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INDUCTIVELY COUPLED RADIO FREQUENCY POWER
TRANSMISSION SYSTEM FOR WIRELESS SYSTEMS
AND DEVICES

Kathleen O'Brien

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List of variables, abbreviations, and indices

$\alpha_{s,rms}$	<i>rms</i> shielding factor
α_{center}	Shielding factor at the center of a system
α^*	Shielding factor at point *
α	Angle between the axis of the source and receiving coils
α_A	Shielding factor at point <i>A</i>
α_B	Shielding factor at point <i>B</i>
α_C	Shielding factor at point <i>C</i>
α_s	Shielding factor
ε	Permittivity
ε_0	Permittivity of free space
Λ_{sr}	Flux linkage between a source and receiving coil
δ	Skin depth
ϕ	Angle between two elements
ϕ_m	Modulation angle
$\phi_{receiver}$	Flux crossing the receiving coil
$\phi_{receivingcoil, max}$	Maximum flux crossing a single receiving coil
ϕ_{source}	Total flux produced by one source coil
ϕ_{sr}	Flux linking source and receiving coils
ϕ_{stray}	Stray flux
ϕ_{Total}	Total flux
ϕ_x	Flux crossing the <i>x</i> -coil
ϕ_y	Flux crossing the <i>y</i> -coil
ϕ_z	Flux crossing the <i>z</i> -coil
$\phi_{rs,max}$	Maximum flux crossing a single receiving coil winding
φ_e	Phase of exciting voltage
ρ	Radial distance from the origin of the coil
ρ	Radius of a conductor

σ	Conductivity
σ_{rc}	Conductivity of a shielding material related to copper
Δf	Difference between frequencies in a system
λ	Wavelength
μ_0	Permeability of free space
μ_{rod}	Effective permeability of a ferrite rod antenna
μ_{rc}	Permeability of a shielding material related to copper
μ	Permeability
μ'	Effective permeability
μ_i	Initial relative permeability
μ_r	Relative permeability
ω	Radian operating frequency
ω_d	Angular frequency of damped resonant circuit
ω_e	Frequency of the exciting voltage
ω_m	Modulation radian frequency
ω_r	Resonant frequency
ω_{r0}	Ideal Resonant Frequency
$\omega_{p,res}$	Resonant frequency in a parallel resonant circuit
ω_r	Resonant frequency
$\omega_{s,res}$	Resonant frequency in a series resonant circuit
θ	Angle of rotation relative to the y-axis
ψ	Angle of rotation relative to the x-axis
A	Area enclosed by one receiving coil
a	Radius of a circular coil
a, b, c	Variables defining position with respect to the x-, y-, and z-axes
$a_k (k=\{x,y,z\})$	Unit vector in the x-, y-, and z-directions
A_{loss}	Absorption loss
A_m	Amplitude of modulation function
A_r	Area encompassed by a receiving coil
B	Magnetic flux density

B_{rect}	Magnetic flux density created by a rectangular coil
B_s	Magnetic flux density created by a source coil
b_w	Bandwidth
B_x	Magnetic flux density in the x -direction
B_y	Magnetic flux density in the y -direction
B_z	Magnetic flux density in the z -direction
C_{ldecay}	Constant in the decaying condition
C_{lrise}	Constant in the rising condition
C_{dc}	dc side capacitor
C_{pr}	Capacitance in a parallel resonant circuit
C_r	Number of receiving coils in a system
C_{res}	Resonant capacitance
C_s	Number of source coils in a system
C_s'	Source side resonant capacitance referred to the receiving side
C_{sr}	Capacitance in a series resonant circuit
d	Distance
D	Distance of separation for source coils
D_{boost}	Duty cycle of boost stage
D_{buck}	Duty cycle of buck stage
d_{coils}	Distance between coils
d_H	Helmholtz distance
D_m	Demagnetization factor
d_s	Thickness of a shield
DSSC	Double Sideband Suppressed Carrier
E	Electric field
E_ϕ	Electric field in the ϕ direction
emf	Electromotive force
f	Frequency
f_{beat}	Beat frequency
$f_{carrier}$	Carrier frequency
f_{cycle}	Cycling frequency

$f_{modulation}$	Modulation frequency
f_x	Frequency of current in the x -axis source coil(s)
f_y	Frequency of current in the y -axis source coil(s)
f_z	Frequency of current in the z -axis source coil(s)
H	Magnetic field intensity
H_ϕ	Magnetic field intensity in the ϕ direction
H_θ	Magnetic field intensity in the θ direction
H_{circ}	Magnetic field intensity created by a single circular coil
H_{min}	Minimum field intensity required to power a receiver
H_P	Magnetic field intensity at point P
H_r	Magnetic field intensity at the location of the receiving coil
H_{rect}	Magnetic field intensity created by a rectangular coil
$H_{rect,axis}$	Magnetic field intensity on the axis of a rectangular coil
H_{res}	Magnetic field intensity vector resulting when a shield is present
H_{rms}	RMS value of magnetic field intensity
H_s	Magnetic field intensity at a point P when shielding is present
H_{set}	The intensity of the field created by a set of rectangular coils
H_{Sp}	Peak value of the source field
H_{sum}	Magnetic field intensity vector
H_u	Magnetic field intensity at a point P when shielding is not present
$H_{unshielded}$	Unshielded magnetic field vector
H_v	System transfer function
H_x	Magnetic field intensity in the x -direction
H_{x_eddy}	Magnetic field intensity created by eddy currents
H_y	Magnetic field intensity in the y -direction
H_z	Magnetic field intensity in the z -direction
I	<i>rms</i> current
i	Current
I_{Cr_k}	Current in receiving coil k ($k=\{1,2,3\}$)
I_{Env}	Envelope function of capacitor current
I_i	Image current

I_{off}	Turn off current
I_{on}	Turn on current
I_{rated}	Rated current
I_{res}	Resonant current
I_s	Current in a source coil
I_{srx}	Current in a series resonant circuit (coil x)
I_{ss}	Amplitude of the steady state current through a load
I_{ss_fb}	Steady state current in a full bridge converter
I_{ss_hb}	Steady state current in a half bridge converter
J	Current density
k	Coupling factor
K	Constant
k_{rr}	Coupling factor between receiving coils
k_{rxy}	Coupling factor between the x^{th} and y^{th} receiving coils
k_{sr}	Coupling factor between source and receiving coils
k_{ss}	Coupling factor between source coils
k_{sxy}	Coupling factor between the x^{th} source and y^{th} receiving coil
k_{sxy}	Coupling factor between the x^{th} and y^{th} source coils
L	Self inductance of a coil
l	Length of a conductor
ℓ_m	Length of a shielding material
L_s'	Source coil inductance referred to the receiving side
L_{12}	Mutual inductance between circuit elements 1 and 2
L_{circle}	Inductance of a circular coil
lcm	Least common multiple
L_{coil}	Inductance of a coil without its core in place
$L_{coil+core}$	Inductance of a coil with its core in place
L_m	Mutual inductance
L_{pr}	Inductance in a parallel resonant circuit
L_r	Self inductance of a receiving coil
L_{rec}	Self inductance of a rectangular coil

L_{res}	Resonant inductance
L_{rl}	Leakage inductance between source and receiving coils
L_{rod}	Inductance of a ferrite rod antenna
L_s	Self inductance of a source coil
L_{sr}	Inductance in a series resonant circuit
M	Mutual Inductance
M'	Mutual inductance referred to the receiving side of the system
M''	Mutual inductance for a fully aligned system
M_{rs}	Mutual inductance between source and receiving coils
MSC	Magnetic Search Coil
M_{sr}	Mutual inductance between a source and receiving coil
n	Number of turns
n	Turns ratio (n_s/n_r)
N_k	Number of coupling factors needed to define a system
n_r	Number of turns on a receiving coil
n_s	Number of turns on a source coil
p	Perimeter
$\vec{P}$	Poynting vector
P	Power
P_{ave}	Average power
P_{Hmin}	Point at which H is at its minimum value
P_r	Power to the load on the receiving coil
p_r	Perimeter of the receiving coil
P_s	Power to the load on the source coil
p_s	Perimeter of the source coil
p_σ	Ohmic power density
$Q_{inductor}$	Quality factor
Q_p	Quality factor for a parallel resonant circuit
Q_r	Quality factor of the reciever
Q_{rod}	Quality factor for a ferrite rod antenna
Q_s	Quality factor for a series resonant circuit

$\mathcal{R}_m$	Reluctance of a shielding material
R	Resistance, radius
r	Distance from an object
R'_s	Source coil resistance referred to the receiving side
R_{ac}	Ac resistance
$R_{ac, input}$	Input impedance
R_c	Core loss resistance
r_c	Radius of a coil
R_{dc}	dc resistance
R_{equ}	Equivalent source/load impedance
RFID	Radio Frequency Identification
R_{loss}	Reflection loss
R_{pr}	Coil resistance in a parallel resonant circuit
r_r	Radius of a receiving coil
R_r	Resistance of a receiving coil
r_s	Radius of a source coil
R_s	Resistance of a source coil
R_{sr}	Coil resistance in a series resonant circuit
R_{srX}	Parasitic resistance in the series resonant circuit of coil x
R_{ss}	Source or sink impedance
R_t	Terminal impedance
r_w	Radius of a wire
$R_{winding}$	Winding loss resistance
$R_{x,y,z}(\psi, \theta, \phi)$	Rotation matrix
S	Cross sectional area
S_p	Peak value of the source field magnitude
S_r	Surface of receiving coil
S_s	Surface of source coil
t_p	Duration that each source coil set is energized
t_r	Duration that each source coil set carries a single frequency
t_{rise}	Rise time

t_{seq}	Sequencing time
t_{ss}	Steady state time
ν	Detuning factor
V	Volume
V_{cmin}	Minimum acceptable receiver capacitor voltage
v_{Cr}	Voltage of the resonant circuit capacitor
V_{Cr_0}	No load voltage of coil
V_{Cr_Env}	Envelope function of capacitor voltage
V_{Cr_k}	Capacitor voltage for receiving coil k
V_{Csrx}	Capacitor voltage in a series resonant circuit of coil x
V_{dc}	dc voltage
v_e	ac voltage
V_{lb}	Lower boundary of receiver capacitor voltage
v_{Lr}	Voltage of the resonant circuit inductor
V_{Lsrx}	Inductor voltage for a series resonant circuit (coil x)
V_{pr}	Equivalent source voltage in a parallel resonant circuit
v_r	Voltage at the terminals of a receiving coil
V_r	<i>rms</i> receiving coil voltage
V_{rated}	Rated voltage
V_{rms}	<i>rms</i> voltage
v_{Rr}	Voltage across the resonant circuit resistor
V_{Rsex}	Voltage across the resistor in a series resonant circuit of coil x
v_s	Voltage at the terminals of a source coil
V_s	<i>rms</i> source coil voltage
$V_{sdc,avg}$	Average output voltage for a two-pulse rectifier
V_{sr}	Equivalent source voltage in a series resonant circuit
V_{srx}	Ac <i>rms</i> voltage
V_{ub}	Upper boundary of receiver capacitor voltage
w	Width
w_e	Electric energy density
w_m	Magnetic energy density

x	Distance along the x -axis
y	Distance along the y -axis
z	Distance along the z -axis
Z_{coil}	Coil impedance
Z_{load_opt}	Load impedance for maximum power transfer
Z_{r0}	Resonant impedance of ideal LC circuit

1 Introduction

Wireless communication technologies have advanced substantially in recent years, enabling a new level of flexibility and efficiency in electronic systems. Parallel advances in the areas of low power electronics and microsystems technology have resulted in the development of many new devices requiring low cost power supplies. The development of Micro-Electro-Mechanical Systems (MEMS) (the integration of mechanical elements, sensors, actuators, and electronics on a common silicon substrate) is a rapidly expanding field [1-1]. Many of these devices will be deployed in areas where wired energy supply is not possible and regular maintenance is not desired or practical. The increasing prevalence of low-cost, low power electronic devices in applications where wired energy transfer is not suitable or even possible calls for power supply options beyond those which are currently offered. In systems involving, for example, robots or robot chains, fully automated production machines, inaccessible or hazardous environments, or applications with high insulation requirements, wired power supplies are often not a viable option. Wires provide opportunities for failure, especially in moving or rotating systems where wire fatigue is a problem and in industrial automation systems with sensor and actuator densities high enough to significantly complicate maintenance.

There are currently several options available when a wired supply is not feasible. Batteries can be an alternative, but require regular recharging and/or replacement at high cost if production must be stopped before maintenance work can begin. Micro fuel cells are also an option, but the technology remains nascent, the reliability is low, and fuel cells also require recharging (albeit less often than batteries). A reliable alternative which requires little maintenance, no regular recharging or replacing of components, and is available at a reasonable cost, is clearly of interest. This thesis presents a novel alternative to the problem of providing power to devices without the use of wires or regular maintenance.

A non-conventional transformer with a large air-gap in the magnetic path can be operated with resonant switch mode power supplies to supply energy to a load [1-2], [1-3], [1-4]. It can enable completely wireless applications by providing auxiliary energy without wires via magnetic fields. The fields can cover distances of up to several meters and volumes from one to several hundred cubic meters. The system consists of one or more source coils encompassing an operating volume and one or more receiving coils located within that volume.

For reliable operation the system must be designed and powered in such a way as to transfer the energy to receivers that may be shifting in position and may be magnetically shielded by metallic objects within the operating volume. The field created by the source coils must be at or above a minimum value required to transfer power to the receivers while remaining within limits set by international standards for magnetic field strength.

This thesis focuses on the development and analysis of methods by which reliable power can be delivered to a load even in highly shielded environments. Systems are developed in which the magnetic fields have components in all three vector directions. This gives the system a high degree of resiliency to shielding, even in cases when a receiver is almost completely encased in a shielding material. Non-field related solutions are also introduced, allowing operation of receivers even under the most difficult conditions.

This work begins with a general description of the system and its operating principles, with reference to existing technologies using similar principles. It continues by exploring various design options for the source side of the system with respect to optimum coil separation distance, uniformity of power transfer, generation of desired magnetic field vectors, and ability to overcome the problems associated with shielding of the magnetic field. Uni-directional and bi-directional systems are discussed, and omni-directional systems are introduced along with several methods for creating them. The effects of shielding by conductive or permeable objects located within the operating volume are then investigated and several methods for the mitigation of this effect are presented. A coupling model is developed and a circuit model is derived. Source and receiver power converter characteristics are investigated and topologies are proposed. Finally, results from laboratory experiments are presented and compared with theoretical results.

2 Theory of Magnetic Power Transfer

This thesis deals with energy transfer using magnetic fields, and with the corresponding ohmic power loss. The system uses a magnetic field generated by a system of source coils. The source coils couple inductively to the receiving coils, forming a system in which energy is transferred from source to receiver via a magnetic field. This chapter describes the method by which this energy transfer takes place. The fundamental characteristics of the system are described, and the operating conditions are defined. Similar systems are then described, and alternatives to inductive power transfer are explored.

2.1 Fundamental considerations

The system is designed to operate in air or other non-conductive power transfer media. It will encompass operation in a vacuum, in air, and in solid bodies such as in systems using sensors embedded in polymers or resin. The effects of various materials in shielding a magnetic field are discussed in Chapter 4.

2.1.1 Transfer of energy in an electromagnetic field

A relation can be derived between the rate of energy transfer by electromagnetic waves in space and the electric and magnetic field intensities associated with traveling electromagnetic waves [2-1]

Beginning with the differential forms of Faraday's and Ampere's laws and assuming a simple medium with permeability μ , conductivity σ , and permittivity ϵ constant over time, the equation

$$\oint (\vec{E} \times \vec{H}) \cdot d\vec{s} = -\frac{\partial}{\partial t} \int \left(\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2 \right) dv - \int \sigma E^2 dv \quad (2-1)$$

can be written. The first and second terms on the right side represent the time-rate of change of the energy stored in the electric and magnetic fields, respectively. The final term represents the ohmic power dissipated in the volume as a result of the flow of conduction current density $\sigma \vec{E}$ in the presence of the electric field. The right side of the equation therefore represents the rate of decrease of the electric and magnetic energies stored, subtracted by the ohmic power dissipated as heat in the volume V . In order to satisfy the law of conservation of energy, this must be equal to the power leaving the volume through its surface. Thus the quantity $\vec{E} \times \vec{H}$ is a vector representing the power flow per unit area.

The Poynting vector describes the power per unit area transported by electric and magnetic fields. It points in the direction of propagation of a traveling electromagnetic wave and has the dimensions of power per area. It is defined as

$$\vec{P} = \vec{E} \times \vec{H} \quad (2-2)$$

Poynting's theorem asserts that the surface integral of the Poynting vector over a closed surface equals the power leaving the enclosed volume.

Equation (2-2) can be written as

$$-\oint_S \vec{P} \cdot d\vec{s} = \frac{\partial}{\partial t} \int_V (w_e + w_m) dv + \int_V p_\sigma dv \quad (2-3)$$

where

$$w_e = \frac{1}{2} \epsilon \vec{E}^2 = \frac{1}{2} \epsilon \vec{E} \cdot \vec{E}^* = \text{Electric energy density}$$

$$w_m = \frac{1}{2} \mu \vec{H}^2 = \frac{1}{2} \mu \vec{H} \cdot \vec{H}^* = \text{Magnetic energy density}$$

$$p_\sigma = \sigma E^2 = J^2 / \sigma = \sigma \vec{E} \cdot \vec{E}^* = \vec{J} \cdot \vec{J}^* / \sigma = \text{Ohmic power density}$$

Thus the total power flowing into a closed surface at any instant equals the sum of the rates of increase of the stored electric and magnetic energies and the ohmic power dissipated inside the enclosed volume.

2.1.2 Near-field operation vs. far-field operation

This work will be limited to an analysis of the power transfer characteristics between distributed sources and sinks (receivers) using magnetic coupling in the near field. Operation in the near field allows significantly more energy to be transferred between source and receiver. In order to achieve operation in the near field, the expected operating volume must be restricted in such a manner as to allow the receivers to maintain a position within the near field region of the magnetic field source or sources.

The near-field region is defined as the area in which angular field distribution is dependent upon distance from the source [2-2]. The electric and magnetic fields produced by a magnetic dipole can be used to understand the division between near and far fields. The fields created by a magnetic dipole are shown in Figure 2-1 can be described by the following equations [2-3]:

$$E_\phi = -j \frac{\omega \mu_0 m \beta}{4\pi} \left[\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} \right] e^{-j\beta r} \sin \theta \quad (2-4)$$

$$H_r = j 2 \frac{\omega \mu_0 m \beta^2}{4\pi \eta_0} \left[\frac{1}{\beta^2 r^2} - \frac{j}{\beta^3 r^3} \right] e^{-j\beta r} \cos \theta \quad (2-5)$$

$$H_{\theta} = j \frac{\omega \mu_0 m \beta^2}{4\pi \eta_0} \left[\frac{j}{\beta r} + \frac{1}{\beta^2 r^2} - \frac{j}{\beta^3 r^3} \right] e^{-j\beta r} \sin \theta \quad (2-6)$$

$$E_r = H_{\phi} = H_{\theta} = 0 \quad (2-7)$$

where $m = I(\pi b^2)$, ω is the frequency of the current in the loop, $\beta = 2\pi/\lambda$, λ is the wavelength of the current, and $\eta_0 = \sqrt{\mu_0 / \epsilon_0}$.

Equations (2-5) and (2-6) show that an infinitesimal loop of wire carrying a sinusoidal current will produce a magnetic field with a $1/r^3$ dependence (magnetic field strength falls off as $1/r^3$, where r is the distance from the loop) and an electric field with a $1/r^2$ dependence. Each term in equations (2-4), (2-5), (2-6), and (2-7) containing the variable ' r ' has the coefficient $\beta = \frac{2\pi}{\lambda}$. The terms

$$\frac{1}{(\beta r)^n} \text{ where } n=1, 2, \text{ or } 3 \text{ can be written as}$$

$$\frac{1}{(\beta r)^n} = \frac{1}{\left(\frac{r}{\lambda/2\pi} \right)^n}.$$

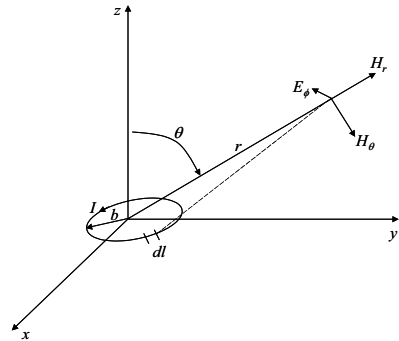


Figure 2-1 Electric and magnetic fields produced by a magnetic dipole

Thus the relative strength of the terms depends on the ratio of the distance from the source to the term $\frac{\lambda}{2\pi}$. When $r = \frac{\lambda}{2\pi}$, all components have the same strength. This is the dividing line between the near- and far-field regions, shown in Figure 2-2.

Power transfer in the systems described here will occur in a narrow frequency band (this requirement is further explained in Chapter 6). The distance from the source coils at which receivers will operate in the near field is shown in Figure 2-3 for a range of frequencies. Figure 2-3 shows that at an operating frequency of 120kHz, the near field will extend approximately 400 meters from the source coils. It is therefore theoretically possible to operate a system at this frequency with a distance of up to 400 meters between sources and receivers. However, the $1/r^3$ dependence of the field strength coupled with legal and safety standards limiting the strength of the field that can be produced generally restrict the maximum distance between source and receiver to several meters.

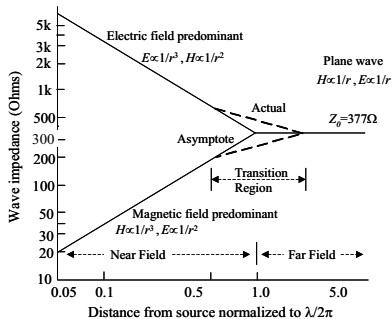


Figure 2-2 Wave impedance vs. distance from field source

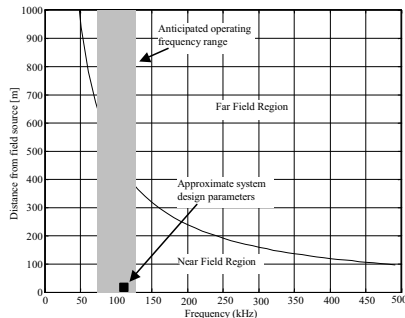


Figure 2-3 Boundary between near and far field areas for a range of magnetic field frequencies

2.2 Method of inductive power transfer

The basic layout of the source and receiving coils is described in this section. System parameters such as the size, dimensions, and configuration of both the source and receiving coils can be modified to fit the desired application. Source coil volumes are generally expected to be in the range of $0.001\text{m}^3 - 1000\text{m}^3$ and receivers have an anticipated volume in the range of $1\text{cm}^3 - 1000\text{cm}^3$.

2.2.1 Sources

The source coils consist of one or more turns of wire around an air-core. These coils can be of any shape and the dimensions of the coils are limited only by legal and safety limits on the magnitude of the magnetic field present in the region directly adjacent to the source coils (this limitation is further discussed below and in Appendix I). In general the coils will be rectangular, square, or circular, with side lengths or radii in the range of one to several meters. The simplest source coil configuration is shown in Figure 2-4. This system will provide a fluctuating magnetic field that can provide power to a receiving coil located close to the source coil and is referred to as a uni-directional system because it produces a magnetic field at the center of the coil, which has vector components in only one direction. This system can be improved by adding an additional source coil placed in space quadrature with that shown in Figure 2-4. The resulting configuration is shown in Figure 2-5 and is referred to as a bi-directional system because it produces a magnetic field at the center of the coil, which has vector components in two directions. This system can produce a magnetic field vector of constant magnitude rotating in a plane (this effect is further explained in Chapter 3) and can provide more power to a secondary coil located close to the source coils than the system shown in Figure 2-4. A logical extension is clearly the addition of a third source coil in the remaining plane (Figure 2-6).

In order to increase the area or volume in which the receiving coils can receive power, the systems described above can be extended such that there are two coils laying in each of the three orthogonal planes (Figure 2-7, Figure 2-8, and Figure 2-9). This configuration will provide a much greater volume in which the receiving coils can obtain power from the source coils. This volume is called the operating volume and its optimization is discussed in Chapter 3.

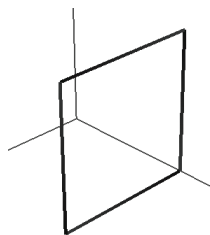


Figure 2-4 Uni-directional system

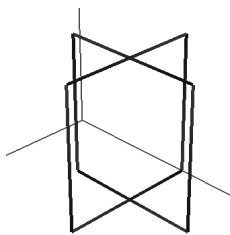


Figure 2-5 Bi-directional system

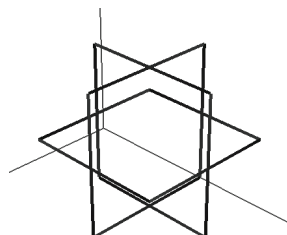


Figure 2-6 Omni-directional system

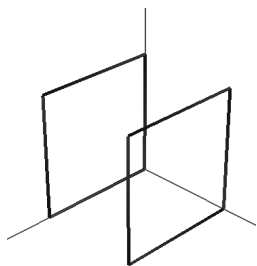


Figure 2-7 Uni-directional system using a coil pair to create a larger operating volume

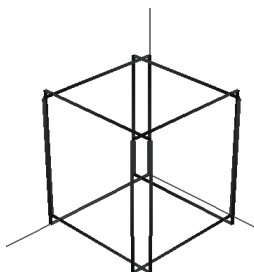


Figure 2-8 Bi-directional system using two coil pairs to create a larger operating volume

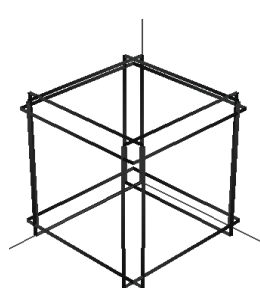


Figure 2-9 Omni-directional system using three coil pairs to create a larger operating volume

2.2.2 Receivers

The system is designed such that the receivers will obtain power from the source coils regardless of their orientation with respect to the source coils and to each other. The receivers consist of a ferrite cube with three mutually orthogonal receiving coils, each wrapped around one of the three axes of the cube. The layout can be seen in Figure 2-10. The circuitry attached to the receiving coils is described in Chapter 6.

Assuming that the unit vector normal to each of the axes of the three coils will rotate through all possible angles with the x -, y -, and z -axes of the system, the unit vectors of each of the coils

$$\begin{aligned}
\hat{a}_x &= [1 \ 0 \ 0]^T \\
\hat{a}_y &= [0 \ 1 \ 0]^T \\
\hat{a}_z &= [0 \ 0 \ 1]^T
\end{aligned} \tag{2-8}$$

are multiplied by the three-dimensional rotation matrix,

$$R_{x,y,z}(\psi, \theta, \phi) = \begin{bmatrix} \cos \theta \cos \phi & \cos \theta \sin \phi & -\sin \theta \\ \sin \psi \sin \theta \cos \phi - \cos \psi \sin \phi & \sin \psi \sin \theta \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \psi \\ \cos \psi \sin \theta \cos \phi + \sin \psi \sin \phi & \cos \psi \sin \theta \sin \phi - \sin \psi \cos \phi & \cos \theta \cos \psi \end{bmatrix} \tag{2-9}$$

where ϕ represents the angle of rotation relative to the z -axis, θ represents the angle of rotation relative to the y -axis, and ψ represents the angle of rotation relative to the x -axis.

The magnitude of the total flux in all three receiving coils is

$$\phi_{Total} = \sqrt{\phi_x^2 + \phi_y^2 + \phi_z^2} \tag{2-10}$$

Because the area enclosed by each of the receiving coils is very small in comparison to that of the source coils, it is possible to assume that the magnitude of the field across this surface is constant. The flux crossing each of the receiving coils can be described as

$$\phi_{x,y,z} = \int \vec{B} \cdot d\vec{s}_{x,y,z} \tag{2-11}$$

Giving a total flux through a receiver of

$$\phi_{total} = A \sqrt{\sum_{i=1}^3 (B_x R_{xi} + B_y R_{yi} + B_z R_{zi})^2} \tag{2-12}$$

where A is the area enclosed by one receiving coil and $R_{ki}(k=x,y,z)$ is the rotation matrix for receiving coils x , y , and z .

This result reduces algebraically to

$$\phi_{total} = A \sqrt{(B_x + B_y + B_z)^2} \tag{2-13}$$

which shows that the total flux crossing the coils comprising of a receiver is not dependent on the orientation of the receiver relative to that of the source field. This result is, of course, valid only for a receiver comprised of identical orthogonal coils. The effect of rotation on the voltage available to the load on a receiver depends on the

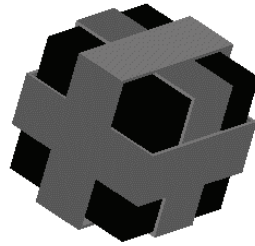


Figure 2-10 Receiver comprised of three receiving coils, each wound around one axis of a ferrite cube

circuit (rectifier, etc.) between the receiving coils and the load. These effects are discussed in Chapter 6. The size of the ferrite core and number of windings on the core determines the power received from the source field. These design parameters are further discussed in Chapters 5, 6, and 7.

2.2.3 Frequency range

The selection of the system's operating frequency (and therefore also the frequency of the magnetic field) is a trade-off between the losses in the power converter, the size of the receiving coil(s), the shielding properties in the application environment, and the consideration of established laws and standards.

The analysis is limited to a narrow-band system operation. Operating both source and receiver at a resonant frequency allows for the compensation of the large inductance inherent in the source coils and increases the voltage at the receiver to a level that permits the use of standard power electronics components. As both the source and receiving sides of the system operate in resonance, a single operating frequency is ideal. This is further discussed in Chapter 3.

The size of the source coils limits the possible operating frequency for such a system. The system must be designed such that the receivers are never located outside of the near field. As the anticipated dimensions of a single source coil do not exceed 10m x 10m, and the distance between source coils is generally smaller than their largest side length (see Chapter 3), receivers located within the operating volume will remain in the near field even beyond an operating frequency in the MHz range.

The systems discussed here generate and radiate electromagnetic waves and are therefore subject to laws and restrictions. Frequencies in the ISM (Industrial-Scientific-Medical) range are available for use in a wireless power system (see Appendix II – Frequency Ranges). However, 13Mhz is the lowest ISM band available worldwide, and the radiated fields produced by large systems at this frequency would produce undesirable EMC effects [1-2]. Frequencies below 135kHz are available and are already used in short-range inductive loop communication systems (SRD – short range devices). Therefore only frequencies below 135kHz will be considered for this system.

Because the large air gap between sources and receivers may be large enough to permit operations personnel to enter the operating volume, adherence to safety standards is required. These limits are available in Appendix I as given in several international standards. The Work Safety Organization (Berufsgenossenschaft) in Germany sets the occupational exposure limit at 41 A/m at 120kHz [2-5], while the IEEE sets the limit at 136 A/m at 120kHz [2-6]. People wearing pacemakers must abide by stricter exposure limits. In this case, the German Electrical Engineering Society (Verband Deutscher Elektrotechniker - VDE) allows a disturbance voltage of only 130mV at 120kHz [2-7]. This would require a person with a pacemaker to maintain a distance of 1.5m from a typical system using coils of 3m x 3m with a current of 24A.

2.3 Review and comparison of systems using inductive power transfer

There are several systems existing today that utilize a method of inductive power transfer. The most widely used systems with similarities to the wireless power system described in this thesis are Radio Frequency Identification (RFID) systems and Magnetic Search Coil (MSC) systems. These are described below with special attention to the similarities and differences between these systems and the wireless power system.

2.3.1 Inductively coupled RFID systems

Inductively coupled RFID systems use a source coil or coils to provide power to receiving coils (also called tags) for the purposes of transmitting information from the tag to the source (also called reader) coil. When a tag enters the magnetic field created by the reader, it modulates the carrier signal sent by the reader according to the data to be sent. The modulated carrier signal is then detected and decoded by the reader [2-9]. The tag coils are used to transmit power and the power available at the tags for this application is generally significantly lower than that which must be provided in a wireless power system. Inductively coupled RFID systems are generally used in applications where the tag coils will be within a definable region and oriented in a definable direction such that the tag coils will be able to receive power from the reader coils. The reader generally consists of one or more coils aligned in the same plane and rotating fields are not used. A wireless power system must, on the other hand, include receiving coils that can operate regardless of their orientation and position relative to that of the source coils and must, therefore, incorporate source coils in more than one axis and use the rotation of the field vector in order to assure that receiving coils will receive power in most cases. Inductively coupled RFID systems are generally used in settings in which the tags will not be placed inside conductive or permeable objects that may damp the source field to the point where the reader cannot read the tags. In cases where RFID systems are used in these environments, the resonant curve of the tags is flattened to allow the tags to be read even when a conductive object is close enough to a tag to raise its inductance and therefore lower its resonant frequency [2-10]. This has the effect of lowering the distance from the reader at which the tags can be read or requiring a higher magnitude of magnetic field created by the reader. These solutions are not optimum for a wireless power system because the resonant curve of the secondary must be narrow in order to optimize the system for power transfer. Likewise, the field created by the source coils is often as high as is legally allowed and cannot be raised significantly to compensate for the presence of conductive objects.

2.3.2 Magnetic search coil (MSC) systems

Magnetic Search Coil Systems can be used to perform quantitative studies of eye and head movements in people and in animals [2-11], [2-12], [2-13], [2-14]. These systems are based on phase-locked amplitude detection of the voltage induced in a search coil by an ac magnetic field. The angular orientation or displacement of the sensing coil in space is detected using a Helmholtz-like arrangement of source coils

arranged in space quadrature on two or three axes [2-15]. Demodulation of the induced signals with respect to the magnetic field directions is obtained on the basis of phase or frequency coding by driving the external magnetic field in phase quadrature or at different frequencies [2-14]. These systems use rotating magnetic fields in two or three dimensions to transfer power to receiving coils. These systems are however, designed to operate at several frequencies and are designed at a much lower power level. Shielding by conductive objects is not a concern for MSC systems as they are used in controlled laboratory settings. Finally, these systems are not optimized to deliver the maximum possible power to the receiving coils in an industrial setting.

2.3.3 Other Systems using inductive coupling

Many medical and industrial applications exist in which energy transfer over an air gap of up to several centimeters is necessary. The charging of batteries powering devices such as heart pumps which are implanted in the human body [2-16], [2-17], [2-18], [2-19], [2-20], and household appliances such as cordless telephones and electric razors [2-21], [2-22] can be accomplished with a contactless power supply. Industrial applications exist such as the contactless charging of large battery systems and power transmission for linear movable systems, robot components, and machine tools [2-23], [2-24]. While these systems use inductive coupling to transmit energy over an air-gap, the gap is limited to the centimeter range (or smaller) in all cases. The efficiency of transmission is, in most cases, highly dependent on the orientation of the primary (source) and secondary (receiver) components. Shielding of the source field is not generally a concern with these systems, as the distance between primary and secondary does not permit the presence of conductive or permeable objects inside the air-gap.

2.4 Alternative power supplies

Reference [1-2] compiled Table 2-1, which provides a rough comparison of alternative autonomous power supply options for industrial sensors. It is included here for reference. 1/3 of a typical application housing was assumed to be usable (a usable surface area of 6cm^2 and usable volume of 2.4cm^3 was assumed). A five-year lifetime was also assumed.

Power sources such as flow, acceleration, and vibration and those related to the sensed qualities are not included in the comparison as they are generally not present in significant quantity and are therefore not suitable for a wide range of applications. Nuclear generators (for example those used in pacemakers) are not considered due to probable industry objections.

Table 2-1 shows the achievable power levels and the energy delivered in a period of five years. A figure of merit is given (figure of merit = energy/cost/risk) for each type of supply. The risks are given in the last column. A high energy density with a low risk and low cost gives the highest figure of merit. The table shows that a power supply using magnetic fields in the range of 100kHz (in the near field) is best suited to the application [1-2].

Table 2-1 Example rough power supply comparison for typical autonomous industrial sensors
(1/3rd of sensor: 6cm² of usable surface, 2.5cm³ of usable volume) [1-2]

Energy Source	Power	Energy (per life of 5 years) Wh	Av. Power w/o service over life in mW	Cost (no service) w/o encaps. USD	Fig of merit Wh/USD/risk	Risks w/o power 1=very low, 5=very high	Risk description
PV1000 Lux a-Si	0.034mW/cm ²	9.0	0.21	0.3	15.0	2	shielding/orient.
PV100 Lux a-Si*	0.0029mW/cm ²	0.8	0.02	0.3	1.3	2	shielding/orient.
PV1000 Lux GaAs*	0.012mW/cm ²	26.9	0.62	3	3.0	3	Costs, shielding/orient.
Thermal (thin film)* dT=10k	0.0063mW/cm ²	1.0	0.02	1	0.3	3	Reliability, costs, feasibility
HF 100kHz (7A/m)	1mW/cm ³	105.7	2.41	1	52.8	2	High Q, E-smog shielding
HF 13MHz	0.23mW/cm ³	24.3	0.55	1.5	8.1	2	High Q, radiation shielding
HF 433 MHz*	0.3mW	13.1	0.30	2	3.3	2	High Q, regulations
Battery LiSoCl2	1.1 Wh/cm ³	2.7	0.06	2	1.3	1	
Battery AlMn	0.32 Wh/cm ³	0.8	0.02	0.2	3.9	1	
Fuel Cell, H2 20MPa	0.25 Wh/cm ³	0.6	0.01	5	0.03	4	Storage, water
Fuel Cell, Hybrid H2	1 Wh/cm ³	2.4	0.06	5	0.16	3	Water, pressure
Fuel Cell, Methanol	2.2 Wh/cm ³	5.3	0.12	2	0.66	4	Water, technology/time

*estimate

2.4.1 Electric vs. magnetic energy transfer

The physics describing electromagnetic phenomena shows that a duality exists between magnetic and electric fields. Energy transfer using a near-field electric field (Figure 2-11) is therefore theoretically possible. Although capacitive systems can be built with the same roll-off behavior as inductive systems, they unfortunately exhibit several problems that make their implementation complex. Moderately conductive objects, including people, severely reshape and attenuate the electric field thus shielding the receiver from the source field and reducing the amount of possible energy transfer [2-25]. The risk of creating a short circuit around large areas of the source field is also high in practical setups. Inductive systems benefit from the use of ferrite cores on the receivers that serve to concentrate the flux passing through the receiving coils, thereby increasing the power available to the load (an in-depth discussion of this effect can be found in Chapter 5). No equivalent benefit exists for

capacitive systems, as the addition of a dielectric material between the plates comprising the receiver will serve to decrease the voltage available to the load.

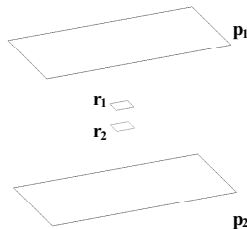


Figure 2-11 Energy transfer system using an electric field. The system consists of two source plates, p_1 and p_2 , and two receiving plates r_1 and r_2 .

The small capacitance between the source plates must be compensated by a large resonant inductor. Although this is theoretically possible, losses in this large inductor would have a negative impact on the efficiency of the system. Because of these problems, the method of inductive power transfer is preferred.

3 Characterization of the Source Field

The source field is described and an optimal design for the source coils is discussed. This chapter also explores various design options for the source coils with respect to uniformity of power transfer, optimum coil separation distance, generation of desired magnetic field vectors, and ability to overcome the problems associated with shielding of the magnetic field. Uni-directional and bi-directional systems are discussed, and omni-directional systems are introduced along with several methods for creating them.

3.1 Basic source coil shapes

The size, shape, and uniformity of the field created by the source coils depend significantly on the coil configuration. Square, rectangular, and circular coils are investigated.

3.1.1 Magnetic fields due to rectangular coils

The magnetic field intensity at an arbitrary point in space $P(x,y,z)$ due to a rectangular coil of side lengths $2l_x$ ($x \in 1-4$) can be expressed as the vector sum of the fields created by each of the straight wires. The field created when current passes through a rectangular coil lying in the yz -plane (Figure 3-1a) where wires 1 and 2 are parallel to the z -axis and wires 3 and 4 are parallel to the y -axis is described in the near field ($2\pi/\lambda \ll 1$, where λ is the wavelength of the current) by

$$\begin{aligned} \vec{H}_{rect} = & \hat{a}_x \left(-\sin\phi_1 \frac{Il_1}{2\pi r_1 \sqrt{l_1^2 + r_1^2}} + \sin\phi_2 \frac{Il_2}{2\pi r_2 \sqrt{l_2^2 + r_2^2}} - \cos\phi_3 \frac{Il_3}{2\pi r_3 \sqrt{l_3^2 + r_3^2}} + \cos\phi_4 \frac{Il_4}{2\pi r_4 \sqrt{l_4^2 + r_4^2}} \right) \\ & + \hat{a}_y \left(\cos\phi_1 \frac{Il_1}{2\pi r_1 \sqrt{l_1^2 + r_1^2}} - \cos\phi_2 \frac{Il_2}{2\pi r_2 \sqrt{l_2^2 + r_2^2}} \right) + \hat{a}_z \left(\sin\phi_3 \frac{Il_3}{2\pi r_3 \sqrt{l_3^2 + r_3^2}} - \sin\phi_4 \frac{Il_4}{2\pi r_4 \sqrt{l_4^2 + r_4^2}} \right) \end{aligned} \quad (3-1)$$

Where I is the current in the wire, r is the distance between the point $P(x,y,z)$ at which the field is being calculated and the current carrying wire, and the angles ϕ_x ($x \in 1-4$) are described as shown in Figure 3-2a. A full derivation of (3-1) can be found in Appendix III.

On the axis of a rectangular coil, the equation describing the magnetic field simplifies to

$$\vec{H}_{rect,axis} = \frac{nIl_x l_y}{\pi \sqrt{r_{cp}^2 + l_x^2 + l_y^2}} \left[\frac{1}{r_{cp}^2 + l_x^2} + \frac{1}{r_{cp}^2 + l_y^2} \right] \quad (3-2)$$

Where $2l_x$ and $2l_y$ are the side lengths of the rectangular coil, r_{cp} is the distance from the center of the coil to the point in question, and n is the number of coil windings.

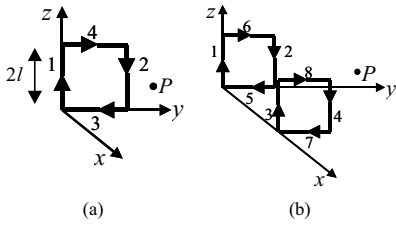


Figure 3-1 (a) Rectangular coil with field intensity described by (3-1) (b) Rectangular coil system with field intensity described by (3-3)

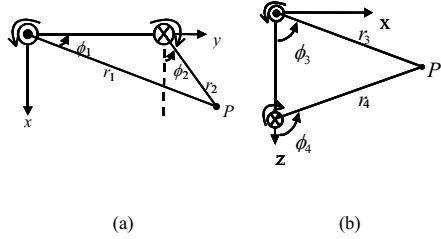


Figure 3-2 (a) Angles used to calculate field strength due to a single coil (xy-axis). (b) Angles used to calculate field strength due to a single coil (xz-axis)

The magnitude of the magnetic field intensity at any point in the system is clearly linearly dependent on the current applied to the coil(s). Assuming that this current is periodic, the root-mean-square magnetic field intensity (H_{rms}^I) can be used to visualize the field created by these currents.

The intensity of the field at an arbitrary point created by a set of rectangular coils can be expressed as the vector sum of the fields created by each of the coils. Numbering the sides of the rectangular coils as in Figure 3-1(b),

$$\vec{H}_{set} = \sum_{i=1}^{2n} [(-1)^i \sin\phi_i X_i \hat{a}_x + (-1)^{i+1} \cos\phi_i X_i \hat{a}_y] + \sum_{i=2n+1}^{4n} [(-1)^{i+1} \sin\phi_i X_i \hat{a}_z + (-1)^i \cos\phi_i X_i \hat{a}_x] \quad (3-3)$$

$$\text{where } X_i = \frac{Il_i}{2\pi r_i \sqrt{L_i^2 + r_i^2}}$$

Equation (3-3) can be extended to find an expression for the magnetic field due to any number and/or arrangement of rectangular coils.

3.1.2 Magnetic field due to circular coils

For a singular circular coil centered on the z-axis (Figure 3-3) the magnetic field intensity created when a current I is applied to the coil can be shown to be:

$$\vec{H} = \hat{a}_\rho \left(\frac{Iz}{2\pi\rho\sqrt{(a+\rho)^2 + z^2}} \left[\frac{a^2 + \rho^2 + z^2}{(a-\rho)^2 + z^2} E(k) - K(k) \right] \right) + \hat{a}_z \left(\frac{\mu_0 I}{2\pi\sqrt{(a+\rho)^2 + z^2}} \left[\frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2} E(k) + K(k) \right] \right) \quad (3-4)$$

¹ The rms value of field strength refers to the root mean square value of the magnetic field and is used only in cases when the magnetic field can be expressed as a periodic function over time.

where z is the distance of the point in question from the z -axis, ρ is the radial distance from the origin of the coil, a is the radius of the coil, and $K(k)$ and $E(k)$ are complete elliptic integrals of the first kind and second kind, respectively.

$$K(k) = \int_0^{\pi/2} \frac{d\Phi}{\sqrt{1 - k^2 \sin^2 \Phi}}, \quad E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \Phi} d\Phi$$

$$\text{and } k = 2 \sqrt{\frac{a\rho}{(a+\rho)^2 + z^2}} \quad (k \in [0,1])$$

On the axis of a circular loop, the equation simplifies to

$$\vec{H}_{\text{circ,axis}} = \hat{a}_z \frac{Ia^2}{2(z^2 + a^2)^{3/2}} \quad (3-5)$$

The magnetic field created by two circular current carrying coils is the vector sum of the field components created by each single coil. The magnitude of the field created by two coils (laying on the z -axis) at an arbitrary point P is

$$\vec{H}_{\text{circ,set}} = \sum_{i=1}^n (\hat{a}_\rho H_{\rho i} + \hat{a}_z H_{zi}) \quad (3-6)$$

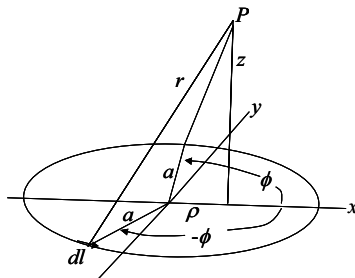


Figure 3-3 Circular coil in xy -plane

3.1.3 Magnetic field at source coil system center points

The field strength at the center of the source coil system is of primary importance in system analysis. Table 3-1 shows equations for the magnetic field strength at the center of the source coil configurations shown in Figures 2-4 – 2-10. The first two rows of equations are derived directly from (3-2), and the second two rows are derived from (3-6). Currents in all coils are assumed to be equal, and coil pairs in bi- and omni-directional systems are equally spaced on all axes. D is the distance of separation for each coil pair.

The equations in Table 3-1 are independent of the phase of the coil currents. The equations are accurate if the *rms* amplitude of the current in all source coils in question are identical. In the case of the omni-directional systems presented in Section 3.6, the field strength at the center of the system must be calculated using (3-2) or (3-6) with the appropriate currents for each source coil pair.

Table3-1 Magnitude of magnetic field at the center of uni-, bi-, and omni-directional systems

	Uni-directional System $H(\text{A/m})$	Bi-directional System $H(\text{A/m})$	Omni-directional System $H(\text{A/m})$
Square - Single Coils*	$\frac{2I\sqrt{2}}{2\pi l_x}$	$\frac{4I\sqrt{2}}{2\pi l_x}$	$\frac{6I\sqrt{2}}{2\pi l_x}$
Square - Coil Set(s)**	$\frac{32Il_x}{\pi(D^2 + 4l_x^2)\sqrt{D^2 + 8l_x^2}}$	$\frac{32Il_x\sqrt{2}}{\pi(D^2 + 4l_x^2)\sqrt{D^2 + 8l_x^2}}$	$\frac{32Il_x\sqrt{3}}{\pi(D^2 + 4l_x^2)\sqrt{D^2 + 8l_x^2}}$
Circular- Single Coils*	$\frac{Ia^2}{2a^3}$	$\frac{Ia^2}{a^3}$	$\frac{3Ia^2}{2a^3}$
Circular- Coil Set(s)**	$\frac{Ia^2}{(D^2 + 4a^2)^{3/2}}$	$8\frac{Ia^2\sqrt{2}}{(D^2 + 4a^2)^{3/2}}$	$8\frac{Ia^2\sqrt{3}}{(D^2 + 4a^2)^{3/2}}$

*In the configuration shown in Figure 2-4 – Figure 2-6.

**In the configuration shown in Figure 2-7 – Figure 2-10.

3.2 Effects of shape and distance between source coils

Fields can be created using source coils of any shape. Circular and square coils are investigated here for the purpose of clarifying the effects of shape and distance between source coils on the field. In practical applications, coil systems may be comprised of any combination of coil shapes, although care must be taken to avoid undesired inductive coupling between source coil pairs.

3.2.1 Coils at Helmholtz distance

Obtaining a configuration in which the greatest possible volume of stable (unchanging) magnetic field exists between two coils is of interest for application where the load on the receiving coils is sensitive to small changes in field strength.

For two equal coils of any shape and size, the optimum distance d_H (also known as Helmholtz spacing) by which the two coils should be separated to achieve the greatest possible area of stable magnetic field can be determined by finding the distance at which the gradient of the magnetic field in the center of the coils is equal to zero [3-1], [3-2], [3-3], [3-4].

Expanding (3-5) to find the magnetic field at the center of two circular coils (Figure 3-4) lying in a plane perpendicular to the z-axis with equal radius $a_1 = a_2 = a$ and at a distance $D = d_H$ apart gives

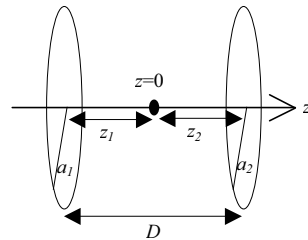


Figure 3-4 Two circular coils with equal radius

$$H = \frac{I}{2} \left[\frac{a_1^2}{\left[a_1^2 + \left(\frac{D}{2} - z_1 \right)^2 \right]^{3/2}} + \frac{a_2^2}{\left[a_2^2 + \left(\frac{D}{2} - z_2 \right)^2 \right]^{3/2}} \right] \quad (3-7)$$

where z is the distance from each coil to the point in question and $z_1 = z_2$ for a point in the center of the coils. The z -axis passes through the center of the coils and the origin of the z -axis is at the point midway between the coils. The gradient of (3-7) is

$$\begin{aligned} \frac{\partial^2 H}{\partial z^2} = \frac{Ia^2}{2} \left(\frac{-3}{2} \right) & \left\{ \left[a^2 + \left(\frac{D}{2} - z \right)^2 \right]^{-5/2} + \left[a^2 + \left(\frac{D}{2} - z \right)^2 \right]^{-7/2} \left(-5 \left(\frac{D}{2} + z \right)^2 \right. \right. \\ & \left. \left. + \left[a^2 + \left(\frac{D}{2} + z \right)^2 \right]^{-5/2} + \left[a^2 + \left(\frac{D}{2} + z \right)^2 \right]^{-7/2} \left(-5 \left(\frac{D}{2} + z \right)^2 \right) \right\} \end{aligned} \quad (3-8)$$

$\left. \frac{\partial^2 H}{\partial z^2} \right|_{z=0} = 0$ when $D = a$. For circular coils d_H is always equal to the radius. For

square coils d_H is always equal to 0.5445 times the length of one side [3-1], [3-2], [3-3]. For rectangular coils d_H varies with the dimensions of the rectangle as shown in Figure 3-5.

The total volume contained within the box created by two square coils at Helmholtz spacing is 1.3866 times larger than the total volume contained within the cylinder created by two circular coils also at Helmholtz spacing. However, this result does not provide adequate information about the volume of stable magnetic field created by these configurations.

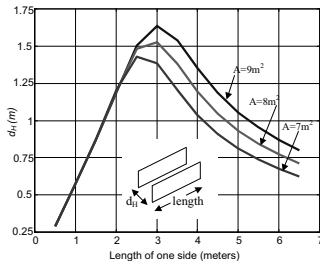


Figure 3-5 Helmholtz distance vs. side length for rectangles of area $A=7, 8$, and 9m^2

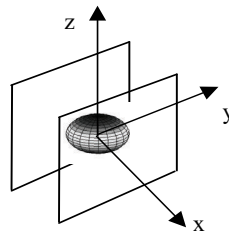


Figure 3-6 Sphere centered at the origin of a system of square coils

The volume in which a stable magnetic field exists when coils are placed at the Helmholtz distance has been investigated in order to find the coil shape providing the largest volume of stable magnetic field.

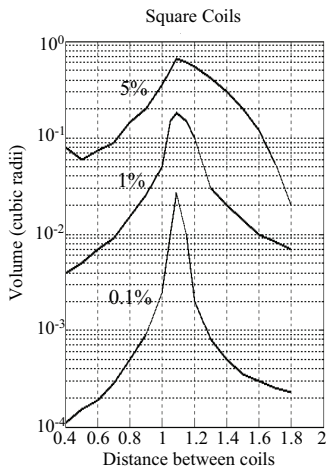


Figure 3-7 Volume of specified field strength within stated percentage of that at the center of the coil pair as a function of spacing for a pair of square coils of side length $l=2$ ($d_H=1.09$).

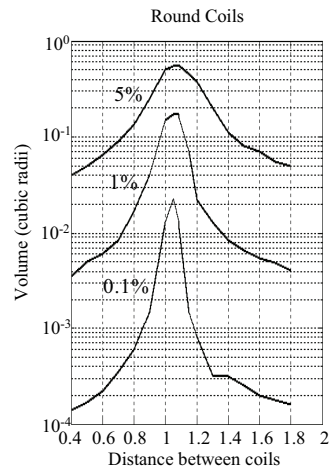


Figure 3-8 Volume of specified field strength within stated percentage of that at the center of the coil pair as a function of spacing for a pair of circular coils of radius $a=1$ ($d_H=1$).

The volume of a sphere centered on the origin of the system (Figure 3-6) in which the magnitude of the magnetic field vector varies by up to a certain percentage from the value at the center of the system is plotted for circular and square coil configurations. The radii of the coils are set to a value of one (the "radii" of the square coils are assumed to be one-half the length of one coil) [3-4]. Figure 3-7 and Figure 3-8 show that while the volume is maximized at or near Helmholtz spacing in all cases, spacing becomes less critical as the percentage of allowable field strength deviation is increased. These results are confirmed by [3-5]. Figure 3-9 shows that for coils with the same radius at the same distance of separation, square coils have a larger volume of stable magnetic field for all investigated percentages of deviation from the center field values. Figure 3-10 shows the trade off between a larger total system volume and a larger volume of stable magnetic field. Although square coils provide the same volume of stable magnetic field with a smaller distance between them than do circular coils, circular coils provide a larger volume of stable magnetic field as a percentage of total volume taken up by the system. However, square coils can in practice be more easily assembled into a modular system. This is a clear advantage when developing scalable systems for industrial use.

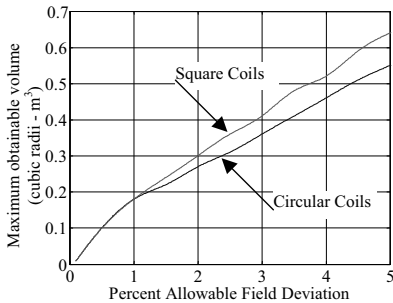


Figure 3-9 Maximum obtainable sphere volume vs. percent allowable deviation for circular and square coils

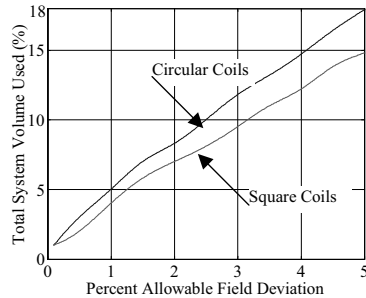


Figure 3-10 Percent of total system volume utilized vs. percent allowable deviation for circular and square coils

Although coils placed at the Helmholtz distance are useful only for specialized applications which require a highly stable region of uniform field strength, it becomes clear that the Helmholtz distance is an excellent initial approximation when investigating optimum distances between coils for other applications.

3.2.2 Coils at other distances

Although some applications may require a highly stable magnetic field magnitude within the system volume, most applications require only that the magnitude of the field strength be at or above a certain minimum value required to provide adequate power to the receiving coils. For the remainder of this work, system volume will be defined as the volume encompassed by the source coil system. Operating volume is defined as the volume in which the field strength created when current flows in the source coil system is at or above the value necessary to provide adequate power to receiving coils located within that volume (H_{min}).

The most important criterion in choosing a coil configuration will involve finding the distance of separation at which the coils will produce the largest possible volume of specified minimum field strength within the system volume while assuring that the shape of this volume is such that receiving coils can operate throughout their range of movement within the system volume. This must also be accomplished while maintaining field values that do not exceed minimum safety standards.

No single ideal distance of separation can be found for all applications. Because each application may require an operating volume of unique shape and dimensions, several methods of finding the optimum distance of separation must be available. These methods are presented in the following discussion and may, in many cases, require further modification depending on system requirements. Several possible basic field distribution requirements are listed:

- i. Systems in which receiving coils are expected to operate in all possible physical positions within the system volume.
- ii. Systems in which the receiving coils are expected to operate within a predefined area or volume within the system volume (i.e. receiving coils attached to the arm of a robot performing assembly line work).
- iii. Systems in which the receiving coils are expected to operate only in regions directly on or very near to the axis of the system; this axis is often much longer than the dimensions of the source coils (i.e. receiving coils moving on a conveyor belt).

3.2.2.1 Uni-directional systems

Although systems consisting of only one source coil set are in most cases the least complicated, obtaining a clear result as to which distance of separation is optimal is complicated. This is due to the lack of symmetry between two of the three axes in the system. When choosing the distance of separation for such a system, several factors must be considered. If the system must operate such that receiving coils can be located at any position within the system volume (type *i* systems), then clearly the operating volume must encompass the entire system volume. In this case, the optimal distance of separation of the two source coils will be determined by the field strength at the points within the system volume at which the field strength is at its lowest value (P_{Hmin}). These points are shown in Figure 3-11.

However, this is an inefficient method of system operation as the magnitude of the magnetic field at all points other than P_{Hmin} is higher than H_{min} , and it is substantially higher than H_{min} in the center of the system volume. This “extra” field strength generated throughout most of the volume is wasted. A uni-directional source coil set is not suited to a type *i* systems.

An alternative method of designing a uni-directional system is to find the maximum volume at the center of the system in which the field has a magnitude of H_{min} or greater.

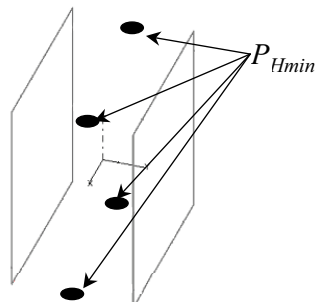


Figure 3-11 Uni-directional system. Points P_{Hmin} are the locations within the system volume with the lowest field strength magnitude.

In this case the operating volume will be confined to an area smaller than, and within, the system volume. The magnitude of the magnetic field at the edges of the system volume will not be at or above H_{min} . The system is then designed such that the receiving coils are located within the operating volume (type *ii* systems). This volume can be optimized using several methods.

3.2.2.1.1 Estimate operating volume as a cube

The simplest method for approximating and optimizing the operating volume at the center of the system volume is shown for square coils. In this case, the operating volume is assumed to be cube shaped with its center at the center of the system volume.

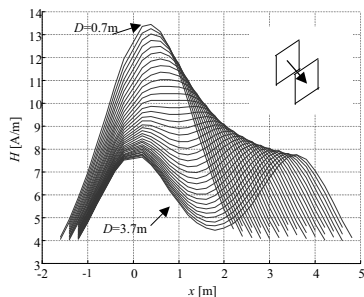


Figure 3-12 Field strength along the axis of a 3m x 3m square coil pair

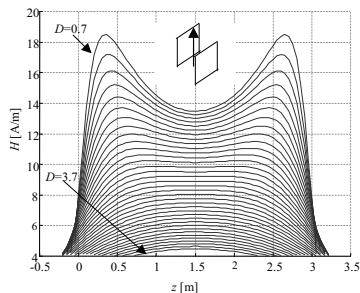


Figure 3-13 Field strength perpendicular to the axis of a 3m x 3m square coil pair

An example system using two coils 3m x 3m and each carrying 24A and centered on the x-axis is used for this analysis. The field strength parallel to the x-axis and through the center of the coils for distances of separation from $D=0.7\text{m} - 3.7\text{m}$ at steps of 0.085m, is shown in Figure 3-12 for field strengths greater than 4A/m. All of the distances of separation considered lead to a field strength of 4A/m or more along the entire path. Figure 3-13 shows the field strength parallel to the z-axis at a distance of $D/2$ for distances of separation from 0.7 to 3.7m. The field strength along this path does not remain at 4A/m or higher along the entire path for all distances of separation. Because of the symmetry of the field created by square coils, the field strength in the y-direction will be the same as that in the z-direction.

The length of each side of the operating volume “cube” is defined as the length along which the field strength is at or above 4A/m. Figure 3-14 shows a plot of the approximate volume in which the field strength is 4A/m or higher inside the cube volume for varying values of D . For 3m x 3m coils carrying 24A, the distance of separation at which the maximum volume within the cube created by the coil pair has a field strength of greater than 4A/m is approximately 2.8m.

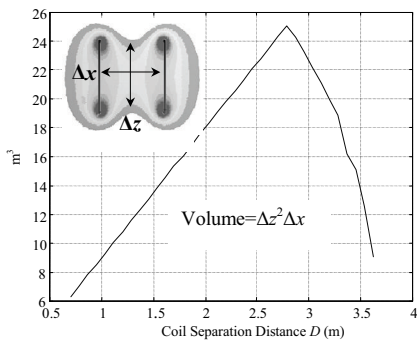


Figure 3-14 Approximate volume of field strength of 4A/m or greater for a single coil pair carrying 24A

The optimum distance of separation for a coil system using this method can be found most easily by first calculating Helmholtz spacing and then varying the distance slightly to achieve the maximum volume of desired field strength.

3.2.2.1.2 Estimate operating volume as a sphere

It may be beneficial to optimize the operating volume by creating an imaginary sphere of infinitely small radius at the center of the system. The radius of the sphere is then expanded until it begins to encompass field strengths of an unacceptably low magnitude. The distance between coils is varied and the system creating the largest of these imaginary spheres is considered to be at the optimum distance.

This method requires a large number of calculations and is not practical for configurations that must be rearranged on a regular basis. It is, however, a highly accurate method for finding the optimum distance between coils for highly sensitive systems in which field strength at the center of the system is of primary importance. This method was used to create Figure 3-7 and Figure 3-8.

3.2.2.1.3 Estimate the operating volume using numerical integration

The operating volume can be maximized by finding the distance between source coils at which the volume of space containing field strength of H_{min} or greater is maximized. However, although the distance between two coils in which the total volume of field strength with magnitude at or above H_{min} is at its maximum value, this solution does not produce a feasible system. When coils are placed such that volume alone is maximized, there is no assurance that the operating volume will exist in the center of the system volume. A volume-optimized system will create a field in which a sphere placed at the center of the system, with a radius such that it encompasses field strength with magnitude at or above H_{min} , would be far smaller than desired. The volume in which a field strength of desired magnitude exists is large, but it exists primarily in the areas close to the source coils, leaving the center of the system with a relatively small volume in which receivers can operate. This is highly undesirable as most systems are expected to have a desired operating volume in or near to the center of the system.

Examining two cross-sections of the field strength created by two coils (Figure 3-15 – Figure 3-18) shows the tradeoffs that must be made to produce a system with a large operating volume of adequate shape. A comparison of Figure 3-15 and Figure 3-16 shows that the maximum operating area does not occur at the same distance of separation for both field cross-sections. A trade-off must be made between these two factors, and this trade-off must be made based on the system being designed. In the case of a type *iii* system, for example, the receiving coils will be close to the axis of the system, and operating at a distance at or near to the maximum shown in Figure 3-15 will be possible. However, type *i* and many type *ii* systems will require distances which produce operating areas that are well off of the maximum points for both axes.

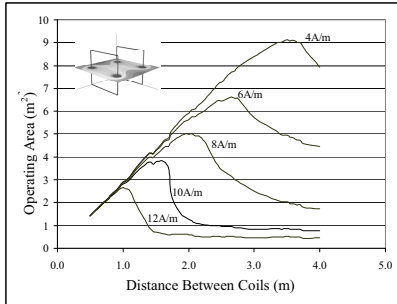


Figure 3-15 Operating area vs. coil separation distance for two coils 3m x 3m carrying 24A at a cross section ($z=1.5\text{m}$)

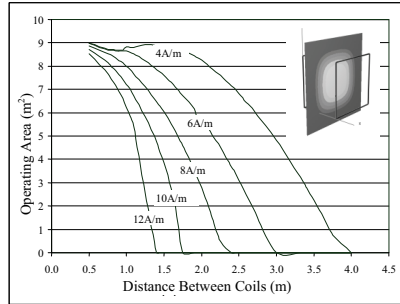


Figure 3-16 Operating area vs. coil separation distance for two coils 3m x 3m carrying 24A at a cross section ($x=D/2\text{m}$)

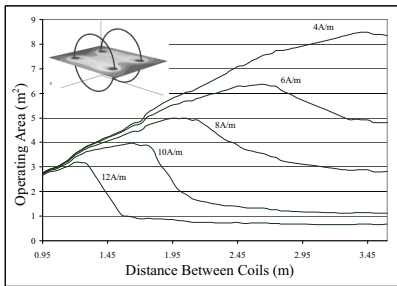


Figure 3-17 Operating area vs. coil separation distance for two coils 3m in diameter carrying 24A at a cross section ($z=1.5\text{m}$)

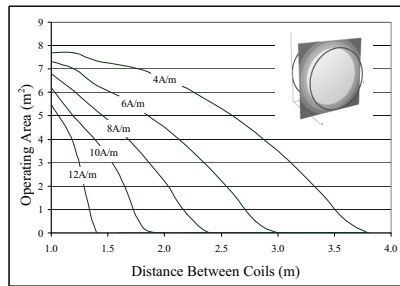


Figure 3-18 Operating area vs. coil separation distance for two coils 3m in diameter carrying 24A at a cross section ($x=D/2\text{m}$)

3.2.2.2 Multi-directional systems

The methods for determining the optimum distance of separation of each set of source coils in a multi-directional system are identical to those used for uni-directional systems. There are, however, several advantages to multi-directional systems that make the determination of optimum separation distance less critical than in the uni-directional case.

For a bi-directional system with coil sets aligned along the x - and y -axes, the operating volume can be made substantially larger; with the limiting point depending upon the distance along the z -axis in which the receivers can be located (Figure 3-20). This is an advantage for type *ii* and/or type *iii* systems. The addition of this second set of source coils in space quadrature with the first naturally creates a more uniform and symmetric field distribution around the edges of the system volume in the xy -plane (Figure 3-19).

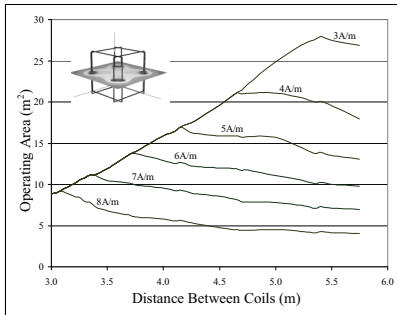


Figure 3-19 Operating area vs. coil separation distance for two sets of coils 3mx3m carrying 24A at a cross section ($z=1.5\text{m}$)

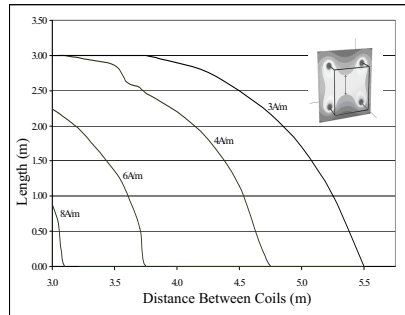


Figure 3-20 Length along the z -axis at which the field is at the specified field strength vs. coil separation distance for two sets of coils 3mx3m carrying 24A at a cross section ($x=D/2\text{m}$)

For bi-directional type i systems, the distance along the z -axis in which the receivers can be located is equal to the height of the source coils (receivers can be located at any point within the system volume). The distance between source coils required for type i systems depends on the magnitude of the magnetic field required to power the load(s) on the receiving coils, on the current in the source coils, and on the dimensions and shape of the source coils.

In order to find the optimum distance of coil separation for a bi-directional type i system, equations (3-3) and/or (3-6) must be solved such that $H(P_{Hmin})=H_{min}$ (P_{Hmin} is shown in Figure 3-21). For two sets of square coils 3m on each side, each carrying 24A and with an H_{min} of 5A/m, the distance of separation between source coils required of a type i system is approximately 2.25m resulting in an operating volume of 27m^3 . Spacing of the source coils becomes even less critical in the omni-directional case. P_{Hmin} in an omni-directional system is located at the center of each of the coils comprising the system. If each of the coil sets is separated by a distance $D=2l_x$, the field strength at P_{Hmin} is approximately 88% of the field strength at the center of the system.

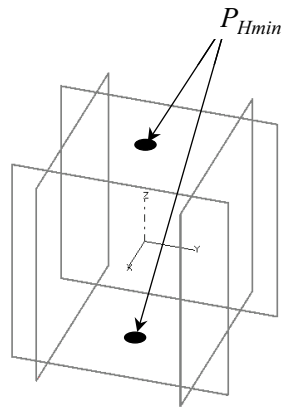


Figure 3-21 Bi-directional system showing the points at which the magnitude of the field strength created when current passes through the source coils is at its minimum value

3.3 Field generation

The magnitude and direction of the magnetic field vectors created when current passes through a source coil can be calculated at any point in the near field by (3-1) or (3-4). The magnetic field vector generated by a single source coil will have a single component along the axis of the source coil. This vector component will have a direction normal to the surface bounded by the source coil. The following discussion will focus on the field vector at the center of the source coil system(s).

Uni-directional systems create a pulsating magnetic field vector and do not provide constant power to the secondary. Bi-directional systems with two coil sets positioned in space quadrature create a rotating field vector of constant length and provide constant power to the secondary when the current applied to the first set of coils is phase shifted by 90° from the current applied to the second set. Omni-directional systems can be created using three coil sets.

Theoretical methods for producing desired magnetic field vectors are presented in the following sections. Methods for creating the currents required to implement these fields are presented in Chapter 6.

3.4 Uni-directional field generation

A uni-directional field can be created using one pair of coils placed on the same axis and energized with the same current. Assuming that a sinusoidal current is applied to coils laying on the x -axis, the field vector produced at the center of the system will be given by

$$\vec{H}_x = \hat{a}_x k I_x \sin \omega t \quad (3-9)$$

$$\text{where } k = 4s \left(\frac{I_x l}{2\pi r \sqrt{l^2 + r^2}} \right)$$

Although easy to implement, a uni-directional system has the following disadvantages:

- i. The field at the center of the source coils system has only one vector component and is therefore not resistant to shielding of the field in the direction of that vector component.
- ii. The magnitude of the unshielded field vector is not constant over time and therefore will not provide constant power to the load.
- iii. Only two coils create the field vector at the center of the system. This limits the magnitude of the vector present at the center of the system due to limits on the field strength allowed in the region directly surrounding each coil. (These limits are discussed in Appendix I).

Implementing a multi-directional system can mitigate the effects of these disadvantages on system performance.

3.5 Bi-directional field generation

Bi-directional systems with coil sets in space quadrature create a rotating field vector of constant length and provide constant power to the secondary when the current applied to the first set of coils is in phase quadrature with the current applied to the second set and the currents have the same magnitude and frequency. Simplifying equation (3-2) shows that the magnitude of the resulting vector in the center of the system has a constant magnitude.

$$\vec{H}_x = \hat{a}_x k I_x \sin \omega t, \quad \vec{H}_y = \hat{a}_y k I_y \cos \omega t \quad (3-10)$$

$$\text{with } k = 4n \left(\frac{I_i l}{2\pi r \sqrt{l^2 + r^2}} \right) \quad i \in ([x, y])$$

$$|H| = \sqrt{H_x^2 + H_y^2} = \sqrt{(k + I_x)^2 + (k + I_y)^2} \quad (3-11)$$

This system has several advantages when compared to a system using uni-directional field generation. The rotating field is beneficial in that it mitigates the problem of shielding by creating a magnetic field with vector components in more than one direction. This effect is further described in Chapter 4. Using coils in space and phase quadrature produces a field vector of constant magnitude and therefore provides constant power to receiving coils in the unshielded case. Finally, the *rms* magnitude of the magnetic field at the center of the system is higher by $\sqrt{2}$ than in the uni-directional case (for identical source coil currents). This allows higher field values in the center of the system without exceeding legal limits on the field created near each of the source coils.

Figure 3-22 shows the rotation of the field vector generated by the bi-directional system described above. The source coils are aligned with the *x*- and *y*-axes, producing a vector that rotates in the *xy*-plane. Figure 3-23 shows the circle traced by the endpoints of the magnetic field vector as it rotates.

A bi-directional system positioned on the *x*- and *y*-planes will not provide adequate power to the receiver(s) if they are placed in an area that is shielded from the source field in both the *x*- and *y*-directions. An omni-directional system is proposed in order to provide power to receiving coils in systems in which shielding can occur in multiple directions.

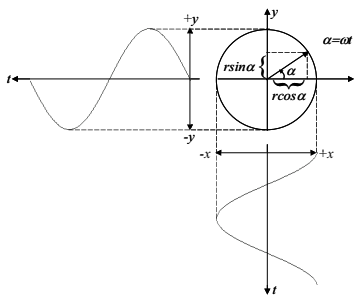


Figure 3-22 Vector rotation for one period of current in the source coils

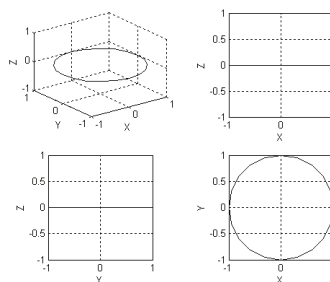


Figure 3-23 Outline of vector rotation for a bi-directional system. Magnitudes of source currents are set to 1. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane

3.6 Omni-directional field generation

Producing field vectors that rotate outside of the plane to which they are restricted in a uni- or bi-directional system is clearly desirable for the purpose of mitigating the effects of shielding within the operating volume. Ideally, the field vector should rotate around all three axes such that it is omni-directional in nature. This will provide the maximum possible protection against the effects of shielding. In order to accomplish this, field vector components must be created in the x -, y -, and z -directions, which requires the addition of source coil(s) centered on the z -axis. The field vector created by coils centered on the z -axis should provide power to the receiving coils even when the field vector components in the x - and y - directions are shielded at the position of the receiving coil. Ideally, the power generated by the x -, y -, and z -centered source coils should be such that equivalent shielding in the x -, y - or z - directions results in the same magnitude of power available to the receiving coils. The frequency of the source field must be maintained at or near the resonant frequency of the receivers in order to assure adequate power transfer between source and receiving coils.

Several methods that can be used to achieve field generation in more than one plane are discussed in the following sections.

3.6.1 Periodic switching of the plane of rotation

Sequential application of a field rotating in a single plane to the xy -, yz -, and xz -planes is a logical extension of the single-plane solution discussed in the previous section. This can be achieved by applying currents in phase quadrature to the x - and y - coils (as in the previous section) and then to the y - and z - coils, and finally to the z - and x - coils. The rate at which switching between the coil pairs occurs must be high enough to maintain adequate power at a receiving coil when it is shielded in two directions (i.e.

in the x -, and y - directions). This rate is then directly related to the size of the energy storage component on the receiving coil and to the size of the load.

In order to examine the constraints on the energy storage devices in the system, several assumptions are made:

- i. A receiving coil operating in an area shielded in both the x - and y -directions will receive 100% of its power from the z -axis source coil.
- ii. Each coil set is energized for the same duration, t_{seq} , so that one period of rotation through all possible coil sets (one rotation cycle) will be $3t_{seq}$ in duration.
- iii. A capacitor is used as the energy storage element. The receiving coil is considered to be capable of operation if the capacitor is charged such that its voltage is at or above a certain minimum value $V_{c_{min}}$ determined by the load requirements.

A receiver located in an area shielded from two of the three available field vectors will receive power for 2/3 of each rotation cycle (rotation sequence is discussed further in Chapter 6). The period of one rotation cycle ($3t_{seq}$) must be determined such that a receiving coil shielded in two directions will be able to maintain sufficient charge in its energy storage element during the 1/3 of the rotation cycle when it receives no power. Figure 3-24 shows a simplified model of the system. V_s represents the voltage induced on the receiving coil due to the field created by the source coils. R_s represents the resistance of the receiving coil, C is the energy storage element of the receiver, and R_l represents the load. The switch is closed when the receiver obtains power in the unshielded direction (duration $2t_{seq}$) and is open when the receiver does not receive power from the source coils (duration t_{seq}). Figure 3-25 shows the charge/discharge characteristic for the energy storage capacitor, C .

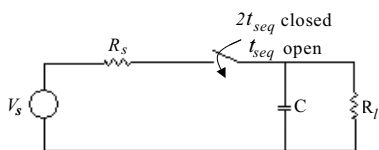


Figure 3-24 Circuit representing the effect of shielding in two-directions on receiver output voltage. The switch is closed when the receiver obtains power in the unshielded direction and is open when the receiver does not receive power from the source coils

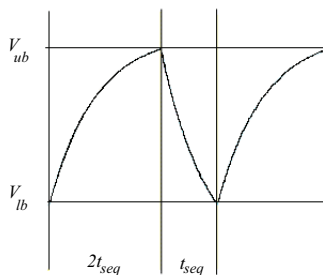


Figure 3-25 Steady-state charging characteristic for the capacitor on a receiving coil that is shielded in two directions

Clearly, V_{lb} must be set at or above the minimum voltage at which the load on the receiver can operate. Equation (3-12) shows the relationship between V_{lb} , C , and t_{seq} .

$$V_{lb} = \frac{R_l V_s (1 - e^{-t_{seq}/\tau_1})}{(e^{t_{seq}/\tau_2} - e^{-t_{seq}/\tau_1})(R_s + R_l)} \quad (3-12)$$

where $\tau_1 = \frac{R_s R_l C}{2(R_s + R_l)}$ and $\tau_2 = R_l C$

Therefore, assuming that V_s , R_s , and R_l are constant for any system, C and t_{seq} can be adjusted to assure that V_{lb} is at or above the minimum voltage required by the load. A full derivation of (3-12) is available in Appendix IV. The maximum possible length of one rotation cycle (and therefore the minimum possible cycling frequency, $f_{cycle,min}$) for any system is therefore set by the energy storage element on the receiver according to (3-12).

This system has the advantage of being relatively uncomplicated and easy to implement. Field vectors are identical to those created in a bi-directional system except that they rotate in three planes rather than in a single plane. Constant power is maintained at the receiving coils in the unshielded case, and no additional frequencies must be introduced into the system. However, this method is not truly omnidirectional because field vectors are created only in three planes. Additional methods must be used to obtain a system in which field vectors in the center of the system rotate outside of these three planes.

3.6.2 Frequency shift

Applying current simultaneously to all three of the source coils can create a field vector that rotates through all possible angles with respect to each of the three axes. However, if the frequency of the current in all three coils is the same ($f_x=f_y=f_z$), the field will rotate in only one plane (the angle of the plane of vector rotation relative to the xy -plane being determined by the relative phase difference between the applied currents). Applying a current of an alternate frequency (f_2) to one or more of the source coils will provide a field vector that rotates in more than one plane (Figure 3-26).

The shape shown in Figure 3-26 is created when a current of frequency f_2 is applied to the z -axis coil set, and is known as a Lissajous knot [3-6], [3-7]. These knots are described using the following parametric equations:

$$\begin{aligned} x(t) &= K_1 \sin(\omega_1 t + \phi_1) \\ y(t) &= K_2 \sin(\omega_2 t + \phi_2) \\ z(t) &= K_3 \sin(\omega_3 t) \end{aligned} \quad (3-13)$$

In the case described here, $\omega_1=\omega_2$, $\phi_1=0$, $\phi_2=\pi/2$ and $K_1=K_2=K_3$ is a constant dependent on the current in the source coils and on their shape and configuration. This results in a magnetic field at the center of the system with a vector rotating in space such that its endpoints trace the lateral surface of a cylinder, forming the shape shown in Figure 3-26. The number and density of the lines shown in Figure 3-26 on the yz -axis is dependent on the ratio of the two frequencies in the system. If the ratio of frequencies is an irrational number, the lines forming the knot will not close on themselves and the cylinder will appear solid as the mesh shown in Figure 3-26 becomes infinitely dense over time. The rotation of the magnetic field vector around the cylinder is not periodic in this case. If the ratio of frequencies is a simple fraction (ω_1 and ω_3 are integers) the lines forming the knot will close on themselves after some time and will form a mesh similar to that shown in Figure 3-26. The rotation of the magnetic field vector is periodic in this case as the lines created by tracing the endpoints of the vector are redrawn in the same position once per rotation period. Finally, if the ratio of frequencies is not a simple fraction (ω_1 and/or ω_3 are not integers) but remains rational, the lines forming the knot will close on themselves after a longer period of time, providing a mesh which is much more dense than that shown in Figure 3-26. In this case, the period of rotation is much longer.

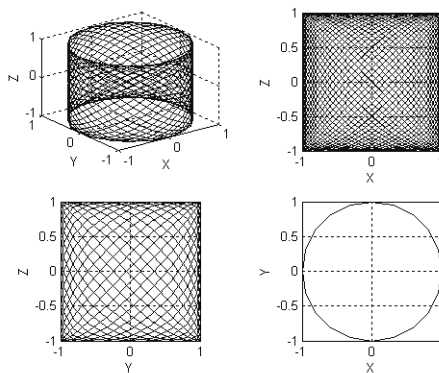


Figure 3-26 Outline of vector rotation for an omni-directional system. Magnitude of source currents are set to 1, and source coil sets x and y are in phase quadrature and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane. $f_x=f_y=120$ kHz, $f_z=124$ kHz

The period of rotation (the amount of time that it takes for a system with a rational frequency ratio to form one complete cylindrical mesh) is equal to the least common multiple of the two system periods ($1/f_1$ and $1/f_2$). Thus a system with $f_1=120$ kHz and $f_2=124$ kHz has a rotation frequency of 4000 Hz. However, a system with $f_1=120$ kHz and $f_2=124.5$ kHz has a rotation frequency of only 1500 Hz. The longer the rotation period, the more dense the mesh. There is therefore a tradeoff between the speed of rotation of the magnetic field vector around the cylinder and the density of the mesh. Although a very dense mesh may be required for specialized applications, such as

those with shielding in all but a single vector direction, a loose mesh is acceptable for most applications.

Clearly the rotation of the field vector shown in Figure 3-26 is not truly omni-directional as directions remain in which there is no field vector (i.e. the z-direction). To alleviate this problem, and to create a truly omni-directional system, the coil carrying the second frequency is changed periodically. The system shown in Figure 3-27 is identical to that of Figure 3-26 except that after a time, t_r , the frequencies of the currents in the x- and z-coils are interchanged. This provides a system in which field vectors are effectively created at all possible angles with respect to the x-, y-, and z-axes of the system.

Currents at the second frequency, f_2 , can be applied sequentially to all three source coil pairs, but this is not necessary as omni-directional coverage is obtained with the application of currents at f_2 to only two of the coil pairs. It is also important to note that the currents in the two coils with frequencies not at f_2 must remain in phase quadrature. The time, t_r , after which current at frequency f_2 should be switched from one coil set to the other is equal to one rotation period as discussed above.

$$t_r = lcm\left(\frac{1}{f_1}, \frac{1}{f_2}\right) \quad (3-14)$$

where f_2 is the frequency of the current in one of the source coil sets and f_1 is the frequency of the current in the remaining two coil sets. After time t_r the field vector has rotated through all possible angles of rotation and has returned to its starting point. The coil set carrying current with frequency f_2 is then changed and the vector again moves through all possible angles of rotation in time t_r .

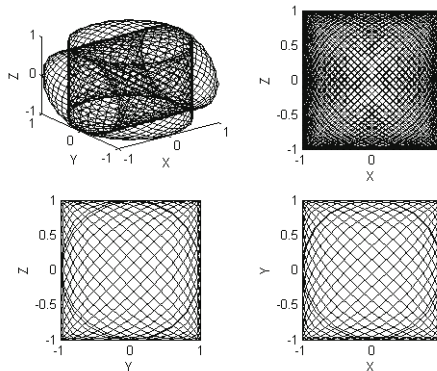


Figure 3-27 Outline of vector rotation for an omni-directional system. Magnitude of source currents are set to 1, and source coil sets x and y are in phase quadrature and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz-plane, xy-plane, and yz-plane. $f_x = f_y = 120$ kHz, $f_z = 124$ kHz followed by $f_z = f_y = 120$ kHz, $f_x = 124$ kHz

The addition of a second frequency to the system means that power will be delivered to the receivers at two frequencies. Because of the narrow-band power transfer characteristic of both source and receiver (see Chapter 6), the frequency f_2 must be chosen such that it is close enough to f to allow a sufficient amount of power to be transferred to the receiving coils even when they are receiving power only from the coil set with current at f_2 .

The magnitude of the field at the center of the system is directly proportional to the coil current in an omni-directional system with currents described by (3-12) where $\omega=\omega_1=\omega_2$, $\phi_1=0$, $\phi_2=\pi/2$ and $K=K_1=K_2=K_3$.

$$H_S \propto \sqrt{x(t)^2 + y(t)^2 + z(t)^2} \quad (3-15)$$

Substituting (3-12) into (3-15) and reducing,

$$H_S \propto \sqrt{K^2 + K^2 \sin^2(\omega_2 t)}$$

$$\text{Therefore, } H_{S \max} \propto \sqrt{2}K, H_{S \min} \propto K \quad (3-16)$$

The magnitude of the magnetic field vector is no longer constant over time, however it does not fall below the magnitude obtained in a bi-directional system. The magnitude of the field vector is a sine function at twice the frequency of f_2 , with a minimum at the peak value of the source field (H_{Sp}), and a maximum at $\sqrt{2}H_{Sp}$. The *rms* field strength available to the receiving coils is then $\sqrt{3/2}$ times that available to the receiving coils in a bi-directional system.

3.6.3 Single-axis amplitude modulation

Amplitude modulation of one or more of the three currents in the source coils will also produce a field vector rotating in more than one plane. Again, by applying current simultaneously to all three of the source coils a field vector is created which rotates through all possible angles with respect to each of the three axes. In this case, a current at an alternate frequency, ω_m , modulates the current applied to one of the coils. The currents applied to each of the coils are of the following form:

$$\begin{aligned} x(t) &= K \sin(\omega t) \\ y(t) &= K \cos(\omega t) \\ z(t) &= K \sin(\omega t) \cos(\omega_m t) \end{aligned} \quad (3-17)$$

where $\omega_m < \omega$. Figure 3-28 shows the shape traced by the endpoints of the field vector as it rotates through one period of the modulation frequency ω_m .

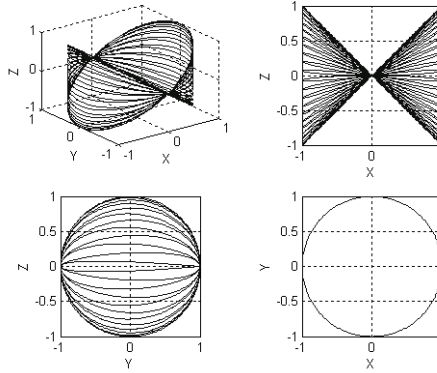


Figure 3-28 Outline of vector rotation for an omni-directional system. Magnitude of source currents are set to 1, and source coil sets x and y are in phase quadrature and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane. $f=120$ kHz, $f_m=5$ kHz

Once again, the rotation of the field vector shown in Figure 3-28 is not truly omni-directional as directions remain in which there is no field vector (i.e. in the z -direction). To alleviate this problem, and to create a truly omni-directional system, the coil carrying the modulated current is changed periodically. The system shown in Figure 3-29 is identical to that of Figure 3-28 except that after a time, t_r , the modulating current is removed from the x -coils and applied to the z -coils. This provides a system in which field vectors are effectively created at all possible angles with respect to the x -, y -, and z -axes of the system.

Modulated currents can be applied sequentially to all three source coil pairs, but this is not necessary as omni-directional coverage is obtained with the application of modulated currents to only two of the coil pairs. It is also important to note that the currents in the two coils carrying unmodulated currents must remain in phase quadrature.

The minimum time, t_r , after which the modulation should be switched from one coil set to the other is equal to the time that it takes for the magnetic field vector to rotate through all possible angles. This is equal to one period of the modulation current.

$$t_r = \frac{2\pi}{\omega_m} \quad (3-18)$$

After time t_r the field vector has rotated through all possible angles of rotation and has returned to its starting point. The coil set carrying current with frequency ω_m is then changed and the vector again moves through all possible angles of rotation in time t_r .

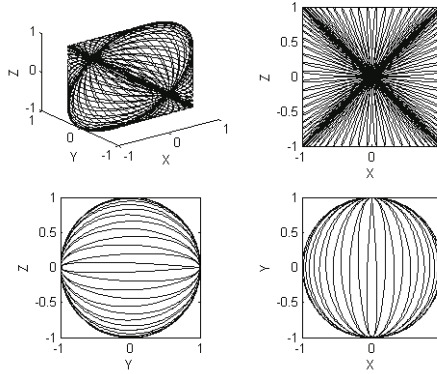


Figure 3-29 Outline of vector rotation for an omni-directional system. Magnitude of source currents are set to 1, and source coil sets x and y are in phase quadrature and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane. $f=120$ kHz, $f_m=5$ kHz. The modulating signal is applied first to the x -coils and then to the z -coils

The addition of a second frequency to the system means that power will be delivered to the receivers at three frequencies. Because of the resonant nature of the power transfer from source to receiver, the frequency ω_m must be chosen such that it will allow a sufficient amount of power to be transferred to the receiving coils even when they are receiving power only from the coil set carrying the modulated current. The modulated current is of the form

$$K \sin(\omega t) \cos(\omega_m t) = \frac{K}{2} [\sin(\omega t - \omega_m t) + \sin(\omega t + \omega_m t)] \quad (3-19)$$

Power is then created at three frequencies: $\omega \pm \omega_m$, and ω .

The magnitude of the magnetic field vector is no longer constant over time, however it does not fall below the magnitude obtained in a bi-directional system. Using the analysis method of the frequency shift method, the field strength at the center of a system using single-axis amplitude modulation has a minimum at the peak value of the source field (H_{sp}), and a maximum at $\sqrt{2}H_{sp}$. The *rms* field strength available to the receiving coils is then $\sqrt{5}/2$ times that available to the receiving coils in a bi-directional system.

A receiver that is shielded from all but the direction in which the modulated field strength is present will receive only 50% of the field strength available in the unshielded condition over one period of the modulation frequency. However, a receiver that is shielded from all but one direction in which unmodulated field strength is present, will receive 70.71% of the field strength available in the unshielded condition. This problem can be compensated by an increase in the magnitude of the

current at the carrier frequency, at the expense of an increase in instantaneous field intensity. This may present a problem in areas immediately surrounding the source coils (see Appendix I – International Standards – Magnetic Field Strength Limits).

3.6.4 Double-axis amplitude modulation

Amplitude modulation can be applied simultaneously to two axes to produce an omni-directional field. This solution forms an omni-directional field with constant magnitude throughout its rotation. A spherical shape is formed by the outline of the endpoint of the vector as it rotates through all possible angles with the three axes, as is shown in Figure 3-30.

The currents applied to each of the source coil sets are described by the parametric equations

$$\begin{aligned}x(t) &= K \cos(\omega t) \cos(\omega_m t) \\y(t) &= K \cos(\omega t) \sin(\omega_m t) \\z(t) &= K \sin(\omega t)\end{aligned}\tag{3-20}$$

Where the azimuthal angle of the vector is determined by ωt and the polar angle by $\omega_m t$.

The field strength at the center of the system is directly proportional to the coil current in an omni-directional system with currents described by (3-20)

Substituting (3-20) into (3-15) shows that the magnitude of the source field at the center of the system is constant.

$$H_s \propto \sqrt{K^2 \cos^2(\omega t) \cos^2(\omega_m t) + K^2 \cos^2(\omega t) \sin^2(\omega_m t) + K^2 \sin^2(\omega t)} = K \tag{3-21}$$

Applying a trigonometric transformation to (3-20), the currents applied to each set of source coils are of the form:

$$\begin{aligned}x(t) &= \frac{K}{2} [\cos(\omega t - \omega_m t) + \cos(\omega t + \omega_m t)] \\y(t) &= \frac{K}{2} [\sin(\omega t - \omega_m t) + \sin(\omega t + \omega_m t)] \\z(t) &= K \sin(\omega t)\end{aligned}\tag{3-22}$$

Where ω is the carrier frequency and ω_m is the modulation frequency. The currents applied to each of the coil sets are shown in Figure 3-31.

It is clear from (3-22) that the field created by this system is at three frequencies ($\omega, \omega \pm \omega_m$). While the field created in the z -direction has components only at the

frequency ω of the receiving coils, the fields created in the x - and y -directions have 50% of their magnitude at $\omega + \omega_m$ and the remaining 50% of their magnitude at $\omega - \omega_m$. The Fourier transform of the modulated waves is identical to that of a Double Sideband Suppressed Carrier (DSSC) amplitude modulated signal used in AM radio transmission.

$$\mathcal{F}\{x(t)\} = \frac{K}{2} [F(\omega + \omega_m) + F(\omega - \omega_m)] \quad (3-23)$$

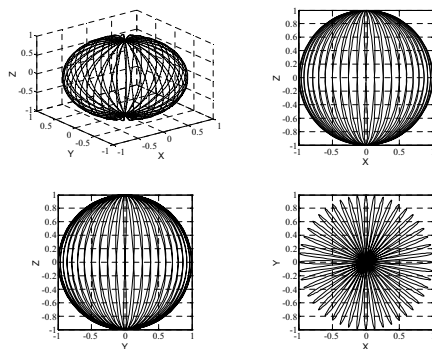


Figure 3-30 Outline of vector rotation for a system using double axis amplitude modulation. Magnitudes are set to 1, and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane. $f=120$ kHz, $f_m=5$ kHz

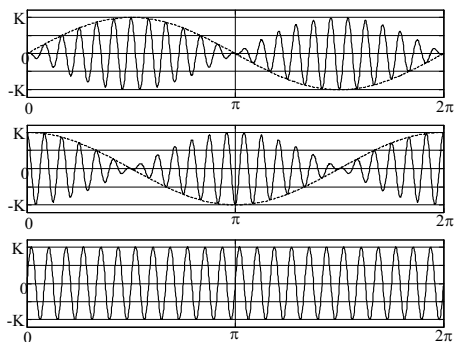


Figure 3-31 Normalized currents in the x -, y -, and z -direction source coils for a system using double axis amplitude modulation. One period of the modulation function is shown. The carrier frequency ($\omega/2\pi$) is 120kHz, and the modulation frequency ($\omega_m/2\pi$) is 5kHz. The modulation function is also shown for the x -, and y -coils

The magnetic field vector completes one full rotation around the sphere shown in Figure 3-30 during each period of the modulating wave. The modulation frequency should be kept as low as possible while maintaining charge in the energy storage capacitor of a receiver shielded in all but one direction. A lower modulation frequency more thoroughly covers the surface of the sphere shown in Figure 3-30, and also provides a smaller difference between the three frequencies at which power is available to the receiving coils. This allows power to be transmitted at adjacent frequencies, which simplifies receiving coil power extraction using a resonant circuit.

As described in the previous section, the field strength available to a receiver which is shielded from all but one of the directions in which the modulated field strength is present will receive only 50% of the field strength available in the unshielded condition over one period of the modulation frequency. However, a receiver that is shielded from all but the direction in which unmodulated field strength is present will receive 70.71% of the field strength available in the unshielded condition. The situation is more important in the case of double axis amplitude modulation because two of the three source coils carry a modulated current. This problem can again be compensated by an increase in the magnitude of the current at the carrier frequency, at the expense of an increase in instantaneous field intensity.

Applying the standard equation

$$F_{rms} = \sqrt{\frac{1}{T} \int_{t_0}^{t_0+T} f(t)^2 dt} \quad (3-24)$$

to the functions $x(t)$ and $y(t)$ described in (3-20) yields

$$rms[x(t), y(t)] = K / 2. \quad (3-25)$$

Further examination shows that

$$rms[\sqrt{2}x(t), \sqrt{2}y(t)] = K / \sqrt{2} \quad (3-26)$$

which is identical to the *rms* value of $z(x)$. Therefore, an increase in magnitude of the carrier-frequency current by a factor of $\sqrt{2}$ will allow a receiver to obtain the same power from any one set of source coils when it is shielded from the other two. This will, however, cause a distortion of the magnetic field vector trace shown in Figure 3-30. The shape of the vector trace will be stretched into an oblate spheroid and the field strength in the center of the system will no longer be constant but will fluctuate at twice the carrier frequency between that produced without the increase in the magnitude of the carrier frequency current and $\sqrt{2}$ times that value. Constant power will no longer be delivered to the receiving coils in this case, but again the available power in an unshielded system will never fall below that available in a bi-directional system.

3.6.5 Wide-band operation

Systems can also be developed in which unmodulated currents of different frequencies are applied to each of the three source coil sets (Figure 3-32). The currents are of the form:

$$\begin{aligned}x(t) &= K \cos(\omega_1 t) \\y(t) &= K \cos(\omega_2 t) \\z(t) &= K \cos(\omega_3 t)\end{aligned}\tag{3-27}$$

The shape formed by the endpoints of the magnetic field vector as it rotates is shown in Figure 3-32. These systems will no longer provide power to receiving coils at or near a single frequency. Wide-band receivers must therefore be used, increasing the cost and complexity of the system.

The *rms* value of the field strength available to receiving coils at the center of the system operating under unshielded conditions is $\sqrt{3}/2$ times that of a bi-directional system operating under the same conditions. This is identical to the *rms* value of the field strength available to receiving coils using the frequency shift method. However, unlike the methods described previously, this system does not maintain a field strength at or above that obtained by a bi-directional system throughout the rotation of the magnetic field vector. At certain times the vector is at a significantly lower strength, which produces effects identical to those in the presence of shielding materials. This may not pose a problem as the field vector may be at a lower than acceptable value for a short enough period of time as to not substantially effect the energy storage element.

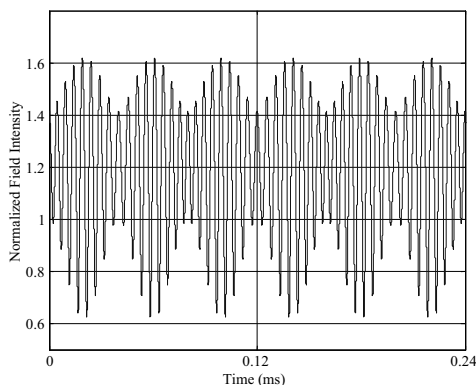


Figure 3-32 Field intensity vs. time for the system shown in Figure 3-33. Magnitudes are set to 1, and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, *xz*-plane, *xy*-plane, and *yz*-plane. $f_x=112.5\text{kHz}$, $f_y=120\text{kHz}$, $f_z=127.5\text{kHz}$. Field intensity of 1.0 is defined as the intensity of the field at the center of the system for a bi-directional system in the unshielded case. This method is the only one in which the field intensity falls below this value during system operation

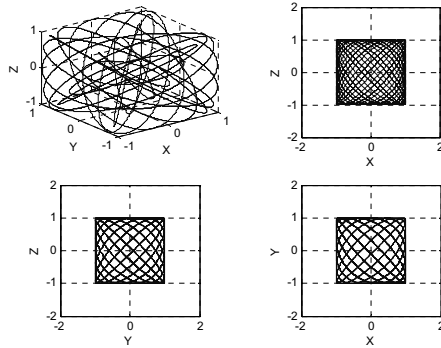


Figure 3-33 Outline of vector rotation for a system using three source coil sets operating at different frequencies. Magnitudes are set to 1, and all three coil sets are in space quadrature. Views shown are (clockwise from upper left) three-dimensional, xz -plane, xy -plane, and yz -plane. $f_x=112.5\text{kHz}$, $f_y=120\text{kHz}$, $f_z=127.5\text{kHz}$

If the three frequencies in the system are equidistant from each other ($|f_x - f_y| = |f_z - f_y|$), the beat frequency shown in Figure 3-32 can be defined as

$$f_{beat} = 2|f_x - f_y| \quad (3-28)$$

With frequencies close enough to each other to allow the use of one resonant circuit at the receiving coils; the magnitude of the magnetic field vector produced is near to zero for an unacceptable duration. Equation (3-28) shows that the beat frequency is reduced as the frequency of the currents in the system come closer to each other in magnitude. A lower beat frequency causes the magnitude of the field vector to fall below that of a bi-directional system for a longer period of time. This effect can be seen in Figure 3-33. During this time, the receiving coils can obtain little or no power from the source coils; effectively creating the same effect as is created by the presence of shielding elements within the source field.

The characteristics of the vector rotation in this case are similar to those of the frequency shift method described previously. If all three frequencies have integer values, the lines forming the vector rotation trace will close on themselves and form a widely spaced pattern. Irrational frequency ratios will form trace patterns that will not close on themselves and will create a mesh pattern that becomes denser over time. If the ratio of frequencies is rational but not a simple fraction, the vector trace will close on itself but will have a much denser pattern than that seen when the frequencies all have integer values. The difference in vector trace patterns between this method and that of the frequency shift is that in this case the frequencies of the currents applied to each of the source coils remains constant over time. This advantage is, however,

negated by the fact that the magnetic field vector falls below the desired value at times, does not deliver constant power to the receiving coils even in the unshielded case, and requires additional circuitry in order to compensate for operation off of a single resonant point.

The added complexity of this method without distinct advantages insures that it would be used only in applications where field vectors must rotate in such a manner as to cover the possible angles of rotation in an evenly distributed pattern without a periodic switching of the current waveforms between source coils.

3.6.6 Comparison of methods for omni-directional field generation

At minimum, a second frequency must be introduced into a system order to create a truly omni-directional magnetic field vector. The introduction of an additional frequency or frequencies complicates a system dependent on power delivery at a single resonant frequency. Due to this complication, none of the proposed solutions is ideal in that none of them are able to provide the same amount of power to a receiving coil, regardless of the direction in which it is shielded from the source field, without increasing the field strength (and therefore the current) produced by one or more of the source coils.

Switching of the plane of rotation is relatively simple and provides power to the receiving coils at only one frequency. It does, however, not produce a truly omni-directional magnetic field vector and is therefore useful only in systems where little shielding is expected to occur or where the shielding which will occur is determined to have characteristics allowing its effects to be mitigated by this method of operation. These conditions may occur, for example, in an assembly line or other factory setting where the receiving coils will move on predetermined paths. This solution is also practical in cases when bi-directional systems are inadequate but field strength limits have been reached with two sets of source coils.

Periodically shifting the frequency in one of the three source coils is a solution that allows the worst-case field strength available to the receiving coils to remain the same as in the bi-directional field generation case. The method does not provide constant power to the receiving coils. It is, however, relatively simple to design and implement and may be especially useful in cases where shielding will occur in areas such that a second frequency is required in only one of the source coil sets. These conditions may exist, for example, when objects are being manipulated randomly on a conveyor belt or assembly line in which shielding may occur in any direction but only within a specified volume. However, in order to achieve vector rotation at all possible angles, the current at the second frequency must be periodically switched between two of the coil sets. As double axis amplitude modulation accomplishes this without the need to periodically switch control signals between source coil sets, the frequency shift method will be considered only when a second frequency is required in only one source coil set.

The modulation of the current in one of the source coil (single-axis amplitude modulation) sets reduces the field strength available to the receivers under both the worst-case and unshielded conditions from that of the frequency shift method. It also requires a periodic shift of the modulated current between two of the source coil pairs and does not provide any additional protection from the effects of shielding. Modulating current in only one source coil set is not substantially simpler than modulating the current in two source coil pairs (see Chapter 6). Periodically shifting the modulated current between two of the coil pairs further complicates the system and makes it less attractive than that of double axis amplitude modulation, which requires no such shifting of currents. This method will therefore receive no further consideration.

The modulation of the current in two of the source coil pairs (double-axis amplitude modulation) provides constant power to the secondary coils and does not require the periodic switching between coil sets of any of the currents. The worst-case power available to the receivers at the center of a system is, however, lower than that obtained using the frequency shift method. Double axis amplitude modulation generates power at three frequencies and thus requires a receiver that is designed to extract sufficient power at these frequencies. However, its high adaptability to changing shielding conditions without the need for switching between source coil sets makes this a very attractive option.

Systems operating with the use of three unmodulated currents at three separate frequencies (wide-band operation) do not provide significant advantage over any of the systems described above. Wide-band power extraction requires additional circuits on each receiving coil and does not provide constant power to the receiving coils. In order to eliminate the need for the additional circuitry, the frequencies must be very close to each other in value, which results in periods where there is little or no field strength produced by the source coils. This method will therefore receive no further consideration. Table 3-2 provides a summary of the characteristics of each of the methods described in this section.

Table 3-1 Omni-directional field generation

	RMS field strength at the center of a symmetrical system*	Vector components possible in all directions	Power generated at two or more frequencies	Worst-case field strength available to receiver**	Effect of frequency shift on power at receiver	Adaptability to changing shielding conditions
Switching the Plane of Rotation	1	No	No	$\frac{\sqrt{2}}{2}$	Not Applicable	Low
Frequency Shift	$\sqrt{\frac{3}{2}}$	Yes	Yes	$\frac{\sqrt{2}}{2}$	Medium	High
Single-Axis Amplitude Modulation	$\frac{\sqrt{5}}{2}$	Yes	Yes	$\frac{1}{2}$	Medium	High
Double-Axis Amplitude Modulation	1	Yes	Yes	$\frac{1}{2}$	High	High
Wide band - Multiple Frequencies	$\sqrt{\frac{3}{2}}$	Yes	Yes	Variable	Very High***	High

*With 1.0 being the field strength at the center of a system using bi-directional field generation and all currents in all coils having a peak magnitude of 1.0.

**When a receiving coil placed at the center of the system is shielded such that it receives power in only one direction. Assuming $I_x=I_y=I_z$

*** Assuming narrow-band operation with a single-tuned receiver.

4 Shielding of the Source Field

Partial or complete shielding of the source field can occur which may effectively prevent the receiving coils from receiving adequate power for operation. Shielding of the source field can occur when conductive or permeable materials are placed in or near to the source field, and its effects on system performance can range from negligible to intolerable.

The amount of shielding that can be tolerated by a system is dependent on the requirements of the load. The *rms* value of the field strength available at the receiving coils must be greater than or equal to the minimum field strength required to power the load. Once the effective field strength at the receiving coils falls below this value, the load must rely on energy storage devices or cease operation. Shielding is clearly a major obstacle that must be dealt with in order to achieve reliable system operation.

The shielding factor (α) is defined as

$$\alpha = 20 \log_{10} \frac{H_u}{H_s} \quad (4-1)$$

where H_u is the unshielded field strength at a particular point and H_s is the shielded field value at that same point.

The field of EMC (electromagnetic compatibility) has extensively studied methods to intentionally shield devices, people, and structures such as buildings from electromagnetic fields. However, little literature exists in the area of preventing this shielding from occurring or in mitigating its effects at a specified physical location.

The effects of shielding on the source field are dependent upon the position of the objects causing the shielding relative to the orientation of the field created by the source coils, the position and orientation of the receiving coil(s), and upon the characteristics of the shielding material. This chapter will illustrate the effects of shielding on the intensity of the magnetic field available to the receiving coils and will propose solutions with respect to a minimization of the effects of shielding materials on system performance. Finally, several solutions will be presented for situations in which the shielding of receivers is unavoidable.

4.1 Shielding of low-impedance electromagnetic waves

Because the system operates in the near-field and uses a magnetic (rather than electric) field source, the wave impedance of the magnetic field is low (the ratio of E to H has a small value). The importance of wave impedance in understanding the effect of shielding materials on the source field can be understood in terms of the effect of a discontinuity in an electromagnetic wave [4-1]. If the magnitude of the wave impedance is much different from the intrinsic impedance at the discontinuity, most of the energy will be reflected rather than being transmitted across the boundary [4-2].

However, most materials have an intrinsic impedance in the milliohm range [4-3]. For low impedance fields (H dominant fields), more energy is absorbed by the shielding material because its impedance is more closely matched to the impedance of the field (Figure 2-2). This causes absorption losses to be the dominant shielding mechanism for near-field magnetic sources at all frequencies, except for very thin materials in which the absorption loss is lower than the reflection loss. Reflection and absorption (per mil thickness) losses for copper and iron in magnetic fields are shown in Figure 4-1 [4-3].

When a physically thin (the absorption factor is less than 15dB) shielding material is present, a correction term must be added to or subtracted from the magnitude of the reflection factor to account for multiple reflections within the shielding material. A secondary reflection occurs on the far side (the side opposite the incident field) of the shield regardless of thickness. A large part of the re-reflected wave can, however, appear on the front surface of a thin shield. This re-reflected wave can add or subtract from the initial reflected wave depending on the phase relationship between them [4-3].

This effect can cause a shielding effect much stronger than that which is estimated using techniques which do not account for it. Calculation of the correction term is complicated as it is a function of the material and dimensional parameters of the shielding material as well as the frequency of the surrounding field. This calculation is beyond the scope of this work, however methods exist to estimate it [4-4] and its effects and an equation describing it are included in the tables showing shielding effectiveness in Appendix V.

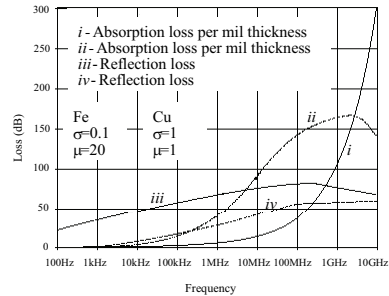


Figure 4-1 Shielding effectiveness of iron and copper barriers to low-impedance waves [4-3].
1mil=0.0254mm

Absorption A and reflection R losses in dB for low impedance waves can be estimated using the following equations respectively [4-4]:

$$A_{\text{loss}} (\text{dB}) = 3.34 \sqrt{\mu_{rc} \sigma_{rc} f d_s} \quad (4-2)$$

$$R_{\text{loss}} (\text{dB}) = 20 \log_{10} \left[\frac{0.462}{r \sqrt{\frac{f \sigma_{rc}}{\mu_{rc}}}} + 0.136 r \sqrt{\frac{f \sigma_{rc}}{\mu_{rc}}} + 0.354 \right] \quad (\text{for } f \ll 2 \times 10^9) \quad (4-3)$$

where μ_{rc} and σ_{rc} are the permeability and conductivity of the shielding material related to copper; d_s is the thickness of the shield, and r is the distance from the shield to the field source.

These equations represent theoretical (ideal) values which are generally very high, unrealistic estimates for real-world shielding situations [4-3]. Tables of measured and calculated effectiveness' of shielding materials against low impedance waves can be found in Appendix V. Figure 4-2 and Figure 4-3 show theoretical values of reflection and absorption losses for iron, copper, and aluminum.

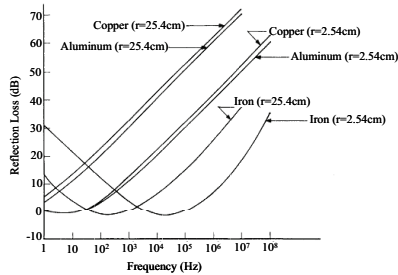


Figure 4-2 Reflection loss for iron, copper and aluminum with a low impedance source (r =distance from source to shield) [4-1]

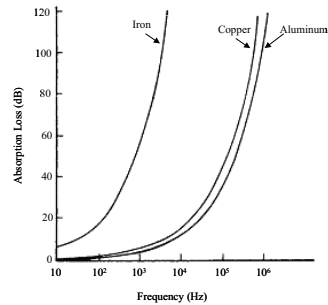


Figure 4-3 Absorption loss for 1mm thick sheets of iron, copper and aluminum [4-1]

As the absorption loss term is dominant in the discussion of the shielding of near-field magnetic field sources, the two basic methods by which this loss takes place must be analyzed. The first method is by the presence of highly conductive materials ($\sigma \gg 1$) within the source field, and the second is by the presence of highly permeable materials ($\mu \gg 1$).

4.2 Shielding by highly conductive materials

The presence of highly conductive materials in or near to the operating volume causes shielding of the source field due to the generation of opposing flux via Faraday's law. This law states that the voltage induced in a loop of conducting material is proportional to the rate of change of magnetic flux passing through it; it is given by the equations

$$emf = n \oint \vec{E} \cdot d\vec{l} = n \frac{d}{dt} \int \vec{B} \cdot d\vec{s} = n \frac{d\phi}{dt} \quad (4-4)$$

where emf is electromotive force and is defined as the line integral of the electric field. The change in flux can be created due to a non-dc current source, or due to motion of the circuit relative to that of the magnetic field.

The eddy currents induced by the source field flow in materials (conducting masses of any shape) with a high conductivity located within the operating volume. The eddy currents induced in these objects will produce a magnetic field that will serve to attenuate the field that created them. This effect is illustrated in Figure 4-4.

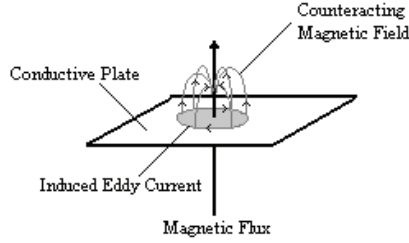


Figure 4-4 Eddy currents induced in a conductive plate by a magnetic field

The effect of eddy currents on the system can be shown by assuming that a conductive shield of thickness d , radius a , and conductivity σ is at the center of and coplanar to a circular source coil of radius R centered on the z -axis. If $d \ll a \ll R$ and the source coil carries current, the magnetic field created by the current in the source coil will induce eddy currents in the shielding material. Because the radius of the shielding material is small in comparison to that of the source coil, the magnetic field is assumed to be uniform over the surface of the shield. Using the expression developed in Chapter 3 for the field created by circular source coils and evaluating at the center of the coil, the flux through a loop of radius R centered on the shield can be described as

$$\phi = \int_0^a B_z 2\pi\rho' d\rho' = \frac{\mu_0 \pi I \rho^2}{2R} \quad (4-5)$$

The *emf* generated is then

$$emf = -\frac{d\phi}{dt} = -\frac{\pi\mu_0\rho^2}{2R} \frac{dI}{dt} \quad (4-6)$$

Using Faraday's law, the *emf* can also be written as

$$emf = \oint \vec{E} \cdot d\vec{l} = 2\pi\rho E_\phi \quad (4-7)$$

The induced electric field can then be written as

$$E_\phi = -\frac{\pi\mu_0\rho}{4R} \frac{dI}{dt} \quad (4-8)$$

The eddy current can then be expressed as $J_\phi = \sigma E_\phi$, and the power dissipated per volume in the shield is $\vec{J} \cdot \vec{E} = \sigma E_\phi^2$. Integrating over the volume of the shield to find the total dissipated power gives

$$P = \int_0^a (\sigma E_\phi^2) d(2\pi\rho d\rho) = \frac{\pi\mu_0^2\sigma da^4}{32R^2} \left(\frac{dI}{dt}\right)^2 \quad (4-9)$$

Averaging the power dissipation over a cycle gives

$$P_{ave} = \frac{\pi\mu_0^2\sigma da^4 R^2 I_0^2 \omega^2}{64}. \quad (4-10)$$

This method for evaluating the effect of eddy currents on the system is straightforward, but is applicable only when the geometry is simple and the skin effect can be neglected. Because more complex geometries must be considered and the effect on the system of shields of various thickness and material properties must be analyzed, this simplified analysis will not provide adequate results in many cases. Analytical methods for solving the diffusion equation and associated field differential equations are highly complex and are limited to simple geometries [4-5], [4-6]. Therefore Finite Element Analysis (FEA) will be used throughout this chapter when the effects of shielding materials of finite conductivity are of interest.

Power dissipation of the source field by shielding materials generally does not produce enough heat to endanger system operation.

4.2.1 Highly conductive objects inside the operating volume

The losses in the system apparent in the undesirable attenuation of the source field are seen in the heat generated by the eddy currents flowing in the conductive shielding material. The magnitude of the losses generated by the presence of these materials within or near to the operating volume is dependent on multiple factors. These are frequency, angle between field vector and shielding material surface, the permeability of the material, and its intrinsic impedance.

A table listing conductivities of commonly found shielding materials can be found in Appendix V. Clearly more conductive materials will produce a higher current given the same voltage (*emf*) induced on the material by the source field. This will increase the attenuation of the source field and therefore also increase the total power losses in the system.

Because of the skin effect, the frequency at which the source field operates will also influence the magnitude of attenuation. The skin depth of a good conductor (defined as a conductor with the property $\omega\varepsilon \ll \sigma$, where ω is the angular frequency of the current in the conductor, ε is the permittivity of the material, and σ is its conductivity) is defined as

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}. \quad (4-11)$$

where f is the frequency of the current flowing in the conductor, and μ is the permeability of the conductor. As the frequency of the source field increases, the skin depth of a conducting material is reduced, leaving a smaller cross-sectional area in

which the induced (eddy) currents can flow. This increases the effective resistance of the conductive material, therefore decreasing the magnitude of current flowing in the material due to the voltage (emf) induced by the source field. A smaller induced current reduces the attenuating effect of the conductive material on the source field and therefore reduces the overall losses in the system.

The angle at which the source field strikes a conductive shield will also affect the amount of attenuation of the source field that takes place. The eddy currents present in a conductive shielding material placed within the source field will themselves act as a field source and can considerably alter both the magnitude and direction of the source field vector. An example of this effect is seen in Figure 4-5.

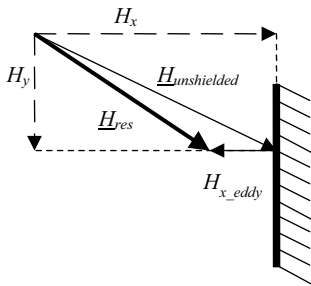


Figure 4-5 Unshielded vector $H_{unshielded}$ and its components. Field vector H_{x_eddy} created by eddy currents in the shielding material, and resulting field vector H_{res}

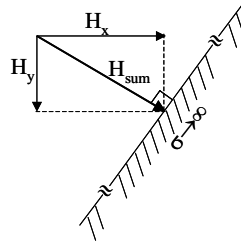


Figure 4-6 Field vector H_x or H_y alone does not induce the maximum shielding effect. The sum of these two vectors will, however, create H_{sum} which is perpendicular to the shield and will induce the maximum shielding effect

With all other variables (shield conductivity, source field frequency, etc.) held constant, the larger the component of the source field normal to the shield, the larger the attenuation of the magnitude of that field. The eddy currents in a shielding material can create magnetic fields with vector components only in the direction normal to the surface in which the eddy currents are flowing. It is clear that no eddy currents are induced in the shielding material shown in Figure 4-5 by the H_y component of the source field (assuming the shield is thin), as it is parallel to the shield and therefore does not create any flux crossing the surface of the shield. Because the H_y component does not induce any eddy currents in the shield, no damping effect on the H_y component is seen.

There are cases where the addition of a field vector in a second direction will increase rather than decrease the effects of shielding when a shielding material is present. Figure 4-6 shows this effect. When H_x alone is present, the field vector is not perpendicular to the conductive shield and shielding is therefore not at its maximum. However, the addition of a vector in the y -direction can, when it is of the correct magnitude, increase the shielding effect by forming a resulting vector in the direction normal to the shield.

The dependence of the magnitude of the source field attenuation on the relative angle between source field and shield is illustrated in Figures 4-7 - 4-12. These figures show a thin plate of conductive shielding material ($\sigma=5.8e7$) lying inside a magnetic field. The plate is lying in the y - z plane and the figures are shown from the perspective of the positive z -axis. The shielding factor, α is given for three points: A , B , and C . These points all lay on a line parallel to and 3cm from the shield. Point A is left of the shield's center, point B is the in center of the shield, and point C is right of center. Negative values of α indicate that the presence of the shield has increased the field strength over that which was present at that point without the presence of the shield. The source field vectors shown in Figure 4-7 are parallel to the surface of the shielding material. Since there is no component of the source field normal to the surface of the conductive plate, no attenuation of the source field, and therefore no shielding, will occur. (Neglecting the relatively narrow width of the plate in the x -direction and the eddy currents that will be produced in this cross section, leading to a small amount of shielding at the edges of the plate).

Figure 4-9 and Figure 4-10 show the same shielding material inside of the same source field. However, the angle between the source field and the shielding material has been altered such that the source field is now normal to the surface of the shield. Clearly as the source field has no components in any direction that do not serve to induce eddy currents in the conductive material, the maximum attenuation of the source field will occur, and the shielding effect will be strongest. When the field vector is normal to the surface of the conductive plate, the magnetic field will dissipate the most power in that material.

Figure 4-11 shows the field distribution around the same shielding material when the source field vector makes an angle of 45° with the shield (an angle where the magnitude of the component of the source field vector normal to the shield is equal to the magnitude of the component parallel to the shield. Here the path of the flux lines has shifted and the areas of highest attenuation of the source field are no longer symmetrical about the shielding material. The magnitude of the attenuation is also lower compared to that of Figure 4-10.

Finally, Figure 4-12 shows the *rms* value of the magnitude of the magnetic field around the shielding material for one full rotation ($-180^\circ \rightarrow 180^\circ$) of the relative angle between the source field and the shield. A comparison of Figure 4-8, Figure 4-10, and Figure 4-12 shows qualitatively that for one full rotation of the relative angle, the attenuation of the source field is present but substantially less than that in the maximally shielded case and greater than that in the unshielded case. Constantly varying the angle between the shield and the source field appears therefore to be an effective method to mitigate the worst-case effects of shielding on the system.

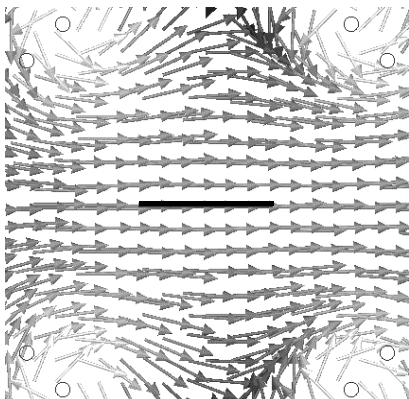


Figure 4-7 Magnetic field vectors around a plate of $\sigma=5.8e7$. The arrow shade indicates relative magnitude (lighter colored arrows have higher magnitude) of the magnetic field. $\alpha_A = -0.174$ dB, $\alpha_B = 0.017$ dB, $\alpha_C = -0.093$ dB

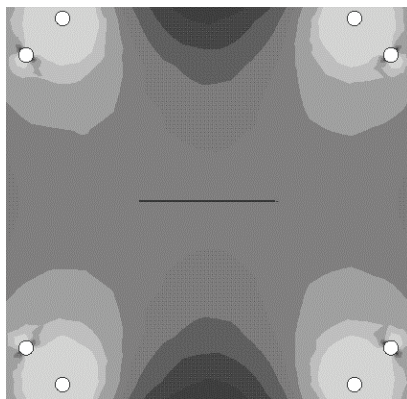


Figure 4-8 Magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors shown in Figure 4-7. Lighter shades indicate a higher magnitude. $\alpha_A = -0.174$ dB, $\alpha_B = 0.017$ dB, $\alpha_C = -0.093$ dB

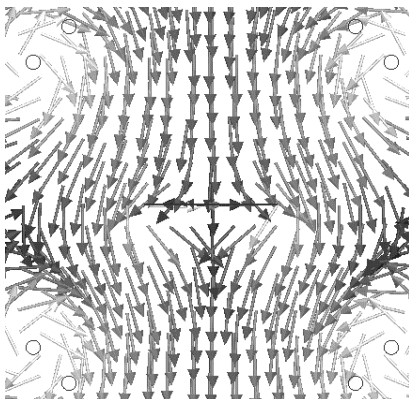


Figure 4-9 Magnetic field vectors around a plate of $\sigma=5.8e7$ with the field vectors normal to the surface of the plate. The arrow shade indicates relative magnitude (lighter arrows have higher magnitude) of the magnetic field. $\alpha_A = -2.097$ dB, $\alpha_B = 13.417$ dB, $\alpha_C = 6.060$ dB

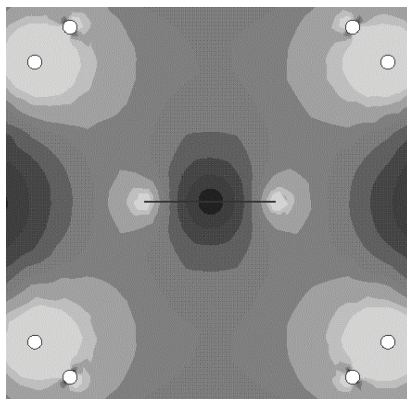


Figure 4-10 Magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors shown in Figure 4-9. Lighter shades indicate a higher magnitude. $\alpha_A = -2.097$ dB, $\alpha_B = 13.417$ dB, $\alpha_C = 6.060$ dB

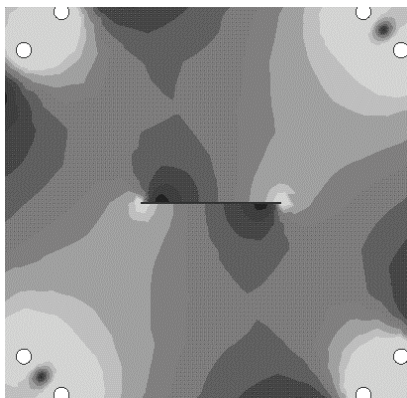


Figure 4-11 Magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors making a 45° angle with the plate. Lighter shades indicate higher field values; note that the magnitude of the field is reduced at point A and is higher at point C. $\alpha_A=14.838$ dB, $\alpha_B=3.126$ dB, $\alpha_C=-3.720$ dB

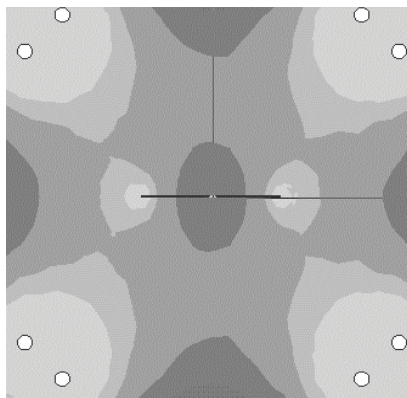


Figure 4-12 *rms* magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors making a 360° rotation. $\alpha_A=-1.1274$ dB, $\alpha_B=0.0160$ dB, $\alpha_C=-0.7422$ dB

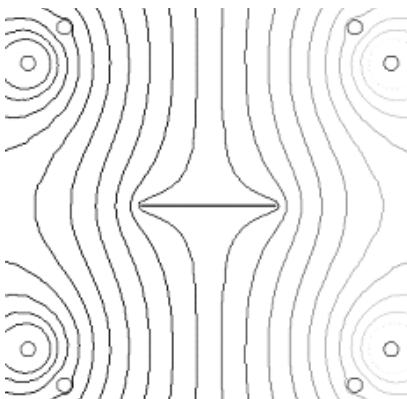


Figure 4-13 Flux lines around a plate of $\sigma=5.8e7$ with the field vectors normal to the surface of the plate. Compare with Figure 4-9 and note that the density of flux lines in any area is directly proportional to field strength

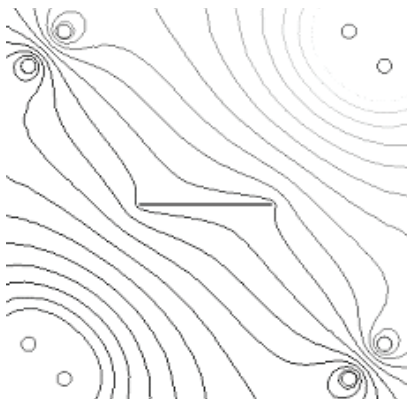


Figure 4-14 Flux lines around a plate of $\sigma=5.8e7$ with the field vectors at 45° to the surface of the plate. Compare with Figure 4-11 and note that the density of flux lines in any area is directly proportional to field strength

Figure 4-10 and Figure 4-11 also show that while the magnitude of the field near the center of the conductive plate is damped in comparison to that of the unshielded case (Figure 4-7), the magnitude of the field on the ends of the plate is increased relative to that of the unshielded case. Flux line diagrams (lines of constant vector potential) corresponding to the cases shown in Figure 4-10 and Figure 4-11 are seen in Figure 4-13 and in Figure 4-14, respectively.

These figures illustrate how the flux lines are diverted around the shielding material and tend to come closer to one another at the edges of the conducting plate. This explains the areas of increased field intensity seen in Figure 4-10 and in Figure 4-11, since magnetic flux density, B , is defined as the number of flux lines per unit area that permeate the magnetic field; and field intensity, H , is directly proportional to B in a linear medium (assumed to be a vacuum in this case). Hence flux lines provide a simple explanation for the shape of the field distribution in Figure 4-10 and in Figure 4-11.

The effect of varying the angle between the source field and the conductive shielding plate can also be seen by examining the shielding factor α along a line parallel to the surface of the shield at a distance of 0.1m from the shield. Figure 4-15 shows that the shielding factor at a 0° angle between the plate and the source field (the field vectors being parallel to the plate) is nearly zero along the entire line. For a 90° angle between the plate and the source field the shielding factor reaches its maximum value at the center of the shielding plate and falls symmetrically towards the edges showing a negative shielding factor at the edges as described above. The shielding factors for angles at 45° and 135° are mirror images of each other as is implied by Figure 4-11 and Figure 4-14. The shielding plate used in Figure 4-15 is identical to that used in Figures 4-8 - 4-15.

For a full rotation of the angle between field and shielding surface the damping value α_{rms} is shown along the line. The value α_{rms} is obtained by comparing the rms value of the magnitude of H from $-180^\circ \rightarrow 180^\circ$ in the unshielded case (no conductive plate present in the field) to the same magnitude in the shielded case. While α_{rms} is slightly higher than the unshielded value it is substantially lower than that of the other angles shown. This result shows that the shielding effect of a conductive object located within the source field can be mitigated by a continual variation of the angle between the conductive object and the source field.

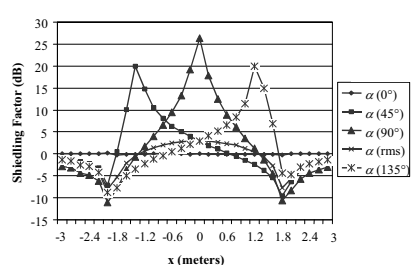


Figure 4-15 Shielding factor along a line parallel to the surface of a conductive shielding plate (3.6m in length) at a distance of 0.1m from the of the plate for various angles between the source field and the plate and an rms value for all angles between 0° and 180°

The discussion thus far has been limited to field vector rotation in a single plane. There are cases in which the *rms* value of the magnetic field in a shielded position is not substantially higher than at the worst-case instantaneous value. This situation is likely to occur when the field vector is rotating in a single plane and the conducting material is shaped such that it can shield a field vector in all directions of rotation. An example of this effect is shown in Figure 4-16. In this case, a copper shield consisting of two plates joined at a 90° angle placed within a magnetic field rotating in the *xy*-plane creates an area of unacceptable field attenuation. The shielding factor at a point inside this area (marked in Figure 4-16 with an *) is 13.9dB. If, however, the field vector rotates outside of the *xy*-plane, the effect of the shielding material in Figure 4-16 will be reduced as the field vectors will no longer be impeding on the shielding material in the worst-case orientation. Figure 4-17 shows the effect of changing the plane of rotation. A third set of source coils was added, shifting the plane of rotation to 45° relative to the *z*-axis. This reduced the shielding factor to 4.9dB at the same point in the system. Several methods for accomplishing field vector rotation outside of a single plane (omni-directional field generation) were discussed in Chapter 3.

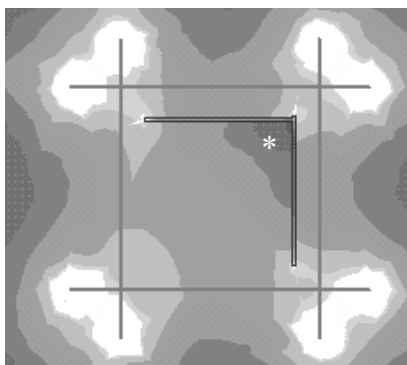


Figure 4-16 *rms* magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors making a 360° rotation in the *xy*-plane.

$$\alpha^* = 13.9 \text{ dB}$$

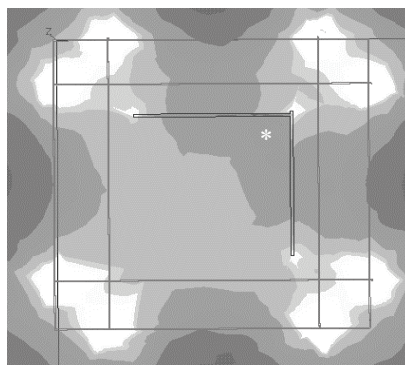


Figure 4-17 *rms* magnitude of the magnetic field around a plate of $\sigma=5.8e7$ with the field vectors making a 360° rotation in the plane 45° from the *xy*-plane and 45° from the *yz*-plane.

$$\alpha^* = 4.9 \text{ dB}$$

4.2.2 Highly conductive objects outside the operating volume

In addition to the shielding of the source field due to conductive objects located within the operating volume, shielding can also occur when the system is located in the vicinity of large objects containing conductive elements (such as walls, floors, ceilings, etc). These objects can cause a reduction in field strength in a significant portion of the operating volume, and the effect can render a system useless if the distance between them and the object is not increased. The effect of these materials on the source field is easily examined using the method of images [2-2], [4-7].

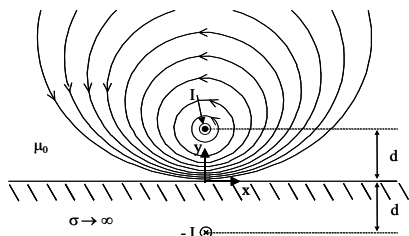


Figure 4-18 Lines of constant vector potential due to a line current a distance d from an infinitely conductive plane. Image current is shown

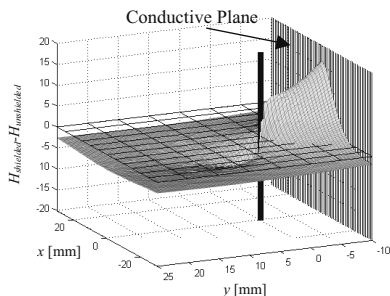


Figure 4-19 Magnetic field intensity around a 1A line current at 10mm above a perfectly conductive plane. Areas above the zero plane indicate areas where the field strength is increased by the addition of the plane. Areas below the zero plane indicate areas where the field strength is decreased by the addition of the plane

The influence of the conducting plane on the magnitude of the field created by the line current is shown in Figure 4-19. The z -axis represents the difference in percent between the magnitude of the magnetic field vectors with and without the addition of the conducting plane. The imaginary plane at $z=0$ is shown for reference as a grid. Figure 4-19 shows that while the presence of a conducting plane increases the field strength (relative to that when there is no conducting plane present) in a small area between the plane and the line current, the field strength in all other areas is depressed.

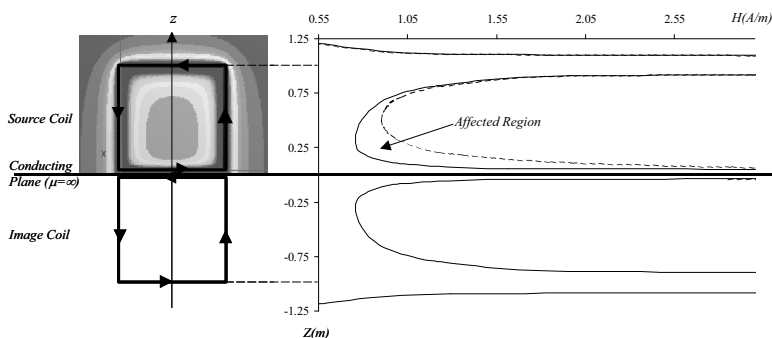


Figure 4-20 The effect of an infinitely conducting plane on the field distribution of a coil. The coil is 1m x 1m and carries 1A. The conducting plane is 1cm from the bottom of the coil is perpendicular to it. The image coil is also shown

This method can be used to provide an accurate worst-case field distribution for coil systems placed on, above, or next to conductive surfaces such as walls or floors. An example of this is shown in Figure 4-20.

The finite conductivity of conducting materials, the skin effect, non-ideal shapes, and other real-world factors must be taken into account. However, this method does provide a simple, reasonable estimate and is often highly valuable in understanding the influence of these materials on field distribution.

4.3 Shielding by highly permeable materials

The magnetic field can also be attenuated when a magnetic material of high permeability ($\mu \gg 1$) and sufficient cross-sectional area is placed inside the field. The effectiveness of such a material as a shield is inversely proportional to both its permeability and its cross-sectional area. Assuming that the field is uniform over the cross section of the shielding material, its reluctance is determined by

$$\mathfrak{R}_m = \frac{\ell_m}{\mu S} \quad (4-12)$$

Where ℓ_m is the length of the material, μ is its permeability, and S is its cross-sectional area.

Because the field created by the source coils will be much lower than 1mT (under normal operating conditions the field in the center of the system is expected to be lower than 50 μ T), only the initial relative permeability μ_i is of importance in examining the effect of a material's permeability on its shielding properties. Initial relative permeability is generally defined as the permeability of a material at an AC flux density of 1mT.

Because the density of the magnetic field in which shielding will occur will be low, it is possible to assume that the permeability of the shielding material will not change from its initial value (the slope of the B-H curve will not be significantly different from that at the origin). The permeability of most magnetic materials declines with frequency and therefore the effect of this type of shielding will tend to decrease with increasing frequency [4-3].

Most materials with extremely high initial permeability are nickel-iron alloys, such as mu-metal and permalloy. These materials are extremely expensive and are used only for intentionally shielding a device from a low-frequency magnetic field. Most of these materials have a relative permeability close to that of cold rolled steel ($\mu \sim 1000$) in the kHz range [4-4]. The presence of these materials inside the field is not likely and will not cause shielding effects beyond those of steel as long as the system frequency remains at or above the kHz range.

The ferromagnetic materials that may be present in the operating volume will most commonly be found in the form of alloys. Soft ferrites, which have high initial

permeability, may be found in the operating volume but generally do not exist in large volumes and their presence is not expected to create large areas of damped source field. Appendix VI lists several magnetic and non-magnetic materials that may be present in the operating volume.

4.3.1 Highly permeable objects inside the operating volume

Permeable materials inside the operating volume provide a low-reluctance path for the magnetic flux and can effectively “guide” the flux around the shielded area. This principle is illustrated in Figure 4-21.

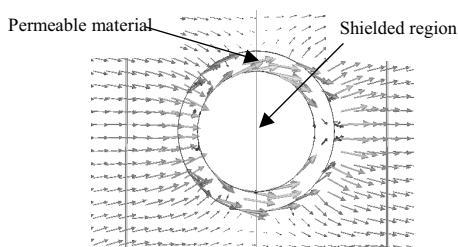


Figure 4-21 A hollow cylinder made of magnetic material with a high permeability reduces the field within the cylinder as the field remains inside the material offering a low-reluctance path

When parallel permeable plates are located within the source field, the attenuation of the field reaches its maximum when the field vectors are parallel to the plates and is at its minimum when the field vectors are normal to the plates. If field lines run parallel to two iron plates, the area between the plates will see a greatly decreased field value because the flux will tend to travel through the low reluctance path provided by the iron rather than through the shielded space between the plates. This effect can be seen in Figure 4-22 and in Figure 4-23. However, if the field lines run perpendicular to the same two iron plates, the effect of the plates on the field between them is negligible, as a low-reluctance path no longer exists. This can be seen in Figure 4-24 and Figure 4-25.

The field between the plates varies with the angle between the plates and the field imposed on them. If the field vectors are not constant but are rotating so that the angle between the plates and the field is constantly changing, the *rms* magnitude of the magnetic field between the plates over one period of the rotation will be significantly higher than that seen in the maximally shielded case. Figure 4-26 shows the magnitude of the field around the plates when the field vectors make a 45° angle with the plates, while Figure 4-27 shows the *rms* magnitude of the magnetic field between the plates over a single period of the rotation of the field vectors.

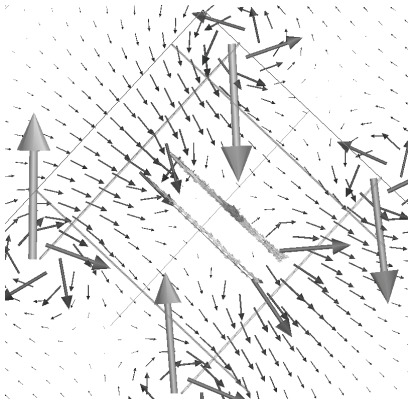


Figure 4-22 Magnetic field around two plates of $\mu_r=4000$. The arrows indicate direction and relative magnitude (arrow size is proportional to magnitude) of the magnetic field. $\alpha_{center}=34.3$ dB

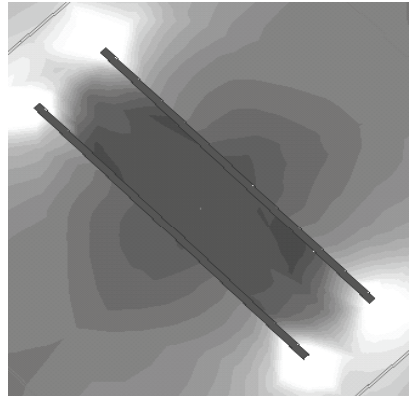


Figure 4-23 Magnitude of the magnetic field around two plates of $\mu_r=4000$ with the field vectors shown in Figure 4-22. $\alpha_{center}=34.3$ dB

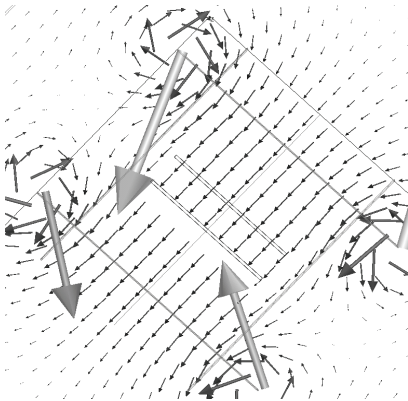


Figure 4-24 Magnetic field around two plates of $\mu_r=4000$. The arrows indicate direction and relative magnitude (arrow size is proportional to magnitude) of the magnetic field. $\alpha_{center}= -0.2$ dB

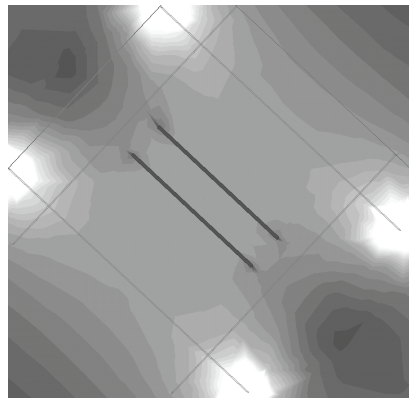


Figure 4-25 Magnitude of the magnetic field around two plates of $\mu_r=4000$ with the field vectors shown in Figure 4-24. Lighter colors indicate higher field values; note that the value in the center of the plates is nearly unchanged from that just outside of the plates. $\alpha_{center}= -0.2$ dB

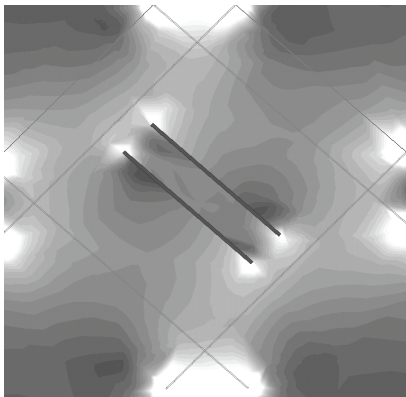


Figure 4-26 Magnitude of the magnetic field around two plates of $\mu_r=4000$ with the field vectors making a 45° angle with the plates. Lighter colors indicate higher field values; note that the value in the center between that of Figure 4-23 and Figure 4-25. $\alpha_{center}=2.79$ dB

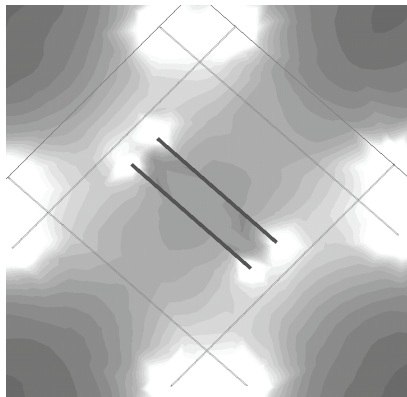


Figure 4-27 *rms* magnitude of the magnetic field around two plates of $\mu_r=4000$ with the field vectors making a 360° rotation. Lighter colors indicate higher field values; note that the value in the center between that of Figure 4-23 and Figure 4-25. $\alpha_{center}=0.74$ dB

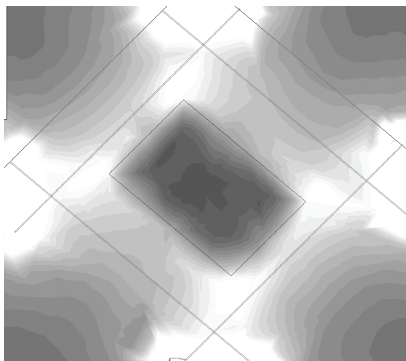


Figure 4-28 *rms* magnitude of the magnetic field around two iron plates positioned in planes parallel to the plane of rotation. $\alpha_{center}=21.74$ dB

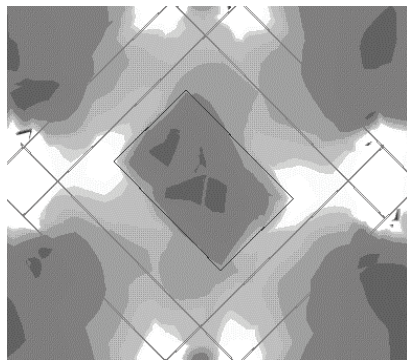


Figure 4-29 *rms* magnitude of the magnetic field around two iron plates with the field vectors making a 360° rotation in the plane 45° from the xy -plane and 45° from the yz -plane. $\alpha_{center}=11.12$ dB

A solution involving a rotating field vector would clearly be useful in dealing with the shielding problem created when a highly permeable magnetic material is placed inside the magnetic field. There are, however, cases in which a vector rotating in only one plane will do nothing to alleviate the problem of shielding. If the plates described above are positioned such that the field is rotating in a plane parallel to those in which the plates lay, the area between the plates will be shielded regardless of the position of the rotating vector. In this case the plates provide a low-reluctance path for the flux throughout the rotation of the field vector and the shielding factor will remain high between the plates. This can be seen in Figure 4-27 where the shielding factor at the center of the system is 21.74dB. If the field vector rotates outside of the xy -plane, the effect of the shielding material in Figure 4-26 will be reduced as the field vectors will no longer be impeding on the shielding material such a way as to provide a low reluctance path around the shielded area throughout the rotation of the field vector. Figure 4-28 shows the effect of changing the plane of rotation. A third set of source coils was added, allowing the plane of rotation to be shifted by 45° relative to the z -axis. This reduced the shielding factor to 11.12dB in the center of the system.

4.3.2 Highly permeable objects outside the operating volume

Permeable objects located outside of the system volume can cause a reduction in field strength in the areas of the operating volume located furthest away from the permeable objects, although the field strength near to the object is increased. The effect of these materials on the source field is generally not such that it prevents receiving coils from operating, and as in the case of conductive objects is easily examined using the method of images [2-2].

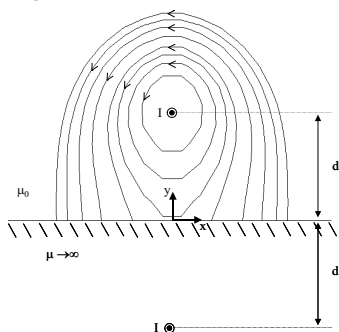


Figure 4-30 Lines of constant vector potential due to a line current a distance d from an infinitely permeable plane. The image current is also shown

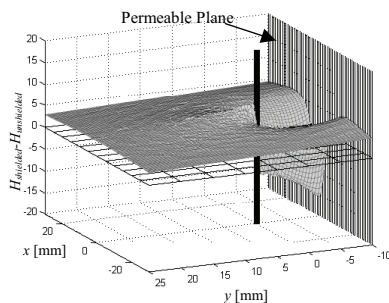


Figure 4-31 Magnetic field intensity around a 1A line current at 10mm above a perfectly permeable plane. Areas above the zero plane indicate areas where the field strength is increased by the addition of the permeable plane. Areas below the zero plane indicate areas where the field strength is decreased by the addition of the permeable plane

The influence of the permeable plane on the magnitude of the field created by the line current is shown in Figure 4-31. The z -axis represents the difference in percent between the magnitude of the magnetic field vectors with and without the addition of the plane. The imaginary plane at $z=0$ is shown for reference as a grid. Figure 4-31 shows that while the presence of a permeable plane decreases the field strength (relative to that when there is no permeable plane present) in a small area between the plane and the line current, the field strength in all other areas is increased.

This method provides an accurate worst-case field distribution for coil systems placed on, above, or next to permeable surfaces such as walls or floors. An example of this is shown in Figure 4-32.

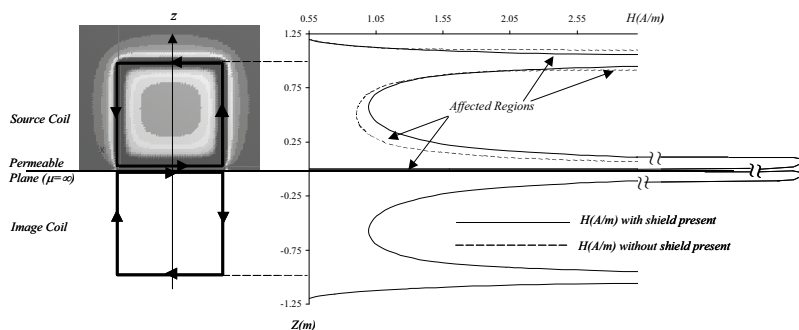


Figure 4-32 The effect of an infinitely permeable plane on the field distribution of a coil. The coil is 1m x 1m and carries 1A. The permeable plane is 1cm from the bottom of the coil is perpendicular to it. The image coil is also shown

When the permeability of the shielding object is known, the image current I_i can be replaced by

$$I_i = \left(\frac{\mu_r - 1}{\mu_r + 1} \right) I \quad (4-13)$$

where I is the current in the source coil [4-8]. This will provide a more accurate calculation of the field inside the operating volume.

4.4 Supplemental solutions to shielding problems

The previous sections have shown that the damping of the source field caused by conductive or permeable objects located within or near to the operating volume can be reduced by the introduction of a rotating magnetic field vector. Although this solution provides a mitigation of the effects of shielding, it does not eliminate them and some cases may exist where further measures must be taken to assure operation of load(s) on the receiver(s).

Several methods can be used to mitigate the problems associated with shielding without changing the method by which the field is generated by the source coils. These solutions focus primarily on canceling the effect of shielding due to walls or floors containing shielding materials. They are considered to be supplemental solutions to be used in special circumstances when the rotation of the source field is not sufficient to maintain adequate field strength in shielded regions. A number of these solutions are discussed in this section.

4.4.1 Incremental increase of the current applied to source coil(s)

If the field value present inside the operating volume is lower than expected because one or more source coils must be placed near to a highly conductive object, the current in the affected source coil can be increased in order to compensate for the field-damping effects of the conductive object.

This solution has several drawbacks. First, the power supplies providing current to the source coils must be designed to constantly handle more than the nominal operating current. Second, increasing the current in one or more of the source coils may increase the magnitude of the field near the coil (specifically between the coil and the conductive material, see Figure 4-19) beyond what is allowed by law (see Appendix I). However, this is a relatively simple solution that can be easily applied to systems in which operation at full power/field strength are not required during normal operation.

4.4.2 Additional coils carrying compensating currents

When coils are placed near to large conductive materials such as reinforced walls or floors, the shielding effects of these objects can also be negated by identifying the affected coil or coils and placing an identical coil on or near to the shielding surface (wall, floor etc.). This additional coil must carry current flowing in the same direction as in the affected coil, and the amount of compensation provided by this solution is directly proportional to the amount of current in the compensating coil and on its position relative to that of the shield and the source coil system.

This provides a solution that is relatively inexpensive initially but may become costly when operated over time because of the need to constantly provide current to an additional coil. For a system with four source coils, the addition of a single compensating coil carrying current identical to that in the source coils will increase the power consumption of the system by 25%. The addition of compensating coils also requires the use of additional active components that will naturally incur significantly higher maintenance costs over time than would a passive system. Finally, there is a limit to how close the compensating coil can be to the source coil system. As the distance between the source coil system and the compensating coil decreases, the field strength between and around the two coils increases and the effect is indistinguishable from an increase in the current in the source coil as described above. If the source coil system is too close to the shielding material, there may not be enough room to add a compensating coil without exceeding the highest permitted field strength magnitude.

This problem is magnified by the fact that the closer the source coil system is to the shielding material, the higher the current in the compensating coil must be to compensate for the effect of the shield. However, if the source coil system is far enough from the shielding material, a compensating coil(s) will not be necessary. Therefore this system will be effective only for a narrow range of distances between the source coil system and the shielding material.

This solution will be most effective as a temporary or short-term solution or as a solution for systems that must be regularly moved or altered in configuration.

4.4.3 Compensation of conductive shielding with permeable materials

In cases where the current in one or more of the source coils cannot be increased, it is possible to install a layer of permeable material over the surface of any conducting object or material in order to negate the damping effects of the conductive shield. Comparing Figure 4-18 to Figure 4-30 it is clear that permeable and conductive materials have opposite effects on flux lines and therefore also on field distribution when in the vicinity of a current-carrying wire.

This solution, while initially very costly, has several advantages. It is passive and therefore incurs very few maintenance costs, if applied correctly it can compensate completely for the effect of a conductive shield, it does not require the modification of any components or controls in the system, and it does not draw additional power from the system and therefore does not increase operating costs over time. However, due to the high cost of ferrite materials this solution is practical only in cases where, for example, the system will be installed in the vicinity of a highly conductive wall and is expected to remain in that setting for a long period of time.

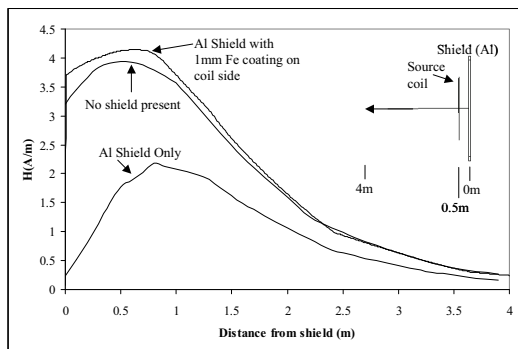


Figure 4-33 An aluminum ($\mu=1.0$, $\sigma=3.8e7$) shield is placed 0.5m from a single square source coil 3m on a side and carrying 15A. The field strength along the axis of the coil is compared to that without the shield present and to that when the shield is covered with a 1mm thick sheet of ferrite ($\mu=1000$, $\sigma=0.01$)

Figure 4-33 shows an example of this solution. An aluminum ($\mu=1.0$, $\sigma=3.8e7$) shield is placed 0.5m from a single square source coil 3m on a side and carrying 15A. The field strength along the axis of the coil is compared to that without the shield present and to that when the shield is covered with a 1mm thick sheet of ferrite ($\mu=1000$, $\sigma=0.01$). The addition of the ferrite counteracts the effect of the aluminum shield, leaving the field strength along the axis of the coil greater than or equal to its value in the unshielded case.

Permeable materials for this application would be chosen based on the material's cost, initial relative permeability at system operating frequency, and characteristics befitting the circumstances, such as durability and ductability.

4.4.4 Ferrite rod antenna

In the event that a receiving coil is placed in a shielded situation for which there is no reasonable method by which it can receive sufficient power at its location, a small tuned ferrite rod antenna can be placed in the vicinity of the receiver in order to concentrate and direct flux lines to the receiver, thereby increasing its available power [2-1]. The antenna is shown in Figure 4-34. The exact size and shape of this antenna will depend on the shielding conditions that it is designed to circumvent (position of the receiving coil, availability of access to locations in the immediate vicinity of the receiving coil, field strength available to the antenna, etc). In general in order to maximize the efficiency of the antenna, the rod should have a high Q value at the operating frequency, which in the kHz range requires a material with a permeability of approximately 1000-2000. The Q factor is described by

$$Q_{rod} = \frac{\omega L_{rod}}{R_{loss} + R_{winding}} \quad (4-14)$$

where ω is the operating frequency of the system, R_{loss} is the core loss resistance, $R_{winding}$ represents resistive losses in the winding, and L_{rod} is defined as

$$L_{rod} = \mu_0 \mu_{rod} n^2 \frac{A}{l} \quad (4-15)$$

where A is the cross-sectional area of the rod, and l is its length. Position of the winding on the rod as well as the ratio of coil length to core length also influence the value of rod inductance by altering the effective length l of the rod [4-9]. The effect of the core dimensions on the flux concentration in the antenna is shown in Figure 4-35. Figure 4-35 shows that the effective permeability ($\mu_{rod} = \phi_{core} / \phi_{air}$) of the ferromagnetic core increases with l/d , but (4-15) shows that the inductance also varies in the inverse with A/l . The result is that L_{rod} decreases slowly with length and increases at a much greater rate with core diameter [4-9].

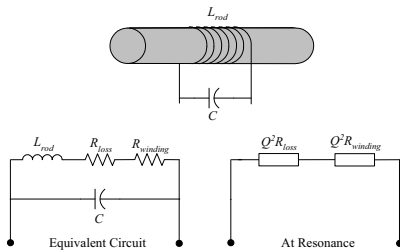


Figure 4-34 Tuned ferrite rod antenna

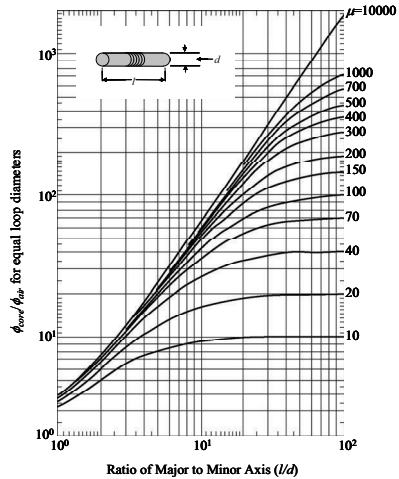


Figure 4-35 Flux concentration in a short loop for a ferromagnetic core as a function of the ratio of major to minor axis [4-10]

These antennas are relatively cheap and easy to manufacture and can be placed inside or near to small, enclosed areas which are otherwise shielded from the source field. They could be used, for example, to supplement the source field for a receiver which must be almost completely encased in a conductive or permeable material.

4.5 Conclusions

A rotating magnetic field vector will act to mitigate the effects of shielding materials located within the operating volume. As was shown in 4.2.1 and 4.3.1, constantly varying the angle between the field vector and the shielding object can provide an *rms* value of the magnetic field in the shielded region above that which exists in an unshielded uni-directional source field. This mitigates the effects of shielding even when the position of the receivers is unknown or varying with respect to the shield.

There are cases (Figure 4-16 and Figure 4-28), where a field vector rotating in a single plane will not reduce the attenuation of the source field caused by the presence of the shield. Most such cases can be helped with a field rotating in multiple planes (an omni-directional field). This field will provide the maximum possible field-related mitigation of the problems of shielding. In cases when the damping of the source field due to shielding cannot be adequately reduced, supplementary solutions exist which will allow the system to remain operational even in shielded regions. Generation of the omni-directional field was discussed in Chapter 3. The omni-directional system will be developed further throughout this work, and results of experimental shielding tests with an omni-directional field are presented in Chapter 7.

5 Equivalent Circuit Representation

This chapter will define the coupling between the source and receiver systems and develop equivalent models which will predict the behavior of a receiver located at any point within the operating volume.

5.1 System layout

A basic block-diagram of the power transmission system is shown in Figure 5-1. The source and receiver side power converters are further investigated in Chapter 6.

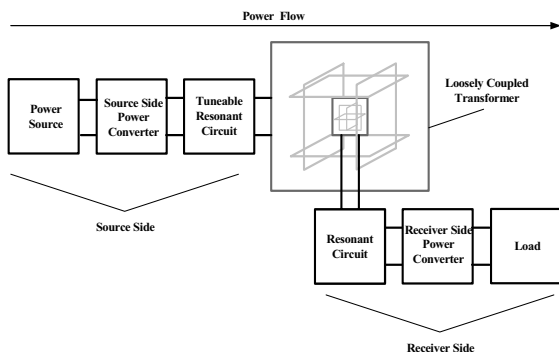


Figure 5-1 Block diagram of a wireless power system

5.2 System description

The coils which form the loosely coupled transformer (source coils being equivalent to primary coils and receiving coils being equivalent to secondary coils) can be represented as a system of loosely coupled inductors with the self-inductances L_s and L_r of each coil coupled with each other coil by $k_{xx'yy'}$, $k_{xx'sy'}$, and/or $k_{rx'xy'}$ (where s represents a source coil, r represents a receiving coil, and x and y represent the x^{th} and y^{th} coil). However, each time that a coil is moved the entire set of coupling factors relating to that coil must be recalculated. This is a complicating factor because the system will be designed such that movement of receiving coils can occur without restriction. The system becomes extremely complex as the number of elements is increased.

The number of coupling factors N_k that must be calculated is given by

$$N_k = C_s C_r + \frac{C_s!}{2(C_s - 2)!} + \frac{C_r!}{2(C_r - 2)!} \quad \text{with } C_s, C_r > 2 \quad (5-1)$$

where C_r is the number of receiving coils and C_s is the number of source coils in the system.

Figure 5-2 shows the coupling factors present in a system of two source coils and three receiving coils.

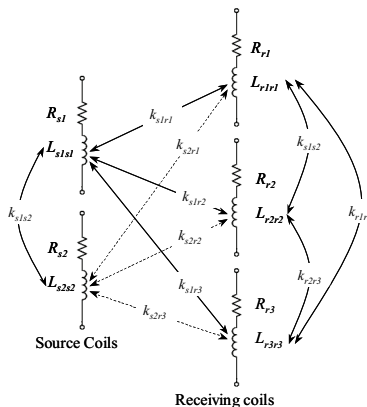


Figure 5-2 Coupling between source and receiving coils

The terminal characteristics of the system of coupled inductors are dependent upon the location and orientation of all of the coils relative to each other; for moving objects both variables are a function of time. Hence they can generally be described by

$$\begin{bmatrix} v_1(t) \\ v_2(t) \\ \vdots \\ v_i(t) \\ \vdots \\ v_n(t) \end{bmatrix} = \begin{bmatrix} L_{11}(t) & L_{12}(t) & \cdots & L_{1k}(t) & \cdots & L_{1m}(t) \\ L_{21}(t) & L_{22}(t) & \cdots & L_{2k}(t) & \cdots & L_{2m}(t) \\ \vdots & \vdots & & \vdots & & \vdots \\ L_{i1}(t) & L_{i2}(t) & \cdots & L_{ik}(t) & \cdots & L_{im}(t) \\ \vdots & \vdots & & \vdots & & \vdots \\ L_{n1}(t) & L_{n2}(t) & \cdots & L_{nk}(t) & \cdots & L_{nm}(t) \end{bmatrix} \cdot \frac{d}{dt} \begin{bmatrix} i_1(t) \\ i_2(t) \\ \vdots \\ i_i(t) \\ \vdots \\ i_n(t) \end{bmatrix} + \begin{bmatrix} R_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & R_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & R_{ik} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & R_{nn} \end{bmatrix} \begin{bmatrix} i_1(t) \\ i_2(t) \\ \vdots \\ i_i(t) \\ \vdots \\ i_n(t) \end{bmatrix} \quad (5-2)$$

For a fixed three-dimensional orthogonal reference system the self inductances ($i=k$) and mutual inductances ($i \neq k$) are a function of

$$L_{ik}(t) = \{a(t), b(t), c(t), \psi(t), \theta(t), \phi(t)\} \quad (5-3)$$

where a, b, c define the position in the system, and ψ, θ, ϕ define the orientation with respect to the x -, y -, and z -axes in the system. This system of equations can also incorporate the effects of conductive elements that influence the system due to shielding of the source field caused by induced eddy currents (this is explained in Chapter 4). These objects are represented by setting $v_i=0$.

The full set of system equations quickly becomes very complex and requires extensive computations. With the following simplifications the number of equations can be reduced considerably without substantially impairing the accuracy of the analysis.

1. Neglecting potential proximity effects that can occur when two or more coils are in very close proximity to each other, the resistance R of the coils can be considered constant for any given operation frequency.
2. As discussed in Chapter 2 each receiver is actually comprised of three coils wrapped around a ferrite core. Ideally, the three coils comprising one receiver are in space quadrature and therefore are not coupled to each other ($k_{pxy}=0$). Practical measurements showed coupling between these three coils of up to 0.6%, which will be neglected.
3. Source coil pairs placed in space quadrature will also be treated as ideally placed and coupling between them will therefore be neglected ($k_{xxy}=0$).
4. The analysis can be further simplified by ignoring the coupling between receivers that are not located within one side-length of the ferrite core of each other as the coupling between cube shaped receivers separated spatially by a distance greater than this has been shown in preliminary tests and simulations to have a negligible effect on the overall performance of the system [5-1].

5.3 Methods of calculation of coil self and mutual inductances

In general, magnetic inductance (both self and mutual) is a function of the geometric dimensions of the inducing coil and the magnitude of flux linkage is a function of the geometric dimensions of the linking coil [5-2]. The inducing coil is defined as the coil creating flux and the linking coil is the coil that defines the surface area through which the flux created by the inducing coil passes.

There are several established formulas for calculating the self- and mutual inductances of conductors [5-2]-[5-5] and [5-15]-[5-18]. Reference [5-2] is the most widely used and provides the most simplified methods of calculation. Mutual inductances of more complex coil arrangements, such as those seen in the system described here require additional calculations and cannot be reduced to simple formulas in most cases.

5.3.1 Flux-linkage per Ampere

The most direct method for finding the self and mutual inductances of conductors involves finding an expression for the flux density created when a current flows in a coil and integrating it over the area in which the flux linking the coil in question is present. The inductance of the coil is then calculated as flux linkage per ampere. The method is used for both internal and external inductances, the inductance of the coil being the sum of these two values. This method is straightforward, and can be simplified by finding equations describing the inductance of ideal elements such as

straight conductors and summing them to form equations describing coils of more complex shapes [5-2].

5.3.2 The Neumann formula

The Neumann formula for the mutual inductance of two circuit elements is given by

$$L_{12} = \frac{\mu_0}{4\pi} \oint \oint \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|} \quad (5-4)$$

where $r_2 - r_1$ is the distance between the two circuit elements and l_1 and l_2 are the lengths of circuit element 1 and 2, respectively. This formula can be derived directly from the definition of inductance as flux-linkage per ampere (the derivation can be found in Appendix AVII). Using the Neumann formula gives a general expression for the mutual inductance between two circuit elements, but in almost every case, the integration must be performed numerically. Weber's formula, Maxwell's formula, and Graneau's formula are equivalent [5-7].

5.4 Self-inductance formulas

The self-inductance of a coil that is comprised of straight elements can be described by the sum of the self-inductance of each straight wire and the mutual inductances of all of the possible wire pairs. The inductance of a coil can be found by starting with the self inductance of a straight single wire given by [5-2].

$$L = \frac{\mu_0 l}{2\pi} \left[\ln \left(\frac{2l}{\rho} - \frac{3}{4} \right) \right] \quad (5-5)$$

where ρ is the radius, l is the length of the conductor, and L is the inductance of the conductor in henries.

Stranded (or Litz) wires will comprise both the source and receiving coils in the system. As the impedance of any one strand in the wire will not involve the same mutual inductances as any other strand, a rigorous solution to the problem is highly complex. However, a good approximation of the inductance of a stranded wire can be made by finding the inductance of a solid wire having the same cross sectional area of copper. This approximation will be the upper limit for the inductance of the stranded wire [5-2].

This approximation does not account for skin- or proximity effects. Calculating the change in inductance due to these effects is beyond the scope of this work. However, [5-2] shows that the maximum fractional change in inductance caused by skin and proximity effects can be approximated by

$$\left(\frac{\Delta L}{L}\right)_{f \rightarrow \infty} \approx \frac{1}{4 \ln \frac{2l}{\rho} - 3} \left[1 + 4 \frac{\rho^2}{d^2} + 6 \frac{\rho^4}{d^4} \right] \quad (5-6)$$

where d is the distance between the centers of the wires.

The mutual inductance between opposite sides of a rectangular coil (parallel conductors of the same length) is given by [5-2] as

$$M_{p_cond} = \frac{\mu_0 l}{\pi} \left[\ln \left(\frac{l}{d} + \sqrt{1 + \frac{l^2}{d^2}} \right) - \sqrt{1 + \frac{d^2}{l^2}} + \frac{d}{l} \right] \quad (5-7)$$

Where l is the length of the conductor, d is the distance between the parallel conductors, and M_{p_cond} is the mutual inductance in henries. In this case the radius of the wires can be neglected because it is very small relative to d [5-2].

Summing the self-inductance (given by (5-5)) of each of the four wires and the mutual inductances (given by (5-7)) between each of the possible wire pairs, the self inductance of a rectangular coil is described by [5-2] as

$$L_{rec} = \frac{\mu_0 n^2}{\pi} \left[\begin{aligned} & l \ln \frac{2l}{\rho} + w \ln \frac{2w}{\rho} + 2\sqrt{l^2 + w^2} - l \sinh^{-1} \frac{l}{w} \\ & - w \sinh^{-1} \frac{w}{l} - 2(l + w) + \frac{1}{4}(l + w) \end{aligned} \right] \quad (5-8)$$

where l and w are the lengths of the sides of the rectangle, n is the number of turns in the coil, and L_{rec} is the inductance of a rectangular coil in henries.

Equation (5-8) provides an approximation of the self-inductance of rectangular coils. However, [5-8] shows that this approximation can suffer from a significant rate of error, especially when the rectangle is small (such as in the receiving coils). In order to provide a reasonable estimate of the self-inductance of a single coil, the result obtained from (5-8) can be averaged with the results obtained using other methods of calculation such as those found in [5-4], [5-16], and [5-18].

5.5 Effect of core magnetization on receiving coil inductance

The nature of the system (see Chapter 2 and Appendix I – International Standards – Magnetic Field Strength Limits) requires that the density of the magnetic field within the operating volume will remain quite small (less than 60 μT in almost all cases and less than 25 μT in most cases). Because of this, the core material around which the receivers are wound will never reach (or even approach) saturation and all operation will occur near the origin of the B - H curve. The initial relative permeability (the relative permeability of a material at very small applied magnetic fields) is therefore of

utmost importance when choosing a core material. Initial permeability for all ferromagnetic materials is a function of frequency and temperature, which clearly must be considered when choosing a core material. High nickel alloys have a high initial permeability but low saturation, whereas the low nickel alloys are lower in initial permeability but higher in saturation induction [5-6]. Manganese Zinc cores are useful due to their high initial permeability (~2000), frequency range of up to 1000kHz, and relatively low cost (as compared to Nickel-Iron-Molybdenum) [5-9]. Core material characteristics can be found in Appendix VIII.

Equation (5-8) provides an approximation of the inductance of the source coils with an error of approximately 1.5%. While the receiving coils are also rectangular in shape, they are wrapped around a ferrite core. The inherent high initial permeability of the core provides a low reluctance path for the flux created by the source coils, thereby concentrating the flux lines within the core and increasing the density of the magnetic field. A concentration of flux in the core (when the core material has $\mu_r > 1$) will increase the inductance of the coil wrapped around that core. The estimate of coil self inductance must be modified due to a flux concentration or demagnetization factor D_m , which accounts for this effect. This factor is based on the construction of the coil, core shape, length-to-diameter ratio of the core, ferrite material characteristics, and the position of the winding relative to the core. Because of these factors, it is generally impossible to calculate an exact value for the demagnetization factor [5-10]. The relationship between effective permeability of the core and the demagnetizing factor is described by [5-11] as

$$\mu' = \frac{1}{\frac{1}{\mu} + \frac{D_m}{4\pi}} \quad (5-9)$$

where μ is the true permeability of the core material and μ' is the effective permeability of the core material.

Although the exact value of D_m is extremely difficult to calculate, [5-10] shows that D_m for prisms with two sides of equal length can be approximated using the value of D_m for a cylinder. The demagnetization factors of cylinders are well known. Reference [5-11] shows that for cylinders having long and short axes of equal length (a good approximation of the ferrite cube used in this system) and a relative permeability in the range of a few thousands, the demagnetization factor is such that the effective permeability of the coil when wrapped around a core is in the range of 3 to 4 times that with an air core. This factor is, of course, lower for materials with initial permeability lower than a few thousands and higher for materials with very high initial permeability.

Although the above discussion provides a reasonable estimate for D_m and μ' , a much more accurate value for effective permeability can be obtained by measuring the inductance of a coil with and without its core and then applying

$$\mu' = \frac{L_{coil+core}}{L_{coil}} \quad (5-10)$$

to find the exact value of apparent permeability. The apparent permeability, μ' , is then used in place of relative permeability when calculating magnetic field intensity in the receiving coils.

5.6 Calculation of mutual inductances

The mutual inductance between source and receiving coils in the system can be found using (5-4). The integral can only be solved numerically. Other methods are also highly computation intensive due to the difference in dimension between source and receiving coils.

As an alternative to methods involving complex integration, the mutual inductance, M_{sr} , between one source coil and one receiving coil (a receiver is comprised of three orthogonal receiving coils) can be calculated by integrating the flux density $\bar{B}_s$ created when current passes through the source coil over the surface r bounded by the receiving coil. This method is made simpler because the flux density at the receiving coil can be assumed to be constant over the small area bounded by the receiving coil.

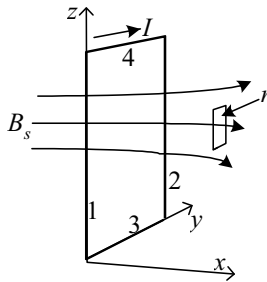


Figure 5-3 Flux density B_s passing through the surface r bounded by the receiving coil

The magnetic field density at any point $P(x,y,z)$ in the near field region when a current is applied to a rectangular current carrying coil lying in the yz -plane (Figure 5-3) where wires 1 and 2 are parallel to the z -axis and wires 3 and 4 are parallel to the y -axis can be found by multiplying (3-1) by the permeability of free space, μ_0 .

$$\begin{aligned}
\vec{B}_{rect} = & \mu_0 \hat{a}_x \left(-\sin \phi_1 \frac{I_1}{2\pi r_1 \sqrt{I_1^2 + r_1^2}} + \sin \phi_2 \frac{I_2}{2\pi r_2 \sqrt{I_2^2 + r_2^2}} \right. \\
& \left. - \cos \phi_3 \frac{I_3}{2\pi r_3 \sqrt{I_3^2 + r_3^2}} + \cos \phi_4 \frac{I_4}{2\pi r_4 \sqrt{I_4^2 + r_4^2}} \right) \\
& + \mu_0 \hat{a}_y \left(\cos \phi_1 \frac{I_1}{2\pi r_1 \sqrt{I_1^2 + r_1^2}} - \cos \phi_2 \frac{I_2}{2\pi r_2 \sqrt{I_2^2 + r_2^2}} \right) \\
& + \mu_0 \hat{a}_z \left(\sin \phi_3 \frac{I_3}{2\pi r_3 \sqrt{I_3^2 + r_3^2}} - \sin \phi_4 \frac{I_4}{2\pi r_4 \sqrt{I_4^2 + r_4^2}} \right)
\end{aligned} \tag{5-11}$$

Integrating the magnetic field density at the location of the receiving coil over the surface of the receiving coil to find the flux linking the coil and assuming that the field is uniform over the surface bounded by the receiving coil

$$\phi_{sr} = \int_{S_r} \vec{B}_s \cdot d\vec{S}_r \tag{5-12}$$

where the subscripts s and r refer to the source and to the receiving coils, respectively, and B_s is the density of source field at the receiver, $\vec{B}_s = \mu' \vec{B}_{rect} = \mu' \mu_0 \vec{H}_{rect}$ gives

$$\phi_{sr} = A_r |\vec{B}_s| \cos \alpha \tag{5-13}$$

where A_r is the area encompassed by a receiving coil, α is the angle between the unit vector normal to the surface bounded by the receiving coil and the field vector created by the source coil.

Flux linkage Λ_{sr} is defined as

$$\Lambda_{sr} = \phi_{sr} n_r \tag{5-14}$$

where n_r is the number of turns on the receiving coil.

The mutual inductance between the source and receiving coil is

$$M_{sr} = \frac{\Lambda_{sr}}{I_s} \tag{5-15}$$

where I_s is the current in the source coil.

Finally, substituting the magnetic field intensity H_{rect} at the receiving coil gives

$$M_{sr} = \frac{n_r A_r \mu_0 \mu' H_{rect}}{I_s} \cos \alpha \quad (5-16)$$

Note that equation (5-16) is for the coupling between one source coil and one receiving coil.

5.7 Coupling factors

The coupling factor k between the source and receiving coils is defined as

$$k = \frac{M_{sr}}{\sqrt{L_s L_r}} \quad (5-17)$$

Where L_s and L_r are the self-inductances of the source and receiving coils (given by equation (5-8)) and M_{sr} is the mutual inductance between the source and receiving coils (given by equation (5-16)).

Figure 5-4 shows the coupling factor in percent between one source and one receiving coil as a function of the distance between the source and receiving coils. The source coil is 3 meters on each side and is lying on the zy -plane. The receiving coil is 19 millimeters on each side and is moved along the three lines parallel to the x -axis. The receiving coil is aligned with the source coil in the zy -plane such that the unit vectors normal to the coils are parallel.

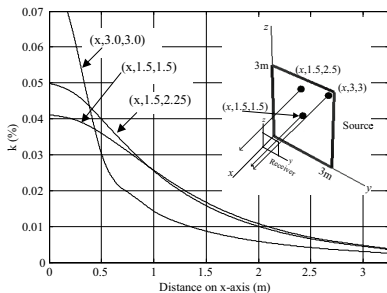


Figure 5-4 Coupling factor in percent between one source and one receiving coil. The source coil is 3 meters on each side, is lying on the zy -plane, and has one turn. The receiving coil is 19mm on each side, has 24 turns, and its position is varied along three lines parallel to the x -axis

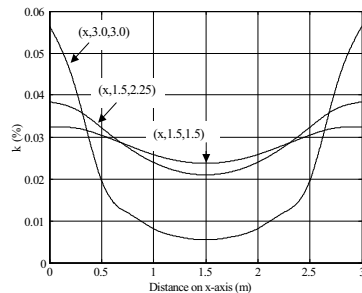


Figure 5-5 Coupling factor in percent between two source and one receiving coil. The source coils are 3 meters on each side and lay on the zy -plane at a distance of 3 meters from each other. The receiving coil is 19mm on each side, has 24 turns, and its position is varied along three lines parallel to the x -axis

A coupling factor inherently exists between only two elements (i.e. one source and one receiving coil). However, if the source coils are considered to be a single coil with two windings placed at different locations, a coupling factor can be found. Figure 5-5 shows the coupling factor between two source coils and one receiving coil. The source coils are 3 meters on each side and lay on the zy -plane at a distance of 3 meters from each other. The receiving coil is 19mm on each side and is moved along the three lines shown parallel to the x -axis. In this case, L_s is the inductance of both source coils and M_{sr} is calculated using the field strength created by both coils.

A comparison of Figure 5-4 and Figure 5-5 shows that the coupling factor is substantially reduced when a second source coil is added and the receiving coil is near to the source coils. This is because the total flux created by the source coil(s) is increased with the addition of the second coil, but the field strength (and thus the flux) available to a receiving coil at a point near to one of the source coils is not substantially increased by the addition of the second source coil. This effect is reduced as the receiving coil moves toward the center of the two-source-coil system. The coupling factor is, of course, symmetrical about the center point of the system shown in Figure 5-6 where it tends toward zero in the single coil system.

Figure 5-6 shows the coupling factor between two sets of source coils and one receiving coil. The first set of source coils are 3 meters on each side and lay on the zy -plane at a distance of 3 meters from each other. The second set of source coils is identical to the first but is in space and phase quadrature with the first set (forming a bi-directional system). The receiving coil is 19mm on each side and is located at the coordinates shown. The source coils are again assumed to form a single coil with spatially separated windings. In this case, L_s is the inductance of one set of source coils and M_{sr} is calculated using the field strength created by both coil sets. As was shown in Chapter 3, the field strength created by a bi-directional system is $\sqrt{2}$ times that created by a uni-directional system (Figure 5-5). This comes at the expense of doubling the number of source coils required and therefore does not substantially increase the coupling factor between source and receiving coils in the center of the system over that of a uni-directional system. Bi- and omni-directional systems are used due to their ability to mitigate the problems associated with shielding of the source field (this is thoroughly addressed in Chapter 4) and their ability to create an increased field strength in the center of the system without surpassing legal limits on the field strength in the area directly surrounding each source coil.

Figure 5-4, Figure 5-5, and Figure 5-6 show the coupling factor for fully aligned systems (a single receiving coil with a normal vector parallel to the field vector created by the source coil system). Figure 5-7 shows the effect of rotation of a single receiving coil with respect to the source coils in a bi-directional system. The receiving coil rotates about the z -axis. k_x is the coupling factor between the receiving coil and the source coils lying on the x -axis, k_y is the coupling factor between the receiving coil and the source coils lying on the y -axis. The total coupling factor for the system, k is calculated by representing the two sets of source coils as a single coil coupling with the receiving coil.

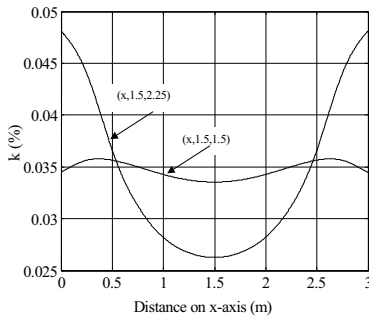


Figure 5-6 Coupling factor between two sets of source coils and one receiving coil. The first set of source coils are 3 meters on each side and lay on the xy -plane at a distance of 3 meters from each other. The second set of source coils is identical to the first but is in space and phase quadrature with it. The receiving coil is 19mm on each side, has 24 turns, and its position is varied along three lines parallel to the x -axis

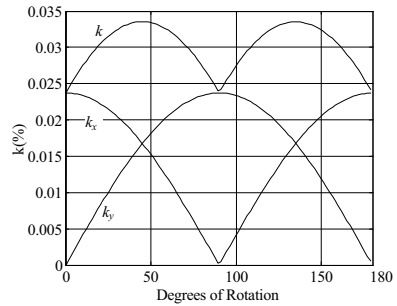


Figure 5-7 Coupling factor in percent for one receiving coil at the center (1.5,1.5,1.5) of the system of Figure 5-6. The receiving coil rotates about the z -axis. k_x is the coupling factor between the receiving coil and the source coils lying on the x -axis, k_y is the coupling factor between the receiving coil and the source coils lying on the y -axis, and k is the total coupling factor for the system

The coupling factor between three sets of source coils and one receiving coil can also be found using the method described above. The coupling factor in this case is not suggestive of the power available at the receiving coil because of the addition of a second or third frequency to the currents in the source coils. As all power will not be transferred at the resonant frequency of the receiving coils, the coupling factor between source and receiving coils will be less representative of the power available to the load than will the Δf between the frequencies of the system and the shape of the resonant curve at the receiving coils. The effect of the second operating frequency on the power available to the load is described in Chapter 6.

5.7.1 Simplifications and approximations of coupling factors

The formula describing the coupling factor between a source and a receiving coil is quite cumbersome. However, some simplifications can be made to make the equation easier to use.

Equation (5-8), describing the self-inductance of rectangular coils, can be simplified using an approximation for the inductance of polygons of round wire [5-2]. The inductance of a polygon of wire with perimeter p and area A is approximated by

$$L_{rec} \approx \frac{\mu_0 p n^2}{2\pi} \left[\ln \left(\frac{2p}{\rho} \right) + .25 - \ln \left(\frac{p^2}{A} \right) \right] \quad (5-18)$$

If the receiving coil is assumed to be near to the axis of a rectangular source coil, equation (5-16) can be approximated by

$$M_{sr} \approx n_r A_r \mu' \mu_0 \frac{n_s l_1 l_2}{2\pi \sqrt{r^2 + l_1^2 + l_2^2}} \left[\frac{1}{r^2 + l_1^2} + \frac{1}{r^2 + l_2^2} \right] \quad (5-19)$$

where l_1 and l_2 are the side lengths of the coil and r is the perpendicular distance from each of the coil wires to the point $P(x, y, z)$.

The error in (5-19) increases as the receiving coil is moved further off axis. The error is proportional to the difference between the field intensity at the actual position of the receiving coil and the field intensity at the position on the axis of the source coil that is closest to the actual position of the receiving coil.

Substituting (5-18) and (5-19) into (5-17) to find an approximation of the coupling factor gives

$$k_{sr} \approx \frac{n_r A_r \mu' \mu_0 \frac{n_s l_1 l_2}{2\pi \sqrt{r^2 + l_1^2 + l_2^2}} \left[\frac{1}{r^2 + l_1^2} + \frac{1}{r^2 + l_2^2} \right]}{\sqrt{\frac{\mu' \mu_0 p_s p_r}{4\pi^2} \left[\left(\ln \left(\frac{2p_s}{\rho_s} \right) + .25 \right) \left(\ln \left(\frac{2p_r}{\rho_r} \right) + .25 - \ln \left(\frac{p_r^2}{A_r} \right) \right) \right] + \ln \left(\frac{p_s^2}{A_s} \right) \left(\ln \left(\frac{p_r}{A_r} \right) - .25 - \ln \left(\frac{2p_r}{\rho_r} \right) \right) }} \quad (5-20)$$

Although the coupling factor k_{sr} between the source and receiving coils is of central importance, an accurate representation of the system will also include coupling factors between the source coils, k_{ss} , and between the receiving coils, k_{rr} .

5.7.2 Coupling factors between like coil types

Finding the mutual inductance between two source coils is difficult using the method of integrating flux density because the flux density created by a source coil cannot be assumed to be constant over the surface bounded by another source coil, as the area of the coils are generally equal or nearly equal and the coils are in close proximity to each other (in the near-field). It is also difficult to find an equation describing the mutual inductance using the method of summation of basic straight elements due to the number of possible combinations of mutual inductances between each of these straight elements. This is especially true when source coil cells are assembled modularly, greatly increasing the number of combinations of straight elements. Equations found using either of these methods are extremely large and nearly impossible to manipulate.

It is possible to find the inductance between any two source coils by numerically integrating the flux density created when current flows in one of the source coils over

the surface bounded by the other coil. The coupling factor between two source coils can then be approximated by

$$k_{s1s2} \approx \frac{n_{s2} \int_{S_{s2}} \vec{B}_{s1} \cdot d\vec{s}_{s2}}{\sqrt{L_{s1}L_{s2}}} \quad (5-21)$$

Where L_{s1} is the self-inductance of source coil 1, n_{s2} is the number of turns on source coil 2, and L_{s2} is the self-inductance of source coil 2.

Source coil systems generally consist of sets of coils operating in phase quadrature. If the ideal case (exactly 90° of separation) is assumed, coupling between source coils is non-existent.

An equation for the coupling between two receiving coils is also difficult to derive by either method proposed in section 5.2.1. Although flux density can normally be considered constant over the relatively small area bounded by the receiving coil, this is not the case when two receiving coils are placed in close proximity to each other. As the case of two receiving coils coming into close proximity is the main reason for studying the coupling between them, this problem cannot be neglected. It is also difficult to use the method of summation of basic straight elements due to the number of possible combinations of mutual inductances between each of these straight elements and the fact that these elements could be at any angle relative to each other, which further complicates the derivation. Numerical integration is therefore also be used to find the coupling factor between two receiving coils. This factor can be approximated by

$$k_{r1r2} \approx \frac{n_{r2} \int_{S_{r2}} \vec{B}_{r1} \cdot d\vec{s}_{r2}}{\sqrt{L_{r1}L_{r2}}} \quad (5-22)$$

Where L_{r1} is the self-inductance of receiving coil 1, n_{r2} is the number of turns on receiving coil 2, and L_{r2} is the self-inductance of receiving coil 2.

Although in practice this value is highly sensitive and dependent not only on the characteristics of the receiving coils but also on the circuitry surrounding them. This approximation is unrealistically low for real-world receivers. As stated in section 5.2, coupling between receivers spatially separated by a distance equal to or greater than the length of one side of the core can be neglected.

As L_s , L_r , k_{sr} , k_{sxsy} , and k_{rxry} have been found, the coils can be represented as a system of coupled inductors with the self-inductances L_s and L_r of each coil coupled with each other coil by k_{sr} , k_{sxsy} , and/or k_{rxry} . The system shown in Figure 5-2 is now fully defined.

5.7.3 Coupling factors with circular source coils

The discussion above involved only square or rectangular coils. Circular coils can, of course, be used as well and the same methods can be used to calculate inductances and coupling factors.

The self-inductance of a circular turn of wire with a circular cross section is approximated by [5-2] as

$$L_{circle} = 4\pi r_c \left(\ln \left(\frac{8\pi}{r_w} \right) - \frac{7}{4} \right) \cdot 10^{-7} \quad (5-23)$$

where r_c is the radius of the coil and r_w is the radius of the wire.

The mutual inductances and therefore the coupling factor can then be determined using the integration of the field produced by the source coil over the area bounded by the receiving coil, as was shown in the previous section.

Reference [5-14] shows that the coupling factor for two coils lying on the same axis and without a the presence of a ferrite core in the receiver is

$$k = \frac{\mu_0 n_s n_r}{2\sqrt{L_s L_r}} \cdot \frac{r_s^2 r_r^2}{\left(\sqrt{r_s^2 + d^2} \right)^3} \quad (5-24)$$

where r_s and r_r are the respective radii of the source and receiving coils, and d is the distance between the coils.

Equation (5-24) obviously becomes much more complex when the effects of a ferrite core are added and when the receiver is not lying on the same axis as the source coil.

5.8 Source and receiving coil characteristics

Inductance and resistance values for various coil dimensions and winding numbers are shown in Figure 5-8 for a square source coil. The inductance was calculated using (5-8). Table 5-1 summarizes typical cable parameter and resistances for high frequency litz wire.

With respect to the choice of the number of strands, the absolute rise of the resistance with frequency depends on the diameter of a single strand, but is independent of the number of strands. Since, however, the direct-current resistance of the conductor is smaller the greater the number of strands, the resistance ratio is greater the greater the number of strands. Reducing the diameter of the strands reduces the resistance ratio, the number of strands remaining unchanged. To obtain a given current-carrying capacity, or a small enough total resistance, the total cross section must not be lowered below a certain limit. Therefore, in general, reducing the diameter of the strands means an increase in the number of strands [5-12].

Table 5-1 Cable parameters for Litz wire [5-13]

Frequency	Strand Cross Sectional Area (mm ²)	Nominal Diameter over Copper	DC Resistance Ω/m (max)	Single Strand R_{ac}/R_{dc}
60 Hz - 1 kHz	0.08	.0126	66.37	1.0000
1 kHz - 10 kHz	0.049	.0100	105.82	1.0000
10 kHz - 20 kHz	0.026	.0071	211.70	1.0000
20 kHz - 50 kHz	0.013	.0050	431.90	1.0000
50 kHz - 100 kHz	0.0078	.0040	681.90	1.0000
100 kHz - 200 kHz	0.0050	.0031	1152.30	1.0000
200 kHz - 350 kHz	0.0028	.0025	1801.0	1.0000
350 kHz - 850 kHz	0.0020	.0020	2873.0	1.0003
850 kHz - 1.4 MHz	0.0013	.0016	4544.0	1.0003

The source coil design remains flexible to allow adaptation of the system to varying application requirements. This leads to a wide range of inductance and resistance values. The quality factor is defined as

$$Q_{inductor} = \frac{2\pi fL}{R} \quad (5-25)$$

The quality factor for a square coil operating at 120kHz is shown in Figure 5-9. Temperature changes in the source coils will cause a variation in the Q -factor, thereby altering the field strength produced by the source coils. Systems installed in high-temperature settings must be designed with this effect in mind.

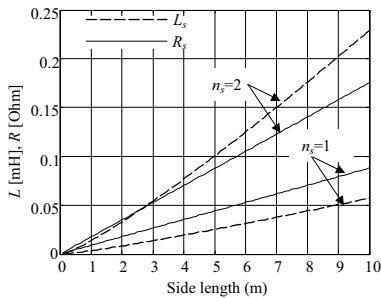


Figure 5-8 Inductance and resistance as function of source coil dimensions (square coil, cable, $f=120\text{kHz}$)

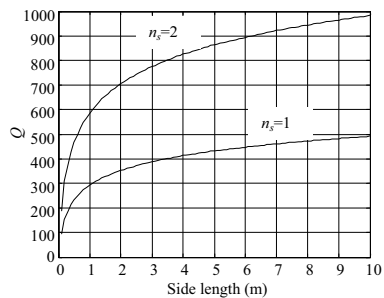


Figure 5-9 Quality factor as function of the source coil dimensions (square coil, cable, $f=120\text{kHz}$)

The shape of the core around which the receiving coils are wound has not proven in practice to have an appreciable effect on the power available to the load [5-1]. Square and round cores were investigated (and were the only possible choices as each of the

receiving coils must be the same size in order to assure that the flux induced in the receiver is constant regardless of orientation). Square cores provide a simpler solution than do circular cores due to the relative ease of winding three orthogonal receiving coils on a square frame. Inductance and resistance of a receiving coil with a core is shown in Figure 5-10 and without a core in Figure 5-11.

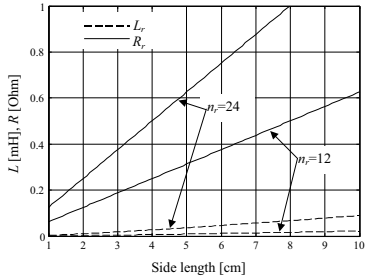


Figure 5-10 Inductance and resistance as function of receiving coil dimensions (square coil, no core)

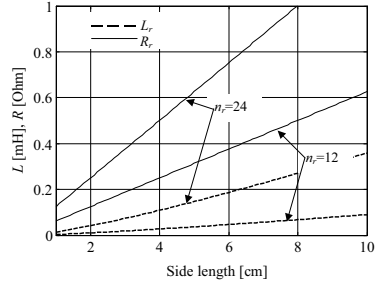


Figure 5-11 Inductance and resistance as function of receiving coil dimensions (square coil, with core)

The quality factor of a receiving coil as a function of coil dimensions is then shown in Figure 5-12. The effect of the core can clearly be seen.

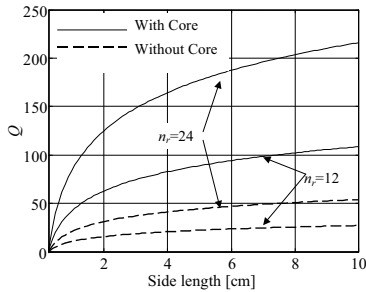


Figure 5-12 Receiving coil - Quality factor as function of the coil dimensions (square coil, with and without core, $f=120\text{kHz}$)

Expanding (5-14) gives the following expression for the flux crossing a single receiving coil with n_r turns,

$$\phi_{sr} = n_r A_r \mu_0 \mu' H \cos \alpha \quad (5-26)$$

Differentiating to find the voltage induced on the coil,

$$V_r = \frac{d\phi_{sr}}{dt} = 2\pi f n_r A_r \mu_0 \mu' H \cos \alpha \quad (5-27)$$

Equation (5-27) shows that the voltage induced is directly proportional to the field strength at the receiving coil. The load resistance on the receiver side must be equal to the receiving coil resistance (assuming a complete cancellation of the reactive terms in the receiving coil) for operation at the maximum power point [6-12]. In this case the theoretical maximum power that can be extracted from the receiving coil is given by

$$P_r = \frac{V_r^2}{4R_{coil}} = \frac{(2\pi f n_r A_r \mu_0 \mu' H \cos \alpha)^2}{4R_{coil}} \quad (5-28)$$

The power transfer efficiency at impedance matching is only $\eta=50\%$. At this point the losses in the coil are identical to the output power. With the extremely low coupling factors the maximum power is essentially independent of the characteristics of the source side power source.

The power delivered to a load attached to a square receiving coil is proportional to the square of the area bounded by the receiving coil. Therefore square receiving coils will observe a rise in power available to a load proportional to l^4 , where l is the side-length of the receiving coil. In order to double the available power, the side length of a receiving coil must be increased by roughly 20%.

As power is proportional to the square of the induced voltage, an increase of field strength by a factor of $\sqrt{2}$ will double the available power. Equation (5-28) also shows that an increase in operating frequency will lead to an increase in power in a quadratic relationship if all other parameters remain constant.

The actual power available to a load attached to a receiver will, however, depend on the design of and the losses in the power conversion circuitry and on the accuracy of the impedance match between load and coil impedance (see Chapter 6).

Using (5-28) in conjunction with (3-1) to determine the field strength from dimensional parameters it is possible to provide a graph summarizing the power transfer capabilities for a uni-directional source coil system. Figure 5-13 shows the correlation between output power multiplied by the coil resistance as a function of source coil parameters for an operating frequency of 120kHz and a receiver core material of $\mu_r=2000$ (a full datasheet describing the core material is available in Appendix VIII). The maximum power available to a receiving coil can be found for any given source coil parameters once the receiving coil winding resistance is established.

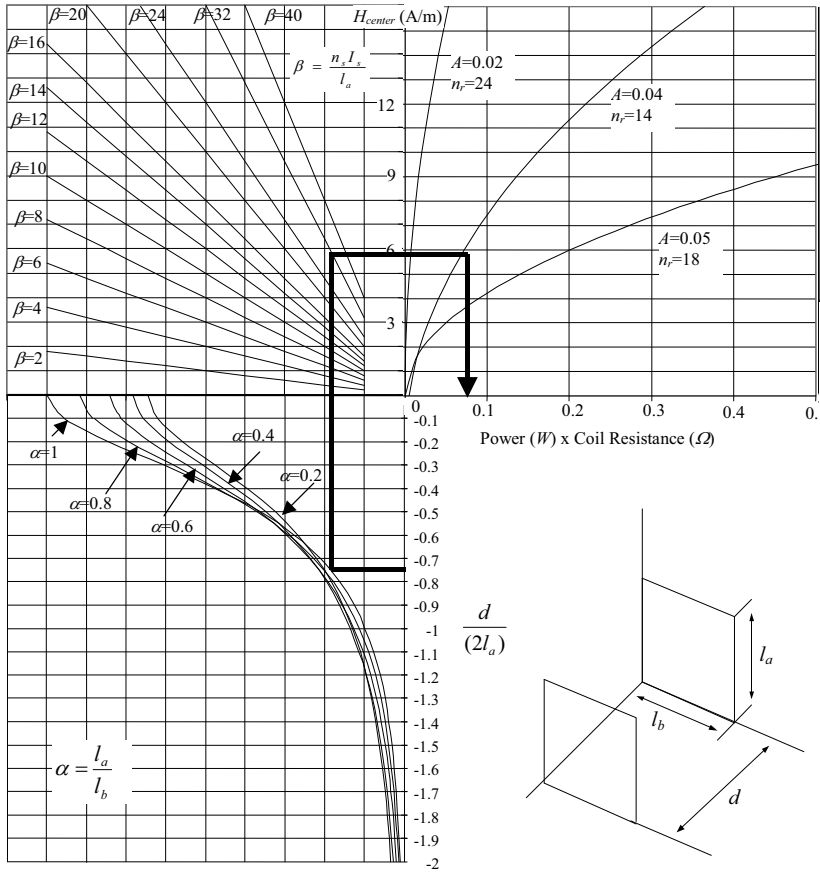


Figure 5-13 Relationship between coil side lengths (l_a , l_b), distance between coils (d), number of source coil turns (n_s), source current (I_s), receiving coil area (A), number of turns on the receiving coil (n_r), receiver side coil resistance and power times coil resistance at receiving coil terminals. Source coils are one set of square coils and the receiving coil is assumed to be at the center of the source coil system and aligned such that its normal is parallel to the normal of the source coil pair. An example is shown for a system of $l_a=l_b=3\text{m}$, $d=4.5\text{m}$, $I_s=16\text{A}$, and $n_s=2$.

5.9 Equivalent circuit model

In order to integrate the coupled inductors shown in Figure 5-2 into a full system description and to allow for system optimization, an equivalent circuit model is developed. A classic single-phase equivalent transformer model is shown in Figure 5-14. This model can be used to represent the energy transfer between one source and one receiving coil.

A wireless power transmission system with its inherent large air-gap is characterized by a small magnetizing inductance and large leakage inductances. The core loss resistance, R_c , can be neglected as the core losses are small enough to be considered negligible for cores in the anticipated size and permeability ranges discussed in Chapter 2.

In the circuit model developed here, the receiving coils on a single receiver are considered to be identical and in phase quadrature. Non-idealities in this arrangement are not considered here as they have been seen in practice to be negligible. The receiving coils are also considered to be conventionally wound. Center tapped windings are not considered here and are not used in practice due to their lower output voltage. Inter-winding capacitance is also neglected in this model as it does not have an appreciable effect on the voltage induced on the receiving coils.

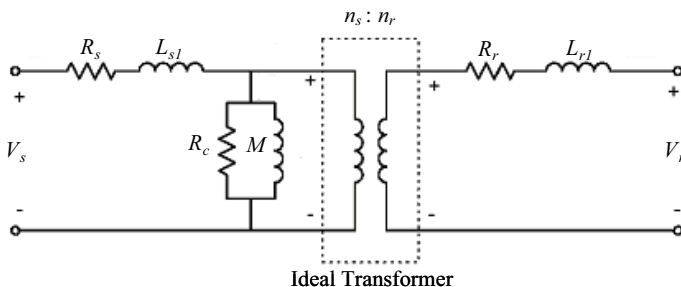


Figure 5-14 Single-phase equivalent transformer model where R_s = source side winding resistance, R_r = receiving side winding resistance, L_{sl} = source side leakage reactance, L_{rl} = receiving side leakage reactance, R_c = Core loss resistance, M = Magnetizing reactance, V_s = Source voltage, n_s = Number of turns on the source coil, n_r = Number of turns on the receiving coil.

The model shown in Figure 5-14 (for one source and one receiving coil) can be simplified if all values are referred to either the source or receiving side of the system. As the characteristics present at the load are of interest, the model will be referred to the receiving side.

The total flux produced by one source coil is equal to the stray flux plus the flux crossing the receiving coil. Multiple windings on the source coil(s) multiply the stray flux (as a function of their arrangement) as well as the flux through the receiving coil.

$$\phi_{source} = \phi_{receiver} + \phi_{stray} \quad (5-29)$$

The maximum flux that can be captured by receiving coil with a single winding is given by the winding area of the receiving coil, A_r , the current in the source coil, I_s , the winding number of the source coil, n_s and the location of the receiving coil relative to that of the source coil.

Assuming that the field strength is constant over the area bounded by the receiving coil, the maximum flux crossing a single receiving coil winding can be described by

$$\phi_{rs,\max} = \mu' \mu_0 H_r A_r \quad (5-30)$$

However, if the receiving coil in question is not aligned with the source field, the flux crossing that coil is described by (5-26) or by

$$\phi_{rs} = A_r (B_x a_{x1} + B_y a_{x2} + B_z a_{x3}) \quad (5-31)$$

where a_{x1} , a_{x2} , and a_{x3} are the x , y , and z components of the unit vector normal to the receiving coil, and B_x , B_y , and B_z are the x , y , and z components of the magnetic flux density vector.

The mutual inductance representing the flux in the receiving coil as a function of the source current is defined by (5-15), which can be written as

$$M = \frac{n_r \phi_{rs}}{I_s} \quad (5-32)$$

where I_s is the current in the source coil.

The inductance of the receiving coil L_r is split into a leakage inductance L_{rl} and a mutual inductance L_m . The total voltage, V_r , at the receiving coil is given by the sum of voltage across the leakage inductance L_{rl} of the receiving coil and the voltage across the mutual inductance L_m .

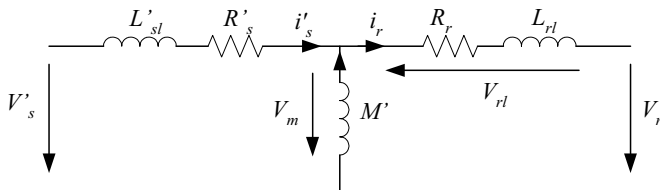


Figure 5-15 Inductance model, L'_{sl} is the leakage inductance of the source coil referred to the receiver side ($L'_{sl} = L_s(n_r/n_s)^2$), R'_s is the resistance of the source coil referred to the receiver side ($R'_s = R_s(n_r/n_s)^2$), I'_s is the source current referred to the receiver side ($I'_s = I_s(n_s/n_r)$), and V'_s is the source voltage referred to the receiver side ($V'_s = V_s(n_r/n_s)$)

The inductance of the source coil L_s is similarly split into mutual and leakage components. The voltage V_s at the source coil can be described as the voltage across the self inductance of the source coil, L_s , minus the voltage across the mutual inductance between the source and receiving coils. Figure 5-15 shows a model of these inductances referred to the receiver side. The voltage at the source and receiving coils can be represented by

$$v_s' / n = (R_s + j\omega L_s)ni_s' - j\omega M_{sr}i_r \quad (5-33)$$

$$v_r = -(R_r + j\omega L_r)i_r + j\omega M_{rs}ni_s'$$

Where $v_s' = nv_s$; $i_s' = i_s / n$, and $n = n_r / n_s$

Multiplying the equation describing the source voltage by n and deliberately extending the equation by $j\omega n M_{sr}i_s'$ without affecting its value gives

$$v_s' = (n^2 R_s + j\omega n^2 L_s)i_s' - j\omega n M_{sr}i_r - j\omega n M_{sr}i_s' + j\omega n M_{sr}i_s' \quad (5-34)$$

The receiving side is similarly extended by $j\omega n M_{rs}i_r$

$$v_r = -(R_r + j\omega L_r)i_r + j\omega n M_{rs}ni_s' - j\omega n M_{rs}i_r + j\omega n M_{rs}i_r \quad (5-35)$$

Introducing $R_s' = n^2 R_s$, $L_s' = n^2 L_s$, $M' = n M_{sr} = n M_{rs}$ and sorting provides circuit equations for the equivalent model representation on the receiving side shown in Figure 5-15.

$$v_s' = (R_s' + j\omega(L_s' - M'))i_s' + j\omega M'(i_s' - i_r) \quad (5-36)$$

$$v_r = -(R_r + j\omega(L_r - M'))i_r + j\omega M'(i_s' - i_r)$$

Therefore in order to integrate the mutual inductance into the equivalent transformer model, M must be multiplied by the turns ratio, n . An expression for the mutual inductance between one source and one receiving coil referred to the receiving side of the equivalent circuit model can be written by multiplying (5-32) by the turns ratio and expanding:

$$M' = M \frac{n_r}{n_s} = \frac{n_r^2 u' \mu_0 H_r A_r}{I_s n_s} \cos \alpha \quad (5-37)$$

where H_r is defined by equation (3-1) (or (3-4) for circular coils) and depends on the relative positions of the source and receiving coils as well as any shielding of the source field by objects located within or near to the operating volume.

5.9.1 Coupling factors

In order to develop an accurate model of the system with all components referred to the receiving side, the coupling factor, k , must be reduced into its component factors.

The coupling factor, k_{rs} , which represents the portion of the flux created by the source coils which induces a current in the receiving coils can be incorporated into the system model. An alternative representation of the mutual inductance can be derived:

$$M = \frac{\Lambda_{rs}}{I_s} = \frac{n_r \phi_{rs}}{I_s} = k_{rs} \frac{n_r \phi_{ss}}{I_s} = k_{rs} \frac{n_r}{n_s} L_s \quad (5-38)$$

Rearranging (5-38), k_{rs} can be expressed as

$$k_{rs} = \frac{M n_s}{L_s n_r} \quad (5-39)$$

Substituting (5-38) into (5-37) gives an expression for the mutual inductance, M' , referred to the receiving side of the model.

$$M' = L_s k_{rs} \left(\frac{n_r}{n_s} \right)^2 \quad (5-40)$$

because $M = M_{rs} = M_{sr}$, the mutual inductance can also be written as

$$M = \frac{\Lambda_{sr}}{I_r} = \frac{n_s \phi_{sr}}{I_r} = k_{sr} \frac{n_s \phi_{rr}}{I_r} = k_{sr} \frac{n_s}{n_r} L_r \quad (5-41)$$

giving an expression for the coupling factor k_{sr} which represents the portion of the flux created by the receiving coils which induces a current in the source coils

$$k_{sr} = \frac{M n_r}{L_r n_s} = \frac{n_r^2 u' \mu_0 H_r A_r \cos \alpha}{I_s L_r n_s} \quad (5-42)$$

Substituting (5-41) into (5-37) gives an alternate expression for the mutual inductance referred to the receiving side of the model:

$$M' = L_r k_{sr} \quad (5-43)$$

The coupling factor, k , between one source and one receiving coil is defined as

$$k = \sqrt{k_{sr} k_{rs}} \quad (5-44)$$

Substituting (5-39) and (5-42) into (5-44) gives

$$k = \frac{M}{\sqrt{L_s L_r}} \quad (5-45)$$

which is the classic expression for the coupling factor, k , between primary and secondary coils in a standard transformer.

A model representing one source and one receiving coil is shown and incorporating the parameters derived above is shown in Figure 5-16. This model also incorporates the resonant circuit on the receiving side and provides an accurate representation of circuit operation.

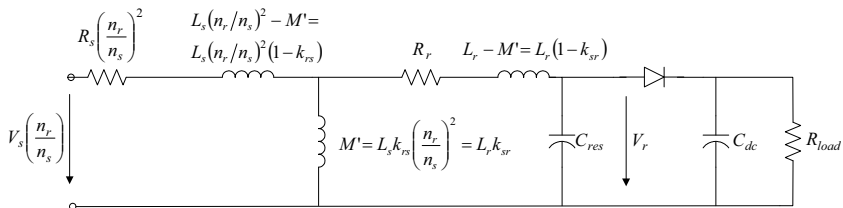


Figure 5-16 Equivalent circuit model referred to the receiving side, extended by a resonant capacitor, rectifier, dc-filter, and load. Winding resistances can be calculated with conventional equations using wire diameter, copper characteristics, number of turns, and the dimensions of the system, taking into account the skin and proximity effects.

5.9.2 Extension of the model for multiple source coils

The above discussion is valid only for systems involving only one source and one receiving coil. Most systems will involve multiple source coils arranged along each of the three axes of a rectangular coordinate system as well as multiple receivers each comprised of three receiving coils. The model described above can be used to represent systems involving multiple source coils on the same axis and one receiving coil. Further models can be developed and extended to model bi- and omni-directional systems with three-coil receivers.

5.9.2.1 Multiple source coils on the same axis – single receiving coil

When source coils are arranged in a uni-directional configuration such that they consist of N coils lying on the same axis, carrying the same current, and separated by a specified distance, the model shown in Figure 5-16 can be used to describe the system. The winding number of the source coils, n_s , now represents the sum of the number of turns on all of the source coils in the system ($n_s = n_{s1} + n_{s2} + \dots + n_{sN}$). The inductance of the source coils, L_s , likewise represents the sum to the inductances of each of the N coils ($L_s = L_{s1} + L_{s2} + \dots + L_{sN}$). The coupling factor(s) must be recalculated accordingly, and the field strength available at the location of the receiving coil is clearly altered with the addition of source coils to the system. With these adjustments, the model is accurate for N source coils lying on the same axis and a receiving coil located anywhere within the near-field range of source coils.

5.9.2.2 Multiple source coils on different axes – single receiving coil

A bi- or omni-directional system consists of multiple coils on the x -, y -, and z - axes. If the current in each of these coils has an identical *rms* value and frequency (regardless of the phase difference between them), the model shown in Figure 5-16 can be used to represent the system. The adjustments to the model parameters are identical to those discussed for multiple source coils on the same axis. This model does not, however, give information regarding individual component stresses and cannot be used for cases involving non-identical source current magnitudes or frequencies. The source coil system is viewed, in this case, as a single coil (with n_s turns) producing the flux linking to the receiving coil.

5.9.2.3 Single source coil/multiple source coils on a single axis and three-coil receiver

A single source coil supplying power to a three-coil receiver can be represented using the model shown in Figure 5-17. The model can be extended to represent multiple source coils on the same axis in the manner shown in section 5.9.2.1.

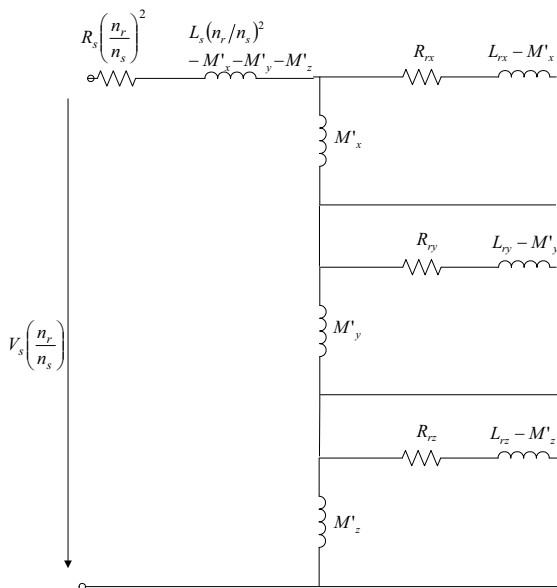


Figure 5-17 Equivalent circuit model multiple coil systems, $M_k(k=\{x,y,z\})$ =Mutual inductance between source coil(s) and receiving coil k . The receiving coils have identical winding numbers (n_r).

5.9.2.4 Multiple source coils – multiple receiving coils

The equivalent circuit model becomes highly complex for multiple source and receiving coils, especially when the system involves power transfer at multiple frequencies. The following matrix equation provides the most reasonable representation of a circuit involving three sets of source coils (omni-directional system) and one receiver (comprised of three orthogonal receiving coils) with source coil currents at a single frequency, ω :

$$\begin{bmatrix} V_{rx} \\ V_{ry} \\ V_{rz} \end{bmatrix} = \begin{bmatrix} R_x + j\omega(L_x - M'_{xx} - M'_{xy} - M'_{xz}) & 0 & 0 \\ 0 & R_y + j\omega(L_y - M'_{yx} - M'_{yy} - M'_{yz}) & 0 \\ 0 & 0 & R_z + j\omega(L_z - M'_{zx} - M'_{zy} - M'_{zz}) \end{bmatrix} \begin{bmatrix} i_{rx} \\ i_{ry} \\ i_{rz} \end{bmatrix} + j\omega \begin{bmatrix} M'_{xx} & M'_{xy} & M'_{xz} \\ M'_{yx} & M'_{yy} & M'_{yz} \\ M'_{zx} & M'_{zy} & M'_{zz} \end{bmatrix} \begin{bmatrix} i'_{sx} \\ i'_{sy} \\ i'_{sz} \end{bmatrix} - j\omega \begin{bmatrix} M'_{xx} & M'_{xy} & M'_{xz} \\ M'_{yx} & M'_{yy} & M'_{yz} \\ M'_{zx} & M'_{zy} & M'_{zz} \end{bmatrix} \begin{bmatrix} i_{rx} \\ i_{ry} \\ i_{rz} \end{bmatrix} \quad (5-46)$$

where

$V_{rk}(k=\{x,y,z\})$ are the voltages induced on receiving coils x , y , and z , $R'_k(k=\{x,y,z\})$ are the resistances (at operating frequency) of receiving coils x , y , and z , $L_k(k=\{x,y,z\})$ are the inductances of receiving coils x , y , and z , $i_{rk}(k=\{x,y,z\})$ are the currents induced in receiving coils x , y , and z , $i'_{sk}(k=\{x,y,z\})$ are the currents in source coils x , y , and z , M'_{rs} is the mutual inductance between receiving coil $r=\{x,y,z\}$ and source coil $s=\{x,y,z\}$.

The equation becomes more complex when the source coils carry currents of frequencies ω_{sx} , ω_{sy} , and ω_{sz} . Expanding equation (5-46) to account for excitation at multiple frequencies provides theoretical insight into the factors affecting the voltages induced on a receiver, but is too cumbersome for practical use. In most cases, ω_{sx} , ω_{sy} , and ω_{sz} are close enough in value to neglect the effect of their difference on the voltages induced. In this case (5-46) can be used as a reasonable approximation.

6 Power Converter Design

Power converter topologies for bi- and omni-directional, narrow-band, wide-range, electro-magnetic power transfer systems are presented and analyzed in this chapter.

6.1 General system design considerations

The power transfer system consists of single or multi-channel source side converter(s) and one or more receiver side power converters. The input of the source side converter(s) can be connected to a variety of power sources including single or three-phase ac power or dc power sources. The number of output channels of the source side converter must be equal to or greater than the field dimensions of the system for full control over the magnetic field. Source and receiver side power converters are inherently galvanically isolated from each other. The need for a galvanic isolation between the source coils and the power source is mainly dependent on the application and the safety norms regulating the application.

The bandwidth of source and receiver converters can be very small as the only purpose is the transfer of power. Frequency selectivity is preferred, suppressing other frequency components or interference outside of a desired operating band [6-1]. Operation at a constant frequency with limited bandwidth also allows the use of load resonant topologies that compensate the strongly reactive nature of the source and receiving coils.

The receiver side system is to be designed for operation at the maximum power point, autonomous start-up, and high reliability. Operation at maximum power point requires impedance matching between source and load impedances. Extremely low receiving coil voltages must be transformed into a stable dc voltage that can operate actuators, sensors, and control equipment. The overall receiver side efficiency cannot exceed 50% for operation at the maximum power point. A high receiver side power conversion efficiency is desired in order to provide the maximum possible power to the load and to allow for the highest possible packaging density.

A common dc bus architecture or multiple independent ac-dc-ac converters can be chosen for a multi-channel source side system. A common dc bus architecture eliminates the need for multiple ac/dc rectifiers. A common dc-bus voltage regulation can be implemented for uni- and bi-directional fields. Each dc/ac converter must be capable of load current regulation at fixed frequency operation for the generation of omni-directional fields. Flexibility, availability, and maintainability requirements may make a system with a separate ac-dc-ac converter for each channel preferable. The control of the omni-directional field will require a communication link among the converters. Cross-coupling in non-ideal source coil arrangements (bi- or omni-directional systems) mandates a bi-directional input stage on the source power converter or a dissipative load connected to the dc bus. A block diagram showing the major elements of an omni-directional system with two receivers is shown in Figure 6-1. Figure 6-2 displays alternatives for single channel ac-dc-ac power conversion.

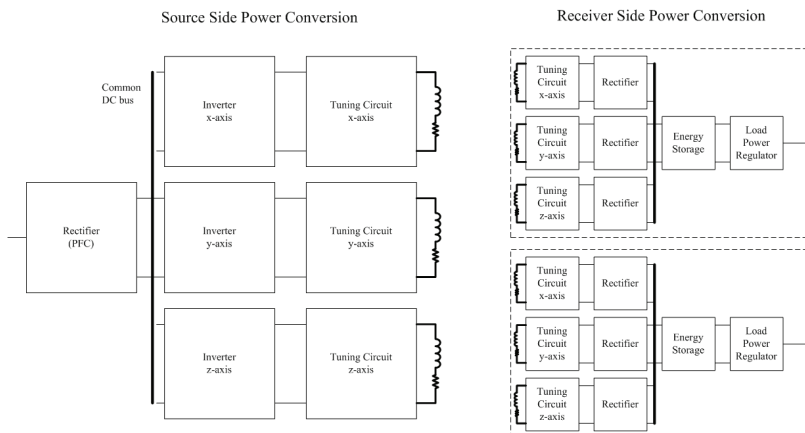


Figure 6-1 Generic block diagram of omni-directional system supplying a two receivers

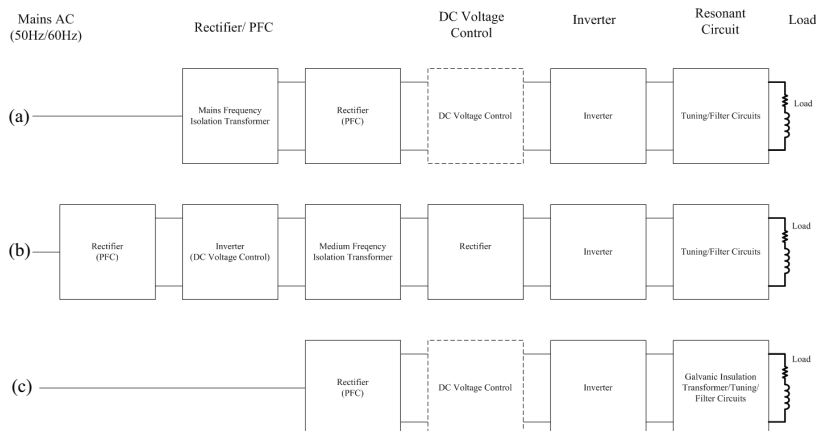


Figure 6-2a - c Selected power converter architectures for single channel ac-dc-ac conversion

Source and receiver side power conversion devices must contain energy storage elements to balance the instantaneous mismatch between available and requested power. Source side ride-through capabilities are needed to accommodate transient power supply outages, and receiver side ride-through capabilities are needed to accommodate temporary shielding of the magnetic field.

The power rating of the source side converter is primarily determined by the losses in the source coils for operation at resonance. Power flow is uni-directional, however a parasitic coupling of undesired magnetic field energy at the system's operating frequency may require bi-directional power flow in the source side converter(s). Cross-coupling among non-orthogonal source coils in bi- or omni-directional systems requires bi-directional power flow capabilities up to the point of common connection.

6.2 Semiconductor technology

State-of-the-art switch-mode power semiconductor devices may be implemented for the power and frequency requirements of the power conversion system. Linear devices designed for operation in the active region are not discussed. The system's target operating frequency is within the limits of the ISM band, i.e. between 30kHz and 400kHz.

For a minimum device switching frequency equal to the system's operating frequency majority carrier devices such as MOSFET and Schottky diodes are superior to minority carrier devices as there are virtually no stored minority carriers to be injected/extracted during a switching transition [6-3]. Further advantages of the majority carrier devices include the purely ohmic on-state voltage of MOSFETs and the low on-state voltage of silicon-based Schottky diodes (0.3 - 0.4 V). State-of-the-art silicon based majority carrier devices are available up to blocking voltages of 1200V (MOSFET) and 200V (Schottky diodes) while emerging silicon carbide Schottky diodes are available for blocking voltages between 300 and 1200 volts (Figure 6-3). A positive temperature coefficient of the majority carrier devices permits a reliable parallel connection of both MOSFETs and Schottky diodes. The introduction of the trench gate technology has further reduced the on-state losses in MOSFETs. Gate drive power for MOSFET structures is small even at high switching frequencies due to the small gate-drain capacitance.

The trade-off between voltage blocking capability and on-state voltage drop favours bipolar devices specifically if blocking voltages above approximately 600V are needed. Although dedicated punch-through IGBTs designed for switching frequencies up to 200kHz [6-24] are available, the charge removal in minority carrier devices is intrinsically slower than in majority carrier devices.

Bipolar p-n diodes also suffer from a substantial reverse recovery period, further impeding switching speed. The optimum structure or combination of structures, such as Si-IGBT and SiC Schottky diode [6-23], depends on the specific application. The details of the selection process are beyond the scope of this work.

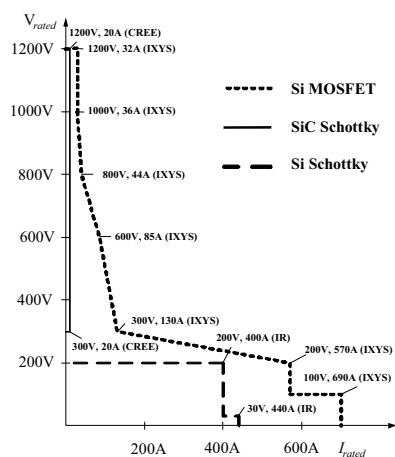


Figure 6-3 Single package semiconductor market survey 10/2004; majority carrier power devices [6-3]

For the further analysis of the power conversion system, mature silicon based majority carrier devices are assumed.

6.3 Tuning circuit characteristics

As the source and receiver sides of the system are tuned to a resonant frequency, and form a transformer with a large air-gap, the characteristics of tuned transformers are investigated in this section.

6.3.1 Generic single tuned circuits

Schematics of the series resonant circuit and the practical parallel resonant circuit are shown in Figure 6-4 Figure 6-5, respectively. The characteristics of the elementary (series and parallel resonant circuit) tuning circuits are well established [6-1], [6-2], [6-4]. A load (sink) or source resistance R_{ss} connected to a resonant circuit will affect the characteristics of both series and parallel resonant circuits differently. For example, an increase in the resistance R_{ss} will increase the quality factor for the parallel resonant circuit while decreasing the quality factor of the series resonant circuit. The presence of R_{pr} in a parallel resonant circuit reduces the frequency at which the voltage and current are in phase to a value below the ideal resonant frequency, ω_{r0} [6-12], [6-2].

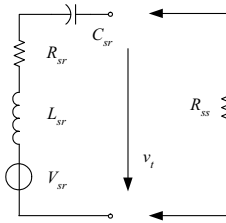


Figure 6-4 Series resonant circuit. L_{sr} , R_{sr} , and C_{sr} are the series resonant inductance, resistance, and capacitance, respectively. R_{ss} is the sink/source resistance

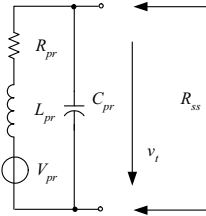


Figure 6-5 Parallel resonant circuit. L_{pr} , R_{pr} , and C_{pr} are the parallel resonant inductance, resistance, and capacitance, respectively. R_{ss} is the sink/source resistance

The parameters of the resonant circuit are defined below:

Resonant Impedance:

$$Z_r = \sqrt{\frac{L}{C}} \quad (6-1)$$

Ideal Resonant Frequency:

$$\omega_{r0} = \frac{1}{\sqrt{L C}} \quad (6-2)$$

De-tuning factor:

$$v = \frac{\omega}{\omega_r} - \frac{\omega_r}{\omega} \quad (6-3)$$

Bandwidth:

$$b_w = \frac{\omega_r}{Q} \quad (6-4)$$

Corner Frequencies (for $Q \gg 1$): $\omega_{\pm 45^\circ} = \omega_r (1 \pm \frac{1}{2Q})$ (6-5)

where $L=L_{pr}$, $C=C_{pr}$, and $L=L_{sr}$, $C=C_{sr}$, for parallel and series resonance, respectively. The quality factors for the series (Q_s) and parallel (Q_p) resonant circuits are defined by:

$$Q_s = \frac{\omega_r L_{sr}}{(R_{sr} + R_{ss})} \quad (6-6)$$

$$Q_p = \frac{\omega_r C_{pr} R_{ss}}{1 + \frac{R_{pr} R_{ss}}{R_r^2 + (\omega_r L_{pr})^2}} \quad (6-7)$$

While the true resonant frequency of the series resonant circuit is identical to the ideal resonant frequency ($\omega_r = \omega_{r0}$), the true resonant frequency of the parallel resonant circuit is given by:

$$\omega_r = \omega_{r0} \sqrt{1 - \left(\frac{R_{pr}}{Z_r} \right)^2} \quad (6-8)$$

6.3.2 Generic double-tuned circuit

The presence of tuned circuits at the source and the receiving side creates a double-tuned circuit. Transfer characteristics of transformer-coupled double-tuned resonant circuits have been discussed in [2-17], [5-14], [6-1], [6-2]. Figure 6-6a shows an example of a double-tuned resonant, series-parallel, transformer coupled circuit. Source and receiver sides are tuned to the same resonant frequency $\omega_r = \omega_{s,res} = \omega_{r,res}$ with the very small mutual inductance M' short-circuited for tuning purposes.

The capacitance C_s' represents the source side resonant capacitance referred to the receiver side and is defined as:

$$C_s' = C_s \left(\frac{n_s}{n_r} \right)^2 \quad (6-9)$$

Figure 6-6b and Figure 6-6c show the equivalent circuit of a double-tuned, transformer-coupled system driven by a voltage and a current source respectively. Replacing the parallel load R_{ss} on the receiver side by an equivalent series resistance $R_{r,tot}$ simplifies the circuit analysis. The conversion of the load resistance into an equivalent series resistance at the resonant frequency is given by

$$R_{r,tot} = R_r + \frac{\omega_r^2 L_r^2}{R_{ss}} \quad (6-10)$$

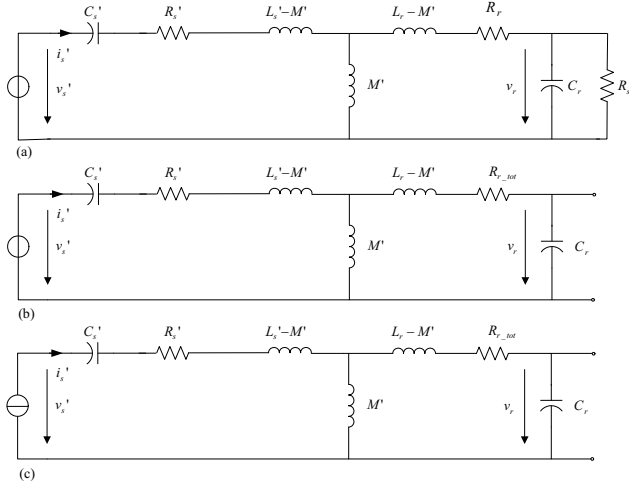


Figure 6-6 Receiver-side based equivalent circuits of a uni-directional system with full receiver alignment: (a) series-parallel resonant configuration, (b) equivalent series-series resonant circuit with a voltage source, (c) equivalent series-series resonant circuit with a current source

The impedance transfer function is shown below for an open-loop, current-controlled system (Figure 6-6c). The characteristics of the receiving coil primarily determine the transfer function,

$$H_i(s) = \frac{v_r(s)}{i_s(s)} = \frac{j\omega M'}{j\omega C_r \left(j\omega L_r + R_{r_tot}(\omega) + \frac{1}{j\omega C_r} \right)} \quad (6-11)$$

A simplified description accurate for small deviations from the resonant frequency is given by

$$H_i(s) = g_i \left(\frac{Q_r k}{jQ_r v + 1} \right) \quad (6-12)$$

where

$$Q_r = \frac{\omega_r L_r}{R_{r_tot}} = \frac{1}{R_{r_tot} \omega_r C_r}, \quad g_i = \omega_r \sqrt{L_s' L_r}, \quad k = \text{coupling factor}$$

A similar analysis for the voltage transfer function of the circuit shown in Figure 6-6b yields the following characteristic equation [6-1]:

$$H_v = \frac{v_r(s)}{v_s(s)} = g_v \left(\frac{Qk}{(jQ v + 1)^2 + Q^2 k^2} \right) \quad (6-13)$$

where $g_v = \frac{Q}{C_r \sqrt{L_s L_r}}$,

and $Q = Q_r = Q_s$, when the quality factors of the source and receiving coils are close enough in magnitude that their difference can be neglected. Reference [6-5] shows that if $Q_r \neq Q_s$, an equivalent quality factor Q can be defined as

$$Q = Q_G (Q_G / Q_A) \quad (6-14)$$

where Q_G is the geometric mean of the quality factors ($Q_G = \sqrt{Q_s + Q_r}$) and Q_A is the arithmetic mean ($(Q_s + Q_r) / 2$).

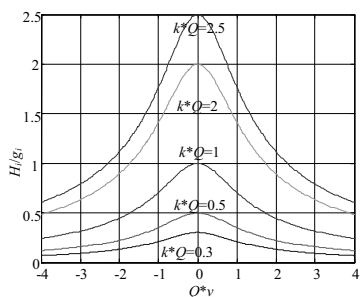


Figure 6-7 Impedance transfer function H_i/g_i of double tuned resonant circuits

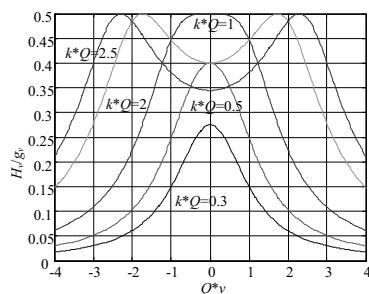


Figure 6-8 Voltage transfer function H_v/g_v of double tuned resonant circuits

Figure 6-7 and Figure 6-8 display the characteristics of the equation for slight variations around the operating frequency. The voltage transfer characteristic may exhibit a double peak depending on the coupling factor. Critical coupling is defined as $k = 1/Q$. For $k < 1/Q$ the system is undercoupled and the peak gain is less than the maximum gain. For values $k > 1/Q$ the circuit is overcoupled and the response shows a double peak characteristic [6-1], [6-2], [6-20].

The system is subject to variable coupling factors (due to variations in location and/or shielding) and variable quality factor (due to load variations). Maximization of the gain at the system's operating frequency is desirable for all uni-, bi- or omni-directional systems. Operation in overcoupling is thus not desired. An exception is in omni-directional systems using multiple frequencies to transfer power for which a wider bandwidth becomes attractive. Gain variations are not expected to introduce control complexities, as the system is controlled open-loop with respect to receiver side quantities.

Coupling factors were thoroughly investigated in Chapter 5, and were shown to have typical values below 0.07% for expected practical systems. Assuming that no system will have a coupling factor over 0.2%, the critical value of $kQ \geq 1$ will be reached only

when $Q \geq 500$. Figure 5-9 and Figure 5-12 show that $Q_s < 1000$ and $Q_r < 250$ for systems of reasonable dimensions. Equation (6-14) shows that $Q=400$ when Q_s and Q_r are simultaneously at these maximum values, the system is therefore not overcoupled even in worst-case situations.

6.3.3 Multiply tuned circuits

Additional control over the quality factor and the width of the pass-band may be obtained by implementing higher order systems using staggered tuned filters.

Only a single-stage resonant circuit per source coil is considered as there is no clear advantage to the use of staggered resonant circuits at the source side of the system. Narrow-band operation is desired with minimum damping introduced by filter components. Slightly varying operation frequencies can be controlled using auto-tuning methods described in Section 6.4.3.2.

Multiple resonant circuits could be used on the receiving side in specific applications where a wide pass-band response or a reduction in the skirt response is desired. As these circuits would, in most cases, add additional component costs and losses to the system, they are not analyzed here. However, a full analysis of multiple or stagger tuned converters is available in [6-1] and [6-6]–[6-8].

6.4 Single channel source side power converter

Single channel power converters will form the building blocks for multi-directional systems. Several topologies are investigated in this section and circuit response time and de-tuning characteristics are described.

6.4.1 Source side load characteristic

The electrical characteristics of the source coils as a function of the coil dimensions were derived in Chapter 5. Parallel and series configurations of coils lying on the same axis and adjustment of winding numbers per coil can be used to adjust the inductance, current, and voltage level for any field strength (Figure 6-9).

Depending on the load power requirements, the dimensions of the source coil system, the source current, and the source coil impedance at operation frequency may lead to large coil voltages. Systems with a nominal *rms* voltage exceeding 1000V (IEC) or 600V (NEMA) at the terminals are classified as medium voltage equipment. This is generally not desirable due to certification requirements for wire insulation, and creepage distances for connectors and circuit boards, as well as for converter enclosures. Depending on the application, the installation of multiple, separate, yet synchronously driven windings on each axis may be advantageous in comparison to using one winding driven by a large current (Figure 6-9c). Typical operating voltages for a single coil set lying on one axis are displayed in Figure 6-10 as function of system dimensions, field strength, and at a frequency of $f_s=120\text{kHz}$.

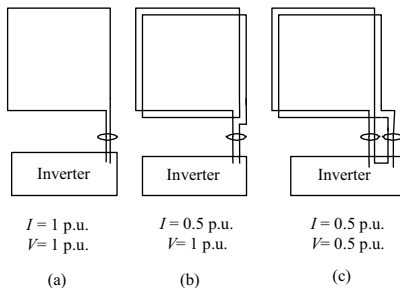


Figure 6-9 Relative voltages and currents at the converter terminal for constant field strength and various coil configurations (inductance changes due to proximity effect neglected)

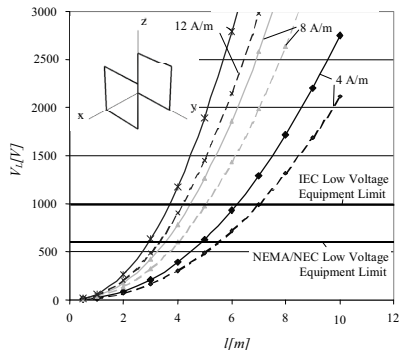


Figure 6-10 Coil voltage of one primary coil set as function of coil side length, spacing, and field strength ($y=z=l$, solid line: $x=l$, dashed line: $x=0.8l$, $f=120\text{kHz}$)

6.4.2 Source side resonant circuit

A series resonant configuration is chosen. The converter provides the full source coil current. The converter output voltage can be substantially smaller than the source coil voltage, thereby simplifying the application of majority carrier devices with low blocking voltage capabilities. Further advantages include direct control over the source coil current and therefore direct control of the magnetic field in the operating volume and the ability to connect the series resonant load directly to a dc voltage-link converter. Short circuits across the load terminals typically lead to a failure of the converter devices. The energy stored in the resonant circuit is independent of the frequency and time-invariant. The characteristic parameters of the series resonant circuit were discussed in Section 6.3.1.

It should be mentioned that if galvanic isolation is required between the converter and the source coils, the fixed frequency operation allows the design of a simple transformer that becomes an integral part of the resonant circuit.

6.4.3 Source side power conversion output stage

A half-bridge voltage source converter is a fundamental topology that links a series resonant load to a dc voltage. Load resonant converters operated at the fundamental system frequency, including fundamental frequency pulse-width modulation [6-15] are classified as class D converters [6-2]. Pulse-width modulated class S converters [6-2] with an operating frequency substantially above the system's operation frequency are outside the scope of this work. The characteristics and control of class D series resonant converters have been discussed in [6-2], [6-9], [6-13].

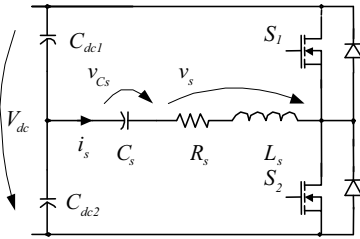


Figure 6-11 Series resonant half-bridge converter with voltage link center-tap ($C_{dc} \gg C_s$)

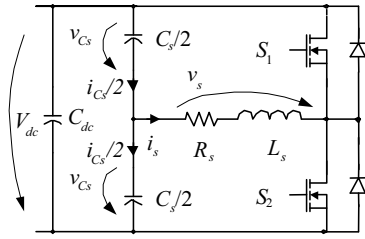


Figure 6-12 Series resonant half-bridge converter with split resonant capacitances

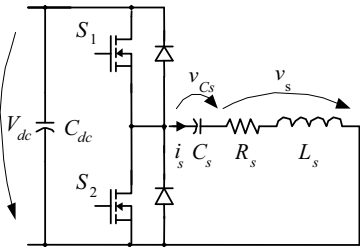


Figure 6-13 Series resonant asymmetric half-bridge converter

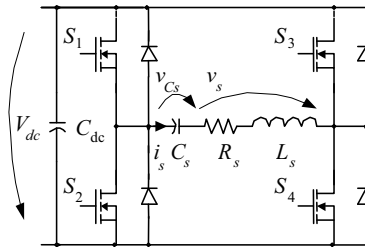


Figure 6-14 Series resonant full-bridge converter

Basic configurations for series resonant, dc/ac class D converters are shown in Figure 6-11 – Figure 6-13 [6-19]. Figure 6-11 and Figure 6-12 depict half-bridge converters using different arrangements for the resonant capacitor. In Figure 6-12, for $C_{s1} = C_{s2}$ the capacitor currents i_{cs1} and i_{cs2} are equal and the effective resonant capacitance is given by $C_s = C_{s1} + C_{s2}$. The dc-link capacitors are not subject to the full load current, which is advantageous if cost-effective electrolytic dc link capacitors are installed. Figure 6-13 shows a half bridge converter with the load connected to the negative dc rail. While a dc offset is present in the output voltage, the infinite impedance of the resonant capacitor at zero frequency effectively blocks the dc component at the load. Figure 6-14 shows a full-bridge class D converter. Twice the output voltage can be achieved at the cost of twice the number of devices and semiconductor losses.

The spectral content of all converter voltages contains the system's operating frequency and its odd harmonics. The spectrum of the circuit in Figure 6-13 contains additional even harmonics. The frequency-dependent impedance of the resonant circuit attenuates undesired spectral content in the load current.

Current control in the series resonant load can be achieved by a variation of the dc voltage, a variation of frequency around the system's operating frequency, or by controlling the effective volt-seconds impressed on the series resonant load per switching cycle. Phase-shift or clamped-mode control, asymmetrical duty-cycle

control, and asymmetrical clamped-mode control are among the proposed methods for the topology shown in Figure 6-14 [6-13]. Half-bridge converters (Figure 6-11, Figure 6-12, and Figure 6-13) are most often controlled with a variable dc-voltage controller. A comparison between a frequency variation control and phase-shift control for the topology in Figure 6-14 is presented in [4-7].

All topologies allow bi-directional power flow. The large source coil may be exposed to magnetic field components foreign to the system under consideration. These field components may be caused by other systems or by a multi-dimensional system with parasitic coupling among the orthogonal coils. Depending on the field frequency relative to the tuned frequency of the series resonant circuit, a notable current may flow from the coil to the dc-link capacitor. For certain applications methods for the dissipation of excess power in the dc-link may be needed if the input stage of the source power converter is designed for uni-directional power flow only.

All voltage source converters require uni-polar blocking, bi-directional conducting three-segment switches [6-10], [6-11] typically formed by a uni-polar blocking, uni-directional conducting active device with an inverse diode. Device choices for these switches were discussed in Section 6.2.

The load characteristics set the control limits for rise and fall times in each of the above topologies. Rise and decay times are minimum if switch states with zero voltage across the load (zero states) are avoided. The minimum decay time of the oscillation can be achieved if all gate signals are inhibited. The inverse diodes then automatically impress the inverse dc voltage onto the load until the load current is fully decayed. Zero states can be used for a controlled rise and decay of the oscillation. The application of a zero state in the full bridge circuit of Figure 6-15 causes the natural decay of the oscillation. The decay of the oscillation in Figure 6-13 is half wave asymmetric if all gate signals are inhibited.

A series resonant circuit excited by an ac voltage source v_e can be described by

$$v_{Ls} + v_{Rs} + v_{Cs} = \hat{v}_e \cos(\omega_e t - \varphi_e) \quad (6-15)$$

or

$$\frac{d^2 v_{Cs}}{dt^2} + \frac{R_r}{L_s} \frac{dv_{Cs}}{dt} + \frac{v_{Cs}}{L_s C_s} = \frac{1}{L_s C_s} \hat{v}_e \cos(\omega_e t - \varphi_e) \quad (6-16)$$

The general solution of the inhomogeneous, second-order differential equation for the capacitor voltage v_{Cs} is

$$v_{Cs} = C_1 e^{-\alpha t} \cos(\omega_d t) + C_2 e^{-\alpha t} \sin(\omega_d t) + A \cos(\omega_e t - \varphi_e) + B \sin(\omega_e t - \varphi_e) \quad (6-17)$$

where

$$\omega_d = \sqrt{\omega_r^2 - \alpha^2}, \quad \alpha = \frac{R_s}{2L_s}, \quad v'_{Cs}(0) = \frac{i_{Cs}(0)}{C_s}$$

$$A = \frac{-B\omega_e R_s C_s}{1 - \omega_e^2 L_s C_s}, \quad B = \frac{\hat{v}_e(1 - \omega_e^2 L_s C_s)}{(1 - \omega_e^2 L_s C_s)^2 + \omega_e^2 R_s^2 C_s^2}$$

$$C_1 = v_{Cs}(0) - A \cos(-\phi_e) - B \sin(-\phi_e)$$

$$C_2 = \frac{v'_{Cs}(0) + C_1 \alpha + A \omega_e \sin(-\phi_e) - B \omega_e \cos(-\phi_e)}{\omega_d}$$

$\hat{v}_e$ peak value of the excitation voltage
 ω_e frequency of the excitation voltage
 ϕ_e phase angle of the excitation voltage

A full derivation of the above equation can be found in Appendix IX.

The envelope functions of the capacitor voltage and current can be described by

$$V_{Cr_Env} = C_1 e^{-\alpha t} + A \quad (6-18)$$

$$I_{Env} = -C_s (C_1 \omega_d e^{-\alpha t} + \omega_e A) \quad (6-19)$$

Figure 6-15 displays the response time for a current control change using the zero states in a full bridge converter and the inverse voltage application in a half bridge converter. The decay time is given by the damping parameters of the resonant circuit and the converter control. Figure 6-16 shows the envelope functions for both the forced (gate signals inhibited) and natural (using zero-states) responses in the full or half-bridge topologies.

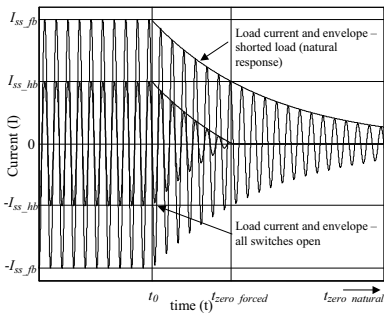


Figure 6-15 Oscillation decay control using zero states or inverted excitation. I_{ss_hb} =steady state current in a half-bridge converter, I_{ss_fb} =steady state current in a full-bridge converter at the same source voltage.

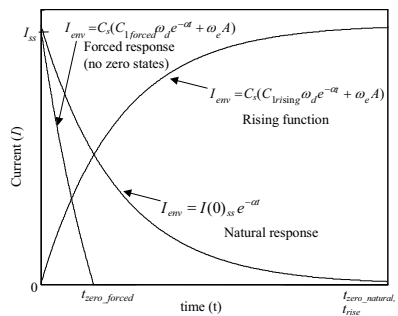


Figure 6-16 Envelopes of source current for a forced response using no zero states (inverted excitation), for a natural response (using zero states), and for forced excitation from zero initial current using no zero states.

6.4.3.1 Tuned operation

The load characteristic becomes purely resistive for ideal tuned operation. The system's operating frequency must be equal to the resonant frequency ω_r of the damped series resonant circuit.

The semiconductor devices in all topologies are switched at the system's operating frequency. The duty cycles of the semiconductors (neglecting dead-time) depend on the control method. The topologies shown in Figure 6-11 and Figure 6-12 require a variable dc voltage or an asymmetrical duty-cycle control for output current control at constant frequency operation. Asymmetrical duty cycle control is also applicable for the topology shown in Figure 6-13 with the duty cycle of the switch S_I defined as $d_I = \pi - 2\gamma$ (Figure 6-17b). The phase-shift control for the topology shown in Figure 6-14 uses a similar duty cycle definition (Figure 6-17a). The variation of the fundamental component of the inverter terminal voltage v_o as a function of the delay angle γ is shown in Figure 6-18. Selected circuit characteristics of the converters and resonant circuits at tuned operation are summarized in Table 6-1. Only the fundamental components of voltages and currents are shown due to the selectivity of a high quality resonant circuit at tuned operation. Odd and even harmonics are suppressed for asymmetric duty cycle control.

Table 6-1 Circuit characteristics for tuned operation

	Figure 6-11/Figure 6-12	Figure 6-13	Figure 6-14
Fundamental Voltage	$V_{rms,1} = \frac{\sqrt{2}}{\pi} V_{dc} \sin \omega t$	$V_{rms,1} = \frac{\sqrt{2}}{\pi} V_{dc} \cos(\gamma) \sin \omega t$	$V_{rms,1} = \frac{2\sqrt{2}}{\pi} V_{dc} \cos(\phi) \sin \omega t$
Fundamental Current	$I_{rms,1} = \frac{\sqrt{2}}{\pi} \frac{V_{dc}}{R_s} \sin \omega t$	$I_{rms,1} = \frac{\sqrt{2}}{\pi} \frac{V_{dc}}{R_s} \cos(\gamma) \sin \omega t$	$I_{rms,1} = \frac{2\sqrt{2}}{\pi} \frac{V_{dc}}{R_s} \cos(\phi) \sin \omega t$
DC Current	$I_{dc} = \frac{2}{\pi^2} \frac{V_{dc}}{R_s}$	$I_{dc} = \frac{2}{\pi^2} \frac{V_{dc}}{R_s} \cos(\gamma)$	$I_{dc} = \frac{8}{\pi^2} \frac{V_{dc}}{R_s} \cos(\gamma)$

The resistance R_s is roughly equivalent to the ac resistance of the source coils (Section 5.8).

If dc voltage control is used, the semiconductor switching losses are close to zero for the topologies shown in Figure 6-11 and Figure 6-12, as switching transients occur at the zero crossing of the load current. For the topologies in Figure 6-13 and Figure 6-14 the switching losses are close to zero for $\gamma=0$. For $\gamma \neq 0$ the semiconductor switching losses vary as the magnitude of the load current and magnitude of the current at the switching instant change with the delay angle. For the topology in Figure 6-13, switch S_I is subject to turn-on and turn-off losses. The turn-on and turn-off currents can be determined. Combining the correlation between current, delay angle, and switching instant it follows that turn-on and turn-off currents I_{on}/I_{off} reach maximum values of 50% of the peak current (which occurs at $\gamma=0$) at a delay angle $\gamma = 45$ degrees (Figure 6-18). The magnitude of the switching loss energies depends on the device technology chosen.

The turn-on and turn-off currents are described by

$$I_{on} = I_{off} = k_t V_{dc} \cos(\gamma) \sin(\gamma) = 0.5 k_t V_{dc} \sin(2\gamma) \quad (6-20)$$

where: $k_t = \frac{2}{\pi} \frac{V_{dc}}{R}$ for Figure 6-13; $k_t = \frac{4}{\pi} \frac{V_{dc}}{R}$ for Figure 6-14

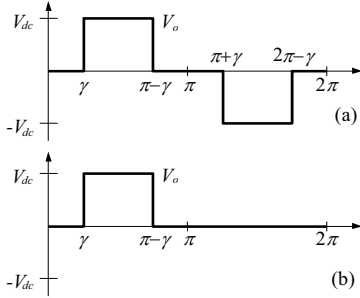


Figure 6-17 Inverter terminal voltage and control angle definition

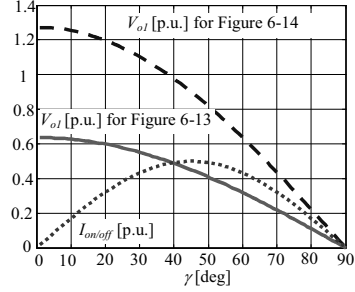


Figure 6-18 Peak fundamental output voltage and device turn-on/turn-off currents as a function of the control angle. 1p.u.= V_{dc} , 1p.p.= I_{peak}

6.4.3.2 De-tuned operation

The essential circuit performance variations at de-tuned operation have been discussed in [6-9]. De-tuned operation may be necessary for the omni-directional systems described in Chapter 3. Undesired de-tuned operation occurs when the coil inductances change due to objects in the vicinity of the system, or due to aging of the series resonant capacitor leading to a loss of capacitance. The variation in the magnitude of the current and the displacement power factor ϕ_l of the series resonant circuit as a function of the de-tuning factor are displayed in Figure 6-19 and Figure 6-20, respectively. These parameters are calculated by:

$$\frac{i}{i_{res}} = \frac{1}{\sqrt{1 + (vQ)^2}} \quad (6-21)$$

$$\phi_l = \arccos \frac{1}{\sqrt{1 + (vQ)^2}} \quad (6-22)$$

If a constant current is desired, the dc voltage power supply must have enough control margin to compensate for the increased impedance during de-tuned operation. The required dc voltage increase is inversely proportional to the current attenuation shown in Figure 6-19. An auto-tuning function may be a desirable alternative allowing a flexible converter design that can provide power to various coil sizes. Tuning in

discrete steps can be achieved by adding capacitors in a series or parallel configuration. A continuous tuning can be achieved by inserting a saturable inductance [2-4], [6-16] or using a variable capacitance switch [6-14]. The relative linear variation of the displacement power factor at small de-tuning values is typically chosen for tuning control. Due to the high Q -factor of the resonant circuits along with the high attenuation if operated in de-tuned mode, only small deviations from the resonant operating point are practically useful.

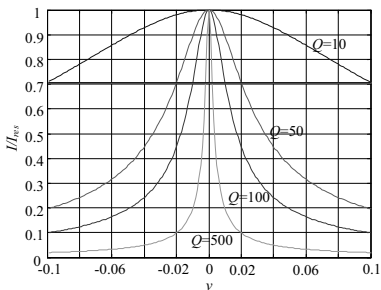


Figure 6-19 Series resonant circuit's sensitivity vs. de-tuning factor for quality factor $Q_s = 10, 50, 100, 500$

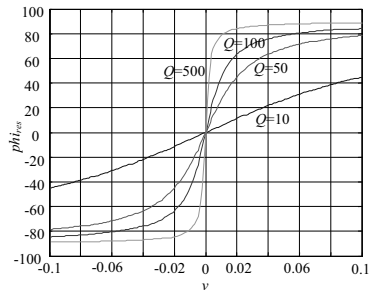


Figure 6-20 Displacement power factor of series resonant circuit vs. de-tuning factor for quality factor $Q_p = Q_s = 10, 50, 100, 500$

The dominant semiconductor losses originate from the switching transitions for operation above and below the resonant frequency of the load circuit. Turn-off losses of the active switches are dominant for operation above the resonant frequency of the load circuit. For operation below the resonant frequency of the circuit, the active switches are subject to turn-on losses only while the inverse diodes contribute additional reverse recovery losses. Detailed analyses are available in [6-17].

6.4.4 Source side power conversion input stage

The design of the input stage of the source side converter depends on the application, the selection of the inverter stage, and the galvanic isolation requirements. Power sources can be dc sources including batteries, uninterruptible power supplies, dc power derived from other industrial power conversion equipment, or conventional ac single-phase or three-phase power supplies. The power flow is uni-directional for an ideal system without interference by undesired coupling between coils or external fields operating at or near to the resonant frequency.

Many dc/dc or ac/dc topologies with uni-directional power flow have been discussed in the literature. Uni-directional, ac/dc power converters with and without power factor control, both directly coupled as well as galvanically isolated topologies, are well established [2-4], [6-4], [6-25]-[6-28]. Similarly, low and medium power dc/dc power conversion topologies are available [6-29], [6-30]. A full assessment of these

topologies is beyond the scope of this work and the reader is referred to the literature referenced.

6.5 Multi-channel source side power converter

Multiple channels with individually controllable currents are required for bi- and omni-directional systems. These converters are described here for bi-directional systems and for several omni-directional systems.

6.5.1 Bi-directional field

The creation of a rotating field vector of constant field magnitude requires a minimum of two currents in phase quadrature and of equal magnitude and frequency applied to two coils in space quadrature (Chapter 3). Figure 6-21 shows a simple architecture based on a parallel arrangement of the two half-bridge channels shown in Figure 6-12. Control signals and a typical current waveform during steady-state operation at resonance are shown in Figure 6-22. The switches S_1 and S_2 in both channels are operated with control signals of equal duty cycle and in phase quadrature. The load current magnitudes can be controlled using the dc bus voltage. The current magnitudes remain constant for a constant field strength. Constant power is drawn from the power source when generating ideal bi-directional fields.

Asymmetric bi-directional fields (where the field vector at the center of the system traces an oval shape) can be created if the current magnitude in each channel is independently controlled. Separate dc buses must then be used unless asymmetric duty cycle control is introduced. Alternatively, an output stage which allows independent control of the current magnitude as shown in Figure 6-13 and Figure 6-14 can be used if a common dc bus is desired.

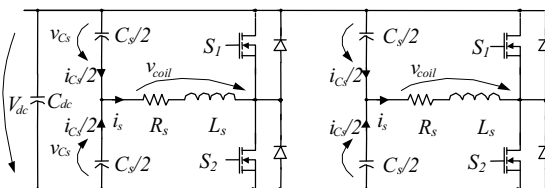


Figure 6-21 Common power supply for bi-directional field creation

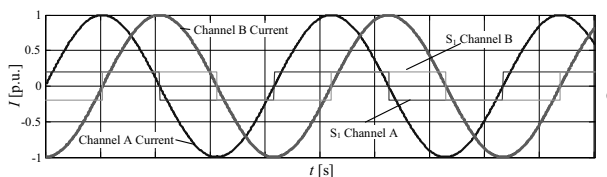


Figure 6-22 Control signals and resonant current for bi-directional field

6.5.2 Omni-directional field

Methods for the creation of an omni-directional field with constant field strength were summarized in Table 3-2. Converter design requirements are discussed below.

6.5.2.1 Periodic switching of the plane of rotation

Two currents in phase quadrature with equal magnitude are applied to two of the three orthogonal source coils or coil sets for this type of operation. A separate source channel is provided for each source coil. Figure 6-23 shows a schematic of a three-channel series resonant converter. A common dc bus control can be applied to create a field of equal magnitude in each direction.

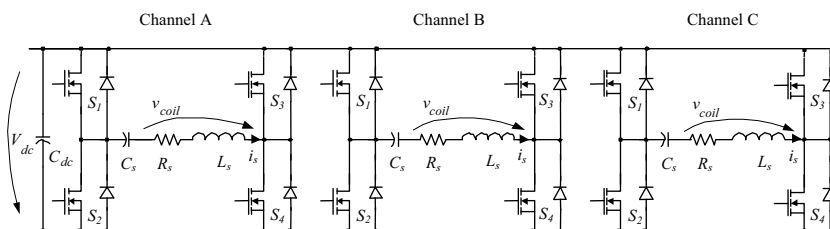


Figure 6-23 Three-channel series resonant converter for sequential bi-directional fields. Constant power is drawn from the source except during transition periods.

An instantaneous turn-on/turn-off transition is impossible due to the rise and decay times of the currents in the series resonant circuits. The rise and decay times of the load current in the various topologies were discussed in Section 6.4.3. The phase reference of the channel with a rising current must be identical to that with a falling current in order to ensure minimum field distortion and the least-complex current control. An optimal pattern in which to energize the source coils is shown in Table 6-2. This assures that the current in the channel being turned off is at the same phase as the current in the channel being turned on, allowing the remaining channel to remain in steady state operation throughout the transition.

Table 6-2 Phase reference sequence for periodic switching of the plane of rotation

Load current at 0°	Load current at 90°	Off
Channel A	Channel B	Channel C
Channel A	Channel C	Channel B
Channel B	Channel C	Channel A
Channel B	Channel A	Channel C
Channel C	Channel A	Channel B
Channel C	Channel B	Channel A

The transition frequency is determined by the likelihood and duration of bi-directional shielding of at least one of the receivers present in the system, and the power requirements and energy storage capacity on the receiver side. The minimum transition frequency was discussed in Section 3.6.1.

The rise and decay times of the circuit will also define an upper boundary on the cycling frequency. Although the decay time can be minimized by impressing a reverse voltage on the load as was discussed in Section 6.4.3, the rise time cannot be decreased beyond the duration of the natural response of the load current. This is therefore the limiting value for sequencing time (t_{seq}). Figure 6-24 shows the first three transitions shown in Table 6-2, assuming that the decay is governed by the natural response. If the circuit is cycles through the required transitions at the maximum possible rate ($t_{ss} \rightarrow 0$), the field vector will rotate through all of its possible angles of rotation in $3t_{seq}$.

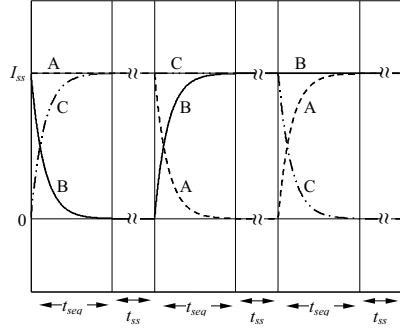


Figure 6-24 Envelopes of the load currents for each channel in Figure 6-23. Three transitions are shown

Using (6-19), the rise time (and decay time assuming a natural response) of the converters shown in Figure 6-11 - Figure 6-14 can be expressed as

$$t_{rise} = t_{seq} = \frac{-1}{\alpha} \ln \left(\frac{I_{ss} - \omega_e A C_s}{C_s C_{lrise} \omega_d} \right) \quad (6-23)$$

whereas the decay time if a reverse voltage is applied across the load is

$$t_{decay_forced} = \frac{-1}{\alpha} \ln \left(\frac{-\omega_e A}{C_{ldecay} \omega_d} \right) \quad (6-24)$$

where I_{ss} is the amplitude of the steady state current through the load, C_s is the source side capacitance, and C_{lrise} and C_{ldecay} are defined as in equation (6-17) for the rise and decay conditions, respectively.

The cycling frequency can then be defined as

$$f_{cycle} = \frac{1}{3(t_{seq} + t_{ss})} \quad (6-25)$$

Giving a maximum when $t_{ss}=0$ of

$$f_{cycle,max} = \frac{1}{3t_{seq}} = \frac{-\alpha}{3[\ln(I_{ss} - \omega_e AC_s) - \ln(C_s C_{1rise} \omega_d)]} \quad (6-26)$$

The field vector at the center of the source coil system will undergo undesired variation in magnitude and direction during the sequencing time (t_{seq}). As the magnitude of the total current in the system is proportional to the length of the magnetic field vector at the center of the system, perturbation in this magnitude is directly reflected in the field vector. Figure 6-25 and Figure 6-26 show this effect.

Using a natural decay and minimum rise time and commencing rise and decay simultaneously provides the minimum perturbation in the magnetic field. As periodic switching of the plane of rotation will be employed most often in cases where maximum allowable field strength is achieved with only two of the three source coil sets energized, commencing rise and decay simultaneously provides the additional advantage of avoiding the generation of a field vector with magnitude greater than that in a bi-directional system.

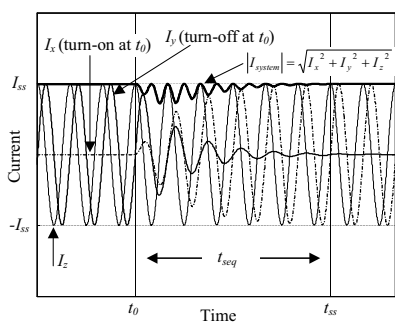


Figure 6-25 Currents in x-, y-, and z-direction source coils during a single transition period, t_{seq} . The transition begins at t_0 , when I_x begins to ramp up and I_y begins to ramp down.

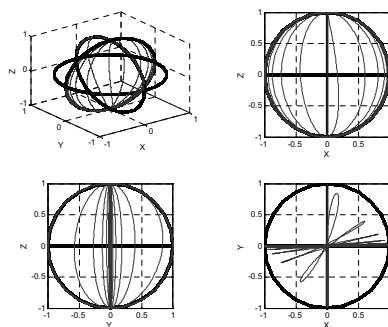


Figure 6-26 Magnetic field rotation for the transition shown in Figure 6-25. Ideal rotation patterns are shown in bold. The non-ideal rotation pattern caused by the transition is shown with a thin line for comparison.

In cases when the field strength limit has not been reached with a bi-directional system, a combination of forced decay and premature ramp-up of the current in the previously non-energized coil set can be used to eliminate field vectors below desired value during the transition period. In this case, the time between the initiation of current ramp-up in one coil set and ramp-down in another is based on maximum tolerable field strength and distortion.

6.5.2.2 Frequency shift

One out three channels operates with a slight frequency offset in this type of operation. For the maintenance of an identical coil current magnitude in all three axes using a common dc bus, the resonant frequency of the load can be adjusted to the shifted frequency using a system of switchable capacitors and/or saturable inductors (Figure 6-27) or a dc/ac converter topology with magnitude control capabilities (e.g. Figure 6-28). The currents of the two channels operating at the system's nominal operating frequency are in phase quadrature. The current of the channel operating at the offset frequency has an arbitrary phase angle with respect to the other two channels. The power drawn from the source is fluctuating with the magnitude and frequency of the offset channel.

The magnetic field vector at the center of the system will rotate through all possible angles with respect to the source coils if at least two of the three channels are sequentially operated at the offset frequency. The selection of the offset frequency is based on the desired vector rotation period and the density of the mesh traced by the field vector. These factors were discussed in Section 3.6.2.

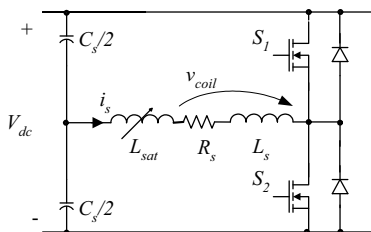


Figure 6-27 Saturable inductance applied to one of three channels

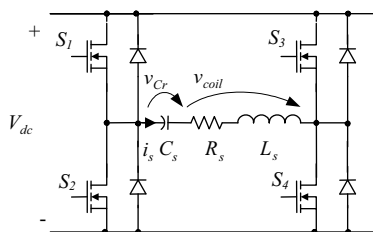


Figure 6-28 One of three converter channels with magnitude control

The addition of the saturable inductor in the circuit of Figure 6-27 is practical in both bi- and omni-directional field generation as the inductor can be used to allow interchangeable source coil systems to be applied to a single power supply. Figure 6-28 is less attractive as the magnitude of the dc voltage required to provide the required current to source coils operating off-resonance limits the possibility of interchanging source coil sets and does not provide any additional advantages to system operation.

As discussed in Chapter 3, the frequency shift method is considered here only for situations in which a fully omni-directional system is not required, but a bi-directional system is insufficient. In this case, only one source coil set carries current at the offset frequency. An extension of the circuit shown in Figure 6-21 with the addition of a saturable inductor can then be used and the circuit will be operated at constant magnitude and frequency. This is a highly simple solution requiring minimal control of the source coil currents during normal operation.

6.5.2.3 Double axis amplitude modulation

Double axis amplitude modulation requires independent control over the magnitude of at least two converter channels. Figure 6-23 shows one possible implementation for a three-channel system fed from one common dc bus and capable of independent control of the currents in each channel. Controlling the current magnitude requires pulse-width control as described in Section 6.4.3.1. The semiconductors are subject to higher switching losses in comparison to those incurred while using block modulation at tuned operation.

Alternatively, double axis modulation can be implemented with multiple single channel power converters with independent dc voltage control. All channels operate at the system's operating frequency. The resonant circuits can be tuned for a given system set-up and auto-tuning circuitry can be avoided. A similar functionality can be provided by a three-channel arrangement based on the half-bridge cell depicted in Figure 6-13.

The response of the series resonant circuit was discussed in Section 6.4.3. The minimum rise and decay times of the oscillation determine the maximum possible modulation frequencies. An upper boundary on the modulation frequency is given by the damping characteristic of the resonant circuit. As Section 6.4.3 shows, the decay of the current can be forced to be significantly faster than the rise time. For this reason, the rising function of the source current determines the upper limit of the modulation frequency.

Using (6-19), the envelope function for the rising current can be described by

$$I_{Env_rise} = C_s A(\omega_d e^{-\alpha t} - \omega_e) \quad (6-27)$$

and the modulation function is

$$I_m = A_m \sin(\omega_m t - \phi_m) \quad (6-28)$$

The maximum possible modulation frequency ω_{m_max} is the greatest ω_m at which $I_m \leq I_{Env_rise}$ for all time t . If the amplitude of the modulation function (A_m) is equal to the steady state current in the system (I_{ss}), ω_{m_max} is at its lowest possible value. As A_m is reduced, ω_{m_max} increases. ω_{m_max} can therefore be controlled using the dc voltage. This is shown in Figure 6-29 and Figure 6-30.

A reasonable estimate of ω_{m_max} can be found by restricting the modulation function such that it reaches one-third of its period when I_{Env_rise} has reached a value of A_m .

Solving

$$A_m = C_s A(\omega_d e^{-\alpha t} - \omega_e) \text{ for } \omega_m \text{ at } t = \frac{\pi}{3\omega_m}$$

gives

$$\omega_{m_max} \approx \frac{-\pi\alpha}{3[\ln(A_m + \omega_e C_s A) - \ln(C_s A \omega_d)]} \quad (6-29)$$

Figure 6-29 shows the envelope of an uncontrolled rise in current through an example load as well as several possible modulation functions. Figure 6-30 shows the relationship between the amplitude of the modulation function and its maximum possible frequency for the same system.

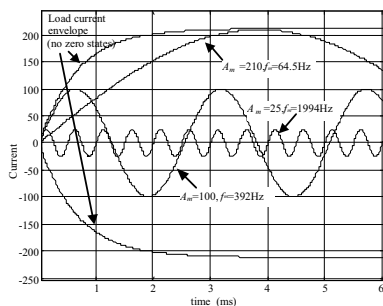


Figure 6-29 Envelope of uncontrolled ramp-up current and the amplitude and frequency of three possible amplitude modulation current envelopes $L_s=20\mu\text{H}$, $C_s=126\text{nH}$, $R_s=0.06\Omega$, $V_{dc}=20\text{V}$.

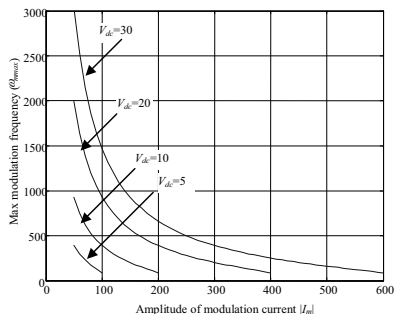


Figure 6-30 Maximum possible load current modulation frequency vs. the amplitude of the load current modulation function for several source voltage conditions. $L_s=20\mu\text{H}$, $C_s=126\text{nH}$, $R_s=0.06\Omega$.

6.6 Receiver side power converter

The receiver side power converter design is a trade-off between cost, reliability, and packaging density vs. the desired functionality, which includes maximum power extraction, dc bus voltage control and autonomous operation and start-up. In contrast to the source side system, operation at the maximum power point is the essential design criterion. The receiver side efficiency at maximum power is 50% [6-12]. An omni-directional receiver design with three coil axes is assumed.

6.6.1 Receiver side winding connection

In general the three identical, orthogonal coils can be connected in series or in parallel (direct or via a converter) to the load. A direct parallel (or delta) connection of the three orthogonal receiving coils is not attractive due to circulating currents in the three receiving coils.

Chapter 2 showed that total flux in a receiver with three orthogonal coils is independent of the orientation for all source fields. Hence a series connection of all

three receiving coil windings will also result in an orientation-independent voltage and power extraction. However, the resistances of the receiver windings in all three coils contribute to the total receiver side source resistance.

It follows from (5-28) that lowering the receiving coil resistances increases the power that can be extracted at any given field strength. An independent extraction of the power from each receiving coil using a parallel connection of the windings after a power conversion stage provides a higher maximum power at the same source field magnitude. It can be shown that there is no orientation dependency for the total power extracted from the three receiving coils for uni-, bi- or omni-directional field excitation.

The receiving coil impedance is strongly dependent on manufacturing tolerances (Table 7-4). Secondary coils with a center-tap are not considered for manufacturing reasons. For the analysis to follow it is assumed that there is no parasitic coupling between the three orthogonal coils on a single receiver.

6.6.2 Receiver side resonant circuit

The reactive component of the receiving coil can be compensated using a series or parallel resonant circuit in a narrow-band system. The extremely low coupling factors along with the field strength limits and winding space constraints lead to a very small induced voltage on the receiving coils. Coil voltages substantially below one volt are typical. A parallel resonant circuit tuned to the system's operating frequency is chosen as it offers a voltage transformation with a terminal voltage magnitude that depends on the quality factor of the resonant circuit. The advantages of using a series connection of all three receiving coils include the reduction of part count, losses, and failure opportunities in the power conversion circuit (Figure 6-32). Only one tuning capacitor and rectifier circuit are needed. The component count triples if a separate channel is used for each receiving coil (Figure 6-31).

The receiver-side resonant capacitance, C_r , is tuned to compensate the self-inductance of the receiving coils. The equivalent receiver side impedance Z_r of the tuned circuit (for a single receiving coil) is given by (6-30). The source coil system can be neglected due to the current-source nature of the source side converter and the extremely low coupling factor ($k < 1\%$). References [6-21] and [2-23] describe the optimum tuning of a resonant receiver considering the source side circuit.

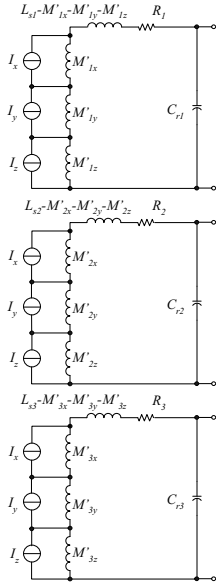


Figure 6-31 Equivalent circuit of receiver side with separate compensation

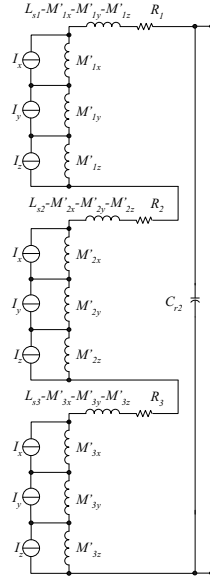


Figure 6-32 Equivalent circuit of receiver side with combined compensation

6.6.3 Receiver side impedance characteristics

The impedance of the load circuit must be matched to the impedance of the resonant circuit to extract maximum power from the parallel resonant circuit(s) ($R_{source}=R_{load}$, $X_{source}=-X_{load}$).

The equivalent source impedance, $Z_{r,res}$, of the receiver side parallel resonant circuit with one coil tuned to the resonant frequency $\omega = \omega_r$ is given by

$$Z_{r,res} = \frac{Z_r^2}{R_r} + \frac{1}{j\omega_r C_r} \quad (6-30)$$

The load impedance for maximum power transfer is given by

$$Z_{load_opt} = R_{load_opt} + jX_{load_opt} \quad (6-31)$$

$$\text{with } R_{load_opt} = \frac{Z_r^2}{R_r} \text{ and } X_{load_opt} = \omega_r L_r$$

The total optimum load impedance must be increased by a factor of three when three identical receiving coils are connected in series. For most applications the optimum

load impedance is primarily set by the real term in (6-31); the imaginary term has a negligible influence on the magnitude of the power transfer, permitting the omission of an additional inductor for loads which are resistive in character. A variation in the operating frequency does not affect the optimum load resistance.

The voltage across the capacitor of the parallel resonant circuit of one receiving coil can be assessed as a function of an ac load with the resistance R_{ss} using V_r from (5-27) for single frequency omni-directional source field. Using the equivalent circuit in (Figure 6-5) the resonant capacitor voltage is given by

$$V_{-Cr} = \frac{R_{ss}}{(j\omega C_r R_{ss} + 1)(j\omega L_r + R_r + \frac{R_{ss}}{j\omega C_r R_{ss} + 1})} V_r \quad (6-32)$$

As shown in (6-7) the load R_{ss} affects the quality factor of the resonant circuit. R_{ss} must be equal to R_{load_opt} for maximum power transfer.

A more general description of the circuit characteristics can be given for multi-frequency excitation. Based on the equivalent circuit in Figure 6-31, the no-load ($R_{load}=\infty$) and load dependent voltages across the resonant capacitor for receiving coil k ($k=\{1,2,3\}$) are given in (6-33) and (6-34), respectively.

$$V_{-Cr_0} = \sum_{i=1}^n j\omega_i C \frac{j\omega_i (M_x I_{xi} + M_y I_{yi} + M_z I_{zi})}{j\omega_i L_r + R_r + 1/(j\omega_i C_r)} \quad (6-33)$$

$$V_{-Cr} = \sum_{i=1}^n \frac{R_{ss} j\omega_i (M_x I_{xi} + M_y I_{yi} + M_z I_{zi})}{(j\omega_i C_r R_{ss} + 1)(j\omega_i L_r + R_r + \frac{R_{ss}}{j\omega_i C_r R_{ss} + 1})} \quad (6-34)$$

Where n is the number of frequencies at which power is transferred from source to receiver, and the currents I_x , I_y , and I_z represent the currents impressed by the current controlled source coils as described in Chapter 3.

For the case in which all three receiving coils are connected in series, the resonant capacitor voltage is given by:

$$V_{-Cr} = \sum_{i=1}^n j\omega_i C \frac{j\omega_i (M''_x I_{xi} + M''_y I_{yi} + M''_z I_{zi})}{3[j\omega_i L_r + R_r] + 1/(j\omega_i C_r)} \quad (6-35)$$

where M is the mutual inductance between source and receiver for a system in which the unit vector normal to the receiving coil is not parallel to the source field vector, and M'' is the mutual inductance between source and receiver for a fully aligned system.

The formulas above are valid for uni-, bi-, and omni-directional excitation with $n=1$ for uni- and bi-directional field generation and for omni-directional field generation using sequential switching of the plane of rotation. $n=2$ using the frequency shift method of

omni-directional field generation, and $n=3$ in the case of single- or double-axis amplitude modulation.

6.6.4 Receiver side converter stage

Many different concepts for the receiver side power converter can be conceived. A passive and an active circuit topology are shown as examples. A detailed circuit analysis is outside the scope of this work.

Figure 6-33 and Figure 6-34 depict a fully controllable active system with individual power extraction for each receiving coil and a completely passive system with a common power extraction, respectively. Advantages of latter design include high reliability, low cost, and high packaging density. The main disadvantage is that the maximum power that can be extracted for a given field strength is lower by a factor of three than in the solution with individual power extraction. The power is transferred directly to the load with a single zener diode providing overvoltage protection. The load voltage is otherwise unregulated.

The parallel resonant circuit represents a voltage-stiff source for the receiver side power converter [6-9]. A passive rectifier is chosen to connect the resonant circuit to the dc-link as uni-directional power flow is sufficient.

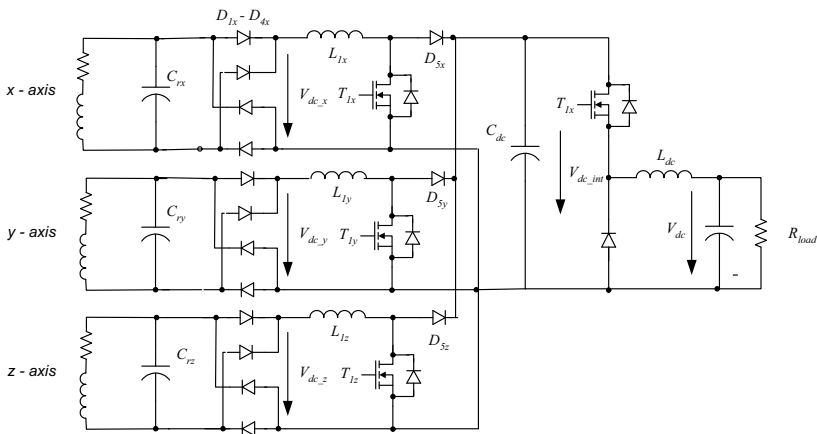


Figure 6-33 Independent impedance matching of coil axes and regulation of dc voltage using a combination of a buck and a boost converter

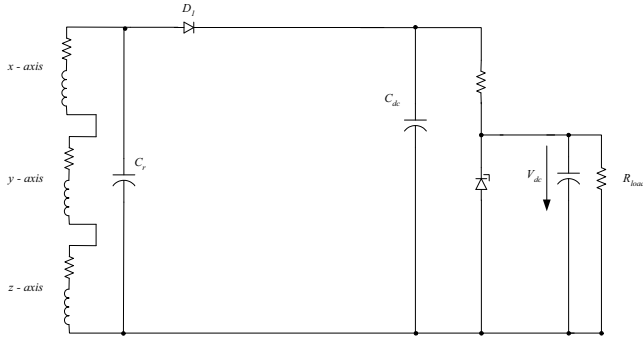


Figure 6-34 Minimum design with no impedance matching and overvoltage limitation of the dc voltage

The circuit shown in Figure 6-33 provides independent impedance matching and regulation of the dc bus voltage. Maximum power can be extracted from the coils in each axis, however, the power conversion stages will reduce the power available to the load. The three independent boost converters must be designed to operate above 1MHz which results in a typical power conversion efficiency of 80-90% for this power range. The buck stage can be operated with moderate switching frequencies assuming that the dc capacitor, C_{dc} , is rated adequately. Boost and buck regulation can be started once the dc link capacitor is charged sufficiently to allow operation of the regulators. Further design aspects of high frequency dc/dc converters were extensively discussed in [6-31], [6-32].

The equivalent ac resistance of the dc load can be calculated for each channel in a lossless system by using the power balance constraint and duty cycle definitions for buck and boost converters. The duty cycles are defined [6-4] as

$$V_{dc} = V_{dc_int} D_{buck} \quad (6-36)$$

and

$$V_{dc_int} = \frac{1}{1 - D_{boost,k}} V_{dc,k} \quad (6-37)$$

V_{dc_int} is the intermediate dc voltage, D_{buck} is the duty cycle of the buck stage, $D_{boost,k}$ is the duty cycle for boost stage k ($k=\{x,y,z\}$), and $V_{dc,k}$ is the average dc voltage at the rectifiers in axis k ($k=\{x,y,z\}$).

Using the power balance constraint and the fundamental frequency resistance transformation across rectifiers described in [6-22], the control circuit can be designed such that maximum power is extracted from each axis by matching the equivalent impedance of load with that of the receiving coil (including the parallel resonant circuit).

For the special case when all three axes contribute equally to the power flow towards the load the equivalent resistance R_{ss} for one axis becomes

$$R_{ac} = 3 \frac{\pi^2}{8} \frac{D_{buck}^2}{(1 - D_{boost})^2} R_{dc} \quad (6-38)$$

where

$$D_{boost} = D_{boost,x} = D_{boost,y} = D_{boost,z}$$

For a completely passive system with the rectifier fed by the parallel resonant circuit and directly supplying a dc capacitor as shown in Figure 6-34, the average dc voltage can be determined as a function of the ac voltage, frequency, dc-bus capacitor, and load resistance [6-18]. The average dc voltage for a single pulse rectifier is determined by

$$V_{dc,avg} = \frac{1 - \cos \delta}{2 \cos \rho} \frac{\sqrt{2}}{\pi} V_{cr} \quad (6-39)$$

where $\rho = \arctan(\omega C_{sdc} R_{sdc})$ and $\sin(\delta + \rho) - \sin \rho \exp\{-(2\pi - \delta) \cot \rho\} = 0$.

Appendix X shows a full derivation of derivation of (6-39). Invoking the power balance criteria, an approximate value for the equivalent ac resistance R_{ss} can be derived for various values of C_{dc} and R_{dc} . The ac resistance is given by

$$R_{ss} = 2 \left(\frac{\pi \cos \rho}{(1 - \cos \delta)} \right)^2 R_{dc} \quad (6-40)$$

The same principle can be applied if the power is extracted from each individual coil through a separate single pulse rectifier and fed to a common dc bus. A single pulse rectifier will introduce orientation dependence for maximum power extraction, which is discussed in Chapter 7. The introduction of a double pulse rectifier connecting the parallel resonant capacitance to the dc capacitor will reduce this dependence.

The completely passive system will not permit an extraction of the maximum power unless the load has a constant impedance. However, determination of the equivalent source resistance of the receiving coil with the parallel resonant circuit in conjunction with the determination of the load impedance range will support circuit design for maximum power transfer.

An energy storage device is required at the receiver side converter to balance short-term power imbalances between the load requirements and the power obtained from the magnetic field. The size of the energy storage device at the secondary side is dependent on the operating frequency and the anticipated shielding at the receiver as discussed in Chapter 3.

7 Experimental Validation

An experimental verification of selected theoretical results discussed in the previous chapters is presented. Field characteristics, shielding effects, and electrical power transfer characteristics were validated for various source and receiver systems.

7.1 Source side systems

A small-scale bi-directional source coil system (*Source Coil System I*, Table 7-1) and a larger omni-directional source coil system (*Source Coil System II*, Figure 7-2) were used to test uni-, bi-, and omni-directional system operation. The nominal operating frequency of both systems was set to 120kHz. For omni-directional systems the source side frequency of the z-axis coil set can be varied between 119kHz and 121kHz. The operating frequency was digitally controlled. The dimensions and electrical parameters of the test systems are summarized in Table 7-1. Source coil inductance predictions based on equation (5-6) match measured values within 7% and 11% for the small and the large coil system, respectively. The characteristics of the litz wire used in *source coil system II* are listed in Table 7-2. Both test-setups suffered from various non-idealities affecting the accuracy of the measurements:

- The three coil sets could not be aligned in perfect space quadrature resulting in a coupling (mutual inductance) between source coil sets and a field strength reduction throughout the operating volume.
- The floor, walls, and equipment located in the vicinity of the source coil system contained metals of unknown permeability and conductivity, reducing the field strength in the operating volume.

Three independent power supplies were used for the tests. The current in each source coil set can be independently controlled. Sinusoidal currents with adjustable magnitudes and frequencies are produced resulting in field strengths up to 15 A/m. The source side series resonant circuit is tuned to resonance using a saturable inductor. A forward converter regulates the intermediary dc-bus voltage magnitude directly affecting the magnitude of the output current. A class D half-bridge converter similar to the one shown in Figure 6-13 provides the ac output voltage. The block diagram of the power supplies is shown in Figure 6-2b, and the hardware set-up is shown in Figure 7-3.

Unless otherwise stated, the measurements referred to in this chapter are taken at the center of the source coil system in question.

Table 7-1 Source coil characteristics

	<i>Source Coil System I</i>	<i>Source Coil System II</i>
Coil axes	bi-directional	omni-directional
Coil side length	300 mm	1500 mm
Same axis coil separation	230mm	1000 mm
Winding number per coil	1	1
Same axis coil winding interconnection	series	series
Winding material	Litz wire	Rupalit Litz wire
Measured <i>x</i> axis winding resistance/inductance	0.1 Ohm/3.035 μ H	0.046 Ohm/ 7.772 μ H
Measured <i>y</i> axis winding resistance/inductance	0.1 Ohm/3.035 μ H	0.046 Ohm/ 7.772 μ H
Measured <i>z</i> axis winding resistance/inductance	-/-	0.046 Ohm/ 7.772 μ H

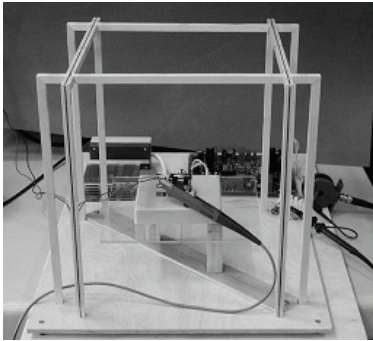


Figure 7-1 *Source Coil System I* and one receiver

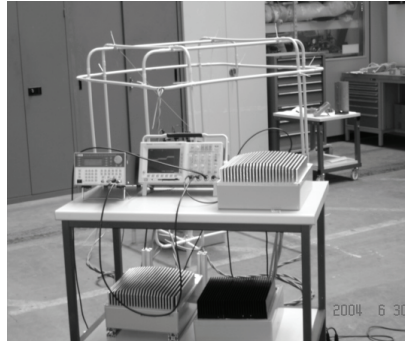


Figure 7-2 *Source Coil System II* and *x*-, *y*-, and *z*-axis power supplies

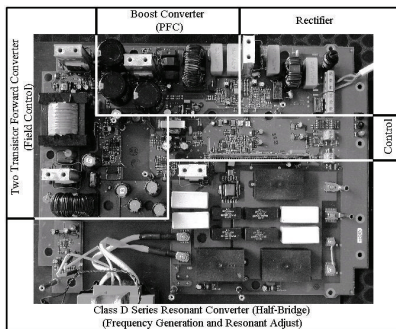


Figure 7-3 Internal structure of the *x*-axis power supply

Table 7-2 Source coil characteristics (*System II*)

Cable Manufacturer / Model	The Deeter Group / Rupalit
Nominal Diameter	4.2 mm
Litz Wire Design	7 x 30 x 40
Strand Nominal Diameter	0.10 mm
Strand Nominal Diameter (including insulation)	0.11 mm
DC Resistance	2.7 m Ω /m
DC Resistance of Cable Connectors	$\leq$ 0.05 m Ω
Resistance at 120kHz	$\leq$ 9 m Ω /m

7.2 Receiver side systems

Three different receiver designs were used in the experimental tests. The electrical and mechanical characteristics of the receivers used in the tests are summarized in Table 7-3.

Table 7-3 Receiver characteristics

Receiver	Type I	Type II	Type III
Coil directions	omni-directional	uni-directional	uni-directional
Coil side length	see Figure 7-4	38 mm	38 mm
Winding number per coil	24	14	14
Measured x axis winding resistance/inductance/capacitance	0.31 Ohm/ 45.08 μ H/39 nF	0.07 Ohm/ 22.76 μ H/ 77.3nF	0.05 Ohm/ 7.402 μ H/ 237n
Measured y axis winding resistance/inductance/capacitance	.32 Ohm/45.46 μ H/39 nF	- /-	- /-
Measured z axis winding resistance/inductance/capacitance	.32 Ohm/46.67 μ H/38 nF	- /-	- /-
Initial Permeability μ_i	2000 $\pm$ 25%	2000 $\pm$ 25%	1

Resistances and inductances are measured values. The tuning capacitors are formed by a parallel connection of two to four individual ceramic capacitors. Despite the best tuning efforts component tolerances lead to variations in the resonant frequency among the three receiving coil axes. The resonant frequencies of the coils on the *type I* receiver were measured at 120.32kHz, 120.46kHz, and 119.61kHz for the receiving coils wound on the *x*, *y*, and *z*-axes, respectively.

An analysis of 119 pre-series *type I* receivers having an optimized winding structure showed that the electrical parameters are strongly influenced by manufacturing tolerances. The results of the resistance and inductance measurements are shown in Table 7-4. *Type I* and *type II* receiver systems use the same ferrite core material, the relevant characteristics of which are shown in Table 7-5 and in Appendix VIII. Figure 7-5 shows the small receiver with its associated power conversion and communication board.

Table 7-4 Manufacturing tolerances for receivers (119 samples)

	L_{ac} [μ H]	R_{ac} [Ohm]
Minimum	34.91	0.25
Average	38.92	0.28
Maximum	40.35	0.34
Standard Deviation	0.840337	0.01230499

Table 7-5 Characteristics of core material used in receivers [7-1]

Initial permeability μ_i at 25°C	2000
Initial permeability μ_i at -20°C	1500
Initial permeability μ_i at 200°C	4700
Frequency range (kHz)	10-200

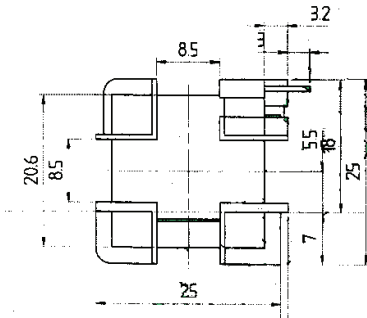


Figure 7-4 Layout of receiver (dimensions are in millimeters)

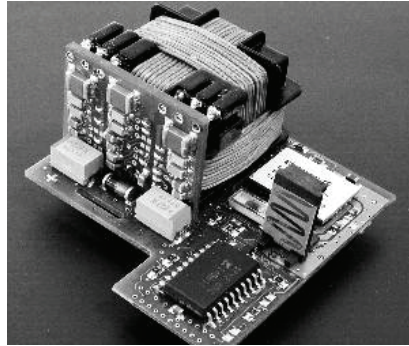


Figure 7-5 Receiver with resonant and communications circuits

Each receiving coil used in the experiments is separately compensated and tuned to the system's operating frequency of 120kHz. Multiple parallel ceramic surface mounted capacitors are used to tune the circuits. Three single pulse Schottky rectifiers connect the ac voltages of the three axes to a common dc-bus (Figure 7-6) for the dc voltage tests. A dc-link capacitor of 100nF is chosen. The communications board (shown as a base board in Figure 7-5) was not connected during the tests.

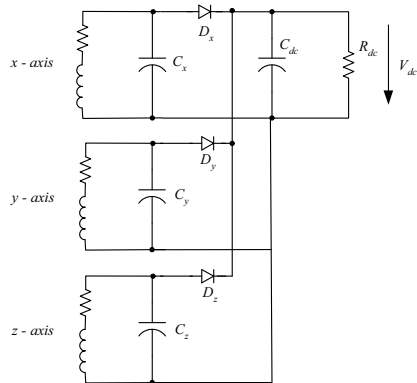


Figure 7-6 Equivalent circuit of receiver used for tests

7.3 Test equipment

The measurement equipment used in conducting all tests is summarized in Table 7-6.

Table 7-6 Test Equipment

Field Sensors	CCITT field probe, calibration factor: 0.0635 A/m per mV 50 Ohm input (FP_1) Kaschke cube, version 2, no. 1, calibration factor: 0.769 A/m per volt
Oscilloscope	Tektronix TDS1314 88520054
Current probe	ILA - UNI Stgt. SMZ200 88511500 Ch3, on x loop supply
Multimeter	Fluke 87 III True RMS EM163

7.4 Magnetic field validation

Expressions describing the field strength created by square coils were developed in Chapter 3. Theoretical values for the field strength created when current passes through the source coils are compared to measured values. These tests were performed using *source coil system II*.

7.4.1 Bi-directional field generation

Figure 7-7 shows the experimental and theoretical data for a field measurement if the field probe is moved away from the center of the system in the x -direction. Figure 7-8 shows the measurement conditions.

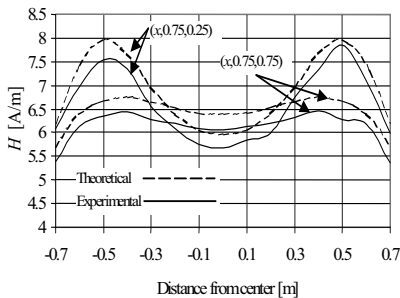


Figure 7-7 Theoretical and experimental results for field strength in a bi-directional system. $I_x=6A$, $f_x=120kHz$, $I_y=6A$, $f_y=120kHz$, $I_z=0A$. Measured in x - and y -directions using field probe i .

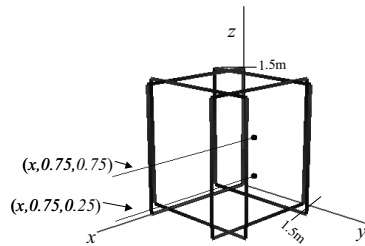


Figure 7-8 Lines along which the field in Figure 7-7 was measured. The center of the system is at $(0.75,0.75,0.75)$.

The predicted and measured values align well. The average error and the maximum error are 4.6% and 6.4%, respectively. Differences between theoretical and experimental results are assumed to be caused by non-idealities in the test setup as were described in section 7.1 above.

7.4.2 Omni-directional field generation

Several methods for creating omni-directional magnetic fields were presented in Chapter 3. The frequency shift method was chosen for laboratory testing because of its relative ease of implementation and because an uncomplicated single-axis frequency shift experiment produces results which give a clear indication of the effectiveness of omni-directional field generation techniques in mitigating the problems created by shielding of the source field in a bi-directional system.

The magnetic field vector produced at the center of the system rotates on the lateral surface of a cylinder as was shown in Chapter 3. The ac voltage induced on the three coils of a *type I* receiver were used for measurements in lieu of a three-dimensional field probe. The voltage induced on each of the receiving coils on a single receiver was sampled as the magnetic field vector generated by the source coils rotated through all of its possible orientations. The receiver was positioned such that each of its receiving coils was aligned with one of the source coils, and the x - and y -axis source coils were supplied with current at a constant 120kHz.

A comparison of the theoretical and experimental results at z -axis source frequencies of 115kHz (Figure 7-9, Figure 7-10) and 119kHz (Figure 7-11, Figure 7-12) confirm the results of Chapter 3 with regard to vector rotation and mesh density. The mesh created at a z -axis frequency of 115kHz is much less tight than that created at 119kHz, and the magnetic field vector rotates through all possible positions on the surface of the cylinder at a much faster rate in the 115kHz case than in the 119kHz case. These results were anticipated by the theoretical results presented in Chapter 3.

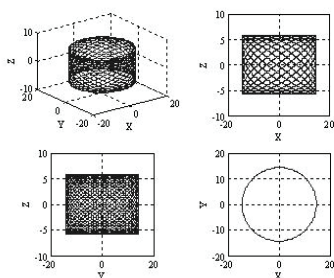


Figure 7-9 Theoretical results showing the voltages produced on x -, y -, and z -axis receiving coils. $f_x=120\text{kHz}$, $f_y=120\text{kHz}$, $f_z=115\text{kHz}$

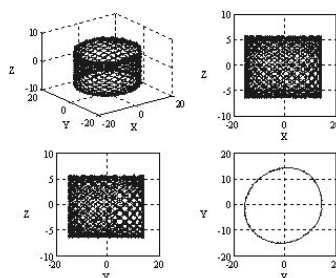


Figure 7-10 Experimental results showing the voltages produced on x -, y -, and z -axis receiving coils. $f_x=120\text{kHz}$, $f_y=120\text{kHz}$, $f_z=115\text{kHz}$

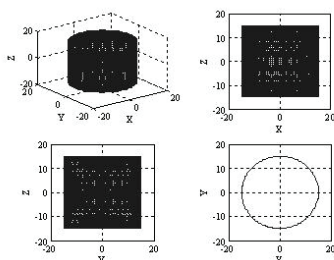


Figure 7-11 Theoretical results showing the voltages produced on x -, y -, and z -axis receiving coils. $f_x=120\text{kHz}$, $f_y=120\text{kHz}$, $f_z=119\text{kHz}$

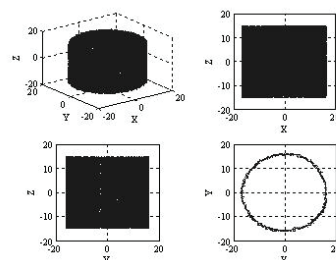


Figure 7-12 Experimental results showing the voltages produced on x -, y -, and z -axis receiving coils. $f_x=120\text{kHz}$, $f_y=120\text{kHz}$, $f_z=119\text{kHz}$

7.4.3 Validation of shielding effects

As a precise expression describing the effects of eddy currents on the source field is difficult to obtain mathematically [4-6], [4-7], a Finite Element Analysis (FEA) tool was used to obtain theoretical values of field attenuation due to the presence of an aluminium shield. Figure 7-13 shows the placement of the shield at the center of *source coil system II*. The shield is made of Aluminum ($\mu_r=1.000021$, $\sigma=3.6e7$), and has a length of 60cm, a width of 30cm, and a thickness of 3mm.

Figure 7-14, Figure 7-15, and Figure 7-16 show a comparison of theoretical and practical results for uni-, bi-, and omni-directional systems. Because of limitations in the FEA tool that was used, the frequency of the z-axis coil in Figure 7-16 could not be varied from 120kHz.

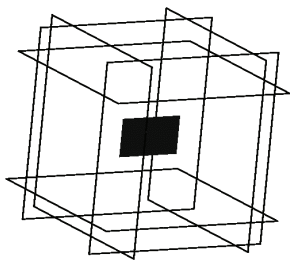


Figure 7-13 *Source coil system II* with shield used in experimental setup (Aluminum shield, $\mu_r=1.000021$, $\sigma=3.6e7$, length=60cm, width=30cm, thickness=3mm)

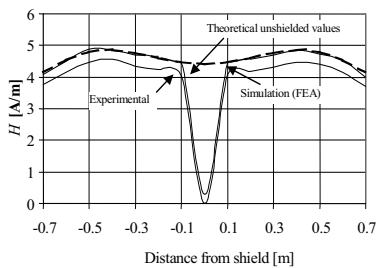


Figure 7-14 Simulated and experimental results for a uni-directional system (the shield is placed as shown in Figure 7-13). $I_x=6A$, $I_y=0A$, and $I_z=0A$. $f_x=120kHz$

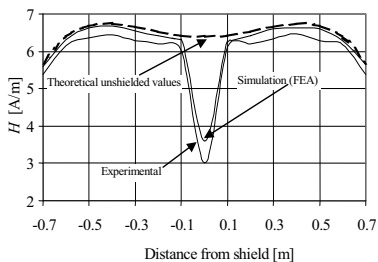


Figure 7-15 Simulated and experimental results for a bi-directional system (the shield is placed as shown in Figure 7-13). $I_x=6A$, $I_y=6A$, and $I_z=0A$. $f_x=120kHz$, $f_y=120kHz$

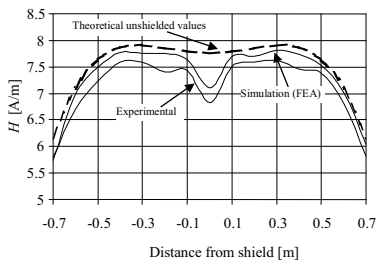


Figure 7-16 Simulated and experimental results for an omni-directional system (the shield is placed as shown in Figure 7-13). $I_x=6A$, $I_y=6A$, and $I_z=6A$. $f_x=120kHz$, $f_y=120kHz$, $f_z=120kHz$

The speed of vector rotation in the omni-directional field is a determining factor only in the sizing of the dc-capacitor on the receiver. Mesh density is relevant only in specialized applications where shielding may occur such that the receiver has access to the source field only at very specific angles with respect to the source coils. The speed of the vector rotation and the density of the mesh created by the rotation of the source field are relatively unimportant in comparison with the effect of the omni-directional field on the receiver voltage in cases where bi-directional shielding is present.

Figure 7-17 and Figure 7-18 show results of tests using aluminum shields ($\mu_r=1.000021$, $\sigma=3.6e7$, length=60cm, width=30cm, thickness=3mm) placed on one or more axes. The dashed lines represent the dc voltage present on a *type I* receiver with a load of 2.25k Ω . A current of 6A was applied to each of the three source coil sets, and the frequency of the x- and y-axis coil sets was 120kHz, while the frequency of the z-axis coil set was varied between 115kHz and 125kHz.

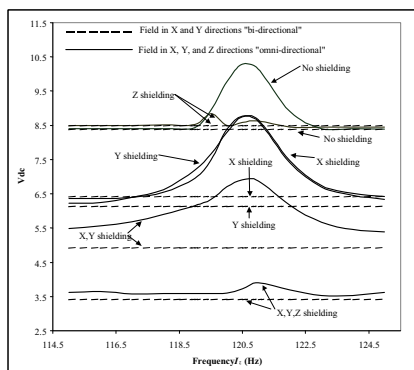


Figure 7-17 Comparison of shielding test results for bi- and omni-directional systems. Bi-directional system $I_x=6A$, $f_x=120kHz$, $I_y=6A$, $f_y=120kHz$. Omni-directional system $I_x=6A$, $f_x=120kHz$, $I_y=6A$, $f_y=120kHz$, $I_z=6A$, $f_z=115kHz-125kHz$

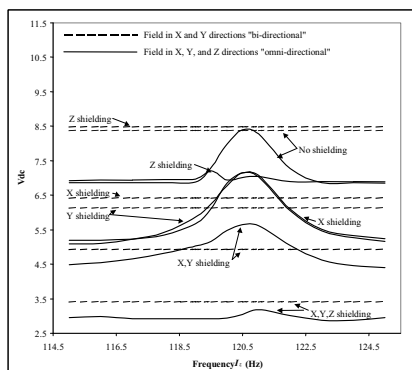


Figure 7-18 Comparison of normalized shielding test results for bi- and omni-directional systems

Figure 7-17 shows that when a system is shielded in the x, y, and z directions, the voltage available to the load is at its lowest value and is nearly the same for bi- and omni-directional source systems. This is, of course, an expected result as the receiver is shielded from the source field in all directions. The shield is not ideal, however, and some voltage is induced. The voltage produced at or near the resonant frequency of 120kHz is slightly higher in the omni-directional case because more energy is present on the source side of the system (three energized coil sets rather than two).

Figure 7-17 also shows that the effect of shielding in the z-direction on a bi-directional system are nearly identical to that of the same system with no shielding present. This is the expected result as the bi-directional system does not produce magnetic field

vector components in the z -direction and therefore shielding in the z -direction is not expected to have an appreciable effect. The slight variation seen here can be attributed to the shield not having been placed exactly perpendicular to the z -axis. Z -axis shielding on an omni-directional system is shown to produce roughly the same voltage as a bi-directional system with no shielding. This is, of course, expected and effectively changes the system from omni- to bi-directional. Shielding in the x - and y -directions are shown to have an identical effect on the load voltage for the omni-directional system. The slight variation seen in Figure 7-17 in load voltage between the x - and y -direction shielding for the bi-directional system can be attributed to imperfect shield placement during the experiment. The load voltage in the x -direction (and likewise in the y -direction) shielding case is significantly higher in the omni-directional system than in the bi-directional case, as shielding in the x - or y -directions effectively creates a uni-directional field in the shielded region of a bi-directional system. The load voltage produced by a bi-directional system shielded in either the x - or y -directions is considered to be sufficient for system operation. However, a bi-directional system shielded in both the x - and y -directions will not supply adequate voltage to the load. Figure 7-17 shows that the voltage available to the load of an omni-directional system shielded in the x - and y -directions is significantly higher (when f_z is at or near the resonant frequency) than that of a similarly shielded bi-directional system. This result shows that the omni-directional system does, in fact, provide an advantage in cases of bi-directional shielding.

Figure 7-18 shows the identical results normalized to assume identical power input in the bi- and omni-directional systems. With this representation the bi- and omni-directional systems produce the same load voltage in an unshielded case. It is clear from Figure 7-18 that the omni-directional system maintains an advantage over the bi-directional system when shielding is placed in both the x - and y -directions.

This test was performed using magnetic field vectors that rotate on the lateral surface of a cylinder (Figure 7-9). As was stated in Chapter 3, in order to form a truly omni-directional system (a system in which the source field rotates through all possible angles with respect to the source coil system), the cylinder must be “rotated” after some time. This is accomplished by alternating the source coil set carrying current at a second frequency. Such a system would, theoretically, further improve the advantages of the omni-directional system seen in Figure 7-17 and Figure 7-18.

7.5 Validation of electrical system characteristic

The electrical transfer characteristics were verified for selected operating conditions. Separately compensated receiving coils linked to a common dc-bus via a single pulse rectifier were used (Figure 7-6).

7.5.1 Uni-directional transfer characteristics for axis alignment

Figure 7-19 shows the measured source coil current, the source coil voltage, the magnitude of the magnetic field, and the receiver side dc voltage for a uni-directional

system using *source coil system I* and a *type I* receiver in axis alignment with the source coil system. An *rms* current of 1.77A was applied to the *x*-axis source coils and the field at the center of the system was 12A/m. The source coils lying on the *y*- and *z*-axes were not excited. The receiver was placed at the center of the source coil system and the voltage at the load was measured. Neglecting the tuning error on the *x*-axis receiving coil ($f_x=120.3\text{kHz}$) the dc voltage is predicted to be 11.43 volts (using (5-27), (6-30) and (6-34)) with the *x*-axis coil parameters in Table 7-3 and a load resistance of $2\text{k}\Omega$. The measured dc voltage is 10.8 volts, which results in an error of 5.8%.

The influence of coil size and core material on power transfer is shown in Figure 7-20 for *type I*, *type II*, and *type III* receivers as defined in Table 7-3. The field strength at the core location is 12A/m for the *type I* and *type III* receiver measurements (small coil and large coil without a core) and 15 A/m for the *type II* receiver (large coil with a core). The predicted output power is calculated using (5-28). The presence of the core material increases the flux density in the coils as anticipated. The presence of the core material also shifts the load impedance at which maximum power is transferred. This is consistent with the derivation of Z_{load_opt} in (6-31) in which a higher coil inductance shifts the optimum load resistance to a higher value.

The variation of the system transfer characteristics with operating frequency is shown in Figure 7-21. Results of two measurements are shown along with the analytical prediction using (5-27), (6-30) and (6-34). With the theoretical resonance curve superimposed on the measurements, the tuning error is clearly visible. The deviation between measured and predicted values is primarily attributed to component tolerances.

A typical variation of an unregulated dc voltage as a function of the load resistance is shown in Figure 7-27 for a uni-directional source coil system. The corresponding power as a function of resistance is shown in Figure 7-28.

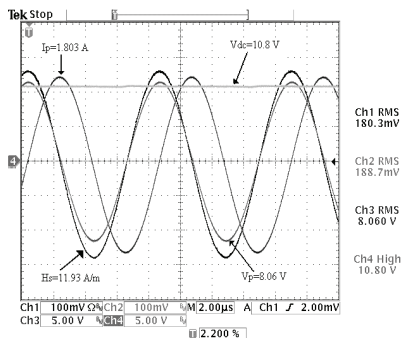


Figure 7-19 Primary terminal voltage (V_p), primary current (I_p), magnetic field intensity at receiving coil position (H_s), dc voltage (V_{dc}) at $R_{load}=2\text{k}\Omega$

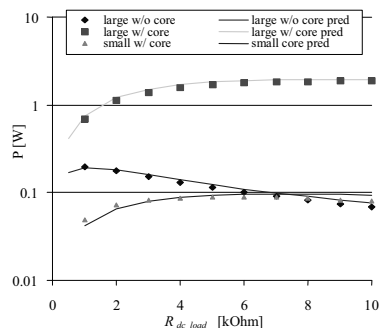


Figure 7-20 Dc power output as a function of core dimension and material for uni-directional system and a single pulse rectifier ($H=12 - 15\text{A/m}$, $f=120\text{kHz}$, Receiving coil *Type I – III*)

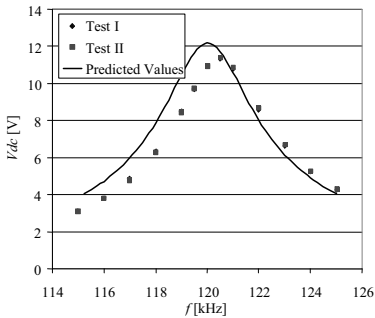


Figure 7-21 Transfer characteristic as function of frequency (*Type I* receiver, $H=8.7$ A/m, $R_{dc}=3.3k\Omega$)

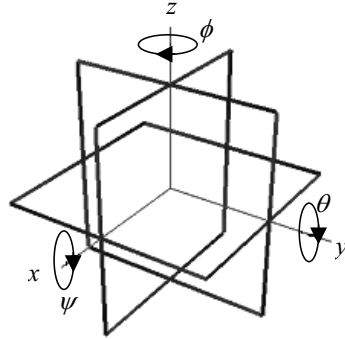


Figure 7-22 Angles of rotation for receiver (shown without core).

7.5.2 Uni and bi-directional transfer characteristics for axis misalignment

The total power transferred to a receiver is independent of its orientation (Figure 7-22) relative to the source coils regardless of the type of receiving coil connection (Chapter 6). However, the application of single pulse rectifiers as they were used in the test set-up (shown in Figure 7-6) causes the power available to a constant load resistor to be dependent upon the orientation of the receiver with respect to the source coil system.

Figure 7-23 shows the steady state dc voltage at various load conditions as a function of the angle of rotation ϕ of the receiver. The dc voltage changes as the receiver is rotated (with a constant load). The conduction duration of the individual single pulse rectifiers on each of the two axes are different for each angle of rotation relative to the source coil system. This leads to an optimum load resistance that varies with the angle of rotation. The dc-link voltage then varies for a constant load resistance.

The variation of the average dc voltage induced on a receiver placed at the center of the source coil system producing a uni-directional field is illustrated in Figure 7-24 for a constant load ($R_{dc}=2k\Omega$, $C_{dc}=100nF$) and at two operating angles ($\phi=0^\circ$ and $\phi=45^\circ$). The average dc voltage is clearly much higher when the axes of the source and receiver are aligned. At a load of $R_{dc}=2k\Omega$ (and at higher load resistances) the maximum power is derived when one of the receiving coil axes is fully aligned with the uni-directional source coil which occurs at angles of rotation of $\phi=0^\circ$ and $\phi=90^\circ$ relative to the source coil system. At $\phi=45^\circ$ the uni-directional field induces voltages in two axes with a magnitude of 70.7% of that of axis alignment. Two axes are feeding the load leading to a slower decay of the dc link voltages when the rectifier diodes are not conducting. This is confirmed by the measurements using uni-directional fields (shown in Figure 7-23). The dc-bus voltage is theoretically identical

at $\phi = n \cdot \pi / 2$ for $n = \{1, 2, 3 \dots\}$, but slight differences in tuning accuracy between the x - and y - axis receiving coils lead to a different dc voltage for $\phi = 0^\circ / 180^\circ$ and $\phi = 90^\circ$.

For a bi-directional source field excitation in the x and y -axis directions, the magnitude of the induced ac voltage at the receiver system shown in Figure 7-6 is identical in the two receiving coils for any angle ϕ , with $\theta = 0$ and $\psi = 0$. However the passive rectifier in conjunction with a bi-directional source field will lead to different dc-bus charging patterns depending on the angle of rotation ϕ . The optimum load resistance then varies with ϕ . The average dc voltage and the power available to the load will vary with ϕ at a constant load resistance. Figure 7-25 and Figure 7-26 show the receiver side dc voltages for identical source coil excitation and at an angle of $\phi = 45^\circ$ and $\phi = 135^\circ$, respectively. The voltages induced by the x - and y -axis source coils add differently for each of these angles such that the resulting receiving coil voltages have different phase angles. For $\phi = 45^\circ$ (and $\phi = 225^\circ$) the voltages induced on the x - and y -axis receiving coils are in phase (Figure 7-25 shows the voltages induced on both the x - and y -axis receiving coils). For $\phi = 135^\circ$ (and $\phi = 315^\circ$) the voltages induced on the x - and y -axis receiving coils are 180° apart (Figure 7-26). If $\phi = n \cdot \pi / 2$ for $n = \{0, 1, 2, 3\}$ the voltages induced on the receiving coils are 90° apart and in phase with the source coil voltages. The difference between the resulting dc voltage (and consequently the difference in output power) for a constant load at $\phi = 45^\circ$ and $\phi = 135^\circ$ are clearly visible in the measurements (Figure 7-23).

The orientation dependence of the dc-voltage is clearly more pronounced when using a single-pulse rectifier as only one half-wave of the ac voltage is rectified. The orientation dependence is also present in receivers with a double pulse characteristic, but the voltage and power variations created when the receiver is rotated are smaller at a given resistive load.

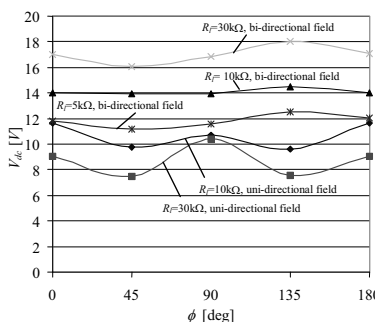


Figure 7-23 Steady state dc voltage induced on a receiver located at the center of the source coil system as a function of variation of the angle ϕ for uni- and bi-directional field excitation (Type 1 receiver, $H = 8.7$ A/m, $f = 120$ kHz)

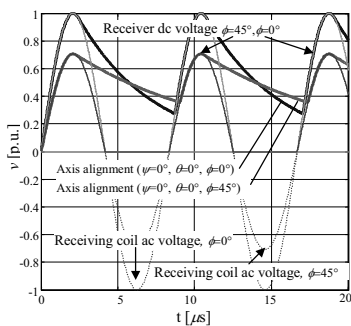


Figure 7-24 Normalized ac and dc voltages for uni-directional field excitation and a single pulse rectifier applied to each individual receiving coil. $\phi = 0$ and $\phi = 45$ ($R_{dc} = 2$ kOhm, $C_{dc} = 100$ nF, $f = 120$ kHz)

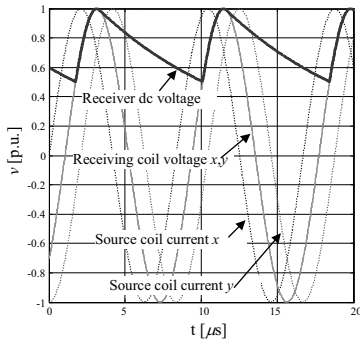


Figure 7-25 Normalized dc voltages and source coil currents for bi-directional field excitation and a single pulse rectifier applied to each individual receiving coil. $\phi=45^\circ$ ($R_{dc}=2k\Omega$, $C_{dc}=100nF$, $f=120kHz$)

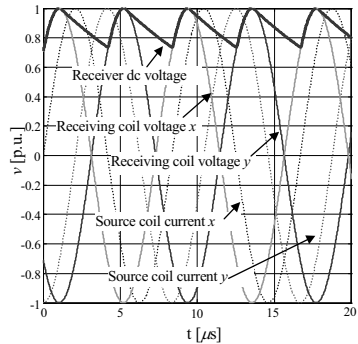


Figure 7-26 Normalized dc voltages and source coil currents for bi-directional field excitation and a single pulse rectifier applied to each individual receiving coil. $\phi=135^\circ$ ($R_{dc}=2k\Omega$, $C_{dc}=100nF$, $f=120kHz$)

7.5.3 Transfer characteristic of aligned omni-directional systems

The measured output voltages and power as a function of the load resistance are shown in Figure 7-27 and Figure 7-28 for uni-, bi- and omni-directional systems. The receiver shown in Figure 7-6 is used. The source and receiving coil axes are aligned ($\phi=0$, $\theta=0$, $\psi=0$).

With a bi-directional source coil system carrying currents in phase quadrature and with identical magnitude, the dc voltage at the receiver side at and around the maximum power point is $\sqrt{2}$ times higher than that of a comparable uni-directional system (Figure 7-27). As predicted by (5-28) the maximum power obtained in a bi-directional system is twice that available in a uni-directional system. The nonlinear characteristic of the single pulse rectifier will not retain this exact ratio over the entire load range. Figure 7-28 shows that the maximum power for the uni-directional and bi-directional system is achieved at different load resistances.

One of the three source coil sets is excited with 119kHz or 121kHz current for omni-directional field excitation. The power available to a load in an omni-directional system is predicted to be 1.5 times higher than that of a comparable bi-directional system. This is shown using (5-28) and Table 3-1. The peak power values measured for the omni-directional system are higher (121kHz) and lower (119kHz) than expected with reference to the bi-directional excitation. The differences are due to the effect of the resonant circuit in the receiver being tuned to a frequency that is closer to 120.5kHz than to 120 kHz (see Figure 7-21), which leads to a higher power at 121 kHz and a lower power at 119kHz. The different frequencies do not effect the equivalent source resistance (described by (6-30)) of the receiver side parallel resonant circuit.

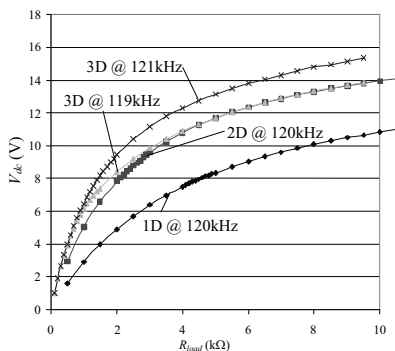


Figure 7-27 Measured voltage for various loads using a *type I* receiver (Uni-directional system (1D), Bi-directional system (2D), Omni-directional system (3D), $f=120\text{kHz}$)

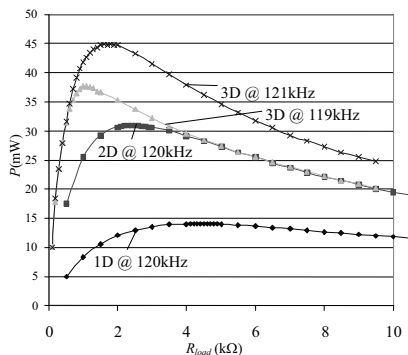


Figure 7-28 Measured power at load resistance (Uni-directional system (1D), Bi-directional system (2D), Omni-directional system (3D), $f=120\text{kHz}$)

7.6 Conclusions

The data presented in this chapter support the theoretical predictions made in the previous chapters. The sensitivity of the system to parameter variations is clearly visible. Specifically, the coil resistances and the tuning accuracy affect the performance. A highly accurate manufacturing process is required to ensure a quality product with consistent performance parameters.

The variation in the coupling factor, the quality factors of the source and receiving coils, the tuning accuracy, and the maximum power point adjustment are the main parameters that influence the power transfer characteristics. A passive receiver side rectifier will introduce orientation dependence to systems employing separate power extraction from each receiving coil.

8 Conclusions

A non-conventional transformer of the type described in this work was first proposed in 2002 [1-2]. These early systems used a bi-directional source field, and most of the research focused on power converter solutions for field generation [2-4] rather than on source and receiver system optimization. This work expanded that research by introducing an omni-directional system, providing a mathematical description of the field characteristics, an optimization of source coil spacing, an explanation of and solutions to problems of shielding of the source field, a circuit model from which output power can be predicted, and an optimized power converter solution for omni-directional field generation.

Bi-directional systems inherently suffer from problems due to shielding of the source field. In order to provide a reliable power supply in an industrial setting, the solution must provide power to receivers even in the presence of conductive or permeable materials. This work shows that the introduction of an omni-directional system will mitigate the problems associated with shielding of the source field.

A system was designed which can provide energy to a load via a magnetic field over distances of up to several meters. The system operates at frequencies below 135kHz and conforms to international standards for magnetic field exposure in an occupational environment.

Source coil systems were thoroughly analyzed with respect to optimum spacing distance, magnetic field uniformity, system and operating volumes, and modularity. The analysis showed that a rectangular source coil system can provide an adequate volume of magnetic field strength and has the advantage of simple assembly into a modular system. Optimum spacing of source coil pairs is application specific, and methods for determining optimum spacing were described for all anticipated application types. Helmholtz spacing is shown to be a reasonable starting point for coil spacing.

Several methods for creating an omni-directional system were developed and analyzed. Periodically switching the plane in which the field vector rotates provides power to the receiving coils at a single frequency. This solution does not provide a truly omni-directional system. It is, however, practical in cases when bi-directional systems are inadequate but field strength limits have been reached with two sets of source coils. Operating one of the three source coils at a slightly offset frequency is relatively simple to implement and is useful in cases where shielding will occur in areas such that a second frequency is required in only one of the source coil sets.

Double-axis amplitude modulation (the modulation of the current in two of the source coil pairs) provides constant power to the secondary coils and does not require the periodic switching between coil sets of any of the currents. The worst-case power available to the receivers at the center of a system is, however, lower than that obtained using the frequency shift method. Double axis amplitude modulation

generates power at the sidebands of the operating frequency and thus requires a receiver that is designed to extract sufficient power at these frequencies. However, its high adaptability to changing shielding conditions without the need for switching between source coil sets makes this a very attractive option.

Potential shielding of the source field by both permeable and conductive elements existing both in and near to the system volume was thoroughly analyzed. Several solutions were presented for circumstances in which an omni-directional source field cannot provide power to a receiver or receivers inside the operating volume. These solutions are generally expensive to implement and may, in some cases, require the generation of a magnetic field with a field strength in excess of that which is allowed by law. The addition of small ferrite rod antennas in the vicinity of a shielded receiver is the most promising solution.

Methods for estimating the inductance of both source and receiving coils were discussed. A general coupling model is developed for source and receiving coils. The coupling between source and receiving coils was thoroughly analyzed, and an example system was presented in which the coupling factor was shown to be approximately 0.03% at the center of the system. Coupling factors for most systems are expected to be in this range. The effect of rotation and orientation of the receiving coils was analyzed and incorporated into a description of the mutual inductances between coils in the system.

Power converter options for both source and receiving side are presented, with emphasis on omni-directional field generation and reception. Series and parallel resonant configurations are found to be optimum for source and receiver sides, respectively. Dynamic limitations of the source side power converters were presented. The sensitivity of the power extraction on the receiver side as function of winding interconnection and converter type were discussed. A series connection of receiving coils with a single resonant capacitor was found to be sufficient in an omni-directional system. A maximum-power-point circuit was designed to enable maximum power extraction.

Experimental results show the benefits of implementing an omni-directional field generation system in a shielded environment. A comparison with experimental test data shows that the model accurately described the behavior of the system. Adding a set of source coils to a bi-directional system to form an omni-directional system is beneficial in that it can substantially increase the power delivered to a load in a shielded environment.

Appendix I

International Standards – Magnetic Field Strength Limits

AI.1 International Commission on Non-Ionizing Radiation Protection (ICNIRP) Guidelines

Table AI-5 Reference levels for occupational exposure to time varying electric and magnetic fields (unperturbed *rms* values) [2-8]

Frequency range	E-field strength (V m ⁻¹)	H-field strength (A m ⁻¹)	B-field (μT)	Equivalent plane wave power density S_{eq} (W m ⁻²)
Up to 1 Hz	-	1.63×10^5	2×10^5	-
1-8 Hz	20000	$1.63 \times 10^5/f^2$	$2 \times 10^5/f^2$	-
8-25 Hz	20000	$2 \times 10^4/f$	$2.5 \times 10^4/f^2$	-
0.025-0.82 kHz	$500/f$	$20/f$	$25/f$	-
0.82-65 kHz	610	24.4	30.7	-
0.065-1 MHz	610	$1.6/f$	$2.0/f$	-
1-10 MHz	$610/f$	$1.6/f$	$2.0/f$	-
10-400 MHz	61	0.16	0.2	10
400-2000 MHz	$3f^{1/2}$	$0.008 f^{1/2}$	$0.1 f^{1/2}$	$F/40$
2-300 GHz	137	0.36	0.45	50

NOTES:

1. f as indicated in the frequency range column
2. Provided that basic restrictions are met and adverse indirect effects can be excluded, field values can be exceeded.
3. For frequencies between 100kHz and 10GHz S_{eq} , E^2 , H^2 , and B^2 , are to be averaged over any 6-min period.
4. For peak values at frequencies up to 100kHz, peak current density values can be obtained by multiplying the rms value by $\sqrt{2}$. For pulses of duration t_p the equivalent frequency to apply in the basic restrictions should be calculated as $f = 1/(2 t_p)$.
5. For peak values at frequencies exceeding 100kHz see figures AI-1 and AI-2. Between 100kHz and 10MHz, peak values for the field strengths are obtained by interpolation from the 1.5-fold peak at 100kHz to the 32-fold peak at 10MHz. For frequencies exceeding 10MHz it is suggested that the peak equivalent plane wave power density, as averaged over pulse width, does not exceed 1,000times the S_{eq} restrictions, or that the field strength does not exceed 32 times the field strength exposure levels given in the table.
6. For frequencies exceeding 10GHz, S_{eq} , E^2 , H^2 , and B^2 , are to be averaged over any $68/f^{1.02}$ -min period (f in GHz).
7. No E-field value is provided for frequencies < 1Hz, which are effectively static electric fields. Electric shock from low impedance sources is prevented by established electrical safety procedures for such equipment.

Table AI-6 Reference levels for general public exposure to time varying electric and magnetic fields
(unperturbed *rms* values) [2-8]

Frequency range	E-field strength (V m ⁻¹)	H-field strength (A m ⁻¹)	B-field (μT)	Equivalent plane wave power density S_{eq} (W m ⁻²)
Up to 1 Hz	-	3.2×10^4	4×10^4	-
1-8 Hz	10000	$3.2 \times 10^4/f^2$	$4 \times 10^4/f^2$	-
8-25 Hz	10000	$4000/f$	$5000/f$	-
0.025-0.8 kHz	$250/f$	$4/f$	$5/f$	-
0.8-3 kHz	$250/f$	5	6.25	-
3-150 kHz	87	5	6.25	-
0.15-1 MHz	87	$0.73/f$	$0.92/f$	-
1-10 MHz	$87f^{1/2}$	$0.73/f$	$0.92/f$	-
10-400 MHz	28	0.073	0.092	2
400-2000 MHz	$1.375 f^{1/2}$	$0.0037 f^{1/2}$	$0.0046 f^{1/2}$	$f/200$
2-300 GHz	61	0.16	0.20	10

NOTES:

1. f as indicated in the frequency range column
2. Provided that basic restrictions are met and adverse indirect effects can be excluded, field values can be exceeded.
3. For frequencies between 100kHz and 10GHz S_{eq} , E^2 , H^2 , and B^2 , are to be averaged over any 6-min period.
4. For peak values at frequencies up to 100kHz, peak current density values can be obtained by multiplying the rms value by $\sqrt{2}$. For pulses of duration t_p the equivalent frequency to apply in the basic restrictions should be calculated as $f = 1/(2 t_p)$.
5. For peak values at frequencies exceeding 100kHz see figures AI-1 and AI-2. Between 100kHz and 10MHz, peak values for the field strengths are obtained by interpolation from the 1.5-fold peak at 100kHz to the 32-fold peak at 10MHz. For frequencies exceeding 10MHz it is suggested that the peak equivalent plane wave power density, as averaged over pulse width, does not exceed 1,000times the S_{eq} restrictions, or that the field strength does not exceed 32 times the field strength exposure levels given in the table.
6. For frequencies exceeding 10GHz, S_{eq} , E^2 , H^2 , and B^2 , are to be averaged over any $68/f^{0.02}$ -min period (f in GHz).
7. No E-field value is provided for frequencies < 1Hz, which are effectively static electric fields. Electric shock from low impedance sources is prevented by established electrical safety procedures for such equipment.

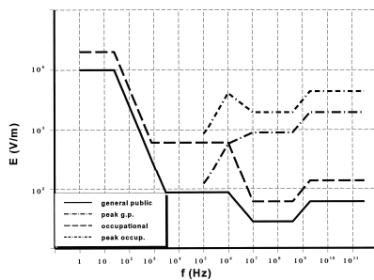


Fig. AI-1 Reference levels for exposure to time varying electric fields (compare table AI-5 and AI-6) [2-8]

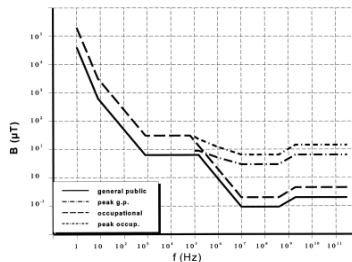


Figure AI-2 Reference levels for exposure to time varying magnetic fields (compare tables AI-5 and AI-6) [2-8]

AI.2 IEEE Standard

Table AI-2 Maximum permissible exposure limits [2-6]

Frequency range (MHz)	Electric field strength (E) (V/m)	Magnetic field strength (H) (A/m)	Power density (S) E-field, H-field (mW/cm ²)	Averaging time $ E ^2$, $ H ^2$ or S (min)
0.003-0.1	614	163		
0.1-3.0	614	$16.3/f$	$(100, 10000/f^2)$	6
3-30	$1842/f$	$16.3/f$	$(900/f^2, 10000/f^2)$	6
3-100	61.4	$16.3/f$	$(1.0, 10000/f^2)$	6
100-300	61.4	0.163	1.0	6
300-3000	-	-	$f/300$	6
3000-6000			10	6
6000-30000			10	$4.11 \cdot 10^6 / f^{4.544}$
30000-300000	-	-	10	$68 / f^{0.477}$
Note – f is the frequency in MHz				

AI.3 Government of Canada

Table AI-3 H-field limits at 10m [AI-1]

Frequency range (MHz)	H-field strength limit (H_f) dBμA/m at 10m
$0.009 \leq f < 0.03$	72 or according to the note
$0.03 \leq f < 0.07$ $0.119 \leq f < 0.135$	72 at 0.03 MHz descending 3 dB/oct or according to the note
$0.05975 \leq f < 0.06025$ $0.07 \leq f < 0.119$	42
$0.135 \leq f < 1.0$	37.7 at 0.135 MHz descending 3 dB/oct
$1.0 \leq f < 4.642$	29 at 1.0 MHz descending 9 dB/oct
$4.642 \leq f < 30.0$	9
$6.765 \leq f < 6.795$ $13.553 \leq f < 13.567$ $26.957 \leq f < 27.283$	42
NOTE: For the frequency ranges 9 to 70 kHz and 119 to 135 kHz, the following additional restrictions apply to the higher limits: - For loop coil antennas with area $\geq 0.16\text{m}^2$ table 2 applies directly; - For loop coil antennas with an area between 0.05m² and 0.16m² table AI-3 applies with a correction factor. The limit is: table value + $10 \cdot \log(\text{area}/0.16\text{m}^2)$; - For loop coil antennas with an area $< 0.05\text{m}^2$ the limit is 10dB below table AI-3	

AI-4 European Telecommunications Standards Institute

Table AI-4 RF/MW occupational exposure limits [AI-2]

Frequency (MHz)	RMS Electric Field Strength (V/m)	RMS Magnetic Field Strength (A/m)	Power Density (mW/cm ²)
0.01 - 1	600	4.9	--
1 - 10	600 / f	4.9 / f	--
10 - 30	60	4.9 / f	--
30 - 300	60	0.163	1
300 - 1500	$3.46 \sqrt{f^2}$	$0.0093 \sqrt{f^2}$	$f / 300$
1500 - 300000	140	0.36	5

Note: RMS = root mean square | f = frequency in MHz | A/m = amperes per meter | mW/cm² = milliwatts per square centimeter | V/m = volts per meter

AI.4 German Work Safety Organization (Berufsgenossenschaft)

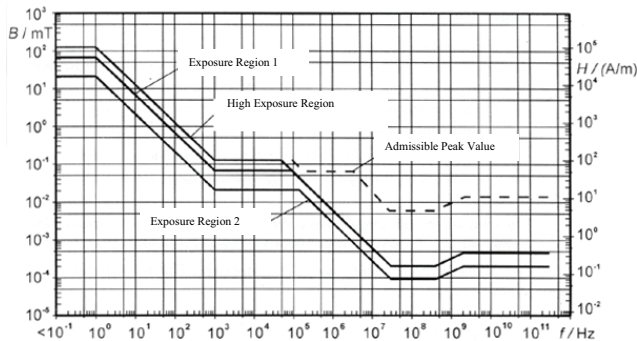


Figure AI-3 Allowable levels for exposure to time varying magnetic fields in different exposure regions [AI-3]

Table AI- 7 Reference levels for durations ≥ 6 minutes in Exposure Region 1² [AI-3]

Frequency Range f / MHz	Effective Electric Field Strength V/m	Effective Magnetic Field Strength A/m	Average Power Density W/m ²
0.091 - 0.576	1066	$4.9/f$	(-)
0.576 - 10	$614/f$	$4.9/f$	(-)
10 - 30	61.4	$4.9/f$	(-)
30 - 400	61.4	0.163	10
400 - 2000	$3.07 \sqrt{f}$	$8.14 \sqrt{f} \cdot 10^{-3}$	$f/40$
2000 - 300000	137.3	0.364	50

² Exposure Region 1 = Controlled areas in which people have access only for a limited time.

Table AI- 8 Reference levels for durations ≥ 6 minutes in Exposure Region 2³ [AI-3]

Frequency Range f / MHz	Effective Electric Field Strength V/m	Effective Magnetic Field Strength A/m	Average Power Density W/m ²
0.091 – 0.14	333.3	16.8	(-)
0.14 – 0.826	333.3	$2.35/f$	(-)
0.826 - 10	$275/f$	$2.35/f$	(-)
10 - 30	27.5	$2.35/f$	
30 - 400	27.5	0.073	2
400 - 2000	$1.375\sqrt{f}$	$3.64\sqrt{f} \cdot 10^{-3}$	$f/200$
2000 - 300000	61.5	0.163	10

Table AI- 9 Peak levels for durations < 6 minutes in High Exposure Regions⁴ [AI-3]

Frequency Range f / MHz	Effective Electric Field Strength V/m	Effective Magnetic Field Strength A/m	Average Power Density W/m ²
0.091 – 0.1	2222	$10/f$	(-)
0.1 – 0.2	$22,222f$	$10/f$	(-)
0.2 - .3	$22,222f$	50	(-)
.3 - 3	6667	50	(-)
3 -10	$20,000f$	$150/f$	(-)
10 - 30	2000	$150/f$	(-)
30 - 400	2000	5	10,000
400 - 2000	$100\sqrt{f}$	$0.25\sqrt{f}$	$25f$
2000 - 300000	4472	11.2	50,000

³ Exposure Region 2 = Generally accessible areas such as offices or social rooms. No time limits.

⁴ Regions of High Exposure = Limits of Exposure Region 1 are exceeded; exposure is allowed only for a limited time. From 29kHz-91kHz < 2 hours. From 91kHz-300GHz < 6 minutes.

Table AI-10 Peak field strength E and H for the protection of people with pacemakers [2-7]

0 Hz ≤ f < 25 Hz	$\frac{H}{1A/m} \leq 800$ $\max\left\{\frac{ H(t) }{1A/m} + \frac{ E(t) }{80V/m}\right\} \leq \frac{5.6kHz}{f}$	$\frac{E}{1V/m} \leq 14140$
25 Hz ≤ f < 1000 Hz	$\max\left\{\frac{ H(t) }{1A/m} + \frac{ E(t) }{80V/m}\right\} \leq \frac{5.6kHz}{f}$	$\frac{E}{1V/m} \leq \frac{353kHz}{f}$
1 kHz ≤ f < 1.72 kHz	$\frac{H}{1A/m} \leq 7.07$ $\max\left\{\frac{ H(t) }{1A/m} + \frac{ E(t) }{80V/m}\right\} \leq 5.6 \frac{f}{1kHz}$	$\frac{E}{1V/m} \leq \frac{353kHz}{f}$
1.72 kHz ≤ f < 3 kHz	$\frac{H}{1A/m} \leq 7.07$	$\frac{E}{1V/m} \leq \frac{353kHz}{f}$
3 kHz ≤ f < 100 kHz		$\frac{E}{1V/m} \leq 123$
100 kHz ≤ f < 150 kHz	$\frac{H}{1A/m} \leq 7.07 \left(\frac{f}{100kHz}\right)^{0.42}$	$\frac{E}{1V/m} \leq 123 \left(\frac{f}{100kHz}\right)^{0.42}$
150 kHz ≤ f < 1000 kHz	$\frac{H}{1A/m} \leq 2.7 \left(\frac{1MHz}{f}\right)^{0.58}$	
1 MHz ≤ f < 5.33 MHz	$\frac{H}{1A/m} \leq 2.7 \left(\frac{1MHz}{f}\right)^{0.62}$	$\frac{E}{1V/m} \leq 322 \left(\frac{1MHz}{f}\right)^{0.12}$
5.33 MHz ≤ f < 10 MHz	$\frac{H}{1A/m} \leq 0.51 \left(\frac{10MHz}{f}\right)$	$\frac{E}{1V/m} \leq 193 \left(\frac{10MHz}{f}\right)^{0.5}$
10 MHz ≤ f < 200 MHz	$\frac{H}{1A/m} \leq 0.51$	$\frac{E}{1V/m} \leq 196$
200 MHz ≤ f < 400 MHz	$\frac{H}{1A/m} \leq 0.51 \left(\frac{f}{200MHz}\right)^{0.66}$	$\frac{E}{1V/m} \leq 196 \left(\frac{f}{200MHz}\right)^{0.66}$
400 MHz ≤ f < 2000 MHz	$\frac{H}{1A/m} \leq 0.82 \left(\frac{f}{400MHz}\right)^{1.16}$	$\frac{E}{1V/m} \leq 304 \left(\frac{f}{400MHz}\right)^{1.16}$

Appendix II

Table AII-1 Frequency ranges for RFID-Systems [AII-1]

Frequency range	Comment	Permitted field strength/ transmission power
< 135 kHz	Low frequency, inductive coupling	72 dB μ A/m
6.765 - 6.795 MHz	Medium frequency (ISM), inductive coupling	42 dB μ A/m
7.400 - 8.800 MHz	Medium frequency, used for EAS (electronic article surveillance) only	9 dB μ A/m
13.553 - 13.567 MHz	Medium frequency (13.56 MHz, ISM), inductive coupling, wide spread usage for contactless smartcards (ISO 14443, MIFARE, LEGIC, ...), smart labels (ISO 15693, Tag-It, I-Code, ...) and item management (ISO 18000-3).	42 dB μ A/m
26.957 - 27.283 MHz	Medium frequency (ISM), inductive coupling, special applications only	42 dB μ A/m
433 MHz	UHF (ISM), backscatter coupling, rarely used for RFID	10 - 100 mW
868 - 870 MHz	UHF (SRD), backscatter coupling, new frequency, systems under development	500 mW, Europe only
902 - 928 MHz	UHF (SRD), backscatter coupling, several systems	4 W - spread spectrum, USA/Canada only
2.400 - 2.483 GHz	SHF (ISM), backscatter coupling, several systems, (vehicle identification: 2.446 - 2.454 GHz)	4 W - spread spectrum, USA/Canada only, 500 mW, Europe
5.725 - 5.875 GHz	SHF (ISM), backscatter coupling, rarely used for RFID	4 W USA/Canada, 500 mW Europe

Table AII-1 Allocation of the Radio Spectrum in the United States of America [AII-2]

9 - 30 kHz	Cable locating equipment [15.213] Power line carrier systems (operated by a power utility for general supervision of the power system) [15.113]
30 - 70 kHz	Cable locating equipment Power line carrier systems (operated by a power utility for general supervision of the power system)
60 kHz	Radio station WWVB (time and frequency signals)
70 - 90 kHz	Cable locating equipment Radiolocation, land or mobile International Fixed Service Maritime Mobile Service Power line carrier systems (operated by a power utility for general supervision of the power system)
90 - 110 kHz	LORAN-C navigation system Cable locating equipment
101.4 kHz	Transmitters used to detect buried electronic markers used by telephone companies Radiolocation, land Power line carrier systems (operated by a power utility for general supervision of the power system)
110 - 130 kHz	Cable locating equipment Radiolocation, land or mobile Power line carrier systems (operated by a power utility for general supervision of the power system) International Fixed Service Maritime Mobile Service
130 - 160 kHz	Cable locating equipment Power line carrier systems (operated by a power utility for general supervision of the power system)

Appendix III

AIII.1 Magnetic flux density created by a circular current carrying element.

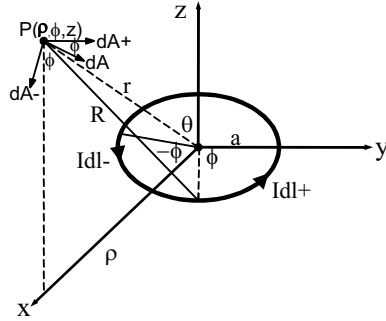


Figure AIII-1 - Circular coil in xy -plane

$$dA = dA_\phi = 2dA_\phi \cos \phi = \frac{\mu_o I a \cos \phi}{2\pi R} d\phi \quad (\text{AIII-1})$$

gives

$$A = \frac{\mu_o I}{2\pi\rho\sqrt{(a+\rho)^2 + z^2}} \left[(a^2 + \rho^2 + z^2)K(k) - ((a+\rho)^2 + z^2)E(k) \right] \quad (\text{AIII-2})$$

Where $K(k)$ is a complete elliptic integral of the first kind

$$K(k) = \int_0^{\pi/2} \frac{d\Phi}{\sqrt{1 - k^2 \sin^2 \Phi}} \quad (\text{AIII-3})$$

And $E(k)$ is a complete elliptic integral of the second kind

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \Phi} d\Phi \quad (\text{AIII-4})$$

with k defined as

$$k = 2\sqrt{\frac{a\rho}{(a+\rho)^2 + z^2}} \quad (k \in [0,1]) \quad (\text{AIII-5})$$

Calculating the curl of the vector magnetic potential A to find B

$$\vec{B} = \nabla \times \vec{A} = \nabla \times (e_\phi A_\phi) = -e_\rho \frac{\partial A}{\partial z} + \frac{e_z}{\rho} \frac{\partial}{\partial \rho} (\rho A) \quad (\text{AIII-6})$$

The components of the field are then

$$B_{\rho} = \frac{-\partial A}{\partial z} = \frac{\mu_0 I z}{2\pi\rho\sqrt{(a+\rho)^2 + z^2}} \left[\frac{a^2 + \rho^2 + z^2}{(a-\rho)^2 + z^2} E(k) - K(k) \right] \quad (\text{AIII-7})$$

$$B_z = \frac{A}{\rho} + \frac{\partial A}{\partial \rho} = \frac{\mu_0 I}{2\pi\sqrt{(a+\rho)^2 + z^2}} \left[\frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2} E(k) + K(k) \right]$$

Giving a magnitude of

$$|B| = \frac{\mu_0 I}{2\pi\sqrt{(a+\rho)^2 + z^2}} \left[\frac{z^2}{\rho^2} \left[\frac{a^2 + \rho^2 + z^2}{(a-\rho)^2 + z^2} E(k) - K(k) \right]^2 + \left[\frac{a^2 - \rho^2 - z^2}{(a-\rho)^2 + z^2} E(k) + K(k) \right]^2 \right]^{1/2} \quad (\text{AIII-8})$$

AIII.2 Magnetic flux density created by rectangular current carrying elements.

The fundamental postulates of magnetostatics in free space

$\nabla \cdot \vec{B} = 0$ and $\nabla \times \vec{B} = \mu_0 \vec{J}$ can be used to derive the Biot-Savart law

$$\vec{B} = \frac{\mu_0 I}{4\pi} \oint_C \frac{d\vec{l} \times \hat{a}_R}{R^2} \quad (\text{AIII-9})$$

which is used to determine B caused by a current I in a closed path C' .

Applying this law to a straight current carrying wire of length $2l$, the equation describing the magnetic field at an arbitrary point in space P can be found.

$$\vec{B}_P = \hat{a}_{\phi} \frac{\mu_0 I l}{2\pi r \sqrt{l^2 + r^2}} \quad (\text{AIII-10})$$

where I is the current in the wire and r is the perpendicular distance between the point P at which the field is being calculated and the current carrying wire.

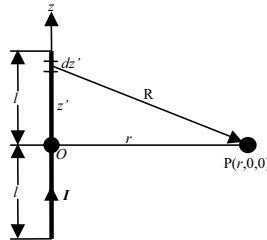


Figure AIII-2 Straight current carrying wire of length $2l$

The radial nature of the field created by one straight current carrying wire naturally leads to the use of a cylindrical coordinate system in order to obtain the simplest description of that field. When examining the field created by multiple wires in one or more planes, it becomes far simpler to use a rectangular coordinate system. Equation (AIII-11) can be expressed as

$$\vec{B}_P = \hat{a}_x \left(-\sin\phi \frac{\mu_0 I}{2\pi r \sqrt{l^2 + r^2}} \right) + \hat{a}_y \left(\cos\phi \frac{\mu_0 I}{2\pi r \sqrt{l^2 + r^2}} \right) \quad (\text{AIII-11})$$

The magnetic field at an arbitrary point in space due to four wires orthogonal to the xy -plane can be expressed as a vector sum of the fields created by each of the straight wires. With $H_{rect} = B_P / \mu_0$,

$$\begin{aligned} \vec{H}_{rect} = & \hat{a}_x \left(-\sin\phi_1 \frac{I_1}{2\pi_1 \sqrt{l_1^2 + r_1^2}} + \sin\phi_2 \frac{I_2}{2\pi_2 \sqrt{l_2^2 + r_2^2}} - \cos\phi_3 \frac{I_3}{2\pi_3 \sqrt{l_3^2 + r_3^2}} + \cos\phi_4 \frac{I_4}{2\pi_4 \sqrt{l_4^2 + r_4^2}} \right) \\ & + \hat{a}_y \left(\cos\phi_1 \frac{I_1}{2\pi_1 \sqrt{l_1^2 + r_1^2}} - \cos\phi_2 \frac{I_2}{2\pi_2 \sqrt{l_2^2 + r_2^2}} \right) + \hat{a}_z \left(\sin\phi_3 \frac{I_3}{2\pi_3 \sqrt{l_3^2 + r_3^2}} - \sin\phi_4 \frac{I_4}{2\pi_4 \sqrt{l_4^2 + r_4^2}} \right) \end{aligned} \quad (\text{AIII-12})$$

Where the subscripted variables r are the perpendicular distances from each of the four wires to the field point in question and the subscripted variables ϕ are the angles between the x -axis and the imaginary line of length r connecting the field point with the wire.

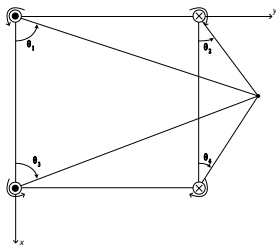


Figure AIII-3 Angles in the xy -plane

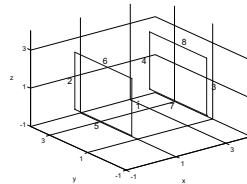


Figure AIII-4 Uni-directional system of (AIII-14)

In this case $\phi_1 = \theta_1, \phi_2 = \theta_2, \phi_3 = \pi - \theta_3, \phi_4 = \pi - \theta_4$

The field at an arbitrary point created by a set of rectangular coils can be expressed as the vector sum of the fields created by each of the coils.

Numbering the sides of the rectangular coils as in Figure AIII-4,

$$\bar{B} = \sum_{i=1}^{2n} [(-1)^i \sin \phi_i X_i \hat{a}_x + (-1)^{i+1} \cos \phi_i X_i \hat{a}_y] + \sum_{i=2n+1}^{4n} [(-1)^{i+1} \sin \phi_i X_i \hat{a}_z + (-1)^i \cos \phi_i X_i \hat{a}_x]$$

(AIII-13)

Where $X_i = \frac{\mu_0 I_i}{2\pi r_i \sqrt{l_i^2 + r_i^2}}$, n =number of coils

Appendix IV

Derivation of the lower-bound voltage for the capacitor charge-discharge characteristic of Figure 3-25.

The source voltage in the circuit of Figure 3-24 can be defined as

$$V_s = R_s i + u_c(t) \quad (\text{AIV-1})$$

with the load current being

$$i_l = \frac{u_c(t)}{R_l} \quad (\text{AIV-2})$$

The source voltage can be rewritten as

$$V_s = R_s \left(i_c + \frac{u_c(t)}{R_l} \right) + u_c(t) \quad (\text{AIV-3})$$

with

$$i_c = C \frac{du_c(t)}{dt}$$

Further simplifications of the source voltage can be made

$$V_s = R_s \left(C \frac{du_c(t)}{dt} + \frac{u_c(t)}{R_l} \right) + u_c(t) \quad (\text{AIV-4})$$

and

$$V_s = R_s C \frac{du_c}{dt} + \left(\frac{R_s}{R_l} + 1 \right) u_c(t) \quad (\text{AIV-5})$$

Converting to the frequency domain

$$\frac{V_s(s)}{s} = R_s C s u_c(s) - R_s C V_0 + \left(\frac{R_s}{R_l} + 1 \right) u_c(s) \quad (\text{AIV-6})$$

where V_0 is the initial voltage across the capacitor.

Solving for the capacitor voltage

$$u_c(s) = \frac{V_s + sR_s C V_0}{s \left(R_s C s + \frac{R_s}{R_l} + 1 \right)} \quad (\text{AIV-7})$$

where $s_0 = 0$ and $s_1 = -\frac{R_s + R_l}{R_s R_l C}$

the voltage at the capacitor can be written as

$$u_c = \frac{L(s)}{sM(s)} \quad (\text{AIV-8})$$

where $L(0) = V_s$, $M(0) = \frac{R_s + R_l}{R_l}$, $L(s_1) = V_s - \frac{R_s + R_l}{R_l} V_0$, and $M'(s_1) = R_s C$

converting to the time domain

$$u_c(t) = \frac{R_l V_s}{R_s + R_l} - \frac{V_s R_l - (R_s + R_l) V_0}{R_s + R_l} e^{-\frac{R_s + R_l}{R_s R_l C} t} \quad (\text{AIV-9})$$

Assuming that the switch is closed for $t = 2t_{seq}$,

$$V_{ub} = \frac{R_l V_s}{R_s + R_l} - \frac{V_s R_l - (R_s + R_l) V_0}{R_s + R_l} a \quad (\text{AIV-10})$$

where

$$a = e^{-2t \frac{R_s + R_l}{R_s R_l C}}$$

V_{lb} is defined as the voltage remaining across the capacitor after a discharge time of $t = t_{seq}$.

$$V_{lb} = V_{ub} e^{-\frac{1}{R_l C} t_{seq}} \quad (\text{AIV-11})$$

which can also be written as

$$V_{ub} = b V_{lb} \quad (\text{AIV-12})$$

where $b = \frac{1}{e^{-\frac{1}{R_l C} t}}$

combining equations gives

$$V_{lb}b = \frac{R_l V_s}{R_s + R_l} - a \frac{V_s R_l - (R_s + R_l) V_{lb}}{R_s + R_l} \quad (\text{AIV-13})$$

which can be simplified to

$$V_{lb}(b-a) = \frac{R_l V_s - R_l V_s a}{R_s + R_l} \quad (\text{AIV-14})$$

Finally, V_{lb} can be written as a function of the circuit parameters

$$V_{lb} = \frac{R_l V_s (1 - e^{-2t_{\text{seq}} \frac{R_s + R_l}{R_s R_l C}})}{(e^{\frac{1}{R_l C} t_{\text{seq}}} - e^{-2t_{\text{seq}} \frac{R_s + R_l}{R_s R_l C}})(R_s + R_l)} \quad (\text{AIV-15})$$

Appendix V

Table AV-1 Measured effectiveness of shielding materials against low-impedance waves [AV-1]

Form		Material	Thick- ness (mils)	Nominal Effectiveness (dB)					
General	Detail			0.1 kHz	1 kHz	10 kHz	85 kHz	1 MHz	10 MHz
Solid sheet		Cu	125	8	22	58			
			63				97		
			31	4	11	29	59	120	
			4.5				34	55	
		Al	125	5	18	50			
			63	1	16	35	78		
			31	1	10	24			
		Steel ($\mu=242$)	63	25	40	80			
			31	4	28	59	94	92	120
		Brass	31				42		
	Clad 2 sides	Cu-clad steel	31				107		
	Clad 1 side		94				103		
Mesh screening	2 layers 1 inch apart	Cu (Oxidized)		2	6	18			
	No. 22	Cu					31	43	43
	No. 16	Bronze					18		
	No. 4	Galvanized steel					10	17	21

Table AV-3 Calculated values of shielding effectiveness (Copper) [AV-3]

Frequency (Hz)	Thickness (cm)	R (dB)	A(dB)	C(dB)	SE=A+R+C (dB)
60	0.0254	22.4	0.026	-22.20	0.23
60	0.254	22.4	0.260	-19.20	3.46
60	7.620	22.0	7.800	0.32	30.12
1000	0.254	34.2	1.060	-10.37	24.89
10k	0.254	44.2	3.340	2.62	50.16
10k	0.762	44.2	10.020	0.58	54.80
150k	0.254	56.0	12.900	0.50	69.40
1M	0.254	64.2	33.400	0.00	97.60
15M	0.254	76.0	129.000	0.00	205.00
100M	0.254	84.0	334.000	0.00	418.00

Table AV-4 Calculated values of shielding effectiveness (Iron) [AV-3]

Frequency (Hz)	Thickness (cm)	R (dB)	A(dB)	C(dB)	SE=A+R+C (dB)
60	0.0254	-0.9	0.33	0.95	0.38
60	0.254	-0.9	3.30	0.78	3.18
60	7.62	-0.9	100.0	0.00	99.10
1000	0.254	0.9	13.70	0.60	14.66
10k	0.254	8.0	43.50	0.00	51.50
10k	0.762	8.0	130.50	0.00	138.50
150k	0.254	19.0	160.00	0.00	179.00
1M	0.254	28.0	363.00	0.00	391.00
15M	0.254	42.0	1060.00	0.00	1102.00
100M	0.254	56.0	1370.00	0.00	1426.00

Table AV-5 Electrical Properties of Shielding Materials at 150kHz [AV-1]

Metal	Relative Conductivity	Relative Permeability	Absorption Loss (dB)	
			1 mm thick	1 mil thick*
Silver	1.05	1	51.96	1.32
Copper, annealed	1.00	1	50.91	1.29
Copper, H. Drawn	0.97	1	49.61	1.26
Gold	0.70	1	42.52	1.08
Aluminum	0.61	1	39.76	1.01
Magnesium	0.38	1	31.10	0.79
Zinc	0.29	1	27.56	0.70
Brass	0.26	1	25.98	0.66
Cadmium	0.23	1	24.41	0.62
Nickel	0.20	1	22.83	0.58
Phosphor-bronze	0.18	1	21.65	0.55
Iron	0.17	1000	665.40	16.90
Tin	0.15	1	19.69	0.50
Steel, SAE 1045	0.10	1000	509.10	12.90
Beryllium	0.10	1	16.14	0.41
Lead	0.08	1	14.17	0.36
Hypernick	0.06	80000	3484.00**	88.50**
Monel	0.04	1	10.24	0.26
Mu-metal	0.03	80000	2488.00**	63.20**
Permalloy	0.03	80000	2488.00**	63.20**
Steel, stainless	0.02	1000	224.40	5.70

*1 mil = 0.0254 millimeters

**With no saturation by incident field

The correction factor, X , is expressed as

$$X = 4 \frac{(1 - m^2)^2 - 2m^2 - j2\sqrt{2}m(1 - m^2)}{[1 + (1 + \sqrt{2}m)^2]^2} \quad (\text{AV-1})$$

Where $m = 0.545 \sqrt{\frac{f\sigma}{\mu}} r$, and r =distance from shield to source

= 1 for $m \ll 1$

= 1 for $m \gg 1$

In practical application, $m \gg 1$ and $X = 1$, except at low frequencies. For m between 0.05 and 20, consult Figure AV-1

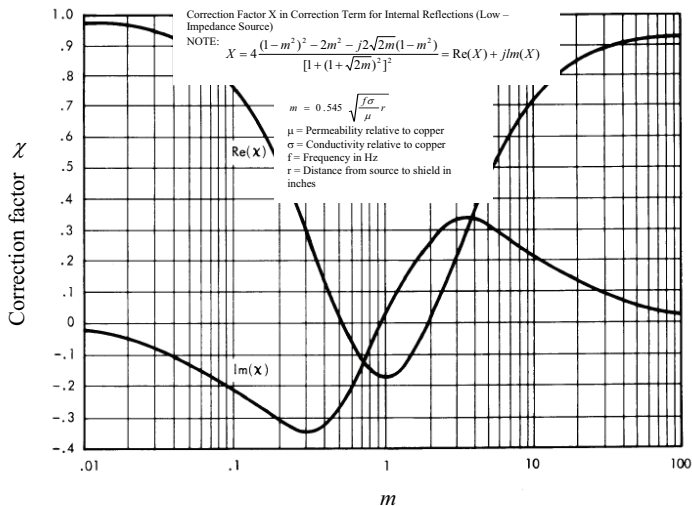


Figure AV-1 Correction factor in correction term for internal reflections

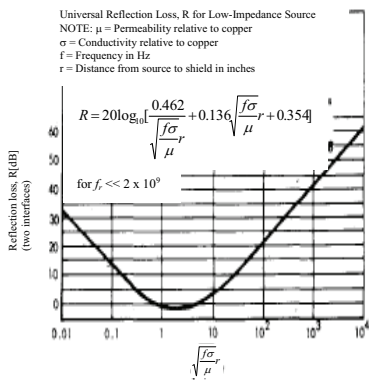


Fig. AV-2 Universal reflection loss for low-impedance source

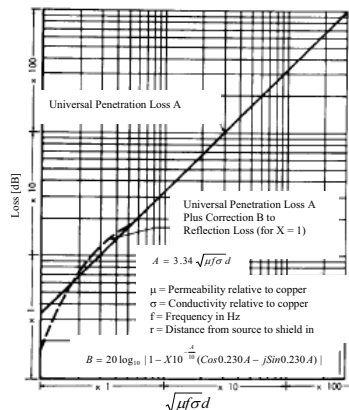


Figure AV-3 Universal penetration-loss curve

Table AV-6 Calculated penetration loss of metal sheet [AV-1]

Frequency	Iron	Penetration loss/mil thickness (dB)	
	μ	Copper $\sigma = 1 \mu = 1$	Iron $\sigma = 0.17$
60 Hz	1000	0.026	0.334
1000 Hz	1000	0.10	1.37
10 kHz	1000	0.334	4.35
10 MHz	1000	1.29	16.9
1 MHz	700	3.34	36.3
15 MHz	400	12.9	106.0
100 MHz	100	33.4	137.0
1500 MHz	10	129.0	168.0
10000 MHz	1	334.0	137.0
NOTE: Other values of μ for Iron are 600 at 3MHz, 500 at 0MHz and 50 at 1000MHz			

Table AV-7 Calculated reflection loss of metal sheet (both interfaces) [AV-1]

Frequency	Iron	Reflection loss ¹ (dB) for given source impedance $\ll 377 \Omega$	
	μ	Copper ² $\sigma = 1 \mu = 1$	Iron ^{2,3} $\sigma = 0.17$
60 Hz	1000	22.4	-0.9
1000 Hz	1000	34.2	0.9
10 kHz	1000	44.2	8.0
10 MHz	1000	56.0	18.7
1 MHz	700	64.2	28.1
15 MHz	400	76.0	42.2
100 MHz	100	84.2	56.5
1500 MHz	10	*	*
10000 MHz	1	*	*
* At these frequencies the fields approach 377 ohms in impedance and become plane waves			

Table AV-8 B-Term correction for single metal sheet [AV-1]

Shield Thickness (mils)	B-term (dB)					
	60 Hz	100 Hz	1 kHz	10 kHz	100 kHz	1 MHz
Copper $\mu = 1$ $\sigma = 1$ Magnetic Fields						
1	-22.22	-24.31	-28.23	-19.61	-10.34	-2.61
5	-21.30	-22.07	-15.83	-6.98	-0.55	+0.14
10	-19.23	-18.59	-10.37	-2.62	+0.57	0
20	-15.35	-13.77	-5.41	+0.13	+0.10	
30	-12.55	-10.76	-2.94	+0.58	0	
50	-8.88	-7.07	-0.58	0		
100	-4.24	-2.74	+0.50			
200	-0.76	+0.50	0			
300	+0.32	+0.53				
Iron $\mu = 1000$ $\sigma = 0.17$ Magnetic Fields						
1	+0.95	+1.23	-1.60	-1.83		
5	+0.93	+0.89	-0.59	0		
10	+0.78	+0.48	+0.06			
20	+0.35	+0.08	0			
30	+0.06	+0.06				
50	0	0				

Appendix VI

Material properties

Table AVI-1 Magnetic Properties of Selected Metals [AVI-1]

Magnetic (μ_r greater than 1.1)	Non-Magnetic (μ_r less than 1.1)
Alloy 225	Alloy 11
Alloy 405	Alloy 203
Alloy 426	Aluminum
Alumel®	Constantan®
Iron	Copper
Kovar®	Chromel®
1010-1020 Low Carbon Steel	Inconel®
Monel®	Molybdenum
Nickel	Platinum
42% Nickel	302-304 Stainless Steel
52% Nickel Iron	Tantalum
430 Stainless Steel	Titanium

Table AVI-2 Composition and initial permeability of Selected Metals [AVI-1]

Material	Approximate composition (percent)					μ_i^*
	Fe	Ni	Co	Mo	Other	
Cold rolled steel	98.5	—	—	—	—	180
Iron	99.91	—	—	—	—	200
Purified Iron	99.95	—	—	—	—	5,000
4% Silicon Iron	96	—	—	—	4 Si	500
45 Permalloy	54.7	45	—	—	0.3 Mn	2,500
Mu metal	18	75	—	—	2 Cr, 5 Cu	20,000
Supermalloy	15.7	79	—	5	0.3 Mn	100,000

*at 2 mT

Appendix VII

Derivation of the Neumann formula for mutual inductance of two circuit elements (C_1 and C_2) from the definition of inductance as flux linkage per ampere.

The mutual inductance between elements one and two is defined as flux linkage per ampere

$$L_{12} = \frac{\Lambda_{12}}{I_1} \quad (\text{AVII-1})$$

Flux linkage Λ_{12} is defined as the portion of the flux created when a current passes through C_1 , which crosses the surface bound by C_2 , multiplied by the number of turns in C_2 .

$$\Lambda_{12} = N_2 \Psi_{12} \quad (\text{AVII-2})$$

Substituting (AVII-2) into (AVII-1)

$$L_{12} = \frac{N_2 \Psi_{12}}{I_1} \quad (\text{AVII-3})$$

The flux Ψ_{12} is the portion of the flux B_1 created when a current passes through C_1 which crosses the surface bound by C_2

$$\Psi_{12} = \int_{S_2} \vec{B}_1 \cdot d\vec{s}_2 \quad (\text{AVII-4})$$

Substituting (AVII-4) into (AVII-3)

$$L_{12} = \frac{N_2}{I_1} \int_{S_2} \vec{B}_1 \cdot d\vec{s}_2 \quad (\text{AVII-5})$$

B can be written as the curl of the magnetic vector potential

$$\vec{B} = \vec{\nabla} \times \vec{A} \quad (\text{AVII-6})$$

Substituting (AVI-6) into (AVI-5)

$$L_{12} = \frac{N_2}{I_1} \int_{S_2} (\vec{\nabla} \times \vec{A}_1) \cdot d\vec{s}_2 \quad (\text{AVII-7})$$

Using stokes theorem: $\int_S (\vec{\nabla} \times \vec{A}) \cdot d\vec{s} = \oint_C \vec{A} \cdot d\vec{l}$

$$L_{12} = \frac{N_2}{I_1} \oint_{C_2} \vec{A}_1 \cdot d\vec{l}_2 \quad (\text{AVII-8})$$

Using the fundamental postulate of magnetostatics: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and (AVI-6)

$$\vec{\nabla} \times \vec{\nabla} \times \vec{A} = \mu_0 \vec{J} \quad (\text{AVII-9})$$

Simplifying to obtain a vector Poisson's equation

$$\vec{\nabla}^2 \vec{A} = -\mu_0 \vec{J} \quad (\text{AVII-10})$$

Solving (AVII-10)

$$\vec{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\vec{J}}{R} dv' \quad (\text{AVII-11})$$

For a thin wire with cross-sectional area S , dv' equals Sdl' so

$$\vec{J} dv' = JSdl' = Idl' \quad (\text{AVII-12})$$

Substituting (AVII-12) into (AVII-11)

$$\vec{A}_1 = \frac{\mu_0 N_1 I_1}{4\pi} \oint_{C_2} \frac{d\vec{l}_1}{R_{12}} \quad (\text{AVII-13})$$

Substituting (AVI-13) into (AVI-8) and noting that the contour integrals are evaluated only once over the periphery of C_1 and C_2 ,

$$L_{12} = \frac{\mu_0 N_1 N_2}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{R_{12}} \quad (\text{AVII-14})$$

Where R_{12} is the distance between $d\vec{l}_1$ and $d\vec{l}_2$.

The Neumann formula for mutual inductance is commonly written as

$$L_{12} = \frac{\mu_0}{4\pi} \oint_{C_1} \oint_{C_2} \frac{d\vec{l}_1 \cdot d\vec{l}_2}{|\vec{r}_2 - \vec{r}_1|} \quad (\text{AVII-15})$$

Appendix VIII

Table AVIII -7 Data for Kaschke K2004 core material (measured on toroids 30/18/7) [7-1]

Material	Manganese Zinc
Initial permeability μ_i	2000 $\pm$ 25%
Flux density B at magnetic field strength H	$\geq 455\text{mT}$, 800A/m
Remanence B_r	$\cong 200\text{mT}$
Coercivity H_c	$\leq 20\text{ A/m}$
Curie-temperature ν_c	$\geq 200^\circ\text{C}$
Relative temperature coefficient $\alpha_f 10^{-6}/\text{K}$ at -25 -25°C $+5 - +25^\circ\text{C}$ $+25 - +55^\circ\text{C}$ $+25 - +80^\circ\text{C}$	 ≤ 3.5 ≤ 5.0 ≤ 6.5 ≤ 6.5
Relative loss factor $\tan\delta/\mu_f 10^{-6}$ At 100kHz 200kHz 500kHz	 ≤ 5 ≤ 7 ≤ 30
Power losses P_f (typical values) At 50kHz, 200mT, 25°C 50kHz, 200mT, 100°C	 500 mW/cm ³ 440 mW/cm ³
Induction B at 100°C , 16kHz, 250A/m	$\geq 330\text{mT}$
Resistivity ρ	$\geq 1\Omega\text{m}$
Sintered density γ	$\geq 4.8\text{ g/cm}^3$

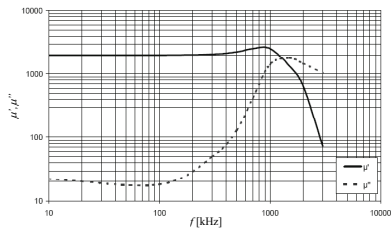


Figure AVIII -29 Complex permeability of K2004 [7-1]

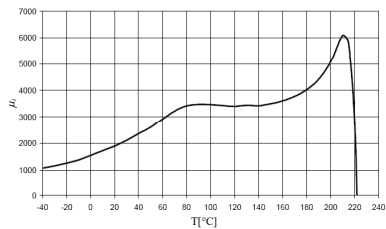


Figure AI-30 Temperature dependence of the initial permeability of K2004 [7-1]

Table AVIII -2 Cross-reference list of ferrite materials [AVIII -2]. The highlighted column represents the material used in experimental validation. Other materials shown may also be suitable.

ISKRA FERITI	25G	15G	45G	35G	65G
SIEMANS	N41	N27	N67	N87	N97
PHILIPS		3C80	3C85	F3	
NEOSID-MMG	F5C	F5	F44		
LLC THOMSON		B3	F1		
KASCHKE		K2004	K2006	K2008	
TRIDELTA	196B	196A MF196	MF198	MF198A	
SAGEM		517			
PRAMET		H21	H24		
EM-VAC		M2TN-B	M2TN-C		
HITACHI/NIPPON	SB-5S				
MAGNETICS	F			R	P
FAIR RITE		77	78		
COSMOFERRITES	CF101	CF196	CF129	CF138	
Initial permeab.	3000	2000	2300	2200	1800
	POWER MATERIALS				

Appendix IX

Opening all four switches in the series resonant full-bridge converter shown in Figure 6-14 causes the inverse diodes to conduct and automatically impress the inverse dc voltage on the load until the load current is fully decayed. Because of the filtering effect of the series RLC load, the circuit can be modeled as a series RLC circuit with a forcing function that is a sine function rather than a square wave, where $\hat{v}_e = \frac{4}{\pi} \hat{v}_{square}$.

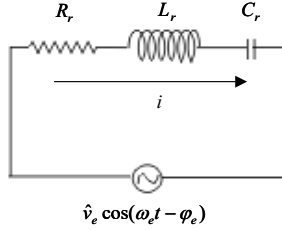


Figure AIX-1 Series RLC circuit

An equation describing the voltages in the circuit can be written

$$\hat{v}_e \sin(\omega_e t - \varphi_e) = L_r \frac{di}{dt} + R_r i + \frac{1}{C_r} \int_0^t i(\tau) d\tau \quad (\text{AIX-1})$$

where $\hat{v}_e$ is the peak value of the excitation voltage, ω_e is the frequency of the excitation voltage, and φ_e is the phase angle of excitation voltage. Because

$$i_{L_r}(t) = C_r \frac{dv_{C_r}(t)}{dt}$$

equation (AIX-1) can be written as

$$\frac{1}{L_r C_r} \hat{v}_e \sin(\omega_e t - \varphi_e) = \frac{d^2 v_{C_r}}{dt^2} + \frac{R_r}{L_r} \frac{dv_{C_r}}{dt} + \frac{v_{C_r}}{L_r C_r} \quad (\text{AIX-2})$$

The initial conditions are

$$v_{C_r}(0) \text{ and } i_{L_r}(0) = C_r \frac{dv(0)}{dt}$$

The homogeneous solution is of the form

$$v_{C_r_h} = C_1 e^{-\alpha t} \cos(\omega_d t) + C_2 e^{-\alpha t} \sin(\omega_d t) \quad (\text{AIX-3})$$

where

$$\omega_d = \sqrt{\omega_{r0}^2 - \alpha^2}$$

$$\alpha = \frac{R_r}{2L_r}$$

The particular solution is assumed to be of the form

$$v_{Cr_p} = A \cos(\omega_e t - \varphi_e) + B \sin(\omega_e t - \varphi_e) \quad (\text{AIX-4})$$

finding

$$v'_{Cr_p} = -A \omega_e \sin(\omega_e t - \varphi_e) + B \omega_e \cos(\omega_e t - \varphi_e) \quad (\text{AIX-5})$$

$$v''_{Cr_p} = -A \omega_e^2 \cos(\omega_e t - \varphi_e) - B \omega_e^2 \sin(\omega_e t - \varphi_e) \quad (\text{AIX-6})$$

and substituting into (AIX-4) and collecting terms gives

$$\begin{aligned} \frac{\hat{v}_e}{L_r C_r} \sin(\omega_e t - \varphi_e) = \cos(\omega_e t - \varphi_e) & \left[\frac{A}{L_r C_r} - A \omega_e^2 + \frac{B \omega_e R_r}{L_r} \right] \\ & + \sin(\omega_e t - \varphi_e) \left[\frac{B}{L_r C_r} - B \omega_e^2 - \frac{A \omega_e R_r}{L_r} \right] \end{aligned} \quad (\text{AIX-7})$$

equating coefficients and solving for A and B gives

$$A = \frac{-B \omega_e R_r C_r}{1 - \omega_e^2 L_r C_r} \quad (\text{AIX-8})$$

$$B = \frac{\hat{v}_e (1 - \omega_e^2 L_r C_r)}{(1 - \omega_e^2 L_r C_r)^2 + \omega_e^2 R_r^2 C_r^2} \quad (\text{AIX-9})$$

The general solution is the sum of the homogeneous and particular solutions

$$\begin{aligned} v_{Cr}(t) = v_{Cr_h} + v_{Cr_p} = C_1 e^{-\alpha t} \cos(\omega_d t) + C_2 e^{-\alpha t} \sin(\omega_d t) \\ + A \cos(\omega_e t - \varphi_e) + B \sin(\omega_e t - \varphi_e) \end{aligned} \quad (\text{AIX-10})$$

Finding the derivative

$$\begin{aligned} v'_{Cr}(t) = -C_1 \alpha e^{-\alpha t} \cos(\omega_d t) - C_1 \omega_d e^{-\alpha t} \sin(\omega_d t) \\ - C_2 \alpha e^{-\alpha t} \sin(\omega_d t) + C_2 \omega_d e^{-\alpha t} \cos(\omega_d t) \\ - A \omega_e \sin(\omega_e t - \varphi_e) + B \omega_e \cos(\omega_e t - \varphi_e) \end{aligned} \quad (\text{AIX-11})$$

Applying the initial conditions and solving for C_1 and C_2 gives

$$C_1 = v_{Cr}(0) - A \cos(-\phi_e) - B \sin(-\phi_e) \quad (\text{AIX-12})$$

$$C_2 = \frac{v'_{Cr}(0) + C_1 \alpha + A \omega_e \sin(-\phi_e) - B \omega_e \cos(-\phi_e)}{\omega_d} \quad (\text{AIX-13})$$

The constants in the solution are now fully defined

$$v_{Cr}(t) = C_1 e^{-\alpha t} \cos(\omega_d t) + C_2 e^{-\alpha t} \sin(\omega_d t) + A \cos(\omega_e t - \phi_e) + B \sin(\omega_e t - \phi_e) \quad (\text{AIX-14})$$

and the current in the circuit can be described as

$$\begin{aligned} i(t) = C \frac{dv_{Cr}}{dt} = C_r (-C_1 \alpha e^{-\alpha t} \cos(\omega_d t) - C_1 \omega_d e^{-\alpha t} \sin(\omega_d t) - C_2 \alpha e^{-\alpha t} \sin(\omega_d t) \\ + C_2 \omega_d e^{-\alpha t} \cos(\omega_d t) - A \omega_e \sin(\omega_e t - \phi_e) + B \omega_e \cos(\omega_e t - \phi_e)) \end{aligned} \quad (\text{AIX-15})$$

Appendix X

Derivation of equivalent ac resistance for a single pulse peak rectifier

Reference [6-18] shows the correlation between average dc voltage $V_{dc,avg}$ and ac rms voltage V_{rms} for a single phase, single diode rectifier (Figure AX-1).

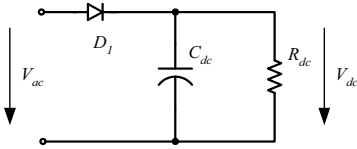


Figure AX-1 Single pulse peak rectifier

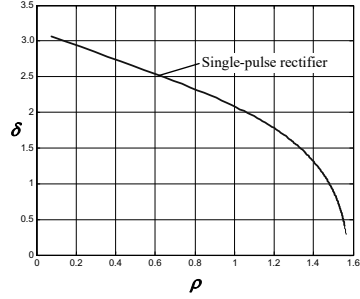


Figure AX-2 δ vs ρ for a single pulse peak rectifier

$$V_{dc,avg} = \frac{1 - \cos \delta}{2 \cos \rho} \frac{\sqrt{2}}{\pi} V_{rms} \quad (AX-1)$$

$$\rho = \arctan(\omega C_{sdc} R_{sdc}) \quad (AX-2)$$

$$\delta = \vartheta_2 - \vartheta_1 \quad (AX-3)$$

$$\vartheta_2 = \pi - \rho \quad (AX-4)$$

$$\sin(\delta + \rho) - \sin \rho \exp\{-(2\pi - \delta) \cot \rho\} = 0 \quad (AX-5)$$

From a power balance consideration:

$$\frac{V_{dc,avg}^2}{R_{sdc}} = \frac{V_{rms}^2}{R_{sac}} \quad (AX-6)$$

$$R_{sac} = 2 \left(\frac{\pi \cos \rho}{(1 - \cos \delta)} \right)^2 R_{sdc} \quad (AX-7)$$

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