

Fundamentals of Noise and Vibration Analysis for Engineers

Noise and vibration affects all kinds of engineering structures, and is fast becoming an integral part of engineering courses at universities and colleges around the world. In this second edition, Michael Norton's classic text has been extensively updated to take into account recent developments in the field. Much of the new material has been provided by Denis Karczub, who joins Michael as second author for this edition.

This book treats both noise and vibration in a single volume, with particular emphasis on wave-mode duality and interactions between sound waves and solid structures. There are numerous case studies, test cases and examples for students to work through. The book is primarily intended as a text book for senior level undergraduate and graduate courses, but is also a valuable reference for practitioners and researchers in the field of noise and vibration.

Fundamentals of Noise and Vibration Analysis for Engineers

Second edition

M. P. Norton

School of Mechanical Engineering, University of Western Australia

and

D. G. Karczub

S.V.T. Engineering Consultants, Perth, Western Australia



CAMBRIDGE UNIVERSITY PRESS
Cambridge, New York, Melbourne, Madrid, Cape Town, Singapore, São Paulo

Cambridge University Press
The Edinburgh Building, Cambridge CB2 8RU, UK

Published in the United States of America by Cambridge University Press, New York

www.cambridge.org
Information on this title: www.cambridge.org/9780521495616

© First edition Cambridge University Press 1989
© Second edition M. P. Norton and D. G. Karczub 2003

This publication is in copyright. Subject to statutory exception
and to the provisions of relevant collective licensing agreements,
no reproduction of any part may take place without the written
permission of Cambridge University Press.

First edition published 1989
Reprinted 1994
Second edition published 2003

A catalogue record for this publication is available from the British Library

ISBN 978-0-521-49561-6 hardback
ISBN 978-0-521-49913-2 paperback

Transferred to digital printing 2007

To
our parents,
the first author's wife Erica,
and his young daughters Caitlin and Sarah

Contents

<i>Preface</i>	<i>page xv</i>
<i>Acknowledgements</i>	xvii
<i>Introductory comments</i>	xviii

1	Mechanical vibrations: a review of some fundamentals	1
1.1	Introduction	1
1.2	Introductory wave motion concepts – an elastic continuum viewpoint	3
1.3	Introductory multiple, discrete, mass–spring–damper oscillator concepts – a macroscopic viewpoint	8
1.4	Introductory concepts on natural frequencies, modes of vibration, forced vibrations and resonance	10
1.5	The dynamics of a single oscillator – a convenient model	12
1.5.1	Undamped free vibrations	12
1.5.2	Energy concepts	15
1.5.3	Free vibrations with viscous damping	16
1.5.4	Forced vibrations: some general comments	21
1.5.5	Forced vibrations with harmonic excitation	22
1.5.6	Equivalent viscous-damping concepts – damping in real systems	30
1.5.7	Forced vibrations with periodic excitation	32
1.5.8	Forced vibrations with transient excitation	33
1.6	Forced vibrations with random excitation	37
1.6.1	Probability functions	38
1.6.2	Correlation functions	39
1.6.3	Spectral density functions	41
1.6.4	Input–output relationships for linear systems	42
1.6.5	The special case of broadband excitation of a single oscillator	50
1.6.6	A note on frequency response functions and transfer functions	52
1.7	Energy and power flow relationships	52

1.8	Multiple oscillators – a review of some general procedures	56
1.8.1	A simple two-degree-of-freedom system	56
1.8.2	A simple three-degree-of-freedom system	59
1.8.3	Forced vibrations of multiple oscillators	60
1.9	Continuous systems – a review of wave-types in strings, bars and plates	64
1.9.1	The vibrating string	64
1.9.2	Quasi-longitudinal vibrations of rods and bars	72
1.9.3	Transmission and reflection of quasi-longitudinal waves	77
1.9.4	Transverse bending vibrations of beams	79
1.9.5	A general discussion on wave-types in structures	84
1.9.6	Mode summation procedures	85
1.9.7	The response of continuous systems to random loads	91
1.9.8	Bending waves in plates	94
1.10	Relationships for the analysis of dynamic stress in beams	96
1.10.1	Dynamic stress response for flexural vibration of a thin beam	96
1.10.2	Far-field relationships between dynamic stress and structural vibration levels	100
1.10.3	Generalised relationships for the prediction of maximum dynamic stress	102
1.10.4	Properties of the non-dimensional correlation ratio	103
1.10.5	Estimates of dynamic stress based on static stress and displacement	104
1.10.6	Mean-square estimates for single-mode vibration	105
1.10.7	Relationships for a base-excited cantilever with tip mass	106
1.11	Relationships for the analysis of dynamic strain in plates	108
1.11.1	Dynamic strain response for flexural vibration of a constrained rectangular plate	109
1.11.2	Far-field relationships between dynamic stress and structural vibration levels	112
1.11.3	Generalised relationships for the prediction of maximum dynamic stress	113
1.12	Relationships for the analysis of dynamic strain in cylindrical shells	113
1.12.1	Dynamic response of cylindrical shells	114
1.12.2	Propagating and evanescent wave components	117
1.12.3	Dynamic strain concentration factors	119
1.12.4	Correlations between dynamic strain and velocity spatial maxima	119
	<i>References</i>	122
	<i>Nomenclature</i>	123

2	Sound waves: a review of some fundamentals	128
2.1	Introduction	128
2.2	The homogeneous acoustic wave equation – a classical analysis	131
2.2.1	Conservation of mass	134
2.2.2	Conservation of momentum	136
2.2.3	The thermodynamic equation of state	139
2.2.4	The linearised acoustic wave equation	140
2.2.5	The acoustic velocity potential	141
2.2.6	The propagation of plane sound waves	143
2.2.7	Sound intensity, energy density and sound power	144
2.3	Fundamental acoustic source models	146
2.3.1	Monopoles – simple spherical sound waves	147
2.3.2	Dipoles	151
2.3.3	Monopoles near a rigid, reflecting, ground plane	155
2.3.4	Sound radiation from a vibrating piston mounted in a rigid baffle	157
2.3.5	Quadrupoles – lateral and longitudinal	162
2.3.6	Cylindrical line sound sources	164
2.4	The inhomogeneous acoustic wave equation – aerodynamic sound	165
2.4.1	The inhomogeneous wave equation	167
2.4.2	Lighthill’s acoustic analogy	174
2.4.3	The effects of the presence of solid bodies in the flow	177
2.4.4	The Powell–Howe theory of vortex sound	180
2.5	Flow duct acoustics	183
	<i>References</i>	187
	<i>Nomenclature</i>	188
3	Interactions between sound waves and solid structures	193
3.1	Introduction	193
3.2	Fundamentals of fluid–structure interactions	194
3.3	Sound radiation from an infinite plate – wave/boundary matching concepts	197
3.4	Introductory radiation ratio concepts	203
3.5	Sound radiation from free bending waves in finite plate-type structures	207
3.6	Sound radiation from regions in proximity to discontinuities – point and line force excitations	216

3.7	Radiation ratios of finite structural elements	221
3.8	Some specific engineering-type applications of the reciprocity principle	227
3.9	Sound transmission through panels and partitions	230
3.9.1	Sound transmission through single panels	232
3.9.2	Sound transmission through double-leaf panels	241
3.10	The effects of fluid loading on vibrating structures	244
3.11	Impact noise	247
	<i>References</i>	249
	<i>Nomenclature</i>	250

4 Noise and vibration measurement and control procedures 254

4.1	Introduction	254
4.2	Noise and vibration measurement units – levels, decibels and spectra	256
4.2.1	Objective noise measurement scales	256
4.2.2	Subjective noise measurement scales	257
4.2.3	Vibration measurement scales	259
4.2.4	Addition and subtraction of decibels	261
4.2.5	Frequency analysis bandwidths	263
4.3	Noise and vibration measurement instrumentation	267
4.3.1	Noise measurement instrumentation	267
4.3.2	Vibration measurement instrumentation	270
4.4	Relationships for the measurement of free-field sound propagation	273
4.5	The directional characteristics of sound sources	278
4.6	Sound power models – constant power and constant volume sources	279
4.7	The measurement of sound power	282
4.7.1	Free-field techniques	282
4.7.2	Reverberant-field techniques	283
4.7.3	Semi-reverberant-field techniques	287
4.7.4	Sound intensity techniques	290
4.8	Some general comments on industrial noise and vibration control	294
4.8.1	Basic sources of industrial noise and vibration	294
4.8.2	Basic industrial noise and vibration control methods	295
4.8.3	The economic factor	299
4.9	Sound transmission from one room to another	301
4.10	Acoustic enclosures	304
4.11	Acoustic barriers	308
4.12	Sound-absorbing materials	313
4.13	Vibration control procedures	320

4.13.1 Low frequency vibration isolation – single-degree-of-freedom systems	322
4.13.2 Low frequency vibration isolation – multiple-degree-of-freedom systems	325
4.13.3 Vibration isolation in the audio-frequency range	327
4.13.4 Vibration isolation materials	330
4.13.5 Dynamic absorption	332
4.13.6 Damping materials	334
<i>References</i>	335
<i>Nomenclature</i>	336

5 The analysis of noise and vibration signals 342

5.1 Introduction	342
5.2 Deterministic and random signals	344
5.3 Fundamental signal analysis techniques	347
5.3.1 Signal magnitude analysis	347
5.3.2 Time domain analysis	351
5.3.3 Frequency domain analysis	352
5.3.4 Dual signal analysis	355
5.4 Analogue signal analysis	365
5.5 Digital signal analysis	366
5.6 Statistical errors associated with signal analysis	370
5.6.1 Random and bias errors	370
5.6.2 Aliasing	372
5.6.3 Windowing	374
5.7 Measurement noise errors associated with signal analysis	377
<i>References</i>	380
<i>Nomenclature</i>	380

6 Statistical energy analysis of noise and vibration 383

6.1 Introduction	383
6.2 The basic concepts of statistical energy analysis	384
6.3 Energy flow relationships	387
6.3.1 Basic energy flow concepts	388
6.3.2 Some general comments	389
6.3.3 The two subsystem model	391

6.3.4	<i>In-situ</i> estimation procedures	393
6.3.5	Multiple subsystems	395
6.4	Modal densities	397
6.4.1	Modal densities of structural elements	397
6.4.2	Modal densities of acoustic volumes	400
6.4.3	Modal density measurement techniques	401
6.5	Internal loss factors	407
6.5.1	Loss factors of structural elements	408
6.5.2	Acoustic radiation loss factors	410
6.5.3	Internal loss factor measurement techniques	412
6.6	Coupling loss factors	417
6.6.1	Structure–structure coupling loss factors	417
6.6.2	Structure–acoustic volume coupling loss factors	419
6.6.3	Acoustic volume–acoustic volume coupling loss factors	420
6.6.4	Coupling loss factor measurement techniques	421
6.7	Examples of the application of S.E.A. to coupled systems	423
6.7.1	A beam–plate–room volume coupled system	424
6.7.2	Two rooms coupled by a partition	427
6.8	Non-conservative coupling – coupling damping	430
6.9	The estimation of sound radiation from coupled structures using total loss factor concepts	431
6.10	Relationships between dynamic stress and strain and structural vibration levels	433
	<i>References</i>	435
	<i>Nomenclature</i>	437

7 Pipe flow noise and vibration: a case study 441

7.1	Introduction	441
7.2	General description of the effects of flow disturbances on pipeline noise and vibration	443
7.3	The sound field inside a cylindrical shell	446
7.4	Response of a cylindrical shell to internal flow	451
7.4.1	General formalism of the vibrational response and sound radiation	451
7.4.2	Natural frequencies of cylindrical shells	454
7.4.3	The internal wall pressure field	455
7.4.4	The joint acceptance function	458
7.4.5	Radiation ratios	460

7.5 Coincidence – vibrational response and sound radiation due to higher order acoustic modes	461
7.6 Other pipe flow noise sources	467
7.7 Prediction of vibrational response and sound radiation characteristics	471
7.8 Some general design guidelines	477
7.9 A vibration damper for the reduction of pipe flow noise and vibration	479
<i>References</i>	481
<i>Nomenclature</i>	483

8 Noise and vibration as a diagnostic tool 488

8.1 Introduction	488
8.2 Some general comments on noise and vibration as a diagnostic tool	489
8.3 Review of available signal analysis techniques	493
8.3.1 Conventional magnitude and time domain analysis techniques	494
8.3.2 Conventional frequency domain analysis techniques	501
8.3.3 Cepstrum analysis techniques	503
8.3.4 Sound intensity analysis techniques	504
8.3.5 Other advanced signal analysis techniques	507
8.3.6 New techniques in condition monitoring	511
8.4 Source identification and fault detection from noise and vibration signals	513
8.4.1 Gears	514
8.4.2 Rotors and shafts	516
8.4.3 Bearings	518
8.4.4 Fans and blowers	523
8.4.5 Furnaces and burners	525
8.4.6 Punch presses	527
8.4.7 Pumps	528
8.4.8 Electrical equipment	530
8.4.9 Source ranking in complex machinery	532
8.4.10 Structural components	536
8.4.11 Vibration severity guides	539
8.5 Some specific test cases	541
8.5.1 Cabin noise source identification on a load–haul–dump vehicle	541
8.5.2 Noise and vibration source identification on a large induction motor	547
8.5.3 Identification of rolling-contact bearing damage	550
8.5.4 Flow-induced noise and vibration associated with a gas pipeline	554

8.5.5 Flow-induced noise and vibration associated with a racing sloop (yacht)	557
8.6 Performance monitoring	557
8.7 Integrated condition monitoring design concepts	559
References	562
Nomenclature	563
Problems	566
Appendix 1: Relevant engineering noise and vibration control journals	599
Appendix 2: Typical sound transmission loss values and sound absorption coefficients for some common building materials	600
Appendix 3: Units and conversion factors	603
Appendix 4: Physical properties of some common substances	605
Answers to problems	607
Index	621

2 Sound waves: a review of some fundamentals

2.1 Introduction

Sound is a pressure wave that propagates through an elastic medium at some characteristic speed. It is the molecular transfer of motional energy and cannot therefore pass through a vacuum. For this wave motion to exist, the medium has to possess inertia and elasticity. Whilst vibration relates to such wave motion in structural elements, noise relates to such wave motion in fluids (gases and liquids). Two fundamental mechanisms are responsible for sound generation. They are:

- (i) the vibration of solid bodies resulting in the generation and radiation of sound energy – these sound waves are generally referred to as structure-borne sound;
- (ii) flow-induced noise resulting from pressure fluctuations induced by turbulence and unsteady flows – these sound waves are generally referred to as aerodynamic sound.

With structure-borne sound, the regions of interest are generally in a fluid (usually air) at some distance from the vibrating structure. Here, the sound waves propagate through the stationary fluid (the fluid has a finite particle velocity due to the sound wave, but a zero mean velocity) from a readily identifiable source to the receiver. The region of interest does not therefore contain any sources of sound energy – i.e. the sources which generated the acoustic disturbance are external to it. A simple example is a vibrating electric motor. Classical acoustical theory (analysis of the homogeneous wave equation) can be used for the analysis of sound waves generated by these types of sources. The solution for the acoustic pressure fluctuation, p , describes the wave field external to the source. This wave field can be modelled in terms of combinations of simple sound sources. If required, the source can be accounted for in the wave field by considering the initial, time-dependent conditions.

With aerodynamic sound, the sources of sound are not so readily identifiable and the regions of interest can be either within the fluid flow itself or external to it. When the regions of interest are within the fluid flow, they contain sources of sound energy because the sources are continuously being generated or convected with the flow (e.g. turbulence, vortices, etc.). These aerodynamic sources therefore have to be included in the wave equation for any subsequent analysis of the sound waves in order that they can

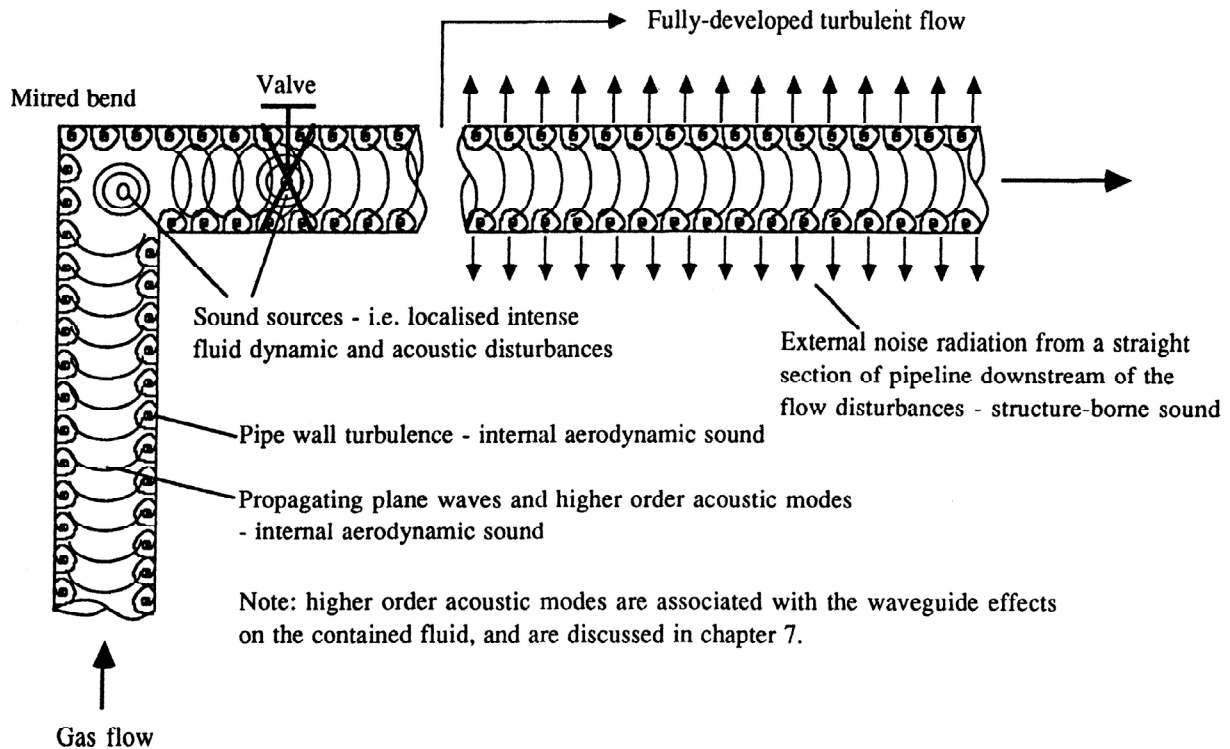


Fig. 2.1. A schematic model of internal aerodynamic sound and external structure-borne sound in a gas pipeline.

be correctly identified. The wave equation is now inhomogeneous (because it includes these source terms) and its solution is somewhat different to that of the homogeneous wave equation in that it now describes both the source and the wave fields.

It is very important to be aware of and to understand the difference between the homogeneous and the inhomogeneous wave equations for the propagation of sound waves in fluids. The vast majority of engineering noise and vibration control relates to sources which can be readily identified, and regions of interest which are outside the source region – in these cases the homogeneous wave equation is sufficient to describe the wave field and the subsequent noise radiation. Most machinery noise, for instance, is associated with the vibration of solid bodies. Engineers should, however, be aware of the existence of the inhomogeneous wave equation and of the instances when it has to be used in place of the more familiar (and easier to solve!) homogeneous wave equation.

A good industrial example that combines both types of noise generation mechanisms is high speed gas flow in a pipeline. Such pipelines are typically found in oil refineries and liquid-natural-gas plants. A typical section of pipeline is illustrated in Figure 2.1. Inside the pipe there are pressure fluctuations which are caused by turbulence, sound waves generated at the flow discontinuities (e.g. bends, valves, etc.), and vortices which are convected downstream of some bluff body such as a butterfly valve splitter-plate. These pressure fluctuations result in internal pipe flow noise which is aerodynamic in

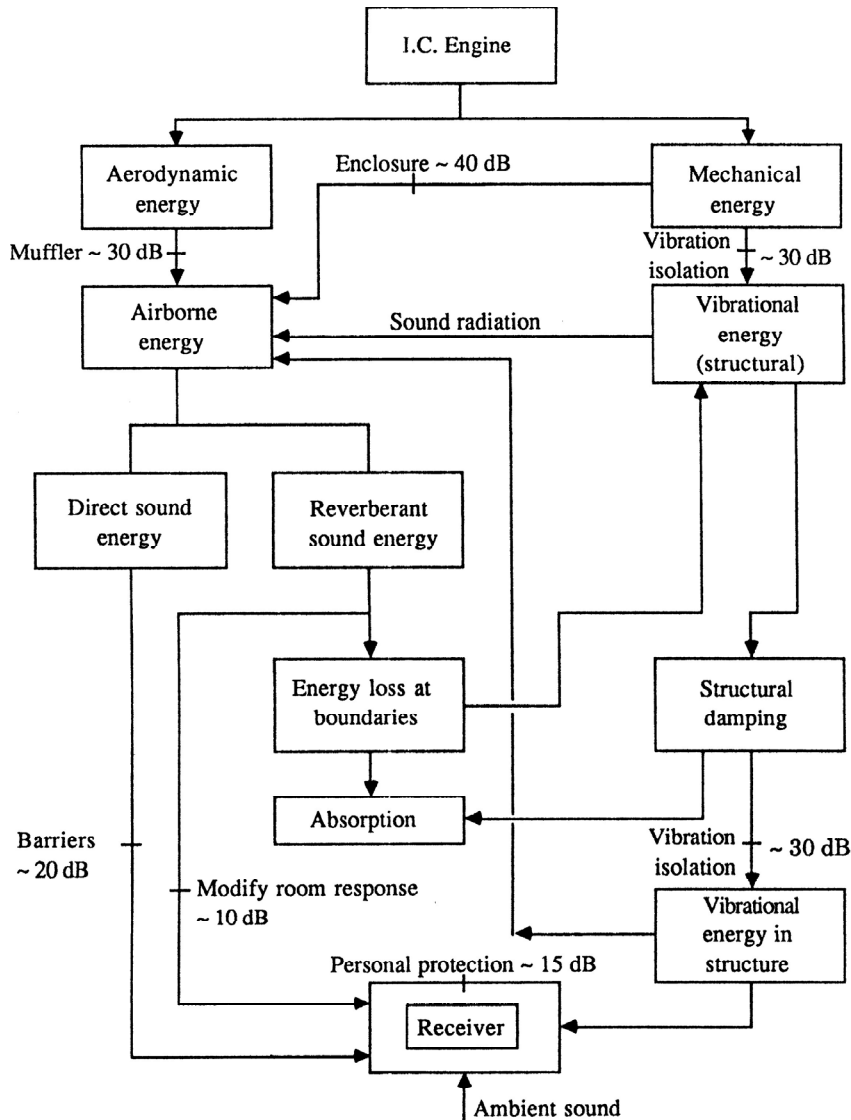


Fig. 2.2. Typical noise and vibrations paths for a machinery source. (Adapted from Pickles^{2,10}.)

nature – the source of sound is distributed along the whole length of the inside of the pipe. If an analysis of the sound sources within the pipe is required, the inhomogeneous wave equation would have to be used. This internal aerodynamic noise excites the structure internally and the vibrating structure subsequently radiates noise to the surrounding external medium. The source of sound is not in the region of space under analysis for the external noise radiation, and this problem can therefore be handled with the homogeneous wave equation. A knowledge of the internal source field (the wall pressure fluctuations) allows for a prediction of the external sound radiation; the converse is, however, not true. A point which is sometimes overlooked is that a description of the external wave field does not contain sufficient information for the source to be identified, but, once the source has been identified and described, the sound field can be predicted. Pipe flow noise is discussed in chapter 7 as a case study.

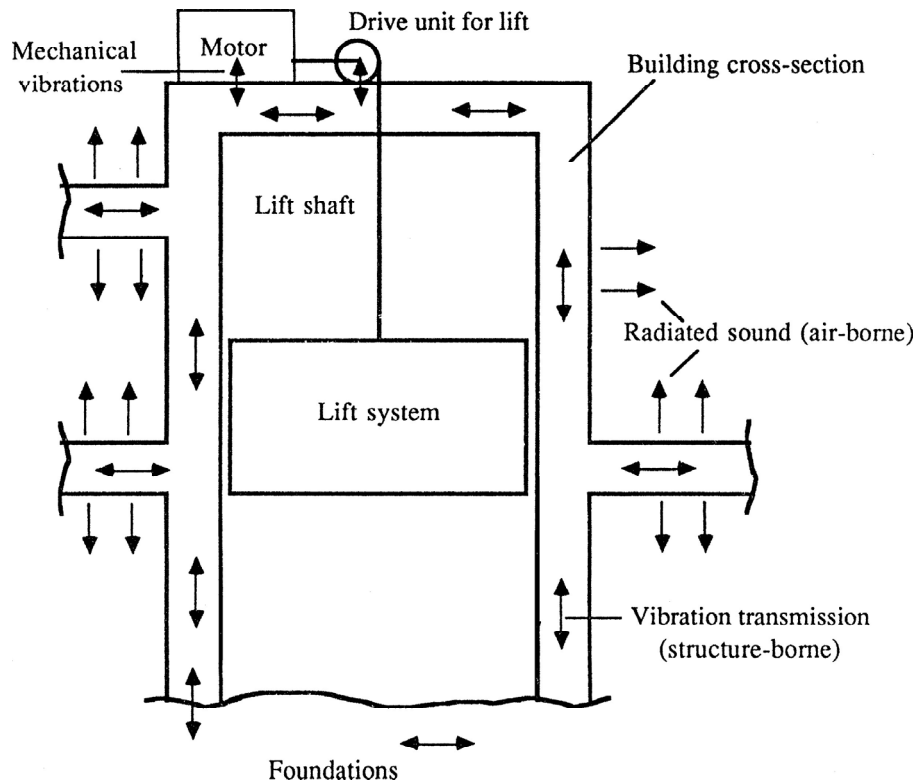


Fig. 2.3. Schematic illustration of structure-borne sound in a building.

Typical machinery noise control problems in industry involve (i) a source, (ii) a path, and (iii) a receiver. There is always interaction and feedback between the three, and there are generally several possible noise and vibration energy transmission paths for a typical machinery noise source. An internal combustion engine, for instance, generates both aerodynamic and mechanical energy, each with several possible transmission paths. This is illustrated schematically in Figure 2.2. The two sources of sound energy are (i) the aerodynamic energy associated with the combustion process and the exhaust system, and (ii) mechanical vibration energy associated with the various functional requirements of the engine. Source modification to reduce the aerodynamic noise component would require changes in the combustion process itself or in the design of the exhaust system. Source modification to reduce the mechanical vibration energy would require a re-design of the moving parts of the engine itself. Various options are open for the reduction of path noise. These include muffling the exhaust noise, structural modification such as adding mass, stiffness or damping to the various radiating panels, providing anti-vibration mounts, enclosing the engine, and providing acoustic barriers. Finally, the receiver could be provided with personal protection such as an enclosure or hearing protectors.

A specific example of structure-borne sound is the vibration and stop-start shocks that can emanate from a lift if it is not properly isolated. This is illustrated in Figure 2.3. In cases such as these, the vibrations are transmitted throughout the building – the waves are carried for large distances without being significantly attenuated. There are

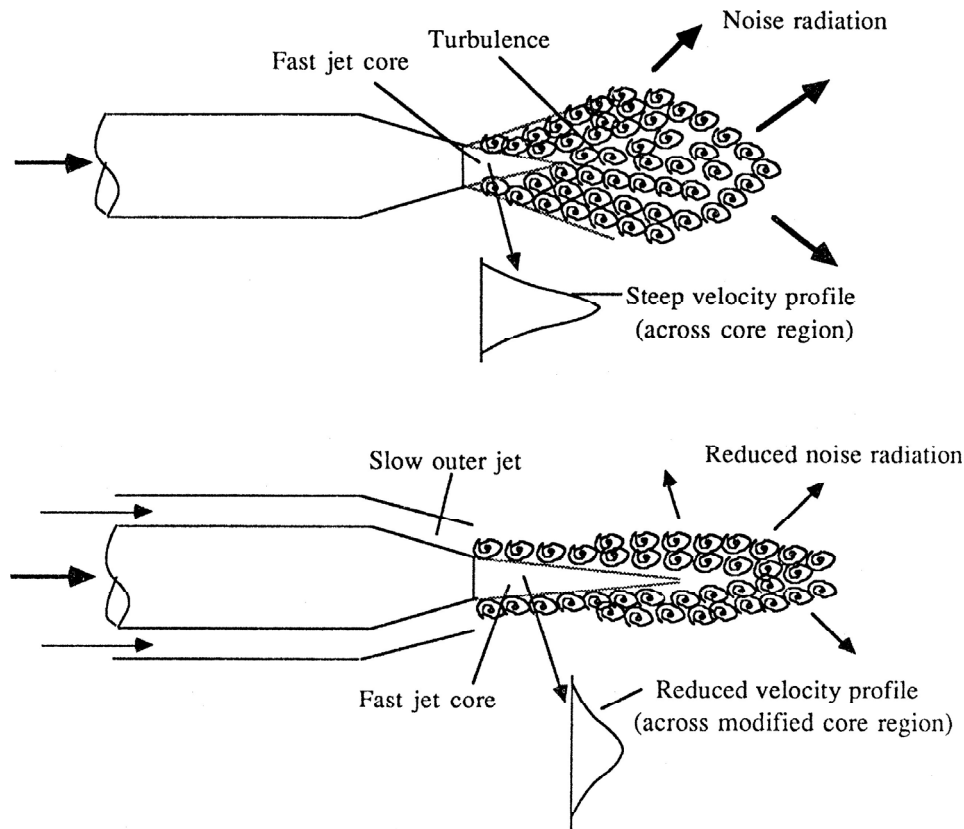


Fig. 2.4. Schematic illustration of aerodynamic sound emanating from a jet nozzle.

no sources of sound in the ambient air in the building, and any acoustic analysis would only require usage of the homogeneous wave equation. The problem could be overcome by isolating the winding machinery from the rest of the structure or by separating the lift shaft and the winding machinery from the remainder of the building.

A specific example of aerodynamic noise is the formation of turbulence in the mixing region at the exhaust of a jet nozzle such as the nozzle of a jet used for cleaning machine components with compressed air. The jet noise increases with flow velocity and the strength of the turbulence is related to the relative speed of the jet in relation to the ambient air. By introducing a secondary, low velocity, air-stream, as illustrated in Figure 2.4, and thus reducing the velocity profile across the jet, significant reductions in radiated noise levels can be achieved. Hence, compound nozzles are sometimes used in industry – here, the velocity of the core jet remains the same but its noise radiating characteristics are reduced by the introduction of a slower outer stream.

2.2 The homogeneous acoustic wave equation – a classical analysis

Three methods are available for approaching problems in acoustics. They are (i) wave acoustics, (ii) ray acoustics and (iii) energy acoustics.

Wave acoustics is a description of wave propagation using either molecular or particulate models. The general preference is for the particulate model, a particle being a fluid volume large enough to contain millions of molecules and small enough such that density, pressure and temperature are constant. Ray acoustics is a description of wave propagation over large distances, e.g. the atmosphere. Families of rays are used to describe the propagation of sound waves and inhomogeneities such as temperature gradients or wind have to be accounted for. Over large distances, the ray tracing procedures are preferred because they approximate and simplify the exact wave approach. Finally, energy acoustics describes the propagation of sound waves in terms of the transfer of energy of various statistical parameters where techniques referred to as statistical energy analysis (or S.E.A.) are used.

The wave acoustics approach is probably the most fundamental and important approach to the study of all disciplines of acoustics. The ray acoustics approach generally relates to outdoor or underwater sound propagation over large distances and is therefore not directly relevant to industrial noise and vibration control. The S.E.A. approach is fast becoming popular for quick and effective answers to complex industrial noise and vibration problems. The wave acoustics approach will thus be adopted for the better part of this book, and this chapter is devoted to some of the more important fundamental principles of sound waves. The subject of ray acoustics is not discussed in this book, but the concepts and applications of statistical energy analysis techniques are discussed in chapter 6.

Sound waves in non-viscous (inviscid) fluids are simply longitudinal waves and adjacent regions of compression and rarefaction are set up – i.e. the particles oscillate to and fro in the wave propagation direction, hence the acoustic particle velocity is in the same direction as the phase velocity. The pressure change that is produced as the fluid compresses and expands is the source of the restoring force for the oscillatory motion. There are four variables that are of direct relevance to the study of sound waves. They are pressure, P , velocity, $\vec{U}$, density, ρ , and temperature, T . Pressure, density and temperature are scalar quantities whilst velocity is a vector quantity (i.e. an arrow over a symbol denotes a vector quantity). Each of the four variables has a mean and a fluctuating component. Thus,

$$P(\vec{x}, t) = P_0(\vec{x}) + p(\vec{x}, t), \quad (2.1a)$$

$$\vec{U}(\vec{x}, t) = \vec{U}_0(\vec{x}) + \vec{u}(\vec{x}, t), \quad (2.1b)$$

$$\rho(\vec{x}, t) = \rho_0(\vec{x}) + \rho'(\vec{x}, t), \quad (2.1c)$$

$$T(\vec{x}, t) = T_0(\vec{x}) + T'(\vec{x}, t). \quad (2.1d)$$

The wave equation can thus be set up in terms of any one of these four variables. In acoustics, it is the pressure fluctuations, $p(\vec{x}, t)$, that are of primary concern – i.e. noise radiation is a fluctuating pressure. Thus it is common for acousticians to solve the wave equation in terms of the pressure as a dependent variable. It is, however, quite valid to solve the wave equation in terms of any of the other three variables. Also,

generally, $\vec{U}_0(\vec{x})$ is zero (i.e. the ambient fluid is stationary) and therefore $\vec{U}(\vec{x}, t) = \vec{u}(\vec{x}, t)$.

As for wave propagation in solids, several simplifying assumptions need to be made. They are:

- (1) the fluid is an ideal gas;
- (2) the fluid is perfectly elastic – i.e. Hooke's law holds;
- (3) the fluid is homogeneous and isotropic;
- (4) the fluid is inviscid – i.e. viscous-damping and heat conduction terms are neglected;
- (5) the wave propagation through the fluid media is adiabatic and reversible;
- (6) gravitational effects are neglected – i.e. P_0 and ρ_0 are assumed to be constant;
- (7) the fluctuations are assumed to be small – i.e. the system behaves linearly.

In order to develop the acoustic wave equation, equations describing the relationships between the various acoustic variables and the interactions between the restoring forces and the deformations of the fluid are required. The first such relationship is referred to as continuity or the conservation of mass; the second relationship is referred to as Euler's force equation or the conservation of momentum; and the third relationship is referred to as the thermodynamic equation of state. In practice, sound waves are generally three-dimensional. It is, however, convenient to commence with the derivation of the above equations in one dimension and to subsequently extend the results to three dimensions.

2.2.1 Conservation of mass

The equation of conservation of mass (continuity) provides a relationship between the density, $\rho(\vec{x}, t)$, and the particle velocity, $\vec{u}(\vec{x}, t)$ – i.e. it relates the fluid motion to its compression.

Consider the mass flow of particles in the x -direction through an elemental, fixed, control volume, dV , as illustrated in Figure 2.5. For mass to be conserved, the time rate of change of the elemental mass has to equal the nett mass flow into the elemental volume. Because the flow is one-dimensional, the vector notation is temporarily dropped. It will be re-introduced later on when the equations are extended to three-dimensional flow. Note that

$$\vec{u} = u_x \vec{i} + u_y \vec{j} + u_z \vec{k},$$

where u_x , u_y and u_z are the particle velocities in the x -, y - and z -directions, respectively. For flow in the x -direction only:

- (i) the elemental mass is $\rho A dx$ (where $A = dy dz$);
- (ii) the mass flow into the elemental volume is $(\rho u A)_x$;
- (iii) the mass flow out of the elemental volume is $(\rho u A)_{x+dx}$.

For the conservation of mass,

$$\frac{\partial(\rho A dx)}{\partial t} = (\rho u A)_x - (\rho u A)_{x+dx}. \quad (2.2)$$

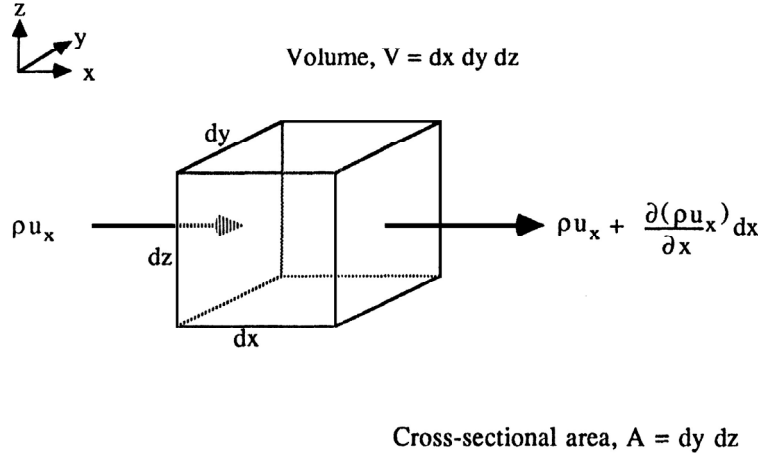


Fig. 2.5. Mass flow of particles in the x -direction through an elemental, fixed, control volume.

Using a Taylor series expansion,

$$\frac{\partial(\rho A \, dx)}{\partial t} = \left\{ (\rho u A)_x - (\rho u A)_x - \frac{\partial(\rho u A)_x}{\partial x} dx \right\}. \quad (2.3)$$

Hence,

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} = 0. \quad (2.4)$$

Equation (2.4) represents the one-dimensional conservation of mass in the x -direction. It can be extended to three dimensions, and the three-dimensional equation of conservation of mass is therefore

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \rho \vec{u} = 0, \quad (2.5)$$

where $\vec{\nabla}$ is the divergence operator, i.e.

$$\vec{\nabla} = \left\{ \frac{\partial}{\partial x} \vec{i} + \frac{\partial}{\partial y} \vec{j} + \frac{\partial}{\partial z} \vec{k} \right\}. \quad (2.6)$$

Equation (2.5) is thus a vector representation for

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_x)}{\partial x} + \frac{\partial(\rho u_y)}{\partial y} + \frac{\partial(\rho u_z)}{\partial z} = 0. \quad (2.7)$$

The equation of conservation of mass (equations 2.5 or 2.7) is a scalar quantity. It is also non-linear because the mass flow terms involve products of two small fluctuating components ($\vec{u}$ and ρ'). These terms have second-order effects as far as the propagation of sound waves is concerned – i.e. the equation can be linearised. Substituting for $\rho(\vec{x}, t) = \rho_0(\vec{x}) + \rho'(\vec{x}, t)$ into equation (2.7) and deleting second- and higher-order terms yields

$$\frac{\partial \rho'}{\partial t} + \rho_0 \frac{\partial u_x}{\partial x} + \rho_0 \frac{\partial u_y}{\partial y} + \rho_0 \frac{\partial u_z}{\partial z} = 0, \quad (2.8)$$

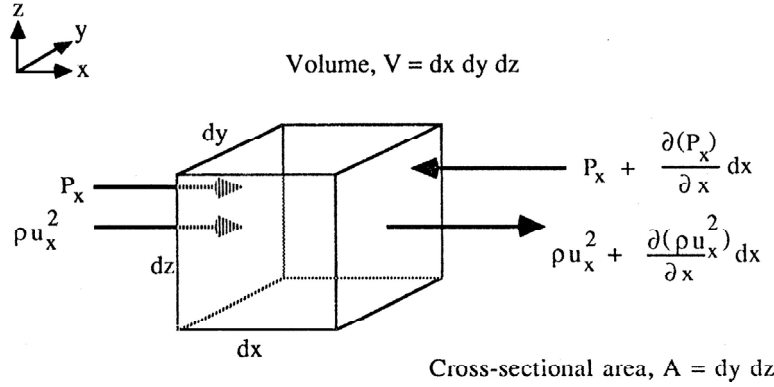


Fig. 2.6. Momentum balance in the x -direction for an elemental, fixed, control volume.

or

$$\frac{\partial \rho'}{\partial t} + \rho_0 \vec{\nabla} \cdot \vec{u} = 0. \quad (2.9)$$

Equation (2.9) is the linearised equation of conservation of mass (continuity).

2.2.2 Conservation of momentum

The equation of conservation of momentum provides a relationship between the pressure, $P(\vec{x}, t)$, the density, $\rho(\vec{x}, t)$, and the particle velocity, $\vec{u}(\vec{x}, t)$. It can be obtained either by observing the stated law of conservation of momentum with respect to an elemental, fixed, control volume, dV , in space, or by a direct application of Newton's second law with respect to the fluid particles that move through the elemental, fixed, control volume. To a purist both procedures are identical! It is, however, instructive to consider them both.

Consider the first approach – consider the momentum flow through an elemental, fixed, control volume, dV , as illustrated in Figure 2.6. For momentum to be conserved the time rate of change of momentum contained in the fixed volume plus the nett rate of flow of momentum through the surfaces of the volume are equal to the sum of all the forces acting on the volume. Once again, because the flow is one-dimensional, the vector notation is temporarily dropped. Also, body forces are neglected – i.e. only pressure forces act on the body. For flow in the x -direction only:

- (i) the momentum of the control volume is $\rho u_x A dx$;
- (ii) the momentum flow into the control volume is $(\rho u^2 A)_x$;
- (iii) the momentum flow out of the control volume is $(\rho u^2 A)_{x+dx}$;
- (iv) the force at position x is $(PA)_x$;
- (v) the force at position $x + dx$ is $-(PA)_{x+dx}$.

For the conservation of momentum

$$\frac{\partial(\rho u_x A) dx}{\partial t} = (\rho u^2 A)_x - (\rho u^2 A)_{x+dx} + (PA)_x - (PA)_{x+dx}. \quad (2.10)$$

If a Taylor series expansion is used for $(\rho u^2 A)_{x+dx}$ and $(PA)_{x+dx}$, equation (2.10) simplifies to

$$\frac{\partial(\rho u_x)}{\partial t} = -\frac{\partial(\rho u_x^2)}{\partial x} - \frac{\partial P}{\partial x}. \quad (2.11)$$

Equation (2.11) can be re-arranged as

$$\rho \frac{\partial u_x}{\partial t} + u_x \left\{ \frac{\partial \rho}{\partial t} + u_x \frac{\partial \rho}{\partial x} + \rho \frac{\partial u_x}{\partial x} \right\} + \rho u_x \frac{\partial u_x}{\partial x} + \frac{\partial P}{\partial x} = 0, \quad (2.12)$$

where the term in brackets is the continuity equation. Thus, equation (2.12) simplifies to

$$\rho \frac{\partial u_x}{\partial t} + \rho u_x \frac{\partial u_x}{\partial x} + \frac{\partial P}{\partial x} = 0. \quad (2.13)$$

Equation (2.13) represents the one-dimensional conservation of momentum in the x -direction. Similar expressions can be obtained for the y - and z -directions. The three-dimensional equation of conservation of momentum is therefore obtained by introducing the divergence operator, $\vec{\nabla}$. It is

$$\rho \left\{ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla}) \vec{u} \right\} + \vec{\nabla} P = 0. \quad (2.14)$$

This equation is the non-linear, inviscid momentum equation or Euler's equation.

Equation (2.14) can also be obtained by a direct application of Newton's second law with respect to the fluid particles that move through the elemental, fixed, control volume. The control volume in Figure 2.6 contains a mass, dm , of fluid at any instant in time. The nett force on the volume element is

$$d\vec{f} = \vec{a} dm, \quad (2.15)$$

from Newton's second law. It is very important to recognise that the acceleration, $\vec{a}$, is the rate of change of velocity of a given fluid particle as it moves about in space and it is not the rate of change of fluid velocity at a fixed point in space.

The particle velocity, $\vec{u}$, is a function of space and time. Thus, at some time t , a particle is at position (x, y, z) and it has a particle velocity $\vec{u}(x, y, z, t)$. At some further time, $t + dt$ the particle is at position $(x + dx, y + dy, z + dz)$ and it has a particle velocity $\vec{u}(x + dx, y + dy, z + dz, t + dt)$. The particle acceleration is

$$\vec{a} = \lim_{dt \rightarrow 0} \frac{\vec{u}(x + dx, y + dy, z + dz, t + dt) - \vec{u}(x, y, z, t)}{dt}. \quad (2.16)$$

The particle velocity at time $t + dt$ can be re-expressed as $\vec{u}(x + u_x dt, y + u_y dt, z + u_z dt, t + dt)$, where $dx = u_x dt$, $dy = u_y dt$, and $dz = u_z dt$. Thus by re-expressing

the particle velocity at time $t + dt$ and using a Taylor series expansion

$$\begin{aligned}\vec{u}(x + dx, y + dy, z + dz, t + dt) = & \vec{u}(x, y, z, t) + \frac{\partial \vec{u}}{\partial x} u_x dt + \frac{\partial \vec{u}}{\partial y} u_y dt \\ & + \frac{\partial \vec{u}}{\partial z} u_z dt + \frac{\partial \vec{u}}{\partial t} dt.\end{aligned}\quad (2.17)$$

Hence the particle acceleration is

$$\vec{a} = \frac{\partial \vec{u}}{\partial t} + u_x \frac{\partial \vec{u}}{\partial x} + u_y \frac{\partial \vec{u}}{\partial y} + u_z \frac{\partial \vec{u}}{\partial z}.\quad (2.18)$$

Equation (2.18) can be re-expressed in vector notation by using the vector operation $\vec{u} \cdot \vec{\nabla}$. Hence,

$$\vec{a} = \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u},\quad (2.19)$$

where

$$\vec{u} \cdot \vec{\nabla} = u_x \frac{\partial}{\partial x} + u_y \frac{\partial}{\partial y} + u_z \frac{\partial}{\partial z}.\quad (2.20)$$

Equation (2.19) is the total acceleration of a fluid particle in an Eulerian frame of reference. It has both a convective part and a local time rate of change when the flow is unsteady.

Neglecting viscosity, the nett force on the elemental fluid volume in the x -direction is

$$d\vec{f} = \left\{ P - \left(P + \frac{\partial P}{\partial x} \right) dx \right\} dy dz = -\frac{\partial P}{\partial x} dV \vec{i}.\quad (2.21)$$

The complete, three-dimensional, vector force is thus

$$-\vec{\nabla} P dV = -\left\{ \frac{\partial P}{\partial x} \vec{i} + \frac{\partial P}{\partial y} \vec{j} + \frac{\partial P}{\partial z} \vec{k} \right\} dV.\quad (2.22)$$

Hence, from Newton's second law,

$$\rho dV \left\{ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} \right\} = -\vec{\nabla} P dV,\quad (2.23)$$

where $\rho dV = dm$. Thus,

$$\rho \left\{ \frac{\partial \vec{u}}{\partial t} + (\vec{u} \cdot \vec{\nabla})\vec{u} \right\} + \vec{\nabla} P = 0.\quad (2.24)$$

Equation (2.24) is identical to equation (2.14) – it is the non-linear, inviscid momentum or Euler's equation. Like the equation of conservation of mass, it can be simplified by linearisation – second- and higher-order terms can be neglected

for the propagation of sound waves. Substituting for $P(\vec{x}, t) = P_0(\vec{x}) + p(\vec{x}, t)$, and $\rho(\vec{x}, t) = \rho_0(\vec{x}) + \rho'(\vec{x}, t)$ and deleting the second- and higher-order terms yields

$$\rho_0 \left\{ \frac{\partial u_x}{\partial t} \vec{i} + \frac{\partial u_y}{\partial t} \vec{j} + \frac{\partial u_z}{\partial t} \vec{k} \right\} + \left\{ \frac{\partial p}{\partial x} \vec{i} + \frac{\partial p}{\partial y} \vec{j} + \frac{\partial p}{\partial z} \vec{k} \right\} = 0, \quad (2.25)$$

or

$$\rho_0 \frac{\partial \vec{u}}{\partial t} + \vec{\nabla} p = 0. \quad (2.26)$$

Equation (2.26) is the linear inviscid force equation (conservation of momentum). Like the linearised equation of conservation of mass, it is valid for small amplitude sound waves ($\sim < 140$ dB).

2.2.3 The thermodynamic equation of state

The thermodynamic equation of state relates the pressure, density and absolute temperature of a fluid. For a perfect fluid it is

$$P = \rho R T_k. \quad (2.27)$$

P is the absolute pressure, ρ is the density of the gas, R is the gas constant, and T_k is the absolute temperature. It should be noted that the universal gas constant, G , is equal to RM , where M is the molecular weight of the particular gas.

The propagation of sound waves in air does not generally produce any significant changes of thermal energy between particles, and the entropy of the gas is constant. In addition, the thermal conductivity of the gas is very small. The propagation of sound waves can therefore be assumed to be nearly adiabatic. This assumption is valid for linear (small amplitude) sound waves within the audio-frequency range. Any loss of thermal energy would result in an attenuation of the sound waves with time and distance. Thus the adiabatic equation of state for a perfect gas is

$$P/P_0 = (\rho/\rho_0)^\gamma, \quad (2.28)$$

where γ is the ratio of specific heats.

For gases which are not perfect, an adiabatic equation of state can be obtained from a Taylor series expansion of an experimentally determined isentropic relationship between the pressure and density fluctuations. Provided that those fluctuations are assumed to be small, a linear relationship can be established between them (Kinsler *et al.*^{2,1}). It is

$$p(\vec{x}, t) = B \left\{ \frac{\rho'}{\rho_0} \right\}, \quad (2.29)$$

where B is the adiabatic bulk modulus – i.e.

$$B = \rho_0 \left\{ \frac{\partial P}{\partial \rho} \right\}_{\rho_0}. \quad (2.30)$$

The partial derivative in equation (2.30) is evaluated for an adiabatic process – i.e. it establishes the adiabatic compression and expansion of the gas about its mean density.

2.2.4 The linearised acoustic wave equation

Equation (2.9) (conservation of mass), equation (2.26) (conservation of momentum) and equations (2.29)–(2.30) can now be combined into a single equation with one dependent variable. The dependent variable of interest in acoustics is the fluctuating pressure.

The time derivative of the equation of conservation of mass is

$$\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \frac{\partial(\vec{\nabla} \cdot \vec{u})}{\partial t} = 0, \quad (2.31a)$$

or

$$\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} = 0. \quad (2.31b)$$

The divergence of the equation of conservation of momentum is

$$\rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} + \nabla^2 p = 0. \quad (2.32)$$

Subtracting equation (2.31) from (2.32) yields

$$\nabla^2 p = \frac{\partial^2 \rho'}{\partial t^2}. \quad (2.33)$$

Equation (2.29) can now be substituted into equation (2.33) to eliminate ρ' . Thus,

$$\nabla^2 p = \frac{\rho_0}{B} \frac{\partial^2 p}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}. \quad (2.34)$$

Equation (2.34) is the linearised, homogeneous acoustic wave equation with the fluctuating pressure, $p(\vec{x}, t)$, as the dependent variable. The constant, c , is the velocity of propagation of the wave and is therefore the speed of sound. It is

$$c = \left(\frac{B}{\rho_0} \right)^{1/2} = \left\{ \left(\frac{\partial P}{\partial \rho} \right)_{\rho_0} \right\}^{1/2}. \quad (2.35)$$

Some useful approximations can now be made in relation to the speed of sound by assuming that the sound propagation medium is a perfect gas. From equation (2.28)

$$P = \frac{P_0 \rho^\gamma}{\rho_0^\gamma}, \quad (2.36)$$

thus

$$\frac{\partial P}{\partial \rho} = \frac{\gamma P}{\rho}. \quad (2.37)$$

Thus, the speed of sound is

$$c = \left(\frac{\gamma P}{\rho} \right)^{1/2}. \quad (2.38)$$

Also, by substituting equation (2.27) into equation (2.38) yields

$$c = (\gamma R T_k)^{1/2}. \quad (2.39)$$

For small fluctuations,

$$c \approx c_0 \approx \left(\frac{\gamma P_0}{\rho_0} \right)^{1/2}. \quad (2.40)$$

The absolute temperature, T_k , in equation (2.39) is in Kelvin, and c_0 is the speed of sound at atmospheric conditions. The gas constant for air is $0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The equations demonstrate that the wave speed is constant for a given pressure and medium.

2.2.5 The acoustic velocity potential

Using vector theory, it can be demonstrated that the acoustic particle velocity, $\vec{u}$, is irrotational. From vector theory it can be shown that if a vector function is the gradient of a scalar function, its curl is the zero vector – i.e. for some scalar function ϕ , $\vec{\nabla} \times \vec{\nabla} \phi = 0$.

The curl of the momentum equation (equation 2.26) is

$$\rho_0 \frac{\partial(\vec{\nabla} \times \vec{u})}{\partial t} + (\vec{\nabla} \times \vec{\nabla} p) = 0, \quad (2.41)$$

and

$$\vec{\nabla} \times \vec{u} = 0, \quad (2.42)$$

since (i) p is a scalar quantity, and (ii) the constant associated with the time integral is zero. The constant has to be zero since the acoustic quantities would disappear if an acoustic disturbance was not present. Equation (2.42) therefore confirms that the acoustic particle velocity, $\vec{u}$, is irrotational. Now, because the acoustic particle velocity is irrotational, it can be expressed as the gradient of a scalar function (if a vector function is the gradient of a scalar function, its curl is the zero vector, i.e. it is irrotational). Hence the introduction of the concept of the acoustic velocity potential, ϕ , and

$$\vec{u} = \vec{\nabla} \phi. \quad (2.43)$$

The above result is a very important one. Its physical interpretation is that the acoustical excitation of an inviscid fluid does not produce rotational flow – i.e. there are no boundary layers, shear stresses or turbulence generated. When the effects of viscosity cannot be completely neglected, the particle velocity is not curl free everywhere and there is some rotational flow. For example, when a flat plate is mechanically excited in the presence of a mean fluid flow, in addition to the plate radiating sound, vorticity is

generated on the surface of the plate (Soria and Norton^{2,2}). These rotational effects are generally confined to the vicinity of boundaries but situations can arise where they exert some influence on sound propagation. These concepts will be discussed later on in this chapter when discussing aerodynamic noise. For the present purposes it is sufficient to assume that, when the mean fluid flow is zero, the particle velocity is irrotational.

Substituting the equation for the velocity potential (equation 2.43) into the momentum equation (equation 2.26) yields

$$\vec{\nabla} \left\{ \rho_0 \frac{\partial \phi}{\partial t} + p \right\} = 0. \quad (2.44)$$

The acoustic quantities inside the brackets have to vanish if there is no acoustic disturbance present, thus the integration constant has to be zero – i.e.

$$p = -\rho_0 \frac{\partial \phi}{\partial t}. \quad (2.45)$$

Substituting for p into the wave equation (equation 2.34) yields

$$\nabla^2 \left\{ \frac{\partial \phi}{\partial t} \right\} = \frac{1}{c^2} \left\{ \frac{\partial^3 \phi}{\partial t^3} \right\}. \quad (2.46)$$

Thus

$$\nabla^2 \phi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}, \quad (2.47)$$

and ϕ satisfies the wave equation.

It is useful to point out that, when rotational flow components (e.g. near boundaries) have to be accounted for, a vector velocity potential, $\vec{\psi}$, can be introduced such that

$$\vec{U} = \vec{\nabla} \phi + \vec{\nabla} \times \vec{\psi}. \quad (2.48)$$

The first term represents the irrotational fluctuating flow and the second term represents the total (mean plus fluctuating) rotational component (see section 2.4.4).

The velocity potential concept is commonly used in fluid dynamics where solutions for the particle velocity are usually sought. The particle velocity for the three-dimensional wave equation is a vector quantity, and the introduction of the scalar velocity potential allows for the wave equation to be solved in terms of a scalar. Acousticians generally use the pressure variable, which is also a scalar quantity. The particle velocity can be related to the acoustic pressure fluctuation via the momentum equation (equation 2.26). Hence,

$$\vec{u} = \vec{\nabla} \phi = - \int \frac{1}{\rho_0} \vec{\nabla} p \, dt. \quad (2.49)$$

2.2.6 The propagation of plane sound waves

Consider a plane, one-dimensional, sound wave propagating in the x -direction. The one-dimensional homogeneous wave equation is

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}, \quad (2.50)$$

where $p = p(\vec{x}, t)$. As was the case in chapter 1, it is convenient to represent the solution to the wave equation in terms of complex, harmonic functions. The general solution involves waves travelling in both the positive and negative x -directions. It is

$$p(\vec{x}, t) = \mathbf{A}_1 e^{i(\omega t - kx)} + \mathbf{A}_2 e^{i(\omega t + kx)}. \quad (2.51)$$

The complex particle velocity, $\vec{u}(\vec{x}, t)$, and the complex velocity potential, $\phi(\vec{x}, t)$, can be obtained by substituting equation (2.51) into equation (2.49). They are

$$\vec{u}(\vec{x}, t) = \left\{ \frac{\mathbf{A}_1}{\rho_0 c} e^{i(\omega t - kx)} - \frac{\mathbf{A}_2}{\rho_0 c} e^{i(\omega t + kx)} \right\} \vec{i}, \quad (2.52)$$

and

$$\phi(\vec{x}, t) = -\frac{\mathbf{A}_1}{i\rho_0 \omega} e^{i(\omega t - kx)} - \frac{\mathbf{A}_2}{i\rho_0 \omega} e^{i(\omega t + kx)}. \quad (2.53)$$

The particle velocity is a vector quantity and the term outside the brackets in equation (2.52) is the unit vector, $\vec{i}$ (i.e. one should not be confused between the complex number i and the unit vector $\vec{i}$). Also, because complex numbers have been introduced, the acoustic variables are now represented as complex numbers. Now, if one lets

$$\mathbf{p}_+ = \mathbf{A}_1 e^{i(\omega t - kx)}, \quad \text{and} \quad \mathbf{p}_- = \mathbf{A}_2 e^{i(\omega t + kx)}, \quad (2.54)$$

then,

$$\vec{u}(\vec{x}, t) = \left\{ \frac{\mathbf{p}_+}{\rho_0 c} - \frac{\mathbf{p}_-}{\rho_0 c} \right\} \vec{i}, \quad (2.55)$$

and

$$\phi(\vec{x}, t) = -\frac{\mathbf{p}_+}{i\rho_0 \omega} - \frac{\mathbf{p}_-}{i\rho_0 \omega}. \quad (2.56)$$

Once again, the term outside the brackets in equation (2.55) is the unit vector, $\vec{i}$.

In acoustics it is common to consider the waves travelling in the positive direction when one is concerned with the propagation of sound waves away from a source into some free field. Both positive and negative travelling waves have to be considered when considering the propagation of sound waves in confined spaces, e.g. ducts, rooms etc. Equation (2.55) illustrates an important point – for a plane, harmonic wave travelling in either the positive or the negative x -directions, the acoustic pressure fluctuations are in phase with the particle velocities. The phase relationship between these two variables is

very important for a fundamental understanding of the propagation of different sound wave-types.

The concepts of impedance were introduced in chapter 1. The ratio of the acoustic pressure fluctuations, $\mathbf{p}(\vec{x}, t)$ in a medium to the associated particle velocity, $\vec{\mathbf{u}}(\vec{x}, t)$ is termed the specific acoustic impedance, $\mathbf{Z}_a$, i.e.

$$\mathbf{Z}_a = \frac{\mathbf{p}(\vec{x}, t)}{\vec{\mathbf{u}}(\vec{x}, t)}. \quad (2.57)$$

For a plane sound wave,

$$\mathbf{Z}_a = Z_a = \pm \rho_0 c. \quad (2.58)$$

The specific acoustic impedance is generally complex – for wave types other than plane sound waves, the acoustic pressure fluctuations and the particle velocities are not always in phase, i.e. the waves diverge. The quantity $\rho_0 c$ is often called the characteristic impedance (resistance) of the medium. For air, at 20 °C and 1 atm, ρ_0 is $\sim 1.21 \text{ kg m}^{-3}$, c is $\sim 343 \text{ m s}^{-1}$ and $\rho_0 c$ is 415 Pa s m^{-1} .

2.2.7 Sound intensity, energy density and sound power

Three additional parameters that play an important role in acoustics are the sound intensity, the sound energy density and the radiated sound power.

The sound intensity is defined as the rate of flow of energy through a unit area which is normal to the direction of propagation. From basic dynamics, power = force $\times$ velocity. For an acoustic process, the instantaneous power is

$$\Pi = \vec{F} \cdot \vec{u}. \quad (2.59)$$

Here, $\vec{F}$ is the vector force acting on a particle and $\vec{u}$ is the associated particle velocity. The power per unit normal area is the instantaneous sound intensity vector, $\vec{I}'$, where

$$\vec{I}' = p \vec{u}. \quad (2.60)$$

The time average of the instantaneous power flow through a unit area is the mean intensity vector, $\vec{I}$, where

$$\vec{I} = \frac{1}{T} \int_0^T p \vec{u} dt = \frac{1}{2} \text{Re}[\mathbf{p} \vec{\mathbf{u}}^*]. \quad (2.61)$$

The second representation of equation (2.61) is used when the acoustic pressure fluctuations and the particle velocities are treated as complex, harmonic variables.

For a plane wave travelling in the positive x -direction

$$p(\vec{x}, t) = \text{Re}[\mathbf{A}_1 e^{i(\omega t - kx)}] = \hat{p} \cos(\omega t - kx), \quad (2.62)$$

and

$$u(x, t) = \operatorname{Re} \left[\frac{\mathbf{A}_1}{\rho_0 c} e^{i(\omega t - kx)} \right] = \frac{\hat{p}}{\rho_0 c} \cos(\omega t - kx). \quad (2.63)$$

The mean sound intensity, I , is obtained by substitution into equation (2.61) and evaluating the integral. Hence,

$$I = \frac{\hat{p}^2}{2\rho_0 c} = \frac{p_{\text{rms}}^2}{\rho_0 c}. \quad (2.64)$$

The vector notation is omitted in the above equations for particle velocity and intensity because of the one-dimensional nature of the travelling wave.

The sound energy density is the sound energy per unit volume of space. The energy transported by a sound wave comprises kinetic energy of the moving particles and potential energy of the compressed fluid. Using the plane wave approximation, a simple relationship can be derived for the sound energy density. It will be seen later on that this approximation is valid because most sound waves approximate to one-dimensional plane waves at large distances from the source.

Consider a fluid element with an undisturbed volume V_0 . Its kinetic energy per unit volume is

$$\frac{T}{V_0} = \frac{1}{2} \rho_0 u^2 = \frac{p^2}{2c^2 \rho_0}, \quad (2.65)$$

since $u = p/(\rho_0 c)$. When the volume changes from V_0 to V_1 there is a change in potential energy. It is

$$U = - \int_{V_0}^{V_1} p \, dV, \quad (2.66)$$

where the negative sign indicates that a positive acoustic fluctuating pressure produces a decrease in the fluid volume. Now, since $\rho = m/V$,

$$d\rho = -\frac{m}{V^2} dV, \quad (2.67)$$

and combining with equation (2.37) yields

$$dV = -\frac{V}{\gamma P} dP. \quad (2.68)$$

For small changes in pressure and volume this approximates to

$$dV = -\frac{V_0}{\gamma P_0} dp. \quad (2.69)$$

Substituting into equation (2.66) and integrating from 0 to p yields the potential energy

per unit volume. It is

$$\frac{U}{V_0} = \frac{p^2}{2\gamma P_0} = \frac{p^2}{2c^2\rho_0}, \quad (2.70)$$

since $c^2 = \gamma P_0/\rho_0$.

The total sound energy per unit volume of space is the sum of the kinetic and potential energies per unit volume. Thus, the instantaneous sound energy density, D' , is

$$D' = \frac{T}{V_0} + \frac{U}{V_0}, \quad (2.71a)$$

$$= \frac{p^2}{\rho_0 c^2}. \quad (2.71b)$$

The mean energy density, D , is obtained by integrating equation (2.71b) with respect to time. It is

$$D = \frac{\hat{p}^2}{2\rho_0 c^2} = \frac{p_{\text{rms}}^2}{\rho_0 c^2}. \quad (2.72)$$

Thus,

$$D = \frac{I}{c}. \quad (2.73)$$

The fluctuating acoustic pressure and the sound intensity decrease with distance from the source (this will become apparent when spherical sound waves are discussed in the next sub-section). It will also be shown that they are a function of the environment – i.e. the reverberant effects of a room enclosure will alter the intensity and sound pressure due to an acoustic source in the room. The sound power of an acoustic source is independent of distance and essentially independent of location – in some instances, the effects of reflecting surfaces have to be accounted for. The sound power, Π , is the integral of the intensity at some point in space over a surface area which is perpendicular to the flow of sound energy. It is

$$\Pi = \int_S \vec{I} \cdot d\vec{S}. \quad (2.74)$$

2.3 Fundamental acoustic source models

This section is devoted to the description of a range of fundamental acoustic source models. Most noise sources that are of concern to engineers (e.g. vehicles, construction equipment, industrial machinery, appliances, flow-duct systems etc.) can be modelled in terms of simple sources such as spheres, pistons in an infinite baffle, cylinders or combinations thereof. It is therefore instructive to analyse the characteristics of some of

these idealised sound sources. (Additional work on monopoles, dipoles and quadrupoles can be found in Norton and Pan^{2,3}.)

2.3.1 Monopoles – simple spherical sound waves

A monopole is a single, spherical sound source which radiates sound waves that are only a function of the radial distance, r , from the source. The wave equation has therefore got to be set up in spherical co-ordinates before any analysis can proceed. In sub-section 2.2.6, the wave equation was solved in terms of the pressure variable, and related to the particle velocity and the velocity potential. For a change, the wave equation will be solved here in terms of the velocity potential, and subsequently related to the pressure fluctuations and the particle velocity.

The one-dimensional spherical wave equation (in terms of the acoustic velocity potential) is

$$\frac{\partial^2(r\phi(r, t))}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2(r\phi(r, t))}{\partial t^2} = 0. \quad (2.75)$$

It represents an omni-directional wave which radiates outwards from or inwards towards a source. The product $r\phi(r, t)$ is treated as a single variable and the equation is of the same form as that for a plane wave. The general solution is

$$r\phi(r, t) = G_1(ct - r) + G_2(ct + r), \quad (2.76)$$

thus

$$\phi(r, t) = \frac{G_1(ct - r)}{r} + \frac{G_2(ct + r)}{r}. \quad (2.77)$$

The first term represents a spherical sound wave travelling radially outwards from the sound source, and the second term represents a spherical sound wave travelling towards the sound source.

In engineering noise and vibration analysis, one is generally only concerned with sound waves that travel away from a source. Hence, one is only concerned with the first part of the solution. In principle, the function G_1 can be any arbitrary function. It is, however, convenient and conventional to assume a complex harmonic solution of the form

$$\phi(r, t) = \frac{\mathbf{A}}{r} e^{i(\omega t - kr)}. \quad (2.78)$$

The term $\mathbf{A}$ is a constant which is determined by the boundary conditions specified at the surface of the monopole.

Consider an oscillating sphere of radius a , with a normal surface velocity $\mathbf{u}_a = U_a e^{i\omega t}$. From equation (2.43),

$$\frac{\partial \phi}{\partial r} = u_r. \quad (2.79)$$

Thus, by substituting equation (2.78) into equation (2.79) with $r = a$ and solving for the constant $\mathbf{A}$ yields

$$\mathbf{A} = -U_a \left\{ \frac{a^2}{1 + ika} \right\} e^{ika}. \quad (2.80)$$

Thus,

$$\Phi(r, t) = -\frac{U_a}{r} \left\{ \frac{a^2}{1 + ika} \right\} e^{i(\omega t - k(r-a))}. \quad (2.81)$$

The concept of a source strength, $Q(t)$, can now be introduced. The source strength of a monopole is defined as its surface area multiplied by its surface velocity. Thus,

$$Q(t) = 4\pi a^2 U_a e^{i\omega t} = Q_p e^{i\omega t}. \quad (2.82)$$

Hence, the complex representation of velocity potential is

$$\Phi(r, t) = -\frac{Q(t)}{4\pi r} \left\{ \frac{1}{1 + ika} \right\} e^{-ik(r-a)}. \quad (2.83)$$

The complex representation of the fluctuating acoustic pressure, $\mathbf{p}(r, t)$ and the particle velocity, $\mathbf{u}(r, t)$, can now be obtained from equation (2.49) (the vector notation is now dropped because of the omni-directional wave propagation) – i.e.

$$p(r, t) = -\rho_0 \{ \partial \Phi / \partial t \}, \quad \text{and} \quad u(r, t) = \partial \Phi / \partial r. \quad (2.84)$$

Thus,

$$\mathbf{p}(r, t) = -ik\rho_0 c \Phi(r, t) = \frac{Q(t)}{4\pi r} \left\{ \frac{ik\rho_0 c}{1 + ika} \right\} e^{-ik(r-a)}, \quad (2.85)$$

and

$$\mathbf{u}(r, t) = -\left\{ \frac{1 + ikr}{r} \right\} \Phi(r, t) = \frac{Q(t)}{4\pi r^2} \left\{ \frac{1 + ikr}{1 + ika} \right\} e^{-ik(r-a)}. \quad (2.86)$$

The specific acoustic impedance, $\mathbf{Z}_a$, can now be evaluated using equation (2.57). It is

$$\mathbf{Z}_a = \frac{i\rho_0 ckr}{1 + ikr} = \rho_0 c \left\{ \frac{k^2 r^2}{1 + k^2 r^2} + i \frac{kr}{1 + k^2 r^2} \right\}. \quad (2.87)$$

Unlike the specific acoustic impedance of a plane wave, the specific acoustic impedance of a spherical wave has both a resistive and a reactive component. When the resistive component dominates, the acoustic pressure fluctuations are in phase with the particle velocities; when the reactive component dominates, they are out of phase with each other. As with energy and power flow in structures (see section 1.7) one would expect that the in-phase components of the sound waves dominate the radiated sound power and that the out-of-phase components produce some near-field reactive exchange of sound energy. These phase relationships are therefore the basis of the far-field/near-field concepts which are so commonly used in noise control engineering.

The sound intensity of a spherical sound wave can now be estimated. The mean radiated intensity is a real quantity and it can be evaluated from equation (2.61). It is

$$I(r) = \frac{Q_p^2 k^2 \rho_0 c}{32\pi^2 r^2 (1 + k^2 a^2)} = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{16\pi^2 r^2 (1 + k^2 a^2)}. \quad (2.88)$$

The sound power radiated by the spherical source can now be evaluated from equation (2.74). It is

$$\Pi = 4\pi r^2 I(r) = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{4\pi (1 + k^2 a^2)}. \quad (2.89)$$

The radiated mean-square acoustic pressure fluctuations can be readily evaluated from equations (2.61) and (2.85). They are

$$p_{\text{rms}}^2 = \frac{1}{2} \text{Re}[\mathbf{p}(r, t) \mathbf{p}^*(r, t)] = \frac{Q_{\text{rms}}^2 k^2 (\rho_0 c)^2}{16\pi^2 r^2 (1 + k^2 a^2)} = I(r) \rho_0 c. \quad (2.90)$$

Thus the relationship between the radiated sound pressure and the mean sound intensity is the same as that for a plane wave (see equation 2.64). It is also important to note that the radiated sound pressure is proportional to the inverse square of the distance from the source.

Returning to equation (2.87), one can see that there are two limiting situations. They are (i) $kr \ll 1$, and (ii) $kr \gg 1$. When $kr \ll 1$,

$$\mathbf{u}(r, t) = \frac{\mathbf{Q}(t)}{4\pi r^2 (1 + ika)} e^{-ik(r-a)}, \quad (2.91)$$

and

$$\mathbf{Z}_a = ikr \rho_0 c. \quad (2.92)$$

Here, the acoustic pressure fluctuations and the particle velocities are out of phase, the impedance is reactive and no sound power is radiated from the source. When $kr \gg 1$,

$$\mathbf{u}(r, t) = \frac{i k \mathbf{Q}(t)}{4\pi r (1 + ika)} e^{-ik(r-a)}, \quad (2.93)$$

and

$$\mathbf{Z}_a = \rho_0 c. \quad (2.94)$$

Here, the acoustic pressure fluctuations and the particle velocities are in phase, the impedance is resistive (i.e. it is equal to the characteristic acoustic impedance) and sound power is radiated from the source. Combining equations (2.89) and (2.90), the radiated sound power can also be obtained in terms of the acoustic pressure fluctuations. It is

$$\Pi = \frac{4\pi r^2 p_{\text{rms}}^2}{\rho_0 c}. \quad (2.95)$$

Equations (2.89) and (2.95) are identical. The former expresses the radiated sound power of a monopole in terms of its source strength, wavenumber, and dimensions,

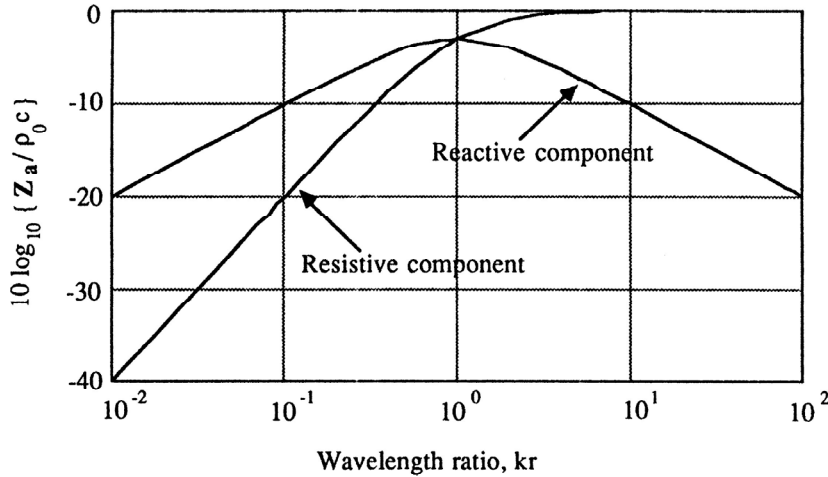


Fig. 2.7. The resistive and reactive components of the specific acoustic impedance of a spherical sound source.

whilst the latter expresses it in terms of the radiated sound pressure at some distance, r , from the source. The former expression is very useful for obtaining a physical understanding of the effects of the source parameters (source strength, wavenumber, and dimensions) on the radiated noise, whilst the latter allows for an estimate of the sound power from a simple measurement of the mean-square pressure fluctuations. A word of caution should be made regarding equation (2.95). It is only valid in the far-field – i.e. at sufficient distances from the source such that the pressure fluctuations and the particle velocities are in phase. It is also only valid in regions of free space where there are no environmental effects such as reverberation (reflected sound) on the measured levels.

It is now instructive to return once again to equation (2.87) to attempt to define a transition point between the two regions ($kr \ll 1$, and $kr \gg 1$). The resistive and reactive components of the specific acoustic impedance can be plotted as a function of kr . This information is presented in Figure 2.7.

The main characteristic of the far-field of a radiating sound source is that $kr \gg 1$ and that Z_a is essentially resistive. From Figure 2.7, the resistive component dominates for $kr \approx 10$. Now, λ is the wavelength of the sound source ($\lambda = 2\pi/k$), thus the transition radius is

$$r \approx 1.6\lambda. \quad (2.96)$$

It is very important to note that this transition point is not a constant but is a function of frequency – i.e. at higher frequencies the transition radius is shorter than at lower frequencies.

The concepts presented in this sub-section form an important basis for the modelling of sound sources. Many practical sound sources, including both vibrating bodies and aerodynamic sources, can be approximated as monopoles. If, for instance, the monopole source was of an aerodynamic nature rather than some vibrating body with a surface

velocity, the radiated sound power could then be related to the mean flow velocity – i.e. the mean addition or subtraction of mass from a source region. To do this, one has to re-consider equation (2.89). The source strength, Q_{rms} , scales as L^2U , where L is a typical dimension of the region of fluid flow, such as the width of a square duct, and U is the mean flow velocity. The characteristic frequency in the flow scales as U/L , and this has to equal the frequency of aerodynamic sound generation for dimensional consistency. Also, the source dimension, a , is assumed to be very much smaller than the geometrical dimension, L , i.e. $a \ll L$. Thus,

$$\Pi \approx \frac{L^4 U^2 \rho_0 c U^2}{4\pi L^2 c^2} \approx \frac{L^2 U^4 \rho_0}{4\pi c}. \quad (2.97)$$

Equation (2.97) suggests that the sound power radiated by an aerodynamically generated monopole scales with the fourth power of the flow velocity. This is a very important statement – one which has significant practical consequences. These practical consequences will be discussed later on in this chapter once some other sound source types have been studied, so that comparisons can be made. It will also be seen later on in this chapter that this scaling law only holds for sub-sonic flows – i.e. where the mean flow velocity is less than the speed of sound, c .

Some typical examples of monopole sound sources associated with vibrating structures are small electric motors, pumps and certain types of traffic noise (at medium distances away from a single vehicle – at larger distances the source can sometimes appear to be a dipole). Aerodynamic monopole sound sources include unsteady combustion from a furnace, sirens, pulsed jets and cavitation.

2.3.2 Dipoles

A dipole is a sound source model that is composed of two monopoles in close proximity to each other. They are also of equal source strength and oscillate 180° out of phase with each other. A nett fluctuating force is produced because of this out-of-phase oscillation. The velocity potential function for a dipole is a function of the polar angle, θ , in addition to the radius, r . It can be represented as a space derivative of the monopole velocity potential function. The analysis for dipoles with finite dimensions is somewhat lengthy and tedious. Numerous permutations are possible and the reader is referred to Pierce^{2.4} or Reynolds^{2.5} for further details. Many sound sources, including aerodynamic dipole sources, can be modelled as point dipoles – i.e. the dimensions of the dipole source are very small ($ka \ll 1$), and, with this assumption, the analysis is somewhat simplified. This section shall therefore be restricted to the special case of far-field sound radiation from point dipole sound sources. The analytical techniques involved here can be readily extended to the cases of both near-field and far-field sound radiation from finite dipole sources.

Consider two point monopoles in free space, separated by some small distance $2d$ as illustrated in Figure 2.8. The total velocity potential at some point, X , in space is the