

Optical perfect shuffle

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Received 25 May 1985.

0003-6935/86/101530-02\$02.00/0.

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Recently some proposals for an optical perfect shuffle (PS) have been made.¹⁻⁴ The reason for this interest is that the PS is a useful concept in computer architecture. The main problem in multiprocessor configurations is the communication between the processing elements. An ideal interconnection network has to be fast and flexible, and the PS is the basis for such a network.

The PS performs a certain permutation of $N = 2^k$ elements (see Fig. 1). If the addresses of the elements are represented by binary numbers, ranging from 0 to $N - 1$, the PS can be described as a cyclical rotation to the left of the address bits. Mathematically the PS is defined by the following expression:

$$n' = 2n + [2n/N] \bmod N; \quad n = 0, 1, \dots, N - 1. \quad (1)$$

The brackets [] indicate the largest integer less than or equal to the arguments.

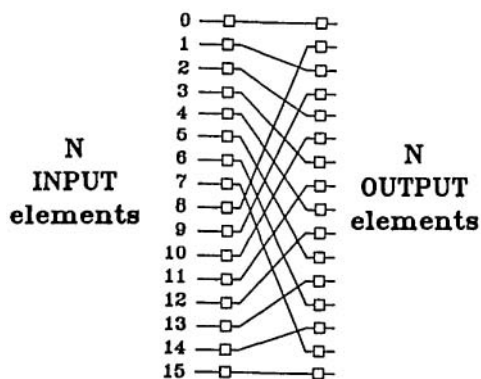


Fig. 1. Perfect shuffle permutation for $N = 16$ elements. The small boxes represent processing elements (PE), numbered from 0 to 15. As an example the output of PE 2 is connected to the input of PE 4.

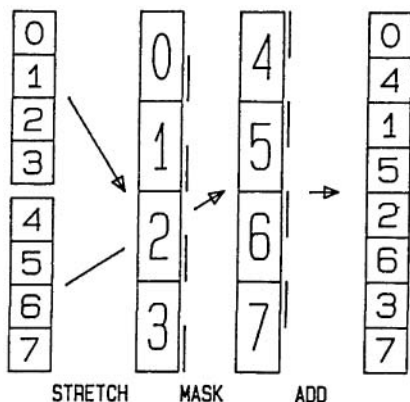


Fig. 2. Basic concept of a PS setup. The PS permutation is achieved by interlacing the upper and lower halves of the input.

This permutation can be performed very fast using classical optics, as described later. But to achieve flexibility the PS has to be supplemented by so-called "exchange boxes."⁵ These boxes offer the facility to exchange two adjacent elements, or not to exchange them, which is usually called bypass. With the PS and $N/2$ exchange boxes it is possible to generate every arbitrary connection permutation in only a few steps, the number of steps being of the order of $\log N$.⁶ Some examples of PS applications like FFT and sorting algorithms are described in Refs. 5 and 7.

In a digital optical computer the PS can be implemented in several ways. One could use optical fibers or integrated optics similar to a conventional electronic computer. But that approach would require a larger number of material connections. Optical shuffles with free space propagation can be implemented by holograms or by classical optics.

The basic principle of our setup, which uses lenses and prisms, is shown in Fig. 2. The first operation is to divide the input elements into upper and lower halves. These two halves have to be stretched in one direction to the size of the original input. An appropriate mask guarantees that there is no overlap of adjacent elements. The final step is the recombination of the two halves. This interlace operation is equivalent to the PS.

Figure 3 shows a simple setup, suitable for collimated illumination. The input object, indicated by the numbers 0-7, may be a spatial light modulator. The first pair of prisms separates the two halves, so that different shifts can be applied to the upper and lower halves of the input. These shifts are caused by a second pair of prisms in the Fourier plane. In the output plane the shuffled version of the input object appears with an overall reversed sequence. This reversal can be compensated if necessary by standard optical means using lenses, mirrors, prisms, or other standard components. Instead of masking out the unwanted light, smaller pixels are used in the input plane which do not overlap after stretching. The smaller pixels may be produced by a 2-D lenslet array.

The output size depends on the ratio of the two focal lengths f_2/f_1 . A magnification of 2 generates an output of the same size as the input in the shuffle direction and of double the size in the orthogonal direction. To avoid this an anamorphic setup can be used. No anamorphic lens system is needed if the PS is generalized to two dimensions. In that case the four quadrants of the input array are to be interlaced.

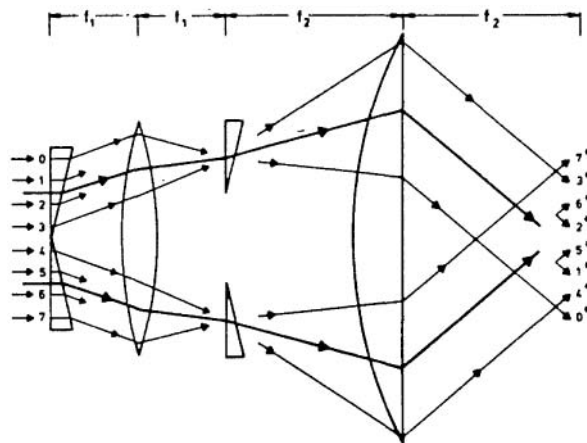


Fig. 3. Simple PS setup with two prism pairs.

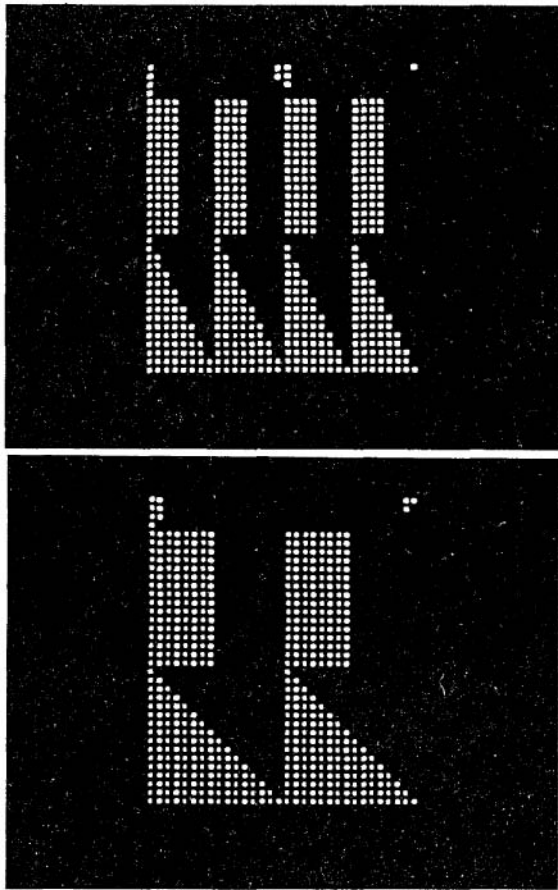


Fig. 4. Input object (upper) and its shuffled version (lower). The columns may represent numbers shuffled in a horizontal direction.

Figure 4 presents a special input object and its optically shuffled version.

This Letter is based on a paper presented at the OSA Topical Meeting on Optical Computing, 18–20 Mar. 1985.

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Conjugate-shift invariant in terms of Gaussian brackets

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Received 29 November 1985.

0003-6935/86/101431-02\$02.00/0.

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Wynne¹ derived the invariant, which corresponds to a finite shift in the position of conjugates and is kept constant before and after the shift. We call it the conjugate-shift invariant. Wynne¹ employed this invariant in aberration theory, and Rayces² used it in the angle solving problem.

Tanaka³ described the Lagrange-Helmholtz invariant in terms of generalized Gaussian constants^{4,5} expressed with Gaussian brackets.^{4,5} Tanaka also derived related invariants for an inhomogeneous optical system,⁶ a Gaussian beam optical system,⁷ a decentered optical system,⁸ and paraxial skew rays.⁹

This Letter derives the general expression of the conjugate-shift invariant in terms of Gaussian brackets and shows that the conjugate-shift invariant can be considered a variation of the Lagrange-Helmholtz invariant.

Let us consider three independent rays propagating through an optical system shown in Fig. 1. Notations used in this Letter are also given in the figure. The generalized Gaussian constants^{4,5} are employed and are defined by ${}^iA_k = [\phi_i, -e'_i, \dots, -e'_{k-1}]$, ${}^iB_k = [-e'_i, \dots, -e'_{k-1}]$, ${}^iC_k = [\phi_i, -e'_i, \dots, -e'_{k-1}, \phi_k]$, and ${}^iD_k = [-e'_i, \dots, -e'_{k-1}, \phi_k]$, where $[\dots]$ indicates Gaussian brackets.^{4,5}

Rayces² expressed the conjugate-shift invariant in the form

$$g = (h^* - h)/\bar{h} = (a^* - a)/\bar{a}, \quad (1)$$

which is constant for any optical component in the system. A paraxial ray-tracing is written in terms of generalized Gaussian constants,⁵ and

$$\begin{pmatrix} h_k \\ a'_k \end{pmatrix} = \begin{pmatrix} {}^iA_k & {}^iB_k \\ {}^iC_k & {}^iD_k \end{pmatrix} \begin{pmatrix} h_i \\ a_i \end{pmatrix}. \quad (2)$$

Applying Eq. (2), corresponding equations for the rays marked by asterisk and bar signs to Eq. (1), and arranging it with the help of the Gaussian brackets formula^{4,5}:

$${}^iG_j^{i+1}G_{j-1} - {}^iG_{j-1}^{i+1}G_j = (-1)^{j-i-1}, \quad (3)$$

where ${}^mG_n = [a_m, a_{m+1}, \dots, a_n]$. The conjugate-shift invariant is finally rewritten in terms of generalized Gaussian constants, and it follows that

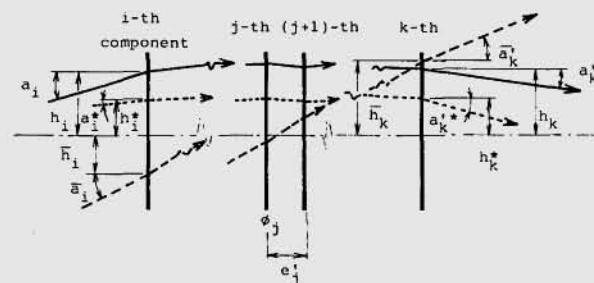


Fig. 1. Notations used in this Letter: h and a are ray height and ray angle, respectively. ϕ means power of a component, and e' denotes separation between consecutive two components.