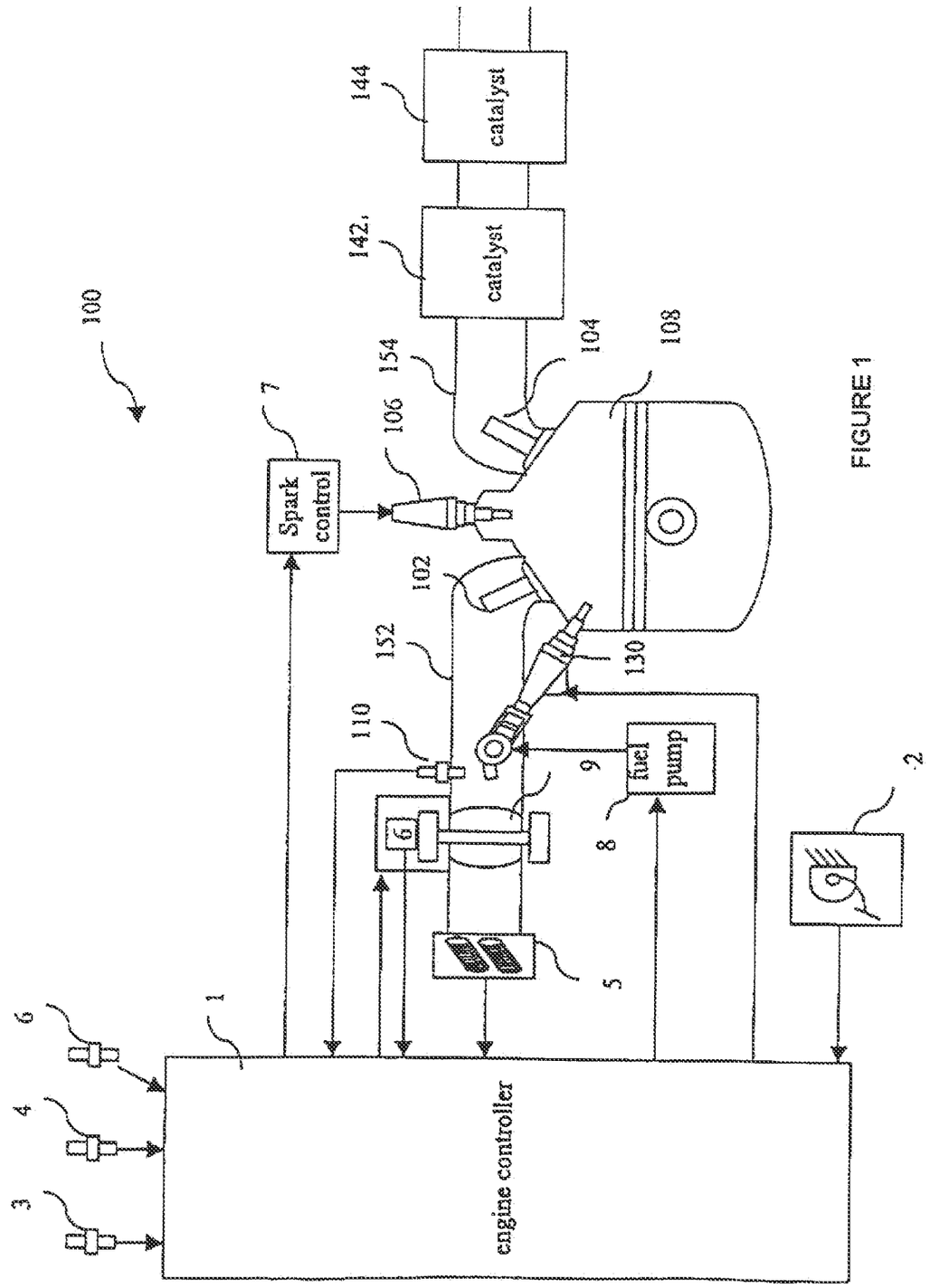

100
144
catalyst
142,
catalyst
154
104
108
7
106
Spark control
102
152
130
110
9
fuel pump
2
8
6
5
1
6
4
3
engine controller

FIGURE 1

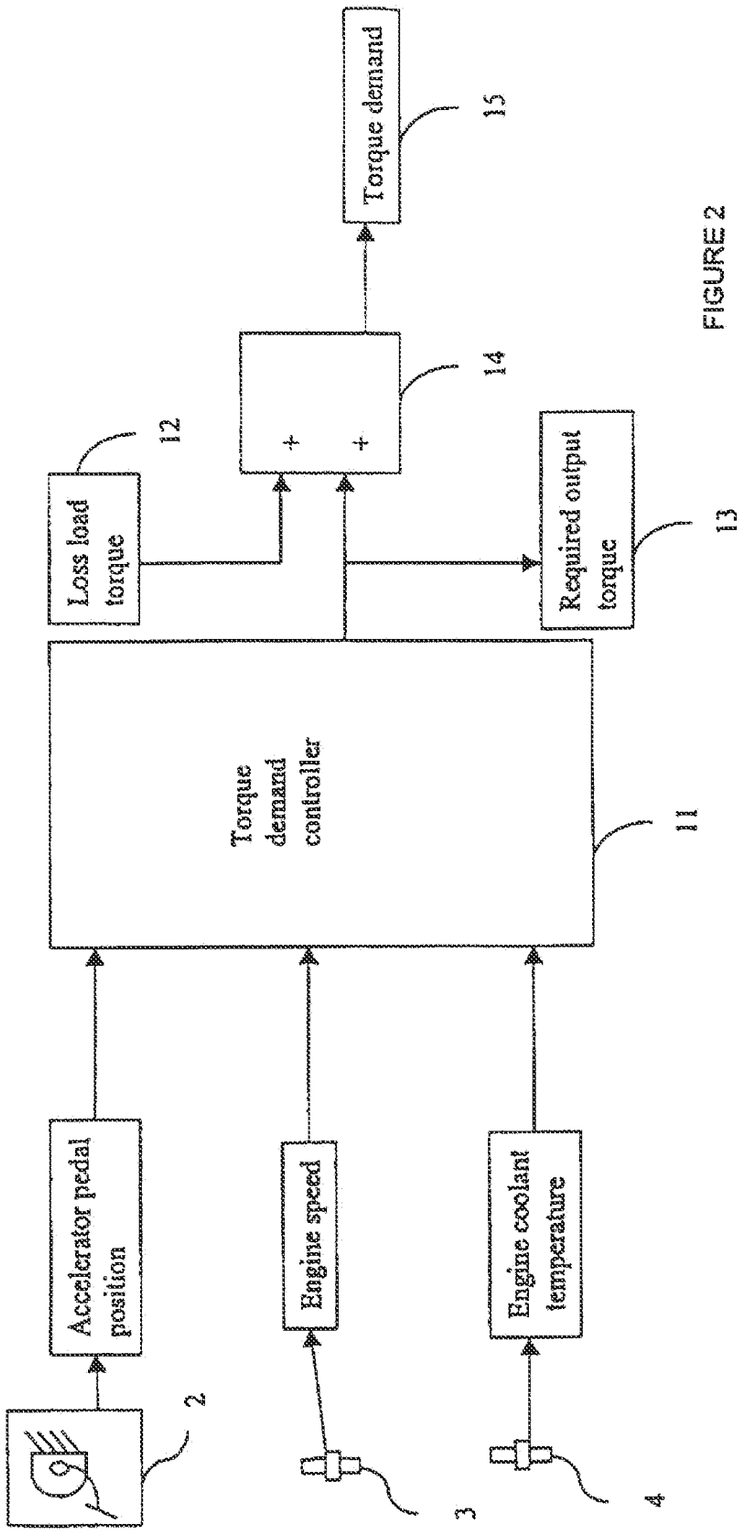

Torque demand
15
14
+
+
12
Loss load torque
Required output torque
13
Torque demand controller
11
Accelerator pedal position
Engine speed
Engine coolant temperature
2
3
4

FIGURE 2

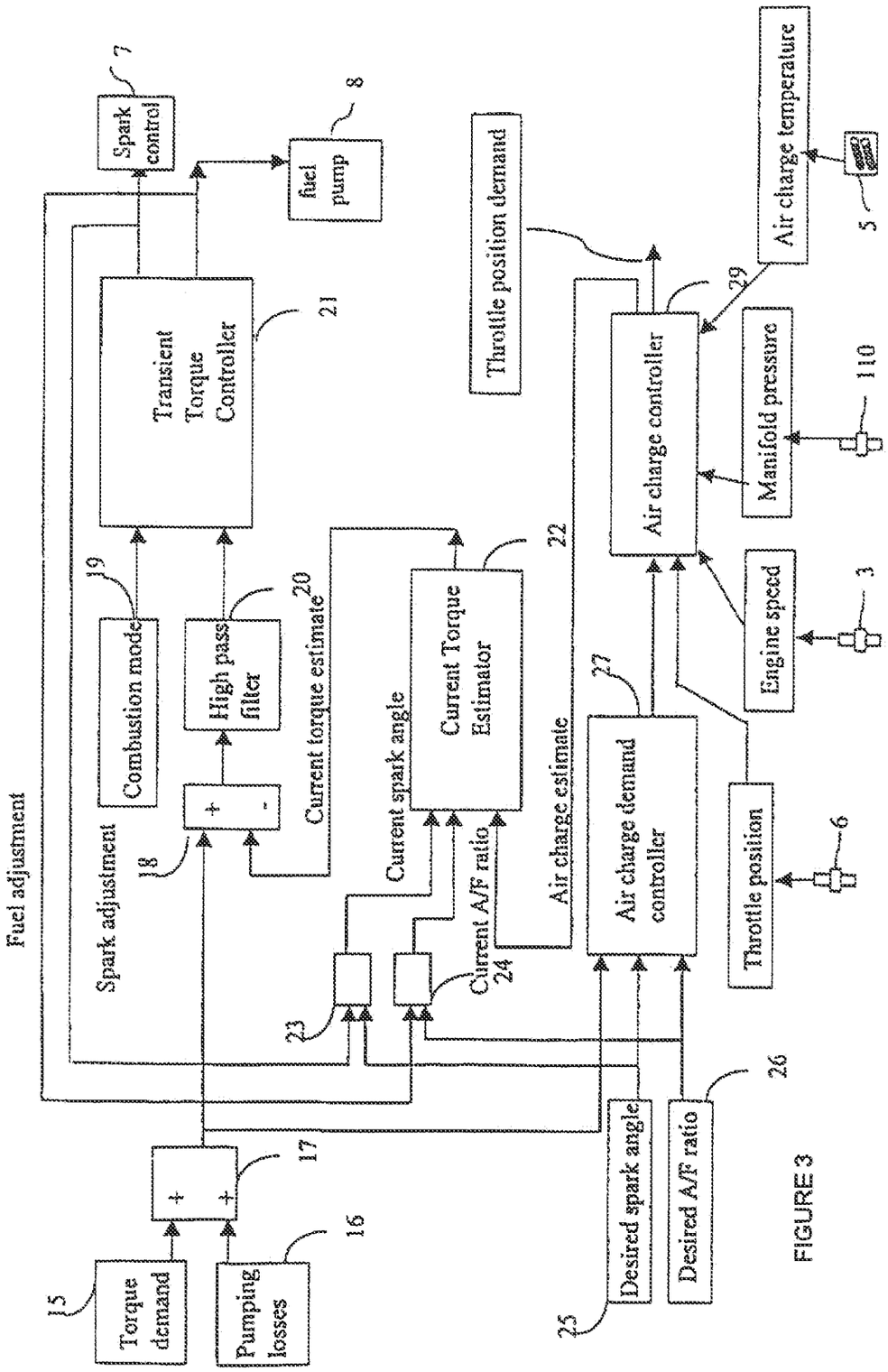

Fuel adjustment
Spark control
7
fuel pump
8
Transient Torque Controller
21
Throttle position demand
Air charge temperature
5
Combustion mode
19
High pass filter
20
Current torque estimate
Current Torque Estimator
22
Air charge controller
29
Manifold pressure
110
Spark adjustment
18
Current spark angle
Engine speed
3
Air charge estimate
27
Air charge demand controller
Current A/F ratio
24
Throttle position
6
23
Torque demand
15
Pumping losses
16
17
Desired spark angle
25
Desired A/F ratio
26

FIGURE 3

ENGINE TORQUE CONTROLLER

TECHNICAL FIELD

This invention relates to an engine torque controller for spark ignition internal combustion engines and more specifically for direct injection engines.

BACKGROUND

An engine torque controller is comprised of a torque demand controller and a torque producer. The torque demand controller determines a required target torque, in accordance with an accelerator pedal position, current engine speed, external loads and other factors. This determined torque is then used by the torque producer to produce the desired torque by controlling the spark angle and the air/fuel ratio.

Direct Injection Spark Ignition (DISI) engines inject fuel directly into cylinders where it is ignited by a spark from a spark plug. DISI engines operate in a stratified mode or a homogenous mode. When a DISI engine is in the stratified mode, the combustion chambers contain stratified layers having different air/fuel mixtures. The strata closest to the spark plug contains a stoichiometric mixture, which is a mixture in which the exact amount of air to combust the amount of fuel is present, i.e. when the combustion leaves no excess oxygen or unburned fuel. Subsequent strata contain progressively leaner mixtures. Operation in a stratified mode occurs at lower speeds and lower load conditions.

When the engine is in a homogenous mode, a homogenous mixture of air and fuel is introduced into the combustion chamber. Homogenous operation may be either lean of stoichiometry (i.e. higher air/fuel ratio), at stoichiometry, or rich of stoichiometry (i.e. lower air fuel ratio).

In engine torque controllers for DISI engines, when the engine is operating in stratified mode, spark angle has little influence on the torque produced. The torque producer modifies the air/fuel in order to control the torque produced. Conversely, when the engine is operating in homogenous mode, the air/fuel ratio is controlled tightly in order to maintain correct operation of the catalytic converter to reduce noxious emission. The torque producer modifies the timing of the spark ignition in order to control the torque produced.

A problem occurs in either of these modes of operation when there is a steady state error between the torque demanded and the estimate of the torque produced. In the stratified mode if a fuel adjustment occurs due to such a steady state error then the air/fuel ratio will not be ideal and fuel economy will suffer and performance of the catalytic converter will deteriorate. In the homogenous mode, if the timing of the spark ignition is altered due to such a steady state torque error then the fuel economy will once again suffer and the engine is more likely to stall when a load is imposed. Therefore, there is a need for a method of correction for a steady state error between the torque demanded and an estimate of the torque produced.

SUMMARY

In a preferred embodiment, the engine is a direct injection spark ignition engine and the transient torque controller is arranged to receive a combustion mode signal indicating whether the engine is operating in a stratified mode or a homogeneous mode. If the signal indicates that the engine is operating in the stratified mode then the fuel and spark controller is arranged to control the fuel adjustment signal. If the signal indicates that the engine is operating in the homogeneous mode then the fuel and spark controller is arranged to control the spark adjustment signal.

Preferably, the controller also has an air charge controller arranged to receive an air charge demand signal, a throttle position signal, an engine speed signal, a manifold pressure signal and an air charge temperature signal and arranged to output the estimated air charge signal. Preferably, there is also an air charge demand controller arranged to receive the desired torque signal, a desired spark angle signal and a desired air/fuel ratio signal and to output the air charge demand signal.

According to another aspect of the invention, there is provided a method of controlling torque for an engine. The method includes estimating a current torque signal in dependence upon a received current spark angle signal, a received current air/fuel ratio signal and a received estimated air charge signal, comparing the estimated current torque signal with a desired torque signal to provide a difference signal, and filtering low frequency components from the difference signal. Finally, controlling a fuel adjustment signal and a spark adjustment signal in dependence upon the filtered difference signal.

These and other aspects and advantages of the present invention will become apparent upon reading the following detailed description of the invention in combination with the accompanying figures.

BRIEF DESCRIPTION OF THE FIGURES

FIG. **1** is a block diagram illustrating part of an engine and an engine controller, in accordance with the present invention;

FIG. **2** is a block diagram of a torque demand controller, in accordance with the present invention; and

FIG. **3** is a block diagram of a torque producer, in accordance with the present invention.

DETAILED DESCRIPTION

FIG. **1** illustrates an embodiment of a direct injection spark ignition engine **100** which has an engine controller **1**. The engine controller **1** receives signals from an accelerator pedal and sensor assembly **2**, an engine speed sensor **3**, an engine temperature sensor **4**, an air charge temperature sensor **5**, a manifold absolute pressure sensor **110** and a throttle position indicator **6**.

A fuel injector **130** injects fuel directly into a combustion chamber **108**. The injected fuel mixes with an air charge which enters through an air intake valve **102** via an air intake manifold **152**. The air charge is controlled by a throttle **9** and the fuel injected is controlled by a fuel pump **8**. A spark control unit **7** controls a spark plug **106**, to generate a spark for ignition of the air/fuel mixture. Exhaust gases from the resulting combustion exit via an exhaust valve **104** into an exhaust manifold **154**. The exhaust manifold **154** has a three way catalytic converter **142** and a Nox trap/catalyst **144**.

FIG. **2** illustrates an embodiment of a torque demand controller **11** that is part of the engine controller **1**. The torque demand controller **11** calculates a required output torque signal **13**, based on an accelerator pedal position signal received from the accelerator pedal and sensor assembly **2**, an engine speed signal received from the engine speed sensor **3** and an engine temperature signal received from the engine temperature sensor **4**. A loss load torque signal **12**, which represents losses due to losses in the engine and powertrain system, is added to the required output torque signal **13** by an adder **14** to generate a torque demand signal **15**.

Referring now to FIG. **3**, an adder **17** receives as inputs the torque demand signal **15** and a pumping losses signal **16**, which represents losses due to the inherent losses in the engine cycle (i.e. due to the energy required to draw air in and to push out exhaust gases). The adder **17** outputs a desired torque signal.

An air charge demand controller **27** receives as inputs the desired torque signal, a desired spark angle signal **25** and a desired air fuel ratio signal **26**. The desired spark angle signal **25** and the desired air fuel ratio signal **26** are calculated elsewhere in the engine controller **1** and depend upon signals such as engine speed, engine load and engine temperature.

The air charge demand controller **27** generates an air charge demand signal that is received by an air charge controller **29**. The air charge controller **29** also receives as inputs a signal indicating throttle position that is received from the throttle position indicator **6** (FIG. **1**), the engine speed sensor **3**, the manifold absolute pressure sensor **110** and the air charge temperature meter **5**. The air charge controller **29** generates a signal indicating desired throttle position that is sent to throttle **9** (FIG. **1**) and a signal representing an estimate of the air charge.

The estimated air charge may be different from the air charge demanded by the air charge demand controller **27** due to delays in the engine **100**, such as the time taken for the throttle **9** to move, the time taken for the pressure in the air intake manifold **152** to rise or fall, or any errors in position of the throttle. The air charge estimate signal is sent to a current torque estimator **22**.

The current torque estimator **22** uses the air charge estimate signal, together with a signal representing the current spark angle and a signal representing the current air/fuel ratio to generate a signal representing an estimate of the current torque.

The estimate of the current torque is compared to the desired torque signal by a comparator **18** to generate an error signal which is then filtered by a high pass filter **20**. The resulting filtered error signal is used by a transient torque controller **21** to generate signals for temporarily adjusting the torque produced by the engine **100**.

A combustion mode signal **19**, which is produced elsewhere in the engine controller **1**, indicates whether the engine **100** is operating in a stratified mode or in a homogenous mode. If the engine **100** is operating in the stratified mode then a fuel adjustment signal is generated and sent to the fuel pump **8** in order to adjust the amount of fuel which is injected into the combustion chamber **108** by the fuel injector **130**. If the engine **100** is operating in the homogenous mode then a spark adjustment signal is generated and sent to the spark control unit **7** to adjust the timing of the ignition spark generated by the spark plug **106**.

The signal representing the current spark angle is calculated by a calculator **23** using the desired spark angle and any spark adjustment signal received from the transient torque controller **21**. The signal representing the current air fuel ratio is calculated by a calculator **24** using the desired air fuel ratio and any fuel adjustment signal received from the transient torque controller **21**. When the engine **100** is operating in stratified mode the current spark angle will be equal to the desired spark angle **25**. When the engine **100** is operating in homogenous mode the current air/fuel ratio will be equal to the desired air fuel ratio **26**.

As any person skilled in the art of systems and methods of controlling the torque output of an engine will recognize from the previous detailed description and from the figures and claims, modifications and changes can be made to the preferred embodiments of the invention without departing from the scope of this invention defined in the following claims.

What is claimed is:

1. A device for controlling a torque output of an engine, the device comprising:

a torque demand controller for generating a torque demand signal; and

a torque producer which receives the torque demand signal, the torque producer includes:

an estimator which receives as inputs a current spark angle signal, a current air/fuel ratio signal and an estimated air charge signal and outputs an estimated torque signal;

a comparator which receives as inputs the estimated torque signal and a desired torque signal and outputs a difference signal;

a high pass filter which receives as an input the difference signal and outputs a filtered difference signal in which low frequency components are absent; and

a transient torque controller which receives as an input the filtered difference signal and outputs a fuel adjustment signal and a spark adjustment signal.

2. The device in claim **1**, wherein the estimated air charge signal is produced by an air charge controller which receives as inputs an air charge demand signal, a throttle position signal, an engine speed signal, a manifold pressure signal and an air charge temperature signal.

3. The device in claim **2**, wherein the air charge demand signal is produced by an air charge demand controller which receives as inputs the desired torque signal, a desired spark angle signal and a desired air/fuel.

4. The device of claim **1**, in which the engine is a direct injection spark ignition engine, wherein the transient torque controller is arranged to receive a combustion mode signal indicating whether the engine is operating in a stratified mode or a homogeneous mode, and

wherein the transient torque controller is arranged to control the fuel adjustment signal when the combustion mode signal indicates that the engine is operating in the stratified mode, and

wherein the transient torque controller is arranged to control the spark adjustment signal when the combustion mode signal indicates that the engine is operating in the homogeneous mode.

5. The device in claim **4**, wherein the estimated air charge signal is produced by an air charge controller which receives as inputs an air charge demand signal, a throttle position signal, an engine speed signal, a manifold pressure signal and an air charge temperature signal.

6. The device in claim **5**, wherein the air charge demand signal is produced by an air charge demand controller which receives as inputs the desired torque signal, a desired spark angle signal and a desired air/fuel.

7. A method for controlling a torque output of an engine, the method comprising:

estimating a current torque signal from a received current spark angle signal, a received current air/fuel ratio signal and a received estimated air charge signal;

comparing the estimated current torque signal with a desired torque signal to output a difference signal;

filtering out a plurality of low frequency components from the difference signal; and

controlling a fuel adjustment signal and a spark adjustment signal in dependence upon a filtered difference signal.

* * * * *

UNITED STATES PATENT AND TRADEMARK OFFICE

CERTIFICATE OF CORRECTION

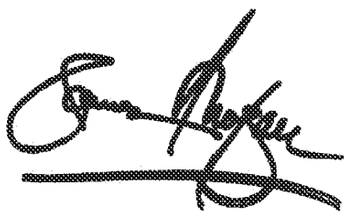
PATENT NO. : 6,581,565 B2
DATED : June 24, 2003
INVENTOR(S) : Garon N. Heslop et al.

It is certified that error appears in the above-identified patent and that said Letters Patent is hereby corrected as shown below:

Column 4,
Lines 30 and 53, after "air/fuel" insert -- ratio -- before "." (period).

Signed and Sealed this

Twenty-eighth Day of October, 2003



JAMES E. ROGAN
Director of the United States Patent and Trademark Office

EXHIBIT 5

US006581574B1

(12) **United States Patent**
Moran et al.

(10) **Patent No.: US 6,581,574 B1**
(45) **Date of Patent: Jun. 24, 2003**

(54) **METHOD FOR CONTROLLING FUEL RAIL PRESSURE**

(75) Inventors: **Kevin David Moran**, Trenton, MI (US); **Guoming George Zhu**, Novi, MI (US)

(73) Assignee: **Visteon Global Technologies, Inc.**, Dearborn, MI (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

(21) Appl. No.: **10/108,093**

(22) Filed: **Mar. 27, 2002**

(51) **Int. Cl.**[7] **F02M 37/04**
(52) **U.S. Cl.** **123/497**; 123/494
(58) **Field of Search** 123/497, 357, 123/456, 494; 417/53

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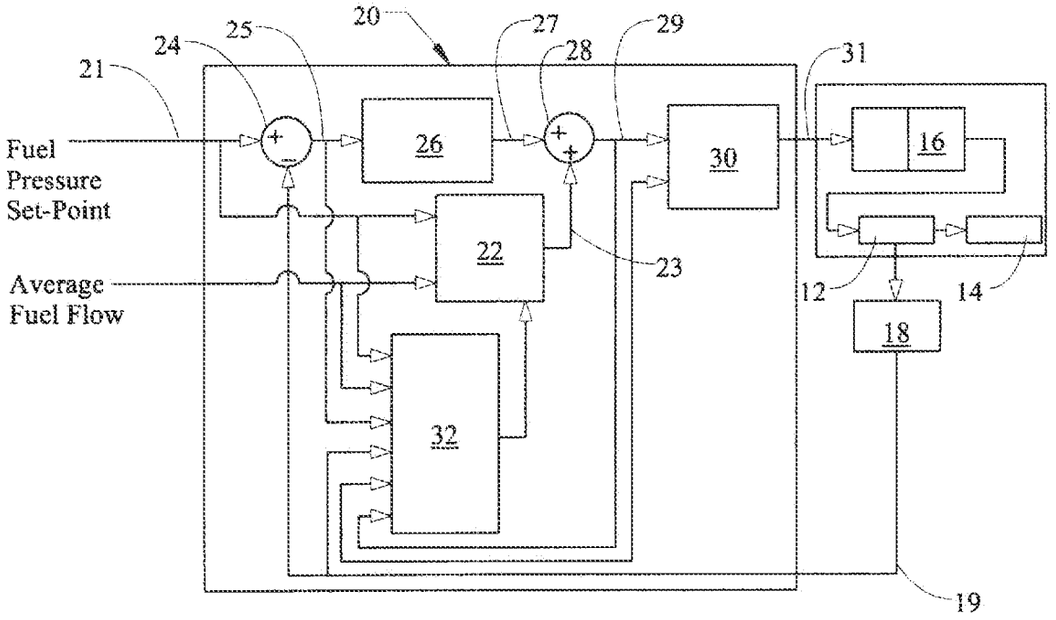
* cited by examiner

Primary Examiner—Carl S. Miller
(74) *Attorney, Agent, or Firm*—Brinks Hofer Gilson & Lione

(57) **ABSTRACT**

A method of controlling the fuel pressure within a fuel delivery system having a fuel pump that delivers fuel to a fuel rail of an internal combustion engine includes providing a set-point fuel pressure, generating a feed forward signal having a set of fuel pump motor control parameters based upon the set-point fuel pressure and the desired fuel flow rate, providing a pressure sensor to measure the fuel rail pressure, comparing the fuel rail pressure to the set-point fuel pressure and generating an error value based upon the difference between the fuel rail pressure and the set-point fuel pressure, providing a feed back controller adapted to receive the error value and to generate a feed back control signal, combining the feedback control signal and the feed forward control signal to generate a motor controller signal.

18 Claims, 3 Drawing Sheets



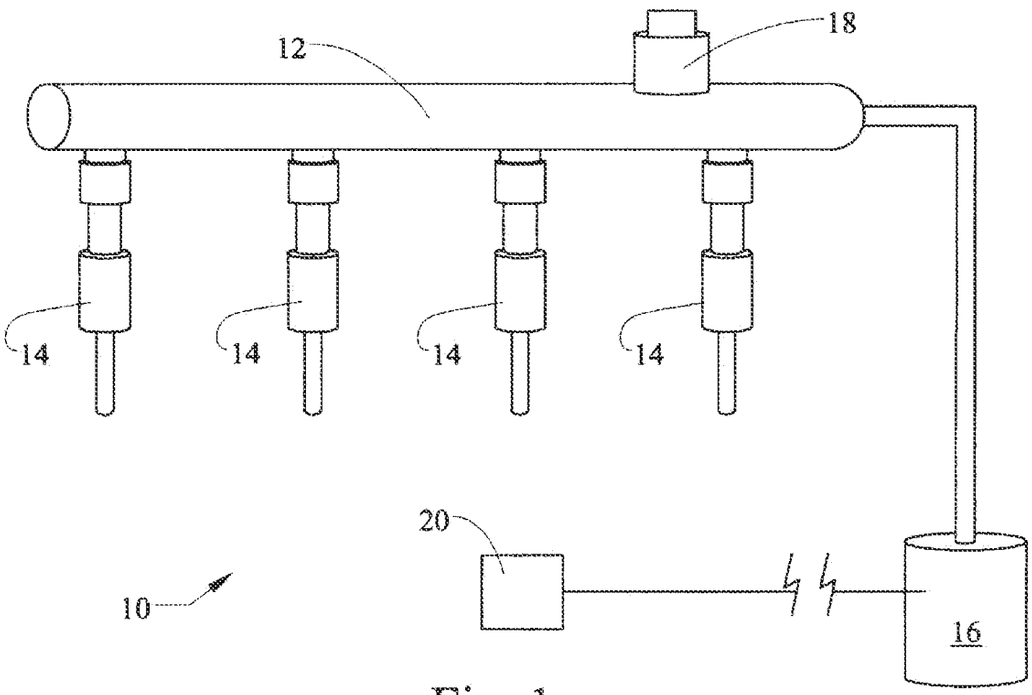


Fig. 1

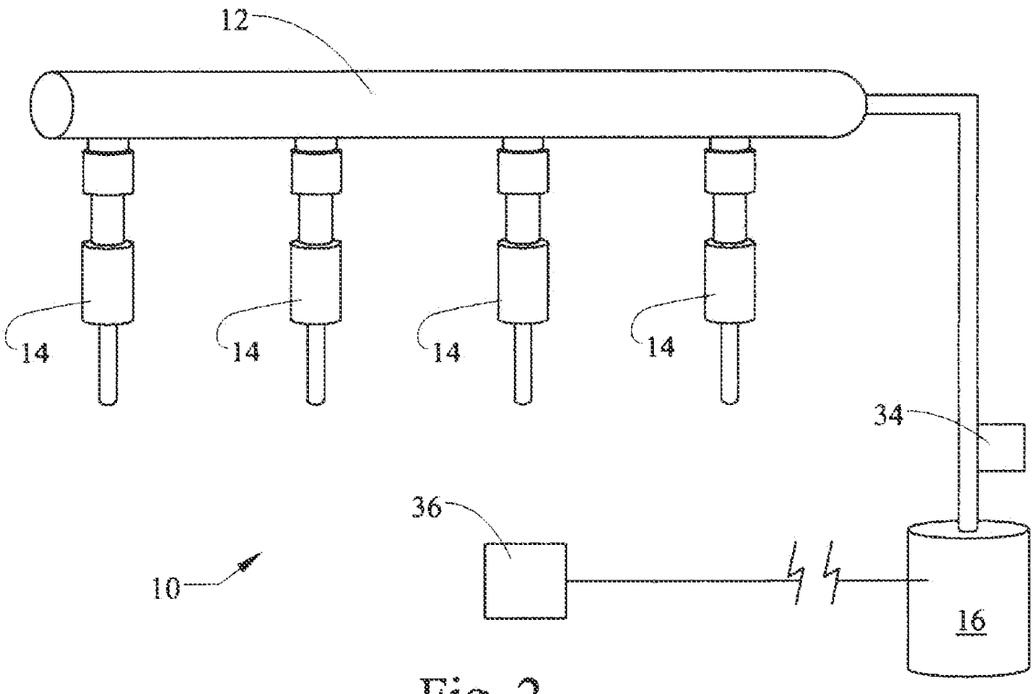


Fig. 2

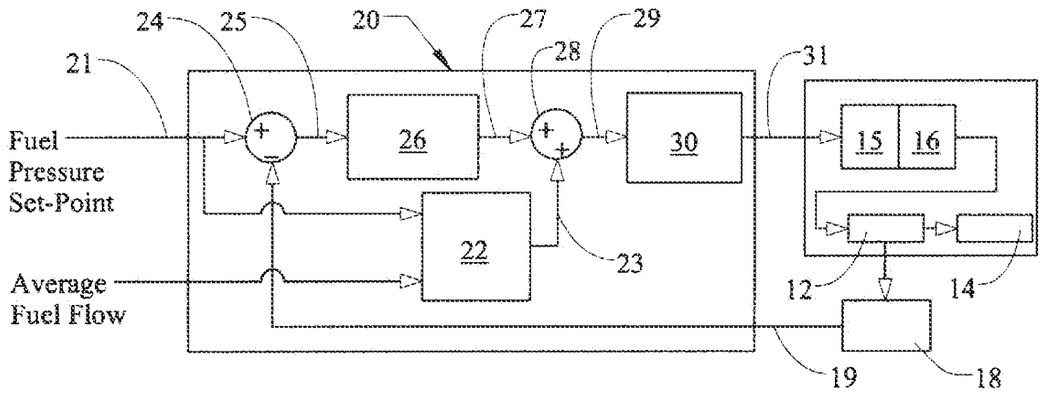

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Fig. 3

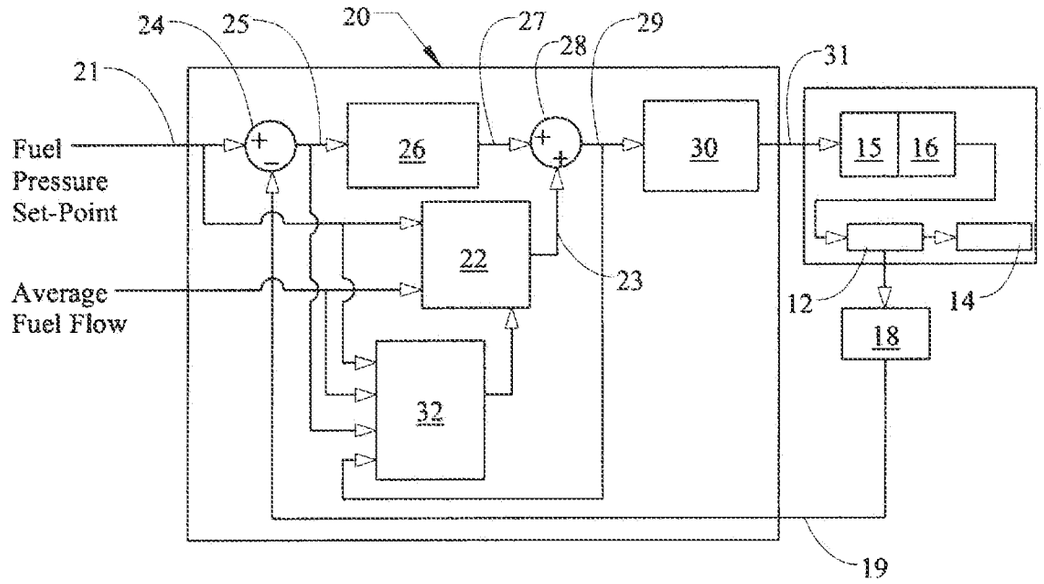

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Fig. 4

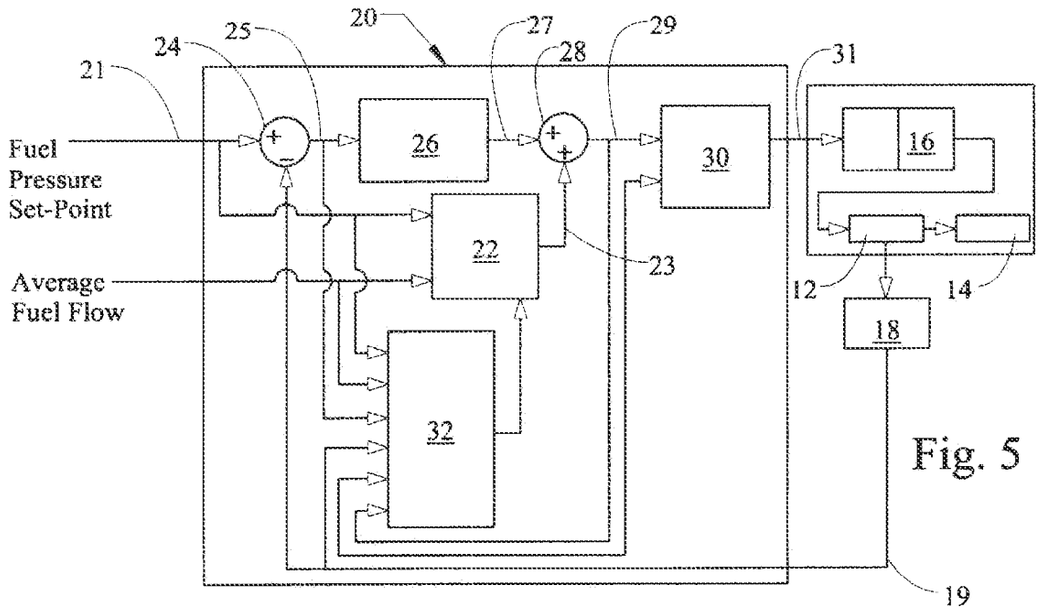

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Fig. 5

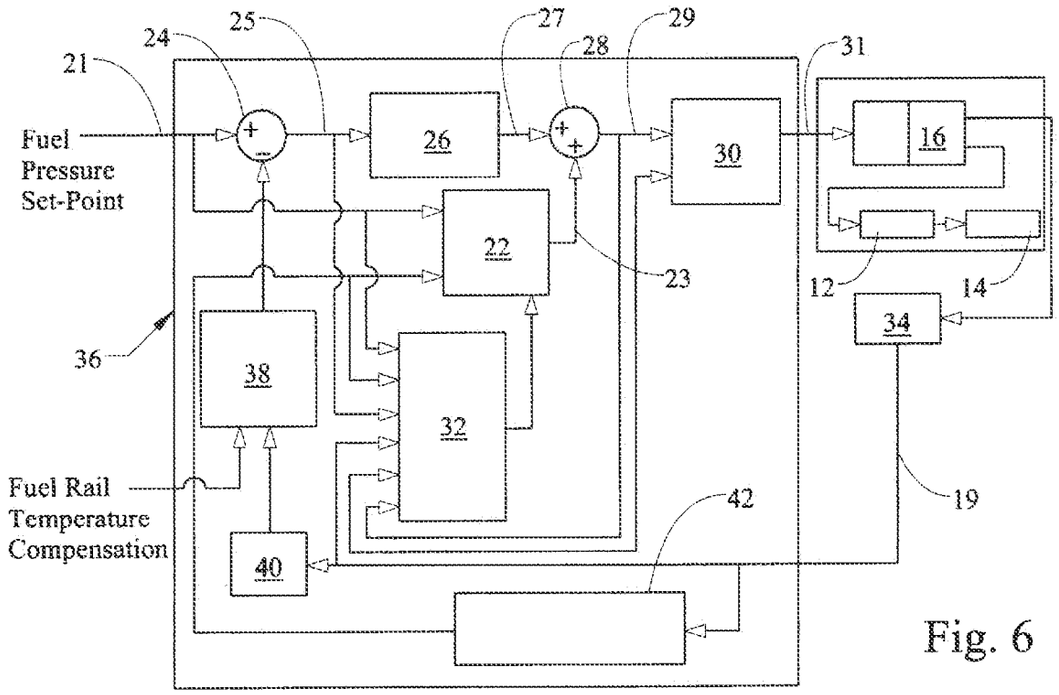

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Fuel Rail
Temperature
Compensation
40
42

Fig. 6

METHOD FOR CONTROLLING FUEL RAIL PRESSURE

TECHNICAL FIELD

The present invention generally relates a method of controlling the fuel pressure within the fuel rail of an internal combustion engine.

BACKGROUND

Within an internal combustion engine fuel delivery system, a fuel rail delivers fuel to fuel injectors that inject the fuel into the cylinders of the engine. The pressure of the fuel being injected through the fuel injectors is critical, therefore it is important to keep the pressure within the fuel rail as consistent as possible. Closed loop control systems for controlling the fuel pressure within fuel rails have been developed, but do not adapt to system variations such as part-to-part tolerance differences and wear. Therefore, there is a need for an improved method of controlling the fuel pressure within the fuel rail of an internal combustion engine.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a schematic view of a first preferred embodiment of a fuel rail delivery system of the present invention;

FIG. **2** is a schematic view of a second preferred embodiment of a fuel rail delivery system of the present invention;

FIG. **3** is a control diagram illustrating the method of controlling the fuel delivery system of the preferred embodiment;

FIG. **4** is a control diagram for a variation of the first preferred embodiment;

FIG. **5** is a control diagram similar to FIG. **4** wherein an adaptive learning algorithm receives additional feedback information; and

FIG. **6** is a control diagram for the second preferred embodiment.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENTS

The following description of the preferred embodiments of the invention is not intended to limit the scope of the invention to these preferred embodiments, but rather to enable any person skilled in the art to make and use the invention.

Referring to FIG. **1**, a fuel delivery system **10** of the present invention includes a fuel rail **12** adapted to deliver fuel to fuel injectors **14** of an internal combustion engine, a fuel pump **16** adapted to deliver fuel to the fuel rail **12**, a fuel pressure sensor **18**, and a fuel pump motor controller **20**.

Referring to FIG. **3**, the fuel pump motor controller **20** is a processor that is adapted to control the fuel pump **16**. Mainly, the fuel pump motor controller **20** controls the speed of a motor **15** that drives the fuel pump **16**. The fuel pressure sensor **18** measures the pressure within the fuel rail **12**. The fuel pump motor controller **20** receives the fuel pressure and calculates the difference between a set-point pressure and the fuel rail pressure. This difference is the fuel rail pressure error signal **25** which is used to control the fuel pump speed.

The set-point pressure is the pressure at which the fuel delivery system **10** works most efficiently. In order to achieve proper fuel injection characteristics such as spray penetration and spray pattern, it is important that the fuel delivered to the fuel injector nozzles **14** remains at the appropriate pressure. This optimum pressure is the set-point pressure, and is pre-determined based upon the injector features and the specifications of the particular application. A set-point pressure signal **21** is sent to the fuel pump motor controller **20**.

The fuel pump motor controller **20** calculates and outputs a motor drive signal **31** to the fuel pump motor **15**. The motor drive signal **31** is calculated from the desired fuel pressure and average fuel flow demand, and attempts to drive the fuel pump motor **15** at a speed that will provide the desired fuel pressure at the desired fuel flow rate.

The fuel pump motor controller **20** includes a feed forward controller **22**, a feedback controller **26**, and a fuel pump motor controller driver **30**. The feed forward controller **22** uses a function or algorithm, such as a look-up table with an interpolation routine, to output a feed forward control signal **23**. The feed forward controller **22** inputs the set-point pressure and the average fuel flow demanded into the function or algorithm and calculates the feed forward control signal **23**.

The fuel rail pressure error signal **25** is calculated by subtracting the fuel pressure sensor signal **19** from the set-point pressure signal **21**. This calculation is performed by a first summing junction **24**. The fuel rail pressure error signal **25** is input to the feed back controller **26**. In turn, the feed back controller **26** calculates a feed back control signal **27**. The feed back controller **26** can be any suitable controller, such as a Proportional Integration and Differential or model based controller.

The feed forward control signal **23** and the feed back control signal **27** are summed in a second summing junction **28**, thereby generating a motor controller signal **29**. The fuel pump motor controller driver **30** receives the motor controller signal **29** and generates a motor drive signal **31** that controls the speed of the fuel pump motor **15**.

In a nominal fuel system with the desired fuel rail pressure at a given fuel flow rate, the feed forward control signal **23** and the motor controller signal **29** would be the same. The feed back controller **26** modifies the feed forward control signal **23** to compensate for system variations due to part to part tolerances and system aging, etc.

A first preferred embodiment of the invention is shown in FIG. **3**, wherein the fuel pressure sensor **18** is a relatively low bandwidth sensor that is mounted onto the fuel rail **12** to directly measure the fuel pressure therein. Referring to FIG. **4**, in a variation of the first preferred embodiment, the fuel pump motor controller **20** further includes an adaptive learning controller **32** which uses an algorithm to monitor the conditions of the fuel delivery system **10** and updates the feed forward controller **22** when the system is operating in a steady state condition.

The algorithm of the adaptive learning controller **32** receives the set-point pressure signal **21**, the average fuel flow, the fuel rail pressure error **25**, and the motor controller signal **29** as inputs. The algorithm then compares the motor controller signal **29** to the feed forward control signal **23** corresponding to the current set-point signal and the average fuel flow within the feed forward controller **22**, and updates the values within the feed forward controller **22** appropriately.

The algorithm of the adaptive learning controller **32** receives the error signal **25** and the feed forward controller **22** is only updated when the error has stabilized and is below a pre-determined threshold. Referring to FIG. **5**, additionally, the speed of the fuel pump **16** and the pressure

from the pressure sensor **18** can also be received by the adaptive learning controller **32** to provide additional parameters for more accurate updating of the feed forward controller **22**.

A second preferred embodiment is shown in FIG. **2**, wherein like elements are numbered the same as the first preferred embodiment. The second preferred embodiment includes a fuel pressure sensor **34** mounted at the fuel pump **16** to measure the fuel pressure within the fuel delivery system **10** at the fuel pump **16**. The fuel pressure sensor **34** of the second preferred embodiment is a relatively wide bandwidth sensor.

Referring to FIG. **6**, a fuel pump motor controller **36** of the second preferred embodiment is similar to the fuel pump motor controller **20** of the first preferred embodiment, however the second preferred embodiment also includes a fuel system model **38**, a low-pass filter **40**, and a device **42** to calculate the average fuel flow. The fuel system model **38** receives the fuel pressure as measured by the pressure sensor **34** at the fuel pump **16** and the fuel system model **38** estimates the fuel rail pressure based upon the pressure at the fuel pump **16**.

The wide bandwidth pressure sensor **34** will measure both the average fuel pump outlet pressure as well as pressure pulses caused by the opening and closing of the fuel injectors **14**. The low-pass filter **40** filters out pulses in the pressure readings due to the opening and closing of the fuel injectors **14**, so the feed back controller **26** does not respond to these injector pulsations. The fuel system model **38** can also include input of the fuel rail **12** temperature. The temperature of the fuel rail **12** influences the fuel rail pressure estimation, so the fuel system model **38** can take this temperature into consideration to more accurately approximate the pressure within the fuel rail **12** based upon the pressure measured at the fuel pump **16**. The wide band pressure from the pressure sensor **34** is also used by the device **42** to calculate the average fuel flow. From the pressure pulsations caused by the opening and closing of the injectors, the injector frequency and on-time duration can be obtained. By using this information along with the injector flow rate, the average fuel flow rate can be calculated, thereby eliminating the need for external average fuel flow information.

It is to be understood, that the processors, sensors, fuel pump, and controllers are conventional devices that are common in the industry and are described herein merely to provide examples of how the method of the present invention can be practiced.

The foregoing discussion discloses and describes two preferred embodiments. One skilled in the art will readily recognize from such discussion, and from the accompanying drawings and claims, that changes and modifications can be made to the preferred embodiments without departing from the true spirit and fair scope of the inventive concepts as defined in the following claims. The preferred embodiments have been described in an illustrative manner, and it is to be understood that the terminology which has been used is intended to be in the nature of words of description rather than of limitation.

We claim:

1. A method of controlling the fuel pressure within a fuel delivery system having a fuel pump which delivers fuel to a fuel rail of an internal combustion engine comprising:

providing a set-point fuel pressure;

providing the estimated average fuel flow through the system;

generating a feed forward control signal based upon the set-point fuel pressure and the average fuel flow;

measuring the fuel rail pressure with a fuel pressure sensor;

filtering the output of the fuel pressure sensor to filter out pulses due to the opening and closing of the fuel injectors;

comparing the fuel rail pressure to the set-point fuel pressure and generating an error signal based upon the difference between the fuel rail pressure and the set-point fuel pressure; and

modifying the feed forward control signal based upon the error signal thereby generating a motor controller signal and sending the motor control signal to the fuel pump motor.

2. The method of claim **1** wherein generating a feed forward control signal includes providing a feed forward controller which provides established fuel pump motor control parameters at particular set-point pressure and average fuel flow rate values, wherein the set-point pressure and average fuel flow are input to the feed forward controller and a feed forward control signal having set of corresponding fuel pump motor control parameters is thereby generated.

3. The method of claim **1** further including:

comparing the motor control signal to the fuel pump motor control parameters within the feed forward controller which correspond to the set-point pressure and the average fuel flow; and

updating the feed forward controller with new fuel pump motor control parameters.

4. The method of claim **3** further including monitoring the error signal, and updating the feed forward controller after the error signal has stabilized and is below a pre-determined level.

5. The method of claim **1** wherein measuring the fuel rail pressure includes measuring the fuel pressure at the fuel pump and estimating the fuel rail pressure based upon the fuel pressure as measured at the fuel pump.

6. The method of claim **1** further including:

detecting pulses in the signal from the fuel pressure sensor to establish when the injectors open and close;

measuring the time between when an injector opens and closes to determine how long the injector is open;

using the injector flow rate and the injector open time to calculate the average fuel flow.

7. A fuel delivery system comprising:

a fuel rail adapted to deliver fuel to fuel injectors of an automotive vehicle, a fuel pump adapted to deliver fuel to said fuel rail, a fuel pressure sensor adapted to measure the fuel rail pressure, a low-pass filter adapted to filter the output of said fuel pressure sensor to filter out pulses due to the opening and closing of the fuel injectors, and a fuel pump motor controller having:

a feed forward controller adapted to provide a feed forward signal having fuel pump motor control parameters based upon a set-point fuel pressure and the average fuel flow through said system;

a first summing junction adapted to compare the fuel rail pressure to the set-point fuel pressure and to generate an error value based upon the difference between the fuel rail pressure and the set-point fuel pressure;

a feed back controller adapted to receive the error value and to generate a feed back control signal;

a second summing junction adapted to receive the feed back control signal from said feed back controller

and the feed forward control signal from the feed forward controller and to modify the fuel pump motor control parameters of the feed forward control signal based upon the feed back control signal to generate a motor controller signal; and

a fuel pump motor controller driver adapted to receive the motor controller signal and to control the speed of said fuel pump based upon the motor controller signal.

8. The fuel delivery system of claim **7** wherein said fuel pressure sensor is mounted at said fuel rail to directly measure the fuel pressure within said fuel rail.

9. The fuel delivery system of claim **7** wherein said feed back controller is a proportional integration and differential closed loop controller.

10. The fuel delivery system of claim **7** further including an adaptive learning controller having an algorithm which is adapted to monitor the conditions of the fuel delivery system and to update said feed forward controller.

11. The fuel delivery system of claim **7** wherein said fuel pressure sensor is mounted at said fuel pump to measure the fuel pressure within the fuel delivery system at said fuel pump.

12. The fuel delivery system of claim **11** further including a fuel system model adapted to estimate the fuel pressure within said fuel rail based upon the fuel pressure as measured at said fuel pump.

13. A method of controlling the fuel pressure within a fuel delivery system having a fuel pump which delivers fuel to a fuel rail and fuel injectors of an internal combustion engine comprising:

providing a set-point fuel pressure;

providing the estimated average fuel flow through the system;

generating a feed forward control signal based upon the set-point fuel pressure and the average fuel flow;

measuring the fuel rail pressure with a fuel pressure sensor;

comparing the fuel rail pressure to the set-point fuel pressure and generating an error signal based upon the difference between the fuel rail pressure and the set-point fuel pressure;

modifying the feed forward control signal based upon the error signal thereby generating a motor controller signal and sending the motor control signal to the fuel pump motor;

detecting pulses in the signal from the fuel pressure sensor to establish when the fuel injectors open and close;

measuring the time between when an injector opens and closes to determine how long the injector is open;

using the injector flow rate and the injector open time to calculate the average fuel flow.

14. The method of claim **13** wherein generating a feed forward control signal includes providing a feed forward controller which provides established fuel pump motor control parameters at particular set-point pressure and average fuel flow rate values, wherein the set-point pressure and average fuel flow are input to the feed forward controller and a feed forward control signal having set of corresponding fuel pump motor control parameters is thereby generated.

15. The method of claim **13** further including:

comparing the motor control signal to the fuel pump motor control parameters within the feed forward controller which correspond to the set-point pressure and the average fuel flow; and

updating the feed forward controller with new fuel pump motor control parameters.

16. The method of claim **15** further including monitoring the error signal, and updating the feed forward controller after the error signal has stabilized and is below a predetermined level.

17. The method of claim **13** wherein measuring the fuel rail pressure includes measuring the fuel pressure at the fuel pump and estimating the fuel rail pressure based upon the fuel pressure as measured at the fuel pump.

18. The method of claim **13** further including filtering the signal sent from the fuel pressure sensor to filter out pulses due to the opening and closing of the fuel injectors.

* * * * *

EXHIBIT 7

US006588260B1

(12) **United States Patent**
Pursifull

(10) **Patent No.: US 6,588,260 B1**
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(54) **ELECTRONIC THROTTLE DISABLE CONTROL TEST SYSTEM**

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(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 70 days.

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(51) **Int. Cl.**[7] **G01M 15/00**
(52) **U.S. Cl.** **73/118.1**
(58) **Field of Search** 73/116, 117.2, 73/117.3, 118.1, 118.2

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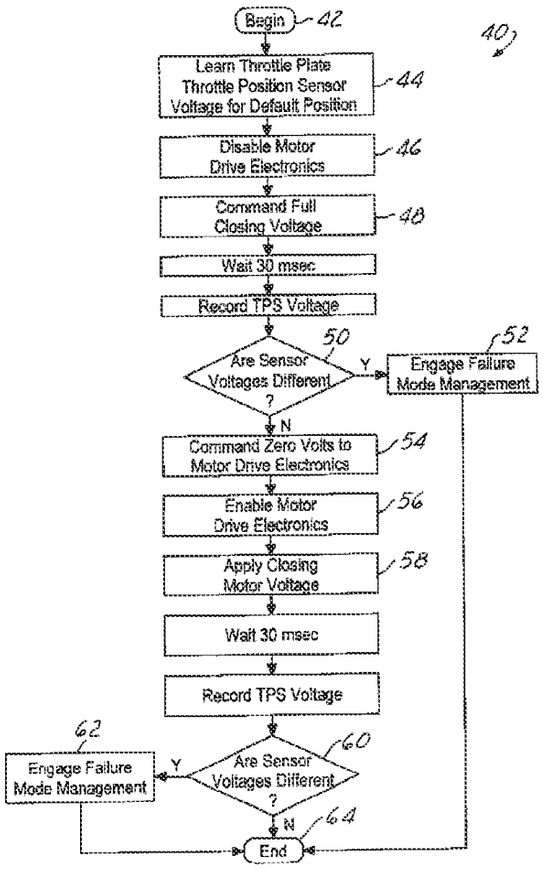
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(57) **ABSTRACT**

A method for testing the integrity of an electronic throttle disable function begins by disabling the drive electronics responsible for controlling the throttle motor. The micro-controller doing the plate position control commands full closing voltage from throttle motor drive electronics (while those electronics are disabled). Then the throttle position voltage is recorded. If the disable system is working, the H-driver will output zero volts even though a full motor closing voltage is requested. The throttle position voltage is then compared to a predetermined default throttle position voltage. If there is a significant difference between the two voltages, then failure mode management is engaged.

17 Claims, 2 Drawing Sheets



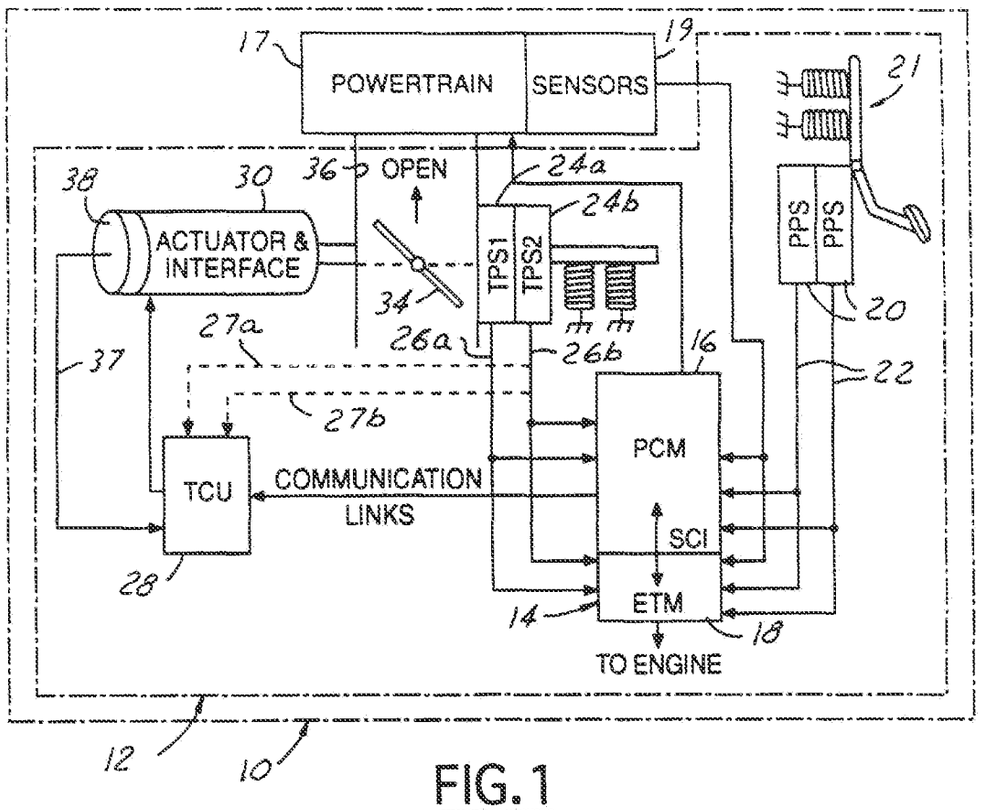

17
POWERTRAIN
SENSORS
19
21
38
30
36
OPEN
24a
24b
ACTUATOR &
INTERFACE
TPS1
TPS2
PPS
PPS
34
20
27a
26a
26b
16
22
37
27b
PCM
TCU
COMMUNICATION
LINKS
SCI
28
ETM
14
18
TO ENGINE
12
10

FIG.1

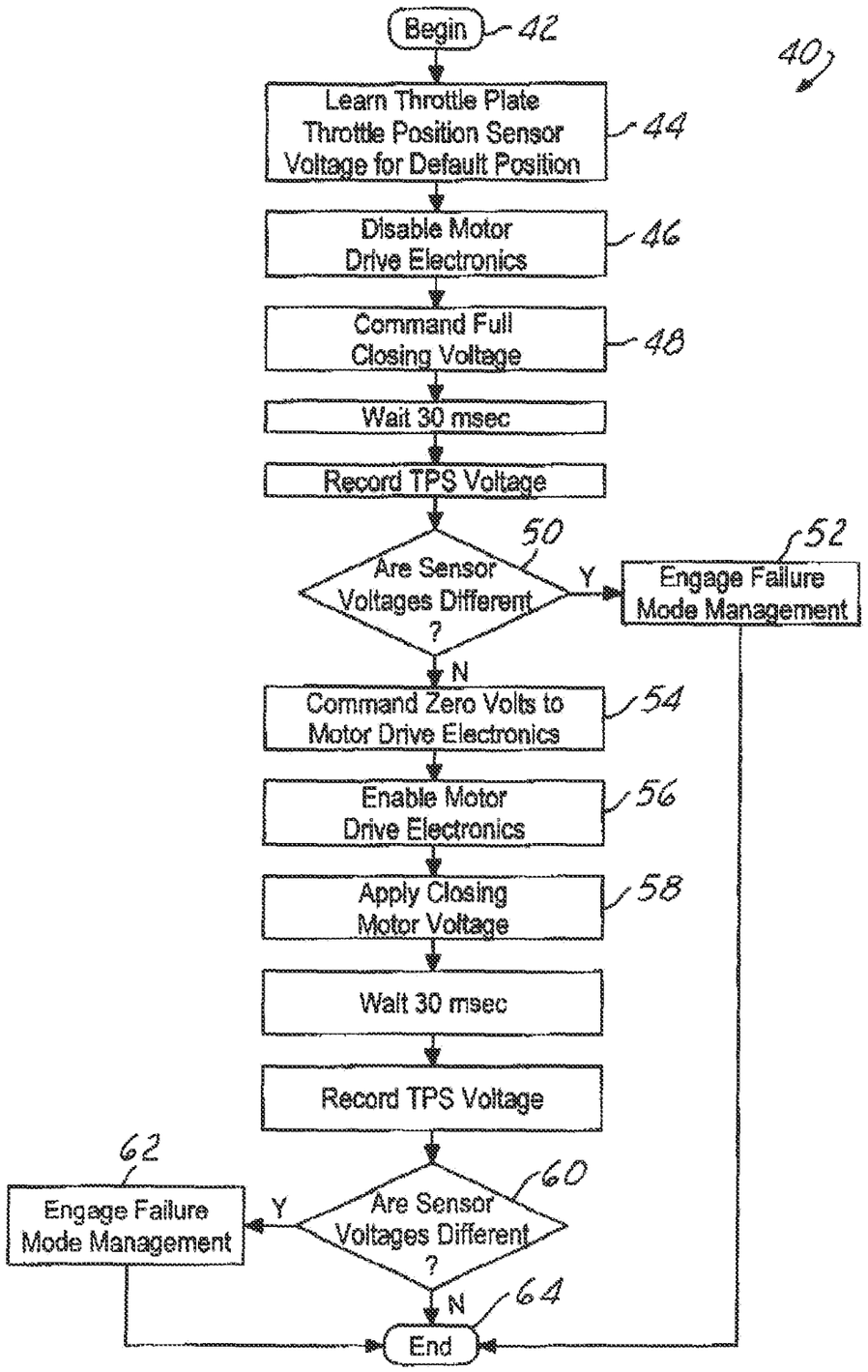

Begin
42
40
Learn Throttle Plate Throttle Position Sensor Voltage for Default Position
44
Disable Motor Drive Electronics
46
Command Full Closing Voltage
48
Wait 30 msec
Record TPS Voltage
50
Are Sensor Voltages Different ?
Y
52
Engage Failure Mode Management
N
Command Zero Volts to Motor Drive Electronics
54
Enable Motor Drive Electronics
56
Apply Closing Motor Voltage
58
Wait 30 msec
Record TPS Voltage
62
Engage Failure Mode Management
Y
Are Sensor Voltages Different ?
60
N
64
End

FIG. 2

ELECTRONIC THROTTLE DISABLE CONTROL TEST SYSTEM

TECHNICAL FIELD

The present invention relates generally to control systems for internal combustion engines, and more particularly, to an electronic throttle disable control test system.

BACKGROUND ART

Many previously known motor vehicle throttle controls have a direct physical linkage between an accelerator pedal and the throttle body so that the throttle plate is pulled open by the accelerator cable as the driver presses the pedal. The direct mechanical linkage includes biasing that defaults the linkage to a reduced operating position, also known as idle, in a manner consistent with regulations. Nevertheless, such mechanisms are often simple and unable to adapt fuel consumption efficiency, minimize regulated emissions, improve driveability to changing traveling conditions, and add significant weight and components to the motor vehicle.

An alternative control for improving throttle control and the efficient introduction of fuel air mixtures into the engine cylinders is presented by electronic throttle control. The electronic throttle control includes a throttle control unit that positions the throttle plate by an actuator controlled by a microprocessor using position feedback sensors. The micro controllers (or micro processors) are often included as part of a powertrain electronic control that can adjust the fuel and air intake and ignition in response to changing conditions of vehicle operation as well as operator control. Protection may be provided so that an electronic system does not misread or misdirect the control and so that unintended operation is avoided when portions of the electronic control suffer a failure.

One previously known type of protection to avoid unintended actuation of excessive throttle is to use a disable function of the electronic throttle drive electronics. The disable function allows the PCM to shut down the electronic throttle drive electronics in the event that the PCM senses a throttle position differing from expected throttle position. Unfortunately, typical electronic throttle systems do not test the integrity of the disable function.

The disadvantages associated with these conventional electronic throttle disable techniques have made it apparent that a new technique for testing the electronic throttle disable function is needed. The new technique should not intrude on normal throttle operation while verifying function integrity at least once each time the vehicle is powered up. The present invention is directed to these ends.

SUMMARY OF THE INVENTION

It is, therefore, an object of the invention to provide an improved and reliable electronic throttle disable control test system. Another object of the invention is to test the electronic throttle driver disable function. And additional object of the invention is to not intrude upon normal throttle operation.

In accordance with the objects of this invention, an electronic throttle disable control test system is provided. In one embodiment of the invention, a method for testing the integrity of an electronic throttle disable function begins by disabling the electronic throttle motor's drive electronics (which is an integrated H-driver in the preferred embodiment). The H-driver is commanded to apply dull closing voltage, however, since the H-driver is disabled, no voltage is applied to the motor and the throttle remains at its default position. The throttle position voltage is then compared to a predetermined default position voltage. If there is a significant difference between the two voltages, then failure mode management is engaged. These voltages can be either "absolute" meaning relative to system ground or "relative" meaning relative to the throttle position sensor voltage output at closed throttle, default position, or open stop position. This allows one to detect the voltage difference between default and closed stop.

At this point, we have verified that the throttle does not move when disabled. Then we have to verify that it does move when enabled.

The present invention thus achieves an improved electronic throttle disable control test system. The present invention is advantageous in that the integrity of the electronic throttle disable function is tested at least once each time the vehicle is powered up.

Additional advantages and features of the present invention will become apparent from the description that follows, and may be realized by means of the instrumentalities and combinations particularly pointed out in the appended claims, taken in conjunction with the accompanying drawings.

BRIEF DESCRIPTION OF THE DRAWINGS

In order that the invention may be well understood, there will now be described some embodiments thereof, given by way of example, reference being made to the accompanying drawings, in which:

FIG. **1** is an electronic throttle disable control test system in accordance with one embodiment of the present invention; and

FIG. **2** is a flow chart for an electronic throttle disable control test in accordance with one embodiment of the present invention.

BEST MODES FOR CARRYING OUT THE INVENTION

In the following figures, the same reference numerals will be used to identify identical components in the various views. The present invention is illustrated with respect to an electronic throttle disable control test system, particularly suited for the automotive field. However, the present invention is applicable to various other uses that may require electronic throttle disable control test systems.

Referring to FIG. **1**, a motor vehicle powertrain system **10** including electronic throttle control system **12** includes an electronic control unit **14**. In the preferred embodiment, the electronic control unit **14** includes a powertrain control module (PCM) **16** including a main processor and an electronic throttle monitor (ETM) **18** including an independent processor. The PCM and ETM share sensors **19** and actuators that are associated with the powertrain system **17** and control module **16**. Preferably, the electronic throttle monitor **18** includes a processor physically located within the powertrain control module housing, although a separate housing, separate locations and other embodiments can also be employed in practicing the invention. Moreover, while the electronic throttle monitor **18** and the powertrain control module **16** have independent processors, they share the inputs and outputs of powertrain sensors **19** and actuators **21** and **34**, respectively, for independent processing.

A wide variety of inputs are represented in the FIG. **1** diagram by the diagrammatic representation of redundant

pedal position sensors **20**. The sensors **20** are coupled through inputs **22** and are representative of many different driver controls that may demonstrate the demand for power. In addition, the electronic control unit **14** includes inputs **26***a* and **26***b* for detecting throttle position. A variety of ways for providing such indications is diagrammatically represented in FIG. **1** by a first throttle position sensor **24***a* and a redundant second throttle position sensor **24***b* to obtain a power output indication. As a result of the many inputs represented at **19**, **22**, **26***a* and **26***b*, the electronic controller **14** provides outputs for limiting output power so that output power does not exceed power demand. A variety of outputs are also diagrammatically represented in FIG. **1** by the illustrated example of inputs to a throttle control unit **28** that in turn powers an actuator and motive interface **30** for displacing the throttle plate **34**. For example, an actuator and interface may comprise redundant drive motors powering a gear interface to change the angle of the throttle plate **34** in the throttle body **36**.

Likewise, the responsive equipment like motors may also provide feedback. For example, the motor position sensor **38** or the throttle position sensors **24***a* and **24***b* may provide feedback to the throttle control unit **28**, as shown at **37**, **27***a* and **27***b*, respectively, to determine whether alternative responses are required or to maintain information for service or repair.

The purpose of the present invention is to establish a method with which to test the ability to take an action (which is taken upon failure detection). For a number of detected electronic control system failures, the throttle plate **34** position control disables the drive electronics for the throttle motor **30**. In normal operation, the system would never take this action.

The challenge is to design a test that does not intrude in normal throttle control operation. Since disabling the driver results in the plate **34** going to default, to test this function non-intrusively, one must test when the system is off-line but powered (initialization or shut-down) or when default is requested anyway. An advantage over previous technology is that the driver disable function (which is a significant contributor to system predictability when a system failure is detected) can now be tested at least once each time the vehicle is powered up.

Referring to FIG. **2**, a flow chart for an electronic throttle disable control test **40** in accordance with one embodiment of the present invention is illustrated. In the present invention, control test **40** begins with step **42** and immediately proceeds to step **44** where PCM **16** learns the TPS output voltage for the default position of throttle plate **36**. (At this point, no motor voltage is applied so that the plate will have attained the default position.) The sequence then proceeds to step **46**.

In step **46**, PCM **16** disables the drive electronics responsible for actuating motor **30**. The sequence then proceeds to step **48** where PCM **16** commands full closing motor voltage. PCM **16** waits approximately 30 milliseconds and then records the resulting throttle position sensor voltage. The sequence then proceeds to step **50**.

In step **50**, the throttle position voltage corresponding to the default position of throttle plate **34**, determined in step **44**, is compared to the throttle position voltage obtained after disabling the drive electronics. If the two voltages are significantly different, it means that the motor drove the plate to a new position and the disable function is not working and then the PCM **16** will engage failure mode management in step **52** and proceed to the end of the sequence **64**. If, however, the two voltages are not significantly different, then the sequence proceeds to step **54**.

In step **54**, PCM **16** commands zero volts and proceeds to step **56** where it enables the H-driver. In step **58** PCM **16** applies approximately five volts in the closing direction to throttle motor **30**. PCM **16** then waits 30 milliseconds before applying full voltage in the closing direction to throttle motor **30**. PCM **16** then waits another 30 milliseconds before learning the output voltage corresponding to a close stop of throttle plate **34**. PCM **16** then records the close stop TPS output voltage and the sequence proceeds to step **60**.

In step **60**, the voltage corresponding to the default position of throttle plate **34**, determined in step **44**, is compared to the close stop voltage obtained in step **58**. If the two voltages are not significantly different, then the sequence proceeds to end step **64**. If, however, the two voltages are significantly different, then throttle plate **34** may be potentially frozen. In this case, PCM **16** engages failure mode management in step **62**, including ice breaking strategy.

While the present invention is shown with respect to testing the integrity of the drive electronics disable function after learning the default voltage corresponding to the default position of throttle plate **34**, one skilled in the art would realize that testing may also occur before learning the default voltage by simply comparing the voltage obtained in step **48** with the close stop voltage. One skilled in the art would also recognize that various routine functions, such as checking memory, may occur before or after the test.

The present invention thus achieves an improved and reliable electronic throttle disable control test system by means to test the disable function without intruding on normal throttle operation. The present invention does this at least once each time the vehicle is powered up.

From the foregoing, it can be seen that there has been brought to the art a new and improved electronic throttle disable control test system. It is to be understood that the preceding description of the preferred embodiment is merely illustrative of some of the many specific embodiments that represent applications of the principles of the present invention. Clearly, numerous and other arrangements would be evident to those skilled in the art without departing from the scope of the invention as defined by the following claims:

What is claimed is:

1. An electronic throttle control apparatus for testing integrity of a motor drive electronics disable feature comprising:

a PCM having drive electronics for controlling a motor coupled to an electronic throttle plate, said PCM having control logic to disable said drive electronics and return said electronic throttle plate to a default position, determine a default throttle position sensor (TPS) output voltage corresponding to said default position, command a full closing motor voltage, compare a full closing TPS output voltage to said default TPS output voltage, and engage failure mode management when said full closing TPS output voltage and said default TPS output voltage are significantly different from each other.

2. The electronic throttle control apparatus of claim **1**, wherein said PCM further includes control logic operative to enable said drive electronics.

3. The electronic throttle control apparatus of claim **1**, wherein said PCM further includes control logic operative to apply close stop voltage to said motor.

4. The electronic throttle control apparatus of claim **3**, wherein said PCM further includes control logic operative to

compare said full closing TPS output voltage to said close stop TPS output voltage.

5. The electronic throttle control apparatus of claim **4**, wherein said PCM includes control logic operative to engage failure mode management when said full closing TPS output voltage and said close stop TPS output voltage are significantly different.

6. The electronic throttle control apparatus of claim **1**, wherein said PCM includes control logic operative to test said motor drive electronics disable feature before leaning said default throttle position sensor output voltage.

7. The electronic throttle control apparatus of claim **1**, wherein said PCM includes control logic operative to test said motor drive electronics disable feature before checking PCM memory.

8. An electronic throttle control test system for an automobile having and internal combustion engine, said system comprising:

a motorized throttle located on the internal combustion engine, said motorized throttle having a throttle plate coupled to a motor for controlling an amount of airflow entering the internal combustion engine; and

a PCM having drive electronics for controlling said motor coupled to said throttle plate, said PCM having control, logic to disable said drive electronics such that said throttle plate returns to or remains at a default position, determine a default throttle position sensor (TPS) output voltage corresponding to said default position, command a full closing motor voltage, compare said full closing TPS output voltage to said default TPS output voltage, and engage failure mode management when said full closing TPS output voltage and said predetermined default TPS output voltage are significantly different.

9. The electronic throttle control test system of claim **8**, wherein said PCM includes control logic operative to test disabling of said drive electronics before learning said default throttle position sensor output voltage.

10. The electronic throttle control test system of claim **8**, wherein said PCM includes control logic operative to test disabling of said drive electronics before checking PCM memory.

11. A method for testing integrity of an electronic throttle plate driver disable function controlled by a powertrain control module (PCM) comprising the steps of:

disabling said driver;

determining a first throttle position value with said driver disabled;

commanding full closing voltage;

determining a second throttle position value at said full closing voltage;

comparing said first and second throttle position values; and

engaging failure mode management when said first and second throttle position values are significantly different.

12. The method of claim **11**, further comprising the step of enabling said driver.

13. The method of claim **11**, further comprising the step of applying close stop output voltage to a motor.

14. The method of claim **13**, further comprising the step of comparing a throttle position sensor (TPS) output voltage at said full closing voltage to a TPS output voltage at said close stop voltage.

15. The method of claim **14**, further comprising the step of engaging failure mode management when said TPS output voltage at said full closing voltage and said TPS output voltage at said close stop voltage are significantly different.

16. The method of claim **11**, wherein said testing occurs before learning default throttle position sensor output voltage.

17. The method of claim **11**, wherein said testing occurs before checking PCM memory.

* * * * *

EXHIBIT 9

US006736122B2

(12) **United States Patent**
West

(10) **Patent No.: US 6,736,122 B2**
(45) **Date of Patent: May 18, 2004**

(54) **MOTOR VEHICLE ENGINE SYNCHRONIZATION**

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(73) Assignee: **Visteon Global Technologies, Inc.**, Dearborn, MI (US)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 138 days.

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(65) **Prior Publication Data**

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(30) **Foreign Application Priority Data**

Mar. 30, 2001 (GB) 0108010

(51) **Int. Cl.**[7] **F02D 41/24**
(52) **U.S. Cl.** ..... **123/690**; 701/115; 73/119 A
(58) **Field of Search** ..... 123/688, 690, 123/691, 692, 693, 703, 704, 480; 701/103, 109, 115; 73/117.1, 117.2, 117.3, 119 A

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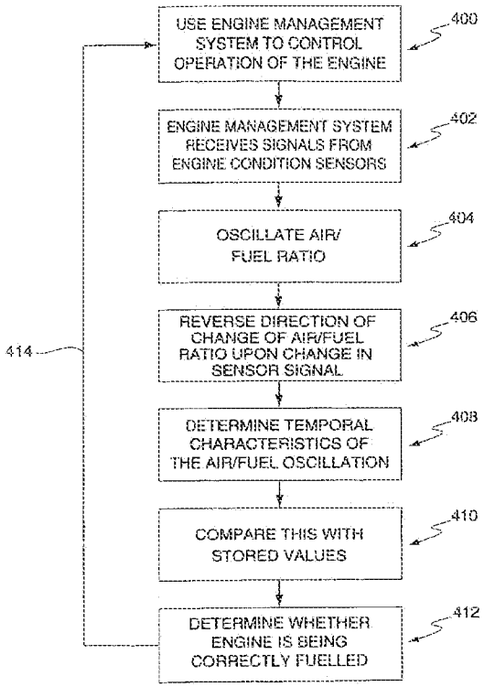
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Primary Examiner—Hai Huynh
(74) *Attorney, Agent, or Firm*—Brinks Hofer Gilson & Lione

(57) **ABSTRACT**

The invention relates to a four stroke internal combustion engine with one or more engine operating condition sensors including an exhaust gas composition sensor in the exhaust conduit. An engine management system is arranged to control the operation of the engine including the fuel injection system and the air/fuel ratio λ for at least one cylinder. The engine management system receives from said sensors respective signals S_i and oscillates the air/fuel ratio λ between a relatively rich level and a relatively lean level, depending on the signal S_i from the exhaust gas sensor. The engine management system determines the temporal characteristics **(142)** of the oscillation in the air/fuel ratio and determines whether or not the engine is being fueled on the correct stroke by comparing **(136)** those temporal characteristics with relevant engine operation data **(120)** stored in the engine management system.

8 Claims, 8 Drawing Sheets



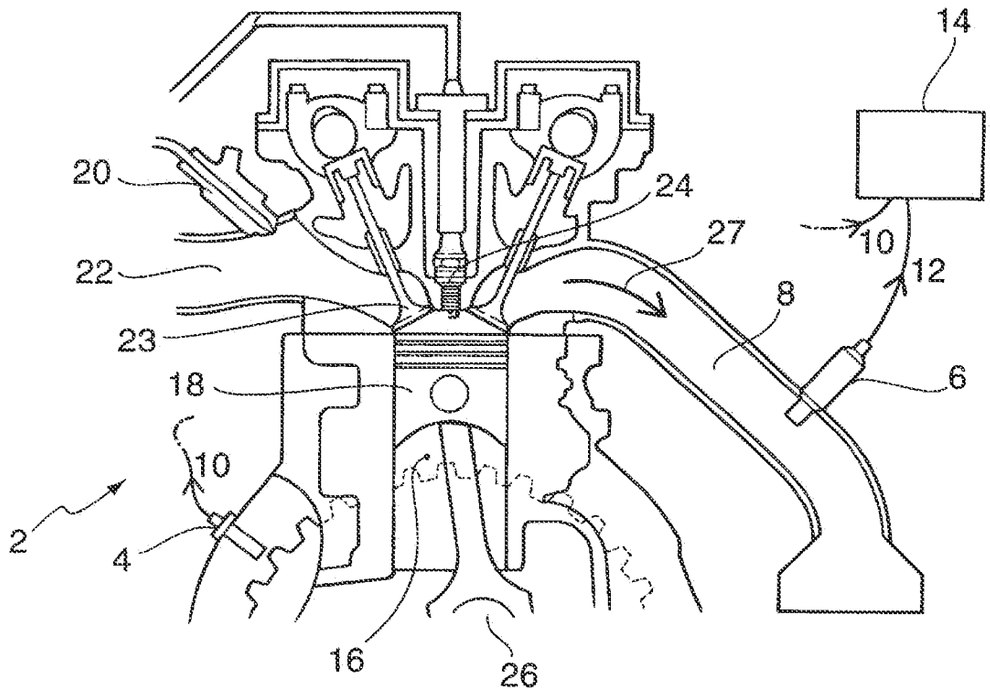


Fig. 1

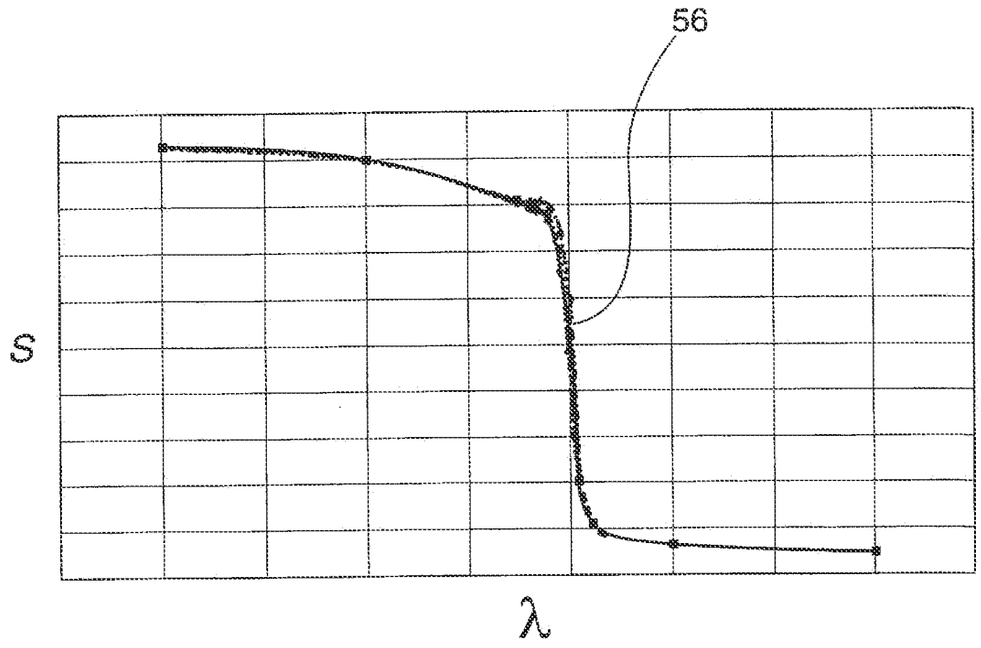


Fig. 2

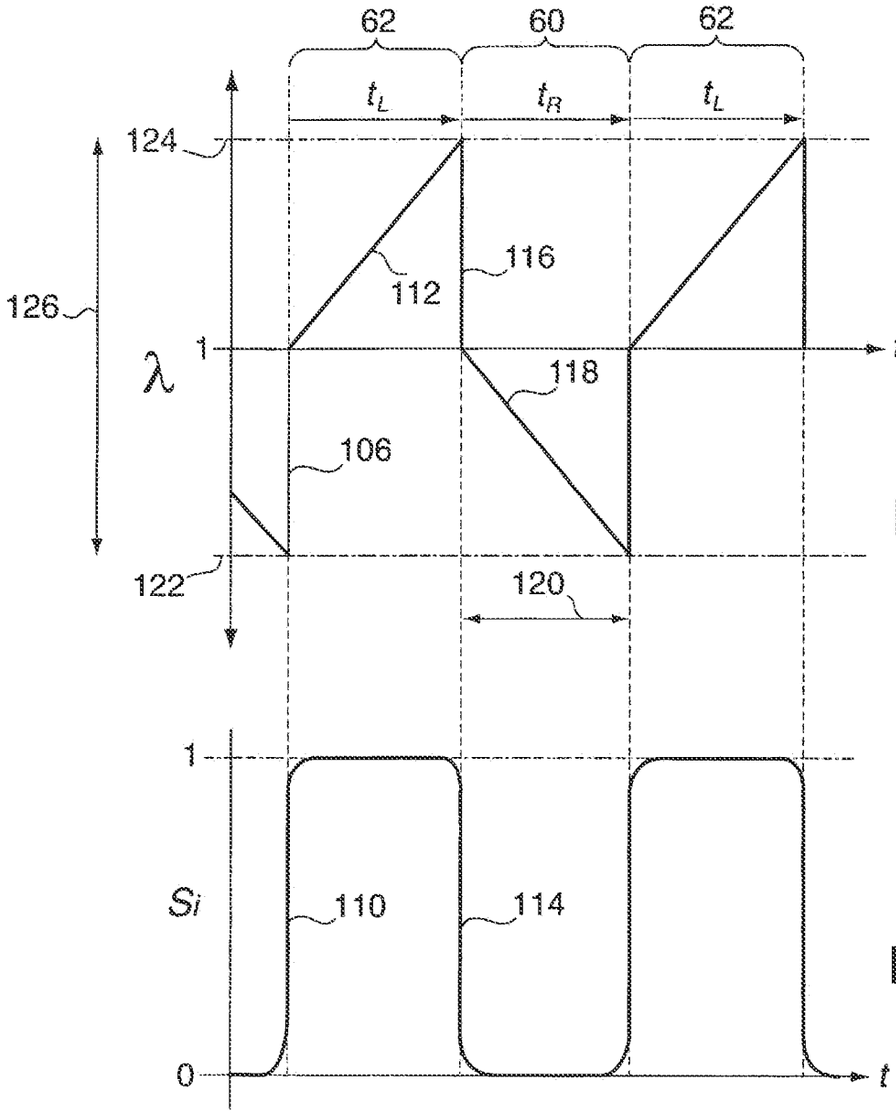

62
60
62
t_L
t_R
t_L
124
126
112
116
1
λ
t
118
106
122
120

Fig. 3

1
Si
110
114
0
t

Fig. 4

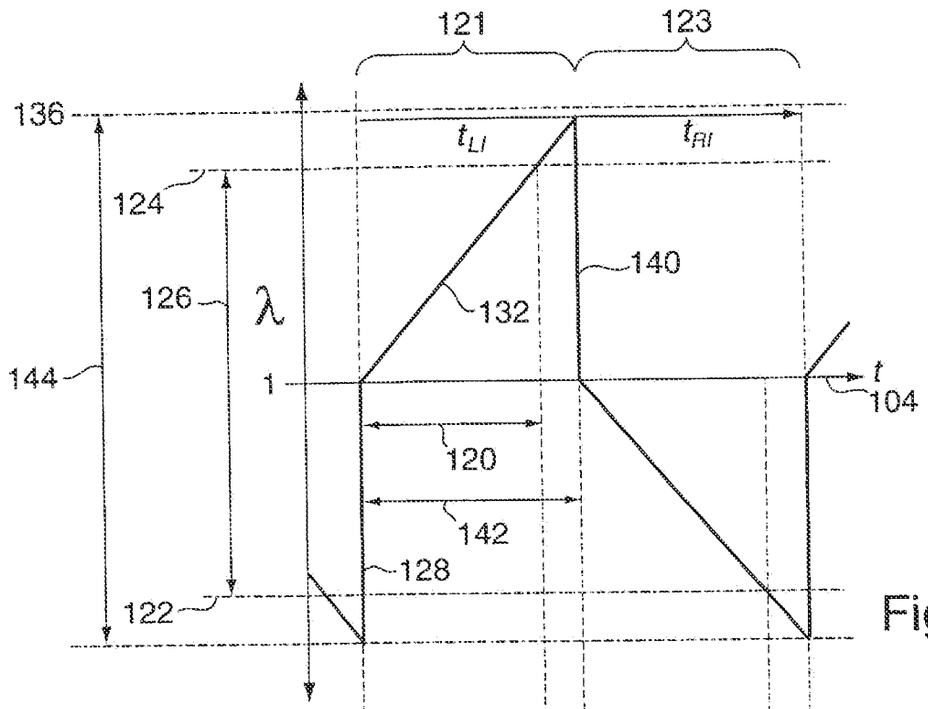

121
123
136
t_{LI}
t_{RI}
124
126
λ
132
140
144
1
t
104
120
142
128
122

Fig. 5

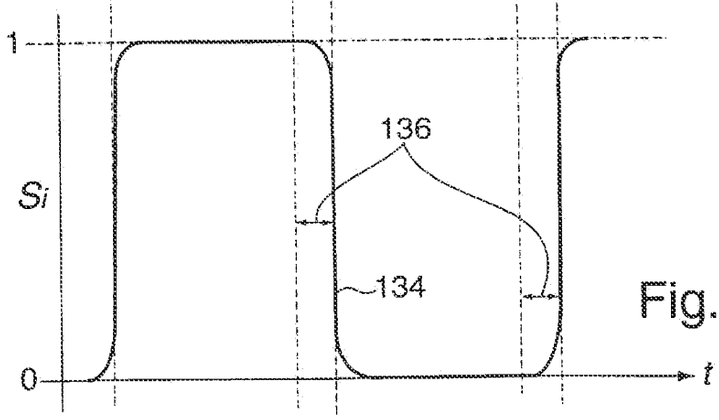

1
136
Si
134
0
t

Fig. 6

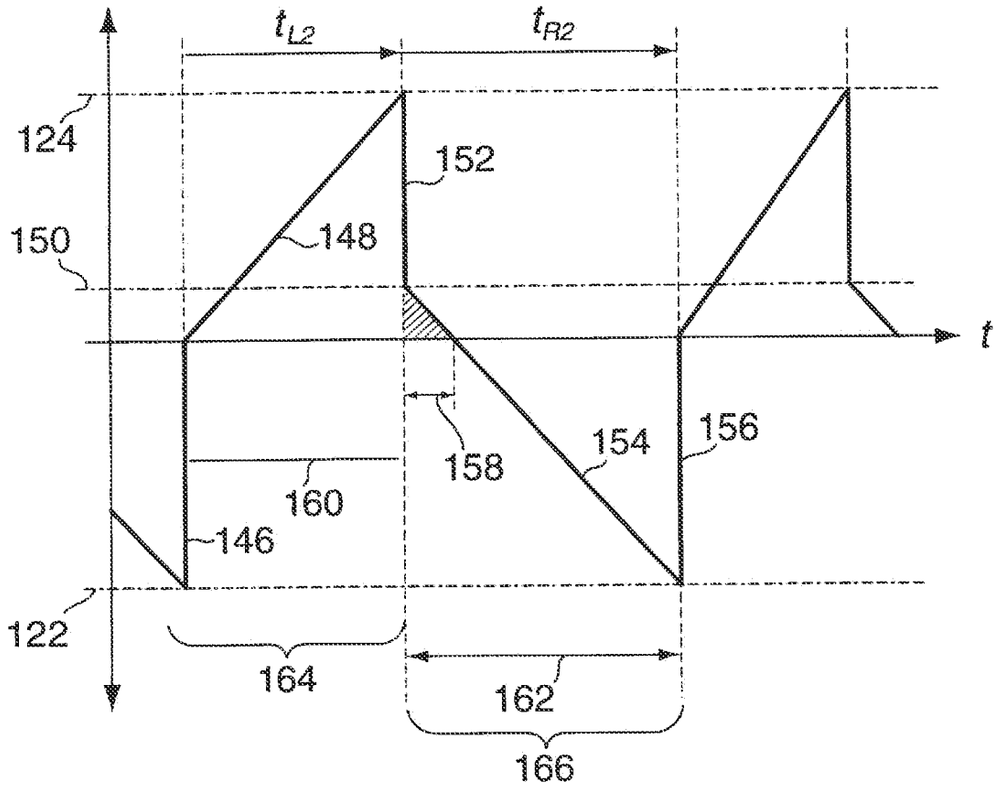


Fig. 7

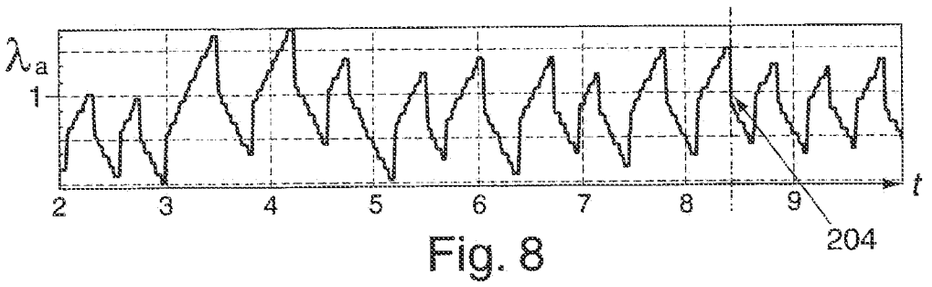


Fig. 8

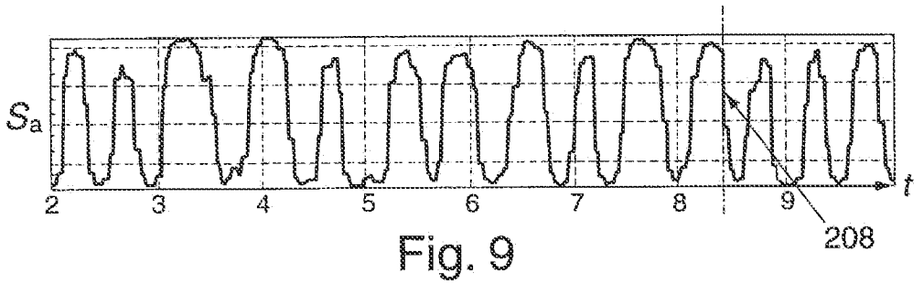


Fig. 9

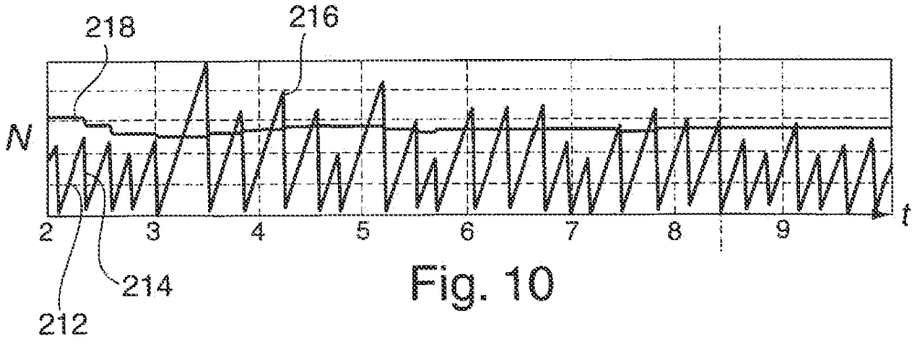


Fig. 10

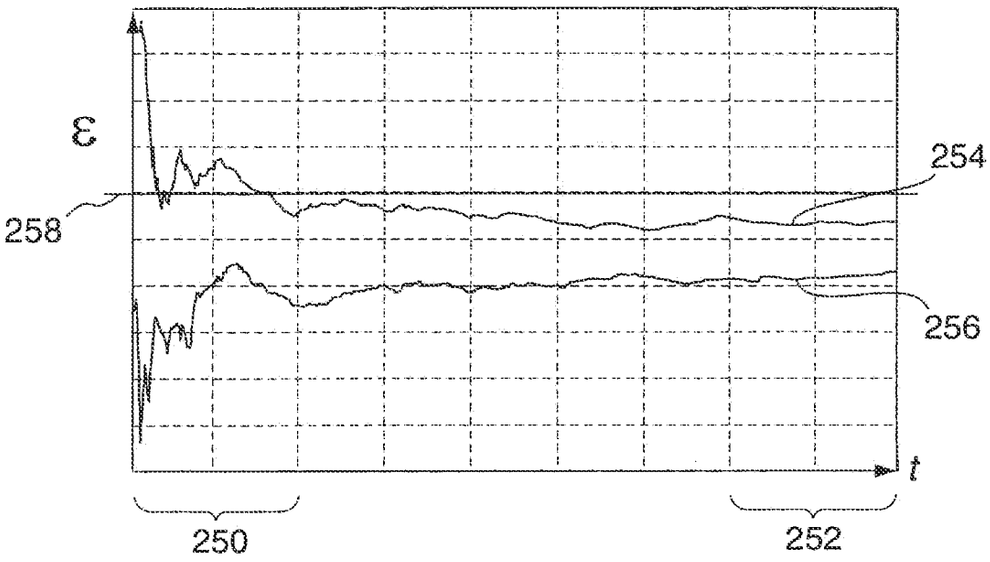


Fig.11

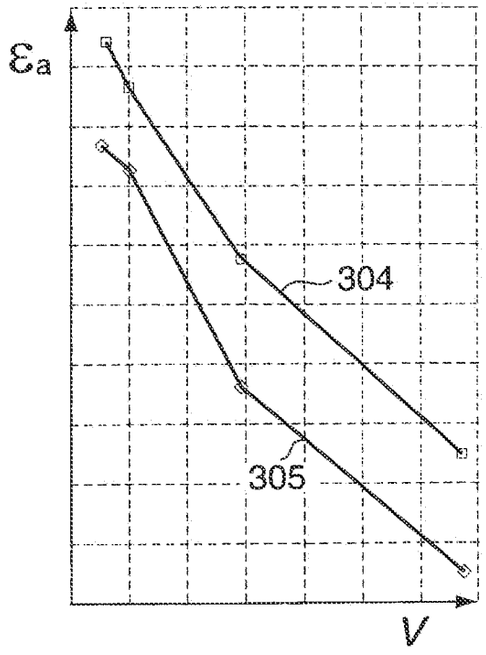


Fig.12

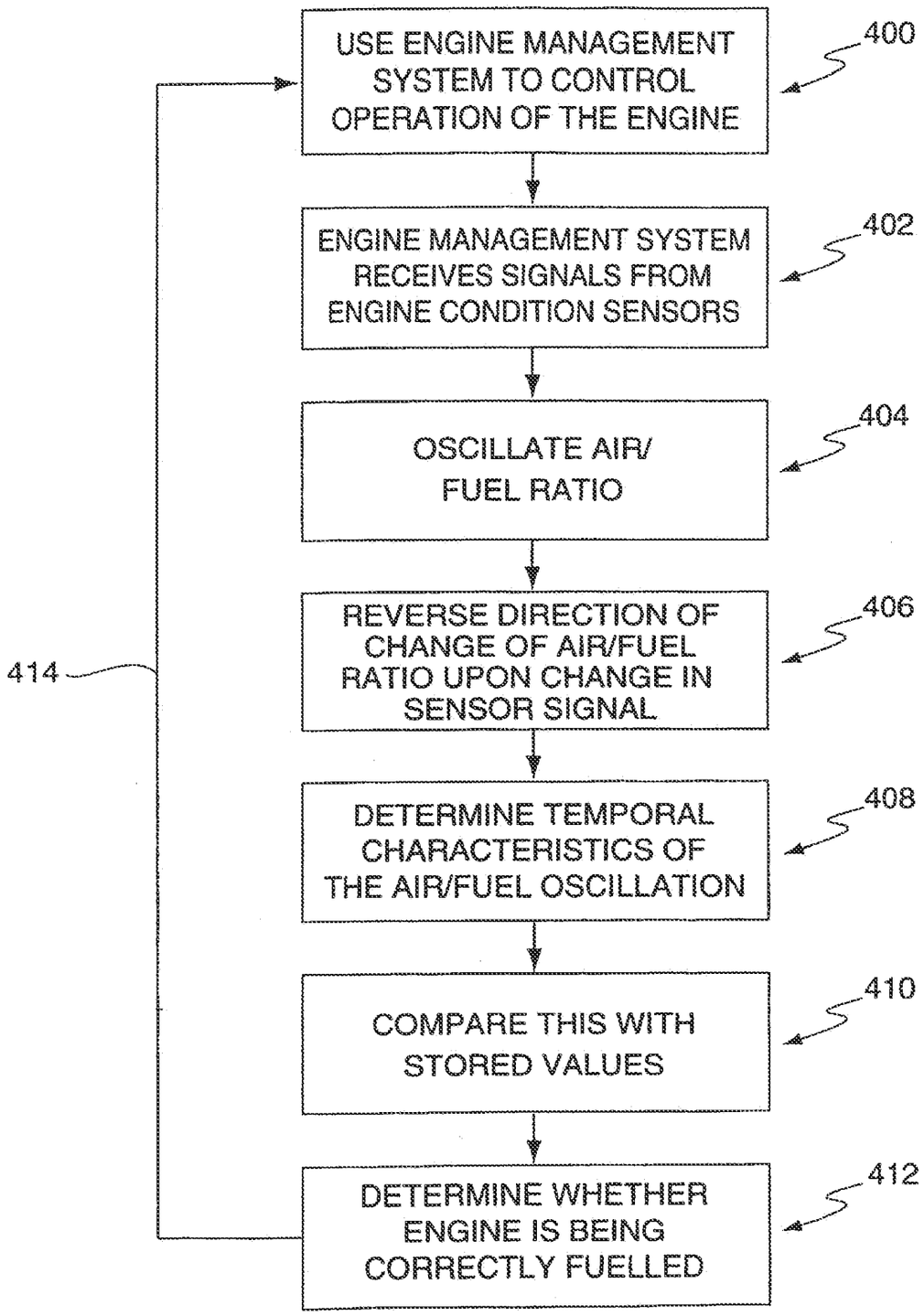

USE ENGINE MANAGEMENT SYSTEM TO CONTROL OPERATION OF THE ENGINE
400
ENGINE MANAGEMENT SYSTEM RECEIVES SIGNALS FROM ENGINE CONDITION SENSORS
402
OSCILLATE AIR/ FUEL RATIO
404
REVERSE DIRECTION OF CHANGE OF AIR/FUEL RATIO UPON CHANGE IN SENSOR SIGNAL
406
414
DETERMINE TEMPORAL CHARACTERISTICS OF THE AIR/FUEL OSCILLATION
408
COMPARE THIS WITH STORED VALUES
410
DETERMINE WHETHER ENGINE IS BEING CORRECTLY FUELLED
412

Fig. 13

MOTOR VEHICLE ENGINE SYNCHRONIZATION

BACKGROUND OF THE INVENTION

This invention relates to an engine management system capable of determining engine phase of a four-stroke internal combustion engine.

When a four-stroke engine is running it is desirable to verify that the fuel is being injected into the cylinder during the correct stroke. If the fuel is injected on the wrong stroke, the engine may still run, but engine emissions will increase and engine efficiency will drop.

Internal combustion engines normally have a crankshaft sensor that provides a rotation signal that can be used to verify piston position over one complete revolution of the engine. However, as the engine takes two full revolutions to complete one cycle of the four strokes, this type of engine sensor cannot distinguish between the two halves of the cycle, sometimes referred to as the charge phase in which the cylinder is charged with air and fuel, and the power phase in which the air fuel charge is ignited and expelled as exhaust gas.

Currently, the most common method of verifying the engine injection stroke is through the use of a camshaft position sensor. However, a camshaft sensor with associated wiring and engine machining adds cost to the engine.

It is possible to achieve engine stoke verification without the use of a camshaft position sensor, for example as described in patent document EP 0 990 787 A. This document describes a method of determining engine stroke in which one cylinder of the engine is made to run for one cycle at non-optimal conditions. This causes a sharp change in the operating conditions for that cylinder, which can then be detected by measuring a momentary change in the composition of the exhaust gas. The time at which the exhaust gas composition changes can then be used to determine the engine stroke on which exhaust gas is expelled by the affected cylinder.

This method suffers from a number of problems, primarily the need to operate the engine in a non-optimal manner and the need for steady state operation before, during and after the test.

SUMMARY OF THE INVENTION

The present invention seeks to provide an improved system for engine injection stroke verification.

According to the present invention there is provided an internal combustion engine, comprising a number of cylinders, the or each cylinder containing a four-stroke reciprocating piston, an exhaust conduit, one or more engine operating condition sensors including an exhaust gas sensor in the exhaust conduit for measuring the composition of the exhaust gas, a fuel injection system, and an engine management system for controlling the operation of the engine including the fuel injection system and the air/fuel ratio for at least one cylinder, wherein the engine management system contains engine operation data, the engine operation data being related to expected engine operation with engine fueling on the correct stroke and/or engine fueling on an incorrect stroke, and the engine management system is arranged to:

a) receive from said sensor(s) respective signal(s);

b) oscillate the air/fuel ratio between a relatively rich level and a relatively lean level, the exhaust gas composition varying depending on the air/fuel ratio;

c) reverse the direction of change of the air/fuel ratio when the exhaust gas composition is sensed as being indicative of rich engine operation or lean engine operation;

d) determine the temporal characteristics of the oscillation in the air/fuel ratio; and

e) determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data.

This procedure provides the advantage that there is no need for a camshaft position sensor, and the engine is no longer required to operate in a non-optimal manner to provide the engine management with the signals required to verify whether or not the engine is being fueled on the correct stroke by comparing.

The term "engine management system" will be understood to mean any electronic system capable of controlling or influencing the operation of the engine.

The term "temporal characteristics" should be understood as meaning period or frequency or any other time dependent characteristic that could be used to characterise an oscillating system.

It should be understood that the term "measure exhaust gas composition" means to measure one or more of the composition characteristics of the exhaust gas composition such as the concentration of one or more of the component gases.

In order to meet modern emission standards, engine control systems utilize a form of closed loop control to control the air/fuel ratio of the cylinder charge. The use of closed loop control is dictated by the use of three way catalytic converters which require very accurate control of the air/fuel ratio around a stoichiometric value in order to operate at their maximum efficiency. Closed loop control uses feedback from a sensor at an output from a system to control an input to the system that affects the signal from the output, sensor. In this case the output sensor measures the exhaust gas composition and the engine management system varies the air/fuel ratio accordingly. In open loop control the engine management system does not take account of the signal from the exhaust gas sensor when setting the air/fuel ratio. Open loop control may occur, for instance, when the driver presses the throttle.

A simplified model of a three way catalyst is that of an oxygen storage device. When the engine air fuel ratio is lean, the exhaust gas is rich in NO_x gases and the oxygen storage sites in the catalyst remove the oxygen in the NO_x gasses to create harmless N_2. Some of the excess oxygen in the exhaust gases is also stored in this phase. When the air fuel ratio is rich, the exhaust gases are rich in hydrocarbons HC, CO and H_2. The oxygen stored on the catalyst is then released to react with these gases to form CO_2 and H_2O.

To measure the exhaust gas composition, it is common to use a Heated Exhaust Gas Oxygen (HEGO) sensor. Commonly available HEGO sensors have a steep change in output around the oxygen concentration resulting from near stoichiometric air/fuel ratio. This results in the HEGO sensor being used in a bi-stable manner to detect either rich or lean combustion, rather than provide an absolute indication of air/fuel ratio.

In a closed loop feedback system this bi-stable HEGO sensor is ideally suited to a limit cycle operation oscillating the air/fuel ratio about the stoichiometric value. The present invention takes advantage of the fact that there is already a slight oscillation in engine operating conditions to determine whether or not the engine is being fueled on the correct stroke. For this invention to be applicable there is no need

for the engine to be fitted with a catalyst, but the invention is particularly suited to an engine that does have a catalyst fitted.

The engine management system may calculate a transport delay time from the period or frequency of the reversal of the oscillation in the air/fuel ratio, this being indicative of the time taken for a change in engine operating conditions to alter the composition of the exhaust gas at the exhaust gas sensor. This transport delay, time may then be compared with the engine operation data stored in the engine management system to determine whether or not the engine is being fueled on the correct stroke.

The exhaust sensor may be an exhaust gas oxygen sensor, but may be any other sensor capable of detecting changes in the composition of the exhaust gases that relate to a change in the air/fuel ratio in the engine.

These types of sensors are common in the field and can be obtained cheaply. It is also usual for the engine to have an exhaust gas oxygen sensor in the exhaust conduit as part of the engine management system and this method would simply make greater use of it.

The period or frequency of the reversal cycle data may be averaged over a plurality of oscillations in the air/fuel ratio. This reduces the effect of signal noise inherent in real systems so that the data used in comparisons is more reliable.

The period or frequency data of an oscillation in the air/fuel ratio may be ignored by the engine management system if said period or frequency is outside a predetermined range. Large signal errors and instances of open loop control can then be ignored by the engine management system so these errors do not alter the final results.

The engine management system may have a range of pre-determined engine operating conditions during which it calculates the period or frequency of the reversal cycle or the transport delay and stores this for future reference. This avoids errors in readings taken at extremes of the operating range of the engine.

The stored, period or frequency of the reversal cycle or the transport delay is used to calculate an average error for those particular engine operating conditions for which data is recorded. The engine management system is then used to determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data. This provides the same benefits as the averaging operation described above.

The fuel injection system may be a direct injection system, in which fuel is injected directly in to the cylinders. In a preferred embodiment of the invention, however, the fuel injection system is an indirect injection system in which fuel is injected into an inlet port for each cylinder.

According to another aspect of the invention there is provided a method of operating an internal combustion engine the engine comprising a number of cylinders, the or each cylinder containing a four-stroke reciprocating piston, one or more engine operating condition sensors including an exhaust gas sensor, a fuel injection system, and an engine management system, wherein the engine management system contains engine operation data, the engine operation data being related to expected engine operation with engine fueling on the correct stroke and/or engine fueling on an incorrect stroke, wherein the method comprises the steps of:

a) using the engine management system to control the operation of the engine including the fuel injection system and the air/fuel ratio for at least one cylinder;

b) sending to the engine management system from said sensor(s) respective signal(s) indicative of engine operating conditions, including exhaust gas composition;

c) oscillating the air/fuel ratio between a relatively rich level and a relatively lean level, the exhaust gas composition varying depending on the air/fuel ratio;

d) using the engine management system to reverse the direction of change of the air/fuel ratio when the exhaust gas composition is sensed as being indicative of rich engine operation or lean engine operation;

e) using the engine management system to determine the temporal characteristics of the oscillation in the air/fuel ratio; and

f) using the engine management system to determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data.

The invention will now be further described, by way of example, with reference to the accompanying drawings, in which:

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** is a schematic view of a four-stroke internal combustion engine according to the invention, having, a crank shaft sensor and an exhaust gas oxygen sensor in an exhaust outlet, both sensors providing a signal to an engine management system;

FIG. **2** is a plot of the output of the exhaust gas sensor against air/fuel ratio;

FIGS. **3** and **4** illustrate a conventional method of controlling air/fuel ratio close to a stoichiometric level for the purposes of emission reduction, showing respectively an idealised cyclic variation in air/fuel ratio and a corresponding ideal signal generated by the exhaust gas sensor;

FIGS. **5** and **6** illustrate how FIGS. **3** and **4** are altered by fueling of the engine on the incorrect engine phase;

FIG. **7** shows how the variation in air/fuel ratio changes when the engine is set to run for lean operation;

FIGS. **8**, **9** and **10** show plots of real data from an internal combustion engine for, respectively, the measured inlet air fuel ratio, the exhaust gas sensor signal, and a count of engine cycles between points where the signal from the exhaust gas sensor alternates;

FIGS. **11** and **12** show plots of real data, respectively, of how the count of engine cycles in FIG. **10** can be averaged over time to distinguish between in-phase and out-of-phase engine operation, and how these counts change with engine speed; and

FIG. **13** shows a flow chart illustrating a method according the invention for determining whether or not the engine is being fueled on the correct stroke.

DETAILED DESCRIPTION OF THE PREFERRED EMBODIMENT

FIG. **1** shows a schematic view of a four-stroke internal combustion engine **2**, having a crankshaft sensor **4** and an exhaust gas oxygen sensor **6** in an exhaust outlet **8**, both sensors providing respective signals **10**, **12** to an engine management system **14**. Also shown in the drawing is a cylinder **16** with an associated piston **18** and indirect fuel injection means **20** for injecting a fine mist of fuel into the inlet conduit **22**. Although in this drawing only one cylinder is shown, it should be understood that there will usually be more than one cylinder.

Air is supplied to the cylinder **16** via an inlet conduit **22**, the air is mixed with fuel from the fuel injection means **20** in a ratio defined by the engine management system **14**. This

air/fuel mixture is then drawn into the cylinder **16**, compressed by the piston **18** and ignited by a spark plug **24**. The ignited air/fuel mixture expands and forces the piston down giving power to a crankshaft **26**. Exhaust gases **27** created by this combustion event are expelled from the cylinder via the exhaust outlet **8**, where the level of oxygen present in the exhaust gases is measured by the exhaust gas oxygen sensor **6**. The exhaust gas oxygen sensor signal **12** is indicative of the level of oxygen in the exhaust gases. The engine management system **14** also receives a signal **10** from the crankshaft sensor **4**, which enables the engine management system **14** to calculate the position of the crankshaft **26** and hence the position of the piston **16**.

FIG. **2** shows a plot **56** of the output signal (S) **12** of the exhaust gas oxygen sensor **6** against air/fuel ratio λ. The air/fuel ratio λ is defined as the actual air/fuel ratio divided by the air/fuel ratio needed for a chemical reaction in the correct stoichiometric ratio. As can be seen from the plot the sensor output signal **12** changes rapidly at an air/fuel ratio λ=1. This behaviour effectively makes the exhaust gas oxygen sensor **6** bi-stable. This means that this exhaust gas oxygen sensor **6** is suited for use in closed loop control of an engine with the air/fuel ratio being oscillated about the stoichiometric value λ=1.

FIGS. **3** and **4** illustrate a conventional method of controlling air/fuel ratio λ close to a stoichiometric level λ=1 for the purposes of emission control, showing respectively an idealised cyclic variation in air/fuel λ against time (t) ratio and a corresponding ideal signal S_i generated by the exhaust gas oxygen sensor **6**. It can be seen from FIG. **3** that the air/fuel ratio λ varies cyclically between a rich phase **60** (λ<1) and a lean phase **62** (λ>1). Starting at the beginning of one of the rich phases **60**, the stoichiometric value is λ=1 and then falls at a constant rate for a time t_R. During this time t_R, the sensor output signal (S_i) **12** is steady at a low level S_i=0, indicating combustion products from earlier lean engine operation are still present in the exhaust outlet **8** around the exhaust gas oxygen sensor **6**.

During time t_R, combustion products from rich operation are transported along the exhaust outlet **8** towards the exhaust gas oxygen sensor **6**. The end of the rich phase occurs when these rich combustion products reach the exhaust gas oxygen sensor **6** the sensor signal S_i rapidly switches from low S_i=0 to high S_i=1. This switch is detected by the engine management system **14** and, in response, the engine management system **14** causes the air/fuel ratio λ to rise suddenly to the stoichiometric value λ=1. The air/fuel ratio λ is steadily increased until the sensor output signal **12** suddenly changes from high S_i=1 to low S_i=0, in response to the combustion products from the lean operation reaching the exhaust gas oxygen sensor **6**. At this point the air/fuel ratio λ is dropped to the stoichiometric value λ=1 and then steadily falls.

This cycle continues while the closed loop control is still in force. The air/fuel ratio λ varies between a relatively rich value **122** of about 0.96 and a relatively lean value **124** of about 1.04. This defines a standard operating range **126** for the air/fuel ratio λ. The time intervals **120** between the sudden changes in sensor output are related to the time delay for the combustion products to reach the exhaust gas oxygen sensor **6** following a change in engine conditions. This time delay **120** is called the transport delay time for the engine.

FIGS. **5** and **6** illustrate how FIGS. **3** and **4** are altered by fueling of the engine on the incorrect engine phase. Starting at the beginning of one of the lean phases **121**, the stoichiometric value is λ=1 and is increased at a constant rate for a time t_{L1}. During this time t_{L1}, the sensor output **12** is steady at a high level S_i=1, indicating combustion products from earlier rich engine operation are still present in the exhaust outlet **8** around the exhaust gas oxygen sensor **6**. When the combustion products from the lean operation reach the exhaust gas oxygen sensor **6**, the sensor output signal S_i will change to low S_i=0 causing a change in the air/fuel ratio λ.

In this case, due to the fueling of the engine on the incorrect stroke, there is an additional time delay **136** between the engine management system **14** altering the air/fuel ratio λ and the combustion products from this altered ratio affecting the sensor output signal **12**. This additional time delay **136** is due to the fuel injection occurring at the incorrect engine phase, for example, near the end of the compression stroke, rather than near the end of the exhaust stroke. As a result, the inlet valve **23** is not open at or shortly after the injection of fuel into the air inlet **22**, and the fuel pools behind the inlet valve **23**. When the inlet valve **23** opens this pool is drawn into the cylinder **16**. Since the fuel is no longer in a fine mist when it enters the cylinder **16**, the combustion event will be less efficient. The additional time delay **136** measured is the time between the injection of the fuel and the introduction of the fuel into the cylinder **16**.

The increased time delay **142** between these sudden changes in engine condition for the incorrectly fueled engine is greater than the time delay **120** in the correctly fueled engine. This leads to the air/fuel ratio λ varying over a range **144** than is the case for a correctly fueled engine.

FIG. **7** shows how the variation in air/fuel ratio λ changes when the engine is set to run for lean operation. Starting at the beginning of one of the lean phases **164**, the air/fuel ratio is the stoichiometric value λ=1 and then rises at a constant rate for a time t_{L2}. When the combustion products from this lean operation reach the exhaust gas oxygen sensor **6** and the sensor signal **12** changes from high S_i=1 to low S_i=0 the engine management system **14** drops the air/fuel ratio to a level above the stoichiometric value λ=1. The air/fuel ratio then falls at a steady rate to begin rich operation of the engine during a time T_{R2} until a further sensor switch is detected. When the air/fuel ratio λ suddenly rises to the stoichiometric level λ=1 and begins to rise steadily.

The extra period of lean fueling **158** gives rise to a measured transport delay time which can take one of two values **160**, **162**. During the cycle with no extra lean fueling **164**, the time delay **160** is less that the time delay **162** due to the period of extra lean fueling **158** occurring during the cycle **166**.

It has been found during test track evaluation of a motor car incorporating the invention, that this lean bias effect does not cause problems in the verification of whether or not the engine is fueled on the correct stroke.

FIG. **8** shows a plot of real data taken during evaluation of the motor car incorporating the invention. The plot shows that the measured air/fuel ratio λ_a against time t has a "saw-tooth" shape. This is to be expected from the conventional method of closed-loop control of an engine outlined above. This plot shows that there is significant noise in the sensor outputs that could potentially affect the results. In this case, it can be seen that the measured air/fuel ratio λ_a does not oscillate symmetrically above and below about the stoichiometric value λ_a=1. A typical jump in the air/fuel ratio is highlighted **204** to show that even with the associated signal noise it is possible to define a point at which the sudden jump in the air/fuel ratio occurs. This then allows the calculation of the time taken between these jumps so that the fueling stroke of the engine can be verified.

FIG. **9** shows the actual exhaust gas sensor output S_a associated with the plot in FIG. **8**. The inherent noise in the system can be seen since the plot is not regular. A typical change in the sensor output is highlighted **208** to show that the sensor switch point can be determined even through signal noise. This allows the engine management system **14** to determine the time at which the switch in engine operating conditions should occur.

FIG. **10** shows the counted number of engine cycles N between the sensor switches rising **212** and being reset **214** when a sudden change in sensor output is detected. The plot **218**, represents the expected number of engine cycles between the sensor switch events. As can be seen, the actual number of engine cycles between sensor switches, defined by the height of the peaks **216** in the plot of engine cycle count N against time t, varies above and below the expected number of cycles **218**. The difference between the peak height **216** and the expected number of engine cycles. **218** defines an error. This error could be due to noise, or due to incorrect fueling of the engine producing an associated extra transport time delay.

FIG. **11** shows how a cumulative average difference λ between the actual number of engine cycles and the expected number of engine cycles between the sudden changes in sensor output, as calculated from FIG. **10**, may be averaged to lessen the effects of noise in the system. There are two plots **254,256** shown in FIG. **11**. The first plot **254** is obtained from an engine being correctly fueled and the second plot **256** is obtained from an engine being incorrectly fueled. These plots show how the average errors, for a test carried out at a constant engine speed, converge over time **201**. Initially, [**250**] the difference **250** varies greatly, but the later difference **250** is relatively steady. The plot **254** from the correctly fueled engine is consistently closer to a zero difference line **258** than the plot **256** from the incorrectly fueled engine. This shows that after averaging the difference between the expected delay time and the actual delay time it is possible to differentiate between a correctly fueled engine and an engine that is being incorrectly fueled, by the mean error away from the zero differences line **258**.

FIG. **12** shows two plots **304,305** of the variation in the average difference λ_a, over a number of readings, between the expected and actual number of engine cycles between the sudden changes in sensor output as shown in FIG. **11** and the engine speed V. Both plots **304,305** show that the average difference λ_a decreases as engine speed V increases. The plot **304** from an engine that is correctly fueled is consistently above the plot **305** from an engine that is being incorrectly fueled. As a result of this distinct difference, an engine can be characterized over a wide range of engine operating conditions in terms of the time taken between the sudden changes in engine output for both correct phase fueling and incorrect phase fueling. Once such a characterisation has been made, it is possible to establish whether or not the engine is being correctly fueled by analysing the frequency or period of the sudden changes in exhaust gas sensor output, and then comparing this with an expected frequency or period.

FIG. **13** shows a flow chart illustrating a method according the invention for determining whether or not the engine is being fueled on the correct stroke.

The engine management system is used to control the operation of the engine **400** including the fuel injection system and the air/fuel ratio for at least one cylinder.

The engine management system receives signals **402** indicative of the engine operating conditions from sensors positioned around the engine, including at least one exhaust gas composition sensor.

The air/fuel ratio is oscillated **404** between a relatively rich level and a relatively lean level, the exhaust gas composition varying depending on the air/fuel ratio.

The direction of change of the air/fuel ratio is reversed **406** when the exhaust gas composition is sensed as being indicative of rich engine operation or lean engine operation.

The temporal characteristics of the oscillation in the air/fuel ratio are then determined **408** and this temporal characteristic is then compared **410** with stored engine operating condition data, in order to determine **412** whether or not the engine is being fueled on the correct stroke. The cycle then returns **414** to the start.

If it is determined that the engine is being incorrectly fueled, the engine management system can then be used to adjust automatically the fueling stroke of the engine.

The invention therefore provides a convenient way to sense and correct fueling on an incorrect stoke of the engine.

What is claimed is:

1. An internal combustion engine, comprising a number of cylinders, the or each cylinder containing a four-stroke reciprocating piston, an exhaust conduit, one or more engine operating condition sensors including an exhaust gas sensor in the exhaust conduit for measuring the composition of the exhaust gas, a fuel injection system, and an engine management system for controlling the operation of the engine including the fuel injection system and the air/fuel ratio for at least one cylinder, wherein the engine management system contains engine operation data, the engine operation data being related to expected engine operation with engine fueling on the correct stroke and/or engine fueling on an incorrect stroke, and the engine management system is arranged to:

a) receive from said sensor(s) respective signal(s);

b) oscillate the air/fuel ratio between a relatively rich level and a relatively lean level, the exhaust gas composition varying depending on the air/fuel ratio;

c) reverse the direction of change of the air/fuel ratio when the exhaust gas composition is sensed as being indicative of rich engine operation or lean engine operation;

d) determine the temporal characteristics of the oscillation in the air/fuel ratio; and

e) determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data.

2. A method of operating an internal combustion engine the engine comprising a number of cylinders, the or each cylinder containing a four-stroke reciprocating piston, one or more engine operating condition sensors including an exhaust gas sensor, a fuel injection system, and an engine management system, wherein the engine management system contains engine operation data, the engine operation data being related to expected engine operation with engine fueling on the correct stroke and/or engine fueling on an incorrect stroke, wherein the method comprises the steps of:

a) using the engine management system to control the operation of the engine including the fuel injection system and the air/fuel ratio for at least one cylinder;

b) sending to the engine management system from said sensor(s) respective signal(s) indicative of engine operating conditions, including exhaust gas composition;

c) oscillating the air/fuel ratio between a relatively rich level and a relatively lean level, the exhaust gas composition varying depending on the air/fuel ratio;

d) using the engine management system to reverse the direction of change of the air/fuel ratio when the exhaust gas composition is sensed as being indicative of rich engine operation or lean engine operation;

e) using the engine management system to determine the temporal characteristics of the oscillation in the air/fuel ratio; and

f) using the engine management system to determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data.

3. A method as claimed in claim 2, comprising the additional steps of calculating the engine management system transport delay time from the period or frequency of the reversal of the oscillation in the air/fuel ratio, this transport delay time being indicative of the time taken for a change in engine operating conditions to alter the composition of the exhaust gas at the exhaust gas sensor, this transport delay time; and

comparing the engine management transport delay with the engine operation data stored in the engine management system to determine whether or not the engine is being fueled on the correct stroke.

4. A method as claimed in claim 2, wherein the exhaust sensor is an exhaust gas oxygen sensor.

5. A method as claimed in claim 2, wherein the period or frequency of the reversal cycle data is averaged over a plurality of oscillations in the air/fuel ratio.

6. A method as claimed in claim 2, wherein the period or frequency data of an oscillation in the air/fuel ratio is ignored by the engine management system if said period or frequency is outside a pre-determined range.

7. A method as claimed in claim 2, wherein the engine management system has a range of pre-determined engine operating conditions during which it calculates the period or frequency of the reversal cycle or the transport delay and stores it for future reference.

8. A method as claimed in claim 7, wherein the stored period or frequency of the reversal cycle or the transport delay is used to calculate an average error for those particular engine operating conditions for which data is recorded and using the engine management system to determine whether or not the engine is being fueled on the correct stroke by comparing said temporal characteristics with said relevant engine operation data.

* * * * *

EXHIBIT 11



US008909482B2

(12) **United States Patent**
Maso Portabella

(10) **Patent No.: US 8,909,482 B2**
(45) **Date of Patent: Dec. 9, 2014**

(54) **DEVICE FOR MEASURING POWER CONSUMPTION AND PERFORMANCE WITH RESPECT TO THE ENVIRONMENT OF A POWER-CONSUMING UNIT**

(76) Inventor: **Alberto Maso Portabella**, Cabrera de Mar (ES)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 932 days.

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G01R 21/00	(2006.01)
G01D 4/00	(2006.01)
G07C 5/02	(2006.01)
G01R 22/10	(2006.01)

(52) **U.S. Cl.**

CPC .. ***G01D 4/00*** (2013.01); ***G07C 5/02*** (2013.01); *G01R 22/10* (2013.01); *Y02B 70/343* (2013.01); *Y04S 20/34* (2013.01)

USPC **702/19**; 702/45; 702/61; 705/7.39

(58) **Field of Classification Search**

CPC G01D 4/00; G01R 22/10; G07C 5/02; Y02B 70/343; Y04S 20/34; G06Q 10/00; G06Q 10/063; G06Q 10/0639; G06Q 10/06393; G06Q 30/02

USPC 702/19, 24, 45, 50, 57, 60, 61, 68, 179, 702/182, 183, 188; 705/7.11, 7.12, 7.39, 10

See application file for complete search history.

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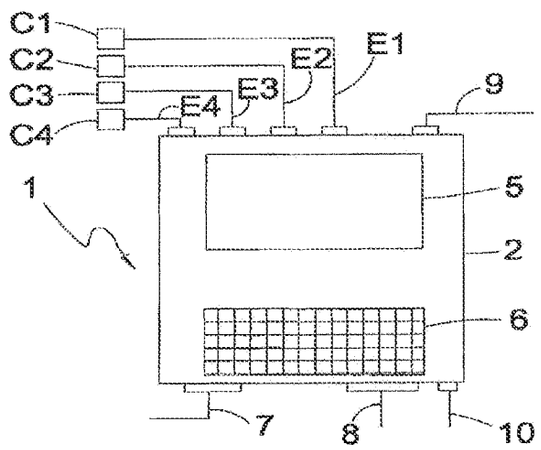
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(57) **ABSTRACT**

A device for measuring power consumption and performance relative to the environment of a power-consuming unit is disclosed. The device includes a data introduction module, a keyboard for inputting data of waste production in the power-consuming unit, a computing device, and a display screen. The computing device receives the data from the data introduction module and the keyboard. The computing device is also provided with a processor that includes a calculation algorithm of an ecological parameter indicative of the performance relative to the environment of the power-consuming unit. The ecological parameter is calculated as an adjusted mean of the sum of the deviations of each power consumption value with respect to a reference value.

16 Claims, 1 Drawing Sheet



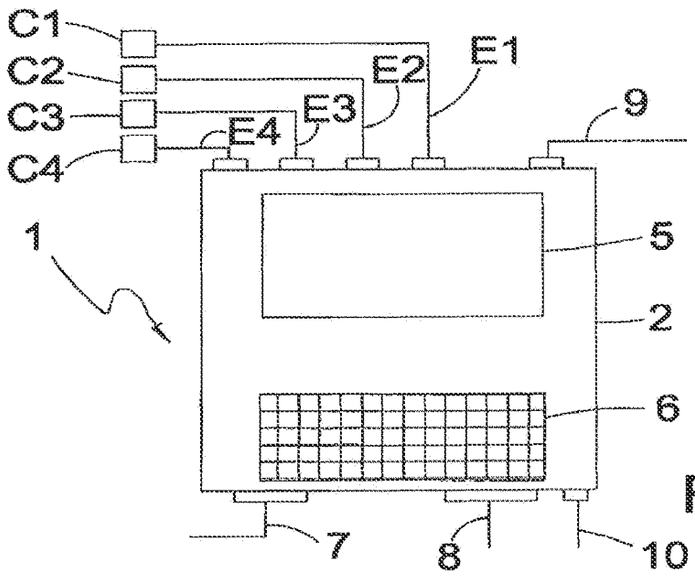


FIG. 1

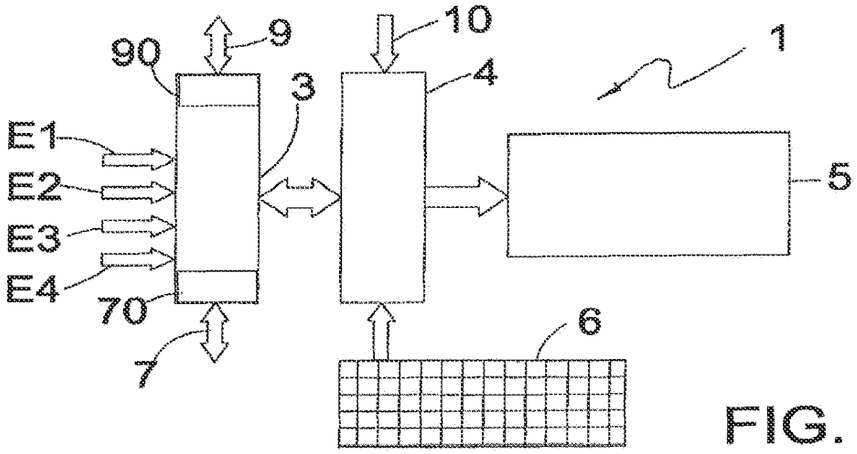


FIG. 2

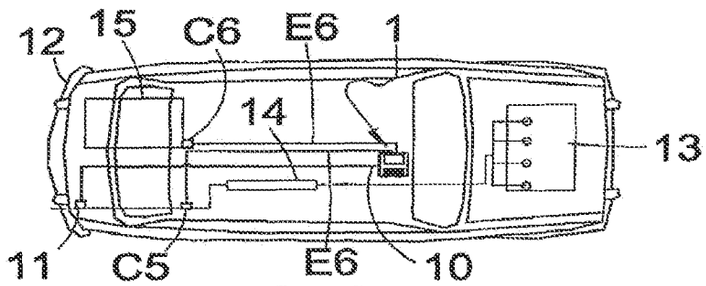


FIG. 3

DEVICE FOR MEASURING POWER CONSUMPTION AND PERFORMANCE WITH RESPECT TO THE ENVIRONMENT OF A POWER-CONSUMING UNIT

CROSS-REFERENCE TO RELATED APPLICATIONS

This application is a U.S. national counterpart application of International Application Serial No. PCT/IB2009/053135 filed Jul. 20, 2009, which claims priority to Spanish Patent Application No. P200802159 filed Jul. 18, 2008. The disclosures of PCT/IB2009/053135 and P200802159 are each hereby incorporated herein by reference.

TECHNICAL FIELD OF THE INVENTION

The invention refers to a device for measuring power consumptions and the respectful performance with the environment, from the measured power consumptions and from the residue production, in a power-consuming unit, a comfort unit (houses, communities, hotels and restaurants, spare time spaces, sport spaces, private and public motor vehicles, and goods vehicles, etc.) or a production unit.

The measurement of this respectful performance or not will be carried out by what we call "ecological parameter", from the measurements of power consumptions (direct consumptions) and of residues and wastes generated (indirect consumptions).

BACKGROUND OF THE INVENTION

A lot of kinds of programmable logic controllers (PLC) are known, in which, from state variables, the outputs of a system are determined (e.g. electrical or pneumatic, etc.) or another kinds of devices that, from inputs, calculate the values of parameters of interest. Examples of these devices can be the known analysers of the electric mains that, from voltage, current intensity and imbalance signals, calculate the electric power consumed by an installation.

However, in the current art no device or apparatus that calculates ecological parameters or magnitudes are known, from power consumption and waste production data in installations, if we define such parameters or magnitudes as a deviation of an optimal work point, defined by an authority. E.g. in which the emissions of COx or NOx are the minimum possible according to its use.

The present invention has the objective to provide an apparatus that fulfils this gap in the current art.

DESCRIPTION OF THE INVENTION

To this end, the object of the present invention is a new measuring device of power consumptions which calculates ecological parameters from the measured power consumptions and from data of waste production, that it is characterised in that it comprises:

- a module for introducing data, provided with at least an input of a value of power consumption of a magnitude, measured in a corresponding measure apparatus of the power-consuming unit;
- a keyboard for introducing data and for selecting menus, in particular data of waste production of the power-consuming unit;
- computing means that receive the data from the module for introducing data and from the keyboard, and which are provided with a processor that incorporates an algorithm for calculating an ecological parameter from the measured consumptions of power and from the introduced data; and
- a display screen, communicated with the computing means, to display the calculated ecological parameter,

said ecological parameter being calculated as an adjusted mean of the sum of the deviations of each value of power consumption measured according to a reference value.

According to another feature of the present invention, the device comprises means for introducing values of waste production.

In preferred variants, the device can comprise an input module of a data bus of a domotic network and/or an USB input for a serial data bus.

In an embodiment of domestic or industrial application, the device comprises at least:

- an input of water flow, from a corresponding first water flow meter;
- an input of combustible gas or liquid flow, from a second flow meter of combustible gases or liquids;
- an input for measuring the electric power, from a counter or analyser of the electric mains; and
- an input for measuring the waste weight from a waste weighting scales.

In an alternative embodiment, for calculating the ecological parameter corresponding to the automobile, the device can comprise at least:

- an input of fuel flow, from a corresponding fuel flow meter of the motor of an automobile; and
- means for introducing the production of wastes of the automobile, selected among: tire rubber wastes, motor oil wastes, coolant liquid wastes, and liquid wastes from closed circuits, not being limitative, by the keyboard of the device.

The device can be provided also with an input for exhaust gases flow, from a corresponding flow meter of exhaust gases of the combustion system of the power-consuming unit, e.g. the motor exhaust; and a gas analyser for measuring the emission levels of gases and particles. The device compares the result obtained with the admitted levels by the current regulations. The incorrect combustion and the subsequent excess of gas (CO_2 and NOx) and particle emission is not a user o producer question, because they do not influence on it, but the direct responsible is the manufacturer of the apparatus or combustion chamber. However, additionally, the repair of said anomaly it is a question of the user or producer. This does not affect only to explosion motors and domestic boilers, but also to production and transformation centres.

Preferably, the device comprises a shell, which encloses the several components of the device, and in whose external part it is the keyboard and the screen.

BRIEF DESCRIPTION OF THE DRAWINGS

In the attached drawings some embodiments of the device object of the present invention are shown, only as a non-limitative example. In said drawings:

FIG. **1** is a diagrammatical frontal elevation view, that shows a device according to the present invention, with its particular connections;

FIG. **2** is a block diagram that shows the internal composition of the device of FIG. **1**; and

FIG. **3** is a diagrammatical view that shows an alternative embodiment of the invention, to be applied to automobiles.

DETAILED DESCRIPTION OF THE DRAWINGS

In said drawings it can be seen the constitution and way of the device for measuring power consumptions and for calcu-

lating ecological parameters from the measured power consumptions, directly and indirectly.

The function of the device of the invention is to measure the respectful performance or behaviour with the environment of a "power-consuming unit". The "power-consuming unit" object of the measure of the possible different power manifestations consumed can be: a house, community (hotel, sports centre, residence, sport spaces, shops, department stores, leisure halls, office buildings and studies, administration centres, etc.), transformation centres, garages, warehouses, motor vehicles or transportation means, private or public.

The basis of the operation will consist in the comparison between the basic actual power flow required by the user for their comfort with the normal consumptions of the current regulations (e.g. UNE, ISO standards, etc.) and/or those recommended by official administrations of the place and/or statistic data of normal power consumptions and wastes (hereinafter "typical consumptions" including flows of power consumptions and wastes by person or by own unit of a preset population).

For this comparison, the proposed system will calculate an ecological parameter as an adjusted mean of the sum of deviations of each value of power consumption measured with respect to its corresponding reference value.

To this end, as seen in FIGS. **1** and **2**, the device **1** of the present invention is constituted by an apparatus that has a carcass or shell **2** that includes the following elements:

- a module for introducing data **3**, provided with inputs of power consumption values E**1**, E**2**, E**3**, E**4**, of magnitudes, measured with corresponding measure apparatuses C**1**, C**2**, C**3**, C**4**;
- a keyboard **6** for introducing data and selecting menus, in particular data of the waste production;
- computing means **4**, which receive the data from the module for introducing data **3** and the keyboard **6**, and which are provided with a processor that includes a calculation algorithm for an ecological parameter from the measured power consumptions and from the introduced data; and
- a display screen **5**, communicated with said computing means **4**, to display the calculated ecological parameter.

The apparatus is provided with an electrical source **8**, as it can be seen in FIG. **1**.

To introduce values of the waste production one of the inputs E**4**, digital o analogical, can be used, from a weight apparatus, or they can be introduced manually through a keyboard **6**.

The device **1** can be complemented with an input module **90** for a data bus **9** of a domotic network, and an USB input **70** for a serial data bus **7**, or equivalent means.

In FIG. **1** a device particularly devised for special application in buildings (houses, hotel, sports centre, residence, sport spaces, shops, department stores, leisure halls, office buildings and studies, administration centres, transformation centres, garages, warehouses, etc.) is shown. The shell **2** comprises at least the following inputs:

- an input for the water flow E**1** from a corresponding first water flow meter C**1**;
- an input for the flow of combustible gases or liquids E**2**, from the second flow meter of combustible gases or liquids C**2**;
- an input for measuring electric power E**3**, from a meter or analyser of electric mains C**3**; and
- an input for measuring the weight of the wastes E**4** from a scale for weighting wastes C**4**,

through the corresponding ports.

The number of inputs will be according to the complexity of the installation. E.g. an industry can be provided with several devices of this kind, each associated with a corresponding section or unit of factory, warehouse, office, etc.

In FIG. **3** is shown an embodiment particularly devised for automobiles, in which the device comprises an input for fuel flow E**6**, from a flow meter C**6** of fuel, associated with the fuel deposit **15**; and an input of exhaust gases flow E**5**, from a corresponding flow meter C**5** of exhaust gases from the exhaust system **14** of the motor **13** of an automobile **12**. In this case, by the keyboard the flows of production of wastes of the automobile are introduced manually, e.g. rubber wastes from tires, oil wastes for the motor, coolant liquid wastes, liquid wastes from closed circuits and metal wastes.

The device has an input **10** to collect, optionally, the information from a gas analyser **11** (FIG. **3**), to measure the emission levels of gases and particles. The device **1** compares the result obtained with the levels admitted by the current regulations.

The algorithm of the computing means **4** calculates the total basic power flow obtained by the evaluation of the flows of the measured power statements E**1**, E**2**, E**3**, E**4**, E**5**, E**6**, etc. referred to the number of people which use the unit to be measured in each trip. The calculation algorithm integrates all the trips of a time period to be defined or the accumulated total.

Each flow has an incidence in the environment: the water consumption reverts in its exploitation, the fuel consumption is associated to the direct emissions of CO2, NOx, . . . , the electric consumption to the indirect emissions of CO2, NOx, . . . , and the waste production with indirect environmental impact, by the quantity of paper, glass, organic material, plastics, oils and greases, rubbers, metals, . . . expressed in kg that the unit consumes per time unit and please who enjoy the comfort of said unit, to be compared with preset consumptions and wastes. These data, once treated by the algorithm, give as result an ecologic parameter (here denoted as "ecocom"), which is the measure of the conduct that respects our environment, to provide integral or combined or individual "ecocoms" of the comfort or production unit.

Regarding the presentation of the results, from said comparison the actual percentage of power consumption with respect to the preset consumption is obtained, indicating if the performance if respectful (equal to zero or higher) or abusive when it is negative. Both deviations are measured as a percentage. The percent unit of each flow is called positive "ecocom", if the result is respectful or negative if it is not. The algorithm of all the flows provides the integral "ecocom" of the unit. As there can be difficulties to receive all the flows, the algorithm of the ecometer could provide only "ecocoms" of water exploitation and/or direct gases emission and/or indirect gases emission and/or waste production. It is also possible to measure only those flows the public administration considers suitable to pay ecotaxes, evaluating a specific pack (which can be called "administrative ecocom").

The results are presented in a permanent way in a screen **5**, indicating the periods of time on which they are based: the day, week, month accumulations, etc. or others, such as those consumed by trip in the case of vehicles. To know better the power exploitation, by a menu the "ecocoms" can be displayed (day, week, month, . . .) of exploitation of the different power sources in the screen or to pass to other successive ones with the direct or indirect emissions or waste production.

As stated previously, if the consumption of only one or several flows are calculated for installation difficulties, the permanent screen shows that they are individual "ecocoms" by power source or waste, combined or integral.

Regarding the installation of different measuring apparatus C1, C2, C3, C4, C5, C6, etc., the water flow meter C1 is placed downstream the company meter and upstream the home key, with the possibility to emit impulses or analogical or digital or light signals. The same applies to C2, flow meter for combustible gases or liquids, to C3, electric measuring equipment in the general board of the unit, to C5, in the case of exhaust gases, and to C6, in the case of fuel flow of an automobile. The same can be said to the flow of waste production E4, which will be from a scale C4. The emission of information can be wireless or wired. In the case that there is no installation, by the keyboard 6 the flow data can be introduced manually.

Algorithm for Calculating the Ecologic Parameter

It results from the adjusted arithmetic mean of all the quotients between the absolute value of actual flows (E1, E2, E3, E4, E5, . . .) that enter in the power-consuming unit object of the measure of the different possible power statements, with respect to their flows or corresponding reference values. The absolute value of the actual flows is taken because the input power consumption has a sign opposed to a waste, which indirectly is associated to another external power consumption;

therefore, as the contributions of each factor is the interesting part, all are taken in flow absolute value, no the quotient of flows, not being compensated to each other.

It is defined:

1—Flow of an actual input power statement (E1, E2, E3, E4, E5, E6 . . . , generally Ei): As the power quantity consumed by the time unit of that power statement, by user of the unit to be measured. E.g. the drinkable consumed in a home is a power flow because it is obtained from the nature with transportation, purification cost . . . ; these power costs are environmental costs associated with that consumed water. The combustible liquid and gas flow consumed is another actual input power flow associated with the direct emissions of CO2 and others to the atmosphere. The electricity consumption is another power flow associated with the indirect emissions of CO2 and other not desired gases. The wastes are also indicative of the environmental impact and they indicate indirectly a power expense. E.g. it is also an input power flow the quantity of glass we waste for identical reasons, but not those that we buy and remains for the development of the activity in the unit. Also with the paper, the organic material, the containers, the spare parts from repairs, etc.

2—Preset flows (Ct): They would be the average power consumption desired or recommended by the public administration of the zone where the activity of each unit to be measured is developed. If there is no environmental policy defined by the administration, standard consumptions or statistic data of consumptions are taken, inputs as consumption and outputs of wastes per resident or per production unit.

3—Adjustment coefficient (Cpd): Each coefficient designs a deviation percentage with respect to the preset consumption of consumed power statement. But not all have the same importance according to:

3.1. Habits regarding the consumptions of the unit. In certain regions there are power statements that have an important weight due to the proximity to the resource, subsidized prices or no other sources . . . . In these situations it must take into account the suitable specific weight that the administration determines or the statistical data. In the case of a motor vehicle, the water consumption will be dismissed, but not the fuel liquids or gases consumptions, the oil and grease wastes and rubber and metal wastes.

3.2. The power policies of the zone. According to the administration of the place and attending if to the environment conduct is associated a tax; these quotients must be adjusted depending on which deviations the administration could reward or punish.

3.3. Practical ends. These coefficients can be zero, because the contribution of said consumption is so low that is dismissed, according to the cost of its measure. E.g. the quantity of organic wastes of an electric train.

The adjusted coefficients are between 0 and 1.

Hence, the ecologic parameter (or "ecocoms", in %), for "n" magnitudes of power consumption or wastes, is calculated applying the formula:

$$Ecocoms(\%) = 1/n \sum_{i=1}^{n} (Cpdx\{1 - \frac{Ei}{p \cdot Ct}\} \times 100$$

Wherein n is the number of power sources or waste variables to be measured (with adjustment coefficient different from zero) and p the people who enjoys the comfort of the unit to be measured.

Interpretation of the Results

The result is a percentage that when is close to zero the performance is desirable, but if it is negative and far away from zero, the performance of this unit is not respectful and the distance to zero is the degree of the separation from the desirable performance, and therefore it can be penalised and the penalization can be measured. Lastly, if it is positive the performance is excellent and the separation from zero is the measure of its environmental excellence.

The device for measuring power consumptions and the respectful performance, according to the present invention, can be used for the calculation of an ecologic parameter, which can be used as basis for the application of called ecotaxes by the administration.

What is claimed is:

1. A device for measuring power consumption and performance with respect to the environment of a power-consuming unit comprising

a data introduction module, which receives at least one power consumption value, measured by a measurement apparatus of the power-consuming unit;

an input device for inputting waste production data;

a display screen; and

a computer with a processor and a memory device having therein a plurality of instructions that, when executed by the processor, cause the processor to

(a) receive the at least one power consumption value from the data introduction module,

(b) receive the waste production data,

(c) calculate an ecological parameter based on the at least one power consumption value and the waste production data, and

(d) operate the display screen to display the calculated ecological parameter,

wherein the calculated ecological parameter is an adjusted mean of the sum of the deviations of the at least one power consumption value with respect to a reference value corresponding to the at least one power consumption value.

2. The device of claim 1 further comprising a means for introducing values of waste production.

3. The device of claim 1 further comprising an input module of a data bus in a domotic network.

4. The device of claim 1 further comprising USB input for a serial data bus.

5. The device of claim 1 wherein the at least one power consumption value includes at least two power consumption values selected from the group of:

a water flow input from a corresponding first water flow meter;

a combustible gas and liquid flow input from a second combustible gas and liquid flow meter;

an electric power measurement input from a meter or analyser of electric mains; and

a waste weight measurement input from a scale for weighting wastes.

6. The device of claim 1 wherein the at least one power consumption value includes a fuel flow input from a corresponding flow meter from the motor of an automobile;

the waste production data corresponds to one of tire rubber wastes, motor oil wastes, coolant liquid wastes, closed circuit liquid wastes and metal wastes, and

the fuel flow input and the waste production data are used to calculate the ecological parameter corresponding to the automobile.

7. The device of claim 6 wherein the at least one power consumption value includes an exhaust gas flow input from a corresponding flow meter of exhaust gases of a combustion system of the automobile.

8. The device of claim 1 further comprising a shell, which encloses the different components of the device, and in whose external part is the input device and the display screen.

9. A method of determining an ecological parameter by processor, comprising:

receiving a first power consumption value of a power consuming unit,

receiving a waste production data value,

determining an ecological parameter based on the first power consumption value and the waste production data value, and

outputting the determined ecological parameter for display on a display screen.

10. The method of claim 9, wherein the first power consumption value corresponds to one of water flow, combustible gas flow, and electric power flow.

11. The method of claim 10, wherein the waste production data corresponds to waste weight.

12. The method of claim 9, wherein the first power consumption value corresponds to one of fuel flow and exhaust gas flow.

13. The method of claim 12, wherein the waste production data corresponds to one of rubber wastes, oil waste, coolant liquid wastes, and metal.

14. The method of claim 12, further comprising generating a positive ecological parameter when a user expends less than a predetermined standard quantity of power consumed in a time period.

15. The method of claim 9, further comprising generating a negative ecological parameter when a user expends more than a predetermined standard quantity of power consumed in a time period.

16. The method of claim 9, wherein the ecological parameter is determined as a percentage

based on an algorithm of the form

$$\text{Parameter}(\%) = 1/n \sum_{p=1}^{n} \{Cpd \times \left(1 - \frac{Ei}{p \cdot Ci}\right)\} \times 100$$

wherein "Ei" is a power quantity consumed in a time period,

"Ci" is a predetermined standard quantity of power consumed in the time period,

"Cpd" is a coefficient corresponding to the predetermined standard quantity of power consumed in the time period,

"n" is a sum of a number of measured power sources and waste variables, and

"p" is a number of users of the measured power sources and waste variables.

* * * * *

PATENT APPLICATION FEE DETERMINATION RECORD
Effective October 1, 2001

Application or Docket Number: 10541-1524

CLAIMS AS FILED - PART I

	(Column 1)	(Column 2)
TOTAL CLAIMS	7	
FOR	NUMBER FILED	NUMBER EXTRA
TOTAL CHARGEABLE CLAIMS	7 minus 20=	*
INDEPENDENT CLAIMS	2 minus 3 =	*
MULTIPLE DEPENDENT CLAIM PRESENT		☐

* If the difference in column 1 is less than zero, enter "0" in column 2

SMALL ENTITY TYPE ☐		OR	OTHER THAN SMALL ENTITY	
RATE	FEE		RATE	FEE
BASIC FEE	370.00	OR	BASIC FEE	740.00
X$ 9=		OR	X$18=	
X42=		OR	X84=	
+140=		OR	+280=	
TOTAL		OR	TOTAL	740

CLAIMS AS AMENDED - PART II

AMENDMENT A

	(Column 1) CLAIMS REMAINING AFTER AMENDMENT		(Column 2) HIGHEST NUMBER PREVIOUSLY PAID FOR	(Column 3) PRESENT EXTRA
Total	*	Minus	**	=
Independent	*	Minus	***	=
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM				☐

SMALL ENTITY		OR	OTHER THAN SMALL ENTITY	
RATE	ADDITIONAL FEE		RATE	ADDITIONAL FEE
X$ 9=		OR	X$18=	
X42=		OR	X84=	
+140=		OR	+280=	
TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

AMENDMENT B

	(Column 1) CLAIMS REMAINING AFTER AMENDMENT		(Column 2) HIGHEST NUMBER PREVIOUSLY PAID FOR	(Column 3) PRESENT EXTRA
Total	*	Minus	**	=
Independent	*	Minus	***	=
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM				☐

RATE	ADDITIONAL FEE		RATE	ADDITIONAL FEE
X$ 9=		OR	X$18=	
X42=		OR	X84=	
+140=		OR	+280=	
TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

AMENDMENT C

	(Column 1) CLAIMS REMAINING AFTER AMENDMENT		(Column 2) HIGHEST NUMBER PREVIOUSLY PAID FOR	(Column 3) PRESENT EXTRA
Total	*	Minus	**	=
Independent	*	Minus	***	=
FIRST PRESENTATION OF MULTIPLE DEPENDENT CLAIM				☐

RATE	ADDITIONAL FEE		RATE	ADDITIONAL FEE
X$ 9=		OR	X$18=	
X42=		OR	X84=	
+140=		OR	+280=	
TOTAL ADDIT. FEE		OR	TOTAL ADDIT. FEE	

* If the entry in column 1 is less than the entry in column 2, write "0" in column 3.
** If the "Highest Number Previously Paid For" IN THIS SPACE is less than 20, enter "20."
***If the "Highest Number Previously Paid For" IN THIS SPACE is less than 3, enter "3."
The "Highest Number Previously Paid For" (Total or Independent) is the highest number found in the appropriate box in column 1.

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10886

Derwent Innovation Patent Export, 2022-09-19 13:50:56 +0000

Search results for: PNS=(US6581565);
Collections searched: DWPI; ARIPO grant; Argentina UM; Argentina app; Armenia grant; Australia UM; Australia app; Australia grant; Austria UM; Austria app; Austria grant; Belarus UM; Belarus grant; Belgium app; Belgium grant; Brazil UM; Brazil app; Brazil grant; Bulgaria UM; Bulgaria app; Bulgaria grant; Canada app; Canada grant; China, Mainland UM; China, Mainland app; China, Mainland grant; Colombia app; Costa Rica UM; Costa Rica app; Croatia UM; Croatia app; Croatia grant; Cuba app; Cuba grant; Czech Republic UM; Czech Republic app; Czech Republic grant; Czechoslovakia grant; Denmark UM; Denmark app; Denmark grant; Estonia UM; Estonia app; Estonia grant; Eurasian (EAPO) grant; Europe (EP) app; Europe (EP) grant; Finland app; Finland grant; France app; France grant; Georgia grant; German Democratic Republic app; German Democratic Republic grant; Germany UM; Germany app; Germany grant; Greece app; Greece grant; Gulf Cooperation Council grant; Hong Kong UM; Hong Kong app; Hong Kong grant; Hungary grant; Iceland grant; India app; India grant; Indonesia UM; Indonesia app; Ireland UM; Ireland app; Ireland grant; Israel grant; Italy grant; Italy grant; Italy grant; Italy grant; Japan UM; Japan app; Japan grant; Korea UM; Korea app; Korea grant; Latvia grant; Lithuania grant; Luxembourgh app; Luxembourgh grant; Malaysia app; Malaysia grant; Mexico app; Mexico grant; Moldova UM; Moldova grant; Monaco grant; Mongolia UM; Mongolia grant; Morocco app; Morocco grant; Netherlands app; Netherlands grant; New Zealand grant; Norway app; Norway grant; OAPI grant; PCT (WO) app; Philippines UM; Philippines app; Philippines grant; Poland UM; Poland grant; Portugal UM; Portugal app; Portugal grant; Romania UM; Romania app; Romania granted; Russia UM; Russia app; Russia grant; Serbia UM; Serbia app; Serbia grant; Singapore app; Singapore grant; Slovakia UM; Slovakia app; Slovakia grant; Slovenia UM; Slovenia grant; South Africa app; Spain UM; Spain app; Spain grant; Sweden app; Sweden grant; Switzerland app; Switzerland grant; Taiwan UM; Taiwan appl; Taiwan grant; Thailand UM; Thailand app; Thailand grant; Tunisia app; Turkey UM; Turkey app; Turkey grant; UK app; UK grant; USSR grant; Ukraine UM; Ukraine grant; United States app; United States grant; Uruguay app; Vietnam UM; Vietnam app; Vietnam grant; Other

Table of Contents

1. US6581565B2 Engine torque controller

Family 1/1

1 record(s) per family

Record 1/1 US6581565B2 Engine torque controller

Title: Engine torque controller

Title - DWPI: Controller for torque output of engine has transient torque control unit which receives filtered difference signal as input, and outputs fuel adjustment signal and spark adjustment signal

Priority Number: EP2001306301A

Priority Date: 2001-07-23

Application Number: US2002191641A

Application Date: 2002-07-09

Publication Date: 2003-06-24

IPC Class Table:

IPC	Section	Class	Subclass	Class Group	Subgroup
F02D004114	F	F02	F02D	F02D0041	F02D004114
F02D004130	F	F02	F02D	F02D0041	F02D004130

IPC Class Table - DWPI:

IPC - DWPI	Section - DWPI	Class - DWPI	Subclass - DWPI	Class Group - DWPI	Subgroup - DWPI
F02B001700 (IPC 1-7)	F	F02	F02B	F02B0017	F02B001700 (IPC 1-7)
F02D004130 (IPC 1-7)	F	F02	F02D	F02D0041	F02D004130 (IPC 1-7)
F02D004114 (IPC 1-7)	F	F02	F02D	F02D0041	F02D004114 (IPC 1-7)
F02D004114	F	F02	F02D	F02D0041	F02D004114
F02D004130	F	F02	F02D	F02D0041	F02D004130

Assignee/Applicant: Visteon Global Technologies Inc.,Dearborn,MI

JP F Terms:

JP FI Codes:

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10888

Assignee - Original: Visteon Global Technologies Inc.

Any CPC Table:

Type	Invention	Additional	Version	Office
Current	**F02D 41/3029**	F02D 2041/1432	20130101	EP
Current	F02D 41/1497	F02D 2200/1004	20130101	EP
Current	F02D 41/3023	F02D 2250/21	20130101	EP

ECLA: F02D004114F | F02D004130C2B | F02D004130C2B2 | R02D004114B20M | R02D02001004 | R02D025021

Abstract:

This invention relates to an engine torque controller for spark ignition internal combustion engines and more specifically for direct injection engines. The invention provides a torque controller and a method of controlling torque for an engine in which torque is controlled in dependence upon a filtered difference signal where the filtered difference signal is the difference between a desired torque signal and a signal representing an estimate of the current torque.

Language of Publication: EN

INPADOC Legal Status Table:

Gazette Date	Code	INPADOC Legal Status Impact
2020-03-03	IPR	-
Description: AIA TRIAL PROCEEDING FILED BEFORE THE PATENT AND APPEAL BOARD: INTER PARTES REVIEW TRIAL NO: IPR2020-00444 2020-01-31		
2020-03-03	IPR	-
Description: AIA TRIAL PROCEEDING FILED BEFORE THE PATENT AND APPEAL BOARD: INTER PARTES REVIEW TRIAL NO: IPR2020-00445 2020-01-31		
2017-08-31	AS	-
Description: ASSIGNMENT MICHIGAN MOTOR TECHNOLOGIES LLC, MICHIGAN ASSIGNMENT OF ASSIGNORS INTEREST; ASSIGNOR:MOBILE AUTOMOTIVE TECHNOLOGIES, LLC; REEL/FRAME:043463/0881 2017-08-28		
2017-08-31	AS	-
Description: ASSIGNMENT MOBILE AUTOMOTIVE TECHNOLOGIES, LLC, ALABAMA ASSIGNMENT OF ASSIGNORS INTEREST; ASSIGNOR:GODO KAISHA IP BRIDGE; REEL/FRAME:043843/0821 2016-11-02		
2017-08-31	AS	-

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10889

Description: ASSIGNMENT MOBILE AUTOMOTIVE TECHNOLOGIES, LLC, ALABAMA ASSIGNMENT OF ASSIGNORS INTEREST; ASSIGNOR:GODO KAISHA IP BRIDGE; REEL/FRAME:043463/0223 2016-09-02		
2015-04-16	AS	-
Description: ASSIGNMENT GODO KAISHA IP BRIDGE 1, JAPAN ASSIGNMENT OF ASSIGNORS INTEREST; ASSIGNOR:VISTEON GLOBAL TECHNOLOGIES INC.; REEL/FRAME:035421/0739 2015-02-13		
2015-02-02	AS	-
Description: ASSIGNMENT VISTEON CORPORATION, MICHIGAN RELEASE OF SECURITY INTEREST IN SPECIFIED PATENTS; ASSIGNOR:CITIBANK, N.A.; REEL/FRAME:034874/0025 2015-02-02		
2015-02-02	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, MICHIGAN RELEASE OF SECURITY INTEREST IN SPECIFIED PATENTS; ASSIGNOR:CITIBANK, N.A.; REEL/FRAME:034874/0025 2015-02-02		
2014-12-05	FPAY	+
Description: FEE PAYMENT FEE PAYMENT YEAR 12		
2014-06-09	AS	-
Description: ASSIGNMENT VC AVIATION SERVICES, LLC, MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON CORPORATION, MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON ELECTRONICS CORPORATION, MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON EUROPEAN HOLDINGS, INC., MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, INC., MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.;		

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10890

REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TREASURY, INC., MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON INTERNATIONAL BUSINESS DEVELOPMENT, INC., RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON INTERNATIONAL HOLDINGS, INC., MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-06-09	AS	-
Description: ASSIGNMENT VISTEON SYSTEMS, LLC, MICHIGAN RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:033107/0717 2014-04-09		
2014-04-18	AS	-
Description: ASSIGNMENT CITIBANK., N.A., AS ADMINISTRATIVE AGENT, NEW YORK SECURITY INTEREST; ASSIGNORS:VISTEON CORPORATION, AS GRANTOR; VISTEON GLOBAL TECHNOLOGIES, INC., AS GRANTOR; REEL/FRAME:032713/0065 2014-04-09		
2011-04-26	AS	-
Description: ASSIGNMENT VC AVIATION SERVICES, LLC, MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON CORPORATION, MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON ELECTRONICS CORPORATION, MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10891

2011-04-26	AS	-
Description: ASSIGNMENT VISTEON EUROPEAN HOLDING, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TREASURY, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON INTERNATIONAL BUSINESS DEVELOPMENT, INC., RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON INTERNATIONAL HOLDINGS, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2011-04-26	AS	-
Description: ASSIGNMENT VISTEON SYSTEMS, LLC, MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317; ASSIGNOR:MORGAN STANLEY SENIOR FUNDING, INC.; REEL/FRAME:026178/0412 2011-04-06		
2010-10-26	FPAY	+
Description: FEE PAYMENT FEE PAYMENT YEAR 8		
2010-10-19	AS	-
Description: ASSIGNMENT MORGAN STANLEY SENIOR FUNDING, INC., AS AGENT, NEW SECURITY AGREEMENT; ASSIGNORS:VISTEON CORPORATION; VC AVIATION SERVICES, LLC; VISTEON ELECTRONICS CORPORATION; AND OTHERS; REEL/FRAME:025241/0317 2010-10-07		
2010-10-19	AS	-
Description: ASSIGNMENT MORGAN STANLEY SENIOR FUNDING, INC., AS AGENT, NEW SECURITY AGREEMENT (REVOLVER); ASSIGNORS:VISTEON CORPORATION; VC AVIATION SERVICES, LLC; VISTEON ELECTRONICS CORPORATION; AND OTHERS; REEL/FRAME:025238/0298 2010-10-01		

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10892

2010-10-07	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS RECORDED AT REEL 022575 FRAME 0186; ASSIGNOR:WILMINGTON TRUST FSB, AS ADMINISTRATIVE AGENT; REEL/FRAME:025105/0201 2010-10-01		
2010-10-06	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, INC., MICHIGAN RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS RECORDED AT REEL 022974 FRAME 0057; ASSIGNOR:THE BANK OF NEW YORK MELLON; REEL/FRAME:025095/0711 2010-10-01		
2009-07-17	AS	-
Description: ASSIGNMENT THE BANK OF NEW YORK MELLON, AS ADMINISTRATIVE AGE ASSIGNMENT OF PATENT SECURITY INTEREST; ASSIGNOR:JPMORGAN CHASE BANK, N.A., A NATIONAL BANKING ASSOCIATION; REEL/FRAME:022974/0057 2009-07-15		
2009-04-21	AS	-
Description: ASSIGNMENT WILMINGTON TRUST FSB, AS ADMINISTRATIVE AGENT, MIN ASSIGNMENT OF SECURITY INTEREST IN PATENTS; ASSIGNOR:JPMORGAN CHASE BANK, N.A., AS ADMINISTRATIVE AGENT; REEL/FRAME:022575/0186 2009-04-15		
2009-04-21	AS	-
Description: ASSIGNMENT WILMINGTON TRUST FSB, AS ADMINISTRATIVE AGENT,MINN ASSIGNMENT OF SECURITY INTEREST IN PATENTS; ASSIGNOR:JPMORGAN CHASE BANK, N.A., AS ADMINISTRATIVE AGENT; REEL/FRAME:022575/0186 2009-04-15		
2009-02-27	AS	-
Description: ASSIGNMENT JPMORGAN CHASE BANK, TEXAS SECURITY INTEREST; ASSIGNOR:VISTEON GLOBAL TECHNOLOGIES, INC.; REEL/FRAME:022368/0001 2006-08-14		
2009-02-27	AS	-
Description: ASSIGNMENT JPMORGAN CHASE BANK,TEXAS SECURITY INTEREST; ASSIGNOR:VISTEON GLOBAL TECHNOLOGIES, INC.; REEL/FRAME:022368/0001 2006-08-14		
2008-02-07	AS	-
Description: ASSIGNMENT JPMORGAN CHASE BANK, N.A., AS ADMINISTRATIVE AGENT SECURITY AGREEMENT; ASSIGNOR:VISTEON GLOBAL TECHNOLOGIES, INC.; REEL/FRAME:020497/0733 2006-06-13		
2006-11-21	FPAY	+
Description: FEE PAYMENT FEE PAYMENT YEAR 4		

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10893

2003-10-28	CC	-
Description: CERTIFICATE OF CORRECTION		
2003-06-05	STCF	-
Description: INFORMATION ON STATUS: PATENT GRANT PATENTED CASE		
2002-07-09	AS	-
Description: ASSIGNMENT VISTEON GLOBAL TECHNOLOGIES, INC., MICHIGAN ASSIGNMENT OF ASSIGNORS INTEREST; ASSIGNORS:HESLOP, GARON NIGEL; DIXON, JON; BLACHFORD, RICHARD STEPHEN; REEL/FRAME:013091/0632 2002-07-09		

Post-Issuance (US): CORR-CERT Certificate of Correction 2003-10-28 2003 2003-11-18 2003 a Certificate of Correction was issued for this patent

Reassignment (US) Table:

Assignee	Assignor	Date Signed	Reel/Frame	Date
MICHIGAN MOTOR TECHNOLOGIES LLC,SYLVAN LAKE,MI,US	MOBILE AUTOMOTIVE TECHNOLOGIES, LLC	2017-08-28	043463/0881	2017-08-31
Conveyance: ASSIGNMENT OF ASSIGNORS INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: TIMOTHY DEVLIN 1306 N. BROOM STREET 1ST FLOOR WILMINGTON, DE 19806				
MOBILE AUTOMOTIVE TECHNOLOGIES LLC,MOBILE,AL,US	GODO KAISHA IP BRIDGE	2016-11-02	043843/0821	2017-08-31
Conveyance: ASSIGNMENT OF ASSIGNORS INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: TIMOTHY DEVLIN 1306 N. BROOM STREET 1ST FLOOR WILMINGTON, DE 19806				
MOBILE AUTOMOTIVE TECHNOLOGIES LLC,MOBILE,AL,US	GODO KAISHA IP BRIDGE	2016-09-02	043463/0223	2017-08-31
Conveyance: ASSIGNMENT OF ASSIGNORS INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: TIMOTHY DEVLIN 1306 N. BROOM STREET 1ST FLOOR WILMINGTON, DE 19806				
GODO KAISHA IP BRIDGE 1,TOKYO,JP	VISTEON GLOBAL TECHNOLOGIES INC.	2015-02-13	035421/0739	2015-04-16
Conveyance: ASSIGNMENT OF ASSIGNORS INTEREST (SEE DOCUMENT FOR DETAILS).				

Corresponent: CPA GLOBAL LIBERATION HOUSE ST HELIER, JE1 1BL JERSEY				
VISTEON CORPORATION,VAN BUREN TOWNSHIP,MI,US	CITIBANK, N.A.	2015-02-02	034874/0025	2015-02-02
VISTEON GLOBAL TECHNOLOGIES,VAN BUREN TOWNSHIP,MI,US	-	-	-	-
Conveyance: RELEASE OF SECURITY INTEREST IN SPECIFIED PATENTS | RELEASE OF SECURITY INTEREST IN SPECIFIED PATENTS				
Corresponent: SKADDEN, ARPS, SLATE, MEAGHER & FLOM LLP FOUR TIMES SQUARE JUSTIN SELLE, LEGAL ASSISTANT NEW YORK, NY 10036 | SKADDEN, ARPS, SLATE, MEAGHER & FLOM LLP FOUR TIMES SQUARE JUSTIN SELLE, LEGAL ASSISTANT NEW YORK, NY 10036				
VISTEON CORPORATION,VAN BUREN TOWNSHIP,MI,US	MORGAN STANLEY SENIOR FUNDING, INC.	2014-04-09	033107/0717	2014-06-09
VC AVIATION SERVICES LLC,VAN BUREN TOWNSHIP,MI,US				
VISTEON ELECTRONICS CORPORATION,VAN BUREN TOWNSHIP,MI,US				
VISTEON GLOBAL TECHNOLOGIES INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON INTERNATIONAL HOLDINGS INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON GLOBAL TREASURY INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON EUROPEAN HOLDINGS INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON SYSTEMS LLC,VAN BUREN TOWNSHIP,MI,US				
VISTEON INTERNATIONAL BUSINESS DEVELOPMENT INC.,VAN BUREN TOWNSHIP,MI,US				
Conveyance: RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY |				

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10895

RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY | RELEASE OF SECURITY INTEREST IN INTELLECTUAL PROPERTY				
Corresponent: SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036 | SKADDEN ARPS SLATE MEAGHER & FLOM LLP FOUR TIMES SQUARE ATTN: REBECCA RODAL, ESQ. NEW YORK, NY 10036				
CITIBANK. N.A. AS ADMINISTRATIVE AGENT,GETZVILLE,NY,US	VISTEON CORPORATION, AS GRANTOR	2014-04-09	032713/0065	2014-04-18
	VISTEON GLOBAL TECHNOLOGIES, INC., AS GRANTOR	2014-04-09	-	-
Conveyance: SECURITY INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: KEN TAN, LEGAL ASSISTANT 80 PINE STREET C/O CAHILL GORDON & REINDEL LLP NEW YORK, NY 10005				
VISTEON CORPORATION,VAN BUREN TOWNSHIP,MI,US	MORGAN STANLEY SENIOR FUNDING, INC.	2011-04-06	026178/0412	2011-04-26
VC AVIATION SERVICES LLC,VAN BUREN TOWNSHIP,MI,US				
VISTEON ELECTRONICS CORPORATION,VAN BUREN TOWNSHIP,MI,US				
VISTEON GLOBAL TECHNOLOGIES INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON INTERNATIONAL HOLDINGS INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON GLOBAL TREASURY INC.,VAN BUREN TOWNSHIP,MI,US				
VISTEON EUROPEAN HOLDING INC.,VAN BUREN				

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10896

TOWNSHIP,MI,US	
VISTEON SYSTEMS LLC,VAN BUREN TOWNSHIP,MI,US	
VISTEON INTERNATIONAL BUSINESS DEVELOPMENT INC.,VAN BUREN TOWNSHIP,MI,US	

Conveyance: RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317 | RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS ON REEL 025241 FRAME 0317

Corresponent: KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654 | KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654

MORGAN STANLEY SENIOR FUNDING INC. AS AGENT,BROOKLYN,NY,US	VISTEON CORPORATION	2010-10-07	025241/0317	2010-10-19
	VC AVIATION SERVICES, LLC	2010-10-07		
	VISTEON ELECTRONICS CORPORATION	2010-10-07		
	VISTEON GLOBAL TECHNOLOGIES, INC.	2010-10-07		
	VISTEON INTERNATIONAL HOLDINGS, INC.	2010-10-07		
	VISTEON GLOBAL TREASURY, INC.	2010-10-07		
	VISTEON EUROPEAN HOLDING, INC.	2010-10-07		
	VISTEON SYSTEMS, LLC	2010-10-07		
	VISTEON INTERNATIONAL BUSINESS DEVELOPMENT,	2010-10-07		

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10897

	INC.			
Conveyance: SECURITY AGREEMENT				
Corresponent: CAROL FRASER PAUL, HASTINGS, JANOFSKY & WALKER LLP 600 PEACHTREE STREET, NE SUITE 2400 ATLANTA, GA 30308				
MORGAN STANLEY SENIOR FUNDING INC. AS AGENT,BROOKLYN,NY,US	VISTEON CORPORATION	2010-10-01	025238/0298	2010-10-19
	VC AVIATION SERVICES, LLC	2010-10-01		
	VISTEON ELECTRONICS CORPORATION	2010-10-01		
	VISTEON GLOBAL TECHNOLOGIES, INC.	2010-10-01		
	VISTEON INTERNATIONAL HOLDINGS, INC.	2010-10-01		
	VISTEON GLOBAL TREASURY, INC.	2010-10-01		
	VISTEON EUROPEAN HOLDINGS, INC.	2010-10-01		
	VISTEON SYSTEMS, LLC	2010-10-01		
	VISTEON INTERNATIONAL BUSINESS DEVELOPMENT, INC.	2010-10-01		
Conveyance: SECURITY AGREEMENT (REVOLVER)				
Corresponent: CAROL FRASER PAUL, HASTINGS, JANOFSKY & WALKER LLP 600 PEACHTREE STREET, NE, SUITE 2400 ATLANTA, GEORGIA 30308				
VISTEON GLOBAL TECHNOLOGIES INC.,VAN BUREN TOWNSHIP,MI,US	WILMINGTON TRUST FSB, AS ADMINISTRATIVE AGENT	2010-10-01	025105/0201	2010-10-07
Conveyance: RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS RECORDED AT REEL 022575 FRAME 0186				
Corresponent: KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654				
VISTEON GLOBAL TECHNOLOGIES INC.,VAN BUREN TOWNSHIP,MI,US	THE BANK OF NEW YORK MELLON	2010-10-01	025095/0711	2010-10-06
Conveyance: RELEASE BY SECURED PARTY AGAINST SECURITY INTEREST IN PATENTS RECORDED AT REEL 022974 FRAME 0057				

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10898

Corresponent: KIRKLAND & ELLIS LLP 300 NORTH LASALLE STREET C/O DONNA GASIOROWSKI, SR. LEGAL ASST. CHICAGO, IL 60654				
THE BANK OF NEW YORK MELLON AS ADMINISTRATIVE AGENT,IRVING,TX,US	JPMORGAN CHASE BANK, N.A., A NATIONAL BANKING ASSOCIATION	2009-07-15	022974/0057	2009-07-17
Conveyance: ASSIGNMENT OF PATENT SECURITY INTEREST				
Corresponent: STEPHEN S. LAPLANTE 150 WEST JEFFERSON SUITE 2500 DETROIT, MI 48226				
WILMINGTON TRUST FSB AS ADMINISTRATIVE AGENT,MINNEAPOLIS,MN, US	JPMORGAN CHASE BANK, N.A., AS ADMINISTRATIVE AGENT	2009-04-15	022575/0186	2009-04-21
Conveyance: ASSIGNMENT OF SECURITY INTEREST IN PATENTS				
Corresponent: ASHLEY B. SCIBELLI ONE FEDERAL STREET BINGHAM MCCUTCHEN LLP BOSTON, MA 02110				
JPMORGAN CHASE BANK,HOUSTON,TX,US	VISTEON GLOBAL TECHNOLOGIES, INC.	2006-08-14	022368/0001	2009-02-27
Conveyance: SECURITY INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: JENNIFER WARD, ESQ. SKADDEN, ARPS, SLATE, MEAGHER & FLOM LLP FOUR TIMES SQUARE NEW YORK, NY 10036				
JPMORGAN CHASE BANK N.A. AS ADMINISTRATIVE AGENT,HOUSTON,TX,US	VISTEON GLOBAL TECHNOLOGIES, INC.	2006-06-13	020497/0733	2008-02-07
Conveyance: SECURITY AGREEMENT				
Corresponent: MINDY M. LOK, ESQ. SIMPSON THACHER & BARTLETT LLP 425 LEXINGTON AVENUE NEW YORK, NY 10017				
VISTEON GLOBAL TECHNOLOGIES INC.,DEARBORN,MI,US	HESLOP, GARON NIGEL	2002-07-09	013091/0632	2002-07-09
	DIXON, JON	2002-07-09	-	-
	BLACHFORD, RICHARD STEPHEN	2002-07-09	-	-
Conveyance: ASSIGNMENT OF ASSIGNORS INTEREST (SEE DOCUMENT FOR DETAILS).				
Corresponent: BRINKS HOFER GILSON & LIONE RAYMOND L. VIVACQUA P.O. BOX 10395 CHICAGO, IL 60610				

Maintenance Status (US): CC

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10899

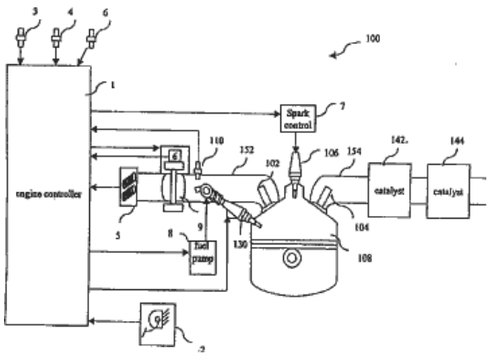
Litigation (US): 2022-07-29 2022 Michigan Motor Technologies LLC Mercedes-Benz USA, LLC N.D. Illinois 1:22cv03957 | 2022-07-22 2022 Michigan Motor Technologies, LLC Bayerische Motoren Werke AG BMW of North America, LLC N.D. Illinois 1:22cv03804 | 2021-07-02 2021 Michigan Motor Technologies LLC Tata Motors Ltd. Jaguar Land Rover North America, LLC E.D. Texas 4:21cv00512 | 2019-02-18 2019 Michigan Motor Technologies LLC Volkswagen Aktiengesellschaft E.D. Michigan 2:19cv10485 | 2017-09-01 2017 Michigan Motor Technologies LLC Hyundai Motor Company Hyundai Motor Group Hyundai America Technical Center, Inc. E.D. Michigan 2:17cv12901

Opposition (EP):

License (EP):

EPO Procedural Status:

Front Page Drawing:



Priority Number - DWPI: EP2001306301A | US2002191641A
Priority Date - DWPI: 2001-07-23 | 2002-07-09
Assignee - Current US: MICHIGAN MOTOR TECHNOLOGIES LLC

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Office of the Commissioner for Patents

ENGINE TORQUE CONTROLLER

PATENT #	APPLICATION #	FILING DATE	ISSUE DATE
6581565	10191641	07/09/2002	06/24/2003

Payment Window Status

No maintenance fees are due.

WINDOW	STATUS	FEES
11.5 Year	Closed	Paid

Window	First Day to Pay	Surcharge Starts	Last Day to Pay	Status	Fees
3.5 Year	06/24/2006	12/27/2006	06/25/2007	Closed	Paid
7.5 Year	06/24/2010	12/28/2010	06/24/2011	Closed	Paid
11.5 Year	06/24/2014	12/25/2014	06/24/2015	Closed	Paid

Patent Holder Information

Customer # 163393

Entity Status UNDISCOUNTED

Phone Number 3024499010

Address Michigan Motor Tech
c/o Devlin Law Firm LLC
1526 Gilpin Ave.
Wilmington, DE 19806
UNITED STATES

PETITIONER'S EXHIBIT 1002
US PATENT NO. 6,581,565
Page 10901