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ELECTRIC CIRCUITS

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ELECTRIC CIRCUITS

EIGHTH EDITION

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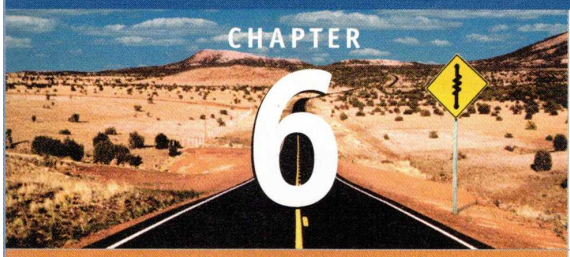
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Inductance, Capacitance, and Mutual Inductance

CHAPTER CONTENTS

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✓CHAPTER OBJECTIVES

1. Know and be able to use the equations for voltage, current, power, and energy in an inductor; understand how an inductor behaves in the presence of constant current, and the requirement that the current be continuous in an inductor.
2. Know and be able to use the equations for voltage, current, power, and energy in a capacitor; understand how a capacitor behaves in the presence of constant voltage, and the requirement that the voltage be continuous in a capacitor.
3. Be able to combine inductors with initial conditions in series and in parallel to form a single equivalent inductor with an initial condition; be able to combine capacitors with initial conditions in series and in parallel to form a single equivalent capacitor with an initial condition.
4. Understand the basic concept of mutual inductance and be able to write mesh-current equations for a circuit containing magnetically coupled coils using the dot convention correctly.

We begin this chapter by introducing the last two ideal circuit elements mentioned in Chapter 2, namely, inductors and capacitors. Be assured that the circuit analysis techniques introduced in Chapters 3 and 4 apply to circuits containing inductors and capacitors. Therefore, once you understand the terminal behavior of these elements in terms of current and voltage, you can use Kirchhoff's laws to describe any interconnections with the other basic elements. Like other components, inductors and capacitors are easier to describe in terms of circuit variables rather than electromagnetic field variables. However, before we focus on the circuit descriptions, a brief review of the field concepts underlying these basic elements is in order.

An inductor is an electrical component that opposes any change in electrical current. It is composed of a coil of wire wound around a supporting core whose material may be magnetic or nonmagnetic. The behavior of inductors is based on phenomena associated with magnetic fields. The source of the magnetic field is charge in motion, or current. If the current is varying with time, the magnetic field is varying with time. A time-varying magnetic field induces a voltage in any conductor linked by the field. The circuit parameter of **inductance** relates the induced voltage to the current. We discuss this quantitative relationship in Section 6.1.

A capacitor is an electrical component that consists of two conductors separated by an insulator or dielectric material. The capacitor is the only device other than a battery that can store electrical charge. The behavior of capacitors is based on phenomena associated with electric fields. The source of the electric field is separation of charge, or voltage. If the voltage is varying with time, the electric field is varying with time. A time-varying electric field produces a displacement current in the space occupied by the field. The circuit parameter of **capacitance** relates the displacement current to the voltage, where the displacement current is equal to the conduction current at the terminals of the capacitor. We discuss this quantitative relationship in Section 6.2.

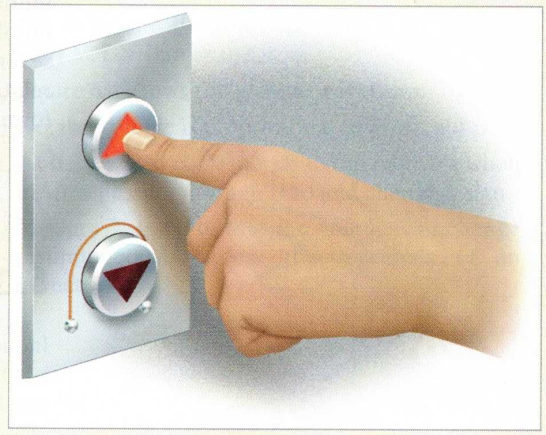
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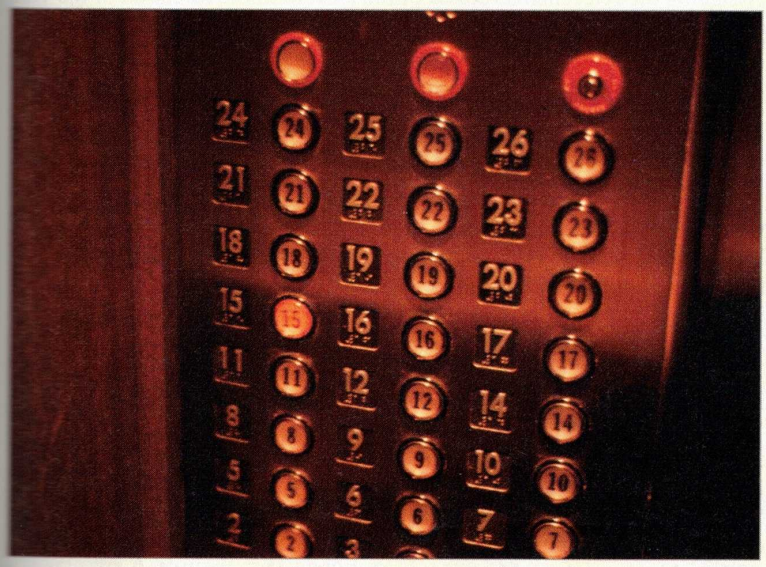
Practical Perspective

Proximity Switches

The electrical devices we use in our daily lives contain many switches. Most switches are mechanical, such as the one used in the flashlight introduced in Chapter 2. Mechanical switches use an actuator that is pushed, pulled, slid, or rotated, causing two pieces of conducting metal to touch and create a short circuit. Sometimes designers prefer to use switches without moving parts, to increase the safety, reliability, convenience, or novelty of their products. Such switches are called proximity switches. Proximity switches can employ a variety of sensor technologies. For example, some elevator doors stay open whenever a light beam is obstructed.

Another sensor technology used in proximity switches detects people by responding to the disruption they cause in electric fields. This type of proximity switch is used in some desk lamps that turn on and off when touched and in elevator buttons with no moving parts (as shown in the figure). The switch is based on a capacitor. As you are about to discover in this chapter, a capacitor is a circuit element whose terminal characteristics are determined by electric fields. When you touch a capacitive proximity switch, you produce a change in the value of a capacitor, causing a voltage change, which activates the switch. The design of a capacitive touch-sensitive switch is the topic of the Practical Perspective example at the end of this chapter.





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Section 6.3 describes techniques used to simplify circuits with series or parallel combinations of capacitors or inductors.

Energy can be stored in both magnetic and electric fields. Hence you should not be too surprised to learn that inductors and capacitors are capable of storing energy. For example, energy can be stored in an inductor and then released to fire a spark plug. Energy can be stored in a capacitor and then released to fire a flashbulb. In ideal inductors and capacitors, only as much energy can be extracted as has been stored. Because inductors and capacitors cannot generate energy, they are classified as **passive elements**.

In Sections 6.4 and 6.5 we consider the situation in which two circuits are linked by a magnetic field and thus are said to be magnetically coupled. In this case, the voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as **mutual inductance**. The practical significance of magnetic coupling unfolds as we study the relationships between current, voltage, power, and several new parameters specific to mutual inductance. We introduce these relationships here and then describe their utility in a device called a transformer in Chapters 9 and 10.

6.1 The Inductor

Inductance is the circuit parameter used to describe an inductor. Inductance is symbolized by the letter L, is measured in henrys (H), and is represented graphically as a coiled wire—a reminder that inductance is a consequence of a conductor linking a magnetic field. Figure 6.1(a) shows an inductor. Assigning the reference direction of the current in the direction of the voltage drop across the terminals of the inductor, as shown in Fig. 6.1(b), yields

The inductor $v - i$ equation ▶

$$v = L\frac{di}{dt}, \tag{6.1}$$

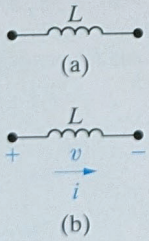


Figure 6.1 ▲ (a) The graphic symbol for an inductor with an inductance of L henrys. (b) Assigning reference voltage and current to the inductor, following the passive sign convention.

where v is measured in volts, L in henrys, i in amperes, and t in seconds. Equation 6.1 reflects the passive sign convention shown in Fig. 6.1(b); that is, the current reference is in the direction of the voltage drop across the inductor. If the current reference is in the direction of the voltage rise, Eq. 6.1 is written with a minus sign.

Note from Eq. 6.1 that the voltage across the terminals of an inductor is proportional to the time rate of change of the current in the inductor. We can make two important observations here. First, if the current is constant, the voltage across the ideal inductor is zero. Thus the inductor behaves as a short circuit in the presence of a constant, or dc, current. Second, current cannot change instantaneously in an inductor; that is, the current cannot change by a finite amount in zero time. Equation 6.1 tells us that this change would require an infinite voltage, and infinite voltages are not possible. For example, when someone opens the switch on an

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inductive circuit in an actual system, the current initially continues to flow in the air across the switch, a phenomenon called *arcing*. The arc across the switch prevents the current from dropping to zero instantaneously. Switching inductive circuits is an important engineering problem, because arcing and voltage surges must be controlled to prevent equipment damage. The first step to understanding the nature of this problem is to master the introductory material presented in this and the following two chapters. Example 6.1 illustrates the application of Eq. 6.1 to a simple circuit.

Example 6.1 **Determining the Voltage, Given the Current, at the Terminals of an Inductor**

The independent current source in the circuit shown in Fig. 6.2 generates zero current for $t < 0$ and a pulse $10te^{-5t}$A, for $t > 0$.

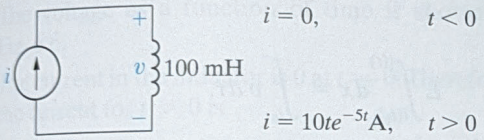


Figure 6.2 ▲ The circuit for Example 6.1.

a) Sketch the current waveform.

b) At what instant of time is the current maximum?

c) Express the voltage across the terminals of the 100 mH inductor as a function of time.

d) Sketch the voltage waveform.

e) Are the voltage and the current at a maximum at the same time?

f) At what instant of time does the voltage change polarity?

g) Is there ever an instantaneous change in voltage across the inductor? If so, at what time?

Solution

a) Figure 6.3 shows the current waveform.

b) $di/dt = 10(-5te^{-5t} + e^{-5t}) = 10e^{-5t}(1 - 5t)$ A/s; $di/dt = 0$ when $t = \frac{1}{5}$ s. (See Fig. 6.3.)

c) $v = Ldi/dt = (0.1)10e^{-5t}(1 - 5t) = e^{-5t}(1 - 5t)$ V, $t > 0$; $v = 0$, $t < 0$.

d) Figure 6.4 shows the voltage waveform.

e) No; the voltage is proportional to di/dt, not i.

f) At 0.2 s, which corresponds to the moment when di/dt is passing through zero and changing sign.

g) Yes, at $t = 0$. Note that the voltage can change instantaneously across the terminals of an inductor.

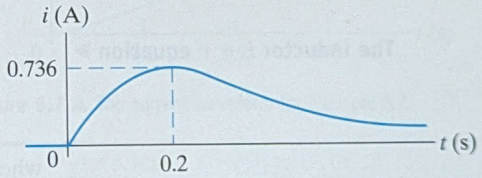


Figure 6.3 ▲ The current waveform for Example 6.1.

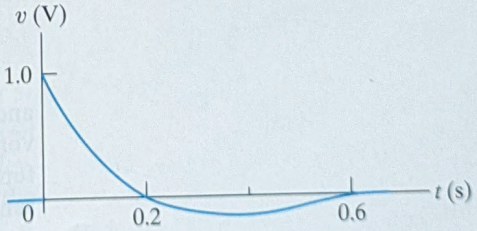


Figure 6.4 ▲ The voltage waveform for Example 6.1.

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Current in an Inductor in Terms of the Voltage Across the Inductor

Equation 6.1 expresses the voltage across the terminals of an inductor as a function of the current in the inductor. Also desirable is the ability to express the current as a function of the voltage. To find i as a function of v, we start by multiplying both sides of Eq. 6.1 by a differential time dt:

$$v\,dt = L\left(\frac{di}{dt}\right)dt. \tag{6.2}$$

Multiplying the rate at which i varies with t by a differential change in time generates a differential change in i, so we write Eq. 6.2 as

$$v\,dt = L\,di. \tag{6.3}$$

We next integrate both sides of Eq. 6.3. For convenience, we interchange the two sides of the equation and write

$$L\int_{i(t_0)}^{i(t)} dx = \int_{t_0}^{t} v\,d\tau. \tag{6.4}$$

Note that we use x and τ as the variables of integration, whereas i and t become limits on the integrals. Then, from Eq. 6.4,

The inductor $i - v$ equation ▶

$$i(t) = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0), \tag{6.5}$$

where $i(t)$ is the current corresponding to t, and $i(t_0)$ is the value of the inductor current when we initiate the integration, namely, t_0. In many practical applications, t_0 is zero and Eq. 6.5 becomes

$$i(t) = \frac{1}{L}\int_{0}^{t} v\,d\tau + i(0). \tag{6.6}$$

Equations 6.1 and 6.5 both give the relationship between the voltage and current at the terminals of an inductor. Equation 6.1 expresses the voltage as a function of current, whereas Eq. 6.5 expresses the current as a function of voltage. In both equations the reference direction for the current is in the direction of the voltage drop across the terminals. Note that $i(t_0)$ carries its own algebraic sign. If the initial current is in the same direction as the reference direction for i, it is a positive quantity. If the initial current is in the opposite direction, it is a negative quantity. Example 6.2 illustrates the application of Eq. 6.5.

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Example 6.2 **Determining the Current, Given the Voltage, at the Terminals of an Inductor**

The voltage pulse applied to the 100 mH inductor shown in Fig. 6.5 is 0 for $t < 0$ and is given by the expression

$$v(t) = 20te^{-10t}\ \text{V}$$

for $t > 0$. Also assume $i = 0$ for $t \leq 0$.

a) Sketch the voltage as a function of time.

b) Find the inductor current as a function of time.

c) Sketch the current as a function of time.

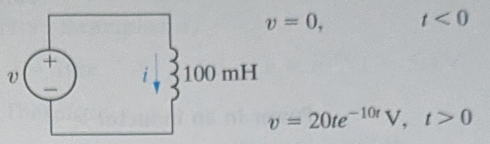


Figure 6.5 ▲ The circuit for Example 6.2.

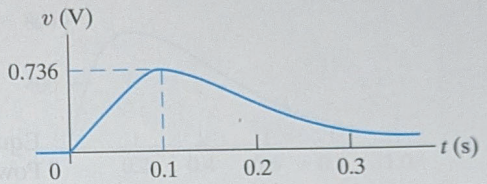


Figure 6.6 ▲ The voltage waveform for Example 6.2.

Solution

a) The voltage as a function of time is shown in Fig. 6.6.

b) The current in the inductor is 0 at $t = 0$. Therefore, the current for $t > 0$ is

$$i = \frac{1}{0.1}\int_0^t 20\tau e^{-10\tau}d\tau + 0$$

$$= 200\left[\frac{-e^{-10\tau}}{100}(10\tau + 1)\right]\Bigg|_0^t,$$

$$= 2(1 - 10te^{-10t} - e^{-10t})\ \text{A}, \qquad t > 0.$$

c) Figure 6.7 shows the current as a function of time.

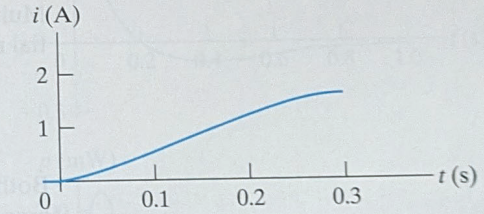


Figure 6.7 ▲ The current waveform for Example 6.2.

Note in Example 6.2 that i approaches a constant value of 2 A as t increases. We say more about this result after discussing the energy stored in an inductor.

Power and Energy in the Inductor

The power and energy relationships for an inductor can be derived directly from the current and voltage relationships. If the current reference is in the direction of the voltage drop across the terminals of the inductor, the power is

$$p = vi. \tag{6.7}$$

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Remember that power is in watts, voltage is in volts, and current is in amperes. If we express the inductor voltage as a function of the inductor current, Eq. 6.7 becomes

Power in an inductor

$$p = Li\frac{di}{dt}. \tag{6.8}$$

We can also express the current in terms of the voltage:

$$p = v\left[\frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)\right]. \tag{6.9}$$

Equation 6.8 is useful in expressing the energy stored in the inductor. Power is the time rate of expending energy, so

$$p = \frac{dw}{dt} = Li\frac{di}{dt}. \tag{6.10}$$

Multiplying both sides of Eq. 6.10 by a differential time gives the differential relationship

$$dw = Li\,di. \tag{6.11}$$

Both sides of Eq. 6.11 are integrated with the understanding that the reference for zero energy corresponds to zero current in the inductor. Thus

$$\int_0^w dx = L\int_0^i y\,dy,$$

Energy in an inductor

$$w = \frac{1}{2}Li^2. \tag{6.12}$$

As before, we use different symbols of integration to avoid confusion with the limits placed on the integrals. In Eq. 6.12, the energy is in joules, inductance is in henrys, and current is in amperes. To illustrate the application of Eqs. 6.7 and 6.12, we return to Examples 6.1 and 6.2 by means of Example 6.3.

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Example 6.3 Determining the Current, Voltage, Power, and Energy for an Inductor

a) For Example 6.1, plot i, v, p, and w versus time. Line up the plots vertically to allow easy assessment of each variable's behavior.

b) In what time interval is energy being stored in the inductor?

c) In what time interval is energy being extracted from the inductor?

d) What is the maximum energy stored in the inductor?

e) Evaluate the integrals

$$\int_0^{0.2} p\,dt \quad \text{and} \quad \int_{0.2}^{\infty} p\,dt,$$

and comment on their significance.

f) Repeat (a)–(c) for Example 6.2.

g) In Example 6.2, why is there a sustained current in the inductor as the voltage approaches zero?

Solution

a) The plots of i, v, p, and w follow directly from the expressions for i and v obtained in Example 6.1 and are shown in Fig. 6.8. In particular, $p = vi$, and $w = (\frac{1}{2})Li^2$.

b) An increasing energy curve indicates that energy is being stored. Thus energy is being stored in the time interval 0 to 0.2 s. Note that this corresponds to the interval when $p > 0$.

c) A decreasing energy curve indicates that energy is being extracted. Thus energy is being extracted in the time interval 0.2 s to ∞. Note that this corresponds to the interval when $p < 0$.

d) From Eq. 6.12 we see that energy is at a maximum when current is at a maximum; glancing at the graphs confirms this. From Example 6.1, maximum current = 0.736 A. Therefore, $w_{max} = 27.07$ mJ.

e) From Example 6.1,

$$i = 10te^{-5t}\ \text{A} \quad \text{and} \quad v = e^{-5t}(1 - 5t)\ \text{V}.$$

Therefore,

$$p = vi = 10te^{-10t} - 50t^2e^{-10t}\ \text{W}.$$

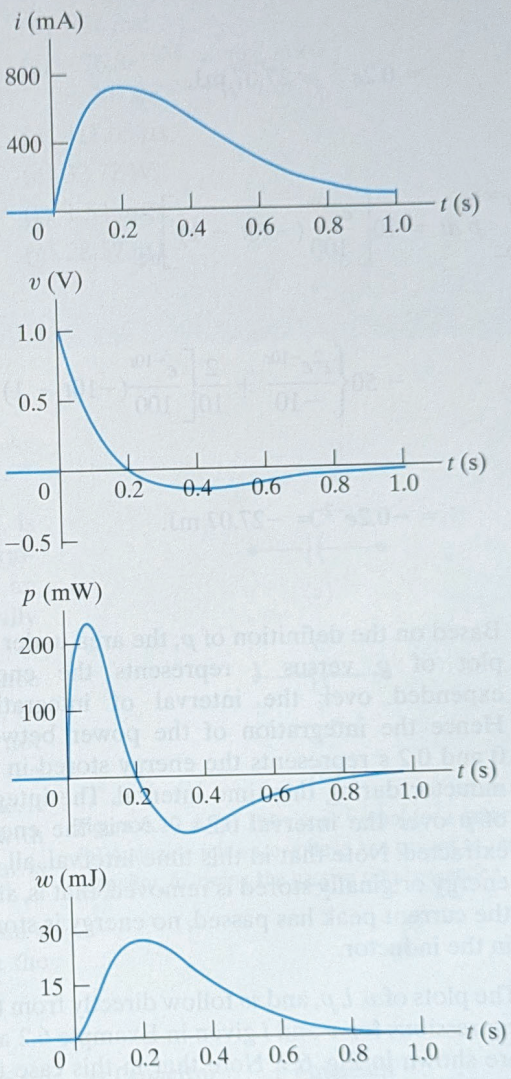


Figure 6.8 ▲ The variables i, v, p, and w versus t for Example 6.1.

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ASSESSMENT PROBLEMS

Objective 3—Be able to combine inductors or capacitors in series and in parallel to form a single equivalent inductor

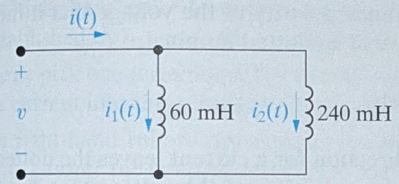
6.4 The initial values of i_1 and i_2 in the circuit shown are + 3 A and −5 A, respectively. The voltage at the terminals of the parallel inductors for $t \geq 0$ is $-30e^{-5t}$ mV.

a) If the parallel inductors are replaced by a single inductor, what is its inductance?

b) What is the initial current and its reference direction in the equivalent inductor?

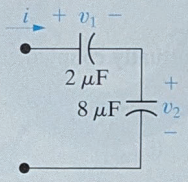
c) Use the equivalent inductor to find $i(t)$.

d) Find $i_1(t)$ and $i_2(t)$. Verify that the solutions for $i_1(t)$, $i_2(t)$, and $i(t)$ satisfy Kirchhoff's current law.



Answer: (a) 48 mH;
(b) 2 A, up;
(c) $0.125e^{-5t} - 2.125$ A, $t \geq 0$;
(d) $i_1(t) = 0.1e^{-5t} + 2.9$ A, $t \geq 0$,
$i_2(t) = 0.025e^{-5t} - 5.025$ A, $t \geq 0$.

6.5 The current at the terminals of the two capacitors shown is $240e^{-10t}\mu$A for $t \geq 0$. The initial values of v_1 and v_2 are −10 V and −5 V, respectively. Calculate the total energy trapped in the capacitors as $t \to \infty$. (*Hint:* Don't combine the capacitors in series—find the energy trapped in each, and then add.)



Answer: 20 μJ.

NOTE: Also try Chapter Problems 6.21, 6.22, 6.26, and 6.27.

6.4 Mutual Inductance

The magnetic field we considered in our study of inductors in Section 6.1 was restricted to a single circuit. We said that inductance is the parameter that relates a voltage to a time-varying current in the same circuit; thus, inductance is more precisely referred to as self-inductance.

We now consider the situation in which two circuits are linked by a magnetic field. In this case, the voltage induced in the second circuit can be related to the time-varying current in the first circuit by a parameter known as mutual inductance. The circuit shown in Fig. 6.19 represents two magnetically coupled coils. The self-inductances of the two coils are labeled L_1 and L_2, and the mutual inductance is labeled M. The double-headed arrow adjacent to M indicates the pair of coils with this value of mutual inductance. This notation is needed particularly in circuits containing more than one pair of magnetically coupled coils.

The easiest way to analyze circuits containing mutual inductance is to use mesh currents. The problem is to write the circuit equations that describe the circuit in terms of the coil currents. First, choose the reference direction for each coil current. Figure 6.20 shows arbitrarily selected reference currents. After choosing the reference directions for i_1 and i_2, sum the voltages around each closed path. Because of the mutual inductance M, there will be two voltages across each coil, namely, a self-induced voltage and a mutually

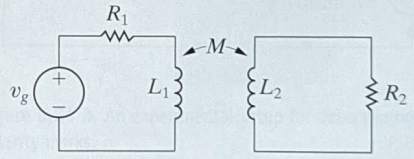


Figure 6.19 ▲ Two magnetically coupled coils.

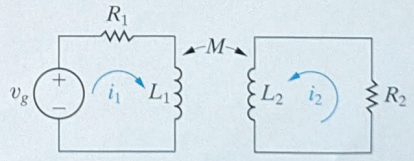


Figure 6.20 ▲ Coil currents i_1 and i_2 used to describe the circuit shown in Fig. 6.19.

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induced voltage. The self-induced voltage is the product of the self-inductance of the coil and the first derivative of the current in that coil. The mutually induced voltage is the product of the mutual inductance of the coils and the first derivative of the current in the other coil. Consider the coil on the left in Fig. 6.20 whose self-inductance has the value L_1. The self-induced voltage across this coil is $L_1(di_1/dt)$ and the mutually induced voltage across this coil is $M(di_2/dt)$. But what about the polarities of these two voltages?

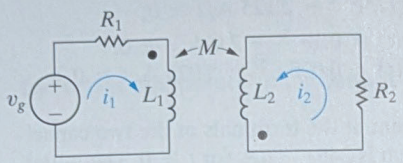


Figure 6.21 ▲ The circuit of Fig. 6.20 with dots added to the coils indicating the polarity of the mutually induced voltages.

Using the passive sign convention, the self-induced voltage is a voltage drop in the direction of the current producing the voltage. But the polarity of the mutually induced voltage depends on the way the coils are wound in relation to the reference direction of coil currents. In general, showing the details of mutually coupled windings is very cumbersome. Instead, we keep track of the polarities by a method known as the **dot convention**, in which a dot is placed on one terminal of each winding, as shown in Fig. 6.21. These dots carry the sign information and allow us to draw the coils schematically rather than showing how they wrap around a core structure.

The rule for using the dot convention to determine the polarity of mutually induced voltage can be summarized as follows:

Dot convention for mutually coupled coils ▶

When the reference direction for a current enters the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is positive at its dotted terminal.

Or, stated alternatively,

Dot convention for mutually coupled coils (alternate) ▶

When the reference direction for a current leaves the dotted terminal of a coil, the reference polarity of the voltage that it induces in the other coil is negative at its dotted terminal.

For the most part, dot markings will be provided for you in the circuit diagrams in this text. The important skill is to be able to write the appropriate circuit equations given your understanding of mutual inductance and the dot convention. Figuring out where to place the polarity dots if they are not given may be possible by examining the physical configuration of an actual circuit or by testing it in the laboratory. We will discuss these procedures after we discuss the use of dot markings.

In Fig. 6.21, the dot convention rule indicates that the reference polarity for the voltage induced in coil 1 by the current i_2 is negative at the dotted terminal of coil 1. This voltage $(M di_2/dt)$ is a voltage rise with respect to i_1. The voltage induced in coil 2 by the current i_1 is $M di_1/dt$, and its reference polarity is positive at the dotted terminal of coil 2. This voltage is a voltage rise in the direction of i_2. Figure 6.22 shows the self- and mutually induced voltages across coils 1 and 2 along with their polarity marks.

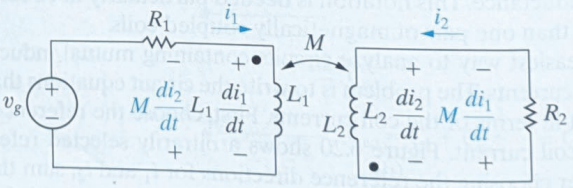


Figure 6.22 ▲ The self- and mutually induced voltages appearing across the coils shown in Fig. 6.21.

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Now let's look at the sum of the voltages around each closed loop. In Eqs. 6.31 and 6.32, voltage rises in the reference direction of a current are negative:

$$-v_g + i_1R_1 + L_1\frac{di_1}{dt} - M\frac{di_2}{dt} = 0, \tag{6.31}$$

$$i_2R_2 + L_2\frac{di_2}{dt} - M\frac{di_1}{dt} = 0. \tag{6.32}$$

The Procedure for Determining Dot Markings

We shift now to two methods of determining dot markings. The first assumes that we know the physical arrangement of the two coils and the mode of each winding in a magnetically coupled circuit. The following six steps, applied here to Fig. 6.23, determine a set of dot markings:

a) Arbitrarily select one terminal—say, the D terminal—of one coil and mark it with a dot.

b) Assign a current into the dotted terminal and label it i_D.

c) Use the right-hand rule[1] to determine the direction of the magnetic field established by i_D *inside* the coupled coils and label this field ϕ_D.

d) Arbitrarily pick one terminal of the second coil—say, terminal A—and assign a current into this terminal, showing the current as i_A.

e) Use the right-hand rule to determine the direction of the flux established by i_A *inside* the coupled coils and label this flux ϕ_A.

f) Compare the directions of the two fluxes ϕ_D and ϕ_A. If the fluxes have the same reference direction, place a dot on the terminal of the second coil where the test current (i_A) enters. (In Fig. 6.23, the fluxes ϕ_D and ϕ_A have the same reference direction, and therefore a dot goes on terminal A.) If the fluxes have different reference directions, place a dot on the terminal of the second coil where the test current leaves.

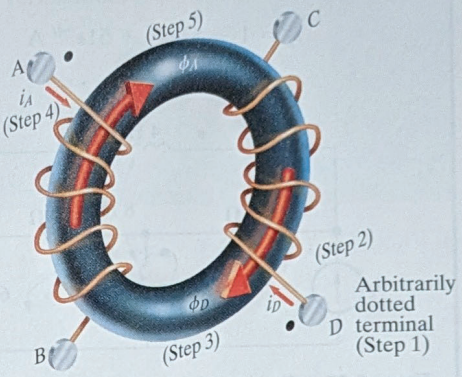


Figure 6.23 ▲ A set of coils showing a method for determining a set of dot markings.

The relative polarities of magnetically coupled coils can also be determined experimentally. This capability is important because in some situations, determining how the coils are wound on the core is impossible. One experimental method is to connect a dc voltage source, a resistor, a switch, and a dc voltmeter to the pair of coils, as shown in Fig. 6.24. The shaded box covering the coils implies that physical inspection of the coils is not possible. The resistor R limits the magnitude of the current supplied by the dc voltage source.

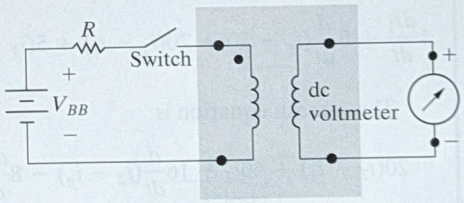


Figure 6.24 ▲ An experimental setup for determining polarity marks.

The coil terminal connected to the positive terminal of the dc source via the switch and limiting resistor receives a polarity mark, as shown in Fig. 6.24. When the switch is closed, the voltmeter deflection is observed. If the momentary deflection is upscale, the coil terminal connected to the positive terminal of the voltmeter receives the polarity mark. If the deflection is downscale, the coil terminal connected to the negative terminal of the voltmeter receives the polarity mark.

Example 6.6 shows how to use the dot markings to formulate a set of circuit equations in a circuit containing magnetically coupled coils.

[1] See discussion of Faraday's law on page 207.

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Example 6.6 Finding Mesh-Current Equations for a Circuit with Magentically Coupled Coils

a) Write a set of mesh-current equations that describe the circuit in Fig. 6.25 in terms of the currents i_1 and i_2.

b) Verify that if there is no energy stored in the circuit at $t = 0$ and if $i_g = 16 - 16e^{-5t}$ A, the solutions for i_1 and i_2 are

$$i_1 = 4 + 64e^{-5t} - 68e^{-4t} \text{ A},$$

$$i_2 = 1 - 52e^{-5t} + 51e^{-4t} \text{ A}.$$

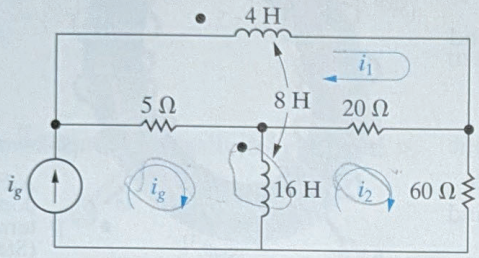


Figure 6.25 ▲ The circuit for Example 6.6.

Solution

a) Summing the voltages around the i_1 mesh yields

$$4\frac{di_1}{dt} + 8\frac{d}{dt}(i_g - i_2) + 20(i_1 - i_2) + 5(i_1 - i_g) = 0.$$

The i_2 mesh equation is

$$20(i_2 - i_1) + 60i_2 + 16\frac{d}{dt}(i_2 - i_g) - 8\frac{di_1}{dt} = 0.$$

Note that the voltage across the 4 H coil due to the current $(i_g - i_2)$, that is, $8d(i_g - i_2)/dt$, is a voltage drop in the direction of i_1. The voltage induced in the 16 H coil by the current i_1, that is, $8di_1/dt$, is a voltage rise in the direction of i_2.

b) To check the validity of i_1 and i_2, we begin by testing the initial and final values of i_1 and i_2. We know by hypothesis that $i_1(0) = i_2(0) = 0$. From the given solutions we have

$$i_1(0) = 4 + 64 - 68 = 0,$$

$$i_2(0) = 1 - 52 + 51 = 0.$$

Now we observe that as t approaches infinity the source current (i_g) approaches a constant value of 16 A, and therefore the magnetically coupled coils behave as short circuits. Hence at $t = \infty$ the circuit reduces to that shown in Fig. 6.26. From Fig. 6.26 we see that at $t = \infty$ the three resistors are in parallel across the 16 A source. The equivalent resistance is 3.75 Ω and thus the voltage across the 16 A current source is 60 V. It follows that

$$i_1(\infty) = \frac{60}{20} + \frac{60}{60} = 4 \text{ A},$$

$$i_2(\infty) = \frac{60}{60} = 1 \text{ A}.$$

These values agree with the final values predicted by the solutions for i_1 and i_2.

Finally we check the solutions by seeing if they satisfy the differential equations derived in (a). We will leave this final check to the reader via Problem 6.37.

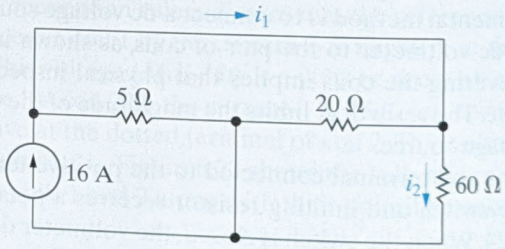


Figure 6.26 ▲ The circuit of Example 6.6 when $t = \infty$.

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✓ ASSESSMENT PROBLEM

Objective 4—Use the dot convention to write mesh-current equations for mutually coupled coils

6.6 a) Write a set of mesh-current equations for the circuit in Example 6.6 if the dot on the 4 H inductor is at the right-hand terminal, the reference direction of i_g is reversed, and the 60 Ω resistor is increased to 780 Ω.

b) Verify that if there is no energy stored in the circuit at $t = 0$, and if $i_g = 1.96 - 1.96e^{-4t}$ A, the solutions to the differential equations derived in (a) of this Assessment Problem are

$$i_1 = -0.4 - 11.6e^{-4t} + 12e^{-5t} \text{ A},$$
$$i_2 = -0.01 - 0.99e^{-4t} + e^{-5t} \text{ A}.$$

NOTE: Also try Chapter Problem 6.34.

Answer: (a) $4(di_1/dt) + 25i_1 + 8(di_2/dt) - 20i_2 = -5i_g - 8(di_g/dt)$

and

$8(di_1/dt) - 20i_1 + 16(di_2/dt) + 800i_2 = -16(di_g/dt)$;

(b) verification.

6.5 A Closer Look at Mutual Inductance

In order to fully explain the circuit parameter mutual inductance, and to examine the limitations and assumptions made in the qualitative discussion presented in Section 6.4, we begin with a more quantitative description of self-inductance than was previously provided.

A Review of Self-Inductance

The concept of inductance can be traced to Michael Faraday, who did pioneering work in this area in the early 1800s. Faraday postulated that a magnetic field consists of lines of force surrounding the current-carrying conductor. Visualize these lines of force as energy-storing elastic bands that close on themselves. As the current increases and decreases, the elastic bands (that is, the lines of force) spread and collapse about the conductor. The voltage induced in the conductor is proportional to the number of lines that collapse into, or cut, the conductor. This image of induced voltage is expressed by what is called Faraday's law; that is,

$$v = \frac{d\lambda}{dt}, \tag{6.33}$$

where λ is referred to as the flux linkage and is measured in weber-turns.

How do we get from Faraday's law to the definition of inductance presented in Section 6.1? We can begin to draw this connection using Fig. 6.27 as a reference.

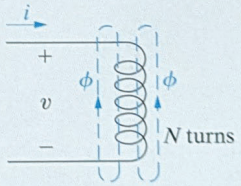


Figure 6.27 ▲ Representation of a magnetic field linking an N-turn coil.

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The lines threading the N turns and labeled ϕ represent the magnetic lines of force that make up the magnetic field. The strength of the magnetic field depends on the strength of the current, and the spatial orientation of the field depends on the direction of the current. The right-hand rule relates the orientation of the field to the direction of the current: When the fingers of the right hand are wrapped around the coil so that the fingers point in the direction of the current, the thumb points in the direction of that portion of the magnetic field inside the coil. The flux linkage is the product of the magnetic field (ϕ), measured in webers (Wb), and the number of turns linked by the field (N):

$$\lambda = N\phi. \tag{6.34}$$

The magnitude of the flux, ϕ, is related to the magnitude of the coil current by the relationship

$$\phi = \mathcal{P}Ni, \tag{6.35}$$

where N is the number of turns on the coil, and $\mathcal{P}$ is the permeance of the space occupied by the flux. Permeance is a quantity that describes the magnetic properties of this space, and as such, a detailed discussion of permeance is outside the scope of this text. Here, we need only observe that, when the space containing the flux is made up of magnetic materials (such as iron, nickel, and cobalt), the permeance varies with the flux, giving a nonlinear relationship between ϕ and i. But when the space containing the flux is comprised of nonmagnetic materials, the permeance is constant, giving a linear relationship between ϕ and i. Note from Eq. 6.35 that the flux is also proportional to the number of turns on the coil.

Here, we assume that the core material—the space containing the flux—is nonmagnetic. Then, substituting Eqs. 6.34 and 6.35 into Eq. 6.33 yields

$$\begin{aligned} v &= \frac{d\lambda}{dt} = \frac{d(N\phi)}{dt} \\ &= N\frac{d\phi}{dt} = N\frac{d}{dt}(\mathcal{P}Ni) \\ &= N^2\mathcal{P}\frac{di}{dt} = L\frac{di}{dt}, \end{aligned} \tag{6.36}$$

which shows that self-inductance is proportional to the square of the number of turns on the coil. We make use of this observation later.

The polarity of the induced voltage in the circuit in Fig. 6.27 reflects the reaction of the field to the current creating the field. For example, when i is increasing, di/dt is positive and v is positive. Thus energy is required to establish the magnetic field. The product vi gives the rate at which energy is stored in the field. When the field collapses, di/dt is negative, and again the polarity of the induced voltage is in opposition to the change. As the field collapses about the coil, energy is returned to the circuit.

Keeping in mind this further insight into the concept of self-inductance, we now turn back to mutual inductance.

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The Concept of Mutual Inductance

Figure 6.28 shows two magnetically coupled coils. You should verify that the dot markings on the two coils agree with the direction of the coil windings and currents shown. The number of turns on each coil are N_1 and N_2, respectively. Coil 1 is energized by a time-varying current source that establishes the current i_1 in the N_1 turns. Coil 2 is not energized and is open. The coils are wound on a nonmagnetic core. The flux produced by the current i_1 can be divided into two components, labeled ϕ_{11} and ϕ_{21}. The flux component ϕ_{11} is the flux produced by i_1 that links only the N_1 turns. The component ϕ_{21} is the flux produced by i_1 that links the N_2 turns and the N_1 turns. The first digit in the subscript to the flux gives the coil number, and the second digit refers to the coil current. Thus ϕ_{11} is a flux linking coil 1 and produced by a current in coil 1, whereas ϕ_{21} is a flux linking coil 2 and produced by a current in coil 1.

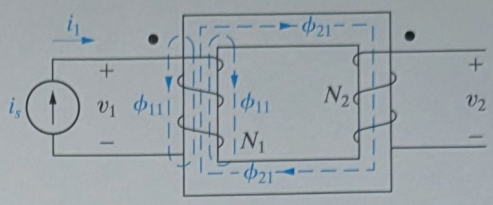


Figure 6.28 ▲ Two magnetically coupled coils.

The total flux linking coil 1 is ϕ_1, the sum of ϕ_{11} and ϕ_{21}:

$$\phi_1 = \phi_{11} + \phi_{21}. \tag{6.37}$$

The flux ϕ_1 and its components ϕ_{11} and ϕ_{21} are related to the coil current i_1 as follows:

$$\phi_1 = \mathcal{P}_1 N_1 i_1, \tag{6.38}$$

$$\phi_{11} = \mathcal{P}_{11} N_1 i_1, \tag{6.39}$$

$$\phi_{21} = \mathcal{P}_{21} N_1 i_1, \tag{6.40}$$

where $\mathcal{P}_1$ is the permeance of the space occupied by the flux ϕ_1, $\mathcal{P}_{11}$ is the permeance of the space occupied by the flux ϕ_{11}, and $\mathcal{P}_{21}$ is the permeance of the space occupied by the flux ϕ_{21}. Substituting Eqs. 6.38, 6.39, and 6.40 into Eq. 6.37 yields the relationship between the permeance of the space occupied by the total flux ϕ_1 and the permeances of the spaces occupied by its components ϕ_{11} and ϕ_{21}:

$$\mathcal{P}_1 = \mathcal{P}_{11} + \mathcal{P}_{21}. \tag{6.41}$$

We use Faraday's law to derive expressions for v_1 and v_2:

$$v_1 = \frac{d\lambda_1}{dt} = \frac{d(N_1\phi_1)}{dt} = N_1\frac{d}{dt}(\phi_{11} + \phi_{21})$$
$$= N_1^2(\mathcal{P}_{11} + \mathcal{P}_{21})\frac{di_1}{dt} = N_1^2\mathcal{P}_1\frac{di_1}{dt} = L_1\frac{di_1}{dt}, \tag{6.42}$$

and

$$v_2 = \frac{d\lambda_2}{dt} = \frac{d(N_2\phi_{21})}{dt} = N_2\frac{d}{dt}(\mathcal{P}_{21}N_1 i_1)$$
$$= N_2 N_1 \mathcal{P}_{21}\frac{di_1}{dt}. \tag{6.43}$$

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The coefficient of di_1/dt in Eq. 6.42 is the self-inductance of coil 1. The coefficient of di_1/dt in Eq. 6.43 is the mutual inductance between coils 1 and 2. Thus

$$M_{21} = N_2 N_1 \mathcal{P}_{21}. \tag{6.44}$$

The subscript on M specifies an inductance that relates the voltage induced in coil 2 to the current in coil 1.

The coefficient of mutual inductance gives

$$v_2 = M_{21}\frac{di_1}{dt}. \tag{6.45}$$

Note that the dot convention is used to assign the polarity reference to v_2 in Fig. 6.28.

For the coupled coils in Fig. 6.28, exciting coil 2 from a time-varying current source (i_2) and leaving coil 1 open produces the circuit arrangement shown in Fig. 6.29. Again, the polarity reference assigned to v_1 is based on the dot convention.

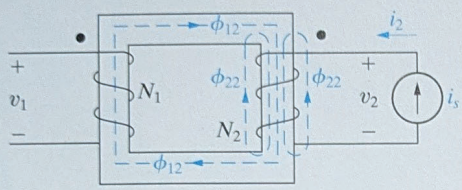


Figure 6.29 ▲ The magnetically coupled coils of Fig. 6.28, with coil 2 excited and coil 1 open.

The total flux linking coil 2 is

$$\phi_2 = \phi_{22} + \phi_{12}. \tag{6.46}$$

The flux ϕ_2 and its components ϕ_{22} and ϕ_{12} are related to the coil current i_2 as follows:

$$\phi_2 = \mathcal{P}_2 N_2 i_2, \tag{6.47}$$

$$\phi_{22} = \mathcal{P}_{22} N_2 i_2, \tag{6.48}$$

$$\phi_{12} = \mathcal{P}_{12} N_2 i_2. \tag{6.49}$$

The voltages v_2 and v_1 are

$$v_2 = \frac{d\lambda_2}{dt} = N_2^2 \mathcal{P}_2 \frac{di_2}{dt} = L_2 \frac{di_2}{dt}, \tag{6.50}$$

$$v_1 = \frac{d\lambda_1}{dt} = \frac{d}{dt}(N_1\phi_{12}) = N_1 N_2 \mathcal{P}_{12}\frac{di_2}{dt}. \tag{6.51}$$

The coefficient of mutual inductance that relates the voltage induced in coil 1 to the time-varying current in coil 2 is the coefficient of di_2/dt in Eq. 6.51:

$$M_{12} = N_1 N_2 \mathcal{P}_{12}. \tag{6.52}$$

For nonmagnetic materials, the permeances $\mathcal{P}_{12}$ and $\mathcal{P}_{21}$ are equal, and therefore

$$M_{12} = M_{21} = M. \tag{6.53}$$

Hence for linear circuits with just two magnetically coupled coils, attaching subscripts to the coefficient of mutual inductance is not necessary.

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Mutual Inductance in Terms of Self-Inductance

The value of mutual inductance is a function of the self-inductances. We derive this relationship as follows. From Eqs. 6.42 and 6.50,

$$L_1 = N_1^2 \mathcal{P}_1, \tag{6.54}$$

$$L_2 = N_2^2 \mathcal{P}_2, \tag{6.55}$$

respectively. From Eqs. 6.54 and 6.55,

$$L_1 L_2 = N_1^2 N_2^2 \mathcal{P}_1 \mathcal{P}_2. \tag{6.56}$$

We now use Eq. 6.41 and the corresponding expression for $\mathcal{P}_2$ to write

$$L_1 L_2 = N_1^2 N_2^2 (\mathcal{P}_{11} + \mathcal{P}_{21})(\mathcal{P}_{22} + \mathcal{P}_{12}). \tag{6.57}$$

But for a linear system, $\mathcal{P}_{21} = \mathcal{P}_{12}$, so Eq. 6.57 becomes

$$L_1 L_2 = (N_1 N_2 \mathcal{P}_{12})^2 \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right)$$

$$= M^2 \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right). \tag{6.58}$$

Replacing the two terms involving permeances by a single constant expresses Eq. 6.58 in a more meaningful form:

$$\frac{1}{k^2} = \left(1 + \frac{\mathcal{P}_{11}}{\mathcal{P}_{12}}\right)\left(1 + \frac{\mathcal{P}_{22}}{\mathcal{P}_{12}}\right). \tag{6.59}$$

Substituting Eq. 6.59 into Eq. 6.58 yields

$$M^2 = k^2 L_1 L_2$$

or

$$M = k\sqrt{L_1 L_2}, \tag{6.60}$$

◀ **Relating self-inductances and mutual inductance using coupling coefficient**

where the constant k is called the **coefficient of coupling**. According to Eq. 6.59, $1/k^2$ must be greater than 1, which means that k must be less than 1. In fact, the coefficient of coupling must lie between 0 and 1, or

$$0 \leq k \leq 1. \tag{6.61}$$

The coefficient of coupling is 0 when the two coils have no common flux; that is, when $\phi_{12} = \phi_{21} = 0$. This condition implies that $\mathcal{P}_{12} = 0$, and Eq. 6.59 indicates that $1/k^2 = \infty$, or $k = 0$. If there is no flux linkage between the coils, obviously M is 0.

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The coefficient of coupling is equal to 1 when ϕ_{11} and ϕ_{22} are 0. This condition implies that all the flux that links coil 1 also links coil 2. In terms of Eq. 6.59, $\mathcal{P}_{11} = \mathcal{P}_{22} = 0$, which obviously represents an ideal state; in reality, winding two coils so that they share precisely the same flux is physically impossible. Magnetic materials (such as alloys of iron, cobalt, and nickel) create a space with high permeance and are used to establish coefficients of coupling that approach unity. (We say more about this important quality of magnetic materials in Chapter 9.)

NOTE: Assess your understanding of this material by trying Chapter Problems 6.42 and 6.43.

Energy Calculations

We conclude our first look at mutual inductance with a discussion of the total energy stored in magnetically coupled coils. In doing so, we will confirm two observations made earlier: For linear magnetic coupling, (1) $M_{12} = M_{21} = M$, and (2) $M = k\sqrt{L_1 L_2}$, where $0 \le k \le 1$.

We use the circuit shown in Fig. 6.30 to derive the expression for the total energy stored in the magnetic fields associated with a pair of linearly coupled coils. We begin by assuming that the currents i_1 and i_2 are zero and that this zero-current state corresponds to zero energy stored in the coils. Then we let i_1 increase from zero to some arbitrary value I_1 and compute the energy stored when $i_1 = I_1$. Because $i_2 = 0$, the total power input into the pair of coils is $v_1 i_1$, and the energy stored is

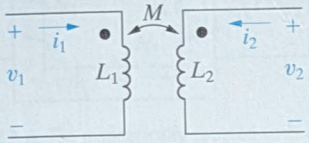


Figure 6.30 ▲ The circuit used to derive the basic energy relationships.

$$\int_0^{W_1} dw = L_1 \int_0^{I_1} i_1 di_1,$$

$$W_1 = \frac{1}{2} L_1 I_1^2. \tag{6.62}$$

Now we hold i_1 constant at I_1 and increase i_2 from zero to some arbitrary value I_2. During this time interval, the voltage induced in coil 2 by i_1 is zero because I_1 is constant. The voltage induced in coil 1 by i_2 is $M_{12} di_2/dt$. Therefore, the power input to the pair of coils is

$$p = I_1 M_{12} \frac{di_2}{dt} + i_2 v_2.$$

The total energy stored in the pair of coils when $i_2 = I_2$ is

$$\int_{W_1}^{W} dw = \int_0^{I_2} I_1 M_{12} di_2 + \int_0^{I_2} L_2 i_2 di_2,$$

or

$$W = W_1 + I_1 I_2 M_{12} + \frac{1}{2} L_2 I_2^2,$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 + I_1 I_2 M_{12}. \tag{6.63}$$

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If we reverse the procedure—that is, if we first increase i_2 from zero to I_2 and then increase i_1 from zero to I_1—the total energy stored is

$$W = \frac{1}{2}L_1I_1^2 + \frac{1}{2}L_2I_2^2 + I_1I_2M_{21}. \tag{6.64}$$

Equations 6.63 and 6.64 express the total energy stored in a pair of linearly coupled coils as a function of the coil currents, the self-inductances, and the mutual inductance. Note that the only difference between these equations is the coefficient of the current product I_1I_2. We use Eq. 6.63 if i_1 is established first and Eq. 6.64 if i_2 is established first.

When the coupling medium is linear, the total energy stored is the same regardless of the order used to establish I_1 and I_2. The reason is that in a linear coupling, the resultant magnetic flux depends only on the final values of i_1 and i_2, not on how the currents reached their final values. If the resultant flux is the same, the stored energy is the same. Therefore, for linear coupling, $M_{12} = M_{21}$. Also, because I_1 and I_2 are arbitrary values of i_1 and i_2, respectively, we represent the coil currents by their instantaneous values i_1 and i_2. Thus, at any instant of time, the total energy stored in the coupled coils is

$$w(t) = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 + Mi_1i_2. \tag{6.65}$$

We derived Eq. 6.65 by assuming that both coil currents entered polarity-marked terminals. We leave it to you to verify that, if one current enters a polarity-marked terminal while the other leaves such a terminal, the algebraic sign of the term Mi_1i_2 reverses. Thus, in general,

$$w(t) = \frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 \pm Mi_1i_2. \tag{6.66}$$

We use Eq. 6.66 to show that M cannot exceed $\sqrt{L_1L_2}$. The magnetically coupled coils are passive elements, so the total energy stored can never be negative. If $w(t)$ can never be negative, Eq. 6.66 indicates that the quantity

$$\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2$$

must be greater than or equal to zero when i_1 and i_2 are either both positive or both negative. The limiting value of M corresponds to setting the quantity equal to zero:

$$\frac{1}{2}L_1i_1^2 + \frac{1}{2}L_2i_2^2 - Mi_1i_2 = 0. \tag{6.67}$$

To find the limiting value of M we add and subtract the term $i_1i_2\sqrt{L_1L_2}$ to the left-hand side of Eq. 6.67. Doing so generates a term that is a perfect square:

$$\left(\sqrt{\frac{L_1}{2}}i_1 - \sqrt{\frac{L_2}{2}}i_2\right)^2 + i_1i_2\left(\sqrt{L_1L_2} - M\right) = 0. \tag{6.68}$$

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The squared term in Eq. 6.68 can never be negative, but it can be zero. Therefore $w(t) \geq 0$ only if

$$\sqrt{L_1 L_2} \geq M, \tag{6.69}$$

which is another way of saying that

$$M = k\sqrt{L_1 L_2} \qquad (0 \leq k \leq 1).$$

We derived Eq. 6.69 by assuming that i_1 and i_2 are either both positive or both negative. However, we get the same result if i_1 and i_2 are of opposite sign, because in this case we obtain the limiting value of M by selecting the plus sign in Eq. 6.66.

NOTE: Assess your understanding of this material by trying Chapter Problems 6.47 and 6.48.

Practical Perspective

Proximity Switches

At the beginning of this chapter we introduced the capacitive proximity switch. There are two forms of this switch. The one used in table lamps is based on a single-electrode switch. It is left to your investigation in Problem 6.50. In the example here, we consider the two-electrode switch used in elevator call buttons.

EXAMPLE

The elevator call button is a small cup into which the finger is inserted, as shown in Fig. 6.31. The cup is made of a metal ring electrode and a circular plate electrode that are insulated from each other. Sometimes two concentric rings embedded in insulating plastic are used instead. The electrodes are covered with an insulating layer to prevent direct contact with the metal. The resulting device can be modeled as a capacitor, as shown in Fig. 6.32.

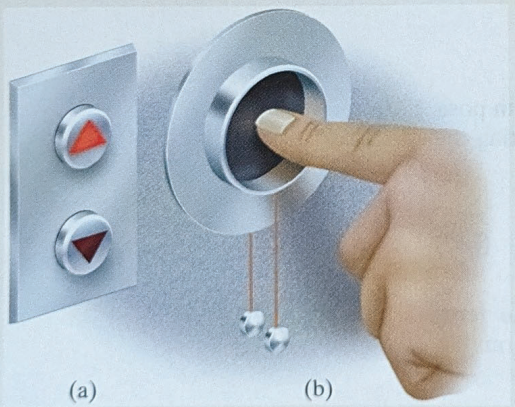


Figure 6.31 ▲ An elevator call button. (a) Front view. (b) Side view.

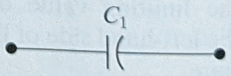


Figure 6.32 ▲ A capacitor model of the two-electrode proximity switch used in elevator call buttons.

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Unlike most capacitors, the capacitive proximity switch permits you to insert an object, such as a finger, between the electrodes. Because your finger is much more conductive than the insulating covering surrounding the electrodes, the circuit responds as though another electrode, connected to ground, has been added. The result is a three-terminal circuit containing three capacitors, as shown in Fig. 6.33.

The actual values of the capacitors in Figs. 6.32 and 6.33 are in the range of 10 to 50 pF, depending on the exact geometry of the switch, how the finger is inserted, whether the person is wearing gloves, and so forth. For the following problems, assume that all capacitors have the same value of 25 pF. Also assume the elevator call button is placed in the capacitive equivalent of a voltage-divider circuit, as shown in Fig. 6.34.

a) Calculate the output voltage with no finger present.

b) Calculate the output voltage when a finger touches the button.

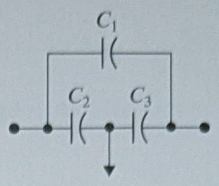


Figure 6.33 ▲ A circuit model of a capacitive proximity switch activated by finger touch.

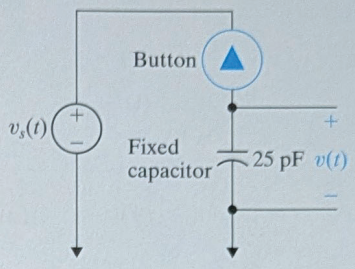


Figure 6.34 ▲ An elevator call button circuit.

Solution

a) Begin by redrawing the circuit in Fig. 6.34 with the call button replaced by its capacitive model from Fig. 6.32. The resulting circuit is shown in Fig. 6.35. Write the current equation at the single node:

$$C_1 = \frac{d(v - v_s)}{dt} + C_2\frac{dv}{dt} = 0. \tag{6.70}$$

Rearrange this equation to produce a differential equation for the output voltage $v(t)$:

$$\frac{dv}{dt} = \frac{C_1}{C_1 + C_2}\frac{dv_s}{dt}. \tag{6.71}$$

Finally, integrate Eq. 6.71 to find the output voltage:

$$v(t) = \frac{C_1}{C_1 + C_2}v_s(t) + v(0). \tag{6.72}$$

The result in Eq. 6.72 shows that the series capacitor circuit in Fig. 6.35 forms a voltage divider just as the series resistor circuit did in Chapter 3. In both voltage-divider circuits, the output voltage does not depend on the component values but only on their ratio. Here, $C_1 = C_2 = 25$ pF, so the capacitor ratio is $C_1/C_2 = 1$. Thus the output voltage is

$$v(t) = 0.5v_s(t) + v(0). \tag{6.73}$$

The constant term in Eq. 6.73 is due to the initial charge on the capacitor. We can assume that $v(0) = 0\ V$, because the circuit that senses the output voltage eliminates the effect of the initial capacitor charge. Therefore, the sensed output voltage is

$$v(t) = 0.5v_s(t). \tag{6.74}$$

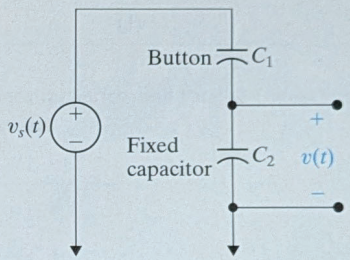


Figure 6.35 ▲ A model of the elevator call button circuit with no finger present.

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Summary

- **Inductance** is a linear circuit parameter that relates the voltage induced by a time-varying magnetic field to the current producing the field. (See page 188.)
- **Capacitance** is a linear circuit parameter that relates the current induced by a time-varying electric field to the voltage producing the field. (See page 195.)
- Inductors and capacitors are passive elements; they can store and release energy, but they cannot generate or dissipate energy. (See page 188.)
- The instantaneous power at the terminals of an inductor or capacitor can be positive or negative, depending on whether energy is being delivered to or extracted from the element.
- An inductor:
  - does not permit an instantaneous change in its terminal current
  - does permit an instantaneous change in its teminal voltage
  - behaves as a short circuit in the presence of a constant terminal current (See pages 188–189.)
- A capacitor:
  - does not permit an instantaneous change in its terminal voltage
  - does permit an instantaneous change in its terminal current
  - behaves as an open circuit in the presence of a constant terminal voltage (See page 196.)
- Equations for voltage, current, power, and energy in ideal inductors and capacitors are given in Table 6.1.
- Inductors in series or in parallel can be replaced by an equivalent inductor. Capacitors in series or in parallel can be replaced by an equivalent capacitor. The equations are summarized in Table 6.2. See Section 6.3 for a discussion on how to handle the initial conditions for series and parallel equivalent circuits involving inductors and capacitors.

TABLE 6.1 Terminal Equations for Ideal Inductors and Capacitors

Inductors	
$v = L\frac{di}{dt}$	(V)
$i = \frac{1}{L}\int_{t_0}^{t} v\,d\tau + i(t_0)$	(A)
$p = vi = Li\frac{di}{dt}$	(W)
$w = \frac{1}{2}Li^2$	(J)
Capacitors	
$v = \frac{1}{C}\int_{t_0}^{t} i\,d\tau + v(t_0)$	(V)
$i = C\frac{dv}{dt}$	(A)
$p = vi = Cv\frac{dv}{dt}$	(W)
$w = \frac{1}{2}Cv^2$	(J)

TABLE 6.2 Equations for Series- and Parallel-Connected Inductors and Capacitors

Series-Connected
$L_{eq} = L_1 + L_2 + \cdots + L_n$
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \cdots + \frac{1}{C_n}$
Parallel-Connected
$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_n}$
$C_{eq} = C_1 + C_2 + \cdots + C_n$

- **Mutual inductance**, M, is the circuit parameter relating the voltage induced in one circuit to a time-varying current in another circuit. Specifically,

$$v_1 = L_1\frac{di_1}{dt} + M_{12}\frac{di_2}{dt}$$

$$v_2 = M_{21}\frac{di_1}{dt} + L_2\frac{di_2}{dt},$$

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