

We denote the DT sequence by $x(n)$ and also the value of a sample of the sequence at a particular value of n by $x(n)$. If a sequence has zero values for $n < 0$, then it is called a *causal sequence*. It is misleading to state that the causal function is a sequence defined for $n \geq 0$, because, strictly speaking, a DT sequence has to be defined for all values of n . Hence it is understood that a causal sequence has zero-valued samples for $-\infty < n < 0$. Similarly, when a function is defined for $N_1 \leq n \leq N_2$, it is understood that the function has zero values for $-\infty < n < N_1$ and $N_2 < n < \infty$. So the sequence $x_1(n)$ in Equation (1.2) has zero values for $2 < n < \infty$ and for $-\infty < n < -3$. The discrete-time sequence $x_2(n)$ given below is a causal sequence. In this form for representing $x_2(n)$, it is implied that $x_2(n) = 0$ for $-\infty < n < 0$ and also for $4 < n < \infty$:

$$x_2(n) = \left\{ \underset{\uparrow}{1} \quad -2 \quad 0.4 \quad 0.3 \quad 0.4 \quad 0 \quad 0 \quad 0 \right\} \quad (1.3)$$

The length of a finite sequence is often defined by other authors as the number of samples, which becomes a little ambiguous in the case of a sequence like $x_2(n)$ given above. The function $x_2(n)$ is the same as $x_3(n)$ given below:

$$x_3(n) = \left\{ \underset{\uparrow}{1} \quad -2 \quad 0.4 \quad 0.3 \quad 0.4 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \right\} \quad (1.4)$$

But does it have more samples? So the length of the sequence $x_3(n)$ would be different from the length of $x_2(n)$ according to the definition above. When a sequence such as $x_4(n)$ given below is considered, the definition again gives an ambiguous answer:

$$x_4(n) = \left\{ \underset{\uparrow}{0} \quad 0 \quad 0.4 \quad 0.3 \quad 0.4 \right\} \quad (1.5)$$

The definition for the length of a DT sequence would be refined when we define the degree (or order) of a polynomial in z^{-1} to express the z transform of a DT sequence, in the next chapter.

To model the discrete-time signals mathematically, instead of listing their values as shown above or plotting as shown in Figure 1.2, we introduce some basic DT functions as follows.

1.3.2 Unit Pulse Function

The unit pulse function $\delta(n)$ is defined by

$$\delta(n) = \begin{cases} 1 & n = 0 \\ 0 & n \neq 0 \end{cases} \quad (1.6)$$

and it is plotted in Figure 1.5a. It is often called the *unit sample function* and also the *unit impulse function*. But note that the function $\delta(n)$ has a finite numerical

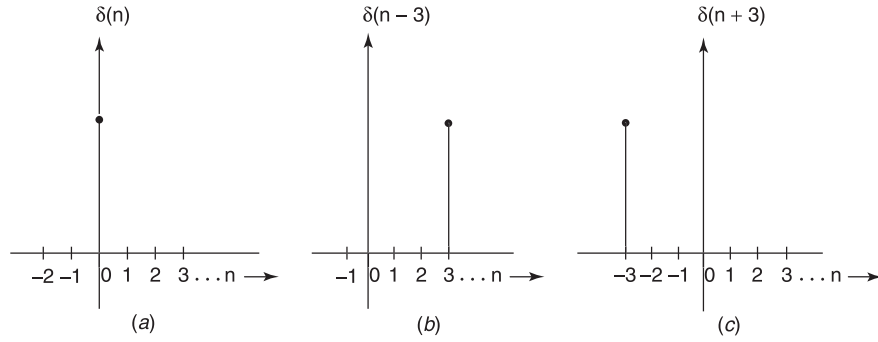


Figure 1.5 Unit pulse functions $\delta(n)$, $\delta(n-3)$, and $\delta(n+3)$.

value of one at $n = 0$ and zero at all other values of integer n , whereas the unit impulse function $\delta(t)$ is defined entirely in a different way.

When the unit pulse function is delayed by k samples, it is described by

$$\delta(n-k) = \begin{cases} 1 & n = k \\ 0 & n \neq k \end{cases} \quad (1.7)$$

and it is plotted in Figure 1.5b for $k = 3$. When $\delta(n)$ is advanced by $k = 3$, we get $\delta(n+k)$, and it is plotted in Figure 1.5c.

1.3.3 Constant Sequence

This sequence $x(n)$ has a constant value for all n and is therefore defined by $x(n) = K$; $-\infty < n < \infty$.

1.3.4 Unit Step Function

The unit step function $u(n)$ is defined by

$$u(n) = \begin{cases} 1 & n \geq 0 \\ 0 & n < 0 \end{cases} \quad (1.8)$$

and it is plotted in Figure 1.6a.

When the unit step function is delayed by k samples, where k is a positive integer, we have

$$u(n-k) = \begin{cases} 1 & n \geq k \\ 0 & n < k \end{cases} \quad (1.9)$$

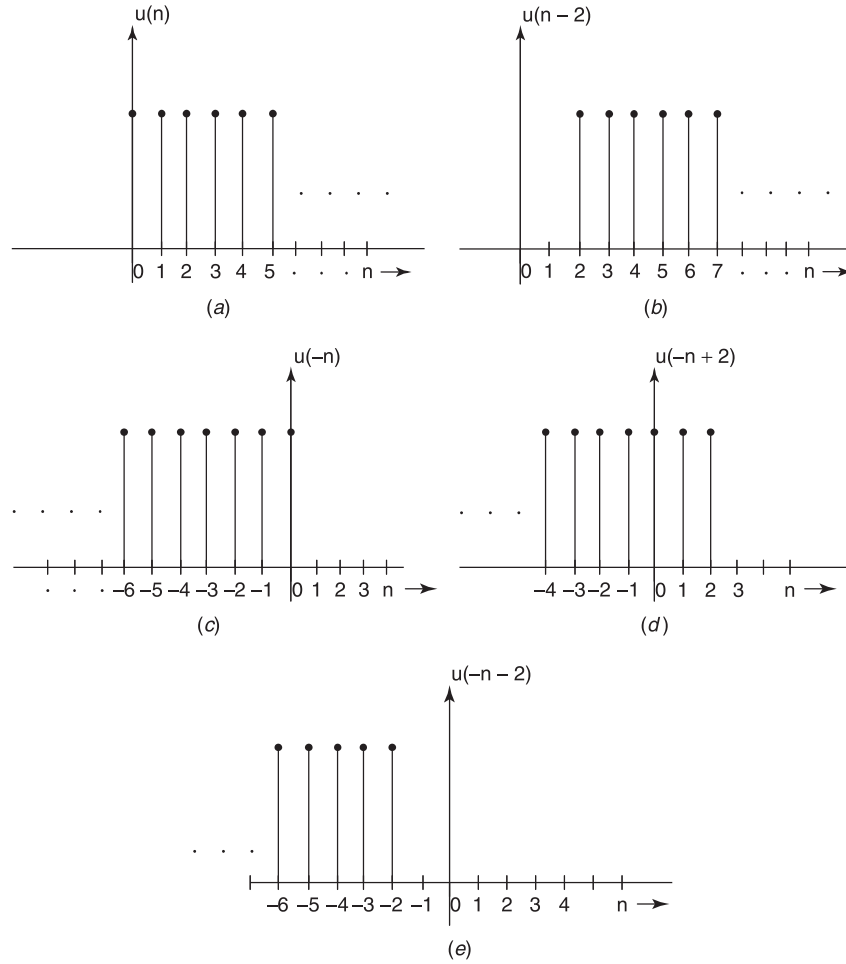


Figure 1.6 Unit step functions.

The sequence $u(n+k)$ is obtained when $u(n)$ is advanced by k samples. It is defined by

$$u(n+k) = \begin{cases} 1 & n \geq -k \\ 0 & n < -k \end{cases} \quad (1.10)$$

We also define the function $u(-n)$, obtained from the time reversal of $u(n)$, as a sequence that is zero for $n > 0$. The sequences $u(-n+k)$ and $u(-n-k)$, where k is a positive integer, are obtained when $u(-n)$ is delayed by k samples and advanced by k samples, respectively. In other words, $u(-n+k)$ is obtained by

delaying $u(-n)$ when k is positive and obtained by advancing $u(-n)$ when k is a negative integer. Note that the effect on $u(-n-k)$ is opposite that on $u(n-k)$, when k is assumed to take positive and negative values. These functions are shown in Figure 1.6, where $k = 2$. In a strict sense, all of these functions are defined implicitly for $-\infty < n < \infty$.

1.3.5 Real Exponential Function

The real exponential function is defined by

$$x(n) = a^n; \quad -\infty < n < \infty \quad (1.11)$$

where a is real constant. If a is a complex constant, it becomes the complex exponential sequence. The real exponential sequence or the complex exponential sequence may also be defined by a more general relationship of the form

$$x(n) = \begin{cases} a^n & k \leq n < \infty \\ b^n & -\infty < n < k \end{cases} \quad (1.12)$$

A special discrete-time sequence that we often use is the function defined for $n \geq 0$:

$$x(n) = a^n u(n) \quad (1.13)$$

An example of $x_1(n) = (0.8)^n u(n)$ is plotted in Figure 1.7a. The function $x_2(n) = x_1(n-3) = (0.8)^{(n-3)} u(n-3)$ is obtained when $x_1(n)$ is delayed by three samples. It is plotted in Figure 1.7b. But the function $x_3(n) = (0.8)^n u(n-3)$ is obtained by chopping off the first three samples of $x_1(n) = (0.8)^n u(n)$, and as shown in Figure 1.7c, it is different from $x_2(n)$.

1.3.6 Complex Exponential Function

The complex exponential sequence is a function that is complex-valued as a function of n . The most general form of such a function is given by

$$x(n) = A\alpha^n, \quad -\infty < n < \infty \quad (1.14)$$

where both A and α are complex numbers. If we let $A = |A| e^{j\phi}$ and $\alpha = e^{(\sigma_0 + j\omega_0)}$, where σ_0 , ω_0 , and ϕ are real numbers, the sequence can be expanded to the form

$$\begin{aligned} x(n) &= |A| e^{j\phi} e^{(\sigma_0 + j\omega_0)n} \\ &= |A| e^{\sigma_0 n} e^{j(\omega_0 n + \phi)} \\ &= |A| e^{\sigma_0 n} \cos(\omega_0 n + \phi) + j |A| e^{\sigma_0 n} \sin(\omega_0 n + \phi) \\ &= x_{\text{re}}(n) + j x_{\text{im}}(n) \end{aligned} \quad (1.15)$$

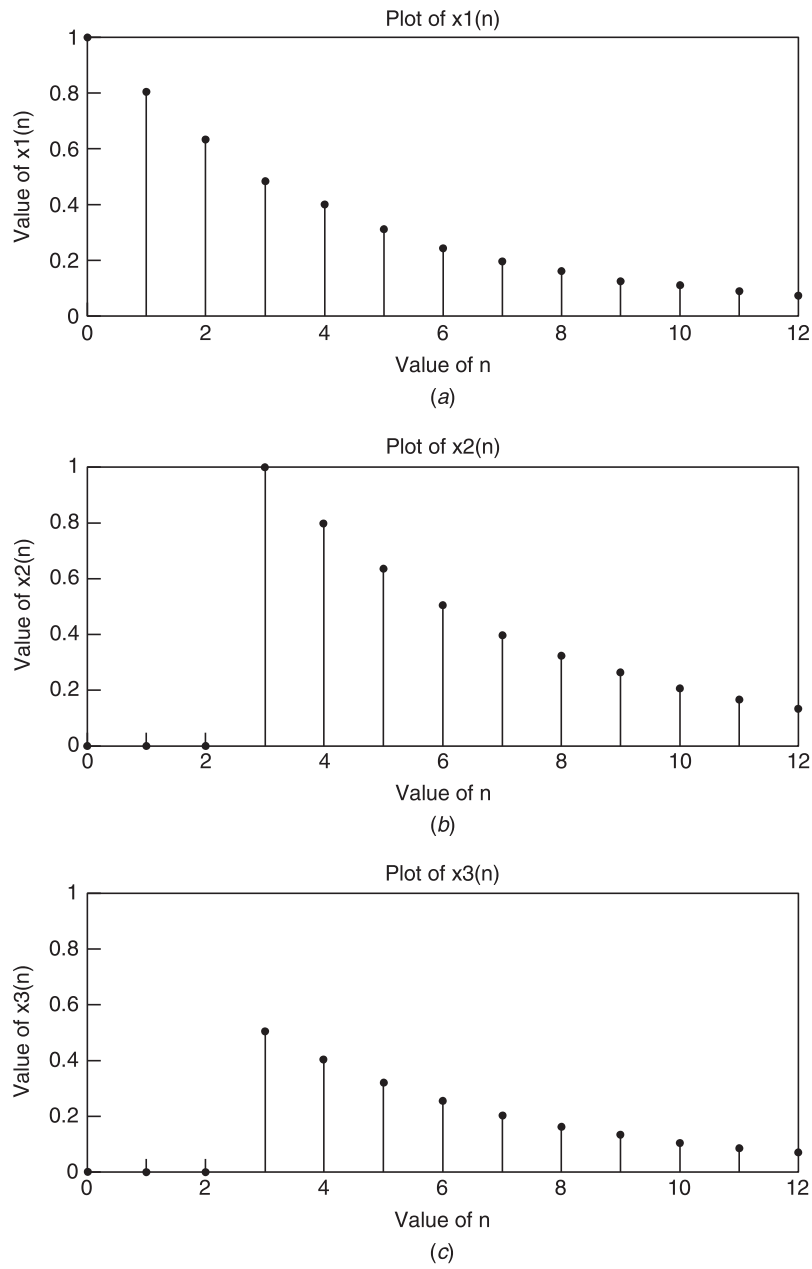


Figure 1.7 Plots of $x_1(n)$, $x_2(n)$, and $x_3(n)$.

When $\sigma_0 = 0$, the real and imaginary parts of this complex exponential sequence are $|A| \cos(\omega_0 n + \phi)$ and $|A| \sin(\omega_0 n + \phi)$, respectively, and are real sinusoidal sequences with an amplitude equal to $|A|$. When $\sigma_0 > 0$, the two sequences increase as $n \rightarrow \infty$ and decrease when $\sigma_0 < 0$ as $n \rightarrow \infty$. When $\omega_0 = \phi = 0$, the sequence reduces to the real exponential sequence $|A| e^{\sigma_0 n}$.

1.3.7 Properties of $\cos(\omega_0 n)$

When $A = 1$, and $\sigma_0 = \phi = 0$, we get $x(n) = e^{j\omega_0 n} = \cos(\omega_0 n) + j \sin(\omega_0 n)$. This function has some interesting properties, when compared with the continuous-time function $e^{j\omega_0' t}$ and they are described below.

First we point out that ω_0 in $x(n) = e^{j\omega_0 n}$ is a frequency normalized by $f_s = 1/T$, where f_s is the sampling frequency in hertz and T is the sampling period in seconds, specifically, $\omega_0 = 2\pi f_0'/f_s = \omega_0' T$, where $\omega_0' = 2\pi f_0'$ is the actual real frequency in radians per second and f_0' is the actual frequency in hertz. Therefore the unit of the normalized frequency ω_0 is radians. It is common practice in the literature on discrete-time systems to choose ω as the normalized frequency variable, and we follow that notation in the following chapters; here we denote ω_0 as a constant in radians. We will discuss this normalized frequency again in a later chapter.

Property 1.1 In the complex exponential function $x(n) = e^{j\omega_0 n}$, two frequencies separated by an integer multiple of 2π are indistinguishable from each other. In other words, it is easily seen that $e^{j\omega_0 n} = e^{j(\omega_0 n + 2\pi r)}$. The real part and the imaginary part of the function $x(n) = e^{j\omega_0 n}$, which are sinusoidal functions, also exhibit this property. As an example, we have plotted $x_1(n) = \cos(0.3\pi n)$ and $x_2(n) = \cos(0.3\pi + 4\pi)n$ in Figure 1.8. In contrast, we know that two continuous-time functions $x_1(t) = e^{j\omega_1 t}$ and $x_2(t) = e^{j\omega_2 t}$ or their real and imaginary parts are different if ω_1 and ω_2 are different. They are different even if they are separated by integer multiples of 2π . From the property $e^{j\omega_0 n} = e^{j(\omega_0 n + 2\pi r)}$ above, we arrive at another important result, namely, that the output of a discrete-time system has the same value when these two functions are excited by the complex exponential functions $e^{j\omega_0 n}$ or $e^{j(\omega_0 n + 2\pi r)}$. We will show in Chapter 3 that this is true for all frequencies separated by integer multiples of 2π , and therefore the frequency response of a DT system is periodic in ω .

Property 1.2 Another important property of the sequence $e^{j\omega_0 n}$ is that it is periodic in n . A discrete-time function $x(n)$ is defined to be periodic if there exists an integer N such that $x(n + rN) = x(n)$, where r is any arbitrary integer and N is the period of the periodic sequence. To find the value for N such that $e^{j\omega_0 n}$ is periodic, we equate $e^{j\omega_0 n}$ to $e^{j\omega_0(n+rN)}$. Therefore $e^{j\omega_0 n} = e^{j\omega_0 n} e^{j\omega_0 rN}$, which condition is satisfied when $e^{j\omega_0 rN} = 1$, that is, when $\omega_0 N = 2\pi K$, where

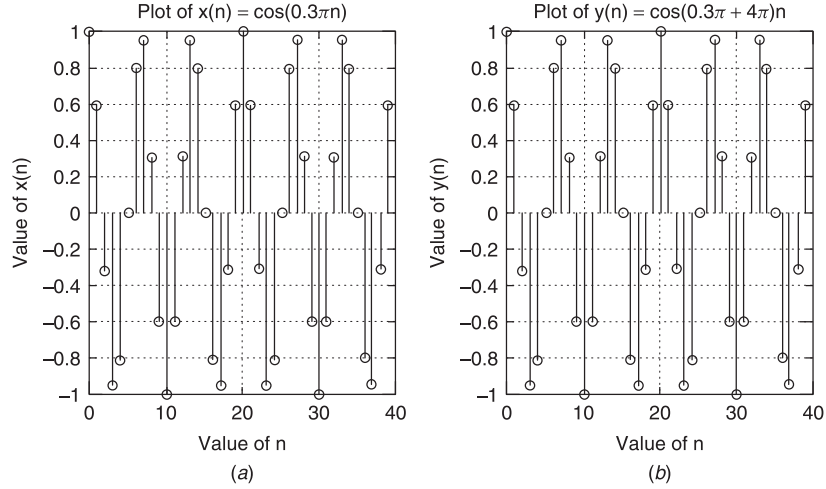


Figure 1.8 Plots of $\cos(0.3\pi n)$ and $\cos(0.3\pi + 4\pi)n$.

K is any arbitrary integer. This condition is satisfied by the following equation:

$$\frac{\omega_0}{2\pi} = \frac{K}{N} \quad (1.16)$$

In words, this means that the ratio of the given normalized frequency ω_0 and 2π must be a rational number. The period of the sequence N is given by

$$N = \frac{2\pi K}{\omega_0} \quad (1.17)$$

When this condition is satisfied by the smallest integer K , the corresponding value of N gives the fundamental period of the periodic sequence, and integer multiples of this frequency are the harmonic frequencies.

Example 1.1

Consider a sequence $x(n) = \cos(0.3\pi n)$. In this case $\omega_0 = 0.3\pi$ and $\omega_0/2\pi = 0.3\pi/2\pi = \frac{3}{20}$. Therefore the sequence is periodic and its period N is 20 samples. This periodicity is noticed in Figure 1.8a and also in Figure 1.8b.

Consider another sequence $x(n) = \cos(0.5\pi n)$, in which case $\omega_0 = 0.5$. Therefore $\omega_0/2\pi = 0.5/2\pi = 1/4\pi$, which is not a rational number. Hence this is not a periodic sequence.

When the given sequence is the sum of several complex exponential functions, each of which is periodic with different periods, it is still periodic. We consider an example to illustrate the method to find the fundamental period in this case.

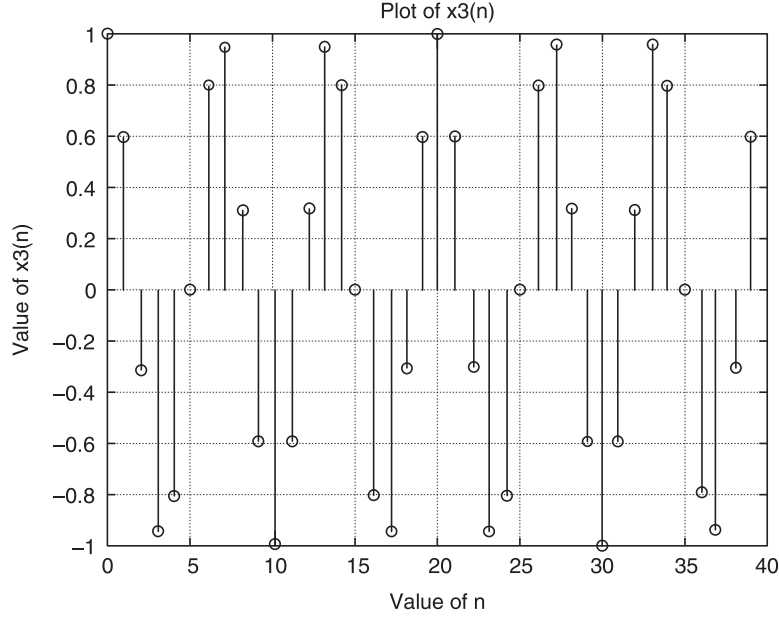


Figure 1.9 Plot of $x_3(n)$.

Suppose $x_3(n) = \cos(0.2\pi n) + \cos(0.5\pi n) + \cos(0.6\pi n)$. Its fundamental period N must satisfy the condition

$$N = \frac{2\pi K_1}{0.2\pi} = \frac{2\pi K_2}{0.5\pi} = \frac{2\pi K_3}{0.6\pi} \quad (1.18)$$

$$= 10K_1 = 4K_2 = \frac{10K_3}{3} \quad (1.19)$$

where K_1, K_2 , and K_3 and N are integers. The value of N that satisfies this condition is 20 when $K_1 = 2$, $K_2 = 5$, and $K_3 = 6$. So $N = 20$ is the fundamental period of $x_3(n)$. The sequence $x_3(n)$ plotted in Figure 1.9 for $0 \leq n \leq 40$ shows that it is periodic with a period of 20 samples.

Property 1.3 We have already observed that the frequencies at ω_0 and at $\omega_0 + 2\pi$ are the same, and hence the frequency of oscillation are the same. But consider the frequency of oscillation as ω_0 changes between 0 and 2π . It is found that the frequency of oscillation of the sinusoidal sequence $\cos(\omega_0 n)$ increases as ω_0 increases from 0 to π and the frequency of oscillation decreases as ω_0 increases from π to 2π . Therefore the highest frequency of oscillation of a discrete-time sequence $\cos(\omega_0 n)$ occurs when $\omega_0 = \pm\pi$. When the normalized frequency $\omega_0 = 2\pi f'_0/f_s$ attains the value of π , the value of $f'_0 = f_s/2$. So the highest frequency of oscillation occurs when it is equal to half the sampling

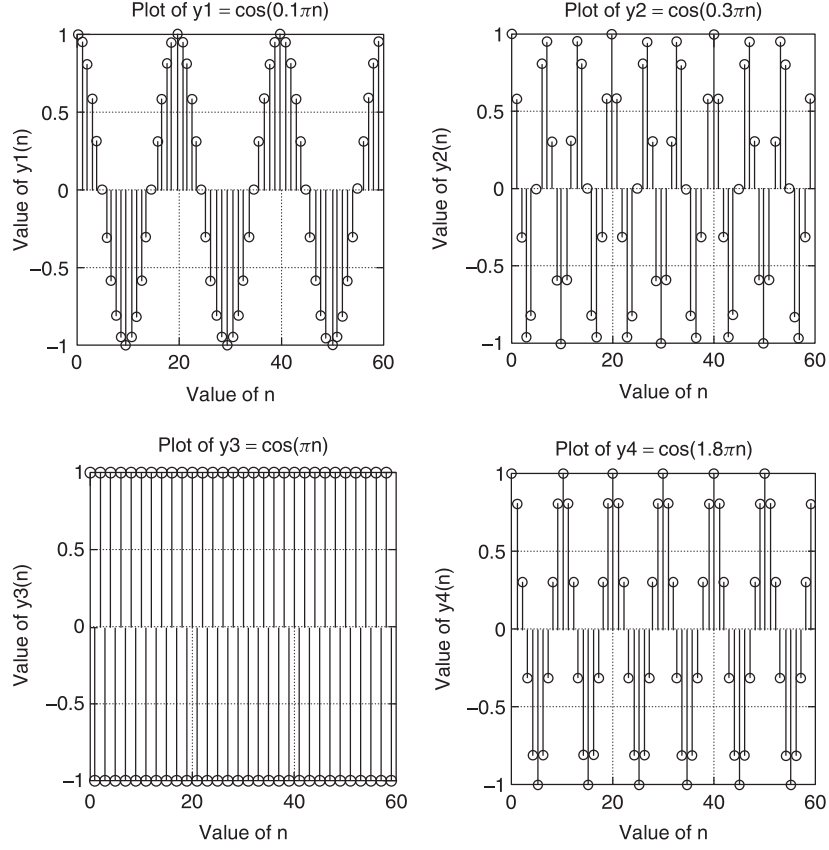


Figure 1.10 Plot of $\cos(\omega_0 n)$ for different values of ω_0 between 0 and 2π .

frequency. In Figure 1.10 we have plotted the DT sequences as ω_0 attains a few values between 0 and 2π , to illustrate this property. We will elaborate on this property in later chapters of the book.

Since frequencies separated by 2π are the same, as ω_0 increases from 2π to 3π , the frequency of oscillation increases in the same manner as the frequency of oscillation when it increases from 0 to π . As an example, we see that the frequency of $v_0(n) = \cos(0.1\pi n)$ is the same as that of $v_1(n) = \cos(2.1\pi n)$. It is interesting to note that $v_2(n) = \cos(1.9\pi n)$ also has the same frequency of oscillation as $v_1(n)$ because

$$v_2(n) = \cos(1.9\pi n) = \cos(2\pi - 0.1\pi n) \quad (1.20)$$

$$= \cos(2\pi n) \cos(0.1\pi n) + \sin(2\pi n) \sin(0.1\pi n) \quad (1.21)$$

$$= \cos(0.1\pi n)$$

$$= v_0(n)$$

$$v_1(n) = \cos(2.1\pi n) = \cos(2\pi n + 0.1\pi n) \quad (1.22)$$

$$= \cos(2\pi n) \cos(0.1\pi n) - \sin(2\pi n) \sin(0.1\pi n) \quad (1.23)$$

$$= \cos(0.1\pi n)$$

$$= v_0(n)$$

We have plotted the sequences $v_1(n)$ and $v_2(n)$ in Figure 1.11, to verify this property.

Remember that in Chapter 3, we will use the term “folding” to describe new implications of this property. We will also show in Chapter 3 that a large class of discrete-time signals can be expressed as the weighted sum of exponential sequences of the form $e^{j\omega_0 n}$, and such a model leads us to derive some powerful analytical techniques of digital signal processing.

We have described several ways of characterizing the DT sequences in this chapter. Using the unit sample function and the unit step function, we can express the DT sequences in other ways as shown below.

For example, $\delta(n) = u(n) - u(n-1)$ and $u(n) = \sum_{m=-\infty}^{m=n} \delta(m)$. A mathematical way of modeling a sequence

$$x(n) = \left\{ \begin{array}{cccccc} 2 & 3 & 1.5 & 0.5 & -1 & 4 \end{array} \right\} \quad (1.24)$$

↑

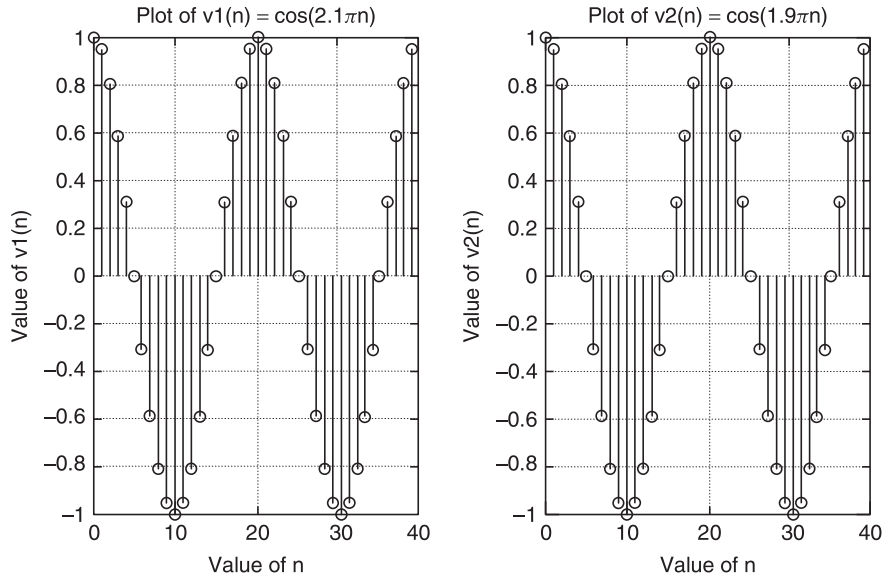


Figure 1.11 Plots of $\cos(2.1\pi n)$ and $\cos(1.9\pi n)$.

is the weighted sum of shifted unit sample functions, as given by

$$x(n) = 2\delta(n+3) + 3\delta(n+2) + 1.5\delta(n+1) + 0.5\delta(n) - \delta(n-1) + 4\delta(n-2) \quad (1.25)$$

If the sequence is given in an analytic form $x(n) = a^n u(n)$, it can also be expressed as the weighted sum of impulse functions:

$$x(n) = \sum_{m=0}^{\infty} x(m)\delta(n-m) = \sum_{m=0}^{\infty} a^m \delta(n-m) \quad (1.26)$$

In the next chapter, we will introduce a transform known as the z transform, which will be used to model the DT sequences in additional forms. We will show that this model given by (1.26) is very useful in deriving the z transform and in analyzing the performance of discrete-time systems.

1.4 HISTORY OF FILTER DESIGN

Filtering is the most common form of signal processing used in all the applications mentioned in Section 1.2, to remove the frequencies in certain parts and to improve the magnitude, phase, or group delay in some other part(s) of the spectrum of a signal. The vast literature on filters consists of two parts: (1) the theory of approximation to derive the transfer function of the filter such that the magnitude, phase, or group delay approximates the given frequency response specifications and (2) procedures to design the filters using the hardware components. Originally filters were designed using inductors, capacitors, and transformers and were terminated by resistors representing the load and the internal resistance of the source. These were called the LC (inductance $\times$ capacitance) filters that admirably met the filtering requirements in the telephone networks for many decades of the nineteenth and twentieth centuries. When the vacuum tubes and bipolar junction transistors were developed, the design procedure had to be changed in order to integrate the models for these active devices into the filter circuits, but the mathematical theory of filter approximation was being advanced independently of these devices. In the second half of the twentieth century, operational amplifiers using bipolar transistors were introduced and filters were designed without inductors to realize the transfer functions. The design procedure was much simpler, and device technology also was improved to fabricate resistors in the form of thick-film and later thin-film depositions on ceramic substrates instead of using printed circuit boards. These filters did not use inductors and transformers and were known as *active-RC* (resistance $\times$ capacitance) filters. In the second half of the century, switched-capacitor filters were developed, and they are the most common type of filters being used at present for audio applications. These filters contained only capacitors and operational amplifiers using complementary metal oxide semiconductor (CMOS) transistors. They used no resistors and inductors, and the whole circuit was fabricated by the

very large scale integration (VLSI) technology. The analog signals were converted to sampled data signals by these filters and the signal processing was treated as analog signal processing. But later, the signals were transitioned as discrete-time signals, and the theory of discrete-time systems is currently used to analyze and design these filters. Examples of an LC filter, an active- RC filter, and a switched-capacitor filter that realize a third-order lowpass filter function are shown in Figures 1.12–1.14.

The evolution of digital signal processing has a different history. At the beginning, the development of discrete-time system theory was motivated by a search for numerical techniques to perform integration and interpolation and to solve differential equations. When computers became available, the solution of physical systems modeled by differential equations was implemented by the digital

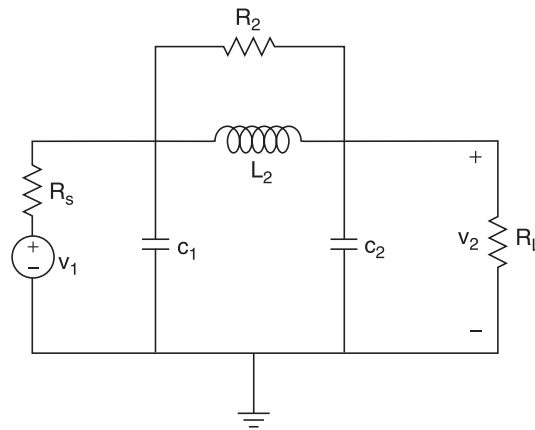


Figure 1.12 A lowpass analog LC filter.

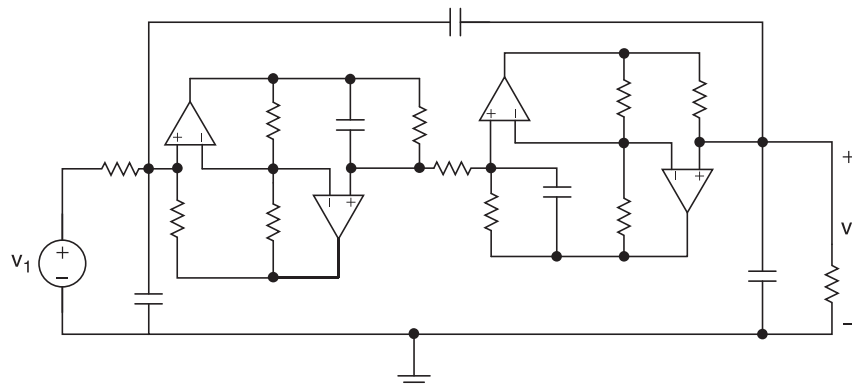


Figure 1.13 An active- RC lowpass analog filter.

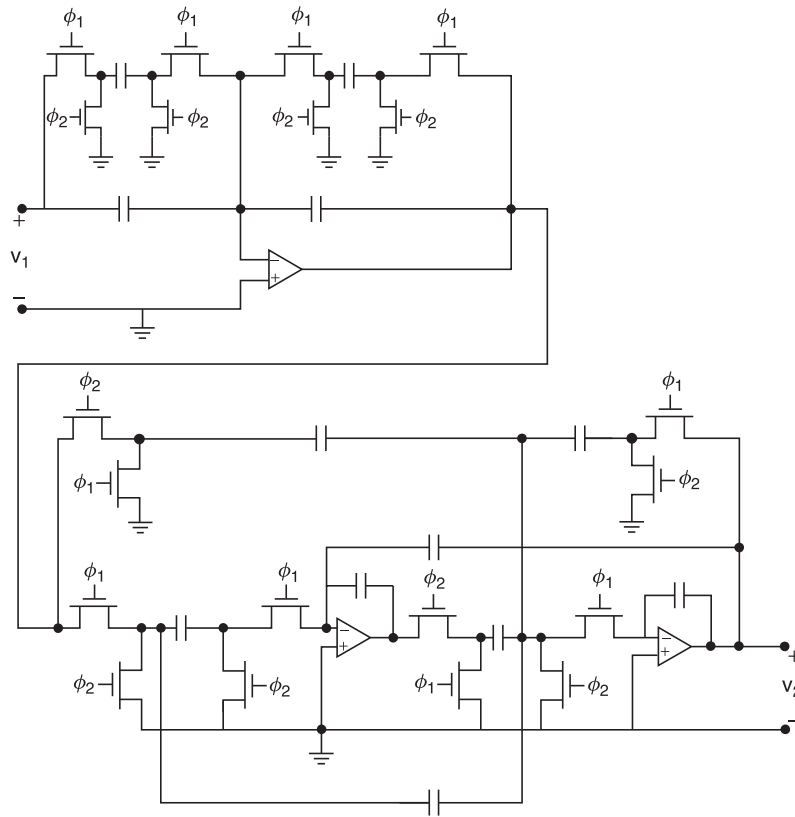


Figure 1.14 A switched-capacitor lowpass (analog) filter.

computers. As the digital computers became more powerful in their computational power, they were heavily used by the oil industry for geologic signal processing and by the telecommunications industry for speech processing. The theory of digital filters matured, and with the advent of more powerful computers built on integrated circuit technology, the theory and applications of digital signal processing has explosively advanced in the last few decades. The two revolutionary results that have formed the foundations of digital signal processing are the Shannon's sampling theorem and the Cooley–Tukey algorithm for fast Fourier transform technique. Both of them will be discussed in great detail in the following chapters. The Shannon's sampling theorem proved that if a continuous-time signal is bandlimited (i.e., if its Fourier transform is zero for frequencies above a maximum frequency f_m) and it is sampled at a rate that is more than twice the maximum frequency f_m in the signal, then no information contained in the analog signal is lost in the sense that the continuous-time signal can be exactly reconstructed from the samples of the discrete-time signal. In practical applications, most of the analog signals are first fed to an analog lowpass filter—known

as the *preconditioning filter* or *antialiasing filter*—such that the output of the lowpass filter attenuates the frequencies considerably beyond a well-chosen frequency so that it can be considered a bandlimited signal. It is this signal that is sampled and converted to a discrete-time signal and coded to a digital signal by the analog-to-digital converter (ADC) that was briefly discussed earlier in this chapter. We consider the discrete-time signal as the input to the digital filter designed in such a way that it improves the information contained in the original analog signal or its equivalent discrete-time signal generated by sampling it. A typical example of a digital lowpass filter is shown in Figure 1.15.

The output of the digital filter is next fed to a digital-to-analog converter (DAC) as shown in Figure 1.17 that also uses a lowpass analog filter that smooths the sampled-data signal from the DAC and is known as the “smoothing filter.” Thus we obtain an analog signal $y_d(t)$ at the output of the smoothing filter as shown. It is obvious that compared to the analog filter shown in Figure 1.16, the circuit shown in Figure 1.17 requires considerably more hardware or involves a lot more signal processing in order to filter out the undesirable frequencies from the analog signal $x(t)$ and deliver an output signal $y_d(t)$. It is appropriate to compare these two circuit configurations and determine whether it is possible to get the output $y_d(t)$ that is the same or nearly the same as the output $y(t)$ shown in Figure 1.16; if so, what are the advantages of digital signal processing instead of analog signal processing, even though digital signal processing requires more circuits compared to analog signal processing?

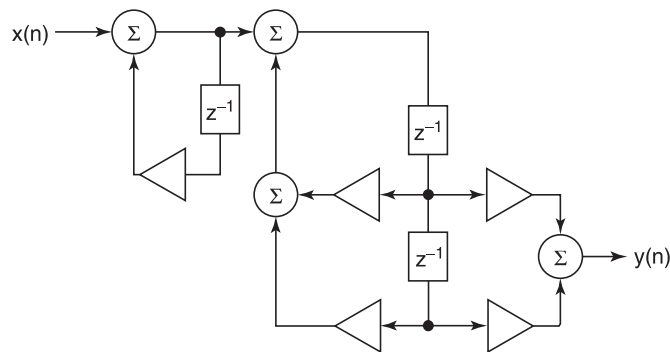


Figure 1.15 A lowpass third-order digital filter.

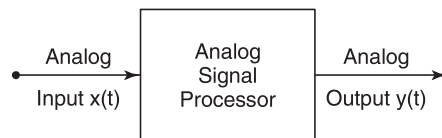


Figure 1.16 Example of an analog signal processing system.

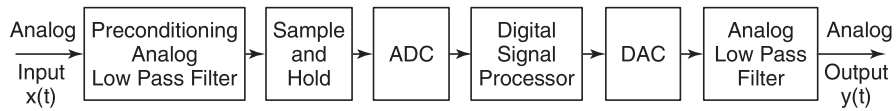


Figure 1.17 Example of a digital signal processing system.

1.5 ANALOG AND DIGITAL SIGNAL PROCESSING

The basic elements in digital filters are the multipliers, adders, and delay elements, and they carry out multiplication, addition, and shifting operations on numbers according to an algorithm determined by the transfer function of the filters or their equivalent models. (These models will be discussed in Chapter 3 and also in Chapter 7.) They provide more flexibility and versatility compared to analog filters. The coefficients of the transfer function and the sample values of the input signal can be stored in the memory of the digital filter hardware or on the computer (PC, workstation, or the mainframe computer), and by changing the coefficients, we can change the transfer function of the filter, while changing the sample values of the input, we can find the response of the filter due to any number of input signals. This flexibility is not easily available in analog filters.

The digital filters are easily programmed to do time-shared filtering under time-division multiplexing scheme, whereas the analog signals cannot be interleaved between timeslots. Digital filters can be designed to serve as time-varying filters also by changing the sampling frequency and by changing the coefficients as a function of time, namely, by changing the algorithm accordingly.

The digital filters have the advantage of high precision and reliability. Very high precision can be obtained by increasing the number of bits to represent the coefficients of the filter transfer function and the values of the input signal. Again we can increase the dynamic range of the signals and transfer function coefficients by choosing floating-point representation of binary numbers. The values of the inductors, capacitors, and the parameters of the operational amplifier parameters and CMOS transistors, and so on used in the analog filters cannot achieve such high precision. Even if the analog elements can be obtained with high accuracy, they are subject to great drift in their value due to manufacturing tolerance, temperature, humidity, and other parameters—depending on the type of device technology used—over long periods of service, and hence their filter response degrades slowly and eventually fails to meet the specifications. In the case of digital filters, such effects are nonexistent because the wordlength of the transfer coefficients as well as the product of addition and multiplication within the filter do not change with respect to time or any of the environmental conditions that plague the analog circuits. Consequently, the reliability of digital filters is much higher than that of analog filters, and this means that they are more economical in application. Of course, catastrophic failures due to unforeseen factors are equally possible in both cases. If we are using computers to analyze,

design, and simulate these filters, we can assume even double-precision format for the numbers that represent filter coefficients and signal samples. We point out that we can carry out the simulation, analysis, and design of any number of filters and under many conditions, for example, Monte Carlo analysis, worst-case analysis, or iterative optimization to test the design before we build the hardware and test it again and again. Of course, we can do the same in the case of analog filters or continuous-time systems also (e.g., analog control systems) using such software as MATLAB and Simulink.² During the manufacture of analog filters, we may have to tune each of them to correct for manufacturing tolerances, but there is no such need to test the accuracy of the wordlength in digital filters.

Data on digital filters can be stored on magnetic tapes, compact disks (CDs), digital videodisks (DVDs), and optical disks for an indefinite length of time. They can be retrieved without any degradation or loss of data; a good example is the music recorded on CDs. In contrast, analog signals deteriorate slowly as time passes and cannot be retrieved easily without any loss. There is no easy way of storing the transfer function coefficients that defines the analog system and feeding the input signals stored on these storage devices to the analog system.

By using digital filters, we can realize many transfer functions that cannot be realized by analog filters. For example, in addition to those already mentioned above, we can realize the following characteristics from digital filters:

1. Transition bands much smaller than what can be achieved from analog filters; an example would be a lowpass filter with a bandwidth of 5000 Hz and a passband ripple of 0.5 dB, and 100 dB attenuation above 5010 Hz. In spectrum analyzers and synthesizers, vocoders (voice recorders), and similar devices, extremely low tolerances on the magnitude and phase responses over adjacent passbands are required, and digital filters can be designed to meet these specifications.
2. Finite duration impulse response and filters with linear phase. Neither of these characteristics can be achieved by analog filters. Digital filters with these characteristics are used extensively in many applications.
3. Bandwidth of the order 5 Hz or even a fraction thereof that are commonly required to process biomedical or seismic signals.
4. Programmable filters, multirate filters, multidimensional filters, and adaptive filters. Programmable filters are used to adjust the frequency-selective properties of the filters. Multirate filters are used in the processing of many complex signals with different rates of fluctuation, whereas two-dimensional digital filters are the filters used in image processing. Adaptive filters are used invariably when the transmission medium between the transmitter and receiver changes—either as the transmission line is switched to

²MATLAB and Simulink are registered trademarks of The MathWorks, Inc. Natick, MA. The software is available from The MathWorks, Inc.

different receivers or as it changes continuously between the transmitter and the receiver. For example, when a telephone conversation is switched from one point to another and the cable or the microwave link changes, or when the mobile phone moves as the talker moves over a wide territory, adaptive filters are absolutely necessary to compensate for the distortion of the signal as it passes through the transmission link.

5. We have chosen filters only as an example to compare digital and analog signal processors. There are many other types of digital signal processing that are feasible and are being used, and these are not possible or very efficient in analog filters. For example, error detection in transmitted signals and correction to reduce the error rate is an advanced technique used in many applications. Another example is our ability to compress the data by a significant factor and receive the input signal at lower cost and very good quality. To point out the power of digital signal processing theory and the digital signal processors available, let us again consider the mobile phone. Bateman and Patterson-Stephans state that “Within the phone, a single DSP device may be performing real-time speech compression, video compression, echo cancellation, noise cancellation, voice recognition, waveform coding, modulation/demodulation, interleaving, multipath equalization, soft decision decoding, convolution, automatic frequency-, power- and gain-control” [3], and all of them done in a triband phone with TDMA, CDMA, and analog signal processing! The mobile phone is just an example to illustrate the large number of digital signal processing techniques that are built into any of the applications described above. But an application such as the mobile phone implements other functions also, and their features are briefly described given below.

1.5.1 Operation of a Mobile Phone Network

Consider a geographic area in which a part of the mobile phone network operates. It is divided into cells as indicated in Figure 1.18. The cells are not really equal in area but could be as small as 300 m where the telephone traffic is high and as large as 35 km in rural areas. The size and shape of each cell is determined by the radiation pattern and the power output of the antenna (and is not hexagonal of equal shape and size) serving the mobile phones. A base station controller (BSC), usually installed on a tower, serves as many as 124 base transceiver stations (BTSs). These stations communicate with all the cell phones that are known to be located within the area covered by the cell. The BTSs operate on different frequencies, also called “channels,” to transmit to and receive signals from the cell phones. Global System for Mobile Communication (GSM) is one of the most widely used mobile cellular phone network systems in the world, and in that system, the frequencies for transmitting from the mobile phone and receiving by the BTS lie in the band 890–915 MHz, and the frequencies for transmitting from the BTS and receiving by the mobile phone lie in the band 935–960 MHz. But in order to utilize the frequency spectrum efficiently, cells

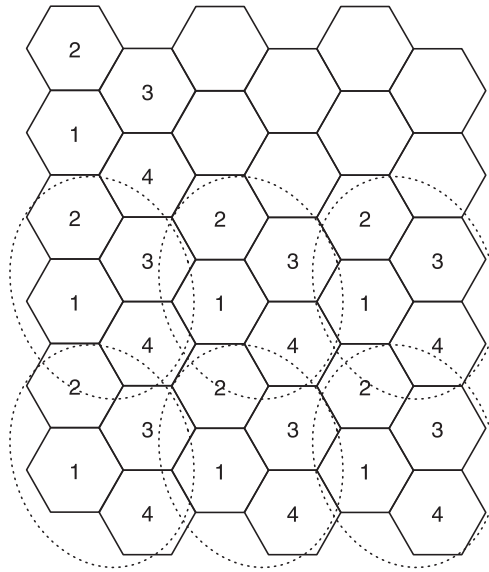


Figure 1.18 Cell repeat pattern in a mobile network coverage area.

using the same frequency are kept sufficiently apart so that there is no cochannel interference; also the frequencies used in a BTS are separated by 200 kHz. The base transceiver stations located on towers over a coverage area are connected by fixed digital lines to a mobile switching center (MSC), and the mobile switching center is connected to the public switched telephone network (PSTN) as shown in Figure 1.19 as well as the Internet, to which other MSCs are also connected.

When a phone initiates a call to send voice, text, an instant message, or other media, it registers with the network via the BTS closest to its location and the BTS tracks its location and passes this information to the mobile switching center (MSC) over fixed digital lines, which updates this information continuously as received from the BTS. Each mobile phone has a home location register (HLR) and a visitor location register (VLR) assigned to it. The HLR contains information such as the identity of the user and phone number assigned to the user in the user's home network, the services to which the user has subscribed, whereas the VLR contains information about the mobile phone when it is used outside the home network. So when a mobile phone initiates a call, it sends the information to the BTS about its identity and so on from the VLR or the HLR depending on the location of the phone at the time the call originates. The mobile switching center checks the data from its HLR or VLR to authenticate the call and gives permission for the phone to access the network. As the caller moves within the cell, the BTS monitors the strength of the signal between the phone and the receiver, and if this falls below a certain level, it may transfer control of the phone to the BTS in the next cell, which may offer a stronger signal. If no such cell is nearby, the caller is cut off (i.e., will not be able to receive or to send

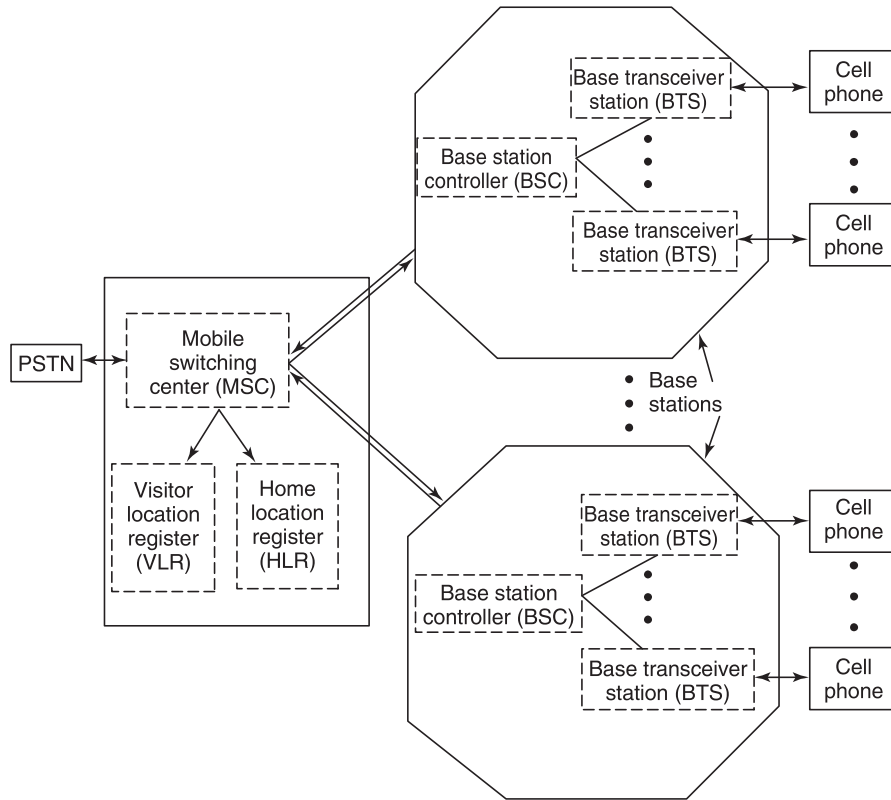


Figure 1.19 Block diagram of a mobile cell phone network.

a call). As the caller moves from one cell to another cell, the BTS serving it will transfer control to the BTS in the cell that it has moved to. This is the main feature that makes mobile telephony possible. All of these operations are carried out by the computers serving the mobile cellular phone network, and that technology is known as *computer networking technology*. It is different from the theory of digital signal processing. This textbook offers an introduction to the fundamental theory of digital signal processing, which is used in such techniques as speech compression, multipath equalization, and echo cancellation, mentioned in the previous section.

There are some disadvantages and limitations in digital signal processing in comparison with analog signal processing. By looking at the two circuit configurations in Figures 1.16 and 1.17, it is obvious that the digital signal processor is a more complex system, because of the additional components of analog low-pass filters (ADC and DAC) on either side of the digital signal processor besides the additional control and programming circuits, which are not shown in the figures. Another disadvantage is that the digital signal processor can process signals within a range of frequencies that is limited mainly by the highest sampling

TABLE 1.3 ADCs Currently Available

Sampling Rate (samples per second)	Resolution (bits)	Maximum Frequency in Input Signal	Power
96,000	24	48 kHz	90 mW
96,000	18	48 kHz	60 mW
96,000	16	48 kHz	40 mW
65,000,000	14	500 MHz	0.6 W
400,000,000	8	1 GHz	3 W

frequency of the ADC and DAC that are available. As the frequency is increased, the wordlength of these devices decreases and therefore the accuracy and dynamic range of the input and output data decrease.

For example, data on a few ADCs currently available are given in Table 1.3 [3].

Hence digital signal processing is restricted to approximately one megahertz, and analog signal processors are necessary for processing signals above that frequency, for example, processing of radar signals. In such applications, analog signal processing is a more attractive and viable choice, and currently a lot of research is being directed toward what is known as mixed-signal processing. Note in Table 1.3, that as the resolution (wordlength) for a given signal decreases, the power consumption also decreases, but that is just the power consumed by the ADCs; the power increases as the sampling frequency increases, even when the resolution is decreased. The digital signal processor itself consumes a lot more power, and hence additional power management circuits are added to the whole system. In contrast, the analog signal processors consume less power. The *LC* filters consume almost negligible power and can operate at frequencies in the megahertz range. The active-*RC* filters and switched-capacitor filters are restricted to the audiofrequency range, but they consume more power than do the *LC* filters. It is expected that mixed-signal processing carried out on a single system or a single chip will boost the maximum frequency of the signal that can be processed, by a significant range, beyond what is possible with a strictly digital signal processing system. Therefore we will see more and more applications of DSP with increasing frequencies because the advantages of DSP outweigh the disadvantages in analog signal processing.

1.6 SUMMARY

In this introductory chapter, we defined the *discrete-time signal* and gave a few examples of these signals, along with some simple operations that can be applied with them. In particular, we pointed out the difference between a sinusoidal signal, which is a continuous-time signal, and a discrete-time signal. We discussed the basic procedure followed to sample and quantize an analog signal

and compared the advantages and disadvantages of digital signal processing with those of directly processing the analog signal through an analog system, taking a filter as an example. In doing so, we introduced many terms or acronyms that we have not explained. Some of them will be explained in great detail in the following chapter. In Chapter 2 we will discuss several ways of modeling a *discrete-time system* and the methods used to find the response in the time domain, when excitation is effected by the discrete-time signals.

PROBLEMS

- 1.1** Given two discrete-time signals $x_1(n) = \{0.9 \quad 0.5 \quad 0.8 \quad \overset{\uparrow}{1.0} \quad 1.5 \quad 2.0 \quad 0.2\}$ and $x_2(n) = \{1.0 \quad 0.3 \quad 0.6 \quad 0.4\}$, sketch each of the following:
 - (a) $y_1(n) = x_1(n) + 3x_2(n)$
 - (b) $y_2(n) = x_1(n) - x_2(n - 5)$
 - (c) $y_3(n) = x_1(n)x_2(n)$
 - (d) $y_4(n) = x_1(-n + 4)$
 - (e) $y_5(n) = y_4(n)x_2(n)$
 - (f) $y_6(n) = x_2(-n - 3)$
 - (g) $y_7(n) = y_4(n)y_6(n)$
- 1.2** Sketch each of the following, where $x_1(n)$ and $x_2(n)$ are the DT sequences given in Problem 1.1:
 - (a) $v_1(n) = x_1(n)x_2(4 - n)$
 - (b) $v_2(n) = \sum_{k=-\infty}^{\infty} x_1(k)x_2(n - k)$
 - (c) $v_3(n) = \sum_{k=-\infty}^{\infty} x_2(k)x_1(n - k)$
 - (d) $v_4(n) = \sum_{n=0}^{n=10} x_2^2(n)$
 - (e) $v_5(n) = x_1(2n)$
- 1.3** Repeat Problem 1.1 with $x_1(n) = \{1.0 \quad 0.8 \quad 0.2 \quad -0.2 \quad -0.5 \quad -0.7\}$ and $x_2(n) = \{0.5 \quad 0.2 \quad 0.1 \quad 0.2 \quad 0.6\}$.
- 1.4** Repeat Problem 1.2 with $x_1(n)$ and $x_2(n)$ as given in Problem 1.3.
- 1.5** Find the even and odd parts of $x_1(n)$ and $x_2(n)$ given in Problem 1.1. Even part of $x_1(n)$ is defined as $[x_1(n) + x_1(-n)]/2$, and the odd part as $[x_1(n) - x_1(-n)]/2$.
- 1.6** Repeat Problem 1.5 with $x_1(n)$ and $x_2(n)$ as given in Problem 1.3.
- 1.7** Find $S_1(k) = \sum_{n=0}^{n=k} n$ and sketch it, for $k = 0, 1, \dots, 5$.
- 1.8** Find and sketch $S_2(k) = \sum_{n=-\infty}^k x_1(n)$ and $S_3(k) = \sum_{n=-\infty}^k x_2(n)$, where $x_1(n)$ and $x_2(n)$ are as given in Problem 1.1.

- 1.9** Repeat Problem 1.8 with $x_1(n)$ and $x_2(n)$ as given in Problem 1.3.
- 1.10** Express the sequences $x_1(n)$ and $x_2(n)$ as a summation of weighted and shifted unit step functions, where $x_1(n)$ and $x_2(n)$ are as given in Problem 1.1.
- 1.11** Repeat Problem 1.10 with the sequences $x_1(n)$ and $x_2(n)$ given in Problem 1.3.
- 1.12** Given $x(n) = [0.5e^{j(\pi/6)}]^n [u(n) - u(n-4)]$, calculate the values of $|x(n)|$ and sketch them. What are the real and imaginary parts of $x(n)$?
- 1.13** Express the real and imaginary parts of $x(n) = \sum_{k=0}^4 [0.5e^{j(\pi/6)}]^k \delta(n-k)$.
- 1.14** What are the real and imaginary parts of $q(n) = \sum_{n=0}^{\infty} (0.3 - j0.4)^n$?
- 1.15** What is the fundamental period of $p_1(n) = e^{j0.3\pi n}$?
- 1.16** Find the fundamental period of $p_2(n) = e^{j0.4\pi n} + 2e^{j0.6\pi n}$.
- 1.17** Find the fundamental period of $p_3(n) = \cos(0.5\pi n) + 4\cos(2.5\pi n)$.
- 1.18** What is the fundamental period of $p_4(n) = \cos(0.2\pi n) + \cos(0.7\pi n) + \cos(\pi n)$?
- 1.19** Find the fundamental period of $p_5(n) = \cos(0.5n) + 3\cos(0.2n)$.
- 1.20** Find the fundamental period of $p_6(n) = p_1(n)p_2(n)$, where $p_1(n)$ and $p_2(n)$ are as given above.
- 1.21** Find the fundamental period of $p_7(n) = \cos(1.2\pi n) + 4\sin(0.2\pi n)$.
- 1.22** What is the fundamental period of $p_8(n) = \cos(0.1\pi n)p_3(n)$?
- 1.23** The sinusoidal sequence $h(n) = A\cos(\omega_0 n + \phi)$ has the following values over one period; find the values of A , ω_0 and ϕ :

$$h(n) = \{2.00 \quad 0.00 \quad -2.00 \quad -0.00\}$$

REFERENCES

1. S. K. Mitra and J. F. Kaiser, *Handbook for Digital Signal Processing*, Wiley-Interscience, 1993, Chapter 1.
2. J. G. Proakis and D. G. Manolakis, *Digital Signal Processing*, Prentice-Hall, 1996.
3. A. Bateman and I. Patterson-Stephans, *The DSP Handbook, Algorithms, Applications and Design Techniques*, Prentice-Hall, 2000.
4. S. K. Mitra, *Digital Signal Processing, A Computer-Based Approach*, McGraw-Hill, 1998.
5. A. V. Oppenheim and R. W. Schaffer, *Discrete-Time Signal Processing*, Prentice-Hall, 1989.

6. R. D. Strum and D. E. Kirk, *Discrete Systems and Digital Signal Processing*, Addison-Wesley, 1989.
7. S. S. Soliman and M. D. Srinath, *Continuous and Discrete Signals and Systems*, Prentice-Hall, 1990.
8. L. R. Rabiner and B. Gold, *Theory and Application of Digital Signal Processing*, Prentice-Hall, 1975.
9. E. C. Ifeachor and B. W. Jervis, *Digital Signal Processing*, Prentice-Hall, 2002.
10. V. K. Ingle and J. G. Proakis, *Digital Signal Processing Using MATLAB^(R) V.4*, PWS Publishing, 1997.