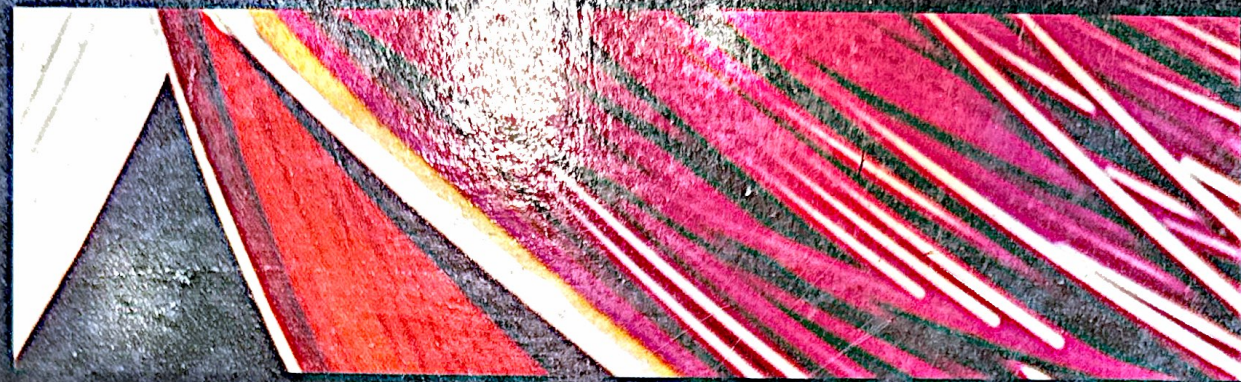
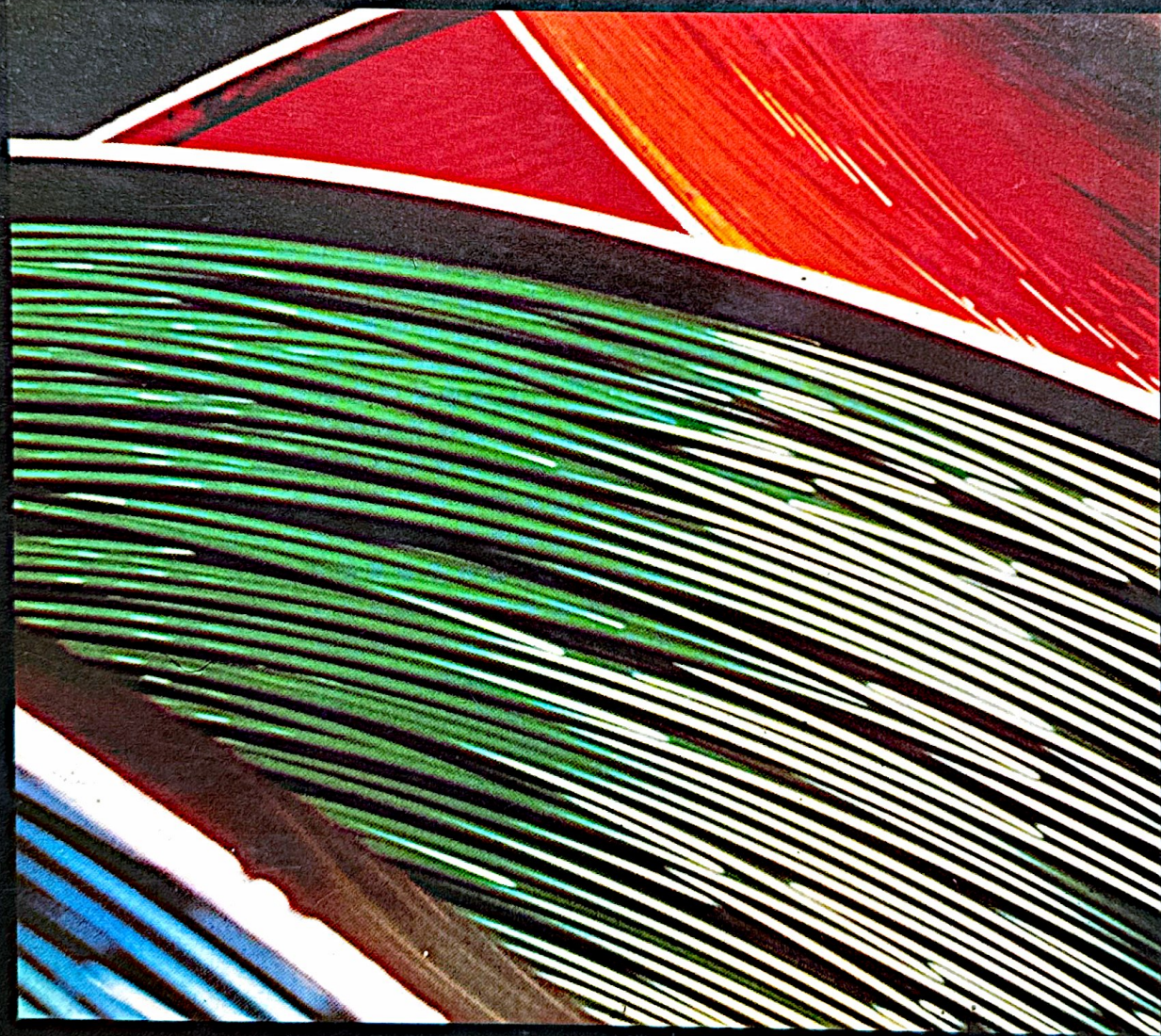




Fiber Optic Communications second edition

Joseph C. Palais



SECOND EDITION

FIBER OPTIC COMMUNICATIONS

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OPTICS REVIEW

This chapter contains basic concepts of classical optics that apply to fiber communications. We call it a review because many of you studied optics in a high school or college physics course. The present material simply consolidates a few fundamentals of rays, waves, and lenses. For those unfamiliar with classical optics, this chapter is a good introduction to some useful topics. The subjects of this chapter are ray theory and the focusing, collimating, imaging, and light-collecting properties of lenses. These subjects apply to the problems of coupling light from sources into fibers and coupling light from one fiber to another.

2-1 RAY THEORY AND APPLICATIONS

A number of optic phenomena (particularly those associated with lenses) are adequately explained by considering light as narrow rays. The theory based on this approach is called *geometrical optics*. These rays obey a few simple rules.

1. In a vacuum, rays travel at a velocity of $c = 3 \times 10^8$ m/s. In any other medium, rays travel at a slower speed, given by

$$v = \frac{c}{n} \quad (2-1)$$

The factor n is the *index of refraction* (or *refractive index*) of the medium. For air and gases, the ray velocity is very close to c , so that $n \approx 1$. At optic frequencies, the refractive index of water is 1.33. Glass has many compositions, each with a slightly different ray velocity. A refractive index of 1.5 is representative for glasses used in fibers. Table 2-1 lists the refractive index for several materials.

2. Rays travel in straight paths unless deflected by some change in the medium.

3. At a plane boundary between two media, a ray is reflected at an angle equal to the angle of incidence, as illustrated in Fig. 2-1. Note that the angles are measured with respect to the boundary normal, that is, the direction perpendicular to the

TABLE 2-1. Index of Refraction for Some Materials

Material	Index of Refraction
Air	1.0
Carbon dioxide	1.0
Ethyl alcohol	1.36
Water	1.33
Fused silica	1.46
Glass	~1.5
Polystyrene	1.59
Silicon	3.5
Germanium	4.0
Sapphire	1.8
Calcite	1.6
Sodium chloride	1.54
Magnesium fluoride	1.38
Zinc sulfide	2.3
Gallium arsenide	3.35
Aluminum gallium arsenide	3.6

surface. This is the conventional notation in optic work. Referring to the drawing, it is clear that

$$\theta_r = \theta_i \quad (2-2)$$

where θ_i is the angle of incidence and θ_r is the angle of reflection.

4. If any power crosses the boundary, the transmitted ray direction is given by *Snell's law*¹

$$\frac{\sin \theta_t}{\sin \theta_i} = \frac{n_1}{n_2} \quad (2-3)$$

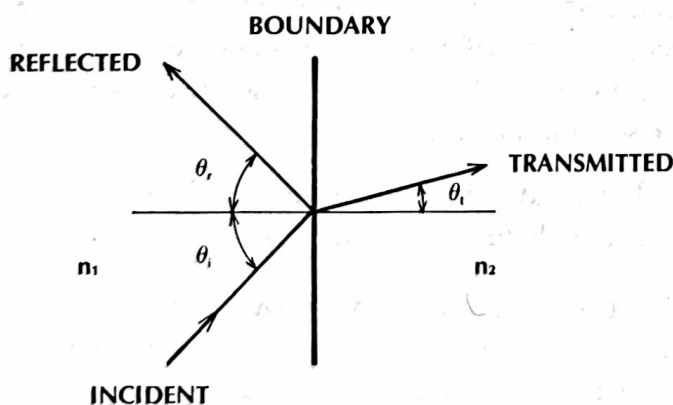


Figure 2-1 Incident, reflected, and transmitted rays at a boundary between two media.

where θ_t is the angle of transmission, and n_1 and n_2 are the refractive indices of the incident and transmission regions, respectively.

The only angles having physical significance in the preceding paragraphs are those lying between 0 and 90°. The trigonometric sine function is plotted in Fig. 2-2 over this range. If n_1 is less than n_2 , then Snell's law predicts that $\sin \theta_t < \sin \theta_i$. Figure 2-2 indicates that smaller angles have lower sine values, so $\theta_t < \theta_i$ in this example. The angle of transmission is less than the angle of incidence. It is helpful when tracing rays from one medium to another to remember this result. We can summarize it as follows:

- (a) The transmitted ray is bent toward the normal when traveling from a medium having a low refractive index into a medium with a higher refractive index.

Figure 2-3 illustrates this situation for a ray entering a glass fiber from air.

If $n_1 > n_2$, Snell's law yields $\sin \theta_t > \sin \theta_i$. So $\theta_t > \theta_i$ and the ray is bent away from the

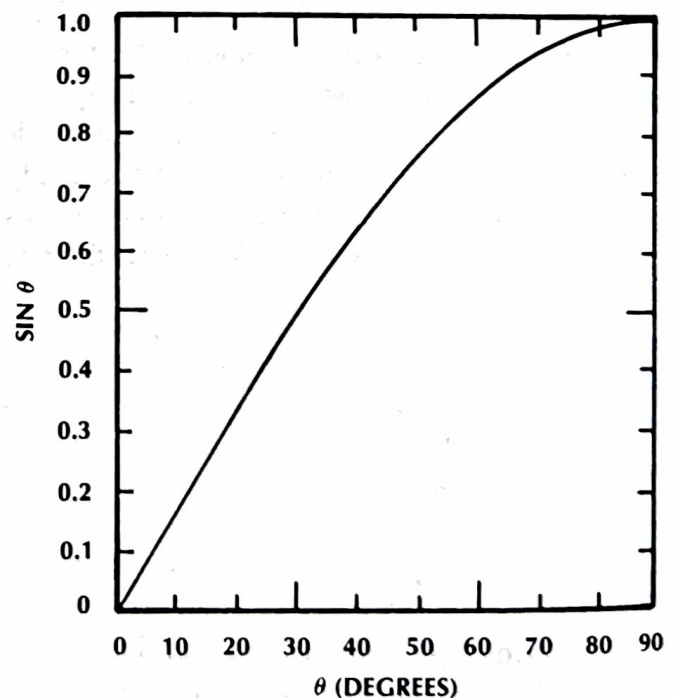


Figure 2-2 The sine function.

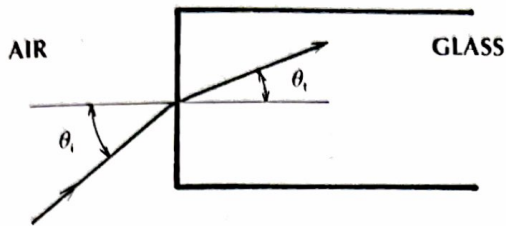


Figure 2-3 Bending of a light ray as it enters a glass fiber.

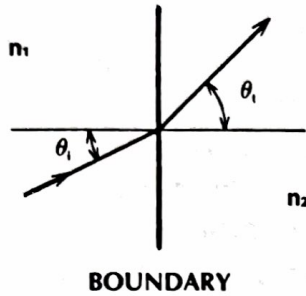


Figure 2-4 When $n_1 > n_2$, the ray is bent away from the normal and toward the boundary surface.

normal, as shown in Fig. 2-4. The result to remember is:

- (b) The transmitted ray is bent away from the normal when traveling from a medium having a high refractive index into a medium with a lower refractive index.

Example 2-1

A light ray proceeds from air ($n_1 = 1$) into glass ($n_2 = 1.5$). Find the transmission angles when $\theta_i = 0^\circ$ (the incident ray is normal to the boundary) and when $\theta_i = 15^\circ$.

Solution:

When the incident angle is 0, then $\sin \theta_i = 0$. Snell's law yields $\sin \theta_r = 0$ and finally $\theta_r = 0^\circ$ itself. The ray is undeflected. When $\theta_i = 15^\circ$, however, $\sin \theta_r = (1/1.5) \sin 15^\circ = 0.17$ and $\theta_r = 9.94^\circ$. As expected, the ray is redirected toward the normal.

Example 2-2

The last ray in Example 2-1 now travels from glass back into air. Assume that this

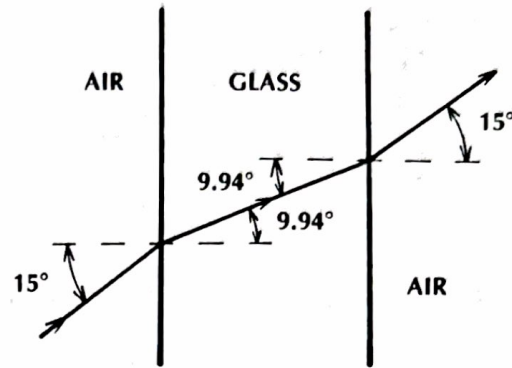


Figure 2-5 A ray is undeflected after transversing a parallel glass plate.

second boundary is parallel to the first one: The new incident angle is 9.94° , as can be determined from Fig. 2-5. Find the direction of the transmitted ray.

Solution:

From Snell's law, $\sin \theta_r = 1.5 \sin 9.94^\circ = 0.259$. Then $\theta_r = 15^\circ$. The transmitted ray is bent away from the normal. In addition, by combining these last two examples, we find that a ray incident on a parallel plate of glass will suffer no net deflection. This will always be true. As illustrated in Fig. 2-5, the ray enters the glass (at 15°), is deflected toward the normal (at 9.94°), and then is deflected back parallel to its original direction (at 15°). The translation (sideways displacement) of this ray will be negligible if the thickness of the glass is small.

2-2 LENSES

Fibers can be tested by sending visible beams of light through them. The simplest tests are those for continuity (checking for a break in the fiber) and more moderate physical damage causing only small losses. A continuity check can be made by observing whether any light emerges from the

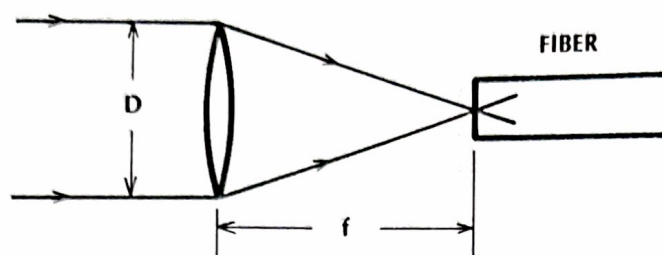


Figure 2-6 Focusing a light beam onto a fiber.

end of the fiber. Cracks or inhomogeneities in a bare fiber (one that is not inside a cable) can be located by observing the scattered light they produce. Gas lasers are convenient for these tests. Because their output beams have diameters on the order of a millimeter (and fibers are much smaller), a lens is used to focus the light onto the fiber end face, as illustrated in Fig. 2-6. To simplify our discussion, we will consider only *thin lenses*. A lens is thin when its thickness is so small that the translation of a ray passing through it is negligible. In other words, rays enter and leave at (approximately) the same distance from the lens axis. We will assume initially that our lenses are ideal, have no absorption or reflection losses, and produce no aberrations. We can add in these complications later if they become important.

In Fig. 2-6, a parallel beam of light (called a *collimated beam*) is focused to a point. This beam is traveling parallel to the lens axis. The incident light is made up of a number of parallel rays. Only the two outermost rays are drawn in the figure. All the rays converge to the position shown, known as the *focal point*. It lies a distance f (called the *focal length*) away from the lens. The plane that passes through the focal point and is perpendicular to the axis of the lens is the *focal plane*. The lens itself has two spherical surfaces. Think of the lens as being constructed by connecting the caps of two solid glass spheres. The radii (or curvatures) of these spheres are R_1 and R_2 . The lens has diameter D and refractive index n . Its focal length is found from²

$$\frac{1}{f} = (n - 1) \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \quad (2-4)$$

The ratio f/D is called the *f-number* of the lens.

A long focal length is easily obtained by having large curvatures (R_1 and R_2), that is, by having fairly flat surfaces for the lens. A small focal length lens is harder to design because the curvatures have to be small, resulting in a small lens. A limiting case illustrates the problem. Consider a lens that is a complete sphere of glass. If the lens diameter is fixed, this design yields the smallest possible curvatures, and thus, the smallest possible focal length. In this example, the radius of the sphere is the lens curvature, and the lens diameter is twice the curvature. That is, $R_1 = R_2 = D/2$. Using these values in Eq. (2-4) yields

$$f = \frac{D}{4(n - 1)}$$

For $n = 1.5$ we conclude that the lens *f-number* is $f/D = 0.5$, showing that a small focal length requires a small lens diameter. A lens such as this would have severe spherical aberrations, causing the focused spot in Fig. 2-6 to smear out considerably. Most lenses have *f-numbers* greater than 0.5 to correct this problem, making it even more difficult to obtain small focal lengths. When a lens is used to couple light from a gas laser beam into a fiber, aberrations may be unimportant. This is because the fiber diameter, although small, is not infinitesimal. For coupling, the beam does not have to be focused to a point; it must only be reduced to a size smaller than the fiber core.

Parallel rays of light that are incident at some angle relative to the lens axis are focused in the focal plane, as shown in Fig. 2-7. The position of the focused point is determined by the intersection of the central ray with the focal plane. The *central ray* (a ray directed toward the center

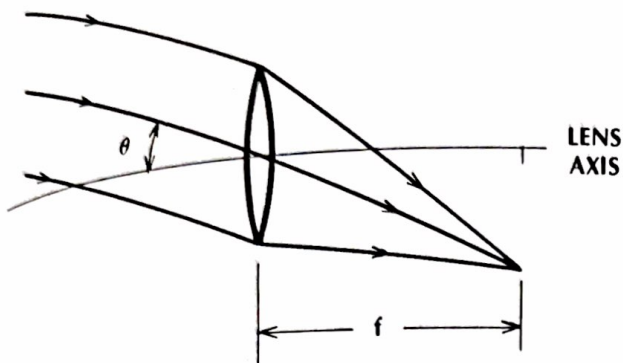


Figure 2-7 Focusing of an off-axis beam.

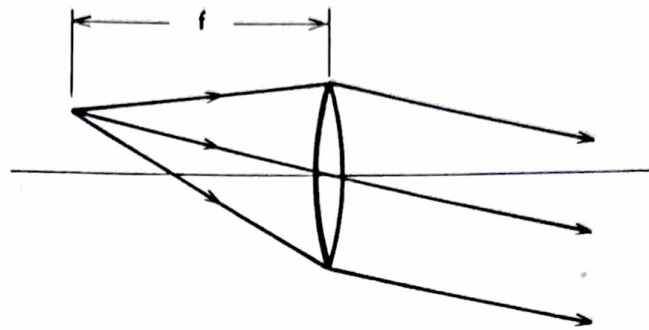


Figure 2-9 Collimating an off-axis point source.

The rules for tracing rays through a thin lens are illustrated in Fig. 2-10. They are:

1. Rays traveling through the center of the lens are undeviated.
2. Incident rays traveling parallel to the lens axis pass through the focal point after emerging from the lens.
3. An incident ray traveling parallel to a central ray intersects that ray in the focal plane after transmission through the lens.
4. An incident ray passing through the focal point travels parallel to the lens axis after it emerges from the lens.

These rules will enable you to trace rays for focusing, collimating, and imaging using thin lenses.

Cylindrical lenses have surfaces that are portions of cylinders, as drawn in Fig. 2-11. This lens only deflects rays in one direction (in Fig. 2-11 this is vertically). The cylindrical lens is a one-dimensional version of the spherical lens. In fact, Eq. (2-4) is true for the cylindrical lens, where R_1 and R_2 are now the curvatures of the cylindrical faces. The focal length f locates a line

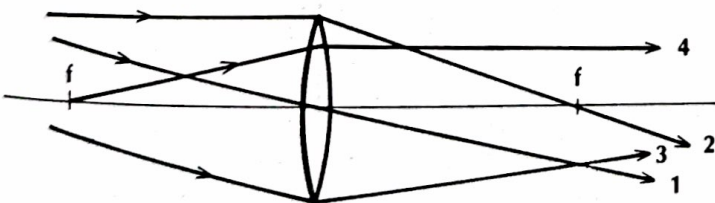


Figure 2-10 Ray paths through a thin lens. The numbers refer to the rules, listed in the text, which they illustrate.

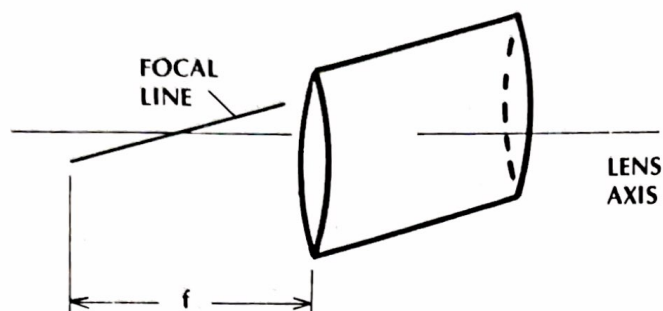


Figure 2-11 Cylindrical lens.

(the *focal line*) that is parallel to the axes of the cylindrical surfaces, a distance f from the lens, and passes through the lens axis (see Fig. 2-11). Light from a line source located along the focal line will be collimated by the lens. The ray paths are drawn in Fig. 2-12. Similarly, a collimated beam of rays entering the lens and traveling parallel to the lens axis will be focused to a line a focal length away from the lens.

It is interesting to consider the effects of a cylindrical lens on a point source located along the focal line. The light emerging from the lens will be collimated vertically but will continue to expand horizontally. Figure 2-13 illustrates this effect. The point to remember is that cylindrical lenses act like spherical lenses in one direction and have no effect in the orthogonal direction. This property is useful for fiber optics, because the light emitted from laser diodes and LEDs is often radiated nonsymmetrically. That is, the emitted light spreads more quickly in one direction (vertically) than the other (horizontally). A

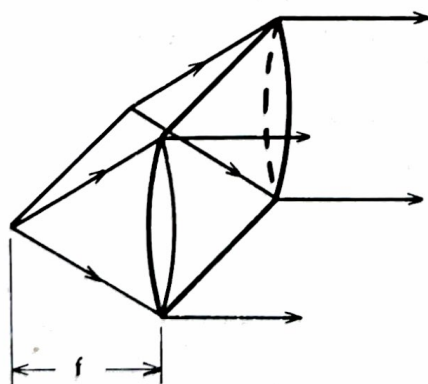


Figure 2-12 Collimating a line source.

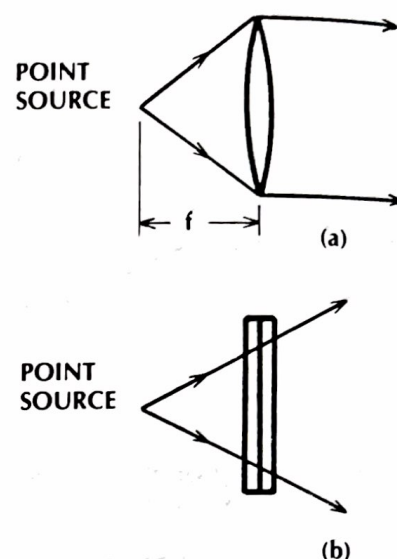


Figure 2-13 Point source and cylindrical lens. (a) Side view, showing collimation. (b) Top view.

cylindrical lens can make the beam spreading more symmetrical by reducing the larger of the two divergence angles. This possibility is sketched in Fig. 2-14; the source emission has negligible beam spread in one of the directions.

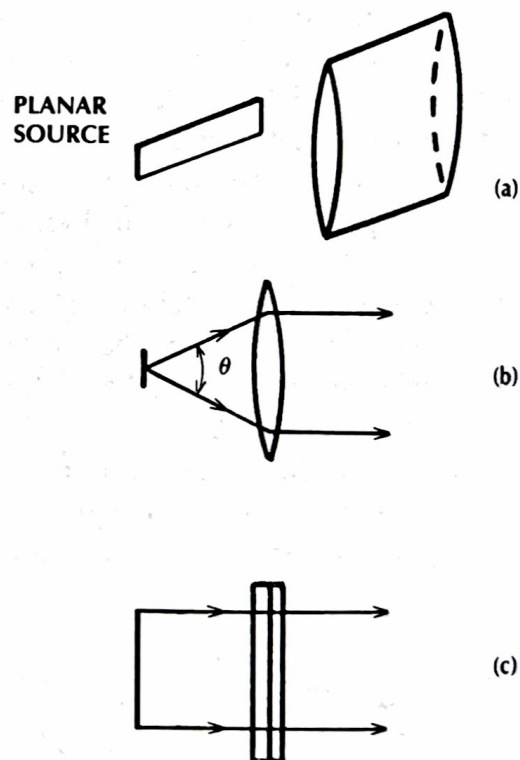


Figure 2-14 (a) Light from an unsymmetrical source is collimated by a cylindrical lens. (b) Side view. (c) Top view.

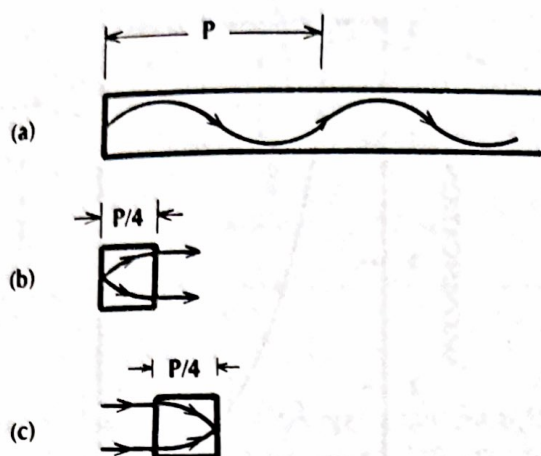


Figure 2-15 Graded-index rod. (a) Typical ray path. (b) A quarter-pitch lens collimates light emerging from a point. (c) A quarter-pitch lens focuses a collimated light beam.

The *graded-index rod lens*, or GRIN rod lens, is a modern development that has been applied to fiber systems in a number of ways.³ The graded-index rod has a refractive index that decreases with distance from its axis. This causes light rays to travel in sinusoidal paths (see Fig. 2-15). (Section 5-2 contains a more extensive discussion of ray travel in a graded-index rod.) The length of one complete cycle is called the lens *pitch* P . Notice what will happen if we cut a length of rod equal to a *quarter-pitch*. The light from a point source located at the center of this rod will be collimated as shown in Fig. 2-15(b). Collimated light entering this lens will be focused as in Fig. 2-15(c). Evidently, the GRIN rod has focusing and collimating properties in common with the classical spherical lens. The GRIN rod is also useful for imaging. The rod is desirable because small focal lengths can be obtained, permitting construction of short, solid optic struc-

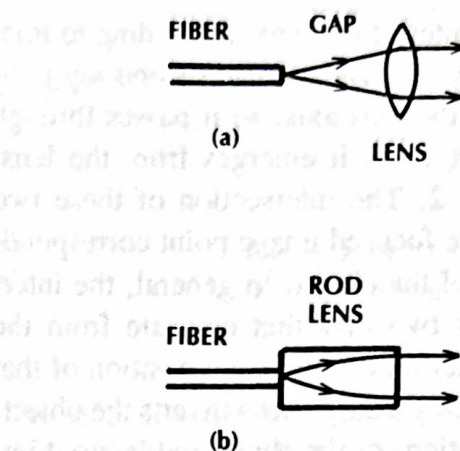
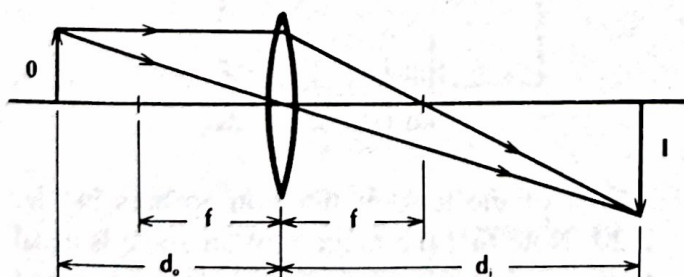


Figure 2-16 Collimating the light radiated from a fiber using (a) a spherical lens and (b) a GRIN rod lens.

tures. For example, the light emitted from the end of a fiber can be collimated by a conventional lens [Fig. 2-16(a)] or by a rod lens [Fig. 2-16(b)]. Using the spherical lens, an air gap exists between the fiber and the lens. The rod lens needs no gap. The fiber can be cemented to the rod, yielding a continuous, solid mechanical structure. The rod collimator would be easier to assemble, align, and maintain than the spherical lens collimator.

2-3 IMAGING

Imaging by a thin lens is illustrated in Fig. 2-17. The object is an arrow with height O , located a distance d_o from the lens. The image is an arrow with height I , a distance d_i from the lens. The position of the image is found by tracing the rays emitted from the tip of the object. One ray passes

Figure 2-17 Image formation by a thin lens.

through the center of the lens. According to tracing rule 1, it is undeviated. The second ray travels parallel to the lens axis, so it passes through the focal point when it emerges from the lens, following rule 2. The intersection of these two rays defines the focused image point corresponding to the tip of the object. In general, the intersection of any two rays that emanate from the same point determines the image position of that point. Notice how a single lens inverts the object.

The positions of the object and focused image are related by the *thin-lens equation*,⁴

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \quad (2-5)$$

The *magnification* M is the ratio of the size of the image to that of the object. It is given by

$$M = \frac{d_i}{d_o} \quad (2-6)$$

The magnification may be greater than, equal to, or less than unity.

Example 2-3

Find the object and image distances if the magnification is unity.

Solution:

If $M = 1$, then $d_i = d_o$. In the thin-lens equation, then,

$$\frac{1}{d_o} + \frac{1}{d_o} = \frac{1}{f}$$

or, $d_o = 2f$. Finally, d_i also equals $2f$.

Equations (2-5) and (2-6) can be combined to show the direct relationship between magnification and object distance. The result is

$$M = \frac{1}{(d_o/f) - 1} \quad (2-7)$$

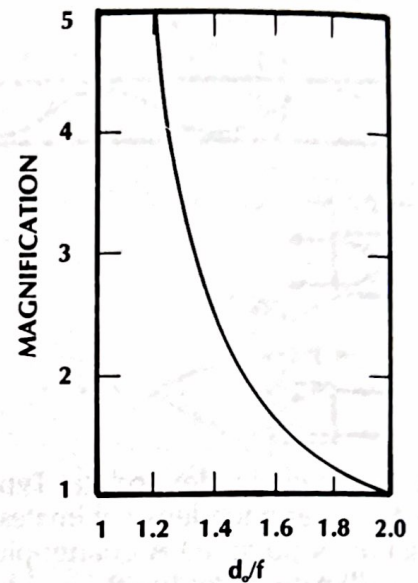


Figure 2-18 Magnification as a function of object position.

This equation is plotted in Fig. 2-18. For magnifications greater than unity, the range of object positions is

$$1 < \frac{d_o}{f} < 2 \quad (2-8)$$

When using lenses to couple light into fibers, it is important to keep track of the angles at which the light rays travel. Referring to Fig. 2-19, we note the angular light spread (α_o) at the object and the resulting angular spread (α_i) at the image for an object point on the lens axis. Figure 2-19 is the same as Fig. 2-17, except now we are considering angular changes rather than size changes. The thin-lens equation still predicts the image position in terms of the object distance and the lens focal length. A little trigonometry shows that

$$\frac{\tan(\alpha_i/2)}{\tan(\alpha_o/2)} = \frac{1}{M} \quad (2-9)$$

A plot of the tangent function appears in Fig. 2-20. Note that the tangent of an angle is equal to the angle itself when that angle is small and

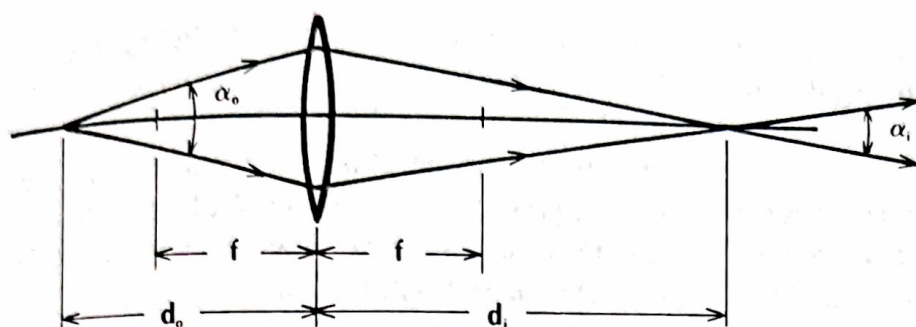


Figure 2-19 Angular changes due to imaging.

expressed in radians. This approximation is very good (less than 4% error) up to 20° (0.35 rad). Conversion between radians and degrees is made using the relationship 1 radian = 57.3 degrees. Assuming small angles, we can replace the tangent functions in Eq. (2-9) with the angles themselves, yielding

$$\frac{\alpha_i}{\alpha_o} = \frac{1}{M} \quad (2-10)$$

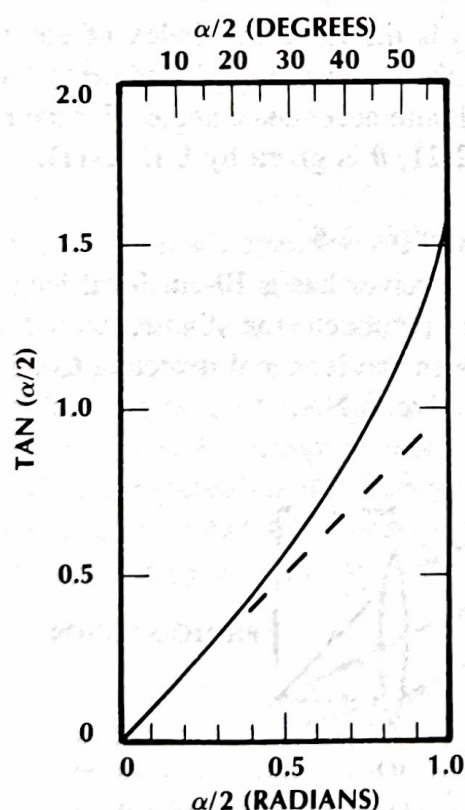


Figure 2-20 The tangent function is shown by the solid line. The dashed line represents the approximation $\tan(\alpha/2) = \alpha/2$.

This equation can be used when the cone half angles are less than 20° , that is, for full angular beam spreads (α_i or α_o) as much as 40° . Although Eq. (2-10) was developed with α_i and α_o measured in radians, it is also correct when α_i and α_o are expressed in degrees.

We may conclude from Eq. (2-10) that an increase in object size due to magnification is accompanied by a decrease of the beam spread. The imaging lens tends to collimate the rays of light radiating from the object. Because laser diodes and LEDs radiate over large angles, and fibers accept rays only over small angles, lenses can increase the efficiency of coupling between sources and fibers.

Example 2-4

A source radiates light uniformly over a region having a 40° full cone angle. The source is a square, planar radiator measuring $20\ \mu\text{m}$ on a side. Design a lens system that will decrease the beam spread to a 10° cone. Determine the image size.

Solution:

The imaging system is the same as that in Fig. 2-19. In this example, $\alpha_o = 40^\circ$ and $\alpha_i = 10^\circ$. We may use the approximate result in Eq. (2-10) because $\alpha_o/2$ is only 20° . Equation (2-10) shows that the magnification is 4, making the planar image $4(20) = 80\ \mu\text{m}$ on a side. For a magnification of 4, Eq. (2-7) or Fig. 2-18 shows that $d_o/f = 1.25$. If we choose a lens hav-

ing focal length 10 cm, then $d_o = 12.5$ cm. Finally, Eq. (2-6) yields the image distance $d_i = Md_o = 50$ cm.

In Example 2-4 a more compact arrangement will be obtained if we choose a much smaller focal length. With $f = 1$ mm, the object and image distances become just 1.25 mm and 5 mm, respectively. An increase in source size from 20 to 80 μm would be acceptable when coupling light into a fiber having a core diameter of 100 μm or more.

Coupling from a source that is larger than the fiber presents a bigger problem. If we attempt to demagnify the source size ($M < 1$), the angular spread will increase as predicted by Eq. (2-10). The fiber might not accept rays over this expanded range.

2-4 NUMERICAL APERTURE

An important characteristic of an optic system is its ability to collect light incident over a wide range of angles. Figure 2-21 illustrates an optic receiver consisting of a lens and a photodetector. The lens is much larger than the detector surface, so it intercepts more rays than the detector would by itself. The lens focuses this light onto the detector. Together, the lens and detector make an efficient collection system. It is easy to locate the

position on the detector where the light is focused by using ray tracing rule 1. Simply extend the incident ray passing through the center of the lens until it hits the detector, as illustrated in the figure. Applying this rule to Fig. 2-21, it is apparent that rays incident at large angles will not strike the detector and will be lost.

Referring to the figure, the *maximum acceptance angle* is determined from

$$\tan \theta = \frac{d}{2f} \quad (2-11)$$

where d is the diameter of the circular photodetector surface and f is the lens focal length. Because of the circular symmetry of the receiver, it will detect light incident within a cone having half angle θ . The *numerical aperture* (NA) is defined to be⁵

$$\text{NA} = n_0 \sin \theta \quad (2-12)$$

where n_0 is the refractive index of the material between the lens and the photodetector, and θ is the maximum acceptance angle. For the receiver in Fig. 2-21, θ is given by Eq. (2-11).

Example 2-5

A receiver has a 10-cm focal length, a 1-cm photodetector diameter, and air between the lens and detector. Compute the receiver's NA.

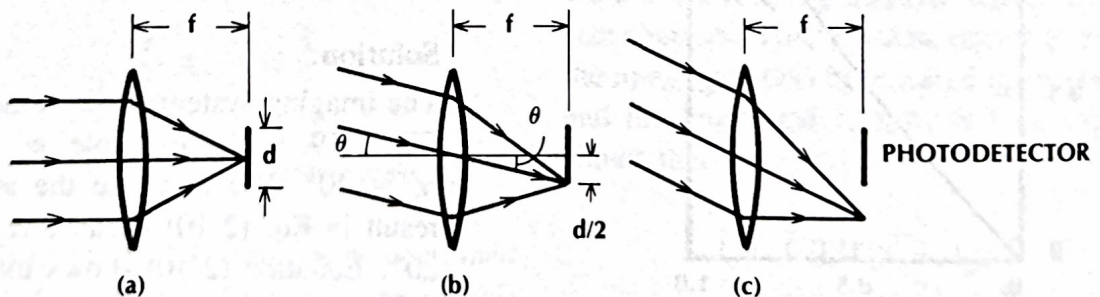


Figure 2-21 An optical receiver with the photodetector placed in the focal plane of the lens. (a) Light is incident parallel to the lens axis. (b) The light rays are at the extreme angle for reception (acceptance angle). (c) The incident rays are beyond the system acceptance angle.

Solution:

Because $d/2f$ is so small, we can use the small-angle approximation $\sin \theta \approx \tan \theta$. (The error is less than 6% if θ is less than 20° .) In this case, with $n_0 = 1$, Eq. (2-12) yields

$$NA = \sin \theta = \frac{d}{2f} = 0.05$$

This corresponds to an acceptance angle θ of 2.87° . The full cone angle is twice this value, 5.74° .

The definition of numerical aperture given in Eq. (2-12) applies to all light-collecting systems, including optic fibers. The collection cone for a fiber is sketched in Fig. 2-22. Light rays incident at angles outside this cone will not propagate along the fiber, but instead will attenuate rapidly. The numerical aperture is usually measured with air in front of the fiber, so $n_0 = 1$ in Eq. (2-12) and

$$NA = \sin \theta \quad (2-13)$$

A plot of this equation appears in Fig. 2-23. A low NA indicates a small acceptance angle. Because of this, coupling to a low-NA fiber is more difficult (mechanical alignment is more sensitive) and less efficient (some of the rays are outside the acceptance angle) than coupling to a high NA fiber. Lenses can be used (as we showed earlier) to reduce the beam spread and, consequently, to improve the coupling efficiency. Typically, fibers

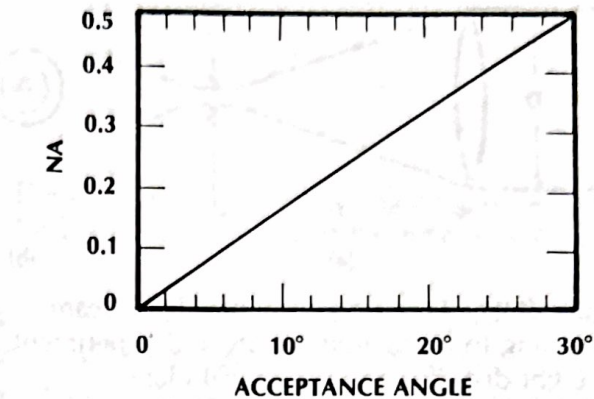
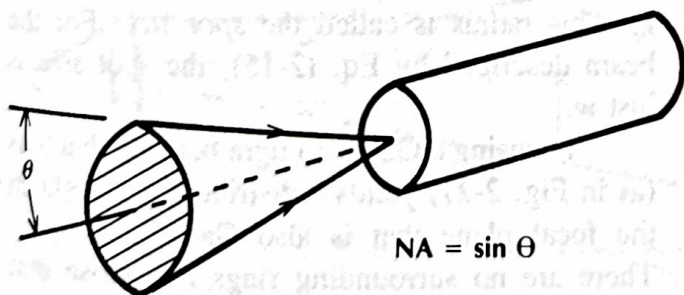


Figure 2-23 Numerical aperture and acceptance angle. $NA = \sin \theta$.

for long-distance communications have numerical apertures from 0.1 to 0.3. Fibers for shorter paths have numerical apertures that are a bit higher (typically, 0.4 to 0.5).

2-5 DIFFRACTION

In some experiments, geometrical optics (ray theory) correctly predicts the gross results but does not agree with the fine details of the observation. In other experiments, even the gross behavior is predicted erroneously. In these instances, a more complete theory based on the wave nature of light is needed to explain the observed phenomena. This theory is called *diffraction* or *physical optics*. We can say that diffraction is the deviation from the predictions of geometrical optics. A few important examples requiring diffraction analysis will be presented in the following paragraphs.

Figure 2-6 shows a lens focusing a uniform light beam to a point. Diffraction theory and care-

Figure 2-22 The fiber will only accept light rays incident within a cone having half-angle θ .

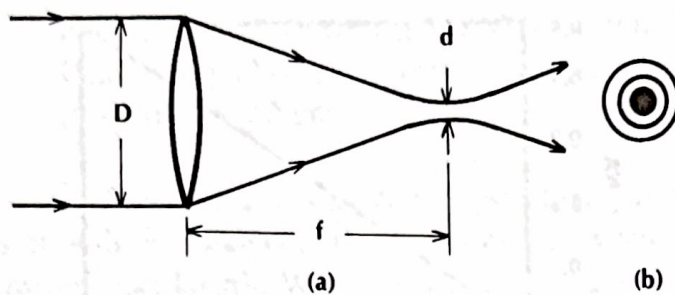


Figure 2-24 Focusing a uniform light beam according to diffraction theory and experiment. (b) Light distribution in the focal plane.

ful experimentation show the beam does not converge to a point, but instead reduces to a central spot of light surrounded by rings of steadily diminishing intensity. The central spot has diameter⁶

$$d = \frac{2.44\lambda f}{D} \quad (2-14)$$

where λ is the wavelength, f is the focal length, and D is the lens diameter. Figure 2-24 illustrates the situation. The central spot is normally pretty small. For example, if $f = 2D$ and the wavelength is $1 \mu\text{m}$, then Eq. (2-14) predicts a spot diameter of $4.88 \mu\text{m}$. This may be negligible in some applications, meaning that the geometrical optics treatment is sufficient. On the other hand, suppose that this beam and lens are used to couple light into a fiber having a diameter less than $4 \mu\text{m}$ (as in Fig. 2-25), or into a glass film with a thickness less than $4 \mu\text{m}$. The coupling efficiency will be low, because the focused spot is larger than the fiber (or the film). Clearly, diffraction theory is required to explain the results of these experiments.

The plane perpendicular to the direction of

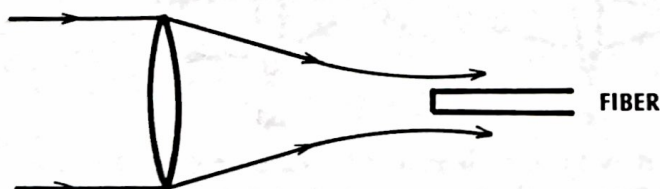


Figure 2-25 Focusing a beam onto a small fiber may result in inefficient coupling.

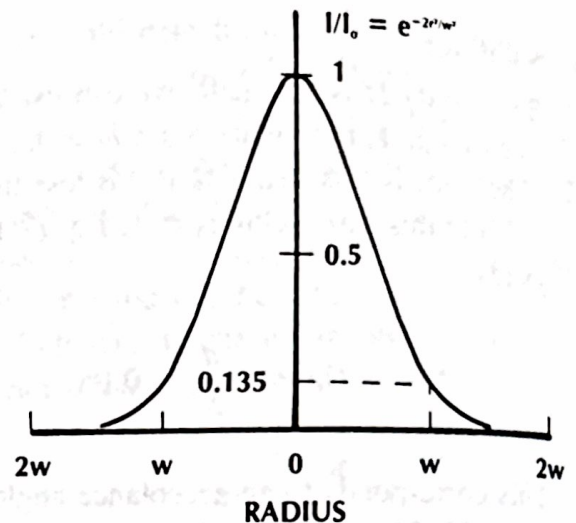


Figure 2-26 The Gaussian intensity distribution.

wave travel is called the *transverse plane*. Actual light sources often produce nonuniform beams. The intensities vary across the transverse plane. A particularly important *transverse pattern* is the *Gaussian distribution*. This is the familiar bell curve drawn in Fig. 2-26. Most gas lasers, and some specially designed laser diodes, radiate in this pattern. Very small fibers (having diameters of a few micrometers) also have their light distributed in this manner. The Gaussian intensity distribution is given mathematically by

$$I = I_0 e^{-2r^2/w^2} \quad (2-15)$$

The number $e = 2.718$ is the base of the natural logarithm. Because $e^0 = 1$, I_0 is the intensity at the center of the beam ($r = 0$). When viewed by the eye, this pattern appears to be a circle of light. The edges of the circle are not sharp, but instead, the light intensity drops off gradually. An accepted definition of the radius of the spot is the distance at which the beam intensity has dropped to $1/e^2 = 0.135$ times its peak value I_0 . This radius is called the *spot size*. For the beam described by Eq. (2-15), the spot size is just w .

Focusing a Gaussian light beam with a lens (as in Fig. 2-27) yields a distribution of light in the focal plane that is also Gaussian shaped. There are no surrounding rings like those that

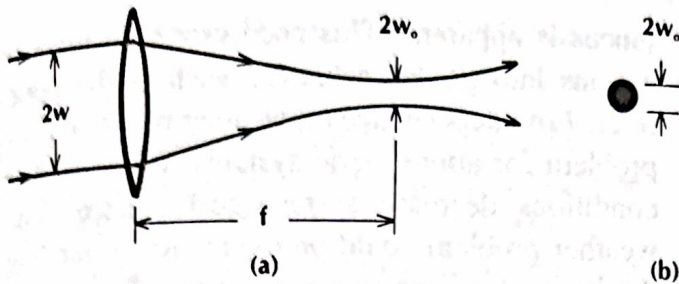


Figure 2-27 (a) Focusing a Gaussian light beam. (b) The spot appearing in the focal plane.

appear when focusing a uniform beam. The spot size in the focal plane is

$$w_o = \frac{\lambda f}{\pi w} \quad (2-16)$$

and the intensity distribution is $I = I'_o \exp(-2r^2/w_o^2)$. The size of the focused Gaussian spot is not much different from the size of the central spot obtained by focusing a uniform beam. This can be seen by writing Eq. (2-16) in the same form as Eq. (2-14). We do this by defining the focused spot diameter as $d = 2w_o$ and the incident beam spot diameter as $D = 2w$. Making these replacements in Eq. (2-16) yields $d = 4\lambda f/\pi D = 1.27 \lambda f/D$, which is comparable to Eq. (2-14). We conclude that the shape of the incident beam does not greatly change the degree to which light can be concentrated.

Next we see what corrections to ray theory are needed when collimating a beam. Referring to Fig. 2-28, a small source is located at the focal point of a lens. Ray theory predicts a parallel beam of light will emerge from the lens. If the light distribution is Gaussian, the beam just to the right of the lens is given by $I = I_o \exp(-2r^2/w^2)$. Diffraction theory agrees with the

geometrical prediction of collimation, if we restrict our observation to regions close to the lens. For longer distances, diffraction theory shows that the beam diverges at a constant full angle given by

$$\theta = \frac{2\lambda}{\pi w} \quad (2-17)$$

where θ is in radians. Experiments verify this result. The radiated field pattern is $I = I'_o \exp(-2r^2/w_o^2)$ where $w_o = \lambda z/\pi w$.

Example 2-6

Consider a Gaussian beam whose spot size is 1 mm when collimated. The wavelength is $0.82 \mu\text{m}$. Compute the divergence angle. Also find the spot size at 10 m, 1 km, and 10 km.

Solution:

The divergence angle is

$$\theta = \frac{2(0.82 \times 10^{-6})}{\pi(10^{-3})} = 0.55 \times 10^{-3} \text{ rad}$$

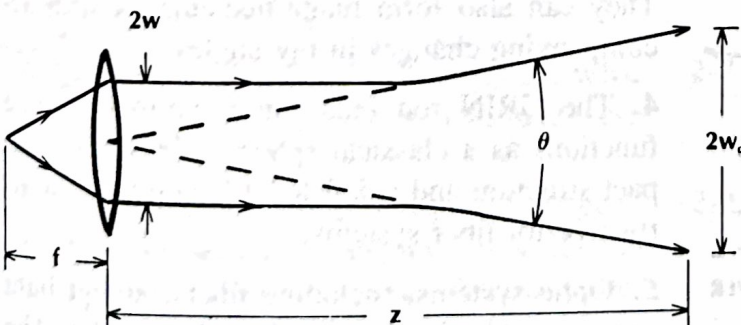


Figure 2-28 Collimating a Gaussian beam.

or $\theta = 0.032^\circ$. The spot size at 10 m is

$$w_o = \frac{(0.82 \times 10^{-6}) 10}{\pi (10^{-3})}$$

$$= 2.6 \times 10^{-3} \text{ m} = 2.6 \text{ mm}$$

At 1 km the spot size is 260 mm and at 10 km it is 2.6 m.

The preceding equations and Example 2-6 illustrate several interesting results. Equation (2-17) shows that very small divergence angles are obtained when the spot size is much larger than the wavelength. Optic wavelengths are so small that this condition is easily achieved. Figure 2-28 is the optic analog of a radio-frequency transmitting antenna. In fact, Eq. (2-17) applies qualitatively to an antenna whose largest dimension is on the order of $2w$. In general, the divergence of a beam radiated at any wavelength is inversely proportional to the size of the radiator as measured in wavelengths. Transmitters that emit narrow beams are many wavelengths long. At radio frequencies, such an antenna must be very large. We conclude that optic transmission provides narrow, highly directed beams.

An atmospheric communication system is drawn in Fig. 2-29. Because of divergence over a long path, the beam at the receiver may be quite large, in fact, a lot larger than the receiving lens itself. Much of the transmitted power will be lost in this case. Although atmospheric systems perform acceptably over short paths, the desirability of more efficient power transfer over long dis-

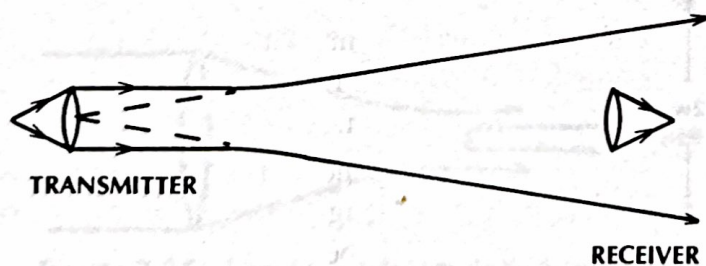


Figure 2-29 Atmospheric transmission link.

tances is apparent. This need prompted investigations into guided schemes, such as the optic fiber. Loss dependence on weather poses another problem for atmospheric systems. Poor weather conditions decrease system performance. The weather problem could be overcome by sending the beam down an evacuated pipe. This would be acceptable if the beam were truly collimated. Since it is not, the expanding beam would strike the sides of the pipe, undergoing losses by absorption, scattering, and imperfect reflection. Using the numbers generated in Example 2-6, a 1-km pipe would need a radius of more than 260 mm to keep the light from contacting its sides. A pipe this large is clearly unacceptable. A short length of tubing (a few centimeters) is practical for protecting a narrow optic beam from its surroundings, because the beam will not expand much over a very short path.

2-6 SUMMARY

This summary contains two lists: one mentions major points we learned from the discussions presented; the other contains new topics suggested by these discussions. First, the list of what we know:

1. Rays travel through a medium at a speed determined by the material's refractive index n .
2. Rays are deflected according to Snell's law when crossing boundaries.
3. Lenses can focus and collimate light beams. They can also form magnified images with accompanying changes in ray angles.
4. The GRIN rod lens can perform the same functions as a classical spherical lens. Its compact structure and small focal length make it attractive for fiber systems.
5. Optic systems, including fibers, accept light only over a limited range of incident angles. The

numerical aperture is a measure of this characteristic.

6. Diffraction tells us that light cannot be focused to an infinitesimally small point and cannot be perfectly collimated. The first of these results applies to attempts at coupling light into very small fibers by focusing, and the second applies to construction of unguided optic communications systems.

7. The Gaussian intensity distribution often occurs in laser and fiber systems. We should be familiar with this pattern.

8. Atmospheric optic communications systems are practical. Over short unobstructed paths, they may be preferable to guided systems. For long paths in which waveguides cannot be installed (for example, communications between satellites), they may provide a viable system. Generally, however, many more applications exist for fiber systems than for unguided systems.

Here is the list of the new topics. These subjects are addressed in succeeding chapters.

1. The propagation properties of light beams within an optic fiber. The velocity given in Eq. (2-1) applies to the speed of light in an unrestricted medium.
2. How much light is transmitted and reflected at a boundary.
3. How to design a lens system for efficient coupling from sources to fibers and between fibers. How to compute the resulting efficiencies.
4. How to design with GRIN rod lenses. What specific devices can profitably include these lenses.
5. Why fibers do not accept rays at all angles. How the numerical aperture of a fiber is computed. How coupling efficiency depends on NA.

These topics are important in the design and evaluation of components, such as couplers, connectors, and multiplexers. These topics will also help us understand how fibers guide beams of light.

PROBLEMS

- 2-1. Image a point source of light with a single lens of focal length f . The point source radiates within a cone having a full angle α_o . Compute the angular light spread (α_i) at the image in terms of the image and object distances and the angular light spread of the source. If $\alpha_o = 40^\circ$ and the magnification is 5, compute α_i .
- 2-2. Plot numerical aperture versus acceptance angle over the range $0 \leq \text{NA} \leq 0.7$. Assume that the refractive index of the surrounding material is 1.0 in the calculation.
- 2-3. Plot magnification versus the normalized object distance d_o/f .
- 2-4. Let the focal length of an imaging lens be 20 mm. Plot the object distance versus the image distance.
- 2-5. A uniform collimated beam is focused by a lens whose focal length is 20 mm and whose diameter is 10 mm. The wavelength is $0.8 \mu\text{m}$. Compute the focused spot size.
- 2-6. A Gaussian beam has a spot size of 1 mm and wavelength of $0.8 \mu\text{m}$. Compute the spot size when focused by a lens whose focal length is 20 mm.
- 2-7. Plot the normalized intensity of a Gaussian beam versus distance from the beam axis if the spot size is 1 mm.
- 2-8. Compute the divergence angle of a Gaussian beam of wavelength $0.8 \mu\text{m}$ and spot size 1 mm. If this beam is aimed at the moon, what is its spot size on the moon's

surface? What is its spot size at distances of 1 km and 10 km?

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