

for the propagation of sound waves. Substituting for $P(\vec{x}, t) = P_0(\vec{x}) + p(\vec{x}, t)$, and $\rho(\vec{x}, t) = \rho_0(\vec{x}) + \rho'(\vec{x}, t)$ and deleting the second- and higher-order terms yields

$$\rho_0 \left\{ \frac{\partial u_x}{\partial t} \vec{i} + \frac{\partial u_y}{\partial t} \vec{j} + \frac{\partial u_z}{\partial t} \vec{k} \right\} + \left\{ \frac{\partial p}{\partial x} \vec{i} + \frac{\partial p}{\partial y} \vec{j} + \frac{\partial p}{\partial z} \vec{k} \right\} = 0, \quad (2.25)$$

or

$$\rho_0 \frac{\partial \vec{u}}{\partial t} + \vec{\nabla} p = 0. \quad (2.26)$$

Equation (2.26) is the linear inviscid force equation (conservation of momentum). Like the linearised equation of conservation of mass, it is valid for small amplitude sound waves ($\sim < 140$ dB).

2.2.3 The thermodynamic equation of state

The thermodynamic equation of state relates the pressure, density and absolute temperature of a fluid. For a perfect fluid it is

$$P = \rho R T_k. \quad (2.27)$$

P is the absolute pressure, ρ is the density of the gas, R is the gas constant, and T_k is the absolute temperature. It should be noted that the universal gas constant, G , is equal to RM , where M is the molecular weight of the particular gas.

The propagation of sound waves in air does not generally produce any significant changes of thermal energy between particles, and the entropy of the gas is constant. In addition, the thermal conductivity of the gas is very small. The propagation of sound waves can therefore be assumed to be nearly adiabatic. This assumption is valid for linear (small amplitude) sound waves within the audio-frequency range. Any loss of thermal energy would result in an attenuation of the sound waves with time and distance. Thus the adiabatic equation of state for a perfect gas is

$$P/P_0 = (\rho/\rho_0)^\gamma, \quad (2.28)$$

where γ is the ratio of specific heats.

For gases which are not perfect, an adiabatic equation of state can be obtained from a Taylor series expansion of an experimentally determined isentropic relationship between the pressure and density fluctuations. Provided that those fluctuations are assumed to be small, a linear relationship can be established between them (Kinsler *et al.*^{2,1}). It is

$$p(\vec{x}, t) = B \left\{ \frac{\rho'}{\rho_0} \right\}, \quad (2.29)$$

where B is the adiabatic bulk modulus – i.e.

$$B = \rho_0 \left\{ \frac{\partial P}{\partial \rho} \right\}_{\rho_0}. \quad (2.30)$$

The partial derivative in equation (2.30) is evaluated for an adiabatic process – i.e. it establishes the adiabatic compression and expansion of the gas about its mean density.

2.2.4 The linearised acoustic wave equation

Equation (2.9) (conservation of mass), equation (2.26) (conservation of momentum) and equations (2.29)–(2.30) can now be combined into a single equation with one dependent variable. The dependent variable of interest in acoustics is the fluctuating pressure.

The time derivative of the equation of conservation of mass is

$$\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \frac{\partial(\vec{\nabla} \cdot \vec{u})}{\partial t} = 0, \quad (2.31a)$$

or

$$\frac{\partial^2 \rho'}{\partial t^2} + \rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} = 0. \quad (2.31b)$$

The divergence of the equation of conservation of momentum is

$$\rho_0 \vec{\nabla} \cdot \frac{\partial \vec{u}}{\partial t} + \nabla^2 p = 0. \quad (2.32)$$

Subtracting equation (2.31) from (2.32) yields

$$\nabla^2 p = \frac{\partial^2 \rho'}{\partial t^2}. \quad (2.33)$$

Equation (2.29) can now be substituted into equation (2.33) to eliminate ρ' . Thus,

$$\nabla^2 p = \frac{\rho_0}{B} \frac{\partial^2 p}{\partial t^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}. \quad (2.34)$$

Equation (2.34) is the linearised, homogeneous acoustic wave equation with the fluctuating pressure, $p(\vec{x}, t)$, as the dependent variable. The constant, c , is the velocity of propagation of the wave and is therefore the speed of sound. It is

$$c = \left(\frac{B}{\rho_0} \right)^{1/2} = \left\{ \left(\frac{\partial P}{\partial \rho} \right)_{\rho_0} \right\}^{1/2}. \quad (2.35)$$

Some useful approximations can now be made in relation to the speed of sound by assuming that the sound propagation medium is a perfect gas. From equation (2.28)

$$P = \frac{P_0 \rho^\gamma}{\rho_0^\gamma}, \quad (2.36)$$

thus

$$\frac{\partial P}{\partial \rho} = \frac{\gamma P}{\rho}. \quad (2.37)$$

Thus, the speed of sound is

$$c = \left(\frac{\gamma P}{\rho} \right)^{1/2}. \quad (2.38)$$

Also, by substituting equation (2.27) into equation (2.38) yields

$$c = (\gamma R T_k)^{1/2}. \quad (2.39)$$

For small fluctuations,

$$c \approx c_0 \approx \left(\frac{\gamma P_0}{\rho_0} \right)^{1/2}. \quad (2.40)$$

The absolute temperature, T_k , in equation (2.39) is in Kelvin, and c_0 is the speed of sound at atmospheric conditions. The gas constant for air is $0.287 \text{ kJ kg}^{-1} \text{ K}^{-1}$. The equations demonstrate that the wave speed is constant for a given pressure and medium.

2.2.5 The acoustic velocity potential

Using vector theory, it can be demonstrated that the acoustic particle velocity, $\vec{u}$, is irrotational. From vector theory it can be shown that if a vector function is the gradient of a scalar function, its curl is the zero vector – i.e. for some scalar function ϕ , $\vec{\nabla} \times \vec{\nabla} \phi = 0$.

The curl of the momentum equation (equation 2.26) is

$$\rho_0 \frac{\partial(\vec{\nabla} \times \vec{u})}{\partial t} + (\vec{\nabla} \times \vec{\nabla} p) = 0, \quad (2.41)$$

and

$$\vec{\nabla} \times \vec{u} = 0, \quad (2.42)$$

since (i) p is a scalar quantity, and (ii) the constant associated with the time integral is zero. The constant has to be zero since the acoustic quantities would disappear if an acoustic disturbance was not present. Equation (2.42) therefore confirms that the acoustic particle velocity, $\vec{u}$, is irrotational. Now, because the acoustic particle velocity is irrotational, it can be expressed as the gradient of a scalar function (if a vector function is the gradient of a scalar function, its curl is the zero vector, i.e. it is irrotational). Hence the introduction of the concept of the acoustic velocity potential, ϕ , and

$$\vec{u} = \vec{\nabla} \phi. \quad (2.43)$$

The above result is a very important one. Its physical interpretation is that the acoustical excitation of an inviscid fluid does not produce rotational flow – i.e. there are no boundary layers, shear stresses or turbulence generated. When the effects of viscosity cannot be completely neglected, the particle velocity is not curl free everywhere and there is some rotational flow. For example, when a flat plate is mechanically excited in the presence of a mean fluid flow, in addition to the plate radiating sound, vorticity is

generated on the surface of the plate (Soria and Norton^{2,2}). These rotational effects are generally confined to the vicinity of boundaries but situations can arise where they exert some influence on sound propagation. These concepts will be discussed later on in this chapter when discussing aerodynamic noise. For the present purposes it is sufficient to assume that, when the mean fluid flow is zero, the particle velocity is irrotational.

Substituting the equation for the velocity potential (equation 2.43) into the momentum equation (equation 2.26) yields

$$\vec{\nabla} \left\{ \rho_0 \frac{\partial \phi}{\partial t} + p \right\} = 0. \quad (2.44)$$

The acoustic quantities inside the brackets have to vanish if there is no acoustic disturbance present, thus the integration constant has to be zero – i.e.

$$p = -\rho_0 \frac{\partial \phi}{\partial t}. \quad (2.45)$$

Substituting for p into the wave equation (equation 2.34) yields

$$\nabla^2 \left\{ \frac{\partial \phi}{\partial t} \right\} = \frac{1}{c^2} \left\{ \frac{\partial^3 \phi}{\partial t^3} \right\}. \quad (2.46)$$

Thus

$$\nabla^2 \phi = \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2}, \quad (2.47)$$

and ϕ satisfies the wave equation.

It is useful to point out that, when rotational flow components (e.g. near boundaries) have to be accounted for, a vector velocity potential, $\vec{\psi}$, can be introduced such that

$$\vec{U} = \vec{\nabla} \phi + \vec{\nabla} \times \vec{\psi}. \quad (2.48)$$

The first term represents the irrotational fluctuating flow and the second term represents the total (mean plus fluctuating) rotational component (see section 2.4.4).

The velocity potential concept is commonly used in fluid dynamics where solutions for the particle velocity are usually sought. The particle velocity for the three-dimensional wave equation is a vector quantity, and the introduction of the scalar velocity potential allows for the wave equation to be solved in terms of a scalar. Acousticians generally use the pressure variable, which is also a scalar quantity. The particle velocity can be related to the acoustic pressure fluctuation via the momentum equation (equation 2.26). Hence,

$$\vec{u} = \vec{\nabla} \phi = - \int \frac{1}{\rho_0} \vec{\nabla} p \, dt. \quad (2.49)$$

2.2.6 The propagation of plane sound waves

Consider a plane, one-dimensional, sound wave propagating in the x -direction. The one-dimensional homogeneous wave equation is

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2}, \quad (2.50)$$

where $p = p(\vec{x}, t)$. As was the case in chapter 1, it is convenient to represent the solution to the wave equation in terms of complex, harmonic functions. The general solution involves waves travelling in both the positive and negative x -directions. It is

$$p(\vec{x}, t) = \mathbf{A}_1 e^{i(\omega t - kx)} + \mathbf{A}_2 e^{i(\omega t + kx)}. \quad (2.51)$$

The complex particle velocity, $\vec{u}(\vec{x}, t)$, and the complex velocity potential, $\phi(\vec{x}, t)$, can be obtained by substituting equation (2.51) into equation (2.49). They are

$$\vec{u}(\vec{x}, t) = \left\{ \frac{\mathbf{A}_1}{\rho_0 c} e^{i(\omega t - kx)} - \frac{\mathbf{A}_2}{\rho_0 c} e^{i(\omega t + kx)} \right\} \vec{i}, \quad (2.52)$$

and

$$\phi(\vec{x}, t) = -\frac{\mathbf{A}_1}{i\rho_0 \omega} e^{i(\omega t - kx)} - \frac{\mathbf{A}_2}{i\rho_0 \omega} e^{i(\omega t + kx)}. \quad (2.53)$$

The particle velocity is a vector quantity and the term outside the brackets in equation (2.52) is the unit vector, $\vec{i}$ (i.e. one should not be confused between the complex number i and the unit vector $\vec{i}$). Also, because complex numbers have been introduced, the acoustic variables are now represented as complex numbers. Now, if one lets

$$\mathbf{p}_+ = \mathbf{A}_1 e^{i(\omega t - kx)}, \quad \text{and} \quad \mathbf{p}_- = \mathbf{A}_2 e^{i(\omega t + kx)}, \quad (2.54)$$

then,

$$\vec{u}(\vec{x}, t) = \left\{ \frac{\mathbf{p}_+}{\rho_0 c} - \frac{\mathbf{p}_-}{\rho_0 c} \right\} \vec{i}, \quad (2.55)$$

and

$$\phi(\vec{x}, t) = -\frac{\mathbf{p}_+}{i\rho_0 \omega} - \frac{\mathbf{p}_-}{i\rho_0 \omega}. \quad (2.56)$$

Once again, the term outside the brackets in equation (2.55) is the unit vector, $\vec{i}$.

In acoustics it is common to consider the waves travelling in the positive direction when one is concerned with the propagation of sound waves away from a source into some free field. Both positive and negative travelling waves have to be considered when considering the propagation of sound waves in confined spaces, e.g. ducts, rooms etc. Equation (2.55) illustrates an important point – for a plane, harmonic wave travelling in either the positive or the negative x -directions, the acoustic pressure fluctuations are in phase with the particle velocities. The phase relationship between these two variables is

very important for a fundamental understanding of the propagation of different sound wave-types.

The concepts of impedance were introduced in chapter 1. The ratio of the acoustic pressure fluctuations, $\mathbf{p}(\vec{x}, t)$ in a medium to the associated particle velocity, $\vec{\mathbf{u}}(\vec{x}, t)$ is termed the specific acoustic impedance, $\mathbf{Z}_a$, i.e.

$$\mathbf{Z}_a = \frac{\mathbf{p}(\vec{x}, t)}{\vec{\mathbf{u}}(\vec{x}, t)}. \quad (2.57)$$

For a plane sound wave,

$$\mathbf{Z}_a = Z_a = \pm \rho_0 c. \quad (2.58)$$

The specific acoustic impedance is generally complex – for wave types other than plane sound waves, the acoustic pressure fluctuations and the particle velocities are not always in phase, i.e. the waves diverge. The quantity $\rho_0 c$ is often called the characteristic impedance (resistance) of the medium. For air, at 20 °C and 1 atm, ρ_0 is $\sim 1.21 \text{ kg m}^{-3}$, c is $\sim 343 \text{ m s}^{-1}$ and $\rho_0 c$ is 415 Pa s m^{-1} .

2.2.7 Sound intensity, energy density and sound power

Three additional parameters that play an important role in acoustics are the sound intensity, the sound energy density and the radiated sound power.

The sound intensity is defined as the rate of flow of energy through a unit area which is normal to the direction of propagation. From basic dynamics, power = force $\times$ velocity. For an acoustic process, the instantaneous power is

$$\Pi = \vec{F} \cdot \vec{u}. \quad (2.59)$$

Here, $\vec{F}$ is the vector force acting on a particle and $\vec{u}$ is the associated particle velocity. The power per unit normal area is the instantaneous sound intensity vector, $\vec{I}'$, where

$$\vec{I}' = p\vec{u}. \quad (2.60)$$

The time average of the instantaneous power flow through a unit area is the mean intensity vector, $\vec{I}$, where

$$\vec{I} = \frac{1}{T} \int_0^T p\vec{u} dt = \frac{1}{2} \text{Re}[\mathbf{p}\vec{\mathbf{u}}^*]. \quad (2.61)$$

The second representation of equation (2.61) is used when the acoustic pressure fluctuations and the particle velocities are treated as complex, harmonic variables.

For a plane wave travelling in the positive x -direction

$$p(\vec{x}, t) = \text{Re}[\mathbf{A}_1 e^{i(\omega t - kx)}] = \hat{p} \cos(\omega t - kx), \quad (2.62)$$

and

$$u(x, t) = \operatorname{Re} \left[\frac{\mathbf{A}_1}{\rho_0 c} e^{i(\omega t - kx)} \right] = \frac{\hat{p}}{\rho_0 c} \cos(\omega t - kx). \quad (2.63)$$

The mean sound intensity, I , is obtained by substitution into equation (2.61) and evaluating the integral. Hence,

$$I = \frac{\hat{p}^2}{2\rho_0 c} = \frac{p_{\text{rms}}^2}{\rho_0 c}. \quad (2.64)$$

The vector notation is omitted in the above equations for particle velocity and intensity because of the one-dimensional nature of the travelling wave.

The sound energy density is the sound energy per unit volume of space. The energy transported by a sound wave comprises kinetic energy of the moving particles and potential energy of the compressed fluid. Using the plane wave approximation, a simple relationship can be derived for the sound energy density. It will be seen later on that this approximation is valid because most sound waves approximate to one-dimensional plane waves at large distances from the source.

Consider a fluid element with an undisturbed volume V_0 . Its kinetic energy per unit volume is

$$\frac{T}{V_0} = \frac{1}{2} \rho_0 u^2 = \frac{p^2}{2c^2 \rho_0}, \quad (2.65)$$

since $u = p/(\rho_0 c)$. When the volume changes from V_0 to V_1 there is a change in potential energy. It is

$$U = - \int_{V_0}^{V_1} p \, dV, \quad (2.66)$$

where the negative sign indicates that a positive acoustic fluctuating pressure produces a decrease in the fluid volume. Now, since $\rho = m/V$,

$$d\rho = -\frac{m}{V^2} dV, \quad (2.67)$$

and combining with equation (2.37) yields

$$dV = -\frac{V}{\gamma P} dP. \quad (2.68)$$

For small changes in pressure and volume this approximates to

$$dV = -\frac{V_0}{\gamma P_0} dp. \quad (2.69)$$

Substituting into equation (2.66) and integrating from 0 to p yields the potential energy

per unit volume. It is

$$\frac{U}{V_0} = \frac{p^2}{2\gamma P_0} = \frac{p^2}{2c^2\rho_0}, \quad (2.70)$$

since $c^2 = \gamma P_0/\rho_0$.

The total sound energy per unit volume of space is the sum of the kinetic and potential energies per unit volume. Thus, the instantaneous sound energy density, D' , is

$$D' = \frac{T}{V_0} + \frac{U}{V_0}, \quad (2.71a)$$

$$= \frac{p^2}{\rho_0 c^2}. \quad (2.71b)$$

The mean energy density, D , is obtained by integrating equation (2.71b) with respect to time. It is

$$D = \frac{\hat{p}^2}{2\rho_0 c^2} = \frac{p_{\text{rms}}^2}{\rho_0 c^2}. \quad (2.72)$$

Thus,

$$D = \frac{I}{c}. \quad (2.73)$$

The fluctuating acoustic pressure and the sound intensity decrease with distance from the source (this will become apparent when spherical sound waves are discussed in the next sub-section). It will also be shown that they are a function of the environment – i.e. the reverberant effects of a room enclosure will alter the intensity and sound pressure due to an acoustic source in the room. The sound power of an acoustic source is independent of distance and essentially independent of location – in some instances, the effects of reflecting surfaces have to be accounted for. The sound power, Π , is the integral of the intensity at some point in space over a surface area which is perpendicular to the flow of sound energy. It is

$$\Pi = \int_S \vec{I} \cdot d\vec{S}. \quad (2.74)$$

2.3 Fundamental acoustic source models

This section is devoted to the description of a range of fundamental acoustic source models. Most noise sources that are of concern to engineers (e.g. vehicles, construction equipment, industrial machinery, appliances, flow-duct systems etc.) can be modelled in terms of simple sources such as spheres, pistons in an infinite baffle, cylinders or combinations thereof. It is therefore instructive to analyse the characteristics of some of

these idealised sound sources. (Additional work on monopoles, dipoles and quadrupoles can be found in Norton and Pan^{2,3}.)

2.3.1 Monopoles – simple spherical sound waves

A monopole is a single, spherical sound source which radiates sound waves that are only a function of the radial distance, r , from the source. The wave equation has therefore got to be set up in spherical co-ordinates before any analysis can proceed. In sub-section 2.2.6, the wave equation was solved in terms of the pressure variable, and related to the particle velocity and the velocity potential. For a change, the wave equation will be solved here in terms of the velocity potential, and subsequently related to the pressure fluctuations and the particle velocity.

The one-dimensional spherical wave equation (in terms of the acoustic velocity potential) is

$$\frac{\partial^2(r\phi(r, t))}{\partial r^2} - \frac{1}{c^2} \frac{\partial^2(r\phi(r, t))}{\partial t^2} = 0. \quad (2.75)$$

It represents an omni-directional wave which radiates outwards from or inwards towards a source. The product $r\phi(r, t)$ is treated as a single variable and the equation is of the same form as that for a plane wave. The general solution is

$$r\phi(r, t) = G_1(ct - r) + G_2(ct + r), \quad (2.76)$$

thus

$$\phi(r, t) = \frac{G_1(ct - r)}{r} + \frac{G_2(ct + r)}{r}. \quad (2.77)$$

The first term represents a spherical sound wave travelling radially outwards from the sound source, and the second term represents a spherical sound wave travelling towards the sound source.

In engineering noise and vibration analysis, one is generally only concerned with sound waves that travel away from a source. Hence, one is only concerned with the first part of the solution. In principle, the function G_1 can be any arbitrary function. It is, however, convenient and conventional to assume a complex harmonic solution of the form

$$\phi(r, t) = \frac{\mathbf{A}}{r} e^{i(\omega t - kr)}. \quad (2.78)$$

The term $\mathbf{A}$ is a constant which is determined by the boundary conditions specified at the surface of the monopole.

Consider an oscillating sphere of radius a , with a normal surface velocity $\mathbf{u}_a = U_a e^{i\omega t}$. From equation (2.43),

$$\frac{\partial \phi}{\partial r} = u_r. \quad (2.79)$$

Thus, by substituting equation (2.78) into equation (2.79) with $r = a$ and solving for the constant $\mathbf{A}$ yields

$$\mathbf{A} = -U_a \left\{ \frac{a^2}{1 + ika} \right\} e^{ika}. \quad (2.80)$$

Thus,

$$\Phi(r, t) = -\frac{U_a}{r} \left\{ \frac{a^2}{1 + ika} \right\} e^{i(\omega t - k(r-a))}. \quad (2.81)$$

The concept of a source strength, $Q(t)$, can now be introduced. The source strength of a monopole is defined as its surface area multiplied by its surface velocity. Thus,

$$Q(t) = 4\pi a^2 U_a e^{i\omega t} = Q_p e^{i\omega t}. \quad (2.82)$$

Hence, the complex representation of velocity potential is

$$\Phi(r, t) = -\frac{Q(t)}{4\pi r} \left\{ \frac{1}{1 + ika} \right\} e^{-ik(r-a)}. \quad (2.83)$$

The complex representation of the fluctuating acoustic pressure, $\mathbf{p}(r, t)$ and the particle velocity, $\mathbf{u}(r, t)$, can now be obtained from equation (2.49) (the vector notation is now dropped because of the omni-directional wave propagation) – i.e.

$$p(r, t) = -\rho_0 \{ \partial \Phi / \partial t \}, \quad \text{and} \quad u(r, t) = \partial \Phi / \partial r. \quad (2.84)$$

Thus,

$$\mathbf{p}(r, t) = -ik\rho_0 c \Phi(r, t) = \frac{Q(t)}{4\pi r} \left\{ \frac{ik\rho_0 c}{1 + ika} \right\} e^{-ik(r-a)}, \quad (2.85)$$

and

$$\mathbf{u}(r, t) = -\left\{ \frac{1 + ikr}{r} \right\} \Phi(r, t) = \frac{Q(t)}{4\pi r^2} \left\{ \frac{1 + ikr}{1 + ika} \right\} e^{-ik(r-a)}. \quad (2.86)$$

The specific acoustic impedance, $\mathbf{Z}_a$, can now be evaluated using equation (2.57). It is

$$\mathbf{Z}_a = \frac{i\rho_0 ckr}{1 + ikr} = \rho_0 c \left\{ \frac{k^2 r^2}{1 + k^2 r^2} + i \frac{kr}{1 + k^2 r^2} \right\}. \quad (2.87)$$

Unlike the specific acoustic impedance of a plane wave, the specific acoustic impedance of a spherical wave has both a resistive and a reactive component. When the resistive component dominates, the acoustic pressure fluctuations are in phase with the particle velocities; when the reactive component dominates, they are out of phase with each other. As with energy and power flow in structures (see section 1.7) one would expect that the in-phase components of the sound waves dominate the radiated sound power and that the out-of-phase components produce some near-field reactive exchange of sound energy. These phase relationships are therefore the basis of the far-field/near-field concepts which are so commonly used in noise control engineering.

The sound intensity of a spherical sound wave can now be estimated. The mean radiated intensity is a real quantity and it can be evaluated from equation (2.61). It is

$$I(r) = \frac{Q_p^2 k^2 \rho_0 c}{32\pi^2 r^2 (1 + k^2 a^2)} = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{16\pi^2 r^2 (1 + k^2 a^2)}. \quad (2.88)$$

The sound power radiated by the spherical source can now be evaluated from equation (2.74). It is

$$\Pi = 4\pi r^2 I(r) = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{4\pi (1 + k^2 a^2)}. \quad (2.89)$$

The radiated mean-square acoustic pressure fluctuations can be readily evaluated from equations (2.61) and (2.85). They are

$$p_{\text{rms}}^2 = \frac{1}{2} \text{Re}[\mathbf{p}(r, t) \mathbf{p}^*(r, t)] = \frac{Q_{\text{rms}}^2 k^2 (\rho_0 c)^2}{16\pi^2 r^2 (1 + k^2 a^2)} = I(r) \rho_0 c. \quad (2.90)$$

Thus the relationship between the radiated sound pressure and the mean sound intensity is the same as that for a plane wave (see equation 2.64). It is also important to note that the radiated sound pressure is proportional to the inverse square of the distance from the source.

Returning to equation (2.87), one can see that there are two limiting situations. They are (i) $kr \ll 1$, and (ii) $kr \gg 1$. When $kr \ll 1$,

$$\mathbf{u}(r, t) = \frac{\mathbf{Q}(t)}{4\pi r^2 (1 + ika)} e^{-ik(r-a)}, \quad (2.91)$$

and

$$\mathbf{Z}_a = ikr \rho_0 c. \quad (2.92)$$

Here, the acoustic pressure fluctuations and the particle velocities are out of phase, the impedance is reactive and no sound power is radiated from the source. When $kr \gg 1$,

$$\mathbf{u}(r, t) = \frac{ik\mathbf{Q}(t)}{4\pi r (1 + ika)} e^{-ik(r-a)}, \quad (2.93)$$

and

$$\mathbf{Z}_a = \rho_0 c. \quad (2.94)$$

Here, the acoustic pressure fluctuations and the particle velocities are in phase, the impedance is resistive (i.e. it is equal to the characteristic acoustic impedance) and sound power is radiated from the source. Combining equations (2.89) and (2.90), the radiated sound power can also be obtained in terms of the acoustic pressure fluctuations. It is

$$\Pi = \frac{4\pi r^2 p_{\text{rms}}^2}{\rho_0 c}. \quad (2.95)$$

Equations (2.89) and (2.95) are identical. The former expresses the radiated sound power of a monopole in terms of its source strength, wavenumber, and dimensions,

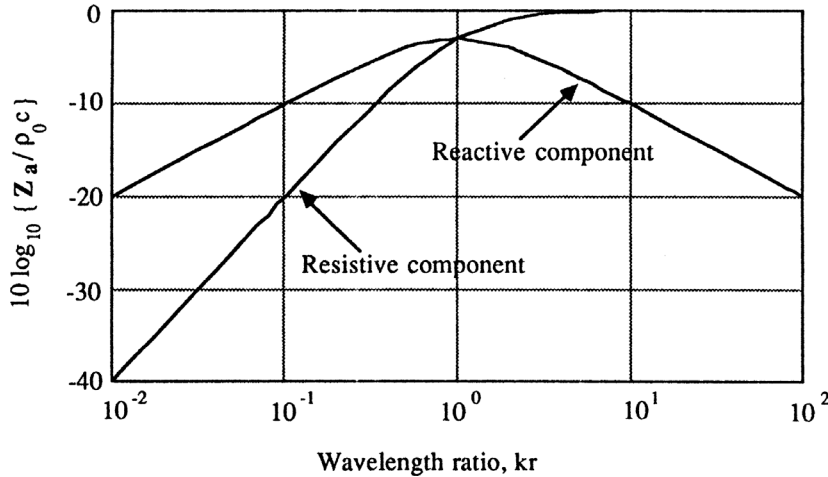


Fig. 2.7. The resistive and reactive components of the specific acoustic impedance of a spherical sound source.

whilst the latter expresses it in terms of the radiated sound pressure at some distance, r , from the source. The former expression is very useful for obtaining a physical understanding of the effects of the source parameters (source strength, wavenumber, and dimensions) on the radiated noise, whilst the latter allows for an estimate of the sound power from a simple measurement of the mean-square pressure fluctuations. A word of caution should be made regarding equation (2.95). It is only valid in the far-field – i.e. at sufficient distances from the source such that the pressure fluctuations and the particle velocities are in phase. It is also only valid in regions of free space where there are no environmental effects such as reverberation (reflected sound) on the measured levels.

It is now instructive to return once again to equation (2.87) to attempt to define a transition point between the two regions ($kr \ll 1$, and $kr \gg 1$). The resistive and reactive components of the specific acoustic impedance can be plotted as a function of kr . This information is presented in Figure 2.7.

The main characteristic of the far-field of a radiating sound source is that $kr \gg 1$ and that Z_a is essentially resistive. From Figure 2.7, the resistive component dominates for $kr \approx 10$. Now, λ is the wavelength of the sound source ($\lambda = 2\pi/k$), thus the transition radius is

$$r \approx 1.6\lambda. \quad (2.96)$$

It is very important to note that this transition point is not a constant but is a function of frequency – i.e. at higher frequencies the transition radius is shorter than at lower frequencies.

The concepts presented in this sub-section form an important basis for the modelling of sound sources. Many practical sound sources, including both vibrating bodies and aerodynamic sources, can be approximated as monopoles. If, for instance, the monopole source was of an aerodynamic nature rather than some vibrating body with a surface

velocity, the radiated sound power could then be related to the mean flow velocity – i.e. the mean addition or subtraction of mass from a source region. To do this, one has to re-consider equation (2.89). The source strength, Q_{rms} , scales as L^2U , where L is a typical dimension of the region of fluid flow, such as the width of a square duct, and U is the mean flow velocity. The characteristic frequency in the flow scales as U/L , and this has to equal the frequency of aerodynamic sound generation for dimensional consistency. Also, the source dimension, a , is assumed to be very much smaller than the geometrical dimension, L , i.e. $a \ll L$. Thus,

$$\Pi \approx \frac{L^4 U^2 \rho_0 c U^2}{4\pi L^2 c^2} \approx \frac{L^2 U^4 \rho_0}{4\pi c}. \quad (2.97)$$

Equation (2.97) suggests that the sound power radiated by an aerodynamically generated monopole scales with the fourth power of the flow velocity. This is a very important statement – one which has significant practical consequences. These practical consequences will be discussed later on in this chapter once some other sound source types have been studied, so that comparisons can be made. It will also be seen later on in this chapter that this scaling law only holds for sub-sonic flows – i.e. where the mean flow velocity is less than the speed of sound, c .

Some typical examples of monopole sound sources associated with vibrating structures are small electric motors, pumps and certain types of traffic noise (at medium distances away from a single vehicle – at larger distances the source can sometimes appear to be a dipole). Aerodynamic monopole sound sources include unsteady combustion from a furnace, sirens, pulsed jets and cavitation.

2.3.2 Dipoles

A dipole is a sound source model that is composed of two monopoles in close proximity to each other. They are also of equal source strength and oscillate 180° out of phase with each other. A nett fluctuating force is produced because of this out-of-phase oscillation. The velocity potential function for a dipole is a function of the polar angle, θ , in addition to the radius, r . It can be represented as a space derivative of the monopole velocity potential function. The analysis for dipoles with finite dimensions is somewhat lengthy and tedious. Numerous permutations are possible and the reader is referred to Pierce^{2.4} or Reynolds^{2.5} for further details. Many sound sources, including aerodynamic dipole sources, can be modelled as point dipoles – i.e. the dimensions of the dipole source are very small ($ka \ll 1$), and, with this assumption, the analysis is somewhat simplified. This section shall therefore be restricted to the special case of far-field sound radiation from point dipole sound sources. The analytical techniques involved here can be readily extended to the cases of both near-field and far-field sound radiation from finite dipole sources.

Consider two point monopoles in free space, separated by some small distance $2d$ as illustrated in Figure 2.8. The total velocity potential at some point, X , in space is the

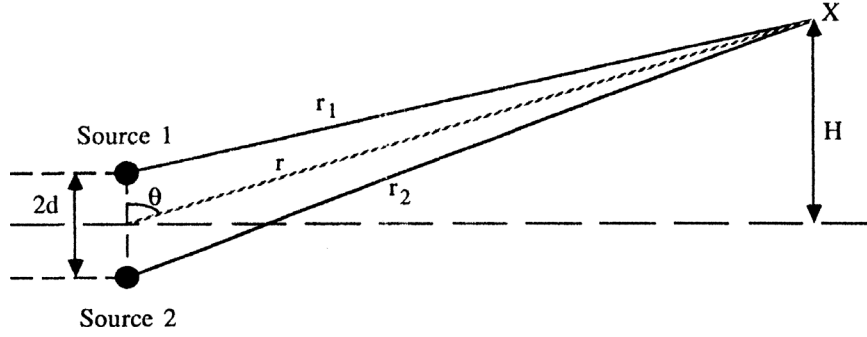


Fig. 2.8. Point dipole model – two point monopoles in free space separated by some small distance $2d$.

sum of the two separate velocity potentials, i.e.

$$\phi(r, t) = \phi_1(r, t) + \phi_2(r, t). \quad (2.98)$$

The separate distances from each of the point sources to the point, X , are given by

$$r_1^2 = (r^2 + d^2 - 2rd \cos \theta), \quad \text{and} \quad r_2^2 = (r^2 + d^2 + 2rd \cos \theta). \quad (2.99)$$

In the far field, $r \gg d$, thus the above geometrical relationships can be simplified to

$$r_1 \approx r - d \cos \theta, \quad \text{and} \quad r_2 \approx r + d \cos \theta. \quad (2.100)$$

The velocity potential for a point monopole can be obtained from equation (2.83) by letting $ka \rightarrow 0$. It is

$$\phi(r, t) = -\frac{\mathbf{Q}(t)}{4\pi r} e^{-ikr}. \quad (2.101)$$

The combined velocity potential at X due to both point monopoles is thus

$$\phi(r, t) = -\frac{\mathbf{Q}_1(t)}{4\pi r_1} e^{-ikr_1} - \frac{\mathbf{Q}_2(t)}{4\pi r_2} e^{-ikr_2}. \quad (2.102)$$

In the far-field, $r \approx r_1 \approx r_2$ and the denominators in the above equation can be replaced by r . The numerators, however, should not be replaced because the phase difference between the two oscillating monopoles has to be accounted for. By substituting equation (2.100) into equation (2.102), the combined velocity potential approximates to

$$\phi(r, \theta, t) \approx -\frac{e^{-ikr}}{4\pi r} \{ \mathbf{Q}_1(t) e^{ikd \cos \theta} + \mathbf{Q}_2(t) e^{-ikd \cos \theta} \}. \quad (2.103)$$

For a point dipole, the two monopole sources are of equal source strength but are out of phase by 180° . Thus

$$\mathbf{Q}_1(t) = Q_p e^{i(\omega t + \pi/2)}, \quad (2.104)$$

and

$$\mathbf{Q}_2(t) = Q_p e^{i(\omega t - \pi/2)}. \quad (2.105)$$

Hence,

$$\phi(r, \theta, t) = -\frac{Q_p e^{i(\omega t - kr)}}{4\pi r} \left\{ e^{i(kd \cos \theta + \pi/2)} + e^{-i(kd \cos \theta + \pi/2)} \right\}, \quad (2.106)$$

or

$$\phi(r, \theta, t) = \frac{Q_p e^{i(\omega t - kr)}}{4\pi r} 2 \sin(kd \cos \theta). \quad (2.107)$$

For $kd \ll 1$ this approximates to

$$\phi(r, \theta, t) = \frac{Q_p e^{i(\omega t - kr)}}{4\pi r} 2kd \cos \theta. \quad (2.108)$$

The sound intensity of a dipole can now be estimated either by following the same procedures adopted in the previous sub-section on monopoles or by recognising that the far-field dipole velocity potential is equivalent to the monopole velocity potential with an additional term which is a function of the polar angle θ – i.e. a directivity factor ($-2kd \cos \theta$). It is

$$I(r, \theta) = \frac{Q_p^2 k^2 \rho_0 c}{32\pi^2 r^2} (2kd \cos \theta)^2. \quad (2.109)$$

Thus,

$$I(r, \theta) = \frac{Q_p^2 k^4 d^2 \rho_0 c}{8\pi^2 r^2} \cos^2 \theta = \frac{Q_{rms}^2 k^4 d^2 \rho_0 c}{4\pi^2 r^2} \cos^2 \theta. \quad (2.110)$$

It is important to point out at this stage that the dipole velocity potential described by equation (2.108) is a far-field approximation. In reality a dipole velocity potential has two terms – one which is associated with the near-field components, and one which is associated with the far-field components. The complete dipole velocity potential can be described in terms of a dipole source strength, $\mathbf{Q}_d$ (see Reynolds^{2.5}, or Dowling and Ffowcs Williams^{2.6}) with units of $m^4 s^{-1}$, where

$$\phi(r, \theta, t) = \frac{\cos \theta}{4\pi} \left\{ \frac{1}{cr} \frac{\partial \mathbf{Q}_d(t - r/c)}{\partial t} + \frac{\mathbf{Q}_d(t - r/c)}{r^2} \right\}. \quad (2.111)$$

The radiated sound field has a $\cos \theta$ dependence – i.e. at 90° to the dipole axis the sound fields cancel each other. Also, the near-field component varies with the inverse square of distance whilst the far-field component varies with the inverse of distance. This point will be discussed again later on in this chapter in relation to flow noise.

The far-field sound power radiated by the dipole can be evaluated by integrating equation (2.110) over an arbitrary spherical surface at a given radius, r . The polar angular dependence has to be accounted for in this integration. The sound power radiated is

$$\Pi = \frac{Q_{rms}^2 k^4 d^2 \rho_0 c}{3\pi}. \quad (2.112)$$

A comparison can be made between the sound radiating efficiencies of monopoles and dipoles by comparing equations (2.89) and (2.112). The ratio of the sound power radiated by a dipole to that radiated by a monopole is

$$\frac{\Pi_D}{\Pi_M} = \frac{4k^2 d^2}{3} \sim \left\{ \frac{d}{\lambda} \right\}^2. \quad (2.113)$$

Π_D is the sound power radiated by the dipole, Π_M is the sound power radiated by the monopole and $k = 2\pi/\lambda$, where λ is the wavelength. It can be seen from the equation that at low frequencies (long wavelengths), the dipole is a very much less efficient radiator of noise than at high frequencies (shorter wavelengths) when compared with a monopole of the same source strength.

A dipole is produced by the fluctuating pressure forces acting along the axis between the two sources. These pressure forces are generated by the out-of-phase oscillations of the two monopoles in close proximity to each other. It is therefore useful to describe the sound power radiated by a dipole in terms of the fluctuating force (along the dipole axis) which is a function of the source strength and the source separation. The derivation of the radiated sound power in terms of the fluctuating force is available in Reynolds^{2,5}. It is estimated by evaluating the r.m.s. fluctuating force, F_{rms} , on a spherical surface containing the dipole, and is

$$\Pi = \frac{3F_{\text{rms}}^2 k^2}{4\pi \rho_0 c}. \quad (2.114)$$

The sound power radiated by aerodynamic dipole sources can be estimated from either equation (2.112) or (2.114) by using the same procedures developed in the previous sub-section for aerodynamic monopoles. The source strength, Q_{rms} , scales as $(2d)^2 U$, where $2d$ is the separation between the two monopole sources in the region of fluid flow, and U is the mean flow velocity. The fluctuating force, F_{rms} , scales as $\rho_0 U^2 d^2$. The characteristic frequency in the flow scales as $U/(2d)$, and this has to equal the frequency of aerodynamic sound generation for dimensional consistency. Thus,

$$\Pi = \frac{Q_{\text{rms}}^2 \omega^4 d^2 \rho_0 c}{3\pi c^4} \approx \frac{(2d)^4 U^2 U^4 d^2 \rho_0 c}{(2d)^4 3\pi c^4} \approx \frac{\rho_0 d^2 U^6}{3\pi c^3}. \quad (2.115)$$

Equation (2.115) suggests that the sound power radiated by an aerodynamically generated dipole scales with the sixth power of the flow velocity (a monopole scales with the fourth power of the flow velocity).

The directivity patterns of monopoles and dipoles in free space are schematically illustrated in Figure 2.9. Whilst the monopoles are omni-directional, the dipoles have a typical ‘figure 8’ sound radiation pattern and there is no sound radiation perpendicular to the dipole axis.

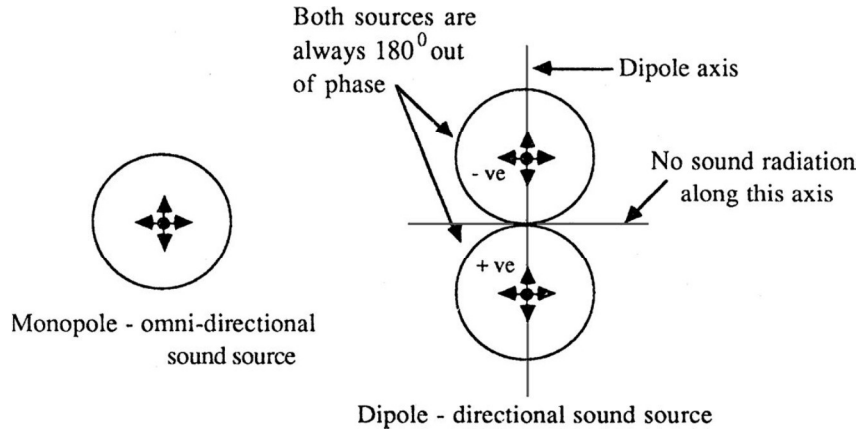


Fig. 2.9. Directivity patterns of monopoles and dipoles in free space.

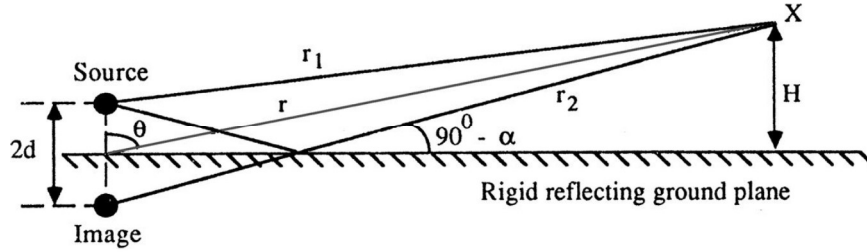
2.3.3 Monopoles near a rigid, reflecting, ground plane

Quite often in practice, most industrial noise sources are mounted on a ground plane or in close proximity to it. In the far-field, they can often be approximated as single point sources. The effects of the ground plane have to be accounted for though. These effects are particularly pronounced when the sound source is less than one acoustic wavelength (λ) from the ground plane. The analysis which follows is not to be confused with the concept of directivity which the reader might be familiar with. The directional effects of floors, intersecting walls, corners etc. on the sound radiation characteristics of an omni-directional noise source are well known and documented – they are reviewed in chapter 4 in this book. This section relates to a point which is often omitted in the literature on noise control engineering – that the sound power of a source can be affected by rigid, reflecting planes. This effect is in addition to the conventional and well known directivity effects.

Consider the case of a monopole near a rigid, reflecting, ground plane as illustrated in Figure 2.10. At some point in the far-field, the sound pressure will be the sum of two sound waves – i.e. a direct and a reflected wave. The reflected wave can be modelled by an image monopole below the reflecting surface. The problem thus reduces to that of two interfering monopoles. In practice, the ground plane will have some finite reflection coefficient (not all the sound will be necessarily reflected) and there will be some finite phase difference between the two waves. If one assumes that the ground plane is a hard reflecting surface, as an upper limit, then the reflection coefficient is unity and the phase difference between the two waves is zero. The problem thus reduces to two in-phase monopoles of equal source strength (a dipole was modelled in the previous sub-section as two out-of-phase monopoles of equal source strength).

The combined velocity potential at the observer position (some point, X , in space) can be obtained from equation (2.103) with $Q_1(t)$ and $Q_2(t)$ being of equal strength and phase. It is

$$\Phi(r, \theta, t) = -\frac{Q_p e^{i(\omega t - kr)}}{4\pi r} 2 \cos(kd \cos \theta). \quad (2.116)$$



$$\Gamma = \frac{Z_s \cos \alpha - \rho_0 c}{Z_s \cos \alpha + \rho_0 c}$$

Γ - reflection coefficient
 Z_s - specific acoustic impedance of reflecting surface
 α - oblique angle of incidence
 $\rho_0 c$ - characteristic acoustic impedance

Fig. 2.10. Monopole near a rigid, reflecting ground plane.

When $d \ll \lambda$, $kd \ll 1$ and the above equation simplifies to

$$\phi(r, t) = -\frac{2Q_p e^{i(\omega t - kr)}}{4\pi r}. \quad (2.117)$$

Equation (2.117) is simply double the far-field velocity potential for a monopole sound source (see equation 2.83)! The hard, reflecting ground plane has resulted in a doubling of the velocity potential. This result is very important. It is often overlooked by noise control engineers when estimating noise radiation levels from sound sources. This doubling of the velocity potential produces a fourfold increase in the sound intensity (equations 2.61, 2.85 and 2.86 with the appropriate velocity potential). In turn, there is a twofold increase in the radiated sound power because the intensity has only got to be integrated over half space (the other half is baffled by the rigid ground plane). The sound intensity is

$$I(r) = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{4\pi^2 r^2}, \quad (2.118)$$

and the radiated sound power is

$$\Pi = 2\pi r^2 I(r) = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{2\pi}. \quad (2.119)$$

The interesting result to come out of this approximate (i.e. limit) analysis is that the sound power of the monopole has been doubled. This is essentially because whilst the strength, Q_{rms} , and the surface vibrational velocity of the source have not changed (from when it is radiating into free space), the reflecting plane has produced a velocity potential, or a pressure, doubling. So, instead of having a constant sound power, the source has a constant volume velocity. These concepts of constant volume velocity sources, as opposed to the more commonly referred to constant power sources, will be discussed from an engineering noise control point of view in chapter 4. They can

be regarded as an upper limit – in practice the effects on non-perfect reflection from the ground plane will reduce the effect of the image source.

Reynolds^{2.5} performs a more rigorous analysis for the effects of a reflecting plane on a monopole and integrates the complete expressions for sound intensity to obtain an expression for the radiated sound power. For the case of perfect reflection and zero phase difference, it is

$$\Pi = \frac{Q_{\text{rms}}^2 k^2 \rho_0 c}{4\pi} \left\{ 1 + \frac{\sin 2kd}{2kd} \right\}. \quad (2.120)$$

For small kd (i.e. the source is located less than an acoustic wavelength from the surface) this reduces to equation (2.119) and for large kd (i.e. the source is located many acoustic wavelengths from the surface) it reduces to a monopole in free space.

Another interesting and somewhat important point to be noted is that at very large distances (typically of the order of several hundred metres) a critical oblique angle of incidence is reached beyond which the reflection coefficient changes sign (Bies^{2.7}). When this happens, the source and its image are out of phase rather than in phase – i.e. the source is now a dipole. This has important consequences for the far-field noise radiation. Referring back to equation (2.110) for a dipole far-field sound intensity, the angle $\cos \theta$ can be approximated by H/r , where H is the distance from the measurement point to the ground plane (see Figure 2.10). Thus the intensity at the point of interest in the far-field is

$$I(r) = \frac{Q_{\text{rms}}^2 H^2 k^4 d^2 \rho_0 c}{4\pi^2 r^4}. \quad (2.121)$$

The important practical point to come out of this is that the intensity of the single sound source now scales as r^{-4} rather than r^{-2} . It will be seen in chapter 4 (where decibels and other noise measurement units are defined) that this produces a 12 dB decay of sound pressure level per doubling of distance rather than the usual 6 dB per doubling of distance for a point source. Bies^{2.7} discusses this phenomenon in some detail.

2.3.4 Sound radiation from a vibrating piston mounted in a rigid baffle

The sound radiation from a vibrating piston mounted in an infinite baffle is a classical problem – one which is covered in numerous books on fundamental acoustics (e.g. Kinsler *et al.*^{2.1}, Reynolds^{2.5}, Ford^{2.8}). The vibrating piston can be either a vibrating surface or a vibrating layer of air. The primary assumption in the analysis (one which is not strictly correct in practice for ‘real’ surfaces) is that all parts of the piston vibrate in phase and with the same amplitude. Its relevance to engineering noise control is that it serves as an introduction to the sound radiation from different types of surfaces, e.g. loudspeakers, open ends of flanged pipes, plates and shells etc.

Consider a flat, circular piston of radius z which is mounted in an infinite, rigid baffle as illustrated in Figure 2.11. The noise radiated by the vibrating piston can be modelled

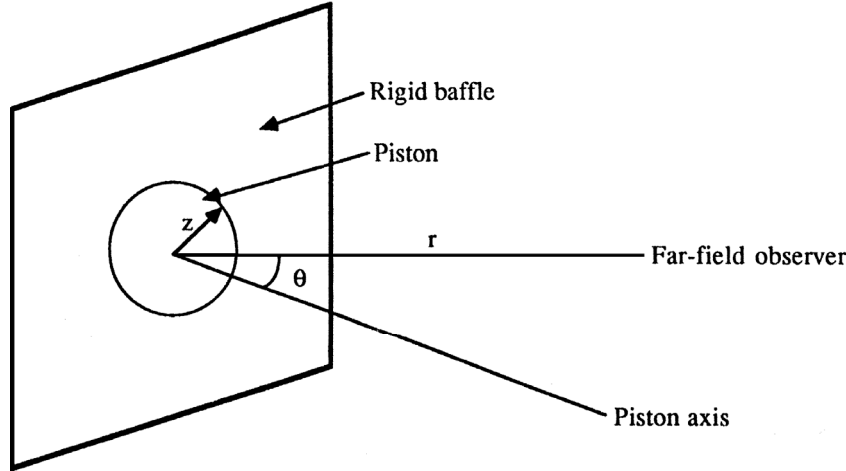


Fig. 2.11. Piston mounted in a rigid baffle.

in terms of numerous point monopoles (monopoles where $ka \ll 1$) radiating together. Each of the monopoles is, however, radiating from a rigid, reflecting, ground plane and not from free space. The sound pressure due to any one of the baffled monopoles is therefore twice that of an equivalent monopole in free space. It is

$$p(r, t) = \frac{ik\rho_0 c}{2\pi r} Q_p e^{i(\omega t - kr)}. \quad (2.122)$$

In this equation, Q_p represents the source strength of the elemental monopole on the piston surface and it is equal to $U_p \delta S$, where U_p is the peak surface velocity of the monopole and δS is the elemental surface area. The total acoustic pressure fluctuations due to the vibrating piston is simply the resultant pressure due to all the point monopoles vibrating in phase and it is obtained by integration over the whole surface area. It is ^{2.1,2.5,2.8}

$$p(r, \theta, t) = \frac{ik\rho_0 c \pi z^2 U_p e^{i(\omega t - kr)}}{2\pi r} \left\{ \frac{2J_1(kz \sin \theta)}{kz \sin \theta} \right\}. \quad (2.123)$$

In the above equation, $U_p e^{i\omega t}$ is the surface velocity of the piston (i.e. each of the monopoles has the same surface velocity and phase). The radiated sound pressure has a similar form to that of a monopole in a reflecting ground plane with the exception of the term in brackets which is a directivity factor. J_1 is the first-order Bessel function and it can be readily evaluated from tables.

The corresponding sound intensity in the far-field ^{2.3}

$$I(r, \theta) = \frac{\rho_0 c k^2 U_{\text{rms}}^2 \pi^2 z^2}{4\pi^2 r^2} \left\{ \frac{2J_1(kz \sin \theta)}{kz \sin \theta} \right\}^2. \quad (2.124)$$

Once again, the sound intensity has a similar form to that of a monopole in a reflecting ground plane with the exception of the term in brackets which is a directivity factor. The form of the directivity factor is presented in Figure 2.12. It is quite clear that the sound

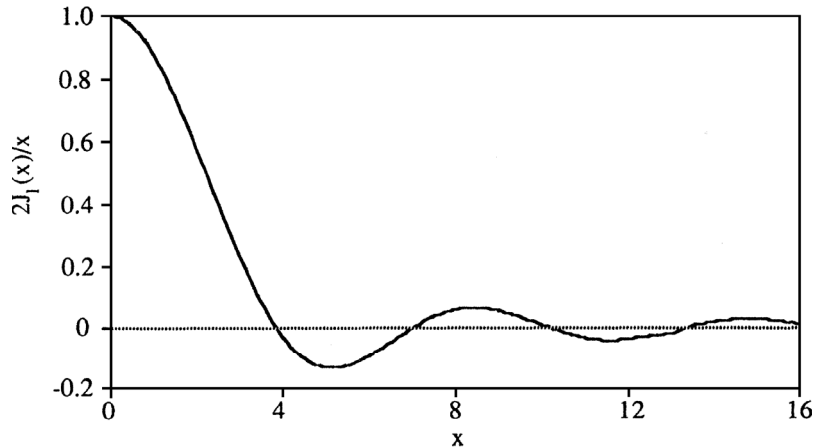


Fig. 2.12. Functional form of the directivity factor for a circular piston in a rigid baffle.

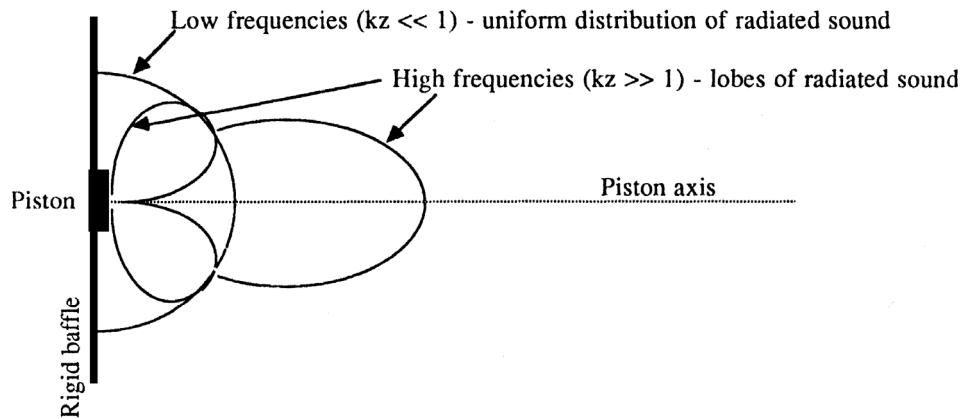


Fig. 2.13. Low and high frequency sound radiation patterns for a circular piston in a rigid baffle.

radiation from the piston is quite directional (except along the axis) and that it increases with frequency. There are several pressure nodes, and this results in a beam pattern of sound radiation. This is schematically illustrated in Figure 2.13. At low frequencies ($kz \ll 1$) the intensity distribution is approximately constant, whereas at high frequencies there are several nodal points and corresponding lobes of radiated sound. Hence, low frequency loudspeakers can be large and still remain omni-directional whereas high frequency loudspeakers need to be small to be relatively omni-directional. The concepts of directivity will be discussed again in chapter 4. This example illustrates how a series of omni-directional sound sources can become directional when combined.

The preceding discussion has been restricted to the acoustic far-field. Now consider an observation point on the piston surface itself. In chapter 1, the concepts of mechanical impedance (force/velocity at point of application of the force) were introduced. In acoustics, when a structure radiates sound due to its vibration, another impedance term has to be included with the mechanical impedance. It is the radiation impedance of the fluid (air) in proximity to the vibrating surface – i.e. the fluid loads the vibrating surface and this alters its vibrational response. The total sound pressure at any arbitrary element on the piston surface is a sum of the pressure due to the vibrating element itself and

the radiated pressures from all the other elements on the piston. The piston velocity, $U = U_p e^{i\omega t}$ is thus given by

$$U = \frac{\mathbf{F}_m}{\mathbf{Z}_m + \mathbf{Z}_r}, \quad (2.125)$$

where $\mathbf{F}_m$ is the applied mechanical force, $\mathbf{Z}_m$ is the mechanical impedance of the piston, and $\mathbf{Z}_r$ is its radiation impedance. $\mathbf{F}_m$ is not to be confused with the force on the piston due to the acoustic pressure fluctuations, $\mathbf{F}_p$. The mechanical impedance is associated with the mechanical driving force and the radiation impedance is associated with the acoustic driving force. It can be seen from equation (2.125) that the radiation impedance ‘fluid-loads’ the surface vibrations of the piston. Fluid loading concepts are very relevant when analysing the vibrational characteristics of structures immersed in fluids – e.g. piping systems filled with liquids, submarines etc. The radiation impedance is thus given by

$$\mathbf{Z}_r = \frac{\mathbf{F}_p}{U}. \quad (2.126)$$

The radiation impedance of a vibrating surface is sometimes defined as the ratio of the sound pressure averaged over the surface to the volume velocity through it (units of $N \text{ s m}^{-5}$ as opposed to $N \text{ s m}^{-1}$ for mechanical impedance) – i.e. some books would define $\mathbf{Z}_r$ as $(F_p/\pi z^2)/\pi z^2 U$ or $F_p/(\pi z^2)^2 U$. In this book, $\mathbf{Z}_r$ is defined, for convenience, in similar units to the mechanical impedance since the radiating surface area, πz^2 , is common to both variables (pressure and volume velocity).

The radiation impedance of a piston can be obtained by integrating the elemental pressure distribution over the surface area of the piston to obtain the total sound pressure at a point and subsequently integrating this again over the surface to obtain the force, $\mathbf{F}_p$. The radiation impedance is thus obtained and given by^{2.1,2.5}

$$\mathbf{Z}_r = \rho_0 c \pi z^2 \{R_1(2kz) + iX_1(2kz)\}, \quad (2.127)$$

where

$$R_1(x) = \frac{x^2}{2 \cdot 4} - \frac{x^4}{2 \cdot 4^2 \cdot 6} + \frac{x^6}{2 \cdot 4^2 \cdot 6^2 \cdot 8} - \dots, \quad (2.128)$$

and

$$X_1(x) = \frac{4}{\pi} \left\{ \frac{x}{3} - \frac{x^3}{3^2 \cdot 5} + \frac{x^5}{3^2 \cdot 5^2 \cdot 7} - \dots \right\}. \quad (2.129)$$

The resistive function, $R_1(x)$, and the reactive function, $X_1(x)$, are plotted in Figure 2.14. The resistive part is real and is due to the radiated sound pressure. The imaginary part is a mass loading term due to the fluid (air or liquid) in proximity to the piston. This mass loading term can become significant when the structure is radiating into liquids.

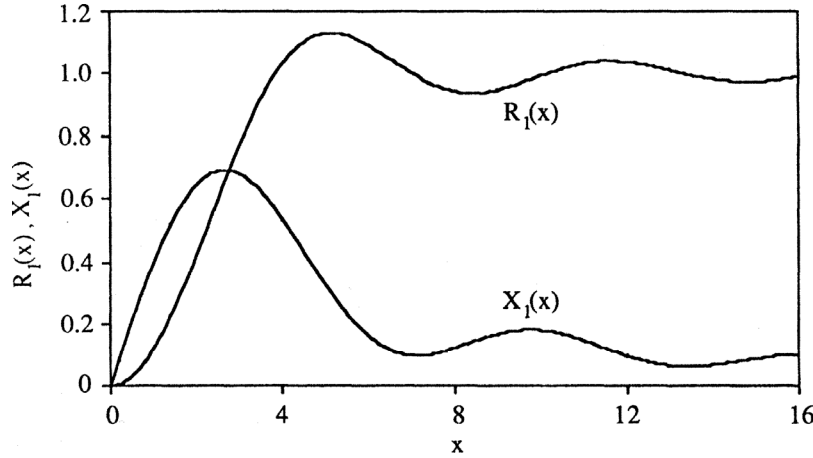


Fig. 2.14. Resistive and reactive functions for the radiation impedance of a circular piston.

The mechanical properties of the piston also affect its sound radiation properties. The equation of motion of the piston is (from Newton's second law)

$$M \frac{\partial U}{\partial t} + C_v U + K_s \int U dt = F_m - F_p, \quad (2.130)$$

where M is the piston mass, C_v is its damping, and K_s is its stiffness. Thus equation (2.125) can be re-written as

$$U = \frac{F_m}{C_v + i(M\omega - K_s/\omega) + \rho_0 c \pi z^2 \{R_1(2kz) + iX_1(2kz)\}}. \quad (2.131)$$

Equation (2.131) clearly illustrates how the vibrational velocity of the piston is a function of (i) its structural damping, (ii) its mass, (iii) its stiffness, (iv) the acoustic radiation resistance, and (v) the acoustic radiation reactance. As mentioned earlier, the acoustic radiation resistance is due to the radiated sound pressure and the acoustic radiation reactance is due to mass loading of the piston by the fluid. For low frequencies, $2kz \ll 1$, thus

$$R_1(2kz) \approx \frac{k^2 z^2}{2}, \quad (2.132)$$

and

$$X_1(2kz) \approx \frac{8kz}{3\pi}. \quad (2.133)$$

For high frequencies, $2kz \gg 1$, thus

$$R_1(2kz) \approx 1, \quad (2.134)$$

and

$$X_1(2kz) \approx \frac{2}{\pi kz}. \quad (2.135)$$

The effects of mass loading can now be estimated. The amplitude of the fluid loaded mass is

$$M_r = \frac{X_r}{\omega} = \frac{\rho_0 c \pi z^2 X_1(2kz)}{\omega} = \frac{\rho_0 \pi z^2 X_1(2kz)}{k}. \quad (2.136)$$

Thus at low frequencies ($2kz \ll 1$)

$$M_r = \frac{8\rho_0 z^3}{3}, \quad (2.137)$$

and at high frequencies ($2kz \gg 1$)

$$M_r = \frac{2\rho_0 z}{k^2}. \quad (2.138)$$

The sound power radiated by the piston can now be estimated from the real part of the radiation impedance – i.e. the acoustic radiation resistance. It could also be obtained by integrating the far-field sound intensity (equation 2.124). From equation (1.162) the real power (rate of energy flow) of the piston is

$$\Pi = \frac{1}{2} U_p^2 \operatorname{Re} [\mathbf{Z}_m + \mathbf{Z}_r]. \quad (2.139)$$

Thus,

$$\Pi = \frac{1}{2} U_p^2 \{C_v + \rho_0 c \pi z^2 R_1(2kz)\}, \quad (2.140)$$

where the first term inside the brackets represents the mechanical power that is dissipated and the second term represents the sound power that is radiated into the surrounding medium. Low and high frequency estimates of the radiated sound power can be readily obtained by substituting equations (2.132) and (2.134) into equation (2.140).

2.3.5 Quadrupoles – lateral and longitudinal

Monopoles were modelled in sub-section 2.3.1 as single oscillating spheres, and dipoles were modelled in sub-section 2.3.2 as two equal spheres oscillating out of phase. A natural extension to these acoustic source models is two dipole sources in close proximity to each other and oscillating 180° out of phase with each other. Such a sound source is called a quadrupole. Whereas a dipole has one axis (i.e. the fluctuating pressure forces act along the axis between the two sources), a quadrupole has two. The two dipoles oscillating out of phase with each other results in no nett addition or subtraction of mass away from the source, and no resultant force – i.e. there is no physical mechanism available for the mass or the momentum to vary. The quadrupole does, however, apply a stress to the medium and it is this fluctuating stress that generates the sound (monopoles generate sound via fluctuating surface velocities or the addition/subtraction of mass from a source region, and dipoles generate sound via fluctuating forces). In gas flows, for instance, quadrupoles are generated by the viscous stresses within the gas.

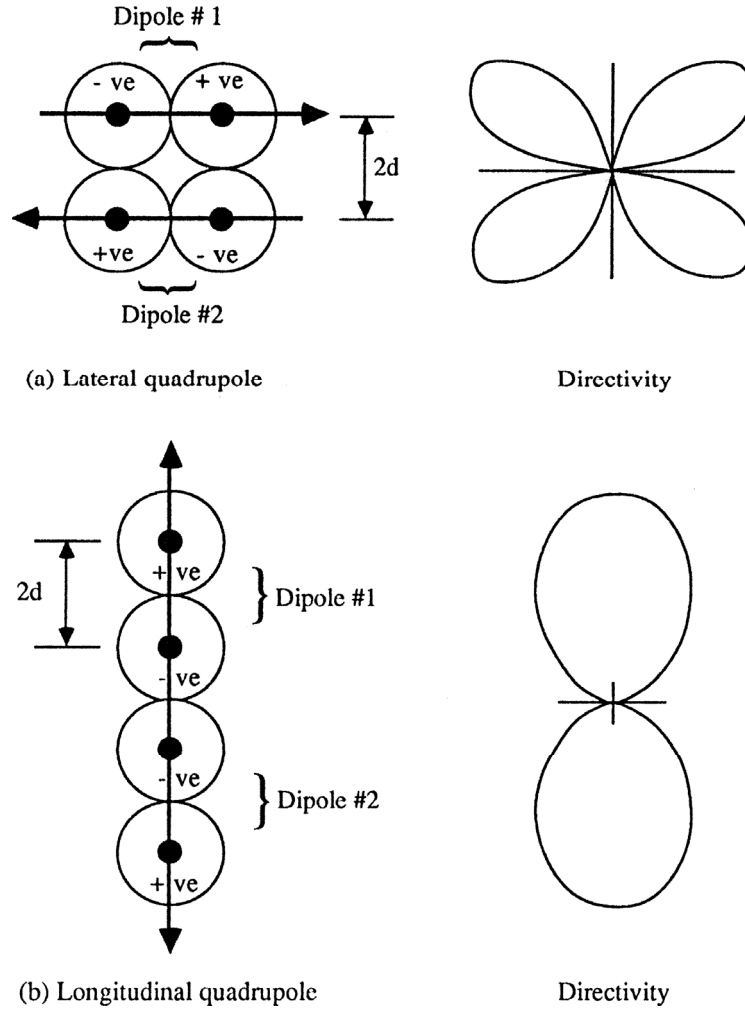


Fig. 2.15. Lateral and longitudinal quadrupoles and their associated directivity patterns.

The relationships for the intensity and sound power radiated by a dipole were obtained by considering the interference between two point monopoles in close proximity to each other. The relationships for the intensity and sound power radiated by a quadrupole can be obtained in a similar manner by considering the interference between two point dipoles in close proximity to each other or by expressing the quadrupole velocity potential as a space derivative of the monopole velocity potential. The mathematical manipulations required to obtain the answers are fairly extensive and are not presented here. The interested reader is referred to Pierce^{2.4} or Reynolds^{2.5}. Two possible combinations of quadrupoles exist. The first is when the two dipole axes do not lie on the same line, and the second is when they both lie on the same line. The former is termed a lateral quadrupole and the latter is termed a longitudinal quadrupole. The two types of quadrupoles and their associated directivity patterns are illustrated in Figure 2.15.

The sound power radiated by a lateral quadrupole is^{2.4,2.5}

$$\Pi = \frac{4Q_{\text{rms}}^2 \rho_0 c d^4 k^6}{15\pi}, \quad (2.141)$$