Resnick · Halliday

Physics

Part I

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Physics

PART I

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Motion in a Plane

CHAPTER 4

4–1 Displacement, Velocity, and Acceleration

In this chapter we return to a consideration of motion in two dimensions taken, for convenience, to be the x-y plane. Figure 4–1 shows a particle at time t moving along a curved path in this plane. Its *position*, or displacement from the origin, is measured by the vector $\mathbf{r}$; its *velocity* is indicated by the vector $\mathbf{v}$ which, as we have seen in Section 3–4, must be tangent to the path of the particle. The *acceleration* is indicated by the vector $\mathbf{a}$; the direction of $\mathbf{a}$, as we shall see more explicitly later, does not bear any unique relationship to the path of the particle but depends rather on the rate at which the velocity $\mathbf{v}$ changes with time as the particle moves along its path.

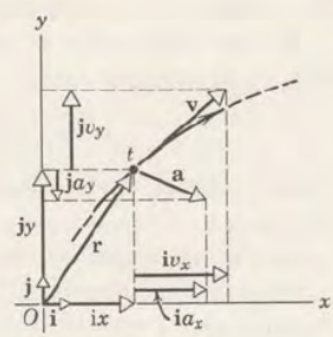

Fig. 4–1 A particle moves along a curved path in the x-y plane. Its position $\mathbf{r}$, velocity $\mathbf{v}$, and acceleration $\mathbf{a}$ are shown at time t, along with the vector components of those vectors. Note that x, y, v_x, v_y, and a_x are positive but that a_y is negative. Compare to Fig. 3-3.

The vectors $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$ are interrelated (see Eqs. 3–4, 3–5, and 3–10) and can be expressed in terms of their components, using unit vector notation, as

$$\mathbf{r} = \mathbf{i}x + \mathbf{j}y, \tag{4–1}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \mathbf{i}v_x + \mathbf{j}v_y, \tag{4–2}$$

and

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \mathbf{i}a_x + \mathbf{j}a_y. \tag{4–3}$$

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These equations can easily be extended to three dimensions by adding to them the terms $\mathbf{k}z$, $\mathbf{k}v_z$, and $\mathbf{k}a_z$, respectively in which $\mathbf{k}$ is a unit vector in the z-direction.

In Chapter 3 we considered the special case in which the particle moved in one dimension only, say along the x-axis, where the vectors $\mathbf{r}$, $\mathbf{v}$, and $\mathbf{a}$ were directed along this axis, either in the positive x-direction or the negative x-direction. The components y, v_y, and a_y were zero and we described the motion in terms of equations relating the scalar quantities x, v_x, and a_x. Or, when the particle moved along the y-axis, the components x, v_x, and a_x were zero and the motion was described in terms of equations relating the scalar quantities y, v_y, and a_y. In this chapter we consider motion in the x-y plane so that, in general, both sets of components have nonzero values.

4–2 Motion in a Plane with Constant Acceleration

Let us consider first the special case of motion in a plane with *constant* acceleration. Here, as the particle moves, the acceleration $\mathbf{a}$ does not vary either in magnitude or in direction. Hence the components of $\mathbf{a}$ in any particular reference frame also will not vary, that is, a_x = constant and a_y = constant. We then have a situation which can be described as the sum of two component motions occurring simultaneously with constant acceleration along each of two perpendicular directions. The particle will move, in general, along a curved path in the plane. This may be so even if one component of the acceleration, say a_x, is zero, for then the corresponding component of the velocity, say v_x, may have a constant, nonzero value. An example of this latter situation is the motion of an artillery shell which follows a curved path in a vertical plane and, neglecting the effects of air resistance, is subject to a constant acceleration $\mathbf{g}$ directed down along the y-axis only.

We can obtain the general equations for plane motion with constant $\mathbf{a}$ simply by setting

$$a_x = \text{constant} \qquad \text{and} \qquad a_y = \text{constant}.$$

The equations for constant acceleration, summarized in Table 3–1, then apply to both the x- and y-components of the position vector $\mathbf{r}$, the velocity vector $\mathbf{v}$, and the acceleration vector $\mathbf{a}$ (see Table 4–1).

The two sets of equations in Table 4–1 are related in that the time parameter t is the same for each, since t represents the time at which the particle, moving in a curved path in the x-y plane, occupied a position described by the position components x and y.

The equations of motion in Table 4–1 may also be expressed in vector form. For example, substituting Eqs. 4–4a, 4a' into Eq. 4–2 yields

$$\begin{aligned}\mathbf{v} &= \mathbf{i}v_x + \mathbf{j}v_y \\ &= \mathbf{i}(v_{x0} + a_x t) + \mathbf{j}(v_{y0} + a_y t) \\ &= (\mathbf{i}v_{x0} + \mathbf{j}v_{y0}) + (\mathbf{i}a_x + \mathbf{j}a_y)t.\end{aligned}$$

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Table 4–1

MOTION WITH CONSTANT ACCELERATION IN THE x-y PLANE

Equation No.	x-Motion Equations	Equation No.	y-Motion Equations
4–4a	$v_x = v_{x0} + a_x t$	4–4a'	$v_y = v_{y0} + a_y t$
4–4b	$x = x_0 + \frac{1}{2}(v_{x0} + v_x)t$	4–4b'	$y = y_0 + \frac{1}{2}(v_{y0} + v_y)t$
4–4c	$x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$	4–4c'	$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$
4–4d	$v_x^2 = v_{x0}^2 + 2a_x(x - x_0)$	4–4d'	$v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$

The first quantity in parentheses is the initial velocity vector $\mathbf{v}_0$ (see Eq. 4–2) and the second is the (constant) acceleration vector $\mathbf{a}$ (see Eq. 4–3). Thus the vector relation

$$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t \tag{4–5a}$$

is equivalent to the two scalar relations Eqs. 4–4a, a' in Table 4–1. It shows simply and compactly that the velocity $\mathbf{v}$ at time t is the sum of the initial velocity $\mathbf{v}_0$ which the particle would have in the absence of acceleration plus the (vector) change in velocity, $\mathbf{a}t$, acquired during the time t under the constant acceleration $\mathbf{a}$. Similarly, the scalar equations 4–4c, c' are equivalent to the single vector equation

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2, \tag{4–5b}$$

which is also easily interpreted. The proof of this and other relations is left to Problem 17.

4–3 Projectile Motion

An example of curved motion with constant acceleration is projectile motion. This is the two-dimensional motion of a particle thrown obliquely into the air. The ideal motion of a baseball, a golf ball, or a bullet is an example of projectile motion.* We assume that the effect the air itself would have on their motions can be neglected.

The motion of a projectile is one of constant acceleration $\mathbf{g}$, directed downward, and thus should be described by the equations in Table 4–1. There is no horizontal component of acceleration. If we choose a reference frame with the positive y-axis vertically upward, we may put $a_y = -g$ and $a_x = 0$ in these equations.

Let us further choose the origin of our reference frame to be the point at which the projectile begins its flight (see Fig. 4–2). Hence the origin will be the point at which the ball leaves the thrower's hand or the fuel in the rocket burns out, for example. In Table 4–1 this choice of origin implies

* See Galileo Galilei, *Dialogues Concerning Two New Sciences*, the "Fourth Day," for a fascinating discussion of Galileo's research on projectiles.

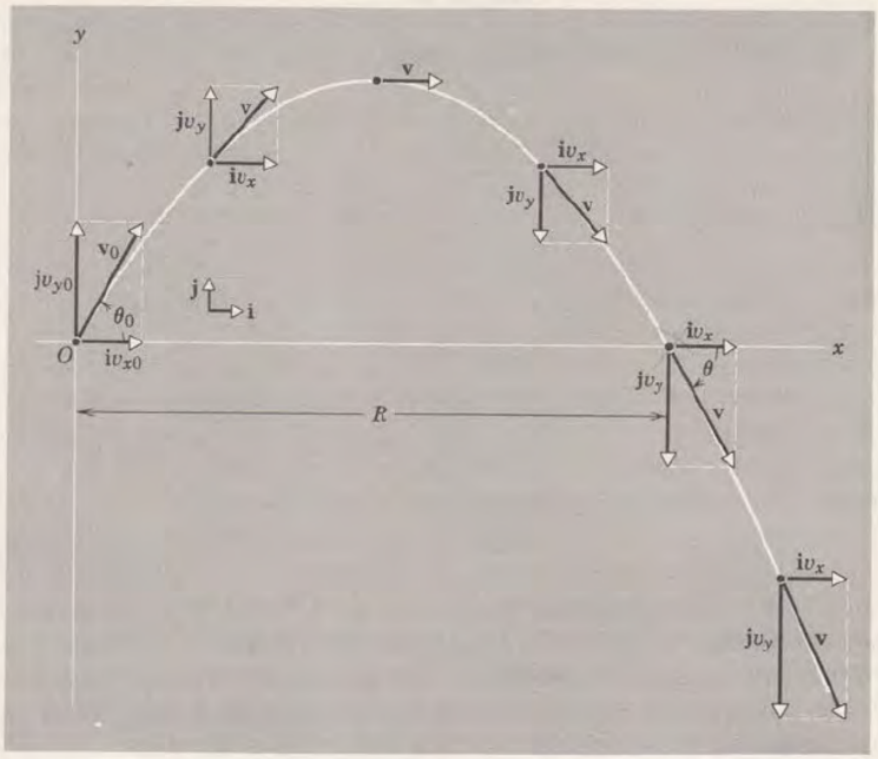

Fig. 4–2 The trajectory of a projectile, showing the initial velocity $\mathbf{v}_0$ and its vector components and also the velocity $\mathbf{v}$ and its vector components at five later times. Note that $v_x = v_{x0}$ throughout the flight. The distance R is the horizontal range.

that $x_0 = y_0 = 0$. The velocity at $t = 0$, the instant the projectile begins its flight, is $\mathbf{v}_0$, which makes an angle θ_0 with the positive x-direction. The x- and y-components of $\mathbf{v}_0$ (see Fig. 4–2) are then

$$v_{x0} = v_0 \cos \theta_0 \qquad \text{and} \qquad v_{y0} = v_0 \sin \theta_0.$$

Because there is no horizontal component of acceleration, the horizontal component of the velocity will be constant. In Eq. 4–4a we set $a_x = 0$ and $v_{x0} = v_0 \cos \theta_0$, so that

$$v_x = v_0 \cos \theta_0. \tag{4–6a}$$

The horizontal velocity component retains its initial value throughout the flight.

The vertical component of the velocity will change with time in accordance with vertical motion with constant downward acceleration. In Eq. 4–4a' we set

$$a_y = -g \qquad \text{and} \qquad v_{y0} = v_0 \sin \theta_0,$$

so that

$$v_y = v_0 \sin \theta_0 - gt. \tag{4–6a'}$$

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The vertical velocity component is that of free fall. Indeed, if we view the motion of Fig. 4–2 from a reference frame that moves to the right with a speed v_{x0}, the motion will be that of an object thrown vertically upward with an initial speed $v_0 \sin \theta_0$.

The *magnitude* of the resultant velocity vector at any instant is

$$v = \sqrt{v_x^2 + v_y^2}. \tag{4–7}$$

The angle θ that the velocity vector makes with the horizontal at that instant is given by

$$\tan \theta = \frac{v_y}{v_x}.$$

The velocity vector is tangent to the path of the particle at every point, as shown in Fig. 4–2.

The x-coordinate of the particle's position at any time, obtained from Eq. 4–4c with $x_0 = 0$, $a_x = 0$, and $v_{x0} = v_0 \cos \theta_0$, is

$$x = (v_0 \cos \theta_0)t. \tag{4–6c}$$

The y-coordinate, obtained from Eq. 4–4c' with $y_0 = 0$, $a_y = -g$, and $v_{y0} = v_0 \sin \theta_0$, is

$$y = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2. \tag{4–6c'}$$

Equations 4–6c, c' give us x and y as functions of the common parameter t, the time in flight. By combining and eliminating t from them, we obtain

$$y = (\tan \theta_0)x - \frac{g}{2(v_0 \cos \theta_0)^2}\, x^2, \tag{4–8}$$

which relates y to x and is the equation of the trajectory of the projectile. Since v_0, θ_0, and g are constants, this equation has the form

$$y = bx - cx^2,$$

the equation of a parabola. Hence the trajectory of a projectile is parabolic.

▶ **Example 1.** A bomber is flying at a constant horizontal velocity of 820 miles/hr at an elevation of 52,000 ft toward a point directly above its target. At what angle of sight ϕ should a bomb be released to strike the target (Fig. 4–3)?

We choose a reference frame fixed with respect to the earth, its origin O being the bomb release point. The motion of the bomb at the instant of release is the same as that of the bomber. Hence the initial projectile velocity $\mathbf{v}_0$ is horizontal and its magnitude is 820 miles/hr or 1200 ft/sec. The angle of projection θ_0 is zero.

The time of fall is obtained from Eq. 4–6c'. With $\theta_0 = 0$ and $y = -52{,}000$ ft, this gives

$$t = \sqrt{-\frac{2y}{g}} = \sqrt{-\frac{2(-52{,}000)\text{ft}}{32\ \text{ft/sec}^2}} = 57\ \text{sec}.$$

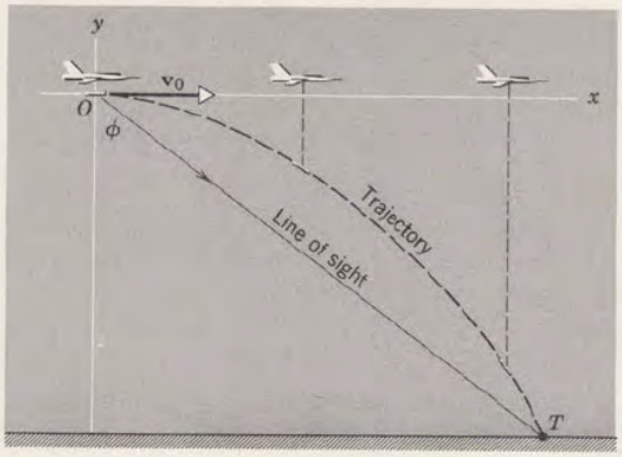

Fig. 4–3 Example 1. A bomb is released from an airplane with horizontal velocity $\mathbf{v}_0$.

Note that the time of fall of the bomb does *not* depend on the speed of the plane for a *horizontal* projection. (See, however, Problem 10.)

The horizontal distance traveled by the bomb in this time is given by Eq. 4–6*c*, $x = (v_0 \cos \theta_0)t$, or $x = (1200 \text{ ft/sec})(57 \text{ sec}) = 68{,}000$ ft so that the angle of sight (Fig. 4–3) should be

$$\phi = \tan^{-1} \frac{x}{|y|} = \tan^{-1} \frac{68{,}000}{52{,}000} = 53°.$$

Does the motion of the bomb appear to be parabolic when viewed from a reference frame fixed with respect to the bomber?

Example 2. A soccer player kicks a ball at an angle of 37° from the horizontal with an initial speed of 50 ft/sec. (A right triangle, one of whose angles is 37°, has sides in the ratio 3:4:5, or 6:8:10.) Assuming that the ball moves in a vertical plane:

(*a*) Find the time t_1 at which the ball reaches the highest point of its trajectory. At the highest point, the vertical component of velocity v_y is zero. Solving Eq. 4–6*a*′ for t, we obtain

$$t = \frac{v_0 \sin \theta_0 - v_y}{g}.$$

With

$$v_y = 0, \qquad v_0 = 50 \text{ ft/sec}, \qquad \theta_0 = 37°, \qquad g = 32 \text{ ft/sec}^2,$$

we have

$$t_1 = \frac{[50(\frac{6}{10}) - 0] \text{ ft/sec}}{32 \text{ ft/sec}^2} = \frac{15}{16} \text{ sec}.$$

(*b*) How high does the ball go? The maximum height is reached at $t = 15/16$ sec. By using Eq. 4–6*c*′,

$$y = (v_0 \sin \theta_0)t - \tfrac{1}{2}gt^2,$$

we have

$$y_{\max} = (50 \text{ ft/sec})(\tfrac{6}{10})(\tfrac{15}{16} \text{ sec}) - \tfrac{1}{2}(32 \text{ ft/sec}^2)(\tfrac{15}{16})^2 \text{ sec}^2 = 14 \text{ ft}.$$

(*c*) What is the horizontal range of the ball and how long is it in the air?

The horizontal distance from the starting point at which the ball returns to its original elevation (ground level) is the *range* R. We set $y = 0$ in Eq. 4–6*c*′ and find the time t_2 required to traverse this range. We obtain

$$t_2 = \frac{2v_0 \sin \theta_0}{g} = \frac{2(50 \text{ ft/sec})(\frac{6}{10})}{32 \text{ ft/sec}^2} = \frac{15}{8} \text{ sec.}$$

Notice that $t_2 = 2t_1$. This corresponds to the fact that the same time is required for the ball to go up (reach its maximum height from ground) as is required for the ball to come down (reach the ground from its maximum height).

The range R can then be obtained by inserting this value t_2 for t in Eq. 4–6*c*. We obtain, from $x = (v_0 \cos \theta_0)t$,

$$R = (v_0 \cos \theta_0)t_2 = (50 \text{ ft/sec})(\tfrac{8}{10})(\tfrac{15}{8} \text{ sec}) = 75 \text{ ft.}$$

(*d*) What is the velocity of the ball as it strikes the ground? From Eq. 4–6*a* we obtain

$$v_x = v_0 \cos \theta_0 = (50 \text{ ft/sec})(\tfrac{8}{10}) = 40 \text{ ft/sec.}$$

From Eq. 4–6*a*′ we obtain for $t = t_2 = \frac{15}{8}$ sec,

$$v_y = v_0 \sin \theta_0 - gt = (50 \text{ ft/sec})(\tfrac{6}{10}) - (32 \text{ ft/sec}^2)(\tfrac{15}{8} \text{ sec}) = -30 \text{ ft/sec.}$$

Hence, from Eq. 4–7,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(40 \text{ ft/sec})^2 + (-30 \text{ ft/sec})^2} = 50 \text{ ft/sec,}$$

and

$$\tan \theta = v_y/v_x = -\tfrac{30}{40},$$

so that $\theta = -37°$, or 37° *clockwise* from the x-axis. Notice that $\theta = -\theta_0$, as we expect from symmetry (Fig. 4–2).

Example 3. In a favorite lecture demonstration a gun is sighted at an elevated target which is released in free fall by a trip mechanism as the bullet leaves the muzzle. No matter what the initial speed of the bullet, it always hits the falling target.

The simplest way to understand this is the following. If there were no accleration due to gravity, the target would not fall and the bullet would move along the line of sight directly into the target (Fig. 4–4). The effect of gravity is to cause each body to accelerate down at the same rate from the position it would otherwise have had. Therefore, in the time t, the bullet will fall a distance $\frac{1}{2}gt^2$ from the position it would have had along the line of sight and the target will fall the same distance from its starting point. When the bullet reaches the line of fall of the target, it will be the same distance below the target's initial position as the target is and hence the collision. If the bullet moves faster than shown in the figure (v_0 larger), it will have a greater range and will cross the line of fall at a higher point; but since it gets there sooner, the target will fall a correspondingly smaller distance in the same time and collide with it. A similar argument holds for slower speeds.

For an equivalent analysis, let us use Eq. 4–5*b*

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2$$

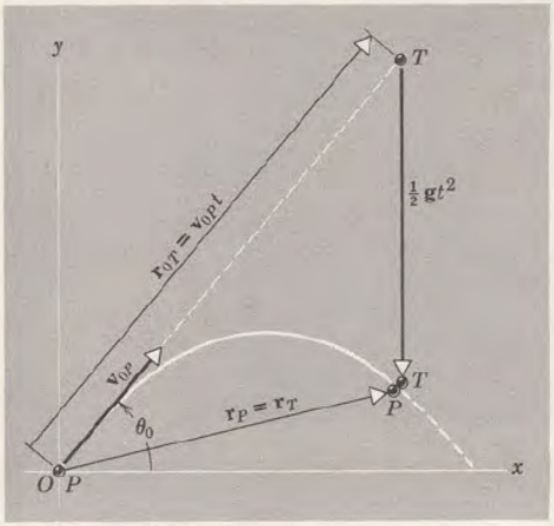

Fig. 4–4 Example 3. In the motion of a projectile, its displacement from the origin at any time t can be thought of as the sum of two vectors: $\mathbf{v}_{0P}t$, directed along $\mathbf{v}_{0P}$, and $\mathbf{g}t^2/2$, directed downward.

to describe the positions of the projectile and the target at any time t. For the projectile P, $\mathbf{r}_0 = 0$ and $\mathbf{a} = \mathbf{g}$, and we have

$$\mathbf{r}_P = \mathbf{v}_{0P}t + \tfrac{1}{2}\mathbf{g}t^2.$$

For the target T, $\mathbf{r}_0 = \mathbf{r}_{0T}$, $\mathbf{v}_0 = 0$, and $\mathbf{a} = \mathbf{g}$, leading to

$$\mathbf{r}_T = \mathbf{r}_{0T} + \tfrac{1}{2}\mathbf{g}t^2.$$

If there is a collision, we must have $\mathbf{r}_P = \mathbf{r}_T$. Inspection shows that this will always occur at a time t given by $\mathbf{r}_{0T} = \mathbf{v}_{0P}t$, that is, in the time t $(= r_{0T}/v_{0P})$ required for the projectile to travel to the target position along the line of sight, assuming that its initial velocity remains unchanged. ◀

4–4 Uniform Circular Motion

In Section 3–6 we saw that acceleration arises from a change in velocity. In the simple case of free fall the velocity changed in magnitude only, but not in direction. In a particle moving in a circle with constant speed, called uniform circular motion, the velocity vector changes continuously in direction but not in magnitude. We seek now to obtain the acceleration in uniform circular motion.

The situation is shown in Fig. 4–5*a*. Let P be the position of the particle at the time t and P' its position at the time $t + \Delta t$. The velocity at P is $\mathbf{v}$, a vector tangent to the curve at P. The velocity at P' is $\mathbf{v}'$, a vector tangent to the curve at P'. Vectors $\mathbf{v}$ and $\mathbf{v}'$ are equal in magnitude, the speed being constant, but their directions are different. The length of path traversed during Δt is the arc length PP', which is equal to $v\,\Delta t$, v being the constant speed.

Now redraw the vectors $\mathbf{v}$ and $\mathbf{v}'$, as in Fig. 4–5*b*, so that they originate at a common point. We are free to do this as long as the magnitude and

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direction of each vector are the same as in Fig. 4–5*a*. This diagram (Fig. 4–5*b*) enables us to see clearly the *change in velocity* as the particle moved from P to P'. This change, $\mathbf{v}' - \mathbf{v} = \Delta\mathbf{v}$, is the vector which must be added to $\mathbf{v}$ to get $\mathbf{v}'$. Notice that it points inward, approximately toward the center of the circle.

Now the triangle OQQ' formed by $\mathbf{v}$, $\mathbf{v}'$, and $\Delta\mathbf{v}$ is similar to the triangle CPP' formed by the chord PP' and the radii CP and CP'. This is so because both are isosceles triangles having the same vertex angle; the angle θ between $\mathbf{v}$ and $\mathbf{v}'$ is the same as the angle PCP' because $\mathbf{v}$ is perpendicular to CP and $\mathbf{v}'$ is perpendicular to CP'. We can therefore write

$$\frac{\Delta v}{v} = \frac{v\,\Delta t}{r}, \qquad \text{approximately,}$$

the chord PP' being taken equal to the arc length PP'. This relation becomes more nearly exact as Δt is diminished, since the chord and the arc then approach each other. Notice also that $\Delta\mathbf{v}$ approaches closer and closer to a direction perpendicular to $\mathbf{v}$ and $\mathbf{v}'$ as Δt is diminished and therefore approaches closer and closer to a direction pointing to the exact center of the circle. It follows from this relation that

$$\frac{\Delta v}{\Delta t} = \frac{v^2}{r}, \qquad \text{approximately,}$$

and in the limit when $\Delta t \to 0$ this expression becomes exact. We therefore obtain

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{v^2}{r} \tag{4–9}$$

as the magnitude of the acceleration. The direction of $\mathbf{a}$ is instantaneously along a radius inward toward the center of the circle.

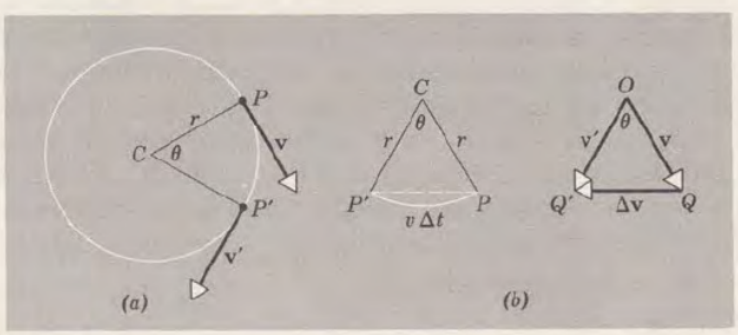

Fig. 4–5 Uniform circular motion. The particle travels around a circle at constant speed. Its velocity at two points P and P' is shown. Its change in velocity in going from P to P' is $\Delta\mathbf{v}$.

Figure 4–6 shows the instantaneous relation between **v** and **a** at various points of the motion. The magnitude of **v** is constant, but its direction changes continuously. This gives rise to an acceleration **a** which is also constant in magnitude (but not zero) but continuously changing in direction. The velocity **v** is always tangent to the circle in the direction of motion; the acceleration **a** is always directed radially inward. Because of this, **a** is called a *radial*, or *centripetal*, acceleration. *Centripetal* means "seeking a center."

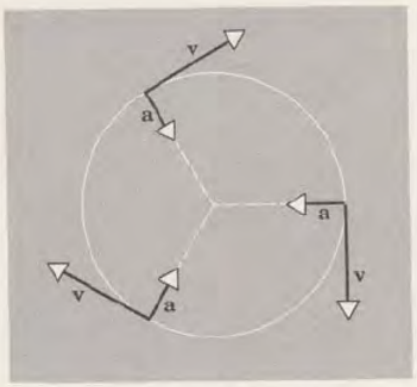

Fig. 4–6 In uniform circular motion the acceleration **a** is always directed toward the center of the circle and hence is perpendicular to **v**.

Both in free fall and in projectile motion **a** is constant in direction and magnitude and we can use the equations developed for constant acceleration (see Table 4–1). We cannot use these equations for uniform circular motion because **a** varies in direction and is therefore not constant.

The units of centripetal acceleration are the same as those of an acceleration resulting from a change in the magnitude of a velocity. Dimensionally, we have

$$\frac{v^2}{r} = \left(\frac{\text{length}}{\text{time}}\right)^2 \Big/ \text{length} = \frac{\text{length}}{\text{time}^2} \qquad \text{or} \qquad \frac{L}{T^2},$$

which are the dimensions of acceleration. The units therefore may be ft/sec^2, meters/sec^2, among others.

The acceleration resulting from a change in direction of a velocity is just as real and just as much an acceleration in every sense as that arising from a change in magnitude of a velocity. By definition, acceleration is the time rate of change of velocity, and velocity, being a vector, can change in direction as well as magnitude. If a physical quantity is a vector, its directional aspects cannot be ignored, for their effects will prove to be every bit as important and real as those produced by changes in magnitude.

It is worth emphasizing at this point that there need not be any motion in the direction of an acceleration and that there is no fixed relation in general between the directions of **a** and **v**. In Fig. 4–7 we give examples in which the angle between **v** and **a** varies from 0 to 180°. Only in one case, $\theta = 0°$, is the motion in the direction of **a**.

▶ **Example 4.** The moon revolves about the earth, making a complete revolution in 27.3 days. Assume that the orbit is circular and has a radius of 239,000 miles. What is the magnitude of the acceleration of the moon toward the earth?

We have $r = 239{,}000$ miles $= 3.85 \times 10^8$ meters. The time for one complete

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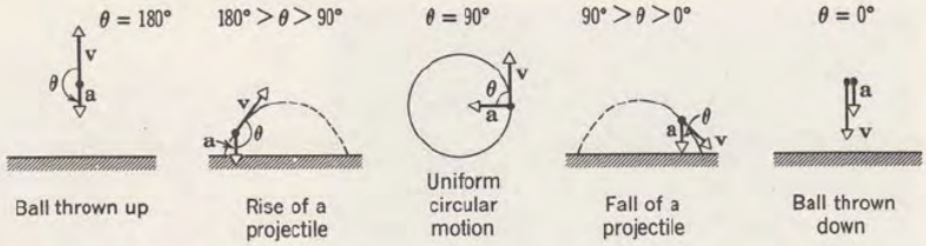

Fig. 4–7 Showing the relation between **v** and **a** for various motions.

revolution, called the period, is $T = 27.3$ days $= 2.36 \times 10^6$ sec. The speed of the moon (assumed constant) is therefore

$$v = 2\pi r/T = 1020 \text{ meters/sec.}$$

The centripetal acceleration is

$$a = \frac{v^2}{r} = \frac{(1020 \text{ meters/sec})^2}{3.85 \times 10^8 \text{ meters}} = 0.00273 \text{ meter/sec}^2, \qquad \text{or} \quad \text{only } 2.78 \times 10^{-4} g.$$

Example 5. Calculate the speed of an artificial earth satellite, assuming that it is traveling at an altitude h of 140 miles above the surface of the earth where $g = 30$ ft/sec^2. The radius of the earth R is 3960 miles.

Like any free object near the earth's surface the satellite has an acceleration g toward the earth's center. It is this acceleration that causes it to follow the circular path. Hence the centripetal acceleration is g, and from Eq. 4–9, $a = v^2/r$, we have

$$g = v^2/(R + h),$$

or

$$v = \sqrt{(R + h)g} = \sqrt{(3960 \text{ miles} + 140 \text{ miles})(5280 \text{ ft/mile})\,(30 \text{ ft/sec}^2)}$$
$$= 2.55 \times 10^4 \text{ ft/sec} = 17{,}400 \text{ miles/hr.}$$ ◀

Let us now derive Eq. 4–9 using vector methods. Figure 4–8a shows a particle in uniform circular motion about the origin O of a reference frame. For this motion the polar coordinates r, θ are more useful than the rectangular coordinates x, y because r remains constant throughout the motion and θ increases in a simple linear way with time; the behavior of x and y during such motion is more complex. The two sets of coordinates are related by

$$r = \sqrt{x^2 + y^2} \qquad \text{and} \qquad \theta = \tan^{-1} y/x \tag{4–10a}$$

or by the reciprocal relations

$$x = r \cos \theta \qquad \text{and} \qquad y = r \sin \theta. \tag{4–10b}$$

In rectangular reference frames we used the unit vectors **i** and **j** to describe motion in the x-y plane. Here we find it more convenient to introduce two new unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$. These, like **i** and **j**, have unit length and are dimensionless; they designate direction only.

The unit vector $\mathbf{u}_r$ at any point is in the direction of increasing **r** at that point; it is directed radially outward from the origin. The unit vector $\mathbf{u}_\theta$ at any point is in the direction of increasing θ at that point; it is always tangent to a circle

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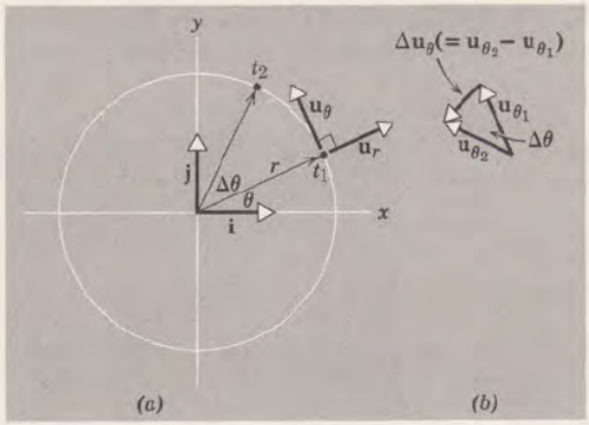

Fig. 4–8 (*a*) A particle moving counterclockwise in a circle of radius r. (*b*) The unit vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ at times t_1 and t_2 respectively, and the change $\Delta\mathbf{u}_\theta$ $(= \mathbf{u}_{\theta_2} - \mathbf{u}_{\theta_1})$.

through the point in a counterclockwise direction. As Fig. 4–8*a* shows, $\mathbf{u}_r$ and $\mathbf{u}_\theta$ are at right angles to each other. The unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$ differ from the unit vectors $\mathbf{i}$ and $\mathbf{j}$ in that the directions of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ vary from point to point in the plane; the unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$ are thus *not constant* vectors.

In terms of $\mathbf{u}_r$ and $\mathbf{u}_\theta$ the motion of a particle moving counterclockwise at uniform speed v in a circle about the origin in Fig. 4–8*a* can be described by the vector equation

$$\mathbf{v} = \mathbf{u}_\theta v. \tag{4–11}$$

This relation tells us, correctly, that the direction of $\mathbf{v}$ (which is the same as the direction of $\mathbf{u}_\theta$) is tangent to the circle and that the magnitude of $\mathbf{v}$ is the constant quantity v (because the magnitude of $\mathbf{u}_\theta$ is unity).

To find the acceleration we combine Eqs. 4–3 and 4–11, yielding

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d\mathbf{u}_\theta}{dt} v. \tag{4–12}$$

Note that v in Eq. 4–11 is a constant, but $\mathbf{u}_\theta$ is not since its direction changes as the particle moves. To evaluate $d\mathbf{u}_\theta/dt$, consider Fig. 4–8*b* which shows the unit vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ corresponding to an elapsed time Δt $(= t_2 - t_1)$ for the moving particle. The vector $\Delta\mathbf{u}_\theta$ $(= \mathbf{u}_{\theta_2} - \mathbf{u}_{\theta_1})$ points radially inward toward the origin in the limiting case as $\Delta t \to 0$. In other words, $d\mathbf{u}_\theta$ at any point has the direction of $-\mathbf{u}_r$. The angle between $\mathbf{u}_{\theta_2}$ and $\mathbf{u}_{\theta_1}$ in the figure is $\Delta\theta$, which is the angle swept out by a radial line from the origin to the particle in time Δt. The magnitude of $\Delta\mathbf{u}_\theta$ is simply $\Delta\theta$; bear in mind that the vectors $\mathbf{u}_{\theta_1}$ and $\mathbf{u}_{\theta_2}$ in Fig. 4–8*b* have the magnitude unity. Thus

$$\frac{d\mathbf{u}_\theta}{dt} = -\mathbf{u}_r \lim_{\Delta t \to 0} \frac{\Delta\theta}{dt} = -\mathbf{u}_r \frac{d\theta}{dt}$$

and, from Eq. 4–12,

$$\mathbf{a} = \frac{d\mathbf{u}_\theta}{dt} v = -\mathbf{u}_r \frac{d\theta}{dt} v. \tag{4–13}$$

Now, $d\theta/dt$ is the uniform angular rotation rate of the particle and is given by

$$\frac{d\theta}{dt} = \frac{2\pi \text{ radians}}{\text{time for one revolution}} = \frac{2\pi}{2\pi r/v} = \frac{v}{r}.$$

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Putting this into Eq. 4–13 leads us finally to

$$\mathbf{a} = -\mathbf{u}_r \frac{v^2}{r} \tag{4–14}$$

which tells us that the acceleration in uniform circular motion has a magnitude v^2/r (see Eq. 4–9) and points radially inward (note the factor $-\mathbf{u}_r$). The vector relation Eq. 4–14 thus tells us *both* the magnitude *and* the direction of the centripetal acceleration $\mathbf{a}$. Note that, as expected, $\mathbf{a}$ has a constant magnitude but changes continually in direction because $\mathbf{u}_r$ changes continually in direction.

4–5 Tangential Acceleration in Circular Motion

We now consider the more general case of circular motion in which the speed v of the moving particle is *not* constant. We shall use vector methods in polar coordinates.

As before, the velocity is given by Eq. 4–11, or

$$\mathbf{v} = \mathbf{u}_\theta v$$

except that, in this case, not only $\mathbf{u}_\theta$ but also v varies with time. Recalling the formula for the derivative of a product, one obtains for the acceleration

$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \mathbf{u}_\theta \frac{dv}{dt} + v\frac{d\mathbf{u}_\theta}{dt} \tag{4–15}$$

In Eq. 4–12 the first term in this equation was not present because, v being there assumed to be constant, dv/dt was zero. The last term in Eq. 4–15 reduces, as we saw in the last section, to $-\mathbf{u}_r(v^2/r)$. We can now write Eq. 4–15 as

$$\mathbf{a} = \mathbf{u}_\theta a_T - \mathbf{u}_r a_R, \tag{4–16}$$

in which $a_T = dv/dt$ and $a_R = v^2/r$ The first term, $\mathbf{u}_\theta a_T$, is the vector component of $\mathbf{a}$ that is tangent to the path of the particle and arises from a change in the magnitude of the velocity in circular motion (see Fig. 4–9). This term and a_T are called the *tangential acceleration*. The second term $-\mathbf{u}_r a_R$ is the vector component of $\mathbf{a}$ directed radially in toward the center of the circle and arises from a

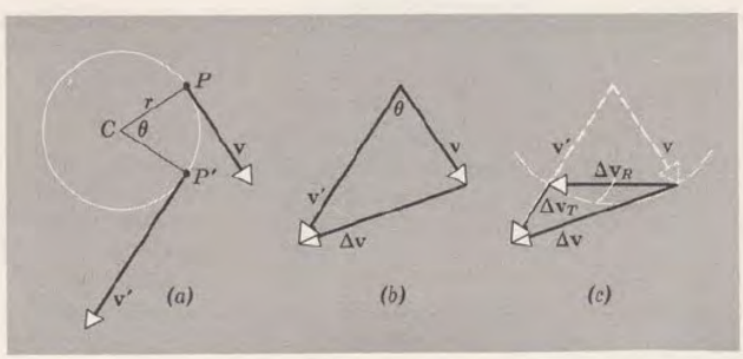

Fig. 4–9 In nonuniform circular motion the speed is variable. The change in velocity $\Delta\mathbf{v}$ in going from P to P' is made up of two parts: $\Delta\mathbf{v}_R$ caused by the change in direction of $\mathbf{v}$, and $\Delta\mathbf{v}_T$ caused by the change in magnitude of $\mathbf{v}$. In the limit as $\Delta t \to 0$, $\Delta\mathbf{v}_R$ points toward the center C of the circle and $\Delta\mathbf{v}_T$ is tangent to the circular path.

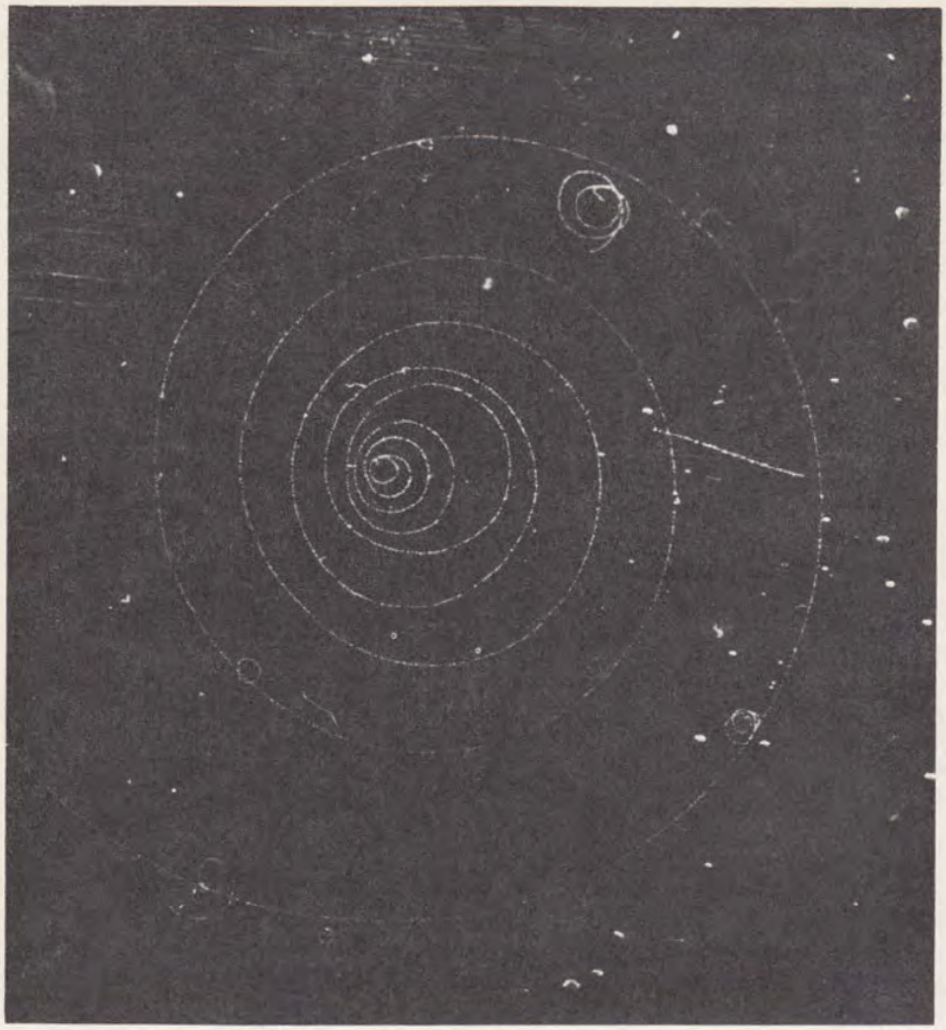

Fig. 4–10 A track left in a 10-in. liquid-hydrogen-filled bubble chamber by an energetic spiralling electron. (Courtesy Lawrence Radiation Laboratory.) This picture is one of a number in a collection prepared for easy stereoscopic viewing and published, with explanatory material, as *Introduction to the Detection of Nuclear Particles in a Bubble Chamber,* The Ealing Press, Cambridge 40, Massachusetts (1964). When viewed stereoscopically the electron is seen to be moving toward the reader as it moves in along the spiral. Its velocity vector at any point, thus, does not lie in the plane of the figure, but tilts up out of it; its motion is thus three-dimensional, rather than two-dimensional as we assumed for other examples in this chapter.

change in the direction of the velocity in circular motion (see Fig. 4–9). This term and a_R are called the *centripetal acceleration.*

The magnitude of the instantaneous acceleration is

$$a = \sqrt{a_T{}^2 + a_R{}^2} \tag{4–17}$$

If the speed is constant, then $a_T = dv/dt = 0$ and Eq. 4–16 reduces to Eq. 4–14. When the speed v is not constant, a_T is not zero and a_R varies from point to point. If the speed changes at a rate that is not constant, then a_T will also vary from point to point.

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If the motion is not circular, the formulas for a_T ($= dv/dt$) and for a_R ($= v^2/r$) can still be applied if instead of using for r the magnitude of the radius vector from the origin we substitute the radius of curvature of the path at the instantaneous position of the particle. Then a_T gives the component of acceleration tangent to the curve at that position, and a_R gives the component of acceleration normal to the curve at that position. Figure 4–10 shows the track left in a liquid-hydrogen-filled bubble chamber by an energetic electron that spirals inward. The electron loses energy as it traverses the liquid in the chamber so that its speed v is being reduced steadily. Thus there is at every point a *tangential acceleration* a_T given by dv/dt. The *centripetal acceleration* a_R at any point is given by v^2/r, where r is the radius of curvature of the track at the point in question; both v and r become smaller as the particle loses energy. The force causing the electron to spiral is produced by a magnetic field present in the bubble chamber and at right angles to the plane of Fig. 4–10 (see Chapter 33).

4–6 Relative Velocity and Acceleration

In earlier sections we considered the addition of velocities in a particular reference frame. Let us now consider the relation between the velocity of an object as determined by one observer S (= reference frame S) and the velocity of the same object as determined by another observer S' (= reference frame S') who is moving with respect to the first.

Consider observer S fixed to the earth, so that his reference frame is the earth. The other observer S' is moving on the earth—for example, a passenger sitting on a moving train—so that his reference frame is the train. They each follow the motion of the same object, say an automobile on a road or a man walking through the train. Each observer will record a displacement, a velocity, and an acceleration for this object measured *relative to his reference frame*. How will these measurements compare? In this section we consider only the case in which the second frame is in motion with respect to the first with a *constant* velocity **u**.

In Fig. 4–11 the reference frame S represented by the x- and y-axes can

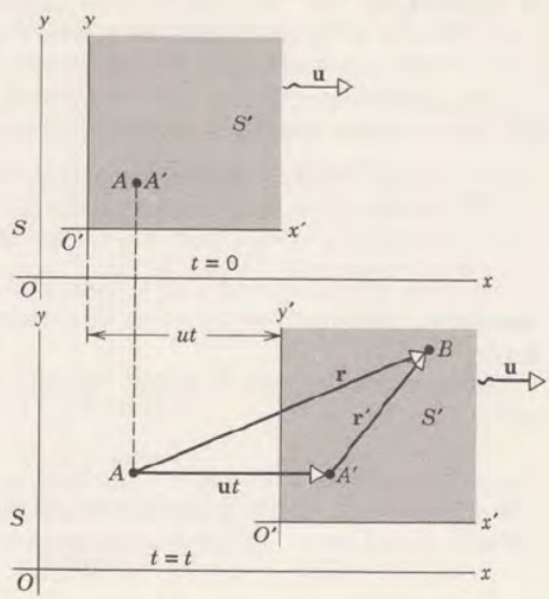

Fig. 4–11 Two reference frames, S ($= x, y$) and S' ($= x', y'$); S' moves to the right, relative to S, with speed u.

be thought of as fixed to the earth. The shaded region indicates another reference frame S', represented by x'- and y'-axes, which moves along the x-axis with a constant velocity $\mathbf{u}$, as measured in the S-system; it can be thought of as drawn on the floor of a railroad flatcar.

Initially, a particle (say a ball on the flatcar) is at a position called A in the S-frame and called A' in the S'-frame. At a time t later the flatcar and its S' reference frame have moved a distance ut to the right and the particle has moved to B. The *displacement* of the particle from its initial position *in the S-frame* is the vector $\mathbf{r}$ from A to B. The *displacement* of the particle from its initial position *in the S′-frame* is the vector $\mathbf{r}'$ from A' to B. These are different vectors because the reference point A' of the moving frame has been displaced a distance ut along the x-axis during the motion. From the figure we see that $\mathbf{r}$ is the vector sum of $\mathbf{r}'$ and $\mathbf{u}t$:

$$\mathbf{r} = \mathbf{r}' + \mathbf{u}t. \tag{4–18}$$

Differentiating Eq. 4–18 leads to

$$\frac{d\mathbf{r}}{dt} = \frac{d\mathbf{r}'}{dt} + \mathbf{u}.$$

But $d\mathbf{r}/dt = \mathbf{v}$, the instantaneous velocity of the particle measured in the S-frame, and $d\mathbf{r}'/dt = \mathbf{v}'$, the instantaneous velocity of the same particle measured in the S' frame, so that

$$\mathbf{v} = \mathbf{v}' + \mathbf{u}. \tag{4–19}$$

Hence the velocity of the particle relative to the S-frame, $\mathbf{v}$, is the vector sum of the velocity of the particle relative to the S'-frame, $\mathbf{v}'$, and the velocity $\mathbf{u}$ of the S'-frame relative to the S-frame.

▶ **Example 6.** (*a*) The compass of an airplane indicates that it is heading due east. Ground information indicates a wind blowing due north. Show on a diagram the velocity of the plane with respect to the ground.

The object is the airplane. The earth is one reference frame (S) and the air is the other reference frame (S') moving with respect to the first. Then

$\mathbf{u}$ is the velocity of the air with respect to the ground.
$\mathbf{v}'$ is the velocity of the plane with respect to the air.
$\mathbf{v}$ is the velocity of the plane with respect to the ground.

In this case $\mathbf{u}$ points north and $\mathbf{v}'$ points east. Then the relation $\mathbf{v} = \mathbf{v}' + \mathbf{u}$ determines the velocity of the plane with respect to the ground, as shown in Fig. 4–12*a*.

The angle α is the angle N of E of the plane's course with respect to the ground and is given by

$$\tan \alpha = u/v'.$$

The airplane's speed with respect to the ground is given by

$$v = \sqrt{(v')^2 + u^2}.$$

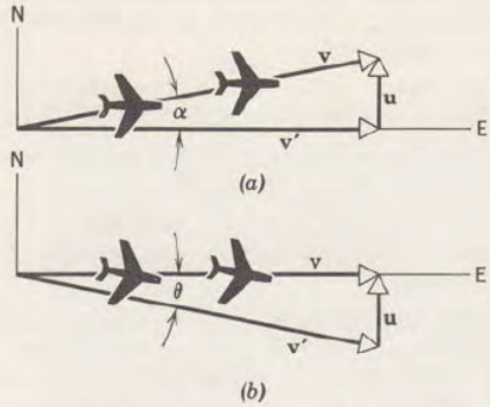

Fig. 4–12 Example 6.

For example, if the air-speed indicator shows that the plane is moving relative to the air at a speed of 200 miles/hr, and if the speed of the wind with respect to the ground is 40.0 miles/hr, then

$$v = \sqrt{(200)^2 + (40.0)^2}\ \text{miles/hr} = 204\ \text{miles/hr}$$

is the ground speed of the plane and

$$\alpha = \tan^{-1}\frac{40.0}{200} = 11°\ 20'$$

gives the course of the plane N of E.

(*b*) Now draw the vector diagram showing the direction the pilot must steer the plane through the air for the plane to travel due east with respect to the ground.

He would naturally head partly into the wind. His speed relative to the earth will therefore be less than before. The vector diagram is shown in Fig. 4–12*b*. The student should calculate θ and v, using the previous data for $\mathbf{u}$ and $\mathbf{v}'$. ◀

We have seen that different velocities are assigned to a particle by different observers when the observers are in relative motion. These velocities *always differ by* the relative velocity of the two observers, which here is *a constant velocity*. It follows that when the particle velocity changes, the *change will be the same* for both observers. Hence they each measure the *same acceleration* for the particle. The acceleration of a particle is the same in *all* reference frames moving relative to one another with constant velocity; that is, $\mathbf{a} = \mathbf{a}'$. This result follows in a formal way if we differentiate Eq. 4–19. Thus $d\mathbf{v}/dt = d\mathbf{v}'/dt + d\mathbf{u}/dt$; but $d\mathbf{u}/dt = 0$ when $\mathbf{u}$ is constant, so that $\mathbf{a} = \mathbf{a}'$.

QUESTIONS

1. In projectile motion when air resistance is negligible, is it ever necessary to consider three-dimensional motion rather than two-dimensional?

2. In broad jumping does it matter how high you jump? What factors determine the span of the jump?

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3. Why doesn't the electron in the beam from an electron gun fall as much because of gravity as a water molecule in the stream from a hose? Assume horizontal motion initially in each case.

4. An aviator, pulling out of a dive, follows the arc of a circle. He was said to have "experienced $3g$'s" in pulling out of the dive. Explain what this statement means.

5. Describe qualitatively the acceleration acting on a bead which moves inward with constant speed along a spiral.

6. Could the acceleration of a projectile be represented in terms of a radial and a tangential component at each point of the motion? If so, is there any advantage to this representation?

7. A boy sitting in a railroad car moving at constant velocity throws a ball straight up into the air. Will the ball fall behind him? In front of him? Into his hand? What happens if the car accelerates forward or goes around a curve while the ball is in the air?

8. A man on the observation platform of a train moving with constant velocity drops a coin while leaning over the rail. Describe the path of the coin as seen by (*a*) the man on the train, (*b*) a person standing on the ground near the track, and (*c*) a person in a second train moving in the opposite direction to the first train on a parallel track.

9. A bus with a vertical windshield moves along in a rainstorm at speed v_b. The raindrops fall vertically with a terminal speed v_r. At what angle do the raindrops strike the windshield?

10. Drops are falling vertically in a steady rain. In order to go through the rain from one place to another in such a way as to encounter the least number of raindrops, should you move with the greatest possible speed, the least possible speed, or some intermediate speed?

11. An elevator is descending at a constant speed. A passenger takes a coin from his pocket and drops it to the floor. What accelerations would (*a*) the passenger and (*b*) a person at rest with respect to the elevator shaft observe for the falling coin?

PROBLEMS

1. Prove that for a vector **a** defined by

$$\mathbf{a} = \mathbf{i}a_x + \mathbf{j}a_y + \mathbf{k}a_z$$

the scaler components are given by

$$a_x = \mathbf{i} \cdot \mathbf{a}, \qquad a_y = \mathbf{j} \cdot \mathbf{a}, \qquad \text{and} \qquad a_z = \mathbf{k} \cdot \mathbf{a}.$$

2. A ball rolls off the edge of a horizontal table top 4.0 ft high. If it strikes the floor at a point 5.0 ft horizontally away from the edge of the table, what was its speed at the instant it left the table?

3. A ball rolls off the top of a stairway with a horizontal velocity of magnitude 5.0 ft/sec. The steps are 8.0 in. high and 8.0 in. wide. Which step will the ball hit first?

4. A shell is fired horizontally from a powerful gun located 144 ft above a horizontal plane with a muzzle speed of 800 ft/sec. (*a*) How long does the shell remain in the air? (*b*) What is its range? (*c*) What is the magnitude of the vertical component of its velocity as it strikes the target?

5. Show that the maximum height reached by a projectile is $y_{\max} = (v_0 \sin \theta_0)^2/2g$.

6. Show that the horizontal range of a projectile having an initial speed v_0 and angle of projection θ_0 is $R = (v_0^2/g) \sin 2\theta_0$. Then show that a projection angle of 45° gives the maximum horizontal range (Fig. 4–13).

7. Find the angle of projection at which the horizontal range and the maximum height of a projectile are equal.

8. In Galileo's *Two New Sciences* the author states that "for elevations (angles of projection) which exceed or fall short of 45° by equal amounts, the ranges are equal . . . ". Prove this statement.

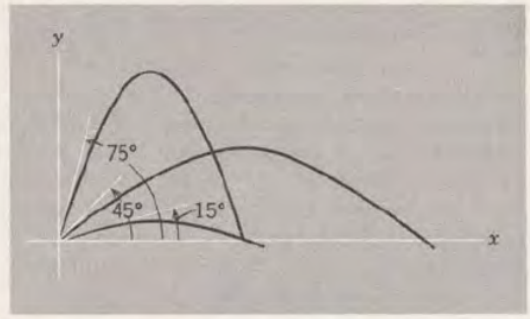

Fig. 4–13

9. A rifle with a muzzle velocity of 1500 ft/sec shoots a bullet at a small target 150 ft away. How high above the target must the gun be aimed so that the bullet will hit the target?

10. A dive bomber, diving at an angle of 53° with the vertical, releases a bomb at an altitude of 2400 ft. The bomb hits the ground 5.0 sec after being released. (*a*) What is the speed of the bomber? (*b*) How far did the bomb travel horizontally during its flight? (*c*) What were the horizontal and vertical components of its velocity just before striking the ground?

11. A batter hits a pitched ball at a height 4.0 ft above ground so that its angle of projection is 45° and its horizontal range is 350 ft. The ball is fair down the left field line where a 24-ft-high fence is located 320 ft from home plate. Will the ball clear the fence?

12. A football is kicked off with an initial speed of 64 ft/sec at a projection angle of 45°. A receiver on the goal line 60 yd away in the direction of the kick starts running to meet the ball at that instant. What must his speed be if he is to catch the ball before it hits the ground?

13. In a cathode-ray tube a beam of electrons is projected horizontally with a speed of 1.0×10^9 cm/sec into the region between a pair of horizontal plates 2.0 cm long. An electric field between the plates exerts a constant downward acceleration on the electrons of magnitude 1.0×10^{17} cm/sec^2. Find (*a*) the vertical displacement of the beam in passing through the plates and (*b*) the velocity of the beam (direction and magnitude) as it emerges from the plates.

14. (*a*) Show that if the acceleration of gravity changes by an amount dg, the range of a projectile (see Problem 6) of given initial speed v_0 and angle of projection θ_0 changes by dR where $dR/R = -dg/g$. (*b*) If the acceleration of gravity changes by a small amount Δg (say by going from one place to another), the range for a given projectile system will change as well. Let the change in range be ΔR. If Δg, ΔR are small enough, we may write $\Delta R/R = -\Delta g/g$. In 1936, Jesse Owens (United States) established a world's running broad jump record of 8.09 meters at the Olympic games at Berlin ($g = 9.8128$ meters/sec^2). By how much would his record have differed if he had competed instead in 1956 at Melbourne ($g = 9.7999$ meters/sec^2)? (In this connection see "Bad Physics in Athletic Measurements," by P. Kirkpatrick, *American Journal of Physics*, February 1944.)

15. Electrons, nuclei, atoms and molecules, like all forms of matter, will fall under the influence of gravity. Consider separately a beam of electrons, of nuclei, of atoms, and of molecules traveling a horizontal distance of 1.0 meter. Let the average speed be

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for an electron 3.0×10^7 meters/sec, for a thermal neutron 2.2×10^3 meters/sec, for a neon atom 5.8×10^2 meters/sec, and for an oxygen molecule 4.6×10^2 meters/sec. Let the beams move through vacuum with initial horizontal velocities and find by how much their paths deviate from a straight line (vertical displacement in 1.0 meter) due to gravity. How do these results compare to that for a beam of golf balls (use reasonable data)? What is the controlling factor here?

16. A radar observer on the ground is "watching" an approaching projectile. At a certain instant he has the following information: (*a*) the projectile has reached maximum altitude and is moving horizontally with a speed v; (*b*) the straight-line distance to the projectile is l; (*c*) the line of sight to the projectile is an angle θ above the horizontal. Find the distance D between the observer and the point of impact of the projectile. Does the projectile pass over his head or strike the ground before reaching him? D is to be expressed in terms of the observed quantities v, l, and θ and the known value of g. Assume a flat earth; assume also that the observer lies in the plane of the projectile's trajectory.

17. Show that Eqs. 4–4*d*, *d*′ in Table 4–1 can be expressed in vector form as

$$\vec{\mathbf{v}} \cdot \vec{\mathbf{v}} = \vec{\mathbf{v}}_0 \cdot \vec{\mathbf{v}}_0 + 2\vec{\mathbf{a}} \cdot (\vec{\mathbf{r}} - \vec{\mathbf{r}}_0),$$

that Eqs. 4–4*b*, *b*′ can be expressed as

$$\mathbf{r} = \mathbf{r}_0 + \tfrac{1}{2}(\mathbf{v}_0 + \mathbf{v})t,$$

and Eqs. 4–4*c*, *c*′ as

$$\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t + \tfrac{1}{2}\mathbf{a}t^2.$$

18. Projectiles are hurled at a horizontal distance R from the edge of a cliff of height h in such a way as to land a horizontal distance x from the bottom of the cliff. If you want x to be as small as possible, how would you adjust θ_0 and v_0, assuming that v_0 can be varied from zero to some maximum finite value and that θ_0 can be varied continuously? Only one collision with the ground is allowed (see Fig. 4–14).

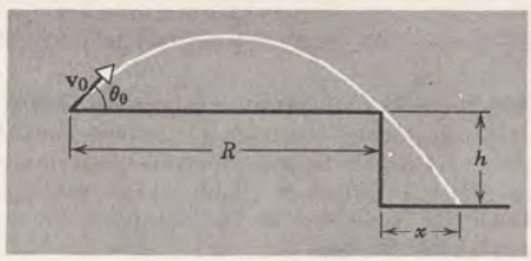

Fig. 4–14

19. Consider a projectile at the top of its trajectory. (*a*) What is its speed in terms of v_0 and θ_0? (*b*) What is its acceleration? (*c*) How is the direction of its acceleration related to that of its velocity? (*d*) Over a short distance a circular arc is a good approximation to a parabola. What then is the radius of the circular arc approximating the projectile's motion near the top of its path?

20. A particle rests on the top of a hemisphere of radius R. Find the smallest horizontal velocity that must be imparted to the particle if it is to leave the hemisphere without sliding down it.

21. A magnetic field will deflect a charged particle perpendicular to its direction of motion. An electron experiences a radial acceleration of 3.0×10^{14} meters/sec^2 in one such field. What is its speed if the radius of its curved path is 0.15 meter?

22. In Bohr's model of the hydrogen atom an electron revolves around a proton in a circular orbit of radius 5.28×10^{-11} meter with a speed of 2.18×10^{6} meters/sec. What is the acceleration of the electron in the hydrogen atom?

23. Find the magnitude of the centripetal acceleration of a particle on the tip of a fan blade, 0.30 meter in diameter, rotating at 1200 rev/min.

24. By what factor would the speed of the earth's rotation have to increase for a body on the equator to require a centripetal acceleration of g to keep it on the earth? Such a body now requires a centripetal acceleration of only about 3.0 cm/sec^2.

25. A particle travels with constant speed on a circle of radius 3.0 meters and completes one revolution in 20 sec (Fig. 4–15). Starting from the origin O, find (*a*) the magnitude and direction of the displacement vectors 5.0 sec, 7.5 sec, and 10 sec later; (*b*) the magnitude and direction of the displacement in the 5.0-sec interval from the fifth to the tenth second; (*c*) the average velocity vector in this interval; (*d*) the instantaneous velocity vector at the beginning and at the end of this interval; (*e*) the average acceleration vector in this interval; and (*f*) the instantaneous acceleration vector at the beginning and at the end of this interval.

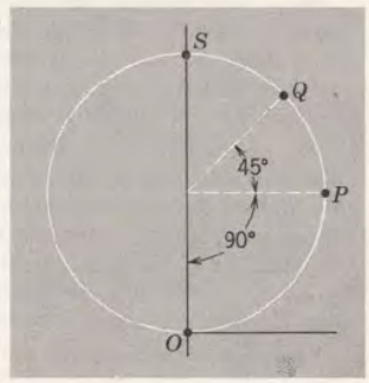

Fig. 4–15

26. An earth satellite moves in a circular orbit 400 miles above the earth's surface. The time for one revolution (the period) is found to be 98 min. Find the acceleration of gravity at the orbit from these data.

27. The earth revolves about the sun in a (nearly) circular orbit with a (nearly) constant speed of 30 km/sec. What is the acceleration of the earth toward the sun?

28. A particle moves in a plane according to

$$x = R \sin \omega t + \omega R t,$$

$$y = R \cos \omega t + R,$$

where ω and R are constants. This curve, called a *cycloid*, is the path traced out by a point on the rim of a wheel which rolls without slipping along the x-axis. (*a*) Sketch the path. (*b*) Calculate the instantaneous velocity and acceleration when the particle is at its maximum and minimum value of y.

29. (*a*) Write an expression for the position vector $\mathbf{r}$ for a particle describing uniform circular motion, using rectangular coordinates and the unit vectors $\mathbf{i}$ and $\mathbf{j}$. (*b*) From (*a*) derive vector expressions for the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$. (*c*) Prove that the acceleration is directed toward the center of the circular motion.

30. Write an expression, using the unit vectors $\mathbf{u}_\theta$ and $\mathbf{u}_r$, for the position vector $\mathbf{r}$ for a particle describing uniform circular motion and from it derive Eq. 4–11, $\mathbf{v} = \mathbf{u}_\theta v$.

31. Express the unit vectors $\mathbf{u}_r$ and $\mathbf{u}_\theta$ in terms of $\mathbf{i}$, $\mathbf{j}$, and the angle θ in Fig. 4–8.

32. A person walks up a stalled escalator in 90 sec. When standing on the same escalator, now moving, he is carried up in 60 sec. How much time would it take him to walk up the moving escalator?

33. Find the speed of two objects if, when they move uniformly toward each other, they get 4.0 meters closer each second, and, when they move uniformly in the same direction with the original speeds, they get 4.0 meters closer each 10 sec.

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34. A man can row a boat 4.0 miles/hr in still water. (*a*) If he is crossing a river where the current is 2.0 miles/hr, in what direction will his boat be headed if he wants to reach a point directly opposite from his starting point? (*b*) If the river is 4.0 miles wide, how long will it take him to cross the river? (*c*) How long will it take him to row 2.0 miles *down* the river and then back to his starting point? (*d*) How long will it take him to row 2.0 miles *up* the river and then back to his starting point? (*e*) In what direction should he head the boat if he wants to cross in the smallest possible time?

35. A man wants to cross a river 500 meters wide. His rowing speed (relative to the water) is 3000 meters/hr. The river flows at a speed of 2000 meters/hr. If the man's walking speed on shore is 5000 meters/hr, (*a*) find the path (combined rowing and walking) he should take to get to the point directly opposite his starting point in the shortest time. (*b*) How long does it take?

36. A train travels due south at 88.2 ft/sec (relative to ground) in a rain that is blown toward the south by the wind. The path of each raindrop makes the angle 21.6° with the vertical, as measured by an observer stationary on the earth. An observer seated in the train, however, sees perfectly vertical tracks of rain on the windowpane. Determine the speed of each raindrop relative to the earth.

37. An airplane has a speed of 135 miles/hr in still air. It is flying straight north so that it is at all times directly above a north-south highway. A ground observer tells the pilot by radio that a 70-miles/hr wind is blowing, but neglects to tell him the wind direction. The pilot observes that in spite of the wind he can travel 135 miles *along* the highway in one hour. In other words, his ground speed is the same as if there were no wind. (*a*) What is the direction of the wind? (*b*) What is the heading of the plane, that is, the angle between its axis and the highway?

38. A pilot is supposed to fly due east from A to B and then back again to A due west. The velocity of the plane in air is $\mathbf{v}'$ and the velocity of the air with respect to the ground is $\mathbf{u}$. The distance between A and B is l and the plane's air speed $\mathbf{v}'$ is constant. (*a*) If $u = 0$ (still air), show that the time for the round trip is $t_0 = 2l/v'$. (*b*) Suppose that the air velocity is due east (or west). Show that the time for a round trip is then

$$t_E = \frac{t_0}{1 - u^2/(v')^2}.$$

(*c*) Suppose that the air velocity is due north (or south). Show that the time for a round trip is then

$$t_N = \frac{t_0}{\sqrt{1 - u^2/(v')^2}}.$$

(*d*) In parts (*b*) and (*c*) one must assume that $u < v'$. Why?