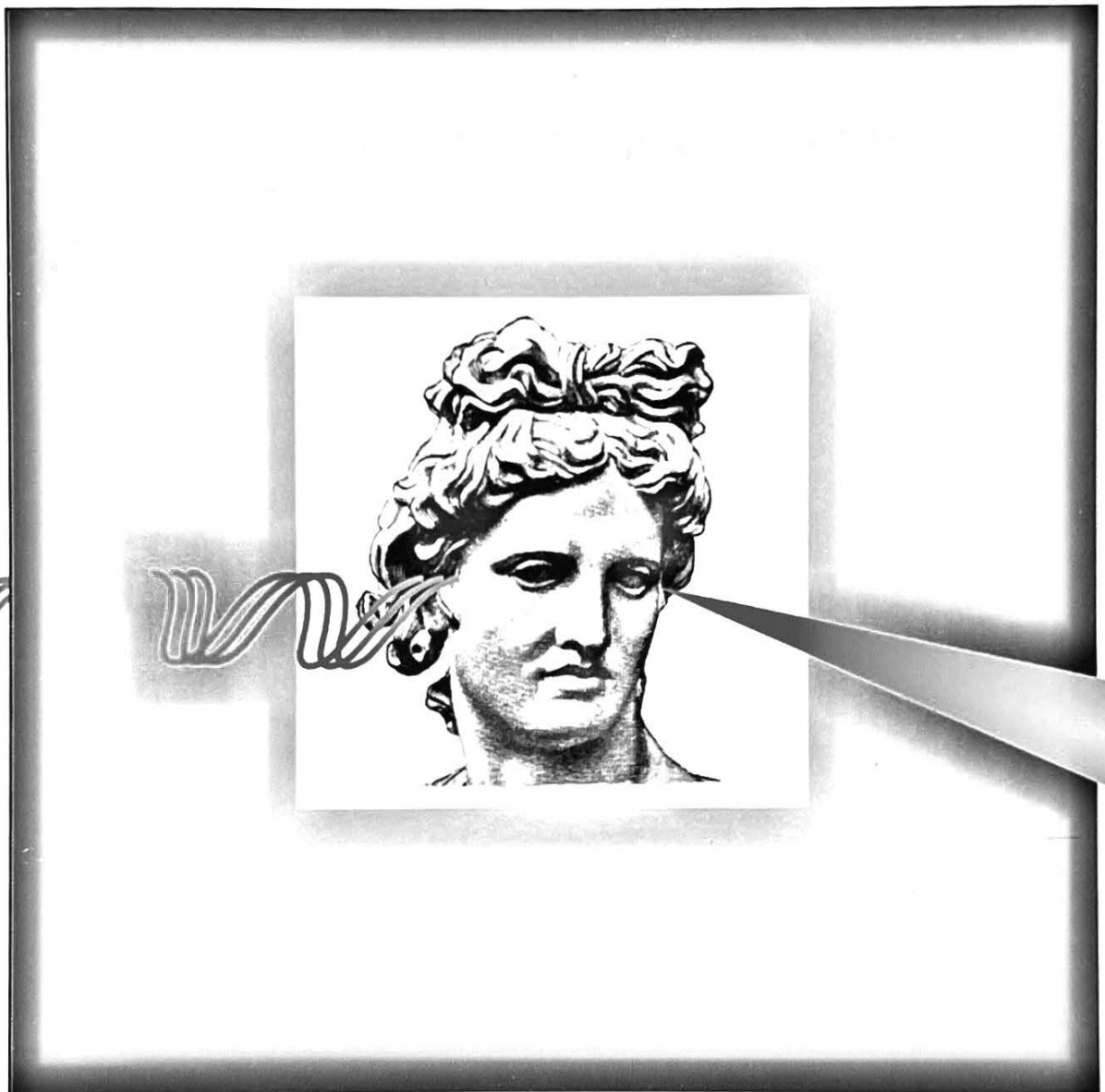


SIGNAL DETECTION THEORY AND PSYCHOPHYSICS



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6.4 REPRESENTATION OF SIGNAL

We assume that the signals we wish to investigate can be represented in the same way as the noise. If the signal is some fixed and definite waveform, such as a sinusoid of fixed amplitude, frequency, and phase, we assume that it can be represented as

$$s(t) = \sum_{n=1}^{WT} \left[a_n \cos \left(\frac{2\pi}{T} n t \right) + b_n \sin \left(\frac{2\pi}{T} n t \right) \right]$$

or

$$s(t) = \sum_{i=1}^{WT} s \left(\frac{i}{2W} \right) \Psi_i(t) \quad (6.19)$$

where s_i is the value of the signal at the i th temporal sample interval, $s_i = s(i/2W)$. If the signal is from some stochastic process, we assume that the sample space of the stochastic process can be specified in terms of the random variables a_n , b_n or in terms of $s(i/2W)$. Since we are assuming a 1-ohm impedance, the energy of the signal, E_s , is given by

$$E_s = \int_0^T s^2(t) dt = \sum_{i=1}^{WT} s_i^2 \Delta t = \frac{1}{2W} \sum_{i=1}^{WT} s_i^2 \quad (6.20)$$

6.5 IDEAL DETECTORS

In this section we derive the optimal detector for several specific detection tasks. That is, the basic paradigm is applied to several specific situations, situations that differ in the amount and kind of information the detector has about the signal. We shall emphasize those aspects of the derivation that are of general interest and, in particular, we emphasize how the detector's uncertainty about the signal parameters affects its performance.

6.5.1 Signal specified exactly

For the case of the signal specified exactly, the observed waveform, $x(t)$, is noise alone, or signal plus noise. The signal, $s(t)$, $0 < t < T$, is specified exactly and is known to the detector in the following sense. Using the temporal representation of the signal, the detector knows that if the signal is sent it will have a certain value $s_i = s(i/2W)$, $i = 1, 2, \dots, 2WT$, at each of the sample points.

The density function for each hypothesis can now be expressed using the temporal representation of the waveform and equal-interval samples.

Consider a single sample point $x_m = x(m/2W)$. Given noise alone,

$$f\{x_m | n\} = \frac{1}{\sqrt{2\pi N_0 W}} \exp\left(-\frac{1}{2} \frac{x_m^2}{N_0 W}\right) \quad (6.21)$$

Given signal plus noise,

$$f\{x_m | s\} = \frac{1}{\sqrt{2\pi N_0 W}} \exp\left[-\frac{1}{2} \frac{(x_m - s_m)^2}{N_0 W}\right] \quad (6.22)$$

The total waveform $x(t)$ may be represented by the collection of sample values x_m , $m = 1, 2, 3, \dots, 2WT$. The entire collection or vector has a probability density, given noise alone, of

$$f\{\hat{x}(t) | n\} = \prod_{m=1}^{2WT} f\{x_m | n\} \quad (6.23)$$

or, given signal plus noise, of

$$f\{\hat{x}(t) | s\} = \prod_{m=1}^{2WT} f\{x_m | s\} \quad (6.24)$$

We multiply the sample densities together because our representation makes each sample point independent of all others. The likelihood ratio is then

$$l[\hat{x}(t)] = \frac{f\{\hat{x}(t) | s\}}{f\{\hat{x}(t) | n\}} = \prod_{m=1}^{2WT} \exp\left(\frac{1}{2} \frac{x_m^2}{N_0 W}\right) \exp\left[-\frac{1}{2} \frac{(x_m - s_m)^2}{N_0 W}\right] \quad (6.25)$$

Taking the natural logarithm of likelihood ratio, and simplifying,

$$\ln l[x(t)] = \frac{1}{N_0 W} \left(\sum_{m=1}^{2WT} x_m s_m - \frac{1}{2} \sum_{m=1}^{2WT} s_m^2 \right) \quad (6.26)$$

Because the term $\sum s_m^2$ is a constant, the cross correlation between the expected sample s_m and the received signal x_m is monotonic with likelihood ratio.* The optimal detection scheme for the signal specified exactly is therefore cross correlation between the expected signal and the received waveform. If the receiver computes the cross correlation and accepts the hypothesis that the signal was present only when the cross correlation is above some criterion value, it will be the optimal detector.

This result does not imply that some other receiver may not also be optimal. In fact, if we had used the Fourier-series representation of $x(t)$, we would have received a different configuration for the optimal detector.

* The cross correlation of two waveforms, $h(t)$, $0 < t < T$, and $g(t)$, $0 < t < T$, is the integral of the product of the two waveforms,

$$\int_0^T h(t)g(t) dt$$

(See Appendix II.)