

Input-Shaping and Model-Following Control of a Helicopter Carrying a Suspended Load

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When a heavy load is suspended from a helicopter, excessive load swing degrades helicopter control and can result in obstacle collisions. This work examines the benefits of combining input-shaping and model-following control to improve performance when carrying a suspended load. Model-following control architectures are used in modern helicopter flight control systems to make the helicopter respond like a prescribed model. On its own, model-following control is ineffective when carrying a suspended load because excessive load swing degrades tracking of the prescribed model dynamics and thus control of the helicopter. Therefore, reducing load swing improves tracking of the prescribed model and increases safety and productivity. Input shaping is a control technique that has been used to control vibration of flexible machines, such as robots, satellites, and cranes. By combining input shaping with model-following control, helicopter payload swing is reduced and tracking of the prescribed model is improved. The design of an attitude-command flight control system that combines input-shaping and model-following control is illustrated using dynamic models of a Sikorsky S-61 helicopter. Simulation results are shown for lateral and longitudinal repositioning movements. These results show that applying input shaping to simulated pilot commands greatly improves helicopter performance when carrying a suspended load.

Nomenclature

A	=	helicopter state matrix
A_D	=	state matrix of helicopter model used in feedforward control element
A_i	=	i th input shaper impulse amplitude
A_M	=	prescribed model state matrix
a	=	vertical distance from the helicopter center of gravity to the main rotor hub, ft
B	=	helicopter input matrix
B_D	=	input matrix of helicopter model used in feedforward control element
B_M	=	prescribed model input matrix
b	=	vertical distance from the helicopter center of gravity to the load suspension point, ft
e	=	model tracking error
G_{FB}, G_{FF}	=	feedback and feedforward control elements
G_{is}	=	generic input shaper transfer function
G_P	=	helicopter plant
G_{zv}	=	zero vibration input shaper transfer function
K	=	state feedback gain matrix
ℓ	=	load suspension cable length, ft
r	=	pilot command
t_i	=	i th input shaper impulse time
t_s	=	settling time, s
u	=	helicopter input vector
u_D	=	output of the feedforward control element
x	=	helicopter state vector
x_D	=	state vector of helicopter model used in feedforward control element
x_H	=	fuselage longitudinal position, ft
x_L	=	load longitudinal position, ft
x_M	=	prescribed model state vector
y_H	=	fuselage lateral position, ft

y_L	=	load lateral position, ft
y_M	=	prescribed model output
θ_c	=	lateral cyclic blade pitch angle, rad
θ_H	=	fuselage pitch attitude, rad
θ_R	=	rotor longitudinal tilt angle, rad
θ_s	=	longitudinal cyclic blade pitch angle, rad
ζ	=	damping ratio
τ	=	time constant of first-order lag, s
ϕ_H	=	fuselage roll attitude, rad
ϕ_R	=	rotor lateral tilt angle, rad
ω_n	=	natural frequency, rad/s

I. Introduction

A HELICOPTER can be used as a “flying crane” by hanging a load (often called a suspended load or sling load) from cables attached to the helicopter. A flying crane is extremely versatile. It can be used to transport timber during remote logging operations, deliver power transmission towers to their installation locations, rescue people stranded in otherwise inaccessible areas, and deliver cargo and supplies to ships at sea, as shown in Fig. 1. These are just a few examples of tasks that are too expensive, too slow, or physically impossible to perform with other types of vehicles.

Unfortunately, the suspended load is a swinging pendulum that adversely affects control of the helicopter. This effect makes efficient and accurate transfer of the load difficult. In fact, accidents can be caused by violent suspended-load swing [1]. The helicopter becomes particularly difficult to control when carrying heavy loads [2].

Guidelines for suspended-load operations suggest that the best way for a pilot to regain control when load swing becomes too large is to slow down the helicopter [1,3]. By trying to actively cancel this swing, the helicopter pilot may actually amplify the problem if his or her control inputs are not in the correct phase relative to the swing [3]. In cases where the swing amplitude becomes extreme and the pilot has difficulty stabilizing the load, the pilot may elect to jettison the load to regain control of the helicopter [1]. Additionally, the suspension cable can get tangled with objects on the ground. This can cause dangerous situations that result in damage to the cargo and helicopter or injury to the pilot and crew. The safety of sling-load operations would be improved significantly by reducing load swing and its effects on helicopters.

Reducing load swing would also increase productivity during load transfer operations. Once a load is positioned above its desired location, it cannot be safely deposited until the swing amplitude settles below an acceptable level or workers on the ground wrestle it

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Fig. 1 Helicopter delivering cargo to a ship at sea. (Photo credit: National Oceanic and Atmospheric Administration, Flying with NOAA Collection.)

to a stop. Keeping the swing at a low amplitude would allow the pilot to transfer and deposit loads more quickly and safely.

Several strategies have been proposed for reducing suspended-load swing. Some control strategies include an actuated suspension point or some other form of active load stabilization [4]. Dukes states that an actively controlled, moving suspension point could be an effective way of damping the load swing [5,6]. However, retrofitting existing heavy-lift helicopters with moving suspension points would be costly compared with small modifications to the digital flight control system. Other proposed control strategies rely on feedback of the suspended-load states [7–9]. Such control algorithms have shown promise because they also allow for control of external disturbances, but they require real-time measurement of the suspension cable angle, which is rarely available in current practice.

Another technique that has been effective at controlling oscillation is input shaping. Input shaping strategically modifies a command by convolving it with a series of impulses, called an *input shaper*. The resulting command induces little or no residual vibration [10,11]. Designing input shapers only requires estimates of the natural frequency and damping ratio of the undesired vibratory mode. A major advantage of this approach is that it does not require real-time measurement or estimation of the load states.

Input shaping has proven effective on many kinds of machines, including cranes [12], robotic arms [13–16], coordinate measuring machines [17,18], and satellites [19–25]. In addition, many studies of crane operators have shown that input shaping can greatly improve performance [26,27]. The primary disadvantages of input shaping are that it cannot reduce vibration caused by external disturbances, and it introduces a small response lag due to the method used to form the shaped commands.

The application of input shaping to helicopters carrying suspended loads has been considered by previous researchers [7,8,28,29]. Bisgaard et al. [7] used a state estimator to measure the natural frequency of the suspended-load oscillation. They then used this measurement to adaptively update an input shaper to prevent oscil-

lation at the measured natural frequency. Ottander and Johnson [8] combined input shaping with delayed feedback control of the payload swing angle. This controller allowed the payload to more accurately track a desired trajectory. Load swing caused by helicopter motion was reduced through input shaping, and external disturbances were canceled using feedback of the load swing angle. Both studies proposed and implemented a vision system to measure the load swing, which would increase complexity and cost.

Potter et al. [28] experimentally verified a simple model of the suspended-load oscillation and used the model to test via simulation the effectiveness and robustness of various types of input shapers. Adams et al. [29] also demonstrated the effectiveness of input-shaping control on an experimental helicopter.

However, input shaping alone is not sufficient for effective control of a helicopter carrying a suspended load. Rather, it should be combined with a feedback controller. Effective design of a helicopter's flight control system is critical to the overall performance of the aircraft. Design requirements such as Aeronautical Design Standard 33E (ADS-33E-PRF [30]) provide specifications on aircraft response characteristics that affect handling qualities ratings. Although ADS-33E-PRF is a military specification, such design requirements give flight control designers guidance on response types favored by pilots. Stability augmentation systems and high authority control augmentation systems assist pilots in maneuvering their aircraft by stabilizing aircraft response, adding damping to oscillatory aircraft modes, and/or introducing different flight characteristics such as altitude or heading holds [31–33]. When carrying a heavy suspended load, the pilot's control task can be challenging despite the assistance of these control systems [1–3].

Although a helicopter's control system would most likely be designed for unloaded flight, mission scenarios where the helicopter needs to carry a suspended load would require a different control system to achieve effective performance. Being able to activate an input-shaping algorithm that is compatible with the baseline flight controller for the unloaded helicopter would be simpler than having an entirely different flight control system designed specifically for loaded flight.

To be effective at preventing suspended-load oscillations, the input-shaping technique must be well integrated with the rest of the helicopter's flight control system. Done properly, this would combine the features of helicopter flight control systems listed earlier with the suspended-load oscillation prevention characteristics of input shaping. The most straightforward location for the input shaper in the controller is immediately after the pilot inputs have been received by the digital flight control system. Such a combination of input shaping with a common flight control architecture will be investigated in this paper.

A control architecture called model-following control (MFC) is used in helicopter flight control systems to force specified helicopter states, such as the attitude, to respond like a prescribed model [34,35]. MFC has become an attractive control technique for helicopter flight control systems. For example, The Boeing Company used a control law architecture consisting of model-following control on several programs in the 1980s and 1990s, including the V-22 and RAH-66 [32].

The MFC architecture combines feedforward and feedback control. The feedback controller is used to stabilize the helicopter system and determines the error dynamics of the model tracking. The feedforward controller uses model-inversion techniques to cancel the undesired helicopter dynamics. However, tracking of the prescribed model may be significantly degraded when the helicopter is carrying a suspended load, particularly when the load is heavy relative to the weight of the helicopter. One way to improve helicopter performance would be to replace the original feedforward model with one that accounts for the sling-load dynamics. A drawback of this method is that the feedback portion of such a controller would require real-time measurement of the load position or angle. The load swing is assumed to be unmeasured in this paper.

In this paper, a near-hover attitude-command model-following controller is designed for a Sikorsky S-61 helicopter with and without a suspended load using linearized dynamic models of this helicopter

obtained from previous studies [36,37]. Simulation results are used to demonstrate the controller's effectiveness on the unloaded helicopter. Next, this controller is applied to the loaded helicopter, and large load oscillations are shown to occur. These oscillations can be dangerous depending on the mission scenario and degrade model tracking performance due to coupling between the load and helicopter fuselage. Additional results show that adding input shaping to the controller 1) reduces swing of the suspended load and 2) improves helicopter tracking of the prescribed model, particularly once the pilot's commands are completed.

An additional novelty in the proposed approach is that input shaping integrates well with the concept of specifying a desired model. MFC is used as the underlying control because it provides good performance for the baseline helicopter dynamics. Although certain performance is desired for the helicopter motion, designers (and pilots) also want the load motion to be critically damped, and so input shaping is added to reduce suspended-load oscillation. In a way, the input-shaping control element is a form of implicit model-following control to achieve damped load motion. The input shaper is essentially part of the prescribed model, which can be referred to as an input-shaped prescribed model. Combining these two relatively simple and commonly used control methods provides a unique solution to this challenging control problem.

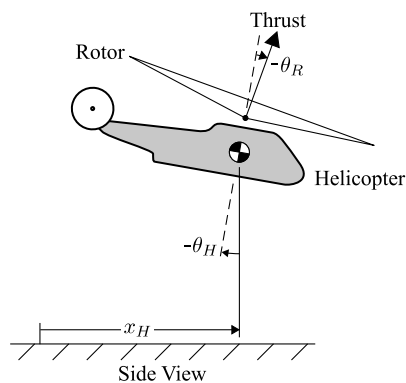
Section II describes dynamic models of the helicopter, and Sec. III gives a brief introduction to input shaping. The MFC architecture is described in Sec. IV. Section V details the MFC implementation used in this paper and presents simulation results using the controller on unloaded and loaded Sikorsky S-61 helicopter models. Section VI shows how the input-shaping technique may be combined with a MFC to improve helicopter performance when carrying a suspended load. Section VII discusses why the approach presented in this paper is effective and why input-shaping and model-following control complement one another when applied to helicopters carrying suspended loads.

II. Helicopter Dynamic Models

Designers of helicopter flight control systems often use helicopter models linearized about a given flight condition. These models can be obtained or improved using flight-test data [38–40]. One such model of a Sikorsky S-61 helicopter in near-hover operation was investigated by Hall and Bryson [36]. Gupta and Bryson [37] combined Hall and Bryson's helicopter model [36] with a linear model of a suspended load to yield a linearized, near-hover model of a Sikorsky S-61 carrying a suspended load. The mass of the helicopter is 13,228 lb, the suspended load is 4409 lb, and the suspension cable length is 65.6 ft mounted 4.9 ft below the helicopter center of mass [37].

The unloaded and loaded Sikorsky S-61 models are state-space models of the form

$$\dot{x} = Ax + Bu \quad (1)$$



Both models use a quasi-static representation of the rotor dynamics [36,37], which assumes that control inputs u cause instantaneous changes in the main rotor disk's longitudinal and lateral angles.

Both the unloaded and loaded models only incorporate the longitudinal and lateral motion of the helicopter. Therefore, it is assumed that the helicopter has well-performing heading- and altitude-hold controllers that maintain a constant direction and vertical position. Because the models are linearized, effects such as air drag on the helicopter and load are weakened by the linearization. The models also neglect the effects of wind and changes in atmospheric properties. Although wind acts as a disturbance on helicopters and suspended loads, this paper focuses only on the effects of helicopter motion on the suspended load.

A. Unloaded Sikorsky S-61 Model

Figure 2 is a sketch of the unloaded helicopter. The state and input vectors are defined as

$$x = [\theta_H \quad \dot{\theta}_H \quad \dot{x}_H \quad \phi_H \quad \dot{\phi}_H \quad \dot{y}_H]^T \quad (2)$$

$$u = [\theta_s \quad \theta_c]^T \quad (3)$$

By appropriately scaling the input matrix B , the input vector given by Eq. (3) becomes identical to the rotor tilt angles [36]:

$$u = [\theta_R \quad \phi_R]^T \quad (4)$$

With all units in feet, seconds, and radians, the state equation for the unloaded S-61 model is [36]

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -0.415 & 0.00338 & 0 & 0.318 & 0.00116 \\ -32.2 & 4.70 & -0.0198 & 0 & -1.02 & -0.0059 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1.23 & 0.00415 & 0 & -1.58 & -0.0124 \\ 0 & -1.02 & 0.0059 & 32.2 & -4.70 & -0.0198 \end{bmatrix} x + \begin{bmatrix} 0 & 0 \\ 6.27 & 0.295 \\ -32.2 & -0.977 \\ 0 & 0 \\ -1.08 & 23.1 \\ -0.977 & 32.2 \end{bmatrix} u \quad (5)$$

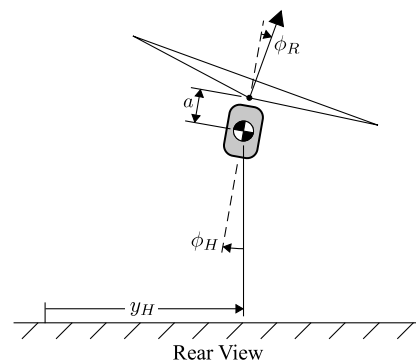


Fig. 2 Helicopter without sling load.

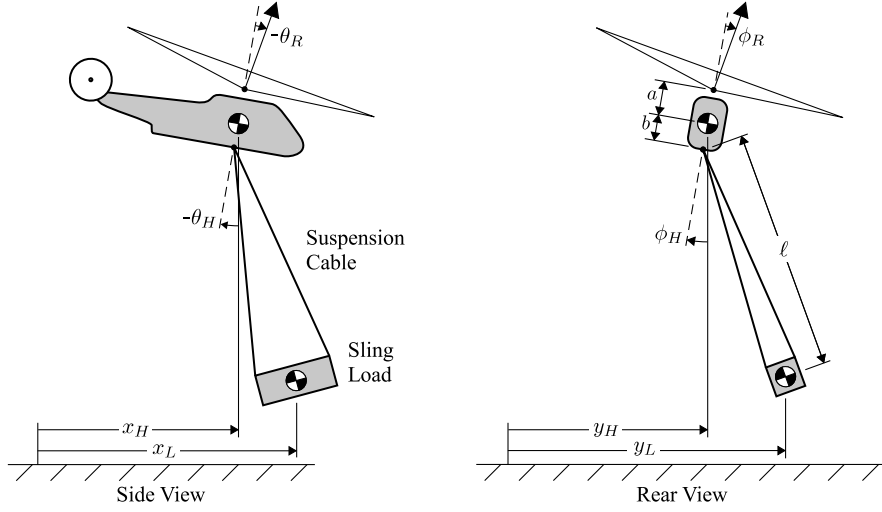


Fig. 3 Helicopter with sling load.

B. Loaded Sikorsky S-61 Model

Gupta and Bryson [37] modified the unloaded Sikorsky S-61 model in Eq. (5) to include suspended-load dynamics. Figure 3 is a sketch of the loaded helicopter. The helicopter state and input vectors for the loaded model are defined as

$$\mathbf{x} = [\theta_H \ \dot{\theta}_H \ x_H \ \dot{x}_H \ x_L \ \dot{x}_L \ \phi_H \ \dot{\phi}_H \ y_H \ \dot{y}_H \ y_L \ \dot{y}_L]^T \quad (6)$$

$$\mathbf{u} = [\theta_R \ \phi_R]^T \quad (7)$$

After converting all units to feet, seconds, and radians, the state matrix A for the loaded S-61 is [37]

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -2.25 & -0.415 & -0.032 & 0.003 & 0.032 & 0 & 0 & 0.318 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -43.6 & 4.69 & -0.164 & -0.020 & 0.164 & 0 & 0 & -1.02 & 0 & -0.006 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2.41 & 0 & 0.491 & 0 & -0.491 & -0.003 & 0 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1.23 & 0 & 0.004 & 0 & 0 & -8.28 & -1.58 & 0.117 & -0.012 & -0.117 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1.02 & 0 & 0.006 & 0 & 0 & 43.6 & -4.69 & -0.164 & -0.020 & 0.164 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -2.41 & 0 & 0.491 & 0 & -0.491 & -0.003 \end{bmatrix}, \quad (8)$$

and the input matrix B is [37]

$$\mathbf{B} = \begin{bmatrix} 0 & 8.38 & 0 & -42.98 & 0 & 0 & 0 & -1.43 & 0 & -1.30 & 0 & 0 \\ \hline 0 & 0.393 & 0 & -1.30 & 0 & 0 & 0 & 30.70 & 0 & 42.98 & 0 & 0 \end{bmatrix}^T \quad (9)$$

Figure 4 shows the magnitude plot for the lateral rotor tilt angle to the roll attitude $\phi_H(s)/\phi_R(s)$. A significant notch is seen in the magnitude at the load swing frequency of approximately 0.9 rad/s. This notch is indicative of degraded control when the pilot's command has components around the load swing frequency.

III. Input Shaping

Input shaping is a command-filtering method that limits unwanted oscillation [10,11,26,27]. Figure 5 illustrates the input-shaping concept. In the top of Fig. 5, an impulse is applied to a flexible system and induces a lightly damped response. A similar response (shown by the dashed line) would result if a second impulse were applied a short time later. The bottom of Fig. 5 shows the response that results from

both impulses. The two responses combine linearly, and the oscillation is eliminated. Furthermore, the two specially timed impulses can be convolved with any arbitrary function, and when used in this way, the resulting function will maintain the oscillation-canceling properties of the original impulses. The series of impulses is called an input shaper.

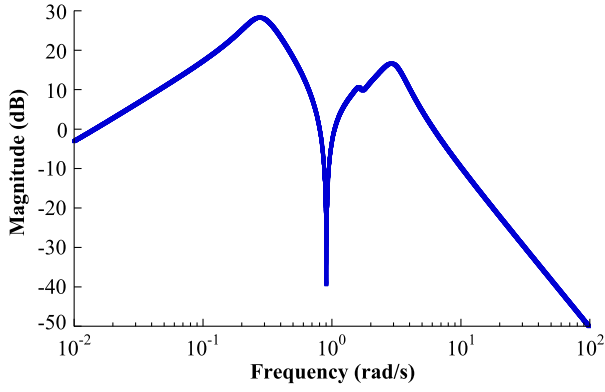


Fig. 4 Sample magnitude and phase plots for the loaded Sikorsky S-61 model.

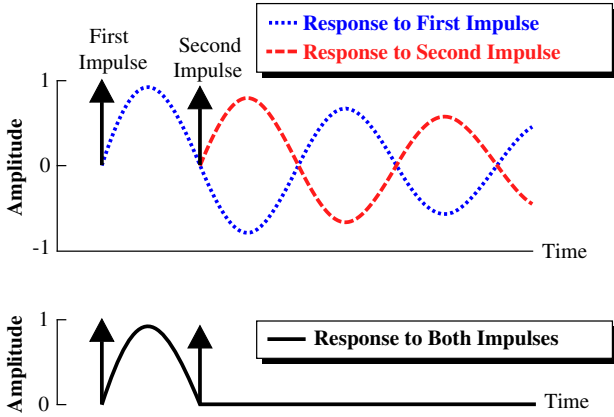


Fig. 5 Input-shaping concept.

Input shapers may have more than two impulses. The transfer function of a generic input shaper with n impulses is

$$\mathbb{G}_{is} = A_1 e^{-t_1 s} + A_2 e^{-t_2 s} + \dots + A_n e^{-t_n s} \quad (10)$$

Without loss of generality, the first impulse time is $t_1 \equiv 0$. The impulse amplitudes and time locations are designed using the estimated natural frequencies and damping ratios of the undesired flexible modes. Input shapers can be made robust to errors and changes in these parameters [10,41]. For this paper, it will be assumed that the natural frequency and damping ratio of the suspended-load oscillation are known accurately, and so robustness to modeling errors will not be addressed.

Different input shapers are designed using different combinations of performance requirements. By constraining the impulses to be all positive and the residual oscillation to be zero when parameter estimates are perfect, a zero vibration (ZV) input shaper [11] is obtained. Its transfer function is

$$\mathbb{G}_{zv} = A_1 + A_2 e^{-t_2 s} \quad (11)$$

where A_1 , A_2 , and t_2 depend on the natural frequency and damping ratio of the flexible mode. Only the ZV input shaper will be used in this paper. For a description of other kinds of input shapers, see [41].

IV. Model-Following Control

Model-following control has become an attractive control technique for helicopter flight controllers. MFC is an effective and viable control architecture because the aircraft response can be specified via a prescribed model to achieve favorable aircraft performance, and the architecture is well suited for full authority fly-by-wire control systems [32].

MFC for aircraft first appeared in the literature in the 1960s and 1970s. Murphy and Narendra [42] adjoined a model that approximates the pilot's inputs to the helicopter equations of motion, and then used optimal control to design a regulator that would track the model response. Winsor and Roy [43] used a combination of partial state feedback and feedforward compensation to achieve good tracking of the prescribed model. More recently, Trentini and Pieper [44] designed a model-following controller for a helicopter in hover to meet handling qualities requirements using an optimal control design approach. MFC has also been synthesized with other types of controllers to achieve better performance in difficult flight conditions or for specific missions, such as gust rejection during shipboard operations [45].

There are two main types of MFC. Implicit MFC uses optimal control to design a feedback controller that yields a closed-loop control system whose dynamics match the dynamics of the prescribed model [43]. The prescribed model itself does not appear in the control law directly; its output is only used in an optimal control performance index used to calculate feedback controller gains [44,46]. Explicit (or real) MFC uses the prescribed model directly in the control system, typically as a feedforward compensator [34,35,44,47]. The control law in explicit MFC is typically constructed using feedforward of the prescribed model states and a feedback controller that uses plant-state feedback [43,46].

Explicit model-following controllers incorporate real-time measurement of the model tracking error, allowing the controller to reject disturbances and to correct the aircraft trajectory in the presence of modeling errors [35,43]. In addition, explicit MFC can be implemented with partial state feedback, and effective model tracking can still be achieved [43]. Most important, the feedback stabilization can be designed independently from the feedforward compensator and prescribed model [32]. For these reasons, explicit MFC was selected for the baseline helicopter flight controller implemented in this paper.

The implemented controller combines feedforward and feedback control. A state feedback controller is used to stabilize the helicopter system. Also, the state feedback control enables the error dynamics of the model tracking to be specified. Based on the recommendation of Tyler [46], the feedback controller is designed as a regulator independent from the rest of the controller using only the helicopter plant model. The regulator is designed using the pole placement state feedback technique. The feedforward controller uses model-inversion techniques to cancel the undesired helicopter dynamics.

A. Controller Description

Figure 6 shows the block diagram of an explicit model-following controller. The structure of this controller is similar to that presented by Osder and Caldwell [35] and Landis et al. [32]. The pilot command is used as the input to a prescribed model $\mathbb{G}_M$. The rest of the controller is designed to force the output x of the helicopter plant $\mathbb{G}_P$ to track the prescribed model output x_M . Generally, helicopters have one or more unstable modes, and so a feedback controller is used to stabilize $\mathbb{G}_P$. The feedback element also enables the controller to reject disturbances that would otherwise degrade the model tracking [35]. Also, the feedback $\mathbb{G}_{FB}$ determines the error dynamics of the model tracking. The feedforward control $\mathbb{G}_{FF}$ is then selected to cancel undesired helicopter dynamics using model inversion techniques.

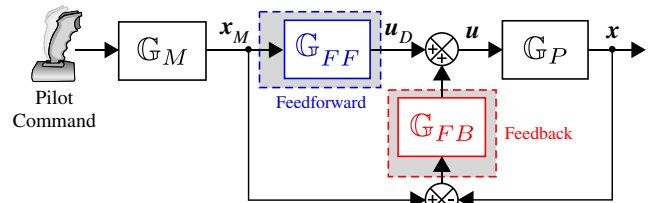


Fig. 6 Block diagram of an explicit model-following control structure.

B. Theoretical Model-Following Controller Performance

1. Steady-State Performance

If the feedforward control $\mathbb{G}_{FF}$ equals the inverse of the helicopter plant $\mathbb{G}_P^{-1}$, then the helicopter output $\mathbf{x}$ asymptotically tracks the model output $\mathbf{x}_M$. The asymptotic model tracking can easily be shown from the block diagram in Fig. 6 using block diagram reduction [35]. The control law that provides asymptotic tracking is given by

$$\mathbf{u} = \mathbb{G}_{FF}\mathbf{x}_M + \mathbb{G}_{FB}(\mathbf{x}_M - \mathbf{x}) \quad (12)$$

where $\mathbb{G}_{FF} = \mathbb{G}_P^{-1}$.

2. Model Tracking Error Dynamics

It can be shown that the design of the feedback controller determines the dynamic characteristics of the model tracking error. First, the helicopter plant is defined as

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \quad (13)$$

and the prescribed model is defined as

$$\dot{\mathbf{x}}_M = \mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M\mathbf{r} \quad (14)$$

The model tracking error is given by

$$\mathbf{e} = \mathbf{x}_M - \mathbf{x} \quad (15)$$

Assuming that all of the helicopter states are measurable, full state feedback may be selected for the feedback controller. Full state feedback calls for feedback control of the form $\mathbf{u} = -\mathbf{K}\mathbf{x}$, where $\mathbf{K}$ is a constant state feedback gain matrix. Therefore, $\mathbb{G}_{FB}$ from Fig. 6 equals the gain matrix $\mathbf{K}$. The control law in Eq. (12) can be rewritten as

$$\mathbf{u} = \mathbf{u}_D + \mathbf{K}(\mathbf{x}_M - \mathbf{x}) \quad (16)$$

Recall that the feedforward block is chosen to be an inverted model of the helicopter plant. A state-space representation of the helicopter plant model to be inverted is given by

$$\dot{\mathbf{x}}_D = \mathbf{A}_D\mathbf{x}_D + \mathbf{B}_D\mathbf{u}_D \quad (17)$$

In practice, this model will not perfectly represent the helicopter. An expression for $\mathbf{u}_D$ is obtained by solving Eq. (17) for $\mathbf{u}_D$, or

$$\mathbf{u}_D = \mathbf{B}_D^\dagger(\dot{\mathbf{x}}_D - \mathbf{A}_D\mathbf{x}_D) \quad (18)$$

where $\mathbf{B}_D^\dagger$ is the pseudoinverse of $\mathbf{B}_D$ and $\mathbf{x}_D$ is the input to the model inverse, which equals the prescribed model output $\mathbf{x}_M$ shown in Fig. 6. The pseudoinverse is required because $\mathbf{B}$ is usually not a square matrix.

Substituting the plant inverse given in Eq. (18) into Eq. (16), and setting $\dot{\mathbf{x}}_D = \dot{\mathbf{x}}_M$ and $\mathbf{x}_D = \mathbf{x}_M$, the control law can be written as

$$\mathbf{u} = \mathbf{B}_D^\dagger(\dot{\mathbf{x}}_M - \mathbf{A}_D\mathbf{x}_M) + \mathbf{K}(\mathbf{x}_M - \mathbf{x}) \quad (19)$$

To analyze the dynamics of the model tracking error, an expression for the derivative of $\mathbf{e}$ must be found. Taking the derivative of Eq. (15) and substituting Eqs. (14) and (13) for $\dot{\mathbf{x}}_M$ and $\dot{\mathbf{x}}$ gives

$$\dot{\mathbf{e}} = \mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M\mathbf{r} - \mathbf{A}\mathbf{x} - \mathbf{B}\mathbf{u} \quad (20)$$

Substituting Eq. (19) into Eq. (20) yields

$$\dot{\mathbf{e}} = \mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M\mathbf{r} - \mathbf{A}\mathbf{x} - \mathbf{B}[\mathbf{B}_D^\dagger(\dot{\mathbf{x}}_M - \mathbf{A}_D\mathbf{x}_M) + \mathbf{K}(\mathbf{x}_M - \mathbf{x})] \quad (21)$$

If the model inversion is exact, or equivalently $\mathbf{A}_D = \mathbf{A}$ and $\mathbf{B}_D = \mathbf{B}$, then Eq. (21) simplifies to

$$\dot{\mathbf{e}} = \mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M\mathbf{r} - \dot{\mathbf{x}}_M + (\mathbf{A} - \mathbf{B}\mathbf{K})(\mathbf{x}_M - \mathbf{x}) \quad (22)$$

Recognizing that $\mathbf{x}_M - \mathbf{x} = \mathbf{e}$ and $\mathbf{A}_M\mathbf{x}_M + \mathbf{B}_M\mathbf{r} = \dot{\mathbf{x}}_M$, Eq. (22) reduces to

$$\dot{\mathbf{e}} = (\mathbf{A} - \mathbf{B}\mathbf{K})\mathbf{e} \quad (23)$$

This result shows that the eigenvalues of $(\mathbf{A} - \mathbf{B}\mathbf{K})$ determine the dynamics of the model tracking error $\mathbf{e} = \mathbf{x}_M - \mathbf{x}$, and designing a controller for the model tracking error reduces to the regulator design problem, as discussed by Tyler [46]. Therefore, using techniques such as pole placement to calculate a state feedback gain $\mathbf{K}$ allows the designer to specify the model tracking error dynamics.

V. Near-Hover Attitude-Command Model-Following Controller

A near-hover attitude-command model-following controller for the unloaded Sikorsky S-61 helicopter was designed using the model discussed in Sec. II. The unloaded S-61 model was used for designing the feedback controller gains, and only the attitude states and translational helicopter velocities are used for feedback. The model inversion is constructed using the unloaded model, which only requires the specified attitudes and velocities as inputs to the inverted model. The inversion of the unloaded model leads to full cancellation of the dynamics of the unloaded helicopter plant.

However, when the helicopter carries a suspended load, the model inversion cancels the baseline helicopter dynamics but not the additional dynamics introduced by the presence of the suspended load. The controller no longer completely cancels the dynamics of the plant, and so error between the actual output and the prescribed model develops. Note that the loaded helicopter is simulated in this paper using the model of the load-carrying Sikorsky S-61 from Gupta and Bryson [37] and discussed in Sec. II.

A. Feedback Controller Design Using Unloaded S-61 Model

The model tracking performance can be specified by selecting eigenvalues of $\mathbf{A} - \mathbf{B}\mathbf{K}$ that give suitable model tracking error dynamics, and then the pole placement technique may be used to calculate $\mathbf{K}$. The selected eigenvalues are $0.8 \pm 0.4i$, $-4.0 \pm 4.6i$, -4.0 , and -28.0 . Using the unloaded Sikorsky S-61 $\mathbf{A}$ and $\mathbf{B}$ matrices results in a gain matrix of

$$\mathbf{K} = \begin{bmatrix} 22.60 & 2.735 & -0.4642 & 1.464 & -0.1871 & 0.2352 \\ 1.339 & 0.1425 & -0.0163 & 2.083 & 0.2455 & 0.0551 \end{bmatrix} \quad (24)$$

The feedback controller could be designed using other techniques, but the pole placement technique proves simple and effective.

B. Prescribed Models

The prescribed model $\mathbb{G}_M$ determines how the designer intends the helicopter to respond to pilot commands. A model that yields the desired helicopter performance should be selected. However, the model should not be too aggressive. An overly aggressive model will lead to commands that the helicopter actuators cannot produce. When the actuators cannot faithfully produce the commanded inputs, the helicopter will not follow the trajectory specified by the prescribed model.

A third-order model was selected that prescribes the desired helicopter attitude and attitude rate. This third-order model is a series combination of an underdamped second-order model and a first-order lag. The first-order lag is included to account for actuator dynamics and effectively smooths the plant command, ensuring that the command can be produced by the helicopter rotor. The majority of the dynamics at the timescale of gross helicopter translation and attitude response is prescribed by the second-order underdamped model.

The prescribed third-order pitch and roll model is given by

$$\dot{\mathbf{x}}_M = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\omega_n^2\tau & -(\omega_n^2 + 2\zeta\omega_n\tau) & -(2\zeta\omega_n + \tau) \end{bmatrix} \mathbf{x}_M + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \mathbf{r}$$

$$\mathbf{y}_M = \begin{bmatrix} \omega_n^2\tau & 0 & 0 \\ 0 & \omega_n^2\tau & 0 \end{bmatrix} \mathbf{x}_M \quad (25)$$

The pilot's attitude command is the input $\mathbf{r}$ to this model. Note that this model was designed to have a steady-state gain of unity, although this can be changed depending on the application and the source of the pilot's attitude command.

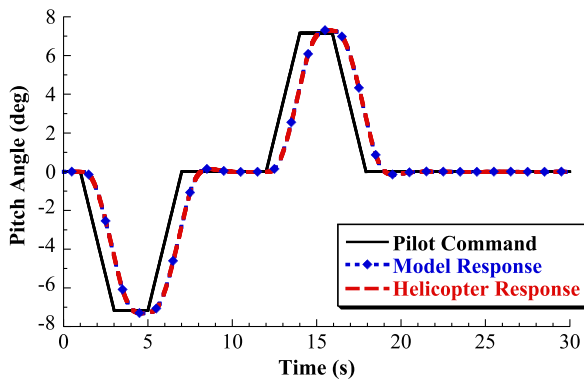
There are two of these third-order models in the control system, one each for the longitudinal and lateral channels. Each model is applied independently to the pilot's attitude command in that channel. The same first-order time constant, natural frequency, and damping ratio are used in the two models to give similar characteristics to the pitch and roll attitude responses.

The model parameters were selected to yield a damping ratio of 0.707 because this provides a good balance between fast rise time and low overshoot. A settling time t_s of 2.5 s was selected. Using this damping ratio and settling time, the natural frequency was found using the approximation

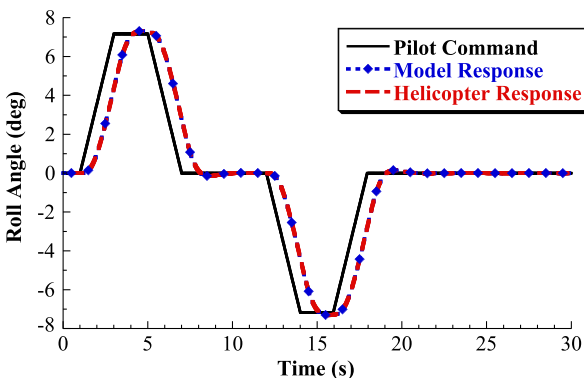
$$t_s \approx \frac{4}{\omega_n\zeta} \quad (26)$$

Solving Eq. (26) with the selected damping ratio and settling time resulted in a corresponding natural frequency of 2.26 rad/s.

A first-order time constant of 0.125 s was selected. This time constant was slow enough to ensure that the helicopter rotor could respond to the commanded rotor tilt angles, while not significantly altering the performance characteristics prescribed by the second-order model.



a) Pitch channel



b) Roll channel

Fig. 7 Pilot command and attitude response for the unloaded Sikorsky S-61.

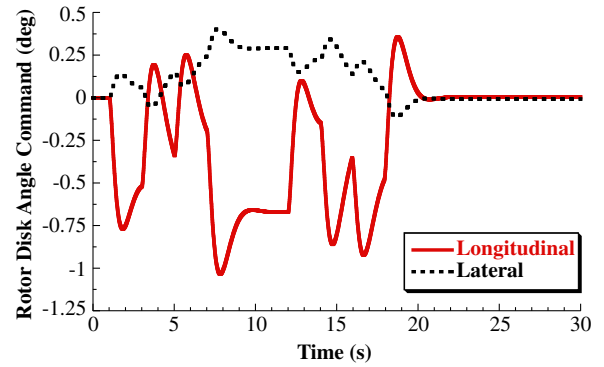


Fig. 8 Longitudinal and lateral rotor disk angles for the unloaded Sikorsky S-61.

The pitch and roll prescribed models are solved in real time by the controller, and both models generate desired attitude and attitude-rate trajectories $\mathbf{y}_M$. The result is pitch attitude, pitch rate, roll attitude, and roll rate trajectories prescribed by the models. Because the helicopter velocities are also needed to form the full state vector for inverting the unloaded model, the measured velocities of the plant are used. The attitude trajectories are combined with the measured longitudinal and lateral velocities to form the complete helicopter state vector. This state vector can be interpreted as the desired state trajectory.

C. Simulation Results and Controller Performance

1. Unloaded S-61 Performance

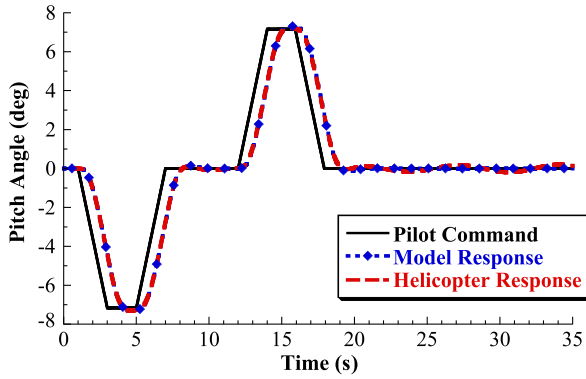
To evaluate the performance of this controller for the unloaded Sikorsky S-61 helicopter, a simultaneous pitch and roll maneuver was simulated. The pilot attitude command was designed to move the helicopter in both the longitudinal and lateral directions to a target location, starting and stopping in hover. The attitude command has a maximum amplitude of 7.16 deg, and this can be considered a small-to-moderate amplitude command.

Figure 7a shows the pilot command, the resulting response of the prescribed model, and the helicopter attitude response for the pitch channel. The pitch attitude tracks the prescribed model almost perfectly. Figure 7b shows the pilot command, the resulting response of the prescribed model, and the helicopter attitude response for the roll channel. As in the pitch channel, the roll attitude tracks the prescribed model almost perfectly. The controller also minimizes the coupling between the longitudinal and lateral motion of the helicopter.

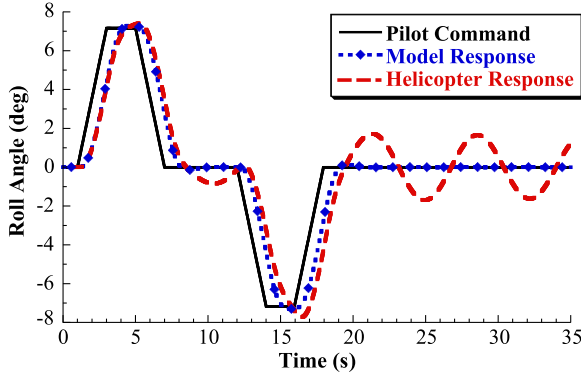
If the control effort required to execute this maneuver is too fast or has too large of an amplitude, then the quasi-static rotor assumption used in the modeling process may be inadequate for approximating the rotor dynamics. In other words, the commanded rotor disk angle may require the helicopter rotor to tilt faster than it can. Because the Sikorsky S-61 dynamic models do not fully model the rotor behavior or account for actuator limitations, the commanded rotor disk tilt angles need to be analyzed to verify that they are feasible. Given the larger inertia of the helicopter about the pitch axis, it is expected that the command in the longitudinal channel will need to have a higher magnitude to produce the required forces and moments for prescribed model tracking and, therefore, the limits of the rotor are most likely to be exceeded in the longitudinal channel.

To investigate these actuator dynamics, the commands sent to the helicopter plant in the longitudinal and lateral directions are shown in Fig. 8. As expected, the longitudinal command is more aggressive than the lateral command because the pitch inertia of the helicopter is larger than the roll inertia. Therefore, a larger-magnitude command is needed to accelerate the pitch states at the same rate as the roll states, as required by the prescribed models.

To verify that the control system requests feasible main rotor tilt angles, the commanded rotor disk angles must be further examined. The response of the rotor to control inputs occurs on the same timescale as the rotor rotation rate. The time to half-amplitude of the rotor tilt is on the order of 0.05 s [31], and the time constant of the rotor response is typically one-quarter to one-half of the rotor rotation



a) Pitch channel



b) Roll channel

Fig. 9 Pilot command and attitude response for the loaded Sikorsky S-61.

period [33]. Half of the rotor rotation period for the Sikorsky S-61 main rotor is approximately 0.15 s [36]. The longitudinal command generated by the controller requires a rotor response with a time to half-amplitude of approximately 0.4 s, which is almost three times slower than the rotor capability. Also, the maximum amplitude of the command is only 1 deg, which is a small rotor tilt angle. These results suggest that the controller is requesting feasible rotor tilt angles.

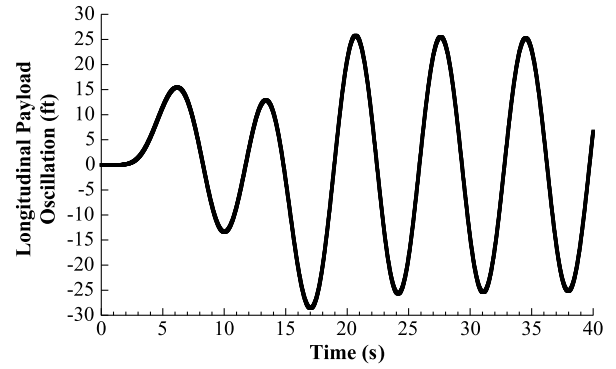
2. Loaded S-61 Performance

The performance of the load-carrying Sikorsky S-61 helicopter was investigated using the same controller and pilot commands as for the unloaded helicopter in the previous section. For feedback, the controller still uses only the unloaded helicopter states (pitch and roll attitudes and attitude rates, and longitudinal and lateral translational velocities). Measurements of the load states are not used by the controller.

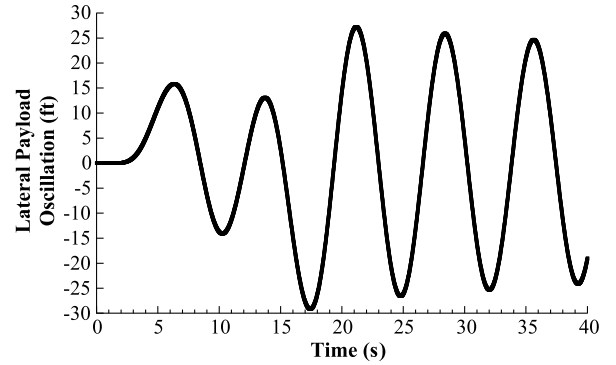
Figure 9 shows the pilot command, the resulting response of the prescribed model, and the helicopter attitude response for the pitch and roll channels. The response in the pitch channel is shown in Fig. 9a. The pitch attitude tracking of the prescribed model is not significantly affected by the addition of a suspended load. This is due to the large pitch inertia of the helicopter resisting the moment applied by the swinging suspended load.

The response in the roll channel is shown in Fig. 9b. The roll attitude does not track the model response very well, particularly following the completion of the command. Following the command, there are residual roll attitude oscillations that have an amplitude of nearly 2 deg. These oscillations are caused by the tension in the suspension cable supporting the load applying an oscillatory torque about the helicopter center of gravity as the load swings. In this sense, the load is acting like a disturbance applied to the helicopter.

According to ADS-33E-PRF design requirements [30], residual attitude oscillations larger than 0.5 deg are considered excessive for any type of maneuver. As seen in Fig. 9a, the pitch attitude residual oscillation amplitude is less than 0.5 deg, and remains within ADS-33E-PRF requirements. However, the residual roll attitude oscillation



a) Longitudinal load swing



b) Lateral load swing

Fig. 10 Suspended-load swing in the a) longitudinal and b) lateral directions.

amplitude is nearly 2 deg, and so it does not satisfy ADS-33E-PRF requirements.

Figure 10 shows the longitudinal and lateral load oscillation caused by the helicopter motion. The 65.6 ft suspension cable leads to a large-amplitude, low-frequency response of the load. The peak-to-peak amplitude of the load oscillation in both the pitch and roll directions is nearly 55 ft. This amount of load swing could be dangerous and hard for the pilot to control following the maneuver. It would also increase the time it takes for the pilot to set down the load, because he or she would have to wait for the load oscillation to damp out or actively move the helicopter to cancel out the swing before setting the load down.

The amount of residual attitude and load oscillation is dependent on properties of the pilot's commands, such as move distance and velocity. This means the amount of residual oscillation will vary depending on the pilot's commands, making the oscillations hard for the pilot to predict and thus difficult to control.

VI. Combining Input-Shaping and Model-Following Control

The model-following controller does not address the load oscillation or its disturbance-like effects on the model tracking error. To reduce the load oscillation, input shaping is added to the model-following controller. The model-following controller block diagram with input shaping is shown in Fig. 11. Input shaping is added to the controller between the pilot command and the prescribed model.

Effectively, this implementation builds input shaping into the prescribed model, and the model output is an input-shaped command that does not excite suspended-load swing. The order of the input shaper and the prescribed model in the controller does not matter because input shapers and the chosen prescribed model are linear.

A. Input Shaper Design

To design an input shaper, the natural frequencies and damping ratios of the oscillation in the longitudinal and lateral directions are required.

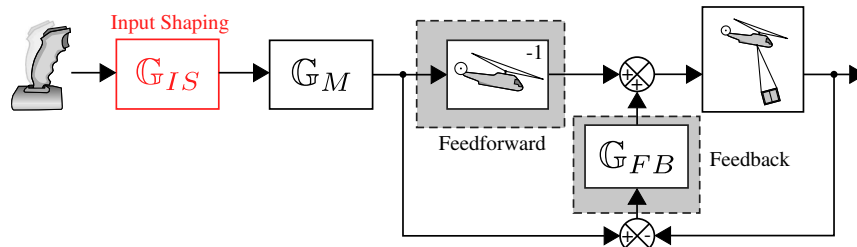


Fig. 11 Combined input shaping and model-following control for the load-carrying Sikorsky S-61 helicopter.

The log decrement method was applied to the data shown in Fig. 10 to determine the natural frequency and damping ratio. The longitudinal load oscillation has a frequency of 0.907 rad/s and a damping ratio of 0.001. The lateral load oscillation has a frequency of 0.869 rad/s and a damping ratio of 0.007. The frequency is slightly different in the two directions because it mildly depends on the helicopter inertia, which is significantly different about the pitch and roll axes.

It is worth noting here that the natural frequency and damping ratio are also partially influenced by the feedback control. The feedback controller tries to drive the model tracking error to zero. However, the load acts like a disturbance on the helicopter motion, as can be seen in Fig. 9, because the load is coupled with the helicopter attitude and translation. This disturbance introduces an error between the trajectory specified by the prescribed model and the actual helicopter states that the feedback controller attempts to eliminate. The feedback controller affects the load oscillation as the controller attempts to eliminate the model tracking error. It does so by adjusting the commands sent to the helicopter plant and therefore affects the motion of the helicopter and the load. This means the poles used to design the feedback controller and calculate the gain matrix K end up having a small effect on the motion of the load and its swing frequencies and damping ratio.

A ZV input shaper was selected for use in this model-following controller. The input shaper amplitudes and times were calculated

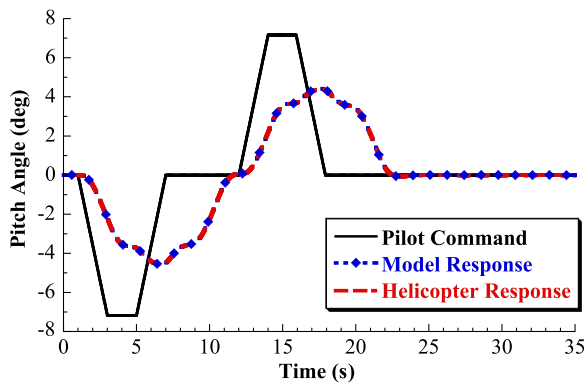
from the load swing natural frequencies and damping ratios. A separate input shaper was used for the longitudinal and lateral load oscillations because their frequencies were somewhat different.

B. Simulation Results and Controller Performance with Input Shaping

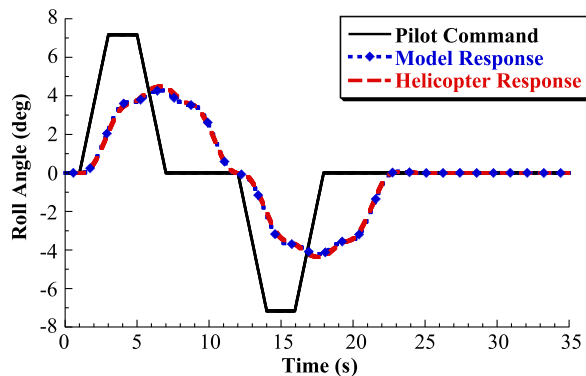
Figure 12 shows the input-shaped pilot command, the resulting response of the prescribed model, and the helicopter attitude response for the pitch and roll channels with input shaping added to the model-following controller. The pitch response is shown in Fig. 12a. The pitch attitude tracking of the prescribed model is not as significantly affected as the roll channel by the addition of a suspended load, as is shown in Fig. 9. Adding input shaping to the controller improves the tracking by reducing the small residual pitch attitude oscillations.

The real benefit, in terms of the attitude response, of adding input shaping to the controller is in the roll dynamics. The roll response is shown in Fig. 12b. The roll attitude tracking of the model response is greatly improved with input shaping added to the controller. As a result, the ADS-33E-PRF requirements for residual attitude oscillation amplitude are satisfied. With the residual attitude oscillations reduced, the addition of input shaping to the controller should make the response of the helicopter more predictable to the pilot.

Figure 13 compares the longitudinal and lateral load oscillation caused by the helicopter motion with and without input shaping.

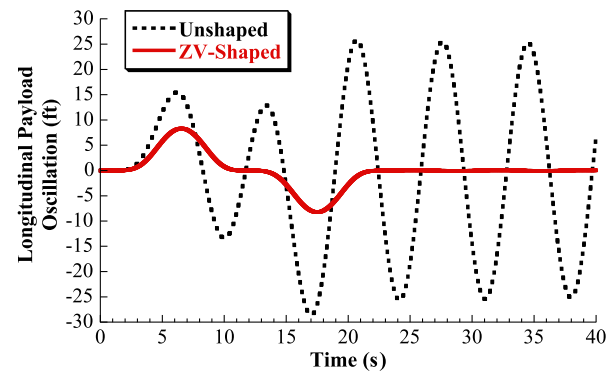


a) Pitch channel

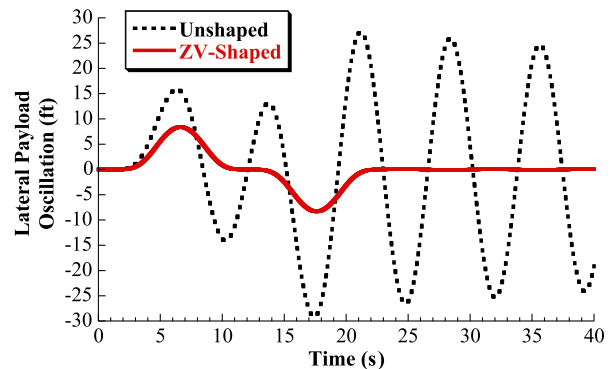


b) Roll channel

Fig. 12 Pilot command and attitude response for the loaded Sikorsky S-61 with input shaping added to the model-following controller.



a) Longitudinal load swing



b) Lateral load swing

Fig. 13 Unshaped and ZV-shaped suspended-load swing in the a) longitudinal and b) lateral directions.

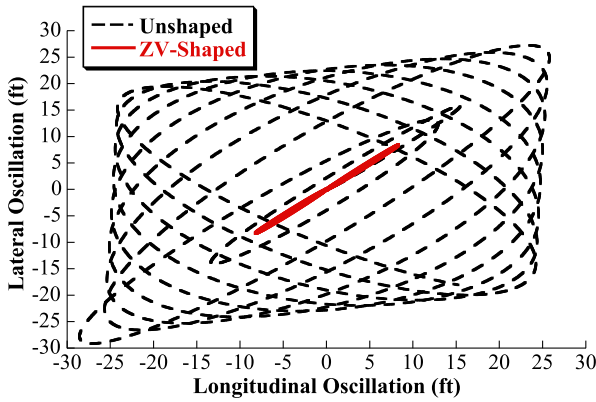
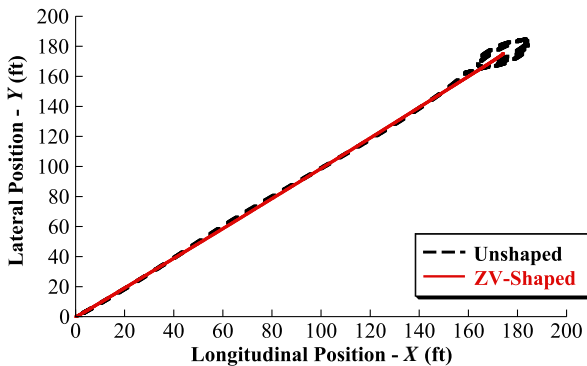


Fig. 14 Unshaped and ZV-shaped two-dimensional load oscillation.

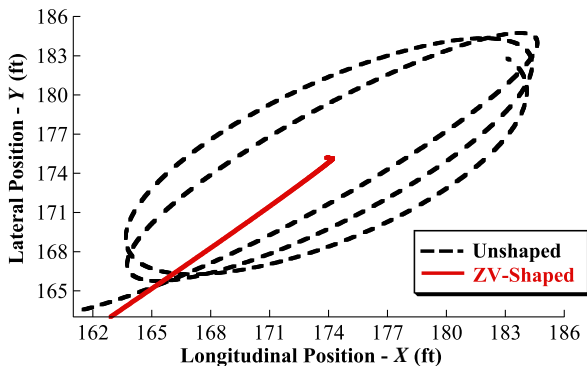
Input shaping significantly reduces the residual load oscillation in both the longitudinal and lateral directions. Note that there is still some transient swing that occurs during the command. Input shapers are only designed to prevent residual oscillation, but they have the added benefit of greatly reducing transient swing as well.

The effectiveness of input shaping is even more clear when looking at the load oscillation in two dimensions. Figure 14 shows the two-dimensional load oscillation. The oscillation is measured relative to the position of the helicopter, and so it is what the pilot would see when looking down on the load. The small amount of swing that occurs in the ZV-shaped case is the transient oscillation.

When using input shaping, the helicopter comes to rest over the desired position at the end of the maneuver because the residual load swing is eliminated. Figure 15 shows the helicopter's position in the horizontal plane as the maneuver is executed with and without input shaping. Figure 15a shows the helicopter traveling in the longitudinal and lateral directions during the maneuver. Figure 15b shows the helicopter's position at the end of the maneuver. In the unshaped case, the residual load swing back drives the helicopter, causing the helicopter to oscillate around the desired position. In the input-shaped

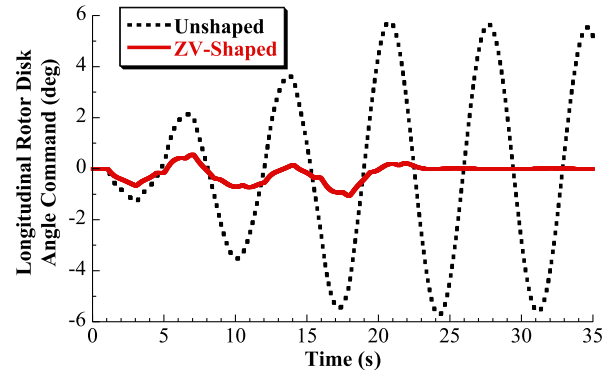


a) Helicopter position during the maneuver

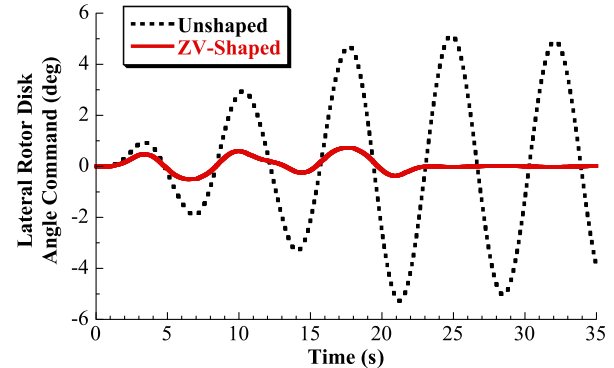


b) Helicopter position at the end of the maneuver

Fig. 15 Helicopter position during and at the end of the maneuver.



a) Longitudinal command



b) Lateral command

Fig. 16 Control effort required by the model-following controller in the a) longitudinal and b) lateral channels for the loaded Sikorsky S-61.

case, the helicopter comes to rest over the desired position. This occurs because the residual load swing has been eliminated, and so the load does not disturb the helicopter at the end of the maneuver. Because input shaping significantly reduces the residual load oscillation and results in the helicopter coming to rest over the desired position, it should decrease the time it takes for the pilot to transfer a load from one location to another.

As with the controller for the unloaded helicopter, the control effort required to execute the maneuver should be examined. The command sent to the helicopter plant in the longitudinal and lateral channels is shown in Fig. 16 with and without input shaping. Figure 16a shows the longitudinal command, and Fig. 16b shows the lateral command. When there is no input shaping, the command is oscillatory and has a larger amplitude compared with the case with input shaping.

As discussed previously, the swing of the load acts like a disturbance applied to the helicopter. This causes error in the model tracking that the feedback controller tries to correct. This corrective action causes the command to have a larger amplitude and to be oscillatory. On the other hand, input shaping proactively eliminates the load swing before it disturbs the helicopter, causing no need for extra corrective action from the feedback controller.

VII. Discussion

Although model-following control has been used in helicopter flight controllers since the 1960s, the implementation presented here for suspended-load control is unique. A model-following controller designed for an unloaded helicopter was combined with an input shaper designed to prevent suspended-load oscillation in the longitudinal and lateral directions. The results show that the combined controller eliminates oscillations of a suspended load and improves tracking of the prescribed model.

This approach is effective because the model-following controller enables the motion of the aircraft to behave like the desired model, whereas the input shaper eliminates the undesirable oscillatory dynamics of the suspended load. In effect, an input-shaped prescribed model is used to achieve the desired helicopter response and critically

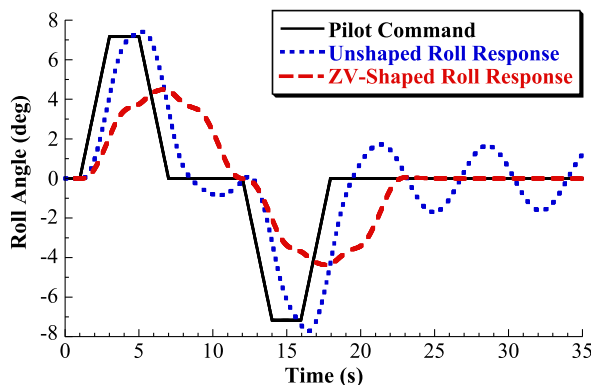


Fig. 17 Comparison of unshaped and ZV-shaped helicopter roll responses.

damped load motion. By using input shaping to prevent load oscillation, the model-following control design can be accomplished using a model of the unloaded helicopter, which is much simpler than a model of a loaded helicopter.

Figure 17 compares the unshaped and input-shaped helicopter roll responses shown in Figs. 9b and 12b. Although the unshaped response appears to more closely track the pilot's command during the transient motion, it results in residual oscillation of the attitude and load. The problematic characteristics of the pilot's command that result in the residual load oscillation are removed by applying ZV input shaping to the command. The addition of input shaping results in the ZV-shaped roll response shown in Fig. 17. Although the response with input shaping does not follow the pilot's transient command as closely, the helicopter completes the maneuver with almost no residual load oscillation, as was shown in Fig. 13. Preventing suspended-load oscillation also improves the residual helicopter attitude response because the effects of the load on the helicopter are reduced when the load swing is reduced. As a result, the maneuver is completed with almost no residual attitude oscillation.

The helicopter's response to pilot commands should be more predictable because the pilot does not need to account for the slow-period and large-amplitude load oscillations and their effects on the helicopter. The transient load oscillations are reduced and the load hangs directly below the helicopter when it stops, as shown in Figs. 13 and 14. This should make transferring a suspended load from one location to another safer, faster, and easier for the pilot. However, the response lag introduced by input shaping could make control more difficult for pilots. Past research on input shaping and operator learning suggests that operators are able to modify their control strategy, become accustomed to the response lag, and improve their overall performance [26,27]. To fully analyze the effectiveness of the proposed control scheme, piloted evaluations and feedback must be obtained using piloted flight simulations to determine the handling qualities. Or, full evaluation of ADS-33E-PRF handling qualities requirements and flight characteristics should be determined to give an indication of the expected piloted performance. However, piloted evaluations would more effectively capture the expected learning effects.

Another significant benefit of this controller is that real-time measurement or estimation of the suspended-load states is not required. Input shaping proactively eliminates the residual vibration of the suspended load, whereas a load-swing feedback controller would have to respond retroactively to measurements in the suspended-load states to damp the load oscillation. The approach presented here simply requires an estimate of the load oscillation frequency and damping ratio in addition to the unloaded helicopter model.

One limitation of this study was the use of a helicopter model that was linearized about hover. Therefore, effects such as air drag, which should be incorporated for forward flight modeling, were weakened by the linearization. The results presented here could be further verified by using a full nonlinear model to simulate the helicopter plant. In addition, suspension cable length, load size and shape, helicopter/load mass ratio, and the location of the suspension point

relative to the helicopter center of gravity affect the natural frequency and damping ratio of load oscillation. Further studies are required to investigate the robustness of the input-shaping and model-following approach to uncertainty in the load oscillation frequency and damping ratio. In addition, the effects of disturbances such as wind on the controller performance should be investigated.

VIII. Conclusions

Applying input shaping to simulated pilot commands greatly reduced oscillation of a helicopter's suspended load. By proactively eliminating load swing using input shaping, the helicopter response itself also improved because the helicopter and load are coupled. The results showed that input shaping can be combined with a typical helicopter flight control system to proactively prevent suspended-load oscillation and improve helicopter flight performance. The unique approach presented here combines the commonly used control methods of model-following control and input shaping in a simple way to address the challenging control problem of suspended-load oscillation. The proposed controller eliminates the slow-period and large-amplitude load oscillations and their effects on the helicopter. The transient load oscillations are reduced, and the load hangs directly below the helicopter when it stops. This should make transferring a suspended load from one location to another safer, faster, and easier for the pilot, but piloted simulations need to be performed to fully verify this expected result.

Acknowledgments

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