

ELECTRONIC AND RADIO ENGINEERING

FOURTH EDITION

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ELECTRONIC AND RADIO ENGINEERING
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PREFACE

This fourth edition has the same objective as the three prior editions, namely, to provide a text and reference book that summarizes in easily understandable terms those principles and techniques which are the basic tools of the electronic and radio engineer. In keeping with current trends, increased emphasis is placed on the general techniques of electronics, without regard to the extent of their use in radio systems. This change is reflected in the new title, "Electronic and Radio Engineering," which is more descriptive of the subject matter actually covered in the present volume than is the previous title, "Radio Engineering."

The keynote continues to be thorough coverage combined with a presentation that allows the reader to study a particular topic without having to read the entire book. The level of presentation, particularly the mathematical level, remains unchanged. Thus the present volume is designed to serve as a text and reference for the same clientele that found the previous editions so useful.

To keep pace with a rapidly advancing technology, new material has been added in practically every chapter. More than half the illustrations are new, and all have been redrawn to conform to new graphic standards. A new chapter dealing with microwave tubes makes available for the first time an explanation in simple language of the basic mechanism of operation of traveling-wave tubes and backward-wave oscillators (carcinotrons). In the treatment of wideband video and tuned amplifiers, primary emphasis is placed on the rise time, overshoot, and sag, since these characteristics are more indicative of the performance under actual conditions than is the older approach in terms of amplitude and phase behavior as a function of frequency. The material on nonlinear waveforms and pulse techniques has been greatly expanded to provide more complete coverage of this important aspect of electronics. The chapter on television has been thoroughly revised, and a compact and simple explanation is given of the system of color television now standard in the United States. Increased attention is also placed on propagation phenomena involving the troposphere.

Of particular importance is the chapter on Transistors and Related Semiconductor Devices, one of the longest in the book. Here is presented a simple, straightforward explanation of the basic phenomena occurring inside the transistor, and of how these phenomena lead to the terminal characteristics. This treatment is such that it can be understood by undergraduate students; at the same time, it is sufficiently com-

plete and fundamental to provide a firm foundation for further study of this new and very important subject.

Special attention has been given to the needs of the teacher. Because of the growth of electronics, it is no longer possible to cover every important topic adequately in a one-year course. "Electronic and Radio Engineering" provides the instructor with an opportunity to select those topics which he himself wishes to emphasize, and at the same time provides the student with a reference book of comprehensive coverage and continuing value. It will be observed that the book breaks down into three distinct parts, namely, a group of chapters dealing with circuits (components, resonant circuits, transmission lines, waveguides, and cavity resonators); a group of chapters concerned with the fundamentals of electronic engineering (vacuum tubes, transistors, amplifiers, oscillators, modulators, detectors, nonlinear waveforms, etc.), which are the heart of the book; and a concluding group of chapters concerned with radio systems and radio engineering (antennas, propagation, transmitters, receivers, television, radar, and radio aids to navigation). Thus an instructor can, if he desires, concentrate on the material concerned with fundamental electronics and regard the remaining subject matter as available to the student, should he need to extend his knowledge at a future date. Alternatively, the instructor can choose to cover a series of selected topics, for example, waveguides, wideband systems, pulse circuits, television, etc. Another possibility is to concentrate on the material concerned primarily with radio systems. Many other combinations, are, of course, possible.

An important feature for the teacher is the more than 1250 Problems and Exercises. Many of these involve numerical calculations, but more than half of them are thought questions that will require the student to give further consideration to topics covered in the text. Such Exercises can be used to extend and solidify the student's knowledge; they are also suggestive of questions suitable for use on examinations. The number of Problems and Exercises is so large that the same problem need not be assigned to a class more often than once every two or three years.

The collaborators listed on the title page have made important contributions to the preparation of this volume. Dr. Helliwell worked on the sections dealing with ionospheric propagation, and Dr. Pettit is in large measure responsible for the general character of the chapter dealing with transistors and semiconductors. The treatment of traveling-wave tubes and backward-wave oscillators is due to Dr. Watkins. William Rambo prepared the background material used in revising the presentation on radar. In addition, acknowledgment is made to Dr. B. H. Wadia, Bruno Ludovici, and Arthur Vassilaides, graduate students at Stanford, for assistance in preparing illustrations.

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CHAPTER 1

THE ELEMENTS OF A SYSTEM OF RADIO COMMUNICATION

1-1. Radio Waves. Electrical energy that has escaped into free space exists in the form of electromagnetic waves. These waves, which are commonly called radio waves, travel with the velocity of light and consist of magnetic and electric fields that are at right angles to each other and also at right angles to the direction of travel. If these electric and magnetic fluxes could actually be seen, the wave would have the appearance indicated in Fig. 1-1. One-half of the electrical energy contained

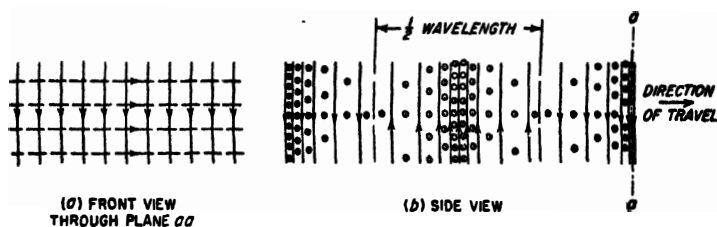


FIG. 1-1. Front and side views of a vertically polarized wave. The solid lines represent electric flux; the dotted lines and the circles indicate magnetic flux.

in the wave exists in the form of electrostatic energy, while the remaining half is in the form of magnetic energy.

The essential properties of a radio wave are the frequency, intensity, direction of travel, and plane of polarization. The radio waves produced by an alternating current will vary in intensity with the frequency of the current and will therefore be alternately positive and negative as shown in Fig. 1-1b. The distance occupied by one complete cycle of such an alternating wave is equal to the velocity of the wave divided by the number of cycles that are sent out each second and is called the wavelength. The relation between wavelength λ in meters and frequency f in cycles per second is therefore

$$\lambda = \frac{300,000,000}{f} \quad (1-1)$$

The quantity 300,000,000 is the velocity of light in meters per second. The frequency is ordinarily expressed in kilocycles, abbreviated kc, or in megacycles, abbreviated Mc. A low-frequency wave is seen from Eq.

(1-1) to have a long wavelength, while a high frequency corresponds to a short wavelength.

The strength of a radio wave is measured in terms of the voltage stress produced in space by the electric field of the wave, and it is usually expressed in microvolts stress per meter. Since the actual stress produced at any point by an alternating wave varies sinusoidally from instant to instant, it is customary to consider the intensity of such a wave to be the effective value of the stress, which is 0.707 times the maximum stress in the atmosphere during the cycle. The strength of the wave measured in terms of microvolts per meter of stress in space is also exactly the same voltage that the magnetic flux of the wave induces in a conductor 1 m long when sweeping across this conductor with the velocity of light.

The minimum field strength required to give satisfactory reception of a wave depends upon a number of factors, such as frequency, type of signal involved, and amount of interference present. Under some conditions radio waves having signal strengths as low as $0.1 \mu\text{v}$ per m are usable. Occasionally signal strengths exceeding $1000 \mu\text{v}$ per m are required to ensure entirely satisfactory reception at all times. In most cases the weakest useful signal strength lies somewhere between these extremes.

A plane parallel to the mutually perpendicular lines of the electric and electromagnetic flux is called the wavefront. The wave always travels in a direction at right angles to the wavefront, but whether it goes forward or backward depends upon the relative direction of the lines of magnetic and electric flux. If the direction of either the magnetic or electric flux is reversed, the direction of travel is reversed; but reversing both sets of flux has no effect.

The direction of the electric lines of flux is called the direction of polarization of the wave. If the electric flux lines are vertical, as shown in Fig. 1-1, the wave is vertically polarized; when the electric flux lines are horizontal and the electromagnetic flux lines are vertical, the wave is horizontally polarized.

Propagation of Radio Waves of Different Frequencies. As radio waves travel away from their point of origin, they become attenuated or weakened. This is due in part to the fact that the waves spread out.

In addition, however, energy may be absorbed from the waves by the ground or by the ionized regions in the upper atmosphere termed the ionosphere, and the waves may also be reflected or refracted by the ionosphere, or by conditions within the lower atmosphere, or by the ground. The resulting situation is quite complex and differs greatly for radio waves of different frequencies, as shown in Table 1-1, which summarizes the behavior of different classes of radio waves.

1-2. Radiation of Electrical Energy. Every electrical circuit carrying alternating current radiates a certain amount of electrical energy in the form of electromagnetic waves, but the amount of energy thus radi-

ated is extremely small unless all the dimensions of the circuit approach the order of magnitude of a wavelength. Thus, a power line carrying 60-cycle current with a 20-ft spacing between conductors will radiate practically no energy because a wavelength at 60 cycles is more than 3000 miles, and 20 ft is negligible in comparison. On the other hand, a coil 20 ft in diameter and carrying a 2000-kc current will radiate a considerable amount of energy because 20 ft is comparable with the 150-m

TABLE 1-1
CLASSIFICATION OF RADIO WAVES

Class	Frequency range	Wavelength range	Propagation characteristics	Typical uses
Very low frequency (VLF)	10-30 kc	30,000-10,000 m	Low attenuation at all times of day and of year; characteristics very reliable	Long-distance point-to-point communication
Low frequency (LF)	30-300 kc	10,000-1000 m	Propagation at night similar to VLF but slightly less reliable; daytime absorption greater than VLF	Long-distance point-to-point service, marine, navigational aids
Medium frequency (MF)	300-3000 kc	1000-100 m	Attenuation low at night and high in daytime	Broadcasting, marine communication, navigation, harbor telephone, etc.
High frequency (HF)	3-30 Mc	100-10 m	Transmission over considerable distance depends solely on the ionosphere, and so varies greatly with time of day, season, and frequency	Moderate and long-distance communication of all types
Very high frequency (VHF)	30-300 Mc	10-1 m	Substantially straight-line propagation analogous to that of light waves; unaffected by ionosphere	Short-distance communication, television, frequency modulation, radar, airplane navigation
Ultra-high frequency (UHF)*	300-3000 Mc	100-10 cm	Same	Short-distance communication, radar, relay systems, television, etc.
Super-high frequency (SHF)*	3000-30,000 Mc	10-1 cm	Same	Radar, radio relay, navigation

* Frequencies higher than about 2000 Mc are frequently referred to as microwave frequencies.

wavelength of this radio wave. From these considerations it is apparent that the size of radiator required is inversely proportional to the frequency. High-frequency waves can therefore be produced by a small radiator, while low-frequency waves require a large antenna system for effective radiation.

Every radiator has directional characteristics as a result of which it sends out stronger waves in certain directions than in others. Directional characteristics of antennas are used to concentrate the radiation toward the point to which it is desired to transmit, or to favor reception of energy arriving from a particular direction.

1-3. Generation and Control of Radio-frequency Power. The radio-frequency power required by a radio transmitter is practically always obtained from a vacuum-tube oscillator or amplifier. Vacuum tubes can convert d-c power into a-c energy for all frequencies from the very lowest up to 30,000 Mc, or even higher. Under most conditions the efficiency with which this transformation takes place is in the neighborhood of 50 per cent or higher. At frequencies up to well over 1000 Mc, the amount of

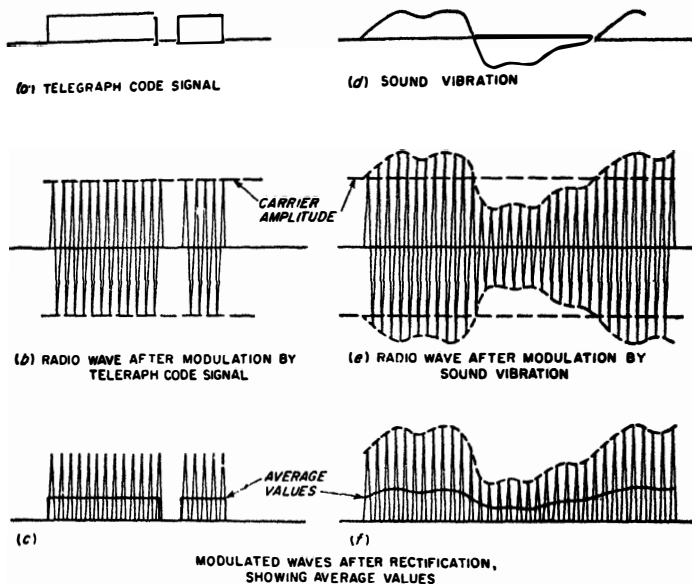


FIG. 1-2. Diagram showing how a signal may be transmitted by modulating the amplitude of a radio wave, and how the original signal may be recovered from the modulated wave by rectification. For the sake of clarity the radio frequency is shown as being much lower than would usually be the case.

power that can be generated continuously by vacuum tubes is of the order of kilowatts.

Modulation. If a radio wave is to convey a message, some feature of the wave must be varied in accordance with the information to be transmitted. One way to do this, termed *amplitude modulation*, consists in varying the amplitude of the radiated wave. In radio telegraphy, this involves turning the radio transmitter on and off in accordance with the dots and dashes of the telegraph code, as illustrated in Fig. 1-2b. In radio-telephone transmission by amplitude modulation the radio-frequency wave is varied in accordance with the pressure of the sound wave being transmitted, as shown in Fig. 1-2e. Similarly in picture transmission, the amplitude of the wave radiated at any one time is made

proportional to the light intensity of the part of the picture that is being transmitted at that instant.

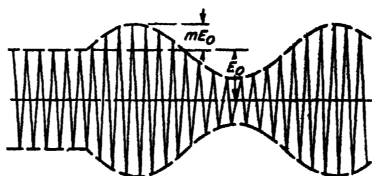
Intelligence may be transmitted by other means than by varying the amplitude. For example, one may maintain the amplitude constant and vary the frequency that is radiated in accordance with the intelligence, thus obtaining *frequency modulation*. This results in a wave such as shown in Fig. 1-3b, which is to be compared with the corresponding amplitude-modulated wave of Fig. 1-3a. Frequency modulation is widely used in very high-frequency communication systems.

1-4. Reception of Radio Signals.

In the reception of radio signals it is first necessary to abstract energy from the radio wave passing the receiving point. Any antenna capable of radiating electrical energy is also able to absorb energy from a passing radio wave. This occurs because the electromagnetic flux of the wave, in cutting across the antenna conductor, induces in the antenna a voltage that varies with time in exactly the same way as does the current flowing in the antenna radiating the wave. This induced voltage, in association with the current that it produces, represents energy that is absorbed from the passing wave.

Since every wave passing the receiving antenna induces its own voltage in the antenna conductor, it is necessary that the receiving equipment be capable of separating the desired signal from the unwanted signals that are also inducing voltages in the antenna. This separation is made on the basis of the difference in frequency between transmitting stations and is carried out by the use of resonant circuits which can be made to discriminate very strongly in favor of a particular frequency. The ability to discriminate between radio waves of different frequencies is called *selectivity* and the process of adjusting circuits to resonance with the frequency of a desired signal is spoken of as *tuning*.

Although intelligible radio signals have been received from radio transmitters thousands of miles distant, using only the energy abstracted from the radio wave by the receiving antenna, much more satisfactory reception can be obtained if the received energy is *amplified*. This amplification may be applied to the radio-frequency currents before detection, in



(a) AMPLITUDE-MODULATED WAVE



(b) SAME INFORMATION TRANSMITTED BY FREQUENCY-MODULATED WAVE

FIG. 1-3. Character of waves produced by amplitude modulation and by frequency modulation, where the modulation is sinusoidal in both cases. For the sake of clarity the radio frequency is shown much lower than would usually be the case.

which case it is called radio-frequency amplification; or it may be applied to the rectified currents after detection, in which case it is called audio-frequency amplification. The use of amplification makes possible the satisfactory reception of signals from waves that would otherwise be too weak to give an audible response. The only satisfactory method of amplifying radio signals that has been discovered is by the use of vacuum tubes or transistors. Before vacuum tubes were discovered, radio reception had available only the energy abstracted from the radio wave by the receiving antenna.

Detection. The process by which the message being transmitted is reproduced from the modulated radio-frequency current present in the receiver is called *detection*, or sometimes *demodulation*. With amplitude-modulated waves, detection is accomplished by rectifying the radio-frequency currents to produce a current that varies in accordance with the modulation of the received wave. Thus, when the modulated wave shown at *e* of Fig. 1-2 is rectified, the resulting current, shown at *f*, is seen to have an average value that varies in accordance with the amplitude of the original signal. In the transmission of code signals by radio, the rectified current reproduces the dots and dashes of the telegraph code, as shown at Fig. 1-2c, and could be used to operate a telegraph sounder. When it is desired to receive the telegraph signals directly on a telephone receiver, it is necessary to break up the dots and dashes at an audible rate in order to give a note that can be heard, since otherwise the telephone receiver would give forth a succession of unintelligible clicks.

The detection of a frequency-modulated wave involves two steps. First, the wave is transmitted through a circuit in which the relative response depends upon the frequency. The wave that then emerges from the circuit is amplitude-modulated, since as the frequency of the constant-amplitude input wave changes, the output amplitude will follow the variation of circuit transmission with frequency. The resulting amplitude-modulated wave is then rectified.

1-5. Nature of a Modulated Wave. A sine wave conveys very little information since it repeats over and over again. When a wave is modulated, either in amplitude or frequency, it is no longer a simple sine wave, but is instead a mixture of several waves of slightly different frequencies superimposed upon each other. The actual nature of a modulated wave can be deduced by writing down the equation of the wave and making a mathematical analysis of the result. Thus, in the case of the simple sine-wave amplitude modulation shown in Fig. 1-3a, the amplitude of the radio-frequency oscillation is given by $E = E_0 + mE_0 \sin 2\pi f_m t$, in which E_0 represents the average amplitude, f_m the frequency at which the amplitude is varied, and m the ratio of amplitude variation from the average to the average amplitude, which is called the *degree of modulation*. The

equation of the amplitude-modulated wave can be hence written as

$$e = E_0(1 + m \sin 2\pi f_m t) \sin 2\pi f t \quad (1-2)$$

in which f is the frequency of the radio oscillation. Multiplying out the right-hand side of Eq. (1-2) gives

$$e = E_0 \sin 2\pi f t + m E_0 \sin 2\pi f_m t \sin 2\pi f t$$

By expanding the last term into functions of the sum and difference angles by the usual trigonometric formula, the equation of a wave with simple sine-wave amplitude modulation can be written in the form

$$e = E_0 \sin 2\pi f t + \frac{m E_0}{2} \cos 2\pi(f - f_m)t - \frac{m E_0}{2} \cos 2\pi(f + f_m)t \quad (1-3)$$

Equation (1-3) shows that the wave with sine-wave modulation consists of three separate waves. The first of these, represented by the term $E_0 \sin 2\pi f t$, is called the *carrier*. Its amplitude is independent of the presence or absence of modulation and is equal to the average amplitude of the wave. The two other components are alike as far as magnitude is concerned, but the frequency of one of them is less than that of the carrier frequency by an amount equal to the modulation frequency, while the frequency of the other is more than that of the carrier by the same amount. These two components, called *sideband frequencies*, carry the intelligence that is being transmitted by the modulated wave. The frequency of the sideband components relative to the carrier frequency is determined by the modulation frequency. The relative amplitude of the sideband components is determined by the extent of the amplitude variations that are impressed upon the wave, i.e., by the degree of modulation.

When the modulation is more complex than the simple sine-wave amplitude variation of Fig. 1-3a, the effect is to introduce additional sideband components. Thus, if the wave of a radio-telephone transmitter is amplitude-modulated by a complex sound wave containing pitches of 1000 and 1500 cycles, the modulated wave will contain one pair of 1000-cycle sideband components and one pair of 1500-cycle sideband components.

The analysis of a frequency-modulated wave is somewhat more complex but leads to an analogous result. The principal difference is that the frequency-modulated wave not only contains the same sideband frequencies as does the corresponding amplitude-modulated wave, but in addition contains higher-order side bands. Thus, if a wave has its frequency varied at a rate of 1000 times per second, the resulting modulated wave will contain not only a pair of 1000-cycle sideband components, but in addition a pair of 2000-cycle sideband components, possibly a pair of 3000-cycle sideband components, etc. The amplitude of these various

sideband pairs will depend upon the extent and upon the rate of frequency variation.

Significance of the Sidebands. The carrier and sideband frequencies are not a mathematical fiction, but have a real existence, as is evidenced by the fact that the various frequency components of a modulated wave can be separated from each other by suitable filter circuits. The sideband frequencies can be considered as being generated as a result of varying the wave. They are present only when the wave is being varied, and their magnitude and frequency are determined by the character of the modulation.

It is apparent that the transmission of intelligence requires the use of a band of frequencies rather than a single frequency. Speech and music of the quality reproduced in standard broadcasting involve frequency components from about 100 cycles up to 5000 cycles; when modulated upon a carrier wave, the total bandwidth involved is therefore 10,000 cycles. If this entire band is not transmitted equally well through space, and by the circuits in both transmitter and receiver through which the modulated wave must pass, then the sideband frequency components that are discriminated against will not be reproduced in the receiving equipment with proper amplitude, and a loss in quality will result. With telegraph signals, the required sideband is relatively narrow because the amplitude of the signals is varied only a few times a second, but a definite frequency band is still required. If some of the sideband components of the code signal are not transmitted, the received dots and dashes tend to be rounded off and run together, and may become indistinguishable.

1-6. The Decibel. The decibel (abbreviated db) is a logarithmic unit used in communication work to express power ratios. If the powers being compared are P_1 and P_2 , then

$$\text{Decibels} = 10 \log_{10} \frac{P_2}{P_1} \quad (1-4)$$

The sign associated with the number of decibels indicates which power is greater; thus a negative sign means P_2 is less than P_1 .

The decibel has no other significance than that given in Eq. (1-4). Thus, if decibels are used to express amplification, this simply means that the presence of the amplification increases the power output by the number of decibels attributed to the amplification. Again, under many conditions relative power is proportional to the square of the voltage E (or current I , or field B , etc.). Under these conditions

$$\text{Decibels} = 20 \log_{10} \frac{E_2}{E_1} = 20 \log_{10} \frac{I_2}{I_1} = 20 \log_{10} \frac{B_2}{B_1}, \text{ etc.} \quad (1-5)$$

These relations must be used with caution, however, as they hold only when the resistance associated with E_2 (or I_2 or B_2) is the same as associated with E_1 (or I_1 or B_1).

TABLE 1-2
(a) POWER, VOLTAGE, AND CURRENT RATIOS FOR ASSIGNED
DECIBEL VALUES

Db	Current and voltage ratio		Power ratio		Db	Current and voltage ratio		Power ratio	
	Gain	Loss	Gain	Loss		Gain	Loss	Gain	Loss
0.0	1.00	1.000	1.00	1.000	10	3.16	0.316	10.00	0.100
0.2	1.02	0.977	1.05	0.955	12	3.98	0.251	15.8	0.063
0.4	1.05	0.955	1.10	0.912	14	5.01	0.200	25.1	0.040
0.6	1.07	0.933	1.15	0.871	16	6.31	0.158	39.8	0.025
0.8	1.10	0.912	1.20	0.832	18	7.94	0.126	63.1	0.016
1.0	1.12	0.891	1.26	0.794	20	10.00	0.100	100.0	0.010
1.5	1.19	0.841	1.41	0.708	25	17.8	0.056	3.16×10^3	3.16×10^{-3}
2.0	1.26	0.794	1.58	0.631	30	31.6	0.032	10^4	10^{-4}
2.5	1.33	0.750	1.78	0.562	35	56.2	0.018	3.16×10^4	3.16×10^{-4}
3.0	1.41	0.708	2.00	0.501	40	100.0	0.010	10^5	10^{-5}
3.5	1.50	0.668	2.24	0.447	45	177.8	0.006	3.16×10^5	3.16×10^{-5}
4.0	1.58	0.631	2.51	0.398	50	316	0.003	10^6	10^{-6}
4.5	1.68	0.596	2.82	0.355	60	1,000	0.001	10^8	10^{-8}
5	1.78	0.562	3.16	0.316	70	3,160	0.0003	10^7	10^{-7}
6	2.00	0.501	3.98	0.251	80	10,000	0.0001	10^9	10^{-9}
7	2.24	0.447	5.01	0.200	90	31,600	0.00003	10^8	10^{-8}
8	2.51	0.398	6.31	0.158	100	100,000	0.00001	10^{10}	10^{-10}
9	2.82	0.355	7.94	0.126	120	1,000,000	0.000001	10^{12}	10^{-12}

(b) DECIBEL EQUIVALENT OF POWER, VOLTAGE, AND
CURRENT RATIOS

Ratio	Db equivalent		Ratio	Db equivalent		Ratio	Db equivalent	
	Power	Voltage or current		Power	Voltage or current		Power	Voltage or current
10^{-8}	-60.00	-120.00	1.2	0.79	1.58	10	10.00	20.00
10^{-6}	-50.00	-100.00	1.4	1.46	2.92	12	10.79	21.58
10^{-4}	-40.00	-80.00	1.6	2.04	4.08	14	11.46	22.92
0.001	-30.00	-60.00	1.8	2.55	5.10	16	12.04	24.08
0.003	-25.23	-50.46	2.0	3.01	6.02	18	12.55	25.10
0.005	-23.01	-46.02	2.5	3.98	7.96	20	13.01	26.02
0.01	-20.00	-40.00	3.0	4.77	9.54	25	13.98	27.96
0.03	-15.23	-30.46	3.5	5.44	10.88	30	14.77	29.54
0.05	-13.01	-26.02	4.0	6.02	12.04	40	16.02	32.04
0.10	-10.00	-20.00	4.5	6.53	13.06	50	16.99	33.98
0.15	-8.24	-16.48	5.0	6.99	13.98	60	17.78	35.56
0.20	-6.99	-13.98	5.5	7.40	14.81	80	19.03	38.06
0.30	-5.23	-10.46	6.0	7.78	15.56	100	20.00	40.00
0.40	-3.98	-7.96	6.5	8.13	16.26	10^3	30.00	60.00
0.50	-3.01	-6.02	7.0	8.45	16.90	10^4	40.00	80.00
0.60	-2.22	-4.44	7.5	8.75	17.50	10^5	50.00	100.00
0.80	-0.97	-1.94	8.0	9.03	18.06	10^6	60.00	120.00
1.00	0.00	0.00	9.0	9.54	19.08	10^7	70.00	140.00

The practical value of the decibel arises from its logarithmic nature. This permits the enormous ranges of power involved in communication work to be expressed in terms of decibels without running into inconveniently large numbers, while at the same time permitting small ratios to be conveniently expressed. Thus, 1 db represents a power ratio of approximately 5:4, while 60 db represents a ratio of 1,000,000:1. The logarithmic character of the decibel also makes it possible to express the ratio of input to output powers of a complicated circuit as the sum of the decibel equivalent of the ratios of the input to output powers of the different parts of the circuit that are in cascade.

Table 1-2 gives a convenient summary of decibel values.

CHAPTER 2

CIRCUIT ELEMENTS

2-1. Inductance. A current flowing in an electrical circuit produces magnetic flux that links with (i.e., encircles) the current. The effect of this flux is expressed in terms of a property of the circuit called the *inductance*.

Inductance can be defined as the flux linkages per ampere of current producing the flux; i.e.,

$$\text{Inductance } L \text{ in henrys} = \frac{\text{flux linkages}}{\text{current (amperes) producing flux}} \times 10^{-8} \quad (2-1)$$

A flux linkage represents one flux line encircling the circuit current once. Thus in Fig. 2-1 flux line *aa* contributes eight flux linkages toward the coil inductance because it circles the current flowing in the coil eight times. On the other hand, flux line *b* of the same coil contributes only one-half a flux linkage toward the coil inductance because this particular line encircles only one-half the coil current.

Calculation of Inductance. The inductance of an electrical circuit is computed by assuming a convenient current flowing in the circuit. The magnetic flux produced by this current is then calculated, and the total number of flux linkages that results is counted. The inductance in henrys is this total number of flux linkages multiplied by 10^{-8} and divided by the circuit current.

Formulas have been derived by this procedure that give the inductance for all commonly used types of air-cored coils.¹ It is thus neither necessary nor desirable to guess at the number of turns and coil dimensions required to obtain a desired inductance. For example, the inductance of a single-layer solenoid, such as shown in Fig. 2-1, is given by the relation

$$\text{Inductance in microhenrys} = Fn^2d \quad (2-2)$$

¹ A comprehensive collection of such formulas is given by F. E. Terman, "Radio Engineers' Handbook," pp. 48-64, McGraw-Hill Book Company, Inc., New York, 1943.

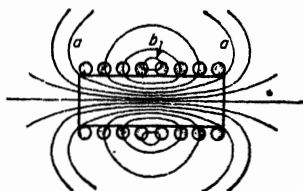


FIG. 2-1. Flux and current distribution in typical single-layer air-cored inductance coil. The current density is indicated by the density of shading.

where n = number of turns

d = diameter of coil measured to center of wire

F = constant that depends only upon the ratio of length to diameter, given in Fig. 2-2

The quantity F depends in a complicated way upon the ratio of coil length to diameter, since the geometrical distribution of the flux produced by the current in the coil does not follow a simple mathematical law. However, once the relationship represented by F has been determined,

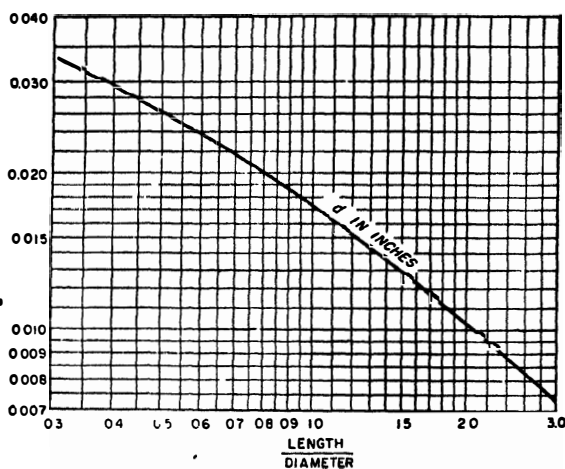


FIG. 2-2. Values of constant F for use in Eq. (2-2), to obtain the inductance of single-layer solenoids.

the value of F can be computed once for all, and presented by a curve, such as Fig. 2-2, or by a table.

The inductance of all coils with air cores is proportional to the square of the number of turns if the dimensions such as length, diameter, depth of winding, etc., are kept constant as the number of turns is altered. The reason for this behavior lies in the fact that, if the coil dimensions are kept constant, the amount of magnetic flux produced by a given coil current and the number of times each flux line links with the coil current are both proportional to the number of turns.

The inductance of all air-cored coils having the same number of turns and the same shape is always proportional to the size (i.e., to a linear dimension, such as length or radius) of the coil. Thus, if two coils have the same number of turns, but one is twice as big as the other in every dimension (such as diameter, length, width, and depth of winding), then the larger coil will have twice the inductance of the smaller one. This rule results from the fact that the cross section of the flux paths is propor-

tional to the square of the linear dimension of the coil, while the length of these paths varies directly as the linear dimension.

In calculating the inductance of coils with magnetic cores, the flux is determined in accordance with the usual methods of making magnetic circuit calculations, taking into account air gaps, leakage and fringing flux, etc. It is also necessary to assume the proper value of permeability, as discussed below.¹ To the extent that the permeability of the core material can be considered as constant, the inductance of a coil with a magnetic core is proportional to the square of the number of turns and to the first power of the size, just as in the air-cored case.

Inductance of a Connecting Wire. The inductance associated with a connecting wire depends on the wire diameter, and can be minimized by making the diameter large. This results from the fact that when the wire diameter is small the length of the flux paths immediately outside of the wire is less than if the diameter is large. As a result the small wire is circled by more flux and hence has higher inductance.

An alternative means of achieving a low-inductance connection consists in employing a conductor comprising two or more spaced wires connected in parallel. If three wires are employed, they should be placed at the corners of an equilateral triangle; in a four-wire system the individual wires would be at the corners of a square, etc. Such arrangements give the first approximation to a solid conductor of large diameter, and will have less inductance the greater the diameter of the individual wires and the greater the spacing between the wires connected in parallel.

Initial and Incremental Permeability; Incremental Inductance. The permeability of a magnetic material is defined as the ratio B/H of the flux density to the magnetizing force, and depends upon the flux and the material. The permeability at very low flux densities, termed the *initial permeability*, is of particular importance in communication systems, where the current is commonly very weak. The initial permeability of magnetic materials is nearly always much less than the permeability at somewhat higher flux densities.

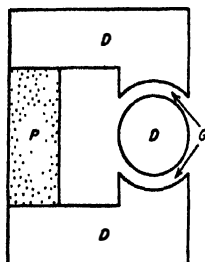
Coils having magnetic cores are frequently used in communication work under conditions where there is a large d-c magnetization upon which is superimposed a small a-c magnetization. Under these conditions, one is interested in the inductance that is offered to the superimposed alternating current. This is called the *incremental inductance*, and the corresponding permeability of the magnetic material is termed the *incremental permeability*.

Incremental permeability, and hence incremental inductance, depend upon the magnitude of both d-c and a-c magnetizations, and upon the previous magnetic history of the core. When a core that has been

¹ Such calculations are discussed in "Components Handbook" (vol. 17, Radiation Laboratory Series), chap. 4, McGraw-Hill Book Company, Inc., New York, 1949.

quencies. The addition of cobalt to nickel-iron alloys introduces the possibility of obtaining substantially constant permeability up to moderate flux densities, combined with extremely low hysteresis loss at low flux densities and almost zero residual induction and coercive force (see page 17). Such alloys are termed *perminvars*, and also possess in large degree the high-permeability features of nickel-iron alloys such as *permalloy*.

Iron-cobalt alloys containing from 36 to 50 per cent cobalt are characterized by a saturation flux density appreciably higher than that of silicon steel. Such alloys also have higher incremental permeability at high d-c magnetizing forces than do other magnetic materials.



P = PERMANENT MAGNET
G = AIR GAP
D = SOFT IRON POLE PIECES

Fig. 2-5. Typical magnetic circuit involving a permanent magnet.

Magnetic cores that are nonconducting have been developed for use in radio-frequency coils.¹ They are composed of mixtures of ferrites, and have a resistivity so high that eddy-current losses are negligible in solid cores even at frequencies higher than 1 Mc. At the same time, such core material has a relatively high initial permeability, a value of 500 being typical. These nonconducting magnetic cores are not suitable for use in power transformers, however, as they saturate at low flux densities.

2-2. Permanent Magnets.² Permanent magnets now find many uses as a result of the development in recent years of improved permanent magnet materials. A typical system involving a permanent magnet is illustrated in Fig. 2-5. Here *P* is the permanent magnet, *G* is an air gap in which it is desired that the permanent magnet produce magnetic flux, and *D* denotes soft-iron pole pieces of low magnetic reluctance. In such an arrangement, the permanent magnet can be thought of as being a generator of magnetomotive force that acts on an external circuit (load) consisting of the magnetic circuit *DGDGD* that is external to the permanent magnet.

Assume that the permanent magnet in the system of Fig. 2-5 is magnetized to saturation and that the magnetizing force is then removed. The resulting situation that exists in the magnet corresponds to a point somewhere on the part of the hysteresis loop lying in the upper left-hand quadrant of Fig. 2-3. This section of the hysteresis curve, shown enlarged in Fig. 2-6, is termed the demagnetization curve, and gives the principal characteristics of the permanent magnet. The flux density *B*,

in the magnet for zero magnetizing force is termed the *residual induction*, while the demagnetizing force H_c which makes the flux density in the magnet zero is termed the *coercive force*.

For an operating condition of the magnet in Fig. 2-6 corresponding to point C' , the flux density in the magnet is B' , and the total flux generated by the permanent magnet is $B'A$ where A is the cross-sectional area of the permanent magnet. Also for the same operating point C' , each unit length of the permanent magnet produces a magnetomotive force H' ; hence the total magnetomotive force that is applied to the external circuit (DGDGD in Fig. 2-5) by the permanent magnet is $H'l$, where l is the length of the permanent magnet. The operating point C' accordingly assumes a position on the demagnetization curve such that $H'l/B'A$ equals the reluctance of the external magnetic circuit.

Design Principles. The magnetic energy developed by the permanent magnet in the external system DGDGD in Fig. 2-5 is proportional to the product $(B'A)(H'l)$ of magnetic flux and magnetomotive force associated with the external circuit. Thus the magnetic energy available in the external circuit per unit volume of the permanent magnet is proportional to the product BH of the demagnetization curve, as plotted in Fig. 2-6.

It is now possible to state the principal design considerations of systems involving permanent magnets. First, the permanent magnet should be operated at a point on the demagnetization curve where the energy product BH is at or near its maximum; this operating point is a characteristic of the magnetic material involved, and defines a magnetomotive force H' per unit length and a flux density B' for the permanent magnet. Next, the cross section A of the magnet is given a value such that, when the flux density in the magnet has the value of B' , the total flux $B'A$ will have the value desired for the external magnetic circuit. Finally, the length l of the permanent magnet is made such that $H'l$ will equal the magnetomotive force required to develop the required flux $B'A$ in the external magnetic circuit.

A permanent magnet operating at a point such as C' in Fig. 2-6 will have the flux density permanently changed when subjected to a transient action that momentarily reduces the flux density below B' . Thus assume that a transient demagnetizing current (or a momentary increase in

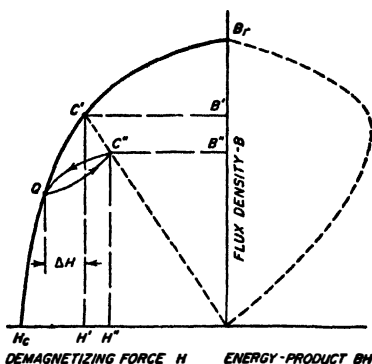


FIG. 2-8. Demagnetization and energy-product curves of a permanent magnet, showing minor hysteresis loop associated with stabilization.

reluctance) shifts the operating point from C' to Q . If this added effect is now removed, the operating point does not return to C' . Rather, it moves to a new point C'' , corresponding to some new value of flux density B'' less than B' , but such that the ratio $H''l/B''A$ still equals the reluctance of the external magnetic circuit; when this reluctance is linear, as when it arises from an air gap, then C'' lies on a straight line joining C' and the origin, as shown in Fig. 2-6. If the transient added force is applied a second time, the operating point will now return to Q , and upon removal of the added force will go back to C'' , following the paths shown.

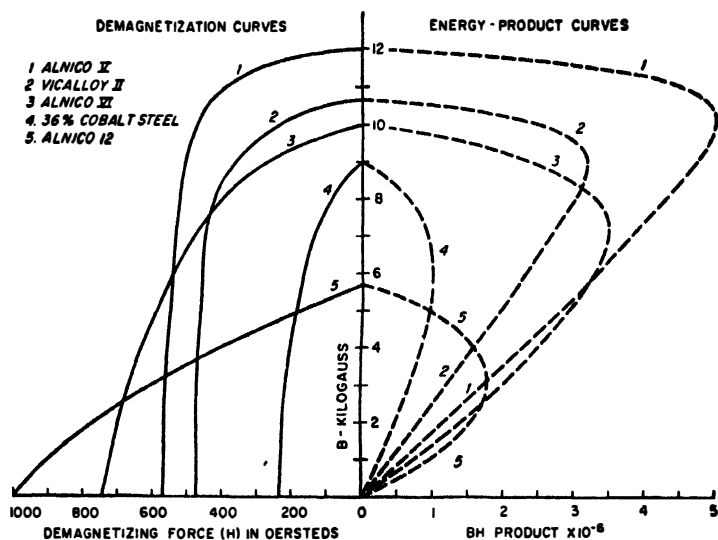


Fig. 2-7. Demagnetization and energy-product curves of typical permanent magnet materials.

Thus a permanent magnet system can be stabilized against added effects by initially subjecting the system to an added demagnetizing force ΔH ; this will reduce the energy that the magnet supports in the system external to the magnet, but it prevents subsequent transient demagnetizing effects from producing a permanent change in the system provided their amplitudes do not exceed the demagnetizing force ΔH used in stabilization.¹ It will be noted that QC'' is a minor hysteresis loop analogous to the right-hand half of loop 1 in Fig. 2-3.

Permanent-magnet Materials.² Many different types of permanent-magnet materials have been developed; the characteristics of representative examples are illustrated in Fig. 2-7. The magnetic properties depend

upon the composition and require proper cold working and heat-treatment to be fully developed. Heat-treatment is sometimes carried out in the presence of a strong magnetizing field; in this case the material when used should be magnetized in the same direction as when heat-treated.

The choice between different materials for a particular application is determined, not only by the energy product, but also by cost, by ease of fabricating, by whether the magnet is to be used in an external system of high or low reluctance, etc. In general, the better permanent-magnet materials are very difficult to work. Thus the alnicos (aluminum-nickel-iron alloys) are hard, weak, and brittle, and are commonly cast to approximate shape and then finished by grinding to exact size; they cannot be machined, drilled, or tapped.

2-3. Mutual Inductance and Coefficient of Coupling. *Mutual Inductance.* When two inductance coils are so placed in relation to each other

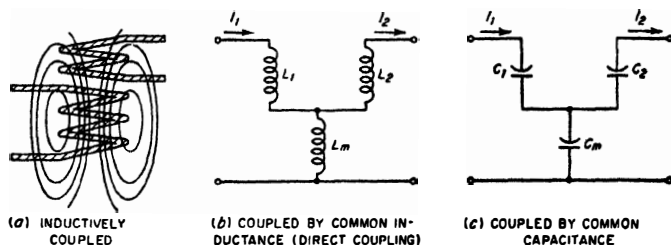


FIG. 2-8. Several simple methods of coupling two circuits.

that flux lines produced by current in one of the coils link with the turns of the other coil as shown in Fig. 2-8a, the two inductances are said to be inductively coupled. The effects that this coupling produces can be expressed in terms of a property called the *mutual inductance*, which is defined by the relation

$$\left. \begin{array}{l} \text{Mutual} \\ \text{inductance} \\ M \text{ in henrys} \end{array} \right\} = \frac{\left\{ \begin{array}{l} \text{flux linkages in second coil} \\ \text{produced by current in first coil} \end{array} \right\}}{\text{current in first coil}} \times 10^{-8} \quad (2-3)$$

$$= \frac{\left\{ \begin{array}{l} \text{flux linkages in first coil} \\ \text{produced by current in second coil} \end{array} \right\}}{\text{current in second coil}} \times 10^{-8} \quad (2-4)$$

Formulas (2-3) and (2-4) are equivalent and give the same value of mutual inductance. The flux linkages produced in the coil that has no current in it are counted just as though there were a current in this coil, so that the number of times a flux line would encircle an imaginary coil current is the number of linkages contributed by this particular line. In adding up the flux linkages it is important to note that different flux lines may conceivably link with the same coil in opposite directions, in which case

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the total number of linkages is the difference between the sums of positive and negative linkages. The mutual inductance may therefore be positive or negative depending upon the direction of the linkages.

The problem of calculating mutual inductance is similar in all respects to the problem of computing inductance, and formulas have been worked out by which the mutual inductance can be calculated with good accuracy in all the ordinary types of configurations.

When two coils of inductance L_1 and L_2 , between which a mutual inductance M exists, are connected in series, the equivalent inductance of the combination is $L_1 + L_2 \pm 2M$. The term $2M$ takes into account the flux linkages in each coil due to the current in the other coil. These mutual linkages may add to or subtract from the self-linkages, depending upon the relative direction in which the current passes through the two coils. Thus, when all linkages are in the same direction, the total inductance of the series combination exceeds by $2M$ the sum of the individual inductances of the two coils.

Coefficient of Coupling. The maximum value of mutual inductance that can be obtained between two coils having inductances L_1 and L_2 is $\sqrt{L_1 L_2}$. The ratio of the mutual inductance M that is actually present to this maximum possible value of mutual inductance is called the *coefficient of coupling*, which can therefore be expressed by the relation

$$\text{Coefficient of coupling} = k = \frac{M}{\sqrt{L_1 L_2}} \quad (2-5)$$

The coefficient of coupling is a convenient constant because it expresses the extent to which the two inductances are coupled, independently of the size of the inductances concerned. In air-cored coils a coupling coefficient of 0.5 is considered high and is said to represent "close" coupling, while coefficients of only a few hundredths represent "loose" coupling.

General Case of Coupled Circuits. Any two circuits so arranged that energy can be transferred from one to the other are said to be coupled, even though this transfer of energy takes place by some means such as a capacitor, resistance, or inductance common to the two circuits rather than by the aid of a mutual inductance. Examples of various methods of coupling are shown in Fig. 2-8. *Any two circuits that are coupled by a common impedance have a coefficient of coupling that is equal to the ratio of the common impedance to the square root of the product of the total impedances of the same kind as the coupling impedance that are present in the two circuits.* That is,

$$k = \frac{Z_m}{\sqrt{Z_1 Z_2}} \quad (2-6)$$

where Z_m is the impedance common to the two circuits, and Z_1 and Z_2 are

the total impedances of the *same kind* in the two circuits. When applied to case *b* in Fig. 2-8, where the coupling is furnished by the common inductance L_m , the total inductances of the two circuits are $L_1 + L_m$ and $L_2 + L_m$, respectively, and Eq. (2-6) reduces to

$$\text{Coefficient of coupling } k \text{ for Fig. 2-8b} = \frac{L_m}{\sqrt{(L_1 + L_m)(L_2 + L_m)}} \quad (2-7)$$

In Fig. 2-8c the coupling element is a common capacitance C_m , and the coefficient of coupling is¹

$$\text{Coefficient of coupling for Fig. 2-8c} = \frac{\sqrt{C_1 C_2}}{\sqrt{(C_m + C_1)(C_m + C_2)}} \quad (2-8)$$

2-4. Skin Effect in Coils and Conductors at Radio Frequencies. The effective resistance offered by conductors to radio frequencies is considerably more than the ohmic resistance measured with direct currents. This is because of an action known as *skin effect*, which causes the current to be concentrated in certain parts of the conductor and leaves the remainder of the cross section to contribute little or nothing toward carrying the current.

A simple example of skin effect, and one that makes its nature clear, is furnished by an isolated round wire. When a current is flowing through such a conductor, the magnetic flux that results is in the form of concentric circles, as shown in Fig. 2-9. It is to be noted that some of this flux exists within the conductor and therefore links with, i.e., encircles, current near the center of the conductor while not linking with current flowing near the surface. The result is that the inductance of the central part of the conductor is greater than the inductance of the part of the conductor near the surface; this is because of the greater number of flux

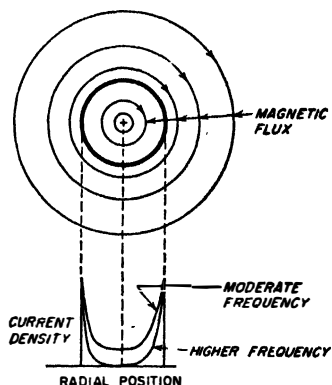


FIG. 2-9. Isolated round conductor, showing magnetic flux paths, and also typical current distributions.

¹ Equation (2-7) is derived as follows: In Fig. 2-8c, the primary circuit has C_1 and C_m in series and so has an equivalent capacitance of $C_1 C_m / (C_1 + C_m)$ while the equivalent capacitance of the secondary is similarly $C_2 C_m / (C_2 + C_m)$. The coupling reactance is $1/\omega C_m$, while the primary and secondary reactances are $(C_1 + C_m)/\omega C_1 C_m$ and $(C_2 + C_m)/\omega C_2 C_m$, respectively. The coefficient of coupling is then

$$k = \frac{1/\omega C_m}{\sqrt{\frac{C_1 + C_m}{\omega C_1 C_m} \frac{C_2 + C_m}{\omega C_2 C_m}}}$$

which reduces to Eq. (2-7).

linkages existing in the central region. At radio frequencies, the reactance of this extra inductance is sufficiently great to affect seriously the flow of current, most of which flows along the surface of the conductor where the impedance is low, rather than near the center where the impedance is high. The center part of the conductor, therefore, does not carry its share of the current and the true or effective resistance is increased, since in effect the useful cross section of the wire is very greatly reduced. The types of current distribution obtained in typical cases of skin effect in a round wire are shown in Fig. 2-9.

When skin effect is present, the current is always redistributed over the conductor cross section in such a way as to make most of the current flow where it is encircled by the smallest number of flux lines. This general principle controls the distribution of current, irrespective of the shape of the conductor involved. Thus, with a conductor consisting of a thin flat strip, such as shown in Fig. 2-10, the current flows primarily along the edges, where it is surrounded by the smallest amount of flux, and the true or effective resistance will be

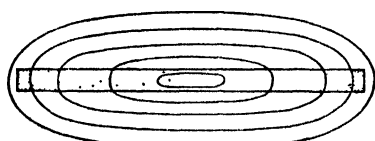


FIG. 2-10. Flux lines and current distribution in a thin strip at radio frequency, showing how skin effect causes the current to concentrate at the edges of the strip. The current density is indicated by the density of the shading.

high because most of the strip carries very little current. This illustration makes clear that it is not the amount of conductor surface that determines the resistance to alternating current, but rather the way in which the conductor material is arranged.

The ratio that the effective a-c resistance bears to the d-c resistance of a conductor is commonly called the *resistance ratio*. It increases with frequency, with conductivity of the conductor material, and with the size of conductor. This results from the fact that a higher frequency causes the extra inductance at the center of the conductor to have a higher reactance. Similarly, a greater conductivity makes the reactance of the extra inductance of more importance in determining the distribution of current, while a greater cross section provides a larger central region. It is to be noted, however, that a larger conductor always has less radio-frequency resistance than a smaller one because, although the ratio of a-c to d-c resistance is less favorable, this is more than made up by the greater amount of conductor cross section present.

*Skin Effect at High Frequencies.*¹ When the frequency is sufficiently high, substantially all of the current in a conductor is confined to a region very close to the surface. The current density then falls off with depth from the surface in accordance with the relation

¹ An excellent discussion of skin effect at very high frequencies is given by Harold A. Wheeler, Formulas for the Skin Effect, *Proc. IRE*, vol. 30, p. 412, September, 1942.

$$\frac{\text{Current at depth } z}{\text{Current at the surface}} = \quad (2-9)$$

Here z and δ are in the same units, and δ is a quantity called the *skin depth* that is given by the equation

$$\delta = 5033 \sqrt{\frac{\rho}{\mu f}} \quad (2-10)$$

where δ = skin depth, cm

ρ = resistivity of conductor, ohms per centimeter cube

f = frequency, cycles

μ = magnetic permeability of core material (permeability of air equals unity), for low flux densities (that is, μ is the initial permeability)

For copper at 20°C this reduces to

$$\text{Skin depth of copper in cm} = \frac{6.62}{\sqrt{f}} \quad (2-11)$$

At 1 Mc the skin depth in copper is thus 0.0066 cm, or 0.0026 in. The phase of the current at depth z lags the current at the surface by z/δ radians. At a depth from the surface corresponding to one skin depth, the current density has dropped to 36.8 per cent of the value at the surface, and the phase of the current lags the current at the surface by 1 radian.

Equation (2-9) is valid whenever the radius of curvature of the conductor surface is at least several times the skin depth, provided the effective thickness of the conductor is at the same time at least three or four skin depths.

The power loss associated with the current flowing under any particular portion of the conductor surface is the same as though this current were uniformly distributed down to a depth δ . Thus, in an isolated round wire, where the current is uniformly distributed over the surface, the effective resistance at high frequencies is the d-c resistance of a hollow cylindrical shell having the same outer diameter as the wire and possessing a thickness δ . The d-c resistance of a strip of surface one skin depth thick, one centimeter long, and one centimeter wide is sometimes called the *surface resistivity*; it is the resistivity that is offered to the flow of current at very high frequencies.

Proximity Effect—Skin Effect in Coils. When two or more adjacent conductors are carrying current as in a coil, the current distribution in any one conductor is affected by the magnetic flux produced by the adjacent conductor as well as by the magnetic flux produced by the current in the conductor itself. This effect, termed *proximity effect*, ordinarily causes the true or effective resistance to be greater than in the case of simple skin effect and is particularly important in radio-frequency inductance coils.

The current distribution under conditions where proximity effect is present follows the same law as for simple skin effect; i.e., the current density is greatest in those parts of the conductor encircled by the smallest number of flux lines. This is illustrated in Fig. 2-1, where the approximate current density is illustrated by relative shading.

Litz Wire. The effective a-c resistance of a conductor can be made to approach the d-c resistance at low and moderate radio frequencies by forming the conductor from a number of strands of small enameled wire connected in parallel at their ends, but insulated throughout the rest of their length and thoroughly interwoven. If the stranding is properly done, each wire will, on the average, link with the same number of flux lines as every other wire, and the current will divide evenly among the strands. If at the same time each strand is of small diameter, it will have relatively little skin effect over its cross section, so all of the material is equally effective in carrying the current. Such a stranded cable is called a *litz* conductor.

Practical litz conductors are very effective at frequencies below about 1000 kc, but as the frequency becomes higher the benefits disappear. This is because irregularities of stranding, and capacitance between the strands, cause a failure to realize the ideal condition at very high frequencies.

2-5. Capacitors and Dielectrics. A capacitor is formed wherever an insulator (i.e., dielectric) separates two conductors between which a difference of potential can exist.

Capacitor Losses and Their Representation. A perfect capacitor when discharged gives up all the electrical energy that was supplied to it in charging. Actual capacitors never realize this ideal perfectly but, rather, dissipate some of the energy delivered to them. Most of the loss in ordinary capacitors occurs in the dielectric, although at very high frequencies skin effect also causes an appreciable loss to occur in the capacitor leads and electrodes. At very high voltages corona may occur and contribute to the loss.

The merit of a capacitor from the point of view of freedom from losses is usually expressed in terms of the power factor of the capacitor.¹ The power factor represents the fraction of the input volt-amperes that is dis-

¹ The merit of a capacitor or of a dielectric can also be expressed in terms of the angle by which the current flowing into the capacitor fails to be 90° out of phase with the applied voltage. This angle is termed the *phase angle* of the capacitor. The power factor is the sine of the phase angle. The tangent of the phase angle is termed the *dissipation factor*. The reciprocal of the dissipation factor is termed the capacitor *Q* and is the ratio of the capacitor reactance to the equivalent series resistance. With ordinary dielectrics, the phase angle is so small that the power factor, the dissipation factor, and the reciprocal of capacitor *Q* are for all practical purposes equal to each other and to the phase angle expressed in radians. Thus a power factor of 0.01 represents a phase angle of 0.573° and a capacitor *Q* of 100.

sipated in the capacitor. To the extent that the losses in the capacitor are a result of dielectric losses, the power factor of the capacitor is also the power factor of the dielectric and is practically independent of the capacitor capacitance, the applied voltage, the voltage rating, or the frequency (unless polar effects are involved). Values of power factor of some typical dielectrics are given in Table 2-1.

TABLE 2-1
CHARACTERISTICS OF TYPICAL DIELECTRICS AT RADIO
FREQUENCIES WITH NORMAL ROOM TEMPERATURE

Material	Dielectric constant	Power factor
Air.....	1.00	0.000
Mica (electrical).....	5-9	0.0001-0.0007
Glass (electrical).....	4.5-7.00	0.002-0.016
Bakelite derivatives.....	4.5-7.5	0.02-0.09
Wood (without special preparation).....	3-5	0.03-0.07
Mycalex.....	8	0.002
Steatite materials.....	6.1	0.002-0.004
Polystyrene.....	2.4-2.9	0.0002
Polyethylene.....	2.3	0.00015-0.0003
Rutile (titanium dioxide).....	90-170	0.0006

Although the power factor of a capacitor is determined largely by the type of dielectric used in the capacitor, it is also affected by the conditions under which the dielectric operates. In particular, the power factor tends to become higher as the temperature is raised, and is likewise adversely affected by high humidity and by the absorption of moisture.

Equivalent Series and Shunt Resistance. The action of a capacitor in an electrical circuit is taken into account by replacing the actual capacitor with a perfect capacitor associated with a resistance. This resistance may be connected in series, as in Fig. 2-11b, or in parallel, as in Fig. 2-11c. The value of the series or shunt resistance is so selected that the power factor of the perfect capacitor associated with the resistance is the same as the power factor of the actual capacitor. The value of the series resistance R_1 can be computed in terms of the power factor, capacitor capacitance C , and frequency f in the usual way, and when the power factor is low (i.e., when $R_1 \ll 1/\omega C$), then R_1 is given to a high degree of accuracy by the equation

$$\text{Series resistance} = R_1 = \frac{\text{power factor}}{2\pi f C} \quad (2-12)$$

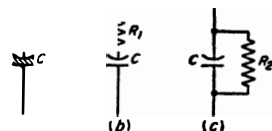


FIG. 2-11. Representation of imperfect capacitor by a perfect capacitor of same capacitance with series resistance, and by a perfect capacitor with shunt resistance.

In the same way, the shunt resistance that can be used to represent the actual losses of the capacitor is related to the power factor, capacitance, and frequency to a high degree of accuracy by the equation

$$\text{Shunt resistance} = R_2 = \frac{1}{(2\pi fC) (\text{power factor})} \quad (2-13)$$

Polar and Nonpolar Dielectrics. Molecules of some dielectrics are polar, while other dielectrics consist of molecules that are not polar. In

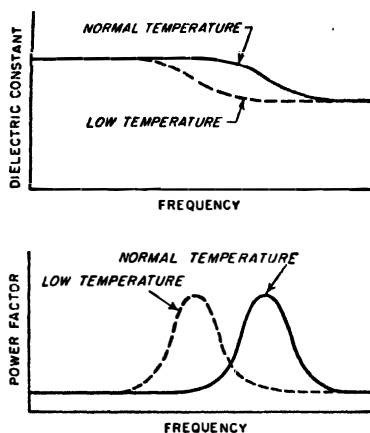


FIG. 2-12. Variation of dielectric constant and power factor of a polar dielectric as a function of frequency, for two temperatures.

the case of polar molecules, the dielectric constant under a-c conditions is increased as a result of the rotation of the polar molecules under the influence of the applied voltage. The extent to which this polar action is effective depends, however, upon the frequency and the temperature. Thus, if the frequency is made sufficiently high, the polar molecules are not able to follow the alternations of the applied field, and the dielectric constant drops. Moreover, the frequency at which this transition occurs is less the lower the temperature. As a result, temperature and frequency affect the capacitance of a capacitor possessing a polar dielectric in the manner shown in Fig. 2-12.

The power factor of a polar dielectric shows a pronounced peak when under conditions where the dielectric constant corresponds to partial polar action, as shown in Fig. 2-12. The power factor of a polar dielectric hence becomes quite large at certain combinations of temperature and frequency.

Nonpolar molecules do not exhibit these changes in dielectric constant under temperature and frequency changes. The power factor of nonpolar dielectrics likewise does not exhibit peaks of loss such as shown in Fig. 2-12.

2-6. Capacitors for Electronics. In electronics the principal uses made of capacitors are for tuning resonant circuits, for blocking d-c voltages from parts of an electrical circuit while permitting alternating voltages to pass through, for obtaining transients with specified time constants, and for by-passing or short-circuiting alternating voltages. By-pass capacitors are frequently but not always subjected to a d-c potential.

A wide variety of dielectrics are used in capacitors designed for radio

work, and new types are continually finding important applications. Among the types of importance are air; solid dielectrics such as mica, plastic films, certain ceramics, and paper; and electrolytic films.

Capacitors with Air Dielectric. Air dielectric finds its principal use in variable capacitors for tuning resonant circuits.

Although air is a perfect dielectric with zero power factor, air capacitors have losses because of the insulating material used to mount the two sets of plates, and also because of the skin-effect resistance of the leads, plates, rods, and washers, through which the capacitor current flows.

An air-dielectric capacitor can be represented by the equivalent electrical network in Fig. 2-13a. Here C is the capacitance of the capacitor

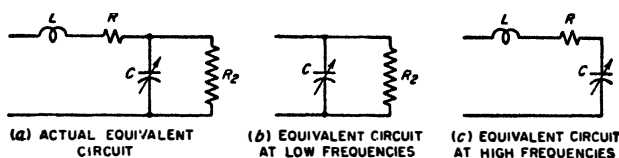


FIG. 2-13. Equivalent electrical circuits of a variable air condenser.

while R_2 is the equivalent shunt resistance introduced by the presence of the solid dielectric. The capacitor inductance L in Fig. 2-13a takes into account the magnetic flux associated with the current flowing in the capacitor; it is proportional to the physical dimensions of the capacitor. The resistance R represents the series resistance of the leads, washers, connecting rods, etc. It increases with frequency as a result of skin effect and is proportional to the square root of the frequency at high frequencies.

At low and moderate frequencies the effects of the inductance L and the series resistance R are negligible, and the capacitor equivalent circuit reduces to Fig. 2-13b. At very high frequencies, on the other hand, the power loss in R becomes very much larger than in R_2 , and the equivalent circuit has the form shown in Fig. 2-13c.

At very high frequencies the reactance of the series inductance L is not negligible compared with the reactance of the capacitor capacitance. This causes the apparent capacitance of the capacitor as observed at the terminals to be greater than the actual capacitance according to the relation¹

$$\text{Apparent capacitance} = \frac{C}{1 - \omega^2 LC} \quad (2-14)$$

where $\omega = 2\pi$ times frequency, and L and C are as shown in Fig. 2-13.

¹ This results from the fact that, neglecting losses,

$$\text{Reactance at terminals} = \frac{1}{\omega C_{app}} = \left(\frac{1}{\omega C} - \omega L \right)$$

Solving for the apparent capacitance C_{app} gives Eq. (2-14).

Capacitors with Solid Dielectrics. Solid dielectrics are used in most fixed and in some adjustable capacitors. The dielectrics most commonly employed include mica, paper, plastic films, and ceramics.

Mica is characterized by low electrical losses, stability, high leakage resistance to d-c voltages, and high voltage strength. It is, however, relatively expensive. Mica capacitors find their chief use in small fixed capacitors for by-passing radio-frequency currents or blocking off d-c voltages, and in resonant circuits or in filters where a stable low-power-factor capacitor is required.

In capacitors employing paper as the dielectric the electrodes are either aluminum foil or are metal films evaporated directly on to the paper. In either case, the assembly is rolled into a bundle which is then vacuum-treated, impregnated with oil or wax, and sealed against moisture. Paper capacitors are inexpensive in proportion to capacitance, and are relatively compact in proportion to voltage rating. Such capacitors are used primarily for by-pass and blocking purposes. The power factor of paper capacitors is of the order of 0.5 per cent, and although the leakage current when subjected to direct voltages is somewhat greater than that of mica capacitors, it is not large.

Thin plastic films have been developed that are suitable for use as a capacitor dielectric in place of paper. Capacitors of this type using polystyrene dielectric have electrical qualities such as power factor, dielectric absorption, and insulation resistance superior even to mica capacitors.

Ceramics based on titanium dioxide mixtures find extensive use as dielectrics of small capacitors.¹ Dielectrics of this type are characterized by a high dielectric constant, a low to very low power factor, and a very high voltage rating. The temperature coefficient of such capacitors depends upon the actual ceramic mixture used and can be made either negative or positive as desired. Ceramic capacitors are used extensively for blocking and by-pass purposes where small mica capacitors have heretofore been employed, and have advantages of compactness and high voltage ratings. Ceramic dielectric capacitors have also found a wide field of usefulness in resonant circuits and other similar applications, where a negative temperature coefficient provided by a ceramic capacitor can be used to compensate for the positive temperature coefficient of associated coils and of capacitors of other types.

Capacitors with solid dielectric can be represented by the same equivalent electrical circuit shown in Fig. 2-13 for air-dielectric capacitors. The only difference is that all the capacitance C in this equivalent circuit is now associated with solid dielectric. As a result, at low and moderate frequencies the capacitor power factor almost exactly equals the power

¹ A survey of such ceramics is given by B. H. Marks, *Ceramic Dielectric Materials*, *Electronics*, vol. 21, p. 116, August, 1948.

factor of the dielectric and is independent of the capacitance of the capacitor and also of the frequency except in so far as polar molecules affect the behavior of the dielectric. At very high frequencies, the power factor increases with increasing frequency as a result of the skin-effect losses in leads and conductors. Also, the apparent capacitance at very high frequencies drops off because of the series inductance, in accordance with Eq. (2-14).

The voltage rating of capacitors with solid dielectrics is subject to two basic limitations: (1) If the applied voltage exceeds the insulation strength of the dielectric, the dielectric will spark through or at least deteriorate rapidly. (2) The temperature of the capacitor must not be permitted to rise excessively as a result of dielectric losses. This second limitation is the ruling one for all except d-c voltages and for very low frequencies. Inasmuch as the relationship between losses and temperature rise depends upon the design of the capacitor with respect to such matters as heat removal, it is not possible to give any general rules regarding voltage ratings. It is to be noted, however, that the voltage rating will drop rapidly as the frequency increases because of the increase in loss with frequency. Thus a particular low-loss air-cooled mica capacitor capable of standing 10,000 volts at low-frequencies was found by test to have a rating of 180 volts at a frequency of 10 Mc. Special cooling methods, such as the use of an air blast, will increase greatly the rating on a capacitor, and water cooling is still more effective.

Electrolytic Capacitors. The electrolytic capacitor makes use of the fact that certain metals, notably aluminum and tantalum, when placed in a suitable solution and made the positive electrodes, form a thin insulating surface film. This film is capable of withstanding considerable voltage and has a high electrostatic capacitance per unit area of film. It is the result of electrochemical action, and is formed by applying positive voltage to the electrode. The thickness of the film, and hence also the capacitance obtained per unit area of surface, depend largely upon the voltage used in this forming process. Typical voltage ratings of electrolytic capacitors range from 25 up to about 500 volts. Constructional details vary but, typically, the electrodes are of etched aluminum foil, thus giving maximum surface area. They are separated by paper or gauze, saturated with an electrolyte that is commonly a fudgelike solid, and the entire assembly is wound into a roll and mounted in a waxed cardboard tube or box.

Electrolytic capacitors are widely used for filter and by-pass purposes in situations where a superimposed d-c voltage is present. Compared with capacitors of the solid dielectric type, electrolytic capacitors have a very high power factor and appreciable leakage conductance to the superimposed d-c potential; they also vary in capacitance and loss with time, frequency, and temperature. However, for many purposes these

features are unimportant and, in proportion to capacitance and voltage rating, electrolytic capacitors are the least expensive and most compact available. They are, however, subject to progressive deterioration with time and so have limited life, and their dependability is appreciably less than that of paper capacitors designed for the corresponding applications.¹

2-7. Coils for Resonant Circuits. Coils intended for use in resonant circuits must have very low losses and small distributed capacitance. Both air-cored and magnetically cored coils are used for resonant circuits, with the choice depending upon circumstances.

Methods of Expressing Coil Losses—Coil Q. The principal causes of energy loss in air-cored coils are skin effect in the conductor, proximity effect resulting from the interaction between nearby turns, dielectric losses associated with the distributed capacitance of the coil, and eddy-current losses in shields and other neighboring metallic objects present within range of the magnetic field of the coil. In the case of coils with magnetic cores, the principal cause of energy loss is usually core loss, although factors such as skin-effect resistance of the wire and also dielectric loss are sometimes likewise of significance.

For purposes of circuit analysis the coil losses are commonly expressed in terms of an equivalent resistance, which when placed in series with the coil inductance will account for all the power losses actually observed. The most convenient way to express the merit of the coil is, however, in terms of the ratio of the reactance ωL of the coil to this equivalent series resistance R . This ratio approximates the reciprocal of the coil power factor, and is usually referred to by the symbol Q ; that is,

$$Q = \frac{\text{coil reactance}}{\text{equivalent series resistance}} = \frac{\omega L}{R} \quad (2-15)$$

It is convenient to express the characteristics of a coil in terms of Q because the Q in the operating range of the coil usually varies only moderately with frequency; moreover, the value of Q corresponding to a good coil is substantially the same irrespective of the frequency for which the coil was designed. The tendency for the coil Q to remain constant with frequency arises from the fact that, as the frequency increases, all the losses also increase, so that the *ratio* of coil reactance to resistance tends to be much more nearly constant with frequency than is either the reactance or the resistance of the coil.

Distributed Capacitance of Coils. In a coil there are small capacitances between adjacent turns, between turns that are not adjacent, between terminal leads, between turns and ground, etc. Some of the different capacitances that may exist in a typical air-cored coil are shown in Fig.

¹ By substituting tantalum for the less expensive aluminum foil electrodes, it is possible to increase greatly the reliability; see M. Whitehead, Tantalum Electrolytic Capacitors, *Bell Lab. Record*, vol. 28, p. 448, October, 1950.

2-14. Each of the various capacitances associated with the coil stores a quantity of electrostatic energy that is determined by the capacitance involved and the fraction of the total coil voltage that appears across it. The total effect that the numerous small coil capacitances have can be represented to a high degree of accuracy by assuming that they can be replaced by a single capacitor of appropriate size shunted across the coil terminals. This equivalent capacitance is called either the distributed capacitance or the self-capacitance of the coil; it causes the coil to show parallel resonance effects under some conditions (see Sec. 3-2).

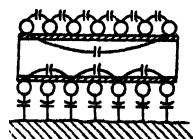


FIG. 2-14. Some of the coil capacitances that contribute to the distributed capacitance of a single-layer coil.

In multilayer coils the distributed capacitance will be high unless arrangements are used that prevent turns from different parts of the winding from being located close to each other. Thus, in the two-layer winding shown in Fig. 2-15a, in which the turns are numbered in order, the first and last turns are adjacent; the capacitance between the

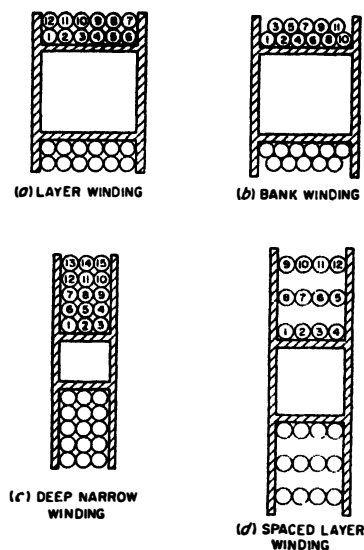


FIG. 2-15. Several types of multilayer windings.

turns at opposite ends of the winding then stores an undesirably large amount of electrostatic energy. This can be avoided by the use of the bank winding, shown at b. Here the adjacent turns represent parts of the coil that are close together electrically, while the ends of the winding, which are far apart electrically, are also far apart physically. Alternative approaches consist in using many layers with few turns per layer, as in Fig. 2-13c, or in spacing the layers, as in Fig. 2-15d. The common "universal" multilayer coil represents a convenient mechanical method of utilizing these principles to achieve low distributed capacitance.

The distributed capacitance of a coil that is to be used in a resonant circuit must be small. This is because the distributed capacitance

limits the highest frequency to which the coil can be tuned, and also introduces losses that become serious at the higher frequencies. These losses are dielectric losses occurring in the coil form, in the wire insulation, and in any other dielectric that may be in the electrostatic fields associated with the coil.

Air-cored Coils for Resonant Circuits. Air-cored coils are widely used in radio receivers and almost universally used for the resonant circuits of radio transmitters. Single-layer coils are generally employed for frequencies above 500 to 1500 kc, while at lower frequencies multilayer coils are typical, as they give the desired inductance compactly. Multilayer coils, generally of the bank-wound type, also find some use at broadcast frequencies (535 to 1600 kc).

In designing a single-layer coil, the highest Q in proportion to size is obtained when the length of the winding is somewhat less than the diameter of the coil.¹ The number of turns required is then determined by

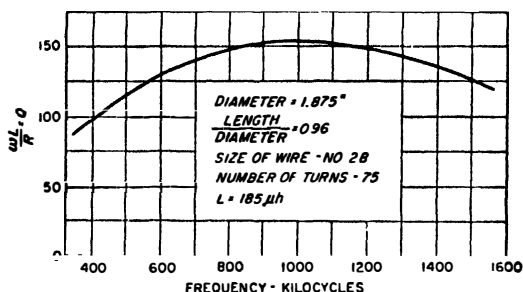


FIG. 2-16. Variation of Q with frequency for a typical air-cored coil.

the exact ratio of length to diameter that is selected, by the diameter, and by the inductance desired; when these factors are all settled, the optimum wire size corresponds to a conductor diameter that is between 0.5 and 0.75 times the distance between the centers of adjacent turns. If one compares the Q of two coils having the same inductance and the same ratio of length to diameter but different physical size, then the coil that is larger will have the highest Q provided it is wound with wire of optimum size.

The design of multilayer coils is more involved than that of single-layer coils because of the increased number of variables. In general, best results are obtained if the coil is relatively "loose," i.e., if the copper occupies only a small fraction of the actual winding cross section. Again, as in the case of single-layer coils, larger physical size will result in a higher Q associated with a given inductance value, and also requires a larger wire.

The Q of a typical air-cored coil varies with frequency in the manner illustrated in Fig. 2-16. With increasing frequency, the Q first rises slowly with frequency, then goes through a broad maximum, and finally drops at very high frequencies. The rise is due to the fact that the

A discussion of coil losses under idealized conditions in which dielectric effects are neglected is given by G. W. O. Howe, *The Q Factor of Single-layer Coils*, *Wireless Eng.*, vol. 26, p. 179, June, 1949. An excellent discussion of coils for high frequencies is given by D. Pollack, *The Design of Inductances for Frequencies between 4 and 25 Megacycles*, *Trans. AIEE*, vol. 56, p. 1169, September, 1937.

inductive reactance of a coil is proportional to frequency, whereas the resistance due to skin effect cannot increase faster than the square root of the frequency; hence the ratio $Q = \omega L/R$ tends to rise with increasing frequency. If skin effect accounted for all the losses, the Q at very high frequencies would be proportional to the square root of frequency. However, dielectric losses arising from the coil form, the cotton or enamel insulation on the wire, etc., give rise to a resistance in series with the coil that is proportional to the cube of the frequency. At very high frequencies these dielectric losses become comparable to the skin-effect losses, and cause the coil Q to drop off.

The best conductor to use in an air-cored coil depends upon the frequency and coil design. In general, solid wire is used for frequencies above 1500 kc. Litz wire will give lower losses than the corresponding solid wire at frequencies below about 500 kc and will give some advantage for small multilayer coils in the frequency range 500 to 1500 kc.

A value of Q in the range 50 to 200 is typical of a good fairly small air-cored coil such as would be used for resonant circuits in a radio receiver. A Q of 10 or 20 is considered to be quite low, while Q values in excess of 300 are high and can ordinarily be achieved only by the use of coils that are physically large, such as are used in radio transmitters. These numbers are applicable for coils of all frequency ranges and inductance values.

Magnetic-cored Coils for Resonant Circuits. Coils with magnetic cores find extensive use at radio frequencies. The principal problem involved in using magnetic cores at radio frequencies is that of preventing eddy-current losses in the core material from becoming excessive. The permeability of magnetic materials does not drop off with frequency until the frequency is of the order of 10^{11} cycles. The hysteresis loss is proportional to the frequency, but since the coil reactance is likewise proportional to the frequency, the Q is not adversely affected by hysteresis loss at radio frequencies. In contrast, the eddy-current loss for a given core is proportional to the square of the frequency, whereas the reactance is proportional only to the frequency. Thus when the frequency becomes sufficiently high, eddy-current losses dominate the situation, and the coil Q drops.

The eddy-current losses can be kept low at high frequencies by arranging the magnetic material in the form of very fine particles or dust, produced either by chemical or mechanical means. These particles are coated with an insulating film, mixed with a suitable proportion of binder, pressed to the desired shape, and baked. In this way one obtains a core in which the individual magnetic particles are very finely subdivided, with resulting low eddy-current losses; such an arrangement is often called a "dust" or powder core. It is possible to make "dust" cores which have low losses at frequencies as high as 150 Mc.

The details of the magnetic core depend upon the application for which the coil is intended, and the frequency range over which the core is to operate. At audio and the lower radio frequencies, the core material is commonly made in the form of rings (toroidal core), so that a closed magnetic path may be obtained. Cores designed for use at these frequencies usually have an effective initial permeability that is quite high, such as 75 to 125, corresponding to a core involving relatively coarse particles combined with a minimum of insulating material and binder. At higher frequencies, the usual practice is to employ a single-layer winding of fine wire on a form that snugly fits an open core made in the form of a cylindrical slug with a large length/diameter ratio, as in Fig. 2-17. Also, as the frequency is increased, the effective permeability that it is practical to employ in a core becomes less because the size of the particles of magnetic material in the core must be reduced, and the proportion of core material to binder and insulation

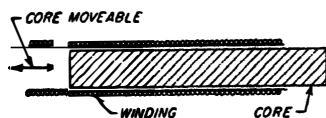


FIG. 2-17. Coil with slug-type magnetic core arranged so that the inductance can be varied by moving the core.

of core material to binder and insulation becomes proportionally less. Thus cores that are suitable for use in coils operating at 100 Mc have, typically, an effective permeability of only 2 to 4, while cores for use at frequencies around 1 Mc have permeabilities from 10 to 30.

The particular magnetic material used likewise depends on frequency. For audio and the lower radio frequencies molybdenum permalloy is common; at the higher radio frequencies it is customary to use iron or magnetite, a natural iron oxide.

An alternative means of obtaining a magnetic core with low eddy-current losses is to employ a nonconducting magnetic material such as mentioned on page 16. Such material is suitable for use up to frequencies above 20 Mc; however, above some limiting frequency the dielectric and residual loss effects in the nonconducting magnetic material may adversely affect the behavior. In frequency ranges for which they are suitable, nonconducting magnetic cores result in coils having Q 's as high as, or higher than, values typical of dust cores; at the same time a nonconducting core possesses considerably greater permeability than can be used in a dust core at the same frequency, and so has the advantage of compactness.¹

Magnetic cores are particularly desirable when it is necessary to obtain a reasonable Q such as 25 to 100 in a very compact coil. They find extensive use in radio receivers. When a magnetic-cored coil is used in a resonant circuit, it is customary to employ a fixed tuning capacitance; the resonant frequency is then adjusted by varying the position of the slug

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¹ A discussion of the properties obtainable in coils employing nonconducting ferrite cores is given by Strutt, *loc. cit.*

core, as indicated in Fig. 2-17. Such *permeability tuning*, as it is called, represents a means of tuning a resonant circuit that is often preferred to the alternative arrangement consisting of a fixed air-cored coil and a variable capacitance.

Radio-frequency Choke Coils. A radio-frequency choke coil is an inductance designed to offer a high impedance to alternating currents over the frequency range for which the coil is to be used. This result is obtained by making the inductance of the coil high and the distributed capacitance low, and by so proportioning that the inductance is in parallel

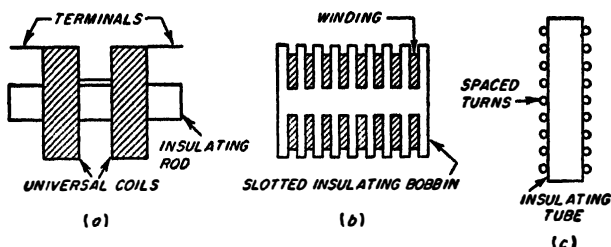


FIG. 2-18. Typical examples of radio-frequency choke coils.

resonance with the distributed capacitance somewhere in the desired operating range of frequencies.

A typical radio-frequency choke coil consists of one or more universal-wound coils mounted on an insulating rod, or of a series of "pies" wound in deep narrow slots in a slotted bobbin. A long single-layer solenoid is likewise sometimes used. Examples of radio-frequency choke coils are shown in Fig. 2-18.

The performance obtainable from a radio-frequency choke can generally be improved by the proper use of slug-type magnetic cores, which increase the inductance, and hence the impedance of the coil, without materially affecting the distributed capacitance.

2-8. Shielding of Magnetic and Electrostatic Fields. Under many conditions it is necessary to confine magnetic and electrostatic fields to a restricted space. This result is accomplished by using a shield composed of suitable material to enclose completely the space to be shielded.

The Shielding of Magnetic Flux at Radio Frequencies; Conducting Shields. The most practical shield for magnetic flux at radio frequencies is made of material having low electrical resistivity, such as copper or aluminum. Magnetic flux in attempting to pass through such a shield induces voltages in the shield which give rise to eddy currents. These eddy currents oppose the action of the flux, and in large measure prevent its penetration through the shield. In this way the flux is restricted to the interior of the shield, as illustrated in Fig. 2-19c.

To be effective a conducting shield should have a thickness a that is at

least several times the skin depth δ as defined by Eq. (2-10). Under these conditions, and assuming that the radius of curvature of the shield is large compared with the skin depth, the ratio of the tangential components of the magnetic field intensities existing on the two sides of the shield is¹

$$\text{Ratio of magnetic fields} = e^{-a/\delta} \quad (2-16)$$

Since the energy associated with a magnetic field is proportional to the square of the field intensity, the attenuation in decibels of the tangential component that is introduced by the conducting magnetic shield is

$$\text{Shield attenuation} = 8.69 \frac{a}{\delta} \quad \text{db} \quad (2-17)$$

Joints which interfere with the eddy currents by adding resistance to the eddy-current paths greatly reduce the effectiveness of a conducting shield. However, a joint parallel to the lines of current flow does not adversely affect the shielding unless it results in an open hole. This is true even if there is failure to make contact, so that the shield lacks continuity. These effects of joints are explained by the fact that the shielding is produced by the eddy currents; if the eddy currents are not disturbed, then the shielding resulting from action is not affected.

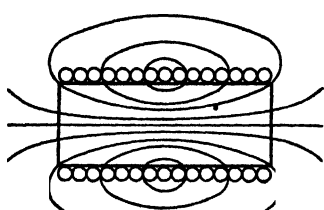
Power is dissipated in a conducting shield because the eddy currents must flow through the resistance of the shield material. When the thickness of the shield is considerably greater than the skin depth, the power loss in the shield can be determined by making use of the fact that the total magnitude I of the eddy currents in a strip of shield 1 cm wide is related to the density B in lines per square centimeter of the tangential component of flux that is adjacent to the surface of that part of the shield according to the relation

$$I = \frac{10B}{4\pi} \quad (2-18)$$

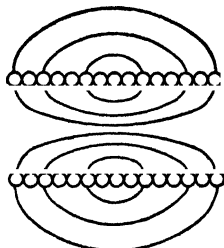
The current I flows along the surface of the shield in a direction that is at right angles to the flux lines adjacent to the shield. For purposes of calculating power dissipation, this current can be considered as uniformly distributed to a thickness of one skin depth; it therefore encounters a resistance that is the surface resistivity of the material as calculated by skin-effect considerations (see page 23). Thus the total power loss in a shield can be obtained by first determining by some means the distribution of the tangential component of the magnetic flux adjacent to the surface of the shield. The distribution of current over the surface of the shield is next obtained with the aid of Eq. (2-18). The energy loss in each square

¹ The effect of a conducting shield on the component of magnetic field that is normal to the shield follows a different law, see B. Boston, Screening at V.H.F., *Wireless Eng.*, vol. 25, p. 221, July, 1948.

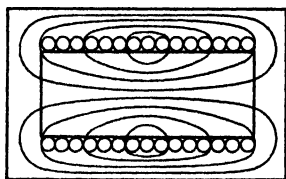
centimeter of shield surface is then determined by assuming that this current flows through a d-c resistance corresponding to a conductor that is one skin depth thick. The power consumed by a conducting shield is derived from the source of energy producing the magnetic field.



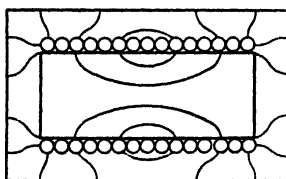
(a) MAGNETIC FLUX-
NO SHIELD



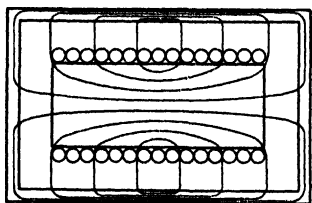
(b) ELECTROSTATIC FLUX-
NO SHIELD



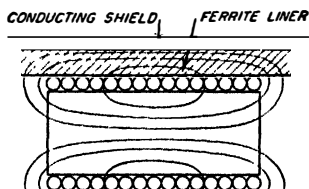
(c) MAGNETIC FLUX-
NON-MAGNETIC SHIELD



(d) ELECTROSTATIC FLUX WITH
CONDUCTING SHIELD



(e) MAGNETIC FLUX-
MAGNETIC SHIELD



(f) MAGNETIC FLUX - CONDUCTING
SHIELD WITH FERRITE LINER

FIG. 2-19. Paths of electrostatic and magnetic-flux lines about the same coil with and without magnetic and nonmagnetic shields.

Shielding of D-C and Low-frequency Magnetic Fields; Magnetic Shields. When shielding against unidirectional magnetic fields is required, a shield composed of magnetic material is employed. Such a shield tends to short-circuit the flux lines which attempt to extend through the shield, as shown in Fig. 2-19e. The effectiveness of a magnetic shield is directly proportional to the thickness of the shield, since the reluctance that the

shield offers to magnetic flux is inversely proportional to thickness. Joints or air gaps which add reluctance to the flux paths must be avoided. The degree of shielding achieved by a given total thickness of material can be increased by dividing the given thickness of magnetic material into two or more concentric shields separated by air spaces.

Magnetic shields must have high initial permeability to be effective. They are accordingly composed of high-permeability alloys such as permalloy; steel or iron is not a satisfactory material because of its low initial permeability. Since the desirable magnetic properties of permalloy and similar materials are adversely affected by mechanical strains, such as are introduced by drilling, punching, bending, etc., magnetic shields must be properly heat-treated *after* fabrication, to relieve these strains and develop the desirable magnetic properties.

Magnetic shields can be used for shielding alternating fields as well as d-c fields. In particular, they find extensive use at audio and power frequencies, particularly 60 cycles, where conducting shields would have to be excessively thick to be effective. The shielding action of a magnetic shield at these lower frequencies is achieved in part because of the short-circuiting action of the magnetic material on magnetic flux and in part because of eddy currents which cause the shield to act simultaneously as a conducting shield.

Magnetic shields of high-permeability material are also more effective at radio frequencies than are copper or aluminum shields. At these higher frequencies they act as conducting shields, but because of their high permeability have less skin depth. Thus the shielding obtained with a given thickness of material is greater. However, conducting material such as copper is less expensive per pound than magnetic material such as permalloy, is easier to fabricate, and requires no heat-treatment. Hence nonmagnetic conducting shields are generally used in preference to magnetic shields for alternating fields when the frequency is high enough so that the required degree of shielding can be obtained with a reasonable thickness of conducting shield; the only practical exception is when a conducting shield employs a liner of nonconducting magnetic material, as discussed below.

Electrostatic Shielding. Electrostatic shielding is obtained by enclosing the space to be shielded by a conducting surface. Accordingly, the magnetic and conducting shields for magnetic flux lines discussed above also serve as electrostatic shields. However, fairly effective electrostatic shielding can be obtained by a metal mesh made of any good to fair electrical conductor, which would be a rather poor shield for magnetic flux.

It is possible to shield electrostatic flux without simultaneously affecting the magnetic field by surrounding the space to be shielded with a conducting cage that is made in such a way as to provide no low-resistance path for the flow of eddy currents, while at the same time offering a

metallic terminal upon which electrostatic flux lines can terminate. Thus, the secondary winding of a transformer may be shielded electrostatically from the primary by a shield having an insulated gap located in such a manner as to prevent the shield from becoming a short-circuited turn. This is illustrated in Fig. 2-20. Another type of electrostatic shield that does not affect the magnetic flux is illustrated below in connection with Prob. 2-45.

Energy loss is associated with electrostatic shielding as a result of the fact that the charging current induced in the shield produces currents that must flow through the surface resistance of the shield. However,

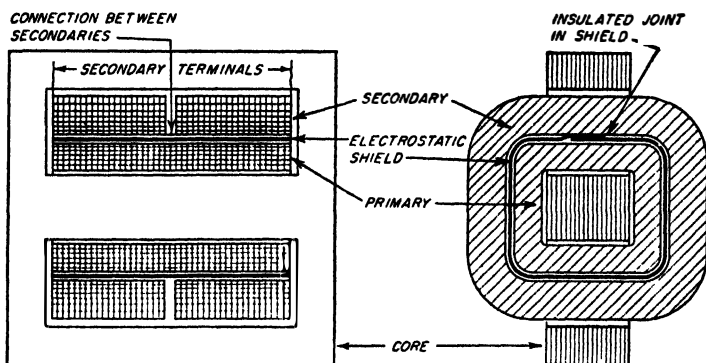


FIG. 2-20. Transformer with electrostatic shield between primary and secondary.

under most conditions these currents are quite small, so that the associated loss is generally insignificant. This is in contrast with shields for alternating magnetic flux, where the shield losses are very often substantial.

Effect of Shielding on Coils.¹ The magnetic and electric fields associated with a coil are frequently confined by placing the coil in a shield can composed of aluminum or copper. Such a shield increases the coil's distributed capacitance and effective resistance, and reduces its inductance. The distributed capacitance is increased as a result of the capacitance between the shield and various parts of the coil (see Fig. 2-19d). The inductance is decreased because the conducting shield restricts the magnetic flux lines to the space within the shield; this decreases the cross section of the magnetic circuit and thus reduces the flux linkages. The effective resistance of the coil is increased because the energy consumed by the eddy currents flowing in the shield is supplied by the coil.

The degree of shielding obtained at radio frequencies by enclosing a coil in a copper or aluminum container is very high, and if a reasonable clear-

¹ The quantitative relations involved are discussed by Howe, *op. cit.*; A. G. Bogle, The Effective Inductance and Resistance of Screened Coils, *J. IEE (Wireless Soc.)*, vol. 15, p. 221, September, 1940.

ance is provided between the shield and the coil, the properties of the coil are not seriously impaired. In general, the clearance between the shield and the coil should everywhere be not less than the coil radius. Under such conditions, the presence of the shield will not reduce the coil Q by more than 20 per cent, while the reduction in inductance will be still less.

A shielded inductance that is very compact can be achieved by making the conducting shield only slightly larger in diameter than the coil, and then using a liner of nonconducting magnetic material to fill the space between coil and shield, as shown in Fig. 2-19f. The high permeability of the magnetic material provides a low-reluctance return path for the magnetic flux outside the coil, with the result that the inductance obtained is greater than when the coil is unshielded as in Fig. 2-19a. This makes it practical to reduce the physical size required to obtain a given inductance at a desired value of Q . In addition, the shielding is more effective than when only a conducting shield is used (Fig. 2-19c), because both magnetic and conducting shielding is simultaneously obtained.

PROBLEMS AND EXERCISES

2-1. If the flux shown in Fig. 2-1 is produced by a current of 0.01 amp, estimate the coil inductance. (Assume that Fig. 2-1 gives a two-dimensional representation of the actual flux lines present in the three-dimensional coil.)

2-2. A single-layer coil is to have an inductance of $220\ \mu\text{h}$ and is to be wound on a form having a diameter of 2 in. If the ratio of length to diameter is 1.5, determine the distance between centers of adjacent turns of the winding.

2-3. A single-layer solenoidal coil having 60 turns on a winding 3 in. long and 3 in. in diameter possesses an inductance of $187\ \mu\text{h}$. Without using Fig. 2-2, determine:

a. How many turns would be required to obtain the same inductance if the core were 2 in. in diameter and 2 in. long.

b. How many turns would be required to obtain an inductance of $400\ \mu\text{h}$ with a winding 4 in. long and 4 in. in diameter.

2-4. a. On a hysteresis loop similar to that of Fig. 2-3, show a minor hysteresis loop originating at point 1 on the main loop, but corresponding to a substantially larger value of alternating magnetization.

b. Repeat for the same alternating magnetization as in (a), but with the d-c magnetization corresponding to point 3, instead of point 1.

2-5. The incremental inductance at low alternating magnetization of a particular iron-cored coil having 1000 turns is 10 henrys with no d-c saturation, and 4 henrys when carrying a d-c magnetizing current of 0.1 amp. When the number of turns is reduced to 500, it is found that the inductance without d-c saturation is reduced to 2.5 henrys, or exactly one-fourth of the previous value, whereas with a d-c magnetizing current of 0.1 amp the incremental inductance is somewhat greater than one-fourth of 4 henrys. Explain.

2-6. A coil uses a silicon-steel core composed of material having the characteristics given in Fig. 2-4, and the core is assembled with negligible air gap. If the incremental inductance is 5.4 henrys with no d-c magnetization and low alternating flux density, what will be the incremental inductances with d-c magnetizations sufficient to produce 1, 2, and 3 ampere turns per cm?

2-7. a. A permanent magnet that is required to produce a large amount of flux in a low-reluctance magnetic circuit will be short and thick. Explain.

b. A permanent magnet that is required to produce a small amount of flux in a high-reluctance circuit will be long and thin. Explain.

2-8. A permanent magnet of Alnico V is required to establish a flux density of 2000 lines per sq cm in an air gap 1.2 cm long and having an effective cross section of 20 sq cm. Determine the length and cross section of the magnet required, assuming stabilization is not necessary.

2-9. Explain why a permanent magnet stabilized as in Fig. 2-6 will have to be both larger in cross section and longer than an unstabilized magnet in order to produce the same flux in a given external circuit.

2-10. A particular permanent-magnet system employs a cylindrical magnet of Alnico V having a diameter of 1.0 in. and a length of 0.6 in. If Alnico XII is used instead, calculate the diameter and length required to give the same result. Assume that the permanent-magnet material is used under optimum conditions in both cases and that stabilization is not required.

2-11. A primary coil having an inductance of $100\ \mu\text{H}$ is connected in series with a secondary coil of $240\ \mu\text{H}$, and the total inductance of the combination is measured as $146\ \mu\text{H}$. Determine (a) the mutual inductance, (b) the coefficient of coupling, and (c) the inductance that would be observed if the terminals of one of the coils were reversed.

2-12. Two circuits are to be coupled by a common capacitor using the circuit of Fig. 2-8c. If the total capacitance required in the primary circuit is $150\ \mu\text{F}$, while the total capacitance required in the secondary circuit is $100\ \mu\text{F}$, determine the value of the common capacitance C_m in Fig. 2-8c to give a coefficient of coupling of 0.02. [Note: In solving this problem do not attempt to use Eq. (2-8).]

2-13. In two circuits coupled as in Fig. 2-8b, $L_1 = 0.05$ henry, $L_2 = 0.08$ henry, and $k = 0.4$. Determine (a) the required value of L_m , and (b) the total primary and total secondary inductances.

2-14. Explain why two coils that have their axes, respectively, parallel to, and at right angles to, the line joining the coil centers will have zero mutual inductance.

2-15. Two single-layer air-cored coils are located coaxially end to end, as illustrated in Fig. 2-8a. It is found that, if a long cylindrical magnetic core is slipped inside of these coils so that it is common to both coils, the mutual inductance is increased more than is the self-inductance of the individual coils. Explain.

2-16. What effect does the redistribution of current associated with skin effect have on the inductance? Explain.

2-17. a. Calculate the skin depth in copper for 1 kc, 1 Mc, and 1000 Mc, and tabulate the results.

b. Repeat for aluminum.

2-18. Parts formed of brass, steel, etc., are sometimes silver- or copper-plated to reduce the effective resistance to radio frequencies. If copper plating is employed, and the part is to be used in the frequency range 5 to 20 Mc, recommend a minimum thickness for this plating, and give the reasoning upon which this recommendation is based.

2-19. Inductances (and also shields) are sometimes plated to reduce corrosion and improve appearance. The resistivity of the plating material suitable for this purpose is usually much higher than the resistivity of the material that is plated. What criterion must the thickness of the plating satisfy if the effective resistance of the plated conductor is to approach closely the resistance obtained without plating?

2-20. A No. 14 copper wire (diameter 0.0641 in.) has a d-c resistance of 0.2525 ohm per 100 ft. Calculate its resistance at 10 Mc, and at 3000 Mc, and tabulate these three values of resistance alongside of one another.

2-21. What diameter must a copper wire have if its resistance is not to exceed 3.0 ohms per 100 ft at 10 Mc?

2-22. A conductor consisting of a thin-walled tube will have much less resistance at very high frequencies than a solid wire of the same d-c resistance. Explain.

2-23. In a conductor consisting of a tube of specified outside diameter, the resistance at very high frequencies will be almost independent of wall thickness if this thickness exceeds several skin depths, but will be roughly inversely proportional to wall thickness when the thickness is small compared with the skin depth. Explain these observations.

2-24. Determine what mathematical approximation is involved in each of the following statements:

- The phase angle in radians is equal to the power factor.
- The reciprocal of capacitor Q is equal to the power factor of the capacitor.
- The dissipation factor is equal to the phase angle.

2-25. On the basis of the information given in Fig. 2-12 and the associated discussion, sketch curves analogous to Fig. 2-12 but showing qualitatively how the dielectric constant and power factor would vary as a function of temperature for (a) a low frequency, and (b) a high frequency.

2-26. a. A mica capacitor with power factor 0.0005 has a capacitance of 0.001 μf . Assuming skin-effect resistance to be negligible, what is the equivalent series resistance of the capacitor at frequencies of 1000, 100,000, and 10,000,000 cycles?
b. What is the equivalent shunt resistance for the same conditions?

2-27. A certain air capacitor employing mycalex insulation has a power factor of 0.0003 at 1000 cycles. What will its power factor be at this same frequency and same capacitance if the mycalex insulation is replaced by polystyrene insulation of the same geometrical configuration?

2-28. The power factor of a capacitor at very high frequencies is roughly proportional to f^n where f is the frequency. What is the value of n ?

2-29. Show that the power factor of a given variable air capacitor at low frequencies is independent of frequency but increases inversely with capacitance setting.

2-30. At very high frequencies, does an increase in frequency cause the power factor of a variable air capacitance for a given capacitance setting to become greater, less, or unchanged? Give an adequate justification for the answer chosen.

2-31. In a variable air capacitor the ratio of the power factor at a given high frequency to the power factor at a given low frequency becomes greater as the capacitance setting increases. Explain.

2-32. In a capacitor having a capacitance of 0.001 μf , the equivalent series inductance of the leads, etc., is 0.1 μh . At what frequency does the apparent capacitance differ from the true capacitance by 10 per cent?

2-33. A certain capacitor having air dielectric with bakelite supports obtains 10 μf of its capacitance through the bakelite dielectric having a power factor of 4 per cent, and the remainder of its capacitance from the air, which has no losses. What is the equivalent series resistance and power factor at 10,000 kc when the total capacitance is 100 μf (90 μf from air and 10 μf from bakelite)? Neglect skin-effect losses.

2-34. A particular mica capacitor having a capacitance of 0.001 μf has a power factor of 0.0005 at a frequency of 1000 cycles, while at 10 Mc the power factor has risen to 0.001. From this information deduce the values of R and R_s applicable in Fig. 2-13 at 10 Mc.

2-35. The capacitor of Prob. 2-26 is able to stand a d-c potential of 5000 volts and is capable of dissipating safely 3 watts of heat.

- At what frequency will heating begin to limit the voltage rating?
- What is the voltage rating at frequencies of 1, 1000, and 10,000 kc?

2-36. Derive an equation giving the exact relation between the Q of a coil and the coil power factor, and from this calculate the error in the approximate relation: power factor = $1/Q$, when $Q = 50$.

2-37. Explain why the distributed capacitance of a coil is always increased by the wax or other coating used for protection against moisture.

2-38. On the basis of proximity and skin effects, explain why it is reasonable to expect that the maximum coil Q would be obtained with a wire not so large as to leave very little clearance between adjacent turns, and not so small as to make this clearance become a large fraction of the spacing between centers of adjacent turns.

2-39. In a coil with a magnetic slug core as in Fig. 2-17, removing the core will reduce the inductance less in a system using a core designed for 100 Mc than in a system using a core designed for 1 Mc. Explain.

2-40. A copper shield is required to reduce the magnetic flux density by 60 db. What shield thickness is required at (a) 1 kc, (b) 1 Mc, and (c) 1000 Mc?

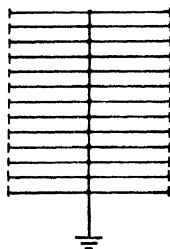
2-41. Derive Eq. (2-17) from Eq. (2-16).

2-42. A particular magnetic shield attenuates d-c magnetic fields by 20 db. What will the attenuation be if the shield thickness is doubled?

2-43. A conducting magnetic shield is composed of permalloy having an initial permeability of 15,000 and a resistivity of $17 \mu\text{ohms}$ per cm cube. Calculate (a) the thickness which this material must have to be 5 skin depths thick at 60 cycles, and (b) the thickness which copper must have to achieve the same degree of shielding.

2-44. Explain why magnetic material in powdered form, such as used in magnetic cores for radio frequencies, is not suitable for use as a shield of alternating magnetic fields.

2-45. A grid of wires such as shown in the accompanying figure will provide electrostatic shielding without magnetic shielding provided the structure (shown dotted in



PROB. 2-45

the illustration) supporting the sides of the shield is an insulator. However, if the material of the supporting structure is a conductor, then magnetic as well as electrostatic fields are shielded at least to some extent. Explain.

2-46. When a nonmagnetic shield can surrounds a solenoidal coil, it is observed that the shielding of the magnetic field is not affected appreciably by a joint in the shield provided this joint is in a plane perpendicular to the axis of the coil, but the effectiveness of the shield is very seriously reduced if the joint is in a plane that contains the axis of the coil. Explain.

2-47. If it is necessary that a magnetic shield for d-c fields have a joint, how should this joint be oriented with respect to the direction of the magnetic flux that is being shielded?

CHAPTER 3

PROPERTIES OF CIRCUITS WITH LUMPED CONSTANTS

3-1. Series Resonance. A circuit consisting of an inductance, capacitance, and resistance all in series, as in Fig. 3-1, is called a series resonant or series tuned circuit. When a constant voltage of varying frequency is applied to such a circuit, the current that flows depends upon frequency in the manner shown in Fig. 3-1. At low frequencies, the capacitive reactance of the circuit is large and the inductive reactance is small. Most of the voltage drop is then across the capacitor, while the current is small and leads the applied voltage by nearly 90° . At high frequencies, the inductive reactance is large and the capacitive reactance low, resulting in a small current that lags nearly 90° behind the applied voltage, and most of the voltage drop is across the inductance. In between these two extremes there is a frequency, called the resonant frequency, at which the capacitive and inductive reactances are exactly equal and, consequently, neutralize each other;

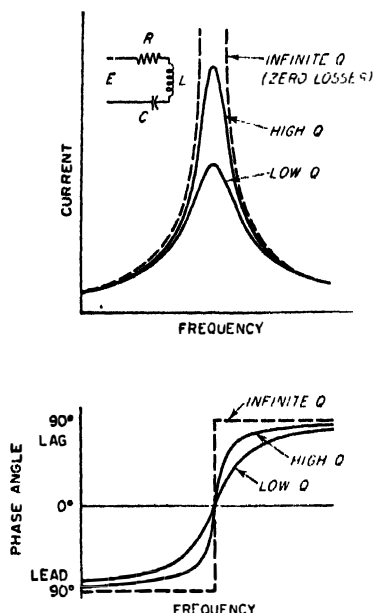


FIG. 3-1. Magnitude and phase angle of current in a series resonant circuit as a function of frequency for constant applied voltage and different circuit Q 's.

there is then only the resistance of the circuit to oppose the flow of current. The current at the resonant frequency is accordingly equal to the applied voltage divided by the circuit resistance, and is very large if the resistance is low.

A resonance curve such as illustrated in Fig. 3-1 finds extensive use in selective systems for separating a desired a-c signal from signals of other frequencies. For frequencies in the vicinity of resonance corresponding to a carrier wave and its sideband frequencies, the response is

nearly uniform and is quite large. However, at frequencies differing greatly from resonance the response is relatively small, with the result that signals of such frequencies, i.e., the unwanted signals, are severely discriminated against.

The characteristics of a series resonant circuit depend primarily upon the ratio of inductive reactance ωL to circuit resistance R , i.e., upon $\omega L/R$. This ratio is frequently denoted by the symbol Q and is called the circuit Q .¹ Most of the loss in the usual resonant circuit is due to coil resistance because the losses in a properly constructed capacitor are small in comparison with those of a coil. The result is that the circuit Q ordinarily approximates the Q of the coil alone, which was discussed in Sec. 2-7.

The general effect of different circuit resistances, i.e., different values of Q , is shown in Fig. 3-1. It is seen that, when the frequency differs appreciably from the resonant frequency, *the actual current is practically independent of the circuit resistance and is very nearly the current that would be obtained with no losses*. On the other hand, the current at the resonant frequency is determined solely by the resistance. The effect of increasing the resistance of a series circuit is, accordingly, to flatten the resonance curve by reducing the current at resonance without significantly affecting the behavior at frequencies differing appreciably from resonance. This broadens the top of the curve, giving a more nearly uniform current over a band of frequencies near the resonant point, but does so by reducing the ability of the circuit to discriminate between voltages of different frequencies.

Analysis of Series Resonant Circuit. The elementary voltage, current, and impedance relations of series resonant circuits are discussed in every book on alternating currents. The basic quantitative relations are listed below for convenient reference.

$$\text{Resonant frequency} = f_0 = \frac{1}{2\pi \sqrt{LC}} \quad (3-2)$$

$$Z_s = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (3-3a)$$

$$|Z_s| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \quad (3-3b)$$

¹ The circuit Q can also be defined as

$$Q = 2\pi \frac{\text{energy stored in circuit}}{\text{energy dissipated in circuit during one cycle}} \quad (3-1)$$

This relation follows from the fact that the energy stored in the inductance L when the current I is maximum (i.e., when all the stored energy is in the inductance) is $I^2 L/2$, where I is the peak current. At the same time, the energy lost per cycle in the circuit resistance R is $I^2 R/2f$, where f is the frequency.

$$\tan \theta = \frac{\omega L - (1/\omega C)}{R} \quad (3-4)$$

$$I = \frac{E}{Z_s} = \frac{E}{R + j[\omega L - (1/\omega C)]} \quad (3-5)$$

$$\text{Current at resonance} = I_0 = \frac{E}{R_0} \quad (3-6)$$

$$\text{Voltage across inductance} = j\omega L I \quad (3-7a)$$

$$\text{Voltage across capacitor} = \frac{-j}{\omega C} I \quad (3-7b)$$

where E = voltage applied to circuit

I = current flowing in circuit, amp

f = frequency, cycles

$\omega = 2\pi f$

$Q = \omega L/R$

R = total series resistance of tuned circuit

L = inductance, henrys

C = capacitance, farads

Z_s = impedance of series circuit

θ = phase angle of impedance

Subscript ₀ denotes values at resonant frequency

At frequencies near resonance the voltages across the capacitor and the inductance will both be very much greater than the applied voltage. This is possible because at frequencies near resonance the voltages across the capacitor and inductance are nearly 180° out of phase with each other and so add up to a value that is much smaller than either voltage alone.

At resonance, where the circuit current is E/R , Eqs. (3-7) show that the voltage across the inductance (or capacitor) is then Q times the applied voltage; i.e., *there is a resonant rise of voltage in the circuit amounting to Q times*. Since a typical value of Q is of the order of 100, a series resonant circuit will thus develop a high voltage even with small applied potentials. At frequencies differing from resonance the voltage developed across the inductance (or capacitor) falls off. In the vicinity of resonance the resulting curve of voltage as a function of frequency has a shape that for all practical purposes can be considered to be the same as the corresponding curve of current as a function of frequency (see Fig. 3-1). The reason for this is that most of the resonance effects exist in a very narrow frequency band, typically representing a frequency variation of only a few per cent. Over this frequency range the term ωL (or $1/\omega C$) in Eqs. (3-7) is so nearly constant that to a first approximation the voltage developed across the circuit can be considered to be proportional to the circuit current.

Universal Resonance Curve. Equations (3-5) and (3-6) can also be rearranged to express the ratio of current actually flowing to the current

at resonance, in terms of the circuit Q and the fractional deviation of the frequency from resonance. This leads to the universal resonance curve of Fig. 3-2.¹

In the universal resonance curve, the frequency is expressed in terms of a parameter a that represents f_0/Q cycles, as defined in Fig. 3-2. Thus $a = 1.0$ when the cycles off resonance equal f_0/Q cycles, $a = 2$ when the number of cycles off resonance is $2f_0/Q$, etc.

The use of Fig. 3-2 in practical calculations can be illustrated by two examples.

Example 1. It is desired to know how many cycles one must be off resonance to reduce the current to one-half the value at resonance when the circuit has a Q of 125 and is resonant at 1000 kc.

Reference to Fig. 3-2 shows that the response is reduced to 0.5 when $a = 0.86$. Hence,

$$\text{Cycles off resonance} = \frac{0.86 \times 1000}{125} = 6.88 \text{ kc}$$

The phase angle of the current as obtained from the curve is 60° .

Example 2. With the same circuit as in the preceding example, it is desired to know what the response will be at a frequency 10,000 cycles below resonance.

To solve this problem it is first necessary to determine a .

$$a = 1\%_{1000} \times 125 = 1.25$$

Reference to Fig. 3-2 shows that for $a = 1.25$ the response is reduced by a factor 0.37 and that the phase of the current is 68° leading.

The only assumption involved in the universal resonance curve is that Q is assumed to be the same at the frequency being considered as at the resonant frequency. *When this is true, the universal resonance curve*

¹ The equation of the universal resonance curve is obtained as follows: The ratio of Eq. (3-5) to Eq. (3-6) gives

$$\frac{\text{Actual current}}{\text{Current at resonance}} = \frac{R_0}{R + j[\omega L - (1/\omega C)]} = \frac{R_0}{R + j[(\omega^2 LC - 1)/\omega C]}$$

Now define the fractional deviation δ of the frequency from resonance, according to the relation

$$\omega = \omega_0(1 + \delta)$$

Substituting this expression for ω and remembering that $\omega_0 L = 1/\omega_0 C$, one obtains

$$\frac{\text{Actual current}}{\text{Current at resonance}} = \frac{R_0}{R + j \left[\frac{(1 + \delta)^2 - 1}{1 + \delta} \right] \omega_0 L} = \frac{1}{\frac{R}{R_0} + jQ\delta \left(\frac{2 + \delta}{1 + \delta} \right)}$$

When Q is constant, the radio-frequency resistance is proportional to frequency so that $R/R_0 = \omega/\omega_0 = (1 + \delta)$, which when substituted yields

$$\frac{\text{Actual current}}{\text{Current at resonance}} = \frac{1}{1 + \delta + jQ\delta \left(\frac{2 + \delta}{1 + \delta} \right)} \quad (3-8)$$

Figure 3-2 is then obtained by substituting $a = Q\delta$.

involves no approximations whatsoever. Over the limited range of frequencies near resonance represented in Fig. 3-2, the variation in Q in practical cases is so small as to introduce negligible (i.e., less than 1 per cent) error from the use of the curve, when the value of Q existing at resonance is used in determining the parameter a .

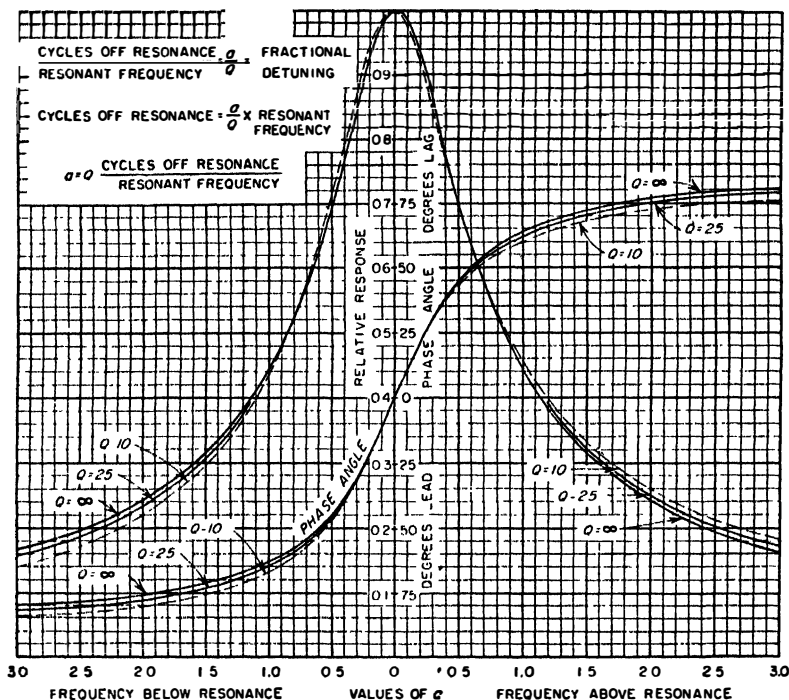


FIG. 3-2. Universal resonance curve for series resonant circuit. This curve can also be applied to the parallel resonant circuit by considering the vertical scale to represent the ratio of actual parallel impedance to the parallel impedance at resonance. When applied to parallel circuits, the angles shown in the figure as leading are lagging, and vice versa.

The universal resonance curve is useful because it is independent of the resonant frequency of the circuit and of the ratio of inductance to capacitance, and because it is substantially independent of circuit Q . It thus follows that *all resonance curves have the same relative shape irrespective of resonant frequency, Q , or ratio of inductance to capacitance of the circuit.*

Working Rules for Estimating Sharpness of Resonance. Since the curves for different values of Q are almost identical in Fig. 3-2, particularly in the neighborhood of the resonant frequency, it is possible to state several easily remembered working rules that will enable one to estimate

the sharpness of any resonance curve with an error of less than 1 per cent when only the Q of the circuit is known.¹ These rules follow:

Rule 1. *When the frequency of the applied voltage deviates from the resonant frequency by an amount that is $1/2Q$ of the resonant frequency, the current that flows is reduced to 70.7 per cent of the resonant current, and the current is 45° out of phase with the applied voltage. Thus the frequency band B over which the response is at least 70.7 per cent of that at resonance (i.e., within 3 db of resonance) is $B = f_0/Q$, where f_0 is the resonant frequency.*

Rule 2. *When the frequency of the applied voltage deviates from the resonant frequency by an amount that is $1/Q$ of the resonant frequency, the current that flows is reduced to 44.7 per cent of the resonant current, and the current is $63\frac{1}{2}^\circ$ out of phase with the applied voltage.*

Thus, in the circuit considered in the above examples, the current would be reduced to 70.7 per cent of the value at resonance when the frequency is $\frac{1}{2}50$ of 1000 kc, or 4000 cycles off resonance, and to 44.7 per cent of the resonant current for a frequency deviation of $\frac{1}{2}25$ of 1000 kc, or 8000 cycles. Since the resonant rise of voltage in this circuit is 125 ($=Q$) times, the rise of voltage is very nearly $0.7 \times 125 = 87.5$ times when the frequency is 4000 cycles off resonance, and is very close to $0.45 \times 125 = 56.25$ times at a frequency 8000 cycles from resonance.

Practical Calculation of Resonance Curves. The proper procedure for calculating a resonance curve is to start by determining the current at resonance, using Eq. (3-6). The working rules can then be applied to obtain the response at frequencies $1/2Q$ and $1/Q$ on either side of resonance. This gives a picture of the sharpness of resonance and is sufficient for many purposes. However, if additional points in the vicinity of resonance are needed, they can be calculated with the aid of Fig. 3-2.

At frequencies too far off resonance to come within the range of the universal resonance curve, the *magnitude* of the current can be determined with an accuracy sufficient for nearly all practical purposes by neglecting the resistance R in Eq. (3-5). The phase angle of the current under these conditions is obtained from Eq. (3-4).

The above procedure for calculating resonance curves is much superior to making calculations based directly upon Eq. (3-5). The use of the universal resonance curve in the vicinity of the resonant frequency not only reduces the amount of labor involved but also greatly improves the accuracy under ordinary conditions. This is because resonant circuit formulas such as Eq. (3-5) contain a term $\left(\omega L - \frac{1}{\omega C}\right)$ which involves the difference of two quantities which near resonance are nearly equal in magnitude. In order to obtain this difference without more than 1 per

¹ An error of 1 per cent is nearly always permissible in calculations of radio-frequency circuits. This is because the effective circuit constants at radio frequencies are very seldom known to an accuracy that involves an error of less than 1 per cent.

cent error, five-place logarithms must ordinarily be employed. Slide-rule calculations are never permissible. Neglecting the resistance at frequencies too far off resonance to come within the range covered by the universal resonance curve enormously reduces the labor involved in calculating magnitudes, and introduces an error of less than 1 per cent of the magnitude at resonance. This accuracy is ample for all ordinary purposes, and the error is undetectable when resonance curves are plotted.

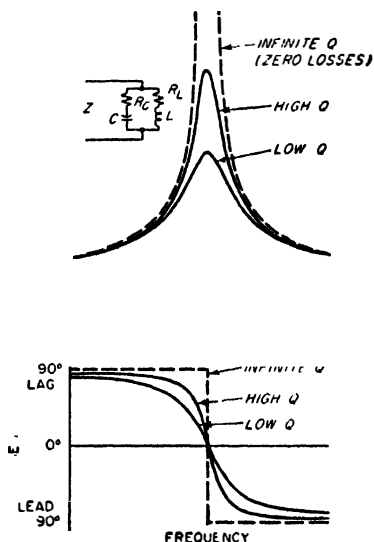


FIG. 3-3. Magnitude and phase angle of impedance of a parallel circuit as a function of frequency for different circuit Q 's.

extremes, there is a frequency at which the lagging current taken by the inductive branch and the leading current entering the capacitance branch are equal; being 180° out of phase, they then neutralize and leave only a small resultant inphase current flowing in the line. The impedance of the parallel circuit will then be a very high resistance, as is brought out in Fig. 3-3.¹

A comparison of Figs. 3-1 and 3-3 shows that the impedance curve of a parallel circuit is similar in character to the current curve of a series circuit. In particular, increasing the resistance of a parallel resonant

¹ In obtaining a parallel resonance curve experimentally by measurements of applied voltage and line current, extreme care must be taken to ensure that the applied voltage contains no harmonics. This is necessary because at resonance the circuit impedance is extremely high to the fundamental component of the applied voltage and very low to the harmonic components, with the result that even a small harmonic-voltage component will cause line currents that mask the small fundamental component.

3-2. Parallel Resonance. A parallel circuit consisting of an inductive branch in parallel with a capacitance branch offers an impedance of the character shown in Fig. 3-3. Such a circuit is termed a parallel resonant or parallel tuned circuit.

When a voltage is applied to such a system, then at very low frequencies, the inductive branch draws a large lagging current while the leading current of the capacitive branch is small, resulting in a large lagging line or circuit current and a low lagging circuit impedance. At high frequencies, the inductance has a high reactance compared with the capacitance, resulting in a large leading line current and a correspondingly low circuit impedance that is leading in phase. In between these two

circuit lowers and flattens the peak of the resonance curve, just as in the analogous series resonance case. This similarity is considered below in greater detail.

The relationship between the line and branch currents in a parallel circuit is illustrated in Fig. 3-4. It will be noted that, unlike the line or circuit current, which shows a resonance effect, the currents in the individual branches of a parallel circuit vary only slightly in the vicinity of resonance and are relatively large. At resonance the two branch currents have similar magnitudes, and being almost (but not quite) out of phase they add up to a very small resultant current, thus giving a high circuit impedance. As the frequency departs from resonance the two branch currents become slightly unequal in magnitude; this causes the line current to increase as shown, which means lowered circuit impedance. It is characteristic of parallel resonant circuits that for frequencies near resonance the current flowing in the branches, commonly referred to as the circulating current, is much larger than the line current, i.e., than the current supplied to the circuit.

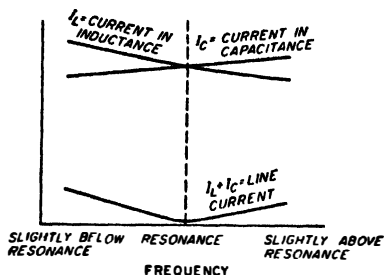


FIG. 3-4. Relationship of line and branch currents in a parallel resonant circuit in the vicinity of resonance.

The fundamental relations of a parallel resonant circuit are derived in every introductory book on a-c circuit theory, and are listed below for convenient reference.

$$\text{Parallel impedance} = Z = \frac{Z_c Z_L}{Z_c + Z_L} = \frac{Z_c Z_L}{Z_s} \quad (3-9)$$

$$\text{Line current} = \frac{E}{Z} \quad (3-10)$$

$$\text{Inductive branch current} = \frac{E}{Z_L} = \frac{E}{R_L + j\omega L} \quad (3-11)$$

$$\text{Capacitive branch current} = \frac{E}{Z_c} = \frac{E}{R_c - (j/\omega C)} \quad (3-12)$$

where E = voltage applied to circuit

$Z_c = R_c - (j/\omega C)$ = impedance of capacitive branch

$Z_L = R_L + j\omega L$ = impedance of inductive branch

$Z_s = Z_c + Z_L$ = series impedance of circuit

Z = impedance of circuit when connected in parallel

$R_s = R_c + R_L$ = total series resistance of circuit

$\omega = 2\pi$ times frequency

$Q = \omega L/R_s$ = circuit Q

These equations are fundamental to every parallel circuit, irrespective of the circuit Q , the frequency, or the division of resistance between the branches.

Quantitative Relations in Parallel Resonant Circuits with Moderate or High Q , and the Use of Universal Resonance Curve. When the Q of a parallel resonant circuit is not too low (e.g., of the order of 10 or more), the quantitative relations become quite simple. To begin with, it is then permissible to assume that the circuit has maximum impedance and unity power factor at the same frequency, which is also the frequency at which the same circuit is in series resonance as given by Eq. (3-2). In contrast, when the circuit Q is low, this is not necessarily the case, as discussed below.

When the circuit Q is not too low, the exact expressions of Eqs. (3-9) and (3-10) can be simplified, without introducing appreciable error, by neglecting the resistance components of the impedances Z_L and Z_C in the numerator of Eq. (3-9). When this is done¹

$$\text{Parallel impedance} = Z = \frac{(\omega_0 L)^2}{Z_s} \quad (3-13)$$

At resonance $Z_s = R_s$, and this becomes

$$\text{Parallel impedance at resonance} = \frac{(\omega_0 L)^2}{R_s} = (\omega_0 L)Q \quad (3-14)$$

In these equations ω_0 is the value of ω at resonance. It will be noted from Eq. (3-14) that *at resonance the impedance of a parallel circuit is a resistance that is Q times the reactance of one of the branches.*² It can, therefore, be said that the parallel arrangement of inductive and capacitive branches causes a resonant rise of impedance of Q times the impedance that would be obtained from either branch alone. It is thus apparent that very high impedances can be developed by parallel resonance. This is one of the most important properties of parallel resonance.

Under conditions where the circuit Q is not too low, the resonance curve of the parallel impedance of a circuit can be considered to have the *same shape as the resonance curve of the series current* in a circuit consisting

¹ This transformation is carried out as follows: If the resistance components in the numerator of Eq. (3-12) are neglected, the product $Z_L Z_C$ becomes $\omega L / \omega C = L/C$. One can now eliminate the capacitance C in this expression by multiplying both numerator and denominator by ω_0 and then noting that $1/\omega_0 C = \omega_0 L$. That is,

$$Z_L Z_C = \frac{L}{C} = \frac{\omega_0 L}{\omega_0 C} = (\omega_0 L)^2$$

² It also follows from Eq. (3-14) and Eqs. (3-10) to (3-12) that at resonance the branch currents are Q times as large as the line current, provided the resistance components in Eqs. (3-11) and (3-12) are small compared with the associated reactive components.

of the same inductance, capacitance, and resistances connected in a series instead of a parallel arrangement. This follows from the fact that a comparison of Eqs. (3-5) and (3-13) shows that both the parallel impedance and the series current are equal to a constant divided by Z_r . Consequently, the universal resonance curve and the working rules that were applied for estimating the sharpness of resonance of the series circuit also apply to the case of parallel resonance when the circuit Q is moderate or high. The only difference is that the signs of the phase angles are now reversed, the phase of the parallel impedance being leading at frequencies higher than resonance and lagging at frequencies below resonance.

The proper procedure for calculating the impedance of a parallel resonant circuit of moderate or high Q is therefore similar to that used

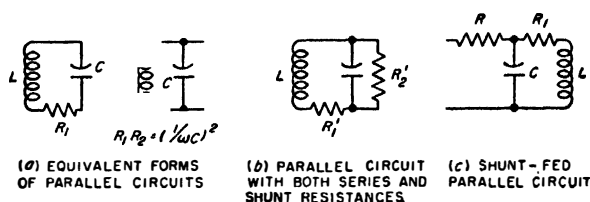


FIG. 3-5. Forms of parallel resonant circuits involving a shunt resistance and shunt feed.

with a series resonant circuit. The first step is to determine the resonant frequency and the impedance at resonance, using Eqs. (3-2) and (3-14). Next, the working rules are applied to obtain the 70.7 and the 44.7 per cent points on either side of resonance. This gives the general picture of the sharpness of resonance and is sufficient for many purposes. If a more complete curve is desired in the vicinity of resonance, one may make use of the universal resonance curve of Fig. 3-2. Finally, at frequencies so far off resonance as to be outside the range of the universal resonance curve, one may determine the magnitude of the impedance by using Eq. (3-13), but neglecting the circuit resistance R when making the calculation. The power-factor angle of the impedance thus obtained is the negative of the corresponding angle for series resonance, as given by Eq. (3-4).

Parallel Resonant Circuits with Shunt Resistances and with Parallel Feed. The two types of parallel resonant circuits shown in Fig. 3-5a are equivalent to each other provided the resistances R_1 and R_2 are properly related, and provided also that the circuit Q is not too low. To determine the relationship that must exist between R_1 and R_2 , one notes that R_1 can be thought of as being associated with capacitance C as its equivalent series resistance, while R_2 can be regarded as an equivalent shunt resistance of the same capacitance C . Assuming that the circuit Q is not too

low [i.e., that $(1/\omega C)/R_1 \gg 1$, and that $R_2/(1/\omega C) \gg 1$], then from Eqs. (2-12) and (2-13) one has

$$R_1 R_2 = \left(\frac{1}{\omega C} \right)^2 \quad (3-15)$$

Although the relationship between R_1 and R_2 is seen by Eq. (3-15) to depend on frequency, it is common practice to determine the relation between R_1 and R_2 at resonance, and then to assume that the values at resonance also hold for all frequencies in the vicinity of resonance. This approximation is equivalent to assuming that the right-hand term of Eq. (3-15) is constant at the value it has at resonance.¹ Since ω changes by only a small percentage in the limited frequency range around resonance, this assumption is not far from the truth, and the error it introduces is quite small.

The parallel resonant circuit of Fig. 3-5b can be transformed to the circuits of Fig. 3-5a by converting R'_2 to an equivalent series resistance R_1 or transforming R'_1 to an equivalent shunt resistance R_2 . By use of Eqs. (2-12) and (2-13), respectively, this leads to the following relations between the circuits of *a* and *b* in Fig. 3-5 for the resonant frequency:

$$\left. \begin{array}{l} \text{Total effective} \\ \text{series resistance} \end{array} \right\} = R_1 = R'_1 + \frac{(\omega_0 L)^2}{R'_2} \quad (3-18a)$$

$$\left. \begin{array}{l} \text{Shunt resistance} \\ \text{equivalent to } R'_1 \end{array} \right\} = R_{eq} = \frac{(\omega_0 L)^2}{R'_1} \quad (3-18b)$$

$$\left. \begin{array}{l} \text{Total effective} \\ \text{shunt resistance} \\ \text{including } R' \text{ and } R_{eq} \end{array} \right\} = R_2 = \frac{R_{eq} R'_2}{R_{eq} + R'_2} \quad (3-18c)$$

The above analysis is of practical importance for two reasons. In the first place, it shows that to a high approximation, the effect produced by shunting a resistance across a parallel resonant circuit is merely to lower the effective Q of the circuit. The resonant frequency is unchanged, however, and the impedance curve still has the shape of a resonance curve as given by the universal resonance curve. In the second place, the analysis provides a simple means of determining the quantitative effect that a shunt resistance produces on the properties of a resonant circuit.

Still another form of parallel resonant circuit that is frequently encountered is shown in Fig. 3-5c, where a resistance R is connected in series with

¹ At resonance, one can write $\omega_0 L = 1/\omega_0 C$, where ω_0 is the value of ω at the resonant frequency. Under these conditions the following useful relations apply to Fig. 3-5a:

$$R_1 R_2 = (\omega_0 L)^2 \quad (3-16)$$

$$\left. \begin{array}{l} Q \text{ of circuit} \\ \text{at resonance} \end{array} \right\} = \frac{\omega_0 L}{R_1} = \frac{R_2}{\omega_0 L} \quad (3-17a)$$

$$\left. \begin{array}{l} \text{Parallel impedance} \\ \text{at resonance} \end{array} \right\} = \omega_0 L Q = R_2 \quad (3-17b)$$

the parallel circuit. The behavior of arrangements of this type is analyzed in Sec. 3-7.

Parallel Circuits with Low Q . The entire discussion of parallel resonance given above except for Eqs. (3-9) to (3-12) assumes that the Q of the parallel circuit is at least reasonably high (i.e., of the order of 10 or more). In the general case when the circuit Q is low, the curve of circuit impedance as a function of frequency still has a shape that resembles a resonance curve unless the circuit Q approaches or is less than unity. However, the maximum impedance no longer necessarily occurs at the frequency of series resonance, and the condition of unity power factor does not necessarily occur either at the frequency of series resonance or when the impedance is a maximum. The actual behavior for any given Q depends upon the division of resistance between the inductive and capacitive branches, as illustrated in Fig. 3-6 for typical cases.

An important consideration in the use of low- Q resonant circuits occurs when such a circuit is tuned to resonance with a given frequency by varying either the inductance or capacitance of the circuit. If, for example, the tuning is accomplished by varying the capacitance, then, if all the circuit losses are in the inductive branch, the capacitance setting that makes the circuit impedance maximum also corresponds to unity power factor. If, however, part or all of the circuit resistance is in the capacitive branch, then the capacitance setting that makes the circuit impedance maximum at an assigned frequency does not correspond to the capacitance setting for which the circuit power factor is unity. This is illustrated in Fig. 3-6. Similarly, if the tuning is accomplished by varying the inductance, then the situation is reversed, and maximum impedance and unity-power-factor conditions coincide only if all the circuit losses are concentrated in the capacitive branch. These properties of parallel resonant circuits with low Q are often of considerable importance in connection with the resonant circuits of Class C amplifiers such as used in radio transmitters.

Components of Parallel Impedance. The parallel impedance as calculated by Eq. (3-9) or (3-13) can be thought of as equivalent to a resistance in series with a reactance, as shown in Fig. 3-7a. When the circuit Q is sufficiently high for Eq. (3-13) to apply, then these resistance and react-

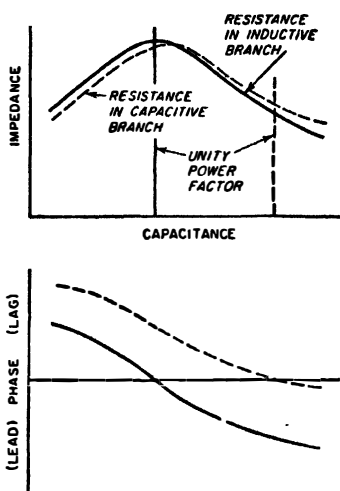


FIG. 3-6. Typical characteristics of parallel resonance circuits having low Q .

ance components will be found to vary with frequency in the manner shown in Fig. 3-7b, which is a universal curve derivable directly from the universal resonance curve of the parallel circuit. It will be noted that the resistance component has a shape superficially similar to that of a resonance curve, but differs in that it has steeper sides. In particular,

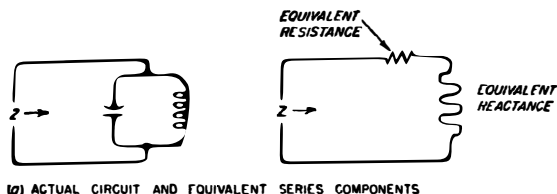
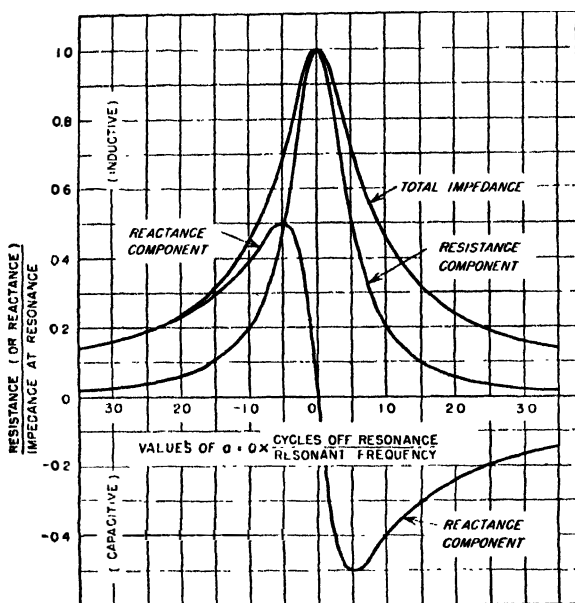


FIG. 3-7. Representation of parallel impedance in terms of equivalent series resistance and reactance components, together with universal curve giving these components as a function of frequency in a parallel resonant circuit having a relatively high Q .

the resistance drops to 50 per cent of the resonant impedance at frequencies corresponding to the 70.7 per cent points of the impedance curve (i.e., when the number of cycles off resonance equals the resonant frequency divided by $2Q$). It will also be noted that the reactance curves are characterized by maxima and minima which occur at the 70.7 per cent points of the resonance curve and which have peak amplitudes that

are exactly 50 per cent of the impedance at resonance as given by Eq. (3-14).

An application of these concepts is supplied by the case of a coil having distributed capacitance. With respect to its terminals, such a coil is represented by the left-hand circuit of Fig. 3-7a, and accordingly behaves as shown in Fig. 3-7b. Below the frequency at which the distributed capacitance is resonant with the inductance, the system is equivalent to a resistance in series with an inductive reactance. The apparent inductance represented by this equivalent reactance depends on frequency, however, rising with frequency until just before resonance is reached, and then dropping rapidly. The apparent inductance becomes zero at the parallel resonant frequency, while for higher frequencies the coil has a capacitive reactance and is therefore equivalent to a small capacitor. The

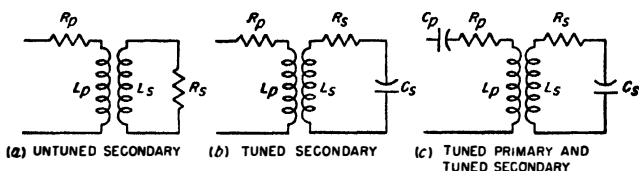


FIG. 3-8. Various types of inductively coupled circuits commonly encountered in electronics.

apparent resistance of the coil increases rapidly with the frequency until a maximum is reached at the resonant frequency, beyond which the resistance rapidly diminishes. These effects are all direct consequences of the properties of parallel resonant circuits, and can be readily deduced by an examination of Fig. 3-7 or of Eqs. (3-13) and (3-14). The behavior of an inductance coil with self-capacitance can accordingly be calculated just as one would determine the characteristics of any other parallel circuit.

3-3. Inductively Coupled Circuits; Theory. When mutual inductance exists between coils that are in separate circuits, these circuits are said to be inductively coupled. The effect of the mutual inductance is to make possible the transfer of energy from one circuit to the other by transformer action. That is, an alternating current flowing in one circuit produces magnetic flux which induces a voltage in the coupled circuit. This results in induced currents and a transfer of energy from the first or primary circuit to the coupled or secondary circuit. Several examples of inductively coupled circuits commonly encountered in electronics are shown in Fig. 3-8.

The behavior of inductively coupled circuits is somewhat complicated, but it can be readily calculated with the aid of the following rules:

Rule 1. As far as the primary circuit is concerned, the effect that the presence of the coupled secondary circuit has is exactly as though an imped-

once $(\omega M)^2/Z_s$, had been added in series with the primary,¹ where M = mutual inductance, $\omega = 2\pi f$, and Z_s = series impedance of secondary circuit when considered by itself. The equivalent impedance $(\omega M)^2/Z_s$, which the presence of the secondary adds to the primary circuit is called the *coupled* (or *reflected*) *impedance* and, since Z_s is a vector quantity having both magnitude and phase, the coupled impedance is also a vector quantity, having resistance and reactance components.

Rule 2. *The voltage induced in the secondary circuit by a primary current of I_p has a magnitude of $\omega M I_p$ and lags behind the current that produces it by 90° . In complex quantity notation the induced voltage is $-j\omega M I_p$.*

Rule 3. *The secondary current is exactly the same current that would flow if the induced voltage were applied in series with the secondary and if the primary were absent.² The secondary current therefore has a magnitude $\omega M I_p/Z_s$, and in complex quantity representation is given by $-j\omega M I_p/Z_s$.*

These three rules hold for all frequencies and all types of primary and secondary circuits, both tuned and untuned. The procedure to follow in computing the behavior of a coupled circuit is (1) to determine the primary current with the aid of Rule 1; (2) to compute the voltage induced in the secondary, knowing the primary current and using Rule 2; and (3) to calculate the secondary current from the induced voltage by means of Rule 3. The following set of formulas will enable these operations to be carried out systematically:

¹ This can be demonstrated by writing down the circuit equations for the primary and secondary. These equations are

$$\begin{aligned} E &= I_p Z_p + j\omega M I_s \\ \text{Induced voltage} &= -j\omega M I_p = I_s Z_s \end{aligned}$$

where Z_p is the series impedance of the primary and E is the voltage applied to the primary. Solving this pair of equations to eliminate I_s gives

$$E = I_p \left[Z_p + \frac{(\omega M)^2}{Z_s} \right] \quad (3-19)$$

This relation shows that the effective primary impedance with secondary present is $Z_p + (\omega M)^2/Z_s$, of which the second term represents the coupled impedance arising from the presence of the secondary.

² Some readers may wonder why it is that, although the secondary circuit couples an impedance into the primary, the primary is not considered as coupling an impedance into the secondary. The explanation for this is as follows: The effect that the secondary really has upon the primary circuit is to induce a back voltage in the primary proportional to the secondary current. This back voltage represents a voltage drop occurring in the primary circuit and is the same voltage drop that results when the primary current is assumed to flow through the hypothetical coupled impedance. The impedance that the secondary couples into the primary is hence a means of taking into account the voltage that the secondary current induces into the primary. The voltage that is induced in the secondary circuit by the primary current is taken into account by Rule 3, so that no coupled impedance need be postulated as present in the secondary to take into account the effect of the primary.

$$\left. \begin{array}{l} \text{Impedance coupled into primary} \\ \text{circuit by secondary} \end{array} \right\} = \frac{(\omega M)^2}{Z_s} \quad (3-20)$$

$$\text{Equivalent primary impedance} = Z_p + \frac{(\omega M)^2}{Z_s} \quad (3-21)$$

$$\text{Primary current} = I_p = \frac{E}{Z_p + (\omega M)^2/Z_s} \quad (3-22)$$

$$\text{Voltage induced in secondary} = -j\omega M I_p \quad (3-23)$$

$$\text{Secondary current} = \frac{-j\omega M I_p}{Z_s} = \frac{-j\omega M E}{Z_p Z_s + (\omega M)^2} \quad (3-24)$$

In these equations Z_p is the series impedance of the primary considered as though the secondary were removed, E is the applied voltage, and the remaining notation is as previously used. The primary and secondary impedances Z_p and Z_s , respectively, are vector quantities, so that Eqs. (3-16) to (3-20) are all vector equations.

Inductively Coupled Circuit as a Transformer. The inductively coupled circuit is a transformer, and the theory of the inductively coupled circuit that is given above is the general theory of transformers. The method commonly used to analyze the behavior of 60-cycle power transformers, which involves the use of leakage inductance, magnetizing current, and turn ratio, is a special form of the general theory that is convenient when the coupling coefficient k between the primary and secondary windings approaches unity. However, when the coupling coefficient k is small, then the use of Eqs. (3-20) to (3-24) is preferable.

The equivalent transformer circuit represented by two coils coupled together with mutual inductance M is shown in Fig. 3-9. Here the total primary inductance L_p is broken up into a leakage inductance L' and a coupled inductance L'_c , while the secondary is likewise broken up into leakage inductance L'' and a coupled inductance L'_c . Each leakage inductance is considered as having no coupling whatsoever to the other winding, while the coupled inductances L'_c and L'_c are taken as having a coefficient of coupling equal to unity. The values of these inductance components in terms of the coefficient of coupling and the primary, secondary, and mutual inductances are given in the figure. In the

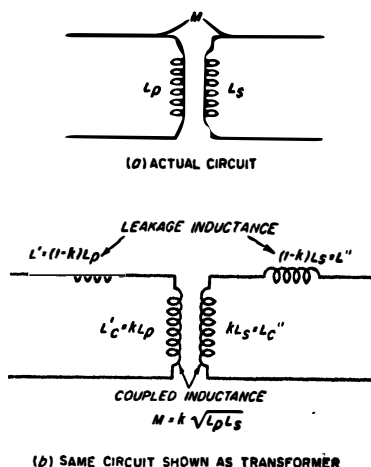


FIG. 3-9. Inductively coupled circuit represented as a transformer with coupled and leakage inductances.

representation of Fig. 3-9b, turn ratio has practical significance only when the coefficient of coupling k approaches unity; when the coefficient of coupling is small, as for example, 0.01, then the primary and secondary inductances are practically entirely leakage inductances. Under these conditions the voltage induced in the secondary may be much smaller than the voltage applied to the primary terminals, even when the secondary winding has many more turns than does the primary.

3-4. Analysis of Some Simple Inductively Coupled Circuits. In this and the next section, the types of coupled circuits most commonly encountered in electronics work will be analyzed by the principles given above.

In studying the behavior of a coupled circuit the first step is always to examine the nature of the coupled impedance $(\omega M)^2/Z_s$. When the coupled impedance is small, then the primary current is very nearly the same as though no secondary were present, and the effects produced in the secondary circuit by the primary current will likewise be small. The coupled impedance will be low if the mutual inductance M is very small (i.e., if there is small coupling), or if the secondary impedance is very high, for example, if the secondary is open-circuited. In contrast, consider the case when the coupled impedance $(\omega M)^2/Z_s$ is large, either because of large M or small Z_s , or both. The voltage and current relations that exist in the primary circuit are then affected to a considerable extent by the presence of the coupled secondary, and a very considerable transfer of energy to the secondary occurs.

When determining the effect produced by the coupled impedance, it is important to note that this impedance has the same phase angle as does the secondary impedance Z_s , but with the exception that the sign of the phase angle is reversed. Thus, if the secondary impedance is inductive and has an angle of 30° , the impedance coupled in series with the primary circuit by the action of the secondary has a capacitive phase angle of 30° . The physical significance of this change in sign of the phase angle becomes apparent from the examples considered below. A particularly important case occurs when the secondary impedance Z_s is a pure resistance; under these conditions the coupled impedance will also be a resistance.

The energy consumed by the secondary circuit is the energy represented by the primary current flowing through the resistance component of the coupled impedance.

Coupled Circuit with an Untuned Secondary Consisting of a Resistance and Inductance. This arrangement is illustrated in Fig. 3-8a, and is the type of coupled circuit that results when a resistance is connected across the terminals of the secondary inductance, or, alternatively, is the case where the secondary load is a resistance and an inductance in series. Such a secondary consists of an inductance L_s in series with a resistance

R_s . The coupled impedance is accordingly

$$\text{Coupled impedance} = \frac{(\omega M)^2}{Z_s} \frac{(\omega M)^2}{R_s + j\omega L_s} \quad (3-25)$$

Multiplying both numerator and denominator by $R_s - j\omega L_s$, gives

$$\text{Coupled impedance} = \frac{R_s}{R_s^2 + (\omega L_s)^2} (\omega M)^2 - j \frac{\omega L_s}{R_s^2 + (\omega L_s)^2} (\omega M)^2 \quad (3-26)$$

Examination of Eq. (3-26) shows that the coupled impedance introduced into the primary circuit by a resistance-inductance secondary consists of a resistance in series with a capacitive reactance. The effect of the coupled resistance is to increase the effective resistance that appears between the primary terminals. The effect of the coupled capacitive reactance is to neutralize a portion of the primary inductance, thereby reducing the equivalent inductance that is observed between the terminals of the primary coil. The physical explanation of the fact that an inductive secondary produces a capacitive coupled reactance is that such a secondary causes some of the inductive reactance already possessed by the primary to be neutralized. This is done electrically by postulating a capacitive reactance of suitable magnitude in series with the primary.¹

A special case of considerable importance is that for which the resistance R_s of the secondary circuit in Fig. 3-8a is negligible compared with the inductive reactance of the secondary. This situation will arise when the secondary coil is short-circuited, or when the secondary load is a low-loss inductance. To the extent that the resistance of the secondary circuit can be neglected, the coupled resistance introduced into the primary by the presence of such a secondary is zero; the only effect produced by the presence of the secondary is then to reduce the effective inductance that exists between the primary terminals. The percentage reduction in the equivalent primary inductance in such a situation depends only upon the coefficient of coupling between the primary and secondary circuits. If $k = 1.0$, the primary inductance is completely neutralized.²

A shield surrounding a coil, or a piece of metal such as a panel located in the magnetic field of a coil, represents a coupled secondary circuit that consists of an inductance in series with a resistance. Such an arrange-

¹ Although the coupled impedance is capacitive and so neutralizes part of the primary inductance, it is impossible to obtain a resultant capacitive reactance in the primary circuit by very large coupling since, with the maximum coupling that can possibly exist ($k = 1$), it will be found that the coupled capacitive reactance can never be greater than the value that will just neutralize all the inductive reactance of the primary.

² For other values of k , it can be shown by manipulating Eqs. (3-21) and (3-26) that the equivalent primary inductance is $L_p(1 - k^2)$.

ment can, accordingly, be analyzed as above. Thus the effect of a shield or metal panel on a coil is to reduce the equivalent inductance and to increase the apparent resistance observed at the coil terminals; these effects, moreover, become greater the larger the coupling, i.e., the smaller the spacing between the primary coil and the metal secondary. It is also to be noted that if the secondary resistance is low, as will be the case if the shield or metal panel is made of a good conductor such as copper or aluminum, then the principal effect produced by the presence of the metal near the coil is to reduce the equivalent inductance of the coil; under these circumstances the increase in equivalent coil resistance is only nominal. It will be noted that these conclusions derived from the viewpoint of coupled circuits are all consistent with the qualitative conclusions stated in Sec. 2-8, relative to the effect that shielding has on the properties of a coil.

Coupled Circuits with Untuned Primary and Tuned Secondary. A circuit of this type is shown in Fig. 3-8b. Here one has

$$\text{Coupled impedance} = \frac{(\omega M)^2}{Z_s} = \frac{(\omega M)^2}{R_s + j[\omega L_s - (1/\omega C_s)]} \quad (3-27)$$

An examination of this expression shows that, in the limited frequency range in which the principal resonance effects take place when the secondary Q is not too low, the numerator is substantially constant, whereas the denominator represents the series impedance of the secondary circuit. This is, therefore, an equation of the same general type as Eq. (3-13) for parallel resonance. *The coupled impedance produced by a tuned secondary circuit consequently varies with frequency according to the same general law as does the parallel impedance of the secondary circuit* (see Fig. 3-3). The absolute magnitude of the curve, however, depends upon the mutual inductance. This arrangement thus provides a means whereby the impedance of a parallel resonant circuit can be transformed in magnitude. Comparison of Eqs. (3-13) and (3-27) shows that the transformed impedance appearing in the primary circuit is $(M/L_s)^2$ times the actual parallel impedance of the resonant secondary circuit.

A special case of the circuit of Fig. 3-8b that is of particular importance occurs when the primary resistance R_p is the plate resistance of a vacuum tube. One then has the equivalent circuit of the transformer-coupled tuned radio-frequency amplifier. In this instance one is interested in the curve showing the variation of the secondary current (or of the voltage developed across the secondary capacitor C_s)¹ as the frequency is varied

¹ The voltage across the secondary capacitor C_s is equal to the product of the secondary current and the reactance $1/\omega C_s$ of this capacitor. In the limited frequency range represented by the vicinity around resonance ω changes very little in comparison with the variation of the secondary current. Hence, to a first approximation the voltage developed across the capacitor can be considered as being equal to

about resonances. When $R_p \gg \omega L_p$, the curve of secondary current (or of voltage across the secondary capacitor) varies with frequency according to a resonance curve having the same resonant frequency as the secondary circuit, but possessing a slightly lower Q . When the reactance ωL_p of the primary inductance is not negligible compared with the primary resistance R_p , the curve of secondary current as a function of frequency still has the shape of a resonant curve. However, the frequency at which the secondary current (or voltage across the secondary capacitor) is maximum is now slightly higher than the resonant frequency of the secondary. A typical example of this is shown by the dotted curve of Fig. 3-10. The analysis that leads to these conclusions is presented in Sec. 3-7.

3-5. Behavior of Systems Involving Resonant Primary and Resonant Secondary Circuits.

Primary and Secondary Circuits Resonant at the Same Frequency and Having Q 's That Are Equal and Not Too Low.

When two resonant circuits having equal Q 's that are not too low are tuned to the same frequency and coupled together, the resulting behavior depends very largely upon the degree of coupling, as seen from Fig. 3-11.¹ When

the coefficient of coupling is small, the curve of primary current as a function of frequency is substantially the series resonance curve of the primary circuit considered alone. The secondary current is small and varies with frequency in such a way as to be much more peaked than the resonance curve of the secondary circuit considered as an isolated circuit. As the coefficient of coupling is increased somewhat, the curve of primary current becomes broader, as a result of a reduction in the primary current at resonance and an increase in the primary current at frequencies

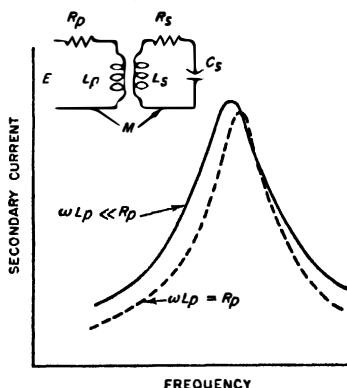


FIG. 3-10. Variation of secondary current as a function of frequency in a coupled system in which the secondary is a resonant circuit and the primary is untuned, showing that the secondary circuit follows a resonance curve, which, however, has a lower Q than that of the secondary circuit taken alone.

the product of the secondary current and a constant. In the immediate vicinity of, resonance the curve of voltage across this secondary capacitor therefore has very nearly the same shape as does the curve of secondary current.

¹ The phase shift is not shown in Fig. 3-11, but varies $\pm 180^\circ$ about the phase at the resonant frequency. Thus the total shift in phase between input voltage and output current as the frequency varies through resonance is 360° . This is in contrast with systems having only one tuned circuit; the total phase shift then varies over the range $\pm 90^\circ$, or a total of 180° .

slightly off resonance. At the same time the secondary-current peak becomes higher and the curve of secondary current somewhat broader.

These trends continue as the coefficient of coupling is increased until the coupling is such that the resistance which the secondary circuit couples into the primary at resonance is equal to the primary resistance. This is called the *critical coupling* and causes the secondary current to

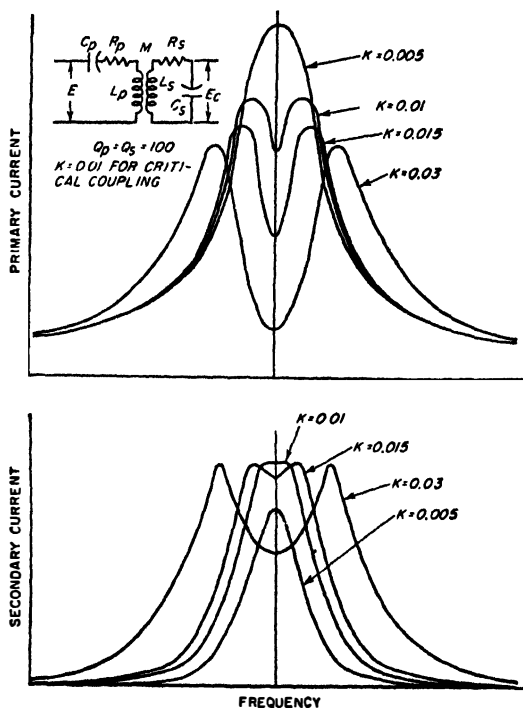


FIG. 3-11. Curves showing variation of primary and secondary currents with frequency for different coefficients of coupling when the primary and the secondary are separately tuned to the same frequency.

have the maximum value it can attain. The curve of secondary current is then somewhat broader than is the resonance curve of the secondary circuit considered alone, and has a relatively flat top. The primary current now has two peaks, being greater at frequencies just off resonance than at the resonant frequency.

As the coefficient of coupling is increased beyond the critical value, the double humps in the primary current become more prominent and the peaks spread farther apart. The curve of secondary current now also begins to display double humps, with the peaks becoming more pronounced and spreading farther apart as the coupling increases. The

value of the primary current at the peaks becomes smaller the greater the coupling, but in the secondary circuit not only do the two peaks have substantially the same height, but this height is also independent of the coefficient of coupling provided only that the coupling is not less than the critical value. The reason for the above behavior centers around the way in which the coupled impedance $(\omega M)^2/Z_1$ varies with frequency. Consider first the total primary-circuit impedance. This consists of the actual self-impedance of the primary plus whatever impedance the secondary circuit couples into the primary. The type of coupled impedance produced by a tuned secondary has already been discussed; it is substantially a parallel resonance curve having a shape corresponding to the Q of the secondary circuit and an amplitude determined by the mutual inductance. The coupled impedance is hence maximum at resonance and is then a resistance. At frequencies below resonance the coupled impedance is inductive and at frequencies above resonance it is capacitive, as shown in Fig. 3-7.

When this coupled impedance is added to the self-impedance of the primary circuit, the effect at resonance is to increase the effective primary resistance above the value that would exist in the absence of the secondary. This causes the primary current at resonance to be reduced in all cases by the presence of the secondary. At frequencies somewhat below resonance the coupled impedance is largely inductive whereas the primary self-impedance is largely capacitive. The coupled inductive reactance then neutralizes some of the primary capacitive reactance, lowering the primary circuit impedance and increasing the primary current. The situation is somewhat similar for frequencies above resonance except that now the coupled reactance is capacitive and neutralizes some of the inductive reactance which the primary circuit otherwise has at frequencies above resonance. Consequently, the net effect of the coupled impedance is to lower the primary current at the resonant frequency and to raise the current at frequencies somewhat off resonance. The magnitude of this effect depends upon the coefficient of coupling, being small when the coupling is small. However, when the coupling is of the order of magnitude of the critical value or greater, the coupled impedance becomes sufficient to be the major factor in determining the impedance of the primary circuit. In particular, at resonance the primary current tends to be relatively small because of the very large coupled resistance, while there is a frequency on each side of resonance at which the coupled reactance exactly neutralizes the primary reactance, giving zero reactance for the total primary circuit impedance and causing the flow of a large primary current. This is the cause of the double-humped curves of primary current for high couplings, such as shown in Fig. 3-11.

The curve of secondary current is determined by the secondary impedance, and by the voltage induced in the secondary by the primary current.

The induced voltage varies with frequency in almost exactly the same way as does the primary current, since the magnitude of the induced voltage is ωMI_p ; and in the limited frequency range in which the resonance effects take place, ω changes very little. As a result of this, the curve of secondary current has a shape that is almost exactly the product of the shape of the curve of primary current and the shape of the resonance curve of the secondary circuit. Since the latter curve is sharply peaked, the secondary current is much more peaked than the primary current, as is clearly evident in Fig. 3-11.

At low coefficients of coupling, the curve of secondary current is particularly sharp, being substantially the product of the resonance curves of the primary and secondary circuits. As the coupling increases, the primary-current curve becomes broader, thereby making the secondary curve less sharp. At the same time, the amplitude of the secondary-current peak increases because of the increased coupling. When the coefficient of coupling reaches the critical value, the secondary current has the maximum value it can attain. Under these conditions the dip in primary current in the vicinity of resonance has a curvature that is exactly opposite from the curvature of the resonance curve of the secondary circuit. The result is that the curve of secondary current now has a very flat top in the immediate vicinity of resonance. As the coupling is increased beyond the critical value, the secondary-current peak splits into two peaks, both of which have amplitudes substantially the same as the secondary-current peak at critical coupling. The separation between these peaks increases with coupling and is substantially the same as the separation of the peaks of primary current when the peaks are pronounced.

The voltage developed across the secondary capacitor is equal to the reactance of this capacitor times the secondary current; thus it can readily be calculated once the current curve is known. For most purposes, it is sufficient to assume that the curve of voltage developed across the capacitor has the same shape as the curve of secondary current. One is interested primarily in the behavior about resonance, and the capacitor reactance changes very little in the limited frequency range consequently involved when the circuit Q 's are not too low.

The exact shapes of curves such as those of Fig. 3-11 can be calculated with the aid of Eqs. (3-20) to (3-24). Such computations are, however, complicated and tedious. The usual practical procedure is accordingly to determine (1) the response at resonance, (2) the frequencies at which the peaks of secondary response occur when this response curve has double humps, (3) the heights of these two peaks, and (4) the response at one or two other frequencies so chosen as to simplify the calculations. In this way, it is possible, with a minimum of work, to obtain a good semiquantitative picture of the behavior. The following nomenclature in addition

to that of Fig. 3-11 will be used in the discussion of the quantitative relations:

- E_c = voltage across secondary capacitor
- E = voltage applied in series with primary
- k = actual coefficient of coupling
- k_c = critical coefficient of coupling
- $Q_p = Q$ of primary circuit
- $Q_s = Q$ of secondary circuit

At resonance, the series impedances of the primary and secondary circuits are resistances, and the response in the secondary is given by the relation^{1,2}

$$\left. \begin{array}{l} \text{Voltage across secondary} \\ \text{capacitor at resonance} \\ \text{Voltage applied in series} \\ \text{with primary} \end{array} \right\} = \frac{E_c}{E} = \frac{\sqrt{L_s}}{\sqrt{L_p}} \frac{k}{k^2 + (1/Q_p Q_s)} \quad (3-28)$$

The secondary response has its maximum value when the coefficient of coupling has a value k_c such that

$$k_c = \frac{1}{\sqrt{Q_p Q_s}} \quad (3-29)$$

This value of coupling is called the *critical coefficient of coupling* and is the condition where the resistance that the secondary circuit couples into the primary circuit at resonance is equal to the resistance of the primary circuit, i.e., when $(\omega M)^2/R_s = R_p$.

When the coefficient of coupling equals the critical value and if $Q_p = Q_s$, then the curve of secondary current (or voltage) as a function of frequency has the maximum flatness that is possible in the vicinity of resonance. The shape of this curve is shown in Fig. 3-12, together with the resonance

¹ This follows from Eq. (3-24) by substituting

$$Z_s = R_s, Z_p = R_p, E_c = I_s/j\omega C_s = -j\omega L_s I_s$$

to give

$$\frac{E_c}{E} = \frac{-j\omega M}{R_p R_s + (\omega M)^2} (-j\omega L_s)$$

Dividing both numerator and denominator by $\omega^2 L_p L_s$ gives

$$\frac{E_c}{E} = \frac{-(M/\sqrt{L_p L_s})}{\frac{R_p}{\omega L_p} \frac{R_s}{\omega L_s} + \frac{M^2}{L_p L_s}} \frac{\sqrt{L_s/L_p}}{1}$$

Equation (3-28) is then obtained by substituting $M^2/L_p L_s = k^2$ and dropping the minus sign.

² It is to be noted that Eqs. (3-28) and (3-29) are not limited to the case where $Q_p = Q_s$, although the rest of the discussion in this section does assume $Q_p = Q_s$.

curve of a simple tuned circuit having the same Q as the primary or secondary. It will be noted that the coupled circuit case has a bandwidth between the 70.7 per cent response points that is $\sqrt{2}$ times as great as for the single tuned circuit. The shapes of the two curves differ greatly, the coupled system being flatter in the center and much deeper on the sides.

When the coefficient of coupling exceeds the critical value, then for $Q_p = Q_s$, double humps will always occur in the secondary response

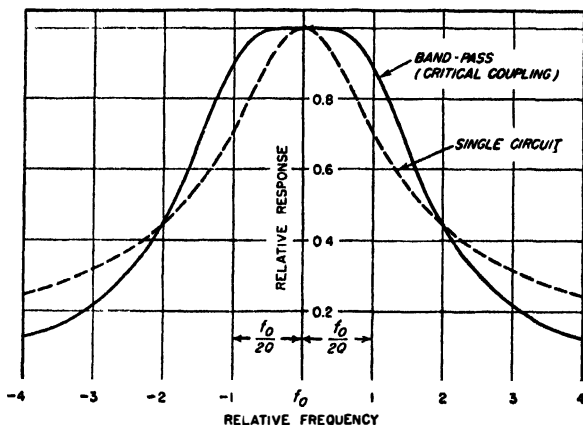


FIG. 3-12. Relative response of a bandpass system provided by two identical circuits critically coupled, together with the resonance curve of a single circuit such as used in the bandpass system.

curve. If the coefficient of coupling is at least several times the critical value these humps are quite pronounced and occur at frequencies that differ from the resonant frequency f_0 by approximately $\pm kf_0/2$ cycles.¹ When the peaks of secondary response are not pronounced, i.e., when the actual coefficient of coupling does not greatly exceed the critical value, then these peaks are somewhat closer together than indicated by this simple relation [see Eq. (3-30) and Fig. 3-16].

When the circuit Q 's are equal and not too low, the peaks of the secondary current for $k > k_c$ will have almost exactly the same height as the resonant peak of secondary current at critical coupling. This relation holds irrespective of the exact location on these peaks provided only that

¹ An analysis that does not contain these restrictions leads to the more precise relation

$$\frac{\text{Frequency at peak of secondary voltage}}{\text{Resonant frequency of tuned circuits}} = \sqrt{1 \pm k \left[1 - \frac{k_c^2}{2k^2} \left(\frac{Q_p}{Q_s} + \frac{Q_s}{Q_p} \right) \right]} \quad (3-30)$$

the coefficient of coupling involved is small compared with unity¹ and that the Q 's are not too low.

When double peaks occur in the secondary response curve, additional information on the shape of the response curve can be easily obtained by taking advantage of the fact, illustrated in Fig. 3-13, that the response equals or exceeds the response at resonance over a frequency band that is $\sqrt{2}$ times the width of the frequency band between coupling peaks, as calculated from Eq. (3-30).

At frequencies that are sufficiently high or low relative to the resonant frequency to lie well on the sides of the response curve, one can neglect the resistances of the primary and secondary circuits when calculating the magnitude of the secondary response. This greatly simplifies calculations while introducing relatively little error in magnitudes.

The Effects Produced by Unequal Q 's. The behavior of two coupled circuits resonant at the same frequency is modified in several respects when $Q_p \neq Q_s$. The secondary response at resonance is still given by Eq. (3-28), and is maximum when the coefficient of coupling has the critical value as defined by Eq. (3-29). However, double peaks do not now appear until the coupling is somewhat greater than the critical value, and the magnitude of the response at the secondary peaks when they do appear is less than the response with critical coupling.

Coupled Resonant Circuits Tuned to Slightly Different Frequencies. Consider the case of two circuits resonant at slightly different frequencies and coupled together. When $Q_p = Q_s$, the response curve of secondary current (or voltage) has almost exactly the same shape as would be obtained if the circuits were both tuned to the same frequency and the coefficient of coupling were increased to a value k_{eq} such that

$$k_{eq} = \sqrt{k^2 + \left(\frac{\Delta}{f_0}\right)^2} \quad (3-31)$$

where k is the actual coefficient of coupling, Δ is the difference between the resonant frequencies of the primary and secondary circuits, and f_0 is the frequency midway between the primary and secondary resonant frequencies. Hence detuning primary and secondary circuits slightly has

¹ If the coefficient of coupling is not small compared with unity, then the relative heights of the individual peaks of voltage developed across the secondary capacitor will be very nearly inversely proportional to the square of the ratio of the frequencies at which the respective peaks occur. Under these conditions the low-frequency peak will be slightly higher than the high-frequency peak, although the average height of the two peaks will still approximate the response with critical coupling.

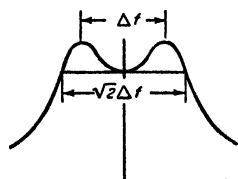


FIG. 3-13. Relationship between bandwidth and width between secondary peaks, existing when two circuits resonant at the same frequency are coupled together.

approximately the same effect on the shape of the secondary-current curve as increasing the coefficient of coupling when there is no detuning.

In the more general case of detuning where the circuit Q 's are not the same, the secondary-response curve is no longer symmetrical about the mean resonance frequency.

Shunt-fed and Shunt-loaded Coupled Circuits. In all the examples of coupled resonant circuits considered so far, the input voltage has been applied in series with the primary circuit. In many practical circumstances, however, the excitation is applied to the system as illustrated in

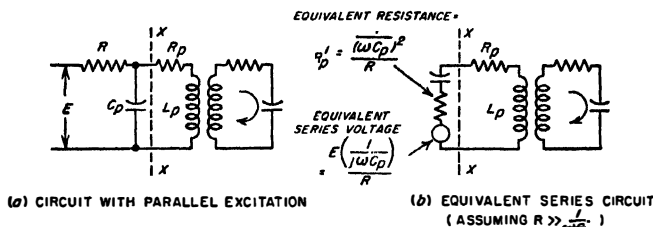


FIG. 3-14. Two coupled resonant circuits in which the primary circuit is excited by shunt feed.

Fig. 3-14a. This arrangement is analogous to the shunt-fed parallel resonant circuit discussed in connection with Fig. 3-5c.

The shunt-fed arrangement of Fig. 3-14a can be reduced to the equivalent series-fed arrangement of Fig. 3-14b by means of Thévenin's theorem, as explained in Sec. 3-7. Examination of the circuit of Fig. 3-14b shows that in a limited frequency range such as represented by the region about resonance, the equivalent voltage acting in series with the circuit is substantially constant. However, there is now an added resistance R'_p in the primary circuit that is equal to the equivalent series resistance that would be obtained by assuming that the resistance R is a shunt resistance for the primary capacitance C_p . The rest of the system is unchanged.

The principal effect of exciting a system of coupled circuits by parallel instead of series feed is accordingly to introduce some added resistance in the primary that lowers the effective value of Q_p . This effect will be slight in the usual case where the resistance R is very large compared with the reactance $1/\omega C_p$ of the capacitor C_p . Under these conditions, shunt feed and series feed accordingly give essentially the same shaped curves of secondary response as a function of frequency.

In systems involving two coupled resonant circuits, resistances are often placed in shunt with the primary and secondary resonant circuits for the purpose of adjusting the effective Q 's of the primary and secondary circuits to desired values. Such resistances are sometimes placed across both primary and secondary circuits, while in other cases they are used only across the primary, or only across the secondary. An example

where a resistance R is shunted across the secondary capacitor is shown in Fig. 3-15. In each case, a resistance in shunt with a particular resonant circuit of a coupled system has the same effect on that resonant circuit as it does when this resonant circuit is isolated, instead of being part of a coupled system. Hence a shunt resistance can be replaced by an equivalent series resistance, such as R'_s in Fig. 3-15. The effect of a shunting resistance is accordingly to lower the effective Q of the resonant circuit with which it is associated, as discussed in connection with Fig. 3-5.

Bandpass Action in Two Coupled Resonant Circuits. When two resonant circuits having $Q_p = Q_s$ are tuned to the same frequency and

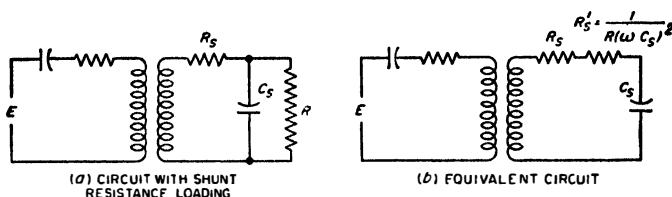


FIG. 3-15. Two coupled resonant circuits in which a shunt resistance loading is used to control secondary Q .

coupled together with critical coupling, the response characteristic of the secondary circuit is as shown in Fig. 3-12. As compared with the response of a simple resonant circuit with the same 70.7 per cent points, the response of the coupled system is found to be much flatter on top, and much steeper on the sides. Such an arrangement is often termed a *bandpass filter* because to a first approximation it responds equally well to a band of frequencies centered on the common resonant frequency, and rather sharply discriminates against frequencies outside of this band. Such bandpass characteristics are particularly desirable when handling modulated waves, because by proper adjustment of the bandwidth of the filter, the response can be made practically the same to the carrier and to all of the important sideband frequencies contained in the wave. In contrast with this, an ordinary resonant circuit has a response that is rounded on top, as shown dotted in Fig. 3-12, and so discriminates against the higher sideband frequencies in favor of the lower sideband frequencies and the carrier.

The bandpass characteristic that is best for most purposes corresponds to a coefficient of coupling equal to the critical value. For this case, still, assuming $Q_p = Q_s$, the design equations giving the required values of k and Q to realize a given bandwidth B are¹

$$k = k_c = \frac{B}{\sqrt{2} f_0} \quad Q_p = Q_s = \frac{1}{k_c} \quad (3-32)$$

¹ With unequal circuit Q 's the formulas will be slightly different for equivalent results, since the curve with flattest top now corresponds to a coefficient of coupling greater than the critical value.

where B = the bandwidth between the 70.7 per cent response points, cycles

f_0 = center frequency of passband (i.e., resonant frequency of tuned circuits)

k_c = critical coefficient of coupling

Effect of Varying Q in Coupled Systems Tuned to the Same Frequency. (Coefficient of Coupling Constant). Additional insight into the characteristics of coupled circuits can be gained by considering what happens to the secondary response curve as the Q 's of the primary and secondary circuits are changed, while keeping the coefficient of coupling constant. The effects observed are illustrated in Fig. 3-16 for a particular case. This example brings out clearly the fact that as the peaks of the response become less pronounced, they tend to move toward each other, and that at frequencies appreciably off resonance, the response differs only negligibly from the response calculated on the assumption of infinite Q (zero circuit loss).

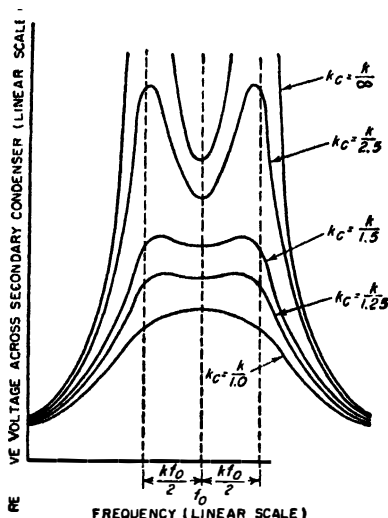


FIG. 3-16. Curves of secondary response when two circuits resonant at the same frequency are coupled together, showing the effect of varying the circuit Q 's while maintaining the coefficient of coupling unchanged.

3-17a the coupling consists of an inductance L_m common to the two circuits; in Fig. 3-17b the coupling is provided by a capacitance C_m common to the two circuits, and in Fig. 3-17c by a capacitance C'_m that connects the two circuits involved. Also, an infinite variety of more complicated coupling systems can be built up from the basic elements of mutual inductance, common inductance, common capacitance, and connecting capacitance. Simple examples of such combined couplings are shown in Fig. 3-17d and e.

The behavior of all these coupled circuits follows the same general character as that discussed for inductive coupling. Thus, the secondary circuit can be considered as producing an equivalent coupled impedance in the primary circuit while the primary circuit can be considered as inducing in the secondary a voltage that gives rise to the secondary current.

The simplest method of analyzing these various forms of coupled circuits is to take advantage of the fact that all of them can be reduced to the simple coupled circuit of Fig. 3-17f, provided suitable values are assigned to Z_p , Z_s , and M . The rules that determine the values of these quantities in the simple equivalent circuit are as follows:

1. The equivalent primary impedance Z_p of the equivalent circuit is the impedance that is measured across the primary terminals of the actual circuit when the secondary circuit has been opened.

2. The secondary impedance Z_s of the equivalent circuit is the impedance that is measured by opening the secondary of the actual circuit and

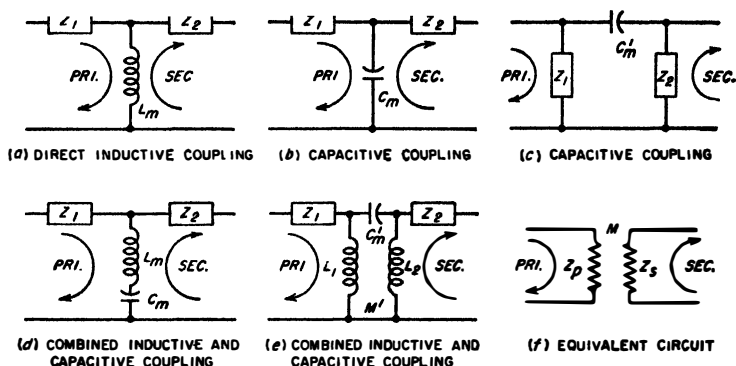


Fig. 3-17. Examples of methods whereby circuits may be coupled.

determining the impedance between these open points when the primary is open-circuited.

3. The equivalent mutual inductance M is determined by assuming a current I_p flowing into the primary circuit. The voltage which then appears across an open circuit in the secondary is equal to $-j\omega MI_p$.

In making use of the equivalent circuit of Fig. 3-17f, it is to be remembered that the values of Z_p , Z_s , and M may all vary with frequency, so that it is generally necessary to determine a new equivalent circuit for each frequency at which calculations are to be made.

After the actual coupled circuit has been reduced by the above procedure to its equivalent form shown in Fig. 3-14f, one can then apply the formulas that have already been derived for inductively coupled circuits, using the appropriate values M , Z_s , Z_p as determined for the equivalent circuit. This procedure has the advantage of using the same fundamental formulas to handle all types of coupling and makes it possible to carry on the analysis in the same manner for all cases. The method is particularly convenient in the handling of complex coupling networks such as illustrated in Fig. 3-17d and e.

The quantity M that appears in the equivalent circuit represents the effective coupling that is present between the primary and secondary

circuits. It is not necessarily a real mutual inductance of the inductive type, but rather a sort of mathematical fiction that gives the equivalent effect of whatever coupling is really present. If the actual coupling is capacitive, the numerical value of M will be found to be negative; if the coupling is of a complex type representing both resistive and reactive coupling, the numerical value of M will be found to have both real and imaginary parts. This need introduce no uncertainty, however, since the proper procedure is to take the value of M as it comes and substitute it with its appropriate sign and phase angle whenever M appears in the expressions previously derived for inductively coupled circuits.

When this analysis is applied to capacitively coupled circuits, such as those illustrated in Fig. 3-18, the results are essentially the same as for

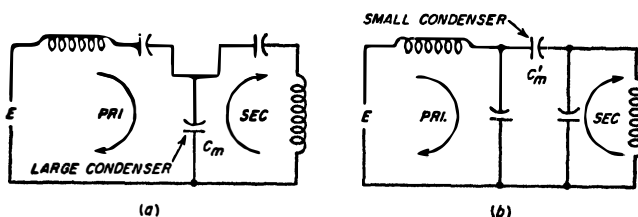


FIG. 3-18. Two methods of capacitively coupling two resonant circuits.

inductive coupling. Thus, when primary and secondary are both tuned to the same frequency, the secondary-current characteristic has two humps if the coupling is large, i.e., if capacitor C_m is small or C'_m large, while there is only one peak of secondary current when the coupling is small, i.e., when capacitor C_m is large.

Circuits having combined electromagnetic and electrostatic coupling, such as those at d and e of Fig. 3-17, behave as ordinary coupled circuits except that the coefficient of coupling varies with frequency. Thus, in the case of circuit d , the circuit is capacitively coupled at low frequencies and inductively coupled at high frequencies because the coupling combination of C_m in series with L_m has capacitive and inductive reactance under these respective conditions. In between, at the resonant frequency of L_m and C_m , there is no coupling and $k = 0$. The arrangement shown at e acts similarly as a circuit with a coefficient of coupling that varies with frequency. Circuits having combined electrostatic and electromagnetic coupling find application where it is desired to obtain a coefficient of coupling that varies with frequency, as is commonly the case in tuned amplifiers and antenna-coupling circuits of radio receivers.

3-7. Thévenin's Theorem. According to Thévenin's theorem, any linear network containing one or more sources of voltage and having two terminals behaves, in so far as a load impedance connected across these terminals is concerned, as though the network and its generators were equivalent to a simple generator having an internal impedance Z and a generated voltage E , where E

is the voltage that appears across the terminals when no load impedance is connected and Z is the impedance that is measured between the terminals when all sources of voltage in the network are short-circuited.^{1,2}

This theorem means that any network and its generators, represented schematically by the block in Fig. 3-19a, can be replaced by the equivalent circuit shown in Fig. 3-19b. The only limitation to the validity of Thévenin's theorem encountered in ordinary practice is that the circuit elements of the network must be linear; i.e., the voltage developed must always be proportional to current.

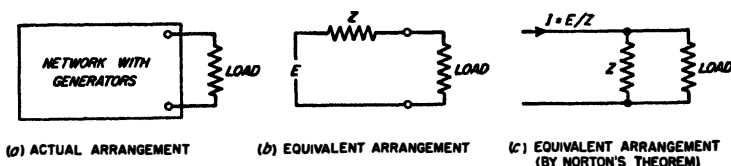


FIG. 3-19. Diagrams illustrating how Thévenin's and Norton's theorems can be used to simplify a complicated network containing generators.

Thévenin's theorem offers a very powerful means of simplifying networks, particularly when a load impedance is connected across the output terminals of a complicated network. Two examples will be used to illustrate this. First, consider the circuit of Fig. 3-10, which is redrawn in Fig. 3-20a. If one takes the secondary capacitor C_2 as the load impedance and applies Thévenin's theorem to the network to the left of C_2 , the result is Fig. 3-20b, in which the equivalent generator voltage is the voltage induced in the secondary inductance L_2 when the secondary is open-circuited, and the equivalent generator impedance consists of the inductance L_2 and the resistance R_2 in series with the impedance which is coupled into L_2 by a secondary circuit consisting of L_1 shunted by the resistance R_1 . The coupled impedance produced by such a secondary circuit has been previously considered; it is equivalent to adding capacitive reactance and resistance in series. The resistance causes the effective Q of the secondary-response curve to be reduced, while the series capaci-

¹ When the sources of energy in the network are constant-current generators instead of constant-voltage generators, the internal impedance Z is the impedance observed between the terminals when all constant-current generators are *open-circuited*. This is due to the fact that a constant-current generator is equivalent to an infinite voltage source having an infinite internal impedance, so that short-circuiting the ultimate source of voltage of the constant-current generator still leaves an infinite impedance in the circuit.

² An alternative circuit that is also equivalent to Fig. 3-19a is given in Fig. 3-19c. Here the network with its generators is replaced by a constant current I that is delivered to a system consisting of the source impedance Z in shunt with which is the load impedance, where I is the output current of the network when the output terminals are short-circuited, and is $I = E/Z$. The equivalence of the arrangements at a and c in Fig. 3-19 is sometimes referred to as *Norton's theorem*.

tive reactance tends to raise the apparent resonant frequency by an amount that becomes greater the higher the ratio $\omega L_1/R_1$. This accounts for the behavior of the curves of Fig. 3-10.

The second example is furnished by Fig. 3-14a. This circuit may be simplified by considering that the load is represented by the circuit to the right of the line xx , the generator being the voltage E acting in series with the resistance R and the capacitance C_p . Such a generator can be reduced immediately by Thévenin's theorem to the form shown to the left of the line xx in Fig. 3-14b. Here it is to be noted that the equivalent generator resistance R'_p is the series resistance equivalent to a shunt resistance R associated with the capacitance C_p , as given by Eqs. (2-12) and (2-13).

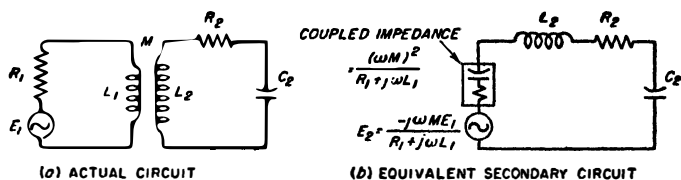


FIG. 3-20. Application of Thévenin's theorem to simplify and explain the behavior of the system of Fig. 3-10, consisting of a tuned secondary and untuned primary circuit coupled together.

3-8. Impedance Matching. A load connected across the output terminals of a network, such as represented schematically by Fig. 3-19a, can be matched to the source of power in either of two ways. When the load impedance has the same magnitude and phase angle as the equivalent generator impedance Z defined by Thévenin's theorem (see Fig. 3-19b), the load is said to be matched to the generator or source of power on an *image-impedance* basis. The term "image" arises from the fact that the impedances on the two sides of the output terminals are images of each other. When the load impedance is not identical with the generator impedance and it is desired to obtain impedance matching on an image basis, it is then necessary to transform the load to the correct impedance to match the generator. This transformation can be accomplished with the aid of an appropriate network of reactances or, in simple cases, by means of a transformer.

Alternatively, a load impedance may be matched to a source of power in such a way as to make the power delivered to the load a maximum.¹ This is accomplished by making the load impedance the conjugate of the generator impedance as defined by Thévenin's theorem. That is, the load impedance must have the same magnitude as the generator impedance, but the phase angle of the load is the negative of the phase angle of the generator impedance. This method of matching is shown schemat-

¹ The power delivered to the load under these conditions is termed the *available power* of the power source.

ically in Fig. 3-21. It will be noted that the reactive component of the load is then in series resonance with the reactive component of the generator impedance; i.e., the load reactance is the correct value to "tune out" the generator reactance. The resistance components of the load and generator impedances are then matched on an image-impedance basis. Such impedance matching to obtain maximum power delivered to the load is a common operation in communication circuits. It is carried out by transforming the equivalent series resistance of the load to a value equal to the resistance component of the generator impedance by the use of suitable networks and transformers, and then adding reactance to the load as required to resonate with the generator reactance.

It will be noted that, when the generator impedance is resistive, the conditions corresponding to matching on an image-impedance basis are identical with those corresponding to matching for maximum power output delivered to the load. Otherwise, the two conditions are not the same, and matching on an image-impedance basis then does not result in maximum possible power being delivered to the load, although it is often still used to maintain appropriate impedance relations in a system of networks.

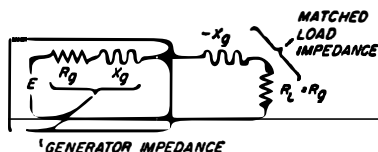


FIG. 3-21. Load impedance matched to generator in such a way as to give maximum power in the load.

PROBLEMS AND EXERCISES

3-1. The coil of Fig. 2-16 is tuned to resonance at 1000 kc by a capacitor having a power factor of 0.001. What is the circuit Q ?

3-2. In Prob. 3-1, what tuning capacitance is required?

3-3. A variable capacitor having a maximum capacitance of 350 μf and a minimum capacitance of 20 μf is used for tuning in a broadcast receiver. The coil and associated wiring have a distributed capacitance of 20 μf .

a. What size inductance coil is required to make the lowest frequency 530 kc?

b. Calculate the exact tuning range with the coil selected.

3-4. A series circuit is resonant at 800 kc and has an inductance of 160 μh and a circuit Q of 75. Calculate and plot the magnitude of the current that flows when 1 volt is applied to the circuit, carrying the curves out to 40 kc on each side of resonance. In making these calculations use the working rules and the universal resonance curve in the range near resonance and neglect the circuit resistance when calculating points too far off resonance to be within the range of the universal resonance curve.

3-5. In Prob. 3-4 calculate the exact response at 40 kc above resonance, taking into account the circuit resistance, and compare the results with those obtained when the circuit resistance is neglected.

3-6. Assume that a series resonant circuit employs the coil of Fig. 2-16, and that the tuning capacitor has negligible losses.

a. Calculate and plot from 500 to 1500 kc the width of the frequency band for

which the tuned circuit response is at least 70.7 per cent of the response at resonance, as a function of resonant frequency.

6. Discuss the results obtained in (a) with respect to the reception of broadcast signals having sideband frequencies extending up to 5000 cycles on each side of the carrier frequency. Consider both the uniformity of response to the different sideband frequencies, and the ability of the circuit to discriminate against undesired signals of other frequencies.

3-7. In a series circuit that is resonant at 1150 kc it is found that when the frequency differs from resonance by 15 kc the current drops to 0.53 of the current at resonance, for the same applied voltage. From this information determine the Q of the circuit.

3-8. A voltage of constant but unknown value is applied to a series circuit resonant at the frequency of this voltage. The circuit current is observed to be I_0 . A known resistance R_1 is then added to the circuit, and it is found that, with the same applied voltage as before, the current is now reduced to I_1 . Derive a formula for the circuit resistance in terms of I_0 , I_1 , and R_1 .

3-9. In variable capacitors used to tune the resonant circuits of radio receivers, it is customary to shape the plates so that the capacitance varies more slowly with angle of rotation at small capacitance settings than at high capacitance settings. Explain why this makes the resonant frequency more nearly linear with respect to the angle of rotation than if semicircular plates were employed.

3-10. What is the highest effective Q that a tuned circuit may have when it must respond to a band of frequencies 10,000 cycles wide (5000-cycle sideband frequencies) with a response always at least 70.7 per cent of the response at resonance, assuming carrier frequencies of 50, 500, 5000, and 50,000 kc?

3-11. a. A tuned circuit having an inductance of 150 μ h and a Q of 70 is adjusted to resonance at 1100 kc. If the circuit is connected for parallel resonance, calculate and plot the magnitude of the parallel impedance as a function of frequency out to 60 kc on each side of resonance. Use the working rules and the universal resonance curve in the region about resonance, and neglect the circuit resistance when calculating the impedance at frequencies too far off resonance to be within range of the universal resonance curve.

b. Repeat (a) for a circuit Q of 40, and plot the results on the same axes as the results of (a).

3-12. Calculate and plot as a function of frequency the parallel impedance at resonance when the coil of Fig. 2-16 is tuned with a capacitor of negligible losses and when the resonant frequency is varied from 500 to 1500 kc.

3-13. A tuned circuit is required to have a parallel impedance of 6000 ohms and a Q of 12. If the resonant frequency is 300 kc determine the inductance, capacitance, and resistance that the circuit must have.

3-14. Using the same tuned circuit as in Prob. 3-4, but connected for parallel resonance, calculate and plot curves as a function of frequency from 760 to 840 kc for (a) magnitude and phase angle of parallel impedance; (b) line current, and current in each branch, when the applied potential is 10 volts (assume that all the circuit resistance is in the inductive branch); and (c) reactance and resistance components of the impedance of (a).

3-15. The coil of Fig. 2-16 is tuned to resonance at 1000 kc with a capacitor having negligible losses. Transform this circuit to the form shown in the right-hand part of Fig. 3-5a by determining R_2 .

3-16. The circuit of Fig. 2-16 is tuned to resonance at 1000 kc with a capacitor having negligible losses, and is then shunted by a resistance R'_1 of 100,000 ohms.

a. Determine the equivalent shunt resistance R_2 for such an arrangement (see Fig. 3-5).

b. Calculate the Q of this system.

3-17. If a parallel resonant circuit is shunted by a resistance R'_2 and if the parallel resonant impedance of the unshunted circuit is R_0 , prove that the shunt resistance R'_2 reduces the equivalent Q of the circuit by the factor $R'_2/(R'_2 + R_0)$.

3-18. In a low- Q parallel circuit in which the losses are all in the inductive branch, prove that, when the capacitance is varied, the capacitance that makes the parallel circuit impedance have unity power factor for a given frequency also makes this impedance have maximum magnitude at this same frequency.

3-19. A particular coil has an inductance of $180 \mu\text{h}$ at 1 kc and an apparent inductance of $200 \mu\text{h}$ at 1400 kc. Determine the distributed capacitance of the coil.

3-20. Primary and secondary coils have inductances of 75 and $300 \mu\text{h}$, respectively, and 1 volt is applied to the primary circuit. Assuming the resistances of the coils are negligible, calculate the voltage induced in the secondary as a function of coefficient of coupling from $k = 0$ to $k = 1.0$.

3-21. Draw an equivalent transformer circuit for the coils of Prob. 3-20, for the case where the mutual inductance is $50 \mu\text{h}$.

3-22. a. Explain the effect of a short-circuited turn upon the inductance and Q observed at the terminals of a coil, using coupled-circuit theory.

b. Indicate qualitatively the differences that would be expected if the short-circuited turn were the end turn of a single-layer solenoid, as in Fig. 2-1, as against being a turn near the center.

3-23. Two identical coils each having $Q = 100$ and an inductance of $200 \mu\text{h}$ are coupled together with a mutual inductance of $50 \mu\text{h}$. If the secondary coil is short-circuited, calculate (a) the coupled resistance and coupled reactance at a frequency of 600 kc, (b) the total resistance and reactance of the primary circuit, and (c) the effective Q of the primary circuit including effect of the coupled impedance.

3-24. Describe a procedure for experimentally determining the coefficient of coupling between a coil and its shield can, assuming that the shield has negligible resistance.

3-25. Derive the formula in the second footnote on page 61 for the equivalent primary-circuit inductance in the presence of an inductive secondary with zero losses.

3-26. An air-cored coil is placed near a brass panel. Describe in a qualitative way the effect that copper plating this panel will have on the inductance and Q observed at the coil terminals.

3-27. The coil of Fig. 2-16 is coupled to a primary coil with a mutual inductance of $50 \mu\text{h}$. If the secondary coil is tuned to resonance by means of a capacitor having negligible loss, calculate and plot the coupled impedance at the resonant frequency of the secondary as this resonant frequency is varied from 500 to 1500 kc.

3-28. The coil of Fig. 2-16 is coupled to a primary circuit having an inductance of $75 \mu\text{h}$, and is tuned to resonance at 1000 kc with a capacitor having negligible losses. Calculate the impedance coupled into the primary circuit at 1000 kc as a function of coefficient of coupling from $k = 0$ to $k = 1.0$.

3-29. In the circuit of Fig. 3-8b, what general effect is produced on the phase and magnitude of the coupled impedance at the resonant frequency of the secondary by shunting the secondary capacitor C_s by a resistance R_s ?

3-30. Explain why in Fig. 3-11 a flat-topped secondary-circuit curve (like $k = 0.01$) can be obtained only if the primary-current curve has pronounced double peaks.

3-31. Derive Eq. (3-29) from Eq. (3-28).

3-32. Two identical circuits resonant at 1000 kc, having $Q = 80$ and inductances of $140 \mu\text{h}$, are coupled together.

a. Calculate the critical coefficient of coupling.

b. Calculate and plot the secondary current at the resonant frequency for 1 volt applied to the primary, as the mutual inductance is varied from zero to twice the critical value.

3-33. The coupling between the circuits of Prob. 3-32 is adjusted to make the coefficient of coupling have a value 0.03, and 1 volt is applied in series with the primary.

a. What will be the approximate frequencies at which the secondary-current peaks will occur?

b. What will be the approximate height of these peaks of secondary current? Assume the two peaks have equal heights.

c. What will be the secondary current at the resonant frequency?

d. Over what frequency range will the secondary response equal or exceed the secondary response at resonance?

e. With the information obtained above, sketch the approximate shape of the secondary-current curve as a function of frequency.

3-34. The circuits of Prob. 3-32 are coupled with a coefficient of coupling of 0.1. Determine the frequencies at which the secondary-current peaks occur, and give the approximate ratio of voltages across the secondary at frequencies corresponding to the low- and high-frequency peaks.

3-35. The two circuits of Prob. 3-32 are coupled with a mutual inductance of $2.8 \mu\text{h}$ ($k = 0.02$).

a. Calculate and plot the resistance and reactance components of the coupled impedance out to 40 kc on each side of resonance.

b. Calculate and plot the resistance and reactance components of the primary circuit when the secondary is removed.

c. Add (a) and (b) to obtain the curve of total primary-circuit resistance and reactance, and convert the results into curves giving the magnitude and phase of the total primary impedance in the presence of the secondary.

3-36. If, in Prob. 3-35, the mutual inductance had a value of $1 \mu\text{h}$, then to what frequencies would it be necessary to tune the primary and secondary circuits in order to obtain the same shape of secondary-response curve as is actually obtained for the conditions given in Prob. 3-35?

3-37. In a shunt-feed circuit such as illustrated in Fig. 3-14, the tuned circuits are the same as in Prob. 3-32, and the shunt-feed resistance R is 100,000 ohms. What is the equivalent primary Q under these conditions?

3-38. The two resonant circuits in Fig. 3-15 are the same as in Prob. 3-32. What value must R have to make the effective Q of the secondary equal to 40?

3-39. A particular bandpass filter is to be used to handle a wave in which the highest modulation frequency is 4000 cycles. The carrier frequency of the wave is 456 kc. If the primary and secondary inductances are both 2 mh and if it is desired just barely to avoid double humps in the response curve, specify the proper coefficient of coupling and the proper circuit Q 's, assuming equal primary and secondary Q 's.

3-40. Two identical tuned circuits are used in a shunt-feed bandpass arrangement. The circuits are resonant at 450 kc, have inductances of 2.0 mh, and Q 's of 80. The shunt-feed resistance has a value of 300,000 ohms. A bandwidth between 70.7 per cent response points of 30 kc is desired.

a. Calculate required values of circuit Q 's, assuming $Q_p = Q_s$.

b. Determine the resistance that must be shunted across the secondary capacitor to make the effective Q of the secondary circuit have the required value.

c. Determine the resistance that must be shunted across the primary capacitor to make the effective Q of the primary have the required value when the effect of both the shunt-feed resistance and the primary-circuit resistance are taken into account.

3-41. According to Fig. 3-16 the response at resonance will increase as the Q is increased while leaving the coefficient of coupling unchanged.

a. Demonstrate that this result is predicted by Eq. (3-28).

b. Determine the ratio of response at resonance for zero circuit losses to the response for $k = 0.01$ when the circuit losses make $k = 0.01$ correspond to critical coupling.

3-42. Calculate the coefficient of coupling in the circuit of Fig. 3-18b when $C_1 = C_2 = 100 \mu\text{f}$, and $C'_m = 1.5 \mu\text{f}$.

3-43. Signals in the frequency range of 550 to 1500 kc are to be handled by means of a bandpass filter. If the circuits are assumed to have $Q = 100$ over this frequency range, and if the adjustment is such that $k = 0.01$ at 1000 kc, discuss how the width and shape of the passband will vary with resonant frequency when the tuning is obtained by varying the primary and secondary capacitors simultaneously and when the coupling is (a) inductive as shown at Fig. 2-8b, and (b) capacitive as shown at Fig. 2-8c. Assume that the circuit elements that provide the coupling do not change as the capacitors are varied. Illustrate the discussion with the aid of sketches showing in a general way the relative character of the response curves to be expected at 550, 1000, and 1500 kc for each type of coupling.

3-44. Explain how the magnitudes of the Thévenin-theorem equivalent voltage and impedance for a complex network can be determined experimentally from an open- and short-circuit test at the output terminals of the network, using only a voltmeter and an ammeter.

3-45. In Fig. 3-20 (also Fig. 3-10) the secondary circuit has an inductance of $150 \mu\text{h}$, and is resonant at 1000 kc. If $R_1 = 10,000$, $L_1 = 150 \mu\text{h}$, and $M = 100 \mu\text{h}$, calculate the frequency at which the peak of secondary response occurs.

3-46. A primary circuit has an inductance of 1 mh and a resistance of 150 ohms connected in series. A secondary coil is coupled to the primary coil and delivers power to a load consisting of the secondary coil, a resistance of 50 ohms, and a tuning capacitance, all in series. If the impedance that the secondary circuit couples into the primary circuit is considered to be the load impedance of the primary circuit, determine the mutual inductance required between the two circuits and the reactance that the secondary circuit must have if the load is to match the generator on a maximum-power basis.

3-47. a. In order to demonstrate impedance matching for maximum-power transfer, write the equation of power P delivered to a rheostat as a function of its resistance R when connected to a d-c generator of internal resistance R_s and open-circuit voltage E_s . Show that this equation has a maximum for $R = R_s$.

b. Plot a graph of the equation of (a), showing P/P_{max} versus R/R_s , where P is the actual power when the load resistance is R , and P_{max} is the power when $R = R_s$. By how many decibels is the power reduced for the following cases of mismatch: (1) $R = 0.5R_s$, and (2) $R = 2R_s$?

CHAPTER 4

TRANSMISSION LINES

4-1. Voltage and Current Relations on Radio-frequency Transmission Lines in Terms of Traveling Waves.¹ Transmission lines find many uses in radio work. They are employed, not only to transmit energy, but also as resonant circuits at very high frequencies, as measuring devices at high frequencies, as aids to obtain impedance matching, etc.

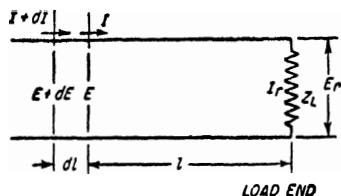


FIG. 4-1. Transmission line, showing elementary length dl .

Basic Transmission-line Equations.

Consider the voltage and current relations that exist in a very short length dl of the transmission line shown in Fig. 4-1. In this short distance the voltage between the wires changes an amount dE as a result of the voltage drop produced by the line current I flowing through the resistance $R dl$ and reactance $j\omega L dl$ of the length dl . Likewise, the current changes a small amount dI in the length as a result of the flow of current between the wires through the capacitance $C dl$ and conductance $G dl$ caused by the voltage that exists between these wires. Referring to Fig. 4-1, one can accordingly write the equations

$$\begin{aligned} dE &= I \times (\text{impedance of length } dl) \\ &= I(R + j\omega L) dl \\ dI &= E \times (\text{admittance of length } dl) \\ &= E(G + j\omega C) dl \end{aligned}$$

¹ This material on transmission lines is a review and summary of those concepts and relations that are most widely used in radio work. It presupposes at least a little previous familiarity with the subject, and therefore should not be regarded as a self-supporting presentation of transmission-line theory. The reader desiring to gain a comprehensive understanding of transmission lines, or desiring the derivation of the equations made use of here, should consult one of the several excellent textbooks that are available on the subject, for example, H. H. Skilling, "Electric Transmission Lines," McGraw-Hill Book Company, Inc., New York, 1951; Walter C. Johnson, "Transmission Lines and Networks," McGraw-Hill Book Company, Inc., New York, 1950. More limited treatments of transmission lines, typically of chapter length, are to be found in most textbooks on communication engineering; these are adequate as an introduction to the material presented here.