

Jacek F. Gieras
Rong-Jie Wang
Maarten J. Kamper

Axial Flux Permanent Magnet Brushless Machines

2nd Edition



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Prof. Jacek F. Gieras
Hamilton Sundstrand, Applied Research
Rockford, Illinois, U.S.A.
University of Technology and Life Sciences
Bydgoszcz, Poland
jgieras@ieee.org

Prof. Maarten J. Kamper
Laboratory for Electrical Machines
and Drives
Department of Electrical
and Electronic Engineering
University of Stellenbosch, South Africa
kamper@sun.ac.za

Dr. Rong-Jie Wang
Laboratory for Electrical Machines
and Drives
Department of Electrical
and Electronic Engineering
University of Stellenbosch, South Africa
rwang@sun.ac.za

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Preface

Growing interest in new topologies of permanent magnet (PM) brushless machines has prompted the authors to update the first edition of *Axial flux permanent magnet brushless machines*, which was originally published in 2004. In the second edition new sections (non-overlap concentrated coil windings, rotor dynamics, miniature axial flux PM motors, in wheel motors), new examples of applications and more numerical examples have been added. On request of engineers involved in axial flux PM motors technology, a CD with all numerical examples solved in Mathcad has been attached.

The importance of PM brushless machines technology and its impact on energy conversion systems are receiving increasing attention each year. While d.c. brush machines production is shrinking, PM brushless machines are replacing d.c. brush and sometimes induction motors in consumer electronics, kitchen and bath equipment, public life, instrumentation and automation system, clinical engineering, industrial electromechanical drives, automobile manufacturing industry, electric and hybrid electric vehicles, marine vessels, toys, more electric aircrafts and many other applications on larger scale. New applications have emerged in distributed generation systems (wind turbine generators, high-speed microturbine generators), miniature power supplies, flywheel energy storages, aircraft and rotorcraft actuators, missile fin actuators, naval integrated motor-propellers (rim driven thrusters). The role of axial flux PM brushless machines is increasing especially in applications where integration of motors with other mechanical parts is imperative.

We believe that the new edition will provide the most up-to-date knowledge on the analysis, design, control and applications of axial flux PM brushless machines. It will increase awareness and stimulate innovations in this field.

*Jacek F. Gieras
Rong-Jie Wang
Maarten J. Kamper*

Contents

1	Introduction	1
1.1	Scope	1
1.2	Features	1
1.3	Development of AFPM Machines	3
1.4	Types of Axial Flux PM Machines	3
1.5	Topologies and Geometries	6
1.6	Rotor Dynamics	10
1.7	Axial Magnetic Field Excited by PMs	12
1.8	PM Eddy-Current Brake as the Simplest AFPM Brushless Machine	14
1.9	AFPM Machines versus RFPM Machines	17
1.10	Power Limitation of AFPM Machines	19
	Numerical Examples	19
2	Principles of AFPM Machines	29
2.1	Magnetic Circuits	29
2.1.1	Single-Sided Machines	29
2.1.2	Double-Sided Machines With Internal PM Disc Rotor	29
2.1.3	Double-Sided Machines With Internal Ring-Shaped Core Stator	31
2.1.4	Double-Sided Machines With Internal Slotted Stator	33
2.1.5	Double-Sided Machines With Internal Coreless Stator	33
2.1.6	Multidisc Machines	34
2.2	Windings	35
2.2.1	Three-Phase Windings Distributed in Slots	35
2.2.2	Toroidal Winding	36
2.2.3	Coreless Stator Winding	36
2.2.4	Non-Overlap (Concentrated Coil) Windings	38
2.3	Torque Production	40
2.4	Magnetic Flux	42

2.5	Electromagnetic Torque and EMF	43
2.6	Losses and Efficiency	45
2.6.1	Stator Winding Losses	45
2.6.2	Stator Core Losses	46
2.6.3	Core Loss Finite Element Model	48
2.6.4	Losses in Permanent Magnets	48
2.6.5	Rotor Core Losses	49
2.6.6	Eddy Current Losses in Stator Conductors	50
2.6.7	Rotational Losses	51
2.6.8	Losses for Nonsinusoidal Current	52
2.6.9	Efficiency	53
2.7	Phasor Diagrams	53
2.8	Sizing Equations	56
2.9	Armature Reaction	58
2.10	AFPM Motor	62
2.10.1	Sine-Wave Motor	62
2.10.2	Square-Wave Motor	63
2.11	AFPM Synchronous Generator	66
2.11.1	Performance Characteristics of a Stand Alone Generator	66
2.11.2	Synchronization With Utility Grid	67
	Numerical Examples	68
3	Materials and Fabrication	79
3.1	Stator Cores	79
3.1.1	Nonoriented Electrical Steels	79
3.1.2	Amorphous Ferromagnetic Alloys	82
3.1.3	Soft Magnetic Powder Composites	84
3.1.4	Fabrication of Stator Cores	84
3.2	Rotor Magnetic Circuits	88
3.2.1	PM Materials	89
3.2.2	Characteristics of PM Materials	94
3.2.3	Operating Diagram	99
3.2.4	Permeances for Main and Leakage Fluxes	102
3.2.5	Calculation of Magnetic Circuits With PMs	105
3.2.6	Fabrication of Rotor Magnetic Circuits	107
3.3	Windings	109
3.3.1	Conductors	109
3.3.2	Fabrication of Slotted Windings	110
3.3.3	Fabrication of Coreless Windings	111
	Numerical Examples	112

4	AFPM Machines With Iron Cores	123
4.1	Geometries	123
4.2	Commercial AFPM Machines With Stator Ferromagnetic Cores	124
4.3	Some Features of Iron-Cored AFPM Machines	125
4.4	Magnetic Flux Density Distribution in the Air Gap	126
4.5	Calculation of Reactances	128
4.5.1	Synchronous and Armature Reaction Reactances	128
4.5.2	Stator Leakage Reactance	128
4.6	Performance Characteristics	132
4.7	Performance Calculation	132
4.7.1	Sine-Wave AFPM Machine	132
4.7.2	Synchronous Generator	135
4.7.3	Square-Wave AFPM Machine	137
4.8	Finite Element Calculations	138
	Numerical Examples	140
5	AFPM Machines Without Stator Cores	153
5.1	Advantages and Disadvantages	153
5.2	Commercial Coreless Stator AFPM Machines	153
5.3	Coreless Stator AFPM Microgenerators	155
5.4	Performance Calculation	155
5.4.1	Steady-State Performance	155
5.4.2	Dynamic Performance	158
5.5	Calculation of Coreless Winding Inductances	160
5.5.1	Classical Approach	160
5.5.2	FEM Approach	161
5.6	Performance Characteristics	165
5.7	Performance of Coreless Non-Overlap Winding AFPM Machines	166
5.8	Eddy Current Losses in the Stator Windings	170
5.8.1	Eddy Current Loss Resistance	170
5.8.2	Reduction of Eddy Current Losses	172
5.8.3	Reduction of Circulating Current Losses	174
5.8.4	Measurement of Eddy Current Losses	175
5.9	Armature Reaction	175
5.10	Mechanical Design Features	179
5.10.1	Mechanical Strength Analysis	179
5.10.2	Imbalanced Axial Force on the Stator	183
5.11	Thermal Problems	183
	Numerical Examples	184

6	AFPM Machines Without Stator and Rotor Cores	193
6.1	Advantages and Disadvantages	193
6.2	Topology and Construction	193
6.3	Air Gap Magnetic Flux Density	195
6.4	Electromagnetic Torque and EMF	198
6.5	Commercial Coreless AFPM Motors	198
6.6	Case Study: Low-Speed AFPM Coreless Brushless Motor	200
6.6.1	Performance Characteristics	200
6.6.2	Cost Analysis	202
6.6.3	Comparison With Cylindrical Motor With Laminated Stator and Rotor Cores	203
6.7	Case Study: Low-Speed Coreless AFPM Brushless Generator	205
6.8	Characteristics of Coreless AFPM Machines	206
	Numerical Examples	208
7	Control	217
7.1	Control of Trapezoidal AFPM Machine	217
7.1.1	Voltage Equations	218
7.1.2	Solid-State Converter	220
7.1.3	Current Control	223
7.1.4	Speed Control	225
7.1.5	High Speed Operation	226
7.2	Control of Sinusoidal AFPM Machine	227
7.2.1	Mathematical Model and dq Equivalent Circuits	227
7.2.2	Current Control	233
7.2.3	Speed Control	233
7.2.4	Hardware of Sinusoidal AFPM Machine Drive	237
7.3	Sensorless Position Control	241
	Numerical Examples	242
8	Cooling and Heat Transfer	251
8.1	Importance of Thermal Analysis	251
8.2	Heat Transfer Modes	251
8.2.1	Conduction	252
8.2.2	Radiation	252
8.2.3	Convection	253
8.3	Cooling of AFPM Machines	256
8.3.1	AFPM Machines With Self-Ventilation	257
8.3.2	AFPM Machines With External Ventilation	265
8.4	Lumped Parameter Thermal Model	269
8.4.1	Thermal Equivalent Circuit	269
8.4.2	Conservation of Energy	271
8.5	Machine Duties	272
8.5.1	Continuous Duty	272

8.5.2	Short-Time Duty	272
8.5.3	Intermittent Duty	273
	Numerical Examples	274
9	Applications	281
9.1	Power Generation	281
9.1.1	High Speed Generators	281
9.1.2	Low Speed Generators	284
9.2	Electric Vehicles	286
9.2.1	Hybrid Electric Vehicles	287
9.2.2	Battery Electric Vehicles	291
9.2.3	Fuel Cell Electric Vehicles	294
9.3	Ship Propulsion	295
9.3.1	Large AFPM Motors	295
9.3.2	Propulsion of Unmanned Submarines	297
9.3.3	Counterrotating Rotor Marine Propulsion System	297
9.4	Electromagnetic Aircraft Launch System	299
9.5	Mobile Drill Rigs	301
9.6	Oil Beam Pumps	302
9.7	Elevators	305
9.8	Miniature AFPM Brushless Motors	308
9.9	Vibration Motors	310
9.10	Computer Hard Disc Drives	312
9.11	Ventricular Assist Devices	313
9.12	Axial Flux Machines with Superconducting Field Excitation System	317
	Numerical Examples	322
	Symbols and Abbreviations	327
	References	335
	Patents	351
	Index	357

Introduction

1.1 Scope

The term *axial flux permanent magnet* (AFPM) machine in this book relates only to permanent magnet (PM) machines with *disc type rotors*. Other AFPM machine topologies, e.g. *transverse flux machines*, have not been considered. In principle, the electromagnetic design of AFPM machines is similar to its radial flux PM (RFPM) counterparts with cylindrical rotors. However, the mechanical design, thermal analysis and assembly process are more complex.

1.2 Features

The AFPM machine, also called the *disc-type* machine, is an attractive alternative to the cylindrical RFPM machine due to its pancake shape, compact construction and high power density. AFPM motors are particularly suitable for electrical vehicles, pumps, fans, valve control, centrifuges, machine tools, robots and industrial equipment. The large diameter rotor with its high moment of inertia can be utilised as a flywheel. AFPM machines can also operate as small to medium power generators. Since a large number of poles can be accommodated, these machines are ideal for low speed applications, as for example, electromechanical traction drives, hoists or wind generators.

The unique disc-type profile of the rotor and stator of AFPM machines makes it possible to generate diverse and interchangeable designs. AFPM machines can be designed as single air gap or multiple air gaps machines, with slotted, slotless or even totally ironless armature. Low power AFPM machines are frequently designed as machines with slotless windings and surface PMs.

As the output power of the AFPM machines increases, the contact surface between the rotor and the shaft in proportion to the power becomes smaller. Careful attention must be given to the design of the rotor-shaft mechanical joint as this is usually the cause of failures of disc type machines.

Table 1.1. Specifications of double-sided disc type AFPM brushless servo motors manufactured by *E. Bautz GmbH*, Weiterstadt, Germany

Quantity	S632D	S634D	S712F	S714F	S802F	S804F
Rated power, W	680	940	910	1260	1850	2670
Rated torque, Nm	1.3	1.8	2.9	4.0	5.9	8.5
Maximum torque, Nm	7	9	14	18	28	40
Standstill torque, Nm	1.7	2.3	3.5	4.7	7.0	10.0
Rated current, A	4.0	4.9	4.9	6.6	9.9	11.9
Maximum current, A	21	25	24	30	47	56
Standstill current, A	5.3	6.3	5.9	7.8	11.7	14.0
Rated speed, rpm	5000	5000	3000	3000	3000	3000
Maximum speed, rpm	6000	6000	6000	6000	6000	6000
Armature constant, V/1000 rpm	23	25	42	42	42	50
Torque constant, Nm/A	0.35	0.39	0.64	0.64	0.64	0.77
Resistance, Ω	2.5	1.8	2.4	1.5	0.76	0.62
Inductance, mH	3.2	2.8	5.4	4.2	3.0	3.0
Moment of inertia, $\text{kgm}^2 \times 10^{-3}$	0.08	0.12	0.21	0.3	0.6	1.0
Mass, kg	4.5	5.0	6.2	6.6	9.7	10.5
Diameter of frame, mm	150	150	174	174	210	210
Length of frame, mm	82	82	89	89	103	103
Power density, W/kg	151.1	188.0	146.8	190.9	190.7	254.3
Torque density, Nm/kg	0.289	0.36	0.468	0.606	0.608	0.809

In some cases, rotors are embedded in power-transmission components to optimise the number of parts, volume, mass, power transfer and assembly time. For electric vehicles (EVs) with built-in wheel motors the payoff is a simpler electromechanical drive system, higher efficiency and lower cost. Dual-function rotors may also appear in pumps, elevators, fans and other types of machinery, bringing new levels of performance to these products.

Most applications use the AFPM machine as a d.c. brushless motor. Encoders, resolvers or other rotor position sensors are thus a vital part of brushless disc motors.

Table 1.1 shows specifications of AFPM brushless servo motors rated up to 2.7 kW, manufactured by *E. Bautz GmbH*, Weiterstadt, Germany.

1.3 Development of AFPM Machines

The history of electrical machines reveals that the earliest machines were axial flux machines (M. Faraday, 1831, anonymous inventor with initials P.M., 1832, W. Ritchie, 1833, B. Jacobi, 1834). However, shortly after T. Davenport (1837) claimed the first patent [P1] for a radial flux machine, conventional radial flux machines have been widely accepted as the mainstream configuration for electrical machines [33, 53].

The first primitive working prototype of an axial flux machine ever recorded was M. Faraday's disc (1831) - see Numerical Example 1.2. The disc type construction of electrical machines also appears in N. Tesla's patents, e.g. U.S. patent No. 405 858 [P2] entitled *Electro-Magnetic Motor* and published in 1889 (Fig. 1.1). The reasons for shelving the axial flux machine were multi-fold and may be summarised as follows:

- strong axial (normal) magnetic attraction force between the stator and rotor;
- fabrication difficulties, such as cutting slots in laminated cores and other methods of making slotted stator cores;
- high costs involved in manufacturing the laminated stator cores;
- difficulties in assembling the machine and keeping the uniform air gap.

Although, the first PM excitation system was applied to electrical machines as early as the 1830s, the poor quality of hard magnetic materials soon discouraged their use. The invention of Alnico in 1931, barium ferrite in the 1950s and especially the rare-earth neodymium-iron-boron (NdFeB) material (announced in 1983) have made a *comeback* of the PM excitation system possible.

It is generally believed that the availability of high energy PM materials (especially *rare earth* PMs) is the main driving force for exploitation of novel PM machine topologies and has thus revived AFPM machines. Prices of rare-earth PMs have been following a descending curve in the last decade of the 20th century with a sharp decline in the last three years. A recent market survey shows that the NdFeB PMs can now be purchased in the Far East for less than U.S.\$ 20 per kilogram. With the availability of more affordable PM materials, AFPM machines may play a more important role in the near future.

1.4 Types of Axial Flux PM Machines

In principle, each type of a radial flux machine should have its corresponding axial flux (disc type) version. In practice, disc type machines are limited to the following three types:

- PM d.c. commutator machines;

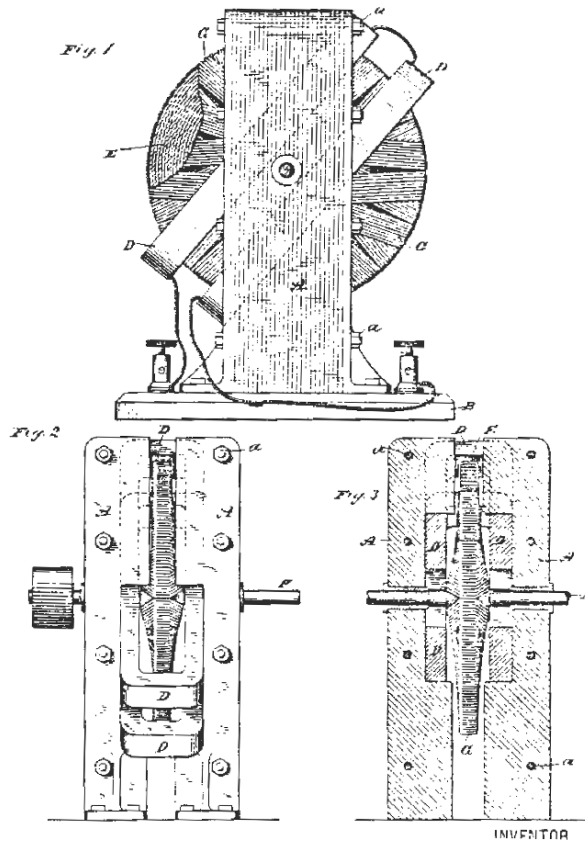


Fig. 1.1. Electro-magnetic motor with disc rotor according to N. Tesla's patent No. 405 858, 1889 [P2].

- PM brushless d.c. and synchronous machines;
- induction machines

Similar to its RFPM counterpart, the AFPM d.c. commutator machine uses PMs to replace the electromagnetic field excitation system. The rotor (armature) can be designed as a *wound rotor* or *printed winding rotor*.

In the wound rotor, the armature winding is made of copper wires and moulded with resin. The commutator is similar to that of the conventional type, i.e. it can be either a cylindrical or radial commutator.

The *disc-type printed armature winding motor* is shown in Fig. 1.2. The rotor (armature) does not have a ferromagnetic core and its winding is similar to the wave winding of conventional d.c. commutator machines. The coils are stamped from pieces of sheet copper and then welded, forming a wave winding.

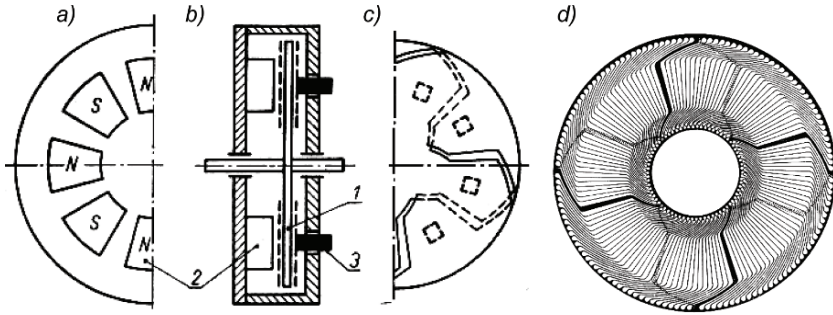


Fig. 1.2. AFMPM 8-pole d.c. commutator motor with printed rotor winding: (a) stator with PMs, (b) cross section, (c) rotor (armature) windings and brushes, (d) construction of $2p = 8$ winding with 145 bars. 1 — rotor with double-sided printed winding, 2 — PMs, 3 — brushes.

When this motor was invented by J. Henry Baudot [18], the armature was made using a similar method to that by which printed circuit boards are fabricated. Hence, this is called the printed winding motor. The magnetic flux of a d.c. printed winding commutator motor with a large air gap can be produced using cost effective Alnico magnets with high remanence.

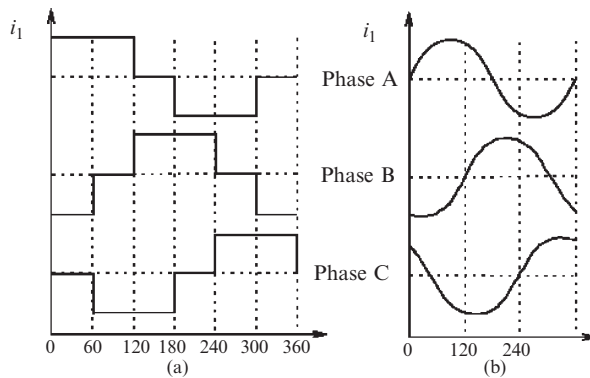


Fig. 1.3. Current waveforms for AFPM brushless machines: (a) square-wave machine, (b) sinewave machine.

AFPM d.c. commutator motors are still a versatile and economical choice for certain industrial, automotive and domestic applications such as fans, blowers, small EVs, power tools, appliances, etc.

Practically, d.c. brushless and a.c. synchronous machines have almost the same structure, though their theory and operation principles are somewhat

different [106, 122, 187]. The main difference is in the shape of the operation current waveform (Fig. 1.3), i.e.:

- the d.c. brushless machine generates a trapezoidal EMF waveform and is operated with a rectangular line current waveform (also called a *square-wave* machine);
- the a.c. synchronous machine generates a sinusoidal EMF waveform and is operated with sinewave currents (also called a *sinewave* machine).

It is difficult to manufacture a laminated rotor with cage winding for a disc-type induction machine [160]. If the cage winding is replaced with a non-magnetic high conductivity (Cu or Al) homogenous disc or steel disc coated with copper layer, the performance of the machine drastically deteriorates. Therefore, there is little interest in disc type induction machines so far [160, 267].

1.5 Topologies and Geometries

From a construction point of view, brushless AFPM machines can be designed as single-sided or double-sided, with or without armature slots, with or without armature core, with internal or external PM rotors, with surface mounted or interior PMs and as single stage or multi-stage machines.

In the case of double-sided configurations, either the external stator or external rotor arrangement can be adopted. The first choice has the advantage of using fewer PMs at the expense of poor winding utilisation while the second one is considered as a particularly advantageous machine topology [38]. The diverse topologies of AFPM brushless machines may be classified as follows:

- single-sided AFPM machines
 - with slotted stator (Fig. 1.4a)
 - with slotless stator
 - with salient-pole stator
- double-sided AFPM machines
 - with internal stator (Fig. 1.4b)
 - with slotted stator
 - with slotless stator
 - with iron core stator
 - with coreless stator (Fig. 1.4d)
 - without both rotor and stator cores
 - with salient pole stator (Fig. 1.5)
 - with internal rotor (Fig. 1.4c)
 - with slotted stator
 - with slotless stator
 - with salient pole stator (Fig. 1.6)
- multi-stage (multidisc) AFPM machines (Fig. 1.7)

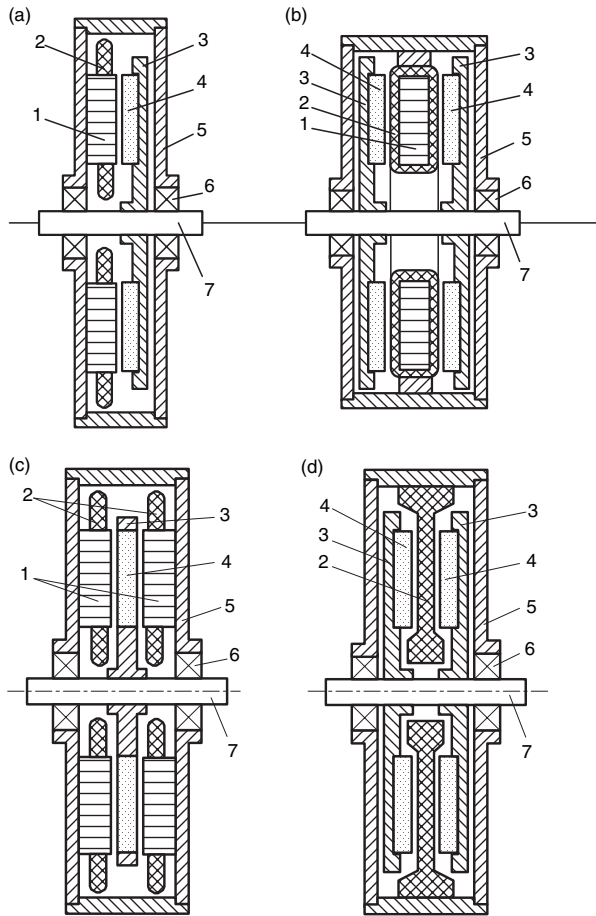


Fig. 1.4. Basic topologies of AFPM machines: (a) single-sided slotted machine, (b) double-sided slotless machines with internal stator and twin PM rotor, (c) double sided machine with slotted stator and internal PM rotor, (d) double-sided coreless motor with internal stator. 1 — stator core, 2 — stator winding, 3 — rotor, 4 — PM, 5 — frame, 6 — bearing, 7 — shaft.

The air gap of the slotted armature AFPM machine is relatively small. The mean magnetic flux density in the air gap decreases under each slot opening due to increase in the reluctance. The change in the mean magnetic flux density caused by slot openings corresponds to a fictitious increase in the air gap [121]. The relation between fictitious g' and physical air gap g is expressed with the aid of Carter coefficient $k_C > 1$, i.e.,

$$g' = gk_C \quad (1.1)$$

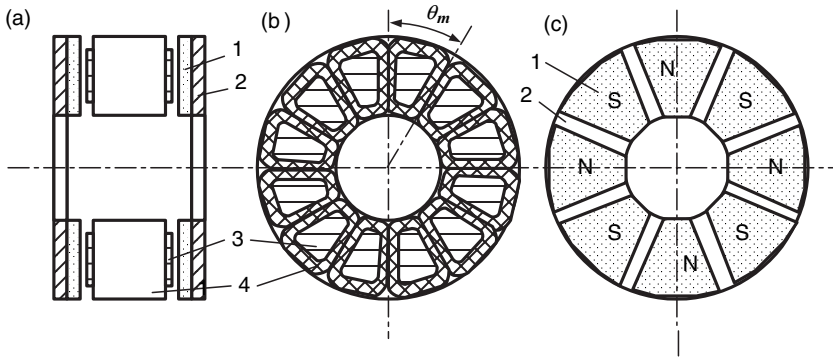


Fig. 1.5. Double-sided AFPM brushless machine with internal salient-pole stator and twin external rotor [185]: (a) construction; (b) stator; (c) rotor. 1 — PM, 2 — rotor backing steel disc, 3 — stator pole, 4 — stator coil.

$$k_C = \frac{t_1}{t_1 - \gamma g} \quad (1.2)$$

$$\gamma = \frac{4}{\pi} \left[\frac{b_{14}}{2g} \arctan \left(\frac{b_{14}}{2g} \right) - \ln \sqrt{1 + \left(\frac{b_{14}}{2g} \right)^2} \right] \quad (1.3)$$

where t_1 is the average slot pitch and b_{14} is the width of slot opening.

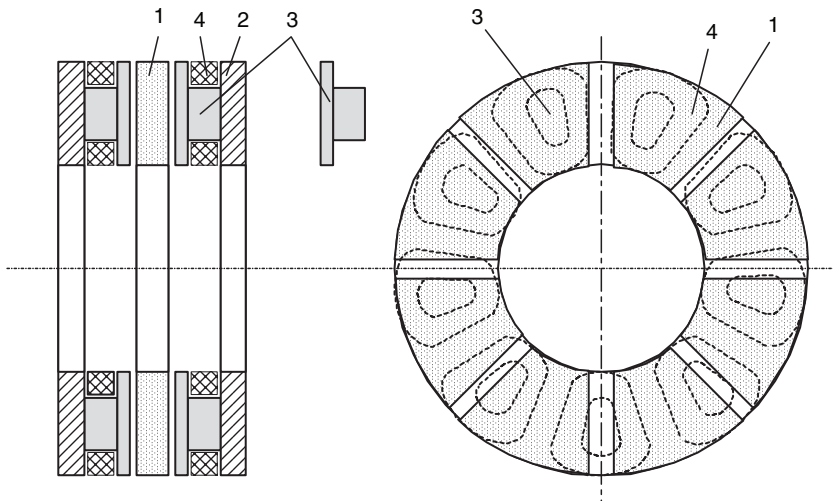


Fig. 1.6. Double-sided AFPM brushless machine with three-phase, 9-coil external salient-pole stator and 8-pole internal rotor. 1 — PM, 2 — stator backing ferromagnetic disc, 3 — stator pole, 4 — stator coil.

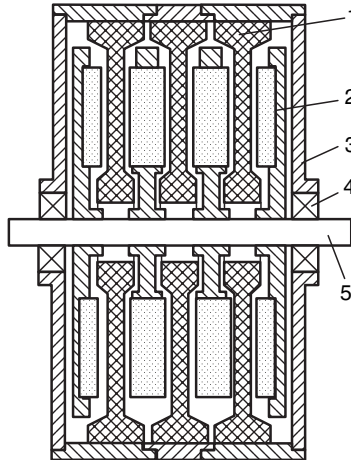


Fig. 1.7. Coreless multidisc AFPM machine with three coreless stators and five PM rotor units: 1 — stator winding, 2 — rotor unit, 4 — PM, 3 — frame, 4 — bearing, 5 — shaft.

For AFPM machines with *slotless windings* the air gap is much larger and equal to the mechanical clearance plus the thickness of all non-magnetic materials (winding, insulation, potting, supporting structure) that is passed by the main magnetic flux. Since there are no slots, Carter coefficient $k_C = 1$. Compared to a conventional slotted winding, the slotless armature winding has advantages such as simple stator assembly, elimination of the cogging torque and reduction of rotor surface losses, magnetic saturation and acoustic noise. The disadvantages include the use of more PM material, lower winding inductances sometimes causing problems for inverter-fed motors and significant eddy current losses in slotless conductors [49].

In the double-sided, salient-pole AFPM brushless machine shown in Fig. 1.5, the stator coils with concentrated parameters are wound on axially laminated poles. To obtain a three-phase self-starting motor, the number of the stator poles should be different from the number of the rotor poles, e.g. 12 stator poles and 8 rotor poles [173, 174, 185]. Fig. 1.6 shows a double-sided AFPM machine with external salient pole stators and internal PM rotor. There are nine stator coils and eight rotor poles for a three-phase AFPM machine.

Depending on the application and operating environment, slotless stators may have ferromagnetic cores or be completely coreless. *Coreless stator* configurations eliminate any ferromagnetic material from the stator (armature) system, thus making the associated eddy current and hysteresis core losses nonexistent. This type of configuration also eliminates axial magnetic attraction forces between the stator and rotor at zero-current state. It is interesting that slotless AFPM machines are often classified according to their winding

arrangements and coil shapes, namely, *toroidal*, *trapezoidal* and *rhomboidal* forms [38, 49, 87].

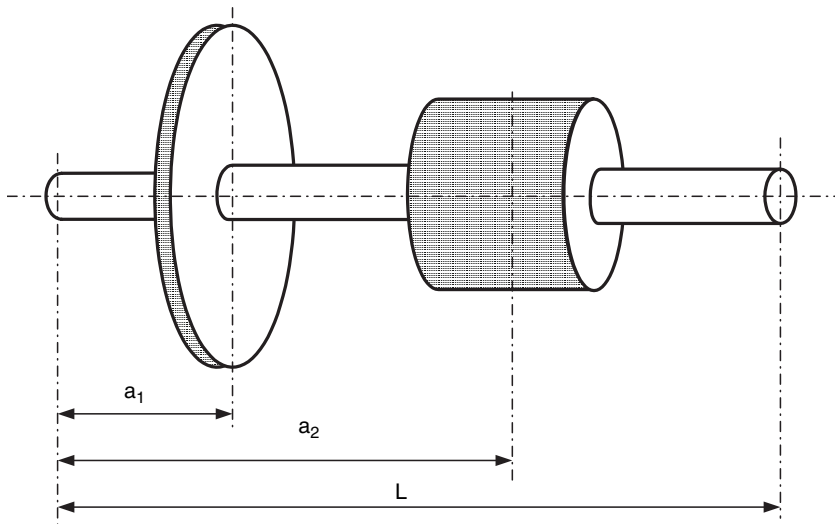


Fig. 1.8. Solid cylindrical shaft loaded with two masses m_1 and m_2 .

1.6 Rotor Dynamics

All spinning shafts, even in the absence of external load, deflect during rotation. Fig. 1.8 shows a shaft with two rotating masses m_1 and m_2 . The mass m_1 can represent a disc-type rotor of an AFPM machine while the mass m_2 can represent a load. The mass of the shaft is m_{sh} . The combined mass of the rotor, load and shaft can cause deflection of the shaft that will create resonant vibration at certain speed called *whirling* or *critical speed*. The frequency when the shaft reaches its critical speed can be found by calculating the frequency at which transverse vibration occurs.

The critical speed in rev/s of the i th rotating mass can be found as [227]

$$n_i = \frac{1}{2\pi} \sqrt{\frac{g}{\sigma_i}} \quad (1.4)$$

where $g = 9.81 \text{ m/s}^2$ is the acceleration due to gravity and δ_i is the static deflection at the i th position of the rotor due to i th rotor only, i.e.,

$$\sigma_i = \frac{m_i g a_i^2 (L - a_i)^2}{3E_i I_i L} \quad (1.5)$$

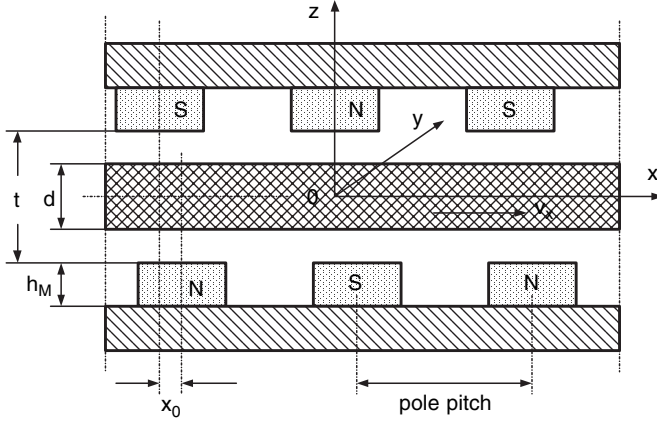


Fig. 1.9. Twin-stator double-sided AFPM machine in Cartesian coordinate system.

In the above equation E_i is the modulus of elasticity (for steel $E = 200 \times 10^9$ Pa), I_i is the area moment of inertia of cross-sectional area, L is the length of the shaft and a_i is the location of the i th rotor from the left end of the shaft (Fig. 1.8). The area moment of inertia can be found as

$$I_i = \frac{\pi D_i^4}{64} \quad (1.6)$$

The resultant angular critical speed $\Omega_{cr} = 2\pi n_{cr}$ for the shaft loaded with i number of rotors ($\Omega_i = 2\pi n_i$) can be found on the basis of Dunkerley's equation [80] as

$$\frac{1}{\Omega_{cr}^2} = \sum_i \frac{1}{\Omega_i^2} \quad (1.7)$$

or Rayleigh's equation [227]

$$\Omega_{cr} = \sqrt{\frac{g \sum_i (m_i \sigma_i)}{\sum_i (m_i \sigma_i^2)}} \quad (1.8)$$

The shaft is also considered a rotor with mass m_{sh} concentrated at $0.5L$ where L is the length of the shaft (bearing-to-bearing).

Dunkerley's empirical method uses the frequencies that each individual load creates when each load acts alone and then combines them to give an approximation for the whole system [80]. Thus eqn (1.7) is an approximation to the *first natural frequency* of vibration of the system which is assumed nearly equal to the critical speed of rotation. Rayleigh's method is based on the fact that the maximum kinetic energy must be equal to maximum potential energy for a conservative system under free vibration [227].

1.7 Axial Magnetic Field Excited by PMs

A double-sided AFPM machine with twin PM rotor in xyz rectangular coordinate system is shown in Fig. 1.9. Assuming that the radius of curvature is higher than the pole pitch and the centre axes of the opposite rotor poles are shifted by a linear distance x_0 , the normal component of the magnetic flux density on the surface of the rotor can be described in the stationary xyz coordinate system by the following equations:

- at $z = 0.5d$

$$B_{mz1}(x, t) = B_0 \sum_{\nu=1}^{\infty} b_{\nu} \cos(\omega_{\nu} t \mp \beta_{\nu} x - \frac{\pi}{2}) \quad (1.9)$$

- at $z = -0.5d$

$$B_{mz2}(x, t) = B_0 \sum_{\nu=1}^{\infty} b_{\nu} \cos \left[\omega_{\nu} t \mp \beta_{\nu} (x - x_0) - \frac{\pi}{2} \right] \quad (1.10)$$

where B_0 is the value of the normal component in the center axis of the North pole, and

$$\beta_{\nu} = \nu \frac{\pi}{\tau} \quad (1.11)$$

$$\omega_{\nu} = \beta_{\nu} v = \beta_{\nu} \pi D n \quad (1.12)$$

$$\begin{aligned} b_{\nu} = \frac{4}{\tau} \left[\frac{c_p}{c_p^2 + \beta_{\nu}^2} \sinh \alpha - 6 \left(\frac{1}{\beta_{\nu}} \right)^4 \frac{1}{b_t^3} \cosh \alpha + 3 \left(\frac{1}{\beta_{\nu}} \right)^2 \frac{2}{\tau - b_p} \cosh \alpha \right] \\ \times \sin(\nu \frac{\pi}{2}) \sin(\nu \frac{\pi b_t}{\tau}) \\ + \frac{4}{\tau} \left[\frac{\beta_{\nu}}{c_p^2 + \beta_{\nu}^2} + 6 \left(\frac{1}{\beta_{\nu}} \right)^3 \frac{1}{b_t^2} - \frac{1}{\beta_{\nu}} \right] \cosh \alpha \sin \left(\nu \frac{\pi}{2} \right) \cos(\beta_{\nu} b_t) \end{aligned} \quad (1.13)$$

$$b_t = \frac{\tau - b_p}{2} \quad c_p = 2 \frac{\alpha}{b_p}$$

In the above eqns (1.9), (1.10), (1.11), (1.12), and (1.13) $v = v_x$ is the linear speed of the rotor in the x -direction, $n = \pi D/v$ is the rotational speed in rev/s and the parameter α depends on the shape of the distribution of the normal component of the magnetic flux density (Fig. 1.10). For the flat-topped

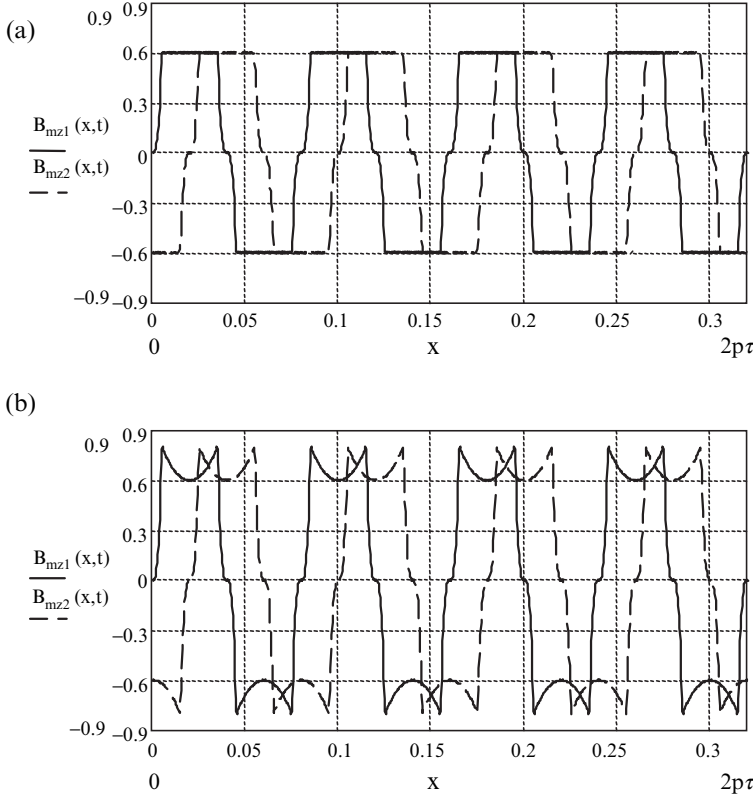


Fig. 1.10. Distribution of the normal components of the magnetic flux density according to eqns (1.9), (1.10) and (1.13) for $\tau = 0.04$ m, $b_p = 0.03$ m, $B_0 = 0.6$ T, $t = 0$, $x_0 = 0.5\tau$: (a) $\alpha = 0$, (b) $\alpha = 0.8$.

curve $\alpha = 0$ and for the concave curve (armature or eddy-current reaction) $0 < \alpha \leq 1$. The coefficient b_ν according to eqn (1.13) has been derived in [103].

The average diameter D of the PM excitation system and corresponding average pole pitch are:

$$D = 0.5(D_{out} + D_{in}) \quad \tau = \frac{\pi D}{2p} \quad (1.14)$$

where D_{in} is the inner diameter of PMs, D_{out} is the outer diameter of PMs and $2p$ is the number of poles.

The electromagnetic field analysis in AFPM brushless machines has been discussed in e.g., [99, 100, 281, 282].

1.8 PM Eddy-Current Brake as the Simplest AFPM Brushless Machine

A double-sided, PM excited *eddy-current brake* with high conductivity non-magnetic disc-type rotor is one of the simplest brushless AFPM machines (Fig. 1.9). In an eddy-current brake the PM excitation system is stationary and the conductive rotor rotates at the speed n . Eqns (1.9) to (1.14) are valid since the stationary PM excitation system and rotating electric circuit (armature) are equivalent to the rotating PMs and the stationary electric circuit.

It is assumed that the eddy currents in the non-magnetic conductive disc flow only in the radial direction, i.e., in the y -direction (Fig. 1.9). Thus, the magnetic vector potential $\mathbf{A}$ in the disc is described by the following scalar equation (2D analysis):

$$\frac{\partial^2 A_{my\nu}}{\partial x^2} + \frac{\partial^2 A_{my\nu}}{\partial z^2} = \alpha_\nu A_{my\nu} \quad (1.15)$$

where

$$\alpha_\nu = \sqrt{j\omega_\nu \mu_0 \mu_r \sigma} = (1 + j)k_\nu \quad (1.16)$$

$$k_\nu = \sqrt{\frac{\omega_\nu \mu_0 \mu_r \sigma}{2}} \quad (1.17)$$

In eqns (1.16) and (1.17) the electric conductivity σ depends on the disc temperature. The relative magnetic permeability of paramagnetic (Al) or diamagnetic (Cu) materials $\mu_r \approx 1$. The angular frequency for higher space harmonics is according to eqn (1.12) or

$$\omega_\nu = 2\pi f_\nu; \quad f_\nu = \nu f; \quad \nu = 1, 3, 5, \dots \quad (1.18)$$

General solution to eqn (1.15) can be written, for example, as

$$A_{my} = \sum_{\nu=1}^{\infty} \sin(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) [A_{1\nu} \exp(-\kappa_\nu z) + A_{1\nu} \exp(\kappa_\nu z)] \quad (1.19)$$

where

$$\kappa_\nu = \sqrt{\alpha_\nu^2 + \beta_\nu^2} = (a_{R\nu} + a_{X\nu})k_\nu \quad (1.20)$$

$$a_{R\nu} = \frac{1}{\sqrt{2}} \sqrt{\sqrt{4 + \left(\frac{\beta_\nu}{k_\nu}\right)^4} + \left(\frac{\beta_\nu}{k_\nu}\right)^2} \quad (1.21)$$

$$a_{X\nu} = \frac{1}{\sqrt{2}} \sqrt{\sqrt{4 + \left(\frac{\beta_\nu}{k_\nu}\right)^4} - \left(\frac{\beta_\nu}{k_\nu}\right)^2} \quad (1.22)$$

Since the currents in the disc flow only in the radial direction y , $E_{mx} = 0$, $E_{mz} = 0$ and $B_{my} = 0$ for $-0.5d \leq z \leq 0.5d$. Using the magnetic vector potential $\nabla \times \mathbf{A} = \mathbf{B}$ and the second Maxwell's equation $\nabla \times \mathbf{E} = -\partial \mathbf{B} / \partial t$, the remaining electric and magnetic components in the disc can be found as

$$E_{my} = -j\omega_\nu A_{my}$$

$$= - \sum_{\nu=1}^{\infty} j\omega_\nu \sin(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) [A_{1\nu} \exp(-\kappa_\nu z) + A_{2\nu} \exp(\kappa_\nu z)] \quad (1.23)$$

$$B_{mx} = -\frac{\partial A_{my}}{\partial z}$$

$$= \sum_{\nu=1}^{\infty} \kappa_\nu \sin(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) [A_{1\nu} \exp(-\kappa_\nu z) - A_{2\nu} \exp(\kappa_\nu z)] \quad (1.24)$$

$$B_{mz} = \frac{\partial A_{my}}{\partial x}$$

$$= \sum_{\nu=1}^{\infty} \beta_\nu \cos(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) (A_{1\nu} \exp(-\kappa_\nu z) + A_{2\nu} \exp(\kappa_\nu z)) \quad (1.25)$$

The integration constants $A_{1\nu}$ and $A_{2\nu}$ can be found on the basis of equality of normal components of the magnetic flux density in the air and in the disc at $z = 0.5d$ and $z = -0.5d$, i.e.

- at $z = 0.5d$

$$\beta_\nu \cos(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) [A_{1\nu} \exp(-\kappa_\nu d/2) + A_{2\nu} \exp(\kappa_\nu d/2)]$$

$$= B_0 b_\nu \cos(\omega_\nu t + \beta_\nu x - \frac{\pi}{2})$$

- at $z = -0.5d$

$$\beta_\nu \cos[\omega_\nu t + \beta_\nu(x - x_0) - \frac{\pi}{2}] [A_{1\nu} \exp(-\kappa_\nu d/2) + A_{2\nu} \exp(\kappa_\nu d/2)]$$

$$= B_0 b_\nu \cos[\omega_\nu t + \beta_\nu(x - x_0) - \frac{\pi}{2}]$$

There is only backward-rotating magnetic field in the air gap of an eddy-current brake, so that the terms $\beta_\nu x$ and $\beta_\nu(x - x_0)$ in eqns (1.9) and (1.10) are with the + sign. Thus,

$$A_{1\nu} = A_{2\nu} = \frac{1}{\beta_\nu} B_0 b_\nu \frac{\sinh(\kappa_\nu d/2)}{\sinh(\kappa_\nu d)} = \frac{1}{2} \frac{1}{\beta_\nu} B_0 b_\nu \frac{1}{\cosh(\kappa_\nu d/2)} \sinh(\kappa_\nu d) \quad (1.26)$$

because $\sinh(2x) = 2 \sinh x \cosh x$. Putting eqn (1.26) into (1.23), (1.24) and (1.25), the particular solution to eqn (1.15) for E_{my} , B_{mx} and B_{mz} components are

$$E_{my} = - \sum_{\nu=1}^{\infty} j\omega_\nu \frac{B_0 b_\nu}{\beta_\nu} \frac{1}{\cosh(\kappa_\nu d/2)} \sin(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) \cosh(\kappa_\nu z) \quad (1.27)$$

$$B_{mx} = \sum_{\nu=1}^{\infty} \kappa_\nu \frac{B_0 b_\nu}{\beta_\nu} \frac{1}{\cosh(\kappa_\nu d/2)} \sin(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) \sinh(\kappa_\nu z) \quad (1.28)$$

$$B_{mz} = \sum_{\nu=1}^{\infty} B_0 b_\nu \frac{1}{\cosh(\kappa_\nu d/2)} \cos(\omega_\nu t + \beta_\nu x - \frac{\pi}{2}) \cosh(\kappa_\nu z) \quad (1.29)$$

The surface wave impedance for the ν th space harmonic is calculated on the basis of eqns (1.27) and (1.28)

$$\begin{aligned} z_\nu = r_\nu + jx_\nu &= -\mu_0 \mu_r \left[\frac{E_{my}}{B_{mx}} \right]_{z=0.5d} = \frac{\alpha_\nu^2}{\sigma} \frac{1}{\kappa_\nu} \coth(\kappa_\nu \frac{d}{2}) \\ &= [(B_{R\nu} A_{R\nu} - B_{X\nu} A_{X\nu}) + j(B_{X\nu} A_{R\nu} - B_{R\nu} A_{X\nu})] \frac{k_\nu}{\sigma} \end{aligned} \quad (1.30)$$

where

$$A_{R\nu} = \frac{\sinh(a_{R\nu} k_\nu d)}{\cosh(a_{R\nu} k_\nu d) - \cos(a_{X\nu} k_\nu d)} \quad (1.31)$$

$$A_{X\nu} = \frac{-\sin(a_{X\nu} k_\nu d)}{\cosh(a_{R\nu} k_\nu d) - \cos(a_{X\nu} k_\nu d)} \quad (1.32)$$

$$B_{R\nu} = \frac{a_{X\nu}}{0.5(a_{R\nu}^2 + a_{X\nu}^2)} \quad B_{X\nu} = \frac{a_{R\nu}}{0.5(a_{R\nu}^2 + a_{X\nu}^2)} \quad (1.33)$$

The impedance of the disc for the ν th space harmonic

$$Z_\nu = z_\nu \frac{0.5(D_{out} - D_{in})}{\tau/\nu} k_{z\nu} = \frac{\alpha_\nu^2}{\sigma} \frac{1}{\kappa_\nu} \frac{0.5(D_{out} - D_{in})}{\tau/\nu} k_{z\nu} \coth(\kappa_\nu \frac{d}{2}) \quad (1.34)$$

where D_{out} is the outer diameter of PMs, D_{in} is the inner diameter of PMs, τ is the average pole pitch (1.14) and $k_{z\nu}$ is the impedance increase factor due to circumferential currents (in the x direction). The impedance increase factor for the ν th harmonic is [69]

$$k_{z\nu} = 1 + \frac{1}{\nu^2} \frac{\tau}{0.5(D_{out} - D_{in})} \quad (1.35)$$

1.9 AFPM Machines versus RFPM Machines

In pace with the application of new materials, innovation in manufacturing technology and improvements in cooling techniques, further increase in the power density (output power per mass or volume) of the electrical machine has been made possible. There is an inherent limit to this increase for conventional radial flux PM (RFPM) machines because of [30, 53, 106, 165, 192]:

- the bottle-neck feature for the flux path at the root of the rotor tooth in the case of induction and d.c. commutator machines or brushless machines with external rotors (Fig. 1.11);
- much of the rotor core around the shaft (rotor yoke) is hardly utilised as a magnetic circuit;
- heat from the stator winding is transferred to the stator core and then to the frame — there is poor heat removal through the stator air gap, rotor and shaft without forced cooling arrangements.

These limitations are inherently bound with radial flux structures and cannot be removed easily unless a new topology is adopted. The AFPM machine, recognised as having a higher power density than the RFPM machine, is more compact than its radial flux counterpart [29, 53, 106, 165].

Moreover, since the inner diameter of the core of an AFPM machine is usually much greater than the shaft diameter (see Fig. 1.4), better ventilation and cooling can be expected. In general, the special properties of AFPM machines, which are considered advantageous over RFPM machines in certain applications, can be summarised as follows [52, 106]:

- AFPM machines have much larger diameter to length ratio than RFPM machines;
- AFPM machines have a planar and somewhat adjustable air gap;
- capability of being designed to possess a higher power density with some saving in core material;
- the topology of an AFPM machine is ideal to design a modular machine in which the number of the same modules is adjusted to power or torque requirements;
- the larger the outer diameter of the core, the higher the number of poles that can be accommodated, making the AFPM machines a suitable choice for high frequency or low speed operations.

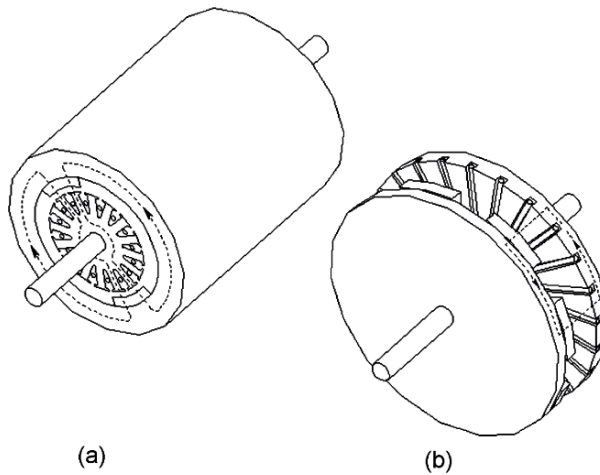


Fig. 1.11. Topologies of (a) RFPM machine (b) AFPM machine.

Consequently, AFPM type machines are particularly suitable for servo, traction, distributed generation and special-purpose applications where their properties offer distinct advantages over their conventional RFPM counterparts.

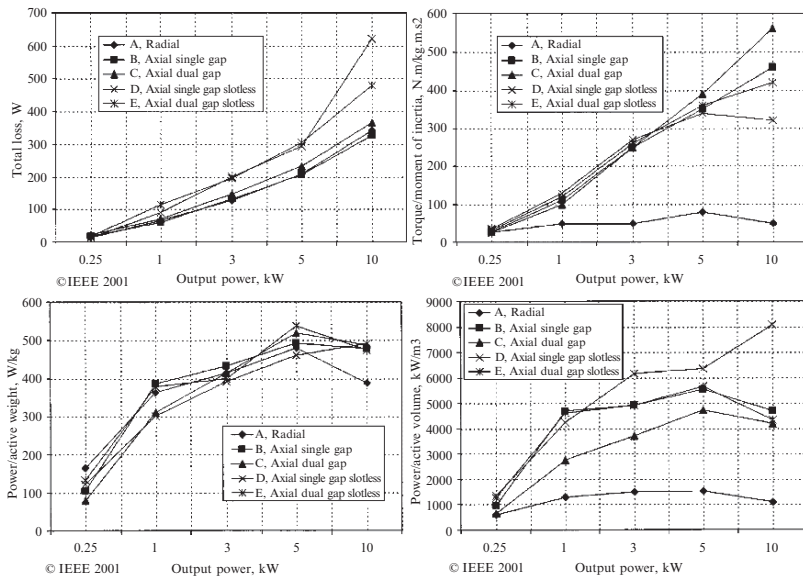


Fig. 1.12. Performance comparison of RFPM and AFPM machines [239].

The quantitative comparison between traditional RFPM machine and AFPM machine is always difficult as it may raise the question of this comparison's fairness. Some published work dealt with quantitative investigations of RFPM and AFPM machine configurations in terms of sizing and power density equations [8, 53, 132, 256, 278]. Fig. 1.12 gives the performance comparison between a conventional RFPM machine and a number of AFPM machines of different configurations at five different power levels [239], which shows that the AFPM machine has a smaller volume and less active material mass for a given power rating than the RFPM machine.

1.10 Power Limitation of AFPM Machines

The power range of AFPM disc-type brushless machines is now from a fraction of a Watt to sub-MW. As the output power of the AFPM machine increases, the contact surface between the rotor and shaft becomes smaller in comparison with the rated power. It is more difficult to design a high mechanical integrity rotor-shaft mechanical joint in the higher range of the output power. A common solution to the improvement of the mechanical integrity of the rotor-shaft joint is to design a multidisc (multi-stage) machine (Fig. 1.7).

Since the scaling of the torque capability of the AFPM machine as the cube of the diameter (see eqn (2.101)) while the torque of a RFPM machines scale as the square of the diameter times the length, the benefits associated with axial flux geometries may be lost as the power level or the geometric ratio of the length to diameter of the motor is increased [189]. The transition occurs near the point where the radius equals twice the length of RFPM machine. This may be a limiting design consideration for the power rating of a single-stage disc machine as the power level can always be increased by simply stacking of disc machines on the same shaft and in the same enclosure.

Numerical Example 1.1

Find the critical speed of rotation of the system consisting of a disc-type rotor (steel disc with PMs), steel shaft and driven wheel. Moduli of elasticity, specific mass densities, diameters and widths (lengths) are as follows:

- (a) $E_1 = 200 \times 10^9$ Pa, $\rho_1 = 7600$ kg/m³, $D_1 = 0.3$ m, $w_1 = 0.01$ m for the disc-type rotor;
- (b) $E_2 = 200 \times 10^9$ Pa, $\rho_3 = 7650$ kg/m³, $D_2 = 0.15$, $w_2 = 0.1$ m for the driven wheel;
- (c) $E_{sh} = 210 \times 10^9$ Pa, $\rho_{sh} = 7700$ kg/m³, $D_{sh} = 0.0245$ m, $L = 0.6$ m for the shaft.

The location of the rotor from the left end of the shaft is $a_1 = 0.2$ m and the location of the driven wheel from the same end of the shaft is $a_2 = 0.35$ m (Fig. 1.8). The acceleration of gravity is 9.81 m/s².

Solution

Mass of rotor

$$m_1 = \rho_1 \frac{\pi D_1^2}{4} w_1 = 7600 \frac{\pi 0.3^2}{4} \times 0.01 = 5.372 \text{ kg}$$

Mass of driven wheel

$$m_2 = \rho_2 \frac{\pi D_2^2}{4} w_2 = 7650 \frac{\pi 0.15^2}{4} \times 0.1 = 13.519 \text{ kg}$$

Mass of shaft

$$m_{sh} = \rho_{sh} \frac{\pi D_{sh}^2}{4} L = 7700 \frac{\pi 0.0245^2}{4} \times 0.6 = 2.178 \text{ kg}$$

Area moment of inertia of the rotor according to eqn (1.6)

$$I_1 = \frac{\pi 0.3^4}{64} = 3.976 \times 10^{-4} \text{ m}^4$$

Area moment of inertia of the driven wheel to eqn (1.6)

$$I_2 = \frac{\pi 0.15^4}{64} = 2.485 \times 10^{-5} \text{ m}^4$$

Area moment of inertia of the shaft according to eqn (1.6)

$$I_{sh} = \frac{\pi 0.0245^4}{64} = 1.769 \times 10^{-8} \text{ m}^4$$

Static deflection of the shaft at position of rotor due to rotor only as given by eqn (1.5)

$$\sigma_1 = \frac{5.372 \times 9.81 \times 0.2^2 (0.6 - 0.2)^2}{3 \times 200 \times 10^9 \times 3.976 \times 10^{-4} \times 0.6} = 2.356 \times 10^{-9} \text{ m}$$

Static deflection of the shaft at position of driven wheel due to driven wheel only as given by eqn (1.5)

$$\sigma_2 = \frac{13.519 \times 9.81 \times 0.35^2 (0.6 - 0.35)^2}{3 \times 200 \times 10^9 \times 2.485 \times 10^{-5} \times 0.6} = 1.135 \times 10^{-7} \text{ m}$$

Static deflection of shaft due to shaft only as given by eqn (1.5)

$$\sigma_{sh} = \frac{2.178 \times 9.81 \times 0.3^2 (0.6 - 0.3)^2}{3 \times 210 \times 10^9 \times 1.769 \times 10^{-8} \times 0.6} = 2.59 \times 10^{-5} \text{ m}$$

where the mid point of the shaft is $0.5L = 0.5 \times 0.6 = 0.3$ m. Thus, critical speeds according to eqn (1.4) are

- Critical speed of the disc-type rotor

$$n_1 = \frac{1}{2\pi} \sqrt{\frac{9.81}{2.356 \times 10^{-9}}} = 10\,269.2 \text{ rev/s} = 616\,150.9 \text{ rpm}$$

- Critical speed of the driven wheel

$$n_2 = \frac{1}{2\pi} \sqrt{\frac{9.81}{1.235 \times 10^{-7}}} = 1479.7 \text{ rev/s} = 88\,780 \text{ rpm}$$

- Critical speed of the shaft

$$n_{sh} = \frac{1}{2\pi} \sqrt{\frac{9.81}{2.59 \times 10^{-5}}} = 97.97 \text{ rev/s} = 5878.4 \text{ rpm}$$

Critical angular speeds for the disc-type rotor is $\Omega_1 = 2\pi 10\,269.3 = 64\,523.2 \text{ rad/s}$, for the driven wheel is $\Omega_2 = 2\pi 1479.7 = 9297 \text{ rad/s}$ and for the shaft is $\Omega_{sh} = 2\pi 97.97 = 615.6 \text{ rad/s}$. According to Dunkerley equation

$$x = \frac{1}{\Omega_1^2} + \frac{1}{\Omega_2^2} + \frac{1}{\Omega_{sh}^2} = \frac{1}{64\,523.2} + \frac{1}{9297} + \frac{1}{615.6} = 2.651 \times 10^{-6} \text{ s}^2/\text{rad}^2$$

Critical angular speed of the system as given by eqn (1.7)

$$\Omega_{cr} = \frac{1}{\sqrt{x}} = \frac{1}{\sqrt{2.651 \times 10^{-6}}} = 614.21 \text{ rad/s}$$

Critical speed of rotation of the system according to Dunkerley equation

$$n_{cr} = \frac{\Omega_{cr}}{2\pi} = \frac{614.21}{2\pi} = 97.75 \text{ rev/s} = 5865.3 \text{ rpm}$$

Critical speed of rotation of the system according to Rayleigh's method - eqn (1.8)

$$\begin{aligned} n_{cr} &= \\ \frac{1}{2\pi} \sqrt{\frac{9.81(5.372 \times 2.356 \times 10^{-9} + 13.519 \times 1.135 \times 10^{-7} + 2.178 \times 2.59 \times 10^{-5})}{5.372 \times (2.356 \times 10^{-9})^2 + 13.519 \times (1.135 \times 10^{-7})^2 + 2.178 \times (2.59 \times 10^{-5})^2}} \\ &= 99.3 \text{ rev/s} = 5958.15 \text{ rpm} \end{aligned}$$

The results according to Dunkerley and Rayleigh are similar.

Numerical Example 1.2

A copper disc with its dimensions as shown in Fig. 1.13 rotates with the speed of 12 000 rpm between U-shaped laminated pole shoes of a PM. A sliding contact consisting of two brushes is used to collect the electric current generated by this primitive homopolar generator: one brush slides on the external diameter $D_{out} = 0.232$ m and the second brush is located directly below one of the poles at the distance of $0.5D_{in} = 0.03$ m from the axis of the disc. The remanent magnetic flux density of the NdFeB PM is $B_r = 1.25$ T, coercivity is $H_c = 950\,000$ A/m and height $2h_M = 0.016$ m. The thickness of the disc is $d = 0.005$ m, one-sided air gap is $g = 0.0015$ m and the width of the pole is $b_p = 20$ mm. The relative magnetic permeability of the laminated core is $\mu_r = 1000$ and the conductivity of the disc is $\sigma = 57 \times 10^6$ S/m at 20°C . The length of the flux path in the laminated core is $l_{Fe} = 0.328$ m. Find:

- the magnetic flux density in the air gap;
- the EMF between brushes;
- the current, if the line resistance is $R_l = 0.001\ \Omega$ and load resistance is $R_L = 0.02\ \Omega$.

The magnetic flux fringing effect in the air gap, the variation of magnetic permeability with the magnetic field intensity in the laminated core and brush voltage drop are neglected.

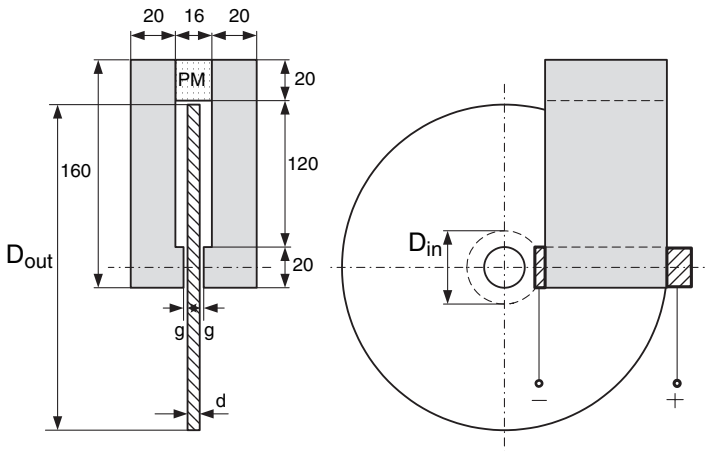


Fig. 1.13. Faraday's disc: a nonmagnetic conductive disc rotating in a stationary magnetic field according to *Numerical example 1.2*.

Solution

This is a homopolar type d.c. generator known as Faraday's disc and can be used as a current source, e.g. for electrolysis.

(a) Magnetic flux density in the air gap

The relative recoil magnetic permeability

$$\mu_{rrec} = \frac{1}{\mu_0} \frac{\Delta B}{\Delta H} = \frac{1}{0.4\pi \times 10^{-6}} \frac{1.25 - 0}{950000 - 0} = 1.047$$

The magnetic flux density in the air gap and saturation factor of the magnetic circuit can be found on the basis of Kirchhoff's magnetic voltage law, i.e.

$$\frac{B_r}{\mu_0 \mu_{rrec}} 2h_M = \frac{B_g}{\mu_0 \mu_{rrec}} 2h_M + \frac{B_g}{\mu_0} 2g + H_{Fe} l_{Fe}$$

$$\frac{B_r}{\mu_0 \mu_{rrec}} 2h_M = \frac{B_g}{\mu_0 \mu_{rrec}} 2h_M + \frac{B_g}{\mu_0} 2g k_{sat}$$

where the saturation factor of the magnetic circuit

$$k_{sat} = 1 + \frac{l_{Fe}}{2\mu_r(g + 0.5d)} = 1 + \frac{l_{Fe}}{2 \times 1000(0.0015 + 0.5 \times 0.005)} = 1.042$$

Thus, the air gap magnetic flux density is

$$B_g = \frac{B_r}{1 + (g + 0.5d)k_{sat}\mu_{rrec}/h_M}$$

$$= \frac{1.25}{1 + (0.0015 + 0.5 \times 0.005)1.042 \times 1000/0.008} = 0.809 \text{ T}$$

(b) EMF between brushes

Since $\mathbf{E}/l = \mathbf{v} \times \mathbf{B}_g$ or $dE = B_g v dr$ where $v = v_x = 2\pi r n$ is the linear speed, r is the radius of the disc and n is the rotational speed in rev/s, $dE = B_g(2\pi r n)dr$. Thus

$$E = \int_{0.5D_{in}}^{0.5D_{out}} B_g(2\pi r n)dr = \frac{\pi n B_g}{4} (D_{out}^2 - D_{in}^2)$$

$$= \frac{\pi 12000}{60} \frac{0.809}{4} (0.232^2 - 0.06^2) = 6.38 \text{ V}$$

(c) Current, if the line resistance is $R_l = 0.001 \Omega$ and load resistance is $R_L = 0.02 \Omega$.

Neglecting the current fringing effect in the disc, the resistance of the disc for induced current at 20°C is

$$R_d \approx \frac{0.5(D_{out} - D_{in})}{\sigma db_p} = \frac{0.5(0.232 - 0.06)}{57 \times 10^6 \times 0.005 \times 0.02} = 0.0000151 \, \Omega$$

The current is

$$I = \frac{E}{R_d + R_l + R_L} = \frac{6.38}{0.0000151 + 0.001 + 0.02} = 303.6 \, \text{A}$$

The terminal voltage is $V = I \times R_L = 303.6 \times 0.02 = 6.1 \, \text{V}$ and the line voltage drop is $\Delta V = I \times R_l = 303.6 \times 0.001 = 0.304 \, \text{V}$.

Numerical Example 1.3

Find the impedance of the aluminum disc of a double-sided eddy current brake at ambient temperature 20°C . The inner diameter of the PMs is $D_{in} = 0.14 \, \text{m}$, the outer diameter of the PMs is $D_{out} = 0.242$, thickness of the disc $d = 3 \, \text{mm}$, number of poles $2p = 16$ and speed $n = 3000 \, \text{rpm}$. Assume the conductivity of the aluminum $\sigma = 30 \times 10^6 \, \text{S/m}$ at 20°C and its relative magnetic permeability $\mu_r \approx 1$.

Solution

The average diameter of the disc

$$D = 0.5(D_{in} + D_{out}) = 0.5(0.14 + 0.232) = 0.191 \, \text{m}$$

The average pole pitch

$$\tau = \frac{\pi D}{2p} = \frac{\pi 0.191}{16} = 0.037 \, \text{m}$$

The frequency and angular frequency of the current in the disc for fundamental space harmonic $\nu = 1$

$$f = pn = 8 \frac{3000}{60} = 400 \, \text{Hz} \quad \omega = 2\pi f = 2\pi 400 = 2513.3 \, 1/\text{s}$$

The attenuation factor for the fundamental space harmonic $\nu = 1$ of the electromagnetic field in the disc according to eqn (1.17) is

$$k = \sqrt{\frac{2513.3 \times 0.4\pi 10^{-6} \times 1.0 \times 30 \times 10^6}{2}} = 217.66 \, 1/\text{m}$$

The coefficients according to eqns (1.11), (1.21), (1.22), (1.31), (1.32) and (1.33) for $\nu = 1$, are respectively

$$\beta = \frac{\pi}{0.037} = 84.865 \text{ 1/m}$$

$$a_R = \frac{1}{\sqrt{2}} \sqrt{\sqrt{4 + \left(\frac{84.865}{217.66}\right)^4} + \left(\frac{84.865}{217.66}\right)^2} = 1.038$$

$$a_X = \frac{1}{\sqrt{2}} \sqrt{\sqrt{4 + \left(\frac{84.865}{217.66}\right)^4} - \left(\frac{84.865}{217.66}\right)^2} = 0.964$$

$$\begin{aligned} A_R &= \frac{\sinh(1.038 \times 217.66 \times 0.003)}{\cosh(1.038 \times 217.66 \times 0.003) - \cos(0.964 \times 217.66 \times 0.003)} \\ &= 1.699 \end{aligned}$$

$$\begin{aligned} A_X &= \frac{-\sin(0.964 \times 217.66 \times 0.003)}{\cosh(1.038 \times 217.66 \times 0.003) - \cos(0.964 \times 217.66 \times 0.003)} \\ &= -1.369 \end{aligned}$$

$$B_R = \frac{0.964}{0.5(1.038^2 + 0.964^2)} = 0.961$$

$$B_X = \frac{1.038}{0.5(1.038^2 + 0.964^2)} = 1.035$$

The surface wave impedance for $\nu = 1$ according to eqn (1.30)

$$r = [0.961 \times 1.699 - 1.035 \times (-1.369)] \frac{217.66}{57 \times 10^6} = 2.2123 \times 10^{-5} \Omega$$

$$x = [1.035 \times 1.699 + 0.961 \times (-1.369)] \frac{217.66}{57 \times 10^6} = 3.216 \times 10^{-6} \Omega$$

$$z = 2.2123 \times 10^{-5} + j3.216 \times 10^{-6} \Omega$$

The impedance increase factor due to circumferential currents (in the x -direction) according to eqn (1.35) is

$$k_{z\nu} = 1 + \frac{1}{1^2} \frac{0.037}{0.5(0.242 - 0.14)} = 1.733$$

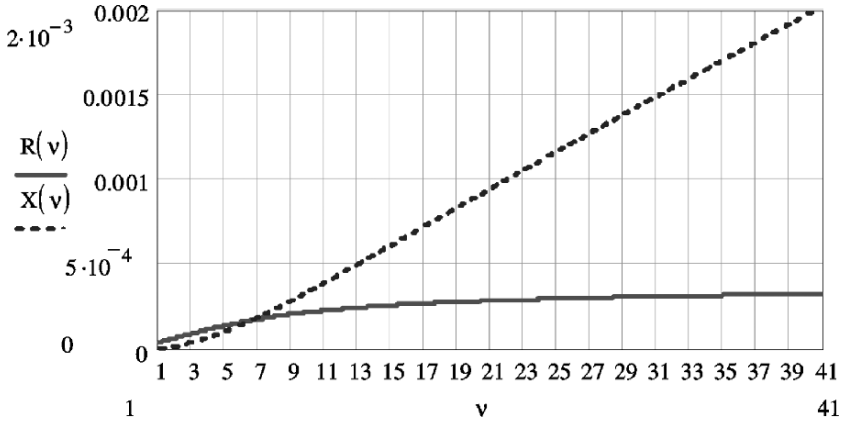


Fig. 1.14. Resistance and reactance of the aluminum disc as a function of higher space harmonics ν according to *Numerical example 1.3*.

The disc impedance per pole for the fundamental space harmonic $\nu = 1$ according to eqn (1.34)

$$\begin{aligned}
 Z &= (2.2123 \times 10^{-5} + j3.216 \times 10^{-6}) \frac{0.242 - 0.14}{0.037} 1.366 \\
 &= 5.231 \times 10^{-5} + j7.605 \times 10^{-6} \Omega
 \end{aligned}$$

$$R = 5.231 \times 10^{-5} \Omega \quad X = 7.605 \times 10^{-6} \Omega$$

The disc resistance R_ν and reactance X_ν for higher space harmonics $\nu > 1$ is plotted in Fig. 1.14.

Numerical Example 1.4

Estimate the cost of materials of a 3-phase, 250 kW, 2300 rpm brushless AFPM machine. The internal stator does not have any ferromagnetic core. The twin-type external rotor (Fig. 1.4d) consists of sintered NdFeB PMs (40 poles) glued to two solid steel discs. The mass of the stator copper conductors is $m_{Cu} = 18.7$ kg, mass of the stator winding insulation (including encapsulation) is $m_{ins} = 1.87$ kg, mass of PMs is $m_{PM} = 32.1$ kg, mass of the twin rotor core is $m_{rc} = 50.8$ kg (two discs) and mass of the shaft is $m_{sh} = 48.5$ kg. The cost of materials in U.S. dollars per kilogram is: copper conductor $c_{Cu} = 10.50$, insulating materials $c_{ins} = 8.50$, sintered NdFeB magnets $c_{PM} = 24.00$ and carbon steel $c_{steel} = 1.40$. The cost of frame, end discs

(bells) and bearings is $C_f = \$164.00$. The cost of components that are independent of the machine geometry (encoder, terminal leads, terminal board, nameplate) is $C_0 = \$140.00$.

Coefficients taking into account the manufacturing and utilisation of steel for the shaft and rotor core (steel discs) are as follows:

- total volume of the steel bar to the volume of the shaft, $k_{ush} = 1.94$
- coefficient taking into account the cost of machining of the shaft, $k_{msh} = 2.15$
- total volume of the steel plate to the volume of the rotor discs, $k_{ur} = 2.1$
- coefficient taking into account the cost of machining of the rotor steel discs, $k_{mr} = 1.8$

Solution

The cost of the copper winding

$$C_w = m_{Cu}c_{Cu} = 18.7 \times 10.50 = \$196.35$$

The cost of the insulation and encapsulation

$$C_{ins} = m_{ins}c_{ins} = 1.87 \times 8.50 = \$15.90$$

The cost of PMs

$$C_{PM} = m_{PM}c_{PM} = 32.1 \times 24.00 = \$770.40$$

The cost of the rotor core (two steel discs)

$$C_{rc} = k_{ur}k_{mr}m_{rc}c_{steel} = 2.1 \times 1.8 \times 50.8 \times 1.40 = \$268.83$$

The cost of the shaft

$$C_{sh} = k_{ush}k_{msh}m_{sh}c_{steel} = 1.94 \times 2.15 \times 48.5 \times 1.40 = \$283.21$$

Total cost of materials

$$C = C_f + C_0 + C_w + C_{ins} + C_{PM} + C_{rc} + C_{sh}$$

$$= 164.00 + 140.00 + 196.35 + 15.90 + 770.40 + 268.93 + 283.21 = \$1838.69$$

Principles of AFPM Machines

In this chapter the basic principles of the AFPM machine are explained in details. Considerable attention is given to the magnetic circuits, windings, torque production, losses, equivalent circuits, sizing procedure, armature reaction and performance characteristics of AFPM machines.

2.1 Magnetic Circuits

2.1.1 Single-Sided Machines

The *single-sided construction* of an axial flux machine is simpler than the double-sided one, but the torque production capacity is lower. Fig. 2.1 shows typical constructions of single-sided AFPM brushless machines with surface PM rotors and laminated stators wound from electromechanical steel strips. The single-sided motor according to Fig. 2.1a has a standard frame and shaft. It can be used in industrial, traction and servo electromechanical drives. The motor for hoist applications shown in Fig. 2.1b is integrated with a sheave (drum for ropes) and brake (not shown). It is used in gearless elevators [113].

2.1.2 Double-Sided Machines With Internal PM Disc Rotor

In the *double-sided machine with internal PM disc rotor*, the armature winding is located on two stator cores. The disc with PMs rotates between two stators. An eight-pole configuration is shown in Fig. 2.2. PMs are embedded or glued in a nonmagnetic rotor skeleton. The nonmagnetic air gap is large, i.e. the total air gap is equal to two mechanical clearances plus the thickness of a PM with its relative magnetic permeability close to unity. A double-sided machine with parallel connected stators can operate even if one stator winding is broken. On the other hand, a *series connection is preferred* because it can provide equal but opposing axial attractive forces.

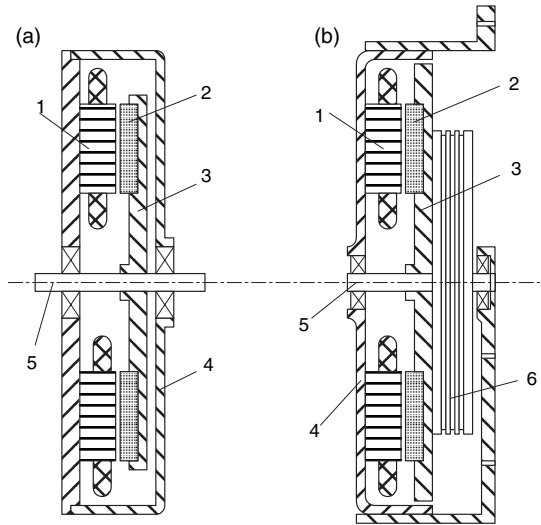


Fig. 2.1. Single sided disc type machines: (a) for industrial and traction electro-mechanical drives, (b) for hoist applications. 1 — laminated stator, 2 — PM, 3 — rotor, 4 — frame, 5 — shaft, 6 — sheave.

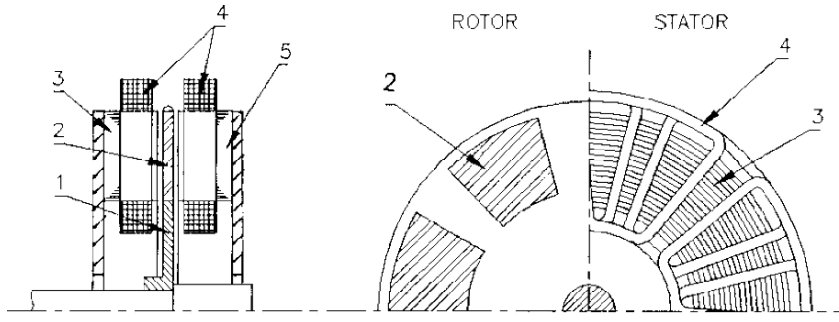


Fig. 2.2. Configuration of double-sided AFPM brushless machine with internal disc rotor: 1 — rotor, 2 — PM, 3 — stator core, 4 — stator winding.

A practical three-phase, 200 Hz, 3000 rpm, double-sided AFPM brushless motor with built-in brake is shown in Fig. 2.3 [157]. The three-phase winding is Y-connected, while phase windings of the two stators are in series. This motor is used as a flange-mounted servo motor. The ratio $X_{sd}/X_{sq} \approx 1.0$ so the motor can be analysed as a cylindrical non-salient rotor synchronous machine [128, 155, 157].

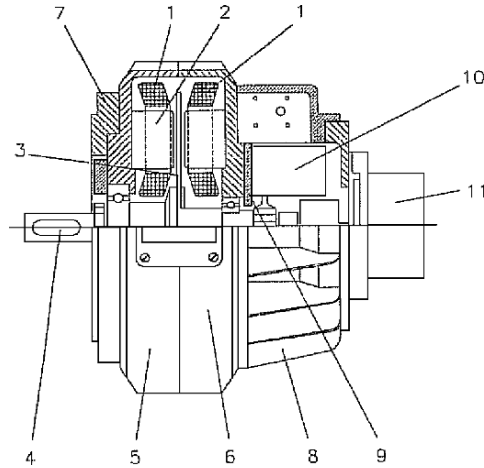


Fig. 2.3. Double-sided AFPM brushless servo motor with built-in brake and encoder: 1 — stator winding, 2 — stator core, 3 — disc rotor with PMs, 4 — shaft, 5 — left frame, 6 — right frame, 7 — flange, 8 — brake shield, 9 — brake flange, 10 — electromagnetic brake, 11 — encoder or resolver. Courtesy of *Slovak University of Technology STU*, Bratislava and *Electrical Research and Testing Institute*, Nová Dubnica, Slovakia.

2.1.3 Double-Sided Machines With Internal Ring-Shaped Core Stator

A *double-sided machine with internal ring-shaped stator core* has a poly-phase slotless armature winding (toroidal type) wound on the surface of the stator ferromagnetic core [101, 173, 244, 276]. In this machine the *ring-shaped stator core* is formed either from a continuous steel tape or sintered powders. The total air gap is equal to the thickness of the stator winding with insulation, mechanical clearance and the thickness of the PM in the axial direction. The double-sided rotor simply called *twin rotor* with PMs is located at two sides of the stator. The configurations with internal and external rotors are shown in Fig. 2.4. The three phase winding arrangement, magnet polarities and flux paths in the magnetic circuit are shown in Fig. 2.5.

The AFPM machines designed as shown in Fig. 2.4a can be used as a propulsion motor or combustion engine synchronous generator. The machine with external rotor, as shown in Fig. 2.4b, has been designed for hoist applications. A similar machine can be designed as an electric car wheel propulsion motor. Additional magnets on cylindrical parts of the rotor are sometimes added [173] or U-shaped magnets can be designed. Such magnets embrace the armature winding from three sides and only the internal portion of the winding does not produce any electromagnetic torque.

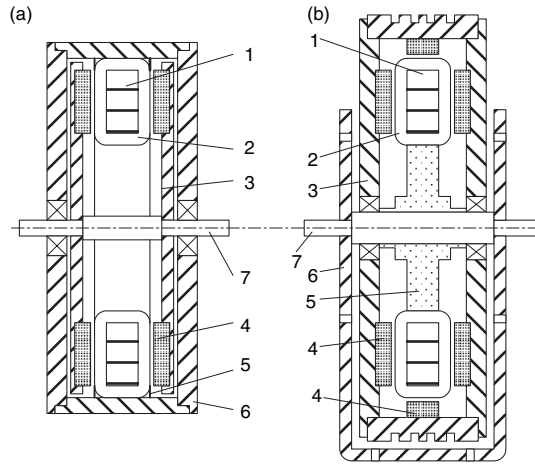


Fig. 2.4. Double-sided machines with one slotless stator: (a) internal rotor, (b) external rotor. 1 — stator core, 2 — stator winding, 3 — steel rotor, 4 — PMs, 5 — resin, 6 — frame, 7 — shaft.

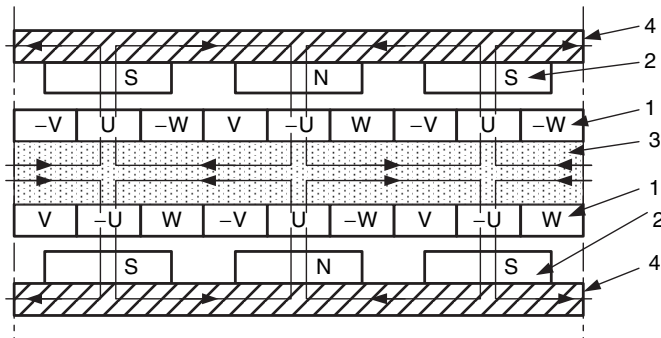


Fig. 2.5. Three phase winding, PM polarities and magnetic flux paths of a double-sided disc machine with one internal slotless stator. 1 — winding, 2 — PM, 3 — stator yoke, 4 — rotor yoke.

Owing to the large air gap the maximum flux density does not exceed 0.65 T. To produce this flux density a large volume of PMs is usually required. As the permeance component of the flux ripple associated with the slots is eliminated, the cogging torque is practically absent. The magnetic circuit is unsaturated (slotless stator core). On the other hand, the machine structure lacks the necessary robustness [244]. Both buried magnet and surface magnet rotors can be used.

There are a number of applications for medium and large power axial flux motors with external PM rotors, especially in electrical vehicles [101,

276]. Disc-type motors with external rotors have a particular advantage in low speed high torque applications, such as buses and shuttles, due to their large radius for torque production. For small electric cars the electric motor mounted directly into the wheel is recommended [101].

2.1.4 Double-Sided Machines With Internal Slotted Stator

The ring-type stator core can also be made with slots (Fig. 2.6). For this type of motor, slots are progressively notched into the steel tape as it is passed from one mandrel to another and the polyphase winding is inserted [276]. In the case of the slotted stator the air gap is small ($g \leq 1$ mm) and the air gap magnetic flux density can increase to 0.85 T [101]. The magnet volume is less than 50% that of the previous design, shown in Figs 2.4 and 2.5.

2.1.5 Double-Sided Machines With Internal Coreless Stator

AFPM machines with *coreless stators* have the stator winding wound on a non-magnetic and non-conductive supporting structure or mould. The stator core losses, i.e. hysteresis and eddy current losses do not exist. The losses in PMs and rotor solid steel disc are negligible. This type of design offers higher efficiency at zero cogging torque. In order to maintain a reasonable level of flux density in the air gap, a much larger volume of PMs in comparison with laminated stator core AFPM machine is required. The stator winding is placed in the air gap magnetic field generated by the PMs mounted on two opposing rotor discs (Fig. 1.4d). When operating at relatively high frequency, significant eddy current losses in the stator winding conductors may occur [263].

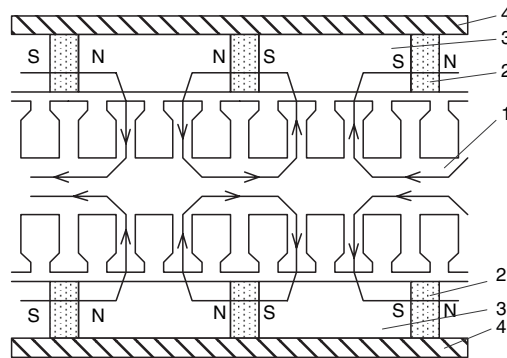


Fig. 2.6. Double-sided machine with one internal slotted stator and buried PMs. 1 — stator core with slots, 2 — PM, 3 — mild steel core (pole), 4 — nonmagnetic rotor disc.

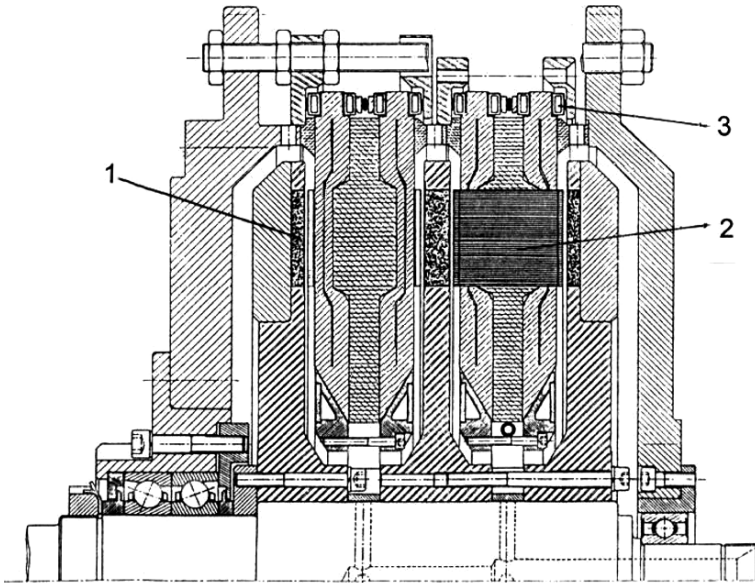


Fig. 2.7. Double stator, triple rotor AFPM brushless motor with water cooling system [59]. 1 — PM, 2 — stator core, 3 — stator winding.

2.1.6 Multidisc Machines

There is a limit on the increase of motor torque that can be achieved by enlarging the motor diameter. Factors limiting the single disc design are:

- (a) axial force taken by bearings;
- (b) integrity of mechanical joint between the disc and shaft;
- (c) disc stiffness.

A more reasonable solution for large torques are *double* or *triple disc motors*.

There are several configurations of *multidisc motors* [5, 6, 7, 59, 81, 82]. Large multidisc motors rated at least 300-kW have a water cooling system (Fig. 2.7) with radiators around the winding end connections [59]. To minimize the winding losses due to skin effect, variable cross section conductors may be used so that the cross section of conductors is bigger in the slot area than in the end connection region. Using a variable cross section means a gain of 40% in the rated power [59]. However, variable cross section of conductors means an increased cost of manufacturing. Owing to high mechanical stresses a titanium alloy is recommended for disc rotors. Multidisc AFPM brushless machines are also proposed for applications such as light-weight high speed generators [P66, 222], propeller-driven stratospheric aircraft [81], and brakes for landing gears of aircrafts [P146, P148].

2.2 Windings

2.2.1 Three-Phase Windings Distributed in Slots

In a single-layer winding, only one coil side is located in a single slot. The number of all coils is $s_1/2$ and the number of coils per phase is $n_c = s_1/(2m_1)$ where s_1 is the number of stator slots and m_1 is the number of phases. In a double-layer winding two sides of different coils are accommodated in each slot. The number of all coils is s_1 and the number of coils per phase is $n_c = s_1/m_1$. The number of slots per pole is

$$Q_1 = \frac{s_1}{2p} \quad (2.1)$$

where $2p$ is the number of poles. The number of slots per pole per phase is

$$q_1 = \frac{s_1}{2pm_1} \quad (2.2)$$

The number of conductors per coil can be calculated as

- for a single-layer winding

$$N_c = \frac{a_p a_w N_1}{n_c} = \frac{a_p a_w N_1}{s_1/(2m_1)} = \frac{a_p a_w N_1}{pq_1} \quad (2.3)$$

- for a double-layer winding

$$N_c = \frac{a_p a_w N_1}{n_c} = \frac{a_p a_w N_1}{s_1/m_1} = \frac{a_p a_w N_1}{2pq_1} \quad (2.4)$$

where N_1 is the number of turns in series per phase, a_p is the number of parallel current paths and a_w is the number of parallel conductors. The number of conductors per slot is the same for both single-layer and double-layer windings, i.e.

$$N_{sl} = \frac{a_p a_w N_1}{pq_1} \quad (2.5)$$

The full coil pitch measured in terms of the number of slots is $y_1 = Q_1$ where Q_1 is according to eqn (2.1). The short coil pitch can be expressed as

$$y_1 = \frac{w_c(r)}{\tau(r)} Q_1 \quad (2.6)$$

where $w_c(r)$ is the coil pitch measured in units of length at a given radius r and $\tau(r)$ is the pole pitch measured at the same radius. The *coil pitch-to-pole pitch* ratio is independent on the radius, i.e.

$$\beta = \frac{w_c(r)}{\tau(r)} \quad (2.7)$$

The *distribution factor* of a polyphase winding for the fundamental space harmonic $\nu = 1$ is defined as the ratio of the *phasor sum—to—arithmetic sum of EMFs induced in each coil* and expressed by the following equation:

$$k_{d1} = \frac{\sin(\pi/2m_1)}{q_1 \sin[\pi/(2m_1q_1)]} \quad (2.8)$$

The *pitch factor* for the fundamental space harmonic $\nu = 1$ is defined as the ratio of the *phasor sum—to—arithmetic sum of the EMFs per coil side* and expressed as:

$$k_{p1} = \sin\left(\beta\frac{\pi}{2}\right) \quad (2.9)$$

The *winding factor* for fundamental is the product of the distribution factor (eqn (2.8)) times pitch factor (eqn (2.9)), i.e.

$$k_{w1} = k_{d1}k_{p1} \quad (2.10)$$

The angle in electrical degrees between neighbouring slots is

$$\gamma = \frac{360^\circ}{s_1}p \quad (2.11)$$

Fig. 2.8 shows a single layer winding distributed in $s_1 = 36$ slots of a three-phase, $2p = 6$ AFPM machine.

2.2.2 Toroidal Winding

Toroidal stator windings are used in twin-rotor double-sided AFPM machines (Fig. 2.4). The toroidal stator winding of a three-phase, six-pole AFPM machine with twin external rotor is shown in Fig. 2.9. Each phase of the winding has an equal number of coils connected in opposition so as to cancel the possible flux circulation in the stator core. Those coils are evenly distributed along the stator core diametrically opposing each other so the only possible number of poles are 2, 6, 10, ... etc. The advantages of the toroidal stator winding [50, 245] are short end connection, simple stator core and easy design of any number of phases.

2.2.3 Coreless Stator Winding

Coreless stator windings are used in twin-rotor double-sided AFPM machines (Fig. 1.4d). For the ease of construction, the stator winding normally consists of a number of single layer trapezoidally shaped coils. The assembly of the stator is made possible by bending the ends of the coils by a certain angle, so that the active conductors lie evenly in the same plane and the end windings

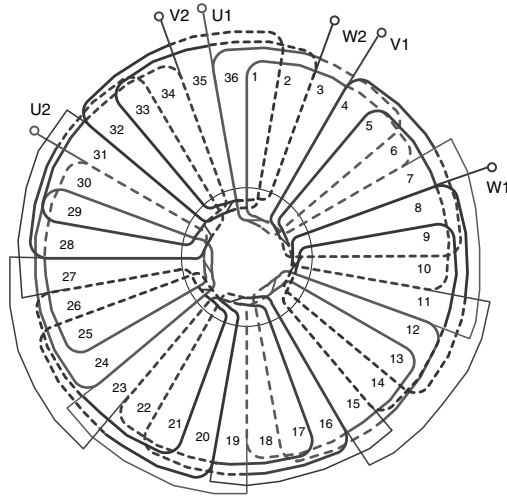


Fig. 2.8. Single-layer winding of an AFPM machine with $m_1 = 3$, $2p = 6$, $s_1 = 36$, $y_1 = Q_1 = 6$ and $q_1 = 2$.

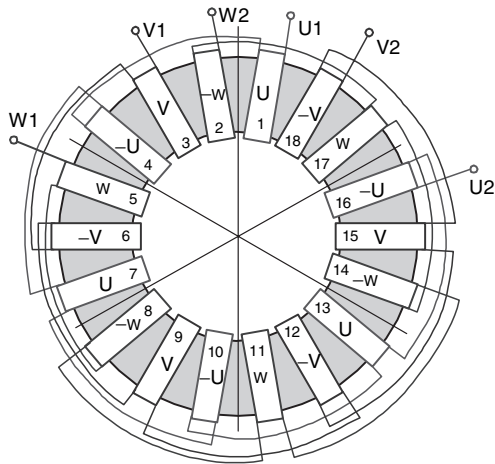


Fig. 2.9. Toroidal winding of a three-phase, six-pole, 18-coil AFPM machine with twin external rotor.

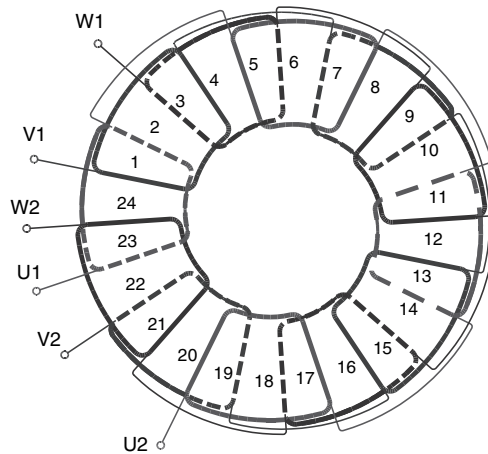


Fig. 2.10. Coreless winding of a three-phase, eight-pole AFPM machine with twin external rotor.

nest closely together. The windings are held together in position by using a composite material of epoxy resin and hardener. Fig. 2.10 shows the coreless stator winding of a three-phase, eight-pole AFPM machine. Obviously, the relations used in the slotted stator windings can be directly used for coreless trapezoidal stator winding with the exception that the term “slot” is replaced by the “coil side”. Another coil profile that has been used in coreless stator AFPM machines is the *rhomboidal* coil. It has shorter end connections than the trapezoidal coils. The inclined arrangement of the coil’s active sides makes it possible to place water cooling ducts inside the stator. The main drawback of rhomboidal coils is the reduction of the torque.

2.2.4 Non-Overlap (Concentrated Coil) Windings

The term *non-overlap windings* defined here refers to the windings of which the coils do not overlap. As with overlap windings non-overlap windings can be of single or double layer, concentrated or distributed, integral or fractional and air-cored or iron-cored. AFPM machines with non-overlap windings can be of single-sided or double-sided construction (Figs. 1.5, 1.6, 2.12).

In double-layer, non-overlap slotted iron-cored windings, two coils are sharing a slot (two coil layers per slot), which means that all teeth are wound. Consequently these windings are sometimes called *tooth windings*. In air-cored, double-layer non-overlap windings, the stator coils lie side-by-side against each other (see e.g. Fig. 1.5b). Note that the term *slot* can also be visualized when applied to air-cored windings.

In single-layer, non-overlap slotted iron-cored windings, only one coil layer is in a slot and therefore every second tooth is wound (*alternate teeth wound*);

air-cored, single-layer non-overlap windings have their coils displaced from each other as shown e.g. in Fig. 2.11. In double-layer, non-overlap windings, the number of stator coils is always equal to the number of stator slots, i.e. $Q_c = n_c m_1 = s_1$, where n_c is the number coils per phase. In single-layer non-overlap windings the number of stator coils is equal to half the number of stator slots i.e. $Q_c = s_1/2$.

In concentrated, non-overlap windings, there is only one coil in a coil group or coil phase belt ($z = 1$), but in distributed non-overlap windings two or more coils are distributed and connected in series to form a coil group ($z = 2, 3, \dots$). In integral non-overlap windings, the coils in a coil group are distributed evenly, but in fractional non-overlap windings they are not.

As with overlap windings, a non-overlap winding can also be divided into a number of sections F that repeats itself (negative or positive periodicity) in the machine every unit or section of poles. Such a pole section, thus, consists of a set of phase coils that completes an m_1 -phase stator coil layout. The number of pole sections F can be determined by taking the greatest common divisor (GCD) of the number of poles and the number of coils, i.e.

$$F = \text{GCD}(2p, Q_c) \quad (2.12)$$

If the number of poles per pole section, $2p/F$, is uneven, then negative periodic boundary conditions can be applied in finite element analysis. If $2p/F$ is even, then positive boundary conditions must be applied. With F known from (2.12), the number of coils in a coil group can be determined by

$$z = \frac{Q_c}{m_1 F} = \frac{n_c}{F} \quad (2.13)$$

There are a set of rules for the pole and slot combinations that make non-overlap windings valid. These are:

- the number of poles must be even;
- the number of slots must be a multiple of the number of phases and must be even in the case of single layer windings;
- the number of coils and slots are equal in double layer windings; in single layer windings the number of coils is equal to half the number of slots;
- the number of coils in a coil group (z) must be an integer;
- the number of slots can not be equal to the number of poles.

Electromagnetic analysis of AFPM machines with non-overlap stator windings are the same as those with overlap windings provided that the winding factors are calculated according to the following equations: the distribution factor for the fundamental space harmonic is calculated by [240]

$$k_{d1} = \frac{\sin(\pi/2m_1)}{z \sin[(\pi/(2m_1 z))]} \quad (2.14)$$

For single and double layer slotted iron-cored non-overlap windings, the pitch factor for the fundamental space harmonic is calculated by using eqn (2.9) or else

$$k_{p1} = \sin\left(\frac{\pi}{2} \frac{2p}{s_1}\right) = \sin(\theta_m/2) \quad (2.15)$$

where θ_m is the slot pitch angle (also corresponding to the coil span angle in the case of double layer windings shown in Fig. 1.5b or corresponding to the coil pitch angle in the case of single layer windings as shown in Fig. 2.11) given by

$$\theta_m = \frac{2\pi p}{s_1} \quad (2.16)$$

For air-cored, non-overlap windings, the pitch factor for the fundamental space harmonic is derived in [146]. For *single layer* non-overlap windings the pitch factor is given by

$$k_{p1} = \sin(\theta_m/2) \frac{\sin(\theta_{re}/2)}{\theta_{re}/2} \quad (2.17)$$

while for *double layer*, non-overlap windings, the pitch factor is given by

$$k_{p1} = \sin[(\theta_m - \theta_{re})/2] \frac{\sin(\theta_{re}/2)}{\theta_{re}/2} \quad (2.18)$$

where θ_{re} is the electrical angle corresponding to the coil-layer width. For example, for a double layer, air-cored non-overlap winding with $m_1 = 3$, $p = 12$, $s_1 = Q_c = 18$ (thus $n_c = 6$) and $\theta_{re} = 0.1 \theta_m$, the number of winding sections is $F = 6$, the number of coils distributed in a coil group is $z = 1$, the angular coil span is 1.333π , the distribution factor is $k_{d1} = 1$ and the pitch factor is $k_{p1} = \sin[(1.333 - 0.1333)\pi/2] \sin(0.1333\pi/2)/(0.1333\pi/2) = 0.944$. Fig. 1.5b shows a double layer, three-phase, 12-coil non-overlap stator winding of a double-sided AFPM machine with $2p = 8$ rotor poles. Figures 1.6 and 2.11 show a single layer, 9-coil ($Q_c = 9$, $s_1 = 2Q_c = 18$) non-overlap winding, while Fig. 2.12 shows a double layer 9-coil ($s_1 = Q_c = 9$) non-overlap stator winding. The difference in the number of the stator coils and rotor poles provides the starting torque for motors and the reduction of torque pulsations.

2.3 Torque Production

Since the dimensions of AFPM machines are functions of the radius, the electromagnetic torque is produced over a continuum of radii, not just at a constant radius as in cylindrical machines.

The *pole pitch* $\tau(r)$ and *pole width* $b_p(r)$ of an axial flux machine are functions of the radius r , i.e.

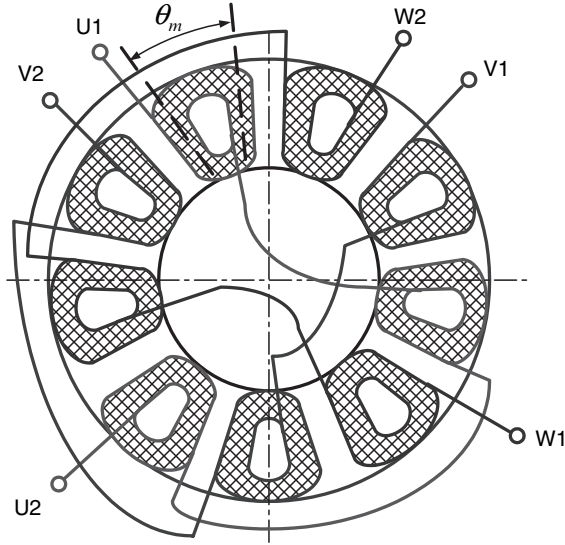


Fig. 2.11. Connection diagram of a three-phase, nine-coil winding of an AFPM brushless machine.



Fig. 2.12. Three-phase, nine-coil stator of a single-sided AFPM brushless machine with salient-pole stator. Photo-courtesy of *Mii Technologies*, Lebanon, NH, U.S.A.

$$\tau(r) = \frac{2\pi r}{2p} = \frac{\pi r}{p} \quad (2.19)$$

$$b_p(r) = \alpha_i \tau(r) = \alpha_i \frac{\pi r}{p} \quad (2.20)$$

where α_i is the ratio of the average B_{avg} to peak value B_{mg} of the magnetic flux density in the air gap, i.e.

$$\alpha_i = \frac{B_{avg}}{B_{mg}} \quad \text{or} \quad \alpha_i = \frac{b_p(r)}{\tau(r)} \quad (2.21)$$

Both the pole pitch $\tau(r)$ and pole width $b_p(r)$ are functions of the radius r , the parameter α_i is normally independent of the radius.

The *line current density* is also a function of the radius r . Thus the peak value of the line current density is

$$A_m(r) = \frac{m_1 \sqrt{2} N_1 I_a}{p\tau(r)} = \frac{m_1 \sqrt{2} N_1 I_a}{\pi r} \quad (2.22)$$

The *tangential force* acting on the disc can be calculated on the basis of Ampere's equation

$$\mathbf{dF}_x = I_a(\mathbf{dr} \times \mathbf{B}_g) = A(r)(\mathbf{dS} \times \mathbf{B}_g) \quad (2.23)$$

where $I_a \mathbf{dr} = A(r) \mathbf{dS}$, $A(r) = A_m(r)/\sqrt{2}$ according to eqn (2.22), $\mathbf{dr}$ is the radius element, $\mathbf{dS}$ is the surface element and $\mathbf{B}_g$ is the vector of the normal component (perpendicular to the disc surface) of the magnetic flux density in the air gap. An AFPM disc-type machine provides $\mathbf{B}_g$ practically independent of the radius r .

Assuming the magnetic flux density in the air gap B_{mg} is independent of the radius r , $dS = 2\pi r dr$ and $B_{avg} = \alpha_i B_{mg}$ according to eqn (2.21), the electromagnetic torque on the basis of eqn (2.23) is

$$dT_d = r dF_x = r[k_{w1} A(r) B_{avg} dS] = 2\pi \alpha_i k_{w1} A(r) B_{mg} r^2 dr \quad (2.24)$$

The line current density $A(r)$ is the electric loading per one stator active surface in the case of a typical stator winding distributed in slots (double-sided stator and internal rotor) or electric loading of the whole stator in the case of an internal toroidal type or coreless stator.

2.4 Magnetic Flux

For sinusoidal distribution of the magnetic flux density waveform excited by PMs, the *average magnetic flux density* is

$$\begin{aligned} B_{avg} &= \frac{1}{\pi/p - 0} \int_0^{\pi/p} B_{mg} \sin(p\alpha) d\alpha = -\frac{p}{\pi} B_{mg} \left[\frac{1}{p} \cos(p\alpha) \right]_0^{\pi/p} \\ &= -\frac{1}{\pi} B_{mg} [\cos \pi - \cos 0] = \frac{2}{\pi} B_{mg} \end{aligned} \quad (2.25)$$

Since the surface element per pole is $2\pi r dr/(2p)$, the magnetic flux excited by PMs per pole for a nonsinusoidal magnetic flux density waveform $B_{avg} = \alpha_i B_{mg}$ is

$$\Phi_f = \int_{R_{in}}^{R_{out}} \alpha_i B_{mg} \frac{2\pi}{2p} r dr = \alpha_i B_{mg} \frac{\pi}{p} \left[\frac{r^2}{2} \right]_{R_{in}}^{R_{out}}$$

$$= \alpha_i B_{mg} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) \quad (2.26)$$

where B_{mg} is the *peak value of the magnetic flux density* in the air gap, p is the number of pole pairs, $R_{out} = 0.5D_{out}$ is the outer radius of the PMs and $R_{in} = 0.5D_{in}$ is the inner radius of the PMs.

It is convenient to use the *inner-to-outer PM radius* or *inner-to-outer PM diameter* ratio, i.e.

$$k_d = \frac{R_{in}}{R_{out}} = \frac{D_{in}}{D_{out}} \quad (2.27)$$

Thus,

$$\Phi_f = \alpha_i B_{mg} \frac{\pi}{2p} [(0.5D_{out})^2 - (0.5D_{in})^2] = \alpha_i B_{mg} \frac{\pi}{8p} D_{out}^2 (1 - k_d^2) \quad (2.28)$$

The same equation for a cylindrical type machine is [106]

$$\Phi_f = \frac{2}{\pi} \tau L_i B_{mg} \quad (2.29)$$

where τ is the pole pitch and L_i is the effective length of the stack.

The *permeance of the air gap* in the d -axis at the radius r is given by $dG_g = \mu_0 \frac{1}{g'} \alpha_i \frac{\pi r}{p} dr$ or

$$\begin{aligned} G_g &= \frac{\mu_0}{g'} \alpha_i \frac{\pi}{p} \int_{R_{in}}^{R_{out}} r dr = \frac{\mu_0}{g'} \alpha_i \frac{\pi}{p} \left[\frac{r^2}{2} \right]_{R_{in}}^{R_{out}} = \mu_0 \frac{1}{g'} \alpha_i \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) \\ &= \lambda_g \alpha_i \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) \end{aligned} \quad (2.30)$$

where $\lambda_g = \mu_0/g'$ is the permeance per unit surface and g' is the equivalent air gap.

2.5 Electromagnetic Torque and EMF

The *average electromagnetic torque* developed by AFPM motor according to eqns (2.22) and (2.24) is

$$dT_d = 2\alpha_i m_1 I_a N_1 k_{w1} B_{mg} r dr$$

If the above equation is integrated from $D_{out}/2$ to $D_{in}/2$ with respect to r , the average electromagnetic torque may be written as

$$\begin{aligned}
T_d &= \frac{1}{4} \alpha_i m_1 I_a N_1 k_{w1} B_{mg} (D_{out}^2 - D_{in}^2) \\
&= \frac{1}{4} \alpha_i m_1 N_1 k_{w1} B_{mg} D_{out}^2 (1 - k_d^2) I_a
\end{aligned} \tag{2.31}$$

where k_d is according to eqn (2.27). Putting eqn (2.28) into eqn (2.31) the average torque is

$$T_d = 2 \frac{p}{\pi} m_1 N_1 k_{w1} \Phi_f I_a \tag{2.32}$$

To obtain the *rms* torque for sinusoidal current and sinusoidal magnetic flux density, eqn (2.32) should be multiplied by the coefficient $\pi\sqrt{2}/4 \approx 1.11$, i.e.

$$T_d = \frac{m_1}{\sqrt{2}} p N_1 k_{w1} \Phi_f I_a = k_T I_a \tag{2.33}$$

where the *torque constant*

$$k_T = \frac{m_1}{\sqrt{2}} p N_1 k_{w1} \Phi_f \tag{2.34}$$

In some publications [92, 244] the electromagnetic force on the rotor is simply calculated as the product of the magnetic and electric loading $B_{avg}A$ and active surface of PMs $S = \pi(R_{out}^2 - R_{in}^2)$, i.e. $F_x = \pi B_{avg}A(R_{out}^2 - R_{in}^2)$, where A is the *rms* line current density at the inner radius R_{in} . For a double-sided AFPM machine $S = 2\pi(R_{out}^2 - R_{in}^2)$. Thus, the average electromagnetic torque of a double-sided AFPM machine is

$$T_d = F_x R_{in} = 2\pi B_{avg}A(R_{out}^2 - R_{in}^2)R_{in} = 2\pi B_{avg}AR_{out}^3(k_d - k_d^3) \tag{2.35}$$

Taking the first derivative of the electromagnetic torque T_d with respect to k_d and equating it to zero, the maximum torque is for $k_d = 1/\sqrt{3}$. Industrial practice shows that the maximum torque is for $k_d \neq 1/\sqrt{3}$.

The *EMF at no load* can be found by differentiating the first harmonic of the magnetic flux waveform $\Phi_{f1} = \Phi_f \sin \omega t$ and multiplying by $N_1 k_{w1}$, i.e.

$$e_f = N_1 k_{w1} \frac{d\Phi_{f1}}{dt} = 2\pi f N_1 k_{w1} \Phi_f \cos \omega t$$

The magnetic flux Φ_f is expressed by eqns (2.26) and (2.28). The *rms* value is obtained by dividing the peak value $2\pi f N_1 k_{w1} \Phi_f$ of the EMF by $\sqrt{2}$, i.e.

$$E_f = \pi\sqrt{2}f N_1 k_{w1} \Phi_f = \pi\sqrt{2}p N_1 k_{w1} \Phi_f n_s = k_E n_s \tag{2.36}$$

where the *EMF constant* (armature constant) is

$$k_E = \pi\sqrt{2}p N_1 k_{w1} \Phi_f \tag{2.37}$$

The same form of eqn (2.36) can be obtained on the basis of the developed torque $T_d = m_1 E_f I_a / (2\pi n_s)$ in which T_d is according to eqn (2.33). For the toroidal type winding the winding factor $k_{w1} = 1$.

2.6 Losses and Efficiency

2.6.1 Stator Winding Losses

The *stator (armature) winding resistance* per phase for the d.c. current is

$$R_{1dc} = \frac{N_1 l_{1av}}{a_p a_w \sigma s_a} \quad (2.38)$$

where N_1 is the number of armature turns per phase, l_{1av} is the average length of turn, a_p is the number of parallel current paths, a_w is the number of parallel conductors, σ_1 is the electric conductivity of the armature conductor at given temperature (for a copper conductor $\sigma_1 \approx 57 \times 10^6$ S/m at 20°C and $\sigma_1 \approx 47 \times 10^6$ S/m at 75°C), and s_a is the conductor cross section. The average length of the armature turn is

$$l_{1av} = 2L_i + l_{1in} + l_{1out} \quad (2.39)$$

in which l_{1in} is the length of the *inner end connection* and l_{1out} is the length of the *outer end connection*.

For a.c. current and stator winding distributed in slots (ferromagnetic core) the stator winding resistance should be divided into the resistance of bars (radial portions of conductors) R_{1b} and resistance of the end connections R_{1e} , i.e.

$$R_1 = R_{1b} + R_{1e} = \frac{N_1}{a \sigma s_a} (2L_i k_{1R} + l_{1in} + l_{1out}) \approx k_{1R} R_{1dc} \quad (2.40)$$

where k_{1R} is the skin-effect coefficient for the stator (armature) resistance. For a double-layer winding and $w_c(r) = \tau(r)$ or $\beta = 1$ [161]:

$$k_{1R} = \varphi_1(\xi_1) + \left[\frac{m_{sl}^2 - 1}{3} - \left(\frac{m_{sl}}{2} \sin \frac{\gamma}{2} \right)^2 \right] \Psi_1(\xi_1) \quad (2.41)$$

where

$$\varphi_1(\xi_1) = \xi_1 \frac{\sinh 2\xi_1 + \sin 2\xi_1}{\cosh 2\xi_1 - \cos 2\xi_1} \quad (2.42)$$

$$\Psi_1(\xi_1) = 2\xi_1 \frac{\sinh \xi_1 - \sin \xi_1}{\cosh \xi_1 + \cos \xi_1} \quad (2.43)$$

$$\xi_1 = h_c \sqrt{\pi f \mu_o \sigma_1 \frac{b_{1con}}{b_{11}}} \quad (2.44)$$

and m_{sl} is the number of conductors per slot arranged above each other in two layers (this must be an even number), γ is the phase angle between the currents of the two layers, f is the input frequency, b_{1con} is the width of all the

conductors in a slot, b_{11} is the slot width and h_c is the height of a conductor in the slot. If there are n_{sl} conductors side by side at the same height of the slot, they are taken as a single conductor carrying n_{sl} -times greater current.

In general, for a three-phase winding $\gamma = 60^\circ$ and

$$k_{1R} = \varphi_1(\xi_1) + \left(\frac{m_{sl}^2 - 1}{3} - \frac{m_{sl}^2}{16} \right) \Psi_1(\xi_1) \quad (2.45)$$

For a chorded winding ($w_c(r) < \tau(r)$) and $\gamma = 60^\circ$

$$k_{1R} \approx \varphi_1(\xi_1) + \left[\frac{m_{sl}^2 - 1}{3} - \frac{3(1 - w_c/\tau)}{16} m_{sl}^2 \right] \Psi_1(\xi_1) \quad (2.46)$$

The skin-effect coefficient k_{1R} for hollow conductors is given, for example, in [161].

- (a) If $m_{sl} = 1$ and $\gamma = 0$, the skin-effect coefficient $k_{1R} = \varphi_1(\xi_1)$ (the same as for a cage winding).
- (b) If $\gamma = 0$, the currents in all conductors are equal and

$$k_{1R} \approx \varphi_1(\xi_1) + \frac{m_{sl}^2 - 1}{3} \Psi_1(\xi_1) \quad (2.47)$$

For small motors with round armature conductors fed from power frequencies of 50 or 60 Hz,

$$R_1 \approx R_{1dc} \quad (2.48)$$

The *armature winding losses* are

$$\Delta P_{1w} = m_1 I_a^2 R_1 \approx m_1 I_a^2 R_{1dc} k_{1R} \quad (2.49)$$

Since the skin effect is only in this part of the conductor which is located in the slot — eqn (2.40), the armature winding losses should be multiplied by the coefficient

$$\frac{k_{1R} + l_{1in}/(2L_i) + l_{1out}/(2L_i)}{1 + l_{1in}/(2L_i) + l_{1out}/(2L_i)} \quad (2.50)$$

rather than by k_{1R} .

2.6.2 Stator Core Losses

The magnetic flux in the stator (armature) core is nonsinusoidal. The rotor PM excitation system produces a trapezoidal shape of the magnetic flux density waveform. The stator windings are fed from switched d.c. sources with PWM or square wave control. The applied voltage thus contains many harmonics which are seen in the stator flux.

The *eddy current losses* can be calculated using the following classical formula

$$\begin{aligned}\Delta P_{eFe} &= \frac{\pi^2}{6} \frac{\sigma_{Fe}}{\rho_{Fe}} f^2 d_{Fe}^2 m_{Fe} \sum_{n=1}^{\infty} n^2 [B_{mxn}^2 + B_{mzn}^2] \\ &= \frac{\pi^2}{6} \frac{\sigma_{Fe}}{\rho_{Fe}} f^2 d_{Fe}^2 m_{Fe} [B_{mx1}^2 + B_{mz1}^2] \eta_d^2\end{aligned}\quad (2.51)$$

where σ_{Fe} , d_{Fe} , ρ_{Fe} and m_{Fe} are the electric conductivity, thickness, specific density and mass of laminations respectively, n are the odd time harmonics, B_{mxn} and B_{mzn} are the harmonic components of the magnetic flux density in the x (tangential) and z (normal) directions and

$$\eta_d = \sqrt{1 + \frac{(3B_{mx3})^2 + (3B_{mz3})^2}{B_{mx1}^2 + B_{mz1}^2} + \frac{(5B_{mx5})^2 + (5B_{mz5})^2}{B_{mx1}^2 + B_{mz1}^2} + \dots} \quad (2.52)$$

is the *coefficient of distortion of the magnetic flux density*. For $\eta_d = 1$, eqn (2.51) expresses the eddy current losses under sinusoidal magnetic flux density.

In a similar way, the *hysteresis losses* can be expressed with the aid of Richter's formula, i.e.

$$\begin{aligned}\Delta P_{hFe} &= \epsilon \frac{f}{100} m_{Fe} \sum_{n=1}^{\infty} n^2 [B_{mxn}^2 + B_{mzn}^2] \\ &= \epsilon \frac{f}{100} m_{Fe} [B_{mx1}^2 + B_{mz1}^2] \eta_d^2\end{aligned}\quad (2.53)$$

where $\epsilon = 1.2$ to $2.0 \text{ m}^4/(\text{H kg})$ for anisotropic laminations with 4% Si, $\epsilon = 3.8 \text{ m}^4/(\text{H kg})$ for isotropic laminations with 2% Si and $\epsilon = 4.4$ to $4.8 \text{ m}^4/(\text{H kg})$ for isotropic siliconless laminations.

Eqns (2.51) and (2.53) exclude the *excess losses* (due to magnetic anomaly) and losses due to metallurgical and manufacturing processes. There is a poor correlation between measured core losses and those calculated using classical methods. The losses calculated according to eqns (2.51) and (2.53) are lower than those obtained from measurements. The coefficient of additional core losses $k_{ad} > 1$ can help to obtain a better agreement of predicted and measured core losses

$$\Delta P_{1Fe} = k_{ad}(\Delta P_{eFe} + \Delta P_{hFe}) \quad (2.54)$$

If the *specific core losses* are known, the stator core losses ΔP_{1Fe} can be calculated on the basis of the specific core losses and masses of teeth and yoke, i.e.

$$\Delta P_{1Fe} = \Delta p_{1/50} \left(\frac{f}{50} \right)^{4/3} [k_{adt} B_{1t}^2 m_{1t} + k_{ady} B_{1y}^2 m_{1y}] \quad (2.55)$$

where $k_{adt} > 1$ and $k_{ady} > 1$ are the factors accounting for the increase in losses due to metallurgical and manufacturing processes, $\Delta p_{1/50}$ is the specific core loss in W/kg at 1 T and 50 Hz, B_{1t} is the magnetic flux density in a tooth, B_{1y} is the magnetic flux density in the yoke, m_{1t} is the mass of the teeth, and m_{1y} is the mass of the yoke. For teeth $k_{adt} = 1.7$ to 2.0 and for the yoke $k_{ady} = 2.4$ to 4.0 [159].

2.6.3 Core Loss Finite Element Model

The core losses within the stator and rotor are calculated using a set of finite element method (FEM) models, assuming constant rotor speed and balanced three-phase armature currents. The eddy current and hysteresis losses, within the cores, in the 2D FEM including distorted flux waveforms can be expressed by eqns (2.51) and (2.53).

The field distribution at several time intervals in the fundamental current cycle is needed to create the magnetic flux density waveforms. This is obtained by rotation of the rotor grid and phase advancement of the stator currents. From a field solution, for a particular rotor position, the magnetic flux density at each element centroid is calculated. Three flux density components in an element can be obtained from a single FEM solution.

2.6.4 Losses in Permanent Magnets

The electric conductivity of sintered NdFeB magnets is from 0.6 to 0.85×10^6 S/m. The electric conductivity of SmCo magnets is from 1.1 to 1.4×10^6 S/m. Since the electric conductivity of rare earth PMs is only 4 to 9 times lower than that of a copper conductor, the *losses in conductive PMs due to higher harmonic magnetic fields* produced by the stator cannot be neglected.

The most important losses in PMs are generated by the fundamental frequency magnetic flux due to the stator slot openings. In practice, those losses are only in AFPM machines with slotted stator ferromagnetic cores. The fundamental frequency of the magnetic flux density component due to the stator slot opening is

$$f_{sl} = s_1 p n \quad (2.56)$$

where s_1 is the number of stator slots, p is the number of pole pairs and n is the rotor speed in rev/s.

The magnetic flux density component due to slot opening is [121]

$$B_{sl} = a_{sl} \beta_{sl} k_C B_{avg} \quad (2.57)$$

where B_{avg} is the mean magnetic flux density over the slot pitch - eqn (2.4), k_C is Carter coefficient (1.2) and

$$a_{sl} = \frac{4}{\pi} \left(0.5 + \frac{\Gamma^2}{0.78 - 2\Gamma^2} \right) \sin(1.6\pi\Gamma) \quad (2.58)$$

$$\beta_{sl} = 0.5 \left(1 - \frac{1}{\sqrt{1 + \kappa^2}} \right) \quad (2.59)$$

$$\Gamma = \frac{b_{14}}{t_1} \quad \kappa = \frac{b_{14}}{g'} \quad (2.60)$$

In the above equations (2.58), (2.59) and (2.60) b_{14} is the stator slot opening, $g' = g + h_M/\mu_{rrec}$ is the equivalent air gap, t_1 is the slot pitch and h_M is the magnet thickness per pole.

Assuming that the relative recoil magnetic permeability $\mu_{rrec} \approx 1$, the power losses in PMs can be expressed by the following equation obtained from a 2D electromagnetic field distribution, i.e.

$$\Delta P_{PM} = \frac{1}{2} a_{R\nu} k_z \frac{|\alpha|^2}{\beta^2} \left(\frac{B_{sl}}{\mu_0 \mu_{rrec}} \right)^2 \frac{k}{\sigma_{PM}} S_{PM} \quad (2.61)$$

where $a_{R\nu}$ is according to eqn (1.21) for $\nu = 1$, $\alpha = (1 + j)k$ is according to eqn (1.16) for $\nu = 1$, k is according to eqn (1.17) for $\nu = 1$, β is according to eqn (1.11) for $\nu = 1$ and $\tau = 0.5t_1$, and σ_{PM} is the electric conductivity of PMs. Eqn (2.61) can also be used to estimate the reactive losses in PMs if $a_{R\nu}$ is replaced with $a_{X\nu}$ according to eqn (1.22).

The coefficient for including the circumferential component of currents induced in PMs can be found as

$$k_z = 1 + \frac{t_1}{D_{out} - D_{in}} \quad (2.62)$$

where $0.5t_1$ is the induced current loop span and $0.5(D_{out} - D_{in})$ is the radial length of the PM — see eqn (1.35). The active surface area of all PMs (single-sided machines) is

$$S_{PM} = \alpha_i \frac{\pi}{4} (D_{out}^2 - D_{in}^2) \quad (2.63)$$

where the average pole pitch τ is according to eqn (1.14).

2.6.5 Rotor Core Losses

The *rotor core losses*, i.e. losses in backing solid steel discs supporting PMs are due to the pulsating flux produced by rapid changes in air gap reluctance as the rotor passes the stator teeth.

The magnetic permeability of a solid steel disc varies with the z axis (normal axis). To take into account the variable magnetic permeability and hysteresis losses in solid ferromagnetic discs, coefficients $a_{R\nu}$, $a_{X\nu}$ according to eqns (1.21) and (1.22) must be replaced by the following coefficients:

$$a_{RFe} = \frac{1}{\sqrt{2}} \left[\sqrt{4a_R^2 a_X^2 + \left(a_R^2 - a_X^2 + \frac{\beta^2}{k^2} \right)^2} + a_R^2 - a_X^2 + \frac{\beta^2}{k^2} \right]^{\frac{1}{2}} \quad (2.64)$$

$$a_{XFe} = \frac{1}{\sqrt{2}} \left[\sqrt{4a_R^2 a_X^2 + \left(a_R^2 - a_X^2 + \frac{\beta^2}{k^2} \right)^2} - a_R^2 + a_X^2 - \frac{\beta^2}{k^2} \right]^{\frac{1}{2}} \quad (2.65)$$

where $a_R = 1.4$ to 1.5 , $a_X = 0.8$ to 0.9 according to Neyman [201], k_ν is according to eqn (1.17) and β is according to eqn (1.11) for $\nu = 1$.

The power losses in solid ferromagnetic discs can be expressed by a similar equation to eqn (2.61), i.e.

$$\Delta P_{2Fe} = \frac{1}{2} a_{RFe} k_z \frac{|\alpha|^2}{\beta^2} \left(\frac{B_{sl}}{\mu_0 \mu_r} \right)^2 \frac{k}{\sigma_{Fe}} S_{Fe} \quad (2.66)$$

where a_{RFe} is according to eqn (2.64) for $\nu = 1$, α is according to eqn (1.16) for $\nu = 1$, k is according to eqn (1.17) for $\nu = 1$, β is according to eqn (1.11) for $\nu = 1$ and $\tau = 0.5t_1$, B_{sl} is according to eqn (2.57), μ_r is the relative magnetic permeability and σ_{Fe} is the electric conductivity of the solid ferromagnetic disc. The frequency in the attenuation coefficient k is according to eqn (2.56). Eqn (2.66) can also be used to estimate the reactive losses in the solid ferromagnetic disc if a_{RFe} is replaced with a_{XFe} according to eqn (2.65). The flux density B_{sl} is according to eqns (2.57) to (2.60). The coefficient k_z is according to eqn (2.62) and the surface of the disc is

$$S_{Fe} = \frac{\pi}{4} (D_{out}^2 - D_{in}^2) \quad (2.67)$$

2.6.6 Eddy Current Losses in Stator Conductors

For slotted AFPM machines the *eddy current losses* in the stator winding are generally ignored as the magnetic flux penetrates through the teeth and yoke and only small leakage flux penetrates through the slot space with conductors.

In slotless and coreless machines, the stator winding is exposed to the air gap magnetic field. The motion of PMs relative to the stator winding produces an alternating field through each conductor and induces eddy currents. In the case of a coreless AFPM machine with a solid ferromagnetic rotor discs, there is also a tangential field component B_{mx} in addition to the axial field component B_{mz} . This can lead to serious additional eddy current loss especially at high frequency. Neglecting the proximity effect, the eddy current loss in the stator winding may be calculated by using a classical equation similar to eqn (2.51) for calculation of the eddy current losses in laminated cores, i.e.

- for round conductors [47]

$$\begin{aligned}\Delta P_e &= \frac{\pi^2}{4} \frac{\sigma}{\rho} f^2 d^2 m_{con} \sum_{n=1}^{\infty} n^2 [B_{mxn}^2 + B_{mzn}^2] \\ &= \frac{\pi^2}{4} \frac{\sigma}{\rho} f^2 d^2 m_{con} [B_{mx1}^2 + B_{mz1}^2] \eta_d^2\end{aligned}\quad (2.68)$$

- for rectangular conductors [47]

$$\begin{aligned}\Delta P_e &= \frac{\pi^2}{3} \frac{\sigma}{\rho} f^2 a^2 m_{con} \sum_{n=1}^{\infty} n^2 [B_{mxn}^2 + B_{mzn}^2] \\ &= \frac{\pi^2}{3} \frac{\sigma}{\rho} f^2 a^2 m_{con} [B_{mx1}^2 + B_{mz1}^2] \eta_d^2\end{aligned}\quad (2.69)$$

where d is the diameter of the conductor, a is the width of the conductor (parallel to the stator plane), σ is the electric conductivity, ρ is specific mass density of the conductor, m_{con} is the mass of the stator conductors without end connections and insulation, f is the stator current frequency, B_{mx} and B_{mz} are the peak values of tangential and axial components of the magnetic flux density, respectively, and η_d is the coefficient of distortion according to eqn (2.52).

2.6.7 Rotational Losses

The *rotational* or *mechanical losses* ΔP_{rot} consist of friction losses ΔP_{fr} in bearings, windage losses ΔP_{wind} and ventilation losses ΔP_{vent} (if there is a forced cooling system), i.e.

$$\Delta P_{rot} = \Delta P_{fr} + \Delta P_{wind} + \Delta P_{vent} \quad (2.70)$$

There are many semi-empirical equations for calculating the rotational losses giving various degrees of accuracy. The *friction losses in bearings* of small machines can be evaluated using the following formula

$$\Delta P_{fr} = 0.06 k_{fb} (m_r + m_{sh}) n \quad \text{W} \quad (2.71)$$

where $k_{fb} = 1$ to $3 \text{ m}^2/\text{s}^2$, m_r is the mass of the rotor in kg, m_{sh} is the mass of the shaft in kg and n is the speed in rev/s.

The *Reynolds number* for a rotating disc with its outer radius R_{out} is

$$Re = \rho \frac{R_{out} v}{\mu} = \frac{2\pi n \rho R_{out}^2}{\mu} \quad (2.72)$$

where ρ is the specific density of the cooling medium, $v = v_x = 2\pi R_{out} n$ is the linear velocity at the outer radius R_{out} , n is the rotational speed and μ is

the dynamic viscosity of the fluid. Most AFPM machines are air cooled. The air density at 1 atm and 20°C is 1.2 kg/m³. The dynamic viscosity of the air at 1 atm and 20°C is $\mu = 1.8 \times 10^{-5}$ Pa s.

The *coefficient of drag* for turbulent flow can be found as

$$c_f = \frac{3.87}{\sqrt{Re}} \quad (2.73)$$

The *windage losses* for a rotating disc are

$$\Delta P_{wind} = \frac{1}{2} c_f \rho (2\pi n)^3 (R_{out}^5 - R_{sh}^5) \quad (2.74)$$

where R_{sh} is the shaft diameter.

AFPM machines are usually designed without a cooling fan so that the ventilation losses $\Delta P_{vent} = 0$.

2.6.8 Losses for Nonsinusoidal Current

Higher time harmonics generated by static converters produce additional losses. The higher time harmonic frequency in the stator winding is nf where $n = 5, 7, 11, \dots$. The armature winding losses, the core losses, and the stray losses are frequency-dependent. The mechanical losses do not depend on the shape of the input waveform.

If the stator winding and stator core losses have been calculated for fundamental frequency, the *frequency-dependent losses* of an inverter-fed motor or generator loaded with a rectifier can be found as

- stator (armature) winding losses:

$$\Delta P_{1w} = \sum_{n=1}^{\infty} \Delta P_{1wn} = m_1 \sum_{n=1}^{\infty} I_{an}^2 R_{1n} \approx m_1 R_{1dc} \sum_{n=1}^{\infty} I_{an}^2 k_{1Rn} \quad (2.75)$$

- stator (armature) core losses

$$\Delta P_{1Fe} = \sum_{n=1}^{\infty} \Delta P_{1Fen} = [\Delta P_{1Fen}]_{n=1} \sum_{n=1}^{\infty} \left(\frac{V_{1n}}{V_{1r}} \right)^2 n^{-0.7} \quad (2.76)$$

where k_{1Rn} is the a.c. armature resistance skin effect coefficient for nf , I_{an} is the higher harmonic *rms* armature current, V_{1n} is the higher harmonic inverter output voltage, V_{1r} is the rated voltage, $[\Delta P_{1Fen}]_{n=1}$ are the stator core losses for $n = 1$ and rated voltage.

2.6.9 Efficiency

The total power losses of an AFPM machine are

$$\Delta P = \Delta P_{1w} + \Delta P_{1Fe} + \Delta P_{2Fe} + \Delta P_{PM} + \Delta P_e + \Delta P_{rot} \quad (2.77)$$

The *efficiency* is

$$\eta = \frac{P_{out}}{P_{out} + \Delta P} \quad (2.78)$$

where P_{out} is the mechanical output power for a motor and the electrical output power for a generator.

2.7 Phasor Diagrams

The *synchronous reactance* of a synchronous machine (sine-wave machine) is defined as the sum of the armature reaction (mutual) reactance X_{ad} , X_{aq} and stator (armature) leakage reactance X_1 , i.e.

- in the d -axis

$$X_{sd} = X_{ad} + X_1 \quad (2.79)$$

- in the q -axis

$$X_{sq} = X_{aq} + X_1 \quad (2.80)$$

When drawing *phasor diagrams* of synchronous machines, two arrow systems are used:

- (a) generator arrow system, i.e.

$$\begin{aligned} \mathbf{E}_f &= \mathbf{V}_1 + \mathbf{I}_a R_1 + j\mathbf{I}_{ad} X_{sd} + j\mathbf{I}_{aq} X_{sq} \\ &= \mathbf{V}_1 + \mathbf{I}_{ad}(R_1 + jX_{sd}) + \mathbf{I}_{aq}(R_1 + jX_{sq}) \end{aligned} \quad (2.81)$$

- (b) consumer (motor) arrow system, i.e.

$$\begin{aligned} \mathbf{V}_1 &= \mathbf{E}_f + \mathbf{I}_a R_1 + j\mathbf{I}_{ad} X_{sd} + j\mathbf{I}_{aq} X_{sq} \\ &= \mathbf{E}_f + \mathbf{I}_{ad}(R_1 + jX_{sd}) + \mathbf{I}_{aq}(R_1 + jX_{sq}) \end{aligned} \quad (2.82)$$

where

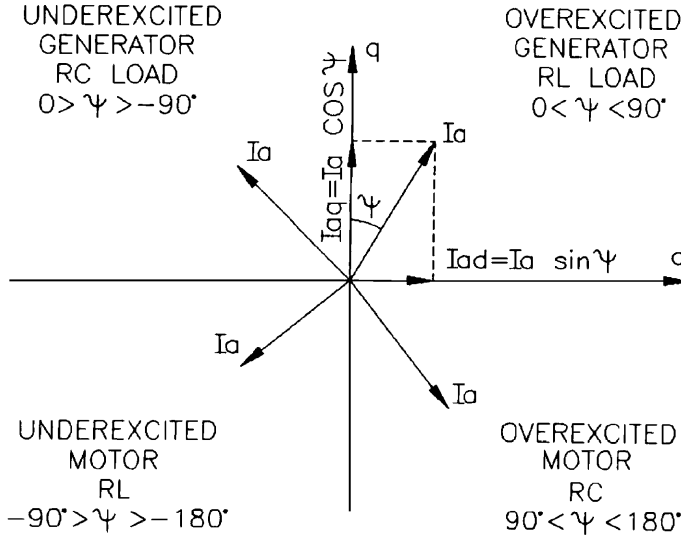


Fig. 2.13. Location of the armature current $\mathbf{I}_a$ in d — q coordinate system.

$$\mathbf{I}_a = \mathbf{I}_{ad} + \mathbf{I}_{aq} \quad (2.83)$$

and

$$I_{ad} = I_a \sin \Psi \quad I_{aq} = I_a \cos \Psi \quad (2.84)$$

The angle Ψ is between the q -axis and armature current $\mathbf{I}_a$. When the current arrows are in the opposite direction, the phasors $\mathbf{I}_a$, $\mathbf{I}_{ad}$, and $\mathbf{I}_{aq}$ are reversed by 180° . The same applies to the voltage drops. The location of the armature current $\mathbf{I}_a$ with respect to the d - and q -axis for generator and motor mode is shown in Fig. 2.13.

Phasor diagrams for synchronous generators are constructed using the generator arrow system. The same system can be used for motors, however, the consumer arrow system is more convenient. An overexcited generator (Fig. 2.14a) delivers both active and reactive power to the load or utility grid. An underexcited motor (Fig. 2.14b) draws both active and reactive power from the line. For example, the load current $\mathbf{I}_a$ (Fig. 2.14b) lags the voltage phasor $\mathbf{V}_1$ by the angle ϕ . An overexcited motor, consequently, draws a leading current from the circuit and delivers reactive power to it.

In the phasor diagrams according to Fig. 2.14 [126] the stator core losses have been neglected. This assumption is justified only for power frequency machines with unsaturated armature cores.

For an *underexcited synchronous motor* (Fig. 2.14b) the input voltage V_1 projections on the d and q axes are

$$V_1 \sin \delta = I_{aq} X_{sq} - I_{ad} R_1$$

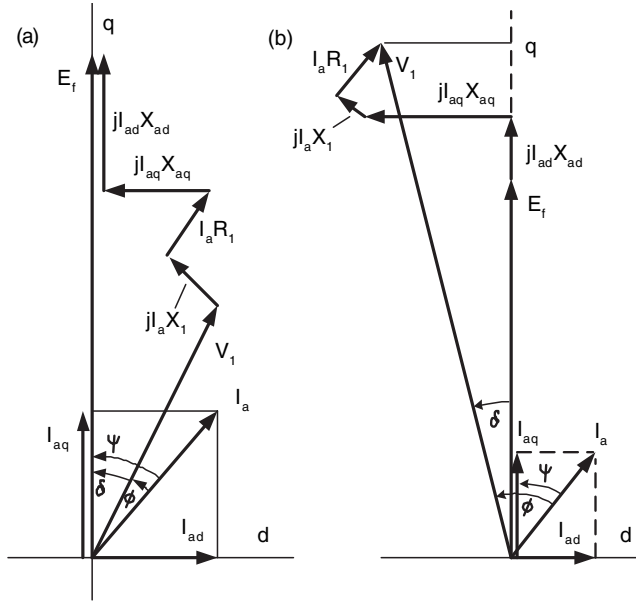


Fig. 2.14. Phasor diagrams of salient-pole synchronous machine: (a) overexcited generator (generator arrow system); (b) underexcited motor (consumer arrow system).

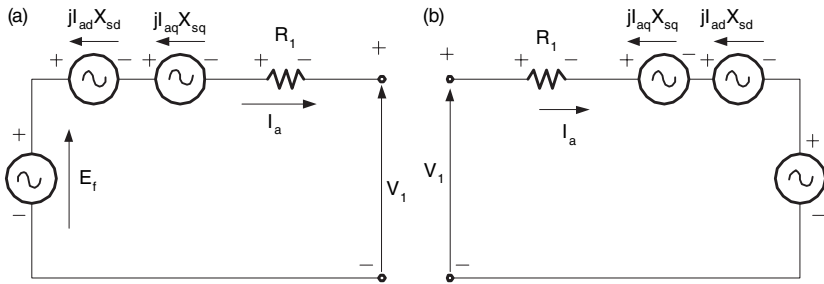


Fig. 2.15. Equivalent circuit per phase of an AFPM synchronous machine: (a) for motoring mode; (b) for generating mode. Stator core losses have been neglected.

$$V_1 \cos \delta = E_f + I_{ad}X_{sd} + I_{aq}R_1 \quad (2.85)$$

where δ is the *load angle* between the voltage V_1 and EMF E_f (q -axis). For an *overexcited motor*

$$V_1 \sin \delta = I_{aq}X_{sq} + I_{ad}R_1$$

$$V_1 \cos \delta = E_f - I_{ad}X_{sd} + I_{aq}R_1 \quad (2.86)$$

The currents of an overexcited motor

$$I_{ad} = \frac{V_1(X_{sq} \cos \delta - R_1 \sin \delta) - E_f X_{sq}}{X_{sd}X_{sq} + R_1^2} \quad (2.87)$$

$$I_{aq} = \frac{V_1(R_1 \cos \delta + X_{sd} \sin \delta) - E_f R_1}{X_{sd}X_{sq} + R_1^2} \quad (2.88)$$

are obtained by solving the set of eqns (2.85). The *rms* armature current as a function of V_1 , E_f , X_{sd} , X_{sq} , δ , and R_1 is

$$I_a = \sqrt{I_{ad}^2 + I_{aq}^2} = \frac{V_1}{X_{sd}X_{sq} + R_1^2} \times \sqrt{[(X_{sq} \cos \delta - R_1 \sin \delta) - E_f X_{sq}]^2 + [(R_1 \cos \delta + X_{sd} \sin \delta) - E_f R_1]^2} \quad (2.89)$$

The phasor diagram can also be used to find the input electric power. For a motor

$$P_{in} = m_1 V_1 I_a \cos \phi = m_1 V_1 (I_{aq} \cos \delta - I_{ad} \sin \delta) \quad (2.90)$$

Thus, the electromagnetic power for a motor mode is

$$\begin{aligned} P_{elm} &= P_{in} - \Delta P_{1w} - \Delta P_{1Fe} \\ &= m_1 [I_{aq} E_f + I_{ad} I_{aq} (X_{sd} - X_{sq})] - \Delta P_{1Fe} \end{aligned} \quad (2.91)$$

On the basis of phasor diagrams (Fig. 2.14), equivalent circuits of a AFPM synchronous machine can be drawn (Fig. 2.15).

2.8 Sizing Equations

The *main dimensions* of a double-sided PM brushless motor with internal disc rotor can be determined using the following assumptions: (a) the electric and magnetic loadings are known; (b) the number of turns per phase per one stator is N_1 ; (c) the phase armature current in one stator winding is I_a ; (d) the back EMF per phase per one stator winding is E_f .

The peak line current density at the average radius per one stator is expressed by eqn (2.22) in which the radius may be replaced by an average diameter

$$D = 0.5(D_{out} + D_{in}) = 0.5D_{out}(1 + k_d) \quad (2.92)$$

where D_{out} is the outer diameter, D_{in} is the inner diameter of the stator core and $k_d = D_{in}/D_{out}$ (according to eqn (2.27)). Thus,

$$A_m = \frac{4\sqrt{2}m_1 I_a N_1}{\pi D_{out}(1 + k_d)} \quad (2.93)$$

The EMF induced in the stator winding by the rotor excitation system, according to eqns (2.36) and (2.28) has the following form

$$E_f = \pi\sqrt{2}n_s p N_1 k_{w1} \Phi_f = \frac{\pi}{4}\sqrt{2}n_s N_1 k_{w1} B_{mg} D_{out}^2 (1 - k_d^2) \quad (2.94)$$

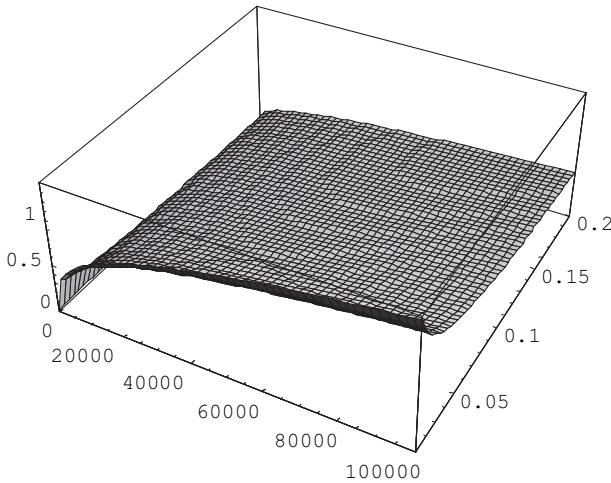


Fig. 2.16. Outer diameter D_{out} as a function of the output power P_{out} and parameter k_D for $\epsilon = 0.9$, $k_{w1}\eta \cos \phi = 0.84$, $n_s = 1000 \text{ rpm} = 16.67 \text{ rev/s}$ and $B_{mg}A_m = 26,000 \text{ TA/m}$.

The apparent electromagnetic power in two stators is

$$\begin{aligned} S_{elm} &= m_1(2E_f)I_a = m_1E_f(2I_a) \\ &= \frac{\pi^2}{8}k_{w1}n_s B_{mg}A_m D_{out}^3 (1 + k_d)(1 - k_d^2) \end{aligned} \quad (2.95)$$

For series connection the EMF is equal to $2E_f$ and for parallel connection the current is equal to $2I_a$. For a multidisc motor the number “2” should be replaced by the number of stators. Putting

$$k_D = \frac{1}{8}(1 + k_d)(1 - k_d^2) \quad (2.96)$$

the apparent electromagnetic power is

$$S_{elm} = \pi^2 k_D k_{w1} n_s B_{mg} A_m D_{out}^3 \quad (2.97)$$

The apparent electromagnetic power expressed in terms of active output power is thus

$$S_{elm} = \epsilon \frac{P_{out}}{\eta \cos \phi} \quad (2.98)$$

where the *phase EMF-to-phase voltage* ratio is

$$\epsilon = \frac{E_f}{V_1} \quad (2.99)$$

For motors $\epsilon < 1$ and for generators $\epsilon > 1$.

In connection with eqns (2.97) and (2.98), the PM outer diameter (equal to the outer diameter of the stator core) is

$$D_{out} = \sqrt[3]{\frac{\epsilon P_{out}}{\pi^2 k_D k_{w1} n_s B_{mg} A_m \eta \cos \phi}} \quad (2.100)$$

The outer diameter of PMs is the most important dimension of disc rotor PM motors. Since $D_{out} \propto \sqrt[3]{P_{out}}$ the outer diameter increases rather slowly with the increase of the output power (Fig. 2.16). This is why small power disc motors have relatively large diameters. The disc-type construction is preferred for medium and large power motors. Motors with output power over 10 kW have reasonable diameters. Also, disc construction is recommended for a.c. servo motors fed with high frequency voltage.

The electromagnetic torque is proportional to D_{out}^3 , i.e.

$$T_d = \frac{P_{elm}}{2\pi n_s} = \frac{S_{elm} \cos \Psi}{2\pi n_s} = \frac{\pi}{2} k_D k_{w1} D_{out}^3 B_{mg} A_m \cos \Psi \quad (2.101)$$

where P_{elm} is the active electromagnetic power and Ψ is the angle between the stator current I_a and EMF E_f .

2.9 Armature Reaction

The *magnetic fluxes produced by the stator* (armature) can be expressed in a similar way as the field excitation flux — eqn (2.26), i.e.

- in the d -axis

$$\Phi_{ad} = \frac{2}{\pi} B_{mad1} \frac{\pi}{p} \frac{R_{out}^2 - R_{in}^2}{2} \quad (2.102)$$

- in the q -axis

$$\Phi_{aq} = \frac{2}{\pi} B_{maq1} \frac{\pi}{p} \frac{R_{out}^2 - R_{in}^2}{2} \quad (2.103)$$

where B_{mad1} is the peak value of the first harmonic of the stator (armature reaction) magnetic flux density in the d -axis and B_{maq1} is the peak value of the first harmonic of the stator magnetic flux density in the q -axis. The *stator linkage* fluxes are

- in the d -axis

$$\Psi_d = \frac{1}{\sqrt{2}} N_1 k_{w1} \Phi_{ad} = \frac{1}{\sqrt{2}} N_1 k_{w1} \frac{2}{\pi} B_{mad1} \frac{\pi}{p} \frac{R_{out}^2 - R_{in}^2}{2} \quad (2.104)$$

- in the q -axis

$$\Psi_q = \frac{1}{\sqrt{2}} N_1 k_{w1} \Phi_{aq} = \frac{1}{\sqrt{2}} N_1 k_{w1} \frac{2}{\pi} B_{maq1} \frac{\pi}{p} \frac{R_{out}^2 - R_{in}^2}{2} \quad (2.105)$$

where N_1 is the number of stator turns per phase and k_{w1} is the winding factor for the fundamental space harmonic.

Neglecting the magnetic saturation, the first harmonic of the stator magnetic flux density normal components are

- in the d -axis

$$B_{mad1} = k_{fd} B_{mad} = k_{fd} \lambda_d F_{ad} = k_{fd} \frac{\mu_0}{g'} \frac{m_1 \sqrt{2}}{\pi} \frac{N_1 k_{w1}}{p} I_{ad} \quad (2.106)$$

- in the q -axis

$$B_{maq1} = k_{fq} B_{mad} = k_{fq} \lambda_q F_{aq} = k_{fq} \frac{\mu_0}{g'_q} \frac{m_1 \sqrt{2}}{\pi} \frac{N_1 k_{w1}}{p} I_{aq} \quad (2.107)$$

where m_1 is the number of stator phases and the permeance per unit surface in the d - and q -axis are

$$\lambda_d = \lambda_g = \frac{\mu_0}{g'} \quad \lambda_q = \frac{\mu_0}{g'_q} \quad (2.108)$$

In the above equations (2.106) and (2.107) the *form factors of armature reaction* are defined as the ratios of the first harmonic amplitude-to-maximum value of normal components of armature reaction magnetic flux density in the d - and q -axis respectively, i.e.

$$k_{fd} = \frac{B_{mad1}}{B_{mad}} \quad k_{fq} = \frac{B_{maq1}}{B_{maq}} \quad (2.109)$$

The equivalent air gaps in the d - and q -axis for surface configuration of PMs are

- for a stator with ferromagnetic core

$$g' = gk_C k_{sat} + \frac{h_M}{\mu_{rrec}} \quad (2.110)$$

$$g'_q = gk_C k_{satq} + h_M \quad (2.111)$$

- for a coreless stator

$$g' = 2 \left[(g + 0.5t_w) + \frac{h_M}{\mu_{rrec}} \right] \quad (2.112)$$

$$g'_q = 2[(g + 0.5t_w) + h_M] \quad (2.113)$$

where t_w is the stator winding axial thickness, h_M is the axial height of the PM and μ_{rrec} is the relative recoil permeability of the PM. To take into account the effect of slots, the air gap (mechanical clearance) for a slotted ferromagnetic core is increased by Carter coefficient $k_C > 1$ according to eqn (1.2). The saturation of the magnetic circuit can be included with the aid of saturation factors $k_{sat} \geq 1$ in the d -axis and $k_{satq} \geq 1$ in the q -axis. For a coreless stator the effect of magnetic saturation of the rotor ferromagnetic discs (core) is negligible.

The MMFs $\mathcal{F}_{ad}$ and $\mathcal{F}_{aq}$ in the d -axis and q -axis are

$$\mathcal{F}_{ad} = \frac{m_1 \sqrt{2}}{\pi} \frac{N_1 k_{w1}}{p} I_{ad} \quad (2.114)$$

$$\mathcal{F}_{aq} = \frac{m_1 \sqrt{2}}{\pi} \frac{N_1 k_{w1}}{p} I_{aq} \quad (2.115)$$

where I_{ad} and I_{aq} are the d - and q -axis stator (armature) currents respectively.

The armature reaction (mutual) inductance is calculated as

- in the d -axis

$$L_{ad} = \frac{\Psi_d}{I_{ad}} = m_1 \mu_0 \frac{1}{\pi} \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{(R_{out}^2 - R_{in}^2)}{g'} k_{fd} \quad (2.116)$$

- in the q -axis

$$L_{aq} = \frac{\Psi_q}{I_{aq}} = m_1 \mu_0 \frac{1}{\pi} \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{(R_{out}^2 - R_{in}^2)}{g'_q} k_{fq} \quad (2.117)$$

For $\mu_{rrec} \approx 1$ and surface configuration of PMs ($k_{fd} = k_{fq} = 1$), the d - and q -axis armature reaction inductances are equal, i.e.

$$L_a = L_{ad} = L_{aq} = m_1 \mu_0 \frac{1}{\pi} \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{(R_{out}^2 - R_{in}^2)}{g'} \quad (2.118)$$

Table 2.1. Armature reaction equations for cylindrical and disc-type machines

Quantity	Cylindrical machine	Disc-type machine
Armature reaction magnetic flux in the d -axis	$\Phi_{ad} = \frac{2}{\pi} B_{mad1} \tau L_i$	$\Phi_{ad} = \frac{2}{\pi} B_{mad1} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2)$
Armature reaction magnetic flux in the q -axis	$\Phi_{aq} = \frac{2}{\pi} B_{maq1} \tau L_i$	$\Phi_{aq} = \frac{2}{\pi} B_{maq1} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2)$
Permeance of the air gap in the d -axis	$\Lambda_d = \frac{\mu_0}{g'} \frac{2}{\pi} \tau L_i$	$\Lambda_d = \frac{\mu_0}{g'} \frac{2}{\pi} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2)$
Permeance of the air gap in the q -axis	$\Lambda_q = \frac{\mu_0}{g_q} \frac{2}{\pi} \tau L_i$	$\Lambda_q = \frac{\mu_0}{g_q} \frac{2}{\pi} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2)$
Permeance of the air gap per surface of one pole in the d -axis	$\lambda_d = \frac{\mu_0}{g'}$	
Permeance of the air gap per surface area of one pole in the q -axis	$\lambda_q = \frac{\mu_0}{g_q}$	
Armature reaction reactance in the d -axis	$X_{ad} = \frac{E_{ad}}{I_{ad}}$ $= 4m_1\mu_0 f \frac{(N_1 k_{w1})^2}{\pi p} \frac{\tau L_i}{g'} k_{fd}$	$X_{ad} = \frac{E_{ad}}{I_{ad}}$ $= 2m_1\mu_0 f \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{R_{out}^2 - R_{in}^2}{g'} k_{fd}$
Armature reaction reactance in the q -axis	$X_{aq} = \frac{E_{aq}}{I_{aq}}$ $= 4m_1\mu_0 f \frac{(N_1 k_{w1})^2}{\pi p} \frac{\tau L_i}{g_q} k_{fq}$	$X_{aq} = \frac{E_{aq}}{I_{aq}}$ $= 2m_1\mu_0 f \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{R_{out}^2 - R_{in}^2}{g_q} k_{fq}$
Armature reaction inductance in the d -axis	$L_{ad} = \frac{\Psi_{ad}}{I_{ad}}$ $= 2m_1\mu_0 \frac{(N_1 k_{w1})^2}{\pi^2 p} \frac{\tau L_i}{g'} k_{fd}$	$L_{ad} = \frac{\Psi_{ad}}{I_{ad}}$ $= m_1\mu_0 \frac{1}{\pi} \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{R_{out}^2 - R_{in}^2}{g'} k_{fd}$
Armature reaction inductance in the q -axis	$L_{aq} = \frac{\Psi_{aq}}{I_{aq}}$ $= 2m_1\mu_0 \frac{(N_1 k_{w1})^2}{\pi^2 p} \frac{\tau L_i}{g_q} k_{fq}$	$L_{aq} = \frac{\Psi_{aq}}{I_{aq}}$ $= m_1\mu_0 \frac{1}{\pi} \left(\frac{N_1 k_{w1}}{p} \right)^2 \frac{R_{out}^2 - R_{in}^2}{g_q} k_{fq}$

For surface configuration of PMs $k_{fd} = k_{fq} = 1$ [106]. For other configurations, equations for MMFs in the d - and q -axis will contain reaction factors $k_{fd} \neq k_{fq}$ [106].

The armature reaction EMFs in the d - and q -axis are

$$E_{ad} = \pi\sqrt{2}fN_1k_{w1}\Phi_{ad} \quad (2.119)$$

$$E_{aq} = \pi\sqrt{2}fN_1k_{w1}\Phi_{aq} \quad (2.120)$$

where the armature fluxes Φ_{ad} and Φ_{aq} are according to eqns (2.102) and (2.103).

The armature reaction reactances can be calculated by dividing EMFs E_{ad} and E_{aq} according to eqns (2.119) and (2.120) by currents I_{ad} and I_{aq} , i.e.

- in the d -axis

$$\begin{aligned} X_{ad} &= 2\pi fL_{ad} = \frac{E_{ad}}{I_{ad}} \\ &= 2m_1\mu_0f \left(\frac{N_1k_{w1}}{p} \right)^2 \frac{(R_{out}^2 - R_{in}^2)}{g'} k_{fd} \end{aligned} \quad (2.121)$$

- in the q -axis

$$\begin{aligned} X_{aq} &= 2\pi fL_{aq} = \frac{E_{aq}}{I_{aq}} \\ &= 2m_1\mu_0f \left(\frac{N_1k_{w1}}{p} \right)^2 \frac{(R_{out}^2 - R_{in}^2)}{g'_q} k_{fq} \end{aligned} \quad (2.122)$$

Table 2.1 compares the armature reaction equations between conventional cylindrical machines and disc-type machines.

2.10 AFPM Motor

2.10.1 Sine-Wave Motor

The three-phase stator winding with distributed parameters produces sinusoidal or quasi-sinusoidal distribution of the MMF. In the case of inverter operation all three solid state switches conduct current at any instant of time. The shape of the stator winding waveforms is shown in Fig. 1.3b. The *sine-wave motor* works as a PM synchronous motor.

For a sinusoidal excitation (synchronous machine) the excitation flux can be found on the basis of eqn (2.26) or (2.28), the EMF per phase induced by PM rotor can be found on the basis of eqn (2.36) and the electromagnetic torque on the basis of eqn (2.33). The EMF constant k_E and torque constant k_T are expressed by eqns (2.37) and (2.34), respectively.

2.10.2 Square-Wave Motor

PM d.c. brushless motors with *square-wave stator current* (Fig. 1.3a) are predominantly designed with large effective pole-arc coefficients, $\alpha_i^{(sq)}$, where $\alpha_i^{(sq)} = b_p(r)/\tau(r)$. Sometimes concentrated stator windings and salient stator poles are used. For the Y-connected windings, as in Fig. 2.17, only two of the three motor phase windings conduct at the same time, i.e. i_{aAB} (T1T4), i_{aAC} (T1T6), i_{aBC} (T3T6), i_{aBA} (T3T2), i_{aCA} (T5T2), i_{aCB} (T5T4), etc. At the on-time interval (120°) for the phase windings A and B, the solid state switches T1 and T4 conduct (Fig. 2.17a). When T1 switches off the current freewheels through diode D2. For the off-time interval both switches T1 and T4 are turned-off and the diodes D2 and D3 conduct the armature current which charges the capacitor C. If the solid state devices are switched at relatively high frequency, the winding inductance keeps the on-off rectangular current waveforms smooth.

For d.c. current excitation $\omega \rightarrow 0$, eqn (2.82) is similar to that describing a steady state condition of a d.c. commutator motor, i.e.

$$V_{dc} = E_{fL-L} + 2R_1 I_a^{(sq)} \quad (2.123)$$

where $2R_1$ is the sum of two-phase resistances in series (for Y-connected phase windings), and E_{fL-L} is the sum of two phase EMFs in series, V_{dc} is the d.c. input voltage supplying the inverter and $I_a^{(sq)}$ is the flat-topped value of the square-wave current equal to the inverter input current. The solid switch voltage drops have been neglected in eqn (2.123). The phasor analysis does not apply to this type of operation since the armature current is nonsinusoidal.

For a rectangular distribution of $B_{mg} = \text{const}$ with the pole shoe width $b_p < \tau$ being included, the excitation flux is

$$\Phi_f^{(sq)} = \alpha_i^{(sq)} B_{mg} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) \quad (2.124)$$

For a square-wave excitation the EMF induced in a single turn (two conductors) is $2B_{mg}L_i v = 4pnB_{mg}L_i\tau$. Including b_p and fringing flux, the EMF for N_1k_{w1} turns $e_f = 4pnN_1k_{w1}\alpha_i^{(sq)} B_{mg} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) = 4pnN_1k_{w1}\Phi_f^{(sq)}$. For the Y-connection of the armature windings, as in Fig. 2.17, two phases are conducting at the same time. The line-to-line EMF of a Y-connected square-wave motor is

$$\begin{aligned} E_{fL-L} &= 2e_f = 8pN_1k_{w1}\alpha_i^{(sq)} B_{mg}(\pi/2p)(R_{out}^2 - R_{in}^2)n \\ &= 8pN_1k_{w1}\Phi_f^{(sq)}n = k_{Edc}n \end{aligned} \quad (2.125)$$

where the *EMF constant* or armature constant k_{Edc} is

$$k_{Edc} = 8pN_1k_{w1}\Phi_f^{(sq)} \quad (2.126)$$

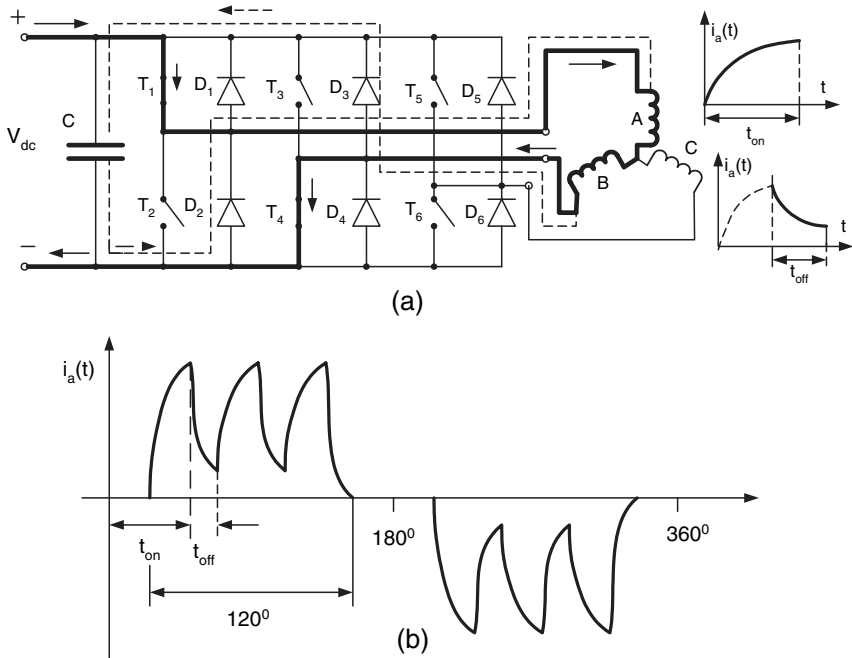


Fig. 2.17. Inverter currents in the stator winding of a PM brushless motor: (a) currents in phases A and B for on-time and off-time intervals, (b) on-off rectangular current waveform.

The *electromagnetic torque* developed by the motor is

$$T_d = \frac{P_{elm}}{2\pi n} = \frac{E_{fL-L} I_a^{(sq)}}{2\pi n}$$

$$= \frac{4}{\pi} p N_1 k_{w1} \Phi_f^{(sq)} I_a^{(sq)} = k_{Tdc} I_a^{(sq)} \quad (2.127)$$

where the *torque constant* of a square-wave motor is

$$k_{Tdc} = \frac{k_{Edc}}{2\pi} = \frac{4}{\pi} p N_1 k_{w1} \Phi_f^{(sq)} \quad (2.128)$$

and $I_a^{(sq)}$ is the flat-top value of the phase current.

The ratio T_d of a square-wave motor-to- T_d of a sinewave motor is

$$\frac{T_d^{(sq)}}{T_d} = \frac{4}{\pi} \frac{\sqrt{2}}{m_1} \frac{\Phi_f^{(sq)}}{\Phi_f} \frac{I_a^{(sq)}}{I_a} \approx 0.6 \frac{\Phi_f^{(sq)}}{\Phi_f} \frac{I_a^{(sq)}}{I_a} \quad (2.129)$$

Assuming the same motor ($\alpha_i^{(sq)} = \alpha_i$) and the same values of air gap magnetic flux densities, the ratio of the square-wave motor flux to sinewave motor flux is

$$\frac{\Phi_f^{(sq)}}{\Phi_f} = \frac{\alpha_i^{(sq)} B_{mg} [\pi/(2p)] (R_{out}^2 - R_{in}^2)}{\alpha_i k_f B_{mg} [\pi/(2p)] (R_{out}^2 - R_{in}^2)} = \frac{1}{k_f} \quad (2.130)$$

where [106]

$$k_f = \frac{4}{\pi} \sin \left(\frac{\alpha_i \pi}{2} \right) \quad (2.131)$$

and

$$\Phi_f = \Phi_{f1} = \alpha_i k_f B_{mg} \frac{\pi}{2p} (R_{out}^2 - R_{in}^2) \quad (2.132)$$

On the basis of eqns (2.125) and (2.127) the torque–speed characteristic can be expressed in the following simplified form

$$\frac{n}{n_0} = 1 - \frac{I_a}{I_{ash}} = 1 - \frac{T_d}{T_{dst}} \quad (2.133)$$

where the no-load speed, locked rotor armature current and stall torque are respectively

$$n_0 = \frac{V_{dc}}{k_E} \quad I_{ash} = \frac{V_{dc}}{R} \quad T_{dst} = k_{Tdc} I_{ash} \quad (2.134)$$

For half-wave operation $R = R_1$ while for full-wave operation $R = 2R_1$. Note that eqn (2.133) neglects the armature reaction, rotational and switching losses.

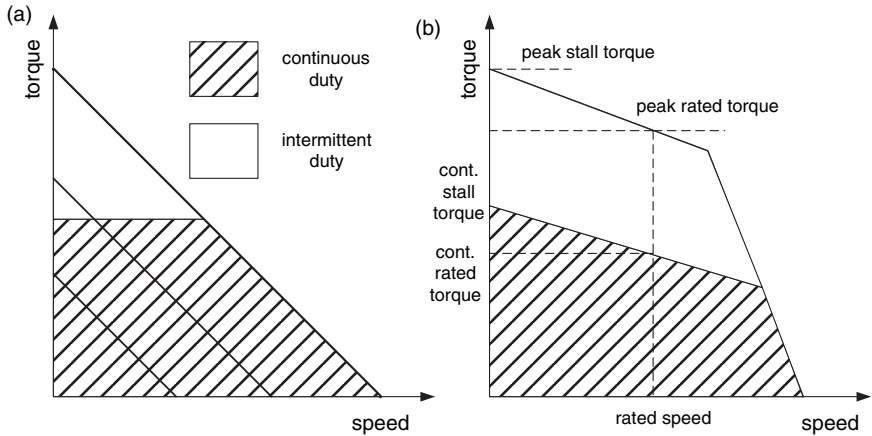


Fig. 2.18. Torque–speed characteristics of a PM brushless motor: (a) theoretical, (b) practical.

The torque-speed characteristics are shown in Fig. 2.18. Eqns (2.133) and (2.134) are very approximate and cannot be used in calculation of performance characteristics of practical PM d.c. brushless motors. Theoretical torque-speed characteristics (Fig. 2.18a) differ from practical characteristics (Fig. 2.18b). The *continuous torque* line is set by the maximum rated temperature of the motor. The intermittent duty operation zone is bounded by the *peak torque* line and the maximum input voltage.

The *rms* stator current of a d.c. brushless motor for a 120° square wave ($T = 2\pi/\omega$) is

$$\begin{aligned} I_a &= \sqrt{\frac{2}{T} \int_0^{T/2} i_a^2(t) dt} = \sqrt{\frac{\omega}{\pi} \int_{\pi/(6\omega)}^{5\pi/(6\omega)} [I_a^{(sq)}]^2 dt} \\ &= I_a^{(sq)} \sqrt{\frac{\omega}{\pi} \left(\frac{5\pi}{6\omega} - \frac{1\pi}{6\omega} \right)} = I_a^{(sq)} \sqrt{\frac{2}{3}} \end{aligned} \quad (2.135)$$

2.11 AFPM Synchronous Generator

2.11.1 Performance Characteristics of a Stand Alone Generator

An AFPM machine driven by a prime mover and connected to an electric load operates as a *stand alone synchronous generator*. For the same synchronous reactances in the *d*- and *q*-axis $X_{sd} = X_{sq} = X_s = \omega L_s$, and load impedance per phase

$$\mathbf{Z}_L = R_L + j\omega L_L - j\frac{1}{\omega C} \quad (2.136)$$

the input current in the stator (armature) winding is

$$I_a = \frac{E_f}{\sqrt{(R_1 + R_L)^2 + [\omega L_s + \omega L_L - 1/(\omega C)]^2}} \quad (2.137)$$

The voltage across the output terminals is

$$V_1 = I_a \sqrt{R_L^2 + \left(\omega L_L - \frac{1}{\omega C} \right)^2} \quad (2.138)$$

Characteristics of EMF per phase E_f , phase voltage V_1 , stator (load) current I_a , output power P_{out} , input power P_{in} , efficiency η and power factor $pf = \cos \phi$ versus speed n of a stand alone AFPM synchronous generator for inductive load $\mathbf{Z}_L = R_L + j\omega L_L$ are plotted in Fig. 2.19.

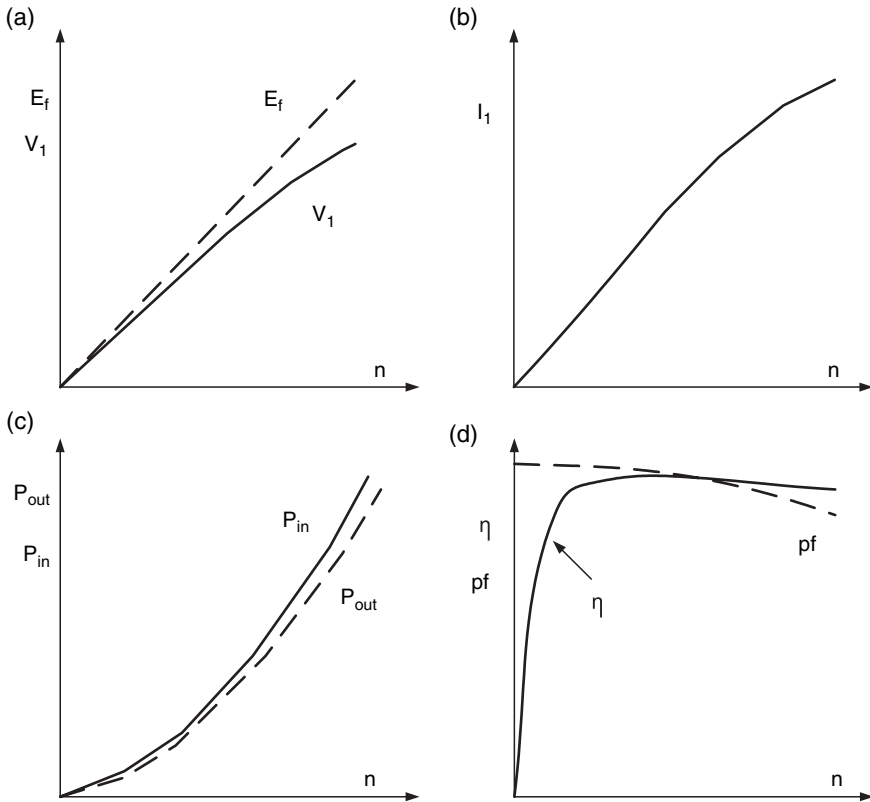


Fig. 2.19. Characteristics of a stand alone AFPM synchronous generator for inductive load $\mathbf{Z}_L = R_L + j\omega L_L$: (a) EMF E_f per phase and phase voltage V_1 versus speed n , (b) load current I_a versus speed n , (c) output power P_{out} and input power P_{in} versus speed n , (d) efficiency η and power factor $pf = \cos \phi$ versus speed n .

2.11.2 Synchronization With Utility Grid

An AFPM synchronous generator can be *synchronized* directly (connected in parallel) with the utility grid. The number of pole pairs is usually minimum $2p = 6$, so that to synchronize a 6-pole generator with a 50 Hz power system (infinite bus), the speed of the prime mover must be $n = 60(f/p) = 60(50/3) = 1000$ rpm. For $2p = 12$ and the same frequency, the speed will drop to $n = 500$ rpm. In general, direct paralleling of AFPM generators requires low speed prime movers. Before connecting the generator to the infinite bus, the incoming generator and the infinite bus must have the same:

- voltage
- frequency

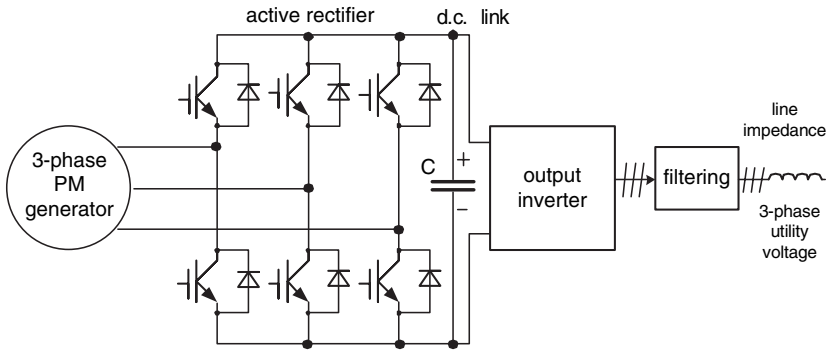


Fig. 2.20. Power circuit of a microturbine driven AFPM generator.

- phase sequence
- phase

In power plants, those conditions are checked using a *synchroscope*.

AFPM synchronous generators have recently been used in distributed generator systems as microturbine-driven high speed self-excited generators. A *microturbine* is a small, single shaft gas turbine of which the rotor is integrated with a high speed electric generator (up to 200 000 rpm), typically rated from 30 to 200 kW. If an AFPM generator is driven by a microturbine, in most cases the speed exceeds 20 000 rpm. The high frequency current of the generator must first be rectified and then inverted to obtain the same frequency as that of the power system. To minimize the higher harmonic contents in the stator windings, an active rather than passive rectifier is used (Fig. 2.20).

Numerical Example 2.1

Find the armature current, torque, electromagnetic power and winding losses of the S802F AFPM d.c. brushless servo motor (Table 1.1) at $n = 3000$ rpm and input voltage 230 V (line-to-line).

Solution

The line to line EMF is

$$E_{fL-L} = \sqrt{3}k_{Edc}n = \sqrt{3} \times \frac{42}{1000} \times 3000 = 218.24 \text{ V}$$

where the EMF constant $k_{Edc} = 42$ V/1000 rpm is for the phase EMF.

Assuming that the d.c. bus voltage is approximately equal to the input voltage, the armature current at 3000 rpm is

$$I_a^{(sq)} \approx \frac{230 - 218.24}{2 \times 0.76} = 7.74 \text{ A}$$

where 0.76Ω is the resistance per phase.

The shaft torque at 3000 rpm is

$$T = k_{Tdc} I_a^{(sq)} = 0.64 \times 7.74 = 4.95 \text{ Nm}$$

where the torque constant is $k_{Tdc} = 0.64 \text{ Nm/A}$.

The electromagnetic power (only two phases conduct current at the same time) is

$$P_{elm} = E_{fL-L} I_a^{(sq)} = 218.24 \times 7.74 \approx 1689 \text{ W}$$

The winding losses for $2 \times 0.76 \Omega$ line-to-line resistance are

$$\Delta P_w = (2 \times 0.76) \times 7.74^2 = 91 \text{ W}$$

Numerical Example 2.2

A three phase, Y-connected, $2p = 12$ pole, AFPM brushless motor has the surface PM of inner radius $R_{in} = 0.06 \text{ m}$, outer radius $R_{out} = 0.11 \text{ m}$ and the pole shoe width-to-pole pitch ratio $\alpha_i = 0.84$ (square wave mode), the number of turns per phase $N_1 = 222$, the winding factor $k_{w1} = 0.96$ and the peak value of the air gap magnetic flux density $B_{mg} = 0.65 \text{ T}$. Neglecting the armature reaction, find approximate values of the EMF, electromagnetic torque developed by the motor and electromagnetic power at $n = 1200 \text{ rpm}$ and *rms* current $I_a = 13.6 \text{ A}$ for:

- (a) sinewave operation ($\Psi = 0^\circ$)
- (b) 120° square wave operation

Solution

(a) sinewave operation at $\Psi = 0^\circ$

The speed in rev/s

$$n = \frac{1200}{60} = 20/\text{rev/s}$$

The form factor of the excitation field

$$k_f = \frac{4}{\pi} \sin \frac{\alpha_i \pi}{2} = \frac{4}{\pi} \times \sin \frac{0.84 \times \pi}{2} = 1.233$$

The excitation flux according to eqn (2.26) is

$$\Phi_f = \Phi_{f1} = \frac{2}{\pi} \times 1.233 \times 0.65 \times \frac{\pi}{12} \times (0.11^2 - 0.06^2) = 0.00114 \text{ Wb}$$

where $B_{mg1} = k_f B_{mg}$. The EMF constant according to eqn (2.37) is

$$k_E = \pi p \sqrt{2} N_1 k_{w1} \Phi_f = \pi \times 6 \times \sqrt{2} \times 222 \times 0.96 \times 0.00114 = 6.452 \text{ Vs}$$

The EMF per phase according to eqn (2.36) is

$$E_f = k_E n = 6.452 \times 20 = 129 \text{ V}$$

The line-to-line EMF for Y connection is

$$E_{fL-L} = \sqrt{3} E_f = \sqrt{3} \times 129 = 223.5 \text{ V}$$

The torque constants according to eqn (2.34) is

$$k_T = \frac{m_1}{\sqrt{2}} p N_1 k_{w1} \Phi_f = \frac{3}{\sqrt{2}} \times 6 \times 222 \times 0.96 \times 0.00114 = 3.08 \text{ Nm/A}$$

The electromagnetic torque developed at 13.6 A according to eqn (2.32) is

$$T_d = k_T I_a = 3.08 \times 13.6 = 41.9 \text{ Nm}$$

The electromagnetic power is

$$P_{elm} = m_1 E_f I_a \cos \Psi = 3 \times 129 \times 13.6 \times 1 = 5264.5 \text{ W}$$

where Ψ is the angle between the EMF E_f and stator current I_a .

(b) 120° square-wave operation

The flat-topped value of the phase current according to eqn (2.135) is

$$I_a^{(sq)} = \sqrt{\frac{3}{2}} I_a = \sqrt{\frac{3}{2}} \times 13.6 = 16.66 \text{ A}$$

The excitation flux according to eqn (2.124) is

$$\Phi_f^{(sq)} = 0.84 \times 0.65 \times \frac{\pi}{12} \times (0.11^2 - 0.06^2) = 0.00122 \text{ Wb}$$

The ratio of the square-wave flux to sinewave flux

$$\frac{\Phi_f^{(sq)}}{\Phi_f} = \frac{0.00122}{0.00114} = 1.07$$

Note that it has been assumed that $\alpha_i = 2/\pi = 0.6366$ for sinewave mode and $\alpha_i^{(sq)} = 0.84$ for square wave mode. For the same α_i the flux ratio is equal to $1/k_f$.

The EMF constant according to eqn (2.126) is

$$k_{Edc} = 8pN_1k_{w1}\Phi_f^{(sq)} = 8 \times 6 \times 222 \times 0.96 \times 0.00122 = 12.43 \text{ Vs}$$

The line-to-line EMF (two phases in series) according to eqn (2.125) is

$$E_{fL-L} = 12.43 \times \frac{1200}{60} = 248.6 \text{ V}$$

The torque constant according to eqn (2.128) is

$$k_{Tdc} = \frac{k_{Edc}}{2\pi} = \frac{12.43}{2\pi} = 1.978 \text{ Nm/A}$$

The electromagnetic torque at $I_a^{(sq)} = 16.66 \text{ A}$ according to eqn (2.127) is

$$T_d = k_{Tdc}I_a^{(sq)} = 1.978 \times 16.66 = 32.95 \text{ Nm}$$

The electromagnetic power is

$$P_{elm} = E_{fL-L}I_a^{(sq)} = 248.6 \times 16.66 = 4140.6 \text{ W}$$

Numerical Example 2.3

Find the main dimensions, approximate number of turns per phase and approximate cross section of the stator slot for a three-phase, double-sided, double-stator disc rotor PM brushless motor with a laminated stator core rated at: $P_{out} = 75 \text{ kW}$, $V_{1L} = 460 \text{ V}$ (Y connection), $f = 100 \text{ Hz}$, $n_s = 1500 \text{ rpm}$. The stator windings are connected in series.

Solution

For $f = 100 \text{ Hz}$ and $n_s = 1500 \text{ rpm} = 25 \text{ rev/s}$ the number of poles is $2p = 8$. Assuming $k_d = D_{in}/D_{out} = 1/\sqrt{3}$, the parameter k_D according to eqn (2.96) is

$$k_D = \frac{1}{8} \left(1 + \frac{1}{\sqrt{3}} \right) \left[1 - \left(\frac{1}{\sqrt{3}} \right)^2 \right] = 0.131$$

For a 75 kW motor the product $\eta \cos \phi \approx 0.9$. The phase current for series connected stator windings is

$$I_a = \frac{P_{out}}{m_1(2V_1)\eta \cos \phi} = \frac{75\,000}{3 \times 265.6 \times 0.9} = 104.6 \text{ A}$$

where $2V_1 = 460/\sqrt{3} = 265.6 \text{ V}$. The electromagnetic loading can be assumed as $B_{mg} = 0.65 \text{ T}$ and $A_m = 40,000 \text{ A/m}$. The ratio $\epsilon = E_f/V_1 \approx 0.9$ and the

stator winding factor has been assumed $k_{w1} = 0.96$. Thus, the stator outer diameter according to eqn (2.100) is

$$D_{out} = \sqrt[3]{\frac{0.9 \times 75\,000}{\pi^2 \times 0.131 \times 0.96 \times 25 \times 0.65 \times 40\,000 \times 0.9}} = 0.452 \text{ m}$$

The inner diameter according to eqns (2.27) is

$$D_{in} = \frac{D_{out}}{\sqrt{3}} = \frac{0.452}{\sqrt{3}} = 0.261 \text{ m},$$

The magnetic flux according to eqn (2.28) is

$$\Phi_f = \frac{2}{\pi} \times 0.65 \times \frac{\pi}{8 \times 4} \times 0.452^2 \left(1 - \frac{1}{(\sqrt{3})^2}\right) = 0.00555 \text{ Wb}$$

The number of stator turns per phase per stator calculated on the basis of line current density according to eqn (2.93) is

$$N_1 = \frac{\pi D_{out}(1 + k_d)A_m}{4m_1\sqrt{2}I_a} = \frac{\pi \times 0.452 \times (1 + 1/\sqrt{3}) \times 40\,000}{4 \times 3\sqrt{2} \times 104.6} \approx 51$$

The number of stator turns per phase per stator calculated on the basis of eqn (2.94) and (2.99) is

$$\begin{aligned} N_1 &= \frac{\epsilon V_1}{\pi\sqrt{2}fk_{w1}\Phi_f} \\ &= \frac{0.9 \times 265.6/2}{\pi\sqrt{2} \times 100 \times 0.96 \times 0.00555} \approx 50 \end{aligned}$$

A double layer winding can be located, say, in 16 slots per phase, i.e. $s_1 = 48$ slots for a three phase machine. The number of turns should be rounded to 48. This is an approximate number of turns which can be calculated exactly only after performing detailed electromagnetic and thermal calculations of the machine.

The number of slots per pole per phase according to eqn (2.2) is

$$q_1 = \frac{s_1}{2pm_1} = \frac{48}{8 \times 3} = 2$$

The number of stator coils (double-layer winding) is the same as the number of slots, i.e. $2pq_1m_1 = 8 \times 2 \times 3 = 48$. If the stator winding is made of four parallel conductors $a_w = 4$, the number of conductors in a single coil for one parallel current path $a_p = 1$ according to eqn (2.4) is

$$N_c = \frac{a_w N_1}{(s_1/m_1)} = \frac{4 \times 48}{(48/3)} = 12$$

The current density in the stator conductor can be assumed $J_a \approx 4.5 \times 10^6$ A/m² (totally enclosed a.c. machines rated up to 100 kW). The cross section area of the stator conductor is

$$s_a = \frac{I_a}{a_w J_a} = \frac{104.6}{4 \times 4.5} = 5.811 \text{ mm}^2$$

The stator winding of a 75 kW machine is made of a copper conductor of rectangular cross section. The slot fill factor for rectangular conductors and low voltage machines can be assumed to be 0.6. The cross section of the stator slot should, approximately, be

$$\frac{5.811 \times 12 \times 2}{0.6} \approx 233 \text{ mm}^2$$

where the number of conductors in a single slot is $12 \times 2 = 24$. The minimum stator slot pitch is

$$t_{1min} = \frac{\pi D_{in}}{z_1} = \frac{\pi \times 0.262}{48} = 0.0171 \text{ m} = 17.1 \text{ mm}$$

The stator slot width can be chosen to be 11.9 mm; this means that the stator slot depth is $233/11.9 \approx 20$ mm, and the stator narrowest tooth width is $c_{1min} = 17.1 - 11.9 = 5.2$ mm. Magnetic flux density in the narrowest part of the stator tooth is

$$B_{1tmax} \approx \frac{B_{mg} t_{1min}}{c_{1min}} = \frac{0.65 \times 17.1}{5.2} = 2.14 \text{ T}$$

This is a permissible value for the narrowest part of the tooth and silicon electrical steel with saturation magnetic flux density of 2.2 T. The maximum stator slot pitch is

$$t_{1max} = \frac{\pi D_{ext}}{z_1} = \frac{\pi \times 0.452}{48} = 0.0296 \text{ m} = 29.6 \text{ mm}$$

The magnetic flux density in the widest part of the stator tooth is

$$B_{1tmin} \approx \frac{B_{mg} t_{1max}}{c_{1max}} = \frac{0.65 \times 29.6}{29.6 - 11.9} = 1.09 \text{ T}$$

Numerical Example 2.4

A 7.5-kg cylindrical core is wound of isotropic silicon steel ribbon. It can be assumed that the magnetic field inside the core is uniform and the vector of the magnetic flux density is parallel to steel laminations. The magnitudes

of magnetic flux density time harmonics are: $B_{m1} = 1.7$ T, $B_{m3} = 0.25$ T, $B_{m5} = 0.20$ T and $B_{m7} = 0.05$ T. Harmonics $n > 7$ are negligible. The electric conductivity of laminations is $\sigma_{Fe} = 3.3 \times 10^6$ S/m, specific mass density $\rho_{Fe} = 7600$ kg/m³, thickness $d_{Fe} = 0.5$ mm and Richter's coefficient of hysteresis losses $\epsilon = 3.8$. Find the core losses at 50 Hz.

Solution

Eddy-current losses

The coefficient of distortion of the magnetic flux density according to eqn (2.52) is

$$\eta_d = \sqrt{1 + \left(3 \times \frac{0.25}{1.7}\right)^2 + \left(5 \times \frac{0.20}{1.7}\right)^2 \times \left(7 \times \frac{0.05}{1.7}\right)^2} = 1.258$$

The eddy current losses according to eqn (2.51) are

- specific losses at sinusoidal magnetic flux density

$$\Delta p_{e\sin} = \frac{\pi^2}{6} \times \frac{3.3 \times 10^6}{7600} \times 50^2 \times 0.0005^2 \times 1.7^2 = 1.29 \text{ W/kg}$$

- specific losses at nonsinusoidal magnetic flux density

$$\Delta p_e = \Delta p_{e\sin} \times \eta_d^2 = 1.29 \times 1.258^2 = 2.04 \text{ W/kg}$$

- losses at sinusoidal magnetic flux density

$$\Delta P_{e\sin} = \Delta p_{e\sin} m_{Fe} = 1.29 \times 7.5 = 9.68 \text{ W}$$

- losses at nonsinusoidal magnetic flux density

$$\Delta P_e = \Delta p_e m_{Fe} = 2.04 \times 7.5 = 15.32 \text{ W}$$

Hysteresis losses

The hysteresis losses may be calculated according to eqn (2.53), in which Richter's coefficient of hysteresis losses $\epsilon = 3.8$

- specific losses at sinusoidal magnetic flux density

$$\Delta p_{h\sin} = 3.8 \times \frac{50}{100} \times 1.7^2 = 5.49 \text{ W/kg}$$

- specific losses at nonsinusoidal magnetic flux density

$$\Delta p_h = \Delta p_{h\sin} \times \eta_d^2 = 5.49 \times 1.258^2 = 8.69 \text{ W/kg}$$

- losses at sinusoidal magnetic flux density

$$\Delta P_{h\sin} = \Delta p_{h\sin} m_{Fe} = 5.49 \times 7.5 = 41.18 \text{ W}$$

- losses at nonsinusoidal magnetic flux density

$$\Delta P_h = \Delta p_h m_{Fe} = 8.69 \times 7.5 = 65.19 \text{ W}$$

Specific eddy current and hysteresis losses plotted against frequency are shown in Fig. 2.21.

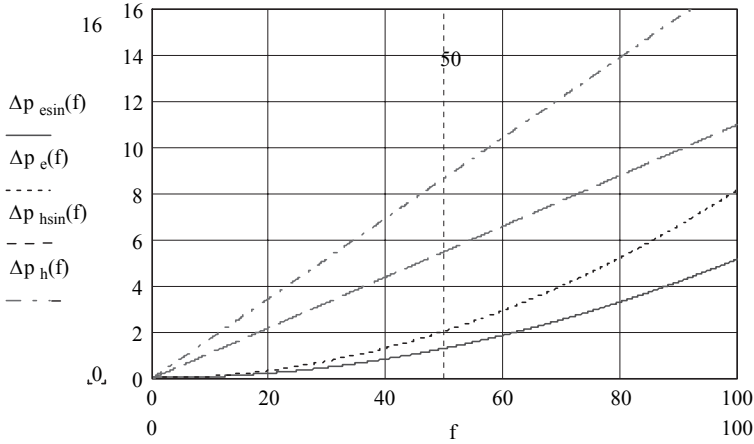


Fig. 2.21. Specific eddy-current and hysteresis losses versus frequency. *Numerical example 2.4.*

Total losses

The eddy current and hysteresis losses at sinusoidal magnetic flux density are calculated as

$$\Delta P_{Fesin} = \Delta P_{esin} + \Delta P_{hsin} = 9.68 + 41.18 = 50.86 \text{ W}$$

The eddy current and hysteresis losses at nonsinusoidal magnetic flux density are

$$\Delta P_{Fe} = \Delta P_e + \Delta P_h = 15.32 + 65.19 = 80.51 \text{ W}$$

The ratio of hysteresis-to-total losses is the same for both sinusoidal and nonsinusoidal magnetic flux density, i.e.

$$\frac{41.18}{50.86} = \frac{65.19}{80.51} = 0.81$$

Numerical Example 2.5

Find the power losses in PMs and solid rotor backing steel disc of a single-sided AFPM with slotted stator at ambient temperature of 20°C. The inner diameter of PMs is $D_{in} = 0.14$ m, the outer diameter of PMs is $D_{out} = 0.242$ m, number of poles $2p = 8$, magnet width-to-pole pitch ratio is $\alpha_i = 0.75$, height of surface permanent magnet is $h_M = 6$ mm, air gap thickness $g = 1.2$ mm, number of slots $s_1 = 36$ and speed $n = 3000$ rpm. Assume the peak value of the air gap magnetic flux density $B_{mg} = 0.7$ T, conductivity of NdFeB PMs $\sigma_{Pm} = 0.8 \times 10^6$ S/m at 20°C, its relative recoil magnetic permeability $\mu_{rrec} = 1.05$, conductivity of solid disc $\sigma_{Fe} = 4.5 \times 10^6$ S/m at 20°C and its relative recoil magnetic permeability $\mu_r = 300$.

Solution

The average magnetic flux density is $B_{avg} = (2/\pi)B_{mg} = (2/\pi) \times 0.7 = 0.446$ T, the average diameter $D = 0.5(D_{out} + D_{in}) = 0.5(0.242 + 0.14) = 0.191$ m, the average pole pitch is $\tau = \pi D/(2p) = \pi 0.191/8 = 0.075$ m, the average slot pitch $t_{s1} = \pi D/s_1 = \pi 0.191/36 = 0.017$ m. The frequency of the fundamental harmonic of the slot component of the magnetic flux density according to eqn (2.56) is

$$f_{sl} = s_1 p n = 36 \times 4 \times \frac{3000}{60} = 7200 \text{ Hz}$$

The corresponding angular frequency is $\omega_{sl} = 2\pi 7200 = 45\,238.9$ rad/s.

The equivalent air gap in the d -axis for $k_C = 1$ according to eqn (2.110) is

$$g' = 0.0012 + \frac{0.006}{1.05} = 0.006914 \text{ m}$$

The parameters $\kappa = 0.434$, $\beta_{sl} = 0.041$, $\Gamma = 0.18$ and $a_{sl} = 0.546$ have been found on the basis of eqns (2.60), (2.59) and (2.58) respectively. The Carter coefficient is $k_C = 1.012$ - eqns (1.2) and (1.3). Thus, eqn (2.57) gives the magnetic flux density component due to slot opening as

$$B_{sl} = 0.546 \times 0.041 \times 1.012 \times 0.446 = 0.01018 \text{ T}$$

The parameters $k = 139.283$ 1/m for $\mu_{rrec} = 1.05$, $\Delta = 1/k = 0.0072$ m, $\alpha = 139.283 + j139.283$ 1/m, $\beta = \pi/(0.5t_{s1}) = \pi/(0.5 \times 0.017) = 377.241$ 1/m, $\kappa = 386.668 + j50.962$ 1/m, $a_{R\nu} = 2.733$ and $a_{X\nu} = 0.366$ for $\nu = 1$ have been found on the basis of eqns (1.17), (1.16), (1.11), (1.17), (1.21) and (1.22) respectively. The transverse edge effect coefficient k_z according to eqn (2.62) is

$$k_z = 1 + \frac{0.01666}{0.242 - 0.14} = 1.163$$

The active surface area of PMs according to eqn (2.63) is

$$S_{PM} = 0.75\pi \times [(0.5 \times 0.242)^2 - (0.5 \times 0.14)^2] = 0.023 \text{ m}^2$$

The active losses in PMs according to eqn (2.61) are

$$\begin{aligned} \Delta P_{PM} &= \frac{1}{2} \times 2.733 \times 1.081 \times \left(\frac{|139.283 + j139.283|^2}{377.241^2} \right) \\ &\times \left(\frac{0.01018}{0.4\pi \times 10^{-6} \times 1.05} \right)^2 \times \frac{139.283}{0.65 \times 10^6} \times 0.023 = 127.2 \text{ W} \end{aligned}$$

The reactive power losses in PMs can be estimated as

$$\Delta Q_{PM} = \frac{a_{X\nu}}{a_{R\nu}} \Delta P_{PM} = \frac{0.366}{2.733} \times 127.2 = 17.03 \text{ VAR}$$

For calculation of power losses in the solid steel disc of the rotor, the magnetic flux density component due to slot openings $B_{sl} = 0.01018 \text{ T}$ is the same. The relative magnetic permeability $\mu_r = 300$ and conductivity of the disc $\sigma_{Fe} = 4.5 \times 10^6 \text{ S/m}$ are different, so that the attenuation factor (1.17) calculated for these values will be $k = 6194.6 \text{ 1/m}$. The coefficients $a_{RFe} = 1.451$ and $a_{XFe} = 0.849$ are calculated using eqns (2.64) and (2.65) for $a_R = 1.45$ and $a_X = 0.85$. The remaining field parameters, i.e. $\alpha = 8982 + j5265 \text{ 1/m}$, $\beta = \pi/(0.5t_{s1}) = \pi/(0.5 \times 0.017) = 377.241 \text{ 1/m}$ and $\kappa = 8988 + j5362 \text{ 1/m}$ have been found from eqns (1.16), (1.11) and (1.17) respectively.

The surface area of the rotor disc according to eqn (2.67) is

$$S_{Fe} = \frac{\pi}{4} \times (0.242^2 - 0.14^2) = 0.031 \text{ m}^2$$

The active power losses in the rotor disc according to eqn (2.66) is

$$\begin{aligned} \Delta P_{Fe2} &= \frac{1}{2} \times 1.451 \times 1.081 \times \left(\frac{|8982 + j5265|^2}{377.241^2} \right) \\ &\times \left(\frac{0.01018}{0.4\pi \times 10^{-6} \times 300} \right)^2 \times \frac{6194.6}{4.5 \times 10^6} \times 0.031 = 19.79 \text{ W} \end{aligned}$$

The coefficients $k_z = 1.081$ is the same as for calculation of losses in PMs. The reactive power losses in the rotor disc can be estimated as

$$\Delta Q_{Fe2} = \frac{a_{XFe\nu}}{a_{RFe\nu}} \Delta P_{Fe2} = \frac{0.849}{1.451} \times 118.2 = 11.6 \text{ VAR}$$

Numerical Example 2.6

Find the rotational loss of a 6000 rpm disc type machine with outer radius of the rotor $R_{out} = 0.14$ m, radius of shaft behind the rotor core $R_{sh} = 0.025$ m, mass of the rotor $m_r = 1.31$ kg and mass of the shaft $m_{sh} = 1.49$ kg. The machine is naturally cooled with air. The ambient temperature is 20°C. The coefficient of bearing friction is $k_{fb} = 1.5$.

Solution

The bearing friction losses according to eqn (2.71) are

$$\Delta P_{fr} = 0.06 \times 1.5 \times (1.31 + 1.49) \times \frac{6000}{60} = 25.23 \text{ W}$$

The Reynolds number according to eqn (2.72) is

$$Re = 1.2 \times \frac{2\pi}{1.8 \times 10^{-5}} \times \frac{6000}{60} \times 0.2^2 = 8.21 \times 10^5$$

where the air density is $\rho = 1.2$ kg/m³ and dynamic viscosity of the air is $\mu = 1.8 \times 10^{-5}$ Pa s. The coefficient of drag according to eqn (2.73) is

$$c_f = \frac{3.87}{\sqrt{8.21 \times 10^5}} = 4.271 \times 10^{-3}$$

The windage losses according to eqn (2.74) are

$$\Delta P_{wind} = \frac{1}{2} \times 4.271 \times 10^{-3} \times 1.2 \times \left(2\pi \times \frac{6000}{60} \right) (0.14^5 - 0.025^5) = 34.18 \text{ W}$$

The rotational losses are thus

$$\Delta P_{rot} = 25.23 + 34.18 = 59.41 \text{ W}$$

Materials and Fabrication

3.1 Stator Cores

Stator cores of AFPM brushless machines are made of laminated steels or soft magnetic powder materials. Soft magnetic powders simplify the manufacturing process and reduce the cost of AFPM machines.

3.1.1 Nonoriented Electrical Steels

Most laminated cores for stators of AFPM brushless machines are made of *nonoriented (isotropic) silicon steel* ribbons with standard thickness from 0.12 to 0.64 mm. Nonoriented steels are Fe-Si alloys with a random orientation of crystal cubes. Magnetic properties are practically the same in any direction in the plane of the sheet or ribbon. A secondary recrystallization process is not needed and high temperature annealing is not essential. Nonoriented grades contain between 0.5% and 3.25% Si with up to 0.5% Al addition to increase the resistivity and lower the temperature of the primary recrystallization.

Nonoriented electrical steels are available as both *fully processed* and *semi-processed* products. Fully processed nonoriented electrical steels are completely processed by the steel manufacturer, ready for use without any additional processing required to achieve the desired magnetic quality. Semi-processed nonoriented electrotechnical steels are those which have not been given the full annealing treatment by the steel producer. In some cases, users prefer to develop the final magnetic quality and achieve relief of fabricating stresses in laminations or assembled cores for small machines.

The most universally accepted grading of electrical steels by core losses is the American Iron and Steel Industry (AISI) system (Table 3.1), the so called “M-grading”. For small and medium power electrical machines (output power less than 75 kW), the following grades can be used: M-27, M-36, M-43, M-45 and M-47.

Laminating a magnetic core is ineffective in keeping excessive eddy currents from circulating within the entire core unless the surfaces of laminations

Table 3.1. The most important silicon steel designations specified by different standards

Europe IEC 404-8-4 (1986)	U.S.A. AISI	Japan JIS 2552 (1986)	Russia GOST 21427 0-75
250-35-A5	M-15	35A250	2413
270-35-A5	M-19	35A270	2412
300-35-A5	M-22	35A300	2411
330-35-A5	M-36	—	—
270-50-A5	—	50A270	—
290-50-A5	M-15	50A290	2413
310-50-A5	M-19	50A310	2412
330-50-A5	M-27	—	—
350-50-A5	M-36	50A350	2411
400-50-A5	M-43	50A400	2312
470-50-A5	—	50A470	2311
530-50-A5	M-45	—	2212
600-50-A5	—	50A600	2112
700-50-A5	M-47	50A700	—
800-50-A5	—	50A800	2111
350-65-A5	M-19	—	—
400-65-A5	M-27	—	—
470-65-A5	M-43	—	—
530-65-A5	—	—	2312
600-65-A5	M-45	—	2212
700-65-A5	—	—	2211
800-65-A5	—	65A800	2112
1000-65-A5	—	65A1000	—

Table 3.2. Specific core losses of *Armco* DI-MAX nonoriented electrical steels M-27, M-36 and M-43 at 60 Hz

Magnetic flux density T	Specific core losses W/kg							
	0.36 mm		0.47 mm			0.64 mm		
	M-27	M-36	M-27	M-36	M-43	M-27	M-36	M-43
0.20	0.09	0.10	0.10	0.11	0.11	0.12	0.12	0.13
0.50	0.47	0.52	0.53	0.56	0.59	0.62	0.64	0.66
0.70	0.81	0.89	0.92	0.97	1.03	1.11	1.14	1.17
1.00	1.46	1.61	1.67	1.75	1.87	2.06	2.12	2.19
1.30	2.39	2.58	2.67	2.80	2.99	3.34	3.46	3.56
1.50	3.37	3.57	3.68	3.86	4.09	4.56	4.70	4.83
1.60	4.00	4.19	4.30	4.52	4.72	5.34	5.48	5.60
1.70	4.55	4.74	4.85	5.08	5.33	5.99	6.15	6.28
1.80	4.95	5.14	5.23	5.48	5.79	6.52	6.68	6.84

Table 3.3. Magnetization curves of fully processed *Armco* DI-MAX nonoriented electrical steels M-27, M-36 and M-43

Magnetic flux density T	Magnetic field intensity A/m		
	M-27	M-36	M-43
0.20	36	41	47
0.40	50	57	64
0.70	74	80	89
1.00	116	119	130
1.20	175	174	187
1.50	859	785	777
1.60	2188	2109	1981
1.70	4759	4727	4592
1.80	8785	8722	8682
2.00	26 977	26 022	26 818
2.10	64 935	65 492	66 925
2.20	137 203	136 977	137 075

are adequately insulated. Types of surface insulation include the natural oxide surface, inorganic insulation, enamel, varnish or chemically treated surface. The thickness of the insulation is expressed with the aid of the *stacking factor*:

$$k_i = \frac{d}{d + 2\Delta} \quad (3.1)$$

where d is the thickness of bare lamination and Δ is the thickness of the insulation layer measured on one side. The stacking factor k_i is typically from 0.94 to 0.97.

Core loss curves of *Armco* DI-MAX nonoriented electrical steels M-27, M-36 and M-43 tested at 60 Hz are given in Table 3.2. Core losses when tested at 50 Hz would be approximately 0.79 times the core loss at 60 Hz. Magnetization curves of the same electrical steels are given in Table 3.3. The specific mass density of DI-MAX M-27, M-36 and M-43 is 7650, 7700 and 7750 kg/m³ respectively. The stacking factor is $k_i = 0.95$ to 0.96. The prefix DI-MAX, e.g. DI-MAX M27, designates a registered trademark with a special strip-annealed process that maximizes *punchability*. DI-MAX grades have superior permeability at high magnetic flux density, low core losses and good gauge uniformity. A smooth surface, excellent flatness and high stacking factor is obtained as a result of cold finishing and strip annealing.

For frequencies exceeding the power frequency of 50 or 60 Hz nonoriented laminations thinner than 0.2 mm must be used. Table 3.4 shows magnetization and specific core losses of nonoriented grades NO 12, NO 18 and NO 20 capable of operating up to 2.5 kHz (*Cogent Power Ltd.*, Newport, UK). Typical chemical composition is 3.0 % Si, 0.4 % Al, 96.6 % Fe. The standard thickness of an inorganic phosphate based insulation is 1 μm (one side). The maximum continuous operating temperature in the air is 230°C, maximum

Table 3.4. Specific core losses and d.c. magnetization curve of nonoriented thin electrical steels manufactured by *Cogent Power Ltd.*, Newport, U.K.

Magn. flux dens. T	Specific core losses W/kg									Magn. field stren. kA/m
	NO 12			NO 18			NO 20			
	0.12 mm			0.18 mm			0.2 mm			
	50 Hz	400 Hz	2.5 kHz	50 Hz	400 Hz	2.5 kHz	50 Hz	400 Hz	2.5 kHz	
0.10	0.02	0.16	1.65	0.02	0.18	2.18	0.02	0.17	2.79	0.025
0.20	0.08	0.71	6.83	0.08	0.73	10.6	0.07	0.72	10.6	0.032
0.30	0.16	1.55	15.2	0.16	1.50	19.1	0.14	1.49	24.4	0.039
0.40	0.26	2.57	2.54	0.26	2.54	31.7	0.23	2.50	40.4	0.044
0.50	0.37	3.75	37.7	0.36	3.86	45.9	0.32	3.80	58.4	0.051
0.60	0.48	5.05	52.0	0.47	5.22	61.5	0.42	5.17	78.4	0.057
0.70	0.62	6.49	66.1	0.61	6.77	81.1	0.54	6.70	103.0	0.064
0.80	0.76	8.09	83.1	0.75	8.47	104.0	0.66	8.36	133.0	0.073
0.90	0.32	9.84	103.0	0.90	10.4	161.0	0.80	10.3	205.0	0.084
1.00	1.09	11.8	156	1.07	12.3	198.0	0.95	12.2	253.0	0.099
1.10	1.31	14.1		1.28	14.9		1.14	14.8		0.124
1.20	1.56	16.7		1.52	18.1		1.36	17.9		0.160
1.30	1.89	19.9		1.84	21.6		1.65	21.4		0.248
1.40	2.29	24.0		2.23	25.6		2.00	25.3		0.470
1.50	2.74	28.5		2.67	30.0		2.40	29.7		1.290
1.60	3.14			3.06			2.75			3.550
1.70	3.49			3.40			3.06			7.070
1.80	3.78			3.69			3.32			13

intermittent operating temperature in an inert gas is 850°C, hardness 180 HV and density 7650 kg/m³.

3.1.2 Amorphous Ferromagnetic Alloys

To minimize core losses at high frequencies, nonoriented electrotechnical steels should be replaced with *amorphous magnetic alloys* (Tables 3.5 and 3.6). Amorphous ferromagnetic alloys, in comparison with electrical steels with crystal structure, do not have arranged in order, regular inner crystal structure (lattice).

Amorphous alloy ribbons based on alloys of iron, nickel and cobalt are produced by rapid solidification of molten metals at cooling rates of about 10⁶°C/s. The alloys solidify before the atoms have a chance to segregate or crystallize. The result is a metal alloy with a glass-like structure, i.e. a non-crystalline frozen liquid.

Application of amorphous alloy ribbons to the mass production of electrical machines is limited by hardness, up to 1100 in Vicker’s scale. Standard cutting methods like a guillotine or blank die are not suitable. The mechanically

Table 3.5. Physical properties of iron based METGLAS amorphous alloy ribbons (*Honeywell*, Morristown, NJ, U.S.A.)

Quantity	2605CO	2605SA1
Saturation magnetic flux density, T	1.8	1.59 annealed 1.57 cast
Specific core losses at 50 Hz and 1 T, W/kg	less than 0.28	about 0.125
Specific density, kg/m ³	7560	7200 annealed 7190 cast
Electric conductivity, S/m	0.813×10^6 S/m	0.769×10^6 S/m
Hardness in Vicker's scale	810	900
Elasticity modulus, GN/m ²	100...110	100...110
Stacking factor	less than 0.75	less than 0.79
Crystallization temperature, °C	430	507
Curie temperature, °C	415	392
Maximum service temperature, °C	125	150

Table 3.6. Specific core losses of iron based METGLAS amorphous alloy ribbons (*Honeywell*, Morristown, NJ, U.S.A.)

Magnetic flux density, <i>B</i> T	Specific core losses, Δp , W/kg			
	2605CO		2605SA1	
	50 Hz	60 Hz	50 Hz	60 Hz
0.05	0.0024	0.003	0.0009	0.0012
0.10	0.0071	0.009	0.0027	0.0035
0.20	0.024	0.030	0.0063	0.008
0.40	0.063	0.080	0.016	0.02
0.60	0.125	0.16	0.032	0.04
0.80	0.196	0.25	0.063	0.08
1.00	0.274	0.35	0.125	0.16

stressed amorphous material cracks. Laser and electro-discharge machining (EDM) cutting methods melt the amorphous material and cause undesirable crystallization. In addition, these methods make electrical contacts between laminations which contribute to the increased eddy-current and additional losses. In the early 1980s chemical methods were used in *General Electric* to cut amorphous materials but these methods were very slow and expensive [191]. The problem of cutting hard amorphous ribbons can be overcome by using a liquid jet [246]. This method makes it possible to cut amorphous materials in ambient temperature without cracking, melting, crystallization and electric contacts between isolated ribbons.

3.1.3 Soft Magnetic Powder Composites

Powder metallurgy is used in the production of ferromagnetic cores of small electrical machines or ferromagnetic cores with complicated shapes. The components of *soft magnetic powder composites* are iron powder, dielectric (epoxy resin) and filler (glass or carbon fibers) for mechanical strengthening. Powder composites for ferromagnetic cores of electrical machines and apparatus can be divided into [265]:

- dielectromagnetics and magnetodielectrics,
- magnetic sinters.

Dielectromagnetics and magnetodielectrics are names referring to materials consisting of the same basic components: ferromagnetic (mostly iron powder) and dielectric (mostly epoxy resin) material [265]. The main tasks of the dielectric material is insulation and binding of ferromagnetic particles. In practice, composites containing up to 2% (of their mass) of dielectric materials are considered as *dielectromagnetics*. Those with a higher content of dielectric material are considered as *magnetodielectrics* [265].

TSC International, Wadsworth, IL, U.S.A., has developed a new soft powder material, *Accucore*, which is competitive to traditional steel laminations [4]. The magnetization curve and specific core loss curves of the non-sintered *Accucore* are given in Table 3.7. When sintered, *Accucore* has higher saturation magnetic flux density than the non-sintered material. The specific density is 7550 to 7700 kg/m³.

Höganäs, Höganäs, Sweden, manufactures soft magnetic composite (SMC) powders that are surface-coated metal powders with excellent compressibility [242]. *SomaloyTM 500* (Tables 3.8, 3.9) has been developed for 3D magnetic circuits of electrical machines, transformers, ignition systems and sensors.

3.1.4 Fabrication of Stator Cores

Fabrication of Laminated Stator Cores

Normally, the stator cores are wound from electrotechnical steel strips and the slots are machined by shaping or planing. An alternative method is first

Table 3.7. Magnetization and specific core loss characteristics of non-sintered *Accucore* (*TSC Ferrite International*, Wadsworth, IL, U.S.A.)

Magnetization curve		Specific core loss curves		
Magnetic flux density, B T	Magnetic field intensity, H A/m	60 Hz W/kg	100 Hz W/kg	400 Hz W/kg
0.10	152	0.132	0.242	1.058
0.20	233	0.419	0.683	3.263
0.30	312	0.772	1.323	6.217
0.40	400	1.212	2.072	9.811
0.50	498	1.742	2.976	14.088
0.60	613	2.315	3.968	18.850
0.70	749	2.954	5.071	24.295
0.80	909	3.660	6.305	30.490
0.90	1107	4.431	7.650	37.346
1.00	1357	5.247	9.039	44.489
1.10	1677	6.129	10.582	52.911
1.20	2101	7.033	12.214	61.377
1.30	2687	7.981	13.845	70.151
1.40	3525	8.929	15.565	79.168
1.50	4763	9.965	17.394	90.302
1.60	6563	10.869	19.048	99.671
1.70	9035	11.707	20.635	109.880
1.75	10,746	12.125	21.407	

Table 3.8. Specific core losses of SomaloyTM 500 +0.5% Kenolube, 800 MPa, treated at 500°C for 30 min in the air, *Höganäs*, Höganäs, Sweden

Magnetic flux density, A/m	Specific losses W/kg					
	50 Hz	100 Hz	300 Hz	500 Hz	700 Hz	1000 Hz
0.4	1.5	3	12	18	27	45
0.5	1.9	3.6	17	27	40	60
0.6	2.7	6	21	34	55	90
0.8	4.6	10	32	52	92	120
1.0	6.8	16	48	80	140	180
2.0	30	50	170	270	400	570

to punch the slots with variable distances between them and then to wind the steel strip into the form of the slotted toroidal core (*R & D Institute of Electrical Machines VÚES* in Brno, Republic of Czech). In addition, this manufacturing process allows for making skewed slots to minimize the cogging torque and the effect of slot harmonics. Each stator core has skewed slots in opposite directions. It is recommended that a wave stator winding should be made to obtain shorter end connections and more space for the shaft. An odd

Table 3.9. Magnetization curves of SomaloyTM 500 +0.5% Kenolube, treated at 500°C for 30 min in the air, Höganäs, Höganäs, Sweden

Magnetic field intensity H A/m	B at density 6690 kg/m ³ T	B at density 7100 kg/m ³ T	B at density 7180 kg/m ³ T
1 500	0.7	0.83	0.87
3 200	0.8	1.13	1.19
4 000	0.91	1.22	1.28
6 000	1.01	1.32	1.38
10 000	1.12	1.42	1.51
15 000	1.24	1.52	1.61
20 000	1.32	1.59	1.69
40 000	1.52	1.78	1.87
60 000	1.65	1.89	1.97
80 000	1.75	1.97	2.05
100 000	1.82	2.02	2.10

number of slots, e.g. 25 instead of 24 can help to reduce the cogging torque (VÚES Brno).

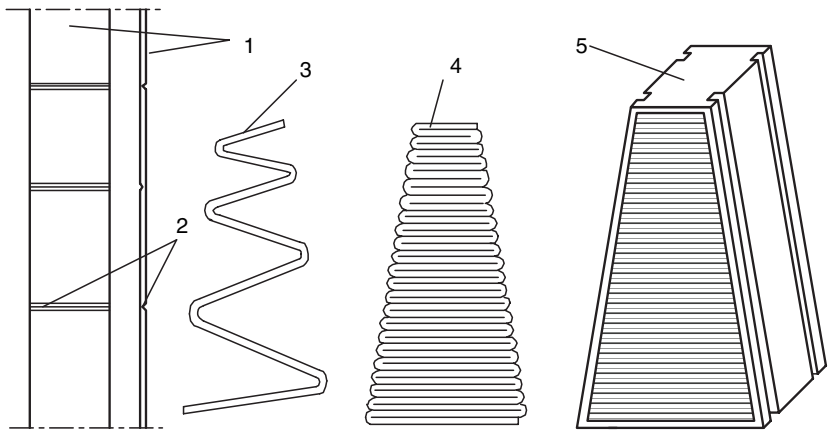


Fig. 3.1. Stator core segment formed from lamination strip: 1 — lamination strip, 2 — groove, 3 — folding, 4 — compressed segment, 5 — finished segment.

Another technique is to form the stator core using trapezoidal segments [257]. Each segment corresponds to one slot pitch (Fig. 3.1). The lamination strip of constant width is folded at distances proportional to the radius. To make folding easy, the strip has transverse grooves on opposite sides of the

alternative steps. The zigzag laminated segment is finally compressed and fixed using a tape or thermosetting, as shown in Fig. 3.1 [257].

Fabrication of Soft Magnetic Powder Stator Cores

The laminated cores of axial flux machines are much more difficult to fabricate than those of radial flux machines. SMC powders simplify the manufacturing process of stator cores with complicated shapes, in general, 3D cores. Mass production of AFPM machines is much more cost effective if soft magnetic powder composites are used as materials for stator cores.

Using SMC powders the stator core of an AFPM machine can be made as a slotted core, slotless cylindrical core and salient pole core with one coil per pole.

The slotted and slotless cylindrical cores for AFPM machines can be made in a *powder metallurgy* process using a ferromagnetic powder with a small amount of lubricants or binders. The powder metallurgy process generally consists of four basic steps, namely: (1) powder manufacture, (2) mixing or blending, (3) compacting and (4) sintering. Most compacting is done with mechanical, hydraulic or pneumatic presses and rigid tools. Compacting pressures generally range between 70 to 800 MPa with 150 to 500 MPa being the most common. The outer diameter of the core is limited by the press capability. Frequently, the stator core must be divided into smaller segments. Most powder metallurgy products must have cross sections of less than 2000 mm². If the press capacity is sufficient, sections up to 6500 mm² can be pressed. Figure 3.2 shows the effect of compacting pressure on the density of *Höganäs* SMC powders.

For *Somaloy*TM500 the heat treatment temperature (sintering) is typically 500°C for 30 min. After heat treatment the compacted powder has much less mechanical strength than solid steel.

The thermal expansion of conductors within the stator slots creates thermal expansion stresses in the stator teeth. The magnitude of these stresses depends upon the difference in the temperature of the winding and core, difference in coefficients of thermal expansion of both materials and slot fill factor. This problem is more important in powder cores than in laminated cores since the tensile stress of powder cores is at least 25 times lower and their modulus of elasticity is less than 100 GPa (versus 200 GPa for steel laminations).

Slotted stators for small disc-type motors fabricated from SMC powders are shown in Fig. 3.3 [160, 265]. SMC powder salient-pole stators for small single-sided AFPM motors manufactured by *Mii Technologies, LLC*, Lebanon, NH, U.S.A. are shown in Fig. 3.4. The three-phase stator has 9 poles. A single SMC powder salient pole manufactured by *Höganäs* is shown in Fig. 3.5.

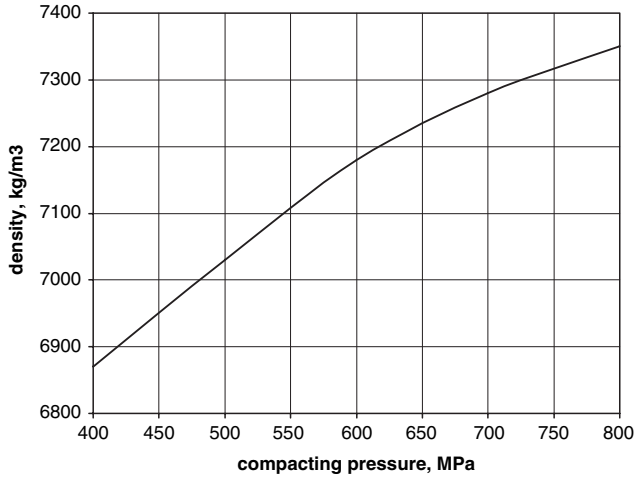


Fig. 3.2. The effect of compacting pressure on the specific mass density of Höganäs soft magnetic composite powders.

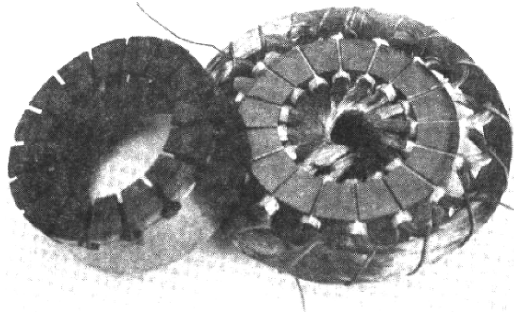


Fig. 3.3. Slotted stators for small single-sided disc-type motors [265].

3.2 Rotor Magnetic Circuits

Magnetic circuits of rotors consist of PMs and mild steel backing rings or discs. Since the air gap is somewhat larger than that in similar RFPM counterparts, high energy density PMs should be used.

Normally, surface magnets are glued to smooth backing rings or rings with cavities of the same shape as magnets without any additional mechanical protection against normal attractive forces. Epoxy, acrylic or silicon based adhesives are used for gluing between magnets and backing rings or between magnets. The minimum required shearing strength of adhesives is 20×10^6 Pa. There were attempts to develop interior PM rotor for AFPM machines. According to [221], rotor poles can only be fabricated by using soft magnetic powders [221]. The main advantage of this configuration is the improved flux

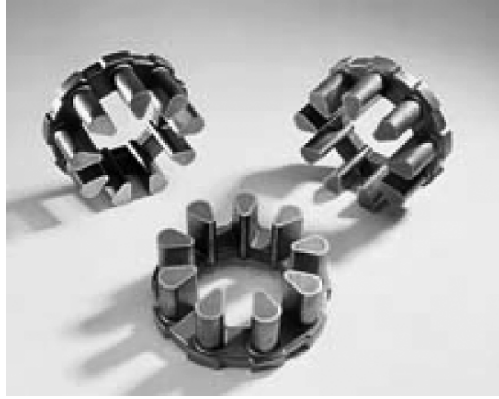


Fig. 3.4. Powder salient pole stators for small single-sided AFPM motors. Courtesy of *Mii Technologies, LLC*, Lebanon, NH, U.S.A.



Fig. 3.5. SMC powder salient pole for small single-sided AFPM motors: (a) single SMC pole; (b) double-sided AFPM motor. Courtesy of *Höganäs*, Höganäs, Sweden.

weakening performance. However, the complexity and high cost of the rotor structure discourage further commercializing development.

3.2.1 PM Materials

A PM can produce magnetic flux in an air gap with no exciting winding and no dissipation of electric power. As any other ferromagnetic material, a PM can be described by its B — H hysteresis loop. PMs are also called *hard magnetic materials*, which means ferromagnetic materials with a wide hysteresis loop.

The basis for the evaluation of a PM is the portion of its hysteresis loop located in the upper left-hand quadrant, called the *demagnetization curve* (Fig. 3.6). If reverse magnetic field intensity is applied to a previously magnetized, say, toroidal specimen, the magnetic flux density drops down to the magnitude determined by the point K . When the reversal magnetic flux density is

removed, the flux density returns to the point L according to a minor hysteresis loop. Thus, the application of a reverse field has reduced the *remanence*, or *remanent magnetism*. Reapplying magnetic field intensity will again reduce the flux density, completing the minor hysteresis loop by returning the core to approximately the same value of flux density at the point K as before. The minor hysteresis loop may usually be replaced with little error by a straight line called the *recoil line*. This line has a slope called the *recoil permeability* μ_{rec} .

As long as the negative value of applied magnetic field intensity does not exceed the maximum value corresponding to the point K , the PM may be regarded as being reasonably permanent. If, however, greater negative field intensity H is applied, the magnetic flux density will be reduced to a value lower than that at point K . On the removal of H , a new and lower recoil line will be established.

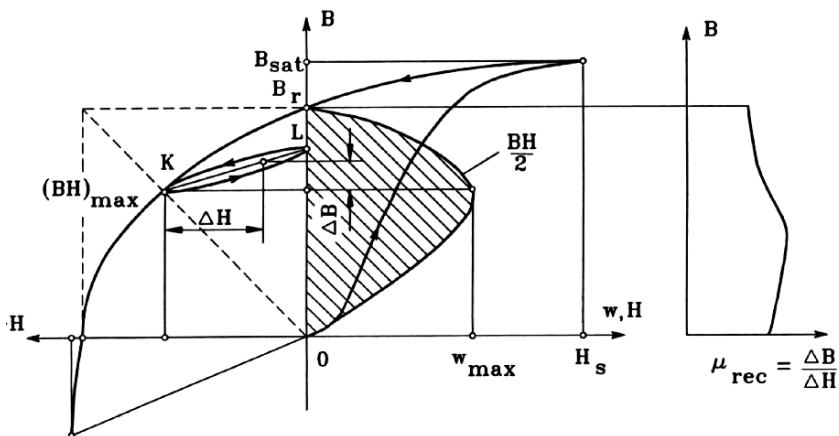


Fig. 3.6. Demagnetization curve, recoil loop, energy of a PM, and recoil magnetic permeability.

Remanent magnetic flux density B_r , or *remanence*, is the magnetic flux density corresponding to zero magnetic field intensity.

Coercive field strength H_c , or *coercivity*, is the value of demagnetizing field intensity necessary to bring the magnetic flux density to zero in a material previously magnetized.

Both B_r and H_c decrease as the magnet temperature increases, i.e.

$$B_r = B_{r20}[1 + \frac{\alpha_B}{100}(\vartheta_{PM} - 20)] \quad (3.2)$$

$$H_c = H_{c20}[1 + \frac{\alpha_H}{100}(\vartheta_{PM} - 20)] \quad (3.3)$$

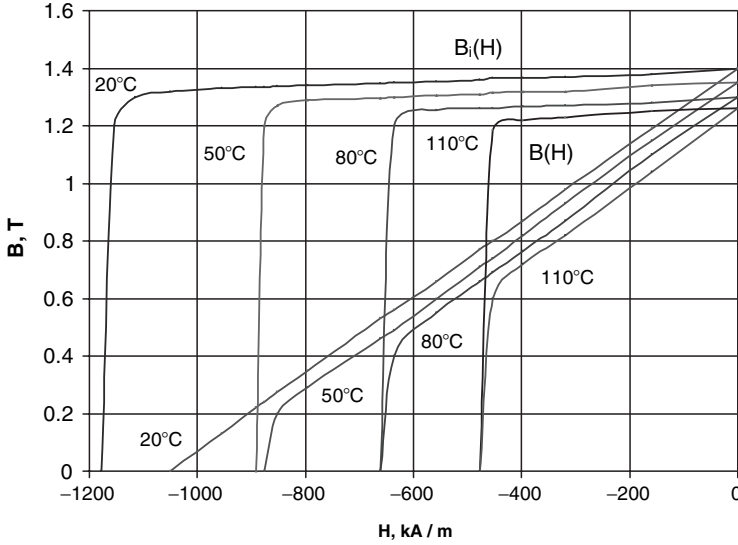


Fig. 3.7. Comparison of B — H and B_i — H demagnetization curves and their variations with the temperature for sintered N48M NdFeB PMs. Courtesy of *ShinEtsu*, Takefu-shi, Fukui Prefecture, Japan.

where ϑ_{PM} is the temperature of PM, B_{r20} and H_{c20} are the remanent magnetic flux density and coercive force at 20°C and $\alpha_B < 0$ and $\alpha_H < 0$ are *temperature coefficients* for B_r and H_c in $\%/^\circ\text{C}$ respectively. Thus, demagnetization curves are sensitive to the temperature (Fig. 3.7).

Intrinsic demagnetization curve (Fig. 3.7) is the portion of the $B_i = f(H)$ hysteresis loop located in the upper left-hand quadrant, where $B_i = B - \mu_0 H$. For $H = 0$ the intrinsic magnetic flux density $B_i = B_r$.

Intrinsic coercivity iH_c is the magnetic field strength required to bring to zero the intrinsic magnetic flux density B_i of a magnetic material described by the $B_i = f(H)$ curve. For PM materials $iH_c > H_c$.

Saturation magnetic flux density B_{sat} corresponds to high values of the magnetic field intensity, when an increase in the applied magnetic field produces no further effect on the magnetic flux density. In the *saturation region* the alignment of all the *magnetic moments of domains* is in the direction of the external applied magnetic field.

Recoil magnetic permeability μ_{rec} is the ratio of the magnetic flux density—to-magnetic field intensity at any point on the demagnetization curve, i.e.

$$\mu_{rec} = \mu_0 \mu_{rrec} = \frac{\Delta B}{\Delta H} \quad (3.4)$$

where the *relative recoil permeability* $\mu_{rrec} = 1 \dots 4.5$.

Maximum magnetic energy per unit produced by a PM in the external space is equal to the maximum magnetic energy density per volume, i.e.

$$w_{max} = \frac{(BH)_{max}}{2} \quad \text{J/m}^3 \quad (3.5)$$

where the product $(BH)_{max}$ corresponds to the maximum energy density point on the demagnetization curve with coordinates B_{max} and H_{max} (Fig. 3.6).

Form factor of the demagnetization curve characterizes the concave shape of the demagnetization curve, i.e.

$$\gamma = \frac{(BH)_{max}}{B_r H_c} = \frac{B_{max} H_{max}}{B_r H_c} \quad (3.6)$$

for a square demagnetization curve $\gamma = 1$ and for a straight line (rare-earth PMs) $\gamma = 0.25$.

The leakage flux causes the magnetic flux to be distributed nonuniformly along the height $2h_M$ of a PM, where h_M is the height per pole. As a result, the MMF produced by the PM is not constant. The magnetic flux is higher in the neutral cross section and lower at the ends, but the behavior of the MMF distribution is the opposite [106].

The PM surface is not equipotential. The magnetic potential at each point on the surface is a function of the distance to the neutral zone. To simplify the calculation, the magnetic flux which is a function of the MMF distribution along the height h_M per pole is replaced by an equivalent flux. This equivalent flux goes through the whole height h_M and exits from the surface of the poles. To find the equivalent leakage flux and the whole flux of a PM, the equivalent magnetic field intensity needs to be found, i.e.

$$H = \frac{1}{h_M} \int_0^{h_M} H_x dx = \frac{\mathcal{F}_M}{h_M} \quad (3.7)$$

where H_x is the magnetic field intensity at a distance x from the neutral cross section and $\mathcal{F}_M$ is the MMF of the PM per pole (MMF = $2\mathcal{F}_M$ per pole pair).

The equivalent magnetic field intensity (3.7) allows the equivalent leakage flux of the PM to be found, i.e.

$$\Phi_{lM} = \Phi_M - \Phi_g \quad (3.8)$$

where Φ_M is the full equivalent flux of the PM and Φ_g is the air gap magnetic flux. The *coefficient of leakage flux* of the PM,

$$\sigma_{lM} = \frac{\Phi_M}{\Phi_g} = 1 + \frac{\Phi_{lM}}{\Phi_g} > 1 \quad (3.9)$$

simply allows the air gap magnetic flux to be expressed as $\Phi_g = \Phi_M / \sigma_{lM}$.

The following leakage permeance expressed in the flux Φ –MMF coordinate system corresponds to the equivalent leakage flux of the PM:

$$G_{lM} = \frac{\Phi_{lM}}{\mathcal{F}_M} \quad (3.10)$$

An accurate estimation of the leakage permeance G_{lM} is the most difficult task in analytical calculation of magnetic circuits with PMs. Using the field approach, e.g. the FEM, the leakage permeance can be found fairly accurately.

The average equivalent magnetic flux and equivalent MMF mean that the magnetic flux density and magnetic field intensity are assumed to be the same in the whole volume of a PM. The full energy produced by the magnet in the outer space is

$$W = \frac{BH}{2} V_M \quad \text{J} \quad (3.11)$$

where V_M is the volume of the PM or a system of PMs.

For a PM circuit with a rectangular cross section, single PM and two mild steel pole shoes, the magnetic flux density B_g in a given air gap volume $V_g = gw_M l_M$ is directly proportional to the square root of the magnetic energy product $(B_M H_M)$ [106], i.e.

$$\begin{aligned} B_g = \mu_0 H_g &= \sqrt{\frac{\mu_0}{\sigma_{lM}} \left(1 + \frac{2H_{Fe} l_{Fe}}{H_g g} \right)^{-1} \frac{V_M}{V_g} B_M H_M} \\ &\approx \sqrt{\mu_0 \frac{V_M}{V_g} B_M H_M} \end{aligned} \quad (3.12)$$

where H_{Fe} is the magnetic field intensity in the mild steel yoke, H_g is the magnetic field intensity in the air gap, $V_M = 2h_M w_M l_M$ is the magnet volume, w_M is the PM width, l_M is the PM length and $2l_{Fe}$ is the length of the magnetic flux path in two mild steel pole shoes. Following the trend to smaller packaging, smaller mass and higher efficiency, the material research in the field of PMs has focused on finding materials with high values of the maximum energy product $(BH)_{max}$.

The air gap magnetic flux density B_g can be estimated analytically on the basis of the demagnetization curve, air gap and leakage permeance lines and recoil lines [106]. Approximately, it can be found on the basis of the balance of the magnetic voltage drops, i.e.

$$\frac{B_r}{\mu_0 \mu_{rrec}} h_M = \frac{B_g}{\mu_0 \mu_{rrec}} h_M + \frac{B_g}{\mu_0} g$$

where μ_{rrec} is the relative permeability of the PM (relative recoil permeability). Hence,

$$B_g \approx \frac{B_r h_M}{h_M + \mu_{rrec} g} = \frac{B_r}{1 + \mu_{rrec} g / h_M} \quad (3.13)$$

The air gap magnetic flux density is proportional to the remanent magnetic flux density B_r and decreases as the air gap g increases. Eqn (3.13) can only be used for preliminary calculations.

For rare-earth PMs the approximation of the demagnetization curve is simple due to practically linear demagnetization curve, i.e.

$$B(H) = B_r \left(1 - \frac{H}{H_c} \right) \quad (3.14)$$

The approximation of more complicated demagnetizations curves (Alnico or ferrites) is given e.g. in [106].

The intersection point of the above demagnetization curve (3.14) and the following line representing the permeance of the air gap

$$B(H) = \mu_0 \frac{h_M}{g} H \quad (3.15)$$

gives a point called the *operating point*. This point corresponds to the air gap magnetic flux density B_g multiplied by the leakage coefficient σ_{lM} according to eqn (3.9).

3.2.2 Characteristics of PM Materials

There are three classes of PMs currently used for electric motors:

- Alnicos (Al, Ni, Co, Fe);
- Ceramics (ferrites), e.g. barium ferrite $\text{BaO} \times 6\text{Fe}_2\text{O}_3$ and strontium ferrite $\text{SrO} \times 6\text{Fe}_2\text{O}_3$;
- Rare-earth materials, i.e. samarium-cobalt SmCo and neodymium-iron-boron NdFeB .

The demagnetization curves of the above three types of permanent magnet materials are given in Fig. 3.8.

Alnico

Alnico magnets dominated the PM motor market in the range from a few watts to 150 kW between the mid 1940s and the late 1960s. The main advantages of Alnico are its high magnetic remanent flux density and low temperature coefficients (Table 3.10). The temperature coefficient of B_r is $-0.02\%/^{\circ}\text{C}$ and maximum service temperature is 520°C . Unfortunately, the coercive force is very low and the demagnetization curve is extremely non-linear. Therefore, it is very easy not only to magnetize but also to demagnetize Alnico. Alnico has been used in PM d.c. commutator motors of the disc type with relatively large air gaps. This results in a negligible armature reaction magnetic flux acting on the PMs. Sometimes, Alnico PMs are protected from the armature flux, and consequently from demagnetization, using additional mild steel pole shoes.

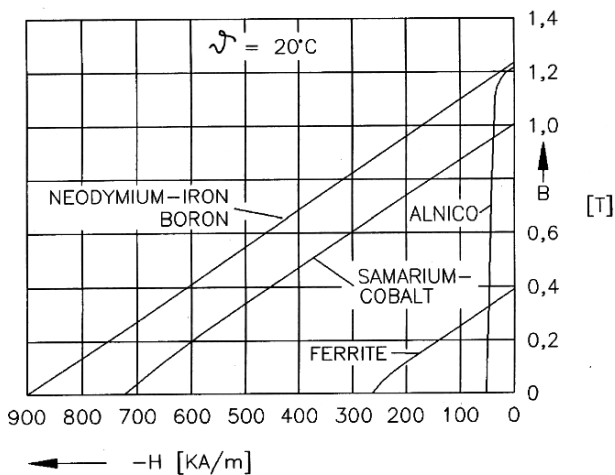


Fig. 3.8. Demagnetization curves for different PM materials.

Table 3.10. Physical properties of a representative selection of PM materials for small motors offered by *Magnaquench GmBH*, Essen, Germany

Property	Alnico sintered Koerzit 500	Hard ferrite bonded Koerox 12/22p	Hard ferrite sintered Koerox 350
Remanent flux density, B_r , T	1.24	0.26	0.39
Coercivity, H_c , kA/m	51	180	270
Intrinsic coercivity, iH_c , kA/m	51	225	310
$(BH)_{max}$, kJ/m ³	41.4	13	30
Relative recoil			
magnetic permeability	3 to 4.5	1.1	1.1
Temperature coefficient α_B of B_r at 20 to 100°C, %/°C	−0.02	−0.2	−0.2
Temperature coefficient α_{iH} of iH_c at 20 to 100°C, %/°C	+0.03 to −0.07	+0.4	+0.3
Curie temperature, °C	850	450	450
Maximum continuous service temperature, °C	500	100 to 200	200
Thermal conductivity, W/(m °C)	10 to 100	–	4
Specific mass density, ρ_{PM} , kg/m ³	7300	3400	4800
Electric conductivity, S/m	1.4 to 2.5 × 10 ⁶	< 0.0001	< 0.0001
Coefficient of thermal expansion at 20 to 100°C, ×10 ^{−6} /°C	11 to 13	20 to 50	12 parallel 8 vertical
Specific heat, J/(°C kg)	350 to 500	–	800

Ferrites

Barium and strontium *ferrites* produced by powder metallurgy were invented in the 1950s. Their chemical formulation may be expressed as $\text{MO} \times 6(\text{Fe}_2\text{O}_3)$, where M is Ba, Sr, or Pb. Ferrite magnets are available in *isotropic* and *anisotropic* grades.

A ferrite has a higher coercive force than Alnico, but at the same time has a lower remanent magnetic flux density (Table 3.10). Temperature coefficients are relatively high, i.e. the coefficient of B_r is $-0.20\%/^{\circ}\text{C}$ and the coefficient of H_c is -0.27 to $-0.4\%/^{\circ}\text{C}$. The maximum service temperature is 450°C . The main advantages of ferrites are their low cost and very high electric resistance, which means practically no eddy-current losses in the PM volume. Ferrite magnets are most economical in fractional horsepower motors. Barium ferrite PMs are commonly used in small d.c. commutator motors for automobiles (blowers, fans, windscreen wipers, pumps, etc.) and electric toys.

Rare-Earth Permanent Magnets

The first generation of *rare-earth permanent magnets*, i.e. alloys based on the composition of SmCo_5 has been commercially produced since the early 1970s (invented in the 1960s). SmCo_5 has the advantage of a high remanent flux density, high coercive force, high energy product, a linear demagnetization curve and a low temperature coefficient (Table 3.11). The temperature coefficient of B_r is -0.02 to $-0.04\%/^{\circ}\text{C}$ and the temperature coefficient of H_c is -0.14 to $-0.40\%/^{\circ}\text{C}$. Maximum service temperature is 300 to 350°C . It is suitable for motors with low volumes and motors operating at increased temperatures, e.g. brushless generators for microturbines. Both Sm and Co are relatively expensive due to their supply restrictions.

With the discovery in the recent years of a second generation of rare-earth magnets on the basis of inexpensive neodymium (Nd), remarkable progress with regard to lowering raw material costs has been achieved. The new generation of rare-earth PMs based on inexpensive neodymium (Nd) was announced by *Sumitomo Special Metals*, Japan, in 1983 at the 29th Annual Conference of Magnetism and Magnetic Materials held in Pittsburgh, PA, U.S.A. The Nd is a much more abundant rare-earth element than Sm. NdFeB magnets, which are now produced in increasing quantities have better magnetic properties (Table 3.12) than those of SmCo, but unfortunately only at room temperature. The demagnetization curves, especially the coercive force, are strongly temperature dependent. The temperature coefficient of B_r is -0.09 to $-0.15\%/^{\circ}\text{C}$ and the temperature coefficient of H_c is -0.40 to $-0.80\%/^{\circ}\text{C}$. The maximum service temperature is 250°C and Curie temperature is 350°C). The NdFeB is also susceptible to corrosion. NdFeB magnets have great potential for considerably improving the *performance-to-cost* ratio for many applications. For this reason they will have a major impact on the development and application of PM machines in the future.

Table 3.11. Physical properties of Vacomax sintered $\text{Sm}_2\text{Co}_{17}$ PM materials at room temperature 20°C manufactured by *Vacuumschmelze GmbH*, Hanau, Germany

Property	Vacomax 240 HR	Vacomax 225 HR	Vacomax 240
Remanent flux density, B_r , T	1.05 to 1.12	1.03 to 1.10	0.98 to 1.05
Coercivity, H_c , kA/m	600 to 730	720 to 820	580 to 720
Intrinsic coercivity, iH_c , kA/m	640 to 800	1590 to 2070	640 to 800
$(BH)_{max}$, kJ/m ³	200 to 240	190 to 225	180 to 210
Relative recoil magnetic permeability	1.22 to 1.39	1.06 to 1.34	1.16 to 1.34
Temperature coefficient α_B of B_r at 20 to 100°C , %/ $^\circ\text{C}$		−0.030	
Temperature coefficient α_{iH} of iH_c at 20 to 100°C , %/ $^\circ\text{C}$	−0.15	−0.18	−0.15
Temperature coefficient α_B of B_r at 20 to 150°C , %/ $^\circ\text{C}$		−0.035	
Temperature coefficient α_{iH} of iH_c at 20 to 150°C , %/ $^\circ\text{C}$	−0.16	−0.19	−0.16
Curie temperature, $^\circ\text{C}$		approximately 800	
Maximum continuous service temperature, $^\circ\text{C}$	300	350	300
Thermal conductivity, W/(m $^\circ\text{C}$)		approximately 12	
Specific mass density, ρ_{PM} , kg/m ³		8400	
Electric conductivity, $\times 10^6$ S/m		1.18 to 1.33	
Coefficient of thermal expansion at 20 to 100°C , $\times 10^{-6}/^\circ\text{C}$		10	
Young's modulus, $\times 10^6$ MPa		0.150	
Bending stress, MPa		90 to 150	
Vicker's hardness		approximately 640	

Chemical reactivity of rare-earth magnets is similar to that of alkaline earth metals, e.g. magnesium. The reaction is accelerated at increased temperature and humidity. The NdFeB alloy if exposed to hydrogen gas, usually at a slightly elevated temperature and/or elevated pressure, becomes brittle and with very little effort, it can be crushed. Diffusion of hydrogen into the alloy causes it literally to fall apart.

Corrosion protective coatings can be divided into metallic and organic. For metallic coatings, e.g. nickel and tin, galvanic processes are used as a rule. Organic coatings include powder coatings applied electrostatically, varnishes and resins.

Nowadays, for the industrial production of rare-earth PMs the powder metallurgical route is mainly used [212]. Apart from some material specific parameters, this processing technology is, in general, the same for all rare-earth magnet materials. The alloys are produced by vacuum induction melting or by a calciothermic reduction of the oxides. The material is then size-reduced

Table 3.12. Physical properties of Hicorex-Super sintered NdFeB PM materials at room temperature 20°C manufactured by *Hitachi Metals, Ltd.*, Tokyo, Japan

Property	Hicorex-Super HS-38AV	Hicorex-Super HS-25EV	Hicorex-Super HS-47AH
Remanent flux density, B_r , T	1.20 to 1.30	0.98 to 1.08	1.35 to 1.43
Coercivity, H_c , kA/m	875 to 1035	716 to 844	1018 to 1123
Intrinsic coercivity, iH_c , kA/m	min. 1114	min. 1989	min. 1114
$(BH)_{max}$, kJ/m ³	278 to 319	183 to 223	342 to 390
Relative recoil magnetic permeability		1.03 to 1.06	
Temperature coefficient α_B of B_r at 20 to 100°C, %/°C		−0.11 to −0.13	
Temperature coefficient α_{iH} of iH_c at 20 to 100°C, %/°C		−0.65 to −0.72	
Curie temperature, °C		≈ 310	
Maximum continuous service temperature, °C	160	180	140
Thermal conductivity, W/(m°C)		≈ 7.7	
Specific mass dens., ρ_{PM} , kg/m ³		7500	
Electric conductivity, ×10 ⁶ S/m		≈ 0.67	
Coefficient of thermal expansion at 20 to 100°C, ×10 ^{−6} /°C		−1.5	
Young’s modulus, ×10 ⁶ MPa		0.150	
Bending stress, MPa		260	
Vicker’s hardness		≈ 600	
Features	High energy product	High temperature	Super high performance

by crushing and milling to a single crystalline powder with particle sizes less than 10 μm . In order to obtain anisotropic PMs with the highest possible $(BH)_{max}$ value, the powders are then aligned in an external magnetic field, pressed and densified to nearly theoretical density by sintering. The most economical method for mass production of simply shaped parts like blocks, rings or arc segments is die pressing of the powders in approximately the final shape.

Researchers at *General Motors*, U.S.A., developed a fabrication method based on the melt-spinning casting system originally invented for the production of amorphous metal alloys. In this technology a molten stream of NdFeCoB material is first formed into ribbons 30 to 50- μm thick by rapid quenching, then cold pressed, extruded and hot pressed into bulk. Hot pressing and hot working are carried out while maintaining the fine grain to provide a high density close to 100% which eliminates the possibility of internal corrosion. The standard electro-deposited epoxy resin coating provides excellent corrosion resistance.

The prices of NdFeB magnets ordered in large quantities are now below US\$20 per kg. Owing to a large supply of NdFeB magnets from China it is expected that the prices will fall further.

3.2.3 Operating Diagram

The energy of a PM in the external space only exists if the reluctance of the external magnetic circuit is higher than zero. If a previously magnetized PM is placed inside a closed ideal ferromagnetic circuit, i.e. toroid, this PM does not show any magnetic properties in the external space, in spite of the fact that there is the magnetic flux

$$\Phi_r = B_r S_M = B_r w_M l_M \quad (3.16)$$

corresponding to the remanent flux density B_r inside the PM.

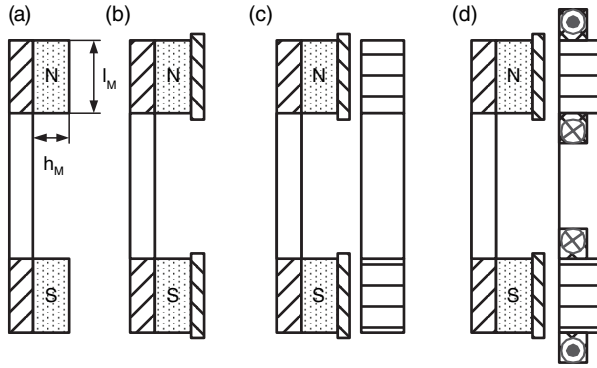


Fig. 3.9. Stabilization of a PM: (a) PM alone, (b) PM with pole shoes, (c) PM inside an external magnetic circuit, (d) PM with a complete external armature system.

A PM previously magnetized and placed alone in an open space, as in Fig. 3.9a, generates a magnetic field. To sustain a magnetic flux in the external open space, an MMF developed by the magnet is necessary. The state of the PM is characterized by the point K on the demagnetization curve (Fig. 3.10). The location of the point K is at the intersection of the demagnetization curve with a straight line representing the permeance of the external magnetic circuit (open space):

$$G_{ext} = \frac{\Phi_K}{\mathcal{F}_K}, \quad \tan \alpha_{ext} = \frac{\Phi_K / \Phi_r}{\mathcal{F}_K / \mathcal{F}_c} = G_{ext} \frac{\mathcal{F}_c}{\Phi_r} \quad (3.17)$$

The permeance G_{ext} corresponds to the flux Φ —MMF coordinate system and is referred to as MMF at the ends of the PM. In the Φ —MMF coordinate

system the remanent flux Φ_r is according to eqn (3.16) and the MMF corresponding to the coercivity H_c is

$$\mathcal{F}_c = H_c h_M \quad (3.18)$$

The magnetic energy per unit produced by the PM in the external space is $w_K = B_K H_K / 2$. This energy is proportional to the rectangle limited by the coordinate system and lines perpendicular to the Φ and $\mathcal{F}$ coordinates projected from the point K . It is obvious that the maximum magnetic energy is for $B_K = B_{max}$ and for $H_K = H_{max}$.

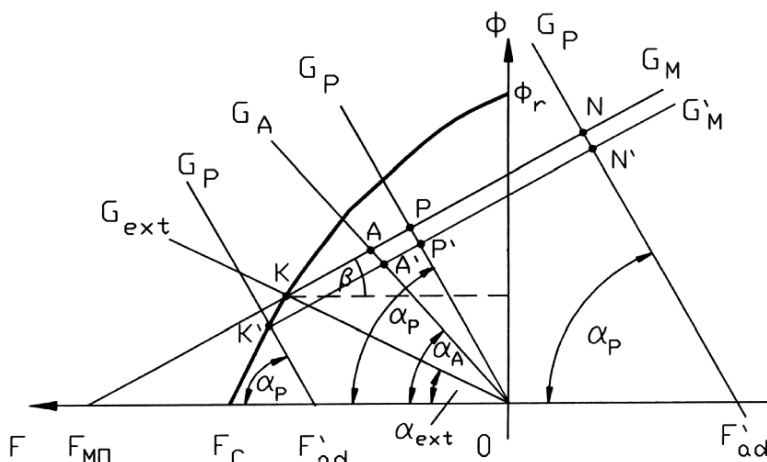


Fig. 3.10. Diagram of a PM for finding the origin of the recoil line and operating point.

If the poles are furnished with pole shoes (Fig. 3.9b) the permeance of the external space increases. The point which characterizes a new state of the PM in Fig. 3.10 moves along the recoil line from the point K to the point A . The recoil line KG_M is the same as the internal permeance of the PM, i.e.

$$G_M = \mu_{rec} \frac{w_M l_M}{h_M} = \mu_{rec} \frac{S_M}{h_M} \quad (3.19)$$

The point A is the intersection of the recoil line KG_M and the straight line OG_A representing the leakage permeance of the PM with pole shoes, i.e.

$$G_A = \frac{\Phi_A}{\mathcal{F}_A}, \quad \tan \alpha_A = G_A \frac{\mathcal{F}_c}{\Phi_r} \quad (3.20)$$

The energy produced by the PM in the external space decreases as compared with the previous case, i.e. $w_A = B_A H_A / 2$.

The next stage is to place the PM in an external ferromagnetic circuit as shown in Fig. 3.9c. The resultant permeance of this system is

$$G_P = \frac{\Phi_P}{\mathcal{F}_P}, \quad \tan \alpha_P = G_P \frac{\mathcal{F}_c}{\Phi_r} \quad (3.21)$$

which meets the condition $G_P > G_A > G_{ext}$. For an external magnetic circuit without any electric circuit carrying the armature current, the magnetic state of the PM is characterized by the point P (Fig. 3.10), i.e. the intersection of the recoil line KG_M and the permeance line OG_P .

When the external magnetic circuit is furnished with an armature winding and when this winding is fed with a current which produces an MMF magnetizing the PM (Fig. 3.9d), the magnetic flux in the PM increases to the value Φ_N . The d -axis MMF $\mathcal{F}'_{ad}$ of the external (armature) field acting directly on the PM corresponds to Φ_N . The magnetic state of the PM is described by the point N located on the recoil line on the right-hand side of the origin of the coordinate system. To obtain this point it is necessary to lay off the distance $O\mathcal{F}'_{ad}$ and to draw a line G_P from the point $\mathcal{F}'_{ad}$ inclined by the angle α_P to the F -axis. The intersection of the recoil line and the permeance line G_P gives the point N . If the exciting current in the external armature winding is increased further, the point N will move further along the recoil line to the right, up to the saturation of the PM.

When the excitation current is reversed, the external armature magnetic field will demagnetize the PM. In this case it is necessary to lay off the distance $O\mathcal{F}'_{ad}$ from the origin of the coordinate system to the left (Fig. 3.10). The line G_P drawn from the point $\mathcal{F}'_{ad}$ with the slope α_P intersects the demagnetization curve at the point K' . This point can be above or below the point K (for the PM alone in the open space). The point K' is the origin of a new recoil line $K'G'_M$. Now if the armature exciting current decreases, the operating point will move along the new recoil line $K'G'_M$ to the right. If the armature current drops down to zero, the operating point takes the position P' (intersection of the new recoil line $K'G'_M$ with the permeance line G_P drawn from the origin of the coordinate system).

On the basis of Fig. 3.10 the energies $w_{P'} = B_{P'}H_{P'}/2$, $w_P = B_P H_P/2$, and $w_{P'} < w_P$. The location of the origin of the recoil line, as well as the location of the operating point, determine the level of utilization of the energy produced by the PM. A *PM behaves differently than a d.c. electromagnet*: the energy of a PM is not constant if the permeance and exciting current of the external armature changes.

The location of the origin of the recoil line is determined by the minimum value of the permeance of the external magnetic circuit or the demagnetization action of the external field.

To obtain the properties of PMs more independent of the external magnetic fields, PMs need to be stabilized. *Stabilization* means the PM is demagnetized up to a value which is slightly higher than the most dangerous demagnetization field during the operation of a system where the PM is installed. In magnetic

circuits with stabilized PMs the operating point describing the state of the PM is located on the recoil line.

More details about how to find the operating point of a PM graphically and analytically can be found in [106].

3.2.4 Permeances for Main and Leakage Fluxes

Permeances of air gaps and permeances for leakage fluxes can be found analytically by dividing the magnetic field into simple solids. Permeances of simple solids shown in Fig. 3.11 can be found using the following formulae:

(a) Rectangular prism (Fig. 3.11a)

$$G = \mu_0 \frac{w_M l_M}{g} \quad \text{H} \quad (3.22)$$

(b) Cylinder (Fig. 3.11b)

$$G = \mu_0 \frac{\pi d_M^2}{4g} \quad (3.23)$$

(c) Half-cylinder (Fig. 3.11c)

$$G = 0.26 \mu_0 l_M \quad (3.24)$$

where the average air gap $g_{av} = 1.22g$ and the average surface $S_{av} = 0.322gl_M$ [15]

(d) One-quarter of a cylinder (Fig. 3.11d)

$$G = 0.52 \mu_0 l_M \quad (3.25)$$

(e) Half-ring (Fig. 3.11e)

$$G = \mu_0 \frac{2l_M}{\pi(g/w_M + 1)} \quad (3.26)$$

For $g < 3w_M$,

$$G = \mu_0 \frac{l_M}{\pi} \ln \left(1 + \frac{2w_M}{g} \right) \quad (3.27)$$

(f) One-quarter of a ring (Fig. 3.11f)

$$G = \mu_0 \frac{2l_M}{\pi(g/c + 0.5)} \quad (3.28)$$

For $g < 3c$,

$$G = \mu_0 \frac{2l_M}{\pi} \ln \left(1 + \frac{c}{g} \right) \quad (3.29)$$

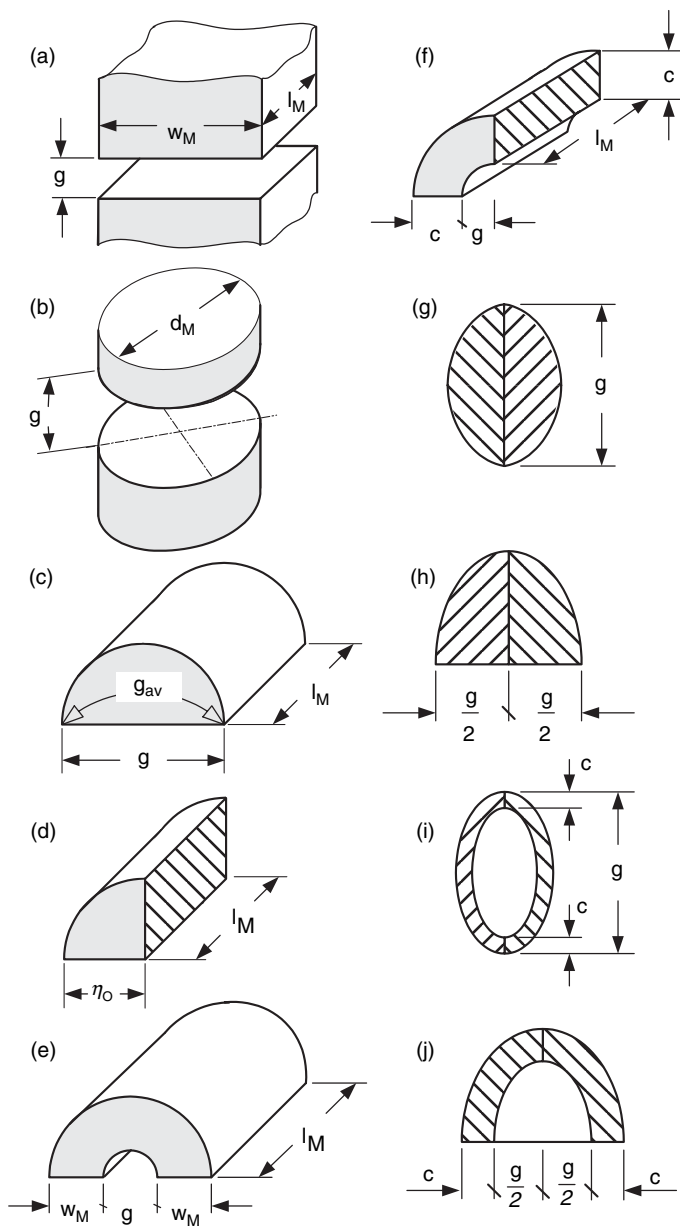


Fig. 3.11. Simple solids: (a) rectangular prism, (b) cylinder, (c) half-cylinder, (d) one-quarter of a cylinder, (e) half-ring, (f) one-quarter of a ring, (g) one-quarter of a sphere, (h) one-eighth of a sphere, (i) one-quarter of a shell, (j) one-eighth of a shell.

(g) One-quarter of a sphere (Fig. 3.11g)

$$G = 0.077\mu_0 g \quad (3.30)$$

(h) One-eighth of a sphere (Fig. 3.11h)

$$G = 0.308\mu_0 g \quad (3.31)$$

(i) One-quarter of a shell (Fig. 3.11i)

$$G = \mu_0 \frac{c}{4} \quad (3.32)$$

(j) One-eighth of a shell (Fig. 3.11j)

$$G = \mu_0 \frac{c}{2} \quad (3.33)$$

Fig. 3.12 shows a model of a flat electrical machine with smooth armature core (without slots) and surface PM excitation system. The armature is of steel laminations. The PMs are fixed to the mild steel yoke.

The pole pitch is τ , the width of each PM is w_M , and its length is l_M . In an AFPM machine

$$l_M = 0.5(D_{out} - D_{in}) \quad (3.34)$$

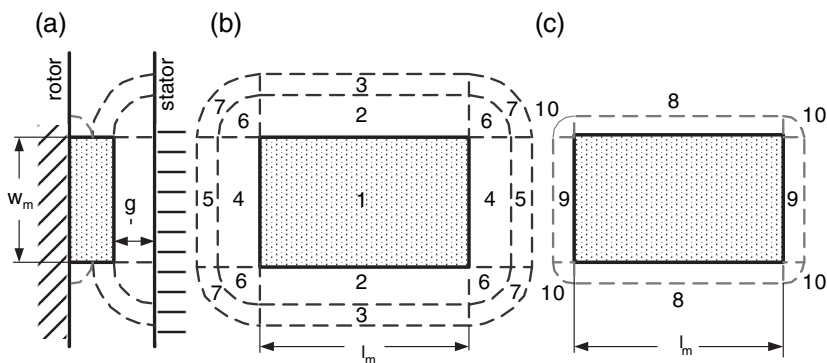


Fig. 3.12. Electrical machine with flat slotless armature and flat PM excitation system — division of the space occupied by the magnetic field into simple solids: (a) longitudinal section, (b) air gap field, (c) leakage field (between the PM and steel yoke).

The space between the pole face and the armature core is divided into a prism (1), four quarters of cylinders (2 and 4), four quarters of rings (3 and 5), four pieces of 1/8 of a sphere (6) and four pieces of 1/8 of a shell (7). Formulae for the permeance calculations have been found on the assumption that the

permeance of a solid is equal to its average cross section area to the average length of the flux line. If we ignore the fringing flux, the permeance of a rectangular air gap per pole (prism 1 in Fig. 3.12) is

$$G_{g1} = \mu_0 \frac{w_M l_M}{g'} \quad (3.35)$$

The equivalent air gap g' is only equal to the nonmagnetic gap (mechanical clearance) g for a slotless and unsaturated armature. To take into account slots (if they exist) and magnetic saturation, the air gap g is increased to $g' = g k_C k_{sat}$, where $k_C > 1$ is Carter's coefficient taking into account slots (1.2), and $k_{sat} > 1$ is the saturation factor of the magnetic circuit defined as the ratio of the MMF per pole pair to the air gap magnetic voltage drop (MVD) taken twice [106].

To take into account the fringing flux it is necessary to include all paths for the magnetic flux coming from the excitation system through the air gap to the armature system (Fig. 3.12), i.e.

$$G_g = G_{g1} + 2(G_{g2} + G_{g3} + G_{g4} + G_{g5}) + 4(G_{g6} + G_{g7}) \quad (3.36)$$

where G_{g1} is the air gap permeance according to eqn (3.35) and G_{g2} to G_{g7} are the air gap permeances for fringing fluxes. The permeances G_{g2} to G_{g5} can be found using eqns (3.25), (3.28), (3.31) and (3.32).

In a similar way the resultant permeance for the leakage flux of the PM can be found, i.e.

$$G_{lM} = 2(G_{l8} + G_{l9}) + 4G_{l10} \quad (3.37)$$

where G_{l8} , G_{l9} (one-quarter of a cylinder) and G_{l10} (one-eighth of a sphere) are the permeances for leakage fluxes between the PM and rotor yoke according to Fig. 3.12c — eqns (3.25) and (3.31).

3.2.5 Calculation of Magnetic Circuits With PMs

The *equivalent magnetic circuit of a PM system* with armature is shown in Fig. 3.13. The reluctances of pole shoes (mild steel) and armature stack (electrotechnical laminated steel) are much smaller than those of the air gap and PM and have been neglected. The “open circuit” MMF acting along the internal magnet permeance $G_M = 1/\mathfrak{R}_{\mu M}$ is $\mathcal{F}_{M0} = H_{M0} h_M$, the d -axis armature reaction MMF is $\mathcal{F}_{ad}$, the total magnetic flux of the PM is Φ_M , the leakage flux of the PM is Φ_{lM} , the useful air gap magnetic flux is Φ_g , the leakage flux of the external armature system is Φ_{la} , the flux in the d -axis produced by the armature is Φ_{ad} (demagnetizing or magnetizing), the reluctance for the PM leakage flux is $\mathfrak{R}_{\mu lM} = 1/G_{lM}$, the air gap reluctance is $\mathfrak{R}_{\mu g} = 1/G_g$, and the external armature leakage reactance is $\mathfrak{R}_{\mu la} = 1/G_{gla}$. The following Kirchhoff's equations can be written on the basis of the equivalent circuit shown in Fig. 3.13

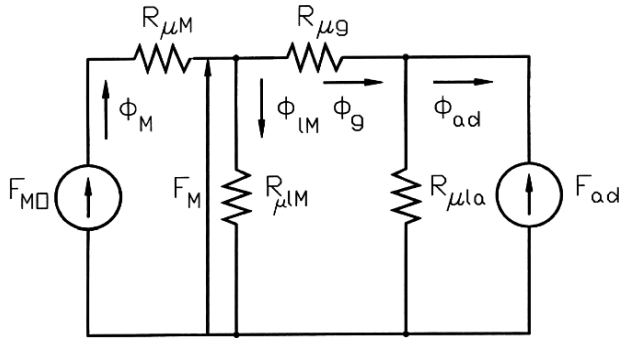


Fig. 3.13. Equivalent circuit (in the d -axis) of a PM system with armature.

$$\Phi_M = \Phi_{lM} + \Phi_g$$

$$\Phi_{la} = \frac{\pm \mathcal{F}_{ad}}{\mathfrak{R}_{\mu la}}$$

$$\mathcal{F}_{M0} - \Phi_M \mathfrak{R}_{\mu M} - \Phi_{lM} \mathfrak{R}_{\mu lM} = 0$$

$$\Phi_{lM} \mathfrak{R}_{lM} - \Phi_g \mathfrak{R}_{\mu g} \mp \mathcal{F}_{ad} = 0$$

The solution to the above equation system yields the air gap magnetic flux:

$$\Phi_g = \left[\mathcal{F}_{M0} \mp \mathcal{F}_{ad} \frac{G_g}{G_g + G_{lM}} \frac{(G_g + G_{lM})(G_M + G_{lM})}{G_g G_M} \right] \frac{G_g G_M}{G_g + G_{lM} + G_M}$$

or

$$\Phi_g = \left[\mathcal{F}_{M0} \mp \mathcal{F}'_{ad} \frac{G_t(G_M + G_{lM})}{G_g G_M} \right] \frac{G_g G_M}{G_t + G_M} \quad (3.38)$$

where the total resultant permeance G_t for the flux of the PM is

$$G_t = G_g + G_{lM} = \sigma_{lM} G_g \quad (3.39)$$

and the direct-axis armature MMF acting directly on the PM is

$$\mathcal{F}'_{ad} = \mathcal{F}_{ad} \frac{G_g}{G_g + G_{lM}} = \mathcal{F}_{ad} \left(1 + \frac{G_{lM}}{G_g} \right)^{-1} = \frac{\mathcal{F}_{ad}}{\sigma_{lM}} \quad (3.40)$$

The upper sign in eqn (3.38) is for the demagnetizing armature flux and the lower sign is for the magnetizing armature flux.

The coefficient of the PM leakage flux (3.9) can also be expressed in terms of permeances, i.e.

$$\sigma_{lM} = 1 + \frac{\Phi_{lM}}{\Phi_g} = 1 + \frac{G_{lM}}{G_g} \quad (3.41)$$

3.2.6 Fabrication of Rotor Magnetic Circuits

Magnetic circuits of rotors of AFPM brushless machines provide the excitation flux and are designed as:

- PMs glued to a ferromagnetic ring or disc which serves as a backing magnetic circuit (yoke);
- PMs arranged into Halbach array without any ferromagnetic core.

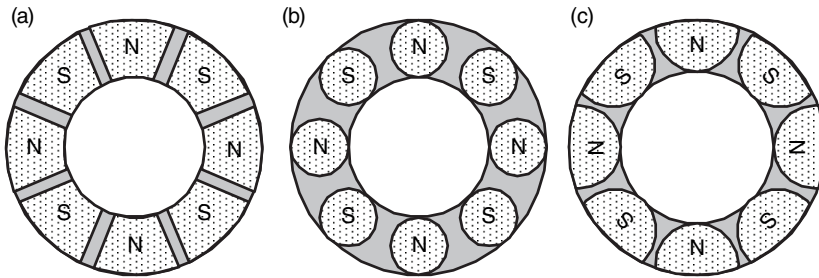


Fig. 3.14. Shapes of PM rotors of disc-type machines: (a) trapezoidal, (b) circular, (c) semicircular.

Shapes of PMs are usually trapezoidal, circular or semicircular (Fig. 3.14). The shape of PMs affects the distribution of the air gap magnetic field and contents of higher space harmonics. The output voltage quality (harmonics of the EMF) of AFPM generators depends on the PM geometry (circular, semicircular, trapezoidal) and distance between adjacent magnets [83].

Since the magnetic flux in the rotor magnetic circuit is stationary, mild steel (carbon steel) backing rings can be used. Rings can be cut from 4 to 6 mm mild steel sheets. Table 3.13 shows magnetization characteristics $B-H$ of a mild carbon steel and cast iron. Electrical conductivities of carbon steels are from 4.5×10^6 to 7.0×10^6 S/m at 20°C .

Halbach Array

Twin rotors of double-sided coreless AFPM machines (Fig. 1.4d) may use PMs arranged in *Halbach array* [114, 115, 116]. The key concept of Halbach array is that the magnetization vector of PMs should rotate as a function of

Table 3.13. Magnetization curves of solid ferromagnetic materials: 1 — carbon steel (0.27%C), 2 — cast iron

Magnetic flux density, B	Magnetic field intensity, H	
	Mild carbon steel 0.27% C	Cast iron
T	A/m	A/m
0.2	190	900
0.4	280	1600
0.6	320	3000
0.8	450	5150
1.0	900	9500
1.2	1500	18,000
1.4	3000	28,000
1.5	1500	
1.6	6600	
1.7	11,000	

distance along the array (Fig. 3.15) [114, 115, 116]. Halbach array has the following advantages:

- the fundamental field is stronger by a factor of 1.4 than in a conventional PM array, and thus the power efficiency of the machine is doubled;
- the array of PMs does not require any backing steel magnetic circuit and PMs can be bonded directly to a non-ferromagnetic supporting structure (aluminum, plastics);
- the magnetic field is more sinusoidal than that of a conventional PM array;
- Halbach array has very low back-side fields.

The peak value of the magnetic flux density at the active surface of Halbach array is

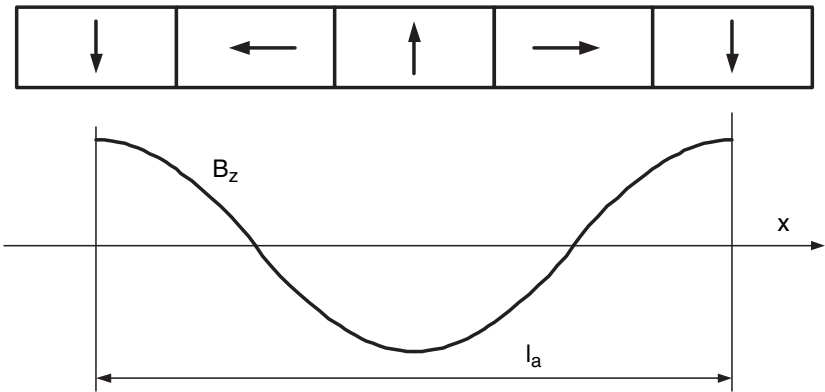


Fig. 3.15. Cartesian Halbach array.

$$B_{m0} = B_r [1 - \exp(-\beta h_M)] \frac{\sin(\pi/n_M)}{\pi/n_M} \quad (3.42)$$

where B_r is the remanent magnetic flux density of the magnet, $\beta = 2\pi/l_a$ - see also eqn (1.11), l_a is the spatial period (wavelength) of the array and n_M is the number of PM pieces per wavelength. For the array shown in Fig. 3.15 $n_M = 4$. For example, assuming $B_r = 1.25$ T, $h_M = 6$ mm, $l_a = 24$ mm, $n_m = 4$ (rectangular PMs), the peak magnetic flux density at the surface of Halbach array $B_{M0} = 0.891$ T.

The tangential B_x and normal B_z components of Halbach array at the distance z from the surface of PMs are

$$B_x(x, z) = B_{m0} \cos(\beta x) \exp(-\beta z) \quad (3.43)$$

$$B_z(x, z) = B_{m0} \sin(\beta x) \exp(-\beta z) \quad (3.44)$$

For a double-sided configuration of Halbach arrays, i.e. twin disc external rotors, the tangential and normal component of the magnetic flux density distribution in the space between discs are

$$B_x(x, z) = B_{m0} \frac{1}{\beta} \cos(\beta x) \frac{1}{\cosh(\beta t/2)} \sinh(\beta z) \quad (3.45)$$

$$B_z(x, z) = B_{m0} \sin(\beta x) \frac{1}{\cosh(\beta t/2)} \cosh(\beta z) \quad (3.46)$$

where B_{m0} is according to eqn (3.42) and t is magnet-to-magnet distance between two halves of the disc. The origin of the $0xyz$ coordinate system is as in Fig. 1.9.

3.3 Windings

3.3.1 Conductors

Stator (armature) windings of electric motors are made of solid *copper conductor wires* with round or rectangular cross sections.

The *electric conductivity* at 20°C of copper wires is $57 \times 10^6 \geq \sigma_{20} \geq 56 \times 10^6$ S/m. For aluminum wires $\sigma_{20} \approx 33 \times 10^6$ S/m. The electric conductivity is temperature dependent and for $\theta - 20^\circ \leq 150^\circ\text{C}$ can be expressed as

$$\sigma = \frac{\sigma_{20}}{1 + \alpha(\theta - 20^\circ)} \quad (3.47)$$

where α is the *temperature coefficient of electric resistance*. For copper wires $\alpha = 0.00393$ 1/°C and for aluminum wires $\alpha = 0.00403$ 1/°C. For $\vartheta - 20^\circ >$

150°C eqn (3.47) contains two temperature coefficients α and β of the electric resistance, i.e.

$$\sigma = \frac{\sigma_{20}}{1 + \alpha(\vartheta - 20^\circ) + \beta(\vartheta - 20^\circ)^2} \tag{3.48}$$

Table 3.14. Maximum temperature rise $\Delta\vartheta$ for armature windings of electrical machines according to IEC and NEMA (based on 40°C ambient temperature)

Rated power of machines, length of core and voltage	Insulation class				
	A °C	E °C	B °C	F °C	H °C
<u>IEC</u>					
a.c. machines < 5000 kVA (resistance method)	60	75	80	100	125
<u>IEC</u>					
a.c. machines $\geq$ 5000 kVA or length of core $\geq$ 1 m (embedded detector method)	60	70	80	100	125
<u>NEMA</u>					
a.c. machines $\leq$ 1500 hp (embedded detector method)	70	–	90	115	140
<u>NEMA</u>					
a.c. machines > 1500 hp and $\leq$ 7 kV (embedded detector method)	65	–	85	110	135

The *maximum temperature rise* for the windings of electrical machines is determined by the temperature limits of insulating materials. The maximum temperature rise in Table 3.14 assumes that the temperature of the cooling medium $\vartheta_c \leq 40^\circ\text{C}$. The maximum temperature of windings is actually

$$\vartheta_{max} = \vartheta_c + \Delta\vartheta \tag{3.49}$$

where $\Delta\vartheta$ is the maximum allowable temperature rise according to Table 3.14. A polyester-imide and polyamide-imide coat can provide an operating temperature of 200°C. The highest operating temperatures (over 600°C) can be achieved using *nickel clad copper* or *palladium-silver* conductor wires and ceramic insulation.

3.3.2 Fabrication of Slotted Windings

Stator windings are usually made of insulated copper conductors. The cross section of conductors can be circular or rectangular. For large AFPM machines a direct water cooling system and consequently hollow conductors can be considered.

It is difficult to make and form stator coils if the round conductor is thicker than 1.5 mm. If the current density is too high, parallel conductor wires of smaller diameter are recommended rather than one thicker wire. Stator windings can also have parallel current paths.

The armature windings can be either *single-layer* or *double layer* (Section 2.2).

After coils are wound, they must be secured in place, somehow, so as to avoid conductor movement. Two standard methods are used to secure the conductors of electrical machines in place:

- *dipping* the whole component into a varnish-like material, and then baking off its solvent,
- *trickle impregnation* method, which uses heat to cure a catalyzed resin which is dripped onto the component.

Polyester, epoxy or silicon resins are used most often as impregnating materials for treatment of stator windings. Silicon resins of high thermal endurance are able to withstand $\vartheta_{max} > 225^{\circ}\text{C}$.

Recently, a new method of conductor securing that does not require any additional material, and uses very low energy input, has emerged [178]. The solid conductor wire (usually copper) is coated with a *heat and/or solvent activated adhesive*. The adhesive which is usually a polyvinyl butyral, utilizes a low temperature thermoplastic resin [178]. This means that the bonded adhesive can come apart after a certain minimum temperature is reached, or it again comes in contact with the solvent. Normally this temperature is much lower than the thermal rating of the base insulation layer. The adhesive is activated by either passing the wire through a solvent while winding or heating the finished coil as a result of passing electric current through it.

The conductor wire with a heat activated adhesive overcoat costs more than the same class of non-bondable conductor. However, a less than two second current pulse is required to bond the heat activated adhesive layer and bonding machinery costs about half as much as trickle impregnation machinery [178].

3.3.3 Fabrication of Coreless Windings

Stator coreless windings of AFPM machines are fabricated as uniformly distributed coils on a disc-type cylindrical supporting structure (hub) made of nonmagnetic and nonconductive material. There are two types of windings:

- (a) winding comprised of multi-turn coils wound with turns of insulated conductor of round or rectangular cross section;
- (b) printed winding also called *film coil winding*.

Coils are connected in groups to form the phase windings typically connected in star or delta. Coils or groups of coils of the same phase can be connected in parallel to form parallel paths.

To assemble the winding of the same coils and obtain high density packing, coils should be formed with *offsetting bends*, as shown in Fig. 3.16. The space between two sides of the same coil is filled with coil sides from each of the adjacent coils.

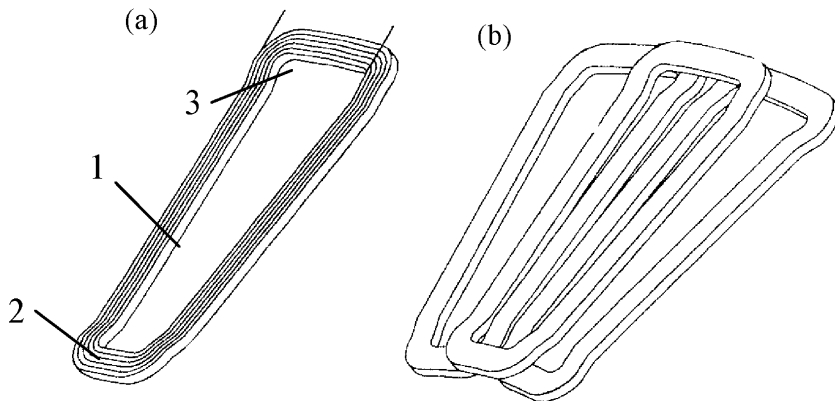


Fig. 3.16. Disc-type coreless winding assembled of coils of the same shape according to U.S. Patent No. 5 744 896 [P83]: (a) single coil; (b) three adjacent coils. 1 — coil side, 2 — inner offsetting bend, 3 — outer offsetting bend.

Coils can be placed in a slotted structure of the mould (Fig. 3.17). With all the coils in position, the winding (often with a supporting structure or hub) is moulded into a mixture of epoxy resin and hardener and then cured in a heated oven. Because of the difficulty of releasing the cured stator from the slotted structure of the mould (Fig. 3.17a), each spacing block that forms a guide slot consists of several removable pins of different size (Fig. 3.17b).

For very small AFPM machines and micromachines printed circuit coreless windings allow for automation of production. Printed circuit windings for AFPM brushless machines fabricated in a similar way as printed circuit boards have not been commercialized due to poor performance. A better performance has been achieved using film coil windings made through the same process as flexible printed circuits [93]. The coil pattern is formed by etching two copper films that are then attached to both sides of a board made of insulating materials (Fig. 3.18). Compact coil patterns are made possible by connecting both sides of coil patterns through holes [93].

Numerical Example 3.1

A simple stationary magnetic circuit is shown in Fig. 3.19. There are two Vacomax 240 HR SmCo PMs (Table 3.11) with $B_r = 1.10$ T, $H_c = 680$ kA/m, temperature coefficients $\alpha_B = -0.03\%/^{\circ}\text{C}$ and $\alpha_H = -0.15\%/^{\circ}\text{C}$ at

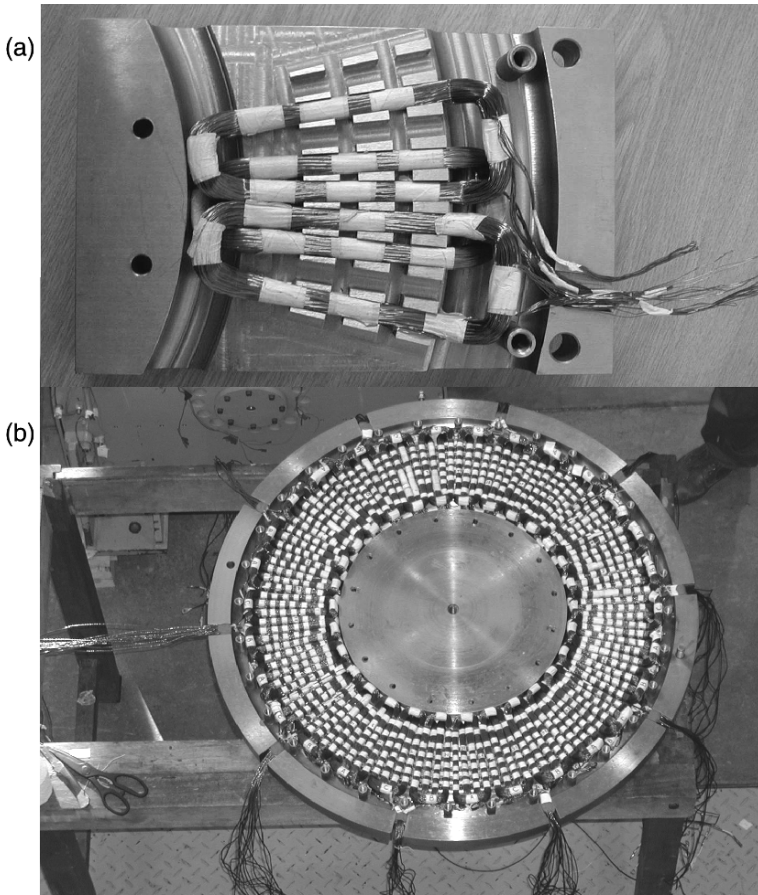


Fig. 3.17. Moulds for positioning the coils: (a) mould with guide slots; (b) mould with guide pins.



Fig. 3.18. Film coils for AFPM micromotors. Courtesy of *EMbest*, Seoul, Korea.

$20 \leq \vartheta_{PM} \leq 100^\circ\text{C}$. The height of the PM per pole is $h_M = 6$ mm and the air gap thickness $g = 1$ mm. The U-shaped and I-shaped (top) ferromagnetic cores are made of a laminated electrotechnical steel. The width of the magnets and cores is 17 mm. Calculate the air gap magnetic flux density, air gap magnetic field strength, the useful energy of PMs and normal attractive force per two poles at: (a) $\vartheta_{PM} = 20^\circ\text{C}$ and (b) $\vartheta_{PM} = 100^\circ\text{C}$. The MVD in the laminated core, leakage and fringing magnetic flux can be neglected.

Solution:

(a) Magnet temperature $\vartheta_{PM} = 20^\circ\text{C}$

The relative recoil magnetic permeability according to eqn (3.4) for a straight line demagnetization curve is

$$\mu_{rrec} = \frac{1}{\mu_0} \frac{\Delta B}{\Delta H} = \frac{1}{0.4\pi \times 10^{-6}} \frac{1.10 - 0}{680,000 - 0} \approx 1.29$$

The air gap magnetic flux density according to eqn (3.13) is

$$B_g \approx \frac{1.10}{1 + 1.29 \times 1.0/6.0} = 0.906 \text{ T}$$

The air gap magnetic field strength according to eqn (3.14) in which $H = H_g$ and $B = B_g$ is

$$H_g = H_c \left(1 - \frac{B_g}{B_r} \right) = 680 \times 10^3 \left(1 - \frac{1.064}{1.10} \right) = 120.12 \times 10^3 \text{ A/m}$$

The useful energy per magnet volume according to eqn (3.5) is

$$w_g = \frac{B_g H_g}{2} = \frac{1.064 \times 120,120}{2} = 54\,395.8 \text{ J/m}^3$$

The useful energy per pole pair is

$$W_g = w_g V_M = 54,395.8 (2 \times 6 \times 15 \times 17 \times 10^{-9}) = 0.166 \text{ J}$$

The normal attractive force per 2 poles is

$$F = \frac{B_g^2}{2\mu_0} (2S_M) = \frac{1.064^2}{0.4\pi \times 10^{-6}} (15 \times 17 \times 10^{-6}) = 166.5 \text{ N}$$

(b) Magnet temperature $\vartheta_{PM} = 100^\circ\text{C}$

The remanent magnetic flux density and coercivity at 100°C according to eqns (3.2) and (3.3) is

$$B_r = 1.10 \left[1 + \frac{-0.03}{100} (100 - 20) \right] = 1.074 \text{ T}$$

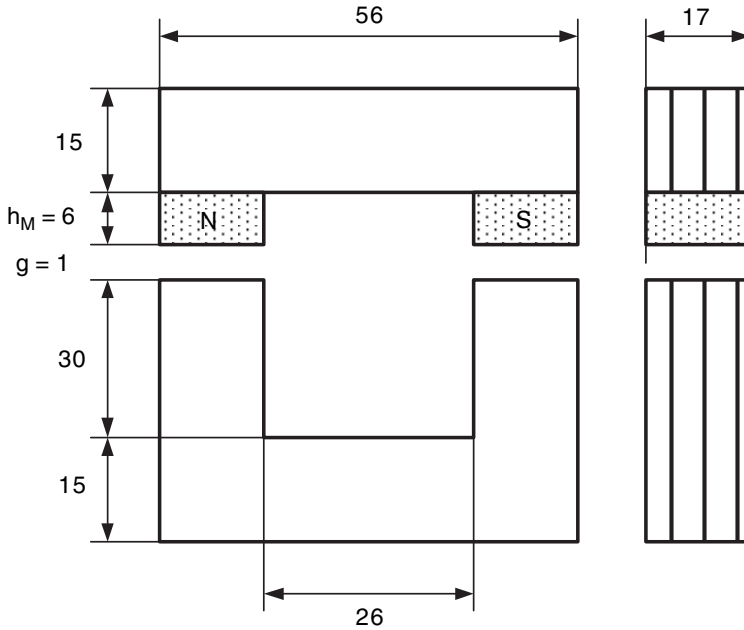


Fig. 3.19. A simple stationary magnetic circuit with PMs and air gap. *Numerical example 3.1.*

$$H_c = 680 \times 10^3 \left[1 + \frac{-0.15}{100} (100 - 20) \right] = 598.4 \times 10^3 \text{ A/m}$$

At $\vartheta_{PM} = 100^\circ\text{C}$ the demagnetization curve is nonlinear. Its linear part is only between 0.5 T and B_r parallel to the demagnetization curve at 20°C . Thus, the relative recoil magnetic permeability μ_{rrec} at 100°C is approximately the same as that at room temperature.

The air gap magnetic flux density according to (3.13) is

$$B_g \approx \frac{1.074}{1 + 1.29 \times 1.0/6.0} = 0.884 \text{ T}$$

The air gap magnetic field strength at 100°C according to eqn (3.14)

$$H_g = 598.4 \times 10^3 \left(1 - \frac{0.884}{1.074} \right) = 105.7 \times 10^3 \text{ A/m}$$

The useful energy per magnet volume is

$$w_g \approx \frac{0.884 \times 105.7 \times 10^3}{2} = 46\,719.5 \text{ J/m}^3$$

The useful energy per pole pair is

$$W_g = 46\,719.5(2 \times 6 \times 15 \times 17 \times 10^{-9}) = 0.143 \text{ J}$$

The normal attractive force per 2 poles is

$$F = \frac{0.884^2}{0.4\pi \times 10^{-6}}(15 \times 17 \times 10^{-6}) = 158.6 \text{ N}$$

Numerical Example 3.2

A single-sided, 8-pole AFPM machine with slotted ferromagnetic stator has the PM outer diameter $D_{out} = 0.22 \text{ m}$ and inner diameter $D_{in} = 0.12 \text{ m}$. The air gap including the effect of slotting (Carter coefficient) is $g = 1.9 \text{ mm}$. Trapezoidal sintered NdFeB magnets have $B_r = 1.15 \text{ T}$ and $H_c = 900 \text{ kA/m}$ at 20°C . The temperature coefficient for B_r is $\alpha_B = -0.15 \% / ^\circ\text{C}$ and the temperature coefficient for H_c is $\alpha_H = -0.64 \% / ^\circ\text{C}$. The coefficient of PM leakage flux is $\sigma_{lM} = 1.15$ and pole width-to-pole pitch coefficient is $\alpha_i = 0.72$.

Find PM dimensions to obtain the air gap magnetic flux density $B_g = 0.64 \text{ T}$ at no-load and temperature $\vartheta_{PM} = 80^\circ\text{C}$. Sketch operating diagrams in the B - H and Φ - MMF coordinate system at no load. Assume that the magnetic circuit is unsaturated.

Solution:

The remanence and coercivity at $\vartheta_{PM} = 80^\circ\text{C}$ according to eqns (3.2) and (3.3) is

$$B_r = 1.15 \left[1 + \frac{-0.15}{100}(\vartheta_{PM} - 20) \right] = 1.046 \text{ T}$$

$$H_c = 900 \left[1 + \frac{-0.64}{100}(\vartheta_{PM} - 20) \right] = 554.4 \text{ kA/m}$$

Approximation of demagnetization curves according to eqn (3.14) at 20°C and 80°C respectively

$$B_{20}(H) = 1.15 \left(1 - \frac{H}{900\,000} \right) \quad B(H) = 1.046 \left(1 - \frac{H}{554\,400} \right)$$

The relative recoil magnetic permeability according to eqn (3.4) is

$$\mu_{rec} = \frac{1.046}{0.4\pi \times 10^{-6} \times 554\,400} = 1.5$$

The axial height of the PM per pole according to eqn (3.13) is

$$h_M = \mu_{rec} \frac{\sigma_{lM} B_g}{B_r - \sigma_{lM} B_g} g = 1.5 \frac{1.15 \times 0.64}{1.046 - 1.15 \times 0.64} 0.0019 = 0.0068 \text{ m}$$

The equivalent air gap is

$$g_{eq} = g + \frac{h_M}{\mu_{rec}} = 1.9 + \frac{6.8}{1.5} = 6.4 \text{ mm}$$

The average diameter, pole pitch (1.14), length (3.34) and width of the magnet are respectively

$$D = 0.5(0.22 + 0.12) = 0.17 \text{ m} \quad \tau = \frac{\pi D}{2p} = \frac{\pi 0.17}{8} = 0.0668 \text{ m}$$

$$l_M = 0.5(0.22 - 0.12) = 0.05 \text{ m} \quad w_M = \alpha_i \tau = 0.72 \times 0.0668 = 0.048 \text{ m}$$

The permeance of the air gap according to eqn (2.30) is

$$\begin{aligned} G_g &= \mu_0 \frac{1}{g} \frac{\alpha_i \pi}{8p} (D_{out}^2 - D_{in}^2) \\ &= 0.4\pi 10^{-6} \frac{1}{0.0019} \frac{0.72\pi}{8 \times 4} (0.22^2 - 0.12^2) = 1.59 \times 10^{-6} \text{ H} \end{aligned}$$

or

$$G_g = \mu_0 \frac{w_M l_M}{g} = 0.4 \times \pi 10^{-6} \frac{0.048 \times 0.05}{0.0019} = 1.59 \times 10^{-6} \text{ H}$$

The total permeance for magnetic flux including leakage permeances is

$$G_t = \sigma_{lm} G_g = 1.15 \times 1.59 \times 10^{-6} = 1.828 \times 10^{-6} \text{ H}$$

Approximation of the total permeance (air gap and leakage) line can be expressed as a linear function of H

$$\lambda_t(H) = G_t \frac{h_M}{w_M l_M} H = 1.828 \times 10^{-6} \frac{0.0068}{0.048 \times 0.05} H = 5.146 \times 10^{-6} H$$

The magnetic field intensity corresponding to the operating point of the magnet is calculated as

$$\begin{aligned} H_M &= \frac{B_r}{G_t h_M / (w_M l_M) + B_r / H_c} \\ &= \frac{1.046}{5.146 \times 10^{-6} + 1.046 / 554,400} = 148\,795 \text{ A/m} \end{aligned}$$

The air gap magnetic flux density as obtained from the operating diagram (Fig. 3.20a) is

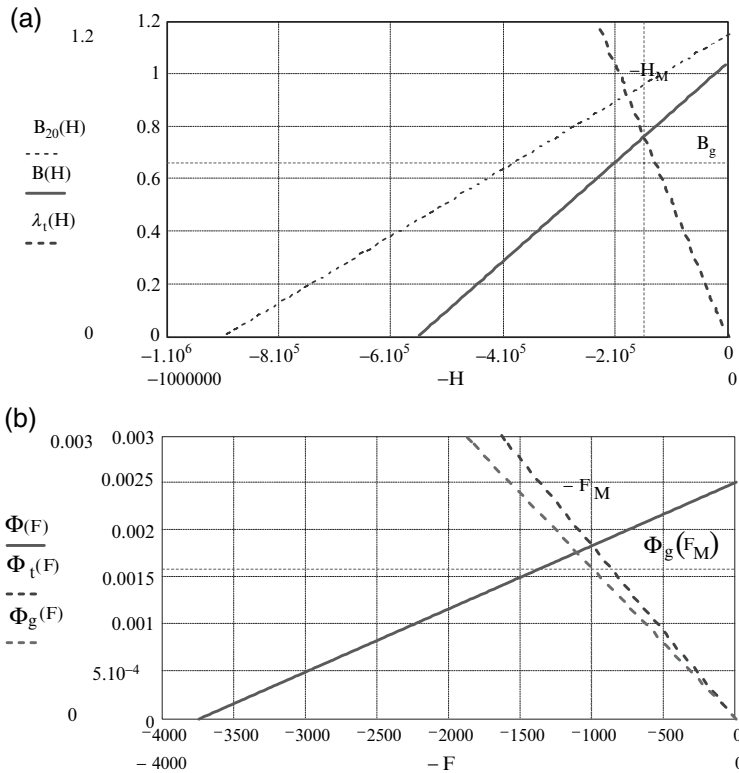


Fig. 3.20. Operating diagram of PM at no load: (a) in $B-H$ coordinate system; (b) in $\Phi-MMF$ coordinate system. Numerical example 3.2.

$$B_g = G_g \frac{h_M}{w_M l_M} H_M = 1.59 \times 10^{-6} \frac{0.0068}{0.048 \times 0.05} 148\,795 = 0.666 \text{ T}$$

To plot the operating diagram in the $\Phi-MMF$ coordinate system, eqns (3.16) and (3.18) are used to find the magnetic flux Φ_r corresponding to B_r and the MMF $\mathcal{F}_c$ per pole corresponding to H_c , i.e.

$$\Phi_r = 1.046 \times 0.048 \times 0.05 = 0.00252 \text{ Wb}$$

$$\mathcal{F}_c = 554\,400 \times 0.0068 = 3750.6 \text{ A}$$

Approximation of the magnetic flux line is

$$\Phi(\mathcal{F}) = \Phi_r \left(1 - \frac{\mathcal{F}}{\mathcal{F}_c} \right) = 0.00252 \left(1 - \frac{\mathcal{F}}{3750.6} \right)$$

The MMF F_M corresponding to the operating point of the magnet in the $\Phi-MMF$ coordinate system (Fig. 3.20b) is

$$\mathcal{F}_M = \frac{\Phi_r}{G_t + \Phi_r/\mathcal{F}_c} = \frac{0.00252}{1.828 \times 10^{-6} + 0.00252/3750.6} = 1006.6 \text{ A}$$

The air gap flux line (Fig. 3.20b) is

$$\Phi_g(\mathcal{F}) = G_g \mathcal{F} = 1.59 \times 10^{-6} \mathcal{F}$$

The total flux line (Fig. 3.20b) is

$$\Phi_t(\mathcal{F}) = G_t \mathcal{F} = 1.828 \times 10^{-6} \mathcal{F}$$

The air gap magnetic flux corresponding to the operating point is $\Phi_g(F_M) = 0.0016 \text{ Wb}$. The air gap magnetic flux density is $B_g = 0.0016/(0.048 \times 0.05) = 0.666 \text{ T}$.

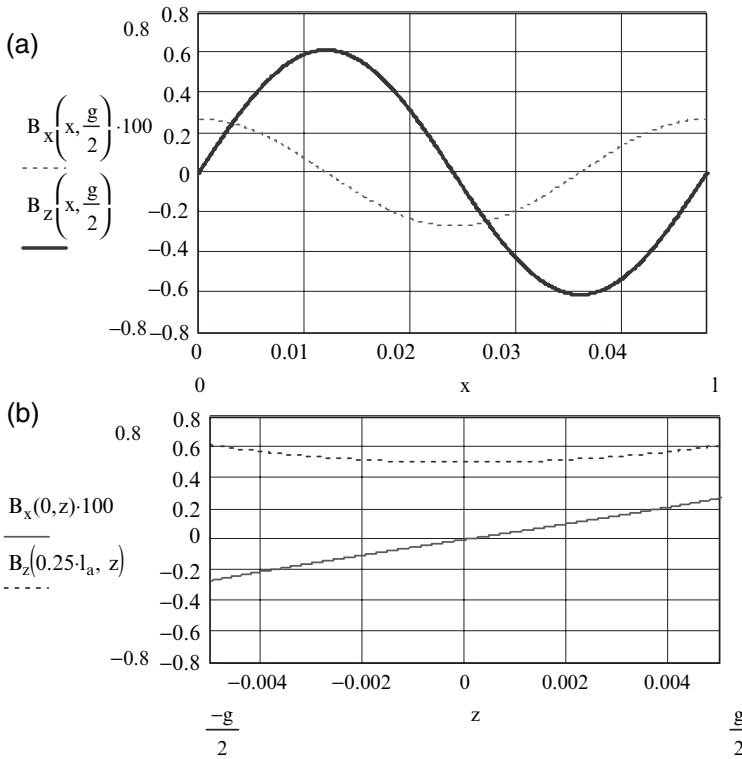


Fig. 3.21. Distribution of B_x and B_z components: (a) in the x direction; (b) in the z direction. Numerical example 3.3.

Numerical Example 3.3

The magnetic field in the air gap of an AFPM machine with coreless stator is excited by sintered NdFeB magnets arranged in Halbach array. The external twin rotors do not have any backing steel discs (Fig. 1.4d). The remanent magnetic flux density is $B_r = 1.25$ T, the height of the PM is $h_M = 6$ mm, the wavelength at the average diameter is $l_a = 2\tau = 48$ mm and the magnet-to-magnet distance is $t = 10$ mm.

Find the distribution of the magnetic flux density in the space between magnets for the 90-degree Halbach array, i.e. $n_M = 4$. Estimate, how the number n_M of PMs per wavelength affects the magnetic flux density at the active surface of PMs.

Solution

The peak value of the magnetic flux density at the active surface of Halbach array according to eqn (3.42) is

$$B_{m0} = 1.25[1 - \exp(-130.9 \times 0.006)] \frac{\sin(\pi/4)}{\pi/4} = 0.612 \text{ T}$$

where $\beta = 2 \times \pi/0.048 = 130.9 \text{ 1/m}$.

The distribution of the tangential component B_x in the space between magnets is described by eqn (3.45) and the distribution of the normal component B_z is described by eqn (3.46). Both components B_x and B_z are plotted in Fig. 3.21.

For the 90-degree Halbach array ($n_M = 4$) the peak value of the magnetic flux density at the active surface of PMs is $B_{m0} = 0.612$ T. Similarly, using eqn (3.42), the peak value B_{m0} can be calculated for other Halbach configurations. For 60-degree Halbach array ($n_M = 6$) $B_{m0} = 0.649$ T and for 45-degree Halbach array ($n_M = 8$) $B_{m0} = 0.663$ T. In general,

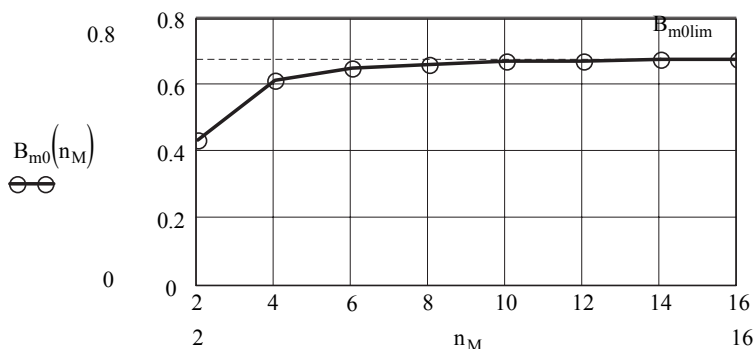


Fig. 3.22. Peak value of magnetic flux density B_m as a function of number n_M of PMs per wavelength. Numerical example 3.3.

$$\lim_{n_M \rightarrow \infty} B_{m0} = B_r [1 - \exp(1 - \beta h_M)] \frac{\sin(\pi/n_M)}{\pi/n_M}$$

$$= 1.25[1 - \exp(130.9 \times 0.006)] \times 1 = 0.68 \text{ T}$$

since $\lim_{x \rightarrow 0} \sin x/x = 1$. The peak value B_{m0} as a function of number n_M of PMs per wavelength is shown in Fig. 3.22.

AFPM Machines With Iron Cores

In Chapter 2, principles of operation, topologies and fundamental equations of a broad family of AFPM machines were discussed. In this chapter, the focus is on those types of AFPM machines that make use of the stator and rotor ferromagnetic cores. The AFPM machines with ferromagnetic cores are designed both as single-sided and double-sided. The stator core can be fabricated using either laminated steels or SMC powders. General equations given in Chapter 2 for the performance calculations will be developed further and adjusted to the construction of AFPM machines with ferromagnetic cores. Application of the FEM analysis to performance calculations is also emphasized.

4.1 Geometries

Single-sided AFPM machines with stator ferromagnetic cores have a single PM rotor disc opposite to a single stator unit consisting of a polyphase winding and ferromagnetic core (Fig. 2.1). The stator ferromagnetic cores can be slotted or slotless. The stator winding is always made of flat wound coils (Fig. 2.8). The PMs can be mounted on the surface of the rotor or embedded (buried) in the rotor disc. In the case of a slotless stator the magnets are almost always surface mounted, while in the case of a slotted stator with a small air gap between the rotor and stator core, the magnets can be either surface mounted on the disc (Fig. 2.1) or buried in the rotor disc (Fig. 2.6). Large axial magnetic forces on bearings are the main drawback of single-sided AFPM machines with ferromagnetic stator cores.

In *double-sided AFPM machines* with ideal mechanical and magnetic symmetry, the axial magnetic forces are balanced. Double-sided AFPM machines with stator ferromagnetic cores have either a single PM rotor disc with iron-cored stators on both sides of the discs (Figs 1.4c and 2.3) or outer PM rotor discs with iron-cored stator fixed in the middle (Figs 2.4, 2.5 and 2.6). As with single-sided AFPM machines the stator ferromagnetic cores can be slotted or slotless, and the rotor magnets can be surface mounted, embedded or buried

(Fig. 2.6). Again, in the case of a slotless stator with a large air gap between the rotor and stator core the magnets are almost always surface mounted. The stator windings of double-sided AFPM machines can be flat wound (slotted or slotless) as shown in Fig. 2.8 or toroidally wound (normally slotless) as shown in Fig. 2.9.

4.2 Commercial AFPM Machines With Stator Ferromagnetic Cores

An example of a commercial double-sided AFPM servo motor with ferromagnetic core is shown in Figs. 4.1 and 4.2. External stators have slotted ring-shaped cores made of nonoriented electrotechnical steel ribbon. The inner rotor does not have any ferromagnetic material. PMs are mounted on a nonmagnetic rotating disc.

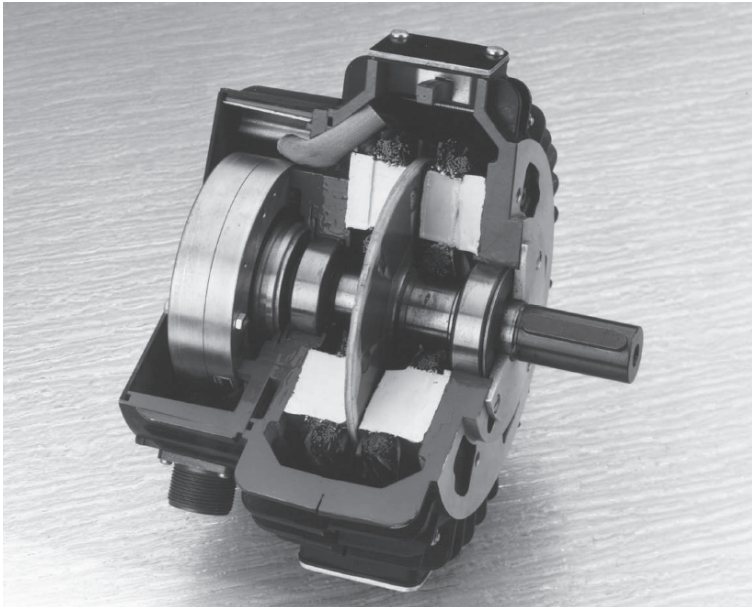


Fig. 4.1. Double-sided AFPM brushless servo motor with the stator ferromagnetic core and built-in brake. Courtesy of *Mavilor*, Barcelona, Spain.

Fig. 4.3 shows a double-sided AFPM synchronous generator with the stator core wound of amorphous alloy ribbon manufactured by *LE Incorporated*, Indianapolis, IN, U.S.A. The volume of *LE* AFPM generators is approximately 60% lower than that of classical synchronous generators of the same rating.

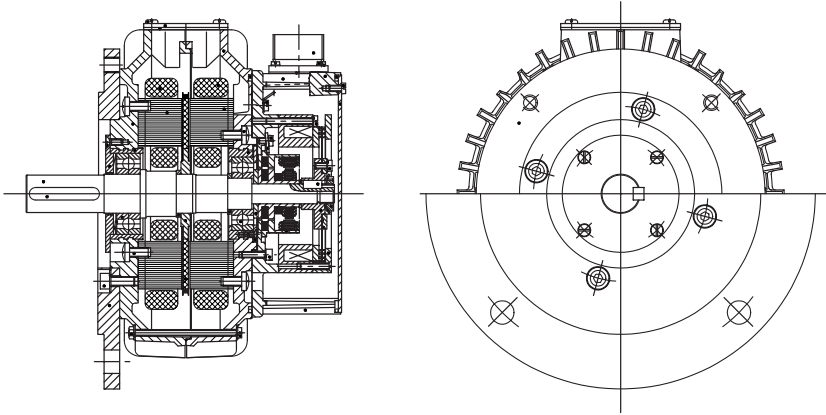


Fig. 4.2. Longitudinal section of the double-sided AFPM brushless servo motor shown in Fig. 4.1. Courtesy of *Mavilor*, Barcelona, Spain.



Fig. 4.3. *GenSmartTM* AFPM synchronous generator with amorphous alloy stator core. Courtesy of *LE Incorporated*, Indianapolis, IN, U.S.A.

4.3 Some Features of Iron-Cored AFPM Machines

Iron-cored AFPM machines are distinguished in two ways from coreless (air-cored) AFPM machines, namely: (i) iron-cored machines have core losses while coreless machines do not and (ii) the per unit values of the synchronous reactances of iron-cored machines are much higher than those of coreless machines.

The core losses are functions of amongst other things the frequency and density of the magnetic flux in the ferromagnetic core – see eqn (2.55). The frequency of the flux variation in the core is in turn determined by the speed

and number of pole pairs. As AFPM brushless machines tend to have a high number of poles (minimum $2p = 6$), the speeds of iron-cored AFPM machines are limited to keep the frequency, and hence the core losses of the machine, within limits; unless the machines are designed to have low flux densities in ferromagnetic cores. Flux frequencies are normally kept below, say, 100 Hz for laminated ferromagnetic cores. If higher frequencies are required, then silicon steel laminations with a thickness of less than 0.2 mm, amorphous alloy ribbons or SMC powders must be used.

The higher synchronous inductance of iron-cored AFPM machines affects negatively the voltage regulation of the machine in generator mode, which might be considered as a disadvantage. However, for solid-state converter-fed applications the higher synchronous inductance of iron-cored AFPM motors is an important advantage as it helps reduce the current-ripple due to converter switching. Thus the relatively low inductance of coreless AFPM machines can be a significant disadvantage in switched solid state converter applications.

4.4 Magnetic Flux Density Distribution in the Air Gap

AFPM brushless machines with ferromagnetic stator and rotor cores can produce strong magnetic flux density in the air gap with a minimum volume of PMs. The distribution of the normal component of the magnetic flux density in the x -direction (circumferential) excited by PMs without armature reaction at the radius corresponding to the average pole pitch τ can be described by the following equation

$$B_g(x) = \frac{1}{k_C} B_{zPM}(x) + B_{sl}(x) \quad (4.1)$$

where the PM excitation flux density for smooth stator core is

$$B_{zPM}(x) = \sum_{\nu=1}^{\infty} \frac{1}{\sigma_{lM}} B_g b_{\nu} \cos \left[\nu \left(\frac{\pi}{\tau} x - \frac{\pi}{2} \right) \right] \quad \nu = 1, 3, 5, \dots \quad (4.2)$$

and the magnetic flux density component due to stator slots is

$$B_{sl}(x) = \lambda_{sl}(x) \frac{g'}{\mu_0} B_{zPM}(x) \quad (4.3)$$

The peak values of the higher harmonics of the magnetic flux density distribution of eqn (4.2) are $B_{mg\nu} = (B_g / \sigma_{lM}) b_{\nu}$.

In the above equations (4.1), (4.2) and (4.3) k_C is Carter's coefficient according to eqn (1.2), b_{ν} is according to eqn (1.13) usually for $\alpha = 0$, τ is the average pole pitch according to eqn (1.14), g' is the equivalent air gap in the d -axis according to eqn (2.110), B_g is the flat-topped value of the magnetic flux density according to eqn (3.13), σ_{lM} is the coefficient of the PM leakage

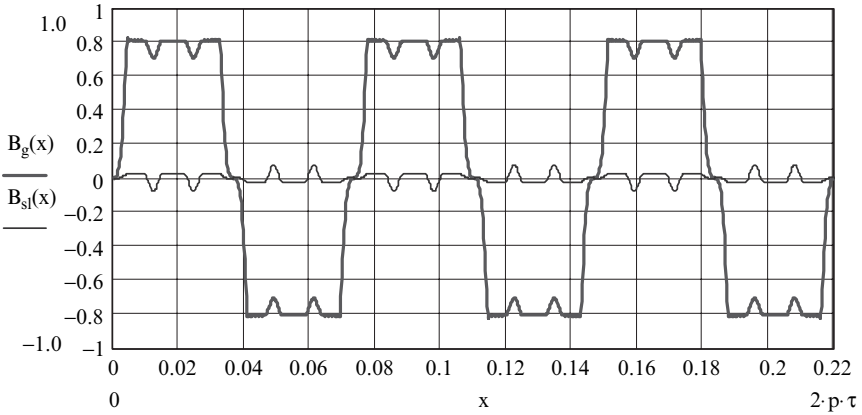


Fig. 4.4. Magnetic flux density distribution in the air gap of an AFPM machine with iron slotted core obtained on the basis of eqn (4.1) for $p = 3$, $s_1 = 18$, $\tau = 36.7$ mm, $\alpha_i = 0.78$, $b_p = \alpha_i \tau$, $B_r = 1.2$ T, $\mu_{rrec} = 1.061$, $g = 1.5$ mm, $b_{14} = 3.5$ mm, $h_M = 5$ mm, $\alpha = 0$ and $\sigma_{IM} = 1.15$.

flux according to eqn (3.9) and λ_{s1} is the relative slot leakage permeance given by

$$\lambda_{sl}(x) = - \sum_{\nu=1}^{\infty} a_{\nu} \cos \left(\nu \frac{2\pi}{t_1} x \right) \quad (4.4)$$

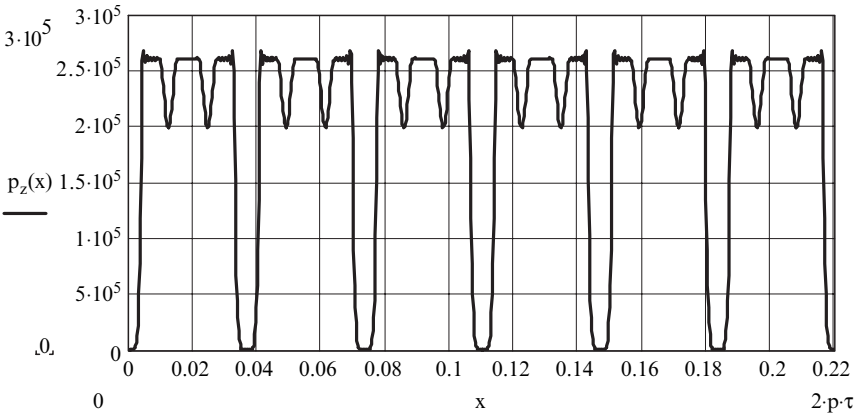


Fig. 4.5. Magnetic pressure distribution in an AFPM machine with iron slotted core obtained on the basis of eqn (4.7) for $p = 3$, $s_1 = 18$, $\tau = 36.7$ mm, $\alpha_i = 0.78$, $b_p = \alpha_i \tau$, $B_r = 1.2$ T, $\mu_{rrec} = 1.061$, $g = 1.5$ mm, $b_{14} = 3.5$ mm, $h_M = 5$ mm, $\alpha = 0$ and $\sigma_{IM} = 1.15$.

in which the average slot pitch $t_1 = 2p\tau/s_1$, s_1 is the number of stator slots and the amplitude of slot harmonics as derived by W. Weber [71, 121] is

$$a_\nu = \mu_0 \frac{\beta(\kappa)}{g'} \frac{1}{\nu} \frac{4}{\pi} \left(0.5 + \frac{\nu^2 \Gamma^2}{0.78 - 2\nu^2 \Gamma^2} \right) \sin(1.6\nu\pi\Gamma) \quad (4.5)$$

The remaining quantities $\beta(\kappa)$, κ and Γ are expressed by the following equations:

$$\beta(\kappa) = \frac{1}{2} \left(1 - \frac{1}{\sqrt{1 - \kappa^2}} \right), \quad \kappa = \frac{b_{14}}{g + h_M/\mu_{rrec}}, \quad \Gamma = \frac{b_{14}}{t_1} \quad (4.6)$$

The distribution of the magnetic flux density in the air gap obtained on the basis of eqn (4.1) is shown in Fig. 4.4. The magnetic pressure on the stator and rotor active surfaces can be found on the basis of the normal component (4.1) of the magnetic flux density distribution in the air gap, i.e.

$$p_z(x) = \frac{1}{2} \frac{B_g(x)^2}{\mu_0} \quad (4.7)$$

The distribution of the magnetic pressure (4.7) is visualized in Fig. 4.5. The magnetic pressure resolved into Fourier series yields harmonics of magnetic forces acting on the stator. It is necessary to calculate those harmonics, e.g. in the noise analysis of electromagnetic origin radiated by the AFPM machine.

4.5 Calculation of Reactances

4.5.1 Synchronous and Armature Reaction Reactances

Calculations of the stator current, other machine parameters and characteristics require knowledge of the *synchronous reactance* per phase which is expressed by eqns (2.79) and (2.80). The synchronous reactance X_{sd} and X_{sq} is the sum of the *armature reaction* (mutual) reactance X_{ad} , X_{aq} and the stator winding *leakage reactance* X_1 . The armature reaction reactances X_{ad} , X_{aq} and armature reaction inductances L_{ad} , L_{aq} can be calculated with the aid of equations given in Section 2.9 and Table 2.1. The analytical approach to calculation of the stator leakage reactance X_1 is discussed in Section 4.5.2.

4.5.2 Stator Leakage Reactance

The armature leakage reactance is the sum of the slot leakage reactance X_{1s} , the end connection leakage reactance X_{1e} , and the differential leakage reactance X_{1d} (for higher space harmonics), i.e.

$$X_1 = X_{1s} + X_{1e} + X_{1d}$$

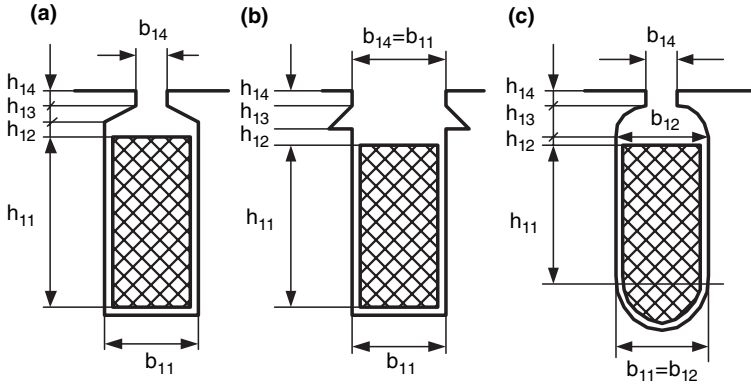


Fig. 4.6. Stator slots of AFPM brushless machines: (a) rectangular semi-open slot; (b) rectangular open slot; (c) oval semi-open slot.

$$= 4\pi f \mu_o \frac{L_i N_1^2}{pq_1} (\lambda_{1s} k_{1X} + \frac{l_{1in}}{L_i} \lambda_{1ein} + \frac{l_{1out}}{L_i} \lambda_{1eout} + \lambda_{1d}) \quad (4.8)$$

where N_1 is the number of turns per phase, k_{1X} is the skin-effect coefficient for leakage reactance, p is the number of pole pairs, $q_1 = s_1/(2pm_1)$ is the number of stator slots s_1 per pole per phase (2.2), l_{1in} is the length of the stator winding inner end connection, l_{1out} is the length of the stator winding outer end connection, λ_{1s} is the coefficient of the slot leakage permeance (slot-specific permeance), λ_{1ein} is the coefficient of the leakage permeance of the inner end connection, λ_{1eout} is the coefficient of the leakage permeance of the outer end connection and λ_{1d} is the coefficient of the differential leakage.

The coefficients of leakage permeances of the slots shown in Fig. 4.6 are:

- for a rectangular semi-open slot (Fig. 4.6a):

$$\lambda_{1s} = \frac{h_{11}}{3b_{11}} + \frac{h_{12}}{b_{11}} + \frac{2h_{13}}{b_{11} + b_{14}} + \frac{h_{14}}{b_{14}} \quad (4.9)$$

- for a rectangular open slot (Fig. 4.6b):

$$\lambda_{1s} \approx \frac{h_{11}}{3b_{11}} + \frac{h_{12} + h_{13} + h_{14}}{b_{11}} \quad (4.10)$$

- for an oval semi-open slot (Fig. 4.6c):

$$\lambda_{1s} \approx 0.1424 + \frac{h_{11}}{3b_{11}} + \frac{h_{12}}{b_{12}} + 0.5 \arcsin[\sqrt{1 - (b_{14}/b_{12})^2}] + \frac{h_{14}}{b_{14}} \quad (4.11)$$

The coefficients of leakage permeances for other shapes of slots than those shown in Fig. 4.6 are given, e.g. in [159]. The above specific-slot permeances

(4.9), (4.10) and (4.11) are for single-layer windings. To obtain the specific permeances of slots containing double-layer windings, it is necessary to multiply eqns (4.9), (4.10) and (4.11) by the factor

$$\frac{3\beta + 1}{4} \quad (4.12)$$

where β is according to eqn (2.7). Such an approach is justified if $2/3 \leq \beta \leq 1.0$.

The specific permeance of the end connection (overhang) is estimated on the basis of experiments. For double-layer, low-voltage, small and medium-power machines the specific permeances of the inner and outer end connections are:

- inner end connection

$$\lambda_{1ein} \approx 0.17q_1 \left(1 - \frac{2}{\pi} \frac{w_{cin}}{l_{1in}}\right) \quad (4.13)$$

- outer end connection

$$\lambda_{1eout} \approx 0.17q_1 \left(1 - \frac{2}{\pi} \frac{w_{cout}}{l_{1out}}\right) \quad (4.14)$$

where l_{1in} is the length of the inner end connection, l_{1out} is the length of the outer end connection, w_{cin} is the inner coil span, and w_{cout} is the outer coil span. The specific permeance of two end connections is a sum of permeances (4.13) and (4.14), i.e.

$$\lambda_{1e} = \lambda_{1ein} + \lambda_{1eout} = 0.34q_1 \left(1 - \frac{1}{\pi} \frac{w_{cin}l_{1out} + w_{cout}l_{1in}}{l_{1in}l_{1out}}\right) \quad (4.15)$$

For $w_{cin} = w_{cout} = w_c$ and $l_{1in} = l_{1out} = l_{1e}$ the specific permeance (see eqn (4.15)) of two end connections has the same form as that for a cylindrical stator, i.e.

$$\lambda_{1e} = 0.34q_1 \left(1 - \frac{2}{\pi} \frac{w_c}{l_{1e}}\right) \quad (4.16)$$

Putting $w_c/l_{1e} = 0.64$, eqn (4.16) also gives satisfactory results for single-layer windings, i.e. the specific permeance of two end connections becomes

$$\lambda_{1e} \approx 0.2q_1 \quad (4.17)$$

For double-layer, high-voltage windings the specific permeances of the inner and outer end connections are:

- inner end connection

$$\lambda_{1ein} \approx 0.21q_1 \left(1 - \frac{2}{\pi} \frac{w_{cin}}{l_{1in}}\right) k_{w1}^2 \quad (4.18)$$

- outer end connection

$$\lambda_{1eout} \approx 0.21q_1 \left(1 - \frac{2}{\pi} \frac{w_{cout}}{l_{1out}}\right) k_{w1}^2 \quad (4.19)$$

where the stator winding factor k_{w1} for the fundamental space harmonic $\nu = 1$ is according to eqn (2.10). The specific permeance of two end connections

$$\lambda_{1e} = \lambda_{1ein} + \lambda_{1eout} = 0.42q_1 \left(1 - \frac{1}{\pi} \frac{w_{cin}l_{1out} + w_{cout}l_{1in}}{l_{1in}l_{1out}}\right) k_{w1}^2 \quad (4.20)$$

In general, the following approximate relation may be used for most of the windings:

$$\lambda_{1e} \approx 0.3q_1 \quad (4.21)$$

The specific permeance of the differential leakage flux is

$$\lambda_{1d} = \frac{m_1 q_1 \tau k_{w1}^2}{\pi^2 g' k_{sat}} \tau_{d1} \quad (4.22)$$

where g' is the equivalent air gap according to eqns (2.110) or (2.111) and k_{sat} is the saturation factor of the magnetic circuit. The differential leakage factor τ_{d1} is

$$\tau_{d1} = \frac{1}{k_{w1}^2} \sum_{\nu > 1} \left(\frac{k_{w1\nu}}{\nu}\right)^2 \quad (4.23)$$

The curves of the differential leakage factor τ_{d1} are given in publications dealing with design of induction motors, e.g. [106, 121, 159].

In practical calculations it is convenient to use the following formula

$$\tau_{d1} = \frac{\pi^2 (10q_1^2 + 2)}{27} \left[\sin \left(\frac{30^\circ}{q_1} \right) \right]^2 - 1 \quad (4.24)$$

The specific permeance between the heads of teeth:

$$\lambda_{1t} \approx \frac{5g/b_{14}}{5 + 4g/b_{14}} \quad (4.25)$$

should be added to the differential specific permeance λ_{1d} of slotted stator windings.

4.6 Performance Characteristics

Specifications of *Mavilor* AFPM motors (Figs. 4.1 and 4.2) are shown in Table 4.1. Torque-speed characteristics of *Mavilor* AFPM motors are plotted in Fig. 4.7. AFPM servo motors MA-3 to MA-30 are designed with Class F insulation and IP-65 protection. Table 4.2 shows specifications of brakes for MA-3 to MA-30 servo motors.

Table 4.3 shows specifications of *GenSmart*TM AFPM synchronous generators with amorphous alloy stator cores manufactured by *LE Incorporated*, Indianapolis, IN, U.S.A. *GenSmart*TM AFPM synchronous generators can be used both as stationary and mobile generators.

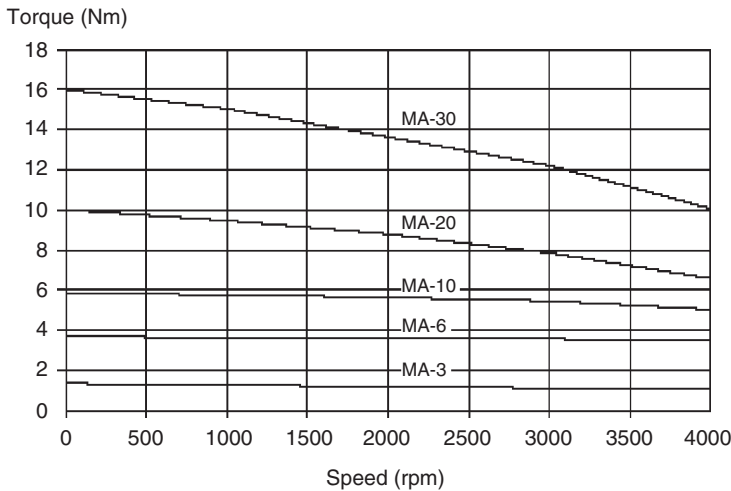


Fig. 4.7. Torque-speed curves of AFPM brushless servo motors manufactured by *Mavilor*, Barcelona, Spain.

4.7 Performance Calculation

4.7.1 Sine-Wave AFPM Machine

The electromagnetic power of a sine-wave (synchronous) AFPM motor is expressed by eqn (2.91) which can also be brought to the following form:

$$P_{elm} = m_1 [I_a E_f \cos \Psi + \frac{1}{2} (X_{sd} - X_{sq}) I_a^2 \sin 2\Psi] \quad (4.26)$$

Hence, the developed torque is

Table 4.1. AFPM brushless servomotors manufactured by *Mavilor*, Barcelona, Spain

	MA-3	MA-6	MA-10	MA-20	MA-30
Maximum speed, rpm	9000	6000	6000	6000	6000
Stall torque, Nm	1.3	3.6	5.8	10.0	16.0
Stall current, A	2.2	4.2	6.8	10.3	16.5
Peak torque, Nm	5.2	28.5	40.7	69.8	96.0
Torque density, Nm/kg	0.7	0.8	1.1	1.2	1.6
EMF constant, Vs/rad	0.3	0.5	0.5	0.6	0.6
Torque constant, Nm/A	0.6	0.9	0.9	1.0	1.0
Cogging torque, Nm	< 0.2	< 0.1	< 0.1	< 0.2	< 0.3
Winding resistance, Ω	10.2	5.3	2.2	1.4	0.5
Winding inductance, mH	25.0	11.6	0.4	7.0	4.0
Rotor moment of inertia, $\text{kgm}^2 \times 10^{-3}$	0.04	0.30	2.10	0.80	1.60
Mechanical time constant, ms	2.0	3.8	3.6	2.1	1.6
Electrical time constant, ms	2.5	2.2	7.5	5.0	7.5
Thermal time constant, s	1500	1500	1800	1500	1500
Thermal resistance, $^{\circ}\text{C}/\text{W}$	1.1	0.6	0.5	0.4	0.4
Mass, kg				34	36
Radial load, N	218	225	225	390	390

Table 4.2. Specifications of built-in brakes for AFPM brushless servomotors manufactured by *Mavilor*, Barcelona, Spain

	MA-3	MA6	MA-10	MA-20	Ma-30
Holding torque, Nm	1	4	4	8	8
d.c. voltage, V	4.8	24	24	24	24
Input power, W	5	22	22	22	22
Moment of inertia, $\text{kgm}^2 \times 10^{-3}$	0.08	0.3	0.3	0.3	0.3
Disengagement response time, ms	7	30	30	30	30
Engagement response time, ms	5	7	7	7	7
Mass, kg	0.3	0.8	0.8	0.8	0.8

$$\begin{aligned}
 T_d &= \frac{P_{elm}}{2\pi n_s} \\
 &= \frac{m_1 p}{2} \left[\sqrt{2} N_1 k_{w1} \Phi_f I_a \cos \Psi + \frac{1}{2} (L_{sd} - L_{sq}) I_a^2 \sin 2\Psi \right] \quad (4.27)
 \end{aligned}$$

For buried and interior PM rotors $X_{sd} > X_{sq}$ and $L_{sd} > L_{sq}$, which implies that an additional reluctance power and torque component is developed by the machine. For surface PM rotors the reluctance dependent components of power and torque are almost not existent as $X_{sd} \approx X_{sq}$ and $L_{sd} \approx L_{sq}$. In eqn (4.26) the EMF E_f can be calculated by means of eqn (2.36). The current I_a and angle Ψ in eqns (4.26) and (4.27) are either specified, or determined according to Section 2.7 using voltage phasor $\mathbf{V}_1$ projections on the d and q

Table 4.3. *GenSmartTM* AFPM synchronous generators manufactured by LE, Inc., Indianapolis, IN, U.S.A.

	28-G22	90-G32	120-G49
Rated output power, kW	28	90	120
Rated speed, rpm		3600	2500
Maximum speed, rpm		4200	3000
Voltage, V		480 or 208	
Stator insulation		Class H	
Efficiency, %	92.2	94.9	95.3
Frame external diameter, mm	216	315	485
Frame external length, mm	150	186	200
Stator core material		Amorphous alloy	
Cooling		liquid or air cooled	
Ambient temperature, °C		-50 to 60	
Maximum allowable temperature, °C		125 (rise from 40°C)	
Enclosure		Totally enclosed	
Mounting		Flange or foot	
Overtemperature control		Embedded thermistor	
Mass, kg	21	44	115
Power density, kW/kg	1.333	2.045	1.043

axes. The reactances and inductances are calculated as described in Sections 2.9 and 4.5.2. Hence, with all the parameters known, the performance of the AFPM brushless motor in terms of the electromagnetic power and developed torque can be calculated. Note that, for the generating mode, the second term in eqns (4.26) and (4.27) (dependent on the difference between the d - and q -axis synchronous inductances) is written with the minus (−) sign.

The electric power available, after losses, of an AFPM machine with the stator ferromagnetic core is calculated as

- the input power for motoring mode

$$P_{in} = P_{elm} + \Delta P_{1w} + \Delta P_{1Fe} + \Delta P_{2Fe} + \Delta P_{PM} + \Delta P_e \quad (4.28)$$

- the output power for generating mode

$$P_{out} = P_{elm} - \Delta P_{1w} - \Delta P_{1Fe} - \Delta P_{2Fe} - \Delta P_{PM} - \Delta P_e \quad (4.29)$$

where P_{elm} is according to eqn (4.26), ΔP_{1w} are the stator winding losses according to eqn (2.49), ΔP_{1Fe} are the stator core losses according to eqn (2.54), ΔP_{2Fe} are the rotor core losses according to eqn (2.66), ΔP_{PM} are the losses in the PMs according to eqn (2.61) and ΔP_e are the eddy current losses in the stator conductors according to eqns (2.68) or (2.69) (only for slotless stators). The shaft power P_{sh} (input power P_{in} for generators, output power P_{out} for motors) is calculated by adding to the electromagnetic power

P_{elm} (generators) or subtracting from the electromagnetic power (motors) the rotational losses ΔP_{rot} according to eqn (2.70).

The shaft torque is

- for a motor (developed by the motor itself)

$$T_{sh} = \frac{P_{out}}{2\pi n} \quad (4.30)$$

- for a generator (developed by the prime mover)

$$T_{sh} = \frac{P_{in}}{2\pi n} \quad (4.31)$$

and the efficiency is

$$\eta = \frac{P_{out}}{P_{in}} \quad (4.32)$$

4.7.2 Synchronous Generator

The phasor diagram of a salient pole synchronous generator with RL load is shown in Fig. 4.8. The input voltage projections on the d and q axes according to Fig. 4.8 are

$$V_1 \sin \delta = I_{aq} X_{sq} - I_{ad} R_1$$

$$V_1 \cos \delta = E_f - I_{ad} X_{sd} - I_{aq} R_1 \quad (4.33)$$

and

$$V_1 \sin \delta = I_{ad} R_L - I_{aq} X_L$$

$$V_1 \cos \delta = I_{aq} R_L + I_{ad} X_L \quad (4.34)$$

where $\mathbf{Z}_L = R_L + jX_L$ is the load impedance per phase across the output terminals. The d - and q -axis currents are

- from eqns (4.33)

$$I_{ad} = \frac{E_f X_{sq} - V_1 (X_{sq} \cos \delta + R_1 \sin \delta)}{X_{sd} X_{sq} + R_1^2} \quad (4.35)$$

$$I_{aq} = \frac{V_1 (X_{sd} \sin \delta - R_1 \cos \delta) + E_f R_1}{X_{sd} X_{sq} + R_1^2} \quad (4.36)$$

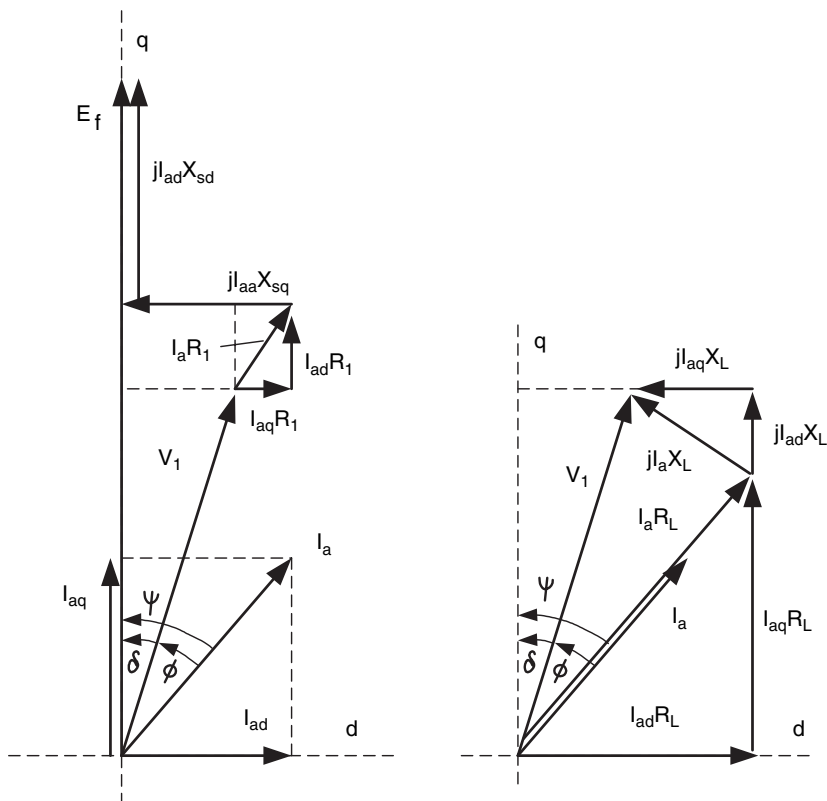


Fig. 4.8. Phasor diagram of an overexcited salient pole synchronous generator for RL load.

- from eqns (4.34)

$$I_{ad} = \frac{V_1(R_L \sin \delta + X_L \cos \delta)}{R_L^2 + X_L^2} \quad (4.37)$$

$$I_{aq} = \frac{V_1(R_L \cos \delta - X_L \sin \delta)}{R_L^2 + X_L^2} \quad (4.38)$$

The load angle δ between the voltage V_1 and EMF E_f can be determined, e.g. from the first eqn (4.34), i.e.

$$\delta = \arcsin \left(\frac{I_{ad}R_L - I_{aq}X_L}{V_1} \right) \quad (4.39)$$

Combining eqns (4.33) and (4.34), the d - and q -axis currents are independent of the load angle δ , i.e.

$$I_{ad} = \frac{E_f(X_{sq} + X_L)}{(X_{sd} + X_L)(X_{sq} + X_L) + (R_1 + R_L)^2} \quad (4.40)$$

$$I_{aq} = \frac{E_f(R_1 + R_L)}{(X_{sd} + X_L)(X_{sq} + X_L) + (R_1 + R_L)^2} \quad (4.41)$$

The angle Ψ between the current I_a and q -axis and the angle ϕ between the current I_a and voltage V_1 are respectively,

$$\Psi = \arccos\left(\frac{I_{aq}}{I_a}\right) = \arccos\left(\frac{I_{aq}}{\sqrt{I_{ad}^2 + I_{aq}^2}}\right) \quad (4.42)$$

$$\phi = \arccos\left(\frac{I_a R_L}{V_1}\right) = \arccos\left(\frac{R_L}{Z_L}\right) \quad (4.43)$$

where $I_a = \sqrt{I_{ad}^2 + I_{aq}^2}$ (see also eqn (2.89)). The output electrical power on the basis of phasor diagram (Fig. 4.8) and eqn (4.33) is

$$\begin{aligned} P_{out} &= m_1 V_1 I_a \cos \phi = m_1 V_1 (I_{aq} \cos \delta + I_{ad} \sin \delta) \\ &= m_1 [E_f I_{aq} - I_{ad} I_{aq} (X_{sd} - X_{sq}) - I_a^2 R_1] \end{aligned} \quad (4.44)$$

Including only the stator winding losses, the internal electromagnetic power of the generator is

$$P_{elm} = P_{out} + \Delta P_{1w} = m_1 [E_f I_{aq} - I_{ad} I_{aq} (X_{sd} - X_{sq})] \quad (4.45)$$

4.7.3 Square-Wave AFPM Machine

The square (trapezoidal) wave AFPM brushless machine has already been introduced in Section 2.10.2. Square-wave machines are characterized by their trapezoidal or quasi-square back EMF waveforms with the conduction angle from 100 to 150 electrical degrees, depending on the construction. The trapezoidal back EMF waveforms can be obtained by proper design of the magnetic and electric circuits. Both slotted and slotless AFPM machines can be designed so as to obtain sinusoidal or trapezoidal back EMF waveforms, except the toroidal AFPM machine of Figs 2.4 and 2.5. The toroidal machine is almost always designed to have a trapezoidal back EMF waveform. The amplitude of the back EMF can be calculated by means of eqn (2.125) for a three phase Y-connected machine with two phase windings active.

The square-wave AFPM machine is always fed by means of a solid state inverter. The inverter supplies d.c. PWM voltages to the machine phase windings during the conduction intervals. The current square waveforms (see Fig. 1.3a) are described by the flat-topped value of the current $I_a^{(sq)}$. The developed torque T_d of the square-wave AFPM brushless machine can be calculated by

means of eqn (2.127). Hence, the electromagnetic power P_{elm} of the machine can be found. Using the same loss and power balance equations as for the sine-wave machine, the input and output power can be calculated with the aid of eqn (4.28) and the output power $P_{out} = P_{elm} - \Delta P_{rot}$. Note that ΔP_{1w} must be calculated by using the true *rms* value of the current given by eqn (2.135).

4.8 Finite Element Calculations

Instead of using analytical or semi-empirical formulae, some of the equivalent circuit parameters and losses of the AFPM machine can be calculated accurately by means of the FEM analysis. The 2D and 3D FEM electromagnetic software packages are nowadays available at affordable prices and the field solution time has become attractively short. For example, most of the 2D FEM magnetostatic problems can be solved in seconds on today's personal computers.

The synchronous inductances L_{sd} and L_{sq} and the EMF E_f of an AFPM brushless machine are the critical parameters used in the performance calculations.

The first step is to specify on a 2D plane the outline, dimensions and materials of the AFPM machine. Due to the symmetry of an AFPM machine, it is usually sufficient to model only one pole pitch of the machine by applying negative periodic boundary conditions [261]. However, for AFPM machines using non-overlap windings, a greater portion of the machine (or sometime the entire machine) needs to be modelled [146]. The cross section is taken at the average radius of the active part of the stator winding, i.e. at $0.25(D_{out} + D_{in})$. Depending on the machine topology and material properties, either Neuman or Dirichlet conditions may be imposed on the remaining boundaries. The length of the winding, which is in the radial direction, is taken as the active length of the stator winding, i.e. $L_i = l_M = 0.5(D_{out} - D_{in})$ — eqn (3.34). After assigning different material properties to various regions of the model, the defined structure can then be meshed.

The second step is to specify the phase currents, i_A , i_B and i_C , in the machine according to the position of the rotor. To calculate the parameter L_{sd} or L_{sq} the phase currents must be assigned in such a way that the stator MMF can be aligned with the d -axis or q -axis respectively. In both cases the PMs have to be kept inactive. In the case of calculation of the EMF E_f equal to the no load voltage in generating mode, the phase currents are set to zero and only PMs are active.

The third step is to run the FEM solver and obtain for each of the three cases described above a solution for the air gap magnetic flux density in the axial direction of the machine. The fundamental component of the air gap magnetic flux density is determined by Fourier expansion of the axial component of the air gap flux density. Hence, the peak values of the fundamental air

gap flux densities can be determined for each of the cases above: (1) B_{mad1} with only the d -axis MMF and thus only the d -axis current I_{ad} , (2) B_{maq1} with only the q -axis MMF and the q -axis current I_{aq} , and (3) B_{mg} with only the PMs active.

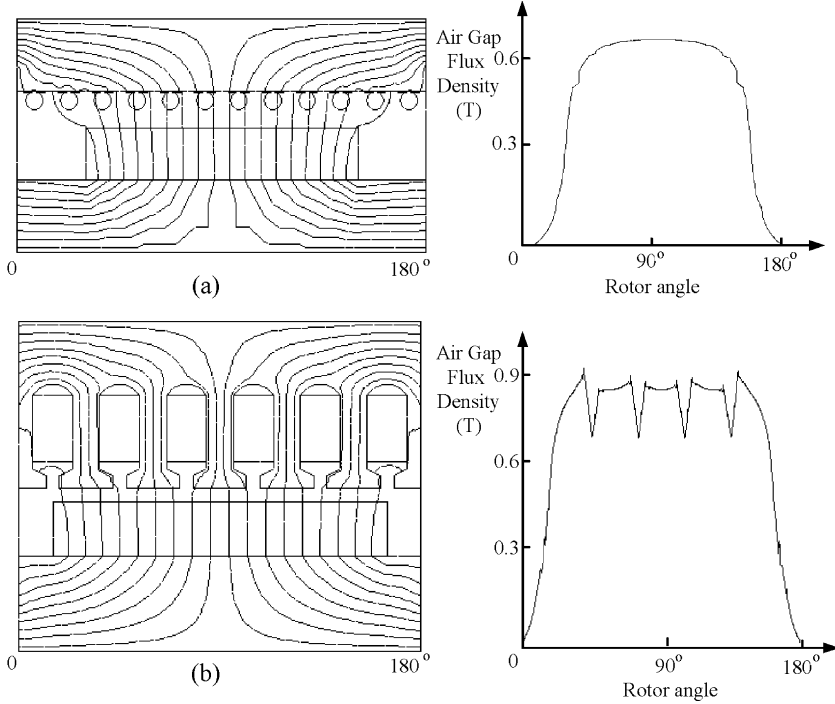


Fig. 4.9. Magnetic flux plot and air gap flux density with only PM excitation of (a) slotless and (b) slotted disc type machine with surface PMs.

Step four is to calculate by means of analytical equations the inductances and induced EMF. With B_{mad1} and B_{maq1} known from the FEM analysis, the flux linkages Ψ_d and Ψ_q can be calculated according to eqns (2.104) and (2.105). The inductances L_{sd} and L_{sq} can be calculated as described in Section 5.5.2. With B_{mg} known from the FEM analysis, the back EMF E_f can be found by first calculating the magnetic flux per pole Φ_f according to eqn (2.29) and then E_f according to eqn (2.36).

Fig. 4.9 shows the FEM flux plots and air gap flux densities due to PM excitation alone for two single-sided AFPM machine configurations with slotted and slotless stators. Fig. 4.10 shows the FEM flux plots and air gap flux densities for a single-sided interior AFPM machine with only the d - and q -axis stator winding excitation.

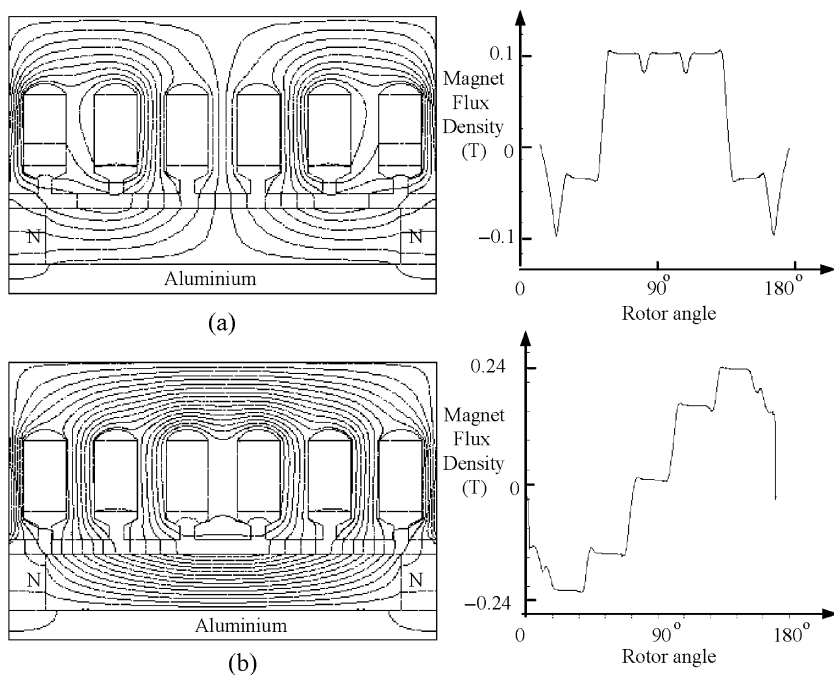


Fig. 4.10. Magnetic flux plot and air gap flux density of interior AFPM machine with (a) only d -axis and (b) only q -axis stator winding excitation.

It should be noted that using the cross section at the average radius of an AFPM machine to represent the entire machine in the FEM modelling has inherent limitations. As explained in [214], for AFPM machines with profiled PM shapes and varied tooth width along the radius the 3D effects become more prominent and thus cannot be disregarded in FEM modelling. A quasi-3D FEM modelling scheme has been proposed in [214], in which the AFPM machine is virtually divided into a number of sub-machines. The overall performance of the machine is obtained by combining the performance of each sub-machine. A similar approach that uses a multilayer and multislice modelling technique for coreless stator AFPM machines has been reported in [262, 263].

Numerical Example 4.1

A three-phase, 2.2-kW, 50-Hz, 380-V (line-to-line), Y-connected, 750-rpm, $\eta = 78\%$, $\cos \phi = 0.83$, double-sided disc PM synchronous motor has the following dimensions of the magnetic circuit: PM outer diameter $D_{out} = 0.28$ m, PM inner diameter $D_{in} = 0.16$ m, thickness of the rotor (PMs) $2h_M = 8$ mm, single-sided mechanical clearance $g = 1.5$ mm. A surface configuration

of uniformly distributed PMs has been used. The rotor does not have any soft ferromagnetic material. The rotor outer and inner diameters correspond to the outer and inner outline of PMs and the stator stack. The dimensions of rectangular semi-closed slots (Fig. 4.6b) are: $h_{11} = 11$ mm, $h_{12} = 0.5$ mm, $h_{13} = 1$ mm, $h_{14} = 1$ mm, $b_{11} = 13$ mm and $b_{14} = 3$ mm. The number of stator slots (one unit) is $s_1 = 24$, the number of armature turns of a single stator per phase is $N_1 = 456$, the diameter of the stator copper bare conductor is 0.511 mm (AWG 24), the number of stator parallel wires is $a_w = 2$ and the air gap magnetic flux density is $B_{mg} = 0.65$ T. The rotational losses are $\Delta P_{rot} = 80$ W and the core and PM losses are $\Delta P_{1Fe} + \Delta P_{PM} = 0.05 P_{out}$. There are two winding layers in each stator slot. The twin-stator Y-connected windings are fed in parallel.

Find the motor performance at the load angle $\delta = 11^\circ$. Compare the calculations obtained from the circuital approach with the FEM results.

Solution

The phase voltage is $V_1 = 380/\sqrt{3} = 220$ V. The number of pole pairs is $p = f/n = 50 \times 60/750 = 4$ and $2p = 8$. The minimum slot pitch is

$$t_{1min} = \frac{\pi D_{in}}{s_1} = \frac{\pi \times 0.16}{24} = 0.0209 \text{ m} \approx 21 \text{ mm}$$

The width of the slot is $b_{12} = 13$ mm, that means the narrowest tooth width $c_{1min} = t_{1min} - b_{12} = 21 - 13 = 8$ mm. The magnetic flux density in the narrowest part of the stator tooth

$$B_{1tmax} \approx \frac{B_{mg} t_{1min}}{c_{1min}} = \frac{0.65 \times 21}{8} = 1.7 \text{ T}$$

which is rather low.

The average diameter and average pole pitch according to eqn (1.14) are

$$D = 0.5(0.28 + 0.16) = 0.22 \text{ m}$$

$$\tau = \frac{\pi \times 0.22}{8} = 0.0864 \text{ m}$$

Because the number of slots per pole per phase (2.2)

$$q_1 = \frac{s_1}{2pm_1} = \frac{24}{8 \times 3} = 1$$

the winding factor $k_{w1} = k_{d1}k_{p1} = 1 \times 1 = 1$ as expressed by eqns (2.8), (2.9) and (2.10).

The magnetic flux according to eqn (2.26) and EMF induced by the rotor excitation system according to eqns (2.36) are respectively

$$\Phi_f = \frac{2}{\pi} \times 0.65 \frac{\pi}{8} [(0.5 \times 0.28)^2 - (0.5 \times 0.16)^2] = 0.002145 \text{ Wb}$$

$$E_f = \pi \sqrt{2} 50 \times 456 \times 1 \times 0.002145 = 217.3 \text{ V}$$

It has been assumed that $B_{mg1} \approx B_{mg}$.

Now it is necessary to check the electric loading, current density and fill (space) factor of the stator slot. For two parallel wires $a_w = 2$, the number of conductors per coil of a double-layer phase winding according to eqn (2.3) is

$$N_c = \frac{a_w N_1}{(s_1/m_1)} = \frac{2 \times 456}{(24/3)} = 114$$

Thus the number of conductors in a single slot N_{sl} is equal to the (number of layers) $\times$ (number of conductors per coil N_c) $= 2 \times 114 = 228$ or according to eqn (2.5)

$$N_{sl} = \frac{1 \times 2 \times 456}{4 \times 1} = 228$$

The rated input current in a single stator is

$$I_a = \frac{P_{out}}{2m_1 V_1 \eta \cos \phi} = \frac{2200}{2 \times 3 \times 220 \times 0.78 \times 0.83} = 2.57 \text{ A}$$

The stator line current density (peak value) on the basis of eqn (2.93) is

$$A_m = \frac{4\sqrt{2} m_1 I_a N_1}{\pi(D_{out} + D_{in})} = \frac{3\sqrt{2} \times 456 \times 2.57}{0.0864 \times 4} = 14\ 452.2 \text{ A/m}$$

which is rather a low value even for small PM a.c. motors. The cross section of the stator (armature) conductor

$$s_a = \frac{\pi d_a^2}{4} = \frac{\pi 0.511^2}{4} = 0.207 \text{ mm}^2$$

gives the following current density under rated conditions

$$j_a = \frac{2.57}{2 \times 0.197} = 6.29 \text{ A/mm}^2$$

This is an acceptable value of the current density for disc-type a.c. machines rated from 1 to 10 kW.

For the class F enamel insulation of the armature conductors, the diameter of the wire with insulation is 0.548 mm. Hence, the total cross sectional area of all conductors in the stator slot is

$$228 \times \frac{\pi \times 0.548^2}{4} \approx 54 \text{ mm}^2$$

The cross section area of a single slot is approximately $h_{11}b_{12} = 11 \times 13 = 143 \text{ mm}^2$. The slot fill factor $54/143 = 0.38$ shows that the stator can be easily wound, since the average slot fill factor for low voltage machines with round stator conductors is about 0.4.

The average length of the stator end connection for a disc rotor a.c. machine is $l_{1e} \approx 0.154 \text{ m}$. The average length of the stator turn according to eqn (2.39) is

$$l_{1av} = 2(L_i + l_{1e}) = 2(0.06 + 0.154) = 0.428 \text{ m}$$

where $L_i = 0.5(D_{out} - D_{in}) = 0.5(0.28 - 0.16) = 0.06 \text{ m}$. The stator winding resistance per phase at a temperature of 75°C (hot motor) according to eqn (2.40) is

$$R_1 = \frac{N_1 l_{1av}}{a_w \sigma s_a} = \frac{456 \times 0.428}{47 \times 10^6 \times 2 \times 0.1965} = 10.57 \Omega$$

Carter's coefficient is calculated on the basis of eqns (1.2) and (1.3), i.e.

$$k_C = \left(\frac{28.8}{28.8 - 0.012 \times 1.5} \right)^2 = 1.001$$

$$\gamma = \frac{4}{\pi} \left[\frac{3}{2 \times 11} \arctan \left(\frac{3}{2 \times 11} \right) - \ln \sqrt{1 + \left(\frac{3}{2 \times 11} \right)^2} \right] = 0.012$$

where $t_1 = \pi D / s_1 = \pi \times 0.22 / 24 = 0.0288 \text{ m} = 28.8 \text{ mm}$. The nonferromagnetic air gap in calculation of Carter's coefficient is $g' \approx 2g + 2h_M = 2 \times 1.5 + 8 = 11 \text{ mm}$. Since there are two slotted surfaces of twin stator cores, Carter's coefficient has been squared.

The stator (one unit) leakage reactance has been calculated according to eqn (4.8) in which $l_{1in} \lambda_{1ein} / L_i + l_{1out} \lambda_{1eout} / L_i \approx l_{1e} \lambda_{1e} / L_i$, i.e.

$$\begin{aligned} X_1 &= 4 \times 0.4\pi \times 10^{-6} \pi \times 50 \times \frac{456^2 \times 0.06}{4 \times 1} \times (0.779 + \frac{0.154}{0.06} \times 0.218 \\ &\quad + 0.2297 + 0.9322) = 6.158 \Omega \end{aligned}$$

where

- the coefficient of slot leakage permeance – eqn (4.9) is

$$\lambda_{1s} = \frac{11}{3 \times 13} + \frac{0.5}{13} + \frac{2 \times 1}{13 + 3} + \frac{1}{3} = 0.779$$

- the coefficient of end connection leakage permeance – eqns (4.16) in which the average coil span $w_c = \tau$ is

$$\lambda_{1e} \approx 0.34 \times 1 \left(1 - \frac{2}{\pi} \frac{0.0864}{0.154} \right) = 0.218$$

- the coefficient of differential leakage permeance – eqns (4.22) and (4.24) is

$$\lambda_{1d} = \frac{3 \times 1 \times 0.0864 \times 1^2}{\pi^2 \times 0.011 \times 1.001} 0.0966 = 0.2297$$

$$\tau_{d1} = \frac{\pi^2(10 \times 1^2 + 2)}{27} \left(\sin \frac{30^\circ}{1} \right)^2 - 1 = 0.0966$$

- the coefficient of tooth-top leakage permeance – eqn (4.25) is

$$\lambda_{1t} = \frac{5 \times 11/3}{5 + 4 \times 11/3} = 0.9322$$

According to eqns (2.121) and (2.122) in which $k_{fd} = k_{fq} = 1$, the armature reaction reactances for surface type PM rotors and unsaturated machines are

$$X_{ad} = X_{aq} = 2 \times 3 \times 0.4 \times \pi \times 10^{-6} \times 50 \times \left(\frac{456 \times 1}{4} \right)^2$$

$$\times \frac{(0.5 \times 0.28)^2 - (0.5 \times 0.16)^2}{1.001 \times 0.011} = 5.856 \, \Omega$$

where the air gap, in the denominator, for the armature flux should be equal to $g' \approx 2 \times 1.5 + 8 = 11$ mm ($\mu_{rrec} \approx 1$). The synchronous reactances according to eqns (2.79) and (2.80) are

$$X_{sd} = X_{sq} = 6.158 + 5.856 = 12.01 \, \Omega$$

The armature currents are calculated on the basis of eqns (2.87), (2.88) and (2.89). For $\delta = 11^\circ$ ($\cos \delta = 0.982$, $\sin \delta = 0.191$) the current components are: $I_{ad} = -1.807$ A, $I_{aq} = 1.96$ A and $I_a = 2.666$ A.

The input power absorbed by one stator is expressed by eqn (2.90). The input power according to eqn (2.90) absorbed by two stators in parallel is twice as much, i.e.

$$P_{in} = 2 \times 3 \times 220 \times (1.88 \times 0.982 - (-1.82) \times 0.191) = 2986.8 \, \text{W}$$

The input apparent power absorbed by two stators is

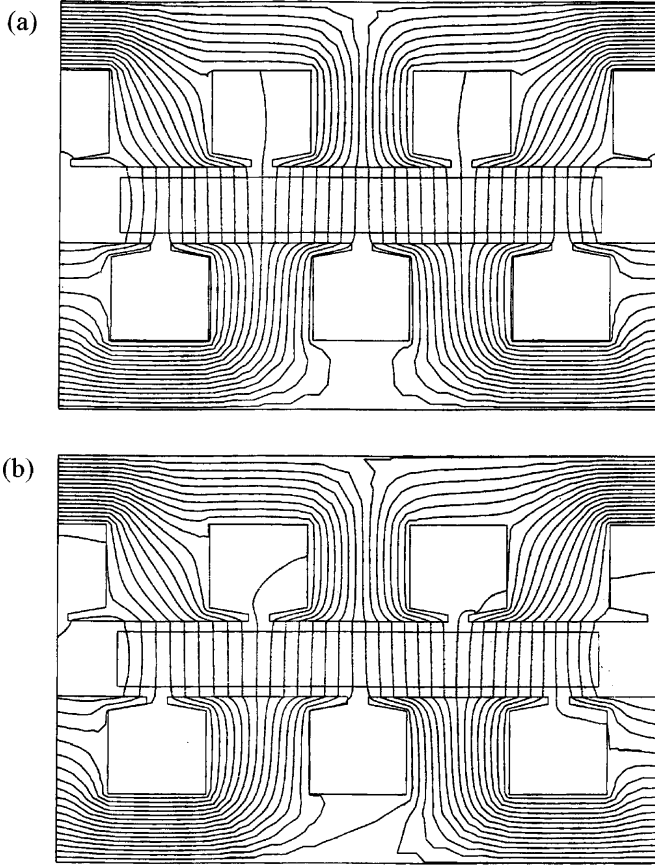


Fig. 4.11. Magnetic flux plots in the double sided AFPM motor: (a) zero armature current, (b) rated armature current. *Numerical example 4.1.*

$$S_{in} = 2 \times 3 \times 220 \times 2.62 = 3509.7 \text{ VA}$$

The power factor is

$$\cos \phi = \frac{2986.8}{3509.7} = 0.851$$

The losses in two stator windings according to eqn (2.49), in which $k_{1R} \approx 1$, are calculated as

$$\Delta P_{1w} = 2 \times 3 \times 2.62^2 \times 10.57 = 431.4 \text{ W}$$

The output power assuming that $\Delta P_{1Fe} + \Delta P_{PM} = 0.05P_{out}$ is

$$P_{out} = \frac{1}{1.05}(P_{in} - \Delta P_{1w} - \Delta P_{rot}) = \frac{1}{1.05}(2986.8 - 431.4 - 80.0) = 2357.5 \text{ W}$$

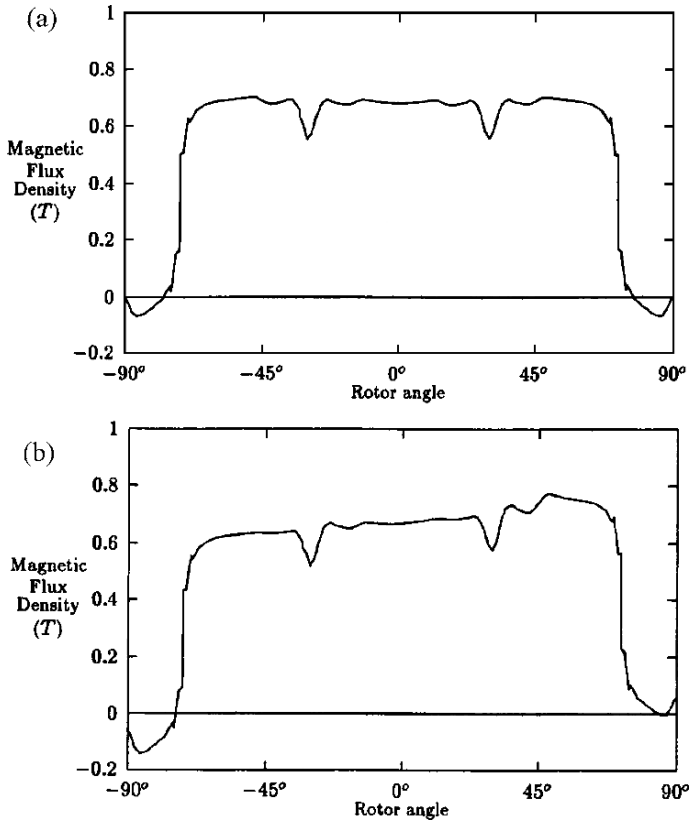


Fig. 4.12. Magnetic flux density distribution along the pole pitch: (a) zero armature current, (b) rated armature current. *Numerical example 4.1.*

Thus

$$\Delta P_{1Fe} + \Delta P_{PM} = 0.05 \times 2357.5 = 117.9 \text{ W}$$

The motor efficiency is

$$\eta = \frac{2357.5}{2986.8} = 0.789 \quad \text{or} \quad \eta = 78.9\%$$

The shaft torque is

$$T_{sh} = \frac{2357.5}{2\pi(750/60)} = 30.02 \text{ Nm}$$

The electromagnetic power is

$$P_{elm} = P_{in} - \Delta P_{1w} - \Delta P_{1Fe} - \Delta P_{PM} = 2986.8 - 431.4 - 117.9 = 2437.5 \text{ W}$$

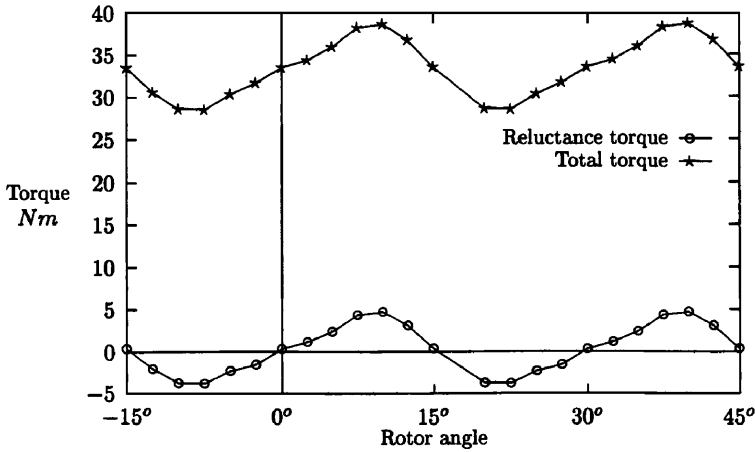


Fig. 4.13. Torque as a function of the disc rotor position. *Numerical example 4.1.*

The electromagnetic torque developed by the motor is

$$T_d = \frac{2437.5}{2\pi \times 750/60} = 31.04 \text{ Nm}$$

The results of the FEM analysis are shown in Figs 4.11, 4.12 and 4.13. The FEM gives higher values of the average developed torque than the analytical approach. Eqns (2.121), (2.122) and (4.8) do not give accurate values of X_{sd} , X_{sq} and X_1 . The electromagnetic torque plotted in Fig. 4.13 against the rotor position has a significant cogging effect component with the period equal to double slot pitch, i.e. 30° .

Numerical Example 4.2

A single-sided AFPM machine has the same stator and dimensions of PMs as those in Numerical example 4.1. The rotor SmCo PMs with $B_r = 1.05 \text{ T}$ and $H_c = 720\,000 \text{ A/m}$ are fixed to a solid steel disc. The air gap (mechanical clearance) is $g = 1.5 \text{ mm}$, the height of PM is $h_M = 4 \text{ mm}$, the coefficient of leakage flux of PM is $\sigma_{lM} = 1.1$, the *pole shoe-to-pole pitch* ratio is $\alpha_i = 0.78$ and the approximation coefficient of the shape of the magnetic flux density under PMs in eqn (1.13) is $\alpha = 0$.

Calculate the magnetic flux density distribution in the air gap at $D = 0.5(D_{in} + D_{out})$ and attractive force between the stator and rotor at zero stator current (no armature reaction).

Solution**1. Air gap permeance**

According to eqns (1.14) and (2.92) the average diameter $D = 0.22$ m and average pole pitch $\tau = 0.086$ m. The number of pole pairs $p = (50/750)/60 = 4$. The average slot pitch $t_1 = 2p\tau/s_1 = 2 \times 4 \times 0.086/24 = 0.0288$ m. The relative recoil permeability expressed by eqn (3.4) is

$$\mu_{rrec} = \frac{1}{\mu_0} \frac{B_r}{H_c} = \frac{1}{\mu_0} \times \frac{1.05}{720\,000} = 1.161$$

The equivalent air gap calculated on the basis of eqn (2.110) is practically independent of Carter's coefficient k_C (large equivalent air gap) and equal to $g' = 4.95$ mm. Carter's coefficient according to eqns (1.2) and (1.3) is very small, i.e., $k_C = 1.01$. The *slot opening-to-the air gap* ratio and *slot opening-to-the slot pitch* according to eqn (2.60) are $\kappa = 0.606$ and $\Gamma = 0.104$.

The relative permeance of smooth air gap is expressed by the same equation as eqn (2.108), i.e., $\lambda_0 = \mu_0/g' = \mu_0/0.00495 = 0.00025$ H/m². Amplitudes of slot harmonics a_ν are given by eqns (4.5) and (4.6). To obtain the relative slot permeance (4.4) it is enough to sum 25 slot harmonics, i.e.,

$$\lambda_{sl}(x) = - \sum_{\nu=1}^{25} a_\nu \cos(\nu \frac{2\pi}{t_1} x)$$

The relative permeance of the air gap is

$$\lambda_g(x) = \lambda_0 + \lambda_{sl}(x)$$

For example, for $x = t_1 = 0.0288$ m the relative permeance of the air gap is $\lambda_{(x=t_1)} = 0.00022$ H/m².

2. Magnetic flux density in the air gap

The normal component of the magnetic flux density excited by PMs in smooth (without slots) air gap can be calculated on the basis of eqn (3.13). Including the PM leakage flux coefficient $\sigma_{IM} = 1.1$ and Carter's coefficient $k_C = 1.01$

$$B_g = \frac{1}{\sigma_{IM}} \frac{B_r}{1 + \mu_{rrec} k_C g / h_M} = \frac{1}{1.1} \frac{1.05}{(1 + 1.161 \times 1.01 \times 0.0015 / 0.004)} = 0.663 \text{ T}$$

The coefficient b_ν of the shape of the magnetic flux density distribution is expressed by eqn (1.13). For example, for $\nu = 1$ $b_1 = 1.228$, for $\nu = 3$ $b_3 = -0.296$ and for $\nu = 5$ $b_5 = 0.064$, where the pole face average width $b_p = \alpha_i \tau = 0.78 \times 0.086 = 0.0674$ m, $c_p = 0$ and $b_t = 0.0095$.

The distribution of the normal component of the magnetic flux density $B_{zPM}(x)$ excited in the smooth air gap by the rotor PMs is expressed by eqn (4.2). It is enough to include maximum $\nu = 1, 3, 5, \dots, 65$ slot harmonics.

The distribution of the magnetic flux density due to slot openings $B_{sl}(x)$ is expressed by eqn (4.3). The resultant magnetic flux density distribution in the air gap including stator slotting $B_g(x)$ is given by eqn (4.1). The flat-top value of the resultant magnetic flux density $B_g(x = 0.5\tau) = 0.671$ T. The difference between $B_g = 0.663$ T according to eqn (3.13) is negligible. Fig. 4.14 shows the magnetic flux density waveforms $B_g(x)$ and $B_{sl}(x)$ and harmonic flux densities $B_{g1}(x)$, $B_{g3}(x)$ and $B_{g5}(x)$.

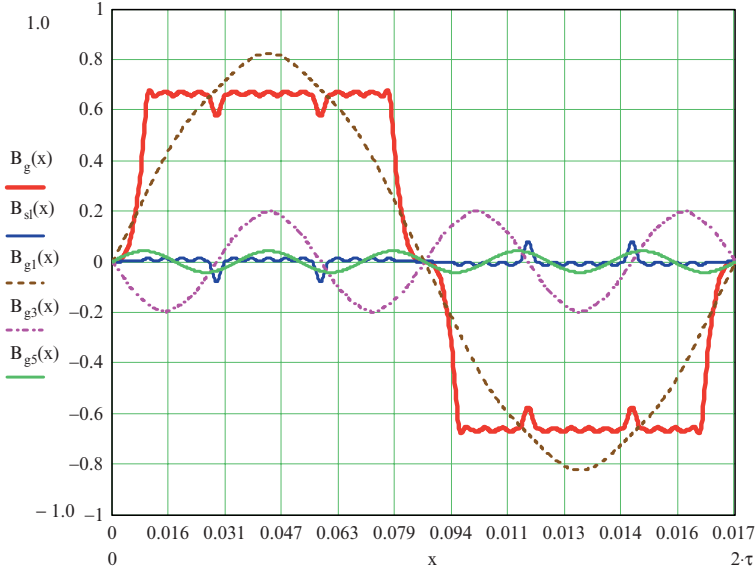


Fig. 4.14. Resultant air gap magnetic flux density waveform, slot component, fundamental, third and fifth harmonics (two pole pitches) *Numerical example 4.2.*

3. Normal attractive force between the stator and rotor

The active surface area of all PMs calculated on the basis of eqn (2.63) is $S_{PM} = 0.0323 \text{ m}^2$. The magnetic pressure on the stator and rotor active surfaces is given by eqn (4.7). Thus, the normal attractive force

$$F_z(x) = p_z(x)S_{PM} = 0.0323p_z(x)$$

For the center axis of PM (d -axis), the normal attractive force is $F_z(x = 0.5\tau) = 5797$ N. The distribution of the normal attractive force $F_z(x)$ at the distance $0.5D$ from the center around the machine circumference $2p\tau$ is plotted in Fig. 4.14.

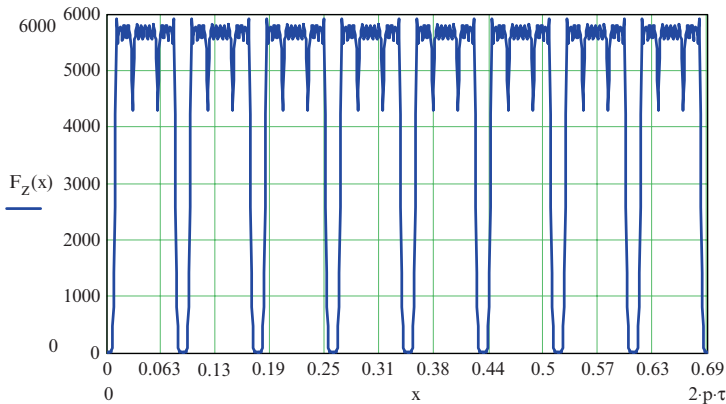


Fig. 4.15. Distribution of normal attractive force between stator and rotor. *Numerical Example 4.2.*

Numerical Example 4.3

A three-phase, Y-connected, 50 Hz, 5.5-kW AFPM synchronous generator with slotted stator has the following parameters of the steady-state equivalent circuit: $R_1 = 0.1 \, \Omega$, $X_{sd} = 2.3 \, \Omega$ and $X_{sq} = 2.2 \, \Omega$. The generator is loaded with a three-phase Y-connected resistance of $R_L = 2.2 \, \Omega$ and inductance of $L_L = 0.0007 \, \text{H}$ connected in series. The EMF per phase at no load induced by the rotor PM excitation system is $E_f = 100 \, \text{V}$ and rotational losses are $\Delta P_{rot} = 90 \, \text{W}$.

Find the stator currents, electromagnetic power, output power and efficiency. Stator core losses and losses in PMs have been ignored.

Solution

The load reactance and module of load impedance are

$$X_L = 2\pi f L_L = 2\pi \times 50 \times 0.0007 = 0.22 \, \Omega$$

$$Z_L = \sqrt{2.2^2 + 0.22^2} = 2.211 \, \Omega$$

The stator currents according to eqns (4.40), (4.41) and (2.89) are

$$I_{ad} = \frac{100 \times (2.2 + 0.22)}{(2.3 + 0.22) \times (2.2 + 0.22) + (0.1 + 2.2)^2} = 21.25 \, \text{A}$$

$$I_{aq} = \frac{100 \times (0.1 + 2.2)}{(2.3 + 0.22) \times (2.2 + 0.22) + (0.1 + 2.2)^2} = 20.20 \, \text{A}$$

$$I_a = \sqrt{21.25^2 + 20.20^2} = 29.32 \text{ A}$$

The terminal voltage (across the load impedance) is

$$V_1 = I_a Z_L = 29.32 \times 2.211 = 64.82 \text{ V}$$

$$V_{1L} = \sqrt{3} V_1 = \sqrt{3} \times 64.82 = 112.3 \text{ V}$$

The power factor, load angle δ and angle Ψ according to eqns (4.43), (4.39) and (4.42) respectively are

$$\cos \phi = \frac{29.32 \times 2.2}{64.82} = 0.995 \quad \phi = 5.71^\circ$$

$$\Psi = \arccos\left(\frac{20.20}{29.32}\right) = 46.45^\circ$$

$$\delta = \arcsin\left(\frac{21.25 \times 2.2 - 20.20 \times 0.22}{64.82}\right) = 40.74^\circ$$

$$\delta = \Psi - \phi = 46.45^\circ - 5.71^\circ = 40.74^\circ$$

The electromagnetic power according to eqn (4.45) is

$$P_{elm} = 3 \times [100 \times 20.20 - 21.25 \times 20.20 \times (2.3 - 2.2)] = 5930.3 \text{ W}$$

The stator winding losses according to eqn (2.49) is

$$\Delta P_{1w} = 3 \times 29.32^2 \times 0.1 = 257.8 \text{ W}$$

The output power is

$$P_{out} = m_1 V_1 I_a \cos \phi = 3 \times 64.82 \times 29.32 \times 0.995 = 5672.5 \text{ W}$$

$$P_{out} = P_{elm} - \Delta P_{1w} = 5930.3 - 257.8 = 5672.4$$

The input power is

$$P_{in} = P_{elm} + \Delta P_{rot} = 5930.3 + 90 = 6020.3 \text{ W}$$

The efficiency is

$$\eta = \frac{5672.5}{6020.3} = 0.957$$

AFPM Machines Without Stator Cores

5.1 Advantages and Disadvantages

Depending on the application and operating environment, stators of AFPM machines may have ferromagnetic cores or be completely coreless. A coreless stator AFPM machine has an internal stator and twin external PM rotor (Fig. 1.4d). PMs can be glued to the rotor backing steel discs or nonmagnetic supporting structures. In the second case PMs are arranged in Halbach array (Fig. 3.15) and the machine is completely coreless. The electromagnetic torque developed by a coreless AFPM brushless machine is produced by the open space current-carrying conductor–PM interaction (Lorentz force theorem).

Coreless configurations eliminate any ferromagnetic material, i.e. steel laminations or SMC powders from the stator (armature), thus eliminating the associated eddy current and hysteresis core losses. Because of the absence of core losses, a coreless stator AFPM machine can operate at higher efficiency than conventional machines. On the other hand, owing to the increased non-magnetic air gap, such a machine uses more PM material than an equivalent machine with a ferromagnetic stator core.

Typical coil shapes used in the winding of a coreless stator are shown in Figs 3.16 and 3.17.

In this chapter AFPM brushless machines with coreless stator and conventional PM excitation, i.e. PM fixed to backing steel discs, will be discussed.

5.2 Commercial Coreless Stator AFPM Machines

Bodine Electric Company, Chicago, IL, U.S.A. manufactures 178-mm (7-inch) and 356-mm (14-inch) diameter *e-TORQ*TM AFPM brushless motors with coreless stator windings and twin external PM rotors with steel backing discs (Fig. 5.1a). The coreless stator design eliminates the so called *cogging (detent) torque*, improves low speed control, yields linear torque-current characteristics due to the absence of magnetic saturation and provides peak torque up to ten