

Calculation of CCT and Duv and Practical Conversion Formulae

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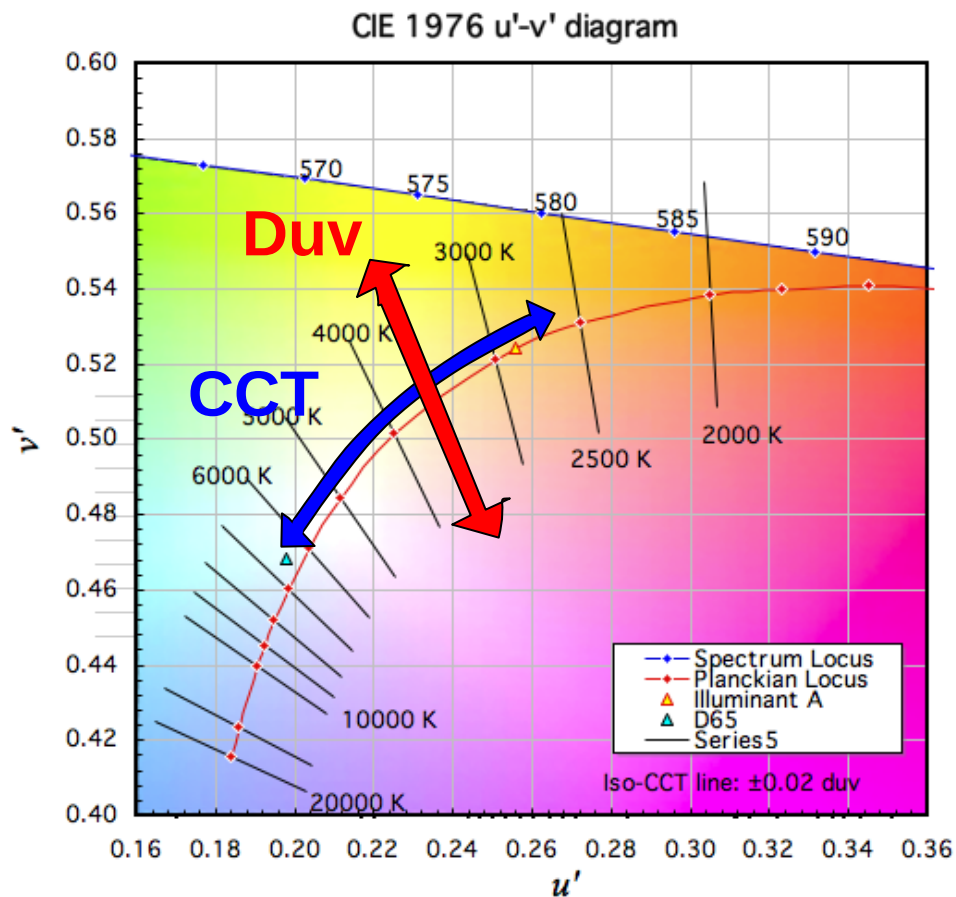
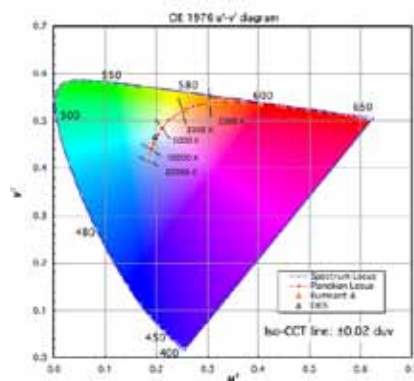
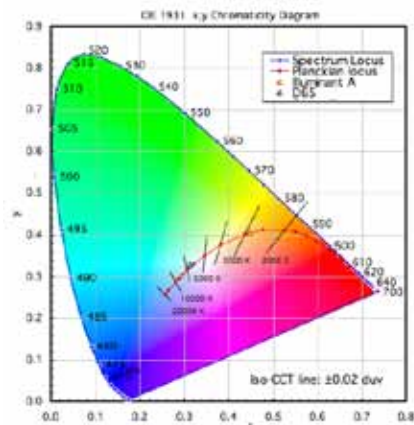
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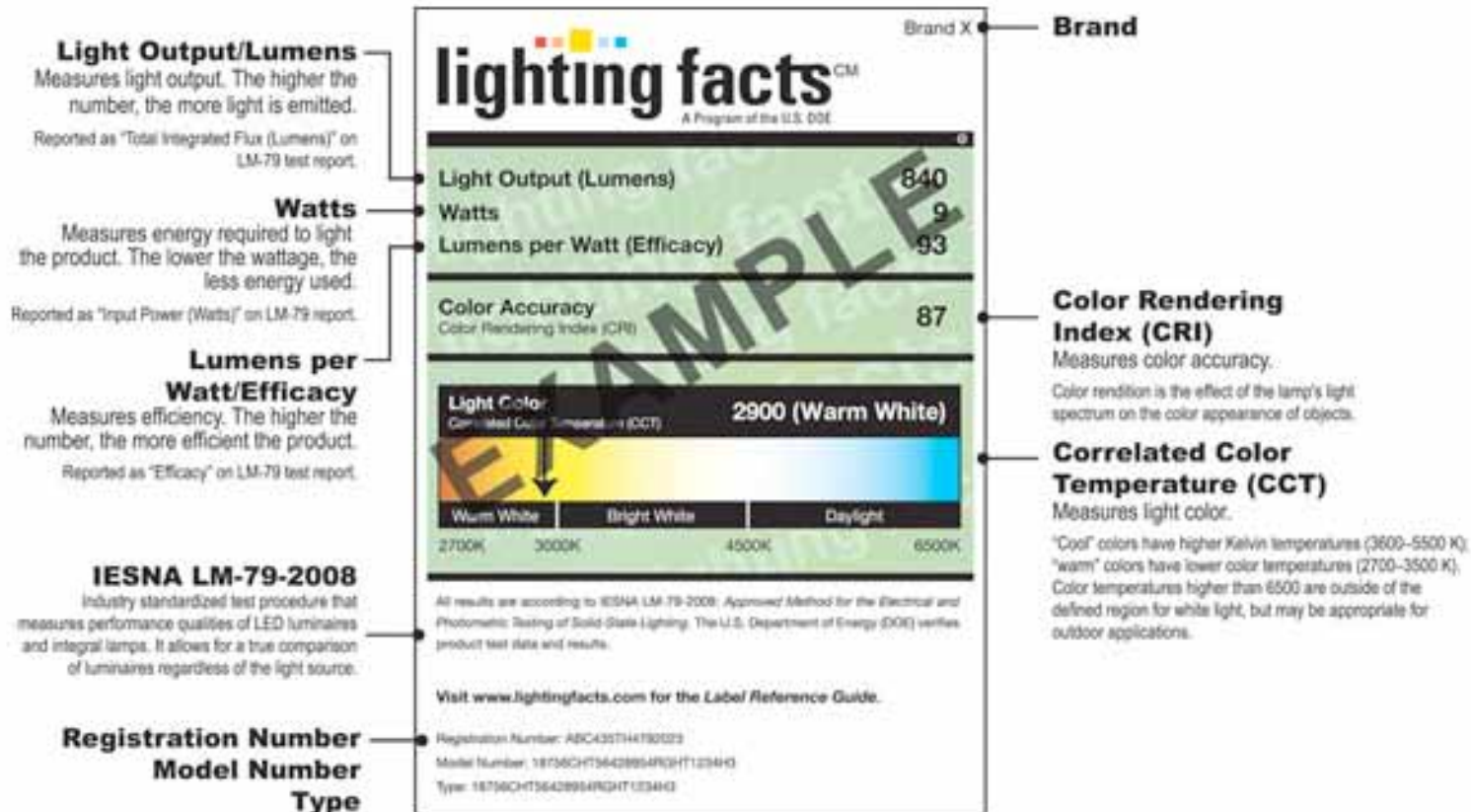
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White Light Chromaticity



Duv often missing

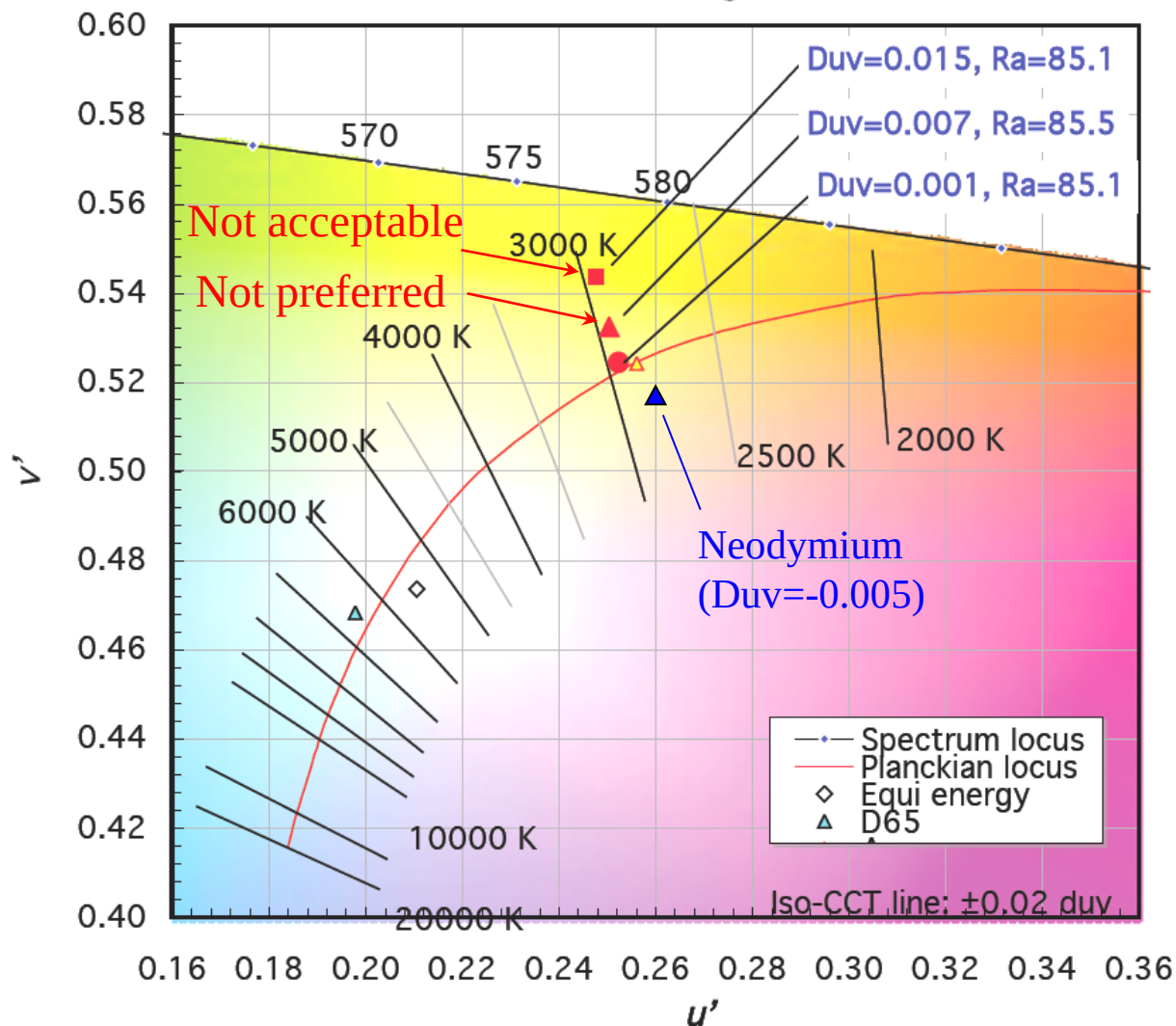
Lighting Facts Label



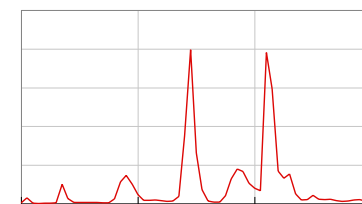
CCT and CRI do not tell the whole story of color quality

CCT and CRI do not tell the whole story

CIE 1976 u' - v' diagram



Triphosphor
FL simulation

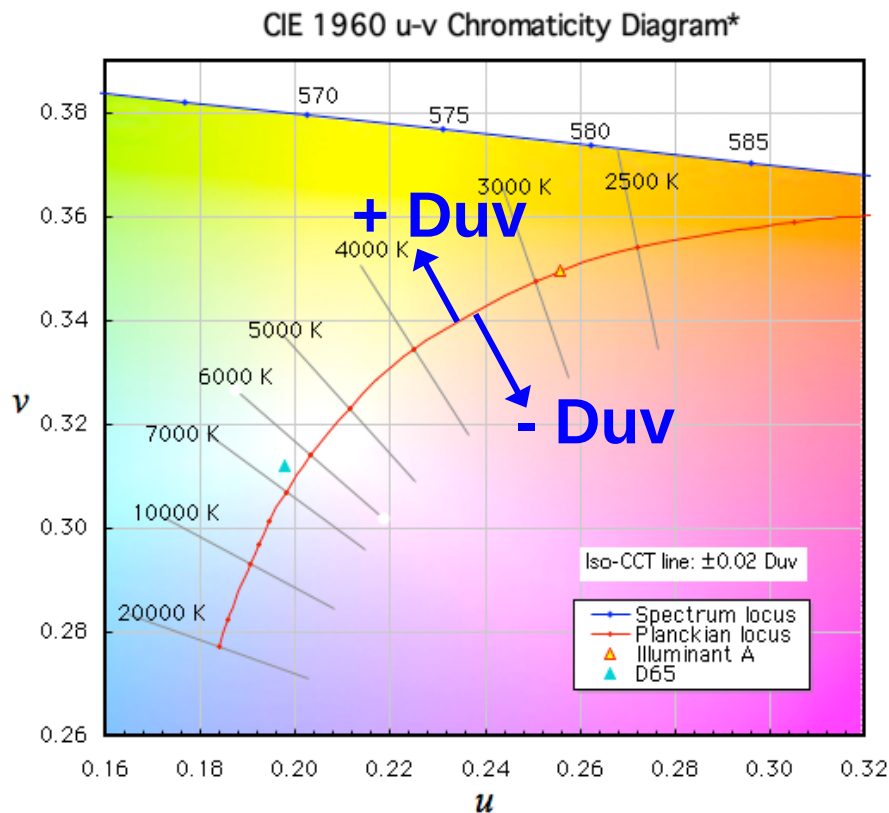


Duv is another
important
dimension of
chromaticity.



Duv defined in ANSI standard

Closest distance from the Planckian locus on the $(u', 2/3 v')$ diagram, with + sign for above and - sign for below the Planckian locus. (ANSI C78.377-2008)



Symbol: D_{uv}

CCT and Duv can specify the chromaticity of light sources just like (x, y) .

The two numbers (CCT, Duv) provides color information intuitively. (x, y) does not.

Duv needs to be defined by CIE.

ANSI C78.377-2008 Specifications for the chromaticity of SSL products

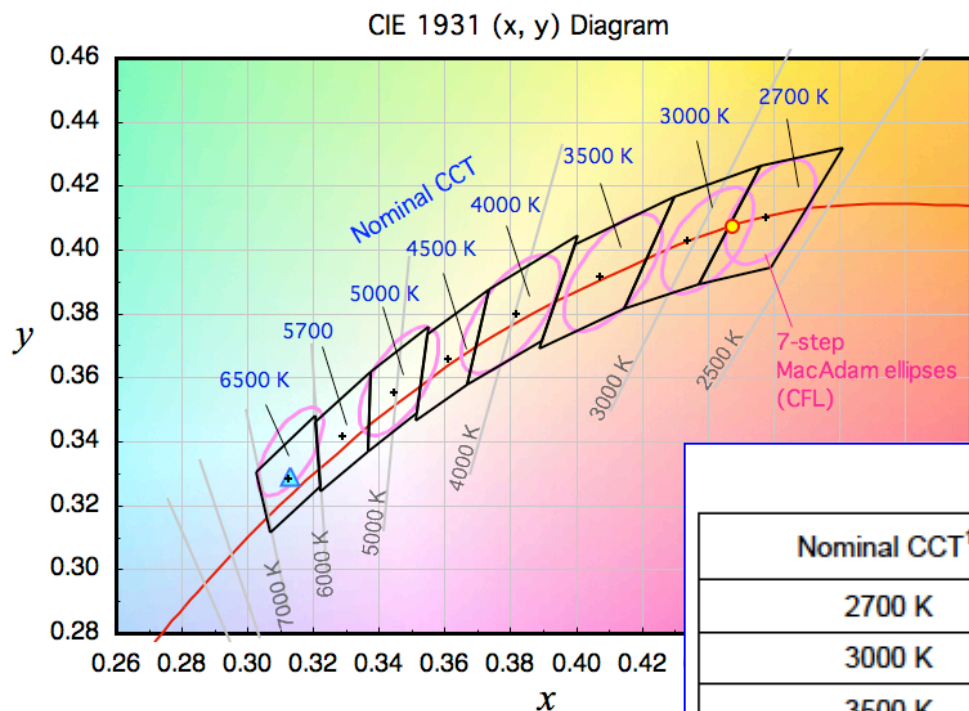
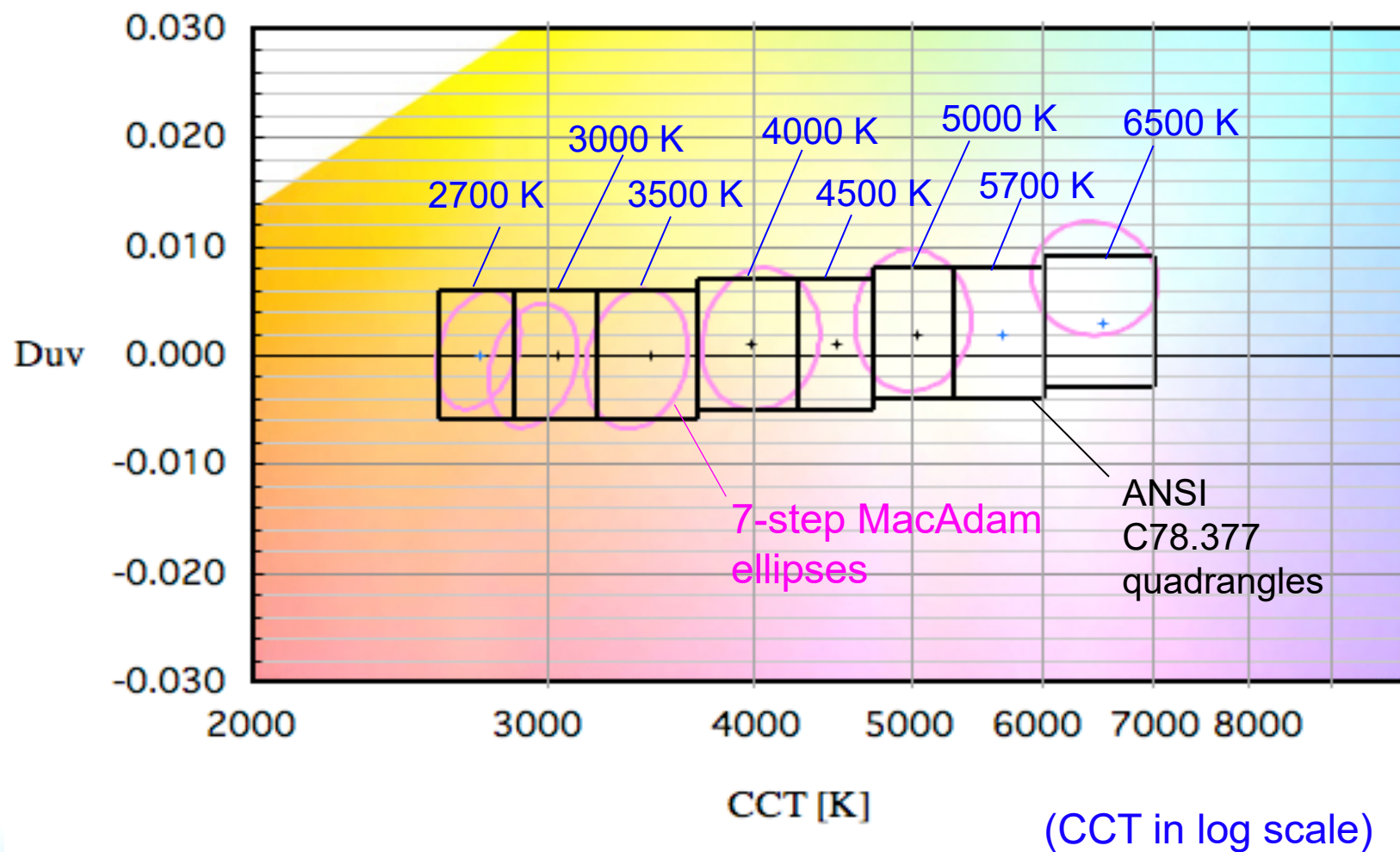


Table 1 - Nominal CCT Categories

Nominal CCT ¹⁾	Target CCT and tolerance (K)	Target Duv and tolerance
2700 K	2725 ± 145	0.000 ± 0.006
3000 K	3045 ± 175	0.000 ± 0.006
3500 K	3465 ± 245	0.000 ± 0.006
4000 K	3985 ± 275	0.001 ± 0.006
4500 K	4503 ± 243	0.001 ± 0.006
5000 K	5028 ± 283	0.002 ± 0.006
5700 K	5665 ± 355	0.002 ± 0.006
6500 K	6530 ± 510	0.003 ± 0.006
Flexible CCT (2700 - 6500 K)	$T^{2)}$ ± $\Delta T^{3)}$	$D_{uv}^{4)}$ ± 0.006

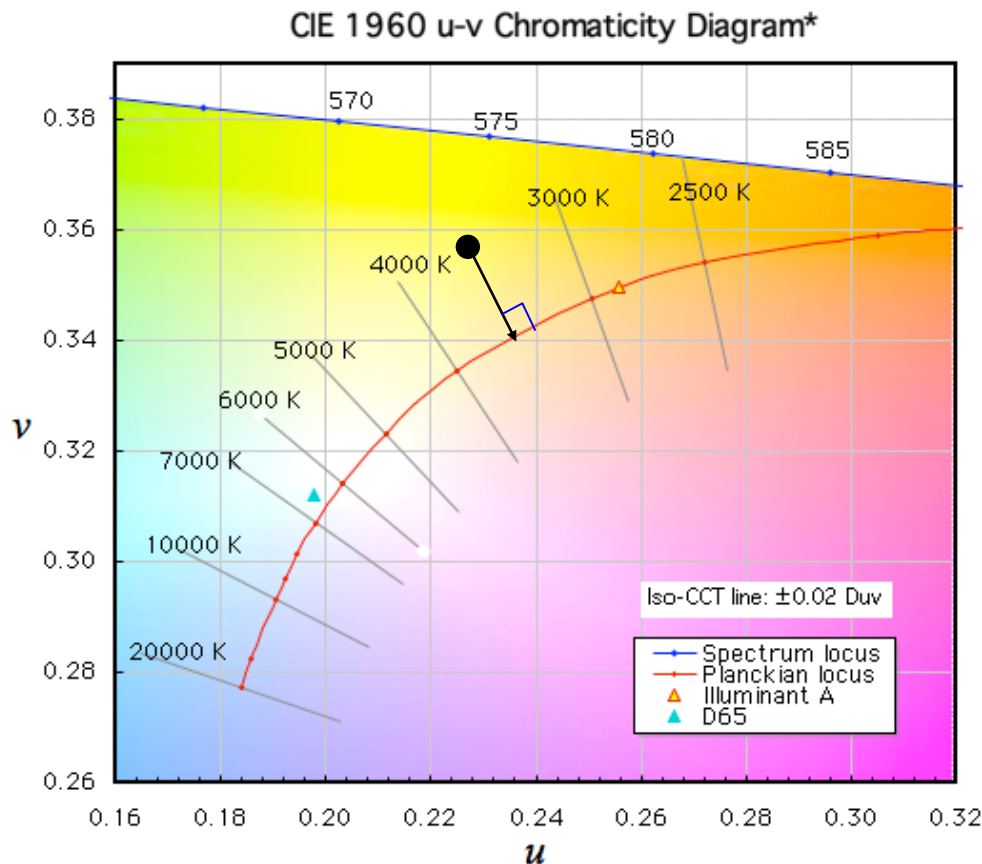
CCT- Duv chart



Correlated Color Temperature (CCT)

Temperature [K] of a Planckian radiator whose chromaticity is closest to that of a given stimulus on the CIE (u' , $2/3 v'$) coordinate.

(CIE 15:2004)



CCT is based on the CIE 1960 (u , v) diagram, which is now obsolete.

CCT is valid within distance 0.05 from the Planckian locus on the (u' , $2/3 \cdot v'$) diagram. (CIE 15: 2004)

CIE 15:2004 Colorimetry, 3rd Edition

APPENDIX E. INFORMATION ON THE USE OF PLANCK'S EQUATION FOR STANDARD AIR

According to the Planck's law, the spectral radiance of a blackbody at thermodynamic temperature T [K] in a medium having index of refraction n is given by

$$L_{e,\lambda}(\lambda, T) = \frac{c_1 n^{-2} \lambda^{-5}}{\pi} \left[\exp\left(\frac{c_2}{n\lambda T}\right) - 1 \right]^{-1} \quad (\text{E.1})$$

where $c_1 = 2\pi hc^2$, $c_2 = hc/k$, h is Planck's constant, c is the speed of light in vacuum, k is the
.....

T should follow the current International Temperature Scale (ITS-90), therefore,

$$c_2 = 1,4388 \times 10^{-2} \text{ m K.}$$

.....

Therefore, in the current recommendation in CIE 15:2004, colour temperature and correlated colour temperature are calculated using Equ. E.1 with $n = 1$ (exactly 1), thus no change from the previous practice. This recommendation may be subject to change in the future.

Robertson (1968)

Computation of Correlated Color Temperature and Distribution Temperature, Journal of the Optical Society of America, 58-11, 1968

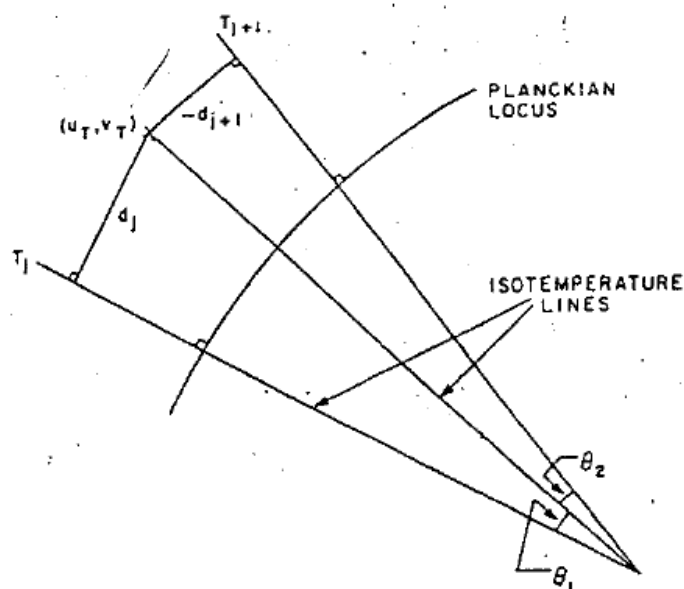


FIG. 1. Method of interpolation to find correlated color temperature.

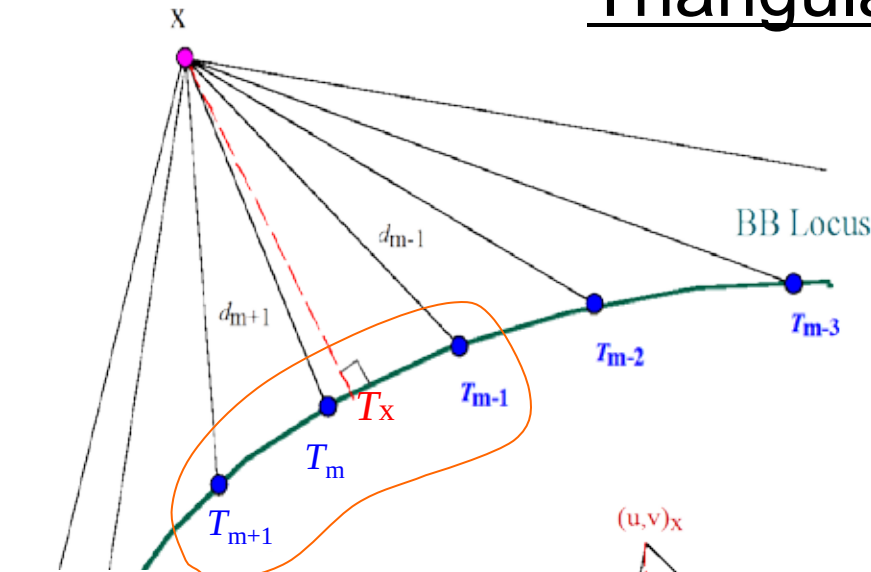
TABLE III. Maximum errors of computed values of correlated color temperature, based on use of the 31 isotemperature lines listed in Table II.

μrd	Range K	Maximum error	
		μrd	K
1-10	1 000 000-100 000	0.11	...
10-20	100 000-50 000	0.09	450
20-30	50 000-33 333	0.08	140
30-40	33 333-25 000	0.08	65
40-50	25 000-20 000	0.07	36
50-60	20 000-16 667	0.05	18
60-70	16 667-14 286	0.04	9.6
70-80	14 286-12 500	0.03	5.4
80-90	12 500-11 111	0.03	3.6
90-100	11 111-10 000	0.03	2.8
100-125	10 000-8 000	0.07	5.9
125-150	8 000-6 667	0.03	1.5
150-175	6 667-5 714	0.05	1.8
175-200	5 714-5 000	0.03	1.0
200-225	5 000-4 444	0.04	1.0
225-250	4 444-4 000	0.05	0.8
250-275	4 000-3 636	0.05	0.7
275-300	3 636-3 333	0.04	0.5
300-325	3 333-3 077	0.03	0.3
325-350	3 077-2 857	0.03	0.2
350-375	2 857-2 667	0.02	0.2
375-400	2 667-2 500	0.03	0.2
400-425	2 500-2 353	0.04	0.2
425-450	2 353-2 222	0.04	0.2
450-475	2 222-2 105	0.05	0.2
475-500	2 105-2 000	0.04	0.2
500-525	2 000-1 905	0.04	0.2
525-550	1 905-1 818	0.05	0.2
550-575	1 818-1 739	0.05	0.2
575-600	1 739-1 667	0.06	0.2

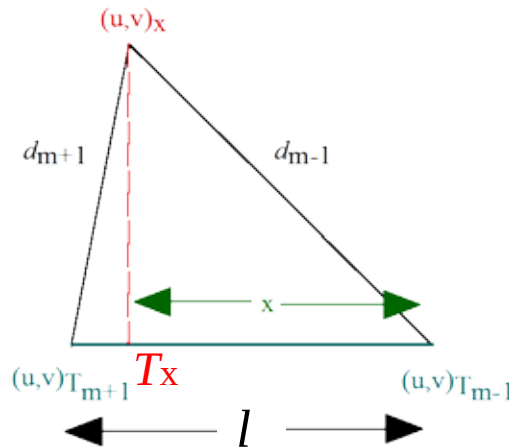
Direct approach (1) to calculate CCT and Duv

Triangular solution

- (1) Create a table of CCT vs distance d_i to BB locus on (u,v) coordinate.
- (2) Find the closest point in the table.
- (3) Solve the triangle for the neighboring 2 points



CCT	u	v	distance d
2705	0.26228	0.35150	0.00772
2732	0.26109	0.35115	0.00650
2759	0.25992	0.35080	0.00529
2787	0.25876	0.35044	0.00411
2815	0.25761	0.35007	0.00297
2843	0.25647	0.34970	0.00193
2871	0.25535	0.34932	0.00122
2900	0.25425	0.34893	0.00144
2929	0.25315	0.34855	0.00232
2958	0.25207	0.34815	0.00337
2988	0.25100	0.34775	0.00446
3018	0.24995	0.34735	0.00556
3048	0.24890	0.34694	0.00666
3078	0.24788	0.34653	0.00776
3109	0.24686	0.34611	0.00885
3140	0.24585	0.34569	0.00994
3172	0.24486	0.34526	0.01101



$$x = \frac{d_{m-1}^2 - d_{m+1}^2 + l^2}{2l}$$

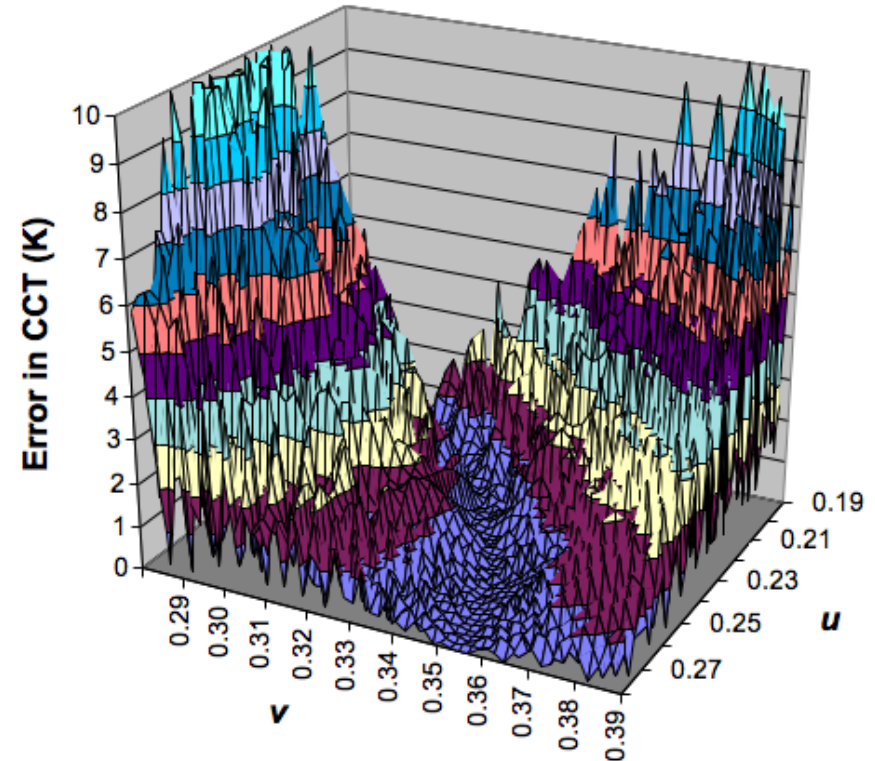
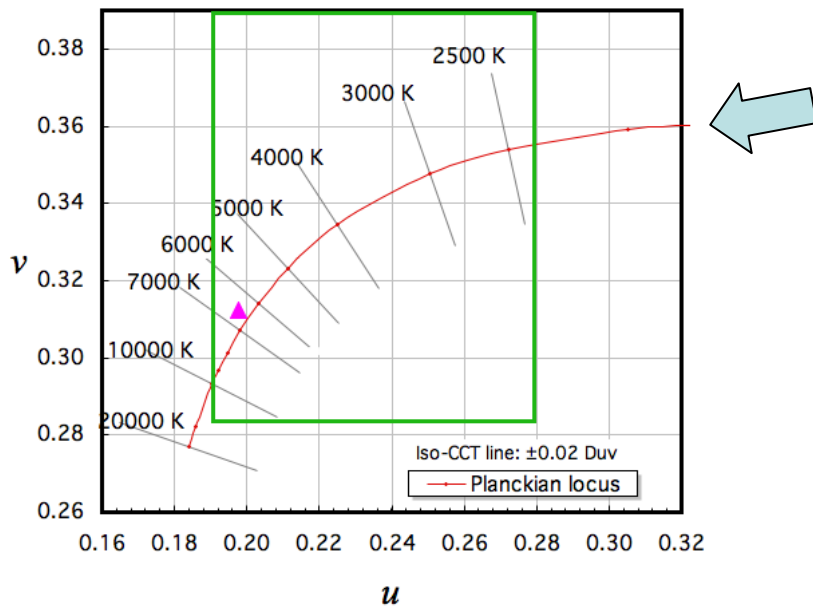
$$T_x = T_{m-1} + (T_{m+1} - T_{m-1}) \cdot \frac{x}{l}$$

$$D_{uv} = [\pm sign] (d_{m-1}^2 - x^2)^{1/2}$$

Use Planck's equation and color matching functions at 1 nm interval.

CCT Error in Triangular Solution

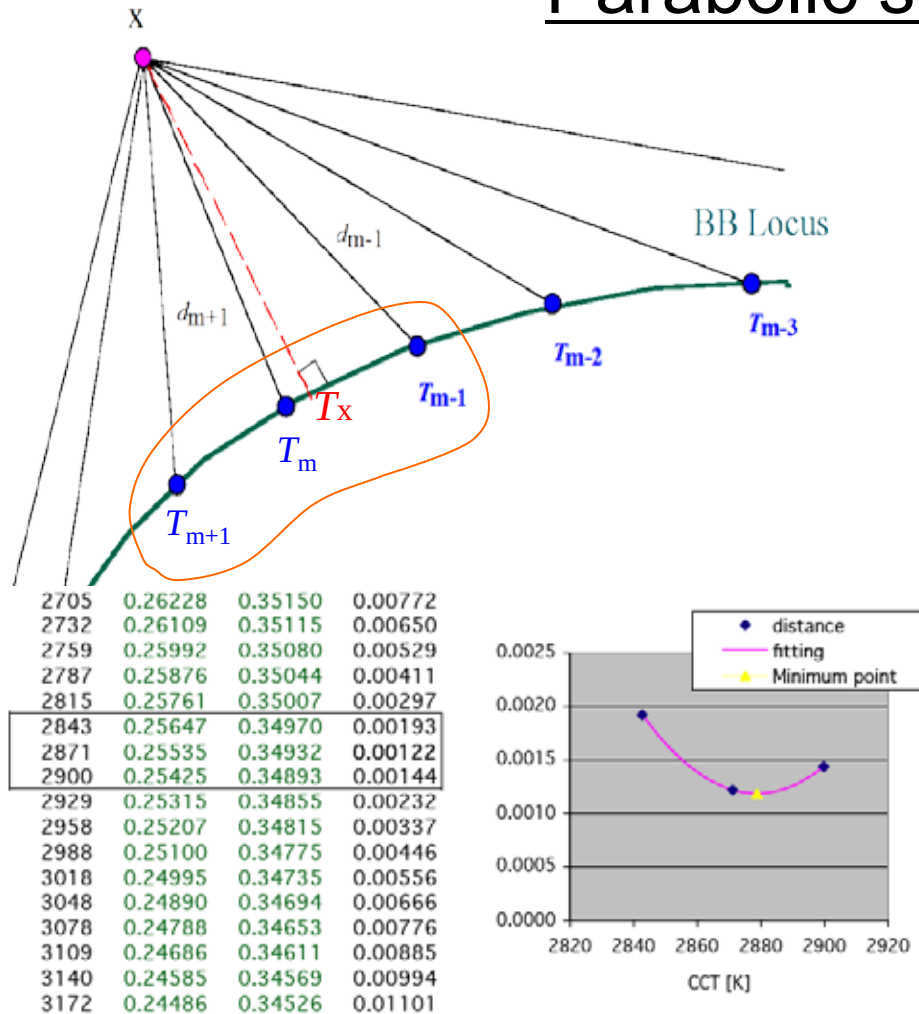
CCT error in Triangular solution (1% step table)



Error increases when the point is far from Planckian locus.

Direct approach (2) to calculate CCT and Duv

Parabolic solution



(1) Create a table of CCT vs distance d_i to BB locus on (u,v) coordinate.

(2) Find the closest point in the table.

(3) Parabolic fit for the neighboring 3 points.

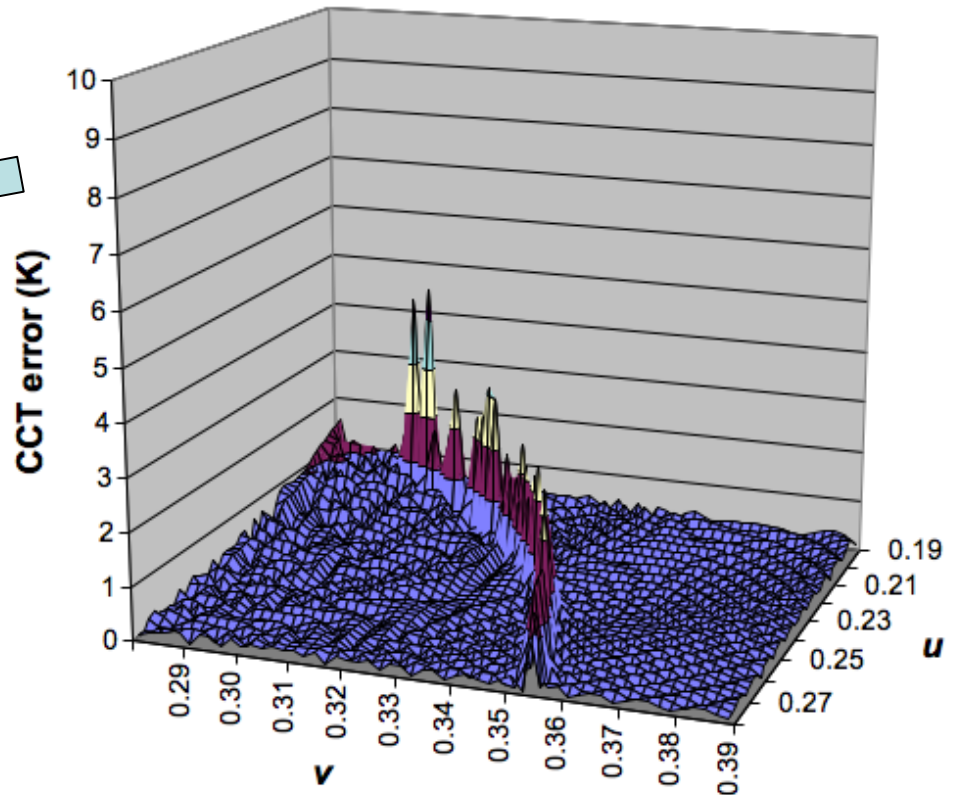
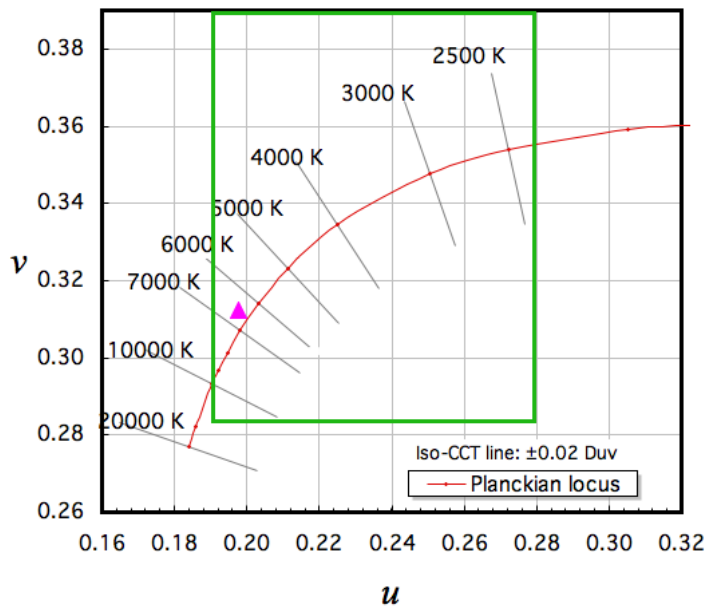
$$d(T) = aT^2 + bT + C$$

$$d(T)' = 2aT_x + b = 0 \quad \therefore T_x = \frac{-b}{2a}$$

$$D_{uv} = [\pm sign] \left(aT_x^2 + bT_x + C \right)$$

CCT Error in Parabolic Solution

CCT error in Parabolic solution (1 % step table)



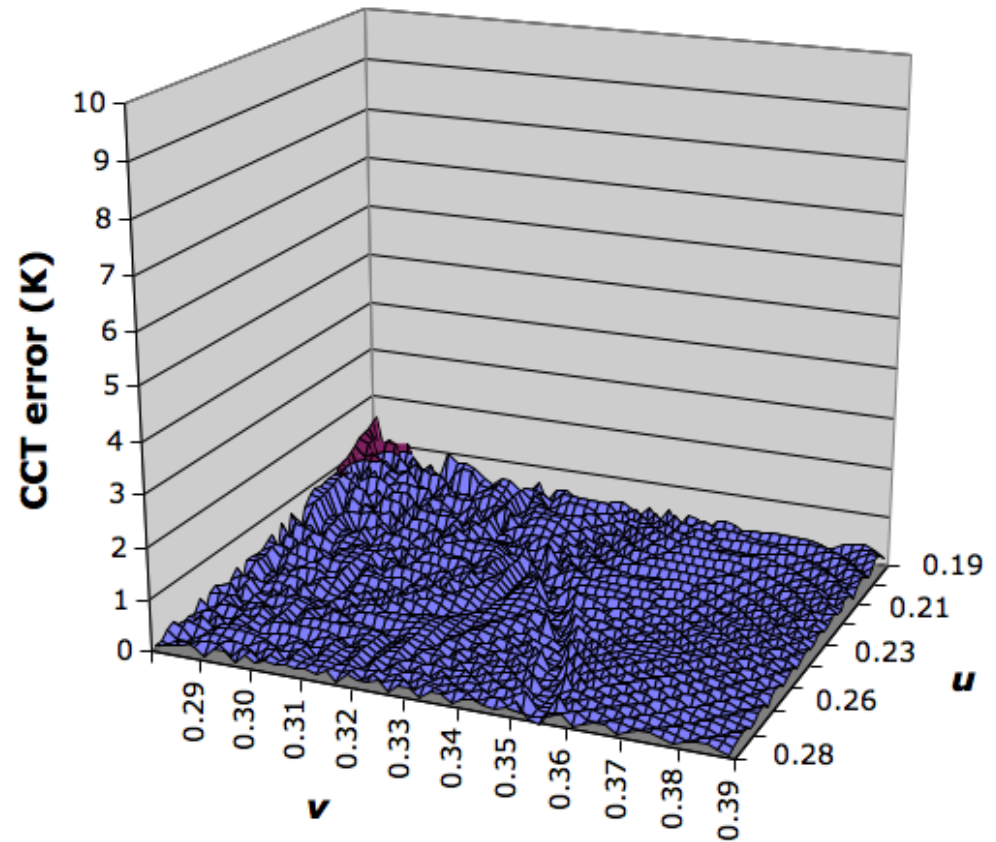
Much better, but the problem is on or very close to Planckian locus.

Combined Solution

Use Parabolic solution
but, take the CCT of
Triangular solution for

$$|D_{uv}| < 0.002$$

Error in Combined solution (1 % step table)



Most Accurate Version (cascade expansion)

Calculation of CCT (Most Accurate version, 1000 K - 20000 K) Y. Ohno

INPUT x 0.3127
y 0.3290
u 0.1978
v 0.3122

CCT 6503.0
Duv 0.0032

15 % step table

Color	Te	u	v	Distance
1000	0.44801	0.35462	0.2537	0
1150	0.41559	0.35725	0.2224	0.1
1323	0.38439	0.35927	0.1924	0.2
1521	0.35512	0.36039	0.1645	0.3
1749	0.32827	0.36036	0.1390	0.4
2011	0.30412	0.35898	0.1161	0.5
2313	0.28281	0.35620	0.0957	0.6
2660	0.26431	0.35208	0.0775	0.7
3059	0.24853	0.34679	0.0614	0.8
3518	0.23527	0.34060	0.0470	0.9
4046	0.22430	0.33382	0.0342	1.0
4652	0.21535	0.32677	0.0228	0.1
5350	0.20813	0.31972	0.0127	0.2
6153	0.20237	0.31292	0.0046	0.3
7076	0.19781	0.30655	0.0057	0.4
8137	0.19423	0.30073	0.0120	0.5
9358	0.19143	0.29552	0.0179	0.6
10761	0.18923	0.29093	0.0230	0.7
12375	0.18751	0.28694	0.0273	0.8
14232	0.18615	0.28350	0.0310	0.9
16367	0.18507	0.28056	0.0341	1.0
18822	0.18420	0.27806	0.0368	
21645	0.18351	0.27593	0.0390	
24891	0.18295	0.27412	0.0409	
28625	0.18249	0.27259	0.0425	
32919	0.18211	0.27129	0.0438	
min	0.00459			
match	14			
distance				
T(m-1)	5350.25	0.01273		
T(m)	6152.79	0.00459		
T(m+1)	7075.71	0.00567		

1.5 % step table

u	v	distances
0.20813	0.31972	0.012732
0.20745	0.31898	0.011750
0.20680	0.31825	0.010802
0.20617	0.31754	0.009886
0.20556	0.31684	0.009005
0.20498	0.31616	0.008158
0.20442	0.31549	0.007350
0.20388	0.31483	0.006582
0.20336	0.31418	0.005860
0.20285	0.31354	0.005191
0.20237	0.31292	0.004588
0.20183	0.31222	0.003997
0.20131	0.31153	0.003548
0.20082	0.31086	0.003279
0.20034	0.31021	0.003217
0.19988	0.30956	0.003356
0.19943	0.30894	0.003659
0.19901	0.30832	0.004077
0.19859	0.30772	0.004571
0.19820	0.30713	0.005109
0.19781	0.30655	0.005672
minimum	0.003217	
match	15	
T(m-1)	6429.7	0.00328
T(m)	6522.0	0.00322
T(m+1)	6614.2	0.00336

Parabolic solution
a 1.925E-07
b -3.78E-07
c 1.97E-07
A 1.181E-08
B -0.000154
C 0.502682
CCT 6504.39
duv 0.00321

Triangular solution
T(m-1) u 0.20082
T(m+1) u 0.19988
T(m-1) v 0.31086
T(m+1) v 0.30956
d 0.00
x 0.00
CCT 6503.70
duv 0.00322

CCT 6504.39

0.2004 0.3103
Sign of Duv 1
Duv 0.00322

0.15 % step table

u	v	Distance
0.20082	0.31086	0.003279
0.20077	0.31080	0.003263
0.20072	0.31073	0.003250
0.20067	0.31066	0.003238
0.20062	0.31060	0.003229
0.20057	0.31053	0.003222
0.20053	0.31047	0.003216
0.20048	0.31040	0.003213
0.20043	0.31034	0.003213
0.20038	0.31027	0.003214
0.20034	0.31021	0.003217
0.20029	0.31014	0.003222
0.20024	0.31008	0.003229
0.20020	0.31001	0.003239
0.20015	0.30995	0.003250
0.20010	0.30988	0.003263
0.20006	0.30982	0.003278
0.20001	0.30976	0.003295
0.19997	0.30969	0.003313
0.19992	0.30963	0.003334
0.19988	0.30956	0.003356
minimum	0.0032125	
match	9	
T(m-1)	6494.3	0.00321
T(m)	6503.5	0.00321
T(m+1)	6512.7	0.00321

Parabolic solution
a 1.886E-05
b -3.77E-05
c 1.886E-05
A 1.223E-08
B -0.000159
C 0.5202378
CCT 6503.05
duv 0.00321

Triangular solution
T(m-1) u 0.20048
T(m+1) u 0.20038
T(m-1) v 0.31040
T(m+1) v 0.31027
d 0.00
x 0.00
CCT 6503.03
duv 0.00321

CCT 6503.05

0.2004 0.3103
Sign of Duv 1
Duv 0.00321

0.015 % step table

u	v	Distance
0.20048	0.31040	0.003213
0.20047	0.31040	0.003213
0.20046	0.31038	0.003213
0.20046	0.31038	0.003213
0.20045	0.31037	0.003213
0.20045	0.31036	0.003213
0.20045	0.31036	0.003213
0.20044	0.31035	0.003213
0.20044	0.31034	0.003213
0.20044	0.31034	0.003213
0.20043	0.31034	0.003213
0.20043	0.31033	0.003213
0.20042	0.31032	0.003213
0.20042	0.31032	0.003213
0.20041	0.31031	0.003213
0.20041	0.31030	0.003213
0.20040	0.31030	0.003213
0.20040	0.31029	0.003213
0.20039	0.31028	0.003213
0.20039	0.31027	0.003214
minimum	0.0032125	
match	10	
T(m-1)	6501.7	0.00321
T(m)	6502.6	0.00321
T(m+1)	6503.5	0.00321

Parabolic solution
a 0.001886
b -0.003772
c 0.001886
A 1.22E-08
B -0.000159
C 0.520744
CCT 6503.03
duv 0.00321

Triangular solution
T(m-1) u 0.20044
T(m+1) u 0.20043
T(m-1) v 0.31035
T(m+1) v 0.31034
d 0.00
x 0.00
CCT 6503.05
duv 0.00321

Final CCT 6503.03

0.2004 0.3103
Sign of Duv 1
Final Duv 0.00321

Used as the reference
for accuracy
verification.

Conversion from (CCT, Duv) back to (x, y)

Input: CCT T (K)
 Duv D_{uv}

- 1) Calculate (u_0, v_0) of the Planckian radiator at T (K).
- 2) Calculate (u_1, v_1) of the Planckian radiator at $T+DT$ (K). $DT=0.01$ (K)
- 3) Calculate

$$du = u_1 - u_0$$

$$dv = v_1 - v_0$$

$$u = u_0 + D_{uv} \cdot \sin \theta$$

$$= u_0 + D_{uv} \cdot dv / \sqrt{du^2 + dv^2}$$

$$v = v_0 + D_{uv} \cdot \cos \theta$$

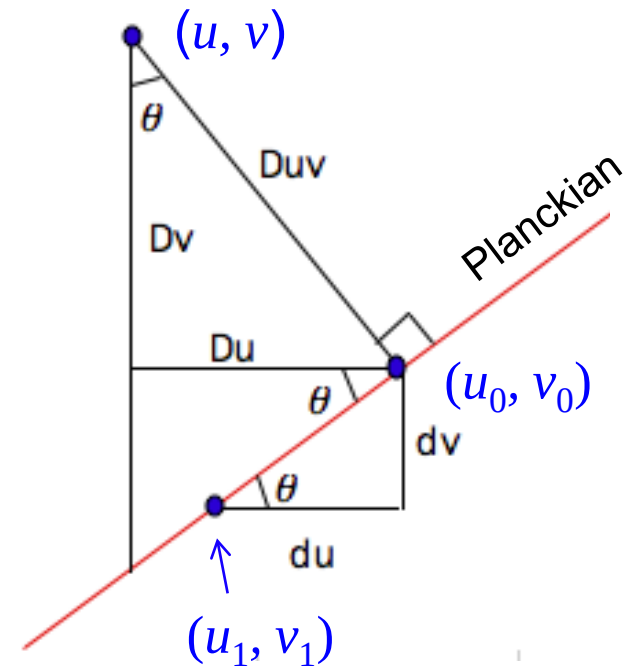
$$= u_0 + D_{uv} \cdot du / \sqrt{du^2 + dv^2}$$

$$u' = u$$

$$v' = 1.5v$$

$$x = 9u' / (6u' - 16v' + 12)$$

$$y = 2v' / (3u' - 8v' + 6)$$



(Included in Revision draft of C78.377)

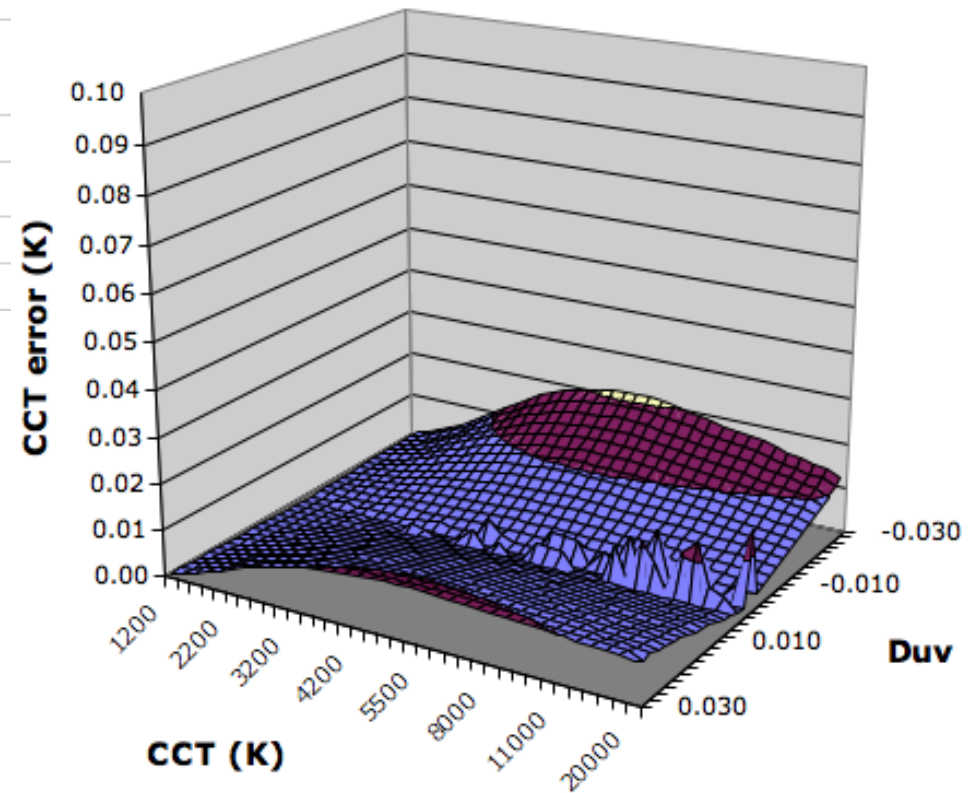
Accuracy of Most Accurate Version (4 stage)

Accuracy verification for 4-stage version

CCT	2900			
Duv	0.0200			
	↓			
x	0.478420	CCT	OUTPUT 2900.006	Error 0.006 K
y	0.473737	Duv	0.020000	0.00000
u	0.247629			
v	0.367808			

CCT 4-stage version

CCT error of 4-stage version



Simple calculation from (x,y) or (u',v') to Duv

Duv is normally calculated in the process of calculating CCT.
Below is a simple approximation formula, without calculation of CCT.

1) Convert (x, y) or (u', v') to (u, v)

$$\begin{aligned} u &= 4x/(-2x + 12y + 3) & \text{or} & & u &= u' \\ v &= 6y/(-2x + 12y + 3) & & & v &= 2v'/3 \end{aligned}$$

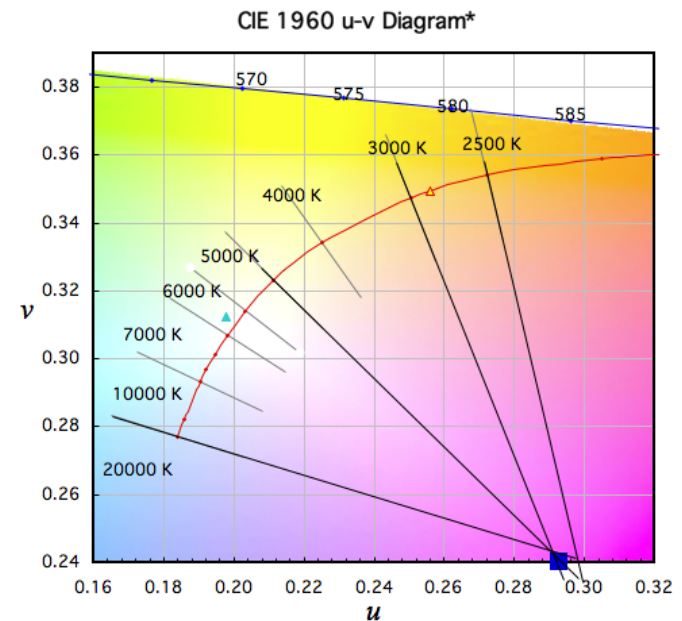
2) Duv is obtained by

$$L_{\text{FP}} = \sqrt{(u - 0.292)^2 + (v - 0.24)^2}$$

$$a = \arccos\left(\frac{u - 0.292}{L_{\text{FP}}}\right)$$

$$L_{\text{BB}} = k_6 a^6 + k_5 a^5 + k_4 a^4 + k_3 a^3 + k_2 a^2 + k_1 a + k_0$$

$$D_{\text{uv}} = L_{\text{FP}} - L_{\text{BB}}$$



k6	-0.00616793
k5	0.0893944
k4	-0.5179722
k3	1.5317403
k2	-2.4243787
k1	1.925865
k0	-0.471106

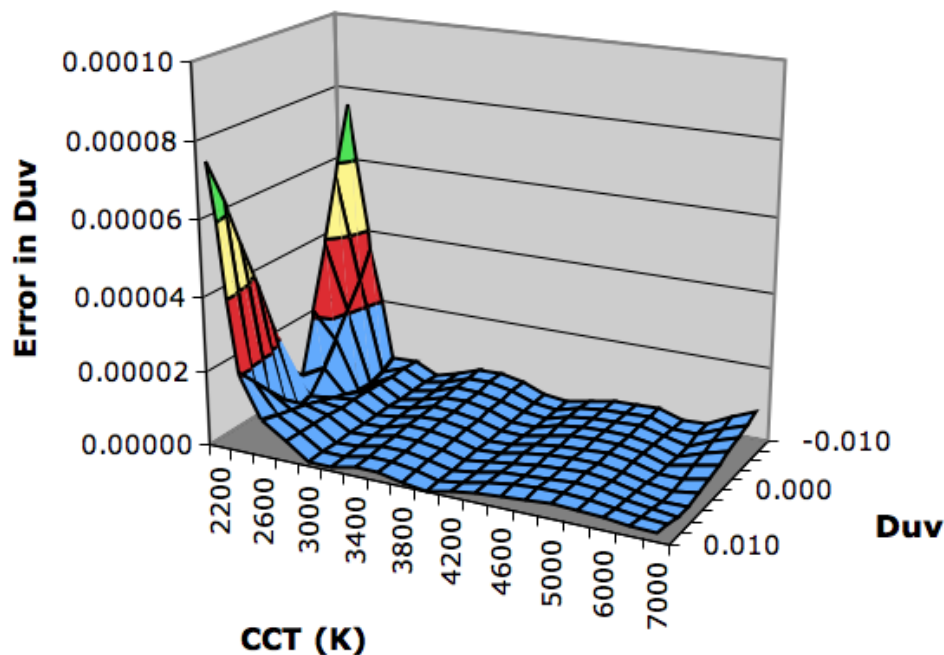
(Included in Revision draft of C78.377)

Simple calculation from (x,y) or (u',v') to Duv

Accuracy of this method

within 0.00001 in the range
from 2600 K to 20000 K and
Duv 0.000 ± 0.010

within 0.0001 in the range
from 2160 K to 20000 K and
Duv 0.000 ± 0.010



(Included in Revision draft of C78.377)

Simple calculation from (x,y) or (u',v') to (CCT, Duv)

$$L_{FP} = \sqrt{(u - 0.292)^2 + (v - 0.24)^2}$$

$$a_1 = \arctan((v - 0.24)/(u - 0.292)), \text{ if } a_1 \geq 0, a = a_1; \text{ if } a_1 < 0, a = a_1 + \pi$$

$$L_{BB} = k_{06} a^6 + k_{05} a^5 + k_{04} a^4 + k_{03} a^3 + k_{02} a^2 + k_{01} a + k_{00}$$

$$D_{uv} = L_{FP} - L_{BB}$$

$$\text{For } a < 2.54; T_1 = 1/(k_{16} \cdot a^6 + k_{15} \cdot a^5 + k_{14} \cdot a^4 + k_{13} \cdot a^3 + k_{12} \cdot a^2 + k_{11} \cdot a + k_{10})$$

$$\text{For } a \geq 2.54; T_1 = 1/(k_{26} \cdot a^6 + k_{25} \cdot a^5 + k_{24} \cdot a^4 + k_{23} \cdot a^3 + k_{22} \cdot a^2 + k_{21} \cdot a + k_{20})$$

$$\text{For } a < 2.54; \Delta T_{c1} = (k_{36} \cdot a^6 + k_{35} \cdot a^5 + k_{34} \cdot a^4 + k_{33} \cdot a^3 + k_{32} \cdot a^2 + k_{31} \cdot a + k_{30}) \cdot (L_{BB} + 0.01) / L_p \cdot D_{uv} / 0.01$$

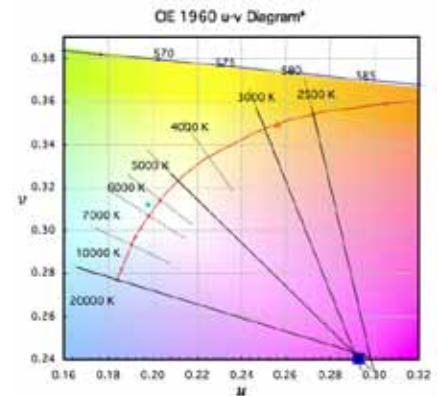
$$\text{For } a \geq 2.54; \Delta T_{c1} = 1/(k_{46} \cdot a^6 + k_{45} \cdot a^5 + k_{44} \cdot a^4 + k_{43} \cdot a^3 + k_{42} \cdot a^2 + k_{41} \cdot a + k_{40}) \cdot (L_{BB} + 0.01) / L_p \cdot D_{uv} / 0.01$$

$$T_2 = T_1 - \Delta T_{c1}, c = \log(T_2)$$

$$\text{For } D_{uv} \geq 0; \Delta T_{c2} = (k_{56} \cdot c^6 + k_{55} \cdot c^5 + k_{54} \cdot c^4 + k_{53} \cdot c^3 + k_{52} \cdot c^2 + k_{51} \cdot c + k_{50})$$

$$\text{For } D_{uv} < 0; \Delta T_{c2} = (k_{66} \cdot c^6 + k_{65} \cdot c^5 + k_{64} \cdot c^4 + k_{63} \cdot c^3 + k_{62} \cdot c^2 + k_{61} \cdot c + k_{60}) \cdot |D_{uv} / 0.03|^2$$

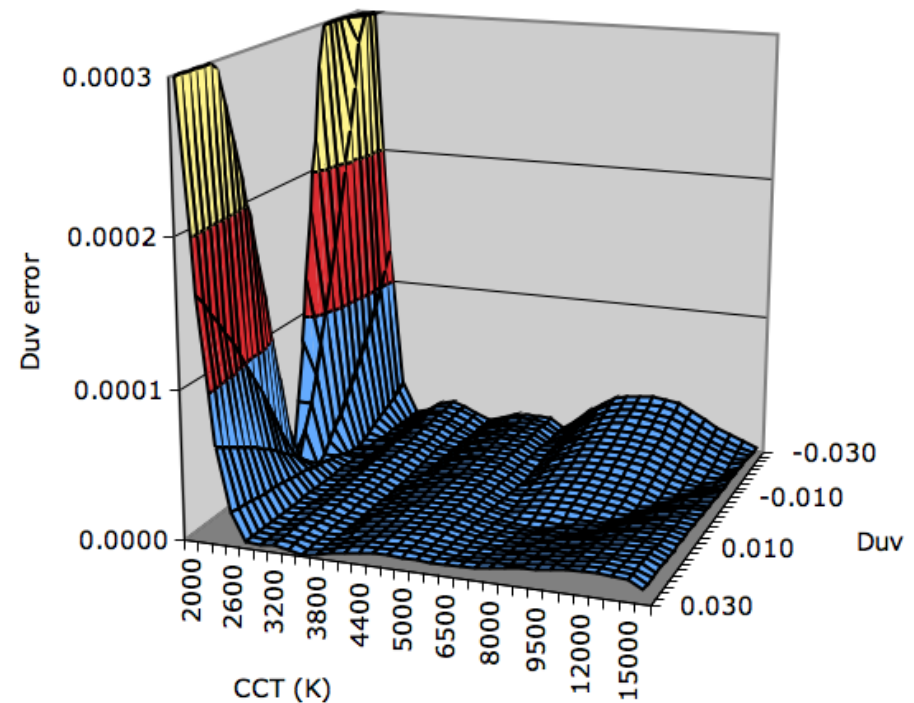
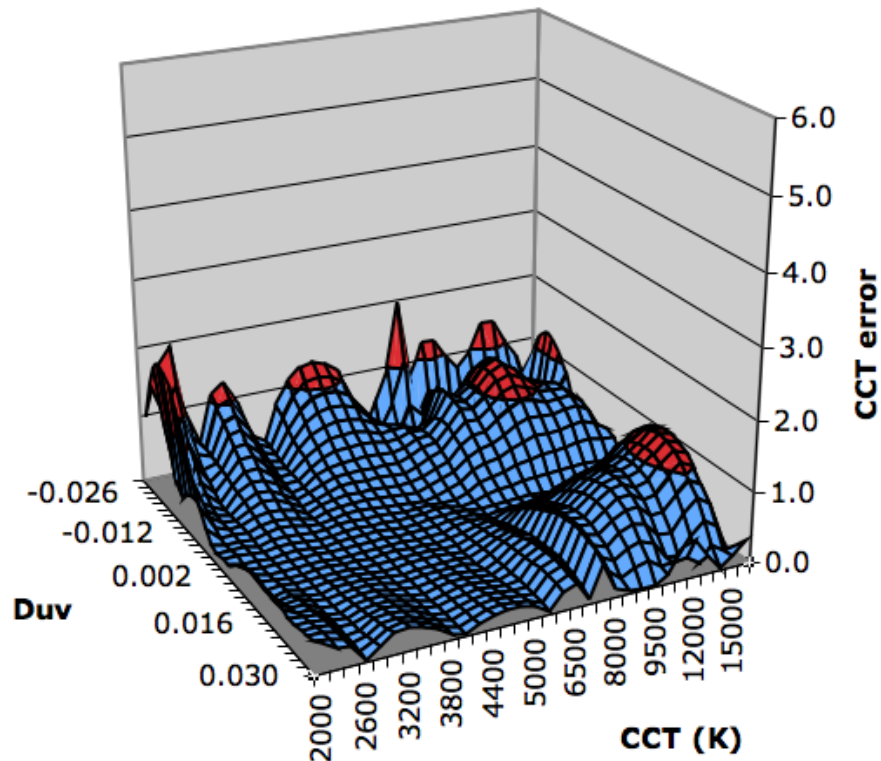
$$T_{FINAL} = T_2 - \Delta T_{c2}$$



i	ki6	ki5	ki4	ki3	ki2	ki1	ki0
0	-3.7146000E-03	5.6061400E-02	-3.307009E-01	9.750013E-01	-1.5008606E+00	1.115559E+00	-1.77348E-01
1	-3.2325500E-05	3.5700160E-04	-1.589747E-03	3.6196568E-03	-4.3534788E-03	2.1595434E-03	5.308409E-04
2	-2.6653835E-03	4.17781315E-02	-2.73172022E-01	9.53570888E-01	-1.873907584E+00	1.964980251E+00	-8.58308927E-01
3	-2.3524950E+01	2.7183365E+02	-1.1785121E+03	2.51170136E+03	-2.7966888E+03	1.49284136E+03	-2.3275027E+02
4	-1.731364909E+06	2.7482732935E+07	-1.81749963507E+08	6.40976356945E+08	-1.27141290956E+09	1.34488160614E+09	-5.926850606E+08
5	-9.4353083E+02	2.10468274E+04	-1.9500061E+05	9.60532935E+05	-2.65299138E+06	3.89561742E+06	-2.3758158E+06
6	5.0857956E+02	-1.321007E+04	1.4101538E+05	-7.93406005E+05	2.48526954E+06	-4.11436958E+06	2.8151771E+06

Simple calculation from (x,y) or (u',v') to (CCT, Duv)

Accuracy of this method



Conclusions

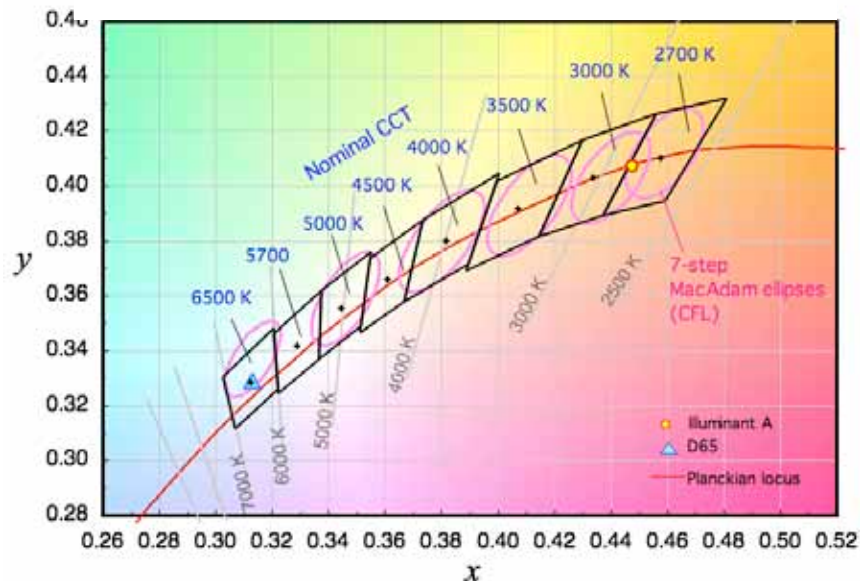
- Practical calculation and conversion formulae for CCT and Duv have been developed.
- Accuracies of some of the formulae will be further improved.
- The use of CCT and Duv (rather than x , y or u' , v' chromaticity coordinates) is recommended to specify the chromaticity of lighting sources.

THANK YOU for your
attention.

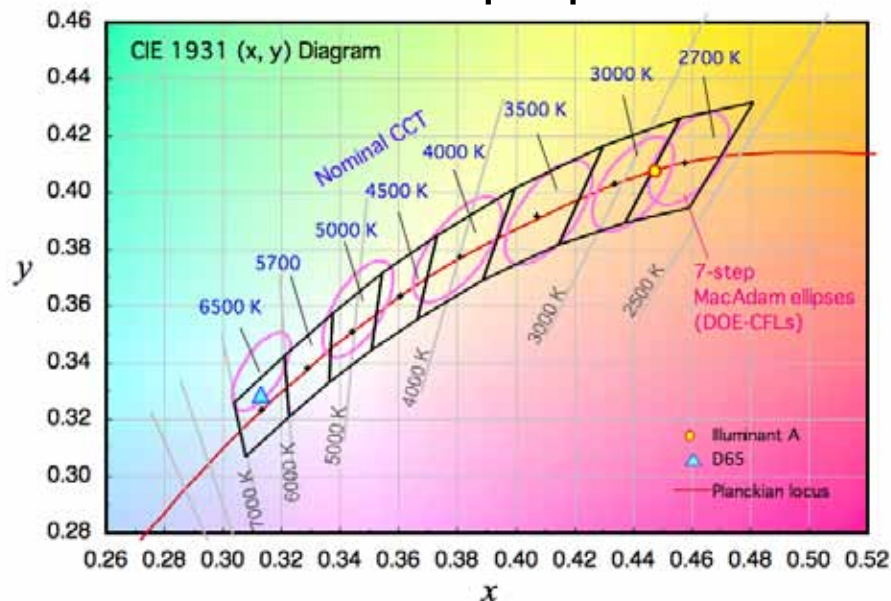
Contact: ohno@nist.gov

Proposed revision of ANSI C78.377

Current version



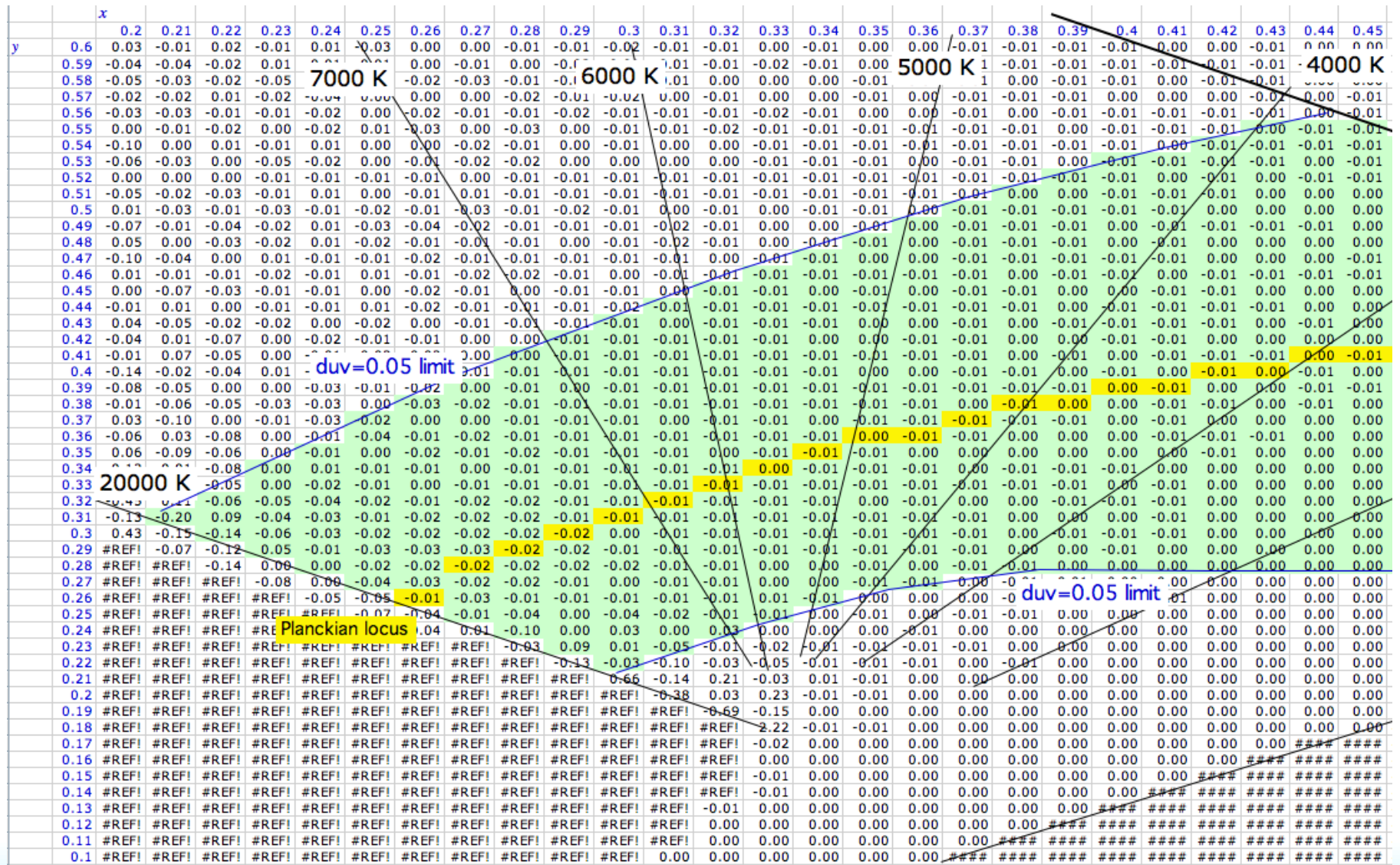
Revision proposal 2



- All center points to be moved onto the Planckian locus.
- This proposal is pending due to a need for vision experiments.
- Anecdotes say people prefer below the Planckian locus.
- NIST is funded by DOE to conduct vision experiments using STLF.



CCT (K) difference between 3rd stage and 5th stage



Summary

- Duv is important for color quality of light sources.
- Duv is often neglected in specifications.
- Parabolic and triangle combined solution works well for CCT calculation.
- 1 % step table provides enough accuracy
(<1 K for 1000 to 10000 K, <2 K up to 20000 K, $D_{uv} \pm 0.03$)
- Most Accurate Version (cascade expansion),
- Conversion from (CCT, Duv) back to (x,y),
- Simple calculation from (x,y) or (u' , v') to Duv,
- Simple calculation from (x,y) or (u' , v') to (CCT, Duv) have been developed.