

A simple adaptive first-order differential microphone

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1. Introduction

As communication devices become more portable and used in any environment, the acoustic pick-up by electroacoustic transducers will require the combination of small compact transducers and signal-processing to allow high quality communication. This paper covers the design and implementation of a novel adaptive first-order differential microphone. The self-optimization is based on minimizing the microphone output under the constraint that the solitary null for first-order systems is located in the rear-half plane. The constraint is simply realized by the judicious subtraction of time-delayed outputs from two closely-spaced omnidirectional microphones. Although the solution presented does not maximize the signal-to-noise ratio, it can significantly improve the signal-to-noise ratio in certain acoustic fields.

2. Derivation of the adaptive first-order array

For a plane-wave signal $s(t)$ with spectrum $S(\omega)$ and wavevector $\mathbf{k}$ incident on a two-element array with displacement vector $\mathbf{d}$ as shown in Fig. 1, the output can be written as,

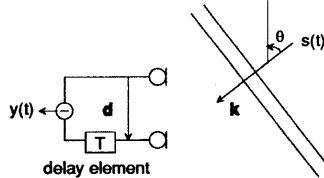


Figure 1: Diagram of first-order sensor composed of two zero-order sensors and a delay.

$$\begin{aligned} Y(\omega, \theta) &= S(\omega) \left(1 - e^{-j(\omega T + \mathbf{k} \cdot \mathbf{d})} \right) \\ &= S(\omega) \left(1 - e^{-j\omega [T + (d \cos \theta)/c]} \right) \end{aligned} \quad (1)$$

where d is the interelement spacing, T is equal to the delay applied to the signal from one sensor and we have made the substitution $|\mathbf{k}| = k = \omega/c$ where c is the speed of sound. Taking the magnitude of Eq. 1 yields,

$$|Y(\omega, \theta)| = 2 \left| S(\omega) \sin \frac{\omega [T + (d \cos \theta)/c]}{2} \right|. \quad (2)$$

If we assume a small spacing and delay ($kd \ll \pi$ and $\omega T \ll \pi$),

$$|Y(\omega, \theta)| \approx \omega |S(\omega) [T + (d \cos \theta)/c]|. \quad (3)$$

As expected, the first-order differential array has a monopole term and a first-order dipole term $\cos \theta$ which resolves the component of the acoustic particle velocity along the sensor axis. One thing to notice from Eq. 3 is that the first-order array has a first-order differentiator frequency dependence (increases linearly with frequency). This frequency dependence is easily compensated in practice by a first-order lowpass filter. The term in the brackets in Eq. 3 contains the array directional response. In the design of differential arrays the array directivity is of major interest. To simplify the analysis for the directivity of the first-order array let us define β such that,

$$\beta = \frac{T}{T + d/c} \quad (4)$$

and,

$$1 - \beta = \frac{d/c}{T + d/c}. \quad (5)$$

The normalized directional response is,

$$Y_N(\theta) = \beta + (1 - \beta) \cos \theta \quad (6)$$

where the subscript N denotes the normalized response of a first-order system, i.e., $Y_N(0) = 1$. If β is constrained to values between $0 \leq \beta \leq 0.5$, then the directional response of the first-order array shown in Eq. 6 contains a single zero (or null) at an angle between $90^\circ \leq \theta \leq 180^\circ$. The null location for values of β between 0 and 1/2 is at:

$$\begin{aligned} \theta_{null} &= \arccos \left(\frac{\beta}{\beta - 1} \right) \\ &= \arccos \left(\frac{-Tc}{d} \right) \end{aligned} \quad (7)$$

An obvious implementation to realize a first-order adaptive differential sensor would be to have a variable time delay T as shown in Fig. 1. However, this solution is not an attractive one as we now require the ability to generate *any* time delay less than or equal to d/c . The computational requirements to realize the general delay and the necessary adaptive algorithm are unattractive for a real-time implementation. Fortunately, there is a much simpler way to implement an adaptive differential array as discussed below.

One very simple possibility for the realization of a general first-order sensor can be seen by looking at the two terms in

Eq. 6. These terms represent a monopole component and a dipole component. The adaptive algorithm to minimize the array output would hinge on finding the appropriate combination of an omnidirectional and dipole sensor such that the mean-square output would be minimized. The minimization could incorporate a constraint on β (or equivalently on T) so that the null is limited to some range of angles.

The dipole directivity pattern can be realized by subtracting two closely-spaced omnidirectional microphones. The omnidirectional pattern can be realized by adding two closely-spaced omnidirectional microphones. Early directional microphones were implemented using the combination of pressure and velocity (or pressure-differential) microphones [1, 2]. Although the combination of the omnidirectional and dipole microphones is simple to implement, there is a problem with the need for a lowpass filter in the path of the dipole output. The filter introduces an interchannel phase shift. This phase shift can result in the inability of the adaptive algorithm to properly steer a null in a noise source direction.

The back-to-back cardioid arrangement can easily be implemented as shown in Fig. 2. The lowpass filter shown following the output $y(t)$ in Fig. 2 is used to compensate the differentiator response of the differential microphone. By setting the sampling period equal to d/c we can form the back-to-back cardioid microphone outputs directly. The back-to-back

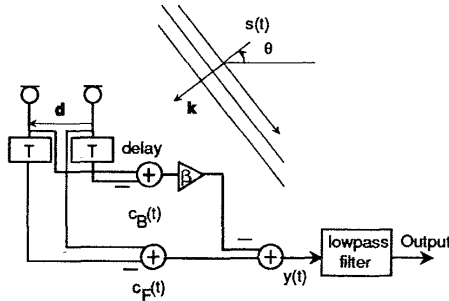


Figure 2: Schematic implementation of an adaptive first-order differential microphone using the combination of a forward and backward facing cardioids.

cardioid configuration is used in some commercial first-order directional microphones [3]. The manufacturer usually gives the user the ability to select one of a few types of preset patterns.

By examining Fig. 2 and Eq. 1 and noting that the sampling period $T = d/c$, we can write the expressions for the forward facing cardioid C_F , and the backwards facing cardioid C_B , (note that to simplify the following expressions the spatial origin is placed at the array center):

$$C_F(\omega, \theta) = 2jS(\omega)e^{-j\omega T/2} \sin \frac{kd(1 + \cos \theta)}{2} \quad (8)$$

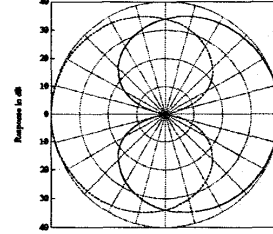


Figure 3: Directional responses of the back-to-back cardioid transducer as shown in Fig. 3.

and,

$$C_B(\omega, \theta) = 2jS(\omega)e^{-j\omega T/2} \sin \frac{kd(1 - \cos \theta)}{2}. \quad (9)$$

Normalizing the output signal by the input spectrum $S(\omega)$ results in,

$$\left| \frac{Y(\omega, \theta)}{S(\omega)} \right| = 2 \left| \sin \frac{kd(1 + \cos \theta)}{2} - \beta \sin \frac{kd(1 - \cos \theta)}{2} \right| \quad (10)$$

3. Frequency response of the array

One of the paramount issues in the design of the adaptive array is the variation of the frequency response for a front-half plane source when the null location varies in the rear. If the frequency response of the resulting adaptive microphone varies greatly with rear null location, the user will hear a modulation distortion of the desired on-axis signal as the rear null varies in position. We therefore develop the analytic expressions for the transducer frequency response.

From Eq. 10, the transfer function from a desired source to the array output can be written as,

$$\frac{Y(\omega, \theta)}{S(\omega)} = (1 + \beta) \sin \left(\frac{kd \cos \theta}{2} \right) \cos \left(\frac{kd}{2} \right) + \frac{1 - \beta}{\cos \theta} \left[\sin \left(\frac{kd}{2} \right) \cos \left(\frac{kd \cos \theta}{2} \right) \cos \theta \right] \quad (11)$$

For small kd the quantity inside the brackets in Eq. 11 is proportional to the first term. Using this approximation yields,

$$\left| \frac{Y(\omega, \theta)}{S(\omega)} \right| \approx \left| \frac{2 - (\beta + 1)(1 - \cos \theta)}{\cos \theta} \sin \left(\frac{kd \cos \theta}{2} \right) \cos \left(\frac{kd}{2} \right) \right| \quad (12)$$

It has been found that the above approximation is very good for array spacings up to $\lambda/4$. One thing to note in Eq. 12 is that the frequency response shape does not vary with β . The only effect of a change in β is to change the amplitude of the response to an off-axis source. For normal incidence ($\theta = 0$), the frequency response is independent of β . Figure 4

shows the frequency response computed by the exact expression from Eq. 11 at incident angle of 0° for an array with 1.5 cm spacing.

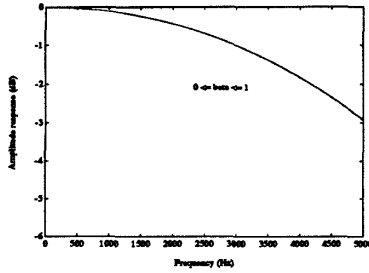


Figure 4: Frequency response of 1.5 cm spaced array with a signal arriving on-axis ($\theta = 0^\circ$), for $0 \leq \beta \leq 1$.

4. Optimum β

The optimum value of β is defined as the value of β which minimizes the mean-square value of the sensor output. Starting with the time-domain representation of the back-to-back cardioid as seen in Fig. 2, we can write,

$$y(t) = c_F(t) - \beta c_B(t). \quad (13)$$

Squaring and taking the expected value yields,

$$E[y^2(t)] = R_{c_F c_F}(0) - 2\beta R_{c_F c_B}(0) + \beta^2 R_{c_B c_B}(0) \quad (14)$$

where, $R_{c_F c_F}(0)$ and $R_{c_B c_B}(0)$ are the powers of the front and back cardioid signals and, $R_{c_F c_B}(0)$ is the cross-power between the front and back cardioid signals. Since the output power $E[y^2(t)]$ is a quadratic function of the variable β , we can be certain that there is only one minimum. The minimum value can be found by taking the derivative of Eq. 14 with respect to β and setting the result to zero. The result is,

$$\beta_{opt} = \frac{R_{c_F c_B}(0)}{R_{c_B c_B}(0)}. \quad (15)$$

The value of β_{opt} is a minimum since the second derivative with respect to β is positive. The second derivative is positive as it has the same sign as an autocorrelation function at zero lag. The autocorrelation function must be positive (or zero) for zero-lag and therefore the error surface $E[y^2(t)]$ is concave with one minimum. The result shown in Eq. 15 is the optimum Wiener filter for a filter of length one.

In an actual DSP implementation we must use estimates of the power and cross-power. This restriction is not as problematic as one may think. In fact, it is desirable to use short-time estimates of the correlation functions since we know a priori that the acoustic fields in which we intend to operate the adaptive microphone are nonstationary.

5. LMS version

The LMS or Stochastic Gradient algorithm is a commonly used adaptive algorithm due to its simplicity and ease of implementation [4]. We therefore develop the LMS algorithm

for the back-to-back cardioid adaptive first-order differential array. To begin, recall that

$$y(t) = c_F(t) - \beta c_B(t). \quad (16)$$

Squaring Eq. 16 results in,

$$y^2(t) = c_F^2(t) - 2\beta c_F(t)c_B(t) + \beta^2 c_B^2(t). \quad (17)$$

The steepest descent algorithm finds a minimum of the error surface $E[y^2(t)]$ by stepping in the direction opposite to the gradient of the surface with respect to the weight parameter β . The steepest descent update equation can be written as,

$$\beta_{t+1} = \beta_t - \mu \frac{d E[y^2(t)]}{d \beta} \quad (18)$$

where, μ is the update step-size and the differential gives the gradient of the error surface $E[y^2(t)]$ with respect to β . The quantity that we want to minimize is the mean of $y^2(t)$ but the LMS algorithm uses the instantaneous estimate of the gradient, i.e., the expectation operation in Eq. 18 is not applied and the instantaneous estimate is used. Performing the differentiation yields,

$$\begin{aligned} \frac{d y^2(t)}{d \beta} &= -2c_F(t)c_B(t) + 2\beta c_B^2(t) \\ &= -2y(t)c_B(t). \end{aligned} \quad (19)$$

Thus, we can write the LMS update equation as,

$$\beta_{t+1} = \beta_t + 2\mu y(t)c_B(t). \quad (20)$$

Typically the LMS algorithm is slightly modified by normalizing the update size so that explicit convergence bounds for μ can be stated that are independent of the input power. The LMS version with a normalized μ is therefore,

$$\beta_{t+1} = \beta_t + 2\mu y(t) \frac{c_B(t)}{\langle c_B^2(t) \rangle} \quad (21)$$

where the brackets indicate a time (or block) average.

6. Implementations and measurements

An experimental adaptive array was realized using two Bruel&Kjaer 4183 microphones spaced at 1.5 cm. Each microphone was fed into a preamp with a gain of 40 dB and then fed into an Ariel DSP-32C PC-DSP board.

Although we have implemented most of the variations that have been discussed in the preceding, we will only discuss the correlation-based optimum β algorithm described in section 5 since this realization was the most robust and simplest implementation. The implementation presently takes less than 40% of an AT&T DSP32C digital signal processing chip. The sampling rate was set to 22 kHz which allows the easy formation of the back-to-back cardioids by appropriately subtracting the present samples from the previous samples. A 5 kHz lowpass filter was utilized to eliminate signals that are above the highest design frequency for the array. One possible way to significantly reduce the amount of real-time DSP cycles would be to sample the input channels close to the Nyquist rate. For the array that we have constructed this would correspond to an 11 kHz sampling rate: one-half of

the present rate. The interpolation for upsampling could be done by the 5 kHz lowpass filter that is presently filtering the input sequences. If the sampling rate is reduced to 11 kHz, we could expect the real-time utilization to drop below 20% of an AT&T DSP 32C.

By constraining the value of β to $0 \leq \beta \leq 1$, we are guaranteed that the null is in the rear-half plane. Of course we are assuming that the sensors are ideal and the source to be nulled is in the farfield. Since the adaptive array presented in this paper is designed to track in nonstationary acoustic sound fields, it is difficult to document the dynamic performance. We can, however, measure the directional responses of the array for various stationary noise fields as well as frequency responses. Figure 5 shows directivity plots measured in the anechoic chamber at Murray Hill. The plots are shown for a 2 kHz sinusoidal signal and for values of β which resulted in nulls being placed in approximately 10° increments.

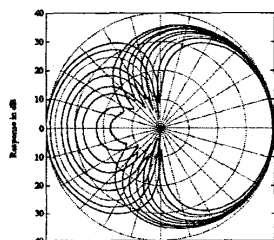


Figure 5: Measured directional responses for the adaptive array for $0 \leq \beta \leq 1$ and β chosen to give nulls in approximately 10° increments. Measured directional responses for the adaptive array for $0 \leq \beta \leq 1$ and β chosen to give nulls in approximately 10° increments.

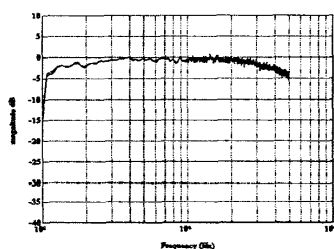


Figure 6: Measured frequency response of the adaptive first-order differential array for on-axis signals ($\theta = 0^\circ$) with null angles at 90° , 135° , and 180° .

Figures 6 and 6 show the frequency response for signals arriving on-axis ($\theta = 0^\circ$) and at $\theta = 45^\circ$. The frequency responses

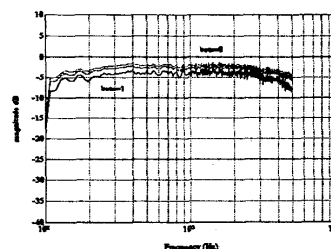


Figure 7: Measured frequency response of the adaptive first-order differential array for signals incident at $\theta = 45^\circ$ with null angles at 90° , 135° , and 180° .

are shown for three different values of β which correspond to the null angles of 90° , 135° , and 180° . The deviation between these responses is barely discernible except for a small amplitude shift for signals impinging from $\theta = 45^\circ$. The highpass characteristic is due to the design of the first-order lowpass filter that is used to correct the differentiator response of the first-order differential sensor. The lowpass response that occurs above 2 kHz is due to the finite difference of two closely-spaced sensors and can also be seen in Fig. 4 which shows the theoretical frequency responses.

7. Conclusions

A novel first-order adaptive differential microphone with a null angle constrained to the rear-half plane has been presented. The back-to-back cardioid was shown to be the best implementation of the simple arrangements presented. An array was constructed and the performance was shown to closely match the theoretical predictions. Two adaptation algorithms were presented: the Wiener optimal solution, and the LMS algorithm.

References

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